SI211 2018 Fall Numerical Analysis Project
Image Registration by using Gauss-Newton Method

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Abstract

In this project, we will solve a image registration problem by using Gauss-Newton method. In this problem, we interested in get transformation parameters from two related images. First, we could get the dataset by using feature matching of these two images. Then, we formula this problem as a least-squared problem and writing the objective function. Last, the problem will be solved by using Gauss-Newton method. We will analysis the converge rate of this problem, besides we also compare with the performance between Gauss-Newton method with Newton method.

1. Introduction

Image registration focus on calculate the transformation between two related images, like Fig. 1. It is a classical domain in image process and computer vision. It is wildly used in many areas. In computer vision, image registration is the basic of calibration and virtual reality. In robotics, it is used to estimated the pose of robot in visual simultaneous localization and mapping(vSLAM). And it is also wildly used in medical image process domain, e.g. cancer detection.

The main algorithm of image registration has three classes. In early, there are many algorithm about blocking matching. In 1972, Barnea et al. propose a blocking matching method to doing satellites images registration [1]. In 1994, Chen et al. propose a algorithm based on Fourier-Mellin transforms [2], they show a frequency-based image registration method could get more robust performance. Now, feature-based is a mainstream image registration method in computer vision domain. It could get more accuracy and faster than other method. Szeliski et al. describe and conclude feature-based method in their book [3].

The different of each classes is using different method to finding overlap area of two images. After getting overlap area, estimating the parameters of transformation is follow

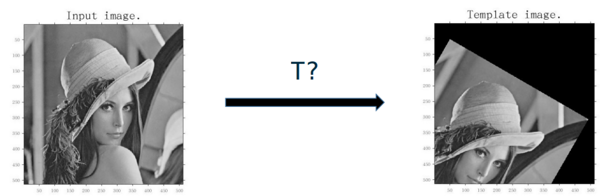


Figure 1: The main task of image registration is estimated the transformation T from a input image and a template image.

steps. In this project, we will implement a feature based image registration algorithm and optimize it by using Gauss-Newton method.

1.1. Report Structure

In section 2, we will discuss how to formula this problem by using a optimization perspective. In section 3 we will introduce how to derivate the update formula according to Gauss-Newton method. In section 4, we will give the result of experiment, and also analysis the convergence rate of Gauss-Newton method. In last section, we will conclude this project.

2. Problem Formula

For solving a image registration problem from a optimization perspective, We need to go through three steps: *matching two images, building objective functions* and *derivate optimization method*. In this section, we concentrate on explain first two steps, and the optimization will be discussed in next section.

2.1. Features Matching

Finding the overlap area always is the first step of image registration problem. In this problem, we need to get sets of data point for modeling images registration problem as a data fitting problem. So, we need find a set of related pixel between two images.

This problem is a classical topics in computer vision domain. In this project, we choose features based method to solving this problem. Because it is fast, robust and wildly used for image registration problem.

Here, we implement this method by using MATLAB Image Process Toolbox. It provide almost all mainstream features matching method. In practice, we choose a corner detection based features matching algorithm for getting a faster speed. The features matching result is shown as Fig. 2. After doing feature matching, we could get a set of matched points $\{x; x'\}$, $x \in \mathbb{R}^{2\times m}$, m is the number of data. That means x' are generated by x through transformation T. We will use this data to build the objective function.

Figure 2: The result after doing feature matching by using corner detection algorithm. The red ∘ are matched features of original image and green + are matched features of template image.

2.2. Objective Function

After feature matching, we could formula this problem as a least-squared problem by using matched data. And our parameter is $p = [t_x, t_y, \theta]^T$. So, the objective function is like formula 1.

$$\min_p = F(x; p) \tag{1}$$

$$F(x; p) = \frac{1}{2}||f(x; p) - x'||_2^2 \tag{2}$$

$$f(x; p) = x' \tag{3}$$

For example, x^i is one data point of data set x, we could derivate the formula of $f(x; p)$ like formula 6.

$$f(x^i; p) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} x_1^i \\ x_2^i \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} x_1^i \cos\theta - x_2^i \sin\theta + t_x \\ x_1^i \sin\theta + x_2^i \cos\theta + t_y \end{bmatrix} \in \mathbb{R}^2 \tag{5}$$

$$f(x; p) = \begin{bmatrix} f(x^0; p) \\ f(x^1; p) \\ \vdots \\ f(x^m; p) \end{bmatrix} \in \mathbb{R}^{2m} \tag{6}$$

Here, we modeling a feature based images registration problem as a least-squared problem. Then, we need to using some optimization algorithm to minimize this objective function and get the accuracy estimation of parameters.

