From Michelson–Morley experiment to length contraction

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Abstract: Michelson–Morley experiment is explained using length contraction. A new approach can explain this experiment while deriving length contraction by theory.

1. Michelson–Morley experiment

Michelson–Morley experiment's setup is a Michelson interferometer formed by 2 arms of equal length which are perpendicular to each other, see Figure 1. At the point O_2 is a light splitter, M_h is the mirror of the horizontal arm and M_v the mirror of the vertical arm. The distances of O_2M_h and O_2M_v equal L_2. The interferometer moves at the velocity v in a stationary frame.

This experiment shows that the journeys of light along the horizontal arm and vertical arm last the same time for whatever the velocity v.

In order to better understand the length contraction effect we will dig deeper into the mechanism of Special Relativity by detailing the derivation of the length contraction law directly from the result of this experiment[1]. The frame of the laboratory is stationary and is the frame 1, its origin is O_1. The frame of the interferometer is moving and is the frame 2, its origin is O_2. O_2 moves at the velocity v in the frame 1 and O_1 moves at the velocity −v in the frame 2. The speed of light is c.

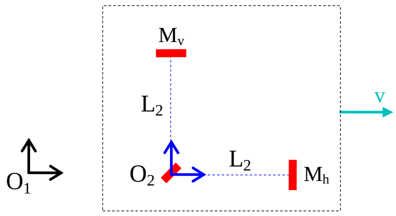

Figure 1

2. Time dilation

Along the vertical arm, a light travels from the light splitter O_2 to the mirror M_v. In the frame of the interferometer, the path of the light is O_2M_v. In the frame of the laboratory the path of the light is the slanted O_2M_v' (see Figure 2). The distance of these 2 paths are L_2 and L_1 respectively. In the same time, the light splitter travels half the path O_2O_2''. Pythagorean Theorem relates L_1, L_2 and O_2O_2'' in equation (1).

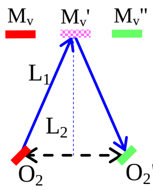

Figure 2

$$L_1{}^2 = L_2{}^2 + \left(\frac{O_2O_2''}{2}\right)^2 \tag{1}$$

$$t_1 = \frac{2L_1}{c}, \qquad t_2 = \frac{2L_2}{c} \tag{2}$$

$$O_2O_2'' = vt_1 \tag{3}$$

$$\left(\frac{ct_2}{2}\right)^2 = \left(\frac{ct_1}{2}\right)^2 - \left(\frac{vt_1}{2}\right)^2 \tag{4}$$

$$t_1 = \frac{t_2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

Then, the mirror reflects the light back to the light splitter. The time of the whole forth and back light journey is t_1 in the frame 1 and t_2 in the frame 2, see equation (2). During t_1, the light splitter travels the path O_2O_2'', see equation (3). The combination of the equations (1), (2) and (3) gives equation (4), which in turn gives the time dilation law in equation (5).

3. Length Contraction Approach

- **Length contraction assumption**

For the horizontal arm, if we compute the light journey using L_2, we would not obtain the result of the Michelson–Morley experiment. So, we assume that in the frame of the laboratory the length of the horizontal arm is contracted to an unknown value which we denote by L_3.

First, a light goes to the mirror along the path A, then is reflected back to the light splitter along the path B. The path A is different from the path B, see Figure 3. We call the light travel along the paths A and B the journeys A and B.

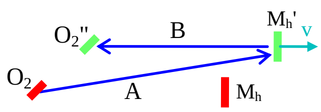

Figure 3

The light takes the time t_a to travel the journey A while the light splitter travels the path O_2O_2', see Figure 4. So, the distance of the journey A equals L_3 plus O_2O_2', see equation (6). Then, the distance of A is expressed in equation (7).

$$A = L_3 + vt_a \qquad A = ct_a \tag{6}$$

$$L_3 = ct_a - vt_a \qquad A = \frac{L_3}{1 - \frac{v}{c}} \tag{7}$$

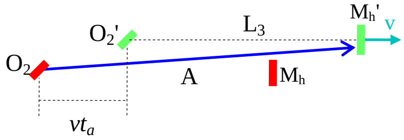

Figure 4

The journey B takes the time t_b while the light splitter travels the path $O_2'O_2''$, see Figure 5. The distance of the journey B equals L_3 minus $O_2'O_2''$, see equation (8). Then, the distance of B is expressed in equation (9).

$$B = L_3 - vt_b \qquad B = ct_b \tag{8}$$

$$L_3 = ct_b + vt_b \qquad B = \frac{L_3}{1 + \frac{v}{c}} \tag{9}$$

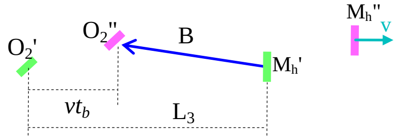

Figure 5

The total distance the light travels forth and back the horizontal arm is A+B which is expressed in equation (10). The time of the journey A+B is t_a+t_b. We derive in equation (11) the expression for t_a+t_b from equation (10). But L_3 is still unknown.

