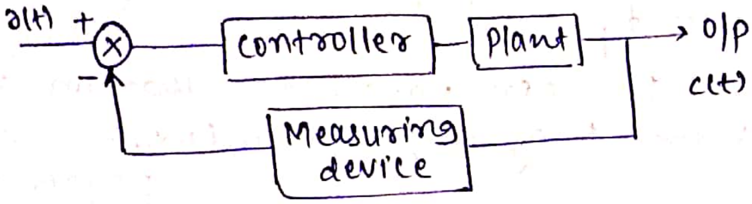
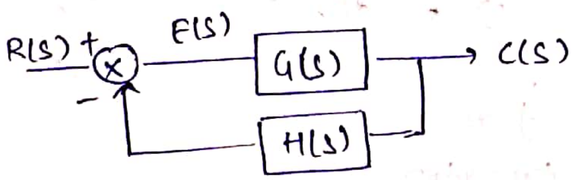
Control System





$C(s) = G(s) \cdot E(s)$ — ①

$E(s) = R(s) - H(s)\, C(s)$ — ②

Put ② in ①

$$\therefore \quad C(s) = G(s)\left[R(s) - H(s)\cdot C(s)\right]$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\, H(s)}}$$ for −ve feedback

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\, H(s)}}$$ for +ve feedback

→ **Benefits of feedback**

1. o/p follow input
2. Accuracy

→ **Reduction of parameter variations of by use of feedback**

- **Sensitivity** : Measures of effectiveness of feedback in reducing the influence of these variations on system performance

↳ **open loop**

$C(s) = G(s) \cdot R(s)$

$G(s) \rightarrow \Delta G(s) \rightarrow G(s) + \Delta G(s)$

$C(s) + \Delta C(s) = G(s) + \Delta G(s)\, R(s)$ $\quad \because \boxed{|G(s)| >> \Delta G(s)}$

$$\boxed{\Delta C(s) = \Delta G(s)\, R(s)}$$

↳ **Closed loop**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$$

$G(s) \rightarrow G(s) + \Delta G(s)$, $C(s) \rightarrow C(s) + \Delta C(s)$

$$= \frac{C(s) + \Delta C(s)}{R(s)} = \frac{G(s) + \Delta G(s)}{1 + (G(s) + \Delta G(s))\, H(s)}$$

$$= C(s) + \Delta C(s) = \frac{G(s) \cdot R(s)}{1 + G(s) \cdot H(s)} + \frac{\Delta G(s) \cdot R(s)}{1 + \Delta G(s)\, H(s)}$$

$$\boxed{\Delta C(s) = \frac{\Delta G(s) \cdot R(s)}{1 + G(s) \cdot H(s)}}$$ $\quad \because \boxed{\Delta G(s)\, H(s) << 1 + G(s)\, H(s)}$

$\therefore$ $\Delta C(s)$ is reduced by a factor of $1+G(s)H(s)$ in closed loop.

* Effect of feedback on sensitivity

 let $P \to$ gain parameter that may vary due to variations in parameters R of the system (i.e., $R \to$ system parameter)

$$\boxed{S = \frac{\%\text{ change in } R}{\%\text{ change in } P}}$$

$$\boxed{S_P^R = \frac{d(\ln R)}{d(\ln P)} = \frac{\delta R/R}{\delta P/P}}$$

consider, $T(s)$ = overall transfer func

$G(s)$ = forward path transfer fun^c

$$\boxed{S_G^T = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{G(s)}{T(s)} \times \frac{\partial T(s)}{\partial G(s)}}$$

for open loop, $T(s) = G(s)$ $\therefore$ $\boxed{S_G^T = \frac{\partial(G(s))/G(s)}{\partial G(s)/G(s)} = 1}$

$G(s) = G(s)$

for closed loop, $T(s) = \dfrac{G(s)}{1+G(s).H(s)}$.

$$\frac{\partial T(s)}{\partial G(s)} = \frac{1+G(s).H(s) - H(s).G(s)}{(1+G(s)H(s))^2} = \frac{1}{(1+G(s)H(s))^2}$$

$$S_G^T = \frac{G(s)/G(s)}{1+G(s)H(s)} \cdot \frac{1}{[1+G(s)H(s)]^2} \qquad \boxed{S_G^T = \frac{1}{1+G(s)H(s)}}$$

o Effect of feedback

$\Rightarrow S_G^T = \dfrac{1}{1+G(s)H(s)}$ $\qquad \Rightarrow S_H^T = \dfrac{H(s)}{T(s)} \cdot \dfrac{\partial T(s)}{\partial H(s)}$

$\Rightarrow T(s) = \dfrac{G(s)}{1+G(s)H(s)}$ $\qquad \Rightarrow \dfrac{\partial T(s)}{\partial H(s)} = \dfrac{-G^2(s)}{(1+G(s)H(s))^2}$

Put in S_H^T

$$S_H^T = \frac{H(s)}{G(s)} \cdot (1+G(s)H(s)) \times \frac{-[G(s)]^2}{[1+G(s)H(s)]^2}$$

$$\boxed{S_H^T = \frac{-G(s)H(s)}{1+G(s)H(s)}}$$

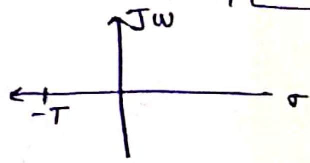
* Effect of feedback on overall gain

$\rightarrow$ open loop $\dfrac{C(s)}{R(s)} = G(s)$ $\longrightarrow$ closed loop $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$ $\longrightarrow [G]$

* Effect of feedback on stability

If all the roots lies in left half of s-plane then stable otherwise unstable if any of roots lies on right side.

↳ open loop $\frac{C(s)}{R(s)} = G(s) = \frac{K}{s+T}$ Poles are $s+T=0$; $\boxed{s=-T}$



↳ closed loop $H(s)=1$, $G(s)/1+G(s)$

* Mathematical Model / Transfer func

Input = $r(t)$, output = $c(t)$

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1}c(t)}{dt^{n-1}} + \cdots + a_1 \frac{dc(t)}{dt} + a_0 c(t)$$

$$= b_n \frac{d^n r(t)}{dt^n} + b_{n-1}\frac{d^{n-1} r(t)}{dt^{n-1}} + \cdots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$$

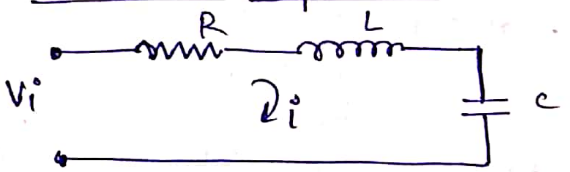
where a & b are constants.

Taking LT on both sides

$$C(s)[a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0] = R(s)[b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0]$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}}$$

* Electrical system



$$V_i = Ri + L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt$$

$$V_o = \frac{1}{C}\int i(t)\,dt$$

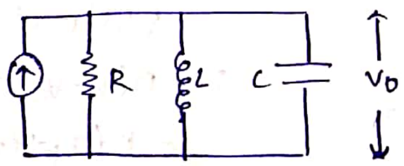
$$V_i(s) = RI(s) + L[sI(s) - i(0)] + \frac{1}{Cs}I(s)$$

$$V_o(s) = \frac{1}{Cs}I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{I(s)/\cancel{sC}}{CRI(s) + LCs^2 I(s) + I(s)\,/\cancel{sC}}$$

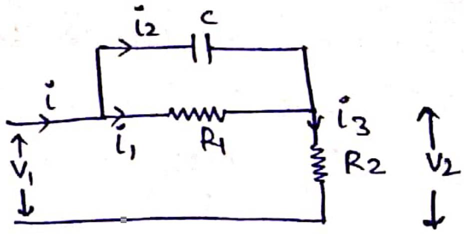
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC + RsC + 1}}$$

↳ In parallel



$$\boxed{i = \frac{1}{L}\int V_o\,dt + C\frac{dV_o}{dt} + \frac{V_o}{R}}$$

* Transfer func of Electrical system



$i = i_1 + i_2$

$i_1 = \frac{V_1 - V_2}{R_1}$, $i_2 = \frac{c\,d(V_1 - V_2)}{dt}$

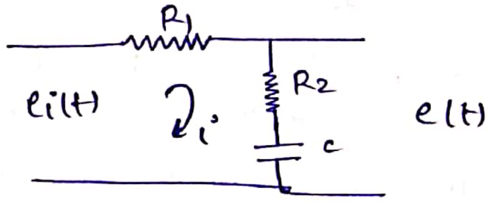
$i = i_B = \frac{V_2}{R_2}$, $i_B = i_1 + i_2$

$\Rightarrow \frac{V_2}{R_2} = \frac{V_1 - V_2}{R_1} + \frac{c\,d}{dt}(V_1 - V_2)$, $\Rightarrow \frac{1}{R_2} V_2(s) = \frac{1}{R_1}[V_1(s) - V_2(s)] + c[sV_1(s)]$

$\frac{V_2(s)}{R_2} + \frac{V_2(s)}{R_1} + sCV_2(s) = \frac{V_1(s)}{R_1} + sCV_1(s)$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{R_2 + R_1R_2Cs}{R_1 + R_2 + R_1R_2Cs}}$$

Q



$e_i(t) = R_1 i(t) + R_2 i(t) + \frac{1}{c}\int i(t)\,dt$ — ①

$e_o(t) = R_2 i(t) + \frac{1}{c}\int i(t)\,dt$ — ②

$e_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{Cs} I(s)$

$e_o(s) = R_2 I(s) + \frac{1}{Cs} I(s)$

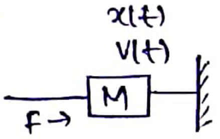
$$\boxed{\frac{e_o(s)}{e_i(s)} = \frac{sCR_2 + 1}{CsR_1 + CsR_2 + 1}}$$

* Transfer func of Mechanical system

Translation Motion	Rotational Motion
i) Force - (N) or (kgm/s^2)	i) Torque (Nm)
ii) Translation momentum $P = \int_{\infty}^{\infty} F\,dt$ (Ns)	ii) Angular momentum $h = \int_{\infty}^{t} T\,dt$
iii) vel. diff. $= (V_1 - V_2)$ m/s	iii) Angular velo. diff $= (\omega_1 - \omega_2)$ (rad/s)
iv) Displacement diff $= (x_1 - x_2)$ m	iv) Angular displn diff $= (\theta_1 - \theta_2)$

* Translational Elements

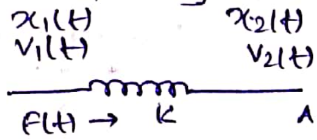
① Mass



$F = ma = m\frac{dv}{dt}$

$F = M\frac{d^2x}{dt^2}$

② spring

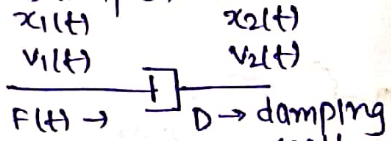


$F \propto$ displn.

$F = Kx = K(x_1 - x_2)$

$F = K\int_{-\infty}^{t}(V_1 - V_2)\,dt = K\int_{-\infty}^{t} V\,dt$

③ Damper



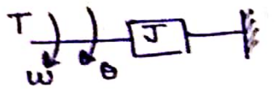
$F \propto$ velocity.

$F = D(V_1 - V_2) = D$

$= D(x_1 - x_2) = Dx$

* Rotational Elements

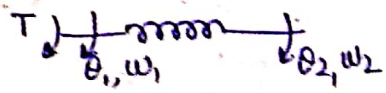
① Torque



$$T = J\frac{dw}{dt}$$

$$= J\frac{d^2\theta}{dt^2}$$

② Tortional spring



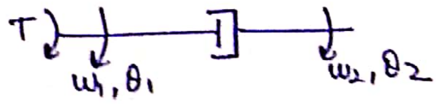
$$T = K\theta$$

$$= K(\theta_1 - \theta_2)$$

$$T = K\int_{-\infty}^{t}(\omega_1 - \omega_2)\,dt$$

$$= K\int_{-\infty}^{t}\omega\,dt$$

③ Dampeu

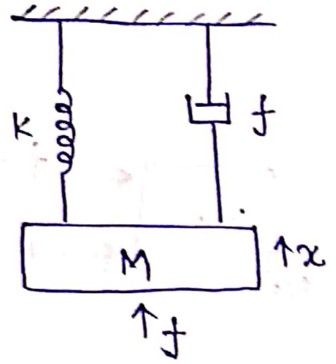


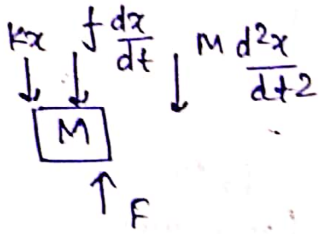
$$T = D\frac{d\theta}{dt} = D(\omega_1 - \omega_2)$$

$$= D\frac{d}{dt}(\theta_1 - \theta_2)$$

* D-Alembert Principle

It states that for any body the algebric sum of externally applied forces & the forces resisting motion in any given dirxn is 0.





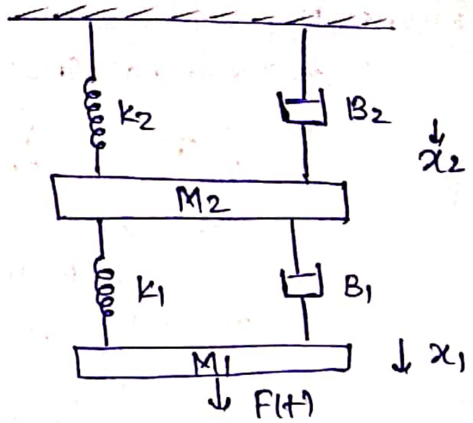
$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

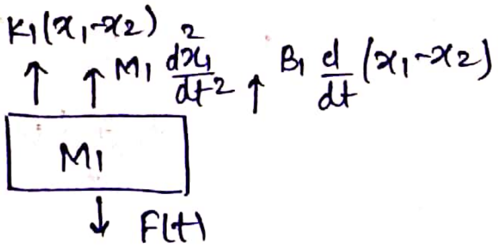
◦ Procedure for Mech. system

1. Assume the system is in equilibrium
2. Assume that the system is given some arbitary displacement if no. of distributing forces are present.
3. Draw the FBD of forces exerted on each mass in the system.
4. Apply N. law of motion to each diagram using the convention that any force acting in the dirxn of displacement is +ve.

