UNIT → 01

SET

#) A well-defined unordered collection of distinct elements is called a 'Set'.

$\{1,1,2\}$ ✗

$\{1,2\}$ ✓ {should be distinct}

Set can be represented in two forms :-

1) Roster or Tabular
2) Set Builder Notation

Roster ⇒ The Set is represented by listing all the elements seperated by the comma (,) and enclosed within braces { }.

$A = \{1, 2, 3, 4\}$

Set builder ⇒ The set is defined by specifying the property that elements of the set having common

$A = \{x : x \in N \text{ and } x > 9 \text{ and } x < 20\}$

set of Natural no.

Ex ⇒ $A = \{a, e, i, o, u\}$

$A = \{x : x$ is a vowel Alphabets$\}$

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N → Set of Natural no.
Z → Set of all integers
Z^+ → Set of all +ve int
Q → Set of all Rational nos.
R → Set of all Real no.
W → Set of all whole no.

Cardinality of Set

The no. of elements present in the Set is called <u>cardinality of Set</u>.

$A = \{1, 2, 3, 4\}$

$|A| = 4$

Ex ⇒ $x = \{1, 2, 3\}$
$y = \{4, 5\}$

$|x| = 3$
$|y| = 2$

$\{|x| \leq |y| \times\}$ $\{|x| > |y| \checkmark\}$

Sets can be classified into many types :-

⇒ Some of which are :-

→ finite
→ infinite
→ empty [or Null Set]
→ Singleton set
→ Equal set
→ Equivalent set
→ overlapping set
→ Disjoint Set

1) finite ⇒ A set which contained a definite no. of elements.

Ex → $S = \{x \mid x \in N \text{ and } 9 < x < 100\}$

2) Infinite ⇒ A set which contained a ~~de~~ infinite no. of elements

3) Empty/Null ⇒ Empty set contains no elements. It is denoted by ϕ
Empty set is the ~~emp~~ ~~ex~~ example of finite set.

→ The cardinality of empty / Null set is zero.

Ex ⇒ $\{x \mid x \in N \text{ and } 9 < x < 10\}$

4) $\{\phi\}$ ⇒ It is a set which contains ϕ.

4) Singleton ⇒ Singleton set contains only one element.

Ex ⇒ $S = \{x \mid x \in N, 7 < x < 9\}$

4) Equal set ⇒ If two sets contain equal set. / same no. of elements.

Ex ⇒ A = {1, 2, 3} A = B

B = {1, 2, 3} |A| = |B|

5) Equivalent set ⇒ If two sets contain unequal set but have same cardinality

Ex ⇒ A = {1, 2, 3}

B = {11, 12, 13}

6) Overlapping ⇒ If the sets that has atleast one common element.

A = {1, 2, 3}
B = {3, 4} } Overlap

7) Disjoint set ⇒ Sets which have no common elements.

B = {1, 2, 3}

A = {4, 5}

Subset

⇒ ✓ If any element of A is also element of B; then A is subset of B.

↳ Denoted by ⊆

B = {1, 2, 3, 4, 5}

A = {1, 2}

A_2 = {3, 4}

A_3 = {1, 5}

$A \subseteq B$, $A_2 \subseteq B$, $A_3 \subseteq B$

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⇒ Trivial Set ⇒ Every set is subset of itself is called 'Trivial set'.
ϕ is also considered as a 'Trivial set'

⇒ Proper subset

Any subset of A which is not a trivial subset of A called Proper subset of A
Denote by $\subset$

$A_1 \subset B$ ⇒
$A_2 \subset B$

⇒ Power Set

If B is a finite set, then a set of all subset of B is called 'Power set of B'.

$B = \{1, 2, 3\} 4, 5$
Power set = T.S. + P.S
↓
$\{\phi\}$
$\{1, 2, 3\}$

$\left\{\begin{matrix}\text{Power} \\ \text{set}\end{matrix} = 2^n\right\}$

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clw

When A & B are two sets, Then every relation from A to B is subsets of Cross product of A & B.

$n \to 2^n$

$|m| \times |n| = 2^{|m| \times |n|}$

Total no. of Relation possible of 2 sets having m & n elements are there.

Reflexive Relation

A Relation ~~on~~ R on set A is said to be reflexive, if

$(x,x) \in R \ \forall \ x \in A$

If every element of set A is related to itself, then it is reflexive relation

$A = \{1, 2, 3\}$

$A = \{1, 2, 3\}$

$$A \times A = \begin{Bmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{Bmatrix}$$

$R_1 = \{(1,1)\ (2,2)\}$ ✗

$R_2 = \{(2,2)\ (3,3)\}$ ✗

$R_3 = \{(1,1)\ (2,2)\ (3,3)\}$ ✓

Ex: $R_1 = \{(x,y) \mid (x-y) \text{ is a integer}\}$ ✓

$R_2 = \{(x,y) \mid (x-y) \text{ is an odd no.}\}$ ✗

$\{(1,1)\ (2,2)\ (3,3) \ldots\}$

$R_1 = 1-1 = 0$ ✓

(0 is an ~~odd~~ even no.

$A = \{(1,2)\}$

$A \times A = \left\{ \begin{matrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{matrix} \right\}$

$R_1 = \{(1,1)(2,2)\}$

$R_2 = \{(1,1)(2,2)(1,2)\}$

$A = \{(1,2,3)\}$

$A \times A = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{matrix} \right\}$

Q) How many no. of reflexive relation are possible on Set A having its cardinality n.

Diagonal elements = n

non-diagonal elements = $n^2 - n$

(Total sets = $2^{|m| \times |n|}$)

Tot

Total no. of non-reflexive = Total Reflex. − Reflex. ~~Relaflexive~~ Relation

Q) What would be the size s of Smallest reflexive relation.

$\rightarrow n$

$\Rightarrow$ Size of largest reflexive relation

$\rightarrow n^2$

$\Rightarrow$ Irreflexive Relation:

$\rightarrow$ A relation R on set A is said to be irreflexive relation.

If

$x \not R x$ { x is not related to x } $\forall x \in A$

Not Reflexive and Irreflexive both.

Ex ⇒ $R = \{(1,2)(2,1)\}$ — Irreflexive

→ Ex ⇒ $R_1 = \{(1,2)(2,1)(1,1)\}$

⇒ In irreflexive relation not even a single diagonal element should exist.

Total size of irreflexive Relation sets $= n^2 - n$

Total no. of Relation $= 2^{n^2 - n}$

⇒ Symmetric Relation

⇒ A Relation R on set A is said to be Symmetric

i.e

if $(x,y) \in R$ then $(y,x) \in R$

$\forall$

$(x,y) \in A$

$R = \{(1,2)(2,1)\}$

$R_2 = \{(1,2),(2,1),(1,1)\}$

(Symmetric)

Anti-Symmetric Relation

→ A Relation R is said to be anti-symmetric if xRy and yRx then $x = y$ $\forall$ $x, y \in A$

i.e. if $(x,y) \in R$ then $(y,x) \in R$ only if $(x = y)$ $\forall$ $x, y \in A$.

