FIRST/SECOND SEMESTER
ELECTRICAL SCIENCE [ES-107/108]

UNIT-I: DC CIRCUITS

Introduction

The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc.

Analysis of electric circuits refers to computations required to determine the unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems, one must acquire the basic knowledge of electric circuit analysis and laws.

Basic Elements

Electrical Network: A combination of various electric elements (such as R, L, C, voltage & current sources) connected in any manner whatsoever is called an electric network.

Circuit Elements

(a) **Active Element:** The elements which supply energy to the network are known as active elements. For example, Voltage sources, dc & ac generators, current sources, transistors.

(b) **Passive Elements:** The components which dissipate or store energy are known as Passive elements. For example, Resistors (dissipate energy), inductors (store energy) and capacitors (store energy).

Difference between Active and Passive Elements:

Refer to Q.1(b), Page: 1 2017.

Bilateral Element: Conduction of current in both direction in an element with same magnitude is termed as bilateral element. For ex. R, L, C

Unilateral Element: Conduction of current in one direction is termed as unilateral element. For ex. Diode, Transistor.

Terminology

Linear Circuits: A circuit whose parameters do not change with voltage or current (Fig. 1). The electric circuits containing only linear resistances are called linear circuits.

Non-linear Circuits: A circuit whose parameters change with voltage or current (Fig. 2). The electric circuits containing resistive elements for which volt-ampere characteristics is other than a straight line are called non-linear circuits. For eg. Tungsten lamps, vacuum tubes, transistors, etc.

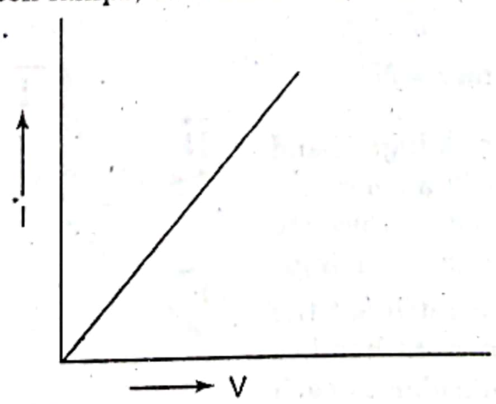


Fig. 1

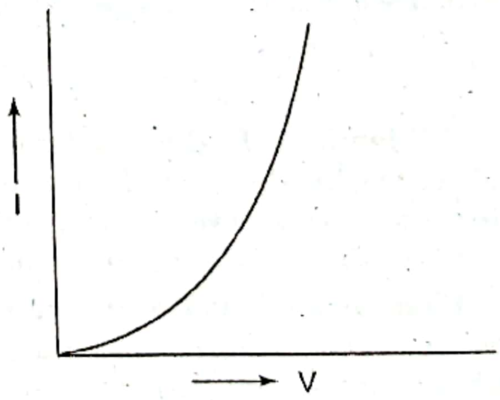


Fig. 2

Potential Energy Difference: The voltage or potential difference between two points in an electric circuit is the amount of energy required to move a unit charge between two points.

Electric Current: It is defined as the time rate of net motion of an electric charge across a cross-sectional boundary. Its unit is **ampere (A)**.

$$i = \text{Rate of transfer of electric charge}$$

$$= \frac{\text{Quantity of electric charge transferred during a given time duration}}{\text{Time duration}}$$

$$= \frac{dQ}{dt}$$

Electromotive Force (EMF): It is the force that causes an electric current to flow in an electric circuit. Its unit is Volt.

Potential Difference: Potential difference between two points in an electric circuit is that difference in their electrical state which tends to cause flow of electric current between them. Its unit is Volt.

Resistance: It may be defined as that property of a substance which opposes the flow of an electric current / electrons through it. Its unit is **ohm** (Ω).

Basic Laws of Electrical Engineering

1. Ohm's Law: The ratio of potential difference applied across a conductor and current flowing through it remains constant provided physical state i.e. temperature etc. of the conductor remains unchanged.

According to ohm's law,

$$\frac{V}{I} = \text{constant} = R$$

where, R is the resistance of the conductor
V is the potential difference across the conductor
I is the current flowing through the conductor

Limitations of Ohm's Law ***(Refer to Q.1(a), Page: 1–2017)***

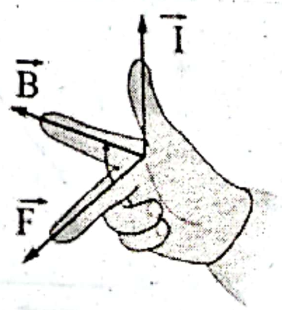
2. Faraday's Law of Electromagnetic Induction: Michael Faraday formulated the two laws of electromagnetic Induction. It is commonly referred as Faraday's Law.

First law states that when a conductor is forcefully moved or rotated in an electromagnetic field, the conductor cuts the magnetic flux, which induces an emf across the conductor.

Second law states that the magnitude of emf induced in a coil is equal to the rate of change of flux that linkages with the coil.

$$e = \frac{d\phi}{dt} \text{ and } \lambda = N\phi$$

3. Flemings' Right Hand rule: Leming's Right-hand Rule shows the direction of induced current in a generator when a conductor moves in a magnetic field. It can be used to determine the direction of current in a generator's windings.



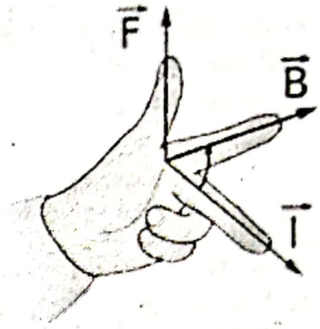
Fleming's Right Hand Rule

Fig. 3

Fleming's Right Hand rule states: "Stretch out the fore finger, middle finger and thumb of your right hand in such a way that they are mutually perpendicular to each other. If the fore finger represents the direction of field (F), the thumb points in the direction of motion or applied force (B), then middle finger points in the direction of the induced current (I)".

4. Flemings' Left Hand rule: Fleming's left-hand Rule shows the direction of motion of motor.

Fleming's Left Hand rule states: "Stretch out the fore finger, middle finger and thumb of your left hand in such a way that they are mutually perpendicular to each other. If the fore finger represents the direction of the field (B) and the middle finger represents that of the current (I), then thumb gives the direction of the force (F)".



Fleming's Left Hand Rule

Fig. 4

5. Lenz's Law: Lenz's law states that "The current flowing in a conductor due to the induced emf always opposes the cause producing it".

According to lenz's law,

$$e = -\frac{d\phi}{dt}$$

6. Laws of Resistance: The resistance of a wire depends upon its length, area of cross-section, type of material, purity & hardness of material of which it is made of and the operating temperature.

Therefore, Resistance of a wire is

(*i*) Directly proportional to its length, L i.e. $R \propto L$

(*ii*) Inversely proportional to its cross-sectional area, A i.e. $R \propto \frac{1}{A}$

Combining the two factors, we have

$$R \propto \frac{L}{A} \text{ or } R = \rho\frac{L}{A}$$

where, ρ is known as Specific resistance or resistivity of the material of the wire. It is named as Rho and is a constant quantity, Its unit is ohm-meters (Ω-m).

SOLVED EXAMPLE

Q.1. ***Refer to Q.1(b), Page: 1–2016***

Temperature Resistance Coefficient: The resistance of all pure metallic conductors increases with the increase in temperature but the resistance of the insulators and non-metallic materials generally decrease with the increase in temperature.

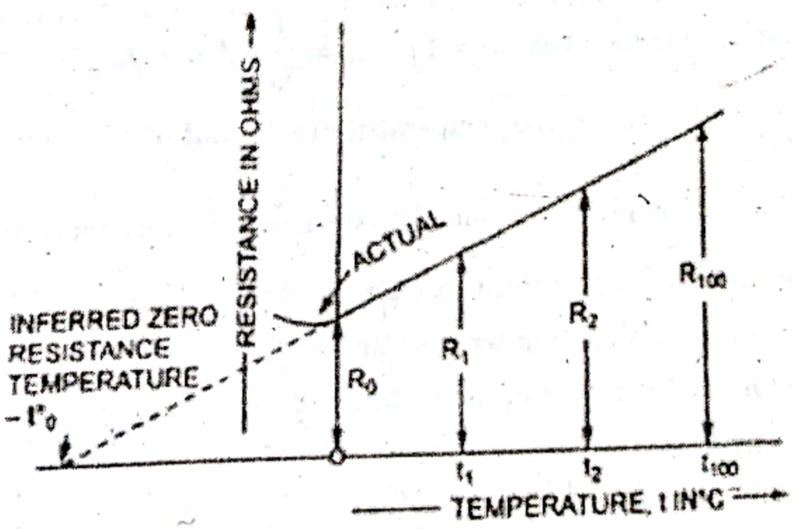


Fig. 5

If the resistance of any pure metal is plotted on a temperature base, it is found that over the range of temperature from 0 to 100 °C the graph is practically a straight line, as illustrate in Fig. 5. If this straight line is extended, it cuts the temperature axis at some temperature, $-t_0$ °C, known as *inferred zero resistance temperature*. This does not mean that the resistance of the metal is actually zero at that temperature, but $-t_0$ °C is the temperature at which the resistance would be zero if the rate of decrease between 100 and 0°C were maintained constant at all temperatures. From the similarity of the triangles in Fig. 5.

$$\frac{R_2}{R_1} = \frac{t_0 + t_2}{t_0 + t_1} \quad ...(1)$$

where R_1 and R_2 are the resistances at temperatures t_1°C and t_2°C respectively. Thus, if the resistance R_1 for any temperature t_1°C is known, then resistance for any other temperature t_2 can be computed from above equation provided that t_0 for that particular material is known.

The variation of resistance with temperature is often utilized in determining temperature variations. For example, in testing of an electric machine, the resistance of the coil is measured both before and after the test run, and the increase in resistance is a measure of the rise in temperature. For computation of temperature rise Eq. 1 may be transposed to the following form

$$t_2 - t_1 = \frac{R_2 - R_1}{R_1}(t_0 + t_1) \quad ...(2)$$

A temperature of 20°C has been adopted as the *standard reference temperature* for measurement of resistance, and the handbooks give the resistance of the various material at that temperature. Consequently, when a designer is computing the resistance of any conductor from its dimensions, the initial temperature t, at which the resistance is known, is generally 20°C.

Temperature Coefficient of Resistance: Let a metallic conductor having a resistance of R_0 at 0°C be heated to t°C and let its resistance at this temperature be R_t, From Eq. (1)

$$\frac{R_t}{R_0} = \frac{t_0 + t}{t_0 + 0}$$

$$\text{or } R_t = R_0 + \frac{1}{t_0} R_o t$$

or change in resistance, $$\Delta R = R_t - R_0 = \frac{1}{t_0} R_0 t = \alpha_0 R_0 t \quad ...(3)$$

where $\alpha_0 = \dfrac{1}{t_0}$ and is called the temperature coefficient of resistance of the material at 0°C

From Eq. (3) it may be concluded that change in resistance due to change in temperature

(a) varies directly as its initial resistance,

(b) varies directly as rise in temperature and

(c) depends on the nature of the material of the conductor.

The Eq. (3) may be rewritten as

$$\alpha_0 = \frac{\Delta R}{R_0 t} \quad ...(4)$$

So that temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree rise of temperature to the original resistance.

If R_0 is the resistance of any conductor at 0°C and α_0 is the temperature coefficient of resistance at 0°C, the resistance at t°C is given as

R_t = Original resistance + increase in resistance $= R_0 + R_0\alpha_0 t = R_0(1 + \alpha_0 t)$ ...(5)

The above expression holds goods for both increase as well as decrease in temperature.

It is to be noted that

(*i*) temperature coefficient of resistance for all pure metallic conductors is positive *i.e.*, the resistance of all pure metallic conductors increases with the increase in temperature, that of non-metallic materials such as of carbon is negative *i.e.*, the resistance of non-metallic materials such as of carbon decrease with the increase in temperature. The temperature coefficient of resistance of alloys like constantan and manganin is negligible.

(*ii*) temperature coefficient of resistance is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then the temperature coefficient of resistance has the value of α_0. At any other temperature t°C, value of temperature coefficient of resistance is α_1 and so on. For any material the temperature coefficient of resistance at 0°C *i.e.*, α_0 has the maximum value.

The temperature coefficient of resistance at any temperature t_1 is given as

$$\alpha_1 = \frac{1}{\frac{1}{\alpha} + t_1} \quad \text{...(6)}$$

The temperature coefficient of resistance at temperature t_2 in term of temperature coefficient of resistance at temperature t_1 is given as

$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)} \quad \text{...(7)}$$

If R_1 is the resistance of any conductor at t_1°C and α_1 is the temperature coefficient of resistance at t_1°C, then resistance of the conductor at t_2°C is given

$$R_2 = R_1[1 + \alpha_1 (t_2 - t_1)] \quad \text{...(8)}$$

(*iii*) As the resistance of the materials change with the change in temperature so it is obvious that resistivity of the material depends on temperature.

Conductance and Conductivity

Conductance is the reciprocal of the resistance and denoted by 'G'. It is defined as the inducement offered by the conductor to the flow of current. Its unit is Siemen (S) or mho (℧).

$$\therefore \quad G = \frac{1}{R} \text{ and } R = \rho\frac{L}{A} \;\dots\rightarrow\; G = \frac{1}{\rho}.\frac{A}{L} \;\dots\rightarrow\; G = \sigma\frac{A}{L}$$

Where, σ is known as Specific Conductance or Conductivity of the material. It is the reciprocal of the resistivity and defined as the conductance between two opposite forces of a unit cube. Its unit is Siemen/metre.

Voltage and Current Sources (Energy Sources)

Independent Sources and Sependent Sources *(Refer to Q.1(b), Page: 2–2018)*

Ideal and Practical Voltage Source

A **Voltage sources** is a two-terminal device whose voltage any instant of time is constant and is independent of the current drawn from it. Such a voltage source is called an **Ideal Voltage Source** and have zero internal resistance.

Practically an ideal voltage source cannot be obtained.

Sources having some amount of internal resistances are known as **practical voltage source**. Due to this internal resistance; voltage drop takes place, and it causes the terminal voltage to reduce. The smaller is the internal resistance (r) of a voltage source, the more closer it is to an Ideal Source.

The symbolic representation of the ideal and practical voltage source is shown below.

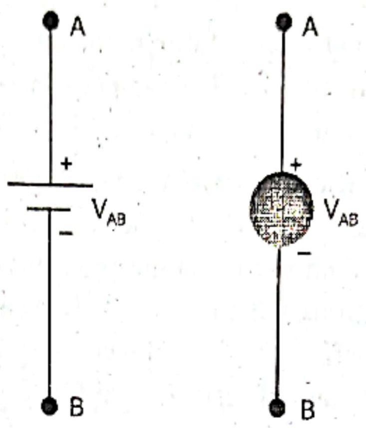


Fig. 6

Figure 7 shown shows the circuit diagram and characteristics of an ideal voltage source and Figure 8 shown gives the circuit diagram and characteristics of Practical Voltage Source. **The example of voltage sources is batteries and alternators.**

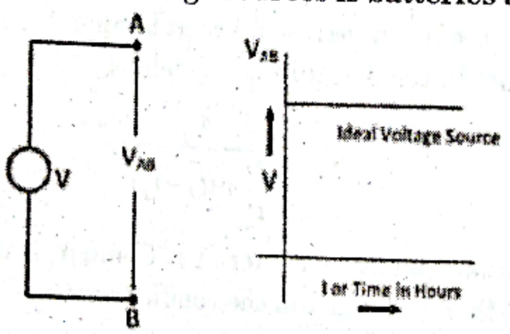


Fig. 7

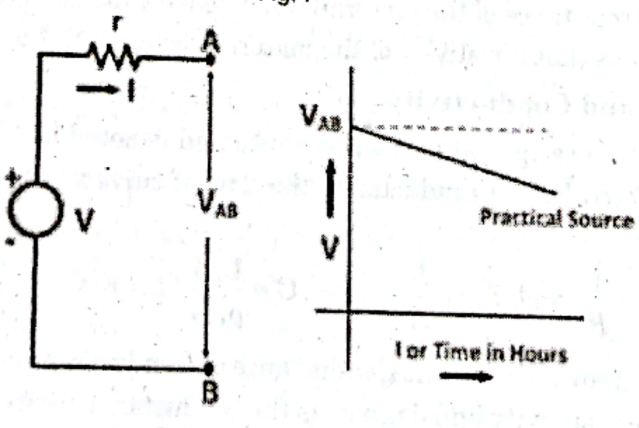


Fig. 8

Ideal and practical current source: The current source are further categorised as Ideal and Practical current source.

An **ideal current source** is a two-terminal circuit element which supplies the same current to any load resistance connected across its terminals. It is important to keep in mind that the current supplied by the current source is independent of the coltage of source terminals. It has infinite resistance.

A practical current source is represented as an ideal current source connected with the resistance in parallel.

The symbolic representation is shown:

Figure 10 shows its characteristics and Figure 11 shown the characteristics of Practical Current Source. **The example of current sources is photoelectric cells, collectors currents of transistors.**

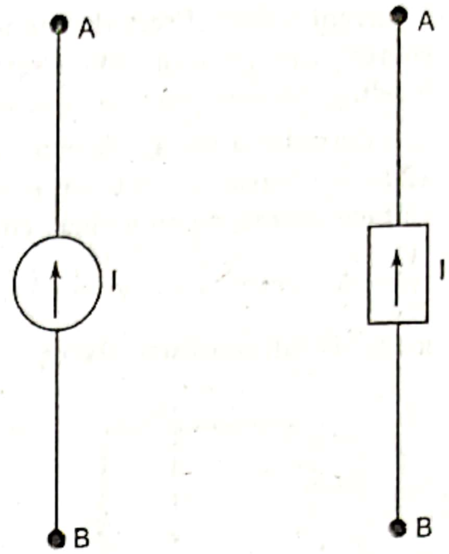


Fig. 9

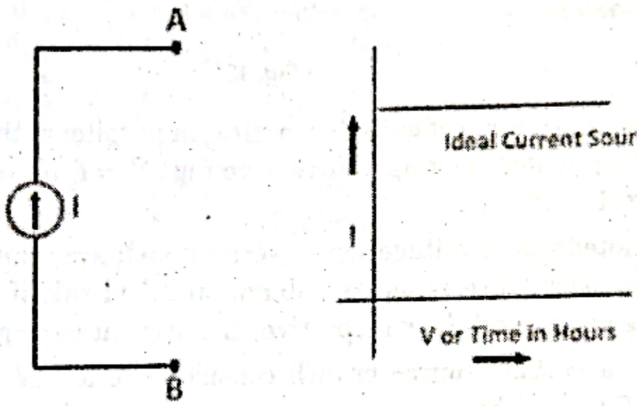


Fig. 10

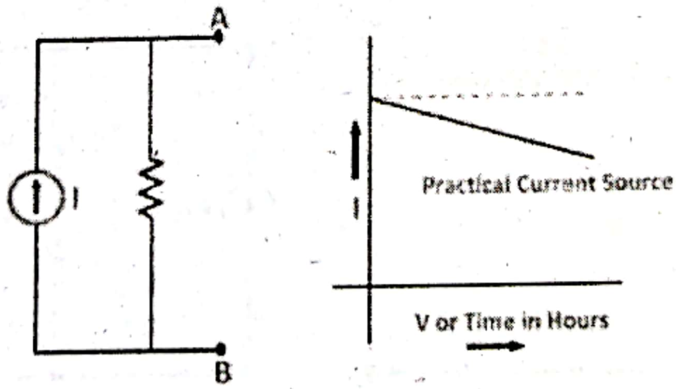
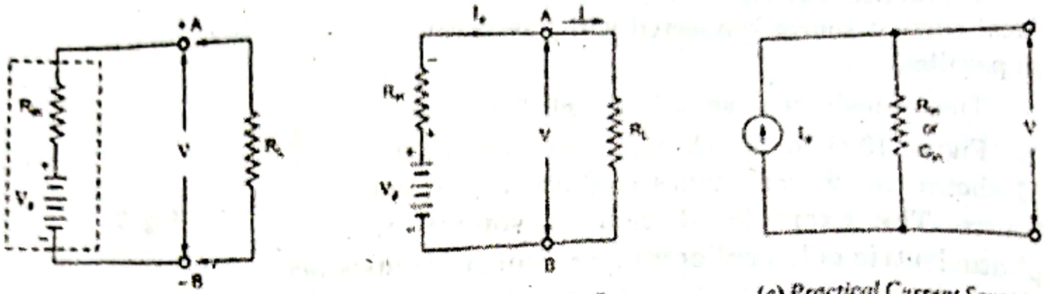


Fig. 11

Source transformation: Practically, a voltage sources is not different from a current source. In fact, a source can either operate as a current source or as a voltage source. It merely depends upon its operating conditions. If load impedance is very large in comparison to internal impedance of the source, it will be advantageous to treat the source as a voltage source. On the other hand, if the load impedance is very small in comparison to the internal impedance of the source, it is better represent the source as a current source. From the circuit point of view it does not matter at all whether the source is treated as a voltage source or a current source. In fact, it is possible to convert a voltage source into a current source and vice versa.

Consider a voltage source of voltage V_1 and internal resistance R_{in} shown in Fig 12(a) for conversion into an equivalent current source. The current supplied by this voltage source, when a short circuit is put across terminals A and B will be equal to $\frac{V_S}{R_{in}}$. A current source supplying this current $I_s = \frac{V_S}{R_{in}}$ and having the same resistance across it will represent the equivalent current source [Fig. 10 (c)].



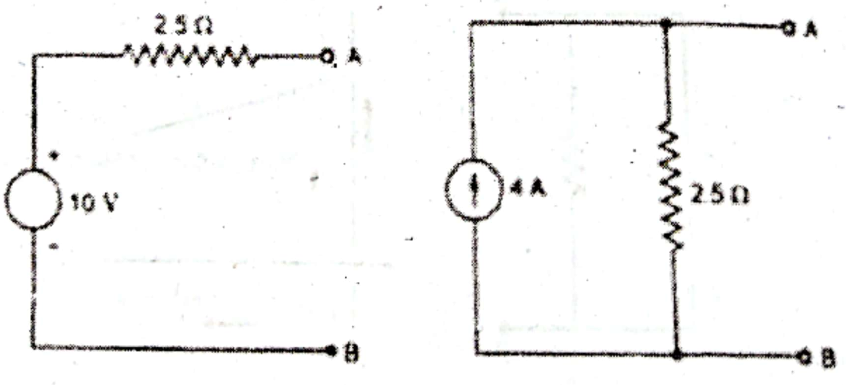
(a) A Practical Voltage Source *(b) Voltage Drop in R_{in}* *(c) Practical Current Source*

Fig. 12

Similarly a current source of output current I_s in parallel with resistance R_{in} can be converted into an equivalent voltage source of voltage $V_s = I_s R_{in}$ and a resistance R_{in} in series with it [Fig. 12 (a)].

It should be noted that a voltage source series resistance combination is equivalent to a current source-parallel resistance combination if and only if their respective open-circuit voltage are equal, and their respective short-circuit currents are equal.

For example, a voltage source branch consisting of a 10V source in series with a resistance of 2.5 Ω may be replaced by a current source branch consisting of a 4 A source in parallel with a 2.5Ω resistance and vice versa as shown in fig. 13(a) and 13(b) respectively.



(a) *(b)*

Fig. 13

SOLVED EXAMPLE

Q.1. ***Refer to Q.1(a), Page: 1–2015***

Q.2. ***Refer to Q.1(b), Page: 1–2015***

Q.3. ***Refer to Q.3(a), Page: 3–2016***

Series and Parallel Circuits

Series Circuits: When the resistors are connected end to end, so that they form only one path for the flow of current , then resistor are said to be connected in series and such circuits are known as *series circuit.*

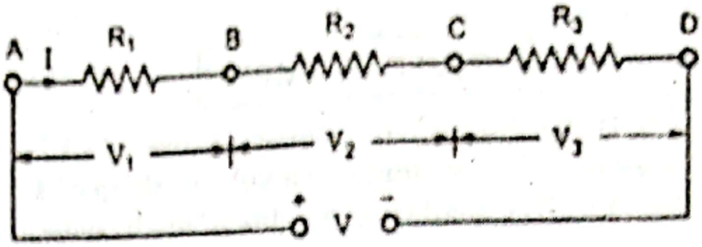


Fig. 14 Series Circuit

Let resistors R_1, R_2 and R_3 be connected in series, as shown in Fig.1, and the potential difference of V volts be applied between extreme ends A and D to cause flow of current of 1 amperes through all the resistors R_1, R_2 and R_3.

Now according to Ohm's law

Voltage drop across resistor R_1, $V_1 = I R_1$

Voltage drop across resistor R_2, $V_2 = I R_2$

Voltage drop acros resistor R_3, $V_3 = I R_3$

Voltage drop across whole circuit,

V = Voltage drop across resistor R_1 +voltage drop across resistor R_2 + voltage drop across resistor R_3

$$\text{i.e., } V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

$$\text{or } \quad \frac{V}{I} = R_1 + R_2 + R_3 \qquad ...(9)$$

and according to Ohm's law $\frac{V}{I}$ gives the whole circuit resistance, say R

∴ Effective resistance of the series circuit,

$$R = R_1 + R_2 + R_3 \qquad ...(10)$$

Thus, when a number of resistor are connected in series, the equivalent resistance is given by the arithmetic sum of their individual resistances.

$$\text{i.e., } \quad R = R_1 + R_2 + R_3 + ...R_n \qquad ...(11)$$

From the above discussions for a series circuit we conculde that

1. same current flows through all parts of the circuit,
2. applied voltage is equal to the sum of voltage drops across the different parts of the circuit,
3. different resistors have their individual voltage drops,
4. voltage drop across individual resistor is directly proportional to its resistance, current being the same in each resistor,
5. voltage drops are additive,
6. resistance are additive,
7. powers are additive.

Series circuit are common in electrical equipment. The tube filaments in small radios are usually in series. Current controlling devices are wired in series with the controlled equipment. Fuses are in series with the equipment they protect. A thermostat switch is in series with the heating element in an electric iron. Automatic house-heating equipment has a thermostat, electromagnet coils, and safety cut-outs in series with a voltage source. Rheostats are placed in series with the coils in large motors for motor current control.

SOLVED EXAMPLE

Q.1. Three resistors are connected in series across a 12 *V* battery. The first resistor has the value of 1 ohm, second has a voltage drop of 4 *V* and third has a power dissipation of 12 *W*. Calculate the value of each resistance and circuit current.

Solution: Let the three resistors be of R_1 (= 1Ω), R_2 and R_3 ohms, current flowing through the three resistors R_1, R_2 and R_3 be of I amperes and voltage drops across resistors R_1, R_2 and R_3 be of V_1, V_2 and V_3 volts respectively. The circuit is show in Fig 15.

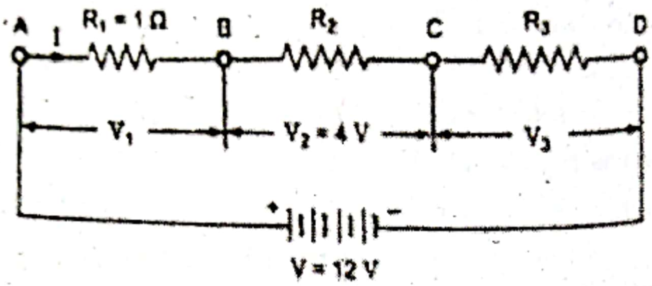


Fig. 15

Now $V_1 = IR_1 = I$ volts

$V_2 = IR_2 = 4$ volts

$V_3 = \dfrac{\text{Power dissipation}}{I} = \dfrac{12}{I}$ volts

since $V = V_1 + V_2 + V_3$

so $12 = I + 4 + \dfrac{12}{I}$

or $I^2 - 8I + 12 = 0$

or $(I - 6)(I - 2) = 0$

or $I = 6$ or $2\,A$ **Ans.**

When $I = 6\,A$; $R_2 = \dfrac{V_2}{I} = \dfrac{4}{6} = \dfrac{2}{3}\Omega$

and $R_3 = \dfrac{P}{I^2} = \dfrac{12}{6^2} = \dfrac{1}{3}\Omega$

i.e., $R_1 = 1\Omega;\ R_2 = \dfrac{2}{3}\Omega$ and $R_3 = \dfrac{1}{3}\Omega$ **Ans.**

When $I = 2\,A$; $R_2 = \dfrac{V_2}{I} = \dfrac{4}{2} = 2\Omega$

and $R_3 = \frac{P}{I^2} = \frac{12}{2^2} = 3\Omega$

i.e., $R_1 = 1\Omega$; $R_2 = 2\Omega$ and $R_3 = 3\Omega$ Ans.

Q.2. A 100 V, 60 watt bulb is to be operated from a 220V supply. What is the resistance to be connected in series with the bulb to glow normally?

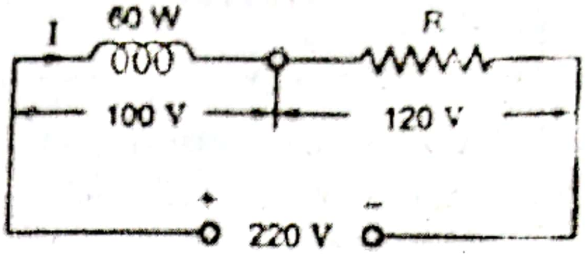


Fig. 16

Solution: Rated power of lamp, P = 60 W

Rated voltage of lamp, $V = 100\ V$

Current drawn by the lamp, when operated on rated voltage, *i.e.*,

rated current, $I = \frac{P}{V} = \frac{60}{100} = 0.6\ A$

Lamp will operate normally on 220V also if the current flowing through the lamp remains the rated current i.e., 0.6 A.

Let the resistance connected in series with the lamp to make it glow normally on 220 V be of R ohms, as shown in Fig. 16.

Now since the resistance R is in series with the lamp, the same current will flow through the resistance R, as in the lamp *i.e.*, 0.6 A and voltage drop across series resistance R will be equal to supply voltage less voltage drop across the lamp (*i.e.*, rated voltage of the lamp) or Voltage drop across the series resistance, IR = supply voltage – rated voltage of the lamp

$$= 220 - 100 = 120\ V$$

or $$R = \frac{120}{I} = \frac{120}{0.6} = 200\Omega \textbf{ Ans.}$$

Parallel Circuits

When a number of resistors are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point, as shown in Fig. 17, then resistors are said to be connected in parallel and such circuit are known as *parallel circuits*. In these circuit current is divided into as many paths as the number of resistances.

Let the resistors R_1, R_2 and R_3 be connected in parallel, as shown in Fig. 17, and the potential difference of V volts be applied across the circuit.

Since potential difference across each resistor is same and equal to potential difference applied to the circuit *i.e.*, V

∴ According to Ohm's law

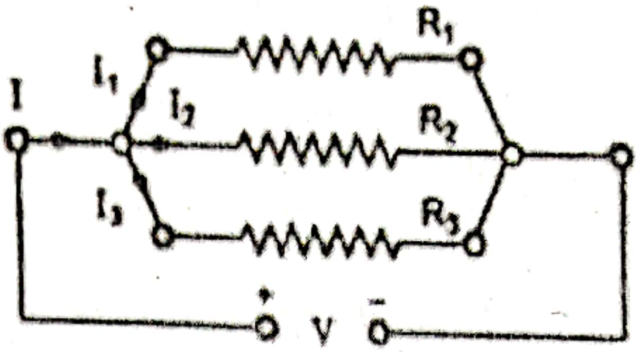


Fig. 17 Parallel Circuit

Current in resistor $R_1, I_1 = \dfrac{V}{R_1}$ ...(1)

Current in resistor $R_2, I_2 = \dfrac{V}{R_2}$ ...(2)

Current in resistor $R_3, I_3 = \dfrac{V}{R_3}$ ...(3)

Adding Eqs. (1), (2) and (3), we have

$$I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad ...(4)$$

and since $I_1 + I_2 + I_3 = I$, the total current flowing through the circuit

so
$$I = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or
$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

and since $\dfrac{I}{V} = \dfrac{1}{R}$ where R is the equivalent resistance of the whole circuit,

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad ...(5)$$

Thus, when a number of resistors are connected in parallel, the reciprocal of the equivalent resistance is given by the arithmetic sum of the reciprocals of their individuals resistances.

In general if n resistors of resistance $R_1, R_2, R_3, \ldots R_n$ are connected in parallel, then equivalent resistance R of the circuit is given by the expression

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \quad ...(6)$$

Also
$$G = G_1 + G_2 + G_3 + \ldots + G_n \quad ...(7)$$

where $G = \dfrac{1}{R}, G_1 = \dfrac{1}{R_1}, G_2 = \dfrac{1}{R_2}, G_3 = \dfrac{1}{R_3}$ and so on.

From the above discussion for a parallel circuit we conclude that

1. same voltage acts across all branches of the circuit,
2. different resistors (or branches) have their individual currents,
3. total circuit current is equal to the sum of individual currents through the various resistors (or branches),

4. branch current are additive,
5. conductances are additive,
6. power are additive
7. the reciprocol of the equivalent or combined resistance is equal to the sum of the sum of the reciprocals of the resistance of the individual branches.

Parallel circuit are very comon in use. Various lamps and appliances in a house are connected in parallel, so that each one can be operated independently. A series circuit is an "all or none" circuit, in which either every thing operates or nothing operates. For individuals control, devices are wired in parallel.

Series-Parallel Circuits: So far, only simple series and simple series and simple parallel circuit have been considered. Practical electric circuit very often consist of combinations of series and parallel resistance. Such circuits may be solved by the proper application of Ohm's law and the rules for series and parallel circuit to the various parts of the complex circuit. There is not definite procedure to be followed in solving complex circuits, the solution depends on the known facts concerning the circuit and the quantities which one desires to find. One simple rule may usually be followed, however–reduce the parallel branches to an equivalent series branch and then solve the circuit as a simple series circuit.

For example, consider a series-parallel circuit shown 18 for solution.

First of all equivalent resistance of all parallel branches are determined separately *e.g.*, of branches AB and CD by the law of parallel circuits.

