# Length, distance and Michelson–Morley experiment

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**Abstract**: There are 2 types of length contraction: Object contraction and Distance contraction. Each has a different physical meaning.

## 1. Distance contraction

There are 2 types of length contraction, the physical meaning of each is explained below with the help of the example shown in Figure 1. The Earth and the star are stationary and the spaceship mobile. The 2 types of length contraction are:

1) Object contraction: In the frame of the Earth the length of the moving spaceship appears shorter than its proper length.
2) Distance contraction: In the frame of the spaceship the distance from the star to the Earth appears shorter than in the frame of the Earth.

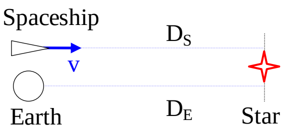

Figure 1

For Object contraction, the shorter length is in the frame of the Earth while for Distance contraction, the shorter distance is in the frame of the spaceship, see annex at the end of this article and section 3 of «Analysis of Einstein's derivation of the Lorentz Transformation».

### a) Measurement of distance by travel

Let us focus on Distance contraction and see why distance is shorter in the moving frame. Suppose that the spaceship coincides with the Earth at time zero and coincides with the star at the end of its travel. The spaceship moves at the velocity v in the frame of the Earth and the star moves at the velocity –v in the frame of the spaceship.

In the frame of the Earth time is $t_E$ at the end of the travel and the spaceship has traveled the distance $v\cdot t_E$ which is the distance from the Earth to the star measured by the travel of the spaceship. This distance is denoted by $D_E$ and expressed by equation (1).

In the frame of the spaceship time is $t_S$ at the end of the travel and the star has traveled the distance $v\cdot t_S$ which is the distance from the Earth to the star measured by the travel of the spaceship. This distance is denoted by $D_S$ and expressed by equation (2). The ratio of $t_S$ to $t_E$ equals $\sqrt{1-\frac{v^2}{c^2}}$, so the ratio of $D_S$ to $D_E$ is also $\sqrt{1-\frac{v^2}{c^2}}$, see equations (3) and (4).

### b) Measurement of distance by light

$$D_E = vt_E \tag{1}$$

$$D_S = vt_S \tag{2}$$

$$\frac{t_S}{t_E} = \sqrt{1-\frac{v^2}{c^2}} \tag{3}$$

$$\frac{D_S}{D_E} = \sqrt{1-\frac{v^2}{c^2}} \tag{4}$$

$$D_O = ct_O \tag{5}$$

$$D_B = ct_B \tag{6}$$

In order to explain Michelson–Morley experiment which uses light, we will compute the distances measured by light. Figure 2 shows the scheme of the measurement. At the moment when the spaceship coincides with the star, a light signal emitted by Earth hits them. In the frame of the Earth, the duration of the signal's travel is $t_O$ and the distance from the Earth to the star equals $c\cdot t_O$, which is $D_O$ expressed by equation (5). $D_O$ is the distance measured by light in the frame of the Earth.

In the frame of the spaceship, the duration of the signal's travel is $t_B$ and the distance from the Earth to the star which coincides with the spaceship equals $c\cdot t_B$, which is $D_B$ expressed by equation (6). $D_B$ is the distance measured by light in the frame of the spaceship, see Figure 2.

The duration of the signal's travel is $t_O$ in the frame of the Earth and $t_B$ in the frame of the spaceship. Because the ratio of $t_O$ to $t_B$ equals $\sqrt{1-\frac{v^2}{c^2}}$, the ratio of $D_O$ to $D_B$ equals also $\sqrt{1-\frac{v^2}{c^2}}$, see equations (7) and (8).