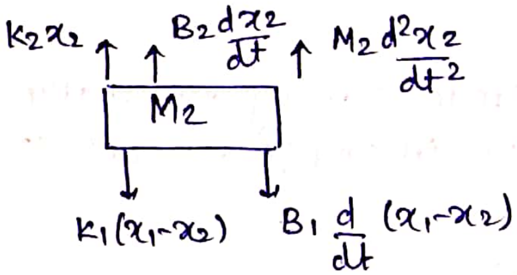
30/07/19.

Q



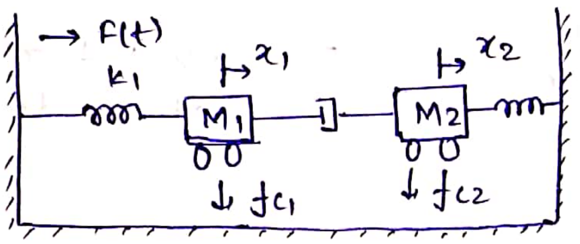


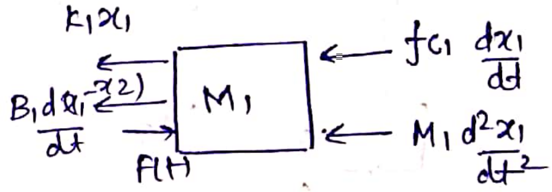
$$F(t) = K_1(x_1 - x_2) + M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d}{dt}(x_1 - x_2)$$



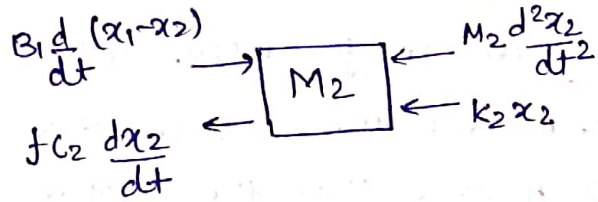
$$K_2x_2 + B_2\frac{dx_2}{dt} + M_2\frac{d^2x_2}{dt^2} = \text{''}$$

Q



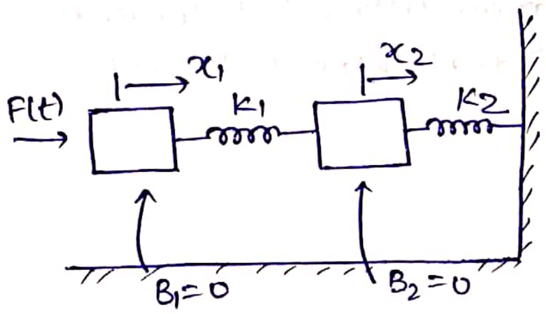


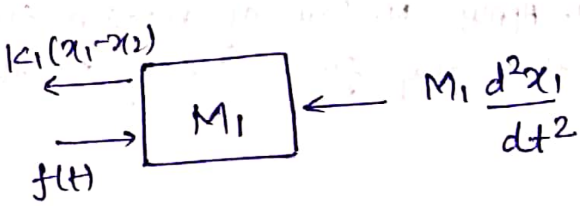
$$F(t) = M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1 - x_2) + f_{c1}\frac{dx_1}{dt} + K_1x_1$$

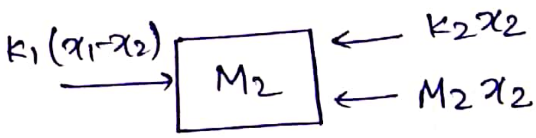


$$B_1\frac{d}{dt}(x_1 - x_2) = f_{c2}\frac{dx_2}{dt} + M_2\frac{d^2x_2}{dt} + K_1x_1$$

Q







$$F(t) = K_1(x_1 - x_2) + M_1x_1$$

$$K_1(x_1 - x_2) = K_2x_2 + M_2x_2$$

taking LT, $F(s) = K_1[x_1(s) - x_2(s)] + M_1 s^2 X_1(s)$ — (i)

$$K_1X_1(s) - K_1X_2(s) = K_2X_2(s) + M_2 s^2 X_2(s) \quad \text{— (ii)}$$

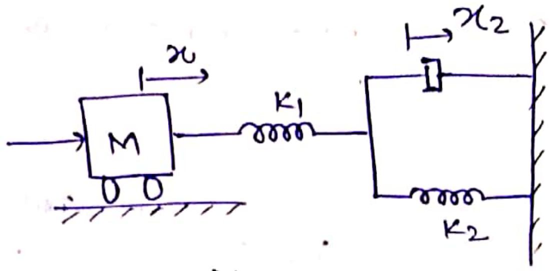
putting (II) in (I)

$$F(s) = [K_2X_2(s) + M_2s^2X_2(s) + K_1X_2(s)] - K_1X_2(s) + M_1s^2\left[\frac{K_2}{K_1}.X_2(s) + \frac{M_2}{K_1}s^2X_2(s) + X_2(s)\right]$$

$$\Rightarrow F(s) = M_2s^2X_2(s) + M_1s^2\frac{K_2}{K_1}X_2(s) + \frac{M_1M_2}{K_1}s^4X_2(s)$$

- Mechanical system

Model system



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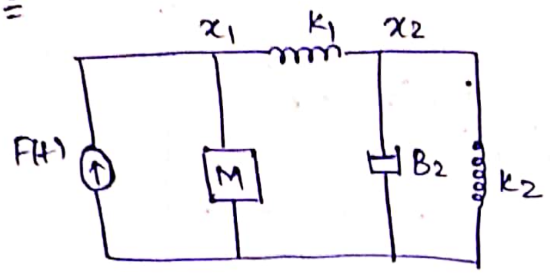
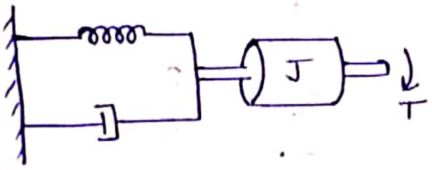
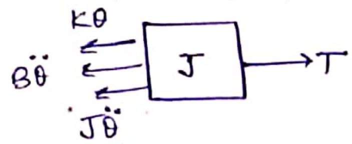


$$F(t) = M\ddot{x}_1 + K_1(x_1 - x_2)$$

$$K_1(x_2 - x_1) + B_2x_1 + K_2x_2 = 0$$

Q



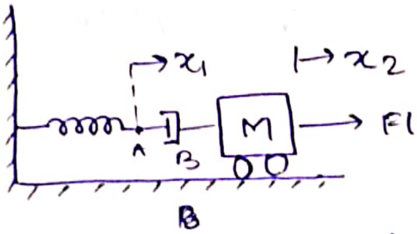


$$T = J\ddot{\theta} + B\dot{\theta} + K\theta$$

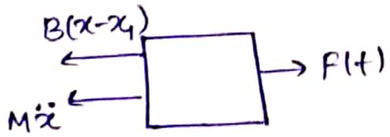
$$T(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + 1}}$$

Q

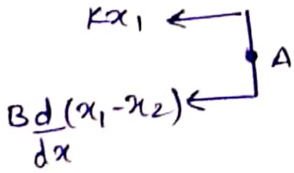


FBD of M



$$F(t) = B(x - x_1) + M\ddot{x}$$

$$F(s) = B[X(s) - X_1(s)] + Ms^2X(s)$$



$BsX_1(s) - BsX(s) + KX_1(s)$ — (2)

$F(s) = X(s)[B + Ms^2] - BX_1(s)$ — (1)

$B\frac{d}{dx}(x - x_1) + Kx_1 = 0$

$[Bs + K]X_1(s) - BsX(s) = 0$ — (2)

$$\boxed{X_1(s) = \frac{BsX(s)}{(Bs + K)}}$$

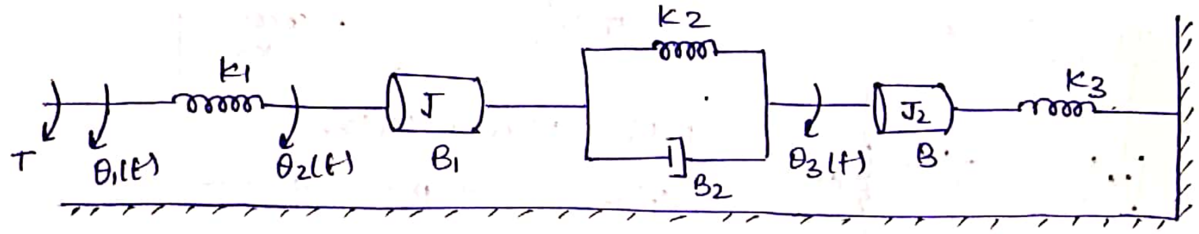
Put $X_1(s)$ in ①, $F(s) = X(s)[B + Ms^2] - B \cdot \frac{BsX(s)}{(Bs+K)}$

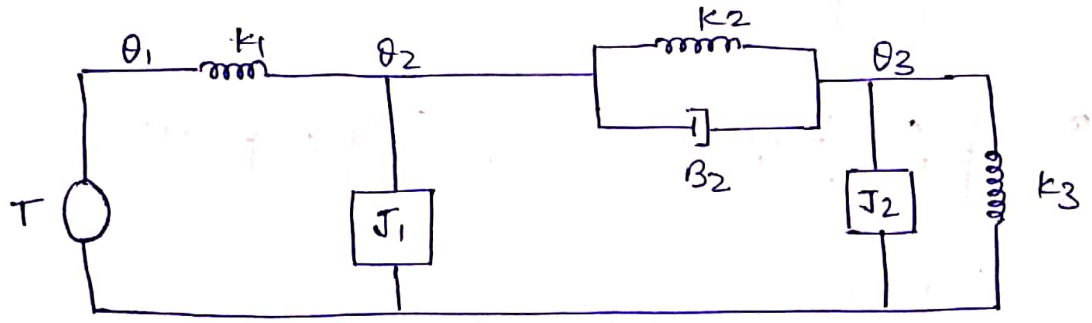
$$\Rightarrow F(s) = X(s)\left[B + Ms^2 - \frac{B^2 s}{(Bs+K)}\right]$$

$$\Rightarrow \frac{F(s)}{X(s)} = \left[\frac{(B+Ms^2)(Bs+K) - B^2 s}{Bs+K}\right] = \frac{\cancel{B^2 s} + BK + MBs^3 + KMs^2 - \cancel{B^2 s}}{Bs+K}$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{Bs+K}{BKs + MBs^3 + KMs^2}}$$

Q





$T = K_1(\theta_1 - \theta_2)$

$$\Rightarrow K_1(\theta_2 - \theta_1) + J_1 \frac{d^2\theta_2}{dt^2} + K_2(\theta_2 - \theta_3) + B_2 \frac{d}{dt}(\theta_2 - \theta_3) = 0$$

$$\Rightarrow K_2(\theta_1 - \theta_2) + K_3\theta_3 + J\frac{d^2\theta_3}{dt^2} + B_2 \frac{d}{dt}(\theta_3 - \theta_2) = 0$$

o Analogous Systems

↳ f-V analogy
T-V

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta \quad ; \quad V = \alpha\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Mechanical	Rotary	Electrical
1. Force, F	1. Torque, T	1. Voltage, V/e
2. Mass, M	2. Inertia, J	2. Inductor, L
3. Viscous velo., f	3. Vis. velo., f	3. Resistor, R
4. spring constt., K	4. spring. constt., K	4. 1/C
5. Displn, x	5. Angular displn, θ	5. charge, q
6. velocity, $\dot{x}$	6. Angular velo., $\dot{\theta}$	6. current, i
7. B, Damper coeff.	7. B, Damper coeff	7. R

↳ f-i analogy / T-i analogy

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx \quad ; \quad T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta$$

$$i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

Mechanical Translation	~~Rotary~~ Mech. Relation	Electrical Sys
F, M, f, K, B,	T, J, f, K, B	i, c, 1/R, 1/L
$x, \dot{x}, \ddot{x}$	$\theta, \dot{\theta}, \ddot{\theta}$	$\phi, \dot{\phi} = V, \ddot{\phi}$

* Steps for force-voltage (f-v) analogy

Step 1: Elements in series in electrical system will have same current conversaly elements having same velocity are in series.

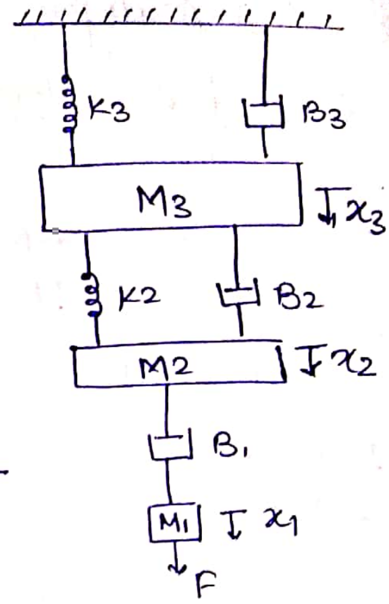
Step 2: In mechanical system every node correspondence to a close loop in electrical system where mass is considered as a node.

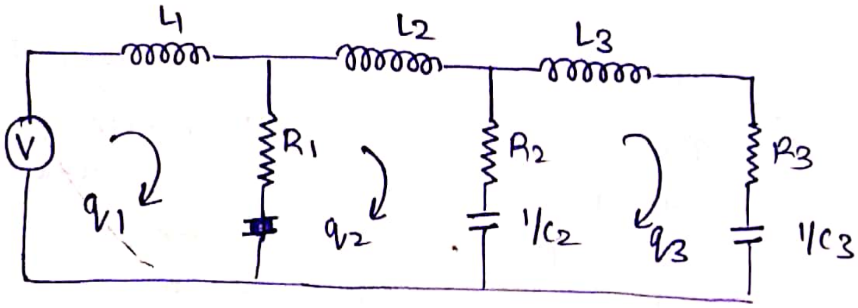
Q.