Equivalent resistance of parallel branches AB,

$$R_{AB} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{R_1+R_2}{R_1+R_2}$$

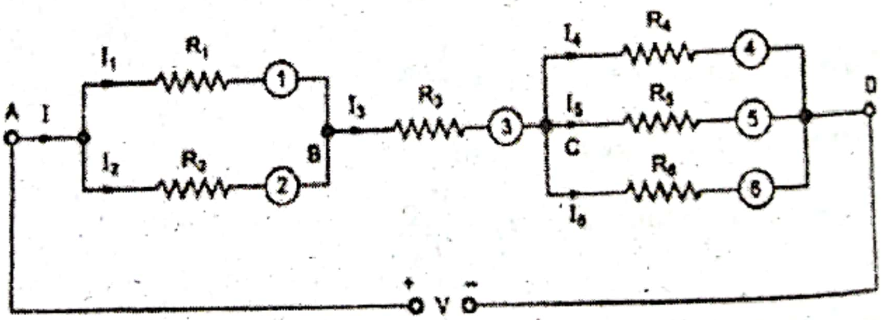


Fig. 18

and equivalent resistance of parallel branch CD

$$R_{CD} = \frac{1}{\frac{1}{\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_6}}} = \frac{R_4R_5R_6}{R_5R_6+R_4R_6+R_4R_5}$$

Now the circuit shown in Fig. 18 gets reduced to a simple series circuit shown in Fig. 19 consisting of three resistance,

$$R_{AB} = \frac{R_1R_2}{R_1+R_2} \cdot R_{BC} = R_3 \quad \text{and} \quad R_{CD} = \frac{R_4R_5R_6}{R_5R_6+R_4R_6+R_4R_5}$$

Total resistance of circuit, $R_T = R_{AB} + R_{BC} + R_{CD}$

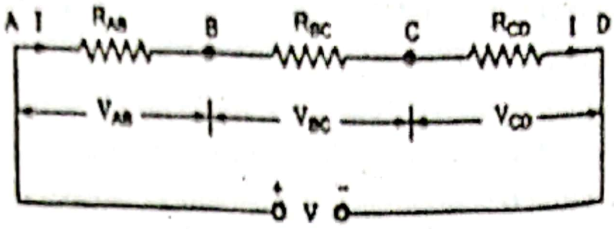


Fig. 19

Now circuit may be determined from the relation

$$I = \frac{V}{R_T}$$

After knowing I, potential difference across branches AB, BC and CD are determined from the relations

PD across branch AB, $\quad V_{AB} = I.R_{AB} = \dfrac{V}{R_T} R_{AB}$

PD across branch BC, $\quad V_{BC} = I.R_{BC} = \dfrac{V}{R_T} R_{BC}$

and PD across branch CD, $V_{CD} = IR_{CD} = \dfrac{V}{R_T} R_{CD}$

After determine of potential difference across each prallel branch, the current in the various resistance are determined from the relations

Current in resistance $R_1 = I_1 = \dfrac{V_{AB}}{R_1}$

Current in resistance $R_2 = I_2 = \dfrac{V_{AB}}{R_2}$

Current in resistance $R_3 = I_3 = I$

Current in resistance $R_4 = I_4 = \dfrac{V_{CD}}{R_4}$

Current in resistance $R_5 = I_5 = \dfrac{V_{CD}}{R_5}$

Current in resistance $R_6 = I_6 = \dfrac{V_{CD}}{R_6}$

Thus, equivalent resistance of the whole circuit, voltage drop across each branch and current in the various resistors may be determined.

SOLVED EXAMPLE

Q.1. Two resistance of 20 Ω and 30 Ω respectively are connected in parallel. These two parallel resistances are further connected in series with a resistance of 15 Ω. If the current through the 15 Ω resistance is 3 A find (a) the current through the 20 Ω and 30 Ω resistances respectively (b) the voltage across the whole circuit (c) the total power consumed.

Solution: Equivalent resistance of branch AB, $R_{AB} = \dfrac{1}{\dfrac{1}{20}+\dfrac{1}{30}} = 12\Omega$

Effective resistance of the circuit, Reff = $R_{AB} + R_{BC} = 12 + 15 = 27\Omega$

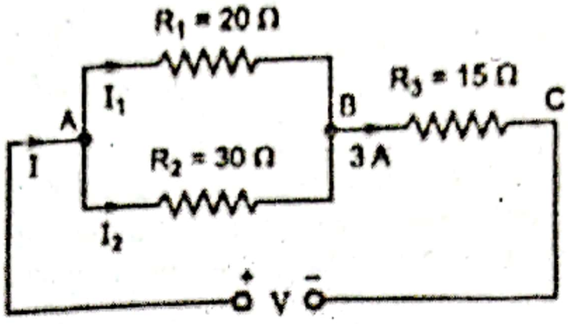


Fig. 20

Circuit current, I = Current through 15 Ω resistance = 3A

(b) Voltage across the whole circuit, $V = I\,R_{eff} = 3 \times 27 = 81\ V$ Ans.

(c) Total power consumed, $P = VI = 81 \times 3 = 243$ watts Ans.

(a) Voltage drop across branch AB, $V_{AB} = I\,R_{AB} = 3 \times 12 = 36\ V$

Current through 20 Ω resistance, $I_1 = \dfrac{V_{AB}}{R_1} = \dfrac{36}{20} = 1.8A$ Ans.

Current through 30 Ω resistance, $I_2 = \dfrac{V_{AB}}{R_2} = \dfrac{36}{30} = 1.2A$ Ans.

Q.2. For the circuit shown in Fig. 21, using the method of series-parallel combination, find V_1 and I_2

Solution: Total resistance of the circuit,

Req = 25 + 2.5 + equivalent resistance of parallel combination of resistors of 10.50 and 20 Ω

$$= 25 + 2.5 + \frac{1}{\dfrac{1}{10}+\dfrac{1}{50}+\dfrac{1}{20}}$$

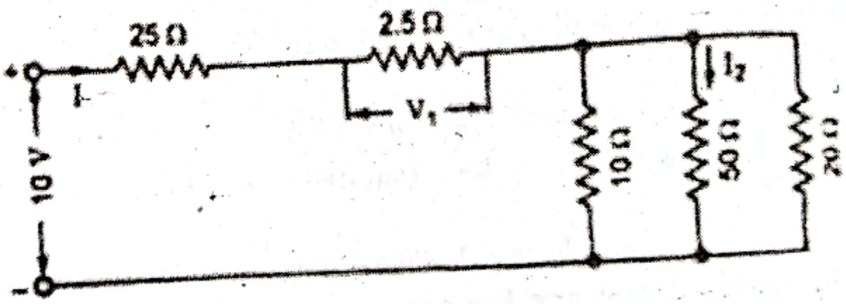


Fig. 21

$$= 25 + 2.5 + \frac{100}{10+2+5} = \frac{1.135}{34}\Omega$$

Applied voltage, $V = 10V$

Current drawn from the supply, $I = \dfrac{V}{R_{eq}} = \dfrac{10}{\dfrac{1,135}{34}} = \dfrac{10 \times 34}{1,135} = \dfrac{68}{227} A$

Voltage drop across resistor of 2.5 Ω $V_1 = I \times 2.5 = \frac{68}{227} \times 2.5 = \frac{170}{227} V$ **Ans.**

Voltage drop across parallel combination, $V_2 = \frac{68}{227} \times \frac{100}{17} = \frac{400}{227} V$

Current through 50 Ω resistor, $I_2 = \frac{V_2}{50} = \frac{400}{227 \times 50} = \frac{8}{227} A$ **Ans.**

Power and Energy

Power is defined as the rate of doing work or the amount of work done in unit time. The MKS or SI unit of power is the joule/second or watt. In practice, the watt is often found to be inconveniently small and so a bigger unit, the kilowatt is *frequently* used.

$$1 \text{ kilowatt} = 1{,}000 \text{ watts}$$

The bigger unit of power most commonly used in engineering practice (not at all SI system) is horse power defined as below:

Metric Horse Power: It is the practical unit of power in MKs system (not in SI system) which according to ISI specifications is equal to 75 kgf-m of work done per second.

Energy is defined as the capacity of doing work. Its units are same as those of work, mentioned above. If a body having mass m, in kg, is moving with velocity v, in meters/second,

$$\text{Kinetic energy} = \frac{1}{2} mv^2 \text{ joules}$$

If a body having mass m, in kg, is lifted vetically through height h, in meters, and if g is the gravitational acceleration, in meters/second^2 in that region, potential energy acquired by the body

= work done in lifting the body = mgh joules = $9.81\, mh$ joules

As already stated, in SI system the unit of energy of all forms is joule. Bigger unit of energy is mega joules (MJ) where 1 MJ = 10^6 J

The thermal units of energy, calorie (gm calorie) and kilocalorie (kilogram calorie), are defined below:

Calorie: It is the amount of heat required to raise the temperature of one gram of water through 1°C.

$$1 \text{ calorie} = 4.18 \text{ J} = 4.2 \text{ J}$$

Kilocalorie: It is the amount of heat required to arise the temperature of 1 kg of water through 1°C.

$$1 \text{ k. calorie} = 1{,}000 \text{ calories} = 4{,}180 \text{ joules} = 4{,}200 \text{ J}$$

Electric Unit of Power and Energy

The unit of work done and of energy expended is joule. It is equal to the energy expended in passing 1 coulomb of charge through a resistance of 1 ohm i.e., the energy expended in passing one ampere current for 1 second through a resistance of one ohm is taken as one joule. It may also be expressed as 1 watt-second i.e., one watt of power consumed for one second.

i.e., $$1 \text{ joule} = 1 \text{ watt-second}$$

The unit of energy, joule or watt-second is too small for pratical purposes, so a bigger unit Mega joule (MJ) or kilowatt-hour (kWh) is used in electrical engineering.

$$1\text{kWh} = 1{,}000 \text{ watt-hour} = 1{,}000\times3{,}600 \text{ watt-second or joules} = 3.6 \text{ MJ}$$

The kWh, also called the *Board of Trade* (BOT) *unit*, is the energy absorbed by supplying a load of 1 KW or 1,000 watts for the period of one hour. This is legal unit on which charges for electrical energy are made, and, therefore, it is called the Board of Trade (BOT) unit.

Watt: It is defined as the power expended when there is an unvarying current of one ampere between two points having a potential difference of one volt. As already stated the bigger unit of power is kW or Megawatt.

$$1 \text{ kW} = 1{,}000 \text{ watts}$$
$$1 \text{ MW} = 1{,}000 \text{ kW} = 1\times10^6 \text{ watts}$$

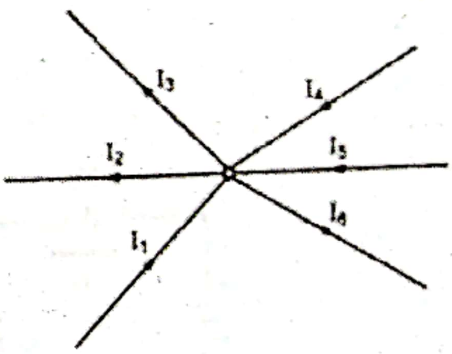
Kirccoff's Laws

- They are used for the systmatic analysis of electric circuits.
- They describe the relationship among circuit voltage and circuit current that must be satisfied.
- they are helpful in determining the equivalent resistance / impedance of a complex network and the current flowing in the various branches of the network.

1. **Kirchhoff's Current Law (KCL) / or Kirchhoff's Point Law:** According to KCL, in any network of wires carrying currents, the algebraic sum of all currents meeting at a point (or junction) is zero.

[OR]

According to KCL, in any network of wires carrying currents, the sum of incoming currents towards any points is equal to the sum of outgoing currents away from that point.



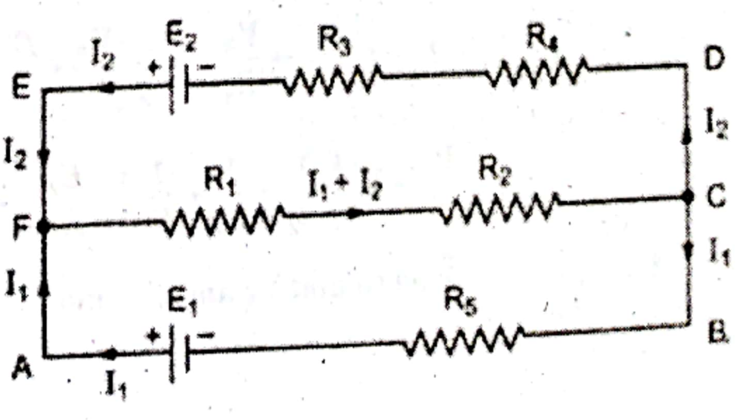
If $I_1, I_2, I_3, I_4, I_5, I_6$ are the currents meeting at junction O as shown, then according to KCL, we get

$$I_1 - I_2 - I_3 + I_4 + I_5 - I_6 = 0$$

OR $$I_1 + I_4 + I_5 = -I_2 - I_3 - I_6$$

2. **Kirchhoff's Voltage Law (KVL) / or Kirchhoff's Second Law / or Kirchhoff's Mesh Law:** According to KVL, in any closed circuit or mesh, the algebraic sum of emfs in that circuit or mesh is equal to the algebraic sum of the products of the currents & resistances of each part of the circuit.

Considering the circuit as shown below,



According to KVL in mesh AFCBA,

$$E_1 - (I_1 + {}_2)R_1 - (I_1 + I_2)R_2 - I_1R_5 = 0$$

According to KVL in mesh EFCDE,

$$E_2 - (I_1 + I_2)R_1 - (I_1 + I_2)R_2 - I_2R_4 - I_2R_3 = 0$$

According to KVL in mesh AEDBA,

$$E_1 - E_2 - I_1R_5 + I_2R_4 + I_2R_3 = 0$$

SOLVED EXAMPLE

Q.1. ***Refer to Q.1 (a), Page: 13–2018***

Q.2. ***Refer to Q.2 (a), Page: 3–2015***

Q.3. ***Refer to Q.2 (b), Page: 5–2018***

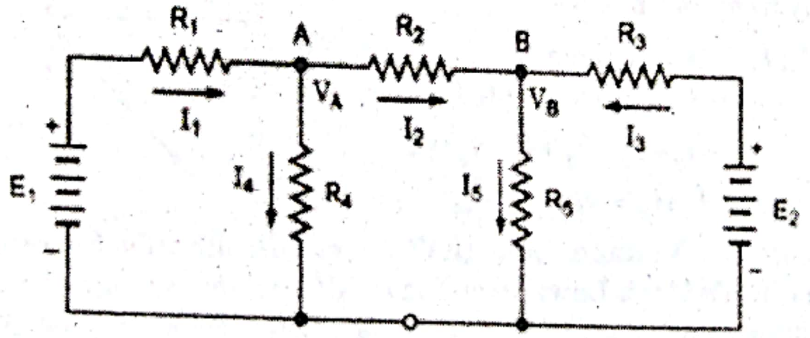
Nodal analysis (Node-Voltage Theorem): According to Nodal Analysis, one of the nodes is taken as refernce node (or zero potential of datum node) and the potential difference between each of the other nodes & the reference node is expressed in terms of an unkonwn voltage and at every node KCL is applied assuing the possible directions of branch currents.

- For 'n' number of nodes, the total number of nodal equation will be $(n - 1)$ in terms $(n - 1)$ number of unkonwn variable of nodal voltages.

Illustration: Considering a two node network as shown. Node C has taken as refernce node. Let V_A and V_B be the voltages at nodes A and B respectively w.r.t node C. Marking the currents arbitrarily, we get

$$I_1 = I_2 + I_4 \quad \ldots \rightarrow \frac{E_1 - V_A}{R_1} = \frac{V_A - V_B}{R_2} + \frac{V_A}{R_4}$$

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} = \frac{E_1}{R_1} \ldots \rightarrow \qquad \text{(i)}$$



Applying KCL at node B, we get

$$I_5 = I_2 + I_3 \quad \ldots \rightarrow \frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3}$$

$$-\frac{V_A}{R_2} + V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) = \frac{E_2}{R_3} \ldots \rightarrow \qquad (ii)$$

Now, the eq. (*i*) and (*ii*) will be solved to find V_A and V_B, and then the values of I_1, I_2, I_3, I_4 & I_5 can be computed easily.

SOLVED EXAMPLE

Q.1. *Refer to Q.3(a), Page: 19–2017*

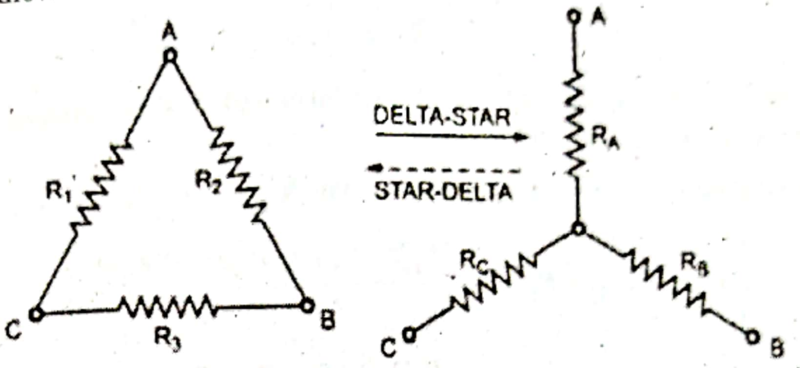
Q.2. *Refer to Q.2, Page: 8–2016*

Q.3. *Refer to Q.2(b), Page: 5–2018*

Q.4. *Refer to Q.2(a), Page: 15–2018*

Network Reduction by Star-Delta

1. Delta-Star Transformation: The replacemnt of delta or mesh by equivalent star system is know as delta-star system.



The two system will be equivalent if the resistance measured between any pair of lines is same in both of the system, when the third line is open.

For terminal B & C,

$$R_{BC} = (R_1 + R_2)\|R_3 \qquad \text{(From Delta System)}$$

$$R_{BC} = \frac{(R_1 + R_2)\times R_3}{R_1 + R_2 + R_3}$$

Also, $$R_{BC} = R_B + R_C \qquad \text{(From Star System)}$$

Since, two system are idential,

$$R_B + R_C = \frac{(R_1 + R_2)\times R_3}{R_1 + R_2 + R_3} \qquad ...(i)$$

Similarly, $$R_{CA} = (R_2 + R_3)\| R_1 \qquad \text{(From Delta System)}$$

$$R_{CA} = \frac{(R_2 + R_3)\times R_1}{R_1 + R_2 + R_3}$$

Also, $$R_{CA} = R_C + R_A \qquad \text{(From Star System)}$$

Since, two system are identical,

$$\therefore \quad R_C + R_A = \frac{(R_2 + R_3)\times R_1}{R_1 + R_2 + R_3} \qquad ...(ii)$$

similarly, $$R_A + R_B = \frac{(R_1 + R_3)\times R_2}{R_1 + R_2 + R_3} \qquad ...(iii)$$

Adding eq. (*i*), (*ii*) & (*iii*), we get,

$$2(R_A + R_B + R_C) = \frac{2(R_1R_3 + R_2R_3 + R_1R_2)}{R_1 + R_2 + R_3}$$

$$R_A + R_B + R_C = \frac{R_1R_3 + R_2R_3 + R_1R_2}{R_1 + R_2 + R_3} \quad ...(iv)$$

Subtracting eq. (i), (ii), & (iii) from eq. (iv), we get respectively,

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3} \quad ...(v)$$

$$R_B = \frac{R_2R_3}{R_1 + R_2 + R_3} \quad ...(vi)$$

$$R_C = \frac{R_3R_1}{R_1 + R_2 + R_3} \quad ...(vii)$$

2. Star-Delta Transformation: the replacement of star network by equivalent delta systemis known as delta-star system.

Multiplying eq. (v) & (vi), (vi)& (vii) and (vii) & (v), we get,

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1R^2{}_2R_3 + R_1R_2R^2{}_3 + R^2{}_1R_2R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1R_2R_3(R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1R_2R_3}{R_1 + R_2 + R_3} \quad ...(viii)$$

Solving eq. (v), (vi), (vii) with eq. (viii), we get

$$R_3 = R_B + R_C + \frac{R_BR_C}{R_A}$$

$$R_1 = R_A + R_C + \frac{R_AR_C}{R_B}$$

$$R_2 = R_A + R_B + \frac{R_AR_B}{R_C}$$

SOLVED EXAMPLE

Q.1. ***Refer to Q.3(b), Page: 20–2017***

Q.2. ***Refer to Q.1(c), Page: 3–2018***

Q.3. ***Refer to Q.3(a), Page: 18–2018***

Q.4. ***Refer to Q.4(b), Page: 6–2016***

Q.5. ***Refer to Q.1(a), Page: 6–2016***

Q.7. ***Refer to Q.1(d), Page: 1–2017***

Q.7. ***Refer to Q.3(b), Page: 29–2015***

Q.8. ***Refer to Q.1(b), Page: 10–2015***

Superposition Theorem ***(Refer to Q.3(b), Page: 22–2014)***

Limitation of Superposition Theorem ***(Refer to Q.1(a), Page: 10–2015)***

SOLVED EXAMPLES

Q.1. *Refer to Q.3(b), Page: 12–2016*

Q.2. *Refer to Q.4(b), Page: 11–2018*

Q.3. *Refer to Q.2(b), Page: 3–2017*

Thevenin's Theorem

(Refer to Q.2(a), Page: 26–2015 and Refer to Q.1(d), Page: 3–2018)

SOLVED EXAMPLES

Q.1. *Refer to Q.2(a), Page: 14–2017*

Q.2. *Refer to Q.3(b), Page: 12–2016*

Q.3. *Refer to Q.4(a), Page: 9–2018*

Q.4. *Refer to Q.1(c), Page: 1–2015*

Norton's Theorem *(Refer to Q.1(d), Page: 3–2018)*

SOLVED EXAMPLES

Q.1. *Refer to Q.4(a), Page: 9–2018*

Q.2. *Refer to Q.3(b), Page: 19–2018*

Q.3. *Refer to Q.2(a), Page: 2–2017*

Q.4. *Refer to Q.3(a), Page: 5–2015*

Maximum Power Transfer Theorem *(Q.2(a), Page: 13–2015)*

• **PLEASE NOTE**

In case of Dependent Source, R_{Th} will be calculated as-

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

SOLVED EXAMPLES

Q.1. *Refer to Q.2(b), Page: 16–2018*

Q.2. *Refer to Q.3(a), Page: 4–2017*

Q.3. *Refer to Q.2(b), Page: 27–2015*

Q.4. *Refer to Q.2(b), Page: 2–2016*

Time Domain Analysis of First Order RC & LC Circuits

Inductance and Capacitance Behaviour at $t = 0^+$

Inductance

$$i_L = \int_0^t v_L dt + i(0)$$

Inductance current, therefore, cannot change suddenly because it would require infinite voltage. Thus at $t = 0^+$, inductance acts as an ideal current source of strength $i_L(0^+) = i_L(0^-)$. If $i_L(0^-) = 0$, the inductance acts as an open-circuit.

This is illustrated in Fig. 1

Capacitance

$$V_C = \int^t_0 i dt + v_c(0)$$

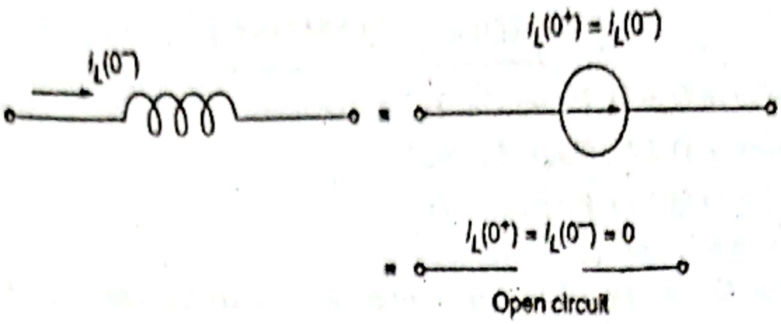


Fig. 1 Inductance behaviour at $t = 0^+$

Capacitance voltage, therefore, cannot change suddenly because of which it acts as an ideal voltage source of strength $v_C(0^+) = v_C(0^-)$. If $v_C(0^-) = 0$ it would act as a short-circuit. This is illustrate in Fig. 2.

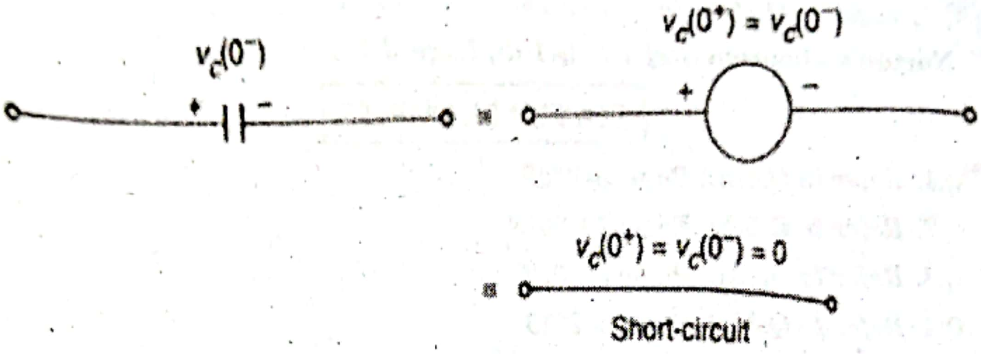


Fig. 2

Inductance and Capacitance Behaviour at $t = \infty$ (Steady State) for dc Excitation

Induction Under steady state; the inductance current reaches a constant value I_C The inductance voltage is then

$$v_L = \frac{dI_L}{dt} = 0$$

Therefore, the inductance acts as short-circuit.

Capacitance Under steady-state, capacitance voltage reaches a constant value V_C The capacitance current is then

$$i_C = C\frac{dV_C}{dt} = 0$$

Therefore, the capacitance acts as an open-circuit.

It is immediately concluded that steady state inductance current and capacitance voltage are determined by the resistive circuit after all inductance have been short-circuit and capacitances open-circuited.

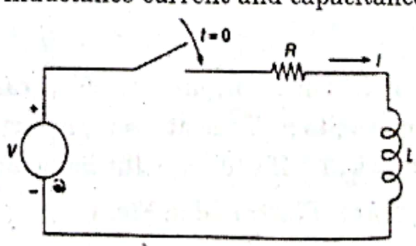


Fig. 3

Step Voltage Response of *RL* Series Circuit: Consider the *RL* series circuit of Fig. 3. Just before the application of voltage (step) the circuit history is represented by the inductor current $i_L(0^-)$ which in this circuit is zero because the circuit is open before the switching-on operation.

The KVL equation of the circuit is

$$R_i + L\frac{di}{dt} = V; \quad t > 0 \tag{1}$$

This is a *nonhomogeneous linear* differential (of first order) with the excitation term appearing on its right hand side. The solution to eq. (1) will have two component, viz. *complimentary function* (natural response, i_n) which should satisfy the equation

$$\frac{di}{dt} + \frac{R}{L}i = 0 \tag{2}$$

and the particular integral (forced response, i_f) which shoul satisfy Eq. (1).

Thus, the complete solution (response) is

$$i = i_n + i_f \tag{3}$$

We already know that the natural response is

$$i_n = Ae^{-(R/L)t} = Ae^{-t/\tau} \tag{4}$$

Let us now discover the forced response (particular integral of eq. (1)). Since the excitation is constant it is intuitively expected that $i_f = I$(a constant) would satisfy Eq. (1). It leads to

$$RI + L\frac{dI}{dt} = V$$

But $$L\frac{dI}{dt} = 0$$

$\therefore$ $$I = \frac{V}{R} = i_f \tag{5}$$

The complete response is then

$$i(t) = Ae^{-(R/L)t} + \frac{V}{R}; t > 0 \tag{6}$$

in which we must determine the arbitrary constant A such that it satisfies the initial condition on inductance current. As already state above

$$i(0^+) = i(0^-) = 0$$

Substituting in Eq. (3.30)

$$0 = A + \frac{V}{R}$$

or $$A = -\frac{V}{R}$$

Hence $$\mathrm{i(t)} = \frac{V}{R}(1 - e^{-(R/L)t}); \; t > 0 \tag{7a}$$

$$= \frac{V}{R}(1 - e^{-t/\tau}); \; t > 0 \tag{7b}$$

From the response, the current rise exponentially from 0 to $I = V/R$ in accordance with the time constant (or natural frequency). It is noticed that the intial (at $t = 0^+$) rate of rise of current is I/τ and the current reaches a vlaue of 63.2% of final value ($I = V/R$) in time of one time constant.

behaviour at $t = 0^+$ and $t = \infty$ stated earlier is confirmed by the response of Eq. (7) from which it follows that

$$i(0^+) = 0$$

i.e., the inductance acts as an open-circuit at $t = 0^+$,

and
$$i(\infty) = \frac{V}{R}$$

i.e., inductance acts as a short-circuit at $t = \infty$ (for dc excitation).

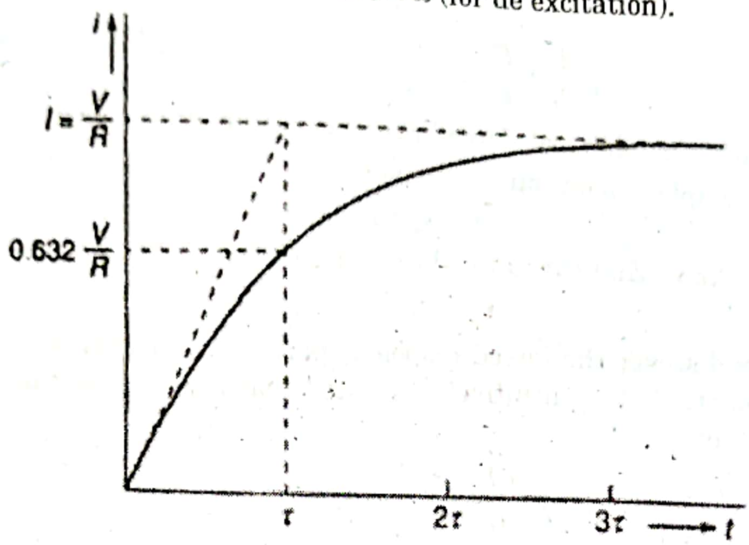


Fig. 4

Consider now the general case with **non-zero initial condition**. then

$$i(0^+) = i(0^-), \text{ non-zero}$$

From Eq. (6) at $t = 0^+$, we find

$$i(0^+) = A + \frac{V}{R} \text{ or } A = i(0^+) - \frac{V}{R}$$

Substituting in Eq. (6), we have the solution

$$i(t) = \left[i(0^+) - \frac{V}{R}\right]e^{-t/\tau} + \frac{V}{R};\ t > 0 \tag{8}$$

We have already shown that

$$i(\infty) = \frac{V}{R}$$

Then eq. (8) is written in the **general form**

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau},\ t > 0 \tag{9}$$

In the **general functional form,** we replace $i(t)$ by $f(t)$. The complete response is

$$f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau},\ t > 0 \tag{10}$$

From Eq. at
$$t = 0^+$$
$$i(t = 0^+) = i(0^+)$$

Which mean that inductance act as a current source

At
$$t = \infty$$
$$i(t = \infty) = \frac{V}{R}$$

which mean that inductance acts as short-circuit in steady-state.

SOLVED EXAMPLES

Q.1. For the circuit shown in Fig. 5, find $i(t)$ after the switch is closed at $t = 0$.

