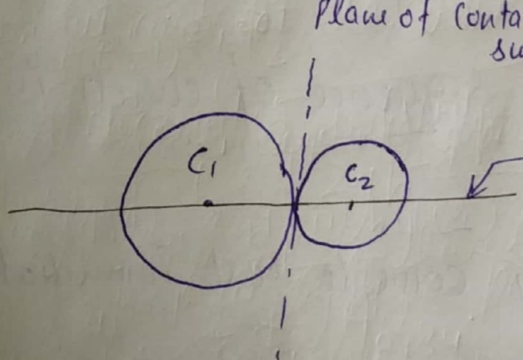
Impact

The phenomenon of collision between two bodies which occurs in a very short interval of time and during which the bodies exert relatively large forces on each other is called an impact.

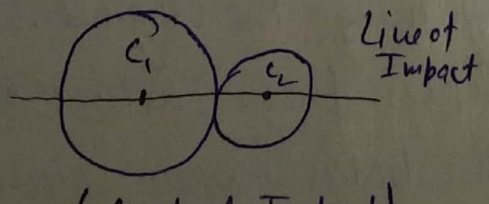
Line of Impact: The line passing through the point of contact is called line of impact.

It is the line which is collinear to the common normal of the surfaces that are closest or in contact during impact. It is the line along which internal force of collision acts during impact.

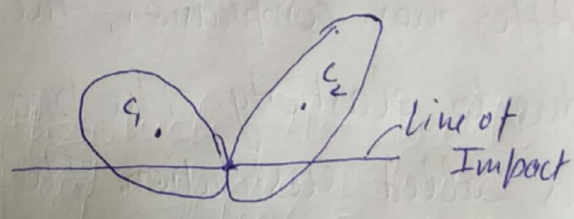


Types of Impact

1) Central and eccentric impact:



(Central Impact)
(mass centre located on line of Impact)



(Eccentric impact)
(mass centre are not located on line of Impact)

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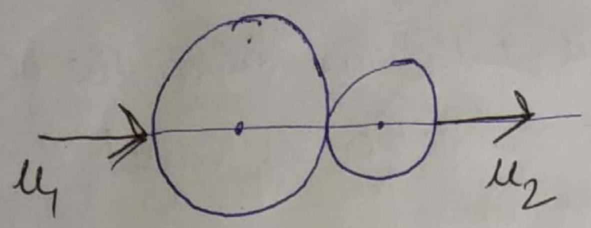
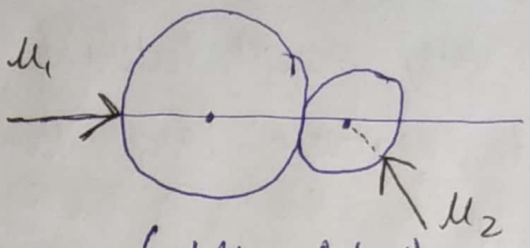
2) Direct and oblique impact



(Direct impact)

[body moves along the line of impact]



(oblique impact)

(if motion of one or both of colliding bodies, before impact is not directed along the line of impact.)

3) Elastic and Inelastic impact

The impact is elastic if the body rebounds after impact.
The impact is inelastic if body does not rebound at all.

The following is the sequence of events in the process of elastic collision.

i) The bodies momentarily come to rest immediately after impact.

ii) The bodies tend to compress each other to a max. value. The time elapses from instant of collision to max. deformation is called <u>period of deformation</u>.

iii) After max. compression, the bodies tend to regain their shapes due to elasticity. The process of regaining the original shape is called restitution. The time elapses from instant of max. deformation to the instant the bodies regain original shape is called period of restitution.

