Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
A		0000000

Enjoy a Cozy and Green Bath

Summary

A traditional bathtub cannot be reheated by itself, so users have to add hot water from time to time. Our goal is to establish a model of the temperature of bath water in space and time. Then we are expected to propose an optimal strategy for users to keep the temperature even and close to initial temperature and decrease water consumption.

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In Question 2,

In Question 3,

Keywords: Heat transfer, Thermodynamic system, CFD, Energy conservation

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3 Assumptions and Justifications

In response to the title of this article, the following hypotheses are proposed:

- **The bath water is incompressible Non-Newtonian fluid**. The incompressible Non-Newtonian fluid is the basis of Navier–Stokes equations which are introduced to simulate the flow of bath water.
- **All the physical properties of bath water, bathtub and air are assumed to be stable**. The change of those properties like specific heat, thermal conductivity and density is rather small according to some studies. It is complicated and unnecessary to consider these little change so we ignore them.
- **There is no internal heat source in the system consisting of bathtub, hot water and air**. Before the person lies in the bathtub, no internal heat source exist except the system components. The circumstance where the person is in the bathtub will be investigated in our later discussion.

- **We ignore radiative thermal exchange**. According to Stefan-Boltzmann's law, the radiative thermal exchange can be ignored when the temperature is low. Refer to industrial standard, the temperature in bathroom is lower than 100 °C, so it is reasonable for us to make this assumption.
- **The temperature of the adding hot water from the faucet is stable**. This hypothesis can be easily achieved in reality and will simplify our process of solving the problem.

4 Notations

Symbol	Description	Unit
h	Convection heat transfer coefficient	W/(m^2· K)
τ	Time	s, min, h
q_m	Mass flow	kg/s
Φ	Heat transfer power	W
T	A period of time	s, min, h
V	Volume	m^3, L
M, m	Mass	kg
A	Area	m^2
a, b, c	The size of a bathtub	m^3

Note: Undefined variables are defined where they first appear.

5 Model Overview

5.1 Data Preprocessing

To simplify the modeling process, we firstly assume there is no person in the bathtub. We regard the whole bathtub as a thermodynamic system and introduce heat transfer formulas. We establish two sub-models: adding water constantly and discontinuously. For the former sub-model, we define the mean temperature of bath water and introduce Newton's cooling formula to determine the heat transfer capacity. After deriving the value of parameters, we deduce formulas to derive results and simulate the change of temperature field via CFD, as described by **anderson2006<empty citation>**.

In our basic model, we aim at three goals: keeping the temperature as even as possible, making it close to the initial temperature and decreasing the water consumption.

We start with the simple sub-model where hot water is added constantly. At first we introduce convection heat transfer control equations in rectangular coordinate system. Then we define the mean temperature of bath water.

Afterwards, we introduce Newton cooling formula to determine heat transfer capacity. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

Secondly, we present the complicated sub-model in which hot water is added discontinuously. We define an iteration consisting of two process: heating and standby. As for heating process, we derive control equations and boundary conditions. As for standby process, considering energy conservation law, we deduce the relationship of total heat dissipating capacity and time.

Then we determine the time and amount of added hot water. After deriving the value of parameters, we get calculating results via formula deduction and simulating results via CFD.

At last, we define two criteria to evaluate those two ways of adding hot water. Then we propose optimal strategy for the user in a bathtub. The whole modeling process can be shown as follows.

