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1) a) $\lim_{x \to 1^+} \frac{2}{3} \ln(x-1)$

$\frac{2}{3} \cdot \lim_{x \to 1^+} \log_e \overset{0^+}{(x-1)} = m$

$\boxed{e^m = 0^+}$

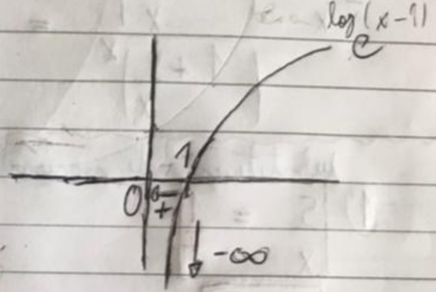
$m = -\infty$

$\lim_{x \to 1^+} \frac{2}{3} \ln(x-1) = -\infty$

Análise do ponto

$x \to 1^+$

$x-1 \to 0^+$



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b) ① $\lim_{x \to -\infty} e^{\sqrt{x^2+1}}$

$\lim_{x \to -\infty} e^{\sqrt{+\infty}}$

③ $\boxed{u = \sqrt{+\infty}}$

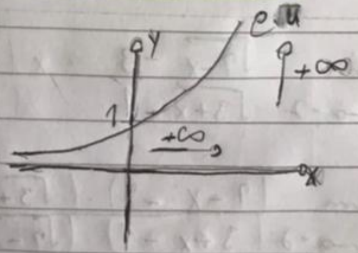
$\lim_{u \to +\infty} e^u = +\infty$

② **Análise do ponto**

$x \to -\infty$

$(-\infty)^2 = +\infty$

$x^2 + 1 \to +\infty$



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c) $\lim\limits_{x \to +\infty} \left(\dfrac{x+5}{x+4}\right)^x = \lim\limits_{x \to +\infty} \dfrac{\left(\dfrac{x}{x} + \dfrac{5}{x}\right)^x}{\left(\dfrac{x}{x} + \dfrac{4}{x}\right)}$

$$= \lim_{x \to +\infty} \frac{\left(1 + \dfrac{5}{x}\right)^x}{\left(1 + \dfrac{4}{x}\right)^x} = \frac{\lim\limits_{t \to +\infty} \left(1 + \dfrac{1}{t}\right)^{5t}}{\lim\limits_{u \to +\infty} \left(1 + \dfrac{1}{u}\right)^{4u}} = \frac{\left[\overbrace{\lim\limits_{t \to +\infty} \left(1 + \dfrac{1}{t}\right)^{t}}^{e}\right]^5}{\left[\underbrace{\lim\limits_{t \to +\infty} \left(1 + \dfrac{1}{u}\right)^{u}}_{e}\right]^4}$$

substituição de variável

1º $\boxed{\dfrac{5}{x} = \dfrac{1}{t}}$ $\boxed{x = 5t}$ $t = \dfrac{x}{5}$ $t \to +\infty$

2º $\boxed{\dfrac{4}{x} = \dfrac{1}{u}}$ $\boxed{x = 4u}$ $u = \dfrac{x}{4}$ $u \to +\infty$

3º $\dfrac{e^5}{e^4} = e^1$

$$\lim_{x \to +\infty} \left(\frac{x+5}{x+4}\right)^x = e^1$$

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d) $\lim\limits_{x \to -2} \dfrac{\sqrt{7-x} - 3}{\sqrt{3+x} - 1} \times \dfrac{(\sqrt{7-x} + 3) \cdot (\sqrt{3+x} + 1)}{(\sqrt{7-x} + 3) \cdot (\sqrt{3+x} + 1)}$

$$\lim_{x \to -2} \frac{(7 - x - 9) \cdot (\sqrt{3+x} + 1)}{(3 + x - 1) \cdot (\sqrt{7-x} + 3)}$$

$$\lim_{x \to -2} \frac{-\cancel{(x+2)}\,(\sqrt{3+x} + 1)}{\cancel{x+2}\,(\sqrt{7-x} + 3} = \lim_{x \to -2} \frac{-1 \cdot (\sqrt{3+x} + 1)}{\sqrt{7-x} + 3}$$

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$$= \lim_{x \to -2} -\left(\frac{\sqrt{3+x}+1}{\sqrt{7-x}+3}\right) = \lim_{x \to -2} -\left(\frac{\sqrt{3-2}+1}{\sqrt{7+2}+3}\right) =$$

$$= -\left(\frac{\sqrt{1}+1}{\sqrt{9}+3}\right) = -\left(\frac{2}{6}\right) = -\frac{1}{3}$$

$$\lim_{x \to -2} \frac{\sqrt{7-x}-3}{\sqrt{3+x}-1} = -\frac{1}{3}$$

2ª Assíntota vertical

$$f(x) = \frac{x^2-2x-3}{x^4-10x^2+9}$$

① $x^4 - 10x^2 + 9$

$\boxed{x^2 = y}$

$y^2 - 10y + 9$

$(y-9)\cdot(y-1)$

$\boxed{y' = 9}$ $\boxed{y'' = 1}$

② $x^2 = 9$

$x = \pm\sqrt{9}$

$x' = +3$

$x'' = -3$

Raízes

$x^2 = 1$

$x = \pm\sqrt{1}$

$x''' = 1$

$x'''' = -1$

③ $f(x) = \dfrac{\cancel{(x-3)}\,\cancel{(x+1)}}{(x+3)\cdot\cancel{(x-3)}\cdot\cancel{(x+1)}\,(x-1)}$

