PSLP akash previous year questions

Guru Gobind Singh Indraprastha University

Syllabus

APPLIED MATHEMATICS-IV
(ETMA-202)

Maximum Marks : 75

Instruction to Paper Setters:

1. *Question No. 1 should be compulsory and cover the entire syllabus. This question should have objective or short answer type questions. It should be of 25 marks.*
2. *Apart from Question No. 1, rest of the paper shall consist of four units as per the syllabus. Every unit should have two questions. However, student may be asked to attempt only 1 question from each unit. Each question should be 12.5 marks*

UNIT-I

Partial Differential Equation: linear partial differential equations with constant coefficient, homogeneous and non homogeneous linear equations. Method of separation of variables. Laplace equation, wave equation and heat flow equation in Cartesian coordinates only with initial and boundary value. **[T1] [No. of Hrs. 12]**

UNIT-II

Probability Theory: Definition, addition law of probability, multiplication law of probability, conditional probability, Baye's theorem, Random variable: discrete probability distribution, continuous probability distribution, expectation, moments, moment generating function, skewness, kurtosis, binomial distribution, Poisson distribution, normal distribution. **[T1, T2] [No. of Hrs. 11]**

UNIT-III

Curve Fitting: Principle of least square Method of least square and curve fitting for linear and parabolic curve, Correlation Coefficient, Rank correlation, line of regressions and properties of regression coefficients. Sampling distribution: Testing of hypothesis, level of significance, sampling distribution of mean and variance, Chi-square distribution, Student's T-distribution, F-distribution, Fisher's Z- distribution. **[T1, T2] [No. of Hrs. 12]**

UNIT-IV

Linear Programming: Introduction, formulation of problem, Graphical method, Canonical and Standard form of LPP, Simplex method, Duality concept, Dual simplex method, Transportation and Assignment problem. **[T1] [No. of Hrs. 11]**

END TERM EXAMINATION [MAY-JUNE 2015]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA-202]

Time. 3 Hours **MM : 75**

Note: *1. Attempt any five quastions including Q. No. 1 which is compulsory. select one question from each unit.*

Q. 1. (a) Write the steady state two dimensional heat flow equation. Find its solution in cartesian coordinates. (7)

Ans. In the steady state two–dimensional heat flow equation is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0 \qquad ...(1)$$

Let $u(x,y) = X(x).\,Y(y)$ be a solution of (1)

When X is a function of x only and y is a function of y only.

$$\frac{\partial^2 u}{\partial x^2} = X''Y \text{ and } \frac{\partial^2 u}{\partial y^2} = XY''$$

Put in equation (1)

$$X''Y = XY''$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{-Y''}{4} = K(\text{say}) \qquad ...(2)$$

(*i*) When K is positive and is equal to p^2, say

$$\Rightarrow \quad X = C_1e^{px} + C_2e^{-px} \text{ and } Y = C_3 \cos py + C_4 \sin py$$

(*ii*) When K is negative and is equal to $-p^2$, say

$$\Rightarrow \quad X = C_5 \cos px + C_6 \sin px,\ Y = C_7e^{py} + C_8e^{py}$$

(*iii*) When $K = 0$

$$\Rightarrow \quad X = C_9x + C_{10} \text{ and } Y = C_{11}y + C_{12}$$

The various possible solution of (1) are

$$u = (C_1e^{px} + C_2e^{-px})(C_3 \cos py + C_4 \sin py) \qquad ...(3)$$

$$u = (C_5 \cos px + C_6 \sin px)(C_7e^{py} + C_8e^{-py}) \qquad ...(4)$$

$$u = (Cx_9 + C_{10})(C_{11}y + C_{12}) \qquad ...(5)$$

Out of these we take that solution which is consistent with the given boundary conditions.

Q. 1. (b) State Baeye's theorem. Design a suitable example and solve that using this theorem.

Ans. Baye's Theorem: Let s be the sample space and let $E_1, E_2, ... E_n$ be n mutually exclusive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or, ... or E_n, then

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)},\ i = 1, 2, ... n.$$

Example: Urn A contains 2 white, 1 black and 3 red balls, Urn B contains 3 white, 2 black and 4 red balls and Urn C contains 4 white, 3 black and 2 red balls. One Urn chosen at random and 2 balls are drawn at random from the Urn. If the chosen balls happen to be red and black what is the probability that both balls come from u_{rn} B?

Let E_1, E_2, E_3 and A denote the following events

E_1 = Urn A is chosen, E_2 =Urn B is chosen, E_3 = Urn C is chosen and A = two balls drawn at random are red and ablack.

Since one of the urns is chosen at randon, therefore

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

It E_1 has already occured, the Urn A has been chosen. Therefore the probability

drawing a red and black ball from u_{rn} $A = \dfrac{{}^3C_1 \times {}^1C_1}{{}^6C_2}$

So, $$P(A/E_1) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$$

Similarly, $$P(A/E_2) = \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{2}{9}$$

and $$P(A/E_3) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} = \frac{1}{6}$$

Required probability $= P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \text{ (Using Baye's theorem)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{9}+\frac{1}{3}\times\frac{1}{6}} = \frac{\frac{2}{9}}{\frac{1}{5}+\frac{2}{9}+\frac{1}{6}} = \frac{20}{53}$$

Q. 1. (c) What is the difference between the problem of correlation and regression? Why are their two regression lines and where do they intersect? Find an expression for the angle of intersection between the two lines. Discuss the special cases. **(7)**

Ans. Correlation: Whenver two variables x and y are so related that an increase in the one is accompained by an increase or decrease in the other, then the variables are said to be correlated.

i.e. Corefficient of correlation 'r' measures the strength of bivariate association.

Regression: It is a method used for estimating the unknown values of one variable corresponding to the known value of another variable.

i.e. the regression line is a prediction equation that estimates the values of y for any given x or x for any given y.

Line of Regression: Let a number of pairs of two correlated variable be (x_1, y_1)... (x_n, y_n). Suppose we have to find out the unknown value of y for a certain value of x, then we have line of regression of y on x *i.e.* $y = a + bx$. (Here y is dependent variable and x is independent variable).

If we have to find out unknown value x for a given value of y, then we have a line of regression of x on y i.e. $x = a + by$ (Here x is dependent variable and y is independent variable).

So, we have two lines of regression.

Yes, they can intersect with each other.

Let θ be the accute angle between the two regression lines.

Equation of lines of regression of y an x and x an y are

$$y-\bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \text{ and } x-\bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

Their slopes are $\quad m_1 = r\frac{\sigma_y}{\sigma_x} \; em_2 = \frac{\sigma_y}{r\sigma_x}$

$$\tan\theta = \pm\frac{m_2-m_1}{1+m_2m_1} = \pm\frac{\frac{\sigma_y}{r\sigma_x}-\frac{r\sigma_y}{\sigma_x}}{1+\frac{\sigma_y^2}{\sigma_x^2}}$$

$$= \pm\frac{1-r^2}{r}\cdot\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}\cdot\frac{\sigma_y}{\sigma_x}$$

$$\tan\theta = \pm\frac{1-r^2}{r}\cdot\frac{\sigma_x\,\sigma_y}{\sigma_x^2+\sigma_y^2}$$

Since $r^2 \le 1$ and σ_x, σ_y are positive

$\therefore$ +ve sign given the acute angle between the lines

Hence $\quad \boxed{\tan\theta = \frac{1-r^2}{r}\cdot\frac{\sigma_x\,\sigma_y}{\sigma_x^2+\sigma_y^2}}$

Cases: (*I*) When $\quad r = 0, \theta = \frac{\pi}{2}$

$\therefore$ The two lines of regression are perpendicular to each other. hence the estimated value of y is the same for all values of x and vice–versa.

(*II*) When $r = \pm 1$, $\tan\theta = 0 \Rightarrow \theta = 0$ or π.

hence the lines of regression coincide and there is perfect correlation between the two variable x and y.

Q. 1. (*d*) Given an example of an *LPP* with no solution at all. Write its dual. What about the solution of the dual? **(5)**

Ans. *LPP* with no solution is

Maximize $\quad z = x_1+x_2$

Subject to constraints

$$x_1+x_2 \le 1$$
$$-3x_1+x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

Standard Primal: Introducing stack variable $s_1 \geq 0$ and surplus variables $s_2 \geq 0$, the standard from of *LPP* is

Maximize $$Z = x_1 + x_2 + 0s_1 + 0s_2$$

Subject to constraints

$$x_1 + x_2 = s_1 + 0s_2 = 1$$
$$-3x_1 + x_2 + 0s_1 - s_2 = 3$$
$$x_1, x_2, s_1, s_2 > 0$$

Dual: Let w_1 and w_2 be the dual variables corresponding to the primal constraints. Then the dual problem will be.

Minimize $$Z^* = w_1 + 3w_2$$

Subject to the constraints:

$$w_1 - 3w_2 \geq 1$$
$$w_1 + w_2 \leq 1$$
$$w_1 + 0w_2 \geq 0 \Rightarrow w_1 \geq 0$$
$$0w_2 - w_2 \leq 0$$

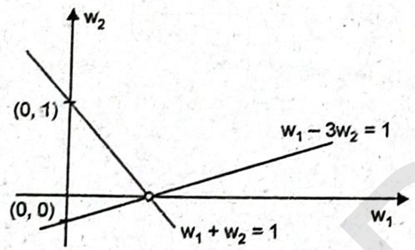
w_1 and w_2 unrestricted (redundant)

Eliminating redundant, the dual problem is

Minimize $$Z^* = w_1 + 3w_2$$

Subject to the constraints.

$$w_1 - 3w_2 \geq 1$$
$$w_1 + w_2 \leq 1$$
$$w_1 \geq 0 \text{ and } w_2 >, 0$$



The solution of dual of on *LPP* with no solution at all is unbounded solution (by duality theorem)

UNIT - I

Q. 2. (a) Solve: $(D^2 - 4DD' + 4D'^2)Z = e^{2x+y}$ **(6)**

Ans. Auxiliary equation is

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$
$$m = 2, 2$$
$$C.F = f_1(y+2x) + xf_2(y+2x)$$
$$P.I = \frac{1}{D^2 - 4DD' + 4D'^2} e^{2e+y}$$
$$= \frac{1}{(D-2D')^2} e^{2x+y}$$

$$= x.\frac{1}{2D - 4D'} e^{2x+y}$$
$$= \frac{x^2}{2}.(e^{2x+y})$$

Hence the solution is $$Z = f_1(y+2x) + xf_2(y+2x) + \frac{x^2}{2}(e^{2x+y})$$

Q. 2. (b) $$(D^2 - D')z = 2y - x^2$$

Ans. $D^2 - D'$ can not be resolved into linear factors in D and D'

Let $$Z = Ae^{hx+ky}$$
$$\Rightarrow \quad D^2Z = Ah^2e^{hx+ky}$$
$$D'Z = AKe^{hx+ky}$$
$$\therefore \quad (D^2 - D')Z = A(h^2 - k)e^{hx+ky}$$
Then $$(D^2 - D')Z = 0$$
$$\Rightarrow \quad A(h^2 - k) = 0$$
$$\Rightarrow \quad h^2 - k = 0 \Rightarrow k = h^2$$

$\therefore C.F = \sum Ae^{hx+h^2y}$ where A and h are arbiitrary constants

$$P.I. = \frac{1}{D^2 - D'}(2y - x^2) = \frac{1}{D^2\left(1 - \frac{D'}{D^2}\right)}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 - \frac{D'}{D^2}\right]^{-1}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 + \frac{D'}{D^2} + \ldots\right](2y - x^2)$$
$$= \frac{1}{D^2}\left[(2y - x^2) + \frac{1}{D^2}\{D'(2y - x^2)\}\right]\ldots$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{1}{D^2}.2\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + x^2\right] = \frac{1}{D^2}.2y = x^2y$$

Hence the required solution is

$$Z = C.F + P.I.$$
$$Z = \left\{Ae^{hx+h^2y} + x^2y,\right.$$

A and h being arbitary constants

Q. 3. (a) A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially in a position given by $u(x) = u_0 \sin^3(\pi x/l)$. if it is released from rest from this position. Find the displacement $u(x,t)$. **(6.5)**

Ans. The equation of the string is

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

The solution of equation (1) is

$$u(x,t) = (C_1 \cos cpt + C_2 \sin cpt)(C_3 \cos px + C_4 \sin px) \quad ...(2)$$

Boundary conditions are

$$u(0,t) = 0 \quad ...(3)$$

$$u(l,t) = 0 \quad ...(4)$$

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \quad ...(5)$$

$$u(x,0) = u_0 \sin^3\left(\frac{\pi x}{l}\right) \quad ...(6)$$

Applying boundary condition in (2)

$$u(0,t) = 0 = (C_1 \cos cpt + C_2 \sin cpt)C_3$$

$\Rightarrow$ $C_3 = 0$

$\therefore$ From (2), $u(x,t) = (C_1 \cos cpt + C_2 \sin cpt)\, C_4 \sin px$

Again, $u(l,t) = 0 = (C_1 \cos cpt + C_2 \sin cpt)\, C_4 \sin pl$

$\Rightarrow$ $\sin pl = 0 = \sin n\pi, x \in I \quad ...(7)$

$\Rightarrow$ $p = \frac{n\pi}{l}$

From (7)

$$u(x,t) = \left(C_1 \cos\frac{n\pi ct}{l} + C_2 \sin\frac{n\pi ct}{l}\right) C_4 \sin\left(\frac{n\pi x}{l}\right) \quad ...(8)$$

$$\frac{\partial u}{\partial t} = \frac{n\pi c}{l}\left(-C_1 \sin\frac{n\pi ct}{l} + C_2 \cos\frac{n\pi ct}{l}\right) C_4 \text{Sin}\left(\frac{n\pi x}{l}\right)$$

At $t = 0$,

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 = \frac{n\pi c}{l} C_2 C_4 \sin\left(\frac{n\pi x}{l}\right)$$

$\Rightarrow$ $C_2 = 0$

From (8)

$$u(x,t) = C_1 C_4 \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) = b_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right)$$

Most general solution is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) \quad ...(9)$$

$$u(x,0) = u_0 \sin^3\frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$\Rightarrow \quad u_0 = \left\{\frac{3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}}{4}\right\}$$

$$= b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + b_3 \sin\frac{3\pi x}{l} + ...$$

On comparing, we get

$$b_1 = \frac{3u_0}{4}, b_2 = 0, b_3 = -\frac{u_0}{4}, u_4 = u_5 = \quad ...(0)$$

Hence ftrom (9),

$$u(x,t) = \frac{3u_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l} - \frac{u_0}{4}\sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{3\pi ct}{l}\right)$$

Q. 3. (b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u : u(0,y) = 0$

$\left(\frac{\partial u}{\partial x}\right)_{(0,y)}$ = it e^{-3y} by the method of separation of variables. **(6)**

Ans. let $U = XY \quad ...(1)$

Where X is a function of x only and Y is a function of y only.

$$\frac{\partial u}{\partial y} = X\frac{\partial Y}{\partial y} = XY' \text{ and } \frac{\partial^2 u}{\partial x^2} = YX''$$

Put in given eq^n.

$$YX'' = XY' + 2XY = X(Y' + 2Y)$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{Y'}{y} + 2 = \text{k (say)}$$

$$\frac{X''}{X} = k$$

$$\Rightarrow \quad X'' - kX = 0$$

Auxilliary equation is

$$m^2 - k = 0$$

$$m = \pm\sqrt{k}$$

$$\text{C.F.} = C_1 e^{\sqrt{kx}} + C_2 e^{-\sqrt{kx}}$$

$$\text{P.I} = 0$$

$$X = C_1 e^{\sqrt{kx}} + C_2 e^{-\sqrt{kx}}$$

and

$$\frac{Y'}{y} + 2 = k$$

$$\frac{Y'}{Y} = k - 2$$

On interpation, we get

$$\log Y = (k-2)y + \log C_3$$

$$Y = C_3 e^{(k-2)y}$$

Hence from (1)

$$u(x,y) = \left(C_1 e^{\sqrt{kx}} + C_2 e^{\sqrt{kx}}\right) C_3 e^{(k-2)y} \quad ...(2)$$

Applying the condition $u(0,y)=0$ in (2), we get

$$0 = (C_1+C_2)\, C_3 e^{(k-2)y}$$

$$\Rightarrow \quad C_1 + C_2 = 0 \Rightarrow C_2 = -C_1$$

From (2), the most general solution is

$$u(x,y) = \sum C_1 C_3 \left(e^{\sqrt{kx}} - e^{-\sqrt{kx}}\right) e^{(k-2)y}$$

$$\frac{\partial u}{\partial x} = \sum C_1 C_3 \sqrt{k}\left(e^{\sqrt{kx}} + e^{-\sqrt{kx}}\right) e^{(k-2)y}$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} = 1 + e^{-3y} = \sum C_1 C_3 \sqrt{k}(2) e^{(k-2)y}$$

$$= \sum b_n e^{(k-2)y}$$

Comparing the coefficients, we get

(i)

$$b_1 = 1,\ k-2 = 0$$

$$2C_1C_3\sqrt{k} = 1,\ k = 2$$

$$C_1C_3 = \frac{1}{2\sqrt{2}}$$

(ii)

$$b_3 = -1,\ k-2 = 3$$

$$2C_1C_3\sqrt{k} = 1,\ k = -1$$

$$C_1C_3 = \frac{1}{2i}$$

hence from (6), the particular solution is

$$u(x,y) = \frac{1}{2\sqrt{2}}\left(e^{\sqrt{2x}} - e^{-\sqrt{2x}}\right) + \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) e^{-3y}$$

$$\Rightarrow \quad \boxed{u(x,y) = \frac{1}{\sqrt{2}} \sin h\sqrt{2x} + e^{-3y}\sin x}$$

UNIT-II

Q. 4. (a) State multiplication rule of probability. From it derive the condition for two events to be independent.

For a system composed of k components in parallel, if p_i independent of others is the probability that the i^{th} component will function $i = 1, 2, \ldots k$, then what is the probability that system will function? (6.5)

Ans. Statement: The probability of the concurrence of two independent events is the product of their separate probabilities *i.e.*

$$P(AB) = P(A).P(B)$$

Proof. Suppose A and B are two independent events.

let A happen in m_1 ways and fail in n_1 ways.

$$\therefore \quad P = \frac{m_1}{m_1+n_1}$$

Also, let B happen in m_2 ways and fail in n_2 ways.

$$\therefore \quad P(B) = \frac{m_2}{m_2+n_2}$$

Now there are four possibilities

(i) A and B both may happen, then the number of ways $= m_1m_2$

(ii) A may happen and B may fail, then the number of ways $= m_1n_2$.

(iii) A may fail and B may happen, then the number of ways $= n_1m_2$

(iv) A and B both may fail, then the number of way $= n_1n_2$.

Thus the total number of ways $= m_1m_2 + m_1n_2 + n_1m_2 + n_1n_2 = (m_1+n_1)(m_2+n_2)$

Hence the probabilities of the happening of both A and B is

$$P(AB) = \frac{m_1m_2}{(m_1+n_1)(m_2+n_2)} = \frac{m_1}{m_1+n_1}.\frac{m_2}{m_2+n_2}$$

$$= P(A).P(B).$$

For a system composed of k components in parallel, if p_i is the probability (indendent of other) that ith component will function, $i = 1, 2, \ldots k$.

Probavility that system will function

$= 1 -$ (all the k components of system is not working)

$$= 1 - [(1-p_1)(1-p_2)(1-p_3)\ldots(1-p_k)].$$

Q. 4. (b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trails with constant probality 2/3 of sucess in each trials? (6)

Ans.

$$E(X) = 1.P(1) + 2.P(2) +$$

$$= 1.\left(\frac{2}{3}\right) + 2\left(1-\frac{2}{3}\right)\left(\frac{2}{3}\right) + 3\left(1-\frac{2}{3}\right)^2\left(\frac{2}{3}\right)$$

$$+ 4.\left(1-\frac{2}{3}\right)^3\left(\frac{2}{3}\right) + \ldots$$

$$= \frac{2}{3}\left[1 + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + \ldots\right]$$

$$= \frac{3}{2}\left[as \sum_{x=1}^{\infty} xp(1-p)^{x-1} = \frac{1}{p}\right]$$

$$\text{Expected value} = \frac{3}{2}$$

Q. 5. (a) Define a binomial variate. What is its mean and variance? By considering an example of your choice illustration its application. (6.5)

Ans. Binomial random variable: A specific type of discrete random variable that counts how aften a particular event occurs in a fixed number of tries or trials.

For a variable to be binomial random variable, all the following conditions must be satisfied.

(1) There are a fixed number of trails (a fixed sample size)

(2) On each trial the event of interest either occurs or does not

(3) The probability of occurrence (or not) is the same on each trial.

(4) trials are independent of one another.

The Binomial probability Distribution is

$$P(X=r) = {}^nC_r p^r q^{n-r}, p+q=1, r=0, 1, 2, \ldots n$$

Where p is the probability of success and X is called binomial variate.

Mean and Variance of the bnomial Distribution.

$$P(r) = {}^nC_r q^{n-r} p^r$$

$$\text{Mean } (\mu) = \sum_{r=0}^{n} r\,p(r) = \sum_{r=0}^{n} r\ {}^nC_r\, q^{n-r} p^r$$

$$= 0 + 1.{}^nC_1 q^{n-1}p + 2^n C_2 q^{n-2}p^2 + 3.{}^nC_3 q^{n-3}p^3 + \ldots + n.{}^nC_n p^n$$

$$= nq^{n-1}p + 2\frac{n(n-1)}{2.1}q^{n-2}p^2 + 3\frac{n(n-1)(n-2)}{3.2.1}q^{n-3}p^3 \ldots\ldots + n.p^n$$

$$= np\left[q^{n-1} + (n-1)q^{n-2}p^2 + \frac{(n-1)(n-2)}{2.1}q^{n-3}p^2 + \ldots + p^{n-1}\right]$$

$$= np\left[{}^{n-1}C_0 q^{n-1} + {}^{n-1}C_1 q^{n-2}p^2 + {}^{n-1}C_2 q^{n-3}p^2 + \ldots\ldots + {}^{(n-1)}C_{n-1}p^{n-1}\right]$$

$$= np(q+p)^{n-1} = np$$

$$\boxed{\text{Mean} = np}$$

Variance,

$$\sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n}[r + r(r-1)]P(r) - \mu^2$$

$$= \sum_{r=0}^{n} rP(r) + \sum_{r=0}^{n} r(r-1)P(r) - \mu^2$$

$$= \mu + \sum_{r=2}^{n} r(r-1)^n c_r q^{n-r} p^r - \mu^2$$

$$= \mu + \left[2.1.{}^n c_2 q^{n-2}p^2 + 3.2^n c_3 q^{n-3}p^3 + \ldots\ldots + n(n-1)^n c_n p^n\right] - \mu^2$$

$$= \mu + \left[2.1\frac{n(n-1)}{2.1}q^{n-2}p^2 + 3.2\frac{n(n-1)(n-2)}{3.2.1}q^{n-3}p^3 + \ldots\ldots + n(n-1)p^n\right] - \mu^2$$

$$= \mu + \left[n(n-1)q^{n-2}p^2 + n(n-1)(n-2)q^{n-3}p^3 + \ldots\ldots + n(n-1)p^n\right] - \mu^2$$

$$= \mu + n(n-1)p^2\left[q^{n-2} + (n-2)q^{n-3}p + \ldots\ldots + p^{n-2}\right] - \mu^2$$

$$= \mu + n(n-1)p^2\left[{}^{n-2}C_0 q^{n-2} + {}^{n-2}C_1 q^{n-3}p + \ldots\ldots + {}^{n-2}C_{n-2}P^{n-2}\right] - \mu^2$$

$$= \mu + n(n-1)p^2(q+p)^{n-2} - \mu^2$$

$$= \mu + n(n-1)p^2 - \mu^2$$

$$= np + n(n-1)p^2 - n^2p^2$$

$$= np[1 + (n-1)p - np]$$

$$= np(1-p) = npq$$

$$\boxed{\text{Variance} = npq}$$

Example: Treatment of kidney cancer: Suppose we have n = 40 patients who will be receiving an experimental therapy which is belived to be better than current treatements which historically have had a 5 year survival rate of 20% *i.e.* the probability of 5-year survival is p = 0.2.

Thus the number of patients out of 40 in our study surviving at least 5-year has a binomial distribution, i.e. X ~ Bin (40, 20)

Suppose that using the new treatment, we find that 16 out of 40 patients survive at least 5-years post diagnosis.

$$P(X=16) = {}^{40}C_{16}(0.2)^{16}(0.8)^{24} = 0.001945$$

The chance that 16 patients out of 40 surviving at least 5-years is very small 0.001945

Q. 5. (b) A manufacturer who produces medicine bottles find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. Find that in 100 such boxes, how many boxes are expected to contain (a) no defective (b) atleast two defectives. **(6)**

Ans. Here N = 100, p = 0.001, n = 500

Mean (m) = np = 500 × 0.001 = 0.5

Let r be the number of defective bottles in a box.

Let $P(r)$ be the number of boxes containing r defective bottles, thes

$$P(r) = N \times \frac{e^{-m}m^r}{r!}$$

(a) P(0) = Number of boxes with no defective bottles

$$= 100 \times \frac{e^{-0.5} \times (0.5)^0}{0!} = 60.65$$

∴ Number of boxes with no defective bottles = 61

(b)
$$P(r \geq 2) = [P(2) + P(3) + \ldots\ldots\ldots + P(500)] \times 100$$

$$= [1-(P(0) + P(1)] \times 100$$

$$= \left\{1 - \left[\left(\frac{e^{-0.5} \times (0.5)^0}{0!} + \frac{e^{-0.5} \times (0.5)^1}{1!}\right)\right]\right\} \times 100$$

$$= 9.02$$

∴ Number of boxes with at least two defectives bottles = 9.

UNIT-III

Q.6. (a) Following data is for the measurement of train resistance R(Ids/ton) with the velocity V(mPh). If $R = a + bV + CV^2$, find a, b, c **(6)**

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Ans. For convenience, let R = y and V = x

∴ the equation is

$$y = a + bx + cx^2$$

Let $$u = \frac{x-80}{h} \text{ and } v = y - 22.8$$

and Let $$h = 20.$$

$$u = \frac{x-80}{20} \text{ and } v = y - 22.8$$

Then the equation is

$$v = a + bu + cu^2$$

x	u	y	v	uv	u^2	u^2v	u^3	u^4
20	–3	5.5	–17.3	51.9	9	–155.7	–27	81
40	–2	9.1	–13.7	27.4	4	–54.8	–8	16
60	–1	14.9	–7.9	7.9	1	–7.9	–1	1
80	0	22.8	0	0	0	0	0	0
100	1	33.3	10.5	10.5	1	10.52	1	1
120	2	46.0	23.2	46.4	4	92.8	8	16
$\Sigma u = -3$			$\Sigma v = -5.2$	$\Sigma uv = 144.1$	$\Sigma u^2 = 19$	$\Sigma u^2v = -115.1$	$\Sigma u^3 = -27$	$\Sigma u^4 = 115$

Normal equations are

$$\Sigma u = na + b\Sigma u + c\,\Sigma u^2$$
$$\Rightarrow \quad -5.2 = 6a + b(-3) + 19c$$
$$\Rightarrow \quad -5.2 = 6a - 3b + 19c \quad ...(1)$$
$$\Sigma uv = a\,\Sigma u + b\Sigma u^2 + c\Sigma u^3$$
$$\Rightarrow \quad 144.1 = -3a + 19b - 27c \quad ...(2)$$
$$\Sigma u^2v = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4$$
$$\Rightarrow \quad -115.1 = 19a - 27b + 115c \quad ...(3)$$

on solving (1), (2) and (3) we get $\begin{bmatrix} a = 4.369 \\ b = -0.00175 \\ c = 0.00287 \end{bmatrix}$

$a = 4.369$, $b = -0.00175$, $c = 0.00287$

∴ $R = 4.369 - 0.00175V + 0.00287V^2$

Q.6. (b) From the following data

x	23	27	28	28	29	30	31	33	35	36
y	18	20	22	27	21	29	27	29	28	29

Estimate y where $x = 32$, by using suitable line of regression **(6.5)**

Solution:

x	y	xy	x^2
23	18	414	529
27	20	540	729
28	22	616	784
28	27	756	784
29	21	609	841
30	29	870	900
31	27	837	961
33	29	957	1089
35	28	980	1225
36	29	1044	1296
Total 300	250	7623	9138

Let y = a + bx be the equation of the line of regrssion of y on x, where a and b are given by the following equations

$$\Sigma y = na + b\Sigma x$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$
$$\Rightarrow \quad 250 = 10a + 300b \quad ...(1)$$
$$7623 = 300a + 9138b \quad ...(2)$$

on solving (1) and (2) we get (Multiplying (1) by 30)

$$7500 = \cancel{300a} + 9000b$$
$$7623 = \cancel{300a} + 9138b$$
$$-123 = -138b$$

$$\Rightarrow \quad b = \frac{123}{138} = 0.89$$

∴ $\boxed{b = 0.89}$

Put in (1), we get

$\boxed{a = -1.74}$

Hence, we get

$\boxed{y = -1.74 + 0.89x}$ is required regression of y on x.

When x = 32

$\boxed{y = 26.74}$

Q.7. (a) A tea company claims that its premium tea brand outsells its normal brand by 10% If it is found that 46 out of a sample of 200 tea-users prefer premium brand and 19 out of another independent sample of 100 tea-users prefer normal brand. Test the validity of the company both at 1% and 5% level of significance. **(6)**

Ans. Here, $n_1 = 200, n_2 = 100$

$$p_1 = \frac{x_1}{n_1} = \frac{46}{200}, n_2 = \frac{19}{100}$$

p = Proportion of premium tea brand in the population = 0.1

$$Q = 1 - P = 0.99$$

Null hypothesis: H_0: The Manufacturer claim is accepted.

Alternative hypothesis H_1: $p \neq 0.1$

Under H_0,
$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.04}{\sqrt{0.1 \times 0.99 \times \frac{3}{200}}} = 26.9$$

Conclusion: Since the calculated value of $|z| > 1.645$ and also $|z| > 2.33$. Hence H_0 is rejected 5% and 1% level of significance, i.e., the proportion of premium tea brand in the population is greater than 10%.

Q.7. (b) A survey of 800 families with four children each, recorded the following data:-

No. of boys	**0**	**1**	**2**	**3**	**4**
No. of Girls	**4**	**3**	**2**	**1**	**0**
No of Families	**32**	**178**	**290**	**236**	**64**

Test the hypothesis that male and female births are equailly likely. **(6.5)**

Ans. Null hypothesis H_0: The data are consistent with the hypothesis of equal probability for male and female births *i.e.* $p = q = \frac{1}{2}$.

We use binomial distribution to calculate theoretical frequency given by:

$$N(r) = N \times P(X = r) = N \times {}^nC_r\, p^r q^{n-r}$$

Where N is the total frequency, N(r) is the number of families with r male children, p and q are possibilities of male and female births respectively, n is the no. of children.

$$N(0) = 800 \times {}^4C_0\left(\frac{1}{2}\right)^4 = 50, N(1) = 200, (2) = 300, N(3) = 200 \text{ and } N(4) = 50$$

Observed frequency O_i	32	178	290	236	64
Expected frequency E_i	50	200	300	200	50
$(O_i - E_i)^2$	324	484	100	1296	196
$\frac{(O_i - E_i)^2}{E_i}$	6.48	2.42	0.333	6.48	3.92

$$\chi^2 = \sum\left[\frac{(O_i - E_i)^2}{E_i}\right] = 19.633$$

Tabulated value of χ^2 at 5% level of significance for 5–1=4 degree of freedem = 9.49

Conclusion: Since the calculated value of χ^2 is greater then the tabulated value, H_0 is rejected, i.e. the data are not consistent with the hypothesis that the binomial law holds and that the chance of a male birth is not equal of that of a female birth.

UNIT-IV

Q.8. Solve the following product mix selection LPP: **(12.5)**

$$\text{Max } w = 4x + 5y + 9z + 11t$$

Subject to constraints

$$\begin{aligned} x + y + z + t &\le 15 \\ 7x + 5y + 3z + 2t &\le 120 \\ 3x + 5y + 10z + 5t &\le 100 \\ x, y, z, t &\ge O \end{aligned}$$

Ans. Introducing slack variables $S_1 \ge 0, S_2 \ge 0, S_3 \ge 0$ in the respective inequalities. So, the standerd LPP is

$$\text{Max w} = 4x + 5y + 9z + 11t + Os_1 + Os_2 + Os_3$$

s.t.c

$$\begin{aligned} x + y + z + t + s_1 &= 15 \\ 7x + 5y + 3z + 2t + s_2 &= 120 \\ 3x + 5y + 10z + 15t + s_3 &= 100 \\ x, y, z, t, s_1, s_2, s_3 &\ge O \end{aligned}$$

The set of constraints can be written in matrix form as:

$Ax = b$

$$\text{Where, } A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 7 & 5 & 3 & 2 & 0 & 1 & 0 \\ 3 & 5 & 10 & 15 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} x \\ y \\ z \\ t \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 15 \\ 120 \\ 100 \end{bmatrix}$$

Inittal basic feasible solution

$$x_B = B^{-1} b \text{ where } B = I_3$$

x_B = basic variable corresponding to columns of basis matrix B(=I)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 120 \\ 100 \end{bmatrix} = \begin{bmatrix} 15 \\ 120 \\ 100 \end{bmatrix}$$

The iterative simple x tables are:

First Table.

			4	5	9	11	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
0	s_1	15	1	1	1	1	1	0	0
0	s_2	120	7	5	3	2	0	1	0
0	s_3	100	3	5	10	$\boxed{15}$	0	0	1
	$z_j - c_j$	=	–4	–5	–9	–11	0	0	0

$z_4 - c_4$ is most negative, so y_4 enters into the basis.

$$\min\left\{\frac{15}{1}, \frac{120}{2}, \frac{100}{15}\right\} = \frac{100}{15}.$$

s_0, s_3 Leaves from the basis.

Second Table:

			4	5	9	11	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
0	s_1	25/3	$\boxed{4/5}$	2/3	1/3	0	1	0	−1/15
0	s_2	320/3	33/5	13/3	5/3	0	0	1	−2/15
11	y_4	100/15	1/5	1/3	2/3	1	0	0	1/15
	$z_j - c_j$	=	−9/5	−4/3	−5/3	0	0	0	11/15

Since $z_1 - c_1$ is most negative, so, y_1 enters into the basis.

$$\min\left\{\frac{25/3}{4/5}, \frac{320/3}{33/5}, \frac{100/15}{1/5}\right\} = \frac{25/3}{4/5}.$$

s_1 leaves from the basis.

IIIrd Table

			4	5	9	11	0	0	0
C_B	Y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
4	y_1	125/12	1	5/6	5/12	0	5/4	0	−1/12
0	s_2	455/12	0	−7/6	−13/12	0	−33/4	/	5/12
11	y_4	55/12	0	1/6	$\boxed{7/12}$	1	−1/4	0	1/12
	$zj - cj =$		0	1/6	−11/12	0	9/4	0	7/12

$z_3 - c_3$ is most negative, so y_3 enter into the basis

$$\text{Min}\left\{\frac{125/12}{5/12}, \frac{55/12}{7/12}\right\} = \frac{55/12}{7/12}$$

So, y_4 leaves from the basis.

IV[th] table

			4	5	9	11	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
4	y_1	50/7	1	5/7	0	−5/7	10/7	0	−1/7
0	s_2	525/7	0	−6/7	0	13/7	−61/7	1	4/7
9	y_3	55/7	0	2/7	1	12/7	−3/7	0	1/7
	$zi - cj =$		0	3/7	0	11/7	13/7	0	5/7

Since all $z_j - c_j = 0$is non-negative we get an optimal solution.
The optimal solution is

$$x = \frac{50}{7}, y = 0, z = \frac{55}{7}, t = 0$$

$$\text{Max } w = \frac{695}{7}$$

Q.9. (a) For the following transportation problem, find the initial BFS using VAM from. (6.5)

	1	2	3	4	5	supply
1	4	2	3	2	6	8
2	5	4	5	2	1	12
3	6	5	4	7	3	14
Demand	4	4	6	8	8	

Ans. Here the total demand is 30 and total supply is 34. Since total demand ≠ total supply. We intorduce a dummy column with is demand as (34-30) *i.e* 4 and take all the cost elements of this column as zero.

Thus the transportation table for the initial basic feasible solution of the given problem is

(I)

							Rowepenalty
	4	2	3	2	6	0	8 (2)
	5	4	5	2	1	0	12 (1)
	6	5	4	7	3	$\boxed{4}$ 0	14 (3)
	4	4	6	8	8	4	
Column Penalty	(1)	(2)	(1)	(0)	(2)	(0)	

						Rewepenalty
	4	$\boxed{4}$ 2	3	2	6	8 (0)
	5	4	5	2	1	12 (1)
	6	5	4	7	3	14 (1)
Column	4	4	6	8	8	
Penalty	(1)	(2)	(1)	(0)	(2)	

(II)

					Rewpenalty
	4	3	2	6	4 (1)
	5	5	2	$\boxed{8}$ 1	12 (1)
	6	4	7	3	10 (1)
Column	4	6	8	8	
Penalty	(1)	(1)	(0)	(2)	

(III)

				Row penalty
	4	3	2	4 (1)
	5	5	$\boxed{4}$ 2	4 (3)
	6	4	7	10 (2)
Column	4	6	8	
penalty	(1)	(1)	(0)	

(IV)

				Row penalty
	4	3	$\boxed{4}$ 2	4 (1)
	6	4	7	10(2)
Column	4	6	4	
Penalty	(2)	(1)	(5)	

(V)

(VI)

	$\boxed{3}$ 4	3	e_1 (1)
	6	4	10(2)
Column	4	6	
Penully	(2)	(1)	

(VII)

			Row Penalty
	$\boxed{4}$ 6	$\boxed{6}$ 4	10
Column Penalty	4	6	

$$\text{Total cost} = 4 \times 0 + 4 \times 2 + 8 \times 1 + 4 \times 2 + 4 \times 2 + 4 \times \varepsilon_1 + 4 \times 6 + 6 \times 4$$
$$= 8 + 8 + 8 + 8 + 24 + 24 + 4\,\varepsilon_1$$
$$= 24 + 24 + 24 + 8 + 4\,\varepsilon_1$$
$$= 80 + 4\,\varepsilon_1$$
$$= 80 \; (\text{as } \varepsilon_1 \to 0)$$

Q. 9.(*b*) An engineer wants to assign 3 Jobs $J_1 J_2, J_3$ to three machines M_1 M_2, M_2 in such a way that each job is assigned to some machine and no machine works on more than one job. The cost matrix is given as follows

	M_1	M_2	M_3
J_1	15	10	9
J_2	9	15	10
J_3	10	12	8

(*i*) Formulate it as LPP

(*ii*) Find the optimal solution using Hungarian method.

