DATE
PAGE NO.

Probability Space :- : A probability space models random events and is made up of three parts: Sample Space, Event space & probability function.
(Ω, A, P)

Sample Space (Ω): the set of all possible outcomes.

Event space (A): the set of all events.

probability function (P):- the assignment of probability to the event.

Q1. Find the prob. space of even no. in a throw of a dice.

Sol. $\Omega = \{1,2,3,4,5,6\}$
$A = \{2,4,6\}$

$P = \cancel{3}/\cancel{6}_2 = \frac{1}{2}$

The required prob. space is (Ω, A, P)
where $\Omega = \{1,2,3,4,5,6\}$
$A = \{2,4,6\}$
$P = \frac{1}{2}$

Q2. If we toss 3 coins simultaneously. Find the probability space of atleast 2 Heads.

Mutually Exclusive Events :- No thing common ~~even~~ in events

Mutually Exhaustive Events :- Something common in events

e.g - 1

E_1 = odd no.
E_2 = Even no.

$E_1 \cap E_2 = \phi$] Mutualy Exclusive

$E_1 \cup E_2 = S$

e.g - 2 $E_1 = \{2, 4, 6\}$
$E_2 = \{5, 3\}$
$E_3 = \{1, 2\}$

$E_1 \cap E_2 \cap E_3 = \phi$

Equally likely events :- whose probability is same.

e.g - E_1 = odd no.
E_2 = Even no.] same probability.

Conditional probability :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B - Already happened.

DATE
PAGE NO.

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

Independent Event :-

$$\boxed{P(A\cap B) = P(A)\cdot P(B)}$$

e.g :- $P(A) = \frac{1}{3}$ → problem solved by A.

$P(B) = \frac{2}{3}$ → problem solved by B.

$P(\text{problem solved}) = P(A)P(\bar{B}) + P(\bar{A})P(B) + P(A)P(B)$

Multiplicative law

$$P(A\cap B) = P(A|B)\,P(B)$$

Baye's Theorem :-

If $E_1, E_2, \ldots E_n$ are mutually exclusive & exhaustive events with $P(E_i) \neq 0$ ($i = 1, 2, \ldots n$) of a random experiment then for any arbitrary event A of the sample space of the above experiment with $P(A) \geq 0$, we have

$$P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A|E_i)}$$

proof :- Let S be the sample space of the random experiment.

The events, $E_1, E_2 — — E_n$ being exhaustive

$$S = E_1 \cup E_2 \cup ---- \cup E_n$$

$$A = A \cap S$$

$$A = A \cap (E_1 \cup E_2 \cup ..... E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) ..... (A \cap E_n)$$

(Distributive Law).

~~$P(A) = P(E_1) P(A|E_1) + P$~~

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + ---- + P(A \cap E_n)$$

$$\Rightarrow P(E_1) P(A|E_1) + P(E_2) P(A|E_2) + ---- + P(E_n) P(A|E_n)$$

$$\Rightarrow \sum_{i=1}^{n} P(E_i) P(A|E_i)$$

$$\left\{ P(A|E_1) = \frac{P(A \cap E_1)}{P(E_1)} \right.$$

$$P(E_i|A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i) P(A|E_i)}{\sum_{i=1}^{n} P(E_i) P(A|E_i)}$$

Q1. In a bolt factory, machines A, B, & C manufacture resp. 25%, 35%, & 40% of the total. On their output 5%, 4% & 2% are defective bolts. A bolt is drawn at random from the product & is found to be defective. What is the probability that it was manufactured by machine B?

Contd...

DATE

PAGE NO.

Sol.

$P(E_1) = \frac{25}{100}$, $P(E_2) = \frac{35}{100}$, $P(E_3) = \frac{40}{100}$

$P(A|E_1) = \frac{5}{100}$, $P(A|E_2) = \frac{4}{100}$,

$P(A|E_3) = \frac{2}{100}$

$$P(E_2|A) = \frac{P(E_2)\,P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

01/09/23

Q1. A university bought 45%, 25%, 30% of computers from HCL, WIPRO and IBM resp. and 2%, 3% & 1% of these were found to be defective. Find the prob. of a comp. selected at random is found to be defective. What is the prob. that the defective comp. is from HCL?

Sol.^{n}

$\underbrace{P(E_1) = \frac{45}{100}}_{HCL}$, $\underbrace{P(E_2) = \frac{25}{100}}_{WIPRO}$, $\underbrace{P(E_3) = \frac{30}{100}}_{IBM}$

$P(A|E_1) = \frac{2}{100}$, $P(A|E_2) = \frac{3}{100}$, $P(A|E_3) = \frac{1}{100}$

$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$

$\Rightarrow \frac{45}{100} \times \frac{2}{100} + \frac{25}{100} \times \frac{3}{100} + \frac{30}{100} \times \frac{1}{100}$

$= \frac{90 + 75 + 30}{10000} = \frac{195}{10000}$

$$P(E_1|A) = \frac{P(A|E_1)\ P(E_1)}{P(A)}$$

$$= \frac{\frac{45}{100} \times \frac{2}{100}}{\frac{195}{10000}} = \frac{\cancel{45}^{\cancel{9}\,3} \times 2}{\cancel{195}_{\cancel{39}\,13}} = \frac{6}{13}$$

$$= 0.46.$$

Q2. Each of the identical Boxes B_1, B_2 & B_3 contain 2 coins → B_1 contain Both gold coins
B_2 contain Both silver coin
B_3 → 1 gold, 1 silver.

If a box is chosen at random & a coin is picked at random & the coin is found to be gold, what is the prob. the other coin in the box is also gold.

Sol. $P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

$P(A)$ = prob. of finding gold

$P(A|E_1) = 1$, $P(A|E_2) = 0$, $P(A|E_3) = \frac{1}{2}$.

$$P(E_1|A) = \frac{\cancel{1/3} \times 1}{\cancel{\frac{1}{3}} \times 1 + \cancel{\frac{1}{3}} \times 0 + \cancel{\frac{1}{3} \times \frac{1}{2}}} = \frac{2}{3}.$$

Imp.

Q3. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a 6. Find the prob. that it is actually six.

Sol. $P(S) = \frac{1}{6}$, $P(S') = \frac{5}{6}$

$P(E|S) = \frac{3}{4}$, $P(E|S') = \frac{1}{4}$.

DATE
PAGE NO.

$$P(S|E) = \frac{P(S)\ P(E|S)}{P(S)\ P(E|S) + P(S')\ P(E|S')}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{3+5} = \frac{3}{8}$$

Imp.

