Time relativity transformation of coordinates

Kuan Peng 彭宽 *titang78@gmail.com*

First published: 3 March 2020, last corrected: 19 June 2020

Abstract: Time relativity transformation transforms coordinates without using length contraction; it explains paradoxes and inexplicable relativistic experiments.

Lorentz transformation is based on length contraction, which can be divided into 2 types: distance contraction and object contraction, see « Length, distance and Michelson–Morley experiment ». But these 2 types contradict each other and cause «Discrepancy of length contraction». In « Length, distance and Michelson–Morley experiment » I have perfectly explained Michelson–Morley experiment without length contraction, solving the discrepancy and allowing us to build a transformation of coordinates without length contraction.

1. Conservation of length

Let us see the example of a spaceship shown in Figure 1. The frame of reference frame 1 is stationary and its origin is O_1. At time zero, the spaceship is stationary, its backend is at O_1 and frontend at A_1. It is accelerated till time t_1 while its backend draws the curve O_1O_2 and its frontend draws the curve A_1A_2. The 2 curves are parallel because these 2 points follow the same acceleration.

At time t_1 the velocity of the spaceship is v, its backend at O_2 and frontend at A_2. O_2 is the origin of the frame 2 which moves with the spaceship at velocity v.

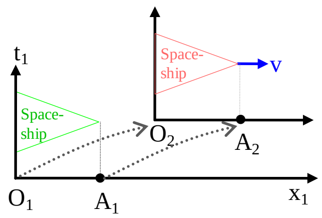

Figure 1

Indices 1 and 2 refer to frame 1 and 2. O_1 and O_2 are the origins of frame 1 and 2, A_1 and A_2 are front of the spaceship, v its velocity.

In frame 1, O_2A_2 is the length of the moving spaceship. Because the curves O_1O_2 and A_1A_2 are parallel, the distance O_2A_2 constantly equals the distance O_1A_1 which is the proper length of the spaceship. As the velocity of the moving spaceship is arbitrary, we conclude that for whatever velocity the length of a moving object constantly equals its proper length, contradicting the length contraction law of Special relativity. Let us name the conservation of length "Law of length conservation", which is stated in mathematical terms below:

Let Δx_2 be the length of a stationary object and Δx_1 its length when moving, Δx_1 always equals Δx_2, see equation (1).

$$\Delta x_1 = \Delta x_2 \tag{1}$$

2. Abscissa transformation

Let frame 1 and frame 2 be 2 frame of reference and frame 2 moves in frame 1. The origin of frame 1 and 2 are respectively O_1 and O_2, the abscissa of a point is x_1 in frame 1 and x_2 in frame 2, see Figure 2. Let us transform x_1 into x_2.

The proper length of the segment O_2x_2 is $\Delta x_2=x_2$, see equation (2). Its moving length in frame 1 is $\Delta x_1=x_1-x_O$, see equation (3). Because of the Law of length conservation, equation (1), O_2x_2 has the same length in frame 1 and 2. So, we equate equations (2) and (3) to obtain equation (4).

x_O is the abscissa of O_2 in frame 1 and equals the time integral of its velocity $v(t_1)$, see equation (5). Then, equation (4) becomes equation (6), which is the general transformation of abscissa from frame 1 to frame 2. If frame 1 and 2 are inertial, the velocity v is constant and the transformation of abscissa for inertial frames is equation (7).

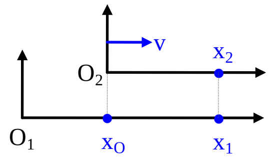

Figure 2

x_1 and x_2 are the abscissas of a point in frame 1 and 2

$$\Delta x_2 = x_2 - 0 = x_2 \tag{2}$$

$$\Delta x_1 = x_1 - x_O \tag{3}$$

$$x_2 = x_1 - x_O \tag{4}$$

$$x_O = \int_0^{t_1} v(t_1)dt_1 \tag{5}$$

$$x_2 = x_1 - \int_0^{t_1} v(t_1)dt_1 \tag{6}$$

$$x_2 = x_1 - vt_1 \tag{7}$$

x transformation from frame 1 to frame 2, t_1 is the time in frame 1.

3. Time transformation

The transformation of time from frame 1 to frame 2 is derived using a light clock like in Special relativity, which is shown in Figure 3. In frame 2 a light signal is sent from O_2 to the mirror M which reflects it back to O_2. The time light takes to complete the journey is $\Delta t_{O2}=2L_2/c$, with c being the speed of light and L_2 the distance from O_2 to M.