$$A + B = \frac{L_3}{1 - \frac{v}{c}} + \frac{L_3}{1 + \frac{v}{c}} = \frac{2L_3}{1 - \frac{v^2}{c^2}} \tag{10}$$

$$A + B = c(t_a + t_b) \qquad t_a + t_b = \frac{2L_3}{c\left(1 - \frac{v^2}{c^2}\right)} \tag{11}$$

- **Resorting to experimental result**

For determining L_3 we use the result of the Michelson–Morley experiment, that is, the time of the journey along the horizontal arm equals that along the vertical arm. That is, t_a+t_b equals t_1 which is given in equation (5). Using equation (12) and (2) we obtain equation (13) which is the length contraction law.

$$t_a + t_b = t_1 \qquad \frac{2L_3}{c\left(1 - \frac{v^2}{c^2}\right)} = \frac{t_2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{12}$$

$$L_3 = L_2\sqrt{1 - \frac{v^2}{c^2}} \tag{13}$$

The scheme of the above derivation is:

- ➢ The time of the journey A+B is computed using a method that does not give directly the correct time.
- ➢ Length contraction was assumed by Hendrik Lorentz to vary the computed time.
- ➢ The Michelson–Morley experiment dictates the ratio of length contraction by equation (12).

Can we find a method that gives directly the correct time?

4. Time Dilation Approach

a) Length or distance

We notice that the arms of the Michelson interferometer are void space except the light splitter and mirrors. Usually the word "length" specifies the size of a material object, for example the length of a ruler. The appropriate word for quantifying void space is "distance", for example we say "the distance between 2 points" when there is nothing between them.

Distance can vary due to time dilation. Figure 6 shows the Earth, a star and a moving spaceship near the Earth. For the Earth, the star is at the distance D_1, while for the spaceship the star is at D_2 which is different. The spaceship would travel the distance D_1 in the time t_1 as measured in the frames of the Earth. But in the frame of the spaceship the same voyage would take the time t_2 which is shorter than t_1 due to time dilation. So, the distance D_2 is shorter than D_1 while both specifying the same amount of space.

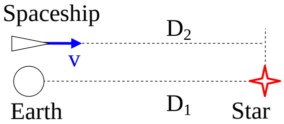

Figure 6

Notice that D_2 is not a length contraction of D_1 because the spaceship is moving while the star is stationary. Length contraction applies to objects whose length is constant, for example a ruler. Below, we use time dilation to derive the duration of the light journey along the horizontal arm and call this method Time Dilation Approach. **We will not assume length contraction.**

b) Time along the horizontal arm

For journey A the light travels the distance D_a, see Figure 7; for journey B the light travels the distance D_b, see Figure 8. Both distances are in the frame of the laboratory. In the frame of the interferometer, the light travels the length of the horizontal arm L_2 in the time t_{h2}, see equation (14). In the frame of the laboratory, t_{h2} is time-dilated to the time t_{h1} during which the light travels the distance L_h, see equation (15). Notice that L_h is not the length of the horizontal arm measured in the frame of the laboratory. Rather, L_h equals L_1 of Figure 2, the distance of the slanted path O_2M_v'.

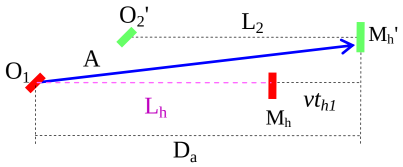

Figure 7

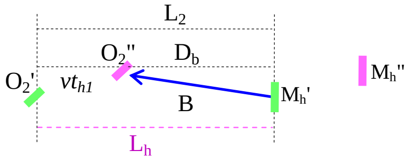

Figure 8

Also during the time t_{h1}, the mirror travels the path M_hM_h' for the journey A, the light splitter travels the path $O_2'O_2''$ for the journey B. So, the distance that the light travels in the frame of the laboratory equals L_h plus the path M_hM_h' for the journey A and L_h minus the path $O_2'O_2''$ for the journey B, see Figure 7 and Figure 8. The distance of M_hM_h' and $O_2'O_2''$ are given in equation (16). Equation (17) derives the final expressions for D_a and D_b with: D_{ab} equals D_a when the sign $\pm$ equals + and D_b when the sign $\pm$ equals $-$.

$$L_2 = ct_{h2} \tag{14}$$

$$L_h = ct_{h1} = \frac{ct_{h2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L_2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

$$M_hM_h' = O_2'O_2'' = vt_{h1} \tag{16}$$

$$\mathrm{D_{ab}} = L_h \pm vt_{h1} = L_h\left(1 \pm \frac{v}{c}\right) = \frac{L_2\left(1 \pm \frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

The total distance that the light travels forth and back the horizontal arm is D_a+D_b and the time of the whole journey is t_d, see equations (18) and (19). Combining equations (19) and (2), we transform t_d in equation (20) and find the time dilation law, see equation (5). So, t_d equals t_1 which is the time that the vertical light takes to travel the vertical arm. This result explains the Michelson–Morley experiment.

$$\mathrm{D_a} + \mathrm{D_b} = L_2\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} + L_2\frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{18}$$

$$t_d = \frac{\mathrm{D_a}+\mathrm{D_b}}{c} = \frac{2L_2}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{19}$$

$$t_d = \frac{t_2}{\sqrt{1-\frac{v^2}{c^2}}} = t_1 \tag{20}$$

5. Comparison of the 2 approaches

We compare the lengths and distances computed with both Approaches. The expression for the distance of the journey A (see Figure 9) according to Length Contraction Approach is given in equation (7) and is transformed in equation (21) using equation (13) for L_3. Equation (21) shows that A equals the distance D_a computed using Time Dilation Approach, see equation (17).

