Unknown properties of magnetic force and Lorentz force law

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Abstract: The 2 experiments presented in this article show unknown properties of magnetic force. The Lorentz force law cannot explain these properties; neither can it compute the magnetic force internal to an open circuit. A correction added to the present magnetic force theory allows explaining these new discoveries.

1 Introduction

The Lorentz force law describes magnetic force, but fails to explain the result of the 2 experiments presented below. A computation of the Lorentz force internal to an open circuit shows a violation of the energy conservation law. These discrepancies will be studied and a correction to the Lorentz force law will explain the experiments and solve the theoretical inconsistency.

2 Parallel action experiment

The first experiment is named "Parallel action experiment" which is shown in this video http://youtu.be/2u5Nadx0mOM[1]. The setup of this experiment is shown in Figure 1 and Figure 2. A test coil is placed in the magnetic field **B** of a magnet to test the force it feels. The plane of the coil is perpendicular to its axle of rotation.

Shown in the video, the coil rotates in its plane proving that the magnetic force's direction is parallel to the current. The light of the led signals the presence of current and the existence of magnetic force.

What is this force? Lorentz force being perpendicular to the current (see Figure 3), the torque about the axle $\boldsymbol{\tau}$ should be zero and the coil should stay immobile, contradicting the experimental result. So, the Lorentz force law cannot explain the rotation of the coil.

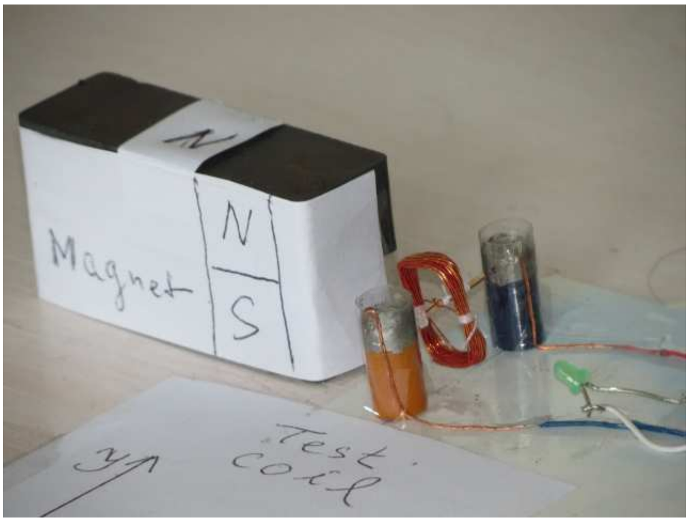

Figure 1

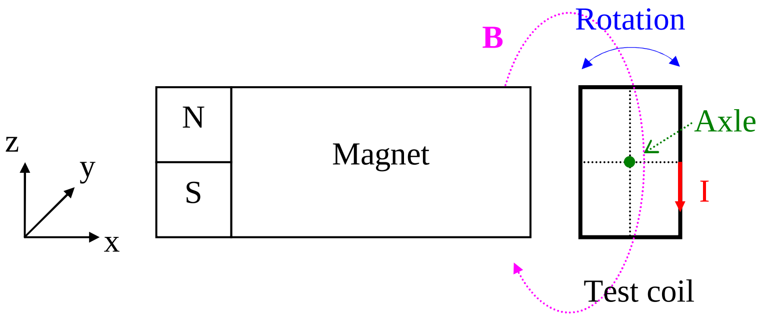

Figure 2

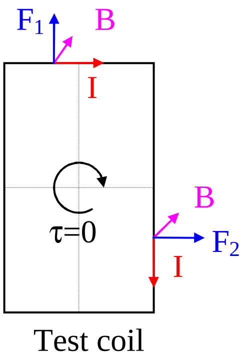

Figure 3

3 Perpendicular action experiment

The second experiment is named "Perpendicular action experiment" which is shown in this video http://youtu.be/uFEIcwePSL0 [2].

The setup of this experiment is shown in Figure 4 and Figure 5. The long rectangular magnet stands vertically, with its north pointing to the left. The test coil is located on the right and is mounted on a horizontal axle that is in the plane of the coil. The magnetic field of the magnet exerts a force on the coil and makes it rotate. The object of this experiment is to test how the coil rotates when the axle changes direction.

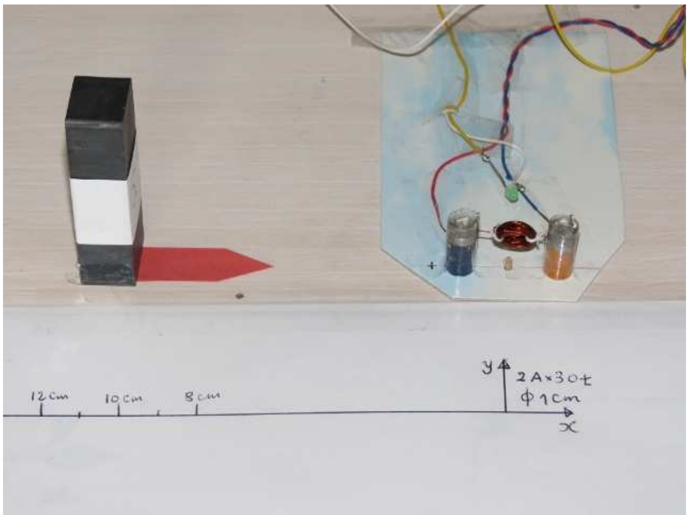

Figure 4

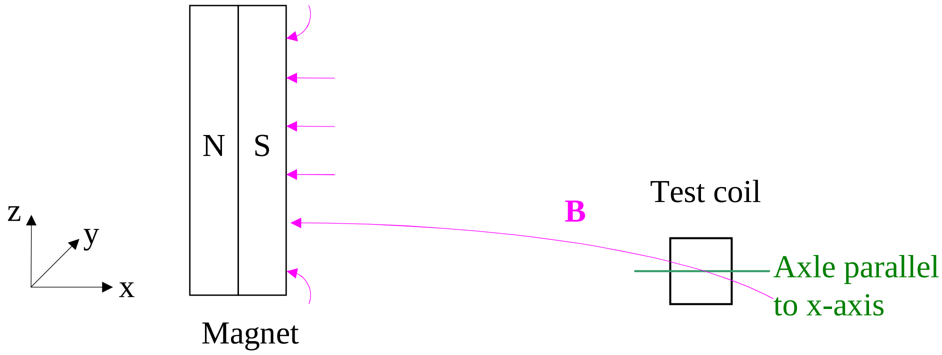

Figure 5

We see in Figure 5 that the magnetic field possesses a z component at the location of the coil, which creates a magnetic force on the current. According to the Lorentz force law, when the axle is parallel to x-axis, the Lorentz force is in y direction; when the axle is parallel to y-axis, the Lorentz force is in x direction (see Figure 6). The video shows that the coil rotates about y-axis, but not about x-axis, meaning that the magnetic force is strong in x direction but very weak in y direction.

