EBERHARD KARLS UNIVERSITÄT TÜBINGEN

PHILOSOPHISCHE FAKULTÄT

Seminar für Sprachwissenschaft

Keno's Logic tutorial 01

2021.11.12

Nr	Date	Content	Main reading	Additional Reading
1	08. Nov	Overview & Sets	Script Sets, ... Section 1	
2	15. Nov	Proofs	Script Sets, ... Section 2	
3	22. Nov	Relations & Functions	Script Sets, ... Sections 3 & 4	
4	29. Nov	Propositional Logic: Syntax & Truth Tables		Gamut 2.1 - 2.3
5	06. Dec	Propositional Logic: Semantics		Gamut 2.5
6	13. Dec	Logical Validity		Gamut 4.2 (up to page 118)
7	20. Dec	Predicate Logic: Syntax & Semantics		Gamut 3.1 - 3.3, 3.6
		XMAS		
8	10. Jan	Predicate Logic: Semantics, Identity & Validity		Gamut 3.6, 3.7, 4.21
9	17. Jan	Natural Deduction: Propositional Logic		Gamut 4.3.1 - 4.3.5
10	24. Jan	Natural Deduction: Predicate Logic; Meta Logic		Gamut 4.3.6 - 4.3.7 & 4.4
11	31. Jan	trade-in or something fun (informal argumentation, probability logic ..)		
12	07. Feb	spill-over, reflection Q&A		
	14. Feb	Exam		

1. Naïve set theory

Key points: Sets, elements, relations between and operations on sets

1.1. Sets and elements

- A *set* is an abstract collection of distinct objects which are called the *members* or *elements* of that set.
  $Y = \{a, b, c, d\}$

  notice:

  - We write *A, B, C* **(Italic)** for sets, and $a, b, c, \ldots$ or sometimes $x, y, z, \ldots$ for members of sets.
  - We use notation with curly braces "$\{\ldots\}$" which means the order of representation of elements is *irrelevant* to represent such a collection.

    If we use **angle bracket "$< \ldots >$"**, which means the order of the elements is *relevant* , in this case $< a, b >= \{\{a\}, \{a, b\}\}$
  - ***singleton*:** A set with only one member.
  - ***empty set*:** a set to have no element at all $\varnothing$.

    $b \in A$ is read as 'b is a member of A'.

    $b \notin A$ is read as 'b is not a member of A'.

Exercise 01: Given the following set:

$A = \{a, b, c, 2, 3, 4\}$

$B = \{a, b\}$

$C = \{c, 2\}$

$D = \{b, c\}$

$E = \{a, b, \{c\}\}$

$F = \emptyset$

$G = \{\{a, b\}, \{c, 2\}\}$

classify each of the following statements as true or false

(a) $c \in A$ (b) $\{c\} \in E$ (c) $\{c\} \in C$

(d) $B \subseteq G$ (e) $D \subseteq G$ (f) $\{\{c\}\} \subseteq E$

Exercise 02: For any arbitrary set *S*,

(a) is *S* a member of {*S*}?

(b) is {*S*} a member of {*S*}?

(c) is {*S*} a subset of {*S*}?

(d) what is the set whose only member is {*S*}?

1.2. Specification of sets

Three main methods for describing sets exist:

(1) **List notation**: by listing all its members.

e.g., the set whose members are the world's longest river, the first president of the United States and the number five {The Amazon River, George Washington, 5}

(2) **Predicate notation**: by stating a property which an object must have to qualify as a member of it

e.g., the predicate 'x is an even number between 3 and 9' = {4, 6, 8}

(3) **Recursive rules**: by defining a set of rules which generate its members.

e.g., the set E = {4,6,8, ...} can be generated by the following rule:

a) $4 \in E$

b) If $x \in E$, then $x + 2 \in E$

c) Nothing else belongs to E.

Exercise 03: Write a specification of the following sets.

(a) $\{5, 10, 15, 20, ...\}$

(b) $\{0, 2, -2, 4, -4, 6, -6, ...\}$

(c) $\{300, 301, 302, ..., 399, 400\}$

1.3. Identity & Cardinality

- **Identity:** two sets are identical if and only if they have exactly the same members.
- **Cardinality:** The number of members in a set A is called the *cardinality* of A, written $|A|$ or $\#(A)$. The cardinality of a finite set is given by one of the natural numbers.

