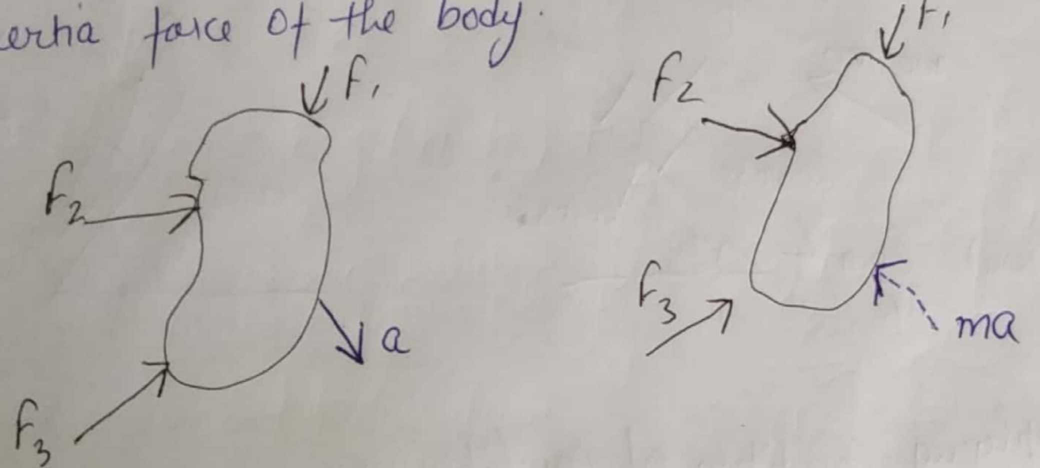
Kinetics

D'Alembert's Principle - "The system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body.



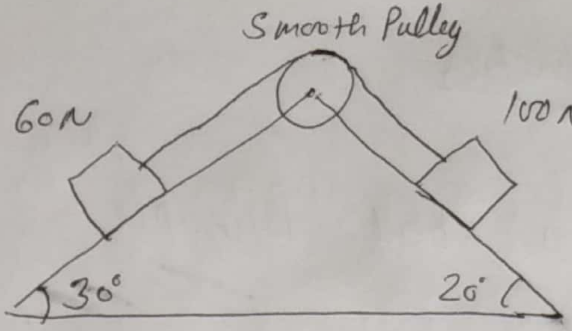
For dynamic equilibrium of the body, the sum of the resultant force and the reversed force should be zero.

$$\Sigma F = 0$$

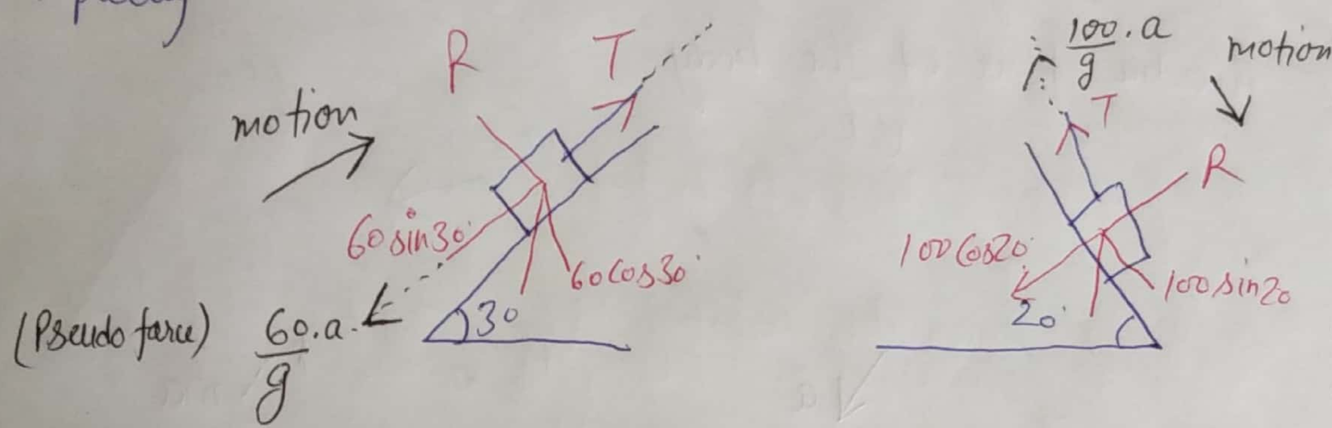
$$R - ma = 0$$

[R = Resultant force
ma = inertia force]

Q Calculate the common acc. of the system; tension in the string and pressure on the pulley.



