Indian Knowledge System

Mathematic and Astronomy in IKS

UNIT 3

By:

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Mathematics

- Geometry is an ancient science in India.
- Just with a pole anchored on the ground and a thread attached to it, Indians were able to generate complex geometrical shapes.
- Indian mathematics is seamless blend of poetry, literature, logic and mathematical thinking weaved into single work.
- The use of sutras and pithy (अर्थपूर्ण) verses is characteristic in the Indian mathematical tradition to convey complex ideas and concepts.

Unique aspects of Indian Mathematics

- There is a popular thinking among many that the world is divided into those who know and love mathematics and those who don’t.
- Mathematics was considered as a part of life. This is why mathematics could be found in temple inscriptions, literary work addressing issues of life and in a discussion on religion or spirituality.
- Bhaskaracarya brings interesting mathematical concepts by composing riddles to a students and solving them.

Unique aspects of Indian Mathematics

- The use of sutras is characteristics in the Indian tradition to convey ideas and concepts.
- These improve the retention of complex ideas and details very easily.
- There has been uninterrupted tradition of mathematical thinking and it has widely spread across the length and breadth of India.
- One of the important characteristics of Indian Mathematics is its algorithmic approach and the fact it allows approximate solutions based on the needs of real life situations.

Great mathematician and their contribution

- **Bhaskara**
- He is also known as Bhaskaracharya. He was born in 1114.
- He was the one who acknowledged that any number divided by zero is infinity and that the sum of any number and infinity is also infinity.
- The famous book “Siddhanta Siromani” was written by him.
- **Aryabhata**
- He was born in 476 CE at Kusumapura.
- He was regarded as the first of the major mathematician-astronomers from the classical age.
- Aryabhaṭiya and Arya-Siddhanta were his known works.
- He worked on the ‘place value system’ using letters to signify numbers and stating qualities.
- He discovered the position of the 9 planets and found that these planets revolve around the sun.
- He also described the number of days in a year to be 365.

Great mathematician and their contribution

- **Brahmagupta**
- He was born in 598 CE near present-day Rajasthan.
- The most important contribution of Brahmagupta to mathematics was introducing the concept and computing methods of zero (0).
- **Srinivasa Ramanujan**

He was born on 1887. His important contributions to this field are

- Hardy-Ramanujan-Littlewood circle method in number theory
- Roger-Ramanujan’s identities in the partition of numbers
- Work on the algebra of inequalities
- Elliptic functions
- Continued fractions
- Partial sums and products of hypergeometric series

Great mathematician and their contribution

- **P.C. Mahalanobis**
- P.C. Mahalanobis was born in 1893.
- He is known for
  - Mahalanobis distance
  - Feldman–Mahalanobis model
- **Calyampudi Radhakrishna Rao**
- R Rao was born in 1920. He is a well-known statistician. He is famous for his ‘Theory of estimation’.
- He is known for
  - Cramer–Rao bound
  - Rao–Blackwell theorem
  - Orthogonal arrays
  - Score test

Great mathematician and their contribution

- **D. R. Kaprekar**
- D. R. Kaprekar was a recreational mathematician.
- He discovered several results in number theory, comprising a class of numbers and a constant named after him.
- **Satyendranath Bose**
- He was born in 1894. He is known for his collaboration with Albert Einstein. He is best known for his work on quantum mechanics.
- He famous contributions are
  - Bose-Einstein correlations
  - Bose-Einstein condensate
  - Bose-Einstein distribution
  - Bose-Einstein statistics
  - Photon gas
  - Ideal Bose equation of state

Great mathematician and their contribution

1. Baudhayana

दीर्घचतुरश्रस्याक्ष्णया रज्जुः पार्श्र्वमानी तिर्यग् मानी-च यत् पृथग् भूते
कुरूतस्तदुभयं करोति ॥

Born: Around 800 BC

Notable Work: Approximation of the square root of 2 and the statement of a version of the Pythagorean Theorem

