`modsem`: An R Package for Estimating Latent Interactions and Quadratic Effects

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`modsem`

- `modsem` is an `R` package for estimating latent interaction and quadratic effects, in *Structural Equation Models* (SEMs).

Frameworks

- Product Indicator (PI) approaches
- Distribution Analytic (DA) approaches

Product Indicator (PI) Approaches

Product Indicator (PI) Approaches

- First attempt at latent interaction in SEMs (Kenny and Judd 1984)
- Creates product indicators, used as indicators for latent interaction terms.
- Has traditionally required manual model specification, and manual construction of product indicators.
- Early approaches involved complicated model constraints.
- Manual specification (partially) led to a continual simplification of constraints.

Product Indicator (PI) Approaches (History)

- Constrained Approach (orthogonal specification) (Kenny and Judd 1984)
- Constrained Approach (oblique specification) (Jöreskog and Yan 1996)
- Constrained Approach (mean-centered indicators) (Algina and Moulder 2001)
- Unconstrained Approach (constrained latent mean) (Martin and Marsh 1999)
- Resdiual Centering Approach (no constraints) (Little, Bovaird, and Widaman 2006)
- Double Centering Approach (no constraints) (Lin et al. 2010)

Product Indicator (PI) Approaches (Software)

- `semTools` offers tools for creating the product indicators, but does not help specify the model.
- Thus the PI approaches largely require manual specification.
- `modsem` automatically handles the creation of product indicators, and model specification.
- Model specification becomes exponentially more complicated for models with more indicators, more interaction terms. Especially for the PI approaches with model constraints.

Example: Double Centering Approach, using `semTools`

```
model <- '
# Measurement Model
  X  =~ x1 + x2 + x3
  Z  =~ z1 + z2 + z3
  Y  =~ y1 + y2 + y3
  XZ =~ x1.z1 + x2.z1 + x3.z1 +
        x1.z2 + x2.z2 + x3.z2 +
        x1.z3 + x2.z3 + x3.z3

# Structural Model
  Y ~ X + Z + XZ

# Residual Covariances
  x1.z1 ~~ x1.z2 + x1.z3 + x2.z1 + x3.z1
  x1.z2 ~~ x1.z3 + x2.z2 + x3.z2
  x2.z1 ~~ x2.z2 + x2.z3 + x3.z1

  x1.z3 ~~ x2.z3 + x3.z3
  x2.z2 ~~ x2.z3 + x3.z2
  x3.z1 ~~ x3.z2 + x3.z3

  x2.z3 ~~ x3.z3
  x3.z2 ~~ x3.z3
'
```

Example: Double Centering Approach, using semTools

```r
library(semTools)
data.prod <- indProd(data = oneInt,
                     var1 = c("x1", "x2", "x3"),
                     var2 = c("z1", "z2", "z3"),
                     match = FALSE)

fit <- sem(model, data = data.prod)
summary(fit)
```

```
Truncated Output:
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X                 0.675    0.027   25.379    0.000
    Z                 0.561    0.026   21.606    0.000
    XZ                0.702    0.027   26.360    0.000
```

Example: Double Centering Approach, using modsem

```
library(modsem)

model <- '
# Measurement Model
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

# Structural Model
  Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt)
summary(fit)
```

```
Truncated Output:
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X                 0.675    0.027   25.379    0.000
    Z                 0.561    0.026   21.606    0.000
    XZ                0.702    0.027   26.360    0.000
```

Distribution Analytic (DA) Approaches

Distribution Analytic (DA) Approaches

- Latent Moderated Structural Equations (LMS) Method, introduced by (A. Klein and Moosbrugger 2000).
- Quasi Maximum Likelihood (QML) Method, introducted by (A. G. Klein and Muthén 2007).
- No formation of product indicators.
- Non-normal distribution of η is explicityly modelled.
- LMS and QML perform very similarily. But QML performs slightly better under normality violations.
- QML is less computationally intensive, with multiple moderators.

