modsem: An R Package for Estimating Latent Interactions and Quadratic Effects

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MODSEM

- `modsem` is an `R` package for estimating latent interaction and quadratic effects in *Structural Equation Models* (SEMs).
- Collection of approaches withing the Product Indicator (PI) and Distribution Analytic (DA) frameworks.
- Based on the `lavaan` package.

Frameworks

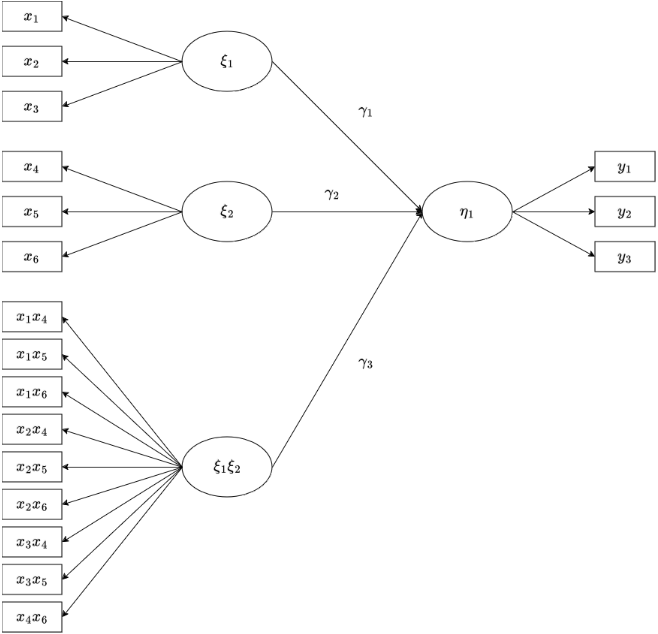
- Product Indicator (PI) approaches
- Distribution Analytic (DA) approaches

Product Indicator (PI) Approaches

Product Indicator (PI) Approaches

- First attempt at latent interaction in SEMs (Kenny and Judd 1984).
- Creates product indicators that serve as indicators for latent interaction terms.
- Traditionally relies on manual model specification and construction of product indicators.
- Early implementations required complicated model constraints.
- Manual specification pressures have gradually simplified those constraints.

Product Indicator (PI) Approaches



Product Indicator (PI) Approaches (History)

- Constrained Approach (orthogonal specification) (Kenny and Judd 1984).
- Constrained Approach (oblique specification) (Jöreskog and Yan 1996).
- Constrained Approach (mean-centered indicators) (Algina and Moulder 2001).
- Unconstrained Approach (constrained latent mean) (Martin and Marsh 1999).
- Residual Centering Approach (no constraints) (Little, Bovaird, and Widaman 2006).
- Double Centering Approach (no constraints) (Lin et al. 2010).

Product Indicator (PI) Approaches (Software)

- `semTools` automates creation of product indicators, but leaves model specification to the user.
- PI approaches therefore still involve error-prone manual setup.
- `modsem` creates the product indicators and model specification automatically.
- The model specification becomes increasingly complex for larger models, and more complicated model constraints.

Example: Double Centering, semTools

```
model <- '
# Measurement Model
  X  =~ x1 + x2 + x3
  Z  =~ z1 + z2 + z3
  Y  =~ y1 + y2 + y3
  XZ =~ x1.z1 + x2.z1 + x3.z1 +
        x1.z2 + x2.z2 + x3.z2 +
        x1.z3 + x2.z3 + x3.z3

# Structural Model
  Y ~ X + Z + XZ

# Residual Covariances
  x1.z1 ~~ x1.z2 + x1.z3 + x2.z1 + x3.z1
  x1.z2 ~~ x1.z3 + x2.z2 + x3.z2
  x2.z1 ~~ x2.z2 + x2.z3 + x3.z1

  x1.z3 ~~ x2.z3 + x3.z3
  x2.z2 ~~ x2.z3 + x3.z2
  x3.z1 ~~ x3.z2 + x3.z3

  x2.z3 ~~ x3.z3
  x3.z2 ~~ x3.z3
'
```

Example: Double Centering, semTools

```r
library(semTools)
data.prod <- indProd(data = oneInt,
                     var1 = c("x1", "x2", "x3"),
                     var2 = c("z1", "z2", "z3"),
                     match = FALSE)

fit <- sem(model, data = data.prod)
summary(fit)
```

```
Truncated Output:
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X                 0.675    0.027   25.379    0.000
    Z                 0.561    0.026   21.606    0.000
    XZ                0.702    0.027   26.360    0.000
```

