Coil and resistor induction paradox

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In my article **Faraday's Law Paradox**, I have explained that in a magnetically induced coil the line integral of electric field is paradoxically zero in contraction with Faraday's law. This paradox has aroused a long discussion which also reveals that the air gap between the coil's terminals confused the understanding of the paradox. The closed coil below makes this paradox sharper. I will give my solution to this paradox.

1. Coil and resistor induction paradox

A thin resistor *R* closes the coil and lets a current *I* to circulate across the electrodes A and B (see Figure 1). Assuming the width of the resistor negligible, Faraday's law and Ohm's law give the voltage and current which are proportional to the variation rate of the magnetic flux Φ:

$$U_B - U_A = -\frac{d\Phi}{dt}, \qquad U_A - U_B = R \cdot I, \qquad I = \frac{1}{R}\frac{d\Phi}{dt} \tag{1}$$

In an electric field, the difference of potential between two points is the negative line integral of the electric field along any path joining the two points. So, by integrating from A to B we obtain the voltage of the point B relative to the points A:

$$U_B - U_A = -\int_A^B \boldsymbol{E} d\boldsymbol{l} \tag{2}$$

If the magnetic field varies linearly with time, the current is constant. Since the electrons in the wire do not accelerate, the resultant force on an electron is zero and then, the electric field around it is zero. So the voltage from A to B integrated via the wire is zero:

$$\boldsymbol{F} = -e\,\boldsymbol{E} = 0 \Rightarrow U_B - U_A = -\int_{A\,wire}^{B\,wire} 0\, d\boldsymbol{l} = 0 \tag{3}$$

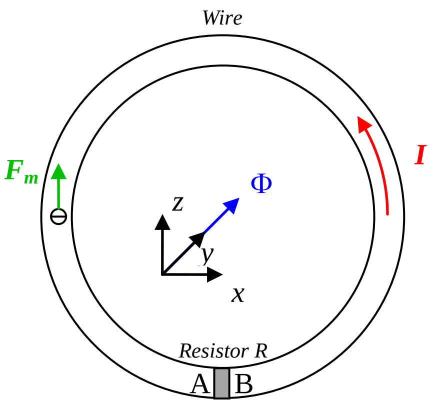

Figure 1

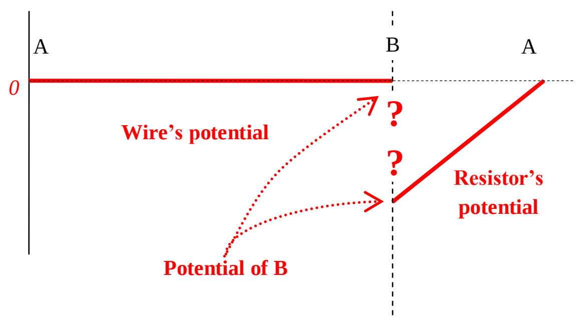

Figure 2

Hence the paradox: two different laws give two different values to the same voltage:

1) For Faraday's law the voltage is: $U_B - U_A = -\frac{d\Phi}{dt}$
2) For line integration via the wire the voltage is: $U_B - U_A = 0$

The potentials in the wire and the resistor are represented by the red lines in Figure 2.

In a non-conservative electric field potential appears ambiguous. So, let us see the energy of an electron traveling in the loop on which the electric work from A to B is:

$$W_{wire} = \int_{A\ wire}^{B\ wire} \boldsymbol{F}\, d\boldsymbol{l} = 0 \tag{4}$$

But when the same electron crosses the resistor, it receives from the electrostatic field this work:

$$W_{Resistor} = \int_{B\ Resistor}^{A\ Resistor} \boldsymbol{F}\, d\boldsymbol{l} = \int_{B\ Resistor}^{A\ Resistor} -e\, \boldsymbol{E}\, d\boldsymbol{l} = e(U_A - U_B) = e\frac{d\Phi}{dt} \tag{5}$$

So, the electric energy an electron receives in one cycle is:

1) A →Wire →B: 0
2) B →Resistor →A: $e\frac{d\Phi}{dt}$

By going from A to B then back to A via the resistor, an electron gets $e\frac{d\Phi}{dt}$ of energy for free which is lost through Joule effect, thus the paradox. One can argue that this energy is given by the magnetic induction. In reality this is true. But in the above derivation the magnetic induction appears nowhere. Thus, the paradox is in the classical theory, not in the physical reality.

