SYLLABUS
DIGITAL SIGNAL PROCESSING
(ECC-303)
Applicable from Batch Admitted in Academic Session 2021-22 Onwards

UNIT I

Review of Discrete Time Fourier Transform, Z- transform and Discrete Fourier Transform, Properties of the DFT: Periodicity, Linearity and Symmetry properties, Multiplication of two DFTs, concept of circular convolution, computation of circular convolution by graphical and matrix form, relationship between linear convolution and circular convolution, computation of linear convolution from circular convolution, linear filtering using DFT, aliasing error, filtering of long data sequences – Overlap-Save and Overlap-Add methods

Efficient computation of the DFT: Complexity analysis of direct computation of DFT, Concept of Fast Fourier transformation, Radix-2 computation of FFT using decimation-in-time and decimation-in-frequency algorithms, signal flow graphs, Butterflies, computations of FFT in one place using both algorithms, bit-reversal process, examples for DIT & DIF FFT Butterfly computations. **[No. of Hrs.: 11]**

UNIT II

Design & structure of FIR filters: Characteristics of practical frequency-selective filters, Basic concepts of IIR and FIR filters, Gibbs Phenomenon, Symmetric and Anti-symmetric FIR filters, Design of Linear-phase FIR filters using windows- Rectangular, Hamming, Hanning, Bartlett windows, FIR differentiator, FIR Hilbert Transformer. Design of FIR filters using frequency sampling method. Structure for FIR Systems: Direct form, Cascade form and Lattice structures. **[No. of Hrs.: 10]**

UNIT III

Design & Structure of IIR filters: Concept of IIR digital filter, recursive and non-recursive system analog to digital domain transformation- Approximation of derivatives, impulse invariant method and bilinear transformation and their properties, limitations of bilinear transformation, frequency warping and prewarping, methods to find out the order of IIR filter, mapping of poles and zeroes of filter in analog domain, computation of filter transfer function in analog domain, digital filter realization techniques, procedure to design Butterworth and Chebyshev digital IIR filters. Direct, Cascade, Parallel , Signal Flow graph and transposed structure, Lattice structures, Lattice and Lattice-Ladder Structures, Schur - Cohn stability Test for IIR filters. **[No. of Hrs.: 11]**

UNIT IV

Quantization Errors in Digital Signal Processing: Fixed point and floating point representation of numbers, Errors resulting from Rounding and Truncation, Digital Quantization of filter coefficients, Round-off effects in digital filters,Dead Band Effects.

Multirate Digital Signal Processing: Decimation, Interpolation, Sampling rate conversion by a rational factor; Frequency domain characterization of Interpolator and Decimator; Polyphase decomposition, Applications of Multirate signal processing.

[No. of Hrs.: 10]

SYLLABUS (2016-17)

DIGITAL SIGNAL PROCESSING
(ETEC-306)

Instructions to Paper Setters:

1. *Question No. 1 should be compulsory and cover the entire syllabus. Thus question should have objective or short answert type questions. It should be 25 marks.*
2. *Apart from Question No.1, rest of the paper shall consists of four units as per the syllabus. Every unit should have two question. However, student may be asked to attempt only 1 question from each unit,. Each question should be of 12.5 marks.*

Objectives: *The aim of this course is to provide in depth knowledge of various digital signal processing techniques and design of digital filters, learn the concept of DFT FFT algorithms, and design of digital filters using different approximations, DSP processor and architecture. The prerequisites of this subject are basic knowledge of signal and systems.*

UNIT–I

Frequency Domain Sampling: The Discrete Fourier Transform, Properties of the DFT, Linear filtering methods based of the DFT.

Efficient computation of the DFT: Principal of FFT, Fast Fourier Transform Algorithms, Applications of FFT Algorithms, A linear filtering approach to computation of the DFT. Application of DFT, Design of Notch filter

[T2, T1][No. of Hours: 11]

UNIT–II

Design & Structure of IIR filters from analog filters: Impulse Invariance; Bilinear transformation and its use in design of Butterworth and Chebyshev IIR Filters; Frequency transformation in Digital Domain, Direct, Cascade, Parallel & transposed structure

Design & structure of FIR filters: Symmetric and anti-symmetric FIR filters; Design of Linear Phase FIR filters using windows, Frequency Sampling Method of FIR design, Direct, Cascade, Frequency Sampling, transposed structure

[T1, T2] [No. of Hours: 11]

UNIT–III

Implementation of Discrete Time Systems: Lattice structures, Lattice and Lattice-Ladder Structures, Schur-Cohn stability Test for IIR filters; Discrete Hilbert Transform.

Linear predictive Coding: Lattice filter design, Levension Darwin Technique, Schur Algorithm

[T1, T2] [No. of Hours: 10]

UNIT–IV

Quantization Errors in Digital Signal Processing: Representation of numbers, Quantization of filter coefficients, Round-off Effects in digital filters.

Multirate Digital Signal Processing: Decimation, Interpolation, Sampling rate conversion by a rational factor; Frequency domain characterization of Interpolator and Decimator; Poly phase decomposition.

[T1, T2][No. of Hours: 10]

New Topics Added from Academic Session 2021-22 Onwards

RSP

FIFTH SEMESTER

DIGITAL SIGNAL PROCESSING [ECC-303]

Unit - I

Q. 1. Discrete-Time Fourier Transform

Ans. A discrete-time signal can be represented in the frequency domain using discrete-time Fourier transform. Therefore, the Fourier transform of a discrete time sequence is called the discrete-time Fourier transform (DTFT).

Mathematically, if **x(n)** is a discrete-time sequence, then its discrete-time Fourier transform is defined as –

$$F[x(n)] = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

The discrete-time Fourier transform $X(\omega)$ of a discrete-time sequence x(n) represents the frequency content of the sequence x(n). Therefore, by taking the Fourier transform of the discrete-time sequence, the sequence is decomposed into its frequency components. For this reason, the DTFT $X(\omega)$ is also called the **signal spectrum.**

Condition for Existence of Discrete-Time Fourier Transform

The Fourier transform of a discrete-time sequence x(n) exists if and only if the sequence x(n) is absolutely summable, i.e.,

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

The discrete-time Fourier transform (DTFT) of the exponentially growing sequences do not exist, because they are not absolutely summable.

Also, the DTFT method of analysing the systems can be applied only to the asymptotically stable systems and it cannot be applied for the unstable systems, i.e., the DTFT can only be used to analyse the systems whose transfer function has poles inside the unit circle.

Numerical Example (1)

Find the discrete-time Fourier transform of the sequence x(n) = u(n).

Solution

The given discrete-time sequence is.

$x(n) = u(n) = \{1$ for $n \geq 0$ 0 for $n < 0$

Now. from the definition of DTFT. we have.

$$F[x(n)] = X(\omega) \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$\therefore \quad F[u(n)] = \sum_{n=-\infty}^{\infty} u(n)e^{-j\omega n} = \sum_{n=0}^{\infty} (1)e^{-j\omega n}$$

$$\Rightarrow \quad F[u(n)] = \frac{1}{1-e^{-j\omega}}$$

Numerical Example (2)

Find the discrete-time Fourier transform x(n) = {1, 3, – 2, 5}

Solution

The given discrete-time sequence is.

$x(n) = \{1, 3, -2, 5, 2\}$

The DTFT of a sequence is defined as-

$$F[x(n)] = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$\Rightarrow \quad X(\omega) = x(0) + x(1)e^{-j\omega} + x(2)e^{-j2\omega} + x(3)e^{-j3\omega} + x(4)e^{-j4\omega}$$

$$\therefore \quad X(\omega) = 1 + 3e^{-j\omega} - 2e^{-j2\omega} + 5e^{-j3\omega} + 2e^{-j4\omega}$$

Q. 2. Z-transform

Ans. The Z-transform may be of two types viz. **unilateral (or one-sided) and bilateral (or two-sided).**

Mathematically, if $x(n)$ is a discrete-time signal or sequence, then its bilateral or two-sided z-transform is defined as-

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Where, z is a complex variable and it is given by.

$$z = r e^{j\omega}$$

Where, r is the radius of a circle.

Also, the unilateral the one-sided z-transform is defined as-

$$Z[x(n)] = X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

Region of Convergence (ROC) of Z-Transform

The set of points in the z-plane, for which the Z-transform of a discrete-time sequence x(n), that is X(z) converges is called the region of convergence (ROC) of the Z-transform X(z).

For any given discrete-time sequence, the Z-transform may or may not converge. If there is no point in the z-plane for which the function X(z) converges, then the sequence x(n) is said to be having no z-transform.

Numerical Example

Find the Z-transform of the following sequence-

$$y(n) = x(n + 3)\,u(n)$$

Solution

The given discrete-time sequence is,

$$y(n) = x(x + 3)\,u(n)$$

From the definition of the Z-transform, we get.

$$Z[y(n)] = Y(z) = Z[(n + 3)\,u(n)]$$

$$Y(z) = \sum_{n=0}^{\infty}[x(n + 3)u(n)]\,z^{-n} = \sum_{n-0}^{\infty} x(n + 3)z^{-n}$$

Let $\quad (n + 3) = m$, then $n = (m - 3)$.

$$\therefore \quad Y(z) = \sum_{n=3}^{\infty} x(m)\,z^{-(m-3)}\,z^3 = \left[\sum_{m=3}^{\infty} x(m)z^{-m}\right]$$

$$Y(z) = z^3\left[\sum_{m=0}^{\infty} x(m)z^{-m} - x(0) - x(1)z^{-1}\right]$$

$$\Rightarrow \qquad Y(z) = z^3 X(z) - z^3 x(0) - z\,x(1)$$

Q.3. Complexity analysis of direct computation of DFT

We now have a way of computing the spectrum for an arbitrary signal: The Discrete Fourier Transform (DFT) computes the spectrum at **N** equally spaced frequencies from a length- **N** sequence. An issue that never arises in analog "computation," like that performed by a circuit, is how much work it takes to perform the signal processing operation such as filtering. In computation, this consideration translates to the number of basic computational steps required to perform the needed processing. The number of steps, known as the **complexity**, becomes equivalent to how long the computation takes (how long must we wait for an answer). Complexity is not so much tied to specific computers or programming languages but to how many steps are required on any computer. Thus, a procedure's stated complexity says that the time taken will be **proportional** to some function of the amount of data used in the computation and the amount demanded.

For example, consider the formula for the discrete Fourier transform. For each frequency we choose, we must multiply each signal value by a complex number and add together the results. For a real-valued signal, each real-times-complex multiplication requires two real multiplications, meaning we have **2N** multiplications to perform. To add the results together, we must keep the real and imaginary parts separate. Adding **N** numbers requires **N-1** additions. Consequently, each frequency requires

$$2N+2(N-1) = 4N-2$$

basic computational steps. As we have N frequencies, the total number of computations is

$$N(4N-2)$$

In complexity calculations, we only worry about what happens as the data lengths increase, and take the dominant term—here the $4N^2$ term—as reflecting how much work is involved in making the computation. As multiplicative constants don't matter since we are making a "proportional to" evaluation, we find the DFT is an $O(N^2)$ computational procedure. This notation is read "order **N**-squared". Thus, if we double the length of the data, we would expect that the computation time to approximately quadruple.

UNIT-II

Q.1. The characteristics listed below that are properties of practical filters:

Ans.

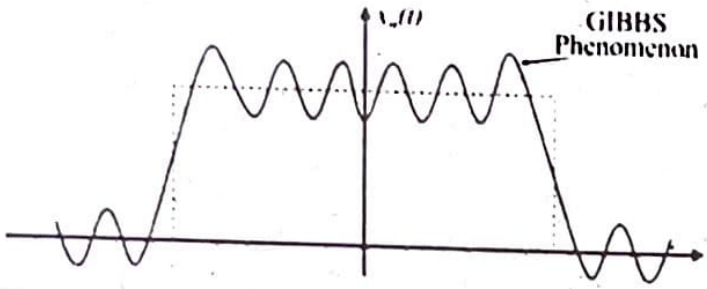
- Passband gain of one for all frequencies.
- Small, but nonzero stopband gain.
- Gradual transition of gain from passband to stopband.
- Zero gain in the stopband.
- Discontinuous magnitude response.

Q. 2. Concepts of FIR and IIR

Ans. If the impulse response of the filter falls to zero after a finite period of time, it is an FIR (Finite Impulse Response) filter. However, if the impulse response exists indefinitely, it is an IIR (Infinite Impulse Response) filter.

Q. 3. Gibbs phenomena

Ans. For a periodic signal with discontinuities, if the signal is reconstructed by adding the Fourier series, then overshoots appear around the edges. These overshoots decay outwards in a damped oscillatory manner away from the edges. This is known as GIBBS phenomenon and is shown in the figure below.



The amount of the overshoots at the discontinuities is proportional to the height of discontinuity and according to Gibbs, it is found to be around 9% of the height of discontinuity irrespective of the number of terms in the Fourier series. The exact proportion is given by the Wilbraham-Gibbs Constant.

$$\frac{1}{\pi}\int_0^{\pi}\frac{\sin t}{t}dt-\frac{1}{2} = 0.089489...$$

It may also be noted that as more number of terms in the series are added, the frequency increases and the overshoots become sharper, but the amplitude of the adjoining oscillation reduces, i.e., the error between the original signal x(t) and the truncated signal $x_n(t)$ reduces except at edges as the n increases. Hence, the truncated Fourier series approaches the original signal x(t) as the number of terms in approximation increases.

UNIT-III

Q. 1. Recursive and non-recursive system

Ans. A recursive system is a system in which current output depends on previous output(s) and input(s) but in non-recursive system current output does not depend on previous output(s). The system with memory is not necessarily a recursive system. For example, in FIR systems for input x[n] and output y[n] if we have. y[n] = y[n-1]+x[n], current output is depended on previous output as well as on current input (generally current and previous inputs). So accumulator is a recursive system.

Q.2. Methods to find out the order of IIR filter

Ans. Order N means length M=N+1 coefficients. You basically have a infinitely long ideal filter, and if you only use the rectangular window of length M to truncate the infinite filter to have only M coefficients then you end up with order N=M-1 filter.

Q.3. Mapping of poles and zeros of filter in analog domain

Ans. Every digital filter can be specified by its poles and zeros (together with a gain factor). Poles and zeros give useful insights into a filter's response, and can be used as the basis for digital filter design. This chapter additionally presents the Durbin step-down recursion for checking filter stability by finding the reflection coefficients, including matlab code.

We can write the general transfer function for the recursive LTI digital filter as

$$H(z) = g\frac{1+\beta_1 z^{-1}+\ldots+\beta M z^{-M}}{1+a_1 z^{-1}+\cdots+aN z^{-N}}$$

We have factored out the leading coefficient b_0 in the numerator (assumed to be nonzero) and called it g. (Here $\beta_i \triangleq b_2/b_0$.) In the same way that $z^2 + 3z + 2$ can be factored into (z + 1) (z + 2), we can factor the numerator and denominator to obtain

$$H(z) = g\frac{(1-q_1 z^{-1})(1-q_2 z^{-1})\ldots(1-qMz^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})\ldots(1-pNz^{-1})}$$

According to New Syllabus (ECC-303)
Digital Signal Processing

UNIT - I

Q. Refer to Q.1,(a) (b). First Term 2016 (Pg. No. 1-2016)
Q. Refer to Q.2 (a) (b) First Term Examination 2016 (Pg. No. 2, 3-2016)
Q. Refer to Q.3 (a) First Term Exam 2016 (Pg. No. 7-2016)
Q. Refer to Q.1 (a) End Term Exam 2016 (Pg. No.21-2016)
Q. Refer to Q.2 (a) (b) End Term Exam 2016 (Pg. No. 25-2016)
Q. Refer to Q.3 (a),(b) End Term Exam 2016 (Pg. No. 27-2016)
Q. Refer to Q.5 Important Questions 2016 (Pg. No. 43-2016)
Q. Refer to Q.1 (a) (b) First Term Exam 2017 (Pg. No. 1-2017)
Q. Refer to Q.2 (a) (b) First Term Exam 2017 (Pg. No. 1-2017)
Q. Refer to Q.3 (a) First Term Exam 2017 (Pg. No. 1-2017)
Q. Refer to Q.2 (a) (b) End Term Exam 2017 (Pg. No. 4,5-2017)
Q. Refer to Q.3 (a) (b) First Term Exam 2017 (Pg. No. 7,8-2017)
Q. Refer to Q.1 (a) (b) (d) First Term Exam 2018 (Pg. No. 1,2-2018)
Q. Refer to Q.2 (a) (b) First Term Exam 2018 (Pg. No. 3-2018)
Q. Refer to Q.3 First Term Exam 2018 (Pg. No. 5-2018)
Q. Refer to Q.1(a),(b) End Term Exam 2018 (Pg. No. 8-2018)
Q. Refer to Q.2 (a) (b) End Term Exam 2018 (Pg. No. 11-2018)
Q. Refer to Q.3 (a) (b) End Term Exam 2018 (Pg. No. 11,12-2018)
Q. Refer to Q.1 (a) (b) First Term Exam 2019 (Pg. No. 1-2019)
Q. Refer to Q.2 (a) (b) First Term Exam 2019 (Pg. No. 2-2019)
Q. Refer to Q.3 (b) First Term Exam 2019 (Pg. No. 3-2019)
Q. Refer to Q.1(b) End Term Exam 2019 (Pg. No. 5-2019)
Q. Refer to Q.2 (a) End Term Exam 2019 (Pg. No. 5-2019)
Q. Refer to Q.3 (a) (b) End Term Exam 2019 (Pg. No. 6-2019)

UNIT - II

Q. Refer to Q.3 (b) First Term Examination 2016 (Pg. No. 8-2016).
Q. Refer to Q.1 (b) Second Term Examination 2016 (Pg. No. 12-2016).
Q. Refer to Q.3 (a) (b) Second Term Examination 2016 (Pg. No. 15,16-2016).
Q. Refer to Q.1 (c) End Term Examination 2016 (Pg. No. 22-2016).
Q. Refer to Q.4 (b) End Term Examination 2016 (Pg. No. 28-2016).
Q. Refer to Q.5 (b) End Term Examination 2016 (Pg. No. 32-2016).
Q. Refer to Q.6 (a) End Term Examination 2016 (Pg. No. 33-2016).
Q. Refer to Q.7 (a) End Term Examination 2016 (Pg. No. 33-2016).
Q. Refer to Q.6 Important Question (Pg. No. 43-2016).
Q. Refer to Q.8 Important Question (Pg. No. 46-2016).
Q. Refer to Q.4 (b) First Term Examination 2017 (Pg. No. 1-2017).
Q. Refer to Q.4 (b) End Term Examination 2017 (Pg. No 10-2017).
Q. Refer to Q.6 (a) End Term Examination 2017 (Pg. No. 12-2017).
Q. Refer to Q.7 (b) End Term Examination 2017 (Pg. No. 14-2017).
Q. Refer to Q.1 (c) First Term Examination 2018 (Pg. No. 2-2018).
Q. Refer to Q.4 First Term Examination 2018 (Pg. No. 5-2018).
Q. Refer to Q.1 (c) End Term Examination 2018 (Pg. No. 9-2018).
Q. Refer to Q.4 (a) (b) End Term Examination 2018 (Pg. No. 13,14-2018).
Q. Refer to Q.1 (c) End Term Examination 2019 (Pg. No. 5-2019).
Q. Refer to Q.4 (b) End Term Examination 2019 (Pg. No. 6-2019).

UNIT - III

Q. Refer to Q.1(c) (d) First Term Examination2016 (Pg. No.1,2-2016).
Q. Refer to Q.4 (a) First Term Examination 2016 (Pg. No. 9-2016).
Q. Refer to Q.2 (b) Second Term Examination 2016 (Pg. No. 14-2016).
Q. Refer to Q.1 (b) End Term Examination 2016 (Pg. No. 21-2016).
Q. Refer to Q.1 (d) End Term Examination 2016 (Pg. No. 24-2016).
Q. Refer to Q.4 (a) End Term Examination 2016 (Pg. No. 28-2016).
Q. Refer to Q.5 (a) End Term Examination 2016 (Pg. No. 30-2016).
Q. Refer to Q.7 (b) End Term Examination 2016 (Pg. No. 34-2016).
Q. Refer to Q.1 Important Question (Pg. No. 39-2016).
Q. Refer to Q.7 Important Question (Pg. No. 44-2016).
Q. Refer to Q.1 (c) (d) First Term Examination 2017 (Pg. No. 1-2017).
Q. Refer to Q.3 (b) First Term Examination 2017 (Pg. No. 1-2017).
Q. Refer to Q.4 (a) First Term Examination 2017 (Pg. No. 1-2017).
Q. Refer to Q.1 (b) (e) End Term Examination 2017 (Pg. No. 2,3-2017).
Q. Refer to Q.4 (a) End Term Examination 2017 (Pg. No. 9-2017).
Q. Refer to Q.5 (b) End Term Examination 2017 (Pg. No. 11-2017).
Q. Refer to Q.1 (e) First Term Examination 2018 (Pg. No. 3-2018).
Q. Refer to Q.5 First Term Examination 2018 (Pg. No. 5-2018).
Q. Refer to Q.1(d) End Term Examination 2018 (Pg. No. 10-2018).
Q. Refer to Q.5 (a), (b) End Term Examination 2018 (Pg. No. 14,15-2018).
Q. Refer to Q.6 (a),(b) End Term Examination 2018 (Pg. No. 16,17-2018).
Q. Refer to Q.7 (b) End Term Examination 2018 (Pg. No. 19-2018).
Q. Refer to Q.1 (c) (d) First Term Examination 2019 (Pg. No. 1,2-2019).
Q. Refer to Q.3 (a) First Term Examination 2019 (Pg. No. 3-2019).
Q. Refer to Q.1(a)(d) End Term Examination 2019 (Pg. No. 5-2019).
Q. Refer to Q.5 (a)(b) End Term Examination 2019 (Pg. No. 7,8-2019).
Q. Refer to Q.6 (b) End Term Examination 2019 (Pg. No. 8-2019).
Q. Refer to Q.7 (a)(b) End Term Examination 2019 (Pg. No. 19-2019).

UNIT - IV

Q. Refer to Q.1.(a)(d) Second Term Examination.2016 (Pg. No. 12-2016)
Q. Refer to Q.4 (a) (b) Second Term Examination 2016 (Pg. No. 16,18-2016)
Q. Refer to Q.1 (e) End Term Exam 2016 (Pg. No. 24-2016)
Q. Refer to Q.8 (a) (b) End Term Exam 2016 (Pg. No. 36-2016)
Q. Refer to Q.9 (a) End Term Exam 2016 (Pg. No. 36-2016)
Q. Refer to Q.2, 3 Important Question (Pg. No. 40,41-2016)
Q. Refer to Q.4 Important Question 2017 (Pg. No.41-2016)
Q. Refer to Q.1 (d) End Term Exam 2017 (Pg. No. 3-2017)
Q. Refer to Q.8 (a) End Term Exam 2017 (Pg. No. 14-2017)
Q. Refer to Q.9 (b) End Term Exam 2017 (Pg. No. 17-2017)
Q. Refer to Q.1 (e) End Term Exam 2018 (Pg. No. 10-2018)
Q. Refer to Q.8 (a)(b) End Term Exam 2018 (Pg. No. 21,22-2018)
Q. Refer to Q.9 (b) End Term Exam 2018 (Pg. No. 23-2018)
Q. Refer to Q.4 First Term Exam 2019 (Pg. No. 4-2019)
Q. Refer to Q.1 (e) End Term Exam 2019 (Pg. No. 5-2019)
Q. Refer to Q.8 (b) End Term Exam 2019 (Pg. No. 8-2019)
Q. Refer to Q.9 (a) (b) End Term Exam 2019 (Pg. No. 23-2019)

FIRST TERM EXAMINATION [FEB-2016]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC-306]

Time : 1½ hrs. **M.M. : 30**

Note: *Q.No.1 is compulsory. Attempt any two more Questions from the rest.*

Q.1.(a) Explain any three properties of DFT. **(3)**

Ans. Refer Q.1. (a) of Few Important Questions

Q.1. (b) What do you mean by twiddle factor? Explain symmetric and periodicity of twiddle factor. **(2)**

Ans. We know that N-point DFT of $x(n)$ is given as

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn};\ K = 0,1,..N-1$$

Here W_N is called twiddle factor and it is defined as

$$W_N = e^{-j2\frac{\pi}{N}}$$

Periodicity property of W_N

$$W_N^{K+N} = W_N^K$$

We know that $W_N = e^{-\frac{j2\pi}{N}}$

$$\therefore \quad W_N^{K+N} = e^{-j\frac{2\pi}{N}(K+N)} = e^{-j\frac{2\pi}{N}} . e^{-j2\pi}$$

$$= \left(e^{-j\frac{2\pi}{N}}\right)^K = W_N^K$$

Symmetry property of W_N

$$W_N^{K+\frac{N}{2}} = -W_N^K$$

We know that, $$W_N = e^{-j2\frac{\pi}{N}}$$

$$W_N^{K+\frac{N}{2}} = e^{-j\frac{2\pi}{N}\left(K+\frac{N}{2}\right)} = e^{\frac{-j2\pi}{N}}K = -W_N^K$$

Q.1. (c) What do you mean by warping and prewarping? **(2)**

Ans. The relation between the analog and digital frequencies in bilinear transformation is given by

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

For smaller values of ω there exist linear relationship between ω and Ω. But for large values of ω the relationship is non-linear. This non-linearity introduces distortion in the frequency axis. This is known as warping effect. This effect compresses the magnitude and phase response at high frequencies.

The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable prescaling, or prewarping the critical frequencies by using the formula.

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

Q.1. (d) How does a butterworth filter differ from Chebyshev filter? **(3)**

Ans. 1. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increase from 0 to ∞, whereas the magnitude response of the Chebyshev filter exhibits ripple in the passband and monotonically decreasing in the stopband.

2. The transition band is more in Butterworth filter compared to Chebyshev filter.

3. The poles of the Butterworth filter lie on a circle whereas the poles of the Chebyshev filter lie on an ellipse.

4. For the same specifications the number of poles in Butterworth filter are more when compared to Chebyshev filter, i.e., the order of the Chebyshev filter is less than that of Butterworth.

Q.2. (a) Compute the DFT of the four point sequence **(5)**

$$x_1(n) = [0123]$$

Ans. Here N = 4. Hence first evaluate W_4^0, W_4^1, W_4^2, and W_4^3. From the cyclic property of W_N we can determine these values. They will be evenly spaced along the unit circle as shown in Fig. 1.

The values given above can be verified easily. For example consider W_4^3.

$$W_4^3 = e^{-j\frac{2\pi}{4}\times 3} = e^{-j\frac{3\pi}{2}}$$

This has magnitude '1' and angle $-\frac{3\pi}{2}$ and,

$$W_4^3 = e^{-j\frac{3\pi}{2}} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = j$$

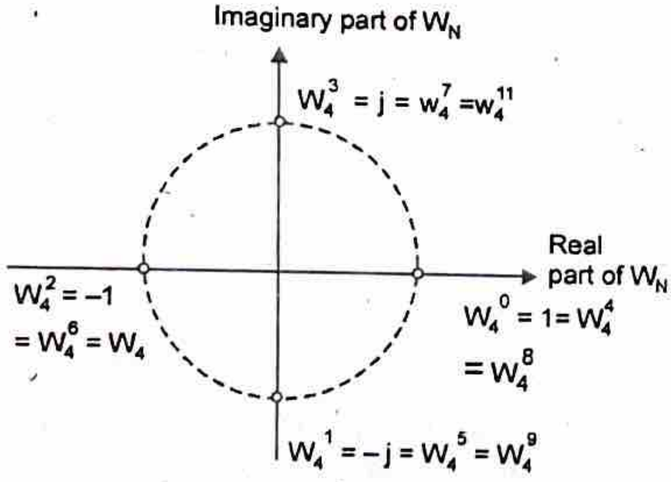


Fig.1. Calculation of W_4^{kn} and its periodic values.

We have the matrix $[W_4]$ of 4×4 size. Its individual elements will be as shown below:

$$[W_4] = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}\begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

In the above matrix individual elements are W_4^{kn} with $k = 0$ to 3 rows and $n = 0$ to 3 columns. From Fig. 1. the values of various elements in above matrix are,

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Also we have $$x_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

from equation we can obtain 4 point DFT as

$$\boxed{X_N = [W_N]x_N}$$

With $N = 4$, $\quad X_4 = [W_4]x_4$

Putting values of W_4 and x_4,

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Thus we obtained 4 point DFT as,

$$X_4 = \begin{bmatrix} X(0) = 6 \\ X(1) = -2+2j \\ X(2) = -2 \\ X(3) = -2-2j \end{bmatrix}$$

This is the required DFT.

Q.2. (b) Perform the circular convolution of the following two sequences. (5)

$$x_1(n) = \underset{\uparrow}{[2121]} \quad x_2[n] = \underset{\uparrow}{[1234]}$$

Ans. Here we have to perform circular convolution of $x_1(n)$ and $x_2(n)$ which is given as,

$$x_3(m) = \sum_{n=0}^{3} x_1(n) x_2((m-n))_4,\ m = 0, 1, 2, 3... \qquad ...(1)$$

To find $x_3(0)$, Put m = 0 in above equation:

Putting m = 0 in above equation we get

$$x_3(0) = \sum_{n=0}^{3} x_1(n) x_2((-n))_4 \qquad ...(2)$$

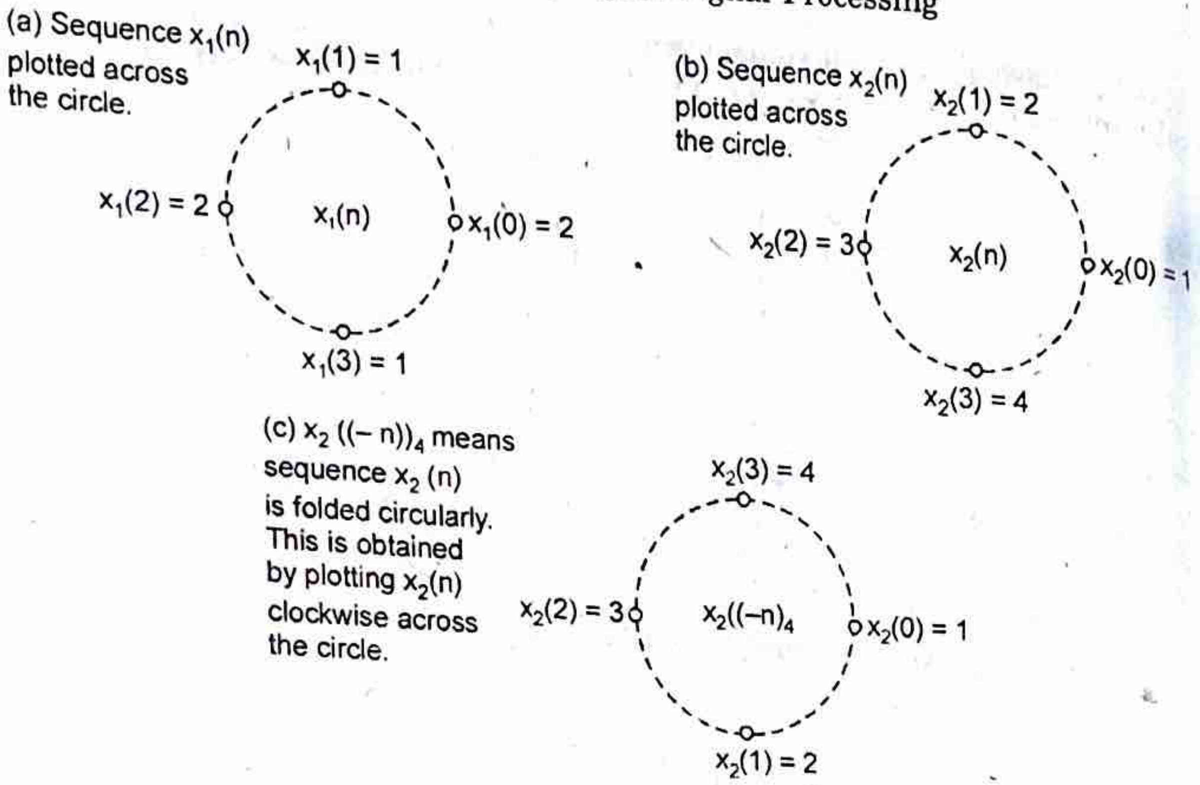


Fig. 1.

$$x_3(0) = 2 + 4 + 6 + 2$$
$$= 14$$

[Thus $x_3(0) = 1$

Because of concentric representation of Fig. 2. The products can be calculated easil

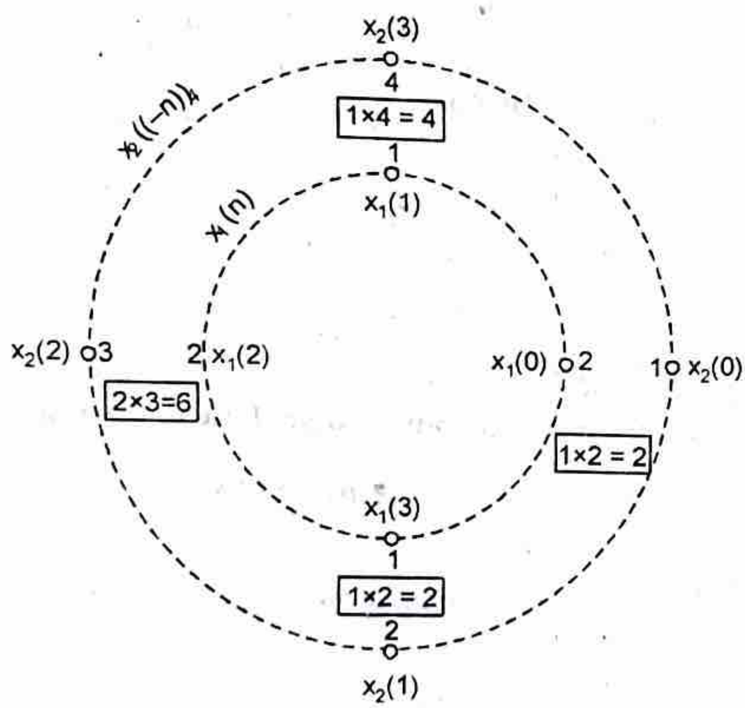


Fig. 2. $x_1(n)$ and $x_2((-n))_4$

To obtain $x_3(1)$, put m = 1 in equation (1).

