

- 4.2 A substance with *fcc* lattice has density  $6250 \text{ kg/m}^3$  and molecular weight 60.2. Calculate the lattice constant  $a$ .

**Hint :**  $a = \left[ \frac{nM}{\rho N} \right]^{1/3} = \left[ \frac{4 \times 60.2}{6250 \times 60.23 \times 10^{26}} \right]^{1/3} = 4 \times 10^{-10} \text{ m} = 4 \text{ \AA}$

- 4.3 Zinc has *hcp* structure. The height of unit cell is  $4.9 \text{ \AA}$ , the nearest neighbour distance is  $2.7 \text{ \AA}$ . Calculate the volume of the unit cell.

**Hint :**  $a = 2r = 2.7 \text{ \AA}$ ,

$$V = \frac{3\sqrt{3}a^2c}{2} = \frac{3\sqrt{3} \times (2.7 \times 10^{-10} \text{ m})^2 \times (4.9 \times 10^{-10} \text{ m})}{2} = 9.4 \times 10^{-29} \text{ m}^3$$

- 4.4 Calculate the lattice constant of NaCl crystal. The density of NaCl crystal is  $2189 \text{ kg/m}^3$  and Avogadro's number  $N$  is  $6.02 \times 10^{26} \text{ kg/molecule}$ .

**Hint :**  $a = \left( \frac{nM}{N\rho} \right)^{1/3}$

$$= \left( \frac{4 \times 58.5}{6.02 \times 10^{26} \times 2189} \right)^{1/3} = (177 \times 10^{-30})^{1/3} = 5.61 \times 10^{-10} \text{ m} = 5.61 \text{ \AA}$$

- 4.5 Obtain lattice constant and radius of the atom having simple lattice and volume density of  $3 \times 10^{22} / \text{cm}^3$  assuming that the atoms are hard sphere with each atom touching its nearest neighbour.

**Hint :**  $a = \left[ \frac{\text{no. of lattice points}}{\text{volume density}} \right]^{1/3}$ , no. of lattice points (*sc*) =  $8 \times \frac{1}{8} = 1$

$$= \left[ \frac{1}{3 \times 10^{22}} \right]^{1/3} = 0.322 \times 10^{-7} \text{ cm} = 3.22 \text{ \AA}$$

- 4.6 An element of atomic weight 60 has density  $6.23 \text{ gm/cc}$ . What is the radius of its atom if it has *fcc* structure?

**Hint :**  $\rho_a = \left[ \frac{nM}{N\rho} \right]^{1/3} = \left[ \frac{4 \times 60}{6.023 \times 10^{23} \times 6.23} \right]^{1/3} = 4.0 \times 10^{-8} \text{ cm}$

Then  $r = \frac{a\sqrt{2}}{4} = 1.414 \text{ \AA}$

- 4.7 Calculate packing factor for chromium metal having *bcc* structure if its density =  $5.96 \text{ g/cc}$  and atomic weight = 50.

[GGSIPU, May 2019 (2.5 marks)]

**Hint :**  $a = \left( \frac{nM}{N\rho} \right)^{1/3} = 5.496 \times 10^{-8} \text{ cm}$ ,

$$r = \frac{a\sqrt{3}}{4} = 2.38 \times 10^{-8} \text{ cm}$$

Packing factor =  $\frac{\text{Volume of atoms in unit cell}}{\text{Volume of unit cell}} = \frac{2 \times \frac{4}{3}\pi r^3}{a^3} = 0.68$

4.8 Sodium is a *bcc* crystal. Its density is  $9.6 \times 10^2 \text{ kg/m}^3$  and atomic weight is 23. Calculate the lattice constant  $a$  for sodium crystal.

Hint : 
$$a = \left( \frac{nM}{N\rho} \right)^{1/3} = \left[ \frac{2 \times 23}{6.023 \times 10^{26} \times 9.6 \times 10^2} \right]^{1/3} = 4.3 \text{ \AA}$$

4.9 The nearest neighbour distance in a silver crystal is  $2.87 \text{ \AA}$ . Silver crystallizes in *fcc* form, determine its density.

