Задание №3

(2)

$y'' + 4y' + 20y = 0$

$k^2 + 4k + 20 = 0$

$D = -64$

$k_{1,2} = \frac{-4 \pm \sqrt{-64}}{2}$

$k_1 = -2 + 4i \qquad k_2 = -2 - 4i$

$y(x) = e^{k_1 x} C_1 + e^{k_2 x} C_2$

$y(x) = e^{-2x}(C_1 \sin 4x + C_2 \cos 4x)$

(1)

$y'' + 4y' + 4y = 0 \qquad y(0) = 1 \qquad y'(0) = -1$

$k^2 + 4k + 4 = 0$

$D = 16 - 4 \cdot 4 = 0$

$k = \frac{-4}{2} = -2$

$y(x) = (C_1 + C_2 x) e^{-2x}$

$\begin{cases} C_2 - 2C_1 = -1 \\ C_1 = 1 \end{cases} \quad \Big| \quad C_1 = C_2 = 1$

$y(x) = e^{-2x}(x + 1)$

Задание 3 не выслал вовремя, высылаю вместе с 4 и 5. Если возможно зачтите.

Задание 4

(1) $y'' - 3y' + 2y = 2x^2 - 3$

$k^2 - 3k + 2 = 0$ | $k_1 + k_2 = 3$, $k_1 k_2 = 2$ | $k_1 = 1$, $k_2 = 2$

$y_0(x) = C_1 e^x + C_2 e^{2x}$

$2x^2 - 3 = Ax^2 + Bx + C = \tilde{y}$

$\tilde{y}' = 2Ax + B$

$\tilde{y}'' = 2A$

$2A - \cancel{B}6Ax - 3B + 2Ax^2 + 2Bx + 2C = 2x^2 - 3$

$$\begin{cases} 2A = 2 & | A = 1 \\ 2A - 3B + 2C = -3 & | -7 + 2C = -3 \quad | C = 2 \\ 2B - 6A = 0 & | B = 3 \end{cases}$$

$\tilde{y} = x^2 + 3x + 2$

$y(x) = C_1 e^x + C_2 e^{2x} + x^2 + 3x + 2$

(2) $y'' - 3y' + 2y = (2x + 4)e^x$

$k_1 = 1 \quad k_2 = 2$

$y_0(x) = C_1 e^x + C_2 e^{2x}$

$\tilde{y} = Axe^x + Be^x \quad |\cdot x \quad = Ax^2 e^x + Bxe^x = e^x(Ax^2 + Bx)$

$\tilde{y}' = e^x(Ax^2 + Bx + 2Ax + B)$

$\tilde{y}'' = e^x(Ax^2 + Bx + 2Ax + B + 2Ax + B + 2A)$

$e^x(\cancel{Ax^2} \cancel{+Bx} + 4Ax + 2B + 2A \cancel{-3Ax^2} \cancel{-3Bx} - 6Ax - 3B \cancel{+2Ax^2} \cancel{+2Bx}) =$

$= e^x(4Ax + 2B + 2A - 6Ax - 3B) = e^x(-2Ax - B + 2A) = e^x(2x + 4)$

$$\begin{cases} 2A - B = 4 & | B = -6 \\ -2A = 2 & | A = -1 \end{cases}$$

$y(x) = C_1 e^x + C_2 e^{2x} - x^2 e^x - 6xe^x$

③ $y'' - 2y' + 2y = 3\cos x - 5\sin 2x$

$k^2 - 2k + 2 = 0 \quad | D = -4$ $\qquad y_0(x) = e^x(C_1\cos x + C_2\sin x)$

$k_{1,2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$\tilde{y} = A_1\cos x + B_1\sin x + A_2\cos 2x + B_2\sin 2x$

