

Antenna And Wave Propagation

UNIT - 1

Introduction of Antenna :-

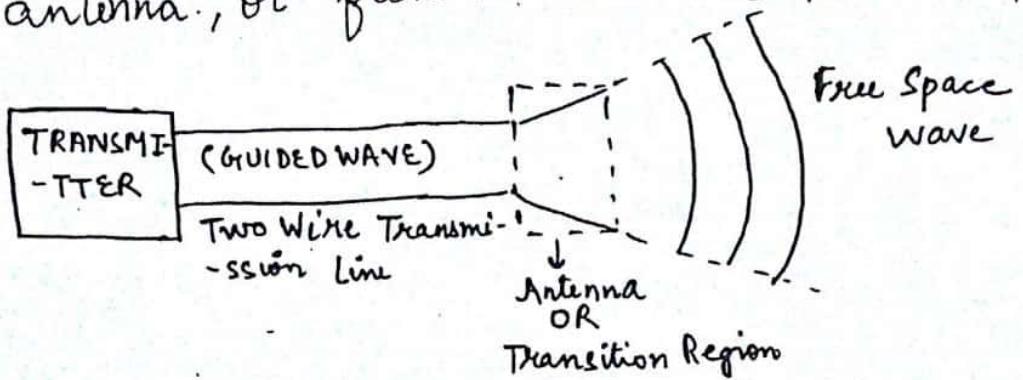
Definitions of Antenna -

Antenna may be defined as:-

- 1) To radiate or receive electromagnetic waves an antenna is required.
- 2. An 'antenna' or ^{aerial} ~~ant~~ is a system of elevated conductors which couples or matches the transmitter or receiver to free space.
- * A transmitting antenna connected to transmitter by a transmission line, forces electromagnetic waves in to free space which travel in space with velocity of light. Similarly,
- * A receiving antenna connected to radio receiver, receives or intercepts a portion of electromagnetic waves travelling through space.
- * Thus the Radio Antenna or Aerial is defined as "a means for radiating or receiving radio waves."
- * The official definition of antenna according to the Institution of Electrical and Electronics Engineers (IEEE)

is simply a means for radiating and receiving transmission waves.

2) Antenna is the transitional structure between free space and a guiding device, as explained in diagram below. The guiding device or transmission line may take the form of a coaxial line or a hollow pipe (waveguide), and it is used to transport electro-magnetic energy from the transmitting source to the antenna, or from the antenna to the receiver.



A transmission line is a device which transfers or guides radio frequency energy from one point to another with minimum attenuation and radiation losses.

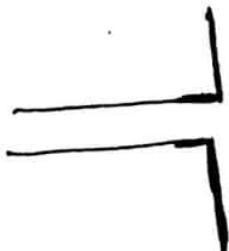
Let us know assume that two wire transmission line is connected with a Transmitter. It is also assumed that transmission line is perfectly matched so that only forward wave (or incident wave) travel in forward direction i.e guided wave, guided by transmission line is travelling along the wire.

Now, if transmission line spreads out gradually till the distance between the two lines is several wavelengths, then, in this region, the wave which is guided by the

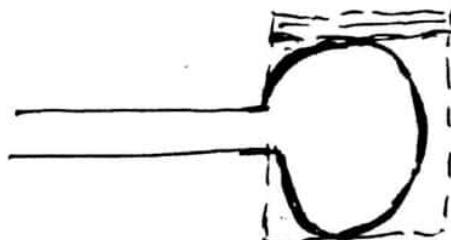
~~Ques.~~ transmission line radiated into free space as free space wave. It is this region of the line b/w guided wave (guided by transmission line) and free space wave, which acts as an antenna.

Types of Antennas

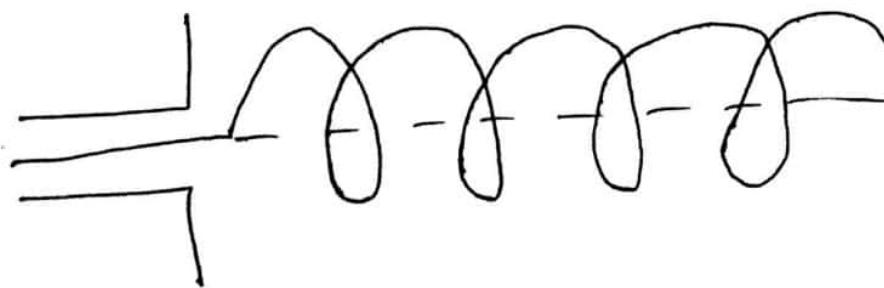
1) Wire Antennas - They are virtually seen everywhere - on automobiles, buildings, ships, aircraft and so on. There are various shapes of wire antennas such as a Straight wire (dipole), loop and helix.



a) Dipole



b) Circular (square loop)



c) Helix.

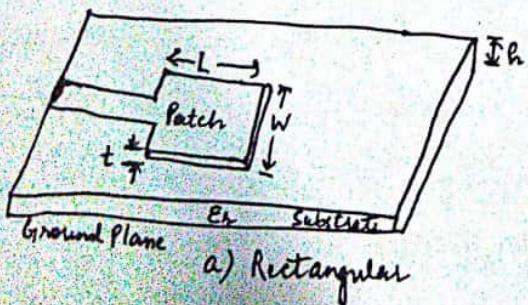
Loop antennas need not be circular, May take form of a rectangle, square, ellipse, or any other. The circular loop is most common because of its simplicity in construction.

2) Aperture Antennas - These antennas are more familiar today than in past because of increasing demand for more sophisticated forms of antennas and the utilization of higher frequencies.

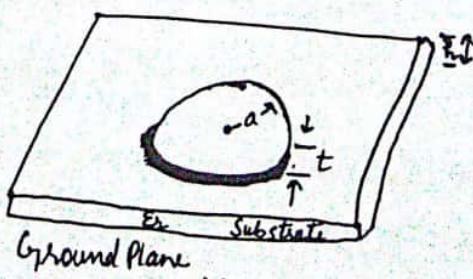
These Antennas are very useful for aircraft and spacecraft applications, because they can be very conveniently mounted on the skin of aircraft or space-craft. They can be covered with dielectric material to protect them from hazardous conditions of the environment.

3) Microstrip Antennas - These antennas became very popular in 1970s for space borne applications. Today they are used for government and commercial applications.

These antennas consist of a metallic patch on a grounded substrate. It can take many different configurations,



a) Rectangular



b) Circular

Rectangular & Circular microstrip (Patch) antennas.

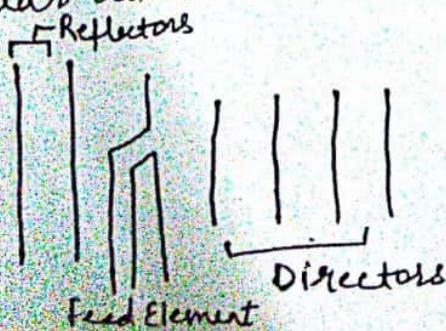
rectangular and circular patches are most popular because of ease of analysis and fabrication, their attractive radiation characteristics.

The microstrip antennas are low profile, comfortable to planar and nonplanar surfaces, simple, inexpensive to fabricate using modern printed-circuits technology, mechanically robust when mounted on rigid surfaces, very versatile in terms of resonant frequency, polarization, pattern and impedance.

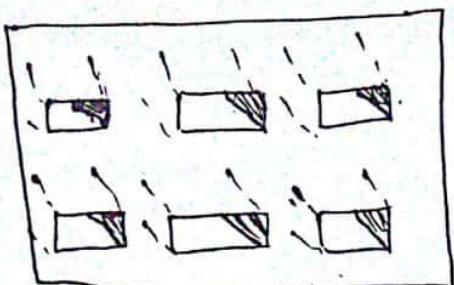
These antennas are mounted on surface of high performance aircraft, spacecraft, satellites; missiles, cars, handheld mobile telephones.

4) Array Antennas - Many applications require radiation characteristics that may not be achievable by a single element but it is however may be possible that an aggregate of radiation elements in an electrical and geometrical arrangement (an array) will result in desired radiation characteristics.

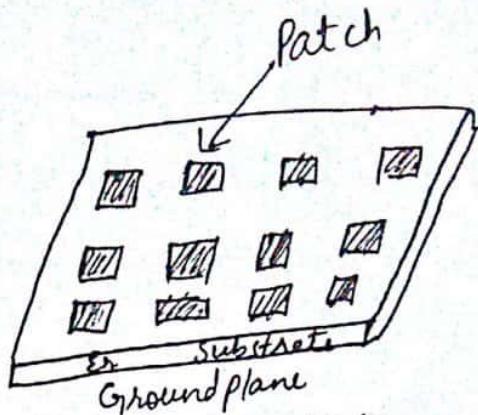
The arrangement of array may be such that radiation from elements adds up to give radiation max. in a particular direction or directions.



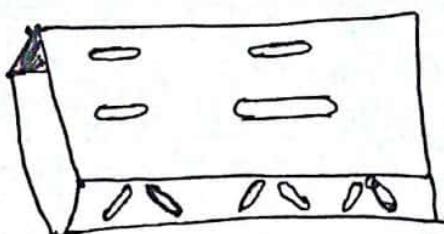
→ a) Yagi-Uda array



b) Aperture Array



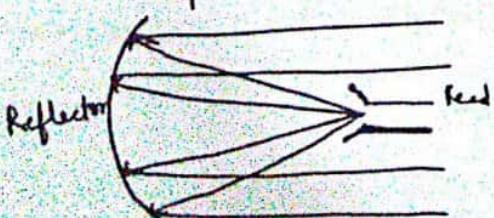
c) Microstrip Antenna
(patch Array)



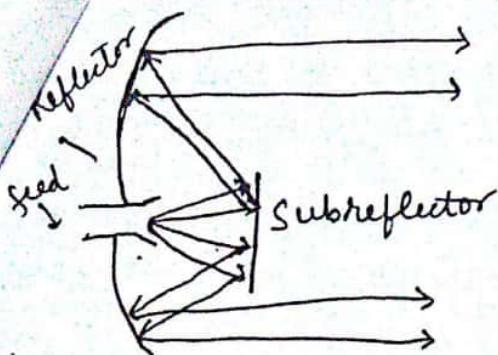
d) slotted - waveguide array

5) Reflector Antennas - Because of the need to communicate over great distances, some forms of antennas had to be used in order to transmit and receive signals that had to travel million of miles. A common antenna form for such application is parabolic reflector. Antennas of this type is built with diameters as large as 305 m.

