From electron to magnetism

7. Longitudinal magnetic force and high field magnet

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Abstract: Theoretical explanation of longitudinal magnetic force and its practical application in high field magnet.

1. What is longitudinal magnetic force?

Longitudinal magnetic force is the force that a current element exerts on a collinear current element. For example, A and B in Figure 1 (a) are 2 current elements belonging to the same wire, so A and B are collinear and the forces $d\boldsymbol{F}_{ab}$ and $d\boldsymbol{F}_{ba}$ that A and B exert on each other are longitudinal magnetic forces. So, we define longitudinal magnetic force as the magnetic force that a current element exerts on a collinear or near collinear current element.

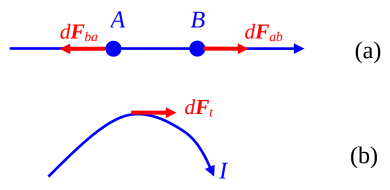

Figure 1

Longitudinal magnetic force is parallel to the current element and thus, is a tangential magnetic force. The term tangential magnetic force is more general and designates magnetic force tangent to a current of arbitrary shape, for example, $d\boldsymbol{F}_t$ in Figure 1 (b). In the following, we will consider only longitudinal magnetic forces, which we will refer to as longitudinal force.

2. Experiments involving longitudinal force

Although longitudinal force does not exist in classical theory, its action has been demonstrated in the past by several experiments.

a) Ampere's hairpin experiment

This experiment was first done by André-Marie Ampère in 1822. Figure 2 shows the setup of this experiment, which consists of a copper wire in the form of a hairpin and 2 troughs of mercury. The 2 legs of the hairpin float in the 2 mercury connecting them electrically. Once a voltage is put on the 2 mercury, current flows from one trough to the other through the hairpin, then the hairpin begins to move.

Figure 2

The experiment in Figure 2 is shown in the video [Floating wire experiment] where the hairpin is propelled to the right. Peter Graneau reported in his article «Electromagnetic jet-propulsion in the direction of current flow» published in Nature V.295, 1982, p.311-312, that mercury was repelled from the rear faces of such hairpin. So, according to Newton's third law, the force propelling the hairpin is the longitudinal force exerted by the current in the mercury.

b) Nasilowski's wire fragmentation

The second experiment was done in 1961 by Jan Nasilowski in Poland. He put a high current in a thin wire, which shattered the wire into small fragments. Figure 3 shows the principle of this experiment. Examination of the microscopic structure of the fracture regions showed that the wire was tore apart in solid state by strong force. The tearing force is parallel to the current and acted by current elements belonging to the same wire. So, it is a longitudinal force. For later use let us call the shattering of straight wire by its proper current Nasilowski effect.

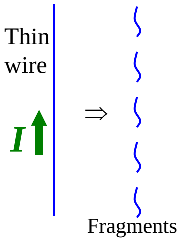

Figure 3

c) Ruscak and Bruce's multi-arc generator

The third experiment was done by Ruscak and Bruce in 1987. Its setup consists of a stack of several conductor rods. When current pulses ranging from 3 to 30 kA passed through the conductor stack, the rods instantly separated, see Figure 4. The force separating the rods is parallel to the current and is acted by the rods themselves, so is a longitudinal force.

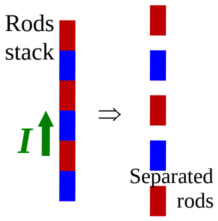

Figure 4

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For more detail of these experiments, follow the links below.

1. Ampere's Force Law: An Obsolete Formula?
2. Experiments
3. Longitudinal electrodynamic forces by Lars Johansson

3. Theory for longitudinal force

The three experiments above show the action of longitudinal force. Coulomb magnetic force law derived in «Coulomb magnetic force» explains longitudinal force in theory. In equation (1) $d\boldsymbol{F}_{cm}$ is the Coulomb magnetic force that the current element $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$. The expressions for the unit vectors $\boldsymbol{e}_r$, $\boldsymbol{e}_a$ and $\boldsymbol{e}_b$ are written in (2) with the base vectors $\boldsymbol{i}$ and $\boldsymbol{j}$, while $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$ being parallel to $\boldsymbol{i}$. Then, the product of unit vectors in (1) equals $\cos\theta\,\boldsymbol{i}$, see (3), and $d\boldsymbol{F}_{cm}$ equals the elementary Lorentz force $d\boldsymbol{F}_{lorentz}$ plus a component parallel to $\boldsymbol{i}$, as expressed in (4).