2. Katyayana

Born: Around 300 BC

Notable Work: Varttika, Vyakarana, later Sulba Sutras

3. Pingala

गायत्रेण प्रति मिमीते अर्कमर्केण साम त्रैष्टुभेन वाकम् ।
वाकेन वाकं द्विपदा चतुष्पदाक्षरेण मिमते सप्त वाणीः ॥२४॥

Born: Around 500 BC

Notable Work: Matrameru, binary numeral system, arithmetical triangle

Great mathematician and their contribution

4. Aryabhata

महान् गणितज्ञः, ज्योतिर्विदः च आसीत्। तस्य जन्म
अश्मकदेशे अभवत्। सः कुसुमपुर्याम् अपठत् अवसत् च। यदा
सः त्रयोविंशतिवर्षीयः तदा सः आर्यभटीयम्अलिखत्।

Born: Between 476-550 CE

Notable works: Aryabhaṭiya, Arya-siddhanta

Notable Ideas: Explanation of lunar eclipse and solar eclipse, rotation of Earth on its axis, reflection of light by moon, sinusoidal functions, solution of single variable quadratic equation, value of π correct to 4 decimal places, circumference of Earth to 99.8% accuracy, calculation of the length of sidereal year.

5 Scientists from Ancient India

5. Varahamihira

कर्मण्येवाधिकारस्ते मा फलेषु कदाचन।
मा कर्मफलहेतुर्भूर्मा ते सङ्गोऽस्त्वकर्मणि॥

Born: Between 505–587 CE

Great mathematician and their contribution

6. Yativrsabha

Born: Around 6th-century CE

Known for: Mathematician and writer of the book Tiloyapannatti which gives various units for measuring distances and time and postulated different concepts about infinity

7. Brahmagupta

Born: Between 598–670 CE

Known for: Zero, Modern number system, Brahmagupta's theorem, Brahmagupta's identity, Brahmagupta's problem, Brahmagupta-Fibonacci identity, Brahmagupta's interpolation formula, Brahmagupta's formula

8. Bhaskara I

$$\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}, \qquad (0 \leq x \leq \frac{\pi}{2})$$

Born: Between 600–680 CE

Known for: Sine approximation formula

Great mathematician and their contribution

9. Shridhara

Born: Between 650–850 CE

Contribution: Gave a rule for finding the volume of a sphere

10. Mahavira

Born: 9th century CE

Notable Work: His work is a highly syncopated approach to algebra and the emphasis in much of his text is on developing the techniques necessary to solve algebraic problems.

5 Facts that you don't know about the Infinity Symbol

11. Pavuluri Mallana

Born: 11th century CE

Notable Work: He translated Ganitasara Samgraham, a mathematical treatise of Mahivaracharya, into Telugu as Sara Sangraha Ganitamu.

Great mathematician and their contribution

12. Bhaskara II
Born: 1114–1185 CE

Known for: Discovery of the principles of differential calculus and its application to astronomical problems and computations

13. Narayana Pandit
Born: Between 1340–1400 CE

Notable Work: Arithmetical treatise called Ganita Kaumudi ; Algebraic treatise called Bijganita Vatamsa

14. Madhava of Sangamagrama
Born: 1340-1350 CE

Known for: Discovery of power series expansions of trigonometric sine, cosine and arctangent functions

Notable work: Golavada, Madhyamanayanaprakara, Venvaroha

15. Parameshvara
Born: Between AD 1360–1455

Notable Work: Discovered drk-ganita, a mode of astronomy based on observations

Great mathematician and their contribution

16. Nilakantha Somayaji

Born: Around AD1444

Notable Work: Golasara, Candrachayaganita, Aryabhatiya-bhashya, Tantrasamgraha Raghunatha Siromani

