Labor Economics Homework 4

Due:

1. Occpational Choice Model

Suppose a worker i chooses an occupation $j \in \{0, 1, \ldots, J\}$ to maximize the utility function

$$u_i(j) = \alpha_j + \epsilon_{ij},$$

where $\epsilon_{ij} \overset{iid}{\sim} T1EV$ across workers and occupations that is observed only by the individual but not us. α_j is the parameter that we want to estimate. We normalize $\alpha_0 = 0$.

1. What is the probability of observing a worker in occupation j?
2. Use the parameters $(\alpha_1, \alpha_2, \alpha_3) = (0.2, 0.3, -0.1)$ to simulate the data for 1000 workers and $J = 3$ occupations.
3. Derive the likelihood function for the parameters α_j.
4. Estimate the parameters α_j via MLE. Can you recover the true parameters?
5. Compute $\Pr(D_i = 1) / \Pr(D_i = 2)$ for the cases $J = 2$ and $J = 3$. What do you find?

Solution. 1.1.

By the $T1EV$ assumption, we have

$$\Pr(D_i = j) = \Pr(\alpha_j + \epsilon_{ij} > \alpha_k + \epsilon_{ik} \text{ for all } k \neq j) = \frac{\exp(\alpha_j)}{\sum_{k=0}^{J} \exp(\alpha_k)}.$$

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Solution. 1.2, 1.4.

See the Julia code file. ■

Solution. 1.3.
The likelihood function is

$$L(\alpha_1, \alpha_2, \alpha_3) = \prod_{i=1}^{1000} \prod_{j=0}^{J} \Pr(D_i = j)^{\mathbb{1}\{D_i = j\}} = \prod_{i=1}^{1000} \prod_{j=0}^{J} \left\{ \frac{\exp(\alpha_j)}{\sum_{k=0}^{J} \exp(\alpha_k)} \right\}^{\mathbb{1}\{D_i = j\}}.$$

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Solution. 1.5.
By 1.1, for $J = 2$, we have

$$\frac{\Pr(D_i = 1)}{\Pr(D_i = 2)} = \frac{\exp(\alpha_1)}{\exp(\alpha_2)} = \exp(\alpha_1 - \alpha_2).$$

For $J = 3$, we have

$$\frac{\Pr(D_i = 1)}{\Pr(D_i = 2)} = \frac{\exp(\alpha_1)}{\exp(\alpha_2)} = \exp(\alpha_1 - \alpha_2).$$

Notice that they are the same. This is called the IIA assumption. One should be careful when using the multinomial logit model despite its convenience. ■

2. Inventory Dynamics

Consider a firm has a warehouse to store its inventory with a capacity of S units. The firm faces demand D_t in each period t. In each period, the firm can choose to restock ($s_t = 1$) the inventory to S units or not ($s_t = 0$). The profit flow is

$$\pi(s_t, I_t, \epsilon_{1t}, \epsilon_{0t}) = \begin{cases} pD_t - cS - h(S - I_t) + \epsilon_{1t} & \text{if } s_t = 1, \\ p \min\{D_t, I_t\} - cI_t + \epsilon_{0t} & \text{if } s_t = 0, \end{cases}$$

where I_t is the inventory level at the beginning of period t, p is the price of the product, c is the cost of holding one unit of inventory, ϵ_{st} are random terms iid across choice and time following $T1EV$ distribution. Notice that ϵ_{st} is observed by the firm but not us. h is the cost of restocking with

$$h(x) = \theta_0 + \theta_1 x.$$

The law of motion for the inventory level is

$$I_{t+1} = \begin{cases} S & \text{if } s_t = 1, \\ \max\{0, I_t - D_t\} & \text{if } s_t = 0. \end{cases}$$

1. Assume that $D_t \overset{iid}{\sim} BetaBinomial(\alpha, \beta, S)$, $S = 20$. Generate the transition array Q for the inventory level I_t.

2. Let the discount factor be $\rho \in (0, 1)$. Write down the Bellman equation for the firm's problem.

3. Given the parameters $(p, c, \theta_0, \theta_1, \alpha, \beta, \rho) = (1.3, 1.0, 3.0, 0.7, 5.0, 2.0, 0.95)$, solve the firm's policy function using the value function iteration.

4. Given the same parameters, solve the firm's policy function using the policy function iteration.
   (Hint: In T_σ step, $v_\sigma = (I - \rho Q_\sigma)^{-1} \pi_\sigma$, where Q_σ is the transition matrix given the policy and π_σ is the profit flow given the policy.)

5. Simulate the data $\{(\min\{D_t, I_t\}, I_t, s_t)\}_{t=1}^{500}$. Estimate the transition matrix parameters α and β for the inventory level I_t using the data.

6. Estimate the parameters $(p, c, \theta_0, \theta_1, \rho)$ using NFXP algorithm.

7. Estimate the parameters $(p, c, \theta_0, \theta_1, \rho)$ using CCP.

8. Simulate and plot the inventory level I_t against t for 100 periods for different price levels $p \in \{1.1, 1.3, 1.5\}$ and storage cost $c \in \{0.8, 1.0, 1.2\}$. Explain your findings.