How galaxies make their rotation curves flat and what about dark matter?

Kuan Peng 彭宽 *titang78@gmail.com*

16 April 2021

Abstract: The rotation curves of disc galaxies are flat and dark matter is speculated as explanation. Alternatively, the gravity of material disk could explain the flat curves. Using the gravitational force that a disk exerts on a body in the disk, we have computed the the rotation curves of disc galaxies and the curve of their mass densities. The numerical result fits the flat curves and the observed mass densities of galaxies. This theory gives a new way to measure the masses of galaxies using their rotation velocities and shape.

1. Rotation curve

In a disc galaxy stars orbit the center of the galaxy at velocities that depend on the radial distance from the centre. The measured orbital velocities of typical galaxies are plotted versus radial distance in Figure 1, which are the rotation curves of these disk galaxies. Figure 1 shows that the observed rotation curves are flat for large radial distance, which means that these velocities are roughly constant with respect to radial distance [1].

The flat aspect of the rotation curves is puzzling because the Newtonian theory of gravity predicts that, like the velocities of the planets in the solar system, the velocity of an orbiting object decreases as the distance from the attracting body increases. Since the centers of galaxies are thought to contain most of their masses, the orbital velocities at large radial distance should be smaller than those near the center. But the observed the rotation curves clearly show the contrary and the observed orbital velocities are bigger than expected.

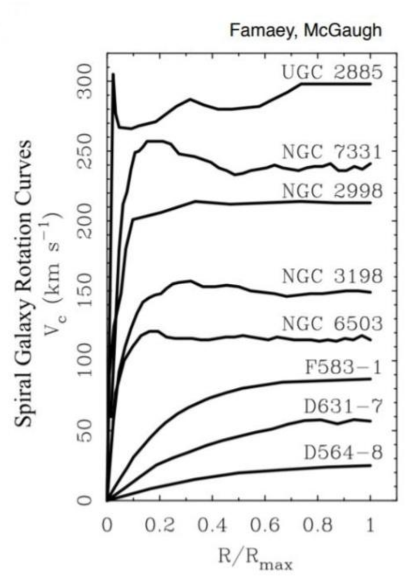

Figure 1: Typical rotation curves of disc galaxies [4]

The masses of galaxies estimated using the luminosity of visible stars are too low to maintain the stars to move at such high speed. So, large amount of matter is needed to explain the observed velocity, but we do not see this matter. The missing matter should act gravitational force because it should hold the flying stars, but should not radiate light because invisible. So, it is dubbed as dark matter. However, dark matter has not been observed directly despite the numerous actively undertaken experiments to detect it. Several alternatives to dark matter exist to explain the rotation curve.

We propose to model galaxy as regular matter disk. Because of its shape, the resultant gravitational force of material disk within the plane of a galaxy is different from that of the galaxy taken as concentrated masses. But, is this resultant force bigger?

2. Gravity of material disk

Galaxies are made of stars that move in circular orbits. Let us take out one circular orbit with all its stars and put them in free space without an attracting body at the center. These stars form a circle and attract each other gravitationally, which would make them to fall toward the center if they were stationary, see Figure 2.

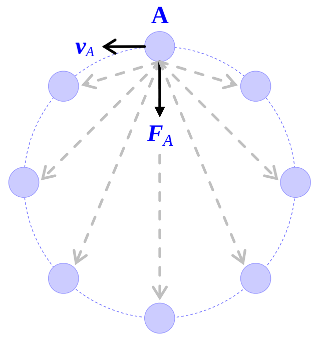

Figure 2: Stars in a circle orbiting under their proper mass

Figure 2 shows a simplified image of stars in a circle with F_A being the combined gravitational force that the other stars act on the star A. For the star A not to fall out of the circle, it must rotate at a nonzero velocity which is labeled as v_A. By symmetry, all the stars feel the same gravitational force and must rotate at the same velocity to stay in the circle. So, this circle of stars should rotate although no attracting body is at the center. It is the proper mass of all the stars of the circle that maintains the stars rotating.

In the explanation, we will often use the case of a single object in a circular orbit around an attracting body. For referring to this case, we give it the following name.

Definition: The Newtonian orbital acceleration and velocity of a single object in a circular orbit around an attracting body are named Single-Orbit acceleration and velocity.

$$\frac{v_s{}^2}{R_s} = \frac{GM}{R_s{}^2} \tag{1}$$

R_s: radius of a circular orbit
v_s: velocity of this orbit

If a star orbits in circle around a central mass M, the gravitational force on it is from M only and is labeled as F_M. The star would move at the Single-Orbit velocity which is v_s, see equation (1).