Distribution Analytic (DA) Approaches (Software)

- `R` package `nlsem`.
- `Mplus`
- `modsem`

`nlsem`

- Pros:
  - Free open source `R` package.
  - Has both LMS and QML approaches.
- Cons:
  - Very slow
  - Hard to use
  - Only handles models with a single endogenous variable (η).
  - Doesn't handle missing data
  - Biased LMS standard errors
  - Unstable estimates
  - Cannot produce standardized estimates
  - Virtually no fit measures
  - Does not handle linear and nonlinear constraints.
  - Generally unpolished

Mplus

- Pros:
  - Extended implementation of LMS approach (multiple endogenous variables, endogenous interactions)
  - Integrates well with other `Mplus` features (e.g., Multilevel SEM)
  - Handles three-way interaction terms
  - Handles higher order models (and interactions)
  - Easy to use
  - Full Information Maximum Likelihood (FIML) estimation.
  - Linear and nonlinear constraints.
- Cons:
  - Scales poorly with multiple interaction effects
  - Does not include QML
  - Proprietary (expensive) software package.
  - No Multigroup LMS.

modsem

- Pros:
  - Free open source R package.
  - Extended implementation of LMS and QML approach (multiple endogenous variables, endogenous interactions).
  - Computational optimized implementation of LMS and QML approaches.
  - Better scaling in LMS approach, compared to Mplus.
  - Handles higher order models (and interactions)
  - Easy to use
  - Full Information Maximum Likelihood (FIML) estimation.
  - Multigroup LMS and QML.
  - Linear and nonlinear constraints.
- Cons:
  - Missing some advanced features available in Mplus (e.g., Multilevel SEM).

Example: LMS approach, using nlsem (213.2 seconds)

```
library(nlsem)
set.seed(3248927)

model <- "
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:Z
"

nlsem_model_1 <- lav2nlsem(model)
oneIntSorted <- oneInt[c("x1", "x2", "x3", "z1", "z2",
                         "z3", "y1", "y2", "y3")]
start <- runif(count_free_parameters(nlsem_model_1))

specs <- as.data.frame(nlsem_model_1)
specs[grepl("tau|alpha", specs$label), "class1"] <- 0
specs[grepl("nu\\.(x|y)", specs$label), "class1"] <- NA

nlsem_model_2 <- create_sem(specs)

fit_nlsem <- em(nlsem_model_2, data = oneIntSorted,
                convergence = 1e-4, max.iter = 500, start = start)
```

Example: LMS approach, using nlsem (213.2 seconds)

```r
summary(fit_nlsem)
```

```
Truncated Output:
        Estimate Std. Error  z value      Pr(>|z|)
Gamma1 0.6750994 0.03021998 22.33950 1.527303e-110
Gamma2 0.5965296 0.02515988 23.70956 2.873341e-124
Omega3 0.7269645 0.02677033 27.15561 2.174013e-162
```

Example: LMS approach, using Mplus (6.2 seconds)

```r
model <- '
# Measurement Model
  X  =~ x1 + x2 + x3
  Z  =~ z1 + z2 + z3
  Y  =~ y1 + y2 + y3

# Structural Model
  Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt, method = "mplus")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  Y ~
    X      (Y<-X)   0.673      0.031   21.710    0.000
    Z      (Y<-Z)   0.569      0.030   18.967    0.000
    X:Z   (Y<-XZ)   0.718      0.028   25.643    0.000
```

Example: LMS approach, using modsem (1.2 seconds)

```r
model <- '
# Measurement Model
  X  =~ x1 + x2 + x3
  Z  =~ z1 + z2 + z3
  Y  =~ y1 + y2 + y3

# Structural Model
  Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt, method = "lms")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  Y ~
    X               0.673      0.031   21.666    0.000
    Z               0.569      0.030   18.710    0.000
    X:Z             0.718      0.028   25.828    0.000
```

