

# modsem: An R Package for Estimating Latent Interactions and Quadratic Effects

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## modsem

- ▶ modsem is an R package for estimating latent interaction and quadratic effects, in *Structural Equation Models* (SEMs).

## Frameworks

- ▶ Product Indicator (PI) approaches
- ▶ Distribution Analytic (DA) approaches

## Product Indicator (PI) Approaches

## Product Indicator (PI) Approaches

- ▶ First attempt at latent interaction in SEMs (Kenny and Judd 1984)
- ▶ Creates product indicators, used as indicators for latent interaction terms.
- ▶ Has traditionally required manual model specification, and manual construction of product indicators.
- ▶ Early approaches involved complicated model constraints.
- ▶ Manual specification (partially) led to a continual simplification of constraints.

## Product Indicator (PI) Approaches (History)

- ▶ Constrained Approach (orthogonal specification) (Kenny and Judd 1984)
- ▶ Constrained Approach (oblique specification) (Jöreskog and Yan 1996)
- ▶ Constrained Approach (mean-centered indicators) (Algina and Moulder 2001)
- ▶ Unconstrained Approach (constrained latent mean) (Martin and Marsh 1999)
- ▶ Residual Centering Approach (no constraints) (Little, Bovaird, and Widaman 2006)
- ▶ Double Centering Approach (no constraints) (Lin et al. 2010)

## Product Indicator (PI) Approaches (Software)

- ▶ `semTools` offers tools for creating the product indicators, but does not help specify the model.
- ▶ Thus the PI approaches largely require manual specification.
- ▶ `modsem` automatically handles the creation of product indicators, and model specification.
- ▶ Model specification becomes exponentially more complicated for models with more indicators, more interaction terms. Especially for the PI approaches with model constraints.

## Example: Double Centering Approach, using semTools

```
model <- '  
# Measurement Model  
X =~ x1 + x2 + x3  
Z =~ z1 + z2 + z3  
Y =~ y1 + y2 + y3  
XZ =~ x1.z1 + x2.z1 + x3.z1 +  
      x1.z2 + x2.z2 + x3.z2 +  
      x1.z3 + x2.z3 + x3.z3  
  
# Structural Model  
Y ~ X + Z + XZ  
  
# Residual Covariances  
x1.z1 ~~ x1.z2 + x1.z3 + x2.z1 + x3.z1  
x1.z2 ~~ x1.z3 + x2.z2 + x3.z2  
x2.z1 ~~ x2.z2 + x2.z3 + x3.z1  
  
x1.z3 ~~ x2.z3 + x3.z3  
x2.z2 ~~ x2.z3 + x3.z2  
x3.z1 ~~ x3.z2 + x3.z3  
  
x2.z3 ~~ x3.z3  
x3.z2 ~~ x3.z3  
'
```

## Example: Double Centering Approach, using semTools

```
library(semTools)
data.prod <- indProd(data = oneInt,
                      var1 = c("x1", "x2", "x3"),
                      var2 = c("z1", "z2", "z3"),
                      match = FALSE)

fit <- sem(model, data = data.prod)
summary(fit)
```

Truncated Output:

Regressions:

	Estimate	Std.Err	z-value	P(> z )
Y ~				
X	0.675	0.027	25.379	0.000
Z	0.561	0.026	21.606	0.000
XZ	0.702	0.027	26.360	0.000

## Example: Double Centering Approach, using modsem

```
library(modsem)

model <- '
# Measurement Model
X =~ x1 + x2 + x3
Z =~ z1 + z2 + z3
Y =~ y1 + y2 + y3

# Structural Model
Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt)
summary(fit)
```

Truncated Output:

Regressions:

	Estimate	Std.Err	z-value	P(> z )
Y ~				
X	0.675	0.027	25.379	0.000
Z	0.561	0.026	21.606	0.000
XZ	0.702	0.027	26.360	0.000

## Distribution Analytic (DA) Approaches

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- ▶ Latent Moderated Structural Equations (LMS) Method, introduced by (A. Klein and Moosbrugger 2000).
- ▶ Quasi Maximum Likelihood (QML) Method, introduced by (A. G. Klein and Muthén 2007).
- ▶ No formation of product indicators.
- ▶ Non-normal distribution of  $\eta$  is explicitly modelled.
- ▶ LMS and QML perform very similarly. But QML performs slightly better under normality violations.
- ▶ QML is less computationally intensive, with multiple moderators.

