Engineering Mechanics

Study of motion of rigid bodies under the action of forces.

- Statics (rest)
  - Force and Moment system
  - Equilibrium
  - Plane Trusses
  - principal of virtual work
- Dynamics (motion)
  - Kinematics ($\vec{r}, \vec{V}, \vec{a}$ etc)
  - Kinetics ($\vec{r}, \vec{V}, \vec{a}$ and $\vec{F}$)

Dynamics

- Rectilinear Translation
- Friction
- circular motion
- Impulse- Momentum equation
- work energy Theorem
- impacts / collisions
- Rotation
- General motion

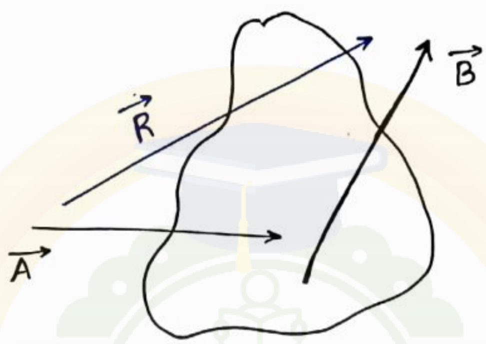
Vector Algebra :-

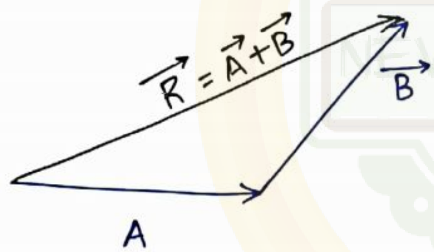
Vector :→ Those physical quantity which have magn. as well as dirn & follows Δ law of vector addn. are called vector quantities.

Ex:- velocity vector $\vec{v}$, $\vec{a}$, $\vec{T}$ etc.

Triangle Law of Vector addition →

$$\vec{R} = \vec{A} + \vec{B}$$

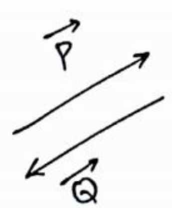




✓ $\vec{A} + \vec{B} + \vec{C} = 0$

$$\vec{C} = -(\vec{A} + \vec{B})$$

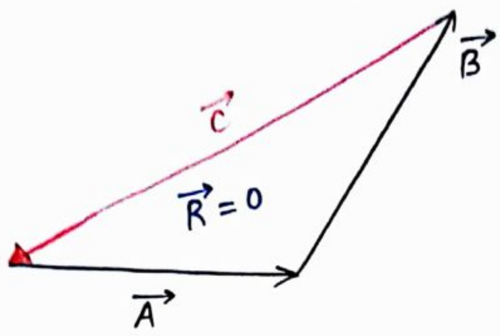
✓



$|\vec{P}| = |\vec{Q}|$ $(P = Q)$

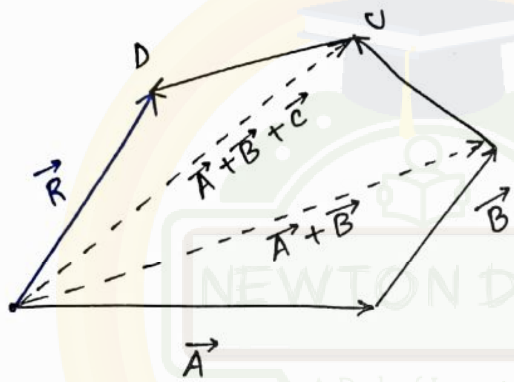
$\vec{P} = -\vec{Q}$

$\vec{P} + \vec{Q} = 0$

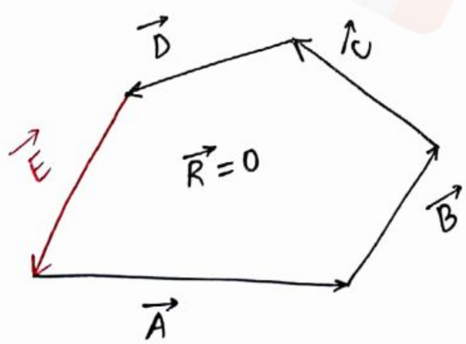


Polygon Law →

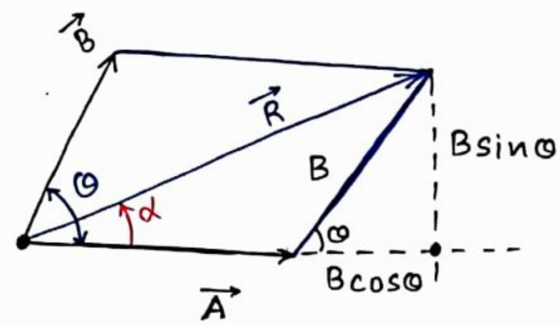
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$



$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = 0$$



Parallelogram Law :-



$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

✓ vector = mag(dirn)

✓ unit vector = 1(dirn.) ≡ dirn.

$\longrightarrow$ $\hat{i}$

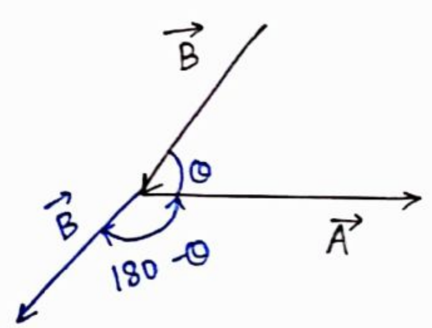
5m/s

$\vec{V} = 5\hat{i}$

✓ $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

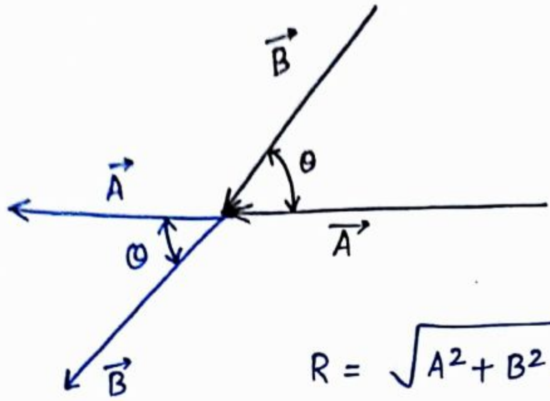
✓



$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ ✗

$R = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$

$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$



$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Product of Two Vectors :-

$\vec{A}$ and $\vec{B}$

1. Dot Product (Scalar Product) →

$\vec{A} \cdot \vec{B} = C$

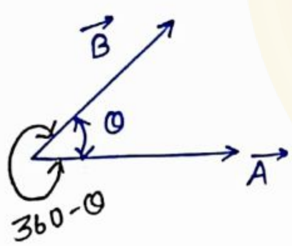
$C = \underline{AB\cos\theta}$

$\vec{B} \cdot \vec{A} = D$

$D = \underline{BA\cos\theta}$

$C = D$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

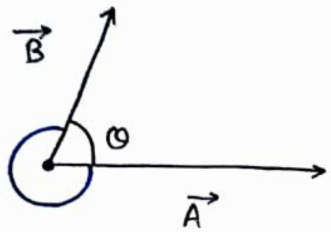
(ii.) Cross Product (vector Product) →

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{C} = |\vec{C}| \, \hat{C}$$

$$C = AB\sin\theta$$

$\theta < 180^\circ$



Right hand thumb Rule,

$\hat{C} \rightarrow \perp r$ to the plane, outward

$$\vec{B} \times \vec{A} = \vec{D}$$

$$\vec{D} = |\vec{D}| \, \hat{D}$$

$$|\vec{D}| = BA\sin\theta$$

$\hat{D} \rightarrow \perp r$, inward

$$\vec{C} = -\vec{D}$$

$$(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Direction of $\vec{C}$ will be $\perp r$ to the plane containing $\vec{A}$ and $\vec{B}$ and will be decided by Right hand thumb rule.

Q The Resultant of 2 vectors $\vec{A}$ & $\vec{B}$ is R. If $\vec{B}$ is doubled, then the new Resultant is ⊥r to $\vec{A}$, then :-

(a) $A = B$

(b) $A = R$

(c) $B = R$

(d) None.

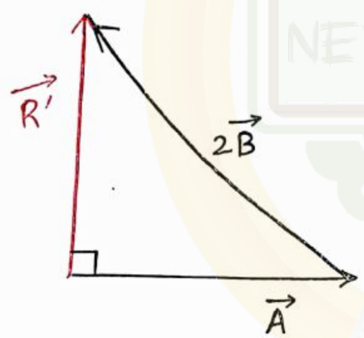
Sol:>

A & $B \longrightarrow R$ (θ)

$A^2 + B^2 + 2AB\cos\theta = R^2$ —— (1)

A and $2B \longrightarrow R'$ is ⊥r to A (θ)

$A^2 + 4B^2 + 4AB\cos\theta = (R')^2$ —— (2)



$A^2 + (R')^2 = 4B^2$ —— (3)

from (2) and (3)

$$A^2 + \cancel{4B^2} + 4AB\cos\theta = \cancel{4B^2} - A^2$$

$$\cancel{4}_{2}\,AB\cos\theta = -\cancel{2}A^2$$

$$2AB\cos\theta = -A^2 \longrightarrow (4)$$

using value of eqn. (4) in eqn. (1)

$$\cancel{A^2} + B^2 - \cancel{A^2} = R^2$$

$$\boxed{B = R}$$

Q The Resultant of 2 vectors when acting at Right angles is 10 KN. If they act as 60°, their resultant is $5\sqrt{6}$ KN. The magnitude of Individual vectors are ?

Sol

A & B ; $\theta = 90°$; $R = 10\,KN$

$$A^2 + B^2 = 10^2 = 100 \longrightarrow (1)$$

A & B ; $\theta = 60°$; $R = 5\sqrt{6}\,KN$

$$A^2 + B^2 + AB = (5\sqrt{6})^2 = 150 \longrightarrow (2)$$

from (1) and (2)

$$AB = 50 \longrightarrow (3)$$

$$(A+B)^2 = A^2 + B^2 + 2AB$$

$$= 100 + 2(50)$$

$$= 200$$

$$A + B = \sqrt{200}$$

$$\boxed{A + B = 10\sqrt{2}}$$

$$(A-B)^2 = A^2 + B^2 - 2AB$$

$$= 100 - 100$$

$$= 0$$

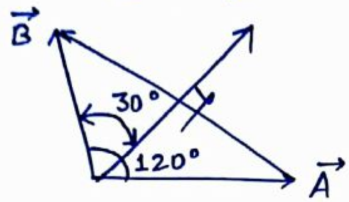
$$A - B = 0$$

$$\boxed{A = B}$$

$$2A = 10\sqrt{2}$$

$$\boxed{A = 5\sqrt{2}\,KN = B}$$

Q. Two forces are acting at a point making an angle of 120° with each other. If the Resultant force is ⊥r to the smaller force (smaller force = 50 N), the larger force is (100 N)



* FORCE :-

to define a force → mag. / dirn. / Point of Application

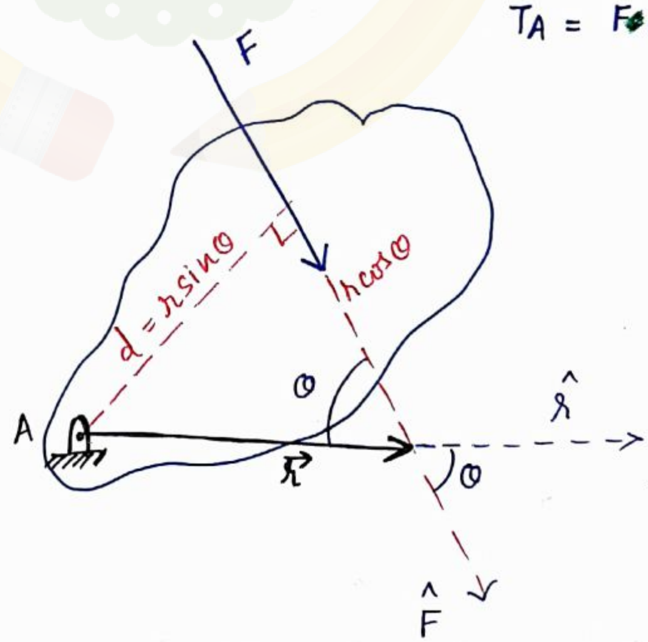
* MOMENT of a force (Torque) :→

$\vec{T_A} = \vec{r} \times \vec{F}$

$T_A = rF\sin\theta$

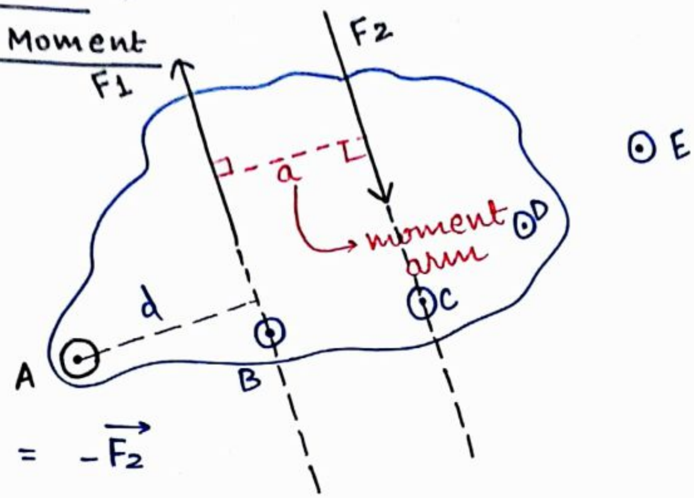
Dirn. → ⊥r inward through A

$T_A = F \cdot d$;(CW) clockwise



→ here $\perp^r$ inward dir^n through A → ↻ ← fingers dir^n.

* COUPLE →
& its Moment



$\vec{F_1} = -\vec{F_2}$

$F_1 = F_2 = F$

$\boxed{\Sigma \vec{F} = 0}$ ← only netforce = 0 but not moment

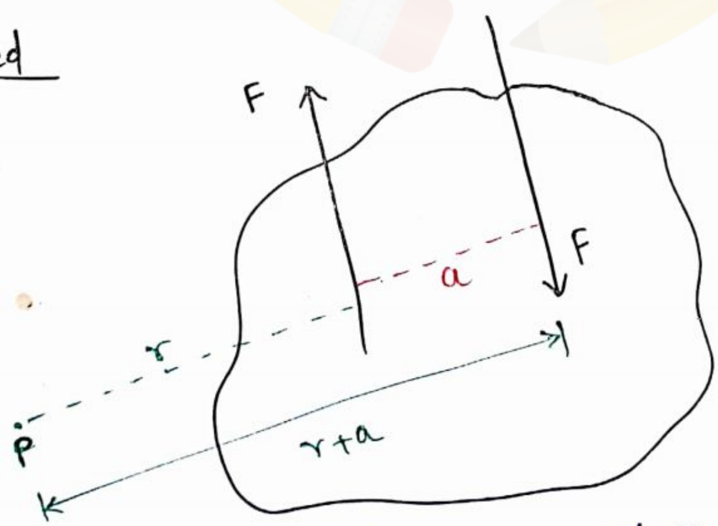
+ inward ; − outward

$M_A = -Fd + F(a+d) = Fa$; cw

$M_B = Fa$; cw

$M_C = Fa$; cw $= M_D = M_E$

Simplified



Moment of a couple = one of the force × moment arm

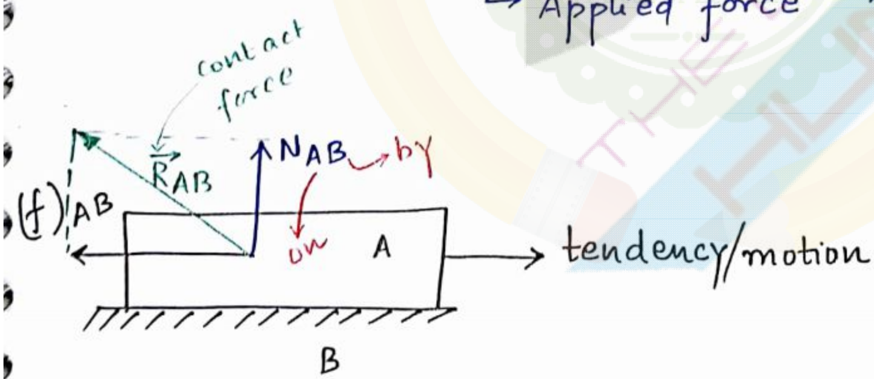
= Fa ; cw → (clockwise)

Couple vector → $\perp^r$ to the plane of couple and will be decided by Right hand thumb rule.

✓ Couple is an arrangement of two equal and opposite forces acting $\parallel$ to each other and whose moment remains uniform throughout on the body on which it is acting.

* Types of Forces :-

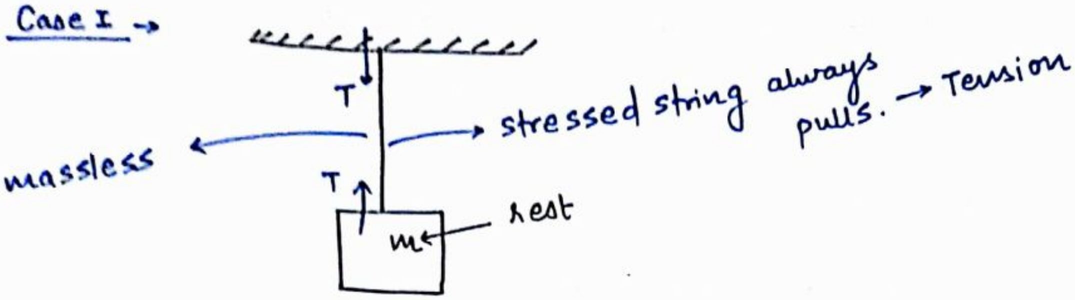
- → Gravity → $[wt = mg\downarrow]$
- → contact force
  - → Normal Rxn
  - → friction
- → Tension
- → Spring force → $F = Kx$.
- → Applied force → given.



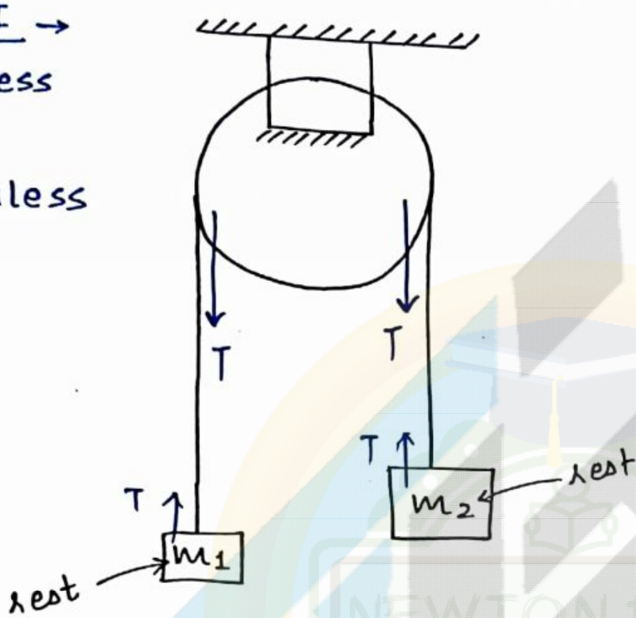
N on A by B

Tension
- case 1.
- case 2.

Case I →



Case II →
massless
and
frictionless



String offered Tension

mass of both not concern

String pulls in stressed condn. → stretched

Load v/s Force

Load → Stress generators

Force → motion generators.

* NEWTON'S 1st LAW :-

for a particle → if Resultant of forces

i.e. if $\vec{F_R} = 0$ then $\vec{a} = 0$

→ Rest

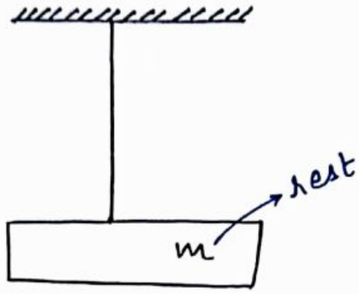
→ uniform velocity

[CM → centre of mass]

for a Rigid Body →

if $(\vec{F_R})_{ext} = 0$ then $\vec{a}_{CM} = 0$

✓ **Case I** :-



(m)

$\vec{a}_{CM} = 0$

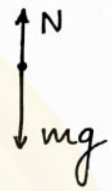
↳ $(\vec{F_R})_{ext} = 0$

$T = mg \longrightarrow 1^{st}$ law

✓ **Case II** :-

(m)

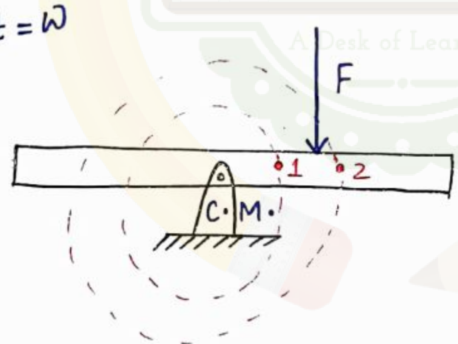
$\vec{a}_{CM} = 0 \Rightarrow (\vec{F_R})_{ext} = 0$



$N = mg \longrightarrow 1^{st}$ law

✓ **Case III** :- Wt = W

Rxn at hinge for the instant shown = ?



$\left.\begin{array}{l} a_1 \neq 0 \\ a_2 \neq 0 \end{array}\right\}$ since N-1st law guarentees for only accl^r. of C.M. not for particle

$a_1 \neq a_2$

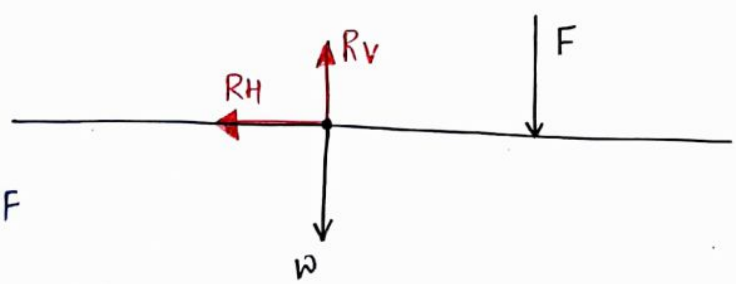
$\vec{a}_{CM} = 0$ [Bcoz C.M. is lying on axis of rotation]

$\Sigma \vec{F_H} = 0$

$R_H = 0$

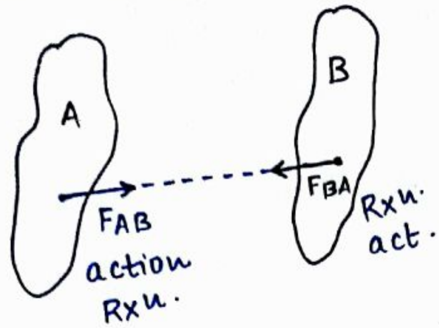
$\Sigma \vec{F_V} = 0$

$R_V = W + F$



$\left.\begin{array}{l} R_V \\ R_H \end{array}\right\rangle$ independent Rxn.

* NEWTON'S 3rd LAW :-



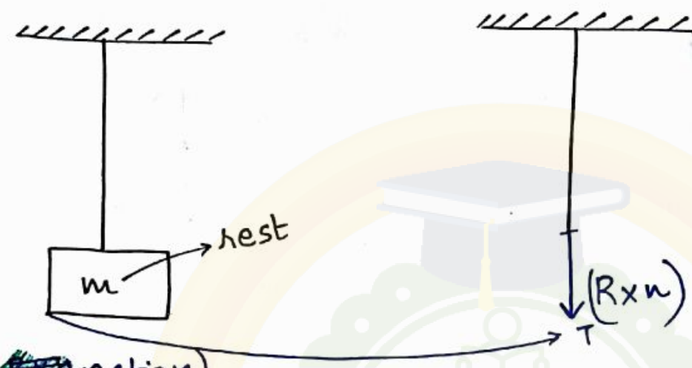
same in nature and opp. dirn

pulling or pushing in both

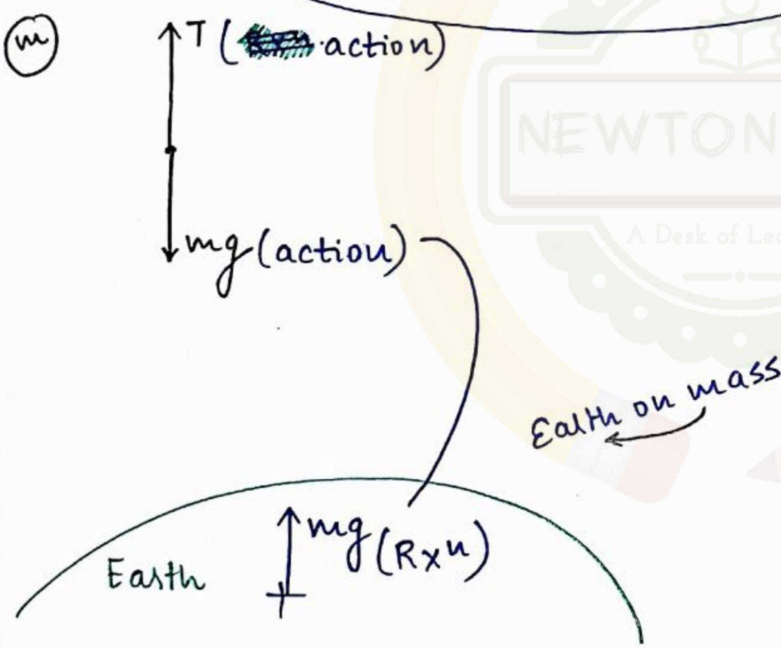
dirn. +, -

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

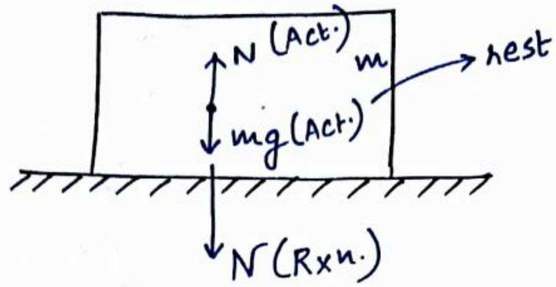
Case . I :→

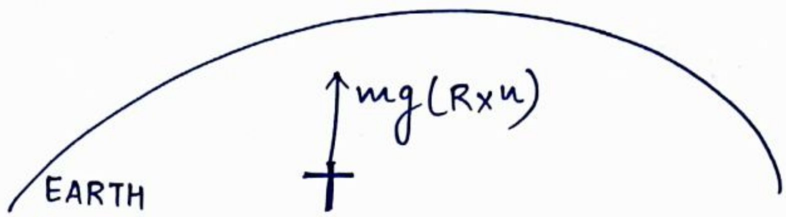


mass pulls the string



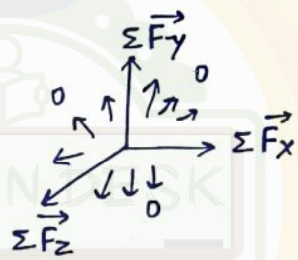
Case II :-





$\ast$ **EQUILIBRIUM** :-
- → rest ✓✓ (mechanics)
- → uniform velocity → (FM)

for a Rigid body



1. $\Sigma\vec{F} = 0$

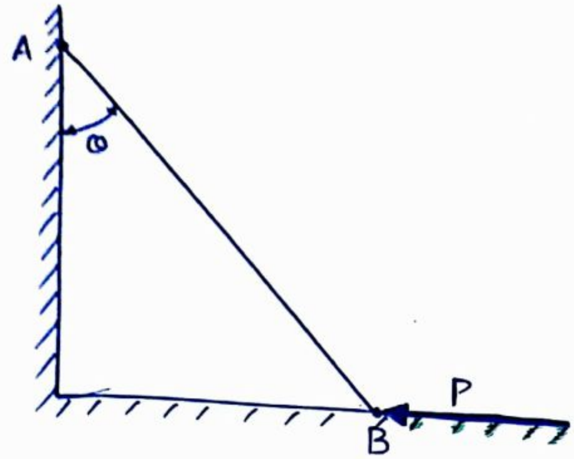
$$\Sigma\vec{F_x} = \Sigma\vec{F_y} = \Sigma\vec{F_z} = 0$$

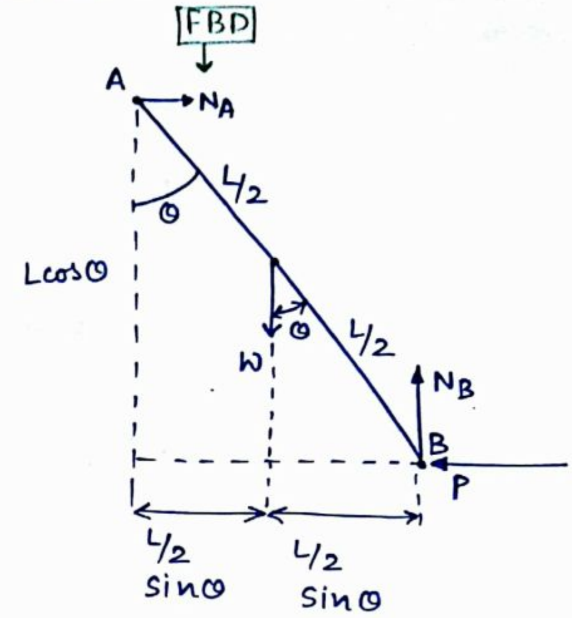
2. $\Sigma\vec{M} = 0$

↳ about any point or line in space.

Q. A ladder AB of weight W and length L is held in eqm. by a horizontal force P as shown in figure. assume ladder to be uniform body and all sufaces smooth. Find P.

Sol





$\sum \vec{F_H} = 0$

$P = N_A \longrightarrow (1)$

$\sum \vec{F_V} = 0$

$W = N_B \longrightarrow (2)$

$\sum \vec{M_B} = 0$

$N_A \times \cancel{L} \cos\theta = W \times \cancel{L}/2 \sin\theta$

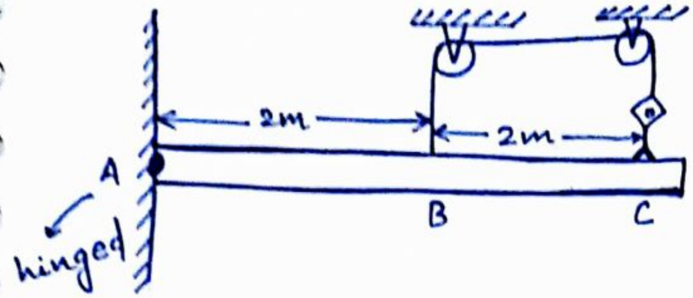
$N_A = \frac{W}{2} \tan\theta = P$

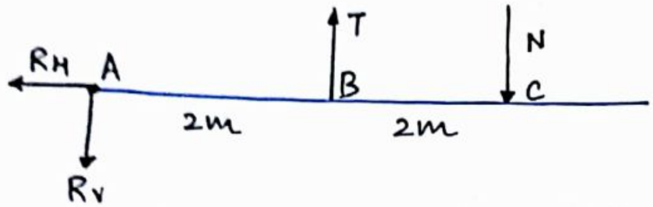
2nd method :- Moment of CW couple = Moment of ACW couple

$P \cancel{L} \cos\theta = \frac{W \times \cancel{L}}{2} \sin\theta$

$$\boxed{P = \frac{W}{2} \tan\theta}$$

Q. A man weighing 600N stands on a horizontal beam of negligible weight at 'C' and holds a string passing over 2 smooth pulleys and attached to the point 'B' on the beam as shown in figure. The tension in the string is?





man

$T + N = wt$

$T + N = 600$

$N = 600 - T$ —— (2)

$\Sigma \vec{M}_A = 0$

$T \times 2 = N \times 4$

$T = 2N$ —— (1)

$T = 2(600 - T)$

$T = 1200 - 2T$

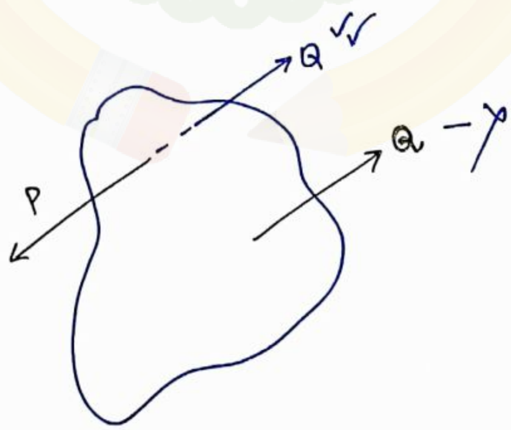
$T = 400N$

* Equilibrium → CASE I → Two-force system →

$\vec{P}$ and $\vec{Q}$

1. $\vec{P} + \vec{Q} = 0$

$\vec{P} = -\vec{Q}$

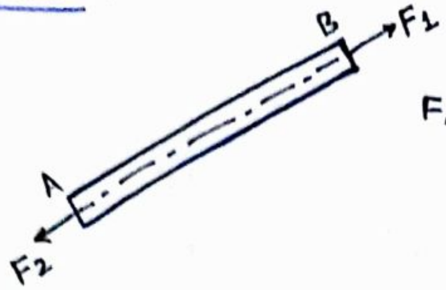


2. $\Sigma \vec{M} = 0$

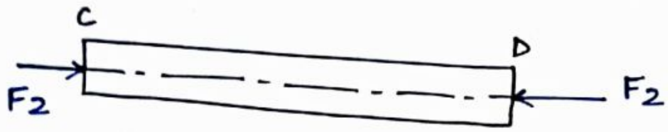
↳ collinear (same line में same points)

To keep 2 forces in equilibrium, they must be equal in magnit. opposite in dirn. and collinear in action.

Application →



$F_{AB} = F_1$ (Tensile)



$F_{CD} = F_2$ (compressive)

Case II :→ Three - force system →

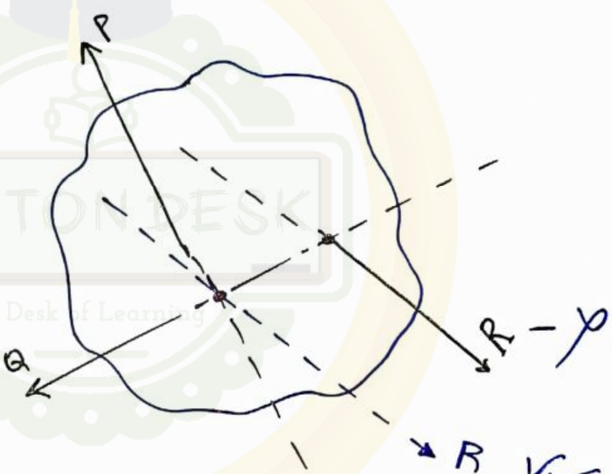
$\vec{P}$, $\vec{Q}$ and $\vec{R}$

1. $\vec{P} + \vec{Q} + \vec{R} = 0.$

↳ coplaner (same plane में)

2. $\Sigma \vec{M} = 0$

↳ concurrent (same point पे)

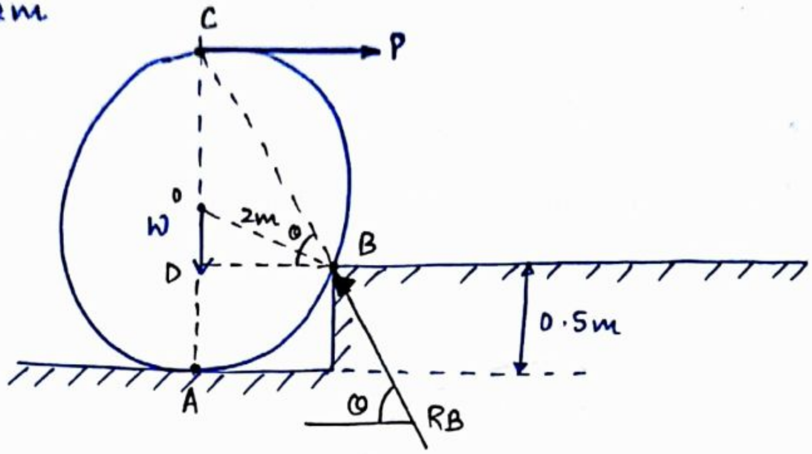


To keep three-forces in equilibrium, they must be coplaner and concurrent.

