Why magnetic field must be a tensor?

Peng Kuan 彭宽 titang78@gmail.com

24 September 2013

1. Magnetic field tensor of a circuit

I have introduced the notion of tensor magnetic field in
Correction to the Biot-Savart law (PDF, word)
and the expression for the magnetic field at the point (x_2, y_2, z_2) in space and originated from the current element $d\boldsymbol{I}_1$ situated at (x_1, y_1, z_1) is this tensor:

$$[d\boldsymbol{M}] = -\frac{\mu_0}{4\pi}\begin{bmatrix} dI_{1,x}\frac{x_2-x_1}{r^3} & dI_{1,x}\frac{y_2-y_1}{r^3} & dI_{1,x}\frac{z_2-z_1}{r^3} \\ dI_{1,y}\frac{x_2-x_1}{r^3} & dI_{1,y}\frac{y_2-y_1}{r^3} & dI_{1,y}\frac{z_2-z_1}{r^3} \\ dI_{1,z}\frac{x_2-x_1}{r^3} & dI_{1,z}\frac{y_2-y_1}{r^3} & dI_{1,z}\frac{z_2-z_1}{r^3} \end{bmatrix} \quad (1)$$

With:

$$r^3 = ((x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2)^{\frac{3}{2}}$$
$$[d\boldsymbol{I}_1] = [dI_{1,x} \quad dI_{1,y} \quad dI_{1,z}]$$

The magnetic force that this field acts on the current element $d\boldsymbol{I}_2$ situated at (x_2, y_2, z_2) is:

$$d^2\boldsymbol{F} = [d\boldsymbol{I}_2]\,[d\boldsymbol{M}]$$
$$\text{With } [d\boldsymbol{I}_2] = [dI_{2,x} \quad dI_{2,y} \quad dI_{2,z}] \quad (2)$$

Let us consider the interaction between a complete circuit c and the current element $d\boldsymbol{I}_2$. The magnetic force that c exerts on $d\boldsymbol{I}_2$ is the integral of $d^2\boldsymbol{F}$ over c:

$$d\boldsymbol{F} = \int_c [d\boldsymbol{I}_2]\,[d\boldsymbol{M}] = [d\boldsymbol{I}_2]\int_c [d\boldsymbol{M}] \quad (3)$$

By considering this force as the product of the current element $d\boldsymbol{I}_2$ and the local magnetic field tensor $[\boldsymbol{M}]$, the expression for the magnetic field from the circuit c is (see the Figure 1):

$$[\boldsymbol{M}] = \int_c [d\boldsymbol{M}] \quad (4)$$

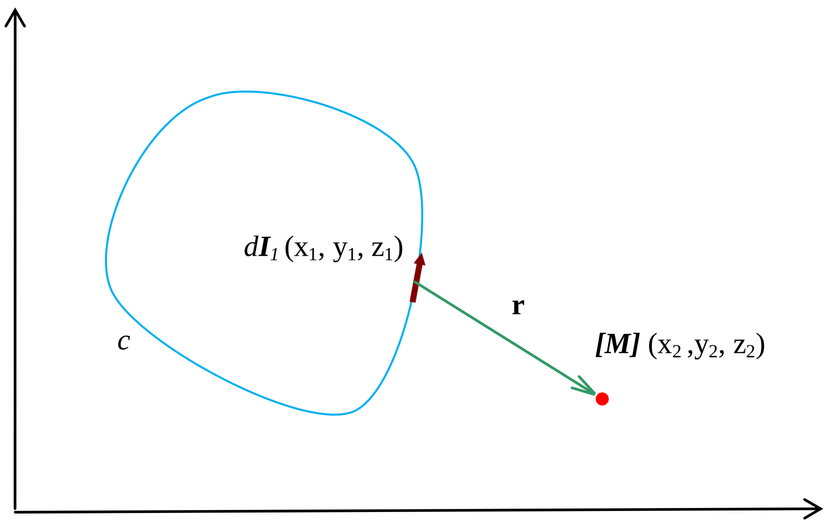

Figure 1

By substituting

$$dI_{1,x} = I_1 dx_1 \quad dI_{1,y} = I_1 dy_1 \quad dI_{1,z} = I_1 dz_1 \tag{5}$$