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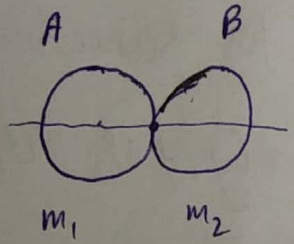
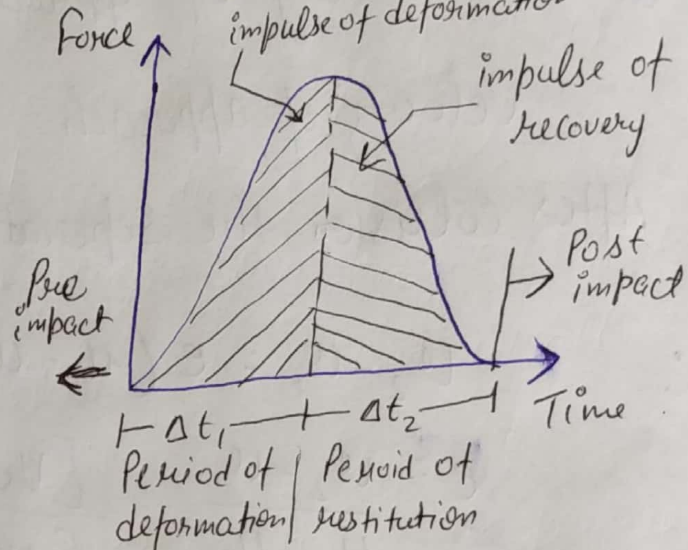
The sum of period of deformation and period of restitution is called the time of collision/period of impact.

Conservation of momentum

Consider two bodies of masses m_1 and m_2 having initial velocity u_1 and u_2 before impact $[u_1 > u_2]$ and v_1 and v_2 after impact.



(impact)



During collision, there is an impulse f.t exerted by body A on body B. This impulse on body B is measured by the change in its momentum.

i.e. Impulse on body B = change in momentum of body B

$$F.t = m_2 v_2 - m_2 u_2 \quad -(i)$$

Also, body B exert force on body A. so impulse on body A is $(-F.t)$

$$\therefore \quad -F.t = m_1 v_1 - m_1 u_1 \quad -(ii)$$

from (i) and (ii)

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$

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⇒ Therefore, the momentum of the system before collision is equal to the momentum of the system after collision.

Newton's Law of Collision: Coefficient of restitution

It states that "when two moving bodies collide with each other, their velocity of seperation bears constant ratio to their velocity of approach."

Velocity of approach = $u_1 - u_2$ [for same direction]

After collision, the seperation will occur only if $v_2 > v_1$

$$\therefore \quad v_2 - v_1 = e(u_1 - u_2)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{[\text{Velocity of seperation}]}{[\text{velocity of approach}]}$$

where e = coefficient of restitution

e lies between 0 to 1.

If e = 0, ~~body~~ bodies are inelastic

e = 1 " " perfectly elastic

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Loss of K.E during impact:

Consider two bodies A and B which experience direct impact.

K.E of masses before impact $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

" " " after " $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Loss in K.E $= \frac{1}{2}\left[(m_1 u_1^2 + m_2 u_2^2) - (m_1 v_1^2 + m_2 v_2^2)\right]$

$$= \frac{1}{2(m_1+m_2)}\left[(m_1+m_2)(m_1 u_1^2 + m_2 u_2^2) - (m_1+m_2)(m_1 v_1^2 + m_2 v_2^2)\right]$$

$$= \frac{1}{2(m_1+m_2)}\left[\{m_1^2 u_1^2 + m_2^2 u_2^2 + m_1 m_2 (u_1^2 + u_2^2)\} - \{m_1^2 v_1^2 + m_2^2 v_2^2 + m_1 m_2 (v_1^2 + v_2^2)\}\right] \quad (i)$$

Also

$$m_1^2 u_1^2 + m_2^2 u_2^2 = (m_1 u_1 + m_2 u_2)^2 - 2 m_1 m_2 u_1 u_2 \quad -(ii)$$

$$m_1 m_2 (u_1^2 + u_2^2) = m_1 m_2 (u_1 - u_2)^2 + 2 m_1 m_2 u_1 u_2 \quad -(iii)$$

Add (ii) & (iii)

$$m_1^2 u_1^2 + m_2^2 u_2^2 + m_1 m_2 (u_1^2 + u_2^2) = (m_1 u_1 + m_2 u_2)^2 + m_1 m_2 (u_1 - u_2)^2 \quad (iv)$$

Similarly

$$m_1 v_1^2 + m_2^2 v_2^2 + m_1 m_2 (v_1^2 + v_2^2) = (m_1 v_1 + m_2 v_2)^2 + m_1 m_2 (v_1 - v_2)^2 \quad (v)$$

