# Circuit & Systems

- **Circuit Elements :-**

(i) Active elements :- Energy Sources ex. battery etc.

(ii) Passive elements :- They consumes energy : ex. R, L, C.

In Resistor → Energy is dissipated.

In Capacitor → Energy is Stored in electrostatic field.

In Inductor → Energy is Stored in magnetic field.

- Circuit :- A closed path formed by various active and passive elements in which electric current can flow.
- Network :- Any possible interconnection of electric circuit element. eg. T network, π network, ladder network.
- System :- A system has many Components which are inter-related and interconnected with each to perform a Specific task. Eg. Power System, Control System, Communication System.

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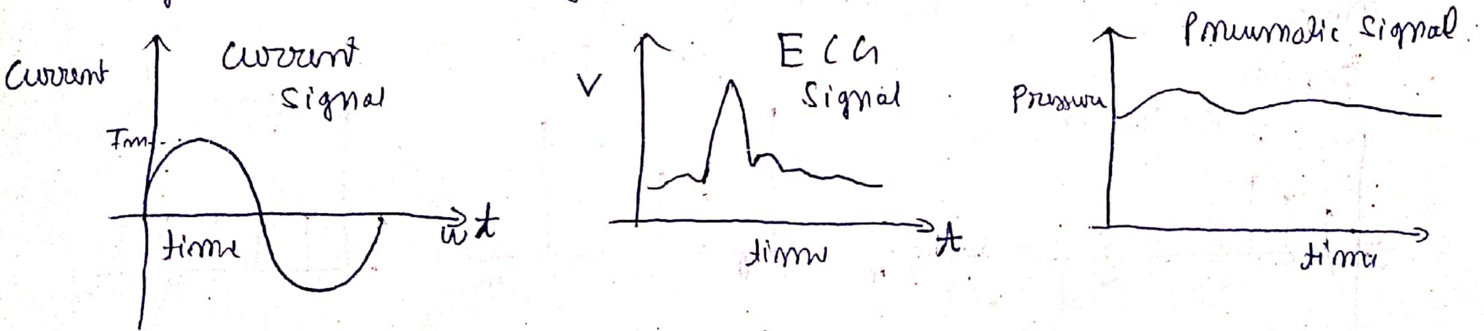
- Signal : Any physical Quantity as a function of time. It represents variation of any physical Quantity with time. It carries useful information about that physical quantity. For ex. electric Signal (current) $I(t) = I_m \sin \omega t$, speech signal, biomedical Signal (ECG signal), pneumatic signal (Pressure as a function of time).

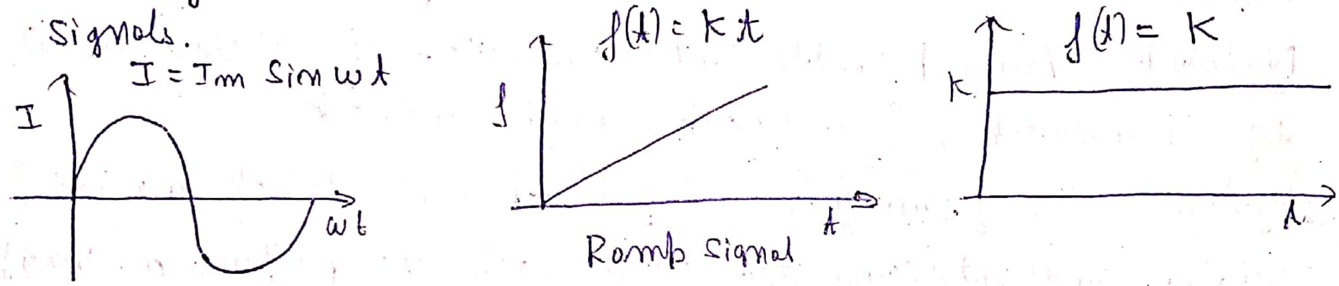



- Electric Signals has prime importance. Any physical Quantity (like pressure) can be converted to equivalent electric current signal by using transducers. This is because electric Signals can be processed easily for control purpose (or for any other purpose).

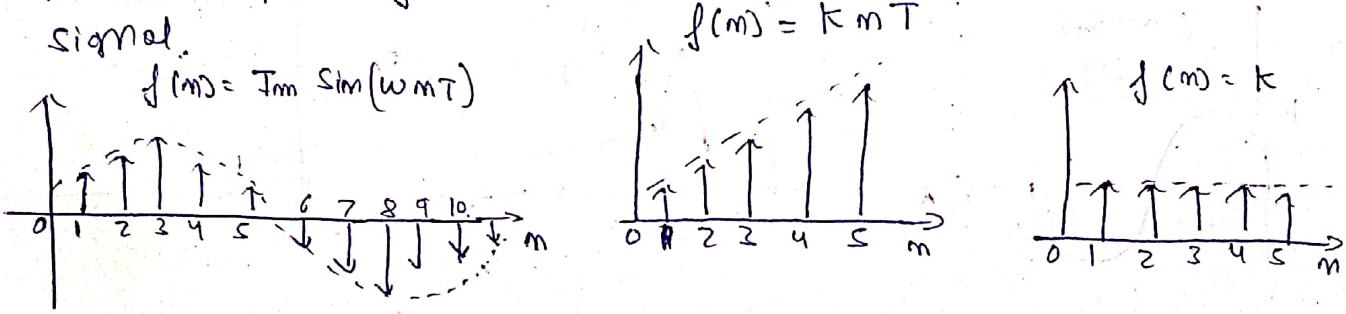
- For Control purpose :-
  (i) Range of current signal : 4 to 20 mA.
  (ii) Range of Pneumatic (Pressure) Signal : 3 to 15 psi.

- Types of Signals :-

(i) Continuous Signal :- These Signals gives variation of any physical Quantity at every instant of time. They are defined at every instant of time. They are Continuous function of time. Such Signals are encountered in Analog devices and System. They are also called Analog signals.




(ii) Discrete Signals :- They are not defined at every instant of time. They gives variation of any physical Quantity for discrete values of time. For example - they are defined at time $t = 0$, $t = 1\,ms$, $t = 2\,ms$, $t = 3\,ms$, $t = 4\,ms$ & so on. They are not defined at defined between 0 to 1 ms &, 1 ms to 2 ms and so on. Such signals are encountered in Digital devices & System. Discrete signal if coded in form of 0's & 1's become digital signal.




* T is sampling time ; n represents the discrete values of time.

- **Periodic Signal** :- Signal which repeats after a particular time, $T_0$. $T_0$ is called the time period of a signal. Signal $x(t)$ which is a function of time is periodic if $\boxed{x(t+T_0) = x(t)}$ $-\infty < t < \infty$

  Examples of periodic signals are $\sin \omega t$, $\cos \omega t$ etc.
  Sine & cosine signal has time period of $2\pi$.

