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Electromagnetic Wave Paradox

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In 2 previous articles, I have shown that the magnetic field generated by displacement current violated energy conservation law. But, this type of magnetic field was involved neither in Lorentz force nor in EMF. Its only use is for electromagnetic wave. Here, we will check the consistency of electromagnetic wave equation.

The general wave equation is:

$$\nabla\times\nabla\times\boldsymbol{H}+\mu_0\varepsilon_0\frac{\partial^2\boldsymbol{H}}{\partial t^2}=0 \tag{1}$$

We study the spherical wave sent by an element of antenna of length *dl* which carries a current *I* (see the Figure 1). As the magnetic field of this element is only in the ϕ direction, in spherical coordinates, the above wave equation simplifies to the scalar equation of the ϕ component (see annex for the derivation):

$$\mu_0\varepsilon_0\frac{\partial^2 H_\phi}{\partial t^2}-\frac{1}{r}\left(\frac{\partial^2 rH_\phi}{\partial r^2}+\frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\left(\frac{\partial rH_\phi\sin\theta}{\partial\theta}\right)\right)\right)=0 \tag{2}$$

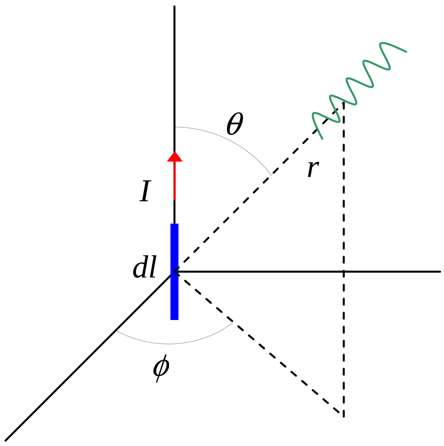

Figure 1

This equation admits the following analytical solution (ref. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.590):

$$H_\phi=\frac{Idl}{4\pi}\beta_0^2\sin\theta\left(\frac{jr^{-1}}{\beta_0}+\frac{r^{-2}}{\beta_0^{\,2}}\right)e^{j(\omega t-\beta_0 r)} \tag{3}$$

In the following we call this solution “antenna wave”. This expression can be simplified by defining an additional phase α as follow:

$$\cos\alpha=\frac{1}{\sqrt{1+\beta_0^{\,2}r^2}},\tan\alpha=\beta_0 r \tag{4}$$

$$H_\phi=\frac{Idl}{4\pi}r^{-2}\sqrt{1+\beta_0^{\,2}r^2}\sin\theta(\cos\alpha+j\sin\alpha)e^{j(\omega t-\beta_0 r)}$$

Then, the equation (3) is transformed into the complex exponential form below:

$$H_\phi = \frac{Idl}{4\pi} r^{-2} \sqrt{1+\beta_0^{\ 2} r^2} \sin\theta e^{j(\omega t - \beta_0 r + \arctan(\beta_0 r))} \tag{5}$$

The definition of β_0 is $\beta_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ and we can write $\beta_0 r$ as a function of wave length λ:

$$\beta_0 = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}, \quad \beta_0 r = 2\pi\frac{r}{\lambda} \tag{6}$$

The equation (5) is then the function of r/λ below:

$$H_\phi = \frac{Idl}{4\pi} r^{-2} \sqrt{1+\left(2\pi\frac{r}{\lambda}\right)^2} \sin\theta e^{j\left(\omega t - 2\pi\frac{r}{\lambda} + \arctan\left(2\pi\frac{r}{\lambda}\right)\right)} \tag{7}$$

The argument of the complex exponential permits us to compute the time a wave takes to cover the distance between the source, $r=0$, and an arbitrary point r. For doing so, we mark a point on the wave by its phase, Φ. Take a point at the source, its phase is:

$$t_0 = 0, r_0 = 0 \Rightarrow \Phi_0 = \omega t_0 - 2\pi\frac{r_0}{\lambda} + \arctan\left(2\pi\frac{r_0}{\lambda}\right) = 0$$

After a period of time t_1, this point arrives at r_1, the phase is:

$$\Phi_1 = \omega t_1 - 2\pi\frac{r_1}{\lambda} + \arctan\left(2\pi\frac{r_1}{\lambda}\right)$$

