Peng Kuan 彭宽 titang78@gmail.com https://twitter.com/pengkuanem

Displacement Current Paradox

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I have exposed 3 inconsistencies of the Lorentz force law in several articles,
Synthesis of the inconsistency of the Lorentz force law,
http://pengkuanem.blogspot.com/2012/04/synthesis.html
B-cutting paradox, http://pengkuanem.blogspot.fr/2012/05/b-cutting.html
Lorentz' EMF paradox, http://pengkuanem.blogspot.fr/2012/05/lorentz-emf.html

These inconsistencies are about Lorentz force, but they can also be about magnetic field. So, let us examine the Maxwell–Ampere equation:

$$Curl\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad (1)$$

The term $\frac{\partial \boldsymbol{D}}{\partial t}$ is displacement current that creates magnetic field in free space. For example, on a conductor sphere been charged by an alternate current (see the Figure 1), the electric charge varies, the electric displacement field $\boldsymbol{D}$ varies and the variation of $\boldsymbol{D}$ creates a magnetic field around the sphere. Let us calculate this magnetic field.

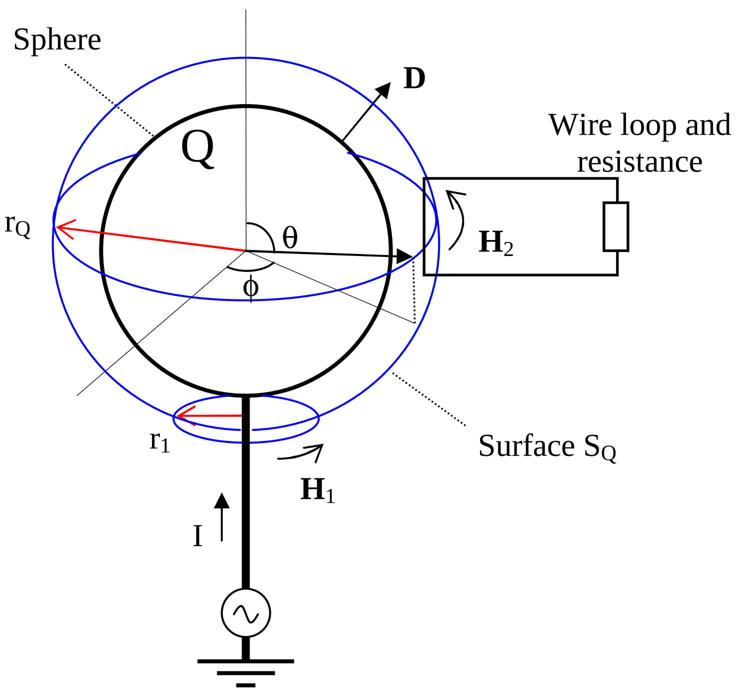

Figure 1

The vector field $\boldsymbol{D}$ in the Figure 1 crosses a closed surface, S_Q. The vector $\boldsymbol{D}$ is radial, $\boldsymbol{D} = D_Q \boldsymbol{e}_r$, and the expression of the flux through S_Q is:

$$\iint_{S_Q} \boldsymbol{D}d\boldsymbol{s} = D_Q S_Q$$

This flux is equal to the charge of the sphere Q:

$$\iint_{S_Q} \boldsymbol{D} d\boldsymbol{s} = Q$$

Then the magnitude of ***D*** is:

$$D_Q = \frac{Q}{S_Q}$$

The surface S_Q is in free space where no current exists, that is $\boldsymbol{J}$=0. According to the Maxwell–Ampere equation, the curl of magnetic field on S_Q is due to displacement current only (see equation (1)):

$$curl\boldsymbol{H} = \frac{\partial \mathbf{D}}{\partial t} = \frac{I}{S_Q}\boldsymbol{e}_r \qquad (2)$$

In spherical coordinates, the expression of this $curl\boldsymbol{H}$ is (with r_Q the radius of the surface S_Q):

$$curl\boldsymbol{H} = \frac{1}{r_Q \sin\theta}\left(\frac{\partial\left(H_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right)\boldsymbol{e}_r + 0\boldsymbol{e}_\theta + 0\boldsymbol{e}_\phi \qquad (3)$$

Because of spherical symmetry, the θ component of ***H*** is constant along ϕ:

$$\frac{\partial H_\theta}{\partial\phi} = 0$$

So, the equation (2) and (3) give:

$$\frac{1}{r_Q \sin\theta}\frac{\partial H_\phi \sin\theta}{\partial\theta} = \frac{I}{S_Q}$$

