Graphic of set counting and infinite number

Kuan Peng, Yvelines, France
彭宽 titang78@gmail.com
14 November 2018

Abstract: When counting a set, we can plot a graphic that represents the members of the set on the plane (x, y) to observe visually the counting. Also, graphic of counting of infinite set helps us to understand infinite natural number.

1. Graphic of set counting

Putting a set in one-to-one correspondence with the set of natural numbers ℕ is the process of set counting. In this article, we will create graphics of set counting by representing the members of a counted set with points in the plane (x, y), which explain visually the countability of a set. For example, the members of an infinite set M are represented by the red squares along the y axis in Figure 1. Because the red squares are in one-to-one correspondence with the set $ℕ_y$ which is the y axis, the set M is graphically shown countable.

For creating the counting graphic of infinite sets that are built with the set of natural numbers ℕ, ℕ is put on the x axis and named $ℕ_x$. For example, ℚ is the set of rational numbers *i/j* and is built with the set of all *i*, which is put on the x axis and named $ℕ_x$.

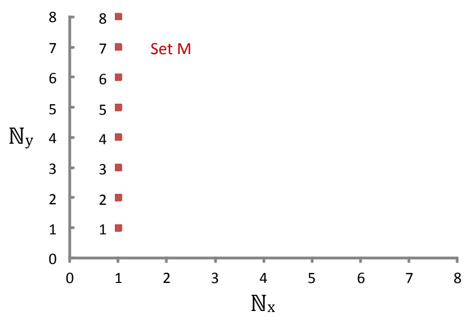

Figure 1

2. Counting curve of ℚ

For counting the members of ℚ we compute the number of the members covered by the counting line shown in Table 1, which equals $i\,(i+1)/2$ at the end of the i^{th} straight line (see section 4.2 "Counting ℚ" of «Building set and counting set»). The member at the end of the i^{th} straight line is represented by the point **(** i, $i\,(i+1)/2$**)**, which are the red dots marked by a cross in Figure 2.

...	8	7	6	5	4	3	2	1	ℕx	ℕy
...	1/8	1/7	1/6	1/5	1/4	1/3	1/2	1/1	1	1
...	2/8	2/7	2/6	2/5	2/4	2/3	2/2	2/1	2	3
...	3/8	3/7	3/6	3/5	3/4	3/3	3/2	3/1	3	6
...	4/8	4/7	4/6	4/5	4/4	4/3	4/2	4/1	4	10
...	5/8	5/7	5/6	5/5	5/4	5/3	5/2	5/1	5	15
...	6/8	6/7	6/6	6/5	6/4	6/3	6/2	6/1	6	21
...	7/8	7/7	7/6	7/5	7/4	7/3	7/2	7/1	7	28
...	8/8	8/7	8/6	8/5	8/4	8/3	8/2	8/1	8	36
...	...	...	...	...	...	...	...	...	...	...

Table 1

The $(i+1)^{th}$ straight line contains several ratios other than the one at the end. They are represented by points with non-whole abscissas x_j which we space equally between i and $i+1$. In Figure 2 these points are the red dots of coordinates **(** x_j, $x_j\,(x_j+1)/2$ **)** between the i^{th} and $(i+1)^{th}$ crosses.

Each red dot represents a member of ℚ, all the red dots form the counting curve of ℚ. The red dots of this curve are in one-to-one correspondence with the set $\mathbb{N}_y$, so ℚ is graphically shown countable. $\mathbb{N}_y$ is called the counting set for ℚ.

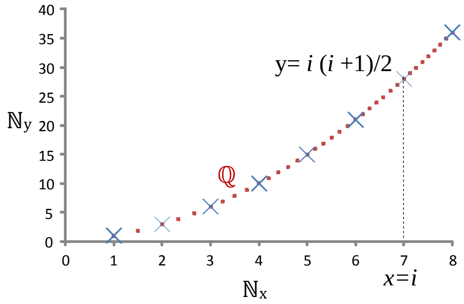

Figure 2

Figure 2 shows that the counting curve of ℚ is always within the range of the y axis. We make then the following observation: as long as all members of a set can be represented within the range of the y axis, the set is countable.

3. Counting curve of ℘(ℕ)

In the same way, we plot the counting curve of ℘(ℕ) which is the power set of the set of natural numbers ℕ. First, let us consider the set N(n) = {1,2,3,…,n-1,n} and its power set ℘(N(n)). The members of ℘(N(n)) are the first 2^n members of ℘(ℕ) in the counting order (see section 3 "Building ℘(ℕ)" of «Building set and counting set»).