$\tilde{y}' = -A_1\sin x + B_1\cos x - 2A_2\sin 2x + 2B_2\cos 2x$

$\tilde{y}'' = -A_1\cos x - B_1\sin x - 4A_2\cos 2x - 4B_2\sin 2x$

$2A_1\cos x + 2B_1\sin x + 2A_2\cos 2x + 2B_2\sin 2x + 2A_1\sin x -$

$-2B_1\cos x + 4A_2\sin 2x - 4B_2\cos 2x - A_1\cos x - B_1\sin x -$

$-4A_2\cos 2x - 4B_2\sin 2x = A_1\cos x + B_1\sin x - 2A_2\cos 2x -$

$-2B_2\sin 2x + 2A_1\sin x - 2B_1\cos x + 4A_2\sin 2x - 4B_2\cos 2x = 3\cos x - 5\sin 2x$

$\begin{cases} A_1 - 2B_1 = 3 \\ B_1 + 2A_1 = 0 \end{cases} \quad \left| \begin{array}{l} A_1 - \cancel{2B_1} = 3 \\ \cancel{2B_1} + 4A_1 = 0 \end{array} \right. \quad \left| \begin{array}{l} 5A_1 = 3 \\ B_1 + \frac{6}{5} = 0 \end{array} \right. \quad \left| \begin{array}{l} A_1 = \frac{3}{5} \\ B_1 = -\frac{6}{5} \end{array} \right.$

$\begin{cases} -2A_2 - 4B_2 = 0 \\ -2B_2 + 4A_2 = -5 \end{cases} \quad \left| \begin{array}{l} -2A_2 - \cancel{4B_2} = 0 \\ \cancel{4B_2} - 8A_2 = 10 \end{array} \right. \quad \left| \begin{array}{l} -10A_2 = 10 \\ -2B_2 - 4 = -5 \end{array} \right. \quad \left| \begin{array}{l} A_2 = -1 \\ B_2 = \frac{1}{2} \end{array} \right.$

$y(x) = e^x(C_1\cos x + C_2\sin x) + \frac{3}{5}\cos x - \frac{6}{5}\sin x - \cos 2x + \frac{1}{2}\sin 2x$

Задание 5

(1) $y'' + y = \frac{1}{\cos x}$

$k^2 = -1 \quad | \quad k_{1,2} = \pm i$

$y_0(x) = C_1 \cos x + C_2 \sin x$

$$\begin{cases} C_1' \cos x + C_2' \sin x = 0 \\ -C_1' \sin x + C_2' \cos x = \frac{1}{\cos x} \end{cases}$$

$$C_1' = \frac{-C_2' \sin x}{\cos x}$$

$$\frac{C_2' \sin^2 x}{\cos x} + C_2' \cos x = \frac{1}{\cos x}$$

$C_2' = 1 \quad | \quad C_2 = \int dx = x + C_3$

$C_1' = -\frac{\sin x}{\cos x} \quad | \quad C_1 = -\int \frac{\sin x}{\cos x} dx = \ln|\cos x| + C_4$

$y(x) = (x + C_3)\sin x + (\ln|\cos x| + C_4)\cos x$

② $y'' - 2y' + y = \frac{e^x}{1+x^2}$

$k^2 - 2k + 1 = 0$

$k_{1,2} = 1$

$y_0(x) = C_1 e^x + C_2 x e^x$

$$\begin{cases} C_1' + C_2' x = 0 \\ C_1' + C_2' x + C_2' = \frac{1}{1+x^2} \end{cases}$$

$C_2 = \int dx \frac{1}{1+x^2} = \operatorname{arctg} x + C_3$

$C_1' = -\frac{x}{1+x^2}$

$C_1 = -\int \frac{x}{1+x^2} = -\frac{1}{2} \int \frac{d(1+x^2)}{1+x^2} = -\frac{1}{2} \ln|1+x^2| + C_4$

$y(x) = xe^x(\operatorname{arctg} x + C_3) + e^x\left(C_4 - \frac{1}{2}\ln|1+x^2|\right)$