

Recurrence Relation: Explain the recurrence relation used in DTW, describing how it facilitates the dynamic programming approach to find the minimal distance.

The recurrence relation used in DTW algorithm in part 1a is

$$\text{costMatrix}[i][j] = \text{costOf}(A[i-1], B[j-1]) + \min(\text{costMatrix}[i-1][j], \text{costMatrix}[i, j-1], \text{costMatrix}[i-1][j-1])$$

This means that the minimum cumulative cost for the cell at i th row and j th column of the cost matrix is calculated by first finding the difference between $(i-1)$ th element in sequence A with $(j-1)$ th element in sequence B, and then add it with the minimum cumulative costs of all possible pathways that can reach to the cell (i, j) . Hence, breaking down the problem of finding the minimum cumulative cost path for all elements in the two time series into the subproblem that finds the minimum cumulative cost path for first i th elements in sequence A and the first j th elements in sequence B. In the algorithm, this dynamic programming approach is captured by the recurrence relation shown above where the value computed for the cell at $\text{costMatrix}[i][j]$ represents the minimum cumulative cost path for first i th elements in sequence A and the first j th elements in sequence B.