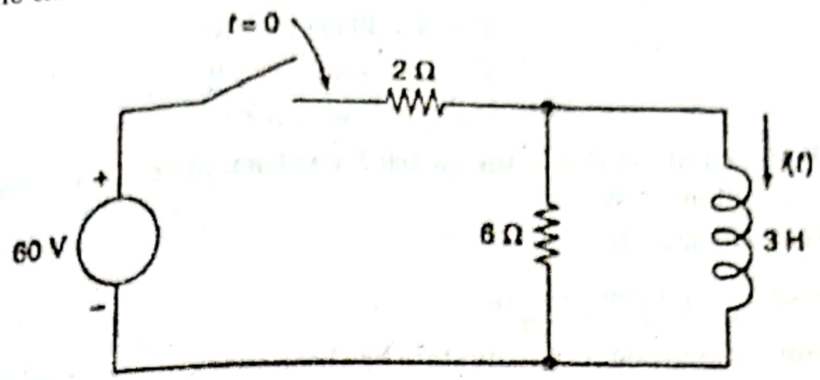


Fig. 5

Solution: Natural Response: After the switch is closed, short-circuiting the voltage source yields the circuit of Fig. 6 from which

$$\tau = L/R_{eq} = 3/1.5 = 2s$$

$$i_n = Ae^{-t/2} \quad \text{...(i)}$$

Forced Responses: With switch closed and $t \to \infty$, the inductance behaves as a short-circuit. The resultant circuit is shown in Fig. 7 from which it follows that

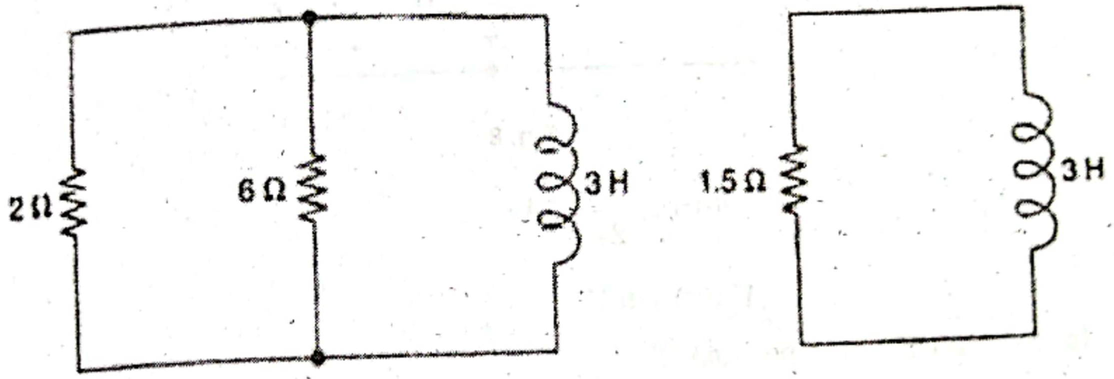


Fig. 6

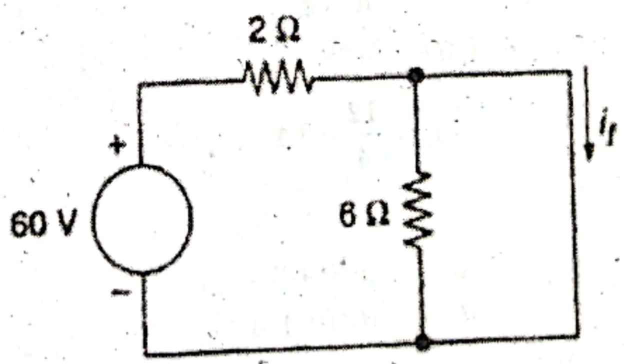


Fig. 7

$$i_f = \frac{60}{2} = 30A \quad \text{...(ii)}$$

Combining

$$i = i_n + i_f$$

$$i(t) = Ae^{-t/2} + 30;\ t > 0 \quad \text{...(iii)}$$

Initial Condition before closing the switch $i(0^-) = 0$

$$\therefore \quad i(0^+) = i(0^-) = 0$$

Substituting in Eq. (iii)

$$0 = A + 30 \text{ or } A = -30$$

$$i(t) = 30(1 - e^{-t/2});\ t > 0 \qquad \text{...(iv)}$$

$$= 30(1 - e^{-t/2})\, u(t) \qquad \text{...(v)}$$

Q.2. In the circuit of Fig. 2, the switch S has been in position '1' for a long time. It is thrown to position '2' at $t = 0$

(a) Find $i(t)$ for $t > 0$

(b) Find $V_L(0^-)$, $V_L(0^+)$ $d\,\dfrac{di}{dt}(0^+)$,

Solution: In position '1', steady state has been reached; inductance acts as a short. Then

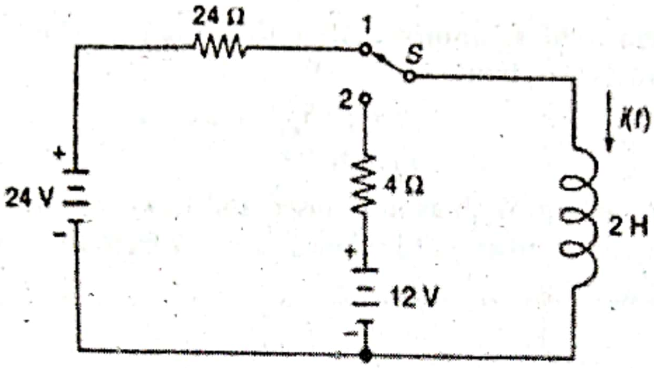


Fig. 8

$$i(0^-) = \frac{24}{24} = 1A$$

$$V_L(0^-) = 0\ V$$

(a) Switch thrown to position '2'

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5s \qquad \text{(i)}$$

$$i_n(t) = Ae^{-2t}$$

$$i_f = \frac{12}{4} = 3A$$

Hence

$$i(t) = Ae^{-2t} + 3 \qquad \text{(ii)}$$

$$i(0^+) = i(0^-) = 1\,A$$

Substituting in Eq. (ii)

$$1 = A + 3 \text{ or } A = -2$$

Hence

$$i(t) = (3 - 2e^{-2t});\ t > 0$$

$$= (3 - 2e^{-2t})\, u(t) \qquad \text{(iii)}$$

Which is plotted in Fig. 9,

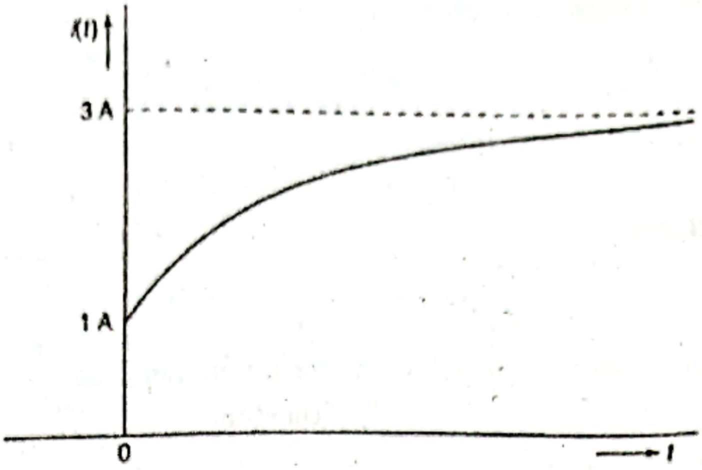


Fig. 9

(b) $v_L(0^-) = 0$, under steady-state inductance acts as a short-circuit. Applying KVL at $t = 0^+$

$$v_L(0^+) + 4i(0^+) = 12$$

$$v_L(0^+) + 4 \times 1 = 12$$

or $$v_L(0^+) = 8\ V$$

As $$v_L = L\frac{di}{dt}$$

$$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$\therefore$ $$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L}$$

$$= \frac{8}{2} = 4A/s$$

Step Voltage Response of RC Series Circuit: Consider the RC series circuit of Fig. 10. Just before the switch is closed, the capacitance is charged to a voltage of V_0. After the switch is closed, the differential equation governing the capacitance voltage is

$$Ri + v_C = V;\ t > 0$$

but $$i = C\frac{dv_C}{dt}$$

$\therefore$ $$RC\frac{dv_C}{dt} + v_C = V;\ t > 0 \qquad (1)$$

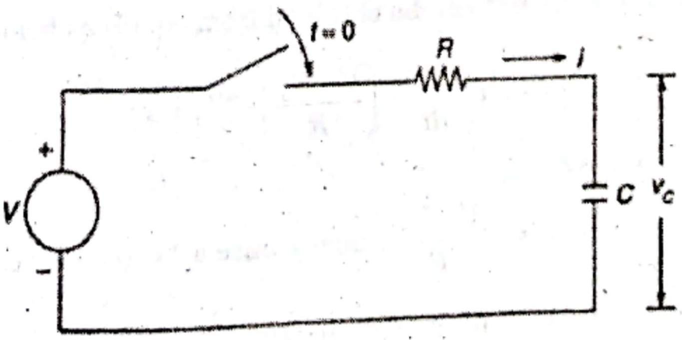


Fig. 10

Natural Response: It is the solution of

$$RC\frac{dv_C}{dt} + v_C = 0$$

which gives

$$V_{Cn} = Ae^{-t/\tau};\ \tau = RC \quad (2)$$

Forced Response

$$RC\frac{dv_C}{dt} + v_C = V$$

Response will have the same form as excitation. Let it be

$$v_{Cf} = V_{Cf}\text{ (constant)}$$

substituting in Eq. (1)

$$V_{Cf} = V \quad (3)$$

Combining Eqs. (2) and (3), the complete response is

$$V_C(t) = Ae^{-t/\tau} + V \quad (4)$$

Substituting the initial condition $v_C(0^+) = V_0$

$$V_0 = A + V \text{ or } A = V_0 - V$$

Hence

$$\begin{aligned} v_C(t) &= (V_0 - V)e^{-t/\tau} + V \\ &= V_0 e^{-t/\tau} + V(1 - e^{-t/\tau});\ t > 0 \quad ...(5) \end{aligned}$$

The plot $V_C(t)$ is shown in Fig. 11.

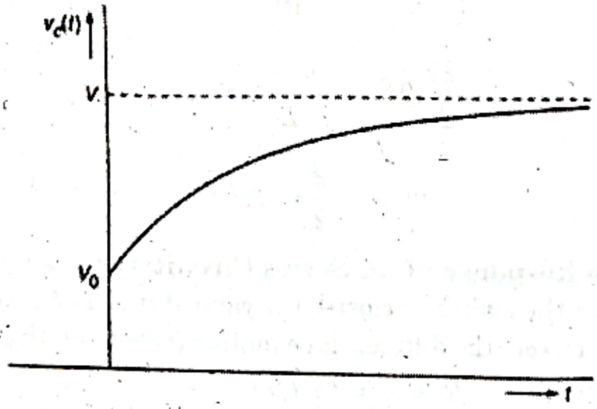


Fig. 11

Observe that $v_C(t)$ of Eq. (5) has the general form

$$f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau};\ t > 0 \quad ...(6)$$

The expression for current can be obtained from eq. (5) as below

$$i(t) = C\frac{dv_C}{dt} = \left(\frac{V - V_0}{R}\right)e^{-t/r};\ t > 0 \quad ...(7)$$

from which if follows that

$$i(0^+) = \frac{V - V_0}{R}; \text{ capacitance acts as a source of voltage} \quad ...(8a)$$

$$i(\infty) = 0; \text{ capacitance acts as an open-circuit} \quad ...(8b)$$

SOLVED EXAMPLES

Q.1. In the circuit of Fig, the switch has been closed for a long time.

Find the expression for v_C as the switch is thrown open. What is the rate of energy consumption in the 400 Ω resistance at $t = 0^+$?

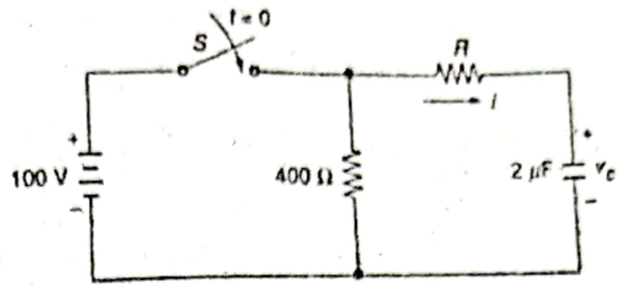


Fig. 12

Solution: It is obvious that $v_C(0^+) = v_C(0^-) = 100V$

After the switch is opened

$$\tau = RC = (400 + 100) \times 2 \times 10^{-6} = 10^{-3} \text{ s}$$

$$\therefore \quad v_C(t) = Ae^{-10^{-3}t}$$

$$v_C(0^+) = A = 100V$$

Hence

$$v_C(t) = 100e^{-10^{-3}t}$$

$$i(0^+) = \frac{100}{500} = 0.2\,A \text{ (capacitance acts as a source of 100 } V)$$

$p(0^+)$ in 400 Ω resistance $= (0.2)^2 \times 400 = 16\ W$

Step Current Response of *RL* Parallel Circuit: Consider the circuit of Fig. 13.

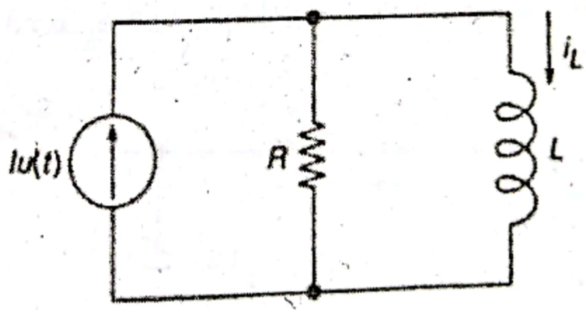


Fig. 13

τ(with current source open-circuited) $= L/R$.

$$\therefore \quad i_{Lf} = Ae^{-t/\tau}$$

i_{Lf}(inductance acts as short-circuit) $= I$

Hence

$$i_L = Ae^{-t/\tau} + I\,;\, t > 0$$

But

$$i_L(0^+) = 0$$

$$\therefore \quad A = -I$$

Finally

$$i_L(t) = I(1 - e^{-t/\tau});\, t > 0$$

Step Current Response of *RC* Parallel Circuit: Consider the circuit of Fig. 14.

τ(with current source open-circuited) = RC

$\therefore$
$$v_{Cn} = Ae^{-t/\tau}$$

Under steady state, capacitance acts as an open-circuit so that all the current passes through R. Therefore

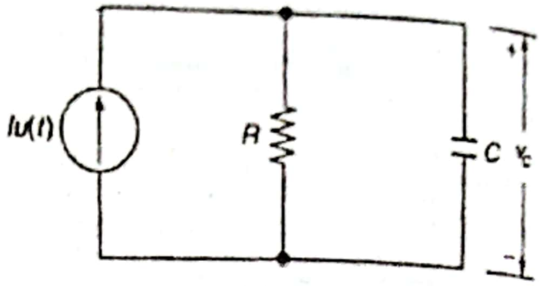


Fig. 14

Hence
$$v_{Cf} = RI$$

But
$$v_C(t) = Ae^{-t/\tau} + RI;\ t > 0$$

$$v_C(0^+) = V_0 \text{ (say)}$$

$\therefore$
$$V_0 = A + RI \text{ or } A = V_0 - RI$$

Hence

$$v_C(t) = (V_0 - RI)e^{-t/\tau} + RI$$
$$= v_0 e^{-t/\tau} + RI\,(1 - e^{-t/\tau});\ t > 0 \qquad (10)$$

SOLVED EXAMPLES

Q.1. In the circuit of Fig. 15, the switch S_1 has been closed for a long time. At $t = 0$, the switch S_2 is closed.

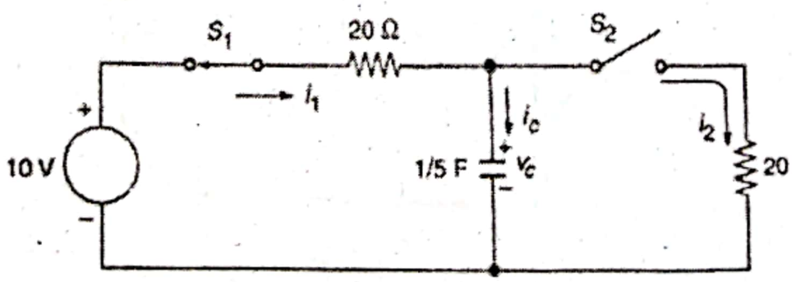


Fig. 15

(a) Without solving for $v_C(t)$, find $v_C(0^+)$, $i_C(0^+)$, $v_C(\infty)$ and $i_C(\infty)$

(b) Derive an expression for $v_C(t)$ and check the result of part (a)

Solution:

(a) Before S_2 is closed, the capacitance is fully charged.

$\therefore$
$$v_C(0^+) = v_C(0^-) = 10 \text{ V}$$

Applying KVL and KCL at $t = 0+$,

KVL (left loop): $-10 + 20\,i_1(0^+) + v_C(0^+) = 0$

or
$$i_1(0^+) = 0$$

KVL (right loop): $-v_C(0^+) + 20\, i_2(0^+) = 0$

or $$i_2(0^+) = \frac{10}{20} = 0.5\,A$$

Using KCL at the node

$\therefore$ $$i_C(0^+) + i_2(0^+) = i_1(0^+) \Rightarrow i_L(0^+) = 0 - 0.5 = -0.5\,A$$

At $t = \infty$, capacitance acts as open-circuit, therefore

$$i_C(\infty) = 0$$

$$v_C(\infty) = 10\times\frac{20}{40} = 5V$$

(b) After closure S_2

To find τ, Short-circuit voltage source. The circuit is shown in Fig. 16.

Then

$$\tau = RC = 10\times\frac{1}{5} = 2s$$

$\therefore$ $$v_{Cn} = Ae^{-t/2}$$

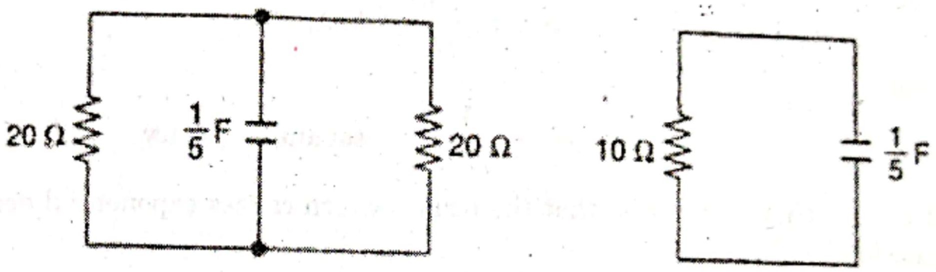


Fig. 16

To find forced response, assume capacitance as open-circuit.

$$v_{Cf} = 10\times\frac{20}{40} = 5V$$

Hence

$$v_C(t) = Ae^{-t/2} + 5;\ t > 0$$

But $$v_C(0^+) = 10V$$

$$10 = A + 5$$

or $$A = 5$$

$\therefore$ $$v_C(t) = 5(1 + e^{-t/2});\ t > 0. \quad (i)$$

Also $$i_C(t) = \frac{1}{5}\times 5\frac{d}{dt}(1+e^{-t/2}) = 0.5e^{-t/2};t > 0 \quad (ii)$$

All the result of part (a) are borne out by Eqs. (i) and (ii).

LC Circuit: If R is assumed infinite in the parallel circuit of RLC or R is assumed zero in the series circuit of RLC, the describing equation of the resulting LC circuit (Fig. 17) is a second order differential equation in which the first order derivative term is absent (case of zero damping). It immediately follows (with $R = \infty$) that

$$\frac{d^2v(t)}{dt^2} + \frac{1}{LC}v(t) = 0 \quad ...(1)$$

Let

$$v(t) = Ae^{st}$$

Which gives

$$Ae^{st}\left(s \quad \overline{LC}\right) = 0 \quad ...(2)$$

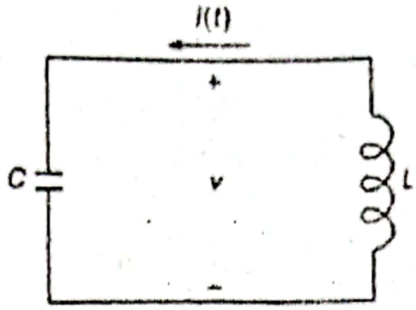


Fig. 17

Non-trivial solution is given by

$$s^2 + \frac{1}{LC} = 0 \quad ...(3)$$

From which

$$S = \pm j\frac{1}{\sqrt{LC}} = \pm j\omega_0 \quad ...(4)$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} = \text{resonant frequency}$$

It is seen that in contrast that the term α which causes exponential decay of the response is absent.

The natural response is now given by

$$v(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$$
$$= (B_1 \cos \omega_0 t + B_2 \sin \omega_0 t) \quad ...(5)$$

Thus $v(t)$ (and so also $i(t)$) is a continuous (non-decaying) sinusoidal oscillation.

In practical circuit, R is never zero so that dissipation occurs and the oscillation cannot be sustained but decays slowly. First few cycles would correspond to nearly sinusoidal oscillation.

Assume LC combination such that

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{13} \quad \text{(i)}$$

Then

$$v(t) = B_1 \cos \sqrt{13}t + B_2 \sin \sqrt{13}t \quad \text{(ii)}$$

Let the capacitor be initially charged so that

$$v_C(0^+) = V_C \quad \text{(iii)}$$

Because the inductance acts as an open-circuit

$$i(0^+) = 0 \quad \text{(iv)}$$

Substituting Eq. (iii) in Eq. (i)

$$V_C = B_1 \quad \text{(v)}$$

From Eq. (ii)

$$i(t) = C\frac{dv(t)}{dt} = C(-\sqrt{13}B_1 \sin \sqrt{13}t + \sqrt{13}\, B_2 \cos \sqrt{13}t) \quad \text{(vi)}$$

By use of Eq. (iv) it follws that

$$B_2 = 0 \tag{vii}$$

Hence

$$v(t) = V_C \cos\sqrt{13}\, t \tag{viii}$$

Then

$$i(t) = C\frac{d}{dt}(V_C \cos\sqrt{13}\, t)$$

$$= -\sqrt{13}\, CV_C \sin\sqrt{13}\, t, \sqrt{13} = \frac{1}{\sqrt{LC}}$$

$$= -\frac{V_C}{\sqrt{L/C}} \sin\sqrt{13}\, t \tag{ix}$$

Step Response: This will be illustrated by means of examples.

Q.1. Solve for $v_C(t)$ in the circuit of Fig. 18. The circuit is initially quiescent (zero initial conditions).

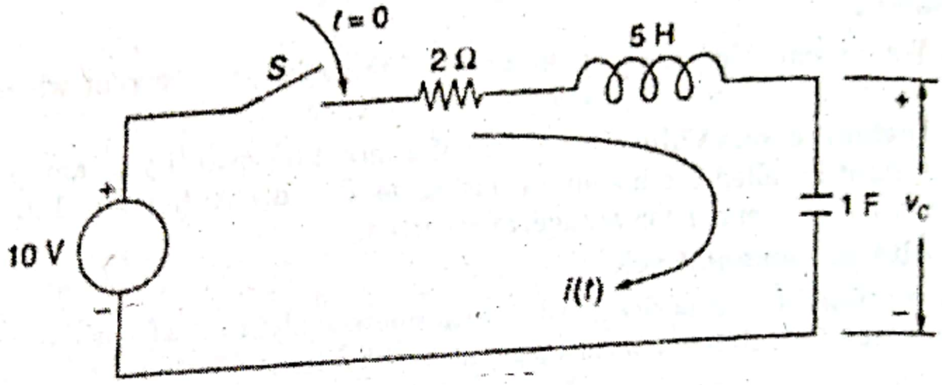


Fig. 18

Solution The describing differential equation of the circuit is

$$2i + 5\frac{di}{dt} + v_c = 10;\ t > 0 \tag{i}$$

But

$$i = 1 \times \frac{dv_C}{dt}$$

and

$$\frac{di}{dt} = \frac{d^2v_C}{dt}$$

Substituting in Eq. (i)

$$2\frac{dv_C}{dt} + 5\frac{d^2vc}{dt^2} + v_C = 10$$

or

$$\frac{d^2v_C}{dt} + \frac{2}{5}\frac{dv_C}{dt} + \frac{1}{5}v_C = 2 \tag{ii}$$

$$\frac{d^2v_C}{dt^2} + \frac{2}{5}\frac{dv_C}{dt} + \frac{1}{5}v_C = 0$$

UNIT - II: AC CIRCUITS

AC Fundamentals

Alternating Quantity: An alternating quantity (V or I) is one which change continuously in magnitude and alternates in direction at regular intervals of time, it rises from zero to maximum positive value, falls to zero increase to maximum value in the reverse direction and falls back to zero again.

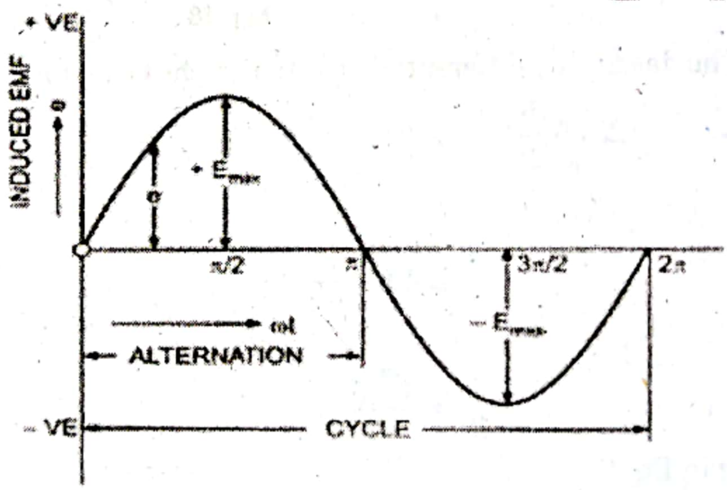
Sinusoidal Quantities (EMF, Voltage or Current): Reasons or advantages of using sinusoidal voltage and current are as follows:

1. The sinusoidal waveform from generation to utilization remains the same.
2. There are no oscillations in develop torque in 3-phase machines and absence of noise in operation.
3. According to Fourier analysis, non-sinusoidal voltages are harmful to the system in term of increased losses in generators, motors, transformers, transmission & distribution system. So, using sinusoidal voltages, the losses can be reduced.
4. Non-sinusoidal voltages provide more interference (or noise) to nearby communication circuits.

Terminology

1. **Waveform:** The shape of the curve of the voltage or current when plotted against time as abscissa is called the waveform.
2. **Instantaneous Value:** The value of alternating quantity at any particular instant is called the instantaneous value. It is denoted by small italic letter such as *e* for emf, *v* for voltage, *i* for current.
3. **Alternation and Cycle**

Alternation: It is a periodic wave with one complete set of positive values or negative values. One alternation corresponds to π radians.



Cycle: It is a periodic wave with one complete set of positive values and negative values. One cycle corresponds to 2π radians.

3. Time Period and Frequency

Time period: Time taken in second by an alternating quantity to complete one cycle is known as Time Period or Periodic Time, denoted by 'T'

Frequency: Number of cycle complete per second by an alternating quantity is known as Frequency, denoted by 'f'. Its unit is Hertz (Hz).

5. **Angular Velocity:** Each cycle spans 2π radians and if this quantity is divided by time period, angular velocity of sine wave is obtained.

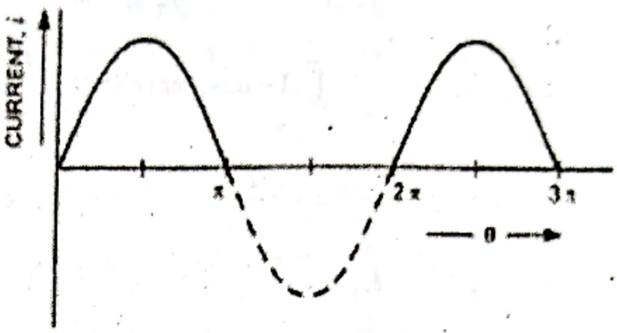
$$\therefore \qquad \omega = \frac{2\pi}{T} = 2\pi f \; radians / \sec$$

6. Electric Time Degrees and Mechanical Degrees

Electrical Time Degrees: In circuit network, one complete cycle of voltage or current is termed as 360 electrical degrees or 2π electrical radians.

Mechanical Degrees: The arc through which a coil of dynamo must rotate in order to generate one cycle of emf is called mechanical degree.

7. Amplitude: The maximu, value, positive or negative, which an alternating quantity attains during one cycle, is called the amplitude of the alternating quantity. The amplitude of an alternating emf is designated by E_{max} or E_m, amplitude of an alternating voltage is designated by V_{max} or V_m, amplitude of an alternating current is designated by I_{max} or I_m.



SOLVED EXAMPLES

Q.1. ***Refers to Q.3(b), Page: 4–2016***

Average and RMS (or Effective) Values of Sinusoidal Quantity:

Refer to Q.4(a), Page- 30, 2015

Form Factor and Peak factor: ***Refer to Q.1(c), Page- 1, 2017***

Significance of From Factor: From factor is a means of relating the mean value with the rms value of alternating quantity. It is useful in determination of rms values of the alternating quantities whose average values over half a period can be determined conveniently.

Significance of Peak Factor: Peak factor of an alternating quantity is very essential in connection with determining the dielectric strength since the dielectric stress developed in an insulating material is proportional to the peak value of the voltage applied to it.

Average and RMS (or Effective) Values of Half-Wave Rectified Alternating Quantity

Average value of Alternating Current or Voltage: Instantaneous values of sinusoidal current is given by

$$i = I_m \sin \omega t$$

Considering the complete cycle, we have

$$I_{avg} = \frac{Area\ of\ complete\ cycle}{2\pi}$$

$$= \frac{1}{2\pi}\int_0^{\pi} i\, d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t\, d(\omega t)$$

$$= \frac{I_m}{2\pi}[-\cos \omega t]_0^{\pi} = \frac{I_m}{\pi}$$

$$\therefore \quad I_{avg} = 0.318 I_m$$

Similarly

$$E_{avg} = 0.318\, E_m$$

RMS Value of Sinusoidal Current or Voltage: Instantaneous value of sinusoidal current is given by

$$i = I_m \sin \omega t$$

$$I^2_{rms} = \frac{\textit{Area of complete cycle of } i^2}{\pi}$$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2\, d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} I^2_m \sin^2 \omega t\, d(\omega t)$$

$$= \frac{I^2_m}{4\pi}\int_0^{\pi}(1-\cos 2\omega t)\, d(\omega t) = \frac{I^2_m}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}$$

$$= \frac{I^2_m}{4\pi} \times \pi = \frac{I^2_m}{4}$$

$$\therefore \quad I_{rms} = \frac{I_m}{2}$$

Similarly

$$E_{rms} = \frac{E_m}{2}$$

Form factor,

$$K_f = \frac{E_{rms}}{E_{avg}} = \frac{E_m/2}{E_m/\pi} = \frac{\pi}{2} = 1.57$$

Peak factor,

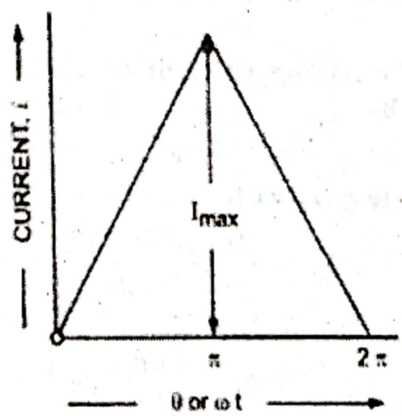
$$K_p = \frac{E_{max}}{E_{rms}} = \frac{E_m}{E_m/2} = 2$$

Average and RMS (or Effective) Values of a Triangular Waveform

Average Value of Alternating Current or Voltage: Let the maximum value of the current be Imax amperes.

The expression for the instantaneous current can be written as

$$i = \frac{I_{max}}{\pi}\theta \qquad \textit{for } 0 < \theta < \pi$$



Considering the half-cycle, we have

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta = \frac{1}{\pi}\int_0^{\pi} \frac{I_{max}}{\pi}\theta\, d\theta = \frac{I_{max}}{\pi^2}\left[\frac{\theta^2}{2}\right]_0^{\pi} = \frac{I_{max}}{2}$$

$$\therefore \quad I_{avg} = \frac{I_{max}}{2}$$

RMS Value of Sinusoidal Current or Voltage

$$I^2_{rms} = \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi}\left(\frac{I_{max}}{\pi}\theta\right) d\theta = \frac{I^2_{max}}{\pi^3}\int_0^{\pi}\theta^2\, d\theta = \frac{I^2_{max}}{\pi^3}\left[\frac{\theta^3}{3}\right]_0^{\pi} = \frac{I^2_{max}}{3}$$

$$\therefore \quad I_{rms} = \frac{I_{max}}{\sqrt{3}}$$

Form factor,
$$K_f = \frac{E_{rms}}{E_{avg}} = \frac{E_m/\sqrt{3}}{E_m/2} = \frac{2}{\sqrt{3}} = 1.155$$

Peak factor,
$$K_p = \frac{E_{max}}{E_{rms}} = \frac{E_m}{E_m/\sqrt{3}} = \sqrt{3} = 1.732$$

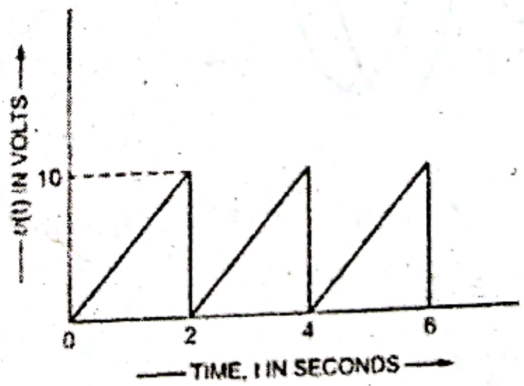
SOLVED EXAMPLE

Q.1. Find average and rms values of the waveform $v(t)$ in Fig

Sol. The given wave is triangular wave of peak value of 10 V.

$$V_{avg} = \frac{V_{max}}{2} = \frac{10}{2} = 5V$$

$$V_{rms} = \frac{V_{max}}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.77V$$



Q.2. ***Refer to Q.4(a), Page: 20-2018***

Q.3. ***Refer to Q.4(a), Page: 30-2015***

Q.4. ***Refer to Q.4(a), Page: 4-2016***

Q.5. ***Refer to Q.3(b), Page: 6-2015***

Phase or Graphical Representation of Alternating Quantities

Phasor: For solution of ac problem, it is advantageous to represent a sinusoidal quantity by a line of definite length rotating in counter-clockwise direction with the same angular velocity as that of the sinusoida; quantity.

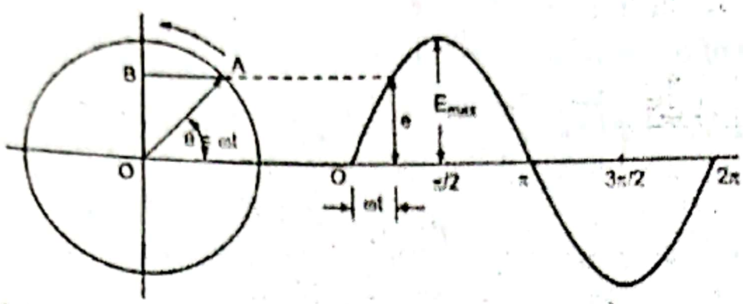
Such a rotating line is called the phasor.

Let $$e = E_{max} \sin \omega t$$

In Fig., $$OA = E_{max}$$

$$OB = Projection\ of\ OA\ on\ y-axis$$

$$= inatananeous\ value\ of\ e$$



If length of the phasor is taken equal to the rms value, then the projection of the rotating phasor on the vertical axis will not give the instantaneous value.

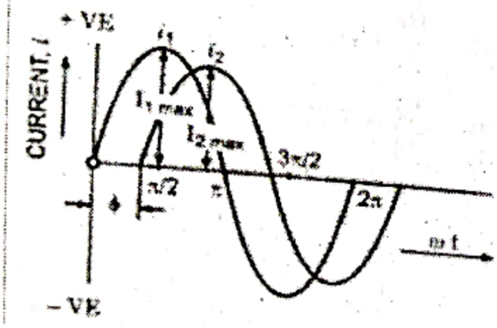
Phase: It is the fraction of the time period of that alternating current that has elapsed since the current last passed through the zero position of reference.

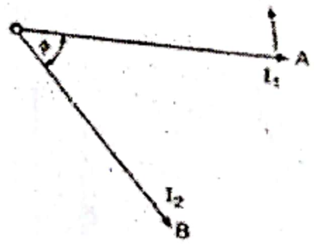
Phase Angle: The angle made by the phasor representing the quantity makes with the reference line.

Phase Difference: It is measured by the angular distance between the points where two curves cross the base or reference line in the same direction.

Case1:

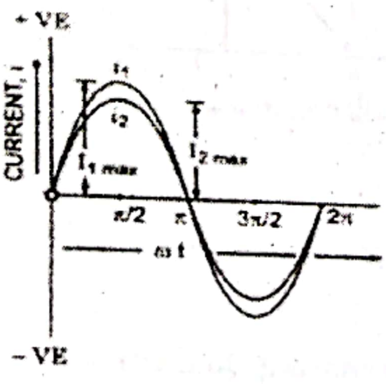
$$i_1 = I_1 \sin \omega t, \qquad i_2 = I_2 \sin (\omega t - \phi)$$

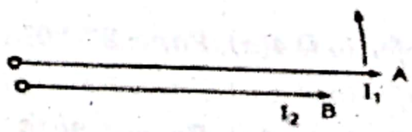




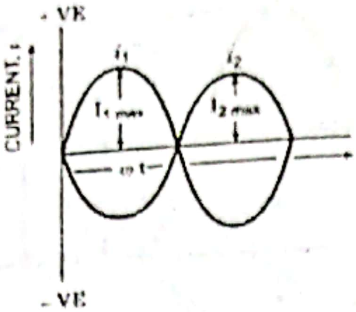
Case 2:

$$i_1 = I_1 \sin \omega t \qquad , i_2 = I_2 \sin \omega t$$





Case 3: $i_1 = I_1 \sin \omega t$, $i_2 = I_2 \sin(\omega t - 180)$



Apparent, Active, Reactive Power Factor ***(Refer to Q.1(c), Page- 10, 2017)***

Power factor: Power factor may be defined as

(i) Cosine of the phase angle between voltage and current, or

(ii) the ratio of the resistance to impedance, or

(iii) The ratio of true power to apparent power.