Ans. (*i*) Linear programing formulation of the given problem is

Minimize the total cost involved, *i.e*,

Minimize $Z = (15x_{11} + 10x_{12} + 9x_{13}) + (9x_{21} + 15x_{22} + 10x_{23}) + (10x_{31} + (2x_{32} + 8x_{33})$

Subject to the constraints:

$$x_{i1} + x_{i2} + x_{i3} = 1; \quad i = 1, 2, 3$$
$$x_{ij} + x_{2j} + x_{3j} = 1; \quad j = 1, 2, 3$$
$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$$

(*ii*) **Step 1:** Let p_i and q_i be row i and calumn j

	M_1	M_2	M_3	Row Minimize
J_1	15	10	9	$p_1 = 9$
J_2	9	15	10	$p_2 = 9$
J_3	10	12	8	$p_3 = 8$

Step 2: We subtract the row minimum form each respective row to obtain the reduced matrix as:

	M_1	M_2	M_3
J_1	6	1	0
J_2	0	6	1
J_3	2	4	0

Minimum $q_1 = 0\; q_2 = 1\; q_3 = 0$

Step 3: We subtract the column minimum from each respective column to obtain the reduced matix as:

	M_1	M_2	M_3
J_1	6	0	0
J_2	0	5	1
J_3	2	3	0

The cells with underscored zero entires provide the optimum solution. This means that job J_1 by M_2 machine and J_2 by M_1 and J_3 by M_3. The total cost is $10 + 9 + 8 = 27$.

FIRST TERM EXAMINATION [FEB. 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 1: 30 hrs. **M.M. : 30**

Note: *Attempt Q. No. 1 which is compulsory and any two more questions from remaining All questions carry equal marks.*

Q.1. (*a*) Solve $\dfrac{\partial z}{\partial x} = 2\dfrac{\partial z}{\partial y} + y\cos x$ **(2.5)**

Ans.
$$\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = y\cos x$$
$$(D - 2D') = y\cos x$$

put $D = m, D' = 1$

A.E $m - 2 = 0$

$$\Rightarrow m = 2.$$
$$\text{C.F.} = f_1(y + 2x).$$
$$\text{P.I.} = \frac{1}{D - 2D'}y\cos x$$
$$= \int (c - 2x)\cos x\, dx$$

Here $m = 2$, where c is replaced by $y + mx = y + 2x$.

$$= (c - 2x)\sin x - (-2)(-\cos x)$$
$$= (c - 2x)\sin x - 2\cos x$$
$$= (y + 2x - 2x)\sin x - 2\cos x$$
$$= y\sin x - 2\cos x$$

$\therefore$ Complete solution is

$$z = f_1(y + 2x) + y\sin x - 2\cos x.$$

Q.1. (*b*) Solve $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x\partial y} + \dfrac{\partial z}{\partial y} = z$ **(2.5)**

Ans. $(D^2 + DD' + D' - 1)z = 0$

A.E $(D^2 - 1) + D'(D + 1) = 0$

$\Rightarrow$ $(D - 1)(D + 1) + D'(D + 1) = 0$

$\Rightarrow$ $(D + 1)(D + D' - 1) = 0$

$$D = -1$$

for $(D + D' - 1), b = 1\; a = -1, c = 1$

C.F $e^{-x}\phi_1(y) + e^{x}\phi_2(y - x)$

Thus, solution is

$$z = e^{-x}\phi_1(y) + e^{x}\phi_2(y - x)$$

Q.1. (c) If the probability that the man aged 60 will live 70 is 0.6, What is the probability that out of 10 men aged 60, 9 men will live upto 70. (2

Ans. Let, probability of success that man will live upto

$$70 = 0.6$$

ie.

$$p = 0.6$$

probability of failure

$$q = 1 - p = 1 - 0.6 = 0.4$$

Let

$$n = 10$$

Let X be the binomial variate

$\therefore$

$$f_X(x) = p[X = x] = n_{C_x} p^x q^{n-x}$$

$$= {}^{10}c_x (0.6)^x (0.4)^{10-x}$$

$$p[X = 9] = {}^{10}c_9 (0.6)^9 (0.4) = 10 \times 0.4 \times (0.6)^9$$

$$= 0.0403$$

Q.1. (d) Determine the value of k, if the probability function of a random variable X is given by

$$p(x) = \begin{cases} \dfrac{kx}{20}, x = 1, 2, 3, 4 \\ 0, \text{other integers} \end{cases}$$ (2

Ans. Since p_X is the probability distribution function

$$\sum p_X(x) = 1$$

$$\Rightarrow \quad \frac{k}{20} + \frac{2k}{20} + \frac{3k}{20} + \frac{4k}{20} = 1$$

$$10k = 20$$

$$k = 2$$

Q.2. (a) Find the solution of the partial differential equation $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$

Ans.

$$(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$$

Replace D by m and D^1 by 1

A.E

$$m^3 - 7m - 6 = 0$$

$$(m + 1)(m^2 - m - 6) = 0$$

$$\Rightarrow \quad (m + 1)(m^2 - 3m + 2m - 6) = 0$$

$$\Rightarrow \quad m = -1, (m + 2)(m - 3) = 0$$

$$\Rightarrow \quad m = -1, -2, 3$$

$$\text{C.F} = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x)$$

$$\text{P.I.} = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \sin(x + 2y)$$

Replace D by 1 and D' by 2

$$= \frac{1}{1 - 28 - 48} \iiint \sin u \, du \, du \, du$$

$$= \frac{-1}{75} \cos u = \frac{-1}{75} \cos(x + 2y)$$

$\therefore$ complete solution is

$$z = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x) \frac{-1}{75} \cos(x + 2y)$$

Q.2. (b) Use the method of separation of variable to solve the partial differential equation **(5)**

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \text{ given } u = 3e^{-y} - e^{-5y} \text{ when } x = 0$$

Ans. Given equation is $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$ ...(A)

Let

$$u = X(x)\,Y(y) = XY$$

$$\therefore \quad \frac{\partial u}{\partial x} = \frac{\partial(XY)}{\partial x} = Y\frac{dX}{dx}$$

and

$$\frac{\partial u}{\partial y} = \frac{\partial(XY)}{\partial y} = X - \frac{dY}{dy}$$

$$\therefore \quad A \Rightarrow \quad 4\frac{YdX}{dx} + X\frac{dY}{dy} = 3XY$$

$$\Rightarrow \quad 4X'Y + XY' = 3XY$$

$$\Rightarrow \quad 4X'Y = (3Y - Y')X$$

$$\Rightarrow \quad \frac{4X'}{X} = \frac{3Y - Y'}{Y}$$

$$\Rightarrow \quad \frac{4X'}{X} = 3 - \frac{Y'}{Y} = a \text{ (say)}$$

As LHS is a function of x and RHS is a function of y only

$$\therefore \quad \frac{4X'}{X} = a \Rightarrow 4\frac{dX}{dx}\cdot\frac{1}{X} = a$$

On integrating,

$$\int\frac{dX}{X} = \int\frac{a}{4}dx$$

$$\Rightarrow \quad \log X = \frac{ax}{4} + \log c_1$$

$$\Rightarrow \quad X = c_1 e^{ax/4} \quad \text{...(1)}$$

Now

$$3 - \frac{Y'}{Y} = a \Rightarrow \frac{Y'}{Y} = 3 - a$$

On integrating,

$$\int\frac{dY}{Y} = \int(3 - a)dy$$

$$\Rightarrow \quad \log Y = (3 - a)y + \log c_2$$

$$\Rightarrow \qquad y = c_2 e^{(3-a)y} \qquad ...(2)$$

$$\text{As} \qquad U = XY = c_1c_2 e^{ax/4} e^{(3-a)y} \qquad ...(B)$$

As given $\qquad U(0,y) = 3e^{-y} - e^{-5y}$

from (B) $\qquad U(0,y) = c_1c_2 e^{(3-a)y}$

comparing two, we get

$$3e^{-y} - e^{-5y} = c_1c_2 e^{(3-a)y}$$

$$c_1c_2 = 3,\ 3 - a = -1 \text{ and } c_1c_2 = -1,\ 3 - a = -5$$

$$\Rightarrow \qquad c_1c_2 = 3,\ a = 4 \Rightarrow c_1c_2 = -1,\ a = 8$$

Now, equation (B) becomes

$$U = 3e^{4x/4} e^{(3-4)y} - e^{8x/4} e^{-5y}$$

$$U = 3e^{x}e^{-y} - e^{2x}e^{-5y}.$$

$$U = 3e^{x-y} - e^{2x-5y}$$

Q.3. (a) Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions

(i) $u \to 0$ as $y \to \infty$ for all x (ii) $u = 0$ at $x = 0$ for all y

(iii) $u = 0$ at $x = l$ for all y (iv) $u = lx - x^2$ if $y = 0$ for all $x \in (0,l)$

Ans. Given equation is $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad ...(A)$

The boundary conditions are

$$\left.\begin{aligned} u(0,y) &= 0 \\ u(l,y) &= 0 \end{aligned}\right\} \text{for all } y$$

$$u(x,\infty) = 0 \ \forall x$$

$$u(x,0) = lx - x^2 \quad 0 < x < l$$

The three possible solutions are

(i) $\qquad u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

(ii) $\qquad u(x,y) = (c_5 \cos px + c_6 \sin px)(c_7 e^{py} + c_8 e^{-py})$

(iii) $\qquad u(x,y) = (c_9 x + c_{10})(c_{11} y + c_{12})$

from the condition that $u \to 0$ as $y \to \infty$ for all value of x, solutions (i) and (iii) lead to trivial solutions and hence (ii) is the only suitable one.

i.e. $u(x,y) = (A \cos px + B \sin px)(Ce^{py} + De^{-py}) \qquad ...(2)$

using boundary conditions $u(0,y) = 0$ in (2) gives

$$0 = A(ce^{py} + De^{-py})$$

$$\Rightarrow \qquad A = 0$$

$\therefore$ (2) reduces to

$$u(x,y) = B \sin px\,(Ce^{py} + De^{-py})$$

$$u(x,y) = \sin px\,(C'e^{py} + D'e^{-py}) \qquad ...(3)$$

Using the condition $\qquad u(l,y) = 0$

$$0 = \sin pl\,(C'e^{py} + D'e^{-py})$$

$$\Rightarrow \qquad \sin pl = 0 \text{ or } p = \frac{n\pi}{l},\ n \text{ being an integer.}$$

Also, using $u(x,\infty) = 0$ in (3), we get $C' = 0$

By (3), we get

$$u(x,y) = \sin\frac{n\pi x}{l} \cdot D e^{-n\pi y/l},\ n \text{ is an integer}$$

$\therefore$ General solution is of the form

$$u(x,y) = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi x}{l} e^{-n\pi y/l} \qquad ...(4)$$

Using, condition $u(x,0) = lx - x^2$, we get

$$lx - x^2 = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi x}{l}\ ,\ 0 < x < l$$

which is half range sine series

$$\therefore \qquad D_n = \frac{2}{l}\int_0^l (lx - x^2)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[\left\{lx\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi} - l\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right\}_0^l\right.$$

$$-\left(x^2\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi} - 2x\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right.$$

$$\left.\left. + 2\cos\frac{n\pi x}{l}\frac{l^3}{n^3\pi^3}\right)_0^l\right]$$

$$= \frac{2}{l}\left[\frac{-l^3}{n\pi}\cos n\pi + \frac{l^3}{n\pi}\cos n\pi - \frac{2l^3}{n^3\pi^3}\cos n\pi + \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{4l^2}{n^3\pi^3}\left[-(-1)^n + 1\right]$$

$$D_n = \begin{cases} \dfrac{8l^2}{n^3\pi^3}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

$$\therefore \qquad u(x,y) = \frac{8l^3}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin\frac{(2n-1)x\pi}{l}e^{(2n-1)\pi_y/l}$$

Q.3. (b) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers.

The probability of an accident involving a scoter driver, a car driver and a truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scoter driver. (5)

Ans. Let E_1, E_2, E_3 denote the events that an insured person at random is scooter, car and truck drivers. respectively.

Let H denote the event person met with an accident.

Then $P(E_1) = \frac{2000}{12000}, P(E_2) = \frac{4000}{12000} P(E_3) = \frac{6000}{12000}$

Probability of insured person met with an accident is scooter driver $P(H/E_1) = 0.01$

similarly $P(H/E_2) = 0.03$

$P(H/E_3) = 0.15$

By Baye's theorem, we have

$$P(E_1/H) = \frac{P(E_1)P(H/E_1)}{P(E_1)P(H)(E_1)+P(E_2)P(H/E_2)+P(E_3)P(H/E_3)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15}$$

$$= \frac{1}{6}\times\frac{6}{52} = \frac{1}{52} = 0.0192$$

Q.4. (a) Find the moment generating function of the distribution $f(x) = \frac{1}{C}e^{-x/c}, 0 \le x < \infty, c > 0$ about origin. Hence find its mean and standard deviation. (5)

Ans. $f(x) = \frac{1}{C}e^{-x/c} \quad 0 \le x < \infty, c > 0$

M.G.F (about origin) $= E[e^{tx}]$

$$= \int_0^{\infty} e^{tx} . f_X(x)\,dx$$

$$= \int_0^{\infty} e^{tx}\frac{1}{c}e^{-x/c}dx$$

$$= \frac{1}{C}\int_0^{\infty} e^{x(t-1/c)}dx$$

$$= \frac{1}{C}\left[\frac{e^{x(t-1/c)}}{t-1/c}\right]_0^{\infty}$$

$$M_X(t) = \frac{-1}{c\frac{(tc-1)}{C}} = \frac{-1}{tc-1} = -(tc-1)^{-1}$$

Now $\quad E[X] = \frac{d}{dt}$ M.G.F.

$$= (tc-1)^{-2}.C$$

Mean $\quad = E[X] = \frac{d}{dt}Mx(t)\Big]_{t=0}$

$$= \left[(tc-1)^{-2}c\right]_{t=0} = 0$$

$$E(X^2) = \frac{d^2}{dt^2}MX(t)\Big|_{t=0}$$

$$= -2c^2(tc-1)^{-3}\Big|_{t=0} = -2C^2$$

$$\text{var } X = E(X^2) - [E(X)]^2 = -2c^2 - c^2$$

$$= -3c^2$$

$$\text{S.D} = \sqrt{\text{var } X} = \sqrt{-3c^2} = c\sqrt{-3}$$

Q.4. (b) A manufacture of pins knowns that on an average 5% of his product is defative. He sells pins in boxes of 100 and gurantees that not more than 4 pins will be defective what is the probability that the box will fail to meet the guaranteed quality? ($e^{-5} = 0.0067$) (5)

Ans. Let X : no of defective pins

$X = p(\lambda)$

Let $\quad p =$ the probability that a pin is defective

$= 5\% = 0.05$

also $\quad n = 100$

$\Rightarrow \quad \lambda = np = 100 \times 0.05 = 5$

$$p.(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}, r = 0,1,2,...$$

The box will meet guarantee if it contains atmost 4 defective pins.

∴ Required probability $= p(X \le 4)$

$$= p[X=0] + p[X=1] + p[X=2] + p[X=3] + p[X=4]$$

$$= e^{-\lambda} + e^{-\lambda}.\lambda + e^{-\lambda}\frac{\lambda^2}{2i} + e^{-\lambda}\frac{\lambda^3}{3i} + e^{-\lambda}\frac{\lambda^4}{4i}$$

$$= e^{-5}\left[1+5+\frac{25}{2}+\frac{125}{6}+\frac{625}{24}\right]$$

$$= e^{-5}(6 + 12.5 + 20.83 + 26.04)$$

$$= e^{-5} \times 65.37 = 0.0067 \times 67.37$$

$$= 0.44$$

Box fails to meet guarantee

$$1 - 0.44 = 0.5619.$$

SECOND TERM EXAMINATION [APRIL 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 1.30 hrs. **M.M. : 30**

Note: *Attempt Q. no. 1 which is compulsory and any two more questions from remaining. All questions carry equal marks.*

Q. 1. (a) Prove that $r = \frac{\text{cov}(x,y)}{\sigma_x\sigma_y}$, where σr and σr are the S.D's of x and y-series respectvely and r is the correlation coefficient. **(2.5)**

Ans. As
$$r = \frac{\sum(x_i - y_i)(y_i - \bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2}}$$

$$= \frac{\frac{1}{n}\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\frac{1}{n}\sum(x_i-\bar{x})^2.\frac{1}{n}\sum(y_i-\bar{y})^2}}$$

As
$$\sigma_x = \sqrt{\frac{1}{n}\sum(x_i-\bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{1}{n}\sum(y_i-\bar{y})^2}$$

$$x = \frac{\text{cov}(x,y)}{\sigma_x\,\sigma_y}$$

Q.1. (b) A random sample of 900 measurem ents from a large population gave a mean value of 64 if this sample has been drawn from a normal population with standard deviation of 20, find the 95% confidence limits for the mean in the population. **(2.5)**

Ans. Here
$$n = 900, \mu = 64, \sigma = 20$$

Test statistic is
$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

At 5% level of significance
$$|Z\alpha| = 1.96$$

95% confidence limit is
$$\bar{x} \pm 1.96\,\sigma/\sqrt{n}$$

$$= 64 \pm 1.96 \times 20/\sqrt{900}$$

$$= 64 \pm 1.96 \times \frac{20}{30} = 64 \pm 1.3066$$

$$= 62.693 < \mu < 65.3066$$

Q.1. (c) A toy company manufactures two types of toys, type A and type B. Each toy of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 toys per day. The supply of plastic is sufficient to produce 1500 toys per day (both A and B combined). Type B toy requires a dress of which there are only 600 per day available. If the company makes a profit of Rs 3 and Rs 5 per toy respectively on type A and B, then how many of each types of toy should be produced per day in order to maximize the total profit. Formulate this problem.

Ans. The key decision is to determine the production of toys of type A and type B respectively. Let x and y denote the number of toys of type A and type B respectively.

Since it is not possible to produce negative quantities of toys, feasible alternative are sets of value of x and y satisfying $x \geq 0, y \geq 0$.

The constraints are

$$x + 2y \leq 2000$$
$$x + y \leq 1500$$

Since only type B requires a dress of which there are only 600 available per day.

$$y \leq 600$$

profit on x toy of type A = Rs. 3x.

profit on y toy of type B = Rs 5y

Total profit on x toys of type A and y toys of type B. = Rs (3x + 5y).

So, A manufactures produce to maximize the profit.

$$z = 3x + 5y$$

mathematical formulation is

$$\text{Max } z = 3x + 5y$$

subject to constraints

$$x + 2y \leq 2000$$
$$x + y \leq 1500$$
$$y \leq 600$$
$$x, y \leq 0.$$

Q.1. (d) Write the dual of the following problem

$$\textbf{Max} z = 3x_1 + 2x_2$$

S.t. $x_1 - x_2 \leq 1, x_1 + x_2 \geq 3, x_1 \geq 0, x_2$ **is unrestricted in sign.**

Ans. Standard primal

Introducing the slack variable $s_1 \leq 0$ and surplus variable $S_2 \geq 0$ and $x_2 = x_2' - x_2''$

The standard L.P.P is

$$\text{Max } z = 3x_1 + 2\left(x_2' - x_2''\right) + 0.s_1 + 0.s_2$$

s.t

$$x_1 - \left(x_2' - x_2''\right) + s_1 = 1$$

$$x_1 + \left(x_2' - x_2''\right) - s_2 = 3.$$

$$x_1 \geq 0, x_2' \geq 0, x_2'' \geq 0\ s_1 \geq 0, s_2 \geq 0.$$

Dual let w_1, w_2 be the dual variables. corresponding to the primal constraints. Then the dual problem will be

$$\text{Min } Z^* = w_1 + 3w_2.$$

subject to constraints

$$w_1 + w_2 \geq 3.$$

$$\left.\begin{aligned} -w_1 + w_2 &\geq 2. \\ w_1 - w_2 &\geq -2 \end{aligned}\right\} w_1 - w_2 = 2$$

$$w_1 + 0.w_2 \geq 0$$
$$0.w_1 - w_2 \geq 0$$
$$w_1 \geq 0, -w_2 \geq 0 \Rightarrow w_2 \leq 0.$$

it is re-written as

minimum $\quad z^* = w_1 + 3w_2$

s.t $\quad w_1 + w_2 \geq 3$

$$w_1 - w_2 = 2$$
$$w_1 \geq 0; w_2 \leq 0.$$

Q.2. (a) By the method of least squares, fit a parabola from the following data. (5)

x :	1	2	3	4	5
y :	2	6	4	5	2

Ans. To find equation of the from

$$y = a + bX + cX^2$$

By least square, normal equations are

$$\sum Y = na + b\sum X + c\sum X^2 \quad \text{...(A)}$$

$$\sum XY = a\sum X + b\sum X^2 + c\sum X^3 \quad \text{...(B)}$$

$$\sum X^2Y = a\sum X^2 + b\sum X^3 + c\sum X^4 \quad \text{...(C)}$$

x	y	x^2	xy	x^2y	x^3	x^4
1	2	1	2	2	1	1
2	6	4	12	24	8	16
3	4	9	12	36	27	81
4	5	16	20	80	64	256
5	2	25	10	50	125	625

$$\sum x = 15, \sum y = 19, \sum x^2 = 55, \sum xy = 56,$$

$$\sum x^2y = 192, \sum x^3 = 225, \sum x^4 = 979, n = 5$$

Equation (A), (B), (C) become

$$19 = 5a + 15b + 55c$$
$$56 = 15a + 55b + 225c$$
$$192 = 55a + 225b + 979c.$$

Multiply (1) by 3. and subtract from (2)

$$57 = 15a + 45b + 165c$$
$$56 = 15a + 55b + 225c$$
$$1 = -10b - 60c.$$

$\Rightarrow$ $$10b + 60c = -1$$

Multiply (2) by 11 and (3) by 3 and subtract

$$616 = 165a + 605b + 2475c$$
$$576 = 165a + 675b + 2937c$$
$$40 = -70b - 462c$$

$\Rightarrow$ $$70b + 462c = -40$$

Multiply (4) by 7

$$70b + 420c = -7$$
$$70b + 462c = -40$$
$$-42c = 33$$

$$c = \frac{33}{42} = -0.785$$

from equation (4), $10b + 60(-0.785) = -1$

$$10b = -1 + 47.1$$

$$b = \frac{46.1}{10} = 4.61$$

By equation (1), $19 = 5a + 15 \times 4.61 + 55(-0.785)$

$$19 = 5a + 69.15 - 43.175$$

$\Rightarrow$ $$5a = -6.975$$

$\Rightarrow$ $$a = -1.395$$

equation is $$y = -1.395 + 4.61x - 0.785x^2.$$

Q.2. (b) The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the regression coefficients byx, bxy and the correlation coefficient r. Also, find the standard deviation of y, if the variance of x is 4. (5)

Ans. Let regression equation of y on x is

$$4x + 3y = -7$$

$$y = \frac{-7}{3} - \frac{4}{3}x$$

$\Rightarrow$ Regression coefficient of y on x is $= byx = -\dfrac{4}{3}$

$\Rightarrow$ Regression equation of x on y is

$$3x + 4y + 8 = 0$$

$\Rightarrow$ $$x = \frac{-8}{8} - \frac{4}{3}y$$

$\Rightarrow$ Regression coefficient of x on $y = bxy = \dfrac{-4}{3}$

Now $$r^2 = byx.\,bxy = \frac{-4}{3} \times \frac{-4}{3} = \frac{16}{9}.$$

But $$r^2 \le 1 \;\therefore$$

Let regression equation of y on x is

$$3x + 4y + 8 = 0 \Rightarrow y = \frac{-8}{4} - \frac{3}{4}x$$

$\therefore$ $$byx = \frac{-3}{4}$$

and regression equation of x on y is

$$4x + 3y + 7 = 0$$

$\Rightarrow$ $$x = \frac{-7}{4} - \frac{3}{4}y$$

$\therefore$ $$b_{xy} = \frac{-3}{4}$$

Now $$r^2 = b_{yx}.\,b_{xy} = \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right) = \frac{9}{16}$$

$$r = \pm 0.75$$

As byx and bxy are negative $$r = -0.75$$

$\therefore$ Regression coefficient of y on x

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{-3}{4},\ \sigma_x = 2$$

$\Rightarrow$ $$-0.75 \times \frac{\sigma y}{2} = \frac{-3}{4}$$

$\Rightarrow$ $$\sigma_y = \frac{3}{2 \times 0.75} = 2$$

Q.3. (a) A random sample of 10 boys had the following *IQ*: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of population mean IQ of 100 at 5% level of significance?

(Given $t_{0.05} = 2.26$ for 9 d.f, $t_{0.05} = 2.23$ for 10 *d.f*, $t_{0.05} = 2.20$ for 11 d.f) (5)

Ans. we are testing

$$H_0: \mu = 100 \ \sqrt{s}\ : H_1 : \mu_1 \ne 100$$

Under $H_0: \dfrac{\bar{X}-\mu}{s1\sqrt{n}} - t_{n-1}$

$$\bar{X} = \frac{\sum x_i}{n} = \frac{972}{10} = 97.2$$

$$s = \sqrt{\frac{1}{n-1}\left(\sum x_i^2 - n\bar{x}^2\right)}$$

$$\sum x_i^2 = 4900 + 14400 + 12100 + 10201 + 7744 + 6889 + 9025 + 9604 + 11449 + 10000 = 96312$$

$$s = \sqrt{\frac{1}{9}(96312 - 10 \times (97.2)^2}$$

$$= \sqrt{\frac{1}{9}(96312 - 94478.4)} = \sqrt{\frac{1833.6}{9}}$$

$$= \sqrt{203.733} = 14.27$$

Test statistic is $\dfrac{97.2-100}{14.27/\sqrt{10}} = \dfrac{-2.8}{4.51257}$

$= -0.62$

$\therefore \quad t = 0.62$

Value of t at 5% level with 9 d.f = 2.26

since calculated value 0.62 < tabulated value, we accept H_0 at 5% level of significance.

Q.3. (b) Solve the following L.P.P max $z = -2x_1 - x_3$ s.t. $x_1 + x_2 - x_3 \geq 5$, $x_1 - 2x_2 + 4x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$ (5)

Ans. $\text{Max } z = -2x_1 - x_3.$

s.t.

$$x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

Introducing slack variable x_4, x_5.

The given L.P.P can be re-written as

$$\text{Max } z = -2x_1 - x_3 + 0.x_4 + 0.x_5$$

s.t.

$$x_1 + x_2 - x_3 - x_4 = 5$$
$$x_1 - 2x_2 + 4x_3 - x_5 = 8.$$

Let us add artifical variables x_6, x_7

So, our L.PP becomes

$$\text{Max } z = -2x_1 - x_3 + 0.x_4 + 0.x_5 - Mx_6 - Mx_7$$

s.t.

$$x_1 + x_2 - x_3 - x_4 + x_6 = 5$$
$$x_1 - 2x_2 + 4x_3 - x_5 + x_7 = 8$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0.$$

Starting table

		$C_j \rightarrow$	-2	0	-1	0	0	$-M$	$-M$
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-M$	x_6	5	1	1	-1	-1	0	1	0
$-M$	$\leftarrow x_7$	8	1	-2	$\boxed{4}$	0	-1	0	1
		$z_j - c_j$	$-2M+2$	M	$-3M+1$ ↑	M	M	0	0

x_3 enters the bases.

$$\min\left\{\frac{xB_i}{x_{i3}}, x_{i3} > 0\right\} = \min\left\{\frac{8}{4}\right\} = 2$$

$\Rightarrow x_7$ leaves the basis

First iteration

			-2	0	-1	0	0	$-M.$
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6
$-M$	$\leftarrow x_6$	7	$\boxed{5/4}$	1/2	0	-1	$-1/4$	1
-1	x_3	2	1/4	$-1/2$	1	0	$-1/4$	0
		$z_j - c_j$	$\frac{-5}{4}M + \frac{7}{4}$ ↑	$-\frac{M}{2} + \frac{1}{2}$	0	M	$\frac{M}{4} + \frac{1}{4}$	0

x_1 enters the basis (most negative)

$$\min\left\{\frac{28}{5}, 8\right\} = \frac{28}{5}.$$

$\therefore x_6$ leave the basis

Second iteration

			-2	0	-1	0	0
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5
-2	$\leftarrow x_1$	28/5	1	$\boxed{2/5}$	0	$-4/5$	$-1/5$
-1	x_3	3/5	0	$-3/5$	1	1/5	$-1/5$
		$z_j - c_j$	0	$-\frac{1}{5}$ ↑	0	$\frac{7}{5}$	$\frac{3}{5}$

Most negative is $x_2 \therefore x_2$ enters the basis

$$\min\{14\} = 14$$

$\therefore x_1$ leaves the basis

Third iteration

			-2	0	-1	0	0
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5
-2	x_2	14	5/2	1	0	-2	$-1/2$
-1	x_3	9	3/2	0	1	-1	$-1/2$
		$z_j - c_j$	1/2	0	0	1	1/2

Since all $z_j - c_j \geq 0$.

$\therefore$ The solution is optimal solution.

$$x_2 = 14, x_1 = 0, x_3 = 9$$

$$\text{Max}\, z = -2 \times 0 - 9 = -9$$

Q.4. (*a*) A car hire company has one car at each of five depots a, b, c, d and e. A customer requires a car in each town, namel A, B, C, D and E. Distances (in kms) between depots (origin) as towns (destinations) are given in the following distance matrix:

	a	*b*	*c*	*d*	*e*
A	**160**	**130**	**175**	**190**	**200**
B	**135**	**120**	**130**	**160**	**175**
C	**140**	**110**	**155**	**170**	**185**
D	**50**	**50**	**80**	**80**	**110**
E	**55**	**35**	**70**	**80**	**105**

How should cars be assigned to customers so as to minimize the distance travelled?

Ans. Here, number of tasks and number of sub-ordinates each equal 4, therefore problem is balanced.

Subtracting smallest element of each row from every element of corres ponding row, reduced matrix is

	a	b	c	d	e
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Subtracting smallest element of each column from every element of corresponding column, reduced matrix is

30	$\boxed{0}$	35	30	15	√
15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	
30	$\cancel{0}$	35	30	20	√
$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5	
20	$\cancel{0}$	25	15	15	√

number of assignments (□) is less than n (order of cost matrix), $\therefore$ optimum solution is not achieved

Mark rows having no assegned zero Mark columns that have zeros in marked rows

Draw lines through all unmarked rows and marked columns.

Revised cost matrix is

min element from reduced matrix is 15.

15	0	20	15	0
15	15	0	10	0
30	15	35	30	20
0	15	20	0	5
5	0	10	0	0

Repeat above process

Reduced matrix is

15	$\cancel{0}$	20	15	$\boxed{0}$
15	15	$\boxed{0}$	10	$\cancel{0}$
15	$\boxed{0}$	20	15	5
$\boxed{0}$	15	20	$\cancel{0}$	5
5	$\cancel{0}$	10	$\boxed{0}$	$\cancel{0}$

Since assignment is equal to n, therefore optimum solution is achieved.

$$A \to e, B \to c, C \to b, D \to a, E \to d$$

Now minimum assignment schedule is

200 + 130 + 110 + 50 + 80 = 570

Q.4. (*b*) Find the initial basic feasible solution the following transportation problem by VAM

From/To	W_1	W_2	W_3	W_4	**Supply**
F_1	11	20	7	8	50
F_2	21	16	10	12	40
F_3	8	12	18	9	70
Demand	30	25	35	40	

Ans. Here demand is 130 and supply is 160.

Since total demand ≠ total supply, we introduce a dummy column with its demand 30.

The transportation table initial *b.f.s* is given as

	W_1	W_2	W_3	W_4	W_5	
F_1	11	20	7	8	0	50 (7)
F_2	21	16	10	12	$\boxed{30}$ 0	40 (10)
F_3	8	12	18	9	0	70 (8)
	30 (3)	25 (4)	35 (3)	40 (1)	30 (0)	

Differences between the smallest and next to smallest in each row and column are given in parenthesis.

Largest of these differences is (10), associated with 2nd row. of the table.

Since least cost is 0 in 2nd row, we allocate

$$x_{25} = \min(40,30) = 30$$

Exhaust 5th column. Reduced table is

	W_1	W_2	W_3	W_4	
F_1	11	20	7	8	50 (1)
F_2	21	16	10	12	10 (2)
F_3	8	$\boxed{25}$ 12	18	9	70 (1)
	30 (3)	25 (4)	35 (3)	40 (1)	

Largest difference is (4), associated with 2nd column.

Least cost is 12, allocate

$$x_{32} = \min(70, 25) = 25$$

Exhaust 2nd column. Reduced table is

	W_1	W_3	W_4	
F_1	11	$\boxed{35}$ 7	8	50 (1)
F_2	21	10	12	10 (2)
F_3	8	18	9	45 (1)
	30 (3)	35 (3)	40 (1)	

Largest difference is (3) associated with 3 column

Least cost is 7, allocate

$$x_{13} = \min(50, 35) = 35.$$

Exhaust 3rd column. Reduced matrix is

11	8	15 (3)
21	$\boxed{10}$ 12	10 (9)
8	9	45 (1)
30 (3)	40 (1)	

Largest difference is (9), associated with 2nd row

Least cost is 12, allocate

$$x_{24} = \min(10, 40) = 10$$

Exhaust 2nd row. Reduced matrix is

11	8	15 (3)
$\boxed{30}$ 8	9	45 (1)
30 (3)	30 (1)	

Largest difference is (3), associated with 1st column Least cost is 8, allocate

$$x_{31} = \min(45, 30) = 30$$

Exhaust 1st column. Reduced matrix is

$\boxed{15}$ 8	15
$\boxed{15}$ 9	15
30	

Now the table is

	W_1	W_2	W_3	W_4	W_5
F_1	11	20	$\boxed{35}$ 7	$\boxed{15}$ 8	0
F_2	21	16	10	$\boxed{10}$ 12	$\boxed{30}$ 0
F_3	$\boxed{30}$ 8	$\boxed{25}$ 12	18	$\boxed{15}$ 9	0

No. of allocated cells is 7, which is same as required $(3 + 5 - 1) = 7$

$\therefore$ Solution is non-degenrate basic feasible

Total cost $= 35 \times 7 + 15 \times 8 + 10 \times 12 + 30 \times 0 + 30 \times 8 + 25 \times 12 + 15 \times 9$

$= 1160$

END TERM EXAMINATION [MAY 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 3 hrs. M.M. : 75

Note: Attempt any five questions including Q. no. 1 which is complusory. Select one question from each unit.

Q.1. (a) Find a particular integral of $(Dx^2 + 3Dx\,Dy + Dx + Dy^2 - 2)\,z = 3x + 4y + y - 2xy$. (7)

Ans. Given quation is

$$(D+D'-1)(D'+2D'+2)z = e^{3x+4y}+(y-2xy)$$

For $(D+D'-1)\ b = 1, a = -1, c = 1$

For $(D+2D'+2)\ b = 1, a = -2, c = -2$

Now, $C.F = e^{x}f_1(y-x)+e^{-2x}f_2(y-2xy)$

$$PI = \frac{1}{(D+D'-1)(D+2D'+2)}e^{3x+4y}+y-2xy$$

Consider
$$\frac{1}{(D+D'-1)(D+2D'+2)}e^{3x+4y}$$

$$= \frac{1}{(3+4-1)(3+8+2)}e^{3x+4y} = \frac{1}{78}e^{3x+4y}$$

and
$$\frac{1}{(D+D'-1)(D+2D'+2)}(y-2xy)$$

$$= \frac{-1}{2[1-(D+D')]\left[1+\left(\frac{D}{2}+D^{1}\right)\right]}(y-2xy)$$

$$= \frac{-1}{2}[1-(D+D')]^{-1}\left[1+\left(\frac{D}{2}+D'\right)\right]^{-1}(y-2xy)$$

$$= \frac{-1}{2}[1+(D+D')]+2DD'\left[1-\frac{D}{2}-D'+DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[1-\frac{D}{2}-D'+DD'+D+D'+2DD'-\frac{DD'}{2}-DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[1+\frac{D}{2}+\frac{3}{2}DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[y-2xy+\frac{1}{2}(-2y)+\frac{3}{2}D(1-2x)\right]$$

$$= \frac{-1}{2}[y-2xy-y-3] = \frac{3}{2}+xy$$

$$P.I = \frac{1}{78}e^{3x+4y}+xy+\frac{3}{2}$$

∴ Complete solution is

$$Z = C.F + P.I$$

$$Z = e^{x}f_1(y-x)+e^{-2x}-f_x(y-2x)+\frac{1}{78}e^{3x+4y}+xy+\frac{3}{2}.$$

Q.1. (b) A die is tossed thrice Getting 2 or 4 on the die in a toss is success. Find the mean and variance of number of success. (6)

Ans. Let X denote the number of success.

i.e. $X = 0,1,2,3$

Probability of success $= \frac{2}{6} = \frac{1}{3}$

Probability of failure $= 1-\frac{1}{3} = \frac{2}{3}$.

