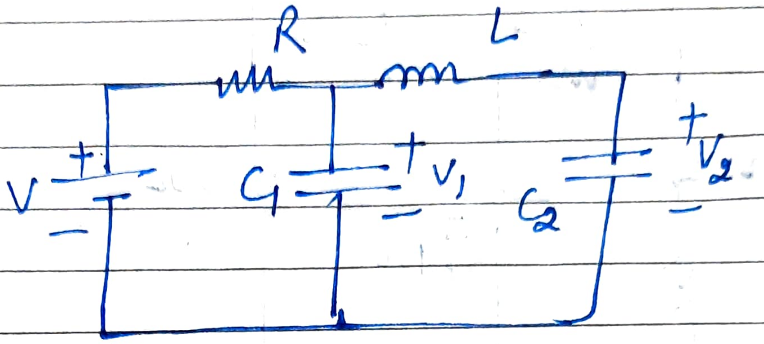
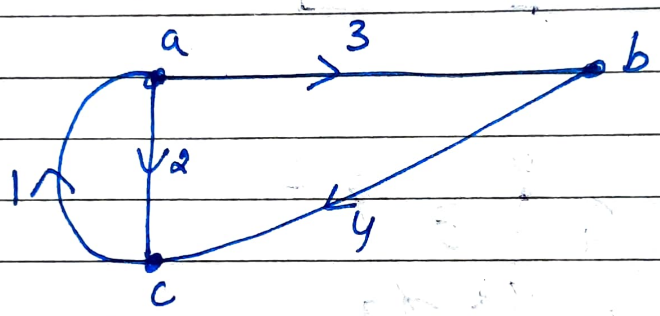
Loop Analysis

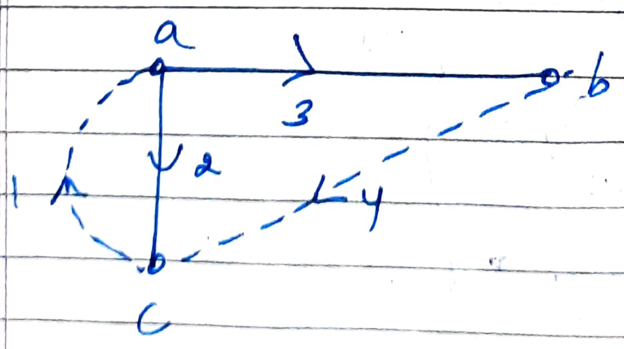
Q1:- write the matrix loop eqn for the n/w shown :- using loop analysis.



Soln:-



$$\text{twig} = n-1 = 3-1 = 2$$
$$\text{links} = b-n+1 = 4-3+1 = 2$$



$$V_\ell = Z_\ell \cdot I_L$$

$$Z_L = B_f \, Z_b \, B_f^T .$$

$$V_\ell = B_f \, Z_b \, I_s - B_f \, V_s$$

Tie set matrix, B_f =

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

Branch Impedance matrix,

$$Z_b = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & \frac{1}{C_1 s} & 0 & 0 \\ 0 & 0 & Ls & 0 \\ 0 & 0 & 0 & \frac{1}{C_2 s} \end{bmatrix}$$

∴ loop impedance matrix, $Z_e = B_f Z_b B_f^T$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & \frac{1}{C_1 s} & 0 & 0 \\ 0 & 0 & Ls & 0 \\ 0 & 0 & 0 & \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$Z_e = \begin{bmatrix} R + \frac{1}{C_1 s} & \frac{-1}{C_1 s} \\ \frac{-1}{C_1 s} & Ls + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{bmatrix} \quad ☆$$

Voltage source matrix =
$$\begin{bmatrix} \text{Voltage source in branch 1} \\ \text{'' '' '' 2} \\ \text{'' '' '' 3} \\ \text{'' '' '' 4} \end{bmatrix}$$

$$V_s = \begin{bmatrix} -V \\ v_1 \\ 0 \\ v_2 \end{bmatrix}$$

$$I_s = \begin{bmatrix} \text{current source in branch 1} \\ \text{'' '' '' 2} \\ \text{'' '' '' 3} \\ \text{'' '' '' 4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_l = B_f Z_b I_s - B_f V_s$$

$$= [B_f Z_b]\, 0 - B_f V_s$$

$$V_l = -B_f V_s$$

$$= -\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -V \\ v_1 \\ 0 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} V - v_1 \\ v_1 - v_2 \end{bmatrix}$$

$\therefore$ The matrix loop eqn can be written as :—

$$\begin{bmatrix} V - v_1 \\ v_1 - v_2 \end{bmatrix} = \begin{bmatrix} R + \frac{1}{C_1 s} & \frac{-1}{C_1 s} \\ \frac{-1}{C_1 s} & Ls + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} I_{l1} \\ I_{l4} \end{bmatrix}$$

② Nodal Analysis :-

The branch I_s are given by eqn (2)

$$I_b = I_s + Y_b(V_b - V_s)$$

Premultiplication by A, gives

$$A I_b = A I_s + A Y_b (V_b - V_s)$$

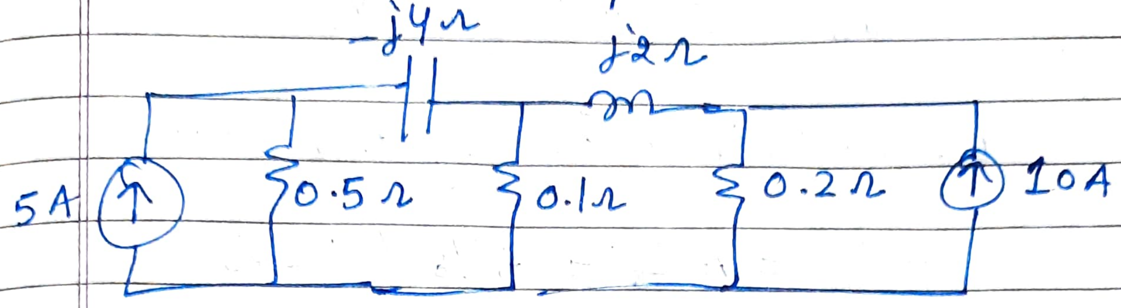
$$0 = A I_s + A Y_b (A^T V_n - V_s)$$

(since $A I_b = 0$ and $V_b = A^T V_n$)

$$\underbrace{\boxed{A Y_b V_s - A I_s}}_{I_n} = \underbrace{A Y_b A^T}_{Y_n} V_n .$$

$$I_n = Y_n V_n .$$

Q2) For the N/w shown. Obtain the incidence matrix, the node admittance matrix and matrix node eqn.



Node matrix :-

$$Y_n = A Y_b A^T$$

$$Y_n = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10+j0.25-j0.5 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix}$$

$$V_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad I_s = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

$$I_n = A Y_b V_s - A I_s$$
$$= 0 - A I_s$$

$$I_n = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$$\therefore I_n = Y_n V_n$$

$$\begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 2+j0.25 & -j0.25 & 0 \\ -j0.25 & 10+j0.25-j0.5 & j0.5 \\ 0 & j0.5 & 5-j0.5 \end{bmatrix}$$

Q 1- Take twig as (2, 4, 6), find $[Q_f]$, $[B_f]$ and Hence show $[Q_f][B_f]^T = 0$