8.1 A. $\int f'(2x)dx = \frac{1}{2}\int f'(2x)d2x = \frac{1}{2}f(2x)+C$

B. $[\int f(2x)dx]' = f(2x)$

8.2 $\int f(x)\,dx = x\cos x + C$

$f(x) = \cos x + (-x\sin x) = \cos x - x\sin x$

C

8.3 $\int \sin[f(x)]dx = x\sin[f(x)] - \int \cos[f(x)]\,dx$

$\int u\,dv = uv - \int v\,du$

$\Downarrow$

$\int uv'dx = uv - \int vu'dx$

记 $u = \sin[f(x)]$, $v' = 1 \Rightarrow u' = f'(x)\cdot\cos[f(x)]$, $v = x$

$\int \sin[f(x)]dx = x\sin[f(x)] - \int f'(x)\cdot x\cdot\cos[f(x)]dx$

$f'(x)\cdot x = 1$, $f'(x) = \frac{1}{x}$

$f(x) = \int \frac{1}{x}dx = \ln|x| + C$

8.4 设$f(x)$的一个原函数为$\frac{\sin x}{x}$，求$\int xf'(x)dx$

解: $\int f(x)dx = \frac{\sin x}{x} + C$

$$\int xf'(x)dx = \int x\,df(x) = xf(x) - \int f(x)dx$$

$$f(x) = \frac{x\cos x - \sin x}{x^2}$$

$$\int xf'(x)dx = \cos x - \frac{\sin x}{x} - \frac{\sin x}{x} + C$$

$$= \cos x - \frac{2\sin x}{x} + C$$

8.5 设xe^{-x}为$f(x)$的一个原函数，求$\int xf'(x)dx$

$$\int f(x)dx = xe^{-x} + C = F(x)$$

$$f(x) = \left(\int f(x)dx\right)' = \left(\frac{x}{e^x}\right)' = \frac{e^x - xe^x}{e^{2x}}$$

$$= \frac{1-x}{e^x}$$

$$I = \int xf'(x)dx = \int x(1-x)e^{-x} = -\int x(1-x)de^{-x}$$

$$= \int x(x-1)\,de^{-x} = x(x-1)e^{-x} - \int e^{-x}(2x-1)dx$$

$$= x(x-1)e^{-x} - 2\int xe^{-x}dx + \int e^{-x}dx \quad = -e^{-x}$$

$$= (x^2-x-1)e^{-x} - 2\int xe^{-x}dx$$

$$\int xe^{-x}dx = -\int x\,de^{-x} = -\left(xe^{-x} - \int e^{-x}dx\right)$$

$$= -(x+1)e^{x}$$

$$I = (x^2-x-1+2x+2)e^{-x} = (x^2+x+1)e^{-x} + C$$

8.6 $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+2n}} + \frac{1}{\sqrt{n^2+3n}} + \cdots + \frac{1}{\sqrt{n^2+n^2}}\right)$

解: $\lim\limits_{n\to\infty}\left(\frac{\frac{1}{n}}{\sqrt{1+\frac{1}{n}}} + \frac{\frac{1}{n}}{\sqrt{1+\frac{2}{n}}} + \cdots + \frac{\frac{1}{n}}{\sqrt{1+\frac{n}{n}}}\right)$

$$= \lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{\sqrt{1+\frac{1}{n}}} + \frac{1}{\sqrt{1+\frac{2}{n}}} + \cdots + \frac{1}{\sqrt{1+\frac{n}{n}}}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\cdot\frac{1}{\sqrt{1+\frac{i}{n}}} = \int_0^1 \frac{1}{\sqrt{1+x}}\,dx$$

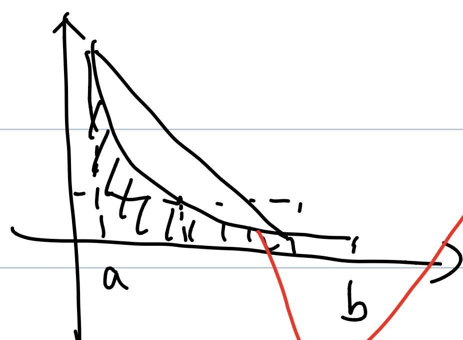
$\int_0^1 \frac{1}{\sqrt{1+x}} dx$ 令 $t=\sqrt{1+x}$, $x=t^2-1$

$dx = 2t\,dt$

$\int_1^{\sqrt{2}} \frac{2t}{t} dt = \int_1^{\sqrt{2}} 2dt = 2t\Big|_1^{\sqrt{2}} = 2(\sqrt{2}-1)$

8.7 在闭区间[a,b]上，设 $f(x)>0$, $f'(x)<0$, $f''(x)>0$,

记 $S_1=\int_a^b f(x)dx$, $S_2=f(b)(b-a)$, $S_3=\frac{1}{2}[f(b)+f(a)](b-a)$



$S_2<S_1<S_3$

D

8.8 设f(x)为[a,b]上的连续函数，[c,d]⊆[a,b]

A

8.9 设$f(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0\end{cases}$，$F(x)=\int_0^x f(t)dt$