Sol:



By applying condition of equilibrium.

$$\cancel{T - W_2} \quad T - 60\sin 30 - \frac{60}{g}\cdot a = 0 \qquad \text{— (i)}$$

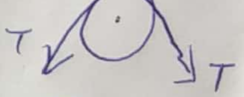
$$T + \frac{100}{g}\cdot a - 100\sin 20 = 0 \qquad \text{— (ii)}$$

from (i) and (ii)

$$\boxed{\begin{aligned} T &= 31.58\,N \\ a &= 6.2575\ m/s^2 \end{aligned}}$$

Pressure on the pulley = Resultant force due to string

$$= \sqrt{T^2 + T^2 + 2T^2\cos\{180 - (\alpha_1 + \alpha_2)\}}$$



$$= \sqrt{2T^2\left[1 - \cos(\alpha_1 + \alpha_2)\right]} = \sqrt{2T^2 \cdot 2\sin^2\frac{\alpha_1 + \alpha_2}{2}}$$

$$= 2T\sin\frac{\alpha_1 + \alpha_2}{2}$$

$$\boxed{P = 26.69\,N}$$

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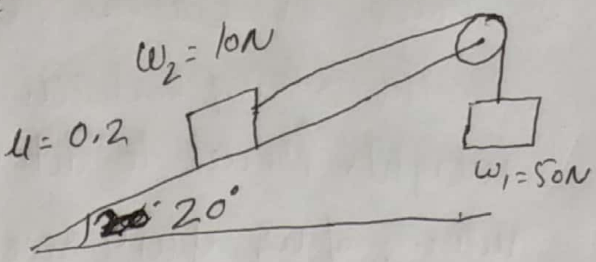
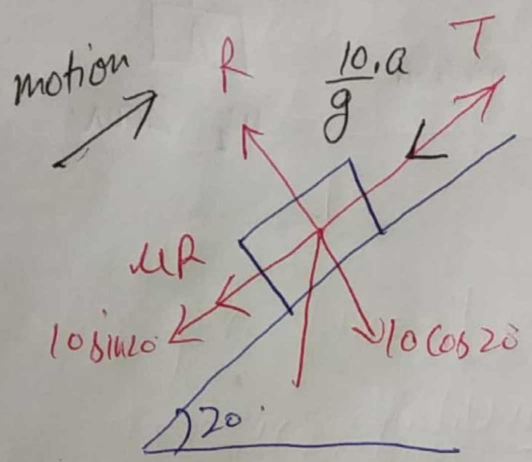
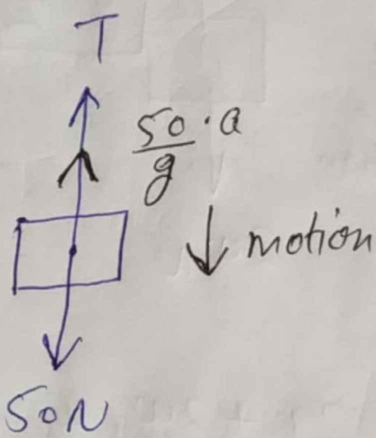
Q Make calculation for the acc. of system, tension in string, reaction at pulley and the distance moved by the body in 3 sec. starting from rest.