Also from Law of conservation of momentum

$$(m_1 u_1 + m_2 u_2)^2 = (m_1 v_1 + m_2 v_2)^2 \quad (vi)$$

From (i), (iv), (v) and (vi)

$$\Delta K.E = \frac{1}{2(m_1+m_2)}\left[m_1 m_2 (u_1 - u_2)^2 - m_1 m_2 (v_1 - v_2)^2\right]$$

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also

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$v_2 - v_1 = e(u_1 - u_2)$$

$$\Delta E = \frac{1}{2(m_1+m_2)} (u_1 - u_2)^2 (1 - e^2)$$

Salient features of diff. types of impacts

Perfectly plastic impact	Partially elastic impact	Perfectly elastic impact	Impact with a fixed surface
- bodies moves together after impact.	bodies seperate after impact	bodies seperate after impact.	-
- momentum is conserved $m_1u_1 + m_2u_2 = (m_1+m_2)v$	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	momentum is not conserved.
- there is loss of K.E.	- there is loss of K.E.	K.E. is conserved	- loss of K.E.
- $e = 0$	$0 < e < 1$	$e = 1$	$e = -\frac{v}{u}$ u = velocity before impact v = " after "

Q A ball of mass 2 kg moving with a velocity of 3 m/s impinges directly on a ball of mass 4 kg at rest. After impact, the 2 kg mass ball comes to rest. Determine the velocity of 4 kg ball after striking and the coefficient of restitution between the two balls.

Sol.

2kg 4kg

$u_1 = 3\ m/s$ $u_1 = 0$

$v = 0$ $v_2 = ?$

By using law of conservation of momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\boxed{v_2 = 1.5\ m/s}$$

As we know

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \boxed{0.5}$$

Q Three perfectly elastic balls A, B and C of masses 2 kg, 4 kg and 8 kg resp. moves in the same direction with velocities 4 m/s, 1 m/s and 0.75 m/s along a straight line. The ball A collides with ball B and subsequently the ball B impinges with ball C. Show that balls B and C will be brought to rest after the impacts.

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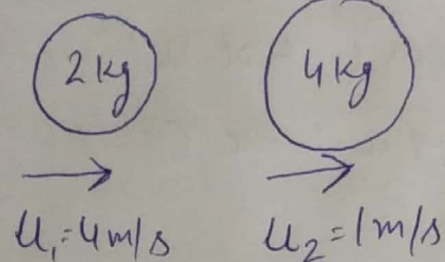
Sol Consider collision b/w balls A and B



By using law of cons. of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$v_1 + 2v_2 = 6 \quad -(i)$$

also $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$\because e = 1$

$$v_2 - v_1 = 3 \quad -(ii)$$

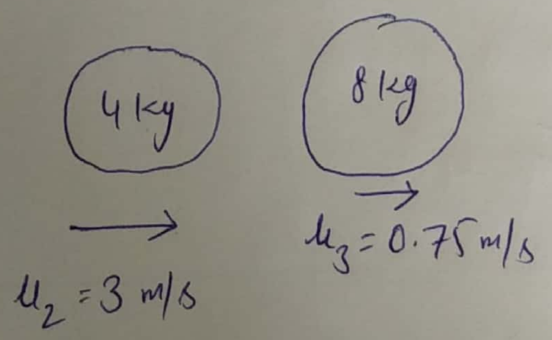
from (i) and (ii)

$$v_2 = 3\,m/s$$

$$v_1 = 0$$

$\therefore$ ball A come to rest after impact.