Physical significance of power factor in AC system

(Refer to Q.1(b), Page: 7-2016)

SOLVED EXAMPLE

Q.1. ***Refer to Q.1(b), Page: 14–2018***

Q.2. ***Refer to Q.3(b), Page: 5–2017***

ANALYSIS OF SINGLE-PHASE AC CIRCUITS

1. Purely resistive Circuits: If the circuit is purely resistive or non-inductive, no reactance emf (self-induced or back emf) is set-up and therefore, whole of the applied voltage is utilized in overcoming the ohmic resistance of the circuit.

Consider an ac circuit containing a resistance R ohms connected across a sinusoidal voltage $v = V_m \sin \omega t$

$$\therefore \quad v = iR$$

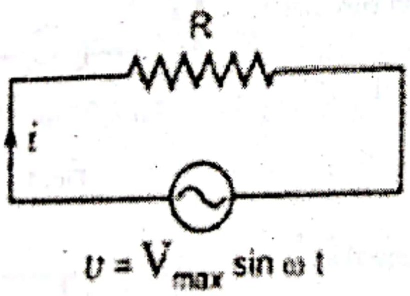


Fig. 1 Circuit Diagram

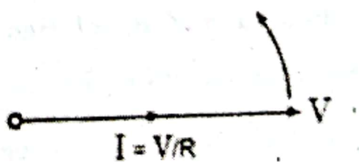


Fig. 2 Phasor Diagram

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \qquad \ldots \rightarrow i = I_m \sin \omega t$$

where, $$I_m = \frac{V_m}{R}$$

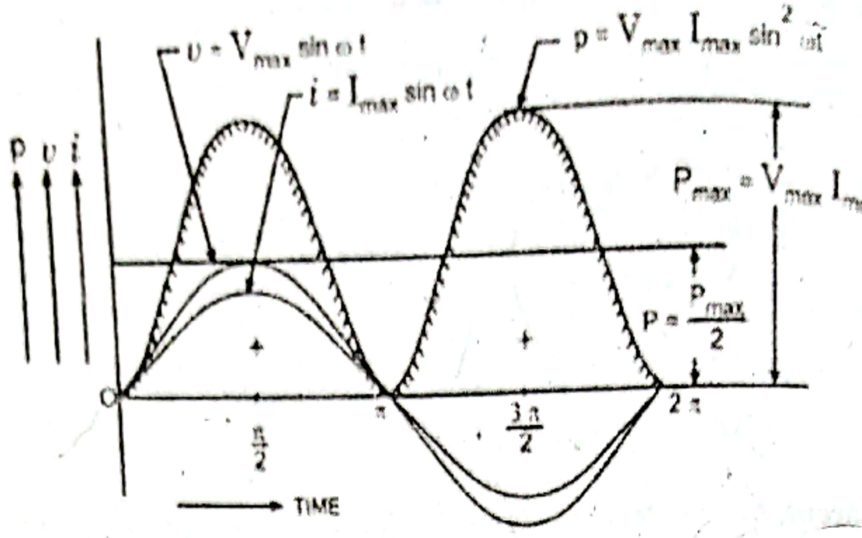


Fig. 3 Wave Diagram

Power in purely Resistive Circuit: Instantaneous Power delivered to the circuit is given as

$$p = v.i = V_m \sin \omega t \,.\, I_m \sin \omega t$$

$$p = V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2}(1-\cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega$$

$$\text{Average power, } P = Avg.of\ \frac{V_m I_m}{2} - Avg.\ of\ \frac{V_m I_m}{2}\cos 2\omega t$$

$$= \frac{V_m I_m}{2} - 0 = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$p = V.I$$

where, V and I are the rms values of applied voltage and current respectively.

2. Purely Inductive Circuits: An inductive is a coil with or without iron core having negligible resistance (practically small resistance). However, a coil of thick copper wire wound on a laminated iron core has negligible resistance is known as choke coil.

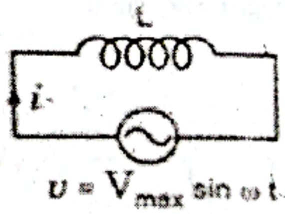


Fig. 1

Let the applied voltage be $v = V_m \sin \omega t$ and

Self-induced of coil $= L$ Henry

Self-induced emf in the coil, $e = -L\dfrac{di}{dt}$

Since, applied voltage at every instant is equal and opposite to the self-induced emf i.e., $v = -e$

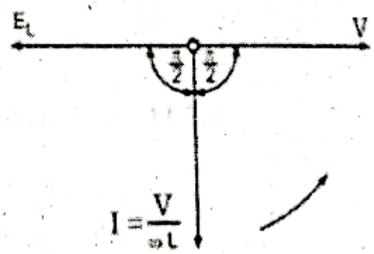


Fig. 2

$$\therefore \quad V_m \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$di = \frac{V_m}{L}\sin \omega t\, dt$$

$$i = \frac{V_m}{\omega L}(-\cos \omega t) + A,$$

where A is constant of integration

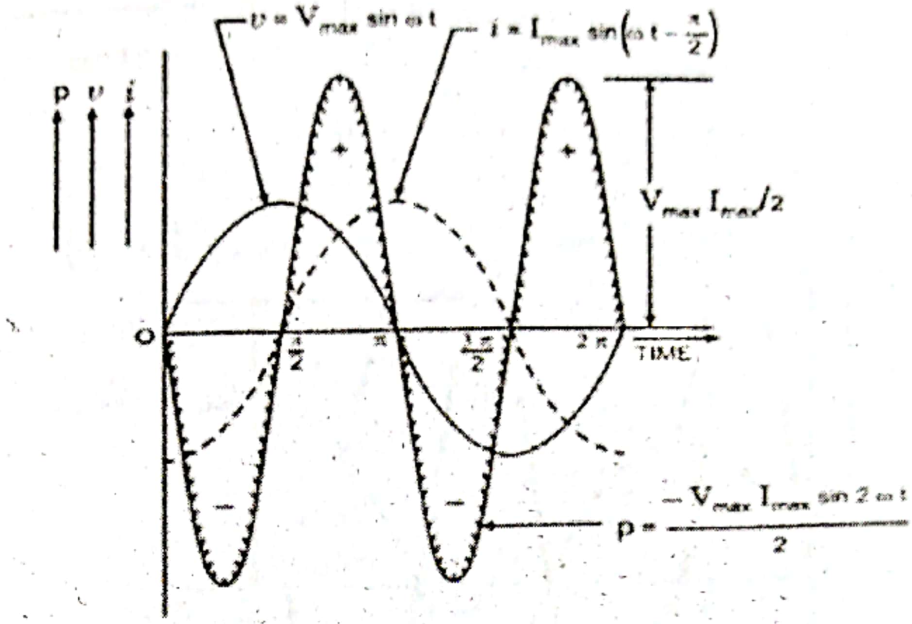
From initial conditions, $A = 0$

$$\therefore \quad i = -\frac{V_m}{\omega L}\cos\omega t = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

At $\omega t = \frac{\pi}{2}$, $i = 0$ *and at* $\omega t = \pi, i = \frac{V_m}{\omega L} = I_m$ (maximum value)

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where, $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$, X_L *is inductive resistance in* Ω



Power in Purely Inductive Circuit

Instanttaneous power, $\quad p = v.\,i = V_m \sin\omega t\,.I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

$$= -V_m I_m \sin\omega t\cos\omega t = -\frac{V_m I_m}{2}\sin 2\omega t$$

The power measured by wattmeter is the average value of 'p' which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence, in a purely inductive circuit, power absorbed is zero

$$P_{avg=0}, \quad \text{Energy stored, } W_L = \frac{1}{2}LI_m^2$$

3. Purely Capacitive Circuits: Let the applied voltage be $v = V_m \sin\omega t$ and Capacitance of a capacitor = C

Farad

The instantaneous charge is –

$$q \quad = C\,.\,v = C\,V_m \sin\omega t$$

As current is rate of change of charge, we have

$$i = \frac{dq}{dt}$$

$$i = \frac{d[CV_m \sin\omega t]}{dt} = \omega C V_m \cos\omega t$$

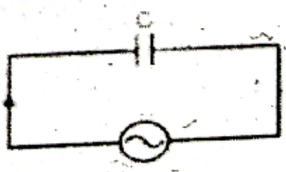


Fig. 1 Circuit Diagram

$$i = \frac{V_m}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots \rightarrow i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $$I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_C}$$

where, X_C is a Capacitive Reactance in Ω

Power in purely Capacitive Circuit

Instantaneous power,

$$p = v.i = V_m \sin\omega t \,.I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t \cos\omega t = \frac{V_m I_m}{2}\sin 2\omega t$$

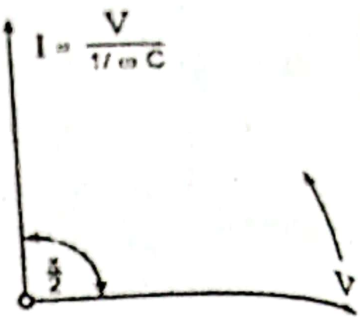


Fig. 2 Phasor Diagram

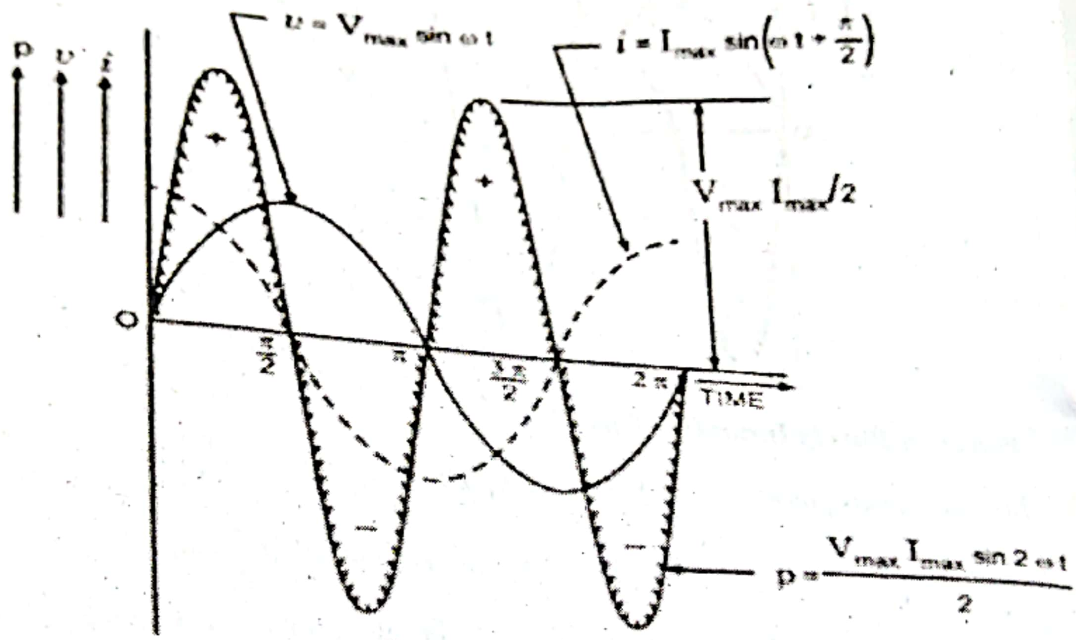


Fig. 3 Wave Diagram

the power measured by wattmeter is the average value of 'p' which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence, in a purely capacitive circuit, power absorbed is zero

$$P_{avg} = 0 \quad , \textit{Energy stored}, W_C = \frac{1}{2}CV^2{}_m$$

SOLVED EXAMPLE

Q.1. *Refer to Q.4(a), Page: 21–2017*

4. Resistance-Inductance (R-L) Series Circuit

Consider an ac circuit consisting of resistance of R ohms and inductance of L henrys connected in series.

Let the supply frequency be 'f' and current flowing through the circuit be of 'I' amperes (rms value).

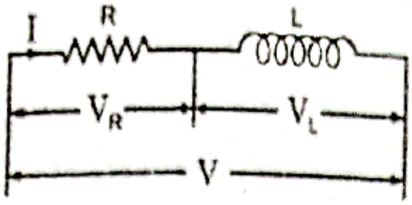


Fig. 1 Circuit Diagram

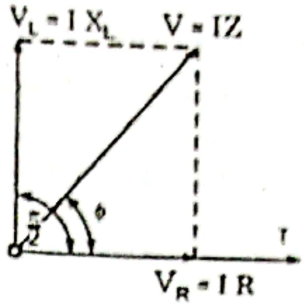


Fig. 2 Phasor Diagram

Voltage drop across resistance, $V_R = I R$ in phase with current.

Voltage drop across inductance, $V_L = I X_L = I\omega L$ leading current I by $\pi / 2$ radians

From the phasor diagrams, the applied voltage is give as-

$$V = \sqrt{V^2{}_R + V^2{}_L} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X^2{}_L} = I.Z$$

where, $Z = Z = \sqrt{R^2 + X^2{}_L}$ and Z is know as impedance

Also, $\tan\phi = \dfrac{V_L}{R_L} = \dfrac{IX_L}{IR} = \dfrac{X_L}{R} = \dfrac{\omega L}{R}$ $\quad \ldots \rightarrow \quad$ $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

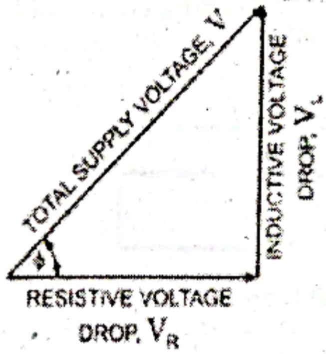


Fig. 3 Voltage Triangle

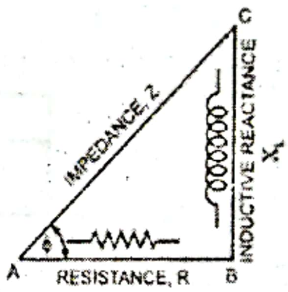


Fig. 4 Impedance Triangle

Let applied voltage voltage be $v = V_m \sin \omega t$,then

$$i = I_m \sin(\omega t - \phi),\ where\ I_m = \frac{V_m}{Z}\ and\ \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

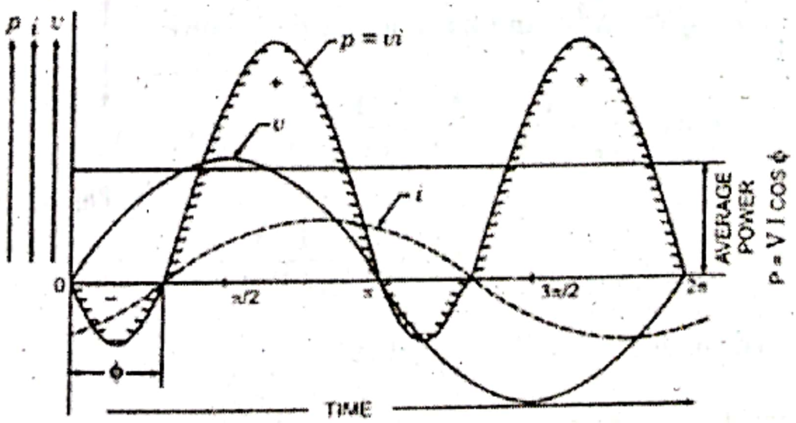


Fig. 5 Wave Diagram

Power in R-L Circuit

instantaneous power,

$p = v.i = v = V_m \sin \omega t . I_m \sin(\omega t - \phi)$

$$\frac{V_m I_m}{2}[2\sin\omega t . \sin(\omega t - \phi)] = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

Average Power, $P = \frac{V_m I_m}{2}\cos\phi - 0 = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$

$$P = VI\cos\phi$$

So, the power in an ac circuit is the product of rms values of voltage and current and cosine of the phase angle between voltage and current.

5. Resistance-Capacitance (R-C) Series Circuit: Consider an ac circuit consisting of resistance of R ohms and capacitance of C farads connected in series.

Let the supply frequency be 'f' and current flowing through the circuit be of 'I' amperes (rms value).

Voltage drop across resistance, $V_R = IR$ in phase with current.

Voltage drop across capacitance, $V_L = IX_C = \frac{I}{\omega C}$ lagging current I by $\frac{\pi}{2}$ radians

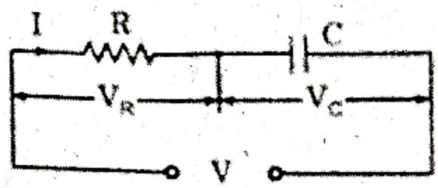


Fig. 1 Circuit Diagram

From the phasor diagram, the applied voltage is give as-

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2} = I.Z$$

where, $Z = \sqrt{R^2 + X_C^2}$ and Z is know as impedance

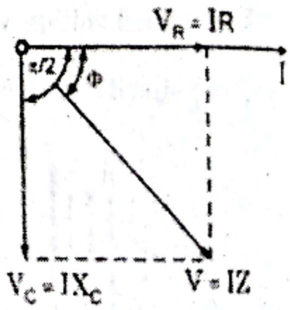


Fig. 2 Phasor Diagram

Also, $\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega CR}$

$$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Let applied voltage be $v = V_m \sin t$, then

$i = I_m \sin(\omega t + \phi)$, *where* $I_m = \frac{V_m}{Z}$ *and* $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

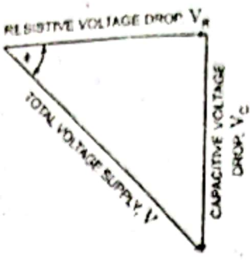


Fig. 3 Voltage Triangle

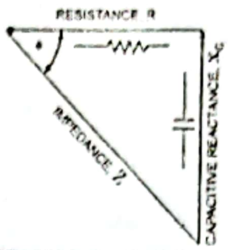


Fig. 4 Impedance Triangle.

Power in R-C Circuit

Instantaneous Power, $p = v.i = v = V_m \sin \omega t \,.\, I_m \sin(\omega t + \phi)$

$$= \frac{V_m I_m}{2}[2\sin \omega t \,.\sin(\omega t + \phi)] = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}(2\omega t + \phi)$$

Average power,

$$p = \frac{V_m I_m}{2}\cos\phi - 0 = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$$

$$P = VI\cos\phi$$

So, the power in an ac circuit is the product of rms values of voltage and current and cosine of the phase angle between voltage and current.

6. **Resistance-Inductance-Capacitance (R-L-C) Series Circuit:** Consider an ac circuit consisting of resistance of R ohms, inductance of L henry and capacitance of C farads connected in series.

Let the supply frequency be 'f' and current flowing through the circuit be of 'I' amperes (rms value)

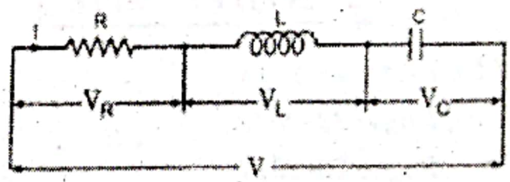


Fig. 1 Circuit Diagram

Voltage drop across resistance, $V_R = I\,R$ in phase with current.

Voltage drop across inductance, $V_L = I\,XL = I\omega L$ leading current I by $\frac{\pi}{2}$ radians

Voltage drop across capacitance, $V_L = I\,XC = \frac{1}{\omega C}$ lagging current I by $\frac{\pi}{2}$ radians

From the phasor diagram, the applied voltage is give as-

(Assuming $V_L > V_C$)

$$V = \sqrt{V^2{}_R + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I.Z$$

where, $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

and Z is known as impedance of the circuit.

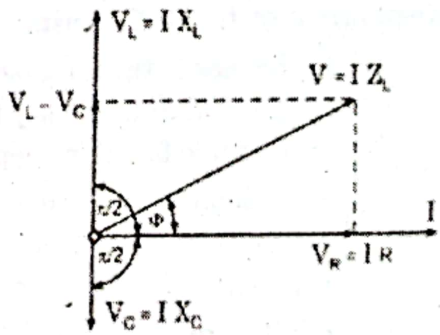


Fig. 2 Phasor Diagram

Also,
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$
$$= \frac{X_L - X_C}{R} = \frac{X}{R}$$
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

ϕ will be positive if $X_L > X_C$ and ϕ will be nagative if $X_L < X_C$

Let applied voltage be $v = V_m \sin \omega t$, then

$i = I_m \sin(\omega t \pm \phi)$, where $I_m = \frac{V_m}{Z}$ *and* $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Power in R-L-C Circuit

instantaneous power, $p = v.i = V_m \sin \omega t \,.\, I_m \sin(\omega t \pm \phi)$

$$= \frac{V_m I_m}{2}[2\sin\omega t.\sin(\omega t + \phi)] = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t \pm \phi)]$$
$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t \pm \phi)$$

Average Power, $p = \frac{V_m I_m}{2}\cos\phi - 0 = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$

$$P = VI\cos\phi$$

So, the power in an ac circuit is the product of rms values of voltage and current and cosine of the phase angle between voltage and current.

SOLVED EXAMPLE

Q.1. ***Refer to Q.4(b), Page: 21–2018***

Q.2. ***Refer to Q.5(a), Page: 22–2018***

Q.3. ***Refer to Q.4(b), Page: 22–2017***

Q.4. ***Refer to Q.5(b), Page: 23–2017***

Q.5. ***Refer to Q.5(b), Page: 33–2015***

Q.6. ***Refer to Q.3(b), Page: 16–2015***

Q.7. ***Refer to Q.3(b), Page: 14–2015***

Q.8. ***Refer to Q.1(d), page: 2–2015***

Q.9. ***Refer to Q.4(b), Page: 6–2017***

Resonance in R-L-C Circuits

- Resonance is the term employed for describing the steady-state operation of a circuit at that frequency for which the resultant response is in the time phase with source function despite the presence of energy storing elements.
- For resonance, there must be two types of independent energy-storing elements capable of intercharging energy between them. For L & C.
- Under the condition of resonance, a network becomes purely resistive in it effects, and the voltage & current in the network are in phase.

1. Series or Voltage Resonance: Under the condition of resonance, a network becomes purely resistive in its effects, and the voltage and current in the network are in phase. For the resonance condition to occur, X_L and X_C should be made equal.

Consider an ac circuit containing a resistance R, an inductance L and a capacitance C connected in series.

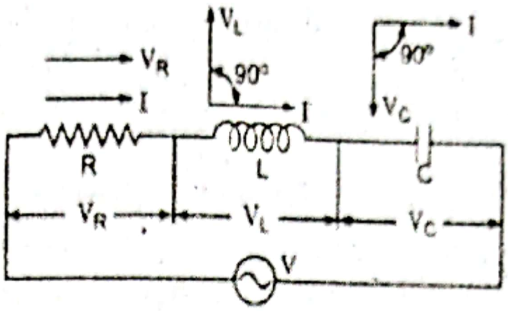


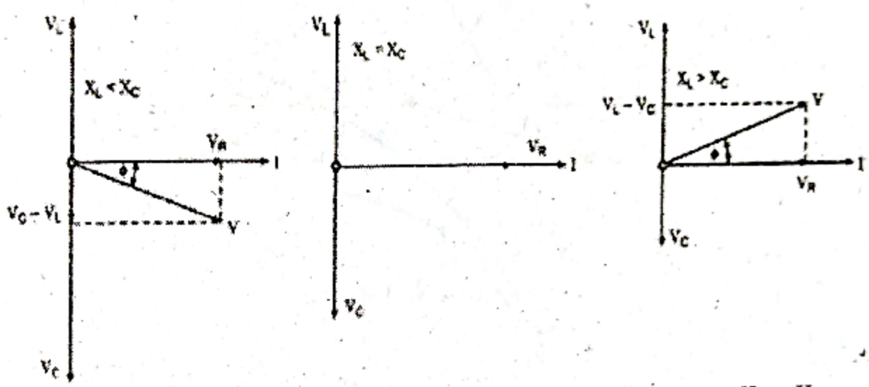
Fig. 1 Circuit Diagram

Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

If for some frequency of applied voltage, $X_L = X_C$ in magnitude, then

i. Net reactance is zero i.e. $X = 0$

ii. Impedance of the circuit, $Z = R$.



(a) When $X_L < X_C$ (b) When $X_L = X_C$ (c) When $X_L > X_C$

Fig. 2. Phasor Diagram

iii. The current flowing through the circuit is maximum and in phase with applied voltage. The magnitude of the current will be equal to $\dfrac{V}{R}$.

iv. The voltage drop across the inductance is equal to the voltage drop across capacitance and is maximum.

v. The power factor is unity.

vi. The power expended = VI watts.

When this condition exists, the circuit is said to be in resonance and the frequency at which it occurs is known as ***resonant frequency***.

Let the resonant frequency be fr, then

$$X_L = \omega L = 2\pi f_r L \quad , \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_r C}$$

$$X_L = X_C \qquad \text{(Condition for resonance)}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C} \quad \ldots \rightarrow \quad f_r = \frac{1}{2\pi\sqrt{LC}} \ or\ \omega_r = \frac{1}{\sqrt{Lc}}$$

Therefore, the value of resonance frequency depends on the parameters of the two energy-storing elements i.e. L and C.

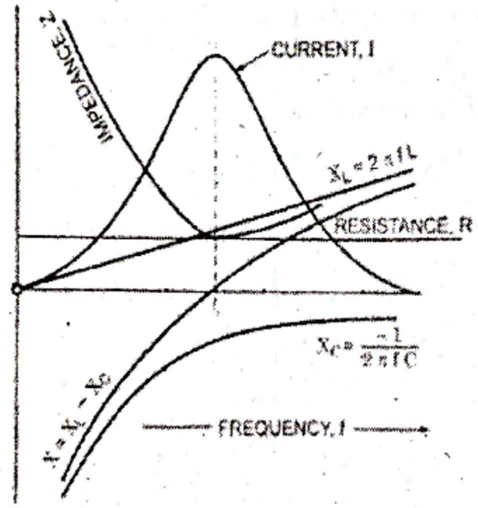
Reason of Voltage Resonance

- When the circuit in resonance, the current is too large and will produce large voltage drop across inductance & capacitance, which will be equal in magnitude but opposite in phase, and each may be several times greater than applied voltage.
- If resistance R would have not been present in the circuit, such a circuit would act like a short-circuit to currents of frequency to which it resonates.
- Since the voltage is maximum, it is called the Voltage Resonance.

Reason of Acceptor Circuit

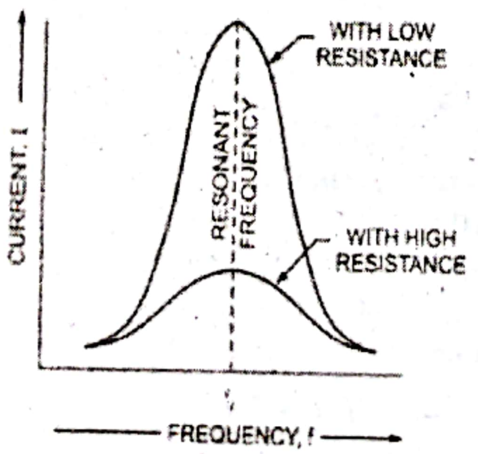
- The series resonance is also called an Acceptor Circuit because such a circuit accepts currents at one particular frequency but rejects currents of other frequencies.
- Such circuits are used in Radio Receivers.

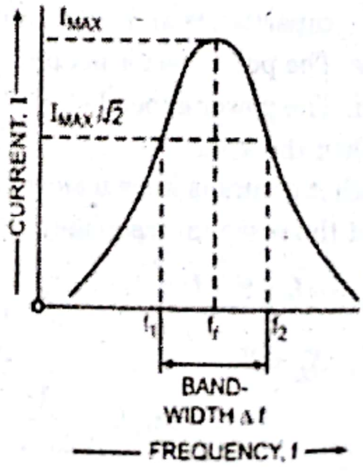
Graphical Representation of Series Resonance



Derivation of bandwidth and quality factor for a series RLC circuit

Bandwidth: The half-wave bandwidth of a circuit is given by the band of frequencies which lies between two points on either side of f_0 where current falls to $\frac{I_0}{\sqrt{2}}$. Narrower the bandwidth, higher the selectivity of the circuit and vice-versa. As shown in Fig., the half power bandwidth is given by





$$\Delta f = f_2 - f_1$$

where, f_1 and f_2 are the corner frequencies.

Actual power input at f_1 and f_2,

$$p = I^2R = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 .R = \frac{I^2{}_{max}R}{2} = \frac{1}{2}P_{max}$$

$$P = \frac{1}{2} \times \textit{power input at resonance}$$

Therefore, f_1 and f_2 at the limits of bandwidth are called half-power points on the frequency scale, and the corresponding value of the bandwidth is termed as half-power bandwidth (B_{hp}) or – 3 dB Bandwidth.

Derivation: The impedance of the turned circuit must be $\sqrt{2}$ times its impedance at resonance so that the current is $\frac{I_{max}}{\sqrt{2}}$. But the impedance at resonance, $X = R$ so at half-power points, the impedance is $\sqrt{2}R$.

$$Z = \sqrt{R^2 + X^2} \quad \ldots \rightarrow \quad \sqrt{2}R = \sqrt{R^2 + X^2} \quad \ldots \rightarrow \quad X = R$$

The reactance at the lower half-power frequency is given as–

$$X_1 = \omega_1 L - \frac{1}{\omega_1 C} = -R$$

Here, minus sign signifies that below resonance, the capacitive reactance exceeds the inductive reactance. The above equation may also be re-written as–

$$\omega^2{}_1 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$\omega 1 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 + \omega^2{}_r}$$

where, $\alpha = \frac{R}{2L}$, $\omega_r = \sqrt{\frac{1}{LC}}$

The equation for the lower half-power frequency is

$$\omega_1 = -\alpha + \sqrt{\alpha^2 + \omega^2{}_r}$$

(negative frequency is meaingless, so discarded)

The reactance at the upper half-power frequency is given as–

$$X_2 = \omega_2 L - \frac{1}{\omega_2 C} = R$$

Here, minus sign signifies that below resonance, the capacitive reactance exceeds the inductive reactance. the above equation may also be re-written as –

$$\omega^2{}_2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \alpha \pm \sqrt{\alpha^2 + \omega^2{}_r}$$

where, $\alpha = \frac{R}{2L}$, $\omega_r = \sqrt{\frac{1}{LC}}$

The equation for the upper half-power frequency is

$$\omega_2 = \alpha + \sqrt{\alpha^2 + \omega^2_r}$$

(*negative frequency is meaningless, so discarded*)

Now, the equation for the bandwidth becomes

$$\omega_{BW} = \omega_2 - \omega_1 = \left[\alpha + \sqrt{\alpha + \omega^2_r}\right] - \alpha\sqrt{\alpha^2 + \omega^2_r} = 2\alpha$$

$$\omega_{BW} = \frac{R}{L}$$

and Bandwidth, $\Delta f = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi} = \frac{R}{2\pi L} HZ$

Lower half-power frequency, $f_1 = f_r - \frac{\Delta f}{2} = f_r - \frac{R}{4\pi L} HZ$

Upper half-power frequency, $f_2 = f_r + \frac{\Delta f}{2} = f_r + \frac{R}{4\pi L} HZ$

Also, $f_1 f_2 = f^2_r$

Quality Factor: It is the voltage magnification that the circuit produces at resonance.

$$Q_r = \frac{Reactive\ Power}{Active\ Power} = \frac{I_{max} X_L}{I_{max} R} = \frac{X_L}{R}$$

$$= \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} \qquad \left(f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$Q_r = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q_r = \frac{\text{Reactive Power}}{\text{Active Power}} = \frac{I_{max} X_C}{I_{max} R} = \frac{X_C}{R}$$

$$= \frac{1}{w_r CR} = \frac{1}{2\pi f_r CR} = \frac{2\pi\sqrt{LC}}{2\pi CR} \qquad \left(f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$Q_r = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The Q-factor may also be defined as–

$$Q - factor = 2\pi \times \frac{\text{Maximum stored energy}}{\text{Energy dissipated per cycle}}$$

$$= 2\pi \times \frac{\frac{1}{2} \times L(I_{max})^2}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R.T_r} = \frac{2\pi L}{R} f_r = \frac{2\pi f_r L}{R}$$

$$Q\text{-}factor = \frac{w_r L}{R}$$

If Q-factor is high, voltage magnification is high and selectivity of tuning coil is also higher.

Q-factor may also be defined as the ratio of resonant frequency to Bandwidth.

i.e.
$$Q_r = \frac{f_r}{\Delta f} = \frac{f_r}{f_2 - f_1}$$

$$= \frac{\dfrac{1}{2\pi\sqrt{LC}}}{\dfrac{R}{2\pi L}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

2. Parallel or Current Resonance ***(Refer to Q.5(b), Page: 22–2018)***

Important Points

(i) Net Susceptance is zero i.e. $\frac{1}{X_C} = \frac{X_L}{Z^2} \ldots \rightarrow \; Z = \sqrt{\frac{L}{C}}$

(ii) Admittance = Conductance

(iii) Reactive component of line current is zero.

(iv) Impedance is purely resistive, maximum in magnitude and equal to L/CR_1

(v) Line current is minimum and equal to *** and is in phase with the applied voltage.

(vi)
$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR_1^2}{L}}$$

Reason of Current Resonance

- Since in parallel resonant circuit, circulating current between the branches is many times the line current.

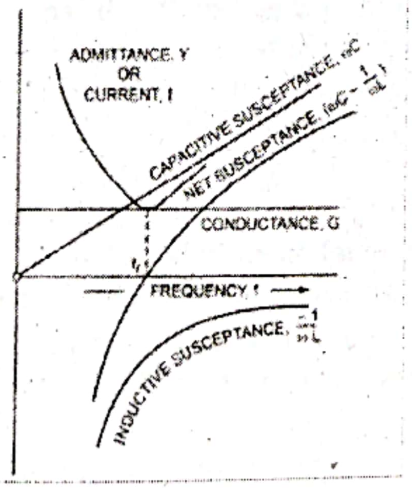
Such type of resonance is sometimes known as Current Resonance.

