Labor Economics Homework 3

Due:

1. Estimation of McCall's Job Search Model

Recall our settings from homework 1. An unemployed worker solves the following Bellman equation:

$$v(w) = \max_s sw + (1-s)\left[\underline{c} + \rho \int v(w')F(dw')\right],$$

where F is the CDF of the wage offer distribution $Beta(\alpha, \beta)$.

1. Use the parameters in homework 1.3 to simulate the data for one period, i.e., $\{(w_i, s_i)\}_{i=1}^{1000}$. Note that we do not observe the wage if $s_i = 0$.

2. Compute the mean wage among those who are employed. Is the mean wage you computed close with the mean wage in the distribution of wage offers?
   (Hint: $X \overset{iid}{\sim} Beta(\alpha, \beta)$ implies $\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta}$.)

3. Estimate the wage distribution parameters α and β by MLE.
   (Hint: The observed wage follows a truncated beta distribution.)

4. We do know $\underline{c} = 0.1$ since it is the unemployment benefit. Estimate ρ.

Solution. 1.1, 1.2, 1.4.
See the Julia code file.

Solution. 1.3.
Notice that the reservation wage $\bar{w}$ is the threshold. The MLE estimator is

$$\arg\max_{\alpha,\beta,\bar{w}} \sum_{i=1}^{1000} s_i \log \frac{f(w_i)}{1-F(\bar{w})} + (1-s_i)\log F(\bar{w}),$$

where f is the PDF of the beta distribution and F is the CDF. Implementation is in the Julia code file.

2. Estimation of Crusoe's Coconuts Model

Recall our settings from homework 2. Crusoe solves the following Bellman equation:

$$v(y) = \max_c u(c) + \beta \int v(y')F(dy' \mid y, c),$$

where $F(dy' \mid y, c)$ is the transition probability from y to y' given c, with the law of motion $y' = zf(y-c)$, $f(k) = k^\alpha$ and $\log z \overset{iid}{\sim} N(0, \sigma^2)$.

1. Use the parameters in homework 2.3 to simulate the time series data $\{(y_t, k_t, c_t)\}_{t=1}^{200}$.
2. Estimate the production process parameters α, μ and σ.
3. Prove that β and γ are not jointly identified.
4. Now suppose that you know $\beta = 0.96$. Estimate γ by SMM. What moment conditions would you use?

Solution. 1.1, 1.2.
See the Julia code file.

Solution. 1.3.
Notice that the Bellman equation is

Solution. 1.4.
There are many possible moment conditions. One possible choice is

$$\mathbb{E}[y_{t+1} - \gamma y_t] = 0.$$

Implementation is in the Julia code file.