Astrophysical jet and length contraction

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Abstract: Astrophysical jets are flows of matter that moves at relativistic speed. They are opportunity to see length contraction in action. An astrophysical jet is analyzed to explain the length contraction effect on it.

1. Astrophysical jet

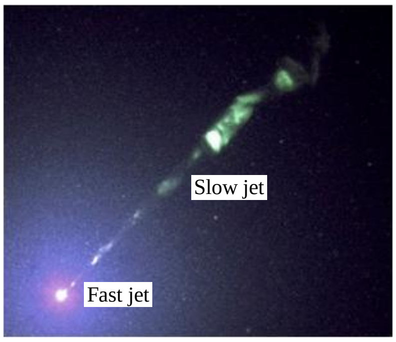

Figure 1

Astrophysical jets are high speed flows of matter ejected from compact objects such as black holes. Figure 1 is a photograph of the jet ejected by the supermassive black hole at the center of the galaxy M87 which stretches over 5 000 light-years. Matter in this jet moves at almost the speed of light and undergoes relativistic effects.

Almost all predictions of Special Relativity are proven by experiment except one: length contraction. Indeed, it is impossible to accelerate chunk of matter to relativistic speed to directly measure length contraction on Earth. Fortunately, length contraction should occur in astrophysical jets where we could finally see this effect for real.

2. Compression approach

Figure 2

How would length-contracted astrophysical jets look like? Commonly, one describes length contraction as the shortening of the length of an object as it accelerates to very high speed. For example, the video Length contraction: the real explanation from the Youtube channel Fermilab illustrates this effect with the image of a ball that becomes more and more flat as it moves at higher and higher speed. Figure 2 is a montage of 2 screenshots from this video: the left image is a stationary ball which is round and the right image is a fast moving ball which is flat. The flat shape is a compressed round shape, so I will refer to the shortening of length as compression approach which we will use to derive the aspect of astrophysical jets under length contraction.

An astrophysical jet is a moving cloud of particles whose velocity varies from almost the speed of light in the ejection region to very slow thousand light-years away. We name the jets in these 2 regions fast jet and slow jet (see Figure 1). The fast jet should be strongly length-contracted while the slow jet should not.

Let us compare a moving cloud of particles with the air inside a ball. The air inside a moving ball moves with it and would seem denser than the air inside the same ball at rest because the flat shape has a smaller surface than the round shape. In the same way, fast jet should seem denser than slow jet. Since the particles of an astrophysical jet emit photons, the fast jet should be brighter than the slow jet.

According to the article «How we discovered the strange physics of jets from supermassive black hole», some jet reaches 99.9% the speed of light. At this speed, length contraction rate reaches 22. That is, the viewed fast jet should contain 22 times more particles per unit length in the stationary frame than the slow jet and thus, the fast jet should be much brighter than the slow jet. As the slowdown of the jet is gradual, we expect that the brightness of a jet decreases gradually from the fast jet to the slow jet in the manner illustrated by Figure 3. However, we do not see such gradual decrease of brightness in Figure 1.

Can temperature and pressure of the jet compensate the effect of length contraction on brightness? Fast jet must carry very high energy in order to escape from the black hole and thus, should have much higher temperature and pressure than the slow jet and the brightness due to temperature and pressure should also gradually decrease from fast jet to slow jet. So, temperature and pressure would not compensate the effect of length contraction.

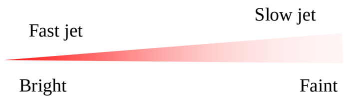

Figure 3

As the fast jet in Figure 1 is not significantly brighter than the slow jet, this jet does not have the aspect derived using the compression approach. Why does the real aspect of the jet contradict the compression approach?

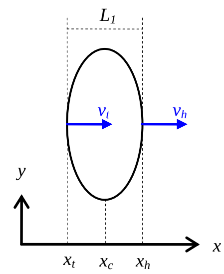

Figure 4

3. Head and Tail's velocities

Let us try to understand with the help of a moving ball which is represented by the ellipse in Figure 4. The length of the ball is contracted in the stationary frame and denoted by L_1. The center is at x_c on the x axis. The points the most to the right and to the left are named the head and tail of the ball which are at x_h and x_t on the x axis and are expressed in equation (1).

$$x_h = x_c + \frac{L_1}{2} \qquad x_t = x_c - \frac{L_1}{2} \tag{1}$$

$$v = \frac{dx_c}{dt} \tag{2}$$

$$v_h = \frac{dx_h}{dt} = v + \frac{1}{2}\frac{dL_1}{dt} \qquad v_t = \frac{dx_t}{dt} = v - \frac{1}{2}\frac{dL_1}{dt} \tag{3}$$

The velocity of the center is v and is expressed in equation (2). The velocities of the head and tail are v_h and v_t and are expressed in equation (3). When the ball moves at constant velocity, v_h and v_t equal v. However, when the ball accelerates, its shape flattens continuously making the head and the tail to move to the center. This effect can be seen in the video [Length contraction: the real explanation]. In approaching the center, the velocity of the head becomes smaller than v and that of the tail bigger than v.

