~~SWITCHING THEORY AND LOGIC DESIGN~~

UNIT - I

CIRCUIT AND SYSTEM:

UNIT - I.

SIGNAL: A signal is defined as a single-valued function of one or more independent variables which contains some information. A signal may be defined as any physical quantity that varies with time, space or any other independent variable. It can be represented in Time Domain or Frequency Domain. If signal depends on only one independent variable, it is called a 1-D signal and if it depends on 2D variable it depends on 2D signal.

Representation of Signal: A signal can be continuous time signal or discrete time signals. Continuous time signals are defined for all instants of time whereas discrete time signal are defined only at discrete instances of time.

Continuous time signal are ~~repre~~ represented by $x(t)$ and discrete time signals are represented by $x(n)$

Continuous - time signal are represented by $x(t)$ and discrete time signals are represented by $x(n)$

Continous Time signals are represented by a function or a graph.

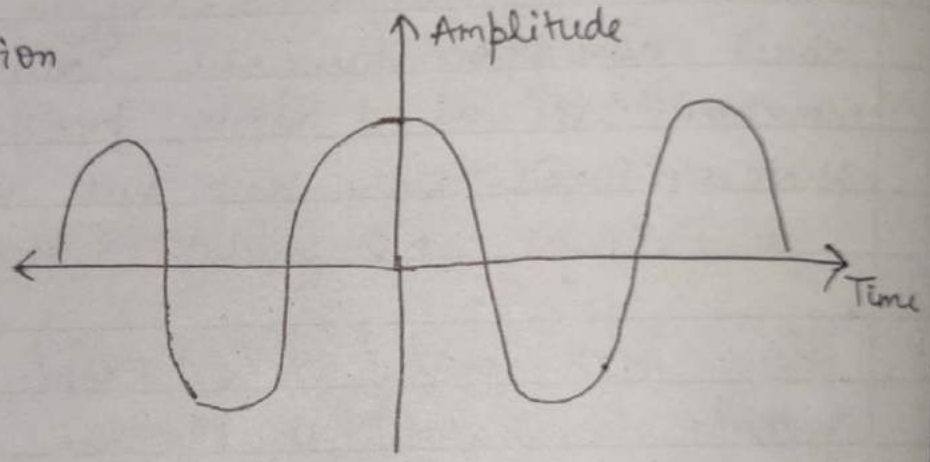
* Classification of SIGNALS:

Signals can be classified based on parameters used to classify them such as:

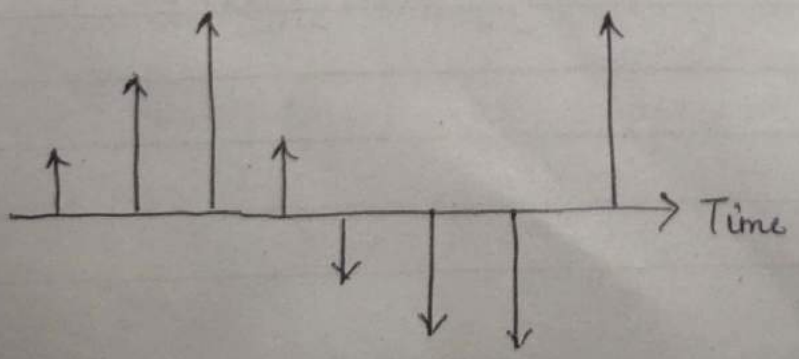
(a) CONTINUOUS TIME and DISCRETE TIME Signals:

A continous time signal is defined for all the continuous values of time variable t. It is one to one mapping of signals for every value of time assumes from $(-\infty, \infty)$, i.e, for every instance of time there exists a unique value of function $f(t)$. The signal can also have continuum of amplitude values. These are also called ANALOG SIGNALS

Eg: Sine, cosine function



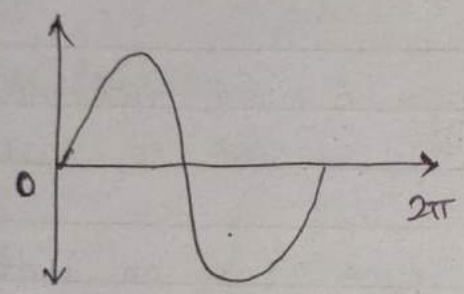
A Discrete time signal is defined only for some specific instant of time variable 'n' signals strictly speaking are all continuous time in nature, in discrete signals we are just ignoring the unwanted info in the signal by taking signal amplitudes at discrete instants of time.



(b) PERIODIC and APERIODIC Signals :

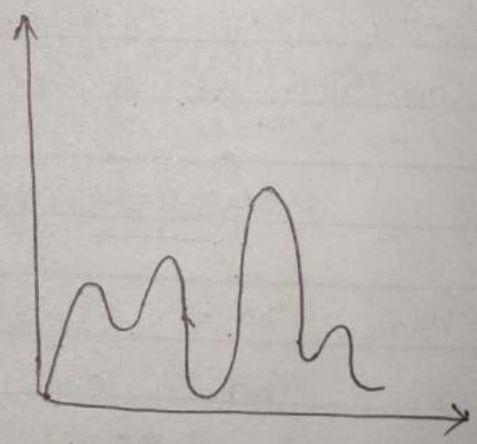
A signal can be classified as periodic signal if it repeats itself after a time interval of T, where T is called period of the signal. Mathematically, a signal f(t) is said to be periodic if $f(t+T) = f(t)$, where T is the smallest +ve non-zero value of T from all possible values for which the equality holds, T is said to be period of ~~time~~. f(t).

Eg Sine wave has period of 2π.



(A sine periodic wave)

A signal which is not periodic is said to be Aperiodic signal. Mathematically, it can be defined as a periodic signal with infinite period. Here infinite period signifies that the signal never repeats itself.



Aperiodic Signal

EVEN and ODD Signals :

- A signal $x(t)$ or $f(t)$ is referred to as an <u>even signal</u> if it is identical to its time reversed counterpart.
- $x(t) = x(-t)$ for all t.
- Even signals are symmetrical or mirror image about y-axis (of x-y plane)
- The area under the even signal is two times its one side area.

  Eg t^n (n is even), t^{2n} (n is integer), $\cos t$, $\sin^2 t$, etc

- A signal is referred to as an odd signal if it is -ve of its reflection $x(-t) = -x(t)$
- Odd signals are antisymmetric about the origin.
- Area under the odd signal is <u>zero</u>
- Odd signal must be zero at $t=0$, so that $x(0) = -x(0)$

  Eg t^n (n is odd), t^{2n+1} [$n \in$ integer], $\sin t$, etc

- Some function (signals) are neither even nor odd: e^t, $t^2 + t$, etc.

→ even + even = even Odd + odd = odd

→ even × even = even Odd × odd = odd

even × odd = odd even + odd = neither even nor odd

STANDARD SIGNALS OR SINGULARITY FUNCTION :

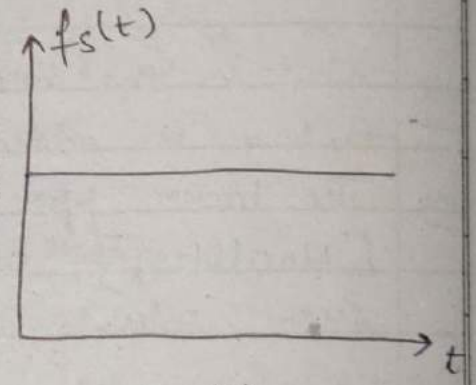
Some standard signals which are reliable in labs are given below :

Singularity function can be obtained from one another by successive differentiation or integration.

1. **STEP SIGNAL** : The step signal $f_s(t)$ is defined by

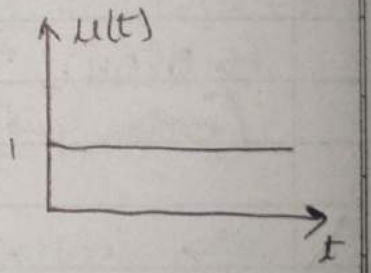
$$f_s(t) = \begin{cases} 0 & , t<0 \\ K & , t>0 \end{cases}$$

where K is the amplitude of step signal. The function is undefined and discontinuous at $t=0$.



UNIT-STEP SIGNAL : If the value of $K=1$ (unity), then $f_s(t)$ is called unit step signal $u(t)$, which is defined by :

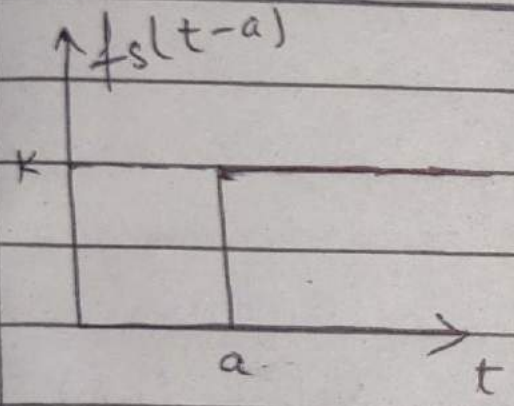
$$u(t) = \begin{cases} 0 & , t<0 \\ 1 & , t>0 \end{cases}$$

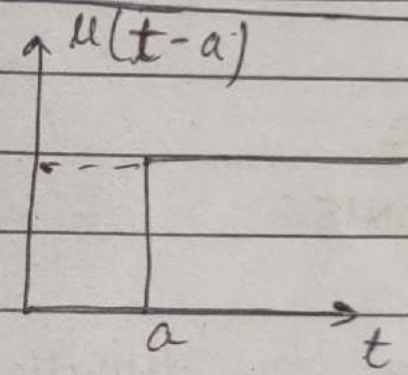


→ $\because$ $f_s(t)$ and $u(t)$ are 0 for $t<0$ and have magnitude K and 1 resp. for $t>0$ (t or any argument)

• this helps in defining the shift or delayed signals as

$$f_s(t-a) = \begin{cases} 0 & , t<a \\ K & , t>a \end{cases}$$





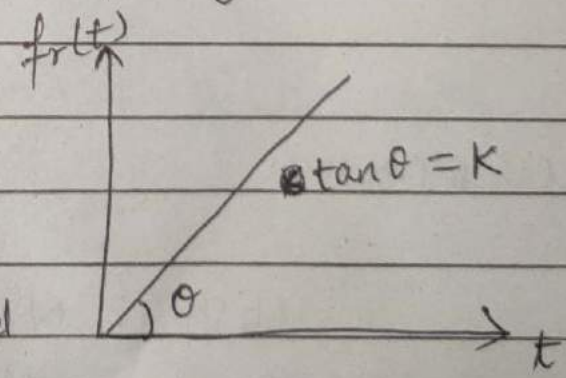
2. <u>RAMP SIGNAL</u> : The ramp signal $f_r(t)$ is defined by

$$f_r(t) = \begin{cases} 0 & , t<0 \\ Kt & , t \geq 0 \end{cases}$$

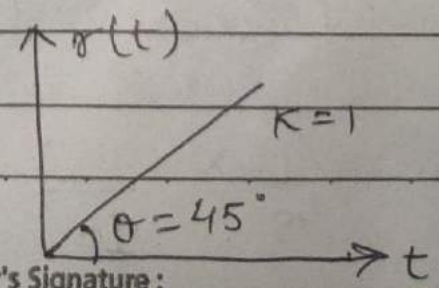
, where K is the slope of ramp signal.

<u>UNIT RAMP SIGNAL</u>:

If the value of $K=1$ then this ramp signal $f_r(t)$ is called unit ramp signal. $r(t)$ which is defined as:



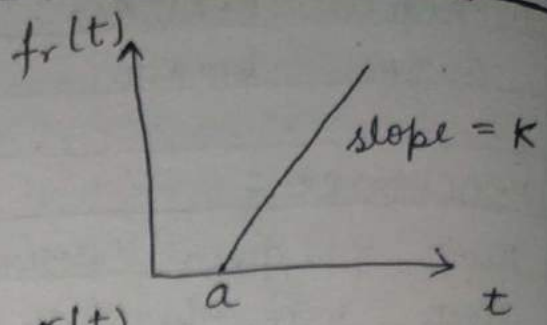
$$r(t) = \begin{cases} 0 & , t<0 \\ t & , t \geq 0 \end{cases}$$



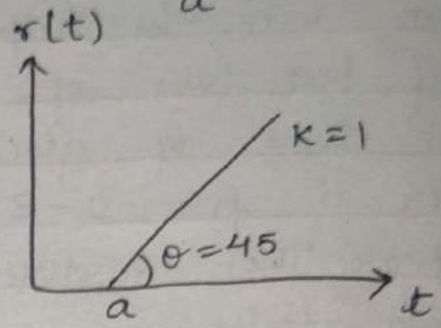
Teacher's Signature : _____

$= 0.015 \times 35.5 \times 1000$
$= 532.5$ ppm.

* **Shifted Ramp Signal :**



$$f_r(t-a) = \begin{cases} 0 & , t < a \\ K(t-a) & , t \geq a \end{cases}$$



$$r(t-a) = \begin{cases} 0 & , t < a \\ (t-a) & , t \geq a \end{cases}$$

NOTE: Unit ramp signal :

$\rightarrow r(t) = t\,u(t)$

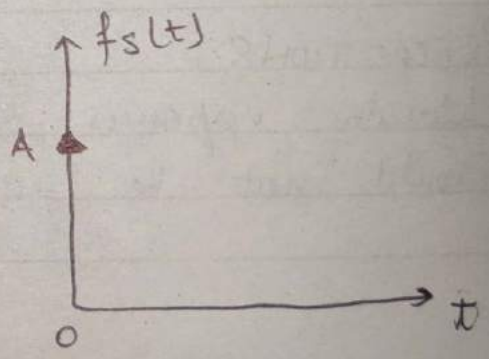
$\rightarrow r(t-a) = (t-a)\,u(t-a)$

Ramp Signal : $f_r(t) = K_r(t) = Kt\,u(t)$

$f_r(t-a) = K_r(t-a) = K(t-a)\,u(t-a)$

3. **IMPULSE SIGNAL :** (Dirac Delta Signal) The impulse signal $f_s(t)$ is defined by :

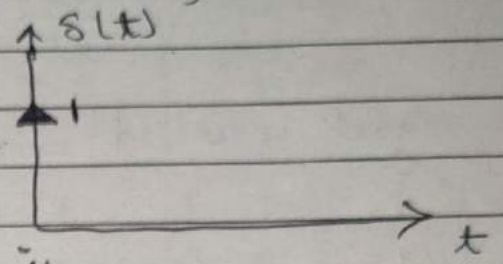
$$f_s(t) = \begin{cases} 0 & , t \neq 0 \\ A & , t = 0 \end{cases}$$



Where A is the area of the impulse signal and sometimes is called <u>Strength of the impulse</u>.

- <u>UNIT IMPULSE SIGNAL</u>: If the value of area $A = 1$, then this impulse signal $f_s(t)$ is called as unit impulse signal $\delta(t)$,

$$\delta(t) = \begin{cases} 0 & , t \neq 0 \\ 1 & , t = 0 \end{cases}$$

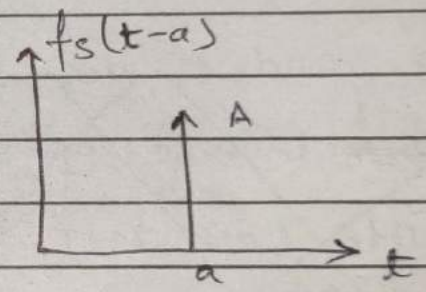


The area of unit impulse signal is

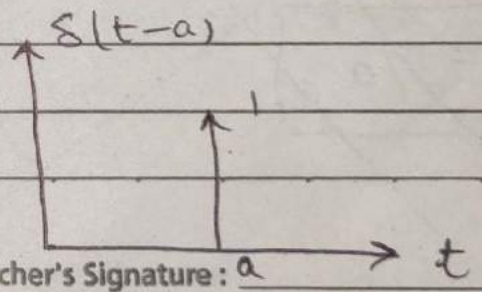
$$A = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

→ Shifted Impulse Signal:

$$f_s(t-a) = \begin{cases} 0 & , t \neq a \\ A & , t = a \end{cases}$$



$$\delta(t-a) = \begin{cases} 0 & , t \neq a \\ 1 & , t = a \end{cases}$$



Teacher's Signature :

NOTE: CONTINUOUS UNIT STEP FUNCTION:

Unit Step function $u(t)$ is running integral of unit impulse function $\delta(t)$.

$$u(t) = \int \delta(t)\, dt$$

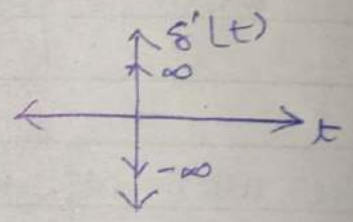
Following this, it can be stated that ~~that~~ unit impulse can be thought as first derivative of continuous-time unit step function.

$$\delta(t) = \frac{d\,u(t)}{dt}$$

4. UNIT DOUBLET SIGNAL: If unit Impulse signal $\delta(t)$ is differential with request to t, we get.

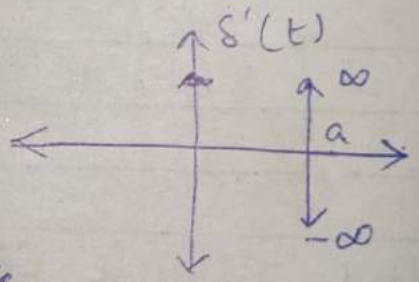
$$\delta'(t) = \frac{d}{dt}(\delta(t)) = \begin{cases} +\infty \text{ and } -\infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

This signal is called unit doublet signal.



→ Shifted Unit doublet signal:

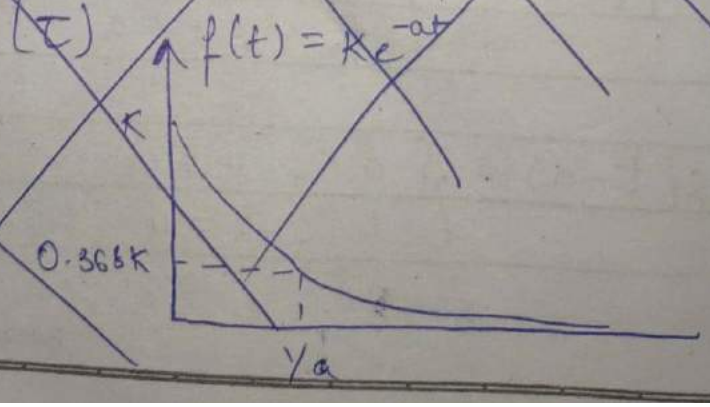
$$\delta'(t-a) = \begin{cases} +\infty \;\&\; -\infty, & t = a \\ 0, & t \neq a \end{cases}$$



Where a and K are real constants.

— The ~~reverse~~ inverse of a has dimension of time and is called TIME CONSTANT (τ)

$$\boxed{\tau = 1/a}$$



5. EXPONENTIAL SIGNAL : Exponential signal $f(t)$ is defined as:

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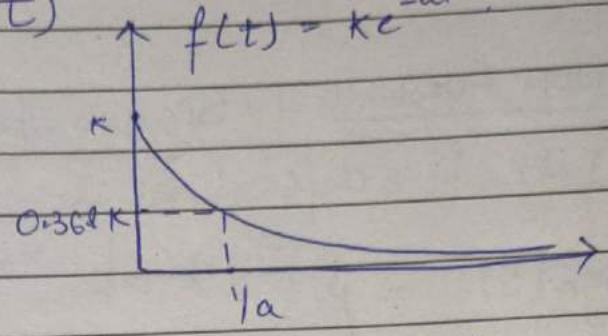
$$f(t) = \begin{cases} 0 & , t \le 0 \\ Ke^{-at} & , t \ge 0 \end{cases}$$

Where a and K are real constants.

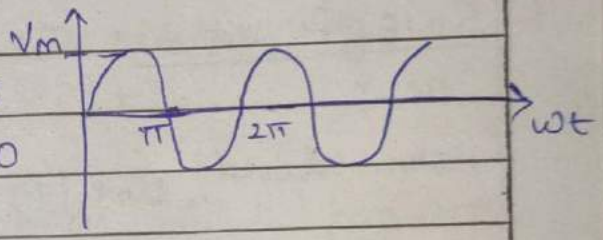
→ The inverse of 'a' has dimension of time and is called TIME CONSTANT (τ)

$$\boxed{\tau = 1/a}$$



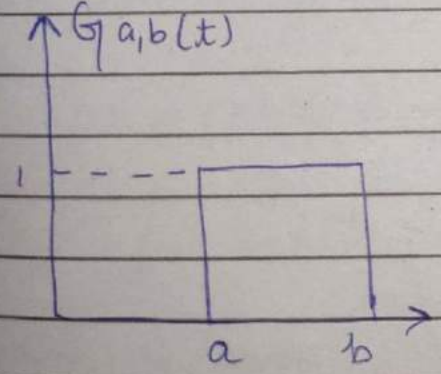
6. SINUSOIDAL FUNCTION : Sinusoidal function $f(t)$ is defined as:

$$f(t) = \begin{cases} 0 & , t < 0 \\ V_m \sin \omega t & , t \ge 0 \end{cases}$$



7. GATE SIGNAL : A rectangular pulse of unit height (i.e., amplitude is one), starting at $t = a$ and ending at $t = b$ and represented as

$$G_{a,b}(t) = u(t-a) - u(t-b)$$ is called gate func



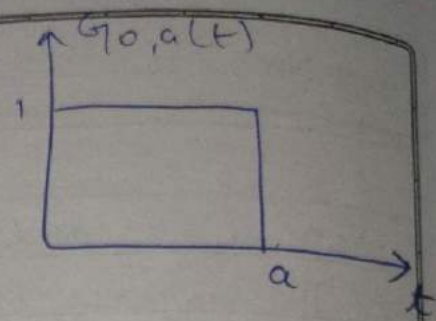
If this gate function start at ~~at~~ origin, then

$$G_{0,a}(t) = u(t) - u(t-a)$$

Teacher's Signature :

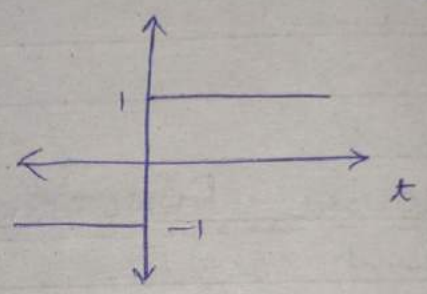
→ Application of Gate Function :

If any function (or signal) is multiplied by a gate function, then that function will be unchanged within the duration of gate and zero outside the duration of gate.



8. SIGNUM FUNCTION : Signum function is denoted as sgn(t) and it is defined as

$$sgn(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

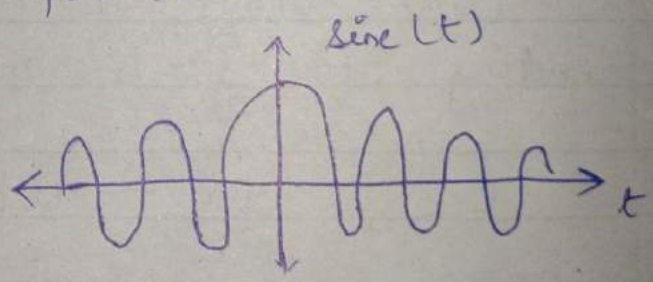


9. SINE FUNCTION : It is denoted as sine(t) and it is defined as :

(normalised) $sine(t) = \frac{\sin \pi t}{\pi t} = 0$ for $t = \pm 1, \pm 2, \pm 3$

$= 1$ for $t = 0$

(unnormalised) $sine(t) = \frac{\sin t}{t}$



SYSTEM: It is a device or combination of devices, which can operate on signals and produce corresponding response. Input to a system is called ~~exception~~ excitation and output from it is called response. Eg - Communication System.

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* Classification of Systems:

(a.) LINEAR AND NON-LINEAR SYSTEMS:

- A system is said to be linear when it satisfies superposition and homogenate principles.

- SUPERPOSITION principle means a system that produces an output $y_1(t)$ for an input $x_1(t)$ and output $y_2(t)$ for input $x_2(t)$ must produce an output $y_1(t) + y_2(t)$ for input $x_1(t) + x_2(t)$.

- HOMOGENEITY property means, a system which produces an output $y(t)$ for input $x(t)$ must produce an output $ay(t)$ for an input $ax(t)$

Eg: Consider a system with 2 inputs $x_1(t)$ and $x_2(t)$ and outputs $y_1(t)$, $y_2(t)$. Then, according to superposition and homogenity

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$

$$\therefore T[a_1x_1(t) + a_2x_2(t)] = a_1y_1(t) + a_2y_2(t)$$

- From the above example, it is clear that response of overall system is equal to response of individual system.

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b. TIME VARIENT and TIME-INVARIENT Systems:

- A system is said to be time varient if its input and output characteristics vary with time.
- A time-invarient system does not depend on time at which input is applied. Also I/o characteristics do not change with time.
- CONDITION for TIME VARIENT System:

$$y(n,t) \neq y(n-t)$$

- CONDITION for TIME-INVARIENT system

$$y(n,t) = y(n-t)$$

where $y(n,t) = T[x(n-t)]$ = Input change

$y(n-t)$ = output change.

c. STATIC and DYNAMIC System:

- Static system is memory-less whereas dynamic system is a memory system.
- A system is said to be static or memory less if response is due to present input only, i.e. the output at any instant t depends only on input applied at that instant t and not on past or future.

Eg $y(t) = x(t)$

for $t=0$, $y(0) = x(0)$

d. CASUAL and NON-CASUAL Systems:

A system is said to be casual if its output depends upon present and past input and does not depend upon future input.

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→ For non-casual system, the output depends upon present, past, and also future input.

e. INVERTIBLE and NON-INVERTIBLE system:

— A system is said to be invertible if the input of the system appears at the output.

→ $y(t) = x(t)$

— If $y(t) \neq x(t)$, then system is said to be non-invertible.

LAPLACE TRANSFORM:

— Transform method is superior to classical method for finding solution of ordinary differential equation.

— To solve the intergo - differential equation by the LAPLACE TRANSFORM Methods:

(i) Transform the time domain equation by the algebric equation in s domain.
→ (Laplace operator)

(ii) find roots of poly equation in s.

(iii) find inverse Laplace transform, to get solution in time domain.

→ FOURIER TRANSFORM is applicable to many functions, however, because of the restriction that

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty$$

Teacher's Signature:

many times function cannot be handled by ... (eg. ramp, periodic) as integral is not converging, hence not fourier transformable.

→ So laplace transform is a general case of fourier transform in which a convergence factor $e^{-\sigma t}$ (σ is a real no.) is introduced to ensure absolute convergence.

→ Laplace Transform ignores the info contained in $f(t)$ prior to $t=0$. No change, since ($f(t)=0$ for $t<0$). So,

$$F(s) = \int_0^\infty f(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t} \cdot dt$$

Taking $\boxed{S = \sigma + j\omega}$ [COMPLEX FREQUENCY]

(*) LAPLACE TRANSFORM:

$$\boxed{F(s) = L(f(t)) = \int_0^\infty f(t) \cdot e^{-st} \cdot dt}$$

where $f(t)$ is the time function which is 0 for $t<0$ and satisfy the condition,

$$\int_0^\infty |f(t) e^{-\sigma t}| \, dt < \infty$$

INVERSE LAPLACE TRANSFORMATION: This converts frequency domain function ~~F(S)~~ $F(S)$ to time-domain function $f(t)$.

$$f(t) = L^{-1} F(s) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) \cdot e^{st} \, ds$$

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LAPLACE TRANSFORM FOR SOME ELEMENTARY FUNCTIONS:

(I) Unit - Impulse function : $L[\delta(t)] = 1$

(II) Unit - Step function : $L[u(t)] = 1/s$, $L[1] = 1/s$

(III) Exponential function : $L[e^{at}] = 1/(s-a)$

$L[u(t-a)] = \frac{1}{s^2} e^{-as}$

(IV) Sine function : $L[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$

(V) $L[t] = \frac{1}{s^2}$, $L[t^n] = \frac{n!}{s^{n+1}}$

(VI) Cosine function : $L[\cos \omega t] = \frac{s}{s^2 + \omega^2}$

(VII) $L[e^{-at} \sin \omega t] = \frac{\omega}{(s+a)^2 + \omega^2}$, $L[e^{at} \sin \omega t] = \frac{\omega}{(s-a)^2 + \omega^2}$

(VIII) $L[e^{-at} \cos \omega t] = \frac{s+a}{(s+a)^2 + \omega^2}$, $L[e^{at} \cos \omega t] = \frac{s-a}{(s-a)^2 + \omega^2}$

(IX) $L[t \cdot e^{-at}] = \frac{1}{(s+a)^2}$

(X) $L[\sin hat] = \frac{a}{s^2 - a^2}$, $L[\cosh at] = \frac{s}{s^2 - a^2}$

(XI) $L[e^{-at} u(t)] = \frac{1}{(s+a)}$

Teacher's Signature :

PROPERTIES OF LAPLACE TRANSFORM:

1. Multiplication By Constant : $L[Kf(t)] = KL[f(t)]$

2. Sum or difference : $L[f_1(t) \pm f_2(t)] = L[f_1(t)] \pm L[f_2(t)]$

3. Time difference (Differentiation wrt. 't')

Let $F(s)$ be the laplace transform of $f(t)$ and let $f(0^+)$ be the value of $f(t)$ as t approaches 0, then

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - \lim f(t)$$
$$= sF(s) - f(0^+)$$

In general:

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} \cdot f(0^+) - s^{n-2} f'(0^+) - \dots - s f^{n-2}(0^+) - f^{n-1}(0^+)$$

4. TIME INTEGRATION (Integration by 't'):

If $L[f(t)] = F(s)$, the laplace transform of first integral of $f(t)$ is,

$$\Rightarrow L\left[\int_0^t f(t) \cdot dt\right] = \frac{F(s)}{s}$$

In general,

$$L\left[\int_0^{t_1}\int_0^{t_2}\int_0^{t_3} \dots \int_0^{t_n} f(t)\, dt_1\, dt_2 \dots dt_n\right] = \frac{F(s)}{s^n}$$

5. FREQUENCY DIFFERENTIATION: (diff. wrt 's')

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Differentiation in the s-domain corresponds to multiplication by t in time domain.

$$L[t \cdot f(t)] = -\frac{dF(s)}{ds}, \quad L[t^n f(t)] = (-1)^n \frac{d^n F(s)}{ds^n}$$

6. FREQUENCY INTEGRATION: (Integration by 's')

Integration of F(s) in s-domain corresponds to division by t in time domain.

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\, ds$$

7. Shifting Theorem:

(a) Time Shifting: Laplace transform of a shifted or delayed function is:

$$L[f(t-a)] = e^{-as} F(s)$$

$$L[f(t+a)] = e^{as} F(s)$$

(b) Frequency Shifting: The laplace transform of e^{-at} times of a function is equal to laplace transform of that function with s replaced by (s+a)

$$L[e^{-at} f(t)] = F(s+a)$$

$$L[e^{at} f(t)] = F(s-a)$$

Teacher's Signature:

8. <u>INITIAL VALUE THEORAM</u>: If $f(t)$ and its first derivative $f'(t)$ are both laplace transformable then,

$$f(0^+) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} [s.F(s)]$$

→ Applicable only when $f(t) = 0$ for $t < 0$

→ $f(t)$ must not contain any impulse or higher order singularities at $t = 0$.

9. <u>FINAL VALUE THEORAM</u>: If the laplace transform of $f(t)$ is $F(s)$ and if $sF(s)$ is analytic on the imaginary axis and in right half of the s-plane, then,

$$\Rightarrow \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

$$f(\infty) = \lim_{s \to 0} sF(s)$$

— Applicable only when $f(t) = 0$, $t < 0$

- $sF(s)$ must have poles in the left half of the s-plane.

10. <u>CONVOLUTION THEORAM</u>: Given two functions $f_1(t)$ and $f_2(t)$ which are 0 (zero) for $t < 0$, If $L[f_1(t)] = F_1(s)$ and $L[f_2(t)] = F_2(s)$

$$\Rightarrow \boxed{L^{-1}[F_1(s).F_2(s)] = f_1(t) * f_2(t)}$$ → is called convolution of $f_1(t)$ & $f_2(t)$

and is equal to

$$\int_0^t f_1(t-\tau)\, f_2(\tau)\, d\tau \text{ or}$$

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$$\int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$\Rightarrow \boxed{L\left[f_1(t) * f_2(t)\right] = F_1(s) \cdot F_2(s)}$$

11. <u>TIME SCALING</u>: If laplace transform of $f(t)$ is $F(s)$ then

$$\boxed{L\left[f(at)\right] = \frac{1}{a} F\left(\frac{s}{a}\right)}$$

12. <u>TIME - REVERSAL</u>: If $L\left[f(t)\right] = F(s)$

$$L\left[f(-t)\right] = F(-s)$$

13. <u>PERIODIC FUNCTION</u>: for a periodic function

$$\boxed{L\left[f(t)\right] = \frac{F_1(s)}{1-e^{-Ts}}}$$

$F_1(s) \rightarrow$ laplace for Ist orde

~~$F_2(s) \rightarrow$~~ $T \rightarrow$ period.

Teacher's Signature :

LTI - System : Linear time - invarient system (LTI system)

are a class of systems that are both linear and time invarient.

- LTI system are used to predict long-term behaviour in a system, eg to model system like power plant.
- Another important application of LTI system is electrical circuits, (circuits made up of inductors, transistors, resistors) which are basis of modern technology.
- Consider $y(t)$ is the output of continuous time LTI system with input $x(t)$ and no initial energy with unit impulse as input $x(t) = \delta(t)$, the output is defined as the IMPULSE RESPONSE ($h(t)$)

* CONVOLUTION : $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

⇒ Convolution Sum :

$$y[n] = x[n] * h[n] = \sum_{i=\infty}^{\infty} x[i]\, h[n-i]$$

The entirety of an LTI system can be described by a single function called its impulse response. This function exists in the time-domain of the system. For an ~~arbita~~ arbitrary input, the output of an LTI system is the CONVOLUTION of input signal with system impulse

- TRANSFER FUNCTION is the laplace transform of impulse response of system.

