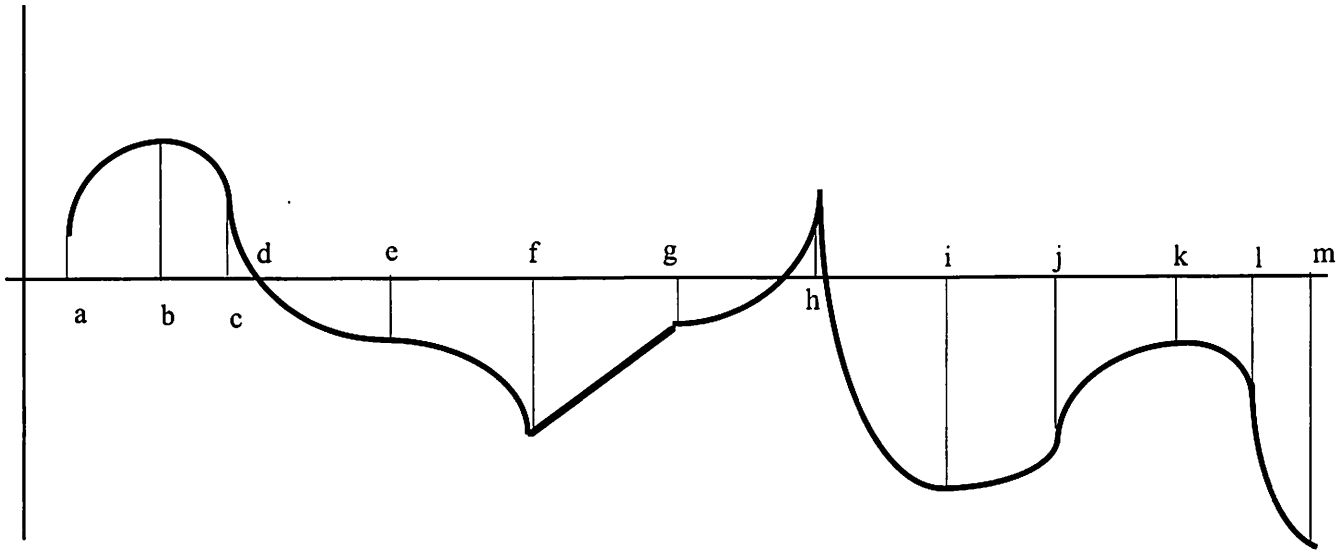
Function Analysis - Homework

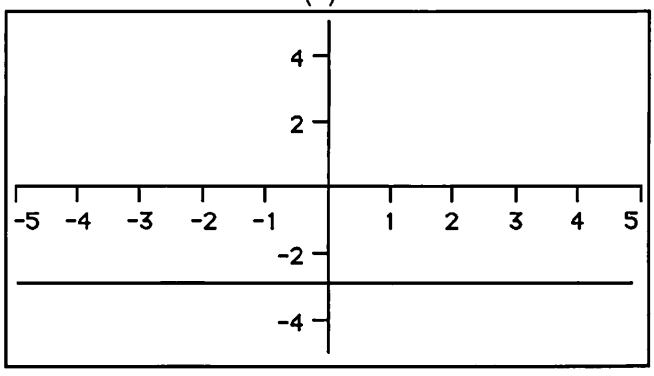


1. For each term, determine if it is applicable at the x-values a - m.

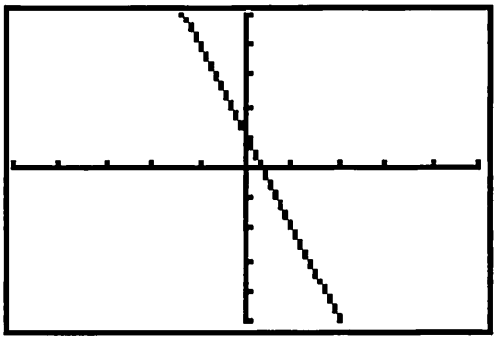
	Critical Point	Relative Minimum	Relative Maximum	Stationary Point	Inflection Point	Absolute Minimum	Absolute Maximum
a	x						
b	x		x	x			x
c	x				x		
d							
e	x			x	x		
f	x	x					
g	x						
h	x		x				
i	x	x		x			
j	x				x		
k	x		x	x			
l	x				x		
m						x	

2) You are given a graph of $f'(x)$. Draw a picture of a possible $f(x)$.

