‘Oumuamua, Pioneer anomaly and solar mass with Time Relativity

Kuan Peng 彭宽 *titang78@gmail.com*

27 April 2020

Abstract: The theory of Time relativity explains well the weird behavior of the interstellar object ‘Oumuamua. I find that the real solar mass is slightly higher than today’s value, which caused the mysterious Speed Boost of which the value should be 0.217 mm/s above the prediction at perihelion. Time relativity confirms that ‘Oumuamua should slow down less than prediction, in proportion of which the difference is 4.28×10^{-8} near the Sun. For Pioneer anomaly I have computed the gap between real and predicted acceleration and found the value 8.70×10^{-10} which is very close to the observation $(8.74\pm1.33)\times10^{-10}$ m/s^2.

Summary

The mysterious behavior of the interstellar object ‘Oumuamua confuses scientists. In the past, the manmade Pioneer spacecrafts were also found to deviate from expected Newtonian trajectory. The theory of Time relativity that I develop provides a theoretical explanation to these phenomena.

In Time relativity the transformation of coordinates is the system (1) which is the adjunction of the system (12) given in « Time relativity transformation of coordinates» with the identity of y and z. The transformation of the x-component of velocity is the equation (2) which is the equation (19) given in «Time relativity transformation of velocity ». Below we will create the transformation of the velocity vector by deriving the transformation of the y and z components of velocity.

$$\begin{aligned} x_2 &= x_1 - ut_1 \\ y_2 &= y_1 \\ z_2 &= z_1 \\ t_2 &= t_1\sqrt{1-\frac{u^2}{c^2}} \end{aligned} \quad (1)$$

$$v_{x,2} = \frac{v_{x,1}-u}{\sqrt{1-\frac{u^2}{c^2}}} \quad (2)$$

1. Relativistic kinematics

a) Y and Z components of velocity

Let frame 1 be a stationary frame of reference in which the inertial frame 2 moves at the velocity u in the x direction, see Figure 1. Let q be an object moving in the frame 2 at the velocity $v_{y,2}$ in the y direction. During an infinitesimal time interval dt_2, q has moved the infinitesimal distance dy_2, see Figure 2. So, the y-component of velocity in frame 2 equals dy_2/dt_2, see equation (3).

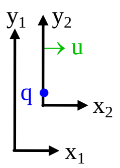

Figure 1

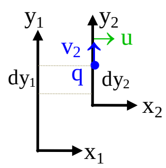

Figure 2

For the same motion, q has moved the infinitesimal distance dy_1 during the time interval dt_1 in frame 1. Because there is not length contraction in the y direction, we have $dy_1=dy_2$ (see Figure 2) and write equation (4) where the term dy_1/dt_1 is the y-component of velocity in frame 1, $v_{y,1}$, see equation (5).

$$v_{y,2} = \frac{dy_2}{dt_2} \tag{3}$$

$$\frac{dy_2}{dt_2} = \frac{dy_1}{dt_2} = \frac{dy_1}{dt_1}\frac{dt_1}{dt_2} \tag{4}$$

$$\frac{dy_1}{dt_1} = v_{y,1} \tag{5}$$

The ratio dt_1/dt_2 is given in equation (6) which is obtained using the time equation of the system (1). We introduce equation (6) in to (4), which becomes equation (7), the relation between the y-components of velocity in frames 1 and 2.

$$\frac{dt_1}{dt_2} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \tag{6}$$

$$v_{y,2} = \frac{v_{y,1}}{\sqrt{1-\frac{u^2}{c^2}}} \tag{7}$$

The z-components of velocity in frame 1 and 2 are $v_{z,1}$ and $v_{z,2}$ and are related in the same way as for y-components, see equation (8).

$$v_{z,2} = \frac{v_{z,1}}{\sqrt{1-\frac{u^2}{c^2}}} \tag{8}$$

b) Velocity vector

Let ***i, j*** and ***k*** be the unit vectors in the x, y and z directions. By convention, vectors are written in boldface letters. The velocity vector of q in frame 2 is written in equations (9) in which we substitute equations (2), (7) and (8) for the x, y and z components and obtain equation (10). The velocity vector of q in frame 1 is written in equations (11), which we substitute for the corresponding terms in equation (10), in which we also substitute **u** for u***i***, with **u** being the velocity vector of the frame 2 in frame 1, see equation (12). Then, equation (10) becomes equation (13).

$$\boldsymbol{v}_2 = v_{x,2}\boldsymbol{i} + v_{y,2}\boldsymbol{j} + v_{z,2}\boldsymbol{k} \tag{9}$$

$$\begin{aligned}\boldsymbol{v}_2 &= \frac{v_{x,1}-u}{\sqrt{1-\frac{u^2}{c^2}}}\boldsymbol{i} + \frac{v_{y,1}}{\sqrt{1-\frac{u^2}{c^2}}}\boldsymbol{j} + \frac{v_{z,1}}{\sqrt{1-\frac{u^2}{c^2}}}\boldsymbol{k} \\ &= \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\left(v_{x,1}\boldsymbol{i} + v_{y,1}\boldsymbol{j} + v_{z,1}\boldsymbol{k} - u\boldsymbol{i}\right)\end{aligned} \tag{10}$$

The velocity vectors of q in frame 1 and 2 are $\mathbf{v_1}$ and $\mathbf{v_2}$ and are related through equation (13).

$$\boldsymbol{v}_1 = v_{x,1}\boldsymbol{i} + v_{y,1}\boldsymbol{j} + v_{z,1}\boldsymbol{k} \tag{11}$$

$$\boldsymbol{u} = u \cdot \boldsymbol{i} \tag{12}$$

$$\boldsymbol{v}_2 = \frac{\boldsymbol{v}_1 - \boldsymbol{u}}{\sqrt{1-\frac{u^2}{c^2}}} \tag{13}$$

c) Acceleration vector

Suppose that the velocity of q is 0 at time t_2 in frame 2 and is **u** in frame 1. After an infinitesimal time interval dt_2, q's velocity becomes $\mathbf{v_2}$ in frame 2. At the same moment, q's velocity becomes $\mathbf{v_1}$ in frame 1.