into the equation (1), we obtain the expression for the magnetic field from the complete circuit *c* at the point (x_2, y_2, z_2), the tensor [$\boldsymbol{M}$]:

$$[\boldsymbol{M}] = -\frac{\mu_0 I_1}{4\pi}\begin{bmatrix} \int_c \frac{x_2 - x_1}{r^3} dx_1 & \int_c \frac{y_2 - y_1}{r^3} dx_1 & \int_c \frac{z_2 - z_1}{r^3} dx_1 \\ \int_c \frac{x_2 - x_1}{r^3} dy_1 & \int_c \frac{y_2 - y_1}{r^3} dy_1 & \int_c \frac{z_2 - z_1}{r^3} dy_1 \\ \int_c \frac{x_2 - x_1}{r^3} dz_1 & \int_c \frac{y_2 - y_1}{r^3} dz_1 & \int_c \frac{z_2 - z_1}{r^3} dz_1 \end{bmatrix} \tag{6}$$

$$\text{With } r^3 = ((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)^{\frac{3}{2}}$$

2. Straight infinite current

Let us see an example of magnetic field tensor, that of an infinite straight current lying on the z axis (see Figure 2). For this current we have:

$$\begin{matrix} dx_1 = 0 & dy_1 = 0 \\ x_1 = 0 & y_1 = 0 \end{matrix} \tag{7}$$

We compute the magnetic field at the point (x_2, y_2, z_2=0) by applying these conditions to the equation (6). We obtain the tensor below:

$$[\boldsymbol{M}] = -\frac{\mu_0 I_1}{4\pi}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \int_c \frac{x_2}{r^3} dz_1 & \int_c \frac{y_2}{r^3} dz_1 & -\int_c \frac{z_1}{r^3} dz_1 \end{bmatrix} \tag{8}$$

Because of symmetry we have:

$$\int_c \frac{z_1}{r^3} dz_1 = 0 \tag{9}$$

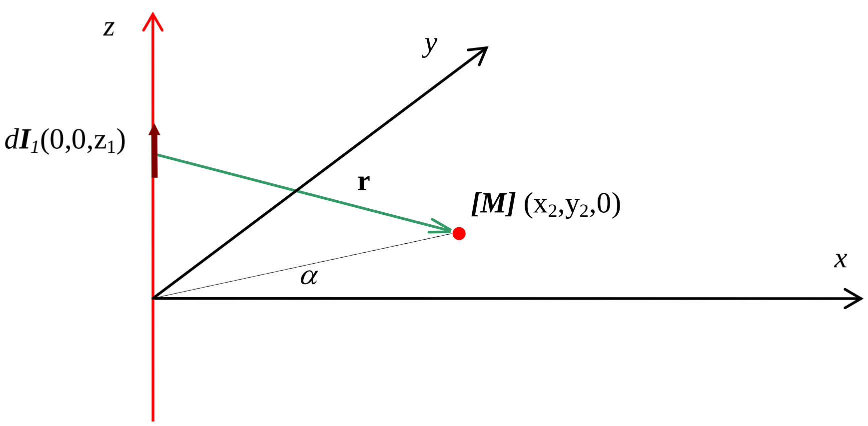

Figure 2

We also know for the other 2 integrals:

$$\int_c \frac{dz_1}{r^3} = \frac{1}{2\sqrt{x_2{}^2 + y_2{}^2}} \tag{10}$$

Finally, by using the functions of the angle α (see Figure 2):

$$\frac{x_2}{\sqrt{x_2{}^2 + y_2{}^2}} = \cos\alpha \,, \frac{y_2}{\sqrt{x_2{}^2 + y_2{}^2}} = \sin\alpha \tag{11}$$

The magnetic field tensor becomes:

$$[\boldsymbol{M}] = -\frac{\mu_0 I_1}{4\pi}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\cos\alpha}{2} & \frac{\sin\alpha}{2} & 0 \end{bmatrix} \tag{12}$$