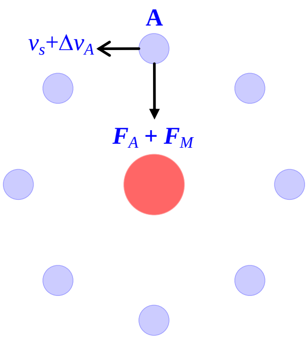

Figure 3: Stars orbiting a central mass M in circle

Now, let us add the mass M at the center of the circle of stars, see Figure 3. The gravitational force acted on the star A by the other stars of the circle is still F_A. But in addition it feels the force F_M from the central mass M. So, the total force on the star A is $F_A + F_M$ which is bigger than F_M. In consequence, to stay in the circle the star A should orbit at a velocity bigger than v_s, say at $v_s + \Delta v_A$ with Δv_A being the contribution of the force F_A. So, if a circle of stars orbit an attracting body at the center, their orbital velocity should be bigger than the Single-Orbit velocity of one star around the same body, which is kind of like the case of the bigger than expected orbital velocity in galaxies.

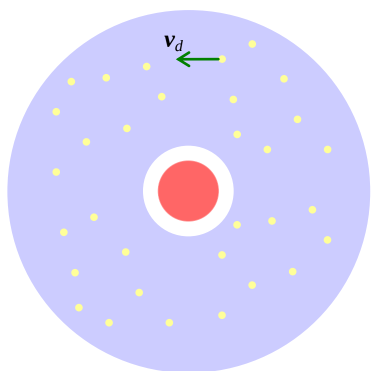

Figure 4: Material disk orbiting a central mass M

Now, let us smear the stars of the circle into a disk of dust around the central mass M which is not held by cohesion but by gravitational force, see Figure 4. Like the stars of the circle, the gravitational force on a dust is from the mass M and the proper mass of the disk. So, the orbital velocity of the dust, v_d in Figure 4, will be bigger than the Single-Orbit velocity around the mass M like the stars in the disk of a galaxy.

This is the working principle of our model that explains the faster than expected orbital velocity in galaxies. Now, let us see if the gravity of material disk could make the orbital velocity constant for large radial distance.

3. Force in a disk

For computing the gravitational force that the entire disk exerts on a chunk of material of mass m_1, we use the Newtonian law of gravitation which expresses the gravitational force that an elementary mass dm exerts on the chunk of material, see equation (2) where $d\boldsymbol{F}_d$ is the gravitational force, G the universal gravitational constant, R_3 the distance between m_1 and dm, $\boldsymbol{e}_3$ the unit vector pointing from m_1 to dm, see Figure 5.

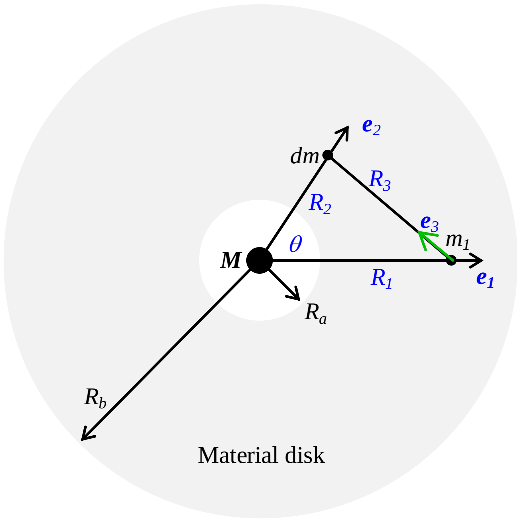

Figure 5: Positions of dm and m_1

As Figure 5 shows, the center of the disk is the point M, the mass m_1 is located by the vector $R_1\boldsymbol{e}_1$, with R_1 being the distance from M to m_1, $\boldsymbol{e}_1$ the unit vector pointing from M to m_1. The elementary mass dm is located by the vector $R_2\boldsymbol{e}_2$, with R_2 being the distance from M to dm, $\boldsymbol{e}_2$ the unit vector pointing from M to dm. The vector $R_3\boldsymbol{e}_3$ joins m_1 to dm.