Ex → $R_1 = \{(1,2)\ (2,2), (2,1)\}$ ✗ Not anti-symmetric

$R_2 = \{(1,2)\ (2,2)\ (2,1)\ (3,1)\}$ Not symmetric, Not anti-symmetric

$R_3 = \{\ \}$ yes, It is irreflexive or anti-symmetric

Q) Pls verify $\leq$ operator is anti-symmetric or not on set of all real nos.

Q) Yes; $\geq$

Asymmetric Relation :-

→ A relation R is said to be Asymmetric. if xRy and yRx then $x = y$ $\forall$ $x, y \in A$.

i.e. if $(x,y) \in R$ then $(y,x) \notin R$ $\forall$ $(x,y) \in A$

Ex = $R_1 = \{(1,2)\ (1,3)\ (1,1)\}$

Symmetric ✗

Anti-symmetric ✓

Asymmetric ✗

Ex = $R_2 = \{(1,2)\ (1,3)\ (2,3)\}$ ✓

Q) Verify T/F Every Asymmetric is Anti-symmetric also?:-

Antisymmetric = Asymmetric + self loop (diagonal elements)

Transitive Relation

→ A Relation R on set A is said to be 'transitive'.

if (xRy) and (yRz) $\Rightarrow (xRz)$

$R_1 = \{(1,1)\ (2,2)\}$ ✓

$R_2 = \{(1,2)\ (2,3)\}$ ✗

$R_3 = \{\ \}$ ✓

$R_4 = \{(1,1)\ (1,2)\ (2,1)\}$ ✗

$\{1, 2, 3\}$

$(1,1), (2,2), (3,3)$

$(1,2); (2,3)\ (1,3)$

→ A Relation R is said to be equivalence - If it is reflexive, symmetric & transitive.

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ ✓

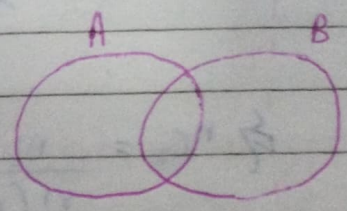
$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (2,1)\ (1,2)\}$ ✓

$\{$Transitive & Symmetry $= \{\ \} = R_3\}$

	(Reflexive)	(Irreflexive)	(Symmetry)	(Antisymmetric)
1) Cardinality of Smallest Rel^n	n	$\{\}, \phi$	ϕ	ϕ
2) Cardinality of Largest Rel^n	n^2	n^2-n	n^2	$\left\{ n + \frac{n^2-n}{2} = \frac{n(n+1)}{2} \right\}$
3) How many total no. of Rel^n possible	2^{n^2-n}	2^{n^2-n}	$2^n \cdot 2^{\frac{n^2-n}{2}}$	$= 2^n \cdot 3^{\frac{n^2-n}{2}}$

	Asymmetric	Transitive
	ϕ	ϕ
	$\frac{n^2-n}{2}$	n^2
	$3^{\frac{n^2-n}{2}}$	

{Principle of Exclusive and Inclusive}



$\Rightarrow$ $|A \cup B| = |A| + |B| - |A \cap B|$

$A \cup B \cup C = \cancel{A \cup B} + \cancel{B \cup C}\ |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

The Pigeon Hole Principle

Q) If $(K+1)$ ~~for~~ more objects are placed into the K boxes then ~~they are it~~ there is atleast one box containing two or more objects.

Q) If there are 367 ~~people~~ (people) Plz verify 1 or 2 of them will have b'day dates.

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Q) If there are 27 Eng. words then prove that 2 words if having atleast starting with a same alphabet.

Generalised Principle of Pigeon hole:

If N objects are placed into the K boxes then K ~~has~~ There is atleast 1 box containing $\left\lceil \frac{N}{K} \right\rceil$ ceiling funⁿ objects.

$\left\lceil \frac{27}{26} \right\rceil = \lceil 1.2 \rceil = 2$

$\lceil 2.3 \rceil = 3$

$\lceil -2.3 \rceil = -2$

Permutation

$$\Rightarrow P(n,r) = \frac{n!}{(n-r)!}$$

Combination

$$\Rightarrow {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$C = \frac{P}{r!}$$

$$\boxed{P = C \times r!}$$

Q) How many ways are there to select 1st Prize, 2nd Prize, 3rd Prize 100 People who has joined the Contest.

$= 100 \times 99 \times 98$

Q) How many permutation of the letter $\boxed{ABCDEFGH}$ contains the string $\boxed{ABC}$

(String)

ABC - 1

1DEFGH = 6!

Q) If there are 10 member, and we have to make a team in such a way that only max. 5 people can be allocated in the team, How many total no. of possible team can be formed?

{Up to the 5 max}

$\hookrightarrow \{{}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \dots {}^{10}C_5\}$

Ans $\Rightarrow$ ${}^{10}C_5$

{Principle of mathematical induction}

Q) Prove that $(n^3 - n)$ is always divisible by 3 where $\{n \geq 1\}$

Basic step → Put $n = 1$;

$$n^3 - n = 1^3 - 1 = 0 \cdot \text{(True)}$$

Inductive step → $P(K) = (K^3 - K)$ is also divisible by 3.

Now; we have to prove this; for $(K+1)$ -

Now → $P(K+1) = (K+1)^3 - (K+1)$

$$= K^3 + \cancel{1} + 3K^2 + 3K - K - \cancel{1}$$

$$= K^3 + 3K^2 + 2K$$

$$= K[K^2 + 3K + 2]$$

$$= K[K^2 + K + 2K + 2]$$

$$K[K(K+1) + 2(K+1)]$$

$$P(K+1) = K(K+2)(K+1)$$

$K^3 - K + 3(K^2 + K)$

Divisible by 3

{It is also Divisible by 3}

Q) Prove that $n^3 + 2n$ is also divisible by 3

⇒ Basic step → Put $n = 0$;

$$n^3 + 2n = 0^3 + 2 \times 0 = 0 \text{ (True)}$$

Inductive step → $P(K) = (K^3 + 2K)$ is also divisible by 3

Now, we have to prove this for $(K+1)$

Now, $P(K+1) = (K+1)^3 + 2(K+1)$

$$= \underbrace{K^3 + 1 + 3K^2 + 3K + 2K}_{} + 2$$

$$= K^3 + 2K + 3K^2 + 3K + 3$$

$$= \underbrace{K^3 + 2K}_{\text{Divisible by 3;}} + \underbrace{3(K^2 + K + 1)}_{}$$

Hence; ~~It~~ is also divisible by 3;

Q) $1 + 2 + 2^2 + \ldots\ldots 2^n = 2^{n+1} - 1$.

for all non-negative integers n.

