A Derivation of Faraday's law from Coulomb's Law and Relativity

1. The Progressing Electric Field Model

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Abstract: Faraday's law is empirically derived and, as such, may be subject to limitations. Notably, it appears to violate the law of conservation of energy in certain contexts. To establish a more robust formulation, it is necessary to derive the law from first principles. In this article, we theoretically derive Faraday's law using only Coulomb's law and special relativity. We present the first stage of this derivation: the construction of the 'Progressing Electric Field Model.' This model determines the curl of the electric field produced by moving charges and calculates the electric potential induced in a wire loop within that field.

1. Introduction

In electromagnetism, Faraday's law defines the electromotive force (EMF) induced in a wire loop by a varying magnetic field. Although considered a cornerstone of classical electromagnetism, it remains an empirical law; consequently, it may be incomplete regarding phenomena not yet captured by experimental observation. To illustrate this, consider the following experimental setup: suppose two coils, A and B, are positioned side by side, with coil B connected to a resistor R, as shown in Figure 1.

Let the current in coil A, denoted as I_a, vary as follows: I_a increases linearly from zero to I_{max}, then decreases linearly back to zero. The duration of each phase is Δt. According to Faraday's law, voltages are induced in coils A and B, which we label V_a and V_b, respectively. Since I_a varies linearly during each phase, V_a and V_b remain constant throughout those intervals. Within resistor R, the voltage V_b generates a current I_b and dissipates electric power equal to $|V_b I_b|$, both of which are constant in each phase. Consequently, the total work performed in R after both phases is $2|V_b I_b|\Delta t$.

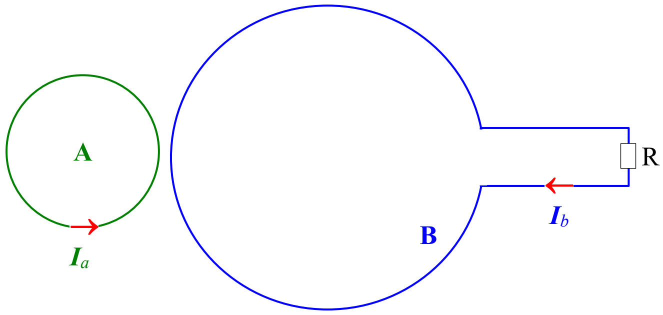


Figure 1

Since I_b is constant, it does not induce a voltage in coil A; therefore, the value of V_a remains unchanged regardless of whether I_b is positive, negative, or zero—just as if coil B were not present. When I_a increases, the voltage in coil A (V_a) is positive, and the electrical work performed in A is given by $\int_o^{\Delta t} V_a I_a dt$. Conversely, when I_a decreases, the voltage in A becomes $-V_a$, and the work equals $-\int_o^{\Delta t} V_a I_a \, dt$. Consequently, the total energy consumption of coil A after both phases equals zero:

$$\int_o^{\Delta t} V_a I_a dt - \int_o^{\Delta t} V_a I_a \, dt = 0 \quad (1)$$

Since the energy consumption in coil A is zero, A does not transfer any energy to coil B. We therefore encounter a case where B performs work equal to $2|V_b I_b|\Delta t$ while receiving no energy from A. This implies that the system consisting of coils A and B performs work without any energy input, which violates the law of conservation of energy.

The cause of this violation is that Faraday's law predicts zero voltage in A when the current in coil B is constant. By theoretically deriving Faraday's law from more fundamental laws, we can not only resolve this inconsistency but also uncover new phenomena and achieve a deeper understanding of nature.
An example of such a phenomenon is demonstrated in my experiment, which reveals a tangential electromotive force not predicted by Faraday's law. You can view the experiment in this video on YouTube:
https://www.youtube.com/watch?v=P33Hgj68G9M

To begin the theoretical derivation of Faraday's law, we must first examine the electric field of a moving electron.

2. The Electric Field of a Moving Electron

a) The Static Case: The Immobile Electron

The electric field $\boldsymbol{E}$ of a stationary electron is static and radiates symmetrically in concentric circles centered on the electron (see Figure 2). Along each of these circles, the intensity of the electric field remains constant. While these circles are typically viewed as equipotential lines in electrostatics, we designate them as 'Iso-intensity circles' to better characterize the field of a moving electron, which is dynamic rather than static.

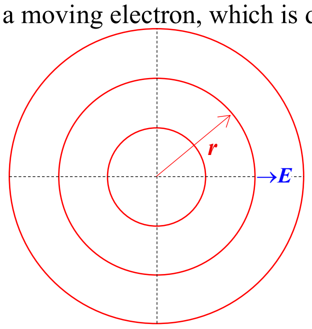


Figure 2

b) The Dynamic Case: The Moving Electron and the Progressing Field

When an electron moves, its electric field varies in space. For explaining this variation, let's take two electrons ⓐ and ⓑ, ⓐ moves at the velocity v and ⓑ is immobile. Let the distance between ⓐ and ⓑ be represented by r.

As ⓐ moves, r varies and subsequently the Coulomb's force that ⓐ exerts on ⓑ varies. This force is represented by $\boldsymbol{F}_b$, see Figure 3.

ⓐ$\rightarrow \boldsymbol{v} \xrightarrow{\boldsymbol{r}}$ ⓑ$\rightarrow \boldsymbol{F}_b$

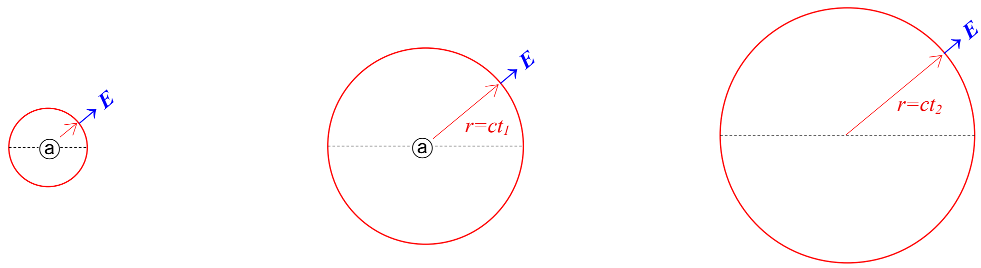
Figure 3

According to special relativity, no information travels faster than the speed of light c. So, we suppose that electric field travels at c, which makes the force $\boldsymbol{F}_b$ to delay with respect to the actual position of ⓐ. This delay is computed as follow. Suppose that at time $t<0$ the electron ⓑ was alone in space. At time $t=0$, the electron ⓐ pops up into existence. At this moment, the force on ⓑ is still zero because the electric field of ⓐ has not reached the electron ⓑ. At time $t= r/c$ the electric field of ⓐ has done the distance r and reached ⓑ. At this time only, the electric force on ⓑ becomes nonzero. Table 1 explains this process.

$t<0$	The charge ⓑ is alone in space	ⓑ
$t=0$	The charge ⓐ pops up into existence at the distance r from ⓑ	ⓐ $\xrightarrow{r}$ ⓑ $F_b=0$
$t= r/c$	The electric field of ⓐ has reached ⓑ and creates the force F_b	ⓐ $\longrightarrow$ ⓑ $\lvert F_b \rvert>0$

Table 1

The electric field of the electron ⓐ evolves in space in iso-intensity circles. Suppose that an iso-intensity circle has left ⓐ at time 0 and expands at the speed of light c. Meanwhile, the electron ⓐ continues to move, changes position and is no longer at the center of the previous iso-intensity circles. So, iso-intensity circles are independent of the electron ⓐ after leaving it. Even if ⓐ disappears in the future, the iso-intensity circles will still exist in space. Figure 4 shows three situations of an iso-intensity circle with respect to the electron.



An iso-intensity circle leaves the electron ⓐ at time t=0.

At time t_1, the radius of the circle equals ct_1.

At time t_2, ⓐ has disappeared but the circle still exists.

Figure 4

The center of an iso-intensity circle is the point from which the circle has left the electron and will stay immobile in space forever. The circle expands with time t and its radius equals ct. As the electron ⓐ moves, it continuously emits iso-intensity circles which leave ⓐ and expand in space on their own. As illustration, we have plotted 3 iso-intensity circles in Figure 5. At time $t=0$ the electron ⓐ was at the point ⓐ$_1$ where the circle 1 was emitted. In the figure, time is t and the radius of the circle 1 is $r_1=ct$. At time t_2, ⓐ was at the point ⓐ$_2$ where the circle 2 was emitted. The radius of the circle 2 is $r_2=c(t-t_2)$. At time t_3, ⓐ was at the point ⓐ$_3$ where the circle 3 was emitted. The radius of the circle 3 is $r_3=c(t-t_3)$.

The center of the circles shifts to the right making the 3 circles tighter on the right. These eccentric circles look like a wave under Doppler effect, except that electric field's intensity is smooth in space instead of wavy.

