Energy density of electromagnetic wave

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Light is an electromagnetic wave whose total energy is the photon's energy *e* multiplied by the number of photons *n*:

$$e = h\nu, \quad E = n \cdot e \tag{1}$$

This leads to constancy of energy inside a cone centered at the source (see Figure 1). Take a steady light source emitting a constant flow of photons. Let *n* be the number of photons emitted during the time *t*, the energy flux is constant and is:

$$\alpha_{flux} = \frac{n \cdot e}{t} \tag{2}$$

The energy within a segment of length *dr* of the cone is:

$$dr = cdt \Rightarrow \Delta E_{dt} = \alpha_{flux} dt = \frac{\alpha_{flux}}{c} dr \tag{3}$$

And the density of energy per unit length of the cone is the following constant:

$$\frac{\Delta E_{dt}}{dr} = \frac{\alpha_{flux}}{c} \tag{4}$$

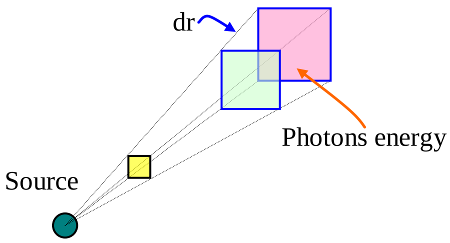

Figure 1

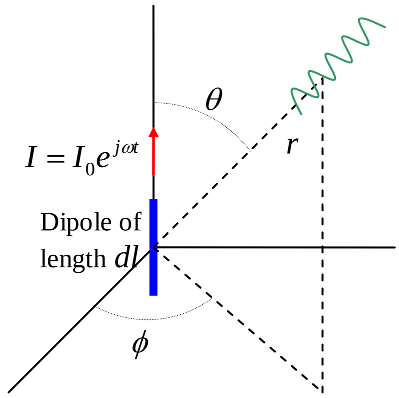

Figure 2

So, whatever the distance from the source, a segment of the cone of unit length contains the same quantity of energy. For an electromagnetic wave, if we know the electromagnetic field, we can also calculate its energy in terms of electric and magnetic fields. This is the case for solutions of the electromagnetic wave equation. Contrary to light, these fields give a variable energy that become infinity near the source, violating physical principles.

Let us calculate the magnetic field energy of a solution wave in an elementary segment of a cone. The space density of magnetic field energy is:

$$\frac{de_H}{dV} = \frac{1}{2}\mu_0 H^2 \tag{5}$$

The volume of an element of a cone is $dV=S \cdot dr$, where S is the surface of the cross section of the cone with solid angle $d\theta d\phi$:

$$S = r^2 \sin\theta d\theta d\phi \tag{6}$$

So, the volume of the elementary segment is:

$$dV = r^2 \sin\theta d\theta d\phi \cdot dr \tag{7}$$

In the plan of θ=π/2, the magnetic field of a sinusoidal wave has only ϕ component. By using the expression below [1,3] (see Figure 2 and the article **Electromagnetic Wave Paradox,** Links: blogspot academia),

$$H_\phi = \frac{I_0 dl}{4\pi r^2}\sqrt{1+\beta_0^{\,2} r^2}\,\sin\theta\cos\left(\omega t - \beta_0 r + \arctan(\beta_0 r)\right),\ \beta_0 = \frac{\omega}{c} \tag{8}$$

And using equations (5) and (7), the expression for the magnetic field energy density is:

$$\frac{d^3 e_H}{r^2 \sin\theta d\theta d\phi \cdot dr} = \frac{1}{2}\mu_0\left(\frac{I_0 dl}{4\pi}\right)^2\left(\frac{1}{r^4}+\frac{\beta_0^{\,2}}{r^2}\right)\sin^2\theta\cos^2\left(\omega t - \beta_0 r + \arctan(\beta_0 r)\right) \tag{9}$$