$$\begin{aligned} d\boldsymbol{F}_{cm} &= d\boldsymbol{F}_{Lorentz} + \frac{1}{4\pi\varepsilon_0 c^2 r^3} d\boldsymbol{I}_a (\boldsymbol{r}\cdot d\boldsymbol{I}_b) \\ &= d\boldsymbol{F}_{Lorentz} + \frac{dI_a dI_b r}{4\pi\varepsilon_0 c^2 r^3} (\boldsymbol{e}_r\cdot\boldsymbol{e}_b)\boldsymbol{e}_a \end{aligned} \tag{1}$$

$$\boldsymbol{e}_r = \cos\theta\,\boldsymbol{i} + \sin\theta\,\boldsymbol{j}, \qquad \boldsymbol{e}_a = \boldsymbol{e}_b = \boldsymbol{i} \tag{2}$$

$$(\boldsymbol{e}_r\cdot\boldsymbol{e}_b)\boldsymbol{e}_a = \big((\cos\theta\,\boldsymbol{i} + \sin\theta\,\boldsymbol{j})\cdot\boldsymbol{i}\big)\boldsymbol{i} = \cos\theta\,\boldsymbol{i} \tag{3}$$

$$d\boldsymbol{F}_{cm} = d\boldsymbol{F}_{Lorentz} + \frac{dI_a dI_b}{4\pi\varepsilon_0 c^2 r^2}\cos\theta\,\boldsymbol{i} \tag{4}$$

$$\theta = 0 \Rightarrow d\boldsymbol{F}_{Lorentz} = 0 \Rightarrow d\boldsymbol{F}_{ba} = \frac{dI_a dI_b}{4\pi\varepsilon_0 c^2 r^2}\boldsymbol{i} \tag{5}$$

$$\begin{cases} \boldsymbol{e}'_r = -\boldsymbol{e}_r \\ (\boldsymbol{e}'_r\cdot\boldsymbol{e}_b)\boldsymbol{e}_a = -\cos\theta\,\boldsymbol{i} \end{cases} \Rightarrow d\boldsymbol{F}_{ab} = -\frac{dI_a dI_b}{4\pi\varepsilon_0 c^2 r^2}\boldsymbol{i} \tag{6}$$

Figure 5 shows how $d\boldsymbol{F}_{cm}$ varies with the angle θ between $d\boldsymbol{I}_b$ and $\boldsymbol{r}$. $d\boldsymbol{I}_b$ is a fixed current element, $d\boldsymbol{I}_c$ is at the distance $\boldsymbol{r}$ from $d\boldsymbol{I}_b$ and stays parallel to it when θ varies. The force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_c$ is $d\boldsymbol{F}_c$ and is expressed in (4). When θ equals zero, $d\boldsymbol{I}_c$ becomes $d\boldsymbol{I}_a$ which is collinear with $d\boldsymbol{I}_b$. In this case, Lorentz force is zero, and the force $d\boldsymbol{F}_c$ becomes $d\boldsymbol{F}_{ba}$ which is expressed in (5). The reaction force that $d\boldsymbol{I}_a$ exerts back on $d\boldsymbol{I}_b$ is $d\boldsymbol{F}_{ab}$ which is obtained in (6) by reversing the unit vector $\boldsymbol{e}_r$.

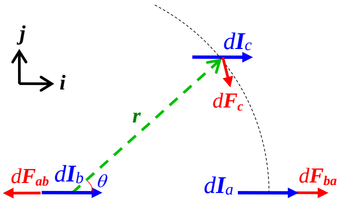

Figure 5

Equations (5) and (6) show that $d\boldsymbol{F}_{ab}$ and $d\boldsymbol{F}_{ba}$ have the same intensity but opposed directions, they are parallel to the collinear current elements $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$. So, $d\boldsymbol{F}_{ab}$ and $d\boldsymbol{F}_{ba}$ are action-reaction forces and longitudinal forces.

Figure 5 shows that $d\boldsymbol{F}_{ab}$ and $d\boldsymbol{F}_{ba}$ are repulsion forces, that is, they tend to push $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$ apart. This explains well the 3 experiments above because the mercury and the hairpin are pushed apart in Ampere's experiment, fragments of the wire are pushed apart in Nasilowski's experiment and conductor rods are pushed apart in Ruscak and Bruce experiment. In consequence, longitudinal force and Coulomb magnetic force law explain correctly these 3 experiments.