$d\boldsymbol{F}_d = \dfrac{Gm_1 dm}{{R_3}^2}\boldsymbol{e}_3$	(2)
$dF_d = d\boldsymbol{F}_d \cdot \boldsymbol{e}_1$ $= Gm_1 dm \dfrac{R_3\boldsymbol{e}_3 \cdot \boldsymbol{e}_1}{{R_3}^3}$ See (2)	(3)
$R_3\boldsymbol{e}_3 = R_2\boldsymbol{e}_2 - R_1\boldsymbol{e}_1$ See Figure 5	(4)
$\boldsymbol{e}_2 \cdot \boldsymbol{e}_1 = \cos\theta$	(5)
Using (4) and (5) for (3) $R_3\boldsymbol{e}_3 \cdot \boldsymbol{e}_1 = (R_2\boldsymbol{e}_2 - R_1\boldsymbol{e}_1) \cdot \boldsymbol{e}_1$ $= R_2\cos\theta - R_1$	(6)

The magnitude of the component of $d\boldsymbol{F}_d$ along the unit vector $\boldsymbol{e}_1$ is dF_d, which equals the dot product of $d\boldsymbol{F}_d$ with $\boldsymbol{e}_1$, see (3). The vector $R_3\boldsymbol{e}_3$ equals the difference of $R_2\boldsymbol{e}_2$ and $R_1\boldsymbol{e}_1$, see (4). The unit vector $\boldsymbol{e}_2$ makes the angle θ with the unit vector $\boldsymbol{e}_1$. Then, the dot product $\boldsymbol{e}_2 \cdot \boldsymbol{e}_1$ equals $\cos\theta$, see (5), and the dot product in (3) is expressed in (6).

For computing R_3, we take the square of (3) in (7). Using (5) in (7), ${R_3}^2$ is expressed in (8). Using (6) and (8) in (3), the force dF_d is expressed in (9). For simplifying this expression, we define the term S_2 in (10). Then, S_2 is further simplified using the ratios δ and σ defined in (11) and is express in (12). Then, dF_d is expressed with S_2 in (13).

dF_d is the elementary force on m_1 exerted by dm. The resultant force that the entire disk exerts on m_1 is F_d and equals the double integral of (13) over the entire disk, see (14). The inner integral $\int_0^{2\pi} Gm_1S_2dm$ is the force exerted on m_1 by one elementary ring of material. The outer integral $\int_{Ra}^{Rb}(\int_0^{2\pi} Gm_1S_2dm)$ runs from R_a to R_b, with R_a being inner radius of the disk and R_b the outer radius of the disk, see Figure 5.

Using (4) ${R_3}^2 = (R_2\boldsymbol{e}_2 - R_1\boldsymbol{e}_1)^2$ $= {R_2}^2 + {R_1}^2 - 2R_2R_1\boldsymbol{e}_2 \cdot \boldsymbol{e}_1$	(7)
Using (5) in (7) ${R_3}^2 = {R_2}^2 + {R_1}^2 - 2R_2R_1\cos\theta$	(8)
Using (6) and (8) in (3) $dF_d = Gm_1dm\dfrac{R_2\cos\theta - R_1}{({R_2}^2 + {R_1}^2 - 2R_2R_1\cos\theta)^{\frac{3}{2}}}$	(9)
Definition $S_2 = \dfrac{R_2\cos\theta - R_1}{({R_2}^2 + {R_1}^2 - 2R_2R_1\cos\theta)^{\frac{3}{2}}}$	(10)

Definition $\delta = \dfrac{R_1}{R_2}, \qquad \sigma = \dfrac{R_2}{2R_1} + \dfrac{R_1}{2R_2}$	(11)
Using (11) in (10) $S_2 = \dfrac{\delta^{\frac{1}{2}}}{2^{\frac{3}{2}}{R_1}^2}\dfrac{\cos\theta - \delta}{(\sigma - \cos\theta)^{\frac{3}{2}}}$	(12)
$dF_d = Gm_1S_2dm$	(13)
$F_d = \iint dF_d = \int_{Ra}^{Rb}\left(\int_0^{2\pi} Gm_1S_2dm\right)$	(14)

$ds_R = 2\pi R\, dR$	(15)
$dm_R = \rho\, ds_R$ $= \rho\, 2\pi R\, dR$ ρ: mass density	(16)
$dm = \rho\, Rd\theta\, dR$	(17)

The surface of an elementary ring of radius R is expressed in (15). The mass of the elementary ring is dm_R and is expressed in (16), with ρ being the surface mass density of the ring. We suppose that the ring is symmetrical about the center M, and its mass density is constant. The elementary mass dm used in the integral (14) is expressed in (17). Using (16) in

Using (16) in (17) $dm = \frac{\rho\, 2\pi R dR\, d\theta}{2\pi} = \frac{dm_R}{2\pi} d\theta$	(18)	Using (18) in (14) $F_d = \int_{Ra}^{Rb}\int_0^{2\pi}\left(Gm_1 S_2 \frac{dm_R}{2\pi} d\theta\right) = \frac{Gm_1}{2\pi}\int_{Ra}^{Rb}\left(\int_0^{2\pi} S_2 d\theta\right) dm_R$	(19)	Using (12) in (19) $F_d = \frac{Gm_1}{2\pi}\int_{Ra}^{Rb}\left(\int_0^{2\pi}\frac{\delta^{\frac{1}{2}}}{2^{\frac{3}{2}}{R_1}^2}\frac{\cos\theta-\delta}{(\sigma-\cos\theta)^{\frac{3}{2}}} d\theta\right) dm_R$	(20)

(17), the elementary mass dm is expressed with the mass of an elementary ring dm_R in (18). Then, introducing (18) into (14) the force F_d is expressed in (19). Using (12) in (19), F_d is expressed in (20).

