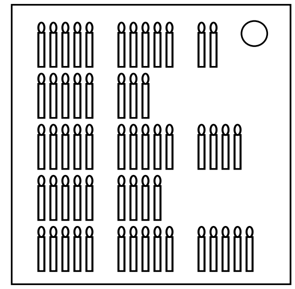
# On the Subject of Nim



*But not too much.*

Beat the module in the match-taking game of Nim.

## How to play

In Nim, two players take turns taking as many matches as they want, from a single row. Take the last match to win (normal play, no misère game).

Click on a row as many times as the number of matches you wish to take. Your turn ends automatically after a short timeout.

When you lose the game, you get a strike and the module resets to a new random configuration.

## How to win

The key to the theory of the game is the binary digital sum, or "bitwise xor", of the row sizes. This is referred to as the nim-sum, and can be written with this symbol: ⊕. An example of the calculation with rows of size 3, 4, and 5:

```
           Decimal    Binary
    Row A        3       011
    Row B        4       100
    Row C        5       101 ⊕
           -----------------
                 2       010
```

The nim-sum of rows A, B, and C is $3 \oplus 4 \oplus 5 = 2$.

The winning strategy is to finish every move with a nim-sum of 0. This is always possible if the nim-sum is not zero before the move. If the nim-sum is zero, then the next player will lose if the other player does not make a mistake. To find out which move to make, let X be the nim-sum of all the row sizes. For each row, let Y be the nim-sum of X and the row-size. If Y is less than the row-size, the winning strategy is to reduce that row to Y. In the example above, with a nim-sum of X=2:

```
    A ⊕ X = 3 ⊕ 2 = 1 (Since binary 011 ⊕ 010 = 001)
    B ⊕ X = 4 ⊕ 2 = 6
    C ⊕ X = 5 ⊕ 2 = 7
```

The only row that is reduced is row A, so the winning move is to reduce the size of row A to 1, by removing two matches. More info on [Wikipedia (https://en.wikipedia.org/wiki/Nim)](https://en.wikipedia.org/wiki/Nim).