Q.> find the horizontal force 'P' required to move the cylinder out of the ~~dish~~ ditch as shown in fig?

Sol $W = 1000N$

$r = 2m$



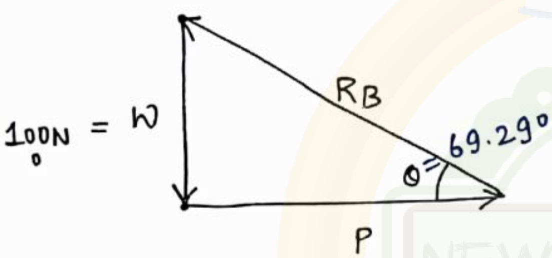
$\vec{W} + \vec{P} + \vec{R_B} = 0$

In ΔODB

$OD^2 + DB = AB^2$

$DB = \sqrt{2^2 - 1.5^2}$

$DB = 1.3228m$



In ΔBCD

$\tan\theta = \frac{CD}{DB} = \frac{3.5}{1.3228}$ ✓

$\theta = 69.29°$

$\frac{W}{P} = \tan\theta = \frac{3.5}{1.3228}$ ✓

$P = \frac{1000 \times 1.3228}{3.5} = 377.9N$

$\frac{W}{R_B} = \sin 69.29°$

$R_B = \frac{1000}{\sin(69.29°)}$

$R_B = 1069.08N$

(or) $\Sigma \vec{M}_B = 0$ — Since B ke about $R_B = 0$ ✓.

$P \times CD = W \times DB$

$P = \frac{1000 \times 1.3228}{3.5} = 377.9N$

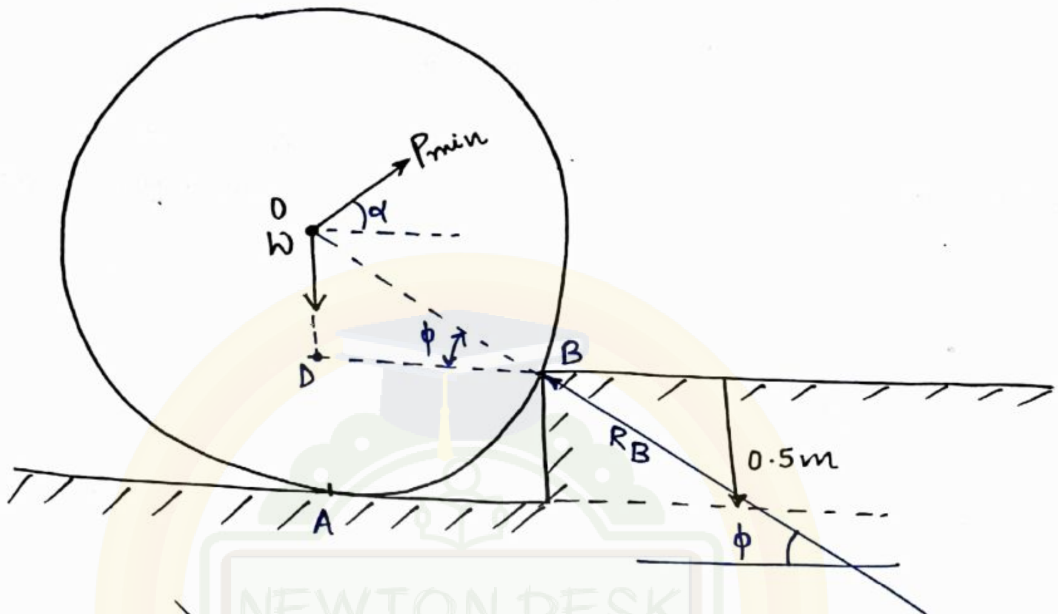
Note → • When the cylinder is about to move out of the ditch it will lose its contact at point A. The only contact will be at point B.

- To keep the cylinder under 'w', 'p' and 'RB', they must meet at a same point c.

Q Find the minimum force 'P' required to move the cylinder out of the ditch (small pit) as shown in the figure.

$W = 1000N$

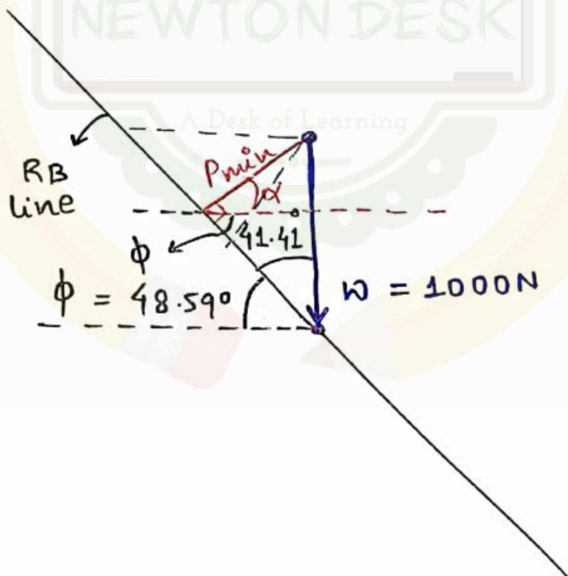
$r = 2m$

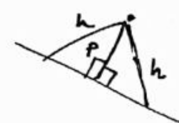


In ΔODB

$$\sin\phi = \frac{OD}{OB} = \frac{1.5}{2}$$

$$\phi = 48.59°$$





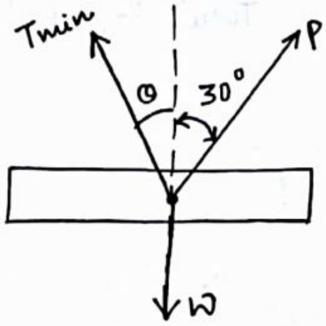
$$\frac{P_{min}}{W} = \sin 41.41°$$

$$P_{min} = 1000 \times \sin 41.41°$$

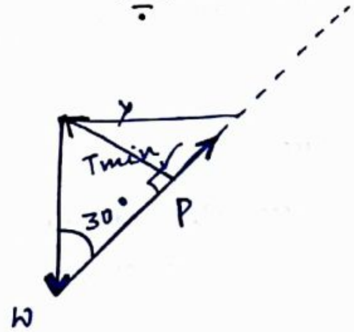
$$\underline{P_{min} = 661.44N}$$

$$\underline{\alpha = 90 - \phi = 41.41°}$$

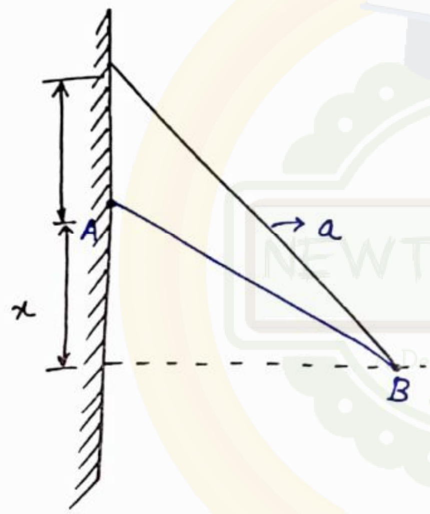
Q.> A Block of weight 300N is supported as shown in figure, for T to be minimum, the value of θ is



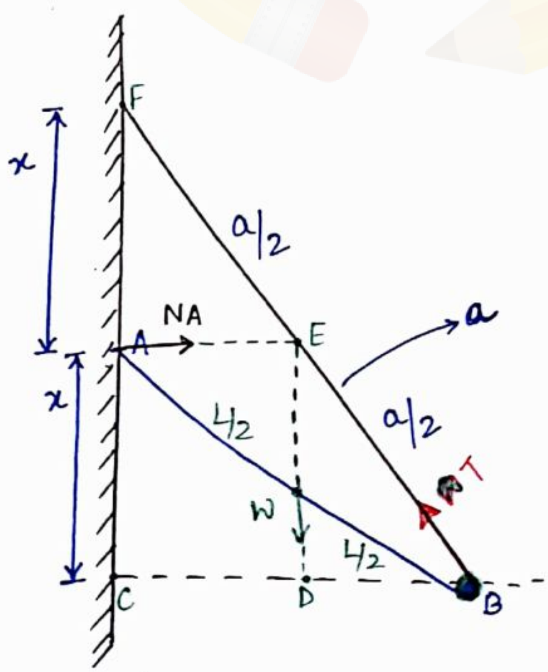
Ans → 60°



Q.> A uniform bar AB of length L is supported against a smooth vertical wall and by a wire of length 'a' as shown in figure. The value of x to keep the ~~syst~~ Bar in equ. is:- ?



Sol.>



AE = CD = DB

CB = 2AE

$$\sqrt{L^2-x^2} = 2\sqrt{a^2/4 - x^2}$$

$$L^2-x^2 = 4\left(a^2/4 - x^2\right)$$

$$\boxed{x = \sqrt{\frac{a^2-L^2}{3}}}$$

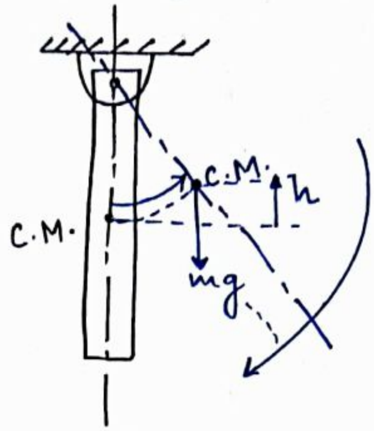
Total Potential Energy = V

* TYPES OF EQUILIBRIUM :-

(i.) Stable →

V = minimum

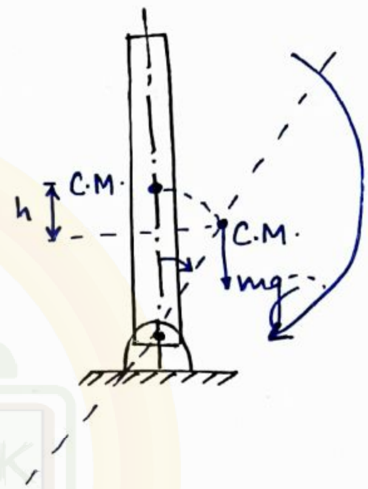
Restoring moments > Disturbing moments



(ii.) Unstable →

V = maximum

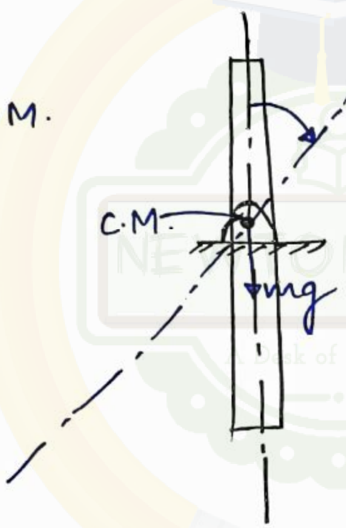
R.M. < D.M.



(iii.) Neutral →

V = const.

R.M. = D.M.

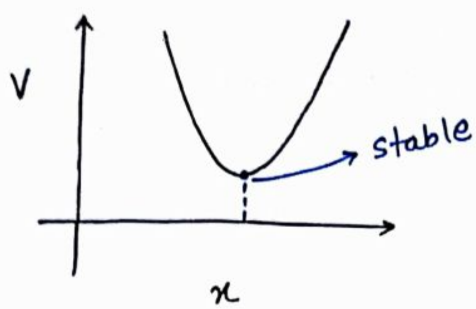


Case I → if system has single DOF

no. of independent variable = 1 (say x)

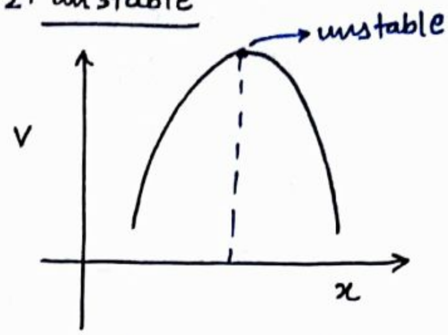
$$V = F(x)$$

1. Stable →



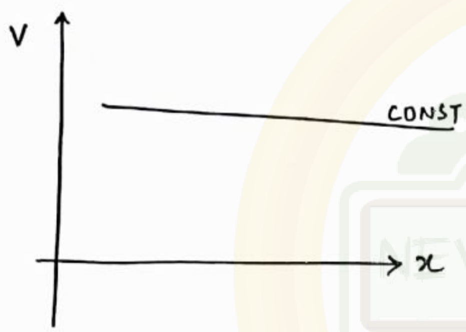
$$\frac{dv}{dx} = 0 \; ; \; \frac{d^2v}{dx^2} > 0$$

2. Unstable



$$\frac{dv}{dx} = 0 \; ; \; \frac{d^2v}{dx^2} < 0$$

3. Neutral



$$\frac{dv}{dx} = \frac{d^2v}{dx^2} = \frac{d^3v}{dx^3} = \text{-------} = 0.$$

Case II → If system has multiple DOF

no. of independent variable > 1 (say x, y, z, ----)

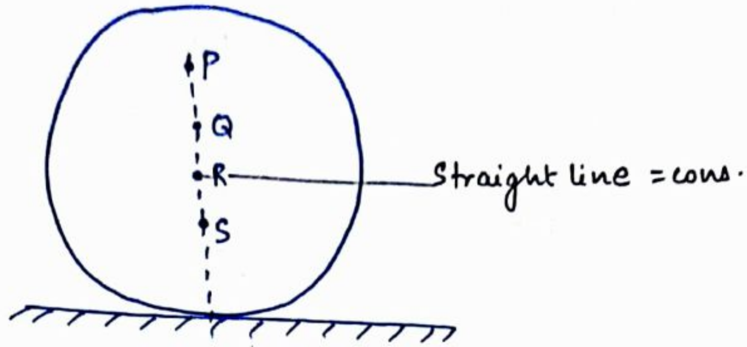
$$V = f(x, y, z, ----)$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0 = \text{---------} = 0 \; [\text{for equ}^m]$$

Q.) for the sphere shown in fig, its C.G. must lie at which point so as to keep the sphere in stable eqm.?

where R is geometric centre.

a) P
b) Q
c) R
d) S



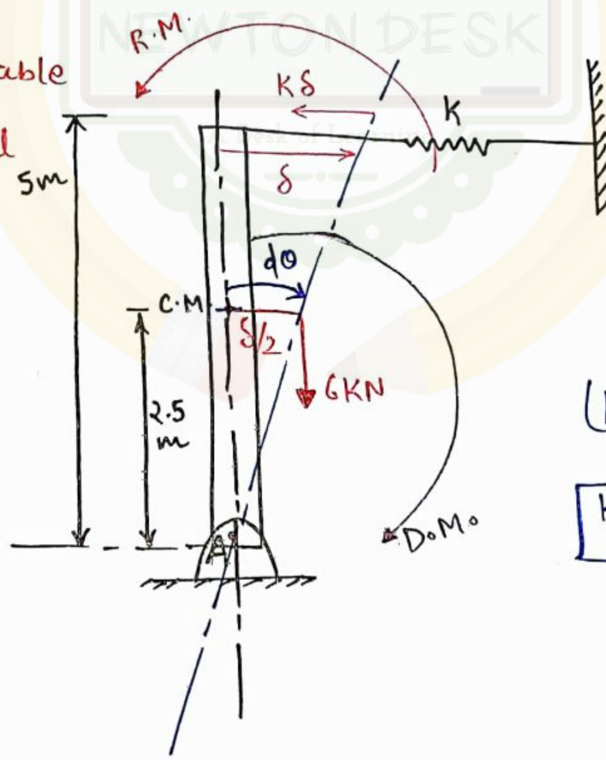
S → stable (V = min)

R → neutral (V = const.)

P, Q → unstable

Q. A column of weight 6 KN and length 5m is hinged at point 'A' and supported by a spring of stiffness 'k' as shown in fig. The value of 'k' to keep it in equilibrium is

a) K > 0.6 KN/m ✓
b) K < 0.6 KN/m → unstable
c) K = 0.6 KN/m → Neutral
d) None



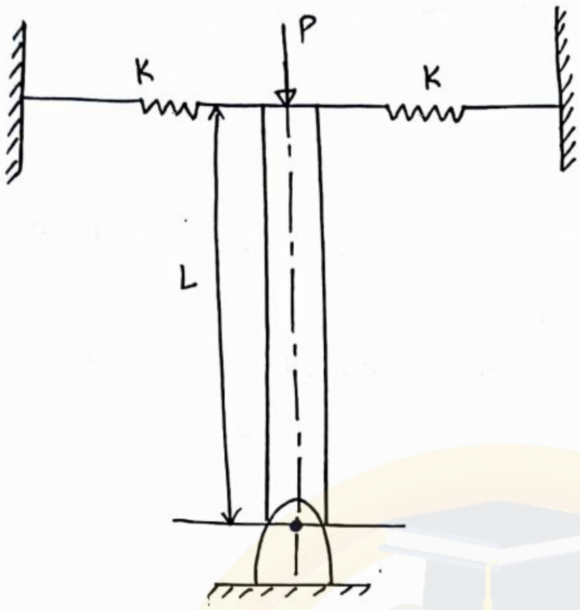
for stable eqm.

R.M. > D.M.

$(K\delta)5 > 6 \times \frac{\delta}{2}$

$\boxed{K > 0.6\,KN/m}$

Q. Find the limiting value of P to keep the system in equilibrium



(a) KL

(b) 2KL

(c) 0

(d) $\frac{KL}{2}$

28/7/16

PLANE TRUSSES

Truss is a Rigid structure in which members are subjected to either axial tensile or axial compressive load only.

Bending moment is 0 everywhere in the structure.

* **Conditions of Truss** :- (a) Members should be pin jointed or hinged only.

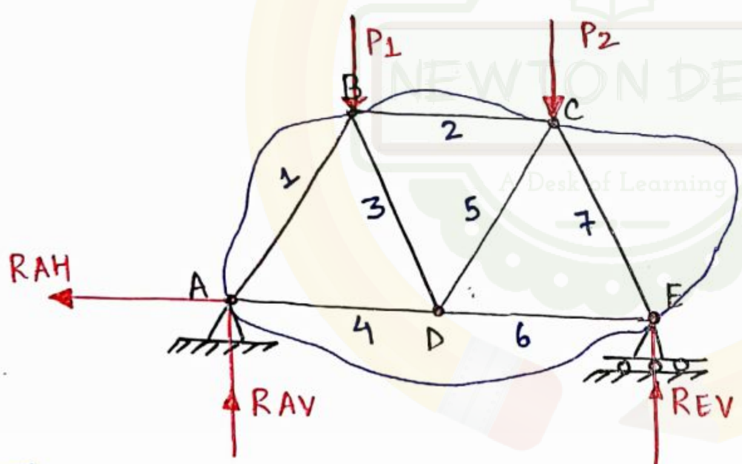
(b) Only concentrated point loads should be apply.

(c) Load should be applied at the joints only.

~~(d)~~

Note :- A member should make joints at its ends only.

* **Truss system** = members + Pins / joints.



$\Sigma \vec{F_H} = 0$

$R_{AH} = 0$

$\Sigma \vec{F_V} = 0$

$R_{AV} + R_{EV} = P_1 + P_2$

m → no. of members.

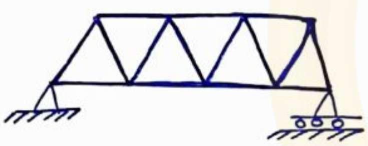
j → no. of joints / pins.

* Types of Plane Trusses [2-D]

Stable/Perfect/determinate

if $\boxed{m = 2j - 3}$

then we will get a stable triangulated truss.



Unstable/collapsible

if $\boxed{m < 2j - 3}$

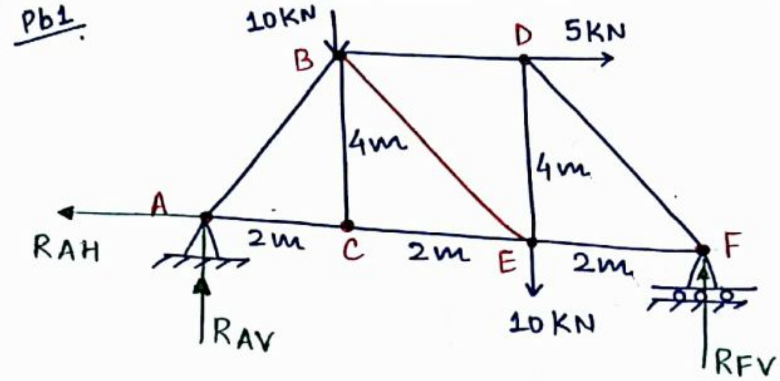
Indeterminate or Indestructible

if $\boxed{m > 2j - 3}$

* Determination of Reactions

Reactions at the supports are calculated by considering equilibrium of the entire truss.

Pb1.



Reactions at the supports?

$\rightarrow \oplus$ $\uparrow \oplus$

$\leftarrow \ominus$ $\downarrow \ominus$

Sol>

$$\boxed{\Sigma \vec{F_H} = 0}$$

$$5 - R_{AH} = 0$$

$$\Rightarrow \boxed{R_{AH} = 5\,KN\,(\leftarrow)}$$

$$\boxed{\Sigma \vec{F_V} = 0}$$

$$R_{AV} + R_{FV} - 20 = 0$$

$$\boxed{R_{AV} + R_{FV} = 20\,KN}\,(\uparrow)$$

$\circlearrowright \oplus$ $\circlearrowleft \ominus$

$$\boxed{\Sigma \vec{M_A} = 0}$$

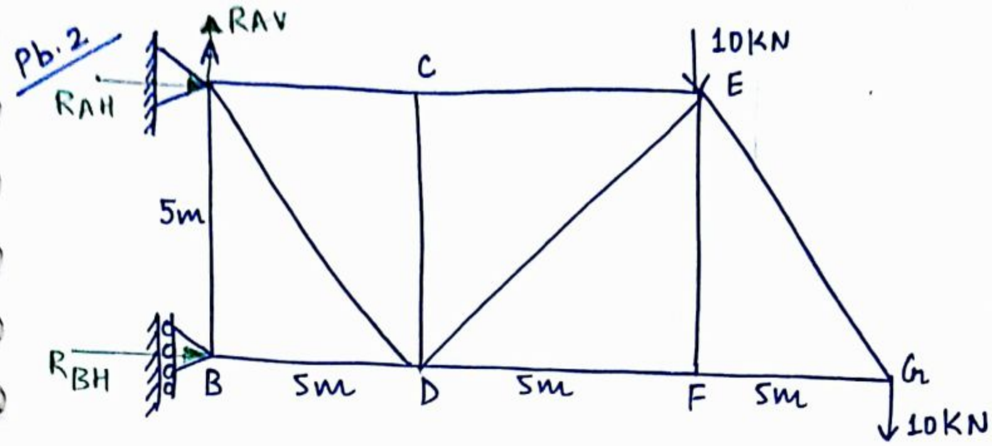
$$\Rightarrow 10 \times 2 + 5 \times 4 + 10 \times 4 - R_{FV} \times 6 = 0$$

$$\Rightarrow \boxed{R_{FV} = 13.33\,KN\,(\uparrow)}$$

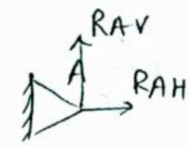
Now,

$$R_{AV} + 13.33 = 20$$

$$\boxed{R_{AV} = 6.67\,KN\,(\uparrow)}$$



Sol:



$\Sigma \vec{F_H} = 0$

$\Rightarrow R_{AH} + R_{BH} = 0$

$\Rightarrow \boxed{R_{AH} = -R_{BH}}$

$\longrightarrow$ $\longleftarrow$ means assumption kisi ek ka wrong hoga

$\Sigma \vec{F_V} = 0$

$\Rightarrow \boxed{R_{AV} = 20\,KN\,(\uparrow)}$

$\Sigma \vec{M_A} = 0$

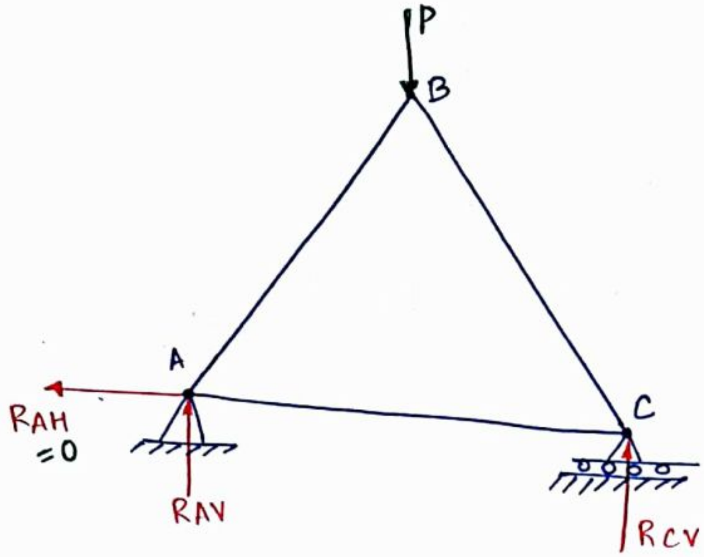
$\Rightarrow 10 \times 10 + 10 \times 15 - R_{BH} \times 5 = 0$

$\Rightarrow \boxed{R_{BH} = 50\,KN\,(\rightarrow)}$

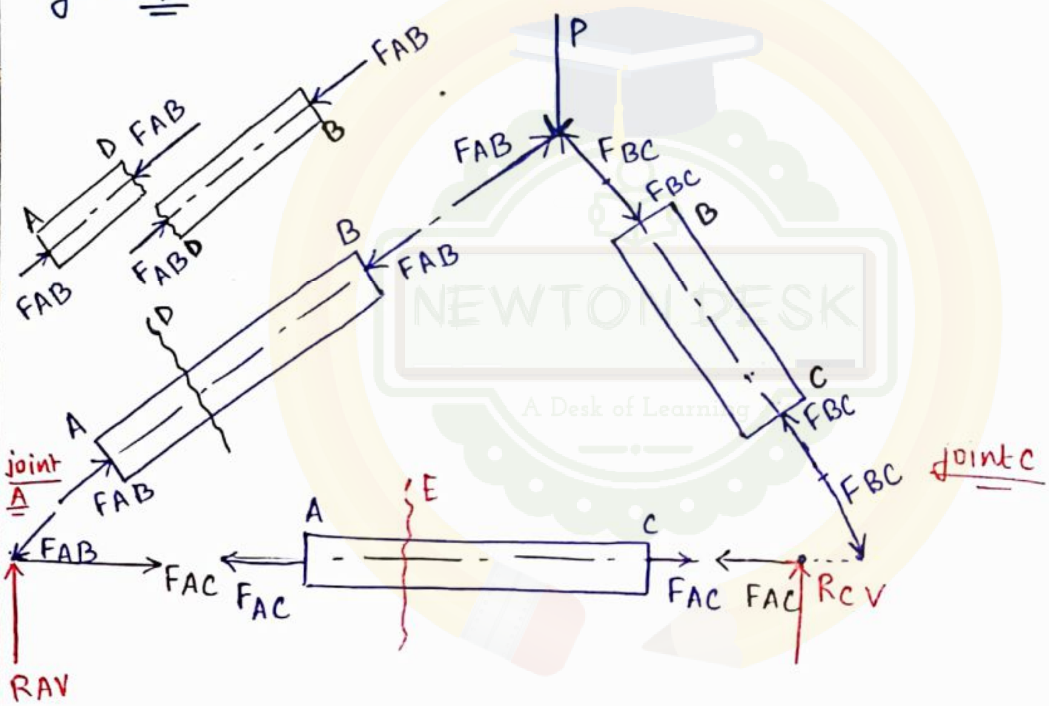
$\boxed{R_{AH} = 50\,KN\,(\leftarrow)}$

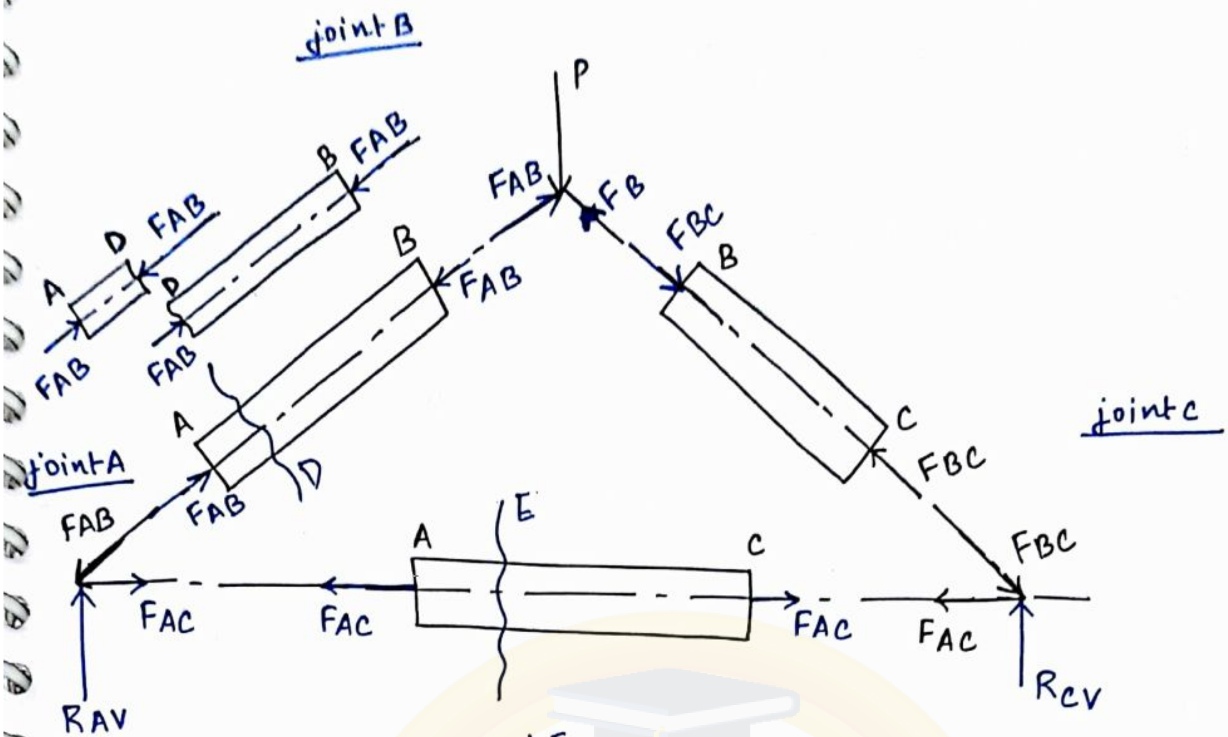
* Interaction of loads, internal forces and Reactions :-

"N.P."

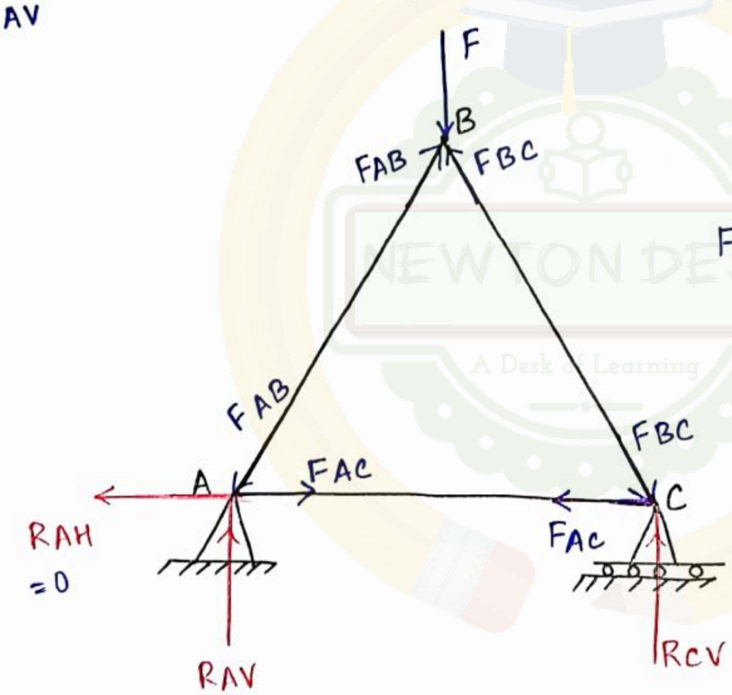


joint B





Forces on the joints



$$\boxed{P = R_{AV} + R_{CV}}$$

Method of joint :- Concept → equilibrium of a joint is considered in method of joint.

Procedure → ① find Reactions at supports.

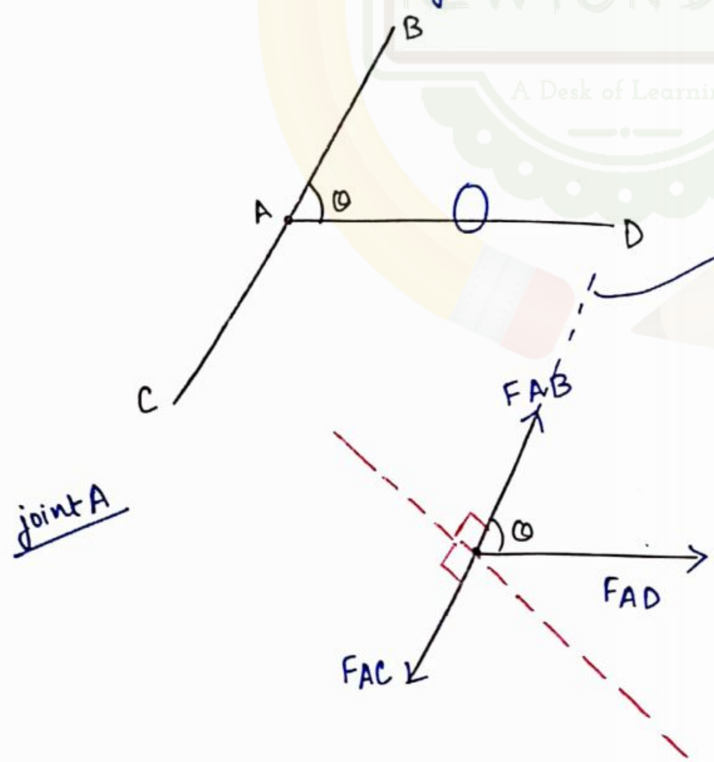
② Consider the equilibrium of a joint where only 2 unknown members are meeting and then use

$$\Sigma \vec{F_x} = \Sigma \vec{F_y} = 0$$. to find the unknowns.

Note :- ① If a member pulls a joint then the member itself will be in tension with the same intensity.

② If a member pushes a joint then the member itself will be in compression with the same intensity.

③ If at a joint 3 members are meeting and 2 are collinear then the force in the IIIrd member will be zero (if there is no load/reaction at that joint).



