Non-loop induced voltage problem

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The Induced conductor net problem could be solved if we can determine the induced voltage in part of the circuit. But is it possible that a voltage exist in no-loop wire? Let us see what the induced voltage is in a circular loop formed with 4 segments separated by resistors (Figure 1).

1. Segmented circular loop

We construct the segmented circular circuit of Figure 1 with 4 identical arc conductor wires, *a, b, c, d* and 4 identical resistors, R_1, R_2, R_3, R_4. The resistance of each resistor is R. This is a closed loop and when induced by a magnetic flux varying at rate Φ', the total induced voltage is well defined by Faraday's law:

$$U = \oint \boldsymbol{E}ds = -\frac{d\Phi}{dt} = -\Phi' \qquad (1)$$

The current is:

$$I = -\frac{\Phi'}{R_1 + R_2 + R_3 + R_4} = -\frac{\Phi'}{4R} \qquad (2)$$

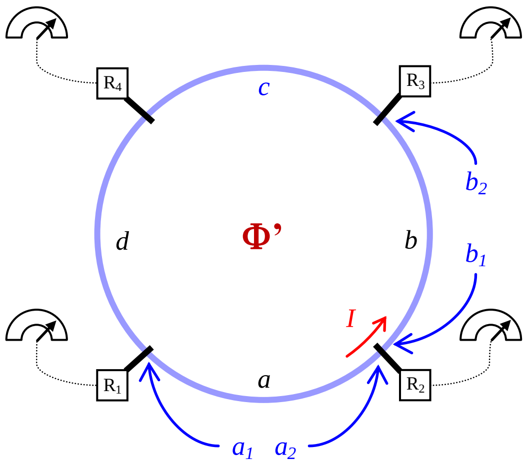

Figure 1

The voltage across each resistor is:

$$U_1 = U_2 = U_3 = U_4 = RI = -\frac{\Phi'}{4} \qquad (3)$$

Induced voltage of a normal loop is only known at the terminals and is the difference of potential of these two points. How the potential is distributed in the loop is not known. But in this segmented circuit, the voltage across each resistor is known and will help us in finding out the distribution of potential.

This circuit is rotational symmetric. When it is given 45 ° rotation, arc *a* occupies the place of arc *b* and the rotated circuit is identical to itself before rotation. So, the potential in arc *a* is identical to that in arc *b*. The curve of potential is shown in Figure 2, the red lines are the potentials in the arcs and the blue lines are the potential falls in the resistors. The potential in arc *a* increases from zero to $-\frac{\Phi'}{4}$, then falls $\frac{\Phi'}{4}$ through the resistor. Then this cycle is repeated 3 times in the following arc-resistor assemblies. So, the induced voltage in each arc is:

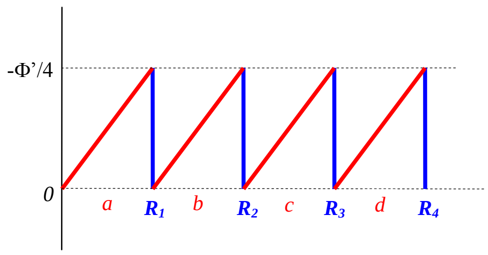

Figure 2

$$U_{arc} = -\frac{\Phi'}{4} \qquad (4)$$

We see that in arc *a* the potential of the start point a_1 and the end point a_2 are different and their difference is the voltage in arc *a*. But, this voltage cannot be measured directly by connecting a voltmeter to these 2 points. However this voltage can be known. Above we have shown that the potential of arc a is identical to that of arc b. So, the potentials of the points a_1 and b_1 , V_{a1} and V_{b1}, are identical and the difference of potential between points b_1 and a_2, V_{b1} and V_{a2}, equals that between a_1 and a_2:

$$U_a = V_{a2} - V_{a1} = V_{a2} - V_{b1} = -\frac{\Phi'}{4} \qquad (5)$$

This conclusion is apparent in Figure 2. V_{a2} - V_{b1} is the voltage across the resistor R_1. So, the measurement of the induced voltage in arc a is equivalent to measuring the voltage across resistor R_1.

2. Isolated wire

Is the voltage in arc a induced in the entire loop or is it induced in the arc by its own? This question is irrational because in open wire there cannot be induction. However, all new discoveries are irrational at the beginning and this question is worth for a reflection.

Let us vary the value of the resistances and see how the voltage varies. Faraday's law is independent to current, that is, the induced voltage is -Φ' whatever the load resistance is. In a loop with a load resistor R of 1 Ω, the induced voltage is -Φ', for R= 1 000 Ω, the voltage is also -Φ'. If this were true for our 4-segments circuit, the induced voltage in each arc would stay constantly $-\frac{\Phi'}{4}$ when the resistance R of the 4 resistors varies from small to very large.

But what will happen if R approaches infinity? If we keep the induced voltage constant, the voltage in each arc will be $-\frac{\Phi'}{4}$ when the current becomes zero. Here, all arcs are isolated wires, having a voltage means induction in open circuit. If we keep the idea of non induction in open wire, then the voltage in the arcs should decrease progressively to zero when the resistance increases to infinity, that is, the induced voltage should depend on current. A current-depending induction will violate Faraday's law because the voltage varies with resistance:

$$U \neq -\Phi' \qquad (6)$$

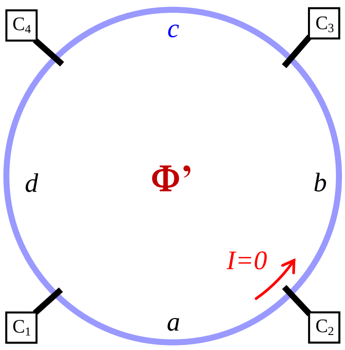

Figure 3

However, in some kind of isolated wires there may be induction. In Figure 3, the 4 resistors are substituted by capacitors and no current passes between arcs. In alternate magnetic field, there should be voltage in the arcs. But in linearly varying magnetic field the voltage is constant and the current is zero. Will there still be voltage in the arcs? If the voltage is zero, how can a low frequency magnetic field induce a voltage in the circuit and charge the capacitors?

The question of induction in open wire is an open question.

3. Comments

This non-loop wire induction problem cannot be solved theoretically. We have to rely on experiment to study this phenomenon. This work can be done by measuring simultaneously the voltage across the resistors of Figure 1 by varying the resistance from small to infinity and see if the voltage decreases for large resistance. Another experiment is designed in **Partial EMF** **measurement** that measures the static distribution of potential in a loop.

Non-Faraday law of induction that will explain induction in lumped circuit should be formulated using experimental data and will be a major advance in physics. Guess who will be the most advantaged for deriving this law? The one who get the data, as Biot, Savart and Faraday were in their times.