IMP: Domínio $x \neq -3$, $x \neq 1$

$$\boxed{f(x) = \frac{1}{(x+3)\cdot(x-1)}}$$

$x' = -3$ $x'' = 1$

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(4) $\lim_{x \to -3} f(x)$

$\to \lim_{x \to -3^+} \dfrac{1}{\underbrace{(x+3)}_{\to 0^+}\underbrace{(x-1)}_{\to -4}} = -\infty$

Análise do ponto

$x \to -3^+$

$x+3 \to 0^+$

$x-1 \to -4$

$\to \lim_{x \to -3^-} \dfrac{1}{\underbrace{(x+3)}_{\to 0^-}\underbrace{(x-1)}_{\to -4}} = +\infty$

Análise do ponto

$x \to -3^-$

$x+3 \to 0^-$

$x-1 \to -4$

$\lim_{x \to 1} f(x) \to \lim_{x \to 1^+} \dfrac{1}{\underbrace{(x+3)}_{\to 4}\underbrace{(x-1)}_{\to 0^+}} = +\infty$

$\to \lim_{x \to 1^-} \dfrac{1}{\underbrace{(x+3)}_{\to 4}\underbrace{(x-1)}_{\to 0^-}} = -\infty$

Análise do ponto

$x \to 1^+$

$x+3 \to 4$

$x-1 \to 0^+$

$x \to 1^-$

$x+3 \to 4$

$x-1 \to 0^-$

Assíntotas verticais: $\boxed{x = 1}$ e $\boxed{x = -3}$

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⑤ Assíntotas horizontais

$$f(x) = \frac{1}{(x+3)(x-1)}$$

$$\boxed{f(x) = \frac{1}{x^2+2x-3}}$$

Assíntota horizontal: $\boxed{y=0}$

⑥ $$\lim_{x \to \pm\infty} \frac{\frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}} = \lim_{x \to \pm\infty} \frac{\cancel{\frac{1}{x^2}}^{0}}{1 + \cancel{\frac{2}{x}}^{0} - \cancel{\frac{3}{x^2}}^{0}} = 0$$

b) $$g(x) = \frac{\sqrt{9x^2+3}}{3x-1}$$

Domínio

$3x-1 \neq 0$

$$\boxed{x \neq \frac{1}{3}}$$

⑦ Assíntota vertical

$$\lim_{x \to \frac{1}{3}} \frac{\sqrt{9x^2+3}}{3x-1}$$

$$\lim_{x \to \frac{1}{3}^+} \frac{\sqrt{3(x^2+1)}}{3x-1} = +\infty \qquad 3x-1 \to 0^+$$

$$\lim_{x \to \frac{1}{3}^-} \frac{\sqrt{3(x^2+1)}}{3x-1} = -\infty \qquad 3x-1 \to 0^-$$

Análise do ponto

$x \to \frac{1}{3}^+$ | $x \to \frac{1}{3}^-$

$(x^2+1) \to 1$ | $(x^2+1) \to 1$

$3x-1 \to 0^+$ | $3x-1 \to 0^-$

Assíntota vertical: $\boxed{x = \frac{1}{3}}$

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2 Assíntota horizontal

$$\lim_{x \to +\infty} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \to +\infty} \frac{\sqrt{\frac{9\cancel{x^2}}{\cancel{x^2}} + \frac{3}{x^2}}}{\frac{3\cancel{x}}{\cancel{x}} - \frac{1}{x}} =$$

$$\boxed{\sqrt{x^2} = x}$$

$$\lim_{x \to +\infty} \frac{\sqrt{9 + \cancel{\frac{3}{x^2}}^{\,0}}}{3 - \cancel{\frac{1}{x}}^{\,0}}$$

$$\lim_{x \to +\infty} \frac{\sqrt{9}}{3} = \frac{3}{3} = 1$$

$$\lim_{x \to -\infty} \frac{\sqrt{9x^2+3}}{3x-1} = \lim_{x \to -\infty} \frac{\frac{\sqrt{9x^2+3}}{-\sqrt{x^2}}}{\frac{3\cancel{x}-1}{\cancel{x}}} =$$

$$\boxed{-\sqrt{x^2} = x}$$

$$\sqrt{x^2} = |x| \begin{cases} \underbrace{x, \text{ se } x>0}_{+\infty} \\ \underbrace{-x, \text{ se } x<0}_{-\infty} \end{cases}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{\frac{9\cancel{x^2}}{\cancel{x^2}} + \cancel{\frac{3}{x^2}}^{\,0}}}{3 - \cancel{\frac{1}{x}}^{\,0}} = \frac{-\sqrt{9}}{3} = -1$$

Assíntotas horizontais: $\boxed{y=1}$ e $\boxed{y=-1}$