Lorentz' EMF paradox

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One basic notion in the electromagnetic theory is the generation of electromotive force (EMF) by a conductor moving in a magnetic field. This EMF is said to be created by Lorentz force. However, I have shown that the energy conservation law is violated by this explanation. The rigorous proof is in the article B-cutting paradox
http://pengkuanem.blogspot.fr/2012/05/b-cutting.html
https://docs.google.com/open?id=0B3YDEaOyRUwcRzRITFgtdzA5VHc

So, there is a need to better understand the mechanism of EMF generation by Lorentz force. Let us look at the Figure 1 which shows a rectangular circuit closed by a conductor bar moving at velocity ***v*** in a magnetic field ***B***.

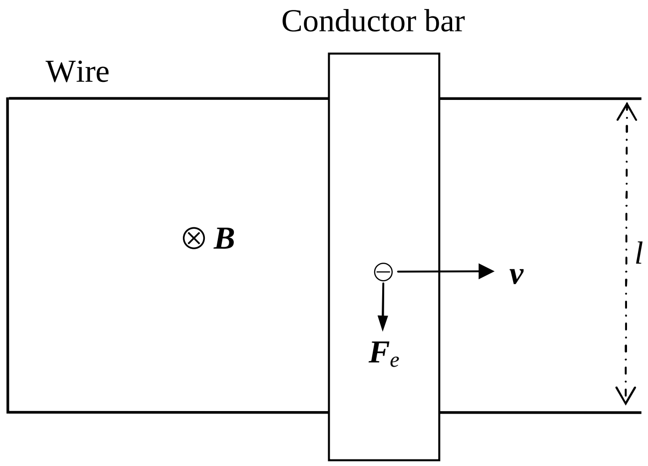

Figure 1

Free electrons are transported by the conductor bar at velocity ***v*** and interact with the magnetic field. According to the Lorentz force law, the Lorentz force on each electron is:

$$\boldsymbol{F}_e = -e\boldsymbol{v}\times\boldsymbol{B}$$

The line integral of this force along a path divided by the charge is the following value:

$$C = -\int_l \frac{1}{e}\boldsymbol{F}_e\cdot d\boldsymbol{l} = \int_l \boldsymbol{v}\times\boldsymbol{B}\cdot d\boldsymbol{l} \qquad (1)$$

EMF is the difference of potential between the 2 contact points of the bar. Let the electric potential be U_1 at the first contact point and U_2 at the second, according to the electromagnetic theory, the difference of potential is equal to C:

$$U_2 - U_1 = v\int_l Bdl$$

But is it true?

Let us look at the Figure 2 where are shown 2 conductors moving at velocity ***v*** in a magnetic field ***B***. The free electrons of the conductors feel the Lorentz force $\boldsymbol{F}_e$. According to electrostatic laws, the free electrons will move in the direction of the force $\boldsymbol{F}_e$ and build a charge distribution up. Finally, the distribution will create an electrostatic field that stops the electrons' move. At equilibrium, on each electron the electrostatic force from the charge distribution is equal but opposed to the Lorentz force $\boldsymbol{F}_e$. In the Figure 2, on the conductor b are shown the separation of charge and the electrostatic field on the surface $\boldsymbol{E}_s$ that is perpendicular to the surface.

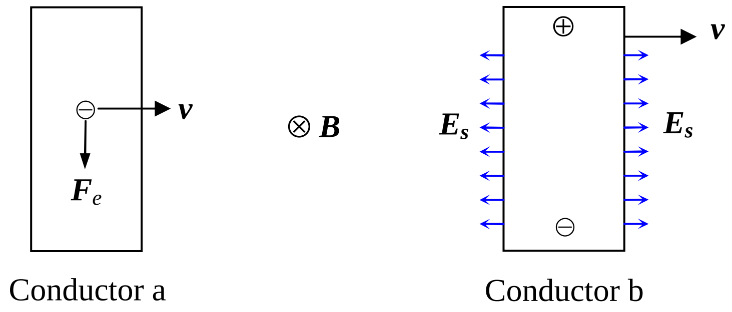

Figure 2

By taking into account the resulting electrostatic field, the resultant force on free electrons is 0 everywhere inside the conductor bar and the surface is perpendicular to the force at any point. So, the line integral of the resultant force in the conductor bar is 0 for any path, and the total difference of potential is 0 but not the value C of the equation (1).

Let us look at the Figure 3 where a conductor bar moves at constant velocity $\boldsymbol{v}$ in a uniform magnetic field $\boldsymbol{B}$. Unlike the rectangular circuit of the Figure 1, the wire is replaced by 2 conductor plates that makes up a capacitor. The conductor bar is in contact with the plate *a* but not with the plate *b*. What will happen if we put the bar in contact with the plate *b*? Nothing, because the force on the free electrons of the bar and the plates is 0, the free electrons will not move. In the contrary, the electromagnetic theory predicts a current driven by the Lorentz force that would charge the capacitor.

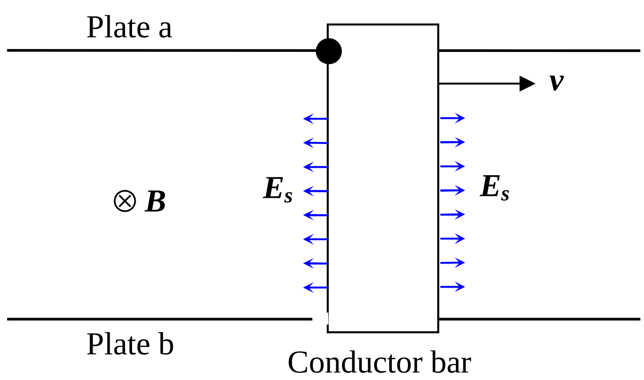

Figure 3

Another explanation is that the potential difference between 2 points is proportional to the energy that a charge acquires when traveling from one point to an other. Along any path inside the conductor bar, the resultant force on the charge is 0, and it gets no energy by moving. If an electron enters into the bar from one contact point and gets out from the second, its energy stays exactly the same because it has not been pushed by any force. Again, the difference of potential between the 2 points of contact is 0.

In conclusion, the EMF in the conductor bar in the presence of a Lorentz force is 0. In other words, Lorentz force does not generate EMF. This is the contradiction: the electromagnetic theory asserts that Lorentz force can generate EMF, but the electrostatic laws make this impossible. I call this inconsistency the "Lorentz' EMF paradox".

In fact, this is not an inconsistency of the Lorentz force law, but a misunderstanding of the EMF phenomenon. Physicists believe that EMF is generated by Lorentz force since Faraday. After he discovered that a wire moving in a magnetic field could create current, this explanation was put foreword and has never been rethought. To correct this inconsistency, we have to admit that EMF is created by flux variation only.