- ~~Examples~~ **Non periodic** :- Such signals ~~A~~ does not have fixed pattern. They donot repeats in time.
  For example :- exponential ($e^t$) is non periodic signal.

- **Even signals** :- Signal $x(t)$ is even if

  $$x(-t) = x(t)$$

  Signal is symmetrical along ordinate axis (y axis).
  For ex. $\cos \omega t$ is even signal

- **Odd signal** :- Signal $x(t)$ is odd if

  $$x(-t) = -x(t)$$

  For ex. $\sin \omega t$ is odd signal.

- Some signals are neither even nor odd. For ex. exponential signal ($e^t$).

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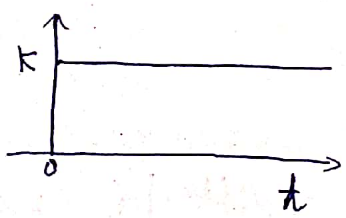
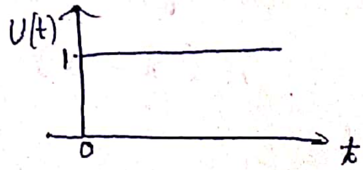
## Standard Signals

1. **Step Signal** :-

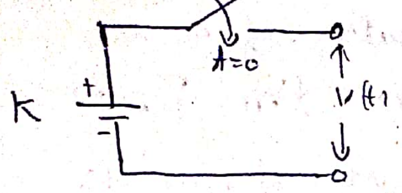


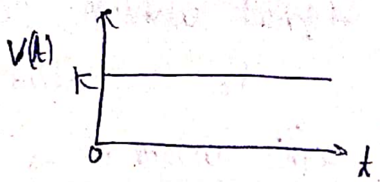
$$\text{Step signal} = \begin{cases} 0 & ; t < 0 \\ K & ; t \geq 0 \end{cases}$$

**Unit step** :- $$U(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



- Step signal can be produced with a circuit having a battery together with switch.

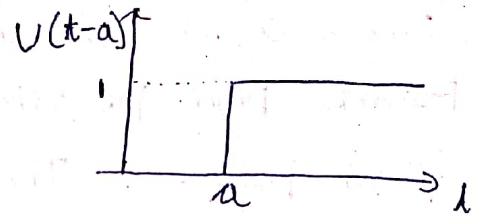



- **Time shifting of step signal**

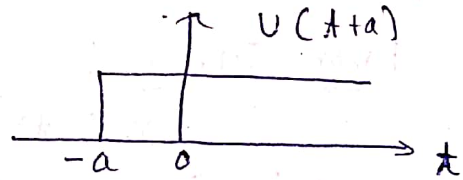
(i) Delayed signal

$$U(t-a) = \begin{cases} 0 & ; t < a \\ 1 & ; t \geq a \end{cases}$$




(ii) Time advanced signal

$$U(t+a) = \begin{cases} 0 & ; t < -a \\ 1 & ; t \geq -a \end{cases}$$

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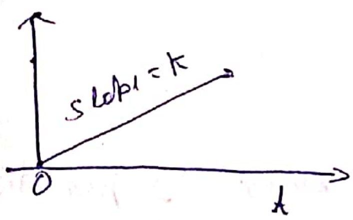
2. **Ramp signal :-**



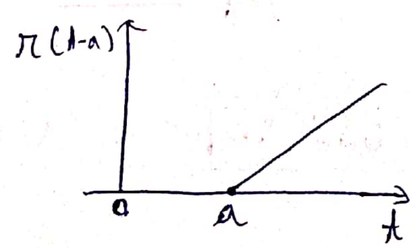

$$\text{Ramp} = \begin{cases} 0 & ; t < 0 \\ Kt & ; t \geq 0 \end{cases}$$

**Unit ramp :-** $$r(t) = \begin{cases} 0 & ; t < 0 \\ t & ; t \geq 0 \end{cases}$$


**Time shifted ramp :-**

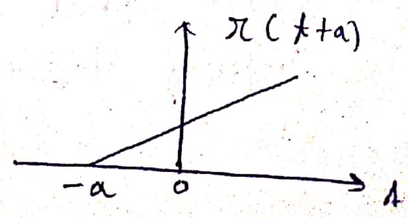
(i) Delayed ramp :-

$$r(t-a) = \begin{cases} 0 & ; t < a \\ (t-a) & ; t \geq a \end{cases}$$



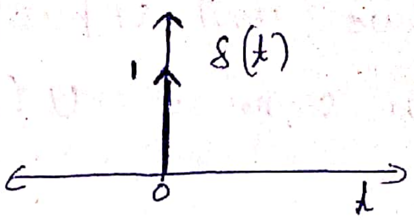

(ii) Time advanced ramp

$$r(t+a) = \begin{cases} 0 & ; t < -a \\ t+a & ; t \geq -a \end{cases}$$



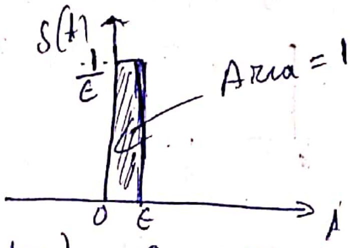
## 3. Unit Impulse ~~function~~ Signal

$$\delta(t) = \begin{cases} 0 & ; t \neq 0 \\ 1 & ; t = 0 \end{cases}$$




Practical unit impulse function :

$$\delta(t) = \begin{cases} \frac{1}{\epsilon} & ; 0 < t \leq \epsilon \\ 0 & ; \text{otherwise} \end{cases}$$



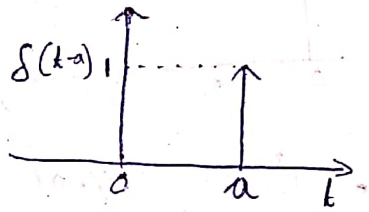

where $\epsilon$ is very small value. (Positive value). $\epsilon \to 0$.

Area of unit impulse $= \epsilon \times \frac{1}{\epsilon} = 1$

Area of unit impulse function is alway one.

- Shifted unit impulse :

$$\delta(t-a) = \begin{cases} 0 & ; t \neq a \\ 1 & ; t = a \end{cases}$$

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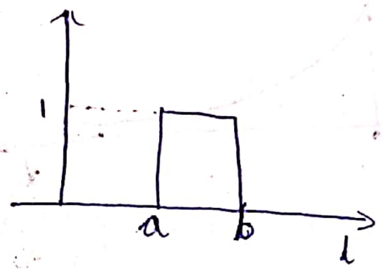
## 4. Gate Signal :-

Gate Signal




- Rectangular pulse of unit amplitude starting at $t=a$ and ending at $t=b$.
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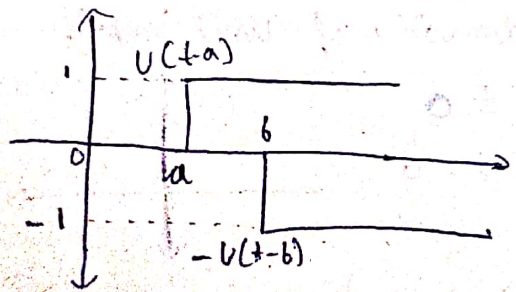
- Gate ~~function~~ signal expressed as the sum of two step ~~f~~ signals.