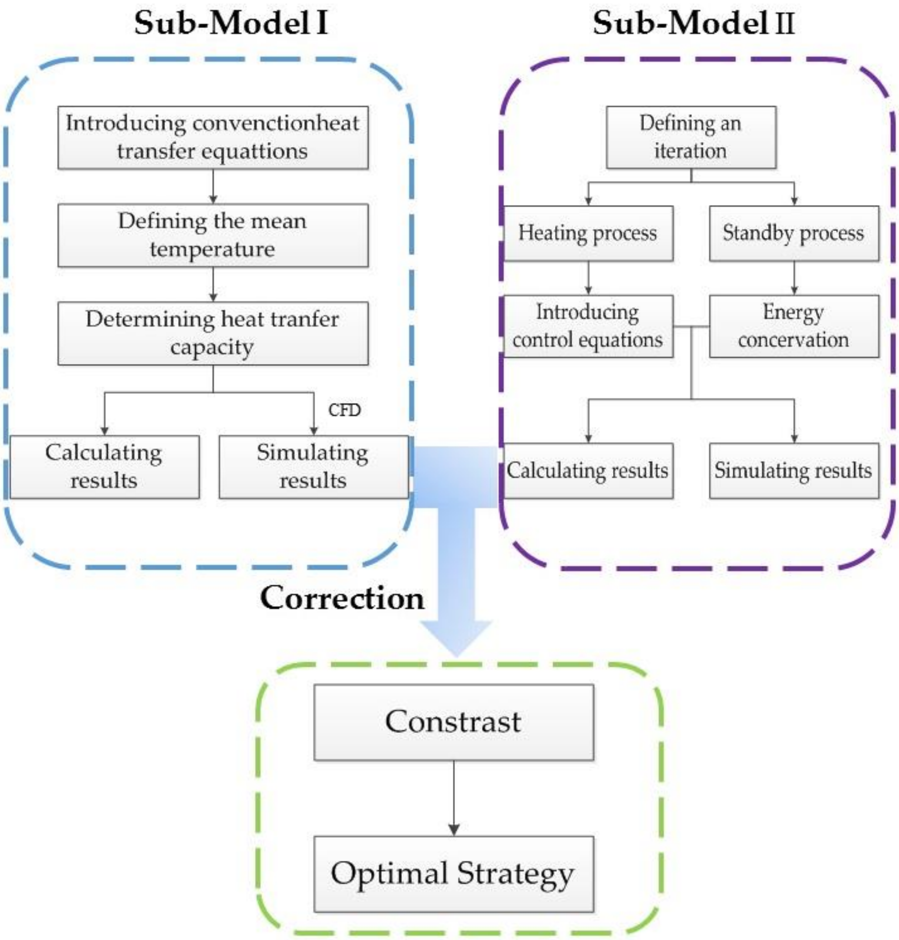

Figure 1: Modeling process

6 Sub-model I : Adding Water Continuously

As for the second sub-model, we define an iteration consisting of two processes: heating and standby. According to the energy conservation law, we obtain the relationship of time and total heat dissipating capacity. Then we determine the mass flow and the time of adding hot water. We also use CFD to simulate the temperature field in the second sub-model, following the techniques outlined by **website2024<empty citation>**.

We first establish the sub-model based on the condition that a person add water continuously to reheat the bathing water. Then we use Computational Fluid Dynamics (CFD) to simulate the change of water temperature in the bathtub. At last, we evaluate the model with the criteria which have been defined before.

6.1 Model Establishment

Since we try to keep the temperature of the hot water in bathtub to be even, we have to derive the amount of inflow water and the energy dissipated by the hot water into the air.

We derive the basic convection heat transfer control equations based on the former scientists' achievement. Then, we define the mean temperature of bath water. Afterwards, we determine two types of heat transfer: the boundary heat transfer and the evaporation heat transfer. Combining thermodynamic formulas, we derive calculating results. Via Fluent software, we get simulation results.

6.1.1 Control Equations and Boundary Conditions

According to thermodynamics knowledge, we recall on basic convection heat transfer control equations in rectangular coordinate system. Those equations show the relationship of the temperature of the bathtub water in space.

We assume the hot water in the bathtub as a cube. Then we put it into a rectangular coordinate system. The length, width, and height of it is a, b and c.

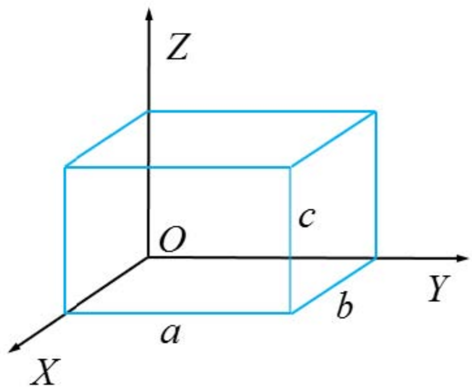

Figure 2: Modeling process

In the basis of this, we introduce the following equations:

- **Continuity equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

where the first component is the change of fluid mass along the X-ray. The second component is the change of fluid mass along the Y-ray. And the third component is the change of fluid mass along the Z-ray. The sum of the change in mass along those three directions is zero.