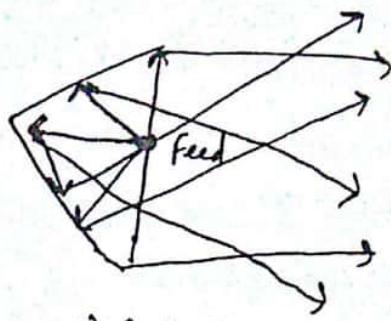
Such large dimensions are needed to achieve high gain required to transmit or receive signals after millions of miles of travel.



a) Parabolic Reflector with front feed.

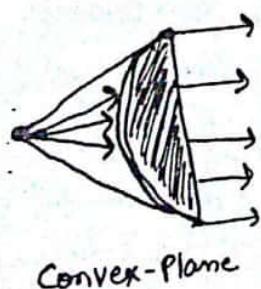


b) Parabolic Reflector
with Cassegrain
Feed

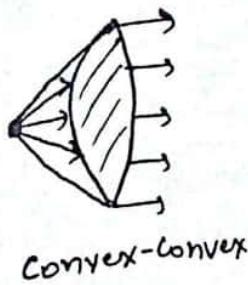


c) Corner
Reflector

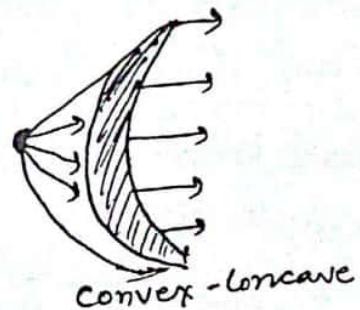
6) Lens Antennas - Lens antennas are classified according to the material from which they are constructed, or according to their geometrical shape.



Convex-Plane

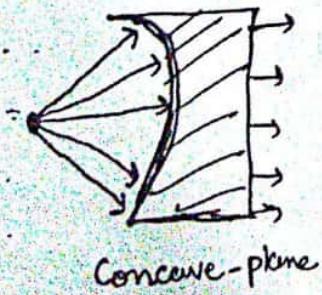


Convex-Convex

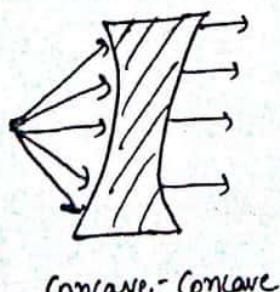


Convex-Concave

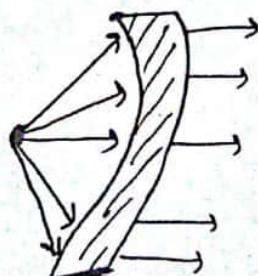
a) Lens antennas with index of refraction $\eta > 1$



Concave-Plane



Concave-Concave



Concave-Convex

b) Lens antennas with index of refraction $\eta < 1$

Structures of Antennas -

- 1) Antenna Sizes - Vary from micro-miniature to giga-watt. Small ones are also known as Brans. Ultra short wavelength antennas are called Millimetre wave antennas.
- Large Antennas are used at short wavelength (high frequencies) to obtain highly directional radiation patterns and high gain in preferred direction.
 - At long wavelengths (low frequencies) very small antenna may be employed for receiving purpose if efficiency is not important.

Antenna of as much as 100λ in any dimensions is considered Electrically Large.

Antenna less than $\lambda/2$ is termed Electrically small.

- 2) Antenna Supports - Antenna should be away from big conductors or absorbing objects.

Antennas are usually supported by devices like masts, towers (are needed when height requirement is more). Masts may also be quite high but they are generally as short as few metres only. Pedestals are base structures of antennas for which height is not important as strength. For Example lens and reflector antennas.

- 3) Sometimes antennas may directly installed on vehicles, ships, aircraft or space craft.

- 3) Antenna Feeders :- are used to connect transmitters and receivers to the antennas. These are connected between antenna G/P terminals and transmitter output (or receiver G/P)

- 4) Antenna Conductors :- Antennas wire of rods are made usually of conducting

Materials like Copper & Aluminium and its alloy are preferred.
Brass also are used for machined parts. Mg is used when ultra light weight is imp. consideration generally with alloy and with protective coating.

When strength becomes primary factor then steel is adequate with or without coating and plating of copper.

The size of antennas depends upon ohmic losses and heating effects.

5) Antenna Insulators - Insulators are employed as spacers supports for two wire open lines and coaxial lines. The materials used for ~~conduct~~ constructing such antenna insulators are - i) glass and ii) ceramics, low loss material such as polystyrene and other plastics are also employed when strength req. is less.

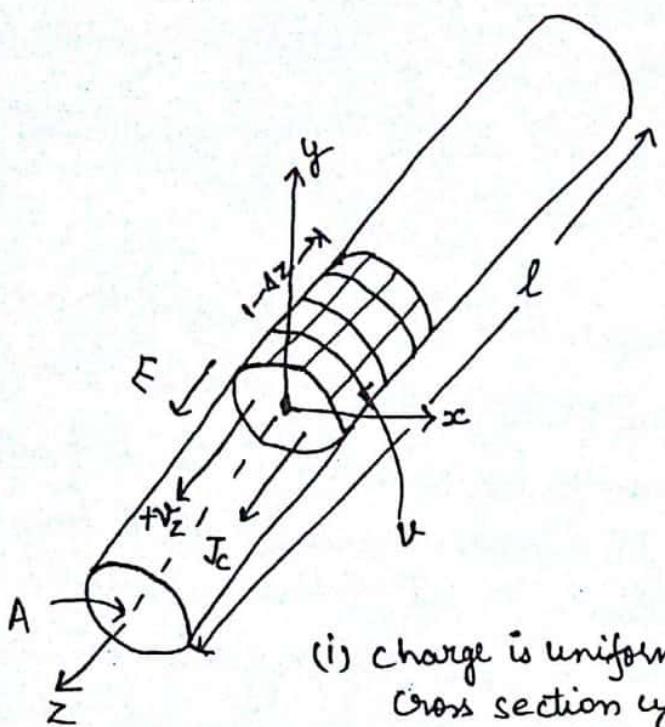
6) Antenna Weather Protection - Antennas are meant usually for 'out doors' and they must be with stand, rain, wind, ice, snow, lightning and corrosive gases etc.

Antennas like rotating paraboloidal reflector or lens are totally enclosed in protective housing of low loss insulating material ie transparent to Electromagnetic radiations. Such housing is known as Radome (Sometimes employed on aircraft antennas)

Radiation Mechanism -

1) Single Wire : - Conducting Wires are material whose characteristic is motion of electric charges and creation of current flow.

Let us assume that an electric volume charge density represented by q_v (coulombs/m³) is distributed uniformly in circular wire of cross sectional area A and volume V.



(i) charge is uniformly distributed in circular cross section cylinder wire.

The total charge Q within volume V is moving in z direction with uniform velocity v_z (m/sec).

Current density J_z (ampere/m²) over cross section of wire is given by $J_z = q_v v_z$ - ①

If the wire is made of ideal electric conductor, current density J_s (ampere/m²) resides on surface of wire and given by $J_s = q_s v_z$ - ②

where q_s (coulombs/m²) is surface charge density. If wire is very thin (ideally zero radius), then the

current in wire is represented by

$$I_z = q_l v_z \quad \text{--- (3)}$$

q_l (coulombs/m) is charge per unit length.

(q_l)

Now we take very thin wire. If the current is time varying, then derivative of current of eq (3) is

$$\frac{dI_z}{dt} = q_l \frac{dv_z}{dt} = q_l a_z \quad \text{--- (4)}$$

Here $dv_z/dt = a_z$ (m/sec²) is the acceleration.

If wire is of length l, then

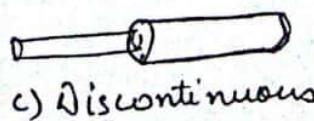
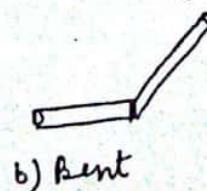
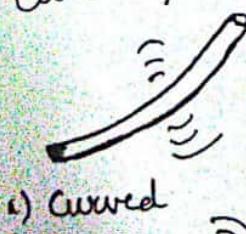
$$l \frac{dI_z}{dt} = l q_l \frac{dv_z}{dt} = l q_l a_z \quad \text{--- (5)}$$

Eq (5) shows basic relation between current and charge and also serve as fundamental relation of Electromagnetic Radiations.

It simply states that to create radiations, there must be a time-varying current or an acceleration (or deceleration) of charge.

To create charge acceleration (or deceleration) wire must be curved, bent, discontinuous, or terminated.

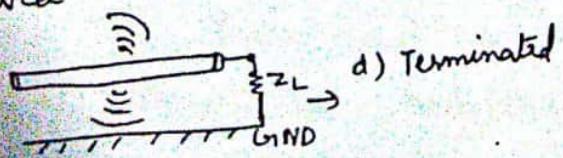
Wire Configurations for Radiation



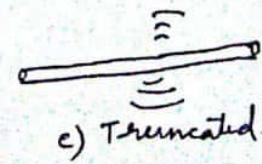
a) Curved

b) Bent

c) Discontinuous



d) Terminated



e) Truncated

fig (a)

- For 1/2 dipole -
- 1) If a charge is not moving, current ^{is zero} & there is no radiation.
 - 2) If charge moving with uniform velocity -
 - i) There is no radiation if wire is straight and infinite
 - (ii) There is radiation if wire is curve, bent, discontinuous, terminated or truncated.
 - 3) If charge is oscillating in time-motion, it radiates even if the wire is straight.