Gate signal $= U(t-a) - U(t-b)$

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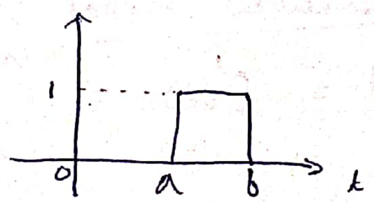
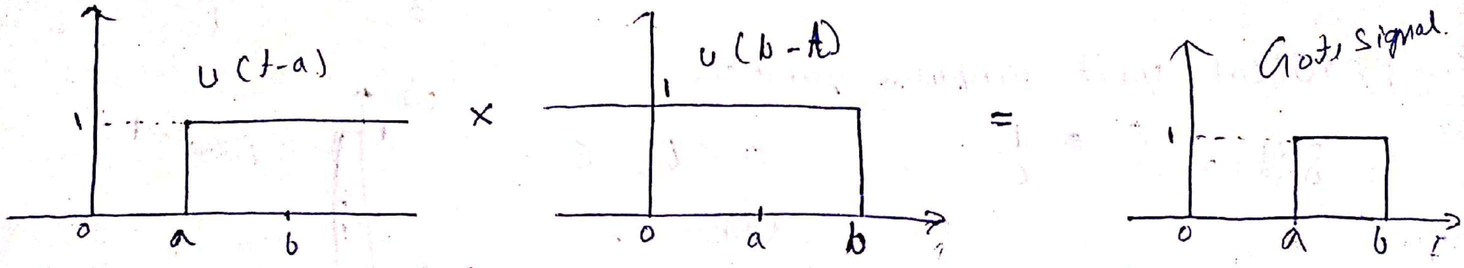


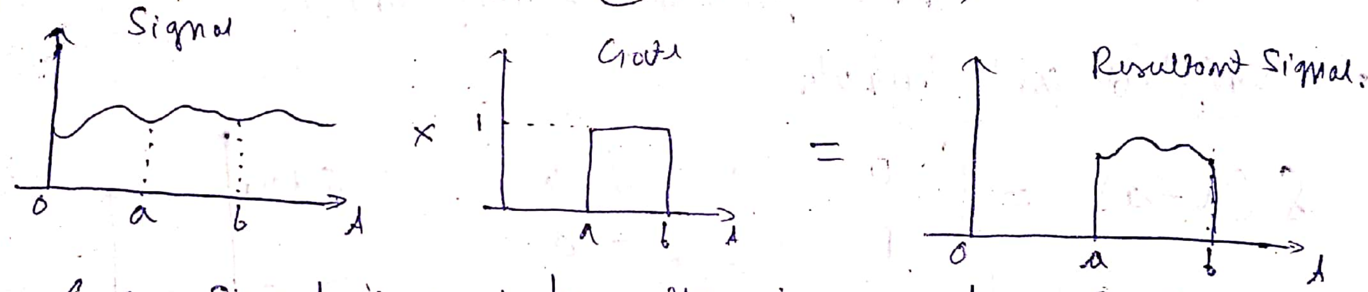
- Gate signal can also be expressed as the product of two unit step signals

Gate signal $= u(t-a) \cdot u(b-t)$




- Let $x(t)$ be any signal

$$x(t) \times \text{Gate Signal} = \begin{cases} x(t) & ; a \leq t \leq b \\ 0 & ; \text{otherwise} \end{cases}$$




- Gate signal is used frequently in wave form synthesis.

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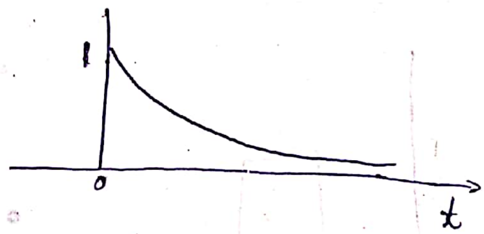
5. <u>Exponential Signal :-</u>

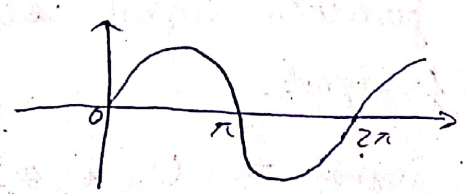
$$f(t) = \begin{cases} 0 & t < 0 \\ e^{-at} & t \geq 0 \end{cases}$$




a is (+)ve real constant.

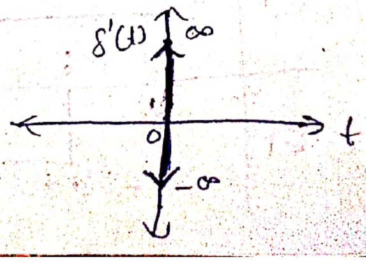
6. <u>Sinusoidal Signal</u>

$$f(t) = \begin{cases} 0 & t < 0 \\ V_m \sin \omega t & t \geq 0 \end{cases}$$




7. <u>Unit Doublet Signal</u> :- It is a derivative of unit impulse.

$$\text{unit Doublet} = \delta'(t) = \begin{cases} 0 & ; t \neq 0 \\ +\infty \ \& -\infty & ; t = 0 \end{cases}$$



- Standard Signals :- Step signal, Ramp Signal & impulse signals are standard ~~function~~ signals. Standard Signals are also called singularity ~~functions~~ Signals because they can be obtained from one another by successive differentiation or integration.
- Differentiation of Ramp Signal = Step signal.
- Differentiation of Step signal = Impulse Signal.

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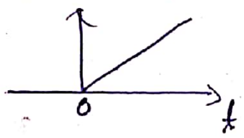
## Signal Synthesis or wave form Synthesis

- Representation of ramp signal in terms of Step signal

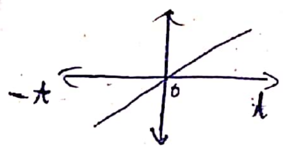
(i) $r(t) = t\,u(t)$ → unit ramp

(ii) $r(t-a) = (t-a)\,u(t-a)$ → Time delayed ramp.