$$L_1 = L_2\sqrt{1 - \frac{v^2}{c^2}} \tag{4}$$

Let us compute v_h and v_t using equations (1), (2), (3) and the length contraction law given in equation (4), where L_2 is the proper length of the ball. The velocities v_h and v_t are derived in equation (5), where v' is the acceleration of the center. v_i will equal v_h when the sign ± equals + and v_t when the sign ± equals −.

$$v_i = v \pm \frac{L_2}{2}\frac{d}{dt}\left(\sqrt{1-\frac{v^2}{c^2}}\right) = v \pm \frac{L_2}{2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left(-\frac{vv'}{c^2}\right) \tag{5}$$

$$v' \approx \frac{\Delta v}{\Delta t} \tag{6}$$

$$\begin{cases} v = 1 \\ \Delta v = 1 \end{cases} \Rightarrow v_i \approx 1 \pm \frac{L_2}{2}\left(-\frac{1}{c^2\Delta t}\right) \tag{7}$$

$$\Delta t < \frac{L_2}{2c^3} \Rightarrow \begin{cases} -v_h > c \\ v_t > c \end{cases} \tag{8}$$

Now we look at the values of v_h and v_t when the ball moves at small velocity, say v=1. In this case, $1 - \frac{v^2}{c^2} \approx 1$. v' is the time derivative of v and is given in equation (6). For small change of v, say Δv=1, if Δt is smaller than $\frac{L_2}{2c^3}$, the velocities v_h and v_t will be bigger than the speed of light c (see equation (8)). This violates the principle that nothing can travel faster than light.

On the other hand, the values of the velocities v_h and v_t are different, which contradicts the fact that the head and tail had been accelerated the same way. Because the flattening of the ball causes the 2 inconsistencies above, the ball should not flatten during acceleration and the compression approach cannot be the correct interpretation of length contraction.

So, the cloud of particles of an astrophysical jet should not seem denser in the fast jet. This explains why the real aspect of the jet contradicts the compression approach.

4. Separation approach

If the compression approach is not correct, the length of a moving object will not shorten in the stationary frame during acceleration. Then, how can the length contraction law still work? This situation has been considered in Bell's spaceship paradox where 2 spaceships A and B are accelerated exactly the same way. In this case, the trajectory of B equals that of A shifted by a distance L, which means that the trajectories of A and B are parallel and the distance between them is always L in the stationary frame. By considering the 2 spaceships as a whole system, the length of this system is L and does not shorten during acceleration.

For reconciling the length contraction law with the constancy of the length of the system, J. S. Bell proposed that the 2 spaceships would rather separate in their proper frame, that is, B will go away from A making the distance between A and B longer. This way, the length of the system will increase in the proper frame rather than will decrease in the stationary frame, making the proper length of the system longer than its length in the stationary frame such that the length contraction law is still respected. I will refer to the separation of the 2 parts of one system as separation approach. I have proposed to test the length contraction law using this approach in «How to test length contraction by experiment?».

Like the 2 spaceships, 2 nearby particles in an astrophysical jet form a system of 2 parts. Because the compression approach is not correct, the length of this system should not shorten in the stationary frame. So, we have to use the separation approach in astrophysical jets, that is, the 2 particles will separate in their proper frame and the distance between them will increase.

The separation of the 2 particles during acceleration is simulated by the system of 2 accelerated electrons between 2 charged plates (see Figure 5). At rest, the 2 electrons are at the distance L_1 from each other. Once the electric field is applied, the 2 electrons are accelerated to the velocity v. According to the separation approach, the distance between the 2 electrons in their proper frame increases gradually and the tailing electron sees the heading electron goes away at a velocity which is the time derivative of L_2. We name this velocity separation velocity, denote it by v_s and derive it in equation (9) by using the length contraction law given in equation (4). In equation (9) v and v' are the velocity and acceleration of the tailing electron.

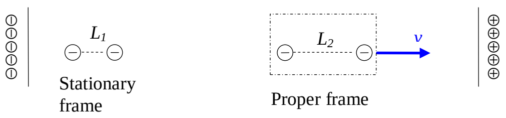

Figure 5

$$v_s = \frac{dL_2}{dt} = -\frac{1}{2}L_1\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(-2\frac{vv'}{c^2}\right) = L_1\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\frac{vv'}{c^2} \quad (9)$$

$$\begin{cases} v = 1 \\ \Delta v = 1 \end{cases} \Rightarrow v_s \approx \frac{L_1}{c^2 \Delta t} \quad (10)$$

$$\Delta t < \frac{L_1}{c^3} \Rightarrow v_s > c \quad (11)$$

What is the value of v_s? When the electrons' velocity is v=1 we have $1-\frac{v^2}{c^2} \approx 1$. v' is given in equation (6). For small change of v, Δv=1, if Δt is smaller than $\frac{L_1}{c^3}$, the velocity v_s is bigger than c (see equation (11)). Again, the separation approach violates the principle that nothing can travel faster than light.