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Expt. No. $H(s) = L(h(t))$ Page No.

- In frequency domain, output is product of transfer function and laplace transform of input.

- In addition to linear and time - invariant, LTI systems are also memory systems, INVERTIBLE, CASUAL, REAL and STABLE

That means, they have memory, they can be inverted, they depend only on current and past events, have fully matched inputs and outputs and produce bounded output, for bounded input.

* PROPERTIES OF LTI SYSTEM [CONVOLUTION]

— DISTRIBUTIVE : $x(t) * [h_1(t) + h_2(t)]$
$\Rightarrow x(t) * h_1(t) + x(t) * h_2(t)$

— COMMUTATIVE : $x(t) * h(t) = h(t) * x(t)$

— ASSOCIATIVE : $x(t) * [h_1(t) * h_2(t)]$
$\Rightarrow [x(t) * h_1(t)] * h_2(t)$

— CONVOLUTION WITH UNIT IMPULSE :

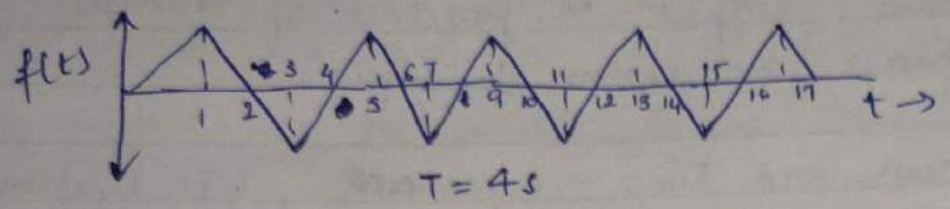
$x(t) * \delta(t) = x(t)$ [∵ unit impulse is 0 everywhere except origin].

Teacher's Signature :

APPLICATION OF LAPLACE TRANSFORM:

Part A: Periodic waveform

Q



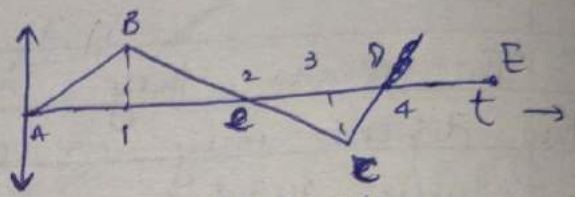
$T = 4s$

$$f(t) = f_1(t) + f_1(t-4) + f_1(t-8) + \dots$$

$$F(s) = \frac{F_1(s)}{1-e^{-sT}}, \quad T = 4$$

periodic function

$$= \frac{F_1(s)}{1-e^{-4s}} \quad \text{--- (1)}$$



$m_{AB} = 1$

$m_{BC} = -1$

$m_{CD} = 1$

$m_{DE} = 0$

where

$$f_1(t) = r(t) - 2r(t-1) + 2r(t-3) - 1r(t-4) \quad \text{--- (2)}$$

Taking laplace tranform.

$$F_1(s) = \frac{1}{s^2} - \frac{2}{s^2} \cdot e^{-s} + \frac{2}{s^2} e^{-3s} - \frac{1}{s^2} e^{-4s}$$

$$F_1(s) = \frac{1}{s^2}\left[1 - 2e^{-s} + 2e^{-3s} - e^{-4s}\right]$$

$$F(s) = \frac{1 - 2e^{-s} + 2e^{-3s} - e^{-4s}}{s^2(1-e^{-4s})}$$

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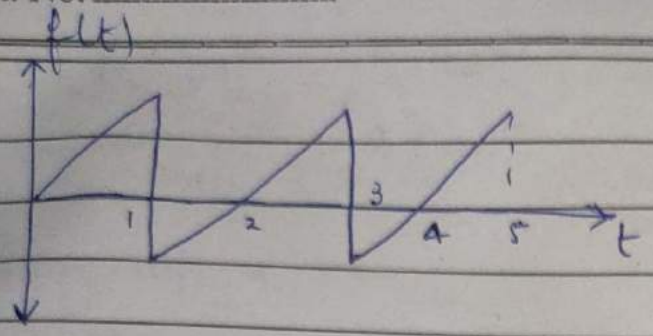
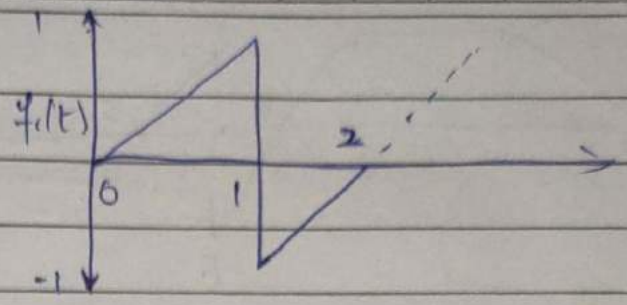
Q





where $f_1(t) = r(t) - u(t-1) - 2u(t-1) - r(t-2)$

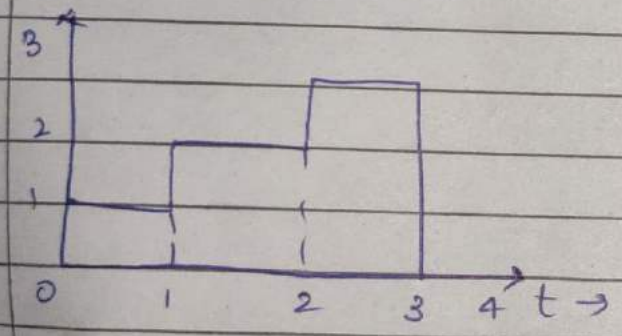
$$F_1(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s} - \frac{1}{s^2} \cdot e^{-2s}$$

$$F(s) = \frac{F_1(s)}{1-e^{-sT}}, \quad \text{here } T = 2$$

$$= \frac{1 - 2se^{-s} - e^{-2s}}{s^2(1-e^{-2s})}$$

Part B : Aperiodic function

Q



$f(t) = u(t) + u(t-1) + u(t-2) - 3u(t-3)$

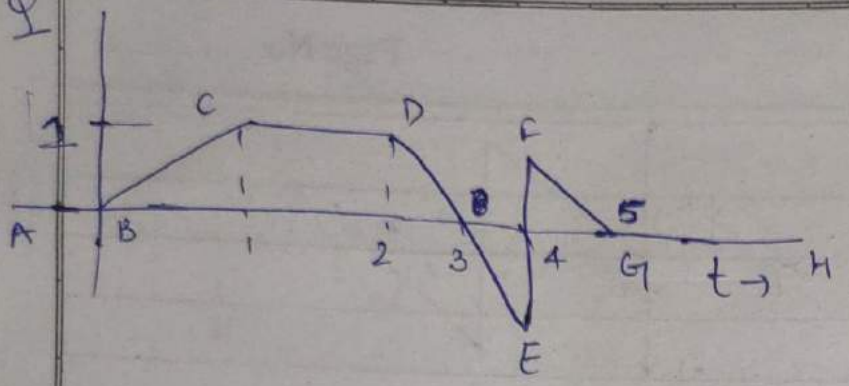
on taking laplace

$$F(s) = \frac{1}{s} + \frac{1}{s}e^{-s} + \frac{1}{s}e^{-2s} - \frac{3e^{-3s}}{s}$$

$$= \frac{1}{s}\left[1 + e^{s} + e^{-2s} - 3e^{-3s}\right]$$

Teacher's Signature :

Q



$m_{AB} = 0$ $\quad m_{DE} = -1$

$m_{BC} = 1$ $\quad m_{FG} = -1$

$m_{CD} = 0$ $\quad m_{GH} = 0$

$$f(t) = r(t) - r(t-1) - r(t-2) + 2u(t-4) + r(t-5)$$

Taking laplace transform.

$$F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^2} + \frac{2e^{-4s}}{s} + \frac{e^{-5s}}{s^2}$$

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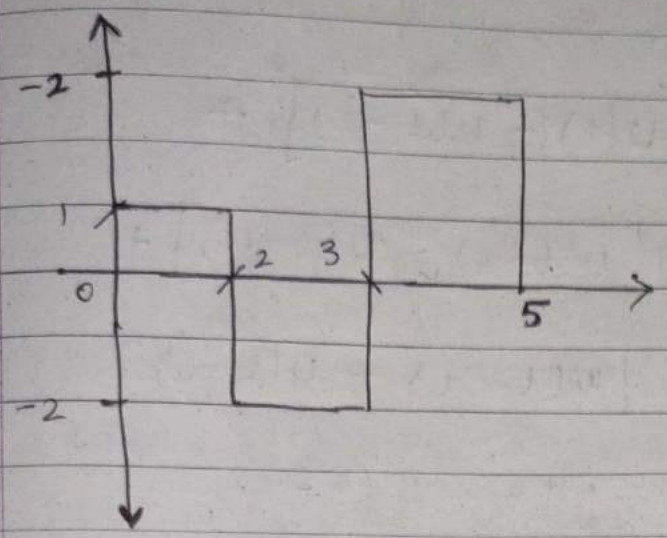
Ques

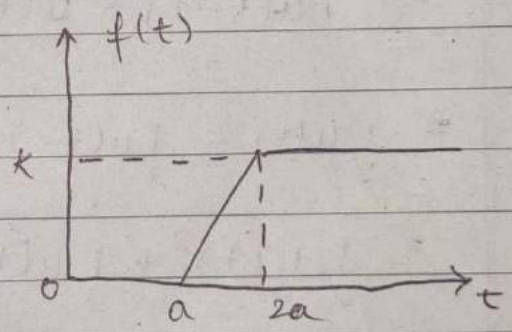


Synthesize the wave function.

$$G(t) = 1[u(t-0) - u(t-2)] + (-2)[u(t-2) - u(t-3)] + 2[u(t-3) - u(t-5)]$$

$$= u(t-0) - 3u(t-2) + 4u(t-3) - 2u(t+5)$$

Ques Synthesize the wave form :



$\cancel{f(t) = \frac{K}{a}(t-a)\,u(t-a)}$

$$f(t) = \frac{K}{a}(t-a)[u(t-a) - u(t-2a)] + K\,u(t-2a)$$

$$= \frac{K}{a}(t-a)\,u(t-a) - \frac{K}{a}(t-a)\,u(t-2a) + K\,u(t-2a)$$

$$= \frac{K}{a}(t-a)\,u(t-a) - K\,u(t-2a)\left[\frac{t-a}{a} - 1\right]$$

from line equation. $y - y_1 = (x - x_1)\,m$ $\left(y = (t-a)\frac{K}{a}\right)$

$$= \frac{K}{a}(t-a).u(t-a) - \frac{K}{a}(t-2a)\,u(t-2a)$$

Q Synthesize the given waveform as shown in figure ~~2.23~~.

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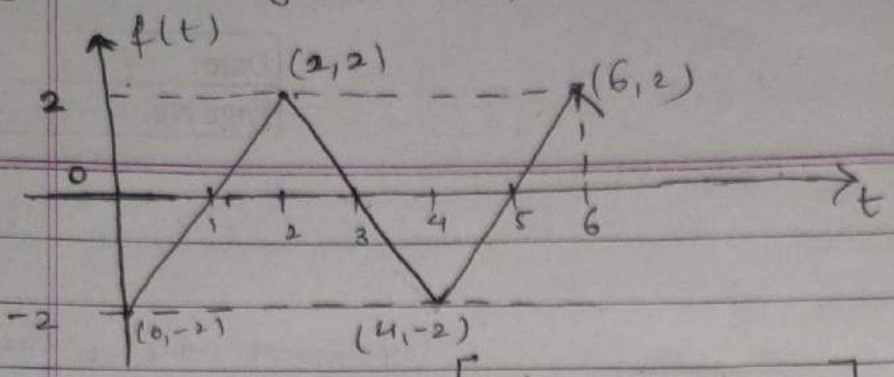


Ans

$$f(t) = (2t-2)[u(t) - u(t-2)] +$$

$$(-2t+6)[u(t-2) - u(t-4)] +$$

$$(2t-10)[u(t-4) - u(t-6)]$$

$$y+2 = \frac{2-(-2)}{6-4}(x-4) \qquad y+2 = \frac{4}{2}(x-4)$$

$$y = 2x-8-2$$

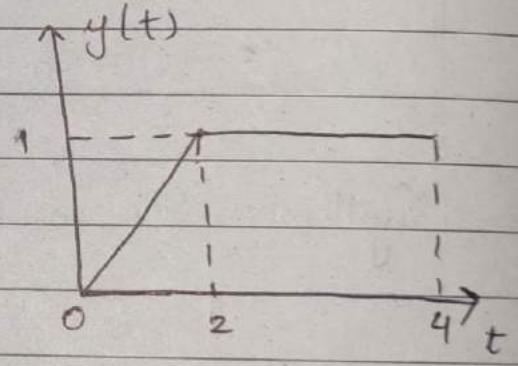
$$= 2x-10$$

Ques Obtain the laplace transform

Soln.

$$y(t) = \frac{1}{2}[u(t) - u(t-2)] +$$

$$[u[t-2] - u(t-4)]$$



$$= \frac{1}{2}u(t) - \frac{1}{2}u(t-2) + u(t-2) - u(t-4)$$

$$= \frac{1}{2}u(t) + \frac{1}{2}u(t-2) - u(t-4)$$

$$= \frac{1}{2}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s^2}e^{-2s} - \frac{1}{s^2}e^{-4s}$$

$$= \frac{1}{2s}\left[1 + \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}\right]$$

System:

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Transfer function: Transfer function is defined as ratio of laplace transformation output to the laplace transform of input, when all ~~the~~ initial condition are assumed to be zero.

$$H(s) = \frac{Y(s)}{X(s)}\Bigg|_{\text{zero initial cond}^n}$$

$$\boxed{Y(s) = H(s)\,X(s)}$$

Impulse Response: It is the output of ~~the~~ a system when we feed an impulse as input.

or

relationship between input & output in time domain.

Ques find the response with e^{-2t} as i/p to the transfer function $H(s) = \frac{1}{s+3}$

Ans

$x(t) = e^{-2t}$

$\Rightarrow x(s) = \frac{1}{(s+2)}$

$$Y(s) = H(s).\,X(s)$$

$$= \frac{1}{(s+3)} \cdot \frac{1}{(s+2)}$$

$$= \frac{A}{s+2} + \frac{B}{s+3} = \frac{1}{s+2} - \frac{1}{s+3}$$

$$Y(s) = \left(e^{-2t} - e^{-3t}\right) u(t)$$

TRANSIENT RESPONSE: Time which system takes after applying input to reach steady state is call TRANSIENT STATE

The response of the system during transient state is transient response.

UNIT - II

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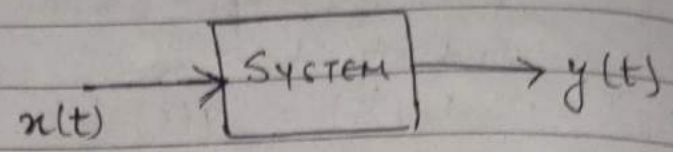
SYSTEM MODELLING: A system is a device or combination of devices that has signal as input and response at output.



- **A CIRCUIT AS A DIFFERENTIAL EQUATION:**

  - An equation involving different coefficients is called differential equation.

- The interconnection of simple electric device in which there is a ~~the~~ at least one closed path for current to flow. is an electric circuit

- The circuits, switch from one condition to another by change in the applied source or a change in the circuit elements. This transition period during which the branch current and voltage changes from their former values to new ones is called TRANSIENT.

- After transient has passed the circuit is said to be steady state.

- A circuit containing inductance (L) or a capacitor (C) and resistor (R) with current and voltage variable is given by a LINEAR DIFFERENTIAL EQUATION:

- The LDE describe that the circuit will have two parts to its solutions:
  - COMPLEMENTARY FUNCTION corresponding to transient state

PARTICULAR INTEGRAL corresponding to steady state.

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- The V-I relation for an inductor and capacitor are differential and they are energy storing elements.

- A circuit ~~containing~~ containing L or C with R, i.e., A RL or RC circuit are first order equations. A RLC circuit are second order.

VOLTAGE - CURRENT RELATIONSHIPS :

I) RESISTOR : By ohm's law, the v-i relationship is $\boxed{V_R = i_R R}$ and it remains same for both time and frequency domain.

II) INDUCTOR : The v-i relationship for an inductor is :

$$V_L(t) = L \cdot \frac{di(t)}{dt}$$ [Voltage is generated by change in current]

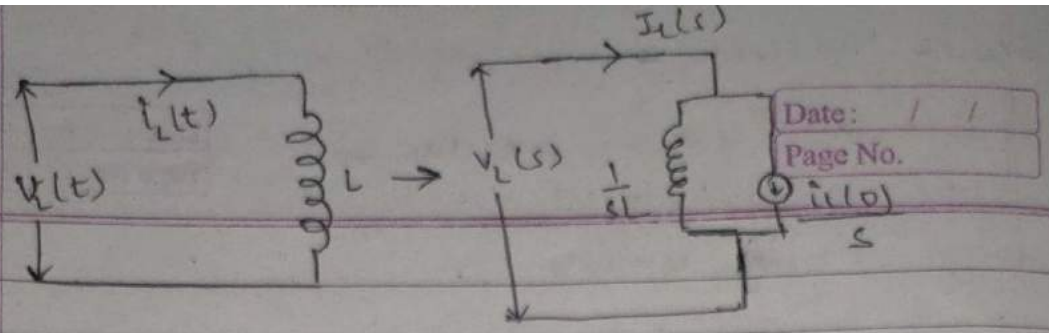
$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$$

$$i_L(t) = \underbrace{\frac{1}{L}\int_{-\infty}^{0} V_L(t)\,dt}_{i(0)} + \underbrace{\frac{1}{L}\int_{0}^{t} V_L(t)\,dt}_{i'(t)}$$

In complex frequency (s) domain, we take laplace transform of function in (t) domain,

$$V_L(s) = sL\,I_L(s) - L\,i_L(0)$$

$$I_L(s) = \frac{1}{sL}V_L(s) + \frac{i_L(0)}{s}$$



INDUCTOR in TIME DOMAIN

INDUCTOR in FREQUENCY DOMAIN.

(III) CAPACITOR: The vi relation in capacitor is given as:

$$i_c(t) = \frac{c\, dv_c(t)}{dt}$$

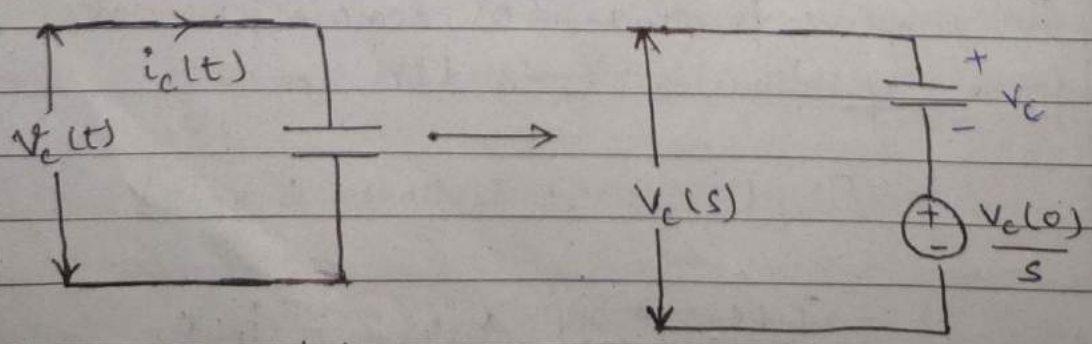
$$v_c(t) = \frac{1}{c}\int_{-\infty}^{t} i_c(t)\, dt$$

$$v_c(t) = \underbrace{\frac{1}{c}\int_{-\infty}^{0} i_c(t)\, dt}_{v_c(0)} + \underbrace{\frac{1}{c}\int_{0}^{t} i_c(t)\, dt}_{v_c'(t)}$$

In frequency (s) domain,

$$I_c(s) = sC V_c(s) - C v_c(0)$$

$$V_c(s) = \frac{1}{sC} I_c(s) + \frac{v_c(0)}{s}$$



Capacitor in time domain

Capacitor in frequency domain

* Two methods of SOLVING DIFFERENTIAL EQUATIONS:

→ CLASSICAL METHOD

→ LAPLACE TRANSFORM.

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(I) CLASSICAL METHOD:

(i) VARIABLE SEPERABLE:

$$\frac{dy(t)}{dt} + Py(t) = 0$$

$$\boxed{y(t) = K \cdot e^{-Pt}} \quad [K \text{ is a constant}]$$

(ii) LINEAR DIFFERENTIAL EQUATION:

$$\frac{dy(t)}{dt} + Py(t) = Q$$

$$y(t) = \frac{Q}{P} + Ke^{-Pt}$$

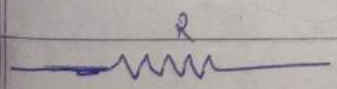
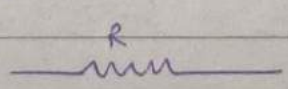
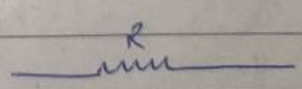
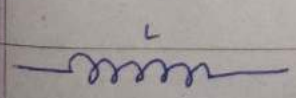
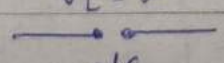
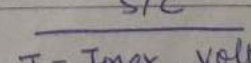
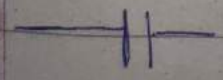
(iii) GENERAL SOLUTION: $A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + Cy(t) = 0$

$$\boxed{y(t) = K_1 e^{P_1(t)} + K_2 e^{P_2(t)}}$$

(K_1 and K_2 are constant), P_1 & P_2 are roots of quad eq^n $P^2A + PB + C = 0$

$\Rightarrow$ Let $\frac{dy(t)}{dt} = P$

→ Initial/ Boundary conditions in a circuit:

Circuit Element	Eq. ckt at $t = 0^+$	Eq. ckt at $t = \infty$
1. 		
2. 	$V_L = \infty$  O/C	S/C  $I = I_{max}$, voltage drop $= 0$
3. 	$I_C = C\frac{dV_c(t)}{dt}$ S/C	O/C

→ ZERO INPUT RESPONSE – The value of current and voltage which results from initial condition when applied input is zero.

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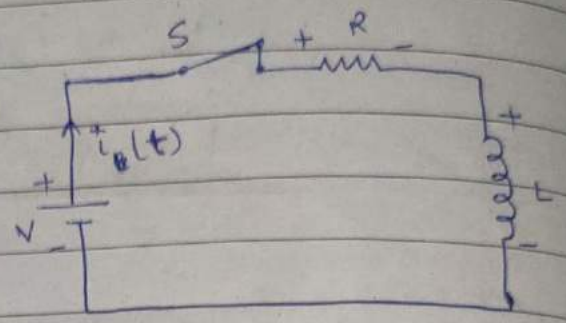
→ ZERO STATE RESPONSE: The values of current and voltage for an input which is applied when all the initial conditions are zero. Such networks/circuits are said to be at rest.

Ques, S is closed at $t=0$. Find $i(t)$ and $v(t)$ for Resistor and inductor.



Sol^n Applying KVL rule

$$V = iR + L\frac{d(i)}{dt}$$

$$\frac{d(i)}{dt} + i\frac{R}{L} = \frac{V}{L}$$

$$i(t) = \frac{V \times L}{L \times R} + K.e^{-Rt/L}$$

$$i(t) = \frac{V}{R} + K.e^{-Rt/L}$$

at time $t=0$, $i(0)=0$ open circuit.

$$\Rightarrow \frac{V}{R} + K.e^{0} = 0$$

$$\boxed{K = -V/R}$$

$$\therefore i(t) = \frac{V}{R} + \left(-\frac{V}{R}\right)e^{-Rt/L}$$

$$= \frac{V}{R}\left[1 - e^{-Rt/L}\right]$$

Voltage across resistor:

$$v_R(t) = \cancel{iR}\, i(t) R$$

$$= R \cdot \frac{V}{R}\left[1 - e^{-Rt/L}\right]$$

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$$v_R(t) = V\left[1 - e^{-Rt/L}\right]$$

Voltage across inductor:

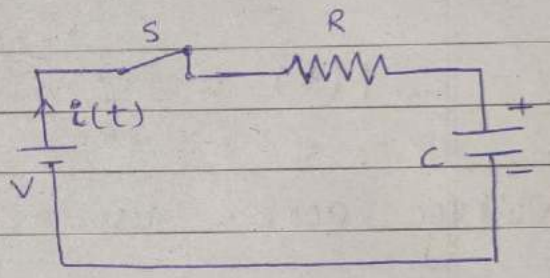
$$v_L(t) = L \cdot \frac{d(i)}{dt}$$

$$v_L(t) = \cancel{L \cdot \frac{V}{R} \cdot \frac{1}{L}}\; L \cdot \frac{V}{R} \cdot \frac{d}{dt}\left(1 - e^{-Rt/L}\right)$$

$$v_L(t) = \frac{VL}{R}\left(\cancel{+}\, \frac{R}{L}\, e^{-Rt/L}\right)$$

$$v_L(t) = V e^{-Rt/L}$$

Ques 2 Switch is closed at $t=0$, find voltage and circuit for resistor and capacitor.



Sol^n applying KVL in circuit, we get

$$V = iR + \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

$$V = i(t)R + \frac{1}{C}\int_{-\infty}^{0} i(t)\, dt + \frac{1}{C}\int_{0}^{t} i(t)\, dt$$

Capacitor was uncharged at $v_c(0)$

$$\Rightarrow \int_{-\infty}^{0} i(t)\, dt = 0$$

$$\left[V = i(t)R + \frac{1}{C}\int_{0}^{t} i(t)\, dt\right]$$

diff. wrt. t on both side.

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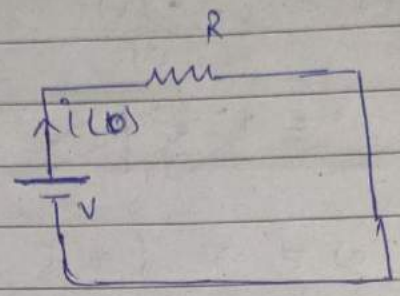
$$R \cdot \frac{di(t)}{dt} + \frac{1}{C} \cdot i(t) = 0$$

$$\frac{di(t)}{dt} + \frac{1}{RC} i(t) = 0$$

$$i(t) = 0 + K \cdot e^{-t/RC}$$

at $t=0$,



$$i(0) = \frac{V}{R}$$

$$K \cdot e^{-0} = \frac{V}{R} \Rightarrow \boxed{K = \frac{V}{R}}$$

$$i(t) = \frac{V}{R} e^{-t/RC}$$

Voltage across resistor will be

$$v_R(t) = R \cdot i(t)$$
$$= R \cdot \frac{V}{R} \cdot e^{-t/RC}$$
$$v_R(t) = V e^{-t/RC}$$

Voltage across capacitor will be :

$$v_C(t) = \frac{1}{C} \int_0^t i(t)\, dt$$

$$= \frac{V}{RC} \int_0^t e^{-t/RC}\, dt$$

$$= \frac{V}{RC} \left[-RC\, e^{-t/RC} + RC \right]$$

Ques 3 In this network, switched at $t=0$ is closed. A steady state current having previously been attained. Solve for current as a function of time

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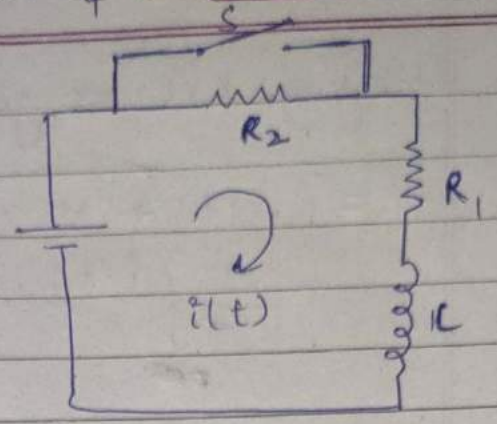

Sol^n Before $t=0$
inductor is short

$$V = R_2\, i(0) + R_1\, i(0)$$

$$i(0) = \frac{V}{R_1 + R_2}$$

after $t>0$, R_2 is short circuited.

$$V = R_1\, i(t) + L \cdot \frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} + \frac{R_1}{L} i(t) = \frac{V}{L}$$

$$i(t) = \frac{V \times L}{L \times R_1} + K \cdot e^{-R_1 t/L}$$

$$i(t) = \frac{V}{R_1} + K \cdot e^{-R_1 t/L}$$

$$i(0) = \frac{V}{R_1} + K \cdot e^{0} = \frac{V}{R_1 + R_2}$$

$$K = \frac{V}{R_1 + R_2} - \frac{V}{R_1} = \frac{\cancel{VR_1} - \cancel{VR_1} - VR_2}{R_1(R_1 + R_2)}$$

$$K = -\frac{VR_2}{R_1(R_1 + R_2)}$$

$$i(t) = \frac{V}{R_1}\left[1 + \frac{R_2}{R_1 + R_2} \cdot e^{-R_1 t/L}\right]$$

Ques 4 Find the value of $V_c(t)$ for $t>0$ in the circuit
Assume $V_c(0^+) = 9V$.

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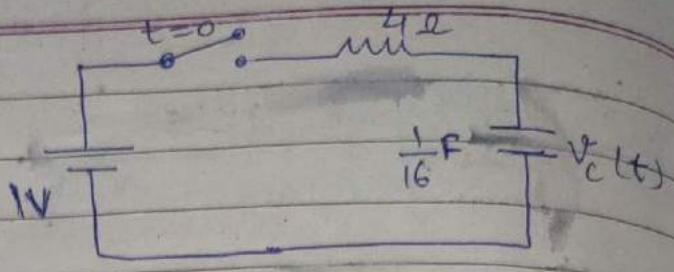
Sol^n Applying KVL we get;



$$V = R.i(t) + V_c$$

$$\frac{1}{C}\int_0^t i(t).dt$$

$$1 = 4\,i(t) + 16\int_0^t i(t)\,dt + 9$$

differentiate wrt t

$$0 = 4.\frac{di(t)}{dt} + 16\,i(t)$$

$$0 = \frac{di(t)}{dt} + 4\,i(t)$$

$$i(t) = K.e^{-4t}$$

at $t=0$ capacitor will get short.

$$V = i(0).R + V_c(0)$$

$$\cancel{i(0) = \frac{V(0)}{R} = \frac{9}{4}}$$

$$1 = i(0).4 + 9$$

$$i(0) = \frac{1-9}{4} = -2$$

$$K = -2$$

voltage across capacitor:

$$v_c(t) = V_c(0) + \frac{1}{C}\int_0^t i(t)\,dt$$

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$$= 9 + 16\int_0^t (-2)e^{-4t}\,dt$$

$$= 9 - 32\left[-\frac{e^{-4t}}{4}\right]_0^t$$

$$= 9 - 8\left[-e^{-4t} - 1\right]$$

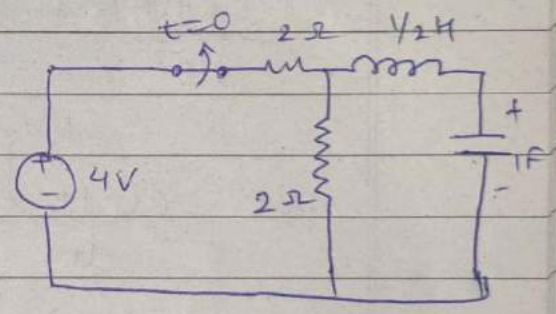
$$= 9 + 8e^{-4t} - 8$$

$$= 1 + 8e^{-4t}.$$

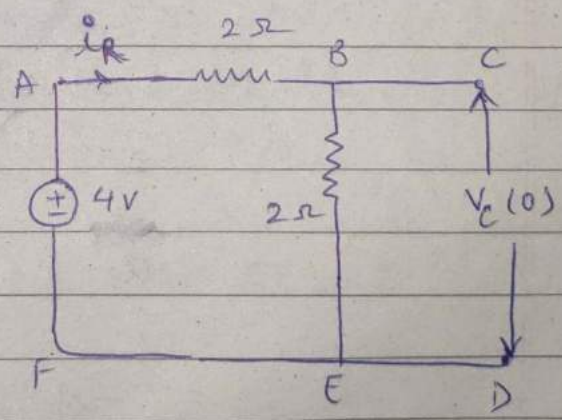
Ques The circuit is in steady state with the switched closed. The switch is opened at $t = 0$. Determine, $i_L(t)$.