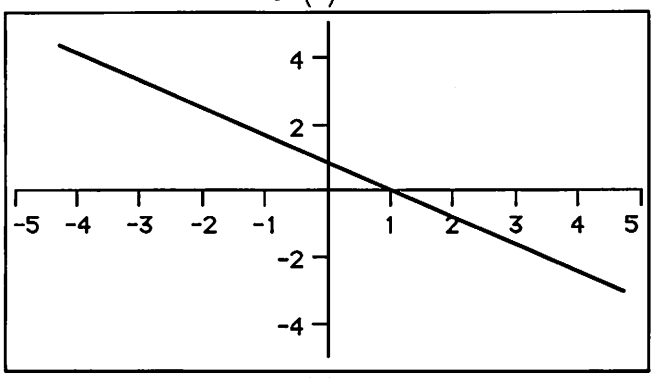
a. $f'(x)$



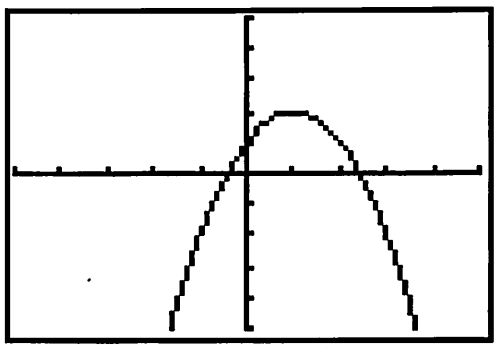
possible $f(x)$



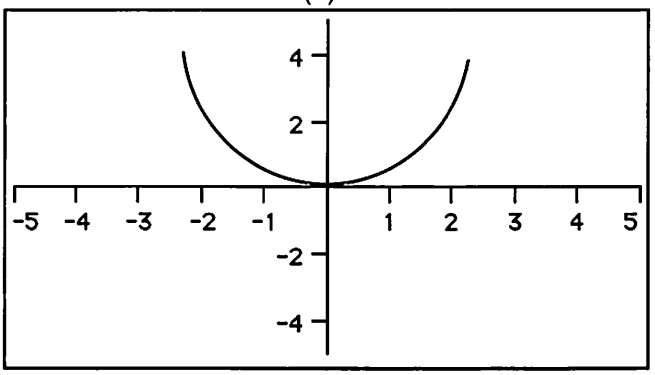
b. $f'(x)$



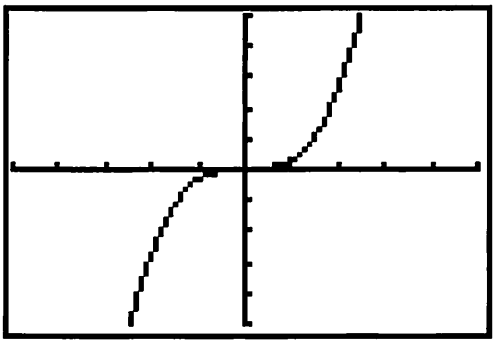
possible $f(x)$



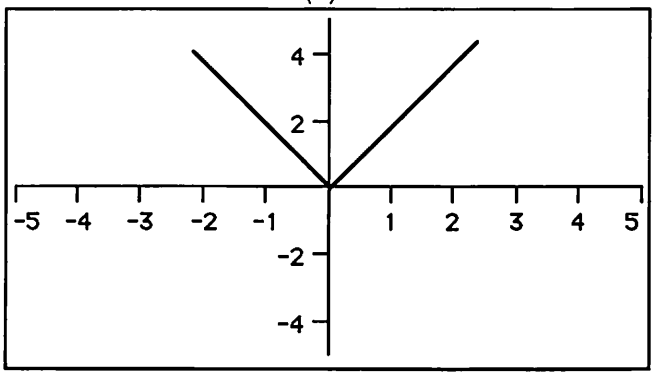
c. $f'(x)$



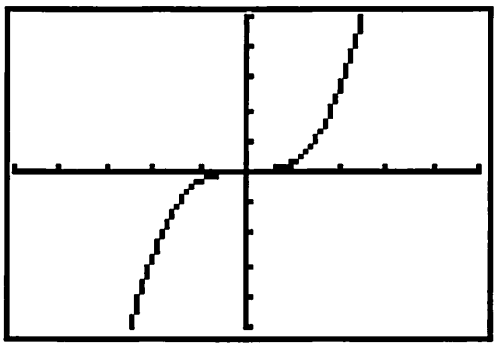
possible $f(x)$