The integration of this equation gives:

$$\frac{\partial H_\phi \sin\theta}{\partial\theta} = \frac{I r_Q}{S_Q}\sin\theta$$

$$H_\phi \sin\theta = -\frac{I r_Q}{S_Q}\cos\theta + c$$

For determining the integration constant c, we notice that for θ=θ_1, where r_1 is the radius of the wire, the value of H_ϕ *is* $H_1 = \frac{I}{2\pi r_1}$. And then:

$$c = \frac{I}{2\pi r_1}\sin\theta_1 + \frac{I r_Q}{S_Q}\cos\theta_1$$

Then the ϕ component of magnetic field vector at θ is:

$$H_\phi = I\left(\frac{r_Q}{S_Q}\frac{\cos\theta_1 - \cos\theta}{\sin\theta} + \frac{1}{2\pi r_1}\frac{\sin\theta_1}{\sin\theta}\right)$$

As an example, for $\theta_1 \approx 0$, $\theta = 90°$, we have:

$$\cos\theta_1 = 1, \sin\theta_1 = 0, \cos\theta = 0, \sin\theta = 1$$

$$H_\phi = I\left(\frac{r_Q}{S_Q}\frac{1-0}{1} + \frac{1}{2\pi r_1}\frac{0}{1}\right) = \frac{I\,r_Q}{S_Q}$$

We see that the magnitude of magnetic field is proportional to the current charging the sphere. For alternate current the charge of the sphere is:

$$Q = -Q_0\cos\omega t \qquad (4)$$

The charging current is:

$$I = \frac{dQ}{dt} = \omega Q_0 \sin\omega t \Rightarrow I = I_0 \sin\omega t, I_0 = Q_0\omega \qquad (5)$$

If there is a wire loop near the sphere, the varying magnetic flux through the loop will generate EMF. Let *L* be the mutual inductance between the sphere and the loop, the induced tension in the loop is (see above for expression of *I*):

$$U = L\frac{dI}{dt}, \frac{dI}{dt} = I_0\omega\cos\omega t \Rightarrow U = LI_0\omega\cos\omega t$$

The loop shown in the Figure 1 has a resistance *R* connected. The induced tension will create a current in the resistance and provide an electric power *P*:

$$P = \frac{U^2}{R} = \frac{(LI_0\omega\cos\omega t)^2}{R} = \frac{(LI_0\omega)^2}{R}\cos^2\omega t$$

The work *W* during one period *T* is the time integral of *P* from 0 to *T*:

$$W = \int_0^T \frac{(LI_0\omega)^2}{R}\cos^2\omega t dt = \frac{\pi\omega(LI_0)^2}{R}$$

This work comes from the magnetic field of the sphere whose alternating potential is:

$$U_s = \frac{Q}{4\pi\varepsilon_0 r_s}$$

The power of the charging current is (see equation (4) and (5)):

$$P = U_s I = -\frac{Q_0\cos\omega t}{4\pi\varepsilon_0 r_s}Q_0\omega\sin\omega t = -\frac{Q_0{}^2\omega}{4\pi\varepsilon_0 r_s}\sin\omega t\cos\omega t$$

The work during one period is:

$$W = \int_0^T P dt = -\frac{Q_0{}^2\omega}{4\pi\varepsilon_0 r_s}\int_0^T \sin\omega t\cos\omega t dt = 0$$

So, the work of the sphere's charging current is 0 during one period. If the sphere does not consume energy, the positive electric work in the loop does not come from a source. Could it be created out of nothing? We notice then the inconsistency: the displacement current of the sphere creates magnetic field, but violates the energy conservation law. I call this inconsistency the "Displacement Current Paradox".

There is a simple explanation to this inconsistency. It is that classic electromagnetism does not provide a law that defines the retroaction of the loop's current on the sphere. Unlike 2 interacting current loops, the induced loop acts on the inducing loop a back EMF according to Faraday's law, the sphere and the charging wire do not constitute a closed loop and, no back EMF exists. The lack of retroaction work destroys the energy balance.

Some readers may ask me: "How did you find all these paradoxes?" My answer is that I have found the correct laws for magnetic force and magnetic field which are different than the classic ones. The correct law for magnetic force that I described in the article Correct differential magnetic force law, http://pengkuanem.blogspot.com/2012/04/correct-law.html has successfully solved the paradoxes about Lorentz force and B-cutting EMF. By contrast with these correct laws, the Lorentz force law and Maxwell–Ampere equation show their inconsistencies up.

One may also ask: "Why are you publishing these paradoxes on the internet?" The reason is that I have proposed my work on the correct law of magnetic force to several physical journals, but they all rejected it because the Maxwell's system is such a basic theory that no objection could get consideration.

In this circumstance, I'm forced to publish it by my self on the internet and I hope that among the readers here some people would carry out the experiments that I propose and show experimental evidences of the inconsistencies and then make the physical community to realize the crisis coming up. Also, if any people could help me in getting my work published, I will be very grateful to them.