Why the forces are so different according to the directions of current? The Lorentz force law says that the magnetic force equals the vector product of magnetic field and current. As the current stays the same when it changes direction, the magnitude of the magnetic force should not change and the coil should rotate with equal intensity about x-axis and y-axis. But this is clearly not the case and we must conclude that the Lorentz force law is inaccurate.

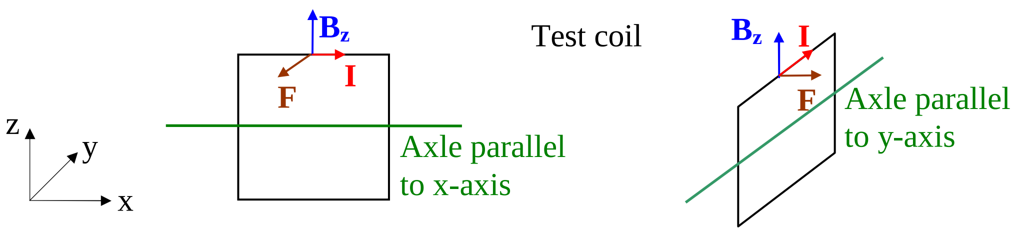

Figure 6

4 Magnetic force internal to an open circuit

The above experiments prove experimentally that the Lorentz force law is inaccurate. In this section, a theoretical inconsistency is also found. Let us compute the Lorentz force that one part of a circuit exerts on another part of the same circuit. Take an angled wire and connect it to a conductor sphere at each end, as shown in Figure 7. The 2 spheres constitute a capacitor that allows an alternating generator to make a current in the wire in spite that it is an open circuit, that is, the wire does not constitute a closed loop.

The arm 1 and 2 are the left and right parts of the wire respectively. The current in the arm 1 is $\mathbf{I}_1$ and that in the arm 2 is $\mathbf{I}_2$. $\mathbf{I}_1$ exerts a Lorentz force on $\mathbf{I}_2$, denoted as $\mathbf{F}_2$, and $\mathbf{I}_2$ exerts on $\mathbf{I}_1$ the Lorentz force $\mathbf{F}_1$. $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only internal forces of this circuit. The resultant force of $\mathbf{F}_1$ and $\mathbf{F}_2$ is:

$$\mathbf{F}_r = \mathbf{F}_1 + \mathbf{F}_2$$

As $\mathbf{F}_1$ and $\mathbf{F}_2$ are at an angle, their resultant force $\mathbf{F}_r$ is not zero. So, there is a net force on this circuit that does not come from the exterior. If this circuit moves in the direction of $\mathbf{F}_r$, a net work would be done and the energy conservation law violated. On the other hand, $\mathbf{F}_r$ violates Newton's third law because there is not a corresponding reaction force.

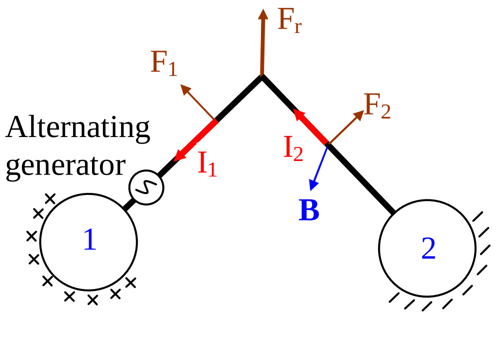

Figure 7

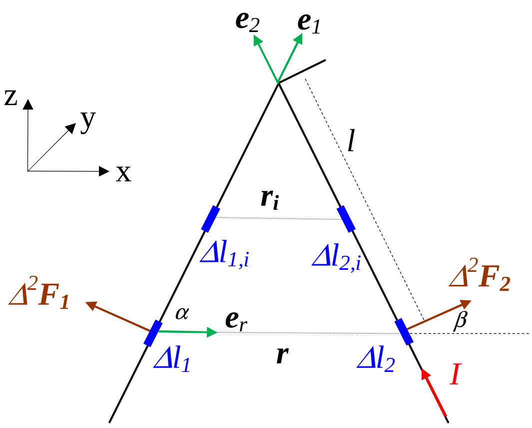

Figure 8

In addition to the nonzero internal force, an incomplete calculation of $\mathbf{F}_r$ shows that its magnitude is infinity. According to the Lorentz force law, the differential Lorentz force between 2 current element vectors $d\mathbf{I}_1$ and $d\mathbf{I}_2$ is the following expression, $\mathbf{r}$ being the distance vector:

$$\boxed{d^2\mathbf{F}_{12} = \frac{\mu_0}{4\pi} d\mathbf{I}_2 \times \left(d\mathbf{I}_1 \times \frac{\mathbf{r}}{r^3} \right)} \qquad (1)$$

Figure 8 shows the Lorentz forces that 2 current segments $\Delta\mathbf{I}_1$ and $\Delta\mathbf{I}_2$ exert on one another. $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_r$ are the directional unit vectors, r, Δl_1 and Δl_2 are the lengths of $\mathbf{r}$, $\Delta\mathbf{I}_1$ and $\Delta\mathbf{I}_2$ which are:

$$\mathbf{r} = r\,\mathbf{e}_r, \quad \Delta\mathbf{I}_1 = I\Delta l_1 \mathbf{e}_1, \quad \Delta\mathbf{I}_2 = I\Delta l_2 \mathbf{e}_2$$

The Lorentz force on $\Delta\mathbf{I}_2$ is computed using equation (1):

$$\Delta^2\mathbf{F}_2 = \frac{\mu_0}{4\pi} I^2 \Delta l_2 \mathbf{e}_2 \times \left(\Delta l_1 \mathbf{e}_1 \times \frac{\mathbf{e}_r}{r^2} \right) = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \mathbf{e}_2 \times (\mathbf{e}_1 \times \mathbf{e}_r)$$