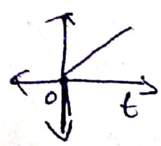
- If we write $r(t)$ in any expression, then it represents



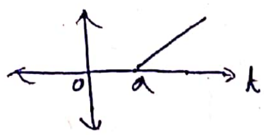
~~If we r~~ we cannot write $\cancel{r(t) = t}$ simply because it represent



But we can write $r(t) = t\,u(t)$. It represents

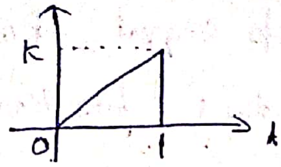


- $r(t-a) = (t-a)\,u(t-a)$ represents

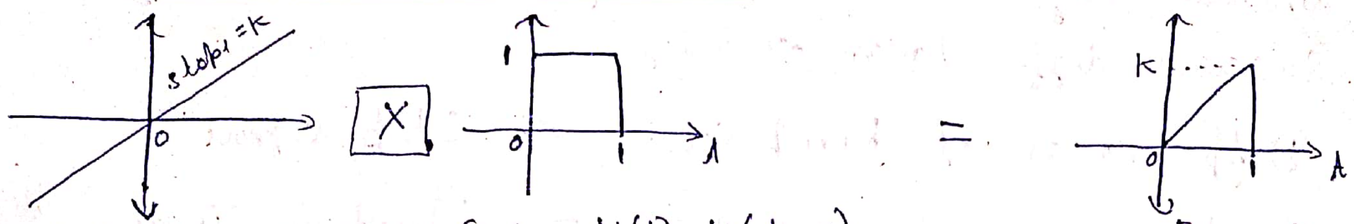


- ~~&~~ Ramp signal is usually expressed in terms of Step i.e $r(t) = t\,u(t)$ & $r(t-a) = (t-a)\,u(t-a)$

Q1 ~~Synthesize~~ Express the given waveform in Standard Signals



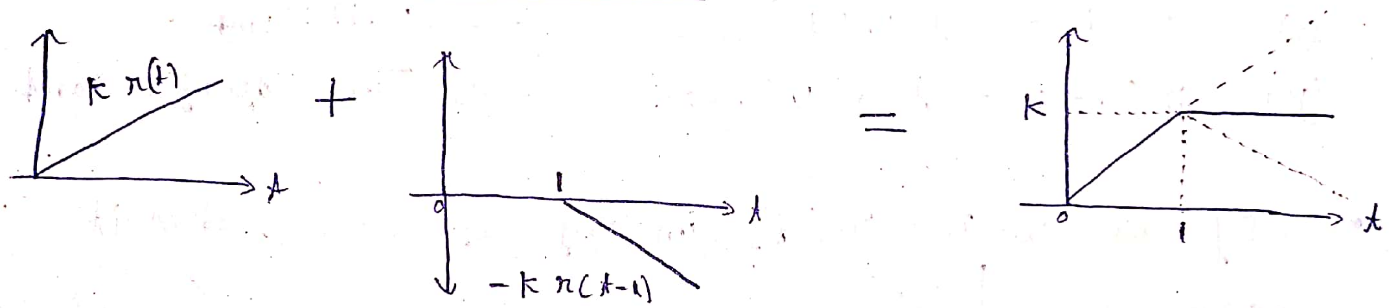

Sol. Method I (Simple technique)

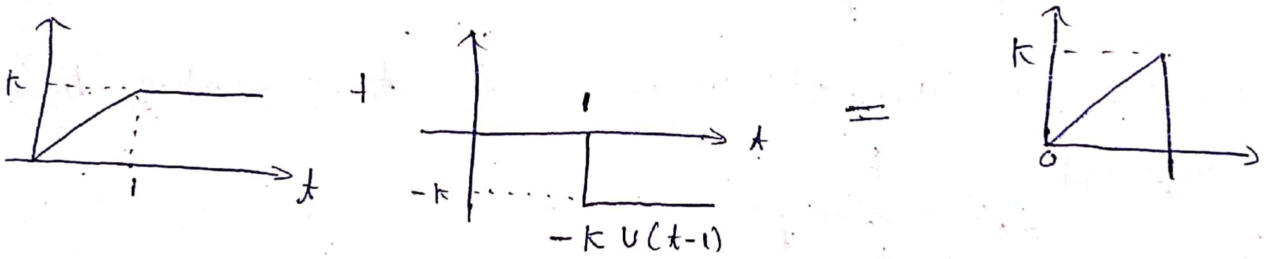



$f_1(t) = kt$     Gate $= u(t) - u(t-1)$     $kt[u(t) - u(t-1)]$

Required waveform $= kt[u(t) - u(t-1)]$

Method II (Complex method)




$\therefore$ Required waveform $= k\,r(t) - k\,r(t-1) - k\,u(t-1)$

$= kt\,u(t) - k(t-1)u(t-1) - k\,u(t-1)$

$= kt\,u(t) - kt\,u(t-1) + \cancel{k\,u(t-1)} - \cancel{k\,u(t-1)}$

$= kt[u(t) - u(t-1)]$

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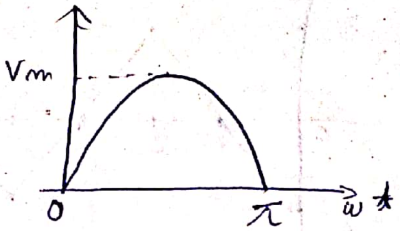
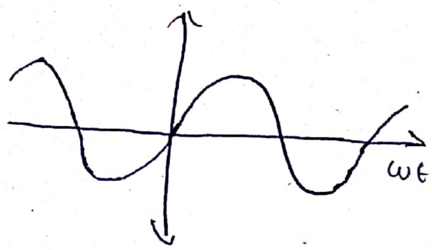
Q2 Synthesize




Sol. :- Method I

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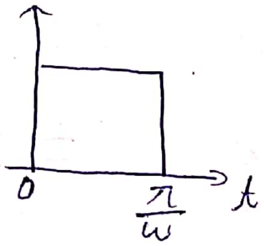


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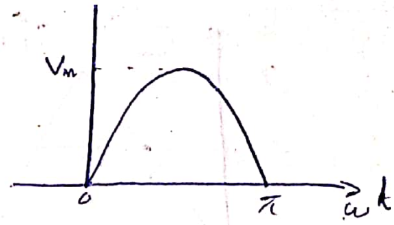



$V_m \sin \omega t$ $\quad u(t) - u(t - \frac{\pi}{\omega})$ $\quad = V_m \sin \omega t \left[u(t) - u(t - \frac{\pi}{\omega})\right]$

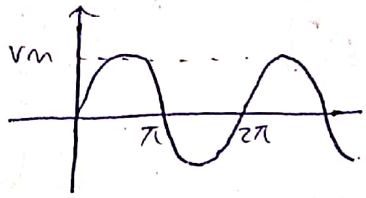
$\therefore$ Required waveform $= V_m \sin \omega t \left[u(t) - u(t - \frac{\pi}{\omega})\right]$

$= V_m \sin \omega t \left[u(t) - u(t - \frac{\pi T}{2\pi})\right]$

$= V_m \sin \omega t \left[u(t) - u(t - \frac{T}{2})\right]$

where T is time period of sine wave.