3. Solution Method

In this project, we need to using Gauss-Newton method estimate the parameters to minimize objective function. In this section, we will derivate the parameter update formula by using Gauss-Newton method firstly. Then, we will analysis the convergence of it.

3.1. Gauss-Newton Method

In last section, we already knew about $F(x; p)$ and $f(x; p)$. Here, we are writing the objective function like formula 9.

$$F(x; p) = \frac{1}{2}||f(x; p) - x'||_2^2 \tag{7}$$

$$g(x; p) = f(x; p) - x' \tag{8}$$

$$F(x; p) = \frac{1}{2}||g(x; p)||_2^2 \tag{9}$$

So, according to the Gauss-Newton method, we could know about the update formula like formula 10.

$$p_{k+1} = p_k - (g'(p_k)^T g'(p_k))^{-1} g'(p_k)^T g(p_k) \tag{10}$$

For using Gauss-Newton method, we have to calculate the gradient of $g(x; p)$. The derivation is shown as below:

$$J(x; p) = g'(x; p) = \nabla_p g(x; p) \tag{11}$$

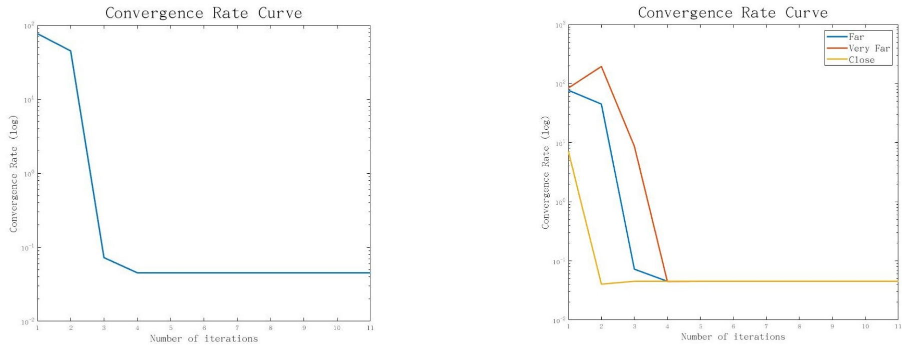
Like the previous derivation, we derivate the gradient of one data point. Then extend it to all data.

$$J(x^i; p) = \nabla_p g(x^i; p) \tag{12}$$

$$\nabla_p g(x^i; p) = \begin{bmatrix} x_1^i \cos\theta + x_2^i \sin\theta & 1 & 0 \\ -x_1^i \sin\theta + x_2^i \cos\theta & 0 & 1 \end{bmatrix} \in \mathbb{R}^{2\times 3} \tag{13}$$

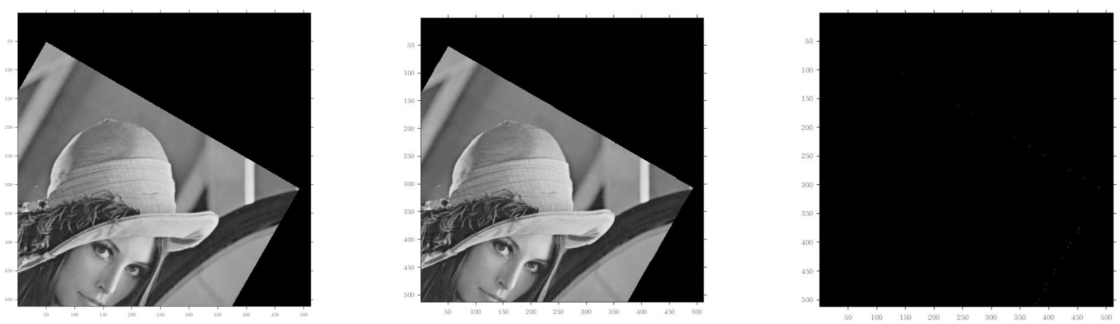
$$J(x; p) = \begin{bmatrix} J(x^0; p) \\ J(x^1; p) \\ \vdots \\ J(x^m; p) \end{bmatrix} \in \mathbb{R}^{2m\times 3} \tag{14}$$