1st-loop $F \to V$, $M_1 \to L_1$, $B_1 \to R_1$

2nd-loop $M_2 \to L_2$, $B_2 \to R_2$, $K_2 \to 1/C_2$

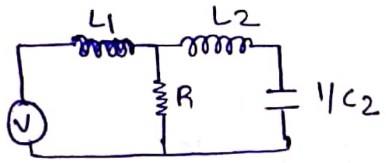
3rd loop $M_3 \to L_3$, $B_3 \to R_3$, $K_3 \to 1/C_3$

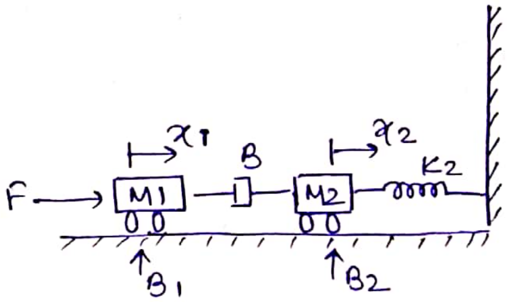




Q

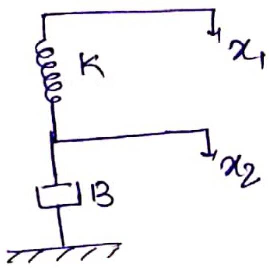
$F \to V$, $M_1 \to L_1$, $M_2 \to L_2$
$K_2 \to 1/C_2$, $B \to R$



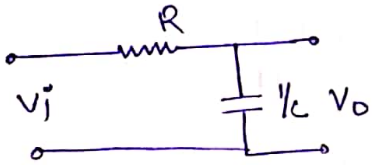


Q * Force-voltage Analogy

Q



$$\boxed{K(\dot{x}_1 - \dot{x}_2) = B\dot{x}_2}$$



$$V_i = iR + \frac{1}{C}\int i\,dt$$

$$V_o = \frac{1}{C}\int i\,dt$$

$$sK[X_1(s) - X_2(s)] = BsX_2(s)$$

$$\cancel{s}KX_1(s) = \cancel{s}X_2(s)[Bs+K)$$

$$\boxed{\frac{X_2(s)}{X_1(s)} = \frac{K}{Bs+K}}$$

taking L.T

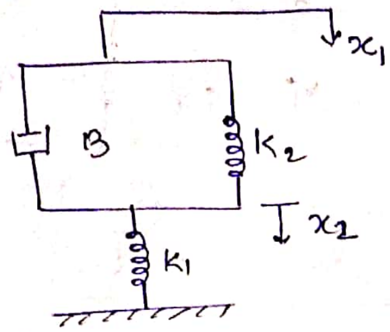
$$V_i(s) = RI(s) + \frac{1}{Cs} \cdot I(s)$$

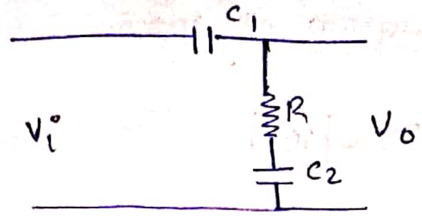
$$V_o(s) = \frac{1}{Cs} I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{I(s)/Cs}{I(s)[R + 1/Cs]}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{RCs+1}}$$

Q





$$K_1 x_2 = B_1(\dot{x}_1 - \dot{x}_2) + K_2(x_1 - x_2)$$

$$K_1 X_2(S) = SB_i X_1(S) - SB_i X_2(S) + K_2 X_1(S) - K_2 X_2(S)$$

$$X_2(S)[K_1 + SB + K_2] = X_1(S)(B + K_2)$$

$$\boxed{\frac{X_2(S)}{X_1(S)} = \frac{SB + K_2}{K_1 + SB + K_2}}$$

$$V_i(S) = \frac{1}{SC_1} I(S) + RI(S) + \frac{1}{SC_2} I(S)$$

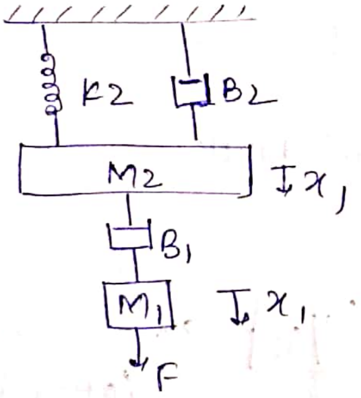
$$V_o(S) = RI(S) + \frac{R}{SC_2} I(S)$$

$$\boxed{\frac{V_o(S)}{V_i(S)} = \frac{(1 + SRC_2).C_1}{SC_2 + SRC_1C_2 + SC_1}}$$

- Force – current Anology: [conversion from mechanical to electrical]

1. Elements in mech. system having the same force are in parallel, similarly elements in elec. system will have same velocity when they are in parallel.
2. Any node i.e., the meeting points of elements in mech. system is analogous to a node in an electrical system a mass is taken as node.
3. Any element connected b/w two nodes i.e., masses in the mechanical system can be represented as a common element b/w two nodes in analogous electrical system.

Q



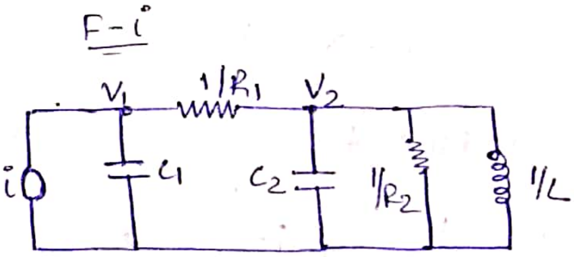
F → current Source

M → capacitance

B → 1/R

K → 1/L

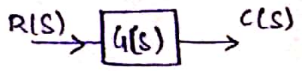
F – i



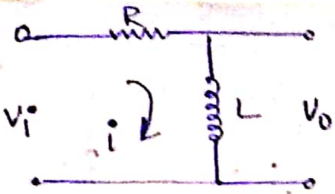
Nodes get connected to velocity

Scanned by CamScanner

- Block diagram algebra :



$$V_i = Ri + L\frac{di}{dt}$$

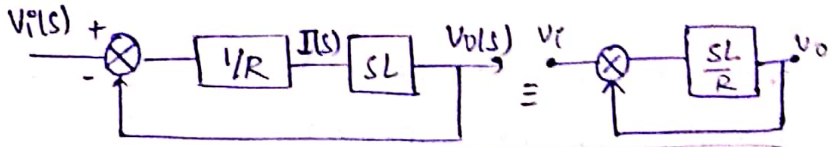


$$V_o = L\frac{di}{dt} = sL\,I(s)$$

$$i = \frac{V_i - V_o}{R}$$

$$\therefore \boxed{\frac{V_o(s)}{V_i(s)} = \frac{sL}{R+sL}}$$

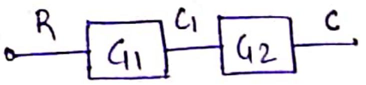
$$I(s) = \frac{1}{R}\left[V_i(s) - V_o(s)\right]$$



$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

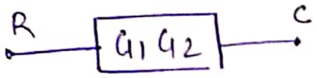
- Block Diagram Reduction

↳ Rule 1 :

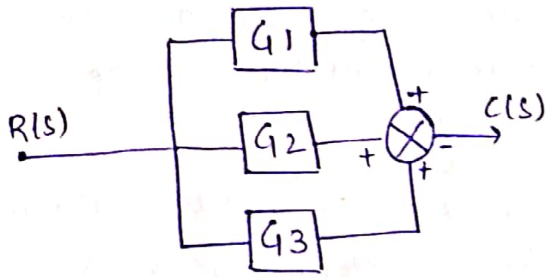


$$\frac{C}{R} = \frac{C}{C_1}\cdot\frac{C_1}{R} = G_1G_2$$

Blocks | components are in series

⇒ 

↳ Rule 2 : Blocks | compnents are in parallel

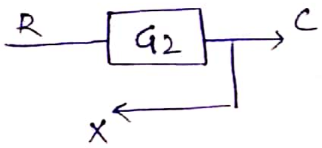


$$C(s) = G_1R + G_2R + G_3R$$

$$C(s) = (G_1 + G_2 + G_3)R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = G_1 + G_2 + G_3}$$

↳ Rule 3 : Moving a take-off point ahead of block



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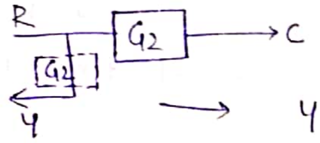
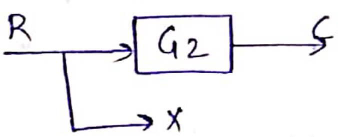


$C = X = G_2R$

$Y = R$ (initially) $\longrightarrow$ $Y = G_2R$ (after shifting)

↳ Rule 4 : Moving a take-off point after block



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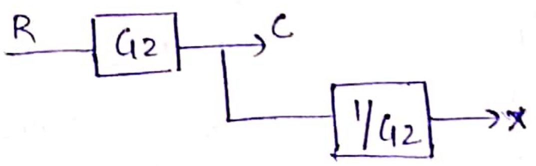
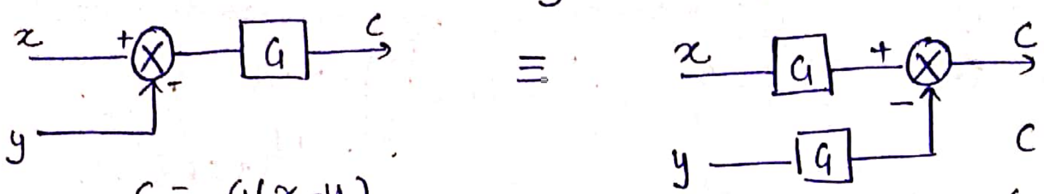


$X = R$ (initially)

$X = R$ (after shifting)

↳ **Rule 5:** Moving a summing point beyond a block

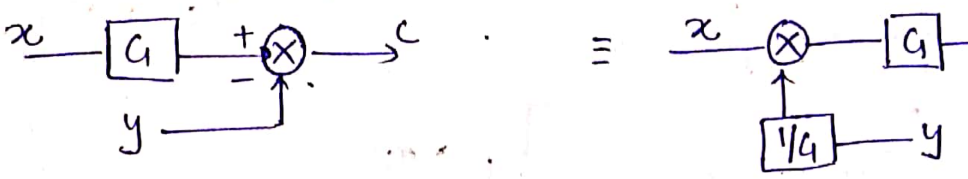


$c = G(x-y)$

$C = G(s)x - G(s)y$

$C = G(s)(x-y)$

↳ **Rule 6:** Moving a summing point ahead of block



$\left(x - \frac{y}{G}\right)G = c$

$c = G(x) - y$

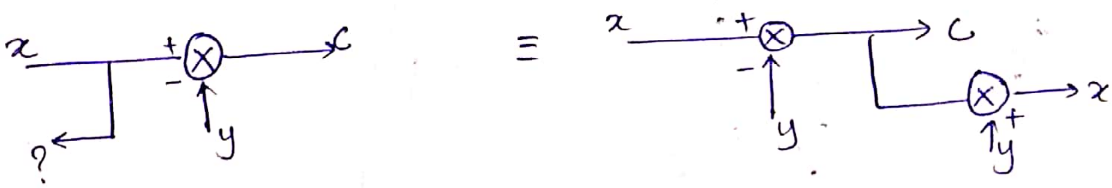
$c = Gx - y$

↳ **Rule 7:** Interchanging summing point

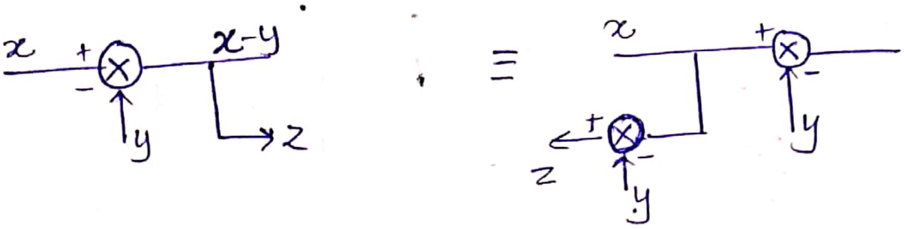
$c = x - y - z$

$c = x - z - y$

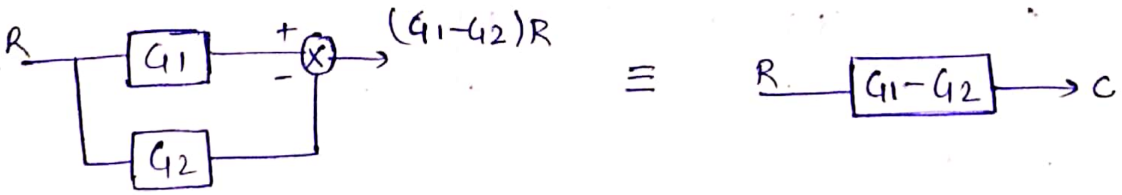
↳ **Rule 8:** Moving a take off point beyond the summing point



↳ **Rule 9:** Moving a take off point ahead the summing point

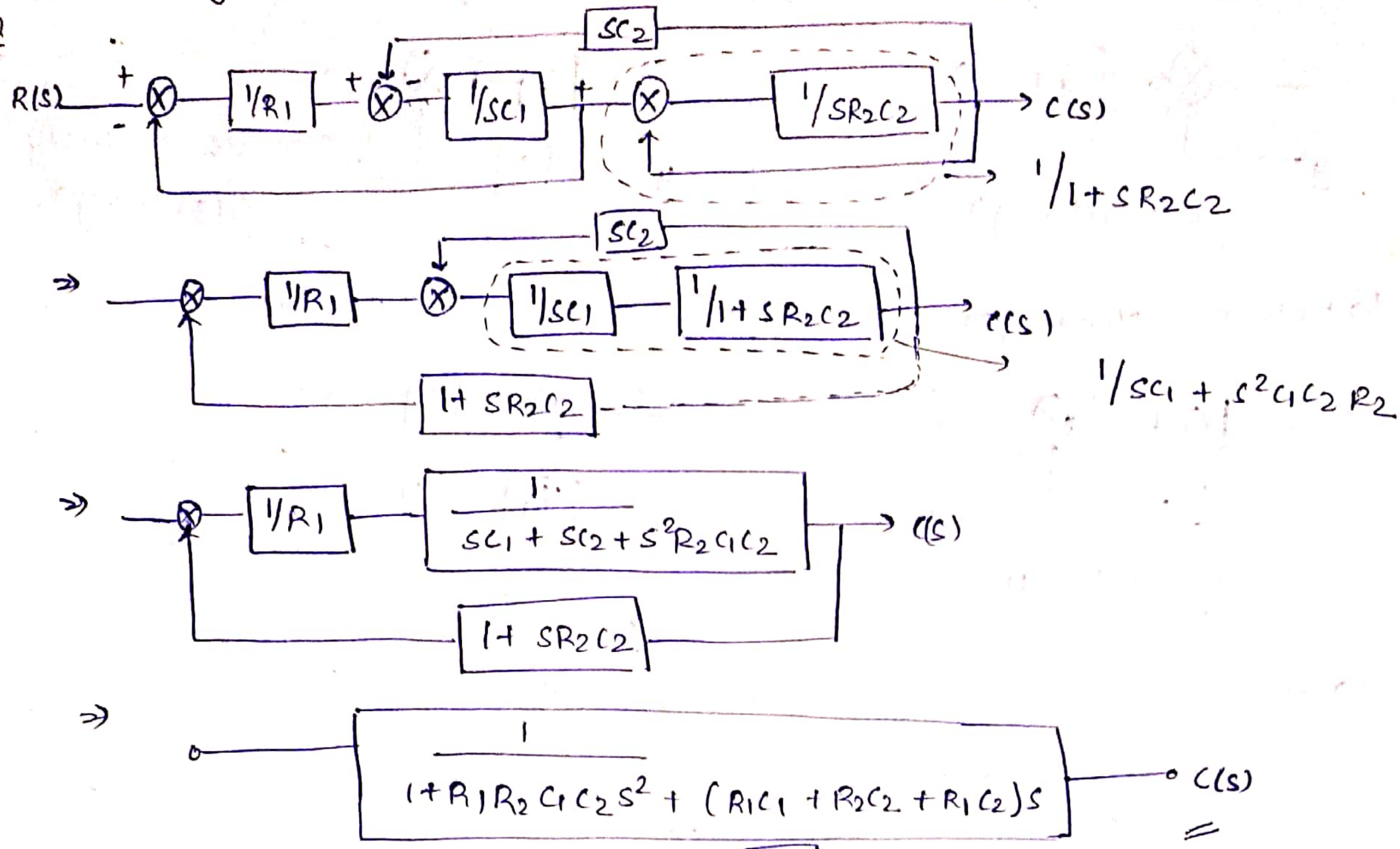


↳ **Rule 10:** Eliminating the forward loop



• Block Diagram Reduction Technique :

