Explaining Oumuamua and Pioneer anomaly using Time relativity

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Warning: I have done a mistake in my previous article, which makes this article wrong. I will give the correct theory later. But I keep this erroneous part below for showing a faithful historical image of my publications.

Abstract: I find that in theory the weird Speed Boost of the interstellar object 'Oumuamua should be 0.217 mm/s above the prediction and that 'Oumuamua should slow down less than prediction, in proportion of which the difference is 4.28×10^{-8} near the Sun. For Pioneer anomaly I have computed the gap between real and predicted acceleration and found the value 8.70×10^{-10} which is very close to the observation $(8.74\pm1.33)\times10^{-10}$ m/s^2.

The mysterious interstellar object 'Oumuamua confuses scientists because of its Speed Boost, which is an excess of velocity with respect to the expected one. In the past, the manmade Pioneer spacecrafts were also found to deviate from expected Newtonian trajectory.

One thinks the velocity of 'Oumuamua is too high because it is faster than the expected velocity that the mass of the Sun allows. But if we have used a mass for the Sun slightly different from the real one, then the expected velocity would be not correct. So, let us see how the mass of the Sun is determined.

1. Solar mass

The mass of the Sun is not measured by weighting, but derived according to Newtonian gravitational theory and using the parameters of Earth's orbit which is nearly circular. Let r_M be the radius of the Earth's orbit around the Sun and u_E its orbital velocity. The orbital centripetal acceleration of the Earth is the α_E given in equation (1) which gives the centripetal force of the Earth expressed in terms of the mass of the Earth m_E, u_E and r_M in equation (2).

$$\alpha_E = \frac{{u_E}^2}{r_M} \tag{1}$$

$$F = m_E \alpha_E = m_E \frac{{u_E}^2}{r_M} \tag{2}$$

$$F = \frac{g M_0 m_E}{{r_M}^2} \tag{3}$$

$$\frac{{u_E}^2 m_E}{r_M} = \frac{g M_0 m_E}{{r_M}^2} \Rightarrow M_0 = \frac{r_M {u_E}^2}{g} \tag{4}$$

The Newtonian gravitational force of the Sun on the Earth is expressed in equation (3), with M_0 being the mass of the Sun and g the gravitational constant. By equating equation (2) with (3), we obtain M_0, the mass of the Sun that we use today, in equation (4).

However, we know that real gravitation is governed by general relativity, which gives slightly different gravitational force on the Earth. As proof, we know that the perihelion of the planet Mercury advances, which the Newtonian theory does not predict. So, the mass of the Sun should be computed using relativistic theory.

$$\alpha_R = \frac{g M_R}{{r_M}^2}\sqrt{1-\frac{u^2}{c^2}} = \frac{{u_E}^2}{r_M} \tag{5}$$

$$M_R = \frac{r_M {u_E}^2}{g\sqrt{1-\frac{{u_E}^2}{c^2}}} \tag{6}$$

$$\frac{M_R}{M_0} = \frac{1}{\sqrt{1-\frac{{u_E}^2}{c^2}}} \approx 1+\frac{{u_E}^2}{2c^2} > 1 \tag{7}$$

$$u_E = 29.8 \text{ km/s} \Rightarrow$$

$$\frac{M_R}{M_0} - 1 \approx \frac{{u_E}^2}{2c^2} = 4.94\times10^{-9} \tag{8}$$

Let M_R be the real mass of the Sun and α_R the orbital centripetal acceleration of the Earth due to M_R. I have derived the expression of α_R in terms of M_R in the equation (33) of « Relativistic kinematics and gravitation », which is given in equation (5) and is equated with equation (1). Then, the real mass of the Sun is derived from equation (5) and expressed in equation (6) in terms of u_E and r_M.

By dividing equation (6) with (4), we obtain the ratio M_R/M_0 in equation (7), which shows that M_R is bigger than M_0, that is, the mass of the Sun used today is smaller than its real mass M_R. The relative gap between M_R and M_0 is defined as $\frac{M_R}{M_0} - 1$, of which the numerical value is computed in equation (8) by using the orbital velocity of the Earth which is 29.8 km/s, and is found to be 4.94×10^{-9}.

Since the real mass of the Sun is bigger than M_0, the real velocity of 'Oumuamua is higher. So, the Speed Boost of 'Oumuamua was unexpected not because it flies too fast, but because the predicted velocity is incorrect.

