Taylor Series:

If function $f(x)$ is continuous and its derivatives upto nth and ~~higher~~ higher order exists, then

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{\lfloor 2} f''(a) + \frac{(x-a)^3}{\lfloor 3} f'''(a)$$

$$+ \cdots + \frac{(x-a)^n}{\lfloor n} f^n(a) + \cdots$$

Rolle's Theorem

If a function $f(x)$ is

(i) continuous is closed interval $[a,b]$

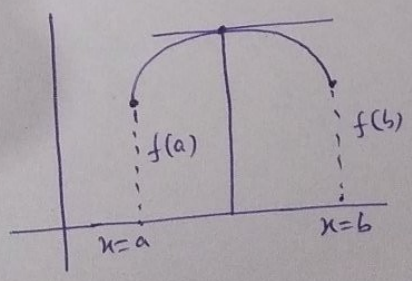
(ii) Differentiable is open interval (a,b)

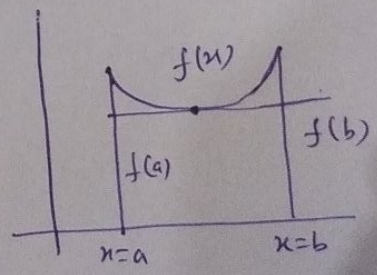
(iii) $f(a) = f(b)$

Then there exists a point $c \in (a,b)$

i.e $(a < c < b)$ such that $f'(c) = 0$

Note:- 1. Meanting of $f'(c) = 0$ i.e tangent is parallel to x-axis





(2) Meaning of Continuous in Closed interval $[a,b]$
i.e graph of the function $f(x)$ is continuous in closed interval $[a,b]$.

(3) Meaning of differentiable in open interval (a,b)
i.e We have tangent of $f(x)$ at every point.

Algebraic Interpretation of Rolle's theorem

Let $f(x)$ be polynomial in x and let $f(x)$ is continuous, then according to Rolle's theorem at least one root of $f'(x) = 0$ lies b/w a and b.

Cauchy Mean Value Theorem (Not is syllabus)

Statement:-

Let $f(x)$ and $g(x)$ be two functions defined on $[a,b]$ such that

(i) $f(x)$ and $g(x)$ are continuous on $[a,b]$

(ii) $f(x)$ and $g(x)$ are differentiable on (a,b)

(iii) $g'(x) \neq 0$ for any x in (a,b)

Then there exist at least one value c of x in open interval (a,b) such that

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}, \quad a < c < b.$$

Mean Value theorem

If function $f(x)$ is

(i) continuous in closed interval $[a, b]$

(ii) differentiable in open interval (a, b)

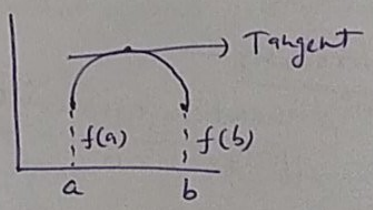
Then there exist at least one point c

$c \in (a, b)$ such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

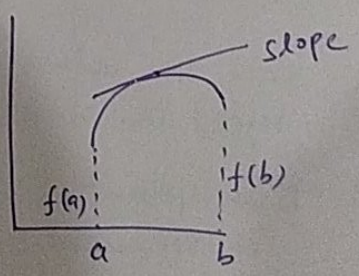
Note: · In Rolle's theorem $f(a) = f(b)$

i.e



In Mean Value Theorem

$f(a) \neq f(b)$



(2) If we want to calculate slope we use Mean Value theorem.

Solution of Algebraic and Transcendental Equations

Determination of roots of algebraic and transcendental equations is a very important problem in science and engineering

A function $f(x)$ is called algebraic, if to get the values of the function starting from the given value of x, we have to perform arithmetic operation between some real numbers and rational power of x. on the other hand transcendental functions include all non-algebraic function i.e, an exponential function, e^x, a^x, a logarithmic function $\log x$, trigonometric function $\sin x$, $\cos x$, $\tan x$, $\cot x$ ... etc. inverse trigonometric functions $\sin^{-1}x$, $\cos^{-1}x$. etc.

The definition of roots of an equation can be given in Two different ways

(i) Algebraically →

A number c is called root of the equation $f(x)=0$ iff $f(c)=0$

(ii) Geometrically →

The real roots of the equation $f(x)=0$ are the values of x where the graph of $y=f(x)$ meet the x-axis.

Note: Development of numerical methods to solve algebraic or transcendental equations are very much essential because the analytic method fails to solve the Polynomial equations of degree greater than four

Order of Convergence

Assume that the sequence $\{x_n\}$ of numbers converges to α and let $\epsilon_n = \alpha - x_n$ for $n \geq 0$

If two positive constants $A \neq 0$ and $p > 0$ exist and

$$\lim_{n \to \infty} \frac{\epsilon_{n+1}}{\epsilon_n^p} = A$$

then the sequence is said to converge to α with order of convergence p. The number A is called the asymptotic error constant.

If $p = 1$, the order of convergence of $\{x_n\}$ is called Linear

If $p = 2$, The order of convergence is called quadratic.

Note:- <u>How to Find number of iterations Required for achieving an accuracy ϵ</u>

$$\boxed{n \geq \frac{\log(b-a) - \log \epsilon}{\log 2}}$$

In Particular, the minimum number of iterations required for converging to a root in the interval $(0,1)$ for a given ϵ are as given

ϵ:	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
n:	7	10	14	17	20	24

Solution of Algebraic And Transcendental Equations

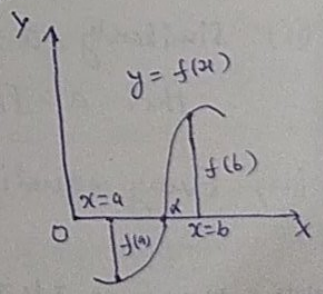
An expression of the form

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

Where a's are constant $a_0 \neq 0$ and n is positive integer, is called a polynomial in x of degree n.

~~If~~ ~~for~~) The polynomial $f(x) = 0$ is called algebraic equations of degree n. If $f(x)$ contains some other functions such as trigonometric, Logarithmic, Exponential etc. Then $f(x) = 0$ is called transcendental equation.

The value of x which satisfies $f(x) = 0$ is called its roots. Geometrically root of $f(x) = 0$ is that value of x where the graph of $y = f(x)$ crosses the x-axis. The Process of finding the roots of an equation is known as the solution of that equation



[According to Algebra, a root is the zero of the function, that is, where the function $f(x)$ is zero, there are three ways to solve the equations, namely Analytically, Graphically, And Numerically]

Intermediate Value Property

If $f(x)$ is continuous in the interval $[a,b]$ and $f(a)$, $f(b)$ have different signs, then $f(x)=0$ has atleast one root between $x=a$ and $x=b$.

