Summary at blogspot, at Academia

Faraday's torque experiment

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Figure 1 shows a charged insulating disc under induction. Will there be a torque on its center? Perhaps one could use Faraday's law to compute the torque. But is Faraday's law valid for this application?

Nevertheless, let us compute with Faraday's law. The point form of Faraday's law is:

$$curl\mathbf{E} = -\frac{d\mathbf{B}}{dt} \tag{1}$$

In Figure 1, a charged insulating disc is placed in uniform magnetic field **B**, which is in the z direction only, $\frac{d\mathbf{B}}{dt} = \frac{dB_z}{dt}\mathbf{e}_z$. $\frac{dB_z}{dt}$ is constant in space. Then, the r and z components of the induced electric field are zero.

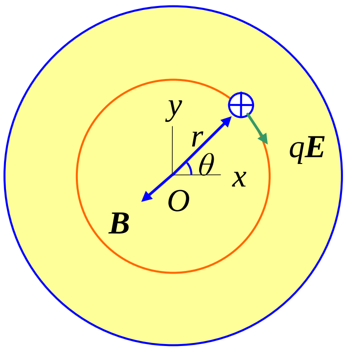

Figure 1

Let us compute the θ component of the induce electric field:

$$curl\mathbf{E} = \frac{1}{r}\frac{\partial rE_\theta}{\partial r}\mathbf{e}_z, E_r = E_z = 0 \Rightarrow \frac{1}{r}\frac{\partial rE_\theta}{\partial r} = -\frac{dB_z}{dt} \tag{2}$$

We obtain (see annex 1 for detail):

$$E_\theta = -\frac{dB_z}{dt}\frac{r}{2} \tag{3}$$

The charge density of the disc is ρ. The force on a surface element $d^2s = rd\theta dr$ is:

$$d^2F_\theta = E_\theta \rho r d\theta dr \tag{4}$$

The torque of this force about the center is:

$$d^2\tau = rd^2F_\theta = -\rho\frac{dB_z}{dt}\frac{r^3}{2}d\theta dr \tag{5}$$

The total torque on a disc of radius *r* is obtained by integration (see annex 2 for detail):

$$\tau = -\pi\rho\frac{dB_z}{dt}\frac{r^4}{4} \tag{6}$$

In Figure 2, we have 2 discs; the radius of the first is R_1, that of the second is R_2=2 R_1. What is the torque on the second disc? According to equation (6), it is the following τ_2:

$$\tau_2 = -\pi\rho \frac{dB_z}{dt} \frac{(2R_1)^4}{4} \tag{7}$$

So, doubling the radius of a disc will increase the torque 16 times:

$$\tau_2 = -16\pi\rho \frac{dB_z}{dt} \frac{R_1^{\ 4}}{4} \Rightarrow \tau(2 * R_1) = 16 * \tau(R_1) \tag{8}$$

This is the result according to Faraday's law. Another way to compute the torque on the second disc is to use the surface ratio of the discs. We cut the first disc into n parts; a part is the red triangle in Figure 2. The torque on this part is $\tau(R_1)/n$. Then, we cut the second disc also into n parts; each part is equivalent to 4 red triangles, as shown by the 4 colors triangle. What is the torque on this 4 times larger part?

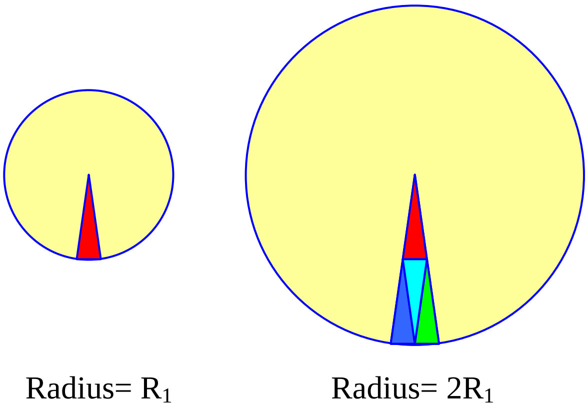

Figure 2

B is uniform in space, and a triangle would feel the same torque everywhere in space. If the origin for the torque is at another place than the tip, the torque is computed as follow. In Figure 3, the grey body creates a torque τ_0 about the point O:

$$\boldsymbol{\tau}_0 = \oiiint \mathbf{r} \times d\mathbf{F} \tag{9}$$

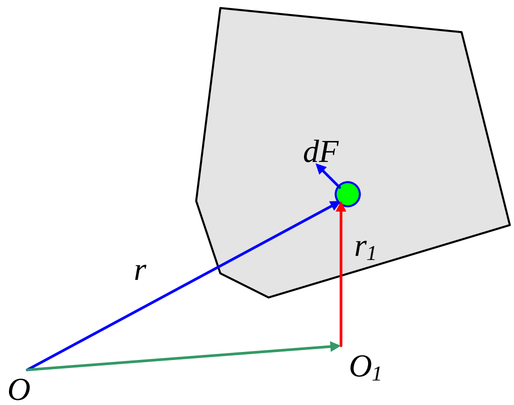

Figure 3

Integration of the above equation with respect to the point O_1 gives the torque of the grey body about O_1 (see annex 3 for detail):

$$\boldsymbol{\tau}_1 = \boldsymbol{\tau}_0 + \overline{OO_1} \times \mathbf{F}_1 \tag{10}$$

Where $\mathbf{F_1}$ is the resultant force on the grey body. The induced electric field does not give rise to any resultant force on charged body, so, $\mathbf{F_1}=0$. Then, the torque on a triangle has the same value about any point in space:

$$\boldsymbol{\tau}_1 = \boldsymbol{\tau}_0 \tag{11}$$

As in the larger disc there are 4 times red triangles than in the small one, its total torque about the center is 4 times that on the smaller one. Thus we have:

$$\boldsymbol{\tau}(2 * R_1) = 4 * \boldsymbol{\tau}(R_1) \tag{12}$$

This is contrary to equation (8), which is obtained by using Faraday's law. Why there are 2 values for the same quantity when we compute with 2 different laws as valid? Which one is good? The final judge is experiment.