Hint : 
$$\rho = \frac{n \times M}{N \times a^3}$$
  

$$= \frac{4 \times 107.68}{6.023 \times 10^{26} \times (4.06 \times 10^{-10})^3} = 1.068 \times 10^4 \text{ kg/m}^3$$

4.10 Sodium chloride has *fcc* structure. Its density is  $2.18 \times 10^3 \text{ kg/m}^3$ . The atomic weights of sodium and chloride are 23 and 35.5, respectively. Calculate the interatomic separation.

Hint : 
$$a = \left( \frac{nM}{N\rho} \right)^{1/3} = 5.63 \text{ \AA}, \text{ the interatomic distance}$$
  

$$r = \frac{a}{2} = 2.81 \text{ \AA.}$$

4.11 Copper has *fcc* structure and its atomic radius is  $1.278 \text{ \AA}$ . Calculate its density. Atomic weight of copper = 63.5, Avogadro's number  $6.023 \times 10^{23}$ .

[GGSIPU, May 2014 (2 marks); May 2018 (2.5 marks)]

Hin : 
$$\rho = \frac{nM}{a^3 N}$$
  

$$= \frac{nM}{\left( \frac{4r}{\sqrt{2}} \right)^3 N} = \frac{4 \times 63.54}{\left( \frac{4 \times 1.278 \times 10^{-8}}{\sqrt{2}} \right)^3 \times (6.02 \times 10^{23})} = 8.98 \text{ gm/cc}$$

4.12 Rubidium (at. mass = 85.5) crystallizes into *bcc* structure. If its density is  $1510 \text{ kg/m}^3$  and radius of the rubidium atom is  $2.48 \text{ \AA}$ , determine Avogadro's number.

Hint : 
$$N = \frac{nM}{\rho a^3},$$
  

$$a = \frac{4r}{\sqrt{3}} = \frac{4 \times 2.48 \times 10^{-10}}{\sqrt{3}} \text{ \AA}$$

$$N = \frac{2 \times 85.5 (\sqrt{3})^3}{1510 \times (4 \times 2.48 \times 10^{-10})^3} = 6.019 \times 10^{26}$$

4.13 Lithium crystallizes in *bcc* structure. Calculate the lattice constant, given that the atomic weight and density for lithium are 6.94 and  $530 \text{ kg/m}^3$  respectively.

Hint. We know  $a^3 = \frac{nM}{\rho N} = \frac{2 \times 6.94}{530 \times 6.023 \times 10^{26}} = 43.50 \times 10^{-30}$

$$\therefore a = 3.517 \times 10^{-10} \text{ m} = 3.517 \text{ \AA}$$

- 4.14 Germanium crystallizes in diamond (form) structure with 8 atoms per unit cell. If lattice constant is 5.62 Å, calculate its density.

[GGSIPU, May 2016 (2.5 marks)]

**Hint:** We know,  $a^3 = \frac{nM}{\rho N}$

or  $\rho = \frac{nM}{Na^3}$

$$= \frac{8 \times 72.59}{(5.62 \times 10^{-10})^3 \times 6.023 \times 10^{26}} = 5434.5 \text{ kg/m}^3 = 5.435 \text{ g/cc}$$

- 4.15 Calculate the density of diamond crystal, given that its lattice parameter 'a' is 3.57 Å and atomic mass A = 12

**Hint:** The effective number of atoms in the diamond cubic unit cell is 8

$$\rho = \frac{Mn}{Na^3}$$

where  $N$  = Avogadro's number

$$\therefore \rho = \frac{8 \times 12}{6.023 \times 10^{26} \times (3.57 \times 10^{-10})^3} = 3540 \text{ kg/m}^3 = 3.54 \text{ g/cc}$$

**Example 5.8** Calculate the smallest glancing angles at which the X-rays of wavelength ( $\lambda = 1.449 \text{ \AA}$ ) will be reflected from a quartz crystal, which has the atomic spacing  $d = 4.255 \text{ \AA}$  [GGSIPU, April 2014 (2 marks)]

**Solution.** Since we know Bragg's relation

$$2d \sin \theta = n\lambda$$

Here  $\lambda = 1.449 \text{ \AA}$  and  $d = 4.255 \text{ \AA}$ .

