

Synchronizing moving GPS clocks

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Abstract: Relativity of simultaneity destroys synchronization of GPS satellites.

This article details the analysis of synchronization of GPS satellites given in the previous article
« [Testing relativity of simultaneity using GPS satellites](#) »

1. Light pulse synchronization

Can we synchronize clocks of a moving frame? Let us see Figure 1 where we have stationary frame F_1 and moving frame F'_2 . The 2 clocks in the frame F_1 are synchronized with the master clock through a light pulse.

The 2 clocks of the frame F'_2 are moving. At the time t_0 in the frame F_1 , the 2 clocks in the moving frame F'_2 coincide with that of the frame F_1 , but due to relativity of simultaneity the time of the moving clocks are different, $\Delta t = t'_2 - t'_1 \neq 0$. So, the light pulse does not synchronize the clocks in the moving frame.

2. Fixed clocks in orbit

GPS satellites are moving, so they suffer from relativity of simultaneity. Let us imagine a disk centered at the center of the Earth whose rim is the GPS orbit and on the rim are equally spaced clocks, see Figure 2. Each fixed clock corresponds one GPS satellite. The disk does not rotate with the Earth and the frame of the disk is the frame F_1 . Because these clocks are fixed they are synchronized with the master clock at the North Pole of the Earth through a light pulse.

At the time t_0 in the frame F_1 , all fixed clocks show the time t_0 , each one is in front of a GPS satellite. But the clocks of the GPS satellites are not synchronized with the master clock because they are moving.

3. Time in GPS satellites

The event the satellite 1 coincides with a fixed clock and the satellite 2 coincides with the next fixed clock are simultaneous in the frame F_1 . But these 2 events are not simultaneous in the frame F'_2 , which is the frame moving with the satellite 1 and containing the satellite 2, see Figure 3.

In the frame F'_2 , the clock of the satellite 1 reads t'_1 and the clock of the satellite 2 reads t'_2 because of non-simultaneity. So, we have the gap of time $\Delta t = t'_2 - t'_1 \neq 0$.

In the same way, the event the satellite 2 coincides with a fixed clock and the satellite 3 coincides with the next fixed clock are simultaneous in the frame F_1 , but not in the frame F'_3 , which is the frame moving with the satellite 2 and containing the satellite 3, see Figure 3. Also for the same reason, the gap of time in the frame F'_3 is $\Delta t = t'_3 - t''_2 \neq 0$.

Note that here in the frame F'_3 , the time of the satellite 2 is denoted by t''_2 with double prime to distinguish it from the time of the satellite 2 in the frame F'_2 .

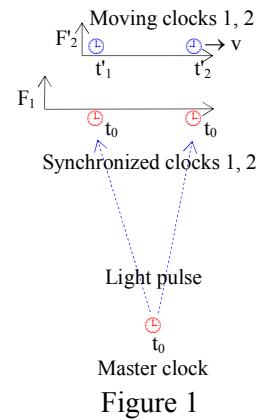


Figure 1

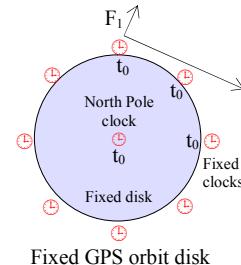


Figure 2

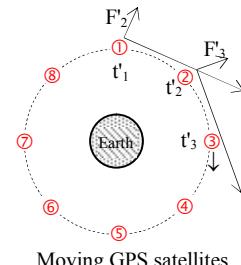


Figure 3

4. Time of the satellite 2

We have 2 different notations of the time of the satellite 2: t'_2 with single prime in the frame F'_2 and t''_2 with double prime in the frame F'_3 . The satellite 2 has only one time which is shown by its clock. For the event the satellite 2 coincides with the fixed clock in Figure 3, its value in the frame F'_2 is t'_2 . Now we ask: does the reading of the clock of the satellite 2 change to a different value t''_2 when seen in the frame F'_3 ? Logically no, because t'_2 is the time of the satellite 2 while t''_2 is the time of the same satellite at the same event. So, we must have $t'_2 = t''_2$. In this case, we have :

$$t'_3 = t''_2 + \Delta t' = t'_2 + \Delta t' = t'_1 + \Delta t' + \Delta t' = t'_1 + 2\Delta t'$$

5. Time of the satellite n

For the same reason, the double-primed t''_3 equals the single-primed t'_3 and each double-primed t''_i equals each corresponding single-primed t'_i . Then, the $n+1^{\text{th}}$ satellite has the time: $t'_{(n+1)} = t'_1 + n\Delta t'$.

Because $n\Delta t'$ is not zero, $t'_{(n+1)} \neq t'_1$ while the satellite $n+1$ being the satellite 1, hence contradiction.

6. If $t'_{(n+1)} = t'_1$?

We can assert that the satellite $n+1$ being the satellite 1 and having the same time, that is, $t'_{(n+1)} = t'_1$. In this case, we first suppose that the single-primed t'_2 does not equal the double-primed t''_2 , but instead their difference cancels $\Delta t'$, that is, $t''_2 = t'_2 - \Delta t'$. So, $t'_3 = t''_2 + \Delta t' = t'_2 - \Delta t' + \Delta t' = t'_1 + \Delta t'$.

In this case, we will have for the $n+1^{\text{th}}$ satellites $t'_{(n+1)} = t'_1 + \Delta t'$.

However, we still do not have $t'_{(n+1)} = t'_1$. For achieving this equality, we have to suppose the difference of time between single-primed and double-primed time to make $t''_2 = t'_2 - \Delta t'(1+1/n)$. This way, we will have at the $n+1^{\text{th}}$ satellite: $t'_{(n+1)} = t'_1 + \Delta t' - \Delta t'*n/n = t'_1$

But this assumption is absurd and cannot be true. So, due to relativity of simultaneity, the satellite $n+1$ and the satellite 1 are the same satellite but cannot have the same time.

The only way for the satellite $n+1$ and the satellite 1 to have the same time is that relativity of simultaneity is zero, that is, $\Delta t' = 0$.