So, the gradient is like formula 14. According to the definition of Gauss-Newton method parameter updating formula, the $J(x)$ must be full-rank. By observing the gradient



(a) (b)

Figure 3: The convergence analysis curve. (a) is clear view of convergence curve. And (b) is the convergence analysis between different initial value.



(a) Template Image (b) Gauss-Newton Method (c) The error between estimation and template.

Figure 4: The final result of image registration by using Gauss-Newton method. (a) is ground truth and (b) is the transformation result by using the estimated parameters. (c) is the error between (a) and (b).

formula, we found J is full-rank at most situation but when $x = 0$. But when all matched features are $x = 0$ also means these two images cannot be registration, it is meaningless for this problem. So, we could think J is full-rank on this problem.

$$p_{k+1} = p_k - (J(p_k)^T J(p_k))^{-1} J(p_k)^T g(x; p_k) \quad (15)$$

After getting the gradient, we could use it to writing the parameter updating formula like formula 15.

3.2. Convergence Rate Analysis

The convergence rate of Newton-typed method like formula 17.

$$||I - M(p_k)^{-1} F'(x; p_k)|| \leq \mathbf{k} \quad (16)$$

$$||p_{k+1} - p^*|| \leq \mathbf{k}||p_k - p^*|| + \frac{w}{2}||p_k - p^*||^2 \quad (17)$$

Here, $M(p_k)$ is the Hessian approximation matrix of $F(p_k)$. According the definition of Gauss-Newton method, we know the $M(p_k) = J(p_k)^T J(p_k)$ and $M(p_k)$ is a positive semi-definite. So, we could make sure the convergence of this updating formula.

And because it is use approximation matrix of Hessian matrix according to Gauss-Newton method, the convergence rate is linear.

4. Numerical Results

Here, we already get the data set, objective function and optimization solver. And we also prove the convergence rate of Gauss-Newton method of this problem. In this section, we will discuss the experiment result of this problem.

4.1. Experiment Setting

In this project, we implement the image registration algorithm by using MATLAB. We use Image Process Toolbox to do feature matching. In this project, we use function of corner detection algorithm to get feature points of two images, and using nearest neighbors algorithm to matching features.

And we are helped with symbolic computation toolbox to calculate gradient. It will make use get Jacobian matrix easily with all data points.

4.2. Experiment Result

For verify the performance and convergence of Gauss-Newton method for image registration, we getting the template image by transform the input images with a designed parameter, and setting this parameter as ground truth. And we use ten as a fixed number of iteration.

The Fig. 3a shows convergence rate of Gauss-Newton method in this problem. We could see it is linear convergent. And we also see it is very fast, even execute the algorithm with different initial value like Fig. 3.

The value of initial values and the final result is shown as Table. 1. We could see the result of Gauss-Newton method is very close to ground truth. We use this estimated parameter to transform input image of Fig. 1, the result is shown as Fig. 4b and the difference of ground truth is very small(Only has a little difference on the edge.)

	θ	t_x	t_y
Far	0	0	0
Very Far	95	10	15
Close	29	45	45
P_{gn}(Far)	29.9957	50.0391	50.0223
ground truth	30	50	50

Table 1: The different initial values setting and experiments result.

5. Conclusion

In this project, we modeling a feature based image registration problem and solving it by using Gauss-Newton method. We prove the convergence of Gauss-Newton method, and get a accuracy result and very fast speed according to experiment. That show the Gauss-Newton method is very suited to image registration domain.

References

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