In frame 1, O_2 coincided with O_1 when the light signal was sent. The light signal is reflected by the moving mirror at M_1, then arrives at O_2. The distance from O_1 to M_1 is L_1 and the time of the journey $O_1M_1O_2$ is $\Delta t_{O1}=2L_1/c$. Because $L_2= L_1\sqrt{1-\frac{v^2}{c^2}}$, Δt_{O1} is related to Δt_{O2} through equation (8).

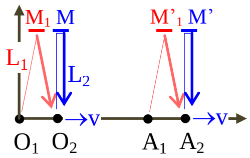

Figure 3

Light clock in frame 2. L_1 and L_2 are the paths of the same light in frame 1 and frame 2.

$$\Delta t_{O2} = \Delta t_{O1}\sqrt{1-\frac{v^2}{c^2}} \qquad (8)$$

$$\Delta t_{A2} = \Delta t_{A1}\sqrt{1-\frac{v^2}{c^2}} \qquad (9)$$

$$\Delta t_{A1} = \Delta t_{O1} \qquad (10)$$

$$\Delta t_{A2} = \Delta t_{O2} \qquad (11)$$

$$dt_2 = dt_1\sqrt{1-\frac{v^2}{c^2}} \qquad (12)$$

t_{O1} and t_{O2} are the time at the origin of frame 1 and frame 2, t_{A1} and t_{A2} the time at the point A_1 and A_2, t_1 and t_2 the time in frame 1 and frame 2.

Suppose that there is an identical light clock A_1 in frame 1, it is at A_2 in frame 2. In frame 1 a light signal will do the journey $A_1M'_1A_2$ in time $\Delta t_{A1}=2L_1/c$, the same light will do the journey $A_2M'A_2$ in time $\Delta t_{A2}=2L_2/c$, see Figure 3. So, Δt_{A1} is related to Δt_{A2} through equation (9), which is identical to equation (8).

In frame 1 the time t_{A1} equals the time t_{O1}, see equation (10). Because equations (8) and (9) are identical, t_{A2} equals t_{O2}, see equation (11), which proves that in frame 2 the time at A_2 is simultaneous with that at O_2. Because the point A_2 can be anywhere in frame 2, we conclude that there is only one time in frame 2 and that relativity of simultaneity defined in Special relativity does not exist. We define t_{O1} and t_{O2} as the time in frame 1 and 2 and denote them by t_1 and t_2.

Because the time intervals Δt_{O1} and Δt_{O2} can be arbitrary small, we will write Δt_{O1} and Δt_{O2} in differential form as dt_1 and dt_2 and equation (8) becomes equation (12). If the velocity of frame 2 is variable, t_2 the time in frame 2 equals the integral of equation (12), which gives equation (13) that expresses t_2 in terms of t_1. Equation (13) is the general transformation of time from frame 1 to frame 2. If v is constant, t_2 is expressed by equation (14).

Time transformation from frame 1 to frame 2.

$$t_2 = \int_0^{t_1}\sqrt{1-\frac{v(t_1)^2}{c^2}}\,dt_1 \qquad (13)$$

$$t_2 = t_1\sqrt{1-\frac{v^2}{c^2}} \qquad (14)$$

4. System of transformation

The general transformation of coordinates from frame 1 to frame 2 is the association of the transformation of abscissa, equation (6), with the transformation of time, equation (13), which makes the system (15). Because the transformation of time is relativistic while that of abscissa is not, I name the system (15) "Time relativity transformation of coordinates" and the theory based on it "Time relativity". If the velocity v is constant, the transformation of coordinates for inertial frames is the system (16).

Transformation of coordinates x and t.

$$\begin{cases} x_2 = x_1 - \int_0^{t_1} v(t_1)dt_1 \\ t_2 = \int_0^{t_1}\sqrt{1-\frac{v(t_1)^2}{c^2}}\,dt_1 \end{cases} \qquad (15)$$

$$\begin{cases} x_2 = x_1 - vt_1 \\ t_2 = t_1\sqrt{1-\frac{v^2}{c^2}} \end{cases} \qquad (16)$$

5. Reverse transformations

We have the transformation of coordinates for inertial frames from frame 1 to frame 2, the system (16). What is the transformation from frame 2 to frame 1? For doing so, we must distinguish 2 cases:

1. Frame 2 moves in frame 1

In this case, we use system (17) which is the inverse rearrangement of system (16).