A third view of this paradox is in the average force on the electrons. Let us admit that energy is conserved in the loop and the work in the wire is opposite to that in the resistor. So, we have:

$$W_{average} = \int_{A\ wire}^{B\ wire} \boldsymbol{F}_{average}\, d\boldsymbol{l} = -\frac{d\Phi}{dt} \tag{6}$$

This implies non-zero average force on the electrons. But the electric field is zero in the wire and there cannot be force on electrons, thus the paradox.

2. Two forces on one electron

In order to solve this paradox, we must find out why the electric field in the wire is zero. In the discussion someone has suggested that the magnetically induced force would move the free electrons in the wire so that an electrostatic field builds up and cancel the force on the electrons at equilibrium. Very good point.

Figure 3 illustrates the electrons distribution in the wire at equilibrium. As the magnetically induced force pushes the electrons in clockwise direction a non constant distribution of free electrons is built, which is schematically represented by the green area in Figure 4. In first approximation, $\Delta\rho$ the density of free electrons relative to a reference value is a straight line with respect to the wire.

As there are more electrons in the right than in the left, an electrostatic field is created and each electron feels an electrostatic force $\boldsymbol{F}_e$ (Figure 3) that cancels the magnetically induced force $\boldsymbol{F}_m$ (Figure 1). The resulting electrostatic forces are schematically represented by the pink arrows in Figure 3 and Figure 4. The electrostatic force $\boldsymbol{F}_e$ is created by the electrostatic field $\boldsymbol{E}_e$ which is determined by the distribution ρ according to Gauss's law:

$$\nabla \cdot (\varepsilon \boldsymbol{E}_e) = \rho \Rightarrow \boldsymbol{F}_e = -e\boldsymbol{E}_e \tag{7}$$

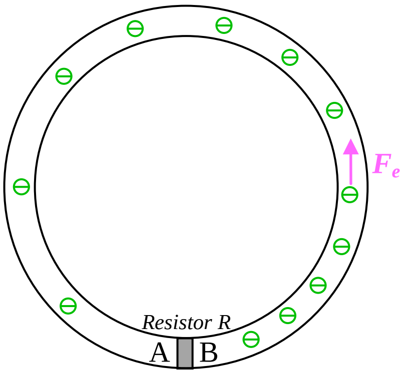

Figure 3

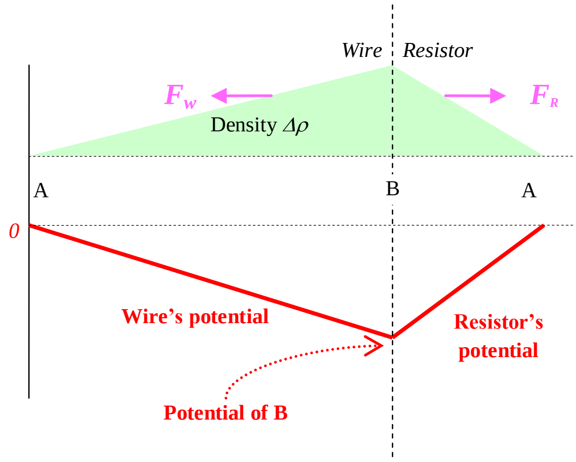

Figure 4

In consequence, there are two fields in the wire: the magnetically induced field $\boldsymbol{E}_m$ and the electrostatic field $\boldsymbol{E}_e$. In classical theory magnetically induced field is thought to be of electric nature because Faraday's law defines $\boldsymbol{E}_m$ as an electric field:

$$\nabla \times \boldsymbol{E}_m = -\frac{\partial \boldsymbol{B}}{\partial t} \Rightarrow \boldsymbol{F}_m = q\boldsymbol{E}_m \tag{8}$$

Adding up directly the magnetically induced field and the electrostatic field makes the total electric field zero. This is how we get zero for the electric field in the wire:

$$\boldsymbol{F}_m + \boldsymbol{F}_e = q(\boldsymbol{E}_m + \boldsymbol{E}_e) = 0 \Rightarrow U_B - U_A = -\int_{A\,wire}^{B\,wire} (\boldsymbol{E}_m + \boldsymbol{E}_e)d\boldsymbol{l} = 0 \tag{9}$$

Remark: The current is constant at equilibrium. As there are more free electrons near B, the velocity of free electrons is lower and the electrons slow down slightly in moving toward B.