Exercise 04: Let $A = \{x | x \text{ is a positive integer less than } 7\}$

i). What is the List notation and Recursive rules of A?

ii). What is the $|A|$?

1.4 Subsets

- $A \subseteq B$: When every member of a set A is also a member of a set B we call A a *subset* of B.

  Note that B may contain other members besides those of A, but this is not necessarily so. Thus the subset relation allows any set to be a subset of itself.
- $A \subset B$: To exclude the case of a set being a subset of itself, the notion is called *proper subset*.
- $A \subsetneq B$: A is not a subset of B, hence that A has at least one member which is not a member of B.

Exercise 05: Classify each of the following statements as true or false:

a) $\{a, b, c\} \subseteq \{s, b, a, e, g, i, c\}$

b) $\{a, b, j\} \subsetneq \{s, b, a, e, g, i, c\}$

c) $\{a, b, c\} \subset \{s, b, a, e, g, i, c\}$

d) $\emptyset \subset \{a\}$

e) $\{a, \{a\}\} \subseteq \{a, b, \{a\}\}$

f) $\{\{a\}\} \subsetneq \{a\}$

g) $\{a\} \subsetneq \{\{a\}\}$

h) $\{a\} \in \{\{a\}\}$

1.5 Power sets

- **Power set:** The set whose members are all the subsets of a given set A, write as $\mathscr{P}(A)$.

  Suppose $A = \{a, b\}$; then the power set of A, $\mathscr{P}(A)$, is $\{\{a\}, \{b\}, \{a, b\}, \emptyset\}$.

  If the cardinality of A is some natural number n, then $\mathscr{P}(A)$ has cardinality 2^n i.e., 2 raised to the n power, or 2 x 2 x 2 x ... x 2 (n times). The power set of A is sometimes denoted as 2^A.

Exercise 06: Specify each of the following sets by listing its members:

(a) $\mathscr{P}\,\emptyset$ (b) $\mathscr{P}\,\{\emptyset\}$ (c) $\mathscr{P}\mathscr{P}\,\{a\}$

1.6 Set operations

- The *union* of A and B

$$A \cup B =_{def} \{x | x \in A \textit{ or } x \in B\}$$

- The *intersection* of A and B

$$A \cap B =_{def} \{x | x \in A \textit{ or } x \in B\}$$

- The *relative complement of* A and B

$$A - B =_{def} \{x | x \in A \textit{ or } x \notin B\}$$

- the *complement* of a set A,

$$A' =_{def} \{x | x \notin A\} = U - A$$

Exercise 07: Let $A = \{a, b, c\}, B = \{c, d\}$ and $C = \{d, e, f\}$

what are (i) $A \cap B$ (ii) $A \cap (B \cup C)$ (iii) $A - B$

1.7 Set-theoretic equalities

1. *Idempotent Laws*
   (a) $X \cup X = X$ (b) $X \cap X = X$

2. *Commutative Laws*
   (a) $X \cup Y = Y \cup X$ (b) $X \cap Y = Y \cap X$

3. *Associative Laws*
   (a) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ (b) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

4. *Distributive Laws*
   (a) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
   (b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

5. *Identity Laws*
   (a) $X \cup \emptyset = X$ (c) $X \cap \emptyset = \emptyset$
   (b) $X \cup U = U$ (d) $X \cap U = X$

6. *Complement Laws*
   (a) $X \cup X' = U$ (c) $X \cap X' = \emptyset$
   (b) $(X')' = X$ (d) $X - Y = X \cap Y'$

7. *DeMorgan's Laws*
   (a) $(X \cup Y)' = X' \cap Y'$ (b) $(X \cap Y)' = X' \cup Y'$

8. *Consistency Principle*
   (a) $X \subseteq Y$ iff $X \cup Y = Y$ (b) $X \subseteq Y$ iff $X \cap Y = X$

Exercise 08. Show that the Distributive Law 4(a) is true by constructing Venn diagrams for

$X \cap (Y \cup Z)$ and $(X \cap Y) \cup (X \cap Z)$.