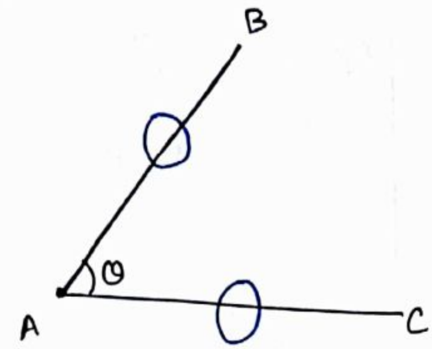
$F_{AC} = F_{AB}$

$F_{AD} \sin\theta = 0$

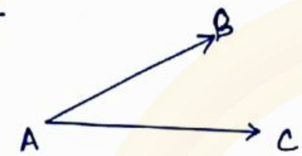
$\sin\theta \neq 0$

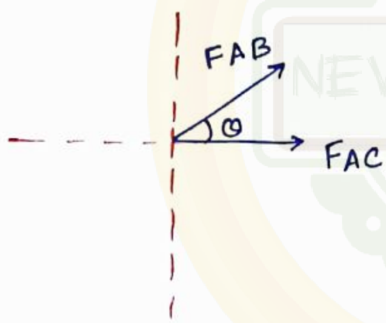
$\boxed{F_{AD} = 0}$

④ If at a joint 2 members are meeting and they are non-collinear then force in both the members are zero (there is no load (or) reaction at that joint).



joint A





$\Sigma \vec{F_V} = 0$

$F_{AB} \sin\theta = 0$

$\sin\theta \neq 0$

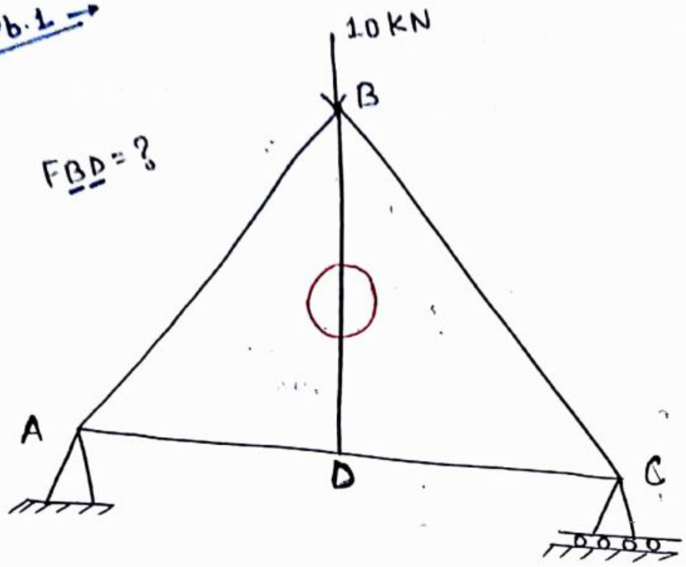
$\boxed{F_{AB} = 0}$

$\Sigma \vec{F_H} = 0$

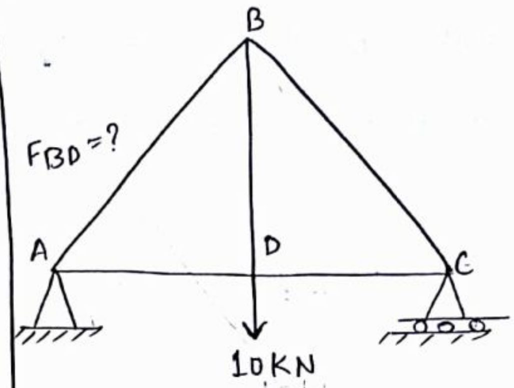
$F_{AC} + \cancelto{0}{F_{AB}\cos\theta} = 0$

$\boxed{F_{AC} = 0}$

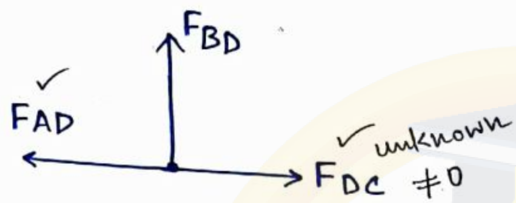
Pb. 1 →



Pb 2 →

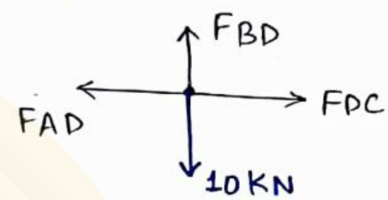


joint D



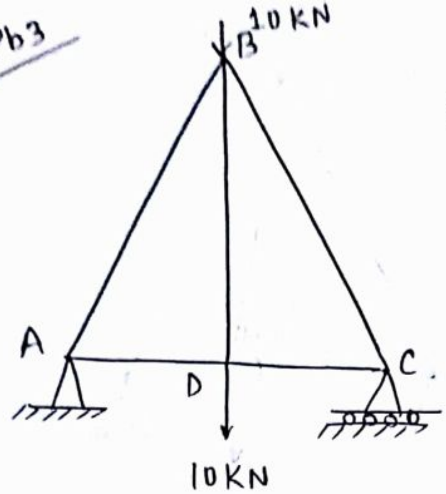
$F_{BD} = 0$

joint D



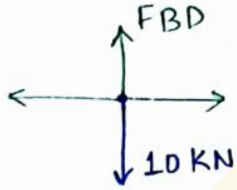
$F_{BD} = 10\,KN$ (Tensile)

Pb3



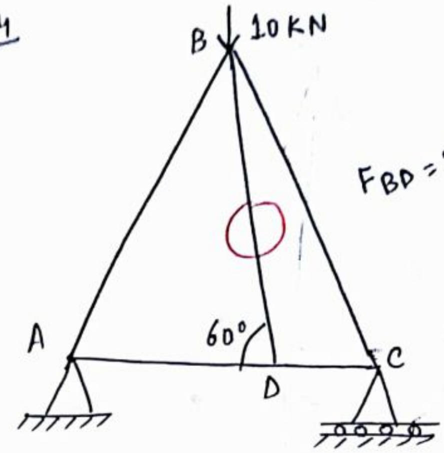
FBD = ?

joint D



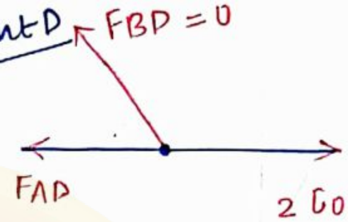
FBD = 10 KN (Tensile)

Pb4



FBD = ?

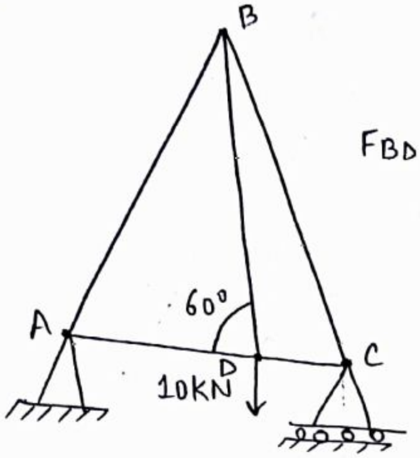
joint D



2 Collinear

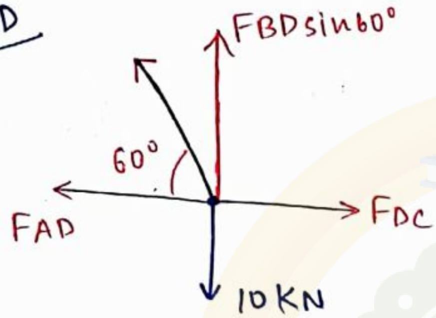
1 non - 11 -

Pb5



$F_{BD} = ?$

joint D



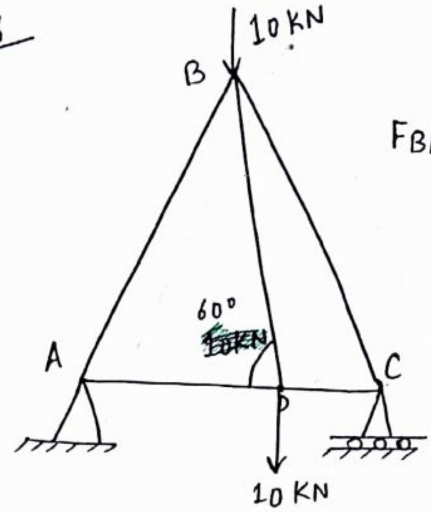
$$F_{BD} \sin 60° = 10$$

$$F_{BD} \frac{\sqrt{3}}{2} = 10$$

$$F_{BD} = \frac{20}{\sqrt{3}}$$

$$\boxed{F_{BD} = 11.547 \text{ KN (Tensile)}}$$

Pb 6



$F_{BD} = ?$

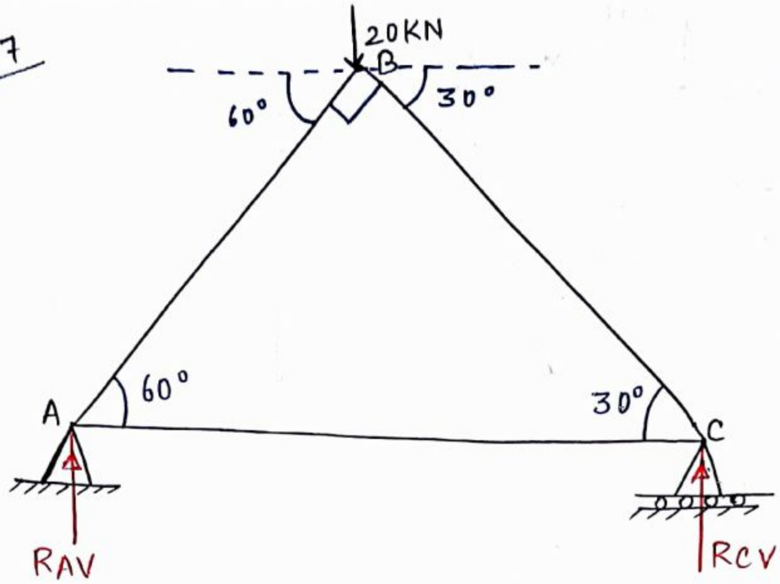
Similarity

Force on BD

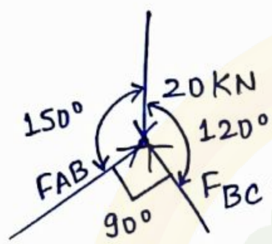
$$\boxed{F_{BD} = 11.547 \text{KN (Tensile)}}$$

Pb. 7

$F_{AC} = ?$



joint B

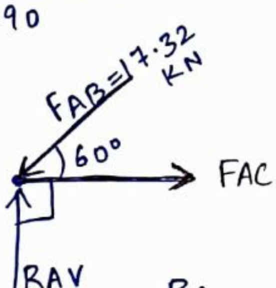


$$\frac{F_{AB}}{\sin 120^\circ} = \frac{F_{BC}}{\sin 150^\circ} = \frac{20\,KN}{\sin 90^\circ}$$

$$F_{AB} = \frac{20 \times \sin 120}{\sin 90} = 17.32\,KN \text{ (comp.)}$$

$$F_{BC} = \frac{20 \times \sin 150}{\sin 90} = 10\,KN \text{ (compr.)}$$

joint A



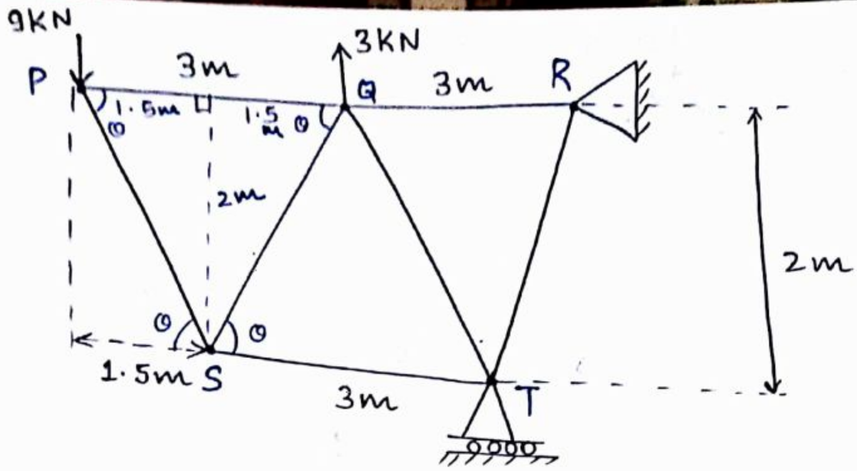
$R_{AV} = F_{AB} \sin 60^\circ = 15\,KN$

$F_{AC} = F_{AB} \cos 60^\circ = 8.66\,KN$ (Tensile)

$R_{CV} = 20 - R_{AV} = 5\,KN\,(\uparrow)$.

NewtonDesk.com

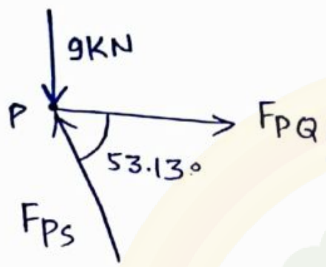
Pb. 8. →



$F_{QS} = ?$

Sol

joint P



$\tan\theta = \frac{2}{1.5}$

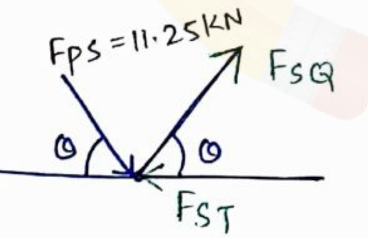
$\theta = 53.13°$

By lami's theorem ← no need.

$9 = F_{PS} \sin 53.13°$

$$\boxed{F_{PS} = 11.25 \text{ KN (comp.)}}$$

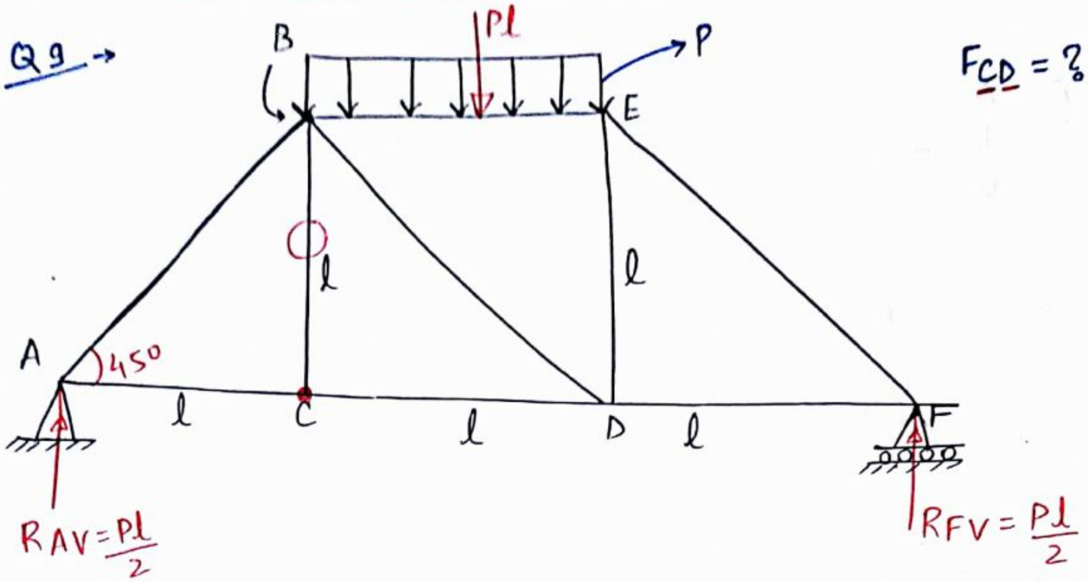
joint S



$F_{PS} \cancel{\sin}\theta = F_{SQ} \cancel{\sin}\theta$

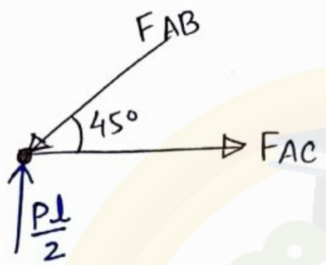
$$\boxed{F_{SQ} = 11.25 \text{ KN (Tensile)}}$$

Q 9 →



$F_{CD} = ?$

joint A



$F_{AB} \sin 45° = \frac{Pl}{2}$

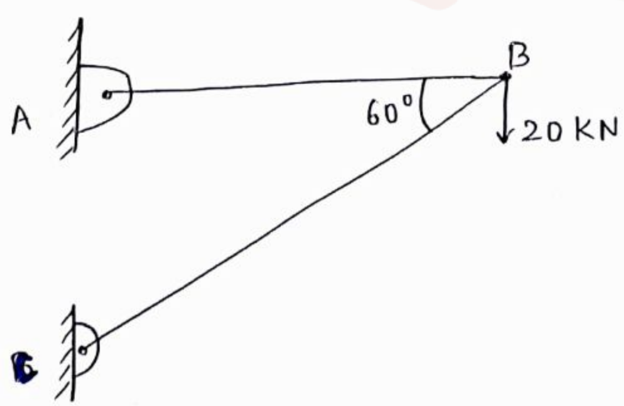
$F_{AB} \cos 45° = F_{AC}$

$F_{AC} = \frac{Pl}{2}$ (Tensile)

joint C

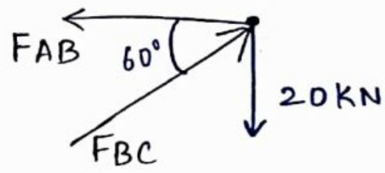
$F_{CD} = \frac{Pl}{2}$ (Tensile)

Pb 10



$(R_c)_H = ?$

joint B

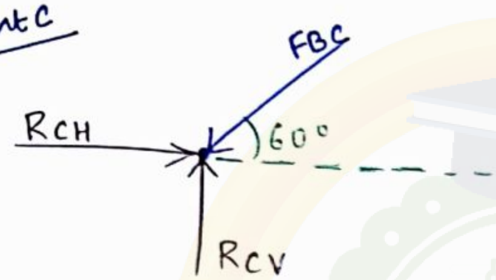


$F_{BC} \sin 60° = 20\,KN$

$\therefore F_{BC} = \dfrac{20}{\sqrt{3}/2} = \dfrac{40}{\sqrt{3}}$

$\boxed{F_{BC} = 23.09\,KN \;\text{(compr.)}}$

Joint C



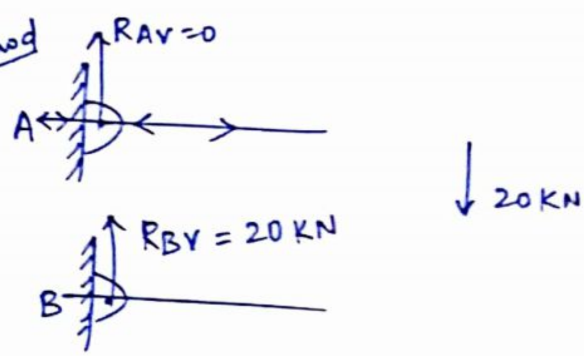
$F_{BC} \sin 60° = 20\,KN = R_{CV}$

$\boxed{R_{CV} = 20\,KN\,(\uparrow)}$

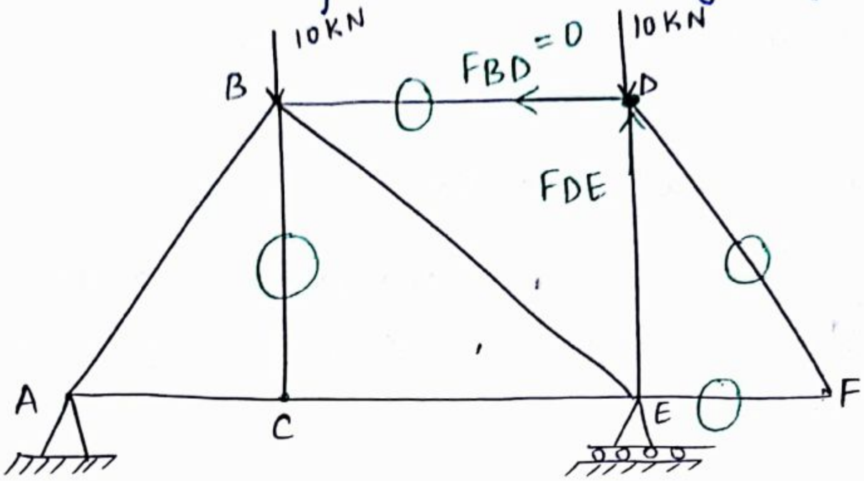
$R_{CH} = F_{BC} \cos 60°$

$\boxed{R_{CH} = 11.5\,KN\,(\rightarrow)}$

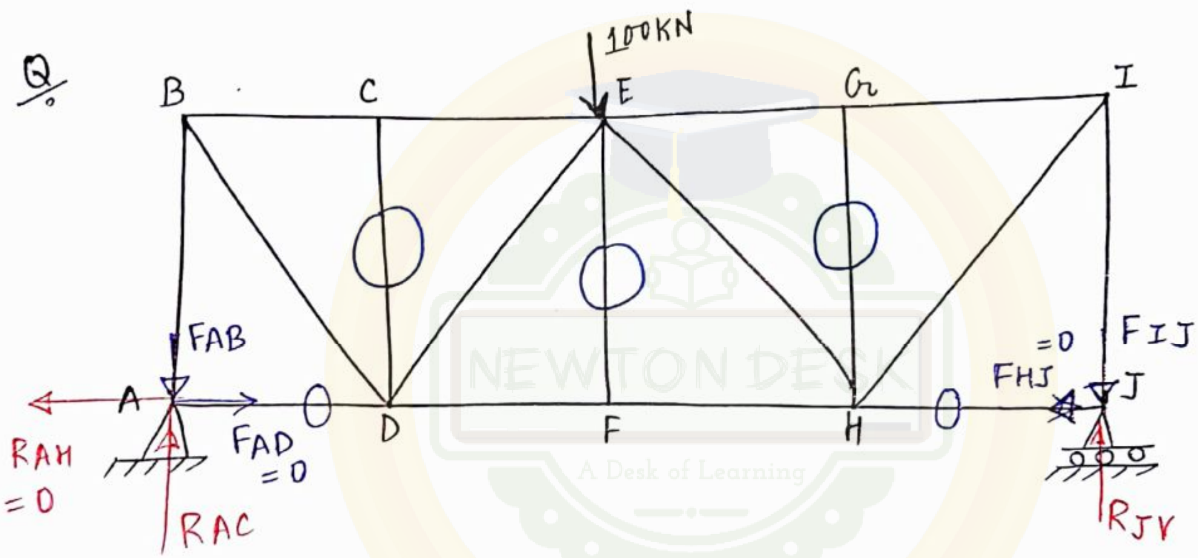
2nd method



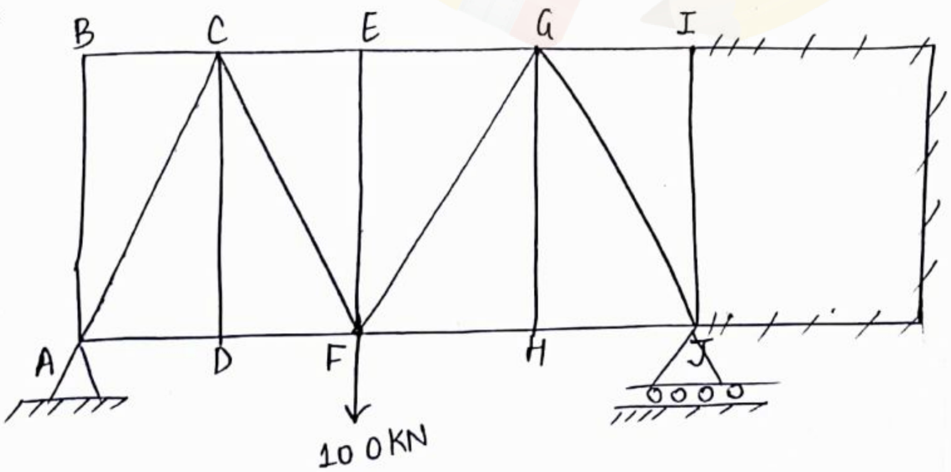
Q. Find the No. of members having 0 force.



Q.



Q.



29/7/16 Method of Section

Concept → eqm. of a section of truss is considered in method of section.

Procedure → (a) find Reactions and supports.

(b) cut the members under consideration by a section (1)- - - - - - - -(1) and consider eqm. of either Left hand side or RHS of section (1)- - -(1) for applied loads, Reactions and forces in the cut members and use

$$\boxed{\Sigma \vec{F_x} = \Sigma \vec{F_y} = \Sigma \vec{Z} = 0}$$ ← not applied (since 2D → Truss)

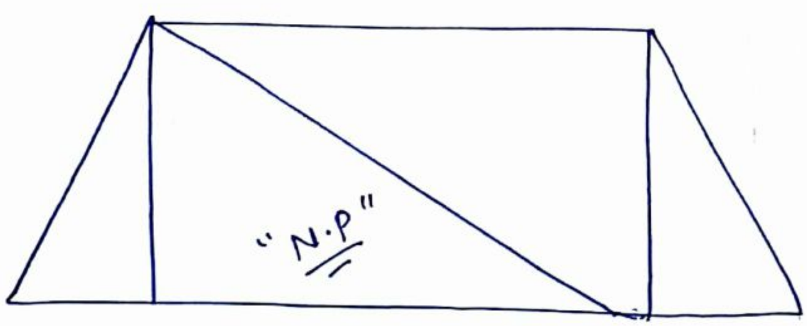
and $\boxed{\Sigma \vec{M} = 0}$

Note :- (1) Advantage of method of section is that force in any intermediate member can be found directly without finding force in any other member.

(2) Cut the members such that entire truss is divided into two seperate parts.

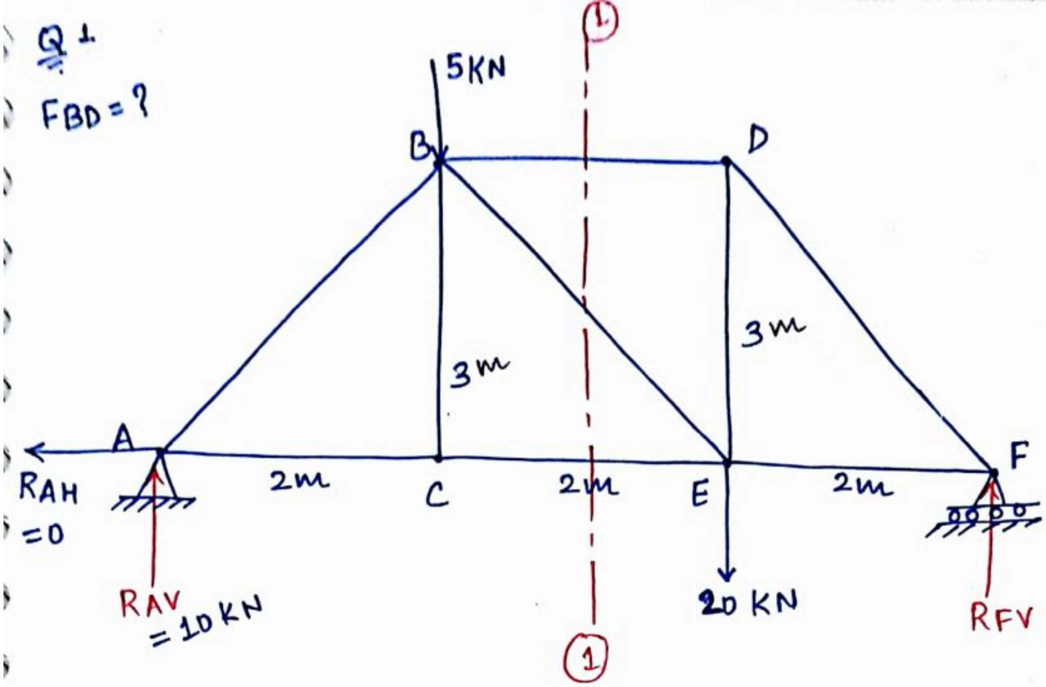
(3) Preferably do not cut more than 3 members because in method of section we have only 3 equations of equilibriums.

(4) Cut the members such that all the cut members do not meet at same joint (If they meet at same joints it becomes of method of joints only).



Q 1.

$F_{BD} = ?$

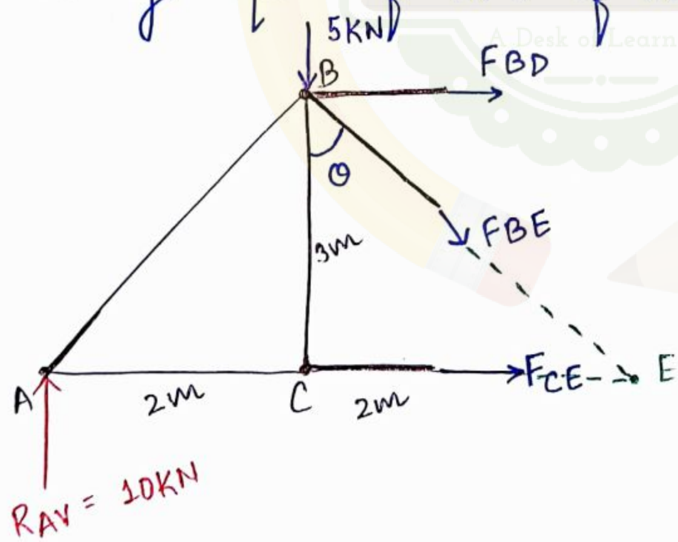


$$\Sigma \vec{M}_F = 0$$

$$R_{AV} \times 6 = 5 \times 4 + 20 \times 2 = 60$$

$$\boxed{R_{AV} = 10 \text{ KN}}$$

Considering equ^m of L.H.S. of section ①------①



$$\tan\Theta = \frac{2}{3}$$

$$\Theta = 33.69°$$

$F_{BD} = ?$

$$\Sigma \vec{M}_E = 0$$

$$\Rightarrow (F_{BD} \times 3) + (10 \times 4) - (5 \times 2) = 0$$

$$\Rightarrow F_{BD} = -10 \text{ KN}$$

$$\Rightarrow \boxed{F_{BD} = 10 \text{ KN (compressive)}}$$

$F_{CE} = ?$

$$\Rightarrow \Sigma \vec{M}_B = 0$$

$$\Rightarrow (10 \times 2) - (F_{CE} \times 3) = 0$$

$$\Rightarrow \boxed{F_{CE} = 6.67 \text{ KN (Tensile)}}$$

$F_{BE} = ?$

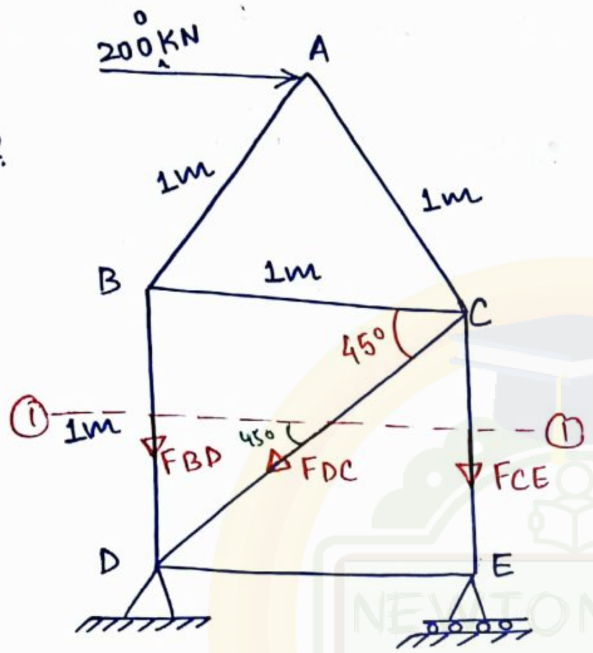
$\Sigma \vec{F_V} = 0$

$\Rightarrow 10 - 5 - F_{BE}\cos\theta = 0$

$\Rightarrow \boxed{F_{BE} = 6\,KN\ (\text{Tensile})}$

Pb. 2 →

$F_{CD} = ?$



Considering upper side of ① - - - - - - ①

$\Sigma \vec{F_H} = 0$

$2000 - F_{DC}\cos 45° = 0$

$\boxed{F_{DC} = 2000\sqrt{2}\ (\text{Tensile})}$

Solⁿ.