The double cross product of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_r$ is a function of the angles α and β:

$$\mathbf{e}_2 \times (\mathbf{e}_1 \times \mathbf{e}_r) = \sin\alpha \left(\cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_y \right)$$

Then:

$$\Delta^2\mathbf{F}_2 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin\alpha \left(\cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_z \right)$$

We compute the Lorentz force on $\Delta\mathbf{I}_1$ in the same way. It is:

$$\Delta^2\mathbf{F}_1 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \sin\alpha \left(-\cos\beta\, \mathbf{e}_x + \sin\beta\, \mathbf{e}_z \right)$$

The sum of $\Delta^2\mathbf{F}_1$ and $\Delta^2\mathbf{F}_2$ is the resultant Lorentz force of these 2 segments:

$$\Delta^2\mathbf{F}_r = \Delta^2\mathbf{F}_1 + \Delta^2\mathbf{F}_2 = \frac{\mu_0}{4\pi} \frac{I^2 \Delta l_2 \Delta l_1}{r^2} \cdot 2\sin\alpha \sin\beta \cdot \mathbf{e}_z \qquad (2)$$

Now we integrate $\Delta^2\mathbf{F}_r$ over the whole 2 arms. For doing so, we cut the arms into small segments $\Delta\mathbf{I}_{1,i}$ and $\Delta\mathbf{I}_{2,i}$, compute the resultant forces $\Delta^2\mathbf{F}_{r,i}$ and sum up $\Delta^2\mathbf{F}_{r,i}$ for all i. Let the lengths of the i^th^ segments be $\Delta l_{1,i}$ and $\Delta l_{2,i}$ respectively, the distance between them be r_i, the distance from the summit to $\Delta\mathbf{I}_{1,i}$ and $\Delta\mathbf{I}_{2,i}$ be l_i. We reduce the length $\Delta l_{1,i}$ and $\Delta l_{2,i}$ progressively to zero as the segments approach the summit in such a way that when i reaches infinity, the sum of $\Delta\mathbf{I}_{1,i}$ and $\Delta\mathbf{I}_{2,i}$ covers the 2 arms entirely. The ratio of reduction of lengths between the i^{th} and the $(i+1)^{th}$ segments is as follow:

$$c = \frac{l_{i+1}}{l_i} = \frac{\Delta l_{1,i+1}}{\Delta l_{1,i}} = \frac{\Delta l_{2,i+1}}{\Delta l_{2,i}} = \frac{r_{i+1}}{r_i}$$

From this relation we deduce that the ratios $\Delta l_{1,i} / r_i$ and $\Delta l_{2,i} / r_i$ are constant for all i:

$$a_1 = \frac{\Delta l_{1,i}}{r_i} = \frac{\Delta l_{1,i+1}}{r_{i+1}}, \quad a_2 = \frac{\Delta l_{2,i}}{r_i} = \frac{\Delta l_{2,i+1}}{r_{i+1}}$$

Then equation (2) becomes a constant for all i:

$$\Delta^2\mathbf{F}_r = \frac{\mu_0}{4\pi} a_1 a_2 \cdot 2\sin\alpha \sin\beta \cdot \mathbf{e}_z \qquad (3)$$

The total resultant force for this scheme is the sum of $\Delta^2\mathbf{F}_{r,i}$ for all i from 1 to infinity, that is, $\Delta^2\mathbf{F}_{r,i}$ times infinity:

$$\mathbf{F}_{r12} = \sum_{i=1}^{\infty} \Delta^2\mathbf{F}_r = \infty \cdot \frac{\mu_0}{4\pi} a_1 a_2 \cdot 2\sin\alpha \sin\beta \cdot \mathbf{e}_z = \infty \cdot \mathbf{e}_z \qquad (4)$$

We arrive at a surprising result: the total resultant force for this scheme is infinity! As this is only one part of the total resultant Lorentz force on the 2 arms, the total Lorentz force internal to this open circuit is also infinity.

As energy cannot be created and Lorentz force cannot be infinity, the Lorentz force law is theoretically inconsistent for this case.

5 Flaw of the Lorentz force law

What is wrong with the Lorentz force law?

Shown in Figure 8, the $\Delta^2\mathbf{F}_1$ and $\Delta^2\mathbf{F}_2$ are action and reaction forces for $\Delta\mathbf{I}_1$ and $\Delta\mathbf{I}_2$, they must respect Newton's third law and their sum must be zero. But they are at an angle to one another and cannot cancel themselves. So, the flaw of the Lorentz force law is that it violates Newton's third law on the differential level. This flaw was well known but no solution was found (See Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, p. 200 [3]).

6 Correction of the Lorentz force law

In order to correct this flaw, we will remove the part of differential Lorentz force that breaks the symmetry of action-reaction forces. Let us calculate the magnetic force between 2 closed coils c_1 and c_2 (see Figure 9) by integrating equation (1).

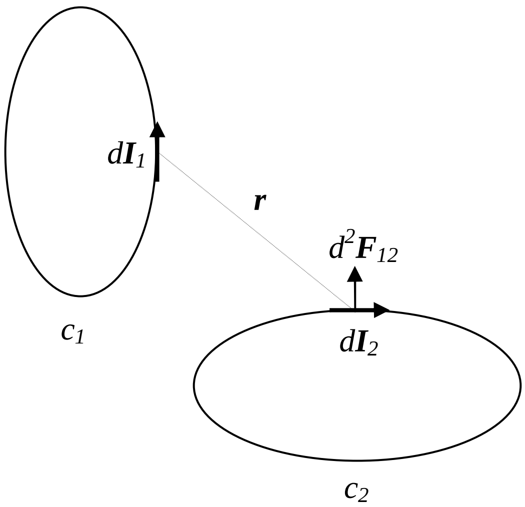

Figure 9

By using the following expansion of double cross product:

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \cdot \boldsymbol{c})\,\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\,\boldsymbol{c}$$

Equation (1) becomes:

$$d^2\mathbf{F}_{12} = \frac{\mu_0}{4\pi}\left[\left(d\mathbf{I}_2 \cdot \frac{\mathbf{r}}{r^3}\right)d\mathbf{I}_1 - (d\mathbf{I}_2 \cdot d\mathbf{I}_1)\frac{\mathbf{r}}{r^3}\right] \tag{5}$$