For first reflection i.e.,  $n = 1$

$$2d \sin \theta_1 = \lambda$$

$$\sin \theta_1 = \frac{\lambda}{2d} = \frac{1.449 \text{ \AA}}{2 \times 4.255 \text{ \AA}} = 0.170$$

$$\theta_1 = \sin^{-1}(0.170) = 9.78^\circ$$

Similarly, for second reflection i.e.,  $n = 2$ .

$$2d \sin \theta_2 = 2\lambda$$

$$\sin \theta_2 = \frac{2 \times 1.449 \text{ \AA}}{2 \times 4.255 \text{ \AA}} = 0.340$$

$$\theta_2 = \sin^{-1}(0.340) = 19.88^\circ.$$

or

Problem 5.1 Find the set of Miller indices for a plane cutting of intercepts  $3a, 2b, 4c$

**Solution.** From the law of rational indices, we may write

$$3a : 2b : 4c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where  $h, k, l$  are the Miller indices.

$$\therefore \frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 3 : 2 : 4 \quad \text{or} \quad h : k : l = \frac{1}{3} : \frac{1}{2} : \frac{1}{4}$$

Converting to smallest whole numbers having the same ratios, we have

$$h : k : l = \frac{4}{12} : \frac{6}{12} : \frac{3}{12} = 4 : 6 : 3$$

Thus, the Miller indices of the planes are 4, 6 and 3 or the plane is (463).

**Problem 5.2** Draw (010), (110) and (222) planes in a cubic crystal.

**Solution.**

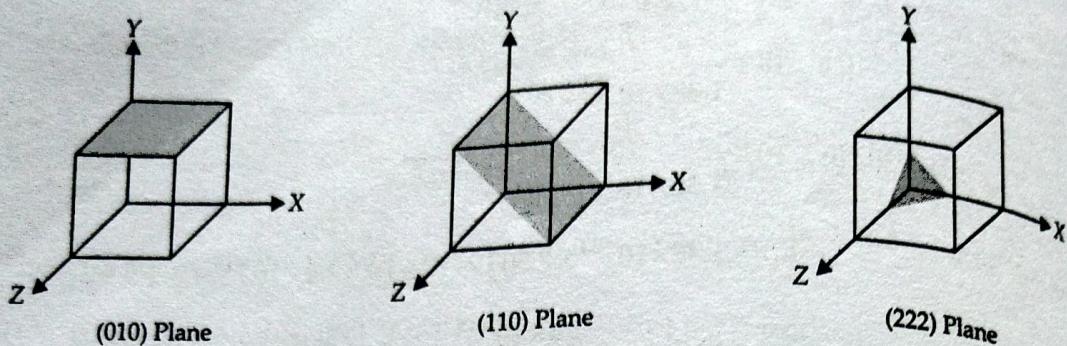


Fig. 5.27

**Problem 5.3** A certain crystal has lattice constant of  $4.24 \text{ \AA}$ ,  $10 \text{ \AA}$  and  $3.66 \text{ \AA}$  on the  $x$ ,  $y$  and  $z$  axes respectively. Determine the Miller indices of the plane of this crystal having  $4.24 \text{ \AA}$ ,  $5 \text{ \AA}$  and  $1.83 \text{ \AA}$  as its  $x$ ,  $y$  and  $z$  intercepts respectively.

[GGSIPU, May 2018 (3 marks)]

**Solution.** Lattice parameters are  $= 4.24 \text{ \AA}$ ,  $10 \text{ \AA}$  and  $3.66 \text{ \AA}$

The intercepts of the given plane  $= 2.12 \text{ \AA}$ ,  $10 \text{ \AA}$  and  $1.83 \text{ \AA}$  i.e., the intercepts are  $0.5$ ,  $1$  and  $0.5$ .

**Step 1.** The intercepts are  $\frac{1}{2}$ ,  $1$  and  $\frac{1}{2}$ .

**Step 2.** The reciprocals are  $2$ ,  $1$  and  $2$ .

**Step 3.** The least common denominator (lcd) is  $2$ .

**Step 4.** Multiplying the lcd by each reciprocal we get,  $4$ ,  $2$  and  $4$ .

**Step 5.** By writing them in parenthesis we get  $(4\ 2\ 4)$ .

Therefore the Miller indices of the given plane is  $(4\ 2\ 4)$  or  $(2\ 1\ 2)$ .

**Problem 5.4** Deduce the Miller indices of a plane which cuts off intercepts in the ratio  $1a : 3b : -2c$  along the three axes.

