(Please write your Exam Roll No.) Exam Roll No. 07476803122

# END TERM EXAMINATION

FIRST SEMESTER [B.TECH] MARCH 2023

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| Paper Code: BS-111 | Subject: Applied Mathematics-I |
| Time: 3 Hours | Maximum Marks: 75 |

**Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit. Assume missing data, if any.**

Q1 Attempt **all** questions:-

(a) If $\int_0^1 x^m dx = \frac{1}{m+1}$, thenfind the value of $\int_0^1 x^m(\log x)\, dx$. (2.5)

(b) If $\vec{F} = (x+y+1)\hat{\imath} + \hat{\jmath} - (x+y)\hat{k}$, thencompute the value of $\vec{F}.\,\mathrm{Curl}(\vec{F})$. (2.5)

(c) Find the particular integral for the linear differential equation: (2.5)

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \sin 3x.$$

(d) Determine the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 1 & 2 & 5 & 2 \\ 2 & 1 & 1 & 3 \end{bmatrix}$. (2.5)

(e) Applying Gauss divergence theorem, find the value of $\int\int_S \vec{F}.\hat{n}\; ds$, for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - xz)\hat{\jmath} + (z^2 - yx)\hat{k}$ and S is the cube $0 \le x \le 1,\; 0 \le y \le 1,\; 0 \le z \le 1.$ (2.5)

(f) Find the stationary values of the function $f(x,y) = x^3y^2(1-x-y)$. (2.5)

## UNIT-I

Q2 (a) If $u = f(r)$ and $x = r\cos(\theta),\; y = r\sin(\theta)$, then prove that (7)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

(b) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}dx$ and hence show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2.$ (8)

Q3 (a) Find the shortest and longest distances from the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24.$ (6)

(b) If $u = x^2 - y^2,\; v = 2xy$ and $x = r\cos(\theta),\; y = r\sin(\theta)$, then find the Jaccobian $J = \frac{\partial(u,v)}{\partial(r,\theta)}$. (4)

(c) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$ (5)

## UNIT-II

Q4 (a) Solve the ordinary differential equation: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$. (8)

(b) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_n(x)$ represents the Bessel function of first kind. (7)

Q5 (a) Let the electric equipotential lines (curves of constant potentials) between two concentric cylinders be given by $x^2 + y^2 = c$, where c is the constant. Find their orthogonal trajectories (known as curves of electric force). (4)

(b) Solve the ODE: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$, by variation of parameters. (6)

(c) Solve the ODE: $2y\,dx + x(2\log x - y)dy = 0$, by choosing suitable method. (5)

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## UNIT-III

Q6 (a) Test the consistency and solve the system of equations: (7)
$3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4.$

(b) Verify Cayley – Hamilton Theorem for the matrix
$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, and hence find $A^{-1}$. (8)

Q7 (a) Check whether the vectors $(2,1,1), (2,0,-1), (4,2,1)$ are Linearly dependent or independent? (6)

(b) Reduce the quadratic form $2xy + 2yz + 2zx$ into the cannonical form and discuss its nature. (9)

## UNIT-IV

Q8 (a) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$, in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1,2,1)$. (7.5)

(b) Apply Stoke's Theorem to evaluate $\int_C (y\,dx + z\,dy + x\,dz)$, where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. (7.5)

Q9 (a) Find the curvature and torsion of the Helix $x = a\cos t,\ y = a\sin t,\ z = bt.$ (7.5)

(b) Prove that $\operatorname{div}(\operatorname{grad} r^n) = \nabla^2(r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{(x^2 + y^2 + z^2)}$. (7.5)

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