17. Shankara Variyar

Born: Around AD1530

Notable Work: Yukti-dipika - an extensive commentary in verse on Tantrasamgraha based on Yuktibhasa; Laghu-vivrti - a short commentary in prose on Tantrasamgraha; Kriya-kramakari - a lengthy prose commentary on Lilavati of Bhaskara II

18. Jyeshtadeva

Born: Around AD1500

Known for: Authorship of Yuktibhaṣa

Notable work: Yuktibhāṣā, Drkkarana

19. Munishvara

Born: Around 17th century

Notable Work: Produced accurate sine tables

Great mathematician and their contribution

20. Kamalakara

Born: Around AD 1657

Notable Work: Combined traditional Indian astronomy with Aristotelian physics and Ptolemaic astronomy as presented by Islamic scientists; Given formulae for sin(A/2) and sin(A/4) in terms of sin(A) and iterative formulae for sin(A/3) and sin(A/5); Given a table for finding the right ascension of a planet from its longitude.

21. Jagannatha Samrat

Born: Around AD1730

Notable Work: Siddhanta-samrat, Yantra-prakara

22. Radhanath Sikdar

Born: Around 1813 AD

Great mathematician and their contribution

23. Pathani Samanta

Born: 11 January, 1835-36 AD

Known for: Naked eye astronomy

24. Ganesh Prasad

Born: 15 November, 1876 AD

Known for: Establishing the culture of organised mathematical research in India

Notable work: A Treatise on Spherical Harmonics and the Functions of Bessel and Lame

25. Srinivasa Ramanujan

Born: 22 December 1887

Known for: Landau–Ramanujan constant; Mock theta functions; Ramanujan conjecture; Ramanujan prime; Ramanujan–Soldner constant; Ramanujan theta function; Ramanujan's sum; Rogers–Ramanujan identities; Ramanujan's master theorem

26. Prasanta Chandra Mahalanobis

Born: 29 June 1893

Known for: Mahalanobis distance Feldman–Mahalanobis model

Arithmetic

- Arithmetic is an elementary part of mathematics that consists of the study of the properties of the traditional operations on numbers—addition, subtraction, multiplication, division, exponentiation, and extraction of roots.
- Arithmetic developed first of all in India and only then came to Western Europe. In the seventeenth century the needs of astronomy, mechanics, and more difficult commercial calculations put before arithmetic new challenges regarding methods of calculation and gave an impetus to further development.

Geometry

- Geometry arose as the field of knowledge dealing with spatial relationships. Geometry was one of the two fields of pre-modern mathematics, the other being the study of numbers. Classic geometry was focused in compass and straightedge constructions.
- In ancient India, geometry was used extensively in constructing fire altars without the use of modern measuring devices.
- By using ropes, they were able to form right angles and various shapes and to transform one shape into another with the same area.

Vedic India geometry

- The Indian Vedic period had a tradition of geometry, mostly expressed in the construction of elaborate altars. Early Indian texts (1st millennium BC) on this topic include the *Satapatha Brahmana* and the *Śulba Sūtras*.
- The Baudhayana Sulba Sutra, the best-known and oldest of the Sulba Sutras (dated to the 8th or 7th century BC) contains examples of simple Pythagorean triples, such as: (3,4,5), (5,12,13), (8,15,17), (7,24,25), and (12,35,37) as well as a statement of the Pythagorean theorem for the sides of a square.

Trigonometry

- In Indian astronomy, the study of trigonometric functions flourished in the Gupta period, especially due to Aryabhata (sixth century CE), who discovered the sine function. During the Middle Ages, the study of trigonometry continued in Islamic mathematics, by mathematicians such as Al-Khwarizmi and Abu al-Wafa.
- The *Siddhantas* and the *Aryabhatiya* contain the earliest surviving tables of sine values and versine (1 − cosine) values, in 3.75° intervals from 0° to 90°, to an accuracy of 4 decimal places.[25] They used the words *jya* for sine, *kojya* for cosine, *utkrama-jya* for versine, and *otkram jya* for inverse sine. The words *jya* and *kojya* eventually became *sine* and *cosine* respectively after a mistranslation described above.