Since the point (t_1, r_1) is the point (t_0, r_0) that has traveled the distance r_1, their phases Φ_0 and Φ_1 have the same value. Then we have:

$$\Phi_0 = \Phi_1 \Rightarrow \omega t_1 - 2\pi\frac{r_1}{\lambda} + \arctan\left(2\pi\frac{r_1}{\lambda}\right) = 0$$

So, the time at which the point arrives at the distance r_1 is:

$$t_1 = \frac{2\pi}{\omega}\frac{r_1}{\lambda} - \frac{1}{\omega}\arctan\left(2\pi\frac{r_1}{\lambda}\right)$$

With $T = \frac{2\pi}{\omega}$ the period of the antenna wave, $t_c = \frac{2\pi}{\omega}\frac{r_1}{\lambda} = \frac{r_1}{c}$ the time that light takes to cover the same distance, the time the wave takes is:

$$t_1 = t_c - \frac{T}{2\pi}\arctan\left(2\pi\frac{r_1}{\lambda}\right) \tag{8}$$

When r_1 tends to infinity, the time becomes t_∞:

$$r_1 \to \infty, \quad \arctan\left(2\pi\frac{r_1}{\lambda}\right) \to \frac{\pi}{2} \quad \Rightarrow \quad t_\infty = t_c - \frac{T}{4} \tag{9}$$

Here we obtain a key result: the antenna wave travels a fourth of a period ahead of light. Can light be overtaken by electromagnetic wave? No. This result is a violation of the relativity principle.

Why does this happen? Let us compute the velocity of the antenna wave. Consider 2 points on the wave, of phases Φ_1 and Φ_2. The expressions of the phases are (see equation (5)):

$$\Phi_1 = \omega t_1 - \beta_0 r_1 + \alpha_1, \quad \Phi_2 = \omega t_2 - \beta_0 r_2 + \alpha_2$$

The point 2 is in fact the point 1 moved to r_2. Thus, $\Phi_1 = \Phi_2$ and we obtain the equation:

$$(\omega t_1 - \beta_0 r_1 + \alpha_1) - (\omega t_2 - \beta_0 r_2 + \alpha_2) = 0$$

So we have:

$$\omega \Delta t - \beta_0 \Delta r + \Delta\alpha = 0$$

$$\omega = \frac{\beta_0 \Delta r - \Delta\alpha}{\Delta t} = \frac{\beta_0 \Delta r}{\Delta t} - \frac{\Delta\alpha}{\Delta r}\frac{\Delta r}{\Delta t}$$

For point 1 very close to point 2, we have:

$$\omega = \left(\beta_0 - \frac{\partial\alpha}{\partial r}\right)\frac{\partial r}{\partial t}$$

The phase velocity of the antenna wave is:

$$v = \frac{\partial r}{\partial t} = \frac{\omega}{\beta_0 - \frac{\partial\alpha}{\partial r}} \tag{10}$$

Using the equation (4) we have:

$$\frac{\partial \tan\alpha}{\partial r} = \frac{1}{\cos^2\alpha}\frac{\partial\alpha}{\partial r} \Rightarrow \frac{\partial\alpha}{\partial r} = \frac{\beta_0}{1 + \beta_0{}^2 r^2}$$

Then the phase velocity of the wave is:

$$v = \frac{\omega}{\beta_0 - \beta_0\left(1 + \beta_0{}^2 r^2\right)^{-1}} = \frac{\omega}{\beta_0}\frac{1}{1 - \left(1 + \beta_0{}^2 r^2\right)^{-1}}$$

$$v = \frac{\omega}{\beta_0}\left(\frac{1}{\beta_0{}^2 r^2} + 1\right)$$

By using the equation (6), the phase velocity becomes:

$$v = c\left(1 + \frac{1}{4\pi^2}\left(\frac{\lambda}{r}\right)^2\right) \tag{11}$$

We see that v is greater than the speed of light c; v tends even to infinity if r tends to 0. This is why antenna wave is in advance of light. We can also wonder about the speed of a real signal, which can be considered as a Fourier series of sinusoidal waves. As the phase velocity of sinusoidal wave of any frequency is higher than that of light (equation (11)), the real signal travels faster than light too.

In order to give a visual feeling of this effect, I have computed the magnitude of antenna wave and drawn it against a light signal carrying a signal of varying intensity at the same frequency. At the source, the antenna wave and the light signal are at the same phase. After 3 periods, antenna wave has already an advance of ¼ of period ahead of the light signal (see Figure 2).