B

8.10 设$f(x)$是连续函数，且$\int_0^{x^3-1} f(t)dt = x-1$，

则$f(7)$

$f(x^3-1)\cdot(3x^2) = 1$

$f(x^3-1) = \frac{1}{3x^2}$，$x=2$

$f(7) = \frac{1}{12}$

8.11 设$f(x)$为连续函数，且$F(x)=\int_{\frac{1}{x}}^{\ln x} f(t)dt$，则$F'(x)$

$F'(x) = f(\ln x)\cdot\frac{1}{x} - f(\frac{1}{x})\cdot(-\frac{1}{x^2})$

$= \frac{f(\ln x)}{x} + \frac{f(\frac{1}{x})}{x^2}$

$$=\frac{f(\ln x)+f(\frac{1}{x})}{x^2}$$

8.12 求连续函数 $f(x)$，使它满足 $\int_0^1 f(tx)dt = f(x)+x\sin x$

解：令 $u=tx$，$t=\frac{u}{x}$　$dt=d\frac{u}{x}=\frac{1}{x}du$

$$\int_0^x f(u)\cdot\frac{1}{x}du=\frac{1}{x}\int_0^x f(u)du$$

$$\int_0^x f(u)du = xf(x)+x^2\sin x$$

$$f(x)= f(x)+xf'(x)+2x\sin x+x^2\cos x$$

$$xf'(x)= -2x\sin x-x^2\cos x$$

$$f'(x) = -2\sin x-x\cos x$$

$$f(x)=-\int 2\sin x+x\cos x\,dx$$

$$=-\left(2\int\sin x\,dx+\int x\cos x\,dx\right)$$

$$= -(-2\cos x) - \int x\cos x\,dx$$

$$= 2\cos x - \int x\,d\sin x$$

$$= 2\cos x - \left(x\sin x - \int \sin x\,dx\right)$$

$$= 2\cos x - (x\sin x + \cos x) + C$$

$$= \cos x - x\sin x + C$$

8.13 设$f(x)$在$[-1,1]$上连续且$f(x)>0$，证明：曲线 $y=\int_{-1}^{1}|x-t|f(t)\,dt$ 在 $-1\le x\le 1$ 上是凹曲线

证明: $y=\int_{-1}^{x}(x-t)f(t)\,dt+\int_{x}^{1}(t-x)f(t)\,dt$

$$y = x\int_{1}^{x}f(t)\,dt - \int_{-1}^{x}tf(t)\,dt + \int_{x}^{1}tf(t)\,dt - x\int_{x}^{1}f(t)\,dt$$

$$y'_x = \int_{1}^{x}f(t)\,dt + \cancel{xf(x)} - \cancel{xf(x)} - xf(x)$$

$-\left(\int_x^1 f(t)dt - xf(x)\right)$

$= \int_{-1}^{x} f(t)dt - \int_x^1 f(t)dt$

$y_x''| = f(x) + f(x) = 2f(x) > 0$

$\therefore y_x''| = 2f(x) > 0$

$\therefore$ 曲线 $y = \int_{-1}^{1} |x-t| f(t)dt$ 在 $-1 \le x \le 1$ 上是凹曲线

8.14 设 $\lim\limits_{n\to\infty}\left(\frac{1+x}{x}\right)^{ax} = \int_0^{+\infty} \frac{x}{1+x^4}dx$，求 a

解：$\lim\limits_{n\to\infty}\left(\frac{1+x}{x}\right)^{ax} = \lim\limits_{n\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right)^a = e^a$

$\int_0^{+\infty} \frac{x}{1+x^4}dx = \frac{1}{2}\int_0^{+\infty} \frac{1}{1+x^4}dx^2 = \frac{1}{2}\arctan x^2\Big|_0^{+\infty}$

$= \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$

$e^a = \frac{\pi}{4}$ $\quad a = \ln\frac{\pi}{4}$

8.15 求反常积分 $\int_0^{+\infty}\frac{1}{(1+x^2)(1+x^\alpha)}dx \quad (\alpha\neq 0)$

解: $\int_0^{+\infty}\frac{1}{(1+x^2)(1+x^\alpha)}dx$

令 $x=\frac{1}{t}$

$$\underbrace{\int_0^{+\infty}\frac{1}{(1+x^2)(1+x^\alpha)}}dx=\int_0^{+\infty}\frac{t^\alpha}{(1+t^2)(1+t^\alpha)}dt$$

$$=\int_0^{+\infty}\underbrace{\frac{x^\alpha}{(1+x^2)(1+x^\alpha)}dx}$$

$$=\frac{1}{2}\int_0^{+\infty}\frac{1+x^\alpha}{(1+x^2)(1+x^\alpha)}dx$$

$$=\frac{1}{2}\int_0^{+\infty}\frac{1}{1+x^2}dx$$

$$=\frac{1}{2}\arctan x\Big|_0^{+\infty}=\frac{\pi}{4}$$

$$I=\int_0^{+\infty}\frac{1}{(1+x^2)(1+x^\alpha)}dx=\int_0^{+\infty}\frac{x^\alpha}{(1+x^2)(1+x^\alpha)}dx$$

$$I=\frac{1}{2}\int_0^{+\infty}\frac{1+x^\alpha}{(1+x^2)(1+x^\alpha)}dx$$