

# Labor Economics Homework 2

Due:

## 1. Robinson Crusoe's Coconuts Revisited

Recall our settings from homework 1. This time, Robinson Crusoe's utility is a CRRA function

$$u(c; \gamma) = \lim_{\gamma' \rightarrow \gamma} \frac{c^{1-\gamma'}}{1-\gamma'}$$

with  $\gamma \geq 0$ . At the beginning of each period  $t$ , Crusoe owns  $y_t$  coconuts. He can either consume them or plant them. If he plants  $k_{t+1}$  coconuts, he will obtain  $y_{t+1} = z_{t+1}f(k_{t+1})$  coconuts in the next period, where  $z_{t+1} \stackrel{iid}{\sim} \text{Lognormal}(\mu, \sigma^2)$ . The coconuts rot after each period, thus  $k_t$  is fully depreciated. We assume that  $f(k) = k^\alpha$  with  $\alpha \in (0, 1)$ . With discount factor  $\beta \in (0, 1)$ , Crusoe's problem is

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 [\beta^t u(c_t)] \quad \text{s.t.} \quad c_t + k_{t+1} = y_t \quad \text{and} \quad y_{t+1} = z_{t+1}f(k_{t+1})$$

1. Write down the Bellman equation and define the Bellman operator.
2. Show that the Bellman operator is a contraction mapping.  
(Hint: Verify the Blackwell's sufficient conditions.)
3. Given  $\alpha = 0.8$ ,  $\beta = 0.96$ ,  $\gamma = 1$ ,  $\mu = 0.0$ ,  $\sigma = 0.3$ . Solve the model by value function iteration for  $y \in (0, 10)$ .  
(Hint: You may use the linear interpolation and monte carlo simulation to approximate the expectation of  $v$ .)
4. The model has a famous closed-form solution with  $\gamma = 1$ :

$$c^*(y) = (1 - \alpha\beta)y.$$

Plot the consumption policies for the analytical and numerical solutions. Does your the numerical solution fit the analytical solution well?

5. An agent is **prudent** if  $u'''(\cdot) > 0$ . Show that an agent with CRRA utility is prudent if  $\gamma > 0$ . Now solve the model for  $\sigma = 0.1, 0.015, 0.2, 0.25$  and  $0.3$  with  $\gamma = 1.5$ . Plot the consumption policies for different  $\sigma$ . What does the prudent preference mean? Explain your findings.