Consider collision between balls B and C



By using law of cons. of momentum.

$$m_2 u_2 + m_3 u_3 = m_2 v_2 + m_3 v_3$$

$$2v_2 + 4v_3 = 9 \quad -(iii)$$

also $e = \dfrac{v_3 - v_2}{u_2 - u_3}$

$$v_3 - v_2 = 2.25 \quad -(iv)$$

from (iii) and (iv)

$$v_3 = 2.25\,m/s$$

$$v_2 = 0$$

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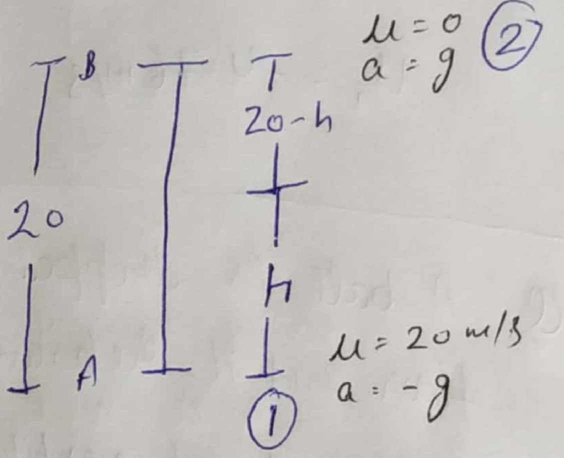
Q. A stone of mass 2 kg is thrown upward from the foot of a tower with a velocity of 20 m/s. One second later, another stone of same mass is dropped from the top of the tower. If the collision between the stones takes place elastically, make calculations for the velocities of the stones just after collision. Take height of tower as 20 m. (5)



Sol. By using relation

$$s = ut + \frac{1}{2}at^2$$

for upward motion

$$h = 20t - 4.905t^2 \quad \text{-(i)}$$

for downward motion

$$20 - h = 4.905(t-1)^2 \quad \text{-(ii)}$$

from (i) & (ii)

$$t = 1.48 \text{ sec}$$

velocity of first stone at collision point

$$v = u - gt = 20 - 9.81 \times 1.48$$
$$= 5.48 \text{ m/s} \uparrow$$

velocity of second stone

$$v = u + g(t-1)$$
$$= 4.71 \text{ m/s} \downarrow$$

Let after collision, v_1 & v_2 be their respective velocities

$$e = \frac{v_2 - v_1}{u_1 - u_2} \; ; \quad v_2 - v_1 = 5.48 - (-4.71)$$
$$v_2 - v_1 = 10.19 \quad \text{-(iii)}$$

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Acc. to principle of conservation of momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$v_1 + v_2 = 5.48 - 4.71 = 0.67$

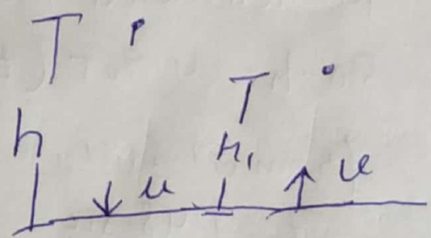
$v_1 + v_2 = 0.67$ (iv)

from (iii) and (iv)

$v_2 = 5.43 \text{ m/s}$ $(\uparrow)$

$v_1 = -4.76 \text{ m/s}$ $(\downarrow)$

Q A ball is dropped from a height of 10m on a smooth floor and it rebounds to a height of 7m. Determine the coefficient of restitution between the ball and floor and expected height of second rebounce