At Steady State

$$\left[\begin{array}{l} L \rightarrow \text{short} \\ C \rightarrow \text{open} \end{array}\right]$$



Applying KVL in ABEFA



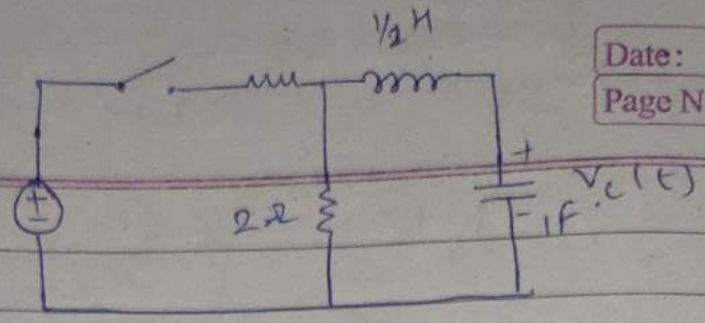
$$2i_R + 2i_R = 4$$

$$i_R = 1A$$

Applying KVL in BCDEB

$$V_c(0) = 2 \cdot i_R = 2V$$

At $t=0$,



$$i_L(t).2 + \frac{1}{2}.\frac{d\,i_L(t)}{dt} + 1\int_0^t i_L(t)\,dt + V_c(0) = 0$$

diff. wrt t

$$\cancel{2\,i_L(t) + 1}$$

$$2\frac{di_L(t)}{dt} + \frac{1}{2}.\frac{d^2 i(t)}{dt^2} + i_L(t) = 0$$

$$\frac{d^2 i_L(t)}{dt^2} + 4\frac{di_L(t)}{dt} + 2i_L(t) = 0$$

put $\frac{di_L(t)}{dt} = p$

$$p^2 + 4p + 2 = 0$$

$$d = 16 - 4\times1\times2 = 16 - 8 = 8$$

$$p_{0} = \frac{-4 \pm \sqrt{8}}{2}$$

$$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

$p_1 = -0.586$ $\quad$ $p_2 = -3.414$

$$i_L(t) = K_1 e^{-0.586t} + K_2 e^{-3.414t}$$

$$i_L(0) = 0$$

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$$K_1 = -K_2$$

diff. wrt. t

$$\frac{di_L(t)}{dt} = K_1 \cdot e^{-0.586t} \cdot (-0.586) + (-K_1) e^{-3.414t} (-3.414)$$

$$\frac{di_L(0)}{dt} = K_1 [-0.586 + 3.414]$$

$$V_L(0^+) = -V_C(0^+) = -2 = -L \frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} = \frac{-2}{L} = \frac{-2}{1/2} = -4$$

$$-4 = K_1 [-0.586 + 3.414]$$

$$-1.414 = K_1$$

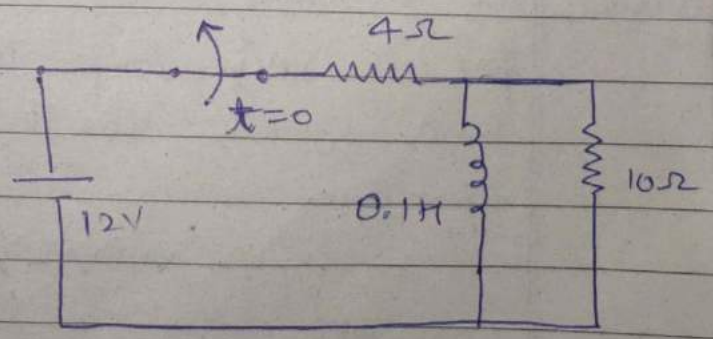
$$K_2 = -K_1 = 1.414$$

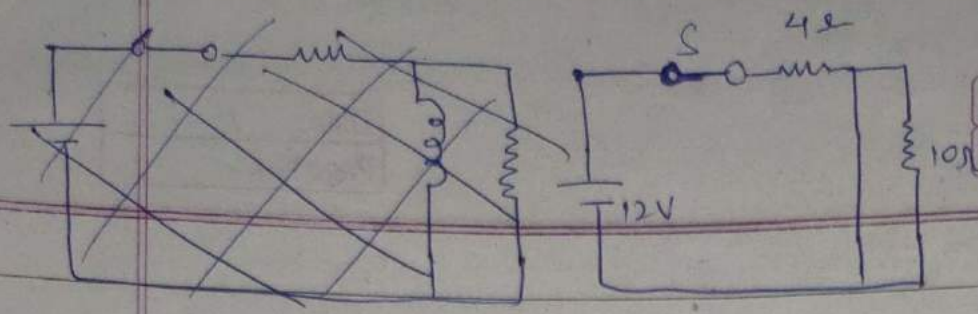
$$i_L(t) = -1.414 [e^{-0.586t} - e^{-3.414t}]$$

Que The 12V battery is disconnected at $t=0$. Find the inductor current and voltage as a function of time.

at steady state

inductor is short.





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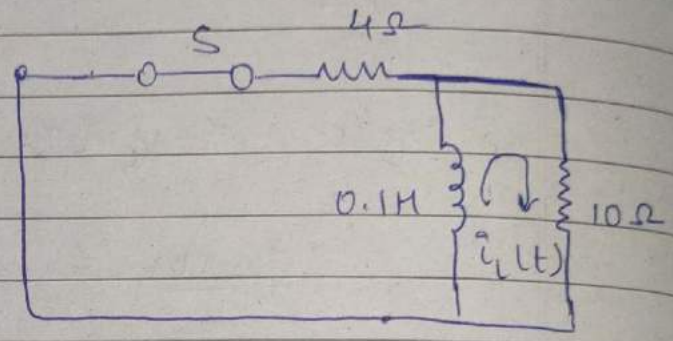
$$V_L(0^-) = 0$$

$$i_L(0^-) = \frac{12}{4} = 3A$$

When battery is removed at $t=0$



$$0.1\frac{di_L(t)}{dt} + 10 i_L(t) = 0$$

$$\frac{di_L(t)}{dt} + 100\, i_L(t) = 0$$

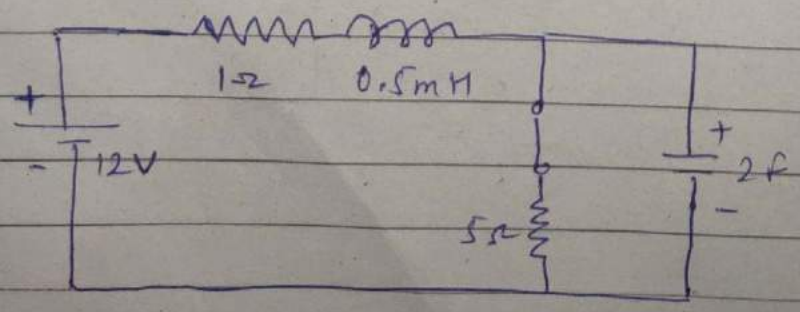
$$i_L(t) = K e^{-100t}$$

$$i_L(0) = 3$$

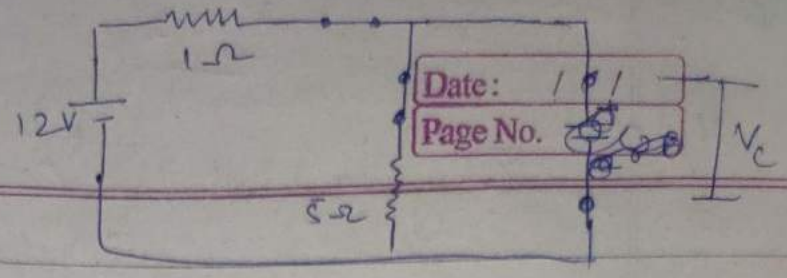
$$K = 3$$

$$i_L(t) = 3e^{-100t}$$

Ques The switching circuit of figure have been closed for a very long time. It opens at $t=0$. Find $v_c(t)$ for $t>0$ using differential equation approach.



Sol^n At steady state,



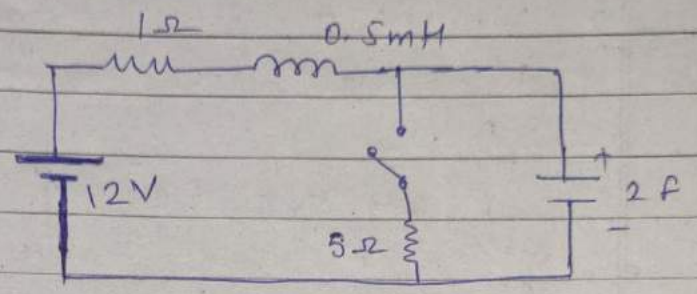
$i_L(0) = \frac{12}{6}$

$= 2A$

$V_c(0) = 5 \times i_L(0)$

$= 5 \times 2 = 10\,V$.

~~At~~ When switch is open at $t=0$



$$1 \cdot i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{2}\int_0^t i(t)\,dt + 10 = 12$$

diff. wrt t

$$0.5\frac{d^2i(t)}{dt} + \frac{di(t)}{dt} + \frac{1}{2}i(t) = 0$$

$$\frac{d^2i(t)}{dt} + 2\frac{di(t)}{dt} + i(t) = 0$$

~~$i(t) = K_1 e$~~ put $p = \frac{di(t)}{dt}$

$$p^2 + 2p + 1 = 0$$

$$p(p+1) + 1(p+1) = 0$$

$$(p+1)^2 = 0$$

~~$p = -1$~~ $p = -1, -1$

$$i(t) = (K_1 + K_2 t)\,e^{-t}$$

At $t = 0^+$ $\quad i(0^+) = \underline{\underline{K_1 = 2}}$.

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$$V_c(t) = \frac{1}{C}\int_{-\infty}^{0} i_0(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

$$V_c(0^+) = \frac{1}{C}\int_{-\infty}^{0} i_0(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

$$\cancel{10} = \cancel{10} + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

$$\int_{0}^{t} (2 + K_2 t)\cdot e^{-t}\Big|_{t=0^+} = 0$$

$$\boxed{K_2 = 2}$$

$$i(t) = (2 + 2t)\,e^{-t}$$

$$\cancel{V_c = V_c(t) = 10 + \frac{1}{2}\int_{0}^{t} (2+2t)e^{-t}\,dt}$$

$$\cancel{= 10 + \int_{0}^{t} e^{-t}\cdot dt + \int_{0}^{t} t\cdot e^{-t}\cdot dt}$$

$$\cancel{= 10 + \left[e^{-t}\right]_0^t + \left[t\,e^{-t} - e^{-t}\right]_0^t}$$

$$\cancel{= 10 + \left[e^{-t} - 1\right] + \left[t\,e^{-t} - e^{-t} - (-1)\right]}$$

$$\cancel{= 10 + e^{-t} - 1 + t\cdot e^{-t} - e^{-t} + 1}$$

$$\cancel{= 10 + t\,e^{-t}}$$

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$$V_c(t) = 10 + \frac{1}{2}\int_0^t (2+2t)\, e^{-t}\, dt$$

$$= 10 + \left[(1+t)\int_0^t e^{-t}\,dt - \int_0^t (1+2t)'\int_0^t e^{-t}\,dt\right]_0^t$$

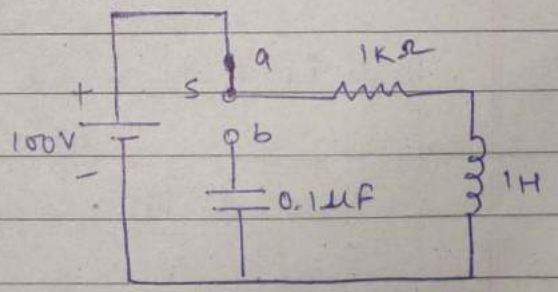
$$= 10 + \left[(1+t)(1-e^{-t}) - \int_0^t 1.(1-e^{-t})\,dt\right]_0^t$$

$$= 10 + \left[1 - e^{-t} + t - te^{-t} - (t + e^{-t} - 1)\right]$$

$$= 10 + 1 - e^{-t} + \cancel{t} - te^{-t} - \cancel{t} - e^{-t} + 1$$

$$V_c(t) = 12 - 2e^{-t} - te^{-t}.$$

Ques In the given circuit as shown in figure switch S is changed from position a to b at $t=0$. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0$.



Soln

Sol^n ~~before~~ at time $t<0$, circuit will be

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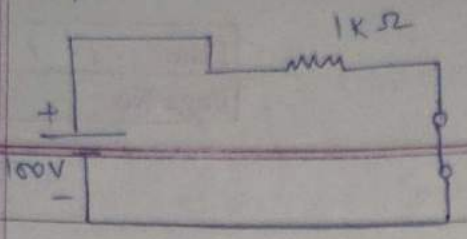
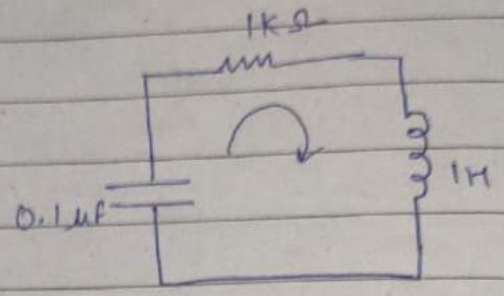


$$i(0^-) = \frac{V}{R} = \frac{100}{1000} = 0.1\,A$$

at time $t=0$,



$$i(t).R + L.\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt = 0$$

$$1000\,i(t) + \frac{di(t)}{dt} + \frac{1}{0.1\times10^{-6}}\int_0^t i(t)\,dt = 0 \quad -(1)$$

$\int_0^t i(t)\,dt = 0$ ↳ uncharged capacitor

~~diff. wrt t~~

$$\cancel{1000\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 10^7\,i(t) = 0}$$

$$\cancel{\frac{d^2i(t)}{dt^2} + \frac{di(t)}{dt} + 10^4\,i(t) = 0}$$

$$1000\,i(t) + \frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} = -1000\,i(t)$$

at $t=0$

$$\frac{di(0^+)}{dt} = -1000\,i(0) = -1000\times0.1$$
$$= -100\,A/s$$

$$\cancel{\frac{d^2 i(t)}{dt^2} = -1000 \frac{di(t)}{dt^2}}$$

$$\cancel{\frac{d^2 i(t)}{dt^2} = -1000 \times 1000}$$

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on diffrentiating eqn ①

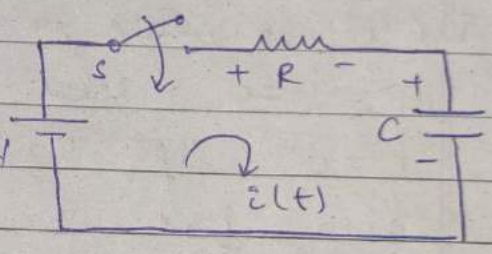
$$1000 \frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} + \frac{i(t)}{0.1 \times 10^{-6}} = 0$$

$$\frac{d^2 i(t)}{dt^2} = -1000 \frac{di(t)}{dt} - \frac{i(t)}{0.1 \times 10^{-6}}$$

$$= -1000 \frac{di(0^+)}{dt} - \frac{i(0^+)}{0.1 \times 10^{-6}}$$

$$= -1000 \times (-100) - (0.1) \times 10^{-7}$$

Ques In the circuit of the figure the switched S is closed at t=0 with the capacitor for uncharged. Find values for i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$ for element values as follows $V = 100V$, $R = 1000\ \Omega$ $C = 1\mu F$



Ans Applying KVL

$$V = R.i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

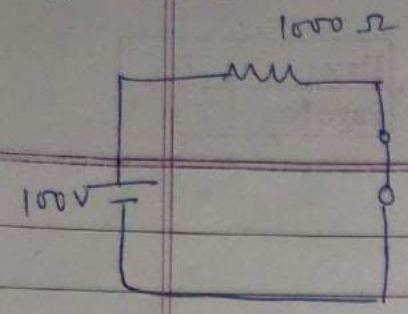
differentiating wrt t

$$0 = 1000 \frac{di(t)}{dt} + 10^6 . i(t) \quad - ①$$

again differentiating wrt t

$$0 = 1000 \frac{d^2 i(t)}{dt^2} + 10^6 \frac{di(t)}{dt} \quad - ③$$

at $t = 0^+$



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$$i(0^+) = \frac{V}{R}$$

$$= \frac{100}{1000} = 0.1A$$

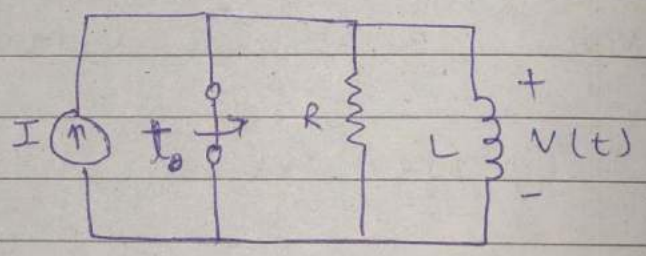
$$\frac{di(t)}{dt} = -1000\, i(t) = -1000 \times 0.1$$
$$= -100 \text{ A/sec}$$

$$\frac{d^2 i(t)}{dt^2} = -1000 \times \frac{di(t)}{dt}$$
$$= -1000 \times -100$$
$$= 10^5 \text{ A/sec}^2$$

Ques The circuit shown in figure has the switch S opened at $t=0$, solve for v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t=0^+$. If $I = 1A$, $R = 100\,\Omega$ and $L = 1H$ also find the expression for $i(t)$.

applying KCL

$$I = \frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$$



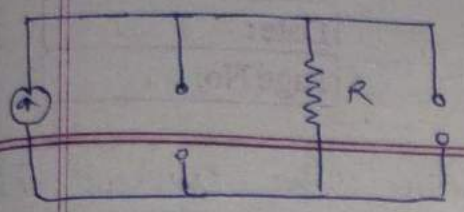
diff wrt t

$$0 = \frac{dv(t)}{dt} \cdot \frac{1}{R} + \frac{1}{L} \cdot v(t) \quad -(1)$$

again diff. wrt t

$$0 = \frac{1}{R}\frac{d^2 v(t)}{dt^2} + \frac{1}{L} \cdot \frac{dv(t)}{dt} \quad -(2)$$

at $t = 0^+$



$v(0^+) = IR$
$= 1 \times 100$
$= 100\,V$

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from ① $\dfrac{dv(t)}{dt} = -\dfrac{1}{L} \cdot v(t) \cdot R$

at $t = 0^+$

$$\frac{dv(0^+)}{dt} = -\frac{100}{1} \cdot 100 = -10^4\ V/sec$$

~~at t~~

from ② $\dfrac{d^2v(t)}{dt^2} = -\dfrac{R}{L}\dfrac{dv(t)}{dt}$

$$= -\frac{100}{1} \times -10^4\ V/sec^2$$

$$= 10^6\ V/sec^2$$

(II) LAPLACE TRANSFORM:

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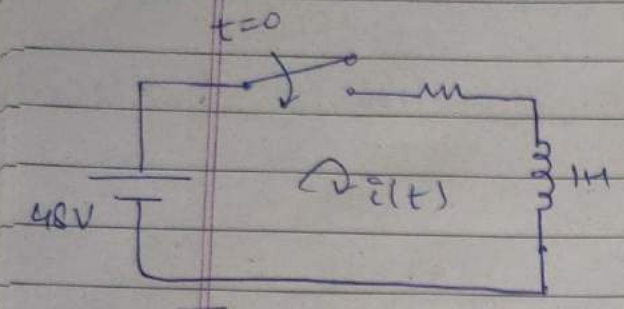
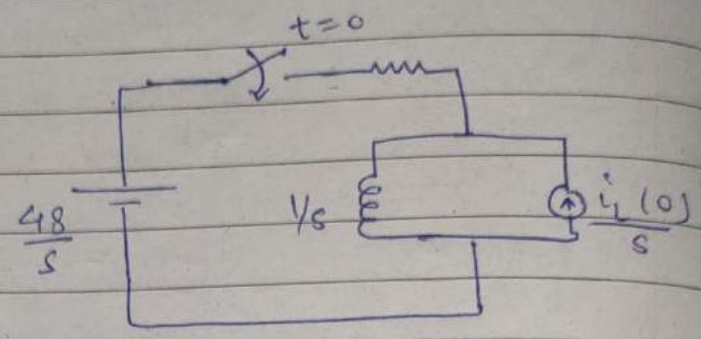
Que Assume initial current through inductor is 3 A using laplace transform determine the current $i(t)$ at $t>0$. Also draw domain representation of circuit. (R-L circuit). $R = 4\,\Omega$, $L = 1H$, $V = 48\,V$



Time Domain



Frequency Domain

Applying KVL,

$$4i(t) + L\frac{di(t)}{dt} = 48$$

Taking Laplace Transform

$$4I(s) + L[sI(s) - i_L(0^+)] = \frac{48}{s}$$

$$4I(s) + sI(s) = \frac{48}{s} + 3 \quad (i_L(0^+) = 3A \text{ given})$$

$$I(s) = \frac{48+3s}{(4+s)s}$$

Taking inverse laplace transform

$$\frac{48+3s}{(4+s)s} = \frac{A}{4+s} + \frac{B}{s}$$

$$As + 4B + Bs = 48 + 3s$$

$$A+B = 3, \quad 4B = 48, \quad B = 12$$

$$A = 3 - 12 = -9$$

$\frac{2}{s}$

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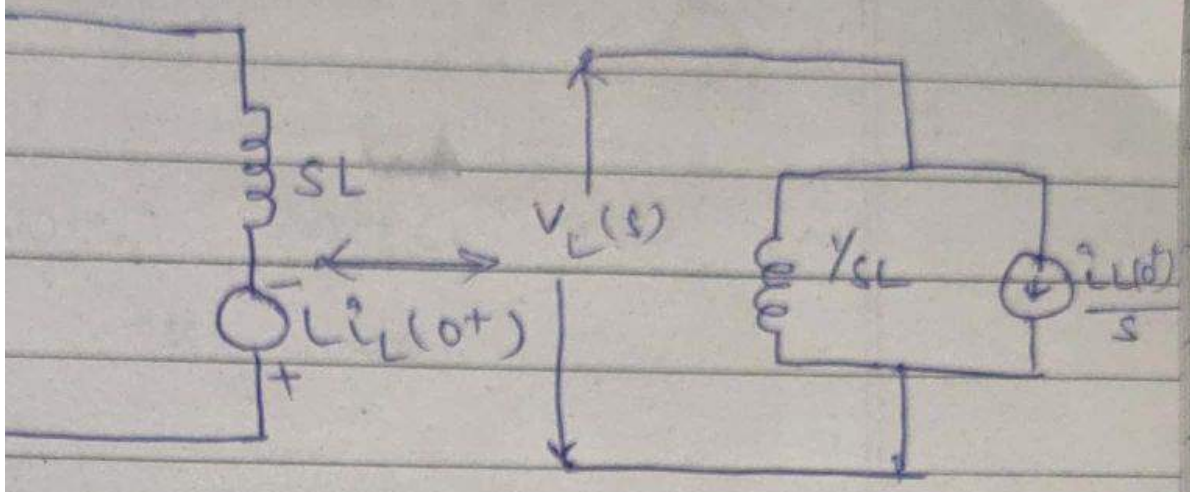
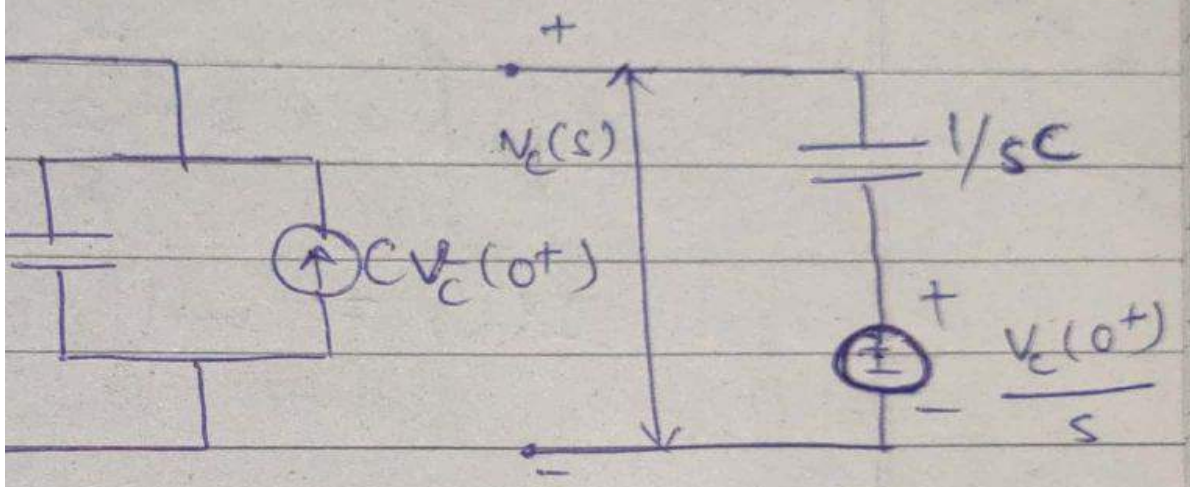
$\frac{1}{4+s}) + 12 L^{-1}(\frac{1}{s})$

12





circuit with the capacitor

Taking laplace transform.

$$R.I(s) + LI(s) - Li_L(0^+) + \frac{1}{sC} I(s) + \frac{V_C(0^+)}{s} = 0$$

$$2I(s) + s.I(s) - i_L(0^+) + \frac{2}{s} I(s) + \frac{1}{s} = 0$$

at the initial condition inductor act as open ckt.

$$\therefore 2I(s) + sI(s) + \frac{2}{s} I(s) + \frac{1}{s} = 0$$

$$I(s)\left[2 + s + \frac{2}{s}\right] = -\frac{1}{s}$$

$$I(s) = \frac{-1}{s} \cdot \frac{s}{[2s + s^2 + 2]}$$

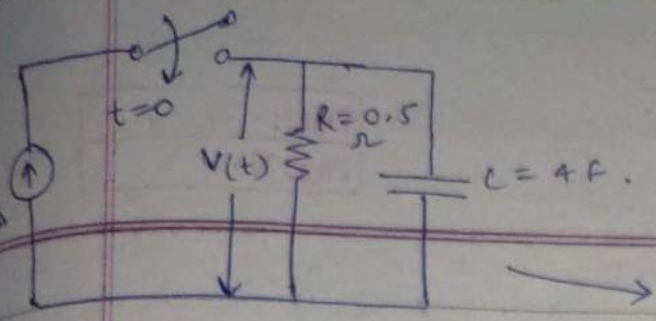
$$= \frac{-1}{(s^2 + 2s + 2)}$$

$$I(s) = \frac{-1}{(s+1)^2 + 1^2}$$

after taking inverse laplace.

$$i(t) = e^{-t} \sin t$$

Ques Consider the R-C parallel circuit shown in fig. with $R = 0.5\,\Omega$ and $C = 4F$ excited by d.c. current source of 10A. Determine the voltage across capacitor by applying laplace transformation. Assume the initial voltage across the capacitor as 2V. also draw s domain transformation.



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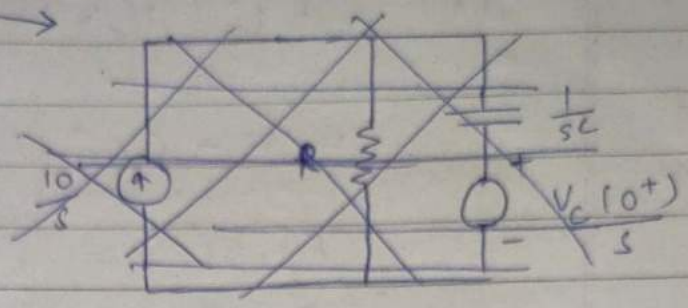
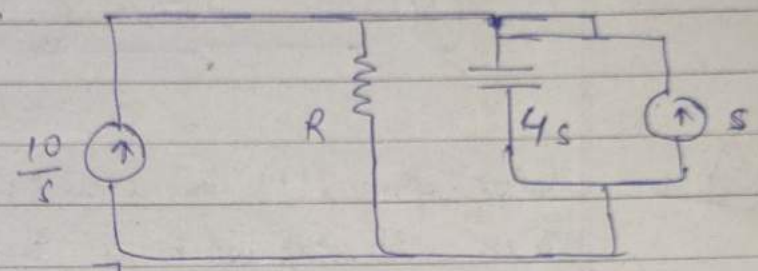


Applying KCL :

$$\frac{V(t)}{R} + C \cdot \frac{dv(t)}{dt} = 10$$

taking laplace transform.



$$\cancel{\frac{V(s)}{R} + \frac{V(s)}{sC} +}$$

$$\frac{V(s)}{R} + C\left[sV(s) - V(0^+)\right] = \frac{10}{s}$$

$$\frac{V(s)}{0.5} + 4s\,V(s) - 4\times 2 = \frac{10}{s}$$

$$2V(s) + 4s\,V(s) = \frac{10+8s}{s}$$

$$V(s) = \frac{10+8s}{s(2+4s)} = \frac{5+4s}{s(1+2s)}$$

using partial fraction :

$$\cancel{V(s)} = \frac{5+4s}{s(1+2s)} = \frac{A}{s} + \frac{B}{1+2s}$$

$$\Rightarrow 5+4s = A + 2As + Bs$$

$\therefore$ $2A + B = 4$ and $A = 5$

$B = 4 - 2\times 5 = 4 - 10$

$B = -6$

$$V(s) = \frac{5}{s} + \frac{-6}{1+2s}$$

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taking inverse laplace

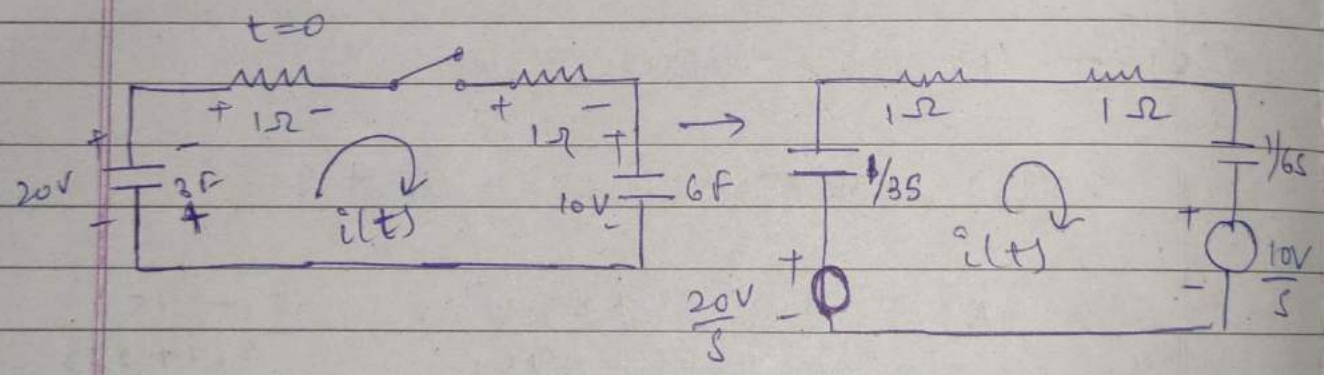
$$V(t) = 5L^{-1}(1/s) - 6L^{-1}\left(\frac{1}{1+2s}\right)$$

$$= 5 - 6L^{-1}\left[\frac{1}{2(0.5+s)}\right]$$

$$= 5 - 3L^{-1}\left[\frac{1}{s+0.5}\right]$$

$$\boxed{V(t) = 5 - 3e^{-0.5t}}$$

Ques Solve for $i(t)$ in circuit as shown in fig. in which 3F capacitor is initially charged to 20V, 6F capacitor to 10V and switch is closed at $t=0$. Also draw transformed circuit.



applying KVL, we get.

$$-20 + \frac{1}{C_1}\int_0^t i(t)\,dt + i(t).1 +$$

$$i(t).1 + 10 + \frac{1}{C_2}\int_0^t i(t)\,dt = 0$$

taking laplace transformation.

$$-\frac{20}{s} + \frac{1}{3s} \cdot I(s) + 2 \cdot I(s) + \frac{10}{s} + \frac{1}{6s} I(s) = 0$$

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$$I(s)\left[\frac{1}{3s} + 2 + \frac{1}{6s}\right] = \frac{10}{s}$$

$$I(s) = \frac{10}{s} \cdot \frac{6s}{2 + 12s + 1}$$

$$I(s) = \frac{60}{12s + 3} = \frac{20}{4s + 1}$$

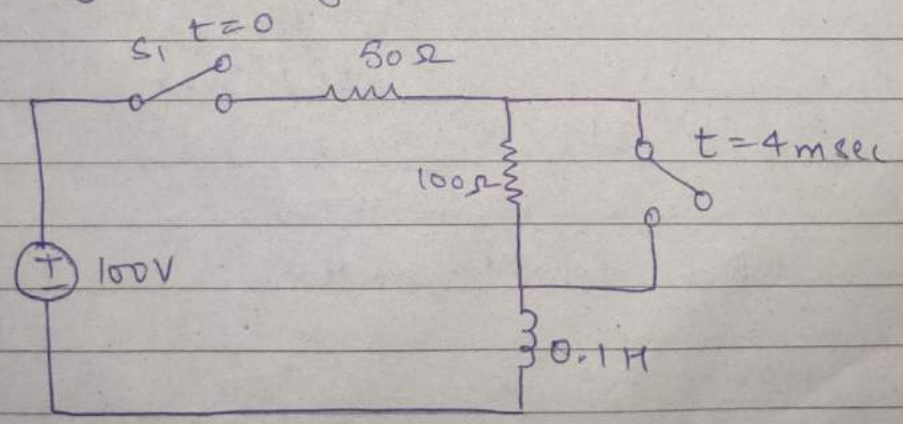
$$I(s) = \frac{5}{s + 1/4}$$

taking inverse laplace.

$$i(t) = 5\, L^{-1}\left[\frac{1}{s + 1/4}\right]$$

$$i(t) = 5\, e^{-t/4}$$

Que In the circuit S_1 is closed at $t=0$ and S_2 is opened at $t = 4$ msec. Determine $i(t)$ for $t > 0$. [Assume inductor is initially ~~de-energ~~ de-energised].



Ans Case 1: when $0 < t < 4$ msec

In this time interval both the switches will be in closed state.