We calculate the mean value averaged over one period T and obtain the average energy density per unit length:

$$\frac{d}{dr}\left(\frac{\int_0^T e_H dt}{T}\right) = \frac{1}{4}\mu_0\left(\frac{I_0 dl}{4\pi}\right)^2\left(\frac{1}{r^2}+\beta_0^{\,2}\right)\sin^3\theta \cdot d\theta d\phi \tag{10}$$

We see that this energy density increases to infinity in proportion to $1/r^2$ when $r \to 0$. This is a physical none sense because wave energy is limited. Let us see the average quantity of energy contained in a cone element:

$$\Delta(\tilde{e}_H) = \frac{1}{4}\mu_0\left(\frac{I_0 dl}{4\pi}\right)^2\left(\frac{1}{r^2}+\beta_0^{\,2}\right)\sin^3\theta \cdot d\theta d\phi \cdot \Delta r,\quad \tilde{e}_H = \frac{\int_0^T e_H dt}{T} \tag{11}$$

An element of length $\Delta r = p * r$, with p a small constant, can approach the source without reaching it while keeping a none zero volume that contains the following quantity of energy:

$$r \to 0,\quad \Delta(\tilde{e}_H) \to \frac{1}{4}\mu_0\left(\frac{I_0 dl}{4\pi}\right)^2 \frac{p}{r}\sin^3\theta \cdot d\theta d\phi \tag{12}$$

As I have said before, this value goes to infinity near the source and thus is inconsistent. For those who want to incorporate the electric field energy, I have put the expression for this energy in the annex, which suffers from the same inconsistency.

Comment

There is no need to calculate a numerical value to show that around the source the "quantity of field energy" could fuel the whole world. Previously I have shown 5 inconsistencies of the wave equation (see chapter "About wave" in **Summary**: blogspot, Academia). This one is the 6th. In the discussion for the last inconsistency, someone has proposed to use Poynting vector to solve the problem. But here, Poynting cannot remedy. As the 6 inconsistencies are independent, they can not be fixed by a single correction. Concretely, near the source no correction can solve the problems all at once for the wave to travel at the speed of light, to conserve the form of signal and to contain constant energy.

With the Poynting example I just want to say that when a patch is put to correct one inconsistency of a wrong law, all others stay. A wrong law can provide a correct result in limited circumstances, but all other predictions are wrong. A plan wave solution is consistent because it is the limit at infinity of the solution of the wave equation. All solutions for elsewhere are wrong.

The wave equation is derived from the Faraday-Maxwell and Ampere-Maxwell equations, which are also shown to be inconsistent (see chapters “About Faraday’s Law” and “About displacement current” in **Summary**: blogspot, Academia). Because the 2 parent laws are wrong, their child law is doubly wrong. The high number of found inconsistencies proves this.

In consequence, the theory of electromagnetism really contains fatal errors and the researchers who want to prove that by experiment will surely succeed. Using my theoretical results, I have proposed several experiment designs (see **Summary**: blogspot, Academia), which aim precisely at the crux of each inconsistency. I cannot do them all. Carrying them out will give you success. So, all interested experimenters are invited to participate and to share the success of correcting electromagnetic theory.

Annex

The θ component of the electric field of a wave is (see **Can EM wave go forward back**: blogspot academia):

$$E_\theta = \frac{I_0 dl}{4\pi r^2}\eta_0\sqrt{1+\left(\beta_0 r-\frac{1}{\beta_0 r}\right)^2}\cos\left(\omega t-\beta_0 r+\arctan\left(\beta_0 r-\frac{1}{\beta_0 r}\right)\right), \eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (13)$$

The field energy density is:

$$\frac{de_E}{d\theta d\phi\cdot dr}=\frac{1}{2}\varepsilon_0\left(\frac{I_0 dl}{4\pi}\eta_0\right)^2\left(\frac{1}{r^2}+\left(\beta_0-\frac{1}{\beta_0 r^2}\right)^2\right)\cos^2\left(\omega t-\beta_0 r+\arctan\left(\beta_0 r-\frac{1}{\beta_0 r}\right)\right)\sin\theta \qquad (14)$$

References

1. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.258

2. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.590

3. **Summary,** links: blogspot, Academia