Peng Kuan 彭宽 titang78@gmail.com https://twitter.com/pengkuanem

Deformation of EM wave signals

14 August 2012

Electromagnetic wave carries signals emitted by antenna into space. The EM field of a wave is mathematically defined by the EM wave equation whose monochromatic solution is the equation (1). A signal is a time varying EM field that can be expressed by Fourier series which is the sum of monochromatic wave functions, that is, a sum of equation (1) of different amplitudes and wave lengths.

We notice that the amplitude and phase of the equation (1) vary with distance and frequency, that is, monochromatic EM waves of different frequencies evolve differently in space. In consequence, the form of the Fourier series is distorted, the traveling signal is deformed. What is the extent of the deformation of EM signal in space?

An alternate current in an antenna emits monochromatic wave. With I_0 the amplitude of the current and ω the angular frequency, λ the wave length, the magnetic field of a monochromatic solution of the wave equation is:

$$H = \frac{\sin\theta \cdot dl}{4\pi r^2}\sqrt{1+\left(2\pi\frac{r}{\lambda}\right)^2}\cdot I_0 e^{j\left(\omega t-2\pi\frac{r}{\lambda}+\arctan\left(2\pi\frac{r}{\lambda}\right)\right)} \tag{1}$$

(See "Why EM wave equation does not conform to relativity?" http://pengkuanem.blogspot.com/2012/08/why-em-wave-equation-does-not-conform.html)

The Fourier series for a variable signal *g(t)* is, with ω_0 the base angular frequency:

$$g(t)=\sum_{n=1}^{\infty}\left(a_n\cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\cos n\omega_0 t\cdot dt,\quad b_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\sin n\omega_0 t\cdot dt$$

Fourier series can also be written as the sum of cosine functions with different coefficients and phases. The signal *g(t)* here is a variable current in an antenna. The nth cosine function is the nth harmonic, whose coefficient is the amplitude of current I_n and whose phase is Φ_n. The expression for the nth harmonic current is:

$$g_n(t)=I_n\cos\left(n\omega_0 t+\Phi_n\right)$$

And the expression for the complete current signal is:

$$g(t)=\sum_{n=1}^{\infty} I_n\cos\left(n\omega_0 t+\Phi_n\right) \tag{2}$$

Into the complex current of the equation (1) we introduce the parameters of the nth harmonic and take the real part,

$$\mathrm{Re}\left(I_n e^{j\left(n\omega_0 t-2\pi\frac{r}{\lambda_n}+\arctan\left(2\pi\frac{r}{\lambda_n}\right)+\Phi_n\right)}\right)=I_n\cos\left(n\omega_0 t-2\pi\frac{r}{\lambda_n}+\arctan\left(2\pi\frac{r}{\lambda_n}\right)+\Phi_n\right)$$

Using this real expression in the equation (1), the expression for the magnetic field of the n[th] harmonic wave is:

$$H_n(t,r)=\frac{\sin\theta\cdot dl}{4\pi r^2}\sqrt{1+\left(2\pi\frac{r}{\lambda_n}\right)^2}\,I_n\cos\left(n\omega_0 t-2\pi\frac{r}{\lambda_n}+\arctan\left(2\pi\frac{r}{\lambda_n}\right)+\Phi_n\right)$$

The complete signal's magnetic field in space is the sum of all H_n:

$$H_{signal}(t,r)=\frac{\sin\theta\cdot dl}{4\pi r^2}\sum_{n=1}^{n=\infty}\left[\sqrt{1+\left(2\pi\frac{r}{\lambda_n}\right)^2}\,I_n\cos\left(n\omega_0 t-2\pi\frac{r}{\lambda_n}+\arctan\left(2\pi\frac{r}{\lambda_n}\right)+\Phi_n\right)\right] \qquad (3)$$

Near the antenna we have:

$$r_0\approx 0,\quad 2\pi\frac{r_0}{\lambda_n}\approx 0,\quad \sqrt{1+\left(2\pi\frac{r_0}{\lambda_n}\right)^2}\approx 1$$

By introducing the above expressions into the equation (3), the signal's magnetic field is then:

$$H_{signal}(t,r_0)=\frac{\sin\theta\cdot dl}{4\pi r_0^2}\sum_{n=1}^{\infty}I_n\cos(n\omega_0 t+\Phi_n)$$

The variable part of this magnetic field is the wave's signal:

$$h(t,r_0)=\sum_{n=1}^{\infty}I_n\cos(n\omega_0 t+\Phi_n) \qquad (4)$$

We verify that this expression is identical to the current signal, the equation (2).

