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Faraday's Law Paradox

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Faraday's law defines how a varying magnetic field creates electric field. Its expression is:

$$\oint_c \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\iint_s \mathbf{B}\cdot d\mathbf{s} \tag{1}$$

Consider a circular wire ABCD, the wire is cut between A and D. In a varying magnetic field **B**, an EMF will appear in the wire (see Figure 1). After a transient phase, the conductor reaches an electrostatic equilibrium, that is, there is not current flowing. Suppose that the rate of variation of magnetic field is constant, the magnitude of electric field E_F in the wire will be constant. The EMF in the whole wire is the difference of potential between the points A and D:

$$U_D - U_A = E_F \cdot l_{ABCD} = -\frac{d\Phi}{dt} \tag{2}$$

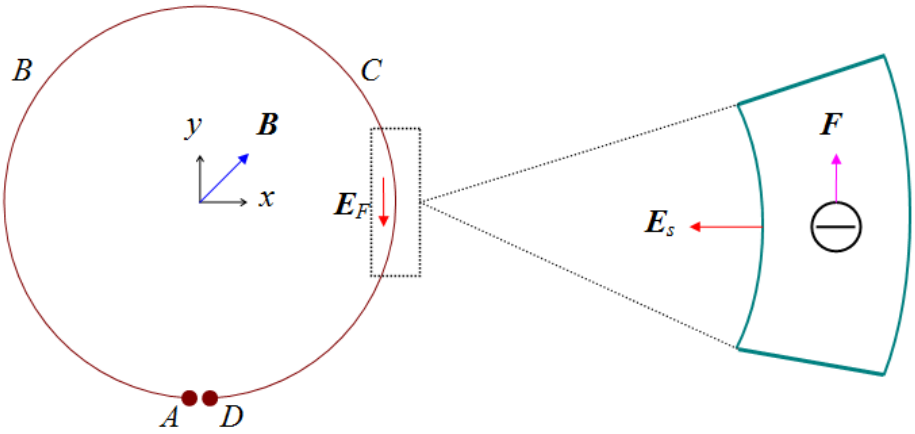

Figure 1

For the potential to be nonzero, E_F must be nonzero. However, if we look at the electric field inside the conductor, it is not so. The right part of the Figure 1 is a magnified view of a segment of the wire where a free electron is shown. As the current is zero, the electrons do not move. According to an electrostatic law, the force on the free electron must be zero; otherwise, the distribution of free electrons will change so that the electric field on the free electron becomes zero. On the surface, the electric field $\mathbf{E}_s$ must be perpendicular to the surface. Considering such electric field in conductor, the potential between the point A and D is zero:

$$\mathbf{F} = 0 \Rightarrow U_D - U_A = \int_A^D \mathbf{E}_s d\mathbf{l} = 0 \tag{3}$$

In this situation, the electric field is static and the surface of the wire is an equipotential. The point A and D are at the same potential and the tension between them must be 0. So, we have a paradox. If we trust the electrostatic law, there should be zero potential difference between the points A and D, that is, EMF is zero. But if we trust Faraday's law, the electric field in the wire is not zero and the free electron must stand against the electrostatic force without moving:

$$\mathbf{E}_F \neq 0 \Rightarrow \mathbf{F} = -e\mathbf{E}_F \neq 0 \tag{4}$$

How can free electrons stay still against nonzero EMF? And how can EMF exist when the electric field on all free electrons is zero or perpendicular to the surface? No explanation

exists now. So, there is a conflict between Faraday's law and the electrostatic law. I call this conflict the Faraday's law Paradox. Faraday's law is the last major law of the electromagnetic theory to fall. In fact, I have given more then one paradox about each major law:

- Lorentz force law violates the energy conservation law (see the article "Lorentz force on open circuit").
- Generalized Ampere's law (displacement current) violates the energy conservation law (see the article "Displacement Current Paradox").
- Electromagnetic wave equation contradicts reality by deforming signal and violates relativity principle (see the article "Electromagnetic Wave Paradox").
- Faraday's law contradicts the electrostatic law (The present article).

You can find the links of all the paradoxes in the page Summary at blogspot or at Academia.

These paradoxes show that in certain circumstances, the dynamic action of magnetic field on electrons, the conversion between electric and magnetic fields and electromagnetic wave are wrongly described theoretically and thus, the electromagnetic theory is in serious trouble.

Although these paradoxes are mutually independent, the connection between them shows a deficiency in the structure of the electromagnetic theory. For example, the paradox of Faraday's law resembles to the B-cutting paradox and the paradox of the wave equation finds its origin in the paradoxes of generalized Ampere's law and Faraday's law.

So, there is something wrong deep in the electromagnetic theory which causes these paradoxes and inconsistency. For fixing this problem, it is crucial to get experimental evidences so that the old theory could be diagnosed; experiments will also search for indications for building correct theory. You can find the links of the experiment designs I propose in the page Summary at blogspot or at Academia. As the theoretical inconsistency is strong, the experiments will succeed and give the most spectacular experimental discovery of physics.