[GGSIPU, May 2016 (2 Marks)]

**Solution.** From the law of rational indices, we may write

$$1a : 3b : -2c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where  $h, k, l$  are the Miller indices

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : 3 : -2 \quad \text{or} \quad h : k : l = 1 : \frac{1}{3} : -\frac{1}{2} = 6 : 2 : -3$$

Thus,  $h = 6$ ,  $k = 2$ ,  $l = -3$ . Hence, the plane is  $(62\bar{3})$ .

**Problem 5.5** Find the Miller indices of a set of parallel planes which make intercepts in the ratio of  $3a : 4b$  on the  $x$  and  $y$  axes and are parallel to the  $z$ -axis.

**Solution.** The parallel planes are parallel to the  $z$ -axis, that is, their intercepts on the  $z$ -axis are infinite. From the law of rational indices, we may write

$$3a : 4b : \infty c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

$$\frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 3 : 4 : \infty$$

$$h : k : l = \frac{1}{3} : \frac{1}{4} : \frac{1}{\infty} = 4 : 3 : 0$$

or  
or  
The Miller indices are [430].

Problem 5.6 X-rays of wavelength  $2 \times 10^{-11}$  m suffer first order reflection from (111) crystal plane at an angle of  $45^\circ$ . What is interatomic spacing of the crystal?

Solution. We know that  $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

where  $a$  = interatomic distance,  $d$  = interplanar distance,  $h, k, l$  = Miller indices

According to problem  $h = k = l = 1$  (being  $d_{111}$ )

$$\theta = 45^\circ, \lambda = 10^{-11} \text{ m}, n = 1$$

From Bragg's law

$$2d \sin \theta = n\lambda$$

We have

$$2 \frac{a}{\sqrt{3}} \times \sin 45^\circ = 2 \times 10^{-11}$$

$$a = \sqrt{6} \times 10^{-11} = 2.45 \times 10^{-11} \text{ m} = 0.25 \text{ \AA}$$

Problem 5.7 A beam of X-ray  $\lambda = 0.842 \text{ \AA}$  is incident on a crystal at a grazing angle of  $8^\circ 35'$  when the first order Bragg's reflection occurs. Calculate the glancing angle for 3rd order reflection.

Solution. According to Bragg's equation,

$$2d \sin \theta = n\lambda$$

For first order,

$$2d \sin 8^\circ 35' = 1 \times 0.842 \times 10^{-10} \quad \dots(i)$$

and for third order,

$$2d \sin \theta_3 = 3 \times 0.842 \times 10^{-10} \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$\frac{\sin \theta_3}{\sin 8^\circ 35'} = 3$$

$$\sin \theta_3 = 3 \sin 8^\circ 35' = 3 \times 0.15$$

$$\theta_3 = \sin^{-1}(0.45) = 26.5^\circ$$

Problem 5.8 Calculate the glancing angle of the (110) plane of simple cubic crystal ( $a = 2.814 \text{ \AA}$ ) corresponding to second order diffraction maxima for the X-rays of wavelength  $0.710 \text{ \AA}$ .

[IGGSIPU, May 2006 (4.5 Marks); May 2019 (3.5 marks)]

Solution. For  $n^{\text{th}}$  order diffraction maximum for X-rays of wavelength  $\lambda$  from lattice planes of spacing  $d$ , the glancing angle  $\theta$  is given by

$$2d \sin \theta = n\lambda \quad \dots(i)$$

The distance between consecutive lattice planes defined by Miller indices  $(hkl)$  in a cubic lattice

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Here  $a = 2.814 \text{ \AA}$ ,  $h = 1$ ,  $k = 1$ ,  $l = 0$

then

$$d_{110} = \frac{2.814}{\sqrt{1^2 + 1^2 + 0}} = \frac{2.814}{\sqrt{2}} \text{ \AA} = 1.989 \text{ \AA}$$

Also we have  $n = 2$  and  $\lambda = 710 \text{ \AA}$

Substituting these values in Eq. (i)

$$2 \times 1.989 \sin \theta = 2 \times 0.710$$

$$\sin \theta = \frac{0.710}{1.989} = 0.3569$$

$$\theta = \sin^{-1}(0.3569)$$

$$\theta = 20^\circ 55'$$

**Problem 5.9** From a powder camera of diameter 114.6 mm using an X-ray beam of wavelength 1.54  $\text{\AA}$ , the following  $S$  values in mm are obtained for material : 86, 100, 148, 130, 188, 232 and 272. Determine the structure and lattice parameter.