3. Solution

Distinguishing magnetically induced field from electrostatic field is the way out of this paradox. Let us see the electrostatic field alone. If the wire in Figure 3 is made of insulating material and charged with the same distribution of electrons, the line integral of its electrostatic field along the wire from A to B is equal to the line integral across the resistor. This is illustrated by the red line in Figure 4.

What will be the value of the line integral for the insulating wire? It equals the negative line integral of the magnetically induced field $\boldsymbol{E}_m$. In fact, equation (9) proves that the line integral of these two fields are exactly opposite:

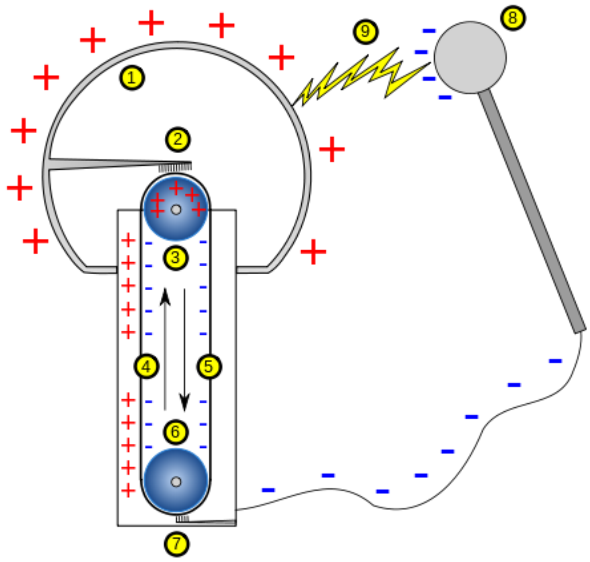

Figure 5

$$\int_{A\,wire}^{B\,wire} \boldsymbol{E}_e d\boldsymbol{l} = -\int_{A\,wire}^{B\,wire} \boldsymbol{E}_m d\boldsymbol{l} \tag{10}$$

But how does magnetic induction work in conductor wire? Let us see the working schema of a **Van de Graaff generator** in Figure 5: The hollow metal sphere (1) is charged to a voltage V; the belt (4) takes a quantity of charge from the lower electrode (7), conveys it to the upper electrode (2). This way, a current circulates against the voltage V. Then, by discharging through the air, the current completes the loop back to the lower electrode (7).

The principle of this generator is that the sphere (1) is always charged at the voltage V and the charge-conveying force is mechanical. Thus, the line integral of the electrostatic field from the lower electrode (7) to the upper electrode (2) equals V through any path. An analogy with **Van de Graaff generator** will help us in understanding the magnetic induction in the coil of Figure 1. The similitude between the two generators:

1) The magnetically induced force corresponds to the belt (4)
2) The electrode B corresponds to the hollow metal sphere (1)
3) The electrode A corresponds to the lower electrode (7)
4) The resistor corresponds to the air gap (9)

And the coil works this way:

1) The electrode B is always charged at the voltage V by the distribution of free electrons
2) The magnetically induced force conveys free electrons against the voltage V to reach B
3) The electrons are pushed by the electrostatic force through the resistor and back to A

From the above working schemas, we can induce the working principle for all generators:

1) The voltage of the electrodes is determined by static charge
2) The current-carrying charges are conveyed by non electric force across the electrodes

This is a general principle for electricity generation. For example, in a battery it is the chemical force that conveys the electrons from the cathode to the anode against the electrostatic field; in solar cells the conveying force is photons; in thermionic converter the conveying force is heat. Because the conveying forces are non-electric, they can make the current flow against the electrostatic field which stays unaltered. Electrostatic force works only in electrical loads such as resistor or motor.

Here is how Faraday's law works: the voltage across the electrodes A and B is the negative line integral of the electrostatic field of the electron distribution, which equals the line integral of the magnetically induced field, which in turn equals the negative variation rate of the magnetic flux through the coil:

$$U_B - U_A = -\int_A^B \boldsymbol{E}_e d\boldsymbol{l} = \int_A^B \boldsymbol{E}_m d\boldsymbol{l} = -\frac{d\Phi}{dt} \tag{11}$$

This way, the voltage obtained from line integral is independent of path and the suffix "wire" in the limits of the integral sign is dropped. The paradox is so solved.