Since $f(x)$ is continuous between a and b, so while x changes from a to b, $f(x)$ must pass through all values from $f(a)$ to $f(b)$. But one of these quantities $f(a)$ or $f(b)$ is positive and the other negative, it follows that atleast for one value of x (say α) lying between a and b, $f(x)$ must be zero. Then α is the required root.

Note

(i) In an equation with real coefficients, imaginary roots occur in conjugate pairs i.e if $\alpha+i\beta$ is a root of the equation $f(x)=0$ then $\alpha-i\beta$ must also be its root.

(ii) Similarly $a+\sqrt{b}$ is an irrational root of an equation, then $a-\sqrt{b}$ must also be its root.

(iii) Every equation of the odd degree has atleast one real root.

Note: How many Iteration is required

As An example, consider $f(x)=x^3+x^2-10$, if we wish to find the root of the equation $f(x)=0$ in the interval $[1,2]$ with an absolute error less than or equal to $.004$, the number of iterations required is 8

$$n = \frac{\ln \frac{1}{.004}}{\ln 2} = 7.966 \simeq 8, \qquad n \geq \frac{\ln\left(\frac{b-a}{\epsilon_a}\right)}{\ln 2}$$

Bisection Method

Bisection method is based on the repeated application of the intermediate value property. Let the function $f(x)$ be continuous between a and b, let $f(a)$ be negative and $f(b)$ be positive. Then the first approximation to the root is

$$x_1 = \frac{a+b}{2}.$$

If $f(x_1) = 0$, then x_1 is a root of $f(x) = 0$. otherwise the root lies between a and x_1 or x_1 and b according as $f(x_1)$ is positive or negative. Then we bisect the interval as before and continue the process until the root is found to desired accuracy.

Q.1 Find a root of the equation $x^3-4x-9=0$, using the Bisection method correct to three decimal places. Also find negative root of the equation $x^3-4x-9=0$

Sol^n Let $f(x) = x^3-4x-9$.

$f(2) = 8-8-9 = -9$ Negative

$f(3) = (3)^3-4(3)-9 = 27-12-9 = 6$ Positive

Since $f(2)$ is $-ve$ and $f(3)$ is $+ve$, Hence Root lies between 2 and 3.

First Approximation to the root is

$$x_1 = \frac{2+3}{2} = 2.5$$

$$f(x_1) = f(2.5) = (2.5)^3 - 4(2.5)-9 = -3.375 \text{ (Negative)}$$

$\therefore$ The root lies between x_1 and 3

Second Approximation to the root is

$$x_2 = \frac{1}{2}(2.5+3) = 2.75$$

$$f(x_2) = f(2.75) = (2.75)^3 - 4(2.75)-9 = .7969 \quad +ve$$

$\therefore$ The root lies between x_1 and x_2.

Thus Third approximation to root is

$$x_3 = \frac{2.5+2.75}{2} = 2.625$$

$$f(x_3) = f(2.625) = (2.625)^3 - 4(2.625)-9 = -1.4121 \text{ (Negative}$$

The root lies between 2.625 and 2.75

Fourth approximation to the root is

$$x_4 = \frac{2.625+2.75}{2} = 2.6875$$

$$\therefore f(x_4) = (2.6875)^3 - 4(2.6875)-9 = -.3391$$

Hence root lies between 2.6875 and 2.75

$x_5 = \frac{2.6875+2.75}{2} = 2.7185$

$f(2.7185) = (2.7185)^3 - 4(2.7185) - 9 = .1216$ (+ve)

Hence Root lies between 2.6875 and 2.7185

$x_6 = \frac{2.6875 + 2.7185}{2} = 2.703$

$f(x_6) = (2.703)^3 - 4(2.703) - 9 = -.063$ Negative

Hence Root lies between 2.703 and 2.7185

$x_7 = \frac{2.703 + 2.7185}{2} = 2.7107$

$f(2.7107) = (2.7107)^3 - 4(2.7107) - 9 = .075$ i.e +ve

Hence root lies between 2.703 and 2.7107

$x_8 = \frac{2.703 + 2.7107}{2} = 2.7068$

$f(2.7068) = (2.7068)^3 - 4(2.7068) - 9 =$ +ve

Hence Root lies between 2.703 and 2.7068

$x_9 = \frac{2.703 + 2.7068}{2} = 2.7049$

$f(2.7049) = -.06$ Negative

i.e Root lies between 2.7049 and 2.7068

$x_{10} = \frac{2.7049 + 2.7068}{2} = 2.7058$

$f(2.7058) = -$ve

i.e Root lies between 2.7058 and 2.7068

$x_{11} = \frac{2.7058 + 2.7068}{2} = 2.706$

$f(2.706) = -ve$

Hence Root lies between 2.706 and 2.7068

$$x_{12} = \frac{2.706 + 2.7068}{2} = 2.706$$

Here x_{11} and $x_{12} = 2.706$ are same

so Desired root is 2.706

Ans

The negative root of the given equation $x^3 - 4x - 9 = 0$ is the Positive root of $x^3 - 4x - 9 = 0$ which we have found above to 2.706

Hence Negative root of the given equation $x^3 - 4x - 9 = 0$ is

-2.706

Note If α, β, γ are the roots of the given equation, then $-\alpha, -\beta, -\gamma$ are the roots of $(-x)^3 - 4(-x) - 9 = 0$ Ans

Order of Convergence of Bisection Method

In Bisection method if root lies between x_0 and x_1

then $x_2 = \frac{x_0 + x_1}{2}$

In general $x_{i+1} = \frac{x_{i-1} + x_i}{2}$

$$e_{i+1} + \alpha = \frac{e_{i-1} + \alpha + e_i + \alpha}{2}$$

$$e_{i+1} + \alpha = \frac{e_{i-1} + e_i}{2} + \alpha$$

$$e_{i+1} = \frac{e_i}{2}\left[1 + \frac{e_{i-1}}{e_i}\right]$$

Since $\frac{e_{i-1}}{e_i}$ is very small so we can neglect it

$$\Rightarrow \quad e_{i+1} = \frac{e_i}{2}$$

$$|e_{i+1}| = \frac{1}{2}|e_i|^1$$

$\Rightarrow p = 1$ [on comparing it by $|e_{i+1}| = k|e_i|^p$

i.e order of convergence of Bisection method is one or linear.

Q.1 Find a real root of $x^3 - 2x - 5 = 0$ by Bisection method.