Visualizing Interaction Effects

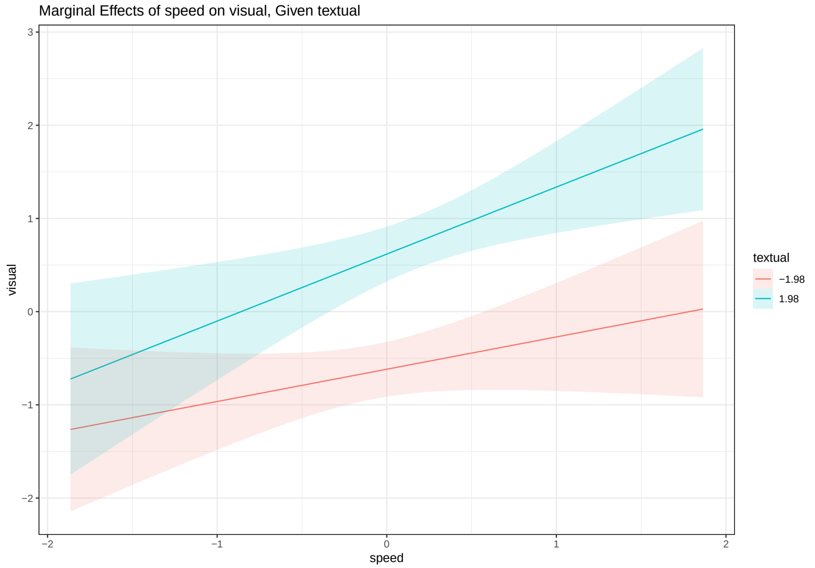
Example Model

```r
model <-  '
  visual  =~ x1 + x2 + x3
  textual =~ x4 + x5 + x6
  speed   =~ x7 + x8 + x9

  visual ~ speed + textual + speed:textual
'

fit <- modsem(model, data = HolzingerSwineford1939, method = "lms")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  visual ~
    textual         0.312      0.076    4.107    0.000
    speed           0.533      0.158    3.366    0.001
    speed:txtl      0.094      0.093    1.016    0.309
```

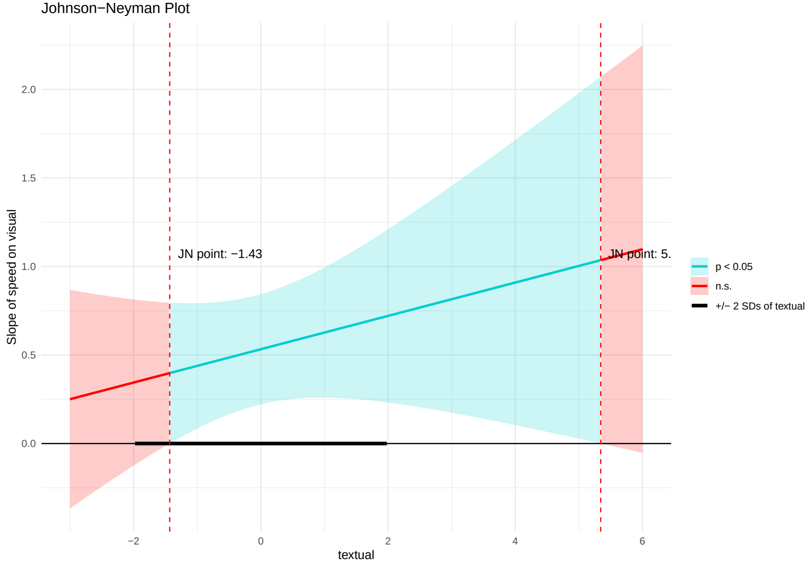
Plotting Margins (Simple Slopes)

```
plot_interaction(x = "speed", z = "textual", y = "visual", model = fit, vals_z
```