Mean of success $= \sum f_i x_i$

Now, distribution table is

$$P(X=0) = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$$

$$P(X=1) = {}^{3}C_{1}\times\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{3!}{2!}\times\frac{4}{27}$$

$$= \frac{12}{27}$$

$$P(X=2) = {}^{3}C_{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3} = \frac{3!}{2!}\times\frac{2}{27} = \frac{6}{27}$$

$$P(X=3) = \frac{1}{27}$$

xi	fi	fixi	xi²	fi xi²
0	8/27	0	0	0
1	12/27	12/27	1	12/27
2	6/27	12/27	4	24/27
3	1/27	3/27	9	9/27

Mean $X = \sum f_i x_i = 0 + \frac{12}{27}+\frac{12}{27}+\frac{3}{27}$

$= 1$

$$\text{var}\, X = \sum fixi^2 - \left(\sum fixi\right)^2$$

$$= \frac{12}{27} + \frac{24}{27} + \frac{9}{27} - 1 = \frac{45}{27} - 1 = \frac{18}{27} = \frac{2}{3}$$

Q.1. (c) Can $y = 5 + 2.8x$ and $x = 3 - 0.5y$ be the estimated regression equations of y on x and x or y respectively? Explain

Ans. Consider $\quad y = 5 + 2.8x$

Here $\quad b_{yx} = 2.8$

and $\quad x = 3 - 0.5y \Rightarrow b_{xy} = -0.5$

Now $\quad r^2 = -1.4$

$\Rightarrow \quad r =$ imaginary

its not feasible, r is imaginary.

$\therefore$ byx and bxy are not feasible.

Q.1. (d) Write the dual of following primal problem

$$\text{Max } Z = 3x_1 + 10x_2 + 2x_3$$

Subject to;

$$2x_1 + 3x_2 + 2x_3 \le 7$$
$$3x_1 - 2x_2 + 4x_3 = 3 \qquad \textbf{(7)}$$

Where $x_1, x_2, x_3 \ge 0$.

Prove that dual of the dual is primal.

Ans. To make it a standard linear form.

Introduce slack variable. S_1

standard form of LPP is

Max $\quad z = 3x_1 + 10x_2 + 2x_3 + 0.S_1$ subject to constraints

$$2x_1 + 3x_2 + 2x_3 + S_1 = 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1, x_2, x_3, S_1 \ge 0$$

dual is

Min $\quad Z^* = 7w_1 + 3w_2$. subject to constraints

$$2w_1 + 3w_2 \ge 3$$
$$3w_1 - 2w_2 \ge 10$$
$$2w_1 + 4w_2 \ge 2$$

$w_1 \ge 0$, w_2 unrestricted

w_1, w_2 are dual variables.

Introduce $S_1 \ge 0$ and $w_2 = w'_2 - w_2''$. $S_2 \ge 0, S_3 \ge 0$

standard form

$$\text{Min } Z^* = 7w_1 + 3(w'_2 - w_2'') + 0.S_1 + 0.S_2 + 0.S_3$$

s.t.
$$2w_1 + 3(w'_2 - w_2'') - S_1 = 3$$
$$3w_1 - 2(w'_2 - w_2'') - S_2 = 10$$
$$2w_1 + 4(w_2' - w_2'') - S_3 = 2$$
$$w_1, w_2', w_2'', S_1, S_2, S_3 \ge 0$$

Dual of dual is

$$\text{Max } Z^{**} = 3x_1 + 10x_2 + 2x_3$$

s.t.
$$2x_1 + 3x_2 + 2x_3 \le 7$$
$$\left.\begin{aligned} 3x_1 - 2x_2 + 4w_3 &\le 3. \\ -3x_1 + 2x_2 - 4w_3 &\le -3 \end{aligned}\right\}$$
$$-x_1 \le 0.$$
$$-x_2 \le 0.$$
$$-x_3 \le 0$$

Primal is

Max $\quad Z = 3x_1 + 10x_2 + 2x_3$

s.t.
$$2x_1 + 3x_2 + 2x_3 \le 7$$
$$3x_1 - 2x_2 + 4x_3 = 3.$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

Thus dual of dual is primal.

UNIT-I

Q.2. (a) Find the general solution of

$(D^3 - 4D^2 D' + 4DD'^2)\, z = \cos(2x + 3y)$ **(6)**

Ans. Consider $\quad D^3 - 4D^2 D' + 4DD'^2 = 0$

A.E. $\quad m^3 - 4m^2 + 4m = 0$

$\Rightarrow \quad m(m^2 - 4m + 4) = 0$

$\Rightarrow \quad m = 0, (m-2)^2 = 0$

$\Rightarrow \quad m = 0, 2, 2$

$$\text{C.F} = f_1(y) + f_2(y + 2x) + xf_3(y + 2x)$$

$$\text{P.I.} = \frac{1}{D^3 - 4D^2D' + 4DD'^2}\cos(2x + 3y)$$

Replace D by 2 and D' by 3

$$= \frac{1}{2^3 - 4\times4\times3 + 4\times2\times9}\iiint \cos u\, du\, du\, du$$

where $\quad u = 2x + 3y$

$$= \frac{1}{8 - 48 + 72}(-\sin u) = \frac{-\sin(2x + 3y)}{32}$$

General solution is

$$Z = f_1(y) + f_2(y + 2x) + xf_3(y + 2x) - \frac{\sin(2x + 3y)}{32}$$

Q.2. (b) Find the complete solution of the equation

$(D^2 + D'^2 + 2DD' + 2D + 2D' + 1)\, z = e^{2x+y}$ **(6.5)**

Ans. Consider. $\quad (D^2 + D'^2 + 2DD' + 2D + 2D' + 1)\, z.$

$\Rightarrow \quad [(D + D')^2 + 2(D + D') + 1]\, z$

A.E. $\quad (D + D')^2 + 2(D + D') + 1 = 0$

$\Rightarrow \quad (D + D' + 1)^2 = 0.$

Here $\quad b = 1, a = -1, c = -1$

$$C.F. = e^{-x}[\Phi_1(y-x) + x\Phi_2(y-x)]$$

$$P.I. = \frac{1}{(D+D'+1)^2}e^{2x+y}$$

Replace D by 2, D′ by 1

$$\Rightarrow \quad \frac{1}{(2+1+1)^2}e^{2x+y} = \frac{1}{16}e^{2x+y}.$$

Complete solution is

$$z = e^{-x}(\Phi_1(y-x) + x\Phi_2(y-x)] + \frac{1}{16}e^{2x+y}$$

Q.3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$, where $u(0,y) = 0$ and $\frac{\partial u}{\partial x}(0, y) = e^{-3y}$ for all y using the method of separation of variables.

Ans. Given equation $\frac{\partial^2 u}{\partial x^2} - 2\cdot\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

Let $\qquad U = X(x)Y(y)$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = X'Y, \frac{\partial u}{\partial y} = XY', \frac{\partial^2 u}{\partial x^2} = X''Y.$$

equation becomes,

$$X''Y - 2X'Y + XY' = 0.$$

$$\Rightarrow \quad XY' = Y(2X' - X'')$$

$$\Rightarrow \quad -\frac{Y'}{Y} = \frac{-2X' + X''}{X} = K \quad \left[\begin{array}{l}\text{As } x \text{ and } y \text{ are}\\ \text{independent variables}\end{array}\right]$$

$$\Rightarrow \quad Y' + KY = 0, X'' - 2X' - KX = 0$$

$$\Rightarrow \quad Y' = -KY, \text{ put } \frac{d}{dx} = m$$

$$\Rightarrow \quad \frac{dY}{dY} = -KY, \Rightarrow (m^2 - 2m - K)X = 0.$$

$$\text{, A.E } m^2 - 2m - K = 0$$

$$\Rightarrow \quad \frac{dY}{Y} = -K\,dy \Rightarrow m = \frac{2 \pm \sqrt{4+4k}}{2}$$

$$\log Y = -Ky + \log c_1, m = 1 \pm \sqrt{1+K}$$

$$\Rightarrow \quad Y = c_1 e^{-Ky}, X = c_2 e^{(1+\sqrt{1+K})x} + c_3 e^{(\sqrt{1+K})}$$

$$\Rightarrow \quad u(x, y) = c_1 e^{-Ky}\left(c_2 e^{1+\sqrt{1+K}x} + c_3 e^{(1-\sqrt{1+K})x}\right)$$

$$\Rightarrow \quad u(x,y) = \left(Ae^{1+\sqrt{1+kx}} + Be^{1-\sqrt{1+kx}}\right)e^{-ky} \quad ...(1)$$

for $\qquad x = 0., y = y$, we get

$$0 = (A+B)e^{-Ky}$$

$$\Rightarrow \quad A + B = 0 \quad (2)$$

Now by (1), we get

$$\frac{\partial u}{\partial x} = e^{-Ky}\left[(1+\sqrt{1+K})Ae^{1+\sqrt{1+Kx}} + (1-\sqrt{1+K})Be^{(1-\sqrt{1+K})x}\right]$$

for $\qquad x = 0, y = y$

$$e^{-3y} = \left[1+\sqrt{1+K}A + 1 - \sqrt{1+K}B\right]e^{-Ky}$$

On comparing

$$e^{-Ky} = e^{-3y}$$

$$\Rightarrow \quad K = 3$$

Now, $\qquad 3A - B = 1$

and $\qquad A + B = 0$

$$\Rightarrow \quad A = -B$$

$$\Rightarrow \quad -3B - B = 1$$

$$\Rightarrow \quad B = -1/4$$

$$\Rightarrow \quad A = 1/4.$$

Now

$$u = \frac{1}{4}\left[e^{3x} - e^{-x}\right]e^{-3y}.$$

Q.3. (b) A long rectangular plate of width π cm with insulated surfaces has its temperature equal to zero on both the long sides and one of the short side so that $u(0, y) = 0$, $u(\pi, y) = 0$, $u(x, \infty) = 0$ and $u(x, 0) = kx$. find the steady state temperature within the plate. (6.5)

Ans.

Consider the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ...(1)$$

$$\left.\begin{array}{l} u(0,y) = 0, 0 < y < \infty \\ u(\pi,y) = 0, 0 < y < \infty \end{array}\right\} B.C.$$

$$u(x,\infty) = 0, 0 < x < \pi$$

$$u(x,\infty) = kx, 0 < x < \pi$$

General solutions of (1) are

$$u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py) \quad ...(2)$$

$$u(x,y) = (c_1 \cos px + c_2 \sin px)(c_3 e^{py} + c_4 e^{-py}) \quad ...(3)$$

$$u(x,y) = (c_1 x + c_2)(c_3 y + c_4) - (4)$$

As solution (2) does not satisfy the boundary conditions for all values of y. Also (4) does not satisfy.

Thus only possible solution is (3), *i.e.* of the form

$$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py}) \quad ...(5)$$

for $u(0,y) = 0$

$\Rightarrow$ $0 = A[Ce^{py} + De^{-py}]$

$\Rightarrow$ $A = 0$

(5) reduces to

$$u(x,y) = B\sin px\,[Ce^{py} + De^{-py}]$$

$$u(x,y) = \sin px\,[C'e^{py} + D'e^{-py}] \quad ...(6)$$

for $u(\pi,y) = 0$

$$\sin p\pi\,(C'e^{py} + D'e^{-py}) = 0.$$

$\Rightarrow$ $\sin p\pi = 0$

$\Rightarrow$ $p\pi = n\pi \Rightarrow p = n$, n being on integer

and $u(x,\infty) = 0.$

$\Rightarrow$ $c' = 0$

$\therefore$ Reduced solution (6) is,

$$u(x,y) = D'\sin nx\,e^{-ny}, n \text{ being an integer.}$$

Adding all solution, we get

$$u = \sum D_n \sin nx e^{-ny} \quad ...(7)$$

Given $u(x,0) = kx.$

$$kx = \sum D_n \sin nx$$

This is half range sine series in interval $(0,\pi)$.

$$D_n = \frac{2}{\pi}\int_0^\pi kx\sin nx\,dx$$

$$= \frac{2k}{\pi}\left[\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^\pi$$

$$= \frac{2k}{\pi}\left[-\frac{\pi}{n}\cos n\pi\right] = \frac{-2k}{n}(-1)^n$$

$$= \frac{2k}{n}(-1)^{n+1}$$

Thus (7) reduces to

$$u(x,y) = 2k\sum\frac{(-1)^{n+1}}{n}\sin nx e^{-ny}.$$

UNIT-II

Q.4. (a) In a bolt factory there are four machines *A, B, C* and *D* manufacturing 20%, 15%, 25% and 40% of the total output respectively. of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen randomly from the factory's production and is found defective What is the probability that the bolt was manufactured by machine *A* or *D* (6)

Ans. Let

B_1 : bolt manufactured by machine A

B_2 : bolt manufactured by machine B

B_3 : bolt manufactured by machine C

B_4 : bolt manufactured by machine D.

E : bolt is defective.

Then

$P(B_1) = 0.20$, $P(E/B_1) = 0.05$

$P(B_2) = 0.15$, $P(E/B_2) = 0.04$

$P(B_3) = 0.25$, $P(E/B_3) = 0.03$

$P(B_4) = 0.4$, $P(E/B_4) = 0.02$

To find $P(B_1/E)$ and $P(B_4/E)$

By Baye's theorem

$$P(B_1/E) = \frac{P(B_1)P(E/B_1)}{\sum P(B_i)P(E/B_i)}$$

$$= \frac{0.20\times0.05}{0.20\times0.05+0.15\times0.04+0.25\times0.03+0.4\times0.02}$$

$$= \frac{0.01}{0.01+0.006+0.0075+0.008}$$

$$= \frac{0.01}{0.0315} = 0.3174$$

$$P(B_4/E) = \frac{P(B_4)P(E/B_4)}{\sum P(B_i)P(E/B_i)}$$

$$= \frac{0.4\times0.02}{0.20\times0.05+0.15\times0.04+0.25\times0.03+0.4\times0.02}$$

$$= \frac{0.008}{0.0315} = 0.253$$

Q.4. (b) Calculate the first four moments for the following frequency distribution about the mean and explain the skewness and kurtosis of the frequency distribution (6.5)

X	:	–4	–3	–2	–1	0	1	2	3	4
f	:	3	4	5	7	12	7	5	4	3

Ans. Let $\mu_1, \mu_2, \mu_3, \mu_4$, are first four moments about the mean. Then by def.

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Where, $\mu_1', \mu_2', \mu_3', \mu_4'$, are first four moments about any point. Consider the table.

Let $A = 0$.

x_i	fi	$fixi$	$fixi^2$	$fixi^3$	$fixi^4$
–4	3	–12	48	–192	768
–3	4	–12	36	–108	324
–2	5	–10	20	–40	80
–1	7	–7	7	–7	7
0	12	0	0	0	0
1	7	7	7	7	7
2	5	10	20	40	80
3	4	12	36	108	324
4	3	12	48	192	768

$$N_1 = \sum fi = 50, \sum fixi = 0, \sum fixi^2 = 222,$$

$$\sum fi\, xi^3 = 0, \sum fixi^4 = 2358,$$

Now,

$$\mu_1' = \frac{1}{N}\sum fi\, xi = \frac{1}{9}\times 0 = 0$$

$$\mu_2' = \frac{1}{N}\sum fi\, xi^2 = \frac{1}{50}\times 222 = \frac{222}{50}$$

$$\mu_3' = \frac{1}{N}\int fixi^3 = \frac{1}{50}\times 0 = 0$$

$$\mu_4' = \frac{1}{N}\sum fi\, xi^4 = \frac{1}{50}\times 2358 = 262$$

Thus, we get

$$\mu_1 = 0$$

$$\mu_2 = \frac{222}{50} - 0 = 4$$

$$\mu_3 = 0 - 3\times\frac{222}{50}\times 0 + 2\times 0 = 0$$

$$\mu_4 = 262 - 4\times 0\times 0 + 6\times\frac{222}{50}\times 0 - 3\times 0$$

$$= 262.$$

Now

$$r_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{0}{\sqrt{4^3}} = 0$$

$$r_2 = \beta_2 - 3 = \frac{\mu 4}{\mu_2^2} - 3 = \frac{262}{4^2} - 3$$

$$= \frac{214}{16} = 13.3$$

$\sigma_1 = 0$, distribution is symmetrical

Q.5. (*a*) Find mean, variance and moment generating function of $f(x)$, where

$$f(x) = \begin{cases} ae^{-ax}, x > 0 \\ 0, x \le 0 \end{cases}$$

(6)

Ans. Given

$$f(x) = \begin{cases} ae^{-ax}, x > 0 \\ 0, x \le 0 \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)dx = \int_0^{\infty} x a e^{-ax} dx$$

$$= a\left[x.\frac{e^{-ax}}{-a}\Big]_0^{\infty} - \int_0^{\infty} -ae^{-ax}dx\right]$$

$$= a\left[\frac{1}{a}\frac{e^{-ax}}{-a}\Big|_0^{\infty}\right] = \frac{-1}{a}(0-1) = \frac{1}{a}$$

$$\text{var} \quad X = \int_{-\infty}^{\infty}(x-\mu)^2 fX(x)dx$$

$$= \int_{-\infty}^{\infty} x^2 f_X(x)dx - [E(X)]^2$$

$$= \int_0^{\infty} x^2 e^{-ax}dx - \frac{1}{a^2}$$

$$= a\left[x^2\frac{e^{-ax}}{-a}\Big|_0^{\infty} - \int_0^{\infty}\frac{2xe^{-ax}}{-a}dx\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\int_0^{\infty} xe^{-ax}dx\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\left|x\frac{e^{-ax}}{-a} - \frac{e^{-ax}}{a^2}\right|_0^{\infty}\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\left(\frac{1}{a^2}\right)\right] - \frac{1}{a^2} = \frac{2}{a^2} - \frac{1}{a^2} = \frac{1}{a^2}$$

M.G.F

$$= \int_{-\infty}^{\infty} e^{tx} f_X(x)dx$$

$$= \int_0^{\infty} e^{tx} a e^{-ax}dx = a\int_0^{\infty} e^{(t-a)x}dx$$

$$= a\left|\frac{e^{(t-a)x}}{t-a}\right|_0^{\infty}$$

$$= a\left(0-\frac{1}{t-a}\right)=\frac{a}{a-t}.$$

Q.5. (b) If the probability that an individual suffers to a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individuals

(i) exactly 3 (ii) more than 2 **(6.5)**

individual will suffer to a bad reaction

Ans. Let 'p' be the probability of success suffering from bad reaction = 0.001

As $n = 2000$ and X be random variable. n is very large and p is small

$\therefore$ $$X \equiv p(\lambda)$$

Here $$\lambda = np = 2000 \times 0.001 = 2$$

Now $$f_X(x) = \frac{e^{-2}(2)^x}{x!}$$

(i) $$P[X=3] = \frac{e^{-2}2^3}{3!} = \frac{0.1353\times 8}{6}$$

$$= 0.1804$$

(ii) $$P[X>2] = 1 - P[X\le 2]$$

$$= 1-[f_X(0)+f_X(1)+f_X(2)]$$

$$= 1-\left[e^{-2}+e^{-2}.2+\frac{e^{-2}2^2}{2!}\right]$$

$$= 1-e^{-2}[1+2+2] = 1-5e^{-2}$$

$$= 1-0.6765 = 0.3235$$

UNIT-III

Q.6. (a) The two regression equation of the variable x and y are $8x-10y+66=0$ and $40x-18y=214$ given that variance of $x=9$. Find **(6)**

(i) Mean of x and y

(ii) The standard deviation of y and

(iii) The coefficient of correlation between x and y.

Ans. Consider $8x-10y+66=0$., $40x-18y-214=0$

(i) As mean values of given series, satisfy the regression lines

$$8\bar{x}-10\bar{y}+66=0$$

$$40\bar{x}-18\bar{y}-214=0$$

On solving them

$$4\bar{x}-5\bar{y}+33=0 \quad ...(1)$$

$$20\bar{x}-9\bar{y}-107=0 \quad ...(2)$$

Multiply (1) by 5 and subtract

$$20\bar{x}-25\bar{y}+165=0$$
$$20\bar{x}-9\bar{y}-107=0$$
$$-16\bar{y}+272=0$$

$\Rightarrow$ $$16\bar{y}=272=0 \quad \Rightarrow \bar{y}=17$$

Now. using (1) $4\bar{x}-85+33=0$

$\Rightarrow$ $$4\bar{x}=52$$

$\Rightarrow$ $$\bar{x}=13$$

(iii) Let $10y = 8x+66$ be regression equation of y on x.

$\Rightarrow$ $$y=\frac{4}{5}x+\frac{33}{5}$$

$\therefore$ $$byx = 4/5$$

and let $40x = 18y + 214$ be regression equation of x and y.

$\Rightarrow$ $$x=\frac{9}{20}y+\frac{107}{20}.$$

$\therefore$ $$bxy = 9/20$$

As $$r^2 = bxy : byx$$

$\Rightarrow$ $$r^2=\frac{9}{20}\times\frac{4}{5}=\frac{9}{25}=0.36$$

$\Rightarrow$ $$r=\pm 0.6$$

As r have same sign as regression coefficients.

$\therefore$ $$r = 0.6$$

(ii) As given $$\sigma_x^2 = 9.$$

$\Rightarrow$ $$\sigma_x = 3.$$

Now $$bxy=\frac{r\sigma_x}{\sigma_y}$$

$\Rightarrow$ $$\frac{9}{20}=\frac{0.6\times 3}{\sigma_y}\Rightarrow \sigma_y=\frac{0.6\times 3\times 20}{9}$$

$\Rightarrow$ $$\sigma_y = 4.$$

Q. 6. (b) The results of measurement of electric resistance R of a copper bar at various temperature t°C are listed below: (6.5)

t:	19	25	30	36	40	45	50
R:	76	77	79	80	82	83	85

if $R = a + bt$, find a and b.

Ans. $R = a + bt$

Normal equations are

$$\sum R = ma + b\sum t \qquad ...(1)$$

$$\sum Rt = a\sum t + b\sum t^2 \qquad ...(2)$$

t	R	Rt	t^2	
19	76	1444	361	
25	77	1925	625	
30	79	2370	900	
36	80	2880	1296,	m = 7
40	82	3280	1600	
45	83	3735	2025	
50	85	4250	2500	

$\sum t = 245, \sum R = 562, \sum Rt = 19884, \sum t^2 = 9307$

substituting values in (1) and (2)

$$562 = 7a + 245b \qquad ...(3)$$

$$19884 = 245a + 9307b \qquad ...(4)$$

Multiply (3) by 35 and subtract

$$19670 = 245a + 8575b$$

$$19884 = 245a + 9307b$$

$$-214 = -732b$$

$$b = 0.2923.$$

By equation (3), we get

$$562 = 7a + 245 \times 0.2923$$

$$\Rightarrow \quad 7a = 490.3865$$

$$\Rightarrow \quad a = 70.05$$

Q.7. (a) Write at least three important properties of regression coefficient and prove that if two variables are uncorrelated then the regression lines are perpendicular to each other (6)

Ans. Three important properties of regression coefficient are:

(1) Correlation coefficient is the geometric mean between the regression coefficients or

$$r^2 = bxy \cdot byx$$

(2) Correlation coefficient and both regression coefficients have same sign.

(3) Arithmetic mean of regression coefficient is greater than the correlation coefficient

i.e. $\dfrac{bxy + byx}{2} > r.$

Since two variables are uncorrelated then $r = 0$.

Equation to the lines of regression of y on x and x on y, are

$$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x}) \text{ and } x - \bar{x} = \frac{r\sigma_x}{\sigma_y}(y - \bar{y})$$

Their slopes are $m_1 = \dfrac{r\sigma_y}{\sigma_x}$ and $m_2 = \dfrac{\sigma_y}{r\sigma_x}$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| = \left|\frac{\dfrac{\sigma_y}{r\sigma_x} - \dfrac{r\sigma_y}{\sigma_x}}{1 + \dfrac{\sigma_y^2}{\sigma_x^2}}\right|$$

$$= \left|\frac{1-r^2}{r} \cdot \frac{\sigma_y}{\sigma_x} \cdot \frac{\sigma_x^2}{\sigma_y^2 + \sigma_x^2}\right| = \frac{1-r^2}{|r|} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Since $r^2 < 1$ and σ_x, σ_y are positive

$$\Rightarrow \quad \tan\theta = \frac{1-r^2}{|r|} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \text{ given } r = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$\therefore$ Two lines of regression are perpendicular to each other.

Q.7. (b) A sample of 10 boxes of chips is drawn in which the mean weight is 490 gm and standard deviation of weight is 9 gm. Can the sample be considered to be taken from a population having mean weight 500 gm where $t_{0.05} = 2.26$? (6.5)

Ans. Given $\quad n = 10, \ \overline{X} = 490, \ S = 9\text{gm}, \ \mu = 500$

$$\therefore \quad s = \sqrt{\frac{n}{n-1}S^2} = \sqrt{\frac{10}{9} \times 9^2}$$

$$= 9.486$$

Null Hypothesis H_0: The difference is not significant

i.e. $\mu = 500$

Alternative Hypothesis $H_1 : \mu \neq 500$. (Two tailed test)

Under $\quad H_0 \ t = \dfrac{\overline{X} - \mu}{S/\sqrt{n}} = \dfrac{490 - 500}{9.486/\sqrt{10}}$

$$= -0.333$$

$$\Rightarrow \quad |t| = 0.333$$

Also $t_{0.05} = 2.26$. for 9 d.f.

Conclusion: Since $\quad |t| = 0.333 > t_{0.05}$

$\therefore$ The hypothesis H_0 is rejected.

Thus, the sample could not have come from the population having mean 500 gm.

UNIT-IV

Q.8. (a) Write the dual of the following problem (6)

$$\text{Min } z = 2x_1 + 3x_2 + 4x_3$$
$$\text{s.t.} = 2x_1 + 3x_2 + 5x_3 = 2$$
$$3x_1 + x_2 + 7x_3 \le 3$$
$$x_1 + 4x_2 + 6x_3 = 5$$

where $x_2, x_3 \ge 0$ and x_1 is unrestricted

Ans. As x_1 is unrestricted

$\therefore \quad x_1 = x_1' - x_1''$

Introducing slack variable $S_1 \ge 0$. the primal probablem is restated as standard primal.

Min $Z \quad = 2(x_1' - x_1'') + 3x_2 + 4x_3 + 0.s_1$

S.t.

$$2(x_1' - x_1'') + 3x_2 + 5x_3 = 2$$
$$3(x_1' - x_1'') + x_2 + 7x_3 + S_1 = 3$$
$$x_1' - x_1'' + 4x_2 + 6x_3 = 5$$
$$x_1' \ge 0, x_1'' \ge 0, x_2 \ge 0, x_3 \ge 0, S_1 \ge 0$$

Let w_1, w_2, w_3 be the dual variables corresponding to the primal constraints.

Dual problem is

$$\text{Max } Z^* = 2w_1 + 3w_2 + 5w_3$$
$$\left.\begin{aligned} \text{S.t. } 2w_1 + 3w_2 + w_3 &\ge 2. \\ -2w_1 - 3w_2 - w_3 &\ge -2 \end{aligned}\right\}$$
$$3w_1 + w_2 + 4w_3 \ge 3$$
$$5w_1 + 7w_2 + 6w_3 \ge 4 \therefore$$
$$w_2 \ge 0$$

$\Rightarrow$

$$\text{Max } Z^* = 2w_1 + 3w_2 + 5w_3$$

S.t.c

$$2w_1 + 3w_2 + w_3 = 2$$
$$3w_1 + w_2 + 4w_3 \ge 3$$
$$5w_1 + 7w_2 + 6w3 \ge 4$$
$$w_2 \ge 0, w_1, w_3 \text{ unrestricted.}$$

Q.8. (b) Using dual simplex method solve the following LPP. (6.5)

$$\text{Max } Z = -3x_1 - 2x_2$$

S.t.

$$x_1 + x_2 \ge 1$$
$$x_1 + x_2 \le 7$$
$$x_1 + 2x_2 \ge 10$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Ans.

$$\text{Max } Z = -3x_1 - 2x_2$$
$$\text{S.t.} -x_1 - x_2 \le -1$$

$$x_1 + x_2 \le 7$$
$$-x_1 - 2x_2 \le 10$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Introducing slack variables $s_1 \ge 0, S_2 \ge 0, S_3 \ge 0, S_4 \ge 0$

$$\text{Max } z = -3x_1 - 2x_2 + S_1 + S_2 + S_3 + S_4.$$
$$-x_1 - x_2 + S_1 = -1$$
$$x_1 + x_2 + S_2 = 7$$
$$-s_1 - 2x_2 + S_3 = 10$$
$$x_2 + S_4 = 3$$
$$x_1, x_2, S_1, S_2, S_3, S_4 \ge 0$$

Dual simplex table is

Intial table:

			−3	−2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
←0	y_3	−1	−1	−1	1	0	0	0
0	y_4	7	1	1	0	1	0	0
0	y_5	10	−1	−2	0	0	1	0
0	y_6	3	0	1	0	0	0	1
	$zj-cj$		3	2 ↑	0	0	0	0

since all $(z_j - c_j) \ge 0$ and $x_{B1} (= S_1) < 0$.

min {−1} = − 1 will leave the basis i.e y_3.

Since $\quad \max \left\{\frac{3}{-1}, \frac{2}{-1}\right\} = -2$

i.e y_2 enters the basis

			−3	−2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
−2	y_2	1	1	1	−1	0	0	0
0	y_4	6	0	0	1	1	0	0
0	y_5	12	1	0	−2	0	1	0
0	y_6	4	−1	0	1	0	0	1
	$zj-cj$		1 ↑	0	2	0	0	0

since all $z_j - c_j \ge 0$. and $x_{B4} (= S_4) < 0$. *i.e.* −2.

min {−2} = −2 will leave the basis ie y_6

since max $\left\{\frac{1}{-1}\right\} = -1$

$\therefore \quad y_1$ enters the basis

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-2	y_2	3	0	1	0	0	0	-1
0	y_4	6	0	0	1	1	0	0
0	y_5	16	0	0	-1	0	1	-1
← -3	y_1	-4	1	0	-1	0	0	-1
		z_j-c_j	0	0	3	0	0	1

since all $z_j - c_j{}^3\ 0$ and $x_{B4}\ (= -4) < 0$ *ie* y_1 leaves the basis

$$\max.\left\{\frac{3}{-1}, \frac{1}{-1}\right\} = -1$$

y_6 enters.

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-2	y_2	7	-1	1	-1	0	0	0
0	y_4	6	0	0	1	1	0	0
0	y_5	12	-1	0	0	0	1	0
0	y_6	4	-1	0	1	0	0	1
			1	0	2	0	0	1

All $z_j - c_j \geq 0$ and $x_B \geq 0$.

$\therefore$ Solution is $x_1 = 0, x_2 = 7$

Q.9. Using VAM method find basic feasible solution of the following transportation problem. Check optimality and hence find the optimal solution. **(12.5)**

From	**A**	**B**	**C**	**D**	**Supply**
I	**21**	**16**	**25**	**13**	**11**
II	**17**	**18**	**14**	**23**	**13**
III	**32**	**27**	**18**	**41**	**19**
Demand	**6**	**10**	**12**	**15**	**43**

Ans.

Here total demand = total supply = 43.

Table for intial b.f. s is

21	16	25	[11] 13	11 (3)
17	18	14	23	13 (3)
32	27	18	41	19 (9)
6 (4)	10 (2)	12 (4)	15 (10)	

Largest of these differenees is (10) associated with 4th column of the table. Min. cost in the 4th column is 13, we allocate $x_{14} = \min(11, 15) = 11$ in cell (1,4) Exhaust first row: Reduceu table is

17	18	14	[4] 23	13 (3)
32	27	18	41	19 (9)
6 (15)	10 (9)	12 (4)	4 (18)	

Largest difference is (18) in 4th column

Min cost is 23, we allocate

$$x_{24} = \min(13,4) = 4 \text{ in cell } (2,4)$$

Exhaust 4th column. Reduced table is

[6] 17	18	14	9 (3)
32	27	18	19 (9)
6 (15)	10 (9)	12 (4)	

Largest difference is (15), associated with first column.

Min cost is 17, we allocate

$x_{21} = \min(9,6) = 6$ in cell (2,1). Exhaust first column

[3] 18	14	3 (4)
27	18	19 (9)
10 (9)	12 (4)	

Largest difference is (9), associated with 2nd column. Min cost is 18, we allocate

$$x_{22} = \min(3, 10) = 3 \text{ on cell } (2,2)$$

Exhaust 2nd row. Reduced table is

[7] 27	[12] 18	19
7	12	

Final table is

	v_1	v_2	v_3	v_4	
u_1	21	16	25	[11] 13	11
u_2	[6] 17	[6] 18	14	[4] 23	13
u_3	32	[7] 27	[12] 18	41	19
	6	10	12	15	

Feasible solution is

$$11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$$
$$= 796$$

optimal solution is

$$u_1 + V_4 = 13$$
$$u_2 + V_1 = 17$$
$$u_2 + V_2 = 18$$
$$u_2 + V_4 = 23$$
$$u_3 + V_2 = 27$$
$$u_3 + V_3 = 18$$

Let $u_1 = 0 \Rightarrow V_4 = 13$

Thus $u_2 = 23 - 3 = 10$

Now $V_1 = 7$

and $10 + V_2 = 18 \Rightarrow V_2 = 8.$

and $U_3 + 8 = 27$

$\Rightarrow U_3 = 19$

and $19 + V_3 = 18$

$\Rightarrow V_3 = -1$

Calculating $w_{ij} = (u_i + v_j) - c_{ij}$

cells	w_{ij}
(1,1)	$(u_1+V_1)-c_{11} = (0+7)-21 = -14$
(1,2)	$(u_1+V_2)-c_{12} = (0+8)-16 = -8$
(1,3)	$(u_1+V_3)-c_{13} = (0-1)-25 = -26$
(2,3)	$(u_2+V_3)-c_{23} = (10-1)-14 = -5$
(3,1)	$(u_3+V_1)-c_{31} = (19+7)-32 = -6$
(3,4)	$(u_3+V_4)-c_{34} = (19+13)-41 = -9$

All w_{ij} are negative, thus solution is optimal

$$z = 11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$$
$$= 796.$$

MID TERM EXAMINATION [FEB. 2017]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA-202]

Time : 1:30 Hrs. **M.M. : 30**

Note: *Q. no. 1 compulsory and attempt any two from remaining questions. All questions carry equal marks.*

Q.1. (a) Find Particular integral of $(4D^2 + 3DD' - D'^2 - D - D')z = 3e^{(x+2y)/2}$.

Ans.
$$\text{P.I.} = \frac{1}{4D^2 + 3DD' - D'^2 - D - D'} 3e^{\left(\frac{x}{2}+y\right)}$$

Replace D by $\frac{1}{2}$, D′ by 1.

$$\Rightarrow \frac{1}{4 \times \frac{1}{4} + \frac{3}{2} - 1 - \frac{1}{2} - 1} 3e^{\left(\frac{x}{2}+y\right)}$$

Case of failure

$$\Rightarrow x \frac{1}{8D + 3D' - 1} 3e^{\left(\frac{x}{2}+y\right)}$$

Replace D by $\frac{1}{2}$ and D′ by 1

$$\Rightarrow x \frac{1}{4+3-1} 3e^{\left(\frac{x}{2}+y\right)}$$

$$\Rightarrow \frac{x}{6}.3e^{\left(\frac{x}{2}+y\right)} = \frac{x}{2}.e^{\left(\frac{x}{2}+y\right)}$$

Thus
$$\text{P.I} = \frac{x}{2}.e^{\left(\frac{x}{2}+y\right)}$$

Q.1. (b) Using the method of separation of variable solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$ where $u(x, 0) = e^{-3x} - 2e^{-x}, x > 0, y > 0$

Ans. Given equation is

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u \quad \ldots(1)$$

Let $U = X(x)\,Y(y) = XY$

$\therefore$
$$\frac{\partial u}{\partial x} = X'Y$$

and
$$\frac{\partial u}{\partial y} = XY'$$

Equating (1) becomes.

$$X'Y = 2XY' + XY$$

$$\Rightarrow \quad X'Y = X(2Y' + Y)$$

$$\Rightarrow \quad \frac{X'}{X} = \frac{2Y'}{Y} + 1$$

Since x and y are independent variables $\therefore$ it is true only when each equation is equal to a constant.

Let $$\frac{X'}{X} = \frac{2Y'}{Y} + 1 = a \text{ (say)}$$

Now $$\frac{X'}{X} = a.$$

$$\Rightarrow \quad \frac{dX}{dx} \cdot \frac{1}{X} = a$$

$$\Rightarrow \quad \frac{dX}{X} = a dx$$

$$\Rightarrow \quad \log X = ax + \log c_1$$

$$\Rightarrow \quad X = C_1 e^{ax}$$

and $$\frac{2Y'}{Y} + 1 = a$$

$$\Rightarrow \quad 2\frac{dY}{dy} \cdot \frac{1}{Y} + 1 = a$$

$$\Rightarrow \quad 2\frac{dY}{Y} = (a-1)dy$$

$$\frac{dY}{Y} = \left(\frac{a-1}{2}\right)dy$$

$$\log Y = \left(\frac{a-1}{2}\right)y + \log C_2$$

$$Y = C_2 e^{\left(\frac{a-1}{2}\right)y}$$

$$U = C_1 C_2 e^{ax} e^{\left(\frac{a-1}{2}\right)y} \quad \text{...(2)}$$

As given $$u(x, 0) = e^{-3x} - 2e^{-x}$$

$$\Rightarrow \quad e^{-3x} - 2e^{-x} = C_1 C_2 e^{ax}$$

Comparing two, we get

$$e^{-3x} = C_1 C_2 e^{ax}$$

Here $$C_1 C_2 = 1, a = -3$$

and $$2e^{-x} = C_1 C_2 e^{ax}$$

$$\Rightarrow \quad C_1 C_2 = 2, a = -1$$

Thus eq^n (2) becomes

$$U(x, y) = e^{-3x} e^{-2y} - 2e^{-x} e^{-y}$$

$$\Rightarrow \quad U = e^{-3x-2y} - 2e^{-x-y}$$

Q.1. (c) Find moment generating function of the following random distribution

x	-1	1
$P(x)$	1/2	1/2

Ans.

$$\text{M.G.F} = E[e^{tx}]$$

$$= \sum e^{tx} \cdot f(x)$$

$$= \frac{e^{-t}}{2} + \frac{e^{t}}{2}$$

$$= \frac{e^{-t} + e^{t}}{2} = \cosh t.$$

Q.1. (d) A can hit a target 3 times in 5 shots, B can hit a target 2 times in 5 shots and C can hit a target 3 times in 4 shots. All of them fire one shot each simultaneously at the target. What is the probability that two shots hit a target.