Faraday's torque experiment

The experiment consists of measuring the torque on a charged insulating disc placed at the center of a solenoid. When the current in it varies, the charge on the disc should feel an electric field and according to Faraday's law, this torque is proportional to R_d^4, with R_d being the radius of the disc (see equation (8)). But this result is contrary to equation (12). Only experiment can determine which one is right.

There are 2 options for this experiment. The first is to induce a charged insulating disc and compare the data with theoretical result. The second is to induce 2 charged insulating discs of different radius and compare the torques on the 2 discs.

There are 3possible outcomes:

1. Confirmation of the result of Faraday's law, this is, equation (8) is right. In this case, one has to explain why we have two theoretical results: 4 times or 16 times when the radius is doubled.
2. Zero torque. In this case, the fact that the torque increases 4 times or 16 times when the radius is doubled is well explained, because 4*0=16*0. But Faraday's law would be shown not to apply to charged insulating surface.
3. Measured torque is neither zero nor Faraday's torque. Equation (12) might be right. In this case, a new law must be found.

Comments

This contradiction does not invalidate Faraday's law, since EMF is measured at the terminal of an induced coil. What is put into question is the mathematical formulation of induction phenomenon in terms of the equation:

$$curl\mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

In reality, what was measured is only the potential difference at the terminals of a induce coil, not electric field in space. Induced electric field **E** is only a theoretical hypothesis that has never been proven experimentally. I have proposed an experiment to measure actually **E** (see **Partial EMF measurement** (Links blogspot academia)

The present contradiction on induced electric field in space is partially due to that appropriate experiment has never been done to confirm it. The lack of thorough experimental study have led to other important contradiction in electromagnetism, such as perpendicular action in Lorentz force law, B-cutting paradox, deformation of transported signals by EM wave equation or non reaction of displacement magnetism. These inconsistencies are explained in the following articles:

Lorentz force on open circuit (Links blogspot academia)
B-cutting paradox (Links blogspot academia)
Deformation of EM wave signals (Links blogspot academia)
Phantom Lorentz force Paradox (Links blogspot academia)

If you have already read these articles and understood them, you could agree that they are rigorously deduced and are real inconsistencies. Electromagnetism was established in the 19th century with limited physical and mathematical knowledge and primitive experimental means. It is natural that it contains errors.

Each time science makes a great step forward, the scientists who promote the new idea and carry out new experiments become themselves eminent ones in the aftermath world. If you are willing to participate in the overhaul of electromagnetism today, take part in the work of promotion and experiment, and you will be the prominent scientists of the 21st century.

Annex

1. Faraday's field

$$\frac{1}{r}\frac{\partial rE_\theta}{\partial r} = -\frac{dB_z}{dt} \Rightarrow \frac{\partial rE_\theta}{\partial r} = -r\frac{dB_z}{dt} \Rightarrow rE_\theta = -\frac{dB_z}{dt}\int_{r=0}^{r} rdr + c$$

$$r = 0, E_\theta = 0 \Rightarrow c = 0, rE_\theta = -\frac{dB_z}{dt}\frac{r^2}{2}, E_\theta = -\frac{dB_z}{dt}\frac{r}{2}$$

2. Faraday's torque

$$d^2\tau = rd^2F_\theta = rd^2qE_\theta, d^2s = rd\theta dr, d^2q = \rho rd\theta dr$$

$$d^2\tau = E_\theta \rho r^2 d\theta dr = -\rho\frac{dB_z}{dt}\frac{r^3}{2}d\theta dr$$

$$d\tau = -2\pi\rho\frac{dB_z}{dt}\frac{r^3}{2}dr = -\pi\rho\frac{dB_z}{dt}r^3 dr, \tau = -\pi\rho\frac{dB_z}{dt}\int_{r=0}^{r=R} r^3 dr + c$$

$$r = 0, \tau = 0 \Rightarrow c = 0, \tau = -\pi\rho\frac{dB_z}{dt}\frac{R^4}{4}$$

3. Axis change for Torque

$$\boldsymbol{\tau}_0 = \oiiint \mathbf{r}\times d\mathbf{F}$$

$$\boldsymbol{\tau}_1 = \oiiint \mathbf{r}_1 \times d\mathbf{F} = \oiiint \left(\mathbf{r} - \overline{OO_1}\right)\times d\mathbf{F} = \oiiint \mathbf{r}\times d\mathbf{f} - \oiiint \overline{OO_1}\times d\mathbf{F}$$

$$\mathbf{F}_1 = \oiiint d\mathbf{F} \Rightarrow \boldsymbol{\tau}_1 = \boldsymbol{\tau}_0 + \overline{OO_1}\times \mathbf{F}_1$$