Solⁿ Let $f(x) = x^3 - 2x - 5$

$f(2) = 8 - 4 - 5 = -1$

$f(2.1) = (2.1)^3 - 2(2.1) - 5 =$

$= +ve$

i.e $f(2) = -ve$ and $f(2.1)$ is positive

Hence Root of the equation lies between 2 and 2.1

First approximation to the root

$$x^{(1)} = \frac{2 + 2.1}{2} = 2.05$$

Now we check

$f(2.05) = (2.05)^3 - 2(2.05) - 5 = -.4848 = -ve$

Hence Root lies between 2.05 and 2.1

Second approximation to the root

$$x^{(2)} = \frac{2.05 + 2.1}{2} = 2.075$$

Now we check

$f(2.075) = (2.075)^3 - 2(2.075) - 5 = -.2158 = -ve$

Hence Root lies between 2.075 and 2.1

Third approximation to the root

$$x^{(3)} = \frac{2.075 + 2.1}{2} = 2.087$$

Now we check

$f(2.087) = (2.087)^3 - 2(2.087) - 5 = -.083$ Negative

i.e Root lies between 2.087 and 2.1

Fourth approximation to the root

$$x^{(4)} = \frac{2.087+2.1}{2} = 2.0935$$

Now Check

$$f(2.0935) = (2.0935)^3 - 2(2.0935) - 5 = -.0011 \text{ Negative}$$

Hence Root lies between 2.0935 and 2.1

$$x^{(5)} = \frac{2.0935+2.1}{2} = 2.0967$$

$$f(2.0967) = (2.0967)^2 - 2(2.0967) - 5 = .024 \text{ Positive}$$
$$= +ve$$

Hence Root lies between 2.0935 and 2.0967

$$\therefore x^{(6)} = \frac{2.0935+2.0967}{2} = 2.0951$$

$$\therefore f(2.0951) = (2.095)^3 - 2(2.0951) - 5 = .00612 = +ve$$

Hence Root lies between 2.0935 and 2.0951

$$x^{(7)} = \frac{2.0935+2.0951}{2} = 2.0943$$

$$\therefore f(2.0943) = (2.0943)^3 - 2(2.0943) - 5 = -.002080 = -ve$$

Hence Root lies between 2.0943 and 2.0951

$$x^{(8)} = \frac{2.0943+2.0951}{2} = 2.0947$$

$$f(2.0947) = (2.0947)^3 - 2(2.0947) - 5 = .001789 = +ve$$

i.e Root lies between 2.0943 and 2.0947

$$x^{(9)} = \frac{2.0943+2.0947}{2} = 2.0945$$

$$f(2.0945) = (2.0945)^3 - 2(2.0945) - 5 = -.00057$$
$$= -ve$$

Hence Root lies between 2.0945 and 2.0947

$$x^{(10)} = \frac{2.0945 + 2.0947}{2} = 2.0946$$

$$f(2.0946) = (2.0946)^3 - 2(2.0946) - 5 = 1.000545 = +ve$$

Hence Root lies between 2.0945 and 2.0946

$$x^{(11)} = \frac{2.0945 + 2.0946}{2} = 2.0945$$

$$f(2.0945) = (2.0945)^3 - 2(2.0945) - 5 = -1.0005745$$

Hence Root lies between 2.094 and 2.094

Hence Root are 2.094

Q.2. Find the real root of the equation, using Bisection method correct to three decimal place.

$$x \log_{10} x = 1.2.$$

Sol^n. Let $f(x) = x \log_{10} x - 1.2$

$$f(2.7) = 2.7 \log_{10} 2.7 - 1.2 = -.0353 \ (-ve)$$

$$f(2.8) = 2.8 \log_{10} 2.8 - 1.2 = .0520 \ (+ve)$$

Hence Root of the equation lies between 2.7 and 2.8

Now First approximation to the root is

$$x^{(1)} = \frac{2.7 + 2.8}{2} = 2.75$$

$$\therefore f(2.75) = 2.75 \log_{10} 2.75 - 1.2 = .00816 \ (+ve)$$

Hence Root lies between 2.7 and 2.75

Second approximation to the root is

$$x^{(2)} = \frac{2.7+2.75}{2} = 2.725$$

$\therefore f(2.725) = 2.725 \log_{10} 2.725 - 1.2 = -.0136$ (-ve)

Hence Root lies between 2.725 and 2.75

Third Approximation to the root is

$$x^{(3)} = \frac{2.725+2.75}{2} = 2.7375$$

$\therefore f(2.7375) = 2.7375 \log_{10} 2.7375 - 1.2 = -.00274$ (-ve)

i.e Root lies b/w 2.7375 and 2.75

Fourth approximation to the root is

$$x^{(4)} = \frac{2.7375+2.75}{2} = 2.7437$$

$f(2.7437) = 2.7437 \log_{10} 2.7437 - 1.2 = .00266$ (+ve)

Hence Root lies b/w 2.7375 and 2.7437

Fifth approximation to the root is

$$x^{(5)} = \frac{2.7375+2.7437}{2} = 2.7406$$

$\therefore f(2.7406) = 2.7406 \log_{10} 2.7406 - 1.2 = -.0000402$ (-ve)

Hence Root lies 2.7406 and 2.7437

$\therefore$ Sixth approximation to the root is

$$x^{(6)} = \frac{2.7406+2.7437}{2} = 2.74215$$

$f(2.74215) = 2.74215 \log_{10} 2.74215 - 1.2 = .0013$ (-ve)

Hence Root lies b/w 2.74215 and 2.743

$$x^{(7)} = \frac{2.74215+2.743}{2} = 2.7425$$

$\therefore f(2.7425) = 2.7425 \log_{10} 2.7425 - 1.2 = .00161$ (+ve)

Hence Roots lies between 2.7421 and 2.742

Hence upto three decimal place i.e 2.742 are same

So Reqd. Root is 2.742 Ans

Q.3. Find the Positive real root of $x - \cos x = 0$ by Bisection method correct ~~to~~ up to three decimal Places between 0 and 1.