0 →

⇒ Base Step

$$p(n) = 1 + 2 + 2^2 + \ldots 2^n = 2^{n+1} - 1$$

$$p(0) = 2^{0+1} - 1 = 2^1 - 1 = 1$$

Inductive Step

$$p(K) = 1 + 2 + 2^2 + \ldots 2^K = 2^{K+1} - 1$$

$$p(K+1) = \underbrace{1 + 2 + 2^2 \ldots\ldots 2^K}_{p(K)} + 2^{K+1} = 2^{K+1+1} - 1$$

$$= 2^{K+2} - 1$$

$$= 2^{K+1} - 1 + 2^{K+1}$$

$$= 2 \cdot 2^{K+1} - 1 = 2^{K+2} - 1 = R.H.S$$

⇒ Principle of complete induction

⇒ Mathematical induction	⇒ complete (strong) induction
1) $(p(0), p(1))$ $p(n)$	1) Base case ✓ $p(1), p(2), p(3) \ldots$
2) Inductive step $P(K)$	2) Inductive case $p(1) \wedge p(2) \wedge p(3) \wedge p(4) \ldots p(K)$
3) $p(K+1)$ verify	3) $p(K+1)$ verify ✓

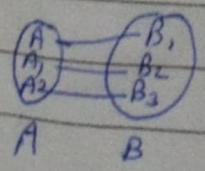
⇒ Principle of well ordering :- (for natural) $\{1, 2 \ldots 10\}$

Every non-empty Every Subset of natural no. has a smallest element, ~~The~~ If 'S' be the non-empty Subset of natural no. then ~~there~~ exist $p \in S$ such that $p \leq q \ \forall \ q \in S$

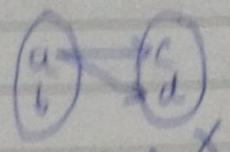
#) functions in Discrete mathematics

A Relation func^n F from a set A to a set B is called func^n if each element belongs to a we can assign unique element of B.

$f: A \to B$

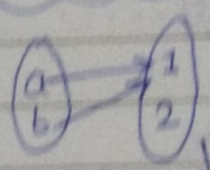


(for ex.→)



(fn^n) ✗

$f: A \to B$

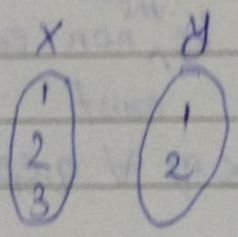


(Domain) Codomain

Range ⊆ codomain

Q How many func^n are possible for 2 set A and B in which A is having x cardinality & B is having y cardinality.

$f: A \to B$ ($|x|$, $|y|$)



$\underline{2}\ \underline{2}\ \underline{2} = 2^{3 \to x} = y^x$ ✓

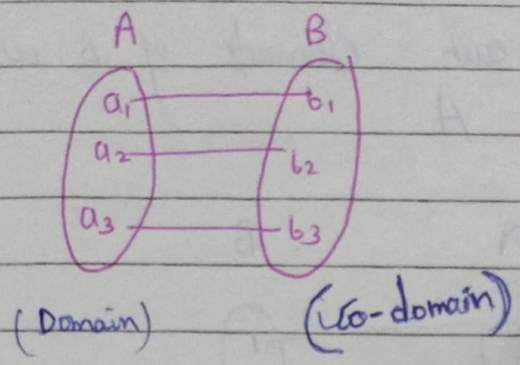
y

Q How many Relations are there which are not func^n

$= 2^{|x| * |y|} - y^x$

Ans →

$f : A \to B$



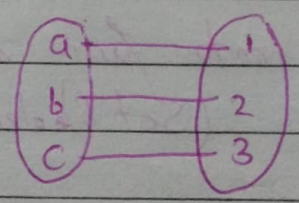
Range $\subseteq$ codomain

Q) How many fnx. are possible on set A & B which is having cardinality x & y resp.

Types of fnx.

⇒ One to one func. (Injection) → A func. f from set A to set B is called one to one func. If no. two elements in A are map to the same element in B

ex →



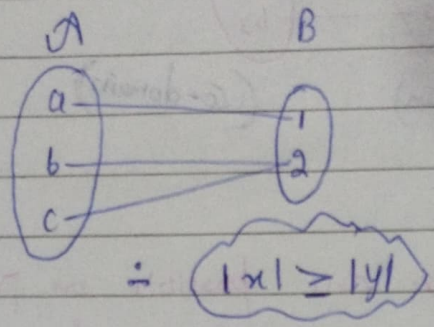
#) for one to one func. a for following condition must match :-

$|x| \leq |y|$

Q) How many one to one func. are possible on a set A and B having a cardinality x and y resp.

⇒ ^yP_x

⇒ onto function (Surjection) → A func. f set A to set B is said to be onto func. if each elements of B is mapped to ~~atleast~~ 1 element of A



Q) If func. is one-to-one then ~~it~~ is it onto as well
T/Fa. (False)

Q) If func. is onto. then is it one-to-one as well
T/F

Q) How many onto func. are there if no. of elements in Set A = (Set B) elements

B
↳ y!

⇒ Bi-jective function (Permutation func.). A func. f from set A to set B is said to be bijective func. if & only if ~~func.~~ if is one-to-one & onto both.

Q) How many total no. of Bijective func. are possible on set A and B having cardinality x & y resp.

⇒ n!

Cyclic permutation :-

Let $x_1, x_2,$ ~~and~~ $, x_3 \ldots x_n$ are the distinct element of the set then permutation on set A X is defined as:

$$p : X \rightarrow X$$

$$p(x_1) = x_2$$
$$p(x_2) = x_3$$
$$p(x_3) = x_4$$
$$\vdots$$
$$p(x_n) = x_1$$

This permutation func^n is called cyclic permutation of length n.

#) Let A be the set $\{1,2,3,4,5\}$. compute $(2,1,3,4)$ using cyclic permutation.

Sol) $p_0 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 2 & 5 \end{pmatrix}$

Q) Compute (5, 4, 2)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 3 & 2 & 4 \end{pmatrix}$$

#) Disjoint cycles ⇒ Two cycles of set A are called Disjoint if they donot have common element.

#) Theorem ⇒ Every permutation of finite set A is a cycle or it can be expressed as product of disjoint cycle.

Q) $A = \{1, 2, 3, 4, 5\}$

compute = (1, 2, 4).
(3, 5)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 1 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$$

Partial ordering Relation (POR) / Composite Relation)

⇒ A Relation R is said to be partial ordering Relation if R is Reflexive, anti-symmetric & transitive.

Q) $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ ✓ composite

Q) $R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (1,3)\}$ ✓

{comp.}

Partial ordered set (POSET)

→ A set A which satify the condition of partial ordering Relation is called POSET.

And it is denoted by $[A; R]$

Q) Show that set A which is a set of Real no.