Q4. The prob. of X, Y, & Z becoming managers of a Company are 4/9, 2/9, 1/3 resp. The prob. that the Bonus scheme will be introduced if X, Y, & Z becomes manager are 3/10, 1/2, 4/5 resp.

(a) find the prob. that bonus scheme will be introduced.

(b) If the bonus scheme has been introduced, find the prob. that manager appointed was X or Y.

Soln. $P(X) = \frac{4}{9}$, $P(Y) = \frac{2}{9}$, $P(Z) = \frac{3}{9}$

$P(B|X) = \frac{3}{10}$, $P(B/Y) = \frac{1}{2} = \frac{5}{10}$, $P(B|Z) = \frac{4}{5} = \frac{8}{10}$

a) $P(B) = \frac{4 \times 3}{90} + \frac{10}{90} + \frac{24}{90} = \frac{\cancel{46}^{23}}{\cancel{90}_{45}} = \frac{23}{45} = 0.52$

(b) $P(X \cup Y/B) = P(X|B) + P(Y/B)$

$$= \frac{P(X)\ P(B|X) + P(Y)\ P(B|Y)}{P(B)}$$

$$= \frac{\frac{4}{\cancel{9}} \times \frac{3}{\cancel{10}} + \frac{2}{\cancel{9}} \times \frac{5}{\cancel{10}}}{\frac{46}{\cancel{90}}}$$

$$= \frac{12+10}{46} = \frac{\cancel{22}^{11}}{\cancel{46}_{23}} = \frac{11}{23}$$

Q5. An insurance company insured 2000 scooters drivers, 4000 car drivers & 6000 truck drivers. The prob. of any accident involving a scooter, car & truck is 0.01, 0.03, 0.15 resp. one of the insured person meets with an accident. what is the prob. that he is a scooter driver?

Soln:- $P(S) = \frac{2000}{12000}$, $P(C) = \frac{4000}{12000}$, $P(T) = \frac{6000}{12000}$

$P(A|S) = \frac{1}{100}$, $P(A|C) = \frac{3}{100}$, $P(A|T) = \frac{15}{100}$

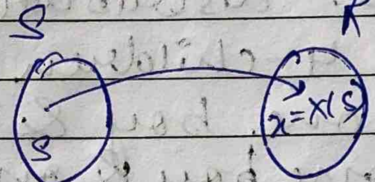
$$P(S|A) = \frac{P(A|S)\,P(S)}{P(A|S)\,P(S) + P(A|C)\,P(C) + P(A|T)\,P(T)}$$

$$\rightarrow \frac{\cancel{\frac{2000}{12000}} \times \cancel{\frac{1}{100}}}{\cancel{\frac{1}{100}} \times \cancel{\frac{2}{12}} + \cancel{\frac{3}{100}} \times \cancel{\frac{4}{12}} + \cancel{\frac{15}{100}} \times \cancel{\frac{6}{12}}} = \frac{2}{14+90}$$

Random variable :- A random variable X on a sample space S is a function

$$X : S \to R$$

$X(s) = x$ to each sample point.



Random variable

Discrete
fixed

Σ

Continuous

X	0	1	2
$P(X)$	$1/4$	$2/4$	$1/4$

→ Discrete prob. Distribution

Properties.

mean $E(x) = \Sigma x . P(x)$

1) $f(x) \geq 0$
2) $\Sigma f(x) = 1$
3) $P(X = x) = f(x)$

Binomial
Poison Dist^n

where $f(x)$ is known as probability Mass function (pmf)

→ Continuous prob. Distribution

properties

$E(x) = \int_{-\infty}^{\infty} x f(x)$

1) $f(x) \geq 0$
2) $\int_{-\infty}^{\infty} f(x) = 1$

Exponential
Normal
Uniform dist^n.

DATE
PAGE NO.

3) $P(a < x < b) = \int_a^b f(x)\,dx$

where, $f(x)$ is known as prob. density func^n. (pdf)

Q1. Find the prob. distribution & prob. that in a family of 4 children there will be (a) atleast one boy &
(b) atleast one boy & one girl, assuming that the prob. of male birth is $\frac{1}{2}$.

Sol^n

X	0	1	2	3	4
$P(X)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

(a) $P(\text{atleast } 1B) = 1 - P(\text{No Boy}) = 1 - \frac{1}{16}$

$= \frac{15}{16}$

(b) $P(\text{atleast } 1B \,\&\, 1G) =$
$P(1) + P(2) + P(3)$
$\Rightarrow \frac{14}{16}$

Q2. Out of 24 eggs, 6 are rotten, If 2 eggs are drawn at random, find the prob. distribution of the no. of rotten eggs that can be drawn.

Sol^n

X	0	1	2
$P(X)$			

$\frac{51}{92}$

$P(X=0) = \frac{{}^{18}C_2}{{}^{24}C_2} \Rightarrow \frac{\frac{18 \times 17 \times \cancel{16!}}{\cancel{16!}}}{\frac{24 \times 23 \times \cancel{22!}}{\cancel{22!}}} \Rightarrow \frac{\overset{3}{\cancel{18}} \times 17}{\underset{4}{\cancel{24}} \times 23}$

DATE ______

PAGE NO. ______

$$P(X=1) = \frac{{}^{18}C_1 \times {}^{6}C_1}{{}^{24}C_2} \Rightarrow \frac{\frac{18 \times \cancel{17!}}{\cancel{17!}} \times \frac{6 \times \cancel{5!}}{\cancel{5!}}}{\frac{24 \times 23 \times \cancel{22!}}{\cancel{22!}}}$$

$$\Rightarrow \frac{18 \times 6}{24 \times 23} = \frac{36}{92}$$

$$P(X=2) = \frac{{}^{6}C_2}{{}^{24}C_2} \Rightarrow \frac{\frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}}}{\frac{24 \times 23 \times \cancel{22!}}{\cancel{22!}}} = \frac{\cancel{6} \times 5}{\cancel{24}_{4} \times 23}$$

$$\Rightarrow \frac{5}{92}$$

Probability Distribution

pmf condition
1) $p_i \geq 0$
2) $\sum p_i = 1$

- Discrete probability Distribution
i) $P(x_i) = f(x_i) \geq 0$
ii) $\sum P(x_i) = 1$

- Continuous probability Distribution.