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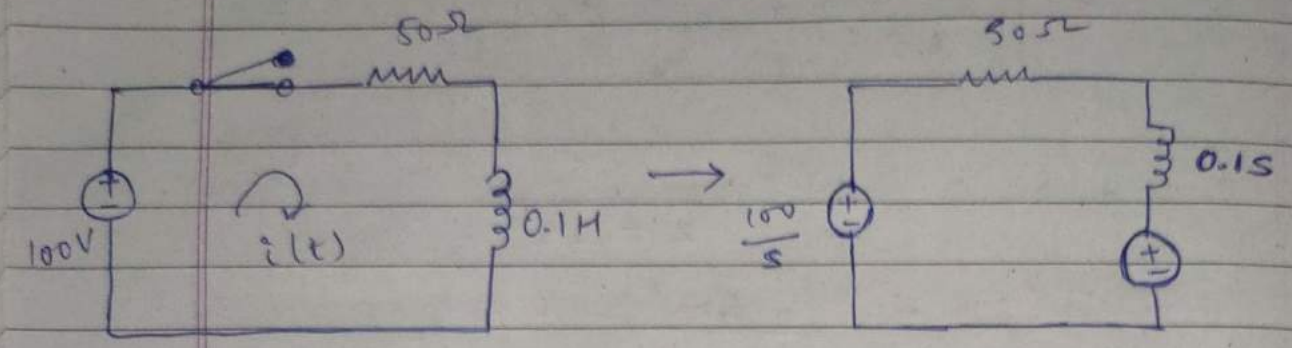

$$i(t).50 + 0.1\frac{di(t)}{dt} = 100$$

taking laplace transform.

$$50\,I(s) + 0.1\left[s\,I(s) - i_L(0^-)\right] = \frac{100}{s}$$

$$50\,I(s) + 0.1\,s\,I(s) = \frac{100}{s}$$

$$I(s)\left[50 + 0.1s\right] = \frac{100}{s}$$

$$I(s)\left[500 + s\right] = \frac{1000}{s}$$

$$I(s) = \frac{1000}{s} \times \cancel{[500+s]}\,\frac{1}{(500+s)}$$

$$= \cancel{\frac{1000}{s(+500s}}$$

$$I(s) = \frac{1000}{s(500+s)}$$

taking partial fraction.

$$\frac{1000}{s(500+s)} = \frac{A}{s} + \frac{B}{s+500}$$

$$1000 = As + 500As + Bs$$

$\Rightarrow$ ~~A0 + B~~ $A + B = 0$ $\quad 500A = 1000$

$A = 2$

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$\Rightarrow B = -A$

$\boxed{B = -2}$

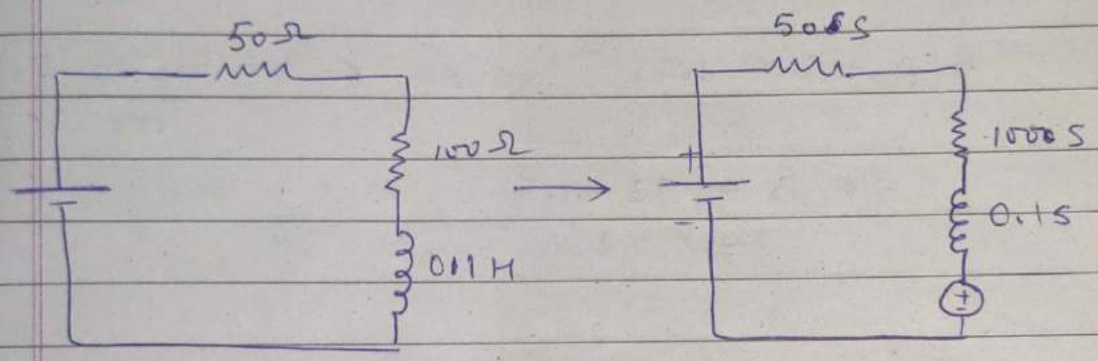
$$I(s) = \frac{2}{s} - \frac{2}{500+s}$$

$$= 2\,L^{-1}\left[\frac{1}{s}\right] - 2\,L^{-1}\left[\frac{1}{s+500}\right]$$

$$= 2 - 2e^{-500t}$$

$$i(t) = 2\left[1 - e^{-500t}\right] A \quad \text{—(1)}$$

case 2: when $4\,msec < t < \infty$ then ckt will be



using (1) $i(4msec) = 2\left[1 - e^{-500(4\times10^{-3})}\right]$

$= 1.73\,A$

applying KVL in ckt.

if $t' = t - 4msec$

then for $t > 4msec$

then $0 \le t' < \infty$

~~$50\,i_2(t') + 100\,i_2(t')$~~

$$50\,i_2(t') + 100\,i_2(t') + 0.1\frac{d\,i_2(t')}{dt} = 100$$

taking laplace

$$150\,I(s) + 0.1\left[s\,I(s) - i_L(4msec)\right] = \frac{100}{s}$$

$$150\,I(s) + 0.1\,s\,I(s) - 0.1\times1.73 = \frac{100}{s}$$

$$I(s)\left[150 + 0.1s\right] = \frac{100 + 0.173s}{s}$$

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$$I(s)\left[1500 + s\right] = \frac{1000 + 1.73s}{s}$$

$$I(s) = \frac{1000 + 1.73s}{s\left[1500 + s\right]}$$

taking ~~inverse laplace~~ Partial fraction

$$\cancel{I(s) =} \frac{1000 + 1.73.s}{s\left[1500 + s\right]} = \frac{A}{s} + \frac{B}{1500 + s}$$

$$\therefore \quad 1000 + 1.73s = 1500A + As + Bs$$

$$A + B = 1.73 \quad \text{and} \quad A = \frac{1500}{1000} = 1.5$$

$$B = 1.73 - 1.5$$
$$= 0.23$$

$$I(s) = \frac{1.5}{s} + \frac{0.23}{s + 1500}$$

take inverse laplace

$$i(t') = 1.5\, L^{-1}\left(\frac{1}{s}\right) + 0.23\, L^{-1}\left[\frac{1}{1500 + s}\right]$$

$$i(t') = 1.5 + 0.23\, e^{-1500t'}$$

$$i(t) = 1.5 + 0.23\, e^{-(1500)(t - 4\times10^{-3})}$$
$$= 1.5 + 0.23\, e^{-1500t + 6}$$
$$= 1.5 + 0.23 \times e^{6} \times e^{-1500t}$$
$$= 1.5 + 92.7\, e^{-1500t}$$

Ques A circuit shown in fig the switch S is changed from position a to position b at time $t=0$. Find out an expression for current $i(t)$, $\frac{di(t)}{dt}$ and $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$

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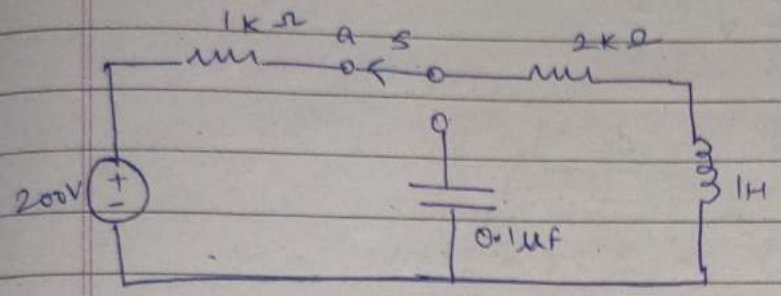
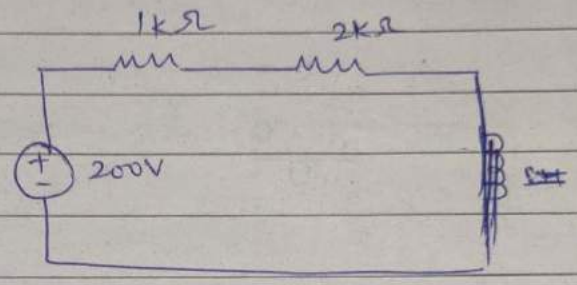


When $t<0$

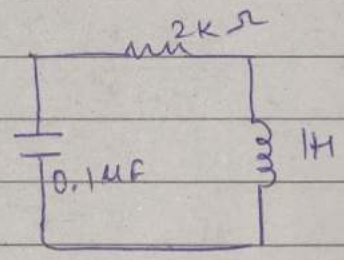
inductor will be short.



$$i(0^-) = \frac{V}{R} = \frac{200}{1000+2000}$$

$$= \frac{200}{3000} = \frac{1}{15} \text{ A}$$

When $t \geq 0$



$$R\,i(t) + L\cdot\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 0$$

$$2000\,i(t) + \frac{di(t)}{dt} + \frac{1}{0.1\times10^{-6}}\int_{0}^{t} i(t)\,dt + 0 = 0 \quad \text{—(1)}$$

$$\int_0^t i(t)\,dt = 0$$

diff wrt t

$$\cancel{2000\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} + 10^7\, i(t) = 0}$$

$$\cancel{\frac{d^2 i(t)}{dt^2} + 2\frac{di(t)}{dt} + 10^4\, i(t) = 0}$$

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~~Put~~ $\cancel{\frac{di(t)}{dt} = P}$

$\cancel{\Rightarrow P^2 + P + 5000P = 0}$

$\cancel{P = -1 \pm \sqrt{1 -}}$

$$2000\, i(t) + \frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} = -2000\, i(t)$$

at $t = 0$

$$\frac{di(0)}{dt} = -2000 \times i(0)$$

$$= -2000 \times \frac{1}{15} = -133.34 \text{ A/sec}$$

differentiating eq. ① wrt t

$$\frac{d^2 i(t)}{dt^2} + 2\frac{di(t)}{dt} + 10^4\, i(t) = 0$$

$$\frac{d^2 i(t)}{dt^2} = -2\frac{di(t)}{dt} - 10^4\, i(t)$$

at $t = 0$

$$\frac{d^2 i(0)}{dt^2} = -2 \times \frac{di(0)}{dt} - 10^4 \times i(t)$$

$$= -2 \times -133.34 - 10^4 \times \frac{1}{15}$$

$$= 266.68 - 666.67$$

$$= -399.99 \text{ A/sec}^2$$

$$\approx -400 \text{ A/sec}^2$$

Ques In the R-L circuit shown in figure, the switch ~~the~~ is in position 1 long enough to establish steady state condition and at t=0 it is switched to position 2. Find the resulting current using classical method.

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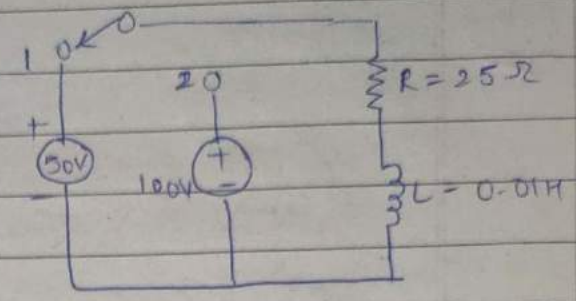
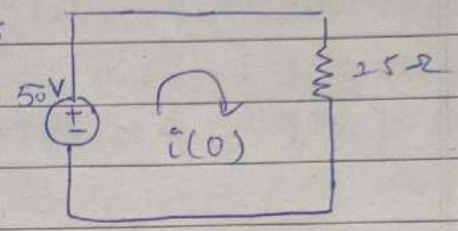


Ans ~~at~~ When $t < 0$ steady state is attained and at this state the inductor get short. ∴ ckt will be

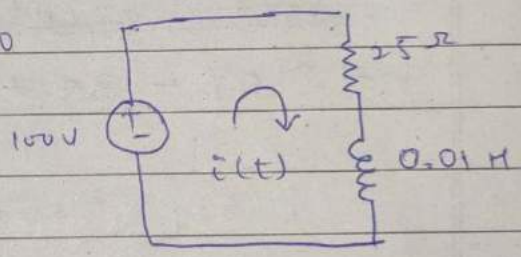
$$i(0) = \frac{V}{R} = \frac{50}{25} = 2A$$



at $t = 0$ when switch position is changed.

$$25\,i(t) + 0.01 \cdot \frac{di(t)}{dt} = 100$$

$$25\,i(t) + \frac{1}{100}\frac{di(t)}{dt} = 100$$



$$\frac{di(t)}{dt} + 2500\,i(t) = 10^4$$

$$i(t) = \frac{10^4}{2500} + K e^{-2500t}$$

$$i(t) = 4 + K e^{-2500t}$$

$$i(0) = 4 + K = 2$$

$$K = 2 - 4$$

$$K = -2$$

$$i(t) = 4 - 2e^{-2500t}$$

$$= 2\left[2 - e^{-2500t}\right]$$

Ques Using laplace transformation, solve the differential equation:

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 8i = 8u(t)$$

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given that $i(0^+) = 3$ and $\frac{di}{dt}(0^+) = -4$

Ans given that,

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 8i = 8u(t)$$

taking laplace tranform

$$L\left[\frac{d^2 i}{dt^2}\right] + 4L\left[\frac{di}{dt}\right] + 8L[i] = 8u(t)$$

$$[s^2 i(s) - s i(0^+) - i'(0^+)] + 4[s i(s) - i(0^+)]$$

$$+ 8[i(s)] = \frac{8}{s}$$

~~$s^2 i(s) - s i(0^+$~~

$$s^2 i(s) - 3s + 4 + 4s i(s) - 12 + 8 i(s) = \frac{8}{s}$$

$$i(s)[s^2 + 4s + 8] - 8 = \frac{8}{s} + 3s$$

$$i(s)[s^2 + 4s + 8] = \frac{8 + 3s^2}{s} + 8$$

$$i(s) = \frac{3s^2 + 8s + 8}{s(s^2 + 4s + 8)}$$

taking partial fraction, we get.

$$\frac{3s^2 + 8s + 8}{s(s^2 + 4s + 8)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 8}$$

$$3s^2 + 8s + 8 = As^2 + 4As + 8A + Bs^2 + Cs$$

$\Rightarrow \quad A + B = 3 \qquad 4A + C = 8$

$8A = 8$

$\Rightarrow A = 1$

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$B = 3 - 1 = 2 \qquad C = 8 - 4 \times 1 = 4$

$$i(s) = \frac{1}{s} + \frac{2s+4}{s^2+4s+8} \cancel{+ \frac{4}{s^2+4s+8}}$$

$$i(s) = \frac{1}{s} + \frac{2s+4}{(s+2)^2+4} \cancel{+ \frac{4}{(s+2)^2+4}}$$

taking inverse laplace transform.

$$i(t) = L^{-1}\left[\frac{1}{s}\right] + 2L^{-1}\left[\frac{s+2}{(s+2)^2+2^2}\right] \cancel{+ 4L^{-1}\left[\frac{1}{(s+2)^2+2^2}\right]}$$

$$i(t) = u(t) + 2 \cdot e^{-2t} \cdot \cos 2t$$

UNIT - III

TWO PORT NETWORK

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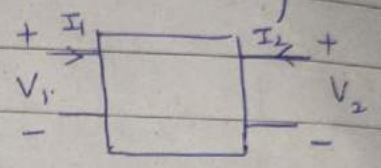
→ Consists of two ports
→ Each port having two terminals - one for entry and other for exit.
→ For a port, the value of current or voltage at entry is equal to that at the exit.

→ There are 4 variables V_1, V_2, I_1 and I_2 in two port network out of these we can choose two variables as independent and another two as dependent. So, we will get six possible pairs of equations, in which we represent dependent variables in terms of independent variables. The coefficients of these independent variables are called PARAMETERS



(I) Z - PARAMETER : V_1 and V_2 are dependent and I_1 and I_2 are independent also called OPEN - CIRCUIT IMPEDENCE PARAMETERS.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$, Input driving-point impedance

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$, Forward transfer impedance

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$, Reverse transfer impedance

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$, output driving-point impedance.

(II) Y- PARAMETERS : I_1 and I_2 are ~~dependent~~ dependent and V_1 and V_2 are independent. Also called as SHORT - CIRCUIT ADMITTANCE PARAMETERS

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$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{and} \quad I_2 = Y_{21}V_1 + Y_{22}V_2$$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$, Input driving point admittance

$Y_{12} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$, Forward transfer admittance

$Y_{21} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$, Reverse Transfer admittance

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$, Output driving point admittance.

(III) T- PARAMETERS : V_1 and I_1 are dependent and V_2 and I_2 are independent. Also called TRANSMISSION PARAMETER or ABCD PARAMETERS

$$V_1 = AV_2 - BI_2 \quad \text{and} \quad I_1 = CV_2 - DI_2$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$, reverse voltage ratio

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, transfer admittance

$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$, transfer impedance

$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$, reverse current ratio.

(IV) T'-PARAMETERS: V_2 and I_2 are dependent and V_1 and I_1 are independent and their coefficients are ~~IVER~~ INVERSE TRANSMISSION PARA.

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or A'B'C'D' PARAMETERS

$V_2 = A'V_1 + B'(-I_1)$ $\quad$ $I_2 = C'V_1 + D'(-I_1)$

$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$, forward voltage ratio

$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$ transfer admittance

$B' = \left.\frac{-V_2}{I_1}\right|_{V_1=0}$, transfer impedance

$D' = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$, forward current ratio

(V) h-parameters: In these equations V_1 & I_2 are dependent V_2 and I_1 are independent. Also called HYBRID PARAMETER.

$V_1 = h_{12} V_2 + h_{11} I_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{22} V_2 + h_{21} I_1 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$, input impedance

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$, forward current gain

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$, reverse voltage gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$, output admittance.

(iii) h'- parameters: In these equations I_1 & V_2 are dependent and I_2 and V_1 are independent.

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Also called INVERSE HYBRID PARAMETER or g- parameter

$$I_1 = g_{11} V_1 + g_{12} I_2 \qquad V_2 = g_{21} V_1 + g_{22} I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$, input admittance

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$, forward voltage gain

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ reverse current gain

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ Output impedance

RECIPROCITY THEORAM: In a linear bilateral network the ratio of response to excitation is constant even though the source is changed from I/P to O/P terminal.

INPUT IMPEDENCE: If a load ~~resistance~~ impedance is connected to output port, then ratio of input voltage to input current is called input impedance.

$$Z_{ip} = \frac{V_1}{I_1} = Z_1$$

OUTPUT IMPEDENCE: If a load impedance (Z_L) is connected to input port then ratio of output voltage to output current is called output impedance

SYMMETRIC COND^n: A two port network is said to be symm. if ports can be interchanged without changing the ports voltages and current.

IMAGE IMPEDENCE: If impedance at input port with impedance Z_{i2} connected across output port be Z_{i1} and impedance at output port with impedence Z_{i1} connected across input port be Z_{i2}, the Z_{i1} and Z_{i2} are IMAGE IMPEDANCES when $Z_{i1} = Z_{i2}$, called characteristics or iterative impedance.

~~DRIVING POINT IMPEDANCE: It is output of a system when we feed an~~

DRIVING POINT IMPEDANCE: It is ~~imp~~ input impedance in frequency domain s. or
Ratio of laplace transform of voltage and current measured at port under initial condition.

IMPULSE RESPONSE: It is output of a system when we feed an impulse as input.

TRANSFER FUNCTION: It is the ratio of the output of system to the input of a system in frequency domain considering initial condition and equilibrium point to be zero.
It is the laplace transform of impulse response
2nd order transfer function.

$$\boxed{T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

ω_n → Natural frequency

ζ → damping factor.

$\omega_n^2 = 1/LC$

$\omega_n = 1/\sqrt{LC}$

$2\zeta\omega_n = R/L$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

NATURAL FREQUENCY : It is a frequency at which a system tends to oscillate in the absence of any driving or damping ~~factor~~ force or it is the frequency at which system resonates.

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DAMPING FACTOR | DAMPING RATIO : Defines the decay in oscillation in a system after some time. Dimensionless.

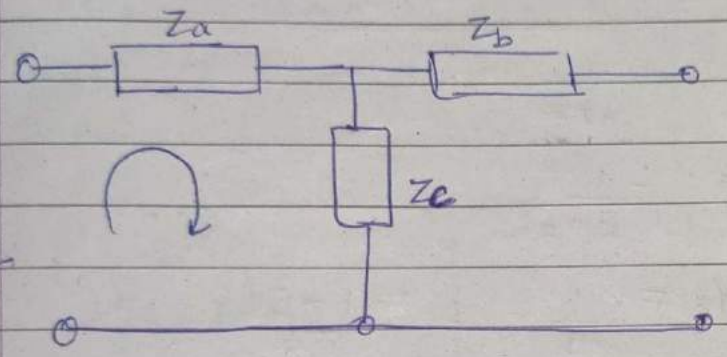
$\zeta = 0 \rightarrow$ ~~underdamped~~ undamped

$\zeta > 0 \rightarrow$ overdamped

$\zeta < 0 \rightarrow$ underdamped

$\zeta = 1 \rightarrow$ critically damped.

(1)

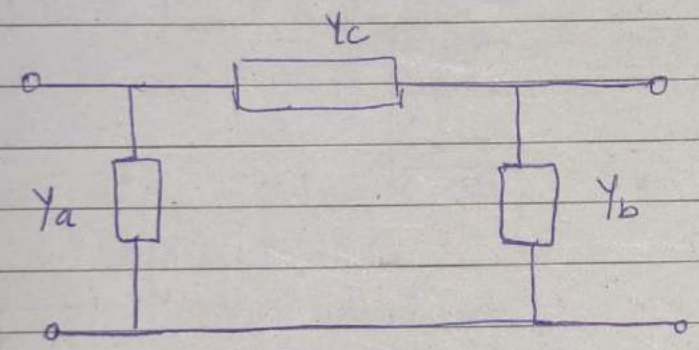


$Z_{11} = Z_a + Z_c$

$Z_{12} = Z_c$

$Z_{21} = Z_c$

$Z_{22} = Z_b + Z_c$



$Y_{11} = Y_a + Y_c$

$Y_{12} = -Y_c$

$Y_{21} = -Y_c$

$Y_{22} = Y_b + Y_c$

$[Z] = [Y]^{-1}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$Y_{11} = \frac{Z_{22}}{\Delta Z}$ $\quad Y_{12} = \frac{-Z_{21}}{\Delta Z}$

$Y_{21} = \frac{-Z_{21}}{\Delta Z}$ $\quad Y_{22} = \frac{Z_{11}}{\Delta Z}$

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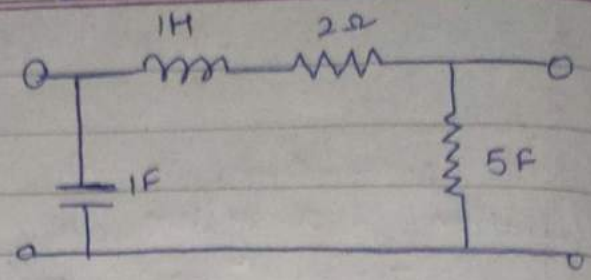
Ques Determine Y parameter



$Y_a = s$

$Y_b = 1/5$

$\cancel{Y_c = \frac{1}{s} + \frac{1}{2}}$ $\quad Y_c = \frac{1}{s+2}$

$Y_{11} = Y_a + Y_c = s + \frac{1}{s+2} = \frac{s^2+2s+1}{(s+2)}$

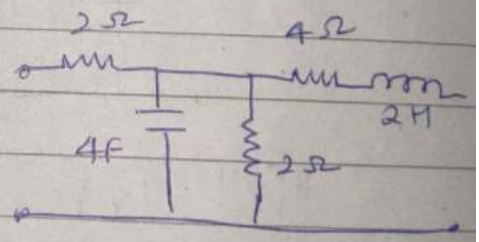
$Y_{12} = -Y_c = -\frac{1}{s+2}$

$Y_{21} = -Y_c = \frac{-1}{s+2}$

$Y_{22} = Y_b + Y_c = \frac{1}{5} + s = \frac{1+5s}{5}$

$$Y = \begin{bmatrix} \frac{s^2+2s+1}{s+2} & \frac{-1}{s+2} \\ \frac{-1}{s+2} & \frac{1+5s}{5} \end{bmatrix}$$

Q Obtain Z parameter of the network



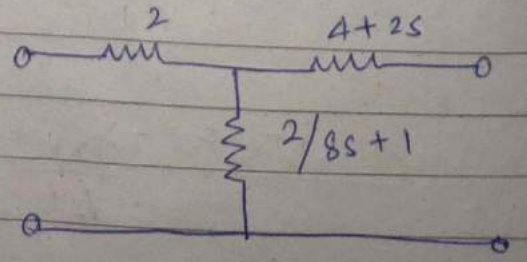
$Z_a = 2$

$Z_b = 4 + 2s$

$\cancel{Z_c = \frac{1}{\frac{1}{4s} + \frac{1}{2}} = 4s + \frac{1}{2}}$

$Z_c = \frac{1}{1/4s + 1/2} = \frac{1}{4s + 1/2} = \frac{2}{8s+1}$

$Z_{11} = Z_a + Z_c = 2 + \frac{2}{8s+1}$



$= \frac{16s+2+2}{8s+1} = \frac{16s+4}{8s+1}$

$Z_{12} = Z_c = \frac{2}{8s+1}$ $\quad Z_{21} = Z_c = \frac{2}{8s+1}$

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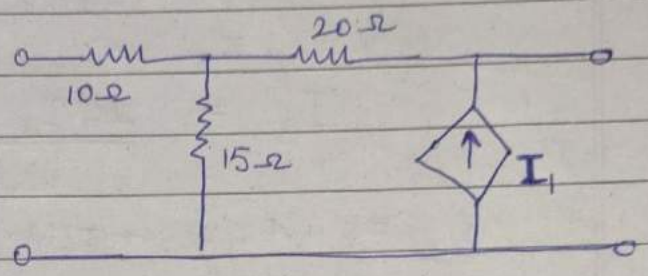
$Z_{22} = Z_b + Z_c = 4 + 2s + \frac{2}{8s+1}$

$= \frac{32s + 4 + 16s^2 + 2s + 2}{8s+1} = \frac{16s^2 + 34s + 6}{8s+1}$

$$[Z] = \begin{bmatrix} \frac{16s+4}{8s+1} & \frac{2}{8s+1} \\ \frac{2}{8s+1} & \frac{16s^2+34s+6}{8s+1} \end{bmatrix}$$

Q Determine Z parameter of network shown in fig.

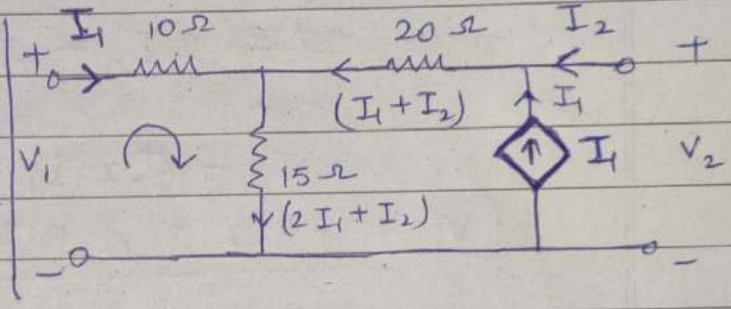


Ans

$V_1 = 10I_1 + 15(2I_1 + I_2)$

$= 10I_1 + 30I_1 + 15I_2$

$V_1 = 40I_1 + 15I_2$



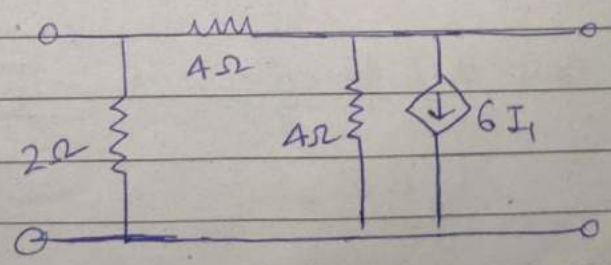
$V_2 = 20(I_1 + I_2) + 15(2I_1 + I_2)$

$= 20I_1 + 20I_2 + 30I_1 + 15I_2$

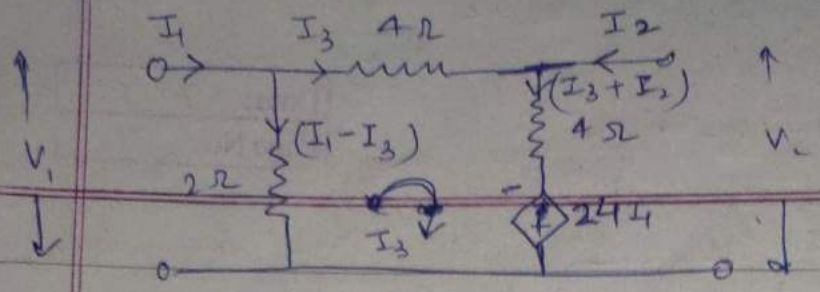
$V_2 = 50I_1 + 35I_2$

$$Z = \begin{bmatrix} 40 & 15 \\ 50 & 35 \end{bmatrix}$$

Q Calculate Z-parameter.



Ans Current source will be converted to equivalent Voltage source.



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$$V_1 = 2(I_1 - I_3)$$

$$24I_1 = 4I_3 - 2(I_1 - I_3) + 4(I_3 + I_2)$$
$$24I_1 = 4I_3 - 2I_1 + 2I_3 + 4I_3 + 4I_2$$
$$24I_1 + 2I_1 - 4I_2 = 10I_3$$

$$I_3 = \frac{26I_1 - 4I_2}{10}$$

$$V_2 = 4(I_3 + I_2) - 24I_1$$

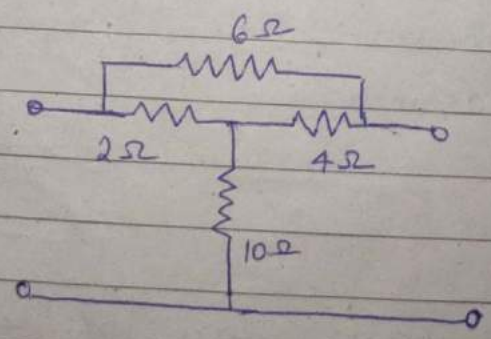
$$V_1 = 2I_1 - 2\times[2.6I_1 - 0.4I_2]$$

$$V_1 = 2I_1 - 5.2I_1 + 0.8I_2$$
$$[V_1 = -3.2I_1 + 0.8I_2]$$

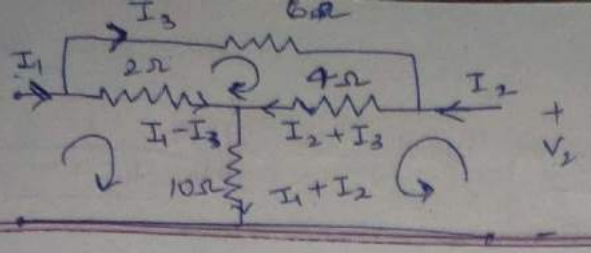
$$V_2 = 4[2.6I_1 - 0.4I_2] - 24I_1$$
$$= -24I_1 + 10.4I_1 - 1.6I_2$$
$$[V_2 = -13.6I_1 - 1.6I_2]$$

$$[Z] = \begin{bmatrix} -3.2 & 0.8 \\ -13.6 & -1.6 \end{bmatrix}$$

Q Obtain open ckt. parameter



Ans



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$$V_1 = 2(I_1 - I_3) + 10(I_1 + I_2)$$
$$V_1 = 2I_1 - 2I_3 + 10I_1 + 10I_2$$
$$V_1 = 12I_1 + 10I_2 - 2I_3 \quad -(1)$$

$$V_2 = 4(I_2 + I_3) + 10(I_1 + I_2)$$
$$= 4I_2 + 4I_3 + 10I_1 + 10I_2$$
$$= 10I_1 + 14I_2 + 4I_3 \quad -(2)$$

$$0 = 6I_3 + 4(I_2 + I_3) - 2(I_1 - I_3)$$
$$0 = 6I_3 + 4I_2 + 4I_3 - 2I_1 + 2I_3$$
$$2I_1 - 4I_2 = 12I_3$$
$$\frac{I_1}{6} - \frac{I_2}{3} = I_3 \quad -(3)$$

put I_3 in (1) & (2)

$$V_1 = 12I_1 + 10I_2 - 2\left[\frac{I_1}{6} - \frac{I_2}{3}\right]$$
$$= 12I_1 + 10I_2 - \frac{I_1}{3} + \frac{2I_2}{3}$$
$$= \frac{36I_1 - I_1}{3} + \frac{30I_2 + 2I_2}{3}$$
$$\boxed{V_1 = \frac{35}{3}I_1 + \frac{32}{3}I_2} \quad (5)$$

$$V_2 = 10I_1 + 14I_2 + 4\left[\frac{I_1}{6} - \frac{I_2}{3}\right]$$
$$= 10I_1 + 14I_2 + \frac{2I_1}{3} - \frac{4I_2}{3}$$
$$= \frac{30I_1 + 2I_1}{3} + \frac{42I_2 - 4I_2}{3}$$
$$\boxed{V_2 = \frac{32}{3}I_1 + \frac{38}{3}I_2} \quad (6)$$

from eq. (5) & (6)

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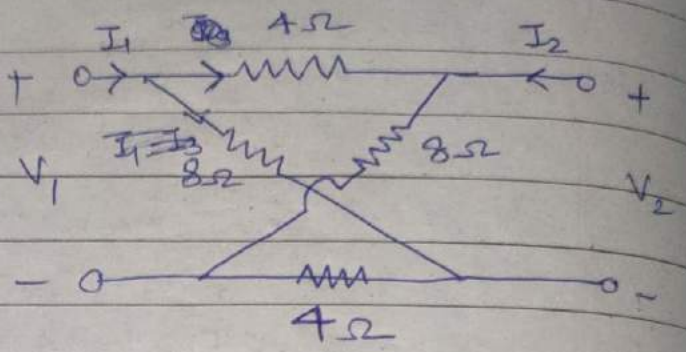
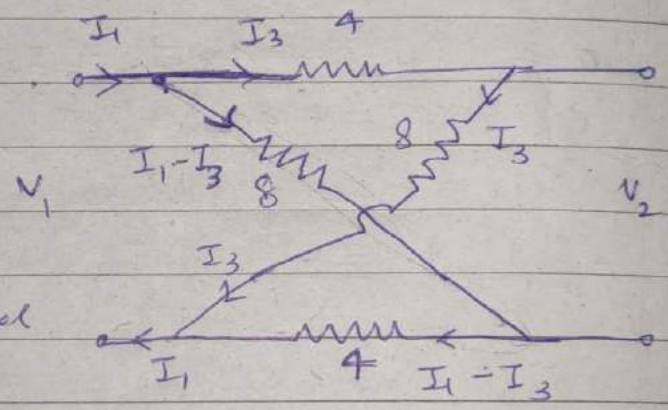
$$[Z] = \begin{bmatrix} \frac{35}{3} & \frac{32}{3} \\ \frac{32}{3} & \frac{38}{3} \end{bmatrix}$$

Ques For Lattice Network find the Z-parameter.