Method II

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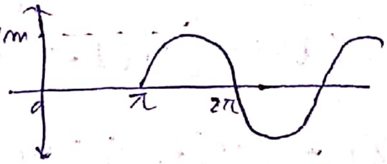


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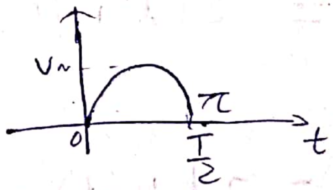


$V_m \sin \omega t \, u(t)$ $\quad V_m \sin \omega(t - \pi) \, u(t-\pi)$

$\therefore$ Required waveform $= V_m \sin \omega t \, u(t) + V_m \sin \omega(t - \pi) \, u(t - \pi)$

$= V_m \left[\sin \omega t \, u(t) + \sin \omega(t - \frac{T}{2}) \, u(t - \frac{T}{2})\right]$

$= V_m \left[\sin \omega t \, u(t) + u(t - \frac{T}{2}) \left(\sin \omega t \cos \frac{\omega T}{2} - \sin \frac{\omega T}{2} \cos \omega t\right)\right]$

$= V_m \left[\sin \omega t \, u(t) + u(t - \frac{T}{2}) \left(\sin \omega t (-1) - (0)\right)\right]$

$= V_m \left[\sin \omega t \, u(t) - \sin \omega t \, u(t - \frac{T}{2})\right]$

$= V_m \sin \omega t \left[u(t) - u(t - \frac{T}{2})\right]$

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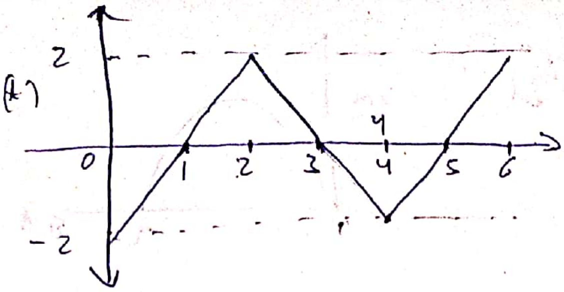
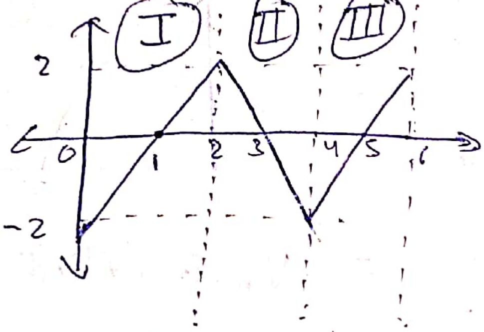
**Q3** Synthesize




**Sol.** Given signal can be divided into ~~two~~ 3 parts

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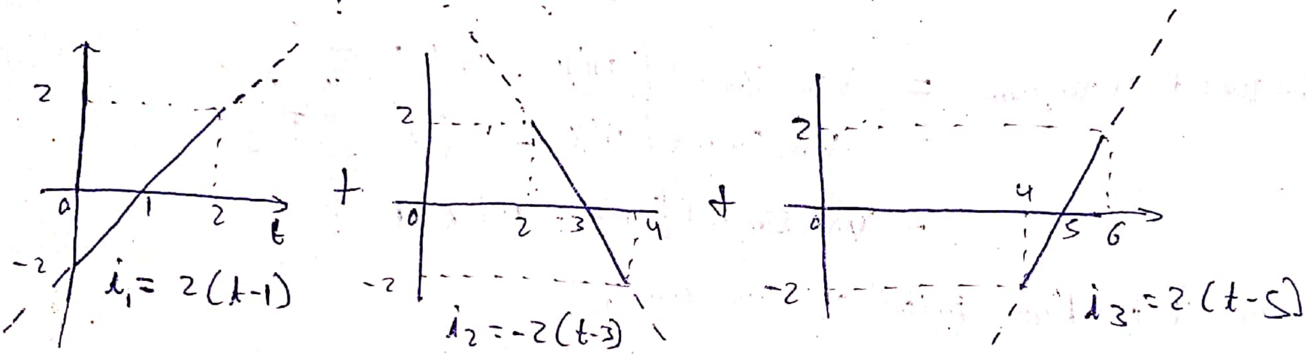


$$= 2(t-1)[u(t)-u(t-2)] + (-2(t-3))[u(t-2)-u(t-4)] + 2(t-5)[u(t-4)-u(t-6)]$$

$$= 2t\,u(t) - 2t\,u(t-2) - 2u(t) + 2u(t-2) - 2t\,u(t-2) + 2t\,u(t-4) + 6u(t-2) - 6u(t-4) + 2t\,u(t-4) - 2t\,u(t-6) - 10u(t-4) + 10u(t-6)$$

$$= (2t-2)u(t) + (-4t+8)u(t-2) + (4t-16)u(t-4) + (-2t+10)u(t-6)$$

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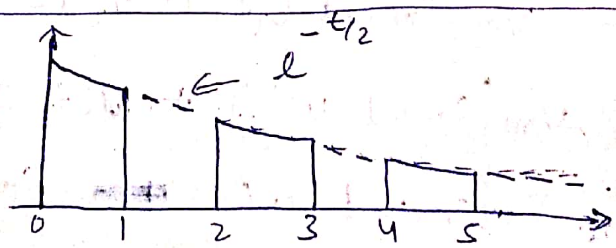
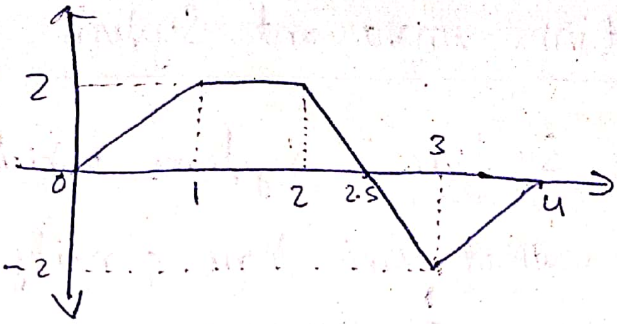
**Q4** Synthesize