$[A; \leq]$

Sol) $A = \{1, 2, 2.5 \cdots\}$

$\{1 \leq 1,\ 2.2 \leq 2.2\}$

Reflexive

$1 \leq 2$ ✓
$2 \leq 1$ (F) } Anti-symmetric

$1 \leq 2,\ 2 \leq 3$

$1 \leq 3$ ✓ (Transitive)

Totally ordered set [TOS]

⇒ A poset[A, R] under the Relation R is called Totally ordered SET; if every ~~all~~ pair of element in A are comparable

$\{\because aRb,\ bRa \quad \forall a, b \in A\}$

⇒ Every TOS is POSET But a POSET may or may not be (TOS). [Every TOS → POSET

⇒ {A is a set of any +ve integer.}

Q) $A = \{+1, +2, +3 \ldots\}$

[A; ÷]

1) ÷ So; it is POSET.

But; it is not TOSS.

Q) Suppose there is set A which is having two element A & B. a and b.

[P(A); ⊆] is TOSS or not.

Sol) $P(A) = \{\phi, \{a\}, \{b\}, \{$

Hasse diagram rules:-

i) ⇒ Every Element of a set is called 'vertex'.

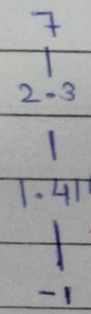
ii) There should be no reflexive ~~loops~~ loops.

iii) → No transitive edge.

iv) No direction should be mention.

Q) $A = [-1, \sqrt{2}, 2.3, 7]$

Sol)

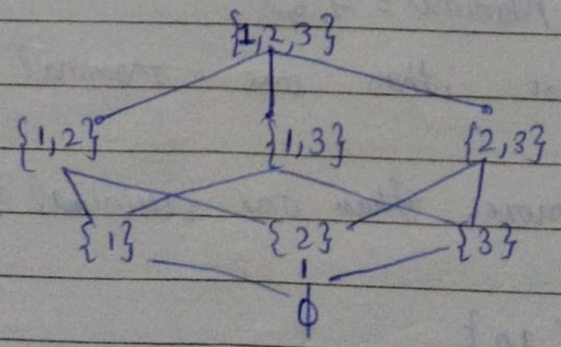


Q) $A = \{1, 2, 3\}$

⇒ $P(A) = \{\phi, \{1\}, \{2\} \{3\} \{1,2\} \{2,3\} \{1,3\}, \{1,2,3\}\}$

$[P(A); \subseteq]$

Sol)

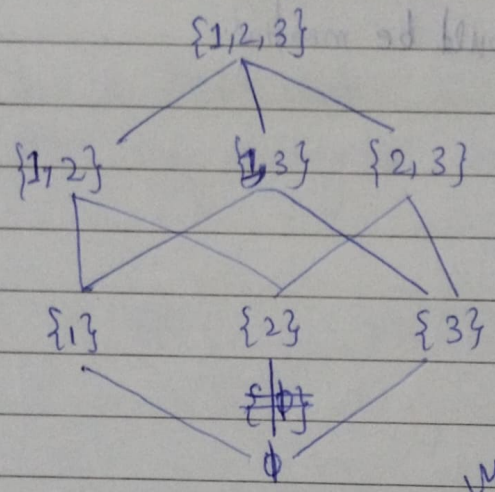


Minimal element :-

In a POSET, no other element is related to this element. It is called minimal element.

Maximal element :-

If in a POSET and element is not related to any other element then it is called maximal element.

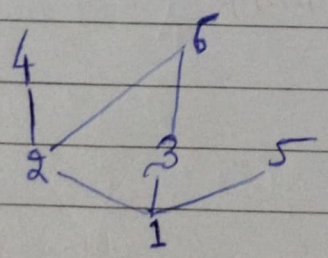


Minimal = ϕ

Maximal = {1,2,3}

Q $A = \{1,2,3,4,5,6\}$

$[A;1]$



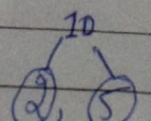
Minimal = 1

Maximal = 4, 6, 5

→ Any POSET can have more than one maximal element.

→ Any POSET can also have more than one minimal element.

$[D;1] = \{2,5,10\}$

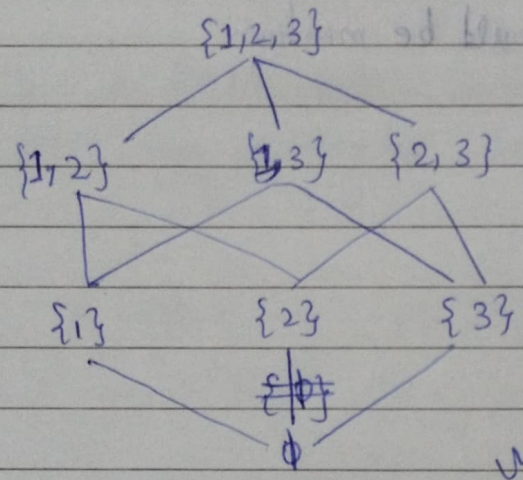


Minimal element :-

In a POSET, no other element is related to this element. It is called minimal element.

Maximal element :-

If in a POSET and element is not related to any other element then it is called maximal element.

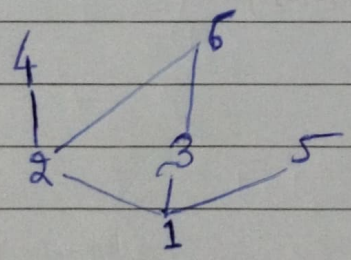


Minimal = ϕ

Maximal = {1,2,3}

Q $A = \{1,2,3,4,5,6\}$

$[A; |]$



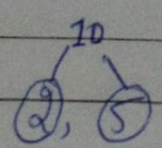
Minimal = 1

Maximal = 4, 6, 5

→ Any POSET can have more than one maximal element

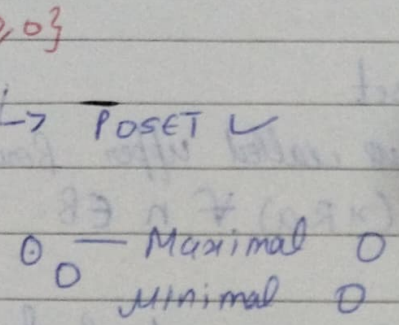
→ Any POSET can also have more than one minimal element.

$[D; |] = \{2, 5, 10\}$

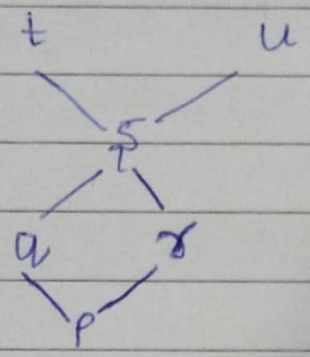


Q) $R = \{0, 0\}$

↳ POSET ✓



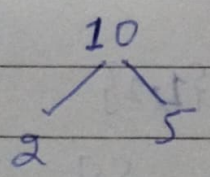
#) Greatest / Least Elements in POSET.