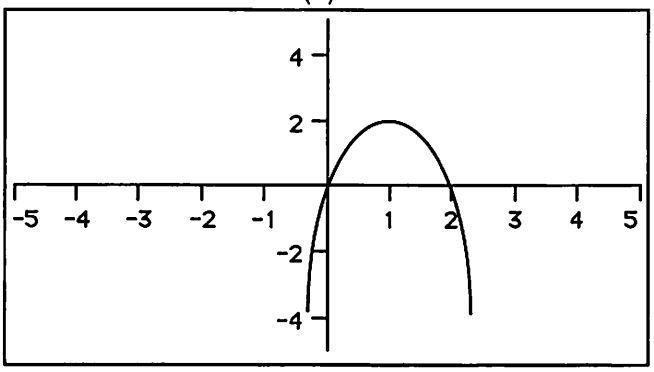
d. $f'(x)$



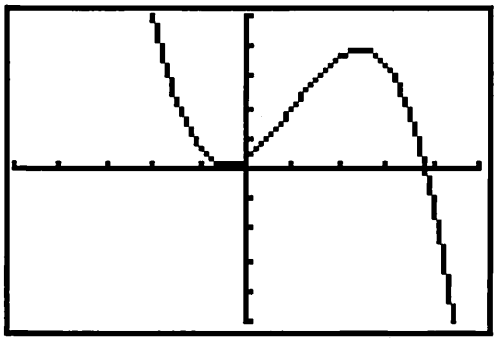
possible $f(x)$



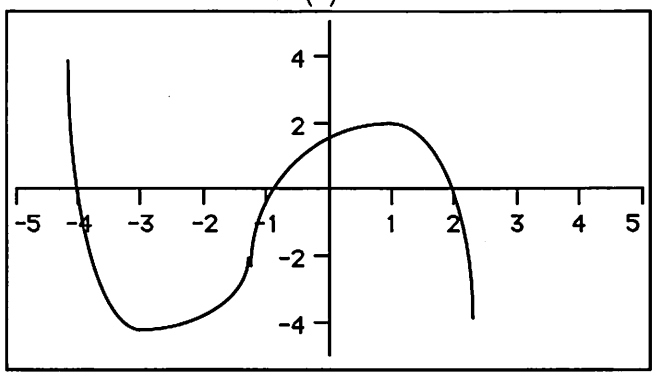
e. $f'(x)$



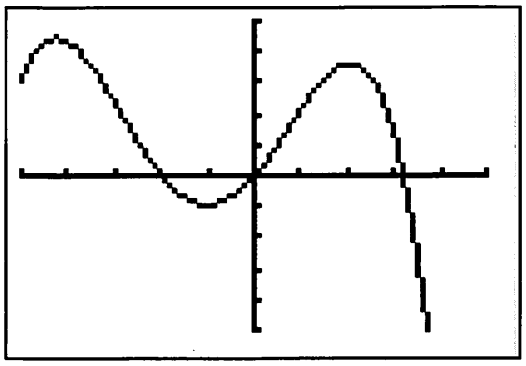
possible $f(x)$



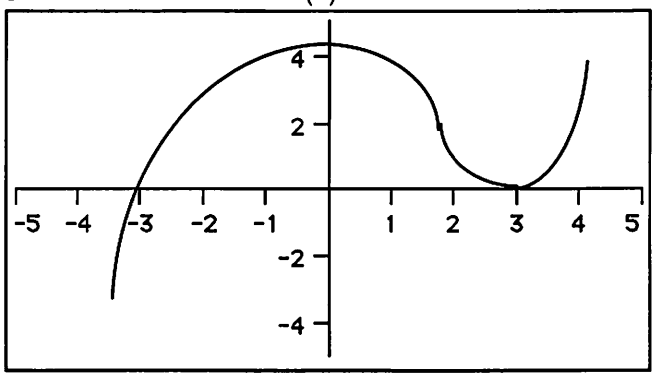
f. $f'(x)$



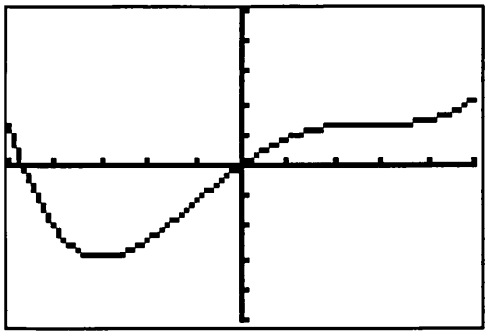
possible $f(x)$.