As these iso-intensity circles expand, the intensity and direction of the electric field at any fixed point in space vary over time. This dynamic behavior distinguishes this phenomenon from a static electric field. Because these circles originate from the retarded positions of the electron, their centers effectively 'move' through space along the electron's trajectory. This progression of the field's origins led us to term the electric field of moving electrons the **'Progressing Electric Field'**, and to designate this framework the **'Progressing Electric Field Model.'**

It should be noted that the Progressing Electric Field is physically identical to the retarded electric field of a moving charge. Our model adopts this terminology to emphasize the dynamic propagation and the 'progression' of the field's origins, which are essential for deriving the non-conservative properties of the field.

In developing the **Progressing Electric Field model**, we recognize that even a static electric field is characterized by a continuous flow through space. In essence, the field is constantly radiated outward from the electron—a fact that only becomes apparent when the electron moves and no longer serves as the common center for the iso-intensity circles it has emitted. A static field appears stationary only because the field elements emitted at different times (t_1 and t_2) happen to reach a specific point with identical intensity and direction. In reality, these are distinct field emissions, differing by the moment of their creation.

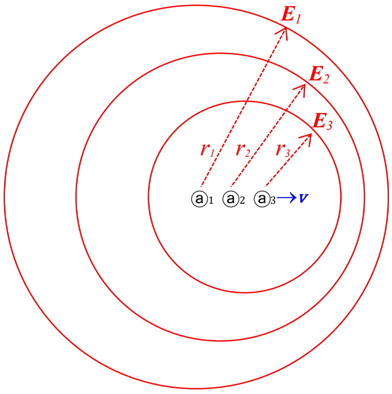


Figure 5

3. Geometry of the Wire Loop and Iso-intensity Circles

a) Partitioning the Wire Loop into Sectors

Faraday's law traditionally defines the electromotive force (EMF) induced by a time-varying magnetic field. However, in this work, I will demonstrate that the EMF arises instead from the iso-intensity circles of the progressing electric field—a concept of my own creation. To determine the nature of this influence, we have plotted four iso-intensity circles in Figure 6, along with their respective centers, indicated by the four red dots. Note that the eccentricity between these circles has been exaggerated for visual clarity. A wire loop has been superimposed over these four circles to evaluate the interaction.

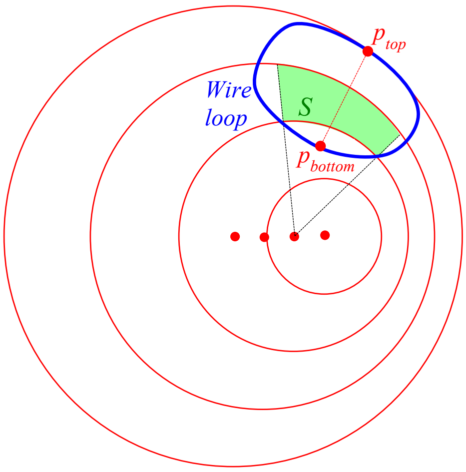


Figure 6

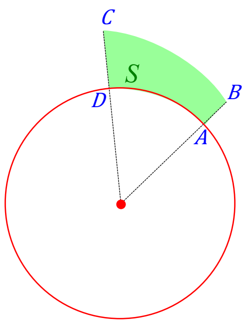


Figure 7

The surface bounded by the wire loop is divided into sectors by three of the iso-intensity circles. To determine the total influence on the entire wire loop, we first analyze the effect of the progressing electric field within a single sector—specifically, the sector highlighted in green and labeled S. This sector S is isolated in Figure 7, with its four corners designated as A, B, C, and D.

The sector S is bounded by the arcs of two successive iso-intensity circles and two segments, AB and CD, which align with the radial vectors of the inner circle (see Figure 7). While AB and CD do not represent the physical wire, they define the geometric boundary of the sector. In Figure 6, small "white spaces" appear between the wire loop and these radial segments. However, since there are infinitely many iso-intensity circles, any two successive circles are infinitely close to one another. In this continuous limit, these white spaces disappear, and the sum of all such sectors completely fills the surface bounded by the wire loop.

In order to calculate the influence within a single sector, we must determine the lengths of the segments AB and CD, as well as the total surface area of sector S.

b) Determining the Lengths of Segments AB and CD

For computing the lengths of the segments AB and CD, we take two iso-intensity circles. The center of the inner circle is O_i, that of the outer circle is O_e, see Figure 8. The segments AB and CD are local clearance between the two circles, which is represented by the letter $\boldsymbol{h}$. O_eO_i is the eccentricity line, its length is l. The radius vector of the outer circle is the vector $\boldsymbol{r}_e$ between O_e and the point p, the length of $\boldsymbol{r}_e$ is r_e. The straight line between O_i and p is composed of the radius vector of the inner circle $\boldsymbol{r}_i$ and the vector $\boldsymbol{h}$; the length of this line equals r_i+h. The lengths of the three sides of the triangle O_eO_ip are r_e, l and r_i+h respectively, the angle at O_e is θ.

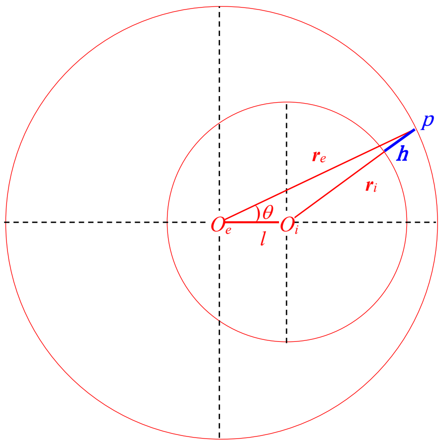


Figure 8

We apply the law of cosines to the triangle O_eO_ip and get h:

$$(r_i+h)^2 = {r_e}^2 + l^2 - 2r_e l\cos\theta$$
$$= {r_e}^2\left(1+\left(\frac{l}{r_e}\right)^2 - 2\frac{l}{r_e}\cos\theta\right) \qquad (2)$$

$$r_i+h = r_e\left(1+\left(\frac{l}{r_e}\right)^2 - 2\frac{l}{r_e}\cos\theta\right)^{\frac{1}{2}} \qquad (3)$$

$$h = r_e\left(1+\left(\frac{l}{r_e}\right)^2 - 2\frac{l}{r_e}\cos\theta\right)^{\frac{1}{2}} - r_i \qquad (4)$$

The iso-intensity circles are emitted by an electron that moves at the velocity v. The outer circle was emitted at time t=0 and the inner circle at t=Δt. These circles expand at the speed of light c and at the time t, the radii r_e and r_i equals:

$$r_e = ct \qquad (5)$$
$$r_i = c(t-\Delta t) \qquad (6)$$

Their difference equals:

$$r_e - r_i = ct - c(t-\Delta t)$$
$$= c\Delta t \qquad (7)$$

The eccentricity line O_eO_i is the distance that the electron moved in Δt, so the length of O_eO_i equals l:

$$l = v\Delta t \qquad (8)$$

The quotient $\frac{l}{r_e - r_i}$ equals:

$$\frac{l}{r_e - r_i} = \frac{v\Delta t}{c\Delta t}$$
$$= \frac{v}{c} \qquad (9)$$

The quotients $\frac{l}{r_e}$ and $\left(\frac{l}{r_e}\right)^2$ equal:

$$\begin{aligned}\frac{l}{r_e} &= \frac{v\Delta t}{ct} \\ &= \frac{v}{c}\frac{\Delta t}{t}\end{aligned} \qquad (10)$$

$$\left(\frac{l}{r_e}\right)^2 = \left(\frac{v}{c}\right)^2\left(\frac{\Delta t}{t}\right)^2 \qquad (11)$$

Because $\frac{v}{c} \ll 1$, $\left(\frac{l}{r_e}\right)^2$ is neglected in (4). In physics, we can use the sign "=" when a really small quantity is neglected. Then, h can be expressed as:

$$h = r_e\left(1 - 2\frac{l}{r_e}\cos\theta\right)^{\frac{1}{2}} - r_i \qquad (12)$$

Because $2\frac{l}{r_e}\cos\theta$ is very small, we expand $\left(1 - 2\frac{l}{r_e}\cos\theta\right)^{\frac{1}{2}}$ into a first-order Taylor series:

$$\left(1 - 2\frac{l}{r_e}\cos\theta\right)^{\frac{1}{2}} = 1 - \frac{l}{r_e}\cos\theta \qquad (13)$$

and h equals, see (12):

$$h = r_e - r_i - l\cos\theta \qquad (14)$$

- Lengths of the segments AB and CD

Let the angles θ_1 correspond the point A and θ_2 to the point D, see Figure 9. the local clearances at A and D equal, see (14):

$$h_A = r_e - r_i - l\cos\theta_1 \qquad (15)$$

$$h_D = r_e - r_i - l\cos\theta_2 \qquad (16)$$

AB equals the local clearance at A, but CD equals the reverse of the local clearance at D which is DC:

$$AB = h_A, CD = -h_D \qquad (17)$$

c) Calculation of the Surface Area of Sector S

The surface of the sector S equals the integral of the elementary surface ds which equals:

$$ds = h \cdot r_i d\theta \qquad (18)$$

with h being the local clearance and $r_i d\theta$ the elementary length tangential to the circle.

