Electromagnetic wave energy flux

Peng Kuan 彭宽 titang78@gmail.com

23 December 2012

EM wave transports energy in space, which is contained in electromagnetic field. The intensity and velocity of wave are computed with the wave equation, which will be used to derive the energy flux of wave. Energy flux must respect the energy conservation law. However, this is not the case for our EM wave equation as shown in the following.

The flux of energy is the time rate of energy that crosses a surface carried by a wave. We consider first a volume that crosses a surface in space carried by a wave and then, the energy it contains. In Figure 1, a wave travels from the left to the right at the velocity $dv=dr/dt$. The volume dV, which is occupied by the wave at the position r, crosses the surface during a time dt. Let S be the surface, the transported volume is:

$$dV = S \cdot dr = S \cdot vdt \tag{1}$$

In this volume, magnetic energy is stored in the magnetic field H and is denoted e_H. Its density per unit volume is [1]:

$$\frac{de_H}{dV} = \frac{1}{2}\mu_0 H^2 \tag{2}$$

Transported by the traveling wave, the volume $dV=S{\cdot}vdt$ crosses the surface S. The energy contained in it, which is the density times the volume, passes through the surface:

$$de_H = \frac{1}{2}\mu_0 H^2 \cdot S \cdot vdt \tag{3}$$

This energy passes during the time dt, and the time rate of the energy's flow is the flux:

$$\frac{de_H}{dt} = \frac{1}{2}\mu_0 H^2 \cdot S \cdot v \tag{4}$$

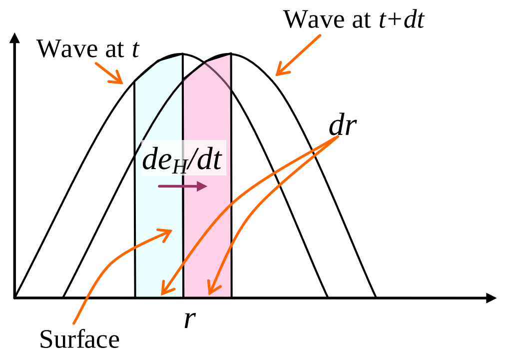

Figure 1

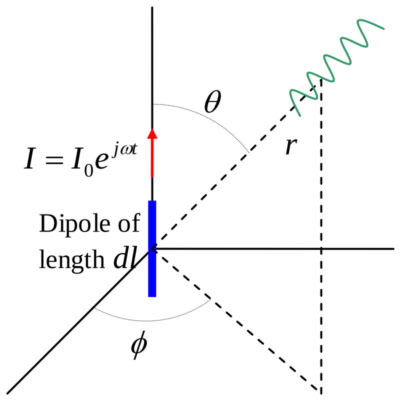

Figure 2

A wave emitted by an electric dipole that carries a sinusoidal current (see Figure 2) is a varying electromagnetic field. Its magnetic part is the solution of the EM wave equation [2]:

$$H_\phi = \frac{I_0 dl}{4\pi}\beta_0^2 \sin\theta\left(j\frac{1}{\beta_0 r} + \frac{1}{{\beta_0}^2 r^2}\right)e^{j(\omega t - \beta_0 r)} \tag{5}$$

In the article **Electromagnetic Wave Paradox** (Links: blogspot academia), the expression with real number amplitude for this wave solution is derived:

$$H_\phi = \frac{I_0 dl}{4\pi r^2}\sqrt{1+\beta_0{}^2 r^2}\sin\theta\cos(\omega t - \beta_0 r + \arctan(\beta_0 r)),\ \beta_0 = \frac{\omega}{c} \tag{6}$$

And the velocity of this wave solution is:

$$v = \frac{\omega}{\beta_0}\left(\frac{1}{\beta_0{}^2 r^2} + 1\right) \tag{7}$$

The magnetic energy flux of this wave solution is computed with the energy density (equation (6)), the wave's velocity (equation (7)) , the expression for energy flux (equation (4)) and the cross section of a cone centered at the dipole:

$$S = r^2 \sin\theta d\theta d\phi \tag{8}$$

We derive the flux of magnetic energy that flows in the cone as below:

$$\frac{de_H}{dt} = \frac{1}{2}\mu_0\left(\frac{I_0 dl}{4\pi r^2}\right)^2 \frac{\omega}{\beta_0{}^3}\left(1+\beta_0{}^2 r^2\right)^2 \cos^2(\omega t - \beta_0 r + \arctan(\beta_0 r))\sin^3\theta d\theta d\phi \tag{9}$$