$F_1 = F_M + F_d$	(21)
$F_1 = -m_1 \frac{v^2}{R_1}$	(22)
$F_M = -\frac{GMm_1}{{R_1}^2}$	(23)

The balance of forces on m_1 is (21), with F_1 being the inertial force of m_1 which moves at the orbital velocity v in the circular orbit of radius R_1, see (22), and F_M being the gravitational force that the mass of the central region of the galaxy M exerts on m_1, see (23). Using (22) and (23) in (21), the balance of forces is expressed in (24).

$\frac{R_1 v^2}{GM} = 1 - \frac{{R_1}^2}{GMm_1} F_d$	(24)
$GM = R_a {v_a}^2$ R_a: inner radius of the disk v_a: Single-Orbit velocity at R_a See (1)	(25)
Left hand member of (24) $\frac{R_1 v^2}{GM} = \frac{R_1 v^2}{R_a {v_a}^2}$	(26)

We will express the balance of forces in dimensionless form. For doing so, we express GM in terms of the Single-Orbit velocity at the inner radius of the disk R_a, see (25) and (1). Then, using (25), the left hand member of (24) becomes (26).

Definition: $\frac{R_b}{R_a}$ is the shape ratio of a disk with R_a being the inner radius of the disk and R_b the outer radius of the disk.

Definition $\beta = \frac{v^2}{{v_a}^2} \Rightarrow \frac{v}{v_a} = \sqrt{\beta}$	(27)
Using (27) in (26) $\frac{R_1 v^2}{GM} = \beta \frac{R_1}{R_a}$	(28)

We define the ratio β in (27) which is used to transform (26) into (28). Using (20) and (28) in (24) we obtain the dimensionless balance of forces in (29). The outer integral of this equation goes from the inner edge to the outer edge of the disk, that is, from the dimensionless inner radius $\frac{R}{R_a} = 1$ to the dimensionless outer radius $\frac{R}{R_a} = \frac{R_b}{R_a}$, and the dimensionless balance of forces is expressed in (30).

Using (20) and (28) in (24) $\beta\frac{R_1}{R_a} + \frac{{R_1}^2}{GMm_1}\frac{Gm_1}{2\pi}\int_{Ra}^{Rb}\left(\int_0^{2\pi}\frac{\delta^{\frac{1}{2}}}{2^{\frac{3}{2}}{R_1}^2}\frac{\cos\theta-\delta}{(\sigma-\cos\theta)^{\frac{3}{2}}} d\theta\right) dm_R = 1$	(29)
Outer integral of (29) from 1 to $\frac{R_b}{R_a}$ $\beta\frac{R_1}{R_a} + \int_1^{\frac{R_b}{R_a}}\left(\int_0^{2\pi}\frac{\delta^{\frac{1}{2}}(\cos\theta-\delta)}{(\sigma-\cos\theta)^{\frac{3}{2}}} d\theta\right)\frac{dm_R}{2^{\frac{5}{2}}\pi M} = 1$	(30)

In equation (30), $\frac{dm_R}{M}$ is the dimensionless mass of a ring and β the dimensionless orbital velocity squared. So, the equation (30) relates the masses of all the rings with the orbital velocity of an object in the disk. Solving the equation (30) will give the orbital velocity at any radius in the disk. But (30) cannot be solved analytically. So, we solve it numerically.

The solution of equation (30) for 4 shapes of disk are represented by the orbital velocity curves plotted in Figure 7 with Single-Orbit velocity as reference. These curves fit pretty well the observed curves shown in Figure 1.

4. Numerical resolution

We split the disk into n discrete rings. The mass of the j^{th} ring is m_j for j running from 1 to n. The radius of a ring is that of the middle circle, see R_j and m_j in Figure 6.

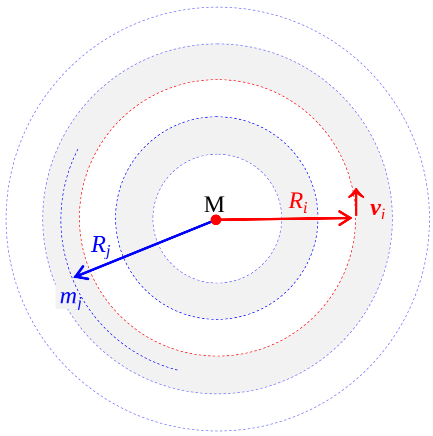

Figure 6: Discretization of a disk into rings.