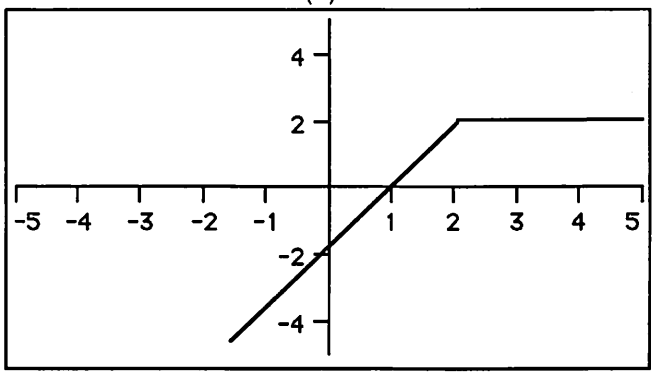
g. $f'(x)$



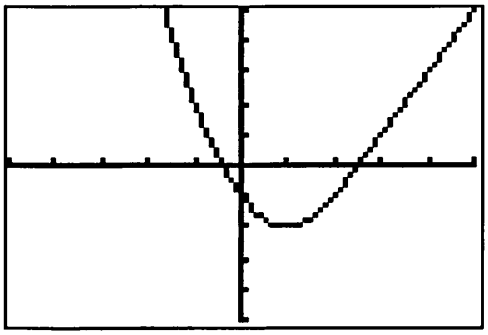
possible $f(x)$.



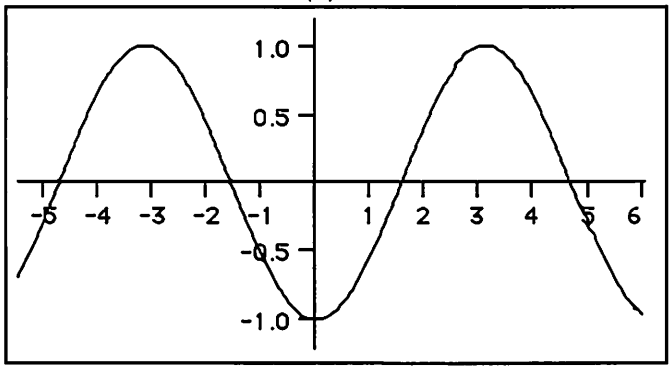
h. $f'(x)$



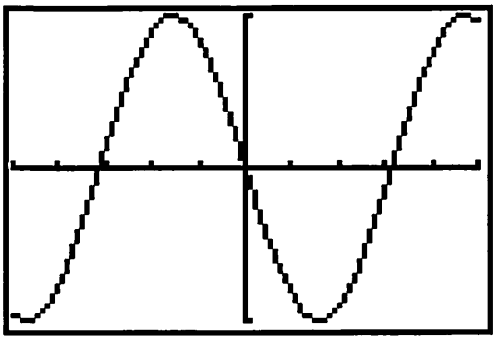
possible $f(x)$.



i. $f'(x)$

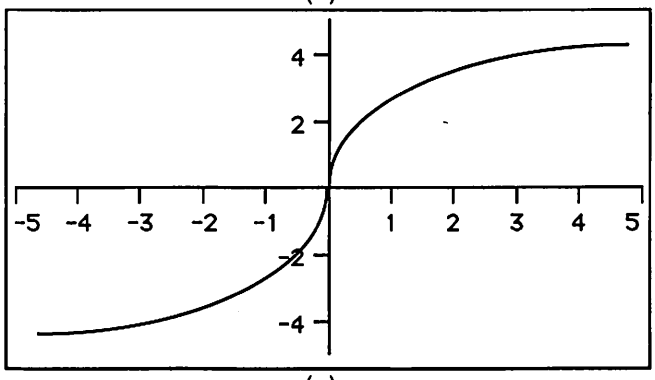


possible $f(x)$.

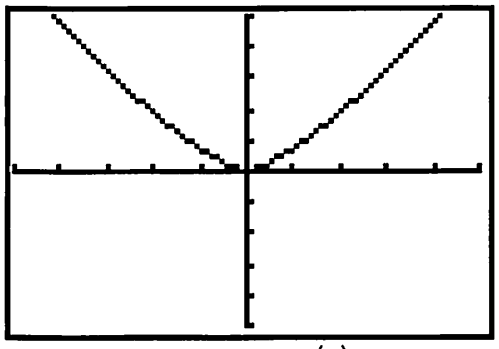


3) You are given a graph of $f'(x)$. Draw a picture of a possible $f(x)$.