Inverse transformation, which is the inverse rearrangement of equation (16)

$$\begin{cases} x_1 = x_2 + v\dfrac{t_2}{\sqrt{1-\frac{v^2}{c^2}}} \\ t_1 = \dfrac{t_2}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases} \qquad (17)$$

2. Frame 1 moves in frame 2

In this case, we use system (18) which is obtained by replacing v with –v and swapping x_1 and t_1 with x_2 and t_2 in system (16).

Back transformation, obtained by replacing v with –v and swapping x_1 and t_1 with x_2 and t_2 in equation (16)

$$\begin{cases} x_1 = x_2 + vt_2 \\ t_1 = t_2\sqrt{1-\frac{v^2}{c^2}} \end{cases} \quad (18)$$

- What is the difference?

We see that system (17) is different from system (18). Why are they different while both transform from frame 2 to frame 1? For explanation, let us see the example of a train moving past a platform. The platform is the frame 1 and the train the frame 2. There is a pendulum named Pla on the platform and a pendulum named Tra in the train. One oscillation of Pla last the time Δt_{p1} on the platform. The oscillation of Pla measured in the train is Δt_{p2}, which we compute using system (16). This is the transformation of time from frame 1 to frame 2.

In case 1, if a traveler in the train measures one oscillation of Pla, he will get Δt_{p2}. If he want to know Δt_{p1}, he will use system (17) because the ratio $\frac{\Delta t_{p2}}{\Delta t_{p1}}$ equals $\sqrt{1-\frac{v^2}{c^2}}$, see equation (8). Notice that Δt_{p2} is not the period of Tra, but that of Pla.

In case 2, frame 1 is still the platform and frame 2 the train. But this time the traveler measures one oscillation of Tra in the train, not that of Pla. Then he computes the period of Tra with respect to the platform. He must use system (18) because the oscillation he computes is that of Tra, not that of Pla.

For distinguishing the 2 transformations, we name the system (17) for case 1 "Inverse transformation" and the system (18) for case 2 "Back transformation". Inverse transformation converts the abscissa and time of Pla, back transformation converts those of Tra. In fact, both computations are done by the traveler in the frame 2. In case 1, he computes the period of Pla, in case 2 that of Tra. Because Pla and Tra are not the same object, their transformations are different, although both transformations are from frame 2 to frame 1.

6. Solving paradoxes

a) Twin paradox

Time transformation from frame 1 to frame 2

$$t_2 = t_1\sqrt{1-\frac{v^2}{c^2}} \quad (19)$$

Inverse time transformation

$$t_1 = \frac{t_2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (20)$$

- The paradox

If a traveler goes for a round travel from Earth to a star then back on Earth, the time of the travel that the traveler will count will be shorter than the time that an earthling will count for the same travel. But for the traveler, the time on Earth should be shorter, hence the twin paradox.

- Solution of the paradox

Let the Earth be the frame 1 and the spaceship the frame 2. For the earthling, the time of the travel is t_1. He transforms t_1 using the system (16), which gives equation (19) and finds that t_2 is shorter than t_1.

At the end of the travel the traveler has actually counted the time t_2 with which he computes the time on Earth. He transforms t_2 using the inverse transformation equation (20), which is the inverse of equation (19). He will find the same t_1 as the earthling because equations (19) and (20) are the same equation. So, he will agree with the earthling and the paradox is solved.

I have painstakingly explained the twin paradox in « Twin paradox when Earth is the moving frame » by introducing a fictive back traveler and a fictive star. The solution with the inverse transformation is much simpler and intuitive.

- Why the paradox?

Lorentz transformation is the system (21) of which the back transformation is the system (22). For computing the time of the traveler the earthling plugs his time t_1 and the position of the traveler $x_1=v\cdot t_1$ into system (21) and finds equation (23) and the time t_2. When the traveler computes the time on Earth, he uses the back Lorentz transformation by plugging his time t_2 and the position of the Earth

x_2=−v·t_1 into system (22). He will find equation (24) and the time t'_1 which is not the time on Earth t_1, see equation (23). In fact, equation (24) is the back transformation of time given in system (18). But we have seen that the correct transformation to use is the inverse transformation system (17) or equation (20), which Lorentz transformation does not possess. So, the twin paradox is caused by the lack of inverse transformation of Lorentz transformation.