Sol.





$$T = \frac{10}{g}.a + \mu R + 10 \sin 20 \quad -(i)$$

$$R = 10 \cos 20° \quad -(ii)$$

Also

$$T + \frac{50}{g}a = 50 \quad -(iii)$$

from (i), (ii), (iii)

$$\boxed{\begin{array}{l} a = 7.31 \ m/s^2 \\ T = 12.74 \ N \end{array}}$$

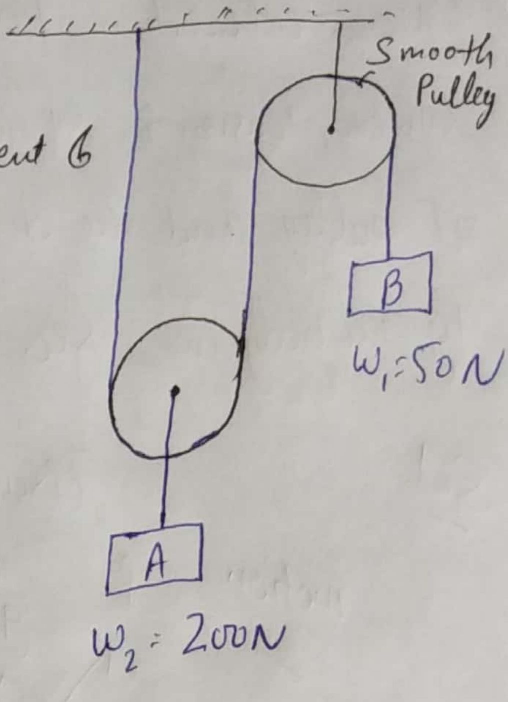
Reaction $= \sqrt{T^2+T^2+2T^2 \cos(90-20)} = \boxed{20.87N}$

By using relation $s = ut + \frac{1}{2}at^2$

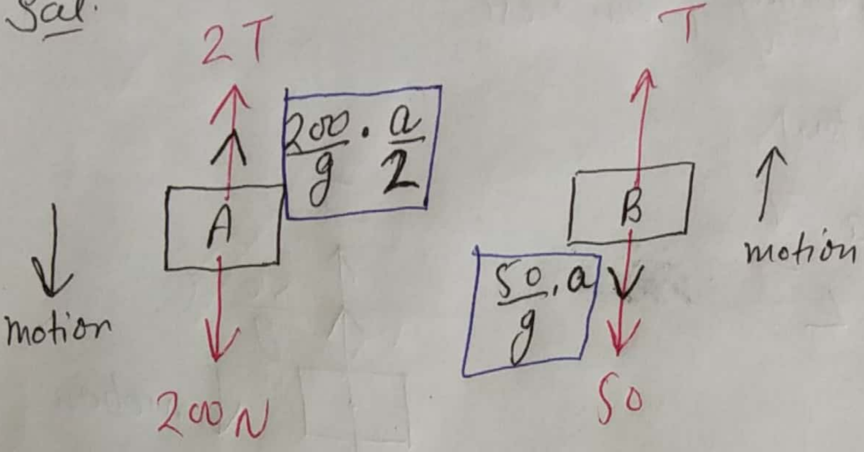
$$\boxed{s = 32.89 \ m}$$

$[u = 0]$

Q. Make calculations for the tension in the string and acc of the weights. Proceed to determine displacement & velocity of w_1 after 4 seconds from start.



Sol.



$$2T + \frac{200}{g}\cdot\frac{a}{2} = 200 \quad \text{-(i)}$$

$$T = \frac{50}{g}\cdot a + 50 \quad \text{-(ii)}$$

from (i) and (ii)

$$\boxed{\begin{aligned} a &= 4.905\ m/s^2 \\ T &= 75\ N \end{aligned}}$$

By using Kinematic relation

$$v = u + at \qquad [\because u = 0]$$

$$v = at = \boxed{19.62\ m/s}$$

$$s = ut + \frac{1}{2}at^2$$

$$\boxed{s = 39.24\ m}$$

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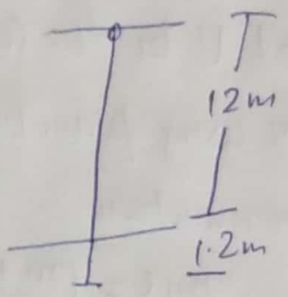
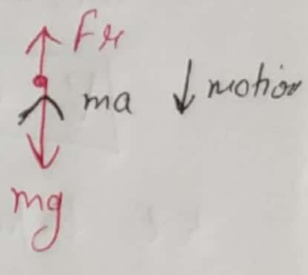
Q A body of weight 100 N falls from a height of 12 m ③
on a sand bed. It is estimated that the body
penerates 1.2 m into sand before coming to rest. Make
calculations for the avg. thrust exerted by the sand
on the body.



