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# Lorentz force on open circuit

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After extensively analyzed EM wave equation, I will explain the core inaccuracy that leads to the inconsistency of the Lorentz force law. In the Figure 1, two metallic spheres are connected together through an angled wire whose angle is 90° and in which circulates an alternate current. While the wire is not a closed loop, the capacitor made up by the two spheres is charged alternately, letting the alternate current circulate.

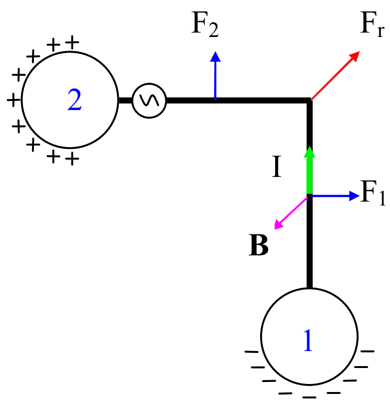


Figure 1

The current creates a magnetic field in space, which exerts a Lorentz force on the wire. The Lorentz force, $\mathbf{F_1}$ and $\mathbf{F_2}$, being perpendicular to the current, is in the x direction on arm 1 and the y direction on arm 2. The 2 arms are of equal length and the forces on them have the same magnitude *F*. So, the resultant force $\mathbf{F_r}$ is:

$$\mathbf{F}_r = \mathbf{F}_1 + \mathbf{F}_2 = F\left(\mathbf{e}_x + \mathbf{e}_y\right)$$

The directions of $\mathbf{F_1}$ and $\mathbf{F_2}$ do not change for alternate currents. The resultant force $\mathbf{F_r}$ is an internal force because the magnetic field is created by the wire itself, but $\mathbf{F_r}$ is nonzero. We can also imagine that the 2 arms are very long. This way, any force between the 2 spheres, between the spheres and the arms become negligible, while the force on the corner $\mathbf{F_r}$ stays nearly constant.

In fact, we will see in the following that the strength of $\mathbf{F_r}$ is infinity. Since any reaction force that opposes $\mathbf{F_r}$ is negligible, the Newton's third law is violated. This system violates the energy conservation law because if it moves at velocity **v**, it will do a work that is not provided by any energy source:

$$W = \mathbf{v} \cdot \mathbf{F_r} \neq 0$$

The Figure 1 gives only a qualitative impression of the resultant force. Its precise value is calculated with the differential Lorentz force law for 2 current elements:

$$d^2\mathbf{F} = \frac{\mu_0}{4\pi} d\mathbf{I}_2 \times \left( d\mathbf{I}_1 \times \frac{\mathbf{r}}{r^3} \right) \tag{1}$$

For the angled wire shown in the Figure 2, whose angle is of any value, the current element $\Delta \boldsymbol{l}_1$ exerts on the current element $\Delta \boldsymbol{l}_2$ the following Lorentz force:

$$\Delta^2\mathbf{F}_2 = \frac{\mu_0}{4\pi} I^2 \Delta l_2 \mathbf{e}_2 \times \left( \Delta l_1 \mathbf{e}_1 \times \frac{\mathbf{e}_r}{r^2} \right) = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \mathbf{e}_2 \times (\mathbf{e}_1 \times \mathbf{e}_r)$$

The double cross product being:

$$\mathbf{e}_2 \times (\mathbf{e}_1 \times \mathbf{e}_r) = \sin\alpha \left( \cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

The expression for the Lorentz force becomes:

$$\Delta^2\mathbf{F}_2 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin\alpha \left( \cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

In the same way, the Lorentz forces on the current elements $\Delta \boldsymbol{l}_1$, $\Delta \boldsymbol{l}_3$ and $\Delta \boldsymbol{l}_4$ are:

$$\Delta^2\mathbf{F}_1 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin(\alpha + 2\beta) \left( -\cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

$$\Delta^2\mathbf{F}_3 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin\alpha \left( -\cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

$$\Delta^2\mathbf{F}_4 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin(\alpha + 2\beta) \left( \cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

