Numerical computation of the Lorentz force internal to an asymmetric coil

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1. Introduction

I have proven theoretically that the Lorentz force internal to a coil is not zero (several proofs are in the references). But theoretical proofs are too long to read and too complex. I give here a numerical computation of the Lorentz force internal to an asymmetrical coil; the computed force is not zero.

The computation uses the differential Lorentz force law below:

$$d^2\mathbf{F} = \frac{\mu_0}{4\pi} d\mathbf{I}_2 \times \left(d\mathbf{I}_1 \times \frac{\mathbf{r}}{r^3} \right) \tag{1}$$

The total Lorentz force internal to a coil is the double integral of this law over the coil:

$$\mathbf{F}_{Lorentz} = \frac{\mu_0}{4\pi} \int_{coil} \int_{coil} d\mathbf{I}_2 \times \left(d\mathbf{I}_1 \times \frac{\mathbf{r}}{r^3} \right) \tag{2}$$

The first integration is to obtain the differential force on a point ‘a’ that the rest of the coil exerts. This integration keeps a point ‘a’ constant and integrates equation (1) over the coil by excluding the point ‘a’, that is, the integration variable s never equals ‘a’:

$$d\mathbf{F}_a = \frac{\mu_0}{4\pi} \int_{\substack{coil \\ s \neq a}} d\mathbf{I}_a \times \left(d\mathbf{I}_s \times \frac{\mathbf{r}}{r^3} \right) \tag{3}$$

d$\mathbf{F}_a$ is the total Lorentz force on the segment of current d$\mathbf{I}_a$ exerted by the rest of the coil. The second integration integrates d$\mathbf{F}_a$ over the entire coil and the result is the total Lorentz force internal to this coil:

$$\mathbf{F}_{\text{internal}} = \int_{coil} d\mathbf{F}_a \tag{4}$$

2. Result of computation

The following computed case is on a coil formed by a half circle on the left and a half ellipse on the right. The equations of the 2 curves are:

$$x^2 + y^2 = 1 \text{ and } \left(\frac{x}{2} \right)^2 + y^2 = 1 \tag{5}$$

The ellipse is two times longer than the circle. Figure 1 shows the coil with discretization marks. The number of segments is 20 for the half circle and 40 for the half ellipse. The result of the computation is a net force in the x direction:

$$F_x = 1.0162\ 10^{-7}\ \text{Newton}$$

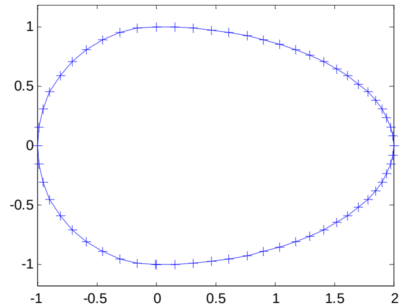

Figure 1

One can ask if this force is due to discretization. To exclude this possibility, I have done several computations with different number of segments. The result of these computations is shown in Table 1 that relates the net force with the total number of segments. We see that the net force stays stable as the number of segments increases. Thus, the obtained net force is a consequence of the Lorentz force law but not a computation error.

Number of segments	F_x 10^{-7} Newton
75	1.0241
120	1.0337
165	1.0370
210	1.0385
255	1.0393

Table 1

We are also interested in how the net force varies with the length of the ellipse. I have computed 4 cases. The shapes of these coils are shown in Figure 2 and the result is in Table 2. We see that the net force increases when the half ellipse lengthens. The net force for a circle is 0.

Number of segments	Major axis of the ellipse	F_x 10^{-7} Newton
40	1	0
60	2	1.0162
80	3	1.7095
100	4	2.2307

Table 2

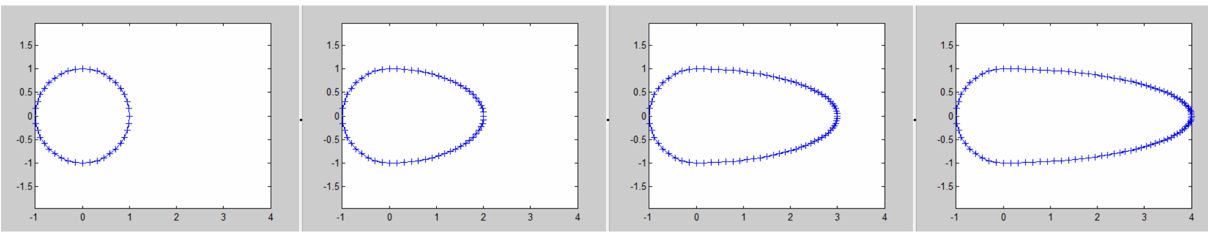

Figure 2

3. Method of computation

In the reference (5), I have proposed a corrected magnetic force law to correct the flaw of the Lorentz force law. For comparing the result of the Lorentz force law and that of the corrected magnetic force law, I use the following expansion of double cross product:

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \cdot \boldsymbol{c})\,\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\,\boldsymbol{c}$$

and transform the differential Lorentz force law, equation (1), into the following expression:

$$d^2\mathbf{F} = \frac{\mu_0}{4\pi}\left[\left(d\mathbf{I}_2 \cdot \frac{\mathbf{r}}{r^3}\right)d\mathbf{I}_1 - (d\mathbf{I}_2 \cdot d\mathbf{I}_1)\frac{\mathbf{r}}{r^3}\right] \tag{6}$$

This expression allows to compute the Lorentz force and the Ampere's force (the corrected magnetic force law) at the same time. The differential Ampere's force is:

$$d^2\mathbf{F}_{amp} = \frac{\mu_0}{4\pi}\left[-(d\mathbf{I}_2 \cdot d\mathbf{I}_1)\frac{\mathbf{r}}{r^3}\right] \tag{7}$$

And the differential Lorentz force is obtained by adding the following term that I call fictive force:

$$d^2\mathbf{F}_{fic} = \frac{\mu_0}{4\pi}\left[\left(d\mathbf{I}_2 \cdot \frac{\mathbf{r}}{r^3}\right)d\mathbf{I}_1\right] \tag{8}$$

The differential Lorentz force is the sum of the two: $d^2\mathbf{F}_{\mathrm{lorentz}} = d^2\mathbf{F}_{\mathrm{amp}} + d^2\mathbf{F}_{\mathrm{fic}}$

For discretization, I cut the curve of the coil into n segments. A current segment vector is defined as:

$$\Delta\mathbf{I}_i = I \cdot \Delta l_i \cdot \mathbf{u}_i \tag{9}$$

i is the index of the segment, Δl_i is the length of the segment *i*, *I* is the current intensity, $\mathbf{u}_i$ is the local unit vector indicating the direction of the current.

The first integration keeps the j^{th} segment fix and sums the differential forces from the other segments, that is, all *i* from 1 to n excluding *j*. We obtain then the Ampere's and fictive forces on the j^{th} segment:

$$\Delta\mathbf{F}_{j,amp} = \frac{\mu_0}{4\pi}\left[-\sum_{i=1\to n}^{i\neq j}(\Delta\mathbf{I}_j \cdot \Delta\mathbf{I}_i)\frac{\mathbf{r}}{r^3}\right],\ \Delta\mathbf{F}_{j,fic} = \frac{\mu_0}{4\pi}\left[\sum_{i=1\to n}^{i\neq j}\left(\Delta\mathbf{I}_j \cdot \frac{\mathbf{r}}{r^3}\right)\Delta\mathbf{I}_i\right] \tag{10}$$

The second integration sums the Ampere's and fictive forces on all segments with *j* going from 1 to n. We obtain then the total forces internal to the coil:

$$\mathbf{F}_{amp} = \sum_{j=1}^{n}\Delta\mathbf{F}_{j,amp},\ \mathbf{F}_{fic} = \sum_{j=1}^{n}\Delta\mathbf{F}_{j,fic},\ \mathbf{F}_{Lorentz} = \mathbf{F}_{amp} + \mathbf{F}_{fic} \tag{11}$$

In the computations, the value of the obtained Ampere's force is always zero and the Lorentz force equals the fictive force.

4. Comment

The existence of a net force internal to a coil is a new strong proof that the Lorentz force law is flawed. I give below my program of computation (executable in Matlab) so that everyone can do the computation.

There have been sufficient strong proofs to topple theoretically the Lorentz force law. But for this result to be accepted, experimental proofs are necessary. All discoveries are accompanied by great experiments and experimenters. A good example is the Michelson–Morley experiment. Without the extraordinary work of Michelson and Morley, Relativity could stay merely a speculation for long time. For the new corrected magnetic force law, we need new Michelson's in the 21st century and I'm sure they will emerge soon.