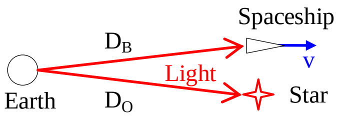

Figure 2

### c) Distance ratio in general

$D_E$ and $D_O$ are the distance from the Earth to the star. Because the star is fixed in the frame of the Earth, we call $D_E$ and $D_O$ "Fixed distances". On the other hand, $D_S$ and $D_B$ are the distance of the same space, but between 2 relatively mobile objects: the star and the moving spaceship for $D_S$ and the Earth and the moving spaceship for $D_B$. So, we call $D_S$ and $D_B$ "Mobile distances".

$$\frac{t_B}{t_O} = \sqrt{1 - \frac{v^2}{c^2}} \qquad (7)$$

$$\frac{D_B}{D_O} = \frac{ct_B}{ct_O} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{D_S}{D_E} \qquad (8)$$

Equation (4) is the ratios of the Mobile distance to the Fixed distance measured by travel, equation (8) is the ratios of the Mobile distance to the Fixed distance measured by light. Since equations (4) and (8) have the same value, we conclude that the ratio of the distances is a general law. This law allows us to derive the Mobile distance using equation (8) if the Fixed distance is known or vice versa. For example, if an object moving at v coincides with an immobile object which is at the Fixed distance $D_O$ from a third immobile object, then the Mobile distance from the moving object to the third object can be derived from $D_O$. Or inversely, if the moving object is at the Mobile distance $D_B$ from the third object, then $D_O$ can be derived from $D_B$.

## 2. Mobile distance in Michelson–Morley experiment

In Michelson–Morley experiment (see Figure 3) a vertical light signal $S_v$ does a round trip from the origin O to the vertical mirror $M_v$ and back to O. The signal $S_v$ takes the time $t_v$ to complete this trip, see equation (9). At the same time, a horizontal light signal $S_h$ does a round trip from the origin O to the horizontal mirror $M_h$ and back to O. The experiment shows that the time of the horizontal trip $t_h$ equals exactly $t_v$.

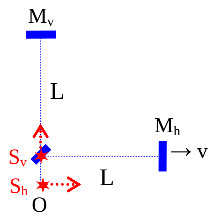


Figure 3

$$t_v = \frac{L}{c} \frac{2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (9)$$

Let us compute $t_h$ using Distance contraction instead of Lorentz transformation. During the forth trip the horizontal light signal S starts from the origin O and reaches the mirror M, see Figure 4. In the frame of M, S takes the time $t_L$ to travel the distance L between O and M, see equation (10). During this time, O has moved to the position O'. The distance from O' to M is denoted by $D_f$ which is expressed by equation (11).

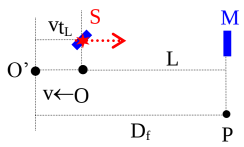


Figure 4

$$L = ct_L \qquad (10)$$

$$D_f = L + vt_L \qquad (11)$$

For the back trip, S begins from the mirror which is now at the origin O, see Figure 5. In the frame of M, S also takes the time $t_L$ to reach the light splitter M, see equation (10). During this time, O has moved to the position O'. The distance from O' to M is denoted by $D_b$ which is expressed by equation (12). $D_f$ and $D_b$ are represented by $D_M$ in equation (13) where $D_M = D_f$ when the sign ± is + and $D_M = D_b$ when the sign ± is -. Because M and O are relatively mobile, $D_M$ is the Mobile distance from M to O.

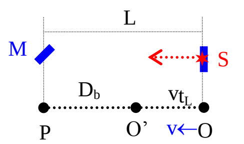


Figure 5

$$D_b = L - vt_L \qquad (12)$$

$$D_M = L \pm vt_L = \left(1 \pm \frac{v}{c}\right) L \qquad (13)$$

In the stationary frame, the point O' is the origin O. At the time when S hits the moving M, M coincides with a point P fixed in the stationary frame. The distance from O to P is the Fixed distance corresponding $D_M$ and is denoted by $D_O$. Using equation (8), we derive $D_O$ from $D_M$ in equation (14). The time that S takes to travel the distance $D_O$ is $t_O$, which is computed in equation (15).

