Labor Economics Homework 2

Due:

1. Robinson Crusoe's Coconuts Revisited

Recall our settings from homework 1. This time, Robinson Crusoe's utility is a CRRA function

$$u(c;\gamma) = \lim_{\gamma' \to \gamma} \frac{c^{1-\gamma'}}{1-\gamma'}$$

with $\gamma \geq 0$. At the beginning of each period t, Crusoe owns y_t coconuts. He can either consume them or plant them. If he plants k_{t+1} coconuts, he will obtain $y_{t+1} = z_{t+1} f(k_{t+1})$ coconuts in the next period, where $z_{t+1} \overset{iid}{\sim} Lognormal(\mu, \sigma^2)$. The coconuts rot after each period, thus k_t is fully depreciated. We assume that $f(k) = k^\alpha$ with $\alpha \in (0,1)$. With discount factor $\beta \in (0,1)$, Crusoe's problem is

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0\left[\beta^t u(c_t)\right] \quad \text{s.t.} \quad c_t + k_{t+1} = y_t \quad \text{and} \quad y_{t+1} = z_{t+1} f(k_{t+1})$$

1. Write down the Bellman equation and define the Bellman operator.

2. Show that the Bellman operator is a contraction mapping.
   (Hint: Verify the Blackwell's sufficient conditions.)

3. Given $\alpha = 0.8$, $\beta = 0.96$, $\gamma = 1$, $\mu = 0.0$, $\sigma = 0.3$. Solve the model by value function iteration for $y \in (0, 10)$.
   (Hint: You may use the linear interpolation and monte carlo simulation to approximate the expectation of v.)

4. The model has a famous closed-form solution with $\gamma = 1$:

   $$c^*(y) = (1 - \alpha\beta)y.$$

Plot the consumption policies for the analytical and numerical solutions. Does your the numerical solution fit the analytical solution well?

5. An agent is called **prudent** if $u'''(\cdot) > 0$. Show that an agent with CRRA utility is prudent if $\gamma > 0$. Now solve the model for $\sigma = 0.1, 0.015, 0.2, 0.25$ and 0.3 with $\gamma = 1.5$. Plot the consumption policies for different σ. What does the prudent preference mean? Explain your findings.