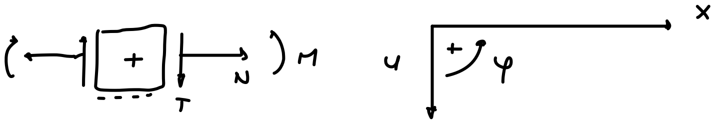
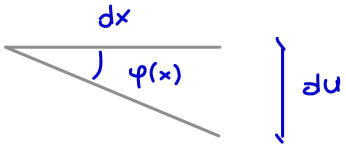
~ Until now

We need to define some relations between momentums, vertical deformation and the rotation of each section:

Let's define some conventions:



We can then say that u is influenced by the momentum and the sheer force. However the sheer force influence can be ignored because its influence is linear while the momentum one is quadratic.



Also because $\varphi(x)$ is small : $\sin(\varphi) \sim \varphi$

$$\Rightarrow \quad du = -\varphi(x) \cdot dx \quad \Rightarrow \quad \frac{du}{dx} = -\varphi(x)$$

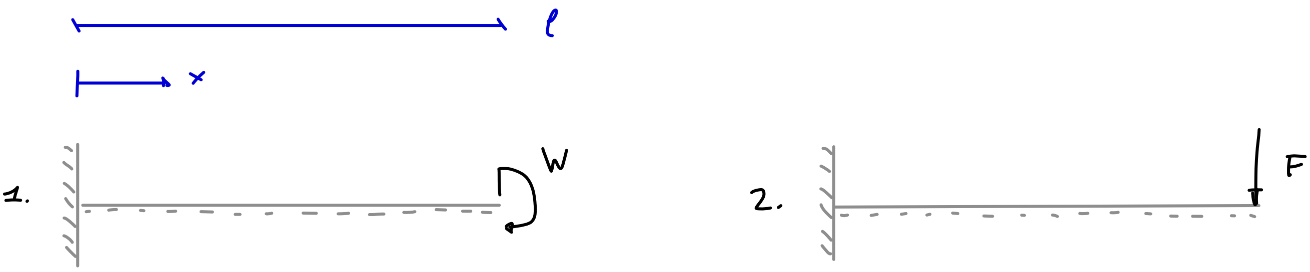
We can then introduce the costitutive bond between momentums and rotation

$$\rightsquigarrow \quad \chi = \frac{d\varphi}{dx} = \frac{M}{EJ} \quad \Longrightarrow \quad \frac{d^2u}{dx^2} = -\frac{M(x)}{EJ}$$

Now given a structure with forces applied and costrains we can find the $u(x)$ as the sum of the effects of simplyer forces.

Particulary for the fixed constrain we can use two border conditions:

$u(x=0) = 0$ and $\varphi(x=0) = du/dx\ (x=0) = 0$



- $M(x) = -W$

$\rightarrow \dfrac{d^2u}{dx^2} = \dfrac{W}{EJ}$

$\rightarrow \dfrac{du}{dx} = \dfrac{Wx}{EJ}$

$\rightarrow u(x) = \dfrac{1}{EJ} \cdot \dfrac{Wx^2}{2}$

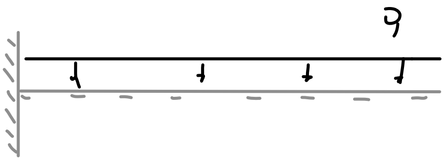
- $M(x) = -F(\ell - x)$

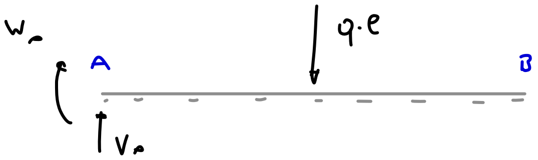
$\rightarrow \dfrac{d^2u}{dx^2} = \dfrac{F(\ell - x)}{EJ}$