## Distribution Analytic (DA) Approaches (Software)

- ▶ R package nlsem.
- ▶ Mplus
- ▶ modsem

## nlsem

- ▶ Pros:
  - ▶ Free open source R package.
  - ▶ Has both LMS and QML approaches.
- ▶ Cons:
  - ▶ Very slow
  - ▶ Hard to use
  - ▶ Only handles models with a single endogenous variable ( $\eta$ ).
  - ▶ Doesn't handle missing data
  - ▶ Biased LMS standard errors
  - ▶ Unstable estimates
  - ▶ Cannot produce standardized estimates
  - ▶ Virtually no fit measures
  - ▶ Does not handle linear and nonlinear constraints.
  - ▶ Generally unpolished

## Mplus

- ▶ Pros:
  - ▶ Extended implementation of LMS approach (multiple endogenous variables, endogenous interactions)
  - ▶ Integrates well with other Mplus features (e.g., Multilevel SEM)
  - ▶ Handles three-way interaction terms
  - ▶ Handles higher order models (and interactions)
  - ▶ Easy to use
  - ▶ Full Information Maximum Likelihood (FIML) estimation.
  - ▶ Linear and nonlinear constraints.
- ▶ Cons:
  - ▶ Scales poorly with multiple interaction effects
  - ▶ Does not include QML
  - ▶ Proprietary (expensive) software package.
  - ▶ No Multigroup LMS.

## modsem

- ▶ Pros:
  - ▶ Free open source R package.
  - ▶ Extended implementation of LMS and QML approach (multiple endogenous variables, endogenous interactions).
  - ▶ Computational optimized implementation of LMS and QML approaches.
  - ▶ Better scaling in LMS approach, compared to Mplus.
  - ▶ Handles higher order models (and interactions)
  - ▶ Easy to use
  - ▶ Full Information Maximum Likelihood (FIML) estimation.
  - ▶ Multigroup LMS and QML.
  - ▶ Linear and nonlinear constraints.
- ▶ Cons:
  - ▶ Missing some advanced features available in Mplus (e.g., Multilevel SEM).

## Example: LMS approach, using nlsem (213.2 seconds)

```
library(nlsem)
set.seed(3248927)

model <- "
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:Z
"
nlsem_model_1 <- lav2nlsem(model)
oneIntSorted <- oneInt[c("x1", "x2", "x3", "z1", "z2",
                           "z3", "y1", "y2", "y3")]
start <- runif(count_free_parameters(nlsem_model_1))

specs <- as.data.frame(nlsem_model_1)
specs[grepl("tau|alpha", specs$label), "class1"] <- 0
specs[grepl("nu\\.\\(x|y\\)", specs$label), "class1"] <- NA

nlsem_model_2 <- create_sem(specs)

fit_nlsem <- em(nlsem_model_2, data = oneIntSorted, m = 16,
                  convergence = 1e-4, max.iter = 500, start = start)
```

## Example: LMS approach, using nlsem (213.2 seconds)

```
summary(fit_nlsem)
```

Truncated Output:

	Estimate	Std. Error	z value	Pr(> z )
Gamma1	0.6750993	0.03022066	22.33900	1.544561e-110
Gamma2	0.5965296	0.02516093	23.70857	2.941752e-124
Omega3	0.7269645	0.02676991	27.15603	2.149299e-162

## Example: LMS approach, using Mplus (4.2 seconds)

```
model <- '
# Measurement Model
X  =~ x1 + x2 + x3
Z  =~ z1 + z2 + z3
Y  =~ y1 + y2 + y3

# Structural Model
Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt, method = "mplus")
summary(fit)
```

Truncated Output:

Regressions:

	Estimate	Std.Error	z.value	P(> z )
Y ~				
X	0.673	0.031	21.666	0.000
Z	0.569	0.030	18.710	0.000
X:Z	0.718	0.028	25.828	0.000

## Example: LMS approach, using modsem (1.2 seconds)

```
model <- '
# Measurement Model
X  =~ x1 + x2 + x3
Z  =~ z1 + z2 + z3
Y  =~ y1 + y2 + y3

# Structural Model
Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt, method = "lms")
summary(fit)
```

Truncated Output:

Regressions:

	Estimate	Std.Error	z.value	P(> z )
Y ~				
X	0.673	0.031	21.666	0.000
Z	0.569	0.030	18.710	0.000
X:Z	0.718	0.028	25.828	0.000

## Visualizing Interaction Effects

# Example Model

```
model <- '
  visual  =~ x1 + x2 + x3
  textual =~ x4 + x5 + x6
  speed   =~ x7 + x8 + x9

  visual ~ speed + textual + speed:textual
'

fit <- modsem(model, data = HolzingerSwineford1939, method = "lms")
summary(fit)
```