The integral of ds from the angle θ_1 to θ_2 equals:

$$\begin{aligned}\int_1^2 ds &= \int_1^2 h \cdot r_i d\theta \\ &= r_i\int_1^2 (r_e - r_i - l\cos\theta)d\theta \\ &= r_i[(r_e - r_i)\theta - l\sin\theta]_1^2 \\ &= r_i[(r_e - r_i)(\theta_2 - \theta_1) - l(\sin\theta_2 - \sin\theta_1)]\end{aligned} \qquad (19)$$

$\int_1^2 ds$ equals the surface of the sector S which we label with s:

$$\begin{aligned}s &= \int_1^2 ds \\ &= r_i(r_e - r_i)\left(\theta_2 - \theta_1 - \frac{l}{r_e - r_i}(\sin\theta_2 - \sin\theta_1)\right)\end{aligned} \qquad (20)$$

4. Potential within the Progressing Electric Field

a) Definition of Potential

The electromotive force (EMF) induced in a wire loop, as predicted by Faraday's law, is fundamentally an electric potential. In a static electric field $\boldsymbol{E}$, the variation in potential is equal to the work performed by the electric force acting on a test charge. If we consider a charge q_b within the field $\boldsymbol{E}$, the electric force $\boldsymbol{F}$ acting upon it is given by:

$$\boldsymbol{F} = q_b\boldsymbol{E} \quad (21)$$

The work ΔW performed by an external force $\boldsymbol{F}$ is equal to the dot product of that force and the displacement of the charge q_b, represented by the vector $\boldsymbol{l}_b$:

$$\Delta W = q_b\boldsymbol{E} \cdot \boldsymbol{l}_b \quad (22)$$

In a static electric field, the variation in electric potential ΔU experienced by the charge q_b is equal to the negative of the work performed, divided by the charge:

$$\Delta U = \frac{\Delta W}{q_b} \qquad (23)$$

$$\Delta U = -\boldsymbol{E} \cdot \boldsymbol{l}_b \qquad (24)$$

In the context of the Progressing Electric Field, however, the situation is different because the motion of the source charge q_b is time-dependent. As q_b moves, the field itself evolves; consequently, the field configuration at the end of the motion is not the same as at the beginning. To avoid the complexities of a time-varying potential during charge displacement, we must not compute the potential variation while the charge is in motion. Instead, we utilize a snapshot of the Progressing Electric Field. Within this snapshot, the iso-intensity circles and the field vectors are spatially fixed. This allows us to compute the variation of the electric potential along a path using equation (24), independent of the concept of mechanical work. Let $\boldsymbol{E}_p$ represent the vector of the Progressing Electric Field and d$\boldsymbol{l}_p$ represent a differential distance vector. The variation of the electric potential along d$\boldsymbol{l}_p$ in this snapshot is defined by the following dot product:

$$dU_p = -\boldsymbol{E}_p \cdot d\boldsymbol{l}_p \qquad (25)$$

Within the snapshot, all values represent instantaneous states at the moment the 'frame' is frozen. Here, $\boldsymbol{E}_p$ denotes the instantaneous Progressing Electric Field, and U_p represents the instantaneous electric potential at a given point relative to a reference point. Consequently, throughout the following sections, the term 'variation of electric potential' specifically refers to the spatial variation of this instantaneous potential within the snapshot.

b) Calculation of Potential around the Boundary of a Sector

The variation of electric potential along a path equals the integral of dU_p along that path. Suppose the path begins at point 1 and ends at point 2. The potential at the point 2 with respect to the point 1 equals:

$$\Delta U_{p12} = -\int_1^2 \boldsymbol{E}_p \cdot d\boldsymbol{l}_p \quad (26)$$

The variation of electric potential around the edge of S equals the integral of dU_p in the counterclockwise direction along the edge of S which is a closed loop, see (25):

$$\begin{aligned}\Delta U_S &= \oint dU \\ &= -\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p\end{aligned} \qquad (27)$$

$\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p$ is a line integral over a closed loop, it is called the circulation of the vector $\boldsymbol{E}_p$ over this loop.

The edge of S is formed by four segments: AB, BC, CD and DA, see Figure 9. We split the closed line integral into the sum of line integrals over the four segments:

$$\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p = \int_A^B \boldsymbol{E}_1 \cdot d\boldsymbol{l}_p + \int_B^C \boldsymbol{E}_2 \cdot d\boldsymbol{l}_p + \int_C^D \boldsymbol{E}_3 \cdot d\boldsymbol{l}_p + \int_D^A \boldsymbol{E}_4 \cdot d\boldsymbol{l}_p \qquad (28)$$

where $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, $\boldsymbol{E}_3$ and $\boldsymbol{E}_4$ are the instantaneous progressing electric field on the four segments.

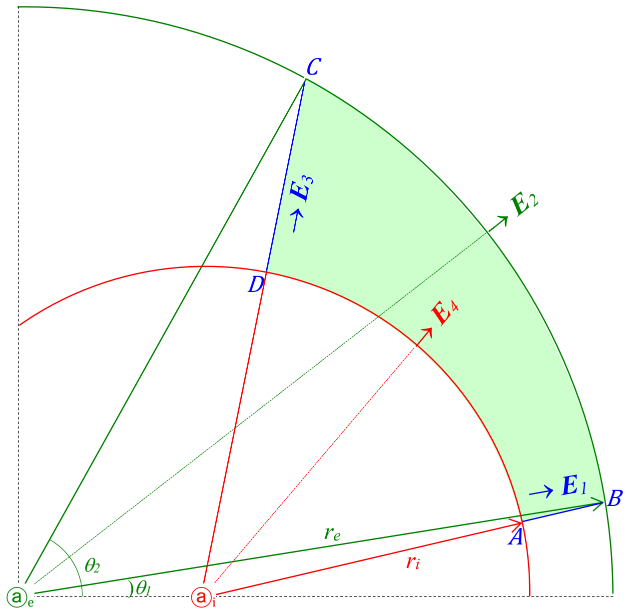


Figure 9

On a snapshot, the progressing electric field on an iso-intensity circle is fixed and defined by Coulomb's law:

$$\boldsymbol{E}_p = \frac{q_a}{4\pi\varepsilon_0 r^2}\boldsymbol{e}_r \quad (29)$$

with $\boldsymbol{e}_r$ being the unit radial vector of the iso-intensity circle.

Remark 1:

The unit vector $\boldsymbol{e}_r$ and the radius r are defined relative to the center of a specific iso-intensity circle. As we transition to a different iso-intensity circle, its center shifts relative to the previous one due to the progression of the field. Consequently, both $\boldsymbol{e}_r$ and r must be redefined with respect to the unique center of each successive circle.

c) Demonstration of Null Potential Variation on Arcs

The progressing electric field on BC is $\boldsymbol{E}_2$ and equals:

$$\boldsymbol{E}_2 = \frac{q_a}{4\pi\varepsilon_0 {r_e}^2}\boldsymbol{e}_r \qquad (30)$$

with r_e being the radius of the outer circle, see Figure 9.

The line integral on BC is along the arc BC and the elementary distance vector $\boldsymbol{dl}_p$ is parallel to the tangential unit vector $\boldsymbol{e}_\theta$:

$$\boldsymbol{dl}_p = dl_p\boldsymbol{e}_\theta \qquad (31)$$

So, the dot product $\boldsymbol{E}_2 \cdot \boldsymbol{dl}_p$ equals:

$$\begin{aligned}\boldsymbol{E}_2 \cdot \boldsymbol{dl}_p &= \frac{q_a}{4\pi\varepsilon_0 {r_e}^2}\boldsymbol{e}_r \cdot \left(dl_p\boldsymbol{e}_\theta\right) \\ &= \frac{q_a dl_p}{4\pi\varepsilon_0 {r_e}^2}\boldsymbol{e}_r \cdot \boldsymbol{e}_\theta\end{aligned} \qquad (32)$$

Because $\boldsymbol{e}_r \cdot \boldsymbol{e}_\theta = 0$, the dot product $\boldsymbol{E}_2 \cdot \boldsymbol{dl}_p$ equals zero:

$$\boldsymbol{E}_2 \cdot \boldsymbol{dl}_p = 0 \quad (33)$$

Then, the line integral on the arc BC equals zero:

$$\int_B^C \boldsymbol{E}_2 \cdot \boldsymbol{dl}_p = 0 \quad (34)$$

On the other hand, the field vector $\boldsymbol{E_4}$ is on the inner circle. For the same reason, the line integral on the arc DA is zero:

$$\int_{D}^{A} \boldsymbol{E}_4 \cdot d\boldsymbol{l}_p = 0 \quad (35)$$

d) Resultant Potential Variation for a Complete Sector

In consequence, the circulation of the vector $\boldsymbol{E_p}$ over the edge of S equals the sum of the nonzero line integrals, see (28):

$$\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p = \int_{A}^{B} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_p + 0 + \int_{C}^{D} \boldsymbol{E}_3 \cdot d\boldsymbol{l}_p + 0 \quad (36)$$

The variation of potential around the edge of S equals $-\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p$, see (27):

$$\Delta U_S = -\left(\int_{A}^{B} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_p + \int_{C}^{D} \boldsymbol{E}_3 \cdot d\boldsymbol{l}_p \right) \qquad (37)$$

Let's compute the value of ΔU_S. Suppose the wire loop intersects the inner and outer circles, see Figure 10. Let the vector $\Delta \boldsymbol{l}_w$ represent the segment of the wire between the intersections and $\boldsymbol{E}_l$ the electric fields on it.