Sol^n Let $f(x) = x - \cos x = 0$

$\cancel{f(.75) = .75 - \cos(57.273 \times .75)} =$

$f(.7) = .7 - \cos(57.273 \times .7) = -ve$

$f(.74) = .74 - \cos(57.273 \times .74) = +ve$

Hence Root lies between .7 and .74

$\therefore$ First approximation to the root is

$x^{(1)} = \frac{.7 + .74}{2} = .72$

$\therefore f(.72) = .72 - \cos(57.273 \times .72) = -.031$ (-ve)

Hence Root lies b/w .72 and .74

$x^{(2)} = \frac{.72 + .74}{2} = .73$

$\therefore f(.73) = .73 - \cos(57.273 \times .73) = -.015$ (-ve)

Hence Root lies b/w .73 and .74

$\therefore x^{(3)} = \frac{.73 + .74}{2} = .735$

$\therefore f(.735) = .735 - \cos(57.273 \times .735) = -ve$

Hence Roots lies b/w .735 and .74

$\therefore x^{(4)} = \frac{.735 + .74}{2} = .7375$

$f(.7375) = .7375 - \cos(57.273 \times .7375) = -ve$

Hence Root lies between .7375 and .74

Now fifth approximation to the root is

$x^{(5)} = \frac{.7375 + .74}{2} = .7387$

$\therefore f(.7387) = .7387 - \cos(57.273 \times .7387) = -ve$

so Root lies b/w .7387 and .74

$x^{(6)} = \frac{.7387 + .74}{2} = .73935 = .7394$

$\therefore f(.7394) = .7394 - \cos(57.273 \times .7394) = +ve$

Hence Root lies between .7387 and .7394

$x^{(7)} = \frac{.7387 + .7394}{2} = .7391$

$\therefore f(.7391) = .7391 - \cos(57.273 \times .7391) = -.00017$
$= -ve$

Hence Root lies between ~~.7387 and~~

.7391 and .7394

$\therefore x^{(8)} = \frac{.7391 + .7394}{2} = .7392$

$\therefore f(.7392) = .7392 - \cos(57.273 \times .7392) = -.000005$
$= -ve$

Hence Root lies b/w .7392 and .7394

$x^{(9)} = \frac{.7392 + .7394}{2} = .7393$

$\therefore f(.7393) = .7393 - \cos(57.273 \times .7393) = +.0002 = +ve$

Hence Root lies b/w $\cdot 7392$ and $\cdot 7393$

$$x^{(10)} = \frac{\cdot 7392 + \cdot 7393}{2} = \cdot 7393$$

$\therefore f(\cdot 7393) = \cdot 7393 - \cos(57.273 \times \cdot 7393) = \cdot 0002$

Hence Root is $\underline{\cdot 7393}$ Ans

Q.④ Find a Positive root of the equation $xe^x = 1$, lying in $(0,1)$ by Bisection method.

Sol^n Let $f(x) = xe^x - 1 = 0$

Put $x = 0$

$f(0) = 0 - 1 = -ve$

$f(\cdot 1) = \cdot 1 e^{\cdot 1} - 1 = -ve$

$f(\cdot 2) = \cdot 2 e^{\cdot 2} - 1 = -ve$

$f(\cdot 3) = \cdot 3 e^{\cdot 3} - 1 = -ve$

$f(\cdot 4) = \cdot 4 e^{\cdot 4} - 1 = -ve$

$f(\cdot 5) = \cdot 5 e^{\cdot 5} - 1 = -ve$

$f(\cdot 6) = \cdot 6 e^{\cdot 6} - 1 = +ve$

Here $f(\cdot 5)$ is Negative and $f(\cdot 6)$ is Positive Hence Root of the equation lies between $\cdot 5$ and $\cdot 6$ (Here $x_0 = \cdot 5$, $x_1 = \cdot 6$)

Now First Approximation to the root is

$$x^{(1)} = \frac{x_0 + x_1}{2} = \frac{\cdot 5 + \cdot 6}{2} = \cdot 55$$

$\therefore f(\cdot 55) = \cdot 55 e^{\cdot 55} - 1 = -\cdot 0467$ (-ve)

Hence Root lies b/w $\cdot 55$ and $\cdot 6$

Second approximation to the root is

$$x^{(2)} = \frac{\cdot 55 + \cdot 6}{2} = \cdot 575$$

$f(\cdot 575) = \cdot 575 e^{\cdot 575} - 1 = \cdot 0218$ (+ve)

Hence Root lies between ($\cdot 55$ and $\cdot 575$)

$\therefore$ Third approximation to the root is

$$x^{(3)} = \frac{.55 + .575}{2} = .5625$$

$$f(.5625) = .5625\, e^{.5625} - 1 = -.0127 = -ve$$

Hence Root lies between .5625 and .575

$\therefore$ Fourth approximation to the root is

$$x^{(4)} = \frac{.5625 + .575}{2} = .5687$$

$$\therefore f(.5687) = .5687\, e^{.5687} - 1 = .00430 = +ve$$

Hence Root lies between (.5625 and .5687)

Fifth approximation to the root is

$$x^{(5)} = \frac{.5625 + .5687}{2} = .5656$$

$$\therefore f(.5626) = .5656\, e^{.5656} - 1 = -ve$$

So root lies between (.5656 and .5687)

Sixth approximation to the root is

$$x^{(6)} = \frac{.5656 + .5687}{2} = .5671$$

$$f(.5671) = .5671\, e^{.5671} - 1 = -ve$$

So Root lies between (.5671 and .5687)

Seventh approximation to the root is

$$x^{(7)} = \frac{.5671 + .5687}{2} = .5679$$

Here Sixth approximation and Seventh approximation are same upto three Places of decimal i.e $x^6 = x^7 = .567$

Hence Required root is $\underline{\underline{.567}}$ Ans

2 (5) Using Bisection method find the approximate root of the equation $\sin x = \frac{1}{x}$ correct to three places of decimal.

Let $f(x) = x \sin x - 1 = 0$

$f(1) = 1 \sin(57.3 \times 1) - 1 = -ve$

$f(1.5) = 1.5 \sin(1.5 \times 57.3) - 1 = +ve$

$f(1.4) = 1.4 \sin(1.4 \times 57.3) - 1 = +ve$

$f(1.3) = 1.3 \sin(1.3 \times 57.3) - 1 = +ve$

$f(1.2) = 1.2 \sin(1.2 \times 57.3) - 1 = +ve$

$f(1.1) = 1.1 \sin(1.1 \times 57.3) - 1 = -ve$

Here $f(1.1)$ is Negative and $f(1.2)$ is Positive so Root of the equation lies between 1.1 and 1.2

First approximation to the root is

$x^{(1)} = \frac{1.1 + 1.2}{2} = 1.15$

$f(1.15) = 1.15 \sin(1.15 \times 57.3) - 1 = .049$ (+ve)

So Root lies between 1.1 and 1.15

Second approximation to the root is

$x^{(2)} = \frac{1.1 + 1.15}{2} = 1.125$

$f(1.125) = 1.125 \sin(1.125 \times 57.3) - 1 = .015$ (+ve)

Third approximation to the root is

$x^{(3)} = \frac{1.1 + 1.125}{2} = 1.1125$

$\therefore f(1.1125) = 1.1125 \sin(1.1125 \times 57.3) - 1 = -ve$

So Root lies between (1.1125 and 1.125)