Considering upper side of ① - - - - ①

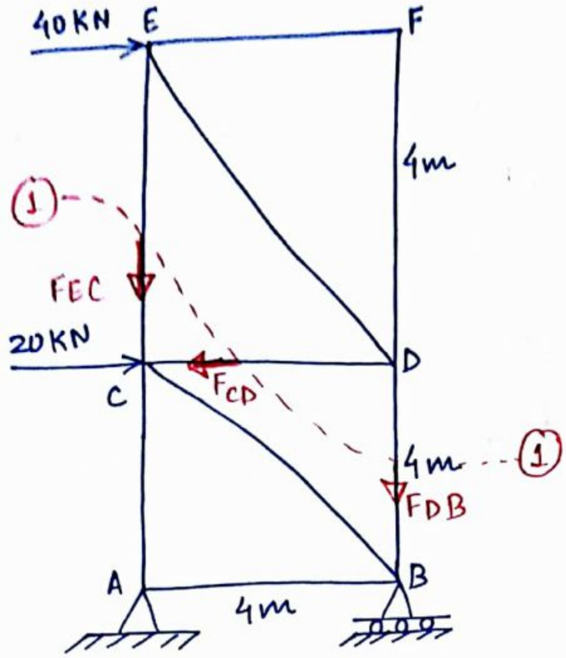
$\Sigma \vec{F_H} = 0$

$40 - F_{CD} = 0$

$\boxed{F_{CD} = 40\ KN\ (\text{Tensile})}$

Pb.3 →

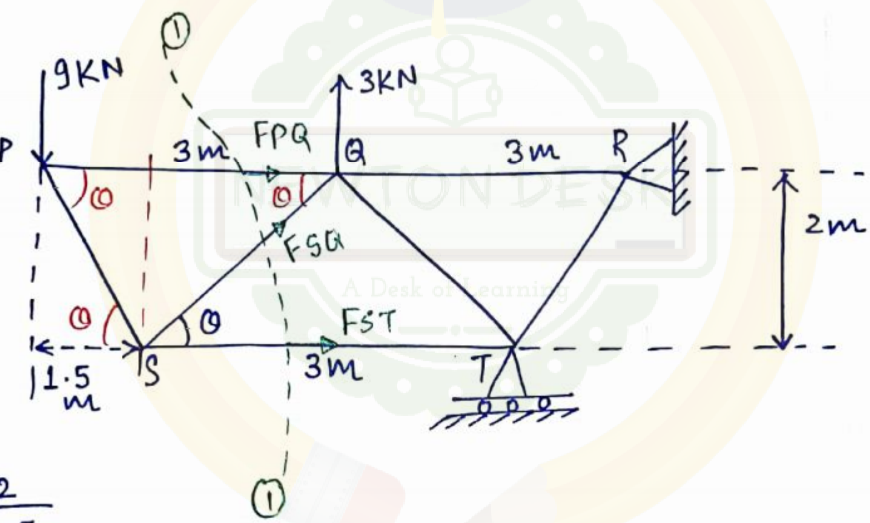
$F_{CD} = ?$



Sol

Pb4 →

$F_{QS} = ?$



$\tan\theta = \frac{2}{1.5}$

$\theta = 53.13°$

Considering LHS of 1 ------ 1

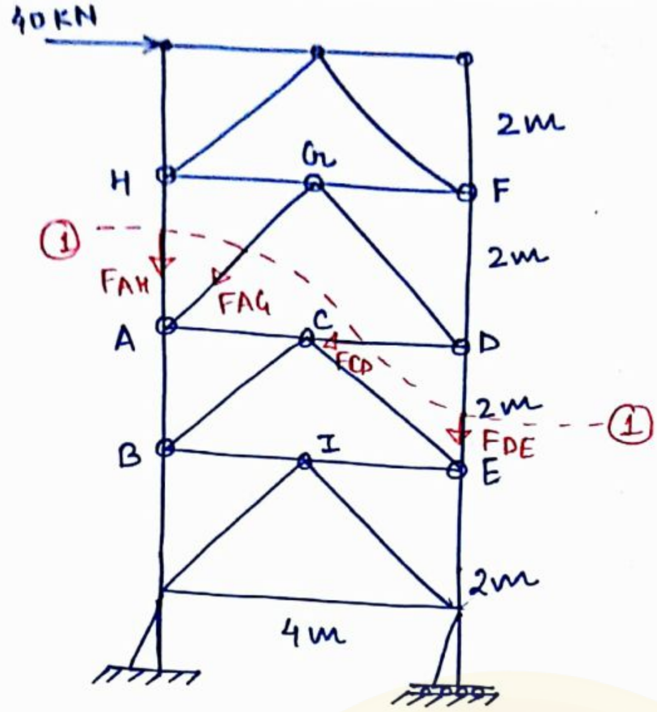
$\Sigma \vec{F_v} = 0$

$\Rightarrow -9 + F_{SQ} \sin\theta = 0$

$\Rightarrow \boxed{F_{SQ} = 11.25 KN \text{ (Tensile)}}$

Pb5

$F_{DE} = ?$



Considering upper side of ① - - - - ①

$\Sigma \overrightarrow{M_A} = 0$

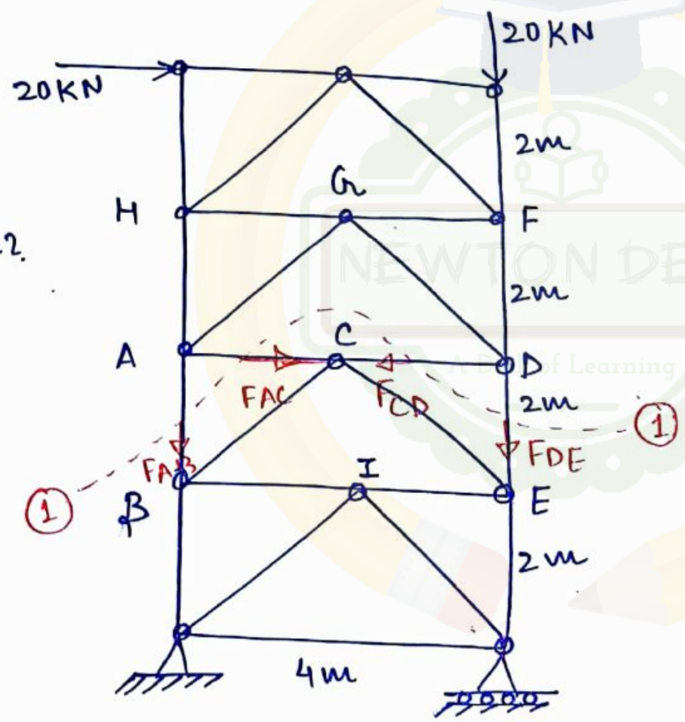
$40 \times 4 + F_{DE} \times 4 = 0$

$F_{DE} = -40KN$

$F_{DE} = 40 KN$ (compressive)

Pb6

$F_{AB} = ?$



Considering upper side of ① - - - - - ①

$\Sigma \overrightarrow{M_D} = 0$

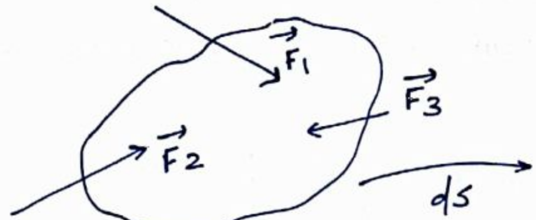
$20 \times 4 - F_{AB} \times 4 = 0$

$F_{AB} = 20KN$ (Tensile)

* WORKDONE

Workdone $= \vec{F} \cdot \vec{ds}$

$= T * d\theta$



$W\vec{F_1} = \vec{F_1} \cdot \vec{ds}$

independent

$W_{F_1} = \vec{F_1} \cdot \vec{ds}$

$W_{F_2} = \vec{F_2} \cdot \vec{ds}$

$W_{F_3} = \vec{F_3} \cdot \vec{ds}$

$T \rightarrow +ve$

$T \rightarrow -ve$

Power $= \dfrac{\vec{F} \cdot \vec{ds}}{dt} = \vec{F} \cdot \vec{V}$

$= \dfrac{T d\theta}{dt} = T\omega$

$= \dfrac{2\pi NT}{60}$

TWD = Total workdone $= W_{F1} + W_{F2} + W_{F3}$

$= (\vec{F_1} + \vec{F_2} + \vec{F_3}) \cdot \vec{ds}$

$$\boxed{TWD = \vec{F_R} \cdot \vec{ds}}$$

If Body is in equilibrium

$\vec{F_R} = 0 \Rightarrow \boxed{TWD = 0}$

* Principle of Virtual (imaginary) WORK :→ [POVW]

It states "if a System is in equilibrium, then the sum of virtual workdone by all the forces will be zero."

Virtual Work = Force (virtual displacement).

Note :- ① Using POVW we can find forces necessary to keep a system in equilibrium.

② If using equ.'s of eq^m. is difficult and time taken, then use POVW to find the unknowns easily.

③ POVW is used in the system when no. of interconnected Rigid bodies have degree of freedom more than 0 but are in equilibrium.

④ In POVW, we do not need to find the Reaction at supports because virtual work of Reactions is always 0.

Procedure :→ ⓐ Take any fixed point in problem as origin, fix coordinate axis and find the coordinates of all the points where forces are acting.

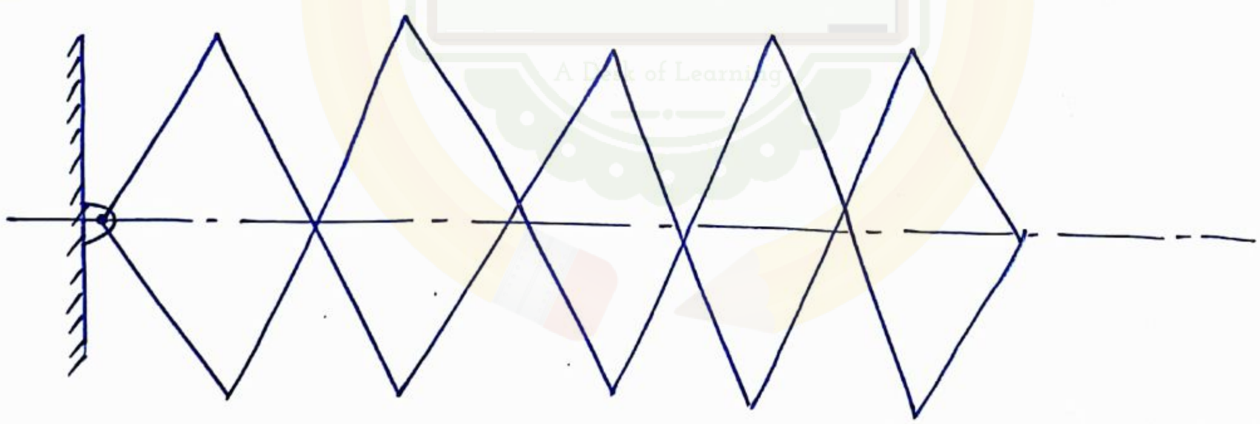
ⓑ find virtual displacements.

ⓒ use POVW to find the unknowns.

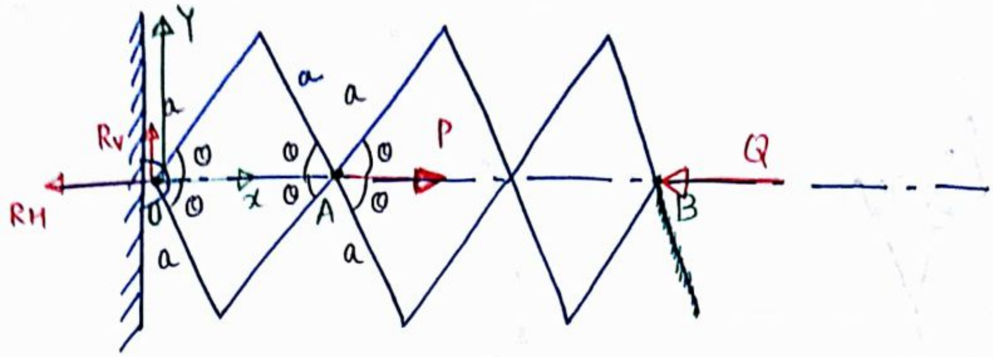
Sign Convention :→ ① Sign convention for the coordinates is choosen based on the quadrant in which they are lying.

② If any force is acting along +ve x-axis or +ve y-axis then take that force as +ve and vice-versa.

Q. For the Lazy tong mechanism shown in figure the Relationship b/w P and Q to keep the system in equilibrium is



"[N.P.]"



$\Sigma \vec{F_V} = 0$

$R_V = 0$ ✓

$\Sigma \vec{F_H} = 0$

$R_H + Q = P$ ✓

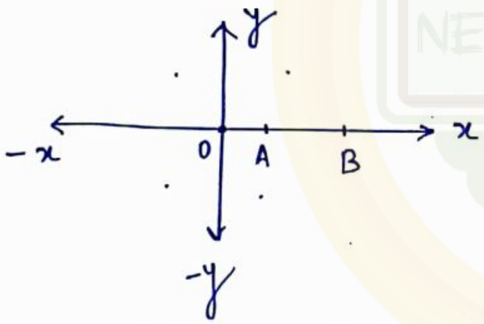
$\Sigma \vec{M_A} = 0$

0 ✓.

No means

PoVW → 1. $x_A = +2a\cos\theta$

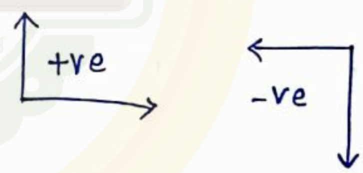
$x_B = +6a\cos\theta$



2. $\partial x_A = -2a\sin\theta\,\partial\theta$

$\partial x_B = -6a\sin\theta\,\partial\theta$

3. $(V.W.)_P + (V.W.)_Q = 0.$

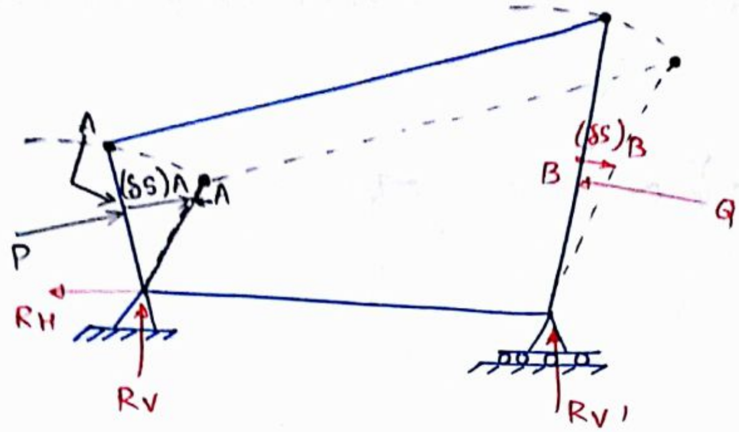


$\Rightarrow (+P)(\partial x)_A + (-Q)(\partial x)_B = 0$

$\Rightarrow +P(-2a\sin\theta\,\partial\theta) + Q(6a\sin\theta\,\partial\theta) = 0$

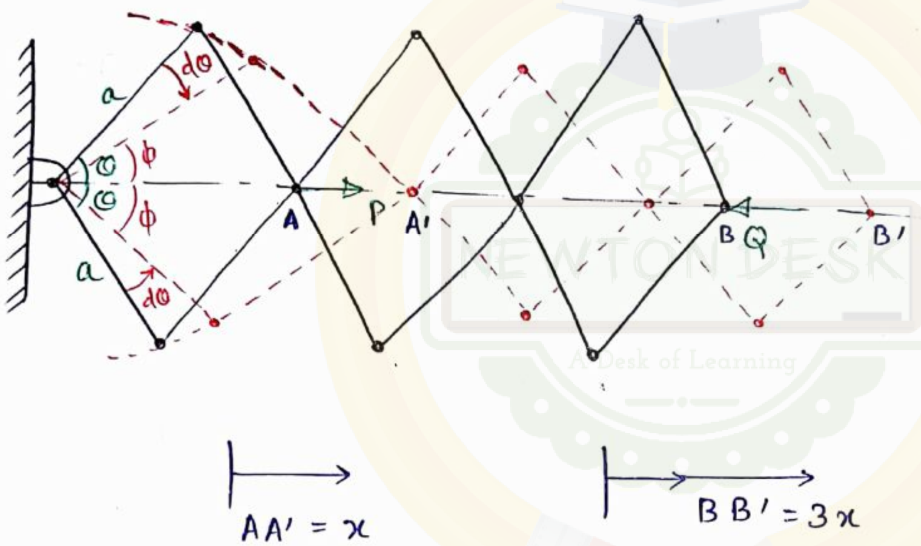
$\Rightarrow \boxed{P = 3Q}$

$\boxed{R_H = P - Q = 2Q}$



$$(VW)_P + (VW)_Q = 0$$

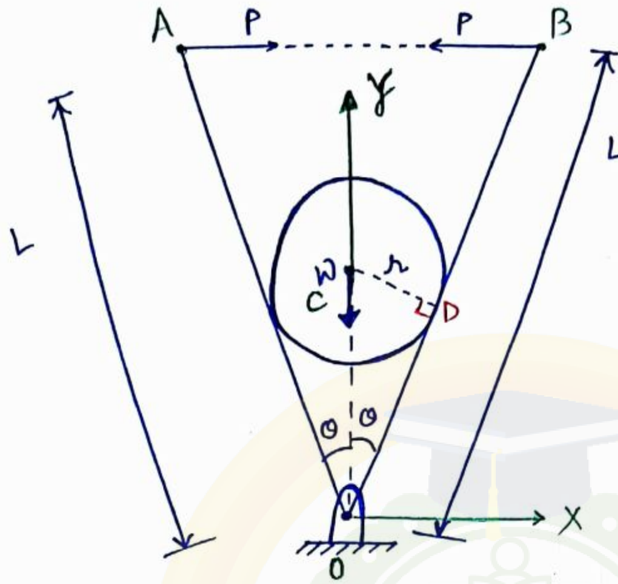
$$P(\delta s)_A + \left(-Q\,(\delta s)_B\right) = 0$$



$$Px + (-Q \times 3x) = 0$$

$$\boxed{P = 3Q}$$

Q. A sphere of weight 'W' and radius 'r' is supported by 2 rods of length 'L' as shown in fig. The value of 'P' to keep it in equilibrium is.



Sol

1. $x_A = -L\sin\theta$

$x_B = +L\sin\theta$

$y_C = +r\,\text{cosec}\,\theta$

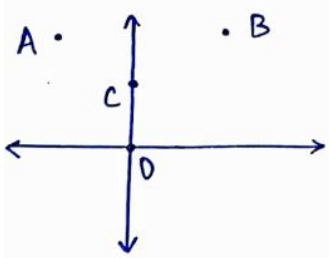
In ΔODC

$\frac{r}{OC} = \sin\theta$

$r = OC\sin\theta$

$\frac{r}{\sin\theta} = y_C$

$r\,\text{cosec}\,\theta = y_C$



2. $\partial x_A = -L\cos\theta\,\partial\theta$

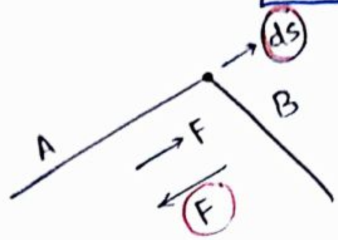
$\partial x_B = +L\cos\theta\,\partial\theta$

$\partial y_C = -r\,\text{cosec}\,\theta\,\partial\theta\,\cot\theta$

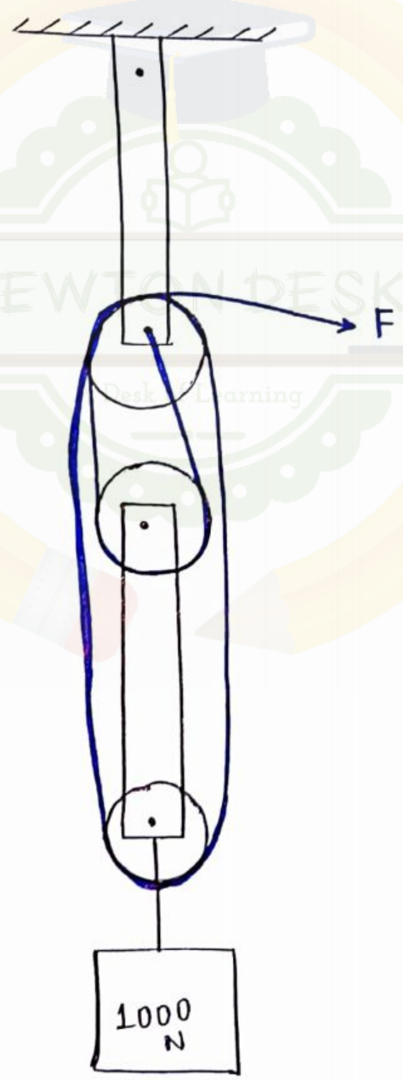
3. $(+P)(-L\cos\theta\,\cancel{2\delta\theta}) + (-P)(L\cos\theta\,\cancel{2\delta\theta}) + (-W)(-r\,\mathrm{cosec}\,\theta\,\cot\theta\,\cancel{2\delta\theta}) = 0$

$$2PL\cancel{\cos\theta} = Wr \times \frac{1}{\sin\theta} \times \frac{\cancel{\cos\theta}}{\sin\theta}$$

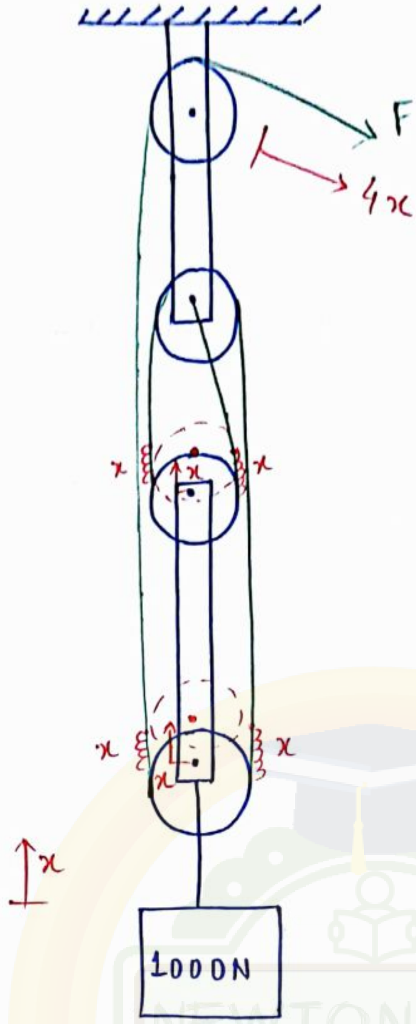
$$\boxed{P = \frac{Wr}{2L\sin^2\theta}}$$



Pb. 7.10



wrong diagram.

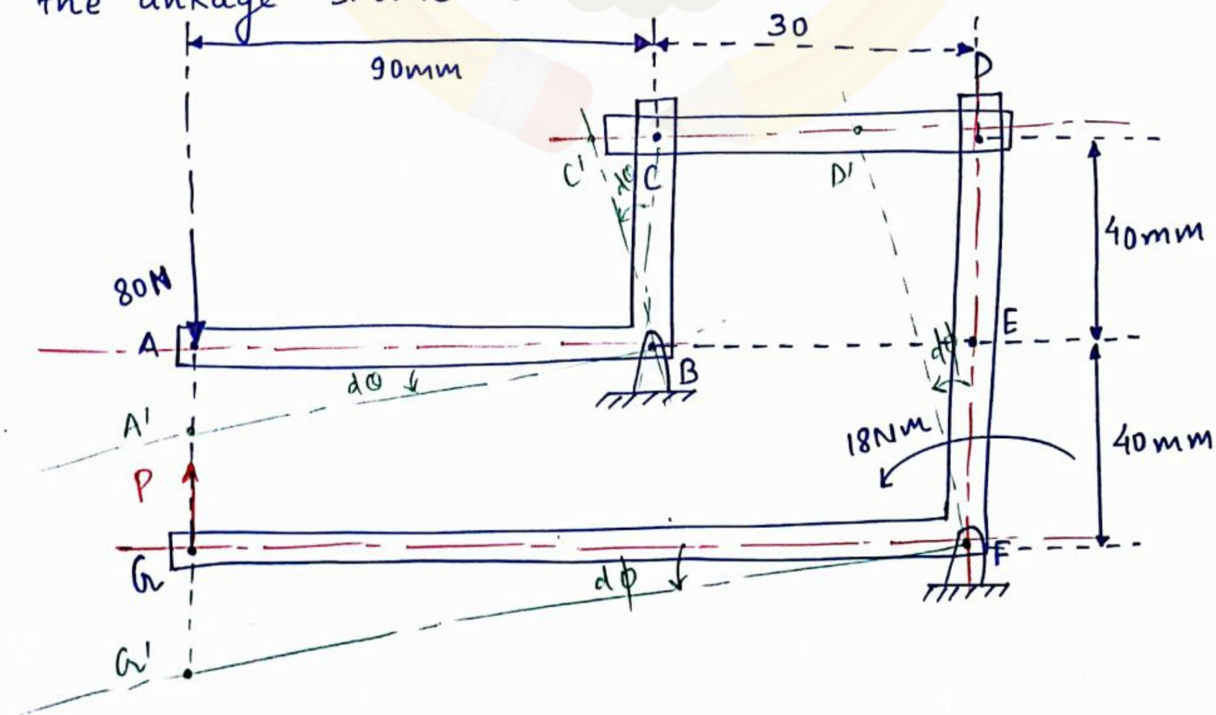


$$-1000 \times x + F \times 4x = 0$$

$$\boxed{F = 250N}$$

Mechanical Advantage = 4.

Q Determine the vertical force at 'G' to maintain equ. of the linkage shown below.



$$CC' = DD'$$

$$40\,d\theta = 80\,d\phi$$

$$\boxed{d\theta = 2d\phi}$$

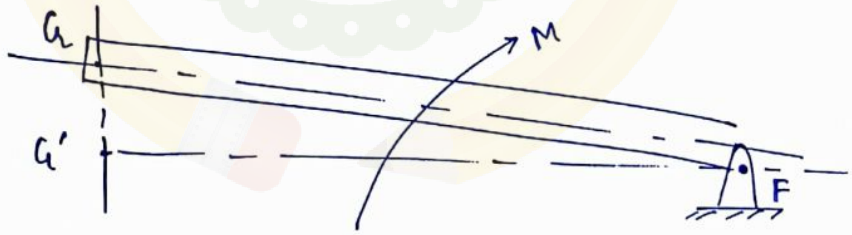
POVW $\Longrightarrow$ $80(AA') + 18\,d\phi - P(GG') = 0$

$$\Rightarrow 80(0.9\,d\theta) + 18\,d\phi - P(0.12\,d\phi) = 0$$

$$\Rightarrow 80 \times 0.9 \times 2\cancel{d\phi} + 18\cancel{d\phi} - P(0.12\cancel{d\phi}) = 0$$

$$\Rightarrow \boxed{P = 270\,N\,(\uparrow)}$$

Pb: Determine the couple M that must be applied on member DEFG instead of vertical force at G so as to maintain equilibrium of previous problem.



$$80(AA') + 18(d\phi) - M\,d\phi = 0$$

$$80(0.9 \times 2\,d\phi) + 18\,d\phi - M\,d\phi = 0$$

$M = 32.4$ Nm ; CW

DYNAMICS

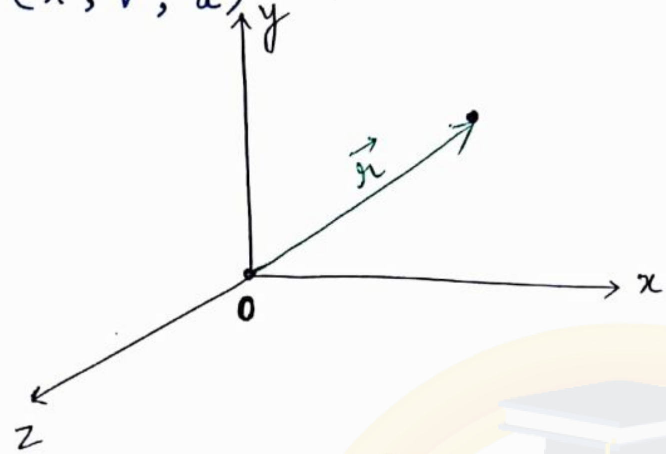
Dynamics

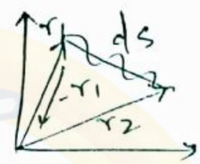
↓

Kinematics

$(\vec{r}, \vec{V}, \vec{a})$

$\vec{r} \rightarrow$ position ~~velocity~~ vector





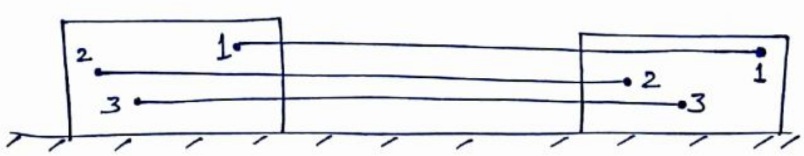
$-r_1 + r_2 = ds$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d\vec{s}}{dt}$$

$$\vec{a} = \frac{d\vec{V}}{dt}$$

Rectilinear Translation

Kinematics



$(\vec{ds})_1 = (\vec{ds})_2 = (\vec{ds})_3 = (\vec{ds})_4 \rightarrow$ during time dt

$$\left.\begin{aligned} \vec{V_1} = \vec{V_2} = \vec{V_3} = \vec{V}_{CM} = \vec{V} \\ \vec{a_1} = \vec{a_2} = \vec{a_3} = \vec{a}_{CM} = \vec{a} \end{aligned}\right\} \rightarrow \text{at any instant}$$

$$\left.\begin{aligned} \vec{V}_{1/2} = \vec{V_1} - \vec{V_2} = 0 \\ \vec{a}_{1/2} = \vec{a_1} - \vec{a_2} = 0 \end{aligned}\right] \rightarrow \text{Relative rest.}$$

if $\vec{a}$ is uniform.

1. $V = u + at$

2. $S = ut + \frac{1}{2}at^2$

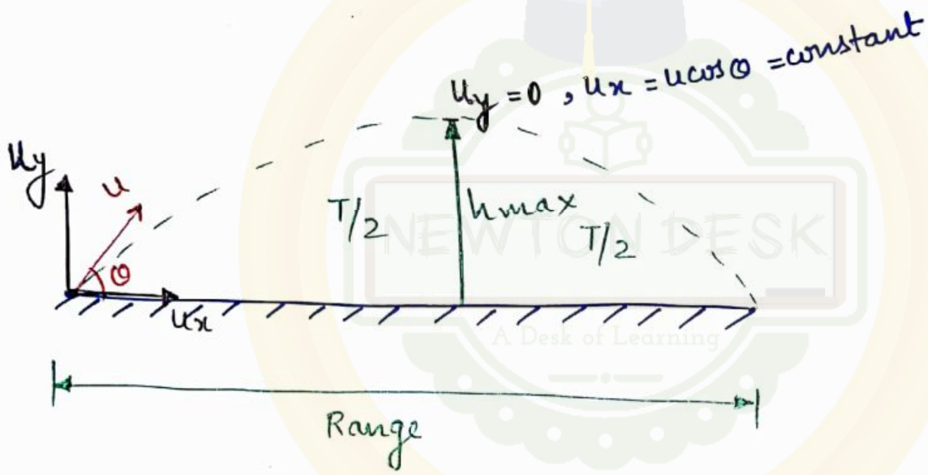
3. $V^2 = u^2 + 2as$

Ex →

Case. 1 :- Free falling bodies

$u = 0 \; ; \; a = g(-\hat{j})$

Case. 2 :- Projectile motion.



(i.) $u_x = u\cos\theta \; ; \; a_x = 0$

u_x = constant $= u\cos\theta$

(ii.) $u_y = u\sin\theta \; ; \; a_y = g(-\hat{j})$

⇒ if $a = F(s)$ — displacement

$a = \frac{dv}{dt} \; ; \; v = \frac{ds}{dt}$

$dt = \frac{dv}{a} \; ; \; dt = \frac{ds}{V}$

$$\boxed{\frac{dv}{a} = \frac{ds}{v}}$$

$$\boxed{a\,ds = v\,dv}$$

$$F(s)\,ds = v\,dv$$

Q. The initial velocity of an object is 40 m/s, acceleration $a = -0.1v$ ($a = -0.1v$) where v is instantaneous velocity, the velocity of the object after 3 sec's is :-

Sol

$v = u + at$

$v = 40 + (-0.1v)3$

$v = 40 - 0.3v$

$v = \frac{40}{1.3}$ m/s

$\cancel{\text{}}$ ~~written~~

SIR $a = -0.1v$

$u_i = 40$ m/s ; $t = 0$

$v_{t=3s} = ?$

$v = u + \underbrace{at}_{\text{constant.}}$

$a = -0.1v$

$$\frac{dv}{dt} = -0.1v = a$$

$$-0.1\int_0^{t=3} dt = \int_{40}^{v_f} \frac{dv}{v}$$

$\Rightarrow -0.1(3-0) = \log_e v_f - \ln 40$

$\Rightarrow -0.3 = \ln\left(\frac{v_f}{40}\right)$

$\Rightarrow \boxed{v_f = 29.63 \text{ m/s}}$

Q. if $s = \frac{t^3}{3} - 36t$ then when does the particle reverses its direction and what is its/the acceleration at that instant?

Sol.

$$V = \frac{ds}{dt} = t^2 - 36 = 0$$

$$\boxed{t = 6s}$$

$$a = \frac{dv}{dt} = 2t$$

$$\boxed{a_{t=6} = 12 m/s^2}$$

* **Kinetics** :-

Newton's 2nd Law →

for a particle →

if $\vec{F_R} \neq 0$ then $\vec{a} \neq 0$

$$\boxed{\vec{a} = \frac{\vec{F_R}}{m}}$$ → dirn. in the $\vec{F_R}$

$\vec{F_R} = m\vec{a}$ —— 2nd law

$m\vec{a}$ → is not a force

$\vec{F_R}$ → Resultant of actual forces

$m\vec{a}$ → Response

for a Rigid Body →

if $\left(\vec{F}_R\right)_{ext} \neq 0$ then $\vec{a}_{CM} \neq 0$

$$\vec{a}_{CM} = \frac{\left(\vec{F}_R\right)_{ext}}{m}$$

not of Body

$$\boxed{\underbrace{\left(\vec{F}_R\right)_{ext}}_{\text{Resultant of actual external forces}} = \underbrace{m\vec{a}_{CM}}_{\text{effect/Response.}}} \longrightarrow \underline{\underline{2^{nd}\ \text{Law}}}$$

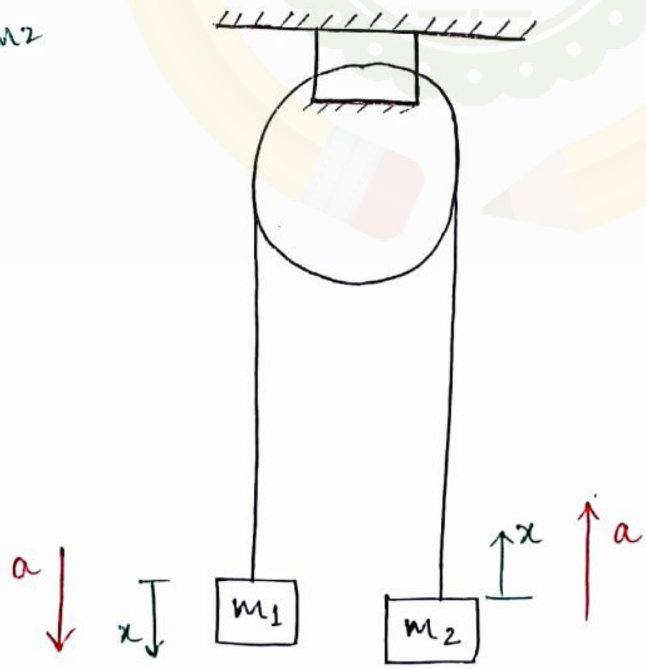
1st, 2nd law → one Body

3rd → 2 Body

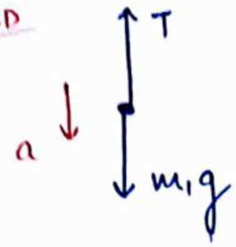
*Rectilinear Translation →

Case. I →

$m_1 \neq m_2$



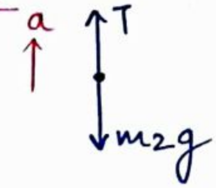
m_1 ← FBD



$$\underbrace{m_1 g - T = m_1 a}_{}\text{ — 2nd Law}$$

response

m_2 ← FBD

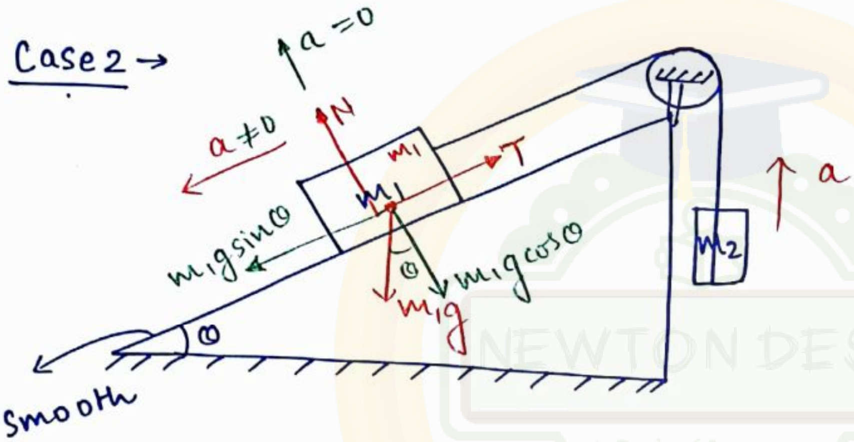


$$T - m_2 g = m_2 a \text{ — 2nd}$$

Response

$\vec{F_R}$ ⟶ effective force — also called

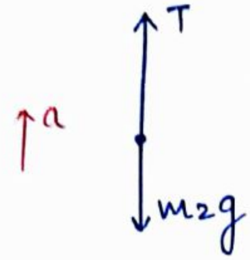
Case 2 →



m_1 $\quad N = m_1 g\cos\theta$ — 1st law

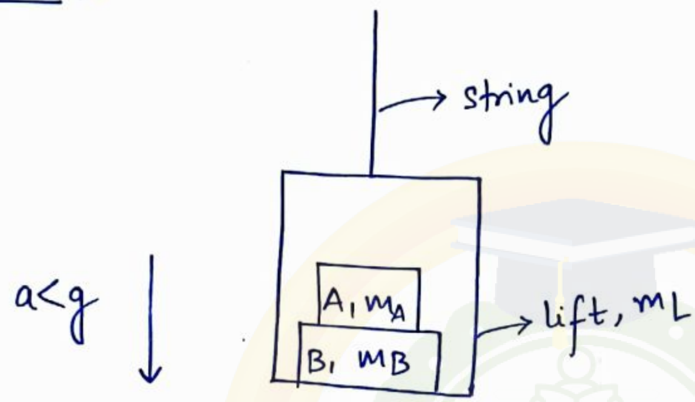
$$\underbrace{m_1 g\sin\theta - T}_{\text{Resultant of actual forces}} = \underbrace{m_1 a}_{\text{Response}} \text{ — 2nd law}$$

(m_2)

