Macroscopic Aharonov–Bohm effect
Experiment and theory
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The Aharonov–Bohm effect is a phenomenon in which an electrically charged particle is affected in a region in which both the magnetic field B and electric field E are zero. Simply speaking, when an electron beam passes around the middle of a long solenoid, the beam is strangely deviated and the interference pattern on the screen slides (Figure 1). As there is not classical force exerted on the electrons, this effect is commonly considered as quantum mechanical. For more detail, see Wikipedia article http://en.wikipedia.org/wiki/Aharonov%E2%80%93Bohm_effect
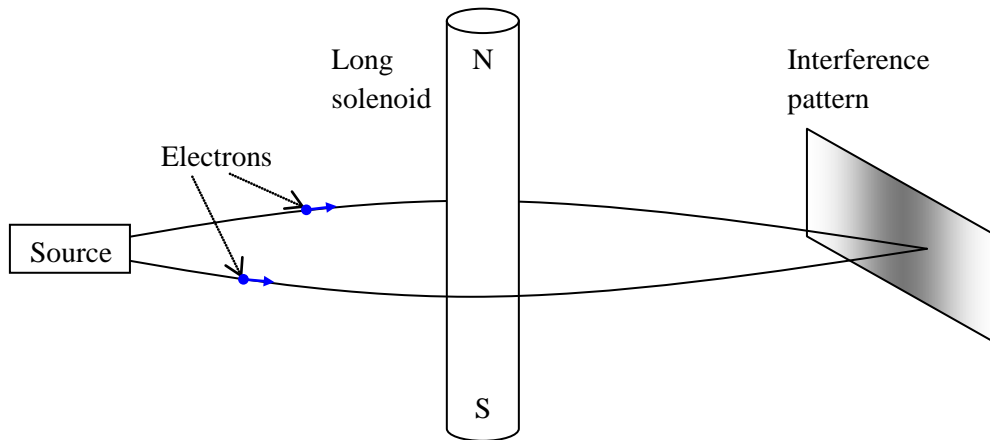Wolfram Demonstrations Project video http://youtu.be/OgDPK5MLVnE

Figure 1

However, the force in the Aharonov–Bohm effect is well explained by the corrected magnetic force law I proposed in "Unknown properties of magnetic force and Lorentz force law"
http://pengkuanem.blogspot.com/2013/04/unknown-properties-of-magnetic-force.html
In order to illustrate this explanation, I have done an experiment that shows this effect in ordinary condition, that is, with a current carrying wire and a bar magnet. The video of this experiment:
http://youtu.be/ugxmtT4FUME

In this experiment, the long solenoid is simulated by a bar magnet whose field is also zero around its middle. The first sequence of this video shows that the iron nails are not attracted by the middle of the bar magnet demonstrating that the magnetic field is well zero there.

The second sequence shows that the current carrying wire is pushed by the zero magnetic field. The red arrow below the wire shows the direction of its current; the white arrow on the bar magnet indicates the direction of the equivalent current producing the field of the magnet. We see that when the 2 arrows are in the same direction, the wire is attracted to the magnet; when the arrows are opposite, the wire is repulsed by the magnet.

Let us explain this force with the corrected magnetic force law that expresses the differential magnetic force as below:

$$d\vec{F} = -\frac{\mu_0}{4\pi}\left(d\vec{I}_1 \cdot d\vec{I}_2\right)\frac{\vec{r}}{r^3} \qquad (1)$$

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For explanation convenience, we take a long rectangular solenoid, outside which the magnetic field is also zero. Figure 2 shows a section ABCD of this solenoid and a current carrying wire. The current in the side AB is in the same direction than that in the wire, the current in CD is opposite to that in the wire. In BC and DA, the currents are perpendicular to the wire.

According to equation (1), 2 currents in same direction attract each other; 2 currents with opposite direction repulse each other; 2 perpendicular currents do not exert any force on each other. So, BC and DA do not exert any force on the wire, AB attracts the wire, CD repulses the wire. As AB is nearer to the wire than CD, the total force is an attraction to the solenoid. This analyze fits the experimental observation.
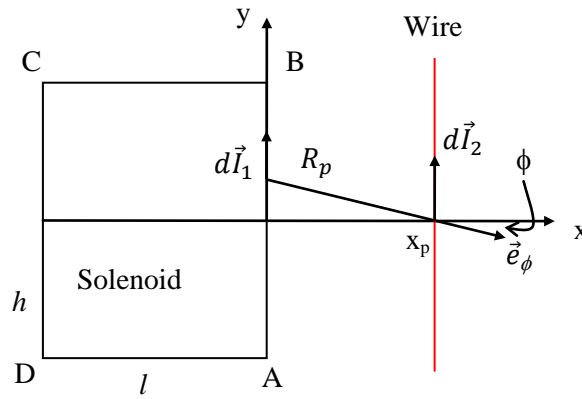

