Self force of a 3D coil

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To my previous proofs of the inconsistency of the Lorentz force law some have objected with dubious arguments such as: magnetic force on displacement current, magnetic force inside a magnetic shield. Also, my mathematical demonstrations using triangular coils are although exact but too hard to follow. So, I propose a much more intuitive and simpler counter-example here, a 3D folded rectangular coil, Figure 1. Figure 2 is the right view showing the vertical sides BC and CD.

ABDE is an originally rectangular coil that is folded at the middle of its long sides making C and F angular corners. The Lorentz forces between different sides summed together is the resultant force this coil exerts on itself, the self force, which must be zero. However, by knowing that the nearer two currents are from each other, the stronger the force between them, we can intuitively figure out that these forces are unbalanced: the sides BC and CD form a sharp angle, they make a strong force; the horizontal side AB feels more moderate force from the other sides. These two forces cannot balance and result in a non-zero self force.

The interaction between BC and CD is denoted as F_C and that on the horizontal side AB as F_{AB}. The z direction component of them are $F_{C,z}$ and $F_{AB,z}$ respectively. Due to the symmetry of the coil, the total vertical force on the coil is: $2F_{C,z} + 2F_{AB,z}$. So only $F_{C,z}$ and $F_{AB,z}$ will be needed in this study.
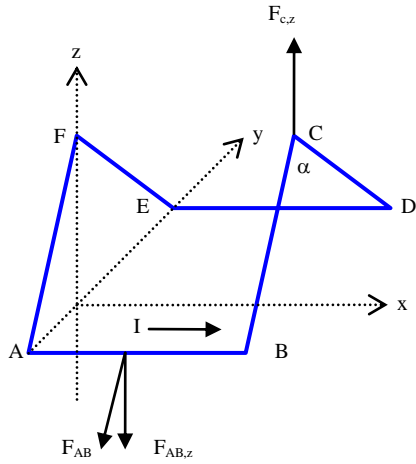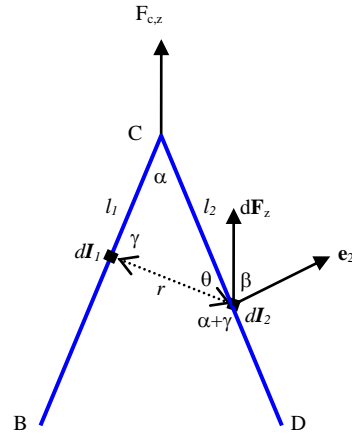

Figure 1

Figure 2

1. Derivation of the 2 z direction forces

Let us derive the force on the corner C. According to the Lorentz force law the elementary Lorentz force on the current element vector dI_2 exerted by dI_1 is:

$$dF = \frac{\mu_0}{4\pi} dI_2 \times \left(dI_1 \times \frac{e_r}{r^2}\right) = \frac{\mu_0}{4\pi} dI_1 dI_2 \frac{\sin\gamma}{r^2} e_2 \tag{1}$$

Its component in z direction is:

$$dF_z = dF \cdot e_z = \frac{\mu_0}{4\pi} dI_1 dI_2 \frac{\sin\gamma \cos\beta}{r^2} \tag{2}$$

The z direction force on the side CD is the double-integral of equation (2) over BC then over CD. The force on corner C is double this integral:

$$F_{c,z} = 2 \int_B^C \int_D^C d\boldsymbol{F_z} = \frac{\mu_0}{2\pi} \int_B^C \int_D^C \frac{\sin \gamma \cos \beta}{r^2} dI_1 dI_2 \tag{3}$$

Because the denominator r^2 reaches zero when the currents elements $d\boldsymbol{I_1}$ and $d\boldsymbol{I_2}$ reach the corner C, this integral's value is infinity. I have shown this in a simple way in <u>Lorentz force on open circuit</u> (equation 2 of that article). The expression of this integral is stated here:

$$F_{C,z} = -\frac{\mu_0 I^2 \sin\frac{\alpha}{2}}{2\pi} ln \left| \frac{\theta_o}{2} \tan \frac{\pi + \alpha}{4} \right| \tag{4}$$

where $\theta_o \nrightarrow 0$

Here θ_o is the angle θ before that $d\boldsymbol{I_1}$ reaches C during integration and can be seen as representing the non-sharpness of real corner. For infinitely sharp corner, $\theta_o = 0$ and the force is infinity, $F_{C,Z} = \infty$. The actual derivation of this expression is put in the Section 4 for those who are interested and me.