Because dt_2 is infinitesimal the vectors $\mathbf{v_2}$ and $\mathbf{v_1}-\mathbf{u}$ are infinitesimal too and can be written in differential form as $d\mathbf{v_2}$ and $d\mathbf{v_1}$ in equations (14), which we introduce into equation (13) to obtain equation (15), of which both sides are divided by dt_2 to make equation (16), in which the ratio dt_1/dt_2 is substituted by equation (6) and then, equation (16) becomes equation (17), in which we substitute $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ for $\frac{dv_1}{dt_2}$ and $\frac{dv_2}{dt_2}$ to get equation (18), the relation between the accelerations of the object q in frame 1 and 2.

$$\begin{aligned} d\boldsymbol{v}_2 &= \boldsymbol{v}_2 - 0 \\ d\boldsymbol{v}_1 &= \boldsymbol{v}_1 - \boldsymbol{u}\end{aligned} \tag{14}$$

$$d\boldsymbol{v}_2 = \frac{d\boldsymbol{v}_1}{\sqrt{1-\frac{u^2}{c^2}}} \tag{15}$$

$$\frac{d\boldsymbol{v}_2}{dt_2} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\frac{d\boldsymbol{v}_1}{dt_1}\frac{dt_1}{dt_2} \tag{16}$$

$$\frac{d\boldsymbol{v}_2}{dt_2} = \frac{1}{1-\frac{u^2}{c^2}}\frac{d\boldsymbol{v}_1}{dt_1} \tag{17}$$

$$\boldsymbol{\alpha}_2 = \frac{\boldsymbol{\alpha}_1}{1-\frac{u^2}{c^2}} \tag{18}$$

d) Derivative of vectors

The velocity vector of q is the time derivative of its position vector, which is $\mathbf{X}_2$ in frame 2, see equation (19). The derivative of $\mathbf{X}_2$ is derived in equation (20), where the coordinates (x_2, y_2, z_2) are written in terms of (x_1, y_1, z_1) the coordinates of q in frame 1 using system (1).

$$\boldsymbol{X}_2 = x_2\boldsymbol{i} + y_2\boldsymbol{j} + z_2\boldsymbol{k} \tag{19}$$

$$\begin{aligned}\frac{d\boldsymbol{X}_2}{dt_2} &= \frac{d(x_2\boldsymbol{i} + y_2\boldsymbol{j} + z_2\boldsymbol{k})}{dt_2} \\ &= \frac{d\big((x_1 - ut_1)\boldsymbol{i} + y_1\boldsymbol{j} + z_1\boldsymbol{k}\big)}{dt_1}\frac{dt_1}{dt_2} \\ &= \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\frac{d(x_1\boldsymbol{i} + y_1\boldsymbol{j} + z_1\boldsymbol{k} - ut_1\boldsymbol{i})}{dt_1}\end{aligned} \tag{20}$$

The position vector of q in frame 1 is $\mathbf{X}_1$, which is given in equation (21), which we substitute for the corresponding terms in equation (20), which then becomes equation (22). Since the derivatives of $\mathbf{X}_1$ and $\mathbf{X}_2$ are the velocity vectors in frame 1 and 2, we write equation (22) into equation (25) which is the equation (13).

$$\boldsymbol{X}_1 = x_1\boldsymbol{i} + y_1\boldsymbol{j} + z_1\boldsymbol{k} \tag{21}$$

$$\frac{d\boldsymbol{X}_2}{dt_2} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\left(\frac{d\boldsymbol{X}_1}{dt_1} - \boldsymbol{u}\right) \tag{22}$$

We can derive the acceleration vector also by taking the time derivative of the velocity vector of q, see equation (23). Because frame 2 is inertial, **u** is constant. The derivative of $\mathbf{v_1}$ and $\mathbf{v_2}$ are the acceleration vector $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$, then, equation (23) becomes (26), which is the equation (18).

$$\frac{d\boldsymbol{v}_2}{dt_2} = \frac{d}{dt_2}\left(\frac{\boldsymbol{v}_1 - \boldsymbol{u}}{\sqrt{1-\frac{u^2}{c^2}}}\right) = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\frac{d\boldsymbol{v}_1}{dt_1}\frac{dt_1}{dt_2} = \frac{1}{1-\frac{u^2}{c^2}}\frac{d\boldsymbol{v}_1}{dt_1} \quad (23)$$

In summary, the transformation of position vector is equation (24) (see system (1)). The derivative of the transformation of position vector is the transformation of velocity vector equation (25); the derivative of the transformation of velocity vector is the transformation of acceleration vector equation (26). We see that the transformations in Time relativity form a consistent vector system. In the contrary, the transformations in Special relativity do not. For example, the velocity-addition formula cannot be directly derived in vector form, neither can give acceleration vector.

$$\boldsymbol{X}_2 = \boldsymbol{X}_1 - \boldsymbol{u}t_1 \quad (24)$$

$$\left.\begin{aligned}\frac{d\boldsymbol{X}_1}{dt_1} &= \boldsymbol{v}_1\\ \frac{d\boldsymbol{X}_2}{dt_2} &= \boldsymbol{v}_2\end{aligned}\right\} \Rightarrow \boldsymbol{v}_2 = \frac{\boldsymbol{v}_1 - \boldsymbol{u}}{\sqrt{1-\frac{u^2}{c^2}}} \quad (25)$$

$$\left.\begin{aligned}\frac{d\boldsymbol{v}_1}{dt_1} &= \boldsymbol{\alpha}_1\\ \frac{d\boldsymbol{v}_2}{dt_2} &= \boldsymbol{\alpha}_2\end{aligned}\right\} \Rightarrow \boldsymbol{\alpha}_2 = \frac{\boldsymbol{\alpha}_1}{1-\frac{u^2}{c^2}} \quad (26)$$