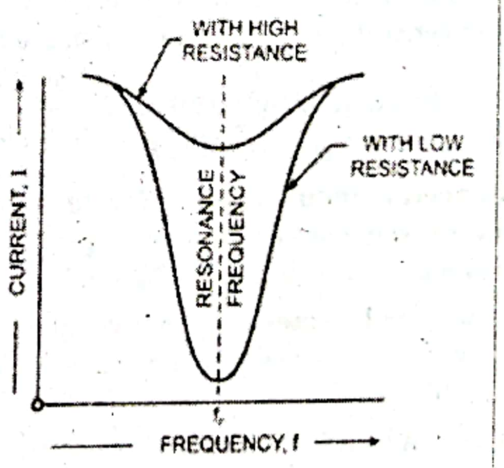
Reason for Rejector Circuit

- Parallel Resonant Circuit is sometimes called the Rejector Circuit because at resonant frequency, the line current is minimum or it almost rejects it.

Tank Circuit: An inductive coil of Inductance L connected in parallel with a Capacitance C is called Tank Circuit.

Graphical Representation of Parallel Resonance





Resonant frequency, Damped resonant frequency and Quality Factor for parallel RLC Circuit

Resonant Frequency: A parallel RLC circuit is said to be in electrical resonance when the reactive component of line current becomes zero. The frequency at which this happens is known as resonant frequency.

Damped Resonant Frequency: The peak resonance frequency depends on the value of the resistor and is described as the damped resonant frequency.

Q-factor: *Refer to Q.5, Page: 16–2016*

Bandwidth:

$$\omega_{BW} = \frac{1}{RC}$$

$$\text{Bandwidth, } \Delta f = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi} = \frac{1}{2\pi RC} \text{Hz}$$

$$\text{Lower half-power frequency, } f_1 = f_r - \frac{\Delta f}{2} = f_r - \frac{1}{4\pi RC} \text{ Hz}$$

$$\text{Upper half-power frequency, } f_2 = f_r + \frac{\Delta f}{2} = f_r + \frac{1}{4\pi RC} \text{ Hz}$$

Also $f_1 f_2 = f^2_r$

SOLVED EXAMPLE

Q.1. ***Refer to Q.5(a), Page: 23–2017***

Q.2. ***Refer to Q.4(a), Page: 14–2016***

Q.3. ***Refer to Q.3(a), Page: 15–2015***

Three-phase Balanced Circuits

- Single-phase systems involve single-phase currents and voltages and they are applicable for domestic applications. For eg, Motors for mixers, fans, air conditioners, refrigerators, etc. However, 1-ϕ system has its own limitation and, therefore, has been replaced by polyphase system.
- For general supply, three-phase (3-ϕ) supply has been universally used. For generation, transmission and distribution of electric power, 3-ϕ system has been universally adopted.
- The 2-ϕ supply and 6-ϕ supply are obtained from 2-ϕsupply.
- A polyphase system is essentially a combination of several 1-ϕ voltage having same magnitude and frequency but displaced from one another by equal angle (electrical), which depends upon the number of phases.

$$\text{Electrical displacement} = \frac{360 \text{ electrical degree}}{\text{Number of Phases}}$$

and Number of phase > 2.

Balanced / Symmetrical System: A supply system is said to be symmetrical when several voltages of the same frequency have equal magnitude and are displaced from one another by equal time. For eg, 3-phase 3-wire or 3-phase 4-wire systems.

Unbalanced System: A 3-phase supply will be unbalanced when either of the 3 phase voltage are unequal in magnitude or the phase angle between these phase is not equal to 120°.

Balanced Load: A load circuit is said to be balanced when the loads (impedances) connected in various phase are same in magnitude as well as in phase.

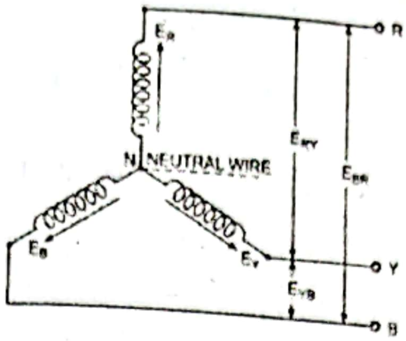
Unbalanced Load: Any 3-phase load in which the impedances in one more phase differ from the impedances of other phases is called unbalanced 3-phase load.

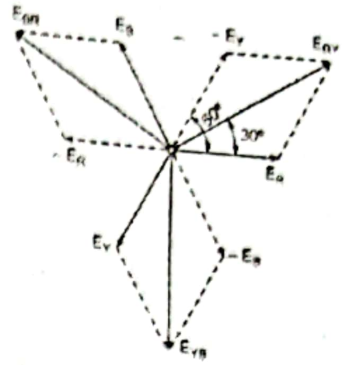
Advantage of 3-phase System over Single Phase System

Refer to Q.5(a), Page: 32–2015

Voltage and Current Relations in Star and Delta Connections

1. STAR / WYE (Y) Connected System





Phase Voltage: Voltage between any line and the neutral point i.e. voltage across the phase winding.

Line Voltage: It is phasor difference of phase emfs of the two phases concerned.

In above Fig.,

- $E_R, E_Y, E_B \rightarrow$ *Phases of induced emf*
- $E_{RN}, E_{YN}, E_{BN} \rightarrow$ *Phase Voltages*
- $E_{RY}, E_{YB}, E_{BR} \rightarrow$ *Line Voltageesn*

(*potential differences between two outers*)

Now,

$$E_{RY} = E_R - E_Y = E_R + (-E_Y)$$

$$E_{RY} = \sqrt{|E_R|^2 + |-E_Y|^2 + 2|E_R||E_Y|\cos 60°}$$

Assuming balanced system, i.e.

$$E_R = E_Y = E_B = E_P$$

$\therefore$

$$E_{RY} = \sqrt{E_P^2 + E_P^2 + 2E_P^2 \times \frac{1}{2}} = \sqrt{3}E_P$$

$\Rightarrow$

$$E_{RY} = E_{YB} = E_{YR} = E_L = \sqrt{3}E_P \qquad \text{and } I_L = I_P$$

Three Phase Power

Instantaneous Phase Voltages are given by –

$$E_R = \sqrt{2}E_P \sin \omega t$$

$$E_Y = \sqrt{2}E_P \sin(\omega t - 120°)$$

$$E_B = \sqrt{2}E_P \sin(\omega t - 240°) = \sqrt{2}E_P \sin(\omega t + 120°)$$

Instantaneous Phase Current are given by –

$$I_R = \sqrt{2}I_P \sin(\omega t - \varnothing)$$

$$I_Y = \sqrt{2}I_P \sin(\omega t - \varnothing - 120°)$$

$$i_B = \sqrt{2}I_P \sin(\omega t - \varnothing - 240°) = \sqrt{2}I_P(\omega t - \varnothing + 120'°)$$

where, $\varnothing$ is the phase angle between E_P and I_P

Total instantaneous power, P is given as –

$$P = E_R I_R + E_Y I_Y + E_B R_B$$

$$= \left[\sqrt{2}E_P \sin \omega t\right]\left[\sqrt{2}I_P \sin(\omega t - \varnothing)\right] + \left[\sqrt{2}E_P \sin(\omega t - 120°)\right]$$
$$\left[\sqrt{2}I_P \sin(\omega t - \varnothing - 120°)\right] + \left[\sqrt{2}E_P \sin(\omega t + 120°)\right]\left[\sqrt{2}I_P \sin(\omega t - \varnothing + 120°)\right]$$

$= E_p I_p [2\sin(\omega t).\sin(\omega t - \varnothing) + 2\sin(\omega t - 120°).\sin(\omega t - \varnothing - 120°) +$
$2\sin(\omega t + 120°).\sin(\omega t - \varnothing + 120°)]$

$= E_p I_p [\cos\varnothing - \cos(2\omega t - \varnothing) + \cos\varnothing - \cos(2\omega t - \varnothing - 240°) +$
$\cos\varnothing - \cos(2\omega t - \varnothing + 240°)]$ $[\because 2\sin A\sin B = \cos(A - B) - \cos(A + B)]$

$P = E_p I_p [3\cos\varnothing - \cos(2\omega t - \varnothing) + \cos(2\omega t - \varnothing - 240°) + \cos(2\omega t - \varnothing + 240°)\}]$

$= E_p I_p [3\cos\varnothing - 0]$ ($\because$ Sum of the three second harmonic oscillating terms which have a progressive phase difference of 120° is zero.)

Thus, the instantaneous power in a 3-Ø balanced system is constant and equal to 3 times the average power per phase.

- Total Circuit Power, $P = 3E_P I_P \cos\varnothing = \sqrt{3}\, E_L I_L \cos\varnothing$
- Reactive Power, $Q = 3E_P I_P \sin\varnothing = \sqrt{3}\, E_L I_L \sin\varnothing$
- Apparent Power, $S = 3 \times$ Apparent power per phase $= 3E_P I_P = \sqrt{3}\, E_L I_L$

Key Points in a Balanced Star-Connected System

1. Line Voltages are 120° apart.
2. Line Voltages are 30° ahead of the respective phase voltages.
3. Line voltages are $\sqrt{3}$ times of the phase voltages.
4. Line currents are equal to phase currents.
5. The angle between line currents and the corresponding line voltages is $(30 \pm \varnothing)$; *+ve for lagging currents and – ve for leading currents.*
6. True Power output $= \sqrt{3}\, E_L I_L \cos\varnothing$, where Ø is the angle between the respective phase current and phase voltage.
7. Apparent Power $= \sqrt{3}\, E_L I_L$.
8. In balanced system, the potential of neutral or star point is zero as

$$E_{NR} + E_{NY} + E_{NB} = 0$$

2. DELTA / MESH (Δ) Connected System: From the fig., it is obvious that line current is phasor difference of phase currents of two phases concerned.

$$\text{Line Current, } I_R = I_{YR} - I_{RB} = I_{YR} + (-I_{RB})$$

Since, phase angle between phase current phasors I_{YR} and $-I_{RB}$ is 60°.

$$I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_{YR}I_{RB}\cos 60°}$$

Assuming balanced load, the phase current in each winding is equal. Let the phase current in each winding be I_P.

$$\therefore \quad I_R = \sqrt{I_P^2 + I_P^2 + 2I_p I_p \times \frac{1}{2}} = \sqrt{3}I_P$$

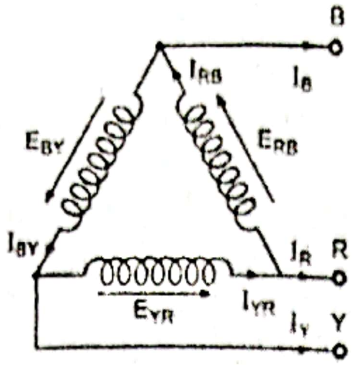
Similarly, Line Current, $I_Y = I_{BY} - I_{YR} = I_{BY} + (-I_{YR})$

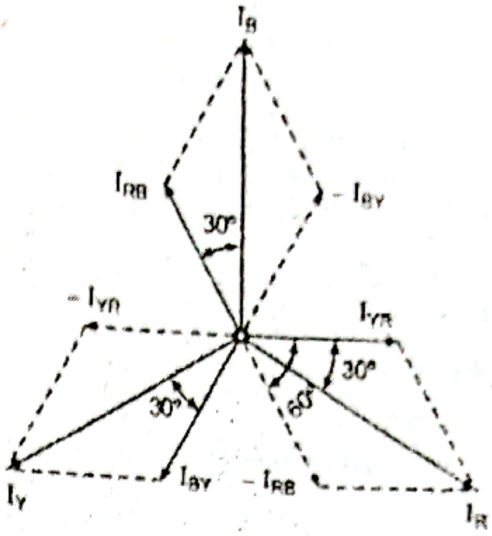
$$I_Y = \sqrt{I_P^2 + I_P^2 + 2I_p I_p \times \frac{1}{2}} = \sqrt{3}I_P$$

And Line Current, $I_B = I_{RB} - I_{BY} = I_{RB} + (-I_{BY})$

$$I_B = \sqrt{I_P^2 + I_P^2 + 2I_p I_p \times \frac{1}{2}} = \sqrt{3}I_P$$

$$\Rightarrow \quad I_R = I_Y = I_B = I_L = \sqrt{3}I_P \quad \text{and} \quad E_L = E_P$$





Three phase power

Instantaneous Phase Voltage are given by –

$$E_R = \sqrt{2}E_P \sin\omega t$$

$$E_Y = \sqrt{2}E_P \sin(\omega t - 120°)$$

$$E_B = \sqrt{2}E_P \sin(\omega t - 240°) = \sqrt{2}E_P \sin(\omega t + 120°)$$

Instantaneous Phase Current are given by –

$$I_R = \sqrt{2}I_P \sin(\omega t - \varnothing)$$

$$I_Y = \sqrt{2}I_P \sin(\omega t - \varnothing - 120°)$$

$$i_B = \sqrt{2}I_P \sin(\omega t - \varnothing - 240°) = \sqrt{2}I_P \sin(\omega t - \varnothing + 120°)$$

where, $\varnothing$ is the phase angle between E_P and I_P

Total instantaneous power, P is given as –

$$P = E_R I_R + E_Y I_Y + E_B R_B$$

$$= \begin{matrix} \left[\sqrt{2}E_P \sin\omega t\right]\left[\sqrt{2}I_P \sin(\omega t - \varnothing)\right] + \left[\sqrt{2}E_P \sin(\omega t - 120°)\right] \\ \left[\sqrt{2}I_P \sin(\omega t - \varnothing - 120°)\right] + \left[\sqrt{2}E_P \sin(\omega t + 120°)\right]\left[\sqrt{2}I_P \sin(\omega t - \varnothing + 120°)\right] \end{matrix}$$

$= E_p I_p[2 \sin(\omega t) . \sin(\omega t - \varnothing) + 2\sin(\omega t - 120°).\sin(\omega t - \varnothing - 120°) +$
$2 \sin(\omega t + 120°) . \sin(\omega t - \varnothing + 120°)]$

$= E_p I_P[\cos \varnothing - \cos (2\omega t - \varnothing) + \cos \varnothing - \cos (2 \omega t - \varnothing - 240°) +$
$\cos \varnothing - \cos (2\omega t - \varnothing + 240°)]$ $\quad [2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$

$P = E_P I_P[3 \cos \varnothing - \cos (2\omega t - \varnothing) + \cos (2\omega t - \varnothing - 240°) + \cos(2\omega t - \varnothing + 240°)\}]$

$= E_p I_P[3 \cos \varnothing - 0]$ ($\because$ Sum of the three second harmonic oscillating terms which have a progressive phase difference of 120° is zero.)

$\therefore P = 3E_P I_P \cos \phi$

Thus, the instantaneous power in a 3-Ø balanced system is constant and equal to 3 times the average power per phase.

- Total Circuit Power, P — $3E_P I_P \cos \varnothing = \sqrt{3}\, E_L I_L \cos \varnothing$
- Reactive Power, $Q = 3E_P I_P \sin \varnothing = \sqrt{3}\, E_L I_L \sin \varnothing$
- Apparent Power, S = 3 × Apparent power per phase = $3E_P I_P = \sqrt{3}\, E_L I_L$

Key Points in a Balanced Star-Connected System

1. Line Voltages are 120° apart.
2. Line Voltages are 30° ahead of the respective phase voltages.
3. Line voltages are $\sqrt{3}$ times of the phase voltages.
4. Line currents are equal to phase currents.
5. The angle between line currents and the corresponding line voltages is $(30 \pm \varnothing)$; *+ve for lagging currents and – ve for leading currents.*
6. True Power output = $\sqrt{3}\, E_L I_L \cos \varnothing$, where $\varnothing$ is the angle between the respective phase current and phase voltage.
7. Apparent Power = $\sqrt{3}\, E_L I_L$.
8. In balanced system, the resultant emf in the closed circuit will be zero.

$$E_{RY} + E_{YB} + E_{BR} = 0$$

Hence, there will be no circulating current in the mesh if no load is connected to the lines.

SOLVED EXAMPLES

Q.1. ***Refer to Q.4(a), Page: 17–2015.***

Q.2 (*a*) State and explain Norton theorem. (6.5)

Ans. Statement: "Any two terminal, linear, bilateral network can be replaced by an equivalent circuit conisting of a current source parallel with the resistance (impedance) seen from that terminals. The equivalent current source, I_N is the short circuit between the terminals and equivalent resistance, R_N, is the ratio of the open circuit voltage to the short circuit current at these terminals."

Explaination:

- Consider a netwoek an fig (a)
- Short circuit terminals AB to find out short circuited current

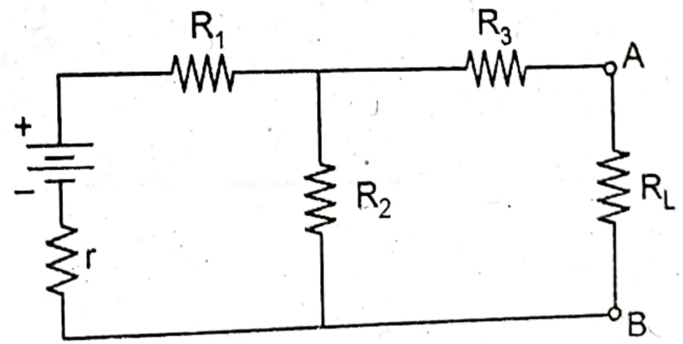


fig. (a)

I_{sc} (By replacing reistance R_L with a zero resistance thick wire) fig (b)

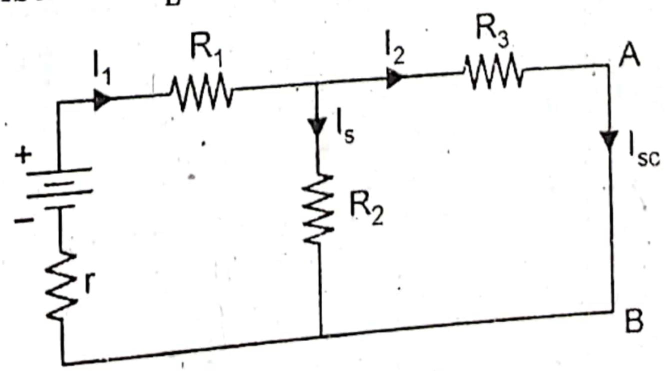


Fig. (b)

– Equivalent resistance of the fig. (c) network, $R_N = \left((R_1 + r) \parallel R_2\right) + R_3$

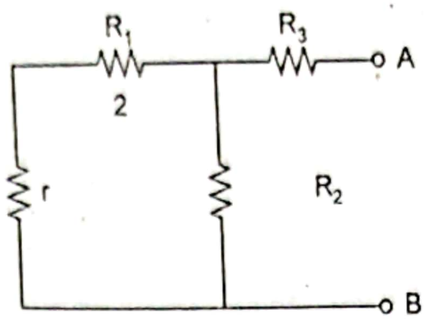


Fig. (c)

– Nortons equivalent circuit will be as per fig (*d*)

$$I_L = \frac{I_{sc} R_N}{R_N + R_L}$$

(applying current dimension in fig. (*d*)

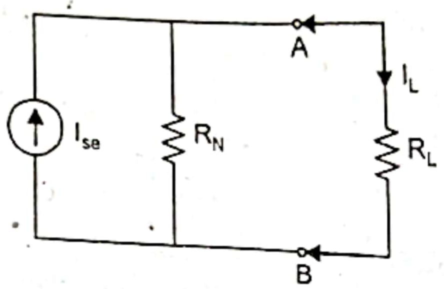
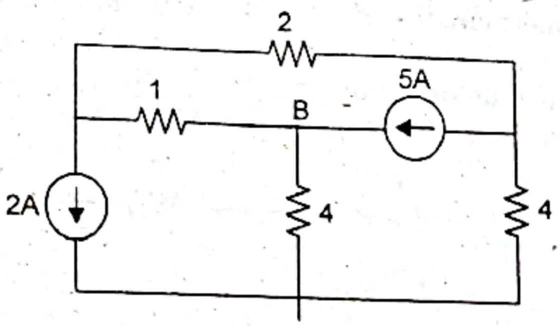
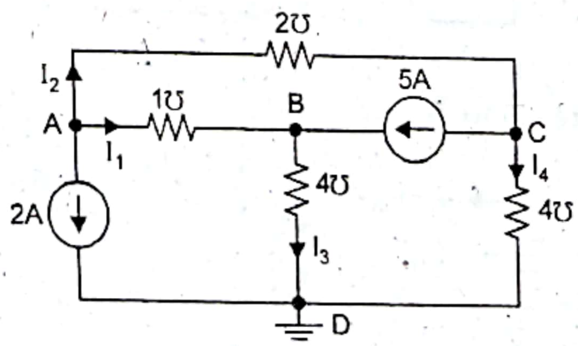


Fig. (d)

Q.2 (*b*) Using the node nodal analysis, find the different branch current in the circuit of figure shown. All Branch conductances are in Siemens. **(6)**



Ans.



At node A, apply KCL/nodal analysis

$$2 + I_1 + I_2 = 0$$

$$\Rightarrow 2 + (V_A - V_B)\,1 + (V_A - V_C)\,2 = 0$$

$$2 + V_A - V_B + 2V_A - 2V_C = 0$$

$$\Rightarrow \quad 2 + 3V_A - V_B - 2V_C = 0$$

$$\Rightarrow \quad 3V_A - V_B - 2V_C = -2 \qquad ....(1)$$

at node B, apply KCL/nodal analysis

$$I_1 + 5 = I_3$$

$$(V_A - V_B)\,1 + 5 = V_B.4$$

$$\Rightarrow \quad V_A - V_B + 5 = 4V_B$$

$$\Rightarrow \quad V_A - V_B - 4V_B = -5$$

$$\Rightarrow \quad V_A - 5V_B = -5 \qquad (2)$$

at node C, apply KCL/nodal analysis

$$I_2 = 5 + I_4$$

$$(V_A - V_C)\,2 = 5 + V_C.4$$

$$\Rightarrow \quad 2V_A - 2V_C = 5 + 4V_C$$

$$\Rightarrow \quad 2V_A - 2V_C - 4V_C = 5$$

$$\Rightarrow \quad 2V_A - 6V_C = 5 \qquad (3)$$

we solve equation (1), (2) & equation (3), then we get

$$V_A = -5/4,\ V_B = 3/4,\ V_C = -5/4$$

$$I_1 = (V_A - V_B)1 = \left(-\frac{5}{4} - \frac{3}{4}\right) 1 = \frac{-8}{4} = -2 \text{ Amp.}$$

$$I_2 = (V_A - V_C)\,2 = \left(-\frac{5}{4} - \left(-\frac{5}{4}\right)\right) 2 = 0 \text{ Amp}$$

$$I_3 = V_B 4 = \frac{3}{4} \times 4 = 3 \text{ amp}$$

$$I_4 = 4V_c = 4 \times \left(-\frac{5}{4}\right) = -5 \text{ amp } \textbf{Ans.}$$

Q.3 (*a*) State and explain maximum power transfer theorem. **(6.5)**

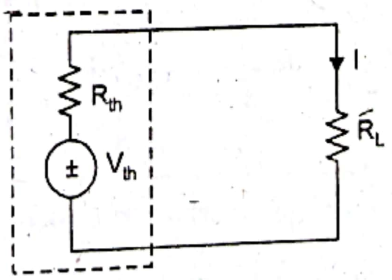
Ans. Statement : For maximum power transfer, load resistance should be equal to the resistance of the Thevenin equivalent (or Norton equialent) of the network to which is connected. However the efficiency of the system is 50%.

Explaination:

– Consider circuit as in fig (*a*).

– For maximum power transfer as per theorem

$$R_L = R_{th}.$$



– Load current I

$$I = \frac{V_{th}}{R_{th} + R_L}$$

– Power consumed by the load Network.
will be –

$$P = I^2 R_L = \frac{V_{th}^2}{(R_{th}+R_L)^2}.R_L$$

– The value of load resistance R_L for the maximum power to be consumed by the load can be obtained as –

$$\frac{\delta P}{\delta R_L} = 0 = \frac{V_{th}^2}{(R_L+R_{th})^2} - \frac{2V_{th}^2}{(R_L+R_{th})^3} R_L = 0$$

or $$R_L + R_{th} - 2 R_L = 0$$

or $$\boxed{R_L = R_{th}}$$

$$P_{max} = \frac{V_{th}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{th}^2}{4R_L}$$

$$P_{lon} = \frac{V_{th}^2}{(R_L+R_L)^2} \times R_{th} = \frac{V_{th}^2}{4R_L}$$

– At maximum power transfer, the efficiency (η) of the system will be

$$\eta = \frac{\text{output}}{\text{input}} = \frac{V_{th}^2/4R_L}{(V_{th}^2/4R_L)} + (V_{th}^2/4R_L)$$

Thus the efficiency of the system is 0.50 or 50%

Q.3 (*b*) State and explian superposition theorem (6)

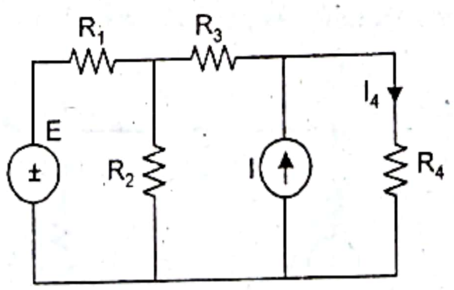
Ans. Statement: The total current or voltage in any part of a linear bilateral network having more than one source is equal to the algebraic sum of the currents or voltage with the sources acting individually while other sources are replaced by their internal resistances.(short circuiting the voltage sources and open circuiting the current sources)

Explaination –

– Consider a network as per fig (*a*)

– Procedure/steps for applying Superposition theorem :

(*a*) Select any one source in the circuit and remove all other sources (replace voltage source by short circuit and current source by open circuit keeping the internal resistances)



(*b*) Calculate the desired voltage or current in an element with only or source selected in the step 1.

(*c*) Repeat the step 1 and step 2 for all the sources one by one.

(*d*) Add all the computed values of elements obtained with each source acting alone. The sum is the actual voltage or current when all the sources are present and acting simultaneously. The polarity of voltage and direction of current must be taken carefully while adding the quantities.

UNIT - II

Q.4 (a) A Coil takes a current of 6 A when connected to a 24 V DC supply. To obtain the same current with a 50 Hz a.c. supply the voltage required was 30 V. Find (i) Inductance of coil (ii) the power factor of the coil. **(6.5)**

Ans. When dc voltage is applied

applied voltage, V_{dc} = 24 volt

current through the coil I_{dc} = 6 A

Resistance of coil, $R = \frac{V_{dc}}{I_{dc}} = \frac{24}{6} = 4\Omega$

when ac voltage is applied

applied voltage V_{ac} = 30 volt

Resistance of coil R = same as when dc is applied i.e. 4 Ω

current, I_{ac} = 6 Amp

coil impedance, $Z = \frac{V_{ac}}{I_{ac}} = 30/6 = 5\ \Omega$

Inductive reactance of coil

$$X_L = \sqrt{Z^2 - R^2}$$

$$= \sqrt{5^2 - 4^2} = 3\ \Omega$$

inductance of coil

$$X_L = 2\pi fL = 3\ \Omega$$

$$\Rightarrow \quad L = 3/2\pi f$$

$$= 3/2\pi \times 50$$

$$= \frac{3}{100\pi} = 9.54 \text{ mH}$$

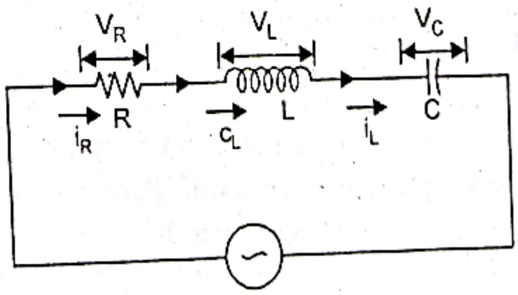
$$\text{power factor} = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{4}{5}$$

$$\cos\phi = 0.8 \text{ (lag).}$$

Q.4 (b) Discuss resonance the Series RLC circuits. How the resonant frequency is calculate. Give graphical representation of resonance. **(6)**

Ans. Consider an ac circuit containing a resistance R, an inductance L and capacitance C connected in series. as shown in figure.



Impedance of the circuit

$$Z = \sqrt{R^2 + (x_L - x_c)^2} = \sqrt{R^2 + \left(wL - \frac{1}{wc}\right)^2}$$

if for some frequency of applied voltage

$x_L = x_c$ in magnitude then

(*i*) Net reactance is zero ie. $X = 0$

(*ii*) Impedance of the circuit $Z = R$

(*iii*) The current flowing through the circuit is maximum and in phase with the applied voltage. The magnitude of the current will be equal to V/R.

(*iv*) $$I x_L = I x_c$$

(*iv*) $$\cos \phi = 1$$

when the above condition exists, the circuit is said to be in resonace and the frequency at which it occurs is known as resonant frequency.

If resonant frequency is denoted by f_r

If, then

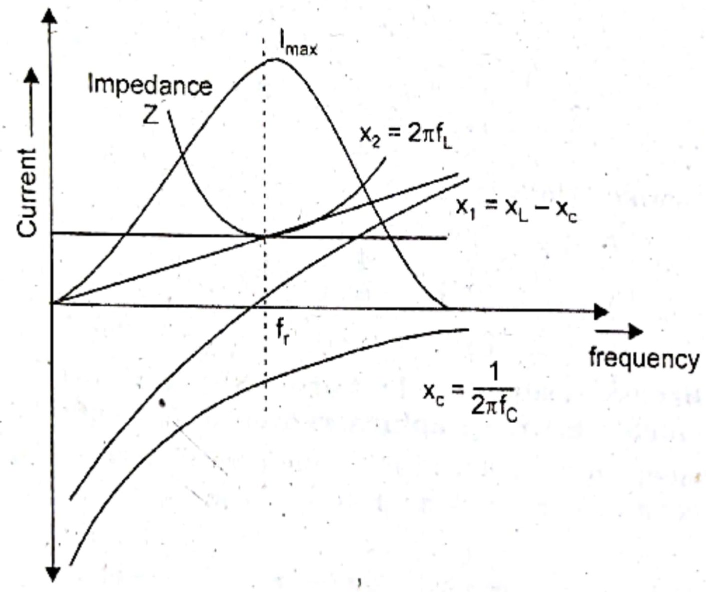
$$x_L = wL = 2\pi f_r L$$

$$x_c = \frac{1}{2\pi f_r c}$$

$x_L = x_c \Rightarrow$ $$2\pi f_r L = \frac{1}{2\pi f_r c}$$

$\Rightarrow$ $$f_r = \frac{1}{2\pi\sqrt{Lc}} \Rightarrow w_r = \frac{1}{\sqrt{Lc}}$$

graphical representation of resonance.



→ The circuit can be made resonant in two different ways namely,

(*i*) by varying L& C paramerer at a constant supply or frequency.

(*ii*) by varying the supply frequency & with Parameters L and C constant.

→ In the above graphical representation, Resistance R is independent of supply frequency, therefore, remains constant. It has been represented by a straight line. i.e. parallel to the X-axis.

$\rightarrow$ $$x_L = \omega_L$$

$\Rightarrow$ $$x_L = 2\pi f_r L$$

$\Rightarrow$ $$f_r \; \alpha \; x_L$$

Due $x_L \alpha f_r$, so it is represented by a straight line passing through the origin. and it lies on the first quadrant. Similary

$$x_c = \frac{1}{2\pi f_r c}$$

$$x_c \alpha (1/fr).$$

→ So x_c inversely proprtional to the resonant frequency and it is represented by a rectangular hyperbola. It lies in the fourth quadrant.

→ The net reactance is the difference of inductive reactance x_L and capacitive reactance x_c and the curve drawn between the net reactance $(x_L - x_C)$ and frequency will be a hyperbola. The frequency at which the reactance curve crosses the frequency axis is called the resonant frequency. (f_r)

→ The impedance of the circuit, Z being equal to $\sqrt{R^2 + (x_L - x_c)^2}$ is minimum at resmant frequncy f_r.

→ At frequencies lower than resonant frequency f_r the impedance Z is large and capacitive as $x_C > x_L$ and the power factor is leading. and at frequencies higher than resonant frequency f_r, the impedance Z is large but $x_L > x_C$ and power factor is lagging. The power factor has the maximum value of unity at resonant frequency.

Q.5 (*a*) A coil of inductance 9 henry and resistance 50 ohm in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current of 1 A occurs at 75 Hz, Find the frequency at which current is 0.5 A. **(6.5)**

Ans. Given

$$\text{Inductance L} = 9 \text{ Henry}$$

$$\text{Inductive reactance, } X_L = 2\pi f L$$

$$\text{Resistance} = 50 \textbf{ ohms}$$

$$\text{maximum current } I_{max} = 1 \text{ A}$$

$$\text{frequency} = 75 \text{ Hz}$$

$$X_L = 2\pi f L = 2\pi \times 75 \times 9 = 4241.15\ \Omega$$

$$I_{max} = \frac{V}{R} \Rightarrow 1 = \frac{V}{50} \Rightarrow V = 50 \text{ Volt.}$$

$$f_r = \frac{1}{2\pi\sqrt{Lc}} \Rightarrow 75 = \frac{1}{2\pi\sqrt{9 \times c}}$$

$$\Rightarrow \quad C = 1/(75 \times 2\pi)^2 . 9$$

$$C = 1/(471.23)^2.9$$

$$C = 5 \times 10^{-7} \text{ Farud}$$

when current is 1 amp

$$V_L = I x_L$$

$$V_L = 1 (2\pi f L)$$

$$= 2\pi \times 75 \times 9$$

$$V_L = 4241.5 \text{ Volt.}$$

when current is 0.5 amp.

$$V_L = Ix_L$$
$$4241.5 = (0.5)(2\pi f L)$$
$$4241.5 = 0.5 \times 2 \times \pi \times f \times 9$$
$$\Rightarrow f = 150.01 \text{ Hz. Ans.}$$

Q.5 (b) Write down the advantges of 3 phase system over single phase. (6)

Ans. Although single phase system is employed for the operation of almost all the domestic & commercial appliances e.g. lamps, fans, electric irons, Tv sets, computers etc. But it has own limitations in the field of generation, transmission, distribution and Industrial applications. Therefore 3-phase system is universalhy adopted for generation, transmission, and distribution of electric power due to the following main advantages over 1 ϕ system.

→ **(1) constant power:** In single phase circuits, the power deliverd is pulsating. Even when the voltage and current are in phase, the power is zero twice in each cycle whereas in 3 ϕ system, power delivered is almost constant when the loads are balanced.

→ **(2) Higher rating:** The rating of 3ϕ supply is nearly 1.5 times the rating of 1ϕ machine of the same size.

→ **(3) Power transmission economics:** To transmit the same power over a fixed distance at a given voltage. 3 ϕ system requires only 75% of the weight of conducting material of that required by 1ϕ system.

