Calculation of the Lorentz' Torque and the Ampere's torque

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In the article «The Lorentz torque experiment», I have proposed an experiment to test the Lorentz force law and the differential Ampere's force law. The theoretical predictions of the torque on a coil according to the Lorentz force law and the differential Ampere's force law give different values. The experimental measurement will confirm one law and refute the other. The curve of the Lorentz' torque and the Ampere's torque are drawn in the Figure 1.

It is important for the experimenters to know how to do this calculation and why the values are so different. Below is the explanation.

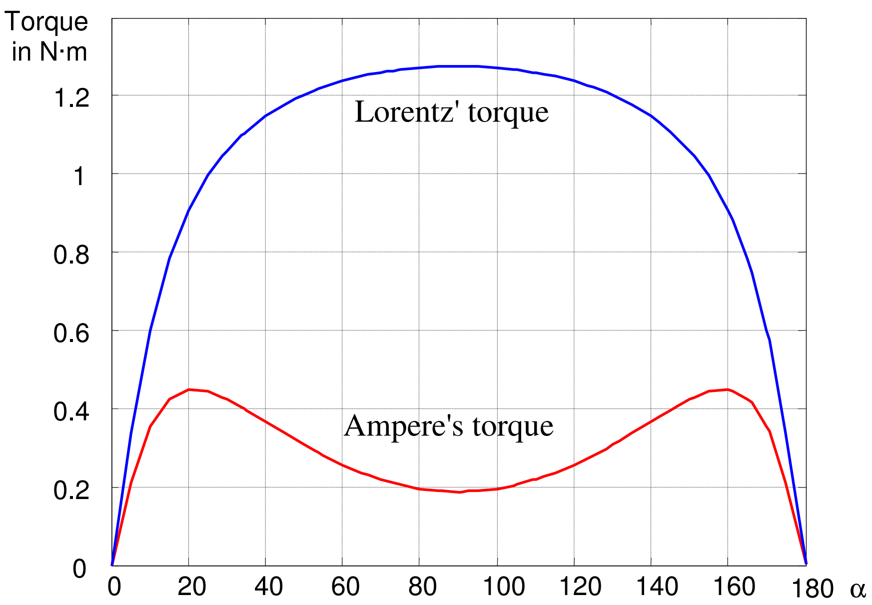

Figure 1

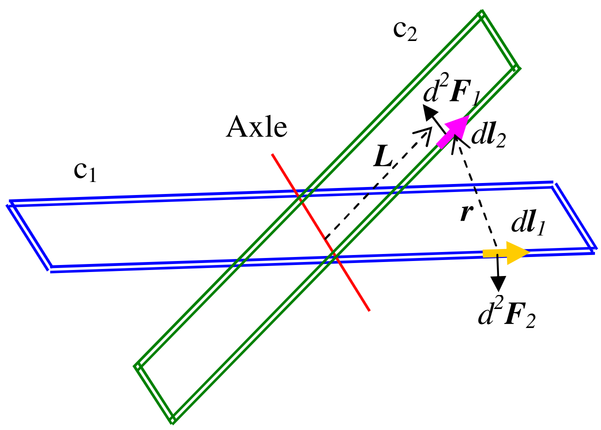

Figure 2

According to the Lorentz force law, the differential force between 2 infinitesimal current elements $d\boldsymbol{I}_1$ and $d\boldsymbol{I}_2$ of 2 interacting coils is (see the Figure 2):

$$d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} d\boldsymbol{I}_2 \times \left(d\boldsymbol{I}_1 \times \frac{\boldsymbol{e}_r}{r^2} \right)$$

The torque of a force about an axle, $\boldsymbol{M}$, is the cross product of the lever vector $\boldsymbol{L}$ with the force $\boldsymbol{F}$:

$$\boldsymbol{M} = \boldsymbol{L} \times \boldsymbol{F}$$

So, the differential torque of the Lorentz force about the axle in the Figure 2 is:

$$d^2\boldsymbol{M}_{12} = \boldsymbol{L} \times d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} \boldsymbol{L} \times \left(d\boldsymbol{I}_2 \times \left(d\boldsymbol{I}_1 \times \frac{\boldsymbol{e}_r}{r^2} \right) \right)$$