2. Anomalies in space

a) ‘Oumuamua

- Speed Boost

The interstellar object ‘Oumuamua was discovered on Oct. 19, 2017, which is 40 days after it has passed the perihelion on Sept. 9, 2017 at the velocity 87.71 km/s. It goes a tiny bit faster than it should have if the only effect on its motion was the Sun’s gravity, which is not predicted by current theory [1][2][3][4][5][6]. But this Speed Boost is predicted by Time relativity.

$$u_R{}^2 = \alpha_R r_c \quad (9)$$

$$u_0{}^2 = \alpha_0 r_c \quad (10)$$

$$\frac{u_R{}^2}{u_0{}^2} = \frac{\alpha_R}{\alpha_0} \quad (11)$$

$$\alpha_R = kM_R \quad (12)$$

$$\alpha_0 = kM_0 \quad (13)$$

$$\frac{u_R{}^2}{u_0{}^2} = \frac{M_R}{M_0} \quad (14)$$

Let u_R be the real orbital velocity of ‘Oumuamua caused by M_R, which is expressed in equation (9) in terms of the real orbital acceleration α_R and r_c the radius of curvature of its orbit at the perihelion. With M_0, the orbital velocity would be u_0 which is expressed in equation (10) in terms of the corresponding orbital acceleration α_0. We divide equation (9) with (10) to obtain the ratio of the 2 velocities squared in equation (11).

$$u_R - u_0 = \frac{u_R{}^2 - u_0{}^2}{u_R + u_0} = \frac{u_0{}^2}{u_R + u_0}\left(\frac{u_R{}^2}{u_0{}^2} - 1\right) \quad (15)$$

The gravitational acceleration is proportional to the mass of the Sun, so, α_R and α_0 can be written as in equations (12) and (13). Introducing equations (12) and (13) in to (11), we get equation (14).

$$u_R \approx u_0 \Rightarrow u_R + u_0 \approx 2u_0 \quad (16)$$

$$u_R - u_0 \approx \frac{u_0}{2}\left(\frac{M_R}{M_0} - 1\right) \quad (17)$$

The difference of velocity $u_R - u_0$ is derived in equation (15). Because the values of u_R and u_0 are very close, we can write $u_R + u_0 \approx 2u_0$ in equation (16), which is introduced along with equation (14) in to equation (15) to get equation (17). Using the numerical values of u_0 and $\frac{M_R}{M_0} - 1$, we find the value of difference of velocity $u_R - u_0 = 0.217$ mm/s in equation (18).

Perihelion

$$u_0 = 87.71 \text{ km/s} \Rightarrow u_R - u_0 = 0.217 \text{ mm/s} \quad (18)$$

If the observed Speed Boost [1][2][3][4][5][6] confirms this theoretical value, then it is caused by a bigger mass of the Sun.

- Resistance to slowdown

‘Oumuamua slows down under the Sun’s gravity, but not as fast as predicted by celestial mechanics [1][2][3][4][5][6]. I call this phenomenon Resistance to slowdown effect, which is predicted by Time relativity because, as relativistic mass is bigger than rest mass, the fast moving ‘Oumuamua carries more kinetic energy than Newtonian theory predicts. So, ‘Oumuamua looks moving slower when carrying the kinetic energy that Newtonian theory predicts.

$$E_N = \frac{m_0 u_N{}^2}{2} \quad (19)$$

$$u_N^{*\,2} - u_N{}^2 = \frac{2}{m_0}(E_N^* - E_N) \quad (20)$$

Let u_N and E_N be the Newtonian velocity and kinetic energy of \`Oumuamua. E_N is expressed in equation (19). When it slows down to the velocity u^*_N, its kinetic energy is E^*_N and the decrease of velocity squared is expressed in equation (20).

$$E_R = m_0 c^2\left(1 - \sqrt{1 - \frac{u_R{}^2}{c^2}}\right) \quad (21)$$

$$\left(1 - \frac{E_R}{m_0 c^2}\right)^2 = 1 - \frac{u_R{}^2}{c^2} \quad (22)$$

The real kinetic energy carried by ‘Oumuamua moving at the real velocity u_R is E_R. According to Time relativity, E_R is given in equation (21), which is the equation (44) in « Velocity, mass, momentum and energy of an accelerated object in relativity ». When ‘Oumuamua slows down to the velocity u^*_R, its kinetic energy is E^*_R. Equation (21) is transformed into equation (22) for deriving the decrease of velocity squared, see equations (23) to (25).