Q



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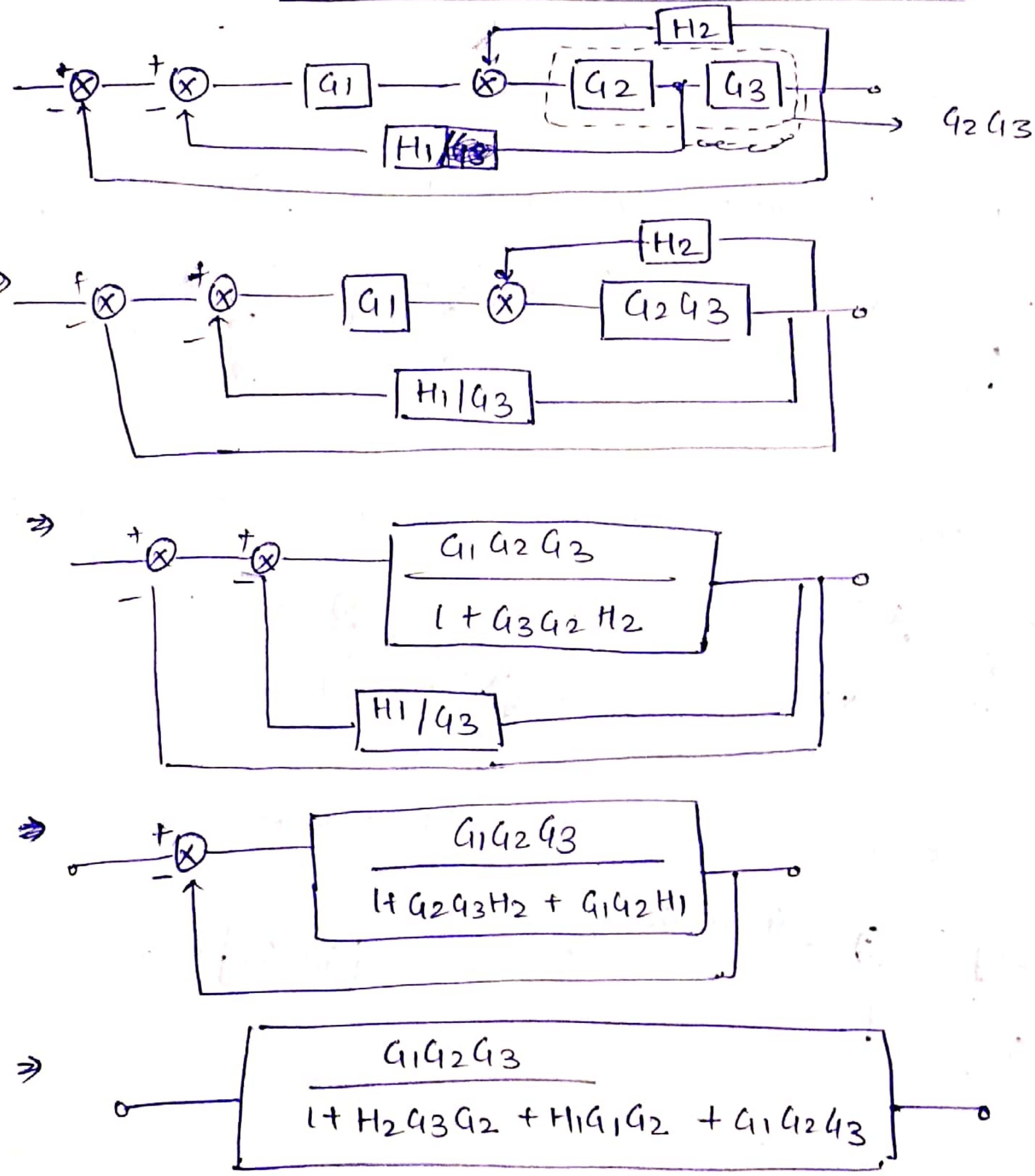
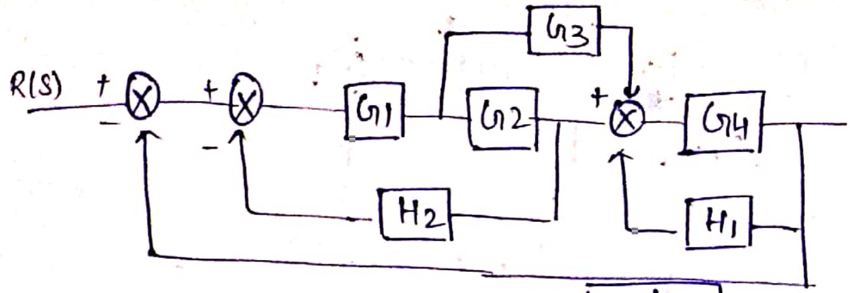
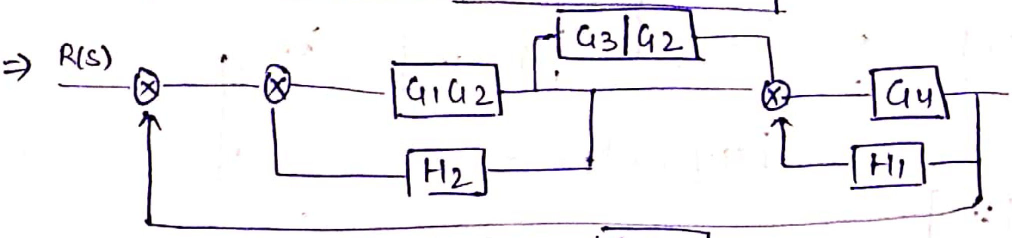
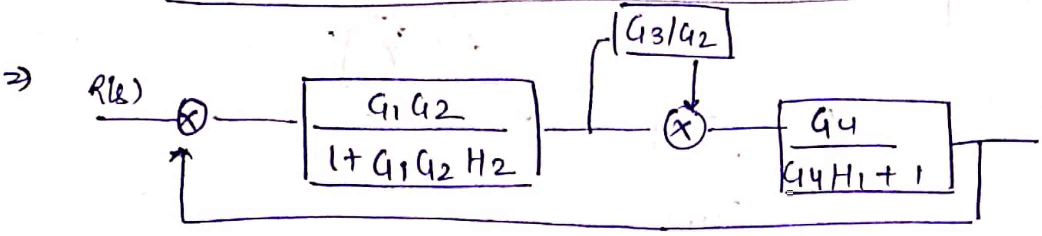
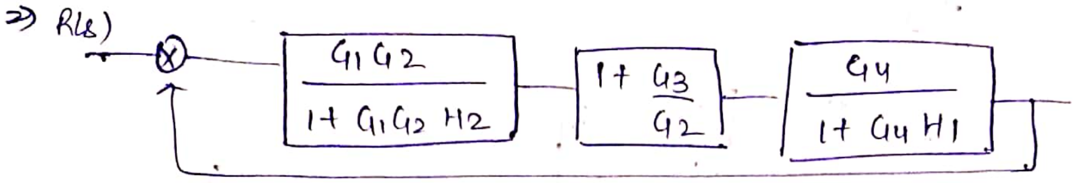


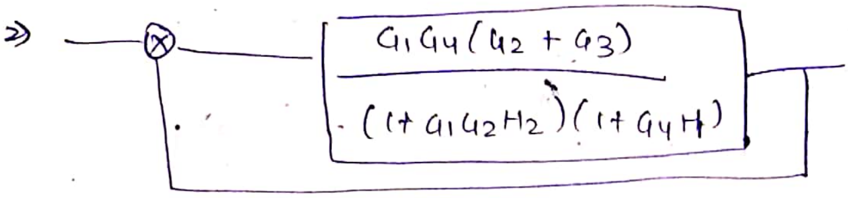
Q

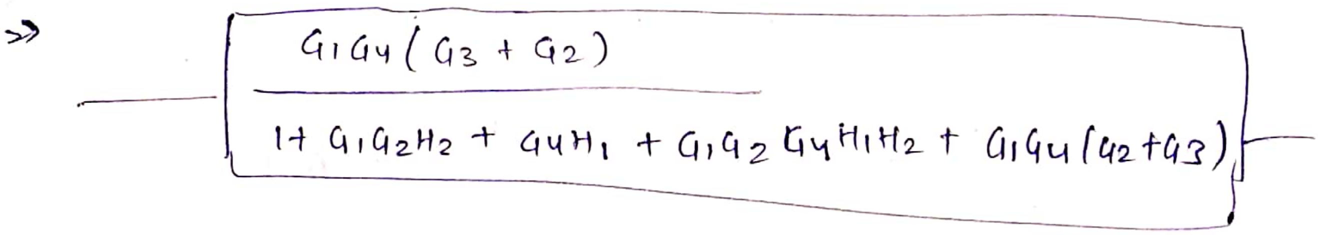

R(S)
+
-
+
-
G1
G2
G3
+
G4
H2
H1


⇒ R(S)
G3/G2
G1G2
G4
H2
H1

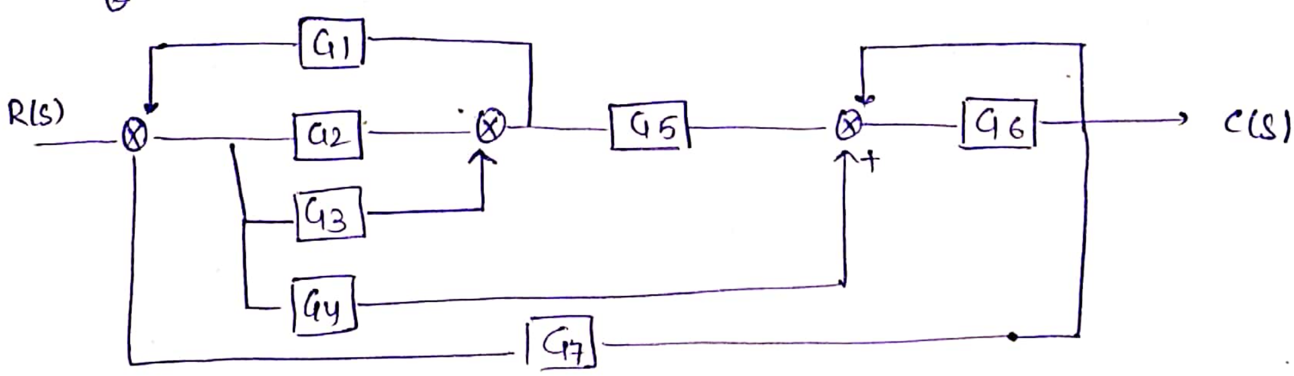

⇒ R(s)
G3/G2
$\frac{G_1G_2}{1+G_1G_2H_2}$
$\frac{G_4}{G_4H_1+1}$

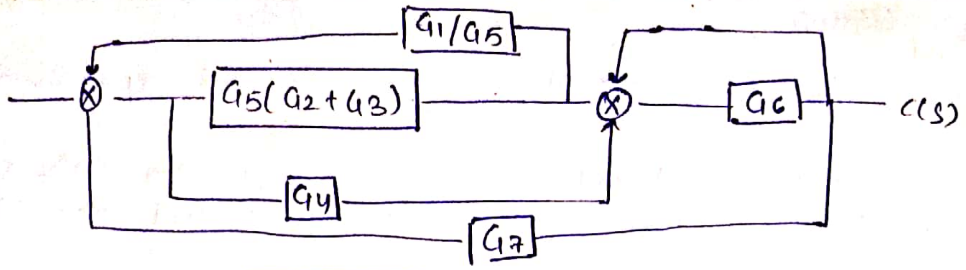

⇒ R(s)
$\frac{G_1G_2}{1+G_1G_2H_2}$
$1+\frac{G_3}{G_2}$
$\frac{G_4}{1+G_4H_1}$

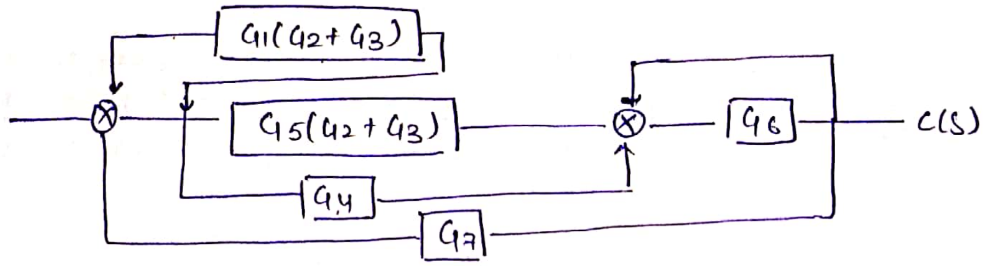

⇒
$\frac{G_1G_4(G_2+G_3)}{(1+G_1G_2H_2)(1+G_4H_1)}$

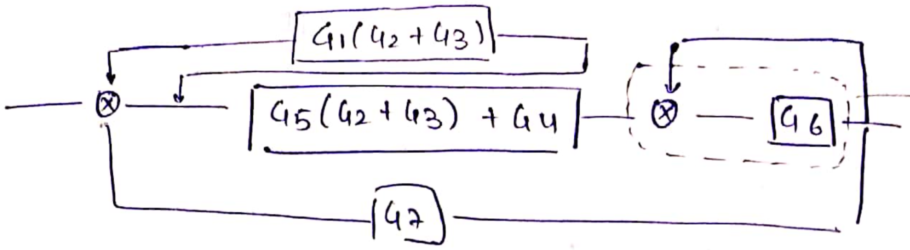

⇒
$\frac{G_1G_4(G_3+G_2)}{1+G_1G_2H_2+G_4H_1+G_1G_2G_4H_1H_2+G_1G_4(G_2+G_3)}$

Q2


R(S)
G1
G2
G3
G4
G5
G6
G7
+
C(S)