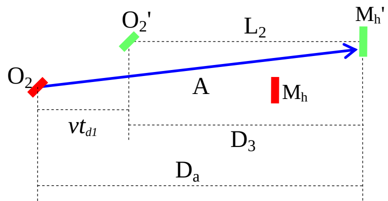

Figure 9

What is the "contracted length" of the horizontal arm for Time Dilation Approach? This length is the distance between the light splitter and the mirror, $O_2'M_h'$ in Figure 9. This distance is denoted by D_3 and equals D_a minus vt_{d1} (see equation (23)) in the frame of the laboratory. t_{d1} is the time that light takes to travel the distance D_a. Then, D_3 is computed in equation (24). By comparing equations (24) and (22), we obtain equation (25), D_3 equals L_3.

$$A = \frac{L_3}{1-\frac{v}{c}} = \frac{L_2}{1-\frac{v}{c}}\sqrt{1-\frac{v^2}{c^2}} = \frac{L_2\left(1+\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \mathrm{D_a} \tag{21}$$

$$A = \frac{L_3}{1-\frac{v}{c}} = \mathrm{D_a} \tag{22}$$

$$\mathrm{D_3} = \mathrm{D_a} - vt_{d1}, \quad t_{d1} = \frac{\mathrm{D_a}}{c} \tag{23}$$

$$\mathrm{D_a} = \frac{\mathrm{D_3}}{1-\frac{v}{c}} \tag{24}$$

$$\mathrm{D_3} = L_3 \tag{25}$$

So, Time Dilation Approach and Length Contraction Approach give the same expression for A and L_3, which proves that these 2 Approaches are equivalent in terms of distance and length except that Time Dilation Approach did not assume length contraction but rather, derived the length contraction ratio theoretically.

6. Acceleration of a ruler

Special Relativity cannot work with acceleration because of length contraction. Let us see the example of the ruler shown in Figure 10. At time 0 the ruler is stationary and its length is L. At time Δt the ruler has moved Δs and keeps moving. So, its length is contracted to $L\sqrt{1-\frac{v^2}{c^2}}$ and its far end is at the point x_f, see equation (26). Because the velocity of the ruler does not depend on its length, length contraction can shrink the ruler such that its far end is nearer than the stationary length L. That is, x_f is smaller than L, see equation (27).

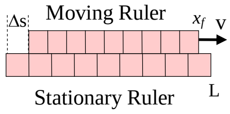

Figure 10

$$x_f = \Delta s + L\sqrt{1-\frac{v^2}{c^2}} \tag{26}$$

$$x_f < L$$

$$\leftrightarrow L > \Delta s + L\sqrt{1-\frac{v^2}{c^2}} \tag{27}$$

$$L > \frac{\Delta s}{1-\sqrt{1-\frac{v^2}{c^2}}} = L_c \tag{28}$$

We derive the critical length L_c in equation (28) and find the criterion for the far end of the ruler to move left: the ruler is longer than the critical length L_c. This is true for whatever Δs and v, even for small velocity.

Let us consider the ruler as the x-axis of a frame in space. The stationary ruler is the x-axis of the frame 1 and the moving ruler that of the frame 2 with x_1 being the abscissa of the frame 1 and x_2 that of the frame 2. Before time 0 the frame 2 is stationary. After time 0, the origin of the frame 2 accelerates from x_1=0 to x_1=Δs/2, then begins to travel at constant velocity.

When passing by the point x_1=Δs, the frame 2 is inertial and the Lorentz transform applies. Let x_2=L, then the abscissa of the point x_2 in the frame 1 equals x_f, see equation (26). Before time 0, the frame 2 is stationary and x_1=x_2. So, the former stationary position of the point x_2 is x_1=L. At this time, all points of the frame 2 such that $x_2 > L_c$ would be on the left of its former stationary position because x_f < L.

7. Comments

The fact that the far part of a moving frame would go in the opposite direction than its origin is rather disturbing. This is a contradiction created by length contraction that makes Special Relativity unable to work with acceleration (see also [2]).

In real world objects accelerate all time, which call for a theory to explain the acceleration of high speed objects. Since Time Dilation Approach does not assume length contraction, it will not create the above contradiction and thus, could in the future extend Special Relativity to include acceleration. This is the real benefit of Time Dilation Approach.

Since Time Dilation Approach does not need length contraction to explain the Michelson–Morley experiment, its base is stronger than that of Length Contraction Approach. The way of physics is always to reduce the number of assumption. The simpler the theory is, the stronger its validity is. So, Special Relativity derived with Time Dilation Approach will be stronger than now.

References:

1) « Introduction to Special Relativity » by James H. Smith
2) « Astrophysical jet and length contraction »