On the other hand, the force pointing downward is only on the side AB and is induced by the sides BC, CD, DE, EF and FA. AB forming a corner with BC, the force on AB around the corner B should be huge as demonstrated above. So, the interaction with remote sides CD, DE, EF and FA are negligible and, F_{AB} lies on the plane ABC and its magnitude is independent of the angle α. We have no need of an exact expression for F_{AB} and just express the z component of this force as:

$$F_{AB,z} = F_{AB} \cos\frac{\alpha}{2} \tag{5}$$

2. Conclusion

Now, we have the expressions for the upward pointing force, equation (4), and the downward pointing force, equation (5). These two equations vary differently, equation (4) is proportional to sin(α/2) and equation (5) to cos(α/2). So they cannot cancel each other and the coil has a left over self force. This can be rigorously proven by contradiction. Suppose these two forces have the same value at a specific angle α_0 and, $F_{C,Z} = -F_{AB,Z}$. When we fold more or less the coil, the angle α has another value. Then, sin(α_o/2) becomes sin(α/2) in the new equation (4) and cos(α_o/2) becomes cos(α/2) in (5). Thus, their values become different and make the new resultant force non-zero. This violates Newton's third law and implies that the Lorentz force law is wrong.

3. Comments

The sheer infinite value of the force around a corner shows that the Lorentz force law is incorrect. I discovered this phenomenon the hard way by doing numerical computation of the force of an angular coil and found that the narrower the discretization, the larger the force. As the computation did not converge, I did a long and tedious search for error to finally understand that this was the theory's error, not mine.

Equation (4) explains why numerical value increases when the discretization becomes narrower. In fact, this formula shows that when you divide the angle θ_o by 2, the value of equation (4) increases by $\frac{\mu_0 I^2 \sin\frac{\alpha}{2}}{2\pi} ln2$. So, when you divide θ_o by 2^n the numerical value will increase $n \frac{\mu_0 I^2 \sin\frac{\alpha}{2}}{2\pi} ln2$, which

reaches infinity for n=∞. So, you can never know what the correct theoretical value is for this force. I have found nothing about this phenomenon in the literature. I think that finding no solution, physical community is forced to ignore it because the Lorentz force law must prevail.

Until now, I have made public many theoretical counter-examples and counter-experiments to the Lorentz force law. They are strong invalidating evidences to this law and no credible opposition has been proposed. But positive reaction is still slow in coming. I'm writing an explanatory summary of my work to make it easier and clearer to understand.

I have been working alone on this theory for 15 years and publishing for nearly 3 years. I have found no support. For the last 7 months I was mostly silent because I was tired and ill. So, please send me support by leaving encouraging comment in my Blogger site. Not only this will keep me motivated, but it will let the world know the number of people who support me. Public opinion does make difference and a small gesture of yours makes sense in science history.

4. Derivation of the expression for $F_{c,z}$

Equation (3):

$$F_{c,z} = \frac{\mu_0}{2\pi} \int_B^C \int_D^C \frac{\sin\gamma\cos\beta}{r^2} dI_1 dI_2$$

Using: $dI_1 = Idl_1, dI_2 = Idl_2$, Sine law: $\frac{l_1}{\sin\theta} = \frac{r}{\sin\alpha} \Rightarrow r = \frac{l_1\sin\alpha}{\sin(\alpha+\gamma)}$

where l_1 is the length between C and dI_1 , l_2 the length between C and dI_2

Equation (3) becomes:

$$F_{c,z} = \frac{\mu_0}{2\pi} \int_B^C \int_D^C \frac{\sin\gamma\cos\beta\sin^2(\alpha+\gamma)}{l_1{}^2\sin^2\alpha} I^2 dl_1 dl_2$$

$$= \frac{\mu_0 I^2 \cos\beta}{2\pi\sin^2\alpha} \int_B^C \left(\frac{1}{l_1{}^2} \int_D^C \sin\gamma\sin^2(\alpha+\gamma)\, dl_2 \right) dl_1$$