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx.$$

i) $0 \leq F(x) \leq 1$
ii) $F'(x) = f(x)$
iii) $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

pdf
condition
1) $f(x) \geq 0, -\infty < x < \infty$
2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$P(a \leq X \leq b) = \int_a^b f(x)\,dx$

Ques: A random variable X has the following probability func^n.

x	0	1	2	3	4	5	6	7
$P(x)$	0	R	$2R$	$2R$	$3R$	R^2	$2R^2$	$7R^2+R$

i) find R
ii) Evaluate $P(x<6)$, $P(X \geq 6)$, $P(3 < x \leq 6)$
iii) Find the minimum value of x so that $P(X \leq x) > \frac{1}{2}$.

Sol^n. $\sum p(x) = 1$

$R + 2R + 2R + 3R + R^2 + 2R^2 + 7R^2 + R = 1$

$9R + 10R^2 = 1$

~~$K(9+10K) = 1$~~

~~$9+10K = 1$~~

$9R + 10R^2 - 1 = 0$

$10R^2 + 9R - 1 = 0$

$10R^2 + 10R - R - 1 = 0$

$10R(R+1) - 1(R+1) = 0$

$(R+1)(10R-1) = 0$

DATE

PAGE NO.

$R = \frac{1}{10}, -1$ $\boxed{R \neq -1}$

so, $\boxed{R = \frac{1}{10}}$

ii) $P(x<6) = P(x) - P \rightarrow 1 - P(x \geq 6)$

$= (1 - 0.19) = 0.81$

$P(x \geq 6) = P(6) + P(7) = 0.02 + 0.17 = 0.19$

$P(3 < x \leq 6) = 0.37$

$P(x \leq 1) = P(x=0) + P(x=1) = 0 + \frac{1}{10} = \frac{1}{10} < \frac{1}{2}$

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2) = 0 + \frac{1}{2} + \frac{2}{10} = \frac{3}{10} < \frac{1}{2}$

$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{8}{10}\frac{1}{2} = \frac{1}{2}$

minimum value of x is 4.

Ques. If the func^n defined as follows a density func^n.

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Distribution func^n

if so, find, $P(1 \leq x \leq 2)$

Sol. $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-x}^{0} 0 \cdot dx + \int_{0}^{\infty} e^{-x} dx = 1$

$\int_{1}^{2} e^{-x} dx = -[e^{-x}]_1^2 = -[e^{-2} - e^{-1}]$

Gamma fun^n $\int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma n}{a^n}$

DATE

PAGE NO.

Ques. 'X' is acts a 'r.v.' with pdf given.

$$f(x) = \begin{cases} Rx, & \text{if } 0 \le x < 2 \\ 2R, & \text{if } 2 \le x < 4. \\ -Rx+6R, & \text{if } 4 \le x < 6. \end{cases}$$

Find R & mean value of X.

Sol^n

$$\int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{-\infty}^{0} 0.dx + \int_0^2 Rx.dx + \int_2^4 2R\,dx + \int_4^6 (-Rx+6R)dx + \int_6^\infty 0.dx = 1$$

$$0 + \left[\frac{x^2}{2}\right]_0^2 + 2R[x]_2^4 + \int_4^6 -Rx\,dx + \int_4^6 6R\,dx = 1$$

$$R[2] + 4R - R\left[\frac{x^2}{2}\right]_4^6 + 6R[x]_4^6 = 1$$

$$2R + 4R - R[18-8] + 6R[x]_4^6 = 1$$

$$2R + 4R - 10R + 12R = 1$$

$$K = \frac{1}{8}.$$

$E(\phi(x)) \longrightarrow$ Expected value or mean

D.R.V: $\sum_x \phi(x).P(x)$

C.R.V: $\int_{-\infty}^{\infty} \phi(x) f(x) dx$

If $\phi(x) = x$,

Mean = $E(x)$

D.R.V: $E(x) = \sum_x x P(x)$

C.R.V: $E(x) = \int_{-\infty}^{\infty} x.f(x).dx$

$E(\bar{x}) = \bar{x}$

variance $= E(x-\bar{x})^2$

$= E(x^2) - [E(x)]^2$

$\downarrow$

σ_X^2

DATE 12/09/23
PAGE NO.

Mean & variance of Random variable:

Discrete prob. Distribution:

X	x_1	x_2	- - -	x_n
P(X)	$P(x_1)$	$P(x_2)$		$P(x_n)$

mean $(\mu) = \dfrac{\Sigma p_i x_i}{\Sigma p_i} = \Sigma p_i x_i$

[Expected value $E(x)$ / Average].

variance $= V(x) = \sigma^2 = \Sigma p_i (x_i - \mu)^2$

if μ is not a whole number then σ^2

$= \Sigma p_i x_i^2 - \mu^2$

Standard Deviation, $\sigma = +\sqrt{var}$.

moments

r^{th} moment about the mean (μ_r).

$\mu_r = \Sigma (x_i - \mu)^r F(x_i)$ where $F(x_i) = p_i$

For $r = 0$, $\mu_0 = \Sigma f(x_i) = \Sigma p_i = 1$

For $r = 1$, $\mu_1 = \Sigma (x_i - \mu) F(x_i) = \Sigma (x_i - \mu) p_i$

$= \Sigma x_i p_i - \mu \Sigma p_i = \mu - \mu = 0$

For $r = 2$, $\mu_2 = \Sigma (x_i - \mu)^2 F(x_i) = \sigma^2$

For $r = 3$, $\mu_3 = \Sigma (x_i - \mu)^3 F(x_i) = \Sigma (x_i - \mu)^3 p_i$

For $r = 4 =$ $\mu_4 = \Sigma (x_i - \mu)^4 F(x_i) = \Sigma (x_i - \mu)^4 p_i$

Mean Deviation about mean $= \Sigma |x_i - \mu| F(x_i)$

Continuous prob. Distribution:-

DATE
PAGE NO.

$$E(x) = \int_{-\infty}^{\infty} x\, F(x)\, dx$$

$$E(x^r) = \int_{-\infty}^{\infty} x^r\, F(x)\, dx$$

$$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2\, F(x)\, dx$$

$$= E(x^2) - [E(x)]^2$$

Mean Deviation about mean $= \int_{-\infty}^{\infty} |x-\mu|\, F(x)\, dx$

$$\mu_r = \int_{-\infty}^{\infty} (x-\mu)^r\, F(x)\, dx$$

if $r = 0$ and if we put it in μ_r then we will get $E(x)$

(Expected value is the moment about origin)

Moment Generating funcn.

$$M_a(t) = \sum p_i\, e^{t(x_i - a)} \qquad M_x(t) = (pe^t + q)^n$$

$$\mu_r = \left[\frac{d^r}{dt^r} M_a(t)\right]_{t=0} \quad r^{th} \text{ moment about } a$$

$$M_a(t) = \sum p_i\, e^{t(x_i-a)} = e^{-at} \sum p_i e^{tx_i} = e^{-at} M_a(t)$$

$$M_a(t) = \int_{-\infty}^{\infty} e^{t(x-a)}\, F(x)\, dx$$

1) $E(ax+b) = aE(x) + b$

2) $Var(ax+b) = a^2\, var(X)$

Relation b/w Central moments (μ_r) & moments about any arbitary original 'a'.

$$\mu_1 = \mu - a$$
$$\mu_2 = (\mu_2') - (\mu_1')^2$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4.$$

14/09/2023

Q. The density function of a random variable x is given by $f(x) = Kx(2-x)$, $0 \le x \le 2$.
Find
(i) K
(ii) Variance
(iii) Mean Deviation about mean.