- **Moment differential equation (N-S equations):**

$$\begin{cases} \rho\Big(u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}\Big)=-\dfrac{\partial p}{\partial x}+\eta\Big(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}\Big) \\ \rho\Big(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+w\dfrac{\partial v}{\partial z}\Big)=-\dfrac{\partial p}{\partial y}+\eta\Big(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}+\dfrac{\partial^2 v}{\partial z^2}\Big) \\ \rho\Big(u\dfrac{\partial w}{\partial x}+v\dfrac{\partial w}{\partial y}+w\dfrac{\partial w}{\partial z}\Big)=-g-\dfrac{\partial p}{\partial z}+\eta\Big(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2 w}{\partial z^2}\Big) \end{cases} \tag{2}$$

- **Energy differential equation:**

$$\rho c_p\Big(u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}+w\frac{\partial t}{\partial z}\Big)=\lambda\Big(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\Big) \tag{3}$$

where the left three components are convection terms while the right three components are conduction terms.

By Equation (3), we have ......

......

On the right surface in Fig. 2, the water also transfers heat firstly with bathtub inner surfaces and then the heat comes into air. The boundary condition here is ......

6.1.2 Definition of the Mean Temperature

......

6.1.3 Determination of Heat Transfer Capacity

......

7 Sub-model II: Adding Water Discontinuously

In order to establish the unsteady sub-model, we recall on the working principle of air conditioners. The heating performance of air conditions consist of two processes: heating and standby. After the user set a temperature, the air conditioner will begin to heat until the expected temperature is reached. Then it will go standby. When the temperature get below the expected temperature, the air conditioner begin to work again. As it works in this circle, the temperature remains the expected one.

Inspired by this, we divide the bathtub working into two processes: adding hot water until the expected temperature is reached, then keeping this condition for a while unless the temperature is lower than a specific value. Iterating this circle ceaselessly will ensure the temperature kept relatively stable.

7.1 Heating Model

7.1.1 Control Equations and Boundary Conditions

7.1.2 Determination of Inflow Time and Amount

7.2 Standby Model

7.3 Results

We first give the value of parameters based on others' studies. Then we get the calculation results and simulating results via those data.

7.3.1 Determination of Parameters

After establishing the model, we have to determine the value of some important parameters.

As scholar Beum Kim points out, the optimal temperature for bath is between 41 and 45°C. Meanwhile, according to Shimodozono's study, 41°C warm water bath is the perfect choice for individual health. So it is reasonable for us to focus on 41°C ~ 45°C. Because adding hot water continuously is a steady process, so the mean temperature of bath water is supposed to be constant. We value the temperature of inflow and outflow water with the maximum and minimum temperature respectively.

The values of all parameters needed are shown as follows:

.....

7.3.2 Calculating Results

8

8.1 1

8.2 2

9

9.1

(1)

(2)

(3)

9.2

(1)

(2)

(3)

9.3

(1)

(2)

(3)

References

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Appendices

Appendix A First appendix

In addition, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI). This letter should be no more than two pages in length.

Here are simulation programmes we used in our model as follow (**Liu02**).

Input matlab source:

```
function [t,seat,aisle]=OI6Sim(n,target,seated)
pab=rand(1,n);
for i=1:n
    if pab(i)<0.4
        aisleTime(i)=0;
    else
        aisleTime(i)=trirnd(3.2,7.1,38.7);
    end
end
```

Appendix B Second appendix

some more text **Input C++ source:**

```
//===================================================================
// Name        : Sudoku.cpp
// Author      : wzlf11
// Version     : a.0
// Copyright   : Your copyright notice
// Description : Sudoku in C++.
//===================================================================

#include <iostream>
#include <cstdlib>
#include <ctime>

using namespace std;

int table[9][9];

int main() {

    for(int i = 0; i < 9; i++){
        table[0][i] = i + 1;
    }

    srand((unsigned int)time(NULL));

    shuffle((int *)&table[0], 9);

    while(!put_line(1))
```

```
    {
        shuffle((int *)&table[0], 9);
    }

    for(int x = 0; x < 9; x++){
        for(int y = 0; y < 9; y++){
            cout << table[x][y] << " ";
        }

        cout << endl;
    }

    return 0;
}
```

Report on Use of AI

1. OpenAI ChatGPT (Nov 5, 2023 version, ChatGPT-4,)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

2. OpenAI Ernie (Nov 5, 2023 version, Ernie 4.0)

   Query1: <insert the exact wording of any subsequent input into the AI tool>

   Output: <insert the complete output from the second query>

3. Github CoPilot (Feb 3, 2024 version)

   Query1: <insert the exact wording you input into the AI tool>

   Output: <insert the complete output from the AI tool>

4. Google Bard (Feb 2, 2024 version)

   Query1: <insert the exact wording of your query>

   Output: <insert the complete output from the AI tool>