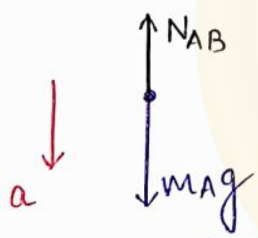


$$T - m_2 g = m_2 a$$ — 2nd Law

Case 3 :→

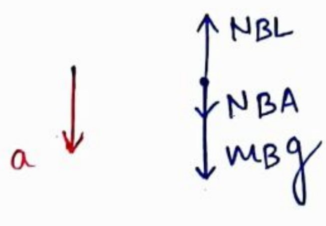


(A)



$$\boxed{m_A g - N_{AB} = m_A a}$$ — 2nd Law

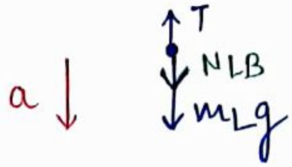
(B)



$$\boxed{m_B g + N_{BA} - N_{BL} = m_B a}$$ — 2nd Law

$$\boxed{N_{BA} = N_{AB}}$$ — 3rd Law

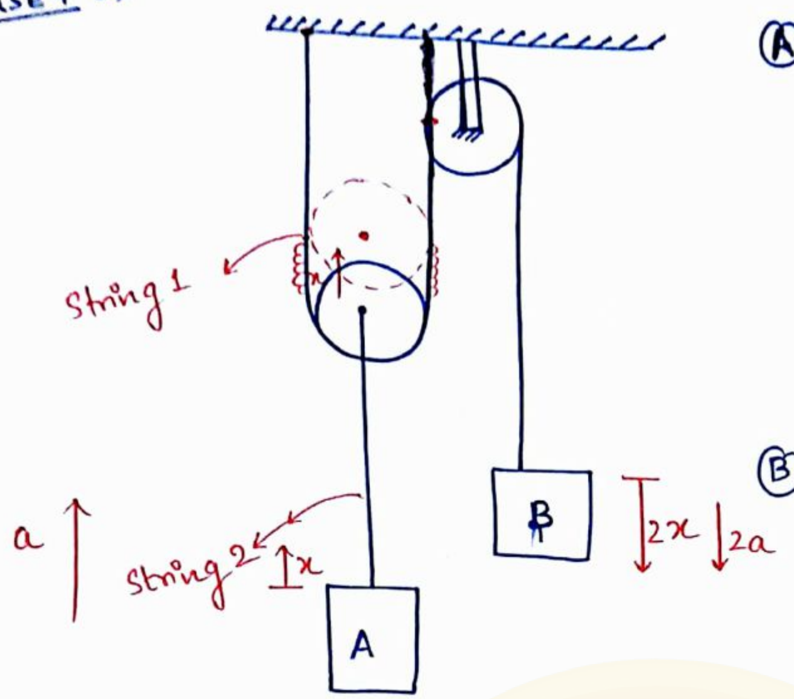
(Lift)



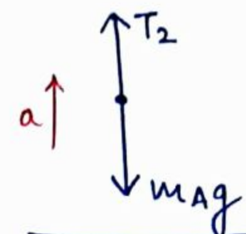
$$\boxed{N_{LB} + m_L g - T = m_L a}$$ — 2nd law

$$\boxed{N_{LB} = N_{BL}}$$ — 3rd Law

Case 4 →

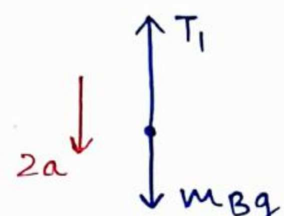


(A)



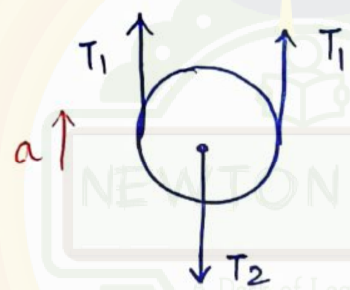
$$\boxed{T_2 - m_A g = m_A a} \text{ — 2nd law}$$

(B)



$$\boxed{m_B g - T_1 = m_B 2a} \text{ — (2) — 2nd law}$$

moving pulley



$$2T_1 - T_2 = (0)a \text{ — 2nd law}$$

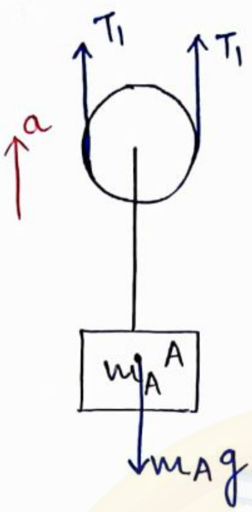
$$\boxed{2T_1 = T_2}$$

put T_2 in 1st equ.

$$\Rightarrow \boxed{2T_1 - m_A g = m_A a} \text{ — (1)}$$

A + String 2 + moving Pulley

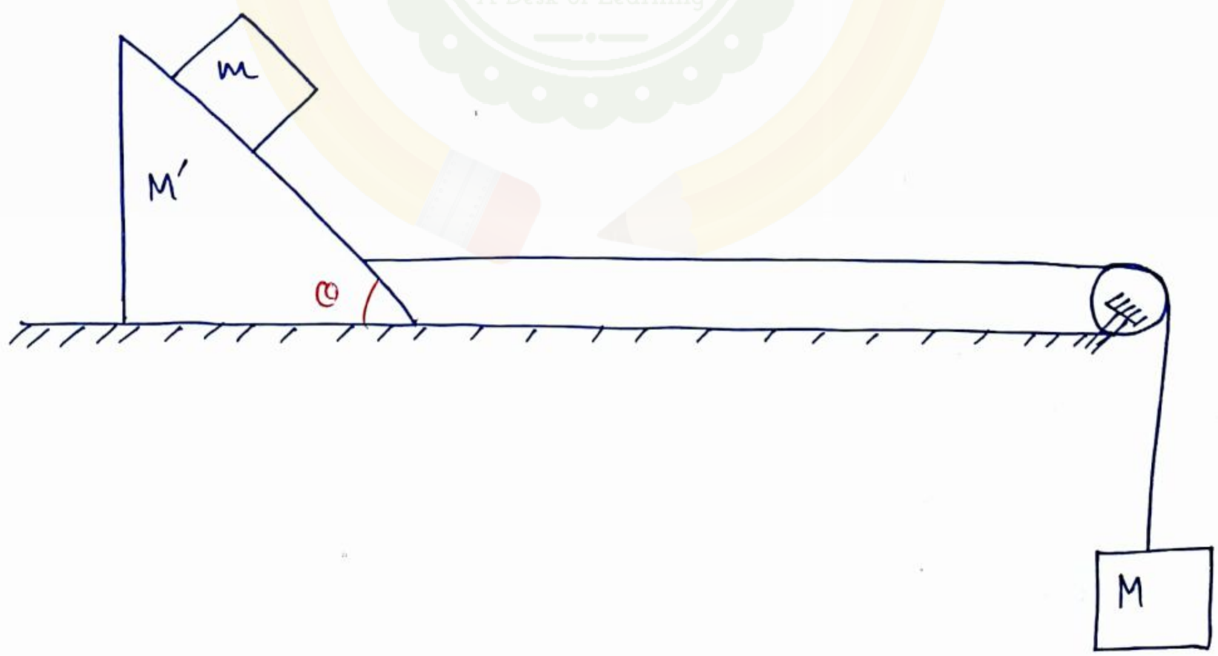
Because $\vec{a}$ vector is same for all.

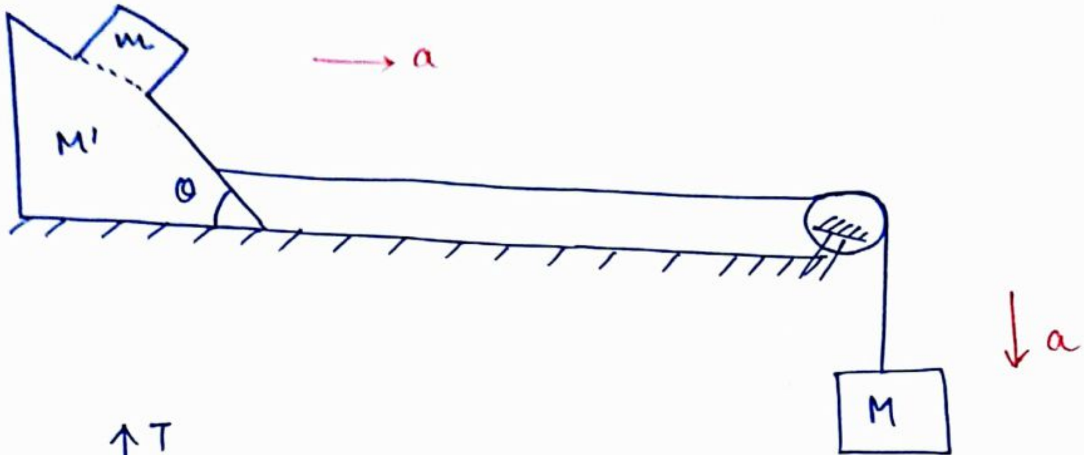


$2T_1 - m_A g = m_A a$ — 2nd Law

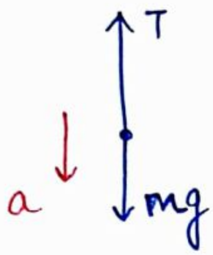
Q. find the mass M of the hanging block as shown in figure so as to prevent the sleeping of smaller Block over the triangulated Block. assume all surfaces smooth.

can motion





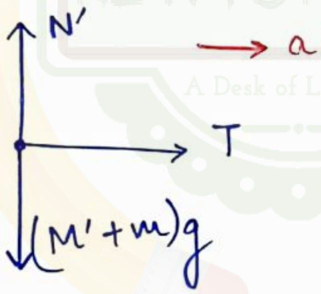
M



$$\boxed{Mg - T = Ma} \quad \text{— 2nd Law} \quad (1)$$

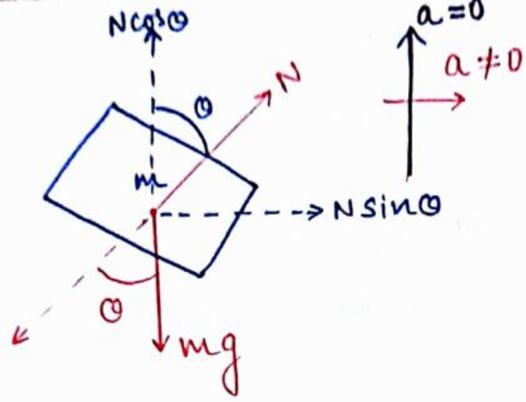
M'+m

$$\vec{a}_{M'} = \vec{a}_m = \vec{a}$$



$$N' = (M'+m)g \quad \text{— 1st Law}$$

$$\boxed{T = \underbrace{(M'+m)}_{\text{Response}}a} \quad \text{— 2nd Law} \quad (2)$$



$N\cos\theta = mg$ — 1st law

$N\sin\theta = \underbrace{ma}_{\text{response}}$ — 2nd law

$\tan\theta = \dfrac{a}{g}$

$\boxed{a = g\tan\theta}$ — (3)

using value of (2), (3) in eqn. (1)

$\Rightarrow Mg - (M' + m)g\tan\theta = Mg\tan\theta$

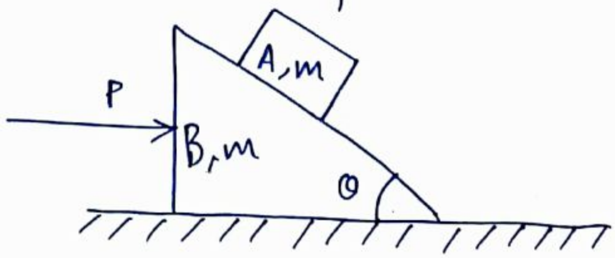
$\Rightarrow M\cancel{g} - M'\cancel{g}\tan\theta - m\cancel{g}\tan\theta = M\cancel{g}\tan\theta$

$\Rightarrow M(1 - \tan\theta) = (M' + m)\tan\theta$

$\Rightarrow M = \dfrac{(M' + m)\tan\theta}{(1 - \tan\theta)}$

$\Rightarrow \boxed{M = \dfrac{M' + m}{\cot\theta - 1}}$

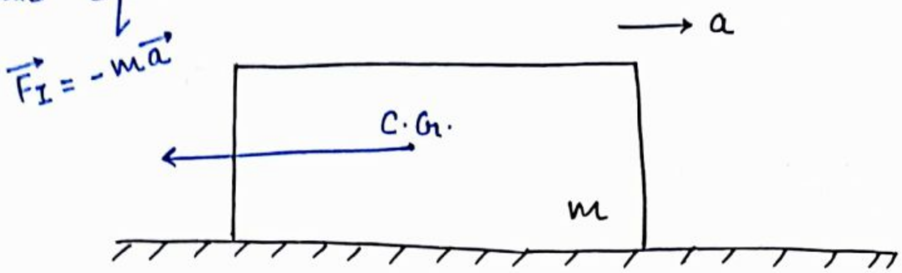
Q. Find P such that there is no slipping between A and B. assume all surfaces smooth.



Ans → $\boxed{P = 2mg\tan\theta}$

30/7/2016

D'Alembert's Principle → It states "under the action of effective force and inertia force body will be in dynamic equilibrium."



$$\vec{F_R} = m\vec{a}$$

$$\underbrace{\vec{F_R}}_{\text{effective force}} + \underbrace{\vec{F_I}}_{\text{inertia force}} = 0$$

If a body of mass 'm' moves rightward with an acc^r. '$\vec{a}$' then it means there is a resultant force $\boxed{\vec{F_R} = m\vec{a}}$ acting rightwards.

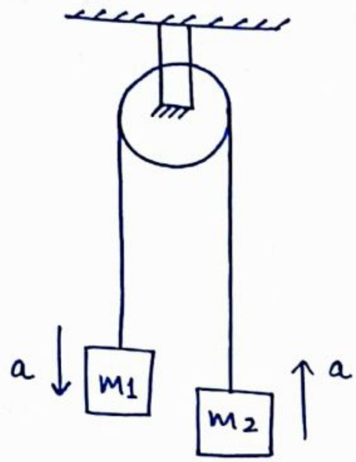
Now If we apply a force $\boxed{F_I = ma}$ acting leftwards at the C.G. of the body then it will bring the body in equilibrium called "DYNAMIC EQUILIBRIUM".

Note :- (1) Inertia force acts opposite to the acceleration but not opposite to the motion.

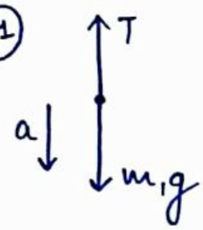
$\rightarrow a$ ✓ $\leftarrow v$ $\rightarrow a$ ✓ $\rightarrow v$

(2) $\boxed{F_I = ma}$ acting leftwards is an imaginary (Pseudo force) which is not actually acting on the body.

Case I

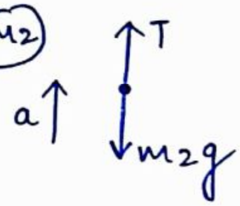


(m1)



$$m_1 g - T = \underbrace{m_1 a}_{\text{effect}} \text{ — } \underline{\underline{2nd}}$$

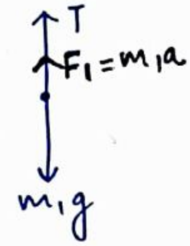
(m2)



$$T - m_2 g = \underbrace{m_2 a}_{\text{effect}} \text{ — } \underline{\underline{2nd}}$$

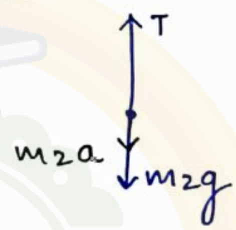
Considering Dynamic equi^m.

(m1)



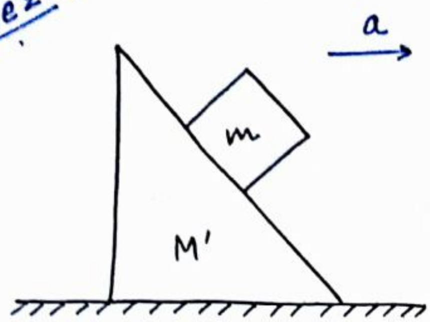
$$T_1 + m_1 a = \underbrace{m_1 g}_{\text{Inertia force}} - \underline{\underline{D'Alemb}}$$

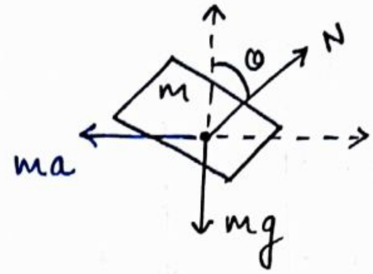
(m2)



$$T = \underbrace{m_2 a}_{\text{Inertia force}} + m_2 g \text{ — } \underline{\underline{D'Alemb}}$$

Case 2 :→





$N\cos\theta = mg$ — 1st

$N\sin\theta = \underbrace{ma}_{\text{Inertia force}}$ — D'Alemberts

* FRICTION :→

Static friction (f_s)

Kinetic friction (f_K)

$0 \le f_s \le (f_s)_{max}$

$(f_s)_{max} \propto N$

$$\boxed{(f_s)_{max} = \mu_s N}$$

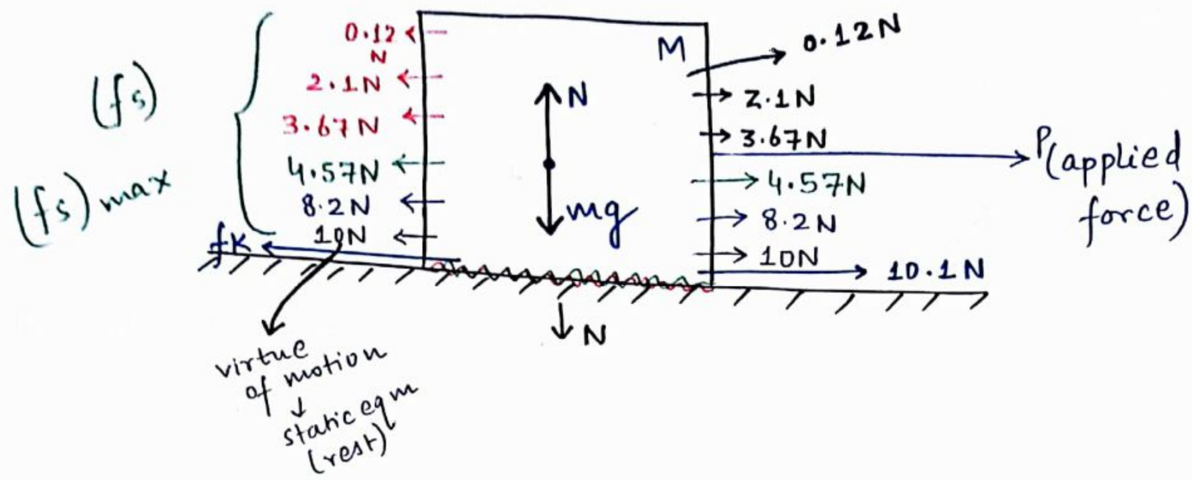
μ_s = coeff. of static friction

f_K = constant $= \mu_K N$

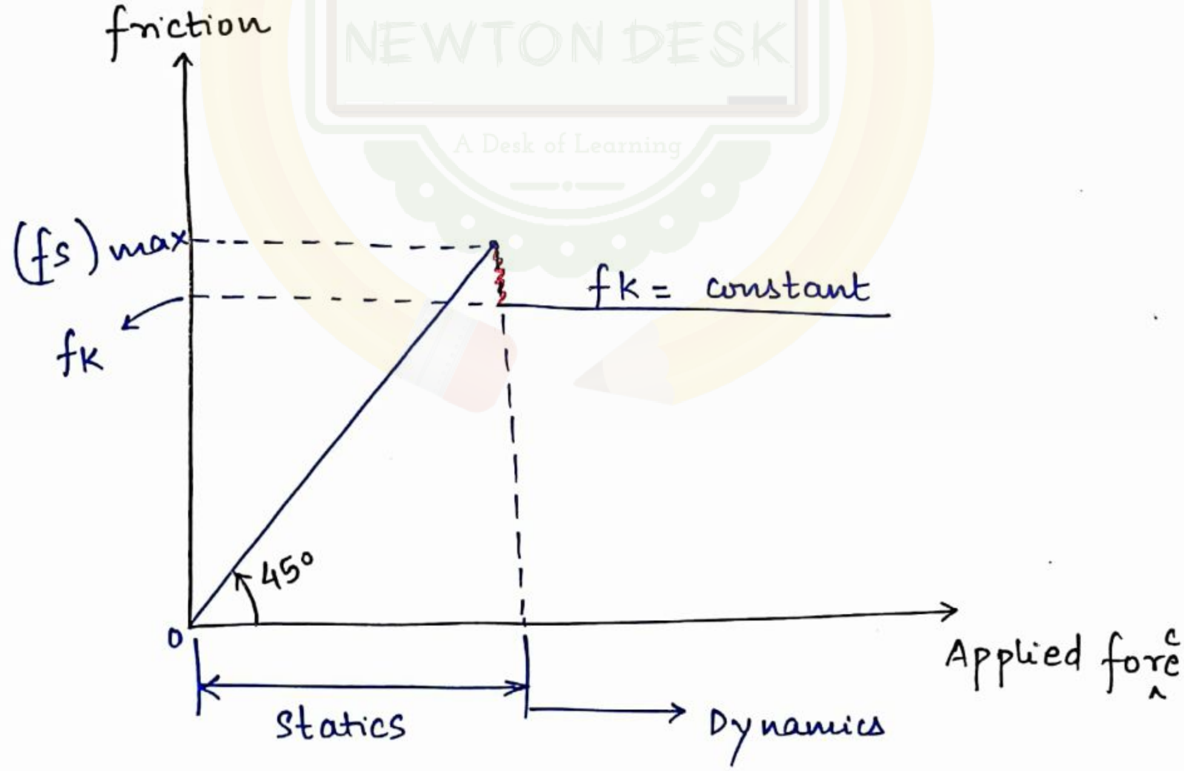
μ_K = coeff. of kinetic friction

$$\boxed{(f_s)_{max} > f_K}$$

$$\boxed{\mu_s > \mu_K}$$



* **Static friction** :- When 2 bodies in contact trying to slide with respect to each other but they are not sliding then there exist static friction whose values varies from 0 to a maximum value $(f_s)_{max}$ depending on the value of applied force.

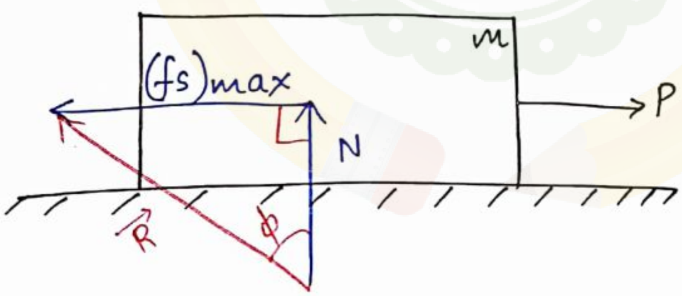


NOTE :- If the value of applied Force is less than (or) equal to $(f_s)_{Max}$, then there exist static friction whose value will be exactly equal to the applied load & will act opposite to the applied Force.

* **Kinetic friction** :→ when there is relative motion, there exist kinetic friction whose value remains constant and equal to $\boxed{\mu_K N}$ and will act opposite to the relative motion.

* **Angle of Friction, ϕ** :→ when the body is at verge of motion ($(f_s)_{max}$ is acting), the angle made by contact force with normal Reaction is called angle of friction.

$$\boxed{\tan\phi = \mu_s}$$



$$\tan\phi = \frac{(f_s)_{max}}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}} = \mu_s.$$

if μ_s and μ_K are not given seperately ← Practical Qn's in Real life ↑✓ ✗ ↑not applied impact force ↓ μ_s in no picture only in gradually applied load for verge of motion.

i) $\mu_s = \mu_K = \mu$

ii) $0 \le f_s \le (f_s)_{max} = \mu N = f_K$

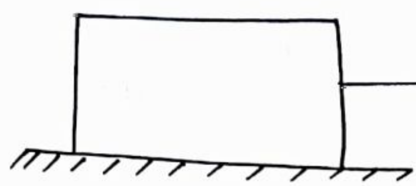
iii) $\tan\phi = \mu$.

Q Find the Frictional Force developed in the system shown in figure below for P = 14Nt, 50Nt and 60Nt :-

wt = 100Nt

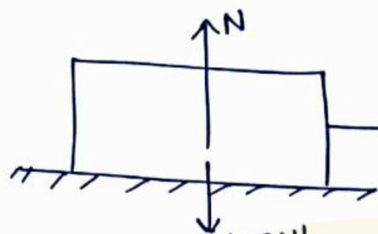
$\mu_s = 0.5$

$\mu_K = 0.3$



Nt → Newton

Sol



$$(f_s)_{Max} = \mu_s N = 0.5(100N) = 50Nt$$

$$f_K = \mu_K N = 0.3(100N) = 30Nt$$

(i.) if P = 40Nt

$P < (f_s)_{max}$

$f = f_s = P = 40Nt$.

(ii.) if P = 50Nt

$P = (f_s)_{max}$ (Body is at verge of motion)

$f = (f_s)_{max} = 50Nt$.

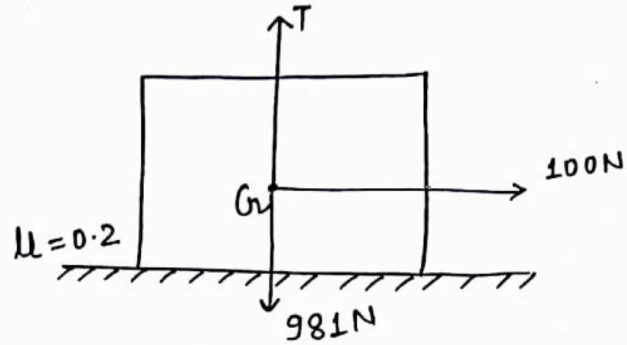
(iii.) if P = 60Nt

$P > (f_s)_{max}$

$f = f_K = \mu_K N = 30Nt$.

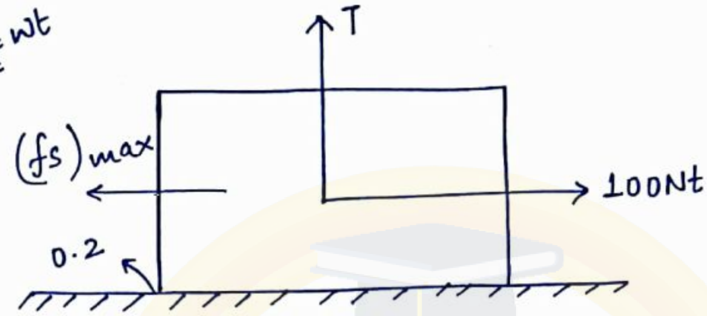
WK 3.10

Pg. 81



Sol

weight = 981 Nt = wt

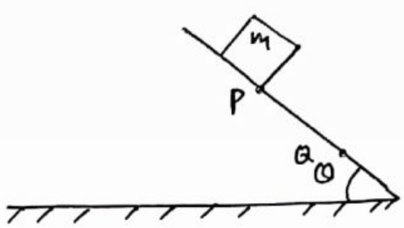


$$100 = (f_s)_{max} = \mu N$$
$$= 0.2 N$$
$$100$$
$$\Rightarrow \boxed{N = 500 Nt} \text{ — } (1)$$

$$T + N = 981 \text{ — } (2)$$

$$T = 981 - N = 981 - 500 = 481 \text{ Nt}.$$

WK Q3.9



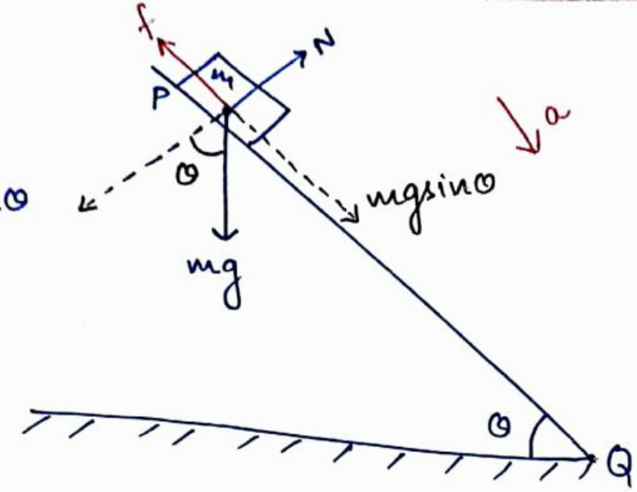
μ , $\mu < \tan\theta$

$P \rightarrow Q$ $\boxed{PQ = s}$

$t = ?$

NewtonDesk.com

Sol



$u=0$

$\mu < \tan\theta$

$PQ = s$

released / rest

$\downarrow$

$u = 0$

$N = mg\cos\theta$ — 1st

$mg\sin\theta \leftarrow$ effective force

$mg\sin\theta > (f_s)_{max} = \mu N$

$\cancel{m}g\sin\theta > \mu \cancel{m}g\cos\theta$

$\boxed{\tan\theta > \mu}$

motion kar raha hai.

$mg\sin\theta - f_K = Ma$ — 2nd

$\cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta = \cancel{m}a$

$$\boxed{a = \underbrace{g\cos\theta\,[\tan\theta - \mu]}_{\text{constant}}}$$

hence, $s = \cancel{ut}^{\,0} + \frac{1}{2}at^2$

$t = \sqrt{\frac{2s}{a}}$

$$\boxed{t = \sqrt{\frac{2s}{g\cos\theta(\tan\theta - \mu)}}}$$

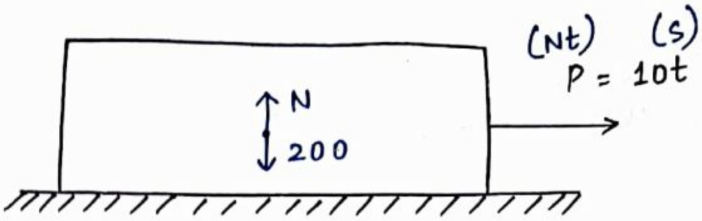
Q3.12

Pg 82

$Wt = 200\,Nt$

$\mu_s = 0.4$

$\mu_K = 0.2$



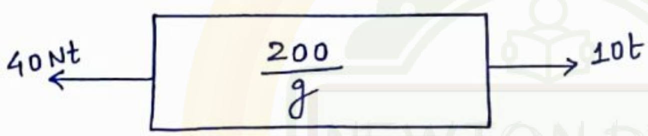
$$(f_s)_{max} = \mu_s N = 0.4 \times 200 = 80\,Nt.$$

$$f_K = \mu_K N = 0.2 \times 200 = 40\,Nt.$$

at t = 8 sec, block will be at verge of motion

$$V_{t=8s} = 0$$

from t = 8s to 10s



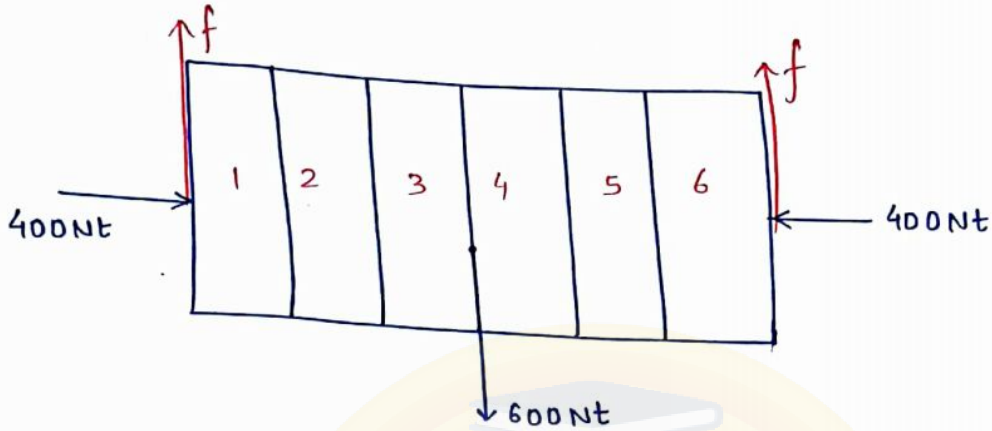
$$a = \frac{10t - 40}{m} = \frac{10t - 40}{\left(\frac{200}{g}\right)} = \frac{dv}{dt}$$

$$\Rightarrow \int_8^{10} (10t - 40)\,dt = \int_0^{V_{10}} \frac{200}{g}\,dv$$

$$\Rightarrow \boxed{V_{10} = 4.905\ m/s}$$

Q 6 identical Books each of weight 100 Nt are lifted with hands by applying compressive force of 400N as shown in fig. if the Books are in the verge of slipping then μ b/w books and hands is?

Sol



$$2f = 600$$

$$f = 300 = \mu N$$

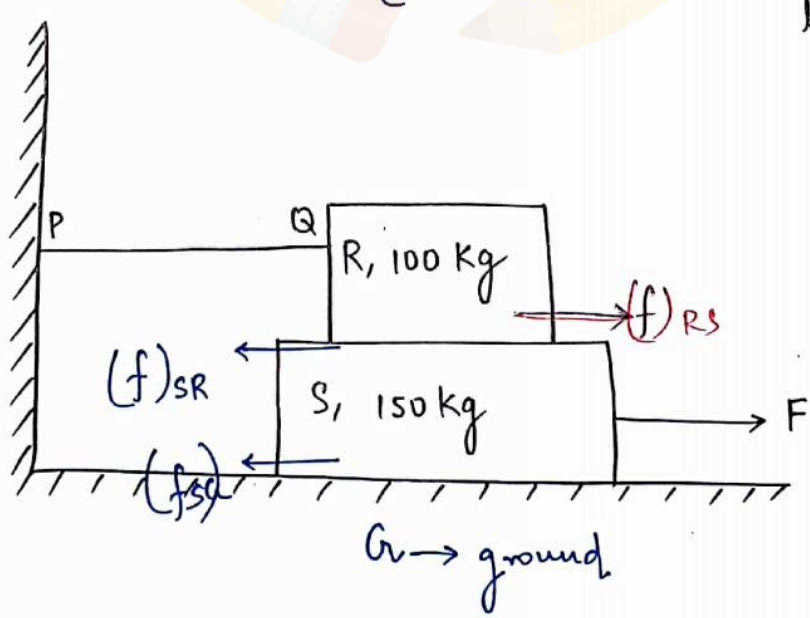
$$\frac{300}{400} = \mu$$

$$\boxed{\mu = 0.75}$$

WK
Q3.13

(massless ↳ इस्तेमाल नहीं.)

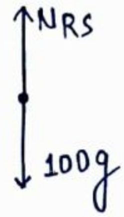
$\mu_s = 0.4$



$$F_{min} = \left[(f_s)_{max}\right]_{SR} + \left[(f_s)_{max}\right]_{SG}$$

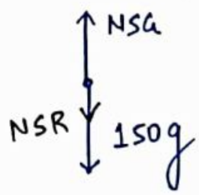
$$F_{min} = \mu_s N_{SR} + \mu_s N_{SG} = 0.4 \times 350 \times 9.81 = 1.37\ KN$$

(R)



$N_{RS} = 100g$ — 1st

(S)

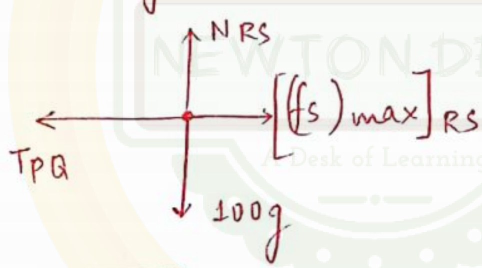


$N_{SR} = N_{RS} = 100g$ — 3rd

$N_{SG} = N_{SR} + 150g$ — 1st

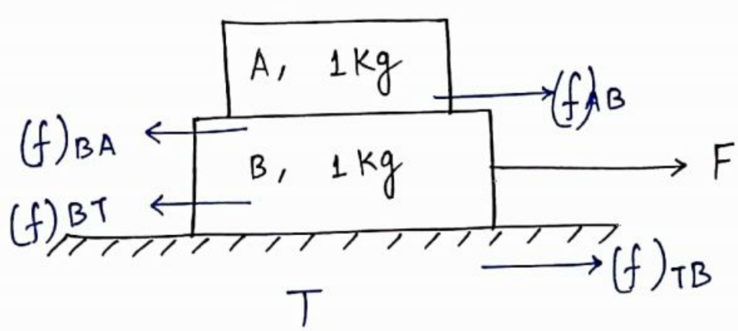
$N_{SG} = 250g$

if Tension in string → ?



$$\boxed{T_{PQ} = \mu \times 100g}$$

Q3.
Q7)



$\mu = 0.3$

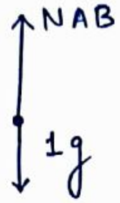
$$\left[(f_s)_{max}\right]_{BA} = \mu N_{BA} = 0.3 \times 10 = \underline{3Nt}$$

$$\left[(f_s)_{max}\right]_{BT} = \mu N_{BT} = 0.3 \times 20 = \underline{6Nt}$$

At $F = 6Nt$, Both A & B together will be at verge of motion.

