B-cutting paradox

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In several articles I have shown one inconsistency of the Lorentz force law, that Lorentz force internal to a coil violates the third Newton's law. Below are some links:
Synthesis of the inconsistency of the Lorentz force law
http://pengkuanem.blogspot.com/2012/04/synthesis.html
Mathematical cause of the existence of the remaining resultant internal Lorentz force
http://pengkuanem.blogspot.com/2012/04/mathematical-cause-of-existence-of.html
Proof of the remaining resultant Lorentz force internal to a triangular coil
http://pengkuanem.blogspot.com/2012/04/analyze-of-lorentz-forces-internal-to.html

The Lorentz force law is also used to interpret the generation of an electromotive force in a coil with a wire moving in a magnetic field. Let us look at the Figure 1 in which a rectangular coil with a movable wire is shown. The movable wire is the bar conductor of length *l* that is constrained to move at the velocity *v*. An electrical generator provides a current *I* in the coil that creates a magnetic field *B*. As the bar conductor moves, it cuts the force lines of the magnetic field *B*. This is the “B-cutting” action referred in the title.

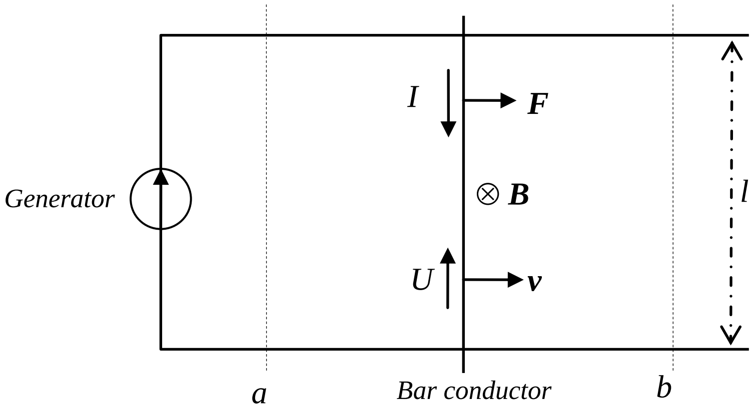

Figure 1

The bar carries with it the free electrons of the conductor which also move at the velocity *v*. According to the Lorentz force law, the free electrons feel a Lorentz force proportional to *B* and *v*. This force integrated along the length of the bar constitutes an electromotive force that is :

$$U = v\int_l Bdl \qquad (1)$$

In order to maintain the current *I* in the circuit, the generator must do an electrical work against the electromotive force *U*. The differential electrical work during a differential time *dt* is:

$$dW = I\,U\,dt \qquad (2)$$

The total electrical work the generator does during the movement of the bar from the position *a* to the position *b* is the integral of the above *dW*:

$$W_{emf} = \int_a^b dW = \int_a^b I\left(\int_l Bdl\right)vdt = \int_a^b I\left(\int_l Bdl\right)dx \quad (3)$$

On the other hand, the current-carrying bar experiences an other Lorentz force F in the magnetic field B (see the Figure 1). This force is :

$$F = I\int_l Bdl \quad (4)$$

As the bar moves, this Lorentz force does a mechanical work against the constraint force. The differential mechanical work is the force multiplies the differential distance dx:

$$dW=F\,dx \quad (5)$$

When the bar moves from the position a to the position b, the total mechanical work is the integral of the above dW:

$$W_{mec} = \int_a^b dW = \int_a^b I\left(\int_l Bdl\right)dx \quad (6)$$

By comparing the expression of W_{emf} and W_{mec}, we see that the mechanical work is equal to the electrical work:

$$W_{emf} = W_{mec} \quad (7)$$

This equality means: the current I in the magnetic field feels the Lorentz force F which is equal to the external constraint force; the bar moves a distance dx, the mass of the bar being neglected, the Lorentz force does the mechanical work Fdx to the external constraint force; this work is lost by the coil; moving at the velocity v, the bar generates the electromotive force U; during the differential time dt, the generator does the electrical work $IUdt$ to the coil; this work is received by the coil; as $IUdt=Fdx$, the energies the coil receives and loses cancel out and no extra energy is left in the coil. This equality holds even I, B and v are variable.

This equality seems to respect perfectly the energy conservation law. But it is not. In fact, the magnetic energy stored in the coil was not taken into account. The magnetic energy is proportional to the magnetic flux through the surface enclosed by the coil. As the bar have moved from the position a to b, this surface becomes greater and so does the magnetic flux and the stored magnetic energy. The magnetic energy stored in the coil is

$$\begin{aligned} &\text{Position } a: \ E_a = \frac{1}{2}L_a I^2 \\ &\text{Position } b: \ E_b = \frac{1}{2}L_b I^2 \end{aligned} \quad (8)$$

L_a and L_b are the self inductances of the coil for position a and b (ref. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, «Introduction to Electromagnetic Fields», Mcgraw-Hill College; 3 Sub edition, p.261). Because the surface for the position b is greater than for the position a, L_b is greater than L_a, and the magnetic energy stored in the coil is greater for the position b than a:

$$L_b > L_a \Rightarrow E_b > E_a \quad (9)$$

This implies that the coil has gained the following extra energy during the process:

$$\Delta E = E_b - E_a > 0 \tag{10}$$

But where does this energy come from? If this were true, we could make a perpetual machine functioning in the following way:

1) The bar at the position *a*, I=0, E_a=0.
2) The current increases to I, the generator does the electrical work $\int_0^t L_a \frac{dI}{dt} I dt$.
3) The magnetic energy stored in the coil is now $E_a = \frac{1}{2} L_a I^2$.
4) The bar slides to the position *b*. According to the Lorentz force law, the electrical generator does the electrical work W_{emf}, the bar does the mechanical work W_{mec}. W_{emf} and W_{mec} canceling out each other and the coil has gained no energy.
5) But, the stored magnetic energy is now $E_b = \frac{1}{2} L_b I^2$.
6) The current decreases to 0, the coil gives the stored magnetic energy back to the generator. In this cycle, the generator has gained the energy $\Delta E = E_b - E_a$.
7) The bar goes back to the position *a* without doing any work because $I=0$.
8) Restart from the step 2).

After *n* cycles, the quantity of energy that the generator would gain is $n\Delta E$, which is provided by no energy source. This is impossible because energy cannot be created. Thus, the principle of this process is incorrect, although it follows rigorously the classical theory. I call this inconsistency the "B-cutting paradox".

In this paradox, the Lorentz force law intervenes 2 times, first for the electromotive force generation, then for the mechanical work. So, this paradox shows another inconsistency of the Lorentz force law completely independent of the previous internal force inconsistency. In consequence, the inaccuracy of the Lorentz force law makes serious problems and needs to be corrected.