To understand radiation mechanism we consider a pulse source attached to open ended conducting wire, which may be connected to ground through load at its open end. When wire is initially energized, charges (free electrons) in wire are set in motion by electrical lines of force created by source.

When charges are accelerated in source end of wire and decelerated (negative acceleration wrt original motion) during reflection from its end, radiation fields are produced at each end and along remaining part of wire.

"Stronger radiation with more broad freq. Spectrum occurs if pulses are shorter or more compact duration while continuous time-harmonic Oscillating charge produces, ideally radiation of single freq."

~~current~~ determined by freq. of oscillation."

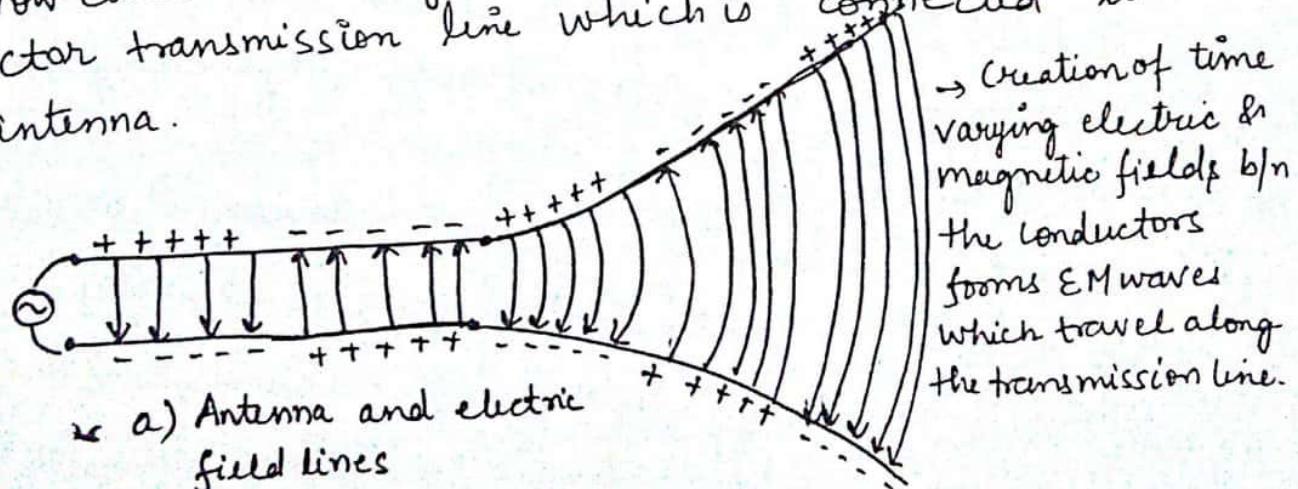
Acceleration of charges is accomplished by external source in which forces are set in motion and produce associated ~~static~~ electric field radiated.

Deceleration of charge at end of wire is accomplished by internal or self forces associated with induced field.

Charge acceleration due to an exciting electric field and deceleration due to impedance discontinuities or smooth curves of wire are mechanism responsible for electromagnetic radiation.

Two-Wires

Now consider a voltage source connected to a two-conductor transmission line which is connected to antenna.



Applying voltage across two-conductor transmission line creates an electric field between the conductors.

The electric field has associated with it electric lines of force which are tangent to electric field at each

point and their strength is proportional to electric intensity.

The electric lines of force have a tendency to act on free electrons associated with each conductor and force them to be displaced. The movement of charges creates a current that in turn creates a magnetic field intensity (Associated with magnetic field intensity are magnetic lines of force which are tangent to magnetic field).

The electric field lines start on positive charges and end on negative charges, can also start on positive charge and end at infinity, start at infinity and end on negative charge, or form closed loops neither starting or ending on any charge.

Magnetic field lines always form closed loops encircling current-carrying conductors because physically there are no magnetic charges.

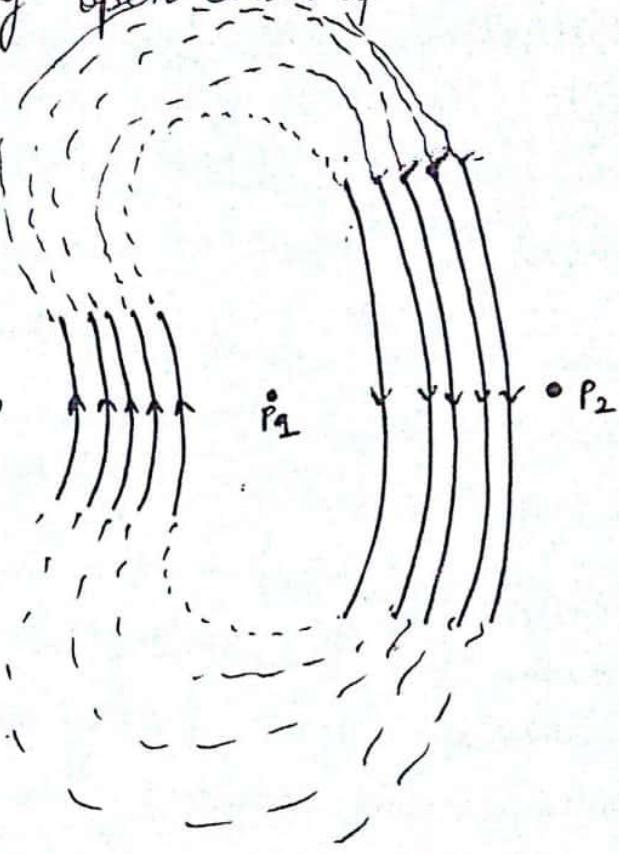
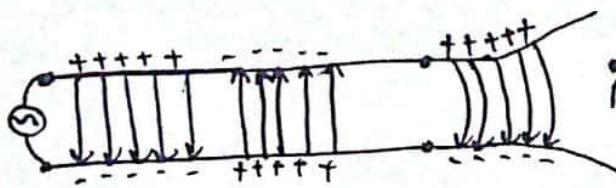
The electric field lines drawn between two conductors help to exhibit distribution of charge.

Assume that voltage source is sinusoidal, we expect electric field between conductors to be sinusoidal with period equal to the applied source.

The relative magnitude of electric field intensity is indicated by density of the lines of force with arrows showing relative direction (positive or negative).

The electromagnetic waves enter the antenna and have associated with them electric charges and corresponding currents.

If we remove part of antenna structure, free space wave can be formed by connecting open ends of electric lines



b) Source, transmission line, antenna and detachment of electric field lines.

Free space waves are also periodic but a constant phase point P₀ moves outwardly with speed of light and travels a distance of $\lambda/2$ (to P₁) in time of onehalf of a period.

→ Close to antenna constant phase point P₀ moves faster than speed of light and approaches speed of light at points far away from antenna.

Now we discuss guided waves are detached from antenna to create free space waves that are indicated as closed loops.

We take example of water waves, these are created by dropping of pebble in calm body of water, waves are created which begin to travel outwardly. If disturbance has been removed, waves do not stop or extinguish themselves but continue their course of travel. If disturbance persists, new waves are continuously created which lag in their travel behind the others.

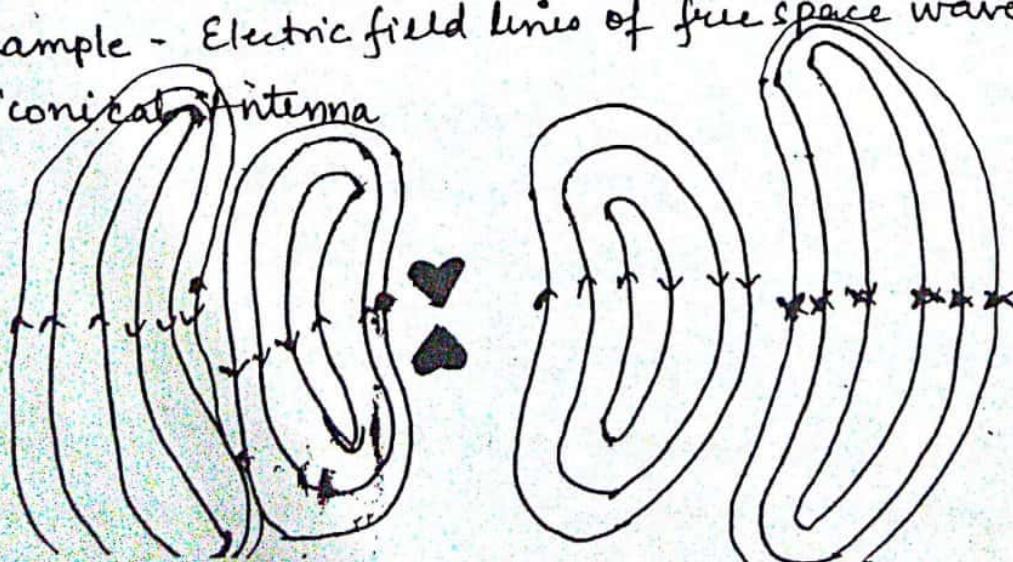
Same is case with EM waves created by electric disturbance.

→ If initially electric disturbance by source is of short duration, the created EM waves travel inside the transmission line, then in to the antenna, and finally are radiated as free space waves, even if electric source has ceased to exist.

→ If electric disturbance is of continuous nature EM waves exist continuously and follow in their travel behind the others.

Example - Electric field lines of free space wave for

Biconical Antenna



Biconical
Antenna

then EM waves within transmission line and antenna
their existence is associated with persistence of charges
inside the conductors.

when waves are radiated, they form closed loops and there
there are no charges to sustain their existence.