4. High field resistive magnet

The action of longitudinal force has been shown in these 3 experiments long time ago, but why is it not recognized in theory? The reason is that in addition to contradicting Lorentz force law, it shows no significant effect on practical devices, so no physicist is interested in exploring these experiments. But I have found a huge effect of longitudinal force in high field resistive magnets.

a) Limit of these magnets

A high field resistive magnet is a stack of big amount of perforated copper sheets called Bitter plate, as Figure 6 shows. Strong current circulates in these plates to produce powerful magnetic field. But, the magnet suffers huge induced magnetic force which can blow it up. Figure 7 shows the explosion of a magnet. So, the best performance of a magnet is the maximal field it can produce without exploding.

Note: Figure 6 and Figure 7 are from the page [Magnets from Mini to Mighty of the National High Magnetic Field Laboratory].

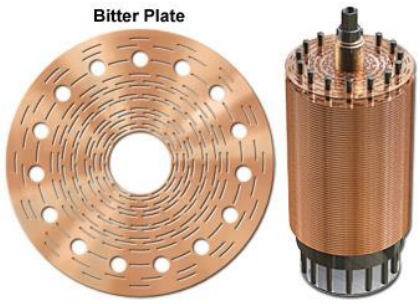

Figure 6

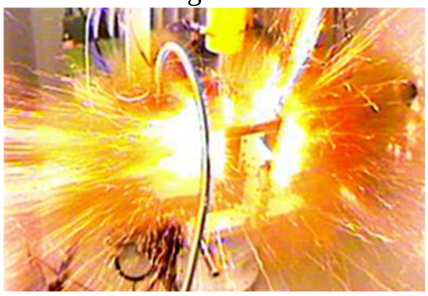
Figure 7

b) Lorentz force in these magnets

Lorentz force is thought to be responsible for the explosion of such magnet. On a round coil Lorentz force acts uniformly and results a tensile stress at all points of the coil. Figure 8 shows the tensile stress at points A and B of a coil. Suppose that B is weaker than A and breaks before A, then the stress on A falls immediately to zero and A will not break. So, if a magnet is broken by Lorentz force, each Bitter plate would have only one fracture, as Figure 8 shows.

For protecting magnets against Lorentz force, they are reinforced with strong housing. However, magnets still explode. Can we reinforce with more and more robust housing for producing ever higher field? Or no matter how robust is the housing, magnets will explode anyway? If this is the case, then another magnetic force is responsible, namely longitudinal force.

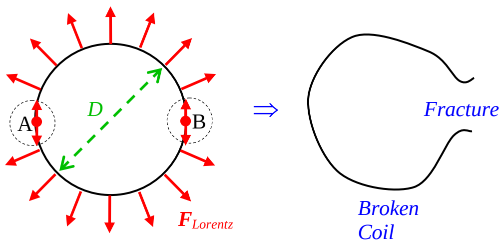

Figure 8

c) Effect of longitudinal force

Nasilowski effect shatters wire into small fragments, see section 1.b). Figure 9 shows the fragmented wires in Nasilowski's experiment. Bitter plates are perforated with long holes between which are bridges of copper sheets. Figure 10 shows these bridges, 3 of them are marked **a, b** and **c**. Currents are concentrated in these narrow bridges creating very strong local longitudinal forces. When the current in a bridge exceed a critical intensity, the bridge will be shattered into small fragments by Nasilowski effect as for the wires in Figure 9.

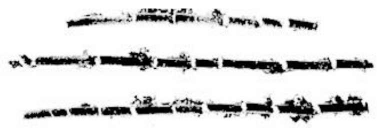
Figure 9

Figure 10

According to equation (6), longitudinal force is proportional to I^2 and increases rapidly with current. If the current in bridge **a** exceeds the breaking intensity, **a** breaks and passes over its current to neighboring bridges. Then the current in **b** increases and exceeds the breaking intensity, **b** breaks. Then **c** and other bridges break instantly. The magnet explodes.

Bridges broken by Nasilowski effect are all in small pieces. So, in the debris of a magnet exploded by longitudinal force there should be lot of small pieces of copper. If a magnet is destroyed by Lorentz force, each broken bridge should have only one fracture because Lorentz force pulls the bridge by the 2 ends. So, just by looking at the debris of the magnet, we will know exactly which force is responsible for the explosion.

In theory, Lorentz force is approximately inversely proportional to the square of the diameter D of the coil, see Figure 8. On the other hand, longitudinal force is inversely proportional to the square of the distance r between 2 adjacent current elements, see equation (5). As r is much smaller than D, $1/r^2 \gg 1/D^2$, then longitudinal force is much stronger than Lorentz force and will destroy the magnet before Lorentz force will.