2. Gravitation

Warning: I have done a mistake in equation (27), which makes the following chapters wrong. I will give the correct theory later. But I keep this erroneous part below for showing a faithful historical image of my publications.

a) Relativistic orbit

The gravitation of a mass M transfers impulse to other mass. According to Newton's gravitational law, the impulse that M transfers to the mass m_0 during an infinitesimal time interval dt_1 in frame 1 is the $d\mathbf{P}_1$ defined by equation (27), with M and m_0 being the respective rest masses, r_M the distance from M to m_0, $\mathbf{e}_r$ the unit vector pointing from M to m_0 and g the gravitational constant.

$$d\boldsymbol{P}_1 = \left(-\frac{gM\cdot m_0}{r_M{}^2}\boldsymbol{e}_r\right)dt_1 \quad (27)$$

Let frame 2 be an inertial frame which moves momentarily with m_0. The impulse that m_0 feels in frame 2 equals $m_0 d\mathbf{v}_2$ with $d\mathbf{v}_2$ being the change of velocity of m_0 in frame 2 and is the $d\mathbf{P}_2$ in equation (28) where we have substituted equation (15) for $d\mathbf{v}_2$.

$$d\boldsymbol{P}_2 = m_0 d\boldsymbol{v}_2 = \frac{m_0 d\boldsymbol{v}_1}{\sqrt{1-\frac{u^2}{c^2}}} \quad (28)$$

The impulse on m_0 in frames 1 and 2 refer to the same impulse. So, $d\mathbf{P}_1 = d\mathbf{P}_2$, see equation (29). Then, we equate equation (27) with (28) and obtain equation (30) which we transform into equation (31).

$$d\boldsymbol{P}_1 = d\boldsymbol{P}_2 \quad (29)$$

$$-\frac{gM\cdot m_0\cdot dt_1}{r_M{}^2}\boldsymbol{e}_r = \frac{m_0 d\boldsymbol{v}_1}{\sqrt{1-\frac{u^2}{c^2}}} \quad (30)$$

$$-\frac{gM}{r_M{}^2}\boldsymbol{e}_r = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\frac{d\boldsymbol{v}_1}{dt_1} \quad (31)$$

$\frac{d\boldsymbol{v}_1}{dt_1}$ is the acceleration of m_0 in frames 1 of which the magnitude is $-\alpha_R$, see equation (32). Combining Equations (31) and (32) we obtain the expression of α_R in equation (33).

$$\frac{d\boldsymbol{v}_1}{dt_1} = \boldsymbol{\alpha}_r = -\alpha_R \boldsymbol{e}_r \quad (32)$$

$$\alpha_R = \frac{gM}{r_M{}^2}\sqrt{1-\frac{u^2}{c^2}} \quad (33)$$

$$\alpha_N = \frac{gM}{r_M{}^2} \quad (34)$$

$$\alpha_R = \alpha_N\sqrt{1-\frac{u^2}{c^2}} \quad (35)$$

The theoretical orbit of m_0 in the gravitational field of M that obeys Newtonian law is called Newtonian orbit. The one that obeys Time relativity is called relativistic orbit. The orbital acceleration of the relativistic orbit is α_R, that of the Newtonian orbit is α_N given in equation (34), which we introduce in to equation (33) to express α_R in terms of α_N in equation (35).

Notice the coefficient $\sqrt{1-\frac{u^2}{c^2}}$ in equation (35), it comes from the conversion of time from frame 1 to 2 and has nothing to do with relativistic mass.

b) Pericenter of orbit

Pericenter is the point of an orbit that is closest to the attracting body. The center and radius of curvature of the orbit at the pericenter are C and r_c which equal that of the circle of curvature at the pericenter, see Figure 3.

m_0 follows momentarily the circle of curvature at the pericenter. On a relativistic orbit, m_0's velocity and acceleration are u_R and α_R which are related by equation (36). On a Newtonian orbit, m_0's velocity and acceleration are u_N and α_N which are related by equation (37). Introducing equations (36) and (37) in to (35), we obtain equation (38), which is then transformed into equation (39) then (40). The solution of equation (40) is given in equation (41), with the negative solution been dropped off because $\frac{u_R^2}{c^2}$ is always positive. Equation (41) expresses the velocity of m_0 at the pericenter of its relativistic orbit in the gravitational field of M.

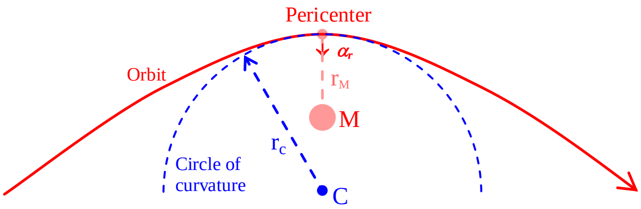

Figure 3

$$\frac{u_R^2}{r_c} = \alpha_R \tag{36}$$

$$\frac{u_N^2}{r_c} = \alpha_N = \frac{gM}{r_M^2} \tag{37}$$

$$\frac{u_R^2}{r_c} = \frac{u_N^2}{r_c}\sqrt{1 - \frac{u_R^2}{c^2}} \tag{38}$$

$$\left(\frac{u_R^2}{c^2}\right)^2 = \left(\frac{u_N^2}{c^2}\right)^2\left(1 - \frac{u_R^2}{c^2}\right) \tag{39}$$

$$\left(\frac{c^2}{u_N^2}\right)^2\left(\frac{u_R^2}{c^2}\right)^2 + \frac{u_R^2}{c^2} - 1 = 0 \tag{40}$$

$$\frac{u_R^2}{c^2} = \frac{\sqrt{1 + 4\left(\frac{c^2}{u_N^2}\right)^2} - 1}{2\left(\frac{c^2}{u_N^2}\right)^2} \tag{41}$$