Maximal = t, u → Greatest ✗

Minimal = p → Least → p

Q)



Minimal = 2, 5

Max = 10 → Greatest

† ⇒ In a POSET, it could be possible that it has not greatest & least element can exist

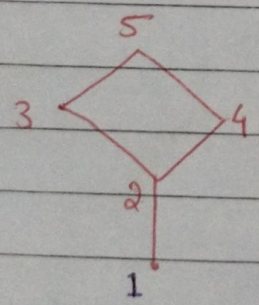
#) Upper Bound

- $[A; R]$ Poset

U.B. ⇒ $a \in A$ is called upper Bound of subset B.
$(B \subseteq A)$ if $(xRa)\ \forall\ n \in B$.

L.B ⇒ $a \in A$ is lower Bound of subset B;
$(B \subseteq A)$ if $aRx\ \forall\ n \in B$.

Q)



$\{4,5\}$

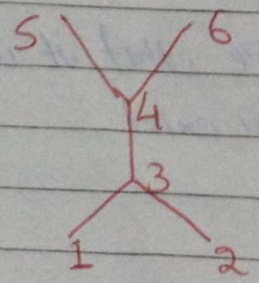
L.B 1, 2, 4

B) $B = \{1,2\}$

L.B = $\{1\}$

U.B = $\{2,3,4,5\}$

Q)

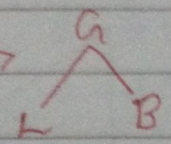


L.B →

U.B. → 4R5
4R6
3R4
3R5
3R6

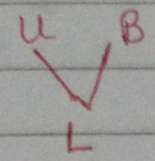
L.B → G.L.B → Infimum → $\wedge$ (meet) →

(max of L.B)

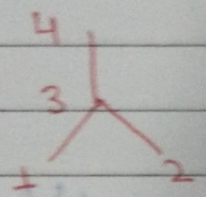


U.B → L.U.B → Supremum → $\vee$ (Join) →

(Least upper Bound / Min)



Q1)

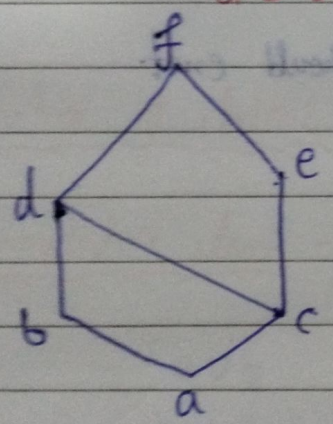


$B = \{1, 2\}$

$L.B. = \phi$ $\quad$ $U.B = \{3, 4\}$

$G.L.B = \phi$ $\quad$ $L.U.B = \{3\}$

Q2)

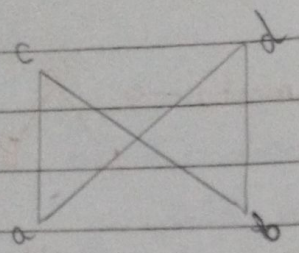


$B = \{c, d\}$

$L.B = \{a, c\}$ $\quad$ $U.B = \{d, f\}$

$G.L.B = \{c\}$ $\quad$ $L.U.B = \{d\}$

Q)



ϕ

$MSL = \phi$

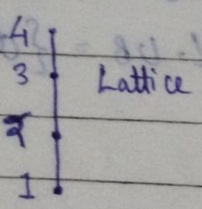
$JSL = \phi$

⇒ Let $[A; R]$ be an POSET, then it is called 'lattice'.

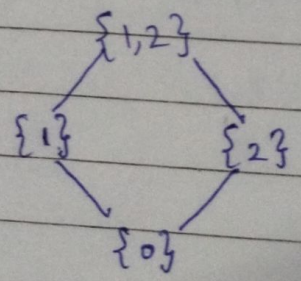
$[A; R]$ $\forall (a,b) = \left.\begin{matrix} JSL \\ MSL \end{matrix}\right\}$ Both should exist.

{Lattice = MSL + JSL}

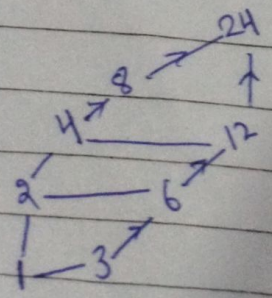
Ex →



Lattice

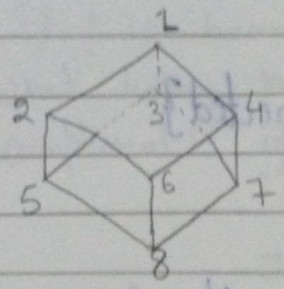


Ex) → $D_{24} = [1, 2, 3, 4, 6, 8, 12, 24]$

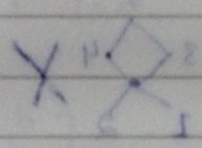


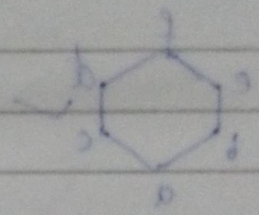
MSL ✓
JSL ✓ → lattice

⇒ Sub lattice :-



$L = \{1, 2, 3, 4, 5, 6, 7, 8\}$





#) Properties of lattice :-

1) Commutative law :-

$a \vee b \qquad b \vee a$

$a \wedge b \qquad b \wedge a$

2) Associative law :-

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

$a \vee (b \vee c) = (a \vee b) \vee c$

3) Idempotent law :-

$a \vee a = a$

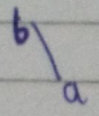
$a \wedge a = a$

4) Absorption law :-

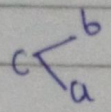
$a \wedge (a \vee b) = a$

$a \vee (a \wedge b) = a$

15) Distributive law (applicable for some lattice).



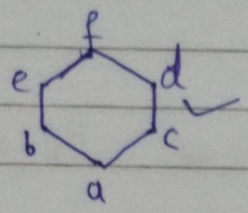
$a \vee b = b$
$a \wedge b = a$

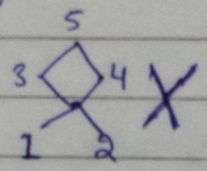


$a \wedge b = b$ {∵ a & b are indirectly connected}
$a \vee b = b$

Complete Lattice :-

⇒ [A; R] is lattice if each non-empty subset of A has a meet and joint

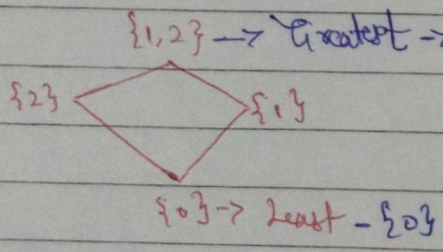




Date → 19/10/22

⇒ BOUNDED LATTICE

⇒ If a lattice has least and greatest element both



$\begin{cases} 0 \wedge \text{element} = 0 \\ 0 \vee \text{element} = \text{element}. \end{cases}$