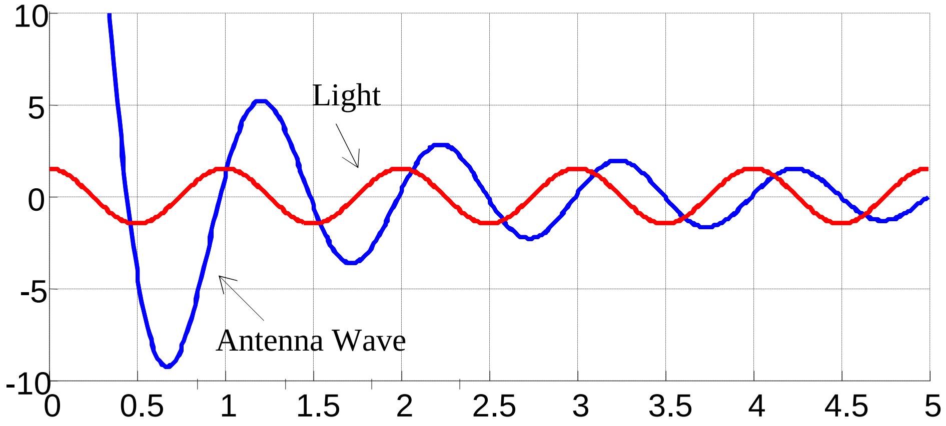

Figure 2

So, the solution of electromagnetic wave equation travels faster than light. Its velocity is even infinitely large near the source, that is, for small *r* (see equation (11)). If the wave equation were exact, the advance of antenna wave with respect to light signal could be measured by experiment. But, do we really need an experiment to verify that the relativity principle is inviolable (remember the neutrinos experiment)?

The only possible explanation of this advance is that the solution of electromagnetic wave equation is wrong, and in consequence, the electromagnetic wave equation is wrong. In fact, this inconsistency comes from that of displacement current that I explained in
Phantom Lorentz force Paradox http://pengkuanem.blogspot.com/2012/07/phantom-lorentz-force-paradox.html
Displacement Current Paradox http://pengkuanem.blogspot.com/2012/07/displacement-current-paradox.html

As the displacement current theory is incorrect, the wave equation that is derived from cannot be correct. Then we can imagine, as wave equation is incorrect, all physical equation and conclusion that are derived from are incorrect. That is, a great amount of conclusion of modern physics and indications the wave equation provides for future research become unreliable. This is a very grave conclusion. As we do not have a correct theory to replace the incorrect one, physics enter into an unprecedented crisis.

I have written the mathematical derivation of the used wave equation (the equation (2)) and the mathematical verification of the analytical solution (the equation (3), that is given in “Introduction to Electromagnetic Fields” of Clayton R. Paul, Keith W. Whites and Syed A. Nasar), in the annex for anyone who doubts the validity of these equations.

Although the wave equation is proven to be incorrect, the electromagnetic wave phenomenon is real. The trouble is only for the mathematical formulation and physical interpretation. If we correct the magnetic field theory, we could find the right wave equation and all would return to normal. More over, we can find new discovery. For example, I have solved the Aharonov–Bohm effect in quantum mechanics.

Unfortunately, my article about the Aharonov–Bohm effect was rejected everywhere, because the physical community opposes all questioning of electromagnetism. This is a collective psychological barrier that impedes progress in physics. In such an atmosphere of hostility, no correction of the present magnetic field theory is possible and because of bad theory, physics would stay at the present state of knowledge forever.

For this not to happen, we have to break the blind trust in classic electromagnetism. The most efficient way is to carry out the experiments I propose in the articles
Non Loop EMF Experiment http://pengkuanem.blogspot.com/2012/06/non-loop-emf-experiment.html
The Lorentz torque experiment http://pengkuanem.blogspot.com/2012/03/lorentz-torque-experiment.html
that would provide experimental evidences of the inconsistencies of classic electromagnetism.

The other thing to do is to convince a great number of physicists, especially those with great influence. For spreading the message, I call for discussion of the paradoxes among readers and your colleagues. The only way that this great progress could break though is to convince the physical community of the failure of classic electromagnetism.