Putting m = 1 in equation (1) we get

$$x_3(1) = \sum_{n=0}^{3} x_1(n) x_2((1-n))_4 \qquad ...(3)$$

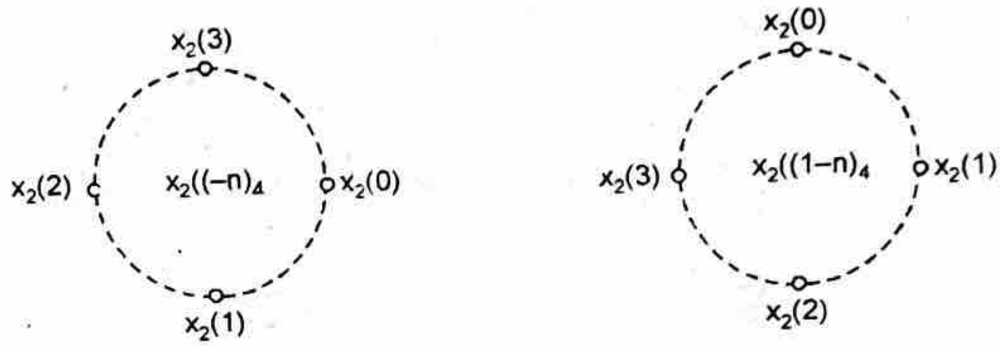


Fig. 3. (*a*) The sequence of x_2 $((-n))_4$ by shifting x_2 $((-n))_4$

(*b*) The sequence x_2 $((1-n))_4$ is obtained anticlockwise by one sample position.

Now we have to obtain products $x_1(n)$ $x_2((1-n))_4$ and their sum. This is obtained easily by plotting $x_1(n)$ and $x_2((1-n))_4$ on concentric circles as shown in Fig.4. Hence $x_3(1)$ of equation 3 becomes.

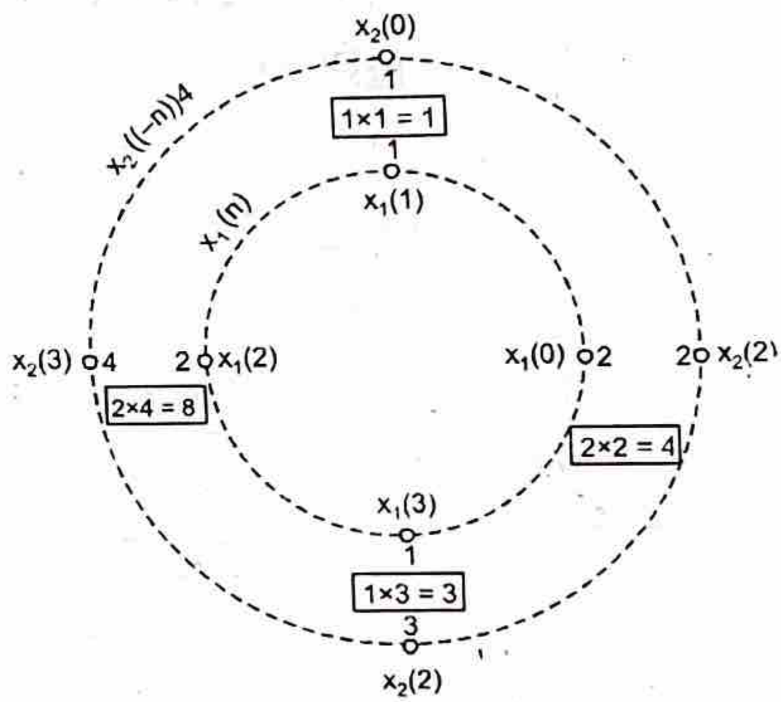


Fig. 4. Sequences $x_1(n)$ plotted on inner circle and x_2 $((1-n))_4$.

$$x_3(1) = 4 + 1 + 8 + 3 = 16$$

Thus $\boxed{x_3(1) = 16}$

To find x_3 (2) put $m = 2$ in equation 1.

Putting $m = 2$ in equation 1 we get

$$x_3(2) = \sum_{n=0}^{3} x_1(n)x_2((2-n))_4 \qquad (4)$$

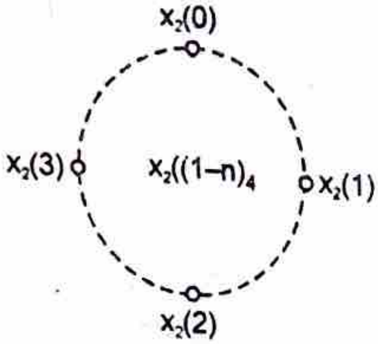
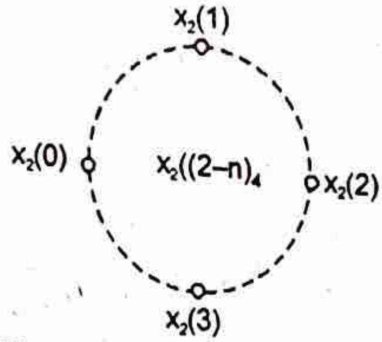


Fig. 5. (*a*) The sequence of $x_2((1-n))_4$



(*b*) The sequence $x_2((2-n))_4$ is obtained by shifting $x_2((1-n))_4$

Now we have to obtain the products $x_1(n)\,x_2((2-n))_4$ and their sum. This is obtained easily by plotting $x_1(n)$ and $x_2((2-n))_4$ on concentric circles as shown in Fig. 6. The point by point products are also shown in the figure.

Thus $x_3(2)$ of equation (4) can be obtained from above fig. 5. as,

$$x_3(2) = 6 + 2 + 2 + 4 = 14$$

Thus

$$\boxed{x_3(2) = 14}$$

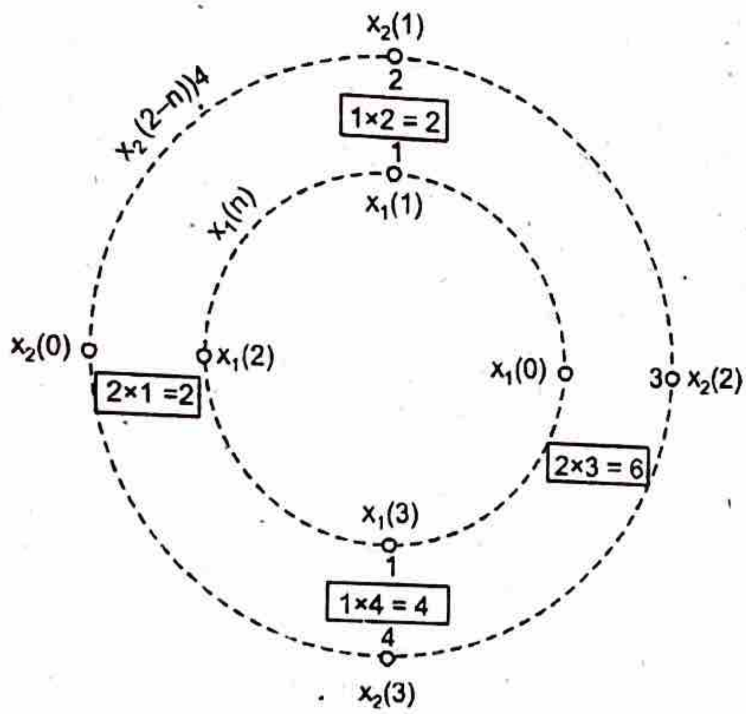


Fig.6. Sequence $x(n)$ plotted on inner circle and $x_2((2-n))_4$ plotted on outer circle.

To find $x_3(3)$, put $m = 3$ in equation (1).

Putting $m = 3$ in equation (1) we get,

$$x_3(3) = \sum_{n=0}^{3} x_1(n)x_2((3-n))_4 \qquad ...(5)$$

Then $x_3(3)$ of equation (5) can be obtained by adding all these products terms, *i.e.*,

$$x_3(3) = 8 + 3 + 4 + 1 = 16$$

Thus

$$\boxed{x_3(3) = 16}$$

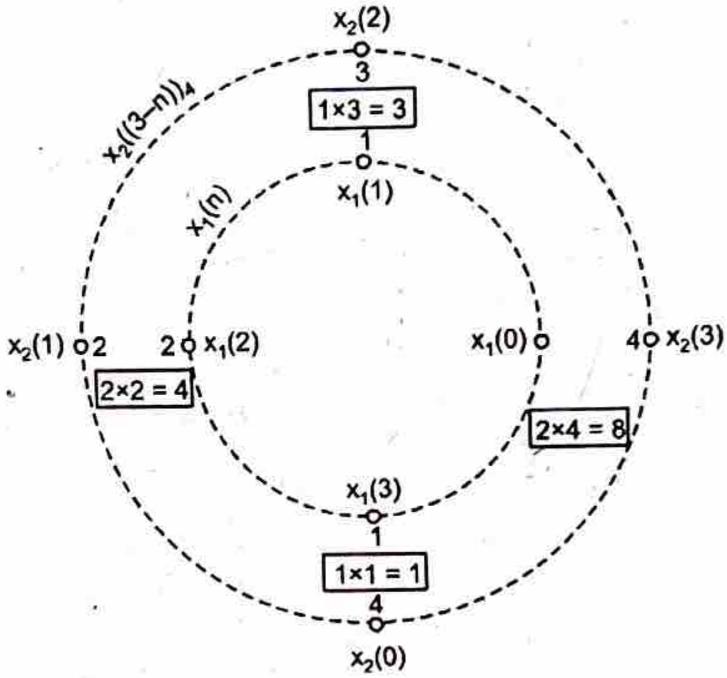


Fig. 7: To obtain $x_3(3)$

$$x_3(n) = \{4,16,14,16\} \qquad ...(6)$$
$\uparrow$

Q.3. (a) An 8 – point sequence is given by x(n) = [2, 2, 2, 2, 1, 1,1]. Compute 8 point DFT of $x(n)$ by radix 2 DIF FFT. (5)

Ans. Given that

$$x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}\ N = 8 \text{ points}$$

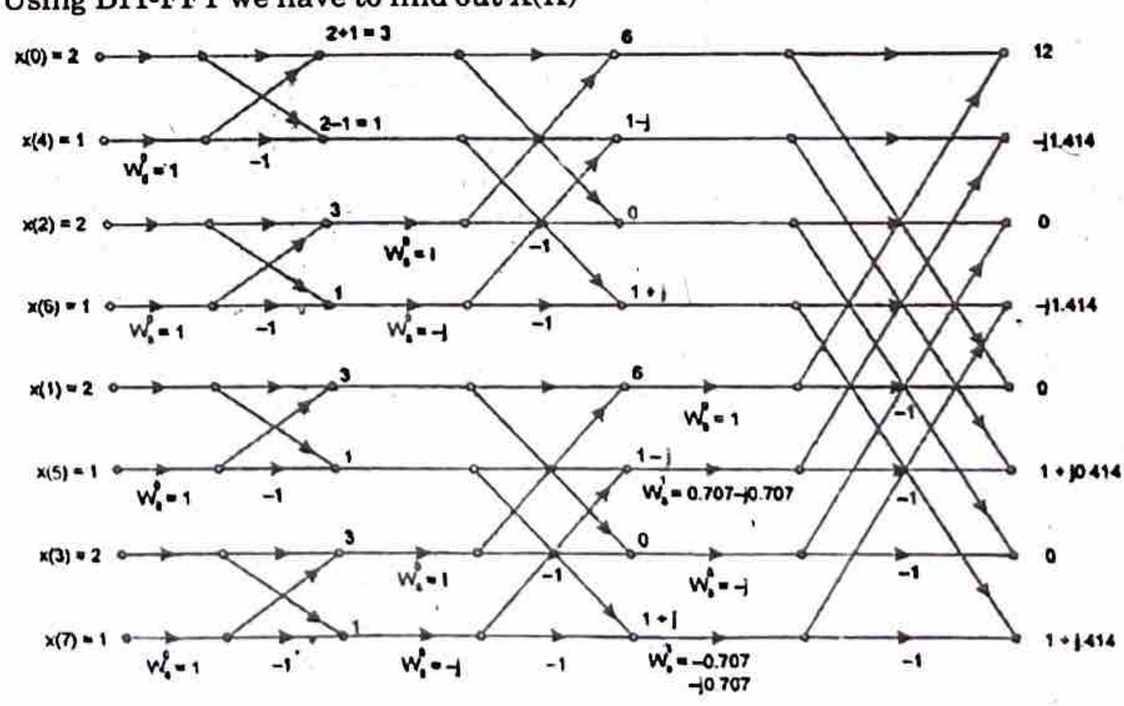
Using bit reversal methods, we get the sequence of $x(n)$ is

$$x(n) = x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7)\}$$

Also we have

$$W_N^K = e^{-j(2\pi/N)K};\ W^0_8 = 1;\ W^1_8 = 0.707 - j0.707$$
$$W_8^2 = -j\ ;\ W^3_8 = -0.707 - j0.707$$

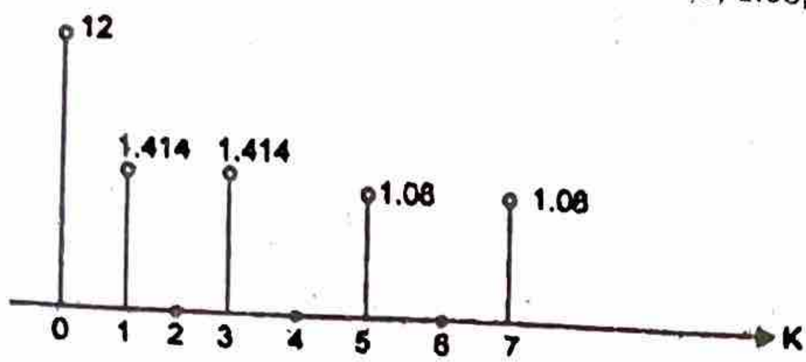
Using DIT-FFT we have to find out X(K)



Hence,

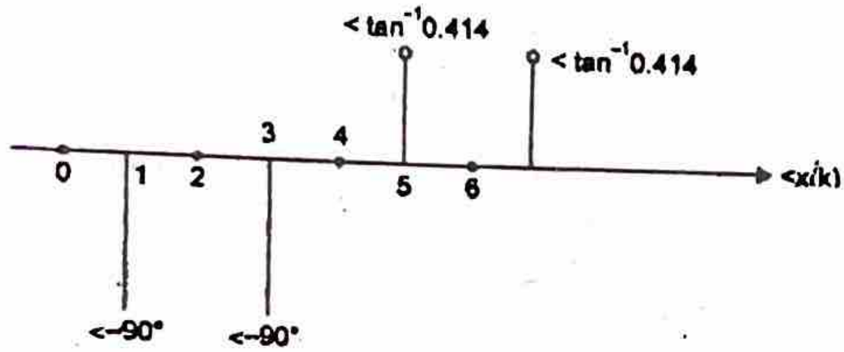
$$X(K) = \{12, -j1.414, 0, -j1.414, 01 + j0.41\ 1 + j0.414\}$$

For magnitude spectrum $|X(k)| = \{12, 1.414, 0, -1.414, 0, 1.08, 0, 1.08\}$



For phase spectrum

$$\angle X(K) = \{\angle 0°, -90°, 0°, \angle \tan^{-1} 0.414, 0°, \angle \tan{-1}\ 0.414\}$$



Q.3. (*b*) Find the digital network in direct and transposed form for system described by the difference equation

$y(n) = x(n) + 0.5\,x(n-1) + 0.4\,x(n-2) - 0.6\,y(n-1) - 0.7y(n-2)$

Ans. Given that

$$y(n) = x(n) + 0.5x(n-1) + 0.4\,x(n-2) - 0.6y(n-1) - 0.7\,y(n$$

Taking Z- transform both sides

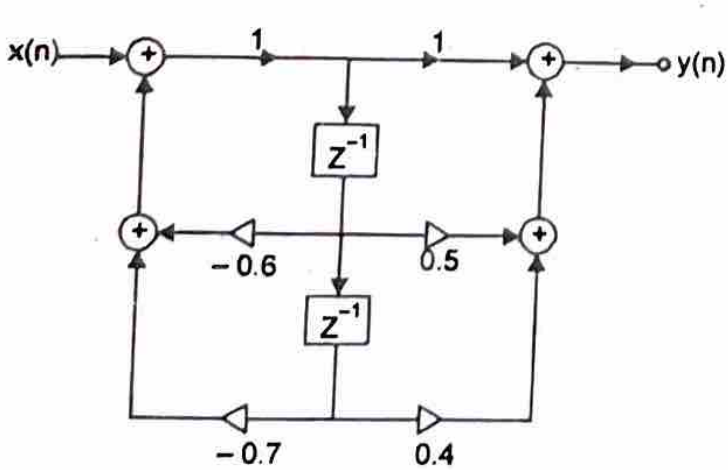
$$Y(z) = X(z) + 0.5\,Z^{-1}\,X(z) + 0.4\,Z^{-2}\,X(z) - 0.6\,Z^{-1}\,Y(z) - 0.7$$

$Y(z)$

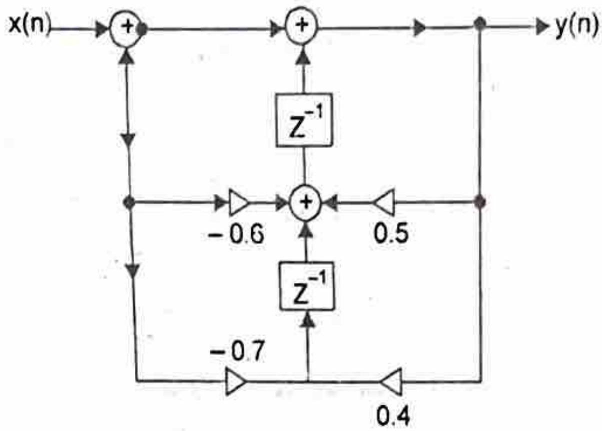
$$\Rightarrow \quad Y(z)\,[1+0.6Z^{-1} + 0.7Z^{-2}] = X(z)\,[1+0.5Z^{-1} + 0.4\,Z^{-2}]$$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1+0.5Z^{-1}+0.4Z^{-2}}{1+0.6Z^{-1}+0.7Z^{-2}}$$

Direct Form:



Transposed Form



Q.4. (*a*) A first order Butterworth low pass transfer function with a 3 *dB* cut off frequency at ΩC is given by

$$H_a(s) = \frac{\Omega c}{s + \Omega c}$$

Design a single low pass with 3 dB bandwidth of 0.2 π using the Bilinear transformation. **(5)**

Ans. Given that $w_c = 0.2\,\pi$ rad/sec.

$$\therefore \qquad \Omega_c = \frac{2}{T}\tan\frac{w_c}{2} = \frac{2}{T}\tan\frac{0.2\pi}{2} = \frac{0.65}{T}$$

Putting the value of Ω_c in given transfer function

$$H_a(s) = \frac{\Omega_c}{S+\Omega_c} = \frac{0.65/T}{S+\dfrac{0.65}{T}}$$

Q.4. (*b*) Design an ideal low pass filter with a frequency response

$$\begin{aligned} H_d(e^{j\omega}) &= 1; -\pi/2 \le |\omega| \le \pi/2 \\ &= 0; \pi/2 \le \omega \le \pi \end{aligned}$$

Find the values of *h*(*n*) for N = 11. **(5)**

Ans. The frequency response of lowpass filter with $\omega_c = \frac{\pi}{2}$ is shown in Fig. 1.

Given

$$\begin{aligned} H_d(e^{j\omega}) &= 1 \text{ for } \frac{-\pi}{2} \le \omega \le \omega \le \frac{\pi}{2} \\ &= 0 \text{ for } \frac{\pi}{2} \le |\omega| \le \pi \end{aligned}$$

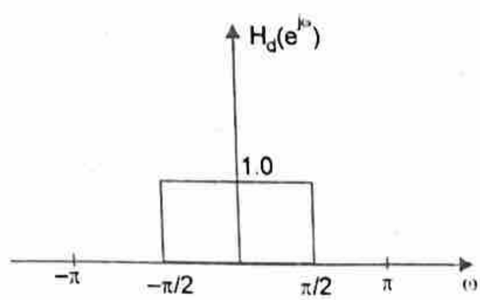


Fig. 1. Ideal frequency Reponse.

From the frequency response we can find that $\alpha = 0$. Therefore, we get a non-causal filter coefficients symmetrical about $n = 0$, i.e. $h_d(n) = h_d(-n)$. The filter coefficients can be obtained by using the formula for zero phase frequency response (or) we can proceed as follows:

We know

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j\omega n}d\omega \quad (1)$$

$$= \frac{1}{2\pi jn}e^{j\omega n}\Big|_{-\pi/2}^{\pi/2} = \frac{1}{\pi n(2j)}[e^{j\pi n/2 - e - j\pi n/2}]$$

$$= \frac{\sin\frac{\pi}{2}n}{\pi n} - \infty \le n \le \infty \quad (2)$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = \frac{\sin\frac{\pi}{2}n}{\pi n} \text{ for } |n| \le 5$$

$$= 0 \text{ otherwise} \quad (3)$$

For $n = 0$ Eq. (3) becomes inderteminate. so

$$h(0) = \lim_{n\to 0}\frac{\sin\frac{\pi}{2}n}{\pi n} = \frac{1}{2}\lim_{n\to 0}\frac{\sin\frac{\pi}{2}n}{\frac{\pi n}{2}} \qquad \boxed{\therefore \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$$

$$= \frac{1}{2}$$

(or) Substitute $n = 0$ in Eq. (1) we get

$$h(0) = h_d(0) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} d\omega = \frac{1}{2\pi}\omega\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2\pi} = \frac{1}{2}$$

For $n = 1$

$$h(1) = h(-1) = \frac{\sin\frac{\pi}{2}}{\pi} = \frac{1}{\pi} = 0.3183$$

Similarly

$$h(2) = h(-2) = \frac{\sin\pi}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = -\frac{1}{3\pi} = -0.106$$

$$h(4) = h(-4) = \frac{\sin\frac{4\pi}{2}}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.06366.$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)(z^n + z^{-n})\right]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)(z^n + z^{-n})]$$

$$= 0.5 + 0.3183\ (z^1 + z^{-1}) - 0.106\ (z^3 + z^{-3}) + 0.06366\ (z^5 + z^{-5})$$

The transfer function of the realizable filter is

$$H'(z) = z^{-(N-1)/2}H(z)$$

$$= z^{-5}\left[0.5 + 0.3183(z + z^{-1}) - 0.106(z^3 + z^{-3}) + 0.06366(z^5 + z^{-5})\right]$$

$$= 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z^{-5} + 0.3183\ z^{-6} - 0.106\ z^{-8} + 0.06366\ z^{-10} \qquad (4)$$

From the above Eq. (4) the filter coefficients of causal filter are given by

$h(0) = h(10) = 0.06366$; $h\ (1) = h\ (9) = 0$; $h\ (2) = h(8) = -0.106$

$h(3) = h\ (7) = 0$; $h\ (4) = h\ (6) = 0.3183$; $h\ (5) = 0.5$.

SECOND TERM EXAMINATION [APRIL–2016]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC–306]

Time : 1½ hrs. **M.M. : 3**

Note: *Q.No.1 is compulsory. Attempt any two more Questions from the rest.*

Q.1. (a) Explain the advantages of multirate sampling in discrete domain. (3)

Ans. Advantages of muitirate sampling in discrete domain are:

(*i*) Computational requirements are less.

(*ii*) Storage for filter coefficients are less.

(*iii*) Finite arithemetic effects are less.

(*iv*) Filter order required in multirate application are low.

(*v*) Sensitivity to filter coefficient lengths are less.

Q.1. (b) Discuss the Lattice Structure for forward and backward linear predictor. (2)

Ans. The forward prediction error in the lattice filter is expressed as

$$f_m(n) = f_{m-1}(n) + K_m g_{m-1}(n-1)$$

The minimization of $E[|f_m(n)|^2]$ with respect to the reflection Coefficient K_m yields the result

$$K_m = \frac{-E[f_{m-1}(n) g^*_{m-1}(n-1)]}{\sqrt{E^f_{m-1} E^b_{m-1}}}$$

Q.1. (c) Write the necessary condition for Toeplitz Matrix and Hermitian Matrix. (2)

Ans. Condition of Toeplitz matrix is

$$\Gamma_P(i,j) = \Gamma_P(i-j)$$

Conditon of Hermitian matrix is

$$\Gamma_p(i,j) = \Gamma_p^*(i,j)$$

Q.1. (d) Plot the signals with Decimation factor D = 2 for Signal 1 and Interpolation I = 2 for Signal 2. (3)

$$\textbf{Signal 1 } x(n) = [3\underset{\downarrow}{2}456421563]$$

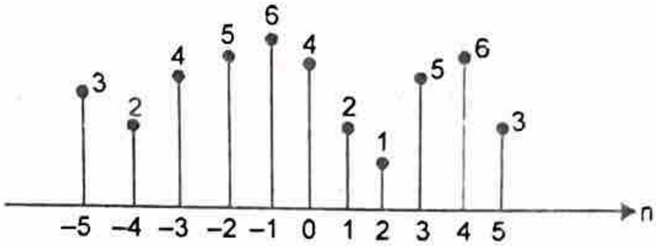
$$\textbf{Signal 2 } x(n) = [45\underset{\uparrow}{6}4215]$$

Ans. Given that

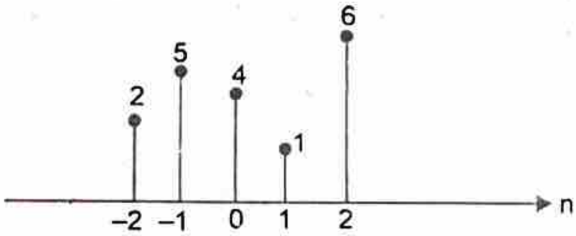
$$x(n) = [3245642\overset{\downarrow}{1}563]$$

$$x(n) = [45\,64\underset{\uparrow}{2}15]$$

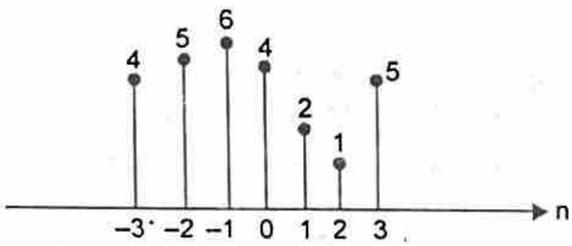
Decimation factor D = 2 for signal ⊥.



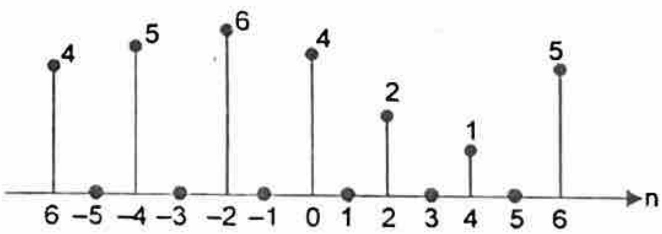
After decimation by a factor 2, we get



Interpolation factor I = 2 for signal 2



After interpolation by 2, we get



Q.2. (*a*) Explain the Lavinson Durbin Algorithm. **(5)**

Ans. Lavinson Durbin Algorithm: It is a computationally efficient algorithm for solving the prediction coefficients. This algorithm exploits the special symmetry in the autocorrelation matrix.

$$p = \begin{bmatrix} \gamma_{xx}(0) & \gamma_{xx}^*(1) & \cdots & \gamma_{xx}^*(p-1) \\ \gamma_{xx}(1) & \gamma_{xx}(0) & \cdots & \gamma_{xx}^*(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{xx}(p-1) & \gamma_{xx}(p-2) & \cdots & \gamma_{xx}(0) \end{bmatrix}$$

The key to the Lavinson-Durbin method of solution, that exploits the Toeplitz property of matrix, is to proceed recursively, beginning with a predictor of order m = 1 and then to increase the order recursively, using the lower-order solutions to obtain the solution to the next-higher order.

Thus the solution to the first-order predictor is

$$a_1(1) = \frac{-\gamma_{xx}(1)}{\gamma_{xx}(0)} \qquad \ldots(1)$$

and the resulting MMSE is

$$E_1^f = \gamma_{xx}(0)+a_1(1)\gamma_{xx}(-1) = \gamma_{xx}(0)[1-|a_1(1)|^2] \quad ...(2)$$

$a_1(1) = K_1$, the first reflection coefficient in the lattice filter. The next step is to solve for the coefficients $\{a_2(1), a_2(2)\}$ of the second-order predictor and express the solution in terms of $a_1(1)$. The two equations are

$$\begin{aligned} a_2(1)\gamma_{xx}(0)+a_2(2)\gamma^*_{xx}(1) &= -\gamma_{xx}(1) \\ a_2(1)\gamma_{xx}(1)+a_2(2)\gamma_{xx}(0) &= -\gamma_{xx}(2) \end{aligned} \quad ...(3)$$

By using equation (1), eliminate $\gamma_{xx}(1)$, we obtain the solution

$$a_2(2) = \frac{\gamma_{xx}(2)+a_1(1)\gamma_{xx}(1)}{\gamma_{xx}(0)[1-|a_1(1)|^2]} = -\frac{\gamma_{xx}(2)+a_1(1)\gamma_{xx}(1)}{E_1^f}.$$

$$a_2(1) = a_1(1) + a_2(2)\, a_1^*(1)$$

Thus we have obtained the coefficients of the second-order predictor. Again, we note that $a_2(2) = k_2$, the second reflection coefficient in the lattice filter.

Proceeding in this manner, we can express the coefficients of the m^{th} order predictor in terms of the coefficients of the (m-1)st order predictor.

Q.2. (b) Determine if the system having the system transfer function

$$H(z) = 1\,(1-7/4z^{-1}-1/2\,Z^{-2})$$

is stable or not using Schur Cohn Stability test. **(5)**

Ans. Given that,

$$H(z) = \frac{1}{1+\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}$$

Let

$$A_N(z) = 1+\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}$$

$$a_2(0) = 1, a_2(1)=\frac{7}{4}, a_2(2)=\frac{1}{2}$$

$\therefore$

$$k_2 = a_2(2) = -\frac{1}{2}$$

$$a_{m-1}(k) = \frac{a_m(k)-a_m(m)a_m(m-k)}{1-a_m^2(m)}$$

We have to find out $a_1(1)$
put $k = 1$ and $m = 2$

$$a_1(1) = \frac{a_2(1)-a_2(2)a_2 1}{1-a_2^2(2)} = \frac{\frac{7}{4}-\left(-\frac{1}{2}\right)\left(\frac{7}{4}\right)}{1-\left(-\frac{1}{2}\right)^2}$$

$$a_1(1) = \frac{\frac{7}{4}+\frac{7}{8}}{1-\frac{1}{4}} = \frac{21}{8}\times\frac{4}{3} = \frac{7}{2}$$

Hence

For system to be stable

$$|k| < 1 \text{ i.e., } -1 < k < 1$$

$$k_1 > 1$$

But system is not stable.

Q.3. (a) Explain Lattice Filter Designing. (5)

Ans. Let us consider an all-pole system with system function

$$H(z) = \frac{1}{1+\sum_{k=1}^{N} aN^{z^{-k}}} = \frac{1}{A_N(z)} \quad ...(1)$$

The difference equation for this IIR system is

$$y(n) = -\sum_{k=1}^{N} a_N(k) y(n-k) + x(n) \quad ...(2)$$

or

$$x(n) = y(n) + \sum_{k=1}^{N} a_N(k) y(n-k) \quad ...(3)$$

For N = 1

$$x(n) = y(n) + a_1(1) y(n-1) \quad ...(4)$$

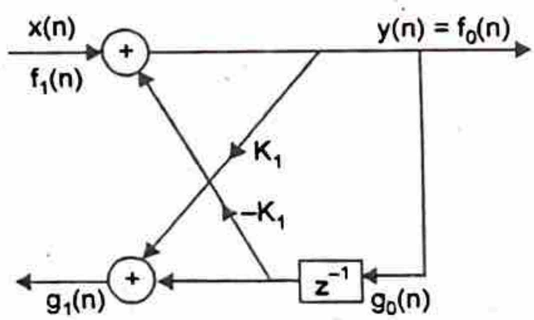


Fig. 1. single stage all-pole lattice filter.