$$\left(1 - \frac{E_R^*}{m_0 c^2}\right)^2 - \left(1 - \frac{E_R}{m_0 c^2}\right)^2 = \left(1 - \frac{u_R^{*\,2}}{c^2}\right) - \left(1 - \frac{u_R{}^2}{c^2}\right) \quad (23)$$

$$c^2\left(1 - \frac{E_R^*}{m_0 c^2} - 1 + \frac{E_R}{m_0 c^2}\right)\left(1 - \frac{E_R^*}{m_0 c^2} + 1 - \frac{E_R}{m_0 c^2}\right) = -u_R^{*\,2} + u_R{}^2 \quad (24)$$

$$u_R^{*\,2} - u_R{}^2 = \frac{E_R^* - E_R}{m_0}\left(2 - \frac{E_R^* + E_R}{m_0 c^2}\right) \quad (25)$$

Let us denote the differences of velocity squared in the form $\Delta(u^2)$, see equation (26).

We compute the ratio of these differences by dividing equation (25) with equation (20), which gives equation (27). Suppose that, when going from one point to another, ‘Oumuamua gains the quantity of potential energy ΔU, which equals the loss of kinetic energy, $E^*_R - E_R$ for Time relativity and $E^*_N - E_N$ for Newtonian theory. Because ΔU is the same for the 2 cases, we have $E^*_R - E_R = E^*_N - E_N$, see equation (28). Also, the values of E^*_R and E_R equal nearly the Newtonian kinetic energy, so we can write equation (29). Introducing equations (28) and (29) in to (27) we get equation (30) which is smaller than 1. So, the decrease of velocity for Time relativity is smaller than the Newtonian one, that is, ‘Oumuamua “resists to slowdown”.

$$\begin{aligned} {u^*_R}^2 - {u_R}^2 &= \Delta({u_R}^2) \\ {u^*_N}^2 - {u_N}^2 &= \Delta({u_N}^2) \end{aligned} \qquad (26)$$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)} = \frac{E^*_R - E_R}{E^*_N - E_N}\left(1 - \frac{E^*_R + E_R}{2m_0c^2}\right) \qquad (27)$$

Gain in potential energy ΔU

$$\left.\begin{aligned} \Delta U &= E^*_R - E_R \\ &= E^*_N - E_N \end{aligned}\right\} \Rightarrow \frac{E^*_R - E_R}{E^*_N - E_N} = 1 \qquad (28)$$

$$\begin{aligned} &E_R \approx m_0\frac{{u_R}^2}{2}, E^*_R \approx m_0\frac{{u^*_R}^2}{2} \Rightarrow \\ &\frac{E^*_R + E_R}{2m_0c^2} \approx \frac{{u^*_R}^2 + {u_R}^2}{4c^2} \end{aligned} \qquad (29)$$

$$\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)} \approx 1 - \frac{{u^*_R}^2 + {u_R}^2}{4c^2} < 1 \qquad (30)$$

We compute this effect for very small decrease of velocity, so we write equation (30) into (31). We differentiate $\Delta({u_R}^2)$ and $\Delta({u_N}^2)$ and simplify the ratio $\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)}$ into $\frac{\Delta u_R}{\Delta u_N}$ because the orbital velocity for Time relativity is very close to the Newtonian one, see equation (32). Then, equation (31) becomes equation (33), which quantifies the relative gap of decrease of velocity between Time relativity and Newtonian predictions.

$$\begin{aligned} &\Delta u_R \text{ small } {u^*_R}^2 \approx {u_R}^2 \Rightarrow \\ &\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)} - 1 \approx -\frac{{u_R}^2}{2c^2} \end{aligned} \qquad (31)$$

$$\begin{aligned} &u_R \approx u_N \Rightarrow \\ &\frac{\Delta({u_R}^2)}{\Delta({u_N}^2)} \approx \frac{2u_R\Delta u_R}{2u_N\Delta u_N} \\ &\qquad\approx \frac{\Delta u_R}{\Delta u_N} \end{aligned} \qquad (32)$$

$$\frac{\Delta u_R}{\Delta u_N} - 1 \approx -\frac{{u_R}^2}{2c^2} \qquad (33)$$

Let us call $\frac{\Delta u_R}{\Delta u_N} - 1$ Resistance to slowdown and compute its value in equation (34). It equals −4.28×10^{-8} at perihelion where the velocity of `Oumuamua is 87.71 km/s. In outer space where the Sun’s gravity is very weak, the velocity of `Oumuamua is 26.33 km/s [1] and its Resistance to slowdown is −3.86×10^{-9}. It will be interesting to compare the theoretical value −4.28×10^{-8} with the observation at perihelion [1][2][3][4][5][6].