The orbiting mass m_1 is on a border of a ring whose radius is R_i, its velocity is v_i, see R_i and v_i in Figure 6. The n rings have $n+1$ borders, so i runs from 0 to n. The balance of forces (30) for the i^{th} border is equation (31). So, $n+1$ borders make $n+1$ equations (31), which form a linear system of $n+1$ equations.

Discretization of (30) for $i = 0 \to n$ $\beta \frac{R_i}{R_a} + \sum_{j=1}^{n} \left(\int_0^{2\pi} \frac{\delta^{\frac{1}{2}}(\cos\theta - \delta)}{(\sigma - \cos\theta)^{\frac{3}{2}}} d\theta \right) \frac{m_j}{2^{\frac{5}{2}}\pi M} = 1$ m_j: mass of the j^{th} ring	(31)

In case where the velocity v_i is constant we have only one β for all i. The masses m_j of the n rings are n unknowns. So, we have n unknown m_j and one unknown β which make the $n+1$ unknowns for the linear system of $n+1$ equations (31), which can be solved.

Equation (32) is the matrix form of the linear system (31). The elements of the matrix $[A_{ij}]$ are given in (33) and (34). In the first column where $j=0$, the elements A_{i0} are the coefficients of β in (31). The elements A_{ij} of the other column are given in (34), see (31), which means that the unknowns for this system are $X_j = \frac{m_j}{2^{\frac{5}{2}}\pi M}$ rather than m_j, see (35).

Matrix form of the linear system (31) $[A_{ij}][X_j] = [1]$	(32)
Elements of the matrix $[A_{ij}]$, see (31) $\begin{cases} i = 0 \to n \\ j = 0 \end{cases}, A_{i0} = \frac{R_i}{R_a}$	(33)
$\begin{cases} i = 0 \to n \\ j = 1 \to n \end{cases}, A_{ij} = \int_0^{2\pi} \frac{\delta^{\frac{1}{2}}(\cos\theta - \delta)}{(\sigma - \cos\theta)^{\frac{3}{2}}} d\theta$	(34)
The unknowns for (32), see (31) $j = 0, \quad X_0 = \beta$ $j = 1 \to n, X_j = \frac{m_j}{2^{\frac{5}{2}}\pi M}$	(35)

Then, solving the linear system (31) will give X_j for $j=1 \to n$ and β which is X_0. Once the X_j are computed, the dimensionless masses of the rings $\frac{m_j}{M}$ are computed using (36).

The total mass of the disk is M_{disk} and the dimensionless mass of the disk equals the sum of all $\frac{m_j}{M}$ which is the ratio τ computed in (37). The value of $\frac{M_{disk,j}}{M}$ in (38) is the dimensionless mass of the inner part of the disk for radius smaller than R_j. The dimensionless mass density of the j^{th} ring is $\frac{\rho}{M}$ and is computed in (39), see (16) and (36).

$\frac{m_j}{M} = 2^{\frac{5}{2}}\pi X_j$ See (35)	(36)
$\tau = \frac{M_{disk}}{M} = \frac{\sum m_j}{M}$ $= 2^{\frac{5}{2}}\pi \sum_1^n X_j$ See (36)	(37)
$\frac{M_{disk,j}}{M} = 2^{\frac{5}{2}}\pi \sum_1^j X_j$ See (36)	(38)

The numerical resolution is quite simple and needs less than 30 lines of code in Scilab which can run on PC. We have used a discretization of 1600 angles θ and 200 rings.

$\frac{\rho}{M} = \frac{1}{M}\frac{dm_R}{ds_R} \approx \frac{m_j}{M}\frac{1}{2\pi R_2 \Delta R_2}$ $= \frac{2^{\frac{3}{2}} X_j}{R_2 \Delta R_2}$ See (16) and (36)	(39)

5. Numerical result

a. Velocity and mass density

We have computed for 4 shapes of disk which are characterized by the following shape ratios: $\frac{R_b}{R_a}$=10, 20, 40 and 80. The dimensionless velocity is $\frac{v}{v_a}$, see (27). The 4 curves of $\frac{v}{v_a}$ are plotted versus radial distance R in Figure 7, where we have plotted the Single-Orbit velocity $\frac{v_s}{v_a}$ as reference, see (40). These 4 curves are so close that they coincide in Figure 7. We see that these 4 curves resemble the observed curves shown in Figure 1 in that they have a bump on the left and are constant on the right. The constant part is well bigger than the Single-Orbit velocity like the observed rotation velocities of galaxies in Figure 1.