On the other hand, imagine that we sit on the tailing electron and we see the heading electron going away. The heading electron is like an elevator that goes up under gravitational acceleration. The elevator gets potential energy because a motor is doing a work against gravitation. The heading electron being under the same acceleration than us, we deduce that it is getting potential energy and some force is doing a work on it. But length contraction does not provide such force and then, the energy of the heading electron is not balanced.

Because the separation approach causes these 2 inconsistencies, the separation approach cannot be the correct interpretation of length contraction and particles in astrophysical jets would not separate either.

5. Energy balance

The problem of energy of the heading electron shows that energy balance is also a concern for the length contraction law. For example, 2 charged particles possess mutual electric potential energy which is a function of the distance between them. If the distance varies, the potential energy will get a variation which is forcefully balanced by an external work. According to the separation approach, the distance between the 2 electrons in Figure 5 varies from L_1 to L_2 due to length contraction. So, their mutual potential energy should vary. But both electrons are under the same electric force from the charged plates and have traveled the same distance in the stationary frame, there is no additional work to balance the variation of their mutual electric potential energy.

To explain Bell's spaceship paradox, J. S. Bell says that if a thread tied the 2 spaceships together, it would break according to the separation approach. However, breaking the thread would consume a quantity of energy that length contraction does not provide.

Also according to the separation approach, the heading electron in Figure 5 moves away from the tailing electron and the heading spaceship in Bell's spaceship paradox moves away from the tailing spaceship. The separation velocity gives a nonzero kinetic energy to the heading objects while the kinetic energy of the tailing objects stays zero. There is no reason that, having done the same distance under the same force, the heading objects get more kinetic energy than the tailing ones.

The above analysis about energy shows that the separation approach violates energy conservation law.

On the other hand, according to the compression approach the velocity of the head of the ball v_h is slower than that of the tail v_t (see section 3. "Head and Tail's velocity"). So, the particles on the head carry less kinetic energy than that on the tail. Again, there is no reason that, having done the same distance under the same force, the particles on the head get less kinetic energy than that on the tail. So, the compression approach violates energy conservation law too.

6. Acceleration

Since object's acceleration was used in the analysis of the length contraction effect and Special Relativity is not valid in accelerated frame, how can this analysis be valid? Suppose that an object moves at velocity v_1, its length is P_1 in its proper frame and S_1 in the stationary frame. After the time Δt, the object moves at velocity v_2, its length is P_2 in its proper frame and S_2 in the stationary frame.

In the analysis, v_1 and v_2, P_1 and P_2, S_1 and S_2 are valid quantities in Special Relativity. The word acceleration indicates that v_1 and v_2 are 2 different instantaneous velocities of the same object which are used to compute P_1 and P_2, S_1 and S_2 by applying the length contraction law without acceleration. So, the above analysis is not in conflict with Special Relativity.

7. Conclusion

The inconsistency of the values of P_1 and P_2, S_1 and S_2 is demonstrated by reductio ad absurdum in the following manner:

1. Suppose that the separation approach is true, then the length of a system would increase in its proper frame, which gives a separation velocity to the heading part of the system.
2. Since the separation velocity can be higher than the speed of light, which is absurd, then the separation approach cannot be true.

In summary, we have exposed the following inconsistencies:

- Compression approach
  1. The velocities of the head and tail of the ball could exceed the speed of light.
  2. These velocities are not equal.
  3. The kinetic energy of the particles on the head and tail of the ball are not equal.
  4. Figure 1 does not show the aspect of jet derived using the compression approach. By the way, this may be the first experimental evidence about the length contraction effect.
- Separation approach
  5. The velocity of the heading part could exceed the speed of light.

6. The mutual electric potential energy of 2 charged particles could vary without work.
7. The heading part would get kinetic energy while the tailing part would not.
8. Bell's spaceship paradox breaks the thread without consuming energy.

These 8 inconsistencies are sufficient to show that the compression approach and the separation approach are inconsistent. Since these 2 approaches are the only possible interpretations of the length contraction effect, the length contraction effect could be inconsistent too.

8. Comments

Why these inconsistencies of the length contraction effect were not noticed before? Because Special Relativity works so well that we have managed to accept this unusual effect. For example, in order to explain why a moving object can be shorter than its proper length without being physically compressed, we have created the compression approach. By doing so, we ignore that different parts of the object could get different velocities and different kinetic energies, that the velocities could exceed the speed of light. In case where the compression approach does not work, we have created the separation approach that conserves length in the stationary frame. By allowing the separation of a system, we ignore that its heading part could travel faster than light and could get energy without work.

So, the compression approach was created only to avoid compressing the moving object and the separation approach was created only to conserve length in the stationary frame. Both approaches are solutions that solve one particular problem while creating others. Solutions of such type are called ad hoc solutions and are arbitrary and dangerous. For example, the compression approach works for a moving ball while the separation approach works for 2 spaceships. How would we explain the case of 2 spaceships tied together with a ball?

Each time an ad hoc solution is used to explain a phenomenon, we show our bad understanding of the phenomenon. So, we will dig deeper into the mechanism of Special Relativity to better understand the length contraction effect.