Annex 1. Wave equations

General wave equation is the equation (1). The expression for curl in spherical coordinates is:

$$\nabla\times\boldsymbol{H}=\frac{1}{r\sin\theta}\left(\frac{\partial H_\phi\sin\theta}{\partial\theta}-\frac{\partial H_\theta}{\partial\phi}\right)\boldsymbol{e}_r+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi}-\frac{\partial rH_\phi}{\partial r}\right)\boldsymbol{e}_\theta+\frac{1}{r}\left(\frac{\partial rH_\theta}{\partial r}-\frac{\partial H_r}{\partial\theta}\right)\boldsymbol{e}_\phi$$

The magnetic field is only in the ϕ direction. So the expression for curl of magnetic field is:

$$H_r=0,\ H_\theta=0\Rightarrow\nabla\times\boldsymbol{H}=\frac{1}{r\sin\theta}\left(\frac{\partial H_\phi\sin\theta}{\partial\theta}\right)\boldsymbol{e}_r-\frac{1}{r}\frac{\partial rH_\phi}{\partial r}\boldsymbol{e}_\theta$$

Spherical symmetry makes:

$$\frac{\partial}{\partial\phi}=0\Rightarrow(\nabla\times\nabla\times\boldsymbol{H})_\phi=-\frac{1}{r}\left(\frac{\partial^2 rH_\phi}{\partial r^2}+\frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\left(\frac{\partial rH_\phi\sin\theta}{\partial\theta}\right)\right)\right)$$

The final expression of the wave equation is:

$$(\nabla\times\nabla\times\boldsymbol{H})_\phi+\mu_0\varepsilon_0\frac{\partial^2 H_\phi}{\partial t^2}=0\Leftrightarrow\mu_0\varepsilon_0\frac{\partial^2 H_\phi}{\partial t^2}-\frac{1}{r}\left(\frac{\partial^2 rH_\phi}{\partial r^2}+\frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\left(\frac{\partial rH_\phi\sin\theta}{\partial\theta}\right)\right)\right)=0$$

Annex 2. Verification of the solution

In order to simplify the derivation, a variable h is used:

$$H_\phi=\frac{Idl}{4\pi}\beta_0^2\sin\theta\cdot h\Rightarrow h=\left(\frac{jr^{-1}}{\beta_0}+\frac{r^{-2}}{\beta_0{}^2}\right)e^{j(\omega t-\beta_0 r)}$$

Introducing the analytical solution (equation (5)):

$$\frac{\partial rh}{\partial r}=\frac{\partial}{\partial r}\left[\left(\frac{j}{\beta_0}+\frac{r^{-1}}{\beta_0{}^2}\right)e^{j(\omega t-\beta_0 r)}\right]=-\frac{r^{-2}}{\beta_0{}^2}e^{j(\omega t-\beta_0 r)}-j\beta_0\left(\frac{j}{\beta_0}+\frac{r^{-1}}{\beta_0{}^2}\right)e^{j(\omega t-\beta_0 r)}$$

$$=\left[-\frac{r^{-2}}{\beta_0{}^2}+\frac{-jj\beta_0}{\beta_0}+\frac{-j\beta_0 r^{-1}}{\beta_0{}^2}\right]e^{j(\omega t-\beta_0 r)}=\left[-\frac{r^{-2}}{\beta_0{}^2}+1-\frac{jr^{-1}}{\beta_0}\right]e^{j(\omega t-\beta_0 r)}$$

$$\frac{\partial^2 rh}{\partial r^2}=\frac{\partial}{\partial r}\left(\left(-\frac{r^{-2}}{\beta_0{}^2}+1-\frac{jr^{-1}}{\beta_0}\right)e^{j(\omega t-\beta_0 r)}\right)$$

$$=\left[2\frac{r^{-3}}{\beta_0{}^2}+\frac{jr^{-2}}{\beta_0}\right]e^{j(\omega t-\beta_0 r)}-j\beta_0\left[-\frac{r^{-2}}{\beta_0{}^2}+1-\frac{jr^{-1}}{\beta_0}\right]e^{j(\omega t-\beta_0 r)}$$

$$=\left[2\frac{r^{-3}}{\beta_0{}^2}+\frac{jr^{-2}}{\beta_0}+j\beta_0\frac{r^{-2}}{\beta_0{}^2}-j\beta_0+j\beta_0\frac{jr^{-1}}{\beta_0}\right]e^{j(\omega t-\beta_0 r)}$$

