How to test length contraction by experiment?

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Abstract: Relativistic length contraction is theoretically predicted but not directly tested, which lead to incorrect interpretation of the theory illustrated by Bell's spaceship paradox and Ehrenfest paradox. But these paradoxes can help us designing experiments to test length contraction.

1. Length contraction

Relativistic length contraction effect is described by equation (1), where l_0 being the rest length, l the contracted length, v the velocity and c the speed of light. A common view of this effect is to think that a round star being squeezed in the direction of its motion, which is illustrated by Figure 1, an image from the Wikipedia page Length contraction .

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \quad (1)$$

Although length contraction is theoretically predicted, it has never been directly measured. Ideal direct experimental proof should contain the following steps:

1. Measure the tested object's length at rest, the value l_0.
2. Put this object in motion.
3. Measure the object's speed, the value v.
4. Measure the object's length in motion, the value l.
5. Check if these 3 values verify equation (1).

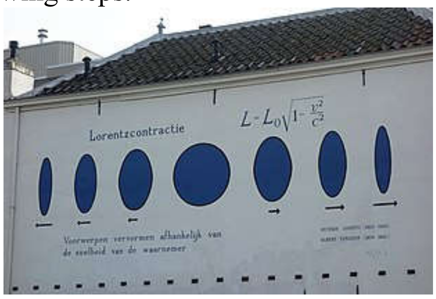

Figure 1

For doing this experiment, the difference of length $l_0 - l$ should be in measurable range. If the object is a chunk of matter, $l_0 - l$ is not measurable. For example, matter objects with the highest speed we can make are satellites, whose speed is generally 7.8 km/s. If a satellite is made of a string of 100 km long, the value of $l_0 - l$ would be 0.03 mm, which is absolutely not measurable from the ground. This is why contraction of length has never been measured.

Below I propose two experiments inspired from Bell's spaceship paradox and Ehrenfest paradox.

2. Bell's spaceship paradox experiment

a. The paradox

Bell's spaceship paradox is a thought experiment originally imagined by E. Dewan and M. Beran in 1959[1] and made famous by J. S. Bell[2]. See Bell's spaceship paradox on Wikipedia. In short, this paradox consists of looking at the variation of the distance between 2 spaceships when brought from rest to high speed.

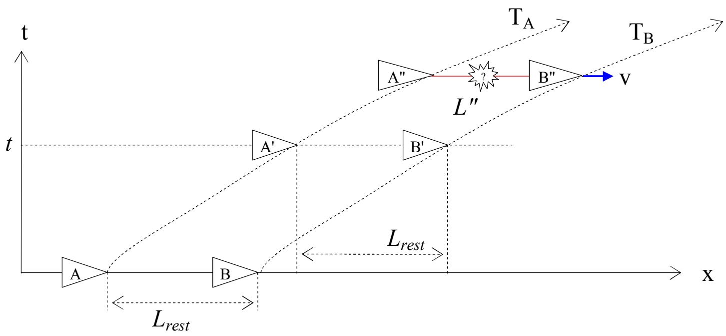

Figure 2

Let A be a spaceship which is brought from rest to high speed. Its motion is described by its trajectory which is the curve T_A in Figure 2. So, its position at any moment is its abscissa in the space (x,t). Let B be an identical spaceship which is also brought from rest to high speed. The motions of A and B are synchronous and identical. At rest the distance between A and B is L_{rest}. So, T_B the trajectory of B is simply T_A shifted to the right by the distance L_{rest}. At time t, A and B are at A' and B'. The abscissa of

A' is x_A. Because T_B is a shift of T_A, the position of B' is simply $x_A + L_{rest}$. So, the distance between the 2 spaceships is constantly L_{rest}.

To illustrate length contraction, E. Dewan and M. Beran imagined that A and B are tied together through a non elastic thread of length L_{rest}. Since the thread is in motion, it is length contracted and appears shorter than L_{rest}. Let L'' be the length of the moving thread when the 2 spaceships are at A'' and B'', see Figure 2. Because the distance between A'' and B'' is L_{rest} and L'' is shorter than L_{rest}, the thread would break, as J. S. Bell insisted. But other physicists like Paul Nawrocki (1962) thought it would not[3]. There is no consensus over this. So, let us do an experiment using this principle.

b. Experimental setup

The only condition in which contraction of length reaches measurable value is when the speed of the object is a fraction of the speed of light. For example, if the speed of the object is ½ that of light, its length would contract by 15%. Only sub-atomic particles can reach such speed. Our experiment will use electrons in cathode ray tube.

But we cannot measure the contraction of the size of an electron. Rather, I propose to measure the distance between two commoving electrons. Like the two spaceships in Bell's paradox, 2 electrons are fired by 2 identical electron guns which are at the distance L_{rest} from one another. The electron guns are connected to the same voltage to imprint the same acceleration to the 2 electrons making their final speeds identical. In order to make the 2 electrons to start exactly at the same time and with the same energy, they are emitted by photoelectric metals which are lighted by one laser source that is equidistant from the 2 electron guns. The location and speed of the 2 electrons with respect to the laboratory are measured by 2 detectors. Figure 3 shows the setup.

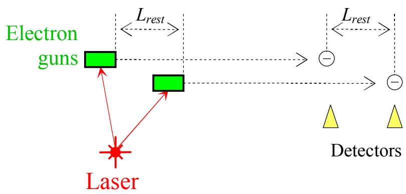

Figure 3

Another and better way to show the distance between the 2 electrons is to shine powerful light on the electrons using stroboscope and to photograph the light scattered from the 2 electrons.

The 2 electrons exert electric force on each other, which would disturb the distance between them. So, the 2 electron guns would be set sufficiently far from each other such that the force on the electrons is small. Also, the detectors would be sufficiently close to the electron guns to minimize the time of action of this force. Under this condition, the effect of this force on the distance can be negligible.