**Solution.** Given, radius of camera =  $\frac{114.6}{2} = 57.3 \text{ mm}$

The Bragg's angles in degrees at which reflections are observed are equal to  $5/4$  and are :  $21.5^\circ$ ,  $25^\circ$ ,  $37^\circ$ ,  $47^\circ$ ,  $58^\circ$  and  $68^\circ$ .

The  $\sin^2 \theta$  values are in the ratio  $0.1346 : 0.1788 : 0.3620 : 0.5003 : 0.5352 : 0.7195 : 0.8596$  within experimental error, these values can be expressed in the ratio of integral numbers

$$3 : 4 : 8 : 11 : 12 : 16 : 19.$$

From the extinction rules, the structure is fcc. The lattice parameter calculated from highest angle is  $3.62 \text{ \AA}$

5.2 X-rays with  $1.54 \text{ \AA}$  are used for calculation of the  $d_{100}$  plane of a cubic crystal. The Bragg's angle of first order reflection is  $10^\circ$ . What is the size of unit cell?

Hint:  $d_{100} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = a \text{ \AA}$

$$2d_{100} \sin \theta = n\lambda \Rightarrow 2a \sin \theta = n\lambda \Rightarrow a = 4.43 \text{ \AA}$$

5.3 For a cubic lattice, calculate the distance of  $(123)$  and  $(234)$  planes from a plane passing through the origin.

Hint:  $d = \frac{a}{(h^2 + k^2 + l^2)^{1/2}}$

$$\Rightarrow d_{123} = \frac{a}{\sqrt{1+4+9}} = \frac{a}{\sqrt{14}} \quad \text{and} \quad d_{234} = \frac{a}{\sqrt{4+9+16}} = \frac{a}{\sqrt{29}}$$

5.4 Deduce the Miller indices for the plane having intercepts  $a, b$  and  $c$  at  $-2, \infty, -2$ . Also draw the plane. [GGSIPU, May 2019 (2.5 marks)]

Hint: Intercept are  $-2, \infty, -2$

The reciprocal are:  $\frac{1}{-2}, \frac{1}{\infty}, \frac{1}{-2}$

LCD  $-2, 0, -2$

Miller indices for the plane is  $(\bar{2}0\bar{2})$

5.5 Deduce the Miller indices of a set of parallel which make intercepts in the ratio of  $a : 2b$  on the  $x$  and  $y$  axes are parallel to  $z$ -axis,  $a, b, c$  being primitive vectors of lattice. Also calculate the interplanar distance  $d$  of the plane taking the lattice to be cube with  $a = b = c = 5 \text{ \AA}$ . [GGSIPU, May 2015 (5 marks)]

Hint:  $a:2b:\infty = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}; h:k:l = 1:\frac{1}{2}:\frac{1}{\infty} = 2:1:0$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{5}{\sqrt{4+1+0}} \text{ \AA} = \sqrt{5} \text{ \AA}$$

5.6 In a simple cubic crystal (i) find the ratio of intercepts of three axes by  $(123)$  plane and (ii) find the ratio of spacing of  $(110)$  and  $(111)$  planes. [GGSIPU, May 2015 (4 marks)]

Hint: Ratio of intercepts:  $\frac{a}{1} : \frac{a}{2} : \frac{a}{3} = 1 : \frac{1}{2} : \frac{1}{3}$

$$d_{110} = \frac{a}{\sqrt{1^2 + 1^2 + 0}} = \frac{a}{\sqrt{2}}; d_{111} = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{a}{\sqrt{3}}$$

$$d_{110}/d_{111} = \frac{\sqrt{3}}{\sqrt{2}} = 1.225$$

5.7 What is the difference between  $(111)$  and  $\langle 111 \rangle$  for Miller indices? [GGSIPU, May 2016 (2.5 marks)]

Hint:  $(111) \rightarrow$  plane and  $\langle 111 \rangle \rightarrow$  direction.