$1 \wedge \text{element} = \text{element}$
$1 \vee \text{element} = 1$

#) Complement in lattice :-

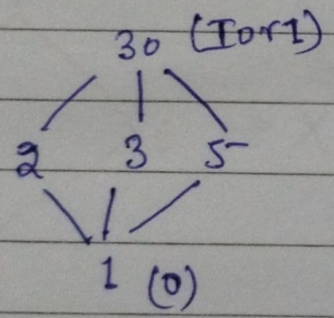
If L is bounded lattice; Suppose a & b are two elements that belong to lattice, then a and b are set to be the complement of each other if & only if

$$a \vee b = I$$
$$a \wedge b = 0$$

then only we can say that

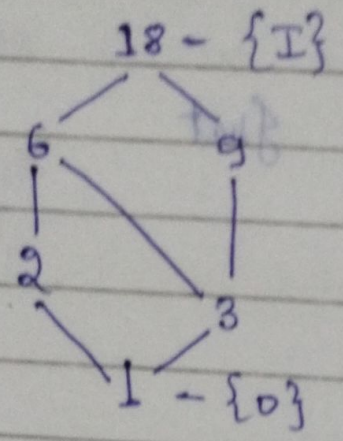
$$\left.\begin{array}{c} a^c = b \\ \text{or} \\ b^c = a \end{array}\right\}$$

Ex → [D, 1] [1, 2, 3, 5, 30]



Q) $[D_{18}, /]$

$[1, 2, 3, 6, 9, 18]$



$(1, 18)$

$1^c = 18$

$18^c = 1$

$(2, 9)$

$= 2 \vee 9 = 18$

$= 2 \wedge 9 = 1 \rightarrow 2^c = 9$

$\rightarrow 9^c = 2$

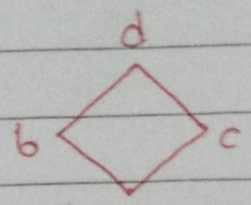
⇒ Distributive lattice ⇒ which follows the distributive property

1) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

2) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

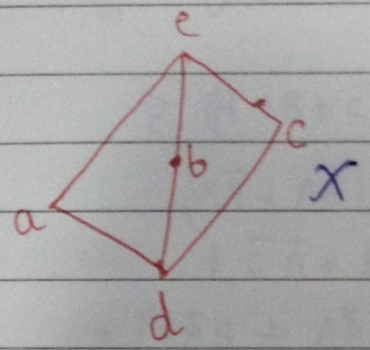
DATE

9)



LHS	RHS
$a \vee a$	$b \wedge c$
$= a$	$= a$

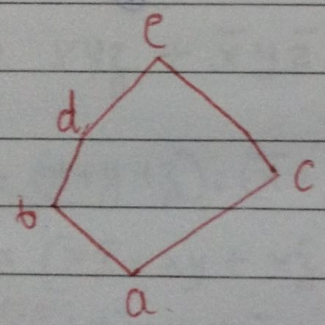
8)



X

→ To check lattice is distributive or not, we check the compliment of an element if exist must be unique for distributive lattice.

11)



→ X Not a distributive lattice.

⇒ (BOOLEAN ALGEBRA)

⇒ SOP → It does not ~~contain~~ need to contain all literals (sum of products).

⇒ 'Canonical SOP'

→ 'Canonical SOP' means it should contain all literals either in the form of complimented or uncomplimented.

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\Rightarrow f = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}z + xyz$$

$$= yz\underbrace{(x+\bar{x})}_{1} + \bar{y}z\underbrace{(x+\bar{x})}_{1}$$

$$= z\underbrace{(y+\bar{y})}_{1}$$

$$f = z$$

Date ⇒ 20/10/22.

Duality Theorem :-

OR ⇔ AND

NOT ⇔ NOT

XOR ⇔ XNOR

NAND ⇔ NOR

0 ⇔ 1

Variable - X
(Literals)

Q) $AB\bar{C} + \bar{A}BC + ABC$

$\Rightarrow (A+B+\bar{C}).(\bar{A}+B+C).(A+B+C)$

$(A\bar{A} + AB + AC + B\bar{A} + B + BC).(A+B+C)$
$+ \bar{C}\bar{A} + B\bar{C} + C\bar{C})$

$= (A\bar{A}A + A\bar{A}B + A\bar{A}C + AAB + ABB + ABC + ACA + ABC + ACC$

$= 0 + 0 + 0 + AB + AB + ABC + AC + ABC + AC$

$= (AB + AC + ABC)$

Q) $xyz + \bar{x}y\bar{z} + \bar{y}z = 1$ (Dual)

$= (x+y+z).(\bar{x}+y+\bar{z}).(\bar{y}+z) = 0$

$= (x\bar{x} + xy + x\bar{z} + \bar{x}y + y + y\bar{z} + \bar{x}z + yz + z\bar{z}).(\bar{y}+z) = 0$

$x\bar{x}\bar{y} + x\bar{x}z + xy\bar{y} + xyz + x\bar{z}\bar{y} + x\bar{z}z + \bar{x}y\bar{y} + \bar{x}yz + y\bar{y} + yz +$
$y\bar{z}\bar{y} + y\bar{z}z + \bar{x}z\bar{y} + \bar{x}zz + yz\bar{y} + yzz$
$z\bar{z}y + z\bar{z}z = xyz + x\bar{y}\bar{z} + \bar{x}yz + yz$ (compl. → Duality + literal change)

Q) $xyz + \bar{x}y\bar{z} + \bar{y}z = 1$ (comp.) +

$= (\bar{x}+\bar{y}+\bar{z}).(x+\bar{y}+z).(y+\bar{z}) = 0$

$(\bar{x}x + \bar{x}\bar{y} + \bar{x}z + \bar{y}x + \bar{y}\bar{y} + \bar{y}z + \bar{z}x + \bar{z}\bar{y} + \bar{z}z)(y+\bar{z}) = 0$

$= (\bar{x}\bar{y}y + \bar{x}\bar{y}\bar{z} + \bar{x}zy + \bar{x}z\bar{z} + \bar{y}x\bar{y} + \bar{y}x\bar{z} + \bar{y}zy + \bar{y}z\bar{z} + \bar{z}xy + \bar{z}x\bar{z} +$
$\bar{z}\bar{y}y + \bar{z}\bar{y}\bar{z})$

$= \bar{x}\bar{y}\bar{z} + \bar{x}zy + \bar{y}x + x\bar{y}\bar{z} + xy\bar{z} + x\bar{z} + \bar{y}\bar{z}$

$= \bar{x}\bar{y}\bar{z} + \bar{x}yz + \bar{y}\bar{z} + x\bar{z} + \bar{y}x$

⇒ Self-dual func^n

→ A func^n is said to be self-dual if & only if its dual is equivalent to given func^n.

$$f(x,y,z) = xy + yz + zx$$

$$= (x+y)\cdot(y+z)\cdot(z+x)$$

$$= (xy + xz + y + yz)\cdot(z+x)$$

$$= (xyz + xy + xz + xz + yz + yx + yz + xyz)$$

$$(xyz + xy + xz + yz)$$

$$\rightarrow (y(x+1+z) + xz)\cdot(z+x)$$

$$(y + xz)(x+z)$$

$$= xy + yz + xz + zx$$

$$= (xy + yz + xz)$$

1) a) How many no. of combination are possible with 1 variable
b) " " no of Boolean exp. " " " " "
c) " " set dual func^n " " " " "

Single Variable

A	f	f	f	f
0	0	1	0	1
1	0	1	1	0

⇒ for 2 variables

Comb. →	4	2^n
Bool. fn. →	16	2^{2^n}
Self dual →	4	$2^{2^{(n-1)}}$

i) K-Map is a graphical representation of simplifing the Boolean exp.

ii) Boolean exp. of n-variable contains the 2^n no. of cells in K-MAP

iii) K-MAP is based on the gray code (unit distance code)

iv) K-MAP is based on the 3 types of ~~input variables~~ values - 0, 1 or don't care cond^n.)

