6/1 a) $yy' = \frac{2-x}{y}$

$y' = \frac{2-x}{y^2}$

$\frac{dy}{dx} = (2-x)y^{-2}$

$\int dy\, y^2 = \int dx\,(2-x)$

$\frac{y^3}{3} = 2\int dx - \int dx\, x$

$\frac{y^3}{3} = 2x - \frac{x^2}{2} + c$

$y^3 = 6x - \frac{3x^2}{2} + c$

$y(x) = \sqrt[3]{6x - \frac{3x^2}{2} + c}$

6/1 б) $xy' + y = y^2$

$$\frac{x\,dy}{dx} = y^2 - y$$

$$\frac{x}{dx} = \frac{y(y-1)}{dy}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y(y-1)}$$

$$\ln|x| = -\int \frac{dy}{y} + \int \frac{dy}{y-1}$$

$$\ln|x| = \ln|y-1| - \ln|y| + C$$

$$\ln|x| = \ln\left|\frac{y-1}{y}\right| + C$$

$$\ln|x| + \ln|C| = \ln \frac{y-1}{y}$$

$$xC = \frac{y-1}{y}$$

$$1 - \frac{1}{y} = xC$$

$$\frac{1}{y} = 1 - xC$$

$$y = \frac{1}{1-xC}$$

6/2) a) $(x+1)(y'+y^2)=-y$ $\quad y(0)=1$

$y'+y^2=-\frac{y}{x+1}$ $\quad | \quad \underline{z=x+1}$

$y'+y^2+\frac{y}{z}=0$ $\quad | \quad y=u\cancel{\cdot}v$

$u'v+v'u+u^2v^2+\frac{uv}{z}=0$ $\quad | \quad y'=u'v+v'u$

$u'v+u(v'+\frac{v}{z})=-u^2v^2$

$\begin{cases} v'+\frac{v}{z}=0 \\ u'v=-u^2v^2 \end{cases}$

$\frac{dv}{dz}=-\frac{v}{z}$

$\int\frac{dv}{v}=-\int\frac{dz}{z}$

$\ln|v|=-\ln|z|$

$v=\frac{1}{z}$

$\frac{u'}{z}=-\frac{u^2}{z^2}$

$\frac{du}{dz\,z}=-\frac{u^2}{z^2}$

$\int\frac{du}{u^2}=\int-\frac{dz}{z}$

$-\frac{1}{u}=-\ln|z|+C$

$u=\frac{1}{\ln|z|+C}$

$y(x)=\frac{1}{z(\ln|z|+C)}=\frac{1}{(x+1)(\ln|x+1|+C)}$

$1=\frac{1}{1(\ln|1|+C)}=\frac{1}{C}$ $\quad C=1$

6/2 б) $2x^3y' - 3x^2y + y^3x^3 = 0$ $\qquad y(0) = 1$

$$2xy' - 3y + y^3x = 0$$

$$2x(u'v + v'u) - 3uv + u^3v^3x = 0$$

$$2xu'v + 2xv'u - 3uv + u^3v^3x = 0$$

$$2xu'v + u(2xv' - 3v) = -u^3v^3x$$

$y = uv$

$y' = u'v + v'u$

$$\begin{cases} 2xv' - 3v = 0 \\ 2xu'v = -u^3v^3x \end{cases}$$

$$\frac{2x\,dv}{dx} = 3v$$

$$\int \frac{dv}{3v} = \int \frac{dx}{2x}$$

$$\frac{1}{3}\ln|v| = \frac{1}{2}\ln|x|$$

$$v^{\frac{1}{3}} = x^{\frac{1}{2}}$$

$$v = x^{\frac{3}{2}}$$

$$2xu'x^{\frac{3}{2}} + u^3(x^{\frac{3}{2}})^3 = 0$$

$$2xu' + u^3x^3 = 0$$

$$\cancel{2dx}\ \frac{2du\,x}{dx} = -u^3x^3$$

$$2\int \frac{du}{u^3} = -\int dx\,x^2$$

$$2\frac{u^{-2}}{-2} = -\frac{x^3}{3} + C$$

$$u^2 = \frac{6}{x^3} + C$$

$$u = \sqrt{\frac{6}{x^3} + C}$$

$$y(x) = x^{\frac{3}{2}}\sqrt{\frac{6}{x^3} + C}$$

$$1 = 0\sqrt{\frac{6}{6} + C}$$

Нет решений

6/3 a) $y'' + y' = 2xe^x$

$k^2 + k = 0 \quad | \quad k_1 = 0, \quad k_2 = -1$

$y_0(x) = C_1 + C_2 e^{-x}$

$\tilde{y} = e^x(Ax + B)$

$\tilde{y}' = Ae^x + e^x(Ax + B)$

$\tilde{y}'' = 2Ae^x + e^x(Ax + B)$

$\underline{\underline{2Ae^x}} + \underline{Axe^x} + \underline{\underline{Be^x}} + \underline{\underline{Ae^x}} + \underline{Axe^x} + \underline{\underline{Be^x}} = 2xe^x$

$3Ae^x + 2Axe^x + 2Be^x = 2xe^x$

$3A + 2Ax + 2B = 2x$

$$\begin{cases} 2A = 2 \\ 3A + 2B = 0 \end{cases} \quad \Bigg| \quad A = 1, \quad B = -\frac{3A}{2} = -\frac{3}{2}$$

$y(x) = C_1 + C_2 e^{-x} + e^x\left(x - \frac{3}{2}\right)$

6/3 б) $y'' - 3y' = 6x + 5\cos x$

$k^2 - 3k = 0 \quad | \quad k_1 = 0, \quad k_2 = 3$

$y_0(x) = C_1 + C_2 e^{3x}$

Для $5\cos x$:

$\tilde{y}_1 = A\sin x + B\cos x$

$\tilde{y}_1' = A\cos x - B\sin x$

$\tilde{y}_1'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 3A\cos x + 3B\sin x = 5\cos x$

$\begin{cases} 3B - A = 0 \\ B + 3A = -5 \end{cases} \quad \Big| \quad \begin{matrix} 3B = A \\ A + 9A = -15 \end{matrix} \quad \Big| \quad \begin{matrix} 3B = -\frac{3}{2} \\ A = -\frac{3}{2} \end{matrix} \quad \Big| \quad B = -\frac{1}{2}$

Для $6x$:

$\tilde{y}_2 = C_1 x^2 + C_2 x$

$\tilde{y}_2' = 2C_1 x + C_2$

$\tilde{y}_2'' = 2C_1$

$2C_1 - 6C_1 x - 3C_2 = 6x$

$\begin{cases} -6C_1 = 6 \\ 2C_1 - 3C_2 = 0 \end{cases} \quad \Big| \quad \begin{matrix} C_1 = -1 \\ C_2 = \frac{2C_1}{3} = -\frac{2}{3} \end{matrix}$

$\tilde{y}_1 = -\frac{3}{2}\sin x - \frac{1}{2}\cos x \quad | \quad \tilde{y}_2 = -x^2 - \frac{2}{3}x$

$y(x) = C_1 + C_2 e^{3x} - \frac{3}{2}\sin x - \frac{1}{2}\cos x - x^2 - \frac{2}{3}x$