Sol.

Velocity of body at ground $= \sqrt{2gh}$
$= 15.34$ m/s



After penetration $v = 0$

$v^2 - u^2 = 2as$

$\boxed{a = -98.05 \text{ m/s}^2}$

Let F_r be avg. resistance

$F_r + ma = mg$

$F_r + \frac{100}{9.81}(-98.05) = 100$

$\boxed{F_r = 1100 \text{ N}}$

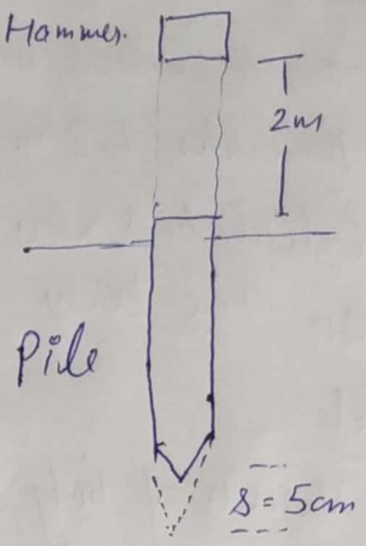
Q A hammer of mass 200 kg is made to fall freely from
2 m height on the head of a pile of 1500 kg mass.
The pile is driven 5 cm into the ground in one blow. Determine
a) the common velocity of pile and hammer after impact
b) the energy lost in the impact.
c) the avg. resistance of the ground to penetration.

• Sol. $m = 200\,kg$
$M = 1500\,kg$

Let v be the velocity of hammer just before impact.

$v^2 - u^2 = 2gh$

$v = 6.26\,m/s$



Let V be the common velocity

By using principle of conservation of momentum.

$mv = (m+M)V$

$\boxed{V = 0.736\,m/s}$

K.E. before impact $= \frac{1}{2} mv^2 = \frac{1}{2} \cdot 200 \cdot (6.26)^2$

$= 3918.76\,Nm$

K.E. after impact $= \frac{1}{2}(m+M)V^2 = 460.44\,Nm$

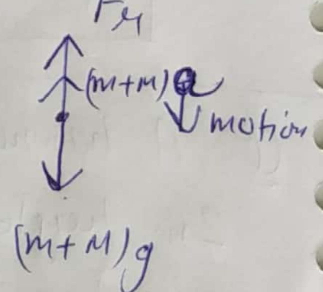
$\boxed{\Delta K.E = 3458.32\,Nm}$

After peneration, final velocity will be zero

$v^2 - u^2 = 2as$

$a = -5.417\,m/s^2$

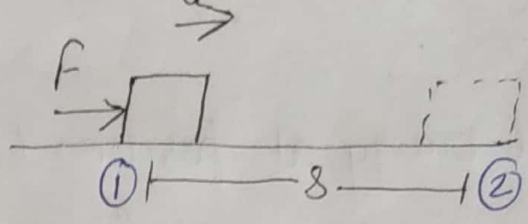
Let F_r be the avg. resistance.



$F_r + (m+M)a = (m+M)g$

$\boxed{F_r = 25885.8N}$

Work Energy Principle:



For an elemental distance ds travelled by the object, the work done would be

$$dw = F \cdot ds$$

But

$$F = ma$$

$$= mv\frac{dv}{ds} \qquad \left[\because a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v\frac{dv}{ds}\right]$$

$$\therefore \quad dw = mv\,dv$$

$$W_{1-2} = m\left|\frac{v^2}{2}\right|_1^2$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\boxed{W_{1-2} = \Delta K.E.}$$

Acc. to this principle, the work done on the objects equals the change in kinetic energy of the object.