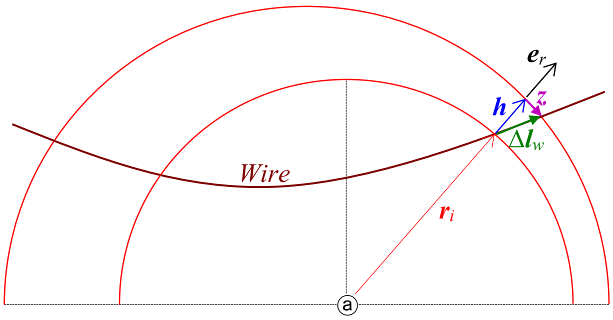


Figure 10

For infinitely small $\Delta \boldsymbol{l}_w$, $\boldsymbol{E}_l$ can be seen as constant and is defined with the mean radius of the sector r_s, we can write, see (29):

$$\boldsymbol{E}_l = \frac{q_a}{4\pi\varepsilon_0 {r_s}^2} \boldsymbol{e}_r \qquad (38)$$

The variation of electric potential along $\Delta \boldsymbol{l}_w$ equals:

$$\begin{aligned} \Delta U_l &= -\boldsymbol{E}_l \cdot \Delta \boldsymbol{l}_w \\ &= -\frac{q_a}{4\pi\varepsilon_0 {r_s}^2} \boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w \end{aligned} \qquad (39)$$

At the point A, $\Delta \boldsymbol{l}_w$ can be decomposed as the sum of two vectors which are the local clearance $\boldsymbol{h}$ and the vector $\boldsymbol{z}$ which is tangential to the outer circle, see Figure 10:

$$\Delta \boldsymbol{l}_w = \boldsymbol{h} + \boldsymbol{z} \quad (40)$$

$\boldsymbol{h}$ is along the unit radial vector of the inner circle $\boldsymbol{e}_r$:

$$\boldsymbol{h} = h\boldsymbol{e}_r \quad (41)$$

Since the inner and outer circles are infinitely close, the unit tangential vectors of the inner and outer circles are the same vector, $\boldsymbol{e}_\theta$:

$$\boldsymbol{z} = z\boldsymbol{e}_\theta \quad (42)$$

Because $\boldsymbol{e}_r \cdot \boldsymbol{e}_r = 1,\ \boldsymbol{e}_r \cdot \boldsymbol{e}_\theta = 0$, the dot product $\boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w$ equals the local clearance h:

$$\begin{aligned} \boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w &= \boldsymbol{e}_r \cdot (\boldsymbol{h} + \boldsymbol{z}) \\ &= h\boldsymbol{e}_r \cdot \boldsymbol{e}_r + z\boldsymbol{e}_r \cdot \boldsymbol{e}_\theta \qquad (43) \\ &= h \end{aligned}$$

and the variation of electric potential along $\Delta \boldsymbol{l}_w$ equals, see (39):

$$\Delta U_l = -\frac{q_a}{4\pi\varepsilon_0 {r_s}^2} h \qquad (44)$$

On the segment AB, we relabel ΔU_l with ΔU_{AB}:

$$\Delta U_{AB} = \Delta U_l \qquad (45)$$

The local clearance h is the length of the segment AB, h_A. In (44), h is replaced with its expression given in (15), $h_{\mathrm{A}} = r_e - r_i - l\cos\theta_1$, then ΔU_{AB} equals:

$$\Delta U_{AB} = -\frac{q_a}{4\pi\varepsilon_0 {r_s}^2}(r_e - r_i - l\cos\theta_1) \qquad (46)$$

At the corner D, the wire segment $\Delta \boldsymbol{l}_w$ goes from the outer circle to the inner circle. So, the dot product $\boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w$ equals negative local clearance $-h_D$:

$$\boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w = -h_{\mathrm{D}} \qquad (47)$$

On the segment CD, we relabel ΔU_l with ΔU_{CD}. In (44), $\boldsymbol{e}_r \cdot \Delta \boldsymbol{l}_w$ is replaced with $-h_D = -(r_e - r_i - l\cos\theta_2)$, see (16):

$$\Delta U_{CD} = \frac{q_a}{4\pi\varepsilon_0 {r_s}^2}(r_e - r_i - l\cos\theta_2) \qquad (48)$$

Finally, the variation of potential around the edge of the sector S equals the sum of those on AB and CD, see (37):

$$\begin{aligned} \Delta U_S &= \Delta U_{AB} + \Delta U_{CD} \\ &= \frac{q_a l}{4\pi\varepsilon_0}\left(\frac{\cos\theta_1 - \cos\theta_2}{{r_s}^2}\right) \qquad (49) \end{aligned}$$

When both angles θ_1 and θ_2 are positive and $\theta_1 < \theta_2$, we have:

$$\cos\theta_1 - \cos\theta_2 > 0 \qquad (50)$$

If $q_a l > 0$, the variation of potential around the edge of the sector S is positive:

$$\Delta U_S = \frac{q_a l}{4\pi\varepsilon_0 {r_s}^2}(\cos\theta_1 - \cos\theta_2) > 0 \qquad (51)$$

Then, the circulation of $\boldsymbol{E}_p$ is negative:

$$\begin{aligned} \oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p &= -\Delta U_S \\ &= \frac{q_a l}{4\pi\varepsilon_0 {r_s}^2}(\cos\theta_2 - \cos\theta_1) < 0 \end{aligned} \qquad (52)$$

5. Potential and Field within the Wire Loop

a) Summation of Individual Sector Influences

In chapter “0.a) Partitioning the Wire Loop into Sectors”, the surface bounded by the wire loop was cut into sectors. So, adding all these sectors together gives the surface back. In the same way, we add the variations of electric potential of all the sectors to get that in the entire wire loop.

Let ΔU_j represent the variations of electric potential on all the sectors, with j being the index of a sector. ΔU_j equals the sum of the variations of electric potential in the segments AB and CD of the sector j, which equals those in the corresponding small parts of the wire loop, see (49). So, adding all ΔU_j is adding the variations of electric potential in these small parts, which gives the variations of electric potential around the entire wire loop, ΔU_C , see (37):

$$\begin{aligned}\Delta U_{\mathrm{C}} &= \sum_{j=1}^{n} \Delta U_j \\ &= -\sum_{j=1}^{n}\left(\int_{A}^{B} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_p + \int_{C}^{D} \boldsymbol{E}_3 \cdot d\boldsymbol{l}_p\right)_j \\ &= -\sum_{j=1}^{n}\int_{A_j}^{B_j} \boldsymbol{E}_{1j} \cdot d\boldsymbol{l}_p - \sum_{j=1}^{n}\int_{C_j}^{D_j} \boldsymbol{E}_{3j} \cdot d\boldsymbol{l}_p\end{aligned} \tag{53}$$

Remark 2:

Because the variations in electric potential along the arcs of iso-intensity circles are zero, the progressing electric field within the surface bounded by the wire loop has no actual influence on the loop itself. This implies that the flux passing through this surface is not responsible for the induced EMF. This conclusion directly contradicts the standard interpretation of Faraday's law, which attributes the induced EMF to the magnetic flux passing through the bounded surface. Consequently, the traditional flux-based model appears to be an incorrect interpretation of the underlying physics of electromotive force.

This realization signals a radical shift in the current physical paradigm. If the electric potential variation along the arcs is zero, then the surface area enclosed by the wire serves no physical purpose in the generation of the electromotive force (EMF). We are compelled to conclude that "magnetic flux" is merely a mathematical proxy—a calculation that yields the correct numerical result but fundamentally misrepresents the underlying physical mechanism. The true source of the EMF resides not in the empty space between the wires, but in the direct interaction between the moving electron—which generates the progressing electric field—and the charges within the conductor itself.