Let us compute the magnetic force on $d\boldsymbol{I}_2$ defined by the following line tensor:

$$[d\boldsymbol{I}_2] = I_2[dx_2 \quad dy_2 \quad dz_2] \tag{13}$$

This force is the product of $[d\boldsymbol{I}_2]$ and $[\boldsymbol{M}]$:

$$\begin{aligned} d\boldsymbol{F} &= [d\boldsymbol{I}_2][\boldsymbol{M}] \\ &= -\frac{\mu_0 I_1 I_2}{4\pi}\left[\frac{\cos\alpha}{2}dz_2 \quad \frac{\sin\alpha}{2}dz_2 \quad 0\right] \end{aligned} \tag{14}$$

This force is directed to the infinite straight current, lies on the z=0 plan and depends only on the z component dz_2. When $d\boldsymbol{I}_2$ is horizontal, dz_2=0 and the force on $d\boldsymbol{I}_2$ is zero (see Figure 3).

Let us see how the Lorentz force behaves. The magnetic field from $d\boldsymbol{I}_2$ is $\boldsymbol{B}_2$ and $\boldsymbol{B}_3$ on the upper and lower half of the infinite straight current respectively. So, the Lorentz forces on the upper and lower half are $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$ (see Figure 3). Because of symmetry, $\boldsymbol{F}_2$ =- $\boldsymbol{F}_3$. So, the resultant Lorentz force on the infinite straight current is zero. But the Lorentz force on $d\boldsymbol{I}_2$ is $\boldsymbol{F}_1$ which is not zero. As there cannot be standalone force in space, these Lorentz forces must be wrong.

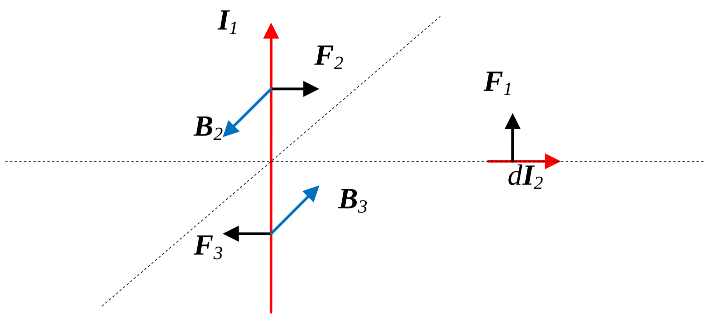

Figure 3

3. Why magnetic field must be a tensor?

The mutual magnetic force on 2 circuits must be oriented toward the other circuit to respect the Newton's third law. Roughly speaking, in the Figure 4 the magnetic force that the loops c_1 exerts on the current element $d\boldsymbol{I}_2$ must be directed toward c_1 and, if c_1 is moved to the position of c_2, the force must be directed toward c_2. The direction of the force is roughly constant when $d\boldsymbol{I}_2$ changes direction. Otherwise, the Newton's third law would be violated.

Mathematically speaking, the computation of the magnetic force on a current element is to transform the current element vector into a force vector through the multiplication with a magnetic field tensor. That is, magnetic field tensor is a linear transformation in affine space. A current element vector is an arbitrary vector in space. As the current intensity, the shape and the position of the loop are arbitrary, the force vector is also an arbitrary vector. In order to transform an arbitrary vector into another arbitrary vector, the linear transformation must have 9 independently variable components.

The Lorentz force law fails because vector magnetic field has only 3 independent components and confines the force vector in the plan perpendicular to the current element, forbidding it to direct freely in space. So, in order to keep the Lorentz force law valid, we are forced to permit it to "legally" violate the Newton's third law.

So, for the sake of Newton's third law and mathematical principle, magnetic field must be a tensor and the Lorentz force law must be wrong.