(1) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ $\quad \int_0^2 Kx(2-x)\,dx = 1$

$$K = 3/4.$$

(2) Variance $= E(x^2) - [E(x)]^2$

$$E(X) = \int_0^2 x\, f(x)\,dx = \frac{3}{4}\int_0^2 x^2(2-x)\,dx = 1$$

$$E(X^2) = \int_0^2 x^2 f(x)\,dx = \frac{3}{4}\int_0^2 x^2(2-x)\,dx = \frac{6}{5}$$

$$\text{variance} = \frac{6}{5} - 1^2 = \frac{1}{5}.$$

(3) Mean deviation about mean

$$= \int_0^2 |x - \bar{x}|\, f(x)\,dx$$

$$= \int_0^2 |x-1|\, f(x)\,dx$$

$$\rightarrow \int_0^1 (1-x) f(x)\,dx - \int_1^2 (x-1) f(x)\,dx$$

$= \frac{3}{8}$

Q. find the MGF of the exponential distribution $f(x) = \frac{1}{c} e^{-x/c}$, $0 \leq x \leq \infty$

$c > 0$, Hence find its mean & SD.

Soln: The MGF about origin is $M_0(t)$.

$$= \int_0^\infty e^{tx} f(x)\, dx = \int_0^\infty e^{tx} \cdot \frac{1}{c} e^{-x/c}\, dx$$

$$= \frac{1}{c} \int_0^\infty e^{(t-\frac{1}{c})x} \cdot dx = \frac{1}{c} \left[\frac{e^{(t-\frac{1}{c})x}}{t - \frac{1}{c}} \right]_0^\infty$$

$$\Rightarrow (1-ct)^{-1}$$

$$= 1 + ct + c^2t^2 + c^3t^3 + - - -$$

$$\mu_1' = \left[\frac{d}{dt} M_0(t) \right]_{t=0} = \left[c + 2c^2 t + 3c^3 t^2 t + — \right] = c$$

Mean $= 0 + \mu_1'$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_0(t) \right]_{t=0} = \left(2c^2 + 6c^3 t + - - - \right) = 2c^2$$

$$E(x^2) - [E(x)]^2$$

Variance $= \sigma^2 = \mu_2 = \mu_2' - (\mu_1')^2$

$$\Rightarrow 2c^2 - c^2 = c^2$$

$$SD = + \sqrt{Va}$$

$$\Rightarrow c.$$

Binomial Distribution $\rightarrow B(n, p)$

Trials $= n$
Probability of success $= p$
" " failure $= q$

$$\boxed{p+q=1}$$

probability mass func^n

$$\left[P(X=r) = {}^nC_r\, p^r q^{n-r}, \quad r = 0, 1, 2, \ldots n \right]$$

Binomial Expansion / Distribution $(p+q)^n$

Recurrence or Recursion formula for $B(n/p)$.

$$P(r) = {}^nC_r\, q^{n-r} p^r = \frac{n!}{r!(n-r)!} q^{n-r} p^r$$

$$P(r+1) = {}^nC_{r+1}\, q^{n-r-1} p^{r+1}$$

$$= \frac{n!}{(n-r-1)!\,(r+1)!} q^{n-r-1} p^{r+1}$$

$$\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \cdot \frac{p}{q}$$

$$P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} P(r)$$

Binomial Distribution $B(n, p)$

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

mean $= np$
variance $= npq$

Recurrence relation of Binomial Distribution.

$$P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} P(r).$$

Imp
Q1. Six dice are thrown 729 times. How many times do you expect atleast 3 dice show a 5 or 6 ?

Solⁿ $n=6$

$p = \frac{2}{6} = \frac{1}{3}$. $\quad q = 1-p = 1-\frac{1}{3} = \frac{2}{3}$

P(at least three success) $= P(r=3) + P(r=4) + P(r=5) + P(r=6)$ (or)

$\Rightarrow 1 - P(r=0) - P(r=1) - P(r=2)$

$\Rightarrow \frac{233}{729} = 0.3196$

Q2. Determine the binomial Distribution whose mean is 9 & S.D is $\frac{3}{2}$.

Solⁿ $\mu = 9 \Rightarrow np = 9$

$\sigma = S.D = \frac{3}{2} = +\sqrt{var}$

Binomial distⁿ ~~of~~ $(p+q)^n$ representation

DATE
PAGE NO.

$\sigma^2 = var = \frac{9}{4}$

~~n = 12~~ $npq = \frac{9}{4}$, $n = 12$

$q = \frac{1}{4} \Rightarrow p = \frac{3}{4}$.

Q3. If the sum of the mean & var. of a binomial distⁿ for 5 trials is 1.8, find the distribution.

Solⁿ. $\mu = np$, $\sigma^2 = npq$, $n = 5$

ATQ

$\mu + \sigma^2 = 1.8$

$np + npq = 1.8$

$np(1+q) = 1.8$

$np(1+1-p) = 1.8$

$np(2-p) = 1.8$

$5p(2-p) = 1.8$

$p = 1.8$ or 0.2

$p \neq 1.8$ as $0 \leq p \leq 1$

$p = 0.2 = \frac{1}{5}$

$q = 0.8 = \frac{4}{5}$.

$\left(\frac{1}{5} + \frac{4}{5}\right)^5$

Q4. Calculate $P(r)$ for $r = 1,2,3,4$ & 5 taking $n=5$, $p = \frac{1}{6}$ with the help of the recurrence formula of the binomial distribution.

Solⁿ $P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} \cdot P(r)$

DATE
PAGE NO.