Ans Case 1: when $I_2 = 0$

$$V_1 = 8(I_1 - I_3) + 4(I_1 - I_3)$$



Since both branches are parallel and have equal resistance. $\therefore I_3 = I_1/2$

$$V_1 = \frac{8I_1}{2} + \frac{4I_1}{2} = 6I_1$$

$$V_2 = \frac{8I_1}{2} + \frac{4I_1}{2} = 6I_1$$

Case 2: when $I_1 = 0$

$$V_1 = \frac{8I_2}{2} + \frac{4I_2}{2} = 6I_2$$

$$V_2 = \frac{8I_2}{2} + \frac{4I_2}{2} = 6I_2$$

$$[Z] = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$$

Lattice Network

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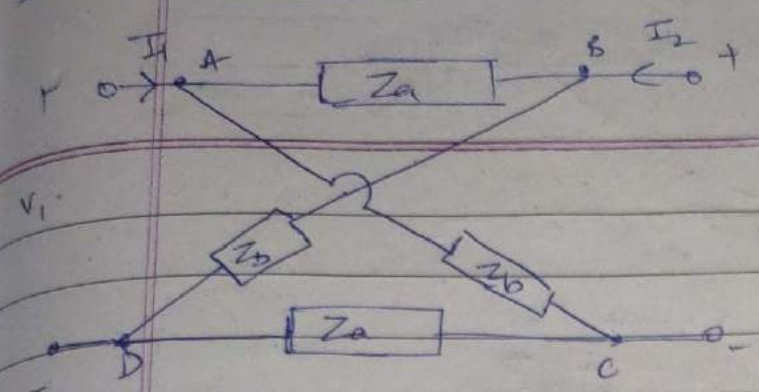
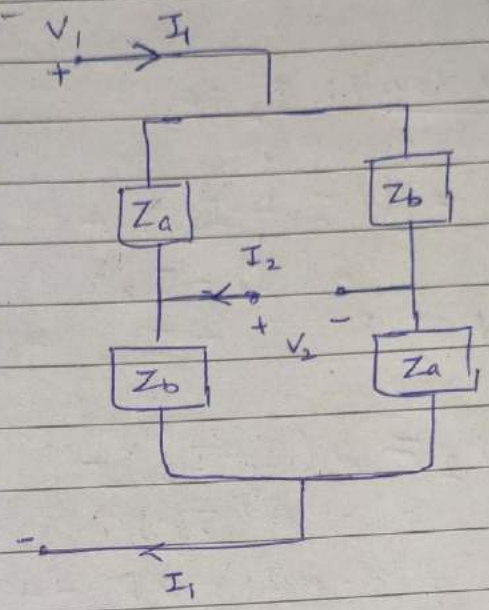


$I_2 = I_4$

Can be redrawn as:-



$$V_1 = \frac{I_1}{2} Z_a - \frac{I_1}{2} \cdot Z_b = 0$$

$$\boxed{\frac{V_1}{I_1} = \frac{Z_a - Z_b}{2}}$$

$$V_2 + \frac{I_1}{2} Z_a - \frac{I_1 Z_b}{2} = 0$$

$$\boxed{\frac{V_2}{I_1} = \frac{Z_b - Z_a}{2}}$$

Calculation of Z_{12} & Z_{22}

$$V_1 + Z_a \cdot \frac{I_2}{2} - \frac{I_2}{2} Z_b = 0$$

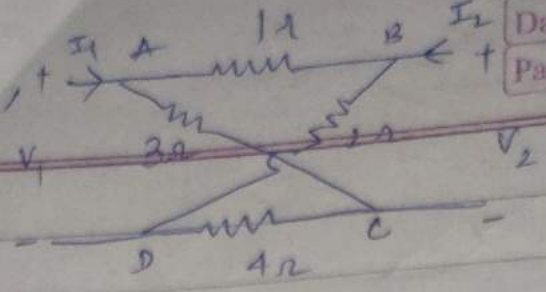
$$\boxed{\frac{V_1}{I_2} = \frac{Z_b - Z_a}{2}}$$

$$V_2 = Z_a \times \frac{I_2}{2} - \frac{I_2}{2} Z_b = 0$$

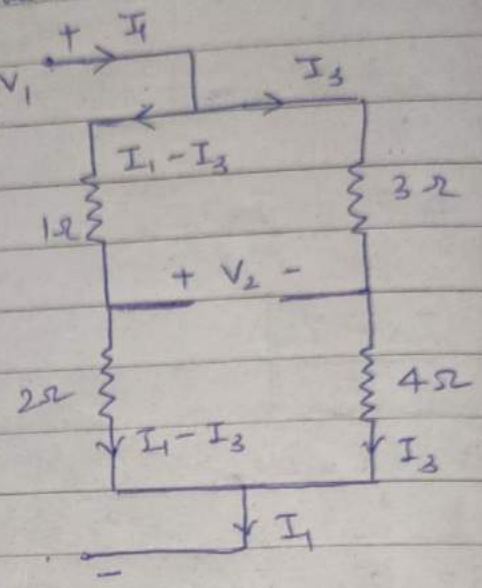
$$\boxed{\frac{V_2}{I_2} = \frac{Z_a + Z_b}{2}}$$

$$[Z]_{2\times2} = \begin{bmatrix} \frac{Z_a + Z_b}{2} & \frac{Z_b - Z_a}{2} \\ \frac{Z_b - Z_a}{2} & \frac{Z_a + Z_b}{2} \end{bmatrix}$$

Q find Z parameter,



Ans The above ckt can be redrawn as



For case 1: $I_2 = 0$

$$V_1 = 1(I_1 - I_3) + 2(I_1 - I_3)$$

$$V_1 = 3(I_1 - I_3) \quad -(1)$$

$$V_2 = 2(I_1 - I_3) - 4(I_3)$$

$$= 2I_1 - 2I_3 - 4I_3$$

$$V_2 = 2I_1 - 6I_3 \quad -(2)$$

$$0 = 3I_3 + 4I_3 - 2(I_1 - I_3) - 1(I_1 - I_3)$$

$$= 7I_3 - 2I_1 + 2I_3 - I_1 + I_3$$

$$3I_1 = 10I_3$$

$$I_3 = \frac{3}{10} I_1$$

$$V_1 = 3\left(I_1 - \frac{3}{10} I_3\right) = \frac{3(10I_1 - 3I_1)}{10}$$

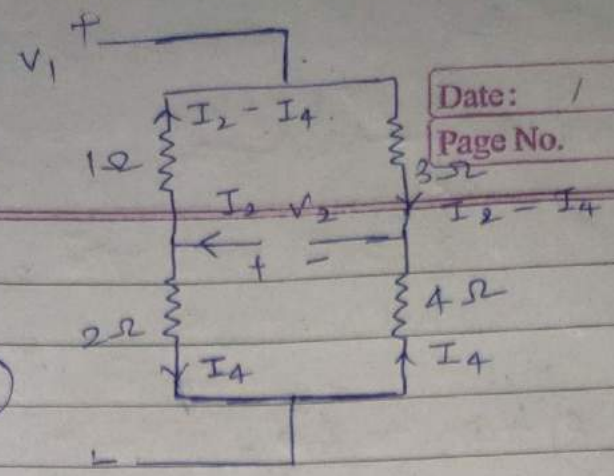
$$V_1 = 3 \times \frac{7I_1}{10}$$

$$\Rightarrow \frac{V_1}{I_1} = \frac{21}{10} = 2.1 = Z_{11}$$

$$V_2 = 2I_1 - 6 \times \frac{3}{10} I_1 = \frac{20I_1 - 18I_1}{10}$$

$$\frac{V_2}{I_1} = \frac{2}{10} = 0.2 = Z_{21}$$

for case 2: When $I_1 = 0$



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$V_1 = -1(I_2 - I_4) + 2I_4$

$V_1 = -I_2 + I_4 + 2I_4$

$V_1 = 3I_4 - I_2$ — (3)

$V_2 = 1(I_2 - I_4) + 3(I_2 - I_4)$

$= 4(I_2 - I_4)$ — (4)

$0 = 1(I_2 - I_4) + 3(I_2 - I_4) - 4I_4 - 2I_4$

$0 = 4(I_2 - I_4) - 6I_4$

$0 = 4I_2 - 4I_4 - 6I_4$

$10I_4 = 4I_2$

$I_4 = \frac{4}{10} I_2$ — (5)

$V_1 = 3 \times \frac{4}{10} I_2 - I_2 = \frac{12I_2 - 10I_2}{10}$

$\frac{V_1}{I_2} = \frac{2}{10} = 0.2 = Z_{12}$

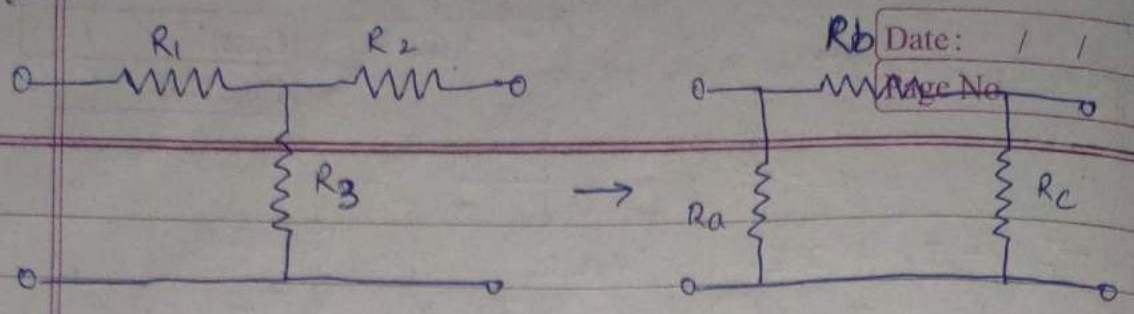
$V_2 = 4\left(I_2 - \frac{4}{10} I_2\right) = 4\left(\frac{10I_2 - 4I_2}{10}\right)$

$V_2 = 4 \times \frac{6I_2}{10}$

$\frac{V_2}{I_2} = \frac{24}{10} = 2.4 = Z_{22}$

$\therefore [Z] = \begin{bmatrix} 2.1 & 0.2 \\ 0.2 & 2.4 \end{bmatrix}$

(*) STAR to Δ :

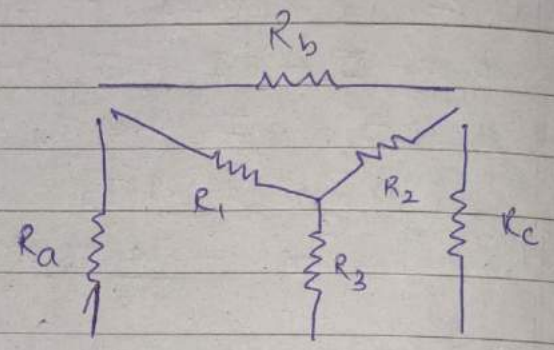


$$R_a = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_b = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_c = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

(*) Δ to STAR :

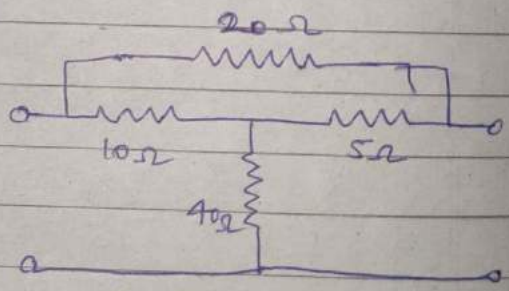


$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b \cdot R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a \cdot R_c}{R_a + R_b + R_c}$$

Ques Find Short circuit parameters.



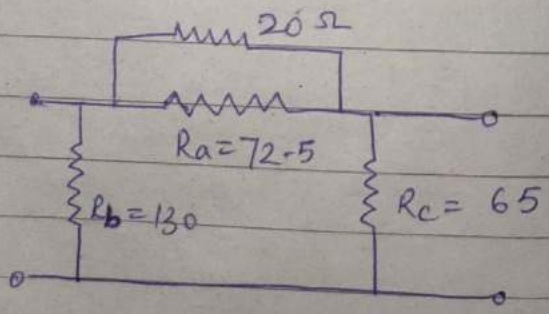
Ans Taking Y to Δ transformation.

$$R_a = 10 + 50 + \frac{10 \times 50}{40}$$

$$= 60 + \frac{\cancel{500}^{25}}{\cancel{40}_{2}}$$

$$= 60 + 12.5$$

$$R_a = 72.5$$



$R_b = 10 + 40 + \frac{\cancel{10}^{2} \times 40}{\cancel{5}}$ $\quad R_c = 40 + 5 + \frac{40 \times 5}{10}$

$= 50 + 80 = 130$ $\quad = 45 + 20$

$R_c = 65$

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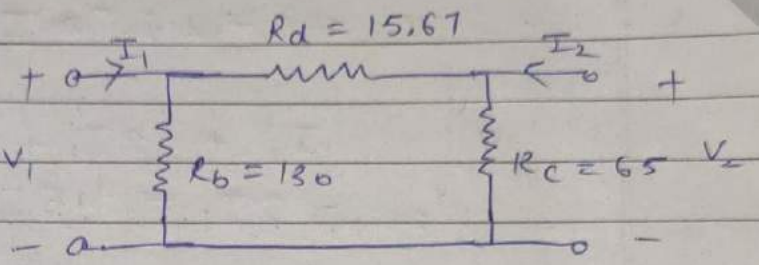
$$R_d = \frac{1}{\frac{1}{20} + \frac{1}{72.5}} = \frac{1450}{72.5 + 20} = \frac{1450}{92.5} = 15.67$$



$Y_{11} = R_b + R_d$

$= 130 + 15.67$

$= 145.67$

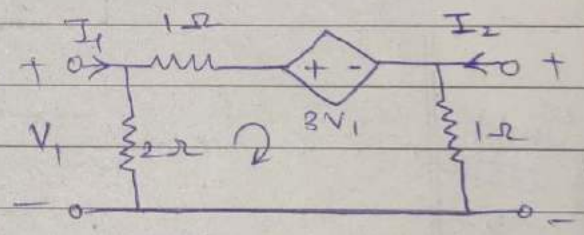
$Y_{21} = -R_d = -15.67$

$Y_{12} = -R_d = -15.67$

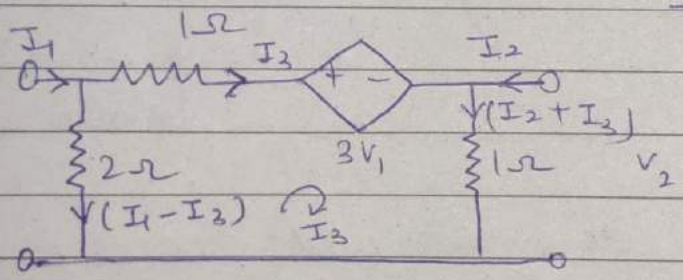
$Y_{22} = R_c + R_d = 65 + 15.67 = 80.67$

$$[Y] = \begin{bmatrix} 145.67 & -15.67 \\ -15.67 & 80.67 \end{bmatrix}$$

Que Find Y and Z parameter for the network



Ans



$V_1 = 2(I_1 - I_3)$ — ①

$V_2 = 1(I_2 + I_3)$ — ②

$I_3 \cdot 1 + 3V_1 + 1(I_2 + I_3) - 2(I_1 - I_3) = 0$

$I_3 + 3V_1 + I_2 + I_3 - 2I_1 + 2I_3 = 0$

$3V_1 = 2I_1 - I_2 - 4I_3$

from ①

$3(2)(I_1 - I_3) = 2I_1 - I_2 - 4I_3$

$4I_3 = 2I_1 - I_2 - 6(I_1 - I_3)$

$4I_3 = 2I_1 - I_2 - 6I_1 + 6I_3$

$4I_3 - 6I_3 = -4I_1 - I_2$

$$-2I_3 = -4I_1 - I_2$$

$$I_3 = 2I_1 + 0.5I_2 \quad -(3)$$

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$$V_1 = 2(I_1 - 2I_1 - 0.5I_2)$$

$$= 2(-I_1 - 0.5I_2)$$

$$V_1 = -2I_1 - I_2$$

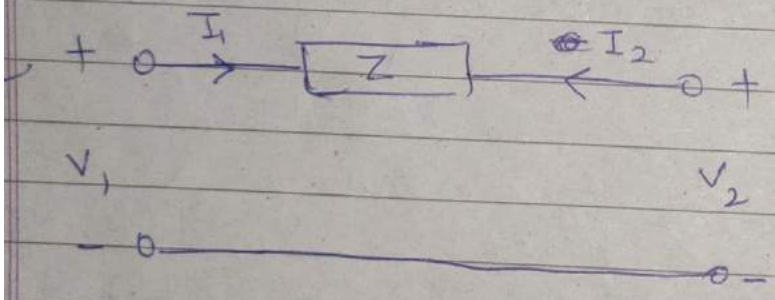
$$V_2 = I_2 + 2I_1 + 0.5I_2$$

$$V_2 = 2I_1 + 1.5I_2$$

$$\cancel{[Z] = \begin{bmatrix} -2 & -01 \\ 1.5 & 2 \end{bmatrix}} \quad [Z] = \begin{bmatrix} -2 & -1 \\ 2 & 1.5 \end{bmatrix}$$

$$[Y] = [Z]^{-1}$$

$$[Y] = \begin{bmatrix} -1.5 & -1 \\ 2 & 2 \end{bmatrix}$$

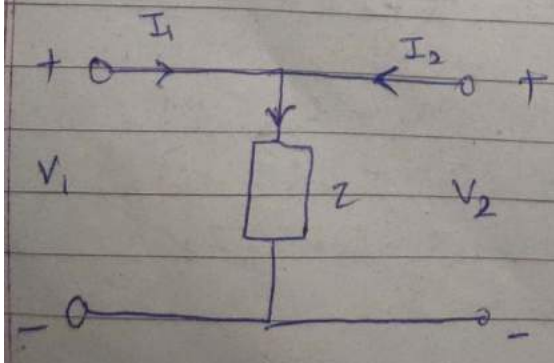


$I_1 = -I_2$

Dependent Variable $= I_1$ & I_2

Z parameter does not exist or

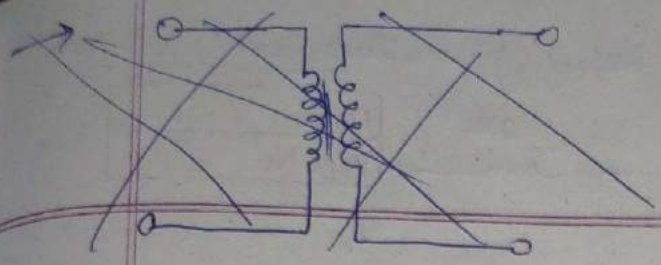
$$Z = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$



$$V_1 = Z(I_1 + I_2)$$

$$V_2 = Z(I_1 + I_2)$$

$$Z = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$



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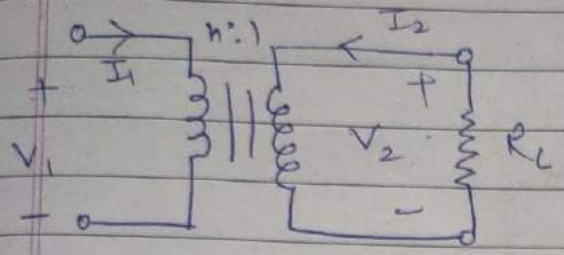
→ 

$$Z = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

~~as I_1 and I_2 are~~

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{0} = \infty$$

$$\frac{V_1}{V_2} = \frac{n}{1}$$

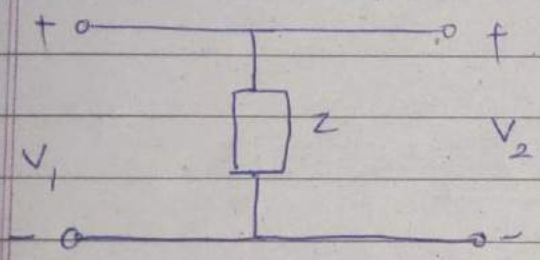
$$-\frac{I_2}{I_1} = \frac{n}{1}$$

$$\left\{\frac{I_2}{I_1} = \frac{n}{1}\right\}$$

as I_1 and I_2 are dependent

Illy, $\frac{V_1}{V_2} = \frac{n}{1}$ and V_1 and V_2 are dependent variable

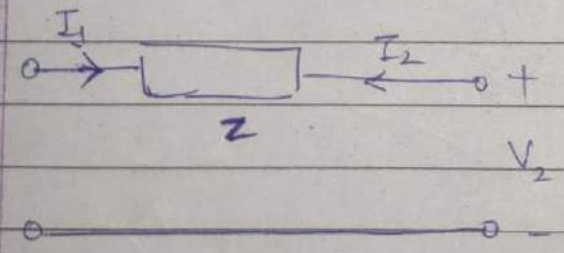
Y does not exist. $\frac{I_1}{I_2} = \frac{1}{n}$

→ 

$V_1 = V_2$

D.V = V_1, V_2

$$[y] = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

→ 

$$I_1 = \frac{V_1 - V_2}{Z}$$

$$I_1 = \frac{V_1}{Z} - \frac{V_2}{Z} \quad -(1)$$

$Y_{11} = 1/Z \quad Y_{12} = -1/Z$

$$I_2 = \frac{V_2 - V_1}{Z} = \frac{-V_1}{Z} + \frac{V_2}{Z}$$

$$[Y] = \begin{bmatrix} 1/Z & -1/Z \\ -1/Z & 1/Z \end{bmatrix}$$

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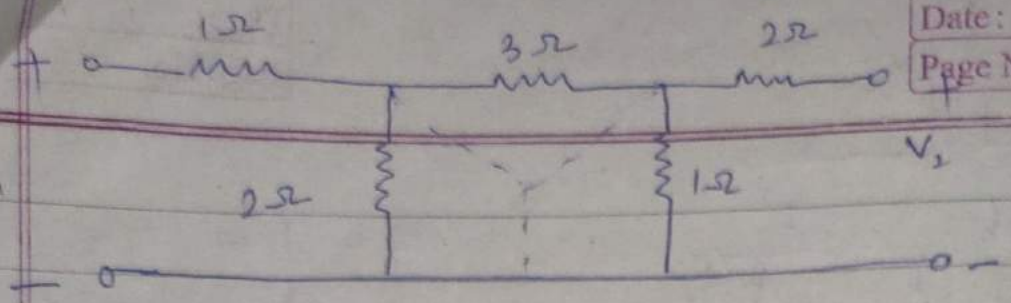
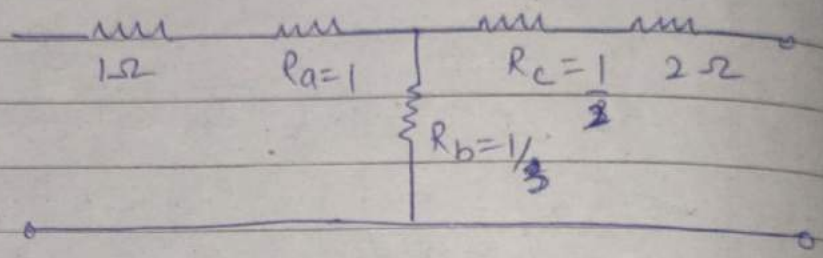
Ques Find Z parameter of the below given two port network.



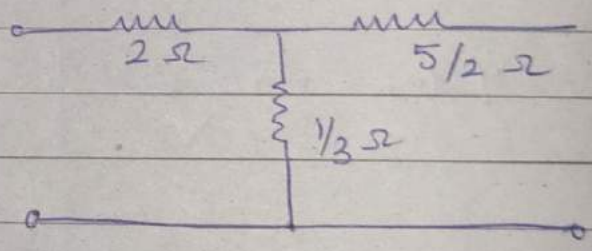
Ans using Δ to Y conversion,



$$R_a = \frac{2 \cdot 3}{2+3+1} = \frac{6}{6} = 1$$

$$R_b = \frac{2 \times 1}{2+3+1} = \frac{1}{3}$$

$$R_c = \frac{3 \times 1}{2+3+1} = 1/2$$



$$Z_{11} = Z_a + Z_c$$
$$= 2 + 1/3 = 7/3$$

$$Z_{12} = Z_c = 1/3 = Z_{21}$$

$$Z_{22} = Z_b + Z_c = \frac{5}{2} + \frac{1}{3} = \frac{15+2}{6} = \frac{17}{6}$$

$$[Z] = \begin{bmatrix} 7/3 & 1/3 \\ 1/3 & 17/6 \end{bmatrix}$$

Z-Parameter Model or Open circuit model

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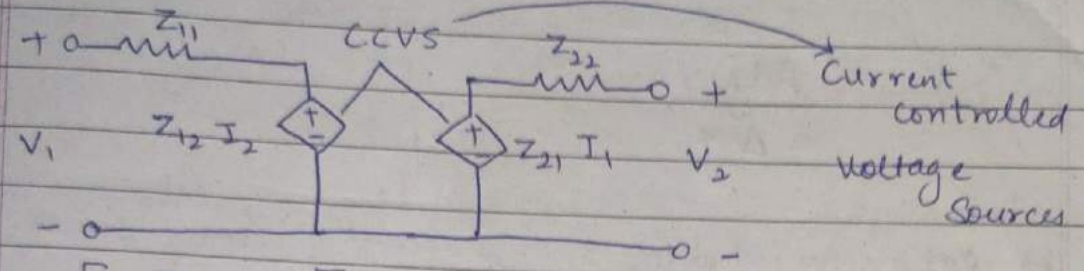
$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ① I/P KVL

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ② O/P KVL



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1}$$

$$[V]_{2\times1} = [Z]_{2\times2} [I]_{2\times1}$$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\text{Adj } Z = \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

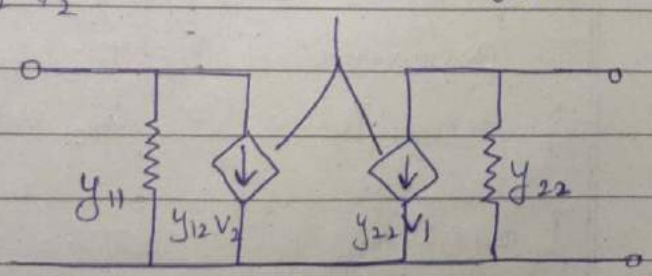
$$Z^{-1} = \frac{\text{Adj } Z}{\Delta Z} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

Admittance Model or Y Parameter Model :

I/P KCL : $I_1 = Y_{11} V_1 + Y_{12} V_2$

O/P KCL : $I_2 = Y_{21} V_1 + Y_{22} V_2$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



$$[I]_{2\times1} = [Y]_{2\times2} [V]_{2\times1}$$

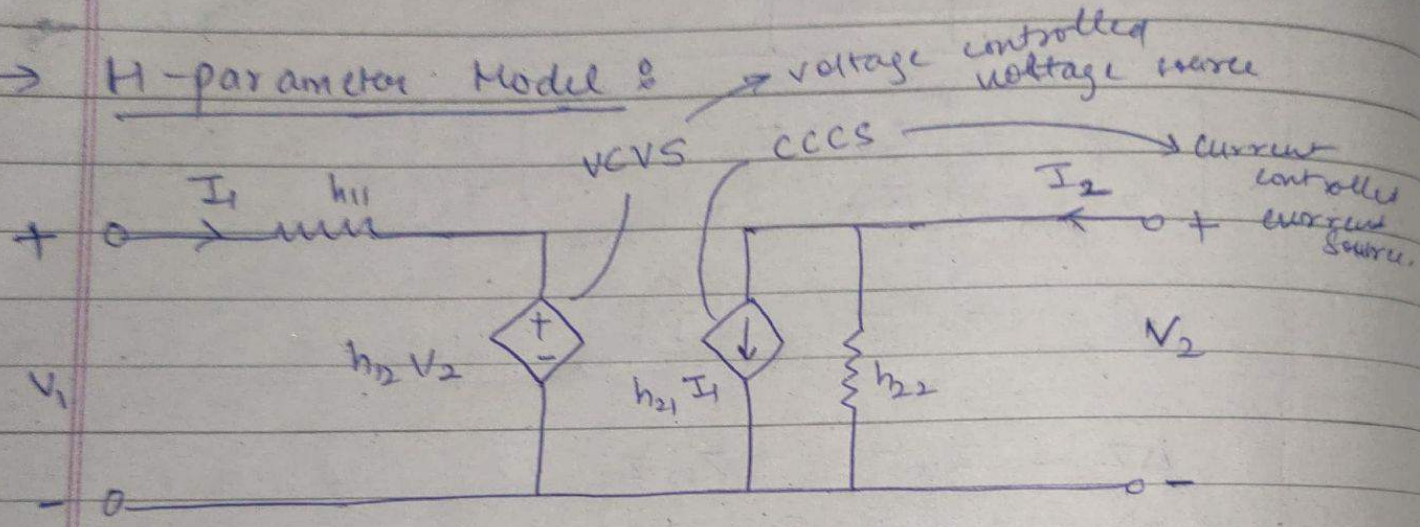
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

$$\text{Adj } Y = \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$[Y]^{-1} = \frac{\text{Adj } Y}{\Delta Y}$$

→ **H-parameter Model :**



$V_1 = h_{11}I_1 + h_{12}V_2$ (I/P KVL) → Z para

$I_2 = h_{21}I_1 + h_{22}V_2$ (O/P KCL) → Y-para

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \qquad \Delta h = h_{11}h_{22} - h_{12}h_{21}$$

→

Parameter	Reciprocity condⁿ	Symmetry condⁿ
Z-parameter	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y-parameter	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h-parameter.	$h_{12} = h_{21}$	$\Delta h = 1$
T-parameter	$\Delta T = 1$	$A = D$
g-parameter	$g_{12} = -g_{21}$	$\Delta g = 1$
T' parameter	$\Delta T' = 1$	$A' = D'$

NOTE: Electrical Modelling of ABCD parameter is not possible because both eqn are related with I/P

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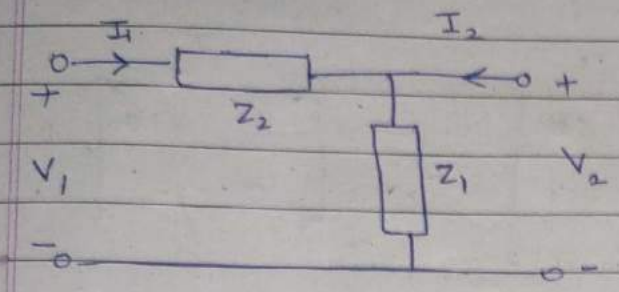

$$V_1 = Z_2 I_1 + Z_1(I_2 + I_1)$$
$$V_1 = Z_2 I_1 + Z_1 I_1 + Z_1 I_2$$
$$V_1 = I_1(Z_1 + Z_2) + I_2 Z_1 \quad (1)$$

$$V_2 = Z_1(I_1 + I_2)$$
$$V_2 = Z_1 I_1 + Z_1 I_2 \quad (2)$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_2 I_1 + Z_1 I_1}{Z_1 I_1}$$
$$= 1 + \frac{Z_2}{Z_1}$$

$$\cancel{= \frac{Z_2 \times 0 + Z_1 I_2}{Z_1 I_2}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{Z_1 I_1} = \frac{1}{Z_1}$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} = \frac{-(Z_2 I_1 + Z_1(I_2 + I_1))}{I_2}$$

$$= -\frac{Z_2 I_1}{I_2} - \frac{Z_1 \cancel{I_2}}{\cancel{I_2}} - \frac{Z_1 I_1}{I_2}$$

$$= -\frac{I_1}{I_2}\left[Z_1 + Z_2\right] - Z_1$$

from (2) $\quad 0 = Z_1 I_1 + Z_1 I_2$

$$Z_1 I_2 = -Z_1 I_1$$
$$I_2 = -I_1$$
$$1 = -\frac{I_1}{I_2}$$

$$C = 1\left[Z_1 + Z_2\right] - Z_1 = Z_2$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = 1 \quad \therefore \left[Z\right] = \begin{bmatrix} 1 + \frac{Z_2}{Z_1} & Z_2 \\ \frac{1}{Z_1} & 1 \end{bmatrix}$$

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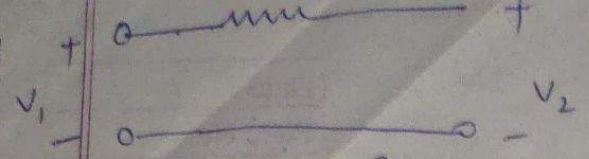
Ques

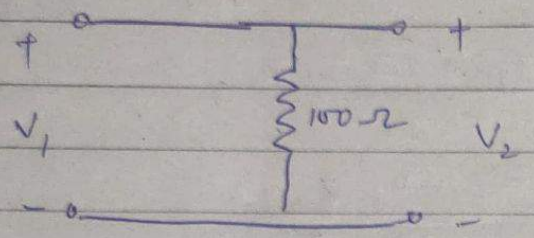


$Z_1 = \infty$

$Z_2 = 10$

$$[T] = \begin{bmatrix} 1 + 10/\infty & 10 \\ \frac{1}{\infty} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

Q



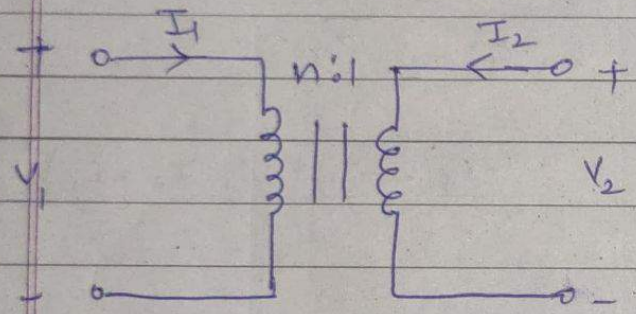
$Z_1 = 100\,\Omega$

$Z_2 = 0$

$$[T] = \begin{bmatrix} 1 + \frac{0}{100} & 0 \\ \frac{1}{100} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{100} & 1 \end{bmatrix}$$