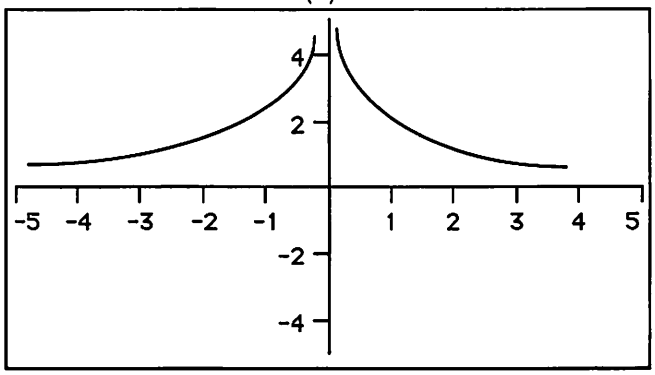
a. $f'(x)$



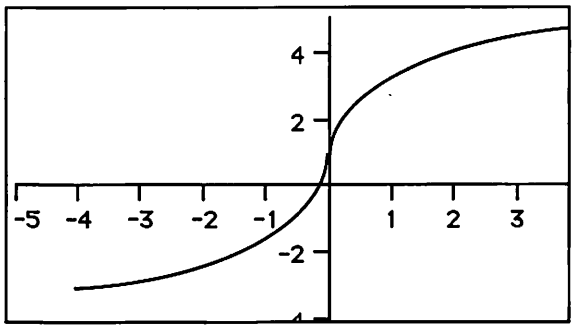
possible $f(x)$



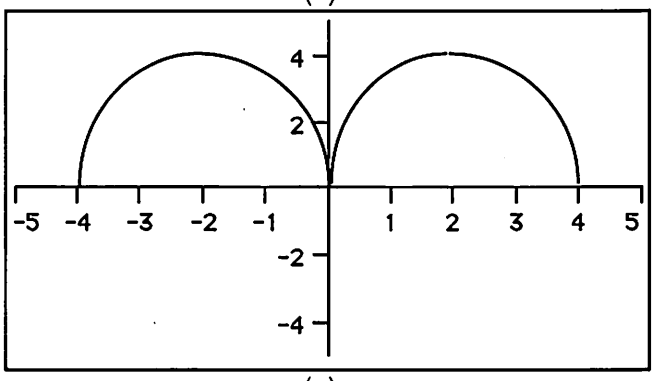
b. $f'(x)$



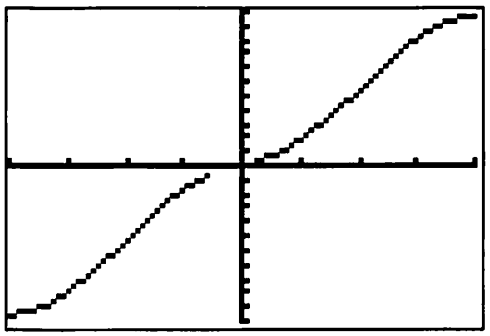
possible $f(x)$



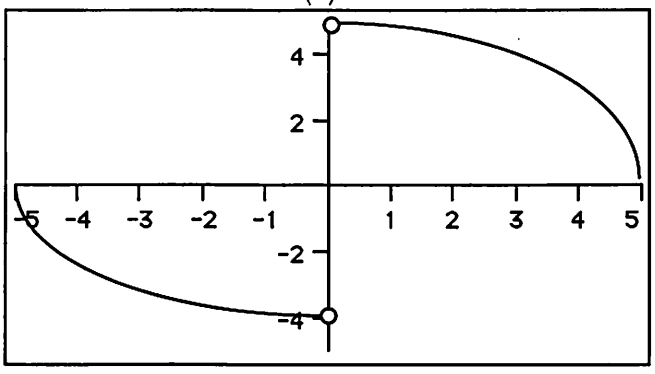
c. $f'(x)$



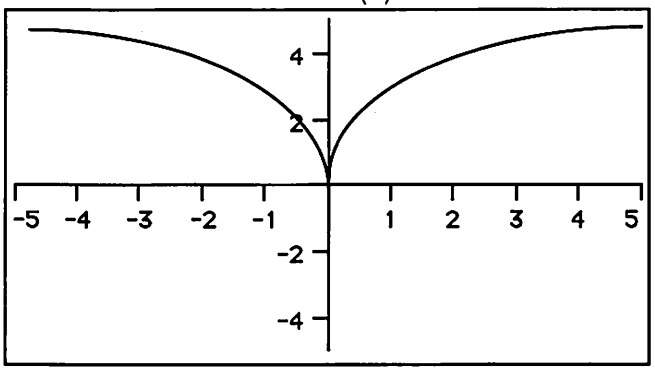
possible $f(x)$



d. $f'(x)$

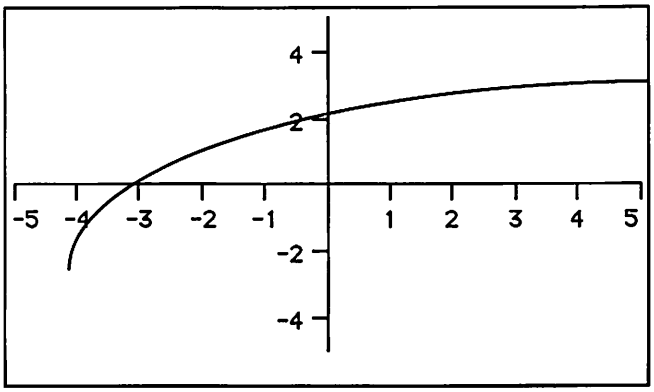


possible $f(x)$

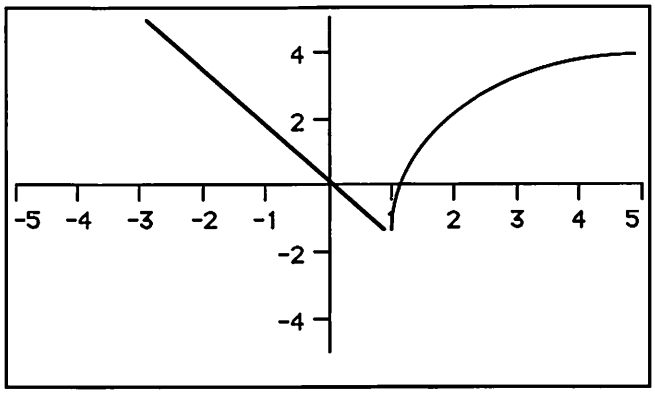


