Time-rate change in relatively moving frames

Kuan Peng 彭宽 *titang78@gmail.com*

4 September 2020

Abstract: To clearly explain the contradiction between the constant flow of ticks delivered by clocks and the relativistic time dilation.

1. Material clock

What is time? This question is tricky because in relativity time-rate changes when frame of reference changes. Time-rate changing is puzzling because it is in conflict with our intuition that time is the flow of ticks of clocks of which the mechanical structure does not change. Then how to clearly explain the contradiction between the constant flow of ticks delivered by clocks and the relativistic time dilation?

In order to grasp the essence of time-rate, we have to understand the fundamental property of time. In still image, there is no time. When we see scenes of cinema time emerges. Time emerges in moving scenes because objects in the scenes change. So, the fundamental property of time is the ability of objects to change state in moving image as well as in reality.

For recording the rate of change of objects, human has invented clock, the work of which is the change of state of the clock itself. For example, in Figure 1 the hands of the clock change position, the pendulum changes position. Clocks record time by counting the number of times that one part passes a specific state, for example, the big hand at the number 12 on the dial. If the big hand has passed n times this state, we say that the recorded time is n hours.

Figure 1

From the principle of work of clock, we extract the fundamental function of clocks: counting the number of times an oscillating object passes by a fixed point in space. This is true for archaic sundial as well as for modern quartz clock which makes a quartz tuning fork to vibrate around its neutral position.

So, all material chocks can be represented by the abstract clock in Figure 2, which is formed by a material point k oscillating between the ends of the short rod a and b. The motion of k is characterized by the length of the rod. Let us refer to this abstract clock as “matter clock”. The time recorded by the matter clock is the number of times that k strikes the point a. We define one tick of time delivered by this clock to be one strike.

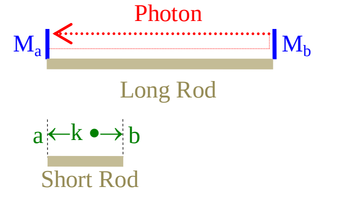

Figure 2
Long rod is a light clock each tick of the photon is a time interval. Short rod is a mechanical clock, k vibrates between a and b. Each strike of k is a tick of the clock. The 2 clocks make a pair of synchronized clocks.

Below, we will show how the time-rate of the matter clock changes while ticking at the same rate. For doing so, we pair it with a light clock.

2. Paired with a light clock

In relativity light is the reference to all motion, so we make the light clock in Figure 2 which is formed by a photon bouncing back and forth between the two mirrors M_a and M_b at the end of the long rod. In order to calibrate the rate of the matter clock, we synchronize it with the light clock by matching the length of the short rod with that of the long rod such that if k starts from the point a simultaneously with the photon from M_a, k gets back to “a” simultaneously with the photon back to M_a. So, the matter clock is synchronous with the light clock and they make one pair of “matter clock - light clock”.

The time recorded by the light clock is the number of strikes the photon makes on the mirror M_a. As matter clock is synchronous with the light clock, the number of strikes k makes on the point a always equals the number of the photon’s strikes. The lengths of the rods and the identically repetitive motion of k stay the same for whatever motion they are in. This way, when the pair of “matter clock - light clock” of Figure 2 is brought into motion, they are always synchronous.

The flow of ticks is the intrinsic tick-rate of a clock. Because the length of the short rod and the motion of k do not change, the intrinsic tick-rate of a material clock does not change either. But the time-rate they show can change due to motion, which we will see below.

3. Time-rate change

Let us take 2 frames of reference frame 1 and frame 2, frame 2 moves at constant speed in frame 1. In order to show the relativistic change of time-rate of frame 2, we will put one pair of “matter clock - light clock” in frame 1 and an identical one in frame 2, see Figure 3. If we stand in frame 1 and look the pair of this frame, then we stand in frame 2 and look the pair of this frame, we will not detect any difference, which shows that material clock does not change when jumping frame.

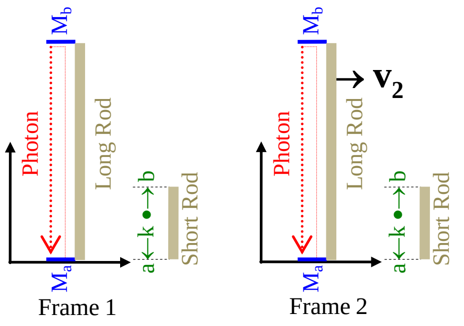

Figure 3

The pair of synchronized clocks in frame 2 moves in frame 1 and its time-rate is compared with that of frame 1.