Ques

Impt.



we know that

$$\frac{V_1}{V_2} = \frac{n}{1}$$

$$\frac{-I_2}{I_1} = \frac{n}{1}$$

$V_1 = nV_2$

$V_1 = AV_2 - BI_2$

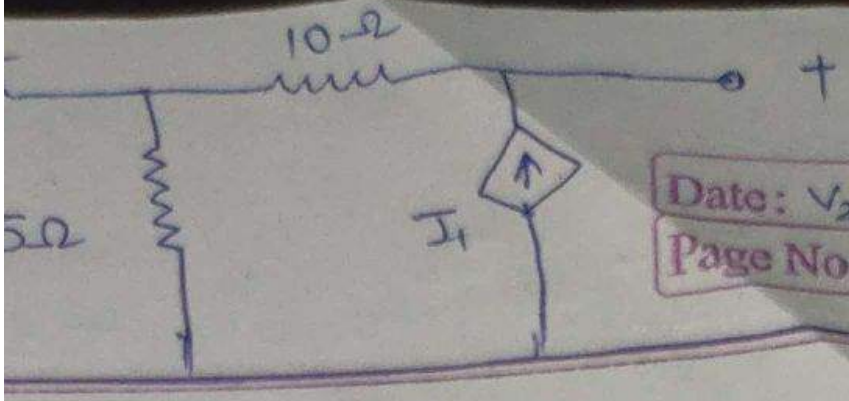
$A = n \quad , B = 0$

$$\frac{-I_2}{n} = I_1$$

$I_1 = CV_2 - DI_2$

$C = 0 \qquad D = 1/n$

$$[T] = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$$



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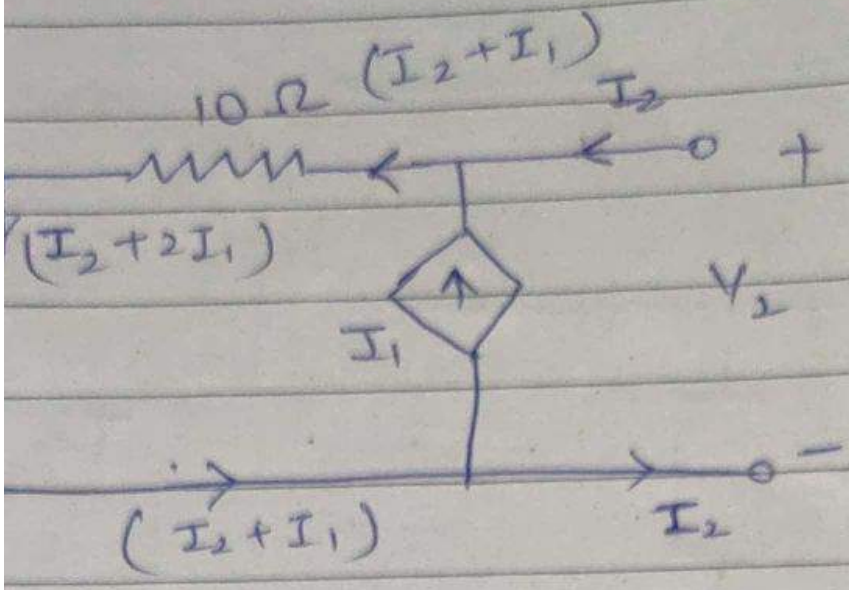


$(I_2 + 2I_1)$

$I_2 + 10I_1$

$5I_2$ — ①

$I_2) + 5(I_2 + 2I_1)$

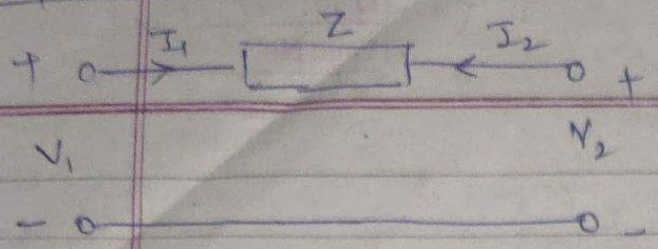
$10I_2 + 5I_2 + 10I_1$

$+ 15I_2$ — ②

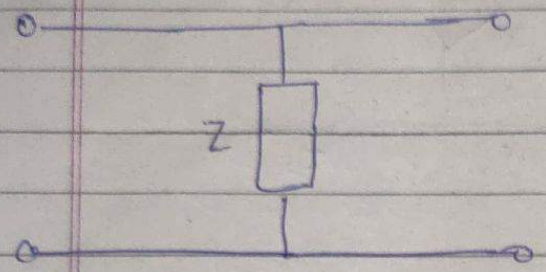
$- 15I_2]$

$- \frac{15}{20} I_2$ — ③

General ckt :



$$T = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

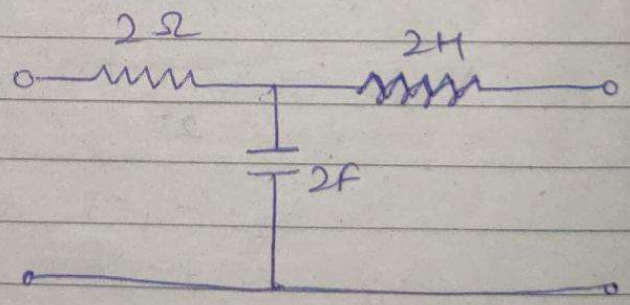


$$T = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$$

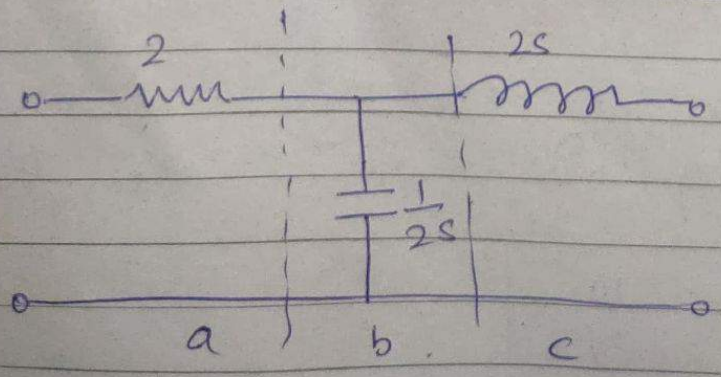
MATRIX FOR INTERCONNECTIONS :

Interconnection	Individual Parameter Matrix	Overall Parameter Matrix
Series - Series	$[Z_a], [Z_b]$	$[Z] = [Z_a] + [Z_b]$
Parallel - Parallel	$[Y_a], [Y_b]$	$[Y] = [Y_a] + [Y_b]$
Cascade or Tandem	$[T_a], [T_b]$	$[T] = [T_a] \cdot [T_b]$
Series - Parallel	$[h_a], [h_b]$	$[h] = [h_a] + [h_b]$
Parallel - Series	$[g_a], [g_b]$	$[g] = [g_a] + [g_b]$

Ques. Determine transmission parameters of a T network shown in fig.



Ans



cascade connection,

$$T_a = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$T_b = \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix} \qquad T_c = \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

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$$[T] = [T_a][T_b][T_c]$$

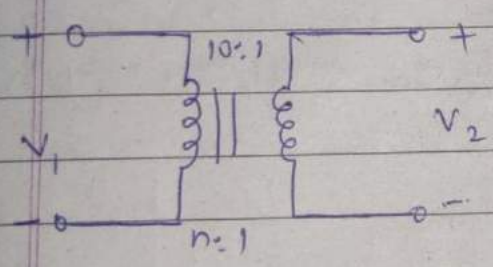
$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix} \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4s & 2 \\ 2s & 2s \end{bmatrix} \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4s & 2s+8s^2+2 \\ 2s & 4s^2+2s \end{bmatrix} = \cancel{\begin{bmatrix} 1+4s & 4+8s \\ 2s & 4s^2+2s \end{bmatrix}}$$

$$= \begin{bmatrix} 1+4s & 8s^2+2s+2 \\ 2s & 4s^2+2s \end{bmatrix}$$

Que



$$\frac{V_1}{V_2} = \frac{n}{1} = \frac{10}{1}$$

$$\frac{-I_2}{I_1} = \frac{10}{1}$$

$V_1 = 10V_2$

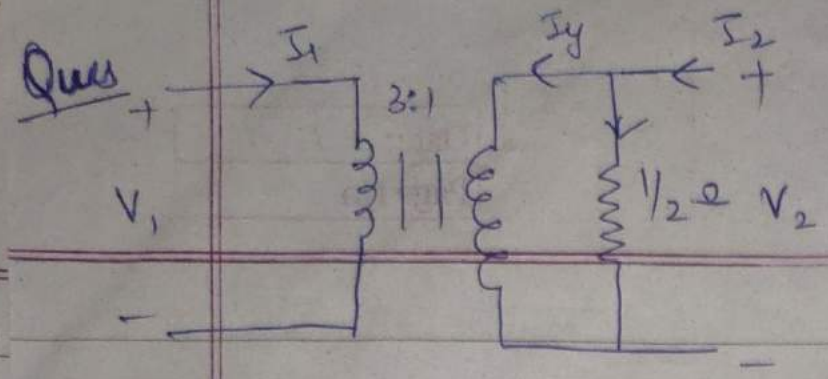
$V_1 = h_{11} I_1 + h_{12} V_2$ $\qquad$ $I_2 = -10 I_1$

$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = 0 \quad h_{12} = n$ $\qquad$ $h_{21} = -10 \quad h_{22} = 0$

$$[h] = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} = \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix}$$

Ques



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$$\frac{V_1}{V_2} = \frac{3}{1} \quad -(1) \qquad \frac{-I_y}{I_1} = \frac{3}{1}$$

$$I_2 = I_y + V_2 \times 1/2 = I_y + \frac{V_2}{1/2}$$

$$I_2 = -3I_1 + \frac{V_2}{1/2}$$

$$2V_2 = I_2 + 3I_1$$

$$V_2 = \frac{I_2}{2} + \frac{3}{2} I_1$$

$$V_2 = 3/2\, I_1 + 1/2\, I_2$$

$$V_1 = 3V_2$$

$$V_1 = 3 \cdot 3/2\, I_1 + 3/2\, I_2 = 9/2\, I_1 + 3/2\, I_2$$

$$[Z] = \begin{bmatrix} 9/2 & 3/2 \\ 3/2 & 1/2 \end{bmatrix}$$

Since V_1, V_2 are dependent variable.

$$\therefore Y = \begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

DRIVING POINT INPUT IMPEDENCE: If a load impedance Z_L is connected to output port, then ~~Zop~~

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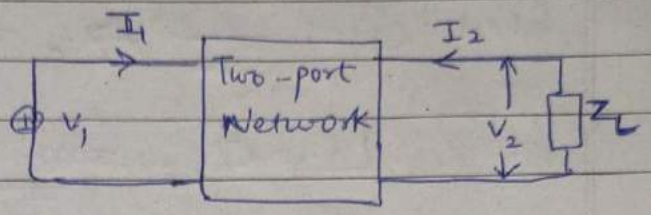
~~Zip~~ Z_{ip} is called input impedance. It can be determined as two port parameter and load impedance Z_L.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

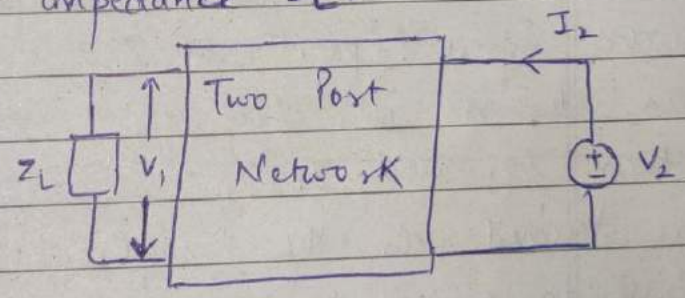


interms of ABCD parameter,

$$\boxed{\frac{V_1}{I_1} = \frac{AZ_L + B}{CZ_L + D} = Z_{ip}}$$

DRIVING POINT OUTPUT IMPEDENCE: If a load impedance Z_L is connected to input port. Then Z_{op} is termed the output impedence. The output impedance can also determined in terms of the two port parameter and load impedance Z_L.

$$Z_{op} = \frac{V_2}{I_2}$$



interms of ABCD parameters,

$$\frac{V_2}{I_2} = \frac{DZ_L + B}{CZ_L + A}$$

Open ckt : $\longrightarrow Z_{10} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{A}{C}$

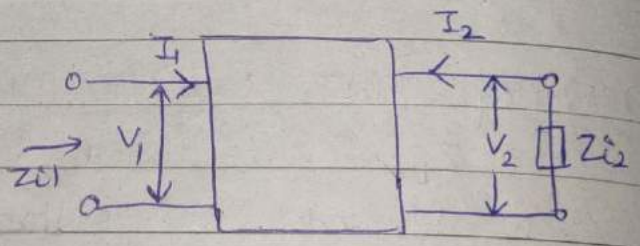
$\longrightarrow Z_{20} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}$

Short ckt : $\longrightarrow Z_{1S} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{B}{D}$

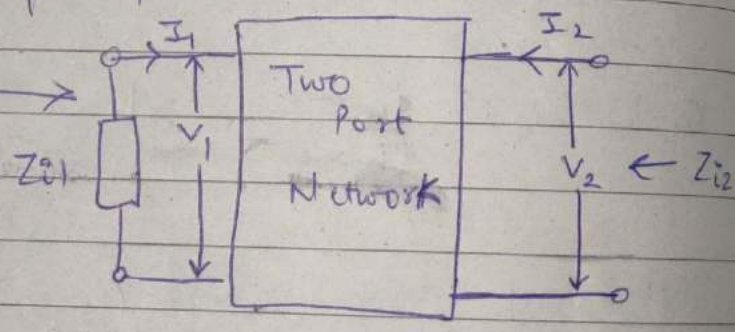
$\longrightarrow Z_{2S} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{B}{A}$

IMAGE IMPEDANCES :

In a two port network, if the impedance at input port with impedance Z_{i2} connected across output port be Z_{i1} as shown in



and the impedance at output port with Z_{i1} connected across input port be Z_{i2} as shown in fig $\longrightarrow$ then Z_{i1} and Z_{i2}, are termed as the <u>image impedances</u> of the network.



For symmetrical network, image impedances are equal to each other, i.e., $Z_{i1} = Z_{i2}$ and is called the <u>characteristic</u> or <u>iterative impedance</u> Z_c.

$Z_{i1} = \frac{V_1}{I_1}$ (driving point impedance at input port)

$Z_{i2} = \frac{V_2}{I_2}$ (driving point impedance at output port)

→ Image impedance in terms of T-parameters :

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$$Z_{i1} = \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad — (1)$$

$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A} \quad — (2)$$

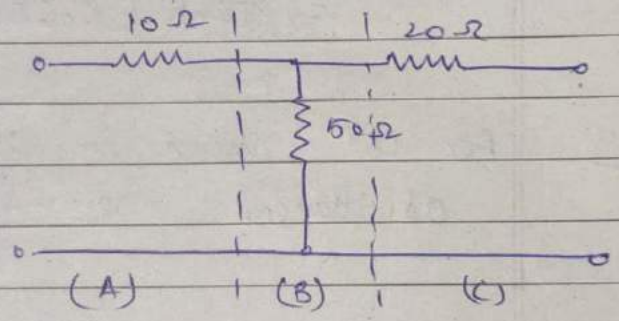
$$Z_{i1} = \sqrt{\frac{AB}{CD}} \quad \& \quad Z_{i2} = \sqrt{\frac{BD}{AC}}$$

→ Image impedance in term of open ckt of open circuit and short circuit impedance :

$$Z_{i1} = \sqrt{\frac{A}{C} \cdot \frac{B}{D}} = \sqrt{Z_{10} Z_{1S}} \qquad \theta = \tanh^{-1}\sqrt{\frac{BC}{AD}}$$

$$Z_{i2} = \sqrt{\frac{B}{A} \cdot \frac{D}{C}} = \sqrt{Z_{20} Z_{2S}}$$

Ques Determine the image parameter of T-Network.



$$T_A = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

$$T_B = \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix}$$

$$T_C = \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$$

$$T = T_A T_B T_C = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/50 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1/5 & 10 \\ 1/50 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6/5 & 10 \\ 1/50 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6/5 & 24+10 \\ 1/50 & 7/5 \end{bmatrix} = \begin{bmatrix} 6/5 & 34 \\ 1/50 & 7/5 \end{bmatrix}$$

$A = \frac{6}{5}$ $B = 34$ $C = 1/50$ $D = 7/5$

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$$Z_{i_1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{6}{5} \times 34 \times 50 \times \frac{5}{7}}$$

$$= 38.17\,\Omega$$

$$Z_{i_2} = \sqrt{\frac{BD}{AC}} = \sqrt{34 \times \frac{7}{5} \times \frac{5}{6} \times 50}$$

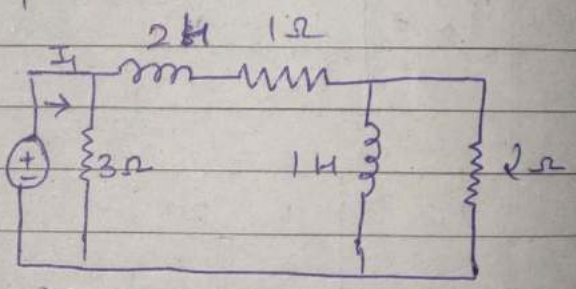
$$= 44.5346\,\Omega$$

$$\theta = \tanh^{-1}\sqrt{\frac{BC}{AD}}$$

$$= \tanh^{-1}\left(\sqrt{34 \times \frac{1}{50} \times \frac{5}{6} \times \frac{5}{7}}\right)$$

$$= \tanh^{-1}(0.6362) = 0.7517.$$

Que For ckt shown in figure obtain driving point admittance. $Y_{11} = I_1/V_1$.



Ans $Z(s) = \left((2\|8) + (1+2s)\right)\|3$

$$= \left[\left(\frac{2 \times s}{2+s}\right) + (1+2s)\right]\|3$$

$$= \left[\frac{2s + (2+s)(1+2s)}{(2+s)}\right]\|3$$

$$= \left[\frac{2s + 2 + 4s + s + 2s^2}{(2+s)}\right]\|3$$

$$Z(s) = \left[\frac{2s^2+7s+2}{2+s}\right] // 3$$

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$$= \frac{\left(\frac{2s^2+7s+2}{2+s}\right) \times 3}{\frac{2s^2+7s+2}{2+s} + 3}$$

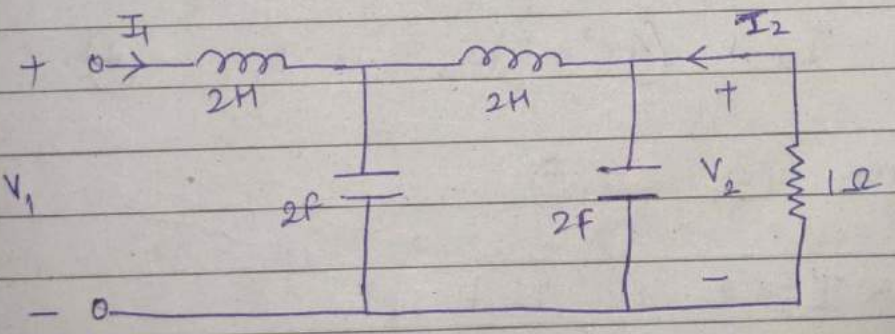
$$= \frac{3(2s^2+7s+2)}{2s^2+7s+2+3(2+s)}$$

$$= \frac{6s^2+21s+2}{2s^2+7s+2+6+3s}$$

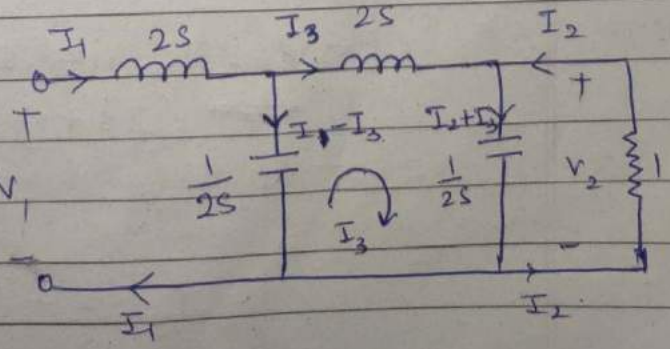
$$= \frac{6s^2+21s+2}{2s^2+10s+8}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{2s^2+10s+8}{6s^2+21s+2}$$

Que For the network shown in figure compute,
(a) $G_{21} = V_2/V_1$ (b) $\alpha_{21} = I_2/I_1$



Solⁿ From the ckt,
applying KVL, we get



$$V_1 = I_1(2s) + \frac{1}{2s}(I_1 - I_3)$$

$$V_1 = 2sI_1 + \frac{I_1}{2s} - \frac{I_3}{2s}$$

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$$V_1 = \left(2s + \frac{1}{2s}\right)I_1 - \frac{1}{2s}\cdot I_3 \quad -(1)$$

$$V_2 = 1(-I_2) = -I_2 \quad -(2)$$

$$V_2 = \frac{1}{2s}(I_2 + I_3) \quad -(3)$$

$$2s\cdot I_3 + \frac{1}{2s}(I_2 + I_3) - \frac{1}{2s}(I_1 - I_3) = 0$$

$$2sI_3 + \frac{1}{2s}I_2 + \frac{1}{2s}I_3 - \frac{1}{2s}I_1 + \frac{1}{2s}I_3 = 0$$

$$-\frac{1}{2s}I_1 + \frac{1}{2s}I_2 + \left(2s + \frac{1}{2s} + \frac{1}{2s}\right)I_3 = 0$$

$$-\frac{1}{2s}I_1 + \frac{1}{2s}I_2 + \left(\frac{4s^2+1+1}{2s}\right)I_3 = 0$$

$$\cancel{-\frac{1}{2s}I_1 + \frac{1}{2s}I_2 +}$$

$$\frac{1}{2s}I_2 + \frac{4s^2+2}{2s}I_3 = \frac{1}{2s}I_1$$

$$I_2 + (4s^2+2)I_3 = I_1 \quad -(4)$$

from (2) and (3)

$$-I_2 = \frac{1}{2s}(I_2 + I_3)$$

$$-2s\cdot I_2 = I_2 + I_3$$

$$(2s+1)I_2 = -I_3$$

$$\cancel{I_2 = -\frac{I_3}{2s+1} \quad -(5)}$$

$$I_3 = -(2s+1)I_2 \quad -(5)$$

~~put (5) in (4)~~

~~$I_1 = I_2 + (4s^2+2) I_2$~~

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put (5) in (4),

$$I_1 = I_2 + (-I_2)(2s+1)(4s^2+2)$$

$$I_1 = I_2 - (8s^3 + 4s + 4s^2 + 2) I_2$$

$$I_1 = (1 - 8s^3 - 4s - 4s^2 - 2) I_2$$

$$I_1 = -(8s^3 + 4s^2 + 4s + 1) I_2 \quad \text{--- (6)}$$

$$\frac{I_2}{I_1} = \frac{-1}{8s^3 + 4s^2 + 4s + 1} = \alpha_{21}$$

putting I_3 value in (1)

$$V_1 = \left(2s + \frac{1}{2s}\right) I_1 - \frac{1}{2s} (-I_2)(2s+1)$$

$$= \left(\frac{4s^2+1}{2s}\right) I_1 + I_2 \frac{(2s+1)}{2s}$$

$$V_1 = \frac{1}{2s}\left[(4s^2+1) I_1 + (2s+1) I_2\right]$$

$$V_1 = \frac{1}{2s}\left[(4s^2+1)(-I_2)(8s^3 + 4s^2 + 4s + 1) + (2s+1) I_2\right] \quad \text{from (6)}$$

$$V_1 = \frac{1}{2s}\left[(2s+1) I_2 - (32s^5 + 16s^4 + 16s^3 + 8s^3 + 4s^2 + 4s + 1) I_2\right]$$

$$V_1 = \frac{I_2}{2s}\left[2s + 1 - 32s^5 - 16s^4 - 24s^3 - 4s^2 - 4s - 1\right]$$

$$= \frac{I_2}{2s}\left[-32s^5 - 16s^4 - 24s^3 - 4s^2 - 2s\right]$$

$$= -I_2 (16s^4 + 8s^3 + 12s^2 + 2s + 1) \quad \text{(7)}$$

$\cancel{V_2 =}$

$$\frac{V_2}{V_1} = \frac{-I_2}{-I_2(16s^4 + 8s^3 + 12s^2 + 4s + 1)}$$

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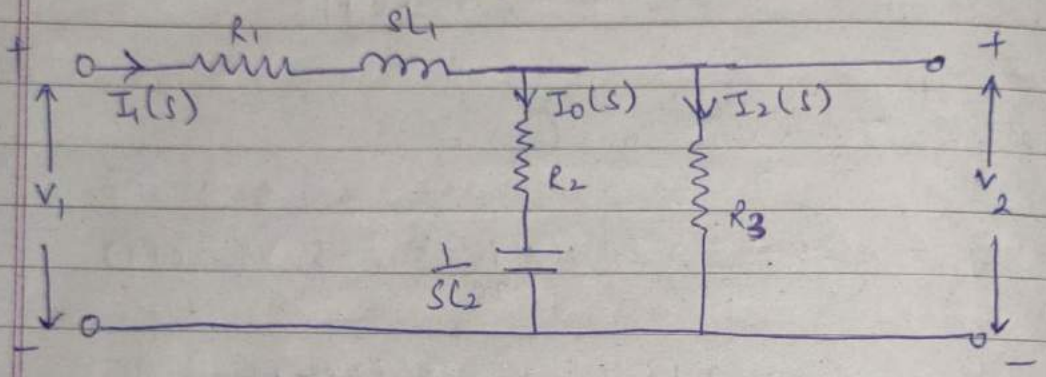
$$\left[G_2 = \frac{V_2}{V_1} = \frac{1}{16s^4 + 8s^3 + 12s^2 + 4s + 1} \right]$$

Ques For the circuit shown in figure determine the network transfer function.



Solⁿ

$$V_2(s) = R_3 \cdot I_2(s) \quad -(1)$$

$$V_2(s) = I_0(s)\left(R_2 + \frac{1}{sC_2}\right) \quad -(2)$$

$$\cancel{V_1(s)}\ I_0(s) = \cancel{I_2(s)}\ \frac{V_2(s)}{\left(R_2 + \frac{1}{sC_2}\right)} \quad -(3)$$

$$I_1(s) = I_0(s) + I_2(s) \quad -(4)$$

$$V_1(s) = I_1(s)\left[R_1 + sL_1\right] + I_0(s)\left[R_2 + \frac{1}{sC_2}\right]$$

$$V_1(s) = I_1(s)\left[R + sL_1\right] + V_2(s) \quad -(5)$$

from (1) & (2),

$$R_3 \cdot I_2(s) = I_0(s)\left[R_2 + \frac{1}{sC_2}\right]$$

$$\cancel{I_0} \Rightarrow \frac{I_2(s)}{I_0(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_3} = \frac{R_2}{R_3} + \frac{1}{sC_2 R_3}$$

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$$\Rightarrow I_0(s) = I_2(s)\left[\frac{sC_2R_3}{sC_2R_2+1}\right]$$

$$I_1 = I_0(s) + I_2(s)$$

$$I_1 = I_0\left[1 + \frac{I_2(s)}{I_0(s)}\right]$$

$$I_1 = I_0\left[1 + \frac{R_2}{R_3} + \frac{1}{sC_2R_3}\right]$$

$$I_1 = I_0\left[\frac{sC_2R_3 + sC_2 + 1}{sC_2R_3}\right]$$

$$V_2(s) = I_0(s)\left[R_2 + \frac{1}{sC_2}\right]$$

$$V_2(s) = I_0(s)\left[\frac{R_2 \cdot sC_2 + 1}{sC_2}\right]$$

(i) $$Z_{21} = \frac{V_2(s)}{I_1(s)}$$

$$Z_{21} = \frac{\cancel{I_0(s)} \cdot \frac{R_2 sC_2 + 1}{\cancel{sC_2}}}{\cancel{I_0(s)} \cdot \left[\frac{sC_2R_3 + sC_2 + 1}{\cancel{sC_2}R_3}\right]}$$

$$= \frac{(sC_2R_2 + 1)R_3}{sC_2R_3 + sC_2 + 1}$$

(ii) $$Y_{21} = \frac{I_2(s)}{V_1(s)}$$

$$V_1(s) = V_2(s) + (R_1 + sL_1)\, I_1(s)$$

$$V_1(s) = R_3 I_2(s) + (R_1 + sL_1)(I_0(s) + I_2(s))$$

$$V_1(s) = R_3 \cdot I(s) + (R + sL_1)\left(\frac{sC_2R_3}{sC_2R}\right)$$

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* TRANFER FUNCTION :

→ It is used to describe networks which have at least two ports.

→ The transfer function relates the transform of a quantity at one port to the transform of another quantity at another port.

→ Thus, transform functions have following possible forms,

(i) ratio of one current to voltage of another or one voltage to another current : $Y(s)$, $Z(s)$

(ii) ratio of ~~one~~ one voltage another voltage, or the VOLTAGE TRANSFER FUNCTION $G(s)$

(iii) ratio of one current to another current, or the CURRENT TRANSFER FUNCTION $\alpha(s)$

→ Transfer function can also be defined as the ratio of an output quantity to an input quantity.

→ Basic transfer functions for two port networks,

(i) TRANSFER IMPEDANCE FUNCTION : $Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$

(ii) TRANSFER ADMITTANCE FUNCTION : $Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$

(iii) VOLTAGE TRANSFER FUNCTION : $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

(iv) CURRENT TRANSFER FUNCTION : $\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$

→ The ratio of an input quantity to an output quantity is termed as INVERSE TRANSFER FUNCTIONS.

T-π Transformation:

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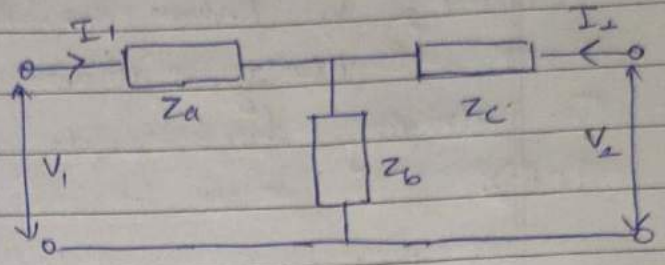
T and π network are two networks which are used to represent the equivalence of transmission lines, filters, etc.

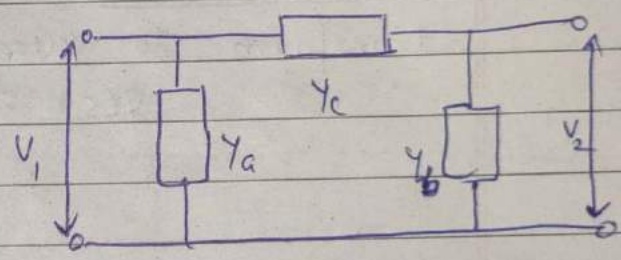
Z_a, Z_b and Z_c can be expressed in terms of Z-parameters.



$$\begin{cases} Z_a = Z_{11} - Z_{12} \\ Z_b = Z_{22} - Z_{12} \\ Z_c = Z_{12} = Z_{21} \end{cases}$$

Also, we can express Y_a, Y_b and Y_c in π-network

$$\begin{cases} Y_a = Y_{11} + Y_{12} \\ Y_b = Y_{22} + Y_{12} \\ Y_c = -Y_{12} \end{cases}$$



NETWORK FUNCTION: A network function is a transfer function of an electrical network where both input and output variable can be voltage and/or current, with all inductors & capacitors having zero initial conditions.

- Network function
  - Immittance function
    - → Impedance func ($Z(s)$)
    - → Addmittance func. ($Y(s)$)
  - Non-Immittance function
    - → Voltage Amplification (G)
    - → Current Amplification (α)

NETWORK FUNCTION

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DRIVING POINT: The pair of terminals connected to an energy source, which is the driving force for the network, is known as the DRIVING POINT of the network. Generally, it is INPUT ~~POLE~~ PORT, being connected to driving force while OUTPUT PORT is connected to LOAD.

TRANSFORM IMPEDANCE: at a port has been defined as the ratio of voltage transform to current transform.

$$Z(s) = \frac{V(s)}{I(s)}$$

TRANSFORM ADMITTANCE is defined as the ratio of current transform to voltage transform.

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

- The impedance or admittance found at the source port or driving point is called DRIVING-POINT IMPEDANCE OR ADMITTANCE. For input & output ports, the term INPUT DRIVING POINT & OUTPUT DRIVING POINT IMPEDANCE or ADMITTANCE is used.

- Because of similarity of impedance & admittance these two quantities are assigned one name IMMITANCE. An IMMITTANCE is thus an impedance or admittance.