$n=5,\ p=\frac{1}{6},\ q=\frac{5}{6}$

$P(0) = {}^5C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^5$

$P(0+1) = P(1) = \frac{5-0}{0+1} \times \frac{1}{5} P(0) = P(0) = \left(\frac{5}{6}\right)^5$

$P(2) = P(1+1) = \frac{(5-1)}{1+1} \times \frac{1}{5} P(1)$

Q. Fit a binomial distribution to the following data:-

x	0	1	2	3	4
f	30	62	46	10	2

Sol^n

x	f	f·x
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8

$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{192}{150}$

$= 1.28$

$np = 1.28$

$4p = 1.28$

$p = 0.32,$

$q = 0.68$

$N = 150.$

Binomial distribution $= N(p+q)^n$

$= 150(0.32+0.68)^4$

DATE 18/9/23
PAGE NO.

Poisson Distribution

A limiting case of binomial distribution.

$$n \to \infty$$
$$p \to 0$$
$$\lambda = np \text{ (fixed)}$$
$$\boxed{\sigma^2 = \mu = \lambda}$$

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Recurrence formula.

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(x+1) = \frac{e^{-\lambda} \lambda^{x+1}}{(x+1)!}$$

$$\boxed{\frac{P(x+1)}{P(x)} = \frac{\lambda}{x+1} \cdot P(x)}$$

$$P(X = r) = {}^{n}C_{r}\, q^{n-r} p^{r}$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)\cancel{(n-r)!}}{r!\,\cancel{(n-r)!}} \times (1-p)^{n-r} p^r$$

$$= \frac{n(n-1)(n-2) - -(n-r+1)}{r!} \times \left(1 - \frac{\lambda}{n}\right)^{n-r} \times \left(\frac{\lambda}{n}\right)^r$$

$\because \lambda = np$

$p = \frac{\lambda}{n}$

{ n should be greater than 25 for poisson distribution }

DATE

PAGE NO.

$$\Rightarrow \frac{\lambda^r}{r!}\left[\left(\frac{n}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)\right]\frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$\Rightarrow \frac{\lambda^r}{r!}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)\right]\frac{\left[\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right]^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^r}$$

As $n\to\infty$, each $(r-1)$ factors $\left(1-\frac{1}{n}\right), \left(1-\frac{2}{n}\right)$ ---- $\left(1-\frac{r-1}{n}\right)$ tends to 1.

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e}$$, then

$$\left[\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right]^{-\lambda}\to e^{-\lambda} \text{ as } n\to\infty.$$

$$\Rightarrow P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

can be x

Q1. Assume that the prob. of an individual coal miner being killed in a mine accident during a year is $\frac{1}{2400}$. Find the prob. that in a mine employing 200 miners there will be atleast one fatal accident a year.

Soln

$n = 200$ $\qquad \lambda = n \times p$

$p = \frac{1}{2400}$ $\qquad = \frac{1}{12}$

$= 0.083$.

DATE

PAGE NO.

$$P(x \geq 1) = 1 - P(X = 0)$$

DIY

Q2. Six coins are tossed 6400 times. Using the Poisson distribution determine the approx. prob. of getting six heads x times.

Solⁿ $n = 6400$

$p = \left(\frac{1}{2}\right)^6$ → six coins.

$\frac{1}{2}$ → probability of getting head.

Q3. Fit a Poisson distribution & also find theoretical frequency.

x	0	1	2	3	4	Total
f	109	65	22	3	1	200 ← N

Solⁿ $\mu = \frac{\Sigma xf}{\Sigma f} = \frac{122}{200} = 0.61$

$\lambda = \mu = 0.61$

$N = 200.$

PTO

$$\boxed{P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}} \Rightarrow \frac{N \cdot e^{-\lambda}\lambda^{r}}{r!}$$

r	$N \cdot P(r)$	Theoretical frequency
0	108.7	109
1	66.3	66
2	20.2	20
3	4.1	4
4	0.7	1
		200

Normal Distribution

Limiting case of Binomial Distribution when $n \rightarrow \infty$, $P \rightarrow \frac{1}{2}$ (P → probability)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$-\infty < x < \infty$

mean $= \mu$ $\quad -\infty < \mu < \infty$

SD $= \sigma$ $\quad \sigma > 0$

$N(\mu, \sigma^2)$

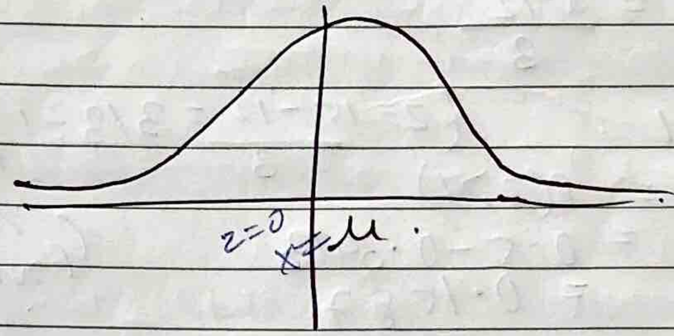
NOTES

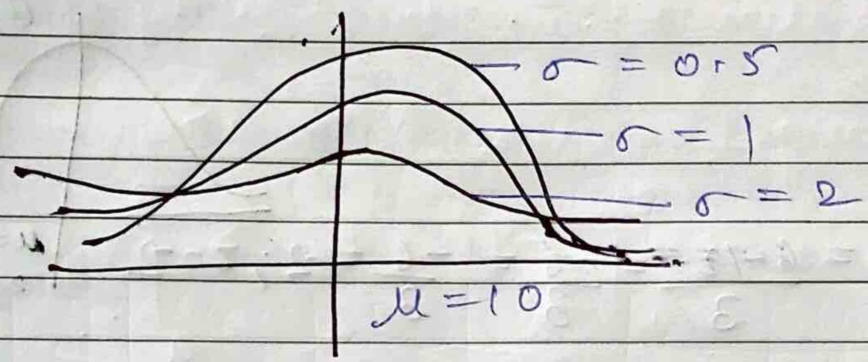
- Normal Curve is a Symmetrical bell shaped Curve.
- Normal Curve is Symmetrical about mean μ;
- Two tails of the normal curve tends to

$-\infty$ & $+\infty$.

- The curve is unimodal, which is nothing but it's mean.
- The median of curve is also coincide with mean.
- Total area is 1.
- standard normal variate.

$$z = \frac{x-\mu}{\sigma}$$





The mean deviation about mean of ND is 4/5 of its SD $\int_{-\infty}^{\infty} |x-\mu| \, f(x) \, dx$.