Lorentz transformation

$$\begin{cases} x_2 = \dfrac{x_1 - vt_1}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ ct_2 = \dfrac{ct_1 - \dfrac{v}{c}x_1}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases} \qquad (21)$$

Back Lorentz transformation

$$\begin{cases} x_1 = \dfrac{x_2 + vt_2}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ ct_1 = \dfrac{ct_2 + \dfrac{v}{c}x_2}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{cases} \qquad (22)$$

Traveler's time computed using Lorentz transformation

$$t_2 = \frac{t_1 - \dfrac{v^2}{c^2}t_1}{\sqrt{1-\dfrac{v^2}{c^2}}} = t_1\sqrt{1-\frac{v^2}{c^2}} \qquad (23)$$

Earth's time computed using back Lorentz transformation

$$t'_1 = \frac{t_2 - \dfrac{v^2}{c^2}t_2}{\sqrt{1-\dfrac{v^2}{c^2}}} = t_2\sqrt{1-\frac{v^2}{c^2}} \qquad (24)$$

b) Other discrepancies

According to the Law of length conservation, the length of a moving object does not shrink, so the distance between 2 moving objects does not shrink either. The Law of length conservation will solve all the discrepancies cited in « Analysis of Einstein's derivation of the Lorentz Transformation »?

- Relativistic jets

The distance between particles in a fast relativistic jet does not contract. So, the fast jet and slow jet should show the same density of particles. The photographs of relativistic jets confirm this prediction and thus, are experimental proofs for the non existence of length contraction.

- Synchronization of clocks of GPS satellites

Since equation (11) proves that time in frame 2 is simultaneous everywhere, relativity of simultaneity should not exist. The correct synchronization of GPS satellites is an experimental proof for the non existence of relativity of simultaneity.

- Jumping rabbits

Since the distance between the 2 jumping rabbits in Figure 4 does not shrink, the right rabbit will not go back, see « Analysis of Einstein's derivation of the Lorentz Transformation », which also solves Bell's spaceship paradox, Ehrenfest paradox, Ladder paradox, Bug-Rivet Paradox.

The distance between jumping rabbit contracts and the right rabbit would jump back

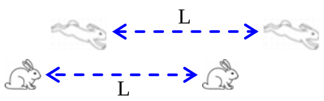

Figure 4

Length contraction between electrons makes the electron train in the left accelerator to shrink in total length.

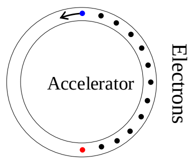

Figure 5

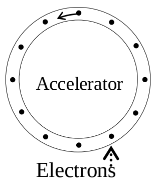

Figure 6

- Particles in circular accelerator

Since the distance between 2 accelerated particles does not shrink, the contraction of the electrons train shown in Figure 5 should not happen, see «Testing relativity of simultaneity using GPS satellites». In reality, particles are evenly spaced in circular accelerators as shown in Figure 6, which experimentally proves the non existence of length contraction.

- Accelerated basketball

Since the length of a basketball does not shrink when accelerated, its edges should not go faster than light, see « Astrophysical jet and length contraction ».

7. Comments

Above are 3 experimental proofs for Time relativity transformation: relativistic jets, synchronization of clocks of GPS satellites, particles in circular accelerator. The Time relativity transformation has solved 7 paradoxes: the twin paradox, accelerated basketball and jumping rabbits along with Bell's spaceship paradox, Ehrenfest paradox, Ladder paradox, Bug-Rivet Paradox. So, the predictions of the Time relativity transformation are correct.

In « Length, distance and Michelson–Morley experiment » the section 3 “Why Object contraction?” shows that length contraction was inferred in the derivation of the Lorentz transformation from Michelson–Morley experiment by applying classical law to light, which flaws this derivation. «Analysis of Einstein's derivation of the Lorentz Transformation» shows that the proportionality assumption in his derivation is not valid. These 2 flaws corroborate each other.

Changing from Lorentz transformation to Time relativity transformation is a great change of paradigm which surely will meet strong resistance. In ancient Greece Pythagorean theory affirmed that all numbers were ratios of integers. But this theory failed to explain the square root of two. Later, this theory was superseded by real numbers theory which is a major progress of mathematics. Facing the defects of Lorentz transformation, physicists should not try to save the failing theory by drowning the discoverer of the flaws like Hippasus, but welcome the new theory that solves old paradoxes. This is how Science progresses.

Correct theory should also give correct velocity and acceleration for describing momentum, mass and energy, which I will do next.