Now we derive the expression for the signal at large distance by tending r to infinity. With λ_0 the base wave length and f_0 the base frequency, we have:

$$r\to\infty\Rightarrow\begin{cases}\sqrt{1+\left(2\pi\frac{r}{\lambda_n}\right)^2}\to\frac{2\pi rnf_0}{c}, & \lambda_n=\frac{c}{f_n}=\frac{c}{nf_0}\\ \arctan\left(2\pi\frac{r}{\lambda_n}\right)\to\frac{\pi}{2}\end{cases}$$

We introduce the above expressions and the following into the equation (3):

$$\cos\left(n\omega_0 t-2\pi\frac{r}{\lambda_n}+\frac{\pi}{2}+\Phi_n\right)=-\sin\left(n\omega_0 t-2\pi\frac{r_\infty}{\lambda_n}+\Phi_n\right)$$

We obtain the expression for the signal at large distance:

$$H_{signal}(t,r_\infty)=-\frac{f_0\sin\theta\cdot dl}{2cr_\infty}\sum_{n=1}^{n=\infty}\left[nI_n\sin\left(n\omega_0 t-2\pi\frac{r_\infty}{\lambda_n}+\Phi_n\right)\right] \qquad (5)$$

The variable part of this expression is the wave's signal:

$$h(t,r_\infty)=-\sum_{n=1}^{n=\infty}[nI_n\sin(n\omega_0 t+\Phi_n)]$$

We verify that it is completely different from the signal near the antenna (see the equation (4)) and in consequence, the signal is deformed at large distance.

In order to get a feeling of the deformation of signal, let us derive the magnetic fields of the wave emitted by the following periodical rectangular current signal (see Figure 1):

$$h(t)=\begin{cases}0, & -T/2<t<-e\\ 1, & -e<t<e\\ 0, & e<t<T/2\end{cases} \tag{6}$$

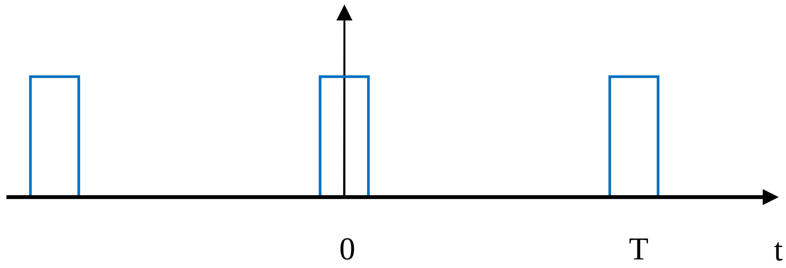

Figure 1

For this signal, the phases of all harmonics are 0: $\Phi_n = 0$

The amplitudes of the harmonics are:

$$I_n = \frac{2}{T}\int_{-T/2}^{T/2} g(t)\cos n\omega_0 t\cdot dt = \frac{2\sin(n\omega_0 e)}{n\pi}$$

Near the antenna the wave's signal is given by the equation (4):

$$h(t,r_0)=\sum_{n=1}^{\infty}\frac{\sin(n\omega_0 e)\cdot\cos(n\omega_0 t)}{n} \tag{7}$$

At large distance, the wave's magnetic field is given by the equation (5),

$$H_{signal}(t,r_\infty)=-\frac{f_0\sin\theta\cdot dl}{2cr_\infty}\sum_{n=1}^{n=\infty}\left[\sin(n\omega_0 e)\cdot\sin\left(n\omega_0 t-2\pi\frac{r_\infty}{\lambda_n}\right)\right]$$

The variable part of this magnetic field is the wave's signal at large distance:

$$h(t,r_\infty)=-\sum_{n=1}^{n=\infty}\left[\sin(n\omega_0 e)\cdot\sin n\omega_0 t\right]$$

This expression is completely different from the signal near the antenna, the equation (7). What is more serious is that this is a divergent Fourier series! This means that the signal at large distance does not have a definite form! If this were true, broadcast by radio wave would be impossible. Fortunately, radio and television function well and signals are not deformed in real world. It is only the mathematical EM wave equation that makes Fourier series divergent at large distance. Thus, the EM wave equation must be wrong.

However, one can object that contrary to the rectangular signal that is emitted directly, radio broadcast uses carrier wave and will not suffer from deformation. In order to verify the deformation of signal, I propose to test signals that are emitted directly.

Experiment design

The experiment consists of sending a signal by an antenna or a spark. The signal is measured near the source at point A and far from the source at point B (sees the Figure 2). There are 2 possible outcomes, the signal is deformed or not deformed.

If the signal is not deformed, the measurement at B is identical to that at A. Then the EM wave equation is wrong because mathematical solution of this equation is deformed.

If the signal is deformed, the signal at B is different from that at A. In this case, all measurement involving EM wave are wrong, current signals are not identical to measured wave signals. We can imagine the consequences, for example, trajectories of charged particles in high energy physics are not what we thought to be, electromagnetic spectrum does not reflect movements and states of atoms and molecules and so on.

What a horrible perspective! But it is improbable that EM signal is deformed and divergent at large distance. In conclusion, the EM wave equation must be wrong.

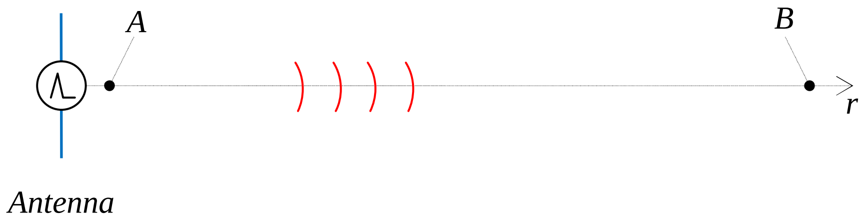

Figure 2