→ **(4) Superiority of 3ϕ induction motors :** The three phase induction motors have wide spread field of applications in the industries because:

3 ϕ Induction motors are self starting

3 ϕ Induction motors have higher power factor and efficiency than that of 1 ϕ.

FIRST TERM EXAMINATION [SEPT. 2015]
FIRST SEMESTER [B.TECH]
ELECTRICAL TECHNOLOGY [ETEE–107]

Time. 1.30 Hours. M.M. : 30

Note: Q No.1 is compulsory. Attempt any two more Questions from the rest.

Q.1. (a) Chose the correct option (Justify your answer) **(2×5 = 10)**

Circuit show in Fig. 1 contains ideal sources. Current flowing through 6 ohm resistor is- (A) 2A (B) 5 A (C) 7A (D) 3A

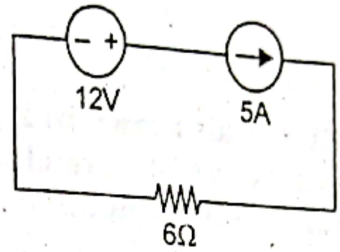


Fig. 1

Ans. From the above circuit diagram, current through '6Ω' resistance is 5 Amp. Because when a voltage source and current source connected in series in a circuit, then current source will dominate. So option 'B' is correct.

Q.1. (b) Rewrite the complete and correct statement: "Internal resistance of ideal voltmeter is....whereas internal resistance of ideal voltage source is....."

Ans. "Internal resistance of ideal voltmeter is "O"... whereas internal resistance of ideal voltage source is "∞" infinite.

Q.1. (c) (i) For a circuit shown in Fig. 2, the reading of ideal voltmeter is.....

(ii) For a network in Fig. 3, if we find the Thevenin's Equivalent Circuit at terminal X & Y then Thevenin's Voltage will be

Ans. 1.(c) (i)

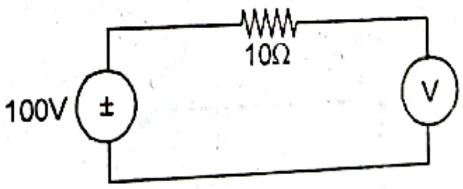


Fig. 2

From the Fig. 2 the reading of ideal voltmeter is 100*V*.

Ans. 1.(c) (ii) For finding the Thevenin's equivalent, short the voltage source and open current source.

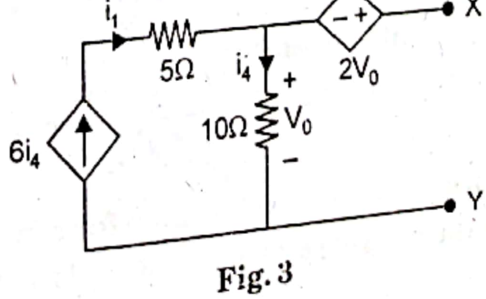


Fig. 3

Now,
$$i_4 = \frac{V_0}{10}$$

$$R_{xy} = 5 + 10 = 15\ \Omega$$

$$6\,i_4 - 2V_o = 0$$

$$\frac{6V_0}{10} - 2V_0 = 0$$

$$6V_0 - 20\ V_0 = 0$$

$$\boxed{V_o = 0}$$

Because there is no independent source.

Q.1. (d) For a series RL circuit shown in Fig 4. Draw a phasor diagram showing all the electrical quantities (Voltage and Current) marked in the circuit. Also write the impedance of circuit in j notation.

Ans.

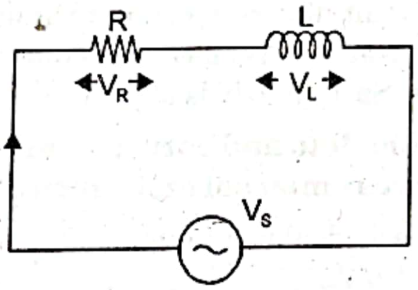
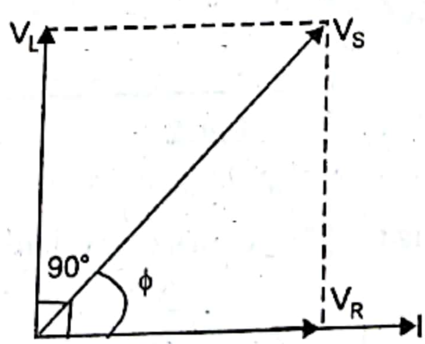


Fig. 4

From the above circuit

$$V_s = \sqrt{V_R^{\,2} + V_L^{\,2}} \qquad ...(1)$$

Phasor diagram of RL circuit



and impedance of RL circuit is given by

$$\boxed{Z = R + jX_L} \qquad ...(2)$$

Q.1. (e) What is a form factor? Give its value for Sinusoidal voltage.

Ans. From factor is the ratio of rms value of wave form to Average value of Wave form.

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}} = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{2}}{2\frac{I_m}{\pi}} = 1.11$$

• "Value of form factor for sinusoid is "1.11."

Q.2. (a) For a network shown in Fig. 5, find all the marked mesh currents.

(6 + 4 = 10)

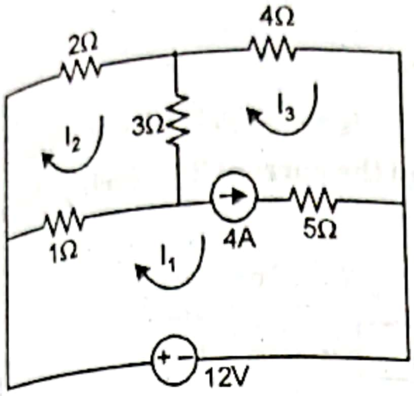


Fig. 5

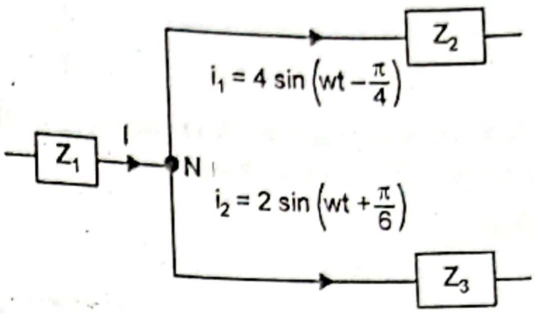
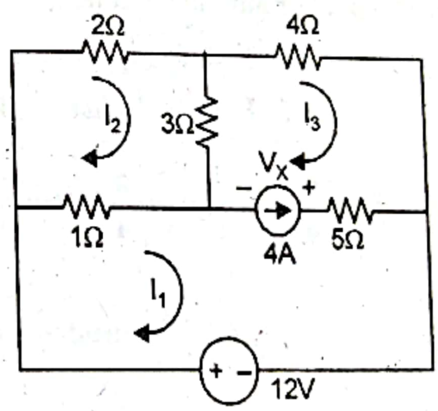


Fig. 6

Ans

$$I_1 = ?$$
$$I_2 = ?$$
$$I_3 = ?$$



(Applying Mesh Analysis)

$$12 + V_x = 6\,I_1 - I_2 - 5\,I_3 \quad ...(1)$$
$$0 = -I_1 + 6\,I_2 - 3\,I_3 \quad ...(2)$$
$$-V_x = -5\,I_1 - 3\,I_2 + 12\,I_3 \quad ...(3)$$

Adding equation (1) to (3), we get

$$12 = I_1 - 4\,I_2 + 7I_3 \quad ...(4)$$

• The relation between the source current and mesh currents is given by

$$I_s = I_1 - I_3$$
$$4 = I_1 - I_3 \quad ...(5)$$

→ Now from equation (3), (4), (5) the value of I_1, I_2 and I_3 can be determine.

→ Multiply equation (3) by 2, equation (4) by 3, and adding the results to eliminate I_2.

We get

$$I_1 + 15\,I_3 = 36$$

$$I_1 - I_3 = 4 \quad ...(6)$$

• From above equation $I_1 = 6\text{ Amp}, I_3 = 2\text{ Amp}$ ...(7)

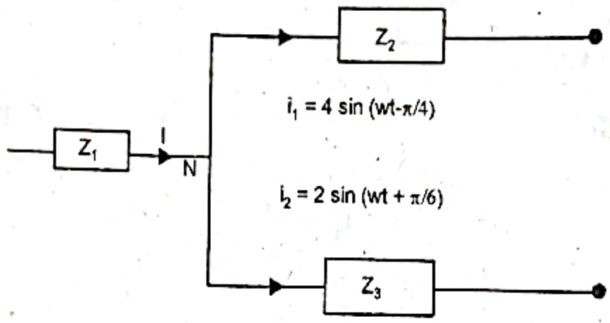
Put the value in equation (3), we get

$$I_2 = 2\text{ Amp}$$

So, $I_1 = 6\text{ Amp}, I_2 = 2\text{ Amp}, I_3 = 2\text{ Amp}$ **Ans.**

Q.2. (b) For a network shown in Fig. 6, find the current 'I' entering the node 'N'. Express 'I' in polar form.

Ans.



• From the above, 'I' is the phasor sum of currents

i.e.

$$I = i_1 + i_2 \quad ...(1)$$

•
$$I = 4\sin(\omega t - \pi/4) + 2\sin(\omega t + \pi/6)$$

$$= 4\sin\omega t.\cos\frac{\pi}{4} - 4\cos\omega t\sin\frac{\pi}{4} + 2\sin\omega t\cos\pi/6 + 2\cos\omega t\sin\pi/6$$

$$= \frac{4}{\sqrt{2}}\sin\omega t - \frac{4}{\sqrt{2}}\cos\omega t + 2.\frac{\sqrt{3}}{2} + 2.\frac{1}{2}\cos\omega t$$

$$I = 4.560\sin\omega t - 1.828\cos\omega t \quad ...(2)$$

Standard form of equation is. $i = I_m \sin(\text{wt} + \alpha)$

$$i = (I_m\cos\alpha)\sin\text{wt} + (I_m\sin\alpha)\cos\text{wt} \quad ...(3)$$

By comparing, we find, $I_m\cos\alpha = 4.560$

$$I_m\sin\alpha = 1.828$$

• Squaring and adding these quantities we get

$$I_m^2\cos^2\alpha + I_m^2\sin^2\alpha = (4.560)^2 + (-1.828)^2$$

$$Im^2(\cos^2\alpha + \sin^2\alpha) = 20.79 + 3.34$$

$$I_m^2 = 24.13$$

$$\Rightarrow \quad I_m = 4.91\text{ Amp.}$$

$$\frac{I_m \sin\alpha}{I_m \cos\alpha} = \frac{-1.828}{4.560} \Rightarrow \tan\alpha = -0.400$$

$\Rightarrow$

$$\boxed{\alpha = -21.8^\circ}$$

$\Rightarrow$ Now standard current equation $- i = I_m \sin(\omega t + \alpha)$

$$i = 4.91 \sin(\omega t - 21.8^\circ) \text{ Ans.}$$

$\Rightarrow$

Q.3. (a) Find the current through R_L in the network shown in Fig. 7 using Norton Theorem. **(5)**

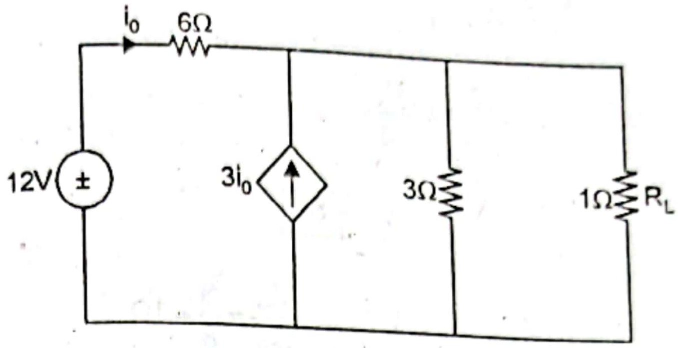
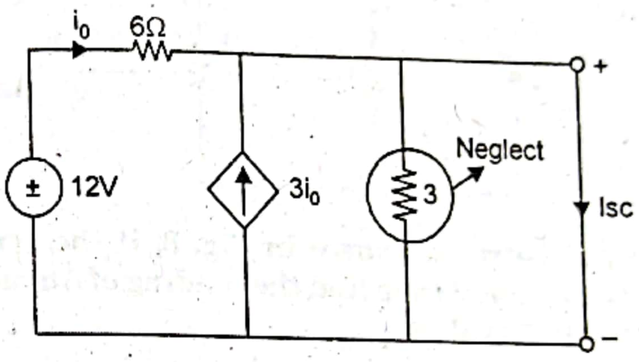


Fig. 7

Ans. First remove R_L and short terminals



$$I_{sc} = 3\,i_o + i_o = 4\,i_o$$

But

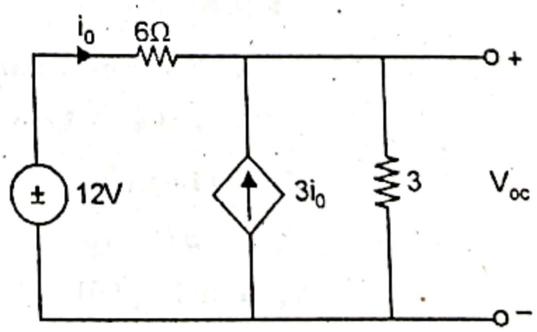
$$i_o = \frac{12}{6} = 2 \text{ Amp}$$

$$I_{sc} = 4 \times 2 = 8 \text{ Amp}$$

$\Rightarrow$

$$\boxed{I_{sc} = I_N = 8 \text{ Amp}}$$

Now open the terminals



Nodal Analysis

$$i_o + 3i_o - \frac{V_{oc}}{3} = 0$$

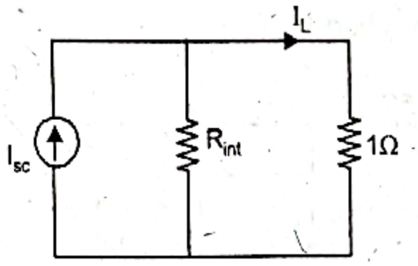
$$\Rightarrow \quad 4i_o = \frac{V_{oc}}{3}$$

$$\Rightarrow \quad 4\left[\frac{12-V_{oc}}{6}\right] - \frac{V_{oc}}{3} = 0$$

$$8 - \frac{2V_{oc}}{3} - \frac{V_{oc}}{3} = 0$$

$$\boxed{V_{oc} = 8V}$$

Now $\quad R_{in} = \frac{V_{oc}}{Isc} = \frac{8}{8} = 1\Omega$



Ans.

Q.3. (*b*) For a AC network shown in Fig. 8, if the reading of voltmeter (indicating RMS value) is 60V then find the reading of Ammeter. Note Ammeter is also indicating the RMS value. (5)

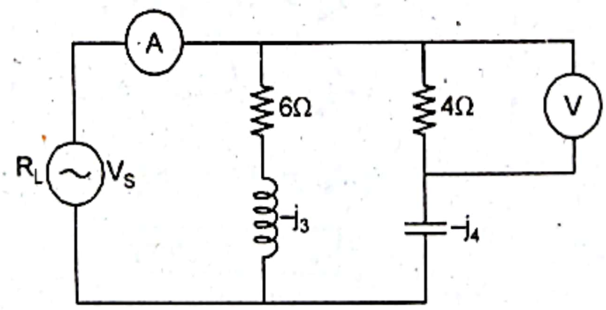


Fig. 8

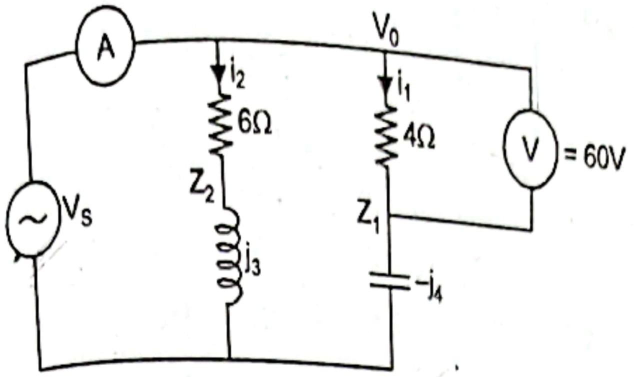
Ans. V = 60V (rms) Ammeter reading

$$i_1 = 60/4 = 15 \text{ Amp}$$

$$Z_1 = (4 - j_4)\ Z_2 = (6 + j_3)$$

$$V_0 \Rightarrow 15\,(4 - j_4)$$

$$V_1 = (60 - j\,60) \text{ volt}$$



$$\boxed{i_2 = \frac{V_1}{Z_2}}$$

Now

Because in parallel voltage is same so it directly appears to Z_2.

$$i_2 = \frac{60 - j60}{6 + j_3} = (10 - j20)$$

Now

$$i = i_1 + i_2$$

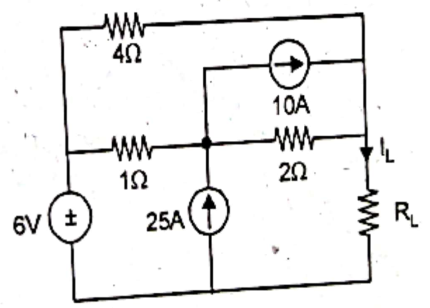
$$= (15 + j0) + (10 - j\,20)$$

$$i = (25 - j20) \text{ Amp}$$

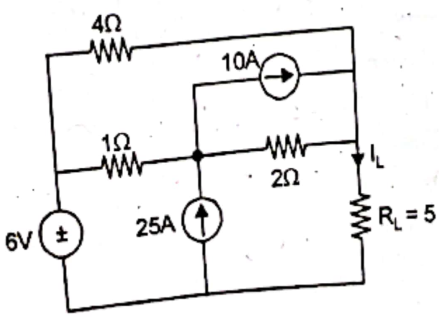
Now Ammeter reads $\boxed{i = 25\text{Amp}}$ **Ans.**

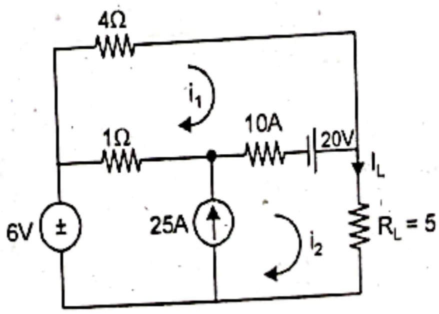
Q.4. (a) For a network shown in Fig. 9, if R_L is load resistance & I_L is current flowing through it then complete the table given below. **(7)**

S.No.	R_L	I_L
1		5 Ohm
2		10 Ohm
3		15 Ohm



Ans.





$$\Rightarrow \quad 4i_1 + 20 + 10(i_1 - i_2) + 1(i_1 - i_2) = 0$$

$$\Rightarrow \quad 15i_1 - 11i_2 = -20 \quad \ldots(1)$$

$$i_2 - i_1 + 10(i_2 - i_1) - 20 + 5i_2 = 0$$

$$-11i_1 + 16i_2 = 20 \quad \ldots(2)$$

$$\begin{aligned} 15i_1 - 11i_2 &= -20 \times 16 \\ \Rightarrow \quad 11i_1 - 16i_2 &= -20 \times 11 \\ \hline 240i_1 - 121i_1 &= -320 + 220 \end{aligned}$$

$$119\,i_1 = -100$$

$$\boxed{i_1 = -0.840 \text{ Amp.}}$$

$$-(15 \times 0.840) = 11\,i_2 = -20$$

$$-12.6 - 11\,i_2 = -20$$

$$-11\,i_2 = -20 + 12.6$$

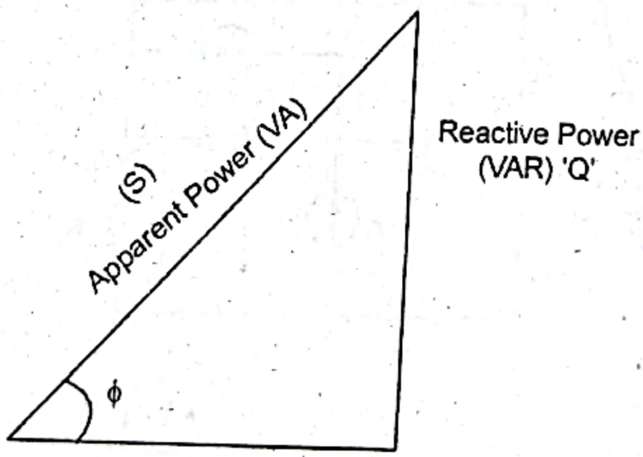
$$-11\,i_2 = -7.4$$

$$\boxed{i_2 = 0.672}$$

$$\boxed{I_1 = I_2 = 0.672 \text{ Amp}}$$

Q.4. (*b*) Draw the Power Triangle. Also indicate different types of powers in AC network along with their units.

Ans. Power triangle is used to define the different types of power.



Real Power

Or

True Power

Or

Active Power

(Watts). 'P'

⇒ Real Power is denoted by 'P' and its unit is "watt".

⇒ Reactive Power is denoted by 'Q' and its unit is "VAR *i.e.* voltage-amp Reactive".

⇒ Apparent Power is denoted by 'S' and its unit is "VA", *i.e.* voltage- Amp.

⇒ From the power triangle

$$\boxed{S^2 = P^2 + Q^2}$$

$\Rightarrow \boxed{N_r = 0}$, motor is at stand still condition

UNIT I

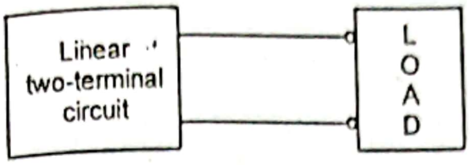
Q.2. (a) State and explain Thevenin's theorem. (6.5)

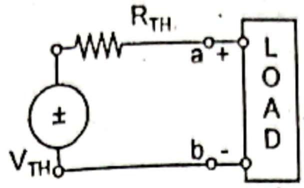
Ans. A linear two-terminal circuit can be replaced by an equivalent circuit consisting of voltage source V_{TH} in series with resistor R_{TH}.

V_{TH} = Open circuit voltage at the terminals

R_{TH} = input or equivalent resistor the terminal.

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$





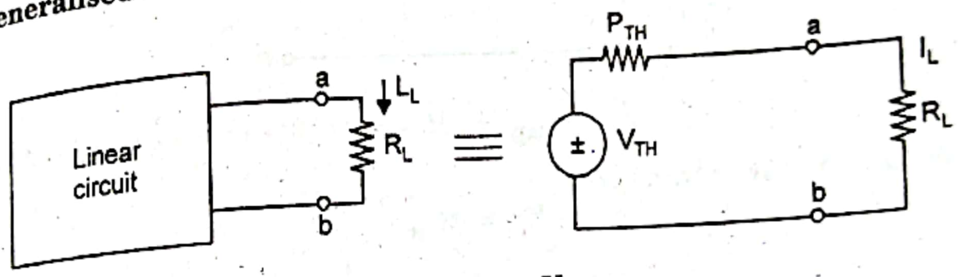
$\Longrightarrow V_L = I_L \cdot R_L$

$$V_L = R_L\left(\frac{V_{TH}}{R_{TH}+R_L}\right)$$

The terminals a - b are made open-ciruited to current flows, so that open circuit voltage across the terminals a-b must be V_{TH}

$$\boxed{V_{TH} = V_{OC}}$$

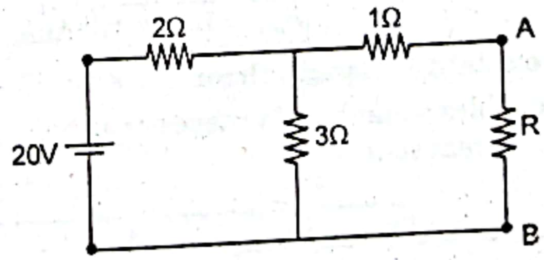
Generalised Form



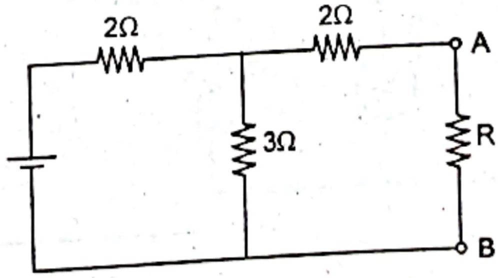
$$I_L = \frac{V_{TH}}{R_{TH}+R_L},$$

$$V_L = I_L R_L = \left[\frac{R_L}{R_{TH}+R_L}\right]V_{TH}$$

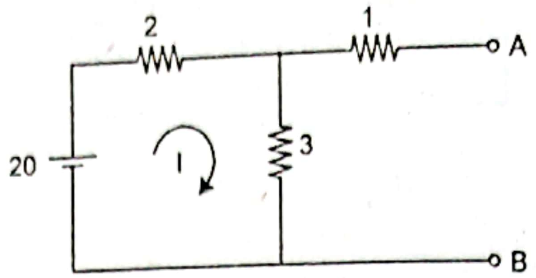
Q.2. (b) Calculate the value of load resistance in branch AB, so that the maximum power is transferred to the load of the circuit shown along side. (6)



Ans.



For V_{TH}:

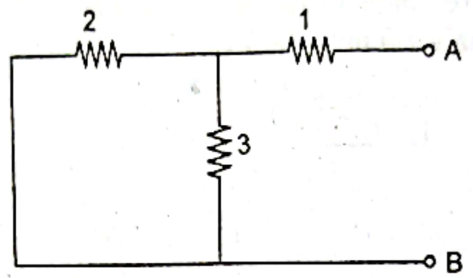


$$2I + 3I = 20$$

$$\Rightarrow \quad \boxed{I = 4\text{ amp}}$$

$$V_{AB} = V_{TH} = V_{OC} = 3.I = 3.4$$
$$= 12\text{ Volt}$$

For R_{TH}:



$$R_{AB} = R_{TH} = (3 \| 2) + 1 = \frac{6}{5} + 1 = 11/_5 = 2.2\Omega$$

For maximum power transfer

$$R_L = R_{TH}$$

$$\Rightarrow \quad \boxed{R_L = R_{TH} = 2.2\Omega}$$

Maximum power

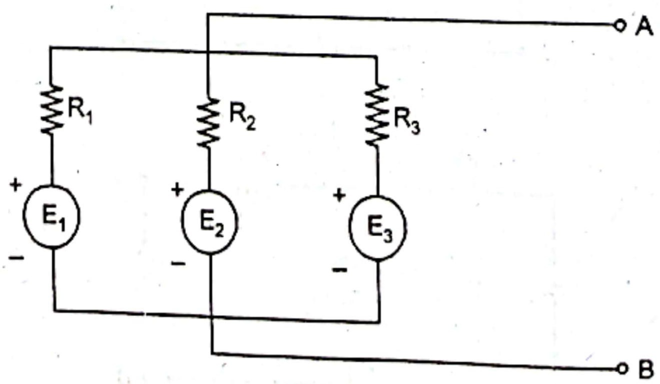
$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

$$P_{max} = \frac{(12^2)}{4 \times 2.2} = \frac{144}{8.8}$$

$$\boxed{P_{max} = 16.3\text{ watt}} \quad \textbf{Ans.}$$

Q.3. (a) State and explain Millman's theorem. (6.5)

Ans. This theorem enables a number of voltage or current sources to be combined into a single voltage and current sources.



$$V_{AB} = \frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\frac{E_3}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{E_1G_1+E_2G_2=E_3G_3}{G_1+G_2+G_3}$$

$$= \frac{\Sigma EG}{\Sigma G}$$

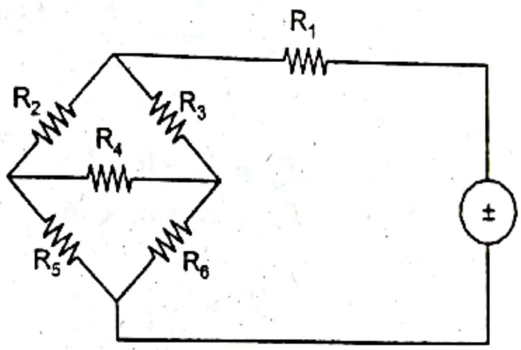
$$R_{AB} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{G_1+G_2=G_3} = \frac{1}{G}$$

$$\rightarrow \quad I_L = \frac{V_T}{R_T=R_L}$$

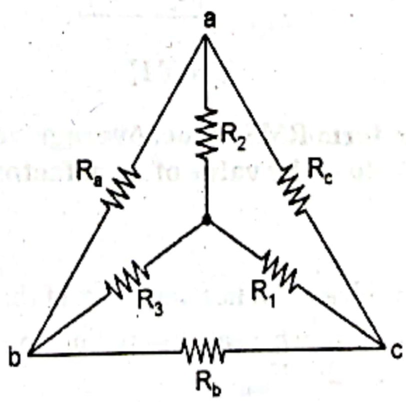
Q.3. (b) Drive expression for converting a star network to a delta equivalent networks. (6)

Ans. γ-Δ Transformation:

→ In circuit Analysis when the resistors are neither in parallel nor in series then we use γ – Δ transformation.



(1) Δ–γ Conversion: It is more convenient to work with γ - Network as compare to Δ- Network.



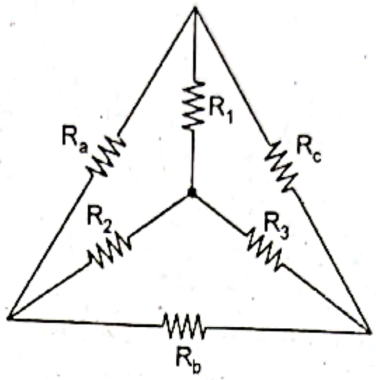
$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$$

$$R_2 = \frac{R_cR_a}{R_a+R_b+R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

(2) $\gamma - \Delta$ Conversion:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$



$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

→ For Balanced system

→ $R_1 = R_2 = R_3 = R\gamma$

→ $R_a = R_b = R_c = R\Delta$

$$\boxed{R\gamma = \frac{R\Delta}{3}}$$

or

$$\boxed{R_\Delta = 3R\gamma}$$

UNIT II

Q.4. (*a*) Explain the term RMS value, Average value and form factor w.r.t alternating quantity. Deduce the value of form factor of a sinusoidal voltage. (6.5)

Ans. Rms Value:

→ Rms value is defined based on heating effect of the wave form.

→ The voltage (A.C) at which heat dissipation in AC circuit is equal to heat dissipation in DC circuit is called V_{rms}.

→ $$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 d\omega t}$$

→ $$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T} V^2 dt}$$

Average Value: Average value is defined based on charge transfer in a circuit.

→ The voltage (A.C) at which charge transfer in AC is equal to charge transfer in DC circuit, called the Average value.

$$\boxed{V_{ag} = \frac{1}{\pi}\int_0^{\pi} V d\omega t}$$

→ **Form factor:** Form factor is the ratio of Rms value of wave form to average value of wave form.

$$\boxed{\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{I_{rms}}{I_{ag}}}$$

Average Value for sinusoidal current or voltage.

$$i = I_m \sin \omega t$$

→ Half cycle i.e. when wt varies from o to π.

$$I_{av} = \frac{\text{Area of first half cycle}}{\pi}$$

$$= \frac{1}{\pi}\int_o^{\pi} i d(\omega t) = \frac{1}{\pi}\int_o^{\pi p} I_{max} \sin \omega t \, d\omega t$$

$$I_{av} = \frac{I_{max}}{\pi}[-\cos \omega t]_0^{\pi} = \frac{2}{\pi} I_{max} = 0.63 I_{max}$$

$$\boxed{I_{av} = 0.637 I_{max}}$$

Rms of effective value for sinusoidal current:

$$i = I_{max} \sin wt$$

$$= \frac{\text{Area of I}^{st}\text{half cycle of } i^2}{\pi}$$

$$= \frac{i}{\pi}\int_0^{\pi} i^2 d\omega t = \frac{1}{\pi}\int_0^{\pi} I_{max}{}^2 \sin^2 \omega t \, d\omega t$$

$$= \frac{I_{max}{}^2}{2\pi}\int_0^{\pi}[1 - \cos 2\omega t] d\omega t$$

$$= \frac{I_{max}{}^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}$$

$$= \frac{I_{max}{}^2}{2\pi} \times \pi = \frac{I^2{}_{max}}{2}$$

$$I_{rms} = I_{max}/\sqrt{2}$$

$$\text{Form factor} = \frac{I_{rms}}{I_{avg}}$$

$$= \frac{I_m/\sqrt{2}}{2I_m/\pi}$$

$\boxed{\text{Form factor} = 1.11}$ **Ans.**

Q.4. (b) Explain the concept of bandwidth and quality factor for a series RLC circuit. Drive their expression.

Ans. Bandwidth: The difference between the two half power frequency

i.e. $$\boxed{B = \omega_2 - \omega_1}$$

where $$\omega_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\boxed{\omega_0 = \sqrt{\omega_1 \omega_2} \text{ rad/sec}}$$

→ 'B' is a half power Bandwidth, because it is the width of frequency band between the half frequencies.

Quality factor: The "sharpness" of the resonance in a resonant is measured quantitatively by the quality factor 'Q'.

→ At resonance the reactive energy in the circuit oscillate between the inductor and the capacitor

→ Quality factor

$$Q = 2\pi\left[\frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}\right]$$

$$\Rightarrow \qquad Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R\left(\frac{1}{f_o}\right)} = \frac{2\pi f_o L}{R}$$

$$\rightarrow \qquad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\rightarrow \qquad \boxed{B = \frac{R}{L} = \frac{\omega_o}{Q}} \text{ Or } \boxed{B = \omega_0^2 CR}$$

→ The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

Q.5. (a) State the advantages of three phase system over single over single phase system. (4.5)

Ans. (1) Primary advantage of a 3-phase system over a poly phase or single phase is the inter connection is possible i.e. windings can be connected either in the form of star or delta.

(2) Nearly all electric power is generated and distributed in three phase system at frequency 50 Hz when are phase or two phase system is required they are taken it separately instead of generating it separetaly.

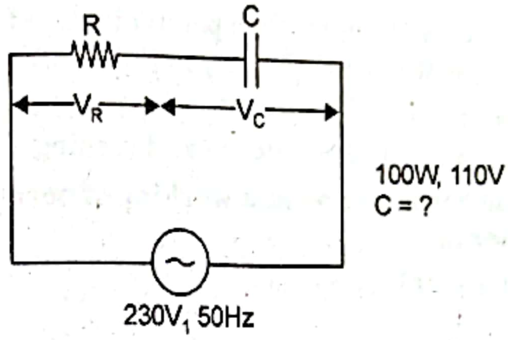
(3) When more than 3- phares are needed such as in aluminium company where 48 phases are required then we use the manipulating of 3-phase system.