The Equation (4) can be realised in lattice structure as shown in Fig. 1. from which we can obtain

$$x(n) = f_1(n) \quad ...(5)$$

$$y(n) = f_0(n) = f_1(n) - k_1 g_0(n-1)$$

$$= x(n) - k_1 y(n-1) \quad (6a)$$

$$\Rightarrow \quad x(n) = y(n) + k_1 y(n-1) \quad (6b)$$

$$g_1(n) = k_1 f_0(n) + g_0(n-1)$$

$$= k_1 y(n) + y(n-1) \quad (7)$$

Comparing Eq. (4) and Eq. (6*b*) we have

$$k_1 = a_1(1) \quad (8)$$

Now, let us consider for the case $N = 2$, then

$$x(n) = f_2(n) \quad (9a)$$

$$y(n) = x(n) - a_2(1) y(n-1) - a_2(2) y(n-2) \quad (9b)$$

This output can also be obtained from a two-stage lattice filter as shown in Fig.2. below from which we have

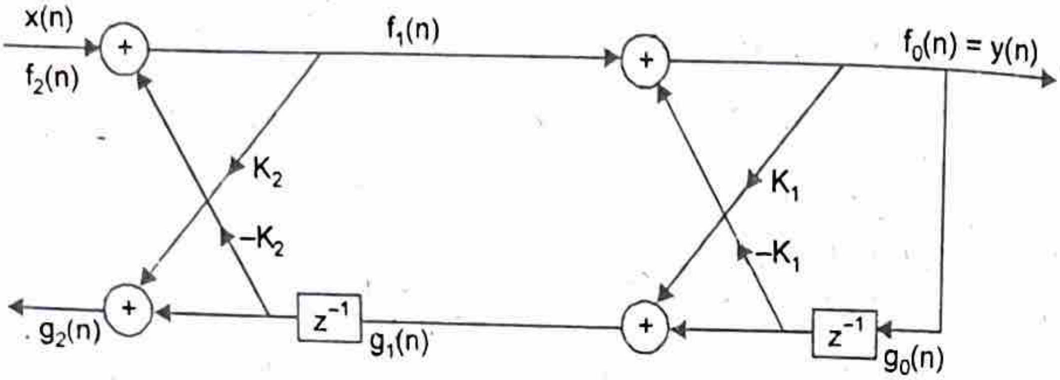


Fig. 2. Two stage all lattice filter.

$$f_2(n) = x(n) \quad ...(10)$$
$$f_1(n) = f_2(n) - k_2 g_1(n-1) \quad ...(11)$$
$$g_2(n) = k_2 f_1(n) + g_1(n-1) \quad ...(12)$$
$$f_0(n) = f_1(n) - k_1 g_0(n-1) \quad ...(13)$$
$$g_1(n) = k_1 f_0(n) + g_0(n-1) \quad ...(14)$$
$$\begin{aligned} y(n) &= f_0(n) = g_0(n) \quad ...(15) \\ &= f_1(n) - k_1 g_0(n-1) \\ &= f_2(n) - k_2 g_1(n-1) - k_1 g_0(n-1) \\ &= f_2(n) - k_2[k_1 f_0(n-1) + g_0(n-2) - k_1 g_0(n-1)] \end{aligned}$$

Using Eq. (15)

$$\begin{aligned} y(n) &= f_2(n) - k_2[k_1 y(n-1)] + y(n-2)] - k_1 y(n-1) \\ &= x(n) - k_1(1+k_2)y(n-1) - k_2 y(n-2) \quad ...(16) \end{aligned}$$

Similarly

$$g_2(n) = k_2 y(n) + k_1(1+k_2)y(n-1) + y(n-2) \quad ...(17)$$

Comparing Eq. (9b) and Eq. (16) we get

$$a_2(0) = 1, a_2(1) = k_1(1+k_2); a_2(2) = k_2 \quad ...(18)$$

Q.3.(*b*) What is Hilbert Transform? (5)

Ans. Refer Q.1. (*b*) of Few Important Questions

Q.4. (*a*) Explain Sampling rate conversion by a rational factor I/D. **(5)**

Ans. We can achieve this sampling rate conversion by first performing interpolation by the factor I and the decimating the output of the interpolator by the factor D. In other words, a sampling rate conversion by the rational factor I/D is accomplished by cascading an interpolator with a decimator, as shown below.

We emphasize that the importance of performing the interpolation first and the decimation second, is to preserve the desired spectral characteristics of $x(n)$. Furthermore, with the cascade configuration illustrated in Fig.(1) the two filters with impulse response $\{h_u(l)\}$ and $\{h_d(l)\}$ are operated at the same rate, namely IFx and hence can be combined into a single lowpass filter with impulse response $h(l)$ as illustrated in

Fig. (2) The frequency response $H(\omega_v)$ of the cobined filter must incorporate the filtering operations for both interpolation and decimation, and hence it should ideally possess the frequency response characteristic.

$$H(\omega_v) = \begin{cases} I, & 0 \le |\omega_v| \le \min(\pi/D, t\pi/I) \\ 0, & \text{otherwise} \end{cases} \quad ...(1)$$

where $\omega_v = 2\pi F/F_v = 2\pi F/I F_x = \omega_x/I.$

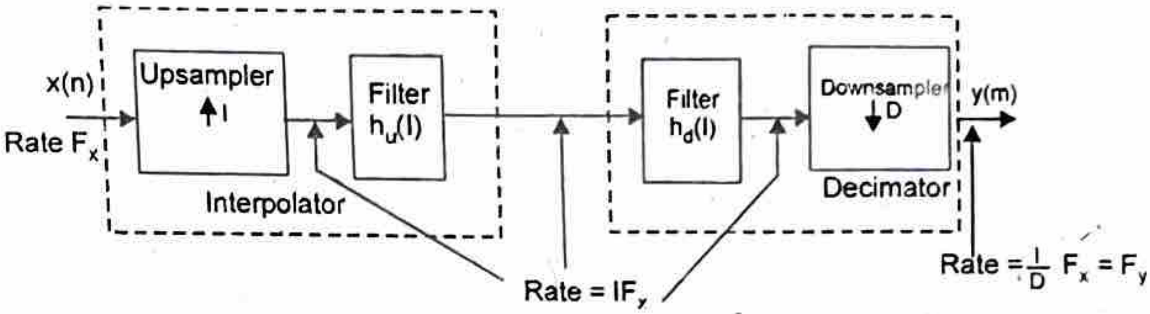


Fig. 1. Method for sampling rate conversion by a factor I/D

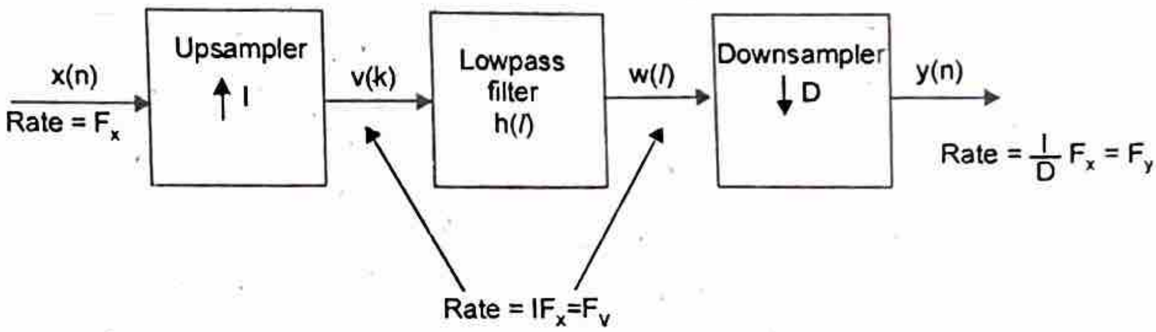


Fig. 2. Method for sampling rate conversion by a factor I/D.

In the time domain, the output of the upsampler is the sequence

$$v(l) = \begin{cases} x(l/I), & l = 0 \pm I, \pm 2I, .. \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

and the output of the linear time-invariant filter is

$$w(l) = \sum_{k=-\infty}^{\infty} h(l-k)v(k) \quad (3)$$

$$= \sum_{k=-\infty}^{\infty} h(l-kI)x(k)$$

Finally, the output of the sampling rate converter is sequence $[y(m)]$, which is obtained by downsampling the sequence $[w(l)]$ by a factor of D. Thus

$$y(m) = w(mD)$$

$$= \sum_{k=-\infty}^{\infty} h(mD-kI)x(k) \quad (4)$$

It is illuminating to express Eq. (4) in a different from by making a change in variable.

Let

$$k = \left[\frac{mD}{I}\right] - n \quad ...(5)$$

Where the notation $[r]$ denoted the largest integer contained in r. With this change in variable, Eq. (4) becomes

$$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \left[\frac{mD}{I}\right]I + nI.\right)x\left(\left\lfloor\frac{mD}{I}\right\rfloor - n\right) \quad ...(6)$$

We note that

$$mD - \left\lfloor\frac{mD}{I}\right\rfloor I = mD \quad \text{modulo } I = (mD)_I$$

Consequently, Eq. (6) can be expressed as

$$y(m) = \sum_{n=-\infty}^{\infty} h\left(nI + (mD)_I\right) x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right) \quad ...(7)$$

It is apparent from this form that the output $y(m)$ is obtained by passing the input sequence $x(n)$ through a time variant filter with impulse response.

$$g(n,m) = h(nI + (mD)_I) \quad -\infty < m,n < \infty \quad ...(8)$$

Where $h(k)$ is the impulse response of the time invariant lowpass filter operating at the sampling rate IF_x. We further observe, that for any integer k,

$$\begin{aligned} g(n, m+kI) &= h(nI + (mD + kDI)_I) \\ &= h(nI + (mD)_I) \\ &= g(n, m) \end{aligned} \quad ...(9)$$

Hence $g(n, m)$ is periodic in the variable m with period I.

The frequency-domain relationships can be obtained by combining the results of the interpolation and decimation process. Thus the spectrum at the output of the linear filter with impulse response $h(l)$ is

$$\begin{aligned} V(\omega_v) &= H(\omega_v) X(\omega_v I) \\ &= \begin{cases} IX(\omega_v I), & 0 \le |\omega_v| \le \min(\pi/D, \pi/I) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad ...(10)$$

The spectrum of the ourput sequence $y(m)$, obtained by decimating the sequence $v(n)$ by a factor of D, is

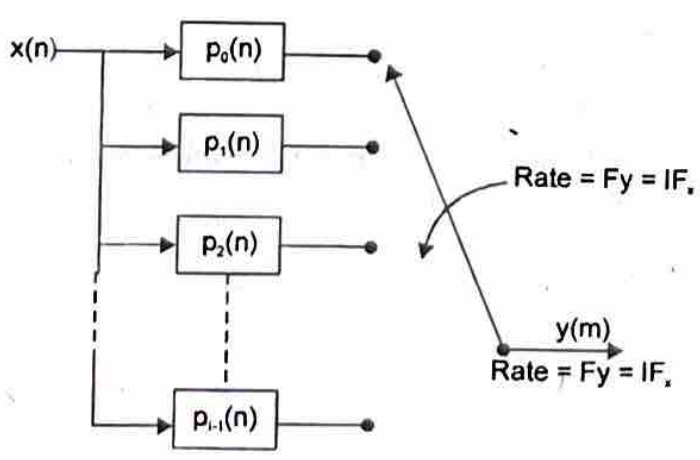
$$Y(\omega_y) = \frac{1}{D} \sum_{k=0}^{D-1} V\left(\frac{\omega_y - 2\pi k}{D}\right) \quad (11)$$

Where $\omega_y = D\omega_v$. Since the linear filter prevents aliasing as implied by Eq. (10), the spectrum of the output sequence given by (11) reduce to

$$Y(\omega_y) = \begin{cases} \frac{I}{D} X\left(\frac{\omega_y}{D}\right) & 0 \le |\omega_y| \le \min\left(\pi, \frac{\pi D}{I}\right) \\ 0, & \text{otherwise} \end{cases} \quad ...(12)$$

Q.4. (*b*) Write short note on any two: 1. Polyphase filter Structures, 2. Quantization of Filter Coefficients, 3. Round off effects in digital filters. (5)

Ans. 1-Polyphase filter structures: The computational efficiency of the filter structure can also be achieved by reducing the large FIR filter of length M into a set of smaller filters of length $K = M/I$, where M is selected to be a multiple of I.



The set of I polyphase filters can be arranged as a parallel realization, and the output of each filter can be selected by a commutator as shown in fig. The rotation of the commutator is in the counter clockwise direction beginning with the point at m= 0. Thus, the polyphase filters perform the computations at the low sampling rate F_x. and the rate conversion results from the fact that I output samples are generated, one from each of the filters, for each input sample.

2. Quantization of filter co-efficient

Digital signal processing algorithms are realized either with special purpose digital hardware or as programs for a general purpose digital computer. In both cases the numbers and coefficients are stored in finite-length registers. Therefore, coefficients and numbers are quantized by truncation or rounding off when they are stored.

The following errors arise due to quantization of numbers.

1. Input quantization error.
2. Product quantization error.
3. Coefficient quantization error.

1. The conversion of a continuous-time input signal into digital value produces an error, which is known as input quantization error. This error arises due to the representation of the input signal by a fixed number of digits in A/D conversion process.

2. Product quantization errors arise at the output of a multiplier. Multiplication of a b bit data with a b bit coefficient results a product having 2b bits. Since a b bit register is used, the multiplier output must be rounded or truncated to b bits which produces an error.

3. The filter coefficients are computed to infinite precision in theory. If they are quantized, the frequency response of the resulting filter may differ from the desired response and sometimes the filter may fail to meet the desired specifications. If the poles of the desired filter are close the unit circle, then those of the filter with quantized coefficients may lie just outside the unit cirde, leading to instability.

The other errors arising from quantization are roundoff noise and limit cycle' oscillations.

Fixed point representation: In fixed point arithmetic the position of the binary point is fixed. The bit to the right represent the fractional part of the number and those to the left represent the integer part. For example, the binary number 01.1100 has the value 1.75 in decimal.

The manner in which negative numbers are represented gives three different form- for fixed-point arithmetic.

(*i*) Sign-magnitude form.

(*ii*) One's-complement form.

(*iii*) Two's-complement form. Floating Point Numbers

In floating point representation a positive number is represented as $F = 2^C . M$, where M, called mantissa, is a fraction such that $1/2 \le M \le 1$ and c the exponent can be either positive or negative.

The decimal numbers 4.5, 1.5, 6.5 and 0.625 have floating point representations as $2^3 \times 0.5625$, $2^1 \times 0.75$, $2^3 \times 0.8125$, $2^{\circ} \times 0.625$ respectively.

Equivalently

$$2^3 \times 0.5625 = 2^{011} \times 0.1001$$
$$2^1 \times 0.75 = 2^{001} \times 0.1100$$
$$2^3 \times 0.8125 = 2^{011} \times 0.1101$$
$$2^{\circ} \times 0.625 = 2^{000} \times 0.1010$$

$$(3)_{10} = (011)_2$$
$$(0.5625)_{10} = (0.1001)_2$$
$$(1)_{10} = (001)2$$
$$(0.75)_{10} = (0.1100)_2$$
$$(0.8125)_{10} = (0.1101)_2$$
$$(0.625)_{10} = (0.1010)_2$$

Negative floating point numbers are generally represented by considering the mantissa as a fixed point number. The sign of the floating point number is obtained from the first bit of mantissa.

3. Round off effects in digital filters: The round off effects in digital filters are:

(*i*) Limit-cycle oscillations in Recursive systems: In recursive systems, the nonlinearities due to the finite precision arithmetic operations often cause periodic oscillations to occur in the output, even when the input sequence is zero or some nonzero constant value, such oscillations in recursive systems are called limit cycles and are directly attributable to round-off errors in multiplication and overflow erros in addition.

(*ii*) Scalling to Prevent overflow: In order to limit the amount of nonlinear distortion, it is important to scale the input signal and the unit sample response, between the input and any internal summing node in the system, such that overflow becomes a rare event.

END TERM EXAMINATION [MAY-JUNE–2016]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC–306]

Time : 3 hrs. **M.M. : 75**

Note: *Attempt any five Question including Q. no. 1 which is compulsory. Select one question from each unit. Assume missing data if any.*

Q.1. (a) Prove the following property of DFT when $X(k)$ is the N-point DFT of sequence $x(n)$.

(i) $X(k)$ is real and even when $x(n)$ is real and even

(ii) $X(k)$ is imaginary and odd when $x(n)$ is real and odd. **(5)**

Ans. We know that, the DFT of $x(n)$ is;

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$X(k) = X_R(k) + jX_I(k)$$

$$x(n) = x_R(n) + jx_I(n)$$

$$X_R(k) + jX_I(k) = \sum_{n=0}^{N-1}[x_R(n) + jx_I(n)]\left[\cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}\right]$$

$$X_R(k) = \sum_{n=0}^{N-1}\left[x_R(n)\cos\frac{2\pi kn}{N} + x_I(n)\sin\frac{2\pi kn}{N}\right]$$

$$X_I(k) = \sum_{n=0}^{N-1}\left[x_I(n)\cos\frac{2\pi kn}{N} - x_R(n)\sin\frac{2\pi kn}{N}\right]$$

(i) If $x(n)$ is real and even

$$x_I(n) = 0 \text{ and } x(n) = x(N\text{-}n) \text{ for } 0 \le n \le N-1$$

Substituting $x_I(n) = 0$ and $x(n) = x(N-n)$ we get

$$X_R(k) = \sum_{n=0}^{N-1} x_R(n)\cos\frac{2\pi kn}{N}$$

$$X_I(k) = 0$$

From which we can find $X(k)$ is real and even.

(ii) If $x(n)$ is real and odd

$$x_I(n) = 0 \text{ and } x(n) = -x(N-n) \text{ for } 0 \le n \le N-1$$

Substituting $x_I(0) = 0$ and $x(n) = -x(N-n)$ we get

$$X_R(k) = 0$$

$$X_I(k) = -\sum_{n=0}^{N-1} X_R(n)\sin\frac{2\pi kn}{N}$$

Hence $X(k)$ is imaginary and odd.

Q.1. (b) Derive the relation for Bilinear Transformation connecting s-domain and the z-domain. Show the mapping of points in s-domain to z-domain.

Ans. Bilinear Transform.

Let the system function of the analog filter be.

$$H(s) = \frac{b}{s+a} \quad ...(1)$$

The differential equation describing the analog filter can be obtained from eqn. (1) as,

$$H(s) = \frac{Y(S)}{X(S)} = \frac{b}{s+a}$$

$$SY(s) + aY(s) = b \times (s) \quad ...(2)$$

Taking Inverse Laplace Transform.

$$\frac{dy(t)}{dt} + ay(t) = bx(t) \quad ...(3)$$

Equation (3) is integrated between the limits (nT – T) and nT

$$\int_{nT-T}^{nT} \frac{dy(t)}{dt} dt + a \int_{nT-T}^{nT} y(t)dt = b \int_{nT-T}^{nT} x(t)dt \quad ...(4)$$

The trapezoidal rule for numeric integration is given by

$$\int_{nT-T}^{nT} a(t)dt = \frac{T}{2}[a(nT) + a(nT-T)] \quad ...(5)$$

Applying equation (5) in equation (4), we get

$$y(nT) - y(nT-T) + \frac{aT}{2} y(nT) + \frac{aT}{2} y(nT-T) = \frac{bT}{2} x(nT) + \frac{bT}{2} x(nT-T)$$

Taking z-Transform, the system function of the digital filter is,

$$H(z) = \frac{Y(Z)}{X(Z)} = \frac{b}{\frac{2}{T}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right) + a} \quad ...(6)$$

Comparing equations (1) and (6), we get

$$S = \frac{2}{T}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right) = \frac{2}{T}\frac{(Z-1)}{(Z+1)}$$

This is called Bilinear Transformation.

Q.1. (c) Derive the frequency response of a linear phase FIR filter with symmetric impulse response and filter length *N* odd.

Ans. Conditions on linear phase FIR system:

The system function

$$H(\omega) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} h(n)[\cos \omega n - j \sin \omega n]$$

$$= \sum_{n=0}^{N-1} h(n) \cos \omega n - j \sum_{n=0}^{N-1} h(n) \sin \omega n$$

Phase of H(ω) is < H(ω), so

$$\angle H(\omega) = \phi(\omega) = -\tan^{-1}\left[\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n}\right]$$

For phase to be linear

$$\phi(\omega) = -\omega\tau$$

$$-\omega\tau = -\tan-1\left[\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n}\right]$$

or,
$$\tan\omega\tau = \left[\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n}\right]$$

or
$$\frac{\sin\omega\tau}{\cos\omega\tau} = \frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n}$$

After cross multiply, we get

$$\sum_{n=0}^{N-1} h(n)\cos\omega n\sin\omega\tau = \sum_{n=0}^{N-1} h(n)\sin\omega n\cos\omega\tau$$

or, $\sum_{n=0}^{N-1} h(n)[\sin\omega\tau\cos\omega n - \cos\omega\tau\sin\omega n] = 0$

τ is constant and 'n' is running variable.

$$\therefore \qquad \tau \neq n$$

Also $h(n) \neq 0; 0 \leq n \leq N-1$

$$h(0)\sin\omega\tau + h(1)\sin(\omega\tau - \omega) + \ldots + h(N-2)\sin[\omega\tau - \omega(N-2)] + h(N-1)\sin[\omega\tau - \omega(N-1)] = 0$$

Consider; $h(0)\sin\omega\tau + h(N-1)\sin[\omega\tau - \omega(N-1)] = 0$

$h(0)\sin\omega\tau + h(N-1)\sin[\omega\tau - \omega(N-1)] = 0$

$\sin\omega\tau\, h(0) = h(N-1)\sin[-\omega\tau + \omega(N-1)]$

if
$$h(0) = h(N-1)$$
$$\sin\omega\tau = \sin(-\omega\tau + \omega(N-1))$$
$$\omega\tau = -\omega\tau + \omega(N-1)$$
$$2\omega\tau = \omega(N-1) \Rightarrow \boxed{\tau = \frac{N-1}{2}}$$

This is the required condition for a linear phase FIR system.

Also if
$$h(0) = h(N-1)$$

$$h(1) = h(N-1-1)$$
$$h(n) = h(N-1-n); 0 \le n \le N-1$$

This is the condition of symmetric.

and
$$h(m) = -h(N-1-n)$$

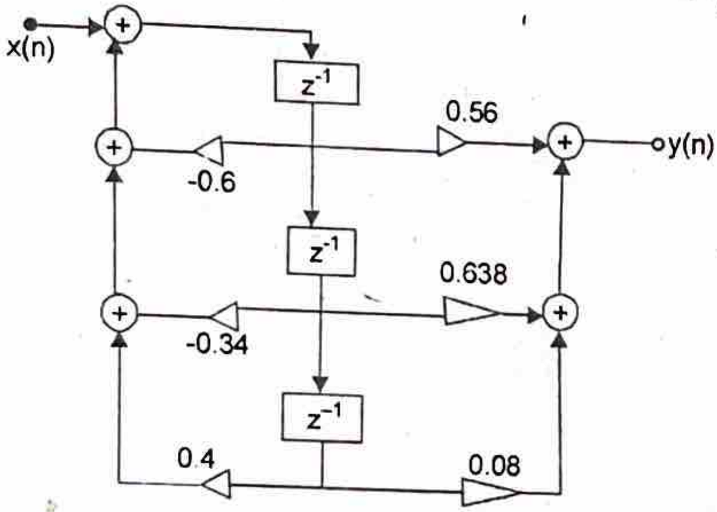
This is the condition of antisymmetric.

Q.1. (*d*) Draw the Direct form-II and its transposed structure for the given transfer function. **(5)**

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

Ans.

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2} = \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$



Q.1. (*e*) Show that the Up-sampler and the Down-Sampler are Linear but Time-varying system. **(5)**

Ans. Consider a factor of L up-sampler defined by

$$y(n) = x\left(\frac{n}{L}\right) \quad ...(1)$$

The output due to delayed input is

$$y(n, k) = x\left(\frac{n}{L} - k\right)$$

The delayed output is

$$y(n-k) = x\left(\frac{n-k}{L}\right)$$

$$\therefore \quad y(n, k) \neq y(n-k)$$

Therefore the up-smapler is a time variant system. Similarly for down-sampler

$$y(n) = x(nM) \quad ...(2)$$
$$y(n, k) = x(nM - k)$$
$$y(n-k) = x(M(n-k))$$
$$y(n, k) \neq y(n-k)$$

Therefore the down-sampler is a time variant system.

We have from equation (1)

$$y(n) = x\left(\frac{n}{L}\right) \quad ...(3)$$

Le $y_1(n)$ and $y_2(n)$ be the output of $x_1(n)$ and $x_2(n)$ respecitvely then, from equation (3)

$$y_1(n) = x_1\left(\frac{n}{L}\right)$$

and $$y_2(n) = x_2\left(\frac{n}{L}\right) \quad ...(2)$$

Then $$ax_1\left(\frac{n}{L}\right)+bx_2\left(\frac{n}{L}\right) = ay_1(n)+by_2(n)$$

This represents up-sampler is linear system.

Similary, from equation (2)

$$y(n) = x(nM)$$

and $$y_1(n) = x_1(nM)$$
$$y_2(n) = x_2(nM)$$

Then $$ax_1(nM)+bx_2(nM) = ay_1(n)+by_2(n)$$

This represents down-sampler is linear system.

UNIT I

Q.2. (a) For a *N*-point periodic sequence *x(n)* with DFT *X(k)*, prove that

$$\sum_{n=0}^{N-1}\left|x(n)\right|^2=\frac{1}{N}\sum_{k=0}^{N-1}X(k)^2 \quad \textbf{(6)}$$

Ans. We have

$$\sum_{n=0}^{N-1}|x(n)|^2=\sum_{n=0}^{N-1}x^*(n)x(n)$$

$$= \sum_{n=0}^{N-1}x^*(n).\frac{1}{N}\sum_{k=0}^{N-1}X(k)W_N^{-nK}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1}x(k).\sum_{n=0}^{N-1}x^*(n)W_N^{-nk}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1}X(k)\left(\sum_{n=0}^{N-1}x(n)W_N^{nk}\right)^* = \frac{1}{N}\sum_{k=0}^{N-1}X(k)X^*(k)$$

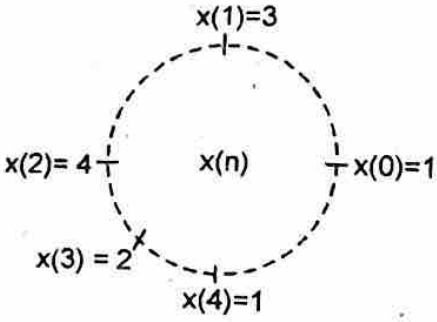
$$= \frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$$

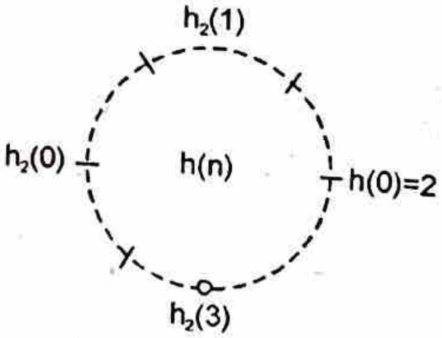
Q.2.(b) Define circular convolution. Evaluate circular convolution of the following sequences. (6.5)

$$\boldsymbol{x(n) = \{1, 3, 4, 2, 1\} \text{ and } h(n) = \{2, 0, 1, 0, 1\}.}$$

Ans. The circular convolution of two aperiodic functions occurs when one of them is convolved in the normal way with a periodic summation of the other function

Given that $$x(n)=\{1, 3, 4, 2, 1\}$$
$$h(n) = \{2, 0, 1, 0, 1\}$$





Circular convolution of two sequences can be written as

$$y(m) = \sum_{n=0}^{N-1} x(n)h(m-n,(\text{mod } N))\ , m = 0, 1,\ldots N-1$$

Here $N = 5$

$$y(m) = \sum_{n=0}^{4} x(n)h(m-n,(\text{mod } 5))\ , m = 0, 1, \ldots 4$$

put $m = 0$

$$y(0) = \sum_{n=0}^{4} x(n)h(-n,(\text{mod } 5)$$

$= x(0)h(0, \text{mod } 5) + x(1)\,h\,(-1, \text{mod } 5) + x\,(2)\,h\,\,(-2, \text{mod } 5) + x(3)\,h\,(-3, \text{mod } 5) + x(4)\,h(-4, \text{mod } 5)$

$= x(0)\,h(0) + x(1)\,h(4) + x(2)\,h(3) + x(3)\,h(2) + x(4)\,h(1)$

$= 1 \times 2 + 3 \times 1 + 4 \times 0 + 2 \times 1 + 1 \times 0 = 2 + 3 + 0 + 2 + 0 = 7$

put $m = 1$

$$y(1) = \sum_{n=0}^{4} x(n)h(-n,(\text{mod } 5)$$

$= x(0)h\,(1, \text{mod } 5) + x(1)\,h(0, \text{and } 5) + x(2)\,h(-1, \text{mod } 5) + x(3)\,h\,(-2, \text{mod } 5) + x(4)\,h(-3, \text{mod } 5)$

$= x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(4) + x(3)\,h(3) + x(4)\,h(2)$

$= 1 \times 0 + 3 \times 2 + 4 \times 1 + 2 \times 0 + 1 \times 1 = 11$

put $m = 2$

$$y(2) = \sum_{n=0}^{4} x(n)h(2-n,(\text{mod } 5)$$

$= x(0)h\,(2, \text{mod } 5) + x(1)\,h(1, \text{and } 5) + x(2)\,h\,(0, \text{and } 5) + x\,(3)\,h\,(-1, \text{mod } 5) + x(4)\,h\,(-2, \text{mod } 5)$

$= x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0) + x(3)\,h(4) + x(4)\,h(3)$

$= 1 \times 1 + 3 \times 0 + 4 \times 2 + 2 \times 1 \times 0 + 6 + 4 + 0 + 1 = 11$

put $m = 3$

$$y(3) = \sum_{n=0}^{4} x(n)h(3-n,(\text{mod } 5)$$

$= x(0)h(3, \text{mod } 5) + x(1)\,h\,(2, \text{mod } 5) + x(2)\,h(1, \text{mod } 5) + x(3)\,h\,(0, \text{mod } 5) + x(4)\,h\,(-1, \text{mod } 5)$

$= x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1) + x(3)\,h(0) + x\,(4)\,h\,(4)$

$$= 1\times 0 + 3\times 1 + 4\times 0 + 2\times 2 + 1\times 1 = 0+3+0+4+1 = 8$$

put $m = 4$

$$y(4) = \sum_{n=0}^{4} x(n)h(4-n_1(\text{mod } 5)$$

$= x(0)h(4, \text{mod } 5) + x(1)\, h(3, \text{mod } 5) + x(2)\, h\,(2, \text{mod } 5) + x(3)\, h\,(1, \text{mod } 5) + x\,(4)\, h(0, \text{mod } 5)$

$= x(0)\, h(4) + x(1)\, h(3) + x(2)\, h(2) + x(3)\, h(1) + x(4)\, h(0)$

$= 1\times 1 + 3\times 0 + 4\times 1 + 2\times 0 + 1\times 2 = 1+0+4+0+2 = 7$

$y(m) = \{7, 11, 11, 8, 7\}$ **Ans.**

Q.3. (*a*) Find inverse DFT of $X_1(k)X_2(k)$. **(3)**

Ans. We know that $X_1(K) = \sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}}$; $0 \le k \le N-1$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n) e^{\frac{-j2\pi kn}{N}} \quad ; 0 \le k \le N-1$$

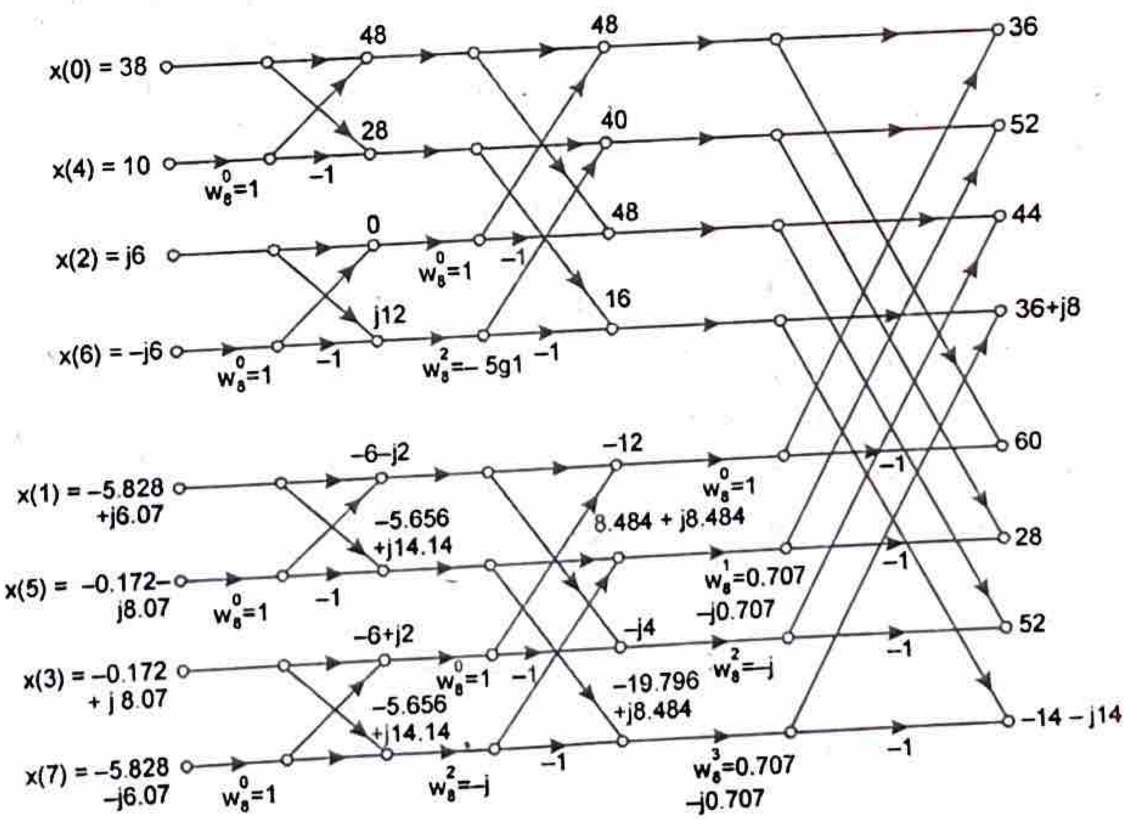
We have,

Multiplication in frequency domain is equal to covolution in time domain. Hence IDFT of $X_1(k)\, X_2(k) = x_1(n) * x_2(n)$

Q.3. (*b*) Calculate the IDFT using Decimation-in-Time FFT structure for the given coefficient. *X(k)* = {38, –5.828 + j6.07, j6, –0.172 + j8.07, 10, –0.172 –*j*8.07, –*g*6, –5.828 –*j*6.07}. **(9.5)**