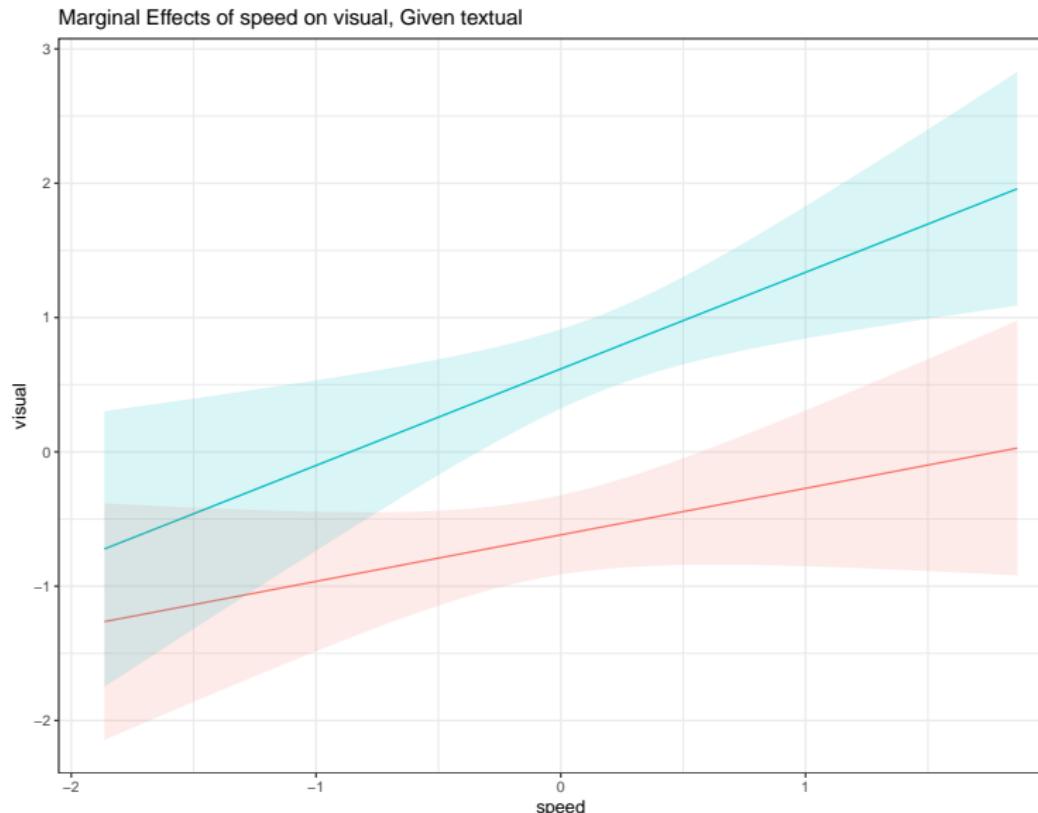
Truncated Output:

Regressions:

	Estimate	Std.Error	z.value	P(> z )
visual ~				
textual	0.312	0.076	4.107	0.000
speed	0.533	0.158	3.366	0.001
speed:txtl	0.094	0.093	1.016	0.309