Ans. Let

$$P(A) = \text{Probability of A hitting target} = \frac{3}{5}$$

$$P(B) = \text{Probability of B hitting target} = \frac{2}{5}$$

$$P(C) = \text{Probability of C hitting target} = \frac{3}{4}$$

Probability two shots hit target, we have

(1) A, B hit target and C misses it

$$= \frac{3}{5} \times \frac{2}{5} \times \left(1 - \frac{3}{4}\right)$$

$$= \frac{6}{25} \times \frac{1}{4} = \frac{6}{100}$$

(2) A misses it and B, C hit target

$$= \left(1 - \frac{3}{5}\right) \times \frac{2}{5} \times \frac{3}{4}$$

$$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{25}$$

(3) B misses it and A, C hit target

$$= \left(1 - \frac{2}{5}\right) \times \frac{3}{5} \times \frac{3}{4}$$

$$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{4} = \frac{27}{100}$$

Since these are mutually exclusive events

$$\therefore \quad \text{Required probability} = \frac{6}{100} + \frac{3}{25} + \frac{27}{100}$$

$$= \frac{45}{100} = \frac{9}{20}$$

Q.2. (a) Find Complete solution of $(D^2-2DD'+D'^2)z = \cos(2x+y)$ (5)

Ans.

$$(D^2-2DD'+D'^2)z = \cos(2x+y)$$

A.E.

$$D^2-2DD'+D'^2 = 0$$

$$m^2-2m+1 = 0$$

$$\Rightarrow \quad m = 1, 1$$

$$\text{C.F.} = f_1(y+x)+xf_2(y+x)$$

$$\text{P.I.} = \frac{1}{D^2-2DD'+D'^2}\cos(2x+y)$$

Here

$$a = 2, b = 1$$

and let

$$u = 2x+y$$

$$\text{P.I.} = \frac{1}{2^2-2\times 2+1}\iint \cos u\, du\, du$$

$$= \int \sin u\, du = -\cos u$$

$$= -\cos(2x+y)$$

∴ Complete solution is

$$Z = f_1(y+x)+xf_2(y+x)-\cos(2x+y)$$

Q.2. (b) Solve $\dfrac{\partial^2 u}{\partial x^2}-\dfrac{\partial^2 u}{\partial y^2}-\dfrac{3\partial u}{\partial x}+\dfrac{3\partial u}{\partial y} = xy$ (5)

Ans.

$$\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-\frac{3\partial u}{\partial x}+\frac{3\partial u}{\partial y} = xy$$

$$(D^2-D'^2-3D+3D')u = xy$$

$$\Rightarrow \quad [(D-D')(D+D')-3(D-D')]u = xy$$

$$\Rightarrow \quad (D-D')(D+D'-3)u = xy$$

For $(D-D'), b = 1, a = 1, c = 0$

For $(D+D'-3)\, b = 1, a = -1, c = 3.$

Now $\text{C.F.} = \phi_1(y+x)+e^{3x}\phi_2(y-x)$

$$\text{P.I.} = \frac{-1}{(D-D')(D+D'-3)}xy$$

$$= \frac{-1}{3\left[1-\left(\frac{D+D'}{3}\right)D\left(1-\frac{D'}{D}\right)\right]}xy$$

$$= \frac{-1}{3D}\left[1-\left(\frac{D+D'}{3}\right)\right]^{-1}\left(1-\frac{D'}{D}\right)^{-1}xy$$

$$= \frac{-1}{3D}\left[1+\frac{D+D'}{3}+\ldots\right]\left[1+\frac{D'}{D}+\ldots\right]xy$$

$$= \frac{-1}{3D}\left[1+\frac{D'}{D}+\frac{D}{3}+\frac{D'}{3}+\frac{D'}{3}+\ldots\right]xy$$

$$= \frac{-1}{3D}\left[xy+\frac{x}{D}+\frac{y}{3}+\frac{x}{3}+\frac{x}{3}\right]$$

$$= \frac{-1}{3D}\left[xy+\frac{x^2}{2}+\frac{y}{3}+\frac{2x}{3}\right]$$

$$= \frac{-1}{3}\left[\frac{1}{D}xy+\frac{1}{D}\frac{x^2}{2}+\frac{1}{D}\frac{y}{3}+\frac{1}{D}\frac{2x}{3}\right]$$

$$= \frac{-1}{3}\left[\frac{x^2y}{2}+\frac{x^3}{6}+\frac{xy}{3}+\frac{x^2}{3}\right]$$

∴ Complete Solution is

$$u = \phi_1(y+x)+e^{3x}\phi_2(y-x)-\frac{1}{3}\left[\frac{x^2y}{2}+\frac{x^3}{6}+\frac{xy}{3}+\frac{x^2}{3}\right]$$

Q.3. (a) A die is tossed twice getting '5' or '6' on a toss is taken as success. Find the probability distribution, the mean and the variance of the number of successes. (6)

Ans. Let p be the probability of success i.e. getting 5 or 6.

$$\therefore \quad n = 2$$

i.e.

$$p = \frac{1}{6}, q = 1-\frac{1}{6} = \frac{5}{6}$$

Let X be Binomial variable

$$\therefore \quad f_x(x) = P[X=x] = {}^nC_x p^x q^{n-x}$$

$$= {}^2C_x\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{2-x}$$

Probability of obtaining success is given by,

P[X = 0] + P[X = 1] + P[X = 2]

$$= {}^2C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2+{}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)+{}^2C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0$$

$$= \frac{2!}{2!}\times\frac{25}{36}+\frac{2!}{1!}\times\frac{5}{36}+\frac{2!}{2!}\times\frac{1}{36}$$

$$= \frac{25}{36}+\frac{10}{36}+\frac{1}{36}$$

$$= \frac{36}{36} = 1$$

Mean of Binomial variable $X = np$

$$= 2\times\frac{1}{6}\times\frac{1}{3}$$

Variance of Binomial variable $X = npq$

$$= 2\times\frac{1}{6}\times\frac{5}{6}$$

$$= \frac{5}{18}$$

Q.3. (b) The first four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. Find the moments about mean β_1 and β_2.

Ans. Let $\mu'_1, \mu'_2, \mu'_3, \mu'_4$ be the first four moments of the given distribution, then

$A = 4, \mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 108$

Let $\mu_1, \mu_2, \mu_3, \mu_4$ be the first four moments about the mean, then by def

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'^2_1 = 17 - (-1.5)^2$$

$$= 17 - 2.25 = 14.75$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$$

$$= -30 - 3 \times 17 \times (-1.5) + 2 \times (-1.5)^3$$

$$= -30 + 76.5 - 6.75$$

$$= 39.75$$

Also
$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$$

$$= 108 - 4 \times (-30) \times (-1.5) + 6 \times 17 \times (-1.5)^2 - 3 \times (-1.5)^4$$

$$= 108 - 180 + 229.5 - 15.1875$$

$$= 142.3125$$

Lastly
$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3}$$

$$= 0.4924$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.3125}{(14.75)^2} = 0.6541$$

Q.4. Solve the boundary value problem $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$, given that $y(0, t) = 0$, $y(5, t) = 0$, $y(x, 0) = 0$ and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$ (10)

Ans. Wave equation is
$$\frac{\partial^2 y}{\partial t^2} = \frac{4\partial^2 y}{\partial x^2} \quad ...(1)$$

Let
$$y = X(x)\,T(t) = XT \quad ...(2)$$

be the solution of (1)

Then
$$\frac{\partial^2 y}{\partial t^2} = XT'' \text{ and } \frac{\partial^2 y}{\partial x^2} = X''T$$

By equation (1), we have

$$XT'' = 4X''T$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{1}{4}\frac{T''}{T} \quad ...(3)$$

Since LHS of (3) is a function of x only and RHS is of t only. As x and t are independent variables. Thus both sides reduce to a constant say $-a^2$.

$\therefore$ (3) reduces to

$$\frac{X''}{X} = -a^2 \text{ and } \frac{1}{4}\frac{T''}{T} = -a^2$$

$$\Rightarrow \quad X'' + a^2 X = 0$$

$$\Rightarrow \quad (D^2 + a^2)X = 0 \Rightarrow D = \pm ia$$

$$X = C_1 \cos ax + C_2 \sin ax$$

and
$$T'' + a^2 4T = 0$$

$$\Rightarrow \quad (D^2 + 4a^2)T = 0$$

$$\Rightarrow \quad D = \pm i2a$$

$$\therefore \quad T = C_3 \cos 2at + C_4 \sin 2at$$

Solution is

$$y = (C_1 \cos ax + C_2 \sin ax)(C_3 \cos 2at + C_4 \sin 2at) \quad ...(4)$$

Now, applying boundary conditions in (4)

$$y(0, t) = 0,\ y(5, t) = 0$$

$$y(0, t) = C_1(C_3 \cos 2at + C_4 \sin 2at)$$

$$0 = C_1(C_3 \cos 2at + C_4 \sin 2at)$$

$$\Rightarrow \quad C_1 = 0$$

$\therefore$ **(4) reduces to**

$$y(x, t) = C_2 \sin ax(C_3 \cos 2at + C_4 \sin 2at) \quad ...(5)$$

Apply $y(5, t) = 0$ in equation (5), we get

$$0 = C_2 \sin 5a\,(C_3 \cos 2at + C_4 \sin 2at)$$

This is satisfied when

$$\sin 5a = 0$$

$$\Rightarrow \quad 5a = n\pi$$

$$\Rightarrow \quad a = \frac{n\pi}{5} \text{ where } n = 1, 2, \ldots\ldots$$

Solution of wave equation reduces to

$$y(x, t) = C_2\left(C_3 \cos_2 \frac{n\pi t}{5} + C_4 \sin \frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5}$$

$$\Rightarrow \quad y(x, t) = \left(a_n \cos\frac{2n\pi t}{5} + b_n \sin\frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5}$$

where
$$a_n = C_2C_3 \text{ and } b_n = C_2C_4$$

Adding solution for different values of n, we get

$$y(x, t) = \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi}{5}t + b_n \sin\frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5} \quad ...(6)$$

Now, applying initial conditions on (6),

$$y(x, 0) = 0 \text{ and } \left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$$

$\therefore$ by (6), we get

$$(y, 0) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{5}.$$

$$0 = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{5}$$

$$\Rightarrow \quad a_n = 0$$

$\Rightarrow$ **Thus, solution reduces to**

$$y(x, t) = \sum_{n=1}^{\infty} b_n \sin\frac{2n\pi}{5}t \sin\frac{n\pi x}{5} \quad ...(7)$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = \sum_{n=1}^{\infty} b_n \cos\frac{2n\pi}{5}t \cdot \frac{2n\pi}{5}\sin\frac{n\pi x}{5}$$

As given $\quad \left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$

$$\Rightarrow \quad \sin \pi x = \frac{2\pi}{5}\sum_{n=1}^{\infty} nb_n \sin\frac{n\pi x}{5}$$

This represents fourier sine series for $\sin \pi x$

$$\Rightarrow \quad \frac{2\pi}{5} nb_n = \frac{2}{5}\int_0^5 \sin \pi x \sin\frac{n\pi x}{5}\,dx$$

We solve for $\quad n = 1$.

$$\frac{2\pi}{5} b_1 = \frac{2}{5}\int_0^5 \sin \pi x \sin\frac{\pi x}{5}\,dx \Rightarrow b_1 = \frac{1}{2\pi}$$

∴ Complete Solution is

$$y = \frac{1}{2\pi}\sin\frac{\pi x}{5}\sin\frac{2\pi t}{5}$$

OR

Q.4. In the normal distribution 7% of the items are under 35 and 89% are 63. Determine the mean and variance of the distribution. Given that P(0 ≤ x ≤ 0.18) = 0.07, P(0 ≤ x ≤ 1.48) = 0.43 and P(0 ≤ x ≤ 1.23) = 0.39

Ans. Let μ and σ be the required mean and standard deviation.

Now 7% of items are under 35. It means area to the left of the ordinate $x = 35$ is 0.07

Also 89% of items are under 63. It means area to the left of the ordinate $x = 63$ is 0.89

Let $\quad z = \frac{x-\mu}{\sigma}$ be the standard normal variate

When $\quad x = 35, z = \frac{35-\mu}{\sigma} = z_1$ (say)

When $\quad x = 63, z = \frac{63-\mu}{\sigma} = z_2$ (say)

Now $\quad P(x < 35) = 0.07$

$\Rightarrow \quad P(z < z_1) = 0.07$

$\Rightarrow \quad 1 - P(z > z_1) = 0.07$

$\Rightarrow \quad P(z > z_1) = 0.93$

$\Rightarrow \quad 0.5 - P(0 < z < z_1) = 0.93$

$\Rightarrow \quad P(0 < z < z_1) = -0.43$

Also $\quad z_1 = 1.48$

$\Rightarrow \quad P(x < 63) = 0.89$

$\Rightarrow \quad P(z < z_2) = 0.89$

$\quad 1 - P(z > z_2) = 0.89$

$\Rightarrow \quad P(z > z_2) = 0.11$

$\Rightarrow \quad 0.5 - P.(0 < z < z_2) = 0.11$

$\quad P(0 < z < z_2) = 0.39$

$\Rightarrow \quad z_2 = 1.23$

Here the values of the ordinate $z = z_1$ and z_2 is negative

i.e. $\quad z_1 = -1.48, z_2 = -1.23$

When $\quad z_1 = -1.48$, then

$$\frac{35-\mu}{\sigma} = -1.48$$

$\Rightarrow \quad 35 - \mu = -1.48\sigma \quad$ ...(1)

$\Rightarrow \quad$ When $z_2 = -1.23$, then

$$\frac{63-\mu}{\sigma} = -1.23$$

$\Rightarrow \quad 63 - \mu = -1.23\sigma \quad$ ...(2)

Solving (1) and (2), we get

$\quad 35 - 63 = (-1.48 + 1.23)\sigma$

$\Rightarrow \quad -28 = (-0.25\sigma)$

$\Rightarrow \quad \sigma = 112$

$\Rightarrow \quad \text{Var} = 12544$

By (1), $\quad 35 - \mu = -1.48 \times 112$

$\quad 35 - \mu = -165.76$

$\Rightarrow \quad \mu = 200.76$

END TERM EXAMINATION [MAY-JUNE 2017]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS–IV
[ETMA–202]

Time : 3 Hrs. **M.M. : 75**

Note: *Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.*

Q.1. (a) Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, given $u=3e^{-y}-e^{-5y}$ when $x=0$, by the method of separation of variables. (3)

Ans.
$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y} = 3u \quad ...(1)$$

Let
$$U = X(x)\,Y(y)$$
$$\frac{\partial u}{\partial x} = X'Y, \quad \frac{\partial u}{\partial y}=XY'$$

By equation (1), we get
$$4X'Y+XY' = 3XY$$
$$\Rightarrow \quad 4X'Y = (3Y-Y')X$$
$$\Rightarrow \quad \frac{4X'}{X} = \frac{3Y-Y'}{Y}$$

Since x and y are independent variables $\therefore$ it is true only when each equation is equal to a constant

$$\Rightarrow \quad \frac{4X'}{X} = 3-\frac{Y'}{Y}=a$$
$$\Rightarrow \quad \frac{4X'}{X} = a \text{ and } 3-\frac{Y'}{Y}=a$$
$$\Rightarrow \quad \frac{4X'}{X} = a \Rightarrow \frac{4dX}{dx}\cdot\frac{1}{X}=a$$
$$\Rightarrow \quad \frac{dX}{X} = \frac{a}{4}dx$$

On integrating, we get

$$\log X = \frac{ax}{4}+\log c_1$$
$$X = c_1 e^{ax/4} \quad ...(2)$$

Now
$$3-\frac{Y'}{Y} = a \Rightarrow \frac{Y'}{Y}=3-a$$
$$\frac{dY}{dy}\cdot\frac{1}{Y} = 3-a$$
$$\frac{dY}{Y} = (3-a)\,dy$$

On integrating, we get

$$\int\frac{dY}{Y} = \int(3-a)dy$$
$$\log Y = (3-a)y+\log c_2$$
$$Y = c_2 e^{(3-a)y}$$

As
$$U = XY \quad ...(3)$$
$$U = c_1 e^{ax/4}\, c_2 e^{(3-a)y}$$
$$U = c_1c_2\, e^{ax/4}\, e^{(3-a)y}$$

As given
$$U(0,y) = 3e^{-y}-e^{-5y} \quad ...(4)$$

By (4)
$$3e^{-y}-e^{-5y} = c_1c_2 e^{(3-a)y}$$

Comparing these two, we get
$$3e^{-y} = c_1c_2e^{(3-a)y}$$
$$c_1c_2 = 3,\ 3-a=-1$$
$$c_1c_2 = 3,\ a=4$$

and
$$-e^{-5y} = c_1c_2e^{(3-a)y}$$
$$c_1c_2 = -1,\ 3-a=-5$$
$$c_1c_2 = -1,\ a=8.$$

Equation (4) becomes
$$U = 3e^{4x/4}e^{-y}-e^{8x/4}e^{-5y}$$
$$U = 3e^{x}e^{-y}-e^{2x}e^{-5y}$$
$$U(x,y) = 3e^{x-y}-e^{2x-5y}.$$

Q.1. (b) Define the skewness and kurtosis. Also write their coefficients. (3)

Ans. Skewness: It refers to asymmetry or lack of symmetry in the shape of frequency distribution. **OR** A distribution is said to be 'skewed' when the mean and median fall at different points in the distribution and the balance is shifted to one side or the other. Karl Pearson coefficient of skewness is

$$S_{kp} = \frac{\text{Mean}-\text{Mode}}{\text{S.D.}},\ \text{where } -1\le S_{kp}\le 1.$$

Also, by using moments, we define measure of skewness

$$\gamma_1 = \sqrt{\beta_1}=\frac{\mu_3}{\sqrt{\mu_2^3}} \text{ and } \gamma_2=\beta_2-3=\frac{\mu_4}{\mu_2^2}-3$$

If $\gamma_1=0$, distribution is symmetrical
If $\gamma_1>0$, distribution is positively skewed
If $\gamma_2<0$, distribution is negatively skewed.

Kurtosis: Kurtosis is defined as degree of flatness or peakedness in the region about the mode of a frequency mode. The degree of kurtosis of a distribution is measured relative to the peakedness of normal curve.

Measure of kurtosis is value of the pearson coefficient β_2, given by $\beta_2=\frac{\mu_4}{\mu_2^2}$

Another measure of kurtosis is $\gamma_2=\beta_2-3$.
For normal curve, $\gamma_2=0$

Q.1. (c) Determine the Bionomial Distribution for which mean = 2 (Variance) and mean + Variance = 3. Also find P(X ≤ 3). (4)

Ans. Let $X\equiv\beta(n,p)$.
$$P(X=x) = {}^nc_x p^x q^{n-x}$$

Given
$$\text{Mean} = np=2=(\text{Variance})\ npq$$

Also
$$np+npq = 3.$$

$$\Rightarrow \quad 2 + 2q = 3$$
$$\Rightarrow \quad q = 1/2$$
$$\Rightarrow \quad p = 1/2$$

Now
$$np = 2 \Rightarrow n \times \frac{1}{2} = 2$$
$$\Rightarrow \quad n = 4$$

Thus, Binomial distribution
$$P(X = x) = {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

For
$$P(X \le 3) = 1 - P(X > 3)$$
$$= 1 - P(X = 4)$$
$$= 1 - {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$$
$$= 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}.$$

Q.1. (d) The first four moments of a distribution about the value '0' are – 0.20, 1.76, –2.36 and 10.88. Find the moments about the mean. (4)

Ans. Moments about the value '0' are
$$\mu_1' = -0.20,\ \mu_2' = 1.76,\ \mu_3' = -2.36,$$
$$\mu_4' = 10.88$$

Then moments about mean, will be
$$\mu_1 = 0$$
$$\mu_2 = \mu'_2 - u_1'^2 = 1.76 - (-0.20)^2 = 1.72$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= -2.36 - 3 \times 1.76 \times (-0.20) + 2 \times (-0.20)^3$$
$$= -2.36 + 1.056 - 0.016$$
$$= -1.32$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$
$$= 10.88 - 4 \times (-2.36) \times (-0.20) + 6 \times 1.76 \times (-0.20)^2 - 3(-0.20)^4$$
$$= 10.88 - 1.888 + 0.4224 - 0.0048$$
$$= 9.4096$$

Q.1. (e) State Baye's theorem
(3)

Ans. Baye's theorem states that

Let $E_1, E_2, \ldots E_n$ be n mutually exclusive and exhaustive events, with non-zero probabilities, of a random experiment. If A be any arbitrary event of the sample space of the above experiment which occurs with E_1 or E_2 or ....or E_n and $P(A) > 0$, then
$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A/E_j)}, 1 \le i \le n.$$

Q.1. (f) The standard weight of a special purpose brick is 5 kg. and it contains two basic ingredients B_1 and B_2. B_1 cost Rs. 5 per kg and B_2 cost Rs. 8 per kg. Strength considerations state that the brick contains not more than 4kg of B_1 and minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find out graphically minimum cost of the brick satisfying the above conditions. (5)

Ans. Let x and y be the weight for B_1 and B_2 respectively.

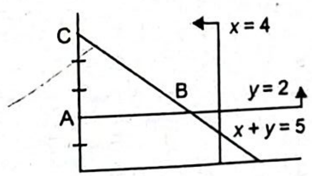
Cost of x bricks = Rs $5x$

Cost of y bricks = Rs 8y

Since constraints are

Not more than 4kg of B_1
$$\Rightarrow \quad x \le 4$$
Minimum of 2kg of B_2
$$\Rightarrow \quad y \ge 2$$
Total weight of brick is 5 kg
$$\Rightarrow \quad x + y = 5$$
Min. cost of the brick is needed
$$\Rightarrow \quad z = 5x + 8y$$
∴ Mathematical formulation of L.P.P. is
$$\text{Min } z = 5x + 8y$$
Subject to constraints
$$x \le 4$$
$$y \ge 2$$
$$x + y = 5$$
$$x \ge 0, y \ge 0$$

Since $x \ge 0$, $y \ge 0$ thus feasible region is in first quadrant only. Plot constraint by first treating it as a linear equation and then using the inequality condition of each constraint, mark the feasible region.



Here feasible region of the LPP is line segment AB with A = (0, 2) and B(3, 2). and C(0, 5)

We find value of objective function
$$z = 5x + 8y \text{ at each corner point}$$
At A(0, 2), $z = 5 \times 0 + 8 \times 2 = 16$

At B(3, 2), $z = 5 \times 3 + 8 \times 2 = 31$

At C(0, 5), $z = 5 \times 0 + 8 \times 5 = 40$

Min. value of z is 16 at (0, 2).

Hence optimal solution of given LPP is $x = 0$, $y = 0$ and Min $z = 16$.

Q.1. (g) Find the dual of the following primal problem: (3)
Min $Z = 3x_1 - 2x_2 + 4x_3$, subject to : $3x_1 + 5x_2 + 4x_3 \ge 7$, $6x_1 + x_2 + 3x_3 \ge 4$, $7x_1 - 2x_2 - x_3 \le 10$, and $x_1, x_2, x_3 \ge 0$.

Ans. Primal problem is
$$\text{Min } z = 3x_1 - 2x_2 + 4x_3$$
S.t
$$3x_1 + 5x_2 + 4x_3 \ge 7$$

$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Let S_1, S_2 and S_3 be the surplus and slack variables, then primal problem becomes

$$\text{Min } z = 3x_1 - 2x_2 + 4x_3 + S_1 + S_2 + S_3$$
$$3x_1 + 5x_2 + 4x_3 - S_1 = 7$$
$$6x_1 + x_2 + 3x_3 - S_2 = 4$$
$$7x_1 - 2x_2 - x_3 + S_3 = 10$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Let w_1, w_2, w_3 be the dual variables corresponding to primal constraints

Dual $\quad \text{Max } z^* = 7w_1 + 4w_2 + 10w_3$

S.t.
$$3w_1 + 6w_2 + 7w_3 \leq 3$$
$$5w_1 + w_2 - 2w_3 \leq -2$$
$$4w_1 + 3w_2 - w_3 \geq 4$$
$$-w_1 \leq 0$$
$$-w_2 \leq 0$$
$$w_3 \geq 0$$

w_1, w_2 and w_3 unrestricted (redundant)

$\Rightarrow \quad \text{Max } z^* = 7w_1 + 4w_2 + 10w_3$

S.t.
$$3w_1 + 6w_2 + 7w_3 \leq 3$$
$$5w_1 + w_2 - 2w_3 \leq -2$$
$$4w_1 + 3w_2 - w_3 \geq 4$$
$$w_1 \geq 0, w_2 \geq 0, w_3 \geq 0.$$

UNIT-I

Q.2. (a) $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. (6.5)

Ans.
$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

A.E.
$$m^2 - m - 2 = 0$$
$$m = 2, -1$$
$$\text{C. F.} = f_1(y+2x) + f_2(y-x)$$
$$\text{P.I.} = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$
$$= \frac{1}{D^2 - DD' - 2D'^2} ye^x - \frac{1}{D^2 - DD' - 2D'^2} e^x$$
$$= \frac{1}{(D-2D')(D+D')} ye^x - \frac{1}{1 - 1.0 - 2.0} e^x$$
$$= \frac{1}{(D-2D')(D+D')} ye^x - e^x$$
$$= \frac{1}{D-2D'} \int (c+x)e^x \, dx - e^x \qquad \text{[Here } m = -1]$$

Where C is replaced by $y + mx$ i.e., $y - x$ after integration.

$$= \frac{1}{D-2D'}\left[(c+x)e^x - \int e^x \, dx\right] - e^x$$
$$= \frac{1}{D-2D'}\left[(y - x + x)e^x - e^x\right] - e^x$$

$$= \frac{1}{D-2D'}(ye^x - e^x) - e^x$$

Now for m = 2

$$= \int (c - 2x - 1)e^x \, dx - e^x$$

where C is replaced by $y + mx = y + 2x$ after integration.

$$= (c - 2x - 1)e^x - \int -2e^x \, dx - e^x$$
$$= (y + 2x - 2x - 1)e^x + 2e^x - e^x$$
$$= ye^x - e^x + e^x$$
$$= ye^x$$

$\therefore$ complete sol^n is

$$z = f_1(y+2x) + f_2(y-x) + ye^x.$$

Q.2. (b) $(D^2 - D')z' = 2y - x^2$. (6)

Ans. Here $(D^2 - D')$ cannot be resolved into linear factors in D and D'

Consider $\quad (D^2 - D')z = 0 \quad$ ...(1)

Let trial solution of (1) be $z = Ae^{hx+ky}$ ...(2)

So, $\quad D^2 z = Ah^2 e^{hx+ky}, D'z = Ake^{hx+ky}$

By (1)
$$A(h^2 - k)e^{hx+ky} = 0$$
$$h^2 - K = 0 \Rightarrow K = h^2$$
$$\therefore \quad \text{C.F.} = \Sigma Ae^{hx+ky} = \Sigma Ae^{hx+h^2y}$$
$$\text{P.I.} = \frac{1}{D^2 - D'}(2y - x^2)$$
$$= \frac{1}{D^2\left(1 - \frac{D'}{D^2}\right)}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 - \frac{D'}{D^2}\right]^{-1}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 + \frac{D'}{D^2} + \ldots\right](2y - x^2)$$
$$= \frac{1}{D^2}\left[(2y - x^2) + \frac{1}{D^2}D'(2y - x^2) + \ldots\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{1}{D^2}.2\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{2}{D}.x\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + x^2\right]$$
$$= \frac{1}{D^2}.2y = 2y.\frac{x^2}{2} = x^2y$$

$\therefore$ Complete sol^n is $\quad z = \Sigma Ae^{h(x+hy)} + x^2y$

A and h are arbitrary Constants.

Q.3. (a) An insulated rod of length l has its end A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t. (6.5)

Ans. The temperature function $u(x, t)$ satisfies the differential equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

Prior to temperture change at end B, when t = 0, the heat flow was independent of time (steady state) i.e., $\frac{\partial u}{\partial t} = 0$.

When temperature u depends upon x and not on t, (1) reduces to $c^2 \frac{\partial^2 u}{\partial x^2} = 0$

$$\Rightarrow \quad \frac{\partial^2 u}{\partial x^2} = 0$$

∴ General solⁿ is $u = ax + b$ ...(2)

Since $u = 0$ for $x = 0$

$u = 100$ for $x = l$

∴ (2) gives

$0 = b$

$\Rightarrow \quad u = ax$

and $100 = al \Rightarrow a = \frac{100}{l}$

∴ Initial condition is $u(x, 0) = \frac{100}{l}x$

Boundary conditions for subsequent flow are $u(0, t) = 0$, $u(l, t) = 0$ for all values of t.

Let $U = X(x)\,T(t) = XT$

$$\Rightarrow \quad \frac{\partial u}{\partial t} = XT', \quad \frac{\partial^2 u}{\partial x^2} = X''T$$

Put these values in equation (1)

$$XT' = c^2 X''T$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T}$$

Now LHS is a function of x only and RHS is a function of t only. Since x and t are independent.

∴ both sides reduce to a constant say 'k':

∴ (3) gives

$$\frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T} = k.$$

Let $k = -p^2$

$$\Rightarrow \quad \frac{X''}{X} = -p^2 \Rightarrow X'' + p^2 X = 0$$

Aux. eqⁿ is $m^2 + p^2 = 0 \Rightarrow m = \pm ip$

$\therefore X = c_1 \cos px + c_2 \sin px$

Also $\frac{T'}{T} = -p^2c^2$

∴ $\log T = -p^2c^2t + \log c_3$

$\Rightarrow \quad T = c_3 e^{-p^2c^2t}$

General solution is

$$u(x, t) = (c_1 \cos px + c_2 \sin px)e^{-p^2c^2t} \quad ...(3)$$

Using condition $u(0, t) = 0$ in (3), we get

$u(0, t) = 0 = c_1 e^{-p^2c^2t}$

$\Rightarrow \quad c_1 = 0$

equation (3) reduces to

$$u(x, t) = c_2 \sin px\, e^{-p^2c^2t} \quad ...(4)$$

Using $u(l, t) = 0$ in (4)

$\Rightarrow \quad u(l, t) = 0 = c_2 \sin pl\, e^{-p^2c^2t}$

$\Rightarrow \quad \sin pl = 0$

$\Rightarrow \quad pl = n\pi$

$\Rightarrow \quad p = \frac{n\pi}{l}, \quad n = 1, 2, ...$

equation (4) reduces to

$$u(x, t) = c_2 \sin\frac{n\pi x}{l} e^{\frac{-n^2\pi^2}{l^2}c^2t}$$

Adding all such solutions for different values of n, we get

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{\frac{-n^2\pi^2c^2t}{l^2}} \quad ...(5)$$

As initial condition is $u(x, 0) = \frac{100x}{l}$

$$\Rightarrow \quad \frac{100x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Which is a fourier sine series for $\frac{100x}{l}$

$$\therefore \quad b_n = \frac{2}{l}\int_0^l \frac{100x}{l}\sin\frac{n\pi x}{l}\,dx$$

$$\Rightarrow \quad b_n = \frac{200}{l^2}\left[x\left(-\cos\frac{n\pi x}{l}\right)\cdot\frac{l}{n\pi} - \left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right]_0^l$$

$$\Rightarrow \quad b_n = \frac{200}{l^2}\left[\frac{-l^2}{n\pi}\cos n\pi\right] = \frac{-200}{n\pi}(-1)^n$$

$$= \frac{200}{n\pi}(-1)^{n+1}$$

$$\therefore \quad u(x, t) = \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi x}{l}e^{\frac{n^2\pi^2c^2t}{l^2}}$$

Q.3. (b) Solve the above problem if the change consists of raising the temperature of A to 20°C and reducing that of B to 80°C. (6)

Ans. Consider equation
$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

As solved earlier solution is
$$u(x,0) = \frac{100x}{l} \quad ...(2)$$

Further B.c's are $\left.\begin{matrix} u(0,t)=20 \\ u(l,t)=80\end{matrix}\right\} - \text{(A)} \ \forall\, t$

Let the required solution be
$$u(x,t) = u_s(x,t) + u_t(x,t) \quad ...(3)$$

Let u_s is steady state Solution and u_t is transient solution given by
$$u_t(x,t) = u(x,t) - u_s(x,t) \quad ...(4)$$

For steady state, solution is gives as
$$\frac{\partial^2 u}{\partial x^2} = 0$$

$\Rightarrow$
$$u_s(x,t) = c_1x + c_2 \quad ...(5)$$

Using condition (A)

(5) $\Rightarrow$ $\quad 20 = c_2$

$\therefore$ $\quad u_s(x,t) = c_1x + 20$

and $\quad 80 = c_1 l + 20$

$\Rightarrow$ $\quad c_1 = \dfrac{60}{l}$

$\therefore$
$$u_s(x,t) = \frac{60x}{l} + 20 \quad ...(6)$$

At $x = 0$, By (4) $\Rightarrow$
$$\begin{aligned} u_t(0,t) &= u(0,t) - u_s(0,t) \\ &= 20 - 20 \\ &= 0. \end{aligned}$$
[By (A) and (6)]

and
$$\begin{aligned} u_t(l,t) &= u(l,t) - u_s(l,t) \\ &= 80 - 80 = 0 \end{aligned}$$

Again by (4)
$$\begin{aligned} u_t(x,0) &= u(x,0) - u_s(x,0) \\ &= \frac{100x}{l} - \frac{60x}{l} - 20 \\ &= \frac{40x}{l} - 20 \end{aligned}$$

Now $\quad u_t(0,t) = 0,\ u_t(l,t) = 0$

and
$$u_t(x,0) = \frac{40x}{l} - 20 \quad ...(7)$$

Then Sol^n of (1) is given by
$$u_t(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{-n^2\pi^2c^2t/l^2}$$

using (7)

$$u_t(x,0) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

This is a fourier sine series for $\left(\dfrac{40x}{l} - 20\right)$

where
$$b_n = \frac{2}{l}\int_0^l \left(\frac{40x}{l} - 20\right)\sin\frac{n\pi x}{l}dx$$

$$b_n = \frac{2}{l}\left[\left(\frac{40x}{l} - 20\right)\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi} - \left(\frac{40}{l}\right)\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right]_0^l$$

$$b_n = \frac{2}{l}\left[-20\cos n\pi.\frac{l}{n\pi} - \frac{20l}{n\pi}\right]$$

$$b_n = \frac{-40}{n\pi}((-1)^n + 1)$$

$$b_n = \begin{cases} \dfrac{-80}{m\pi}, & \text{n is odd} \\ & \text{n is even}\end{cases}$$

$$b_n = \begin{cases} 0, & \text{n is odd} \\ \dfrac{-80}{m\pi}, & \text{n = 2m (even)}\end{cases}$$

$$u_t(x,t) = \frac{-80}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{2m\pi x}{l}e^{-\frac{\pi^2c^24m^2t}{l^2}}$$

Using (3), (6) and (8), we get
$$u(x,t) = \frac{60x}{l} + 20 - \frac{80}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{2m\pi x}{l}e^{-\frac{4m^2\pi^2c^2t}{l^2}}$$

UNIT – II

Q.4. (a) Three balls are drawn successively from a box containing 6 red balls, 4 white balls, and 5 blue balls. Find the probability that they are drawn in the order red, white and blue if each ball is (i) replaced and (ii) not replaced. (6.5)

Ans. We have 6R, 4W, 5B

Total number of balls = 15

(i) We have to draw red, white and blue Probability of drawing 3 balls with replacement

First draw having 3 red balls = $\dfrac{^6C_3}{^{15}C_3}$

Second draw having 1 white balls $= \dfrac{^4C_1}{^{15}C_1}$

Third draw having 1 blue balls $= \dfrac{^5C_1}{^{15}C_1}$

$$\text{Required prob.} = \frac{^5C_1}{^{15}C_1} \times \frac{^4C_1}{^{15}C_1} \times \frac{^5C_1}{^{15}C_1}$$

(ii) Not replaced.