Fourth approximation to the root is

$x^{(4)} = \frac{1.1125 + 1.125}{2} = 1.11875$

$\therefore f(1.11875) = 1.11875 \sin(1.11875 \times 57.3) - 1 = +ve$

i.e root lies between 1.1125 and 1.11875

Fifth approximation to the root is

$$x^{(5)} = \frac{1.1125 + 1.11875}{2} = 1.115625$$

$\therefore f(1.1156) = 1.1156 \sin(1.1156 \times 57.3) - 1 = +ve$

i.e root lies between 1.1125 and 1.1156

Sixth approximation to the root is

$$x^{(6)} = \frac{1.1125 + 1.1156}{2} = 1.11405$$

$\therefore f(1.11405) = 1.11405 \sin(1.11405 \times 57.3) - 1 = -ve$

Hence Root lies between 1.11405 and 1.1156

$\therefore$ Seventh app. to the root is

$$x^{(7)} = \frac{1.11405 + 1.1156}{2} = 1.1148$$

Here $x^{(6)} = x^{(7)} = 1.114$ upto three places of decimal so root of the equation is $\underline{\underline{1.114}}$ Ans

<u>Regula Falsi Method or Method of False Position</u>

Let a and b are two Point,
and f(a) is Negative, f(b) is Positive, then Point c between a and b, we find as

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$a \to f(a)$
$b \to f(b)$

$\therefore f(.7393) = .4373 - \ldots$

Newton Raphson Method

Let $x = x_0$ be an approximate root of the equation $f(x) = 0$ and let $x_1 = x_0 + h$ be the correct root of the equation so that $f(x_1) = 0$, where h is very small.

$\therefore f(x_1) = 0$

$f(x_0 + h) = 0$

Expend by Taylor' Series Method.

$$f(x_0) + h f'(x_0) + \frac{h^2}{\lfloor 2} f''(x_0) + \cdots + \cdots = 0$$

Here h is very small so $h^2, h^3, \ldots$ become very very small

so we neglecting $h^2, h^3, \ldots$

$$\therefore f(x_0) + h f'(x_0) = 0$$

$$h = -\frac{f(x_0)}{f'(x_0)}$$

$\because x_1 = x_0 + h$, putting the value of h

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

similarly $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$\vdots$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Rate of Convergence of Newton Raphson Method

Let x_n be different from the root 'α' of $f(x)=0$ by a small quantity e_n so that

$$\left.\begin{aligned} x_n &= \alpha + e_n \\ x_{n+1} &= \alpha + e_{n+1} \end{aligned}\right\} \longrightarrow (1)$$

by Newton Raphson method we know that

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \longrightarrow (2)$$

Put the value of x_n and x_{n+1} from (1) in (2)

$$\cancel{\alpha} + e_{n+1} = \cancel{\alpha} + e_n - \frac{f(x_n)}{f'(x_n)}$$

$$e_{n+1} = e_n - \frac{f(\alpha+e_n)}{f'(\alpha+e_n)}$$

Using Taylor theorem

$$= e_n - \frac{f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{\lfloor 2} f''(\alpha) + \cdots}{f'(\alpha) + e_n f''(\alpha) + \frac{e_n^2}{\lfloor 2} f'''(\alpha) + \cdots} \quad \because f(\alpha)=0$$

$$= e_n - \frac{0 + e_n f'(\alpha) + \frac{e_n^2}{\lfloor 2} f''(\alpha) + \cdots}{f'(\alpha) + e_n f''(\alpha) + \frac{e_n^2}{\lfloor 2} f''(\alpha) + \cdots}$$

$$= \frac{\cancel{e_n f'(\alpha)} + e_n^2 f''(\alpha) + \frac{e_n^3}{\lfloor 2} f''(\alpha) - \cancel{e_n f'(\alpha)} - \frac{e_n^2}{\lfloor 2} f''(\alpha)}{f'(\alpha) + e_n f''(\alpha) + \frac{e_n^2}{\lfloor 2} f''(\alpha)}$$

$$= \frac{\frac{1}{2} e_n^2 f''(\alpha)}{f'(\alpha) + e_n f''(\alpha)} \quad \text{Neglecting higher order of } e_n$$

$$= \frac{e_n^2 f''(\alpha)}{2 f'(\alpha)\left[1 + e_n \frac{f''(\alpha)}{f'(\alpha)}\right]}$$

$$= \frac{e_n^2 f''(\alpha)}{2 f'(\alpha)} \left[1 + e_n \frac{f''(\alpha)}{f'(\alpha)}\right]^{-1}$$

$$= \frac{e_n^2 f''(\alpha)}{2 f'(\alpha)} \left[1 - e_n \frac{f''(\alpha)}{f'(\alpha)} \cdots\right]$$

$$= \frac{e_n^2 f''(\alpha)}{2 f'(\alpha)} - \frac{e_n^3}{2}\left[\frac{f''(\alpha)}{f'(\alpha)}\right]^2$$

Neglecting e_n^3 term

$$e_{n+1} = \frac{e_n^2 f''(\alpha)}{2 f'(\alpha)}$$

$$\boxed{e_{n+1} \propto e_n^2}$$

This shows that subsequent error at each step is proportional to the square of the previous error. Hence the convergence is quadratic.

Q. Find the real root of $x \log_{10} x = 1.2$ correct to four decimal places using Newton Raphson method.

Note (i)

$$\frac{d}{dx} \log_a x = \frac{1}{x} \log_a e$$

$$= \frac{1}{x} \log_a e$$

$$\boxed{e = 2.718}$$

Solution. Let $f(x) = x \log_{10} x - 1.2$

$f(2.7) = 2.7 \log_{10} 2.7 - 1.2 = -.035$ Negative

$f(2.8) = 2.8 \log_{10} 2.8 - 1.2 = .0520$ Positive

Hence Root of $f(x)$ lies b/w 2.7 and 2.8

So Here we take $x_0 = 2.7$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n \log_{10} x_n - 1.2}{\log_{10} e + \log_{10} x_n}$$

Here $f(x_n) = x_n \log_{10} x_n - 1.2$

$$\cancel{f'(x_n) = x_n \frac{1}{x_n} \log_{10} e}$$

$$f'(x_n) = x_n \cdot \frac{1}{x_n} \log_{10} e + \log_{10} x_n$$

$$= \log_{10} e + \log_{10} x_n$$

Q.1

$$= \frac{x_n \log_{10} e + \cancel{x_n \log_{10} x_n} - \cancel{x_n \log_{10} x_n} + 1.2}{\log_{10} e + \log_{10} x_n}$$

$$x_{n+1} = \frac{x_n \log_{10} e + 1.2}{\log_{10} e + \log_{10} x_n}$$

Here $\log_{10} e = \log_{10} 2.718$
$= .43424$

$$x_{n+1} = \frac{x_n (.43424) + 1.2}{.43424 + \log_{10} x_n}$$

Putting $n = 0$

$$x_1 = \frac{.43424(x_0) + 1.2}{.43424 + \log_{10} x_0} = \frac{.43424(2.7) + 1.2}{.43424 + \log_{10} 2.7} = \frac{2.372448}{.8656037} = 2.74080$$

$$x_2 = \frac{.43424(x_1) + 1.2}{.43424 + \log_{10} x_1} = \frac{.43424(2.74080) + 1.2}{.43424 + \log_{10}(2.74080)}$$

$$x_2 = \frac{2.39016}{.8721173} = 2.7406$$

$$x_3 = \frac{.43424(x_2) + 1.2}{.43424 + \log_{10} x_2}$$

$$x_3 = \frac{.43424(2.7406) + 1.2}{.43424 + \log_{10}(2.7406)} = \frac{2.39007}{.8720856} = 2.7406$$

$x_2 = x_3 = 2.7406$ upto four decimal Places.