$\frac{v_s}{v_a} = \sqrt{\frac{R_a}{R_s}}$ See (1)	(40)

The computed orbital velocity decreases sharply when the dimensionless radial distance is 1, see the shaded part in Figure 7. This is because the central region of the galaxy is not in the model. For model that takes the central region into account, the computed orbital velocity would go smoothly to zero at the center of the galaxy.

The dimensionless mass density is $\frac{\rho}{M}$, see (39). The 4 curves of $\frac{\rho}{M}$ are plotted versus R in Figure 8. These curves fit well the observed mass density of the Milky Way galaxy shown in Figure 9. The 4 curves of dimensionless mass of the inner part of the disks $\frac{M_{disk,j}}{M}$ are plotted versus R in Figure 10, see (38). These 4 curves increase from 0 to the maximums at the outer edge of the disks, which are the total dimensionless masses of the disks.

We give the computed masses of disk for 5 shape ratios in Table 1. For $\frac{R_b}{R_a}$=10, the total mass of the disk weights 3.3 times the mass of the central region M, while for $\frac{R_b}{R_a}$= 1000 the disk weights 416 times M. So, the total dimensionless mass of a disk increases as its shape ratio increases. For large shape ratio, the mass of the central region of a galaxy is small with respect to the mass of the disk. This could explain the observed case of galaxy which misses supermassive black hole in the center, see the example of the galaxy in the center of the galaxy cluster Abell 2261 which is enormous [2].

Shape ratio $R_b\,/R_a$	10	20	40	80	1000
Mass of disk $M_{disk}\,/M$	3.3	7.4	16	32	416
Orbital velocity v/v_a	0.82	0.81	0.81	0.81	0.81

Table 1

In Table 1, we also give the dimensionless orbital velocities in the disks. For all shapes the orbital velocities are about 0.81 times the Single-Orbit velocity at the inner radius of the disks. As R increases, the Single-Orbit velocity decreases rapidly to smaller than 0.81 while the orbital velocities stay constant and become much bigger than Single-Orbit velocity, see Figure 7.

The Milky Way galaxy has a diameter of about 150 000 to 200 000 light-years, its central bulge measures 10 000 light-years across [3]. So, the shape ratio of the Milky Way galaxy is about 20, which correspond to the second column of Table 1 and the mass of the disk would weights 7.4 times that of its bulge. The orbital velocity of its disk is 0.81 times the Single-Orbit velocity at the inner

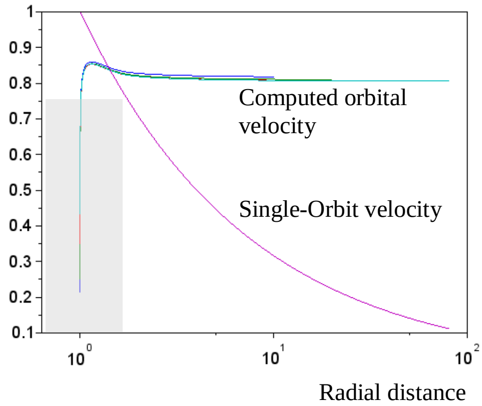

Figure 7: Orbital velocity for R_b/R_a=10, 20, 40,80

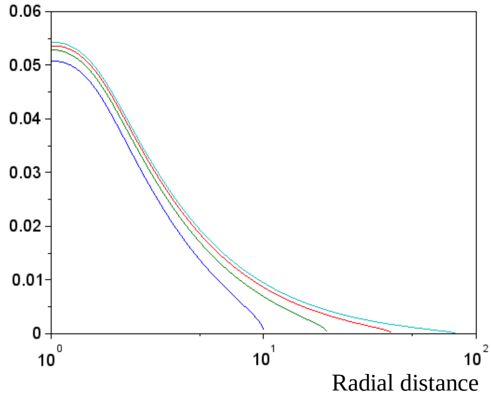

Figure 8: Mass density of rings for R_b/R_a =10, 20,40,80

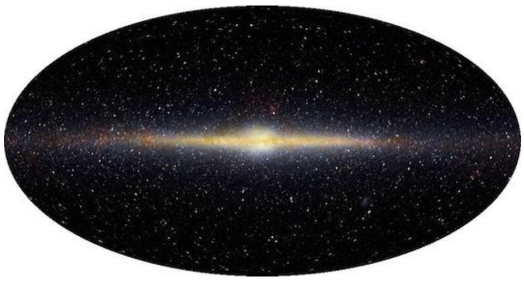

Figure 9: Cross section of the Milky Way galaxy [5]