$P(z_1 \le z \le z_2)$
$P(z_1 < z \le z_2)$
$P(z_1 \le z < z_2)$
$P(z_1 \le z < z_2)$

All are same

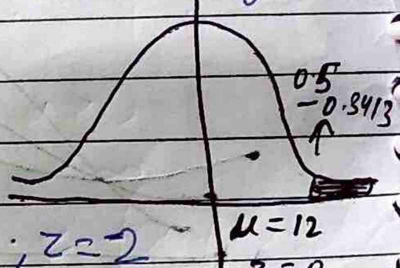
Q A sample of 100 dry battery cells tested to find the length of life produced the following result $\bar{x} = 12$ hours, $\sigma = 3$ hours.
Assuming the data to be normal distributed, what percentage of battery cells are expected to have life
(i) More than 15 hours
(ii) Less than 6 hours $P(x<6)$
(iii) Between 10 & 14 hours $P(10 < x < 14$

Sol^n $z = \frac{x - \bar{x}}{\sigma} = \frac{x-12}{3}$

(i) For $x = 15$, $z = 1$ $\left(z = \frac{15-12}{3} = 3/3 = 1\right.$

$P(x>15) = P(z>1)$
$= 0.5 - 0.3413$ (from table) Normal distribution table Google
$= 0.1587$

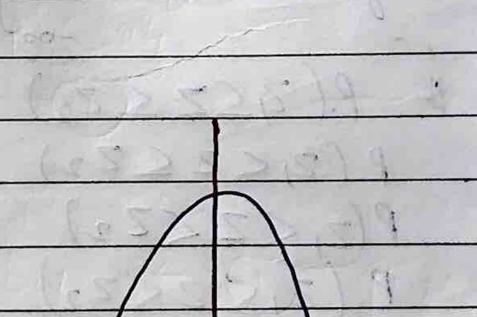
$\boxed{\% = 15.87\%}$



(ii) For $x = 6$, $z = \frac{6-12}{3} = \frac{-6}{3} = \frac{-6}{3} = -2$; $z = -2$

$P(x<6) = P(z<-2)$
$P(x<6) = P(z>2)$
$= 0.5 - P(z=2)$
$= 0.5 - 4.772$
$= 0.0228$

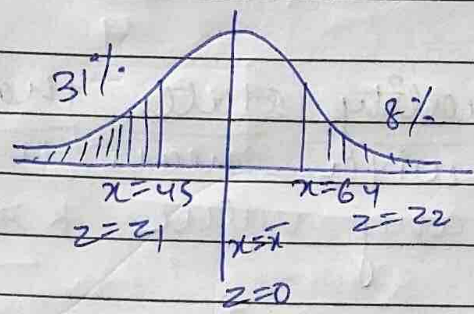
$\boxed{\% = 2.28\%}$



DATE 22/09/23
PAGE NO.

Q1. In a normal distribution 31% of the items are under 45 & 8% are over 64. Find the mean & S.D of the Distribution.

Sol^n Let $\bar{x}$ & σ be the mean & S.D resp.



when $x = 45$, let $z = z_1$

$P(z_1 < z < 0) = 0.5 - 0.31 = 0.19$

From the table the value of z corresponding to the area is 0.5.

$z_1 = -0.5 \quad (\because z_1 < 0)$

when $x = 64$ let $z = z_2$

$P(0 < z < z_2) = 0.5 - 0.08 = 0.42$

From table

$z_2 = 1.4$

Since, $z = \dfrac{x - \bar{x}}{\sigma}$

$-0.5 = \dfrac{45 - \bar{x}}{\sigma}$ & $1.4 = \dfrac{64 - \bar{x}}{\sigma}$

$45 - \bar{x} = -0.5\sigma$ ——— (1)

$64 - \bar{x} = 1.4\sigma$ ——— (2)

Solving (1) & (2), we get

$\bar{x} = 50, \ \& \ \sigma = 10.$

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DATE 25/09/23
PAGE NO.

Chebyshev's Inequality

→ limited case of Markov's inequality

$\sigma \to \infty$

⇒ prob. of getting values farther away from μ.

The result is helpful when actual distribution of X is not know.

Chebyshev's inequality states that:-
If X is a r.v with mean μ & variance σ^2 then for any value $K>0$.

$$P: \{|x-\mu| \geq K\} \leq \frac{\sigma^2}{K^2}$$

proof:- we prove it for the case when X is continuous with density funcⁿ $f(x)$

for a non-negative r.v X & $a>0$.

$$E(X) = \int_0^\infty x f(x).dx + \int_a^\infty x f(x).dx \geq \int_a^\infty x f(x) dx$$

$$\geq a\int_a^\infty f(x) dx$$

$$= a P\{x \geq a\}$$

Thus, $\underbrace{P\{x \geq a\} \leq \frac{E(X)}{a}}_{\text{Markov's inequality}}$ —— ①

Replacing x by $(X-\mu)^2$ & a by K^2, ① gives

$$P\{(X-\mu)^2 \geq K^2\} \leq \frac{E(X-\mu)^2}{K^2}$$

DATE ________
PAGE NO. ________

(or)

$$P\{|x-\mu| \geq K\} \leq \frac{\sigma^2}{K^2}$$ H.P

1) By replacing k by $K\sigma$ in chebyshev's inequality can be expressed as.

$$P(|x-\mu| \geq K\sigma) \leq \frac{1}{K^2}$$ → Imp.

2) Sometimes result is applicable in the form $P\{|x-\mu| \leq K\sigma\} \geq 1 - \frac{1}{K^2}$

3) The inequality $P(X \geq a) \leq \frac{E(X)}{a}$ is known as Markov's inequality & is of practical importance.

Q1. The no. of items cleared by an assembly line during a week is a r.v with mean 50 & variance 25.

(a) what is the prob. that this week items ~~closed~~ cleared will exceed 75 ?

(b) what can be said about the prob. that this week's clearance will be b/w 40 to 60.

Solⁿ:- Let x be the r.v denoting the no. of items cleared in a week.

$\therefore E(X) = 50$ & $var(X) = 25$

(a) As $P\{x \geq a\} \leq \frac{E(X)}{a}$; $a > 0$.

$$P(X > 75) \leq \frac{50}{75} = \frac{2}{3}$$

(b)

DATE
PAGE NO.

