EBERHARD KARLS UNIVERSITÄT TÜBINGEN

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Seminar für Sprachwissenschaft

Kuan's Logic Tutorial 04

2021.12.03

Nr	Date	Content	Main reading	Additional Reading
1	08. Nov	Overview & Sets	Script Sets, ... Section 1	
2	15. Nov	Proofs	Script Sets, ... Section 2	
3	22. Nov	Relations & Functions	Script Sets, ... Sections 3 & 4	
4	29. Nov	Propositional Logic: Syntax & Truth Tables		Gamut 2.1 - 2.3
5	06. Dec	Propositional Logic: Semantics		Gamut 2.5
6	13. Dec	Logical Validity		Gamut 4.2 (up to page 118)
7	20. Dec	Predicate Logic: Syntax & Semantics		Gamut 3.1 - 3.3, 3.6
		XMAS		
8	10. Jan	Predicate Logic: Semantics, Identity & Validity		Gamut 3.6, 3.7, 4.21
9	17. Jan	Natural Deduction: Propositional Logic		Gamut 4.3.1 - 4.3.5
10	24. Jan	Natural Deduction: Predicate Logic; Meta Logic		Gamut 4.3.6 - 4.3.7 & 4.4
11	31. Jan	trade-in or something fun (informal argumentation, probability logic ..)		
12	07. Feb	spill-over, reflection Q&A		
	14. Feb	Exam		

Review exercises of Module 1 (Tutorial 01 – 03): Sets, Proofs, Relations and Functions

1. Consider the following sets:

$$\begin{aligned} A_1 &= \{\emptyset, \{B\}, \{\emptyset, B\}\} \\ A_2 &= \{B\} \\ A_3 &= \{\emptyset\} \\ A_4 &= \{\emptyset, \{\emptyset, B\}\} \\ A_5 &= \{\{B\}, \{\emptyset, B\}\} \\ A_6 &= \{\emptyset\} \end{aligned} \tag{1}$$

Determine the following sets:

(a) $A_1 \cap A_4$

(b) $(A_2 \cup A_3) - A_6$

(c) $\mathscr{P}(A_6) \cap A_5$

(d) $A_4 - A_2$

2. On the integers specify a relation which is :
   (a) symmetric and irreflexive
   (b) transitive and asymmetric

3. Consider the set N of all natural numbers in set-theoretic representation.
   (a) Let R be the subset relation on N. Is R symmetric and/or transitive and/or reflexive?
   (b) If $n \in N$ and $x \in n$, is necessarily $x \subset n$? Proof your answer.

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4. Propositional Logic I

Key points:

- Syntax tree
- Truth tables

4.1. Basic concepts

- The languages we speak and use naturally to communicate with each other are what we call ***natural* languages**.
- **Formal languages** are usually designed by people for a clear, particular purpose, but although these languages are constructed, in use they may to a certain extent change and evolve.

- **Propositional logic (PropLog)** studies how propositions are combined by logical operators
- A **proposition** in the sense of PropLog is a minimal unit of thought which can be evaluated as true or false independently of other propositions.
- The language of PropLog is formed by:

  (i) a set of ***proposition letters*** $\mathscr{P} = \{p, q, r, s, p_1, q_{27}, \ldots\}$ and

  (ii) a set of ***sentential connectives*** $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$

name	paraphrase	symbol
negation	"not"	$\neg$
conjunction	"and"	$\wedge$
disjunction	"or"	$\vee$
implication	"if ..., then ..."	$\rightarrow$
equivalence	"if and only of"	$\leftrightarrow$

$$\underbrace{\textit{The earth is round}}_{p}\ \underbrace{\textit{and}}_{\wedge}\ \underbrace{\textit{the moon is made of cheese}}_{q} \tag{2}$$

4.2. Syntax

- The syntax of statement logic: we assume an infinite basic vocabulary of ***atomic statements*** represented by the symbols p, q, r, s, ..., with primes or subscripts added as needed.

4.2.1 Formulas

The language $\mathscr{L}$ of PropLog is the set of all *formulas* which are recursively defined as follows:

- (i) Every proposition letter is a formula.
- (ii) If φ is a formula,so is $\neg\varphi$.
- (iii) If φ and ψ are formulas, so are: a. $(\varphi \wedge \psi)$ b. $(\varphi \vee \psi)$ c. $\varphi \rightarrow \psi$ d. $\varphi \leftrightarrow \psi$
- (iv) Anything that cannot be constructed by (i)–(iii) is not a formula.