$$D_O = \frac{D_M}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L\left(1 \pm \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (14)$$

$$t_O = \frac{D_O}{c} = \frac{L}{c} \frac{1 \pm \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (15)$$

$$t_h = t_f + t_b \qquad (16)$$

The total time of the horizontal round trip $t_h$ equals the sum of the time of the forth trip $t_f$ and that of the back trip $t_b$. Equation (15) equals $t_f$ when the sign ± is + and $t_b$ when the sign ± is -. Then, $t_h$ is computed in equation (16) and is found to be identical to $t_v$ which is given by equations (9). So, the $t_h$ computed using Distance contraction agrees with the Michelson–Morley experiment.

$$= \frac{L}{c}\left(\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{L}{c}\frac{2}{\sqrt{1-\frac{v^2}{c^2}}} = t_v$$

The theory of Distance contraction has been presented in section 4 of «From Michelson–Morley experiment to length contraction ».

## 3. Why Object contraction?

For Object contraction the length of a moving object shrinks in the stationary frame. How was Object contraction derived? Let us see Figure 6 which shows the horizontal arm of Michelson interferometer which moves at the velocity v.

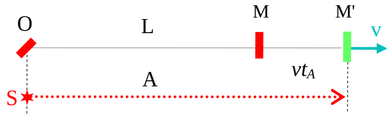


Figure 6

In the stationary frame the light signal S travels from the origin O to the mirror M which moves to the position M' when S hits it. The distance of the travel is denoted by A and lasts the time $t_A$. A is expressed in equation (17). The distance between M and M' is $vt_A$.

$$A = ct_A \tag{17}$$
$$A = L + vt_A \tag{18}$$
$$t_A = \frac{L}{c-v} \tag{19}$$
$$A = \frac{L}{1-\frac{v}{c}} \tag{20}$$
$$L' = L\sqrt{1-\frac{v^2}{c^2}} \tag{21}$$
$$t_B = \frac{L}{v_B - v} \tag{22}$$

On the other hand, A is the distance between O and M'. If the distance between O and M is L, A equals L plus $vt_A$ as shown in equation (18). Substituting $c \cdot t_A$ for A in equation (18), we obtain $t_A$ in equation (19). Combining equations (17) and (19) the distance A is obtained in equation (20). This computation can be seen in section 4-2 «Introduction to Special Relativity» by James H. Smith.

However, the so computed $t_A$ does not agree with Michelson–Morley experiment. In order to create a theoretical time that agrees with the experiment, $t_A$ has to be reduced by the factor $\sqrt{1-\frac{v^2}{c^2}}$, which is achieved by contracting the length of the horizontal arm from L to L', see equation (21). Object contraction is so created.

Why does not $t_A$ agree with the experiment? Because in the above derivation S was handled with classical laws while being light and obeying relativistic laws. Let us imagine that S is a ball moving at velocity $v_B$ toward the mirror, see Figure 6. The ball starts from O and hits the mirror at M', which lasts $t_B$ and is expressed in equation (22), which is the expression of $t_A$ except that c is replaced by $v_B$. So, $t_A$ is computed by taking c the speed of light like the velocity of a slowly moving ball. This is why the so computed $t_A$ is wrong and must be artificially corrected with the factor $\sqrt{1-\frac{v^2}{c^2}}$.

How to correctly compute the position of a light signal? Suppose a light signal S hits an object in a stationary frame. If the object is moving, the hitting point is not determined. If the object is fixed the hitting point is precisely the position of the object. Suppose that at time t S hits a fixed object located at P, then the position of S at time t is precisely P.

In section 2, the distance between the origin O and the position of S hitting the mirror M was not computed directly in the stationary frame where M is moving. Instead, I determined the hitting point between S and M in the frame of M, which is always M. Then I determined that the position of the origin at this moment is O' and the distance between the origin and M is $D_f$, see equation (11). $D_f$ is the Mobile distance because the origin and M are relatively moving.

In the stationary frame, the fixed point that coincides with the mirror when S hits is P, see Figure 4 and Figure 5. So, S has traveled from O to P in the stationary frame and the distance of this travel is denoted by $D_O$. Because P is fixed, $D_O$ is the Fixed distance corresponding to $D_f$ which is known.