I have already done 2 experiments that have given good results. But my condition is bad and the quality of the result poor. One valid experiment is to measure the magnetic field at small spots in space, then measure the torque about 3 axes on a small coil at these spots. This technique is suitable to measure the magnetic force parallel to current, as I have done with my parallel action experiment (see ref. 5). The comparison of the measurement with the theoretical predictions by the Lorentz force law and the corrected magnetic force law can give conclusive result even on a small area.

For simpler manipulation, one can determine a field line and measure the direction of the magnetic force on a current. When the current is in a plane perpendicular to the field line, the Lorentz forces on currents in all directions have constant magnitude; but the Ampere's force has not the same magnitude for narrow magnet. My perpendicular action experiment is of this type (see ref. 5). As its result was positive, one can just redo it with better material and precision on a larger area. The ratio of the magnitude of the forces in x and y directions should be 1 for the Lorentz force law and variable for the corrected magnetic force law.

Based on the results of the numerical computation and my 2 experiments, the new experiments can only succeed. It is now a competition of speed for experimenters.

5. The program of computation

```
% Internal force of a coil
a=1;
na=4;xdisp=zeros(na,4);
a1=2;a2=1;
for ia=1:na
a1=ia; np=20;%a1=ia*0.5;np=10+ia*10;
mp=np*a1/a2;
tm=-1:2/mp:1;tm=tm*pi/2;
s1=[a1*cos(tm);sin(tm);zeros(1,mp+1)];
tn=-1:2/np:1;tn=tn*pi/2;
s2=[cos(tn+pi)*a2;sin(tn+pi);zeros(1,np+1)];
lc=[s1,s2(:,2:np+1)];m=length(lc)-1;
x1=(lc(:,2:m+1)+lc(:,1:m))/2;dI1=lc(:,2:m+1)-lc(:,1:m);%
n=m;x2=x1;dI2=dI1;

ddfamp=zeros(3,m,n);ddffic=ddfamp;
dfamp=zeros(3,n);dffic=dfamp;dcplamp=dfamp;dcplfic=dfamp;
%Differential force
for j=1:n
for i=1:m
   if i==j;
   ddfamp(:,i,j)=0; ddffic(:,i,j)=0;
   else
   r12=x2(:,j)-x1(:,i);vr=r12/norm(r12)^3; %radius coefficient
   ddfamp(:,i,j)=-dot(dI2(:,j),dI1(:,i))*vr; %diff force ampere
   ddffic(:,i,j)=dot(dI2(:,j),vr)*dI1(:,i); %diff force fictive
```

```
    end
end
end
% First integral
for j=1:n
trp=[ddfamp(:,:,j),ddfamp(:,1,j)];dfamp(:,j)=trapz(trp,2);
trp=[ddffic(:,:,j),ddffic(:,1,j)];dffic(:,j)=trapz(trp,2);
end
% Second integral
trp=[dfamp,dfamp(:,1)];famp=trapz(trp,2)*a; %force ampere
trp=[dffic,dffic(:,1)];ffic=trapz(trp,2)*a; %force fictive
florentz=famp+ffic; %force  lorentz
famficlor_t=[famp,ffic,florentz];
xdisp(ia,:)=[np+mp,a1/a2,florentz(1),famp(1)];
figure,plot(lc(1,:),lc(2,:),'-+');axis equal;
end
xdisp,figure,plot(xdisp(:,1),xdisp(:,3));
return
```

6. References

1) Proof of the remaining resultant Lorentz force internal to a triangular coil
   http://pengkuanem.blogspot.com/2012/04/analyze-of-lorentz-forces-internal-to.html
2) Mathematical cause of the existence of the remaining resultant internal Lorentz force
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3) Equilateral triangle coil case
   http://pengkuanem.blogspot.com/2012/03/lorentz-forces-internal-to-equilateral.html
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   http://pengkuanem.blogspot.com/2012/03/lorentzforce-internal-to-coil-analyze.html
5) Unknown properties of magnetic force and Lorentz force law
   http://pengkuanem.blogspot.com/2013/04/unknown-properties-of-magnetic-force.html