The sum of the above 4 forces gives the resultant force on the 4 current elements $\Delta \boldsymbol{l}_1$, $\Delta \boldsymbol{l}_2$, $\Delta \boldsymbol{l}_3$ and $\Delta \boldsymbol{l}_4$, which is in the y direction only because the x components cancel out:

$$\Delta^2\mathbf{F}_{res} = \Delta^2\mathbf{F}_1 + \Delta^2\mathbf{F}_2 + \Delta^2\mathbf{F}_3 + \Delta^2\mathbf{F}_4 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \cdot 2\left(\sin\alpha + \sin(\alpha + 2\beta)\right)\sin\beta \cdot \mathbf{e}_y$$

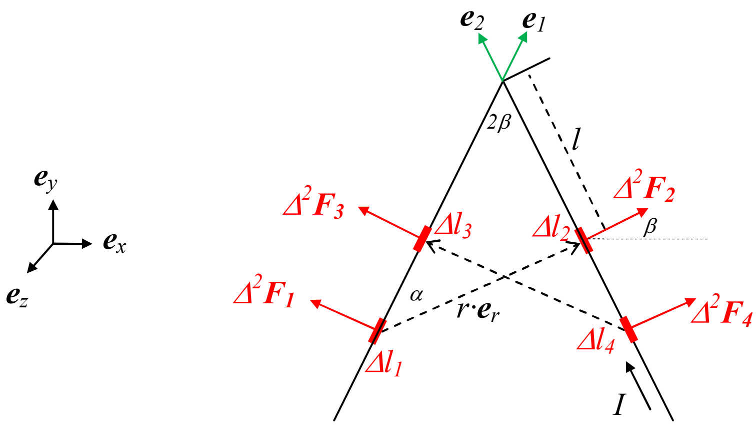


Figure 2

To compute the total resultant force on the wire, we create a series of $(\Delta \boldsymbol{l}_1, \Delta \boldsymbol{l}_2, \Delta \boldsymbol{l}_3, \Delta \boldsymbol{l}_4)_i$ with index $i$. The distance $l$ from the summit (see the Figure 2) and the lengths $(\Delta \boldsymbol{l}_1, \Delta \boldsymbol{l}_2, \Delta \boldsymbol{l}_3, \Delta \boldsymbol{l}_4)$ of the $i^{th}$ and $(i+1)^{th}$ terms are such that they cover contingent segments of the wire without overlapping. The distance $l$ of the $i^{th}$ term is $l_i$, that of the $(i+1)^{th}$ term is:

$$l_{i+1} = l_i - \Delta l_{2,i}$$

The positions and lengths of ($\Delta l_1$, $\Delta l_2$, $\Delta l_3$, $\Delta l_4$) of the $i^{th}$ and $(i+1)^{th}$ terms respect geometric similarity. So, the $(i+1)^{th}$ term is defined by the following relation:

$$c = \frac{l_{i+1}}{l_i} = \frac{\Delta l_{1,i+1}}{\Delta l_{1,i}} = \frac{\Delta l_{2,i+1}}{\Delta l_{2,i}} = \frac{\Delta l_{3,i+1}}{\Delta l_{3,i}} = \frac{\Delta l_{4,i+1}}{\Delta l_{4,i}} = \frac{r_{i+1}}{r_i}$$

The ratio between the elements' lengths and the distance $r$ is then constant for all terms. We define 2 constants as follow:

$$a_1 = \frac{\Delta l_{1,i}}{r_i} = \frac{\Delta l_{1,i+1}}{r_{i+1}}, a_2 = \frac{\Delta l_{2,i}}{r_i} = \frac{\Delta l_{2,i+1}}{r_{i+1}}$$

The sum of $\Delta^2 \mathbf{F}_{res}$ of the first $n$ terms of the series is:

$$\mathbf{F}_n = \sum_{i=1}^{i=n} \Delta^2 \mathbf{F}_{res} = n \frac{\mu_0}{4\pi} I^2 a_1 a_2 \cdot 2(\sin\alpha + \sin(\alpha + 2\beta))\sin\beta \cdot \mathbf{e}_y \qquad (2)$$

Near the corner, when $i$ increases, the distance $l_i$ tends to 0 but never reaches 0:

$$\begin{bmatrix} 0 < c < 1 \\ l_{i+1} = c \cdot l_i \end{bmatrix} \Rightarrow \begin{cases} l_n > 0 \\ l_n = l_1 c^{n-1} \underset{n\to\infty}{\to} 0 \end{cases}$$

