Peng Kuan 彭宽 titang78@gmail.com https://twitter.com/pengkuanem

Phantom Lorentz force Paradox

9 September 2012

I have exposed an inconsistency of Ampere-Maxwell equation, in the article "Displacement Current Paradox" http://pengkuanem.blogspot.com/2012/07/displacement-current-paradox.html
This equation states that displacement current creates magnetic field and EMF. However, "Displacement Current Paradox" shows that this EMF would violate the energy conservation law.

Magnetic field has 2 properties, EMF and Lorentz force. Let us study the Lorentz force created by magnetic field associated to displacement current. The Figure 1 shows a round plate capacitor charged by an alternate current I_c, and a wire loop in which circulates a constant I_l. The varying charge of the capacitor creates a displacement current and then a magnetic field, which in turn, exerts a Lorentz force on the current loop.

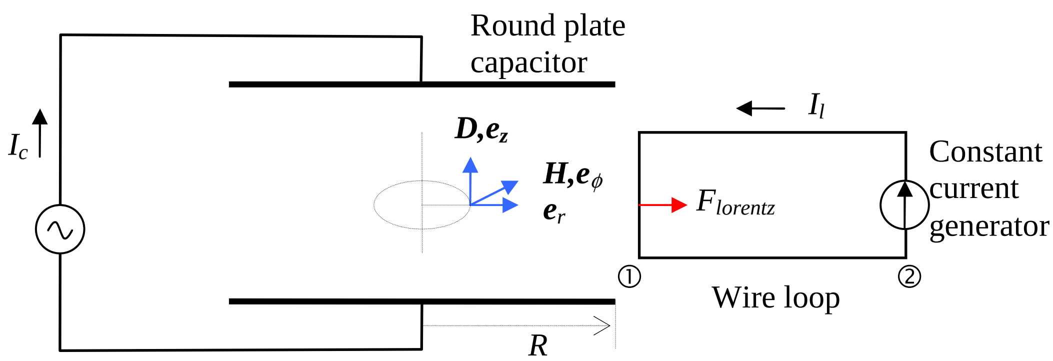

Figure 1

The magnetic field is computed using Ampere-Maxwell equation:

$$curl\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \quad (1)$$

The radius of the capacitor is R, its charge is Q. Between the plates, the electric displacement field $\boldsymbol{D}$ and the displacement current $\frac{\partial \boldsymbol{D}}{\partial t}$ are:

$$\boldsymbol{D} = \frac{Q}{\pi R^2}\boldsymbol{e}_z, \quad \frac{\partial \boldsymbol{D}}{\partial t} = \frac{I_c}{\pi R^2}\boldsymbol{e}_z, \quad I_c = \frac{\partial Q}{\partial t}$$

In cylindrical coordinate, the curl of the magnetic field $\boldsymbol{H}$ is:

$$curl\boldsymbol{H} = 0\boldsymbol{e}_r + 0\boldsymbol{e}_\phi + \frac{1}{r}\left(\frac{\partial (rH_\phi)}{\partial r} - \frac{\partial H_r}{\partial \phi}\right)\boldsymbol{e}_z \quad (2)$$

Because of cylindrical symmetry we have: $\frac{\partial H_r}{\partial \phi} = 0$

Then, the equation (2) becomes: $\frac{1}{r}\frac{\partial (rH_\phi)}{\partial r} = \frac{I_c}{\pi R^2}$

Integration of the above equation gives:

$$rH_\phi = \int \frac{I}{\pi R^2} r dr = \frac{I_c}{\pi R^2}\frac{r^2}{2} + c_0$$

For determining the integration constant c_0, we use the fact that:

$$r = 0, H_\phi = 0$$

So, c_0 is 0, and finally the magnitude field within the capacitor is:

$$H_\phi = \frac{I_c r}{2\pi R^2} \tag{3}$$

Outside the capacitor, displacement current is 0 and Ampere-Maxwell equation gives (see equation (2)):

$$\frac{1}{r}\left(\frac{\partial(rH_\phi)}{\partial r} - \frac{\partial H_r}{\partial \phi}\right)\boldsymbol{e}_z = 0$$

$$\frac{\partial H_r}{\partial \phi} = 0 \Rightarrow \frac{\partial(rH_\phi)}{\partial r} = 0$$

Integration of this equation gives: $rH_\phi = c_1$

For determining the integration constant c_1, we use the magnetic field on the edge of the capacitor $H_\phi(r=R)$. From above we have (see equation (3)):

$$r = R \Rightarrow R \cdot H_\phi(R) = \frac{I_c}{2\pi} \Rightarrow c_1 = \frac{I_c}{2\pi}$$

So, outside the capacitor the magnitude of magnetic field is:

$$H_\phi = \frac{I_c}{2\pi r}$$

This magnetic field connects the energy of the loop to that of the capacitor. With C the capacity and U_c the tension, the power of the charging current is:

$$P = U_c I_c = \frac{Q}{C}\frac{\partial Q}{dt}, \quad U_c = \frac{Q}{C}, \quad I_c = \frac{\partial Q}{dt}$$

The electric work of the current is the time integral of P:

$$W_c = \int P dt = \int \frac{Q}{C}\frac{\partial Q}{dt} dt = \int \frac{Q}{C} dQ = \frac{Q^2}{2C} + c_2$$