- Large r_M

When $\left(\frac{2c^2}{gM}\frac{r_M^2}{r_c}\right)^2$ is much larger than 1, the square root can be simplified as in equation (42), which makes equation (41) to become (43). We see that, when $r_M \to \infty$, u_R^2 reduces to the Newtonian solution while being always smaller than it, see equation (44).

$$\left(\frac{2c^2}{u_N^2}\right)^2 = \left(\frac{2c^2}{gM}\frac{r_M^2}{r_c}\right)^2 \gg 1 \Rightarrow \sqrt{\left(\frac{2c^2}{u_N^2}\right)^2 + 1} \approx \frac{2c^2}{u_N^2} \tag{42}$$

$$\frac{u_R^2}{c^2} \approx \frac{\frac{2c^2}{u_N^2} - 1}{2\left(\frac{c^2}{u_N^2}\right)^2} = \frac{u_N^2}{c^2} - \frac{1}{2}\left(\frac{u_N^2}{c^2}\right)^2 \tag{43}$$

$$u_R \xrightarrow[r_M \to \infty]{} u_N \tag{44}$$

- Small r_M

When $\left(\frac{2c^2}{gM}\frac{r_M^2}{r_c}\right)^2$ is much smaller than 1, the square root can be simplified as in equation (45), which makes equation (41) to become equation (46), which shows that, when $r_M \to 0$, $\frac{u_R^2}{c^2}$ reduces to nearly 1 while being always smaller than 1. So, when r_M the distance between M and m_0 decreases towards zero, the orbital velocity of m_0 tends to the speed of light c without ever reaching it, see equation (47).

$$\left(\frac{2c^2}{u_N^2}\right)^2 = \left(\frac{2c^2}{gM}\frac{r_M^2}{r_c}\right)^2 \ll 1 \Rightarrow \sqrt{1 + \left(\frac{2c^2}{u_N^2}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{2c^2}{u_N^2}\right)^2 - \frac{1}{8}\left(\frac{2c^2}{u_N^2}\right)^4 \tag{45}$$

$$\frac{u_R^2}{c^2} \approx \frac{\frac{1}{2}\left(\frac{2c^2}{u_N^2}\right)^2 - \frac{1}{8}\left(\frac{2c^2}{u_N^2}\right)^4}{2\left(\frac{c^2}{u_N^2}\right)^2} = 1 - \left(\frac{c^2}{u_N^2}\right)^2 < 1 \tag{46}$$

$$u_R < c, \qquad u_R \xrightarrow[r_M \to 0]{} c \tag{47}$$

- Circular orbit

M coincides with C in Figure 3 and $r_c = r_M$ in the above solutions.

3. Solar mass

The mass of the Sun is not measured by weighting, but derived from the parameters of Earth's orbit which is nearly circular. Let r_M be the radius of the Earth's orbit and u_E its orbital velocity. By equating the orbital acceleration of the Earth (see equation (36)) with its Newtonian gravitational acceleration (see equation (34)), we obtain equation (48) which gives the today's used value of the mass of the Sun, M_0, in equation (49).

$$\frac{u_E^2}{r_M} = \frac{gM_0}{r_M^2} \tag{48}$$

$$M_0 = \frac{r_M u_E^2}{g} \tag{49}$$

According to Time relativity, the orbital acceleration of the Earth is given by equation (33). So, by equating equation (36) with (33), we obtain equation (50) which gives the mass of the Sun M_R in equation (51), which is derived using Time relativity.

$$\frac{u_E^2}{r_M} = \frac{gM_R}{r_M^2}\sqrt{1 - \frac{u_E^2}{c^2}} \tag{50}$$

$$M_R = \frac{r_M u_E^2}{g\sqrt{1 - \frac{u_E^2}{c^2}}} \tag{51}$$

The ratio M_R/M_0 is given in equation (52), which shows that M_R is bigger than M_0, that is, the mass of the Sun derived using Time relativity is bigger than its today's value. The relative gap between M_R and M_0 is defined as $\frac{M_R}{M_0} - 1$ of which the numerical value is computed in equation (53) using the orbital velocity of the Earth 29.8 km/s, which gives 4.94×10^{-9}.

$$\frac{M_R}{M_0}=\frac{1}{\sqrt{1-\frac{u_E^2}{c^2}}}\approx 1+\frac{u_E^2}{2c^2}>1 \quad (52)$$

Since bigger mass makes bigger orbital velocity, the unexpected Speed Boost of 'Oumuamua could be explained by a bigger mass of the Sun.

$$u_E = 29.8\text{ km/s} \Rightarrow$$

$$\frac{M_R}{M_0}-1\approx\frac{u_E^2}{2c^2} = 4.94\times10^{-9} \quad (53)$$

4. Anomalies in space

a) 'Oumuamua

- Speed Boost

The interstellar object 'Oumuamua was discovered on Oct. 19, 2017, which is 40 days after it has passed the perihelion on Sept. 9, 2017 at the velocity 87.71 km/s. It goes a tiny bit faster than it should have if the only effect on its motion was the Sun's gravity, which is not predicted by current theory [1][2][3][4][5][6]. But this Speed Boost is predicted by Time relativity.