Ans

Q.1 Find the root of equation x^2-5x+2 correct to four decimal places which is near to $x=4$ by Newton Raphson method.

Sol^n Let $f(x) = x^2-5x+2$

$\therefore f(x_n) = x_n^2 - 5x_n + 2$

$f'(x_n) = 2x_n - 5$

By Newton Raphson method we know that

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2-5x_n+2}{2x_n-5} = \frac{2x_n^2-\cancel{5x_n}-x_n^2+\cancel{5x_n}-2}{2x_n-5}$$

$$x_{n+1} = \frac{x_n^2-2}{2x_n-5} \rightarrow (1) \quad \text{Given } x_0 = 4$$

For First approximation put $n=0$

$$x_1 = \frac{x_0^2-2}{2x_0-5} = \frac{4^2-2}{2\times4-5} = \frac{14}{3} = 4.66666$$

For Second approximation put $n=1$ in (1)

$$x_2 = \frac{x_1^2-2}{2x_1-5} = \frac{(4.66666)^2-2}{2(4.66666)-5} = \frac{19.7777}{4.3333} = 4.56408$$

Third approximation put $n=2$ in (1)

$$x_3 = \frac{x_2^2-2}{2x_2-5} = \frac{4.56408^2-2}{2(4.56408)-5} = \frac{18.83082}{4.12816} = 4.56155$$

For Fourth approximation put $n=3$ in (1)

$$x_4 = \frac{x_3^2-2}{2x_3-5} = \frac{4.56155^2-2}{2(4.56155)-5} = \frac{18.807738}{4.1231} = 4.56155$$

Here x_3 and x_4 are same

$x_3 = x_4 = 4.56155$

So Required Root is 4.56155 Ans

Q.2 Using Newton Raphson method obtain real root of the equation $3x - \cos x - 1 = 0$

Let $f(x) = 3x - \cos x - 1$

$f(.6) = 3(.6) - \cos(.6 \times 57.3) - 1 = -.0253$ Negative

$f(.7) = 3(.7) - \cos(.7 \times 57.3) - 1 = .3351$ Positive

Hence Root lies between .6 and .7

So we take $x_0 = .6$

$f(x_n) = 3x_n - \cos x_n - 1$

$f'(x_n) = 3 + \sin x_n$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

$$= \frac{\cancel{3x_n} + x_n \sin x_n - \cancel{3x_n} + \cos x_n + 1}{3 + \sin x_n}$$

$$x_{n+1} = \frac{x_n \sin x_n + \cos x_n + 1}{3 + \sin x_n} \longrightarrow (1)$$

For First approximation put $n = 0$

$$x_1 = \frac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0}$$ Here $x_0 = .6$

$$= \frac{.6 \sin(.6 \times 57.3) + \cos(.6 \times 57.3) + 1}{3 + \sin(.6 \times 57.3)} = \frac{.338807 + .825310 + 1}{3.564678}$$

$= .607100$

For Second approximation put $n=1$ in ①

$\cancel{x_2 = x_1 - x_1 \sin x_1}$.

$$x_2 = \frac{x_1 \sin x_1 + \cos x_1 + 1}{3 + \sin x_1} \qquad \text{Put } x_1 = .607100$$

$$x_2 = \frac{.607100 \sin(57.3 \times .607100) + \cos(57.3 \times .607100) + 1}{3 + \sin(57.3 \times .607100)}$$

$$x_2 = \frac{.3463656 + 1.821280372}{3.57052409} = \frac{2.1676459}{3.57052409} = .6071$$

Here $x_1 = x_2 = .6071$

So Root is $\underline{\underline{.6071}}$ Ans

Q.3. Find the fifth root of 35, using Newton Raphson method.

Sol Let $x = (35)^{1/5}$

$x^5 = 35$

$f(x) = x^5 - 35$

$f(2) = 2^5 - 35 = -3$

$f(3) =$ Positive

$f(2.5) =$ Positive

Hence Root lies b/w 2 and 2.5

We take $x_0 = 2$

$f(x_n) = x_n^5 - 35$

$f'(x_n) = 5x_n^4$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^5 - 35}{5x_n^4}$$

$$x_{n+1} = \frac{5x_n^5 - x_n^5 + 35}{5x_n^4} = \frac{4x_n^5 + 35}{5x_n^4} \longrightarrow (1)$$

For First approximation put $n = 0$

$$x_1 = \frac{4x_0^5 + 35}{5x_0^4} = \frac{4(2)^5 + 35}{5(2)^4} = \frac{163}{80} = 2.0375$$

For second approximation put $n = 1$ in (1)

$$x_2 = \frac{4x_1^5 + 35}{5(x_1)^4} = \frac{4(2.038)^5 + 35}{5(2.038)^4} = 2.03617$$

For Third approximation put $n = 2$ in (1)

$$x_3 = \frac{4x_2^5 + 35}{5x_2^4} = \frac{4(2.03617)^5 + 35}{5(2.03617)^4} = 2.03617$$

Here x_2 and x_3 are same

i.e $x_2 = x_3 = 2.03617$

Hence Root is 2.03617 Ans.

Q.4. Find the root of the equation $\cos x - x e^x = 0$ using Newton Raphson method.

