

# Axiomatic Formulation of

## «A Derivation of Faraday's law from Coulomb's Law and Relativity»

Kuan Peng [titang78@gmail.com](mailto:titang78@gmail.com)

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### Abstract

We present an axiomatic formulation of electromagnetic induction based solely on electric charge, Coulomb interaction, and relativistic causality. The theory is built upon a single fundamental object—the **Progressing Electric Field**—defined as the causal propagation of electric influence from the past positions of charges at finite speed. Magnetic fields, magnetic flux, and Faraday's law are not assumed as primitives but emerge as derived, effective quantities. This work establishes the axioms, primitive definitions, and core propositions of the theory, providing a foundation for re-deriving induction phenomena without postulating magnetic fields as fundamental entities.

### The original article:

Kuan Peng, 2026, « [A Derivation of Faraday's law from Coulomb's Law and Relativity / 1.The Progressing Electric Field Model](#) »

[https://www.academia.edu/146009113/A\\_Derivation\\_of\\_Faradays\\_law\\_from\\_Coulombs\\_Law\\_and\\_Relativity\\_1\\_The\\_Progressing\\_Electric\\_Field\\_Model](https://www.academia.edu/146009113/A_Derivation_of_Faradays_law_from_Coulombs_Law_and_Relativity_1_The_Progressing_Electric_Field_Model)

<https://pengkuanem.blogspot.com/2026/01/a-derivation-of-faradays-law-from.html>

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### 1. Scope and Intent

This article does not seek to reinterpret Maxwell's equations within their own conceptual framework. Instead, it proposes an alternative foundational structure for electromagnetic phenomena. The objective is not to negate existing theories but to identify a deeper causal layer from which standard electromagnetic laws arise as effective descriptions.

The present work constitutes the *foundational stage* of the theory. Its purpose is to define axioms, construct the primary field object, and demonstrate its essential properties, notably its non-conservative nature and its capacity to induce electromotive effects in conductors.

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### 2. Primitive Ontology

The theory is built upon the following primitive entities:

- **Electric charge**: a scalar quantity carried by particles.
- **Spacetime**: a relativistic spacetime endowed with a universal invariant speed  $c$ .
- **Progressing Electric Field (PEF)**: a causal distribution of electric influence propagated from charges.

No magnetic field, magnetic flux, vector potential, or electromagnetic energy density is introduced at the axiomatic level.

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### 3. Axioms

#### Axiom A1 — Charge as the Sole Source

Electric charge is the only fundamental source of electromagnetic interaction.

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#### Axiom A2 — Finite Causal Propagation

Any influence produced by a charge propagates through space at a finite invariant speed  $c$ . No instantaneous action at a distance exists.

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#### Axiom A3 — Retarded Local Interaction

At any spacetime point, the electric influence due to a charge depends exclusively on the charge's position at the corresponding retarded time.

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#### Axiom A4 — Local Coulomb Structure

The magnitude and radial orientation of the electric influence emitted from a charge at its retarded position obey Coulomb's law locally with respect to that retarded center.

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#### Axiom A5 — Superposition

Electric influences from distinct charges combine linearly.

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## Axiom A6 — Snapshot Field Representation

At any instant of time, the total electric influence in space may be represented as a spatial field constructed from all retarded contributions corresponding to that instant.

This representation is purely kinematic and does not imply the existence of a global scalar potential.

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## 4. Definition of the Progressing Electric Field

### Definition 1 (Progressing Electric Field).

The Progressing Electric Field  $\mathbf{E}_p(x, t)$  is the spatial field obtained at time  $t$  by superposing all Coulomb-type electric influences emitted by charges from their retarded positions.

The field is structured by a continuous family of expanding iso-intensity surfaces whose centers coincide with the past positions of the source charges.

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## 5. Fundamental Properties of the PEF

### Proposition 1 — Non-Conservativity

The Progressing Electric Field is not, in general, a conservative vector field.

*Justification.* Because the centers of its iso-intensity surfaces vary in space due to charge motion, the circulation of  $\mathbf{E}_p$  around a closed loop does not vanish.