In Figure 3, the last member of ℘(N(n)) is represented by the red dot marked by a cross of coordinates **(**n, 2^n**)**. $\mathbb{N}_x$ is the set of all n and is the building set of ℘(ℕ).

The power set ℘(N(n+1)) has 2^n more members than ℘(N(n)). These members are indexed with i and represented by the points (x_i, y_i). The abscissas x_i are equally spaced between n and n+1, the ordinates are $y_i = 2^{x_i}$. The points (x_i, y_i) are the red dots between the n^{th} and $(n+1)^{th}$ crosses in Figure 3.

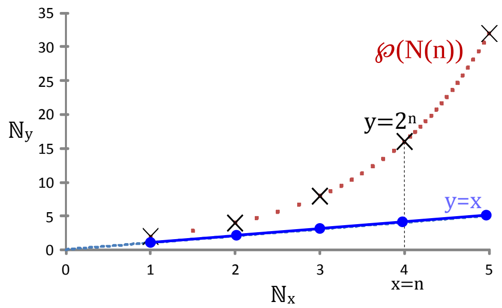

Figure 3

When n increases endlessly, the red dots represent all the members of ℘($\mathbb{N}_x$) and form the counting curve of ℘($\mathbb{N}_x$). The ordinates of the crosses are 2^n and are within the range of the y axis for all n. Indeed, there cannot be a number n above which 2^n stops to be a natural number. So, the red dots of the counting curve are in one-to-one correspondence with $\mathbb{N}_y$. In consequence, ℘($\mathbb{N}_x$) is in one-to-one correspondence with $\mathbb{N}_y$ and ℘($\mathbb{N}_x$) is graphically shown countable. $\mathbb{N}_y$ is the counting set for ℘($\mathbb{N}_x$).

4. Graphical method of counting

Let us compare the graphical counting of ℘($\mathbb{N}_x$) in Figure 3 with that of ℚ in Figure 2. For ℚ, each member of ℚ is represented by a red dot of the counting curve when the abscissa x=i increases to infinity. For ℘($\mathbb{N}_x$), each member of ℘($\mathbb{N}_x$) is represented by a red dot of the counting curve when the abscissa x=n increases to infinity. So, ℚ and ℘($\mathbb{N}_x$) are shown countable by the one-to-one correspondence between the red dots and the set $\mathbb{N}_y$.

We then define the graphical method of counting as: If a set can be represented by a discrete function which monotonically increases with the members of its building set $\mathbb{N}_x$ and is in one-to-one correspondence with the counting set $\mathbb{N}_y$, then the counted set is countable.

5. Discussion

1) About Cantor's theorem

In Figure 3, the members of $\wp(N(n))$ are represented by the red dots, the members of the set N(n) are represented by the blue dots on the straight line y = x. For each n, there are more red dots than blue dots. For n increasing endlessly, the red dots represent all the members of $\wp(\mathbb{N}_x)$ and the blue dots represents all the members of $\mathbb{N}_x$. In this case, the red dots are more numerous than the blue dots, which means that $\wp(\mathbb{N}_x)$ is bigger than $\mathbb{N}_x$ in size, confirming the conclusion of Cantor's theorem.

However, Cantor's theorem does not refute the one-to-one correspondence between $\wp(\mathbb{N}_x)$ and $\mathbb{N}_y$ and thus, does not refute that $\wp(\mathbb{N}_x)$ is countable.

2) Finitely many digits

In an online discussion about my article «Building set and counting set», one person objected that if $\wp(\mathbb{N})$ were countable, there should be a binary number that corresponds to the set of all even natural numbers. I replied: "I think that the set of all even natural numbers corresponds to the natural number ...10101010." Then, another person replied: "That is not a number because it has an infinite number of digits" (see this discussion behind the link).

To argue with this objection, we must stick to their definition for infinite numbers, which is: natural numbers have finitely many digits. The set of all even numbers from the finite set N(n) corresponds to the binary number K_e = 1…1010 (n or n-1digits). K_e is a natural numbers for all n, thus K_e has finitely many digits when n increases endlessly. In this case, K_e corresponds to the set of all even natural numbers and has finitely many digits.

This result is confirmed by Figure 3 where K_e corresponds to a red dot. When n increases endlessly, K_e increases toward infinity without jumping out of $\mathbb{N}_y$, keeping the number of digits finite. So, the above objection does not refute the countability of $\wp(\mathbb{N})$.