Sol^n Let $f(x) = \cos x - x e^x$

$$f(1) = \cos(1 \times 57.3) - e^1 = -ve$$
$$f(1.5) = \cos(1.5 \times 57.3) - e^{1.5} \cdot (1.5) = -ve$$
$$f(2) = \cos(2 \times 57.3) - 2e^2 = -ve$$
$$f(0) = \cos 0 - 0 = 1 \; +ve$$

Hence Root lies between 0 and 1

Here $f(x) = \cos x - x e^x$

$$f(x_n) = \cos x_n - x_n e^{x_n}$$

$$f'(x_n) = -\sin x_n - [x_n e^{x_n} + e^{x_n}]$$

$$f'(x_n) = -\sin x_n - x_n e^{x_n} - e^{x_n}$$

By Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\cos x_n - x_n e^{x_n}}{-\sin x_n - x_n e^{x_n} - e^{x_n}}$$

$$= \frac{-x_n \sin x_n - x_n^2 e^{x_n} - \cancel{x_n e^{x_n}} - \cos x_n + \cancel{x_n e^{x_n}}}{-\sin x_n - x_n e^{x_n} - e^{x_n}}$$

$$= \frac{+[x_n \sin x_n + x_n^2 e^{x_n} + \cos x_n]}{+[\sin x_n + x_n e^{x_n} + e^{x_n}]}$$

$$x_{n+1} = \frac{x_n \sin x_n + x_n^2 e^{x_n} + \cos x_n}{\sin x_n + x_n e^{x_n} + e^{x_n}} \longrightarrow (1) \qquad x_0 = 0$$

Put $n=0$ For First approximation

$$x_1 = \frac{x_0 \sin x_0 + x_0^2 e^{x_0} + \cos x_0}{\sin x_0 + x_0 e^{x_0} + e^{x_0}} = \frac{1}{1} = 1$$

Put $n=1$ in (1)

$$x_2 = \frac{x_1 \sin x_1 + x_1^2 e^{x_1} + \cos x_1}{\sin x_1 + x_1 e^{x_1} + e^{x_1}} = \frac{\sin(1\times 57.3) + 1 e^1 + \cos(1\times 57.3)}{\sin(1\times 57.3) + e^1 + e^1}$$

$$x_2 = \frac{2.55793}{6.27807} = .4074$$

Put $n=2$ in (1)

$$x_3 = \frac{x_2 \sin x_2 + x_2^2 e^{x_2} + \cos x_2}{\sin x_2 + x_2 e^{x_2} + e^{x_2}}$$

Put $x_2 = .4074$

$$\therefore x_3 = \frac{.4074 \sin(.4074 \times 57.3) + (.4074)^2 e^{.4074} + \cos(.4074 \times 57.3)}{\sin(.4074 \times 57.3) + .4074 e^{.4074} + e^{.4074}}$$

$$= \frac{1.3290}{2.51143} = .5291$$

Q. Find the smallest Positive root of $x^3-5x+3=0$ by Newton Raphson method.

Sol. Let $f(x) = x^3-5x+3$

$f(0) = 3$ +ve

$f(1) = 1-5+3 = -1$ (-ve)

$f(.5) = (.5)^3 - 5(.5)+3 =$ +ve

$f(.6) = (.6)^3 - 5(.6)+3 =$ +ve

$f(.7) = (.7)^3 - 5(.7)+3 =$ -ve

Hence Root lies b/w .6 and .7

take $x_0 = .6$

$\because f(x) = x^3 - 5x + 3$

$f(x_n) = x_n^3 - 5x_n + 3$

By Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 5x_n + 3}{3x_n^2 - 5}$$

$$x_{n+1} = \frac{3x_n^3 - \cancel{5x_n} - x_n^3 + \cancel{5x_n} - 3}{3x_n^2 - 5} = \frac{2x_n^3 - 3}{3x_n^2 - 5} \longrightarrow (1)$$

For First approximation put $n=0$ in (1) put $x_0 = .6$

$$x_1 = \frac{2x_0^3 - 3}{3x_0^2 - 5} = \frac{2(.6)^3 - 3}{3(.6)^2 - 5} = \frac{-2.568}{-3.92} = .65510$$

For Second approximation put $n=1$ in (1)

$$x_2 = \frac{2(.65510)^3 - 3}{3(.65510)^2 - 5} = \frac{-2.437719}{-3.7125319} = .6566$$

For Third approximation put $n=2$ in (2)

$$x_3 = \frac{2(.6566)^3 - 3}{3(.6566)^2 - 5} = \frac{-2.4338485}{-3.70662932} = .6566$$

$\therefore x_2 = x_3 = .6566$ so root is .6566 Ans

Q Solve $(30)^{-1/5}$ by Newton Raphson method upto three decimal places.

Solⁿ Let $x = 30^{-1/5} \Rightarrow x^5 = \frac{1}{30}$

Let $f(x) = 30x^5 - 1$

$f(0) = -ve$

$f(.5) = -ve$, $f(1) = +ve$

i.e Root lies between .5 and 1, take $x_0 = .5$

$f(x_n) = 30x_n^5 - 1$

$f'(x_n) = 150x_n^4$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{30x_n^5 - 1}{150x_n^4}$$

$$x_{n+1} = \frac{150x_n^5 - 30x_n^5 + 1}{150x_n^4} = \frac{120x_n^5 + 1}{150x_n^4}$$

For first approximation put $n=0$

$$x_1 = \frac{120x_0^5 + 1}{150x_0^4} = \frac{120(.5)^5 + 1}{150(.5)^4} = \frac{4.75}{9.357} = .50676$$

For second Approximation put $n=1$

$$x_2 = \frac{120x_1^5 + 1}{150x_1^4} = \cancel{\frac{120(.5076)^5 + 1}{150(.5076)^4} = \frac{5.043796}{9.458120} = .50}$$

$$= \frac{120(.5067)^5 + 1}{150(.5067)^4} = .5065$$

For third approximation put $n=2$

$$x_3 = \frac{120x_2^5 + 1}{150x_2^4} = \frac{120(.5065)^5 + 1}{150(.5065)^4} = .5065$$

Hence $x_2 = x_3 = .5065$

So $(30)^{-1/5}$ is $.5065$ Ans

to three decimal

Q. Find an iterative formula to find $\sqrt{N}$ (Where N is a Positive number) and hence find $\sqrt{5}$.

Solution Let $\sqrt{N} = x$

$$x = \sqrt{N}$$

$$x^2 - N = 0$$

Let $f(x) = x^2 - N$

$$f'(x) = 2x$$

$\because$ $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ by Newton Raphson method.

$$= x_n - \frac{x_n^2 - N}{2x_n}$$

$$x_{n+1} = \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{\cancel{2}x_n^2}{\cancel{2}x_n} - \frac{x_n^2}{2x_n} + \frac{N}{2x_n}$$

$$= x_n - \frac{1}{2}x_n + \frac{N}{2x_n}$$

$$= \frac{1}{2}x_n + \frac{N}{2x_n}$$

$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ is the Iterative formula to find $\sqrt{N}$.

