# Construction of the diagonal flipped number

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## 1. Cantor's two proofs

We summarize Cantor's diagonal argument as the 4 steps deduction below:

1. Assumption A: All real numbers are in a list.
2. $s_{out}$ is constructed using diagonal and is a real number.
3. $s_{out}$ is not in the list, contradicting thus the Assumption A.
4. Conclusion: the set of all real numbers cannot be put into a list.

See «Examination of Cantor's proofs for uncountability and axiom for counting infinite sets»[1].

## 2. Construction of the list L

We write the natural numbers 1,2,3, … in column 1, see Table 1,write them in binary form in column 2, invert the bits of all the numbers of column 2, the leftmost bit becomes the rightmost bit etc, then add 0. on the left of each inverted number in the column 3 to make them smaller than 1. The column 3 is the list L and does not end. As the bits of all these numbers do not ends, each number in the list L is a real number. So, L contains all the real numbers in the interval [0, 1].

| Column 1 | Column 2 | Column 3 |
|---|---|---|
| 0 | 0 | 0.00000000000… |
| 1 | 1 | 0.10000000000… |
| 2 | 10 | 0.01000000000… |
| 3 | 11 | 0.11000000000… |
| 4 | 100 | 0.00100000000… |
| 5 | 101 | 0.10100000000… |
| 6 | 110 | 0.01100000000… |
| 7 | 111 | 0.11100000000… |
| ⋮ | ⋮ | ⋮ |
| ⋮ | …1111111111 | 0.11111111111… |

**Table 1**

I have put the table here for easier reading.
https://www.academia.edu/86917528/Construction_of_the_diagonal_flipped_number

## 3. Construction of the diagonal flipped number

We notice that the bits of the diagonal of the column 3 are all zero. Indeed, the number of the cell column 2 line n is n-1, the number of bits of this number is $\log_2(n)$. As $\log_2(n)<n$, the $n^{th}$ bit of the $n^{th}$ line column 3 is always zero. Flipped 0 is 1, then the diagonal flipped number is 0.11111… which is put in the last line of the Column 3.

0.11111… does not equal any entry of the column 3 except itself. The column 3 is a list of smaller than 1 real numbers which we name L. The list in the column 3 is then in one-to-one correspondence with the set of natural numbers ℕ which is in column 1.

References

[1] Kuan Peng, 2022, «Examination of Cantor's proofs for uncountability and axiom for counting infinite sets», https://www.academia.edu/86410224/Examination_of_Cantors_proofs_for_uncountability_and_axiom_for_counting_infinite_sets