Q) $f = \Sigma m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$

AB \ CD	00	01	11	10
00	1		X	1
01		1	1	
11	X			X
10	X	1	1	X

$= A\bar{B} + \bar{A}BD + \bar{A}\bar{B}\bar{D}$

Q) $F(A,B,C,D) = \Sigma m(0,1,2,5,7,8,9,10,13,15)$

AB \ CD	00	01	11	10
00	1	1		1
01		1	1	
11		1	1	
10	1	1		1

$= BD + \bar{C}D + B'D'$

⇒ Ans

Q) $f(A,B,C,D) = \Sigma m(0,1,3,5,7,8,9,11,13,15)$

AB \ CD	00	01	11	10
00	1	1	1	
01		1	1	
11		1	1	
10	1	1	1	

$(= D + \bar{B}\bar{C})$

⇒ Ans

Q) $F(A,B,C,D) = \Sigma m(1,3,4,6,8,9,11,13,15) + \Sigma d(0,2,14)$

AB \ CD	00	01	11	10
00	X	1	1	X
01	1			1
11		1	1	X
10	1	1	1	

$= AD + \bar{B}D + \bar{A}\bar{D} + \bar{B}\bar{C}$

⇒ Ans

→ The integer n is even if there exist an integer K such that $n = 2K$.

→ Note that every integer is either even or odd but no integer can be odd and even both at a time

⇒ $p \to q$ can be proof by showing that if p is true then q is also true.

→ Give a direct proof of theorem if n is odd then n^2 is also odd.

Q) $n \to 2K+1 \ (K \in I)$

Let $(2K+1)^2$

$= 4K^2 + 1 + 4K$

$= 2(2K^2 + 2K) + 1$

$n^2 = 2M + 1 \quad \because \text{let } (M = 2K^2 + 2K)$

So, If n is odd; Then n^2 is also odd.

If p is true then q is also true

(Th)

⇒ Product of 2 ~~odd~~ odd integer is odd

Let x & y are two odd no.

$= \underset{(2K+1)}{x} * \underset{(2n+1)}{y} \qquad (K \,\&\, n \in I)$

$= (2K+1) \times (2n+1)$

$= 4nK + 2K + 2n + 1$

$= \cancel{2n(2K+1)+(2K+1)}$

$= \cancel{(2K+1)[2n+1]} = 2[nK + K + n] + 1$

$= \underbrace{2M + 1}_{\text{odd}} \; [M \in I]$

⇒ Indirect proof (contraposition).

⇒ An implication -

$P \to q$ is equivalent to

Therefore; we can proof $\neg q \to \neg P$; whenever q is false; then P is also false.

Q) $\underbrace{\text{If } 3n+2 \text{ is odd}}_{P}$ then $\underbrace{n \text{ is odd}}_{q}$.

$\neg q \to n$ is even

$(n = 2K)$

$3(n) + 2 \Rightarrow 3(2K) + 2 = 6K + 2$

$= 2(3K+1) = \neg P$

$= 2M$

⇒ This is even

Q) $\underbrace{\text{If } n = ab}_{P}$, $a, b \in$ Positive integer, then $\underbrace{a \le \sqrt{n} \text{ or } b \le \sqrt{n}}_{q}$

$\neg q = a > \sqrt{n}$ And $b > \sqrt{n}$

$a \cdot b > \sqrt{n}\sqrt{n}$

$ab > n$

$P: n = ab$

$\neg P: n \ne ab$

$\neg q \to \neg P$

$\Rightarrow$ Contradiction

A) we assume that q is false. ($\sim q$ is true)

then by logical argument we arrive at at situation where $\sim q \xrightarrow{\text{implies}}$ contradiction.

This can only happen when $\sim q$ is false which implies that q must be true.

Q) $\underbrace{\text{If } 3n+2 \text{ is odd}}_{p}$ then $\underbrace{n \text{ is odd}}_{q}$.

$q \rightarrow n$ is odd

$\sim q \rightarrow n$ is even

$\therefore n = 2K$

$3n+2$

$= 3(2K)+2$

$(6K+2)$

$= 2(3K+1)$

$P: 2M =$ even

So, it contradict that $2M$ is even but it must be odd; So;

⇒ Predicate

↳ Property of Subject

→ Predicate is the expression that contains variables

→ $x > 5$

→ Predicate is also called proposition func^n;

If value is assigned to variable x then $P(x)$ becomes a proposition.

Q ⇒ We have a proposition func^n $Q(x, y, z)$

i.e. $x + y = z$;

what is the truth value of $Q(2,3,5)$; $Q(0,2,3)$

$Q(2,3,5) \rightarrow T$; $Q(0,2,3) \rightarrow F$

→ Predicate can't be T or F until value is not assigned to the variable

#) Quantifier

→ Universal $\forall x P(x)$

→ Existential

→ A Quantifier is an operator used to create a proposition from a propositional func.

Q) what is the truth value of $\forall x P(x)$ where $P(x)$ is the statement "$x^2 < 10$". & the universe of discourse (Range) consist of (the integers) not exceeding 4.

⇒ $[0, 1, 2, 3, 4]$

$\underset{T}{0^2<10}$; $\underset{T}{1^2<10}$; $\underset{T}{2^2<10}$ $\underset{T}{3^2<10}$ $\underset{F}{4^2<10}$

Date - 2/Nov/22

→ We have two types of normal forms :-

1) Disjunctive normal form (DNF) (SOP)
2) Conjuctive Normal form (CNF)

A logical expression is formed if it is sum of elementary form (product).