Perihelion

$$\begin{aligned} &u_R = 87.71\ \text{km/s} \Rightarrow \\ &\frac{\Delta u_R}{\Delta u_N} - 1 = -4.28\times10^{-8} \end{aligned}$$

Outer space

$$\begin{aligned} &u_R = 26.33\ \text{km/s} \Rightarrow \\ &\frac{\Delta u_R}{\Delta u_N} - 1 = -3.86\times10^{-9} \end{aligned} \qquad (34)$$

I have also computed the gap between the total decreases of velocity for Time relativity and Newtonian theory, see equations (35) to (38), which shows that changing its velocity from 87.71 km/s to 26.33 km/s, `Oumuamua would move 3.1 mm/s faster than predicted in outer space.

Equation (30) ⇒

$$\begin{aligned} &\Delta({u_R}^2) - \Delta({u_N}^2) \\ &\approx -\frac{{u^*_R}^2 + {u_R}^2}{4c^2}\Delta({u_N}^2) \\ &= -\frac{{u^*_R}^2 + {u_R}^2}{4c^2}({u^*_N}^2 - {u_N}^2) \end{aligned} \qquad (35)$$

$$\begin{aligned} &u_R = u_N \Rightarrow \\ &\Delta({u_R}^2) - \Delta({u_N}^2) \\ &= ({u^*_R}^2 - {u_R}^2) \\ &\quad -({u^*_N}^2 - {u_N}^2) \\ &= {u^*_R}^2 - {u^*_N}^2 \\ &= (u^*_R - u^*_N)(u^*_R + u^*_N) \end{aligned} \qquad (36)$$

Equations (35) = (36)⇒

$$\begin{aligned} &u^*_R - u^*_N \\ &\approx -\frac{{u^*_R}^2 + {u_R}^2}{4c^2}\frac{{u^*_N}^2 - {u_N}^2}{u^*_R + u^*_N} \end{aligned} \qquad (37)$$

$$\left.\begin{aligned} u_N &\approx u_R \approx 87.71\ \text{km/s} \\ u^*_N &\approx u^*_R \approx 26.33\ \text{km/s} \end{aligned}\right\} \Rightarrow \qquad (38)$$

In outer space

$$u^*_R - u^*_N \approx 3.1\ \text{mm/s}$$

b) Pioneer anomaly

The Wikipedia page Pioneer anomaly [7] explains that while the spacecrafts Pioneer 10 and Pioneer 11 are escaping the Solar System, their navigational data show that they are slowing slightly more than expected, they get an additional extremely small acceleration towards the Sun which is Δα=(8.74±1.33)×10^{-10} m/s^2 [7][8][9][10][11][12][13][14][15][16].

Tangential acceleration

$$\alpha_R = \frac{\Delta u_R}{\Delta t}, \alpha_N = \frac{\Delta u_N}{\Delta t} \qquad (39)$$

Let us compute the Resistance to slowdown for Pioneer by using the tangential orbital acceleration which equals the change of magnitude of velocity Δu divided by time interval Δt, see equation (39) where the index R and N indicate Time relativity and Newtonian theory. We divide α_R with α_N in equation (40) which shows that the ratio of acceleration equals that of the change of velocity. Introducing equation (40) in to equation (33), we get equation (41).

Equation (32) ⇒

$$\frac{\alpha_R}{\alpha_N} = \frac{\Delta u_R}{\Delta u_N} \qquad (40)$$

Equation (31) ⇒

$$\frac{\alpha_R}{\alpha_N} - 1 \approx -\frac{{u_R}^2}{2c^2} \qquad (41)$$

Using the velocity of Pioneer 10 on December, 30, 2005 which is 12.51 km/s, equation (41) gives the Resistance to slowdown for Pioneer: *-8.70×10^{-10}*, see equation (42), which is amazingly close to the value of $\Delta\alpha$ given in the Wikipedia page Pioneer anomaly [7]: $(8.74\pm1.33)\times10^{-10}$ m/s^2.

$$u_R = 12.51 \text{ km/s}$$

$$\frac{\alpha_R}{\alpha_N} - 1 \approx -8.70 \times 10^{-10} \tag{42}$$

However, there is a glitch: the quantity $\frac{\alpha_R}{\alpha_N} - 1$ is dimensionless while $\Delta\alpha$ has the dimension m/s^2. I suspect that there may be more than a pure coincidence.