Figure 2

Let us compute the force this solenoid exerts on the central segment of the wire. The length of the segment is Δy_2. By integrating equation (1) over the solenoid, we obtain the total force on the segment:

$$\vec{F}_2 = \int\limits_{all\ turns} \int\limits_{z=-\infty}^{\infty} -\frac{\mu_0}{4\pi}(d\vec{I}_1 \cdot d\vec{I}_2)\frac{\vec{r}}{r^3}$$

The current element of the segment is: $\qquad d\vec{I}_2 = I_2\Delta y_2 \vec{e}_y$

For a section of the solenoid of thickness dz, the current is $J_1 dz$, with J_1 the current density. In the side AB, a current element of length dy is:

$$d\vec{I}_1 = J_1 dz dy \vec{e}_y$$

And we have: $\qquad d\vec{I}_1 \cdot d\vec{I}_2 = J_1 dz dy I_2 \Delta y_2$

The force that AB exerts on $d\vec{I}_2$ is integrated over AB:

$$\vec{F}_{AB} = \int\limits_{AB} \int\limits_{z=-\infty}^{\infty} -\frac{\mu_0}{4\pi}(J_1 dz dy I_2 \Delta y_2)\frac{\vec{r}}{r^3}$$

$$\vec{F}_{AB} = -\frac{\mu_0}{4\pi}J_1 I_2 \Delta y_2 \int\limits_{AB}\left(\int\limits_{z=-\infty}^{\infty}\frac{\vec{r}dz}{r^3}\right)dy$$

We consider mathematically the solenoid as infinitely long in z direction. With R_p been the x-y plane distance between $d\vec{l}_1$ and the segment (see Figure 2), the integral over z becomes:

$$\int_{z=-\infty}^{\infty} \frac{\vec{r}dz}{r^3} = \frac{2}{R_p}\vec{e}_\phi$$

Then we have:

$$\vec{F}_{AB} = -\frac{\mu_0}{4\pi}J_1I_2\Delta y_2 \int_{AB}\left(\frac{2}{R_p}\vec{e}_\phi\right)dy$$

The unit vector is:

$$\vec{e}_\phi = \cos\phi\,\vec{e}_x - \sin\phi\,\vec{e}_y$$

By symmetry the y component vanishes:

$$\int_{AB}(\sin\phi\,\vec{e}_y)dy = 0$$

The force is then in x direction only:

$$\vec{F}_{AB} = -\frac{\mu_0}{4\pi}J_1I_2\Delta y_2 \int_{AB}\left(\frac{2}{R_p}\cos\phi\,\vec{e}_x\right)dy$$

By using:

$$\cos\phi = \frac{x_p}{R_p} \text{ and } R_p = \sqrt{x_p^2 + y^2}$$

The force from AB becomes:

$$\vec{F}_{AB} = -\frac{\mu_0}{4\pi}J_1I_2\Delta y_2 \int_{AB}\left(\frac{2x_p}{R_p^2}\vec{e}_x\right)dy$$

$$\vec{F}_{AB} = -\frac{\mu_0}{\pi}J_1I_2\Delta y_2 \left(\tan^{-1}\frac{h}{2x_p}\right)\vec{e}_x$$

where h is the height of the solenoid.

For the side CD, the current element is: $\quad d\vec{l}_1 = -I_1 dy\,\vec{e}_y$

And: $\quad d\vec{l}_1 \cdot d\vec{l}_2 = -I_1 dy I_2 \Delta y_2$

Then, we derive the force from CD in the same way:

$$\vec{F}_{CD} = \frac{\mu_0}{\pi}J_1I_2\Delta y_2 \left(\tan^{-1}\frac{h}{2(x_p + l)}\right)\vec{e}_x$$

where l is the width of the solenoid.

The total force the solenoid exerts on $d\vec{l}_2$ is then:

$$\vec{F}_2 = \frac{\mu_0}{\pi}J_1I_2\Delta y_2 \left(\tan^{-1}\frac{h}{2(x_p + l)} - \tan^{-1}\frac{h}{2x_p}\right)\vec{e}_x \tag{2}$$

For a round solenoid, the force is computed referring to Figure 3. The derivation is in the annex. The expression for this force is:

$$\vec{F}_2 = -\frac{\mu_0}{2\pi}J_1I_2\Delta y_2 \int_{\theta=0}^{2\pi} \frac{\left(\frac{x_p}{R} - \cos\theta\right)\cos\theta}{\sin^2\theta + \left(\frac{x_p}{R} - \cos\theta\right)^2}d\theta \tag{3}$$

