

# What is the thickness of the event horizon of a black hole?

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## 1. For Newtonian mechanics

The Schwarzschild radius is the radius of the event horizon of a black hole. It can be computed with Newtonian mechanics from the gravitational acceleration  $\alpha$  of the mass of the black hole  $M$  at the distance  $r$  from  $M$ , see equation (1) and figure 1 where  $m$  is a small mass. The Schwarzschild radius of  $M$  is the  $r_s$  shown in (2) and is derived in the paper «[Radius of a black hole for relativity and Newtonian mechanics](https://www.academia.edu/84798805/Radius_of_a_black_hole_for_relativity_and_Newtonian_mechanics)» linked below:  
[https://www.academia.edu/84798805/Radius\\_of\\_a\\_black\\_hole\\_for\\_relativity\\_and\\_Newtonian\\_mechanics](https://www.academia.edu/84798805/Radius_of_a_black_hole_for_relativity_and_Newtonian_mechanics)

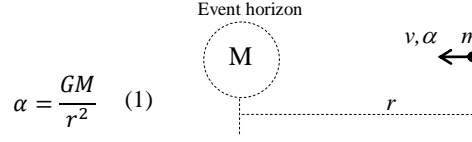


Figure 1

$$\text{Schwarzschild radius } r_s = \frac{2GM}{c^2} \quad (2)$$

The event horizon is a sphere of radius  $r_s$  where the velocity of a falling object reaches the speed of light  $c$ . But, this object's velocity should be 0 at infinitely far. What will be the "Schwarzschild radius" for a falling object whose initial velocity  $v_0$  is not 0 at  $r=\infty$ ? In this case, when  $v = c$  the radius is  $r_c$  which does not equal  $r_s$  given by equation (2).

Let us derive  $r_c$  for initial condition where the velocity of  $m$  is  $v_0$  at the radial distance  $r_0$ . The equation (6) of «[Radius of a black hole for relativity and Newtonian mechanics](https://www.academia.edu/84798805/Radius_of_a_black_hole_for_relativity_and_Newtonian_mechanics)» is given below:

$$\frac{v_2^2 - v_1^2}{2} = \frac{GM}{r_2} - \frac{GM}{r_1} \quad (3)$$

Applying  $v_0 = v_1$  and  $r_0 = r_1$  into (3) and let  $v_2 = c$ , we have (4), then (5).

$$\frac{c^2 - v_0^2}{2} = \frac{GM}{r_2} - \frac{GM}{r_0} \quad (4)$$

$$r_c = r_2 = \frac{2GM}{c^2} \frac{1}{1 + \frac{2}{c^2} \left( \frac{GM}{r_0} - \frac{v_0^2}{2} \right)} = \frac{r_s}{1 + \frac{2}{c^2} \left( \frac{GM}{r_0} - \frac{v_0^2}{2} \right)} \quad (5)$$

In (5)  $r_c$  is different for different  $v_0$  and  $r_0$ . That is, the velocity  $v_2$  reaches  $c$  at different radial distances  $r_c$ .

- If  $\frac{v_0^2}{2} > \frac{GM}{r_0}$ , then  $r_c$  is bigger than the Schwarzschild radius  $r_s$ .
- If  $\frac{v_0^2}{2} < \frac{GM}{r_0}$ , then  $r_c$  is smaller than the Schwarzschild radius  $r_s$ .

As the event horizon of a black hole is where  $v = c$  and the value of  $r_c$  is spread off, we get the idea that the event horizon is not a boundary but has a thickness.

However, I think that a "thick event horizon" is physically absurd. But, as general relativity is derived from the equation (1) which belongs to Newtonian mechanics, general relativity would give "thick event horizon" too.

- Velocity at the Schwarzschild radius for  $v_0 = v_1$  at  $r_1 = \infty$ .

Using the equation (2), we write (6) and apply it to (3) which gives (7) and (8). So, the velocity of a particle starting with velocity  $v_1$  at infinitely far will be the  $v_2$  given by (8) which is bigger than  $c$  and violates relativistic principle.

$$r_2 = r_s = \frac{2GM}{c^2}, \quad r_1 = \infty \quad (6)$$

$$\frac{v_2^2 - v_1^2}{2} = \frac{GM}{\frac{2GM}{c^2}} - \frac{GM}{\infty} \quad (7)$$

$$v_2^2 = c^2 + v_1^2 \quad (8)$$

## 2. For relativity

Let us see what would be the thickness of the event horizon when we apply relativistic principle to equation (1). In «[Radius of a black hole](#) for relativity and [Newtonian mechanics](#)», [link](#), we have obtained (9).

$$d \ln \left( 1 - \frac{v^2}{c^2} \right) = -d \left( \frac{2GM}{c^2 r} \right) \quad (9)$$

which gives (10)

$$\frac{1 - \frac{v_2^2}{c^2}}{1 - \frac{v_1^2}{c^2}} = e^{-\left( \frac{2GM}{c^2 r_2} - \frac{2GM}{c^2 r_1} \right)} \quad (10)$$

When  $r_2 = 0$ , we have  $v_2 = c$  for any initial velocity  $v_1$  at any radial distance  $r_1$ , see (11).

$$1 - \frac{v_2^2}{c^2} = \left( 1 - \frac{v_1^2}{c^2} \right) e^{-\infty} = 0 \Rightarrow v_2^2 = c^2 \quad (11)$$

So, when relativistic principle is applied correctly, the radius of the event horizon of a black hole is  $r_2 = 0$  and the event horizon does not have thickness, which makes sense in physics.