Sol. Here

velocity before impact $= \sqrt{2gh}$

" after impact $= \sqrt{2gh_1}$

$e = \sqrt{\frac{h_1}{h}}$

$\boxed{h_1 = e^2 h}$

$e = 0.837$

also, in second rebounce

$h_2 = h_1 \cdot e^2$

$\boxed{h_2 = 4.9 \text{ m}}$

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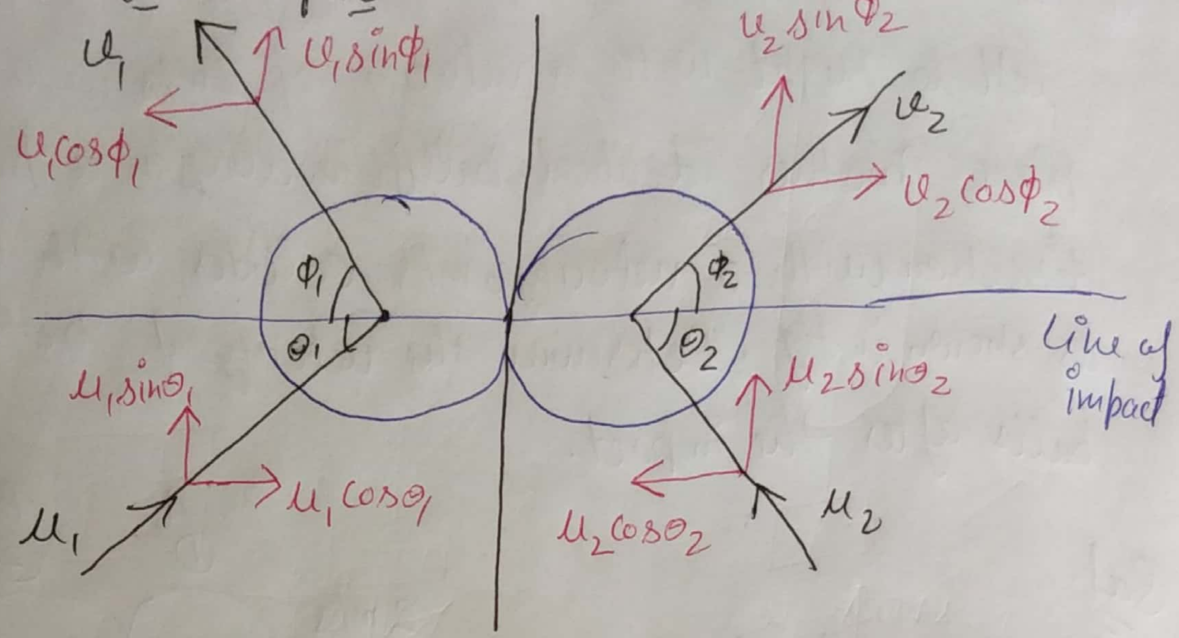
Oblique central impact



Motion along line of impact (x-axis)

$$\boxed{m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\phi_1 + m_2 v_2 \cos\phi_2} \quad \text{—(i)}$$

also
$$\boxed{e = \frac{v_2 \cos\phi_2 - v_1 \cos\phi_1}{u_1 \cos\theta_1 - u_2 \cos\theta_2}} \quad \text{—(ii)}$$

Motion along normal to line of impact (y-axis)

The momentum is conserved in y direction for each body individually.

$$\therefore \quad m_1 u_1 \sin\theta_1 = m_1 v_1 \sin\phi_1$$

$$\boxed{u_1 \sin\theta_1 = v_1 \sin\phi_1} \quad \text{—(iii)}$$

$$m_2 u_2 \sin\theta_2 = m_2 v_2 \sin\phi_2$$

$$\boxed{u_2 \sin\theta_2 = v_2 \sin\phi_2} \quad \text{—(iv)}$$

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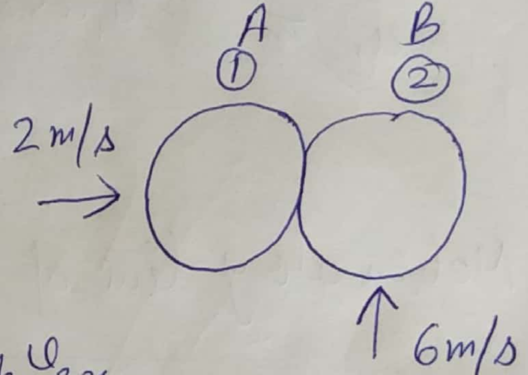
Q. A smooth spherical ball of 120 gms is moving from left to right with a velocity of 2 m/s in a horizontal plane. Another identical ball B travelling in a perpendicular direction with a velocity 6 m/s collides with ball A as shown in fig. Determine the velocity of both the balls after the impact.



Sol.

From principle of Conservation of momentum (x-axis)

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$0.12 \times 2 + 0.12 \times 0 = 0.12 \times v_{1x} + 0.12 v_{2x}$$

$$v_{1x} + v_{2x} = 2 \quad \text{— (i)}$$

also $e = \dfrac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$; $1 = \dfrac{v_{2x} - v_{1x}}{2 - 0}$

$$v_{2x} - v_{1x} = 2 \quad \text{— (ii)}$$

from (i) and (ii)

$$v_{1x} = 0 \; ; \; v_{2x} = 2 \text{ m/s}$$

For motion in y direction:

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

$$0.12 \times 0 + 0.12 \times 6 = 0.12 v_{1y} + 0.12 v_{2y}$$

$$v_{1y} + v_{2y} = 6 \quad \text{—(iii)}$$

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$$\frac{v_{2y} - v_{1y}}{u_{1y} - u_{2y}} = 1 \; ; \; \frac{v_{2y} - v_{1y}}{0 - 6} = 1$$

$$v_{2y} - v_{1y} = -6 \quad \text{— (iv)}$$

from (iii) and (iv)

$v_{1y} = 6 \, m/s$; $v_{2y} = 0$

$v_{1x} = 0$; $v_{2x} = 2 \, m/s$

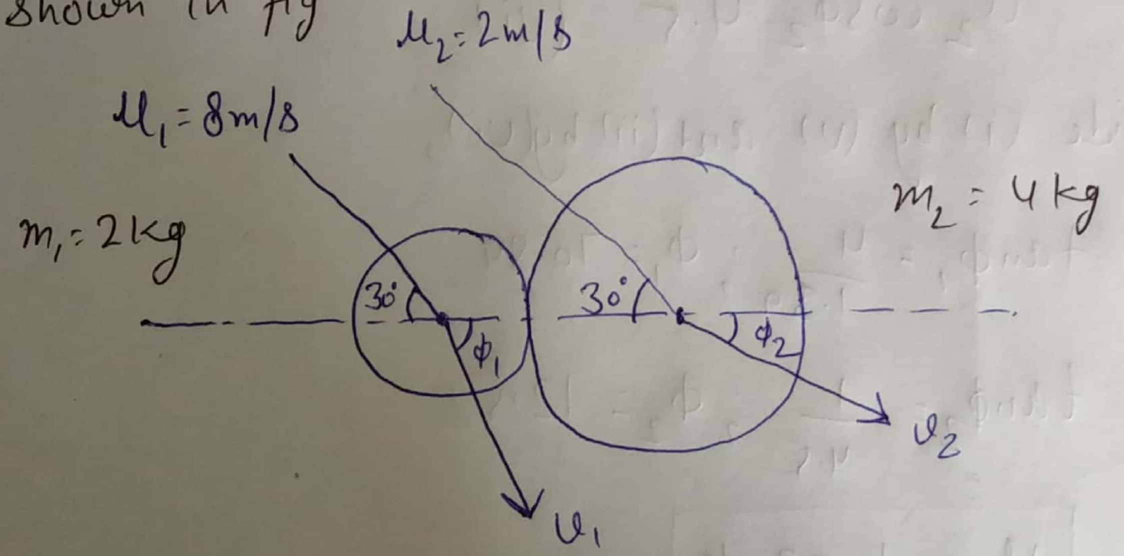
$v_1 = \sqrt{0^2 + 6^2} = 6 \, m/s$

$\alpha_1 = \tan^{-1}\left(\frac{6}{0}\right) = \tan^{-1}\infty = \frac{\pi}{2}$

$v_2 = \sqrt{2^2 + 0^2} = 2 \, m/s$

$\alpha_2 = \tan^{-1}\left(\frac{0}{2}\right) = 0'$

Q There occurs a collision between two balls as shown in fig



Determine the velocity with which the balls would move after the impact. Take $e = 0.6$

Motion along y axis

$8 \sin 30 = u_1 \sin\phi_1$ | $2 \sin 30 = v_2 \sin\phi_2$

$u_1 \sin\phi_1 = 4$ — (i) | $v_2 \sin\phi_2 = 1$ — (ii)

Motion along x direction

$$m_1 u_1 \cos\theta_1 + m_2 u_2 \cos\theta_2 = m_1 v_1 \cos\phi_1 + m_2 v_2 \cos\phi_2$$

$$v_1 \cos\phi_1 + 2v_2 \cos\phi_2 = 10.39 \quad -(iii)$$

$$e = \frac{v_2 \cos\phi_2 - v_1 \cos\phi_1}{u_1 \cos\theta_1 - u_2 \cos\theta_2}$$

$$v_2 \cos\phi_2 - v_1 \cos\phi_1 = 3.12 \quad -(iv)$$

from (iii) and (iv), we get

$$v_1 \cos\phi_1 = 1.39 \quad (v)$$

$$v_2 \cos\phi_2 = 4.5 \quad (vi)$$

Divide (i) by (v) and (ii) by (vi),

$$\tan\phi_1 = \frac{4}{1.39} ; \quad \phi_1 = 70.84^\circ$$

$$\tan\phi_2 = \frac{1}{4.5} ; \quad \phi_2 = 12.52^\circ$$

$$\boxed{\begin{aligned} v_1 &= 4.23 \text{ m/s} \\ v_2 &= 4.61 \text{ m/s} \end{aligned}}$$