Probability of drawing 3 red at first trial is

$$= \frac{^5C_3}{^{15}C_3} = \frac{5 \times 5 \times 4}{^{15}C_3}$$

Probability of drawing 1 white in second trial is

$$= \frac{^4C_1}{^{12}C_1} = \frac{4}{^{12}C_1}$$

Probability of drawing 3 blue in third trial is

$$= \frac{^5C_3}{^9C_3} = \frac{5 \times 4}{9 \times 8 \times 7 \times 6 \times 5 \times 4}$$

$$= \frac{1}{3024}$$

$$\text{Required prob. is } \frac{120}{^{15}C_3} \times \frac{4}{^{12}C_1} \times \frac{1}{3024}$$

$$= \frac{120}{^{15}C_3 \times {}^{12}C_1 \times 756}$$

$$= \frac{30}{^{15}C_3 \times {}^{12}C_1 \times 189}$$

Q.4. (b) Find (i) E(X), (ii) E(X²), (iii) E[(X – $\bar{X}$)²] for the probability distribution shown in the following table:

X	8	12	16	20/2	24
P(X)	1/8	1/6	3/8	1/4	1/12

Ans. Given

X	P(X)	$xP(x)$	$x^2P(x)$
8	1/8	1	8
12	1/6	2	24
16	3/8	6	96
202	1/4	101/2	10201
24	1/12	2	48

$$E(X) = \Sigma x. P(x) = 1 + 2 + 6 + \frac{101}{2} + 2$$

$$= 11 + \frac{101}{2} = \frac{123}{2}$$

$$E(X_2) \quad \Sigma x^2. P(x) = 8 + 24 + 96 + 10201 + 48$$

$$= 10377$$

$$E[(X - \bar{X})^2] = E(X^2) - [E(x)]^2$$

$$= 10377 - \frac{(123)^2}{4}$$

$$= 10377 - \frac{15129}{4}$$

$$= \frac{26379}{4}$$

Q.5. (a) A continuous distribution of a variable x in the range (–3, 3) is defined

$$f(x) = \frac{1}{16}(3 + x)^2, \quad -3 \le x < -1.$$

$$= \frac{1}{16}(2 - 6x^2), \quad -1 \le x < 1.$$

$$= \frac{1}{16}(3 - x)^2, \quad 1 \le x \le 3.$$

Verify that the area under the curve is unity. Show that the mean is zero. **(6)**

Ans. To show $\displaystyle\int_{-3}^{3} f_x(x)\, dx = 1$

$$\text{Consider } \int_{-3}^{3} f_x(x)\, dx = \int_{-3}^{-1} \frac{1}{16}(3 + x)^2 dx + \int_{-1}^{1} \frac{1}{16}(2 - 6x^2) dx + \int_{1}^{3} \frac{1}{16}(3 - x)^2 dx$$

$$= \frac{1}{16}\left[\int_{-3}^{-1}(9 + x^2 + 6x)dx + \int_{-1}^{1}(2 - 6x^2)dx + \int_{1}^{3}(9 + x^2 - 6x)dx\right]$$

$$= \frac{1}{16}\left[\left|9x + \frac{x^3}{3} + 3x^2\right|_{-3}^{-1} + \left|2x - 2x^3\right|_{-1}^{1} + \left|9x + \frac{x^3}{3} - 3x^2\right|_{1}^{3}\right]$$

$$= \frac{1}{16}\left[-9 - \frac{1}{3} + 3 + 27 + \frac{27}{3} - 27 + 2 - 2 + 2 - 2 + 27 + \frac{27}{3} - 27 - 9 - \frac{1}{3} + 3\right]$$

$$= \frac{1}{16}\left[\frac{16}{3}\right] = 1/3.$$

$$E(X) = \int_{-3}^{3} x.f_x(x)\,dx$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} x(3+x)^2 dx + \int_{-1}^{1} x(2-6x^2)dx + \int_{1}^{3} x(3-x)^2 dx\right]$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} x(9+x^2+6x)dx + \int_{-1}^{1}(2x-6x^3)dx + \int_{1}^{3}(9x+x^3-6x^2)dx\right]$$

$$= \frac{1}{16}\left[\left|\frac{9x^2}{2}+\frac{x^4}{4}+2x^3\right|_{-3}^{-1} + \left|x^2-\frac{6x^4}{4}\right|_{-1}^{1} + \left|\frac{9x^2}{2}+\frac{x^4}{4}-2x^3\right|_{1}^{3}\right]$$

$$= \frac{1}{16}\left[\frac{9}{2}+\frac{1}{4}-2-\frac{81}{2}-\frac{81}{4}+54+1-\frac{6}{4}-1+\frac{6}{4}+\frac{81}{2}+\frac{81}{4}-54-\frac{9}{2}-\frac{1}{4}+2\right]$$

$$= \frac{1}{16}[0] = 0$$

Q.5. (b) If the heights of 300 students are normally distribution with mean 68.0 inch and standard deviation 3.0 inch, how many students have heights. (i) greater than 72 inch. (ii) between 65 and 71 inch. **(6.5)**

Ans. Given $n = 300, \mu = 68, \sigma = 3$

Let X denote the height of students, following the normal distribution

Let $Z = \frac{X-\mu}{\sigma}$ be the standard normal variate.

$$Z = \frac{z-68}{3}$$

When $X > 72$ inch.

$$Z = \frac{72-68}{3} = \frac{4}{3} = 1.33$$

$$P(X > 72) = P(z > 1.33)$$
$$= 0.5 - P(0 < z < 1.33)$$
$$= 0.5 - 0.4082$$
$$= 0.0918$$

(ii) When $65 < X < 71$

for $X = 65$,

$$Z = \frac{65-68}{3} = \frac{-3}{3} = -1$$

for $X = 71$,

$$Z = \frac{71-68}{3} = \frac{3}{3} = 1$$

$$P(65 < X < 71) = P(-1 < z < 1)$$
$$= 2P(0 < z < 1)$$
$$= 2 \times 0.3413 = 0.6826$$

UNIT - III

Q.6. (a) Fit a second degree parabola to the following data using method of least squares. **(6.5)**

X	10	12	15	23	20
Y	14	17	23	25	21

Ans. Let $y = a + bx + cx^2$ be the second degree parabola. Then normal equation are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

Now $\Sigma x = 80, \Sigma y = 100, \Sigma x^2 = 1398, n = 5.$

Let $\bar{x} = 16, \bar{y} = 20$

Let $A = 16, B = 20$

x	y	$u = x - A$	$v = y - B$	u^2	u^3	u^4	uv	u^2v
10	14	–6	–6	36	–216	1296	36	–216
12	17	–4	–3	16	–64	256	12	–48
15	23	–1	3	1	–1	1	–3	3
23	25	7	5	49	343	2401	35	245
20	21	4	1	16	64	256	4	16

$\Sigma u = 0, \Sigma v = 0, \Sigma u^2 = 118, \Sigma u^3 = 126, \Sigma u^4 = 4210, \Sigma uv = 84, \Sigma u^2 v = 0$

$$0 = 5a + 0 + 118c \quad ...(1)$$
$$84 = 0 + 118b + 126c \quad ...(2)$$
$$0 = 118a + 126b + 4210c \quad ...(3)$$

$(1) \Rightarrow$
$$a = \frac{-118c}{5}$$

By $(3) \Rightarrow$
$$0 = 118\left(\frac{-118c}{5}\right) + 126b + 4210c$$

$$0 = -\frac{13924}{5}c + 126b + 4210c$$

$$0 = \frac{7126}{5}c + 126b$$

$$\Rightarrow \quad 630b + 7126c = 0 \quad ...(4)$$
$$\Rightarrow \quad 118b + 126c = 84 \quad ...(5)$$
$$\Rightarrow \quad 315b + 3563c = 0 \Rightarrow c = -\frac{315}{3563}b$$
$$\Rightarrow \quad 59b + 63c = 42$$
$$\Rightarrow$$

$$\Rightarrow \quad 59b + 63\left(\frac{-315}{3563}\right)b = 42$$

$$\Rightarrow \quad 59b - \frac{19845}{3563}b = 42$$

$$\Rightarrow \quad \frac{190372}{3563}b = 42$$

$$\Rightarrow \quad b = 0.7861$$

$$\Rightarrow \quad C = \frac{-315}{3563} \times 0.7861 = -0.0695$$

$$\Rightarrow \quad a = -\frac{118}{5} \times (-0.0695)$$

$$\Rightarrow \quad a = 1.6402$$

Thus

$$\Rightarrow \quad y = 1.6402 + 0.7861x - 0.0695\,x^2$$

Q.6. (b) For 10 observations on price (x) and supply (y), the following data were obtained $\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$, $\Sigma xy = 3467$.

Obtained the two lines of regression, correlation coefficient and estimate the supply when the price is 16 units.

Ans. Given $n = 10$, $\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$, $\Sigma xy = 3467$

$$r_{xy} = \frac{\frac{1}{n}\Sigma xy - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{1}{n}\Sigma x^2 - \bar{x}^2\right)\left(\frac{1}{n}\Sigma y^2 - \bar{y}^2\right)}} \quad ...(1)$$

Now

$$\bar{x} = \frac{\Sigma x}{n} = \frac{130}{10} = 13$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{220}{10} = 22$$

Now

$$r_{xy} = \frac{\frac{3467}{10} - 13 \times 22}{\sqrt{\left(\frac{2288}{10} - 13^2\right)\left(\frac{5506}{10} - 22^2\right)}}$$

$$= \frac{346.7 - 286}{\sqrt{(228.8 - 169)(550.6 - 484)}}$$

$$= \frac{60.7}{\sqrt{59.8 \times 66.6}} = \frac{60.7}{63.1085} = 0.9618$$

Now

$$\sigma_x^2 = \frac{1}{n}(\Sigma x^2) - \bar{x}^2 = \frac{1}{10} \times 2288 - 13^2$$

$$= 228.8 - 169 = 59.8$$

$$\Rightarrow \quad \sigma_x = 7.73$$

and

$$\sigma_y^2 = \frac{1}{n}(\Sigma y^2) - \bar{y}^2 = \frac{1}{10}(5506) - 22^2$$

$$= 66.6$$

$$\Rightarrow \quad \sigma_y = 8.16$$

Two regression lines are:

Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{0.9618 \times 8.16}{7.73} = 1.015$$

$$\Rightarrow \quad y - 22 = 1.015\,(x - 13)$$

$$\Rightarrow \quad y = 1.015x - 13.195 + 22$$

$$\Rightarrow \quad y = 1.015x + 8.805 \quad ...(1)$$

Regression line x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{r\sigma_x}{\sigma_y} = \frac{0.9618 \times 7.73}{8.16}$$

$$= 0.911$$

$$\Rightarrow \quad x - 13 = 0.911\,(y - 22)$$

$$\Rightarrow \quad x = 0.911y - 7.042 \quad ...(2)$$

To find y when $x = 16$

Using equation (1) we get

$$y = 1.015 \times 16 + 8.805 = 25.045.$$

$$\Rightarrow \quad y \approx 25$$

Q.7. (a) A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B, recorded the following increase in weight: (6.5)

Diet A(gm.)	5	6	8	1	12	4	3	9	6	10
Diet B(gm.)	2	3	6	8	10	1	2	8	-	-

Does it show that superiority of diet A over that of B.

Ans. Here $n_1 = 10, n_2 = 8$,

Null hypothesis: $H_0 : \bar{x}_A = \bar{y}_B$

i.e., there is no significant difference between the two diets.

Alternative hypothesis: $H_1 : \bar{x}_A > \bar{y}_B$ (one tailed test)

Using t-test, we have

$$t = \frac{\bar{x}_A - \bar{y}_B}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$$

Here

$$\bar{x}_A = \frac{\Sigma x}{n_1} = \frac{64}{10} = 6.4$$

$$\bar{y}_B = \frac{\Sigma y}{n_2} = \frac{40}{8} = 5$$

Consider the table

x_A	y_B	$(x_A - \bar{x}_A)^2$	$(y_B - \bar{y}_B)^2$
5	2	1.96	9
6	3	0.16	4
8	6	2.56	1
1	8	29.16	9
12	10	31.36	25
4	1	5.76	16
3	2	11.56	9
9	8	6.76	9
6		0.16	
10		12.96	

Now $\Sigma(x_A - \bar{x}_A)^2 = 102.4$

and $\Sigma(y_B - \bar{y}_B)^2 = 82$

Now
$$S^2 = \frac{\Sigma(x_A - \bar{x}_A)^2 + \Sigma(y_B - \bar{y}_B)^2}{n_1 + n_2 - 2}$$

$$= \frac{102.4 + 82}{16} = \frac{184.4}{16} = 11.525$$

$\Rightarrow$ $s = 3.39$

Consider
$$t = \frac{6.4 - 5}{3.39\sqrt{\frac{1}{10} + \frac{1}{8}}} = \frac{1.4}{3.39\sqrt{0.225}}$$

$$= \frac{1.4}{1.6080} = 0.8706$$

The value of t at 10% level of significance for 16 d.f is 1.75

Since $|t| = 0.8706 < 1.75$

$\therefore$ The hypotheis H_0 is accepted

i.e., there is no significant difference between the two diets.

Q.7. (b) Fit a Poisson distribution to the following data and test for its goodness of fit at 0.05 level of significance.

X:	0	1	2	3	4
F:	419	352	154	56	19

Ans. Null hypotheis H_0: Poisson fit is good fit to data.

Mean of the given distribution,

$$\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\lambda = \frac{0 + 352 + 308 + 168 + 76}{1000}$$

$$\lambda = \frac{904}{1000} = 0.904$$

By Poisson distribution, the frequency of r success is

$$N(r) = N \times e^{-\lambda} \cdot \frac{\lambda^r}{r!}, \text{ N is the total frequency}$$

Now
$$N(0) = 1000 \times \frac{e^{-0.904}}{0!} = 404.94$$

$$N(1) = 1000 \times e^{-0.904} \times \frac{0.904}{1!} = 366.071$$

$$N(2) = 1000 \times e^{-0.904} \times \frac{(0.904)^2}{2!} = 165.464$$

$$N(3) = 1000 \times e^{-0.904} \times \frac{(0.904)^3}{3!} = 49.859$$

$$N(4) = 1000 \times e^{-0.904} \times \frac{(0.904)^4}{4!} = 11.27$$

X	0	1	2	3	4
O_i	419	352	154	56	19
E_i	404.94	366.071	165.464	49.859	11.27

Now $\frac{(Oi - Ei)^2}{Ei}$

for X = 0, $\frac{(419 - 404.94)^2}{404.94} = 0.4882$

for X = 1, $\frac{(352 - 366.071)^2}{366.071} = 0.5409$

for X = 2, $\frac{(154 - 165.464)^2}{165.464} = 0.7943$

for X = 3, $\frac{(56 - 49.859)^2}{49.859} = 0.7564$

for X = 4, $\frac{(19 - 11.27)^2}{11.27} = 5.3019$

$$\chi^2 = \Sigma \frac{(Oi - Ei)^2}{Ei}$$

$= 0.4882 + 0.5409 + 0.7943 + 0.7564$
$+ 5.3019$
$= 7.8817$

The calculated value of χ^2 is 7.8817. Tabulated value of χ^2 at 5% level of significance for

$$v = 5 - 2 = 3 \text{ d.f. is } 7.815$$

Since the calculated value of χ^2 is more than that of tabulated. i.e., $7.8817 > 7.815$

$\Rightarrow H_0$ is rejected

i.e., Poisson distribution does not provide good fit to the data.

UNIT-IV

Q.8. Use penalty (or Big-M) method to solve the probelm:

Max. $z = 6x_1 + 4x_2$ subject to $2x_1 + 3x_2 \le 30$, $3x_1 + 2x_2 \le 24$, $x_1 + x_2 \ge 3$, and $x_1, x_2 \ge 0$. Is the solution is unique? If not, give two different solutions. (12.5)

Ans. The given L.P.P can be written as

$$\text{Max } z = 6x_1 + 4x_2 + 0.x_3 + 0.x_4 + 0.x_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 = 3$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Let us add an artifical variable x_6 to the 3rd constraint, so that our L.P.P. becomes

$$\text{Max } z = x_1 + 5x_2 + 0.x_3 + 0.x_4 + 0.x_5 - Mx_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 + x_6 = 3$$

$x_1, x_2, x_3, x_4, x_5 \ge 0$ and $x_6 \ge 0$ is an artifical variable and $M > 0$ is very very large.

Starting Table

			6	4	0	0	0	–M
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_3	30	2	3	1	0	0	0
0	y_4	24	3	2	0	1	0	0
–M	y_6	3	1	$\boxed{1}$	0	0	–1	1
		$z_j - c_j$	–M–6	–M–4	0	0	M	0

y_2 enters the basis

$$\min\left\{\frac{x_{Bi}}{yi_2}, yi_2 > 0\right\} = \text{Min}\left\{\frac{30}{3}, \frac{24}{2}, \frac{3}{1}\right\} = 3$$

$\Rightarrow y_6$ leaves the basis.

First Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_3	21	–1	0	1	0	$\boxed{3}$
0	y_4	18	1	0	0	1	2
4	y_2	3	1	1	0	0	–1
		$z_j - c_j$	–2	0	0	0	–4

$\therefore y_5$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_5}, yi_5 > 0\right\} = \text{Min}\left\{\frac{21}{3}, \frac{18}{2}\right\} = 7$$

$\Rightarrow y_3$ leaves the basis

Second Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	7	–1/3	0	1/3	0	1
0	y_4	4	$\boxed{5/3}$	0	–2/3	1	0
4	y_2	10	2/3	1	1/3	0	0
		$z_j - c_j$	–10/3	0	4/3	0	0

$\Rightarrow y_1$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_1}, yi_1 > 0\right\} = \text{Min}\left\{\frac{4}{5/3}, \frac{10}{2/3}\right\} = \frac{12}{5}$$

$\Rightarrow y_4$ leaves the basis

Third Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	39/5	0	0	1/5	1/5	1
6	y_1	12/5	1	0	–2/5	3/5	0
4	y_2	42/5	0	1	3/5	–2/5	0
		$z_j - c_j$	0	0	0	2	0

Since all $z_j - c_j \ge 0$.

Thus sol^n is optimal.

$$x_1 = \frac{12}{5}, \; x_2 = \frac{42}{5}$$

$$\text{and Max } z = 6 \times \frac{12}{5} + 4 \times \frac{42}{5} = \frac{240}{5} = 80.$$

Q.9. (a) A method Engineer wants to assign four new methods to three work centers. The assignment of the new methods will increase production and they are given below. If only one method can be assigned to a work center, determine the optimum assignement. **(6)**

Increase in Production (unit)

Methods	Works Centers A	B	C
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

Ans. Since number of tasks and number of subordinates are not equal. We introduce a dummy column.

$$\begin{array}{|cccc|}10 & 7 & 8 & 0\\ 8 & 9 & 7 & 0\\ 7 & 12 & 6 & 0\\ 10 & 10 & 8 & 0\end{array}$$

Locate smallest element from each row and subtract from it.

$$\begin{array}{|cccc|}10 & 7 & 8 & 0\\ 8 & 9 & 7 & 0\\ 7 & 12 & 6 & 0\\ 10 & 10 & 8 & 0\end{array}$$

Locate smallest element from each column and subtract from it

$$\begin{array}{|cccc|}3 & \boxed{0} & 2 & \cancel{0}\\ 1 & 2 & 1 & \boxed{0}\\ \boxed{0} & 5 & \cancel{0} & \cancel{0}\\ 3 & 3 & 2 & \cancel{0}\end{array}$$

Since assigned zero (3) < 4 (order of matrix) optimum solution is not reached.
For optimum solution

$$\begin{array}{|cccc|c}3 & [0] & 2 & 0 & \\ 1 & 2 & 1 & \boxed{0} & \surd\\ [0] & 5 & 0 & 0 & \\ 3 & 3 & 2 & 0 & \surd\\ \hline & & & \surd & \end{array}$$

Since minimum number of lines so drawn is 3, which is less than the order of the cost matrix. To increase minimum number of lines, we generate new zeros in the modified matrix. Smallest element not covered by the lines is 1. Subtracting this element from all the uncovered elements and adding the same to all the element lying at the intersection of the lines, we obtain new reduced cost matrix as:

$$\begin{array}{|cccc|}3 & 0 & 2 & 1\\ 0 & 1 & 0 & 0\\ 0 & 5 & 0 & 1\\ 2 & 2 & 1 & 0\end{array}$$

Repeating the whole procedure, we get:

$$\begin{array}{c|cccc|}& A & B & C & D\\ \hline 1 & 3 & \boxed{0} & 2 & 1\\ 2 & \boxed{0} & 1 & \cancel{0} & \cancel{0}\\ 3 & \cancel{0} & 5 & \boxed{0} & 1\\ 4 & 2 & 2 & 1 & \boxed{0}\\ \hline\end{array}$$

Since each row and each column has one and only one assignement, an optimal solution is reached.

The optimum assignement is:

$1 \to B,\ 2 \to A,\ 3 \to C,\ 4 \to D$

The minimum assignment given to a work center

$$= 7 + 8 + 6 + 0 = 21$$

Q.9. (b) Solve the following Transportation problem: **(6.5)**

Suppliers / Consumers	A	B	C	Available
I	6	8	4	14
II	4	9	8	12
III	1	2	6	5
Required	6	10	15	31

Ans. Since demand and availability = 31

There exists a feasible solution to the transportation problem. we solve by VAM.

$$\begin{array}{|c|c|c|l}6 & 8 & 4 & 14\ (2)\\ \hline 4 & 9 & 8 & 12\ (4)\\ \hline \cancel{1} & \boxed{5}\ \cancel{2} & \cancel{6} & 5\ (1)\\ \hline 6 & 10 & 15 & \\ (3) & (6) & (2) & \end{array}$$

Largest of these differeneces is (6), associated with 2nd column of the table.

Since min. cost in 2nd column is 2, we allocate

$$x_{32} = \min(5, 10) = 5$$

Exhaust 3rd row. Reduced transportation table is

$$\begin{array}{|c|c|c|l}6 & 8 & 4 & 14\ \ (2)\\ \hline \boxed{6}\ \cancel{4} & 9 & 8 & 12\ \ (4)\\ \hline 6 & 5 & 15 & \\ (2) & (1) & (4) & \end{array}$$

Largest of these differences is (4), associated with 2nd row. Since min. cost in 2nd. row is 4, we allocate

$$x_{21} = \min(12, 6) = 6$$

Exhaust Ist column. Reduced table is

8	$\boxed{14}$ ~~4~~	14	(4)
9	8	6	(1)
5	15		
(1)	(4)		

Largest of these differences is (4) in 3rd column. Since min cost in 3rd column is (4) we allocate

$$x_{13} = \min(14, 15) = 14$$

Exhaust Ist row. Reduced table is

$\boxed{5}$ 9	$\boxed{1}$ 8	6
5	1	

Basic feasible solution is

6	8	$\boxed{14}$ 4
$\boxed{6}$ 4	$\boxed{5}$ 9	$\boxed{1}$ 8
1	$\boxed{5}$ 2	6

The transportation cost is

$$\begin{aligned} &= 14\times4+6\times4+5\times9+1\times8+5\times2 \\ &= 56+24+45+8+10 \\ &= 143. \end{aligned}$$

FIRST TERM EXAMINATION [FEB. 2018]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS–IV [ETMA–202]

Time : 1.5 hrs. **M.M. : 30**

Note: *Attempt three questions including Question No. 1. which is compulsory.*

Q. 1. (a) Solve the partial differential equation.

$$\frac{\partial^3 z}{\partial x^3}-6\frac{\partial^3 z}{\partial x^2\partial y}+11\frac{\partial^3 z}{\partial x\partial y^2}-6\frac{\partial^3 z}{\partial y^3}=0 \quad (2)$$

Ans. $\dfrac{\partial^3 z}{\partial x^3}-6\dfrac{\partial^3 z}{\partial x^2\partial y}+11\dfrac{\partial^3 z}{\partial x\partial y^2}-6\dfrac{\partial^3 z}{\partial y^3}=0$

$$D^3-6D^2D'+11DD'^2-6D'^3 = 0$$

Replace D by m and D′ by 1

$$\begin{aligned} \Rightarrow \quad m^3-6m^2+11m-6 &= 0 \\ \Rightarrow \quad (m-1)(m^2-5m+6) &= 0 \\ \Rightarrow \quad (m-1)(m-2)(m-3) &= 0 \\ \Rightarrow \quad m &= 1, 2, 3 \end{aligned}$$

C.F. $= f_1(y+x)+f_2(y+2x)+f_3(y+3x)$

$\therefore$ sol^n is $\quad z = f_1(y+x)+f_2(y+2x)+f_3(y+3x)$

Q. 1. (b) Use the method of separation of variables to solve the equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u. \quad (3)$$

Ans. $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y}+2u.$$

Let $$u = X(x)Y(y)$$

$$\frac{\partial^2 u}{\partial x^2} = X''Y, \quad \frac{\partial u}{\partial y}=XY'$$

$$\begin{aligned} \Rightarrow \quad X''Y &= XY'+2XY \\ \Rightarrow \quad X'' &= X(Y'+2Y) \\ \Rightarrow \quad \frac{X''}{X} &= \frac{Y'}{Y}+2 \end{aligned}$$

* Since x and y are independent variables

$\therefore$ It is true only when each equation is equal to a constant

$$\Rightarrow \quad \frac{X''}{X} = \frac{Y'}{Y}+2=k^2 \text{ (say)}$$

Now $$\frac{X''}{X} = k^2, \quad \frac{Y'}{Y}+2=k^2$$

$$\Rightarrow \quad X''-k^2x = 0, \quad \frac{dy}{Ydy}=k^2-2$$

$$\Rightarrow \quad (D^2-k^2)X = 0, \quad \frac{dy}{Y}=(k^2-2)dy$$

$$D^2 - k^2 = 0 \quad \text{, on integrating}$$

$\Rightarrow$ $$D = \pm k = 0 \quad , \log Y = (k^2 - 2)y + \log C_3$$

$\Rightarrow$ $$X = C_1 e^{kx} + C_2 e^{-kx},\ Y = C_3 e^{(k^2-2)y}.$$

$\Rightarrow$ $$u = (C_1 e^{kx} + C_2 e^{-kx})\, C_3 e^{(k^2-2)y}$$

$\therefore$ $$u = (Ae^{kx} + Be^{-kx})\, e^{(k^2-2)y}$$

$\Rightarrow$

Q. 1. (c) Define r^{th} order moment of random variable X about the point a and find relation between first four order moment of X about mean and origin. (3)

Ans. Moment of random variable X about the point 'a'. Let X be a random variable. Its r^{th} moment about 'a' is defined as

$$V_r(a) = E[(X-a)^r],\ a \in R, r = 1, 2, 3, \ldots.$$

To obtain moments about mean, we consider

$$\mu_1 = \mu_1' - \mu_1' = 0,\ \mu_2 = \mu_2^1 - (\mu_1^1)^2$$
$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$
$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2(\mu_2') - 3(\mu_1')^4$$

Relationship between moments about mean and in terms of moments about any point and conversely

$$\mu_r = \frac{1}{N}\Sigma(X_i - \bar{X})^r,\ \mu_1' = \frac{1}{N}\Sigma(X_i - a)^2$$

$$X_i - \bar{X} = (X_i - a) - (\bar{X} - a)$$

$$\mu_r = \frac{1}{N}\Sigma(X_i - d)^r$$

where $X_i = x_i - a, d = \bar{X} - a$

Using Bionomial theorem

$$= \frac{1}{N}[\Sigma X_i^r - {}^rC_1 d\Sigma Z X_i^{r-1} + {}^rC_2 d^2 \Sigma X_i^{r-2} + \ldots\ldots + {}^rC_{r-1}(-d)^{r-1}\Sigma X_i + (-d)^r]$$

$$= \mu_r' - {}^rC_1 du_{r-1}' + {}^rC_2 d^3 \mu_{r-2}' + \ldots + (-1)^{r-1} d^{r-1}\mu_1' + (-1)^r d^r$$

$$\mu_r = \mu_r' - {}^rC_1 \mu'_{1r-1} + {}^rC_2 \mu_1'^2\, \mu_{r-2}' \lim_{x\to\infty} + (-1)^{r-1}(r-1)\mu_1'^r$$

Putting $r = 1, 2, 3, 4$

$$\mu_1 = \mu'_1 - \mu'_1 = 0$$
$$\mu_2 = \mu'_2 - (\mu'_1)^2$$
$$\mu_3 = \mu_3^1 - 3\mu'_1\mu'_2 + 2(\mu'_1)^3$$
$$\mu_4 = \mu'_4 - 4\mu'_1\mu'_3 + 6(\mu'_1)^2\mu'_2 - 3(\mu'_1)^4$$

Conversely

$$\mu_r' = \frac{1}{N}\Sigma(X_i - a)^r = \frac{1}{N}\Sigma(X - \bar{X} + \bar{x} - a)^r = \frac{1}{N}\Sigma(x_1' - d)^r$$

where $x_i' - x_i - \bar{x}$ and $d = \bar{x} - a$

$$= \frac{1}{N}\left[\Sigma x_i'^r + rC_1 d\Sigma x_i'^{r-1} + {}^rC_2 d^2 \Sigma x_i'^{r-2} + \ldots + {}^rC_{r-1} d^{r-1}\Sigma x_1' + d^r\right]$$

$$= \mu_r + {}^rC_1\mu_{r-1} + d^r C_2 \mu_{r-2} d^2 + \ldots + d^r$$

In particular

$$\mu_2' = \mu_2 + d^2,\ \mu_3' = \mu_3' - 3d\mu_2 + d^3$$
$$\mu_4' = \mu_1 + 4d\,\mu_3 + 6d^2\mu_2 + d^4$$

Moments about zero.

$$\upsilon_1 = \frac{\Sigma fx}{N},\ \upsilon_2 = \frac{\Sigma fx^2}{N},\ \upsilon_3 = \frac{\Sigma fx^3}{N}$$

$$\upsilon_4 = \frac{\Sigma fx^4}{N}$$

or $$\upsilon_1 = A + \mu_1',\ \upsilon_2 = \mu_2 + (\upsilon_1)^2,\ \ \upsilon_3 = \mu_3 + 3\upsilon_1\upsilon_2 - 2\upsilon_1^3$$
$$\upsilon_4 = \mu_4 + 4\upsilon_1\upsilon_3 - 6\upsilon_1^2\upsilon_2 + 3\upsilon_1^4$$

Q. 1. (d) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson Distribution, with mean 1.5. Calculate how many days, in a year, car is not used. (2)

Ans. Let $X \approx \rho(\lambda)$

Here $\lambda = 1.5$

P(x = 0) *i.e.*, car is not used. $= \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-1.5} = 0.2231$

No. of days, car is not used = 0.2231 × 365 = 81.44 days.

Q. 2. (a) Solve the linear partial differential equation.

$$2\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x\partial y} + 2\frac{\partial^2 z}{\partial y^2} = 5\sin(2x+y) \quad (5)$$

Ans. Given equation is

$$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x+y)$$

A.E. $$2m^2 - 5m + 2 = 0$$
$$2m^2 - 4m - m + 2 = 0$$
$$2m(m-2) - 1(m-2) = 0$$

$\Rightarrow$ $$m = 2, \frac{1}{2}$$

$$CF = f_1(y+2x) + f_2\left(y + \frac{x}{2}\right) = f_1(y+2x) + f_2(2y+x)$$

P.I, $$= \frac{1}{2D^2 - 5DD' + 2D'^2} 5\sin(2x+y)$$

Here a = 2, b = 1, let u = 2x + y

$$= \frac{1}{2\times 2^2 - 5\times 2\times 1 + 2\times 1} 5\iint \sin u\, du\, du$$

$$= \frac{1}{8 - 10 + 2} 5\iint \sin u\, du\, du$$

Case of failure

$\therefore$ $$\text{P.I.} = x\frac{1}{4D - 5D'} 5\sin(2x+y)$$

Replace D by 2, D′ by 1

$$= x\frac{1}{8-5} 5\int \sin u\, du = \frac{-5}{3}x\cos(2x+y)$$

Complete sol^n is $$z = f_1(y+2x) + f_2(2y+x) - \frac{5}{3}x\cos(2x+y)$$

Q. 2. (b) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-\frac{3\partial z}{\partial x}+\frac{3\partial z}{\partial y}=e^{3x+2y}$$ (5)

Ans. Given equation is

$$(D^2-D'^2-3D+3D')z = e^{3x+2y}$$

$$\Rightarrow\quad [(D-D')(D+D')-3(D-D')]z = e^{3x+2y}$$

$$\Rightarrow\quad [(D-D')(D+D'-3)]z = e^{3x+2y}$$

for $D-D'$ $\quad b = 1,\ a=1,\ c=0$

for $D+D'-3$ $\quad b = 1,\ a=-1,\ c=3.$

Now $\quad \text{C.F.} = \phi_1(y+x)+e^{3x}\phi_2(y-x)$

$$\text{P.I.} = \frac{1}{(D-D')(D+D'-3)}e^{3x+2y}$$

Replace D by 3, D′ by 2

$$\therefore\quad \text{P.I} = \frac{1}{(3-2)(3+2-3)}e^{3x+2y} = \frac{1}{2}e^{3x+2y}$$

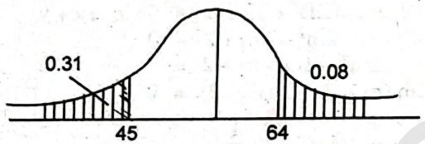
Complete solⁿ is $\quad z = \phi_1(y+x)+e^{3x}\phi_2(y-x)+\dfrac{e^{3x+2y}}{2}$

Q. 3. (a) In a normal Distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. (5)

Ans. Let μ and σ be the required mean and standard deviation.

31% are under 45.

Area to left of ordinate $x = 45$ is 0.31



Area to right of ordinate $x = 64$ is 0.08.

Let $\quad z = \dfrac{x-\mu}{\sigma}$ be the standard normal.

when $\quad x = 45,\ z = \dfrac{45-\mu}{\sigma} = z_1$

$\quad x = 64,\ z = \dfrac{64-\mu}{\sigma} = z_2$

Now $\quad P(x<45) = 0.31 = P(z<z_1)$

$\Rightarrow\quad 0.5-P(z_1<z<0) = 0.31$

$\Rightarrow\quad P(z_1<z<0) = 0.19$

$\Rightarrow\quad z_1 = -0.5$ ...(1) ($z_1<0$)

Also $\quad P(x>64) = 0.08$

$\Rightarrow\quad P(z>z_2) = 0.08$

$\Rightarrow\quad 0.5-P(0<z<z_2) = 0.08$

$\Rightarrow\quad P(0<z<z_2) = 0.5-0.08 = 0.42$

$\Rightarrow\quad z_2 = 1.4$ ...(2)

Thus $\quad \dfrac{45-\mu}{\sigma} = -0.5$

$\Rightarrow\quad 45-\mu = -0.5\sigma$ ...(3)

and $\quad \dfrac{64-\mu}{\sigma} = 1.4$

$\Rightarrow\quad 64-\mu = 1.4\sigma$ ...(4)

Solving (3) and (4), we get

$\quad -19 = -1.9\sigma$

$\Rightarrow\quad \sigma = \dfrac{19}{1.9} = 10$

By (3), we get

$\quad 45-\mu = -0.5\times 10$

$\Rightarrow\quad \mu = 45+5 = 50$

Q. 3. (b) The random variable X can assume the values – 1 and 1 with probability $\frac{1}{2}$ each. Find

(i) Moment generating function

(ii) Skewness and kurtosis (2 + 3)

Ans. Given $\quad P(X=1) = \dfrac{1}{2},\ P(X=-1)=\dfrac{1}{2}$

M.G.F $\quad = E[e^{tX}] = \sum_{x=1,-1} e^{tX}.f_X(x)$

$$M_x(t) = \frac{e^t}{2}+\frac{e^{-t}}{2} = \frac{e^t+e^{-t}}{2} = \cosh t$$

Now

$$E(X) = \frac{d}{dt}M_X(t)\Big|_{t=0} = \frac{d}{dt}\left(\frac{e^t+e^{-t}}{2}\right)\Big|_{t=0} = \frac{e^t-e^{-t}}{2}\Big|_{t=0} = 0$$

$$E(X^2) = \frac{d^2}{dt^2}M_X(t)\Big|_{t=0} = \frac{d^2}{dt^2}\frac{e^t+e^{-t}}{2}\Big|_{t=0} = \frac{e^t+e^{-t}}{2}\Big|_{t=0} = 1$$

$$E(X^3) = \frac{d^3}{dt^3}M_X(t)\Big|_{t=0} = \frac{d}{dt}\frac{e^t+e^{-t}}{2}\Big|_{t=0} = \frac{e^t-e^{-t}}{2}\Big|_{t=0} = 0$$

$$E(X^4) = \frac{d^4}{dt^4}M_X(t)\Big|_{t=0} = \frac{d}{dt}\frac{e^t-e^{-t}}{2}\Big|_{t=0} = \frac{e^t+e^{-t}}{2}\Big|_{t=0} = 1$$

Now,

$$\mu_1 = 0$$

$$\mu_2 = E(X^2)-[E(X)]^2 = 1-0 = 1$$

$$\mu_3 = E[X^3]-3\bar{X}E[X^2]+2(\bar{X})^3 = 0$$

$$\mu_4 = E[(X-\bar{X})^4]$$

$$= E[(X^2+\bar{X}^2-2X\bar{X})^2]$$

$$= E[(X^2+\bar{X}^2)^2+4X^2\bar{X}^2-4X\bar{X}(X^2+\bar{X}^2)]$$

$$= E[X^4+\bar{X}^4+2X^2\bar{X}^2+4X^2\bar{X}^2-4X^3\bar{X}+4X\bar{X}^3]$$

$$= E[X^4+\bar{X}^4+6X^2\bar{X}^2-4X^3\bar{X}+4X\bar{X}^3]$$

$$= E[X^4]+E[\bar{X}^4]+6E(X^2\bar{X}^2)-4E(X^3\bar{X})+4E(X\bar{X}^3)$$

$$= E[X^4]+\bar{X}^4+6\bar{X}^2E(X^2)-4\bar{X}E(X^3)+4\bar{X}^3E(X)$$

$$= 1+0+0-0+0=1$$

Measure of skewness, $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3} = 0$

Measure of kurtosis, $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = \dfrac{1}{1^2} = 1$

Q. 4. An insulated rod of length l is initially at uniform temperature μ_0. Its ends are suddenly cooled at 0°C and kept at that temperature. Find the temperature distribution function u(x, t) **(10)**

Ans. The temperature function $u(x, t)$ satisfies the differential equation

$$\frac{\partial u}{\partial t} = C^2\frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

Let $\quad u = X(x)\,T(t) = XT \quad ...(2)$

(1) becomes

$$XT' = C^2X''T$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{1}{C^2}\frac{T'}{T}$$

Since x and t are independent variables

$$\therefore \quad \frac{X''}{X} = \frac{1}{C^2}\frac{T'}{T} = k\text{(say)}$$

$$\Rightarrow \quad \frac{X''}{X} = k,\ \frac{1}{C^2}\frac{T'}{T} = k$$

$$\Rightarrow \quad X''-kX = 0,\ \frac{dT}{Tdt} = kC^2$$

$$\Rightarrow \frac{dT}{T} = kC^2dt$$

(i) When k is positive $= p^2$(say)

$$\Rightarrow \quad X''-p^2X = 0,\ \int\frac{dT}{T} = \int p^2c^2dt$$

$$(D^2-p^2)X = 0,\ \log T = p^2c^2t+\log c_1$$

A.E. $\quad D^2-p^2 = 0,\ T = C_3e^{p^2c^2t}$

$$D = \pm p$$

$$\therefore \quad X = c_1e^{px}+c_2e^{-px}$$

(ii) When k is negative $= -p^2$ (say)

$$\Rightarrow \quad X''+p^2X = 0,\ \int\frac{dT}{T} = -\int p^2c^2dt$$

$$(D^2+p^2)X = 0,\ \log T = -p^2c^2t+\log C_6$$

A.E. $\quad D^2 = -p^2,\ T = C_6e^{-p^2c^2t}$

$$\Rightarrow \quad D = \pm ip$$

$$X = (C_4\cos px + C_5\sin px)$$

(iii) When K is zero

$$X'' = 0,\ T' = 0$$

$$\Rightarrow \quad X = C_7x+C_8,\ T = C_9$$

Various possible solution of (1) are

$$u(x,t) = (C_1e^{px}+C_2e^{-px})\,C_3e^{-p^2c^2t}$$

$$u(x,t) = (C_4\cos px + C_5\sin px)\,C_6e^{-p^2c^2t}$$

$$u(x,t) = (C_7x+C_8)\,C_9$$

Out of these solution which is consistent with physical nature of problem is

$$u(x,t) = (C_1\cos px + C_2\sin px)\,e^{-p^2c^2t} \quad ....(3)$$

Since, the ends $x = 0$ and $x = l$ are cooled at 0°C and kept at that temperature throughout, the boundary conditions are $u(0, t) = u(l, t) = 0\ \forall\ t$.