Taking $g = 10\, m/s^2$

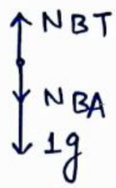
(A)



$N_{AB} = 10\,Nt$ — $\underline{\underline{1st}}$

$N_{BA} = N_{AB} = 10\,Nt$ — $\underline{\underline{3rd}}$

(B)

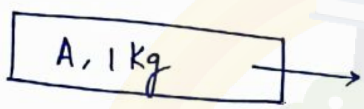


$N_{BT} = N_{BA} + 1g$

$N_{BT} = N_{BA} + 10$ — $\underline{\underline{1st}}$

$N_{BT} = 20\,Nt.$

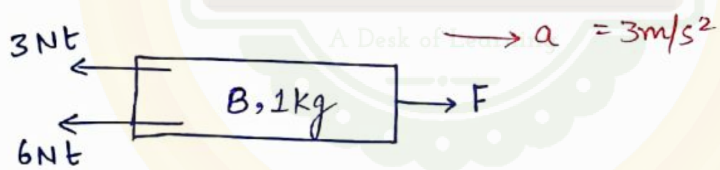
(A)



$3\,Nt = [(f_s)_{max}]_{AB}$

$(a_A)_{max} = 3/1 = 3\, m/s^2$

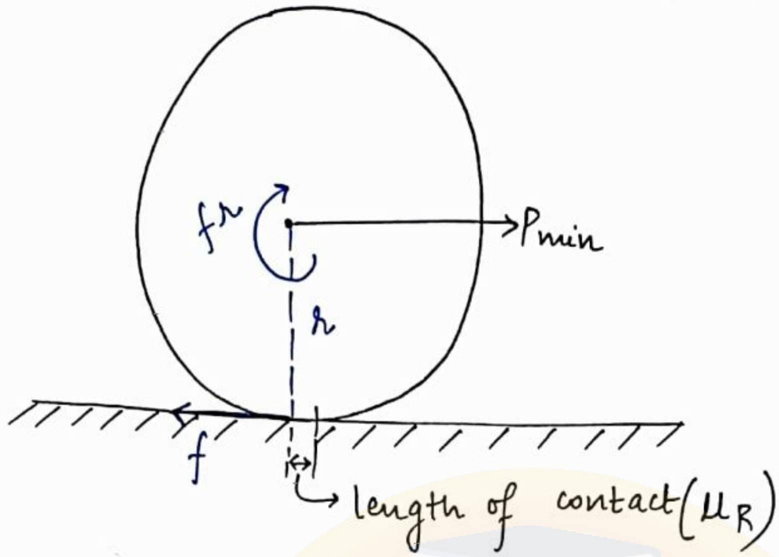
(B)



$F - 9 = 1 \times 3$

$\boxed{F = 12\,Nt}$

* Rolling Resistance (f_R) →



$$P_{min} = f_R = \frac{\mu_R W}{R}$$

W → weight 'or' load on Roller

R → Radius of Roller

μ_R → coefficient of Rolling resistance (mm)

Q. A wheel of radius 600 mm diameter on a horizontal steel rail carries a load of 500N, if coefficient of rolling resistance is 0.3 mm, the force in Nt necessary to roll the wheel along the rail is ?

Sol

$D = 600mm$; $R = 300mm$

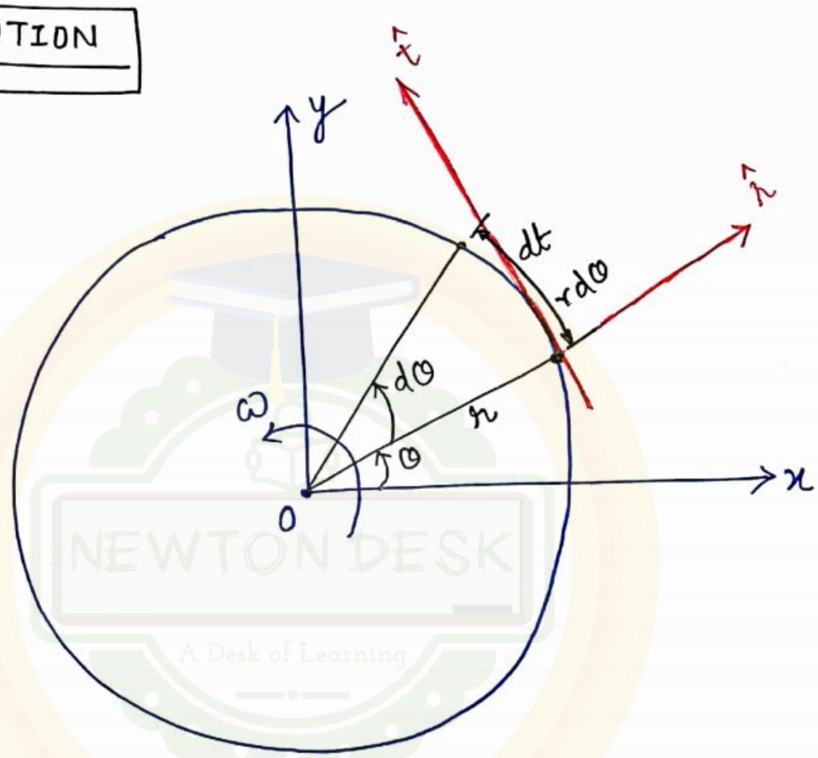
$\mu_R = 0.3mm$

$W = 500N$

$$P_{min} = f_R = \frac{\mu_R W}{R} = \frac{0.3 \times 500}{300} = 0.5 Nt$$

* **CIRCULAR MOTION**

Kinematics



ω = angular velocity

$$= \frac{d\theta}{dt} \left(rad/s\right)$$

$$\alpha = \frac{d\omega}{dt} \left(rad/s^2\right)$$

$\vec{\omega}$ and $\vec{\alpha}$ → vector quantities

Dirn → $\vec{\omega}$ and $\vec{\alpha}$ will be ⊥r to plane of ~~circle~~ circle, will pass through centre ~~to~~ and will be given by Right hand thumb rule.

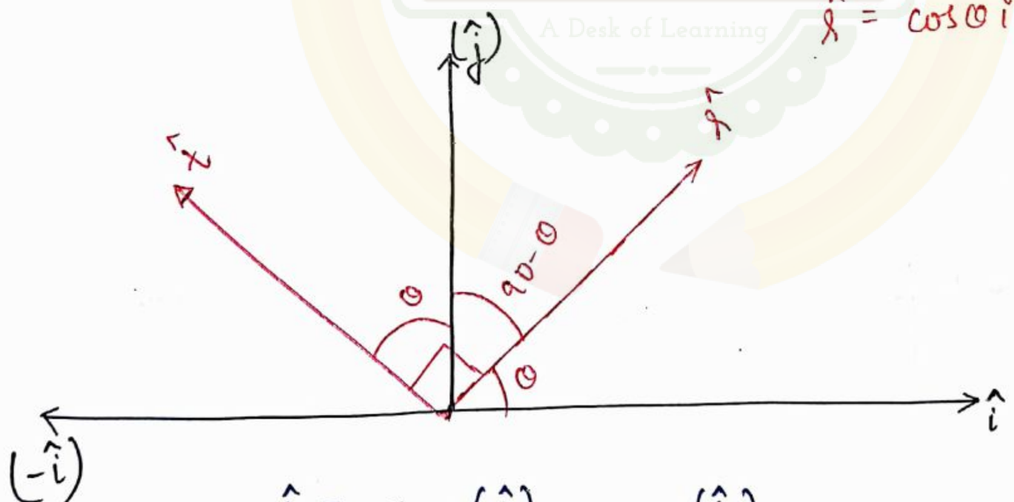
Linear speed $= \dfrac{\text{Distance}}{\text{time}} = \dfrac{r\,d\theta}{dt} = r\omega \,(m/s)$

If $\vec{\alpha}$ is constant :→

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$\boxed{(2\pi)\left(\frac{N}{60}\right) = \omega}$$

$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$



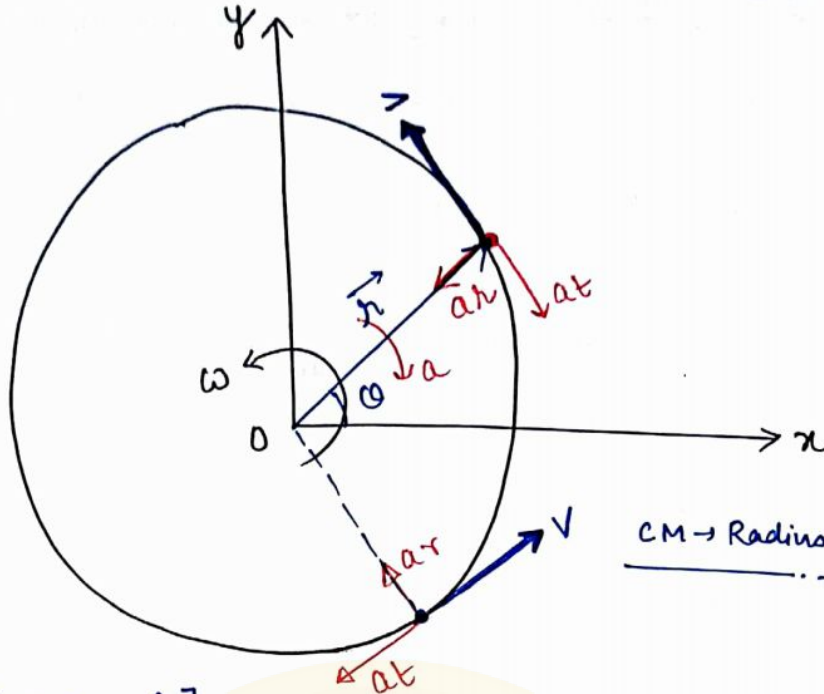
$\hat{t} = \sin\theta(-\hat{i}) + \cos\theta(\hat{j})$

[tangential dirn. $\vec{n}$ unit vector]

$\vec{v}, \vec{a}$

$\vec{v} = \dfrac{d\vec{r}}{dt}$

$\vec{r} = |\vec{r}|\,\hat{r}$



CM → Radius(same/const)

$$\vec{r} = r\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\left[-\sin\theta\frac{d\theta}{dt}\,\hat{i} + \cos\theta\frac{d\theta}{dt}\,\hat{j}\right]$$

$$\vec{v} = r\omega\left[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right]$$

$$\boxed{\vec{v} = r\omega\,(\hat{t})}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\left[\omega\left(-\cos\theta\frac{d\theta}{dt}\,\hat{i} - \sin\theta\frac{d\theta}{dt}\,\hat{j}\right) + \left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\frac{d\omega}{dt}\right]$$

$$\vec{a} = r\omega^2\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right] + r\alpha\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

$$\vec{a} = r\omega^2(-\hat{r}) + r\alpha(\hat{t}).$$

$$\vec{a} = \vec{a}_r + \vec{a}_t$$

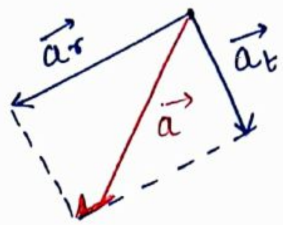
$\vec{a}_t$ → causes change in mag. of vel.

$\vec{a}_r$: causes change in direction of velocity

$a_r = r\omega^2 = \frac{v^2}{r}$

$v = r\omega$

$\frac{v}{r} = \omega$



$|\vec{a}| = \sqrt{a_r^2 + a_t^2}$

* Uniform Circular Motion:→

$v = r\omega = \text{Constant} \Rightarrow \omega = \text{constant} \Rightarrow \alpha = 0 \Rightarrow \vec{a_t} = 0$

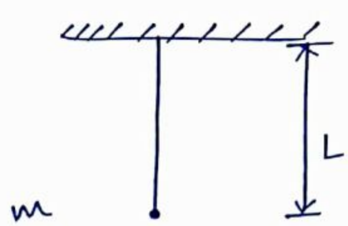
$$\boxed{\vec{a} = \vec{a_r}}$$

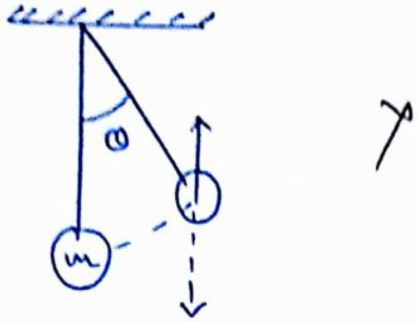
~~Note:~~

Q A particle of mass 'm' is suspended from a ceiling through a string of length 'L' and moves in a horizontal circle of Radius 'r'. Find :- (a) speed of the particle.

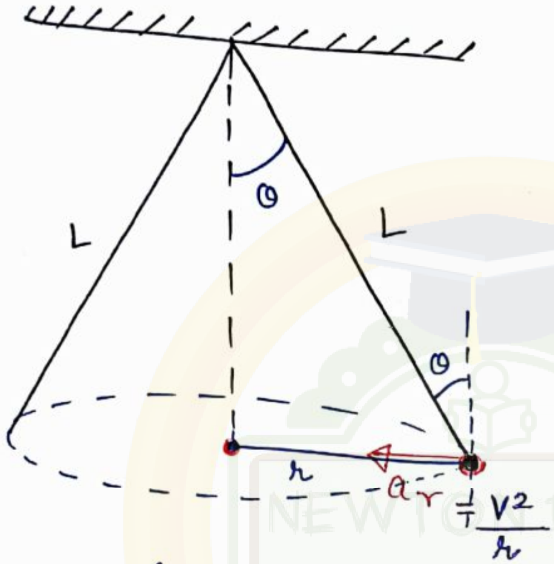
(b) Tension in the string.

Sol

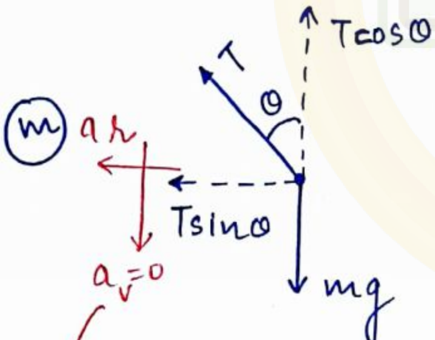




Sol :-



$\sin\theta = \frac{r}{L}$



accr. in vertical

$T\cos\theta = mg$ — 1st

$T\sin\theta = m|\vec{a}_r| = \frac{mv^2}{r}$ — 2nd

FR (net Resultant force)

effect

centripetal force

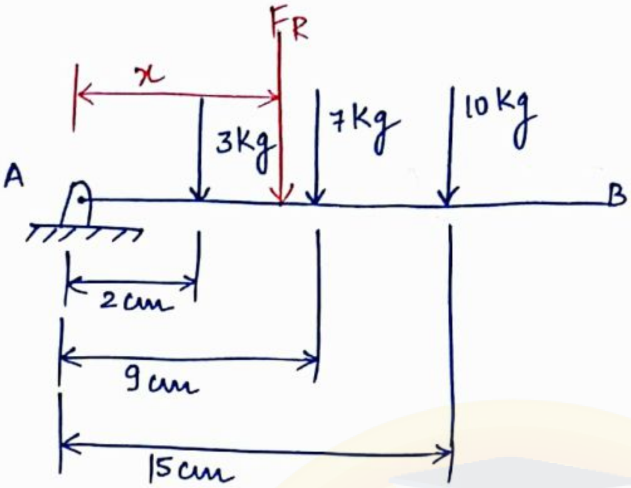
actual forces ka Resultant

$$\boxed{T = \frac{mg}{\cos\theta}}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\Rightarrow \boxed{v = \sqrt{rg\tan\theta}}$$

Q1.3



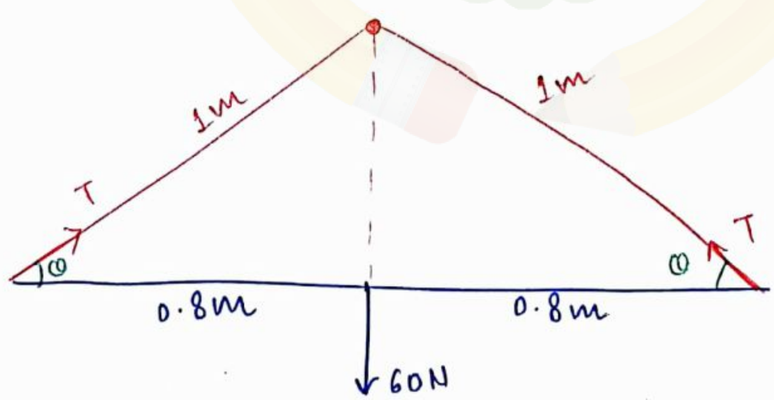
$F_R = 3+7+10 = 20\,kg$

$\Sigma \vec{M}_A = 3\times 2 + 7\times 9 + 10\times 15 = F_R \times x$

$6 + 63 + 150 = F_R x = 20 \times x$

$x = 10.95\,cm$

Q1.8



$2T\sin\theta = 60N$

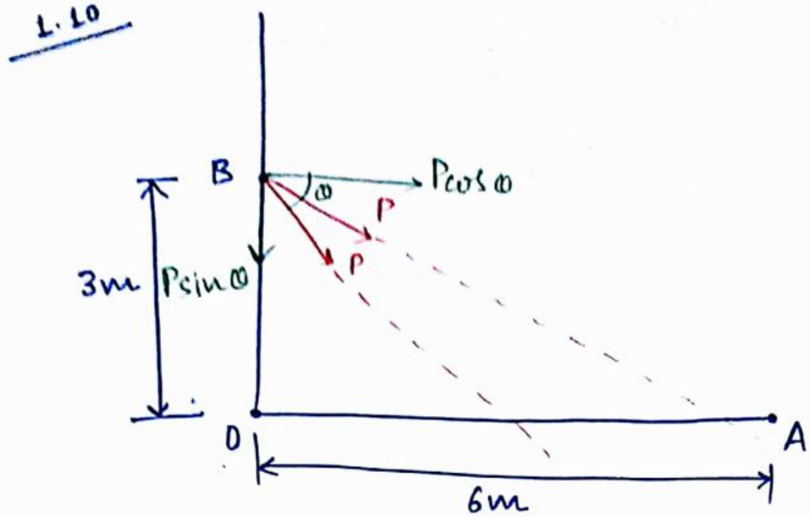
$T = 30/\sin\theta$

$\cos\theta = \frac{0.8}{1} = 0.8$

$\sin\theta = \sqrt{1 - 0.8^2}$
$= 0.6$

$\boxed{T = 50}$

1.10



$M_O = 180 = 3P\cos\theta$ —— ①

$M_A = 90 = 6P\sin\theta - 3P\cos\theta$ —— ②

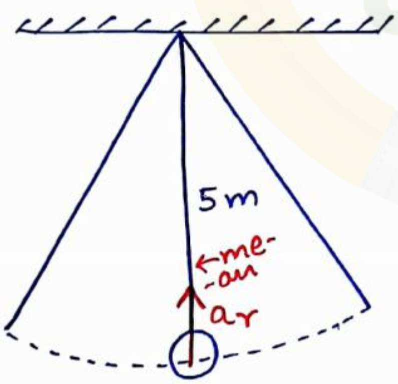
$M_O = 180$ Nm; CW

$M_A = 90$ Nm; ACW

$M_B = 0$

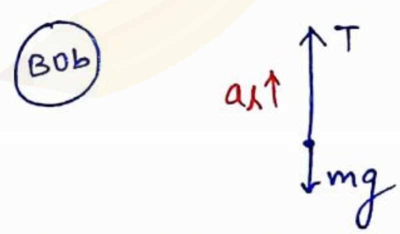
6/10/2016 Q> A simple pendulum of length 5m with a bob of mass 1Kg is in simple harmonic motion as it passes through its mean position, the bob has a speed of 5m/sec. The net force on the bob at the mean position is?

Sol>



← mean position

FBD of Bob



Newton's 2nd Law of motion ← $\underbrace{T - mg}_{\text{centripetal force}} = ma_r$ —— $\underline{\underline{2^{nd}}}$

$$T - mg = \frac{mv^2}{r}$$

$$F_{net} = \frac{mv^2}{r} = \frac{1 \times 5^{\cancel{2}}}{\cancel{5}}$$

$$= 5 \text{ Newton (Nt)}$$

Q.> A stone of mass 'm' is tied to the end of a string of length 'l' and whirled in a vertical circle at constant speed. The Tension in the string is max^m. when the stone is ___

ⓐ at top of circle.

ⓑ at bottom of circle.

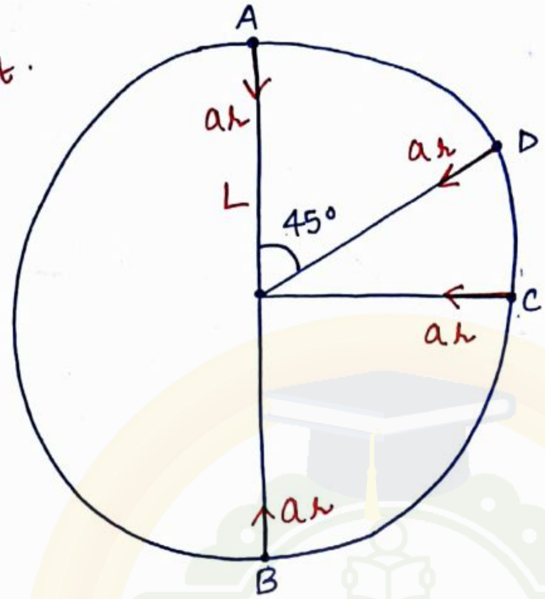
ⓒ half way down from top.

ⓓ Quarter way down from top.

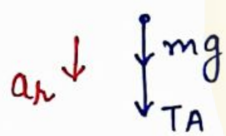
Ans.

$V = \text{const.}$

$a_r = \dfrac{V^2}{L} = \text{const.}$



Ⓐ

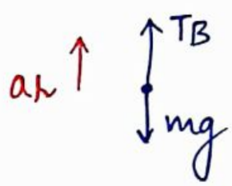


$$T_A + mg = ma_r \text{ — } \underline{2^{nd}}$$

$$T_A + mg = \frac{mV^2}{L}$$

$$\boxed{T_A = \frac{mV^2}{L} - mg} = T_{min}$$

Ⓑ

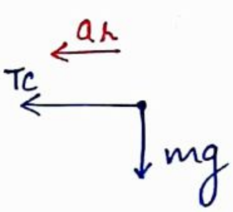


$$T_B - mg = ma_r \text{ — } \underline{2^{nd}}$$

$$T_B - mg = \frac{mV^2}{L}$$

$$\boxed{T_B = \frac{mV^2}{L} + mg} = T_{max}$$

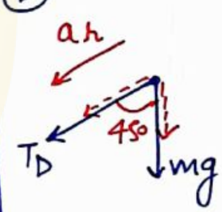
Ⓒ



$$T_C = ma_r \text{ — } \underline{2^{nd}}$$

$$\boxed{T_C = \frac{mV^2}{L}}$$

Ⓓ

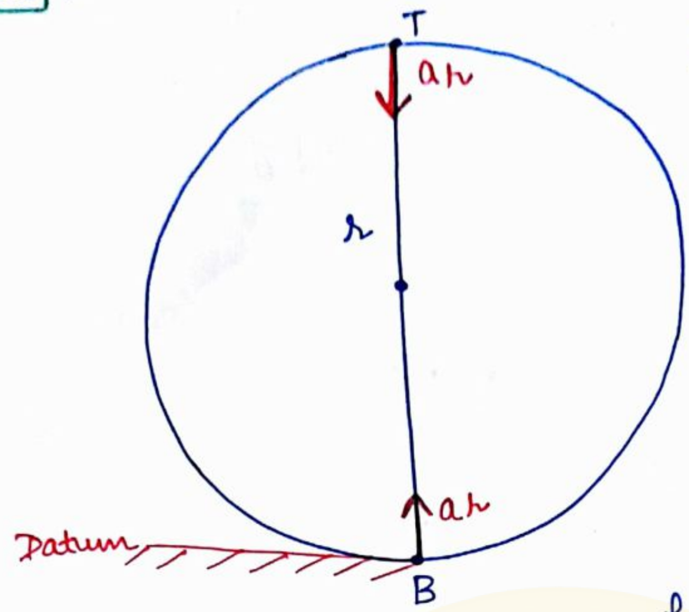


$$T_D + mg\cos 45° = ma_r \text{ — } \underline{2^{nd}}$$

$$T_D + mg\cos 45° = \frac{mV^2}{L}$$

$$\boxed{T_D = \frac{mV^2}{L} - mg\cos 45°}$$

* **Min^m. velocities at Top and Bottom to complete a vertical circle** :-



Top

$a_r \downarrow \quad \downarrow mg \quad \downarrow T$

$$T + mg = m a_r \text{ — } \underline{2^{nd}}$$

radial accⁿ.

$$T + mg = \frac{mv^2}{r}$$

$$\underbrace{A}_{\text{min}} + \underbrace{B}_{\text{const.}} = C \rightarrow \text{min}$$

for velocity to be minimum, $\boxed{T=0}$

$$\cancel{m}g = \frac{\cancel{m}v^2_{min}}{r}$$

$$\boxed{v_{min} = \sqrt{gr}}$$

Now, Bottom

$$(GPE)_B + (K.E)_B = (GPE)_T + (KE)_T$$

gravitational

$$0 + \tfrac{1}{2}\cancel{m}v_B^2 = \cancel{m}g(2r) + \tfrac{1}{2}\cancel{m}v_T^2$$

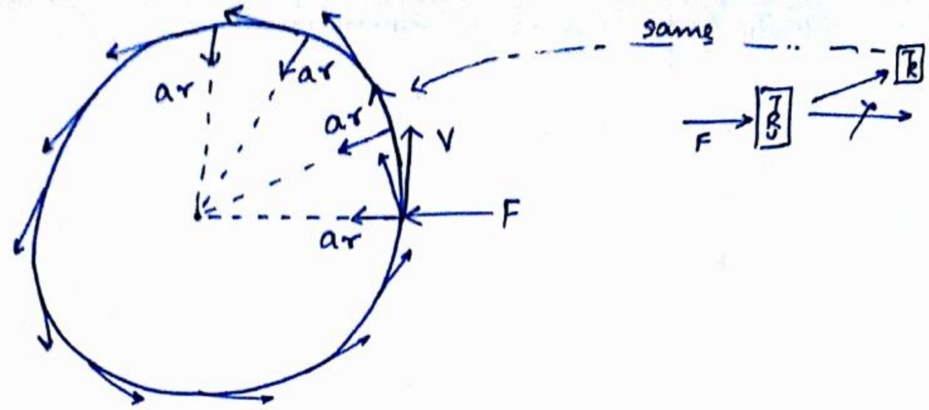
$$v_B^2 = 4gr + v_T^2$$

$$\underbrace{C}_{\text{min}} = \underbrace{A}_{\text{const.}} + B \rightarrow \text{min}$$

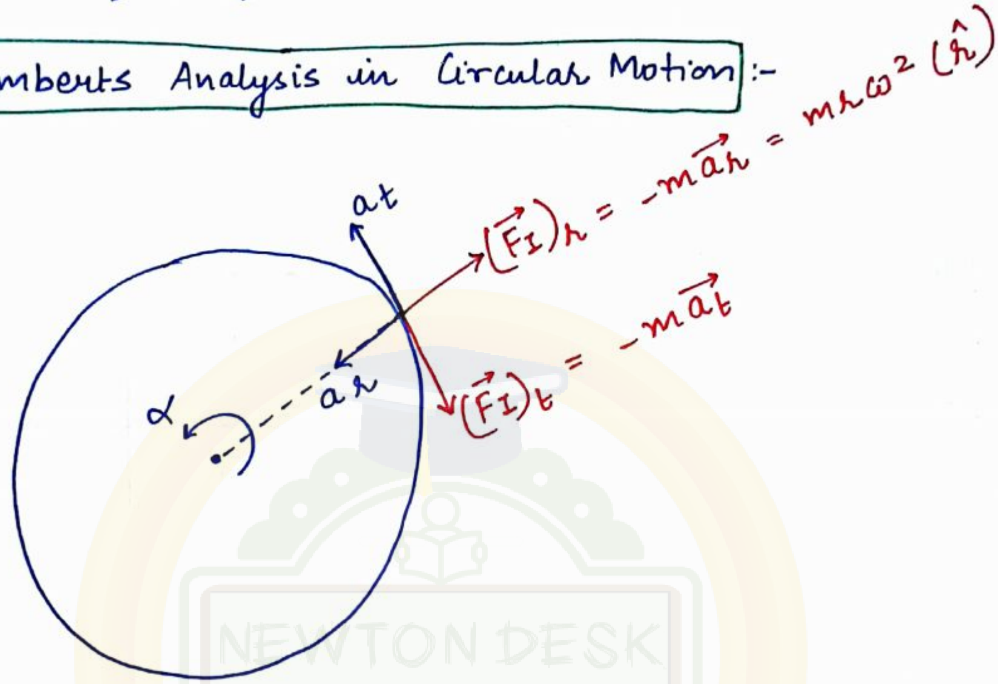
$$v_B^2 = 4gr + gr$$

$$\boxed{(v_B)_{min} = \sqrt{5gr}}$$

NOTE :-



* D' Alemberts Analysis in Circular Motion :-



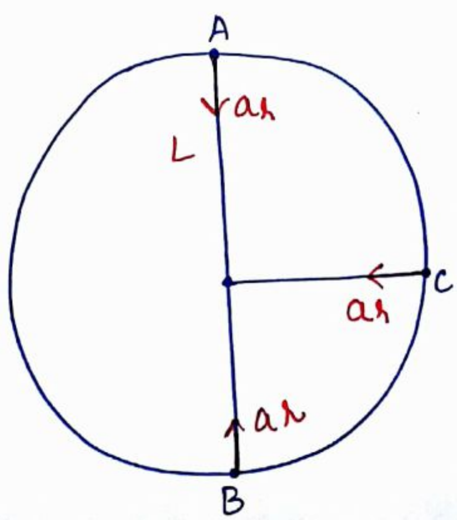
$$\underbrace{(\vec{F_R})_r}_{\text{centripetal force}} + \underbrace{(\vec{F_I})_r}_{\text{centrifugal force}} = 0$$

$$(\vec{F_R})_t + (\vec{F_I})_t = 0$$

Case →

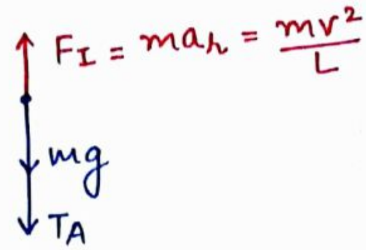
$v = \text{const.}$

$a_r = \frac{v^2}{L} = \text{const.}$



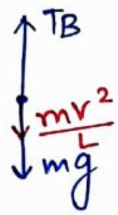
Considering dynamic equilibrium

(A)



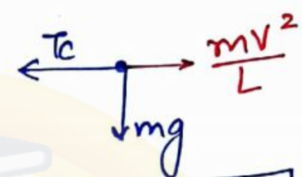
$$T_A + mg = \underbrace{\frac{mv^2}{L}}_{\text{Inertia force}} \quad \text{— D'Alemb}$$

(B)



$$T_B = mg + \frac{mv^2}{L}$$

(C)



$$\boxed{T_C = \frac{mv^2}{L}} \longrightarrow \text{D'Alemb}$$

* Linear Momentum, $\vec{P} = m\vec{V}_{cm}$

$$\frac{d\vec{P}}{dt} = \frac{d(m\vec{V}_{CM})}{dt} = m\frac{d\vec{V}_{CM}}{dt} = m\vec{a}_{CM} = (\vec{F}_R)_{ext}$$

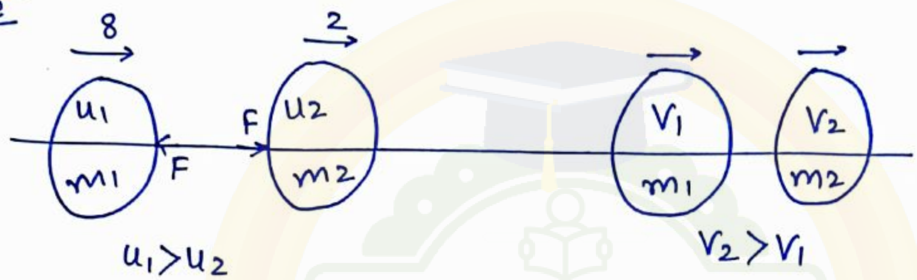
2nd

$$\boxed{\frac{d\vec{P}}{dt} = (\vec{F}_R)_{ext}} \rightarrow \underline{2nd}$$

* Conservation of $\vec{P} \Rightarrow \vec{P} =$ constant $\Rightarrow d\vec{P} = 0$

only when $(\vec{F}_R)_{ext} = 0$

Example :-



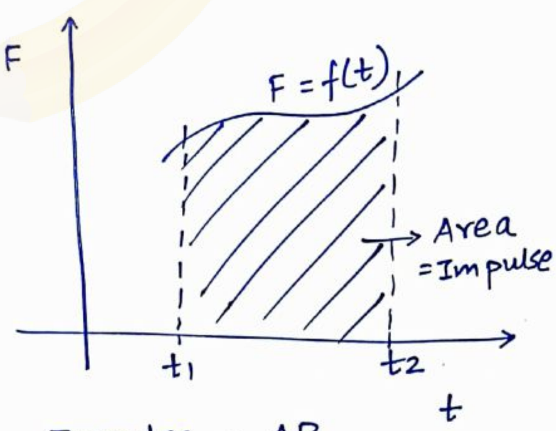
$m_1 + m_2$

$$\boxed{m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2} \leftarrow \text{vector form}$$

* Impulse →

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$\underbrace{\vec{F}\,dt}_{\text{Impulse (vector)}} = \underbrace{d\vec{P}}_{\text{change in } \vec{P}}$$



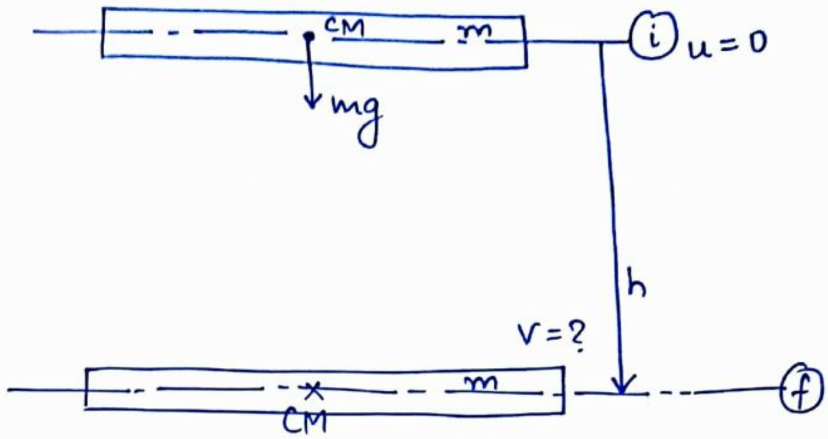
Impulse $= \Delta P$

$$\int_{t_1}^{t_2} F\,dt = \Delta P$$

* Work - Energy Theorem →

Total workdone $= \Delta(K.E.) = (K.E.)_f - (K.E.)_i$

Case 1 :→



m

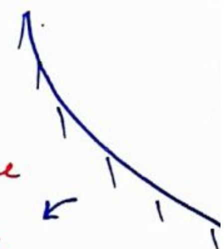
$TWD = \Delta(K.E.)$

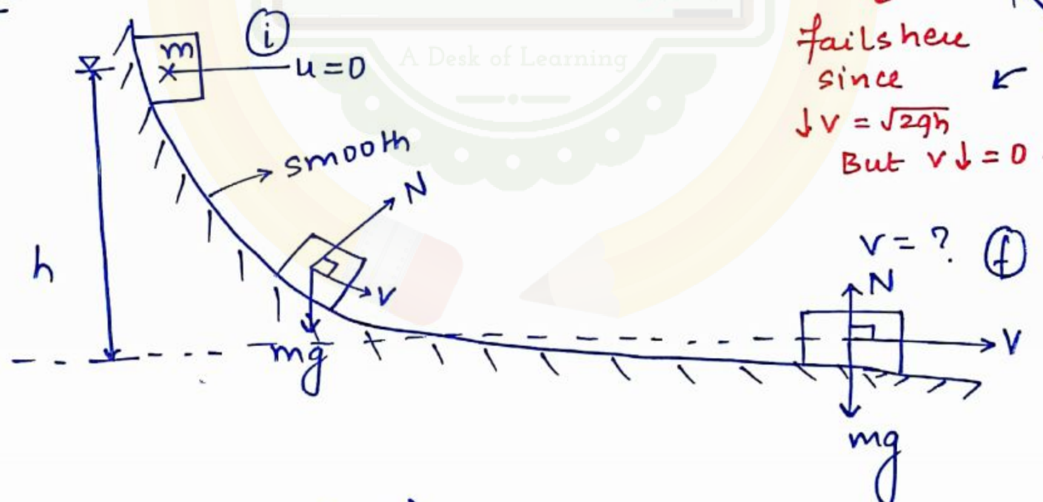
$W_{mg} = \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 - 0$

$\boxed{v = \sqrt{2gh}}$

Case 2 :-

$v^2 = u^2 + 2as$ fails here since $\downarrow v = \sqrt{2gh}$ But $v\downarrow = 0$.