Ans. Given that

$X(k)$ = (38, –5.828 + j6.07, j6, – 0.172 + j 8.07, 10, – 0.172 – j8.07, –j6, –5.828 – j6.07)



Given that N = 8

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_8^0 = 1$$

$$W_8^1 = 0.707 - j0.707$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j0.707$$

UNIT-II

Q.4.(*a*) Convert the analog filter the system function $H_a(s) = \dfrac{s+0.2}{(s+0.2)^2+9}$ into a digital IIR filter using Impulse Invariant technique. Assume $T = 1$ sec. (6)

Ans. The system response of the analog filter is of the standard form

$$H(s) = \frac{s+a}{(s+a)^2+b^2}$$

where $a = 0.2$ and $b = 3$. The system response of the digital filter can be obtained by

$$H(z) = \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$= \frac{1-e^{-0.2T}(\cos 3T)z^{-1}}{1-2e^{-0.2T}(\cos 3T)z^{-1}+e^{-0.4T}z^{-2}}$$

Taking $T = 1\text{s},$

$$H(z) = \frac{1-(0.8187)(-0.99)z^{-1}}{1-2(0.8187)(-0.99)z^{-1}+0.6703z^{-2}}$$

$$= \frac{1+(0.8105)z^{-1}}{1+1.6210z^{-1}+0.6703z^{-2}}$$

Q.4.(*b*) Design a FIR filter with the following desired frequency response,

$$H_d(e^{j\omega}) = \begin{cases} 1 & \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & |\omega| \le \frac{\pi}{4} \end{cases}$$

Using Hamming window for $N = 7$. (6.5)

Ans. Given that

$$H_d(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \pi \\ 0, & |\omega| \le \frac{\pi}{4} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} 1e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} 1 ej^{\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{-\frac{\pi}{4}} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{\pi}$$

$$= -\frac{1}{\pi n}\left[\frac{e^{\frac{jn\pi}{4}} - e^{-jn\pi}}{2j}\right] + \frac{1}{\pi n}\left[\frac{e^{jn\pi} - e^{j\frac{\pi}{4}n}}{2j}\right]$$

$$= -\frac{1}{\pi n}\left[\frac{e^{j\pi/4n} - e^{-j\pi/4n}}{2j}\right] + \frac{1}{\pi n}\left[\frac{e^{j\pi n}e^{-j\pi n}}{2j}\right]\Bigg\}$$

$$= \frac{1}{\pi n}\left\{\left[-\sin\frac{\pi}{4}n\right] + \sin(\pi.n)\right]\right\}$$

$$= \frac{1}{\pi n}\left[\sin\pi n - \sin\frac{\pi}{4}n\right], n \neq 0$$

To find $h_d(0)$ use L. Hospital rule

$$h_d(0) = \underset{n\to 0}{lt}\frac{\pi\cos\pi n - \frac{\pi}{4}\cos\frac{\pi}{4}n}{\pi} = \frac{\pi - \frac{\pi}{4}}{\pi} = \frac{3}{4}$$

$$h_d(1) = \frac{1}{\pi}\left[\sin\pi - \sin\frac{\pi}{4}\right] = \frac{1}{\pi}\left[0 - \frac{1}{\sqrt{2}}\right] = -\frac{1}{\pi\sqrt{2}}$$

$$h_d(2) = \frac{1}{2\pi}\left[\sin 2\pi - \sin\frac{\pi}{4}\right] = \frac{1}{2\pi}[0-1] = \frac{-1}{2\pi}$$

$$h_d(3) = \frac{1}{3\pi}\left[\sin 3\pi - \sin\frac{3\pi}{4}\right] = \frac{1}{3\pi}\left[0 - \frac{1}{\sqrt{2}}\right] = -0.075$$

$$h_d(4) = \frac{1}{4\pi}[\sin 4\pi - \sin\pi] = 0$$

$$h_d(5) = \frac{1}{5\pi}\left[\sin 5\pi - \sin\frac{5\pi}{4}\right] = \frac{1}{5\pi}\left[0 - \left(\frac{1}{\sqrt{2}}\right)\right] = 0.045$$

$$h_d(6) = \frac{1}{6\pi}\left[\sin 6\pi - \sin\frac{6\pi}{4}\right] = \frac{1}{6\pi}[0-(-1)] = 0.053$$

The Hamming window function is,

$$w(n) = \begin{cases} 0.54 - 0.46\cos\frac{2\pi n}{M-1}, 0 \le n \le M-1 \\ 0, \quad \text{otherwise} \end{cases}$$

Given that $M = 7$

$$w(0) = 0.54 - 0.46 = 0.08\ ;\ \omega(4) = 0.77$$

$$w(1) = 0.54\text{-}0.46 \cos \frac{2\pi}{6} = 0.31;\ \omega(5) = 0.31$$

$$w(2) = 0.77;\ \omega(6) = 0.08$$

$$w(3) = 1;$$

The filter coefficient of the resultant filter are then,

$$h(n) = h_d(n).\ w(n),\ n = 0, 1, 2, 3, 4, 5, 6$$

Therefore,

$$h(0) = h_d(0).\ \omega\ (0) = 0.75 \times 0.08 = 0.06$$

$$h(1) = h_d(1).\ \omega\ (1) = -0.22 \times 0.31 = -0.069$$

$$h(2) = h_d(2).\ \omega\ (2) = -0.159\ 5\ 0.77 = -0.122$$

$$h(3) = h_2(3).\ \omega\ (3) = -0.075 \times 1 = -0.075$$

$$h(4) = h_d(4).\ \omega\ (4) = 0 \times 0.77 = 0$$

$$h(5) = h_d(5).\ \omega\ (5) = 0.045 \times 0.31 = 0.013$$

$$h(b) = h_d(b)\ .\ \omega\ (6) = 0.053 \times 0.08 = 0.004$$

The frequency response is given by

$$H(e^{jw}) = \sum_{n=0}^{6} .h(n) e^{-j\omega n}$$

$$= h(0) + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j\omega n} + h(4)\,e^{-j4\,\omega} + h(5)\,e^{-j5\omega} + h(6)\,e^{-i6\,\omega}$$

$$H(e^{jw}) = 0.06 + (-0.069)e^{-j\,\omega} + (-0.122)\,e^{-j2\,\omega} + (-0.075)\,e^{-j3\,\omega} + 0 + 0.013\,e^{-j5\,\omega} + 0.004\,e^{-j6\,\omega}$$

$$H(e^{j\omega}) = 0.06 - (0.069)e^{-j\omega} - (0.122)e^{-j2\omega} - (0.075)\,e^{-j3\omega} + 0.013\,e^{-5\omega} + 0.004\,e^{-j6\,\omega}$$

Q.5.(*a*) Using Bilinear Transformation, design a Butterworth HPF to meet the following specifications:

$$\mathbf{0.8 \le |H(e^{j\omega})| \le 1\ ;\ 0.6\pi \le |\omega| \le \pi}$$

$$\mathbf{|H(e^{j\omega})| \le 0.2\ ;\ 0 \le |\omega| \le 0.2\pi}$$ **(6.5)**

Ans. Given that

$$0.8 \le |H(e^{j\omega})| \le 1 \quad , \quad 0.6\pi \le |\omega| \le \pi$$

$$|H(e^{jw})| \le 0.2 \quad , \quad 0 \le |\omega| \le 0.2\pi$$

Given that $\omega_1 = 0.4\ \pi$ and $\omega_2 = 0.8\ \pi\ ;\ \delta_1 = 0.8,\ \delta_2 = 0.2$

Assume $T = 1$ sec.

Step I: Analog filters edge frequencies

$$\Omega_1 = \frac{2}{T} \tan \frac{\omega_1}{2} = 2 \tan \frac{0.4\pi}{2} = 1.453$$

$$\Omega_2 = \frac{2}{T} \tan \frac{\omega_2}{2} = 6.155$$

Step II. Order of the filter

$$N \geq \frac{1}{2} \frac{\log\left\{\left[\left(\frac{1}{\delta_2^2}-1\right) \Big/ \left(\frac{1}{\delta_1^2}-1\right)\right]\right\}}{\log\left(\frac{\Omega_1}{\Omega_2}\right)}$$

$$N \geq 1.29 \text{ i.e. } \boxed{N=2}$$

Step III: Determination of – 3dB cut-off frequency

$$\Omega C = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{1/2N}} = \frac{1.453}{[0.5625]^{1/4}} = 1.677$$

1.677

Step IV: Determination of $H_a(s)$

$$H(s) = \prod_{k=1}^{N/2} \frac{B_k \Omega_c^2}{S^2 + b_k \Omega_C S + C_K \Omega_c^2} \quad ...(1)$$

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right] \text{ and } C_K = 1 \quad ...(2)$$

$$A = \prod_{K=1}^{N/2} B_K \quad ...(3)$$

From (1)

$$H(s) = \prod_{k=1}^{2/2} \frac{B_k \Omega_c^2}{S^2 + b_k \Omega_C S + C_K \Omega_c^2}$$

$$= \frac{B_1 \Omega_c^2}{S^2 + b_1 \Omega_C S + C_1 \Omega_c^2} \quad ...(4)$$

From (2) $C_1 = 1$

$$b_1 = 2\sin\frac{\pi}{4} = 2 \times 0.707 = 1.414$$

From (3)

$$A = \prod_{k=1}^{2/2} B_K = B_1 \quad \therefore B_1 = 1$$

From eqn (4)

$$H(s) = \frac{\Omega_c^2}{S^2 + 1.414\Omega_C S + \Omega_c^2}$$

$$\boxed{H(s) = \frac{2.812}{S^2 + 2.364S + 2.812}}$$

Step V: Determination of H(z)

$$H(z) = H(s)\Big|_{S=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{2.812}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 2.364 \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 2.812}$$

$$= \frac{2.812(1+z^{-1})^2}{4(1-z^{-1})^2 + 4.729(1-z^{-2}) + 2.812(1+z^{-1})^2}$$

Step VI: Convert LPF into HPF

We have,

$$Z^{-1} \to \frac{-Z^{-1}+a}{1+aZ^{-1}}$$

Where,

$$a = -\frac{\cos\left[(\omega_c - \omega_c^*)/2\right]}{\cos[\omega_c + \omega_c^*)/2]}$$

Q.5.(b) Write the various window functions used for FIR filter design. Compare their important performance parameter.

Ans. Window Functions:

1. Rectangular Window function

The weighting function for the rectangular window is given by

$$w_R(n) = \begin{cases} 1, \text{for } |n| \le \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

II. Hamming window function:

The cansal hamming window function is expressed by

$$W_H(n) = \begin{cases} 0.25 - 0.46\cos\frac{2\pi n}{M-1}; 0 \le n \le M-1 \\ 0, \quad \text{otherwise} \end{cases}$$

The non-causal Hamming window function is given by

$$W_H(n) = \begin{cases} 0.25 - 0.46\cos\frac{2\pi n}{M-1}; for\ |n| \le \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

III. Hanning window function

The window function of a causal hanning window is given by

$$W_{\text{Hann}}(n) = \begin{cases} 0.25 - 0.5\cos\frac{2\pi n}{M-1}; for\ |n| \le M-1 \\ 0, \quad \text{otherwise} \end{cases}$$

The window function of a non-causal hanning window is expressed by

$$W_{\text{Hann}}(n) = \begin{cases} 0.5 - 0.5\cos\frac{2\pi n}{M-1}; 0 < |n| < \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

IV. Blackman window function: The window function of a causal blackman window is expressed by

$$W_B(n) = \begin{cases} 0.45 - 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}; \le n \le M-1 \\ 0, \quad otherwise \end{cases}$$

The window function of a non-causal Blackman window is given by

$$W_B(n) = \begin{cases} 0.42 - 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}; \text{ for } |n| < \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

V. Bartlett Window Function: The window function of a non-casual bartlelt window is expressed by.

$$W_{Bart}(n) = \begin{cases} 1+n, -\frac{M-1}{2} < n < 1 \\ 1-n, \ 1 < n < \frac{M-1}{2} \end{cases}$$

UNIT-III

Q.3.(b) Discuss and explain Hilbert Transform? (5)

Ans. Refer Q.1. (b) of Few Important Questions

Q.6.(b) Explain the Levinson Durbin Algorithm in detail. (6)

Ans. Refer question no. 2.(a) Second Term Examination 2016.

Q.7. (a) Draw and explain the Lattice realization of an all-pole filter function.

$$H(z) = \frac{1}{1 - 0.2z^{-1} + 0.4z^{-2} + 0.6z^{-3}} \quad (6)$$

Ans. Given that

$$H(z) = \frac{1}{1 - 0.2z^{-1} + 0.4z^{-2} + 0.6z^{-3}}$$

It is the transfer function of an all-pole IIR filter we have

$$A_N(z) = 1 - 0.2z^{-1} + 0.4z^{-2} + 0.6z^{-3}$$

$$a_3(0) = 1; a_3(1) = -0.2; a_3(2) = 0.4; a_3(3) = 0.6$$

$$k_3 = a_3(3) = 0.6$$

We know that

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)a_m(m-k)}{1 - a_m^2(m)}$$

For $m = 3$ and $k = 1$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1 - a_3^2(3)} = \frac{-0.2 - (0.6)(0.4)}{1 - (0.6)^2}$$

$$= \frac{-0.44}{0.64} = -0.6875$$

For $m = 3$ and $k = 2$

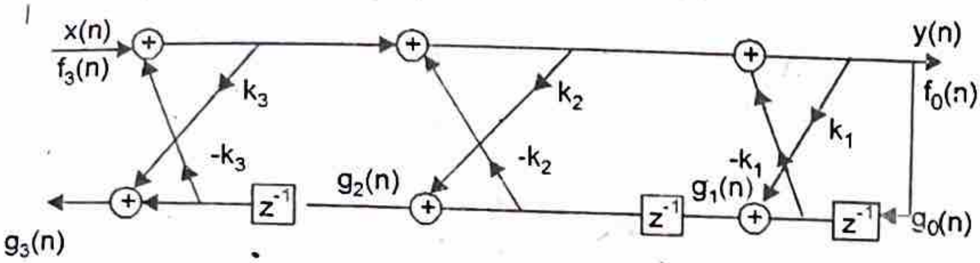
$$k_2 = a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1 - a_3^2(3)}$$

$$= \frac{0.4 - (0.6)(-0.2)}{1 - (0.6)^2} = \frac{0.52}{0.64} = 0.8125$$

For $m = 2$ and $k = 1$

$$k_1 = a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1 - a_2^2(2)} = \frac{-0.6875 - (0.8125)(-0.685)}{1 - (0.8125)^2}$$

$$k_1 = a_1(1)\frac{-0.1309}{0.3398} = -0.385$$



Q.7.(b) Without factoring any polynomial, determine whether or not the following filter function is stable.

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}} \quad \textbf{(6.5}$$

Ans. Given that

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}}$$

without factoring any polynomial, the stability can be checked by Schur Coehn stability test for an all pole IIR filter. *i.e.*

$$H(z) = \frac{1}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}}$$

We have

$$A_N(z) = 1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4\,z^{-4}$$
$$a_4(0) = 1;\ a_4(1) = 1.58;\ a_4(2) = 1.638;\ a_4(3) = 1.556;$$
$$a_4(4) = 0.4 = k_4$$

We have to find out

$$a_3(3) = k_3;\ a_2(2) = k_2\ ;\ a_1(1) = k_1$$

We know that

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)a_m(m-k)}{1 - a_m^2(m)}$$

For $m = 4$, $m = 3$

$$a_3(3) = \frac{a_4(3) - a_4(4)a_4(1)}{1 - a_4^2(4)} = \frac{1.556 - (0.4)(1.58)}{1 - (0.4)^2}$$

$$a_3(3) = \frac{0.924}{0.84} = 1.1 = k_3$$

For $m = 3$, $k = 2$

$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1 - a_3^2(3)} \qquad \text{...(1)}$$

To find the value of $a_3(2)$

For $m = 4, k = 2$

$$a_3(2) = \frac{a_4(2) - a_4(4)a_4(2)}{1 - a_4^2(4)} = \frac{1.638 - (0.4)(1.638)}{1-(0.4)^2}$$

$$= \frac{0.9828}{0.84} = 1.17$$

From (1)

$$a_2(2) = \frac{1.17 - (1.1)(1.638)}{1-(1.1)^2} = \frac{-0.6318}{-0.21} = 3$$

$$a_2(2) = 3 = k_2$$

For $m = 2, k = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1 - a_2^2(2)} \qquad ...(2)$$

To find the value of $a_2(1)$

For $m = 3, k = 1$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1 - a_3^2(3)} \qquad ...(3)$$

To find the value of $a_3(1)$

For $m = 4, k = 1$

$$a_3(1) = \frac{a_4(1) - a_4(4)a_4(3)}{1 - a_4^2(4)} = \frac{1.58 - (0.4)(1.556)}{1-(0.4)^2}$$

$$a_3(1) = \frac{0.9576}{0.84} = 1.14$$

From (3)

$$a_2(1) = \frac{1.14 - (1.1)(1.17)}{1-(1.1)^2} = \frac{-0.147}{-0.21} = 0.7$$

From eqn (2)

$$a_1(1) = \frac{0.7 - (3)(0.7)}{1-(3)^2} = \frac{0.7 - 2.1}{-8} = 0.175$$

$$a_1(1) = k_1 = 0.175$$

Therefore the coefficients are

$$K_4 = 0.4$$
$$K_3 = 1.1$$
$$K_2 = 3$$
$$K_1 = 0.175$$

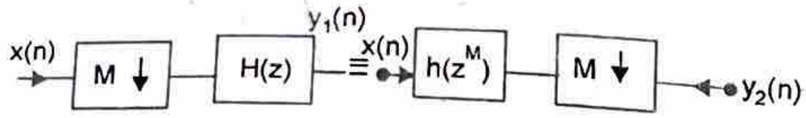
For stable system $|K_m| < 1$ for all $m = 1, 2, \ldots N$

But $K_2 = 3 > 1$.

Hence system is unstable

UNIT–IV

Q.8. (a) Prove the identity. (6)



Ans.

$$Y_1(z) = X(z)\,H(z^M)$$

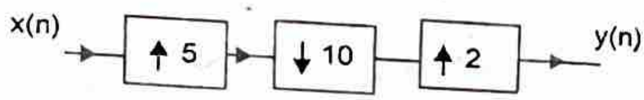
$$Y(z) = \frac{1}{M}\sum_{K=0}^{M-1} H(z)X\left(e^{-j2\pi k/M} Z^{\frac{1}{M}}\right)$$

$$= H(z)\frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{-j2\pi k/M} Z^{\frac{1}{M}}\right)$$

$$X_1(z) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{-j2\pi k/M} Z^{\frac{1}{M}}\right)$$

$$Y(z) = H(z)\,\frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{-j2\pi k/M} Z^{\frac{1}{M}}\right)$$

Q.8. (b) Develop an expression for the output $y(n)$ as a function of input for a given multi-rate system. (6.5)



Ans.

$$x(n) \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 10} \rightarrow \boxed{\uparrow 2} \rightarrow y(n)$$

$$\equiv x(n) \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 2} \rightarrow y(n)$$

$$\equiv x(n) \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 2} \rightarrow y(n)$$

$$x(n) \rightarrow \boxed{\downarrow 2} \xrightarrow{x_1(n)} \boxed{\uparrow 2} \rightarrow y(x)$$

$$x_1(n) = x(2n)$$

and $y(n) = x_1\left(\frac{n}{2}\right)$ for $n = 2k$

$= 0$; otherwise

$\Rightarrow$ $y(n) = x(n)$ for $n = k$

$= 0$; otherwise.

Q.9. (a) A cascaded realization of the two-first order system is given by $H(z) = H_1(z) \cdot H_2(z)$. Find out the overall output noise power of the system. Where

$$H_1(Z) = \frac{1}{1-0.9Z^{-1}} \text{ and } H_2(Z) = \frac{1}{1-0.8Z^{-1}} \quad (6)$$

Ans. Given

$$H_1(z) = \frac{1}{1-0.9z^{-1}} \text{ and } H_2(z) = \frac{1}{1-0.8z^{-1}}$$

We have $\sigma_{err}^2 = \sigma_{01}^2 + \sigma_{02}^2$ where $\sigma_{01}^2 = \sigma_e^2 = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n)$ and $\sigma_{02}^2 = \sigma_e^2 \sum_{n=0}^{\infty} h_2^2(n)$

Let us first determine $h(n)$ and $h_2(n)$.

$$H(z) = \frac{1}{(1-0.9z^{-1})(1-0.8z^{-1})}$$

$$= \frac{z^2}{(z-0.9)(z-0.8)}$$

Therefore

$$\frac{H(z)}{z} = \frac{z}{(z-0.9)(z-0.8)} = \frac{9}{(z-0.9)} - \frac{8}{(z-0.8)}$$

and $$H(z) = \frac{9}{(1-0.9z^{-1})} - \frac{8}{(1-0.8z^{-1})}$$

Taking inverse z–transform,

$$h(n) = [9\,(0.9)^n - 8(0.8)^n]\,u(n)$$

and

$$h^2(n) = [9\,(0.9)^n - 8(0.8)\,n]^2 u(n)$$

Similarly,

$$h_2(n) = Z^{-1}[H^2(z)] = Z^{-1}\left[\frac{1}{1-0.8z^{-1}}\right] = (0.8)^n u(n)$$

and $$h_2^2(n) = (0.8)^{2n}\,u(n)$$

Therefore,

$$\sigma_{01}^2 = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n) = \sigma_e^2 \sum_{n=0}^{\infty} [9(0.9)^n - 0(0.8)^n]^2$$

$$= \sigma_e^2 \sum_{n=0}^{\infty} [81(0.81)^n + 64(0.64)^n - 144(0.72)^n)$$

Q.9. (*b*) What do you understand by Zero input limit cycle oscillations. Explain with the help of an example. (6.5)

Ans. ***LIMIT CYCLE OSCILLATIONS***

Let the input sequence traverses several quantisation levels between two successive samples and so the samples of the round-off noise sequence were uncorrelated with each other and with the input signal. These assumptions are invalid in cases such as a constant or zero input to a digital filter. In such cases, the input signal remains constant during successive samples and does not traverse several quantisation levels. There are two types of limit cycles, namely, zero input limit cycle and overflow limit cycle. Zero input limit cycles are usually of lower amplitudes in comparison with overflow limit cycles. Let us consider a system with the difference equation

$$y(n) = 0.8y(n-1) + x(n) \qquad ...(1)$$

with zero input, i.e., $x(n) = 0$ and initial condition $y(-1) = 10$. A comparison between the exact values of $y(n)$ as given by Eq. using unquantised arithmetic and the rounded values of $y(n)$ as obtained from quantised arithmetic are given in Table below.

Table 1. A Comparison of exact y (n) and rounded y (n)

n	$y(n)$-unquantised	$y(n)$-quantised
– 1	10.0	10
0	8.0	8
1	6.4	6
2	5.12	5
3	4.096	4
4	3.2768	3
5	2.62144	2
6	2.0972	2
7	1.6772	2

From above Table it can be observed that for zero input, the unquantised output $y(n)$ decays exponentially to zero with increasing n. However, the rounded-off (quantised) output $y(n)$ gets stuck at a value of two and never decays further. Thus, the output is finite even when no input is applied. This is referred to as zero input limit cycle effect. It can also be seen that for any value of the input condition $|y(-1)| \leq 2$, the output $y(n) = y(-1), n \geq 0$, when the input is zero. Thus, the deadband in this case is the interval [-2, 2].

IMPORTANT QUESTIONS

Q.1. Design a Butterworth filter to satisfy the following contraints, using

$$0.707 \le |H(e^{j\omega})| \le 1 \quad 0 \le \omega \le 0.2\ \omega$$

(6.5)

bilinear Transformation $|H(e^{j\omega})| \le 0.1 \quad 0.5\ \pi \le \omega \le \pi$

Ans. Given that

$$0.707 \le |H(e^{j\omega})| \le 1 \quad 0 \le \omega \le 0.2\ \pi$$

$$|H(e^{j\omega})| \le 0.1 \quad 0.5 \le \omega \le \pi$$

given that

$$\delta_1 = 0.707; \delta_2 = 0.1$$

$$\omega_1 = 0.2\pi; \omega_2 = 0.5. \text{ Assume } T = 1 \text{ sec.}$$

Step I : Determination of the analog filter's edge frequencies.

$$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2} = \frac{2}{T}\tan 0.1\pi = 0.64$$

$$\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2} = \frac{2}{T}\tan 0.25\pi = 1$$

Therefore

$$\frac{\Omega_2}{\Omega_1} = 1.5625$$

Step II: Determination of the order of the filter.

$$N \ge \frac{1}{2}\frac{\log\left\{\left(\left(\frac{1}{\delta_2^2}\right)-1\right)/\left(\left(\frac{1}{\delta_1^2}\right)-1\right)\right\}}{\log\left(\frac{\Omega_2}{\Omega_1}\right)}$$

$$= \frac{1}{2}\frac{\log\left\{\frac{99}{1}\right\}}{2\log(1.5625)} = \frac{1}{2}\times\frac{2}{0.1938} = 5.15$$

Let

$$N = 5$$

Step III: Determination of $-3dB$ cut-off frequency.

$$\Omega_c = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{\frac{1}{2N}}} = \frac{0.64}{0.10} = 6.4$$

Step IV : Determination of $H_a(S)$:

$$H(S) = \frac{B_0\Omega_c}{S + C_0\Omega_c}\prod_{k=1}^{(N-1)/2}\frac{B_K\Omega_c^2}{s^2 + b_k\Omega_c s + c_k\Omega_c^2} \quad \text{for } N = 3, 5, 7, ..$$

$$H(S) = \frac{B_0\Omega_c}{S + C_0\Omega_c}\prod_{k=1}^{2}\frac{B_K\Omega_c^2}{s^2 + b_k\Omega_c s + c_k\Omega_c^2}$$

$$= \frac{B_0\Omega_c}{S + C_0\Omega_c}\left(\frac{B_1\Omega_c^2}{s^2 + b_1\Omega_c s + C_1\Omega_c^2}\right)\left(\frac{B_2\Omega_c^2}{s^2 + b_2\Omega_c s + c_2\Omega_c^2}\right)$$

The coefficients b_k and c_k are given by.

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right] \text{ and } c_k = 1$$

$$b_1 = 2\sin\left[\frac{\pi}{10}\right] = 0.61$$

$$b_2 = 2\sin\left[\frac{3\pi}{10}\right] = 1.61$$

and $\quad c_1 = c_2 = 1$ Also $c_0 = 1$

B_K can be obtained from

$$A = \prod_{k=1}^{(N-1)/2} B_K, \text{ for } N \text{ odd}$$

$$A = \prod_{K=1}^{2} B_K = B_1 B_2$$

Let filter gain $A = 1$

So, $\quad B_1 = B_2 = 1$ Also, $B_0 = 1$

Therefore,

$$H(s) = \left(\frac{\Omega_c}{s+\Omega_c}\right)\left(\frac{\Omega_c^2}{s^2+0.61\Omega_c s+\Omega_c^2}\right)\left(\frac{\Omega_c^2}{s^2+1.61\Omega_c s+\Omega_c^2}\right)$$

$$= \left(\frac{6.4}{s+6.4}\right)\left(\frac{41}{s^2+4s+41}\right)\left(\frac{41}{s^2+10s+41}\right)$$

Step (V): Determination of $H(Z)$

$$H(Z) = H(S)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left(\frac{6.4}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+6.4}\right)\left(\frac{41}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+8\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+41}\right)\left[\frac{41}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+20\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+41}\right]$$

Q.2. What is dead band effect? Determine the dead band range of the filter characterised by y(n) = 0.95 y (n – 1) + x(n).

Ans. The limit cycles occur as result of the quantization effects in multiplications. The amplitudes of the output during a limit cycle are confined to a range of values that is called the dead band of the filter.

A comparison between exact values of $y(n)$ and rounded values of $y(n)$ is given below

we have $\quad y(n) = 0.95\, y(n-1) + x(n)$

Let $x(n) = 0$ and $y(-1) = 12$.

n	y(n) – exact	y(n) – rounded
–1	12	12
0	11.4	11
1	11.4	11
2	10.83	11
3	10.28	10
4	9.77	10
5	9.28	9
6	8.82	9
7	8.38	8
8	7.96	8
9	7.56	8

From the above table, it is seen that for any value of $|y(-1)| \le 8$, $y(n)= y(-1), n \le 0$ for zero input. Thus the deadband is the interval [–8,8].

Q.3. Write a short note on frequency sampling realization. **(4)**

Ans. Frequency sampling realization: We have the frequency sampling of designing of FIR filters is given by.

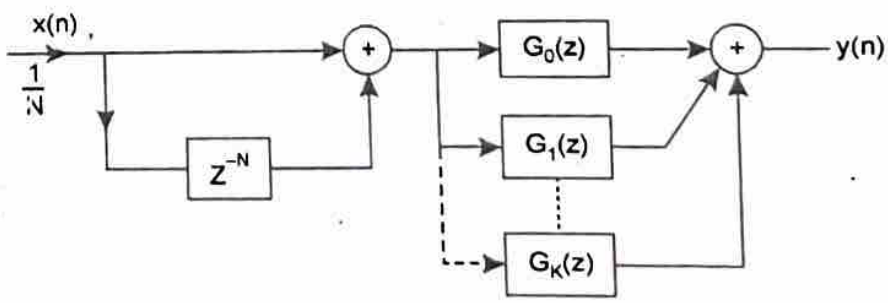
$$H(z) = \frac{1-Z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(K)}{1-e^{j2\pi K/N}Z^{-1}} \tag{1}$$

This can be written as,

$$H(z) = \frac{1-Z^{-N}}{N}\sum_{k=0}^{N-1}G_k(z)$$

where
$$G_K(z) = \frac{H(K)}{1-e^{\frac{j2\pi k}{N}}z^{-1}} \quad 0 \le k \le N-1$$

is the transfer function of the first order FIR filters, where poles lie on the unit circle at equdistant point. The equation (2) can be realized as shown in fig. below.



$G_K(Z)$ are sometimes called resonant filters, because they are resonant at the sample values of K^{th} frequency.

Q.4. Determine the variance of the round-off noise at the output of the two cascade realization of the filter with system function.

$$\mathbf{H(z) = H_1(z).\,H_2(z)}$$

Where
$$\mathbf{H_1(z) = \left(\frac{1}{1-0.5z^{-1}}\right) \text{ and } H_2(z) = \frac{1}{1-0.25z^{-1}}}$$

Ans. Given that

$$H(z) = H_1(z).H_2(z)$$

where $$H_1(z) = \frac{1}{1-0.5z^{-1}}$$

and $$H_2(z) = \frac{1}{1-0.25z^{-1}}$$

The noise transfer function seen by $e_1(n)$ is $H_1(z)$ and $e_2(n)$ is $H_2(z)$. The overall output noise power is given by

$$\sigma^2_{err} = \sigma^2_{01} + \sigma^2_{02}$$

We have, $$\sigma^2_{01} = \frac{\sigma_e^2}{2\pi j}\oint_c H(z)H(z^{-1})z^{-1}dz$$

$$= \frac{\sigma_e^2}{2\pi j}\oint_c \frac{1}{(1-0.5z^{-1})(1-0.25z^{-1})} \times \frac{1}{(1-0.5z)(1-0.25z)} \times z^{-1}dz$$

$$= \frac{\sigma_e^2}{2\pi j}\oint_c \frac{z}{(z-0.5)(z-0.25)(1-0.5z)(1-0.25z)}dz$$

$$\sigma^2_{01} = \sigma^2_c \times I_1$$

where $$I_1 = \frac{1}{2\pi j}\oint_c \frac{z}{(z-0.5)(z-0.25)(1-0.5z)(1-0.25z)}dz$$

The integral I_1 can be solved by the method of resid.
I_1 = sum of the resid at the poles within the unit circle.
I_1 = (residue at z = 0.5) + (residue at z = 0.25)

$$= \cancel{(z-0.5)}\frac{z}{\cancel{(z-0.5)}(z-0.25)(1-0.5z)(1-0.25z)}\Bigg|_{z=0.5}$$

$$+ \cancel{(z-0.25)}\frac{z}{(z-0.5)\cancel{(z-0.25)}(1-0.5z)(1-0.25z)}\Bigg|_{z=0.5} = 0$$

Therefore, $$\sigma^2_{01} = \sigma^2_e \times I_1 = 0$$

Similarly, $$\sigma^2_{02} = \frac{\sigma_e^2}{2\pi j}\oint_c H(z)H(z^{-1})z^{-1}dz = \frac{\sigma_e^2}{2\pi j}\oint_c \frac{1}{(1-0.25^{z-1})} \times \frac{1}{1-0.25z)} \times z^{-1}$$

$$= \frac{\sigma_e^2}{2\pi j}\oint_c \frac{1}{(z-0.25)(1-0.25z)}$$

$$\sigma^2_{02} = \sigma_e^2 \times I_2 \text{ where, } I_2 = \frac{1}{2\pi j}\oint_c \frac{dz}{(z-0.25)(1-0.25z)}$$

The integral I_2 can be solved by the method of residnes.

I_2 = sum of the residnes at poles within the unit circle
= residue at z = 0.25

$$= \cancel{(z-0.25)} \times \frac{1}{\cancel{(z-0.25)}(1-0.25z)}\Bigg|_{z=0.25} = 2.667$$

Therefore, $\sigma^2_{02} = \sigma^2_e \times I_2 = 2.667\sigma^2_e$

The overall output noise power is

$$\sigma^2_{err} = \sigma^2_{01} + \sigma^2_{02} = \sigma_e^2(0+2.667)$$
$$= 2.667\,\sigma^2_e$$

Substituting $\sigma_e^2 = \frac{z^{-2B}}{12}$, we get

$$\sigma^2_{err} = 2.667 \times \frac{2^{-2B}}{12}$$

$$\sigma^2_{err} = 0.22225 \times 2^{-2B}$$

Q.5. Explain Overlap add method and Overlap save method. **(5)**

Ans. Overlap-Save method: In this method the data sequence is divided into N-point sections $x_i(n)$. Each section contains the last M-1 data points of the previous section followed by L new data points to form a data sequence of length N = L + M – 1. If we take N-point circular convolution of $x_i(n)$ with $h(n)$, the first M-1 points will not agree with the linear convolution of $x_i(n)$ and $h(n)$ because of aliasing; the remaining $(N-M+1)$ points however will agree with the linear convolution. Hence we discard the first $(M-1)$ points of filter section $x_i(n)$ Ⓝ $h(n)$. This process is repeated for all sections and the filtered sections are abutted together.