Example: Double Centering, modsem

```r
library(modsem)

model <- '
# Measurement Model
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

# Structural Model
  Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt)
summary(fit)
```

```
Truncated Output:
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X                 0.675    0.027   25.379    0.000
    Z                 0.561    0.026   21.606    0.000
    XZ                0.702    0.027   26.360    0.000
```

Distribution Analytic (DA) Approaches

Distribution Analytic (DA) Approaches

- Latent Moderated Structural Equations (LMS) method introduced by (A. Klein and Moosbrugger 2000).
- Quasi Maximum Likelihood (QML) method introduced by (A. G. Klein and Muthén 2007).
- Avoids creating product indicators.
- Estimators in PI approaches (incorrectly) assume normally distributed interaction terms,
- and latent endogenous variables $\boldsymbol{\eta}$.
- Explicitly models the non-normal distribution of $\boldsymbol{\eta}$.
- LMS and QML perform similarly, although QML is slightly more robust under normality violations.
- QML scales better computationally when multiple moderators are involved.

Distribution Analytic (DA) Approaches (Software)

- `R` package `nlsem`.
- `Mplus`.
- `modsem`.

NLSEM

- Pros:
  - Free, open-source R package.
  - Has both LMS and QML approaches.
- Cons:
  - Very slow.
  - Hard to use.
  - Only handles models with a single endogenous variable (η).
  - Doesn't handle missing data.
  - Biased LMS standard errors.
  - Unstable estimates, sensitive to starting estimates.
  - Cannot produce standardized estimates.
  - Virtually no fit measures.
  - Does not support linear and nonlinear constraints.
  - Generally unpolished.

Mplus

- Pros:
  - User friendly.
  - Extended LMS implementation covering multiple endogenous variables and endogenous interactions.
  - Integrates with other `Mplus` features (e.g., multilevel SEM).
  - Handles three-way interactions and higher-order models.
  - Provides Full Information Maximum Likelihood (FIML) estimation.
  - Supports linear and nonlinear constraints.
- Cons:
  - Proprietary (expensive) software package.
  - Scales poorly with multiple interaction effects.
  - Does not include QML.
  - No multigroup LMS.

MODSEM

- Pros:
  - Free, open-source `R` package.
  - User friendly.
  - Extended LMS and QML implementation (multiple endogenous variables, endogenous interactions).
  - Computationally optimized LMS and QML estimation.
  - Better LMS scaling relative to `Mplus`.
  - Supports higher-order models and interactions.
  - Provides Full Information Maximum Likelihood (FIML) estimation.
  - Supports linear and nonlinear constraints.
  - Offers multigroup LMS and QML.
- Cons:
  - Missing advanced features available in `Mplus` (e.g., multilevel SEM).

Example: LMS, nlsem (213.2 seconds)

```r
library(nlsem)
set.seed(3248927)

model <- "
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:Z
"

nlsem_model_1 <- lav2nlsem(model)
oneIntSorted <- oneInt[c("x1", "x2", "x3", "z1", "z2",
                         "z3", "y1", "y2", "y3")]
start <- runif(count_free_parameters(nlsem_model_1))

specs <- as.data.frame(nlsem_model_1)
specs[grepl("tau|alpha", specs$label), "class1"] <- 0
specs[grepl("nu\\.(x|y)", specs$label), "class1"] <- NA

nlsem_model_2 <- create_sem(specs)

fit_nlsem <- em(model = nlsem_model_2, data = oneIntSorted,
                convergence = 1e-4, max.iter = 500, start = start)
```

Example: LMS, nlsem (213.2 seconds)

```r
summary(fit_nlsem)
```

```
Truncated Output:
        Estimate Std. Error  z value      Pr(>|z|)
Gamma1 0.6750994 0.03021998 22.33950 1.527303e-110
Gamma2 0.5965296 0.02515988 23.70956 2.873341e-124
Omega3 0.7269645 0.02677033 27.15561 2.174013e-162
```

Example: LMS, Mplus (4.2 seconds)

```
ANALYSIS:
  ESTIMATOR = ML;
  TYPE = RANDOM;
  ALGORITHM = INTEGRATION;

MODEL:
  X BY x1-x3;
  Z BY z1-z3;
  Y BY y1-y3;
  Y ON X Z XZ;
  XZ | X XWITH Z;
```

```
MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
 Y          ON
    X                  0.673      0.031     21.674      0.000
    Z                  0.569      0.030     18.723      0.000
    XZ                 0.718      0.028     25.831      0.000
```