With equations (2) and (3), we can predict the displacement of the interference pattern in real Aharonov–Bohm effect experiment. For a beam of electron, I_2 is negative, so the force $\vec{F}_2$ is in positive x direction. When the current in the solenoid increases, $\vec{F}_2$ increases and the interference

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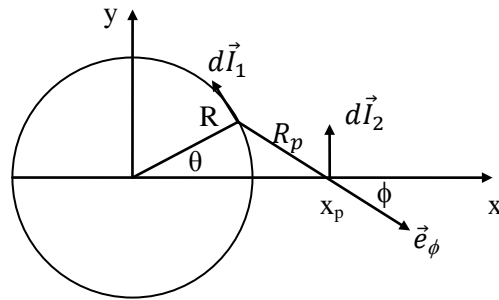pattern would move to the right in Figure 1. This is confirmed by the video of Wolfram Demonstrations Project http://youtu.be/OgDPK5MLVnE.

Figure 3

Comment

The Aharonov–Bohm effect is very important in physics, it is seen as an experimental proof of the "Spooky action at a distance" in quantum mechanics because the magnetic field is inside the solenoid but the force is outside. My experiment and theoretical explanation show that it is not a force out of zero field, but a real force between currents. This work has two important consequences: it explains correctly the Aharonov–Bohm effect and proves that the Lorentz force law fails to explain magnetic force in regions where magnetic field is zero.

In the future, the intensity of this force must be measured in more precise experiments and compared with prediction using equation (2) and (3). It will also be necessary to do an experiment with electron beam in cathode ray tube (CRT) with 2 solenoids on both sides to influence the beam. If you have a long CRT and 2 long solenoids, do not hesitate to do this experiment. You are also invited to do the precise measurement of the force. The Aharonov–Bohm effect is so famous that any experiment about this mystery will get great attention.

$$d\vec{I_1} = J_1 dz R d\theta \vec{e}_\theta = J_1 dz R d\theta \left(-\sin\theta\, \vec{e}_x + \cos\theta\, \vec{e}_y \right)$$

$$d\vec{I_2} = I_2 \Delta y_2 \vec{e}_y$$

$$d\vec{I_1} \cdot d\vec{I_2} = J_1 dz R d\theta \left(-\sin\theta\, \vec{e}_x + \cos\theta\, \vec{e}_y \right) \cdot I_2 \Delta y_2 \vec{e}_y = J_1 dz R d\theta \cdot I_2 \Delta y_2 \cos\theta$$

$$\vec{F_2} = -\frac{\mu_0}{4\pi} \int_{\theta=0}^{2\pi} \int_{z=-\infty}^{\infty} J_1 dz R d\theta \cdot I_2 \Delta y_2 \cos\theta \frac{\vec{r}}{r^3} = -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} \left(\int_{z=-\infty}^{\infty} \frac{\vec{r} dz}{r^3} \right) \cos\theta\, d\theta$$

$$\vec{F_2} = -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} \left(\frac{2\cos\theta}{R_p} \vec{e}_\phi \right) d\theta$$

$$= -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} \frac{2\cos\theta}{R_p} \left(\cos\phi\, \vec{e}_x - \sin\phi\, \vec{e}_y \right) d\theta$$

$$= -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} \frac{2\cos\theta \cos\phi}{R_p} d\theta$$

$$= -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} 2 \frac{\left(x_p - R\cos\theta \right)\cos\theta}{R_p{}^2} d\theta$$

$$= -\frac{\mu_0}{4\pi} J_1 R \cdot I_2 \Delta y_2 \int_{\theta=0}^{2\pi} 2 \frac{\left(x_p - R\cos\theta \right)\cos\theta}{(R\sin\theta)^2 + \left(x_p - R\cos\theta \right)^2} d\theta$$

$$\vec{F_2} = -\frac{\mu_0}{2\pi} J_1 I_2 \Delta y_2 \int_{\theta=0}^{2\pi} \frac{\left(\frac{x_p}{R} - \cos\theta \right)\cos\theta}{\sin^2\theta + \left(\frac{x_p}{R} - \cos\theta \right)^2} d\theta$$

$\displaystyle\int_{z=-\infty}^{\infty} \frac{\vec{r} dz}{r^3} = \frac{2}{R_p} \vec{e}_\phi$
$\vec{e}_\phi = \cos\phi\, \vec{e}_x - \sin\phi\, \vec{e}_y$
$\displaystyle\int_{\theta=0}^{2\pi} \cos\theta \sin\phi\, d\theta = 0$
$\cos\phi = \dfrac{x_p - R\cos\theta}{R_p}$
$R_p = \sqrt{(R\sin\theta)^2 + \left(x_p - R\cos\theta \right)^2}$