Comparison of the 2 magnetic force laws

Peng Kuan 彭宽
titang78@gmail.com
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I have given the correct magnetic force law in the article «Correct differential magnetic force law », named the differential Ampere's force law:

$$d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{e}_r}{r^2}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)$$

Is this law really correct? The unique way to certainty is experiment. However, we can still do theoretical analysis to get more confidence. For example, comparing the comportment of the 2 laws in some particular cases. Let us analyze the magnetic force internal to a triangular coil and near the summits (see the Figure 1).

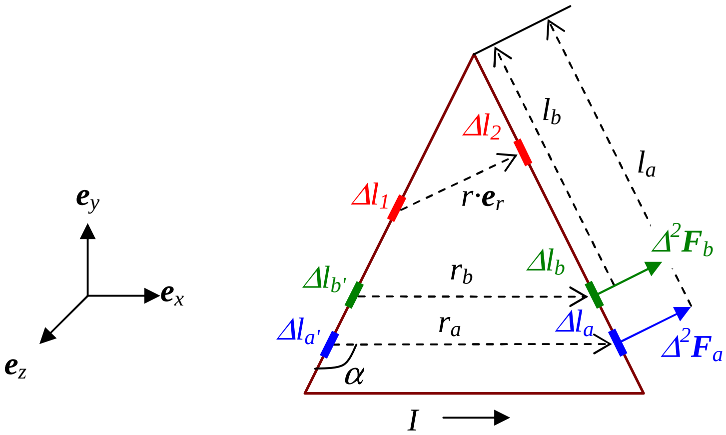

Figure 1

According to the differential Ampere's law, the magnetic force that the left current element $I\Delta\boldsymbol{l}_1$ exerts on the right one $I\Delta\boldsymbol{l}_2$ is:

$$\Delta^2\boldsymbol{F}_{amp}(1 \to 2) = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{e}_r}{r^2}I^2\left(\Delta\boldsymbol{l}_2 \bullet \Delta\boldsymbol{l}_1\right)$$

The force that the right current element $I\Delta\boldsymbol{l}_2$ exerts on the left one $I\Delta\boldsymbol{l}_1$ is:

$$\Delta^2\boldsymbol{F}_{amp}(2 \to 1) = -\frac{\mu_0}{4\pi}\frac{\left(-\boldsymbol{e}_r\right)}{r^2}I^2\left(\Delta\boldsymbol{l}_1 \bullet \Delta\boldsymbol{l}_2\right)$$

We see that this pair of differential Ampere's forces have the same magnitude and opposite sign. Thus, their sum is 0:

$$\Delta^2\boldsymbol{F}_{amp}(1 \to 2) + \Delta^2\boldsymbol{F}_{amp}(2 \to 1) = 0$$

The global internal force is the double integral of the differential force:

$$\boldsymbol{F}_{\text{internal}} = \int_{coil}\int_{coil} d^2\boldsymbol{F}_{amp} = \lim_{\substack{m\to\infty \\ n\to\infty}}\sum_{j=1}^{n}\sum_{i=1}^{m}\Delta^2\boldsymbol{F}_{amp}(i, j)$$

The last double sum can be written as the sum of pairs of differential Ampere's forces:

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\Delta^2\boldsymbol{F}_{amp}(i,j)=\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\Delta^2\boldsymbol{F}_{amp}(i,j)+\Delta^2\boldsymbol{F}_{amp}(j,i)\right]=0$$

So, the global internal force according to the differential Ampere's law is 0 whatever the form of the coil is:

$$\boldsymbol{F}_{\text{internal}}=0$$

According to the Lorentz force law, the differential magnetic force is a double cross product (see the Figure 1):

$$\Delta^2\boldsymbol{F}_a=\frac{\mu_0}{4\pi}I^2\Delta\boldsymbol{l}_{a'}\times\left(\Delta\boldsymbol{l}_a\times\frac{\boldsymbol{e}_r}{r_a^2}\right)$$

The expression of the right cross product is:

$$\Delta\boldsymbol{l}_a\times\frac{\boldsymbol{e}_r}{r_a^2}=\frac{\Delta l_a}{r_a^2}\sin\alpha\ \boldsymbol{e}_z$$