Let u_R be the real orbital velocity of 'Oumuamua caused by M_R, which is expressed in equation (54) in terms of the real orbital acceleration α_R and r_c the radius of curvature of its orbit at the perihelion. With M_0, the orbital velocity would be u_0 which is expressed in equation (55) in terms of the corresponding orbital acceleration α_0. We divide equation (54) with (55) to obtain the ratio of the 2 velocities squared in equation (56).

$$u_R^2=\alpha_R r_c \quad (54)$$

$$u_0^2=\alpha_0 r_c \quad (55)$$

$$\frac{u_R^2}{u_0^2}=\frac{\alpha_R}{\alpha_0} \quad (56)$$

$$\alpha_R = kM_R \quad (57)$$

$$\alpha_0 = kM_0 \quad (58)$$

$$\frac{u_R^2}{u_0^2}=\frac{M_R}{M_0} \quad (59)$$

The gravitational acceleration is proportional to the mass, so, α_R and α_0 can be written as in equations (57) and (58). Introducing equations (57) and (58) in to (56), we get equation (59).

The difference of velocity u_R-u_0 is derived in equation (60). Because the values of u_R and u_0 are very close, we can write $u_R+u_0\approx2u_0$ in equation (61), which is introduced along with equation (59) in to equation (60) to get equation (62). Using the numerical values of u_0 and $\frac{M_R}{M_0}-1$, we find the value of difference of velocity u_R-u_0=0.217 mm/s in equation (63).

$$u_R-u_0=\frac{u_R^2-u_0^2}{u_R+u_0}=\frac{u_0^2}{u_R+u_0}\left(\frac{u_R^2}{u_0^2}-1\right) \quad (60)$$

$$u_R\approx u_0\Rightarrow u_R+u_0=2u_0 \quad (61)$$

$$u_R-u_0\approx\frac{u_0}{2}\left(\frac{M_R}{M_0}-1\right) \quad (62)$$

If the observed Speed Boost [1][2][3][4][5][6] confirms this theoretical value, then it is caused by a bigger mass of the Sun.

Perihelion

$$u_0 = 87.71\text{ km/s}\Rightarrow u_R-u_0 = 0.217\text{ mm/s} \quad (63)$$

- Resistance to slowdown

'Oumuamua slows down under the Sun's gravity, but not as fast as predicted by celestial mechanics [1][2][3][4][5][6]. I call this phenomenon Resistance to slowdown effect, which is predicted by Time relativity.

In fact, as relativistic mass is bigger than rest mass, the fast moving 'Oumuamua carries more kinetic energy than Newtonian theory predicts. So, 'Oumuamua looks moving slower when carrying the kinetic energy that Newtonian theory predicts.

Let u_N and E_N be the Newtonian velocity and kinetic energy of `Oumuamua. E_N is expressed in equation (64). When it slows down to the velocity u^*_N, its kinetic energy is E^*_N and the decrease of velocity squared is expressed in equation (65).

$$E_N=\frac{m_0u_N^2}{2} \quad (64)$$

$$u_N^{*2}-u_N^2=\frac{2}{m_0}(E_N^*-E_N) \quad (65)$$

The real kinetic energy carried by 'Oumuamua moving at the real velocity u_R is E_R. According to Time relativity, E_R is given in equation (66), which is the equation (44) in « Velocity, mass, momentum and energy of an accelerated object in relativity ». When 'Oumuamua slows down to the velocity u^*_R, its kinetic energy is E^*_R.

$$E_R=m_0c^2\left(1-\sqrt{1-\frac{u_R^2}{c^2}}\right) \quad (66)$$

$$\left(1-\frac{E_R}{m_0c^2}\right)^2=1-\frac{u_R^2}{c^2} \quad (67)$$

Equation (66) is transformed into equation (67) for deriving the decrease of velocity squared, see equations (68) to (70).

$$\left(1-\frac{E_R^*}{m_0c^2}\right)^2-\left(1-\frac{E_R}{m_0c^2}\right)^2=\left(1-\frac{{u_R^*}^2}{c^2}\right)-\left(1-\frac{{u_R}^2}{c^2}\right) \quad (68)$$

$$c^2\left(1-\frac{E_R^*}{m_0c^2}-1+\frac{E_R}{m_0c^2}\right)\left(1-\frac{E_R^*}{m_0c^2}+1-\frac{E_R}{m_0c^2}\right) = -{u_R^*}^2+{u_R}^2 \quad (69)$$

$${u_R^*}^2-{u_R}^2=\frac{E_R^*-E_R}{m_0}\left(2-\frac{E_R^*+E_R}{m_0c^2}\right) \quad (70)$$

Let us denote the differences of velocity squared in the form $\Delta(u^2)$, see equation (71). We compute the ratio of these differences by dividing equation (70) with equation (65), which gives equation (72).

$$\begin{aligned}{u_R^*}^2-{u_R}^2&=\Delta({u_R}^2)\\ {u_N^*}^2-{u_N}^2&=\Delta({u_N}^2)\end{aligned} \quad (71)$$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}=\frac{E_R^*-E_R}{E_N^*-E_N}\left(1-\frac{E_R^*+E_R}{2m_0c^2}\right) \quad (72)$$

Suppose that, when going from one point to another, 'Oumuamua gains the quantity of potential energy ΔU, which equals the loss of kinetic energy, $E^*_R-E_R$ for Time relativity and $E^*_N-E_N$ for Newtonian theory. Because ΔU is the same for the 2 cases, we have $E^*_R-E_R=E^*_N-E_N$, see equation (73). Also, the values of E^*_R and E_R equal nearly the Newtonian kinetic energy, so we can write equation (74). Introducing equations (73) and (74) in to (72) we get equation (75) which is smaller than 1. So, the decrease of velocity for Time relativity is smaller than the Newtonian one, that is, 'Oumuamua "resists to slowdown".