Overlap-add method: In this method the size of the input data block $x_i(n)$ is L. To each data block, we append M-1 zeros and perform N-point $(N = L + M - 1)$ circular convolution of $x_i(n)$ with $h(n)$. Since each data block is terminated with M-1 zeros, the last M-1 points from each output block must be overlapped and added to first M-1 points of the succeeding block. Hence, this method is called overlap-add method.

Q.6. The desired response of a low pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-3j\omega}; & -3\pi/4 \le |\omega| \le 3\pi/4 \\ 0; & 3\pi/4 \le |\omega| \le \omega \end{cases}$$

Determine $H(e^{j\omega})$ for M = 7 using a hanning window.

Ans. The filter coefficients are given by

$$H_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3\omega n}e^{j\omega n}d\omega$$

$$H_d(n) = \frac{\sin 3\pi(n-3)/4}{\pi(n-3)}, n \ne 3$$

and $$H_d(3) = \frac{3}{4}$$

The filter coefficients are,

$$H_d(0) = 0.0750,\ H_d(1) = 0.1592$$
$$H_d(2) = 0.2251,\ H_d(3) = 0.75$$
$$H_d(4) = 0.2251,\ H_d(5) = -0.1592$$
$$H_d(6) = 0.0750$$

The Hamming window function is

$$\omega(n) = \begin{cases} 0.54 - 0.46\cos\dfrac{2\pi n}{M-1}, & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, with

$$M = 7$$

$$\omega(0) = 0.08,\ \omega(1) = 0.31, \omega(2) = 0.77,\ \omega(3)=1$$

$$\omega(4) = 0.77$$

$$\omega(5) = 0.31,\ \omega(6) = 0.08$$

The filter coefficients of the resultant filter are then

$$H(n) = H_d(n), \omega(n)\ n = 0, 1, 2, 3, 4, 5, 6$$

Therefore

$$H(0) = 0.006,\ H(1) = -0.0494,\ H(2) = 0.1733$$

$$H(3) = 0.75$$

$$H(4) = 0.1733,\ H(5) = -0.0494 \text{ and } H(6) = 0.006$$

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0}^{6} H(n)e^{-j\omega n}$$

$$= e^{-j3\omega}\left[H(3) + 2H(0)\cos 3\omega + 2H(1)\cos 2\omega + 2H(2)\cos\omega\right]$$

$$= e^{-j3\omega}\left[0.75 + 0.3466\cos\omega - 0.0988\cos 2\omega + 0.012\cos\omega\right]$$

Q.7. Explain with an example the Schur Cohn Stability Test for systems.

Ans. The Schur-Cohn Stability Test: We have stated previously that the stability of a system is determined by the position of the poles. The poles of the system are the roots of the denominator polynomial of H(z), namely

$$A(z) = 1 + a_1z^{-1} + a_2z^{-2} + \ldots + a_Nz^{-N} \quad \ldots(1)$$

When the system is causal all the roots of A(z) must lie inside the unit circle for the system to be stable.

There are several computational procedures that aid us in determining if any of the roots of A(z) lie outside the unit circle. These procedures are called stability criteria, we describe the Schur-Cohn test procedure for the stability of a system characterized by the system function H(z) = B(z)/A(z).

Before we describe the Schur-Cohn Test we need to establish some useful notation. We denote a polynomial of degree m by.

$$A_m(z) = \sum_{k=0}^{m} a_m(k)z^{-k}$$

$$a_m(0) = 1$$

The reciprocal or reverse polynomial $B_m(z)$ of degree m is defined as

$$B_m(z) = z^{-m}A_m(z^{-1})$$

$$= \sum_{k=0}^{m} a_m(m-k)z^{-k} \quad (3)$$

We observe that the coefficients of $B_m(z)$ are the same as those of $A_m(z)$, but in reverse order.

In the Schur-Cohn stability test, to determine if the polynomial A(z) has all its roots inside the unit circle, we compute a set of coefficients, called reflection coefficients, K_1, K_2....K_N from the polynomials $A_m(z)$. First, we set

$$A_N(z) = A(z)$$

and
$$K_N = a_N(N) \tag{4}$$

Then we compute the lower-degree polynomials $A_m(z)$, $m = N, N-1, N-2, \ldots 1$ according to the recursive equation.

$$A_{m-1}(z) = \frac{A_m(z) K_m B_m(z)}{1 - K_m^2} \tag{5}$$

where the coefficients K_m are defined as

$$K_m = a_m(m) \tag{6}$$

The Schur-Cohn stability test states that the polynomial A(z) given by (1) has all its roots inside the unit circle if and only if the coefficients K_m satisfy the condition $|K_m| < 1$ for all $m = 1, 2, \ldots N$.

Example. Determine if the system having the system function.

$$H(z) = \frac{1}{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}} \text{ is stable.}$$

Solution. We begin with $A_2(z)$, which is defined as

$$A_2(z) = 1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}$$

Hence
$$K_2 = -\frac{1}{2}$$

Now
$$B_2(z) = -\frac{1}{2} - \frac{7}{4}z^{-1} + z^{-2}$$

and
$$A_1(z) = \frac{A_2(z) - K_2 B_2(z)}{1 - K_2^2}$$

$$= 1 - \frac{7}{2}z^{-1}$$

Therefore,
$$K_1 = -\frac{7}{2}$$

Since $|K_1| > 1$ it follows that the system is unstable. This fact is easily established in this example, since the denominator is easily factored to yield the two poles at $p_1 = -2$ and $p_2 = \frac{1}{4}$. However, for higher-degree polynomials, the Schur-Cohn test provides a similar test for stability than direct factoring of H(z).

The Schur-Cohn stability test can be easily programmed in a digital computer and it is very efficient in terms of arithmetic operations. Specifically, it requires only N^2 multiplications to determine the coefficients $[K_m]$, $m = 1, 2, \ldots, N$. The recursive equation in (5) can be expressed in terms of the polynomial coefficients by expanding the polynomials in both sides of (5) and equating the coefficients corresponding to equal powers. Indeed, it is easily established that (5) is equivalent to the following algorithm: Set

and
$$a_{m-1}(k) = \frac{a_m(k) - K_m b_m(k)}{1 - K_m^2}$$

$$k = 1,2,\ldots,m-1$$

where $$b_m(k) = a_m(m-k) \quad k = 0,1,\ldots,m$$

This recursive algorithm for the computation of the coefficients $[K_m]$ finds applic in various signal processing problems, especially in speech signal processing.

Q.8. Realize the following using ladder structure.

$$\mathbf{H(z) = \frac{2+8z^{-1}+6z^{-2}}{1+8z^{-1}+12z^{-2}}}$$

Ans. Given that
$$H(z) = \frac{2+8z^{-1}+6z^{-2}}{1+8z^{-1}+12z^{-2}}$$

For the given system, obtain the Routh array

z^{-2}	6	8	2
z^{-2}	12	8	1
z^{-1}	4	3/2	
z^{-1}	7/2	1	
1	5/14	0	
1	1		

The ladder structure parameters are

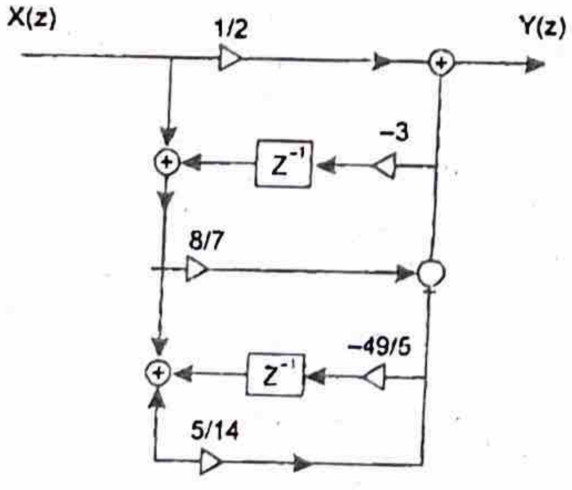
$$\alpha_0 = \frac{1}{2}, \beta_1 = 3$$

$$\alpha_1 = \frac{8}{7}, \beta_2 = \frac{49}{5}$$

$$\alpha_2 = \frac{5}{14}$$

$$H(z) = \frac{1}{2} + \cfrac{1}{3z-1+\cfrac{1}{\frac{8}{7}+\cfrac{1}{(49/5)z^{-1}+\cfrac{1}{5/14}}}}$$

The ladder structure is shown in Fig. below.



FIRST TERM EXAMINATION [FEB. 2017]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING
[ETEC–306]

Time : 1.5 hrs. **M.M. : 30**

Note: *Q. No 1 is compulsory. Attempt any two more Questions from the rest.*

Q.1. (a) Explain any three properties of DFT. **(3)**

Q.1. (b) What do you mean by twiddle factor? Explain symmetric and periodicity of twiddle factor. **(2)**

Q.1. (c) What do you mean by warping and prewarping? **(2)**

Q.1. (d) How does a butterworth filter from Chebyshev filter? **(3)**

Q.2. (a) Compute the DFT of the four point sequence. **(5)**

$$x(n) = [0\ 1\ 2\ 3]$$

Q.2. (b) Perform the circular convolution of the following two sequence. **(5)**

$$x_1(n) = [\underset{\uparrow}{2}\ 1\ 2\ 1] \qquad x_2[n] = [\underset{\uparrow}{1}\ 2\ 3\ 4]$$

Q.3. (a) An 8-point sequence is given by x(n) = [2, 2, 2, 2, 1, 1, 1, 1]. Computing 8 point DFT of x(n(by raix 2 DIF FFT. **(5)**

Q.3. (b) Find the digital network in direct and transposed from for the system described by the difference equation.

$$y(n) = x(n) + 0.5\,x(n-1) + 0.4\,x(n-2) - 0.6\,y(n-1) - 0.7\,(n-2)$$ **(5)**

Q.4. (a) A first order Butterworth low pass transfer function with a 3 dB cut off frequency at Ωc is given by

$$H_a(s) = \frac{\Omega c}{s + \Omega c}$$

Design a single pole low pass with 3 dB bandwidth of 0.2 π using the Bilinear transformation. **(5)**

Q.4. (b) Design an ideal low pass filter with a frequency response

$$H_d(e^{j\omega}) = 1;\ -\pi/2 \le |\omega| \le \pi/2$$
$$= 0;\ \pi/2 \le \omega \le \pi$$

Find the value of h(n) for N = 11. **(5)**

ANSWERS

All the questions are repeated in First Term 2016
Same question paper is repeated.

END TERM EXAMINATION [MAY-JUNE 2017]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING
[ETEC-306]

Time : 3 hrs. **M.M. : 75**

Note: *Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.*

Q.1. Attempt all the questions of the following:

Q.1. (a) Compare the computation cost of DFT and radix 2 DIF FFT for computing 16 point DFT. **(3)**

Ans. For a direct computation of an N-point DFT, without exploiting symmetry, N^2 complex multiplications and N(N – 1) Complex additions are required.

Hence for 16-point, there are 256 complex multiplications and 240 complex additions are required. The radix-2 DIF FFT algorithm requires $\left(\frac{N}{2}\right)\log_2 N$ complex multiplications and $N\log_2 N$ complex additions.

Hence for 16-point

The complex multiplications requires

$$\frac{N}{2}\log_2 N = 8\log_2 16 = 8\log_2(2)^4 = 32$$

and complex additions requires

$$N\log_2 N = 16\log_2 16 = 16\log_2(2)^4 = 64$$

Q.1. (b) What is Warping effect? Explain pre-warping filter. **(3)**

Ans. The relation between the analog and digital frequencies in bilinear transformation is given by

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

For smaller values of ω there exist linear relationship between ω and Ω. But for large values of ω the relationship is non-linear. This non-linearity introduces distortion in the frequency axis. This is known as warping effect. This effect compresses the magnitude and phase response at high frequencies.

The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable prescaling, or prewarping the critical frequencies by using the formula.

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

Q.1. (c) Write the necessary condition for Toeplitz Matrix and Hermitian Matrix. **(3)**

Ans. Condition of Toeplitz matrix is

$$\Gamma_P(i,j) = \Gamma_P(i-j)$$

Conditon of Hermitian matrix is

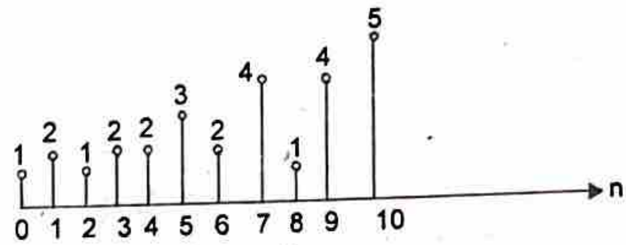
$$\Gamma_p(i,j) = \Gamma_p^*(i,j)$$

Q.1. (d) $x(n) = \{1, 2, 1, 2, 2, 3, 2, 4, 1, 4, 5\}$ Perform (i) decimation by 2, and (ii) interpolation by 3 on the above signal $x(n)$. **(3)**

Ans. Given that $x(n) = \{1, 2, 1, 2, 2, 3, 2, 4, 1, 4, 5\}$

(i) Decimation by 2

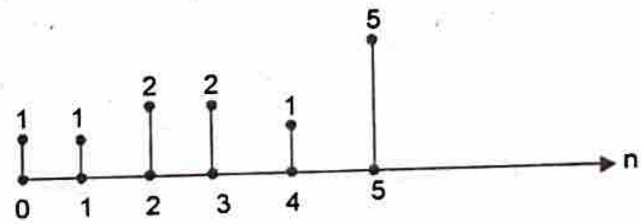
$x(n)$ is shown below



After decimation by a factor of 2, we have

$$x(n) \rightarrow \boxed{\downarrow 2} \rightarrow y(n) \;=\; \{1, 1, 2, 2, 1, 5\}$$

Since every integer point is divided by 2, then we get only the samples at points 0,2, 4, 6, 8 and 10. since it is divided by 2 then we have the points 0,1,2,3,4,5

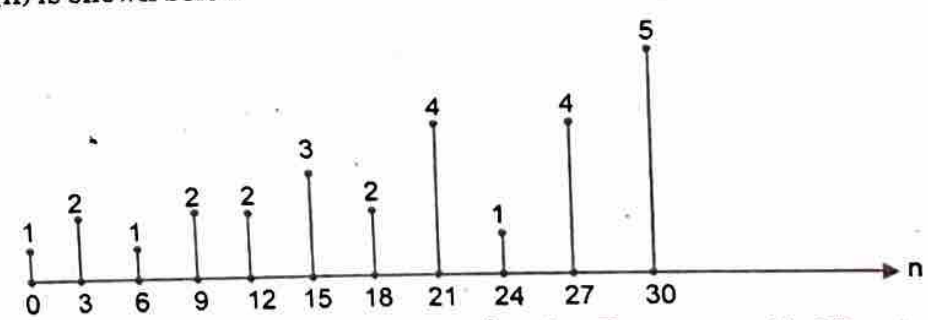


(ii) Similarly interpolation by 3

$$x(n) \rightarrow \boxed{\uparrow 3} \rightarrow y(n)$$

y(n) exists at the points n = 0, 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30 which means zero samples are added into the original sequences.

The y(n) is shown below



Q.1. (e) Determine the poles of the 2nd order Butterworth filter. **(3)**

Ans. To obtain the system function $H_a(s)$, we have to determine the poles first. At s = jω, the magnitude of $H_a(s)$ and $H_a(-s)$ is same i.e.

$$H_a(s).H_a(-s) \;=\; |H_a(\Omega)|^2 \qquad ...(1)$$

we know that

$$|H_a(\Omega)|^2 \;=\; \frac{1}{1+\left(\dfrac{\Omega^2}{\Omega_c^2}\right)^N} \qquad ...(2)$$

At $s = j\Omega$, $\Omega^2 = -s^2$. Hence putting this value in above equation and from (1)

$$H_a(s).H_a(-s) = \frac{1}{1+\left(\frac{-s^2}{\Omega_c^2}\right)^N} \qquad ...(3)$$

The poles of $H_a(s).H_a(-s)$ can be obtained by finding roots of denominator in above equation i.e.,

$$1+\left(\frac{-s^2}{\Omega_c^2}\right)^N = 0 \therefore \left(\frac{-s^2}{\Omega_c^2}\right)^N = -1$$

$$\therefore \qquad \frac{-s^2}{\Omega_c^2} = (-1)\frac{1}{N}$$

Let us consider $e^{j(2k+1)\pi}$ for $K = 0,1,.....N-1$

$e^{j(2k+1)\pi} = \cos(2k+1)\pi + j\sin(2k+1)\pi$

Since $(2k+1)\pi = \pi, 3\pi, 5\pi,...$

we have $\frac{-s^2}{\Omega_c^2} = \left[e^{j(2k+1)\pi}\right]^{\frac{1}{N}}$ for $k = 0,1,2,....N-1$

$= e^{j(2k+1)\pi/N}$ for $k = 0, 1, 2,....N-1$

$\therefore s^2 = -\Omega_c e^{j(2k+1)\pi/N} = (-1)\Omega^2{}_c e^{j(2k+1)\pi/N}$

Taking square root of both sides we get poles

$$= \pm\sqrt{(-1)}.\Omega_c.\left[e^{j(2k+1)\pi/N}\right]^{\frac{1}{2}}$$

Hence $p_k = \pm j\Omega_c. e^{j(2k+1)\pi/2N}$ for $k = 0,1,2,....N-1$

Putting $j = e^{j\frac{\pi}{2}}$ in above equation, we get

$$\left.\begin{aligned} p_k &= \pm\Omega_c e^{j\frac{\pi}{2}} e^{j(2K+1)\frac{\pi}{2}N} \\ &= \Omega_c e^{j\left[\frac{\pi}{2}+\frac{(2k+1)\pi}{2N}\right]} \\ &= \pm\Omega_c e^{j(N+2k+1)\pi/2N} \end{aligned}\right\} K = 0,1....N-1$$

UNIT-I

Q.2. (a) Compute radix 2 DIF FFT of the following signal $x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\}$. **(8)**

Ans. Given that $x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\}$

Here N = 8

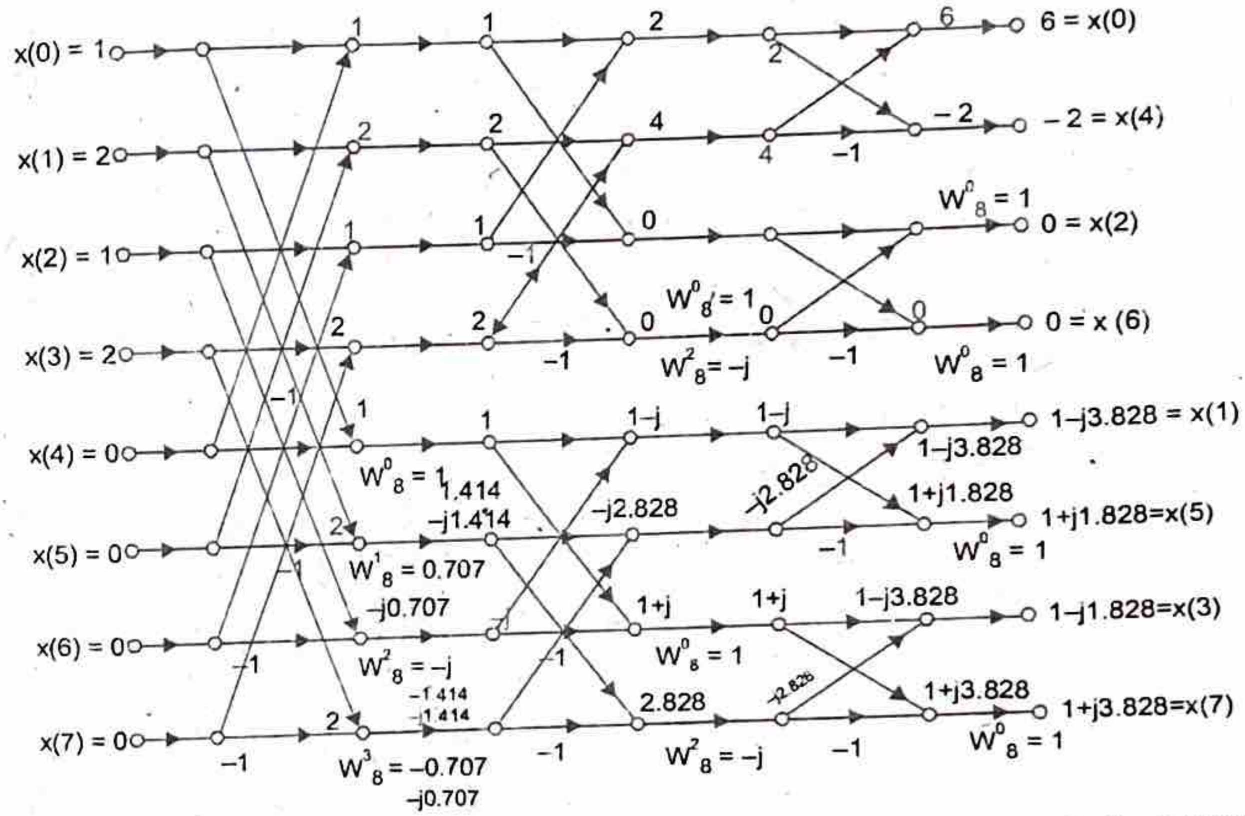
We know that $W_N^K = e^{\frac{-j(2\pi k)}{N}}$

Hence $W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



$x(n) = \{6, 1-j3.828, 0, 1-j1.828, -2, 1+j1.828, 0, 1+j3.828\}$

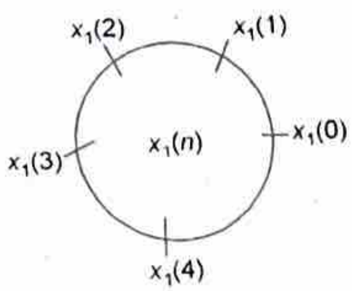
Q.2. (b) Perform the circular convolution of the following signals $x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{1, 0, 2, 1, 5\}$. (7)

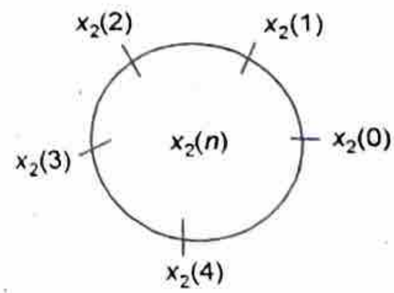
Ans. The given sequence are not same, so add one zero sample to $x_1(n)$, we have $x_1(n) = \{1, 2, 3, 1, 0\}$ and $x_2(n) = \{1, 0, 2, 1, 5\}$

Here N = 5

We know that, $x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2[m-n, (\text{mod } N)]$, $m = 0, 1, \ldots N-1$

$$x_3(m) = \sum_{n=0}^{4} x_1(n)x_2((m-n))_5$$





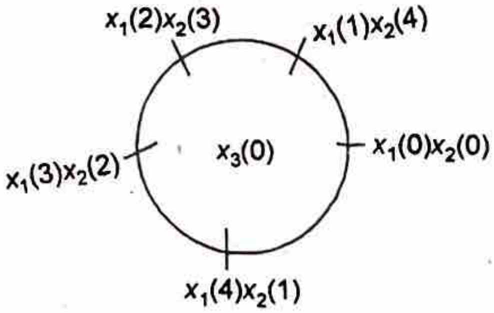
$$= x_1(0)\, x_2((m-0))_5 + x_1(1) x_2\, ((m-1))_5 + x_1(2)\, x_2((m-2))_5$$
$$+ x_1(3)\, x_2((m-3))_5 + x_1(4) x_2\, ((m-4))_5 \quad \ldots(1)$$

Case I when m = 0

From equation (1)

$x_3(0) = x_1(0)x_2(0) + x_1(1)x_2(4) + x_1(2)x_2(3)+x_1(3)x_2(2)+x_1(4)x_2(1)$

$= 1 \times 1 + 2 \times 5 + 3 \times 1 + 1 \times 2 + 0 \times 0 = 1 + 10 + 3 + 2 + 0 = 16$

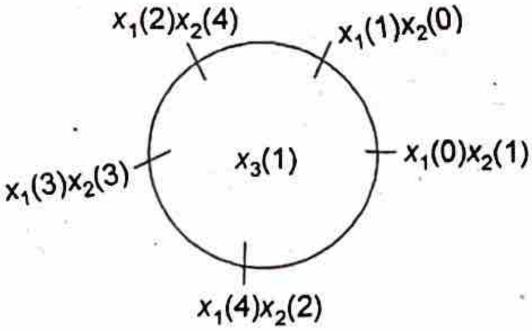


Case II when m = 1

From eqnation (1)

$x_3(1) = x_1(0)x_2(1)+ x_1(1)\, x_2\, (0) + x_1\, (2)\, x_2(4) + x_1(3)x_2(3) + x_1\, (4)x_2\, (2)$

$= 1 \times 0 + 2 \times 1 + 3 \times 5 + 1 \times 1 + 0 \times 2 = 0 + 2 + 15 + 1 + 0 = 18$

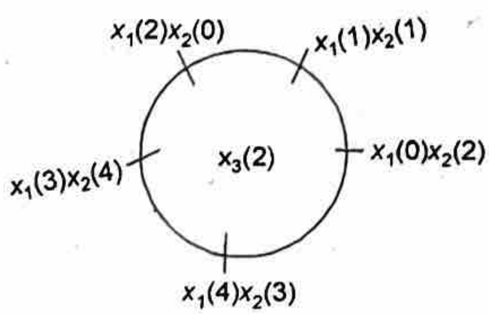


Case III When m = 2

From equation (1)

$x_3(2) = x_1(0)\, x_2(2) + x_1(1)\, x_2(1) + x_1(2)\, x_2(0) + x_1(3)\, x_2(4) + x_1(4)\, x_2\, (3)$

$= 1 \times 2 + 2 \times 0 + 3 \times 1 + 1 \times 5 + 0 \times 1 = 2 + 0 + 3 + 5 + 0 = 10$

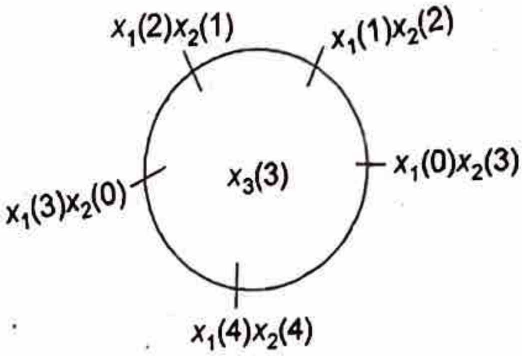


Case IV When m = 3

From equation (1)

$x_3(3) = x_1(0)\, x_2(3) + x_1(1)\, x_2(2) + x_1(2)\, x_2(1) + x_1(3)\, x_2(0) + x_1(4)\, x_2\, (4)$

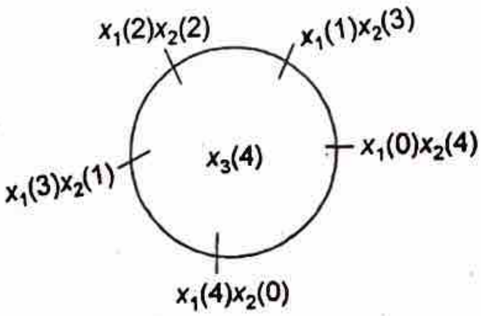
$= 1 \times 1 + 2 \times 2 + 3 \times 0 + 1 \times 1 + 0 \times 5 = 1 + 4 + 0 + 1 + 0 = 6$



Case V When m = 4

From equation (1)

$x_3(4) = x_1(0)x_2(4) + x_1(1)x_2(3) + x_1(2)x_2(2) + x_1(3)x_2(1) + x_1(4)x_2(0)$
$= 1 \times 5 + 2 \times 1 + 3 \times 2 + 1 \times 0 + 0 \times 1 = 5 + 2 + 6 + 0 + 0 = 13$



Q.3. (a) Compute 8 pt. DFT of the following signal using twiddle factor property : **(10)**

$x(n) = \{1, 1, 0, 0, 1, 0, 1, 0\}$.

Ans. Given that

$$x(n) = \{1, 1, 0, 0, 1, 0, 1, 0\}$$
$$N = 8$$

We know that

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \; k = 0, 1, \ldots N-1$$

or

$$X(K) = \sum_{n=0}^{7} x(n) W_8^{kn}, \; k = 0, 1, \ldots 7$$

In matrix form

$$X_N = [W_N]\, x_N$$

with $N = 8, X_8 = [W_8]\, x_8$
$[W_8]$ can be written as

$$[W_8] = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix}$$

We have $W_N^{Kn} = e^{+j\frac{2\pi}{N}Kn}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & +0+0+1+0+1+0 \\ 1+\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & +0+0-1+0+j+0 \\ 1-j & +0+0+1+0-1+0 \\ 1-\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & +0+0-1+0-j+0 \\ 1-1 & +0+0+1+0+1+0 \\ 1-\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & +0+0-1+0+j+0 \\ 1+j & +0+0+1+0-1+0 \\ 1+\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & +0+0-1+0-j+0 \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{1}{\sqrt2}+j\left(1-\frac{1}{\sqrt2}\right) \\ 1-j \\ \frac{-1}{\sqrt2}-j\left(1+\frac{1}{\sqrt2}\right) \\ 2 \\ \frac{-1}{\sqrt2}+j\left(1+\frac{1}{\sqrt2}\right) \\ 1+j \\ \frac{1}{\sqrt2}+j\left(\frac{1}{\sqrt2}-1\right) \end{bmatrix}$$

Q.3. (*b*) Compute linear convolution of the following sequences using overlap add method $x_1(n) = \{1, 2, 3, 4\}, x_2(n) = \{1, 2, 0, 1\}$. (5)

Ans. Given that $x_1(n) = \{1,2,3,4\}$

$x_2(n) = \{1,2,0,1\}$

The sequence is divided into blocks of data size having length L and M–1 zeros are appended to it to make the data size of L + M – 1

$$L + M - 1 = 4 + 4 - 1 = 7$$

Thus $$y(n) = x_1(n) \, (N) \, x_2(n)$$

$$y(n) = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+8+0+2 \\ 2+2+0+3 \\ 3+4+0+4 \\ 4+6+0+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 7 \\ 11 \\ 11 \end{bmatrix}$$

UNIT - II

Q.4. (a) Design a Butterworth digital IIR LPF using bilinear transformation by taking T = 0.1 second, to satisfy the following specification

$0.6 \le |H(e^{j\omega})| \le 1.0$; for $0 \le \omega \le 0.35\pi$

$|H(e^{j\omega})| \le 0.1$; for $0.7\pi \le \omega \le \pi$ **(10)**

Ans. Given that

$0.6 \le |H(e^{j\omega})| \le 1.0$; for $0 \le \omega \le 0.35\pi$

$|H(e^{j\omega})| \le 0.1$; for $0.7\pi \le \omega \le \pi$

and T = 0.1 second

we have, $\delta_1 = 0.6$; $\delta_2 = 0.1$; $\omega_1 = 0.35\pi$; $\omega_2 = 0.7\pi$

Step I: Analog filter's edge frequencies

$$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2} = \frac{2}{0.1}\tan\left(\frac{0.35\pi}{2}\right) = 12.24$$

$$\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2} = \frac{2}{0.1}\tan\left(\frac{0.7\pi}{2}\right) = 39.19$$

Therefore
$$\frac{\Omega_2}{\Omega_1} = \frac{39.19}{12.24} = 3.20$$

Step II: Order of the filter

$$N \ge \frac{1}{2}\frac{\log\left\{\left(\left(\frac{1}{\delta_2^2}\right)-1\right)\Big/\left(\left(\frac{1}{\delta_1^2}\right)-1\right)\right\}}{\log\left(\frac{\Omega_2}{\Omega_1}\right)}$$

$$N \ge 1.72 \text{ i.e. } N = 2$$

Step III: –3dB cut-off frequency

$$\Omega_C = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{\frac{1}{2N}}} = \frac{12.24}{[1.777]^{\frac{1}{4}}} = 10.606$$

Step IV: Determination of $H_a(S)$:

$$H(S) = \prod_{k=1}^{N/2}\frac{B_k\Omega_C^2}{S^2 + b_k\Omega_C^S + C_k\Omega_C^2}$$

Since N = 2

Hence,
$$H(S) = \frac{B_1\Omega_C^2}{S^2 + b_1\Omega c S + C_1\Omega_C^2}$$

$$b_1 = 2\sin\left[(2.1-1)\frac{\pi}{2\times 2}\right] = 2\sin\frac{\pi}{4} = 1.68$$

and $c_1 = 1$

$B_1 = 1$

$$\therefore \qquad H(S) = \frac{1 \times (10.606)^2}{S^2 + 1.68 \times 10.606S + 1 \times (10.606)^2}$$

Therefore $$H(S) = \frac{112.48}{S^2 + 17.81S + 112.48}$$

Step V: Determinaton of H(z)

$$H(Z) = H(S)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = H(S)\Big|_{s=20\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(Z) = \frac{112.48}{\left\{20\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\}^2 + 17.81 \times 20\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 112.48}$$

$$= \frac{112.48}{400\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 356\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 112.48}$$

Q.4. (*b*) Explain the characteristics of ideal window for FIR filter design. (5)

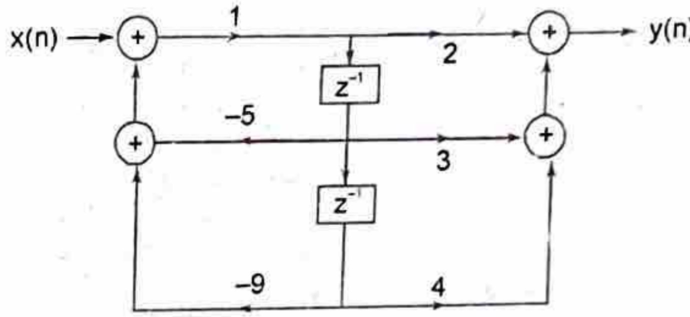
Ans. Design using Window Functions: Different types of window functions are available which reduce ringing effect. The particular window is selected depending upon the application. Table shown below lists the time domain equation for different type of windows.