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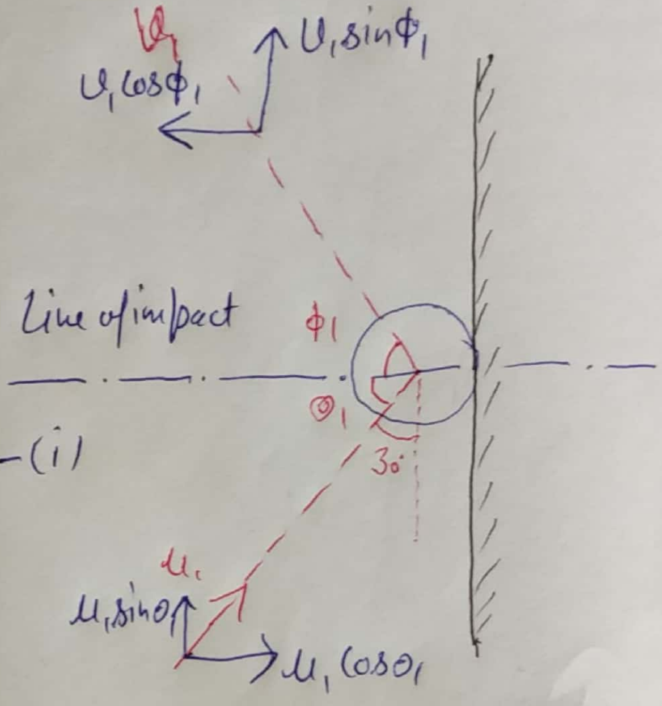
Q A ball is thrown against a wall with 25 m/s (8) and making an angle of 30° with the wall as shown in fig. Determine the velocity of ball after impact.

Take $e = 0.6$



Sol.

Motion in y direction

$u_1 \sin\theta_1 = v_1 \sin\phi_1$

$v_1 \sin\phi_1 = 21.65 \quad [\theta_1 = 60°] \quad -(i)$

Motion in x direction

$$e = \frac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}} = \frac{0 - v_1 \cos\phi_1}{u_1 \cos\theta_1 - 0}$$

$v_1 \cos\phi_1$ and $u_1 \cos\theta_1$ are in opposite direction

$$e = \frac{v_1 \cos\phi_1}{u_1 \cos\theta_1}$$

$v_1 \cos\phi_1 = 7.5 \quad -(ii)$

Divide (i) by (ii)

$$\boxed{\phi_1 = 70.89°}$$

$$\boxed{v_1 = 22.91 \, m/s}$$

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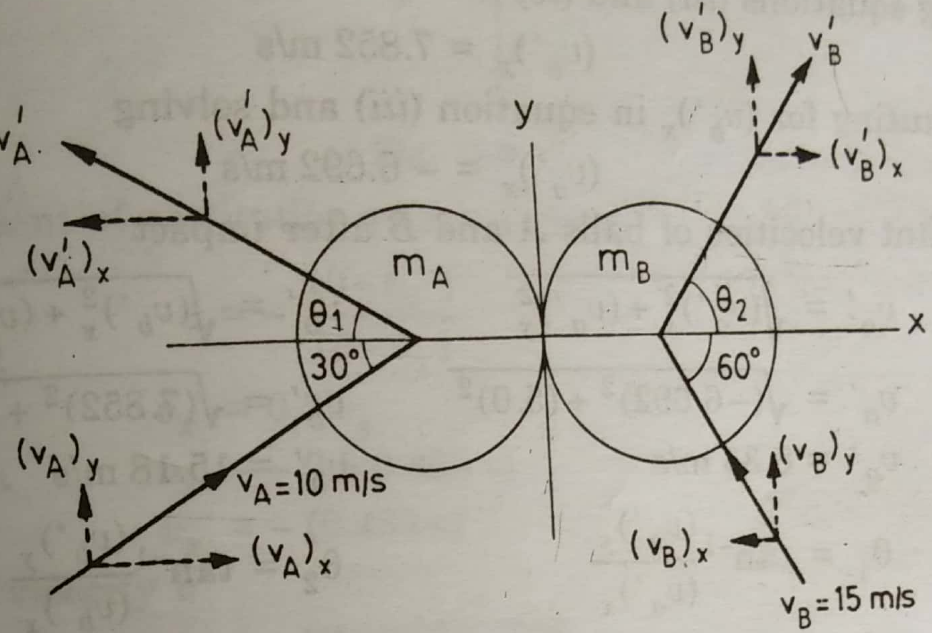