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### Proposition 2 — Vanishing Tangential Contribution on Iso-Intensity Surfaces

On any iso-intensity surface associated with a given retarded emission, the tangential line integral of  $\mathbf{E}_p$  vanishes.

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### Proposition 3 — Radial Asymmetry and Net Circulation

Non-zero circulation of  $\mathbf{E}_p$  arises exclusively from radial asymmetries between neighboring iso-intensity surfaces.

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## 6. Induced Potential in a Conductor

### Definition 2 — Collected Potential Variation

The potential variation collected by a closed conducting loop is defined as the negative circulation of  $\mathbf{E}_p$  along the loop at a fixed temporal snapshot:

$$\Delta U = -\oint \mathbf{E}_p \cdot d\mathbf{l}$$

This quantity is operationally measurable and does not require the existence of a scalar potential field.

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### Proposition 4 — Sector Decomposition

The circulation of  $\mathbf{E}_p$  may be computed by partitioning the surface bounded by the loop into infinitesimal sectors defined by adjacent iso-intensity surfaces.

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## 7. Average Induced Electric Field

### Definition 3 — Average Induced Field

The average induced electric field within a conducting loop of length  $\ell$  is defined by:

$$\bar{\mathbf{E}} = -\Delta U / \ell$$

This field exists solely within the conductor and governs charge transport.

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### Proposition 5 — Directionality (Lenz-Type Behavior)

The induced field  $\bar{\mathbf{E}}$  is oriented so as to oppose the causal progression of the source charges.

This opposition is a geometric consequence of retarded interaction and requires no independent postulate.

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## 8. Curl of the Progressing Electric Field

### Definition 4 — Local Curl

The curl of  $\mathbf{E}_p$  at a point is defined operationally as the limit of circulation per unit area over shrinking sectors.

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## Proposition 6 — Existence of Non-Zero Curl

The Progressing Electric Field possesses a non-zero local curl whenever its source charges exhibit motion.

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## 9. Emergence of Magnetic Quantities

### Definition 5 — Derived Magnetic Field

A vector field  $\mathbf{B}$  is defined implicitly by the local relation:

$$\mathbf{B}_p = -(\mathbf{r}_s / c) \cdot \text{curl}(\mathbf{E}_c)$$

where  $r_s$  denotes the retarded distance between the source charge element and the field point,  $c$  is the invariant propagation speed, and  $\mathbf{E}_c$  is the progressing electric field generated by moving charges.

This definition introduces  $\mathbf{B}$  as a *derived geometric quantity* encoding the local curl structure of the progressing electric field, rather than as a fundamental physical field.

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## Proposition 7 — Stationary-Current Limit

In the steady-motion limit, the derived field  $\mathbf{B}$  satisfies the Biot–Savart relation.

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## 10. Energy and Work (Foundational Statement)

The work performed on charges in a conductor is determined by the induced field  $\tilde{\mathbf{E}}$ . Mechanical work performed by source charges is transmitted through the causal deformation of the Progressing Electric Field.

A full quantitative energy-density formulation is deferred to subsequent work.

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## 11. Status of Classical Electromagnetic Laws

Within this framework:

- Faraday's law appears as an effective integral relation.
- Magnetic flux is a calculational construct derived from  $\mathbf{B}$ .
- Lorentz forces arise from interactions mediated by the curl structure of  $\mathbf{E}_p$ .

None of these quantities are fundamental.

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## 12. Conclusion

We have established an axiomatic foundation for electromagnetic induction based on charge, causality, and Coulomb interaction alone. The Progressing Electric Field emerges as the central physical object, from which induction phenomena follow geometrically. Classical electromagnetic laws arise as secondary descriptions within this deeper causal structure.

This formulation defines a program rather than a completed theory. Its completion requires a rigorous energetic formulation and experimental confrontation, which will be addressed in subsequent work.

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This Axiomatic Formulation benefited from AI assistance in terms of structure and editing.