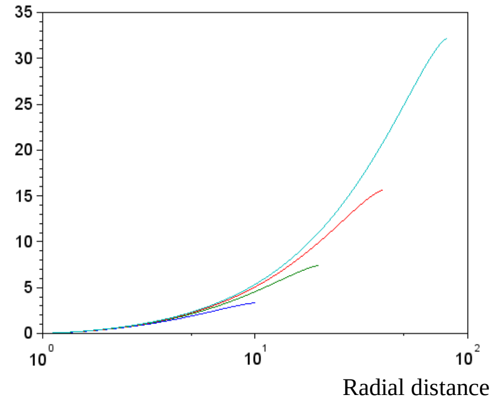

Figure 10: Mass of inner disk for R_b/R_a =10, 20,40,80

radius of the disk.

b. Technique of resolution

We do the computation in 2 steps. In the first the step, orbital velocity is kept constant but with unknown value. For a given shape ratio $\frac{R_b}{R_a}$, we solve the linear system (31) and obtain the value of the orbital velocity. We plot the obtained dimensionless mass density in Figure 11, see (39). This curve does not fit the real mass density of galaxy, see Figure 9. The difference between curve in Figure 11 and the mass density of real galaxy is that the computed curve decreases from the maximum to zero as the dimensionless radial distance approaches 1. This unnatural decrease is caused by the boundary condition that the disk abruptly stops at its inner radius, $\frac{R}{R_a}=1$, while real galaxies do not.

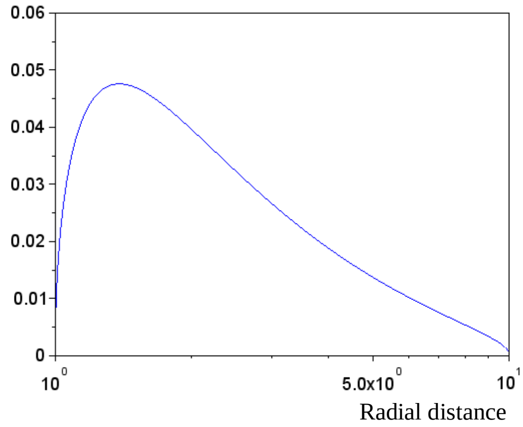

Figure 11: Mass density obtained with constant orbital velocity

So, in the second step we modify the mass density curve to fit the real mass density by making the curve to increase to the maximum for $\frac{R}{R_a}=1$, see Figure 12. The modified curve is the solid one whose right part is kept identical to the dashed one which is the curve obtained in the first step because this part makes the orbital velocity constant for large radials distance.

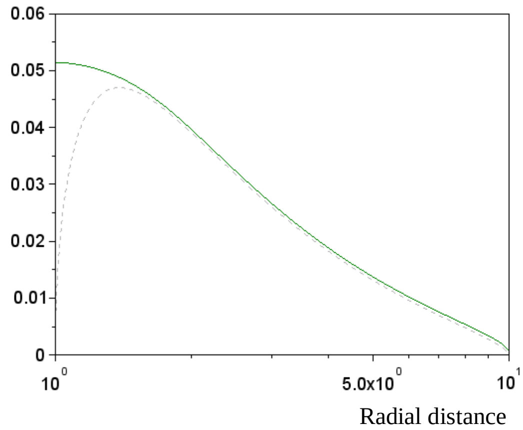

Figure 12: Modified mass density

For getting the correct dimensionless orbital velocity which is $\frac{v}{v_a}=\sqrt{\beta_i}$, we re-inject the modified mass density into the system (41) which is the system (31). $\frac{v}{v_a}$ are the curves plotted in Figure 7. These curves present a bump near the inner edge of the disks and are constant for large radial distance, which fit pretty well the observed orbital velocity shown in Figure 1.

Using (31)	
$$\beta_i\frac{R_i}{R_a}=1-\sum_{j=1}^{n}\left(\int_0^{2\pi}\frac{\delta^{\frac{1}{2}}(\cos\theta-\delta)}{(\sigma-\cos\theta)^{\frac{3}{2}}}d\theta\right)\frac{m_j}{2^{\frac{5}{2}}\pi M}$$	(41)

The 4 curves of dimensionless mass density plotted in Figure 8 are the modified mass density with which the 4 curves of velocity are computed. So, each curve of orbital velocity in Figure 7 and the corresponding mass density curve in Figure 8 make an exact solution to the system (31).

6. How to measure the masses of galaxies

The shape of a galaxy and the orbital velocity of its disk can be measured. Using the equations above, the total mass of the galaxy can be derived from these measurements just like the mass of the Sun is derived with the measured velocity of a planet and the radius of its orbit.