To find $\sqrt{5}$, put $N = 5$

Also $x = \sqrt{5}$ lies between 2 and 3. Take $x_0 = 2$

$$\therefore x_1 = \frac{1}{2}\left[x_0 + \frac{5}{x_0}\right] = \frac{1}{2}\left[2 + \frac{5}{2}\right] = 2.25$$

$$x_2 = \frac{1}{2}\left[x_1 + \frac{5}{x_1}\right] = \frac{1}{2}\left[2.25 + \frac{5}{2.25}\right] = 2.236111$$

$$x_3 = \frac{1}{2}\left[x_2 + \frac{5}{x_2}\right] = \frac{1}{2}\left[2.236111 + \frac{5}{2.236111}\right] = 2.236067$$

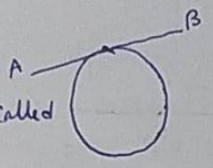
$$x_4 = \frac{1}{2}\left[x_3 + \frac{5}{x_3}\right] = \frac{1}{2}\left[2.236067 + \frac{5}{2.236067}\right] = 2.23606$$

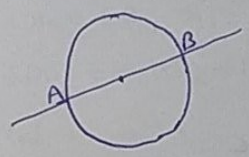
Hence $x_3 = x_4 = 2.23606$, so Root of $\sqrt{5}$ is 2.23606 Ans

Secant Method

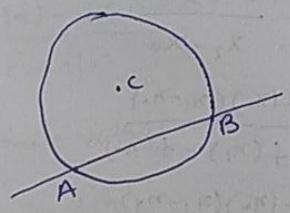
Tangent

Line AB is called tangent





If Line ~~the~~ touches the circle at two Point A and B and it Passes through centre of circle this line is called Diameter



If Line touches the circle at two Point A and B but it does not Pass through centre of circle this line AB is called Chord or Secant line.

Q.1 Find the real root of the equation $x^3 - x - 1 = 0$

Sol^n Let $f(x) = x^3 - x - 1$

$f(1.3) = -.103$

$f(1.4) = .344$

Hence root will lie between 1.3 and 1.4

So we take $x_0 = 1.3$, $x_1 = 1.4$

$f(x_0) = -.103$, $f(x_1) = .344$

by secant method

First Approximate value

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(1.3)(.344) - 1.4(-.103)}{.344 + .103} = \frac{.5914}{.447} = 1.32304$$

Now second Approximate value

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

Here $x_1 = 1.4$, $f(1.4) = .344$

$x_2 = 1.32304$, $f(x_2) = -.00714$

$$x_3 = \frac{1.4(-.00714) - 1.32304(.344)}{-.00714 - .344} = \frac{-.45189136}{-.35114}$$

Q1 Compute the real root of $x^3-5x+3=0$ in the interval $[1,2]$ by the secant method by performing four iterations

Solⁿ. Let $f(x) = x^3-5x+3$

$f(1) = 1-5+3 = -1$, -ve

$f(2) = 8-10+3 = 1$ +ve

Hence one root lies between 1 and 2

take $x_0 = 1$, $f(x_0) = -1$

$x_1 = 2$, $f(x_1) = 1$

First Approximation by secant method is

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{1(1) - 2(-1)}{1+1} = \frac{1+2}{1+1} = \frac{3}{2}$$

$x_2 = 1.5$

$\therefore f(x_2) = f(1.5) = (1.5)^3 - 5(1.5)+3 = -1.125$ -ve

For second Approximation we take

$x_0 = 1.5$, $f(1.5) = -1.125$

$x_1 = 2$, $f(2) = 1$

$$\therefore x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{1.5 \times 1 - 2(-1.125)}{1+1.125} = 1.76471$$

$\therefore f(x_2) = (1.76471)^3 - 5(1.76471)+3 = -.32788 = -ve$

For Third Approximation, we take

$x_0 = 1.76471$, $f(x_0) = -.32788$

$x_1 = 2$, $f(2) = 1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{1.76471(1) - 2(-.32788)}{1+.32788} = 1.02281$$

$\therefore f(1.02281) = (1.02281)^3 - 5(1.02281)+3 = -.05751 = -ve$

For Fourth Approximation

take $x_0 = 1.82281$, $f(1.82281) = -.05751$

$x_1 = 2$, $f(2) = 1$

$$\therefore x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{1.82281(1) - 2(-.05751)}{1 + .05751} = 1.83245$$

Ans

Q.2. Find the root of the equation $x = .5 + \sin x$, using secant method to four decimal places.

Sol: Let $f(x) = .5 + \sin x - x = 0$

$f(1) = .5 + \sin(57.3 \times 1) - 1 = .3415$ +ve

$f(1.5) = .5 + \sin(57.3 \times 1.5) - 1 = -.002497$

i.e Root lies between 1 and 1.5

take $x_0 = 1$, $f(1) = .3415$

$x_1 = 1.5$, $f(1.5) = -.002497$

First Approximation, By secant method

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{1(-.002497) - 1.5(.3415)}{-.002497 - .3415}$$

$x_2 = 1.4963$

$f(x_2) = f(1.4963) = .5 + \sin(1.4963 \times 57.3) - 1.4963$

$= .0009$

For Seccond Approximation,

$x_0 = 1.4963$, $f(x_0) = .0009$

$x_1 = 1.5$, $f(x_1) = -.002497$

$$\therefore x_2 = \frac{1.4963(-.002497) - 1.5(.0009)}{-.002497 - .0009} = 1.497$$

Hence Root is 1.497

Q.3. Find the root of the equation $xe^x = \cos x$ between .5 and 1 using the secant method correct to four decimal places.

Sol^n. Let $f(x) = \cos x - xe^x = 0$

$f(.5) = \cos(.5 \times 57.3) - .5e^{.5} = .0532$ +ve

$f(1) = \cos(1 \times 57.3) - 1e^1 = -2.1780 =$ -ve

take $x_0 = .5$, $f(.5) = .0532$

$x_1 = 1$, $f(1) = -2.1780$

By secant method

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{.5(-2.1780) - 1(.0532)}{-2.1780 - .0532}$$

$= .51192$

$f(.51192) = \cos(.51192 \times 57.3) - .51192e^{.51192}$

$= .01765 = .0177 =$ +ve

For second Approximation

take $x_0 = 1$, $f(x_0) = -2.1780$

$x_1 = .51192$, $f(x_1) = .0177$

$$\therefore x_2 = \frac{1 \times .0177 - .51192(-2.1780)}{.0177 + 2.1780} = .5158$$

$f(.5158) = \cos(.5158 \times 57.3) - .5158e^{+.5158} = .0058$

For Third Approximation

$x_0 = .51192$, $f(x_0) = .0177$

$x_1 = .5158$, $f(x_1) = .0058$

$$x_2 = \frac{.51192(.0058) - .5158(.0177)}{.0058 - .0177} = .5178$$

If

$f(.5178) = \cos(57.3 \times .5178) - .5178e^{.5178} = .0001 \approx 0$

i.e $.5178$ is the root of $\cos x - xe^x = 0$ Ans