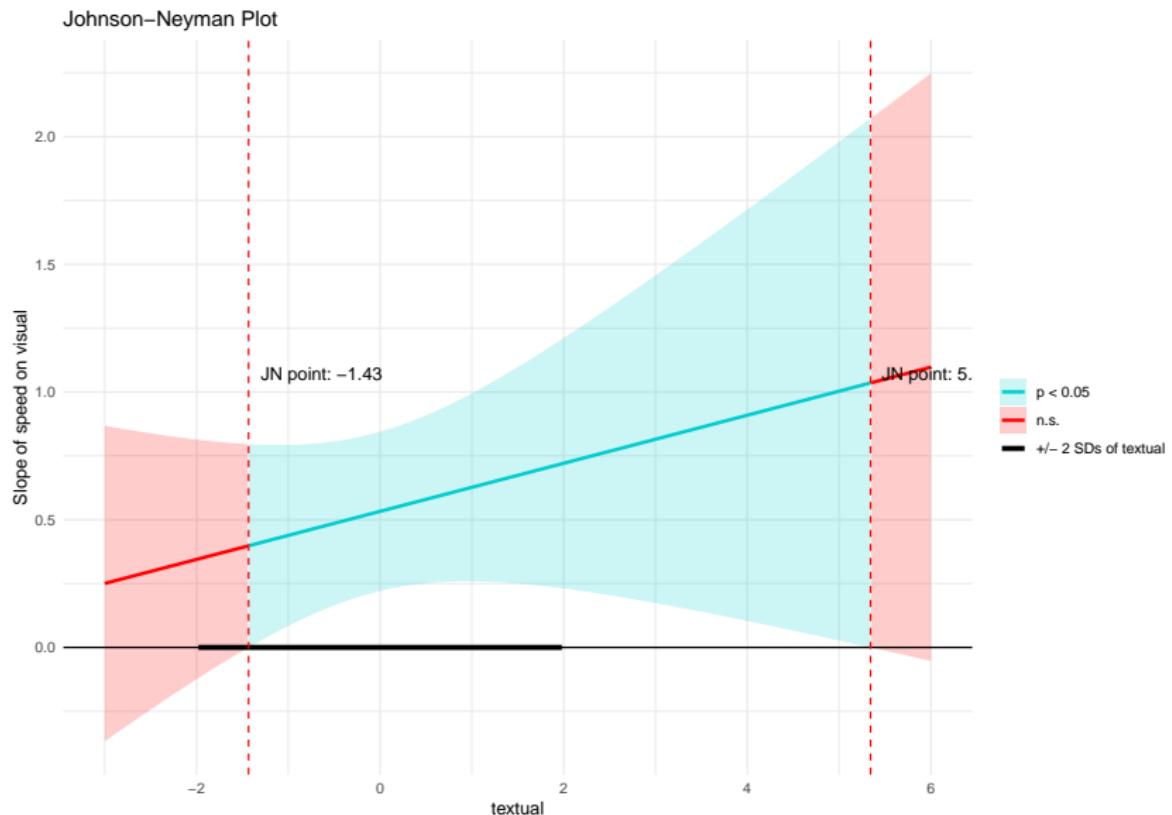
# Plotting Margins (Simple Slopes)

```
plot_interaction(x = "speed", z = "textual", y = "visual", model = fit, vals_z
```



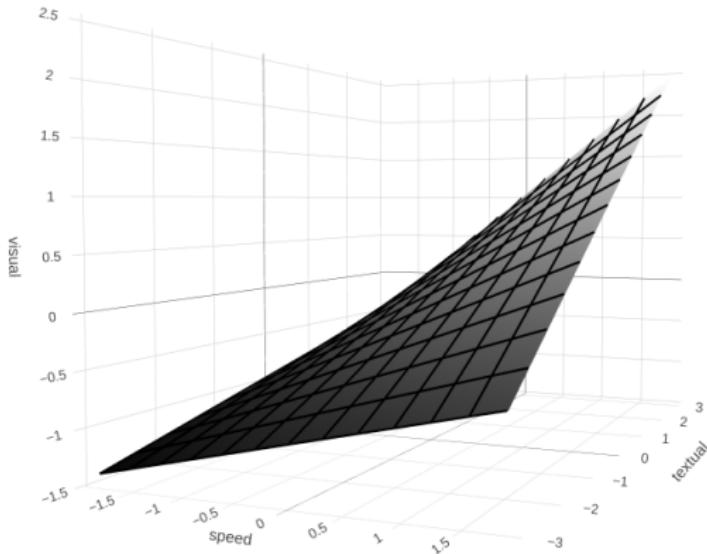
# Johnson-Neyman Plot

```
plot_jn(x = "speed", z = "textual", y = "visual", model = fit, max_z = 6)
```



# Surface Plot

```
plot_surface(x = "speed", z = "textual", y = "visual", model = fit,  
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



## Visualizing Models With Quadratic Effects

# Example Model

```
model <- '
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:X + Z:Z + X:Z
'