Because longitudinal force is unknown today, no measure protects high field magnets against Nasilowski effect, which leaves us a way to increase the intensity of allowable current and improve the performance of high field magnet by addressing longitudinal force.

d) Addressing longitudinal force

One way to better resist longitudinal force is to make Bitter plate tougher and harder. Tougher material withstands higher stress, harder plate deforms less avoiding weak points to appear.

But the biggest weak point of Bitter-plate-stack magnet is that the plates have plenty of holes. So, I propose to build magnet without punching holes in the conductor sheet. For example, the magnet shown in Figure 11 (a) is constructed by rolling the long conductor sheet having spaced thin ribs which is shown in Figure 11 (b). Once rolled the ribs create the void spaces between the layers letting cooling fluid to flow from one end to the other of the magnet cooling every layer.

Note that the resistivity of conductor increases with temperature and if the temperature of the sheet increases from one end to the other, current would be stronger at the cooler end. So, for maintaining the density of current constant throughout the sheet, the temperature of the cooling water should stay constant. For doing so, the cooling water should be saturated, that is, at boiling temperature. This way, evaporation will keep the cooling water at this temperature throughout the magnet.

This type of magnet is more resilient to stress because no hole harms the integrity of the material, no conductor bridge creates concentration of current and high local longitudinal force. This construction is easy to realize because the sheet is continuously rolled with no need of connecting layers. So, I think this type of magnet will be high performing and cheap.

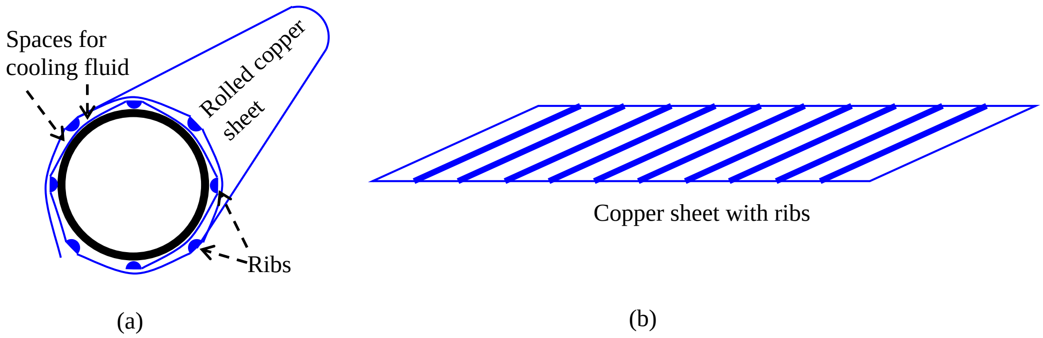

Figure 11

5. Comment

So, Nasilowski effect is strongly suspected to cause the explosion of high field resistive magnets. Considering the importance of such equipment, it is really worth researching extensively about this forgotten effect. For example, what is the relation between current, stress and material? How to compute the stress from current intensity? How do materials behave under longitudinal force? Why do wires fragment into small pieces while the current is already stopped by the first fracture?

Lorentz force law is unable to explain Nasilowski effect. In fact, Lorentz force law was established by experiment and thus, is a law of observable forces on current carrying wires. As longitudinal force and tangential force were not observable, they are naturally not included in this law. This has some similarity with Ptolemy's geocentric model in astronomy. Geocentric model was established by reproducing in geometry the visual motion of the Sun, the Moon, Mercury, Venus, Mars, Jupiter, Saturn and the fixed Stars. But it cannot predict the positions of Uranus and Neptune because they were not visible at the time.

Laws are valid only within the domains where they are established. Extrapolation outside these domains is unreliable. Like Ptolemy's geocentric model does not exclude the existence of Uranus and Neptune, Lorentz force law does not exclude the existence of longitudinal force and tangential force which are outside the domain of it.

On the contrary, laws based on deeper nature of the observed phenomena have the power to explain beyond the observable domain. For example, Copernicus' heliocentric model is based on the nature of the solar system: the Sun is the center of the trajectories of all planets, including the Earth. So, it can also explain the trajectories of Uranus and Neptune even though they were invisible for Copernicus. Similarly, Coulomb magnetic force law is based on the deep nature of magnetic force: Coulomb force from moving electric charges. This is why it can explain not only Lorentz force, but also longitudinal force and tangential force.

However, great theories gain recognition not because they are correct, but because of their spectacular success. For example, Copernicus' heliocentric model gained its glory because of Newton's theory of gravity. Nobody knows what innovation in physics Coulomb magnetic force law will inspire, but the one who finds it will be great. For now, I have proposed new techniques for improving high field magnet and plasma confinement, see «Plasma under Coulomb magnetic force» and hope that these techniques will succeed.