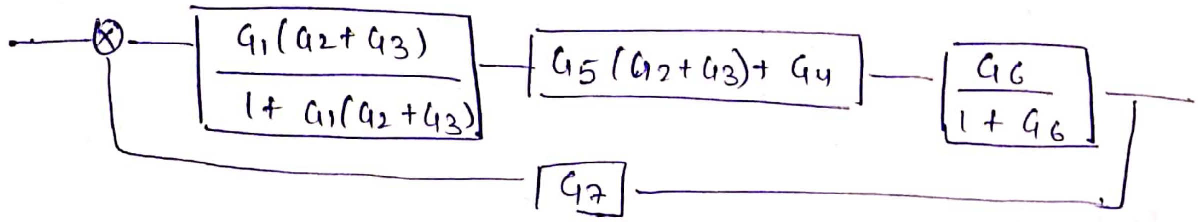
⇒


⇒


⇒


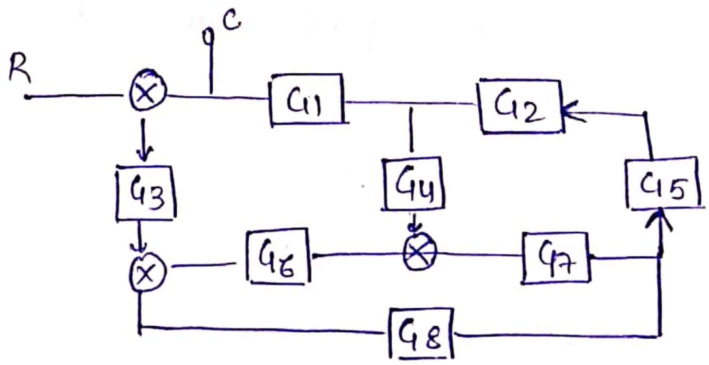
$\therefore \dfrac{G_6}{1+G_6}$

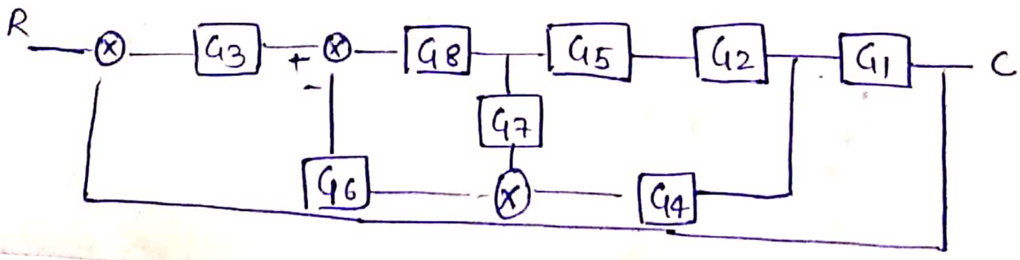
$\therefore \dfrac{G_1(G_2+G_3)}{1+G_1(G_2+G_3)}$

⇒


⇒

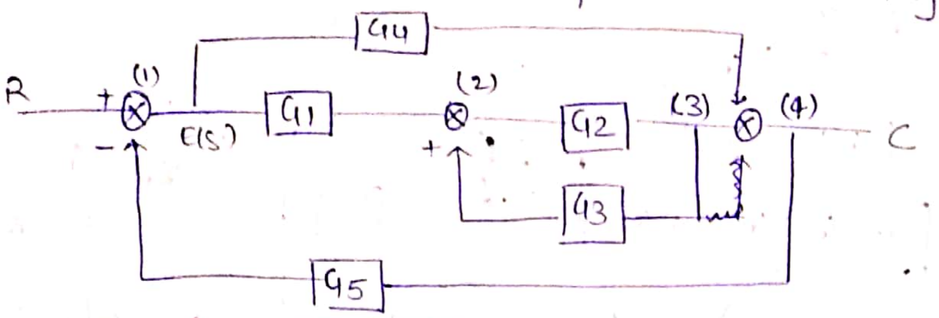
$$\frac{G_4G_6 + G_2G_5G_6 + G_3G_5G_6}{1 + G_6 + G_1G_2 + G_1G_3 + G_1G_2G_6 + G_1G_3G_6 + G_4G_6G_7 + G_2G_5G_6G_7 + G_3G_6G_5G_7}$$

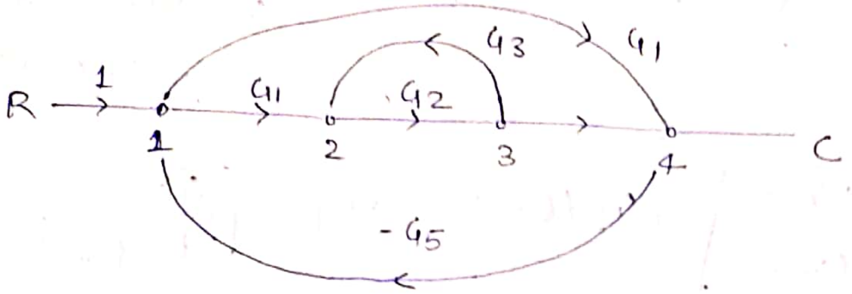
Q


⇒


- Signal Flow Graph . .

1. All variables summing points and take off pts are represented by nodes.
2. If the summing pts is placed before a take off pt in the dirn of signal flow in such case represent the summing pt and take off pt. by a single node.
3. If a summing pt. is placed after a take off pt in the dirn of signal flow in such case represent the summing pt & take off pt. by seperate node connected by a branch having transmittance unity.





- Mason's Gain Rule / Formula

$$\boxed{T = \frac{\sum g_k \Delta k}{\Delta}}$$

T → transfer func

g_k → gain of the k^{th} fwd path

Δk → part of Δ not touching the k^{th} forward path

Δ = 1 - [sum of all individual loop gain]
+ [sum of all possible gain products of two non-touching loop]
- [three non touching loop] + [ ] - ....

$g_1 = G_1G_2$; $g_2 = G_4$

$L_1 = G_2G_3$; $L_2 = -G_1G_2G_3$; $L_3 = -G_4 \cdot G_5$

Non-touching loop, $L_1L_3 = -G_2G_3G_4G_5$

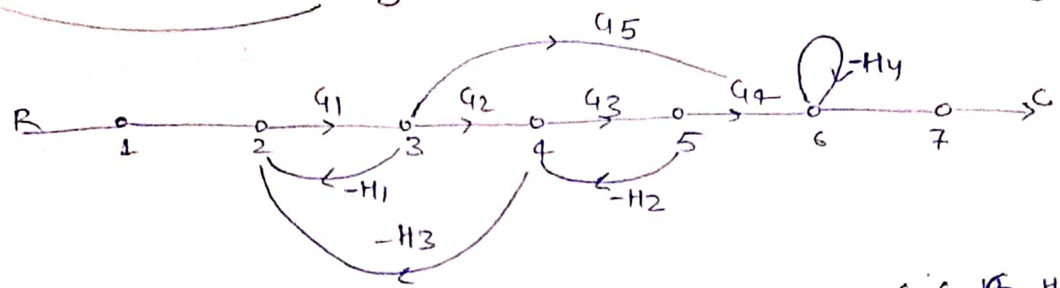
$\Delta 1 = 1 - 0 = 1$

$\Delta 2 = 1 - G_2G_3$

$$T = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta}$$

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 \cdot L_3]$$
$$= 1 - [G_1G_3 - G_1G_2G_5 - G_4G_5] - G_2G_3G_4G_5$$
$$\Delta = 1 - G_2G_3 + G_1G_2G_3 + G_4G_5 - G_2G_3G_4G_5$$
$$T = \frac{(G_1G_2).1 + (G_4)\cdot(1 - G_2G_3)}{\Delta} = \frac{G_1G_2 + G_4 - G_2G_3G_4}{\Delta}$$

Eg.



$g_1 = G_1G_2G_3G_4$, $g_2 = G_1G_5$, $L_3 = -G_1G_2G_3H_3$

$L_1 = -G_1H_1$, $L_2 = -G_3H_2$, $L_4 = -H_4$

2 non touching loops

$L_1L_2 = G_1H_1G_3H_2$; $L_1L_4 = G_1H_1H_4$; $L_2L_4 = G_3H_2H_4$; $L_3L_4 = G_1G_2G_3H_3H_4$

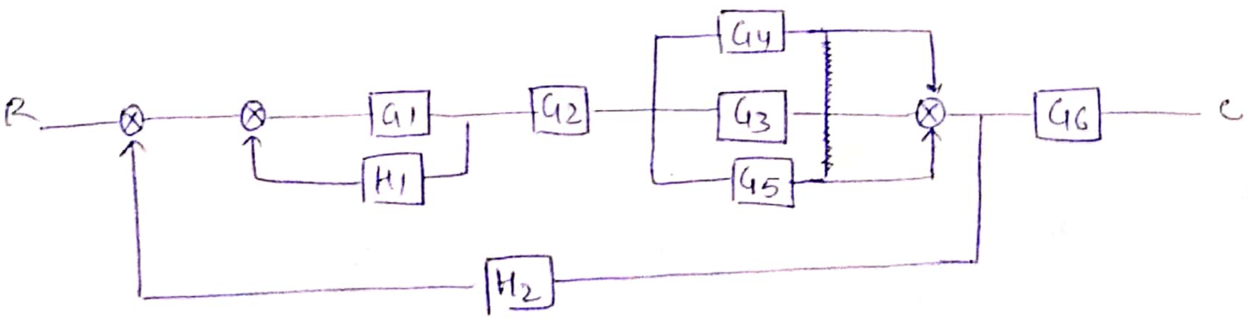
3 non-touching loops

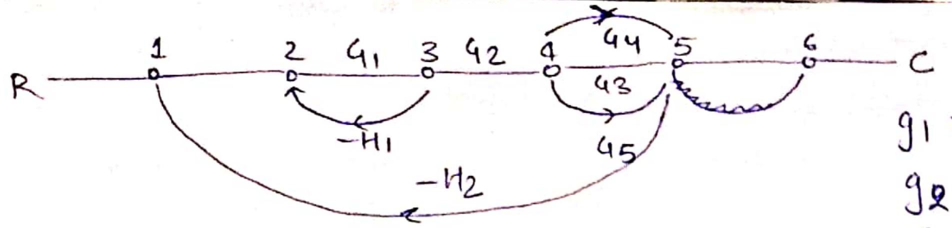
$L_1L_2L_4 = -G_1G_3H_1H_2H_4$

$\Delta 1 = 1 - 0 = 1$, $\Delta 2 = 1 - (-G_3H_2) = 1 + G_3H_2$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1L_2 + L_1L_4 + L_2L_4 + L_3L_4] - L_1L_2L_4$$
$$= 1 + [G_1H_1 + G_1H_1H_4 + G_3H_2H_4 + G_1G_2G_3H_3H_4]$$
$$- [-G_1H_1 - G_3H_2 - G_1G_2G_3H_3 - H_4] - [-G_1H_1G_3H_2H_4]$$
$$= 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + H_4 + G_1H_1 + G_1H_1H_4 + G_3H_2H_4$$
$$+ G_1G_2G_3H_3H_4 + G_1G_3H_1H_2H_4$$
$$T = 1 + 2G_1H_1 + G_3H_2(1 + H_4) + G_1G_2G_3H_3(1 + G_4) + H_4(1 + G_1G_3H_1H_2)$$
$$T = \frac{1.(G_1G_2G_3G_4) + (1 + G_3H_2)(G_1G_3)}{1 + 2G_1H_1 + G_3H_2(1 + H_4) + G_1G_2G_3H_3(1 + G_4) + H_4(1 + G_1G_3H_1H_2)}$$

Q





$g_1 = G_1G_2G_3G_6$

$g_2 = G_1G_2G_4G_6$

$g_3 = G_1G_2G_5G_6$

$L_1 = -G_1H_1$, $L_3 = -G_1G_2G_4H_2$

$L_2 = -G_1G_2G_3H_2$, $L_4 = -G_1G_2G_5H_2$

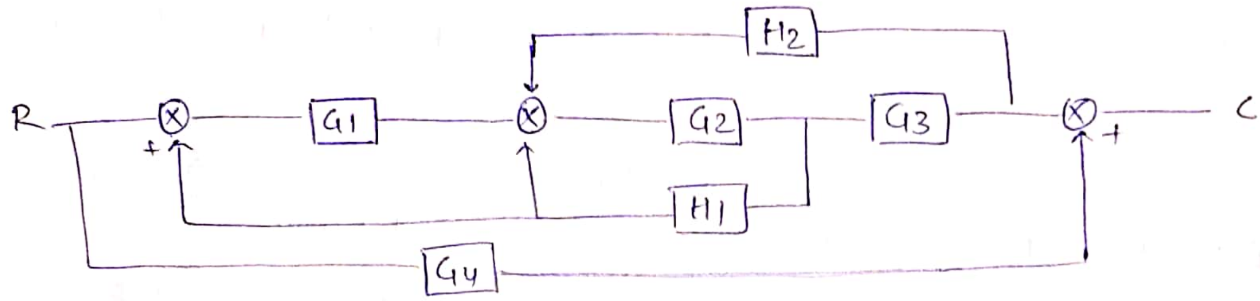
$\Delta 1 = 1-0 = 1$, $\Delta 2 = 1-0 = 1$, $\Delta 3 = 1-0 = 1$

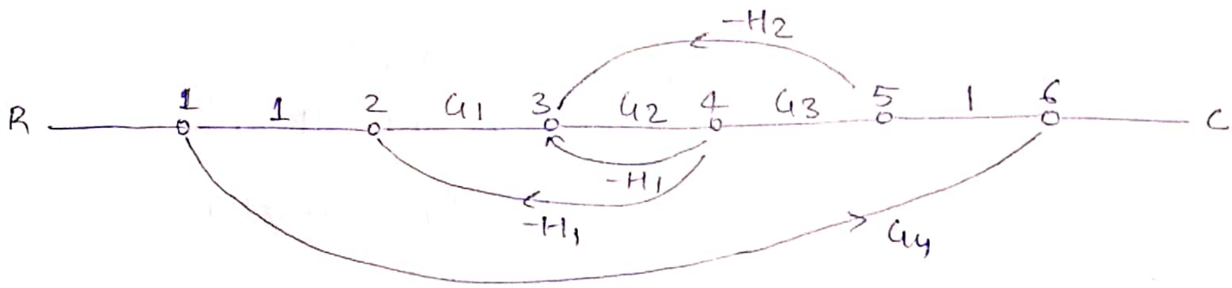
$\Delta = 1-[L_1+L_2+L_3+L_4]$

$$T = \frac{g_1\Delta 1 + g_2\Delta 2 + g_3\Delta 3}{\Delta}$$

$$T = \frac{G_1G_2G_3G_6 + G_1G_2G_4G_6 + G_1G_2G_5G_6}{1+[G_1H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_1G_2G_5H_2]}$$