Using the ratio of Mobile distance to Fixed distance given by equation (8), I derived $D_O$ from $D_f$ and computed $t_h$.

$t_h$ is computed by respecting relativistic principle and agrees with Michelson–Morley experiment, which validates the theory of Distance contraction. By the way, A' the length-contracted distance equals $D_O$ given by equation (14) when the sign ± is set to +, see equations (23). So, Object contraction is no longer useful.

$$A' = \frac{L'}{1-\frac{v}{c}} = \frac{L\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \frac{L\left(1+\frac{v}{c}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = D_O \quad (23)$$

## 4. Comments

Next, I will present a relativistic transformation of coordinates based on Distance contraction. My preliminary study indicates that this transformation explains relativistic momentum, $E=mc^2$ and creation of mass much better than Lorentz transformation does while solving all the discrepancies cited in section 3 of « Analysis of Einstein's derivation of the Lorentz Transformation ». Using this transformation, I also find that dark matter is in fact a relativistic phenomenon. I have put the main idea concerning dark matter in « Magnetism and dark matter ».

Reference

« Introduction to Special Relativity » by James H. Smith

# Annex

## Discrepancy of length contraction answer to

https://pengkuanonphysics.blogspot.com/2020/02/drawing-relativity.html
Discussion concerning « Length, distance and Michelson–Morley experiment »
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Object contraction is shown in figure 1. Frame $O_2$ is the Spaceship P moving at v with respect to frame $O_1$. Its length is $L_2$. In frame $O_1$ P is seen as P'. P is length contracted and its length is $L_1=L_2/\gamma$, shorter than $L_2$. Time is t=0

Distance contraction is shown in figure 2. Frame $O_2$ moves $L_1$ in frame $O_1$. Time is $t=L_1/v$. The backend of the spaceship is at $O_1$. In Frame $O_2$, $O_1$ moves $L_1/\gamma$, shorter than $L_1$. So, $O_1$ in Frame $O_2$ is at the position $O'_1=-L_1/\gamma=-L_2/\gamma^2$

But because the backend of the spaceship is at $O_1$, $O_1$ should coincide with the backend of the spaceship in Frame $O_2$. That is, at $O''_1=-L_2$

The question is, what is the position of $O_1$ in Frame $O_2$? Is it $-L_2$? Or $-L_2/\gamma^2$?

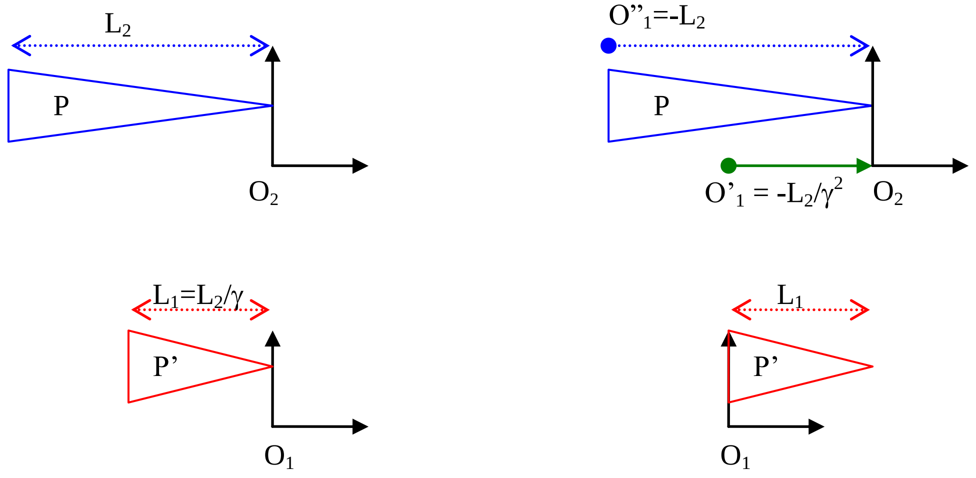


**Figure 1 t=o** **Figure 2 t=L1/v**