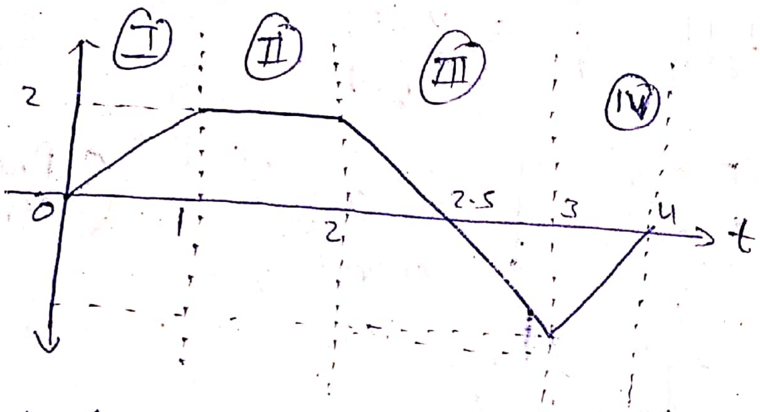



**Sol:** $= e^{-t/2}[u(t)-u(t-1)] + e^{-t/2}[u(t-2)-u(t-3)] + e^{-t/2}[u(t-4)-u(t-5)]$

**Q 5** Synthesize




**Sol.**




Eq. of line in region I : $V_1 = 2t$

" " " " " II : $V_2 = 2$

" " " " " III : $V_3 = -4t + 10$

" " " " " IV : $V_4 = 2(t-4)$

Required waveform =

$$= 2t[u(t) - u(t-1)] + 2[u(t-1) - u(t-2)] + (-4t+10)[u(t-2) - u(t-3)] + 2(t-4)[u(t-3) - u(t-4)]$$

$$= 2t\,u(t) - 2t\,u(t-1) + 2u(t-1) - 2u(t-2) - 4t\,u(t-2) + 4t\,u(t-3) + 10u(t-2) - 10u(t-3) + 2t\,u(t-3) - 2t\,u(t-4) - 8u(t-3) + 8u(t-4)$$

$$= 2t\,u(t) + (-2t+2)u(t-1) + (-4t+8)u(t-2) + (6t-18)u(t-3) + (-2t+8)u(t-4)$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 4(t-2)u(t-2) + 6(t-3)u(t-3) - 2(t-4)u(t-4)$$

$$= 2r(t) - 2r(t-1) - 4r(t-2) + 6r(t-3) - 2r(t-4)$$

# Linear time invariant System (LTI System)

- **Linear System**: Systems which obeys the principle of Superposition and homogeneity are linear.

(i) $x(t) \xrightarrow{} \boxed{\text{LTI System}} \xrightarrow{} y(t)$

Input → output

$a\,x(t) \xrightarrow{} \boxed{\text{Same LTI System}} \xrightarrow{} a\,y(t)$ (Principle of Homogeneity)

Input → output

(ii) $x_1(t) \xrightarrow{} \boxed{\text{LTI System}} \xrightarrow{} y_1(t)$

$x_2(t) \xrightarrow{} \boxed{\text{LTI System (Same)}} \xrightarrow{} y_2(t)$

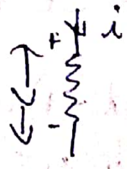
$[x_1(t)+x_2(t)] \xrightarrow{} \boxed{\text{Same LTI System}} \xrightarrow{} [y_1(t)+y_2(t)]$ (Principle of Superposition)

- If a system does not follows principle of Superposition & homogeneity, then it is non linear.
- Example of linear System :- $y(t) = m\,x(t)$
- Example of Non linear System :- $y(t) = m\,x(t) + c$ ; $y(t) = x^2(t)$

- R, L & C are linear elements.

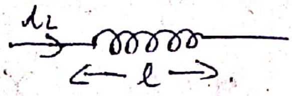
(i) $V \xrightarrow{} \boxed{\frac{1}{R}} \xrightarrow{} i$

(Excitation) (Response)




$$i = \frac{1}{R} V$$

(ii)

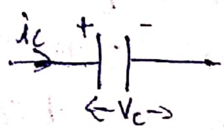



$$\underset{\text{(Excitation)}}{\frac{di_L}{dt}} \longrightarrow \boxed{L} \longrightarrow \underset{\text{(Response)}}{e}$$

(Change in inductor current cause an induced emf across L.)

$$e = L\frac{di_L}{dt}$$

(iii)




$$\underset{\text{(Excitation)}}{\frac{dV_c}{dt}} \longrightarrow \boxed{C} \longrightarrow \underset{\text{(Response)}}{i_c}$$

(Change in voltage across capacitor cause capacitor current)

$$i_c = C\frac{dV_c}{dt}$$

---

- **Time Invariant System :-** System in which input – output relationship does not change with time.
- Electric system (electric network) consisting of resistor, inductor and capacitor is time invariant if parameters of circuit (R, L, C) does not change with time.
- An electric system containing thermistor is time variant system. Value of resistance of thermistor changes with temperature. And temperature changes with time.
- Mathematically, a system is time invariant if a time shift in the input signal results in an identical time shift in the output signal

$$x(t) \longrightarrow \boxed{\text{LTI system}} \longrightarrow y(t)$$

$$x(t-a) \longrightarrow \boxed{\text{Same LTI system}} \longrightarrow y(t-b) \qquad \text{Time Invariant}$$

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- Example of Time invariant System : $y(t) = \sin[x(t)]$
- Example of Time Variant System : $y(t) = x(2t)$

**Proof:** $y(t) = \sin[x(t)]$

$$x(t) \rightarrow \boxed{\text{sine of input}} \rightarrow \sin[x(t)]$$

Let $x(t) = t$

$y(t) = \sin(t)$

$x(t-2) = t-2$

$$x(t-2) \rightarrow \boxed{\text{sine of input}} \rightarrow \sin(t-2)$$

$y(t-2) = \sin(t-2)$ shifted o/p

Shifted output = output of shifted input

$\therefore$ System is Time invariant

**Proof** : $y(t) = x(2t)$

$$x(t) \rightarrow \boxed{\text{Data Compressor}} \rightarrow x(2t)$$

Let $x(t) = t$

$y(t) = 2t$

$x(t-2) = t-2$ → New shifted I/P $x(t)$

$$\underset{\uparrow \text{ New } x(t)}{x(t-2)} \rightarrow \boxed{x(2t)} \rightarrow \underset{\uparrow \text{ Corresponding new output}}{(2t)-2}$$

$y(t-2) = 2(t-2) = 2t-4$ → shifted o/p

Shifted output $\neq$ output of shifted input

$\therefore$ System is Time Variant

---

## Causal System:

- Output of Causal System at any time depends on value of present input and past input values. Output of causal input does not depends on future input.

- For ex. $y(t) = \underset{\uparrow \text{ Present input}}{x(t)} + 4\underset{\uparrow \text{ Past input}}{x(t-1)}$ is Causal System.

- For ex. $y(t) = \underset{\uparrow \text{ Present input}}{2x(t)} + \underset{\uparrow \text{ Future input}}{x(t+2)}$ is Non Causal System.