Gain in potential energy ΔU

$$\left.\begin{aligned}\Delta U&=E_R^*-E_R\\&=E_N^*-E_N\end{aligned}\right\}\Rightarrow\frac{E_R^*-E_R}{E_N^*-E_N}=1 \quad (73)$$

$$E_R\approx m_0\frac{{u_R}^2}{2}, E_R^*\approx m_0\frac{{u_R^*}^2}{2}\Rightarrow \quad \frac{E_R^*+E_R}{2m_0c^2}\approx\frac{{u_R^*}^2+{u_R}^2}{4c^2} \quad (74)$$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}\approx 1-\frac{{u_R^*}^2+{u_R}^2}{4c^2}<1 \quad (75)$$

We compute this effect when the decrease of velocity is very small, so we write equation (75) into (76). We differentiate $\Delta({u_R}^2)$ and $\Delta({u_N}^2)$ and simplify the ratio $\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}$ into $\frac{\Delta u_R}{\Delta u_N}$ because the orbital velocity for Time relativity is very close to the Newtonian one, see equation (77). Then, equation (76) becomes equation (78), which quantifies the relative gap of decrease of velocity between Time relativity and Newtonian predictions.

Δu_R small ${u_R^*}^2\approx{u_R}^2\Rightarrow$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}-1\approx-\frac{{u_R}^2}{2c^2} \quad (76)$$

$u_R\approx u_N\Rightarrow$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}\approx\frac{2u_R\Delta u_R}{2u_N\Delta u_N}\approx\frac{\Delta u_R}{\Delta u_N} \quad (77)$$

$$\frac{\Delta u_R}{\Delta u_N}-1\approx-\frac{{u_R}^2}{2c^2} \quad (78)$$

Let us call $\frac{\Delta u_R}{\Delta u_N}-1$ Resistance to slowdown and compute its value in equation (79). It equals -4.28×10^{-8} at perihelion where the velocity of `Oumuamua is 87.71 km/s. In outer space where the Sun's gravity is very weak, the velocity of `Oumuamua is [26.33 km/s](#) [1] and its Resistance to slowdown is -3.86×10^{-9}. It will be interesting to compare the theoretical value -4.28×10^{-8} with the observation at perihelion [1][2][3][4][5][6].

Perihelion

$$u_R=87.71\text{ km/s}\Rightarrow\quad\frac{\Delta u_R}{\Delta u_N}-1=-4.28\times10^{-8}$$

Outer space

$$u_R=26.33\text{ km/s}\Rightarrow\quad\frac{\Delta u_R}{\Delta u_N}-1=-3.86\times10^{-9} \quad (79)$$

Equation (75) ⇒

$$\Delta({u_R}^2)-\Delta({u_N}^2)\approx-\frac{{u_R^*}^2+{u_R}^2}{4c^2}\Delta({u_N}^2)=-\frac{{u_R^*}^2+{u_R}^2}{4c^2}({u_N^*}^2-{u_N}^2) \quad (80)$$

$u_R=u_N\Rightarrow$

$$\begin{aligned}\Delta({u_R}^2)-\Delta({u_N}^2)&=({u_R^*}^2-{u_R}^2)-({u_N^*}^2-{u_N}^2)\\&={u_R^*}^2-{u_N^*}^2\\&=(u_R^*-u_N^*)(u_R^*+u_N^*)\end{aligned} \quad (81)$$

Equations (80) = (81)⇒

$$u_R^*-u_N^*\approx-\frac{{u_R^*}^2+{u_R}^2}{4c^2}\frac{{u_N^*}^2-{u_N}^2}{u_R^*+u_N^*} \quad (82)$$

$$\left.\begin{aligned}u_N&\approx u_R\approx 87.71\text{ km/s}\\u_N^*&\approx u_R^*\approx 26.33\text{ km/s}\end{aligned}\right\}\Rightarrow$$

In outer space

$$u_R^*-u_N^*\approx 3.1\text{ mm/s} \quad (83)$$

I have also computed the gap between the total decreases of velocity for Time relativity and Newtonian theory, see equations (80) to (83), which shows that changing its velocity from 87.71 km/s to 26.33 km/s, `Oumuamua would move 3.1 mm/s faster than predicted in outer space.

b) Pioneer anomaly

The Wikipedia page [Pioneer anomaly](#) [7] explains that while the spacecrafts [Pioneer 10](#) and [Pioneer 11](#) are escaping the Solar System, their navigational data show that they are slowing slightly more than expected, they get an additional extremely small acceleration towards the Sun which is $\Delta\alpha$=[(8.74±1.33)×10^{−10} m/s²](#) [7][8][9][10][11][12][13][14][15][16].

Let us compute the Resistance to slowdown for Pioneer by using the tangential orbital acceleration which equals the change of magnitude of velocity Δu divided by time interval Δt, see equation (84) where the index R and N indicate Time relativity and Newtonian theory. We divide α_R with α_N in equation (85) which shows that the ratio of acceleration equals that of the change of velocity. Introducing equation (85) in to equation (78), we get equation (86).

Tangential acceleration

$$\alpha_R = \frac{\Delta u_R}{\Delta t}, \alpha_N = \frac{\Delta u_N}{\Delta t} \tag{84}$$

Equation (77) ⇒

$$\frac{\alpha_R}{\alpha_N} = \frac{\Delta u_R}{\Delta u_N} \tag{85}$$

Using the velocity of Pioneer 10 on December, 30, 2005 which is 12.51 km/s, equation (86) gives the Resistance to slowdown for Pioneer: *−8.70×10⁻¹⁰*, see equation (87), which is amazingly close to the value of Δα given in the Wikipedia page Pioneer anomaly [7]: (8.74±1.33)×10^{-10} m/s^2.