Example: LMS, modsem (1.2 seconds)

```r
model <- '
# Measurement Model
  X  =~ x1 + x2 + x3
  Z  =~ z1 + z2 + z3
  Y  =~ y1 + y2 + y3

# Structural Model
  Y ~ X + Z + X:Z
'

fit <- modsem(model, data = oneInt, method = "lms")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  Y ~
    X               0.673      0.031   21.666    0.000
    Z               0.569      0.030   18.710    0.000
    X:Z             0.718      0.028   25.828    0.000
```

Visualizing Interaction Effects

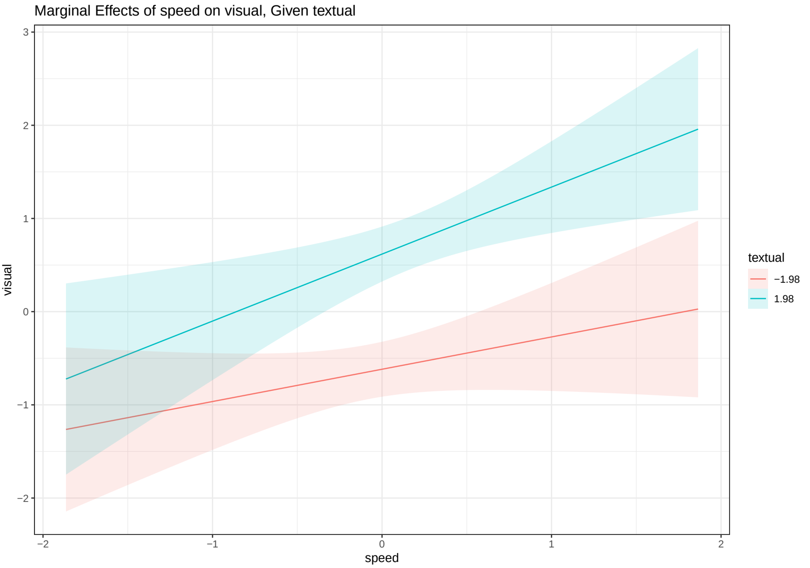
Example Model

```r
model <-  '
  visual  =~ x1 + x2 + x3
  textual =~ x4 + x5 + x6
  speed   =~ x7 + x8 + x9

  visual ~ speed + textual + speed:textual
'

fit <- modsem(model, data = HolzingerSwineford1939, method = "lms")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  visual ~
    textual         0.312      0.076    4.107    0.000
    speed           0.533      0.158    3.366    0.001
    speed:txtl      0.094      0.093    1.016    0.309
```

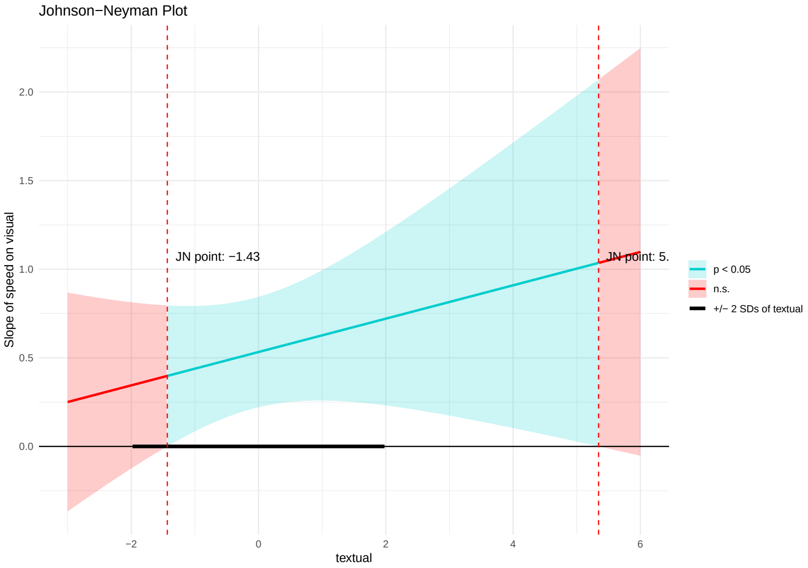
Plotting Margins (Simple Slopes)

```
plot_interaction(x = "speed", z = "textual", y = "visual", model = fit, vals_z
```