$\rightarrow \dfrac{du}{dx} = \dfrac{F}{EJ}\left(\ell x - \dfrac{x^2}{2}\right)$

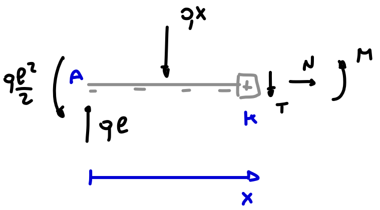
$\rightarrow u(x) = \dfrac{F}{EJ}\left(\dfrac{\ell x^2}{2} - \dfrac{x^3}{6}\right)$

3.





$$\begin{cases} +\uparrow\ \sum F_y = V_a - q\ell = 0 & \rightarrow\ V_a = q\ell \\ \circlearrowleft\!+\ \sum M_{(A)} = -q\ell\cdot\frac{\ell}{2} - W_a = 0 & \rightarrow\ M_a = -q\frac{\ell^2}{2} \end{cases}$$



$$\rightsquigarrow \begin{cases} \xrightarrow{+}\ \sum F_x = N = 0 & \\ +\uparrow\ \sum F_y = -q\cdot x + q\ell - T = 0 & \rightarrow\ T = q(\ell - x) \\ \circlearrowleft\!+\ \sum M_K = M + q\frac{\ell^2}{2} - q\ell x + qx\cdot\frac{x}{2} = 0 & \end{cases}$$

$$\Rightarrow M = q\ell x - q\frac{\ell^2}{2} - q\frac{x^2}{2} = q\left(\ell x - \frac{\ell^2}{2} - \frac{x^2}{2}\right) = -\frac{q}{2}\left(\ell - x\right)^2$$

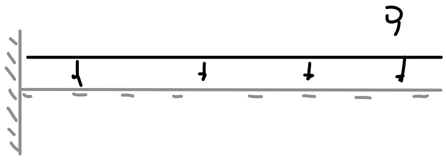
$$\rightarrow \frac{d^2u}{dx^2} = -\frac{M(x)}{EJ} = \frac{1}{EJ}\cdot\frac{q}{2}\left(\ell - x\right)^2$$

$$\rightarrow \frac{du}{dx} = \frac{1}{EJ}\cdot\frac{q}{6}\left(\ell - x\right)^3 + C_1 \quad \xrightarrow{x=0} \quad \frac{1}{EJ}\cdot\frac{q}{6}\ell^3 + C_1 = 0 \ \rightarrow\ C_1 = -\frac{1}{EJ}\cdot\frac{q}{6}\ell^3$$

$$\rightarrow u(x) = -\frac{1}{EJ}\cdot\frac{q}{24}\left(\ell - x\right)^4 + C_1\cdot x + C_2 \quad \xrightarrow{x=0} \quad C_2 = +\frac{1}{EJ}\cdot\frac{q}{24}\ell^4$$

$$\rightsquigarrow \frac{du}{dx} = \frac{1}{EJ}\cdot\frac{q}{6}\cdot\left[\left(\ell - x\right)^3 - \ell^3\right]$$

$$\rightsquigarrow u(x) = \frac{q}{EJ}\left[-\frac{1}{24}\left(\ell - x\right)^4 - \frac{1}{6}\cdot\ell^3 x + \frac{1}{24}\ell^4\right]$$



- $M(x) = -\frac{q}{2}\left(\ell - x\right)$

$$\rightarrow \frac{d^2u}{dx^2} = \frac{q}{EJ}\cdot\frac{q}{2}\left(\ell - x\right)^2$$

$$\rightarrow \frac{du}{dx} = \frac{1}{EJ}\cdot\frac{q}{6}\cdot\left[\left(\ell - x\right)^3 - \ell^3\right]$$

$$\rightarrow u = \frac{q}{EJ}\left[-\frac{1}{24}\left(\ell - x\right)^4 - \frac{1}{6}\cdot\ell^3 x + \frac{1}{24}\ell^4\right]$$