First, we measure the inner radius and the outer radius of the disk, the ratio of which is the shape ratio of the disk $\frac{R_b}{R_a}$ with which we solve the system (31). The solution will give the ratio β in (27) and the value of τ of the galaxy in (37).

β is the ratio $\frac{v^2}{v_a{}^2}$ with v being the value of the orbital velocity in the constant part of the curve and v_a the Single-Orbit velocity at the inner radius of the disk. So, we measure v and derive v_a with β using (42). We derive the mass of the central region of the galaxy M with v_a in (43), see (25). The mass of the disk M_{disk} equals $M{\cdot}\tau$, see (37).

The total mass of the galaxy equals the sum of the mass of the central region and that of the disk: $M + M_{disk}$. Finally, the total mass of the galaxy is given in (44).

$v_a{}^2 = \frac{v^2}{\beta}$ See (27)	(42)
$M = \frac{R_a v_a{}^2}{G}$ See (25)	(43)
$M + M_{disk} = (1+\tau)\frac{R_a v_a{}^2}{G}$ See (37)	(44)

This measurement of the mass of a galaxy is independent from the classical estimation of mass using mass-luminosity ratio [0]. Because $M + M_{disk}$ is computed using the system (31) which is the balance of force, $M + M_{disk}$ is the mass that maintains gravitationally the orbital velocity of the galaxy, which encompass the mass of stars, dust, gas, planets, black holes and all real material in the galaxy. That is, $M + M_{disk}$ is the mass of real matter only and does not contain dark matter because the system (31) does not use dark matter.

7. Discussion

By applying Newton's law of gravitation to material disk, we were able to compute the orbital velocity in the disks of galaxies and their mass densities. As these results fit pretty well observation, our model of material disk for galaxy can explain the observed rotation curves of disk galaxies without dark matter. So, our model seems to rule out dark matter for explaining rotation curves of disk galaxies.

Our model gives a new way to measure the real gravitational mass of a galaxy. Since the ratio τ is bigger than 1, the masses of galaxies are not concentrated in their bulges as thought before, but are mostly in their disk.

The curve of mass density shows that the mass density at the outer edge of the disk tends to zero, but the curve of orbital velocity is constant in this region, which explains well that in the outskirt of galaxies gas still rotates around the center but there is no star in this region because the density of gas is too low to form star.

Our model gives the relation between the mass density and the orbital velocity, but it does not explain how orbital velocity manages to stay constant. Why is stellar material distributed in this particular way that makes the curve of orbital velocity flat? This is the next problem to solve.

Currently, the orbital velocity of galaxies can be measured accurately. So, we can compute the real mass density in the disk by introducing the curve of the observed velocity into the system (31) and compute the mass density of the rings. By comparing this mass density with the local brightness of the rings, we can derive the real mass-luminosity ratio of a galaxy. This measurement of mass-luminosity ratio would be reasonably reliable.

If we worked out the global mass-luminosity ratio of a galaxy and knowing that the mass of a galaxy is related to its shape, it would be possible to work out the shape-luminosity relation of galaxies. As the visual shape of a galaxy can be measured, it would be possible to measure the distance of the galaxy using this shape-luminosity relation, which would extend the cosmic distance ladder.

Our model is symmetrical about the center of galaxies and thus, is a one dimensional model. As galaxies are 3D objects, 2D and 3D models will give much better the orbital velocity in galaxies. In 2D model orbital velocity can vary in the angular direction. As the arms of galaxies are regions where the mass density is higher, orbital velocity would accelerate in entering the arms and slow down in leaving the arms. 3D model could give variation of orbital velocity in the vertical direction. We think that vertical velocity would oscillate at the frequency of one period per rotation in galaxies with 2 arms and the phase of this oscillation would shift backward in the radial direction such that the arms of galaxies tilt backward. The vertical velocity would be zero in the arms which are concentration of mass that stars orbit around. So, 3D model could provide explanation to the formation of the arms.

In conclusion, our model not only explains the rotation curves of galaxies, it could also ushers many new explanations of phenomena in galaxies.

References

1. https://en.wikipedia.org/w/index.php?title=Galaxy_rotation_curve&oldid=1012870049
2. https://www.nasa.gov/mission_pages/chandra/images/on-the-hunt-for-a-missing-giant-black-hole.html
3. https://en.wikipedia.org/w/index.php?title=Milky_Way&oldid=1015572980
4. https://mosphys.ru/indico/event/5/contributions/298/attachments/238/386/Vikman_ITEP_2020_III.pdf
5. https://imagine.gsfc.nasa.gov/science/objects/milkyway1.html
   https://ned.ipac.caltech.edu/level5/Sept14/Courteau/Courteau2.html