Also $\quad u(x, 0) = u_0$ is the initial condition.

Since $\quad u(0, t) = 0$ from (3), we have

$$0 = C_1e^{-c^2p^2t} \Rightarrow C_1 = 0$$

$\therefore$ (3) reduces to

$$u(x,t) = C_2\sin px\,e^{-p^2c^2t} \quad ...(4)$$

Since u $(l$, t) 0, by (4), we get

$$0 = C_2\sin pl\,e^{-p^2c^2t}$$

$$\Rightarrow \quad \sin pl = 0 \Rightarrow pl = n\pi.$$

$$\therefore \quad p = \frac{n\pi}{l},\ \text{n being an integer.}$$

$\therefore$ equation (4) reduces to

$$u(x,t) = C_2\sin\frac{n\pi x}{l}e^{\frac{-c^2n^2\pi^2t}{l^2}},\ n = 1,2,..$$

By adding all solution for n = 1, 2, ..., we get general solution as

$$u(x,t) = \sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{l}e^{\frac{-c^2n^2\pi^2t}{l^2}} \quad ...(5)$$

Since $\quad u(x, 0) = u_0$, we have

$$u_0 = \sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{l}$$

which is half range sine series.

$$\therefore \quad b_n = \frac{2}{l}\int_0^l u_0\sin\frac{n\pi x}{l}dx = \frac{2}{l}u_0\left|-\cos\frac{n\pi x}{l}.\frac{l}{n\pi}\right|_0^l$$

$$= \frac{2\mu_0 l}{l n\pi}[-\cos n\pi+1] = \frac{2u_0}{n\pi}[1-(-1)^n]$$

$$\Rightarrow \quad b_n = \begin{cases} 0, & \text{n is even} \\ \dfrac{4u_0}{n\pi}, & \text{n is odd.}\end{cases}$$

Hence the temperature function is

$$u(x,t) = \frac{4\mu_0}{\pi}\sum_{n=1,3,\ldots}\frac{1}{n}\sin\frac{n\pi x}{l}e^{-c^2n^2\pi^2t/l^2}$$

$$= \frac{4\mu_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\frac{\sin(2n-1)x\pi}{l}e^{-c^2\pi^2t(2n-1)^2/l^2}$$

Or

Q. 4. Find the constant C such the the function $f(x) = \frac{C}{x^2+1}$, where $-\infty < x < \infty$ is a density function of continuous random variable x and compute $P\left\{\frac{1}{3} < x < 1\right\}$ and distribution function F(X).

Ans. Since X is a continuous random variable, then

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{C}{x^2+1}dx = 1$$

$$\Rightarrow C\left|\tan^{-1}x\right|_{-\infty}^{\infty} = 1$$

$$\Rightarrow C\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1$$

$$\Rightarrow \pi C = 1 \Rightarrow C = 1/\pi.$$

Now, c.d. f

$$F_x(x) = \int_{-\infty}^{x}\frac{c}{x^2+1}dx = \frac{1}{\pi}\int_{-\infty}^{x}\frac{dx}{x^2+1} = \frac{1}{\pi}\left|\tan^{-1}x\right|_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left(\tan^{-1}x+\frac{\pi}{2}\right)$$

Now $$P\left\{\frac{1}{3} \le X \le 1\right\} = \int_{1/3}^{1} f(x)dx = \int_{1/3}^{1}\frac{C}{x^2+1}dx$$

$$= \frac{1}{\pi}\int_{1/3}^{1}\frac{dx}{x^2+1} = \frac{1}{\pi}\left|\tan^{-1}x\right|_{1/3}^{1}$$

$$= \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}\frac{1}{3}\right]$$

END TERM EXAMINATION [MAY-JUNE 2018]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS–IV [ETMA-202]

Time : 3 hrs. **M.M. : 75**

Note: *Attempt five questions including Question No. 1. which is compulsory. Select one question from each unit.*

Q. 1. (a) Find particular integral of $(4D^2 + 3DD' - D'^2 - D - D')Z = 3e^{(x+2y)/2}$. **(6)**

Ans. Given $(4D^2 + 3DD' - D'^2 - D - D')Z = 3e^{(x+2y)/2}$.

$$\text{P.I.} = \frac{1}{4D^2+3DD'-D'^2-D-D'}3e^{\left(\frac{x}{2}+y\right)}$$

$$= 3\frac{1}{4\times\frac{1}{4}+\frac{3}{2}-1-\frac{1}{2}-1}e^{\left(\frac{x}{2}+y\right)}$$

Case of failure

$$= 3x\frac{1}{8D+3D'-1}e^{\left(\frac{x}{2}+y\right)} = x\frac{3}{8\times\frac{1}{2}+2}e^{\left(\frac{x}{2}+y\right)} = \frac{x}{2}e^{\left(\frac{x}{2}+y\right)}$$

Q. 1. (b) A die is tossed twice. Getting '5' or '6' on a toss is taken as success. Find the probability distribution, the mean and the variance of the number of successes.

Ans. Let X = No. of success.

i.e., X takes the value 0, 1, 2

Let E be the event of getting 5 or 6 on a toss.

$$\therefore \quad P(E) = \frac{2}{6} = \frac{1}{3}$$

$$P(\bar{E}) = 1 - P(E) = 1 - \frac{1}{3} = \frac{2}{3}$$

Now

$$P(X=0) = \frac{2}{3}\times\frac{2}{3} = \frac{4}{9}$$

$$P(X=1) = \frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3} = \frac{2}{9}+\frac{2}{9} = \frac{4}{9}$$

$$P(X=2) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$$

Probability distribution is as follows

$X = x$	$P(X) = p$	px	px^2
0	4/9	0	0
1	4/9	4/9	4/9
2	1/9	2/9	4/9

$$\text{Mean} = \Sigma px = \frac{4}{9}+\frac{2}{9}=\frac{6}{9}=\frac{2}{3}$$

$$\text{Variance} = \Sigma px^2 - (\Sigma px)^2 \qquad = \left(\frac{4}{9}+\frac{4}{9}\right)-\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Q. 1. (c) Prove that coefficient of correlation is geometrical mean of coefficients of regression but arithmetic mean of coefficients of regression is greater than the coefficient of correlation. **(6)**

Ans. The regression coefficients are $\frac{r\sigma_y}{\sigma_x}$ and $\frac{r\sigma_x}{\sigma_y}$

G.M between them $= \sqrt{\frac{r\sigma_y}{\sigma_x}\times\frac{r\sigma_x}{\sigma_y}} = \sqrt{r^2}$

$= r =$ correlation coefficient.

Now, to show that

$$\frac{b_{yx}+b_{xy}}{2} > r$$

or
$$\frac{\frac{r\sigma_y}{\sigma_x}+\frac{r\sigma_x}{\sigma_y}}{2} > r$$

$\Rightarrow$ $\sigma_y^2+\sigma_x^2 > 2\,\sigma_x\sigma_y$

$\Rightarrow$ $\sigma_y^2+\sigma_x^2 - 2\,\sigma_x\sigma_y > 0$

$\Rightarrow$ $(\sigma_y - \sigma_x)^2 > 0$ which is true

Q. 1. (d) For the following transportation problem and find the basic Feasible solution using VAM.

	1	2	3	4	5	Supply
1	4	2	3	2	6	8
2	5	4	5	2	1	12
3	6	5	4	7	3	14
	4	4	6	8	8	

Ans. Since Demand = 30 and supply = 34

Here Demand < Supply

	1	2	3	4	5	6		
1	4	2	3	2	6	0	8	(2)
2	5	4	5	2	1	0	12	(1)
3	6	5	4	7	3	$\boxed{4}$ 0	14	(3)
	4	4	6	8	8	4	34	
	(1)	(2)	(1)	(0)	(2)	(0)		

Largest difference is (3) associated with third row. Since least cost in 3rd row is 0 in 6th column

allocate $x_{36} = \min(14, 4) = 4$

Exhaust 6th column

	1	2	3	4	5		
1	4	$\boxed{4}$ 2	3	2	6	8	(0)
2	5	4	5	2	1	12	(1)
3	6	5	4	7	3	10	(1)
	4	4	6	8	8		
	(1)	(2)	(1)	(0)	(2)		

Largest difference is (2) associated with 2nd column.
Since least cost is 2 in 2nd column, allocate

$$x_{12} = \min(8, 4) = 4$$

Exhaust 2nd column.

	1	3	4	5		
1	4	3	2	6	4	(1)
2	5	5	2	$\boxed{8}$ 1	12	(1)
3	6	4	7	3	10	(1)
	4	6	8	8		
	(1)	(1)	(0)	(2)		

Largest difference is (2) associated with 5th column. Since least cost is 1 in 5th column, allocate

$$x_{25} = \min(12, 8) = 8$$

Exhaust 5th column.

	1	3	4		
1	4	3	2	4	(1)
2	5	5	$\boxed{4}$ 2	4	(3)
3	6	4	7	10	(2)
	4	6	8		
	(1)	(1)	(0)		

Largest difference is (3) associated with 2nd row. Least cost is 2 is 2nd row, allocate

$$x_{24} = \min(4, 8) = 4$$

Exhaust 2nd row.

	1	3	4		
1	4	3	$\boxed{4}$ 2	4	(1)
3	6	4	7	10	(2)
	4	6	4		
	(2)	(1)	(5)		

Largest difference is (5) associated with 4th column.
Least cost is 2 in 4th column, allocate

$$x_{14} = \min(4, 4) = 4$$

Exhaust 4th column

	1	3	
1	[0] 4	3	0 (1)
3	6	4	10 (2)
	4	6	
	(2)	(1)	

Largest difference is (2), associated in 1 column. Least cost is 4, allocate x_{11} = min (0, 4) = 0

Exhaust 1st row.

	1	3	
3	[4] 6	[6] 4	10
	4	6	

Final table is

	1	2	3	4	5	6
1	[0] 4	[4] 2	3	[4] 2	6	0
2	5	4	5	[4] 2	[8] 1	0
3	[4] 6	[6] 5	4	7	3	[4] 0

As occupied cell = 8

Required cell = (6 + 3 – 1) = 8.

∴ Optimal solution is

$0 \times 4 + 4 \times 2 + 4 \times 2 + 4 \times 2 + 8 \times 1 + 4 \times 6 + 6 \times 5 + 4 \times 0 = 80$

UNIT-I

Q. 2. (a) Find complete solution of

$$\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial x^2\partial y}+5\frac{\partial^3 z}{\partial x\partial y^2}-2\frac{\partial^3 z}{\partial y^3}=e^{y+2x}+(y+x)^{1/2}$$ **(8.5)**

Ans. $(D^3-D^2D'+5DD'^2-2D'^3)Z=e^{y+2x}+(y+x)^{1/2}$

Replace $D \to m$, $D' \to 1$

$$(m^3-4m^2+5m-2)\,Z = e^{y+2x}+(y+x)^{1/2}$$

A.E. $\quad m^3-4m^2+5m-2 = 0$

$\Rightarrow \quad (m-1)(m^2-3m+2) = 0$

$\Rightarrow \quad (m-1)^2(m-2) = 0$

$\Rightarrow \quad m = 1, 1, 2$

$$\text{C.F.} = f_1(y+x)+xf_2(y+x)+f_3(y+2x)$$

$$\text{P.I.} = \frac{1}{D^3-4D^2D'+5DD'^2-2D'^3}e^{y+2x}+(y+x)^{1/2} \quad ...(1)$$

Now
$$\frac{1}{D^3-4D^2D'+5DD'^2-D'^3}e^{y+2x}$$

$$=\frac{1}{2^3-4\times 2^2+5\times 2-2}e^{y+2x} \quad \text{(case of failure)}$$

$$= x\frac{1}{3D^2-8DD'+5D'^2}e^{y+2x}$$

$$=\frac{1}{3\times 4\times 8\times 2+5}e^{y+2x}$$

$$=\frac{x}{12-16+5}e^{y+2x}=xe^{y+2x}.$$

Also
$$\frac{1}{D^3-4D^2D'+5DD'^2-2D'^3}(y+x)^{1/2}$$

$$=\frac{1}{1-4+5-2}(y+x)^{1/2}$$

(case of failure)

$$= x\frac{1}{3D^2-8DD'+5D'^2}(y+x)^{1/2}=x\frac{1}{3-8+5}(y+x)^{1/2}$$

(case of failure)

$$= x^2\frac{1}{6D-8D'}(y+x)^{1/2}$$

$$=\frac{-x^2}{2}\int V^{1/2}dV=\frac{-x^2}{2}\cdot\frac{2}{3}(y+x)^{3/2}=\frac{-x^2}{3}(y+x)^{3/2}$$

Complete solⁿ is

$$Z=f_1(y+x)+xf_2(y+x)+f_3(y+2x)+xe^{y+2x}-\frac{x^2}{3}(y+x)^{3/2}$$

Q. 2. (b) Using the method of separation of variable solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$, where $u(x,0)=4e^{-x}$, $x>0$, $y>0$ **(4)**

Ans. Given equation is $3\frac{\partial u}{\partial x}+\frac{2\partial u}{\partial y}=0$ ...(1)

Let
$$U = X(x)\,Y(y) = XY$$

$$\therefore \quad \frac{\partial u}{\partial x} = X'Y,\ \frac{\partial u}{\partial y}=XY'$$

Thus (1) $\Rightarrow 3X'Y+2XY'=0$

$$\Rightarrow \quad \frac{3X'}{X}=-\frac{2Y'}{Y}$$

As L.H.S is a function of x only and RHS is a function of y only.

$$\Rightarrow \quad \frac{3X'}{X}=\frac{-2Y'}{Y}=a \text{ (say)}$$

$$\Rightarrow \quad \frac{3X'}{X}=a,\ -\frac{2Y'}{Y}=a$$

$$\Rightarrow \quad \frac{dX}{X}=\frac{a}{3}dx,\ \frac{dY}{Y}=-\frac{a}{2}dy$$

$$\log X = \frac{ax}{3} + \log C_1,\ \log Y = -\frac{a}{2}y + \log C_2$$

$\Rightarrow$ $$X = C_1 e^{ax/3},\ Y = C_2 e^{-ay/2}$$

$\Rightarrow$ $$U = C_1C_2 e^{ax/3} e^{-ay/2} \qquad ...(2)$$

$\therefore$ $$y = 0$$

for $$u(x,0) = 4e^{-x} = C_1C_2 e^{ax/3}$$

On comparing, we get $$4e^{-x} = C_1C_2 e^{ax/3}$$

$\Rightarrow$ $$C_1C_2 = 4,\ \frac{a}{3} = -1 \Rightarrow a = -3.$$

$\therefore$ Complete sol^n is $$U = 4e^{-x} e^{3y/2}.$$

Q. 3. (a) Solve $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} - \dfrac{3\partial u}{\partial x} + \dfrac{3\partial u}{\partial y} = e^{(2x+3y)} + \sin(x+2y)$ **(6)**

Ans. $D^2 - D'^2 - 3D + 3D' = e^{(2x+3y)} + \sin(x+2y)$

$(D-D')(D+D') - 3(D-D') = e^{2x+3y} + \sin(x+2y)$

$\Rightarrow (D-D')(D+D'-3) = e^{2x+3y} + \sin(x+2y)$

for $D - D'$, $b = 1, a = 1, c = 0$

for $D + D' - 3$, $b = 1, a = -1, c = 3$

$\therefore$ $\quad$ C.F $= f_1(y+x) + e^{3x} f_2(y-x)$

P.I. $\dfrac{1}{D^2 - D'^2 - 3D + 3D'} e^{2x+3y} + \sin(x+2y)$

$$\Rightarrow \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{2x+3y} + \frac{1}{D^2 - D'^2 - 3D - 3D'} \sin(x+2y)$$

$$\Rightarrow \frac{1}{4-9-6+9} e^{2x+3y} + \frac{1}{-1+4-3D+3D'} \sin(x+2y)$$

$$\Rightarrow \frac{-1}{2} e^{2x+3y} + \frac{1}{3D'-3D+3} \sin(x+2y)$$

$$\Rightarrow \frac{-1}{2} e^{2x+3y} + \frac{1}{3+3(D'-D)} \sin(x+2y)$$

$$\Rightarrow \frac{-1}{2} e^{2x+3y} + \frac{3(D'-D)-3}{[3(D'-D)+3][3(D'-D)-3]} \sin(x+2y)$$

$$\Rightarrow -\frac{1}{2} e^{2x+3y} + \frac{3(D'-D)-3}{9(D'^2+D^2-2DD')-9} \sin(x+2y)$$

$$\Rightarrow -\frac{1}{2} e^{2x+3y} + \frac{3D'\sin(x+2y) - 3D\sin(x+2y) - 3\sin(x+2y)}{9(-4-1+4)-9}$$

$$\Rightarrow -\frac{1}{2} e^{2x+3y} + \frac{6\cos(x+2y) - 3\cos(x+2y) - 3\sin(x+2y)}{(-18)}$$

$$\Rightarrow -\frac{1}{2} e^{2x+3y} + \frac{3\cos(x+2y) - 3\sin(x+2y)}{(-18)}$$

$$\Rightarrow -\frac{1}{2} e^{2x+3y} + \frac{\sin(x+2y) - \cos(x+2y)}{6}$$

$\therefore$ Complete sol^n is

$$Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{2} e^{2x+3y} + \frac{\sin(x+2y) - \cos(x+2y)}{6}$$

Q. 3. (b) Solve the partial differential equation $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$ **representing the vibration of a string of length l, fixed at both ends and satisfies the boundary conditions $y(0, t) = 0$, $y(l, t) = 0$ and initial conditions $y = y_0 \sin \dfrac{\pi x}{l}, t = 0$ and** $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} = 0.$ **(6.5)**

Ans. Wave equation is given by

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \qquad ...(1)$$

Let $$Y = X(x)\,T(t) = XT$$

$\Rightarrow$ $$XT'' = a^2 X''T$$

$\Rightarrow$ $$\frac{X''}{X} = \frac{1}{a^2}\frac{T''}{T} \qquad ....(2)$$

Since x and t are independent functions

$\Rightarrow$ (2)... $$\frac{X''}{X} = \frac{1}{a^2}\frac{T''}{T} = -p^2 \text{ (say)}$$

$\Rightarrow$ $$X'' + p^2 X = 0,\ T'' + p^2a^2 T = 0$$

$\Rightarrow$ $$(D^2 + p^2)X = 0,\ (D^2 + p^2a^2)T = 0$$

A.E. $$D^2 + p^2 = 0,\ \text{A.E. } D^2 + p^2a^2 = 0$$

$\Rightarrow$ $$D = \pm ip \ \Rightarrow\ D = \pm ip\,a$$

$\therefore$ $$X = C_1 \cos px + C_2 \sin px,\ T = C_3 \cos p\,at + C_4 \sin p\,at$$

$\Rightarrow$ $$Y = (C_1 \cos px + C_2 \sin px)(C_3 \cos p\,at + C_4 \sin p\,at) \qquad ...(3)$$

Since boundary conditions are

$$y(0,t) = 0 = y(l,t)$$

At $$x = 0,\ y = 0$$

$\Rightarrow$ (3)... $$0 = C_1(C_3 \cos p\,at + C_4 \sin p\,at)$$

$\Rightarrow$ $$C_1 = 0$$

Thus (3) reduces to

$$Y = C_2 \sin px\,(C_3 \cos apt + C_4 \sin apt) \qquad ...(4)$$

As $$y = 0 \text{ at } x = l$$

$\therefore$ (4)... $\Rightarrow$ $$0 = C_2 \sin pl\,(C_3 \cos apt + C_4 \sin apt)$$

This is satisfied when $\sin pl = 0$

$\Rightarrow$ $$pl = n\pi$$

$$\Rightarrow \qquad p = \frac{n\pi}{l} \text{ where } n = 1, 2, 3.....$$

Thus solution of (4) is

$$y = C_2\left(C_3 \cos\frac{n\pi a}{l}t + C_4 \sin\frac{n\pi a}{l}t\right)\sin\frac{n\pi x}{l}$$

$$\Rightarrow \qquad y = \left(a_n \cos\frac{n\pi a}{l}t + b_n \sin\frac{n\pi a}{l}t\right)\sin\frac{n\pi x}{l}$$

where $a_n = C_2C_3, b_n = C_2C_4$

Adding solution for different values of n.

$$y = \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi at}{l} + b_n \sin\frac{n\pi at}{l}\right)\sin\frac{n\pi x}{l} \qquad ...(5)$$

Given $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ and $y(x, 0) = y_0 \frac{\sin \pi x}{l}$.

Consider $\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty}\left(-a_n \frac{n\pi a}{l}\sin\frac{n\pi a}{l}t + bn\frac{n\pi a}{l}\cos\frac{n\pi at}{l}\sin\frac{n\pi x}{l}\right)$

At $t = 0$

$$0 = \sum_{n=1}^{\infty} b_n \frac{n\pi a}{l}\sin\frac{n\pi x}{l} \Rightarrow b_n = 0 \,\forall n$$

∴ (5) reduces to

$$y = \sum_{n=1}^{\infty} a_n \cos\frac{n\pi a}{l} t \sin\frac{n\pi x}{l} \qquad ...(6)$$

Also at $t = 0, y = y_0 \sin\frac{\pi x}{l}$.

(6) gives

On $y_0 \sin\frac{\pi x}{l} = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{l}$

This represents fourier sine series for $y_0 \frac{\sin \pi x}{l}$

$$\therefore \qquad a_n = \frac{2}{l}\int_0^l y_0 \sin\frac{\pi x}{l}\sin\frac{n\pi x}{l}dx$$

We solve for $n = 1$.

(all other terms vanish)

$$\therefore \qquad a_1 = \frac{2y_0}{l}.\int_0^l \sin^2\frac{\pi x}{l}dx = \frac{2y_0}{l} - \frac{1}{2}\int_0^l\left(1-\cos\frac{2\pi x}{l}\right)dx$$

$$= \frac{y_0}{l}\left[x - \frac{l}{2\pi}\sin\frac{2\pi x}{l}\right]_0^l = \frac{y_0}{l}[l - 0 - 0 + 0] = y_0$$

Thus (6) reduces to

$$y = y_0 \cos\frac{\pi at}{l}\sin\frac{\pi x}{l}.$$

UNIT-II

Q. 4. (a) A die is tossed thrice. Getting '5' or '6' on a toss is taken as success. Find the probability distribution, the mean and the variance of the number of successes. (6)

Ans. Let X ≡ no. of successes.

i.e., X = 0, 1, 2, 3

Let E be the event of getting '5' or '6' on toss.

$$P(E) = \frac{2}{6} = \frac{1}{3} \text{ and } P(\bar{E}) = 1 - P(E) = \frac{2}{3}.$$

Now

$$P(X = 0) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

$$P(X = 1) = 3C_1 \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{12}{27}$$

$$P(X = 2) = 3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{6}{27}$$

$$P(X = 3) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}.$$

Probability distribution is as follows

$X = x$	$P(x) = p.$	px	px^2
0	8/27	0	0
1	12/27	12/27	12/27
2	6/27	12/27	24/27
3	1/27	3/27	9/27

$$\text{Mean} = \Sigma px = \frac{12}{27} + \frac{12}{27} + \frac{3}{27} = \frac{27}{27} = 1$$

$$\text{Variance} = \Sigma px^2 - (\Sigma px)^2$$

$$= \frac{12}{27} + \frac{24}{27} + \frac{9}{27} - 1 = \frac{45}{27} - 1 = \frac{5}{3} - 1 = 2/3$$

Q. 4. (b) A random variable X has the density function $f(x) = \frac{C}{x^2+1}$, where $-\infty < x < \infty$. **(1.5 + 2.5 + 2.5 = 6.5)**

(i) Find the value of the constant C.

(ii) Find the distribution function corresponding to the given density.

(iii) Find the probability that x lies between $\frac{1}{3}$ and 1. (6.5)

Ans. Given $f(x) = \frac{C}{x^2+1}, -\infty < x < \infty$

(i) As $f_x(x) = 1$

$$\Rightarrow \qquad \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} \frac{C}{x^2+1} dx = 1$$

$$\Rightarrow \quad C \int_{-\infty}^{\infty} \tan^{-1} x \, dx = 1$$

$$\Rightarrow \quad C\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 1 \Rightarrow C = \frac{1}{\pi}$$

(ii) For continuous random variable

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t) dt$$

$$\Rightarrow \quad F_X(x) = \int_{-\infty}^{x} \frac{1}{\pi(x^2+1)} dx = \frac{1}{\pi} |\tan^{-1} x|_{-\infty}^{x}$$

$$F_X(x) = \frac{1}{\pi}\left(\tan^{-1} x + \frac{\pi}{2}\right)$$

(iii) $$P\left(\frac{1}{3} < X < 1\right) = \int_{1/3}^{1} f(x) dx = \int_{1/3}^{1} \frac{1}{\pi(x^2+1)} dx$$

$$= \frac{1}{\pi}(\tan^{-1} x)_{1/3}^{1} = \frac{1}{\pi}\left[\tan^{-1} 1 - \tan^{-1} \frac{1}{3}\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \tan^{-1} \frac{1}{3}\right]$$

Q. 5. (a) In the normal distribution 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution. (6)

Ans. Let μ and σ be the required mean and standard deviation of the distribution.

7% are cover under 35.

Area to left of ordinate $x = 35$ is 0.07

Area to right of ordinate $x = 63$ is 0.89

Let $Z = \frac{x-\mu}{\sigma}$ be the standard normal.

when $x = 35, \; z = \frac{35-\mu}{\sigma} = Z_1$

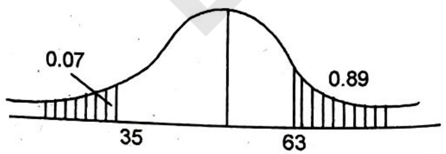
$\Rightarrow \quad P(x < 35) = 0.07$

$\Rightarrow \quad P(Z < Z_1) = 0.07$

$\Rightarrow \quad 0.5 - P(Z_1 < Z < 0) = 0.07$

$\Rightarrow \quad P(Z_1 < Z) = 0.43$

$\Rightarrow \quad Z_1 = -1.48$ ...(1)



When $x = 63, Z = \frac{63-\mu}{\sigma} = Z_2$

$\Rightarrow \quad P(x > 63) = 0.89$

$\Rightarrow \quad P(Z > Z_2) = 0.89$

$\Rightarrow \quad 0.5 - P(0 < Z < Z_2) = 0.89$

$\Rightarrow \quad P(0 < Z < Z_2) = 0.5 - 0.89 = -0.39$

$\Rightarrow \quad Z_2 = 1.23$ ...(2)

Thus $\frac{35-\mu}{\sigma} = -1.48$

$\Rightarrow \quad 35 - \mu = -1.48\,\sigma$ ...(3)

and $\frac{63-\mu}{\sigma} = 1.23$

$\Rightarrow \quad 63 - \mu = 1.23\,\sigma$ ...(4)

Solving (3) and (4) we get

$2.71\,\sigma = 28$

$\Rightarrow \quad \sigma = \frac{28}{2.71} = 10.33$

By equation(3) $35 - \mu = -1.48 \times 10.33$

$\Rightarrow \quad \mu = 50.29$

Mean $= 50.29$

Variance $= (10.33)^2 = 106.71$

Q. 5. (b) If the variance of the Poisson's distribution is 2 then find the probability for r = 1, 2, 3 and 4. Also find $P(r \le 4)$ (6.5)

Ans. Given $\lambda = 2$

and $X \equiv P(\lambda)$

$$\Rightarrow \quad P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!} = \frac{2^r e^{-2}}{r!}$$

Recurrence relation for Poisson distribution

$$P(r+1) = \frac{\lambda}{r+1} P(r) = \frac{2}{r+1} P(x) \quad ...(1)$$

for $r = 0, \; P(X = 0) = P(0) = e^{-2} = 0.1353$

put $r = 0, 1, 2, 3$ in (1)

$P(1) = 2P(0) = 2 \times 0.1353 = 0.2706$

$P(2) = \frac{2}{2} P(1) = 0.2706$

$P(3) = \frac{2}{3} P(2) = \frac{2}{3} \times 0.2706 = 0.1804$

$P(4) = \frac{2}{4} P(3) = \frac{1}{3} \times 0.1804 = 0.0902$

Now $P(r \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$= 0.1353 + 0.2706 + 0.2706 + 0.1804 + 0.0902 = 0.9471$

UNIT-III

Q. 6. (a) Following data is for the measurement of train resistance R(1 ds/ton) with the velocity (V/mph). If $R = a + bV + cV^2$, find a, b and c. (8.5)

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46

Ans. Given $R = a + bV + cV^2$

Then normal equations are

$\Sigma R = na + b\Sigma V + c\Sigma V^2$

$\Sigma VR = a\Sigma V + b\Sigma V^2 + c\Sigma V^3$

$$\Sigma V^2R = a\Sigma V^2 + b\Sigma V^3 + c\Sigma V^4$$

V	R	VR	V^2	V^3	V^4	V^2R
20	5.5	110	400	8000	160000	2200
40	9.1	364	1600	64000	2560000	14560
60	14.9	894	3600	216000	12960000	53640
80	22.8	1824	6400	512000	40960000	145920
100	33.3	3330	10000	1000000	100000000	333000
120	46	5520	14400	1728000	207360000	662400

$$\Sigma V = 420,\ \Sigma R = 131.6,\ \Sigma VR = 12042$$
$$\Sigma V^2 = 36400,\ \Sigma V^3 = 3528000,\ \Sigma V^4 = 364000000$$
$$\Sigma V^2R = 1211720$$

Now
$$131.6 = 6a + 420b + 36400c \quad ...(1)$$
$$12042 = 420a + 36400b + 3528000c \quad ...(2)$$
$$1211720 = 36400a + 35280000b + 364000000c \quad ...(3)$$
$$\Rightarrow \quad 65.8 = 3a + 210b + 18200c \quad ...(4)$$
$$6021 = 210b + 18200b + 1764000c \quad ...(5)$$
$$151465 = 4550a + 441000b + 4550000c \quad ...(6)$$

Q. 6. (b) Following table shows how 10 students were ranked according to their achievements in both the practical and theory of a mathematics course. Find the coefficient of rank correlation. (4)

Practical	8	3	9	2	7	10	4	6	1	5
Theory	9	5	10	1	8	7	3	4	2	6

Ans.

Practical (R_1)	Theory (R_2)	$R_1 - R_2 = d$	d^2
8	9	–1	1
3	5	–2	4
9	10	–1	1
2	1	1	1
7	8	–1	1
10	7	3	9
4	3	1	1
6	4	2	4
1	2	–1	1
5	6	–1	1

Now $\Sigma d^2 = 24,\ n = 10$

Coefficient of rank correlation

$$r = \frac{1-6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 24}{10(100-1)}$$

$$\Rightarrow \quad r = 1 - \frac{144}{990} = \frac{846}{990} = 0.8545$$

Q. 7. (a) The following table gives the number of accidents took place in an area during various days of the week. Test if accidents are uniformly distributed over the week.

Days	Mon	Tue	Wed	Thus	Fri	Sat
No. of accidents	14	18	12	11	15	14

(6)

Ans. Null hypothesis H_0: The accidents are uniform distributed over the week.

Under this H_0, the expected frequencies of the accidents on each of these days $= \frac{84}{6} = 14$

Observed freq. (O_i)	14	18	12	11	15	14
Expected freq. (E_i)	14	14	14	14	14	14
$(O_i - E_i)^2$	0	16	4	9	1	0

$$\therefore \quad \chi^2 = \frac{\Sigma(O_i - E_i)^2}{E_i} = \frac{16+4+9+1}{14} = \frac{30}{14}$$
$$= 2.1428$$

Tabular value of χ^2 at 5% level for $(6-1) = 5$ d.f is 11. 09.

Calculated value of χ^2 is less than the tabulated values, H_0 is accepted i.e. the accidents are uniformly distributed over the week.

Q. 7. (b) A manufacturer claims that only 4% of the products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of the manufacturer. (6.5)

Ans. Let p = Observed proportion of success.

i.e. p = Proportion of defective products in the sample $= \frac{36}{600} = 0.06$

P = Proportion of defectives in the population = 0.04

$$Q = 1 - P = 0.96$$

Null hypothesis H_0; $P = 0.04$ is true

i.e., the claim of the manufacturer is accepted.

Alternative hypothesis H_1: $P \neq 0.04$ (two tailed test)

Under H_0,
$$Z = \frac{p-P}{\sqrt{PQ/n}} = \frac{0.06-0.04}{\sqrt{\frac{0.04\times 0.96}{600}}} = 2.5$$

Conclusion. Since $|z| = 2.5 > 1.96$, we reject the hypothesis H_0 at 5% level of significance i.e. the manufacturer's claim is not acceptable.

UNIT-IV

Q. 8. (a) Convert the following L.P.P into the standard form

maximize $Z = 2x_1 + 3x_2 + 6x_3$

Subject to
$$x_1 + 2x_2 \leq 7$$
$$3x_1 + 4x_2 - 6x_3 \geq 10$$

$$4x_1 + 3x_2 \le 5 \qquad (5)$$

and $x_1, x_2 \ge 0$, x_3 unrestricted

Ans. x_3 is unrestricted

$$x_3 = x_3' - x_3''.$$

$$\Rightarrow \quad \text{Max } Z = 2x_1 + 3x_2 + 6(x_3' - x_3'')$$

s.t.

$$x_1 + 2x_2 \le 7$$

$$3x_1 + 4x_2 - 6x_3' + 6x_3'' \ge 10$$

$$4x_1 + 3x_2 \le 5$$

Introduce slack and surplus variables x_4, x_5, x_6

$$\text{Max } Z = 2x_1 + 3x_2 + 6x_3' - 6x_3'' + 0.x_4 + 0.x_5 + 0.x_6$$

s.t.

$$x_1 + 2x_2 + x_4 = 7$$

$$3x_1 + 4x_3 - 6x_3' + 6x_3'' - x_5 = 10$$

$$4x_1 + 3x_2 + x_5 = 5.$$

$$x_1, x_2, x_3', x_3'', x_4, x_5, x_6 \ge 0$$

Q. 8. (b) An engineer wants to assign 3 jobs to three machines M_1, M_2 and M_3 in such a way that each jobs is assigned to some machine and no machine works on more than one job. The cost matrix is given as follows. (7.5)

	M_1	M_2	M_3
J_1	15	10	9
J_2	9	15	10
J_3	10	12	8

Find the job and corresponding machine so as to minimize the total cost.

Ans. Since number of sources is equal to the number of destinations.