Q





$g_1 = G_1G_2G_3$, $g_2 = G_4$

$L_1 = -H_1G_2$, $L_2 = -G_2G_1H_1$, $L_3 = -H_2G_2G_3$

NTL $= 0$

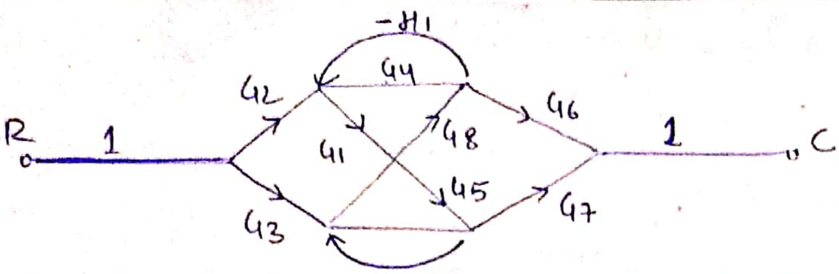
$\Delta 1 = 1-0 = 1$, $\Delta 2 = 1-[-G_2H_1 - G_1G_2H_1 - G_2G_3H_2]$

$\Delta = 1-[L_1+L_2+L_3]$

$T = g_1\Delta 1 + g_2\Delta 2 / \Delta$

$$T = \frac{G_1G_2G_3 + G_4[1+G_2H_1 + G_1G_2H_1 + G_2G_3H_2]}{1+H_1G_2 + G_2G_1H_1 + H_2G_2G_3}$$

8



$g_1 = G_2 G_4 G_6$ $\qquad$ $g_2 = G_3 G_5 G_7$

$g_3 = G_2 G_1 G_7$ $\qquad$ $g_4 = G_3 G_8 G_6$

$g_5 = -G_3 G_8 H_1 G_1 G_2$ $\qquad$ $g_6 = -G_2 G_1 H_2 G_8 G_6$

$L_1 = -G_4 H_1$; $L_2 = -G_5 H_2$; $L_3 = G_1 H_2 G_3 H_1$

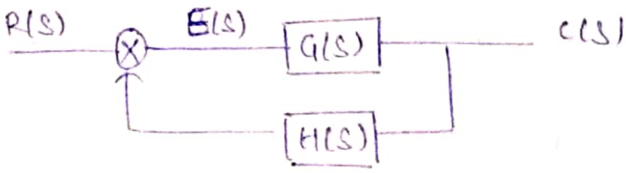
NTL , $L_1 L_2 = G_4 H_1 G_5 H_2$

$\Delta 1 = 1 - (-G_5 H_2)$; $\Delta 2 = 1 + G_4 H_1$; $\Delta 3 = 1$; $\Delta 4 = \Delta 5 = \Delta 6 = 1$

$$\therefore \frac{C}{R} = \frac{g_1 \Delta_1 + g_2 \Delta_2 + g_3 \Delta_3 + \dots + g_6 \Delta 6}{1 - [L_1 + L_2 + L_3] + [L_1 L_2]}$$

$=$

- Time Domain Analysis



$C(S) = G(S)\, E(S)$ —— ①

$E(S) = R(S) - H(S)\, C(S)$ —— ②

$= R(S) - H(S)\, G(S)\, E(S)$

$E(S)\left[1 + H(S) G(S)\right] = R(S)$

$$\boxed{\frac{E(S)}{R(S)} = \frac{1}{1 + H(S) G(S)}}$$

Steady state error

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} S E(S)$$

$$\boxed{e_{ss} = \lim_{S \to 0} \frac{S \cdot R(S)}{1 + G(S) H(S)}}$$

→ Test Signals

1. Step Signal $= A/s$
2. Ramp Signal $= A/s^2$
3. Parabolic Signal $= A/s^3$
4. Impulse Signal $= 1$

Eg ↳ If $R(S) = A/s$

$$e_{ss} = \lim_{s \to 0} \frac{S \cdot R(S)}{1 + G(S) H(S)} = \lim_{S \to 0} \frac{S \cdot A}{S(1 + G(S) H(S))} = \lim_{S \to 0} \frac{A}{1 + G(S) H(S)}$$

$$= \frac{A}{1 + \lim_{s \to 0} G(S) H(S)} = \frac{A}{1 + k_p}$$

Position error coeff. $\boxed{K_p = \lim_{s\to 0} G(s)H(s)}$

↳ If $R(s) = A/s^2$ take $A=1$.

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)H(s)} = \lim_{s\to 0} \frac{\cancel{s}\cdot 1}{s^{\cancel{2}}[1+G(s)H(s)]} = \lim_{s\to 0} \frac{1}{s+sG(s)H(s)}$$

$$= \frac{1}{K_V}$$

velocity error coeff. $\boxed{K_V = \lim_{s\to 0} sG(s)H(s)}$

↳ If $R(s) = A/s^3$

$$e_{ss} = \frac{\cancel{\lim} \; 1}{\lim_{s\to 0} s^2 G(s)H(s)} = \frac{1}{K_a}$$

acc. error coeff. $\boxed{K_a = \lim_{s\to 0} s^2 G(s)H(s)}$

o Effect of $G(s)H(s)$ on e_{ss}

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{s^j(1+sT_a)(1+sT_b)\ldots} \quad \text{(time constant)}$$

if $j=0$ → type zero system (i.e., poles lies on origin)

$j=1$ → type one system

① $R(s) = \frac{1}{s}$ — type 0, type 1, type 2

$$e_{ss} = \frac{1}{1+K_p}, \quad K_p = \lim_{s\to 0} G(s)H(s)$$

for zero degree $j=0$, $K_p = 1$

$$\boxed{e_{ss} = \frac{1}{1+K}}$$

for 1 degree, $j=1$, $K_p = \infty$

$\boxed{K_p = \infty}$ $\boxed{e_{ss} = 0}$

② $R(s) = \frac{1}{s^2}$

$$e_{ss} = \frac{1}{K_V}, \quad K_V = \lim_{s\to 0} G(s)H(s)$$

for type 0, $j=0$, $K_V = 0$

$\boxed{e_{ss} = \infty}$

for type 1, $j=1$, $K_V = K$

$\boxed{e_{ss} = \frac{1}{K}}$

for type 2, $j=2$, $K_V = \infty$

$\boxed{e_{ss} = 0}$

(3) $R(s) = 1/s^3$ $e_{ss} = \frac{1}{K_a}$ $K_a = \lim_{s\to 0} s^2 G(s)H(s)$

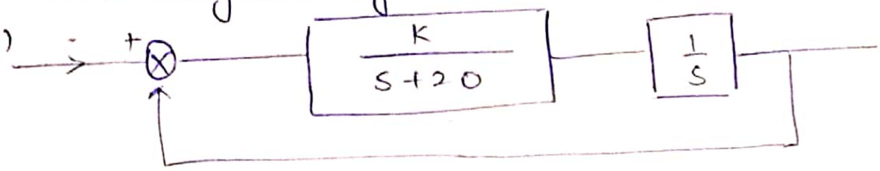
for type 0, $K_a = 0$, $\boxed{e_{ss} = \infty}$ | for type 1, $K_a = 0$ | for type 2, $K_a = K$, $\boxed{e_{ss} = \frac{1}{K}}$

Q. The open loop transfer func of unity feedback system is given by $G(s) = \frac{50}{(1+0.1s)(s+10)}$. Determine the static error coeff.

Ans. $K_p = \lim_{s\to 0} \frac{50}{(1+0.1s)(s+10)} = 5$

$K_v = \lim_{s\to 0} \frac{s \cdot 50}{(1+0.1s)(s+10)} = 0$, $K_a = \lim_{s\to 0} \frac{s^2 \cdot 50}{(1+0.1s)(s+10)} = 0$

Q The block diagram of electronic pacemaker is shown as



Determine the steady state error for unit ramp i/p when gain is 400. Also determine the value of K for which the steady state error to a unit ramp error by 0.02.

Ans. $K = 400$ $e_{ss} = 0.02$ $G(s) = \frac{K}{s(s+20)}$

Unit ramp, type 1

$e_{ss} = \frac{1}{K_v}$ ~~lim~~ $K_v = \lim_{s\to 0} \frac{\cancel{s}\, K}{\cancel{s}(s+20)} = \lim_{s\to 0} \frac{K}{s+20} = \frac{400}{20} = 20$

$\boxed{e_{ss} = \frac{1}{20} = 0.05}$ now $K = ?$ when $e_{ss} = 0.02$

$\therefore 0.02 = \frac{1}{\lim_{s\to 0} \frac{s \cdot K}{s(s+20)}} = \frac{1}{\lim_{s\to 0} \frac{K}{s+20}}$

$0.02 = \frac{1}{K/20}$ $\boxed{K = 1000}$

Q The open loop transfer func of a unity fbk func is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$. Find the static error coeff. and steady state error when subjected to ~~an~~ i/p given by $r(t) = 2 + 5t + 2t^2$.

Ans. $H(s) = 1$, $G(s) = \dfrac{108}{s^2(s+4)(s^2+3s+12)}$

$$H(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3} \quad \left[\frac{n!}{s^{n+1}}\right]$$

type 2 system

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+H(s)G(s)} \quad , \quad K_p = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)} = \infty$$

$K_v = \infty$, $K_a = 2.25$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3} \qquad = \frac{2}{1+K_p} + \frac{5}{K_v} + \frac{4}{K_a}$$

$$= \frac{2}{1+\infty} + \frac{5}{\infty} + \frac{4}{2.25} = 1.77$$

- **Time Response of First Order Signal systems:**

$H(S) = 1 \qquad G(S) = \frac{1}{ST}$

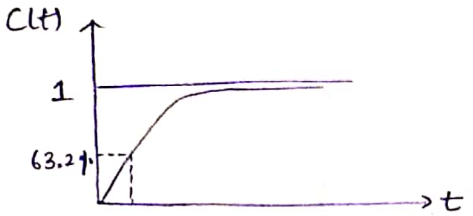
$\therefore \frac{C(S)}{R(S)} = \frac{1}{ST+1}$

→ Step signal $\quad R(S) = \frac{1}{S}$; $\quad C(S) = \frac{1}{S(ST+1)} = \frac{A}{S} + \frac{B}{ST+1}$

$\therefore A = 1 \text{ \& } B = -T$

$= \frac{1}{S} - \frac{T}{ST+1}$

$\therefore \boxed{c(t) = 1 - e^{-t/T}}$



put $t = T$

$\therefore c(t) = 1 - e^{-1} = 1 - 0.368 = 63.2\%$

error, $e(t) = r(t) - c(t) = 1 - (1 - e^{-t/T})$

$\boxed{e(t) = e^{-t/T}}$

Steady state error, $e_{ss} = \lim_{t\to\infty} e(t)$

$\boxed{e_{ss} = 0}$

→ Ramp signal $\quad R(S) = \frac{1}{S^2}$; $\quad \frac{C(S)}{R(S)} = \frac{1}{ST+1} = \frac{1}{S^2(ST+1)}$

$= \frac{A}{S^2} + \frac{B}{S} + \frac{C}{ST+1}$

$\therefore A = 1,\ B = -T,\ C = T^2$

$\therefore C(S) = \frac{1}{S^2} - \frac{T}{S} + \frac{T^2}{ST+1}$

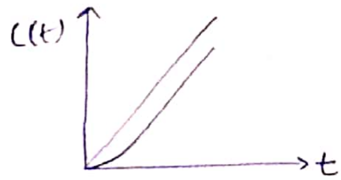
$c(t) = t - T + \frac{T^2}{T(S+\frac{1}{T})}$

$\boxed{c(t) = t - T + Te^{-t/T}}$

error,

$e(t) = r(t) - c(t) = t - t + T - Te^{-t/T} = T(1 - e^{-t/T})$

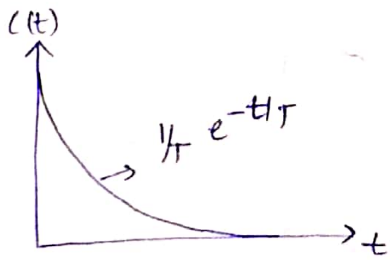
Steady state error, $e_{ss} = \lim_{t\to\infty} e(t) = T$



→ Impulse signal $\quad R(S) = 1$

$\therefore C(S) = \frac{1}{ST+1}$

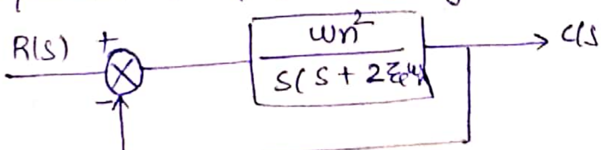
$\therefore c(t) = \frac{1}{T} e^{-t/T}$



error, $e(t)$ = can't find in this case

- **Time Response of Second Order System:** (only for step signal)

$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$



tendency to oppose oscill. $\leftarrow \xi \rightarrow$ damping ratio, $\omega_n \rightarrow$ natural freq. of oscill.