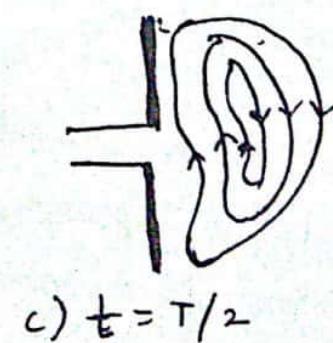
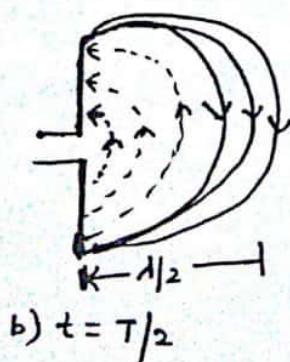
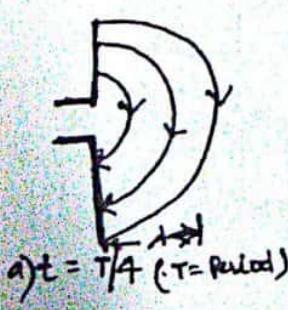
so it's concluded that electric charges are required
to excite the fields but are not needed to sustain
them and may exist in their absence.

Dipole → a pair of equal and oppositely charged or
magnetized poles separated by a distance

→ An aerial or antenna consisting of horizontal metal rod
with a connecting wire at its centre.

We will explain mechanism by which electric lines of
force are detached from antenna to form the free
space waves with help of dipole.

We take ex. of dipole (small antenna) where time of
travel is negligible.



Formation and detachment of electric field lines for short dipole

Above diagram shows the lines of force created between the arms of small centred dipole in first quarter of the period during which time charge has reached its ~~max~~ value (assuming sinusoidal time variation) and lines travel outwardly a radial distance of $1/4$.

for Ex-

Assume that number of lines formed are three.

During next quarter of period, original three lines travel an additional $1/4$ (a total of $1/2$ from the initial point) and charge density on conductors begin to diminish.

This can be accomplished by introducing opposite charges which at the end of first half of period have neutralized the charges on conductors.

The lines of forces created by opposite charge are three and travel a distance $1/4$ during second quarter of first half and they are shown in above diagram.

The end result is that there are three lines of force pointed upward in first $1/4$ distance and same no. of lines of directed downward in second $1/4$.

Since there is no net charge on antenna, lines of force must have been detach themselves

join the conductors and unite together to form closed loop.

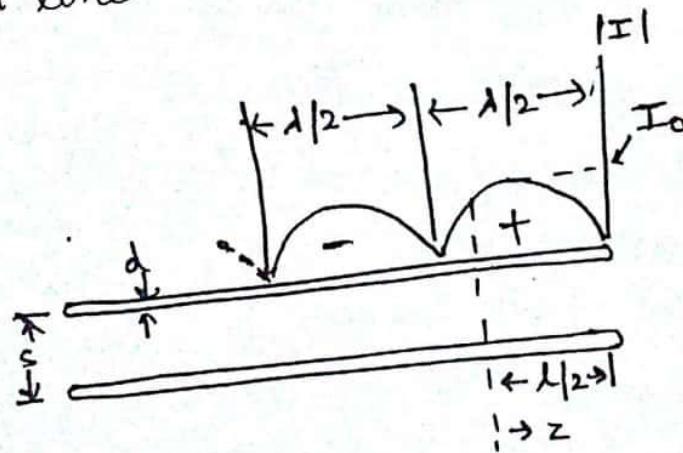
The dig. shown above in (c) we see loop.

In remaining second half of period, same procedure is followed but in opposite direction.

The process is repeated and continues indefinitely and electric field patterns are formed.

Current distribution of thin Wire Antenna

Let's consider geometry of lossless two-wire transmission line



a) Two-wire transmission line

The movement of charges creates a travelling wave current of magnitude $I_0/2$, along each of wires. When current arrives at end of each of wires, it undergoes complete reflection (equal magnitude and 180° phase reversal).

The reflected travelling wave, when combined with incident travelling wave, forms in each wire pattern

of sinusoidal form as shown above in fig.

Current in each wire undergoes 180° phase reversal between adjoining half-cycles.

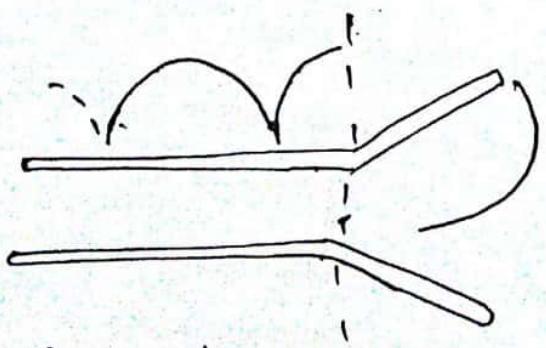
Radiation from each wire individually occurs because of time-varying nature of current and termination of wire.

for two-wire balanced (symmetrical) transmission line, current in a half cycle of one wire is of same magnitude but 180° out of phase from that in corresponding half cycle of other wire.

If in addition spacing between two wires is very small ($s \ll \lambda$), fields radiated by current of each wire are cancelled by those of other.

The net result is almost ideal (desired) non radiating transmission line.

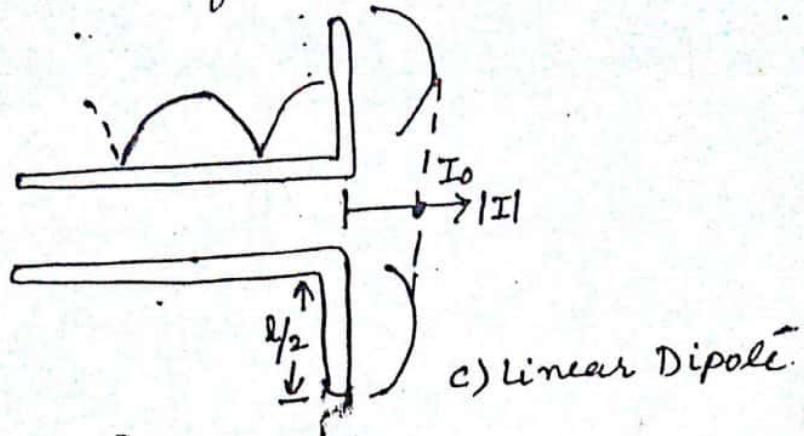
Transmission line between $0 \leq z \leq l/2$ begins to flare.



b) Flared
Transmission line.

Because of two wires of flared section are not necessarily close to each other, field radiated by one do not cancel those of other line can tar

those of other. Ultimately flared section of transmission line can take form of shown in (c)



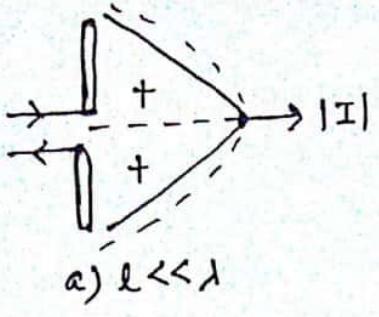
This geometry is widely used dipole antenna also classified standing wave antenna (because of standing wave current pattern).

→ If $l < \lambda$, phase of current standing wave pattern in each arm is same throughout its length.

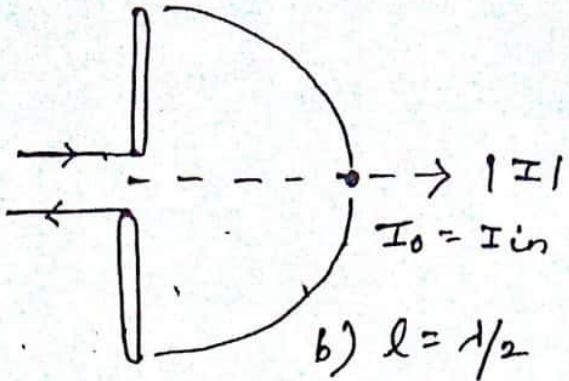
→ If diameter of each wire is very small ($d \ll \lambda$), the ideal standing wave pattern of current along arms of dipole is sinusoidal with a null at the end, its overall form depends on the length of each arm.

→ For center-fed dipoles with $l \ll \lambda$, $l = \lambda/2$, $\lambda/2 < l < \lambda$ and $\lambda < l < 3\lambda/2$

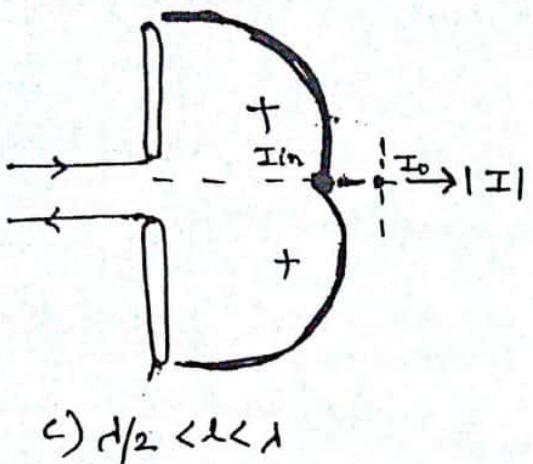
Current patterns are shown in diagram below-



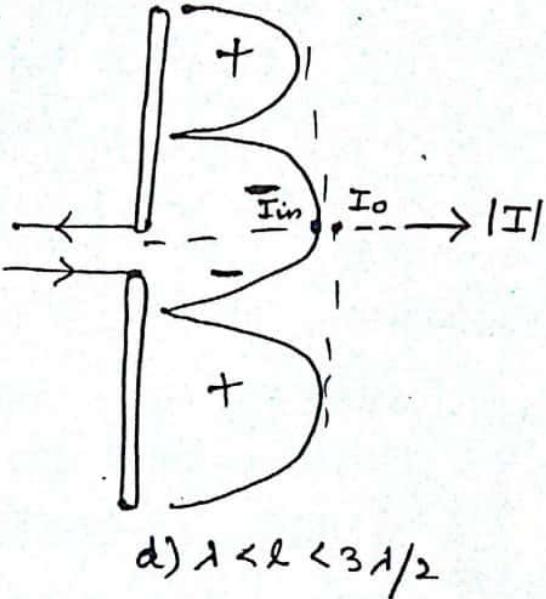
$$a) l \ll \lambda$$



$$b) l = \lambda/2$$



$$c) \lambda/2 < l < \lambda$$



$$d) \lambda < l < 3\lambda/2$$

Current distribution on linear dipoles.