At the beginning and the end of one period of the alternate current, the charge of the capacitor has the same value, that is, $Q_0 = Q_T$. So, the electric work during one period is 0:

$$W_c = \int_0^T P dt = \frac{{Q_T}^2}{2C} - \frac{{Q_0}^2}{2C} = 0 \tag{4}$$

As the charge of the capacitor varies, the magnetic flux passing through the wire loop varies and creates EMF (see Figure 1) that in turn, will produce electric power on the constant current circulating in the loop. With U_l the EMF, L the inductance of the loop, Φ the magnetic flux, I_l the current in the loop, the electric power is:

$$P = I_l U_l = I_l \frac{d\Phi}{dt}, \quad U_l = \frac{d\Phi}{dt}$$

The magnetic flux has the same value at the beginning and the end of one period, that is, $\Phi_0=\Phi_T$, the electric work during one period is then 0:

$$W_l = \int_0^T P dt = \int_0^T I_l \frac{d\Phi}{dt} dt = I_l \int_0^T d\Phi = I_l \left(\Phi_T - \Phi_0\right) = 0 \qquad (5)$$

Now, let us consider the Lorentz force on the wire loop. On a vertical side with height h the Lorentz force is:

$$F = \mu_0 H_\phi I_l h = \frac{\mu_0 h I_c I_l}{2\pi r}$$

The resultant Lorentz force on the loop is the difference of that on the vertical side ① and ②. With r_1 and r_2 the radial positions of the 2 vertical sides, the resultant force is:

$$F = \frac{\mu_0 h I_c I_l}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

The expression of the alternate current I_c is:

$$I_c = I_m \sin \omega t$$

The resultant Lorentz force on the loop is:

$$F = \frac{\mu_0 h I_l I_m}{2\pi} \sin \omega t \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

We make the loop do a reciprocating movement in synchronism with the Lorentz force F, its velocity is:

$$v = v_m \sin \omega t$$

And the work done by the Lorentz force during one period is:

$$W_F = \int_0^T F v dt = \int_0^T \left[\frac{\mu_0 h I_l I_m}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \sin \omega t \cdot v_m \sin \omega t\right] dt$$

$$W_F = \frac{\mu_0 h I_l I_m v_m}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (6)$$

The loop must move in space for the Lorentz force to do work. If the capacitor is fixed, the computation would be complicated. So, we let the system capacitor-loop move as a whole. This way the capacitor and the loop are stationary relative to each other and the magnetic fields will be the same. This way, the physical phenomenon is much simpler.

The energy that the system capacitor-loop exchanges with the exterior is the sum of the above electric works and that done by the Lorentz force (the equation (6)). The electric work in the capacitor and the loop being 0 (see equations (4) and (5)), the balance is.

$$\Delta W = W_c + W_l + W_F = \frac{\mu_0 h I_l I_m v_m}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

So, there is a net output of work! Is it created energy? No, it is an inconsistency. If there were real current at the place of the displacement current, it would bear the same Lorentz force than the loop and it would provide the reaction work that balances energy on the system. But, displacement current is in a void and void cannot bear force, thus the lack of reaction work and unbalanced energy. To correct this inconsistency, we need a force on a void, that is, a “Phantom Lorentz force”.

Void also explains the inconsistency of “Displacement current paradox”. EMF can do work because electrons of the wire are pushed by EMF. In displacement current there is no electron to be pushed, no reaction work can be done. Thus, energy exchange is unbalanced.

Until now, I have provided 5 inconsistencies. Normally, only one counterexample is necessary to invalidate a theory. For example, to prove 231 is not prime, it is sufficient to show that 3 is a divisor. Saying also that 7 is a divisor, 11 is a divisor, 33 is a divisor does not make sense. Why am I providing relentlessly more and more paradoxes?

In fact, it is not difficult to invalidate a theory. The most difficult is to make people aware that the theory they trust is wrong. People always search farfetched justification against objections. For example, it is known for long time that Lorentz force does not respect Newton’s third law. But one has pretended that Lorentz force law is valid only for closed loop and ignored this inconsistency. When I showed the B-cutting paradox to a physicist, he said to me: “you must be wrong” and went away. When I presented a numerical computation of the resultant Lorentz force internal to a coil that was not null, a physicist has contended that my mathematics were wrong, even though he did not read my work.

We see that if only one counterexample is presented, it is abruptly rejected. Now, I have rigorously proven that, the resultant Lorentz force internal to a closed coil is not null (1), that the EMF of B-cutting violates energy conservation law (2,3), that displacement current violates energy conservation law (4), etc. These independent inconsistencies can no longer be rejected with a simple “you did it wrong” and force physicists to seriously consider that the laws they use are actually wrong.

(1) Synthesis of the inconsistency of the Lorentz force law http://pengkuanem.blogspot.com/2012/04/synthesis.html
(2) B-cutting paradox http://pengkuanem.blogspot.fr/2012/05/b-cutting.html
(3) Lorentz' EMF paradox http://pengkuanem.blogspot.fr/2012/05/lorentz-emf.html
(4) Displacement Current Paradox http://pengkuanem.blogspot.com/2012/07/displacement-current-paradox.html