Dividing by the solid angle $d\theta \times d\phi$, we obtain the flux of energy per unit solid angle. Using β_0 of equation (6) we express this flux as a function of r, ω and c:

$$\frac{d^2}{d\theta d\phi}\left(\frac{de_H}{dt}\right) = \frac{1}{2}\mu_0\left(\frac{I_0 dl}{4\pi r^2}\right)^2 \frac{c^3}{\omega^2}\left(1+\frac{\omega^2}{c^2}r^2\right)^2 \cos^2(\omega t - \beta_0 r + \arctan(\beta_0 r))\sin^3\theta \tag{10}$$

We note that this flux is variable with respect to the distance from the dipole r. Its property near the dipole is obtained by approaching r to zero. When $r \to 0, \arctan(\beta_0 r) - \beta_0 r \to 0$, and the flux of energy becomes:

$$\frac{d^2}{d\theta d\phi}\left(\frac{de_H}{dt}\right) = \frac{1}{2}\frac{\mu_0 c^3}{\omega^2}\left(\frac{I_0 dl}{4\pi r^2}\right)^2 \sin^3\theta\cos^2(\omega t) \tag{11}$$

This expression is proportional to $1/r^4$ and increases to infinity when r tends to 0. In order to get an idea of this flux, let us compute the numerical value of the energy emitted by a current of amplitude $I_0 = 1\ A$, of pulsation $\omega = 2\pi \cdot 1000/s$ from a dipole of length 0.01 m. At the distance $r = 0.1\ m$, in the plan perpendicular to the dipole, $\theta = \frac{\pi}{2}$, and knowing the values $c = 3\cdot 10^8 \text{m/s}, \mu_0 = 4\pi 10^{-7}$, the energy flux within a solid angle $\Delta\theta = 2\pi 10^{-3}, \Delta\phi = 2\pi 10^{-3}$ is:

$$\Delta^2 \frac{de_H}{dt} = \cos^2(\omega t)\cdot 100KW$$

This is such a huge value that would roast any bird flying by into fried chicken.

At long distance, $r \to \infty$, the amplitude of equation (10) becomes constant:

$$\frac{d^2}{d\theta d\phi}\left(\frac{de_H}{dt}\right) = \frac{1}{2}\frac{\mu_0}{c}\left(\frac{I_0 dl}{4\pi}\right)^2 \cos^2\left(\omega t - \beta_0 r + \frac{\pi}{2}\right)\sin^3\theta,\ \arctan(\beta_0 r) \to \frac{\pi}{2} \tag{12}$$

One can argue that the electric field energy of the EM wave is not taken into account. I have derived the expression for this energy flux in the annex, which is even stranger. The electric field energy of the θ component near the dipole is (see equation (18)):

$$\frac{d^2}{d\theta d\phi}\left(\frac{de_E}{dt}\right)=-\frac{1}{4}\frac{c^5\mu_0}{\omega^4 r^2}\left(\frac{I_0 dl}{4\pi r^2}\right)^2\sin^2(\omega t)\sin\theta \tag{13}$$

In the plan perpendicular to the dipole, $\theta=\pi/2$, the r component of the electric field is zero. And the flux of the total energy e_H+e_E is the sum of equations (11) and (13):

$$\frac{d^2}{d\theta d\phi}\frac{d(e_H+e_E)}{dt}=\frac{1}{2}\frac{\mu_0 c^3}{\omega^2}\left(\frac{I_0 dl}{4\pi r^2}\right)^2\cos^2(\omega t)-\frac{1}{4}\frac{c^5\mu_0}{\omega^4 r^2}\left(\frac{I_0 dl}{4\pi r^2}\right)^2\sin^2(\omega t) \tag{14}$$

This expression is variable with respect to r and implies that the total energy that enters the cone at the distance r and that leaves it at the distance $r+\Delta r$ do not have the same value. So, the energy conservation law is violated.