With Sine law: $\frac{l_1}{\sin\theta} = \frac{l_2}{\sin\gamma} \rightarrow l_2 = \frac{l_1\sin\gamma}{\sin(\alpha+\gamma)}$

the expression of dl_2 is derived:

$$dl_2 = l_1 \left(\frac{\sin\gamma}{\sin(\alpha+\gamma)} \right)' d\gamma = l_1 \left(\frac{\cos\gamma}{\sin(\alpha+\gamma)} - \frac{\sin\gamma\sin'(\alpha+\gamma)}{\sin^2(\alpha+\gamma)} \right) d\gamma$$

$$dl_2 = l_1 \left(\frac{\cos\gamma}{\sin(\alpha+\gamma)} - \frac{\sin\gamma\cos(\alpha+\gamma)}{\sin^2(\alpha+\gamma)} \right) d\gamma$$

The inner integral over CD is:

$$\int_D^C \sin\gamma\sin^2(\alpha+\gamma)\,dl_2 = \int_D^C \sin\gamma\sin^2(\alpha+\gamma)\left(\frac{\cos\gamma}{\sin(\alpha+\gamma)} - \frac{\sin\gamma\cos(\alpha+\gamma)}{\sin^2(\alpha+\gamma)}\right)l_1\,d\gamma$$

$$= l_1\int_D^C \sin\gamma(\sin(\alpha+\gamma)\cos\gamma - \cos(\alpha+\gamma)\sin\gamma)\,d\gamma$$

$$= l_1\int_D^C \sin\gamma\sin(\alpha+\gamma-\gamma)\,d\gamma = l_1\int_D^C \sin\gamma\sin\alpha\,d\gamma$$

$$= l_1\sin\alpha\left(\sin\gamma_C - \sin\gamma_D\right) = -l_1\sin\alpha\sin\gamma_D\,,\qquad \gamma_C = 0$$

where γ_C is the angle γ when $d\boldsymbol{I_2}$ is at C, γ_D when $d\boldsymbol{I_2}$ is at D.

Then:

$$F_{C,z} = \frac{\mu_0 I^2\cos\beta}{2\pi\sin^2\alpha}\int_B^C\left(\frac{-l_1\sin\alpha\sin\gamma_D}{l_1^{\,2}}\right)dl_1 = -\frac{\mu_0 I^2\cos\beta}{2\pi\sin\alpha}\int_B^C\frac{\sin\gamma_D}{l_1}\,dl_1$$

Using Sine law: $\dfrac{l_1}{\sin\theta} = \dfrac{l_{2,0}}{\sin\gamma_D} \to l_1 = \dfrac{l_{2,0}\sin(\alpha+\gamma_D)}{\sin\gamma_D}$

where $l_{2,0}$ is the length of CD

We have:

$$dl_1 = l_{2,0}\left(\frac{\sin(\alpha+\gamma_D)}{\sin\gamma_D}\right)'d\gamma_D = l_{2,0}\left(\frac{\cos(\alpha+\gamma_D)}{\sin\gamma_D} - \frac{\sin(\alpha+\gamma_D)\cos\gamma_D}{\sin^2\gamma_D}\right)d\gamma_D$$