So, when the number of terms tends to infinity, the whole wire is entirely covered by the current elements. But, we arrive at a mathematical problem, the force $\mathbf{F_n}$ tends to infinity when $n$ increases to infinity (see the equation (2)):

$$\boldsymbol{F}_n \xrightarrow[n\to\infty]{} \infty$$

As this is for one α only, the sum of $\mathbf{F_n}$ for all α that is the total Lorentz force will be a larger value. The computation of the precise value of the resultant Lorentz force for angled wire gives a non-definite value. One can argue that real wires do not have infinitely sharp tip and the series will not have infinite terms. But what will be the normal number n? Should it be 100, 1000, or $10^{10}$? As $\mathbf{F_n}$ is proportional to n, the value of $\mathbf{F_n}$ is quite different if n is different.

We see that, because 2 perpendicular currents exert a Lorentz force on each other, which are perpendicular to each other, the resultant internal Lorentz force on angled wire cannot be zero. I call this property the **perpendicular action**. It is this incorrectness that makes the Lorentz force law inconsistent because non-straight current involve perpendicular action in any circuit.

The system of the Figure 1 is a direct illustration of perpendicular action. But it is an open circuit, that is, the current does not circulate in a loop. In the articles
Mathematical cause of the existence of the remaining resultant internal Lorentz force
http://independent.academia.edu/KuanPeng/Papers/1869003/Mathematical_cause_of_the_existence_of_the_remaining_resultant_internal_Lorentz_force
Proof of the remaining resultant Lorentz force internal to a triangular coil
http://independent.academia.edu/KuanPeng/Papers/1868998/Proof_of_the_remaining_resultant_Lorentz_force_internal_to_a_triangular_coil
I have provided 2 rigorous and independent mathematical demonstrations of nonzero resultant internal Lorentz force for triangle coil. Triangle coil is the simplest case of closed loop and can be analyzed directly using perpendicular action. The equation (2) shows that Lorentz force near a corner is very large and is in the direction indicated by the tip. Thus, the forces on the 3 corners do not cancel out for general triangle and the resultant internal Lorentz force is nonzero. In consequence, perpendicular action between the sides of the triangles is the cause of nonzero resultant force.

In experiment, perpendicular action will provide evidence for inconsistency. The experiment I propose in Lorentz torque experiment
http://independent.academia.edu/KuanPeng/Papers/1869001/Lorentz_torque_experiment
makes 2 perpendicular coils interact. The Figure 3 shows the crossing sides of the 2 coils. Perpendicular action creates a torque on the coil 2. The measurement of this torque will confirm or invalidate the Lorentz force law.

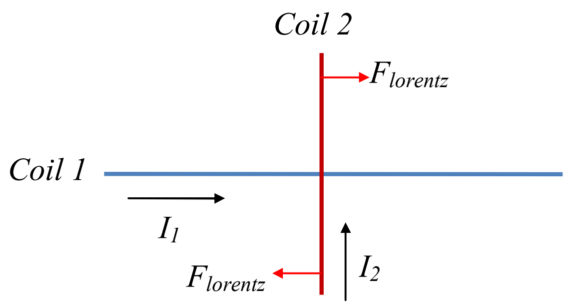


Figure 3

The origin of the inconsistency being identified, the Lorentz force law can be easily corrected by removing the part of force that causes perpendicular action. In the article Correct differential magnetic force law , http://independent.academia.edu/KuanPeng/Papers/1868992/Correct_differential_magnetic_force_law
I propose the following correct law for magnetic force:

$$d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{r}}{r^3}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)$$

This article proves that this law gives the same magnetic force than the Lorentz force law for closed loop while avoiding perpendicular action.

Numerical computations for the proposed experiment are given in the article Curve shape of the magnetic torques http://independent.academia.edu/KuanPeng/Papers/1869006/Curve_shape_of_the_magnetic_torques using the method given in Calculation of the Lorentz' Torque and the Ampere's torque
http://independent.academia.edu/KuanPeng/Papers/1869005/Calculation_of_the_Lorentz_Torque_and_the_Amperes_torque

For the case where the dimension ratio between the 2 coils is 0.18, the curves of the magnetic torque according to the Lorentz force law and the correct law are shown in the Figure 4. When the angle between the 2 coils is 90°, the torque computed with the Lorentz force law (red) is maximal and that computed with the correct law (green) is minimal, showing therefore that perpendicular action is eliminated in the correct law.

