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Can EM wave go forward back?

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This is a bizarre question, but the solution of the EM wave equation goes this way. Take an element of antenna in which an alternate current emits a wave, as shown in the Figure 1. The electric field of this wave is given by the equations (1). With the emitting current being:

$$I = I_0 e^{j\omega t}$$

The electric field solution of the EM wave equation is:

$$\begin{aligned} E_r &= 2\frac{I_0 dl}{4\pi}\eta_0\beta_0^2\cos\theta\left(\frac{1}{\beta_0{}^2 r^2} - j\frac{1}{\beta_0{}^3 r^3}\right)e^{j(\omega t-\beta_0 r)} \\ E_\theta &= \frac{I_0 dl}{4\pi}\eta_0\beta_0^2\sin\theta\left(j\frac{1}{\beta_0 r} + \frac{1}{\beta_0{}^2 r^2} - j\frac{1}{\beta_0{}^3 r^3}\right)e^{j(\omega t-\beta_0 r)} \\ E_\phi &= 0 \end{aligned} \tag{1}$$

The constants in the equations (1) are defined as below:

$$\beta_0 = \omega\sqrt{\mu_0\varepsilon_0}\ ,\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{2}$$

(Ref. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.590).

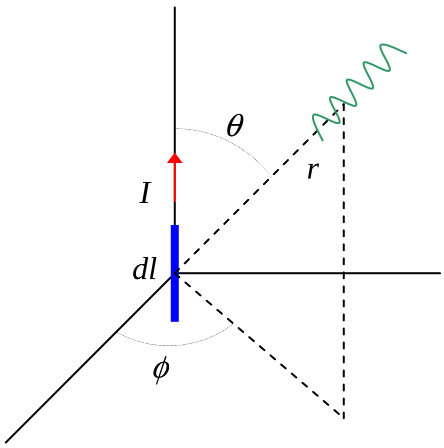

Figure 1

Let us compute the phase velocity of the θ component E_θ. By putting $\frac{1}{\beta_0{}^2 r^2}$ out of the parentheses, the expression for E_θ of the equations (1) becomes:

$$E_\theta = \frac{I_0 dl}{4\pi r^2}\eta_0\sin\theta\left(1 + j\left(\beta_0 r - \frac{1}{\beta_0 r}\right)\right)e^{j(\omega t-\beta_0 r)} \tag{3}$$

We define an angle α as follow,

$$\tan\alpha = \beta_0 r - \frac{1}{\beta_0 r}, \alpha = \arctan\left(\beta_0 r - \frac{1}{\beta_0 r}\right)$$

$$\cos\alpha = \frac{1}{\sqrt{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}}, \sin\alpha = \frac{\beta_0 r - \frac{1}{\beta_0 r}}{\sqrt{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}} \tag{4}$$

And write the complex number in trigonometric form:

$$1 + j\left(\beta_0 r - \frac{1}{\beta_0 r}\right) = \sqrt{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}\left(\cos\alpha + j\sin\alpha\right)$$

The expression for E_θ is then exponential, and whose argument will give the phase velocity of the electric field wave E_θ:

$$E_\theta = \frac{I_0 dl}{4\pi r^2}\eta_0 \sin\theta \sqrt{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}\, e^{j(\omega t - \beta_0 r + \alpha)} \tag{5}$$

A point traveling with the wave keeps the same phase, that is, the difference of the point's phase at different time is 0:

$$\Delta(\omega t - \beta_0 r + \alpha) = 0,\ \omega - \beta_0\frac{\Delta r}{\Delta t} + \frac{\Delta\alpha}{\Delta t} = 0$$

For Δt tending to 0, ω becomes a function of derivatives:

$$\omega = \left(\beta_0 - \frac{\partial\alpha}{\partial r}\right)\frac{\partial r}{\partial t}$$

Then, the velocity of the point is:

$$v = \frac{\partial r}{\partial t} = \frac{\omega}{\beta_0 - \frac{\partial\alpha}{\partial r}}$$

The derivative of α is computed as follow (see the equation (4)):

$$\frac{\partial\tan\alpha}{\partial r} = \frac{1}{\cos^2\alpha}\frac{\partial\alpha}{\partial r} \Rightarrow \frac{\partial\alpha}{\partial r} = \cos^2\alpha\frac{\partial\tan\alpha}{\partial r}$$

$$\frac{\partial\alpha}{\partial r} = \frac{1}{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}\frac{\partial}{\partial r}\left(\beta_0 r - \frac{1}{\beta_0 r}\right) = \frac{1}{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}\left(\beta_0 + \frac{1}{\beta_0 r^2}\right)$$

The velocity's expression is then:

$$v = \frac{\omega}{\beta_0 - \frac{1}{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}\left(\beta_0 + \frac{1}{\beta_0 r^2}\right)} = \frac{\omega}{\beta_0}\frac{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2 - \left(1 + \frac{1}{\beta_0^2 r^2}\right)}$$

$$v = \frac{\omega}{\beta_0}\frac{1+\left(\beta_0 r - \frac{1}{\beta_0 r}\right)^2}{(\beta_0 r)^2 - 2}$$

Using the equation (2) we write $\beta_0 r$ as a function of wave length λ:

$$\beta_0 = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}, \quad \beta_0 r = 2\pi \frac{r}{\lambda} \tag{6}$$

The velocity's expression becomes:

$$v = c\frac{\left(\frac{2\pi r}{\lambda} - \frac{\lambda}{2\pi r}\right)^2 + 1}{\left(\frac{2\pi r}{\lambda}\right)^2 - 2}$$

Strangely, v is negative for small distance; for $x = \frac{2\pi r}{\lambda}$ between 0 and $\sqrt{2}$, the wave goes backward! This is non sense because real waves travel forward, from antenna into space. This is why I said "EM wave goes forward back". The curve of v against r is drawn in the Figure 2.

