Rolle's and the Mean Value Theorem - Homework

For the exercises below, determine whether Rolle's theorem can be applied to the function in the indicated interval. If Rolle's Theorem can be applied, find all values of x that satisfy Rolle's Theorem.

1. $f(x) = x^2 - 4x$ on $[0,4]$

$$\boxed{\begin{array}{l} 2x - 4 = 0 \\ x = 2 \end{array}}$$

2. $f(x) = x^2 - 11x + 30$ on $[5,6]$

$$\boxed{\begin{array}{l} 2x - 11 = 0 \\ x = \dfrac{11}{2} \end{array}}$$

3. $f(x) = (x-2)(x-3)(x-4)$ on $[2,4]$

$$\boxed{\begin{array}{l} x^3 - 9x^2 + 26x - 24 = 0 \\ 3x^2 - 18x + 26 = 0 \\ x = \dfrac{18 \pm \sqrt{12}}{6} = 3.577, 2.423 \end{array}}$$

4. $f(x) = (x+4)^2(x-3)$ on $[-4,3]$

$$\boxed{\begin{array}{l} x^3 + 5x^2 - 8x - 48 = 0 \\ 3x^2 + 10x - 8 = 0 \\ (x+4)(3x-2) = 0 \Rightarrow x = \dfrac{2}{3} \end{array}}$$

5. $f(x) = 4 - |x-2|$ on $[-2,2]$

$$\boxed{\text{Not differentiable at } x = 0}$$

6. $f(x) = \sin x$ on $[0, 2\pi]$

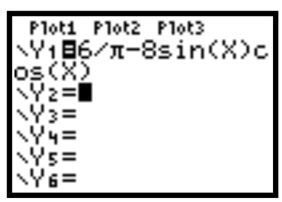
$$\boxed{\begin{array}{l} \cos x = 0 \\ x = \dfrac{\pi}{2}, \dfrac{3\pi}{2} \end{array}}$$

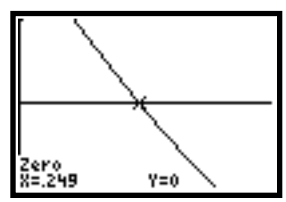
7. $f(x) = \cos 2x$ on $\left[\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right]$

$$\boxed{\begin{array}{l} -2\sin 2x = 0 \\ 2x = 0, \pi, \ldots \\ x = \dfrac{\pi}{2} \end{array}}$$

8. $f(x) = \dfrac{6x}{\pi} - 4\sin^2 x$ on $\left[0, \dfrac{\pi}{6}\right]$

$$\boxed{\dfrac{6}{\pi} - 8\sin x \cos x = 0}$$





For the exercises below, apply the mean value theorem to $f(x)$ on the indicated interval. Find all values of c which satisfy the mean value theorem.

9. $f(x) = x^2$ on $[-1,2]$

$$\boxed{\begin{array}{l} \dfrac{4-1}{3} = 2x \\ x = \dfrac{1}{2} \end{array}}$$

10. $f(x) = x^3 - x^2 - 2x$ on $[-1,1]$

$$\boxed{\begin{array}{l} \dfrac{-2-0}{2} = 3x^2 - 2x - 2 \\ 3x^2 - 2x - 1 = 0 \\ (3x+1)(x-1) = 0 \\ x = \dfrac{-1}{3} \end{array}}$$

11. $f(x)=\dfrac{x+2}{x}$ on $\left[\dfrac{1}{2},2\right]$

$$\boxed{\begin{array}{l}\dfrac{2-5}{1.5}=\dfrac{-2}{x^2}\\ -2x^2=-2\\ x=\pm 1, x=1\end{array}}$$

12. $f(x)=\sqrt{x-3}$ on $[3,7]$

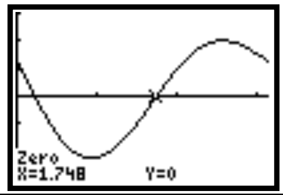
$$\boxed{\begin{array}{l}\dfrac{2-0}{4}=\dfrac{1}{2\sqrt{x-3}}\\ 4\sqrt{x-3}=4\\ (x-3)=1\\ x=4\end{array}}$$

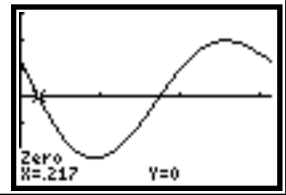
13. $f(x)=x^3$ on $[0,1]$

$$\boxed{\begin{array}{l}\dfrac{1-0}{1}=3x^2\\ x^2=\dfrac{1}{3}\\ x=\pm\sqrt{\dfrac{1}{3}}\Rightarrow x=\sqrt{\dfrac{1}{3}}\end{array}}$$

14. $f(x)=2\cos x+\cos 2x$ on $[0,\pi]$

$$\dfrac{-1-3}{\pi}=-2\sin x-2\sin 2x$$





15. A trucker handed in a ticket at a toll booth showing that in 2 hours the truck had covered 159 miles on a toll road in which the speed limit was 65 mph. The trucker was cited for speeding. Why?

Average velocity is 79.5 mph. There must be some time when the trucker was traveling at 75 mph.

16. A marathoner ran the 26.2 mile New York marathon in 2 hours, 12 minutes. Show that at least twice, the marathoner was running at exactly 11 mph.

Avg vel $=\dfrac{26.2}{2.2}=11.909$ mph. Runner had to reach 11 mph at least twice, once when speeding up to 11.909, once when slowing down

17. The order and transportation cost C of bottles of Pepsi is approximated by the function:

$$C(x)=10{,}000\left(\frac{1}{x}+\frac{x}{x+3}\right)$$ where x is the order size of bottles of Pepsi in hundreds.

According to Rolles's Theorem, the rate of change of cost must be zero for some order size in the interval [3,6]. Find that order size.

$$\frac{0-0}{6-3}=10000\left(\frac{-1}{x^2}+\frac{3}{(x+3)^2}\right)\Rightarrow\frac{1}{x^2}=\frac{3}{(x+3)^2}$$

$3x^2=x^2+6x+9\Rightarrow 2x^2-6x-9=0\Rightarrow x=4.098\ldots$order size $=$ 410

18. A car company introduces a new car for which the number of cars sold S is the function:

$$S(t)=300\left(5-\frac{9}{t+2}\right)$$ where t is the time in months.

a) Find the average rate of cars sold over the first 12 months. $\boxed{\text{96.421 cars}}$

b) During what month does the average rate of cars sold equal the rate of change of cars sales?

$$\boxed{\frac{2700}{(t+2)^2}=96.421\Rightarrow t=3.291-\text{April}}$$