We convert addition into integral:

$$\begin{aligned}\sum_{j=1}^{n}\int_{A_j}^{B_j} \boldsymbol{E}_{1j} \cdot d\boldsymbol{l}_p &= \int_{P_{bottom}}^{P_{top}} \boldsymbol{E}_{right} \cdot d\boldsymbol{l}_p \\ \sum_{j=1}^{n}\int_{C_j}^{D_j} \boldsymbol{E}_{3j} \cdot d\boldsymbol{l}_p &= \int_{P_{top}}^{P_{bottom}} \boldsymbol{E}_{left} \cdot d\boldsymbol{l}_p\end{aligned} \tag{54}$$

p_{bottom} is the point of the wire loop that touches the smallest iso-intensity circle and p_{top} the point that touches the largest iso-intensity circle. $\int_{P_{bottom}}^{P_{top}} \boldsymbol{E}_{right} \cdot d\boldsymbol{l}_p$ is the line integral of $\boldsymbol{E}_p$ on the right part of the wire loop, $\int_{P_{top}}^{P_{bottom}} \boldsymbol{E}_{left} \cdot d\boldsymbol{l}_p$ is that on the left part, see Figure 6. So, ΔU_C equals:

$$\begin{aligned}\Delta U_{\mathrm{C}} &= -\int_{P_{bottom}}^{P_{top}} \boldsymbol{E}_{right} \cdot d\boldsymbol{l}_p - \int_{P_{top}}^{P_{bottom}} \boldsymbol{E}_{left} \cdot d\boldsymbol{l}_p \\ &= -\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p\end{aligned} \tag{55}$$

Then, ΔU_C is the potential collected by the wire loop in the progressing electric field. Let's express ΔU_C as a function of radii and angles of the sectors using (49):

$$\begin{aligned}\Delta U_{\mathrm{C}} &= \sum_{j=1}^{n} \Delta U_j \\ &= \sum_{j=1}^{n} \frac{q_a l}{4\pi\varepsilon_0}\left(\frac{\cos\theta_1 - \cos\theta_2}{{r_s}^2}\right)_j\end{aligned} \tag{56}$$

Then, ΔU_C equals:

$$\Delta U_{\mathrm{C}} = \frac{q_a l}{4\pi\varepsilon_0} \sum_{j=1}^{n} \left(\frac{\cos\theta_{1j} - \cos\theta_{2j}}{{r_{sj}}^2} \right) \qquad (57)$$

Let the angles θ_{1j} and θ_{2j} be both positive and $\theta_{1j} < \theta_{2j}$. Then, the values of $\cos\theta_{1j} - \cos\theta_{2j}$ are all positive. Let $q_a l > 0$, then the variation of electric potential in a wire loop is positive, see (57):

$$\begin{cases} \cos\theta_{1j} - \cos\theta_{2j} > 0 \\ q_a l > 0 \end{cases} \Rightarrow \Delta U_{\mathrm{C}} > 0 \qquad (58)$$

Because equations (55) and (57) both express ΔU_C, the circulation of the instantaneous progressing electric field $\boldsymbol{E}_p$ over the wire loop equals :

$$\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p = -\frac{q_a l}{4\pi\varepsilon_0} \sum_{j=1}^{n} \frac{\cos\theta_{1j} - \cos\theta_{2j}}{{r_{sj}}^2} \qquad (59)$$

So, the circulation of $\boldsymbol{E}_p$ over a closed loop is nonzero and the progressing electric field is not a conservative vector field.

Remark 3:

The Maxwell–Faraday equation relies on the curl of a non-conservative electric field—a quantity that is not directly measured in experiments but is inferred to satisfy the mathematical framework. Since the progressing electric field is inherently non-conservative, it possesses the exact physical properties required to account for this non-conservative behavior. At the time of James Clerk Maxwell, the only well-understood electric field was the static (Coulombic) field. Consequently, Maxwell's theory lacked this fundamental building block, leading to a reliance on the magnetic field as a proxy for the dynamic effects of moving charges.

b) Characterization of the Electric Field in the Wire Loop

In a wire loop, it is the induced electric field that creates current, not the potential. We can get the average value of this induced electric field from the variation of electric potential. Let E_{avg} be the average electric field in a wire loop and l_w the length of the wire loop. E_{avg} is defined by the equation below:

$$\Delta U_{\mathrm{C}} = -E_{avg} \cdot l_w \qquad (60)$$

Then, E_{avg} equals, see (57):

$$\begin{aligned} E_{avg} &= -\frac{\Delta U_{\mathrm{C}}}{l_w} \\ &= \frac{q_a}{4\pi\varepsilon_0} \frac{l}{l_w} \sum_{j=1}^{n} \frac{\cos\theta_{2j} - \cos\theta_{1j}}{{r_{sj}}^2} \end{aligned} \qquad (61)$$

Because ΔU_C is positive, E_{avg} is negative:

$$E_{avg} < 0 \qquad (62)$$

Remark 4:

The potential differences $\Delta \boldsymbol{U}_{Sj}$ were integrated in the counterclockwise direction; consequently, their summation follows the same orientation. A positive net variation, $\Delta \boldsymbol{U}_C$, indicates that the electric potential within the wire loop increases in the counterclockwise direction. It follows that the average induced electric field, $\boldsymbol{E}_{avg}$, is negative, meaning it is oriented in the clockwise direction. This result is fundamentally consistent with Lenz's Law, as the induced field opposes the progression of the source field.

Note that E_{avg} is not the progressing electric field $\boldsymbol{E}_p$ which is in space. E_{avg} is in the wire of the loop and is parallel to the wire, see Figure 11.

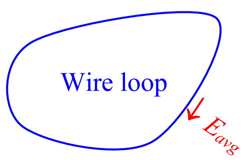


Figure 11

Because $\Delta U_C = -\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p$ and $E_{avg} = -\frac{\Delta U_C}{l_w}$, see (55) and (61), the average electric field in the wire loop equals the circulation of the progressing electric field $\boldsymbol{E}_p$ over a wire loop divided by the length of the wire loop:

$$E_{avg} = \frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}}{l_w} \quad (63)$$

c) Application of Stokes' Theorem to the Induced Field

Since $\boldsymbol{E}_p$ is a nonconservative vector field, according to Stokes' theorem, it possesses nonzero curl which satisfies the equation below:

$$\oint \boldsymbol{E}_p \cdot d\boldsymbol{l} = \oiint curl(\boldsymbol{E}_p)\, ds \quad (64)$$

Faraday's law in integral form is:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\oiint \frac{\partial \boldsymbol{B}}{\partial t} d\boldsymbol{s} \quad (65)$$

where $\boldsymbol{E}$ is the nonconservative electric field used in Faraday's law and $\boldsymbol{B}$ magnetic field.

By comparing (64) with (65), we notice that $\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}$ is similar to $\oint \boldsymbol{E} \cdot d\boldsymbol{l}$ and $curl(\boldsymbol{E}_p)$ to $-\frac{\partial \boldsymbol{B}}{\partial t}$, which indicates the intimate relation between progressing electric field $\boldsymbol{E}_p$ and magnetic field $\boldsymbol{B}$.

6. The Influence of Electric Current

The variation of electric potential computed above is the influence of a single moving charge. What is the influence of several moving charges situated at different points? For example, the charges in a current. Will their influences add together or mutually destroy? Let's see the influence of the moving charge ⓐ on the four wire loops numbered 1, 2, 3 and 4 in Figure 12. ⓐ moves to the right, when passing at each small blue circle, it emits an iso-intensity circle. We suppose that the charge of ⓐ is positive.

The wire loops 1 and 2 are above the trajectory of ⓐ. According to Remark 4, the direction of the average electric fields E_{avg} is in the clockwise direction. The red arrows indicate the direction of E_{avg} and the blue arrow indicates the direction of motion of ⓐ. Notice that the red arrows oppose the blue arrow.

The wire loops 3 and 4 are below the trajectory of ⓐ. If we flip the figure upside down, the wire loops 3 and 4 become the wire loops 1 and 2. So, the E_{avg} in the loops 3 and 4 are in the counterclockwise direction. We notice then, whether a wire loop is above or below the trajectory of ⓐ, the direction of E_{avg} opposes the direction of the motion of the charge ⓐ.

