Date...............

## ▪ Half Range Series

① Half Range Sine Series
② Half Range Cosine Series

* for Limits $(-l, l)$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$\Rightarrow a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)\,dx \Rightarrow \frac{1}{l}\int_{0}^{l} f(x)\,dx$$

$$l = f - I$$

$$\Rightarrow a_n = \frac{2}{l}\int_{0}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

### • Half Range Cosine Series

$$\Rightarrow a_0 = \frac{1}{l}\int_{0}^{l} f(x)\,dx$$

$$\Rightarrow a_n = \frac{2}{l}\int_{0}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

### • Half Range Sine Series

$$b_n = \frac{2}{l}\int_{0} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

Date...............

## ⇒ Half Range Cosine Series

**Q.** Find a cosine series of period $2\pi$ to represent $\sin x$ in $0 \leq x \leq \pi$.

**Sol→** $f(x) = \sin x \; \ldots\ldots \quad 0 \leq x \leq \pi$

$l = F - I = \pi - 0 = \pi$

∴ Half Range cosine series will be

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$\Rightarrow a_0 = \frac{1}{l}\int_0^l f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} \sin x \, dx \Rightarrow \frac{1}{\pi}\left(-[\cos x]_0^{\pi}\right)$$

$$\Rightarrow -\frac{1}{\pi}[\cos\pi - \cos 0] = -\frac{1}{\pi}[-1-1] \rightarrow \frac{2}{\pi}$$

$$\Rightarrow a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$\Rightarrow \frac{2}{\pi}\int_0^{\pi} \sin x \cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$\Rightarrow \frac{2}{\pi}\int_0^{\pi} \sin(x)\cos(nx)\,dx$$

$$\left(\sin A \sin B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]\right)$$

Date...............

$$\Rightarrow \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin(x+nx)+\sin(x-nx)\right]dx$$

$$\Rightarrow \frac{1}{\pi}\int_0^{\pi}(\sin(1+n)x+\sin(1-n)x)\,dx$$

$$\Rightarrow \frac{1}{\pi}\left[\frac{-\cos(1+n)x}{(1+n)}-\frac{\cos(1-n)x}{(1-n)}\right]_0^{\pi}$$

$$\Rightarrow -\frac{1}{\pi}\left[\frac{\cos(1+n)x}{(1+n)}+\frac{\cos(1-n)x}{(1-n)}\right]_0^{\pi}$$

$$\Rightarrow -\frac{1}{\pi}\left[\left(\frac{\cos(1+n)\pi}{(1+n)}+\frac{\cos(1-n)\pi}{(1-n)}\right)-\left(\frac{\cos 0}{(1+n)}+\frac{\cos 0}{(1-n)}\right)\right]$$

n is even → $-1$

n is odd → $0$

$\Rightarrow$ If n is even

$$a_n = -\frac{1}{\pi}\left[\frac{-1}{1+n}-\frac{1}{1-n}-\frac{1}{1+n}-\frac{1}{1-n}\right]$$

$$\Rightarrow -\frac{1}{\pi}\left[\frac{-2}{1+n}-\frac{2}{1-n}\right] \Rightarrow \frac{2}{\pi}\left[\frac{1}{1+n}+\frac{1}{1-n}\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{1-n+1+n}{1-n^2}\right] \Rightarrow \frac{4}{\pi}\left(\frac{1}{1-n^2}\right)$$

$\Rightarrow$ If n is odd

$$\boxed{a_n = 0}$$

Date...............

$$\therefore f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$\Rightarrow \boxed{\sin x = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n^2-1}\right)\cos nx}$$ Ans

## ⇒ Half Range Sine Series

Q Obtain Half range sine series in $(0, \pi)$ for $x(\pi - x)$.

Sol→ $f(x) = x(\pi - x)$ ------ $(0, \pi)$

$l = F - I = \pi - 0 = \pi$

$a_0 = a_n = 0$

⇒ Half Range Sine Series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$\Rightarrow b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

Date...............

$$\Rightarrow b_n = \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\sin\left(\frac{n\pi x}{\pi}\right)dx = \frac{2}{\pi}\int_0^{\pi}(x\pi - x^2)\sin nx\,dx$$

$$\Rightarrow \frac{2}{\pi}\left[(x\pi - x^2)\left(-\frac{\cos nx}{n}\right) - (\pi - 2x)\left(-\frac{\sin nx}{n^2}\right) - 2\frac{\cos nx}{n^3}\right]_0^{\pi}$$

$$\Rightarrow \frac{2}{\pi}\left[\left(0 - \frac{2\cos n\pi}{n^3}\right) - \left(0 - \frac{2\cos 0}{n^3}\right)\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{-2(-1)^n}{n^3} + \frac{2}{n^3}\right] = \frac{4}{\pi}\left[\frac{1 - (-1)^n}{n^3}\right]$$

$\Rightarrow$ So, Half Range sine series will be:

$$\boxed{x(\pi - x) = \sum_{n=1}^{\infty} \frac{4}{n^3\pi}\left[1 - (-1)^n\right]\sin nx}$$ Ans

---