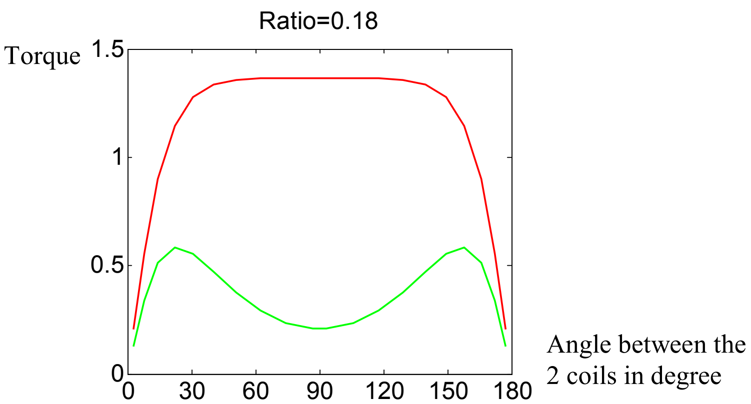


Figure 4

One can argue: "Why should perpendicular action be wrong? After all, the Lorentz force law have resisted about 150 years." But, if perpendicular action exists, there would be funny consequences. For example, we could use the resultant internal Lorentz force and put the system of the Figure 1 under the hood of a car to spare horse power (see Figure 5) or to propel spacecrafts to visit nice distant galaxies.

One can say that displacement current creates the reaction force. No. I have shown in the article Phantom Lorentz force Paradox, http://independent.academia.edu/KuanPeng/Papers/1868631/Phantom_Lorentz_force_Paradox that displacement current cannot bear reaction force. Then, near the corner the magnetic field from the current is very large whereas the displacement current is very weak (see the Figure 1). The 2 magnetic fields cannot cancel out there, and nonzero B creates Lorentz force.

One can also say that some energy source should be ignored in my argument. For example, the alternate current emits an electromagnetic wave that can provide the energy. But, this energy is not equal to the work done by the Lorentz force. When the system is fixed, the power that the resultant Lorentz force does is 0 because the velocity is 0, while the radiated energy power is $E_{radio}$. When the system moves, the radiated power is still $E_{radio}$, but this time the power of the Lorentz force is $\mathbf{v}\cdot\mathbf{F}$. $E_{radio}$ cannot be equal to $\mathbf{v}\cdot\mathbf{F}$ and 0 at the same time. So, the energy flow is not balanced.

The violation of the energy conservation law is the contradiction that proves by Reductio ad absurdum that the assumption, that perpendicular action exists, is wrong. So, be reassured experimenters, the experiment "Lorentz torque experiment " will surely show the non existence of perpendicular action.

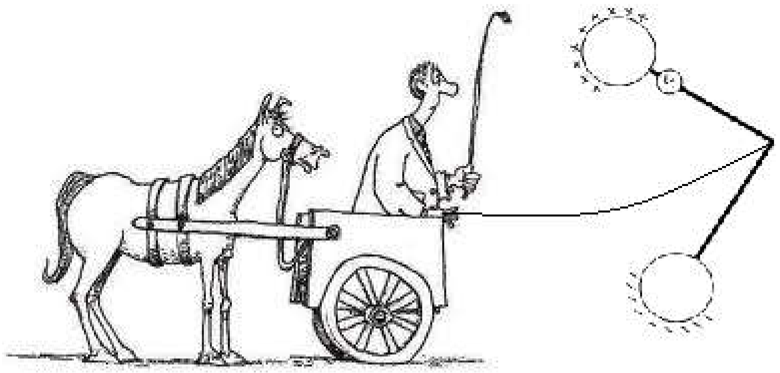

Figure 5

In conclusion, perpendicular action is the cause of nonzero resultant internal Lorentz force; it can be tested by experiment and is eliminated in the correct law. The theoretical predictions by the 2 laws are clearly different. The work left is to effectively carry the experiment out to see which of the 2 expected outcomes will show up. It is now up to you, experimenters.