(4) The instantaneous power in a 3-phase system can be constant (not pulsating). This results in uniform power transmission and less vibration, of 3-phase machine.

(5) For the same amount of power, the three phase system is more economical than the single phase. The amount of wire required for a-3- phase system is less than that required for an equivalent signle-phase system.

Q.5. (b) Find the capacitance which must be connected in series with a 100 W, 110 V lamp in order that the lamp may draw its normal current when the combination is connected to a 230 V 50 Hz supply. (8)

Ans.



$$\Rightarrow \quad P = V.i \Rightarrow i = \frac{P}{V} = \frac{100}{110} = 0.909$$

$$\Rightarrow \quad V = \sqrt{V_R^2 + V_C^2} \Rightarrow V_C^2 = V^2 - V_R^2$$

$$V_C^2 = (230)^2 - (110)^2 = (52900)^2 - (12100)^2$$

$$V_C^2 = 40800$$

$$\boxed{V_c = 20.19} \text{ Volt}$$

$$\Rightarrow \quad V_C = \text{I.2 } X_c$$

$$\Rightarrow \quad X_C = \frac{V_c}{I} = \frac{20.19}{0.909} = 22.21$$

$$\rightarrow \quad Z = R + jx_c$$

$$\rightarrow \quad X_C = \frac{1}{2\pi fc} = 22.21 = \frac{1}{2\times 3.14\times 50\times C}$$

$$C = \frac{1}{6977.477}$$

$$\boxed{C = 1.4331\times 10^{-4} \text{ Farad.}}$$

FIRST TERM EXAMINATION [SEPT. 2016]
FIRST SEMESTER [B.TECH]
ELECTRICAL TECHNOLOGY [ETEE-107]

Time : 1½ hrs. M.M. : 30

Note: Q.No.1 is compulsory. Attempt any two more Questions from rest.

Q.1. (a) What are limitations of Ohm's law? (5×2)

Ans. The limitation of Ohm's law are as follow:

1. This law can not be applied to unilateral networks, like diode transistors etc, which do not have same voltage-current relation for both directions of current.

2. It is not applicable for Non-linear elements or systems.

Q.1. (b) If the length of a wire of resistance R is uniformly stretched to 'n' times its original value, what will be its new resistance?

Ans. Wire resistance = R

length = l

stretched length = n.l

$$R = \frac{\rho l}{A} \Rightarrow R \propto l$$

$$= \boxed{R \propto n.l}$$

New resistance ⇒ R = n.l Ohms

Q.1. (c) What is rms value of an alternating current?

Ans. Rms value is defined as the heating of the quantity or parameter in AC as well DC system.

- Rms value for Alternating current is

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Where I_m is the maximum value of current.

Q.1. (d) What is the significance of peak factor?

Ans. Peak factor is defined as the ratio of maximum value to the Rms value of Alternating quantity.

- The alternating quantities can be voltage or current.
- Maximum value is the peak value of the voltage or current and the root mean square value is the amount of heat produced by the alternating current.

$$\text{Peak factor} = \frac{I_m}{I_{rms}} \text{ or } \frac{E_m}{E_{rms}}$$

Q.1. (e) What do you mean by phase and phase difference?

Ans. Phase: It is a definition of the position of a point in time (instant) on a wave form cycle.

Phase difference: It is the difference in phase angle between two sinusoids or phasors.

Example: In 3-Phase system, Phase difference is 120°

Q.2. (a) State and derive Maximum power transfer theorem? (4)

Ans. Maximum Power Transfer Theorem

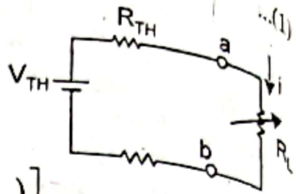
Maximum Power is transferred to the load when the load resistance is equal to the thevenin's resistance *i.e.*

$$\boxed{R_L = R_{TH}}$$

→ Thevenin's equivalent is useful in finding the maximum power a linear circuit can deliver to a load.

→ If Entire circuit is replaced by its thevenin's equivalent except for the load then.

$$P = i^2.R_L\left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2.R_L \quad ...(1)$$



$$\Rightarrow \qquad P = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2.R_L$$

$$\frac{dP}{dR_L} = V_{TH}{}^2\left[\frac{(R_{TH}+R_L)^2-2R_L(R_{TH}+R_L)}{(R_{TH}+R_L)^4}\right]$$

$$V_{TH}^2\frac{[R_{TH}+R_L-2R_L]}{(R_{TH}+R_L)^3} = 0$$

$$\Rightarrow \qquad \boxed{R_{TH}=R_L} \text{ Proved.}$$

Q.2. (*b*) Find the value of R in the ckt of fig (1) such that maximum power transfer takes place. And how much maximum power delivered to load, also indicate in graph? (6)

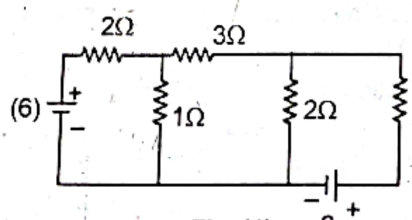
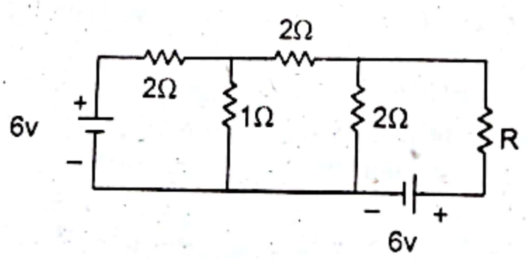


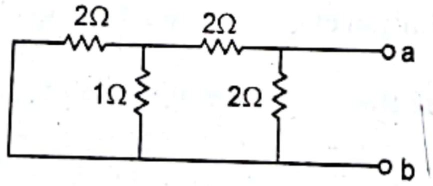
Fig. (1)

Ans.



$$R = ?$$

⇒ For 'R'

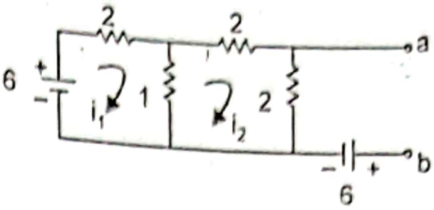


$$R_{ab} = R_{TH} = (2\Omega \,||\, 1\Omega)+2\Omega+2\Omega$$

$$= \frac{2}{3}+4 = \frac{14}{3}\Omega$$

For maximum Power, $\boxed{R_L = R_{TH} = \frac{14}{3}\Omega}$

Maximum Power $P_{max} = \frac{V_{TH}^2}{4R_{TH}}$



For V_{TH}

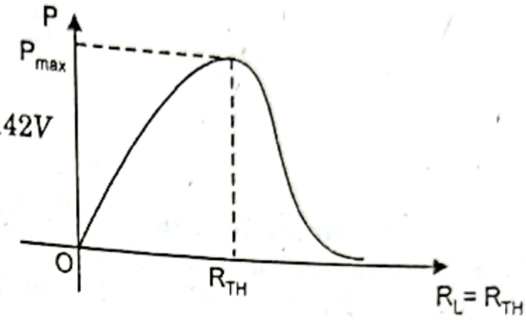
$$= 2i_1 + (i_1 - i_2) = 6$$
$$= 3i_1 - i_2 = 6 \quad ...(1)$$
$$= 2i_2 + 2i_2 + i_2 - i_1 = 0$$
$$5i_2 = i_1 \quad ...(2)$$

Put value of i_1 in equation (1), we get $3.5\, i_2 - i_2 = 6$

$14\, i_2 = 6 \Rightarrow i_2 = \frac{6}{14} Amp$

Vab = $V_{ab} = V_{TH} = 2.i_2 - 6$

$$= \frac{2.6}{14} - 6 = \frac{12}{14} - 6 = -5.142V$$



$$P_{max} = \frac{V_{TH}^2}{4.R_{TH}}$$

$$= \frac{(-5.142)^2}{4 \times 4.666} = \frac{26.440}{18.664}$$

$\boxed{P_{max} = 1.416\, Watt}$ Ans.

Q.3. (a) State thevenin theorem. Find thevenin equivalent voltage and thevenin equivalent resistance shown in fig. (2) and draw Norton equivalent circuit? (6)

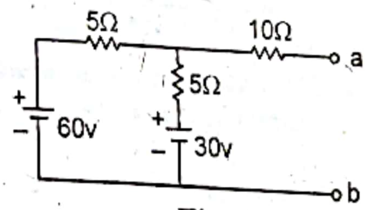
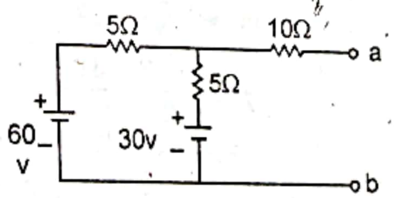


Fig. 2.

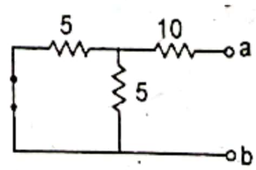
Ans. A linear two-terminal circuit can be replaced by an equivalent circuit consisting of voltage source V_{TH} is series with resistor R_{TH}.

Where R_{TH} = Equivalent Resistance

V_{TH} = Equivalent Voltage.



$R_{ab} = R_{TH} \Rightarrow$

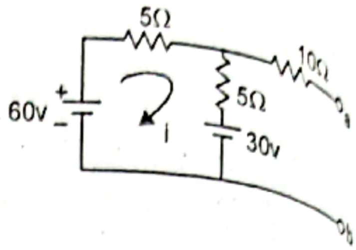


$$R_{ab} = R_{TH} = (5 \| 5) + 10$$
$$= 12.5\ \Omega$$

For V_{TH}

$= 5i + 5i + 30 = 60$

$10i = 30$



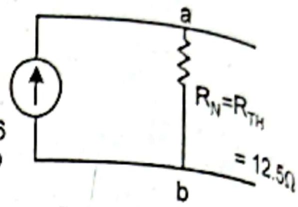
$$\boxed{i = 3amp}$$

$V_{ab} = V_{TH} = 10 \times 0 + 5.i + 30$

$= 0 + 5 \times 3 + 30$

$$\boxed{V_{ab} = V_{TH} = 45V}$$

For Norton's Equivalent



$\Rightarrow$

$$I_N = \frac{V_{TH}}{R_{TH}}$$

$$I_N = \frac{45}{12.5} = 36\,amp \text{ Ans.}$$

Q.3. (*b*) A supply voltage of 230V, 50 Hz is fed to a residential building. Write down its equation for instantaneous value? (4)

Ans. $V_m = 230V$, F = 50Hz instantaneous equation-?

Instantaneous equation of Alternating voltage

$$V = V_m(\sin wt + \phi)$$

Phase difference $\phi = 0$

Now $w = 2\pi f$

$= 2\pi \times 50 = 100\pi$

$$\boxed{w = 314}$$

Equation is given by

$$\boxed{V = 230 \sin 314t} \text{ Ans.}$$

Q.4. (a) The equation of an alternating current is i=42.42 Sin 628t. Determine (*i*) its maximum value (*ii*) frequency (*iii*) rms value (*iv*) average value (*v*) Form factor (*vi*) Peak factor. (6)

Ans. $i = 42.42 \sin 628t$

(*i*) Compare the equation with

$$i = I_m \sin wt$$

$$\boxed{I_m = 42.42} \text{ amp}$$

(ii). $wt = 628t$

$$w = 2\pi f = 628$$

$$f = \frac{628}{2\pi} = \frac{314}{\pi}$$

$$\boxed{f = 100\,Hz}$$

(iii). Rms value

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I^2 dwt} = I_m/\sqrt{2}$$

$$I_{rms} = \frac{42.42}{\sqrt{2}} = 29.99\,Amp$$

(iv) Average value

$$I_{avg} = \frac{2I_m}{\pi} = 0.637 \times 42.42$$
$$= 27.02 \text{ Amp}$$

(v) Form factor

$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = \frac{29.99}{27.02}$$
$$= 1.11$$

(vi) Peak factor

$$= \frac{I_m}{I_{rms}} = \frac{42.42}{29.99} = 1.414 \text{ Ans.}$$

Q.4. (b) Find the current I in the network shown in fig (3) using star-delta transformation. **(4)**

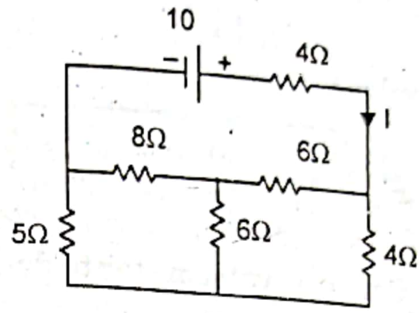
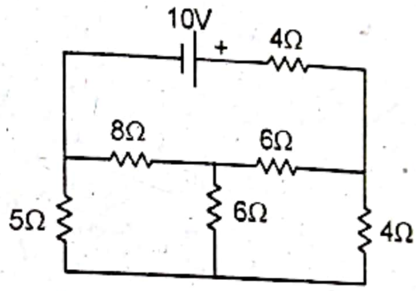


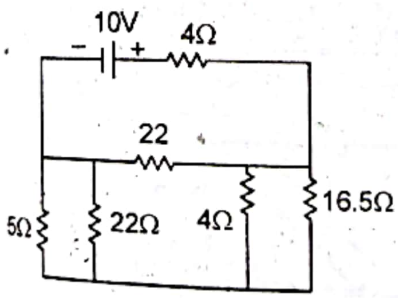
Fig. 3.

Ans.

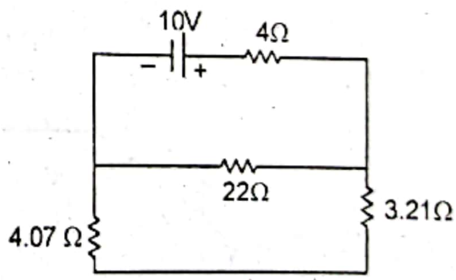


Converting star in to delta

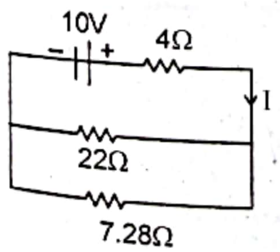
$$\Rightarrow R_{12} = \frac{8+6+8\times6}{6} = 22\Omega; \quad R_{23} = \frac{6+6+6\times6}{8} = 16.5\Omega; \quad R_{31} = \frac{8+6+8\times6}{6} = 22\Omega$$



$\Rightarrow$



$\Rightarrow$



$$\Rightarrow I = \frac{10}{4+4.69} = 1.150\,Amp \text{ Ans.}$$

FIRST TERM EXAMINATION [SEPT. 2017]
FIRST SEMESTER [B.TECH]
ELECTRICAL TECHNOLOGY [ETEE–107]

Time : 1½ hrs. M.M. : 30

Note: *Q.No 1 is compulsory. Attempt any two more questions from the rest.*

Q.1. (a) What are the limitation of Ohm's law? (2)

Ans. Limitation of Ohm's law are

(*i*) Ohm's law is applicable only to bilateral elements.

(*ii*) It can not be applied to circuits consisting of non-linear elements such as powdered carbon, thyrite etc.

Q.1. (b) Write down the difference between Active and Passive elements. (2)

Ans.

Active Elements	Passive Elements
(*i*) Elements which supply energy to the network are known as active elements	(*i*) Components that dissipate energy are known as passive components.
(*ii*) Examples Batteries, DC generators, AC generators photoelectric cell etc.	(*ii*) Examples are Resistor, Inductor, capacitor etc.

Q.1. (c) Difference between Form factor and Peak factor. (2)

Ans. Form factor: The ratio of effective value to the average or mean value of periodic wave is known as form factor

$$\text{Mathematically, form factor} = \frac{\text{Effective value}}{\text{Average value}}$$

$$\text{For sinusoidal wave } K_f = \frac{E_{rms}}{E_{av}} = \frac{E_{\max}/\sqrt{2}}{E_{\max}/\pi/2} = 1.11$$

Peak factor: Peak/crest/amplitude factor of a periodic wave is defined as the ratio of maximum or peak of the effective or rms value of the wave

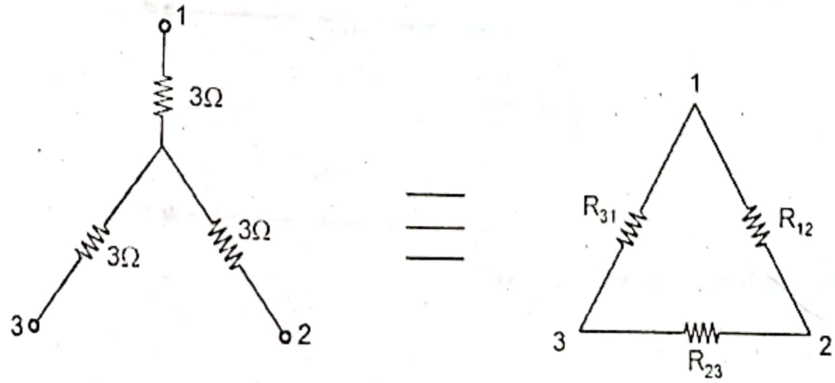
$$K_p = \frac{\text{Maximum value}}{\text{Effective value}}$$

For sinusoidal wave

$$K_p = \frac{E_{\max}}{E_{\text{rms}}} = \sqrt{2} \cong 1.414$$

Q.1. (d) Three equal resistance of 3 ohm are connected in star. What is the resistance in one of the arms in an equivalent delta circuit? (2)

Ans.



$$R_{31} = R_{23} = R_{12} = 3 + 3 + \frac{3 \times 3}{3} = 9\,\Omega$$

Q.1. (e) For 100 volts RMS value Triangular-Wave. What is the peak voltage?

Ans. For triangular wave

$$I_{rms} = \frac{I_{max}}{\sqrt{3}}$$

$$\Rightarrow \qquad I_{max} = \sqrt{3}\, I_{rms} = \sqrt{3} \times 100 = 100\sqrt{3}\ \text{V}$$

Q.2. (a) Find the Thevenin's and Norton's Equivalents for the circuit shown in fig 1 with respect to terminals *ab*.

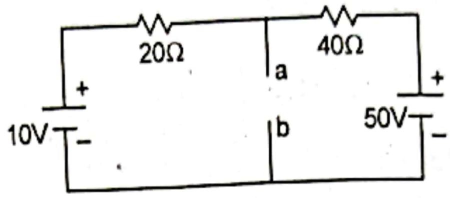


Fig 1.

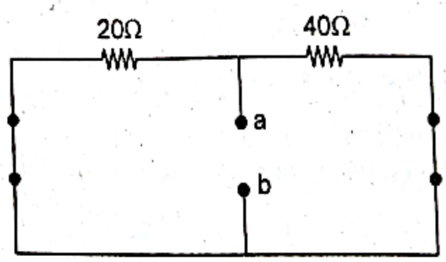
Ans. (a) Thevenin's Equivalent circuit

Step (*i*)

$$i = \frac{50 - 10}{20 + 40} = \frac{40}{60} = \frac{2}{3} A$$

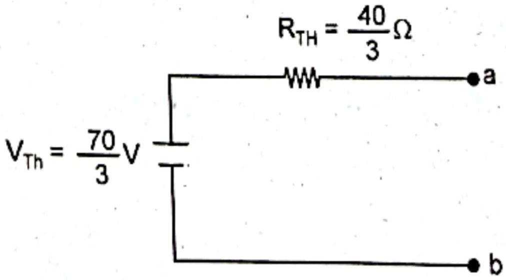
$$V_{th} = V_{ab} = 50 - 40 \times \frac{2}{3} = \frac{70}{3} V$$

Step (*ii*) For R_{Th}



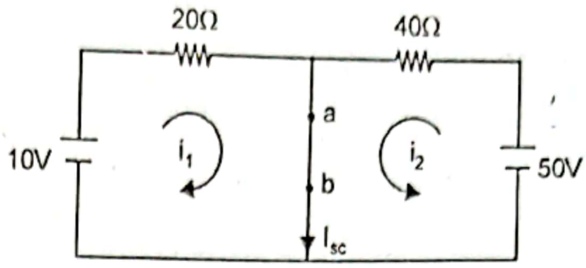
$$R_{Th} = 20\,\Omega \,||\, 40\,\Omega = \left(\frac{1}{20} + \frac{1}{40}\right)^{-1} = \frac{40}{3}\,\Omega$$

Step (*iii*) Thevenin equivalent circuit



(b) Norton's Equivalent circuit:

Step (*i*) For finding I_{SC}



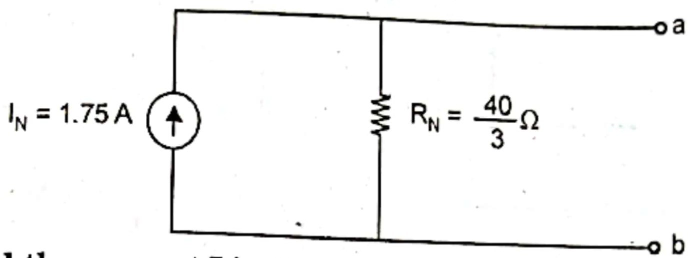
$$i_1 = \frac{10}{20} = 0.5\,A;\ i_2 = \frac{50}{40} = 1.25\,A$$

$$I_{SC} = i_1 + i_2 = 0.5 + 1.25 = 1.75\,A$$

Step (*ii*) For finding R_N

$$R_N = R_{th} = \frac{40}{3}\,\Omega$$

Step (*iii*) Norton's Equivalent circuit



Q.2. (*b*) Find the current I in the circuit as shown in fig 2 by using Super position Theorem. **(5)**

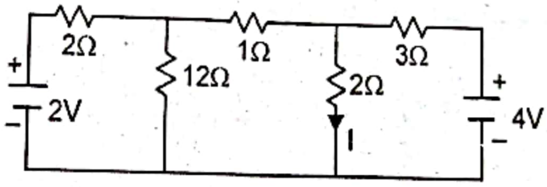
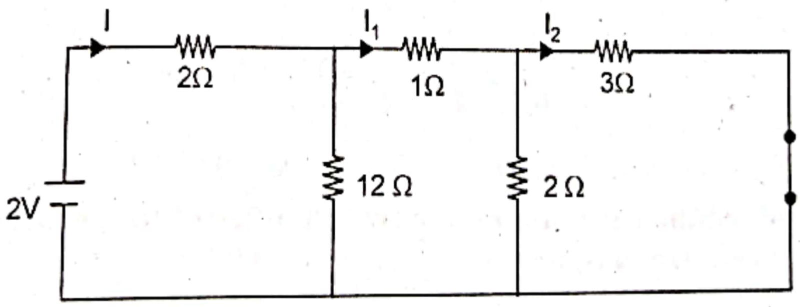


Fig. 2

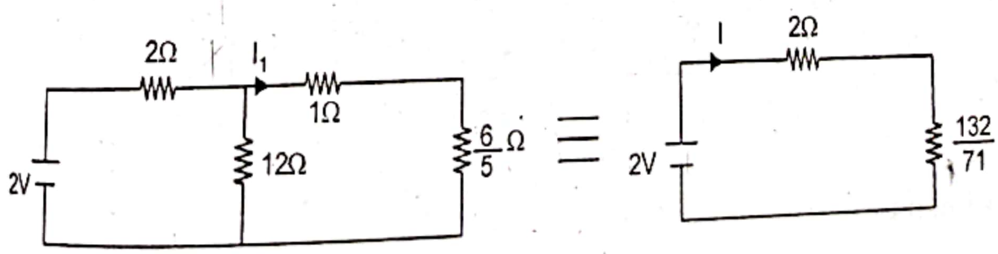
Ans.

Step (*i*) Considering 2*V* voltage source



$$R_{eq} = (((3\Omega \,||\, 2\Omega) \text{ series } 1\Omega) \,||\, 12\Omega) \text{ series } 2\Omega$$

$$= \frac{274}{71}\,\Omega$$

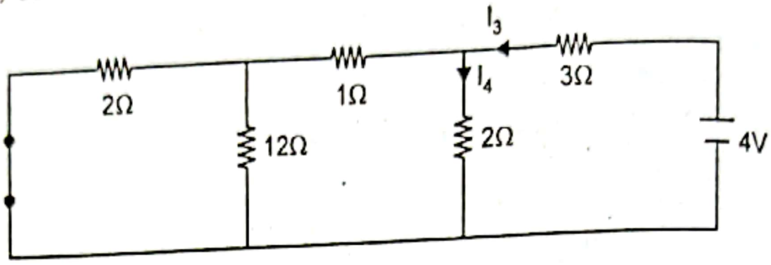


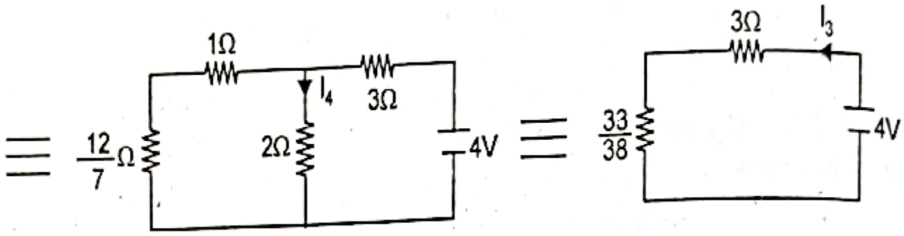
$$I = \frac{V}{R_{eq}} = \frac{2}{\frac{274}{71}} = \frac{71}{137}$$

$$I_1 = \frac{71}{137} \times \frac{12}{12 + \frac{11}{5}} = \frac{60}{137} A$$

$$I_2 = \frac{60}{137} \times \frac{3}{3+2} = \frac{36}{137} A = 0.263\ A$$

Step (*ii*) Considering $4V$ voltage source





$$I_3 = \frac{4}{\frac{33}{38} + 3} = \frac{4 \times 38}{33 + 3 \times 38} = \frac{152}{147}$$

$$I_4 = \frac{152}{147} \times \frac{\frac{19}{7}}{\frac{19}{7} + 2}$$

$$= \frac{152}{147} \times \frac{19}{\cancel{7}} \times \frac{\cancel{7}}{33} = 0.595\ A$$

$$I_1' = I_2 + I_4 = 0.263 + 0.595 = 0.858\ A$$

Q.3. (*a*) Determine the maximum power delivered to the Load Resistance in the circuit as shown in figure. (5)

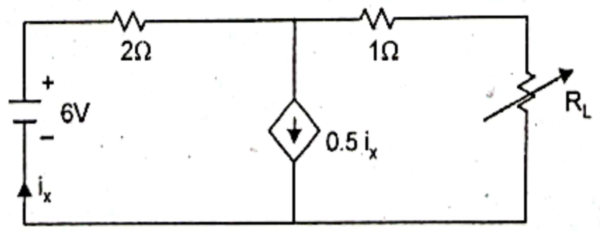
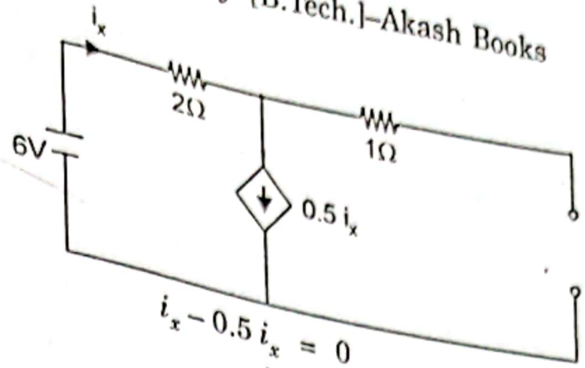


Fig 3.

Ans.

For finding V_{Th}

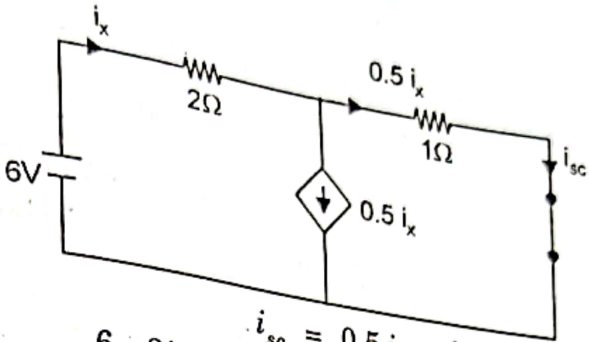


$$\Rightarrow \quad i_x - 0.5\, i_x = 0$$

$$i_x = 0$$

$$V_{Th} = V = 6V$$

For finding I_{SC}



$$i_{sc} = 0.5\, i_x$$

$$6 - 2i_x - 0.5\, i_x = 0$$

$$\Rightarrow \quad i_x = \frac{6}{2.5} = 2.4\, A$$

$$\Rightarrow \quad I_{SC} = 2.4 \times 0.5 = 1.2\, A$$

$$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{6}{1.2} = 5\, \Omega$$

For maximum power transfer $R_{Th} = R_L = 5\Omega$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{6}{5+5} = 0.6\, A$$

$$\text{Power} = I_L^2 R_L = 0.6^2 \times 5 = 1.8\, W.$$

Q.3. (b) A current of (15 + j8) A flows in a circuit which has a supply voltage of (20 + j10), find out the circuit parameters, impedance, power-factor, active power, reactive power, apparent power? **(5)**

Ans. Circuit Voltage $V = (20 + j10)V = 22.36 \angle 26.56°$

Circuit current $I = (15 + j8)A = 17\angle 28°$

Circuit impendance $Z = \frac{V}{I} = \frac{22.36\angle 26.56}{17\angle 28°} = 1.32 \angle -1.44°$

Power factor $\cos\phi = \cos(1.44) = 0.99$ (leading)

$P = VI\cos\phi = 22.36 \times 17 \times 0.99 = 376.32$ W

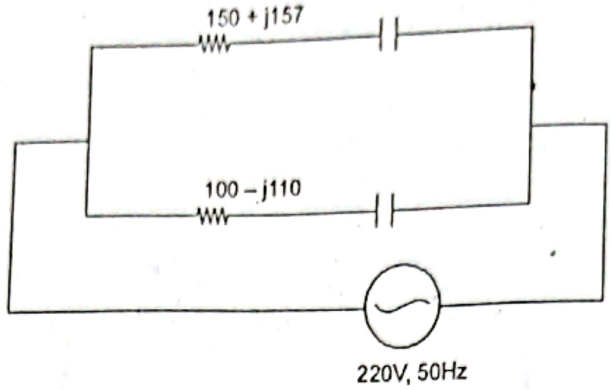
$Q = VI\sin\phi = 22.36 \times 17 \times 0.02 = 7.60$ VAr

Apparent power $S = \sqrt{P^2 + Q^2}$

$$= \sqrt{376.32^2 + 7.6^2} = 376.4\ W$$

Q.4. (a) Two impedances (150+j 157) ohm and (100–j 110) ohm connected in parallel across a 220volt, 50 Hz AC supply. Find out the equivalent impedance, total current and power factor. (

Ans.



$$Z_1 = 150 + j157 = 217.17\angle 46.31$$
$$Z_2 = 100 - j110 = 148.66\angle -47.73$$

$$Z = \left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)^{-1} = \left(\frac{1}{(150+j157)}+\frac{1}{(100-j110)}\right)^{-1}$$
$$= 124.09 - j\,26.53 = 126.9\ \angle -12.07\ \Omega$$

$$\text{Current } I_1 = \frac{V}{Z_1} = \frac{200\ \angle 0}{217.14\ \angle 46.31}$$
$$= 0.92\ \angle -46.31\text{A}$$

$$\text{Current } I_2 = \frac{V}{Z_2} = \frac{200\angle 0}{148.66\ \angle -47.73} = 1.345\angle 47.73\ \text{A}$$

$$I = I_1 + I_2$$
$$= 0.92\ \angle -46.31 + 1.345\ \angle 47.73$$
$$= 0.6355 - j\,0.6652 + 0.9047 + j\,0.9953$$
$$= 1.5402 + j\,0.33 = 1.575\ \angle 11.83\ \text{A}$$

Power factor = $\cos\phi = \cos(11.83) = 0.979.$

Q.4. (b) A series R-C circuit takes a power of 7000watt, when connected to 200 volt, 50Hz supply. The voltage across the resistor is 130volt. Calculate the value of R, I, C, Z, Power factor? (5)

Ans. Supply Voltage $V = 200$ V

Voltage across resistor, $V_R = 130$ V

$$\text{Pour factor, } \cos\phi = \frac{R}{Z} = \frac{I\times R}{I\times Z} = \frac{V_R}{V_z}$$

$$= \frac{130}{200} = 0.65\ \text{(leading)}$$

$$\text{Current } I = \frac{P}{V \cos\phi} = \frac{7000}{200 \times 0.65} = 53.846\, A$$

$$\text{Resistance } R = \frac{V_R}{I} = \frac{130}{53.846} = 2.41\, \Omega$$

$$\text{Impedance } Z = \frac{V}{I} = \frac{200}{53.846} = 3.714\, \Omega$$

$$\text{Capacitive Reactance } X_C = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(3.714)^2 - (2.41)^2} = 2.826\, \Omega$$

$$\text{Capacitance, } C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times \pi \times 50 \times 2.826} = 1126\, \mu F.$$

FIRST TERM EXAMINATION [SEPT. 2018]
FIRST SEMESTER [B.TECH]
ELECTRICAL TECHNOLOGY [ETEE–107]

Time : 1.5 hrs. **M.M. : 30**

Note: Q. 1. is compulsory. Attempt any two questions from the rest.

Q.1. Answer the following questions.

(a) State and explain reciprocity theorem. **(2.5)**

Ans. Reciprocity Theorem

Reciprocity Theorem states that: The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured. The reciprocity theorem is applicable only to single-source networks.

| OR |

In any linear and bilateral network, if a source of emf E in any branch produces a current I in any other branch, then after interchanging the positions of E and I, the same emf E acting in the second branch would produce the same current I in the first branch.

Illustration: Consider a network as shown in Fig. 1.(a). In the network, the current due to the voltage source E is to be determined. If the position of each is interchanged as shown in Fig. 1(b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network of Fig. 2, in which values for the elements of Fig. 1.(a) have been assigned.

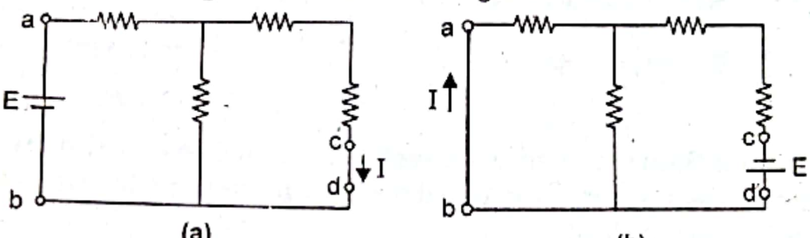


Fig. 1 Demonstrating the reciprocity theorem.