First, we integrate $d^2\mathbf{F}_{12}$ over c_2:

$$d\mathbf{F}_{1-c2} = \int_{c2} d^2\mathbf{F}_{12} = \frac{\mu_0}{4\pi}\left[\int_{c2}\left(d\mathbf{I}_2 \cdot \frac{\mathbf{r}}{r^3}\right)d\mathbf{I}_1 - \int_{c2}(d\mathbf{I}_2 \cdot d\mathbf{I}_1)\frac{\mathbf{r}}{r^3}\right] \tag{6}$$

The first integral on the right side can be written as follow, with dl_2 being the length of $d\mathbf{I}_2$, $\mathbf{n}_2$ the directional unit vector and $d\mathbf{I}_2 = I_2\, dl_2\, \mathbf{n}_2$:

$$\int_{c2}\left(d\mathbf{I}_2\cdot\frac{\mathbf{r}}{r^3}\right)d\mathbf{I}_1 = I_2 d\mathbf{I}_1\int_{c2}\frac{\mathbf{r}}{r^3}\cdot\mathbf{n}_2\,dl_2 = 0 \tag{7}$$

This is the circulation of a conservative field over the closed loop c_2 and its value is zero. Then, equation (6) becomes:

$$d\mathbf{F}_{1-c2} = -\frac{\mu_0}{4\pi}\int_{c2}\frac{\mathbf{r}}{r^3}\left(d\mathbf{I}_2\cdot d\mathbf{I}_1\right) \tag{8}$$

Now, we integrate $d\mathbf{F}_{1\text{-}c2}$ over c_1 and obtain the total magnetic force c_1 exerts on c_2:

$$\mathbf{F}_{c1-c2} = \int_{c1} d\mathbf{F}_{1-c2} = \int_{c1}\int_{c2}\left(-\frac{\mu_0}{4\pi}\frac{\mathbf{r}}{r^3}d\mathbf{I}_2\cdot d\mathbf{I}_1\right) \tag{9}$$

We define the integrand of this double integral as the differential magnetic force between the current element vectors $d\mathbf{I}_1$ and $d\mathbf{I}_2$:

$$d^2\mathbf{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\mathbf{r}}{r^3}d\mathbf{I}_2\cdot d\mathbf{I}_1 \tag{10}$$

Because equation (10) is proportional to the dot product of 2 current element vectors, $d\mathbf{I}_1\cdot d\mathbf{I}_2$, it resembles the Ampere's force between 2 parallel currents. Then, this differential magnetic force is given the name “differential Ampere's force” and equation (10) is named “Corrected magnetic force law” in the following.

7 Properties of the Corrected magnetic force law

From the derivation, we know that equation (9) equals the double integral of equation (1) which is the integrated Lorentz force law:

$$\mathbf{F}_{c1-c2} = \int_{c1}\int_{c2}\left(-\frac{\mu_0}{4\pi}\frac{\mathbf{r}}{r^3}d\mathbf{I}_2\cdot d\mathbf{I}_1\right) = \int_{c1}\int_{c2}\frac{\mu_0}{4\pi}d\mathbf{I}_2\times\left(d\mathbf{I}_1\times\frac{\mathbf{r}}{r^3}\right) \tag{11}$$

In consequence, the double integral of equation (10) equals the total Lorentz force the coil c_1 exerts on the coil c_2 and the first property of the corrected magnetic force law is that it gives the same total magnetic force than the Lorentz force law for closed coils:

$$\int_{c1}\int_{c2} d^2\mathbf{F}_{amp} = \int_{c1}\int_{c2} d^2\mathbf{F}_{Lorentz} \tag{12}$$

This property means that the corrected magnetic force law is identical to the Lorentz force law on macroscopic level.

But on differential level, differential Ampere's force is different from differential Lorentz force (equation (1)); it respects Newton's third law. In fact, the expression for the reverse force of $d^2\mathbf{F}_{amp}$ is obtained by simply reversing the radial vector $\boldsymbol{r}$ and interchanging the current element vectors $d\mathbf{I}_1$ and $d\mathbf{I}_2$ in equation (10):

$$d^2\mathbf{F}_{rev} = -\frac{\mu_0}{4\pi}\frac{-\mathbf{r}}{r^3}\left(d\mathbf{I}_1\cdot d\mathbf{I}_2\right) = \frac{\mu_0}{4\pi}\frac{\mathbf{r}}{r^3}\left(d\mathbf{I}_2\cdot d\mathbf{I}_1\right) = -d^2\mathbf{F}_{amp}$$

This change reflects exactly the physical process of changing view, that is, $d^2\mathbf{F}_{amp}$ is the force when one regards $d\mathbf{I}_2$ from $d\mathbf{I}_1$, and $d^2\mathbf{F}_{rev}$ is the force when one regards $d\mathbf{I}_1$ from $d\mathbf{I}_2$. Like reversing the physical view produces the reaction force, reversing the vectors $\boldsymbol{r}$, d$\mathbf{I}_1$ and d$\mathbf{I}_2$ in equation (10) produces $d^2\mathbf{F}_{rev}$ that has the same magnitude and the opposite sign than $d^2\mathbf{F}_{amp}$, respecting mathematically Newton's third law.

In the contrary, when we reverse $\boldsymbol{r}$, d$\mathbf{I}_1$ and d$\mathbf{I}_2$ in equation (1), we do not obtain a reaction force, proving that the Lorentz force law violates Newton's third law:

$$\frac{\mu_0}{4\pi} d\mathbf{I}_1 \times \left(d\mathbf{I}_2 \times \frac{-\mathbf{r}}{r^3} \right) \neq -\frac{\mu_0}{4\pi} d\mathbf{I}_2 \times \left(d\mathbf{I}_1 \times \frac{\mathbf{r}}{r^3} \right)$$

Then, the second property of the corrected magnetic force law is that it respects Newton's third law on the differential level in both physical and mathematical senses. Differential Ampere's force and differential Lorentz force are drawn in Figure 10 for comparison. The difference between them is the part of differential Lorentz force that breaks the symmetry of action-reaction forces, that is, the term of equation (7) that we have removed.