5.8 Sodium crystallizes in bcc structure. If the radius of the sodium atom is  $1.55 \text{ nm}$ , compute the spacing between the  $(111)$  planes. [GGSIPU, May 2018 (2 marks)]

Hint:  $a = \frac{4}{\sqrt{3}}r = \frac{4}{\sqrt{3}} \times (1.55 \times 10^{-9}) \text{ m}$ . The spacing  $d_{111} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

$$d_{111} = \frac{4}{\sqrt{3}} \times \frac{1.55 \times 10^{-9}}{\sqrt{3}} = \frac{6.20}{3} \times 10^{-9} \text{ m} = 3.1 \times 10^{-9} \text{ m.}$$

- 5.9 Find the Miller indices of a plane that makes intercepts 1 on  $a$ -axis, 2 on  $b$ -axis and is parallel to  $c$ -axis.
- Hint:** Intercepts 1, 2,  $\infty$ ; reciprocal  $1, \frac{1}{2}, 0$ ; smallest integer having same ratio 2, 1, 0,

Miller indices (2 1 0).

- 5.10 Obtain the Miller indices of planes have intercepts  $\frac{a}{2}$ ,  $b$  and  $2c$  in a simple cubic cell. Draw Miller indices.

**Hint:**  $\frac{1}{2}, 1, 2$ ; Reciprocal  $2, 1, \frac{1}{2}$ ; LCM 2 and Miller Indices (4 2 1)

- 5.11 Copper has fcc structure and atomic radius is 0.127 nm. Find the ratio of the spacing of (111) and (123) planes. [GGSIPU, May 2017 (2.5 marks)]

**Hint:**  $a = 2\sqrt{2}r = 2\sqrt{2} \times (0.127 \times 10^{-9}) \text{ m} \Rightarrow \frac{d_{111}}{d_{123}} = \sqrt{\frac{(1+4+9)}{(1+1+1)}} = \sqrt{\frac{14}{3}} = 2.16$

- 5.12 If  $a, b$  and  $c$  are the primitive vectors of the unit cell a plane of Miller indices (310) cut the crystal lattice, find the intercepts of the plane along the three axes. [GGSIPU, May 2017 (3 marks)]

**Hint:** LCD = 3; Reciprocal :  $1, \frac{1}{3}, 0$  and Intercepts : 1, 3, 0

- 5.13 Lead has fcc structure and its body diagonal is 0.86 nm. When X-rays of wavelength 0.71 Å undergoes diffraction from (110) plane to produce second order maxima, calculate the glancing angle. [GGSIPU, May 2017 (2.5 marks)]

**Hint:**  $4r = 0.86 \times 10^{-9} \text{ m}, r = 0.215 \times 10^{-9} \text{ m};$

$$d_{110} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{2\sqrt{2} \times 0.215 \times 10^{-9}}{\sqrt{1^2 + 1^2 + 0}} = 0.43 \times 10^{-9} \text{ m}; 2d \sin \theta = n\lambda$$

$$\Rightarrow \sin \theta = \frac{n\lambda}{2d} = \frac{2 \times (0.71 \times 10^{-10})}{0.86 \times 10^{-9}} = 0.65 \Rightarrow \theta = \sin^{-1} \left( \frac{2 \times (0.71 \times 10^{-10})}{0.86 \times 10^{-9}} \right) = 9.49^\circ$$

- 5.14 The first order maxima in Bragg's diffraction patterns by a crystal is observed at  $28^\circ$  when X-rays of wavelength of 0.32 nm are used. Find the distance between atomic planes.

**Hint:**  $n = 1, \theta = 28^\circ, \lambda = 0.32 \text{ nm}$

$$2d \sin \theta = n\lambda \Rightarrow d = \frac{n\lambda}{2 \sin \theta} = \frac{1 \times 0.32 \times 10^{-9}}{2 \times \sin 28^\circ} = 0.34 \times 10^{-9} = 0.34 \text{ \AA}$$

- 5.15 If X-rays of wavelength 0.5 Å are diffracted at an angle of  $5^\circ$  in the first order, what is the spacing between the adjacent planes of the crystal? At what angle will second maximum occur?