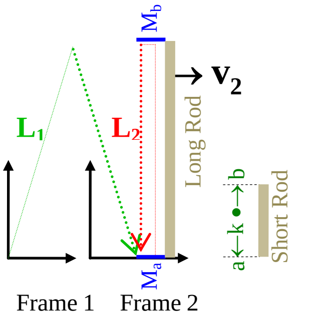

Figure 4

The distance the photon does in frame 2 is L_2, it does L_1 in frame 1.

Then, why is the time-rate of frame 2 different from that of of frame 1? Let us see Figure 4 in which a pair “matter clock - light clock” moves with frame 2 in frame 1. In frame 2 the photon goes straight upward. But due to the motion of the light clock, the path of the same photon is slanted in frame 1. Let us denote the length of the path (back and forth) in frame 1 with L_1 and that in frame 2 with L_2. Because the path in frame 1 is slanted, L_1 is longer than L_2.

One strike of the photon indicates that it has done the distance L_2 once in frame 2. Meanwhile, the same photon has done the distance L_1 in frame 1, see Figure 4. Suppose that we have counted n_2 strikes, then the photon has done n_2 times the distance L_1 in frame 1, which makes the length of its total path to equal $S_1=n_2L_1$, see equation (1).

For counting the time passed in frame 1 during the n_2 strikes, we count the ticks given by the identical pair “matter clock - light clock” in frame 1, see Figure 3. Within the same frame, light travels simultaneously the same distance in all direction. Then, during the n_2 strikes the photon of frame 1 will also do the distance S_1. Because the length of the long rod is also L_2 in frame 1, this photon will strike $n_1=S_1/L_2$ times and S_1 also equals n_1L_2, see equation (2). Then, we find in equation (2) that $n_1 = n_2\ L_1/L_2$. As $L_1>L_2$, the number n_1 is bigger than n_2.

Total distance the photon does in frame 1 during n_2 ticks. (1)

$$S_1 = n_2 L_1$$

The light clock of frame 1 ticks n_1 times during the n_2 ticks of the light clock of frame 2. (2)

$$S_1 = n_1 L_2 \Rightarrow n_1 = n_2 \frac{L_1}{L_2}$$

$$L_1 > L_2 \Rightarrow n_1 > n_2 \quad (3)$$

One tick is one strike by the photon. (4)

$$\frac{L_2}{c} = 1 \text{ tick}$$

Time in frame 1 is n_1 ticks. (5)

$$t_1 = \frac{S_1}{c} = n_1 \frac{L_2}{c} = n_1 \text{ ticks}$$

Simultaneously, time in frame 2 is n_2 ticks. (6)

$$t_2 = n_2 \frac{L_2}{c} = n_2 \text{ ticks}$$

Notice that n_1 and n_2 concern only the length of the photon’s paths, not time. For knowing the time-rate in frame 1 and 2, we define the quantity of time passed as the number of ticks delivered by light clocks which equals the number of strikes by their respective photons. As n_2 ticks is delivered by the one of frame 2, the quantity of time passed in frame 2 equals n_2 ticks. Simultaneously, the photon of the light clock of frame 1 has struck n_1 times, so the quantity of time passed in frame 1 equals n_1 ticks, see equation (5) and (6).

So, when the light clock of frame 2 delivers n_2 ticks, simultaneously the light clock of frame 1 delivers n_1 ticks. If 2 clocks deliver different number of ticks simultaneously, we say that the one that delivers fewer ticks is slower. Using this image, we say that time is slower in frame 2 than in frame 1 because n_2 is smaller than n_1. But "time slowing" is only an image to describe this phenomenon, it is not an appropriate term and it confuses people for understanding relativity.

Notice this difference: the n_2 ticks are delivered by the light clock of frame 2 but we count them in frame 1, the n_1 ticks are delivered by the light clock of frame 1 and also counted in frame 1.

4. Moving material clock

What about the moving matter clocks? As it is synchronized with the paired light clock, the number of ticks it delivers equals that of the paired light clock and the matter clock of frame 2 delivers fewer ticks than that of frame 1 too, although the 2 "matter clocks" are identical, which means that material clock shows slower time-rate when moving while keeping the same mechanical structure.

If we really want to find what object causes time to slow, we would say the culprit is our standpoint. The path of the photon is straight upward when we see it in frame 2. The path of the same photon is slanted when we see it in frame 1. So, it is our standpoint that makes the path to appear slanted and longer, which makes it to contain more ticks. In consequence, the intrinsic mechanical structure of clocks and time itself do not change, only their appearance changes depending on our standpoint.