

Gabriel Catayan Toris Oliveira - 20.1.4004 - Turma 95

1) a)  $\lim_{x \rightarrow 1^+} \frac{2 \ln(x-1)}{3}$

$\frac{2}{3} \cdot \lim_{x \rightarrow 1^+} \ln(x-1) = m$

$e^m = 0^+$

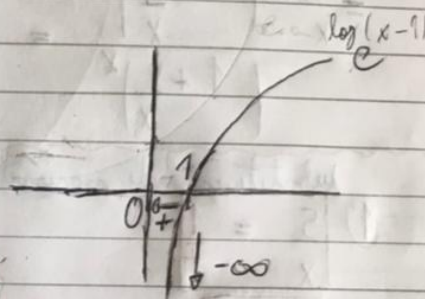
$m = -\infty$

$\lim_{x \rightarrow 1^+} \frac{2 \ln(x-1)}{3} = -\infty$

Análise do ponto

$x \rightarrow 1^+$

$x-1 \rightarrow 0^+$



b) 1)  $\lim_{x \rightarrow -\infty} e^{\sqrt{x^2+1}}$

$\lim_{x \rightarrow -\infty} e^{\sqrt{x^2+1}}$

3)  $u = \sqrt{x^2+1}$

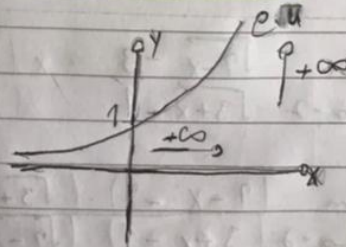
$\lim_{u \rightarrow +\infty} e^u = +\infty$

2) Análise do ponto

$x \rightarrow -\infty$

$\sqrt{x^2+1} \rightarrow +\infty$

$x^2+1 \rightarrow +\infty$



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$$c) \lim_{x \rightarrow +\infty} \left( \frac{x+5}{x+4} \right)^x = \lim_{x \rightarrow +\infty} \left( \frac{\frac{x+5}{x}}{\frac{x+4}{x}} \right)^x$$

$$= \lim_{x \rightarrow +\infty} \left( 1 + \frac{5}{x} \right)^x = \lim_{t \rightarrow +\infty} \left( 1 + \frac{1}{t} \right)^{5t} = \left[ \lim_{t \rightarrow +\infty} \left( 1 + \frac{1}{t} \right)^t \right]^5$$

$$= \lim_{x \rightarrow +\infty} \left( 1 + \frac{4}{x} \right)^x = \lim_{u \rightarrow +\infty} \left( 1 + \frac{1}{u} \right)^{4u} = \left[ \lim_{u \rightarrow +\infty} \left( 1 + \frac{1}{u} \right)^u \right]^4$$

substituição de variável

$$\textcircled{1} \begin{cases} 5 = \frac{1}{t} \\ x \rightarrow +\infty \\ x = 5t \end{cases}$$

$$\textcircled{2} \begin{cases} 4 = \frac{1}{u} \\ x \rightarrow +\infty \\ x = 4u \end{cases}$$

$$t = \frac{x}{5}$$

$$x = 4u$$

$$u = \frac{x}{4}$$

$$t \rightarrow +\infty$$

$$u \rightarrow +\infty$$

$$\textcircled{3} \frac{e^5}{e^4} = e$$

$$\lim_{x \rightarrow +\infty} \left( \frac{x+5}{x+4} \right)^x = e$$

$$d) \lim_{x \rightarrow -2} \frac{\sqrt{7-x} - 3}{\sqrt{3+x} - 1} \cdot \frac{x(\sqrt{7-x} + 3) \cdot (\sqrt{3+x} + 1)}{(\sqrt{7-x} + 3) \cdot (\sqrt{3+x} + 1)}$$

$$\lim_{x \rightarrow -2} \frac{(7-x-9) \cdot (\sqrt{3+x} + 1)}{(3+x-1) \cdot (\sqrt{7-x} + 3)}$$

$$\lim_{x \rightarrow -2} \frac{-(x+2) \cdot (\sqrt{3+x} + 1)}{x+2 \cdot (\sqrt{7-x} + 3)} = \lim_{x \rightarrow -2} \frac{-1 \cdot (\sqrt{3+x} + 1)}{\sqrt{7-x} + 3}$$