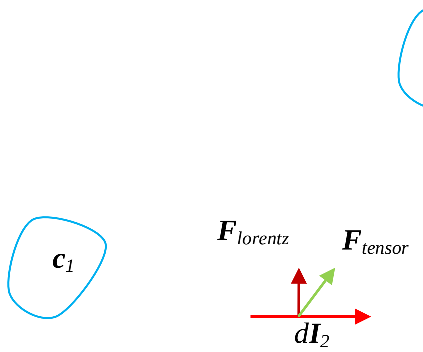

Figure 4

4. Warning

As I call for experimenter, I want to warn them about a pitfall in which I fell into: the magnetization of the wire by the current it carries. See:
Success of the modified Lorentz perpendicular action experiment (blogspot academia)

Before my experiments, I thought naively that magnetic force came from currents exclusively and if I put two currents perpendicular to each other, they would not interact. I was gravely shocked when the test coil turned under perpendicular current. Why!!!!! I wondered. Have I made an error in my theory? I had found none. Finally, I realized that the wires are magnetized by the current within! This phenomenon was unknown and it is not surprising that I was trapped.

We know nothing about this effect. I guess that when the wires are magnetized, they act like perpendicular magnet strings and attract each other. I have deduced that the magnetic field of a segment of magnetized wire would have a magnitude inversely proportional to the 4th power of the distance. So, if the magnitude is m_v at distance 1, its magnitude at r should be:

$$M_v = \frac{m_v}{r^4} \tag{15}$$

On the other hand, the magnitude of the magnetic field from the current in this segment is inversely proportional to the distance squared. If the magnitude is m_t at distance 1, its magnitude at r should be:

$$M_t = \frac{m_t}{r^2} \tag{16}$$

There is a distance from which M_v becomes negligible before M_t. The location of measurement of magnetic force must be farther than that distance. How far? Only experiments can tell.

5. Comment

Now, the tensor law of magnetic force is stated. It remains to convince the physics community that it is the right theory. Let us summarize the evidences we have:

1) Magnetic field must be a tensor for the sake of Newton's third law and mathematical principle.
2) The Lorentz force law gives paradoxical net force to triangular and non symmetric loops and open circuit (See Summary).
3) The experiments of parallel action and macroscopic Aharonov-Bohm effect indicate that magnetic field is of tensor nature and contradict the Lorentz force law.
   See Consequences of macroscopic Aharonov-Bohm effect (PDF, word)
   Current and parallel action (PDF, word)
4) The tensor law of magnetic force gives the same value to the magnetic force for closed loop than the Lorentz force law.

Each of these 4 evidences taken alone may be weak, but taken together they are sufficiently strong to break the Lorentz force law down. They are also strong indication that magnetic field is really tensorial.

Many times people argue that the classical electromagnetic theory cannot be wrong because it has successfully resisted experimental test for 150 years. Really? What about the parallel action and macroscopic Aharonov-Bohm effect experiments? Why these experiments were not imagined before? In fact, Aharonov-Bohm effect experiment was actually carried out in 1960, but on microscopic level. Had they gone ahead to test on the macroscopic level and the history of science could be rewritten. But the classical electromagnetic theory was believed flawless and this idea was just skipped. The classical electromagnetic theory was successful for 150 years not because it is perfect, but because it has not been tested with the right experiment.

The example of Aharonov-Bohm effect experiment shows how a bad theory impedes the advance of science. A handful contradictory evidences exits in the classical electromagnetic theory. Professor Richard Feynman has cited some in his Lectures on Physics. But they are just ignored. All the elements for the tensor magnetic force law were present back in the beginning of the 20th century, but the wrong law still reigns now. It is not surprising that physics has not had great breakthrough for at least 50 years.

Now, the tensor theory for electromagnetism is born, the paradigm of Maxwell's theory will be overturned and with new insight in physics we will see interest on ideas that would be simply rejected before. For example, as quantum and particle physics are based on vector electromagnetic theory, new discovery will soon be coming. I can already show some examples:

1) Maxwell's equations show a sort of symmetry between electric and magnetic vector fields and a counterpart for electron is suspected to exist for magnetism: magnetic monopole. But tensor

magnetic field breaks down this symmetry. So, we can trash all researches relative to magnetic monopole.

2) The macroscopic Aharonov-Bohm effect experiment shows that this effect is not a quantum effect.
3) Magnetization of wire by the current within has caused the failure of my experiment. But it is also the first unknown effect discovered by the tensor theory, while it looked just well for the Lorentz force law.

The tensor theory of electromagnetism is in its very early age, but discoveries already flourish. So, if you want to do discovery in these fields, you should be better to master this theory. The best way to learn this theory is to do experiments for it right now.