The current pattern of very small dipole ($\lambda/50 < l \leq \lambda/10$) can be approximated by ~~triangular~~ triangular distribution since $\sin(kl/2) \approx kl/2$ when $kl/2$ is very small.

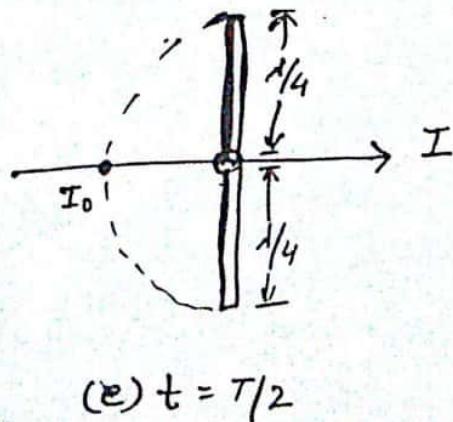
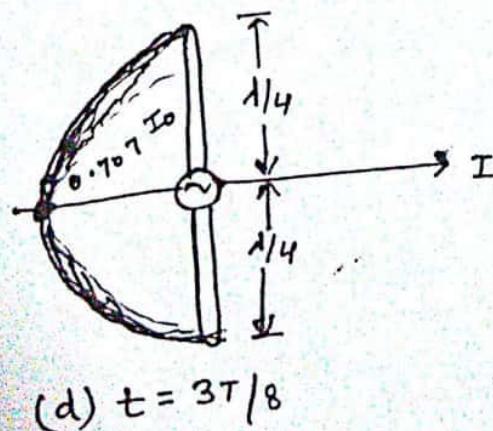
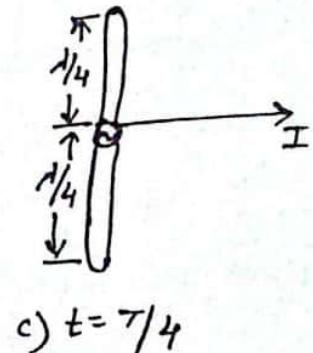
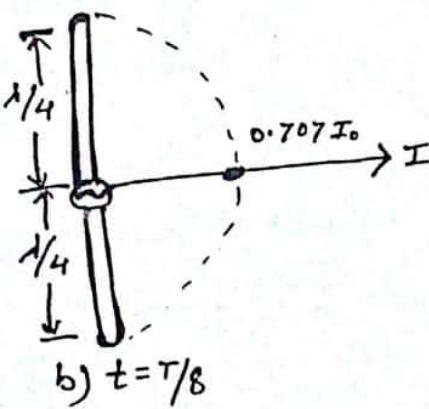
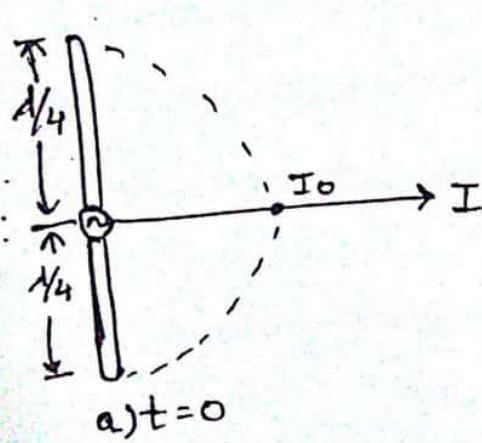
Current standing wave pattern of dipole longer than λ ($l > \lambda$) undergoes 180° phase reversals between adjoining half-cycles.

Therefore current in all parts of dipole does not have same phase, shown in (d) above.

or time harmonic varying system of radian frequency $\omega = 2\pi f$, current standing wave patterns of above fig. represent max. current excitation for any time.

The current variations, as function of time, on $1/2$ center-fed dipole is shown below for $0 \leq t \leq T/2$ where T is period.

These variations can be obtained by multiplying current standing wave pattern shown in (b) above ($l=1/2$) by $\cos(\omega t)$



Current distribution on $1/2$ wire antenna for different time

Standing wave - also called stationary wave,
is a wave in ^{med. in} which each point on axis of wave
has associated constant amplitude.

locations at which amplitude is minimum are
called nodes and max. are antinodes.

~~Radiation /
Isotropic
ray!~~

Radiation Pattern & Isotropic Radiator

Isotropic Radiator is fictitious radiator, a radiator which radiates uniformly in all directions.

- called as Isotropic source or Omnidirectional radiator or unipole.
- Hypothetical lossless antenna by which practical antennas are compared.
- Half-wave dipole also used as reference antenna.

Let us imagine isotropic radiator is situated at centre of sphere of radius (r), Power radiated (Energy) from it, must pass over surface area of sphere $4\pi r^2$.

Pointing vector or power density P at any point on sphere is power radiated per unit area in any direction.

Radiated power from isotropic source flows in Radial lines, for isotropic radiator, magnitude of Pointing vector P is equal to radial component only ($P_\theta = P_\phi = 0$)

$|P| = P_r$, Total power radiated by source is integral over the surface of sphere of radial component P_r

$$W_t = \iint P \cdot dS = \iint P_r \cdot dS = P_r \iint dS \quad [\theta = \phi = 0 \text{ for isotropic}]$$

$$W_t = P_r 4\pi r^2 \quad \therefore \int P = P_r \int dS = 4\pi r^2 - \text{area of sphere}$$

$$P_r = \frac{W_t}{4\pi r^2} \text{ watts/m}^2$$

W_t - Total Power Radiated

dS = infinitesimal element of area of sphere of

$$\text{radius } r = r^2 \sin \theta d\theta d\phi$$

Radiation Pattern

- Energy radiated from an antenna is not of same strength in all directions, it is more in one direction and less or zero in other direction.
 - Energy radiated by antenna in particular direction is measured in terms of FIELD STRENGTH at a point which is at a particular ~~directed~~ distance from antenna.
 - Calculation of field strength, voltage at two points on electric lines of force are taken and divided by distance between two points. Unit is Volt/metre, milli-volt/metre.
 - It determines distribution of radiated energy in space.
 - It is graph which shows variation in actual field strength of electromagnetic field at all points which are at equal distance from antenna.
 - Field strength are measured at every point, treating antenna as centre.
 - Patterns are different for different antenna, affected by location of antenna w.r.t ground.
 - Radiation of antenna as function of direction
 - Expressed in terms of E (Field Strength), pattern is called field strength pattern
 - Radiation in given direction expressed in terms of power per unit solid angle called Power Pattern
 - Power Pattern is proportional to square of field strength
 - Direction of E for radiation field is always tangent to spherical surface of sphere (imaginary sphere of radius r), vertical Dipole E is in dirⁿ θ
 - Horizontal loop E in dirⁿ ϕ
- Five
- $$E = \sqrt{E_0^2 + E_\phi^2}$$
- Total Electric field Amplitude of θ component
Amplitude of ϕ component

Fundamental parameters of Antenna

o Gain :- Gain is closely associated with directivity and directivity itself dependent entirely upon the shape of radiation pattern

Gain of Antenna may be defined by :-

1. The ratio of maximum radiation intensity in given direction to the maximum radiation intensity from a reference antenna produced in the same direction with same power input i.e

$$\text{Gain } (G) = \frac{\text{Max. Radiation Intensity from subject or Test Antenna}}{\text{Max. Radiation Intensity from a reference antenna with same power Input}}$$

Effect of losses are involved in both test or reference antenna.

when reference antenna is taken as isotropic antenna
(100% efficiency)

Gain of subject antenna is denoted by G_0 is known as Gain w.r.t isotropic Antenna.

$$G_0 = \frac{\text{Max. Radiation Intensity from test Antenna}}{\text{Radiation Intensity from Isotropic antenna (lossless) with same power Input}}$$

$$G_0 = \frac{\phi_m}{\phi_0}$$

ϕ_m = Max. Radiation from test antenna

ϕ_0 = Radiation Intensity from lossless Antenna.

Gain equals to Directivity provided antenna efficiency is 100%.

The gain in a direction from which the radiation Intensity ϕ is not max. may be designated by specifying angle ϕ at which it is measured.

$$G_0(\theta, \phi) \cdot \frac{\phi_m}{\phi} = G_0$$

$$G_0(\theta, \phi) = \frac{\phi_m}{\phi} \cdot G_0$$

ϕ_m = Max. Radiation Intensity

ϕ = Radiation Intensity in direction of θ and ϕ .

2. Signal Power Received by a receiver at a distant point in the direction of max. Radiation, Gain is defined as

$$\text{Gain } (G_1) = \frac{\text{Max. Power Received from given antenna } (P_1)}{\text{Max. Power Received from Reference antenna } (P_2)}$$

from the same input power in both cases of subject antenna and reference antenna

$$G_1 = \frac{P_1}{P_2}$$

Signal power is proportional to square of radiation field at the point.

③ Let certain amount of power is supplied to an isotropic antenna and assume that it produces a certain field strength (E) at a given ~~distance~~ distance from the antenna.

Now if this isotropic antenna is replaced by any practical antenna under the same condition, then practical antenna will also radiate same power as was being radiated by isotropic (reference) antenna.

Intensity of Radiation is zero in many directions. will be smaller than field strengths of isotropic antenna at same ~~distance~~ distance.