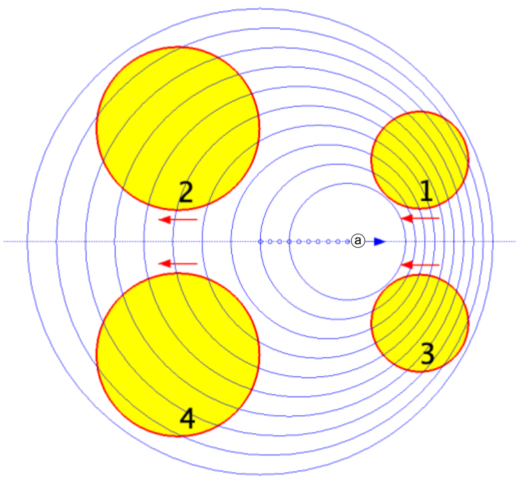


Figure 12

For determining the influence in a wire loop of two moving charges, let's see Figure 13. In this figure we have the wire loop B, two positive moving charges, ⓐ$_l$ on its left and ⓐ$_r$ on its right. The position of ⓐ$_l$ with respect to B is like that of ⓐ with respect to the wire loop 1 in Figure 12. Then, the E_{avg} due to ⓐ$_l$ points in the clockwise direction. The position of ⓐ$_r$ with respect to B is like that of ⓐ with respect to the wire loop 2 in Figure 12. Then, the E_{avg} due to ⓐ$_r$ points in the clockwise direction too. So, the influence of ⓐ$_l$ and ⓐ$_r$ add together in the wire loop B. In consequence, if the horizontal line were a current, all the charges in the horizontal current move in the same direction and their influences add together in the wire loop B.

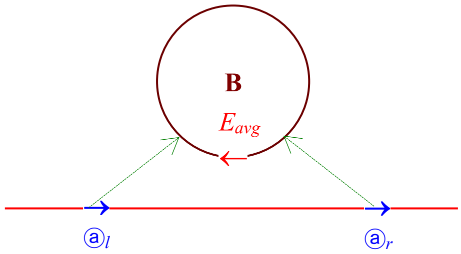


Figure 13

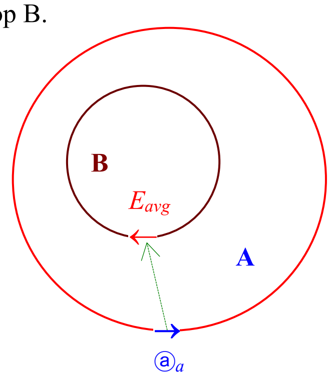


Figure 14

Figure 14 shows the wire loop B inside the wire loop A. Let ⓐ$_a$ represent one of the moving charges in A. The position of ⓐ$_a$ with respect to B is like the position of ⓐ with respect to the wire loops 1 or 2 in Figure 12. If the wire loop A carried a current, its influence on the wire loop B is the sum of those given by all the moving charges in the current.

For the same reason as in Figure 12, the red arrows in Figure 13 and Figure 14 oppose the blue arrows, which means that the induced electric fields E_{avg} oppose the direction of the currents that create progressing electric field. This result is in conformation with Lenz's law, although the currents here do not necessarily vary.

Remark 5:

> The average induced electric field within the wire loop is found to be oriented in direct opposition to the direction of the currents generating the progressing electric field. This result is fundamentally consistent with Lenz's Law, providing a causal, force-based explanation for the counter-electromotive force that arises in dynamic electromagnetic systems.

7. The Curl of the Progressing Electric Field

a) Calculation of the Curl for a Single Moving Charge

Faraday's law expressed in differential form is the equality between the curl of a non-conservative electric field $\boldsymbol{E}$ and the negative rate of change of a magnetic field $\boldsymbol{B}$:

$$curl(\boldsymbol{E}) = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (66)$$

Curl is a property of vector field, see the mathematical definition in this Wikipedia article Curl (mathematics). Progressing electric field $\boldsymbol{E}_p$ is non-conservative and thus, has nonzero curl which we compute with the quotient between the circulation of $\boldsymbol{E}_p$ around a sector and the surface of the sector. This curl equals the limit of this quotient as the sector shrinks towards a point.

The circulation of $\boldsymbol{E}_p$ around a sector is given in (52), the surface of the sector is given in (20). The quotient of them equals:

$$\frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s} = \frac{\frac{q_a}{4\pi\varepsilon_0}\frac{l}{{r_s}^2}(\cos\theta_2 - \cos\theta_1)}{r_i(r_e - r_i)\left(\theta_2 - \theta_1 - \frac{l}{r_e - r_i}(\sin\theta_2 - \sin\theta_1)\right)} \quad (67)$$

We had in (9):

$$\frac{l}{r_e - r_i} = \frac{v}{c} \quad (68)$$

Because $\frac{v}{c} \ll 1$ we have:

$$\left|\frac{l}{r_e - r_i}(\sin\theta_2 - \sin\theta_1)\right| = \frac{v}{c}|\sin\theta_2 - \sin\theta_1| \ll |\theta_2 - \theta_1| \quad (69)$$

Then, $\frac{l}{r_e - r_i}(\sin\theta_2 - \sin\theta_1)$ is neglected before $\theta_2 - \theta_1$ in (67) which simplifies the quotient $\frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s}$ into:

$$\frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s} = \frac{q_a}{4\pi\varepsilon_0 {r_s}^2}\frac{1}{r_i}\frac{l}{r_e - r_i}\frac{\cos\theta_2 - \cos\theta_1}{\theta_2 - \theta_1} \quad (70)$$

When $\theta_2 - \theta_1 \rightarrow 0$, the limit of the last quotient in (70) is:

$$\lim_{\theta_2 \to \theta_1}\frac{\cos\theta_2 - \cos\theta_1}{\theta_2 - \theta_1} = -\sin\theta_1 \quad (71)$$

Then, we replace $\frac{\cos\theta_2 - \cos\theta_1}{\theta_2 - \theta_1}$ with $-\sin\theta_1$ in (70):

$$\lim_{\theta_2 \to \theta_1}\frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s} = -\frac{q_a}{4\pi\varepsilon_0 {r_s}^2}\frac{1}{r_i}\frac{l}{r_e - r_i}\sin\theta_1 \quad (72)$$

Because $c\Delta t = r_e - r_i$ and $v\Delta t = l$, see (7) and (8), we have $\frac{l}{r_e - r_i} = \frac{v}{c}$. When $r_e - r_i \rightarrow 0$, we replace $\frac{l}{r_e - r_i}$ with $\frac{v}{c}$ in (72):

$$\lim_{r_e \to r_i}\left(\lim_{\theta_2 \to \theta_1}\frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s}\right) = -\frac{1}{r_i}\frac{q_a}{4\pi\varepsilon_0 {r_s}^2}\frac{v}{c}\sin\theta_1 \quad (73)$$

When both $\theta_2 - \theta_1 \to 0$ and $r_e - r_i \to 0$, the sector shrinks towards the point ($r=r_i$, $\theta=\theta_1$) and equation (73) gives the curl of the progressing electric field $\boldsymbol{E}_p$ at that point:

$$\begin{aligned} curl(\boldsymbol{E}_p) &= \lim_{r_e \to r_i}\left(\lim_{\theta_2 \to \theta_1} \frac{\oint \boldsymbol{E}_p \cdot d\boldsymbol{l}_p}{s}\right) \\ &= -\frac{1}{r_i}\frac{q_a}{4\pi\varepsilon_0 {r_s}^2}\frac{v}{c}\sin\theta_1 \end{aligned} \qquad (74)$$

b) Calculation of the Curl due to a Macroscopic Current

Above, $\boldsymbol{E}_p$ is the progressing electric field of a single charge. What is the curl of the progressing electric field of a current? Let's derive it from (74). Let Δl_a be the distance that a charge q_a moves within the time Δt. Its velocity v equals then:

$$v = \frac{\Delta l_a}{\Delta t} \qquad (75)$$

The product $q_a v$ equals:

$$\begin{aligned} q_a v &= q_a \frac{\Delta l_a}{\Delta t} \\ &= \frac{q_a}{\Delta t}\Delta l_a \end{aligned} \qquad (76)$$

The quotient $\frac{q_a}{\Delta t}$ equals the current created by the moving charge q_a, which is labeled with I_a:

$$\frac{q_a}{\Delta t} = I_a \qquad (77)$$

Equation (76) becomes:

$$q_a v = I_a \Delta l_a \qquad (78)$$

The product $I_a\Delta l_a$ is conventionally called elementary current and is labeled with ΔI_a:

$$\Delta I_a = I_a \Delta l_a \qquad (79)$$

Equation (78) becomes:

$$q_a v = \Delta I_a \qquad (80)$$

We replace $q_a v$ with ΔI_a in (74) and obtain:

$$curl(\boldsymbol{E}_p) = -\frac{1}{r_i}\frac{1}{4\pi\varepsilon_0 {r_s}^2}\frac{\Delta I_a}{c}\sin\theta_1 \qquad (81)$$

We need the speed of light squared c^2 in the denominator in (81), so we multiply the numerator and denominator with c. Because this equation expresses the curl of the progressing electric field due to a current, the field vector $\boldsymbol{E}_p$ will be relabeled with $\boldsymbol{E}_c$. Then:

$$curl(\boldsymbol{E}_c) = -\frac{c}{r_i}\frac{1}{4\pi\varepsilon_0 c^2}\frac{\Delta I_a}{{r_s}^2}\sin\theta_1 \qquad (82)$$