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$$= \lim_{x \rightarrow -2} - \left| \frac{\sqrt{3+x} + 1}{\sqrt{7-x} + 3} \right| = \lim_{x \rightarrow -2} - \left| \frac{\sqrt{3-2} + 1}{\sqrt{7+2} + 3} \right| =$$

$$= - \left| \frac{\sqrt{1} + 1}{\sqrt{9} + 3} \right| = - \left| \frac{2}{6} \right| = \left( -\frac{1}{3} \right)$$

$$\lim_{x \rightarrow -2} \frac{\sqrt{7-x} - 3}{\sqrt{3+x} - 1} = \frac{-1}{3}$$

2o Assintota Vertical

$$f(x) = \frac{x^2 - 2x - 3}{x^4 - 10x^2 + 9}$$

1°  $x^4 - 10x^2 + 9$

$$x^2 = y$$

$$y^2 - 10y + 9$$

$$(y - 9) \cdot (y - 1)$$

$$\begin{matrix} \downarrow & \downarrow \\ y^2 = 9 & y^2 = 1 \end{matrix}$$

2°  $x^2 = 9$

$$x = \pm \sqrt{9}$$

$$x^1 = +3$$

$$x^1 = -3$$

Raiz

$$x^2 = 1$$

$$x = \pm \sqrt{1}$$

$$x^1 = 1$$

$$x^1 = -1$$

3°  $f(x) = \frac{(x-3)(x+1)}{(x+3)(x-3)(x+1)(x-1)}$

$$(x+3) \cdot (x-3) \cdot (x+1) \cdot (x-1)$$

IMP: Domínio  
 $x \neq -3, x \neq 1$

$$f(x) = \frac{1}{(x+3) \cdot (x-1)}$$

$$x^0 = -3 \quad x^0 = 1$$

FORONI



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$\lim_{x \rightarrow -3} f(x) \rightarrow \lim_{x \rightarrow -3^+} \frac{1}{(x+3)(x-1)} = -\infty$   
 $\rightarrow 0^+$   $\rightarrow -4$

Análise do ponto  
 $x \rightarrow -3^+$   
 $x+3 \rightarrow 0^+$   
 $x-1 \rightarrow -4$

$\lim_{x \rightarrow -3^-} \frac{1}{(x+3)(x-1)} = +\infty$   
 $\rightarrow 0^-$   $\rightarrow -4$

Análise do ponto  
 $x \rightarrow -3^-$   
 $x+3 \rightarrow 0^-$   
 $x-1 \rightarrow -4$

$\lim_{x \rightarrow 1} f(x) \rightarrow \lim_{x \rightarrow 1^+} \frac{1}{(x+3)(x-1)} = +\infty$   
 $\rightarrow 4$   $\rightarrow 0^+$

$\lim_{x \rightarrow 1^-} \frac{1}{(x+3)(x-1)} = -\infty$   
 $\rightarrow 4$   $\rightarrow 0^-$

Assíntotas verticais:  $x=1$  e  $x=-3$

Análise do ponto  
 $x \rightarrow 1^+$   
 $x+3 \rightarrow 4$   
 $x-1 \rightarrow 0^+$   
  
 $x \rightarrow 1^-$   
 $x+3 \rightarrow 4$   
 $x-1 \rightarrow 0^-$

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5) Assintota horizontal

$$f(x) = \frac{1}{(x+3)(x-1)}$$

$$f(x) = \frac{1}{x^2 + 2x - 3}$$

Assintota horizontal:  $x=0$

$$\textcircled{6} \lim_{x \rightarrow \pm\infty} \frac{1}{x^2} = \lim_{x \rightarrow \pm\infty} \frac{1}{1 + 2\frac{0}{x} - 3\frac{0}{x^2}} = \textcircled{0}$$

$$b) g(x) = \frac{\sqrt{9x^2+3}}{3x-1}$$

Domínio  
 $3x-1 \neq 0$   
 $x \neq \frac{1}{3}$

Assintota vertical

$$\lim_{x \rightarrow \frac{1}{3}^+} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \rightarrow \frac{1}{3}^+} \frac{\sqrt{3(x^2+1)}}{3x-1} = +\infty$$

$$\lim_{x \rightarrow \frac{1}{3}^-} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \rightarrow \frac{1}{3}^-} \frac{\sqrt{3(x^2+1)}}{3x-1} = -\infty$$

Análise de pontos

$$x \rightarrow \frac{1}{3}^+$$

$$x \rightarrow \frac{1}{3}^-$$

$$\begin{aligned} (x^2+1) &\rightarrow 1 \\ 3x-1 &\rightarrow 0^+ \end{aligned}$$

$$\begin{aligned} (x^2+1) &\rightarrow 1 \\ 3x-1 &\rightarrow 0^- \end{aligned}$$

Assintota vertical:  $x = \frac{1}{3}$

FORONI

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## ② Assíntota horizontal

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{9x^2+3}{x^2}}}{\frac{3x-1}{x}} =$$

$$\boxed{\sqrt{x^2} = x}$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{9 + \frac{3}{x^2}}}{3 - \frac{1}{x}} =$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{9}}{3} = \frac{3}{3} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2+3}}{-\sqrt{x^2}} =$$

$$\boxed{-\sqrt{x^2} = x}$$

$$\sqrt{x^2} = |x| \begin{cases} x, & \text{se } x > 0 \\ -x, & \text{se } x < 0 \end{cases} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2+3}}{-\sqrt{x^2}} = \frac{-\sqrt{9}}{-1} = 1$$

Assíntota horizontal:  $y = 1$  e  $y = -1$