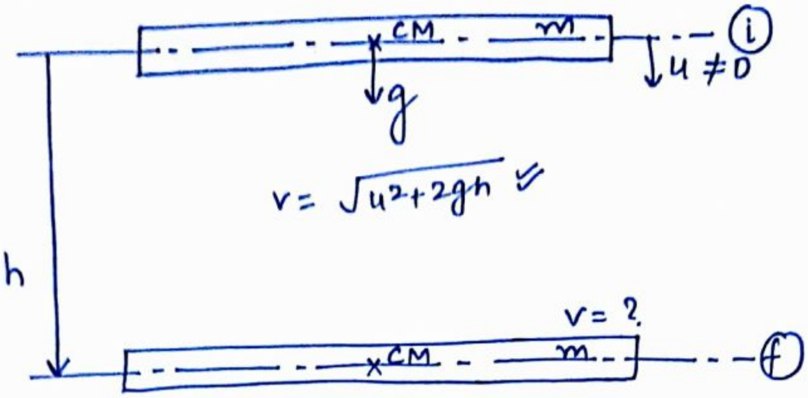
m

$TWD = \Delta(K.E.)$

$W_{mg} + \cancel{W_N}^{\,0} = \cancel{m}gh = \frac{1}{2}\cancel{m}v^2 - 0$

$\boxed{\sqrt{2gh} = v}$

Case 3 :-

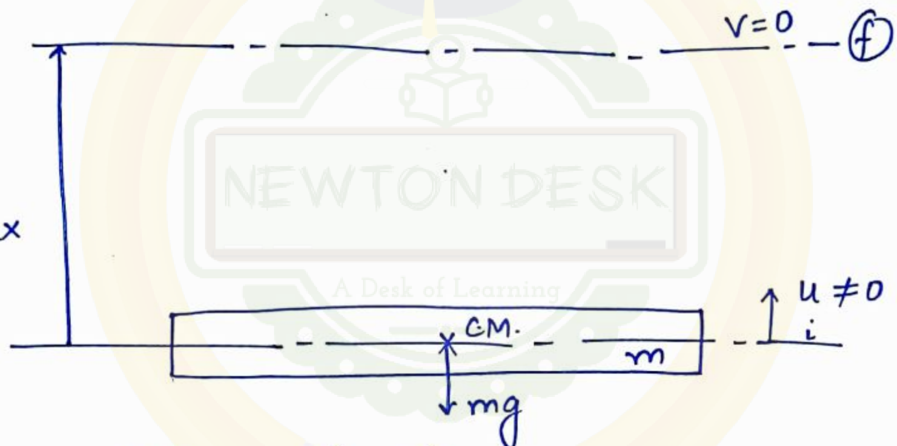


$$TWD = W_{mg} = \Delta(K.E.)$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}u^2$$

$$\boxed{v = \sqrt{u^2 + 2gh}}$$

Case 4 :-

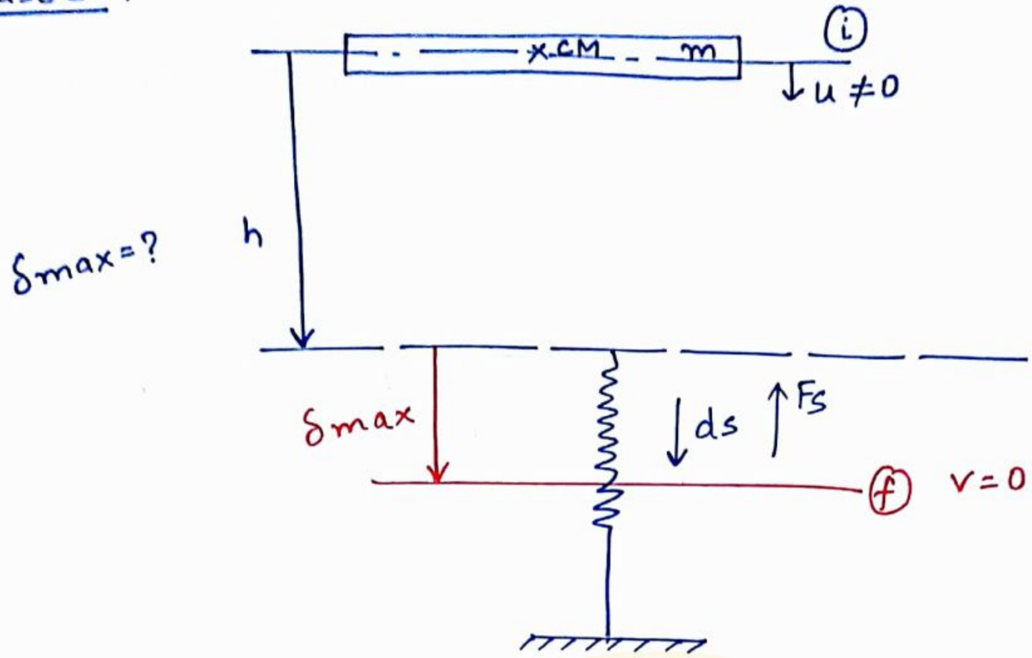


$$TWD = \Delta(K.E.)$$

$$W_{mg} = -\cancel{m}gh_{max} = 0 - \frac{1}{2}\cancel{m}u^2$$

$$\boxed{h_{max} = \frac{u^2}{2g}}$$

Case 5 :-



(m)

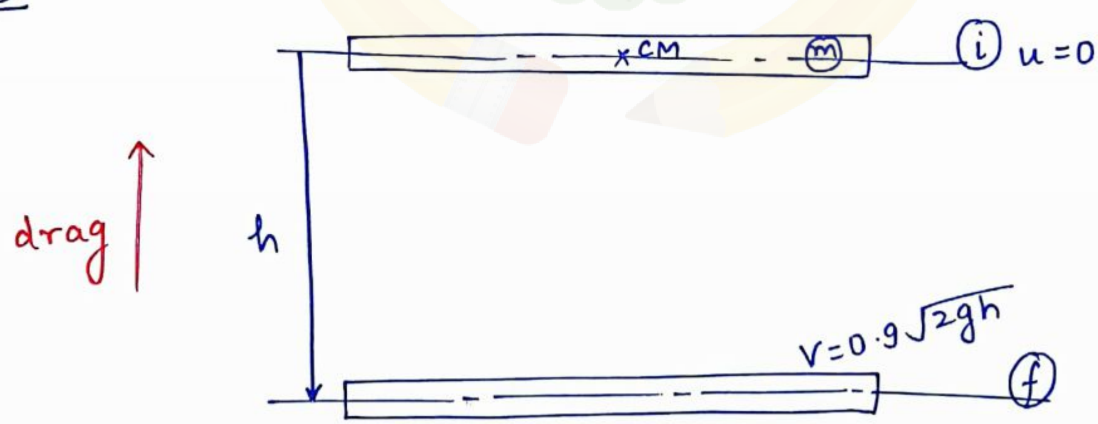
$$TWD = \Delta(K.E.)$$

$$\underbrace{W_{mg} + W_{F_S}}_{} = mg(h+\delta_{max}) - \frac{1}{2}K\delta_{max}^2 = 0 - \frac{1}{2}mu^2$$

spring force

$$\Rightarrow \boxed{mg(h+\delta_{max}) + \frac{1}{2}mu^2 = \frac{1}{2}K\delta_{max}^2}$$

Pr (1)



$W_{drag} = ?$

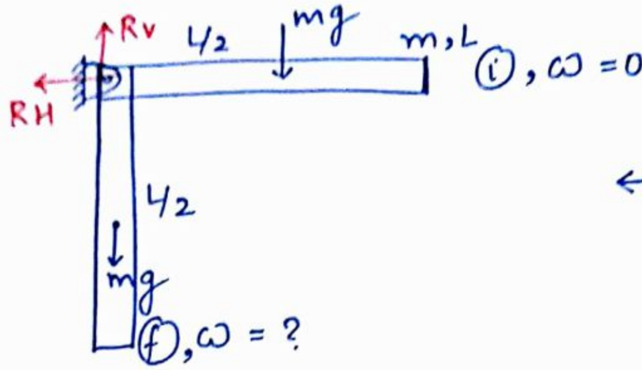
Sol.

$$TWD = W_{mg} + W_{drag}$$

$$= mgh + W_{drag} = \underbrace{\frac{1}{\cancel{2}} m(0.81)\times\cancel{2}gh}_{\frac{1}{2}mv^2} - 0$$

$$\boxed{W_{drag} = -0.19\,mgh}$$

Pb ②



← Pure Rotational Motion

Sol

$$TWD = \Delta(K.E.)$$

$$W_{mg} = mg \times L/2 = \frac{1}{2} I\omega^2 - 0$$

$$\cancel{m}g\,\cancel{L}/\cancel{2} = \frac{1}{\cancel{2}} \times \frac{\cancel{m}L^{\cancel{2}}}{3} \times \omega^2$$

$$\boxed{\omega = \sqrt{\frac{3g}{L}}}$$

* COLLISIONS / Impacts :→

It is a short time phenomenon in which momentum of the individual Body undergoing collision changes significantly.

① Perfectly Elastic $(e = 1)$.

② Perfectly Inelastic / Plastic $(e = 0)$.

③ Partially Elastic $(0 < e < 1)$.

① Perfectly Elastic ($e = 1$) :→ recovery of shape

when the initial K.E. = final K.E., such a collision is called perfectly elastic collision.

{collide → deform → Regain} so K.E. conserved

$u_1 > u_2$ $\qquad$ $v_2 > v_1$

$m_1 + m_2$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{——} ①$$

1st order eqn.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ②$$

2nd order eqn.

~~equating or~~ from ① & ②

$$u_1 - u_2 = v_2 - v_1 \quad \text{——} ③$$

velocity of approach before collision $\qquad$ velocity of seperation after collision

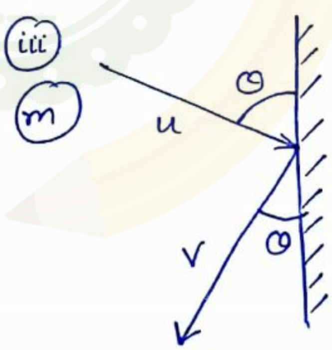
Special cases for $e = 1$

ⓘ if $m_1 = m_2$

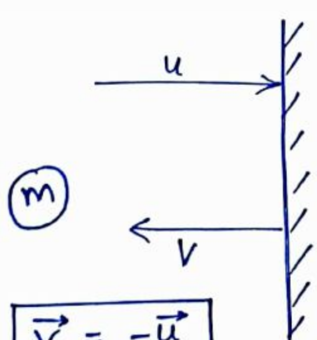
$\vec{u_1} = \vec{v_2}$

$\vec{u_2} = \vec{v_1}$

(iii)



(ii)

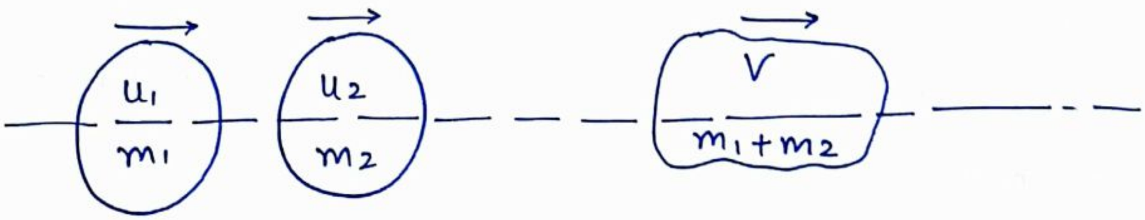


$\boxed{\vec{v} = -\vec{u}}$

② Perfectly Inelastic / Plastic ($e = 0$) :→

When two perfectly inelastic bodies moving along the same line collide, they stick to each other.

collide → deform (plastic)



m_1+m_2

$$\boxed{m_1u_1 + m_2u_2 = (m_1+m_2)V}$$

$$\boxed{(K.E.)_{loss} = (K.E.)_i - (K.E.)_f}$$

(iii) Partially Elastic ($0<e<1$) :→

$u_1 > u_2$

m_1+m_2

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2 \quad \text{—— (1)}$$

Vel. of sep. = e (Vel. of approach)

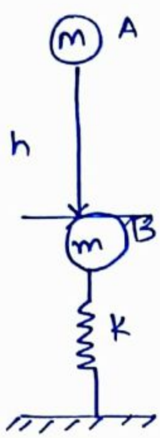
$$V_2 - V_1 = e(u_1 - u_2) \quad \text{—— (2)}$$

$\Rightarrow$ e = coefficient of restitution.

$\Rightarrow$ $e = \dfrac{\text{Velocity of separation}}{\text{Velocity of approach}}$.

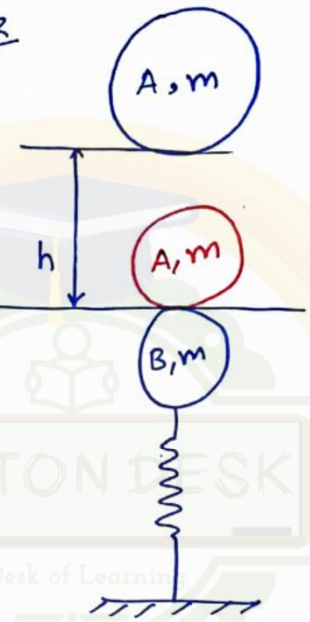
Q A Ball 'A' of mass 'm' falls under gravity through a height h and strikes another Ball 'B' of mass 'm' which is supported at rest by a spring of stiffness 'K'. The velocity of Ball 'A' immediately after the collision is (assume perfect elastic collision) ——.

Sol



SIR

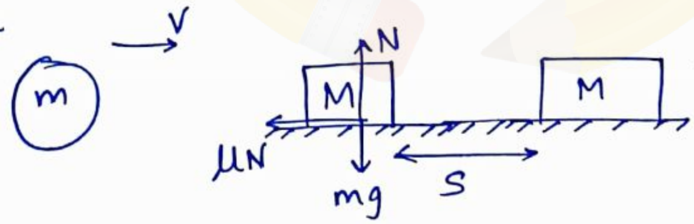
$e = 1$



$u_A = \sqrt{2gh} = V_B$

$u_B = 0 = V_A$ ✓ **Ans** ∴

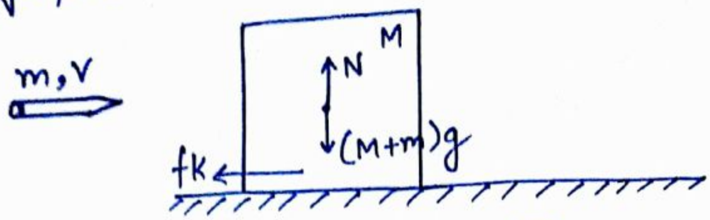
WB 5.09

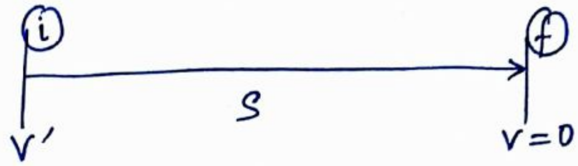


µ

Sol

$\frac{1}{2}mv^2 \neq \frac{1}{2}(M+m)(v')^2$





m + M

$mv + 0 = (M+m)v'$

$v = \left(\frac{M+m}{m}\right)v'$ —— (1)

$TWD = \Delta(K.E.)$

$TWD = \cancel{W_{(M+m)g}}^{0} + \cancel{W_N}^{0} + W_{f_K} = -f_K s$

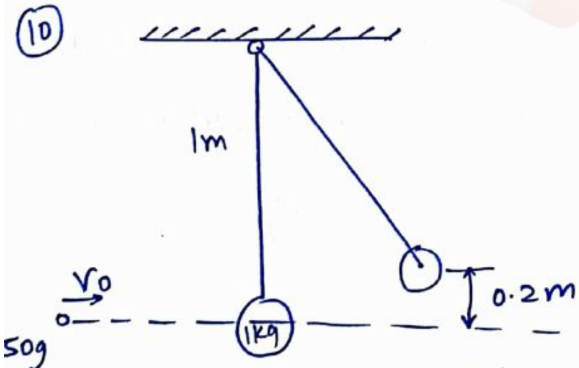
$-\mu(\cancel{M+m})gs = 0 - \frac{1}{2}(\cancel{M+m})(v')^2$

$v' = \sqrt{2\mu gs}$ —— (2)

from (1) & (2)

$$\boxed{v = \left(\frac{M+m}{m}\right)\sqrt{2\mu gs}}$$

(10)



$g = 10\ m/sec^2$

$v_0 = ?$

m + M

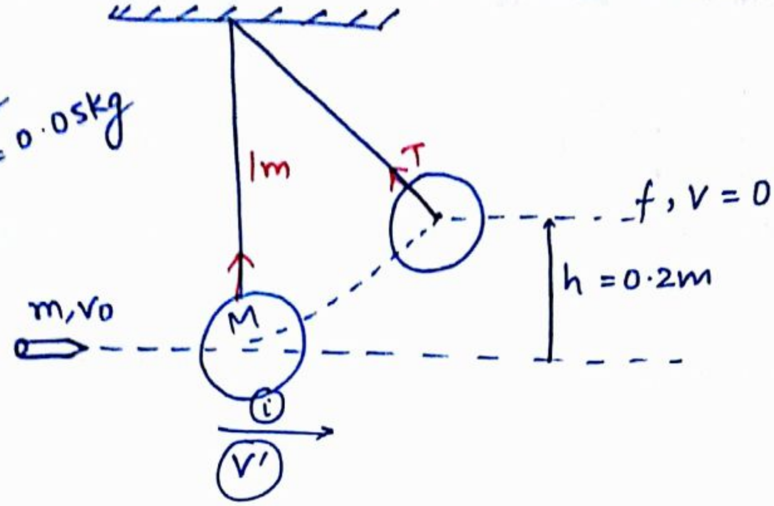
$mv + 0 = (M+m)v'$

$v = \left(\frac{M+m}{m}\right)v'$

Q 5.10 →

$M = 1kg$

$m = 50g = 0.05kg$



M+m

$$mV_0 + 0 = (M+m)V'$$

$$V_0 = \left(\frac{M+m}{m}\right)V' \quad \text{--- (1)}$$

$$TWD = W_{(M+m)g} + \cancel{W_T}^{\,0} = \Delta(K.E.)$$

$$-\cancel{(M+m)}gh = 0 - \frac{1}{2}\cancel{(M+m)}(V')^2$$

$$V' = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2m/s \quad \text{--- (2)}$$

from (1) & (2)

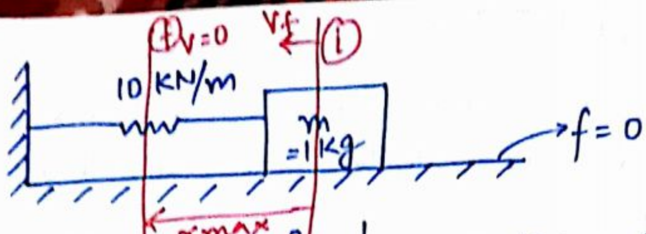
$$V_0 = \frac{1.05}{0.05} \times 2 = 42\ m/s.$$

Q> A single degree of freedom having mass 1kg and stiffness 10 KN/m, initially at rest is subjected to an impulsive force of 5KN for 10^{-4} sec. The amplitude in mm of the resulting free vibrn. is ___.

Sol

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$F = 5\,KN$
$t = 10^{-4}\,sec$



$F_{im} = 5\times10^{3}$

$t = 10^{-4}\,sec$

$\text{Impulse} = Ft = 5\times10^{+3}\times10^{-4}$

$= \Delta P = P_f - \cancel{P_i} \to 0$

$\Rightarrow \boxed{0.5 = mV_f}$

Before collision

$\frac{0.5}{1} = V_f = 0.5\,(m/sec)$

$$\frac{1}{2} mV_f^2 = \frac{1}{2} Kx_{max}^2$$

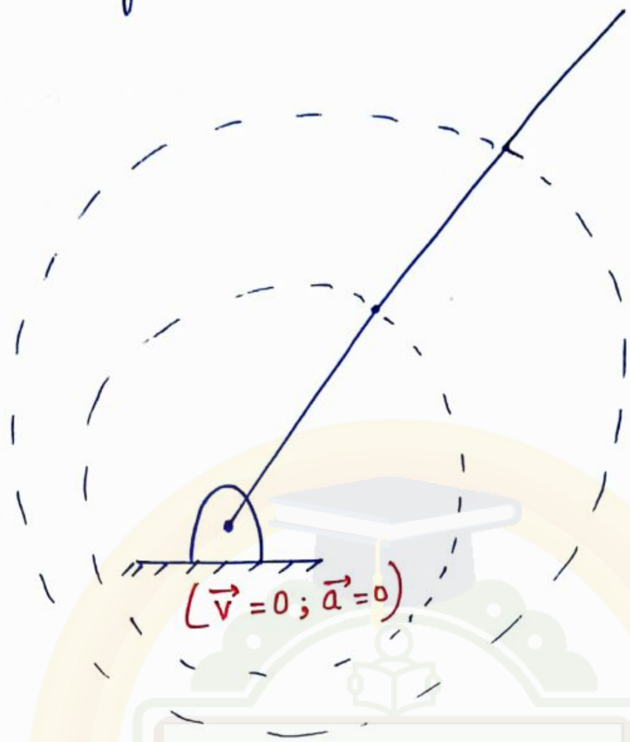
$$1\times0.5^2 = 10\times10^3\times x_{max}^2$$

$$x_{max} = 0.005\,m = \underline{\underline{5\,mm}}$$

07/10/2016

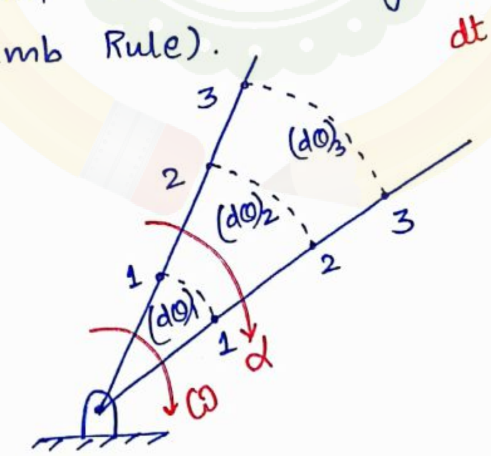
Rotation

In Rotation, all the particles will be in circular motion and their centres will lie on a line which will / should remain fixed called axis of Rotation.



Kinematics :→ of a Rotating Body

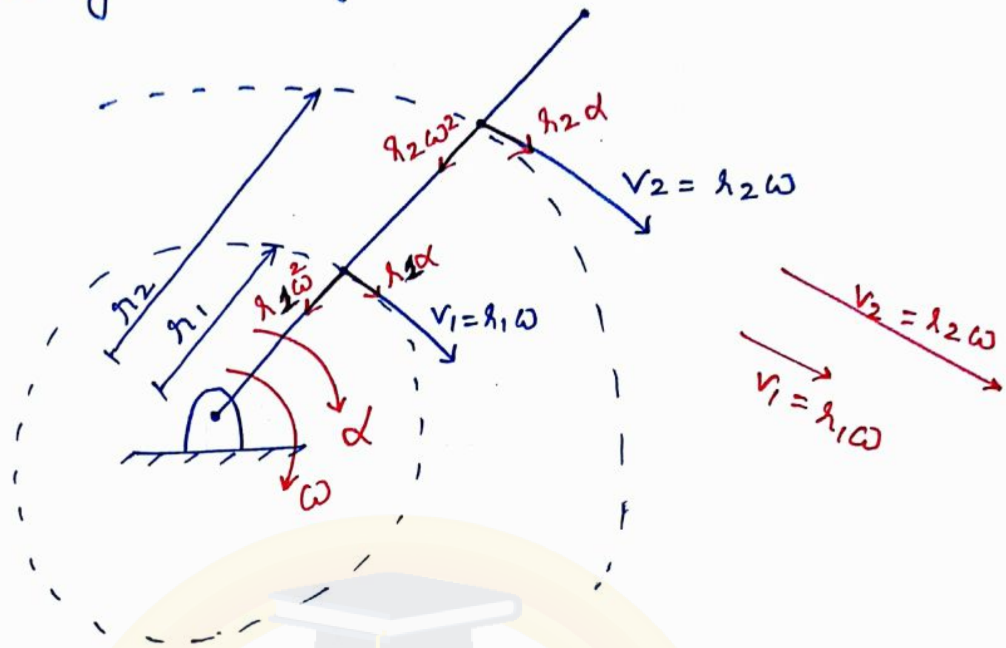
$\vec{\omega}$ and $\vec{\alpha}$ of each and every particle on a Rotating Body remains same and will be along axis of rotation (decided by Right hand thumb Rule).



$(d\theta)_1 = (d\theta)_2 = (d\theta)_3 = d\theta \rightarrow$ during time dt

$$\left.\begin{array}{l} \vec{\omega_1} = \vec{\omega_2} = \vec{\omega_3} = \omega \\ \vec{\alpha_1} = \vec{\alpha_2} = \vec{\alpha_3} = \alpha \end{array}\right]\text{ at an instant}$$

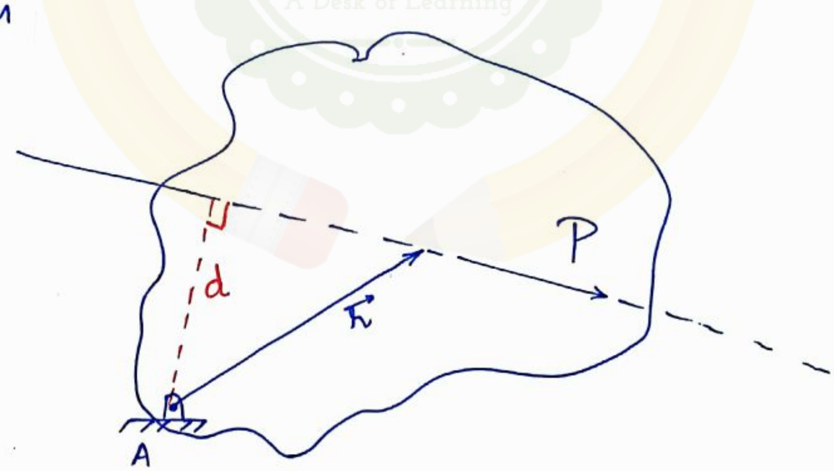
velocity vector and accl^r vector ($\vec{V}$ & $\vec{a}$) of a particle on a rotating Body is dependent on its location from axis of rotation and can be calculated by considering its circular motion.



* Moment of Linear Momentum (Angular Momentum) : $\vec{L} \Rightarrow$

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{P} = m\,\overrightarrow{V_{CM}}$$



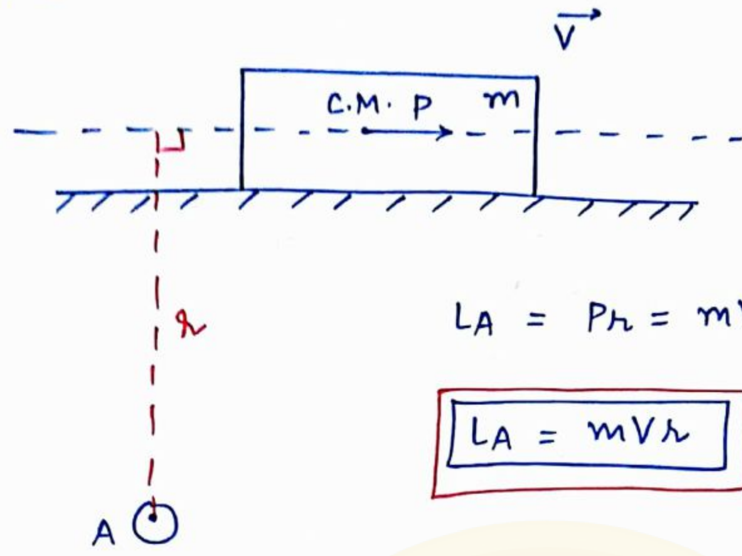
$$\overrightarrow{L_A} = \vec{r} \times \vec{P}$$

$$L_A = (P)d = m V_{CM}\, d$$

Dir^n $\Rightarrow$ $\perp^r$ inward through A

(i) Recti. Trans.

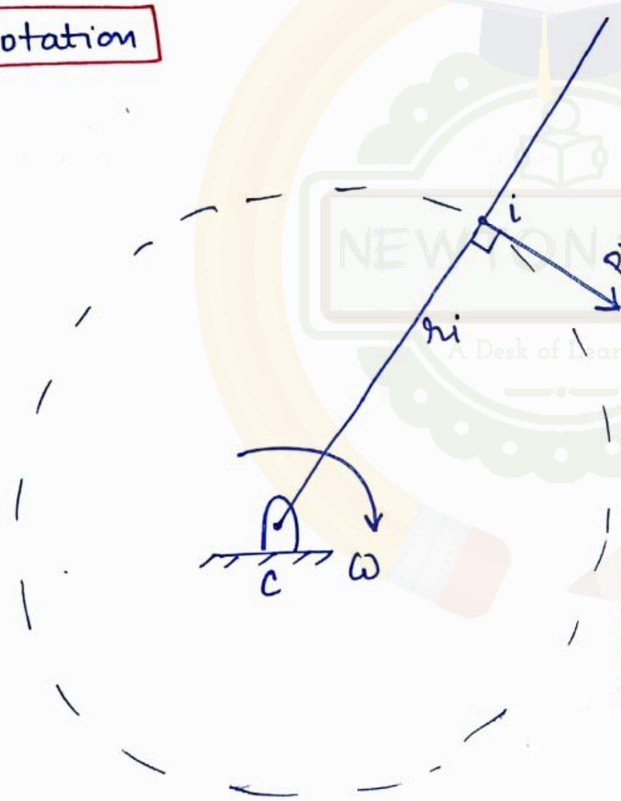
$$\vec{P} = m\vec{V}_{CM} = m\vec{V}$$



$$L_A = Pr = mV_{CM}r$$

$$\boxed{L_A = mVr}$$

(ii) Rotation



$$[\vec{L}_i]_c = \vec{r}_i \times \vec{P}_i \quad \Big| \quad \vec{P}_i = m\vec{v}_i$$

$$[L_i]_c = P_i r_i = m_i v_i r_i$$

$$= m_i r_i^2 \omega$$

$$\sum_i [L_i]_c = [L_{Body}]_{axis}$$

$$= \sum m_i r_i^2 \omega$$

$$\boxed{(\vec{L})_{axis} = I_{axis}\vec{\omega}}$$

$$\left[\because I_{axis} = \sum m_i r_i^2\right]$$

* **Kinetic Energy of a Rotating Body** :-

$$(KE)_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

$$\sum_i (KE)_i = (KE)_{Body} = \frac{1}{2}\omega^2 \sum_i m_i r_i^2$$

$$\boxed{KE = \frac{1}{2} I\omega^2}$$

Similarily, $(\overrightarrow{T_{axis}})_{ext.} = I_{axis}\vec{\alpha}$

$$\frac{d\vec{L}}{dt} = \frac{d(\vec{r}\times\vec{P})}{dt} = \frac{d\vec{r}}{dt}\times\vec{P} + \vec{r}\times\frac{d\vec{P}}{dt}$$

$$\frac{d\vec{L}}{dt} = \cancelto{0}{\vec{v}\times\vec{P}} + \vec{r}\times\vec{F} = \vec{T}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{T} = \vec{T}_{ext}}$$

Conservation of $\vec{L}$ $\Rightarrow$ $\vec{L}$ = constant $\Rightarrow d\vec{L} = 0$

only when $\vec{T} = 0$

$$\Rightarrow \vec{T} = \frac{d\vec{L}}{dt}$$

$$\Rightarrow \boxed{\underbrace{\vec{T}\,dt}_{\text{Angular Impulse}} = \underbrace{d\vec{L}}_{\text{change in } \vec{L}}}$$

Power = rate of doing work

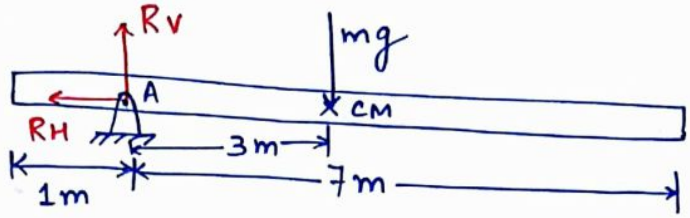
$$= \vec{F}\cdot\left(\frac{d\vec{s}}{dt}\right) = \boxed{\vec{F}\cdot\vec{V}}$$

and

$$= \frac{T\,d\theta}{dt} = T\omega = \boxed{\frac{T\,2\pi N}{60}}$$

Q A uniform slender rod of mass 3 Kg and length 8 m rotates in a vertical plane hinged at 1 m from one of its end as shown in figure. Find angular acceleration of rod at the position shown in figure.

