DH

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First utilize DH representation:

1. Z^K : each Z always align with the axis of joint
2. $origin^K$: intersection of Zs, or norm of Zs if parallel
3. X^K: to be orthogonal to Zs. away from prev Z
4. Y^K: right handed

as for the tool tip:
Z: r3 approach
Y:r2 sliding
X:r1 normal

Second generate 4tuples for all joints(except joint0):

θ_k: Xs angle about prev Z
d_k: Xs dist about prev Z
a_k: Zs dist about now X
α_k :Zs angle about now X

Third use forward kinematics to determine T^k_{k-1}

Transferring Frame *k-1* to Frame *k*

Operation		Description
1	**Rotate**	L_{k-1} **about** z^{k-1} **by** θ_k.
2	**Translate**	L_{k-1} **along** z^{k-1} **by** d_k.
3	**Translate**	L_{k-1} **along** x^{k-1} **by** a_k.
4	**Rotate**	L_{k-1} **about** x^{k-1} **by** α_k

$$T^k_{k-1}(\theta_k, d_k, a_k, \alpha_k) = Rot(\theta_k, 3)Tran(d_k i^3)Tran(a_k i^1)Rot(\alpha_k, 1)$$

so we get `Link-Coordinate Transformation`

$$T_{k-1}^{k} = \begin{pmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse of T is:

$$T^{-1} = \begin{pmatrix} & R^T & & -R^T p \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Its inverse transformation which maps link *k-1* coordi
given as:

$$T_k^{k-1} = \begin{pmatrix} C\theta_k & S\theta_k & 0 & -a_k \\ -C\alpha_k S\theta_k & C\alpha_k C\theta_k & S\alpha_k & -d_k S\alpha_k \\ S\alpha_k S\theta_k & -S\alpha_k C\theta_k & C\alpha_k & -d_k C\alpha_k \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and we also define

$$q_k = \xi_k \theta_k + (1-\xi_k) d_k.$$

because a joint can either but not both be revolute and prismatic

Arm Equation

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$$T_{base}^{tool}(q) = \begin{pmatrix} & R(q) & & p(q) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

as for the p(q), each dim denotes an axis

as for the R(q), each column denotes an axis rotation

Direct Kinematics

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$$R_1(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

$$R_2(\phi) = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$R_3(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The rotation matrix must be orthogonal matrix
thus we have

$$Rot^{-1}(\phi,k) = Rot(-\phi,k) = Rot^{T}(\phi,k) \text{ for } 1 \le k \le 3$$

Inverse Homogeneous Transformation

The inverse of T is:

$$T^{-1} = \begin{pmatrix} & R^T & & -R^T p \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Normal, Sliding, and Approach

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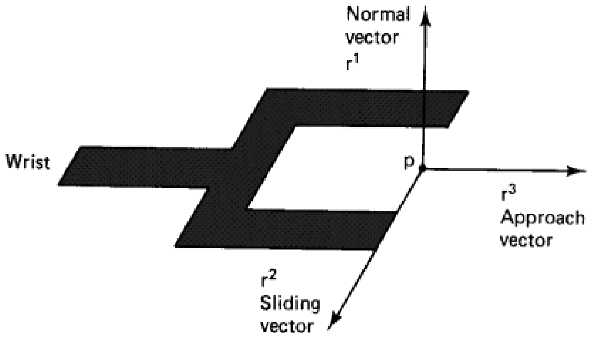
2.5.2 Normal, Sliding, and Approach Vectors



The orientation of the tool can be expressed in rectangular coordinates by a rotation matrix $R = [r^1, r^2, r^3]$, where the three columns of R correspond to the

- normal vector,
- sliding vector, and
- approach vectors,

respectively. The origin of the $\{r^1, r^2, r^3\}$ frame is usually placed at the tool tip.

Tool-Configuration

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$$w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{pmatrix} = \begin{pmatrix} p \\ exp(q_n / \pi)\ r^3 \end{pmatrix}$$

$$q_n = \pi\, ln(w_4^2 + w_5^2 + w_6^2)^{1/2}$$