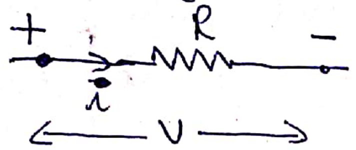
---

## Behaviour (or Nature) of Circuit elements

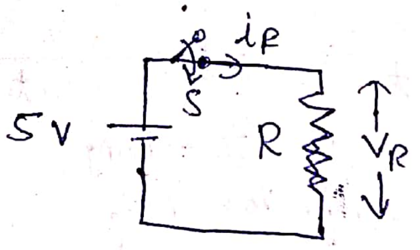
1 **Resistor**

- Property of resistance : It opposes the flow of free electrons due to the presence of atom in the lattice. When a source of emf is applied, free electrons collide with atoms in the lattice structure and hence produce irreversible heat ($I^2R$ heat loss).

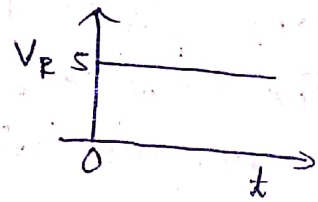
As a result, temperature of conducting material increases. (8)
This process of increase in temperature continues. Then after some time a limit is reached when temperature does not increase further. Then it dissipate the heat to the surrounding in the form of $I^2R$ loss.

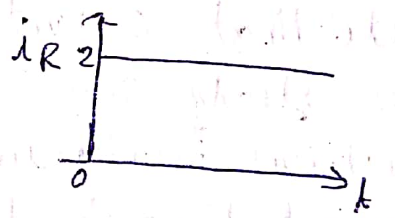



$i = \frac{1}{R} \cdot V$ (ohm's law)




Let switch S be closed at time $t=0$




When a switch is closed at $t=0$, $V_R$ changes from 0 to 5V in 0 seconds (Theoretically). Also $I_R$ changes from 0 to 2A in 0 (zero) second.

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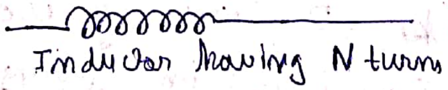
## Inductor

- Property of inductance :- It opposes any change in current (inductor current) by inducing an emf in itself.

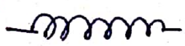
$\phi \propto I$

When I changes, $\phi$ also change.
Change in flux causes an induced emf (Acc. to electro magnetic induction principle).

- $N\phi = Li$



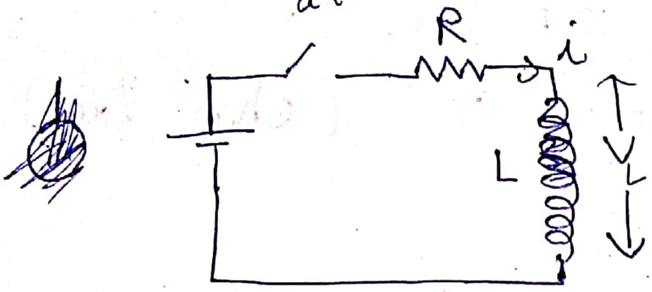

$L = \frac{\text{Total flux linkage}}{\text{current}}$

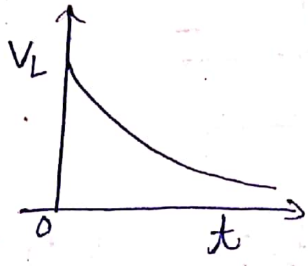
- Inductance of straight wire : ——— straight wire  L = Very low
- Inductance of solenoid :  L = Large value because total flux linkage increases due to N turns.

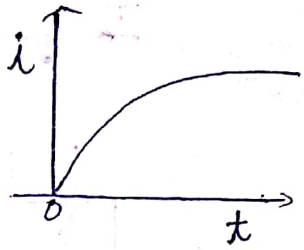
$$\phi = Li$$

$$\frac{d\phi}{dt} = L\frac{di}{dt}$$

$$e = L\frac{di}{dt} \Rightarrow e = V_L = L\frac{di}{dt}$$




- Note that current takes finite time in reaching the steady state. This is because inductor has electrical inertia which does not allow sudden change of current through it.

Proof: $V_L = L\frac{di}{dt} = L\frac{\text{Steady state value of } I}{\text{Zero time}} = \infty$ (Practically Not possible)

For a minute change in current within zero time ($dt=0$), gives an infinite voltage across the inductor which is physically not at all feasible.

- Inductor is an energy storage device: It stores energy in the form of magnetic field.

  Energy of inductor $= \frac{1}{2}LI^2$

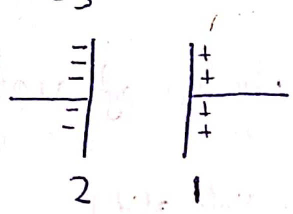
- Inductor takes some finite time in going from Zero energy level to full energy level ($\frac{1}{2}LI^2$).

  It is just like, a 2 litres of empty water bottle takes atleast 1 minute to 1.5 minutes in getting fully filled.

  This is the reason behind the electrical inertia of an inductor.

# Capacitor

- Capacitance :- It is the capability of ~~an~~ a capacitor to store electric charge within it.
- Capacitor stores electric energy in the form of electric field being established by the two polarity of charges on the two electrodes ( or plates) of a capacitor.
- $Q \propto V$



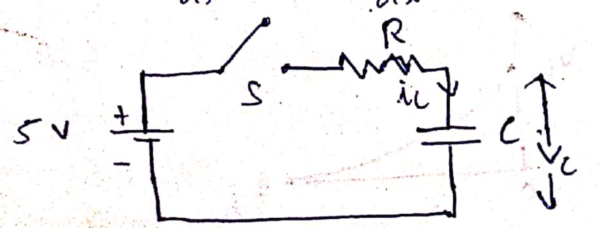

Let a unit (−)ve charge is transferred from plate 1 to plate 2 .

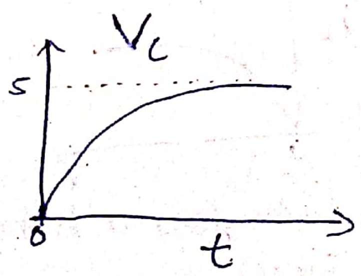
Plate 1 is (+)ve charged . Plate 2 is (−) ve charged. During the transfer of (−)ve charge from plate 1 to plate 2, Plate 1 will attract the (−) charge towards itself and Plate 2 will repel the (−)ve charge away from itself. So work is to ~~be~~ be done against the electrostatic forces. Potential difference (V) is defined as work done in moving a unit charge (+)ve) from one point to another point. So if more charge is to be transferred, more potential difference across the plate is required.

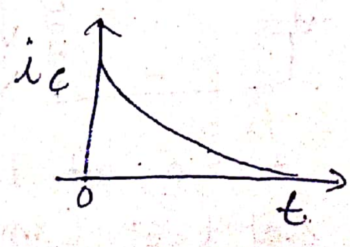
- $Q = CV$

$$\text{Capacitance} = \frac{\text{charge}}{\text{potential difference}}$$

$$\frac{dQ}{dt} = C\frac{dv}{dt} \Rightarrow i_c = C\frac{dv}{dt}$$




Switch S is turned on at time $t = 0$.

