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Can EMF's location be known?

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The expression for Faraday's law is:

$$\mathrm{EMF} = -\frac{d}{dt}\iint_s \mathbf{B}\cdot d\mathbf{s} \tag{1}$$

EMF is the generated voltage in a loop under the induction of a varying magnetic field **B**. But the location in space of EMF is not known. The present study may give some hints for it.

The Figure 1 shows a hexagonal conductor loop, constituted with 6 straight superconductor wires (blue straights) that are connected to their neighbor through resistances (red squares). Closing the loop a load resistance is connected to the terminals, the points A and D. There are 5 nodes of connection 1, 2, 3, 4 and 5. The body of the resistances stays outside the loop and the thickness of the connecting wires is negligible (see the right part of the Figure 1), near the load resistance the magnetic field is 0. Faraday's law applies.

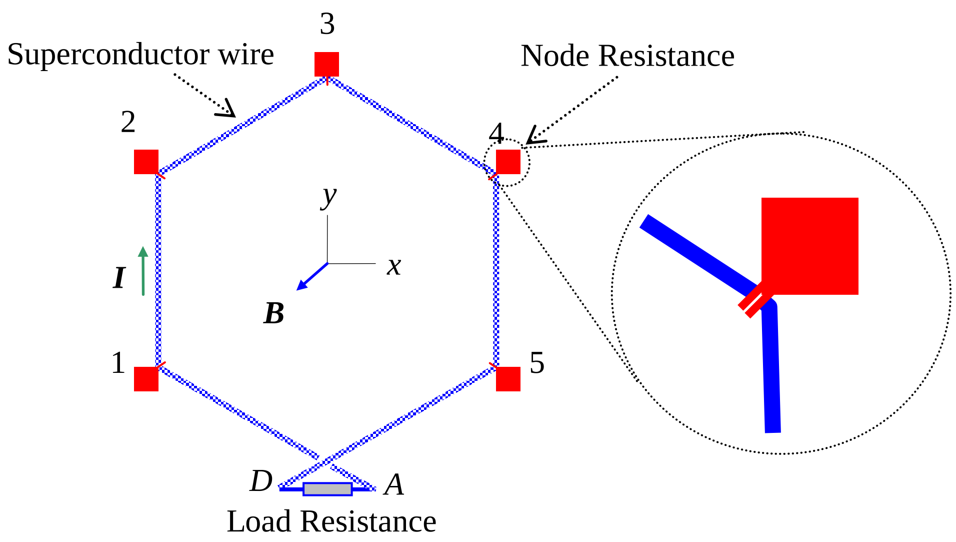

Figure 1

The magnetic field **B** varies, a current I flows in the loop. What is the voltage between A and D by considering the EMF distributed along the loop? Let us calculate the voltage by integrating the electric field along the loop:

$$V_D - V_A = \int_A^D E_{total} dl \tag{2}$$

The total electric field is the sum of induced E field and electrostatic field. The variation of the magnetic field is maintained constant so that the current through the loop is constant (Eddy current is negligible), and the electrons being not accelerated, the total electric field in the superconductor wires is 0:

$$\mathrm{E_{wire}}=0 \tag{3}$$

Then the integral of the total electric field in each wire is 0:

$$\int_{wire} E_{total} dl = 0 \tag{4}$$

Around the resistances, the voltage is negative because along the current it decreases:

$$V_{\mathrm{Resistance}} = -R_{node} \cdot I \tag{5}$$

The current equals EMF divided by the total resistance of the circuit:

$$I = \frac{\mathrm{EMF}}{R_{load} + 5R_{node}} \tag{6}$$

So, the total integral from A to D through the loop is:

$$V_D - V_A = \int_A^D E_{total} dl = 6*0 - 5R_{node} \cdot I = -5R_{node} \frac{EMF}{R_{load} + 5R_{node}} \leq 0 \tag{7}$$

The voltage around the load resistance is:

$$V_D - V_A = R_{load} \frac{EMF}{R_{load} + 5R_{node}} \geq 0 \tag{8}$$

Surprisingly, we have obtained 2 different values for the voltage between the same 2 points A and D, one positive and one negative. For the special case where $R_{node}=0$, we have:

Integrated along the loop: $V_D - V_A = 0$

Around the load resistance: $V_D - V_A = EMF$

In fact, the two ways of calculation are to follow one part of the loop and the other part of the same loop. Take an electron to go through the wire from A to D. It acquires energy that equals the work done on it by the EMF, W_{emf}. Then, it goes from D to A through the load resistance and loses energy in heat. The lost energy equals W_{emf} because it returns to the point A with the same state of energy. In this process, the work done on the electron through the wire is the same than that lost through the load resistance; that is, the integral of work done through the wire equals the work lost in the load resistance. But in the above example, the work done by the EMF on the electron is

$$-e \cdot EMF \frac{5R_{node}}{R_{load} + 5R_{node}} \tag{9}$$

The work lost in the load resistance is

$$e \cdot EMF \frac{R_{load}}{R_{load} + 5R_{node}} \tag{10}$$

And the electron violates the energy conservation law. The right value of the voltage is the one around the load resistance as proved by experiment. However, the integration of the **E** field along the loop is also a correct computation. Why its result is wrong?

The point is that there should be something mysterious about the nature of induced EMF. The difference between the results could be linked to its unknown location in space. Since a loop in resistive conductor is equivalent to the example loop with large number of nodes, this study may be helpful for understanding what EMF's hidden nature is and finding its location in the wire; a long time mystery for physics.