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Considere a relação de recorrência $T(n) = 2 \cdot T(n/2) + 2$, com caso base $T(1) = 2$. Encontre a forma fechada de $T(n)$.

1ª

k

1 $T(n) = 2T(n/2) + 2$

$\bullet\ T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + 2$

2 $T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + 2\right) + 2$

$= 2^2 T\left(\frac{n}{2^2}\right) + 2^2 + 2$

$\bullet\ T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + 2$

3 $T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + 2\right) + 2^2 + 2$

$= 2^3 T\left(\frac{n}{2^3}\right) + 2^3 + 2^2 + 2$

$\vdots$

k $T(n) = 2^k T\left(\frac{n}{2^k}\right) + \sum_{i=1}^{k} 2^i$

$$\boxed{\sum_{i=1}^{k} a r^i = a\left(\frac{1-r^k}{1-r}\right)}$$

$2^k T\left(\frac{n}{2^k}\right) + \left(\frac{1-2^k}{1-2}\right) \cdot 2$

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2º $\frac{n}{2^k} = 1$ $\quad T(n) = 2^k T\left(\frac{n}{2^k}\right) + \left(\frac{1-2^k}{1-2}\right) 2$

$\log_2 n = k$

$= 2^k T\left(\frac{n}{2^k}\right) - (1-2^k) \cdot 2$

$= 2^k T\left(\frac{n}{2^k}\right) - (1-2^k) \cdot 2$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) - 2 + (2^k \cdot 2)$

3º $T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) - 2 + 2^{\log_2 n} \cdot 2$

$T(n) = n \cdot T\left(\frac{n}{n}\right) - 2 + n \cdot 2$

$T(n) = n \cdot T(1) - 2 + 2n$

$T(n) = 2n - 2 + 2n$

$T(n) = 4n - 2$

$\boxed{\text{Resposta: } T(n) = 4n - 2}$