Subtracting smallest element of each row, we get

$$\begin{bmatrix} 6 & 1 & 0 \\ 0 & 6 & 1 \\ 2 & 4 & 0 \end{bmatrix}$$

Subtract smallest element from each column.

$$\begin{bmatrix} 6 & \boxed{0} & \cancel{0} \\ \boxed{0} & 5 & 1 \\ 2 & 3 & \boxed{0} \end{bmatrix}$$

Since number of assignment = 3 = order of matrix

Thus optimum solution is

	M_1	M_2	M_3
J_1	15	$\boxed{10}$	9
J_2	$\boxed{9}$	15	10
J_3	10	12	$\boxed{8}$

$$\Rightarrow \quad J_1 \to M_2, J_2 \to M_1, J_3 \to M_3.$$

$$10 + 9 + 8 = 27$$

Q. 9. (a) Solve the following LPP using simplex method

$$\text{Max } Z = 107x + y + 2z$$

Subject to

$$14x + y - 6z + 3t = 7$$

$$16x + 0.5y - 6z \le 5$$

$$3x - y - z \le 0$$

$$x, y, z \ge 0 \qquad (8.5)$$

Ans.

$$\text{Max } z = 107\,x + y + 2z$$

s.t.

$$\frac{14}{3}x + \frac{y}{3} - 2z + t = 7/3$$

$$16x + 0.5y - 6z \le 5$$

$$3x - y - z \le 0$$

$$x, y, z \ge 0$$

Introduce slack variables $u \ge 0$, $v \ge 0$

$$\text{Max } z = 107\,x + y + 2z + 0.t + 0.u + 0.V$$

s.t.

$$\frac{14}{3}x + \frac{y}{3} - 2z + t = \frac{7}{3}$$

$$16x + 0.5\,y - 6z + u = 5$$

$$3x - y - z + v = 0$$

$$x, y, z, t, u, v \ge 0$$

			107	1	2	0	0	0
C_B	y_B	x_B	x	y	z	t	u	✓
0	t	7/3	14/3	1/3	–2	1	0	0
← 0	u	5	$\boxed{16}$	0.5	–6	0	1	0
0	v	0	3	–1	–1	0	0	1
$Z_j - C_j$			$\boxed{-107}$ ↑	–1	–2	0	0	0

Most negative is –107, x will enter the basis.

$$\text{Min}\left\{\frac{7/3}{14/3}, \frac{5}{16}\right\} = 5/16, \ u \text{ leaves the basis.}$$

Ist iteration

			107	1	2	0	0	0
C_B	y_B	x_B	x	y	z	t	u	v
0	t	7/8	0	9/48	–1/4	1	–14/48	0
← 107	x	5/16	1	0.5/16	–3/8	0	1/16	0
0	v	–15/16	0	–35/32	1/8	0	–3/16	0
$Z_j - C_j$			0	75/32	–337/8 ↑	0	107/16	0

Z will enter the basis

$$\text{Min}\left\{\frac{1}{8}\right\} = \frac{1}{8}, \text{ V will leave the basis.}$$

			107	1	2	0	0	0
C_B	y_B	x_B	x	y	z	t	u	v
0	t	–1	0	–2	0	1	–2/3	0
107	x	–5/2	1	–13/4	0	0	–1/2	0
2	z	–15/2	0	–35/4	1	0	–3/2	0
$Z_j - C_j$			0	–1465/4	0	0	–113/2	0

↑

y will enter the basis

Non of y_{i1} is positive

Thus solution is unbounded.

Q. 9. (b) Write the dual of the following LPP. **(4)**

$$\text{Min } z = 2x_1 + 3x_2 + x_3$$

Sub to

$$x_1 - x_2 - x_3 \geq 2$$
$$2x_1 + 5x_2 \leq 3$$

Ans. Given

$$\text{Min } z = 2x_1 + 3x_2 + x_3$$

s.t.

$$x_1 - x_2 - x_3 \geq 2$$
$$-2x_1 - 5x_2 \geq -3$$

Introducing surplus variables $s_1, s_2 \geq 0$.

standard L.P.P. is

$$\text{Min } Z = 2x_1 + 3x_2 + x_3 + 0.s_1 + 0.s_2$$

s.t.

$$x_1 - x_2 - x_3 - s_1 = 2$$
$$-2x_1 - 5x_2 - s_2 = -3$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Dual

Let w_1 and w_2 be the dual variables corresponding to the primal constraints.

Then dual problem will be

$$\text{Max } z^* = 2w_1 - 3w_2$$

s.t.

$$w_1 - 2w_2 \leq 2$$
$$-w_1 - 5w_2 \leq 3$$
$$-w_1 \leq 1$$
$$-w_1 \leq 0$$
$$-w_2 \leq 0$$

$\Rightarrow$

$$\text{Max } z^* = 2w_1 - 3w_2$$

s.t

$$w_1 - 2w_2 \leq 2$$
$$w_1 + 5w_2 \geq -3$$
$$w_1 \geq -1$$
$$w_1 \geq 0, w_2 \geq 0$$

w_1 and w_2 are unrestricted.

FIRST TERM EXAMINATION [FEB. 2019]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS–IV
[ETMA–202]

Time: 1½ hrs. **M.M. : 30**

Note: *Attempt Q.No. 1 which is compulsory and any two more questions from remaining. All questions carry equal marks.*

Q.1.(a) Find the particular integral of $(D - 4D')^2 z = 6x \tan(y + 4x)$. **(2.5)**

Ans. $(D - 4d')^2 z = 6x \tan(y + 4x)$

$$\text{P.I.} = \frac{1}{(D - 4D')^2} 6x \tan(y + 4x)$$

$$= 6\frac{1}{(D - 4D')(D - 4D')} x \tan(y + 4x)$$

$$= 6\frac{1}{D - 4D'} \int x \tan(c - 4x + 4x)\, dx$$

$$= 6\frac{1}{D - 4D'} \int x \tan c\, dx$$

$$= 6\frac{1}{D - 4D'} \frac{x^2}{2} \tan c$$

$$= 3\frac{1}{D - 4D'} x^2 \tan(y + 4x)$$

$$= 3\int x^2 \tan(c - 4x + 4x)\, dx$$

$$= 3.\int x^2 \tan c\, dx = 3.\frac{x^3}{3} \tan c$$

$\Rightarrow$ $$\text{P.I.} = x^3 \tan(y + 4x)$$

Q.1. (b) Using the method of separation of variables, solve. **(2.5)**

$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0, \quad u(0, y) = 2e^{-y}$$

Ans.

$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0$$

Let

$$U = XY$$

$$\frac{\partial u}{\partial x} = X'Y, \frac{\partial u}{\partial y} = XY'$$

$\therefore$

$$2X'Y + 3XY' = 0$$

$\Rightarrow$

$$\Rightarrow \quad \frac{2X'}{X} = -\frac{3Y'}{Y}$$

Since x and y are independent variables $\therefore$ it is true only when both are equal to a constnat say 'c'.

$$\Rightarrow \quad \frac{2X'}{X} = c, \quad -\frac{3Y'}{Y} = c$$

$$\Rightarrow \quad \frac{dX}{X} = \frac{c}{2}dx, \quad \frac{dY}{Y} = \frac{-c}{3}dy$$

On integrating, we get

$$\Rightarrow \quad \int\frac{dX}{X} = \int\frac{c}{2}dx, \quad \int\frac{dY}{Y} = -\int\frac{c}{3}dy$$

$$\Rightarrow \quad \log X = \frac{c}{2}x + \log a_1, \quad \log Y = -\frac{c}{3}y + \log a_2$$

$$\Rightarrow \quad X = a_1 e^{c/2x}, \quad Y = a_2 e^{-c/3y}$$

$$\therefore \quad U = a_1 a_2 e^{\frac{c}{2}x - \frac{c}{3}y} \quad ...(1)$$

At $x = 0$, $U = 2e^{-y}$, By (1) we get

$$2e^{-y} = a_1 a_2 e^{-c/3y}$$

On comparing, we get $\quad a_1 a_2 = 2$ and $\frac{-c}{3} = -1$

$$\Rightarrow \quad c = 3$$

$\therefore$ (1) gives $\quad U(x,y) = 2e^{\frac{3}{2}x - y} = 2e^{(3/2x-y)}$

Q.1. (c) A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. What is the probability that the problem would be solved? (2.5)

Ans. The random experiments of trying the problem by the given students are independent.

Let A, B, C be the event of first, second, third student solving the problem.

$$\therefore \quad P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

$\therefore$ P (the problem would be solved) $= P$ (A or B or C)

$$= 1 - P(\bar{A}\bar{B}\bar{C})$$

$$= 1 - P(\bar{A}) \cdot P(\bar{B}) P(\bar{C})$$

As

$$P(A) = \frac{1}{2}, P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B) = \frac{1}{3}, P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(C) = \frac{1}{4}, P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ P (the problem would be solved) $= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{1-1}{4} = \frac{3}{4}$

Q.1. (d) A random variable X has probability density function $f(x) = \begin{cases} e^{-x}, x \geq 0 \\ 0, \text{ otherwise} \end{cases}$. Find the moment generating function about origin. (2.5)

Ans.

$$M_X(t) = E[e^{tX}]$$

$$= \int_0^\infty e^{tx} e^{-x} dx = \int_0^\infty e^{(t-1)x} dx = \left.\frac{e^{(t-1)x}}{t-1}\right|_0^\infty$$

$$= \frac{-1}{t-1} = \frac{1}{1-t}$$

Q.2. (a) Find the solution of the partial differential equation $(D^3 - 4D^2D' + 4DD'^2)z = 2\cos(3x + 2y)$. (5)

Ans. $\quad (D^3 - 4D^2D' + 4DD'^2)z = 2\cos(3x + 2y)$

Replace $D \to m, D' = 1$

A.E. $\quad m^3 - 4m^2 + 4m = 0$

$$m(m^2 - 4m + 4) = 0$$

$$\Rightarrow \quad m = 0, (m-2)^2 = 0$$

$$\Rightarrow \quad m = 0, 2, 2.$$

$$\text{C.F.} = f_1(y) + f_2(y + 2x) + x f_3(y + 2x).$$

$$\text{P.I.} = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 2\cos(3x + 2y)$$

$= $ Replace $D = 3, D' = 2, 3x + 2y = u$

$$= 2\frac{1}{27 - 4\times9\times2 + 4\times3\times4}\iiint \cos u \, du\, du\, du$$

$$= \frac{2}{3}\iint \sin u\, du\, du = \frac{-2}{3}\int \cos u\, du = -\frac{2}{3}\sin(3x + 2y)$$

Complete solution is

$$z = f_1(y) + f_2(y + 2x) + x f_3(y + 2x) - \frac{2}{3}\sin(3x + 2y).$$

Q.2. (b) Find the solution of the partial differential equation $(4D^2 + 3DD' - D'^2 - D - D')z = 3e^{(x+2y)/2}$. (5)

Ans. $(4D^2 + 3DD' - D'^2 - D - D')z = 3e^{(x+2y)/2}$

Consider $4D^2 + 3DD' - D'^2 - D - D'$

$\Rightarrow 3D^2 + D^2 + 3DD' - D'^2 - (D + D')$

$\Rightarrow 3D(D + D') + (D^2 - D'^2) - (D + D')$

$\Rightarrow (D + D')[3D + D - D' - 1]$

$\Rightarrow (D + D')(4D - D' - 1)$

For $D + D' : b = 1, a = -1, c = 0$

For $4D - D' - 1 : b = 4, a = 1, c = -1$

$$\therefore \quad \text{C.F.} = \phi_1(y - x) + e^{x/4}\phi_2(4y + x)$$

$$\text{P.I.} = \frac{1}{(D + D')(4D - D' - 1)} 3e^{(x+2y)/2}$$

$$\Rightarrow \quad \frac{2X'}{X} = -\frac{3Y'}{Y}$$

Since x and y are independent variables $\therefore$ it is true only when both are equal to a constnat say 'c'.

$$\Rightarrow \quad \frac{2X'}{X} = c, \quad -\frac{3Y'}{Y} = c$$

$$\Rightarrow \quad \frac{dX}{X} = \frac{c}{2}dx, \quad \frac{dY}{Y} = \frac{-c}{3}dy$$

On integrating, we get

$$\Rightarrow \quad \int\frac{dX}{X} = \int\frac{c}{2}dx, \quad \int\frac{dY}{Y} = -\int\frac{c}{3}dy$$

$$\Rightarrow \quad \log X = \frac{c}{2}x + \log a_1, \quad \log Y = -\frac{c}{3}y + \log a_2$$

$$\Rightarrow \quad X = a_1 e^{c/2x}, \quad Y = a_2 e^{-c/3y}$$

$$\therefore \quad U = a_1 a_2 e^{\frac{c}{2}x - \frac{c}{3}y} \quad \ldots(1)$$

At $x = 0$, $U = 2e^{-y}$, By (1) we get

$$2e^{-y} = a_1 a_2 e^{-c/3y}$$

On comparing, we get $\quad a_1 a_2 = 2$ and $\dfrac{-c}{3} = -1$

$$\Rightarrow \quad c = 3$$

$\therefore$ (1) gives $\quad U(x, y) = 2e^{\frac{3}{2}x - y} = 2e^{(3/2x - y)}$

Q.1. (c) A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. What is the probability that the problem would be solved? (2.5)

Ans. The random experiments of trying the problem by the given students are independent.

Let A, B, C be the event of first, second, third student solving the problem.

$$\therefore \quad P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

$\therefore$ P (the problem would be solved) $= P(A \text{ or } B \text{ or } C)$

$$= 1 - P(\bar{A}\bar{B}\bar{C})$$

$$= 1 - P(\bar{A}) \cdot P(\bar{B}) P(\bar{C})$$

As

$$P(A) = \frac{1}{2}, P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B) = \frac{1}{3}, P(\bar{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(C) = \frac{1}{4}, P(\bar{C}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ P (the problem would be solved) $= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{1-1}{4} = \frac{3}{4}$

Q.1. (d) A random variable X has probability density function $f(x) = \begin{cases} e^{-x}, x \geq 0 \\ 0, \text{ otherwise} \end{cases}$. Find the moment generating function about origin. (2.5)

Ans.

$$M_X(t) = E[e^{tX}]$$

$$= \int_0^\infty e^{tx} e^{-x} dx = \int_0^\infty e^{(t-1)x} dx = \left.\frac{e^{(t-1)x}}{t-1}\right|_0^\infty$$

$$= \frac{-1}{t-1} = \frac{1}{1-t}$$

Q.2. (a) Find the solution of the partial differential equation $(D^3 - 4D^2D' + 4DD'^2)z = 2\cos(3x + 2y)$. (5)

Ans. $\quad (D^3 - 4D^2D' + 4DD'^2)z = 2\cos(3x + 2y)$

Replace $D \to m, D' = 1$

A.E. $\quad m^3 - 4m^2 + 4m = 0$

$$m(m^2 - 4m + 4) = 0$$

$$\Rightarrow \quad m = 0, (m-2)^2 = 0$$

$$\Rightarrow \quad m = 0, 2, 2.$$

$$\text{C.F.} = f_1(y) + f_2(y + 2x) + x f_3(y + 2x).$$

$$\text{P.I.} = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 2\cos(3x + 2y)$$

$$= \text{Replace } D = 3, D' = 2, 3x + 2y = u$$

$$= 2\frac{1}{27 - 4 \times 9 \times 2 + 4 \times 3 \times 4}\iiint \cos u \, du \, du \, du$$

$$= \frac{2}{3}\iint \sin u \, du \, du = \frac{-2}{3}\int \cos u \, du = -\frac{2}{3}\sin(3x + 2y)$$

Complete solution is

$$z = f_1(y) + f_2(y + 2x) + x f_3(y + 2x) - \frac{2}{3}\sin(3x + 2y).$$

Q.2. (b) Find the solution of the partial differential equation $(4D^2 + 3DD' - D'^2 - D - D')z = 3e^{(x+2y)/2}$. (5)

Ans. $(4D^2 + 3DD' - D'^2 - D - D')z = 3e^{(x+2y)/2}$

Consider $4D^2 + 3DD' - D'^2 - D - D'$

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For $D + D' : b = 1, a = -1, c = 0$

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$$\therefore \quad \text{C.F.} = \phi_1(y - x) + e^{x/4}\phi_2(4y + x)$$

$$\text{P.I.} = \frac{1}{(D + D')(4D - D' - 1)} 3e^{(x+2y)/2}$$

$$= \frac{1}{(D+D')(4D-D'-1)} 3e^{x/2+y} \text{ (Case of failure)}$$

$$= 3\frac{1}{(4D-D'-1)}\left[\frac{1}{\left(\frac{1}{2}+1\right)} e^{x/2+y}\right]$$

$$= 3\cdot\frac{2}{3}\frac{1}{4D-D'-1} e^{x/2+y}\cdot 1$$

$$= 2e^{x/2+y}\frac{1}{\left[4\left(D+\frac{1}{2}\right)-(D'+1)-1\right]}.1$$

$$= 2e^{x/2+y}\left[\frac{1}{4D-D'}\cdot 1\right]$$

$$= \frac{-2e^{x/2+y}}{D'}\left(\frac{1}{1-\frac{4D}{D'}}\right).1 = -2e^{x/2+y}\left[\frac{1}{D'}\left(\frac{1-4D}{D'}\right)^{-1}.1\right]$$

$$= -2e^{x/2+y}\left[\frac{1}{D'}\left(\frac{1+4D}{D'}\right).1\right]$$

$$= -\frac{2e^{x/2+y}}{D'}.1 = -2e^{x/2+y}\int 1.dy = -2y\,e^{x/2+y}$$

Complete solution is

$$z = \phi_1(y-x) + e^{x/4}\phi_2(4y+x) - 2ye^{x/2+y}.$$

Q.3. (a) Three bags contains 2 white, 3 black; 3 white, 2 black and 4 white and 1 black balls respectively. There is equal probability of each bag being chosen. One ball is drawn from a bag chosen at random what is the probability that a white ball is drawn? Also, find the probability that it is drawn from the second bag. (5)

Ans. Let E_1, E_2, E_3 be the events of choosing bag 1, 2, 3 respectively

$$\therefore \quad P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$$

Let A be the event of getting a white ball.

$$P(A/E_1) = \frac{2}{5}$$

$$P(A/E_2) = \frac{3}{5},\quad P(A/E_3) = \frac{4}{5}$$

Probability of getting white ball

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

$$= \frac{1}{3}\times\frac{2}{5} + \frac{1}{3}\times\frac{3}{5} + \frac{1}{3}\times\frac{4}{5}$$

$$= \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = \frac{9}{15} = \frac{3}{5}$$

Probability that the white ball is drawn from the second bag.

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(A)} \text{ (By Baye's theorem)}$$

$$= \frac{\frac{1}{3}\times\frac{3}{5}}{\frac{3}{5}} = \frac{1}{5}\times\frac{5}{3} = \frac{1}{3}$$

Q.3. (b) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 50 respectively. Find the mean and all the four moments about the mean. Also, comment upon skewness and kurtosis. (5)

Ans. Let u_1', u_2', u_3', u_4' be the first four moments about $x = 5$.

$\Rightarrow u_1' = 2, u_2' = 20, u_3' = 40, u_4' = 50, A = 5.$

To find u_1, u_2, u_3, u_4

By definition $u_1 = 0$

$$u_2 = u_2' - u_1'^2 = 20 - 4 = 16$$

$$u_3 = u_3' - 3u_2' + 2u_1'^3$$

$$= 40 - 3\times 20 + 2\times 8 = -4$$

$$u_4 = u_4' - 4u_3'u_1' + 6u_2'u_1' - 3u_1'^4$$

$$= 50 - 4\times 40\times 2 + 6\times 20\times 2 - 3\times 16$$

$$= 50 - 320 + 240 - 48 = -78.$$

Mean $\quad \bar{X} = u_1' + A = 2 + 5 = 7$

Skewness $\quad \gamma_1 = \frac{u_3}{\sqrt{u_2^3}} = \frac{-4}{\sqrt{16^3}} = -0.313$

Kurtosis $\quad \beta_2 = \frac{u_4}{\sqrt{u_2^2}} = \frac{-78}{16^2} = -0.3047$

Curve is platy kurtic.

Q.4. A thin uniform tightly stretched vibrating string fixed at the points $x = 0$ and $x = l$ satisfies the equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$: $y(x, 0) = y_0\sin^3\left(\frac{\pi x}{l}\right)$ and released from rest from this position. Find the displacement y (x, t) at any x and any time t. (10)

Ans. The wave equation is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} \quad ...(1)$$

Let $\quad Y = X(x)T(t) = XT$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = XT'', \frac{\partial^2 y}{\partial x^2} = X''T$$

$\therefore$ By (1) $\quad XT'' = c^2X''T$

$$\Rightarrow \quad \frac{X''}{X} = \frac{1}{c^2}\frac{T''}{T}$$

$$= \frac{1}{(D+D')(4D-D'-1)} 3e^{x/2+y} \text{ (Case of failure)}$$

$$= 3\frac{1}{(4D-D'-1)}\left[\frac{1}{\left(\frac{1}{2}+1\right)}e^{x/2+y}\right]$$

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$$= 2.e^{x/2+y}\frac{1}{\left[4\left(D+\frac{1}{2}\right)-(D'+1)-1\right]}.1$$

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Ans. The wave equation is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2} \quad \text{...(1)}$$

Let $\quad Y = X(x)T(t) = XT$

$$\frac{\partial^2 y}{\partial t^2} = XT'', \frac{\partial^2 y}{\partial x^2} = X''T$$

$\Rightarrow$

$\therefore$ By (1) $\quad XT'' = c^2X''T$

$\Rightarrow \quad \frac{X''}{X} = \frac{1}{c^2}\frac{T''}{T}$

Since it is a function of x only and t only. As x and t are independent variables. Thus, both sides reduce to a constant say 'a'

$$\Rightarrow \quad \frac{X''}{X} = a, \; \frac{1}{c^2}\frac{T''}{T} = a$$

$$\text{Let} \quad a = -p^2 \text{(say)}$$

$$\Rightarrow \quad X'' + p^2 X = 0$$

$$\Rightarrow \quad (D^2+p^2)X = 0$$

$$\Rightarrow \quad D^2+p^2 = 0$$

$$\Rightarrow \quad D = \pm ip$$

$$\therefore \quad X = c_1 \cos px + c_2 \sin px$$

$$\text{and} \quad T'' + p^2c^2 T = 0$$

$$\Rightarrow \quad (D^2+p^2c^2)T = 0$$

$$\Rightarrow \quad D = \pm ipc$$

$$\therefore \quad T = c_3 \cos pct + c_4 \sin pct.$$

Thus $y = (C_1 \cos px + C_2 \sin px)(C_3 \cos pct + C_4 \sin pct)$ ...(2)

Since boundary conditions are

$$y(0, t) = 0 \text{ and } y(l, t) = 0$$

Since initial transverse velocity at any point of the string is zero

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \text{ and } y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$$

At $x = 0, y = 0$, ...(2) gives

$$0 = c_1(c_3 \cos pct + c_4 \sin pct)$$

$$\Rightarrow \quad c_1 = 0$$

$\therefore$ (2) reduces to

$$y(x, t) = c_2 \sin px(c_3 \cos pct + c_4 \sin pct) \quad ...(3)$$

$$\text{At} \quad x = l, y = 0$$

$$\Rightarrow \quad 0 = c_2 \sin pl(c_3 \cos pct + c_4 \sin pct)$$

It is satisfied if $\sin pl = 0$

$$\Rightarrow \quad pl = n\pi$$

$$\Rightarrow \quad p = \frac{n\pi}{l}, n = 1,2,3...$$

Thus (3) reduces to

$$y(x, t) = c_2 \sin \frac{n\pi}{l} x \left(c_3 \cos \frac{n\pi ct}{l} + c_4 \sin \frac{n\pi ct}{l}\right)$$

$$\Rightarrow \quad y = \left(a_n \cos \frac{n\pi ct}{l} + b_n \sin \frac{n\pi ct}{l}\right) \sin \frac{n\pi x}{l}$$

where $a_n = c_2c_3, b_n = c_2c_4$

Adding solution for different values of n, we get

$$y = \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi ct}{l} + b_n \sin \frac{n\pi ct}{l}\right) \sin \frac{n\pi x}{l} \quad ...(4)$$

Applying initial condition

$$\frac{\partial y}{\partial t} = 0 \text{ at } t = 0$$

$$\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty}\left(-a_n \sin \frac{n\pi ct}{l} \cdot \frac{n\pi c}{l} + b_n \cos \frac{n\pi ct}{l} \cdot \frac{n\pi c}{l}\right) \sin \frac{n\pi x}{l}$$

$$\frac{\partial y}{\partial t} = 0 \text{ at } t = 0$$

$$\Rightarrow \quad 0 = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cdot \frac{n\pi c}{l}$$

$$\Rightarrow \quad b_n = 0, \forall n \in I.$$

By (4), we get

$$y(x, t) = \sum_{n=1}^{\infty} a_n \cos \frac{n\pi ct}{l} \sin \frac{n\pi x}{l}, 0 < x < l \quad ...(5)$$

Now at $t = 0, y = y_0 \sin^3 \frac{\pi x}{l}$.

$\therefore$ (5) gives

$$y_0 \sin^3 \frac{\pi x}{l} = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{l}$$

$$\text{since} \quad \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$\Rightarrow \frac{y_0}{4}\left(3 \sin \frac{\pi x}{l} - \sin \frac{3\pi x}{l}\right) = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{2\pi x}{l} + ...$$

Comparing the coefficients of $\frac{\sin \pi x}{l}, \frac{\sin 3\pi x}{l}, ...$

$$\frac{3y_0}{4} = a_1, a_2 = 0, a_3 = -\frac{y_0}{4}, a_4 = 0$$

$$\Rightarrow \quad a_n = 0 \; \forall n \text{ except } n = 1, 3.$$

$\therefore$ (5) reduces to

$$y(x, t) = a_1 \cos \frac{\pi ct}{l} \sin \frac{\pi x}{l} + a_3 \cos \frac{3\pi ct}{l} \sin \frac{3\pi x}{l}$$

$$= \frac{3y_0}{4} \cos \frac{\pi ct}{l} \sin \frac{\pi x}{l} - \frac{y_0}{4} \cos \frac{3\pi ct}{l} \sin \frac{3\pi x}{l}$$

$$= \frac{y_0}{4}\left(3 \cos \frac{\pi ct}{l} \sin \frac{\pi x}{l} - \cos \frac{3\pi ct}{l} \sin \frac{3\pi x}{l}\right)$$

OR

Q.4. Show that poisson distribution is a limiting case of Binomial distribution when n is very large and p is small such that np is fixed. Also, using poisson distribution, find the chance that there will be less than three accidents in a day if 10 accidents took place in a span of 50 days in a town. (10)

Ans. For a binomial distribution

$$P(X = r) = {}^nCr\, q^{n-r} p^r$$

$$= \frac{n(n-1)(n-2)...(n-r+1)}{r!} \times (1-p)^{n-r} \cdot p^r$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}\left(1-\frac{\lambda}{n}\right)^{n-r}\left(\frac{\lambda}{n}\right)^{r}$$

(since $np = \lambda$)

$$= \frac{\lambda^r}{r!} \times \frac{n(n-1)(n-2)\ldots(n-r+1)}{n^r} \frac{\left(1-\frac{\lambda}{n}\right)^{n}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

$$= \frac{\lambda^r}{r!}\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\ldots\left(\frac{n-r+1}{n}\right) \times \frac{\left(1-\frac{\lambda}{n}\right)^{n}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

$$= \frac{\lambda^r}{r!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{(r-1)}{n}\right) \times \frac{\left[\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right]^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

As $n \to \infty$, each of the $(r-1)$ factors

$\left(1-\frac{1}{n}\right), \left(1-\frac{2}{n}\right), \ldots \left(1-\frac{r-1}{n}\right)$ tends to 1. Also $\left(1-\frac{\lambda}{n}\right)^{r} \to 1$.

Since $\lim_{x \to \pm\infty}\left(1+\frac{1}{x}\right)^{x} = e$,

$\therefore \sin^3 \frac{\pi x}{l}\left[\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right]^{-\lambda} \to e^{-\lambda}$ as $n \to \infty$

Hence in the limiting case, when $n \to \infty$, we have

$$P(X=r) = \lambda^r \frac{e^{-\lambda}}{r!} \quad (r = 0, 1, 2, \ldots) \qquad \ldots(1)$$

where λ is a finite number $= np$ and (1) represents probability distribution.

Let x be the poisson variable "no. of accidents in a day".

Average No. of accidents in 50 days = 10

Average No. of accidents in1 day $= \frac{10}{50} = 0.2$

$$\therefore \quad \lambda = 0.2$$

$$\therefore \quad P(X=r) = \frac{e^{-0.2}(0.2)^r}{r!}$$

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= e^{-0.2} + e^{-0.2}\frac{0.2}{1!} + e^{-0.2}\frac{(0.2)^2}{2!}$$

$$= e^{-0.2}[1 + 0.2 + 0.02] = 0.8187 \times 1.22 = 0.9988.$$

END TERM EXAMINATION [MAY 2019]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV
[ETMA-202]

Time: 3 hrs. **M.M. : 75**

Note: *Attempt five questions in all including Q.no.1 which is compulsory. Select one question from each unit. Assume missing data, if any.*

Q.1. (a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$ **(3)**

Ans.

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \qquad \ldots(1)$$

Let $U = XT$

Here X is a function of x only and T is a function of t only.

$$\Rightarrow \quad \frac{\partial u}{\partial x} = X'T, \frac{\partial u}{\partial t} = XT'$$

Given equation becomes

$$X'T = 2XT' + XT$$

$$\Rightarrow \quad X'T = (2T' + T)X$$

$$\Rightarrow \quad \frac{X'}{X} = \frac{2T' + T}{T} \qquad \ldots(2)$$

Since x and t are independent variables. Therefore, equation (2) can hold only when each side is equal to the same constant, say 'a'.

$$\therefore \quad \frac{X'}{X} = a \Rightarrow \frac{dX}{X} = a\,dx$$

On integrating, we get

$$\log X = ax + \log c_1$$

$$\Rightarrow \quad X = C_{1e}^{ax}$$

and

$$\frac{2T' + T}{T} = a$$

$$\Rightarrow \quad \frac{T'}{T} = \frac{(a-1)}{2}$$

$$\Rightarrow \quad \frac{dT}{T} = \left(\frac{a-1}{2}\right)dt$$

$$\Rightarrow \quad \log T = \frac{1}{2}(a-1)t + \log c_2$$

$$\Rightarrow \quad T = C_2 e^{1/2(a-1)t}$$

$$\therefore \quad U(x,t) = C_1 C_2 e^{ax} \cdot e^{\left(\frac{a-1}{2}\right)t} \quad ...(3)$$

Since $u(x, 0) = 6e^{-3x}$

$\therefore$ $C_1 C_2 e^{ax} = 6e^{-3x}$

$\Rightarrow$ $C_1 C_2 = 6, a = -3$

Thus (3) gives

$$u(x,t) = 6e^{-3x}e^{-2t}$$

$$u(x,t) = 6e^{-(3x+2t)}$$

Q.1. (b) A book on statistics is independently reviewed by three reviewers favourably with probabilities $\frac{3}{5}, \frac{4}{7}$ and $\frac{2}{5}$ respectively. What is the probability that of the reviews, at least two reviews will be favourable? (3)

Ans. Let A = reviewed by first reviewer

B = reviewed by second reveiwer

C = reviewed by third reviewer

Given $P(A) = \frac{3}{5}$, $P(B) = \frac{4}{7}$, $P(C) = \frac{2}{5}$

All three events are independent.

Probability that at least two reviews will be favourable.

$= P(A \cap B \cap \bar{C} \text{ or } A \cap \bar{B} \cap C \text{ or } \bar{A} \cap B \cap C \text{ or } A \cap B \cap C)$

$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(B)P(C) + P(A)P(B)P(C)$ ...(1)

Now $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{5} = \frac{2}{5}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{4}{7} = \frac{3}{7}$

$P(\bar{C}) = 1 - P(C) = 1 - \frac{2}{5} = \frac{3}{5}$

By (1), we get

$$= \frac{3}{5}\cdot\frac{4}{7}\cdot\frac{3}{5} + \frac{3}{5}\cdot\frac{3}{7}\cdot\frac{2}{5} + \frac{2}{5}\cdot\frac{4}{7}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{4}{7}\cdot\frac{2}{5}$$

$$= \frac{36}{175} + \frac{18}{175} + \frac{16}{175} + \frac{24}{175} = \frac{94}{175}.$$

Q.1. (c) The diameter of an electric cable is assumed to be a continuous variate with p.d.f. $f(x) = 6x(1-x), 0 \le x \le 1$. Verify that the above is a p.d.f. Also find the mean. (5)

Ans. $f(x) = 6x(1-x), 0 \le x \le 1$

If $f(x)$ is a p.d.f. it must satisfy

$$f(x) \ge 0 \text{ and } \int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Now $\int_0^1 f(x)\,dx = \int_0^1 (6x - 6x^2)\,dx$

$$= \left|3x^2 - 2x^3\right|_0^1 = 3 - 2 = 1$$

$\therefore f(x)$ is a p.d.f.

Now mean of $f(x)$ is

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^1 x(6x - 6x^2)\,dx$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx$$

$$= \left|2x^3 - \frac{6x^4}{4}\right|_0^1 = 2 - \frac{3}{2} = \frac{1}{2}$$

Q.1. (d) If X is a Poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$. Find the variance of X. (3)

Ans. Let $P(X=r) = \frac{e^{-m}\cdot m^r}{r!}, r = 0, 1, 2...$

We have $P(X=2) = 9P(X=4) + 90P(X=6)$

$$\Rightarrow \quad \frac{e^{-m}\cdot m^2}{2!} = 9\cdot\frac{e^{-m}\cdot m^4}{4!} + 90\cdot\frac{e^{-m}\cdot m^6}{6!}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{3}{8}m^2 + \frac{1}{8}m^4$$

$\Rightarrow$ $m^4 + 3m^2 - 4 = 0$

$\Rightarrow$ $x^2 + 3x - 4 = 0$ Let $x = m^2$

$\Rightarrow$ $x^2 - x + 4x - 4 = 0$

$x(x-1) + 4(x-1) = 0 \Rightarrow (x-1)(x+4) = 0$

$x = 1, -4$

$\Rightarrow$ $m^2 = 1, -4$

$\therefore$ $m = \pm 1, m = \sqrt{-4}$ (Not possible).

$\Rightarrow$ $m = 1$ (as $m = -1$ is not possible).

$\Rightarrow$ Thus variance is $m = 1$.

Q.1. (e) If θ is the angle between the two regression lines, show that

$$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right). \quad \textbf{(3)}$$

Ans. Equations to the lines of regression of y on x and x on y are

$$y-\bar{y} = \frac{r\sigma_y}{\sigma_x}(x-\bar{x}) \text{ and } (x-\bar{x}) = \frac{r\sigma_x}{\sigma_y}(y-\bar{y})$$

Their slopes are $m_1 = \dfrac{r\sigma_y}{\sigma_x}$ and $m_2 = \dfrac{\sigma_y}{r\sigma_x}$

$$\therefore \quad \tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \frac{\left|\dfrac{\sigma_y}{r\sigma_x} - \dfrac{r\sigma_y}{\sigma_x}\right|}{\left|1+\dfrac{\sigma_y{}^2}{\sigma x^2}\right|}$$

$$= \left|\frac{1-r^2}{r}\cdot\frac{\sigma_y}{\sigma_x}\cdot\frac{\sigma_x^2}{\sigma y^2+\sigma x^2}\right|$$

$$= \frac{1-r^2}{|r|}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

Since $r^2 < 1$ and σ_x, σ_y are positive.

Hence $$\tan\theta = \frac{1-r^2}{|r|}\,\frac{\sigma_x\cdot\sigma_y}{\sigma_x^2+\sigma_y^2}.$$

Q.1. (f) In two large populations, there are 40% and 30% respectively, are blue eyed people. Is the difference likely to be hidden in sample of 1,000 and 800 respectively from the two populations? **(3)**

Ans. p_1 = proportion of blue eyed people in first population.

p_2 = proportion of blue eyed people in second population.

$$\therefore \quad p_1 = 40\% = 0.4,\ p_2 = 30\% = 0.3$$
$$n_1 = 1000,\ n_2 = 800$$

Null Hypothesis H_0 : sample proportions are equal i.e. the difference in population proportions is likely to be hidden in sampling.

Alternative Hypothesis $H_1 : p_1 \neq p_2$

$$\therefore \quad P = \frac{n_1p_1+n_2p_2}{n_1+n_2} = \frac{1000\times 0.4+800\times 0.3}{1000+800}$$

$$= \frac{400+240}{1800} = 0.3556$$

$$Q = 1-P = 1-0.3556 = 0.6444$$

Under H_0,
$$z = \frac{p_1-p_2}{\sqrt{PQ\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}} = \frac{0.4-0.3}{\sqrt{0.3556\times 0.6444\left(\dfrac{1}{1000}+\dfrac{1}{800}\right)}}$$

$$= \frac{0.1}{\sqrt{0.2291(0.001+0.0013}} = \frac{0.1}{0.023} = 4.3478$$

Since $|Z| > 1.96 \therefore H_0$ is rejected at 5% level of significance.

These samples will reveal the difference in the population proportions.

Q.1. (g) A company makes two kinds of leather belts. Belt A is a high quality belt, and belt B is of lower quality. The respective profits are Rs 4.00 and Rs 3.00 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requries a fancy buckle and only 400 per day are available. There are only 700 buckles a day available for belt B. Formulate this as a L.P.P. and solve it by graphical method. **(5)**

Ans. Let x and y denote respectively the number of belts of type A and B to be produced per day.

Since the number of belts cannot be negative, we have

$$x \geq 0, \quad y \geq 0.$$

Since the rate of producing type B belts is 1000 per day, the total time taken to produce y belts of type B is $\dfrac{y}{1000}$.

Also each belt of type A requries twice as much time as a belt of type B, the rate of producing type A belts is 500 per day and the total time taken to produce x belts of type A is $\dfrac{x}{500}$.