(b) $P\{|X-\mu| \geq K\} \leq \frac{\sigma^2}{K^2}$

$K = 10 \longrightarrow$ $50 - 10 = 40$, $50 + 10 = 60$

$P\{|X-50| \geq 10\} \leq \frac{25}{100} = \frac{1}{4}$

$P\{|X-50| < 10\} \geq \frac{3}{4}$

$P\{40 < X < 60\} \geq \frac{3}{4} = 0.75$

Exponential Distribution → Depends on only 1 parameter

A continuous random variable X is ~~standard~~ said to be have exponential (negative exponential) distribution with parameter 'a', if it assumes only real positive values and its p.d.f is given by.

$$f(x) = \begin{cases} ae^{-ax} & , \text{ if } x > 0 \\ 0 & , \text{ otherwise.} \end{cases}$$

where, $a > 0$ is the prescribed parameter.

mean $= 1/a$ & variance $= \frac{1}{a^2}$

Q) The time b/w breakdowns of a machine follows an exponential distribution with a mean of 17 days. Find the prob. that a machine breaks down in a 15 days period?

Soln

$\mu = 17$

$\frac{1}{a} = 17$

$a = 1/17$

DATE
PAGE NO.

$$pdf = ae^{-ax} = \frac{1}{17} e^{-t/17}, \quad t \geq 0$$

$$p(0 \leq t \leq 15) = \int_0^{15} f(t)\, dt$$

$$= \int_0^{15} \frac{1}{17} e^{-t/17}\, dt$$

$$= \left[-e^{-t/17}\right]_0^{15} = 1 - e^{-15/17}$$

$\Rightarrow$ 0.5862

$\Rightarrow$ There is 58.62% chances.

Gamma Distribution.

A cts. r.v X defined over the range $x > 0$. s.t.b Gamma distributed if the p.d.f is of the form

$$f(x) = \frac{\lambda^r x^{r-1}}{\Gamma(r)} e^{-\lambda x}, \quad x \geq 0$$

where $r > 0$ & $\lambda > 0$ & $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx$.

$\Gamma(r) = (r-1)$ If r has an integer value.

CORRELATION →

↳ coeff. of correlation (r) $[-1 \leq r \leq 1]$

↳ types of correlation

1. +ve $0 > r > 1$
2. strictly +ve $r = +1$
3. −ve $-1 < r < 0$
4. strictly −ve $r = -1$
5. No correlation. $r = 0$

Karl person's coeff. of correlation.

$$r = \frac{\text{covariance}(x, y)}{\sqrt{\text{var.}\, x}\ \sqrt{\text{var}\, y}} = \frac{\text{cov}(x, y)}{\sigma_x\, \sigma_y} = \frac{\Sigma xy}{(\Sigma x^2)(\Sigma y^2)}$$

$X = x - \bar{x}$ $\text{cov} = \frac{\Sigma XY}{N}$

$Y = y - \bar{y}$

Total no. of terms.

For assumed mean.

$x' = x - a$

$y' = y - b$.

$$r = \frac{N \Sigma x'y' - \Sigma x' \Sigma y'}{\sqrt{N[\Sigma (x')^2] - (\Sigma x')^2}\ \sqrt{N[\Sigma (y')^2] - (\Sigma y')^2}}$$

Q1. Calculate the correlation coeff. b/w the following data:-

x	5	9	13	17	21
y	12	20	25	33	35

Sol^n. $\bar{x} = 13$; $\bar{y} = 25$

$X = x - \bar{x}$, $Y = y - \bar{y}$

x	y	X	Y	X^2	Y^2	XY
5	12	-8	-13	64	169	104
9	20	-4	-5	16	25	20
13	25	0	0	0	0	0
17	33	4	8	16	64	32
21	35	8	10	64	100	80
				160	358	236

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \Sigma Y^2}} = \frac{236}{\sqrt{160 \times 358}} = \frac{236}{239.33} = 0.986.$$

Q2.

x	21	23	30	54	57	58	72	78	87	90
y	60	71	72	83	110	84	100	92	113	135

$\bar{x} = 57$, $\bar{y} = 92$

$X = x - \bar{x}$, $Y = y - \bar{y}$

x	y	X	Y	X^2	Y^2	XY
21	60	-36	-32	1296	1024	1152
23	71	-34	-21	1156	441	714
30	72	-27	-20	729	400	840
54	83	-3	-9	9	81	27
57	110	0	18	0	324	0
58	84	1	-8	1	64	-8
72	100	15	8	225	64	120
78	92	21	0	441	0	0
87	113	30	21	900	441	630
90	135	33	43	1089	1849	1419
				5842	4688	4594

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = \frac{4594}{5233.29}$$

$$r = 0.8778.$$

Q.

x	3	7	5	4	6	8	2	7
y	7	12	8	8	10	13	5	10

Let $a = 5$, $b = 10$.

DATE 06/10/23
PAGE NO.

Spearman's Rank correlation coefficient

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

For equal ranks $\qquad d = R_1 - R_2$

$$r = 1 - \frac{6\left[\sum d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots\right]}{n(n^2-1)}$$

Q: obtain the rank correlation co-eff for the following data.

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

Soln =

x	y	R_1	R_2	d	d^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
					72

If same ranking then takes average of their ranks

$n = 10$

$\sum d^2 = 72$

$m_1 = 2 \quad (75)$

$m_2 = 2 \quad (68)$

$m_3 = 3 \quad (64)$

DATE
PAGE NO.

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3)\right]}{n(n^2-1)}$$

$= 0.545$

Regression $\rightarrow$ Relation

Line of Regression X on Y.

$$X - \overline{X} = b_{xy}(Y - \overline{Y})$$

$b_{xy} \rightarrow$ Coefficient of regression

Line of Regression Y on X

$$Y - \overline{Y} = b_{yx}(X - \overline{X})$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma Y^2 - (\Sigma Y)^2}$$

$$= \frac{\Sigma xy}{\Sigma y^2}$$

$x = X - \overline{X}$

$y = Y - \overline{Y}$

$$r^2 = b_{yx} \cdot b_{xy}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$$

8). From the following data obtain the two regression lines :-

Capital Employed (Rs. in lakh)	7	8	5	9	12	9	10	15
Sales volume (Rs. in lakh)	4	5	2	6	9	5	7	12

Solⁿ

X	Y	XY	X^2	Y^2
7	4			
8	5			
5	2			
9	6			
12	9			
9	5			
10	7			
15	12			

$X = 3.2406 + 0.9815\,Y$

$Y = -3.1784 + 1.0057\,X.$

DATE 9/10/23
PAGE NO.

