Correct differential magnetic force law

Peng Kuan 彭宽
titang78@gmail.com
Tuesday, May 01, 2012

The Internal Lorentz force inconsistency is illustrated by the many contradictions I have provided (read the following documents)
Synthesis of the inconsistency of the Lorentz force law
Mathematical cause of the existence of the remaining resultant internal Lorentz force
Proof of the remaining resultant Lorentz force internal to a triangular coil
All Documents Links

These contradictions are sufficient evidences to break the Lorentz force law down. But how would physics be without the Lorentz force law? In fact, the correct magnetic force law is already there and the above contradictions were demonstrated to clear the way. Indeed, the Lorentz force law is so strong in the mind of physicists that no one accepts a new law without the old one proven to be false.

The principal cause of this inconsistency is the violation of the third Newton's law. The correct magnetic force law solves this problem by defining a differential magnetic force that respects it. Let us calculate the Lorentz force between 2 closed coils c_1 and c_2 (see Figure 1) without using magnetic field, but instead, by integrating the differential force between infinitesimal current elements (ref. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, pp. 199-200).

A current element vector is defined as the product of the current I, the length dl and the unit vector $\boldsymbol{n}$:

$$d\boldsymbol{I} = I\, dl\, \boldsymbol{n}$$

The 2 current element vectors taken from the coils c_1 and c_2 are denoted by the subscript 1 and 2 (see Figure 1):

$$d\boldsymbol{I}_1 = I\, dl_1\, \boldsymbol{n}\ ,\ d\boldsymbol{I}_2 = I\, dl_2\, \boldsymbol{n} \qquad (1)$$

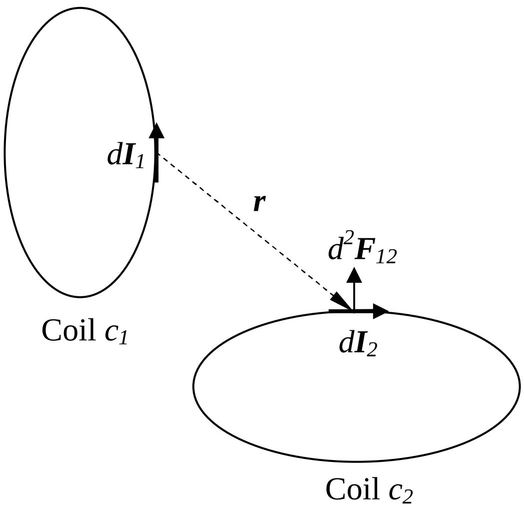

Figure 1

According to the Lorentz force law, the differential Lorentz force that $d\boldsymbol{I}_1$ exerts on $d\boldsymbol{I}_2$ is:

$$d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} d\boldsymbol{I}_2 \times \left(d\boldsymbol{I}_1 \times \frac{\boldsymbol{r}}{r^3} \right) \tag{2}$$

We expand this double cross product by using the well known relation:

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \bullet \boldsymbol{c})\, \boldsymbol{b} - (\boldsymbol{a} \bullet \boldsymbol{b})\, \boldsymbol{c}$$

Then the equation (2) becomes:

$$d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} \left[\left(d\boldsymbol{I}_2 \bullet \frac{\boldsymbol{r}}{r^3} \right) d\boldsymbol{I}_1 - \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right) \frac{\boldsymbol{r}}{r^3} \right] \tag{3}$$

First, we integrate $d^2\boldsymbol{F}_{12}$ over c_2:

$$d\boldsymbol{F}_{1-c2} = \int_{c2} d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} \left[\int_{c2} \left(d\boldsymbol{I} \bullet \frac{\boldsymbol{r}}{r^3} \right) d\boldsymbol{I}_1 - \int_{c2} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right) \frac{\boldsymbol{r}}{r^3} \right] \tag{4}$$

By using the equation (1), the first integral becomes:

$$\int_{c2} \left(d\boldsymbol{I}_2 \bullet \frac{\boldsymbol{r}}{r^3} \right) d\boldsymbol{I}_1 = \int_{c2} \left(I_2 dl_2 \boldsymbol{n}_2 \bullet \frac{\boldsymbol{r}}{r^3} \right) d\boldsymbol{I}_1$$

$d\boldsymbol{I}_1$ and I_2 being constant, they are put out of the integral sign. Like an electrostatic field whose circulation over closed path is 0, the integral over the coil c_2 of the vector field $\frac{\boldsymbol{r}}{r^3}$ is its circulation, thus the first integral is 0:

$$\int_{c2} \left(d\boldsymbol{I}_2 \bullet \frac{\boldsymbol{r}}{r^3} \right) d\boldsymbol{I}_1 = I_2 d\boldsymbol{I}_1 \int_{c2} \frac{\boldsymbol{r}}{r^3} \bullet \boldsymbol{n}_2 dl_2 = 0$$

So, the equation (4) becomes:

$$d\boldsymbol{F}_{1-c2} = -\frac{\mu_0}{4\pi} \int_{c2} \frac{\boldsymbol{r}}{r^3} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right)$$

Now, we integrate $d\boldsymbol{F}_{1-c2}$ over the coil c_1 and obtain the total Lorentz force the coil c_1 exerts on the coil c_2:

$$\boldsymbol{F}_{c1-c2} = \int_{c1} d\boldsymbol{F}_{1-c2} = \int_{c1} \int_{c2} \left(-\frac{\mu_0}{4\pi} \frac{\boldsymbol{r}}{r^3} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right) \right) \tag{5}$$

The integrand of this double integral is defined as a differential magnetic force:

$$d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi} \frac{\boldsymbol{r}}{r^3} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right) \tag{6}$$

This force is proportional to the dot product of 2 current element vectors and resembles the Ampere's force between 2 parallel currents. So, it is given the name "differential Ampere's force " and the equation (6) the "differential Ampere's force law".

The equation (5) and (6) are interesting in that they prove that the differential Ampere's force law gives an integral force identical to the Lorentz force while having different value on the differential level:

$$d^2\boldsymbol{F}_{amp} \neq d^2\boldsymbol{F}_{12} \qquad\qquad \int_{c1}\int_{c2} d^2\boldsymbol{F}_{amp} = \int_{c1}\int_{c2} d^2\boldsymbol{F}_{12}$$

Because the integral forces are identical between the 2 laws, the differential Ampere's force law satisfies all experiments that have served to define the Lorentz force law. Even more, the differential Ampere's force on infinitesimal current elements respects the third Newton's law. Indeed, the reaction force of $d^2\boldsymbol{F}_{amp}$ is obtained by reversing the radial vector $\boldsymbol{r}$ in the equation (6), and thus has the same magnitude and the opposite sign.

The correct differential magnetic force law is then:

$$\boxed{d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{r}}{r^3}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)}$$

And the correct expression of magnetic force between 2 closed coils is:

$$\boxed{\boldsymbol{F}_{c1-c2} = \int_{c1}\int_{c2}\left(-\frac{\mu_0}{4\pi}\frac{\boldsymbol{r}}{r^3}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)\right)}$$

The theoretical prediction of the magnetic torque in the experiment proposed in Lorentz torque experiment is calculated with this law. Because the differential Ampere' force respects the third Newton's law and is identical to the Lorentz force for integral force, I'm confident that this experiment will succeed and I invite all experimenters to make this great contribution to physics.