P	Q	R	F(P, Q, R)
F	F	F	T ✓
F	F	T	F
F	T	F	T ✓
F	T	T	F
T	F	F	T ✓
T	F	T	F
T	T	F	F
T	T	T	T ✓

$= \{ (\sim P \wedge \sim Q \wedge \sim R) .$
$+ (\sim P \wedge Q \wedge \sim R)$
$+ (P \wedge \sim Q \wedge \sim R)$
$+ (P \wedge Q \wedge R) \}$

Q) $\underbrace{\sim(P \vee Q)}_{A} \Leftrightarrow \underbrace{(P \wedge Q)}_{B}$

P	Q	$\sim(P \vee Q)$	$(P \wedge Q)$	$\sim(P \vee Q) \Leftrightarrow (P \wedge Q)$
F	F	T	F	F
F	T	F	F	T ✓
T	F	F	F	T ✓
T	T	F	T	F

$= (\sim(P \vee Q) \wedge (P \sim Q) \vee ((P \vee Q) \wedge \sim(P \wedge Q))$
$((\sim P \wedge \sim Q) \wedge (P \wedge Q)) \vee ((P \vee Q) \wedge (\sim P \vee \sim Q))$
P

Q) $P \Rightarrow ((P \Rightarrow q) \wedge \sim(\sim q \vee \sim p))$

$= \sim P \vee ((P \Rightarrow q) \wedge \sim(\sim q \vee \sim p))$
$= \sim P \vee ((\sim P \vee q) \wedge \sim(\sim q \vee \sim p))$
$= \sim P \vee ((\sim P \vee q) \wedge (q \wedge p))$
$= \sim P$

$Q \leftarrow (Q \leftarrow q) \wedge q$

$Q \leftarrow q$
q
$\therefore Q$

Sub all two q

⇒ If a form ~~of~~ is product of elementary sum then it is conjunctive normal form.

Ex → → $p \wedge q$

→ $(\sim p) \wedge (q \vee r)$

1) $p \wedge (p \Rightarrow q)$

→ $p \wedge (\sim p \vee q)$

2) $(q \vee (p \wedge r)) \wedge \sim((p \vee r) \wedge q)$

$(q \vee (p \wedge r)) \wedge ((\sim p \wedge \sim r) \vee (\sim q))$

$= ((q \vee p) \wedge (q \vee r)) \wedge ((p q \vee r q))$

$= ((q \vee p) \wedge (q \vee r)) \wedge ((\sim p \wedge \sim q) \vee (\sim r \wedge \sim q))$

$((q \vee p) \wedge (q \vee r)) \wedge ((\sim p \wedge \sim q) \vee (\sim r \wedge \sim q))$

Date → 04/Nov/22

Rule of inference

⇒ If you have password then you can log in the system.

P Q

S S

P P

Modus Ponens

$P \to Q$

P

$\therefore Q$

$P \wedge (P \to Q) \to Q$

T

⇒ 1st Rule

$P_1 \wedge P_2 \wedge P_3 \ldots\ldots P_n = C_n (C)$ {Rule of Inference System}

Suppose there are n Premises $P_1, P_2, P_3 \ldots P_n$ which leads to a conclusion C then our conclusion can be 'valid or invalid'.

2nd Rule (MODUS TOLLEN)

$$\begin{array}{l} \neg Q \\ P \rightarrow Q \\ \hline \therefore \neg P \end{array}$$

$$\boxed{\neg Q \wedge (P \rightarrow Q) \longrightarrow \neg P}$$

Assume That condition Statement

i) → If tanu gets a 1st class then tanu will get a good job.

and its hypothesis tanu gets a 1st class is true. then what do you inference from This Statement?

⇒ MODUS PONIS

Q2) Determine whether the argument is valid or not.

if $\sqrt{3} > \frac{7}{4}$ then $(\sqrt{3})^2 > \left(\frac{7}{4}\right)^2$

consequently ; $3 > \frac{49}{16}$

⇒ By MODUS Tollen.

⇒ Hypothetical Syllogism

$p \to q$
$q \to r$
$\therefore p \to r$

for ex ⇒ consider a fro
If Hari's studies sincerely, Hari will obtain a 1st class; if Hari will obtain a 1st class, Hari will get a good job.
check the validity whether Hari will get a good job or not.

⇒ Disjunctive Syllogism

$p \vee q$
$\sim p$
$\therefore q$

Ex ⇒ consider a proposition "Either it is raining or it is windy the proposition expresses that raining is not happening therefore it is windy. ~~Prov~~ check the validity of this proposition.

⇒ By Prove of Disjunctive Syllogism.
⇒ It is true.

Genereting fn.

⇒ Let A be any sequence with terms $a_0, a_1, a_2, \cdots$

G.F. (A, z) of a seq^n A is infinite series.

$$G(A,z) = \sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + \cdots \infty$$

① $a_n = c : n \geq 0$

$$= \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} c z^n$$

$$= c \sum_{n=0}^{\infty} z^n$$

$$= c(z^0 + z^1 + z^2 + z^3 \cdots \infty)$$

$$= c\left(\frac{z^0}{1-z}\right) = \frac{c \cdot 1}{1-z} = \left(\frac{c}{1-z}\right) \quad \left\{ \because G.P(\infty) = \frac{a}{1-r} \right\}$$

② ~~br~~ $a_n = b^n ; \; n \geq 0$

$$= \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} b^n z^n$$

$$= \sum_{n=0}^{\infty} (bz)^n$$

$$= (bz)^0 + (bz) + (bz)^2 + \cdots (bz)^n$$

$$= \frac{1}{(1-bz)}$$

(3) $a_n = cb^n; \quad n \geq 0$

$$= \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} cb^n z^n$$

$$= c\left(\frac{1}{1-bz}\right)$$

$$= \frac{c}{(1-bz)}$$

(4) $a_n = n; \quad n \geq 0$

$$G(A,z) = \sum_{n=0}^{\infty} a_n z^n$$

$$\sum_{n=0}^{\infty} n z^n$$

$$= 0 + 1z + 2z^2 + 3z^3 \ldots \infty$$

$$= z(1 + 2z^1 + 3z^2 \ldots \infty)$$

$$= \frac{z}{(1-z)^2}$$

Ref. Relation

Let given R.R is

$$c_0 a_n + c_1 a_{n-1} + \ldots c_K a_{n-K} = 0$$

for $n \geq K$.

Step 1;

Multiply both sides by z^n & sum up from $n=K$ to ∞

Step 2;

Write each term in terms of $G(A,z)$.

Step 3;

Solve $G(A,z)$ then using standard Generating f^n Seq^n a_n can be found.

Q) Find G.F & seq^n of R.R.

$$a_n + 2a_{n-1} = 0 \; ; \; a_0 = 5$$

$$a_1 = -10 \; , \; a_2 = 20$$

a

$5, -10, 20, -40, 80 \cdots$

$5[1, -2, 4, -8, 16$

$[1 + (-2)^1 + 2^2 + (-2)^3 + 2^4$

$\cancel{A(z) + 2[a_0 + a_1 z + a_2 z^2 + a_3 z^3 \cdots]}$

$$(a_1 z + a_2 z^2 \cdots) + 2(a_0 + a_1 z + a_2 z^2 \cdots) = 0$$

$$A(z) - a_0 + 2(A(z)) = 0$$

$$3A(z) = 5$$

$$\boxed{A(z) = \frac{5}{3}}$$