Algebra

- The subject of Algebra originated in India. Its origin can be traced back to the Shatapatha Brahmana (शतपथब्राह्मण) (2000 BCE) and the Sulba sutras (800-500 BCE). Algebra was used to design and construct the vedis. The Indian name for algebra is Bijaganita (बीजगणित).
- Bija (बीज) means analysis and ganita (गणित) means “the science of calculation”. Thus Bijaganita literally means “the science of analytical calculation”. Algebra is also called अव्यक्त गणित (avyakta ganita), that is the “the science of calculation with unknowns” (avyakta means unknown). Whereas Arithmetic is व्यक्त गणित (vyakta ganita), that is the “the science of calculation with knowns”.

Binary Mathematics and Combinatorial Problems in Chandah-śāstra of Pingala

- Acharya **Pingala** (*piṅgala*; c. 3rd–2nd century BCE) was an ancient Indian poet and mathematician, and the author of the *Chandaḥśāstra* (also called the *Pingala-sutras*), the earliest known treatise on Sanskrit prosody.
- he *Chandaḥśāstra* is a work of eight chapters in the late Sūtra style, not fully comprehensible without a commentary. It has been dated to the last few centuries BCE. In the 10th century CE, Halayudha wrote a commentary elaborating on the *Chandaḥśāstra*. Pingala Maharshi was also said to be the brother of Pāṇini, the famous Sanskrit grammarian, considered the first descriptive linguist.

Binary Mathematics and Combinatorial Problems in Chandah-śāstra of Pingala

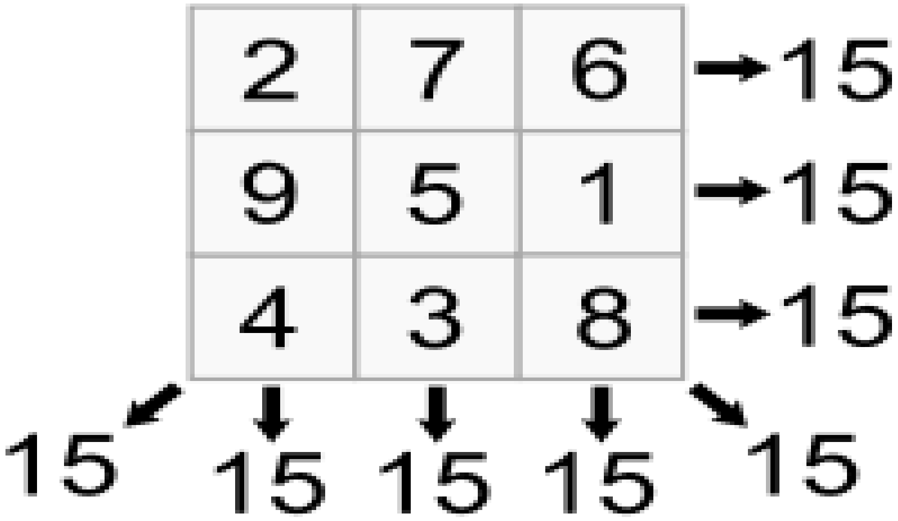
- The Chandaḥśāstra presents the first known description of a binary numeral system in connection with the systematic enumeration of meters with fixed patterns of short and long syllables. The discussion of the combinatorics of meter corresponds to the binomial theorem.
- Use of zero is sometimes ascribed to Pingala due to his discussion of binary numbers, usually represented using 0 and 1 in modern discussion, but Pingala used light (laghu) and heavy (guru) rather than 0 and 1 to describe syllables.
- As Pingala's system ranks binary patterns starting at one (four short syllables—binary "0000"—is the first pattern), the nth pattern corresponds to the binary representation of n-1 (with increasing positional values).
- Pingala is credited with using binary numbers in the form of short and long syllables (the latter equal in length to two short syllables), a notation similar to Morse code. Pingala used the Sanskrit word śūnya explicitly to refer to zero.