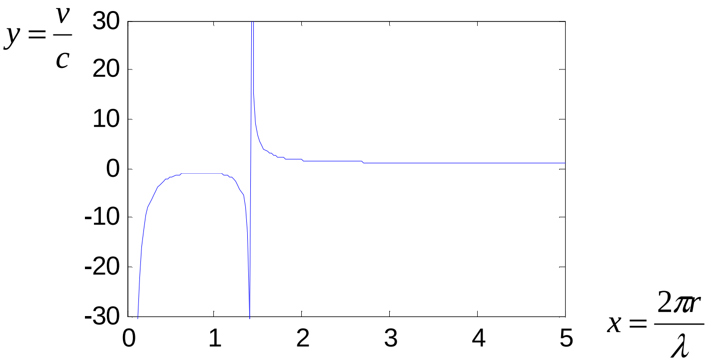

Figure 2

Note that the value of the velocity is not constant. The value of v is negative infinity near x=0, then increases to a maximum of negative value and decreases to negative infinity again near $x = \sqrt{2}$. For $x > \sqrt{2}$, v decreases from infinity to c, the speed of light.

Let us look at the time at which a point arrives at distance r. Take a point whose phase is 0 at the antenna. Arriving at distance r, its phase stays the same (see equation (5)):

$$\omega t - \beta_0 r + \arctan\left(\beta_0 r - \frac{1}{\beta_0 r}\right) = 0$$

And the time is:

$$t = \frac{1}{\omega}\left[\beta_0 r - \arctan\left(\beta_0 r - \frac{1}{\beta_0 r}\right)\right]$$

Using the equation (6), the time is expressed in terms of wave length λ and period T:

$$t = \frac{T}{2\pi}\left[\frac{2\pi r}{\lambda} - \arctan\left(\frac{2\pi r}{\lambda} - \frac{\lambda}{2\pi r}\right)\right]$$

When r approaches 0, $r \to 0 \Rightarrow \arctan\left(\frac{2\pi r}{\lambda} - \frac{\lambda}{2\pi r}\right) \to -\frac{\pi}{2}$

The time of the point is:

$$t_{r\to 0} = \frac{T}{4}$$

We emphasize that this is not the time at which the point leaves the antenna, but that at which it arrives there by traveling backward from the distance $r = \frac{\lambda}{\sqrt{2\pi}}$. In other words, a wave leaves the antenna at time t=0, but does not appear in space near the antenna. By magic, it emerges at the distance $r = \frac{\lambda}{\sqrt{2\pi}}$, and then travels backward to the antenna. This is really bizarre. Is the wave in a wormhole in space-time?

Let us see the curve of the point's time against the distance, drawn in the Figure 3. We see that for small distance the wave time is greater than light time, that is, the wave arrives later than light. At large distances, the wave time has an advance a quarter of a period ahead of light time, that is, the wave travels faster then light:

$$t_{r\to\infty} = \frac{r}{c} - \frac{T}{4}$$

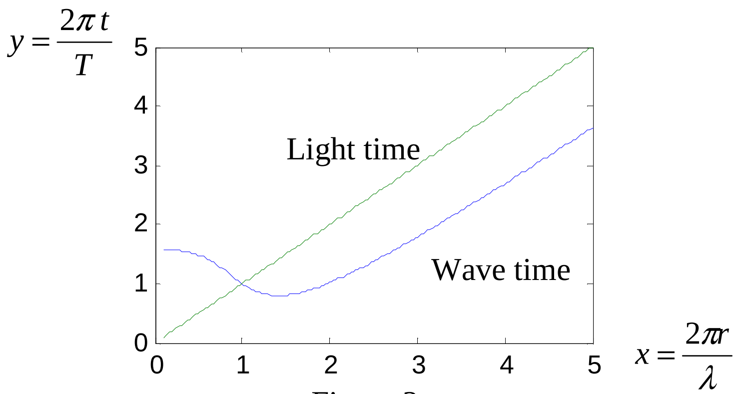

Figure 3

Backward traveling wave that goes forward at the same time, velocity that goes to infinity 3 times, the wave disappears near the antenna only to go back later, all these weird properties are not natural. A law is right only if its prediction is in accordance with reality. The EM wave equation is definitely not. But, it is shocking to say plainly that it is wrong. The EM wave equation is correctly derived from the Maxwell equations which are proven theory. The solution of the EM wave equation we analyzed above is mathematically exact. See <<Why EM wave equation does not conform to relativity?>> for demonstration http://independent.academia.edu/KuanPeng/Papers/1869009/Why_EM_wave_equation_does_not_conform_to_relativity But, it is contrary to physical phenomenon.

So, I ask this question: what is wrong?