In equation (82) we have the inverse of the quotient $\frac{r_i}{c}$ which is the time t_i that electric field takes to travel the radius of the inner iso-intensity circle:

$$t_i = \frac{r_i}{c} \qquad (83)$$

By replacing $\frac{c}{r_i}$ with $\frac{1}{t_i}$ in (82) we obtain the expression for the curl of the progressing electric field due to a current:

$$curl(\boldsymbol{E}_c) = -\frac{1}{t_i}\frac{1}{4\pi\varepsilon_0 c^2}\frac{\Delta I_a}{{r_s}^2}\sin\theta_1 \qquad (84)$$

c) Connection to the Biot–Savart Law

Let's name the constant $\frac{1}{\varepsilon_0 c^2}$ as μ_0 and $curl(\boldsymbol{E}_c)$ becomes:

$$curl(\boldsymbol{E}_c) = -\frac{1}{t_i}\frac{\mu_0}{4\pi}\frac{\Delta I_a}{{r_s}^2}\sin\theta_1 \quad (85)$$

In this equation we recognize the magnetic field at a point created by ΔI_a. Its magnitude B_b is given by Biot–Savart law:

$$\boldsymbol{B}_b = \frac{\mu_0}{4\pi}\frac{r_s \times \Delta\boldsymbol{I}_a}{{r_s}^3} \quad (86)$$

$$\left|\frac{r_s \times \Delta\boldsymbol{I}_a}{{r_s}^3}\right| = \frac{\Delta I_a}{{r_s}^2}\sin\theta_1 \quad (87)$$

$$B_b = |\boldsymbol{B}_b| = \frac{\mu_0}{4\pi}\frac{\Delta I_a}{{r_s}^2}\sin\theta_1 \quad (88)$$

By comparing (85) with (88), we find that $curl(\boldsymbol{E}_c)$ equals:

$$curl(\boldsymbol{E}_c) = -\frac{1}{t_i}B_b \quad (89)$$

$$-t_i curl(\boldsymbol{E}_c) = B_b \quad (90)$$

We use (83) to replace t_i with $\frac{r_i}{c}$ and have :

$$B_b = -\frac{r_i}{c}curl(\boldsymbol{E}_c) \quad (91)$$

At the limit of $r_e=r_i$, we have s=0 and $r_i=r_s$, the magnitude B_b given by Biot–Savart law equals :

$$B_b = -\frac{r_s}{c}curl(\boldsymbol{E}_c) \quad (92)$$

Of course, we knew the famous relation $\varepsilon_0\mu_0 = \frac{1}{c^2}$.

Remark 6:

The relationship $\varepsilon_0\mu_0 = \frac{1}{c^2}$ was inadvertently discovered by James Clerk Maxwell through numerical coincidence. Historically, the constant μ_0 was treated as a value to be determined solely through empirical measurement. In this work, however, μ_0 is derived theoretically. This derivation demonstrates that the magnetic constant is not an independent physical parameter, but a structural consequence of the interaction between Coulomb's Law and the finite speed of the progressing electric field.

Remark 7:

In the classical Biot–Savart law, the $sin\ \theta_1$ term is an empirical factor derived from experimental observation to account for the angular dependency of the magnetic effect. In contrast, in Equation (85) of this work, this term emerges naturally from a rigorous theoretical derivation. This suggests that the angular distribution of the force is not a standalone property of a "magnetic field," but rather a geometric necessity of how the progressing electric field interacts with a target charge at a given angle.

d) Connection to the Lorentz Force

The force that the elementary current ΔI_a exerts on another elementary current ΔI_b is a Lorentz force. For deriving this Lorentz force from (85), let ΔI_b be produced by another moving charge q_b which moves at the velocity v_b. Suppose that q_b travels the distance Δl_b in the time t_i expressed in (83). The velocity of q_b equals:

$$v_b = \frac{\Delta l_b}{t_i} \quad (93)$$

To introduce the elementary current ΔI_b we multiply (89) with $q_b \Delta l_b$:

$$\begin{aligned} q_b \Delta l_b \cdot curl(\boldsymbol{E}_c) &= -\frac{q_b \Delta l_b}{t_i} B_b \\ &= -q_b v_b B_b \\ &= -\Delta I_b B_b \end{aligned} \qquad (94)$$

where the elementary current ΔI_b being $q_b v_b$, see (80).

In this equation we recognize the magnitude of the Lorentz force which is $\Delta I_b B_b$:

$$q_b \Delta l_b \cdot curl(\boldsymbol{E}_c) = -F_{\text{Lorentz}} \qquad (95)$$

Remark 8:

The significance of Equation (95) lies in its direct relation of the Lorentz force to $curl(\boldsymbol{E}_c)$, the latter representing the induced electromotive force. In the case of a current-carrying wire loop moving through the magnetic field of another loop, the Lorentz force performs mechanical work. Simultaneously, the motion of the first loop generates electrical work within the second. The principle of conservation of energy dictates that this mechanical work must equal the electrical work—the fundamental operating principle of the electric generator. While Maxwell's theory utilizes both concepts, it does not provide an explicit equation directly bridging them in this manner. Equation (95) serves as this essential relation, unifying the mechanical and electrical aspects of the interaction.

Remark 9:

Faraday's Law establishes a relationship between the induced electromotive force and the rate of change of magnetic flux, yet it lacks a direct mathematical link to the Lorentz Force. This conceptual separation is likely the reason that Faraday's Law, in its standard form, does not explicitly manifest the law of conservation of energy. By treating induction as a flux-dependent phenomenon rather than a force-driven interaction, the classical framework obscures the mechanical-to-electrical energy conversion. In contrast, by deriving the induced field directly from the progressing electric field, the Lorentz Force and the EMF are shown to be two manifestations of the same underlying physical process, ensuring that energy conservation is respected by design rather than by postulate.

8. Implications for a Theoretical Faraday's Law

Above, we have derived $curl(\boldsymbol{E}_c)$ which is induced by a current I_a that does not necessarily vary. This may seem not relevant to Faraday's law or even impossible. Let's make I_a vary and suppose that the current is I_{a1} at time t_1, and I_{a2} at time t_2. Then, the progressing electric field and magnetic field of I_{a1} are $\boldsymbol{E}_{c1}$ and B_{b1} at time t_1, those of I_{a2} are $\boldsymbol{E}_{c2}$ and B_{b2} at time t_2. Equation (89) for these two times are:

$$curl(\boldsymbol{E}_{c1}) = -\frac{1}{t_i} B_{b1} \qquad (96)$$

$$curl(\boldsymbol{E}_{c2}) = -\frac{1}{t_i} B_{b2} \qquad (97)$$

Let's subtract these two equations:

$$curl(\boldsymbol{E}_{c2}) - curl(\boldsymbol{E}_{c1}) = -\left(\frac{1}{t_i} B_{b2} - \frac{1}{t_i} B_{b1}\right) \qquad (98)$$

$$curl(\boldsymbol{E}_{c2} - \boldsymbol{E}_{c1}) = -\frac{B_{b2} - B_{b1}}{t_i} \qquad (99)$$

Let's suggest that between B_{b1} and B_{b2} the lasted time is t_i, then:

$$\frac{B_{b2} - B_{b1}}{t_i} = \frac{\partial B}{\partial t} \qquad (100)$$

By introducing equation (100) into (99) we obtain:

$$curl(\boldsymbol{E}_{c2} - \boldsymbol{E}_{c1}) = -\frac{\partial B}{\partial t} \qquad (101)$$

This equation looks a lot more like the Maxwell–Faraday equation which reads:

$$curl(\boldsymbol{E}) = -\frac{\partial B}{\partial t} \qquad (102)$$

Also, equation (63) shows that the average electric field collected by a wire loop E_{avg} is proportional to the circulation of $\boldsymbol{E}_p$. So, $curl(\boldsymbol{E}_{c2} - \boldsymbol{E}_{c1})$ is a sort of E_{avg} and equation (101) shows a close relation between the average electric field collected by a wire loop and the rate of change of magnetic field.

9. Discussion

This article presents the first stage of a derivation of Faraday's law based solely on Coulomb's law and special relativity. We demonstrate that the retarded electric field of a moving electron—which we term the 'Progressing Electric Field'—is a non-conservative vector field. We have derived the mathematical expression for its curl and have found that several properties of this field are analogous to those of the magnetic field.

We have derived the electric potential induced in a wire loop by a progressing electric field, which shares certain properties with the electromotive force defined by Faraday's law. However, because this potential is not yet proportional to the rate of change of the magnetic field, it cannot be classified as electromotive force at this stage. Several additional steps of theoretical derivation are required to fully arrive at Faraday's law.