**Hint:**  $2d \sin \theta = n\lambda \Rightarrow d = \frac{n\lambda}{2 \sin \theta} = 2.87 \text{ \AA}; 2d \sin \theta' = 2\lambda$

$$\Rightarrow \theta' = \sin^{-1} \frac{\lambda}{d} = 10.03^\circ$$

- 5.16 An electron initially at rest is accelerated through a p.d. of 5000 V. Calculate (i) momentum, (ii) de-Broglie wavelengths (iii) wave number. Also calculate the Bragg's angle for first order reflection from (111) plane which are 0.2 nm apart.

[GGSIPU, May 2014-reappear (6 marks)]

Hint :

(i)  $p^2 = 2 \text{ meV} \Rightarrow p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 5000} = 3.816 \times 10^{-23} \text{ kg m/s}$

(ii)  $\lambda = \frac{h}{p} = \frac{6.623 \times 10^{-34}}{3.186 \times 10^{-23}} = 1.735 \times 10^{-11} = 0.1735 \text{ Å}$

(iii)  $k = \frac{2\pi}{\lambda} = \frac{2 \times 3.14}{0.1735 \times 10^{-10}} = 3.62 \times 10^{11} / \text{m}$

(iv)  $2d \sin \theta = n\lambda \Rightarrow \theta = \sin^{-1} \left( \frac{n\lambda}{2d} \right) = \sin^{-1} \left[ \frac{1 \times 0.01735}{0.2} \right] = 4.9^\circ$

- 5.17 From the following data, calculate the wavelength of neutron beam and its speed. Spacing between successive (100) planes = 3.84 Å ; grazing angle 30° ; order of the Bragg's reflection = 1

Hint :  $2d \sin \theta = n\lambda \Rightarrow \lambda = \frac{2d \sin \theta}{n} = \frac{2 \times 2.84 \times 10^{-10} \sin 30^\circ}{1} = 3.84 \times 10^{-10} \text{ m}$

As per de-Broglie relation,  $\frac{h}{mv} = \lambda \Rightarrow v = \frac{h}{m\lambda} = \frac{6.623 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.84 \times 10^{-10}} = 1.03 \times 10^3 \text{ m/s}$

- 5.18 Calculate the glancing angles on the cube (100) of a rock salt ( $a = 2.814 \text{ Å}$ ) corresponding to 2nd order diffraction maxima for X-rays of wavelength 0.710 Å.

Hint :  $2d \sin \theta = n\lambda \quad \text{and} \quad d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

We have,  $\frac{2a}{\sqrt{h^2 + k^2 + l^2}} \sin \theta = n\lambda$

$$\frac{2 \times 2.814}{\sqrt{1}} \sin \theta = 2 \times 0.710 \Rightarrow \sin \theta = 0.2523 \Rightarrow \theta = 14^\circ 36' 40''$$

- 5.19 For a certain bcc crystal the (110) planes have a separation 1.81 Å. These (110) planes are indicated with X-rays of wavelength 1.54 Å. How many order of Bragg's reflection can be ?

Hint :  $n = \frac{2d \sin \theta}{\lambda} = \frac{2 \times 1.81 \times 10^{-10} \times \sin 90^\circ}{1.54 \times 10^{-10}} = 1.53 \approx 1.$

- 5.20 The powder of BCC structure crystal is studied with X-rays of wavelength 2 Å. The (210) reflection is observed at Bragg's angle 35°. Calculate lattice parameter.

Hint :  $2d \sin \theta = n\lambda \Rightarrow d = \frac{n\lambda}{2 \sin \theta}$

$$a = d \sqrt{h^2 + k^2 + l^2} = \frac{n\lambda}{2 \sin \theta} \sqrt{h^2 + k^2 + l^2} = 3.89 \text{ Å}$$

- 5.21 Calculate the glancing angle of the (111) plane of simple cubic structure (atomic radius 1.404 Å) corresponding to second order diffraction maxima for X-rays of wavelength 1 Å.

[GGSIPU, May 2014 (3.5 marks)]

Hint :  $2d \sin \theta = n\lambda, d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{2.808}{\sqrt{3}} \text{ Å}$

Then  $\theta = \sin^{-1} \left( \frac{n\lambda}{2d} \right) = \sin^{-1} \left( \frac{2 \times 1 \times 10^{-10} \times \sqrt{3}}{2.808 \times 10^{-10}} \right) = \sin^{-1}(1.23)$

Note. Answers is wrong, it will be correct for  $n = 1$