4) Sketch a possible $f(x)$ given the following information.

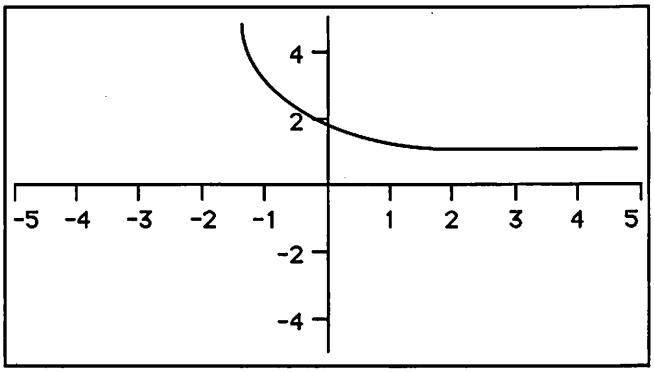
a. $f'(x) > 0, \quad f''(x) < 0$
$f(0) = 2$



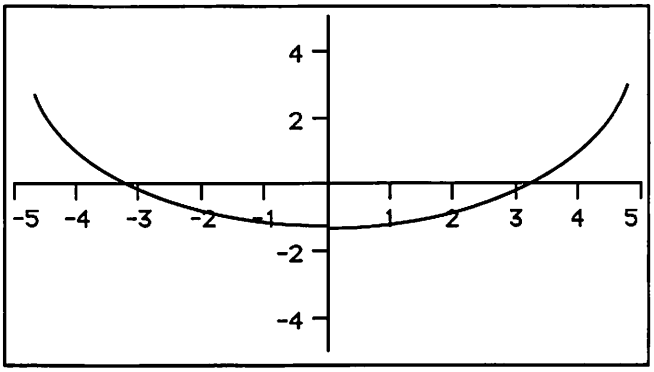
b. $f'(x) > 0, x > 1, \quad f'(x) = -1, x < 1$
$f(1) = -1 \quad \lim_{x \to \infty} f(x) = 4$



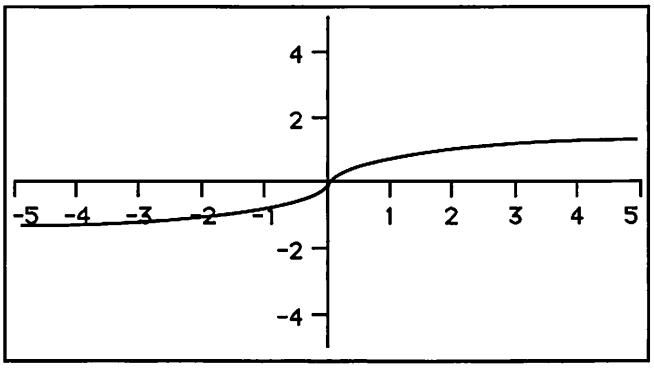
c. $f'(x) < 0, x < 2 \quad f''(x) > 0, x < 2$
$f(x) = 1, x \geq 2 \quad y - \text{intercept} = 2$