Window	Transition width the main lobe	Minimum stopband attenuation	Relative amplitude of sidelobe
Rectangular	$\frac{4\pi}{M+1}$	– 21 dB	– 13 dB
Bartlett	$\frac{8\pi}{M}$	– 25 dB	– 25 dB
Hanning	$\frac{8\pi}{M}$	– 44 dB	– 31 dB
Hamming	$\frac{8\pi}{M}$	– 53 dB	– 41 dB
Blackman	$\frac{12\pi}{M}$	– 74 dB	– 57 dB

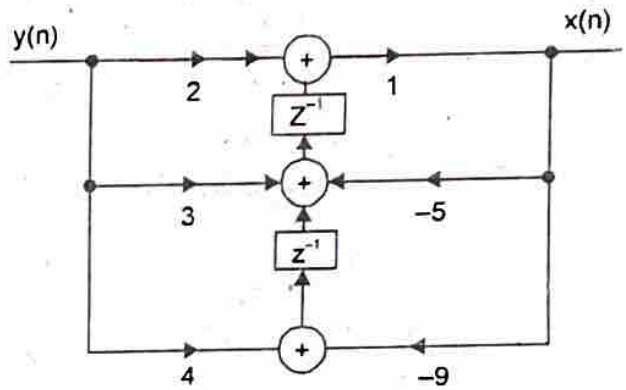
Q.5. (*a*) Draw the canonical structure and transpose structure for the following transfer function $H(z) = \frac{2 + 3z^{-1} + 4z^{-2}}{1 + 5z^{-1} + 9z^{-2}}$ (7)

Ans. Given that $$H(z) = \frac{2 + 3z^{-1} + 4z^{-2}}{1 + 5z^{-1} + 9z^{-2}}$$

In the canonical structure the system order (here 2) equals the number of delay units in the filter. The canonical structure of the above transfer function is shown below



The transposed structure for the given transfer function is shown below



Q.5. (b) Determine H(z) for the following transfer function using impulse invariance method if (i) T = 0.1 sec, (ii) T = 1 sec.
$H(s) = 2/(s^2 + 3s + 2)$ **(8)**

Ans. Given that
$$H(s) = \frac{2}{s^2 + 3s + 2}$$

(i) For T = 0.1 sec

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \left.\frac{2}{(s+2)}\right|_{s=-1} = \frac{2}{-1+2} = 2$$

$$B = \left.\frac{2}{(s+2)}\right|_{s=-2} = \frac{2}{-1} = -2$$

$$\therefore \quad H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

Using impulse invariance method, we have

$$\frac{1}{s-p_k} \to \frac{1}{1-e^{p_K T} z^{-1}}$$

$$\therefore \quad \frac{2}{s+1} \to \frac{2}{1-e^{-1\times 0.1} z^{-1}} = \frac{2}{1-e^{-0.1} z^{-1}}$$

$$\frac{2}{s+2} \to \frac{2}{1-e^{-2\times 0.1} z^{-1}} = \frac{2}{1-e^{-0.2} z^{-1}}$$

$$\therefore \qquad H(z) = \frac{2}{1-e^{-0.1}z^{-1}} - \frac{2}{1-e^{-0.2}z^{-1}}$$

(ii) For T = 1 sec.

$$\frac{2}{s+1} \rightarrow \frac{2}{1-e^{-1\times 1}z^{-1}} = \frac{2}{1-e^{-1}z^{-1}}$$

$$\frac{2}{s+2} \rightarrow \frac{2}{1-e^{-2\times 1}z^{-1}} = \frac{2}{1-e^{-2}z^{-1}}$$

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} - \frac{2}{1-e^{-2}z^{-1}}$$

UNIT–III

Q.6. (*a*) Find the direct form coefficients from the Lattice coefficients $K_1 = 1/4, K_2 = 1/2, K_3 = 1/3$. Write an application of Lattice structure. **(7)**

Ans. Given that $\qquad K_1 = \frac{1}{4};\ K_2 = \frac{1}{2};\ K_3 = \frac{1}{3}$

From the given data, we can find that

$$\alpha_3(0) = 1; \alpha_3(3) = K_3 = \frac{1}{3}$$

$$\alpha_1(1) = K_1 = \frac{1}{4}; \alpha_2(2) = K_2 = \frac{1}{2}$$

We know that

$$\alpha_m(K) = \alpha_{m-1}(K) + K_m\,\alpha_{m-1}(m-K)$$

for m = 2 and $K = 1$

$$\alpha_2(1) = \alpha_1(1) + K_2\alpha_1(1)$$

$$= \frac{1}{4} + \frac{1}{2}\times\frac{1}{4} = \frac{1}{4} + \frac{1}{8} = \frac{2+1}{8} = \frac{3}{8}$$

for m = 3 and $K = 1$

$$\alpha_3(1) = \alpha_2(1) + \alpha_3(3)\,\alpha_2(2)$$

$$= \frac{3}{8} + \frac{1}{3}\times\frac{1}{2} = \frac{3}{8} + \frac{1}{6} = \frac{9+4}{24} = \frac{13}{24}$$

for m = 2 and $K = 2$

$$\alpha_3(2) = \alpha_2(2) + \alpha_3(3)\,\alpha_2(1)$$

$$= \frac{1}{2} + \frac{1}{3}\times\frac{3}{8} = \frac{1}{2} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

$$\Rightarrow \qquad \alpha_3(0) = 1; \alpha_3(1) = \frac{13}{24};\ \alpha_3(2) = \frac{5}{8};\ \alpha_3(3) = \frac{1}{3}$$

Applications of lattice structure

(i) It can be used to implement FIR and IIR filters.

(ii) It can be used in linear prediction for speech processing.

(iii) It can be used to model the vocal tract with an all-pole structure.

(iv) It is the most efficient structure for generating at the same time the forward and backward prediction errors.

(v) In lattice filter implementations of fixed-point IIR filters, stability and frequency responses are less sensitive to round off errors of the coefficients compared to classic implementations.

(vi) Lattice IIR filters produce very small limit cycles much smaller than in classic implementations.

Q.6. (*b*) Explain Lavinson Durbin Algorithm in detail. (8)

Ans. Lavinson Durbin Algorithm: It is a computationally efficient algorithm for solving the prediction coefficients. This algorithm exploits the special symmetry in the autocorrelation matrix.

$$p = \begin{bmatrix} \gamma_{xx}(0) & \gamma_{xx}^*(1) & ..... & \gamma_{xx}^*(p-1) \\ \gamma_{xx}(1) & \gamma_{xx}(0) & .... & \gamma_{xx}^*(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{xx}(p-1) & \gamma_{xx}(p-2) & .... & \gamma_{xx}(0) \end{bmatrix}$$

The key to the Lavinson-Durbin method of solution, that exploits the Toeplitz property of matrix, is to proceed recursively, beginning with a predictor of order m = 1 and then to increase the order recursively, using the lower-order solutions to obtain the solution to the next-higher order.

Thus the solution to the first-order predictor is

$$a_1(1) = \frac{-\gamma_{xx}(1)}{\gamma_{xx}(0)} \qquad ...(1)$$

and the resulting MMSE is

$$E_1{}^f = \gamma_{xx}(0) + a_1(1)\gamma_{xx}(-1) = \gamma_{xx}(0)[1 - a_1(1)|^2] \qquad ...(2)$$

$a_1(1) = K_1$, the first reflection coefficient in the lattice filter. The next step is to solve for the coefficients $\{a_2(1), a_2(2)\}$ of the second-order predictor and express the solution in terms of $a_1(1)$. The two equations are

$$\begin{aligned} a_2(1)\gamma_{xx}(0) + a_2(2)\gamma_{xx}^*(1) &= -\gamma_{xx}(1) \\ a_2(1)\gamma_{xx}(1) + a_2(2)\gamma_{xx}(0) &= -\gamma_{xx}(2) \end{aligned} \qquad ...(3)$$

By using equation (1), eliminate $\gamma_{xx}(1)$, we obtain the solution

$$a_2(2) = \frac{\gamma_{xx}(2) + a_1(1)\gamma_{xx}(1)}{\gamma_{xx}(0)[1 - |a_1(1)|^2]} = -\frac{\gamma_{xx}(2) + a_1(1)\gamma_{xx}(1)}{E_1^f}$$

$$a_2(1) = a_1(1) + a_2(2)\, a_1^*(1)$$

Thus we have obtained the coefficients of the second-order predictor. Again, we note that $a_2(2) = k_2$, the second reflection coefficient in the lattice filter.

Proceeding in this manner, we can express the coefficients of the m^{th} order predictor in terms of the coefficients of the (m-1)st order predictor.

Q.7. (*a*) Explain linear forward and backward prediction in detail. (8)

Ans. A forward linear predictor is a filter that attempts to predict the u(n) sample from the previous m samples. Forward predictors are causal, which means they only act on previous and present results.

Backward prediction is similar to forward prediction, they are closely related mathematically. Backward prediction is the process of trying to determine the u(n + M– 1) element of a signal given the next M elements. In other words, the backward predictor is an attempts to "remember" what a past value was , given later values.

Q.7. (*b*) Compute the Lattice coefficients from the Direct form Coefficients $\alpha_3(3) = 1/3$, $\alpha_3(2) = 5/8$, $\alpha_3(1) = 13/24$ and draw the Lattice structure. (7)

Ans. Given that
$$\alpha_3(3) = \frac{1}{3}, \alpha_3(2) = \frac{5}{8}, \alpha_3(1) = \frac{13}{24}$$

We know that
$$\alpha_{m-1}(K) = \frac{\alpha_m(K) - \alpha_m(m)\alpha_m(m-K)}{1-\alpha_m^2(m)}$$

For m = 3 and K = 1
$$\alpha_2(1) = \frac{\alpha_3(1) - \alpha_3(3)\alpha_3(2)}{1-\alpha_3^2(m)} = \frac{\frac{13}{24} - \frac{1}{3}\left(\frac{5}{8}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{3}{8}$$

For m = 3 and K = 2
$$\alpha_2(2) = \frac{\alpha_3(2) - \alpha_3(3)\alpha_3(1)}{1-\alpha_3^2(3)} = \frac{\frac{5}{8} - \frac{1}{3}\left(\frac{13}{24}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{1}{2}$$

for m = 2 and K = 1
$$\alpha_1(1) = \frac{\alpha_2(1) - \alpha_2(2)\alpha_2(1)}{1-\alpha_2^2(2)} = \frac{\frac{3}{8} - \frac{1}{2}\left(\frac{3}{8}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{4}$$

Therefore,
$$K_1 = \alpha_1(1) = \frac{1}{4};\ K_2 = \alpha_2(2) = \frac{1}{2};\ K_3 = \alpha_3(3) = \frac{1}{3}$$

UNIT-IV

Q.8. (a) What do you understand by Polyphase decomposition? Compute section 2 and section 4 Polyphase decomposition of the following transfer functions

(a)
$$\mathbf{H(z) = 0.2 + 0.7z^{-1} + 0.8z^{-2} + 0.5z^{-3} + 0.1z^{-4} + 0.2z^{-5} + 0.1z^{-6} + 0.6z^{-7} + 0.7z^{-8}}$$ (15)

Ans. Polyphase decomposition results in reduction of computation complexity in realisation.

The z-transform of a filter with impulse response h(n) is given by
$$H(z) = h(0) + z^{-1}h(1) + z^{-2}h(2) + ...$$

Rearranging the above equation we get,
$$H(z) = h(0) + z^{-2}h(2) + z^{-4}h(4) + ... + z^{-1}(h(1) + z^{-2}h(3) + z^{-4}h(5) + ...)$$

Given that

$$H(z) = 0.2 + 0.7z^{-1} + 0.8z^{-2} + 0.5z^{-3} + 0.1z^{-4} + 0.2z^{-5} + 0.1z^{-6} + 0.6z^{-7} + 0.7z^{-8}$$

Rearranging the above equation we get,

$$H(z) = 0.2 + 0.8z^{-2} + 0.1z^{-4} + 0.1z^{-6} + 0.7z^{-8} + z^{-1}(0.7 + 0.5z^{-2} + 0.2z^{-4} + 0.6z^{-6}.)$$

Q.8. (*b*) $H(Z) = \dfrac{1 + 3z^{-1}}{1 - 2z^{-1}}$

Ans.

$$H(z) = \frac{1+3z^{-1}}{1-2z^{-1}}$$

We have,

$$H(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

$$H(z) = \frac{1+3z^{-1}}{1-2z^{-1}} \times \frac{1+2z^{-1}}{1+2z^{-1}} = \frac{1+5z^{-1}+6z^{-2}}{1-4z^{-2}}$$

$$= \frac{1+6z^{-2}}{1-4z^{-2}} + z^{-1}\frac{5}{1-4z^{-2}}$$

The polyphase components are,

$$E_0(z^2) = \frac{1+6z^{-2}}{1-4z^{-2}} \text{ and } E_1(z^2) = \frac{5}{1-4z^{-2}}$$

Q.9. (*a*) Write short note on (i) Limit Cycle Oscillations (ii) Coefficient Quantization. (8)

Ans. (i) Limit Cycle Oscillations

Let the input sequence traverses several quantisation levels between two successive samples and so the samples of the round-off noise sequence were uncorrelated with each other and with the input signal. These assumptions are invalid in cases such as a constant or zero input to a digital filter. In such cases, the input signal remains constant during successive samples and does not traverse several quantisation levels. There are two types of limit cycles, namely, zero input limit cycle and overflow limit cycle. Zero input limit cycles are usually of lower amplitudes in comparison with overflow limit cycles. Let us consider a system with the difference equation

$$y(n) = 0.8y(n-1) + x(n) \quad \text{...(1)}$$

with zero input, i.e., $x(n). = 0$ and initial condition $y(-1) = 10$. A comparison between the exact values of $y(n)$ as given by Eq. using unquantised arithmetic and the rounded values of $y(n)$ as obtained from quantised arithmetic are given in Table below.

Table 1. A Comparison of exact y (n) and rounded y (n)

n	$y(n)$-unquantised	$y(n)$-quantised
−1	10.0	10
0	8.0	8
1	6.4	6
2	5.12	5
3	4.096	4
4	3.2768	3
5	2.62144	2
6	2.0972	2
7	1.6772	2

From above Table it can be observed that for zero input, the unquantised output $y(n)$ decays exponentially to zero with increasing n. However, the rounded-off (quantised) output $y(n)$ gets stuck at a value 'of two and never decays further. Thus, the output is finite even when no input is applied. This is referred to as zero input limit cycle effect. It can also be seen that for any value of the input condition $|y(-1)| \leq 2$, the output; $y(n) = y(-1)$, $n \geq 0$, when the input is zero. Thus, the deadband in this case is the interval [-2, 2].

(ii) Quantization of filter co-efficient

Digital signal processing algorithms are realized either with special purpose digital hardware or as programs for a general purpose digital computer. In both cases the numbers and coefficients are stored in finite-length registers. Therefore, coefficients and numbers are quantized by truncation or rounding off when they are stored.

The following errors arise due to quantization of numbers.

1. Input quantization error.
2. Product quantization error.
3. Coefficient quantization error.

1. The conversion of a continuous-time input/signal into digital value produces an error, which is known as input quantization error. This error arises due to the representation of the input signal by a fixed number of digits in A/D conversion process.

2. Product quantization errors arise at the output of a multiplier. Multiplication of a b bit data with a b bit coefficient results a product having 26 bits. Since a b bit register is used, the multiplier output must be rounded or truncated to b bits which produces an error.

3. The filter coefficients are computed to infinite precision in theory. If they are quantized, the frequency response of the resulting filter may differ from the desired response and sometimes the filter may fail to meet the desired specifications. If the poles of the desired filter are close the unit circle, then those of the filter with quantized coefficients may lie just outside the unit cirde, leading to instability.

The other errors arising from quantization are roundoff noise and limit cycle oscillations.

Fixed point representation: In fixed point arithmetic the position of the binary point is fixed. The bit to the right represent the fractional part of the number and those to the left represent the integer part. For example, the binary number 01.1100 has the value 1.75 in decimal.

The manner in which negative numbers are represented gives three different form for fixed-point arithmetic.

(*i*) Sign-magnitude form.

(*ii*) One's-complement form.

(*iii*) Two's-complement form. Floating Point Numbers

In floating point representation a positive number is represented as $F = 2^C \cdot M$, where M, called mantissa, is a fraction such that $1/2 \leq M \leq 1$ and c the exponent can be either positive or negative.

The decimal numbers 4.5, 1.5, 6.5 and 0.625 have floating point representations as $2^3 \times 0.5625$, $2^1 \times 0.75$, $2^3 \times 0.8125$, $2^\circ \times 0.625$ respectively.

Equivalently

$$\begin{aligned} 2^3 \times 0.5625 &= 2^{011} \times 0.1001 \\ 2^1 \times 0.75 &= 2^{001} \times 0.1100 \\ 2^3 \times 0.8125 &= 2^{011} \times 0.1101 \end{aligned}$$

$$2^{\circ} \times 0.625 = 2^{000} \times 0.1010$$

$$\boxed{\begin{aligned} (3)_{10} &= (011)_2 \\ (0.5625)_{10} &= (0.1001)_2 \\ (1)_{10} &= (001)2 \\ (0.75)_{10} &= (0.1100)_2 \\ (0.8125)_{10} &= (0.1101)_2 \\ (0.625)_{10} &= (0.1010)_2 \end{aligned}}$$

Negative floating point numbers are generally represented by considering the mantissa as a fixed point number. The sign of the floating point number is obtained from the first bit of mantissa.

Q.9. (*b*) Find o/p of the following system with i/p $x(n) = \{1, 2, 3, 1, 2, 3, 0, 0, 1\}$ (7)

$$\boxed{x(n)} \rightarrow \boxed{5 \uparrow} \rightarrow \boxed{20 \downarrow} \rightarrow \boxed{4 \uparrow}$$

Ans. Given that

$$\boxed{x(n)} \rightarrow \boxed{5 \uparrow} \rightarrow \boxed{20 \downarrow} \rightarrow \boxed{4 \uparrow} \rightarrow y(n)$$

$$= \boxed{x(n)} \rightarrow \boxed{5 \uparrow} \rightarrow \boxed{\downarrow 4} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{4 \uparrow} \rightarrow y(n)$$

$$= \boxed{x(n)} \rightarrow \boxed{\downarrow 4} \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 4} \rightarrow y(n)$$

$$= \xrightarrow{x(n)} \boxed{\downarrow 4} \xrightarrow{x_1(n)} \boxed{\uparrow 4} \rightarrow y(n)$$

$$x_1(n) = x(2n)$$

and
$$y(n) = x_1\left(\frac{n}{2}\right) \text{ for } n = 2K$$
$$= 0; \text{otherwise}$$

$\Rightarrow$
$$y(n) = x(n) \text{ for } n = K$$
$$= 0; \text{otherwise}$$

$\Rightarrow$
$$y(n) = \{1, 2, 3, 1, 2, 3, 0, 0, 1\} \text{ for } n = K$$
$$= 0; \text{otherwise.}$$

FIRST TERM EXAMINATION [FEB. 2018]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC–306]

Time : 1.5 hrs. **M.M. : 30**

Note: Attempt question No. 1 which is compulsory and two more questions, one from each sections..

Q. 1. (a) The first five points of the eight point DFT of a real valued sequence are {0.25, 0.125-j0.3018,0,0.125-j0.0518,0}. Determine the remaining three points. **(2)**

Ans. We have given that,

$$X(0) = 0.25;\quad X(1) = 0.125 - j0.3018;\quad X(2) = 0;$$
$$X(3) = 0.125 - j0.0518;\quad X(4) = 0$$

Given sequence is a real valued sequence. According to symmetry property we have,

$$X^*(k) = X(N-k) \text{ or } X(k) = X^*(N-k)$$

This is 8 – point DFT, hence $N = 8$.

Thus, $X(k) = X^*(8-k)$

Put K = 5, we get

$$X(5) = X^*(8-5) = X^*(3)$$

We have

$X(3) = 0.125 - j0.0518$ then $X^*(3) = 0.125 + j0.0518$

Put K =6, we get

$$X(6) = X^*(8-6) = X^*(2)$$

We have $X(2) = 0$ then $X^*(2) = 0$. Hence $X(6) = 0$

Put K =7, we get

$$X(7) = X^*(8-7) = X^*(1)$$

We have, $X(1) = 0.125 - j0.3018$ then $X^*(1) = 0.125 + j0.3018$

Hence, $X(7) = 0.125 + j0.3018$

Q. 1. (b) State and prove the Circular Convolution property of DFT. **(3)**

Ans. Circular Convolution

Let $x_1(n)$ and $x_2(n)$ are finite duration sequences both of length N with DFTs $X_1(k)$ and $X_2(k)$. Now we find a sequence $x_3(n)$ for which the DFT is $X_3(k)$

Where

$$X_3(k) = X_1(k)\, X_2(k)$$

We have

$$x_{3p}(n) = \sum_{m=0}^{N-1} x_{1p}(m) x_{2p}(n-m)$$

Or

$$x_3((n))_N = \sum_{m=0}^{N-1} x_1((m))_N\, x_2((n-m))_N$$

For $0 \le n \le N-1$; $x_3((n))_N = x_3(n)$. Similarly $x_1((m))_N = x_1(m)$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2((n-m))_N$$

Above equition represents the circular convolution of $x_1(n)$ and $x_2(n)$ represented as

$$x_3(n) = x_1(n) \text{ (N) } x_2(n)$$

Hence, we find that

$$\text{DFT}\,[x_1(n) \text{ (N) } x_2(n)] = X_1(k)\,X_2(k)$$

Q.1. (c) Realize an FIR filter with impulse response $h(n)$ given by **(3)**

$$h(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n-5)]$$

Ans. Given that $h(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n-5)]$

$$H(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

$$= \sum_{n=0}^{4}\left(\frac{1}{2}\right)^n z^{-1} = 1 + \frac{1}{2}z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \left(\frac{1}{2}\right)^4 z^{-4}$$

$$= 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} + \frac{1}{16}z^{-4}$$

Q.1. (d) Find the frequency response of the Type- 1 FIR filter and prove that zeros of Type-1 filter occurs in reciprocal pairs. **(3)**

Ans. For Type – I design

The samples are taken at the frequency

$$w_k = \frac{2\pi K}{M};\ K = 0, 1, ..., M-1$$

The samples of the desired frequency response at these frequencies are given by

$$\widetilde{H}(k) = H_d(e^{j\omega})\big|_{\omega=\omega_k};\ k = 0, 1, ..., M-1$$

$$= H_d(e^{j2\pi K/M}),\ k = 0, 1, ..., M-1$$

This set of points can be considered as DFT samples, then the filter coefficients $h(n)$ can be computed using the IDFT,

$$h(n) = \frac{1}{M}\sum_{K=0}^{M-1} \widetilde{H}(K)e^{j2\pi nK/M};\ n = 0, 1, ..., M-1$$

Q.1. (e) Explain Impulse Invariant method for design of IIR filter. **(3)**

Ans. Impulse Invariant Method: In this method, the design starts from the specifications of analog filter. Here, we have to replace analog filter by digital filter. This is achieved if impulse response of digital filter resembles the sampled version of impulse response of analog filter. If impulse response of both, analog and digital filter matches then, both filters perform in a similar manner.

In this method, we shall use the following different notations:

$h_a(t)$ = Impulse response in time domain

$H_a(s)$ = Transfer function of analog filter, here 's' is Laplace operator

$h_a(nT)$ = Sampled version of $h_a(t)$, obtained by replacing t by nT

$H(z)$ = z-transform of $h(nT)$. This is response of digital filter

Ω = Analog frequency

ω = Digital frequency

SECTION-I

Q. 2. Consider the sequences and their 5-points DFTs **(8)**

$x_1(n)=\{0,1,2,3,4\}$ $\quad x_2(n)=\{0,1,0,0,0\}$ $\quad s(n)=\{1,0,0,0,0\}$

(a) Determine a sequence $y(n)$ so that $Y(k) = X_1(k)X_2(k)$

(b) Is there a sequence $x_3(n)$ such that $S(k) = X_1(k)X_3(k)$?

Ans. We have given that

$x_1(n) = \{0, 1, 2, 3, 4\}$; $x_2(n) = \{0, 1, 0, 0, 0\}$, $S(n) = \{1, 0, 0, 0, 0\}$

(a) The N-point DFT of the sequence $x(n)$ is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}\ ;\ k = 0, 1, ..., N-1$$

So,
$$X_1(k) = \sum_{n=0}^{5-1} x(n)e^{-j2\pi nk/5}\ ;\ k = 0, 1, ..., 4$$

$$= \sum_{n=0}^{4} x(n)e^{-\frac{j2\pi nk}{5}}$$

$$= x(0) + x(1)e^{-\frac{j2\pi k}{5}} + x(2)e^{-\frac{j2\pi\times 2k}{5}} + x(3)e^{-\frac{j2\pi k\times 3}{5}} + x(4)e^{-\frac{j2\pi k\times 4}{5}}$$

$$X_1(k) = 0 + e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j4\pi k}{5}} + 3e^{-\frac{j6\pi k}{5}} + 4e^{-\frac{j8\pi k}{5}}$$

Put $k = 0$

$$X_1(0) = 1 + 2 + 3 + 4 = 10$$

Put $k = 1$

$$X_1(1) = e^{-\frac{j2\pi}{5}} + 2e^{-\frac{j4\pi}{5}} + 3e^{-\frac{j6\pi}{5}} + 4e^{-\frac{j8\pi}{5}}$$

$$= \cos\frac{2\pi}{5} - j\sin\frac{2\pi}{5} + 2\left\{\cos\frac{4\pi}{5} - j\sin\frac{4\pi}{5}\right\} + 3\left\{\cos\frac{6\pi}{5} - j\sin\frac{6\pi}{5}\right\} + 4\left\{\cos\frac{8\pi}{5} - j\sin\frac{8\pi}{5}\right\}$$

$$= 0.30 - j0.95 + 2\{-0.80 - j0.58\} + 3\{-0.80 + j0.58\} + 4\{0.30 + j0.95\}$$

$$= 0.30 - j0.95 - 1.6 - j1.16 - 2.4 + j1.74 + 1.20 + j3.8$$

$$= -2.5 + j3.43$$

Put $k = 2$

$$X_1(2) = e^{-\frac{j4\pi}{5}} + 2e^{-\frac{j8\pi}{5}} + 3e^{-\frac{j12\pi}{5}} + 4e^{-\frac{j16\pi}{5}}$$

$$= \{-0.80 - j0.58\} + 2\{0.30 + j0.95\} + 3\{0.30 - j0.95\} + 4\{-0.80 + j0.58\}$$

$$= -0.80 - j0.58 + 0.60 + j3.9 + 0.90 - j2.85 - 3.2 + j2.32$$

$$= -0.9 + j2.79$$

Put $k = 3$

$$X_1(3) = e^{-\frac{j6\pi}{5}} + 2e^{-\frac{j12\pi}{5}} + 3e^{-\frac{j18\pi}{5}} + 4e^{-\frac{j24\pi}{5}}$$

$$= -0.80 + j0.58 + 2\{0.30 - j0.95\} + 3\{0.30 + j0.95\} + 4\{-0.80 - j0.58\}$$

$$= -0.80 + j0.58 + 0.60 - j3.9 + 0.90 + j2.85 - 3.20 - j2.32$$

$$= -0.9 - j2.79$$

Put $k = 4$

$$X_1(4) = e^{-\frac{j8\pi}{5}} + 2e^{-\frac{j16\pi}{5}} + 3e^{-\frac{j24\pi}{5}} + 4e^{-\frac{j32\pi}{5}}$$

$$= 0.30 + j0.95 + 2\{-0.80 + j0.58\} + 3\{-0.80 - j0.58\} + 4\{0.30 - j0.95\}$$

$$= 0.30 + j0.95 - 1.6 + j1.16 - 2.4 - j1.74 + 1.20 - j3.8$$

$$= -2.5 - j3.43$$

Hence $X_1(k) = \{10, -2.5 + j3.43, -0.9 + j2.79, -0.9 - j2.79, -2.5 - j3.43\}$

Similarly

$$X_2(k) = \sum_{n=0}^{4} x(n) e^{-\frac{j2\pi kn}{5}}$$

$$= 0 + 1e^{-\frac{j2\pi k}{5}} + 0 + 0 + 0 = e^{-\frac{j2\pi k}{5}} \text{; K} = 0, 1, \ldots 4$$

Put $k = 0$

$$X_2(0) = e^0 = 1$$

Put $k = 1$

$$X_2(1) = e^{-\frac{j2\pi}{5}} = 0.30 - j0.95$$

Put $k = 2$

$$X_2(2) = e^{-\frac{j4\pi}{5}} = -0.80 - j0.58$$

Put $k = 3$

$$X_2(3) = e^{-\frac{j6\pi}{5}} = -0.80 + j0.58$$

Put $k = 4$

$$X_2(4) = e^{-\frac{j8\pi}{5}} = 0.30 + j0.95$$

$$X_2(k) = \{1,\ 0.30 - j0.95,\ -0.80 - j0.58,\ -0.80 + j0.58,\ 0.30 + j0.95\}$$

(*b*) $S(k) = \sum_{n=0}^{4} x(n) e^{-\frac{j2\pi kn}{5}}$; $k = 0, 1, ..., 4 = 1$

Hence, $S(k) = \{1, 1, 1, 1, 1\}$

Q. 3 Compute the 8-point DFT of the following sequence using DIF FFT algorithm. (8)

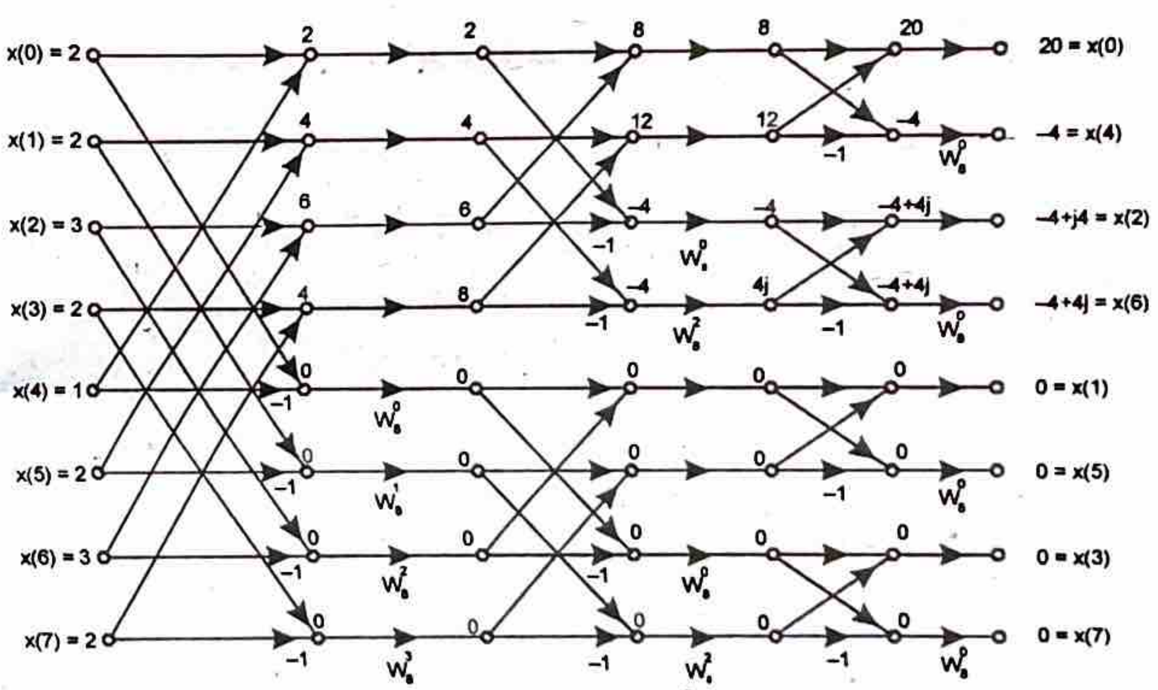
$$x(n) = \{1,2,3,2,1,2,3,2\}$$

Ans. Given that

$x(n) = \{1, 2, 3, 2, 1, 2, 3, 2\}$

We know that

$W_N^K = e^{-j\left(\frac{2\pi}{N}\right)K}$; Hence $W_8^0 = 1$; $W_8^1 = 0.707 - j0.707$, $W_8^2 = -j$; $W_8^3 = -0.707 - j0.707$



Hence, $X(K) = \{20, 0, -4 + j4, 0, -4, 0, -4 + j4, 0\}$

SECTION-II

Q. 4. Design a linear phase FIR LPF with the following desired frequency response: (8)

$$H_d(e^{j\omega}) = \begin{cases} e^{-3j\omega}, & 0 \le |\omega| \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Use a Hamming window.