Suppose E_1 is field strength at a given distance from Practical Antenna, and E_2 is field strength

from an isotropic Antenna -

$$G_d = \frac{E_1}{E_2}$$

Expressed in decibel Ratio ↗

$$\text{db gain} = 10 \log_{10} G_d$$

Grain of
an antenna

directive gain
Doesn't have a
lens

② Directive Gain -

Ratio of radiation Intensity in particular direction to average radiated Power.

Directive Gain is function of angles (θ and ϕ)

$\Phi(\theta, \phi)$ = Radiation Intensity

Φ_{av} = Average Radiation Intensity = $\frac{W_r}{4\pi}$

Directive Gain (G_d) = Radiation Intensity in particular direction

Average Radiated Power

$$G_d(\theta, \phi) = \frac{\Phi(\theta, \phi)}{\Phi_{av}} = \frac{\Phi(\theta, \phi)}{W_r/4\pi}$$

$$= \frac{4\pi \Phi(\theta, \phi)}{W_r}$$

$$G_d(\theta, \phi) = \frac{4\pi \Phi(\theta, \phi)}{\int \Phi d\Omega}$$

$$\text{db } G_d = 10 \log_{10} G_d(\theta, \phi) - (\text{in decibels})$$

Directive Gain depends upon \rightarrow distribution of Radiated Power in space.

Does not depend on power Input to antenna, antenna losses or power consumed in terminating resistance

Directive Gain of an antenna is defined as in particular direction "as ratio of power density in particular direction at given distance, to power density that would be radiated at same distance by an isotropic antenna, radiating same total power.

$$G_{d} = \frac{\text{Power density (PD) radiated in particular dir}^n \text{ by Subject Antenna}}{\text{PD radiated in that particular dir}^n \text{ by Isotropic Antenna}}$$

for same total radiated power and at same distance.

(3) Power Gain - Power Gain compares radiated power density of actual Antenna and that of isotropic antenna on basis of same Input Power to both ie

$$G_p = \frac{\text{PD radiated in particular direction by Subject Antenna}}{\text{PD radiated in that direction by Isotropic Antenna}}$$

$$G_p = \eta G_d$$

η = Efficiency factor lies b/w 0 to 1

$$\eta = 1, G_p = G_d$$

For VHF or UHF etc.

② Power Gain in given direction is also defined as ratio of radiation intensity in that direction to average total input power

$$G_p = \frac{\text{Radiation Intensity in given direction}}{\text{Average total Input Power}}$$

$$G_p = \frac{\Phi(\theta, \phi)}{W_T / 4\pi}$$

$$W_T = W_r + W_i = \text{Total SPP Power}$$

$$W_i = \text{Ohmic loss in antenna}$$

$$G_p = \frac{4\pi \Phi(\theta, \phi)}{W_T}$$

③ In terms of Power Input, Power gain is defined as -

$$G_p = \frac{\text{Power SPP supplied to subject antenna in dir}^n \text{ of max. Radiation}}{\text{Power SPP supplied to reference Antenna}}$$

for same field strength at same given point.

Power gain depends upon 1) Sharpness of Lobe \rightarrow Sharper lobe higher Power gain

2) Volume of solid Radiation Pattern,

$$G_p (\text{dB}) = 10 \log_{10} G_p$$

$$G_p (\text{dB}) = 10 \log_{10} \frac{P_1}{P_2}$$

Directivity (D) - Maximum directive gain is called as directivity of an antenna denoted by D.

In particular dirⁿ Directivity D is a constant.

- ① Defined as Ratio of Max. radiation intensity to its average radiation Intensity ie

$$D = \frac{\text{Max. Radiation Intensity of test Antenna}}{\text{Average Radiation Intensity of test Antenna}}$$

$$D = \frac{\phi(\theta, \phi)_{\max}}{\phi_{\text{av}}} \quad \text{both of test Antenna}$$

- ② D is defined as ratio of max. radiation Intensity of subject antenna to radiation Intensity of an isotropic or reference antenna radiating the same total power

$$D = \frac{\text{Max. Radiation Intensity of subject or test Antenna}}{\text{Radiation Intensity of an isotropic Antenna}}$$

$$D = \frac{\phi(\theta, \phi)_{\max} (\text{test antenna})}{\phi_0 (\text{isotropic Antenna})}$$

- ③ In terms of total Radiated Power,

Ratio of total Radiated power by subject Antenna to power radiated by isotropic antenna for same radiation intensity

For same radiation Intensity -

$$D = \frac{W' \text{ (for test antenna)}}{W'' \text{ (for isotropic antenna)}}$$

Average Radiation Intensity (Φ_{av}) is obtained by dividing total Power radiated W by 4π steradian.

$$D = \frac{\Phi(\theta, \phi)_{max}}{W/4\pi}$$

$$D = \frac{4\pi \Phi(\theta, \phi)_{max}}{W}$$

$$\boxed{D = \frac{4\pi (\text{max. Radiation Intensity})}{\text{Total Radiated Power}}}$$

$$\text{Directivity } D(\theta, \phi) = \frac{\Phi_{max}}{\Phi} = D$$

$$\boxed{D(\theta, \phi) = \frac{\Phi_{max}}{\Phi}}$$

Φ_{max} = max. radiation intensity

Φ = Radiation Intensity in direction of (θ, ϕ)

$$\boxed{D(dB) = 10 \log_{10} D}$$

D always lies b/w 1 and ∞ i.e
 $1 \leq D \leq \infty$

Radiation Intensity is function of θ and ϕ

$$\Phi = \Phi_a f(\theta, \phi) \quad - \textcircled{1}$$

Φ_a = a constant

Φ = Radiation Intensity

Max. Value of Radiation Intensity

$$\Phi_{\max} = \Phi_a f(\theta, \phi)_{\max}. \quad - \textcircled{2}$$

For an isotropic antenna $f(\theta, \phi)_{\max} = 1$

$$\Phi_{\max} = \Phi_a \quad - \textcircled{3}$$

Put Eq \textcircled{3} in \textcircled{1} $\Phi = \Phi_m f(\theta, \phi)$ - \textcircled{4}

Average Radiation Intensity Φ_{av} given by total power radiated divided by 4π steradians.

$$\Phi_{av} = \frac{W_r}{4\pi} = \frac{\iint \Phi d\Omega}{4\pi}$$

$$\Phi_{av} = \frac{\iint \Phi_a f(\theta, \phi) \cdot d\Omega}{4\pi}$$

$$\frac{f(\theta, \phi)}{f(\theta_{\max}, \phi)} = f_n(\theta, \phi)$$

= normalized Power Pattern

$d\Omega = \sin\theta d\theta d\phi$ elements of solid angle.

$$D = \frac{\text{Max. Radiation Intensity } (\Phi_{\max})}{\text{Avg. R.I. } (\Phi_{av})} = \frac{\Phi_a f(\theta, \phi)_{\max}}{\frac{\iint \Phi_a f(\theta, \phi) d\Omega}{4\pi}}$$

$$= \frac{4\pi f(\theta, \phi)_{\max}}{\iint f(\theta, \phi) d\Omega} = \frac{4\pi}{\frac{\iint f(\theta, \phi) d\Omega}{f(\theta, \phi)_{\max}}} = \frac{4\pi}{\iint f_n(\theta, \phi) d\Omega}$$

$$D = \frac{4\pi}{B.A.}$$

$$B.A. = \frac{\iint f(\theta, \phi) d\Omega}{f(\theta, \phi)_{\max}} = \text{Beam Area.}$$

$$D = \frac{4\pi}{\Omega_A}$$

Ω_A = Beam Solid Angle

$$\Omega_A = \frac{\iint f(\theta, \phi) d\Omega}{f(\theta, \phi)_{\max}}$$

\therefore Directivity is nothing but solid angle of a sphere (4π steradian) divided by antenna beam solid angle Ω_A .

$$D = \frac{\Phi_{\max.}}{\Phi_{\text{av}}} = \frac{4\pi}{\Omega_A}$$

$$4\pi \Phi_{\text{av}} = \Phi_{\max} \Omega_A$$

$$4\pi \cdot \frac{W_R}{4\pi} = \Phi_{\max} \Omega_A$$

$$\Phi_{\text{av}} = \frac{W_R}{4\pi}$$

$$W_R = \Omega_A \Phi_{\max}$$

W_R = Total Power Radiated.

Antenna \rightarrow 100% efficient, directivity or gain are same

$$G_0 = K D$$

K: Efficiency factor = 1 for 100% efficiency
 < 1 if losses present.

ext max.
 2nd max.
 antenna

Let max. radiation intensity from test antenna is Φ_{\max} and max. radiation intensity Φ_0^{\max} of 100% efficient antenna.

$$\boxed{\Phi_{\max} = k \Phi_0^{\max}} \quad 0 \leq k \leq 1$$

R.I. \rightarrow Radiation Intensity

Gain w.r.t an isotropic Antenna

$$G_0 = \frac{\Phi_{\max}}{\Phi_0}$$

Φ_0 = R.I. from lossless isotropic Antenna

$$G_0 = \frac{k \Phi_{\max}}{\Phi_0}$$

$$\boxed{G_0 = k D} \quad , \quad \boxed{D = \frac{\Phi_{\max}}{\Phi_0}}$$

Directive Gain and Gain or Power Gain

① Directive Gain G_d is given by

$$G_d = \frac{\Phi(\theta, \phi)}{W_r / 4\pi} = \frac{4\pi \Phi(\theta, \phi)}{W_r}$$

$$G_d = \frac{\text{R.I.}}{\text{Avg. Radiated Power}}$$

$$\text{Power gain or gain} = \frac{\text{R.I.}}{\text{Total G/P Power}} \quad G_p = \frac{\Phi(\theta, \phi)}{W_T / 4\pi}$$

$$G_p = \frac{4\pi \Phi(\theta, \phi)}{W_T} \quad W_T = W_r + W_e$$

Total Power = Radiated Power + Power loss in ohmic resistance

② Directive gain depends on distribution of radiated power in space, gain or power gain depends upon sharpness of lobe and volume of Solid Radiation pattern.

