First Semester 2022-2023
COMPREHENSIVE EXAMINATION

Course No. : MTH ZG531
Course Title : Advanced Mathematics
Nature of Exam : Closed Book
Weightage : 40%
Duration : 2½ Hours
Date of Exam : Sunday, 27/11/2022 (FN)
No. of Pages = 2
No. of Questions = 3

Q.1 Answer the following questions related to Linear Algebra:
(a) Given a matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, compute its determinant.
(b) If and $\mathbf{v}_2 = (4, 5, 6)$, find their dot product and cross product.
(c) Solve the system $\mathbf{v}_1 = (1, 2, 3)$of equations using Gaussian elimination: i'm not abl [15]

Q.2 Answer the following questions on Calculus and Differential Equations:
(a) Evaluate the integral $\int_0^{\pi} x \sin x \, dx$.
(b) Solve the differential equation $\frac{dy}{dx} + y = e^x$ using the integrating factor method.
(c) Find the Taylor series expansion of $f(x) = e^x$ around $x = 0$ up to the fourth term. [15]

Q.3 State True or False providing due justification for your answers:
(a) If a function $f(x)$ is continuous at $x = a$, then it must be differentiable at $x = a$.
(b) The eigenvalues of a symmetric matrix are always real.
(c) The rank of a matrix A is equal to the number of nonzero rows in its row echelon form.
(d) The Laplace transform of $f(t) = t^2$ is $\frac{2}{s^3}$.
(e) If a sequence $\{a_n\}$ is bounded, then it must be convergent. [10]