Finding the current I due to a source E

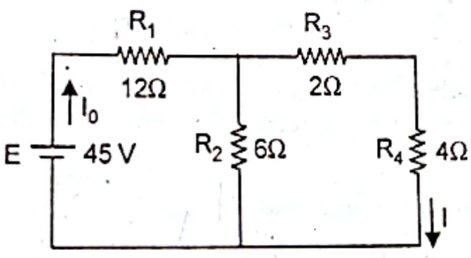


Fig. 2.

The total resistance is

$$R = R_1 + [R_2 \| (R_3 + R_4)]$$

$$= 12 + [6 \| (2 + 4)] = 12 + (6 \| 6)$$

$$= 12 + 3 = 15\ \Omega$$

$$I_S = \frac{E}{R} = \frac{45}{15} = 3A$$

and

$$I = \frac{1}{2} \times I_S = \frac{3}{2} = 1.5A$$

[∵ I_S is divided into 2 equal half in 6 Ω and (2 + 4) Ω branch]

Interchanging the location of E and I of Fig. 2 to demonstrate the validity of the reciprocity theorem

For the network of Fig. 3, which corresponds to that of Fig. 1(b),

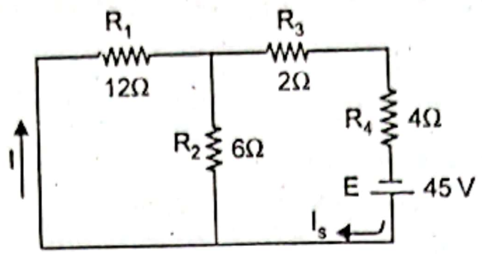


Fig. 3.

$$R = R_4 + R_3 + (R_1 \| R_2) = 4 + 2 + (12 \| 6) = 6 + 4 = 10\,\Omega$$

$$I_S = \frac{E}{R} = \frac{45}{10} = 4.5A$$

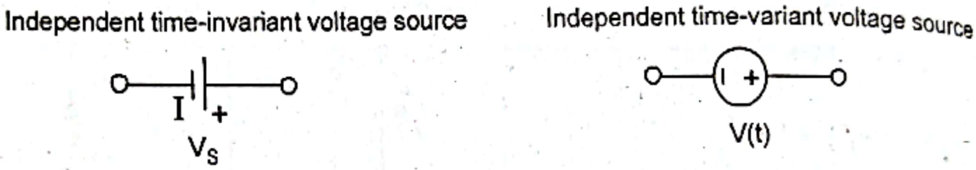
$$I = \frac{R_2}{R_2 + R_1} \times I_S = \frac{6}{6+12} \times 4.5 = \frac{6 \times 4.5}{18} = \frac{4.5}{3} = 1.5A$$

Q. 1. (b) What do you understand by "Dependent" and "Independent" energy sources? (2.5)

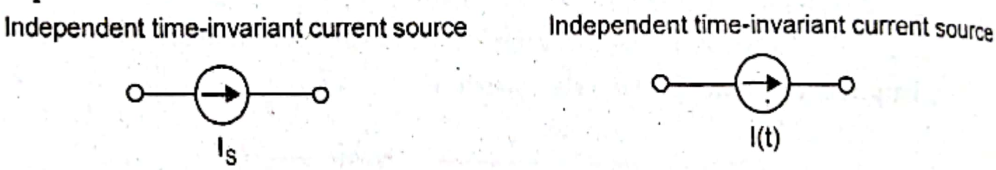
Ans. Energy Sources (Voltage and Current Sources)

Independent Sources

1. Voltage Source: An ideal voltage source is a device that produces a constant voltage across its terminals, no matter what current is draw from it.

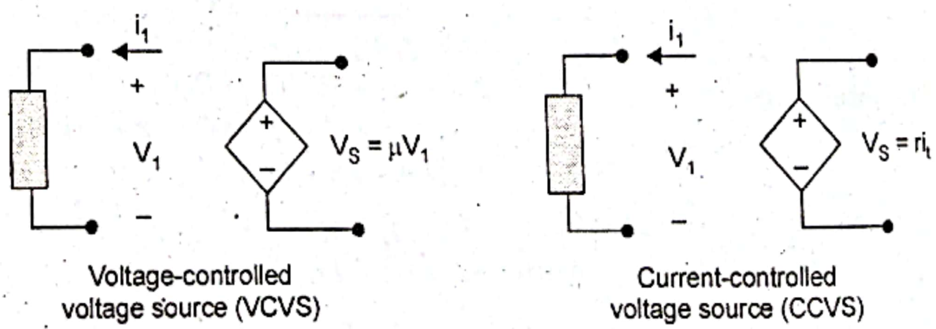


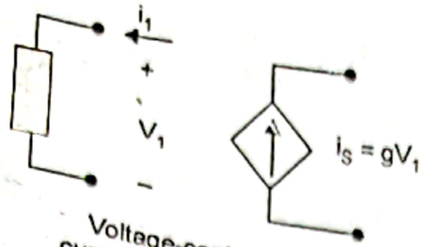
2. Current Source: An ideal current source is a device that delivers a constant current to any load resistance connected across it, no matter what the terminal voltage is developed across the load.



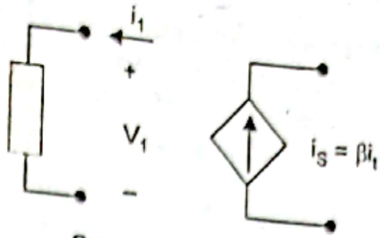
Dependent Sources: The voltage and current sources may be dependent and either may be controlled by a voltage or current. A dependent source is represented by a diamond-shaped symbol.

They are categorized into four sources.



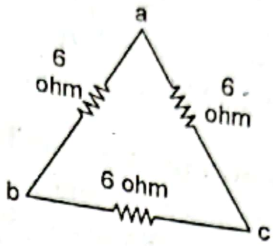


Voltage-controlled current source (VCCS)

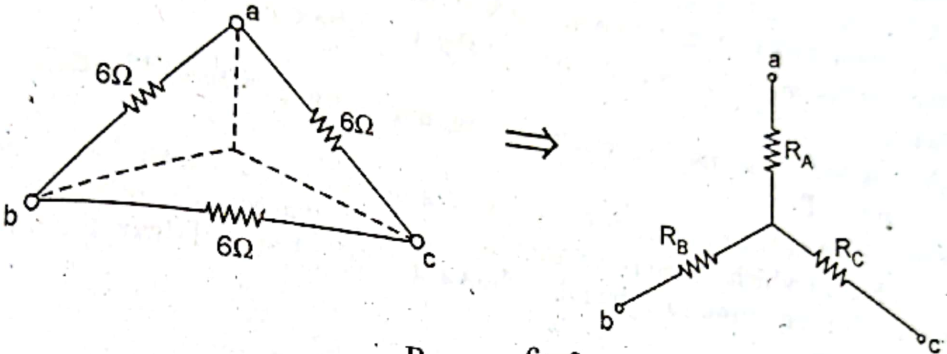


Current-controlled current source (CCCS)

Q. 1. (c) Convert delta network given in fig. in equivalent star network. **(2.5)**



Ans.



$$R_A = \frac{6\times6}{6+6+6} = \frac{36}{18} = 2\Omega$$

$$R_B = \frac{6\times6}{6+6+6} = \frac{36}{18} = 2\Omega$$

$$R_C = \frac{6\times6}{6+6+6} = \frac{36}{18} = 2\Omega$$

Q. 1. (d) Give statements of Thevenin's and Norton's theorem. **(2.5)**

Ans. Thevenin's Theorem: Thevenin's theorem states that any two-terminal, linear bilateral dc network can be replaced by an- equivalent circuit consisting of a voltage source, E_{Th} (or V_{Th}) and a series resistor, R_{Th} as shown in Fig. 1.

Here, V_{Th} is voltage across'two terminals (load terminals). It is also known as V_{oc} (open circuit voltage). R_{Th} is internal resistance of the network as viewed back into the open circuited network from terminals a and b with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

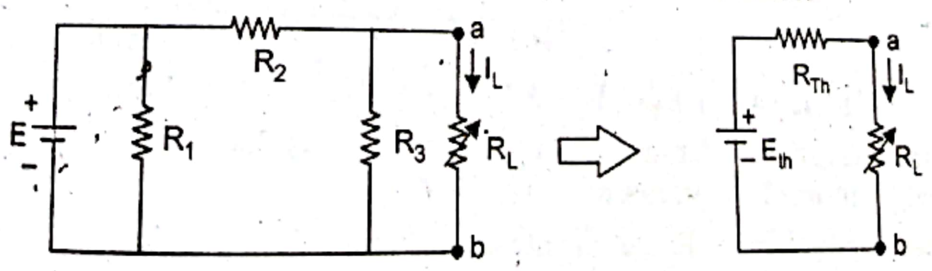


Fig. 1. Illustration of Thevenin's Theorem

Norton's Theorem: Norton Theorem states that any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source, I_N (or I_{sc}) and a parallel resistor, R_N as shown in Fig. 2.

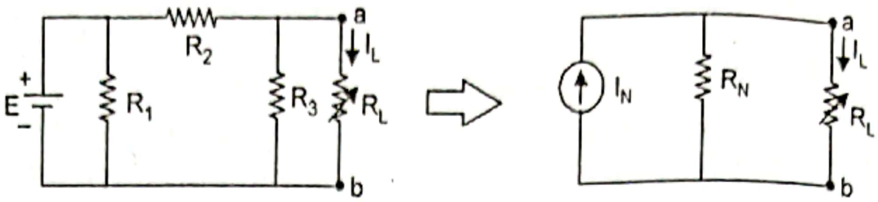


Fig. 2. Illustration of Norton's Theorem

Here, I_N is the constant current equal to the current which would flow in a short-circuit placed across the terminals a and b. R_N is internal resistance of the network as viewed back into the open circuited network from terminals a and b with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

Q. 2. (a) State and prove max power transfer theorem. **(4)**

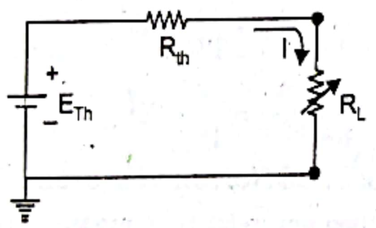
Ans. The maximum power transfer theorem states that a load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as "seen" by the load.

For the Thevenin equivalent circuit of Fig. maximum power will be delivered to the load when $R_L = R_{Th}$

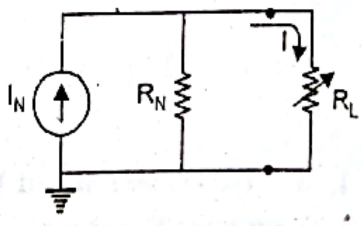
For the Norton equivalent circuit of Fig. maximum power will be delivered to the load when $R_L = R_N$

Consider a network consists of a generator of emf E and internal resistance R_{Th} as shown in Fig. in which a load resistance R_L is connected across the terminals A and B of a network. It is required to determine the value of R_L that will draw maximum power from the source.

The current delivered to the load resistance, $I = \dfrac{E}{(R_{Th} + R_L)}$



Thevenin Equivalent Circuit



Norton Equivalent Circuit

Power delivered to the load resistance, $P = I^2 R_L = \dfrac{E^2 R_L}{(R_{Th} + R_L)^2}$

Differentiating the above expression w.r.t. R_L and equate to zero, we get

$$\frac{dP}{dR_L} = \frac{E^2(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)E^2}{(R_{Th}+R_L)^4} = 0$$

$$E^2(R_{Th}+R_L)[R_{Th}+R_L-2R_L] = 0 \quad .... \rightarrow R_L = R_{Th}$$

Thus, the condition for maximum power transfer is that the load resistance R_L shall be equal to internal resistance or

Thevenin resistance R_{Th}, of the network.

The value of the maximum power transferred is -

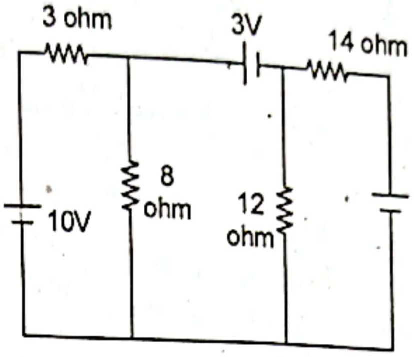
$$P_{max} = \frac{E^2}{4R_{Th}} \text{ or } P_{max} = \frac{V_{Th}^2}{4R_{Th}} \text{ or } P_{max} = \frac{V_{OC}^2}{4R_{Th}}$$

Similarly, for the current source, the power transferred will be maximum when load conductance- is equal to the source conductance i.e. $G_L = G_N$

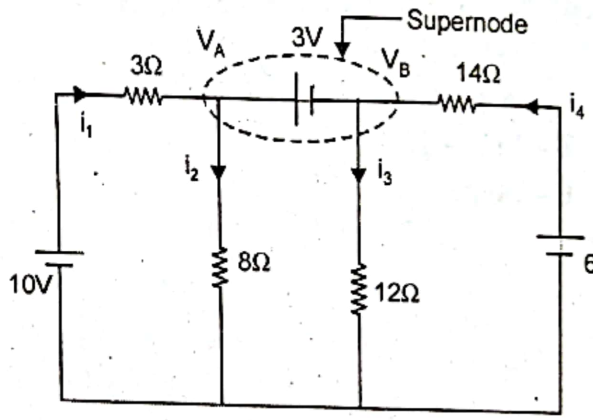
The value of the maximum power transferred is -

$$P_{max} = \frac{I_N^2}{4G_N} \text{ or } P_{max} = \frac{I_{SC}^2}{4G_N}$$

Q. 2. (b) For the circuit shown in fig. Find current across 8 ohm resistor by nodal analysis OR by mesh analysis. (6)



Ans. By Nodal Analysis: From the given circuit, it is clear that it is the case of super node.



Let the direction of current in different branches be i_1, i_2, i_3 and i_4 as shown in the modified circuit diagram.

By nodal analysis, we have

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{10 - V_A}{3} + \frac{6 - V_B}{14} = \frac{V_A}{8} + \frac{V_B}{12}$$

$$\frac{(140 - 14V_A + 18 - 3V_B)}{3 \times 14} = \frac{3V_A + 2V_B}{24}$$

$$4(158 - 14V_A - 3V_B) = 7(3V_A + 2V_B).$$

$$632 - 56V_A - 12V_B - 21V_A - 14V_B = 0$$

$$77 V_A + 26 V_B = 632 \quad \text{...(1)}$$

From the super node, we have

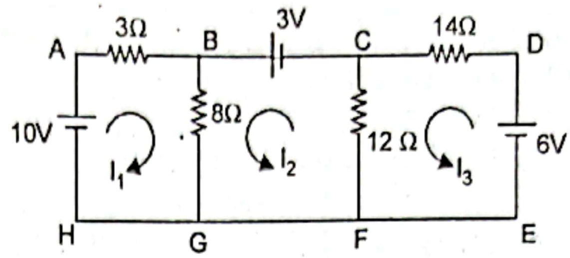
$$V_A - V_B = 3 \quad \text{....(2)}$$

$$V_B = V_A - 3$$

$$77\,V_A + 26\,(V_A - 3) = 632$$
$$77\,V_A + 26\,V_A - 78 = 632$$
$$V_A = \frac{710}{103} = 6.893\,V$$

$\therefore$ The current through 8Ω resistor $= i_2 = \dfrac{V_A}{8} = \dfrac{6.893}{8} = 0.862\text{ A}$

By Mesh Analysis



From loop ABGH, we get

$$10 - 3I_1 - 8(I_1 + I_2) = 0$$
$$10 - 3I_1 - 8I_1 - 8I_2 = 0$$
$$11I_1 + 8I_2 = 10 \quad ...(1)$$

From loop, BCFG, we get

$$3 - 8\,(I_1 + I_2) - 12\,(I_2 - I_3) = 0$$
$$3 - 8I_1 - 8I_2 - 12I_2 + 12I_3 = 0$$
$$8I_1 + 20I_2 - 12I_3 = 3 \quad ...(2)$$

From loop CFDE, we get

$$6 - 14\,I_3 + 12\,(I_2 - I_3) = 0$$
$$6 - 14I_3 + 12I_2 - 12I_3 = 0$$
$$12\,I_2 - 26\,I_3 = -6 \quad ...(3)$$

$$\Rightarrow \quad I_3 = \frac{6 + 12I_2}{26}$$

From eq. (2) we have

$$8I_1 + 20I_2 - 12\left(\frac{6+12I_2}{26}\right) = 3$$
$$208I_1 + 520I_2 - 72 - 144\,I_2 = 3 \times 26$$
$$208I_1 + 376I_2 = 150 \quad ...(4)$$

From eq. (1) we have $\quad I_2 = \dfrac{10 - 11I_1}{8}$

From eq. (4) we have

$$208\,I_1 + 376\left(\frac{10-11I_1}{8}\right) = 150$$
$$208\,I_1 + 47\,(10 - 11I_1) = 150$$
$$208\,I_1 + 470 - 517\,I_1 = 150$$
$$309\,I_1 = 320$$
$$\boxed{I_1 = 1.035A}$$

$$\Rightarrow \quad I_2 = \frac{10 - 11 \times 1.035}{8} \Rightarrow \boxed{I_2 = -0.173A}$$

∴ Current though 8Ω resistor = $I_1 + I_2$

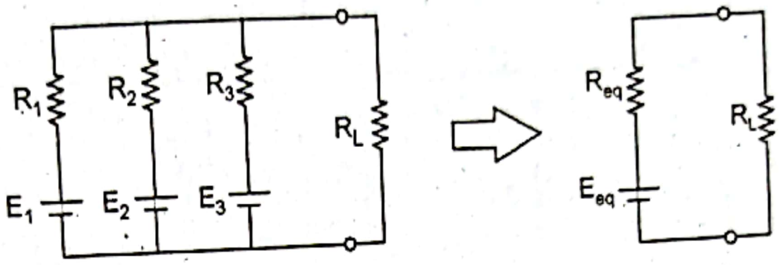
$$= 1.035 - 0.173 = 0.862 \text{ A}$$

Q. 3. (a) Explain Mill- man's theorem. **(3)**

Ans. Millman's Theorem is stated as:

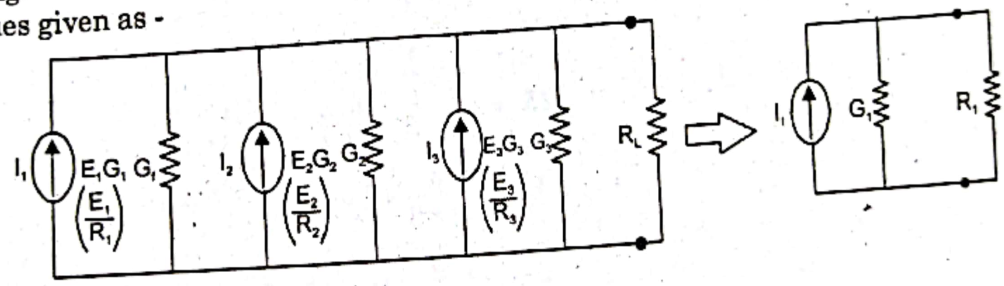
(i) As Applicable to Voltage Sources: If'n' voltage sources $E_1, E_2, \ldots E_n$ having internal resistances $R_1, R_2, \ldots R_n$ are in parallel, then these sources may be replaced by single voltage source V_{eq} having a series resistance R_{eq}, such that V_{eq} and Z_{eq} has the values given as -

$$V_{eq} = \frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\ldots+\frac{E_n}{R_n}}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_n}} = \frac{E_1G_1+E_2G_2+\ldots+E_nG_n}{G_1+G_2+\ldots+G_n} = \frac{\Sigma EG}{\Sigma G}$$



$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_n}} = \frac{1}{G_1+G_2+\ldots+G_n} = \frac{1}{\Sigma G}$$

(ii) As Applicable to Current Sources: If'n' current sources $I_1, I_2, \ldots I_n$, having internal admittances $G_1, G_2, \ldots G_n$, are in series, then these sources may be replaced by a single current source I_{eq} having a parallel admittance G_{eq}, such that I_{eq} and G_{eq} has the values given as -

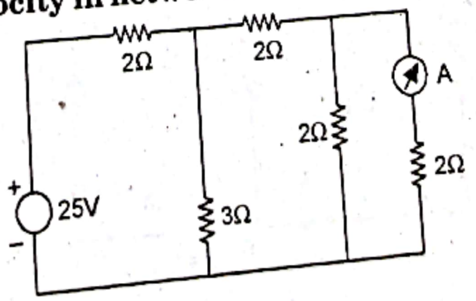


$$I_{eq} = \frac{\frac{I_1}{G_1}+\frac{I_2}{G_2}+\ldots+\frac{I_n}{G_n}}{\frac{1}{G_1}+\frac{1}{G_2}+\ldots+\frac{1}{G_n}} = \frac{I_1R_1+I_2R_2+\ldots+I_nR_n}{R_1+R_2+\ldots+R_n} = \frac{\Sigma IR}{\Sigma R}$$

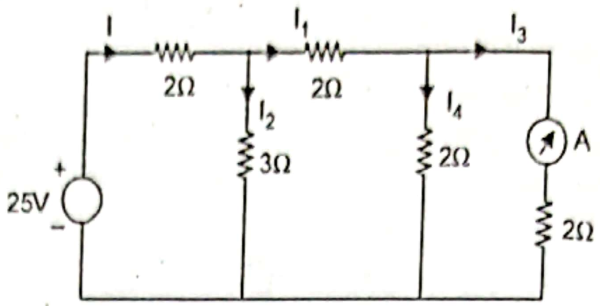
$$G_{eq} = \frac{1}{\frac{1}{G_1}+\frac{1}{G_2}+\ldots+\frac{1}{G_n}} = \frac{1}{R_1+R_2+\ldots+R_n} = \frac{1}{\Sigma R}$$

(7)

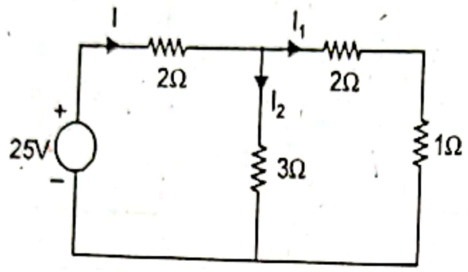
Q. 3. (b) Verify reciprocity in network as shown in figure.

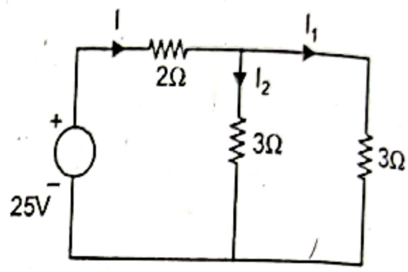


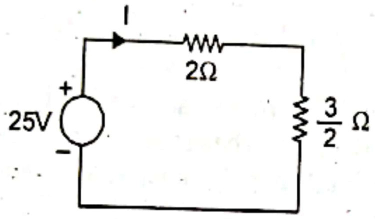
Ans.



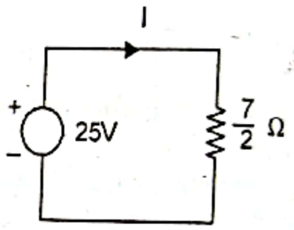
$$R_{eq} = \{[(2 \parallel 2) + 2] \parallel 3\} + 2$$
$$= [(1 + 2) \parallel 3] + 2$$
$$= \frac{3}{2} + 2 = \frac{7}{2}\,\Omega$$







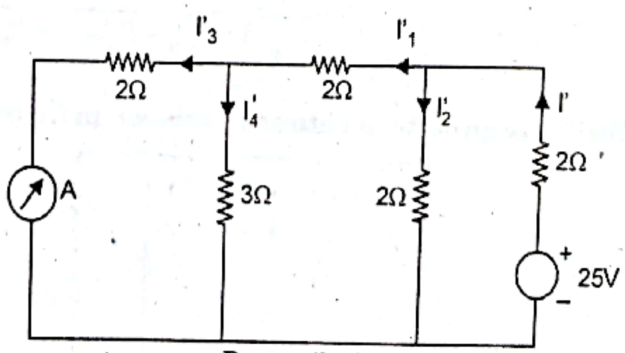
$\Rightarrow$



$$V = I R_{eq}$$
$$25 = I \times \frac{7}{2} \Rightarrow I = \frac{50}{7} A$$
$$I_1 = \frac{3}{3+3} \times I = \frac{1}{2} \times \frac{50}{7} = \frac{25}{7} A$$
$$I_3 = \frac{2}{2+2} \times I_1 = \frac{1}{2} \times \frac{25}{7} = \frac{25}{14} A$$

$\therefore$ Ratio $\quad \dfrac{V}{I_3} = \dfrac{25}{25/14} = 14$

Now, interchanging the position of source voltage and ammeter, we get the modified circuit diagram.

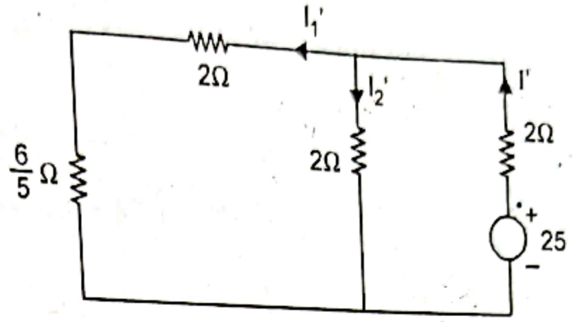


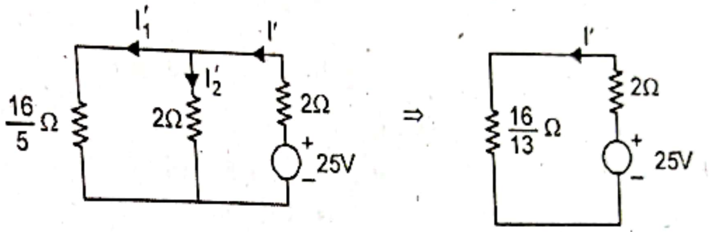
$$R_{eq} = \{[(2 \parallel 3) + 2] \parallel 2\} + 2$$

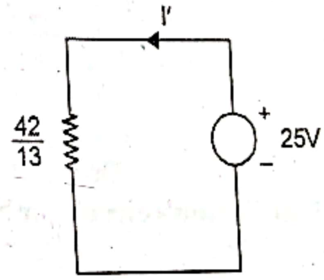
$$= \left[\left(\frac{6}{5}+2\right) \| 2\right]+2$$

$$= \left(\frac{16}{5} \| 2\right)+2$$

$$= \frac{\frac{16}{5}\times 2}{\frac{16}{5}+2}+2 = \frac{16}{13}+2 = \frac{42}{13}$$







$$V = I'R_{eq}$$

$$25 = I'\times\frac{42}{13}$$

$$I' = \frac{325}{42}\text{A}$$

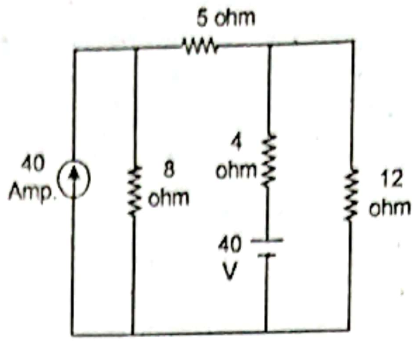
$$I'_1 = \frac{2}{2+\frac{16}{5}}\times I' = \frac{2\times 5}{26}\times\frac{325}{42}$$

$$I'_3 = \frac{3}{3+2}\times I'_1 = \frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{2}\times\cancel{5}}{\cancel{26}_{13}}\times\frac{325}{\cancel{42}_{14}} = \frac{25}{14}\text{A}$$

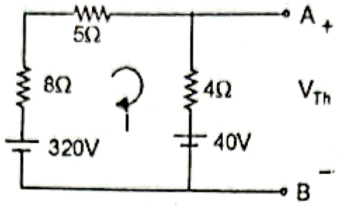
∴ Ratio $\qquad \frac{V}{I_3'} = \frac{25}{25/14} = 14$

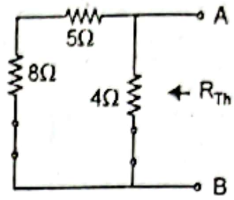
Hence, Reciprocity Theorem is verified.

Q. 4. (a) Draw equivalent Thevenin's circuit for Network shown in fig. Take 12 Ohm as load resistance. (5)



Ans. By Thevenin's Theorem





$$R_{Th} = (5+8) \,||\, 4 = 13 \,||\, 4 = \frac{13 \times 4}{17} = \frac{52}{17}\Omega$$

$$320 - 8I - 5I - 4I - 40 = 0$$

$$17I = 280$$

$$I = \frac{280}{17} = 16.47A$$

$$\Rightarrow \quad 40 + 4I - V_{Th} = 0$$

$$V_{Th} = 40 + 4 \times 16.47$$

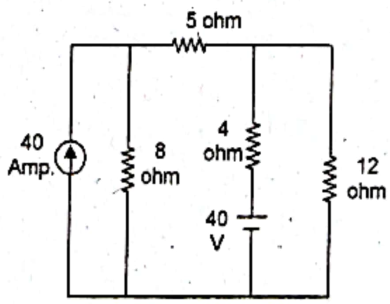
$$= 105.\,88 \text{ V}$$

$$\therefore \quad I_L' = \frac{V_{Th}}{R_{Th} + R_L} = \frac{105.88}{12 + \frac{52}{17}}$$

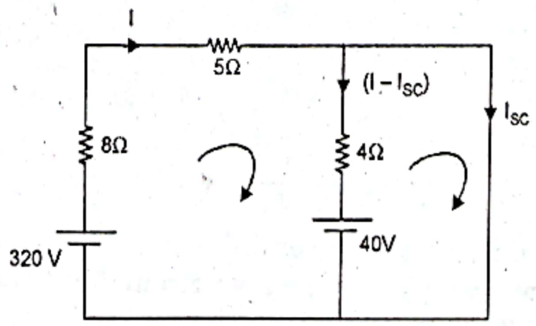
$$= 7.03 \text{ A}$$

Or

Q. 4. (a) Draw equivalent Norton's circuit for Network shown in fig. Take 12 Ohm as load resistance. **(5)**



Ans. By Norton's Theorem



$$R_N = R_{Th} = \frac{52}{17} = 3.06\,\Omega$$

$$320 - 13I - 4(I - I_{sc}) - 40 = 0$$

$$280 - 17I + 4I_{sc} = 0$$

$$17I - 4I_{sc} = 280 \quad ...(1)$$

$$40 + 4(I - I_{sc}) = 0$$

$$I - I_{sc} = -10$$

$$I = I_{sc} - 10 \quad ...(2)$$

From eq. (1) and (2), we get

$$17(I_{sc} - 10) - 4I_{sc} = 280$$

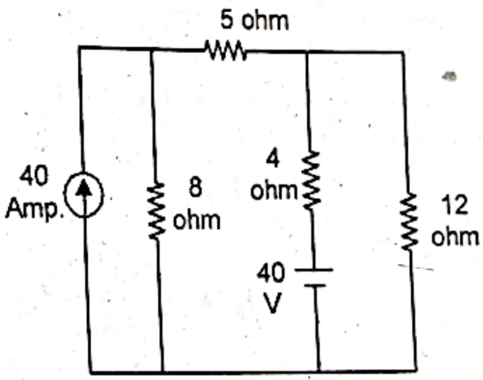
$$13I_{sc} = 450$$

$$I_{sc} = \frac{450}{13} = 34.61A$$

$$\therefore \quad I_L = \frac{R_N}{R_L + R_N} \times I_{sc} = \frac{3.06}{12 + 3.06} \times 34.61$$

$$= 7.03\text{ A}$$

Q. 4. (b) Find current across. 12 ohm resistor in fig. by superposition theorem. (5)

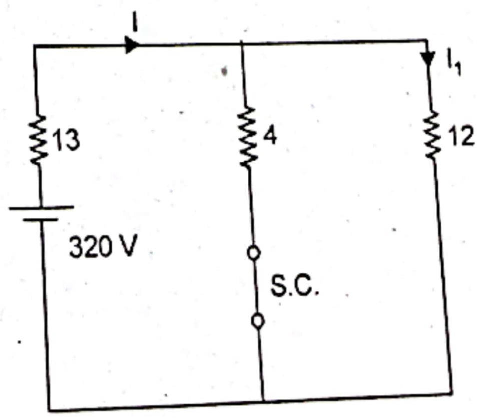


Ans. By Super position Theorem

Case 1: Considering the 320 V source voltage only.

$$R_{eq} = (12 \| 4) + 13$$

$$= \frac{12 \times 4}{16} + 13 = 16\,\Omega$$



$$V = IR_{eq}.$$

$$320 = I \times 16$$

$$I = \frac{320}{16} = 20A$$

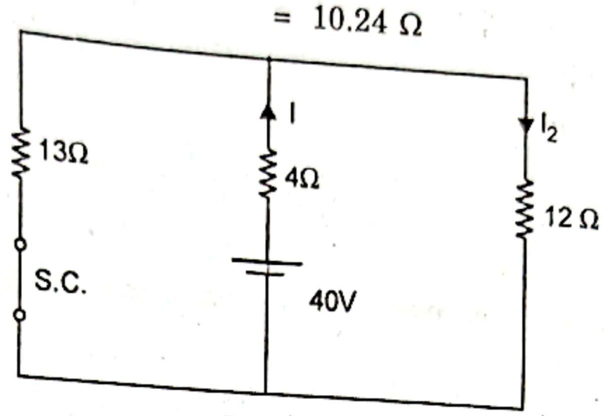
$$I_1 = \frac{4}{4+12} \times I = \frac{4}{16} \times 20 = 5A$$

Case 2: Considering the 40 V source voltage only

$$R_{eq} = (13 \parallel 12) + 4$$

$$= \frac{13 \times 12}{25} + 4$$

$$= 10.24\ \Omega$$



$$V = I R_{eq}.$$

$$40 = I \times 10.24$$

$$I = \frac{40}{10.24} A$$

$$I_2 = \frac{13}{13+12} \times I = \frac{13}{25} \times \frac{40}{10.24} = 2.03A$$

$\therefore$ Total current through 12 Ω resistor $= I_1 + I_2$

$$= 5 + 2.03$$

$$= 7.03\ A$$