$w_d \to$ damped freq. of oscillation

$$C(s) = \frac{w_n^2}{s(s^2+2\xi_e w_n s + w_n^2)} = \frac{a_1}{s} + \frac{a_2 s + a_3}{s^2 + 2\xi_e w_n s + w_n^2}$$

$$a_1 = 1, \quad a_2 = -1, \quad a_3 = -2\xi_e w_n$$

$$\therefore C(s) = \frac{1}{s} + \frac{-s-2\xi_e w_n}{s^2+2\xi_e w_n s + w_n^2} = \frac{1}{s} - \left[\frac{s+2\xi_e w_n}{s^2+2\xi_e w_n s + w_n^2}\right]$$

$$= \frac{1}{s} - \left[\frac{s+\xi_e w_n + \xi_e w_n}{s^2+2\xi_e w_n s + \xi_e^2 w_n^2 - \xi_e^2 w_n^2 + w_n^2}\right]$$

$$= \frac{1}{s}\left[\frac{s+\xi_e w_n + \xi_e w_n}{(s+\xi_e w_n)^2 + \underbrace{w_n^2 - \xi_e^2 w_n^2}_{w_d^2}}\right]$$

$$= \frac{1}{s} - \left[\frac{s+\xi_e w_n}{(s+\xi_e w_n)^2 + w_d^2} + \frac{\xi_e w_n \cdot w_d}{w_d (s+\xi_e w_n)^2 + w_d^2}\right]$$

$$\because \left\{ e^{-at}\cos wt = \frac{s+a}{(s+a)^2+w^2} \; ; \; e^{-at}\sin wt = \frac{w}{(s+a)^2+w^2} \right\}$$

$$\therefore c(t) = 1 - \left[e^{-\xi_e w_n t}\cos w_d t + \frac{\xi_e w_n}{w_d} \cdot e^{-\xi_e w_n t}\sin w_d t\right]$$

$$= 1 - \left[e^{-\xi_e w_n t}\cos w_d t + \frac{\xi_e \cancel{w_n}}{\cancel{w_n}\sqrt{1-\xi_e^2}} \cdot e^{-\xi_e w_n t}\sin w_d t\right] \quad \because \left[w_d = w_n\sqrt{1-\xi_e^2}\right]$$

$$= 1 - e^{-\xi_e w_n t}\left[\cos w_d t + \frac{\xi_e}{\sqrt{1-\xi_e^2}}\sin w_d t\right]$$

$$= 1 - \frac{e^{-\xi_e w_n t}}{\sqrt{1-\xi_e^2}}\left[\sqrt{1-\xi_e^2}\cos w_d t + \xi_e \sin w_d t\right]$$

Put $\sqrt{1-\xi_e^2} = \sin\phi$, $\cos\phi = \xi_e$

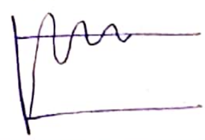
$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\xi_e^2}}{\xi_e}, \quad \phi = \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}$$

$$c(t) = 1 - \frac{e^{-\xi_e w_n t}}{\sqrt{1-\xi_e^2}}\left[\sin(w_d t + \phi)\right]$$

$$c(t) = 1 - \frac{e^{-\xi_e w_n t}}{\sqrt{1-\xi_e^2}}\sin\left[w_n\sqrt{1-\xi_e^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$$

$$e(t) = r(t) - c(t) = 1 - \frac{1 + e^{-\xi_e w_n t}}{\sqrt{1-\xi_e^2}}\sin\left[w_n\sqrt{1-\xi_e^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$$

$c(t)$



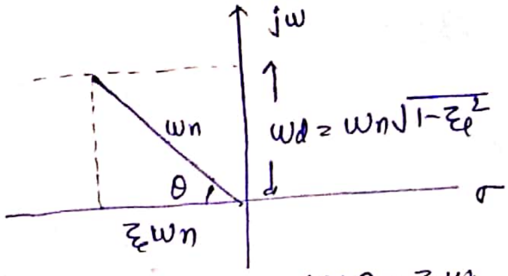
$$C_{ss} = \lim_{t\to\infty} c(t) = 1$$

- **Time Response of second order system:**

$$c(t) = 1 - \frac{e^{-\xi_e \omega_n t}}{\sqrt{1-\xi_e^2}} \sin\left[\omega_n\sqrt{1-\xi_e^2} + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$$

since,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi_e\omega_n s + \omega_n^2}$$



Characteristic eq^n, $s^2 + 2\xi_e\omega_n s + \omega_n^2 = 0$

$$s_1, s_2 = \frac{-2\xi_e\omega_n \pm \sqrt{4\xi_e^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$\cos\theta = \frac{\xi_e\omega_n}{\omega_n} = \xi_e$$

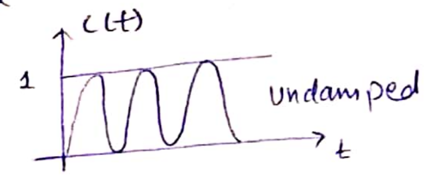
$$\theta = \cos^{-1}\xi_e$$

$$= -\xi_e\omega_n \pm \omega_n\sqrt{\xi_e^2 - 1}$$

For $R(s) = \frac{1}{s}$

↳ Case 1: $\xi_e = 0$, $s_1, s_2 = \pm j\omega_n$

$$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}, \quad c(t) = 1 - \sin\left(\omega_n t + \frac{\pi}{2}\right) = 1 - \cos\omega_n t$$

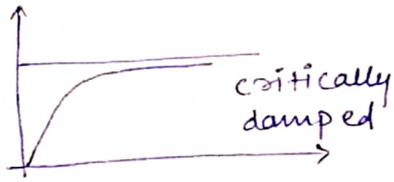


↳ Case 2: $\xi_e = 1$, $s_1, s_2 = -\omega_n$

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$A = 1$, $B = -1$, $C = -\omega_n$

$$C(s) = \frac{1}{s} + \frac{-1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2} = 1 - e^{-\omega_n t}(1 + \omega_n t)$$



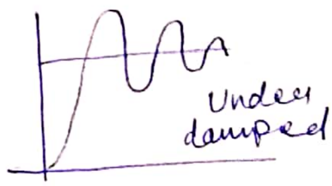
$$\left\{ \frac{\text{Actual damping}}{\text{Critical damping}} = \frac{\xi_e\omega_n}{\omega_n} = \xi_e \right\}$$

↳ Case 3: $0 < \xi_e < 1$, $s_1, s_2 = -\xi_e\omega_n \pm j\omega_n\sqrt{1-\xi_e^2}$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\xi_e\omega_n s + \omega_n^2} = \frac{a_1}{s} + \frac{a_2 s + a_3}{s^2 + 2\xi_e\omega_n s + \omega_n^2}$$

$a_1 = 1$, $a_2 = 1$, $a_3 = -2\xi_e\omega_n$, $c(t) =$

$$c(t) = 1 - \frac{e^{-\xi_e\omega_n t}}{\sqrt{1-\xi_e^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right)$$



(ii)

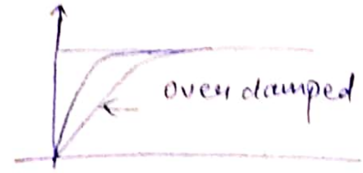
↳ Case 4: $\xi_e > 1$ $S_1, S_2 = -\xi_e\omega_n \pm \omega_n\sqrt{\xi_e^2-1}$

$$C(s) = \frac{1}{s}\cdot\frac{\omega_n^2}{s^2+2\xi_e\omega_n s+\omega_n^2} = \frac{A_1}{s} + \frac{A_2}{s+\omega_n(\xi_e+\sqrt{\xi_e^2-1})} + \frac{A_3}{s+\omega_n(\xi_e-\sqrt{\xi_e^2-1})}$$

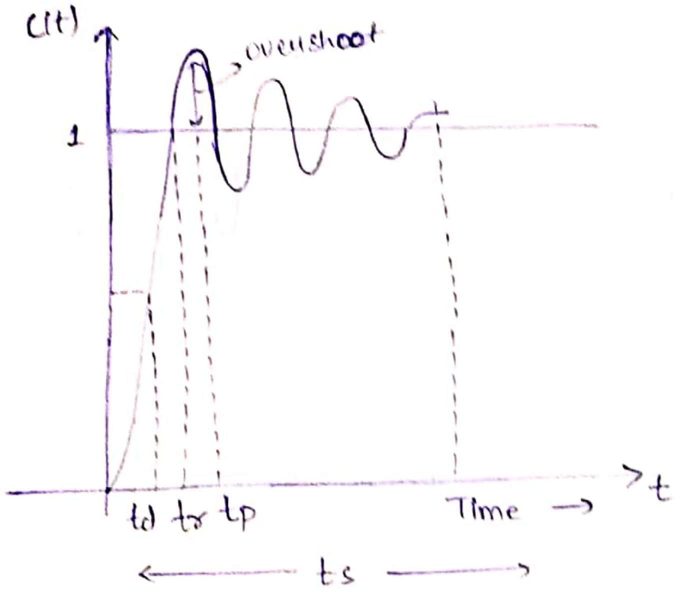
$A_1 = 1$, $A_2 = \dfrac{1}{2(\xi_e^2-1+\xi_e\sqrt{\xi_e^2-1})}$

$A_3 = \dfrac{1}{2(\xi_e^2-1-\xi_e\sqrt{\xi_e^2-1})}$

$$\therefore c(t) = 1 + \frac{e^{-\omega_n(\xi_e+\sqrt{\xi_e^2-1})t}}{2(\xi_e^2-1+\xi_e\sqrt{\xi_e^2-1})} + \frac{e^{-\omega_n(\xi_e-\sqrt{\xi_e^2-1})t}}{2(\xi_e^2-1-\xi_e\sqrt{\xi_e^2-1})}$$



○ Time Response Specification



$0 < \xi_e < 1$

① Delay time (td)
② Rise time (tr)
③ Peak time (tp)
④ Overshoot
⑤ Settling time (ts)
⑥ Steady state error

↳ Peak time : when response reaches maximum value.

↳ Overshoot : Peak overshoot indicates the normalized diff. between the time response peak & the steady output.

$$\%\ \text{Overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\ \%$$

↳ Settling time (ts) : when the oscill. attains a steady state. which have 2% - 5% tolerance wrt req. signal.

↳ Steady state error : diff. b/w discussed o/p & actual o/p.

$$\boxed{e_{ss} = \lim_{t\to\infty} (r(t) - c(t))}$$

↳ Delay time (td) : i/p reaches 50% of discussed o/p

→ Rise time (tr) : time req. for the response to rise from 10-90% of the final value for overdamped system & 0-100% of the final value for underdamped system.

- **Rise time (t_r) :**

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t_r + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] = \cancel{1}$$

for rise time, t_r, $c(t) = 1$

$$-\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t_r + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] = 0$$

$e^{-\xi\omega_n t_r} \neq 0$, $\sin(\omega_n\sqrt{1-\xi^2}\, t_r + \phi) = 0 = \sin n\pi$

for $n = 1$, $t_r = \dfrac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}$

$$\boxed{t_r = \frac{\pi - \tan^{-1}\left[\sqrt{1-\xi^2}\,\right]\xi}{\omega_n\sqrt{1-\xi^2}}}$$

- **Peak time (t_p) :** when slope $\dfrac{d\,c(t)}{dt} = 0$

$$0 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(-\xi\omega_n)\cdot\sin\left[\omega_n\sqrt{1-\xi^2}\cdot t_r\right] - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos\left[\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] \times \frac{\cos\sqrt{1-\xi^2}}{-2\sqrt{1-\xi^2}}$$

$$= -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\xi\omega_n \sin\left[\omega_n\sqrt{1-\xi^2}\cdot t_r + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] + \cos\left[\omega_n\sqrt{1-\xi^2}\cdot t\right]\cdot\omega_n\sqrt{1-\xi^2}\,\right]$$

$$= \sin(\omega_n\sqrt{1-\xi^2}\,t + \phi)\cos\phi - \cos(\omega_n\sqrt{1-\xi^2}\,t + \phi)\sin\phi = 0$$

$$= \sin(\underbrace{\omega_n\sqrt{1-\xi^2}\,t + \phi}_{A} - \underbrace{\phi}_{B}) = 0 \approx \sin(A - B) = 0$$

$$= \sin(\omega_n\sqrt{1-\xi^2}\cdot t) = 0 , \quad \omega_n\sqrt{1-\xi^2}\, t = 0, \pi, 2\pi, \ldots$$

$$\boxed{t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$n = 1$, 1^{st} overshoot

$n = 2$, ~~1st~~ undershoot

$n = 3$, 2^{nd} overshoot

- **Overshoot** :

$$M_p = c(t_p) - 1$$

$$M_p = 1 - \frac{e^{-\omega_n \xi_e t_p}}{\sqrt{1-\xi_e^2}} \sin\left[\omega_n\sqrt{1-\xi_e^2}\, t_n + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right] - 1$$

$n = 1$

$$= -\frac{e^{-\xi_e \omega_n . \frac{n\pi}{\omega_n\sqrt{1-\xi_e^2}}}}{\sqrt{1-\xi_e^2}} . \sin\left[\omega_n\sqrt{1-\xi_e^2} . \frac{n\pi}{\sqrt{1-\xi_e^2}} + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$$

$$= -\frac{e^{-n\pi\xi_e/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}} . \sin\left[n\pi + \tan^{-1}\frac{\sqrt{1-\xi_e^2}}{\xi_0}\right]$$

$$= -\frac{e^{-\pi\xi_0/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}} \sin[\pi + \phi] = \frac{e^{-\pi\xi_e/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}} \sin\phi$$

$$= \frac{e^{-\pi\xi_0/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}} . \sqrt{1-\xi_e^2} \qquad \boxed{M_p = e^{-\pi\xi_0/\sqrt{1-\xi_e^2}}}$$

- **Settling time** : (t_s)

$$1 - e^{-\xi_e\omega_n t_s} = 0.98$$

$$\Rightarrow -e^{-\xi_e\omega_n t_s} = 0.98 - 1$$

$$\Rightarrow e^{-\xi_0\omega_n t_s} = 0.02 \quad ; \quad -\xi_e\omega_n t_s = \ln(0.02)$$

$$t_s = \frac{-3.912}{-\xi_e\omega_n} \simeq \frac{4}{\xi_e\omega_n} \qquad \boxed{t_s = \frac{4}{\xi_0\omega_n}}$$

9/09/18

Q 2nd order ~~Time~~ Transfer Func $G(s) = 25/s^2+8s+25$. If the system initially at rest is subjected to a unit step i/p at $t=0$ the 2nd peak in the response will occur at

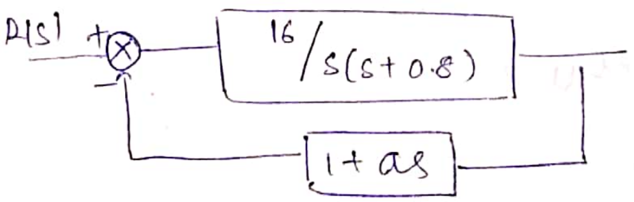
Sol^n

$$t = n\pi/\omega_n\sqrt{1-\xi_e^2} \leftarrow \text{Compairing with} \rightarrow \omega_n^2/s^2 + 2\xi_e\omega_n s + \omega_n^2 \qquad n=3$$

$\omega_n = 5$, $\xi_e = 4/5 = 0.8$

$$t = 3\pi/5\sqrt{1-0.8^2} = 3\pi/5\sqrt{1-0.64} = \pi$$

Q



Determine the value of 'a' such that the $\xi_e = 0.5$ also obtain the value of rise time & max. overshoot for its step response.

$$\frac{C(s)}{R(s)} = \frac{16/s(s+0.8)}{1+ 16/s(s+0.8) \times (1+as)} = \frac{16}{s^2 + (0.8+16a)s + 16}$$

on compairing Dr with 0

$\therefore\ s^2 + (0.8 + 16a)s + 16 = 0$

$\therefore\ \omega_n = 4,\quad 2\xi_e\omega_n = (0.8+16a)$

$\therefore\ \underline{a = 0.2}$

$$t_r = \frac{\pi - \tan^{-1}\left[\sqrt{1-\xi_e^2}/\xi_e\right]}{\omega_n\sqrt{1-\xi_e^2}}$$

$$= \frac{\pi - \tan^{-1}(\sqrt{0.5}/0.5)}{4\sqrt{0.5}} = \frac{\pi - \pi/4 \times 0.5}{4\sqrt{0.5}} = \frac{3.5\pi}{16\sqrt{0.5}} \times 100 = 0.61\%.$$

$$M_p = e^{-\pi \times 0.5/\sqrt{1-0.5^2}} \times 100 = 16.31\%.$$

Q The open loop time func of a unity FB system is given by $G(s) = \frac{K}{s(1+sT)}$ where K & T are -ve constants. By what factor should the ~~com~~ amplifier gain be reduced so that the peak overshoot of unit step response of the system is reduced from 75% to 25%.