$m = 3\,Kg$



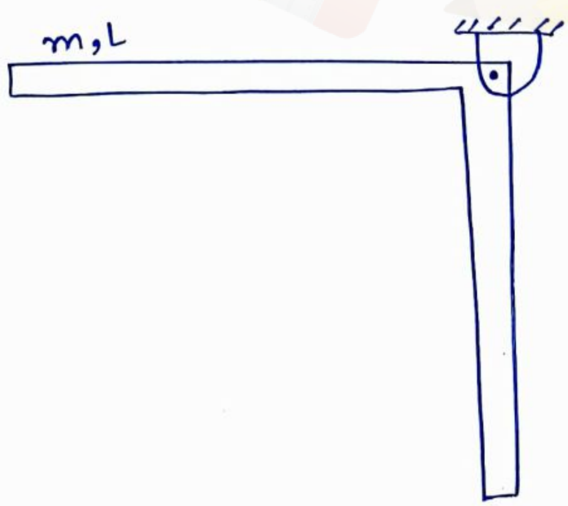
$$\tau_A = I_A \alpha$$

$$mg \times 3 = \left[I_{CM} + m \times 3^2 \right] \times \alpha$$

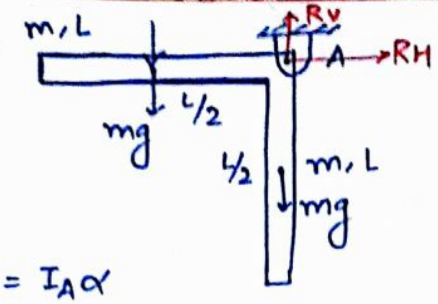
$$3 \times 9.81 \times 3 = \left[\frac{3 \times 8^2}{12} + 3 \times 3^2 \right] \times \alpha$$

$$\Rightarrow \boxed{\alpha = 2.05\ rad/sec^2}$$

Q. A uniform L shaped member with each of its limb has mass 'm' and length 'l' as shown in figure. Find angular accl^r. of the member at the position shown.



Sol



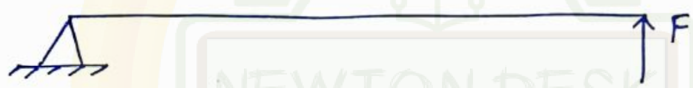
$T_A = I_A \alpha$

$\Rightarrow mg \times L/2 = \left[\frac{mL^2}{3} + \frac{mL^2}{3}\right] \times \alpha$

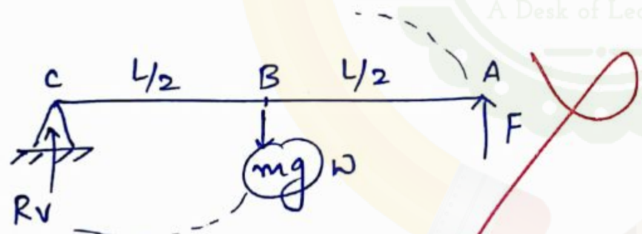
$\Rightarrow \cancel{m}g \times \cancel{L}/2 = \frac{2\cancel{m}L^{\cancel{2}}}{3} \times \alpha$

$\Rightarrow \boxed{\alpha = \frac{3g}{4L}}$

Q> A uniform beam of weight 'W' and Length 'L' is supported as shown in figure. The moment force 'F' is removed, the Rxn at hinge is :-



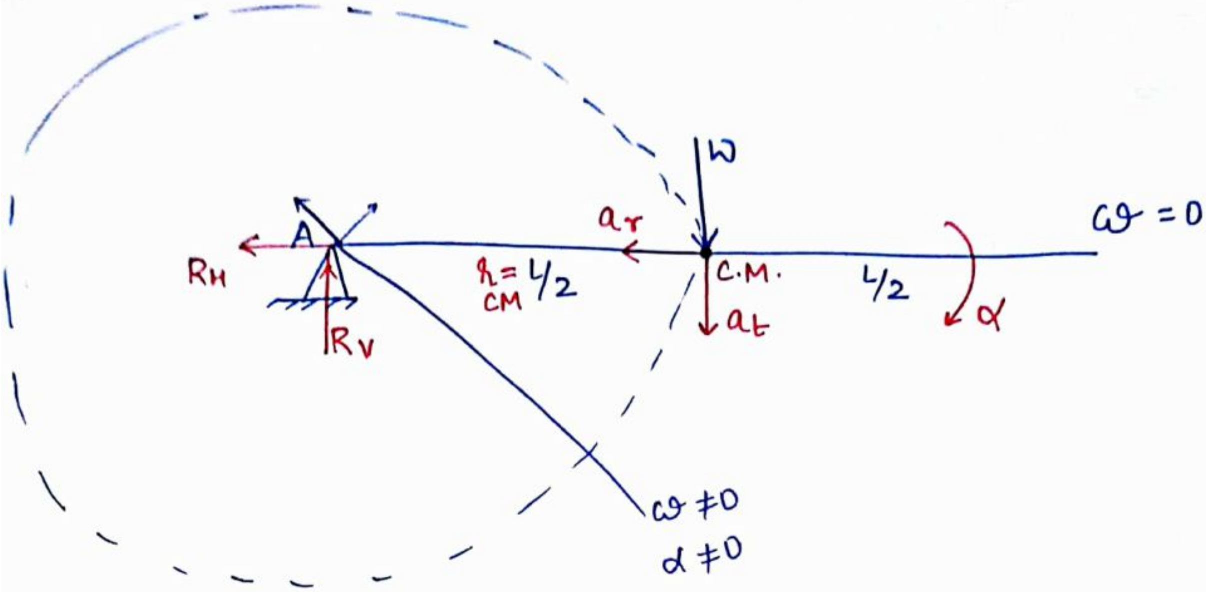
Sol



$WL/2 + RL = 0$

$W\cancel{L}/2 = R\cancel{L}$

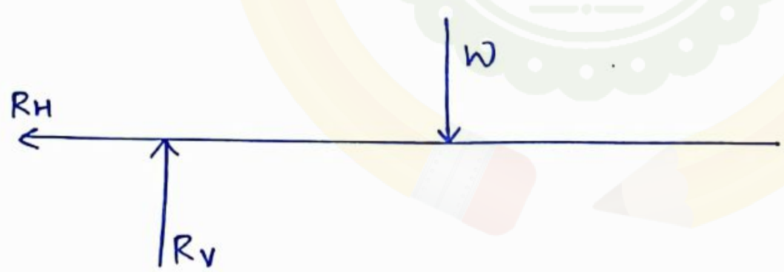
$R = W/2$



$$T_A = I_A \alpha$$

$$\cancel{W} \times \cancel{L}/2 = \frac{\cancel{W}}{g}\frac{L^{\cancel{2}}}{3} \times \alpha$$

$$\boxed{\alpha = \frac{3g}{2L}}$$



$$W - R_V = m\, a_{CM(-y)} \text{ — } \underline{2nd}$$

$$W - R_V = \frac{W}{g}(a_{CM})_t = \frac{W}{g} r_{CM}\alpha = \frac{W}{g} \times \cancel{L}/2 \times \frac{3\cancel{g}}{2\cancel{L}} = \frac{3W}{4}$$

$$R_V = W - \frac{3W}{4} = W/4$$

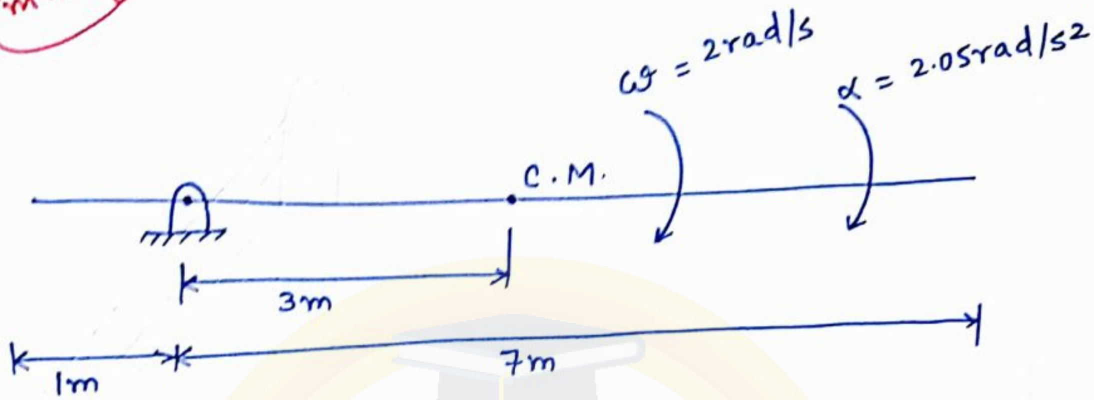
$$R_H = m\,a_{CM}(-x) \quad \text{— } \underline{2^{nd}}$$

$$R_H = m(a_{CM})_r = m r_{CM}\,\omega^2$$

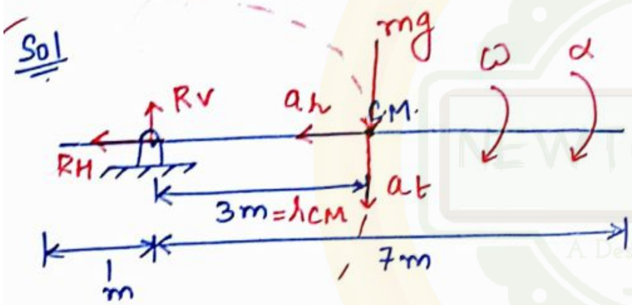
$$R_H = 0 \quad [\because \omega = 0]$$

Q. Find Reaction at hinge for the system shown in figure.

$m = 3\,Kg$



Sol



$\left.\begin{array}{l}\omega = 2\,rad/s \\ \alpha = 2.05\,rad/s^2\end{array}\right\} \rightarrow$ at any instant.

$$mg - R_V = m\,a_{CM}(-y) \quad \text{— } \underline{2^{nd}}$$

$$mg - R_V = m(a_{CM})_t = m r_{CM}\,\alpha$$

$$3 \times 9.81 - R_V = 3 \times 3 \times 2.05 \Rightarrow R_V = 10.98\,N$$

$$R_H = m\,a_{CM}(-x) \quad \text{— } \underline{2^{nd}}$$

$$R_H = m(a_{CM})_r = m r_{CM}\,\omega^2 = 3 \times 3 \times 2^2 = 36\,N$$

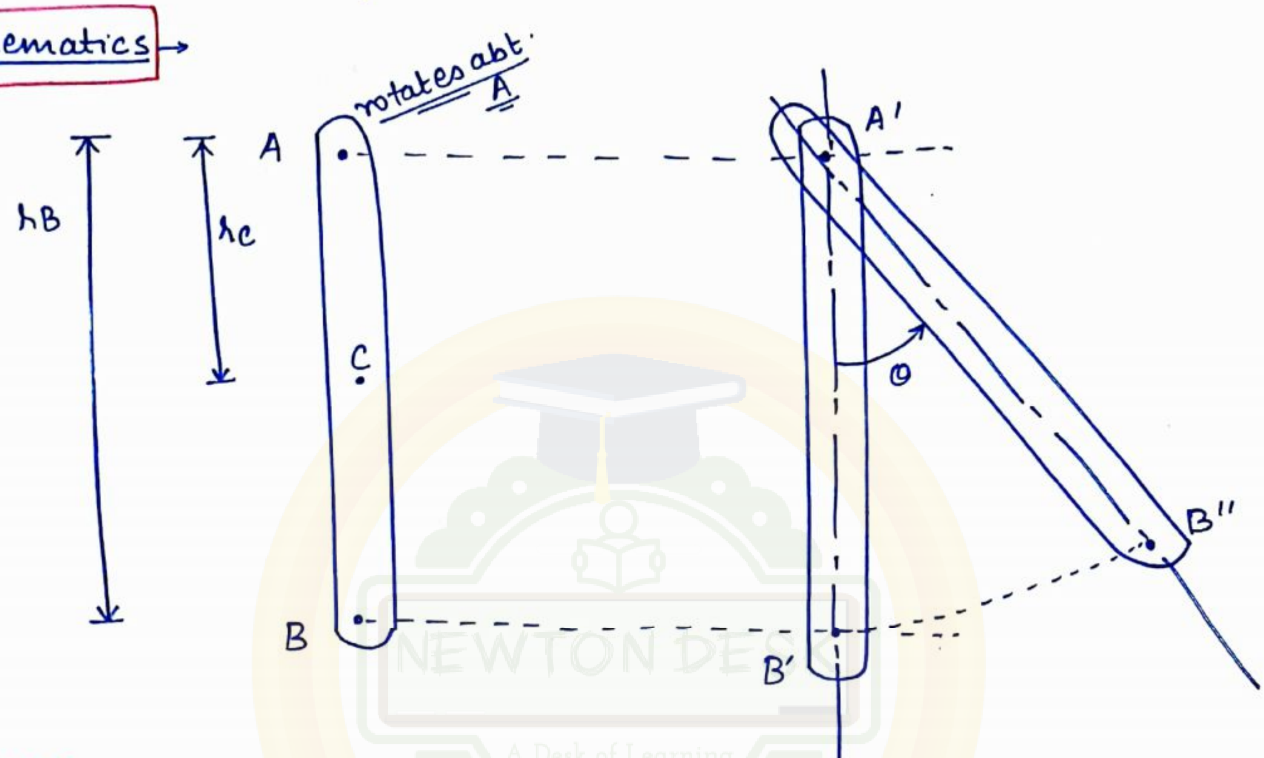
* **GENERAL MOTION** :- If a Rigid body rotates as well as translates simultaneously, such a motion is called General Motion.

Example :- (1) Rolling Motion.

(2) Motion of connecting rod.

(3) Motion of ladder, etc.

Kinematics →



Hence,

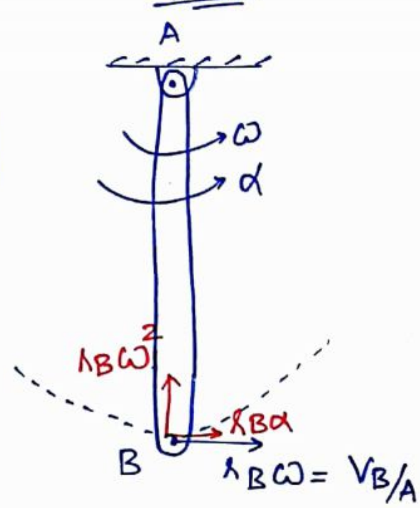
$$[\vec{S}, \vec{V}, \vec{a}]_{\text{General motion}} = [\vec{S}, \vec{V}, \vec{a}]_{\text{Trans.}} + [\vec{S}, \vec{V}, \vec{a}]_{\text{Rotation}}$$

$$\vec{S_B} = \vec{S_A} + \vec{S_{B/A}} \quad [\vec{S_{B/A}} = r_B \vec{\theta}]$$

$$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}} \quad [\vec{V_{B/A}} = r_B \vec{\omega}]$$

$$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}} \quad [\vec{a_{B/A}} = r_B \vec{\omega^2} + r_B \vec{\alpha}]$$

wrt A



Simil.

$$\vec{S_C} = \vec{S_A} + \vec{S_{C/A}} \quad [\vec{S_{C/A}} = r_C \vec{\theta}]$$

$$\vec{V_C} = \vec{V_A} + \vec{V_{C/A}} \quad [\vec{V_{C/A}} = r_C \vec{\omega}]$$

$$\vec{a_C} = \vec{a_A} + \vec{a_{C/A}} \quad [\vec{a_{C/A}} = r_C \vec{\omega^2} + r_C \vec{\alpha}]$$

Rolling :-

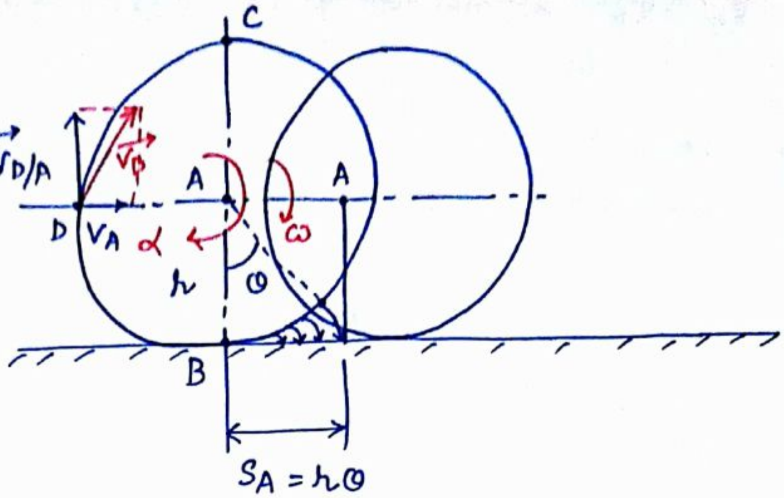
$\vec{S_A} = r\theta\,\hat{i}$

$\vec{V_A} = r\dfrac{d\theta}{dt}\,\hat{i}$

$\vec{V_A} = r\omega\hat{i}$

$\vec{a_A} = r\dfrac{d\omega}{dt}\,\hat{i}$

$\vec{a_A} = r\alpha\,\hat{i}$



$\vec{V_B} = \vec{V_A} + \vec{V}_{B/A} = r\omega\hat{i} + r\omega(-\hat{i}) = 0$

$\vec{V_C} = \vec{V_A} + \vec{V}_{C/A} = r\omega\hat{i} + r\omega(\hat{i}) = 2r\omega(\hat{i})$

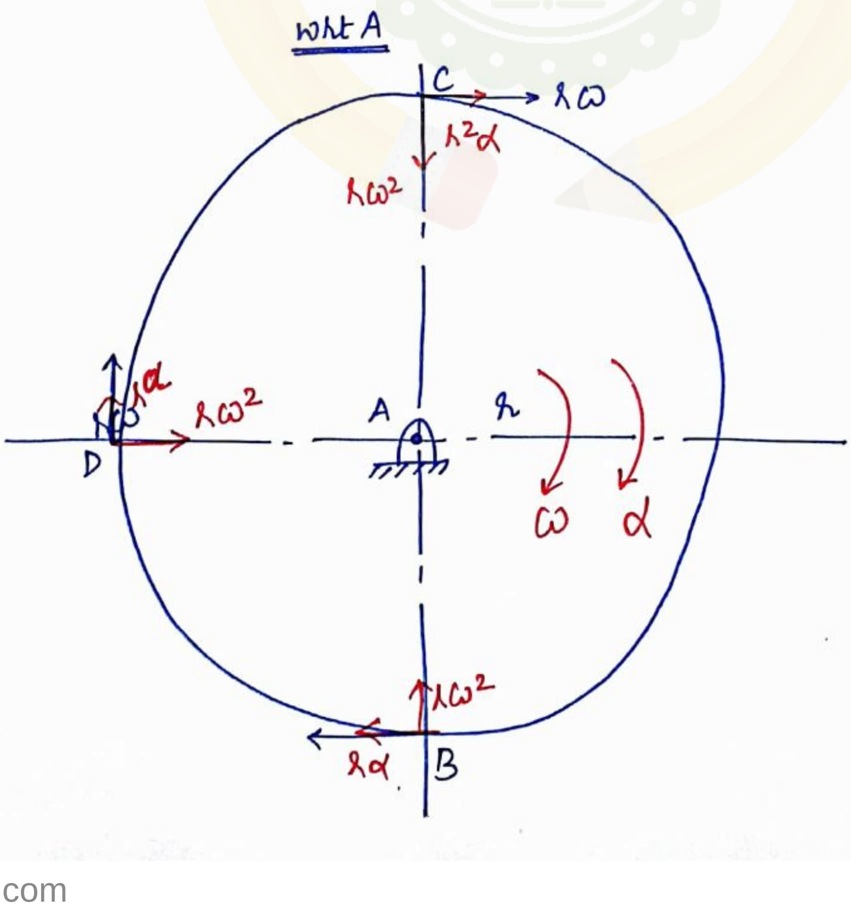
$\vec{V_D} = \vec{V_A} + \vec{V}_{D/A} = r\omega\hat{i} + r\omega(\hat{j})$

$\vec{a_B} = \vec{a_A} + \vec{a}_{B/A} = r\alpha\hat{i} + r\alpha(-\hat{i}) + r\omega^2(\hat{j}) = r\omega^2(\hat{j}) \neq 0$

$\vec{a_C} = \vec{a_A} + \vec{a}_{C/A} = r\alpha\hat{i} + r\alpha(\hat{i}) + r\omega^2(-\hat{j}) = 2r\alpha\hat{i} + r\omega^2(\hat{j})$

$\vec{a_D} = \vec{a_A} + \vec{a}_{D/A} = r\alpha\hat{i} + r\omega^2(\hat{i}) + r\alpha(\hat{j})$

<u>wrt A</u>



(i.) if $S_A = r\omega \rightarrow$ pure rolling (DOF = 1) $\rightarrow$ free Rolling

In 1 rev, $\omega = 2\pi$, $S_A = 2\pi r$

(ii) if $S_A > r\omega \rightarrow$ skidding (DOF = 2)

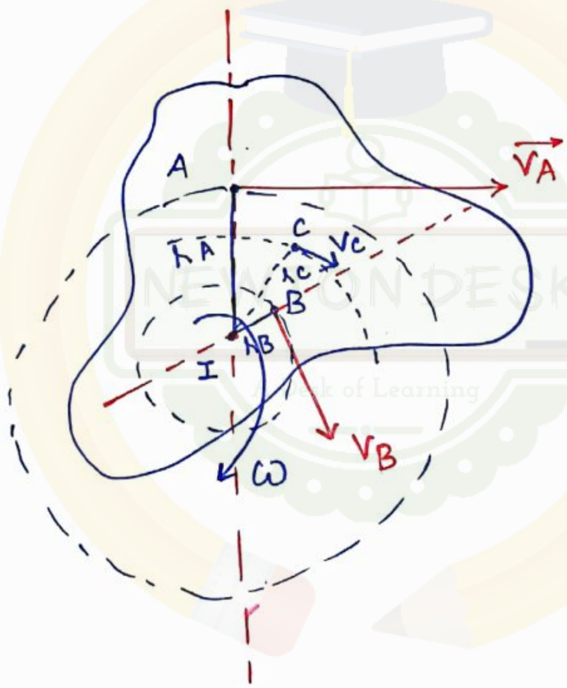
(iii) if $S_A < r\omega \rightarrow$ slipping (DOF = 2)

* $\boxed{\text{Instantaneous Centre}}\ \boxed{\text{/Axis}}$ → It is a point / line in space about which a Body in general motion can be assumed as in pure rotation to find the velocities.

$\boxed{\text{Note}}$:- (i) I-centre has 0 velocity But not 0 acceleration.

* $\boxed{\text{Location of I-Cs}}$:→

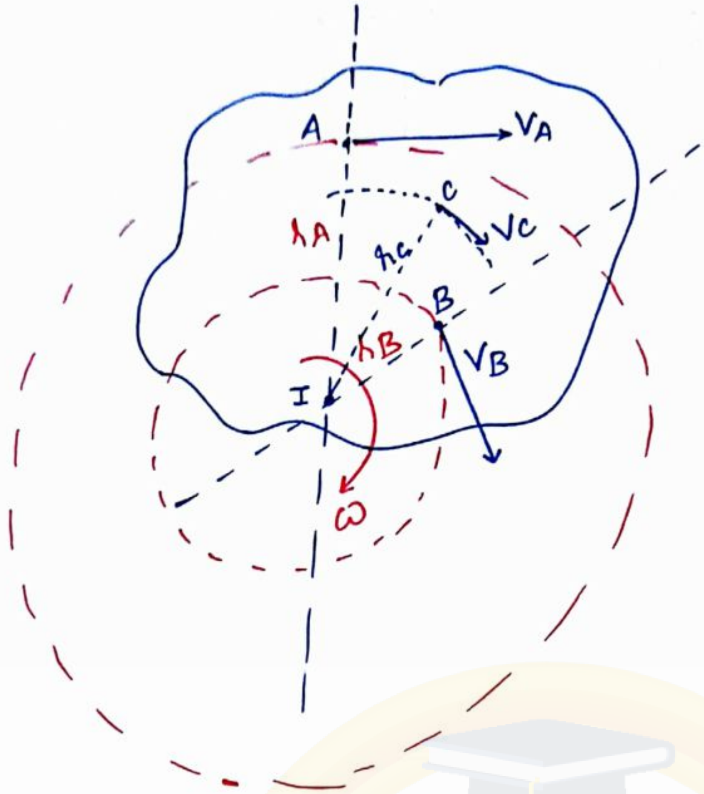
$\boxed{\text{Case 1}}$:-



$V_A = r_A\,\omega$

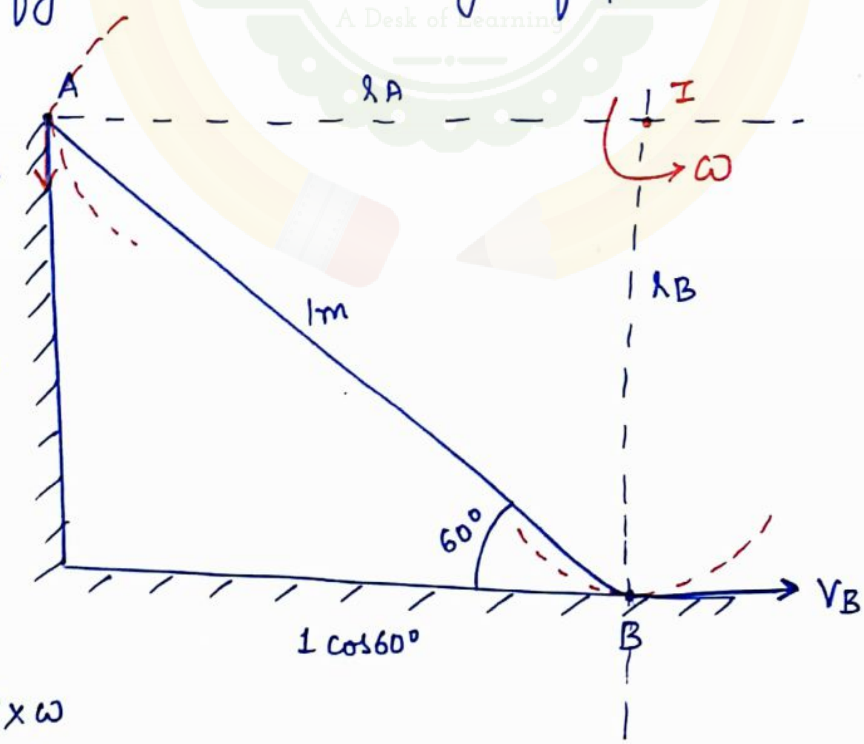
$V_B = r_B\,\omega$

$V_C = r_C\,\omega$



Q. A ladder AB of length 1m is sliding in a corner as shown in figure. at an instant; when the ladder makes an angle 60° with the horizontal, velocity of pt. A is 1m/sec as shown in figure. Then the velocity of pt. B is

Sol



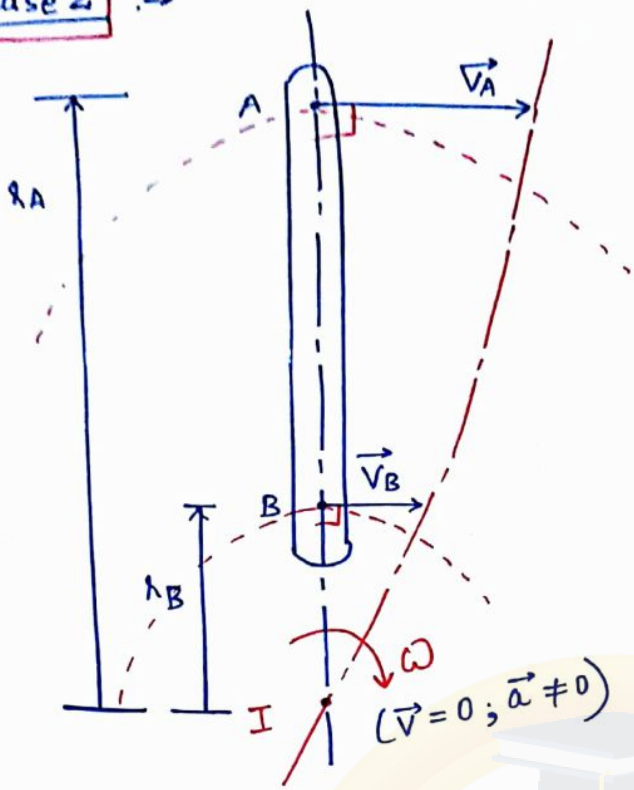
$V_A = r_A \omega$

$1 = 1 \times \cos 60° \times \omega$

$\omega = 2 rad/s$; ACW

$V_B = r_B \omega = 1 \times \sin 60° \times 2 = \sqrt{3}\ m/s$

Case 2 :→



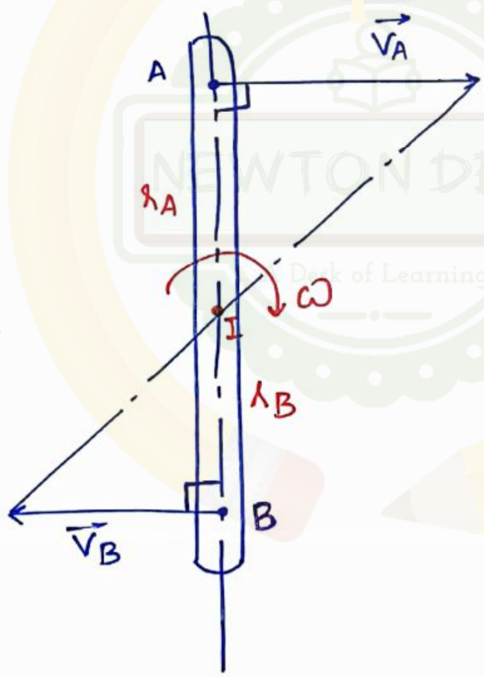
$V_A = r_A \omega$

$V_B = r_B \omega$

$$\frac{V_A}{V_B} = \frac{r_A}{r_B} \quad \text{——} (1)$$

$$AB = r_A - r_B \quad \text{——} (2)$$

Case 3 :→



$V_A = r_A \omega$

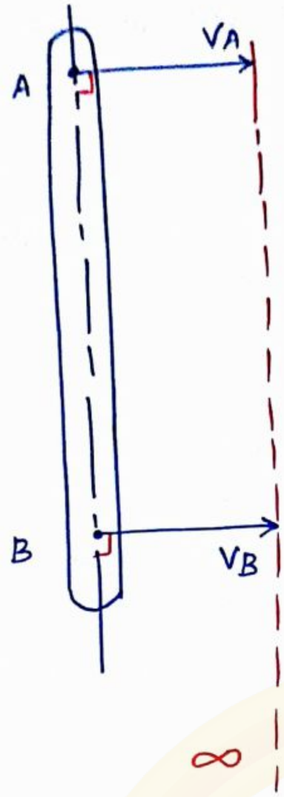
$V_B = r_B \omega$

$$\frac{V_A}{V_B} = \frac{r_A}{r_B} \quad \text{——} (1)$$

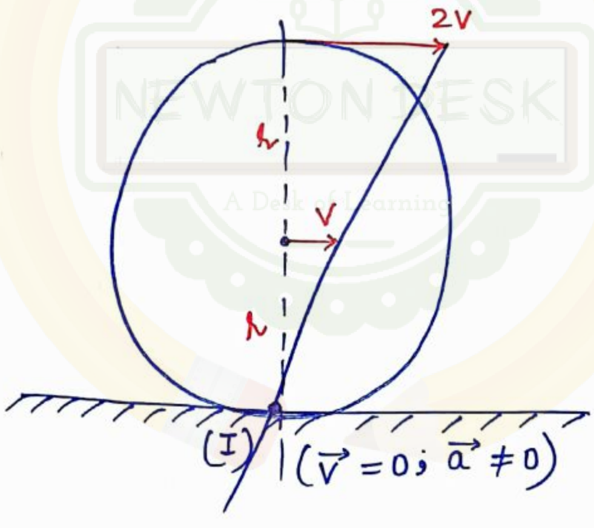
$$AB = r_A + r_B \quad \text{——} (2)$$

Case 4 :→

$\vec{V_A} = \vec{V_B}$



Application :→

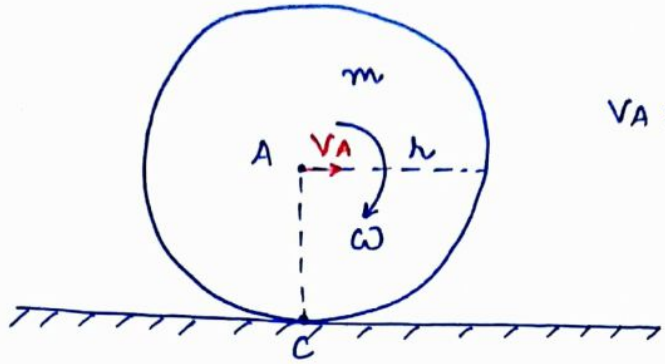


$(\vec{v} = 0;\ \vec{a} \neq 0)$

$(KE)_{Rolling} = (KE)_{trans.} + (KE)_{Rotation}$

"(N.P.)"

$$(K.E.)_{Rolling} = (K.E.)_{Trans.} + (KE)_{Rotation}$$



$V_A = r\omega = V$

$$(KE) = \frac{1}{2}mV^2 + \frac{1}{2}I_A\omega^2$$

$$= \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I_A\omega^2$$

$$= \frac{1}{2}\omega^2\left[mr^2 + I_A\right]$$

$$= \frac{1}{2}I_C\omega^2$$

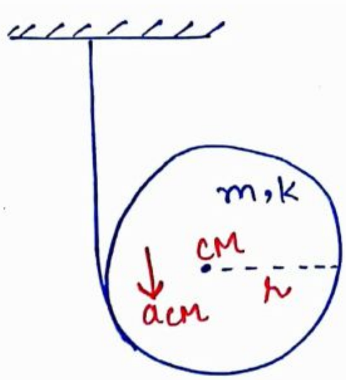
$$(KE)_{rolling} = \frac{1}{2}I_C\omega^2$$

$$L_C = I_C\omega$$

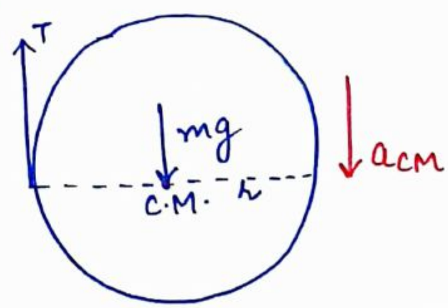
<u>Q</u> A reel of mass 'm', radius 'r' & radius of gyration 'K' is rolling down smoothly from rest, with one end of the thread wound on it held in the ceiling as shown in fig. Neglect wrapped thickness of thread. Find :- (a) Tension in the thread.

(b) acceleration of COM of reel.

<u>Sol</u>



wrapped



$mg - T = ma_{CM}(-y)$ — <u>2nd</u>

$mg - T = mr\alpha$ —— ①

$[Torque]_{CM} = I_{CM}\alpha$

$$Tr = mk^2\alpha$$

$$\alpha = \frac{Tr}{mk^2} \quad \text{—— (2)}$$

from (1) & (2)

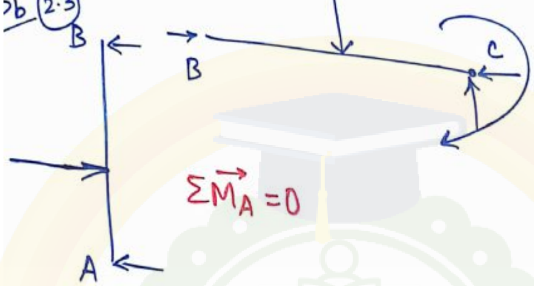
$$mg - T = \frac{\cancel{m} r^2 T}{\cancel{m} k^2}$$

$$\boxed{T = \frac{mgk^2}{r^2 + k^2}}$$

$$a_{CM} = r\alpha = \frac{Tr^2}{mk^2} = \frac{\cancel{m} g \cancel{k^2}}{r^2 + k^2} \times \frac{r^2}{\cancel{m}\cancel{k^2}}$$

$$\boxed{a_{CM} = \frac{gr^2}{r^2 + k^2}}$$

Pb (2.3)



$\Sigma \vec{M}_A = 0$