Magic Squares in Indian Mathematics

- The oldest datable magic square in India occurs in Varāhamihira's encyclopedic work on divination, Bṛhatsaṃhitā (ca. AD 550). He utilized a modified magic square of order four in order to prescribe combinations and quantities of ingredients of perfume.
- It consists of two sets of the natural numbers 1–8, and its constant sum (p) is 18. It is, so to speak, pan-diagonal, that is, not only the two main diagonals but also all “broken” diagonals have the same constant sum. Utpala, the commentator (AD 967), also points out many other quadruplets that have the same sum.

Magic Squares in Indian Mathematics

- In recreational mathematics, a square array of numbers, usually positive integers, is called a **magic square** if the sums of the numbers in each row, each column, and both main diagonals are the same.



Astronomy

- Some of the earliest roots of Indian astronomy can be dated to the period of Indus Valley civilization or earlier. Astronomy later developed as a discipline of Vedanga, or one of the "auxiliary disciplines" associated with the study of the Vedas, dating 1500 BCE or older.
- Indian astronomy flowered in the 5th–6th century, with Aryabhata, whose work, *Aryabhatiya*, represented the pinnacle of astronomical knowledge at the time. The Aryabhatiya is composed of four sections, covering topics such as units of time, methods for determining the positions of planets, the cause of day and night, and several other cosmological concepts.

Unique aspects of Indian Astronomy

- The Vedanga Jyotisha describes rules for tracking the motions of the Sun and the Moon for the purposes of ritual. According to the Vedanga Jyotisha, in a yuga or "era", there are 5 solar years, 67 lunar sidereal cycles, 1,830 days, 1,835 sidereal days and 62 synodic months.
- The divisions of the year were on the basis of religious rites and seasons (*Rtu*). The duration from mid March—mid May was taken to be spring (*vasanta*), mid May—mid July: summer (*grishma*), mid July—mid September: rains (*varsha*), mid September—mid November: autumn (*sharad*), mid November—mid January: winter (*hemanta*), mid January—mid March: the dews (*shishir*).

Unique aspects of Indian Astronomy

- In the *Vedānga Jyotiṣa*, the year begins with the winter solstice. Hindu calendars have several eras:
  - The Hindu calendar, counting from the start of the Kali Yuga, has its epoch on 18 February 3102 BCE Julian (23 January 3102 BCE Gregorian).
  - The Vikram Samvat calendar, introduced about the 12th century, counts from 56 to 57 BCE.
  - The "Saka Era", used in some Hindu calendars and in the Indian national calendar, has its epoch near the vernal equinox of year 78.
  - The Saptarishi calendar traditionally has its epoch at 3076 BCE.

Historical Development of Astronomy in India

- **The Rigveda (**c1700-1100 BCE), one of Hinduism's primary and foremost texts, contains the first records of sophisticated astronomy in India dating back to at least 2000 BCE.
- According to the Rigveda, the Indians divided the year into 360 days, which were then divided into 12 months of 30 days.
- Two intercalary periods were added every 5 years to bring the calendar back in line with the solar year, ensuring that years averaged 366 days.
- The Indian year, however, migrated four days every five years, and Indian astronomers constantly tweaked and adjusted their calendars over millennia.
- The text also demonstrates that the Indians used four cardinal points to ensure that altars were properly oriented.

Historical Development of Astronomy in India

- **The Jyotisa Vedanga**, the first Vedic text to mention astronomical data, records events as far back as 4000 BCE, though many archaeo astronomers believe it may include observations as far back as 11 000 BCE.
- They note that some of the records may have been copied from earlier manuscripts, but more research is needed in this area because many of the references are unclear and couched in religious terminology.
- Many advances in measuring time and the progression of the heavens occurred during this period, as well as a few proto-theories about the structure of the universe.
- More importantly, this time period witnessed the exchange of ideas between Indians, Babylonians, Greeks, and Persians. This exchange of theories and philosophy was critical to the advancement of astronomy.