Soln

feedback $H(s) = 1$, $G(s) = K/s(1+sT)$

$$\frac{C(s)}{R(s)} = \frac{K/s(1+sT)}{1+K/s(1+sT)} = K/s+s^2T+K = \frac{K/T}{s^2+\frac{s}{T}+\frac{K}{T}}$$

$\therefore\ \omega_n = \sqrt{K/T}$, $\xi_e = \frac{1}{2T}\sqrt{\frac{T}{K}} = \frac{1}{2\sqrt{KT}}$

$$M_{p_1} = e^{-\pi\xi_{e1}/\sqrt{1-\xi_{e1}^2}} = 0.75$$

$$M_{p_2} = e^{-\pi\xi_{e2}/\sqrt{1-\xi_{e2}}} = 0.25$$

$$\therefore\ \frac{-\pi\xi_{e1}}{\sqrt{1-\xi_{e1}^2}} = \log(0.75),\quad = \frac{\pi\xi_{e1}}{\sqrt{1-\xi_{e1}^2}} = 0.12$$

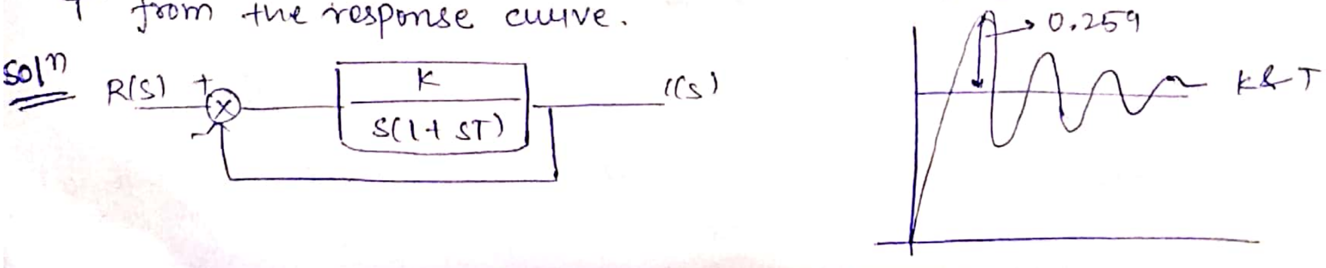
$\pi^2\xi_{e1}^2 = 0.0144(1-\xi_{e1})$, $\pi^2\xi_{e1}^2 = 0.0144 - 0.0144\xi_{e1}$ $\boxed{\xi_{e1} = 0.091}$

$M_p = e^{-\pi\xi_{e2}/\sqrt{1-\xi_{e2}^2}} = 0.25$ $\boxed{\xi_{e2} = 0.4}$

$\xi_{e1} = \frac{1}{2\sqrt{K_1T}}$ $\quad \xi_{e2} = \frac{1}{2\sqrt{K_2T}}$ $\quad \therefore\ \frac{\xi_{e2}}{\xi_{e1}} = \frac{0.4}{0.091}$

Q For the system shown when subjected to a unit step i/p gives a waveform as shown. Determine the value of given K & time constant T from the response curve.

Soln



7.

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + \frac{s}{T} + \frac{K}{T}}$$

$\therefore \omega_n = \sqrt{K/T}$ $\quad \xi = \frac{1}{2\sqrt{KT}}$

$2\xi\omega_n = \frac{1}{T}$ $\Rightarrow 2 \times 0.4 \times 1.14$ $\quad \underline{T = 1.09}$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.254$$

$$\boxed{\xi = 0.4}$$

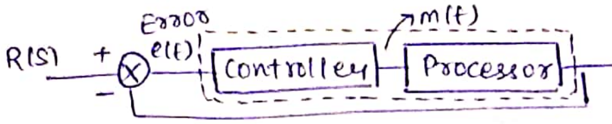
$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 3 \Rightarrow \frac{3.14}{3\sqrt{1-0.16}} = \omega_n \quad \therefore \boxed{\omega_n = 1.14}$$

$$\omega_n = \sqrt{\frac{K}{T}} \Rightarrow \sqrt{K} = 1.14\sqrt{1.09} \quad \underline{K = 1.42}$$

x —— x —— x —— x —— x —— x —— x —— x

(13/09/19)

- **Controller** < Analog / Digital



↳ Analog controller Types
A ① Proportional
A ② Integral
A ③ Derivative

↳ Digital controller Types
① Microprocessor
② Microcontroller
③ PLC
④ PC

A① $m(t) \propto e(t)$, $\boxed{m(t) = K_p\, e(t)}$ — K_p → proportional gain

A② $\frac{d}{dt} m(t) = K_i\, e(t)$, $m(t) = K_i \int e(t)\,dt$, $\frac{M(s)}{E(s)} = \frac{K_i}{s}$

A③ $m(t) = K_d \frac{d}{dt} e(t)$, $\frac{M(s)}{E(s)} = sK_d$
K_d ↓ differential gain

- Only proportional controller is used alone while other analog controller (integral, directive) are always used in combinations with proportional.
- Analog controller are used as
  ① PI – proportional integral controller
  ② PD – " derivative "
  ③ PID – " integral derivative controller – one of best controller

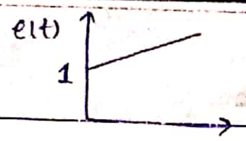
- **For ① PI controller :** $m(t) = K_p e(t) + K_p K_i \int_0^t e(t)\,dt$

$$\frac{M(s)}{E(s)} = K_p\left[1 + \frac{K_i}{s}\right] , \quad T_i = \frac{1}{K_i} = \text{integral time}$$

$$= K_p\left[1 + \frac{1}{sT_i}\right] \qquad \text{Reset Rate} = \frac{1}{T_i}$$

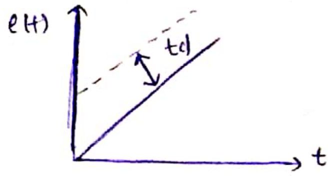
- Integral time defined as time of change of o/p is unit change of actuating error signal.
- Reset Rate defined as no. of times per min. that the proportional part of the response is duplicated.

Output from PI controller



• **For PD controller :** $m(t) = K_p e(t) + K_p T_d \frac{d}{dt} e(t)$ $T_d \rightarrow$ derivative time

T_d is the time interval by which the rate action advances the effect of proportional controller.



$$\frac{M(s)}{E(s)} = K_p(1 + sT_d)$$

• PD controller reduces the rise time faster response improves the bandwidth & damping ratio.

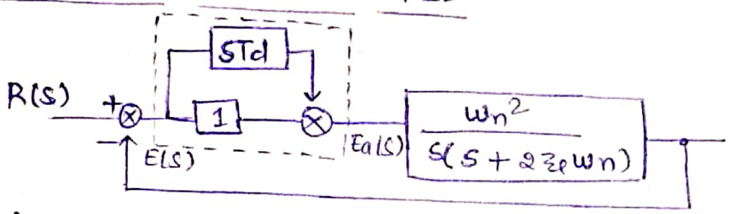
• **For PID controller :** $m(t) = K_p e(t) + K_p T_d \frac{d}{dt} e(t) + \frac{K_p}{T_i}\int_0^t e(t)\,dt$

$$M(s) = K_p E(s) + K_p T_d E(s) + \frac{K_p}{sT_i} E(s)$$

$$\frac{M(s)}{E(s)} = K_p\left[1 + sT_d + \frac{1}{sT_i}\right]$$

Response with PI, PI & PID

→ **PD**



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\omega_n^2(1+sT_d)}{s^2 + (2\xi\omega_n + \omega_n^2 T_d)s + \omega_n^2}$$

$$E_a(s) = (1 + sT_d)E(s)$$

Characteristic eqn. $= s^2 + (2\xi\omega_n + \omega_n^2 T_d)s + \omega_n^2 = 0$

Standard eqn, $= s^2 + 2\xi'\omega_n s + \omega_n^2 = 0$

$$2\omega_n\xi' = 2\xi\omega_n + \omega_n^2 T_d$$

$$\xi' = \xi + \frac{\omega_n T_d}{2}$$

↳ Effect of PD controller

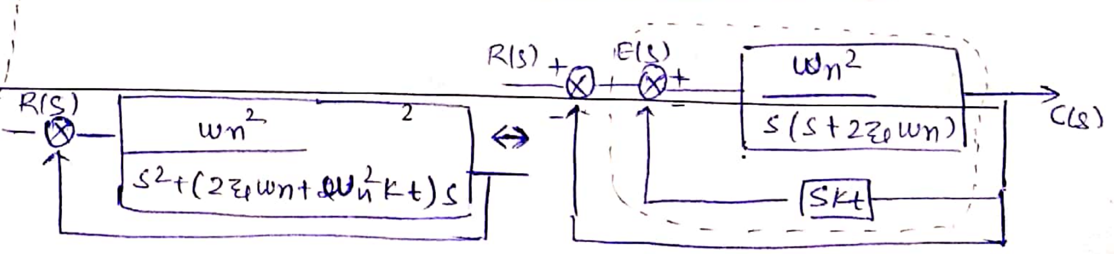
$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1 + \frac{\omega_n^2(1+sT_d)}{s(s+2\xi\omega_n)}}$$

let $R = \frac{1}{s^2}$ $\therefore$ $E(s) = \frac{1}{s^2}\left(\frac{s(s+2\xi\omega_n)}{s^2 + (2\xi\omega_n + \omega_n^2 T_d)s + \omega_n^2}\right.$

as $e_{ss} = \lim_{s\to 0} sE(s)$

$$\therefore \boxed{e_{ss} = \frac{2\xi}{\omega_n}}$$

• **Derivative Feedback Controller (Rate controller)**



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{\omega_n^2}{s^2 + (2\xi\omega_n + \omega_n^2 K_t)s + \omega_n^2}.$$

$s^2 + 2\xi_e' \omega_n s + \omega_n^2 = 0 \Rightarrow 2\omega_n \xi_e' = 2\xi_e \omega_n + \omega_n^2 K_t$

$\frac{E(S)}{R(S)} = \frac{1}{1+G(S)}$ take $R(S) = \frac{1}{S^2}$

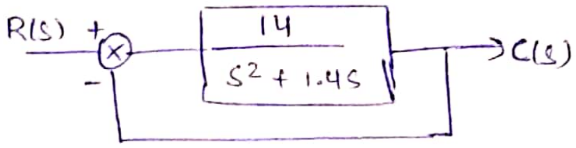
$$\boxed{\xi_e' = \xi_e + \frac{\omega_n K_t}{2}}$$

$$E(S) = \frac{1 \cdot (s^2 + (2\xi_e\omega_n + \omega_n^2 K_t)s)}{s^2(s^2 + 2\xi_e\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

$$e_{ss} = \lim_{s \to 0} sE(S) = \frac{2\xi_e}{\omega_n} + K_t$$

Q A closed loop system with unity feedback shown in fig. having value $\frac{14}{s^2+1.4s}$ By using derivative control the damping ratio is to be made 0.7. Determine the value of Td (derivative time), also determine t_r, t_p, and max. overshoot without derivative controlled & with derivative control. Input to the system is unit step.

Ans
$$\frac{C(S)}{R(S)} = \frac{14}{s^2+1.4s+14}$$



$s^2 + 2\xi_e\omega_n s + \omega_n^2 = 0$

$2\xi_e\omega_n = 1.4 \qquad \omega_n^2 = 14$

$\xi_e = 0.18 \qquad \xi_e' = \xi_e + \frac{\omega_n T_d}{2} = 0.7$ (given)

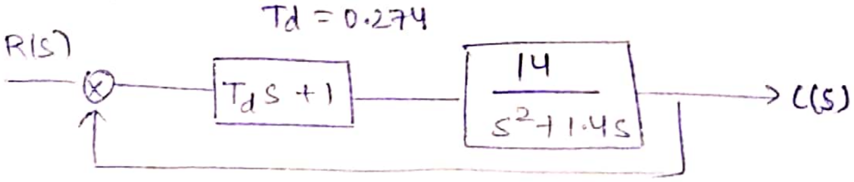
$$\boxed{T_d = 0.27}$$

as $t_r = \frac{\pi - \tan^{-1}\sqrt{1-\xi_e^2}/\xi_e}{\omega_n\sqrt{1-\xi_e^2}} = 0.47 \qquad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi_e^2}} = 0.85$

$M_p = e^{-\pi\xi_e/\sqrt{1-\xi_e^2}} \times 100 = 55\%$

$T_d = 0.274$

$$\frac{C(S)}{R(S)} = \frac{14(1+0.274S)}{s^2+5.236S+14}$$



let $R(S) = \frac{1}{S}$

$$C(S) = \frac{1}{S}\left[\frac{14 + 3.836S}{S^2 + 5.236S + 14}\right] = \frac{1}{S} \cdot \frac{14}{S^2+5.236S+14} + \frac{1}{S} \cdot \frac{3.836S}{S^2+5.236S+14}$$

$$= \frac{A}{S} + \frac{BS+C}{S^2+5.236S+14} + \frac{3.836}{S^2+5.236S+14}$$

$A = 1 ; B = -1 ; C = -5.2$

$$C(t) = 1 - e^{-2.618t}\cos 2.673t + 0.455\, e^{-2.618t}\sin 2.673t$$