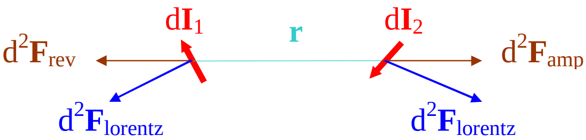

Figure 10

We rewrite here the corrected magnetic force law:

$$\boxed{d^2\mathbf{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\mathbf{r}}{r^3}(d\mathbf{I}_2 \cdot d\mathbf{I}_1)} \tag{13}$$

8 Solution for the inconsistency of open circuit

By applying the corrected magnetic force law to the open circuit, the pair of differential Ampere's force, $d^2\mathbf{F}_{rev}$ and $d^2\mathbf{F}_{amp}$, cancel out one another (see Figure 11):

$$d^2\mathbf{F}_{amp} + d^2\mathbf{F}_{rev} = 0$$

Thus, the resultant force is zero for all pairs of segments and for the whole circuit. The inconsistency of nonzero resultant internal force and infinitely large Lorentz force disappear.

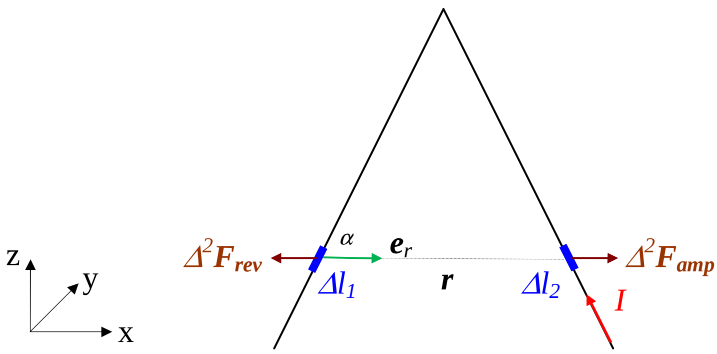

Figure 11

9 Explanation of the parallel action experiment

The corrected magnetic force law expresses magnetic force as a direct interaction between 2 currents. In the parallel action experiment the magnetic force is exerted by the magnet. For computing this magnetic force using the corrected magnetic force law, we will replace the magnet with an equivalent current loop $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_4$, as shown in Figure 12, which creates exactly the same magnetic field.

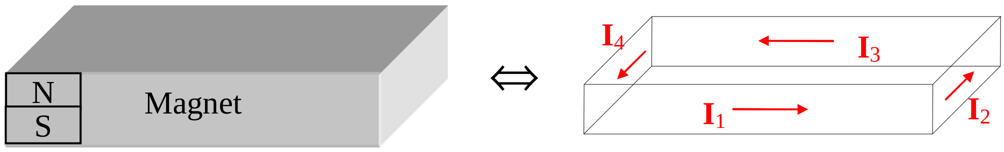

Figure 12

9.1 Torques

Currents $\mathbf{I}_2$ and $\mathbf{I}_4$ are perpendicular to the coil. According to equation (13), the force between 2 perpendicular currents is zero and then, $\mathbf{I}_2$ and $\mathbf{I}_4$ do not exert any force on the coil.

Currents $\mathbf{I}_1$ and $\mathbf{I}_3$ are parallel to currents $\mathbf{I}_5$ and $\mathbf{I}_7$ and exert a force on them. The force that $\mathbf{I}_1$ exerts on $\mathbf{I}_5$ is $\mathbf{F}_1$ and its x component is positive (see Figure 13):

$$\mathbf{I}_1 \cdot \mathbf{I}_5 < 0 \Rightarrow \mathbf{F}_1 \cdot \mathbf{e}_x > 0 \tag{14}$$

The force that $\mathbf{I}_1$ exerts on $\mathbf{I}_7$ is $\mathbf{F}_2$ and its x component is negative:

$$\mathbf{I}_1 \cdot \mathbf{I}_7 > 0 \Rightarrow \mathbf{F}_2 \cdot \mathbf{e}_x < 0 \tag{15}$$

So, the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ create the torque $\boldsymbol{\tau}_1$ that is counter-clockwise:

$$\Rightarrow \boldsymbol{\tau}_1 \cdot \mathbf{e}_y < 0 \tag{16}$$

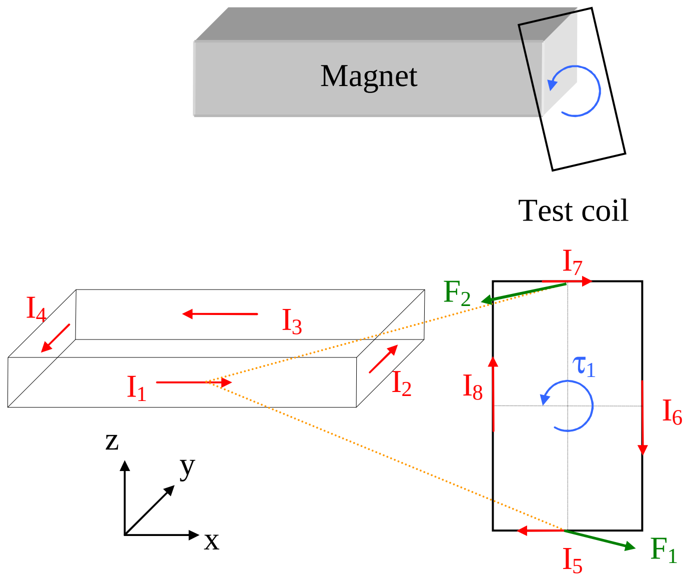

Figure 13

Similarly, $\mathbf{I}_3$ exerts the force $\mathbf{F}_3$ on $\mathbf{I}_5$ and the force $\mathbf{F}_4$ on $\mathbf{I}_7$. The forces $\mathbf{F}_3$ and $\mathbf{F}_4$ create the torque $\boldsymbol{\tau}_2$ that is clockwise (see Figure 14):

$$\begin{gathered}\mathbf{I}_3\cdot \mathbf{I}_5>0 \Rightarrow \mathbf{F}_3\cdot\mathbf{e}_x <0 \\ \mathbf{I}_3\cdot \mathbf{I}_7<0 \Rightarrow \mathbf{F}_4\cdot\mathbf{e}_x >0 \\ \Rightarrow \boldsymbol{\tau}_2\cdot\mathbf{e}_y >0\end{gathered} \tag{17}$$