Moments

r^{th} moment about any pt. $x = a$.

$$\boxed{\mu'_r = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - a)^r} \; ; \; N = \sum_{i=1}^{n} f_i$$

r^{th} central movement

$$\boxed{\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^r} \; ; \; N = \sum_{i=1}^{n} f_i$$

Relation b/w μ_r & μ'_r

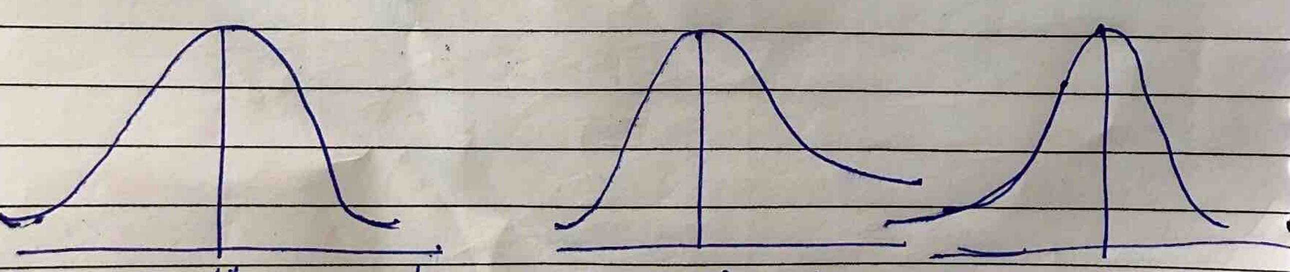
$$\boxed{\mu_r = \mu'_r - {}^{r}C_1 \mu'_{r-1} \mu'_1 + {}^{r}C_2 \mu'_{r-2} (\mu'_1)^2 + \cdots + (-1)^r (\mu'_1)^r}$$

→ Pearson's β & γ co-eff.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \, , \quad \gamma_1 = \sqrt{\beta_1} \, , \quad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_2 = \beta_2 - 3$$

Skewness → measure of asymmetry.



Mean = Median = Mode　　Mode < Median < Mean　　Mean < Median < Mode no. skewed +vely skewed -vely skewed

karl pearson's coeff. of skewness = $\frac{\text{mean} - \text{mode}}{S.D}$

Measure of skewness = β_2

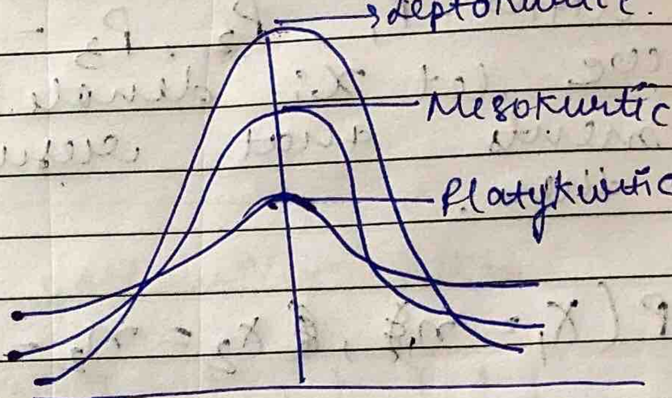
Kurtosis

normally distributed

$\underbrace{\beta_2 = 3 \text{ or } \gamma_2 = 0 \text{ for } M}$

$\beta_2 > 3$ or $\gamma_2 > 0$ for L

$\beta_2 < 3$ or $\gamma_2 < 0$ for P



Q. Calculate the first 4 ~~numbers~~ moments for the following frequency distribution about the mean & comment upon these.

x	-4	-3	-2	-1	0	1	2	3	4
f	3	4	5	7	12	7	5	4	3

$\mu_1 = 0$, $\mu_2 = 44.4$, $\mu_3 = 0$, $\mu_4 = 47.16$

$\beta_1 = 0$, $\beta_2 = 2.39 < 3$

DATE 16/10/23
PAGE NO.

Multidimensional Distribution

A joint distribution, which arises when a sequence of n independent and identical experiments is performed. Suppose that each experiment can result in any one of r possible outcomes, with respective probabilities $P_1, P_2, P_3 ---- P_r$, $\sum_{i=1}^{r} p_i = 1$. If we let x_i denote the no. of experiments that result in outcome no. i, then.

$$P(X_1 = n_1, X_2 = n_2 \ldots X_r = n_r)$$

$$\Rightarrow \frac{n!}{n_1!\, n_2! \cdots n_r!} \; {}^{n_1}P_1 \; {}^{n_2}P_2 \; {}^{n_3}P_3 \cdots {}^{n_r}P_r$$

$$\sum_{i=1}^{r} n_i = n$$

Q1. If a fair die is rolled 9 times. find the prob. that one appears 3 times, 2 & 3 appears twice each, 4 & 5 one each and 6 not at all.

Soln. Req. probability $= \frac{9!}{3!\,2!\,2!\,1!\,1!\,0!} \left(\frac{1}{6}\right)^3 \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^0$

$\frac{1}{6}$

$1 \rightarrow 3$
$2 \rightarrow 2$
$3 \rightarrow 2$
$4 \rightarrow 1$
$5 \rightarrow 1$
$6 \rightarrow 0$

$n = 9$
$n_1 = 3$
$n_2 = 2$
$n_3 = 2$

DATE
PAGE NO.

$n_4 = 1$
$n_5 = 1$
$n_6 = 0$

$\Rightarrow \dfrac{9!}{3!\,2!\,2!\,1!\,1!\,0!} \times \left(\frac{1}{6}\right)^9$

$\Rightarrow \dfrac{7 \times 5}{3 \times 6^5} = \dfrac{35}{23328}$

Q1. Calculate r for the following data.

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

Q2. Find rank correlation

x	56	42	72	36	63	47	55	49	38
y	147	125	160	118	149	128	150	145	115

Q3. X is normally distributed $N(30, 5)$
find probability
(a) $x \geq 45$
(b) $26 \leq X \leq 40$
(c) $|X - 30| > 5$

A: Here, $\bar{x} = 5$
$\bar{y} = 12$.

X	Y	X^2	Y^2	XY
4	3	16	9	12
3	4	9	16	12
2	2	4	4	4
1	1	1	1	1
0	-1	0	1	0
-1	0	1	0	0
-2	-2	4	4	4
-3	-4	9	16	12
-4	-3	16	9	12
		60	60	57

34
16
50

$$r = \frac{57}{\sqrt{60 \times 60}} \Rightarrow \frac{57}{\sqrt{3600}} = \frac{\overset{0.9}{\cancel{570}}}{\cancel{60}}$$

$= 0.9$