- Note that Capacitor Voltage $V_C$ takes finite time (some time) in reaching the steady state value.
- It takes some time to get charged (Charges take some time in jumping (or get transferred) from one plate to another plate).
  In other words, it get stored with energy in some finite time.
- Energy $= \frac{1}{2} CV^2$
- Capacitor does not allow sudden change of voltage across it.

Proof : $i_C = C\frac{dV}{dt} = C\frac{(\text{steady state value of } V)}{\text{zero time}} = \infty$ (Practically Not possible)

For a minute change in voltage across capacitor within zero time ($dt=0$), gives an infinite capacitor current which is practically not at all feasible.

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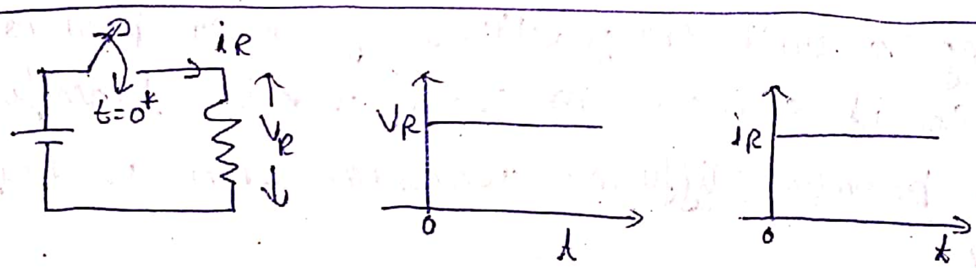
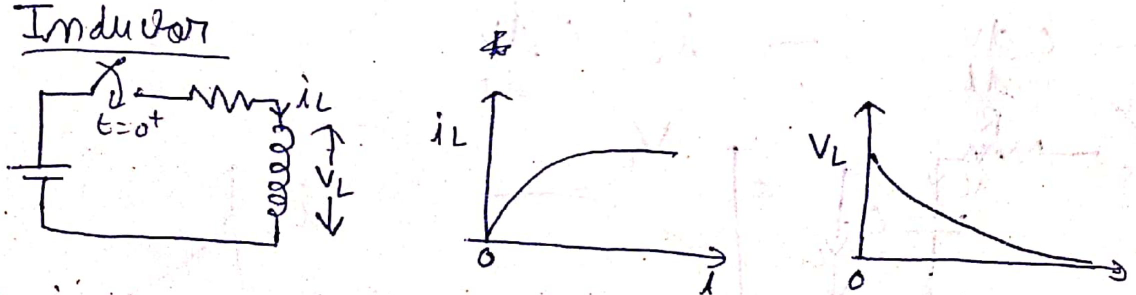
How elements behave in initial state (i.e at $t=0^+$)

1. Resistor




At the time of switching on, at time $t=0^+$, Resistances behave as usual (neither short nor open).

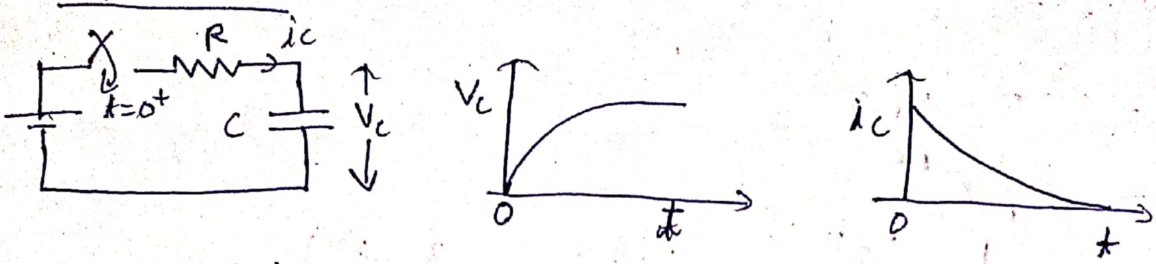
2. Inductor




(i) $i_L = 0$ at $t=0^+$ therefore it behaves as open circuit at $t=0^+$ (initial state)

(ii) $V_L = 0$ at $t=\infty$ (steady state), therefore it act as short circuit under steady state (only in case of DC excitation).

## 3 Capacitor




(i) At $t = 0^+$, $V_c = 0$ therefore it act as short circuit at $t = 0^+$ (initial stage).

(ii) At $t = \infty$ (Steady state), $i_c = 0$, therefore it act as open circuit under steady state (only for DC excitation).

| Element | $t = 0^+$ (Initial) | $t = \infty$ (Steady state) |
|---|---|---|
| 1. R | R | R |
| 2. L | OC | $j\omega L = j2\pi f L$ |
| 3. C | SC | $j\frac{1}{\omega C} = j\frac{1}{2\pi f C}$ |
| 4. L with $\rightarrow I_0$ | $I_0$ | $j\omega L$ |
| 5. C with + $V_0$ | − + V | $j\frac{1}{\omega C}$ |

### Differential equation

- $\frac{d\,y(t)}{dt} + P\,y(t) = 0 \Rightarrow (D + P)\,y(t) = 0$

$y(t) = C.F + P.I$

$P.I = 0 \longrightarrow$ Particular Integral (P.I.).

$C.F \Rightarrow D + P = 0 \Rightarrow D = -P$

$\therefore C.F = C\,e^{-Pt} \longrightarrow$ Complementary Function (C.F)

$y(t) = C\,e^{-Pt}$

- $$\frac{d\,y(t)}{dt} + P\,y(t) = Q$$

$$y(t) = C.F. + P.I.$$

$$C.F = C_1 e^{-Pt}$$

$$P.I = \frac{1}{D+P}[Q] \qquad \text{where } \frac{d}{dt} \equiv D$$

$$P.I = e^{-Pt} \int Q\, e^{Pt}\, dt$$

$$y(t) = C_1 e^{-Pt} + e^{-Pt} \int Q\, e^{Pt}\, dt$$

- $$A\frac{d^2 y(t)}{dt^2} + B\frac{dy(t)}{dt} + C\,y(t) = 0$$

$$(AD^2 + BD + C)\,y(t) = 0 \qquad D \equiv \frac{d}{dt}$$

$$\cancel{D_1, D_2}\; p_1, p_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

(i) $y(t) = C_1 e^{p_1 t} + C_2 e^{p_2 t}$ when $p_1 \neq p_2$

(ii) $y(t) = C_1 e^{p_1 t} + C_2 t\, e^{p_1 t}$ when $p_1 = p_2$

(iii) $y(t) = e^{at}[C_1 \cos Bt + C_2 \sin Bt]$ when $p_1 = a + jB$, $p_2 = a - jB$