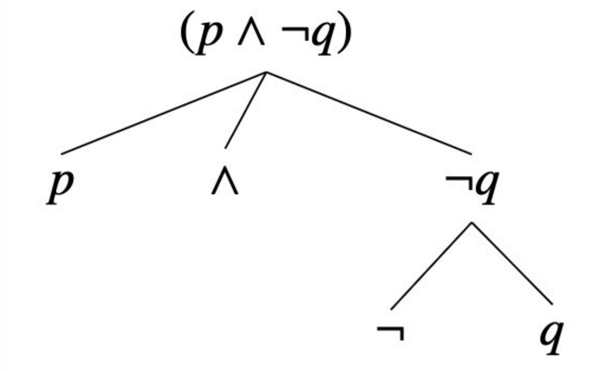
Exercise 1. Determine which of the following strings are formulas of propositional logic. For any formula, determine its main operator.

a). q_{12} b). $p, q \wedge r$ c). $(p) \wedge q$

d). $p \rightarrow (p \wedge p)$ e). $(p \rightarrow)(p \wedge p)$ f). $(p \vee \neg q) \leftrightarrow (r \rightarrow (\neg(p \vee \neg p)))$

4.2.2 Syntactic trees

- A *syntactic tree* is a useful visual illustration of the internal structure of a formula.



- **Atomic formula**: A formula of PropLog which consist of a single proposition letter.
- **Complex formula:** Any formula of PropLog which is not atomic. Each complex formula has a ***main connective***. The main connective is the last sentential connector introduced during the construction of the formula.

Exercise 2. Draw the syntactic tree for each of the following formulas.

a. $p \leftrightarrow q$

b. $\neg p \wedge p$

c. $p \to \neg(q \wedge r)$

d. $(\neg p \vee \neg q) \wedge (r \to p)$

4.3. Semantics

- The *semantics* of PropLog gives a meaning to each formula. More specifically, *truth-conditional meaning*.
- That PropLog only considers two truth values is also referred to as the *principle of bivalence*

4.3.1 Truth tables

- To give a truth-conditional meaning to the operator $\neg$ we need to define whether $\neg\varphi$ is true or false for each case: when φ is true, and when φ is false.
- Semantics of negation

φ	$\neg\varphi$
1	0
0	1

- Semantics of binary connectives

ϕ	ψ	$\phi \wedge \psi$	$\phi \vee \psi$	$\phi \rightarrow \psi$	$\phi \leftrightarrow \psi$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Exercise 3. Please draw the syntax tree and truth table of $(\varphi \wedge \psi) \rightarrow \neg\chi$

4.3.2 *Contingencies, tautologies & contradictions*

- **Contingency**: A formula which can become true and false for different assignments of truth values to its components.
- **Tautology**: A formula which is always true under any constellation of truth values for its components. A tautology is a formula that is *necessarily true*, or *true by logical necessity*.
- **Contradiction**: A formula which is false for all ways of assigning truth values to its subformulas. A contradiction can be said to be *necessarily false*, or *false by logical necessity*.

Exercise 4: Please draw the syntax tree and truth table for $\varphi \vee \neg\varphi$ and $\varphi \wedge \neg\varphi$

4.3.3. Logical equivalence

- Two formulas φ and ψ are logically equivalent if they have exactly the same truth value no matter how we assign meanings to all subformulas of φ and ψ, as long as we assign any subformula that occurs in both φ and ψ the same truth value.
  - the equivalence of $\varphi \wedge \neg\psi$ and $\neg(\varphi \to \psi)$

φ	ψ	$\neg\psi$	$\varphi \wedge \neg\psi$	$\varphi \to \psi$	$\neg(\varphi \to \psi)$
1	1	0	0	1	0
1	0	1	1	0	1
0	1	0	0	1	0
0	0	1	0	1	0

- If two formulas φ and ψ are *not* logically equivalent, there must be at least one row in which their truth values differ.
  - non-equivalence of $\varphi \to \psi$ and $\varphi \leftrightarrow \psi$

φ	ψ	$\varphi \to \psi$	$\varphi \leftrightarrow \psi$
1	1	1	1
1	0	0	0
0	1	1	0
0	0	1	1

Exercise 5. For each of the following formulas, try to intuit whether it is a tautology, a contradiction or a contingency. Write down your best guess. Then use the truth-table method to find out for certain.

a. $\varphi \to \neg\varphi$ b. $\varphi \leftrightarrow (\varphi \wedge \psi)$ c. $(\varphi \wedge \neg\varphi) \to \psi$ d. $(\varphi \wedge \neg\psi) \leftrightarrow (\varphi \to \psi)$

Exercise 6. For each of the following pairs of formulas, try to intuit whether they are logically equivalent of not. Write down you guess. Then use truth tables to determine whether they are. Highlight the relevant columns in your truth table. Mark a row as a counterexample if they are not.

a. $\varphi, \neg\neg\varphi$ b. $\varphi \leftrightarrow \varphi, \varphi \wedge \psi$ c. $\neg\varphi \vee \psi, \varphi \to \psi$ d. $\varphi \vee (\chi \to \neg\varphi),\ \ \psi \vee (\chi \to \neg\psi)$