Ans. We have given that the desired frequency response is

$$H_d(e^{jw}) = \begin{cases} e^{-j\omega} & 0 \le |\omega| \le \dfrac{\pi}{4} \\ 0, & \dfrac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Therefore,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\pi/4} e^{-j2\omega}e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\pi/4} e^{j\omega(n-2)}d\omega$$

$$= \frac{1}{\pi(n-2)}\left[\frac{e^{j(n-2)\frac{\pi}{4}} - e^{-j(n-2)\frac{\pi}{4}}}{2j}\right] = \frac{1}{\pi(n-2)}\sin\frac{\pi}{4}(n-2)\,,\ n \ne 2$$

For $n = 2$, the filter coefficient can be obtained by L' Hospital rule to the above expressions. Thus,

$$h_d(2) = \frac{1}{4}$$

The other filter coefficients are given by

$$h_d(0) = \frac{1}{2\pi} = h_d(4) \text{ and } h_d(1) = \frac{1}{\sqrt{2}\pi} = h_d(3)$$

The hamming window function is,

$$\omega(n) = \begin{cases} 0.54 - 0.46\cos\dfrac{2\pi n}{M-1}; & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

Hence, $\omega(0) = 0.08$, $\omega(1) = 0.31$, $\omega(2) = 0.77$, $\omega(3) = 1$, $\omega(4) = 0.77$

The filter coefficents of the resultant fillter are then

$$h(n) = h_d(n).\omega(n)$$

Therefore, $h(0) = h_d(0).\omega(0) = \dfrac{1}{2\pi} \times 0.08$

$$h(1) = h_d(1).\omega(1) = \frac{1}{\sqrt{2}\pi} \times 0.31$$

$$h(2) = h_d(2).\omega(2) = \frac{1}{4} \times 0.77$$

$$h(3) = h_d(3).\omega(3) = \frac{1}{\sqrt{2}\pi} \times 1$$

$$h(4) = h_d(4).\omega(4) = \frac{1}{2\pi} \times 0.77$$

Q. 5. Design an IIR low-pass Butterworth filter using bilinear transformation for the following specifications: (8)

Passband: $0.8 \leq |H(e^{j\omega})| \leq 1$ $\quad |\omega| \leq 0.2\pi$

Stop band: $|H(e^{j\omega})| \leq 0.2$ $\quad 0.6\pi \leq |\omega| \leq \pi$

Assume T = 1 sec

Ans. Refer Q.no. 5(*b*) End Term Examination 2018.

END TERM EXAMINATION [MAY-JUNE 2018]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC-306]

Time : 3 hrs. **M.M. : 75**

Note: Attempt any five questions including Q. No. 1 which is compulsory. Assume missing data if any.

Q. 1. (a) Explain DFT. Give matrix relations for computing DFT and IDFT. **(5)**

Ans. As the name implies, the Discrete Fourier Transform (DFT) is purely discrete: discrete-time data sets are converted into a discrete-frequency representation. This is in contrast to the DTFT that uses discrete time, but converts to continuous frequency. Since the resulting frequency information is discrete in nature, it is very common for computers to use DFT(Discrete fourier Transform) calculations when frequency information is needed.

The DFT can be calculated much more easily using a matrix equation:

$$X = D_N X$$

The little "X" is the time domain sequence arranged as a N_{X1} vertical vector. The big "X" is the resulting frequency information, that will be arrange as a vertical vector (as per the rules of matrix multiplication). The "D_N" term is a matrix defined as such:

$$D_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

In a similar way, the IDFT can be calculated using matrices as follows:

$$x = D_N^{-1} X.$$

And we will define D_N^{-1} as the following:

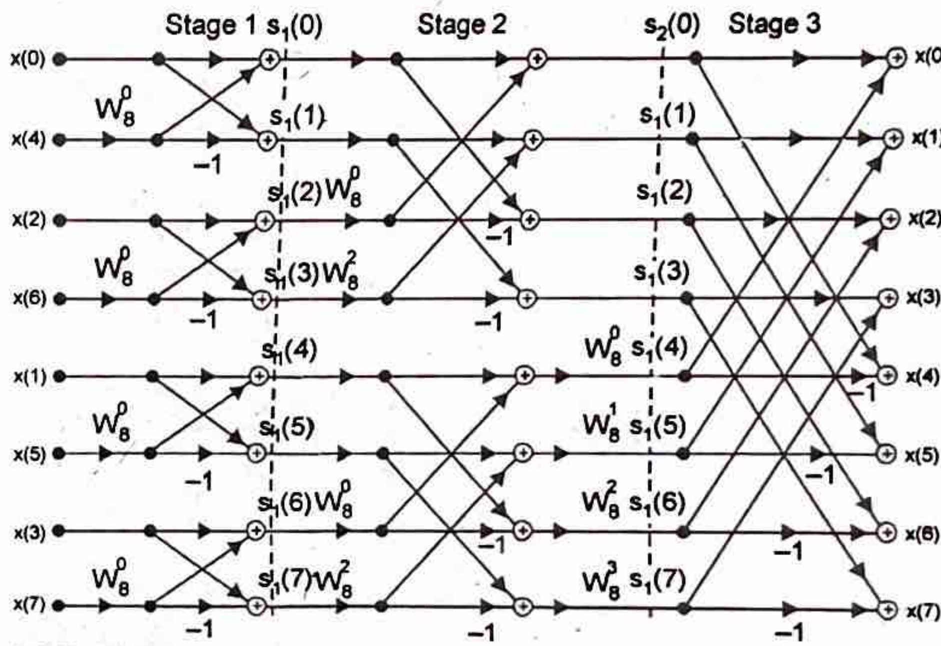
$$D_N^{-1} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{-1} & W_N^{-2} & \cdots & W_N^{-(N-1)} \\ 1 & W_N^{-2} & W_N^{-4} & \cdots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \cdots & W_N^{-(N-1)(N-1)} \end{bmatrix}.$$

Q.1. (b) Why FFT is so important? What are its advantages? Draw complete flow diagram of DIT FFT algorithm, taking sequence length $N = 8$. **(5)**

Ans. The direct evaluation of DFT using the formula $X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$ requires N^2 complex multiplications and N(N–1) complex additions. Thus for reasonably large values of N (in order of 1000) direct evaluation of the DFT requires an inordinate amount of computation. By using FFT algorithms the number of computations can be reduced. For example, for an N-point DFT, the number of complex multiplications required using FFT is $\frac{N}{2}\log_2 N$. If $N = 16$, the number of complex multiplications required for direct evaluation of DFT is 256, whereas by using DFT only 32 multiplications are required.

FFT reduces the computation time required to compute discrete Fourier transform.

The total signal flow graph is obtained by interconnecting all stages of decimation. In this case, it is obtained by interconnecting first and second stage of decimation. But, the starting block is the block used to compute 2-point DFT(butterfly structure). The total signal flow graph has been shown in figure below.



Q.1. (c) Derive the frequency response of a linear phase FIR filter with anti-symmetric impulse response and filter length N even. (5)

Ans. We know that,

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{N-2}{2}} h(N-1-n)e^{-j\omega(N-1-n)}$$

We have $h(n) = -h(N-1-n)$, therefore

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega n} - \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega(N-1/2)}\left[\sum_{n=0}^{\frac{N-2}{2}} h(n)e^{j\omega[(N-1)/2-n]} - \sum_{n=0}^{\frac{N-2}{2}} h(n)e^{-j\omega[(N-1)/2-n]}\right]$$

$$= e^{-j\omega(N-1)/2}e^{j\pi/2}\left[\sum_{n=1}^{N/2} 2h\left(\frac{N}{2}-n\right)\sin\omega(n-1/2)\right]$$

$$= e^{-j\omega(N-1)/2}e^{j\pi/2}\left[\sum_{n=1}^{N/2} d(n)\sin\omega\left(n-\frac{1}{2}\right)\right]$$

where $$d(n) = 2h\left(\frac{N}{2}-n\right)$$

$$H(e^{j\omega}) = e^{-j\omega(N-1)/2}e^{j\pi/2}\overline{H}\left(e^{j\omega}\right)$$

$$H(e^{j\omega}) = \overline{H}\left(e^{j\omega}\right)e^{j\theta(\omega)} = \overline{H}\left(e^{j\omega}\right)e^{j\left(\frac{\pi}{2}-\alpha\omega\right)}.$$

Q.1. (*d*) Define the Chebyshev polynomial $C_N(x)$. Obtain the recursive relation to build up higher order Chebyshev polynomials. (5)

Ans. There are two types of Chebyshev filters. Type I Chebyshev filters are all-pole filters that exhibit equiripple behaviour in the passband and a monotonic characteristics in the stopband. On the other hand, the family of type II Chebyshev filter contains both poles and zeros and exhibits a monotonic behaviour in the passband and an equiripple behaviour in the stopband as shown in figure.

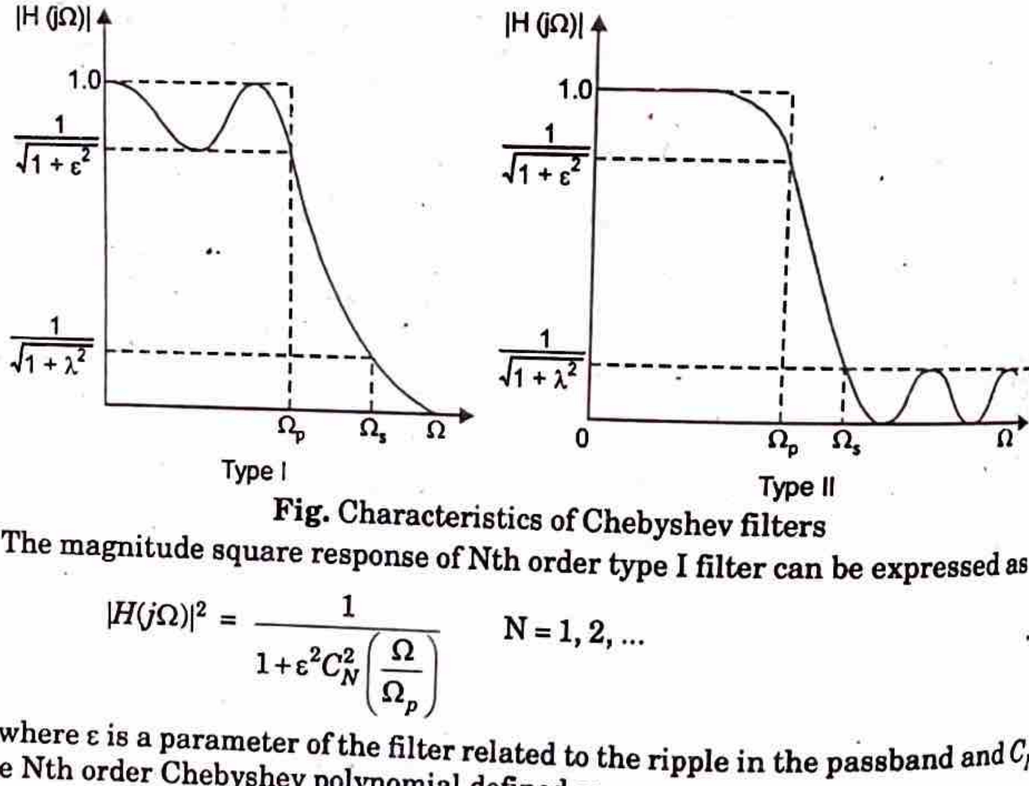


Fig. Characteristics of Chebyshev filters

The magnitude square response of Nth order type I filter can be expressed as

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2\left(\dfrac{\Omega}{\Omega_p}\right)} \qquad N = 1, 2, \ldots \qquad \ldots(1)$$

where ε is a parameter of the filter related to the ripple in the passband and $C_N(x)$ is the Nth order Chebyshev polynomial defined as

$$C_N(x) = \cos(N\cos^{-1}x),\ |x| \le 1 \qquad \text{(Passband)} \qquad \ldots(2)$$

and $$C_N(x) = \cosh(N\cosh^{-1}x),\ |x| > 1 \qquad \text{(Stopband)} \qquad \ldots(3)$$

The Chebyshev polynomial is defined by the recursive formula

$$C_N(x) = 2xC_{N-1}(x) - C_{N-2}(x),\ N > 1 \qquad \ldots(4)$$

where $C_0(x) = 1$ and $C_1(x) = x$.

Q.1. (*e*) What are the advantages of poly-phase decomposition? (5)

Ans. Advantages of polyphase decomposition are

(*i*) Polyphase is a way of doing sampling rate conversion that leads to very efficient implementations.

(*ii*) It leads to very general viewpoints that are useful in building filter banks.

(*iii*) When hardware resources are limited, the polyphase approach replaces a L tap filter with N filter sets of L/N taps.

(*iv*) A significant reduction in the computational complexity is achieved.

(*v*) If $H(z)$ is preceded by a factor of M upsampler, we can rewrite the system function in terms of its polyphase components, $P_K(Z^M)$.

Q.2. (*a*) Prove the following property of DFT when $X(k)$ is the N-point DFT of sequence $x(n)$. (6.5)

(*i*) $X(k)$ is real and even when $x(n)$ is real and even.

(*ii*) $X(k)$ is imaginary and odd when $x(n)$ is real and odd.

Ans. We know that, the DFT of $x(n)$ is;

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$X(k) = X_R(k) + jX_I(k)$$

$$x(n) = x_R(n) + jx_I(n)$$

$$X_R(k) + jX_I(k) = \sum_{n=0}^{N-1}\left[x_R(n) + jx_I(n)\right]\left[\cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}\right]$$

$$X_R(k) = \sum_{n=0}^{N-1}\left[x_R(n)\cos\frac{2\pi kn}{N} + x_I(n)\sin\frac{2\pi kn}{N}\right]$$

$$X_I(k) = \sum_{n=0}^{N-1}\left[x_I(n)\cos\frac{2\pi kn}{N} - x_R(n)\sin\frac{2\pi kn}{N}\right]$$

(*i*) If $x(n)$ is real and even

$$x_I(n) = 0 \text{ and } x(n) = x(N-n) \text{ for } 0 \le n \le N-1$$

Substituting $x_I(n) = 0$ and $x(n) = x(N-n)$ we get

$$X_R(k) = \sum_{n=0}^{N-1} x_R(n)\cos\frac{2\pi kn}{N}$$

$$X_I(k) = 0$$

From which we can find $X(k)$ is real and even.

(*ii*) If $x(n)$ is real and odd

$$x_I(n) = 0 \text{ and } x(n) = -x(N-n) \text{ for } 0 \le n \le N-1$$

Substituting $x_I(0) = 0$ and $x(n) = -x(N-n)$ we get

$$X_R(k) = 0$$

$$X_I(k) = -\sum_{n=0}^{N-1} X_R(n)\sin\frac{2\pi kn}{N}$$

Hence $X(k)$ is imaginary and odd.

Q.2. (*b*) Prove the circular convolution property of DFT. (6)

Ans. Refer to Q.1. (b) of First Term Examination 2018.

Q. 3 (*a*) If $X(k)$ is an N-point DFT of $x(n)$ and if $x(n) = -x(N-1-n)$. Then show that $X(0) = 0$. (3)

Ans. Since $X(k)$ is an N-point DFT of $x(n)$.

$$X(k) = \sum_{n=0}^{\frac{N-2}{2}} x(n)e^{-j\pi kn/N} + \sum_{n=0}^{\frac{N-2}{2}} x(N-1-n)e^{-j2\pi k(N-1-n)/N}$$

Given that, $x(n) = -x(N-1-n)$

$$X(k) = \sum_{n=0}^{\frac{N-2}{2}} x(n)e^{-j\pi kn/N} - \sum_{n=0}^{\frac{N-2}{2}} x(n)e^{-j2\pi k(N-1-n)/N}$$

For $k = 0$

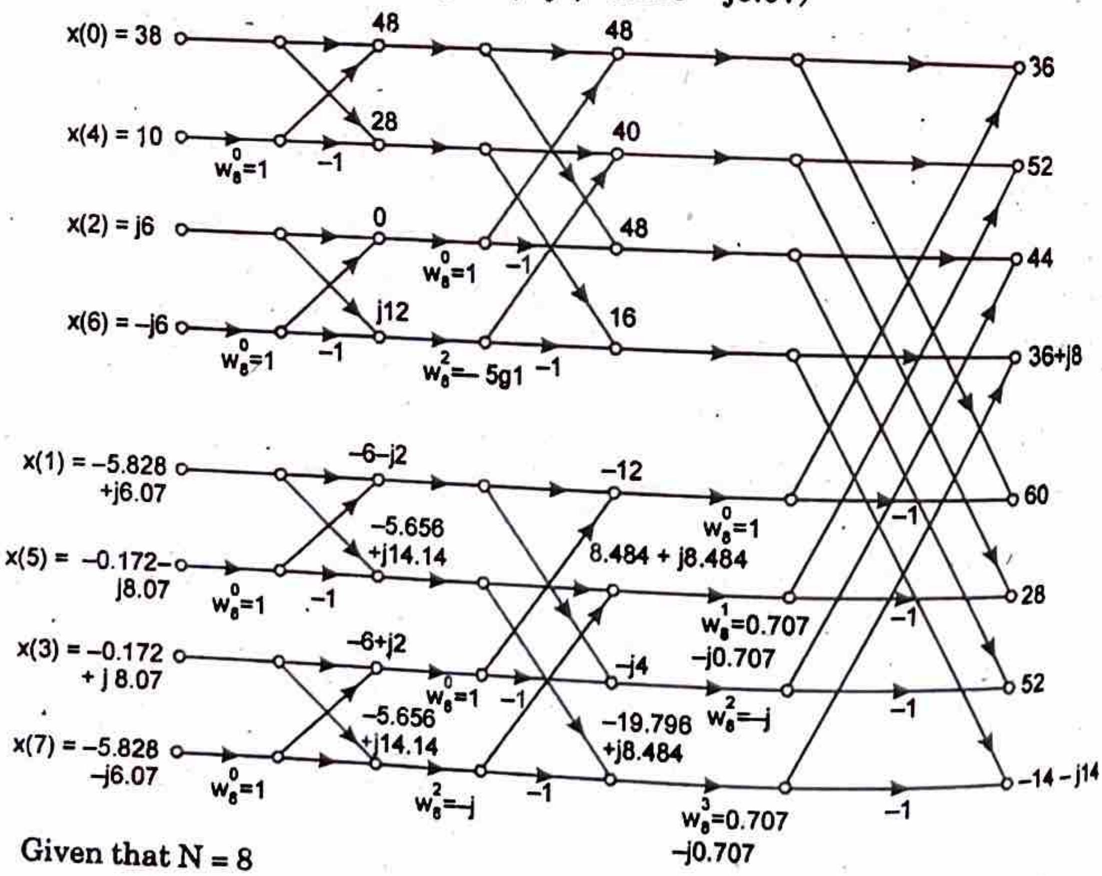
$$X(0) = \sum_{n=0}^{\frac{N-2}{2}} x(n) - \sum_{n=0}^{\frac{N-2}{2}} x(n) = 0.$$

Q.3. (b) Calculate the IDFT using Decimation-in-Frequency FFT structure for the given coefficient. **(9.5)**

$X(k)$ =(38, –5.828 + j6.07, j6, –0.172 + j8.07, 10, –0.172 – j8.07 – j6, –5.828 – j6.07)

Ans. Given that

$$X(k) = (38, -5.828 + j6.07, j6, -0.172 + j\,8.07, 10, -0.172 - j8.07, -j6, -5.828 - j6.07)$$



Given that N = 8

$$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$$

$$W_8^0 = 1$$

$$W_8^1 = 0.707 - j0.707$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j0.707$$

Q.4. (a) A requirement exists for a low pass FIR filter satisfying the following specifications: **(6.5)**

Pass band	**0–5 KHz**
Sampling Frequency	**18 KHz**
Filter Length	**9**

Obtain the filter coefficients using Frequency Sampling method.

Ans. We have given that

$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_{c_1}}{F} = \frac{2\pi(0)}{F} = 0$$

$$\omega_{c_2} = 2\pi f_{c_2} T = \frac{2\pi f_{c_2}}{F} = \frac{2\pi(5000)}{18000} = \frac{5\pi}{9}$$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{9}K} ; k = 0, 1, 2, ... 8$$

$$|H(k)| = 0 \text{ for } k = 0, 4$$

$$= 1 \text{ for } k = 1, 2, 3$$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \quad \text{for} \quad 0 \le k \le \frac{N-1}{2}$$

$$= \frac{-8}{9}\pi k \quad \text{for} \quad 0 \le k \le 4$$

$$H(k) = 0 \quad \text{for} \quad k = 0, 4$$

$$= e^{-j\frac{8\pi k}{9}} \quad \text{for} \quad k = 1, 2, 3$$

The filter coefficients are given by

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \text{Re}(H(k)e^{j2\pi kn/N})\right]$$

$$= \frac{1}{9}\left[2\sum_{k=1}^{4} \text{Re}(e^{-j8\pi k/9}e^{j2\pi kn/9})\right]$$

$$= \frac{1}{9}\left[2\sum_{k=1}^{2} \cos\frac{2\pi k}{9}(4-x)\right]$$

$$= \frac{2}{9}\left[\cos\frac{2\pi}{9}(4-x) + \cos\frac{4\pi}{9}(4-x)\right]$$

Hence

$$h(0) = -0.03856$$
$$h(1) = -0.2222$$
$$h(2) = -0.1702$$
$$h(3) = 0.2088$$
$$h(4) = 0.4444$$
$$h(5) = 0.2088$$
$$h(6) = -0.1702$$
$$h(7) = -0.1702$$
$$h(8) = -0.03856$$

Q.4.(*b*) Write the various window functions used for FIR filter design. Compare their important performance parameter. (6)

Ans. Refer to Q.5. (b) of End Term Examination 2016.

Q. 5 (*a*) Derive the relation for Impulse – Invariant Technique connecting *s*-domain and the *z*-domain. Show the mapping of points in s-domain to z-domain. (6)

Ans. Let
$$H_a(s) = \sum_{i=1}^{N} \frac{A_i}{s-p_i} \quad ...(1)$$

where p_1 are the poles of $H_a(s)$ and A_i are the partial fraction coefficients, if

$$H_a(s) = \frac{N(s)}{(s-p_1)(s-p_2)...(s-p_N)} \quad ...(2)$$

Taking inverse Laplace transform of equation (1), we obtain

$$h_a(t) = \sum_{i=1}^{N} A_i e^{p_i t} \quad ...(3)$$

$$h_a(nT) = h(n) = \sum_{i=1}^{N} A_i e^{p_i nT} \qquad \text{[replacing } t \text{ by } nT \text{ in equation 3]}$$

Taking *z*-transform on both the sides of equation (3), we get

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} \qquad \text{(by definition)}$$

$$= \sum_{n=0}^{\infty}\left(\sum_{i=1}^{N} A_i e^{p_i nT}\right) z^{-n} \text{ for causal } h(n)$$

$$= \sum_{i=1}^{N}\sum_{n=0}^{\infty} A_i e^{p_i nT} z^{-n}$$

$$H(z) = \sum_{i=1}^{N} \frac{A_i}{1-e^{p_i T} z^{-1}} \quad ...(4)$$

which is the required digital filter $H(z)$ obtained from $H_a(s)$.

Q.5. (b) Using Bilinear Transformation, design a Butterworth LPF to meet the following specifications: **(6.5)**

$0.8 \le |H(e^{j\omega})| \le 1 \qquad 0 \le |\omega| \le 0.2\pi$

$|H(e^{j\omega})| \le 0.2 \qquad 0.6\pi \le |\omega| \le \pi$

Ans. Given that

$$0.8 \le |H(e^{ju\omega})| \le 1 \quad , \quad 0.6\pi \le |\omega| \le \pi$$

$$|H(e^{jw})| \le 0.2 \quad , \quad 0 \le |\omega| \le 0.2\pi$$

Given that $\omega_1 = 0.4\,\pi$ and $\omega_2 = 0.8\,\pi$; $\delta_1 = 0.8$, $\delta_2 = 0.2$

Assume $T = 1$ sec.

Step I: Analog filters edge frequencies

$$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2} = 2\tan\frac{0.4\pi}{2} = 1.453$$

$$\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2} = 6.155$$

Step II. Order of the filter

$$N \ge \frac{1}{2}\frac{\log\left\{\left[\left(\frac{1}{\delta_2^2}-1\right)\Big/\left(\frac{1}{\delta_1^2}-1\right)\right]\right\}}{\log\left(\frac{\Omega_1}{\Omega_2}\right)}$$

$$N \ge 1.29 \text{ i.e. } \boxed{N=2}$$

Step III: Determination of – 3dB cut-off frequency

$$\Omega_c = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{1/2N}} = \frac{1.453}{[0.5625]^{1/4}} = 1.677$$

Step IV: Determination of $H_a(s)$

$$H(s) = \prod_{k=1}^{N/2}\frac{B_k\Omega_c^2}{S^2+b_k\Omega_C S+C_K\Omega_c^2} \qquad ...(1)$$

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right] \text{ and } C_K = 1 \qquad ...(2)$$

$$A = \prod_{K=1}^{N/2} B_K \qquad ...(3)$$

From (1)

$$H(s) = \prod_{k=1}^{2/2}\frac{B_k\Omega_c^2}{S^2+b_k\Omega_C S+C_K\Omega_c^2}$$

$$= \frac{B_1\Omega_c^2}{S^2+b_1\Omega_C S+C_1\Omega_c^2} \qquad ...(4)$$

From (2)

$$C_1 = 1$$

$$b_1 = 2\sin\frac{\pi}{4} = 2 \times 0.707 = 1.414$$

From (3)

$$A = \prod_{k=1}^{2/2} B_K = B_1 \therefore B_1 = 1$$

From eqn (4)

$$H(s) = \frac{\Omega_c^2}{S^2 + 1.414\Omega_C S + \Omega_c^2}$$

$$\boxed{H(s) = \frac{2.812}{S^2 + 2.364S + 2.812}}$$

Step V: Determination of H(z)

$$H(z) = H(s)\Big|S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{2.812}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 2.364 \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 2.812}$$

$$= \frac{2.812(1+z^{-1})^2}{4(1-z^{-1})^2 + 4.728(1-z^{-2}) + 2.812(1+z^{-1})^2}$$

Step VI: Convert LPF into HPF

We have,

$$Z^{-1} \rightarrow \frac{-Z^{-1}+a}{1+aZ^{-1}}$$

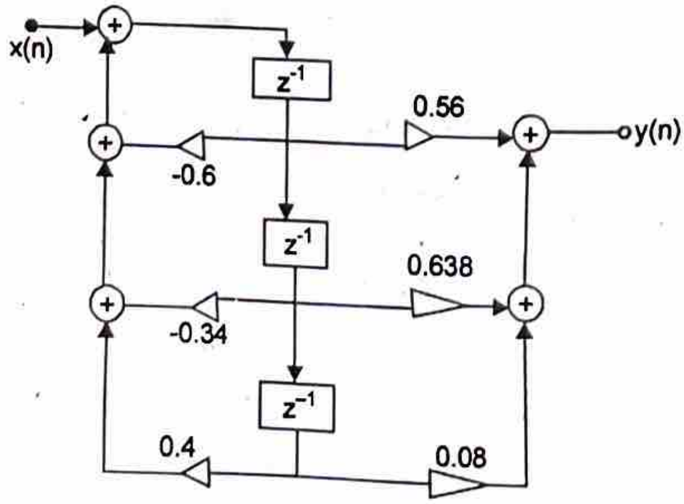
Where,

$$a = -\frac{\cos[(\omega_c - \omega_c^*)/2]}{\cos[(\omega_c + \omega_c^*)/2]}$$

Q.6. (*a*) Draw the Direct form – II and its transposed structure for the given transfer function (6

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

Ans. $H(z) = \dfrac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2} = \dfrac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$



Q.6. (b) Draw and explain the Ladder-Lattice realization of the following transfer function. **(6.5)**

$$H(z) = \frac{0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}}{1 - 0.0010z^{-1} + 0.6546z^{-2} - 0.0775z^{-3}}$$

Ans. Given that
$$H(z) = \frac{0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}}{1 - 0.0010z^{-1} + 0.6546z^{-2} - 0.0775z^{-3}}$$

So,
$$b_M(z) = 0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}$$
$$A_N(z) = 1 - 0.0010z^{-1} + 0.6546z^{-2} - 0.0775z^{-3}$$
$$a_3(0) = 1;\ a_3(1) = -0.0010;\ a_3(2) = 0.6546;\ a_3(3) = -0.0775$$
$$k_3 = a_3(3) = -0.0775$$

We know that,

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)a_m(m-k)}{1 - a_m^2(m)}$$

For $m = 3$ and $k = 1$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1 - a_3^2(3)} = \frac{-0.0010 - (-0.0775)(0.6546)}{1 - (-0.775)^2}$$

$$= \frac{0.0497}{0.3993} = 0.1244.$$

For $m = 3$ and $k = 2$

$$k_2 = a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1 - a_3^2(2)} = \frac{0.6546 - (-0.0775)(-0.0010)}{1 - (0.6546)^2}$$

$$= \frac{0.6545}{0.5714} = 1.1454.$$

For $m = 2$ and $k = 1$

$$k_1 = a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1 - a_2^2(2)} = \frac{0.1244 - 1.1454 \times (0.1244)}{1 - (1.1454)^2}$$

$$= \frac{-0.0180}{-0.3119} = 0.0577.$$

Hence, for lattice structure

$$k_1 = 0.0577;\ k_2 = 1.1454;\ k_3 = -0.0775$$

For ladder structure

$$c_m = b_m - \sum_{i=m+1}^{M} c_i a_i(i-m);\ m = M, M-1, \dots 0$$

$$c_3 = b_3 = 1$$
$$c_2 = b_2 - c_3 a_3(1) = 2 - 0.5121(-0.0010) = 2$$

$$c_1 = b_1 - \sum_{i=2}^{3} c_i a_i(i-m);$$

$$= b_1 - [c_2 a_2(1) + c_3 a_3(2)]$$

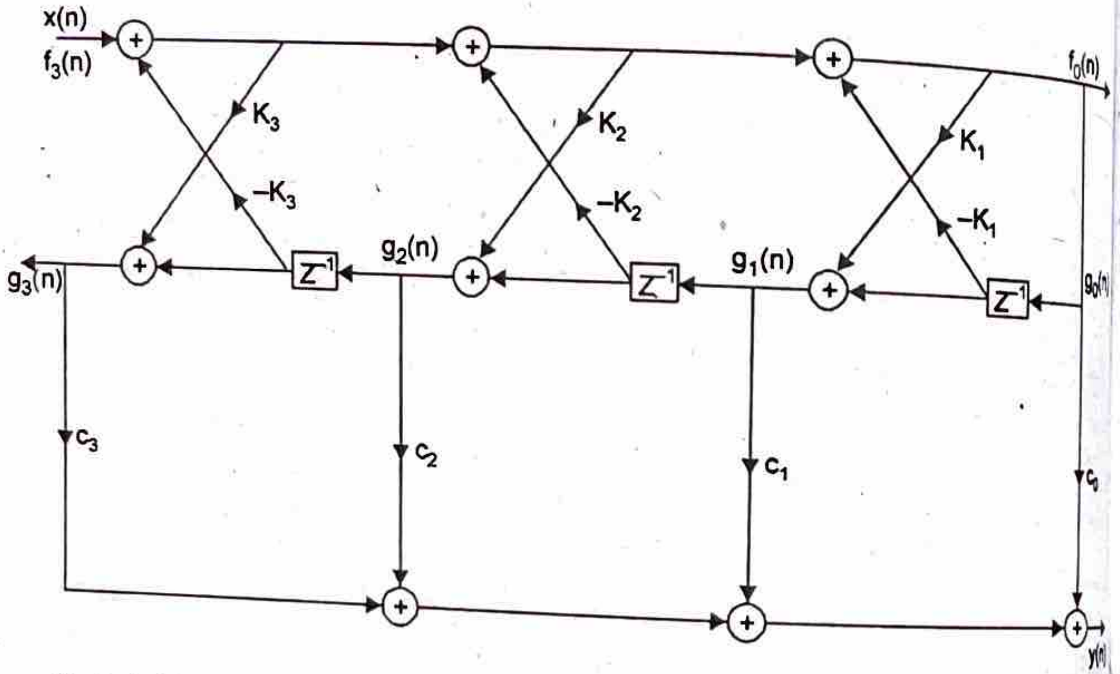
$$= 0.5121 - [2 \times 0.1244 + 0.2759 \times 0.6546] = 0.0827.$$

$$c_0 = b_0 - \sum_{i=1}^{3} c_i a_i(i-m)$$

$$= b_0 - [c_1 a_1(1) + c_2 a_2(2) + c_3 a_3(3)]$$

$$= 0.2759 - [0.0827 \times 0.0577 + 2 \times 1.1454 + 1 \times (-0.0775)]$$

$$= 0.2759 - [0.0105] = 0.2654$$



Q. 7. (a) Explain the Lavinson Durbin Algorithm in detail. (6)

Ans. Lavinson Durbin Algorithm: It is a computationally efficient algorithm for solving the prediction coefficients. This algorithm exploits the special symmetry in the autocorrelation matrix.

$$p = \begin{bmatrix} \gamma_{xx}(0) & \gamma_{xx}^*(1) & \cdots & \gamma_{xx}^*(p-1) \\ \gamma_{xx}(1) & \gamma_{xx}(0) & \cdots & \gamma_{xx}^*(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{xx}(p-1) & \gamma_{xx}(p-2) & \cdots & \gamma_{xx}(0) \end{bmatrix}$$

The key to the Lavinson-Durbin method of solution, that exploits the Toeplitz property of matrix, is to proceed recursively, beginning with a predictor of order $m = 1$ and then to increase the order recursively, using the lower-order solutions to obtain the solution to the next-higher order.