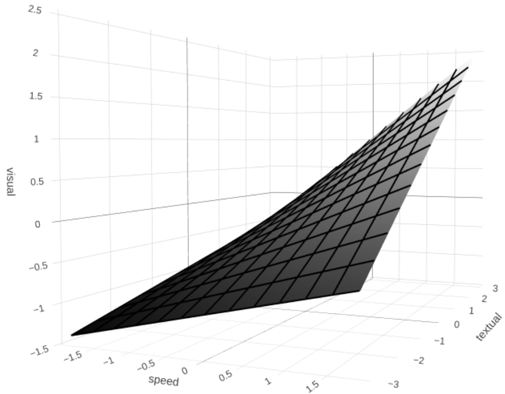
Johnson-Neyman Plot

```
plot_jn(x = "speed", z = "textual", y = "visual", model = fit, max_z = 6)
```



Surface Plot

```
plot_surface(x = "speed", z = "textual", y = "visual", model = fit,
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



Visualizing Models With Quadratic Effects

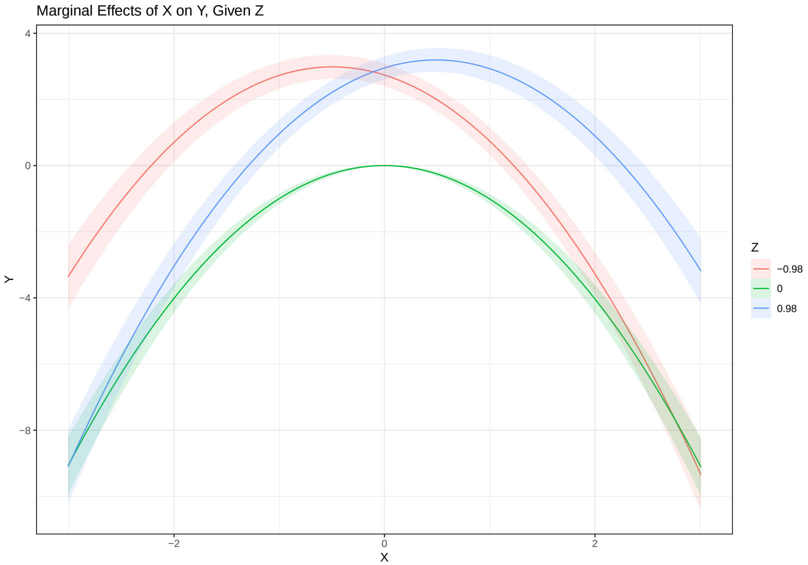
Example Model

```
model <- '
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:X + Z:Z + X:Z
'

fit <- modsem(model, data = data.quadratic, method = "qml")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  Y ~
    X              -0.006      0.067   -0.088    0.930
    Z               0.110      0.163    0.674    0.501
    X:X            -1.005      0.044  -23.046    0.000
    X:Z             1.011      0.080   12.563    0.000
    Z:Z             2.969      0.079   37.366    0.000
```

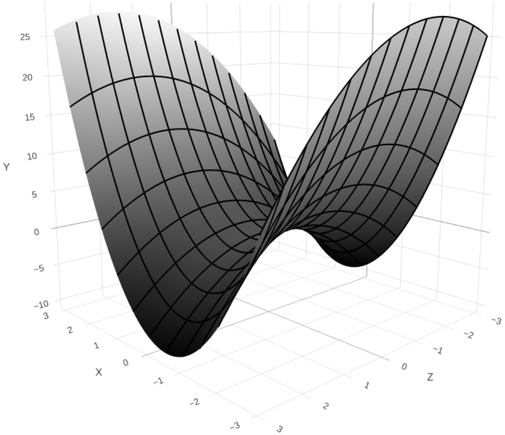
2D Plot

```
plot_interaction(x = "X", z = "Z", y = "Y", model = fit, vals_z = c(-1, 0, 1))
```



3D (Response Surface) Plot

```
plot_surface(x = "X", z = "Z", y = "Y", model = fit,
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



References

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