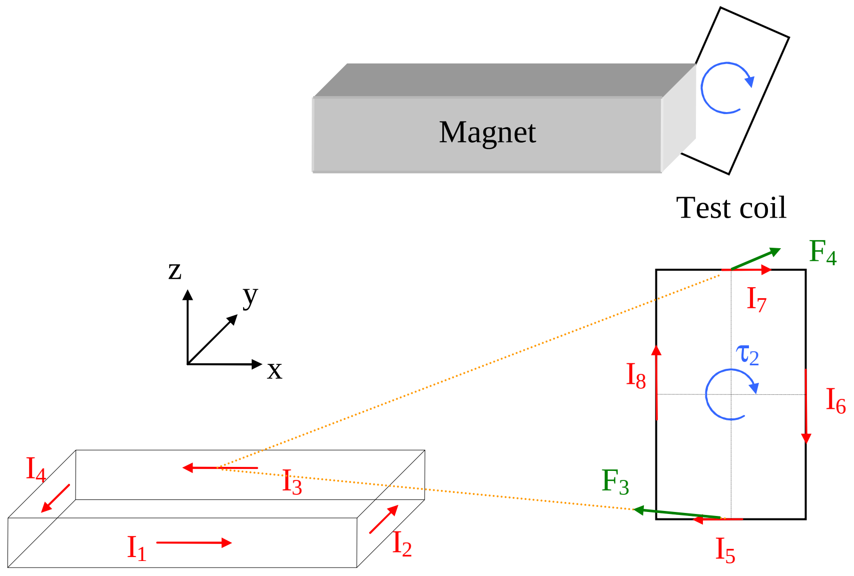

Figure 14

9.2 Rotation of the coil

The experiment tests the rotation of the coil in 2 positions: first, the magnet is behind the coil and the coil rotates counter-clockwise (Figure 13); second, the magnet is before the coil and the coil rotates clockwise (Figure 14).

In the first case, the front current $\mathbf{I}_1$ is nearer to the coil than the rear current $\mathbf{I}_3$ (Figure 13). So, the torque $\boldsymbol{\tau}_1$ is stronger than $\boldsymbol{\tau}_2$ and the test coil rotates in the direction of $\boldsymbol{\tau}_1$, that is, counter-clockwise. In the second case, the rear current $\mathbf{I}_3$ is nearer to the coil than the front current $\mathbf{I}_1$ (Figure 14). So, the torque $\boldsymbol{\tau}_2$ is stronger than $\boldsymbol{\tau}_1$ and the test coil rotates in the direction of $\boldsymbol{\tau}_2$, that is, clockwise. This theoretical prediction fits well the experimental result.

9.3 Equilibrium inclination

The video shows that for constant current, the test coil reaches an equilibrium position and stays inclined. This equilibrium can be explained by the corrected magnetic force law. In fact, when the coil is inclined, the current in the long side possesses an x component that feels a magnetic force and creates a torque.

Figure 15 shows the situation where the magnet is before the test coil. In this case, the magnetic force from the rear current $\mathbf{I}_3$ is stronger. The long side of the test coil being inclined, its lower half is nearer to $\mathbf{I}_3$ than the upper half and the force $\mathbf{F}_5$ is stronger than $\mathbf{F}_6$. So, $\mathbf{F}_5$ and $\mathbf{F}_6$ create a counter-clockwise torque $\boldsymbol{\tau}_3$ which strengthens with the inclination.

The original torque $\boldsymbol{\tau}_2$ is clockwise, which weakens as the angle of inclination increases. The coil rotates first clockwise, but then is stopped by the counter torque $\boldsymbol{\tau}3$. The angle of the equilibrium inclination is such that the torque $\boldsymbol{\tau}_3$ balances exactly the original torque $\boldsymbol{\tau}2$ and the resultant torque becomes zero:

$$\boldsymbol{\tau}_2 + \boldsymbol{\tau}_3 = 0 \tag{18}$$

For the case where the magnet is behind the test coil, the analysis is similar and we reach a counter-clockwise inclination at equilibrium. This theoretical explanation of the equilibrium inclination in both cases fits well the experimental result and will also allow computing the angle of inclination with future measurement data.

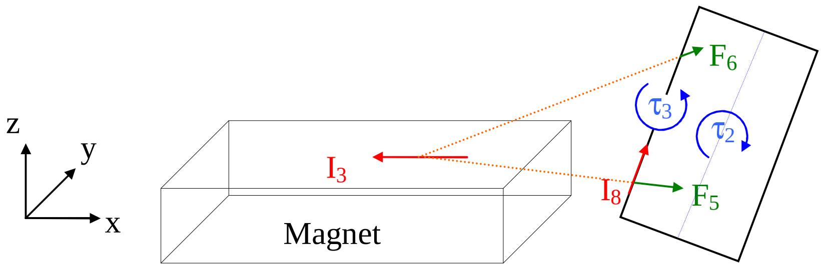

Figure 15

The Lorentz force law says that magnetic force is perpendicular to the current. Using the corrected magnetic force law and experiment, we have proven that parallel to current magnetic force exists.

10 Explanation of the perpendicular action experiment

Like in the precedent section, we replace the magnet with an equivalent current loop which is ABCD in Figure 16, so the force on the test coil abcd can be computed using equation (13).

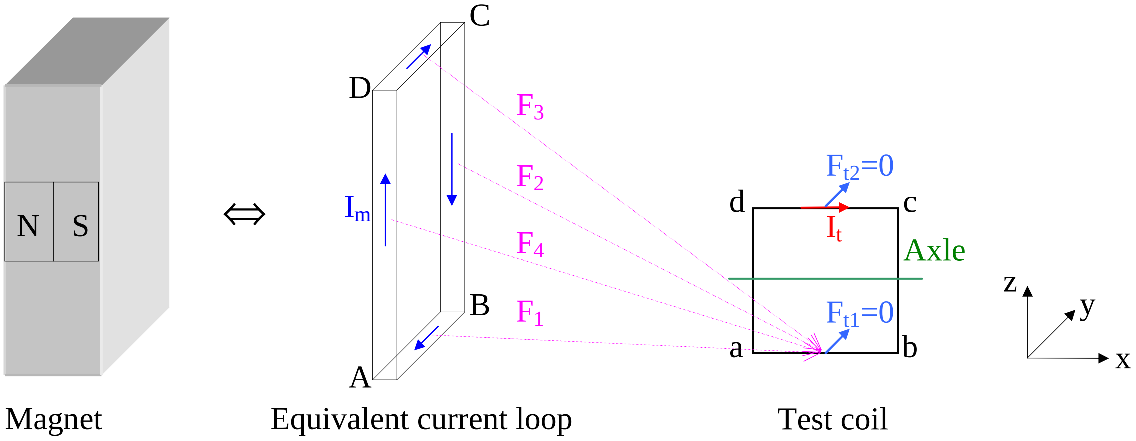

Figure 16

10.1 Case 1: Axle parallel to x-axis

The test coil will rotate when a torque exists, which is created by the y-component of the forces on the segment ab, bc, cd and da (see Figure 16). In this section, I will compute these forces and show that the torque is zero.