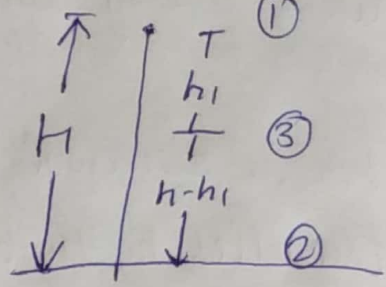
Conservation of mechanical energy

"The energy can neither be created nor destroyed but it can be transformed from one form to another."

OR

The total energy possessed by an object remains constant provided no energy is added or subtracted from it.

Proof:



Total energy at position 1

$= mgh$ $[K.E = 0]$

Consider position 3.

Total energy $= mg(h-h_1) + \frac{1}{2} m v_3^2$ — (i)

By using kinematic relation $v_3^2 = 2gh_1$ —(ii)

from (i) and (ii)

Total energy $= mgh - mgh_1 + mgh_1$

$= mgh$

Consider position 2.

Total energy $= \frac{1}{2} m v_2^2$ $[P.E = 0]$

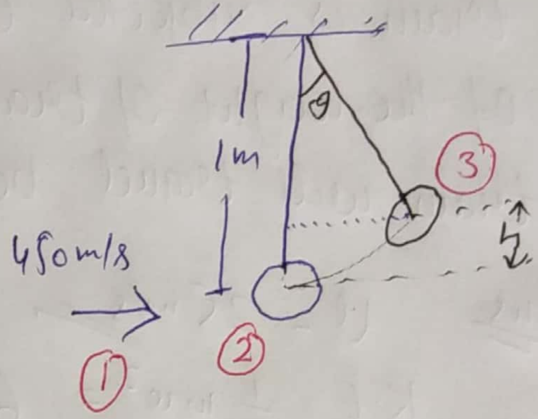
$v_2^2 = 2gh$

Total energy $= \frac{1}{2} m(2gh)$

$= mgh$

It illustrate that the sum of K.E and P.E remains constant throughout the motion.

Q A body of mass 10 kg is suspended by a string of length 1m. It is struck by a bullet travelling horizontally with a velocity of 450 m/s. The bullet weighs 30 gm and gets embedded into the body after striking it. Determine the maximum angle through which the body swings.



Sol.

By using the principle of Conservation of momentum

$$m_1 u_1 = (m_1 + m_2) V$$

$$\boxed{V = 1.346 \text{ m/s}}$$

Now from principle of Conservation of energy for position 2 and 3.

$$\frac{1}{2}(m_1 + m_2)V^2 = (m_1 + m_2) g h$$

$$\boxed{h = 0.0923 \text{ m}}$$

by using geometry of diagram

$$\cos\theta = \frac{l - h}{l}$$

$$\boxed{\theta = 24.81^\circ}$$

Q A train weighing $2 \times 10^6 N$ starts from rest with an acceleration of $0.8 \ m/s^2$ and acquire a speed of 90 Km/hr. Determine the kinetic energy corresponding to final speed and the avg. power required.

Subsequently the power is shut off and the train is subjected to a retarding force equal to 8% of the weight of train. Calculate the distance the train will travel before coming to rest.

Ans. $v = 25 \ m/s$

$K.E = \frac{1}{2} m v^2 = 63.71 \times 10^6 \ Nm$

As we know

$v = u + at$

$\boxed{t = 31.25 \ sec}$ $[u = 0]$

Work done = Change in K.E.