③ Directive Gain is qualitative measure of total Power radiated concentrated in one direction and does not depends on power G/P to antenna and antenna losses like dielectric losses, CV losses and power consumed in terminating resistance etc.

Power gain depends on efficiency as well as its directional properties.

④ Directive Gain and Directivity

$$① G_d = \frac{\phi(\theta, \phi)}{W_r / 4\pi} = \frac{4\pi \phi(\theta, \phi)}{W_r}$$

$$G_d = \frac{\phi(\theta, \phi)}{W_r / 4\pi} -$$

$$\text{Directivity } D = \frac{\text{Max. R.I of test Antenna}}{\text{Avg. R.I of test Antenna}}$$

$$D = \frac{\phi(\theta, \phi)_{\max} (\text{test Antenna})}{\phi_{\text{av}} (\text{test Antenna})}$$

OR

$$D = \frac{\phi(\theta, \phi)_{\max} (\text{test Antenna})}{\phi_0 (\text{isotropic Antenna})}$$

Ans: ② For lossless Isotropic Antenna, directive Gain and directivity is same.

Radiation Efficiency $K=1$

$$G_D = K D$$

$$G_D = D \text{ if } K=1$$

③ Numerical value of directive gain may lie b/n 0 and ∞ , Directivity b/n 1 and ∞ . (not less than 1)

④ Directive gain depends upon distribution of radiated power in space, value of directivity depends upon solid angle of far field pattern.

⑤ Directive gain does not depend on power G/P to antenna and antenna losses and is true for directivity.

⑥ Numerical value of directivity of current element and half-wave dipole is 1.5 (1.76 db) and 1.64 (or 2.15 db)

Directive Gain of half wave dipole over current element = $(2.15 - 1.76) \text{ db} = 0.39 \text{ db}$.

⑤ Antenna Efficiency (η)

Efficiency of an antenna is defined as ratio of Power Radiated to total Input power supplied to an antenna and denoted by η or k .

$$\boxed{\eta = \frac{\text{Power Radiated}}{\text{Total Input Power}}}$$

$$\begin{aligned}\eta &= \frac{W_r}{W_T} = \frac{W_r}{W_r + W_e} = \frac{W_r}{W_T} \times \frac{4\pi \phi(\theta, \phi)}{4\pi \phi(\theta, \phi)} \\ &= \frac{4\pi \phi(\theta, \phi)}{W_T} \cdot \frac{W_r}{4\pi \phi(\theta, \phi)} = G_P \cdot \frac{1}{G_d} = \frac{G_P}{G_d}\end{aligned}$$

$$\boxed{\eta = \frac{G_P}{G_d}} = \frac{W_r}{W_r + W_e}$$

W_r = Power Radiated, W_e = ohmic losses.

If current flowing in Antenna is I , $\eta = \frac{I^2 R_r}{I^2 (R_r + R_e)}$

$$\boxed{\eta \cdot 100 = \frac{R_r}{R_r + R_e} \times 100}$$

R_r = Radiation Resistance,

R_e = ohmic loss of antenna conductor

$R_r + R_e$ = Total Effective Resistance

Loss Resistances consist of following :-

- ① Ohmic loss in antenna conductor
- ② Dielectric loss
- ③ I^2R loss in antenna and ground system
- ④ Loss in earth connections
- ⑤ Leakage loss in insulation

η represents fraction of total energy supplied to antenna which is converted into Electromagnetic Waves

⑥ Antenna BandWidth

- ① Band-Width over which gain is higher than some acceptable value
- ② Band-width over which given $\frac{\text{Front to Back Ratio}}{\text{FBR}}$ is achieved

$$\text{FBR} = \frac{\text{Power Radiated in desired direction}}{\text{Power Radiated in opposite direction}}$$



Antenna Band Width is a width (range) of frequency over which antenna maintains certain required characteristics like gain, FBR or SWR pattern (shape or direction), polarization and impedance.

Basically BW of Antenna depends on Impedance and pattern.

Let two frequency limits (i.e ω_2 and ω_1)

$$\Delta\omega = \omega_2 - \omega_1 = \frac{\omega_r}{Q} = BW$$

$$\boxed{\Delta\omega = \frac{\omega_r}{Q}}$$

$$\omega_r = 2\pi f_r$$

$$\boxed{\Delta f = \frac{f_r}{Q}}$$

$$\Delta\omega = 2\pi\Delta f$$

$$\Delta f \propto \frac{1}{Q} \quad f_r = \text{Centre or resonant or design frequency.}$$

$$Q = \frac{2\pi (\text{Total Energy stored by Antenna})}{(\text{Energy Dissipated or radiated per cycle})}$$

Lower 'Q' of Antenna, higher the BW and vice versa.

Frequency - Independent Antennas like log-periodic antennas, which have unlimited BW where lower and upper frequencies limits are specified independently. In such cases, bandwidth is represented by ratio of highest to lowest operating frequency.

Ex - BW of Broadband antennas like log periodic 20:1 is attained with ease and 100:1 with careful design.

Bandwidth generally of low and moderate values are expressed in terms of Percentage of centre frequency

$$B.W.\% = \frac{\text{Operating Range}}{\text{Centre frequency}} \times 100$$

Antenna operates b/w minimum frequency of 98 MHz to max. frequency of 102 MHz, then $BW = 4 \text{ MHz}$.

$$B.W. = \frac{4 \text{ MHz}}{100 \text{ MHz}} \times 100 = 4\%$$

⑦ Antenna Beam-Width is measure of directivity of an Antenna.

"Antenna Beam-width is angular width in degrees, measured on radiation pattern (major lobe) between points where radiated power has fallen to half of its max. value.

This is called 'beam width' between half power point or half power beam width (HPBW) because power at half power points is just half.

HPBW is also known as 3-dB beam width because

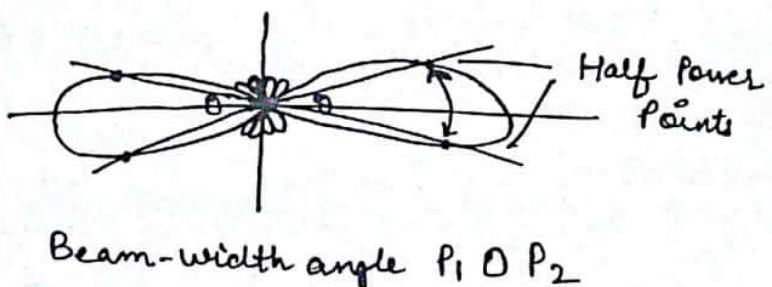
at half power points, power is 3-db down of max.
Power value of major lobe.

radiation pattern

'Half Power Point' the field intensity (ie Voltage) equals $1/\sqrt{2}$ or 0.707 times its max. value or 3-db down from max. value.

Consider radiation pattern in below dig. and let P_1 and P_2 be the half power points, angle $P_1 O P_2$ is beam width of the antenna.

Antenna beamwidth can be defined as "The angular width (in degrees) of major lobe between the two directions at which the radiated or received power is one half the max. power".



Radiation Pattern terms of angular width b/n first nulls or first side lobes, known as beam - width b/n first nulls and is abbreviated as (BW FN) or beam width -10 db down from pattern maximum.

Directivity (D) related with beam solid angle (ΔA) or Beam Area (B)

$$D = \frac{4\pi}{\Delta A} = \frac{4\pi}{B}$$

Radiation pattern or lobe is 3-dimensional, major lobe area approx. may be given by product of beam width in horizontal and vertical planes or E plane and H planes i.e.

$$B \approx (\text{HPBW}) \text{ in Horizontal Plane} \times (\text{HPBW}) \text{ in vertical plane} \dots \text{square Radians.}$$

$$\approx (\text{HPBW}) \text{ in EPlane} \times (\text{HPBW}) \text{ in H Plane.}$$

$$B \approx \theta_E \times \theta_H \dots \text{square radians if } \theta_E \text{ and } \theta_H \text{ in radians}$$

$$D = \frac{4\pi}{\theta_E \theta_H} \quad \boxed{\theta_E, \theta_H \text{ in radians}} \quad \therefore 1 \text{ rad.} = 57.3$$

$$D = \frac{4\pi \times (57.3)^2}{\theta_E \theta_H} \text{ square degrees}$$

$$\boxed{D = \frac{41,257}{\theta_E \times \theta_H}}$$

Factors affecting beamwidth of an antenna are -

- ① Shape of Radiation Pattern
- ② wavelength
- ③ Dimensions (e.g. radius of aperture)

Beamwidth of antenna is given in two planes perpendicular to each other, when beam is linearly polarized, E-plane and H-plane are used, called E-plane beamwidth and H-plane beamwidth.

Antenna Beam Efficiency $\rightarrow (BE)$

BE is parameter that is frequently used to judge quality of Transmitting & Receiving antennas.