Equation (76) ⇒

$$\frac{\alpha_R}{\alpha_N} - 1 \approx -\frac{u_R{}^2}{2c^2} \tag{86}$$

$u_R = 12.51$ km/s

$$\frac{\alpha_R}{\alpha_N} - 1 \approx -8.70 \times 10^{-10} \tag{87}$$

However, there is a glitch: the quantity $\frac{\alpha_R}{\alpha_N} - 1$ is dimensionless while Δα has the dimension m/s^2. I suspect that there may be more than a pure coincidence.

There is a second glitch. $\frac{\alpha_R}{\alpha_N} - 1 < 0$ means that α_R is smaller than α_N in magnitude. Because the tangential acceleration points towards the Sun while the velocity away from the Sun, a smaller acceleration means that the spacecrafts are less pulled towards the Sun. So, the resistance to slowdown effect makes the spacecrafts to slow less than expected rather than "*slowing slightly more than expected*". Also, because the thermal radiation pressure proposed by the cited papers [7][8][9][10][11][12][13][14][15][16] pushes towards the Sun it acts against the resistance to slowdown effect.

M=1.99×10^{30} kg
g=6.67× 10^{-11} m^3/kg/s^2
u_E=29.8 km/s
r_M=89.7 AU

$$|\alpha_0| = \frac{gM}{r_M{}^2} = 7.37\times10^{-7} \text{m}/s^2 \tag{88}$$

Equation (57) and (58) ⇒

$$\frac{\alpha_R}{\alpha_0} = \frac{M_R}{M_0} \tag{89}$$

$$\frac{\alpha_R}{\alpha_0} - 1 \approx 4.94\times10^{-9} \tag{90}$$

$$|\alpha_R| - |\alpha_0| \approx 4.94\times10^{-9}|\alpha_0| = 3.64\times10^{-15} \text{m}/s^2 \tag{91}$$

Anyway, because the resistance to slowdown effect is observed with `Oumuamua and the theoretical value *−8.70×10⁻¹⁰* is so close to the observational value (8.74±1.33)×10^{-10} m/s^2, it will be very interesting to look again into the navigational data of Pioneer 10 and Pioneer 11.

Just for comparison, the total acceleration due to the Sun is 7.37×10^{-7} m/s^2, see equation (88). Due to Speed Boost effect (bigger solar mass, see section 4. a) 'Oumuamua •Speed Boost) the relative increase of gravitational acceleration is 4.94×10^{-9}, see equation (90). The increased of acceleration is 3.64×10^{-15} m/s^2 towards the Sun, see equation (91).

c) Flyby anomaly

Flyby is a common gravity assist maneuver to accelerate a spacecraft using the gravity of the Earth. It was noticed that in multiple cases, spacecrafts have been observed to gain more speed than scientists had predicted. This phenomenon is called flyby anomaly [17][18][19][20][21].

This gain in speed may have the same origin than the Speed Boost of 'Oumuamua, except that the attracting body is the Earth. And indeed, the mass of the Earth was derived using the orbit of the Moon of which the velocity is 1.022 km/s, which makes the mass of the Earth (1+ 5.81×10^{-12}) times more massive than the today's used value, see section 3 "Solar mass".

5. Planets and Parker Solar Probe

The above cases of speed boost make reevaluating the mass of the Sun important. For doing so, we can compute the quantity $\mu = \alpha \cdot r_M{}^2$ for all the planets, with α being the orbital acceleration of the planet and r_M its distance from the Sun, α and r_M being instantaneous values.

If μ is the same for all the planets, then the mass of the Sun is consistent with Newtonian theory as given in equation (92). If μ varies according to equation (93) (see equation (33)), with i indicating instantaneous orbital values for the i^{th} planet, then the mass of the Sun is now undervalued and μ will give the real mass of the Sun.

Newtonian

$$\mu = \alpha \cdot {r_M}^2 = gM \tag{92}$$

Time relativity

$$\mu = \alpha \cdot {r_{M,i}}^2 = gM \sqrt{1 - \frac{{u_i}^2}{c^2}} \tag{93}$$

The orbital data of Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune exist in the archives, but I doubt that they are sufficiently precise for this work. So, new measurement is necessary. To quickly obtain high precision orbital data around the Sun, the navigation data of the Parker Solar Probe are a good source because they are precise measurement and its orbit is very elongated, which makes the value of r_M and u to vary in a large range.

Also, the orbit of the Parker Solar Probe is suitable for verifying the Resistance to slowdown effect. Indeed, its orbital velocity and tangential acceleration vary in a large range.

References

`Oumuamua

[1] Bill Gray, “"Pseudo-MPEC" for A/2017 U1 = 1I = ‘Oumuamua” https://projectpluto.com/temp/2017u1.htm#speed

[2] Micheli, Marco; Farnocchia, Davide; Meech, Karen J.; Buie, Marc W.; Hainaut, Olivier R.; Prialnik, Dina; Schörghofer, Norbert; Weaver, Harold A.; Chodas, Paul W.; Kleyna, Jan T.; Weryk, Robert; Wainscoat, Richard J.; Ebeling, Harald; Keane, Jacqueline V.; Chambers, Kenneth C.; Koschny, Detlef; Petropoulos, Anastassios E. (2018). "Non-gravitational acceleration in the trajectory of 1I/2017 U1 (‘Oumuamua)". Nature. **559** (7713): 223–226. Bibcode:2018Natur.559..223M. doi:10.1038/s41586-018-0254-4. PMID 29950718.

[3] Williams, Matt (2 November 2018). "Could Oumuamua Be an Extra-Terrestrial Solar Sail?". Universe Today.

[4] Bialy, Shmuel; Loeb, Abraham (26 October 2018). "Could Solar Radiation Explain ‘Oumuamua's Peculiar Acceleration?". The Astrophysical Journal. **868**: L1. arXiv:1810.11490. doi:10.3847/2041-8213/aaeda8.