And the global Lorentz' torque on the coil 2, $\boldsymbol{M}_{c1\text{-}c2}$, is the double integral of $d^2\boldsymbol{M}_{12}$ over the coil c_1 and c_2:

$$\boxed{M_{c1-c2} = \int_{c1}\int_{c2} \frac{\mu_0}{4\pi} \boldsymbol{L} \times \left(d\boldsymbol{I}_2 \times \left(d\boldsymbol{I}_1 \times \frac{\boldsymbol{e}_r}{r^2} \right) \right)}$$

In the article «Correct differential magnetic force law », I have given the correct magnetic force law, the differential Ampere's force law:

$$\boxed{d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi} \frac{\boldsymbol{e}_r}{r^2} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right)}$$

In the same way, the global torque of the differential Ampere's force, $\boldsymbol{M}_{amp}$, is the double integral of the cross product of the lever vector $\boldsymbol{L}$ with the differential Ampere's force over the 2 coils:

$$\boxed{\boldsymbol{M}_{amp} = \int_{c1}\int_{c2} \left(-\frac{\mu_0}{4\pi} \boldsymbol{L} \times \frac{\boldsymbol{e}_r}{r^2} \left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1 \right) \right)}$$

The Figure 1 shows that the Lorentz' torque is a single-humped curve, while the Ampere's torque is a double-humped curve with lower values. Why does this difference exist? Let us see the Figure 3, where the differential Lorentz force, $d^2\boldsymbol{F}_{12}$, and the differential Ampere's force, $d^2\boldsymbol{F}_{amp}$, are drawn.

When the current elements $d\boldsymbol{I}_1$ and $d\boldsymbol{I}_2$ are perpendicular, the differential Lorentz force is not null, and so is the differential torque:

$$d^2\boldsymbol{F}_{12} = \frac{\mu_0}{4\pi} d\boldsymbol{I}_2 \times \left(d\boldsymbol{I}_1 \times \frac{\boldsymbol{e}_r}{r^2} \right) \neq 0 \Rightarrow d^2\boldsymbol{M}_{12} \neq 0$$

When the angle increases, the global Lorentz' torque increases due to the force between perpendicular components of the current elements and achieves its maximum at 90°.

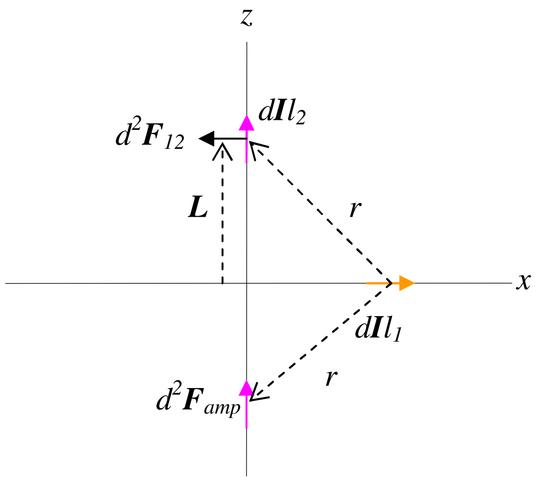

Figure 3

In the contrary, the differential Ampere's force is proportional to the dot product of the current elements, which is 0 for perpendicular currents:

$$\boxed{d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{e}_r}{r^2}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)}$$

Thus, in the Figure 3 the differential Ampere's force is 0, and so is the differential torque:

$$\boxed{d^2\boldsymbol{F}_{amp} = 0 \Rightarrow d^2\boldsymbol{M}_{amp} = 0}$$

When the angle increases, the Ampere's torque decreases after a maximum due to the decrease of the force between perpendicular components of the current elements which becomes 0 at 90°, where the global torque achieves its minimum at 90°. The decrease of the global torque makes the double humps of the curve.