3) Infinitely many digits

In another online discussion someone objected: "You are enumerating the finite sets of $\wp(\mathbb{N})$, which are indeed countable. You are missing the infinite sets, which are not countable" (see this discussion behind the link).

ℕ	… 8 7 6 5 4 3 2 1
B	… 1 0 0 1 0 1 1 1
S	… 8 · · 5 · 3 2 1

Table 2

I have explained in «Building set and counting set» that a subset of ℕ is equivalent to a binary number. For example, in Table 2 the 1's of the binary number B select numbers from the set ℕ to create the subset S below. If B can only have finitely many digits, infinite subsets of ℕ do not have corresponding binary numbers. But if natural numbers can have infinitely many digits, the set $\mathbb{N}_y$ in Figure 3 contains binary numbers with infinitely many digits. Because each of these numbers corresponds to an infinite subset of ℕ, all infinite subsets of ℕ are counted by $\mathbb{N}_y$ and this objection would not refute the countability of $\wp(\mathbb{N})$.

However, can infinite number have infinitely many digits?

6. Infinite numbers

Let us determine if infinite number have infinitely many digits or not. Take the number K defined by equation (1), which is the sum of infinitely many 1. K is infinitely big but has finitely many digits. Indeed, if K-1 has finitely many digits, K cannot have infinitely many digits and this is true for K increasing to infinity.

$$K = 1 + 1 + 1 + \cdots \quad (1)$$

$$L = 10 \cdot 10 \cdot 10 \cdot \ldots \quad (2)$$

$$9 \ldots 999 = L - 1 \quad (3)$$

$$3 \ldots 333 = (L-1)/3 \quad (4)$$

$$I = a_0 10^0 + a_1 10^1 + a_2 10^2 \ldots = \sum_{i=0}^{\infty} a_i 10^i \quad (5)$$

Now, let us see the number L defined by equation (2). When the 10's are repeated indefinitely, L has infinitely many digits. Since 10 is a natural number, 10^2 is a natural number, 10^3 is a natural number and so on. There is no number *i* for which 10^i stops to be a natural number. So, L is a natural number with infinitely many digits. Equations (3) and (4) are examples of numbers with infinitely many digits created from L using arithmetic operations. As equation (5) shows, L makes possible to create any number with infinitely many digits.

The existence of number with infinitely many digits is graphically confirmed by Figure 4 where the y axis is in logarithm scale. The red dots represent the members of ℘(ℕ) and form a straight line due to the logarithm scale. The logarithm of the number of members of the set ℘(N(n)) increases linearly without jumping out of the range of the y axis.

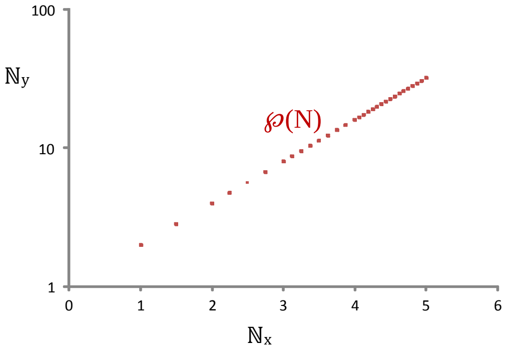

Figure 4

The value of a graduation of ordinates equals the previous graduation multiplied by 10. For counting the red dots we just have to count in binary 10 dots, then 10·10 dots and so on till infinity. At the end of the y axis, we will obtain a number with infinitely many digits which is the number of members of ℘(ℕ).

Finally, the definition of the number L is similar to that of K in that L equals infinitely repeated "10·" while K equals infinitely repeated "1+". Let us call K additive expression of infinite number and L multiplicative expression of infinite number. The set of natural numbers can be written as ℕ = {1,2,3,…,K-1,K, …} or as ℕ = {1,2,3,…,L-1,L, …}. The first set contains only natural number with finitely many digits, while the second contains also natural numbers with infinitely many digits. In consequence, natural number with infinitely many digits exists and one can no longer say that number with infinitely many digits is not a natural number.

Note

I think that the ideas in this article are important and correct. But I lack the training for theoretical mathematics to write in suitable form for publishing in mathematical journals. Nevertheless, these ideas are worth spreading because if accepted, new field of set theory will emerge, which will call for new studies and papers. So, I authorize you, mathematician, to rewrite this article in suitable form for publishing under your name, at the condition that you cite my article as the original source and give in your publication the internet links to my article which are:
https://pengkuanonmaths.blogspot.com/2018/11/graphic-of-set-counting-and-infinite.html
https://www.academia.edu/37766761/Graphic_of_set_counting_and_infinite_number