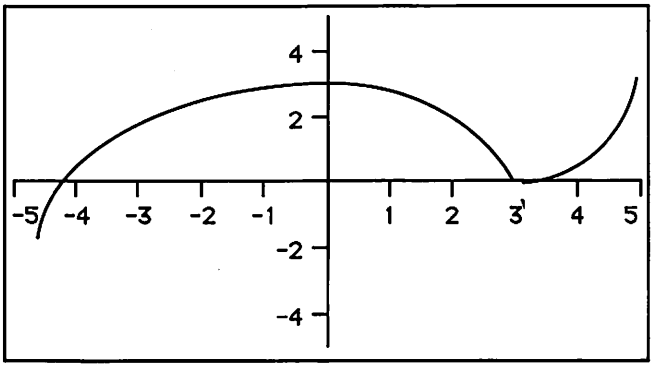
d. $f'(x) < 0, x < 0 \quad f'(x) > 0, x > 0$
$f''(x) > 0 \quad f(0) = -1$



e. $f'(x) > 0 \quad f(0) = 0$
$\lim_{x \to \infty} f(x) = 1 \quad \lim_{x \to -\infty} f(x) = -1$



f. $f'(x) > 0, x < 0 \quad f'(x) > 0, x > 3 \quad f'(x) < 0, 0 < x < 3$
$f'(0) = 0 \quad f(0) = 3 \quad f(3) = 0$
$f''(x) < 0, x < 3 \quad f''(x) > 0, x > 3$



$f'(x)<0, x<0 \quad f'(x)<0, x>3 \quad f'(x)>0, 0<x<3$

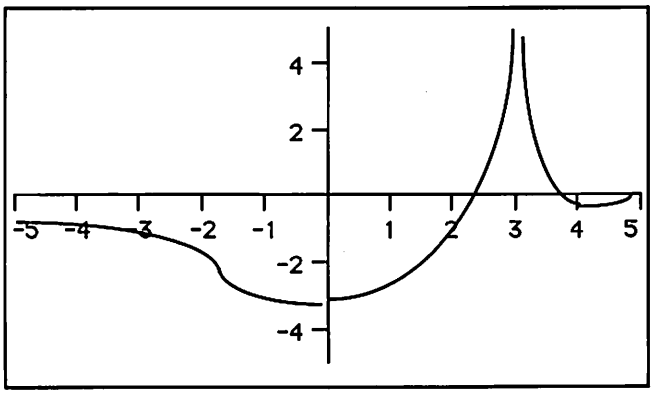
g. $f'(0)=0 \quad f(0)=-2 \quad f''(-2)=0$

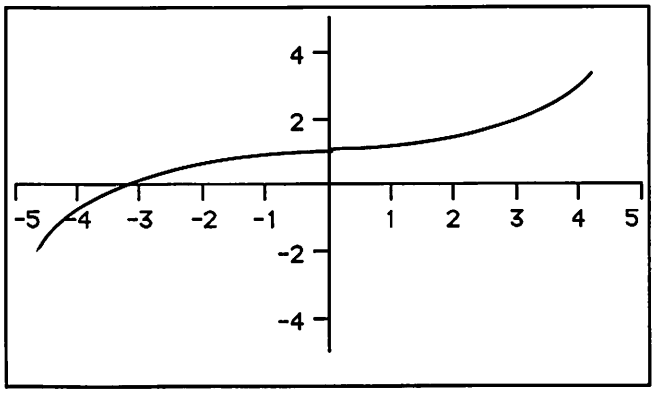
$\lim_{x\to\pm\infty} f(x)=0 \quad \lim_{x\to 3} f(x)=\infty$