IMMITANCE Of circuit elements :

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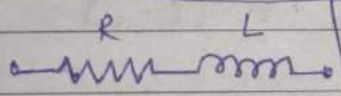
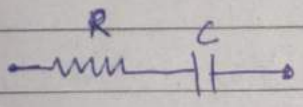
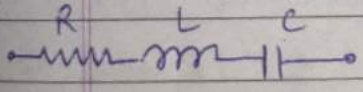
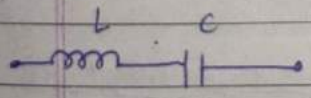
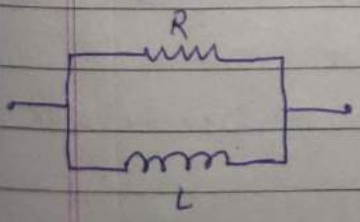
Elements	Z(s)	Y(s)
Resistance (Ω)	R	1/R
Inductance (H)	sL	1/sL
Capacitance (C)	1/sC	sC

$\Rightarrow Z(s) \equiv R \equiv sL \equiv \frac{1}{sC}$, $\Rightarrow Y(s) \equiv \frac{1}{R} \equiv \frac{1}{sL} \equiv sC$

→ For series circuit : $Z(s) = Z_1(s) + Z_2(s) + \dots$

→ For parallel circuit : $Y(s) = Y_1(s) + Y_2(s) + \dots$

- **IMMITANCE FUNCTIONS FOR SOME NETWORKS :**

Networks	Z(s)	Y(s)
	$R+sL$	$\frac{1}{R+sL}$
	$R+\frac{1}{sC}$	$\frac{sC}{sRC+1}$
	$\frac{sRC+s^2LC+1}{sC}$	$\frac{sC}{sRC+s^2LC+1}$
	$\frac{s^2LC+1}{sC}$	$\frac{sC}{s^2LC+1}$
	$\frac{sRL}{sL+R}$	$\frac{sL+R}{sRL}$

• CALCULATION OF NETWORK FUNCTIONS:

~~FOR~~ FOR LADDER NETWORK

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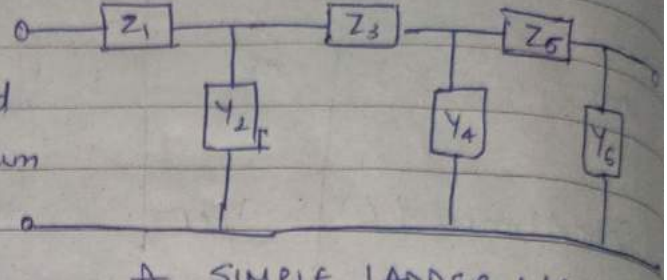
Computation begins from Y_6, which is first inverted and combined with Z_5. Next, the sum is inverted & combined with Y_4. This process is continued.



A SIMPLE LADDER NETWORK

The driving point impedance or transform impedance

$$Z = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6 + \dots}}}}}$$

This eq^n is K/a CONTINUED FUNCTION EQN (CFE)

• POLES & ZEROES OF Network Functions:

— In linear RLC network, all networks functions $T(s)$ are the rational function of s and may be expressed as the ratio of two polynomials as:

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

Where $N(s) \rightarrow$ Numerator and $D(s) \rightarrow$ Denominator

$$T(s) = \frac{N(s)}{D(s)} = \frac{K.(s^n + c_{n-1}.s^{n-1} + \dots + c_1 s + c_0)}{(s^m + d_{m-1}.s^{m-1} + \dots + d_1 s + d_0)}$$

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where $K = \frac{a_n}{b_m}$, is a positive constant k/a SCALAR FACTOR

→ The polynomial $N(s) = 0$ has n roots, called zeroes. The roots of polynomial $N(s)$ are called ~~ZEROES~~ ZEROS of a network function $T(s)$.

→ The polynomial $D(s) = 0$ has m roots, called poles. So, The roots of the polynomial $D(s)$ are called POLES of $T(s)$.

$$T(s) = K.\frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

z_i → zeroes of $T(s)$, p_j → Poles of $T(s)$

→ The value poles and zeroes and their location in s-plane completely specify the network ~~funt~~ function except for scaler factor K.

→ In the complex s-plane, a pole is denoted by a small cross (X) & a zero by a small circle (o)

- When r poles ($r < m$) or r zeroes ($r < n$) have same value, the pole or zero is said to be of MULTIPLICITY. r.
- When $n > m$, we have poles at infinity of multiplicity (or degree) of $(n-m)$
- When $n < m$, we then have ZEROES at infinity of multiplicity (or degree) of $(m-n)$

DRIVING POINT IMITTANCE FUNCS

s) & D(s)

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) & D(s) must be real and

be conjugate if imaginary or

e poles and zeroes must be -ve

part is zero, then that pole

SIMPLE, i.e., all roots of N(s) = 0

n left half of s-plane and

lie on the imaginary or

s) may not have missing terms

& lowest degrees, unless all even

missing.

s) & D(s) may differ by zero. or

N(s) & D(s) may differ by

unless all even or odd terms are missing.

5. The poly. $N(s)$ may have terms missing and some of the coeff. may be -ve.

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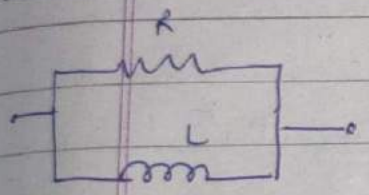
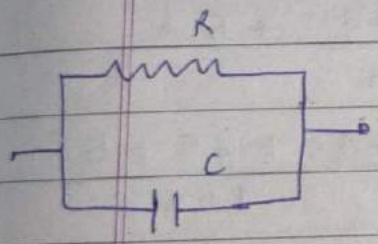
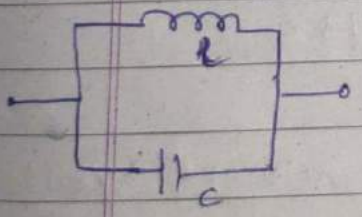
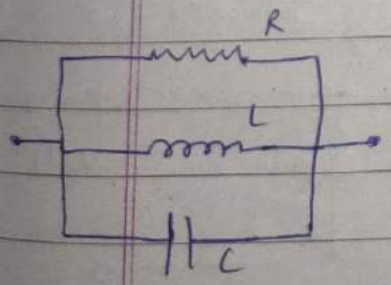
6. The degree of $N(s)$ may be small as zero independent of degree of $D(s)$.

7. (a) For G & α : max. degree of $N(s)$ = degree of $D(s)$

(b) For Z & Y : max. degree of $N(s)$ = degree of $D(s) + 1$

INMITANCE FUNCTION FOR SOME SIMPLE NETWORKS :

Networks	Impedance fn. $Z(s)$	Admittance fn. $Y(s)$
	$\frac{sRL}{sL+R}$	$\frac{sL+R}{sRL}$
	$\frac{R}{sRC+1}$	$\frac{sRC+1}{R}$
	$\frac{sL}{s^2LC+1}$	$\frac{s^2LC+1}{sL}$
	$\frac{sRL}{s^2RLC+sL+R}$	$\frac{s^2RLC+sL+R}{sRL}$

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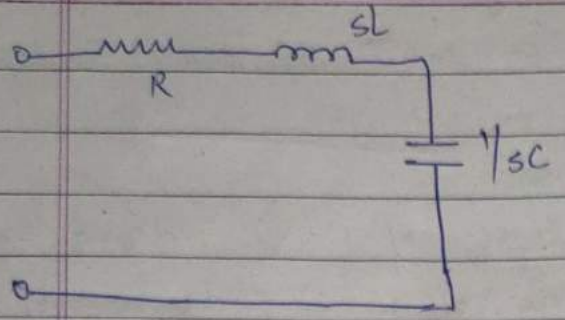
Ques Obtain the driving point impedance function or transfer ~~function~~ impedance $Z(s)$ for the N/W.

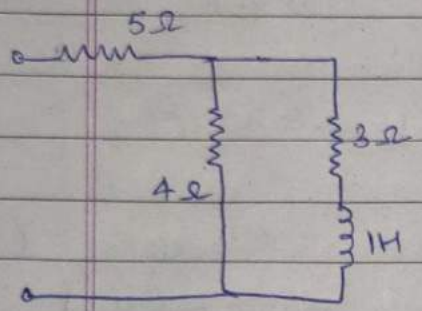
(i)



$$Z(s) = R + sL + \frac{1}{sC}$$

$$= \frac{sRC + s^2LC + 1}{sC}$$

(ii)



$$Z(s) = \left[(3+s) \parallel 4\right] + 5$$

$$= \frac{1}{\frac{1}{3+s} + \frac{1}{4}} + 5$$

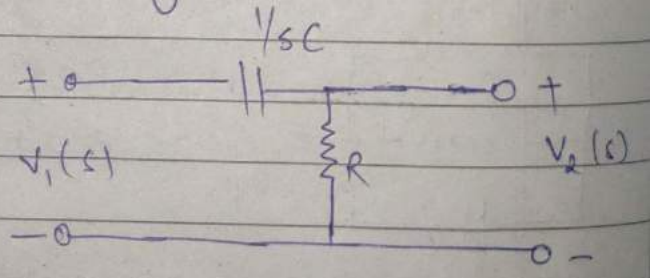
$$= \frac{(3+s)4}{4+3+s} + 5$$

$$= \frac{12 + 4s}{7+s} + 5$$

$$= \frac{12 + 4s + 35 + 5s}{7+s}$$

$$= \frac{47 + 9s}{7+s}$$

Ques Find out $G = \frac{V_2(s)}{V_1(s)}$ for the given network below:



Ans By voltage Division formula,

$$\frac{V_2(s)}{V_1(s)} = \frac{R}{R + \frac{1}{sC}}$$

$$= \frac{sCR}{sCR + 1} = \left(\frac{s}{s + 1/RC}\right)$$

$s = 0 \Rightarrow$ Zero

$s = \frac{-1}{RC} \Rightarrow$ Pole

Ques Find out $\alpha = \frac{I_2(s)}{I_1(s)}$ for the given network:

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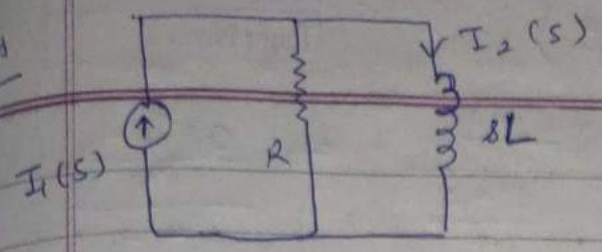
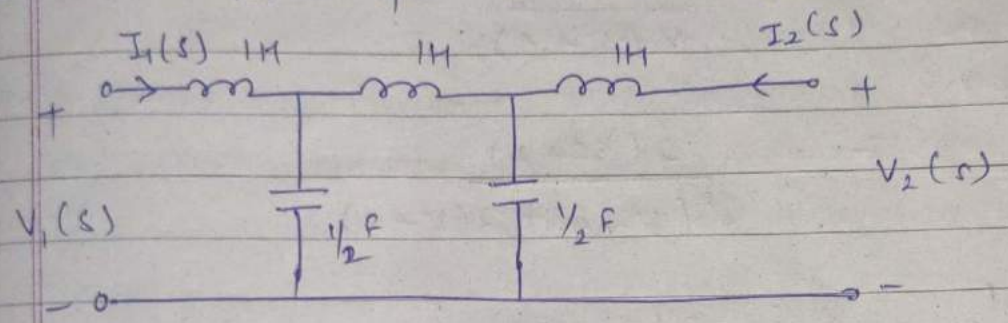
Ans



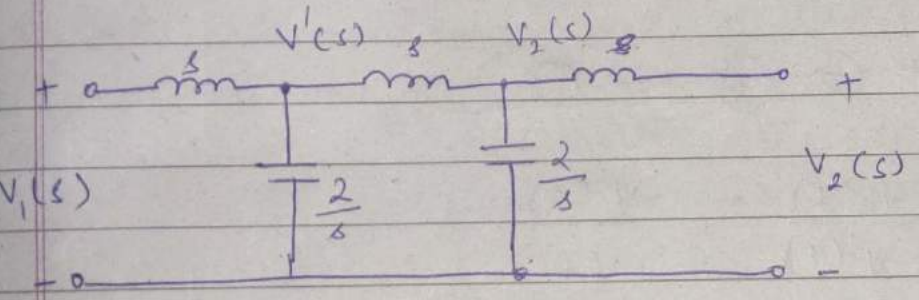
$$\frac{I_2(s)}{I_1(s)} = \frac{R}{R+sL} = \frac{R/L}{s+R/L}$$

No zeros
pole is $-R/L$

Ques Find $G_{21}(s)$ for the circuit below:

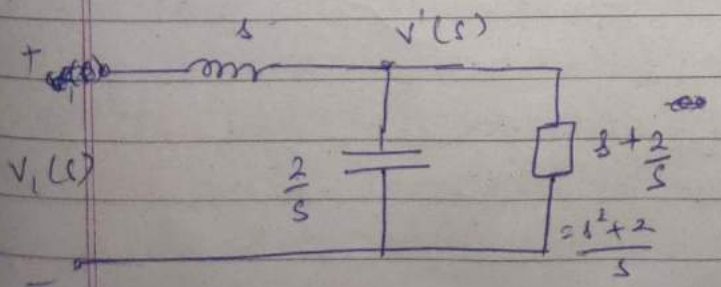


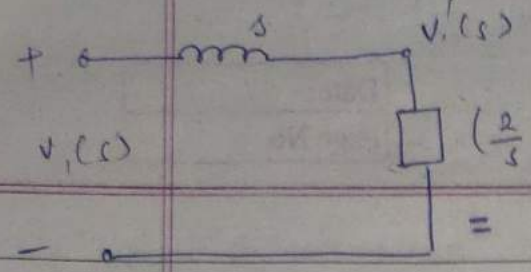
Ans $I_2(s)$ will be zero.



$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_2(s)}{V'(s)} \times \frac{V'(s)}{V_1(s)} \quad -(1)$$

$$\frac{V_2(s)}{V'(s)} = \frac{2/s}{s+2/s} = \frac{2}{s^2+2} \quad -(2)$$





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$$\left(\frac{2}{s}\right) \| \left(\frac{s^2+2}{s}\right)$$

$$= \frac{\frac{2}{s} \times \frac{s^2+2}{s}}{\frac{2}{s} + \frac{s^2+2}{s}} = \frac{\frac{2s^2+4}{s^2}}{\frac{s^2+4}{s}}$$

$$= \frac{2(s^2+2)}{s(s^2+4)}$$

So, $$\frac{V'(s)}{V_1(s)} = \frac{\frac{2(s^2+2)}{s(s^2+4)}}{s + \frac{2(s^2+2)}{s(s^2+4)}}$$

$$= \frac{2(s^2+2)}{s^2(s^2+4) + 2(s^2+2)}$$

$$\frac{V'(s)}{V_1(s)} = \frac{2(s^2+2)}{s^4+4s^2+2s^2+4} = \frac{2(s^2+2)}{s^4+6s^2+4} \quad -③$$

put ②, ③ in ①

$$G_{21} = \frac{V_2(s)}{V'(s)} \times \frac{V'(s)}{V_1(s)}$$

$$= \frac{2}{\cancel{(s^2+2)}} \times \frac{2\cancel{(s^2+2)}}{s^4+6s^2+4} = \frac{4}{s^4+6s^2+4}$$

HURWITZ POLYNOMIAL:

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- for a network function, $T(s) = \frac{N(s)}{D(s)}$ or $\frac{Q(s)}{P(s)}$

Where $P(s)$ is the Hurwitz Polynomial if it has following basic properties:

- The poly $P(s)$ is real when 's' is real
- The roots of the poly $P(s)$ have real parts which are either zero or -ve.

→ Properties of Hurwitz polynomial

1. If $P(s)$ can be written as
$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0,$$
where $a_n, a_{n-1}, \dots$, are coefficients, then all coefficients must be POSITIVE.

2. Both odd and even parts of $P(s)$ have roots on $j\omega$ axis only.
$$P(s) = \underbrace{M(s)}_{\text{EVEN}} + \underbrace{N(s)}_{\text{ODD}}$$

3. Continued fraction expansion of ratio $\psi(s)$ of a hurwitz poly. yields all positive quotient.
$$\psi(s) = \frac{M(s)}{N(s)} \text{ or } \frac{N(s)}{M(s)}$$

4. If $P_1(s) = P(s) \cdot W(s)$, where if $P(s)$ is Hurwitz, then $P_1(s)$ will be Hurwitz only if $W(s)$ is hurwitz.

5. If poly. is either even or odd. It is not possible to obtain CFE. $P(s)$ is hurwitz if ratio of $P(s)$ & $P'(s)$ give CFE (continued fraction expansion)

ROUTH HURWITZ ARRAY METHOD:

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- Another method for testing a given polynomial for hurwitz is called ROUTH HURWITZ array method.

- In this method, an array is constructed using the coefficients of a given poly. in a specific way.

- In the array, first row consists of all the coeffs of alternate powers of s starting from n, while the next row consist of all the coeffs. of alternate powers of s. starting from (n-1). Thus, all coeffs are accomodated in first two rows. So, each next rows is generated from coeffs. of two previous rows. So, third row from (n-2) is,

$$b_n = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}},$$

$$b_{n-1} = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

So, fourth row from (n-3) is,

$$c_n = \frac{b_n \cdot a_{n-3} - a_{n-1} b_{n-1}}{b_n},$$

$$c_{n-1} = \frac{b_n a_{n-5} - a_{n-1} b_{n-2}}{b_n}$$

Process is continued till 0,

s^n	a_n	a_{n-2}	a_{n-4} ...
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5} ...
s^{n-2}	b_n	b_{n-1}	b_{n-2} ...
$\vdots$	c_n	c_{n-1}	c_{n-2} ...
s^0	a_0		

For Given Poly $P(s)$ to be Hurwitz,

- All elements in the first column should be non zero

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- There should not be any sign change in the first column.

Ques Check wheather the given polynomial

$P(s) = s^4 + s^3 + 5s^2 + 3s + 4$ is Hurwitz or not.

Sol^n $P(s) = s^4 + s^3 + 5s^2 + 3s + 4$

$M(s) = s^4 + 5s^2 + 4$ even power

$N(s) = s^3 + 3s$

$$
\begin{array}{l}
s^3 + 3s \,\big)\, s^4 + 5s^2 + 4 \,\big(\, s \\
\qquad\quad \underline{-s^4 - 3s^2} \\
\qquad\qquad 2s^2 + 4 \,\big)\, s^3 + 3s \,\big(\, s/2 \\
\qquad\qquad\qquad \underline{-s^3 - 2s} \\
\qquad\qquad\qquad\quad s \,\big)\, 2s^2 + 4 \,\big(\, 2s \\
\qquad\qquad\qquad\qquad \underline{2s^4} \\
\qquad\qquad\qquad\qquad\quad 4 \,\big)\, s \,\big(\, s/4 \\
\qquad\qquad\qquad\qquad\qquad \underline{-s} \\
\qquad\qquad\qquad\qquad\qquad\quad \times
\end{array}
$$

All quotient are +ve

$\therefore$ $P(s)$ is Hurwitz

Q Test ~~the~~ whether the following polynomial is Hurtoitz

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$$P(s) = s^4 + 3s^2 + 2$$

Sol^n $P'(s) = 4s^3 + 6s$

$$4s^3+6s \,\big)\, s^4 + 3s^2 + 2 \,\big(\, s/4$$
$$\underline{s^4 + 3s^2/2}$$
$$\frac{3s^2}{2}+2 \,\big)\, 4s^3 + 6s \,\big(\, 8/3\, s$$
$$\underline{4s^3 + 16s/3}$$
$$\frac{2s}{3} \,\big)\, \frac{3s^2}{2} + 2 \,\big(\, \frac{9}{4} s$$
$$\underline{\frac{3s^2}{2}}$$
$$2 \,\big)\, 2s/3 \,\big(\, s/3$$
$$\underline{2s/3}$$
$$\times$$

∴ All coffients are positive and real.
∴ Hurtoitz polynomial.

Ques Check whether given polynomial is hurwitz or not.

$$P(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$$

$$M(s) = 2s^6 + s^4 + 8s^2 + 4$$
$$N(s) = s^7 + 2s^5 + 4s^3 + 8s$$

$$2s^6 + s^4 + 8s^2 + 4 \,\big)\, s^7 + 2s^5 + 4s^3 + 8s \,\big(\, s/2$$
$$\underline{s^7 + s^5/2 + 4s^3 + 2s}$$
$$3/2\, s^5 + 6s \,\big)\, 2s^6 + s^4 + 8s^2 + 4 \,\big(\, \frac{2}{3} \cdot 2s$$
$$\underline{2s^6 + 8s^2}$$
$$s^4 + 4 \,\big)\, \frac{3}{2} s^5 + 6s \,\big(\, 3/2\, s$$
$$\underline{3/2\, s^5 + 6s}$$
$$\times$$

$P_1(s) = s^4 + 4$

$= (s^2+2)^2 - 4s^2$

$= (s^2+2+2s)(s^2+2-2s)$

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Hurwitz — Not Hurwitz

∴ It is clear that $P(s)$ is not Hurwitz

using Routh Hurwitz method.

Que $D(s) = s^4 + 2s^3 + 8s^2 + 4s + 3$

s^4	1	8	3
s^3	2	4	0
s^2	$\frac{2\times8-1\times4}{2}$		
s^1			
s^0			

s^4	1	8	3
s^3	2	4	0
s^2	$\frac{2\times8-1\times4}{2} = 6$	$\frac{4\times3-8\times0}{4} = 3$	
s^1	$\frac{6\times4-2\times3}{6} = 3$		
s^0	$\frac{3\times3-6\times0}{3} = 3$		

4
6
18

entry in first coloumn are +ve

∴ system is stable.

$-4 \quad -4 \quad -8$

Ques $D(s) = s^4 + 2s^3 + s^2 + 4s + 2$

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s^4	1	1	2
s^3	2	4	0
s^2	$\frac{2\times1 - 4\times1}{2} = -1$	$\frac{4\times2 - 1\times0}{4} = 2$	
s^1	$\frac{-1\times4 - 2\times2}{-1} = 8$		
s^0	$\frac{8\times2 - (-1)\times0}{8} = 2$		

In first coloumn sign changes

$\therefore$ system is unstable.

GRAPH THEORY

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Collection of nodes and branches, Directed graph.

→ **Oriented Graph**: A graph with directions imparted to every branch is oriented graph.

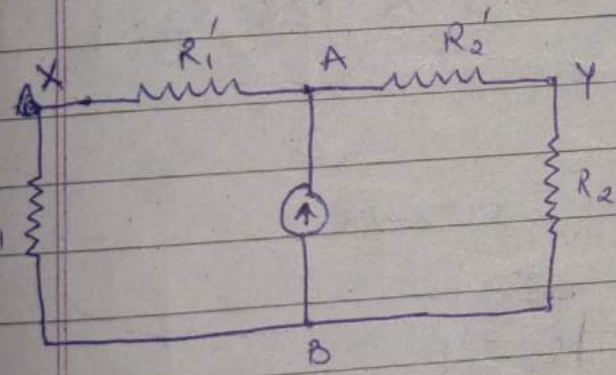
→ **Tree**: A sub-graph containing all Nodes but not all branches, without forming a mesh or loop.

→ **Mesh**: The most elementary form of a loop, which cannot be further sub-divided.

→ **Loop**: Any closed circuit is a loop.

→ **Node & Junction**: All nodes are junctions but all junctions are not nodes.

[Note: All meshes are loops but all loops are not mesh]

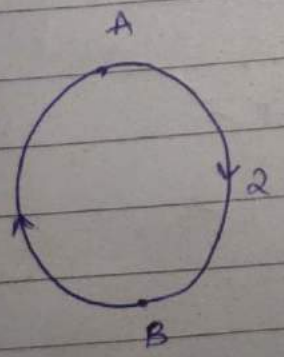


A & B are nodes

x, y, A, B are junctions

R_1 & R_1' are in series, so same sub script 1.

R_2 & R_2' are also in series so same sub-script.



current source is not shown in graph.

Example :

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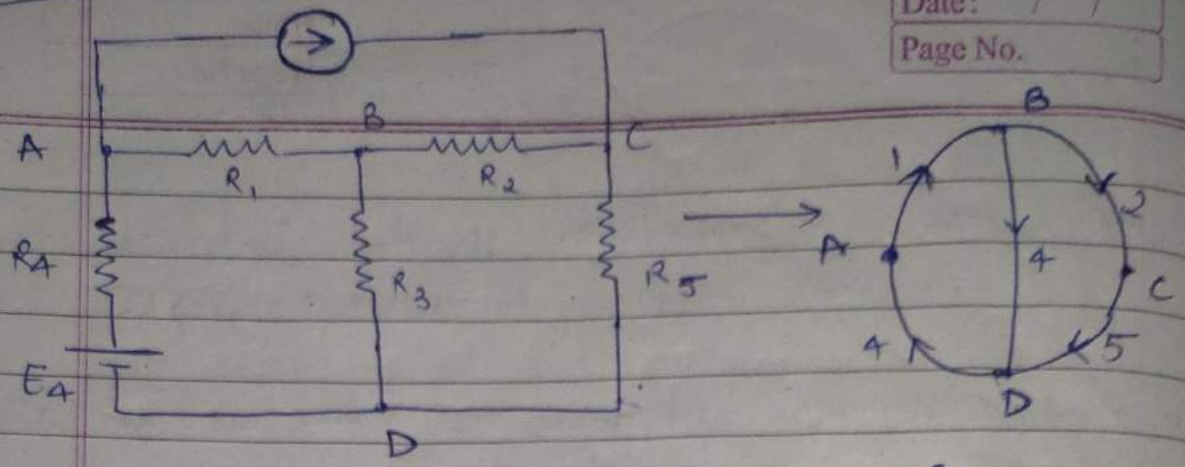
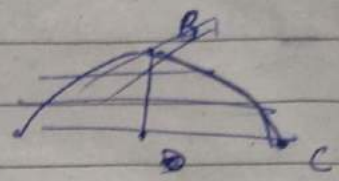
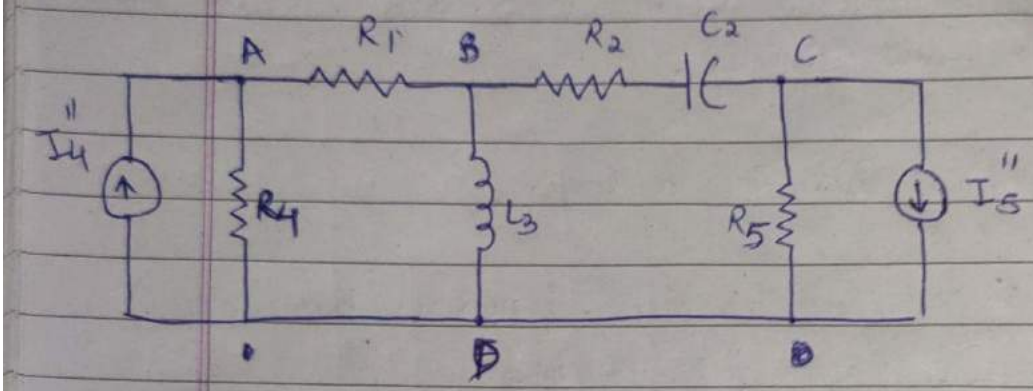
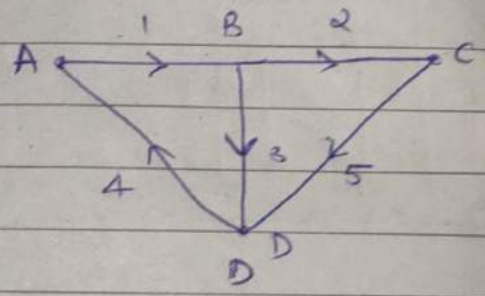
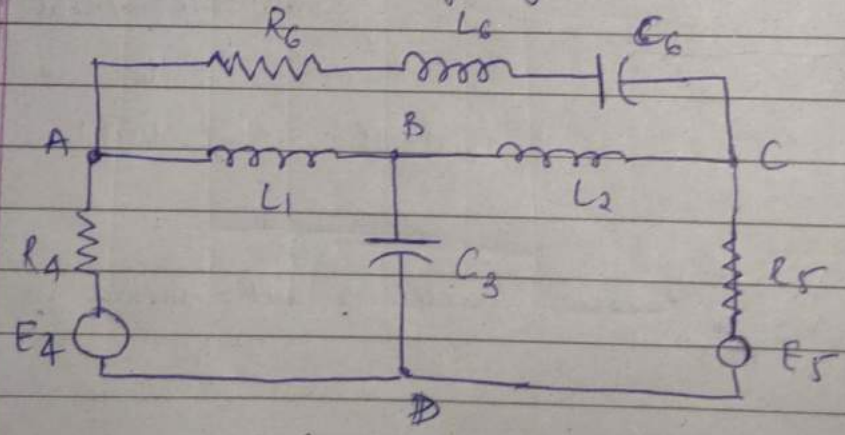


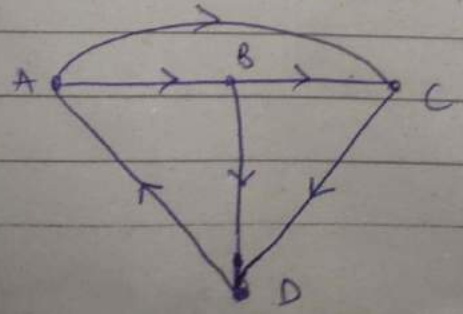






Ques Draw the graph of given network :





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- **Co-tree**: The complement of Tree is known as a co-tree (A co-tree have mesh/loop)

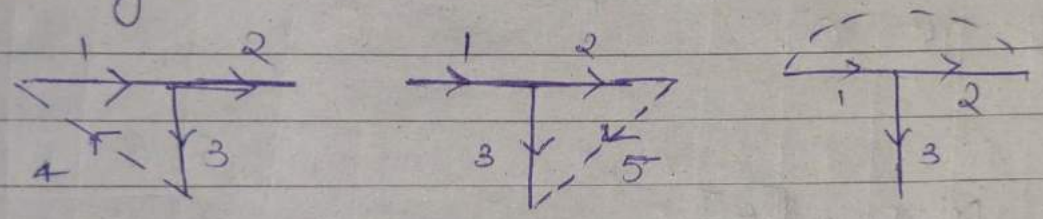
- **Twigs**: The branches of a tree is called "Twigs".

- **Links**: The branches of a co-tree are called "Links".

[Twigs: generally represented by bold lines
Links: generally represented by dotted lines]

- **Fundamental Tree**: one in which, on adding a link at a time, always we obtain 'mesh'.

For eg:



- **Tie-Sets**: combination of ONE link & several twigs to form mesh or loop ~~(as~~ Eg - (4, 1, 3), (5, 3, 2) and (6, 2, 1) ~~and t~~ are tie set for above graph.

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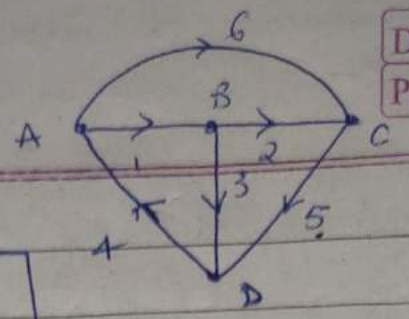
INCIDENCE MATRIX [A] :



NODE	1	2	3	4	5	6
A	1	0	0	-1	0	1
B	-1	1	1	0	0	0
C	0	-1	0	0	1	-1
D	0	0	-1	1	-1	0

→ denoted by "A" known as complete incidence matrix

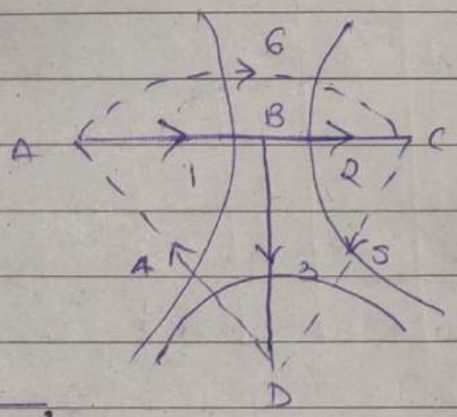
→ out going direction will get +1

→ incoming direction will get -1

→ remaing get 0.

$A = [A_t : A_l]$

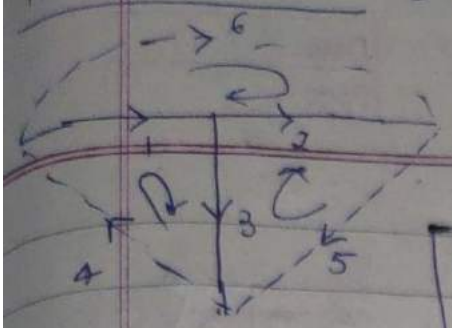
CUTSET MATRIX [Q] :



CUT SETS	TWIGS			LINKS		
	1	2	3	4	5	6
(1, 4, 6) →	1	0	0	-1	0	1
(2, 5, 6) →	0	1	0	0	-1	1
(3, 4, 5)	0	0	1	-1	1	0

$[Q] = [U : Q_L]$

TIESET MATRIX [B]:



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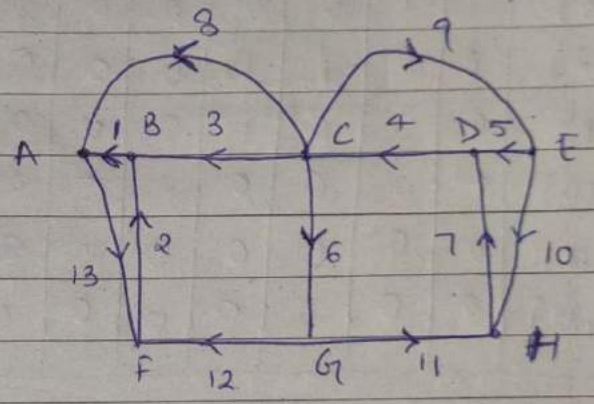
TIE SETS	TWIGS			LINKS		
	1	2	3	4	5	6
(4,1,3)	1	0	1	1	0	0
(5,3,2)	0	1	-1	0	1	0
(6,2,1)	-1	-1	0	0	0	1

Ques Find incidence, cutset and tieset matrix of the graph given below.