Power $= \frac{W}{t} = \frac{63.71 \times 10^6}{31.25} = \boxed{2.039 MW}$

Retarding force $= 8\%$ of 2×10^6

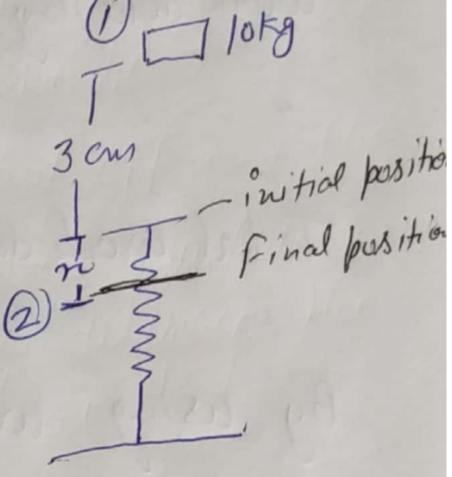
$= 0.16 \times 10^6 N$

By using work energy principle

$(0.16 \times 10^6) \cdot S = 63.71 \times 10^6$

$\boxed{S = 398.19 m}$

Q A body of mass 10 kg is made to fall 3 cm height on a spring of stiffness 120 N/cm. Find the displacement of spring.



Sol.

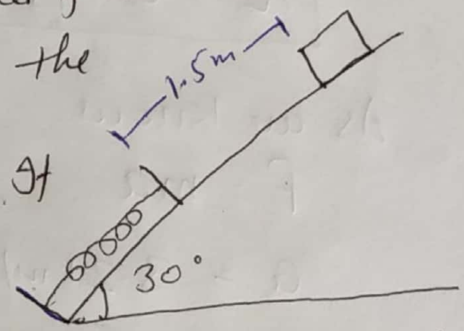
By using principle of conservation of energy

$$mg(3+x) = \frac{1}{2} K x^2$$

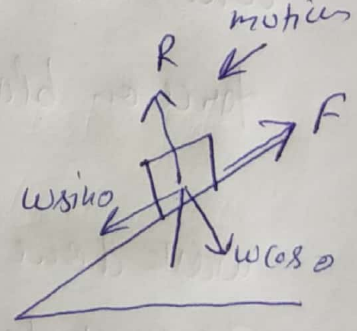
$$\boxed{x = 5.178\ cm}$$

$\mu = 0$
$m = 50\ kg$

Q A block of 50 kg mass is released from rest. After sliding 1.5m, the block hits the spring of spring constant 25 N/mm. If the $\mu = 0.2$, make calculation for the max. deformation of the spring and max. velocity of the block. Also find the distance the block will move up the plane due to rebound.



Sol.



The net force on the block when it starts moving downwards

$= W\sin\theta - F$

$= W\sin\theta - \mu R = 160.29\ N$

let spring compressed by x m

$\therefore$ distance moved by block $= (1.5 + x)$

work done by block $= 160.29(1.5 + x)$ — (i)

" " " spring $= -\frac{1}{2}Kx^2 = -12.5 \times 10^3 x^2$ — (ii)

Total work done $= 160.29(1.5 + x) - 12.5 \times 10^3 x^2$

By using work energy principle.

$160.29(1.5 + x) - 12.5 \times 10^3 x^2 = 0$ [K.E = 0]

$\boxed{x = 0.145 \text{ m}}$

As we know

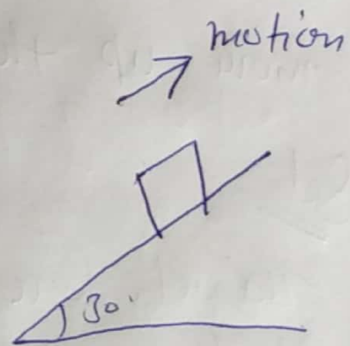
$F = ma$

$a = 3.21 \text{ m/s}^2$

$v^2 - u^2 = 2as$

$\boxed{v = 3.1 \text{ m/s}}$ [at the time of hit]

when the block rebounds



force on block $= W\sin\theta + f$

$= 330.21$ N

work done $= 330.21 \times s$

The work done equals the energy stored in the spring.

$\frac{1}{2}Kx^2 = 330.21 \times s$

$\boxed{s = 0.796 \text{ m}}$

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Conservative force : If the work of force in moving an object between two positions is independent of the path followed by the object and can be expressed as a change in its potential energy, then such a force is called a conservative force.

eg - gravity force, elastic force, spring force etc.