10.1.1 Forces on the segments ab and cd

As Figure 16 shows, the equivalent current loop is composed of 4 segments, AB, BC, CD and DA. The force on the segment ab is the sum of the forces exerted by these segments, respectively denoted by $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$. As AB, BC, CD and DA are all perpendicular to ab, these 4 forces are all zero:

$$d\mathbf{I}_m \cdot d\mathbf{I}_t = 0 \Rightarrow \mathbf{F}_1 = \mathbf{F}_2 = \mathbf{F}_3 = \mathbf{F}_4 = 0$$

where the index *m* indicates the equivalent current loop and *t* the test coil.

In consequence, the resultant force on ab is zero:

$$\mathbf{F}_{t1} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 = 0 \tag{19}$$

In the same way for the segment cd, AB, BC, CD and DA are perpendicular to cd and the force on it is zero too:

$$\mathbf{F}_{t2} = 0 \tag{20}$$

10.1.2 Forces on the segments ad and bc

As Figure 17 shows, the force on the segment ad is $\mathbf{F}_{t3}$ which is the sum of the forces from AB, BC, CD and DA, denoted by $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$:

$$\mathbf{F}_{t3} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$$

AB and CD are perpendicular to ad and the forces $\mathbf{F}_1$ and $\mathbf{F}_3$ are zero:

$$d\mathbf{I}_m \cdot d\mathbf{I}_t = 0 \Rightarrow \mathbf{F}_1 = \mathbf{F}_3 = 0 \tag{21}$$

BC and DA are parallel to ad and their forces are not zero. The resultant force on ad is:

$$\mathbf{F}_{t3} = \mathbf{F}_2 + \mathbf{F}_4$$

On the segment bc, the force is also not zero and is denoted as $\mathbf{F}_{t4}$.

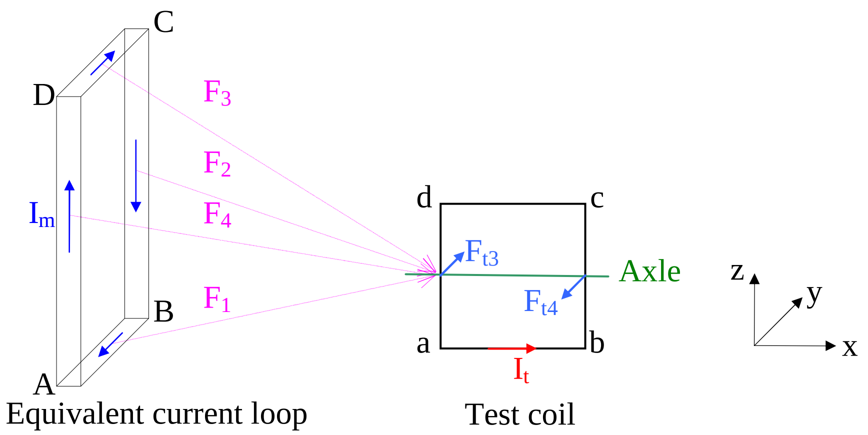

Figure 17

10.1.3 Torque about x-axis

The torque of a force is the vector product of the force with its lever arm. The lever arms of the forces $\mathbf{F}_{t1}$, $\mathbf{F}_{t2}$, $\mathbf{F}_{t3}$ and $\mathbf{F}_{t4}$ are respectively:

$$-\frac{ad}{2}\mathbf{e_z},\ \frac{ad}{2}\mathbf{e_z},\ 0,\ 0$$

The resultant torque is then:

$$\boldsymbol{\tau}_x = -\frac{ad}{2}\mathbf{e_z}\times\mathbf{F}_{t1} + \frac{ad}{2}\mathbf{e_z}\times\mathbf{F}_{t2} + 0\times\mathbf{F}_{t3} + 0\times\mathbf{F}_{t4}$$

Since $\mathbf{F}_{t1}$ and $\mathbf{F}_{t2}$ are zero, the torque is zero: $\boldsymbol{\tau}_x=0$

So, the corrected magnetic force law predicts that for this case the torque is zero and the coil will not rotate about x-axis. This prediction fits the experimental result.

10.2 Case 2: Axle parallel to y-axis

The axle is parallel to y-axis and the torque is created by the x-component of the forces on the segment ab, bc, cd and da (see Figure 18). In this section, I will show that these forces create the torque that makes the test coil rotate.

10.2.1 Forces on the segments ab and cd

Like in above sections the force exerted by AB, BC, CD and DA are respectively denoted by $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$. BC and DA are perpendicular to ab and the forces they exert are zero:

$$d\mathbf{l}_m \cdot d\mathbf{l}_t = 0 \Rightarrow \mathbf{F}_2 = 0,\ \mathbf{F}_4 = 0$$

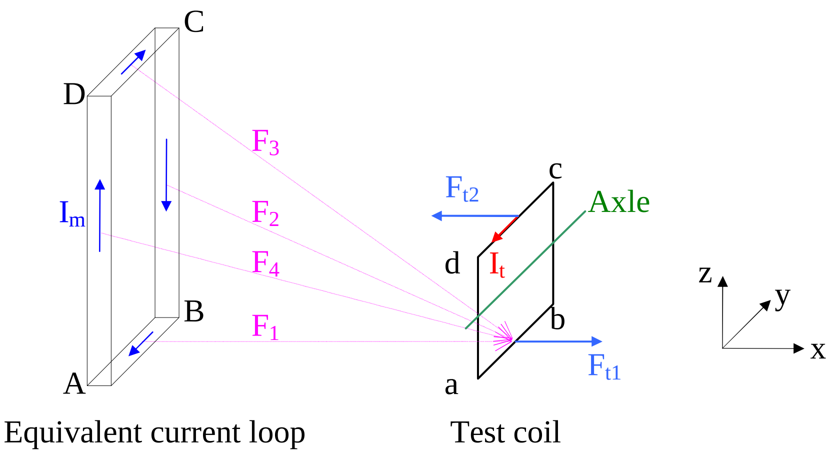

Figure 18

The segment AB is parallel to ab, the force $\mathbf{F}_1$ is computed using equation (13):

$$\mathbf{F}_1 = -\frac{\mu_0 I_m I_t}{4\pi}\int_{AB}\int_{ab}\left(\frac{\mathbf{r}}{r^3}d\mathbf{l}_m\cdot d\mathbf{l}_t\right) \qquad (22)$$