[A] =

NODE	1	2	3	4	5	6	7	8	9	10	11	12	13
A	-1	0	0	0	0	0	0	-1	0	0	0	0	1
B	1	-1	-1	0	0	0	0	0	0	0	0	0	0
C	0	0	1	-1	0	1	0	1	1	0	0	0	0
D	0	0	0	1	-1	0	-1	0	0	0	0	0	0
E	0	0	0	0	1	0	0	0	-1	1	0	0	
F	0	1	0	0	0	0	0	0	0	0	0	-1	-1
G	0	0	0	0	0	-1	0	0	0	0	1	1	0
H	0	0	0	0	0	0	1	0	0	-1	-1	0	0

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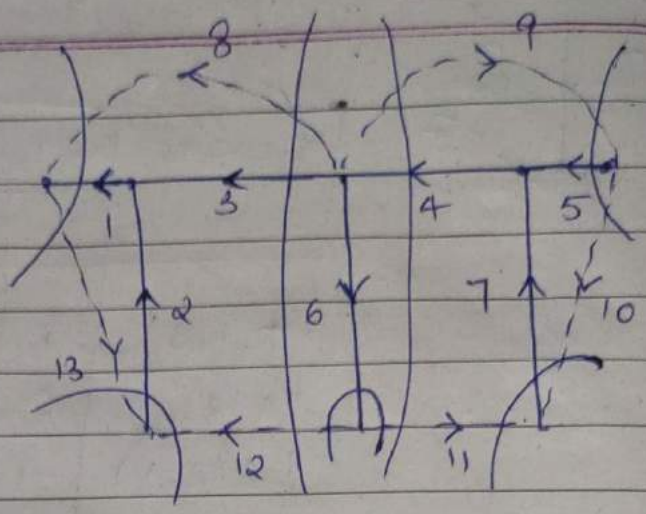
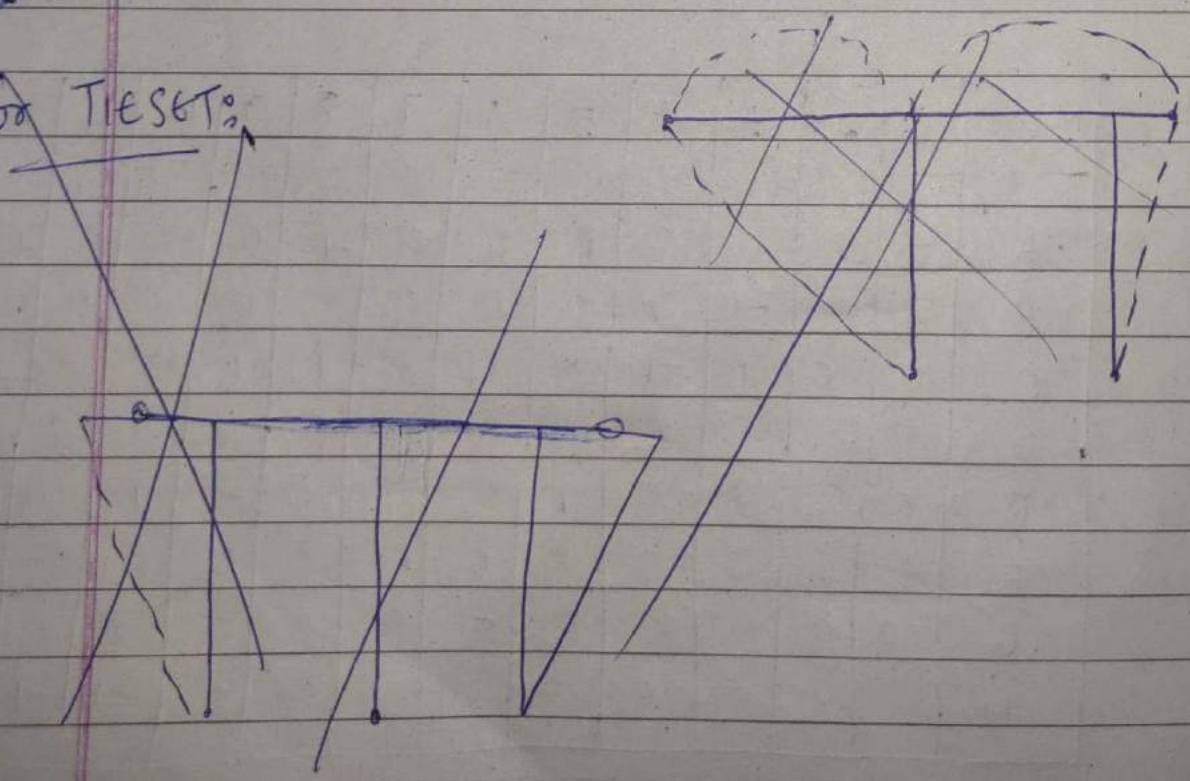
for CUTSET :

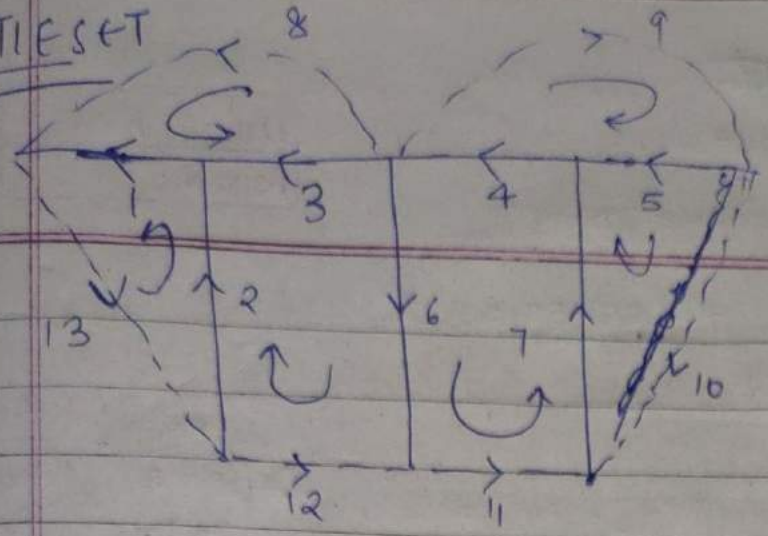


CUTSET	TWIGS							LINKS					
	1	2	3	4	5	6	7	8	9	10	11	12	13
(1,8,13)	1	0	0	0	0	0	0	1	0	0	0	0	-1
(2,12,13)	0	1	0	0	0	0	0	0	0	0	0	-1	-1
(3,8,12)	0	0	1	0	0	0	0	1	0	0	0	1	0
(4,9,11)	0	0	0	1	0	0	0	0	-1	0	-1	0	0
(5,9,10)	0	0	0	0	1	0	0	0	-1	1	0	0	0
(6,11,12)	0	0	0	0	0	1	0	0	0	0	-1	-1	0
(7,10,11)	0	0	0	0	0	0	1	0	0	-1	-1	0	0

for TIESET:



for TIESET



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TIESET	TWIGS							LINKS					
	1	2	3	4	5	6	7	8	9	10	11	12	13
(8,1,3)	-1	0	-1	0	0	0	0	1	0	0	0	0	0
(9,5,4)	0	0	0	1	1	0	0	0	1	0	0	0	0
(10,7,5)	0	0	0	0	-1	0	1	0	0	1	0	0	0
(11,7,4,6)	0	0	0	1	0	1	1	0	0	0	1	0	0
(12,2,3,6)	0	1	-1	0	0	1	0	0	0	0	0	1	0
(13,2,1)	1	1	0	0	0	0	0	0	0	0	0	0	1

UNIT - IV

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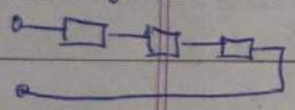
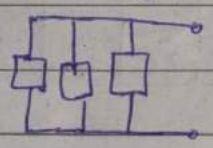
~~NETWORK FUNCTIONS~~

NETWORK - SYNTHESIS

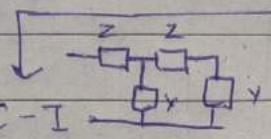
- **FOSTER'S FORM** (Partial fraction Expansion)
  - **F-I** for $Z(s)$ only



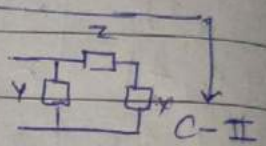
  - **F-II** for $Y(s)$ only



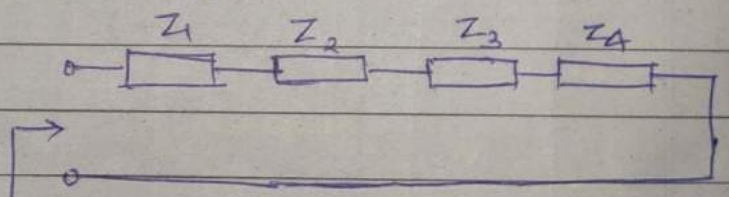
- **CAUER'S FORM** (Ladder form) (Long division method)
  - **C-I** when long division is done with highest power of 's' first [Both $Z(s)$ & $Y(s)$]



  - **C-II** when Long Division is done with lowest power of 's' first [Both $Z(s)$ & $Y(s)$]



(I) FOSTER'S - I or IMPEDANCE FORM:



$$Z(s) = Z_1(s) + Z_2(s) + Z_3(s) + Z_4(s) \ldots$$

Each block is — Either R, sL & $\frac{1}{sC}$

or Parellel R-L, Parallel RC & Pallel LC

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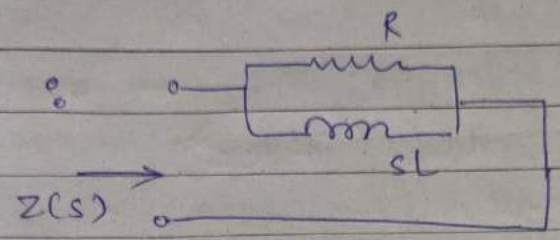
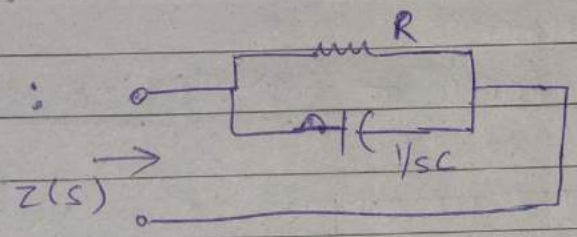
(ii) Parallel R-L :



$$Z(s) = \frac{R \cdot sL}{R + sL} = \frac{sRL}{L(s + R/L)} = \frac{sR}{s + R/L}$$

in form $\left(\frac{sA}{s+B}\right)$

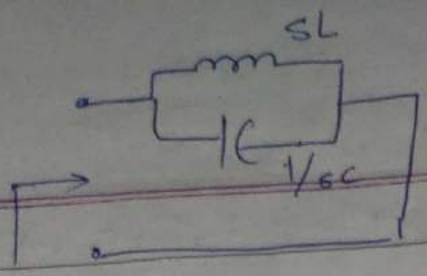
(iii) Parallel R-C :



$$Z(s) = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{sCR + 1}$$

$$= \frac{R}{CR\left(s + \frac{1}{RC}\right)} = \frac{1/C}{s + 1/RC}$$ in form $\left(\frac{A}{s+B}\right)$

• Parallel L-C



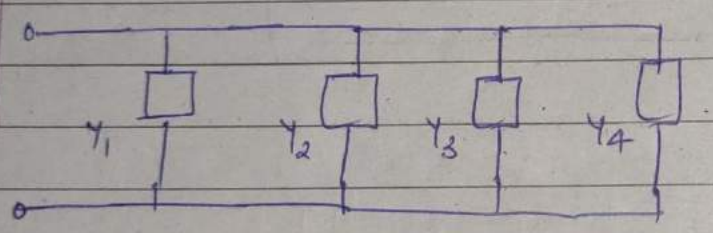
Z(s)

$$Z(s) = \frac{sL \cdot 1/sC}{sL + 1/sC} = \frac{sL}{s^2LC + 1}$$

$$= \frac{sL}{LC(s^2 + 1/LC)}$$

$$= \frac{s\,(1/C)}{s^2 + 1/LC} \quad \text{in form} \quad \frac{sA}{s^2 + B}$$

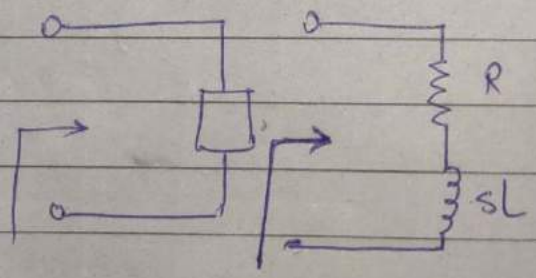
II. FOSTER - II FORM or ADMITTANCE FORM:



$$Y(s) = Y_1(s) + Y_2(s) + Y_3(s) + Y_4(s) + \ldots$$

Each block is — Either $(1/R)$, $(1/sL)$ or sC
— or series R-L, series RC & series LC

(i) Series R-L



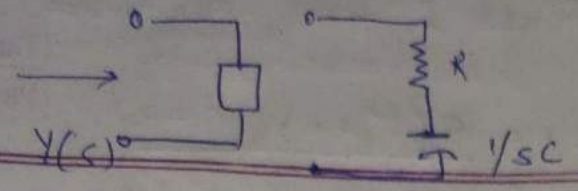
Y(s)

$$Y(s) = \frac{1}{R + sL}$$

$$= \frac{1/L}{s + R/L}$$

$$= \frac{A}{s + B}$$

ii) Series R-C



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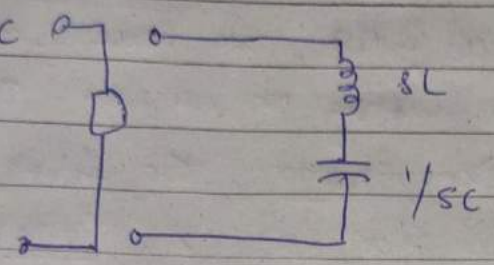
$$Y(s) = \frac{1}{R + 1/sC}$$

$$= \frac{sC}{sCR + 1} = \frac{(1/R)\, s}{(s + 1/RC)} = \frac{sA}{s+B}$$

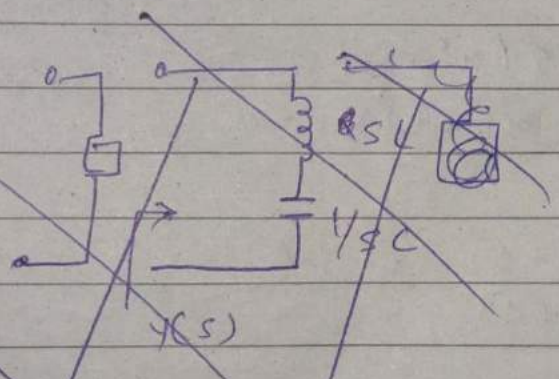
iii) Series L-C



$$Y(s) = \frac{1}{sL + 1/sC} = \frac{sC}{s^2LC + 1}$$

$$= \frac{(1/L)\, s}{s^2 + 1/LC} \text{ in form of } \frac{sA}{s^2 + B}$$

iv) Series L-C



$$Y(s) = \frac{1}{sL + \frac{1}{sC}} = \frac{sC}{s^2LC + 1}$$

$$= \frac{(1/L)\, s}{s^2 + 1/C} \text{ in } y$$

Ques (a) Using the Foster I form, synthesize the impedance function

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$$Z(s) = \frac{8(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

(b) Using Foster II form, synthesize the admittance function

$$Y(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$$

Sol^n Since in both (a) & (b), we have missing terms (all odd terms missing or all even terms missing) in Numerator or Denominator, So first step is to write down $\frac{Z(s)}{s}$ for (a) & $\frac{Y(s)}{s}$ for b. The second step is to replace s^2 by 'x' and then find out partial ~~functions~~ fractions.
Finally get Z(s) & Y(s) terms and replace by L-C networks.

(a) $$\frac{Z(s)}{s} = \frac{8(s^2+1)(s^2+3)}{s^2(s^2+2)(s^2+4)}$$

putting $s^2 = x$

$$\frac{Z(s)}{s} = \frac{8(x+1)(x+3)}{x(x+2)(x+4)}$$

let $$\frac{8(x+1)(x+3)}{x(x+2)(x+4)} = \frac{A}{x} + \frac{B}{x+2} + \frac{D}{x+4}$$

$$A = \frac{8 \times 1 \times 3}{2 \times 4} = 3$$

$$B = \frac{8 \times (-1) \times 1}{(-2) \times 2} = 2$$

$$D = \frac{8 \times (-3) \times (-1)}{(-4) \times (-2)} = 3$$

$$\frac{Z(s)}{s} = \frac{3}{x} + \frac{2}{x+2} + \frac{3}{x+4}$$

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$$\frac{Z(s)}{s} = \frac{3}{s^2} + \frac{2}{s^2+2} + \frac{3}{s^2+4}$$

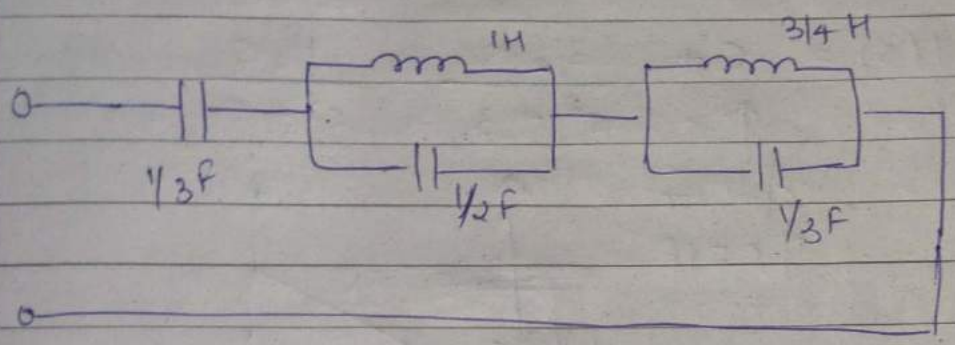
$$Z(s) = \frac{3}{s} + \frac{2s}{s^2+2} + \frac{3s}{s^2+4}$$

Capacitor of 3F

$$\frac{(1/c)\,s}{s^2+\frac{1}{LC}} \qquad \frac{(1/c)\,s}{s^2+1/LC}$$

$C = 1/2\ F$ $\qquad$ $C = 1/3\ F$

$L = 1H$ $\qquad$ $L = 3/4\ H$



(b) $$\frac{Y(s)}{s} = \frac{(s^2+4)(s^2+6)}{(s^2+3)(s^2+5)}$$

put $s^2 = x$

$$\frac{Y(s)}{s} = \frac{(x+4)(x+6)}{(x+3)(x+5)} = \frac{x^2+10x+24}{x^2+8x+15}$$

$$\begin{array}{r|l} x^2+8x+15 & x^2+10x+24 \quad (1 \\ & \underline{x^2+8x+15} \\ & \quad 2x+9 \end{array}$$

$$\frac{Y(s)}{s} = 1 + \frac{2x+9}{x^2+8x+15} = 1 + \frac{2x+9}{(x+3)(x+5)}$$

$$\frac{2x+9}{(x+3)(x+5)} = \frac{A}{x+3} + \frac{B}{x+5}$$

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$$A = \frac{2\times -3+9}{2} = 3/2$$

$$B = \frac{2\times -5+9}{-2} = \frac{-1}{-2} = 1/2$$

$$\frac{Y(s)}{s} = 1 + \frac{3/2}{x+3} + \frac{1/2}{x+5}$$

$$\frac{Y(s)}{s} = 1 + \frac{3/2}{s^2+3} + \frac{1/2}{s^2+5}$$

$$Y(s) = s + \frac{3/2\, s}{s^2+3} \quad \frac{1/2\, s}{s^2+5}$$

Capacitor of $C = 1F$

$\cancel{C = 3/2}$

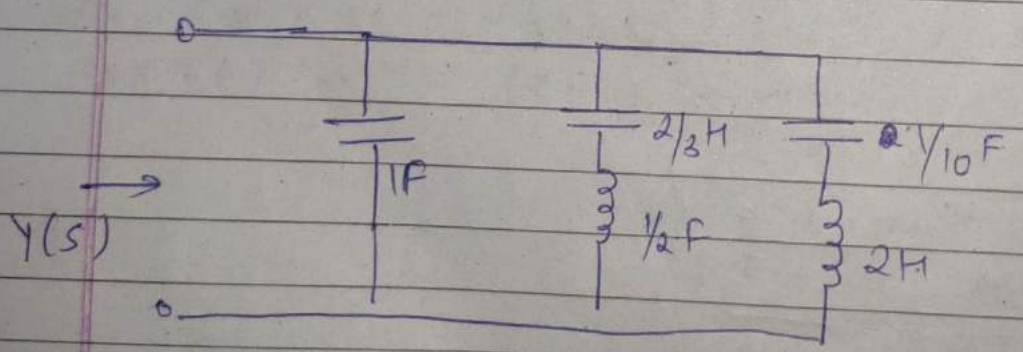
$$\frac{(1/L)s}{s^2 + 1/LC}$$

$L = 2/3 H$

$C = 1/2 F$

$L = 2H$

$C = \frac{1}{10} F$



III CAUER'S I FORM : (LONG DIVISION)

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→ Applicable for both $Z(s)$ & $Y(s)$ forms

→ long division (Repeated one) is done, starting from higher powers of s.

Ques Realize the function $Z(s) = \dfrac{(s+1)(s+4)}{s(s+2)(s+5)}$ in C-I form

Sol^n

$$Z(s) = \frac{s^2+5s+4}{s^3+7s^2+10s}$$

$$s^3+7s^2+10s \,\big)\, s^2+5s+4 \,\big(\, 1/s$$
$$\underline{s^2+7s+10}$$
$$-2s-6$$

(-ve) so no need to proceed

In such cases, we shall take reciprocal & proceed for : $Y(s) = \dfrac{s^3+7s^2+10s}{s^2+5s+4}$

$$s^2+5s+4 \,\big)\, s^3+7s^2+10s \,\big(\, s \longrightarrow Y_1(s) \rightarrow C = 1F$$
$$\underline{s^3+5s^2+4s}$$

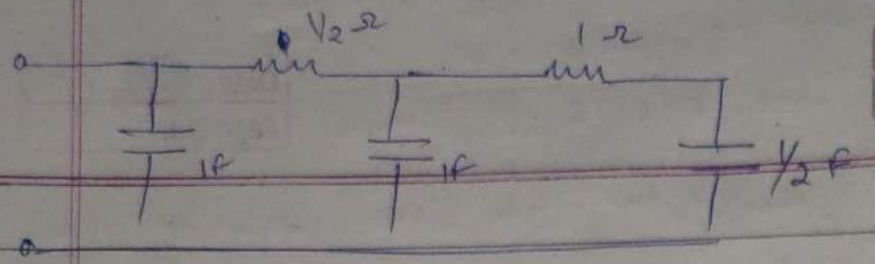
$R = 1/2\ \Omega$

$$2s^2+6s \,\big)\, s^2+5s+4 \,\big(\, 1/2 \rightarrow Z_2(s)$$
$$\underline{s^2+3s}$$

$C = 1F \leftarrow Y_3(s)$

$$2s+4 \,\big)\, 2s^2+6s \,\big(\, s$$
$$\underline{2s^2+4s}$$
$$2s$$

$$2s \,\big)\, 2s+4 \,\big(\, 1 \rightarrow Z_4(s) \rightarrow R = 1\Omega$$
$$\underline{2s}$$
$$4 \,\big)\, 2s \,\big(\, s/2 \rightarrow Y_5(s) \rightarrow C = 1/2\,F$$
$$\underline{2s}$$
$$0$$



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IV CAUER'S - II FORM:

- Again this is also applicable for both $Z(s)$ & $Y(s)$
- BUT, now long division is done, starting from lower power of s.

Ques 2 Realize the function $Z(s) = \dfrac{(s+1)(s+4)}{s(s+2)(s+5)}$ in C-II form

Soln

$$Z(s) = \frac{s^2+5s+4}{s^3+7s^2+10s} = \frac{4+5s+s^2}{10s+7s^2+s^3}$$

$$10s+7s^2+s^3 \overline{)\,4+5s+s^2\,(} \cancel{2/5s}\ \frac{2}{5s} \rightarrow Z_1(s) \rightarrow C = 5/2$$

$$\frac{4}{-} + \frac{14s}{5} + \frac{2s^2}{-\ 5}$$

$$\frac{11s}{5} + \frac{3s^2}{5} \overline{)\,10s+7s^2+s^3\,(} \frac{50}{11} \rightarrow Y_2(s) \quad R = \frac{11}{50}$$

$$10s + \frac{30s^2}{11}$$

$$\frac{47s^2}{11} + s^3 \overline{)\,\frac{11s}{5} + \frac{3}{5}s^2\,(} \frac{125}{235s} \rightarrow Z_3(s) \quad C = \frac{235}{125}$$

$$\frac{11}{5}s + \frac{121}{235}s^2$$

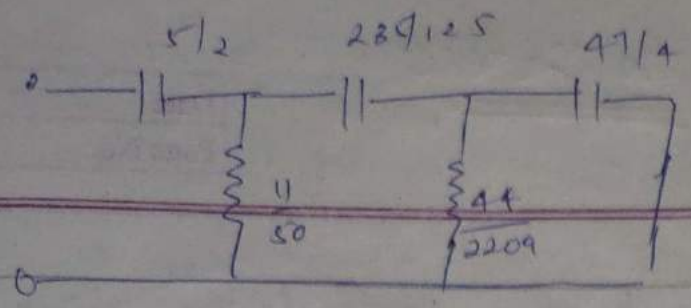
$$\frac{4}{47}s^2 \overline{)\,\frac{47s^2}{11} + s^3\,(} \frac{2209}{44} \rightarrow Y_4(s) \quad R = \frac{4}{2209}$$

$$-\ \frac{47s^2}{11}$$

$$s^3 \overline{)\,4/47\,s^2\,(} \frac{4}{47s} \rightarrow Z(s)$$

$$4/47\,s^2 \quad C = \frac{47}{4}$$

X



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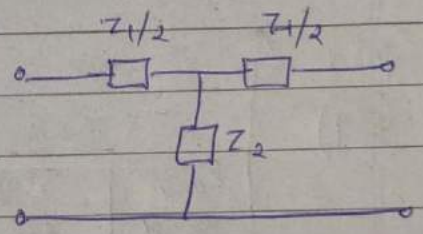
FILTERS

→ A filter is a circuit capable of passing or amplyfying certain frequencies while attenuating other frequencies.

→ An ideal filter must have zero attenuation in the pass band or transmission band. This can happen if the elements of a filter are dissipationless i.e purely reactive.

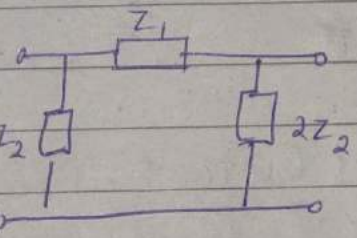
→ characteristic Impedance, Z_0

Its value for symmetrical N/w



is $\boxed{\sqrt{\frac{Z_1^2}{4} + Z_1 \cdot Z_2}}$

and for π-network



is $\frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$

Thus, a filter can extract important & desired frequencies from the signals and leaving behind those which are undesired.

TYPES OF FILTERS

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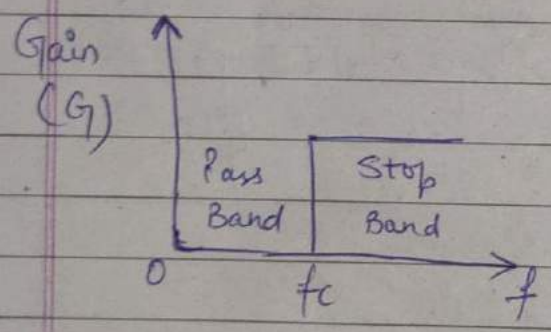
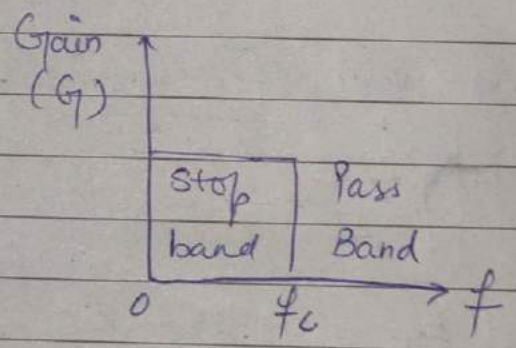
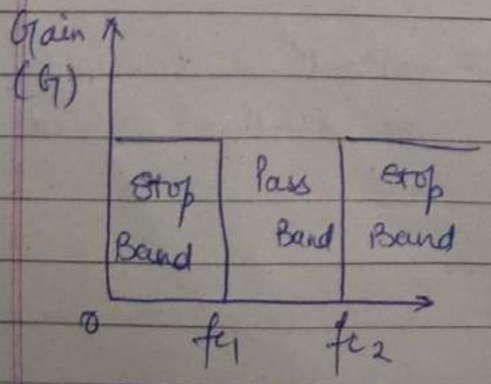
① **LOW PASS FILTERS:** These ~~band~~ filter reject all frequencies above cut-off frequency, f_c. Thus the pass band or transmission band for the LP filter is the frequency range 0 to f_c and the stop band or attenuation band is the frequency range above f_c.

② **HIGH PASS FILTER:** These filter rejects all frequency below cut off frequency, f_c. Thus the pass band or transmission band for LP filter is the frequency range 0 to f_c and the stop band or attenuation band is the frequency range above f_c.

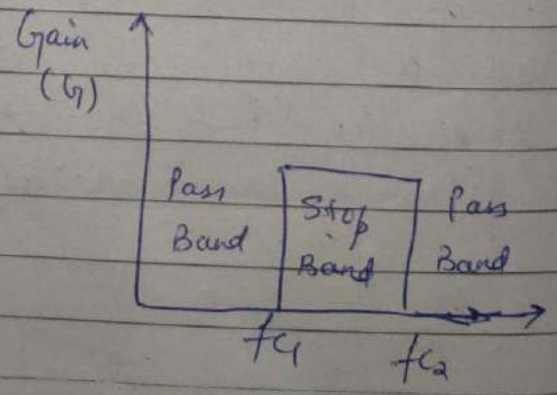




BAND PASS FILTER: These filter allow transmission of frequencies between two designated cut off frequencies and reject all other frequencies.



Band Pass



Band Stop.

Band Stop or Band Elimination Filter

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These filters pass all frequencies lying outside a certain range, while it attenuates all frequencies between the two frequencies f_{c_1} & f_{c_2}.

Ques A series LCR type Band Pass filter has $L = 50mH$, $C = 130nF$ & $R_p = 80\Omega$. Determine
(i) resonant frequency
(ii) Bandwidth
(iii) Cut off frequencies.

Sol (i) $f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{50\times10^{-3}\times130\times10^{-9}}} = 1970\ Hz$$

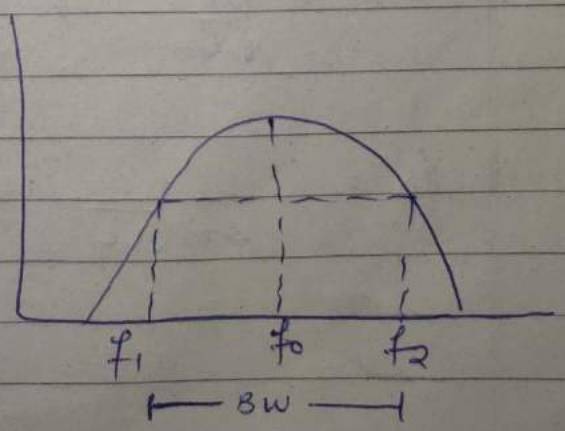
(ii) Bandwidth, $BW = \frac{R}{2\pi L} = \frac{80}{2\times\pi\times50\times10^{-3}}$

$$= 254.61\ Hz$$

(iii) Cut off frequencies, f_1 & f_2

$$f_1 = f_0 - \frac{BW}{2} = 1970 - \frac{254.6}{2} = 1842.7\ Hz$$

$$f_2 = f_0 + \frac{BW}{2} = 1970 + \frac{254.6}{2} = 2097.3\ Hz$$



Q2 Determine the cut off frequency for the HIGH PASS filter as shown in fig (a) & fig (b) below.

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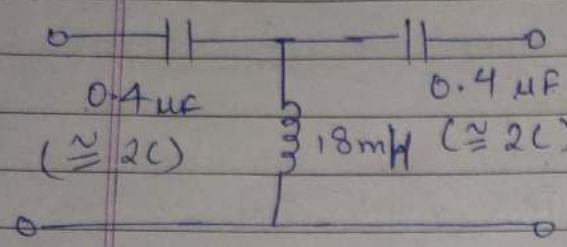

fig A

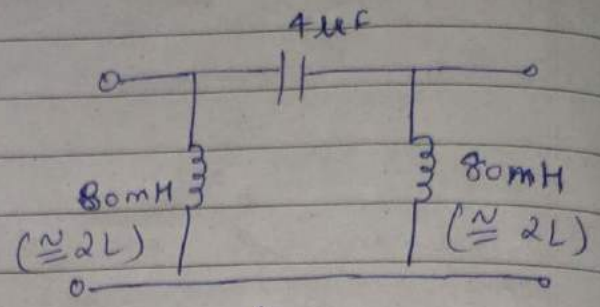

fig B

Soln We know $K = \sqrt{\frac{L}{C}}$

& $f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{K}{4\pi L}$

(a) $C = \frac{0.4}{2} = 0.2\ \mu F$

$L = 18\ mH$ $\quad K = \sqrt{\frac{18 \times 10^{-3}}{0.2 \times 10^{-6}}} = 300\ \Omega$

$f_c = \frac{K}{4\pi L}$

$= \frac{300}{4\pi \times 15 \times 10^{-3}} = 1326\ Hz$

(b) $L = \frac{80}{2} = 40\ mH$, $C = 4 \times 10^{-6}\ F$

$\cancel{K = \sqrt{\frac{L}{C}} = \sqrt{\frac{40 \times 10^{-3}}{4}}}$

$f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4 \times \pi \times \sqrt{40 \times 10^{-3} \times 4 \times 10^{-6}}}$

$= 199\ Hz$