The 2 electrons are in a vacuum and no chunk of mater shows its length. Can length contraction be detected with void space? In fact, length contraction is about space, not about matter. For example, the arms of the LIGO interferometer are vacuum tubes, but the space in the arms is stretched by gravitational waves allowing their detection. The distance between the 2 electrons is a measure of space which will be affected by length contraction effect.

First let us consider one electron. Before being kicked off by the laser, its position is in the electron gun. Once emitted, it is accelerated by the high voltage and exits with relativistic speed. Its trajectory is like that of the spaceship A, T_A in Figure 2. The second electron is kicked off by the same laser at exactly the same time and accelerated by the same voltage, so its trajectory is parallel to that of the first electron, except being shifted to the right by the distance L_{rest}.

At rest the distance between the 2 electrons is L_{rest}. In motion the distance between them is L_{move} which carries the information of length contraction. This experiment consists of measuring L_{move} and comparing L_{move} with L_{rest}. Let us analyze the 2 possible outcomes of this experiment.

c. Will the distance contract?

If the measured L_{move} is shorter than L_{rest}, then L_{move} is the contracted distance between the 2 electrons and L_{rest} the proper distance. In consequence, when the speed of the electrons increases the proper distance stays constant and the contracted distance would decrease. If this were true, the thread of Bell's spaceship paradox will not break contradicting J. S. Bell. Also the common view that a round moving star would appear elliptical would be true, as shown in Figure 1.

However, if this were really true, then we can shrink the contracted distance to zero by firing the electrons near the speed of light. In the extreme limit, if the 2 electrons were 2 photons, we would see only one photon because the distance separating them is zero. Then, all the light emitted by a star would shrink into one unique flash. This is not consistent.

d. Will not the distance contract?

On the other hand, since the trajectories of the 2 electrons are parallel and the electron on the right equals that on the left shifted by L_{rest}, we would expect that the distance in the frame of the laboratory equals constantly the rest distance L_{rest}. Then, the proper distance between the 2 electrons would increase with speed due to length contraction. If this were true, the thread of Bell's spaceship paradox will break. Also, a moving star will appear round whatever its speed is, contradicting the common view shown in Figure 1.

If we fire the electrons near the speed of light, the proper distance will increase to infinity. Why would an electron drift away from the other without any action of force? How can an electron drift infinite distance in finite time as time in the moving frame shortens? This is also inconsistent.

3. Ehrenfest paradox experiment

a. The paradox

Ehrenfest paradox is a thought experiment imagined by Paul Ehrenfest in 1909, see Ehrenfest paradox on Wikipedia. In short, take a disk of radius R, when it is immobile, its circumference is $2\pi R$. When it rotates its perimeter moves at the velocity v and is length contracted. So, the circumference of a rotating disk is shorter than $2\pi R$ while its radius is still R. This is paradoxical.

b. Experimental setup

The experiment consists of accelerating several electrons in a circular particle accelerator such as a synchrotron. The electrons will first travel at the stable speed v_1, then they are accelerated to the higher speed v_2. The distance between 2 successive electrons and their velocity are measured using detectors. The distances under speeds v_1 and v_2 are compared to show the effect of length contraction. Figure 4 shows a circular accelerator in which 12 equidistant electrons circulate.

In real experiment the electrons are not necessarily equidistant. In fact, only 2 electrons are necessary to give valid measurement.

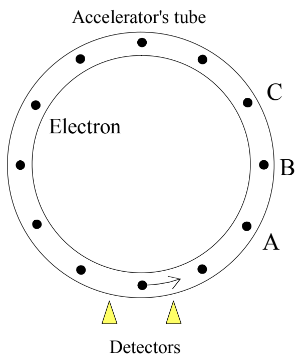

Figure 4

c. How will distance contract?

Let AB_1 be the distance between the electrons A and B which move at the speed v_1. When they are accelerated to the speed v_2, the distance between them is AB_2. AB_2 should appear shorter than AB_1 due to length contraction.

If AB_2 is really shorter than AB_1, then the distance between other electrons would lengthen because of the circular shape of the accelerator. For example, the distance between B and C would be longer while being accelerated to a higher speed. This contradicts special relativity. So, for whatever speed the distance between adjacent electrons should stay constant in the frame of the laboratory. For the case of Figure 4, this distance is $2\pi R/12$.

On the other hand, for an observer riding with the electrons, the latter rests on an immobile circle of radius R. Because they are equidistant, the distance between adjacent electrons is also $2\pi R/12$. Then, the observer would conclude that the distance between adjacent electrons is not length contracted. This is not allowed by special relativity.

4. Comments

Direct test of length contraction effect consists of measuring the length of a static thing and the length of the same thing in motion. This was never done before. I think the experiments described above could be the first feasible such experiments in history. As many synchrotrons are functioning in the world, the laboratories that possess this equipment could easily perform these experiments and be the first to really test length contraction effect.

The above analysis shows that all outcomes of these experiments will be controversial. So, it is very interesting to see what a real measurement of length contraction will be. These controversial measurements will question our interpretation and application of special relativity in real world while the theory itself is no doubt exact. In fact, time dilation is successfully used in many domains because it is proven by experiments. But length contraction finds no application and is not directly tested by experiment, which impedes our understanding of the theory. This why we have so many paradoxes like Bell's spaceship paradox, Ehrenfest paradox, Supplee's paradox and the ladder paradox.

These paradoxes show that we cannot correctly predict relativistic effects in real world and illustrate our misconception of length contraction. Real experimental measurement will give us elements to learn the exact meaning of the length contraction effect and only after, we will be able to correctly interpret and apply the theory of relativity and solve these paradoxes.

References

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