[5] Cofield, Calla; Chou, Felicia; Wendel, JoAnna; Weaver, Donna; Villard, Ray (27 June 2018). "Our Solar System's First Known Interstellar Object Gets Unexpected Speed Boost". NASA. Retrieved 27 June 2018.

[6] Rafikov, Roman R. (20 September 2018). "Spin Evolution and Cometary Interpretation of the Interstellar Minor Object 1I/2017 ‘Oumuamua". arXiv:1809.06389v2 [astro-ph.EP].

Pioneer anomaly

[7] Pioneer anomaly https://en.wikipedia.org/wiki/Pioneer_anomaly

[8] Nieto, M. M.; Turyshev, S. G. (2004). "Finding the Origin of the Pioneer Anomaly". Classical and Quantum Gravity. 21(17): 4005–4024. arXiv:gr-qc/0308017. Bibcode:2004CQGra..21.4005N. CiteSeerX 10.1.1.338.6163. doi:10.1088/0264-9381/21/17/001.

[9] Benny Rievers, Claus Lämmerzahl (2011). "High precision thermal modeling of complex systems with application to the flyby and Pioneer anomaly". Annalen der Physik. 523 (6): 439. arXiv:1104.3985. Bibcode:2011AnP...523..439R. doi:10.1002/andp.201100081.

[10] Edward M. Murphy. (1999). "A Prosaic explanation for the anomalous accelerations seen in distant spacecraft". Physical Review Letters. 83 (9): 1890. arXiv:gr-qc/9810015. Bibcode:1999PhRvL..83.1890M. doi:10.1103/PhysRevLett.83.1890.

[11] Katz, J. I. (1999). "Comment on "Indication, from Pioneer 10/11, Galileo, and Ulysses data, of an apparent anomalous, weak, long-range acceleration"". Physical Review Letters. 83 (9): 1892. arXiv:gr-qc/9809070. Bibcode:1999PhRvL..83.1892K. doi:10.1103/PhysRevLett.83.1892.

[12] Scheffer, Louis K. (2003). "Conventional forces can explain the anomalous acceleration of Pioneer 10". Physical Review D. 67(8): 084021. arXiv:gr-qc/0107092. Bibcode:2003PhRvD..67h4021S. doi:10.1103/PhysRevD.67.084021.

[13] Anderson, John D.; Laing, Philip A.; Lau, Eunice L.; Liu, Anthony S.; Nieto, Michael Martin; Turyshev, Slava G. (2002). "Study of the anomalous acceleration of Pioneer 10 and 11". Physical Review D. 65 (8): 082004. arXiv:gr-qc/0104064. Bibcode:2002PhRvD..65h2004A. doi:10.1103/PhysRevD.65.082004.

[14] Bertolami, O.; Francisco, F.; Gil, P. J. S.; Páramos, J. (2008). "Thermal analysis of the Pioneer anomaly: A method to estimate radiative momentum transfer". Physical Review D. 78 (10): 103001. arXiv:0807.0041.

[15] Turyshev, S. G; Toth, V. T.; Kellogg, L.; Lau, E.; Lee, K. (2006). "A study of the pioneer anomaly: new data and objectives for new investigation". International Journal of Modern Physics D. 15 (1): 1–55. arXiv:gr-qc/0512121. Bibcode:2006IJMPD..15....1T. doi:10.1142/S0218271806008218.

[16] Fienga, A., Laskar, J., Kuchynka, P., Manche, H., Gastineau, M. and Leponcin-Lafitte, C. (2009). "Gravity tests with INPOP planetary ephemerides" (PDF). Proceedings of the Annual Meeting of the French Society of Astronomy and Astrophysics: 105–109. Bibcode:2009sf2a.conf..105F. Archived from the original (PDF) on 2011-07-20. Also published in Proceedings of the International Astronomical Union. 5: 159–169. 2010. arXiv:0906.3962. Bibcode:2010IAUS..261..159F. doi:10.1017/S1743921309990330.

Flyby anomaly

[17] C, Edwards, J. Anderson, P, Beyer, S. Bhaskaran, J. Borders, S. DiNardo, W. Folkner, R. Haw, S. Nandi, F. Nicholson, C. 0ttenhoff, S. Stephens (1993). "Tracking galileo at earth-2 perigee using the tracking and data relay satellite system". CiteSeerX 10.1.1.38.4256

[18] Thompson, Paul F.; Matthew Abrahamson; Shadan Ardalan; John Bordi (2014). Reconstruction of Earth flyby by the Juno spacecraft. 24th AAS/AIAA Space Flight Mechanics Meeting. Santa Fe, NM: AAS. pp. 14–435.

[19] Anderson, John D.; James K. Campbell; Michael Martin Nieto (July 2007), "The energy transfer process in planetary flybys", New Astronomy, 12 (5): 383–397, arXiv:astro-ph/0608087, Bibcode:2007NewA...12..383A, doi:10.1016/j.newast.2006.11.004

[20] Páramos, Jorge; Hechenblaikner, G. (2013). "Probing the Flyby Anomaly with the future STE-QUEST mission". Planetary and Space Science. 79-80: 76–81. arXiv:1210.7333. Bibcode:2013P&SS...79...76P. doi:10.1016/j.pss.2013.02.005.

[21] John D. Anderson, James K. Campbell, John E. Ekelund, Jordan Ellis, and James F. Jordan (7 March 2008), "Anomalous Orbital-Energy Changes Observed during Spacecraft Flybys of Earth"(PDF), Phys. Rev. Lett., 100 (9): 091102, Bibcode:2008PhRvL.100i1102A, doi:10.1103/physrevlett.100.091102, PMID 18352689.