The 2 element vectors $\Delta\boldsymbol{l}_a$ and $\Delta\boldsymbol{l}_{a'}$ having the same length Δl_a, the expression of the double cross product is:

$$\Delta\boldsymbol{l}_{a'}\times\left(\Delta\boldsymbol{l}_a\times\frac{\boldsymbol{e}_r}{r_a^2}\right)=\left(\frac{\Delta l_a}{r_a}\right)^2\sin\alpha\ (\sin\alpha\,\boldsymbol{e}_x+\cos\alpha\,\boldsymbol{e}_y)$$

By summing the forces on the right and left elements, the x components cancel and the resultant is:

$$\Delta^2\boldsymbol{F}_a+\Delta^2\boldsymbol{F}_{a'}=\frac{\mu_0}{4\pi}I^2\left(\frac{\Delta l_a}{r_a}\right)^2\sin 2\alpha\,\boldsymbol{e}_y$$

For the *b* pair of current elements $\Delta\boldsymbol{l}_b$ and $\Delta\boldsymbol{l}_{b'}$, we have the same expression of the resultant:

$$\Delta^2\boldsymbol{F}_b+\Delta^2\boldsymbol{F}_{b'}=\frac{\mu_0}{4\pi}I^2\left(\frac{\Delta l_b}{r_b}\right)^2\sin 2\alpha\,\boldsymbol{e}_y$$

We extend the number of element to $n+1$ pairs and replace the indices by i spanning from 0 to n. The sum of the forces on the $n+1$ pairs is:

$$\boldsymbol{F}_n=\sum_{i=0}^{i=n}\left(\Delta^2\boldsymbol{F}_i+\Delta^2\boldsymbol{F}_{i'}\right)=\sum_{i=0}^{i=n}\frac{\mu_0}{4\pi}I^2\left(\frac{\Delta l_i}{r_i}\right)^2\sin 2\alpha\,\boldsymbol{e}_y \qquad (1)$$

The distance from the summit is l_i for the i^{th} pair. We make l_i and l_{i+1} related as follow:

$$l_{i+1}=k\cdot l_i$$

Also, we choose the length of the elements related the same way:

$$\Delta l_{i+1}=k\cdot\Delta l_i$$

So, the distance and length for the i^{th} pair are:

$$l_i=k^i\cdot l_0\ ,\quad \Delta l_i=k^i\cdot\Delta l_0$$

With k<1, the distance decreases to 0 but the elements never reaches the summit. The original length Δl_0 is small so that the elements never overlap. With this series of distance and length, we have for all *i*:

$$\frac{\Delta l_i}{r_i} = \frac{\Delta l_0}{r_0}$$

And the equation (1) becomes:

$$\boldsymbol{F}_n = (n+1)\frac{\mu_0}{4\pi} I^2 \left(\frac{\Delta l_0}{r_0}\right)^2 \sin 2\alpha \, \boldsymbol{e}_y$$

This equation tends to infinity when *n* increases:

$$\boldsymbol{F_n} \xrightarrow[n\to\infty]{} \infty$$

The Lorentz force that is greater than $\boldsymbol{F}_n$, a part force, will be infinity near the summit. Let us examine the Lorentz force on a side of a triangular coil. The extremities of a side are 2 summits. As the magnitude of the force tends to infinity on the summits, the Lorentz force on the side tends also to infinity.

I have proven in the following articles that the Lorentz force internal to a triangular coil does not respect the third Newton's law,
Synthesis of the inconsistency of the Lorentz force law
Mathematical cause of the existence of the remaining resultant internal Lorentz force
Proof of the remaining resultant Lorentz force internal to a triangular coil

The present study shows that the Lorentz force has even not a definite value. Indeed, the force has a definite value only on part of a side and increases to infinity when the extremities of the part approach the 2 summits. This phenomenon puzzled me when I did my numerical computations. I have found that the numerical value of Lorentz force did not converge to a finite limit, but instead increased when the length of the discrete elements decreased. I have had to stop the computation before the sides reached their full lengths to get a reasonable value. Actually, the finite limit does not exist and the above study gives the theoretical explanation.

In conclusion, for magnetic force internal to a coil, the Lorentz force law is definitely false. This will reassure experimenters, because they will surely find a great result: that the Lorentz force law will be refuted.

For the differential Ampere's law, we know that it respects the third Newton's law, that the differential force is proportional to the currents and lengths of the 2 current elements, that the double integral of the law gives the same result than the Lorentz force law for interactions between 2 or more coils. These properties are strong and mutually corroborating hints, and there is little doubt that this law will be proven by experiments.