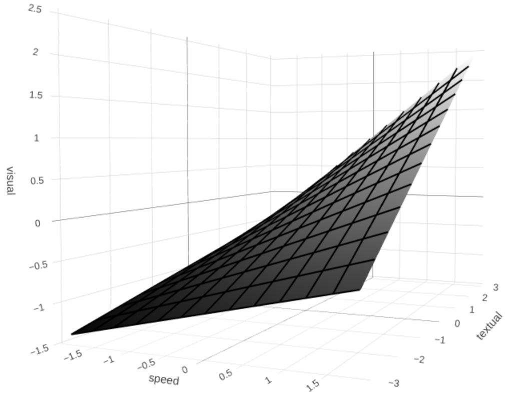
Johnson-Neyman Plot

```
plot_jn(x = "speed", z = "textual", y = "visual", model = fit, max_z = 6)
```



Surface Plot

```r
plot_surface(x = "speed", z = "textual", y = "visual", model = fit,
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



Visualizing Models With Quadratic Effects

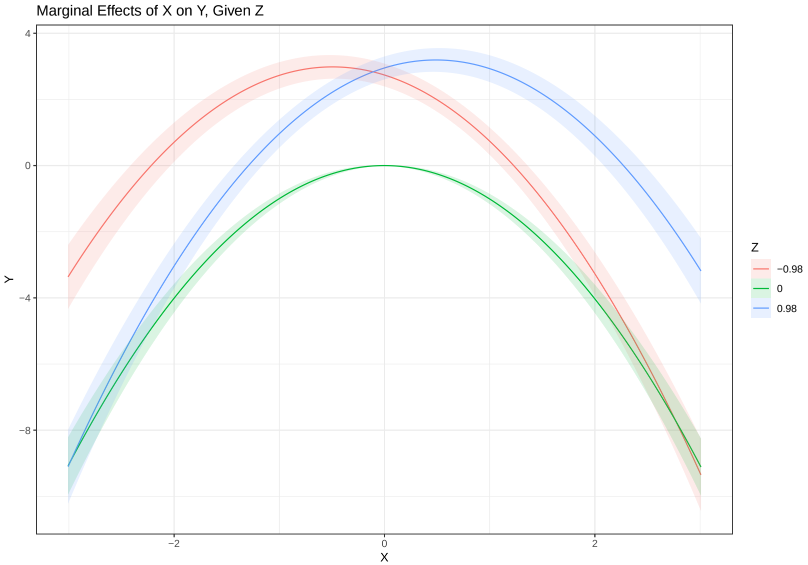
Example Model

```r
model <- '
  X =~ x1 + x2 + x3
  Z =~ z1 + z2 + z3
  Y =~ y1 + y2 + y3

  Y ~ X + Z + X:X + Z:Z + X:Z
'

fit <- modsem(model, data = data.quadratic, method = "qml")
summary(fit)
```

```
Truncated Output:
Regressions:
                 Estimate  Std.Error  z.value  P(>|z|)
  Y ~
    X              -0.006      0.067   -0.088    0.930
    Z               0.110      0.163    0.674    0.501
    X:X            -1.005      0.044  -23.046    0.000
    X:Z             1.011      0.080   12.563    0.000
    Z:Z             2.969      0.079   37.366    0.000
```

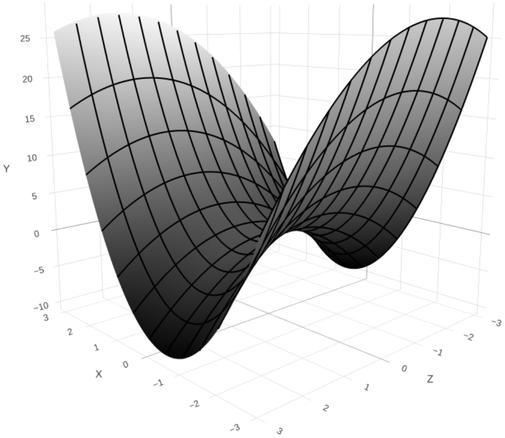
2D Plot

```
plot_interaction(x = "X", z = "Z", y = "Y", model = fit, vals_z = c(-1, 0, 1))
```



3D (Response Surface) Plot

```
plot_surface(x = "X", z = "Z", y = "Y", model = fit,
             colorscale = "Greys", grid = TRUE, grid_color = "black")
```



References

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