The outer integral over BC is derived by replacing l_1 and dl_1 in the integral:

$$F_{C,z} = -\frac{\mu_0 I^2\cos\beta}{2\pi\sin\alpha}\int_B^C\frac{\sin\gamma_D\sin\gamma_D}{l_{2,0}\sin(\alpha+\gamma_D)}l_{2,0}\left(\frac{\cos(\alpha+\gamma_D)}{\sin\gamma_D} - \frac{\sin(\alpha+\gamma_D)\cos\gamma_D}{\sin^2\gamma_D}\right)d\gamma_D$$

$$= -\frac{\mu_0 I^2\cos\beta}{2\pi\sin\alpha}\int_B^C\frac{\sin\gamma_D\cos(\alpha+\gamma_D) - \sin(\alpha+\gamma_D)\cos\gamma_D}{\sin(\alpha+\gamma_D)}d\gamma_D = -\frac{\mu_0 I^2\cos\beta}{2\pi\sin\alpha}\int_B^C\frac{\sin(-\alpha)}{\sin(\alpha+\gamma_D)}d\gamma_D$$

$$= \frac{\mu_0 I^2\cos\beta}{2\pi}\int_B^C\frac{d(\alpha+\gamma_D)}{\sin(\alpha+\gamma_D)} = \frac{\mu_0 I^2\cos\beta}{2\pi}\left[ln\left|\tan\frac{\alpha+\gamma_D}{2}\right|\right]_B^C$$

$$= \frac{\mu_0 I^2\cos\beta}{2\pi}\left[ln\left|\tan\frac{\alpha+\gamma_{D,C}}{2}\right| - ln\left|\tan\frac{\alpha+\gamma_{D,B}}{2}\right|\right]$$

where $\gamma_{D,B}$ is the angle γ when $d\boldsymbol{I_1}$ is at B and $d\boldsymbol{I_2}$ at D, $\gamma_{D,C}$ when $d\boldsymbol{I_1}$ is at C and $d\boldsymbol{I_2}$ at D.

Then, because $\gamma_{D,B} = \frac{\pi-\alpha}{2}$, $\alpha+\gamma_{D,C} = \pi$ the force on the corner C is:

$$F_{C,z} = \frac{\mu_0 I^2\cos\beta}{2\pi}\left[ln\left|\tan\frac{\pi}{2}\right| - ln\left|\tan\frac{\pi+\alpha}{4}\right|\right]$$

In order to avoid infinity, we integrate till an angle $\frac{\alpha+\gamma_{D,C}}{2} = \frac{\pi}{2} - \varepsilon$ where ε is a small angle that represents the non-sharpness of the corner C, then:

$$F_{C,z} = \frac{\mu_0 I^2 \cos\beta}{2\pi} \left[ln \left| \tan\left(\frac{\pi}{2} - \varepsilon\right) \right| - ln \left| \tan\frac{\pi + \alpha}{4} \right| \right]$$

For ε small, $\cos\varepsilon \cong 1$, $\sin\varepsilon \cong \varepsilon$,

$$ln \left| \tan\left(\frac{\pi}{2} - \varepsilon\right) \right| = ln|\cot\varepsilon| = ln \left| \frac{\cos\varepsilon}{\sin\varepsilon} \right| \cong -ln(\varepsilon)$$

As $\theta = \pi - \alpha - \gamma_{D,C}$, ε defines the upper integration bound of the angle θ:

$$\frac{\alpha + \gamma_{D,C}}{2} = \frac{\pi}{2} - \varepsilon \rightarrow \pi - \alpha - \gamma_{D,C} = \theta_o = 2\varepsilon \rightarrow \varepsilon = \frac{\theta_o}{2}$$

So, this integral before reaching infinity is:

$$F_{C,z} = \frac{\mu_0 I^2 \cos\beta}{2\pi} \left[-ln\left(\frac{\theta_o}{2}\right) - ln \left| \tan\frac{\pi + \alpha}{4} \right| \right] = -\frac{\mu_0 I^2 \cos\beta}{2\pi} ln \left| \frac{\theta_o}{2} \tan\frac{\pi + \alpha}{4} \right|$$

With $= \frac{\pi - \alpha}{2} \Rightarrow \cos\beta = \sin\frac{\alpha}{2}$, We have:

$$F_{C,z} = -\frac{\mu_0 I^2 \sin\frac{\alpha}{2}}{2\pi} ln \left| \frac{\theta_o}{2} \tan\frac{\pi + \alpha}{4} \right|$$

When the integral is complete, $\theta_o=0$, $ln 0 = -\infty$, the Lorentz force between BC and CD is infinity:
$$F_{c,y} = \infty$$