Fig. 18.8

Example 18.4 Two identical frictionless balls strike each other as shown in Fig. 18.8. Assuming $e = 0.90$, determine the magnitude and direction of the velocity of the each ball after the impact.

Solution. Components of the initial velocity of ball *A*

$$(v_a)_x = v_a \cos 30° = 10 \times \cos 30°$$
$$(v_a)_x = +\,8.66 \text{ m/s}$$
$$(v_a)_y = v_a \sin 30° = 10 \times \sin 30°$$
$$(v_a)_y = +\,5 \text{ m/s}$$

Components of the initial velocity of ball *B*

$$(v_b)_x = -\,v_b \cos 60° = -\,15 \cos 60°$$
$$(v_b)_x = -\,7.5 \text{ m/s}$$
$$(v_b)_y = +\,v_b \sin 60° = 15 \cos 60°$$

$$(v_b)_y = +\ 12.99 \text{ m/s}$$
$$m_a = m_b = m$$

Motion in y-direction : Velocity of *each ball* normal to the line of impact remains unchanged

Ball A $\quad (v_a')_y = (v_a)_y = 5 \text{ m/s}$

Ball B $\quad (v_b')_y = (v_b)_y = 12.99 \text{ m/s}$

Motion in x-direction : Conservation of momentum gives,

$$m_a(v_a)_x + m_b(v_b)_x = m_a(v_a')_x + m_b(v_b')_x$$

Or $\quad 8.66 + (-7.5) = (v_a')_x + m_b(v_b')_x$

Or $\quad (v_a')_x + (v_b') = 1.16 \text{ m/s}$

The coefficient of restitution relation gives

$$e = -\frac{(v_b')_x - (v_a')_x}{(v_b)_x - (v_a)_x}$$

$$-e[(v_b)_x - (v_a)_x] = (v_b')_x - (v_a')_x$$
$$-0.9\,[(-7.5) - 8.66] = (v_b')_x - (v_a')_x$$
$$(v_b')_x - (v_a')_x = 14.544$$

Adding equations (*iii*) and (*iv*)

$$(v_b')_x = 7.852 \text{ m/s}$$

Substituting for $(v_b')_x$ in equation (*iii*) and solving

$$(v_a')_x = -\ 6.692 \text{ m/s}$$

Resultant velocities of balls A and B after impact

$$v_a' = \sqrt{(v_a')_x^2 + (v_a')_y^2} \qquad v_b' = \sqrt{(v_b')_x^2 + (v_b')_y^2}$$

$$v_a' = \sqrt{(-6.692)^2 + (5.0)^2} \qquad v_b' = \sqrt{(7.852)^2 + (12.99)^2}$$

$$v_a' = 8.35 \text{ m/s} \qquad v_b' = 15.18 \text{ m/s} \quad \textit{Ans.}$$

$$\theta_1 = \tan^{-1}\frac{(v_a')_y}{(v_a')_x} \qquad \theta_2 = \tan^{-1}\frac{(v_b')_y}{(v_b')_x}$$

$$\theta_1 = \tan^{-1}\frac{5.0}{(-6.692)} \qquad \theta_2 = \tan^{-1}\frac{12.99}{7.852}$$

$$\theta_1 = 36.76° \qquad \theta_2 = 58.84° \quad \textit{Ans.}$$

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