fit <- modsem(model, data = data.quadratic, method = "qml")
summary(fit)
```

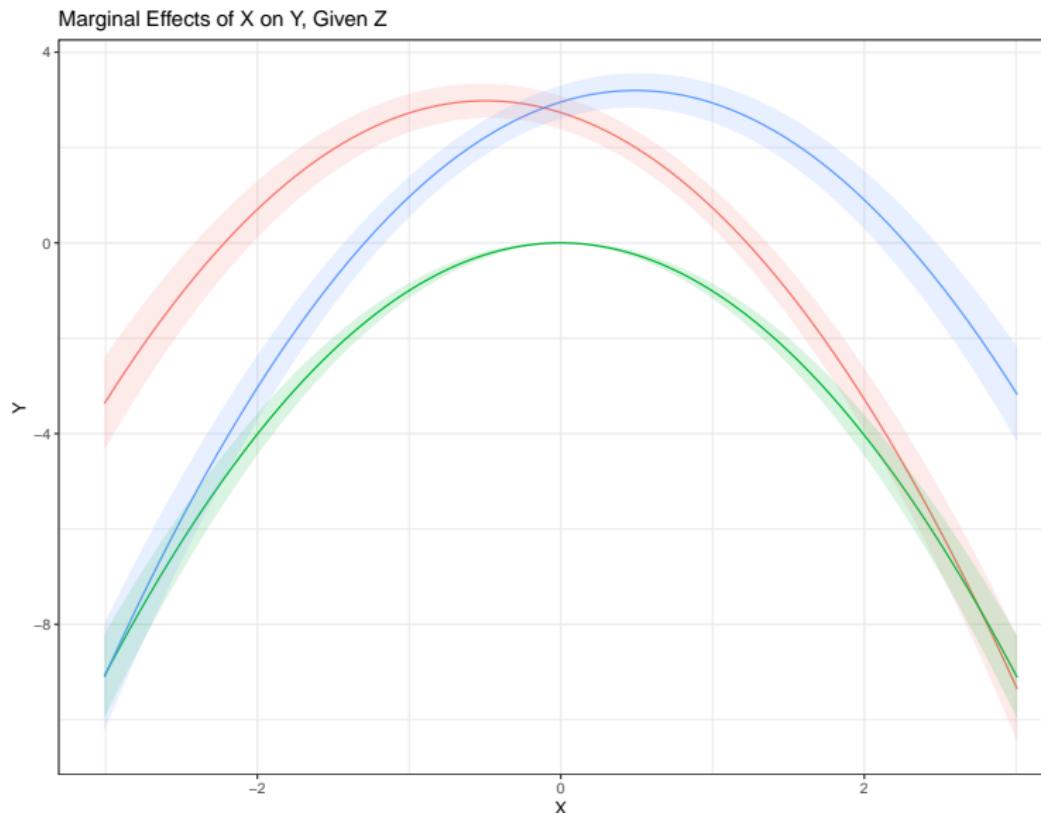
Truncated Output:

Regressions:

	Estimate	Std.Error	z.value	P(> z )
Y ~				
X	-0.005	0.067	-0.081	0.935
Z	0.113	0.163	0.693	0.488
X:X	-1.006	0.044	-23.040	0.000
X:Z	1.011	0.080	12.571	0.000
Z:Z	2.969	0.079	37.362	0.000

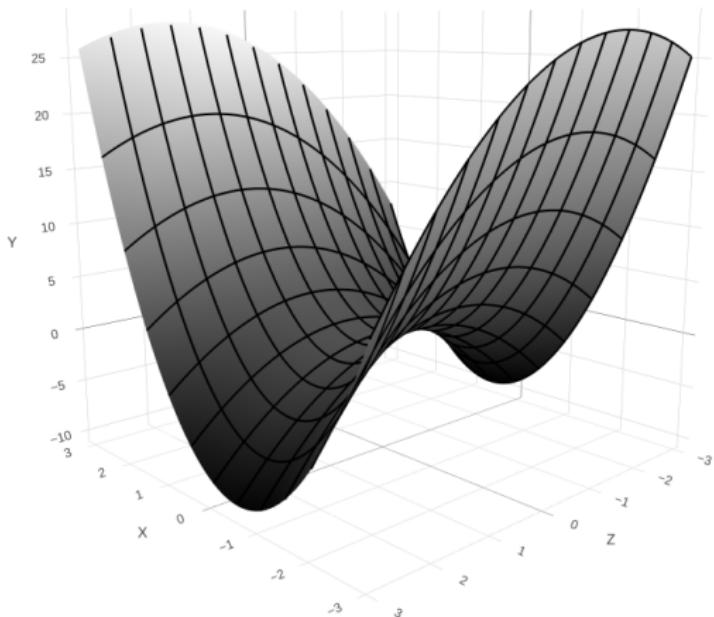
## 2D Plot

```
plot_interaction(x = "X", z = "Z", y = "Y", model = fit, vals_z = c(-1, 0, 1))
```



# 3D (Response Surface) Plot

```
plot_surface(x = "X", z = "Z", y = "Y", model = fit,  
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



## References

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- Kenny, David A., and Charles M. Judd. 1984. "Estimating the Nonlinear and Interactive Effects of Latent Variables." *Psychological Bulletin* 96 (1): 201–10.  
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