Thus the solution to the first-order predictor is

$$a_1(1) = \frac{-\gamma_{xx}(1)}{\gamma_{xx}(0)} \quad ...(1)$$

and the resulting MMSE is

$$E_1^f = \gamma_{xx}(0) + a_1(1)\gamma_{xx}(-1) = \gamma_{xx}(0)[1 - |a_1(1)|^2] \qquad ...(2)$$

$a_1(1) = K_1$, the first reflection coefficient in the lattice filter. The next step is to solve for the coefficients $\{a_2(1), a_2(2)\}$ of the second-order predictor and express the solution in terms of $a_1(1)$. The two equations are

$$\begin{aligned} a_2(1)\gamma_{xx}(0) + a_2(2)\gamma_{xx}^*(1) &= -\gamma_{xx}(1) \\ a_2(1)\gamma_{xx}(1) + a_2(2)\gamma_{xx}(0) &= -\gamma_{xx}(2) \end{aligned} \qquad ...(3)$$

By using equation (1), eliminate $\gamma_{xx}(1)$, we obtain the solution

$$a_2(2) = \frac{\gamma_{xx}(2) + a_1(1)\gamma_{xx}(1)}{\gamma_{xx}(0)[1 - |a_1(1)|^2]} = -\frac{\gamma_{xx}(2) + a_1(1)\gamma_{xx}(1)}{E_1^f}$$

$$a_2(1) = a_1(1) + a_2(2)\, a_1^*(1)$$

Thus we have obtained the coefficients of the second-order predictor. Again, we note that $a_2(2) = k_2$, the second reflection coefficient in the lattice filter.

Proceeding in this manner, we can express the coefficients of the m^{th} order predictor in terms of the coefficients of the (m-1)st order predictor.

Q.7. (b) Without factoring any polynomial, determine whether or not the following filter function is stable. (6.5)

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 0.58.z^{-1} + 1.638z^{-2} + 1.556z^{-2} + 0.4z^{-4}}$$

Ans. Given that

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}}$$

without factoring any polynomial, the stability can be checked by Schur Coehn stability test for an all pole IIR filter. *i.e.*

$$H(z) = \frac{1}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}}$$

We have

$$A_N(z) = 1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}$$

$$a_4(0) = 1;\ a_4(1) = 1.58;\ a_4(2) = 1.638;\ a_4(3) = 1.556;$$

$$a_4(4) = 0.4 = k_4$$

We have to find out

$$a_3(3) = k_3;\ a_2(2) = k_2;\ a_1(1) = k_1$$

We know that

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)a_m(m-k)}{1 - a_m^2(m)}$$

For $m = 4, m = 3$

$$a_3(3) = \frac{a_4(3) - a_4(4)a_4(1)}{1 - a_4^2(4)} = \frac{1.556 - (0.4)(1.58)}{1 - (0.4)^2}$$

$$a_3(3) = \frac{0.924}{0.84} = 1.1 = k_3$$

For $m = 3, k = 2$

$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1 - a_3^2(3)} \qquad ...(1)$$

To find the value of $a_3(2)$

For $m = 4, k = 2$

$$a_3(2) = \frac{a_4(2) - a_4(4)a_4(2)}{1 - a_4^2(4)} = \frac{1.638 - (0.4)(1.638)}{1 - (0.4)^2}$$

$$= \frac{0.9828}{0.84} = 1.17$$

From (1)

$$a_2(2) = \frac{1.17 - (1.1)(1.638)}{1 - (1.1)^2} = \frac{-0.6318}{-0.21} = 3$$

$$a_2(2) = 3 = k_2$$

For $m = 2, k = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1 - a_2^2(2)} \qquad ...(2)$$

To find the value of $a_2(1)$

For $m = 3, k = 1$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1 - a_3^2(3)} \qquad ...(3)$$

To find the value of $a_3(1)$

For $m = 4, k = 1$

$$a_3(1) = \frac{a_4(1) - a_4(4)a_4(3)}{1 - a_4^2(4)} = \frac{1.58 - (0.4)(1.556)}{1 - (0.4)^2}$$

$$a_3(1) = \frac{0.9576}{0.84} = 1.14$$

From (3)

$$a_2(1) = \frac{1.14 - (1.1)(1.17)}{1 - (1.1)^2} = \frac{-0.147}{-0.21} = 0.7$$

From eqn (2)

$$a_1(1) = \frac{0.7 - (3)(0.7)}{1 - (3)^2} = \frac{0.7 - 2.1}{-8} = 0.175$$

$$a_1(1) = k_1 = 0.175$$

Therefore the coefficients are

$$K_4 = 0.4$$
$$K_3 = 1.1$$
$$K_2 = 3$$
$$K_1 = 0.175$$

For stable system $|K_m| < 1$ for all $m = 1, 2, ...N$

But $K_2 = 3 > 1.$

Hence system is unstable

Q. 8 (a) Show that the Up - sampler and the Down - Sampler are Linear but Time - varying system. (6)

Ans. Consider a factor of L up-sampler defined by

$$y(n) = x\left(\frac{n}{L}\right) \quad ...(1)$$

The output due to delayed input is

$$y(n, k) = x\left(\frac{n}{L} - k\right)$$

The delayed output is

$$y(n-k) = x\left(\frac{n-k}{L}\right)$$

$$\therefore \quad y(n,k) \neq y(n-k)$$

Therefore the up-smapler is a time variant system. Similarly for down-sampler

$$y(n) = x(nM) \quad ...(2)$$
$$y(n,k) = x(nM-k)$$
$$y(n-k) = x(M(n-k))$$
$$y(n,k) \neq y(n-k)$$

Therefore the down-sampler is a time variant system.

We have from equation (1)

$$y(n) = x\left(\frac{n}{L}\right) \quad ...(3)$$

Le $y_1(n)$ and $y_2(n)$ be the output of $x_1(n)$ and $x_2(n)$ respecitvely then, from equation (3)

$$y_1(n) = x_1\left(\frac{n}{L}\right)$$

and $$y_2(n) = x_2\left(\frac{n}{L}\right) \quad ...(2)$$

Then $$ax_1\left(\frac{n}{L}\right) + bx_2\left(\frac{n}{L}\right) = ay_1(n) + by_2(n)$$

This represents up-sampler is linear system.

Similary, from equation (2)

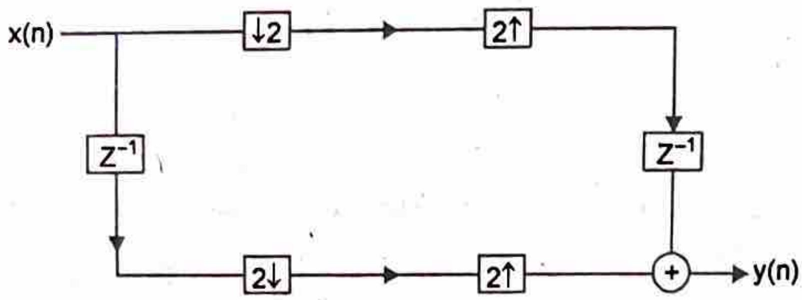
$$y(n) = x(nM)$$

and $$y_1(n) = x_1(nM)$$
$$y_2(n) = x_2(nM)$$

Then $$ax_1(nM)+bx_2(nM) = ay_1(n)+by_2(n)$$

This represents down-sampler is linear system.

Q.8. (*b*) Express the output $y(n)$ of Figure (*a*) as a function of input $x(n)$. By simplifying the expression derived show that $y(n) = x(n-1)$. **(6.5)**



Ans.

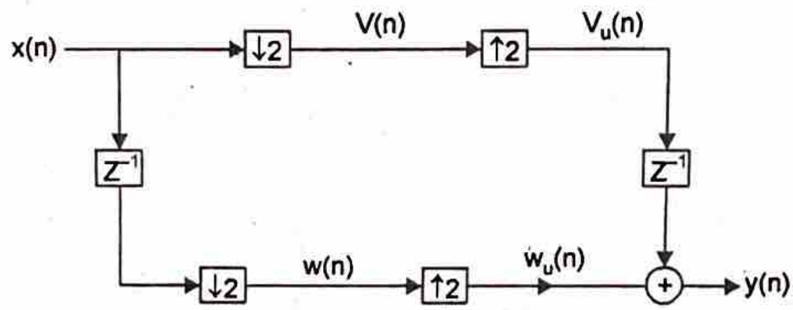


Figure (*a*)

We have $$V(z) = \frac{1}{2}X\left(z^{\frac{1}{2}}\right)+\frac{1}{2}X\left(-z^{\frac{1}{2}}\right)$$

$$W(z) = \frac{z^{-\frac{1}{2}}}{2}X\left(z^{\frac{1}{2}}\right)-\frac{z^{-\frac{1}{2}}}{2}X\left(-z^{\frac{1}{2}}\right)$$

The outputs of the up sampler are

$$V_u(z) = \frac{1}{2}X(z)+\frac{1}{2}X(-z)$$

$$W_u(z) = \frac{z^{-1}}{2}X(z)-\frac{z^{-1}}{2}X(-z)$$

$Y(z)$ is given by

$$Y(z) = z^{-1}V_u(z) + W_u(z)$$

$$= \frac{z^{-1}}{2}\{X(z)+X(-z)\}+\frac{z^{-1}}{2}\{X(z)-X(-z)\} = z^{-1}X(z)$$

Hence, $y(n) = x(n-1)$.

Q.9 (*a*) What do you understand by Zero input limit cycle oscillations. Explain with the help of an example. **(6.5)**

Ans. ***LIMIT CYCLE OSCILLATIONS***

Let the input sequence traverses several quantisation levels between two successive samples and so the samples of the round-off noise sequence were uncorrelated with each other and with the input signal. These assumptions are invalid in cases such as a constant or zero input to a digital filter. In such cases, the input signal remains constant during successive samples and does not traverse several quantisation levels. There are two types of limit cycles, namely, zero input limit cycle and overflow limit cycle. Zero input limit cycles are usually of lower amplitudes in comparison with overflow limit cycles. Let us consider a system with the difference equation

$$y(n) = 0.8y(n-1) + x(n) \quad ...(1)$$

with zero input, i.e., $x(n) = 0$ and initial condition $y(-1) = 10$. A comparison between the exact values of $y(n)$ as given by Eq. using unquantised arithmetic and the rounded values of $y(n)$ as obtained from quantised arithmetic are given in Table below.

Table 1. A Comparison of exact y (n) and rounded y (n)

n	*y (n)*-unquantised	*y (n)*-quantised
−1	10.0	10
0	8.0	8
1	6.4	6
2	5.12	5
3	4.096	4
4	3.2768	3
5	2.62144	2
6	2.0972	2
7	1.6772	1

From above Table it can be observed that for zero input, the unquantised output $y(n)$ decays exponentially to zero with increasing n. However, the rounded-off (quantised) output $y(n)$ gets stuck at a value 'of two and never decays further. Thus, the output is finite even when no input is applied. This is referred to as zero input limit cycle effect. It can also be seen that for any value of the input condition $|y(-1)| \leq 2$, the output ;

$y(n) = y(-1), n \geq 0$, when the input is zero. Thus, the deadband in this case is the interval [-2, 2].

Q.9. (b) Prove that $\sum_{n=0}^{\infty} x^2(n) = \frac{1}{2\pi j} \oint_c X(z)X(z^{-1})z^{-1}dz$ **(6)**

Ans. We know that, the z-transform of $x(n)$ is,

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \quad ...(1)$$

Taking the z-transform of $x^2(n)$, we have

$$z[x^2(n)] = \sum_{n=0}^{\infty} x(n)x(n)z^{-n} = \sum_{n=0}^{\infty} x^2(n)z^{-n} \quad ...(2)$$

The integral formula for the inverse z-transform is given by

$$z^{-1}[X(z)] = x(n) = \frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz$$

Using the above formula for one of the $x(n)$ is equation (2), we get

$$z[x^2(n)] = \sum_{n=0}^{\infty} x^2(n)z^{-n} = \sum_{n=0}^{\infty} \left[\frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz\right]x(n)z^{-n}$$

Interchanging the order of summation and integration,

$$\sum_{n=0}^{\infty} x^2(n)z^{-n} = \frac{1}{2\pi j}\oint_c X(z)\left[\sum_{n=0}^{\infty} x(n)z^{-1}\right]dz$$

i.e., $$\sum_{n=0}^{\infty} x^2(n) = \frac{1}{2\pi j}\oint_c X(z)\left[\sum_{n=0}^{\infty} x(n)(z^{-1})^{-n}\right]z^{-1}dz$$

Using the definition of z-transform, the above expression can be written as,

$$\sum_{n=0}^{\infty} x^2(n) = \frac{1}{2\pi j}\oint_c X(z)X(z)^{-1}z^{-1}dz$$

Thus, the expression is obtained, the above expression is a form of the Parseval's relation.

FIRST TERM EXAMINATION [FEB. 2019]
SIXTH SEMESTER [B.TECH]
DIGITAL SIGNAL PROCESSING [ETEC–306]

Time : 1.5 hrs. **M.M. : 30**

Note :- *Q 1. is compulsory. Attempt any two more questions from the rest.*

Q.1. (a) Explain the circular symmetry and complex conjugate properties of DFT. **(2.5)**

Ans. Circular symmetry property of DFT

A discrete time signal x(n) is symmetric if

$$x(n) = x(-n)$$

But if $x(n)$ is an N-point signal $\{x(n), n \in Z_N\}$, then the sample $x(-n)$ falls outside the range. The definition of circular symmetry evaluates the index modulo N. An N-point signal $x(n)$ is circularly symmetric if

$$x(n) = x(<-n>_N)$$

Complex Conjugate property of DFT

If DFT [x(n)] = X(k) then

$$\text{DFT}[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) e^{-j2\pi kn/N}$$

$$= \left[\sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}\right]^* = \left[\sum_{n=0}^{N-1} x(n) e^{-j2\pi n(N-k)/N}\right]^*$$

$$= X^*(N-k)$$

$$\text{DFT}[x^*(N-n)] = X^*(k)$$

Q. 1. (b) Why is FFT important? Explain. **(2.5)**

Ans. For large values of N, the direct evaluation of the DFT requires an inordinate amount of computation. By using FFT algorithms the number of computations can be reduced. For example, for an N-point DFT, the number of complex multiplications required using FFT is $\frac{N}{2}\log_2 N$. If N = 16, the number of complex multiplications required for direct evaluation of DFT is 256, whereas by using DFT only 32 multiplications are required.

Q. 1. (c) What are the necessary conditions fpr. the design of digital IIR filters from analog filters? **(2.5)**

Ans. The necessary conditions for the design of digital IIR filters from analog filters are–

(i) Map the desired digital filter specification into those for an equivalent analog filter.

(ii) Derive the analog transfer function for the analog prototype.

(iii) Transform the transfer function of the analog prototype into an equivalent digital filter transfer function.

Q. 1. (d) Determine the Direct Form-II realization for a third order IIR transfer function. **(2.5)**

$$\mathbf{H(z) = \frac{0.14z^2 + 0.813z + 0.02}{0.2z^3 + 0.1z^2 - 0.80z + 0.4}}$$

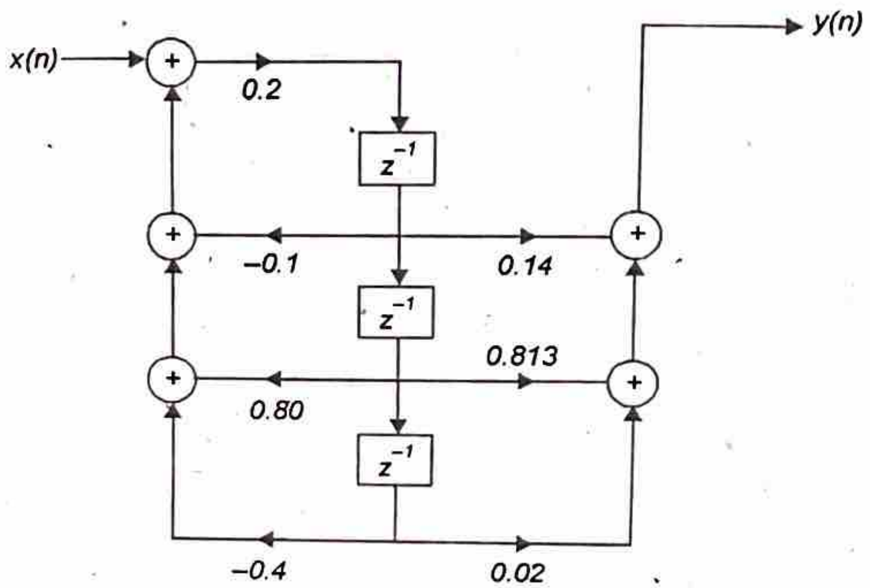
Ans. We have given that,

$$H(z) = \frac{0.14z^2 + 0.813z + 0.02}{0.2z^3 + 0.1z^2 - 0.80z + 0.4}$$

Divide numerator and denominator by z^3, we get

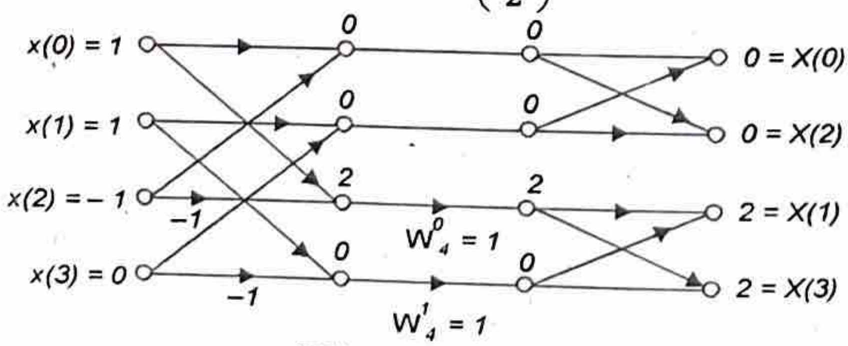
$$H(z) = \frac{0.14z^{-1} + 0.813z^{-2} + 0.02z^{-3}}{0.2 + 0.1z^{-1} - 0.80z^{-2} + 0.4z^{-3}}$$

Direct from-II realization is



Q. 2. (a) Compute 4 point DFT of the sequence $\mathbf{x(n) = \cos\left(\frac{n\pi}{2}\right)}$ using DIF FFT algorithm. **(5)**

Ans. Given that, $x(n) = \cos\left(\frac{n\pi}{2}\right) = [\{1, 0, -1, 0\}$



$\therefore \quad X(k) = \{0, 2, 0, 2\}$

Q. 2. (b) Perform the circular convolution of the following two sequences $\mathbf{x_1(n) = \{1, -1, 2, 3\}}$ and $\mathbf{x_2(n) = \{1, 2, 0, 4, 3\}}$ **(5)**

Ans. We have given thar, $x_1(n) = \{1, -1, 2, 3\}$ and $x_2(n) = \{1, 2, 0, 4, 3\}$

To perform circular convolution, both sequences should be equal in length. But here sequence of $x_1(n)$ is of length 4 and $x_2(n)$ is of length 5. Hence with adding zeros we can get equal length. Hence,

$$x_1(n) = \{1, -1, 2, 3, 0\} \text{ and } x_2(n) = \{1, 2, 0, 4, 3\}$$

By matrix multiplications method, we have

$$x_1(n) \times x_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 & -1 \\ -1 & 1 & 0 & 3 & 2 \\ 2 & -1 & 1 & 0 & 3 \\ 0 & 2 & -1 & 1 & 0 \\ 0 & 3 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+0+0+.8+(-3) \\ -1+2+0+12+6 \\ 2+(-2)+0+0+9 \\ 3+4+0+4+0 \rightarrow \\ 0+6+0+)-4)+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ 9 \\ 11 \\ 5 \end{bmatrix}$$

$$\therefore \quad x_1(n) \times x_2(n) = \{6, 19, 9, 11, 5\}$$

Q. 3. (a) Convert the analog filter with system function.

$$\mathbf{H_a(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}}$$

into a digital IIR filter by means of Impulse invariance method. Assume T = 0.5s **(5)**

Ans. Given that,
$$H_a(s) = \frac{s + 0.1}{(s + 0.1)^2 + 9}$$

Here $a = 0.1$, $b = 3$ and given that $T = 0.5$ s.

The system response of the digital filter can be obtained by

$$H(z) = \frac{1 - e^{-aT}(\cos bT)^{z-1}}{1 - 2a^{-aT}(\cos bT)z^{-1} + e^{-2aT}z^{-2}}$$

$$= \frac{1 - e^{-(0.1)(0.5)}\cos(3 \times 0.5)z^{-1}}{1 - 2e^{-(0.1)(0.5)}\cos(3 \times 0.5)z^{-1} + e^{-2\times 0.1\times 0.5}z - 2}$$

$$= \frac{1 - e^{0.05}\cos(1.5)z^{-1}}{1 - 2e^{-0.05}\cos(1.5)z^{-1} + e^{-0.1}z - 2} = \frac{1 - 0.95z^{-1}}{1 - 1.9z^{-1} + 0.90z^{-2}}$$

Q. 3. (b) Explain overlap-add method with the help of an example. **(5)**

Ans. Overlap-add method: In this method the size of the input data block $x_i(n)$ is L. To each data block, we append M–1 zeros and perform N-point (N – L + M – 1) circular convolution of $x_i(n)$ with $h(n)$. Since each data block is terminated with M-1 zeros, the last M–1 points from each output block must be overlapped and addded to first M–1 points of the succeeding block. Hence, this method is called overlap add method.

$$x_1(n) = \{1, 2, 3, 4\}$$

The sequence is divided into blocks of data size having length L and M–1 zeros are appended to it to make the data size of L + M – 1

$$L + M - 1 = 4 + 4 - 1 = 7$$

Thus
$$y(n) = x_1(n) \, Ⓝ \, x_2(n)$$

$$y(n) = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+8+0+2 \\ 2+2+0+3 \\ 3+4+0+4 \\ 4+6+0+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 7 \\ 11 \\ 11 \end{bmatrix}$$

Q. 4. The. desired response of a low pass filter is

$$\mathbf{H_d(e^{jw}) = \begin{cases} e^{-j3w}, & -3\pi/4 \le w \le 3\pi/4 \\ 0, & 3\pi/4 < |w| \le \pi \end{cases}}$$

Determine $\mathbf{H(e^{jw})}$ for M = 7 using a Hamming window. (10)

Ans. The filter coefficients are given by

$$H_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3\omega n}e^{j\omega n}d\omega$$

$$H_d(n) = \frac{\sin 3\pi(n-3)/4}{\pi(n-3)}, n \neq 3$$

and $$H_d(3) = \frac{3}{4}$$

The filter coefficients are,

$$H_d(0) = 0.0750, \quad H_d(1) = 0.1592$$
$$H_d(2) = 0.2251, \quad H_d(3) = 0.75$$
$$H_d(4) = 0.2251, \quad H_d(5) = -0.1592$$
$$H_d(6) = 0.0750$$

The Hamming window function is

$$\omega(n) = \begin{cases} 0.54 - 0.46\cos\dfrac{2\pi n}{M-1}, & 0 \le 0 \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, with $M = 7$

$$\omega(0) = 0.08, \quad \omega(1) = 0.31, \quad \omega(2) = 0.77, \quad \omega(3) = 1$$
$$\omega(4) = 0.77, \quad \omega(5) = 0.31, \quad \omega(6) = 0.08$$

The filter coefficients of the resultant filter are then

$$H(n) = H_d(n)\,\omega(n)$$

where $n = 0, 1, 2, 3, 4, 5, 6$

Therefore

$$H(0) = 0.006, H(1) = -0.044, \quad H(2) = 0{,}1733$$
$$H(3) = 0.75$$
$$H(4) = 0.1733, H(5) = -0.0494 \text{ and } H(6) = 0.006$$

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0}^{6} H(n)e^{-j\omega n}$$

$$= e^{-j3\omega}[H(3) + 2H(0)\cos 3\omega + 2H(1)\cos 2\omega + 2H(2)\cos\omega)]$$
$$= e^{-j3\omega}[0.75 + 0.3466\cos\omega - 0.0988\cos 2\omega + 0.012\cos\omega)]$$

END TERM EXAMINATION [MAY. 2019]

Time : 3 hrs. **M.M. : 75**

Note :- *Attempt five questions in all including Q.No. 1. which is compulsory. Assume missing data if any.*

Q.1. (a) Explain Warping effect in IIR filter. **(5)**

Ans. The relation between the analog and digital frequencies in bilinear transformation is given by

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

For smaller values of ω there exist linear relationship between ω and Ω. But for large values of ω the relationship is non-linear. This non-linearity introduces distortion in the frequency axis. This is known as warping effect. This effect compresses the magnitude and phase response at high frequencies.

The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable prescaling, or prewarping the critical frequencies by using the formula.

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

Q.1. (b) Why FFT is so important? What are it advantages? Draw the complete flow diagram of DIT FFT algorithm, taking sequence .length N = 8. **(5)**

Ans. Refer Q. 1. (b) of End Term Exam 2018. (Page No. 8-2018)

Q.1. (c) Derive the frequency response of a linear phase FIR filter with anti-symmetric impulse response and filter length N even. **(5)**

Ans. Refer Q. 1. (c) of End Term Exam 2018. (Page No. 9 and 10-2018)

Q.1. (d) Define the Chebyshev polynomial $C_N(x)$, Obtain the recursive relation to build up higher order Chebyshev polynomials. **(5)**

Ans. Refer Q. 1. (d) of End Term Exam 2018. (Page No. 10-2018)

Q.1. (e) What are the advantages of poly-phase decomposition?

Ans. Refer Q. 1. (e) of End Term Exam 2018. (Page No. 10 and 11-2018)

Q. 2. (a) The first five points of the 8 point DFT of a real valued sequence are {0.25,0.125-j0.03018,0,0.125-j0.0518,0}. Determine the remaining three values. **(6.5)**

Ans. We have given that

$$X(k) = \{0.25,\ 0.125-j0.03018,\ 0, 0.125-j0.0518, 0\}.$$

Given N = 0. The first five samples of $X(k)$ are given as

$$X(0) = 0.25;\ X(1) = 0.125 - j.03018;\ X(2) = 0.$$

$$X(3) = 0.125 - j0.0518$$

$$X(4) = 0$$

We know that, $X(k) = X^*(N-k)$

$\therefore$

$$X(5) = X^*(8-5) = X^*(3) = 0.125 + j0.0518$$

$$X(6) = X^*(9-6) = X^*(2) = 0$$

$$X(7) = X^*(8-7) = X^*(1) = 0.125 + j0.03018$$

Q.2. (b) Prove the circular convolution property of DFT.

Ans. Refer Q. 1. (b) of First Term Exam 2018. (Page No. 1&2-2018).

Q.3. (a) If X(k) is an N-point DFT of $x(n)$ and if $x(n) = -x(N-1-n)$. Then show that X(0) = 0.

Ans. Refer Q. 3. (a) of End Term Exam 2018. (Page No. 12-2018).

Q.3. (b) Calculate the IDFT using Decimation-in-Frequency FFT structure for the given coefficient.

X(k) = {38,–5.828 + j6.07, j6, –0.172 + j8.07, 10,–0.172–j8.07, – j6,–5.828 -5.828- j6.07}

Ans. Refer Q.3. (b) of End Term Exam 2018. (Pg-12 & 13-2018).

Q.4. (a) A low pass filter has the desired frequency response as given below.

$$H_d(e^{j\omega}) = \begin{cases} e^{-3j\omega} & 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h(n)$ for $M = 7$, using Frequency Sampling Technique.

Ans. The samples of the given frequency response is taken uniformly at $\omega_k = 2\pi k/M$.

For $0 \le \omega \le \frac{\pi}{2}$, the values of $k = 0, 1$. For $\frac{\pi}{2} \le \omega \le \frac{3\pi}{2}$, $k = 2,3,4,5$. Thus, the sampled frequency response is given by

$$\tilde{H}(k) = \begin{cases} e^{-j6\pi k/7}, & k = 0,1 \\ 0, & k = 2,3,4,5 \\ e^{-j6\pi k/7}, & k = 6 \end{cases}$$

The filter coefficients hd(n) are given by the inverse discrete Fourier transform,

$$h_d(n) = \frac{1}{M}\sum_{k=0}^{M-1} \tilde{H}(k)e^{j2\pi kn/M} = \frac{1}{7}\sum_{k=0}^{M-1} \tilde{H}(k)e^{j2\pi kn/7}$$

$$= \frac{1}{M}\left[\sum_{k=0}^{1} e^{-j6\pi k/7}e^{-j6\pi kn/7} + e^{-j6\pi k/7}e^{-j6\pi k/7}\Big|_{k=6}\right]$$

$$= \frac{1}{7}\left[\sum_{k=0}^{1} e^{-j6\pi k(n-3)/7} + e^{j12\pi(n-3)/7}\right]$$

$$= \frac{1}{7}\left[1 + e^{j2\pi(n-3)/7} + e^{j12\pi(n-3)/7}\right]$$

Since $\tilde{H}^*(M-k)$, we have $e^{j12\pi(n-3)/7} = e^{-j2\pi(n-3)/7}$.

Therefore, $$h_d(n) = \frac{1}{7}\left[1 + e^{j2\pi(n-3)/7} = e^{-j2\pi(n-3)/7}.\right]$$

$$h_d(n) = 0.1429 + 0.2857\cos[0.898\,(n-3)]$$

Q. 4. (b) Write the various window functions used for FIR filter design. Compare their important performance parameter, (6)

Ans. Window Functions:

I. Rectangular Window function: The weighting function for the rectangular window is given by

$$w_R(n) = \begin{cases} 1, \text{for } |n| \le \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

II. Hamming window function: The cansal hamming window function is expressed by

$$W_H(n) = \begin{cases} 0.25 - 0.46\cos\frac{2\pi n}{M-1}; 0 \le n \le M-1 \\ 0, \quad \text{otherwise} \end{cases}$$

The non-causal Hamming window function is given by

$$W_H(n) = \begin{cases} 0.25 - 0.46\cos\frac{2\pi n}{M-1}; for\ |n| \le \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

III. Hanning window function: The window function of a causal hanning window is given by

$$W_{\text{Hann}}(n) = \begin{cases} 0.25 - 0.5\cos\frac{2\pi n}{M-1}; for\ |n| \le M-1 \\ 0, \quad \text{otherwise} \end{cases}$$

The window function of a non-causal hanning window is expressed by

$$W_{\text{Hann}}(n) = \begin{cases} 0.5 - 0.5\cos\frac{2\pi n}{M-1}; 0 < |n| < \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

IV. Blackman window function: The window function of a causal blackman window is expressed by

$$W_B(n) = \begin{cases} 0.45 - 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}; \le n \le M-1 \\ 0, \quad \textit{otherwise} \end{cases}$$

The window function of a non-causal Blackman window is given by

$$W_B(n) = \begin{cases} 0.42 - 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}; \text{ for } |n| < \frac{M-1}{2} \\ 0, \quad \text{otherwise} \end{cases}$$

V. Bartlett Window Function: The window function of a non-casual bartlelt window is expressed by.

$$W_{\text{Bart}}(n) = \begin{cases} 1+n, -\frac{M-1}{2} < n < 1 \\ 1-n, \ 1 < n < \frac{M-1}{2} \end{cases}$$

Q. 5. (a) Derive the relation for Impulse-Invariant Technique connecting s-domain and z-domain. Show the mapping of points in s-domain to z- domain. (6.5)

Ans. Refer Q. 5. (a) of End Term Exam 2018. (Page No. 14-2018)

Q. 5. (b) Using Bilinear transformation, design a Butterworth LPF to meet the following specifications: **(6.5)**

$$0.8 \le |H(e^{j\omega})| \le 1 \qquad 0 \le |\omega| \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \qquad 0.6\,\pi \le |\omega| \le \pi$$

Ans. Refer Q. 5. (b) of End Term Exam 2018. (Page No. 15&16-2018)

Q. 6. (a) Explain the Lavinson Durbin Algorithm in detail. **(6)**

Ans. Refer Q. 7. (a) of End Term Exam 2018. (Page No. 18 & 19-2018)

Q. 6. (b) Draw and explain the Ladder-Lattice realization of the following transfer function. **(6.5)**

$$H(z) = \frac{0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}}{1 - .0.0010z^{-1} + 0.6546z^{-2} - 0.0775z^{-3}}$$

Ans. Refer Q. 6. (b) of End Term Exam 2018.(Page No. 17&18-2018)

Q. 7. (a) Draw the Direct from-II and its transposed structure for the given transfer function. **(6)**

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5x^3 + 0.3z^2 + 0.17z - 0.2}$$

Ans. Refer Q.6. (a) of End Term Exam 2018. (Page No. 16-2018)

Q. 7. (b) Without factoring any polynomial, determine whether or not the following filter function is stable. **(6.5)**

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556\,z^{-3} + 0.4z^{-4}}$$

Ans. Refer Q. 7. (b) of End Term Exam 2018. (Page No. 19,20,21-2018)

Q. 8. (a) What do you understand by Zero input limit cycle oscillations. Explain with the help of an example. **(6.5)**

Ans. Refer Q. 9. (a) of End Term Exam 2018. (Page No. 22 and 23-2018)

Q. 8. (b) Express the output $y(n)$ of Figure (a) as a function of input $x(n)$. By simplifying the expression derived show that $y(n) = x(n-1)$. **(6)**

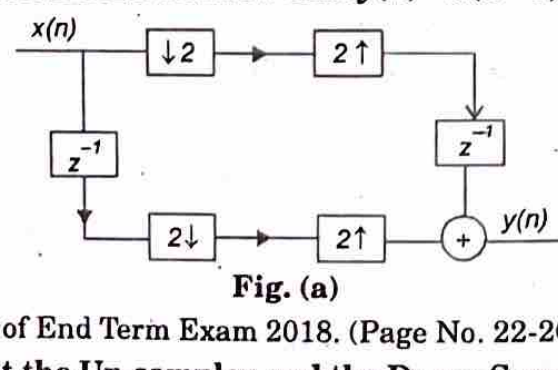


Fig. (a)

Ans. Refer Q. 8. (b) of End Term Exam 2018. (Page No. 22-2018)

Q. 9. (a) Prove that the Up-sampler and the Down-Sampler are Linear but Time-varying system. **(6.5)**

Ans. Refer Q. 8. (a) of End Term Exam 2018. (Page No. 23 and 24-2018)

Q. 9. (b) Prove that $\sum_{n=0}^{\infty} x^2(n) = \frac{1}{2\pi j}\oint_e X(z)X(x^{-1})z^{-1}dx$. **(6)**

Ans. Refer Q. 9. (b) of End Term Exam 2018.