$f'(x)>0, x\neq 0 \quad f'(0)=0$

h. $f''(x)<0, x<0 \quad f''(x)>0, x>0$

$f(0)=1$





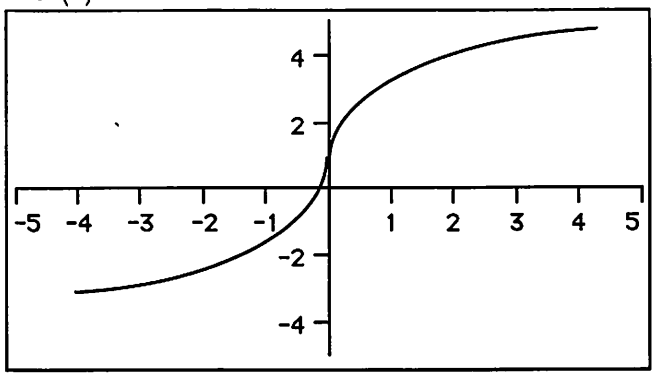
$f'(x)>0, x\neq 0 \quad f'(0)$ DNE

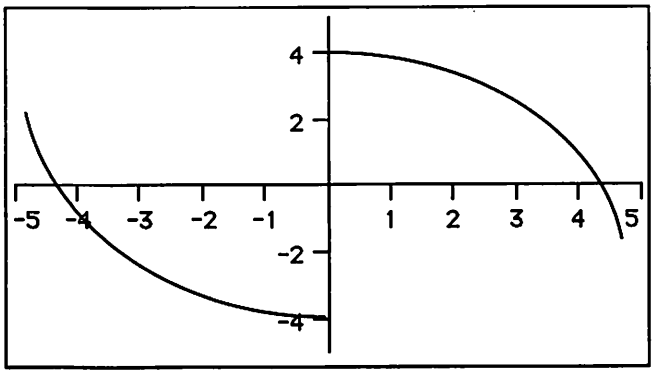
i. $f''(x)>0, x<0 \quad f''(x)<0, x>0$

$f(0)=1$

$f'(x)<0, x>0 \quad f''(x)<0, x>0$

j. $\lim_{x\to 0^+} f(x)=4 \quad f$ is symmetric to the origin





5) Find all points of relative maximum and relative minimum and points of inflection if any. Justify your answers. Confirm by calculator.

a. $f(x)=x^2-8x+4$

$f'(x)=2x-8$

$2x-8=0 \Rightarrow x=4$

$(4,-12)$ - rel min

$f''(x)=2>0$

no inflection pts

b. $f(x)=1+12x-3x^2-2x^3$

$f'(x)=12-6x-6x^2$

$-6(x^2+x-2)=0 \Rightarrow x=1,-2$

$(-2,-19)$ - rel min

$(1,8)$ - rel max

$f''(x)=-6-12x$

$6x=12 \Rightarrow x=-5$

$(-5,-55)$ - infl. pt.

c. $f(x)=(2x-5)^3$

$f'(x)=6(2x-5)^2$

$6(2x-5)^2=0 \Rightarrow x=\frac{5}{2}$

No extrema

$f''(x)=24(2x-5)^2$

$24(2x-5)^2=0 \Rightarrow x=\frac{5}{2}$

$\left(\frac{5}{2},0\right)$ - infl. pt.

d. $f(x)=3\sqrt[3]{x}-2$

$f'(x)=\dfrac{1}{x^{2/3}}$

$x^{2/3}=0\Rightarrow x=0$

No extrema

$f''(x)=\dfrac{2}{3x^{5/3}}$

$3x^{5/3}=0\Rightarrow x=0$

$(0,-2)$ - infl. pt.

e. $f(x)=\dfrac{x^2}{x^2-4}$ (don't do inflection pts)

$f'(x)=\dfrac{-8x}{(x^2-4)^2}$

$-8x=0\Rightarrow x=0$

$(0,0)$ - rel max

Don't do inflection pts

f. $f(x)=\sin^2 x+\sin x \quad [0,2\pi]$

(don't do inflection pts)

$f'(x)=2\sin x\cos x+\cos x$

$\cos x(2\sin x+1)=0\Rightarrow x=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{7\pi}{6},\dfrac{11\pi}{6}$

$\left(\dfrac{\pi}{2},2\right),\left(\dfrac{3\pi}{2},0\right)$ - rel max

$\left(\dfrac{7\pi}{6},-\dfrac{1}{4}\right),\left(\dfrac{11\pi}{6},-\dfrac{1}{4}\right)$ - rel min

g. $f(x)=x-\cos x \quad [0,2\pi]$

$f'(x)=1+\sin x$

$1+\sin x=0\Rightarrow x=\dfrac{3\pi}{2},\dfrac{7\pi}{2},\dfrac{11\pi}{2}$K

No relative extrema

$f''(x)=\cos x$

$\cos=0\Rightarrow x=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2}$

$\left(\dfrac{\pi}{2},\dfrac{\pi}{2}\right),\left(\dfrac{3\pi}{2},\dfrac{3\pi}{2}\right),\left(\dfrac{3\pi}{2},\dfrac{3\pi}{2}\right)$ - infl. pt.

h. $f(x)=x\sqrt{x+1}$ (don't do inflection pts)

$f'(x)=\dfrac{3x+2}{x\sqrt{x+1}}$

$3x+2=0\Rightarrow x=\dfrac{-2}{3}$

$\left(\dfrac{-2}{3},\dfrac{-2}{3}\sqrt{\dfrac{1}{3}}\right)=\left(\dfrac{-2}{3},-.385\right)$, rel min

i. $f(x)=(x^2-16)^{2/3}$ (don't do inflection pts)

$f(x)=(x^2-16)^{2/3}$

$f'(x)=\dfrac{4x}{3(x^2-16)^{1/3}}$

$4x=0\Rightarrow x=0$

$(x^2-16)=0\Rightarrow x=\pm 4$

$\left(0,(-16)^{2/3}\right)=(0,6.346)$ - rel max

$(4,0),(-4,0)$ - rel min