The essential steps of the derivation presented in this article are as follows:

1. **Propagation at the speed of light**: In accordance with special relativity, the electric field propagates at the speed of light, *c*.
2. **Iso-intensity circles**: The electric field of a moving electron radiates in 'iso-intensity circles.' The centers of these circles correspond to the retarded positions of the electron.
3. **Application of Coulomb's law**: The intensity of the electric field on an iso-intensity circle is defined by Coulomb's law relative to the circle's center.
4. **Non-conservative nature**: The 'Progressing Electric Field' is a deformation of the static electric field and is inherently non-conservative.
5. **Instantaneous curl calculation**: The Progressing Electric Field is analyzed within a single temporal snapshot, allowing its curl to be computed using instantaneous values.
6. **Superposition of charges**: The curl of the Progressing Electric Field for a steady current is derived by integrating the fields of individual moving charges.
7. **Induction and Lenz's Law**: The Progressing Electric Field induces an electric field within a wire loop; the calculated average value of this field is shown to be consistent with Lenz's law.

We have constructed our theory upon the Progressing Electric Field, a concept that may initially appear to lack direct experimental verification. One might ask: what if this field is merely a theoretical construct? In reality, experimental evidence for its existence already exists; however, it has historically been overlooked or reinterpreted through the lens of well-established theories.

By applying the 'Progressing Electric Field Model,' we will provide a new and comprehensive explanation for this experimental evidence in a forthcoming article.

In summary, the 'Progressing Electric Field Model' demonstrates that the induction phenomena traditionally attributed to Faraday's law can be derived from the relativistic motion of electric charges. By accounting for the propagation delay of the field at speed *c*, we establish that the resulting electric field is non-conservative and possesses a non-zero curl. While this initial stage of the derivation produces results consistent with Lenz's law, further refinement is required to achieve full mathematical proportionality with the rate of change of magnetic flux. This theoretical framework not only aligns with the principle of energy conservation but also opens the door to predicting electromagnetic phenomena beyond the reach of classical empirical laws.

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Letter to the Readers

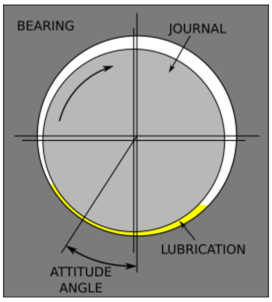
I began my work on electromagnetism in 1997, after I succeeded in partially deriving the Lorentz force using only Coulomb's law and special relativity. The core idea of 'iso-intensity circles' came to me later, in 2005. However, I waited twenty years before finally putting it all down on paper. Why?

First, while investigating the Lorentz force law, I felt it was necessary to conduct experiments to demonstrate its flaws. I have since performed numerous other experiments to test my ideas and refine my theory, all of which required a significant investment of time.

If you are interested in the empirical side of this work, you can watch the videos of these experiments on my YouTube channel here:
https://www.youtube.com/watch?v=h7BZEYVLNIk&list=PLhXr3fc4dvSaWGHhHe3ix6SRwoPfDBIII&pp=gAQB

Secondly, I struggled with deep depression, which made it impossible for me to work full-time. During those years, I submitted my articles to numerous physics journals, only to have them all rejected without review. I became so disillusioned that I stopped working on electromagnetism for a long time and turned to mathematical subjects instead. Although I obtained interesting results—for example, « N-complex number, N-dimensional polar coordinate and 4D Klein bottle with 4-complex number»,
https://www.academia.edu/120524016/N_complex_number_N_dimensional_polar_coordinate_and_4D_Klein_bottle_with_4_complex_number
https://pengkuanonmaths.blogspot.com/2024/06/n-complex-number-n-dimensional-polar.htmlon N-complex
I did not succeed in publishing in mathematics either. These repeated rejections were deeply painful, and each one pushed me further into depression. They cost me a great deal of time before I finally found the strength to return to Faraday's law.

To derive Faraday's law, I use iso-intensity circles to explain how moving electrons give rise to an electromotive force in a wire loop without the intermediary of a magnetic field. I would like to share how I arrived at this idea. First, I asked myself: What does the electric field of a moving electron actually look like? Since special relativity dictates that nothing travels faster than the speed of light, I imagined the electric field of a moving electron taking the form of Doppler-like circles. The eccentricity of these successive circles reminded me of my doctoral thesis, during which I worked on journal bearings.
A journal bearing consists of a cylindrical bearing surface and a shaft. The radius of the bearing surface is slightly larger than that of the shaft; the shaft sits within the bearing surface, but its center is positioned lower than that of the outer surface. The figure below illustrates a cross-section of a journal bearing. The space between these two eccentric circles is filled with oil, referred to as the oil film. The thickness of this film is calculated using the same logic I applied to compute the local clearance *h*, (see equation (14)).



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Since the two circles shown in the figure represent two successive iso-intensity circles, I imagined a particle within the oil film being acted upon by a force perpendicular to the surfaces. I applied this same principle to two successive iso-intensity circles of an electron, performed a quick calculation of the curl of the electric field, and successfully derived the Biot–Savart law.

This result reinforced the conclusion I reached while studying the Lorentz force law: that the force between electrons must be transmitted directly, without the intermediary of a magnetic field. Consequently, it must be possible to express both the Lorentz force and the electromotive force using only Coulomb's law and special relativity—which define the electric force and the influence of motion, respectively. I am now confident that all laws of magnetism can be expressed solely through Coulomb's law and special relativity.

The Evolution of the Progressing Electric Field Model

1. **Questioning the Status Quo**: I began by asking: "What does the electric field of a moving electron actually look like?" Without questioning established assumptions, one can never find a new answer.
2. **Visualizing the Disturbance**: I imagined the shape of an electric field being "disturbed" by the motion of the source charge, behaving in a manner similar to the Doppler effect.
3. **Applying Physical Constants**: I used the speed of light, *c*, to determine the geometry of the iso-intensity circles relative to the retarded positions of the charge.
4. **Cross-Disciplinary Synthesis**: I applied knowledge from a seemingly unrelated field—the geometry of journal bearings from my doctoral thesis—to the problem of Faraday's law.
5. **Rigorous Verification**: I tested the idea by computing the curl of these iso-intensity circles. Even though the approach seemed unconventional at first, the mathematical results confirmed the theory.

How did I come to conduct research in electromagnetism? After completing my PhD, I began working as an engineer, but I lost the capacity to work in a traditional environment after the onset of depression in 1995. This is a struggle that continues to this day. During the years that followed, I dedicated my time to studying relativity and became intrigued by Albert Einstein's observation that "Maxwell's equations do not change under Lorentz transformations." I began to ask: "Why are Maxwell's equations invariant?"

In 1997, I discovered that the Lorentz force occurs because the density of a moving electric charge increases due to length contraction. This led me to the insight that the Lorentz force is a direct electron-to-electron interaction. Since then, despite the ongoing challenges of my health, I have been committed to completely deriving both the Lorentz force law and Faraday's law without the intermediary of a magnetic field.

In fact, I completed the derivation of the Lorentz force law in 2018. You can find the detailed explanations in the following articles:

- «Coulomb magnetic force»
  http://pengkuanem.blogspot.com/2018/03/coulomb-magnetic-force.html
  https://www.academia.edu/36278169/Coulomb_magnetic_force
- «Changing distance effect»
  https://www.academia.edu/36272940/Changing_distance_effect
  https://www.academia.edu/36272940/Changing_distance_effect
- «Length-contraction-magnetic-force between arbitrary currents»
  http://pengkuanem.blogspot.com/2017/05/length-contraction-magnetic-force.html
  https://www.academia.edu/32815401/Length-contraction-magnetic-force_between_arbitrary_currents

Since my illness prevented me from applying for a formal research position, I have conducted my work independently—outside of any institution and without any funding or economic resources.

You may ask: why didn't I seek help? I did seek help multiple times from highly competent professionals in physics, some of whom were even my friends. But they all turned me down because 1% of them thought I was wrong, and the other 99% thought I was insane. I eventually realized that asking a physicist to help me show that Maxwell's theory is flawed was like asking a priest to help tear down his own church. For them, it was simply unthinkable.

In such harsh conditions, with little hope of publication, why do I keep writing? I do so because of a singular conviction: my theories are correct. I am now 66 years old, and I know that my time is limited. Even if no one currently understands the significance of my work, and even if no journal will publish it, I must continue. This is the only way to preserve these ideas for the future.

History shows that many great scientific theories were ignored upon their arrival because even the brightest minds of the era could not yet perceive their true value. I hold onto the hope that, in the future, someone will understand what I have found. I will continue to post my work online progressively as I write it, ensuring that these ideas are available to the world without wasting any more time battling with publishers.

Physics has not seen a major breakthrough for a long time. What if the direction of modern research has been led astray by these fundamental flaws? If you find that you understand my theories and recognize their true value for science, I ask for your help. Please forward my articles to the physics community.

Thank you.

Lists of my articles:

- On Electromagnetism https://pengkuanem.blogspot.com/2023/10/my-work-on-electromagnetism.html
- On Relativity https://pengkuanonphysics.blogspot.com/2023/10/my-work-on-relativity.html
- On Mathematics https://pengkuanonmaths.blogspot.com/2023/10/my-work-on-mathematics.html