

$$y' + \frac{y}{x} = \ln x$$

$$\left| \begin{array}{l} y = uv \\ y' = u'v + uv' \end{array} \right|$$

$$u'v + uv' + \frac{uv}{x} = \ln x$$

$$u'v + u(v' + \frac{v}{x}) = \ln x$$

$$\cancel{\frac{1}{x}} = \cancel{\frac{1}{x}}$$

$$y = \frac{1}{x} \left(\frac{x^2(2\ln x - 1) + C}{4} \right)$$

$$y = \frac{x^2(2\ln x - 1) + C}{4x}$$

$$\left| \begin{array}{l} v' + \frac{v}{x} = 0 \\ \int \frac{dv}{v} = - \int \frac{dx}{x} \\ \ln|v| = -\ln|x| \\ v = x^{-1} \end{array} \right|$$

$$u' \frac{1}{x} = \ln x$$

$$\cancel{u'} \frac{du}{x dx} = \ln x$$

$$\int du = \int x \ln x dx$$

$$u = \frac{x^2 \ln x}{2} - \int \frac{x}{2} dx$$

$$\frac{1}{2} \int x dx$$

$$u = \frac{x^2 \ln x}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + C$$

$$u = \frac{2 \cdot x^2 \ln x - x^2}{4} + C$$

$$u = \frac{x^2(2\ln x - 1) + C}{4}$$

$$y' + \frac{1}{x} y = x y^2$$

$$y = uv$$

$$u'v + uv' + \frac{uv}{x} = x(uv)^2$$

$$y' = u'v + uv'$$

$$u'v + u\left(v' + \frac{v}{x}\right) = x(uv)^2$$

$$v' + \frac{v}{x} = 0$$

$$v = x^{-1}$$

$$\frac{u'}{x} = x(u x^{-1})^2$$

$$u' = x^2 u^2 x^{-2}$$

$$\frac{du}{dx} = u^2$$

$$\int dx = \int \frac{du}{u^2}$$

$$x = -\frac{1}{u} + C$$

$$C - x = \frac{1}{u}$$

$$u = \frac{-1}{C+x}$$

$$y = uv = x^{-1} \left(-\frac{1}{x+C} \right)$$

$$y = -\frac{1}{x(x+C)}$$