4. Comments

This paradox about Faraday's law is solved by discarding the idea that magnetically induced field is an electric field. In fact, Michael Faraday never said that induced voltage arose from an electric field. It was James Maxwell who invented non-conservative electric field out of the need of linking magnetic and electric fields. But he did not have any direct experimental evidence for his assertion that magnetically induced field is equivalent to an electric field. So, Faraday's law is correct for the voltage across the electrodes A and B but its mathematical formulation in space is not.

In consequence, these two fields are of different nature and we are not allowed to add them up any more. Another consequence is that, since the quantity $\boldsymbol{E}_m$ is not an electric field, we can no more write Faraday's law in its classical form below:

$$\nabla \times \boldsymbol{E}_m = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{12}$$

And then, the classical wave equation becomes incorrect since it is derived from the above equation. This fact corroborates the irrational properties of classical wave equation that I have demonstrated in the following articles:

1) **Energy density of electromagnetic wave:** EM wave equation gives a solution whose angular density of energy goes to infinity at r=0.
2) **Electromagnetic wave energy flux:** EM wave equation gives a solution whose energy density is variable with distance and goes to infinity at r=0.
3) **Can EM wave go forward back?:** EM wave equation gives a solution whose velocity is variable with distance and goes to infinity at some points and becomes negative near the source.
4) **Electromagnetic Wave Paradox:** EM wave equation gives a solution whose velocity is variable with distance from the source.

In the discussion about my previous article, bja...@teranews.com said: "the free negative ones move so as cancel the induced E field. Hence there is a gradient in the electron charge density in the wire from one end to the other". This may be the first time in history that such electrostatic field is mentioned. Before my **Faraday's Law Paradox,** the voltage across the terminals of an induced coil is equalized to the line-integral of the induced field in all text books, suggesting this field to be the sole field in the wire. It is a major advance to understand that there are two fields in the wire: induced and electrostatic. My solution to this paradox complete the second step: the induced field is not an electric field. Indeed, if it were of electric nature, the total field is still zero in the wire and the voltage would be zero too.

Maxwell's electromagnetism is a pillar of today's science because its predictions are correct until now. However, it is not correct outside its domain of validity which is still unknown. The Faraday's law paradoxes, wave equation's weird properties and the paradoxes about Lorentz force law (see **Explanatory summary for my studies about the Lorentz force law**) are outside this domain and, the predictions of classical electromagnetism are naturally wrong.

Finding the limit of validity of a theory and analyzing the error off the limit of this theory do not destroy the old theory but give birth to a new one that is correct in a much larger domain. This was what Copernicus, Newton and Einstein realized in their times and the purpose of my work on electromagnetism. My principle of electric generator and the corrected magnetic force law in **Unknown properties of magnetic force and Lorentz force law** are correct in regions where electromagnetism becomes invalid.

I encourage all physicists to contribute in this enterprise with theoretical research or experiment. Your participation will give you the excitement of being hit in the head by an apple or seeing the stars out of their places on the solar eclipse photograph exactly as predicted by General Relativity.

5. For discussion

F_A and F_B are reaction forces from the current in the resistor. F_A is zero; F_B is the driving force that makes current flow in the resistor:

$$W_{resistor} = UI = -\Phi' * \rho v = F_B * \frac{dl}{dt}$$

So, the reaction force from the current at the point B is F_B :

$$F_B = -\Phi' * \rho$$

In the conductor loop, the average force on the electrons is zero, F_e=0. So, the action force at the current at the point B is zero. Thus, the paradox is: there is no force that pushes the current into the resistor, but the resistor pushes back on the loop's electrons.

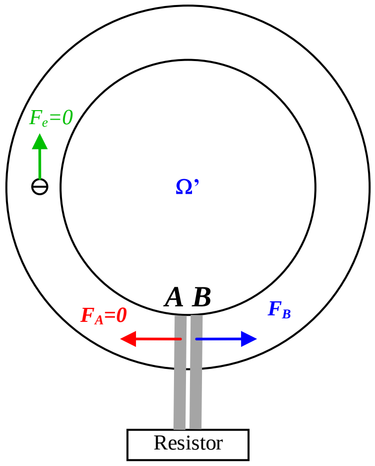

Figure 6