The second term of the equation (2) becomes then:

$$\frac{\partial^2 rH_\phi}{\partial r^2}=\frac{Idl}{4\pi}\beta_0^2\sin\theta\left[2\frac{r^{-3}}{\beta_0{}^2}+2\frac{jr^{-2}}{\beta_0}-j\beta_0-r^{-1}\right]e^{j(\omega t-\beta_0 r)}$$

Derivation of the third term in the equation (2):

$$\frac{\partial rH_\phi\sin\theta}{\partial\theta}=\frac{\partial}{\partial\theta}\left(\frac{Idl}{4\pi}\beta_0^2\sin^2\theta\left(\frac{j}{\beta_0}+\frac{r^{-1}}{\beta_0{}^2}\right)e^{j(\omega t-\beta_0 r)}\right)=\left(\frac{j}{\beta_0}+\frac{r^{-1}}{\beta_0{}^2}\right)\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\cdot 2\sin\theta\cos\theta$$

$$\frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\left(\frac{\partial rH_\phi\sin\theta}{\partial\theta}\right)\right)=\frac{\partial}{\partial\theta}\left(\frac{1}{r^2\sin\theta}\left(\frac{j}{\beta_0}+\frac{r^{-1}}{\beta_0{}^2}\right)\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\cdot 2\sin\theta\cos\theta\right)$$

$$=\frac{\partial}{\partial\theta}\left(\left(\frac{jr^{-2}}{\beta_0}+\frac{r^{-3}}{\beta_0{}^2}\right)\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\cdot 2\cos\theta\right)=-\left(\frac{jr^{-2}}{\beta_0}+\frac{r^{-3}}{\beta_0{}^2}\right)\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\cdot 2\sin\theta$$

The expression of $(\nabla\times\nabla\times\boldsymbol{H})_\phi$ (see equation (1) and (2)) is then:

$$(\nabla\times\nabla\times\boldsymbol{H})_\phi=-r^{-1}\left(\begin{array}{l}\left(2\frac{r^{-3}}{\beta_0{}^2}+2\frac{jr^{-2}}{\beta_0}-j\beta_0-r^{-1}\right)\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\\-\left(\frac{jr^{-2}}{\beta_0}+\frac{r^{-3}}{\beta_0{}^2}\right)\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}\cdot 2\sin\theta\end{array}\right)$$

$$=-r^{-1}\left(\left(2\left(\frac{r^{-3}}{\beta_0{}^2}+\frac{jr^{-2}}{\beta_0}\right)-j\beta_0-r^{-1}\right)-2\left(\frac{jr^{-2}}{\beta_0}+\frac{r^{-3}}{\beta_0{}^2}\right)\right)\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}$$

$$(\nabla\times\nabla\times\boldsymbol{H})_\phi=-r^{-1}\left(-j\beta_0-r^{-1}\right)\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}=\left(j\beta_0r^{-1}+r^{-2}\right)\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}$$

The term of time derivative is:

$$\frac{\partial^2 H_\phi}{\partial t^2}=\frac{Idl}{4\pi}\beta_0^2\sin\theta\left(\frac{jr^{-1}}{\beta_0}+\frac{r^{-2}}{\beta_0{}^2}\right)\left(-\omega^2\right)e^{j(\omega t-\beta_0 r)}$$

Introducing the above 2 expressions into the equation (2):

$$(\nabla\times\nabla\times\boldsymbol{H})_\phi+\mu_0\varepsilon_0\frac{\partial^2 H_\phi}{\partial t^2}=\left(j\beta_0r^{-1}+r^{-2}\right)\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}-\omega^2\mu_0\varepsilon_0\frac{Idl}{4\pi}\beta_0^2\sin\theta\left(\frac{jr^{-1}}{\beta_0}+\frac{r^{-2}}{\beta_0{}^2}\right)e^{j(\omega t-\beta_0 r)}$$

By using $\beta_0=\omega\sqrt{\mu_0\varepsilon_0}=\frac{\omega}{c}$, the analytical solution makes the wave equation (2) equal to 0:

$$(\nabla\times\nabla\times\boldsymbol{H})_\phi+\mu_0\varepsilon_0\frac{\partial^2 H_\phi}{\partial t^2}=\left[\left(j\beta_0r^{-1}+r^{-2}\right)-\beta_0^2\left(\frac{jr^{-1}}{\beta_0}+\frac{r^{-2}}{\beta_0{}^2}\right)\right]\sin\theta\frac{Idl}{4\pi}\beta_0^2e^{j(\omega t-\beta_0 r)}=0$$