B-Cutting Solution

Peng Kuan 彭宽 titang78@gmail.com
Monday, June 18, 2012

Curiously, the Lorentz' EMF paradox permits to solve the B-cutting paradox. I have proven that EMF generation by cutting magnetic field line violated the energy conservation law and that a wire moving in a magnetic field did not create EMF. The proofs are in these articles:
B-cutting paradox http://pengkuanem.blogspot.fr/2012/05/b-cutting.html
Lorentz' EMF paradox http://pengkuanem.blogspot.fr/2012/05/lorentz-emf.html
Lorentz' EMF Experiment
http://pengkuanem.blogspot.com/2012/06/lorentz-emf-experiment.html

The Figure 1 shows the B-cutting paradox. A current loop closed by a moving conductor bar exchanges energy, mechanical work done by the Lorentz force W_{mec} , electric work done by the current against EMF W_{emf} , variation of magnetic energy stored in the loop *ΔE:*

$$W_{mec} = \int_a^b I\left(\int_l Bdl\right)dx\,,\ \ W_{emf} = -\int_a^b I\left(\int_l Bdl\right)dx\,,\ \ \Delta E = \frac{1}{2}I^2\left(L_b - L_a\right)$$

The B-cutting paradox is that, as W_{mec} and W_{emf} cancel each other, there is a net increase of energy equal to the variation of stored magnetic energy, destroying the balance of energy that must be 0.

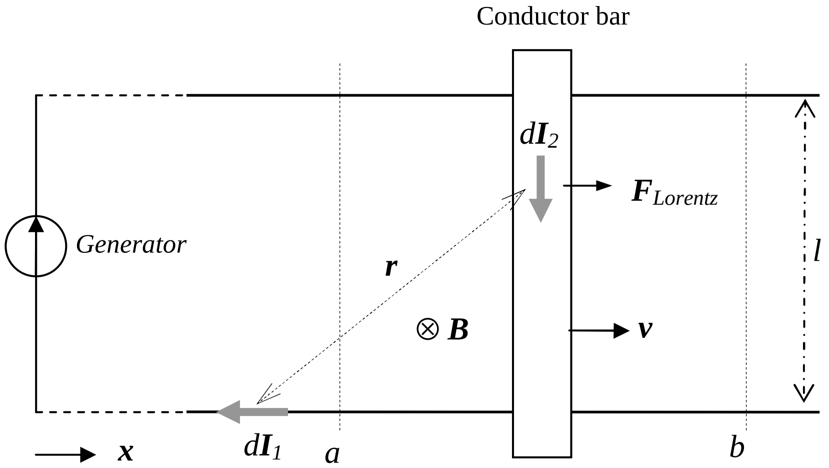

Figure 1

The Lorentz' EMF paradox shows that the moving conductor bar does not create EMF. However, EMF does exist when the bar moves. This EMF is in fact created by the variation of magnetic flux. According to Faraday's law, the tension generated by flux variation is:

$$U_{emf} = -\frac{d\Phi}{dt}$$

And the electric work done by the current *I* during the time interval *dt* is:

$$dW_{emf} = -IU_{emf}\,dt = -I\frac{d\Phi}{dt}dt$$

So, when the bar moves from the position *a* to *b*, the electric work for constant *I* is:

$$W_{emf} = -\int_a^b I d\Phi = -\left(I\Phi_b - I\Phi_a\right)$$

On the other hand, the magnetic energy stored in the loop is $E_a = I\Phi_a$ when the bar is at position a and $E_b = I\Phi_b$ when the bar is at b (ref. Feynman, R. The Feynman Lectures on Physics. Vol. II, p. 15-6). So, the increase of stored magnetic energy is:

$$E_b - E_a = I\Phi_b - I\Phi_a = -W_{emf} \qquad (1)$$

For the mechanical work, we use the correct law of magnetic force that I propose in the article
Correct differential magnetic force law
http://pengkuanem.blogspot.com/2012/04/correct-law.html

This law gives the magnetic force that 2 current elements exert on each other:

$$d^2\boldsymbol{F}_{amp} = -\frac{\mu_0}{4\pi}\frac{\boldsymbol{r}}{r^3}\left(d\boldsymbol{I}_2 \bullet d\boldsymbol{I}_1\right)$$

For the simple case of a very long loop (see the Figure 1), the magnetic field near the bar is created by the horizontal wires only, that from the left vertical wire is negligible. As the current element vectors $d\boldsymbol{I}_1$ and $d\boldsymbol{I}_2$ are perpendicular to one another, their dot product is 0. So, the mechanical work is 0:

$$d\boldsymbol{I}_1 \bullet d\boldsymbol{I}_2 = 0 \Rightarrow d^2\boldsymbol{F}_{amp} = 0$$

$$dW_{mec} = d^2\boldsymbol{F}_{amp} \bullet d\boldsymbol{x} = 0$$

The balance of energy is then (see the equation (1)):

$$E_b - E_a + W_{emf} + W_{mec} = 0$$

The energy conservation law is respected when the correct law of magnetic force and Faraday's law are applied instead of the Lorentz force law and Lorentz' EMF. This simple case illustrates the solution of the B-cutting paradox.

The solution of the B-cutting paradox solves the puzzle of Richard P. Feynman who has written in his "The Feynman Lectures on Physics"(Feynman, R. The Feynman Lectures on Physics. Vol. II, p. 17-2):

> We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of two different phenomena.

The truth is that there is not "two different phenomena" of EMF generation, but only one, that of flux variation. The remark of Richard P. Feynman was judicious.