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The Celestial Coordinate System

- To understand the concept of celestial coordinates, we must first learn about the celestial sphere.
- Whenever we look upwards, all the heavenly bodies appear to lie on the inner surface of a large imaginary sphere, with us being at center of that imaginary sphere.
- They appear to lie in the same way as the artificial stars lie on the dome of a planetarium. This large imaginary sphere is known as the celestial sphere.

The Celestial Coordinate System

- With the basic information about the celestial sphere in hand, we are now in a position to understand the three basic celestial coordinate systems. So, let’s proceed, beginning with the Horizontal system as follows:
    1. The Horizontal Coordinate System (Altitude and Azimuth)
    2. The Equatorial Coordinate System (Right Ascension and Declination)
    3. The Ecliptic Coordinate System (Celestial Latitude and Longitude)

Elements of the Indian Calendar

- The Hindu calendar is based on a geocentric model of the solar system. A large part of this calendar is defined based on the movement of the sun and the moon around the earth (saura māna and cāndra māna respectively).
- Furthermore, it includes synodic, sidereal, and tropical elements.
- Many variants of the Hindu calendar have been created by including and excluding these elements (solar, lunar, lunisolar etc.) and are in use in different parts of India.

Elements of the Indian Calendar

Elements of the Hindu calendar

	synodic elements	sidereal elements	tropical elements
saura māna		rāśi, sauramāsa, varṣa	uttarāyaṇa, dakṣiṇāyana, devayāna, pitṛyāṇa, ṛtu
cāndra māna	tithi, pakṣa, candramāsa, varṣa		
nākṣatra māna		dina, ghaṭikā (aka nāḍī), vighaṭikā (aka vināḍī), prāṇa (aka asu)	
sāvana māna	dina		

Indian Astronomy and the Siddhantic Era

- During this time, a new branch of astronomy emerged that diverged from the Vedas. The Siddhantic Era began with the Siddhanat, or 'Solutions,' a series of books that charted the solar year, including solstices, equinoxes, lunar periods, solar and lunar eclipses, and planetary movements.
- The Siddhantic Era saw three great Indian astronomers, who are sadly little known in the west despite their significant contributions.
- At a time when the Greeks were still using celestial crystal spheres to explain the cosmos, Indian astronomers proposed that the stars were exactly like the sun but much further away by the first century CE.
- They also recognised that the earth was spherical, and Indian astronomers attempted to calculate the planet's circumference.

Aryabhata

- In his magnum opus Aryabhatiya (499), Aryabhata (476-550) proposed a computational system based on a planetary model in which the Earth was assumed to be spinning on its axis and the periods of the planets were given with respect to the Sun.
- Many astronomical constants, such as the periods of the planets, times of solar and lunar eclipses, and the instantaneous motion of the Moon, were precisely calculated by him.
- **Varahamihira, Brahmagupta, and Bhaskara II** were among the early followers of Aryabhata's model.

Astronomy During Shunga Empire

- During the Shunga Empire, astronomy advanced, and many star catalogues were created. The Shunga period is known as India's "Golden Age of Astronomy."
- It saw the development of calculations for various planets' motions and positions, their rising and setting, conjunctions, and eclipse calculations.
- By the sixth century, Indian astronomers believed that comets were celestial bodies that reappeared on a regular basis.
- This was the view expressed by the astronomers Varahamihira and Bhadrabahu in the sixth century, and the 10th-century astronomer Bhattotpala listed the names and estimated periods of certain comets, but it is unclear how these figures were calculated or how accurate they were.

Significance of astronomy in ancient India

- Although Indian astronomy was heavily influenced by their religious and spiritual worldview, it contained many accurate observations of phenomena.
- This acted as a catalyst for the growth of mathematics in the subcontinent, and is considered one of India's greatest legacies to the Western world.

THANKS