$\therefore$ The time constraint is $\dfrac{x}{500}+\dfrac{y}{1000} \leq 1$

or $$2x+y \leq 1000$$

The constraint imposed by supply of leather is

$$x+y \leq 800$$

The constraint imposed by supply of fancy buckles is

$$x \leq 400$$

The constraint imposed by supply of buckles for type B belt is

$$y \leq 700.$$

The objective is to maximize the profit Z (in Rs.) given by

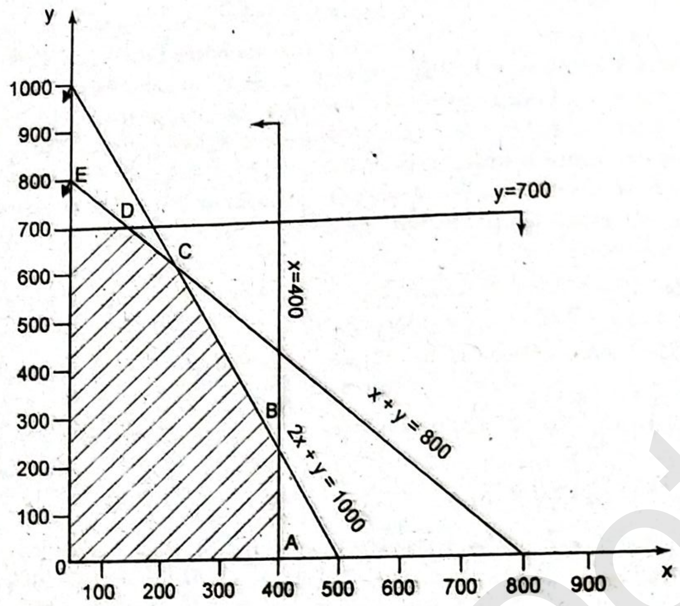
$$Z = 4x+3y$$

The mathematical formulation of the problem is

$$\text{Max } Z = 4x + 3y$$

Subject to constraints

$$2x + y \leq 1000$$
$$x + y \leq 800$$
$$x \leq 400$$
$$y \leq 700$$
$$x \geq 0, y \geq 0$$



The feasible region is shown by the shaded area $OA\,BC\,DE$ in the figure.

Since the optimal value of the objective function occurs at one of the corners of the feasible region, we determine their coordinates.

Here $O = (0, 0), A = (400, 0), B = (400, 200), C = (200, 600), D = (100, 700), E = (0, 700)$.

Corner point	Coordiantes (x, y)	Value of objective function $z = 4x + 3y$
O	(0, 0)	0
A	(400, 0)	$4 \times 400 + 0 = 1600$
B	(400, 200)	$4 \times 400 + 3 \times 200 = 2200$
C	(200, 600)	$4 \times 200 + 3 \times 600 = \boxed{2600}$
D	(100, 700)	$4 \times 100 + 3 \times 700 = 2500$
E	(0, 700)	$0 + 3 \times 700 = 2100$

The maximum value of z is 2600 at (200, 600).

UNIT I

Q.2. (a) Solve $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2.$ **(6)**

Ans. Given equation is

$$(D^2 + 2DD' + D'^2)z = x^2 + xy + y^2$$

Replace D by m and D' by 1

A.F $\quad m^2 + 2m + 1 = 0$

$\Rightarrow \quad (m+1)^2 = 0$

$\Rightarrow \quad m = -1, -1$

$$\text{C.F.} = f_1(y - x) + x f_2(y - x)$$

$$\text{P.I.} = \frac{1}{D^2 + 2DD' + D'^2}(x^2 + xy + y^2)$$

$$= \frac{1}{(D + D')^2}\left(x^2 + xy + y^2\right)$$

$$= \frac{1}{D^2\left(1 + \frac{D'}{D}\right)^2}(x^2 + xy + y^2)$$

$$= \frac{1}{D^2}\left[1 + \frac{D'}{D}\right]^{-2}(x^2 + xy + y^2)$$

$$= \frac{1}{D^2}\left[1 - \frac{2D'}{D} + \ldots\right](x^2 + xy + y^2)$$

$$= \frac{1}{D^2}\left[\left(x^2 + xy + y^2\right) - \frac{2}{D}D'\left(x^2 + xy + y^2\right)\right]$$

$$= \left[\frac{1}{D^2}\left(x^2 + xy + y^2\right) - \frac{2}{D}(x + 2y)\right]$$

$$= \frac{1}{D^2}\left[x^2 + xy + y^2 - 2\int(x + 2y)\,dx\right]$$

$$= \frac{1}{D^2}\left[\left(x^2 + xy + y^2\right) - 2\left(\frac{x^2}{2} + 2xy\right)\right]$$

$$= \frac{1}{D^2}\left[x^2 + xy + y^2 - x^2 - 4xy\right]$$

$$= \frac{1}{D}\int\left(y^2 - 3xy\right)dx$$

$$= \frac{1}{D}\left(xy^2 - \frac{3x^2}{2}y\right)$$

$$= \int \left(xy^2 - \frac{3}{2}x^2 y \right) dx = \frac{x^2y^2}{2} - \frac{x^3y}{6}$$

Complete solution is

$$Z = f_1(y-x) + x f_2(y-x) + \frac{x^2y^2}{2} - \frac{x^3y}{6}.$$

Q.2. (b) Solve $(2DD' + D'^2 - 3D')\, Z = 3\cos(3x - 2y)$. **6.5**

Ans. Given $2DD' + D'^2 - 3D'$

$D'(2D + D' - 3)$

for $D' : b = 0, a = -1, c = 0$

or $2D + D' - 3 : b = 2, a = -1, c = 3$

$$\text{C.F.} = \phi_1(-x) + e^{3x/2}\phi_2(2y - x)$$

$$\text{P.I.} = \frac{1}{2DD' + D'^2 - 3D'} 3\cos(3x - 2y)$$

Here $D^2 = -9, D'^2 = -4, DD' = 6$

$$= 3\frac{1}{2\times 6 - 4 - 3D'}\cos(3x - 2y) = 3\frac{1}{8 - 3D'}\cos(3x - 2y)$$

$$= 3\frac{(8 + 3D')}{(8 - 3D')(8 + 3D')}\cos(3x - 2y)$$

$$= 3\frac{(8 + 3D')}{64 - 9D'^2}\cos(3x - 2y)$$

$$= 3\frac{(8 + 3D')}{64 + 36}\cos(3x - 2y)$$

$$= \frac{3}{100}[8\cos(3x - 2y) + 3D'\cos(3x - 2y)]$$

$$= \frac{3}{100}[8\cos(3x - 2y) - 3\sin(3x - 2y)(-2)]$$

$$= \frac{12\cos(3x - 2y) + 3\sin(3x - 2y)}{50}$$

Complete solution is

$$Z = \phi_1(-x) + e^{3x/2}\phi_2(2y - x) + \frac{12\cos(3x - 2y) + 3\sin(3x - 2y)}{50}$$

Q.3. A tightly stretched string with fixed end points at $x = 0$ and $x = 1$, is initially in position given by

$$f(x) = \begin{cases} x, & 0 \le x < 1/2 \\ 1 - x, & 1/2 < x \le 1 \end{cases} \qquad \textbf{(12.5)}$$

If it is released from this position with velocity a, perpendicular to the x-axis, find the displacement $u(x, t)$ at any point x of the string at any time $t > 0$.

Ans. The wave euation is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad \text{...(1)}$$

B.C. $u(0, t) = 0$

B.C. $u(1, t) = 0$ ...(2)

I.C. $u(x, 0) = \begin{cases} x, & 0 \le x < 1/2 \\ 1 - x, & 1/2 < x \le 1 \end{cases}$ ...(3)

I.C. $\left(\frac{\partial u}{\partial t}\right)_{t=0} = a$

Let $U = X(x)\,T(t) = XT$

$\Rightarrow \quad \frac{\partial^2 u}{\partial t^2} = XT'', \ \frac{\partial^2 u}{\partial x^2} = X''T$

$\therefore$ By (1)

$$XT'' = c^2 X''T$$

$\Rightarrow \quad \frac{X''}{X} = \frac{1}{c^2}\frac{T''}{T}$

Since it is a function of x only and t only. As x and t are independent variables. Thus, both sides reduce to a constant say 'b',

$\Rightarrow \quad \frac{X''}{X} = b \quad \frac{1}{C^2}\frac{T''}{T} = b$

Let $b = -p^2$ (say)

$\Rightarrow \quad X'' + p^2X = 0, T'' + p^2c^2T = 0$

$\Rightarrow \quad (D^2 + p^2)X = 0, (D^2 + p^2c^2)T = 0$

$\Rightarrow \quad D^2 + p^2 = 0, D^2 + p^2c^2 = 0$

$D = \pm ip, D = \pm ipc$

$\therefore \quad X = C_1\cos px + C_2\sin px$

and $T = C_3\cos pct + C_4\sin pct$

Thus, $u = (C_1\cos px + C_2\sin px)(C_3\cos pct + C_4\sin pct)$ ...(4)

As $u(0, t) = 0$ and $u(1, t) = 0$

At $x = 0, u = 0$, (4) gives

$0 = C_1(C_3\cos pct + C_4\sin pct)$

$\Rightarrow \quad C_1 = 0$

$\therefore$ (4) reduces to.

$u(x, t) = C_2\sin px\,(C_3\cos pct + C_4\sin pct)$ ...(5)

At $x = 1, u = 0$

$\Rightarrow \quad 0 = C_2\sin p.(C_3\cos pct + C_4\sin pct).$

It is satisfied if $\sin p = 0$.

$\Rightarrow \quad p = n\pi, n = 1, 2, 3...$

Thus (5) reduces to

$u(x, t) = C_2\sin n\pi x\,(C_3\cos n\pi ct + C_4\sin n\pi ct)$

$\Rightarrow \quad u(x, t) = (a_n\cos n\pi ct + b_n\sin n\pi ct)\sin n\pi x.$

Where $a_n = C_2C_3, b_n = C_2C_4.$

Adding solution for different values of n, we get

$$u = \sum_{n=1}^{\infty}(a_n\cos n\pi ct + b_n\sin n\pi ct)\sin n\pi x \qquad \text{...(6)}$$

Applying initial condition

$$\frac{\partial u}{\partial t} = a \text{ at } t = 0$$

$$\frac{\partial u}{\partial t} = \sum_{n=1}^{\infty}(-a_n \sin n\pi ct \cdot n\pi c + b_n \cos n\pi ct \cdot n\pi c)\sin n\pi x$$

$$\Rightarrow \quad a = \sum_{n=1}^{\infty} b_n \cdot n\pi c \cdot \sin n\pi x$$

$$\Rightarrow \quad b_n = \frac{a}{n\pi c}$$

$\therefore$ (6) becomes

$$u = \sum_{n=1}^{\infty}\left(a_n \cos n\pi ct + \frac{a}{n\pi c}\sin n\pi ct\right)\sin n\pi x \quad ...(7)$$

At $t = 0$

$$f(x) = \sum_{n=1}^{\infty} a_n \sin n\pi x$$

This is a fourier sine series.

$$\therefore \quad a_n = \frac{2}{1}\int_0^1 f(x)\sin n\pi x\,dx$$

$$\Rightarrow \quad a_n = 2\left[\int_0^{1/2} x\sin n\pi x\,dx + \int_{1/2}^{1}(1-x)\sin n\pi x\,dx\right]$$

$$\Rightarrow \quad a_n = 2\left[\left\{x\left(\frac{-\cos n\pi x}{n\pi}\right) + \frac{\sin n\pi x}{n^2\pi^2}\right\}_0^{1/2} + \left\{(1-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - (-1)\left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right\}_{1/2}^{1}\right]$$

$$= 2\left[\frac{-1}{2}\frac{\cos n\pi/2}{n\pi} + \frac{\sin n\pi/2}{n^2\pi^2} + \frac{\cos n\pi/2}{2n\pi} + \frac{\sin n\pi/2}{n^2\pi^2}\right]$$

$$= 2\left[\frac{2\sin n\pi/2}{n^2\pi^2}\right] = \frac{4\sin n\pi/2}{n^2\pi^2}$$

$\therefore$ (7) becomes

$$u(x,t) = \sum_{n=1}^{\infty}\left(\frac{4\sin n\pi/2}{n^2\pi^2}\cos n\pi ct + \frac{a}{n\pi c}\sin n\pi ct\right)\sin n\pi x$$

UNIT-II

Q.4. (a) The contents of three urns are: 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. The two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. **(6)**

Ans. Let E_1, E_2, E_3 be the events of choosing urn I, urn II, urn III respectively.

$$\therefore \quad P(E_1) = \frac{1}{3},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{3}$$

Urn I: 1W, 2R, 3G

Urn II : 2W, 1R, 1G

Urn III: 4W, 5R, 3G

Let A be the event of drawing 2 balls (1W, 1G) when 2 balls are drawn from an urn.

$\therefore$ $P(A|E_1)$ = Probability that 1W and 1G balls are drawn, when 2 balls are drawn simultaneously from urn I.

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1\times\frac{3!}{2!}}{\frac{6!}{2!4!}}$$

$$= \frac{3\times 2}{6\times 5} = \frac{1}{5}$$

$$P(A|E_2) = \frac{{}^2C_1\times{}^1C_1}{{}^4C_2} = \frac{2\times 1}{\frac{4!}{2!\times 2!}} = \frac{2\times 2}{4\times 3} = \frac{1}{3}$$

$$P(A/E_3) = \frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2} = \frac{\frac{4!}{3!}\times\frac{3!}{2!}}{\frac{12!}{2!\,10!}} = \frac{4\times 3\times 2}{12\times 11} = \frac{2}{11}$$

To find $P(E_3/A)$

By Baye's theorem

$$P(E_3/A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{11}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{2}{33}}{\frac{1}{15}+\frac{1}{9}+\frac{2}{33}} = \frac{\frac{2}{33}}{\frac{33+55+30}{495}}$$

$$= \frac{2}{33}\times\frac{495}{118} = 0.2542$$

Q.4. (b) A random variable given measurements X between 0 and 1 with a probability function **(6.5)**

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x, & 0 \le x \le 1 \\ 0 & \end{cases}$$

Find (i) $P(X < 1/2)$ and $P(X > 1/2)$

(ii) Find number K such that $P(X \le K) = \frac{1}{2}$.

Ans. (i)
$$P(X \le 1/2) = \int_0^{1/2} f(x)\,dx$$
$$= \int_0^{1/2} (12x^3 - 21x^2 + 10x)\,dx$$
$$= \left[3x^4 - 7x^3 + 5x^2\right]_0^{1/2}$$
$$= \left[\frac{3}{16} - \frac{7}{8} + \frac{5}{4}\right] = \frac{3 - 14 + 20}{16} = \frac{9}{16}$$
$$P(X > 1/2) = 1 - P(X < 1/2)$$
$$= 1 - \frac{9}{16} = \frac{7}{16}$$

(ii)
$$P(X \le K) = 1/2$$
$$\Rightarrow \int_0^K f(x)\,dx = \frac{1}{2}$$
$$\Rightarrow \int_0^K (12x^3 - 21x^2 + 10x)\,dx = \frac{1}{2}$$
$$\Rightarrow 3x^4 - 7x^3 + 5x^2\Big]_0^K = \frac{1}{2}$$
$$\Rightarrow 3K^4 - K^3 + 5K^2 = \frac{1}{2}$$
$$\Rightarrow K^2(3K^2 - 7K + 5) = \frac{1}{2}$$
$$\Rightarrow K^2 = \frac{1}{2}, 3K^2 - 7K + 5 = \frac{1}{2}$$
$$\Rightarrow K = \pm\frac{1}{\sqrt{2}}, 3K^2 - 7K + \frac{9}{2} = 0$$
$$, 6K^2 - 14K + 9 = 0$$
$$K = \frac{14 \pm \sqrt{-20}}{12} = \frac{7 \pm i5}{6}$$

Q.5. (a) The first four moments of a distribution about the vlaue '0' are –0.20, 1.76, –2.36 and 10.88. Find the moments about the mean and kurtosis of the distribution. (6)

Ans. Let μ_1', μ_2', μ_3', μ_4' be the first four moments of the given distribution then $\mu_1' = -0.2$, $\mu_2' = 1.76$, $\mu_3' = -2.36$, $\mu_4' = 10.88$

Let μ_1, μ_2, μ_3, μ_4 be the first four moments about the mean

$$\Rightarrow \mu_1 = 0$$
$$\mu_2 = \mu_2' - \mu_1'^2 = 1.76 - (-0.2)^2 = 1.72$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= -2.36 - 3 \times 1.76 \times (-0.2) + 2(-0.2)^3$$
$$= -2.36 + 1.056 - 0.016$$
$$= -1.32$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 10.88 - 4(-2.36)(-0.2) + 6 \times 1.76(-0.2^2) - 3(-0.2)^4$$
$$= 10.88 - 1.888 + 0.4224 - 0.0048 = 9.4096$$
$$\text{Kutosis, } \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9.4096}{1.72^2} = 3.1806$$

Q.5. (b) The marks of 1000 engineering students in a college are found to be normally distributed with mean 70 and variance 25. Find the number of students whose marks will be between (6.5)

(i) 60 and 75

(ii) more than 75

Ans. Let x be the random variable denoting the marks of students.

Here mean $\mu = 70$, variance $\sigma_2 = 25 \Rightarrow \sigma = 5$.

Let $z = \frac{x - \mu}{\sigma}$ be the standard normal variate

(i) When $x = 60$, $z = \frac{60 - 70}{5} = -2$

When $x = 75$, $z = \frac{75 - 70}{5} = 1$

$$\therefore \quad P(60 < x < 75) = P(-2 < z < 1)$$
$$= P(-2 < z < 0) + P(0 < z < 1).$$
$$= P(z < 2) + P(z < 1) \qquad \text{(due to symmetry)}$$
$$= 0.4772 + 0.3413 = 0.8185$$

$\therefore$ Required number of students $= 0.8185 \times 1000$
$$= 818.5 \approx 819.$$

(ii) $P(x > 75)$

When $x = 75$, $z = 1$

$$\therefore \quad P(x > 75) = P(z > 1)$$
$$= P(0 < z < \infty) - P(0 < z < 1)$$
$$= 0.5 - 0.3413 = 0.1587$$

$\therefore$ Required number of students
$$= 0.1587 \times 1000 = 158.7 \approx 159$$

UNIT-III

Q.6. (a) By the method of least squares, fit a parabola of the form $y = a + bx + cx^2$, to the following data: (6)

x	2	4	6	8	10
y	6.07	12.85	31.47	57.38	91.29

Ans.

$$y = a + bx + cx^2$$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2 \quad ...(1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad ...(2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \; ...(3), n = 5$$

x	y	x^2	x^3	x^4	xy	x^2y
2	6.07	4	8	16	12.14	24.28
4	12.85	16	64	256	51.4	205.6
6	31.47	36	216	1296	188.82	1132.92
8	57.38	64	512	4096	459.04	3672.32
10	91.29	100	1000	10000	912.9	9129

$$\Sigma y = 199.06, \Sigma x = 30, \Sigma x^2 = 220, \Sigma x^3 = 180$$

$$\Sigma x^4 = 15664, \Sigma xy = 1624.3, \Sigma x^2 y = 14164.12$$

Now

$$199.06 = 5a + 30b + 220c \quad ...(4)$$

$$1624.3 = 30a + 220b + 1800c \quad ...(5)$$

$$14164.12 = 220a + 1800b + 15664c \quad ...(6)$$

By Gauss elimination

$$\left[\begin{array}{ccc|c} 5 & 30 & 220 & 199.06 \\ 30 & 220 & 1800 & 1624.3 \\ 220 & 1800 & 15664 & 14164.12 \end{array}\right]$$

Operating $R_2 \to R_2 - 6R_1$, $R_3 \to R_3 - 44R_1$

$$\sim \left[\begin{array}{ccc|c} 5 & 30 & 220 & 199.06 \\ 0 & 40 & 480 & 429.94 \\ 0 & 480 & 5984 & 5405.48 \end{array}\right]$$

Operating $R_3 \to R_3 - 12R_2$

$$\left[\begin{array}{ccc|c} 5 & 30 & 220 & 199.06 \\ 0 & 40 & 480 & 429.94 \\ 0 & 0 & 224 & 246.2 \end{array}\right]$$

On solving

$$5a + 30b + 220c = 199.06$$

$$40b + 480c = 429.94$$

$$224c = 246.2$$

$\Rightarrow$

$$c = 1.0991$$

and

$$40b + 480 \times 1.0991 = 429.94$$

$\Rightarrow$

$$b = -2.4408$$

Also $5a + 30\,(-2.4408) + 220 \times 1.0991 = 199.06$

$\Rightarrow$

$$a = 6.0964$$

$\therefore y = 6.0964 - 2.4408x + 1.0991x^2$

Q.6. (b) Find two lines of regression and coefficient of correlation for the data given below:

$n = 18, \Sigma x = 12, \Sigma y = 18, \Sigma x^2 = 60, \Sigma y^2 = 96, \Sigma xy = 48.$ **(6.5)**

Ans. Let r_{xy} be the coefficient of correlation between the two vriables x and y, then

$$r_{xy} = \frac{\frac{1}{n}\Sigma x_i y_i - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{1}{n}\Sigma_i^2 - \bar{x}^2\right)\left(\frac{1}{n}\Sigma y_i^2 - \bar{y}^2\right)}}$$

Here

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12}{18} = 0.6667$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{18}{18} = 1$$

$\therefore$

$$r_{xy} = \frac{\frac{1}{18} \times 48 - 0.6667 \times 1}{\sqrt{\left(\frac{1}{18} \times 60 - 0.6667^2\right)\left(\frac{1}{18} \times 96 - 1\right)}}$$

$$= \frac{2.6667 - 0.6667}{\sqrt{(3.3333 - 0.4445)(5.3333 - 1)}}$$

$$= \frac{2}{3.5381} = 0.5653$$

Also,

$$\sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$$

$$= \frac{1}{n}\Sigma x_i^2 - \bar{x}^2 = \frac{1}{18} \times 60 - 0.6667^2 = 2.8888$$

$\Rightarrow$

$$\sigma_x = 1.6996$$

$$\sigma_y^2 = \frac{1}{n}\Sigma y_i^2 - \bar{y}^2 = \frac{1}{18} \times 96 - 1 = 4.3333$$

$\Rightarrow$

$$\sigma_y = 2.0817$$

Let bxy and byx are the regression coefficients

$$bxy = \frac{r_{xy}\sigma_x}{\sigma_y} = \frac{0.5653 \times 1.6996}{2.0817}$$

$$bxy = 0.4615$$

$$byx = \frac{r_{xy}\sigma_y}{\sigma_x} = \frac{0.5653 \times 2.817}{1.6996} = 0.6924$$

Line of regression of y on x is

$$y - \bar{y} = byx(x - \bar{x})$$

$\Rightarrow$

$$y - 1 = 0.6924\,(x - 0.6667)$$

$\Rightarrow 0.6924x - 0.4616 - y + 1 = 0$

$\Rightarrow 0.6924x - y + 0.5384 = 0$

Line of regression of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$\Rightarrow$

$$x - 0.6667 = 0.4615\,(y - 1)$$

$\Rightarrow$

$$x - 0.4615y - 0.2052 = 0$$

Q.7. (a) Fit a poisson distribution to the following data and test for its goodness of fit at 5% level of significance **(6)**

x	0	1	2	3	4
f	419	352	154	56	19

Ans. Null Hypothesis H_0: Poisson fit is a good fit to the data. Mean of the given distribution

$$= \frac{\Sigma f_i x_i}{\Sigma f_i}$$

x	f	fx
0	419	0
1	352	352
2	154	308
3	56	168
4	19	76

$$\Sigma f = 1000, \Sigma fx = 904$$

$$\therefore \quad \text{Mean} = \frac{904}{1000} = 0.904$$

To fit a poisson distribution we, require m.

Parameter $m = \bar{x} = 2$.

By poisson distributon, the frequency of r success is

$$N(r) = N \times e^{-m} \frac{m^r}{r!}, \ N \text{ is total frequency}$$

$$N(0) = 1000 \times e^{-0.904} \cdot \frac{(0.904)^0}{0!} = 404.946$$

$$N(1) = 1000 \times e^{-0.904} \cdot \frac{(0.904)^1}{1!} = 366.071$$

$$N(2) = 1000 \times e^{-0.904} \cdot \frac{(0.904)^2}{2!} = 165.071$$

$$N(3) = 1000 \times e^{-0.904} \frac{(0.904)^3}{3!} = 49.359$$

$$N(4) = 1000 \times e^{-0.904} \frac{(0.904)^4}{4!} = 11.268$$

X	0	1	2	3	4
O_i	419	352	154	56	19
E_i	404.946	366.071	165.464	49.859	11.268
$\frac{(O_i - E_i)^2}{E_i}$	0.4878	0.5409	0.7943	0.7564	5.3056

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 7.885$$

Tabulated value of χ^2 at 5% level of significance for

$5 - 2 = 3$ d.f. is 7.815

Since calculated value 7.885 > 7.815 (Tabulated). H_0 is rejected

$\therefore$ Poisson does not provide a good fit.

Q.7. (b) In one sample of 8 observations, the sum of squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether the difference is significant at 5% level of significance. Give [$F_{.05}$ (7, 9) = 3.29] **(6.5)**

Ans. Null Hypothesis $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma^2$

i.e. there is no significant difference between population variance.

$$\text{Under } H_0 : F = \frac{S_1^2}{S_2^2} \sim F(V_1, V_2 \text{ d.f.})$$

Where

$$V_1 = n_1 - 1, n_1 = 8$$

$$V_2 = n_2 - 1, n_2 = 10$$

$$\Sigma(X_1 - \bar{X}_1)^2 = 84.4, \ \Sigma(X_2 - \bar{X}_2)^2 = 102.6$$

Here

$$S_1^2 = \frac{\Sigma(X_1 - \bar{X}_1)^2}{n_1 - 1} = \frac{84.4}{7} = 12.057$$

$$S_2^2 = \frac{\Sigma(X_2 - \bar{X}_2)^2}{n_2 - 1} = \frac{102.6}{9} = 11.4$$

$$F = \frac{S_1^2}{S_2^2} (S_1^2 > S_2^2)$$

$$= \frac{12.057}{11.4} = 1.0576$$

The tabulated value of F at 5% level of significance for (7, 9) d.f. = 3.29.

$\therefore F_{0.05} = 3.29$ and $|F| = 1.0576 < 3.29$.

Thus H_0 is accepted.

i.e. There is no significant difference between variance of the population.

UNIT IV

Q.8. (a) Use Big-M method to solve the following LPP. **(10)**

Maximize $z = 6x_1 + 4x_2$ subject to constraints

$$2x_1 + 3x_2 \le 30; \quad 3x_1 + 2x_2 \le 24$$

$$x_1 + x_2 \ge 3 \quad x_1 > 0, x_2 \ge 0$$

Ans. The given L.P.P can be written as

$$\text{Max } z = 6x_1 + 4x_2 + 0.x_3 + 0.x_4 + 0.x_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 = 3$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Let us add an artifical variable x_6 to the 3rd constraint, so that our L.P.P. becomes

$$\text{Max } z = x_1 + 5x_2 + 0.x_3 + 0.x_4 + 0.x_5 - Mx_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 + x_6 = 3$$

$x_1, x_2, x_3, x_4, x_5 \geq 0$ and $x_6 \geq 0$ is an artifical variable and $M > 0$ is very very large.

Starting Table

		c_j	6	4	0	0	0	–M
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_3	30	2	3	1	0	0	0
0	y_4	24	3	2	0	1	0	0
–M	y_6	3	1	$\boxed{1}$	0	0	–1	1
		$z_j - c_j$	–M–6	–M–4	0	0	M	0
				↑				

y_2 enters the basis

$$\min\left\{\frac{x_{Bi}}{yi_2}, yi_2 > 0\right\} = \text{Min}\left\{\frac{30}{3}, \frac{24}{2}, \frac{3}{1}\right\} = 3$$

$\Rightarrow y_6$ leaves the basis.

First Iteration

		c_j	6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_3	21	–1	0	1	0	$\boxed{3}$
0	y_4	18	1	0	0	1	2
4	y_2	3	1	1	0	0	–1
		$z_j - c_j$	–2	0	0	0	–4
							↑

$\therefore y_5$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_5}, yi_5 > 0\right\} = \text{Min}\left\{\frac{21}{3}, \frac{18}{2}\right\} = 7$$

$\Rightarrow y_3$ leaves the basis

Second Iteration

		c_j	6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	7	–1/3	0	1/3	0	1
← 0	y_4	4	$\boxed{5/3}$	0	–2/3	1	0
4	y_2	10	2/3	1	1/3	0	0
		$z_j - c_j$	–10/3	0	4/3	0	0
			↑				

$\Rightarrow y_1$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_1}, yi_1 > 0\right\} = \text{Min}\left\{\frac{4}{5/3}, \frac{10}{2/3}\right\} = \frac{12}{5}$$

$\Rightarrow y_4$ leaves the basis

Third Iteration

		c_j	6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	39/5	0	0	1/5	1/5	1
6	y_1	12/5	1	0	–2/5	3/5	0
4	y_2	42/5	0	1	3/5	–2/5	0
		$z_j - c_j$	0	0	0	2	0

Since all $z_j - c_j \geq 0$.

Thus sol^n is optimal.

$$x_1 = \frac{12}{5}, \; x_2 = \frac{42}{5}$$

$$\text{and Max } z = 6 \times \frac{12}{5} + 4 \times \frac{42}{5} = \frac{240}{5} = 80.$$

Q.8. (b) Find the dual of the following L.P.P. **(2.5)**

Maximize $z = 2x_1 + x_2$ **Subject to constraints**

$$x_1 + 5x_2 \leq 10$$
$$x_1 + 3x_2 \geq 6$$
$$2x_1 + 2x_2 \geq 8$$

x_1 unrestricted, $x_2 \geq 0$.

Ans. x_1 is unrestricted

$$\therefore \quad x_1 = x_1' - x_1''$$

$$\Rightarrow \quad \text{Max } z = 2(x_1' - x_1'') + x_2$$

S.t.

$$x_1' - x_1'' + 5x_2 \leq 10$$
$$x_1' - x_1'' + 3x_2 \geq 6$$
$$2x_1' - 2x_1'' + 2x_2 \geq 8$$

$$x_1', x_1'', x_2 \ge 0$$

Introduce slack and surplus variables s_1, s_2, s_3

$$\text{Max } Z = 2x_1' - 2x_1'' + x_2 + 0.s_1 + 0.s_2 + 0.s_3$$

S.t.

$$x_1' - x_1'' + 5x_2 + s_1 = 10$$

$$x_1' - x_1'' + 3x_2 - s_2 = 6$$

$$2x_1' - 2x_1'' + 2x_2 - s_3 = 8$$

$$x_1' - x_1'', x_2, s_1, s_2, s_3 \ge 0$$

Dual

If w_1, w_2, w_2 be the dual variables corresponding to each primal constraints, the problem will be

$$\text{Min } z^* = 10w_1 + 6w_2 + sw_3$$

S.t.

$$\left.\begin{array}{l} w_1 + w_2 + 2w_3 \ge 2 \\ -w_1 - w_2 - 2w_3 \le -2 \end{array}\right\} - w_1 - w_2 - 2w_3 = 2$$

$$5w_1 + 3w_2 + 2w_3 \le 1$$

$$-w_2 \le 0, -w_3 \le 0$$

$$\Rightarrow \quad w_2 \ge 0, w_3 \ge 0$$

w_1 is unrestricted.

Q.9. (a) Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method. Also find the optimum solution. (6.5)

	D_1	D_2	D_3	D_4	Suppply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

Ans. Since demand = total supply = 10.

Table for initial b.f.s. is

	D_1	D_2	D_3	D_4	
S_1	3	7	6	4	5 (1)
S_2	2	4	$\boxed{2}$ 3	$\boxed{\varepsilon_1}$ 2	2 (0)
S_3	4	3	8	5	3 (1)
	3 (1)	3 (1)	2 (3)	2 (2)	

Largest of these difference is (3) associated with 3rd column of the table.

Least cost is 3, allocate $x_{23} = \min(2, 2) = 2$

Arbitrarly cross 3rd column and ε_1 as small quantity to s_2.

Reduced table is

	D_1	D_2	D_4	
S_1	3	7	4	5 (1)
S_2	2	4	$\boxed{\varepsilon_1}$ 2	0 (0)
S_3	4	3	5	3 (1)
	3 (1)	3 (1)	2 (2)	

Largest difference is (2), associated with 4th column of the table.

Least cost is 2, allocate $x_{24} = \min(0, 2) = 0$

	D_1	D_2	D_4	
S_1	3	7	4	5 (1)
S_3	4	$\boxed{3}$ 3	5	3 (1)
	3 (1)	3 (4)	2 (1)	

Largest difference is (4), associated with 2nd column. Least cost is 3, allocate $x_{32} = \min(3, 3) = 3$

Eliminate 2nd column.

Reduced table is

	D_1	D_4	
S_1	$\boxed{3}$ 3	4	5 (1)
S_3	4	5	0 (1)
	3 (1)	2 (1)	

allocate $x_{11} = \min(3, 5) = 3$

eliminate first column

	D_4	
S_1	$\boxed{2}$ 4	2
S_3	$\boxed{\varepsilon_2}$ 5	0
	2	

Final table is

	D_1	D_2	D_3	D_4
S_1	$\boxed{3}$ 3	7	6	$\boxed{2}$ 4
S_2	2	4	$\boxed{2}$ 3	$\boxed{\varepsilon_1}$ 2
S_3	4	$\boxed{3}$ 3	8	$\boxed{\varepsilon_2}$ 5

$$\text{Total cost} = 3\times3+2\times4+2\times3+2\varepsilon_1+3\times3+5\varepsilon_2$$
$$= 32+2\varepsilon_1+5\varepsilon_2$$
$$= 32 \text{ as } \varepsilon_1\to \text{ and } \varepsilon_2\to 0$$

Let u_i and v_j represents shadow for supply and demand centres respectively.

Compute u_i $(i, 1, 2, 3)$ and $v_j (j=1,2,3,4)$ using successively the equation $u_i+v_j=C_{ij}$ for all occupied cells.

Let
$$v_4 = 0$$
$$u_1+v_4 = 4\Rightarrow u_1=4$$
$$u_2+v_4 = 2\Rightarrow u_2=2$$
$$u_3+v_4 = 5\Rightarrow u_3=5$$

Now
$$u_1+v_1 = 3\Rightarrow v_1=-1$$
$$u_2+v_3 = 3\Rightarrow v_3=1$$
$$u_3+v_2 = 3\Rightarrow v_2=-2$$

Net evaluations of unoccupied cells are
$$z_{12}-c_{12} = u_1+v_2-c_{12}=4-2-7=-5$$
$$z_{13}-c_{13} = u_1+v_3-c_{13}=4+1-6=-1$$
$$z_{21}-c_{21} = u_2+v_1-c_{21}=2-1-2=-1$$
$$z_{22}-c_{22} = u_2+v_2-c_{22}=2-2-4=-4$$
$$z_{31}-c_{31} = u_3+v_1-c_{31}=5-1-4=0$$
$$z_{33}-c_{33} = u_3+v_2-c_{33}=5+1-8=-2$$

Since all $z_{ij}-c_{ij}>0$

$\therefore$ Basic feasibe solution is optimum.

Thus optimal solution is

$x_{11}=3, x_{14}=2, x_{23}=2, x_{24}=\varepsilon_1, x_{32}=3, x_{34}=\varepsilon_2$

Transportation cost is
$$= 3\times3+2\times4+2\times3+2\varepsilon_1+3\times3+5\varepsilon_2$$
$$= 32$$

Q.9. (*b*) Four professors are each capable of teaching any one of the four different courses. Class preparation time in hours for different topics varies from professor to professor and given in the table below. Each professor is assigned only one course. Determine an assignment schedule so as to minimize the total course preparation time for all courses. (6)

Professor	Linear Programming	Queing Theory	Dynamic Programming	Regression analysis
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

Ans. Here number of tasks = number of subordinates = 4.

Subtracting smallest element of each row.

Reduced matrix is

$$\begin{array}{|cccc|}0 & 8 & 7 & 5\\ 11 & 0 & 10 & 4\\ 2 & 3 & 5 & 0\\ 0 & 11 & 9 & 5\end{array}$$

Subtracting smallest element of each column.

Reduced matrix is

$$\begin{array}{|cccc|}0 & 8 & 2 & 5\\ 11 & 0 & 5 & 4\\ 2 & 3 & 0 & 0\\ 0 & 11 & 4 & 5\end{array}$$

Starting with first row, enrectangle (☐) single zero, if any and cross (X) all other zeros in the column.

$$\begin{array}{|cccc|}\boxed{0} & 8 & 2 & 5\\ 11 & \boxed{0} & 5 & 4\\ 2 & 3 & 0 & 0\\ \cancel{0} & 11 & 4 & 5\end{array}$$

Repeat the procedure for each column.

$$\begin{array}{|cccc|}\boxed{0} & 8 & 2 & 5\\ 11 & \boxed{0} & 5 & 4\\ 2 & 3 & \boxed{0} & \cancel{0}\\ \cancel{0} & 11 & 4 & 5\end{array}$$

Since number of assigned values is less than 4.

(1) Mark row 4 (✔)

$$\begin{array}{|cccc|l}\boxed{0} & 8 & 2 & 5 & ✔\\ 11 & \boxed{0} & 5 & 4 & \\ 2 & 3 & \boxed{0} & \cancel{0} & \\ \cancel{0} & 11 & 4 & 5 & ✔\end{array}$$

(2) Mark 1st column (✔) of marked row.

(3) Mark rows (✔) havind assigned zeroes of the marked column.

(4) Draw straight lines through all unmarked rows and marked columns.

Minimum number of line is less than order of the matrix.

New modified matrix is

smallest element not covered by lines is 2. Subtracting 2 from all the uncovered elements and adding same to all elements lying at intersection of lines.

Reduced cost matrix is

$$\begin{bmatrix} 0 & 6 & 0 & 3 \\ 13 & 0 & 5 & 4 \\ 4 & 3 & 0 & 0 \\ 0 & 9 & 2 & 3 \end{bmatrix}$$

Repeat steps of enrectangling and crossing cross

$$\begin{bmatrix} \cancel{0} & 6 & \boxed{0} & 3 \\ 13 & \boxed{0} & 5 & 4 \\ 4 & 3 & \cancel{0} & \boxed{0} \\ \boxed{0} & 9 & 2 & 3 \end{bmatrix}$$

Each row and each column has one assigned.

Zero ∴ Optimal solution is reached.

Optimum assignment is

	E	*F*	*G*	*H*
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

$A \to G$, $B \to F$, $C \to H$, $D \to E$

Min. time required is $= 9 + 4 + 11 + 4 = 28$ hrs.