$$\begin{aligned} &M=1.99\times10^{30} \text{ kg} \\ &g=6.67\times 10^{-11} \text{ m}^3/\text{kg/s}^2 \\ &u_E=29.8 \text{ km/s} \\ &r_M=89.7 \text{ AU} \end{aligned} \tag{43}$$

There is a second glitch. $\frac{\alpha_R}{\alpha_N} - 1 < 0$ means that α_R is smaller than α_N in magnitude. Because the tangential acceleration points towards the Sun while the velocity away from the Sun, a smaller acceleration means that the spacecrafts are less pulled towards the Sun. So, the resistance to slowdown effect makes the spacecrafts to slow less than expected rather than "*slowing slightly more than expected*". Also, because the thermal radiation pressure proposed by the cited papers [7][8][9][10][11][12][13][14][15][16] pushes towards the Sun it acts against the resistance to slowdown effect.

$$\begin{aligned} |\alpha_0| &= \frac{gM}{{r_M}^2} \\ &= 7.37\times10^{-7} \text{m}/s^2 \end{aligned} \tag{44}$$

Equation (12) and (13) $\Rightarrow$

$$\frac{\alpha_R}{\alpha_0} = \frac{M_R}{M_0} \tag{45}$$

$$\frac{\alpha_R}{\alpha_0} - 1 \approx 4.94\times10^{-9} \tag{46}$$

$$\begin{aligned} |\alpha_R| - |\alpha_0| &\approx 4.94\times10^{-9}|\alpha_0| \\ &= 3.64\times10^{-15} \text{m}/s^2 \end{aligned} \tag{47}$$

Anyway, because the resistance to slowdown effect is observed with `Oumuamua and the theoretical value *-8.70×10^{-10}* is so close to the observational value $(8.74\pm1.33)\times10^{-10}$ m/s^2, it will be very interesting to look again into the navigational data of Pioneer 10 and Pioneer 11.

Just for comparison, the total gravitational acceleration from the Sun is 7.37×10^{-7} m/s^2, see equation (44). Due to Speed Boost effect (bigger solar mass, see section 2. a) 'Oumuamua •Speed Boost) the relative increase of gravitational acceleration is 4.94×10^{-9}, see equation (46). The increased acceleration is 3.64×10^{-15} m/s^2 towards the Sun, see equation (47).

c) Flyby anomaly

Flyby is a common gravity assist maneuver to accelerate a spacecraft using the gravity of the Earth. It was noticed that in multiple cases, spacecrafts have been observed to gain more speed than scientists had predicted. This phenomenon is called flyby anomaly [17][18][19][20][21].

This gain in speed may have the same origin than the Speed Boost of 'Oumuamua, except that the attracting body is the Earth. And indeed, the mass of the Earth was derived using the orbit of the Moon of which the velocity is 1.022 km/s, which makes the mass of the Earth ($1+ 5.81\times10^{-12}$) times more massive than the today's used value, see section 1 "Solar mass".

3. Planets and Parker Solar Probe

The above cases of speed boost make reevaluating the mass of the Sun important. For doing so, we can compute the quantity $\mu = \alpha \cdot {r_M}^2$ for all the planets, with α being the orbital acceleration of the planet and r_M its distance from the Sun, α and r_M being instantaneous values.

If μ is the same for all the planets, then the mass of the Sun is consistent with Newtonian theory as given in equation (48). If μ varies according to equation (49) (see equation (5)), with i indicating instantaneous orbital values for the i^{th} planet, then the mass of the Sun is now undervalued and μ will give the real mass of the Sun.

Newtonian

$$\mu = \alpha \cdot {r_M}^2 = gM \tag{48}$$

Time relativity

$$\mu = \alpha \cdot {r_{M,i}}^2 = gM\sqrt{1 - \frac{{u_i}^2}{c^2}} \tag{49}$$

The orbital data of Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune exist in the archives, but I doubt that they are sufficiently precise for this work. So, new measurement is necessary.

To quickly obtain high precision orbital data around the Sun, the navigation data of the Parker Solar Probe are a good source because they are precise measurement and its orbit is very elongated, which makes the value of r_M and u to vary in a large range.

Also, the orbit of the Parker Solar Probe is suitable for verifying the Resistance to slowdown effect. Indeed, its orbital velocity and tangential acceleration vary in a large range.

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`Oumuamua

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