The force from the segment CD is computed in the same way, its expression is the above equation with the domain of integration changed from AB to CD:

$$\mathbf{F}_3 = -\frac{\mu_0 I_m I_t}{4\pi} \int_{CD} \int_{ab} \left(\frac{\mathbf{r}}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \tag{23}$$

And the resultant force on the segment ab is the sum of the above 2 forces:

$$\mathbf{F}_{t1} = \mathbf{F}_1 + \mathbf{F}_3 = -\frac{\mu_0 I_m I_t}{4\pi} \int_{AB+CD} \int_{ab} \left(\frac{\mathbf{r}}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \tag{24}$$

The force on the segment cd is derived in the same way. Its expression is the equation (24) with the domain of integration changed from ab to cd:

$$\mathbf{F}_{t2} = -\frac{\mu_0 I_m I_t}{4\pi} \int_{AB+CD} \int_{cd} \left(\frac{\mathbf{r}}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \tag{25}$$

10.2.2 Forces on the segments ad and bc

The forces on the segments ad and bc are computed in the same way than in section 10.1.2. The segments AB and CD are perpendicular to the segment da (see Figure 19). So, the forces $\mathbf{F}_1$ and $\mathbf{F}_3$ on da are zero:

$$d\mathbf{I}_m \cdot d\mathbf{I}_t = 0 \Rightarrow \mathbf{F}_1 = 0,\ \mathbf{F}_3 = 0$$

But the forces from BC and DA are not zero and the resultant force on da is:

$$\mathbf{F}_{t3} = \mathbf{F}_2 + \mathbf{F}_4$$

On the segment bc, the resultant force is denoted as $\mathbf{F}_{t4}$.

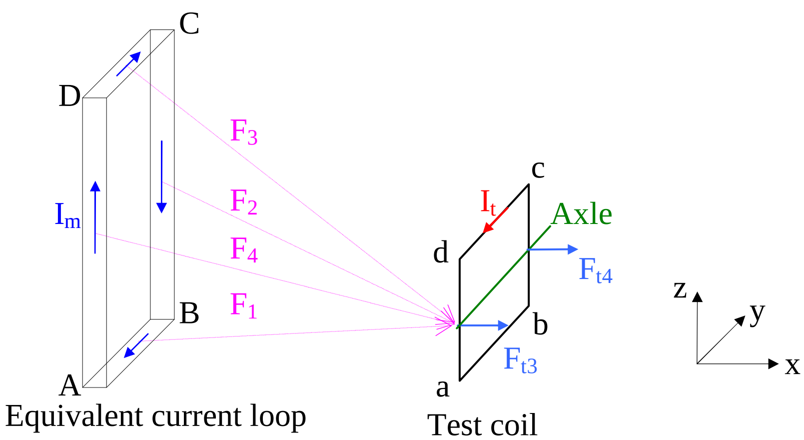

Figure 19

10.2.3 Torque about y-axis

The lever arms of the forces $\mathbf{F}_{t1}$, $\mathbf{F}_{t2}$, $\mathbf{F}_{t3}$ and $\mathbf{F}_{t4}$ are respectively:

$$-\frac{ad}{2}\mathbf{e}_z,\ \frac{ad}{2}\mathbf{e}_z,\ 0,\ 0$$

So, the resultant torque is the following sum of vector products:

$$\boldsymbol{\tau}_y = -\frac{ad}{2}\mathbf{e}_z \times \mathbf{F}_{t1} + \frac{ad}{2}\mathbf{e}_z \times \mathbf{F}_{t2} + 0 \times \mathbf{F}_{t3} + 0 \times \mathbf{F}_{t4}$$

From above we obtain the x components of the forces $\mathbf{F}_{t1}$ and $\mathbf{F}_{t2}$:

$$F_{t1,x} = -\frac{\mu_0 I_m I_t}{4\pi} \int_{AB+CD} \int_{ab} \left(\frac{\Delta x}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right), \; F_{t2,x} = -\frac{\mu_0 I_m I_t}{4\pi} \int_{AB+CD} \int_{cd} \left(\frac{\Delta x}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \qquad (26)$$

where Δx is the x-component of the radial vector **r**

So, the total torque about y-axis is:

$$\boldsymbol{\tau}_y = -\frac{\mu_0 I_m I_t}{4\pi} \frac{ad}{2} \left(\left| \int_{AB+CD} \int_{ab} \left(\frac{\Delta x}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \right| + \left| \int_{AB+CD} \int_{cd} \left(\frac{\Delta x}{r^3} d\mathbf{l}_m \cdot d\mathbf{l}_t \right) \right| \right) \mathbf{e}_y \qquad (27)$$

The corrected magnetic force law predicts that the torque about y-axis is not zero and the test coil will rotate about y-axis, fitting the experimental result.

The Lorentz force law says that magnetic force has the same magnitude in all direction, that is, magnetic force is isotropic. Using the corrected magnetic force law and experiment, we have proven that magnetic force's magnitude varies according to the direction of current, that is, magnetic force is anisotropic.

11 Conclusion

This study has proven experimentally and theoretically that in certain cases the Lorentz force law's predictions are wrong. The parallel action experiment finds a magnetic force that is parallel to current whereas the Lorentz force law says there is not. The perpendicular action experiment finds no magnetic force in a case where the Lorentz force law says there is one. The study about the open circuit shows that the Lorentz force law leads to an energy creating force with infinite magnitude.

Derived from the Lorentz force law, the corrected magnetic force law solves the open circuit inconsistency by respecting Newton's third law on differential level in both physical and mathematical senses. It explains correctly the parallel to current magnetic force and the anisotropy of magnetic force. We conclude that the right law is the corrected magnetic force law but not the Lorentz force law.

2 new properties of magnetic force have been discovered: anisotropy, parallel to current magnetic force. These new properties and the corrected magnetic force law need to be studied with precise measurement in future and open new fields of research.

References

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3. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, p. 200