Comment

Unlike Poynting's energy flux theory, the present study uses the abnormal energy flux to show the inconsistency of the wave equation. The fact that the energy flux is proportional to $1/r^4$ near the antenna is due to that the amplitude of magnetic field and the velocity of wave both are proportional to $1/r^2$. These are contrary to physical principles. So, the EM wave equation does not describe real EM wave.

The wave equation is derived from the Ampere-Maxwell and Faraday-Maxwell equations. Beside the wave equation, I have also given several direct proofs of the inconsistency of these Maxwell equations [3]. No one has proposed remedy for these inconsistencies. It is necessary to correct these laws, as I have done by proposing **Correct differential magnetic force law** (Links: blogspot academia). But, no new theory can be implemented unless the physical community accepts that the old one is wrong. The mainstream physicists oppose hardest resistance to consider the classical EM theory to be wrong, arguing that it has stood for 150 years and was repeatedly proven right.

So, with the aid of the new theory I have designed new experimental conditions that have never been used before, and the multiple experiments I propose will test the exactness of the Lorentz force law, Faraday's law and the Ampere-Maxwell equation in these conditions [3]. I have even launched a competition for the **Lorentz perpendicular action experiment** (Links: blogspot academia), which makes the whole thing more exciting. And I solicit all experimenters all over the world to participate by doing these experiments.

To those who think that their participation is useless because others are doing the experiments, I would say that every experiment is truly useful. In fact, one unique experimental proof is not sufficient to change the mind of physicists; they could always contend that it is wrongly performed or just ignore it as to the homopolar motor experiment. Only after that multiple experiments in different laboratories have given the same result in the same conditions that the mainstream thinking could be changed.

The new theory is a fundamental change in physical thinking, in the same way as Quantum mechanics or Relativity. This is why it is so hard to overcome the resistance of old belief. But, it is also an extraordinary chance that this once in a century revolution of idea occurs now, that its experiments are so simple, and everybody can participate. If you do, you will be able to proudly say to your children: "Hey, I did it. I participated in this revolution".

So, by doing the experiments, you make history. Do not miss it.

Annex

The θ component of electric field of the wave and its speed (see **Can EM wave go forward back?** Links: blogspot academia):

$$E_\theta = \frac{I_0 dl}{4\pi r^2}\eta_0\sqrt{1+\left(\beta_0 r-\frac{1}{\beta_0 r}\right)^2}\cos\left(\omega t-\beta_0 r+\arctan\left(\beta_0 r-\frac{1}{\beta_0 r}\right)\right), \eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{15}$$

$$v=\frac{\omega}{\beta_0}\frac{1+\left(\beta_0 r-\frac{1}{\beta_0 r}\right)^2}{(\beta_0 r)^2-2} \tag{16}$$

Electric field energy flux of the θ component:

$$de_E=\frac{1}{2}\varepsilon_0 E_\theta{}^2 r^2\sin\theta d\theta d\phi\cdot vdt \tag{17}$$

$$\frac{d^2}{d\theta d\phi}\left(\frac{de_E}{dt}\right)=\frac{1}{2}\frac{c\mu_0}{r^2}\left(\frac{I_0 dl}{4\pi}\right)^2\frac{\left(1+\left(\beta_0 r-\frac{1}{\beta_0 r}\right)^2\right)^2}{(\beta_0 r)^2-2}\cos^2\left(\omega t-\beta_0 r+\arctan\left(\beta_0 r-\frac{1}{\beta_0 r}\right)\right)\sin\theta \tag{18}$$

References

1. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.258

2. Clayton R. Paul, Keith W. Whites and Syed A. Nasar, Introduction to Electromagnetic Fields, Mcgraw-Hill College; 3 Sub edition (December 9, 1997), p.590

3. **Summary,** links: blogspot, Academia