Задание 4

1. $y'' - 3y' + 2y = 2x^2 - 3$

$k^2 - 3k + 2 = 0$

$(k-2)(k-1) = 0$

$k_1 = 2$

$k_2 = 1$

$\bar{y} = C_1 e^{2x} + C_2 e^{x}$

$y^* = Ax^2 + Bx + C$

$y^{*\prime} = 2Ax + B$

$y^{*\prime\prime} = 2A$

$\cancel{2A - 3Ax^2 - 3Bx + 8C}$

$2A - 3(2Ax + B) + 2(Ax^2 + Bx + C) = 2x^2 - 3$

$2A - 6Ax - 3B + 2Ax^2 + 2Bx + 2C =$

$= 2x^2 - 3$

$2A = 2$

$A = 1$

$-6A + 2B = 0$

$$2B = 6A$$
$$B = 3A = 3$$
$$2A - 3B + 2C = -3$$
$$2 - 9 + 2C = -3$$
$$2C = 4$$
$$C = 2$$
$$y^* = x^2 + 3x + 2$$
$$y = \bar{y} + y^*$$
$$y = C_1 e^{2x} + C_2 e^x + x^2 + 3x + 2$$

② $y'' - 3y' + 2y = (2x+4)e^x$

$$\bar{y} = C_1 e^{2x} + C_2 e^x$$
$$y^* = (Ax^2 + Bx)e^x$$
$$y^{*'} = (Ax^2 + Bx + 2Ax + B)e^x$$
$$y^{*''} = (2Ax + B + 2A + Ax^2 + Bx + 2Ax + B)e^x$$

$$\cancel{Ax^2} + 4Ax + 2A + \cancel{Bx} + 2B - \cancel{3Ax^2} - 3\cancel{Bx} - 6Ax - 3B + \cancel{2Ax^2} + \cancel{2Bx} = 2x + 4$$

$$\begin{cases} -2A = 2 \\ -B + 2A = 4 \end{cases} \Rightarrow \begin{cases} A = -1 \\ -B - 2 = 4 \end{cases}$$

$$\begin{cases} A = -1 \\ B = -6 \end{cases}$$

$$y^* = \cancel{Ax} \left(-x^2 - 6x\right) e^x$$

$$y = \bar{y} + y^*$$

$$y = C_1 e^{2x} + C_2 e^x + \left(-x^2 - 6x\right) e^x$$

3 $y'' - 2y' + 2y = 3\cos x - 5\sin 2x$

$$k^2 - 2k + 2 = 0$$

$$D = (-2)^2 - 4 \cdot 2 = 4 - 8 = -4$$

$$k_{1,2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$$\bar{y} = e^x \left(C_1 \cos x + C_2 \sin x\right)$$

$$y^* = A_1 \cos x + B_1 \sin x +$$
$$+ A_2 \cos 2x + B_2 \sin 2x$$

$y^{*'} = -A_1 \sin x + B_1 \cos x - 2A_2 \sin 2x +$

$+ 2B_2 \cos 2x$

$y^{*''} = -A_1 \cos x - B_1 \sin x - 4A_2 \cos 2x -$

$-4B_2 \sin 2x$

$-A_1 \cos x - B_1 \sin x - 4A_2 \cos 2x -$

$-4B_2 \sin 2x + 2A_1 \sin x - 2B_1 \cos x +$

$+ 4A_2 \sin 2x - 4B_2 \cos 2x +$

$+ 2A_1 \cos x + 2B_1 \sin x + 2A_2 \cos 2x +$

$+ 2B_2 \sin 2x = 3 \cos x - 5 \sin 2x$

$$\begin{cases} A_1 - 2B_1 = 3 \\ B_1 + 2A_1 = 0 \\ -2A_2 - 4B_2 = 0 \\ 4A_2 - 2B_2 = -5 \end{cases}$$

$$\begin{cases} A_1 - 2B_1 = 3 \\ 4A_1 + 2B_1 = 0 \end{cases}$$

$5A_1 = 3$ $\qquad$ $B_1 = -2A_1 = \frac{-6}{5}$

$A_1 = 3/5$

$$\begin{cases} A_2 + 2B_2 = 0 \\ 4A_2 - 2B_2 = -5 \end{cases} \Rightarrow \quad 5A_2 = -5$$

$A_2 = -1$

$$B_2 = -\frac{1}{2} \quad A_2 = \frac{1}{2}$$

$$y^* = \frac{3}{5}\cos x - \frac{6}{5}\sin x -$$

$$-\cos 2x + \frac{1}{2}\cdot\sin 2x$$

$$y = \bar{y} + y^*$$

$$y = e^x(C_1\cos x + C_2\sin x) +$$

$$+\frac{3}{5}\cos x - \frac{6}{5}\sin x - \cos 2x +$$

$$+\frac{1}{2}\sin 2x$$

Задание5

① $y'' + y = \frac{1}{\cos x}$

$k^2 + 1 = 0$

$k^2 = -1$

$k_{1,2} = \pm i$

$y = C_1 \cos x + C_2 \sin x$

$$\begin{cases} C_1' \cos x + C_2' \sin x = 0 \\ -C_1' \sin x + C_2' \cos x = \frac{1}{\cos x} \end{cases} \Rightarrow$$

$C_1' = \frac{-C_2' \sin x}{\cos x}$

$C_2' \cdot \frac{\sin x \cdot \sin x}{\cos x} + C_2' \cos x = \frac{1}{\cos x}$

$C_2' = 1$

$C_2 = \int dx = x + C_3$

$C_1' = -\frac{\sin x}{\cos x} \Rightarrow C_1 = -\int \frac{\sin x}{\cos x} dx =$

$$= \int \frac{d(\cos x)}{\cos x} = \ln|\cos x| + C_4$$

$$y = (\ln|\cos x| + C_4)\cos x +$$
$$+ (x + C_3)\sin x$$

(2) $y'' - 2y' + y = \frac{e^x}{1+x^2}$

$$k^2 - 2k + 1 = 0$$
$$(k-1)^2 = 0$$
$$k_{1,2} = 1$$

$$y = C_1 e^x + C_2 x e^x$$

$$\begin{cases} C_1' e^x + C_2' x e^x = 0 \\ C_1' e^x + C_2' x e^x + C_2' e^x = \frac{e^x}{1+x^2} \end{cases}$$

$$\Rightarrow \begin{cases} C_1' + C_2' x = 0 \\ C_1' + C_2' x + C_2' = \frac{1}{1+x^2} \end{cases}$$

$$C_2' = \frac{1}{1+x^2}$$

$$C_2 = \int \frac{dx}{1+x^2} = \operatorname{arctg} x + C_3$$

$$C_1' = -C_2' x$$

$$C_1' = -\frac{x}{1+x^2}$$

$$C_1 = -\int \frac{x}{1+x^2} dx = -\frac{1}{2} \int \frac{d(1+x^2)}{1+x^2} =$$

$$= -\frac{1}{2} \ln|1+x^2| + C_4$$

$$y = (\operatorname{arctg} x + C_3) e^x +$$

$$+ \left(-\frac{1}{2} \ln|1+x^2| + C_4\right) x e^x$$