BE = Power transmitted (or received) within cone angle θ_1

Power transmitted (or received) by antenna

θ_1 = half angle of cone within which percentage of total power is to be found.

$$BE = \frac{\int_0^{\theta_1} \int_0^{2\pi} \Phi(\theta, \phi) \sin \theta d\theta d\phi}{\int_0^{\pi} \int_0^{2\pi} \Phi(\theta, \phi) \sin \theta d\theta d\phi} \quad - (1)$$

Beam Area (Ω_A), beam efficiency is defined as "the ratio of main beam area (Ω_M) to total beam area (Ω_A).

$$BE \text{ or } (\epsilon_M) = \frac{\Omega_M}{\Omega_A} = \frac{\text{Main beam area}}{\text{Total beam area}}$$

Total beam area = Main beam area + Minor lobe area.

$$\Omega_A = \Omega_M + \Omega_m \quad - (2)$$

Divide Eq (2) by Ω_A

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$1 = \epsilon_M + \epsilon_m \quad - (3)$$

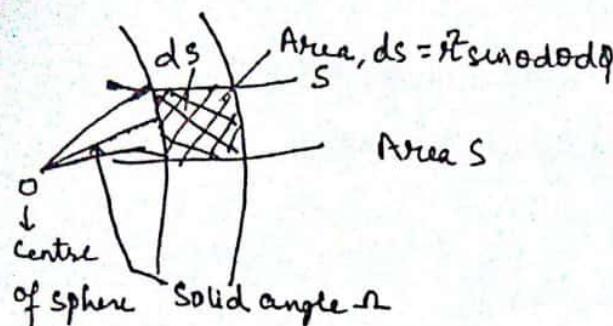
$$\epsilon_m = \frac{\Omega_m}{\Omega_A} = \text{stray factor} = \frac{\text{Minor lobe area}}{\text{Total beam area}}$$

$$\epsilon_M = \frac{\Omega_M}{\Omega_A} = \text{Beam Efficiency}$$

Antenna Beam Area OR Beam Solid Angle Ω

An area ds of surface of sphere seen from centre of sphere subtends solid angle $d\Omega$.

Total solid angle subtended by sphere is 4π steradians (sr)



def. \rightarrow Solid Angle Ω subtended by Area S .

$$ds = (r \sin \theta d\phi) (r d\theta)$$

$$= r^2 \sin \theta d\theta d\phi$$

$$d\Omega = r^2 \cdot d\theta \cdot d\phi \text{ m}^2$$

$d\Omega$ = solid angle subtended by area ds .

$$= d\Omega = \frac{ds}{r^2} \text{ Sr}$$

$$= d\Omega = \frac{4\pi r^2}{r^2} \text{ Sr} , \boxed{d\Omega = 4\pi}$$

$$1 \text{ steradian} = \frac{d\Omega}{4\pi} = \frac{\text{Solid angle of sphere}}{4\pi}$$

$$= 1 (\text{radian})^2 = \left(\frac{180}{\pi}\right)^2 (\text{degrees})^2$$

$$= \frac{180 \times 180}{3.141 \times 3.14} (\text{deg.})^2 = \frac{32,400}{3.141 \times 3.14} (\text{deg.})^2$$

$$= \boxed{1 \text{ Sr} = 3282.7909 (\text{deg.})^2}$$

$$= \pi \text{ radian} = 180^\circ$$

$$= 1 \text{ radian} = \left(\frac{180}{\pi}\right)$$

$$\therefore 4\pi \text{ Steradians} = 3282.7909 \times 4\pi (\text{deg.})^2$$

$$= 41252.861 (\text{deg.})^2$$

$$\boxed{4\pi \text{ Sr} \approx 41253 (\text{deg.})^2 = \text{Solid angle in sphere}}$$

Beam Area (or beam Solid angle) Ω_A for antenna is given by integral of normalized power pattern over a sphere (4π Sr)

$$\Omega_A = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega \text{ Sr}$$

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} P_n(\theta, \phi) \cdot \sin \theta d\theta d\phi \quad S_R.$$

where $P_n(\theta, \phi)$ = Normalised Power Pattern

$$P_n(\theta, \phi) = \frac{P(\theta, \phi)}{P(\theta, \phi)_{\max}}$$

Solid Angle is described in terms of angles subtended by
Half-Power Points of main lobe in principal planes

$$\Omega_A = \Theta_{HP} \Phi_{HP} (S_R)$$

Θ_{HP} = HPBW in E-Plane or θ Plane

Φ_{HP} = HPBW in H-Plane or ϕ Plane

⑨ Antenna Radiation Efficiency

Conduction-dielectric efficiency ϵ_{cd} is defined as ratio of power delivered to radiation resistance R_r to power delivered to R_r and R_L

Radiation Efficiency is

$$\epsilon_{cd} = \left[\frac{R_r}{R_L + R_r} \right] \text{ (dimensionless)}$$

for a metal rod of length l and uniform cross sectional Area A , dc resistance is

$$R_{dc} = \frac{1}{\sigma} \frac{l}{A} \text{ (ohms)}$$

skin depth δ [$\delta = \sqrt{2/(\omega \mu_0 \sigma)}$] of metal is very small compared to smallest diagonal of cross section of rod, current is confined to thin layer near conductor surface.

High-frequency resistance can be written based upon uniform current distribution

$$R_{hf} = \frac{1}{P} R_s = \frac{1}{P} \sqrt{\frac{\omega \mu_0}{2\sigma}} \text{ (ohms)}$$

P → Perimeter of cross section of rod.

($P = C = 2\pi b$ for circular wire of radius b)

R_s is conductor surface resistance

ω is angular frequency

μ_0 is permeability of free space

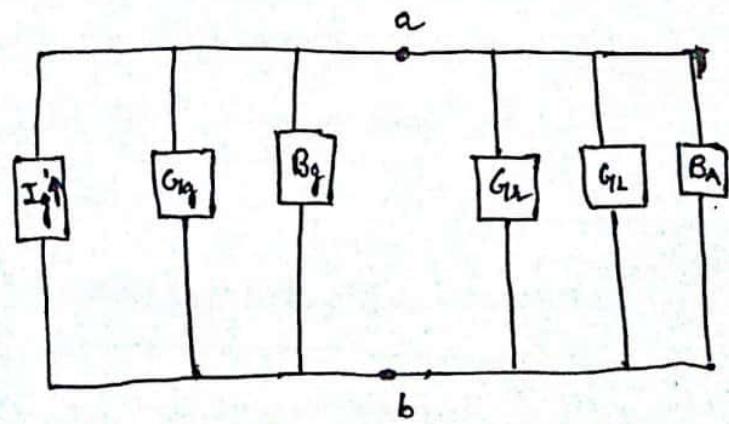
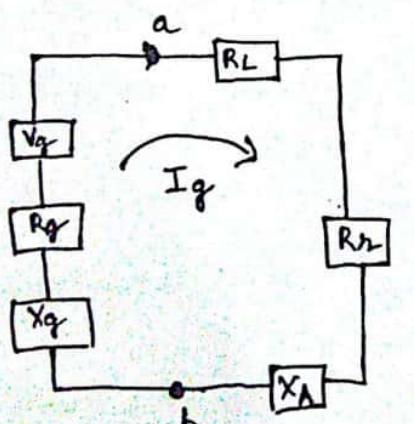
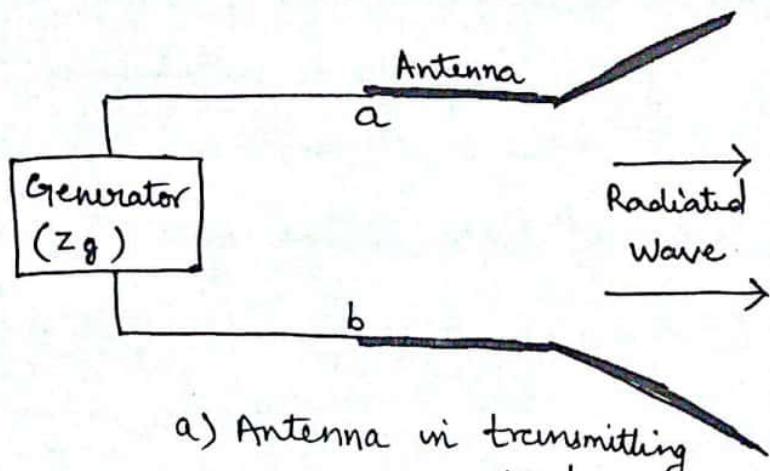
σ is conductivity of metal.

⑩ Input Impedance defined as "the impedance presented by an antenna at its terminals or the ratio of voltage to current at a pair of terminals or the ratio of appropriate components of electric to magnetic fields at a point".

Let 9/P terminals of antenna designated by a-b

The ratio of voltage to current at these terminals, with no load attached, defines impedance of the antenna as

$$Z_A = R_A + j X_A$$



Z_A = Antenna Impedance at terminals a-b (ohms)

R_A = antenna resistance at terminals a-b (ohms)

X_A = antenna reactance at terminals a-b (ohms)

Resistive part consist of two components

$$R_A = R_r + R_L$$

R_r = Radiation resistance of antenna

R_L = Loss resistance of antenna

We assume antenna is attached to generator with internal impedance

$$Z_g = R_g + j X_g$$

R_g = Resistance of generator impedance (ohms)

X_g = Reactance of generator impedance (ohms)

Amount of Power delivered to R_r for radiation and amount dissipated to R_L as heat ($I^2 R_L / 2$).
We find current developed within loop which is given by

$$\text{by } I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_g + Z_t} = \frac{V_g}{(R_r + R_L + R_g) + j(X_A + X_g)} \quad (\text{A})$$

and its magnitude by

$$|I_g| = \frac{|V_g|}{\sqrt{(R_r + R_L + R_g)^2 + (X_A + X_g)^2}} \quad (\text{B})$$

$V_g \rightarrow$ peak generator voltage.

Power delivered to antenna for radiation is given by

$$P_r = \frac{1}{2} |I_g|^2 R_r = \frac{|V_g|^2}{2} \left[\frac{R_r}{(R_r + R_L + R_g)^2 + (X_A + X_g)^2} \right] \quad (\text{C})$$

and that dissipated as heat by

$$P_L = \frac{1}{2} |I_g|^2 R_L = \frac{|V_g|^2}{2} \left[\frac{R_L}{(R_r + R_L + R_g)^2 + (x_A + x_g)^2} \right] \text{(w)}$$

Remaining Power dissipated as heat on internal resistance

R_g of generator is given by

$$P_g = \frac{|V_g|^2}{2} \left[\frac{R_g}{(R_r + R_L + R_g)^2 + (x_A + x_g)^2} \right] \text{(w)}$$

Max. Power delivered to antenna occurs when we have conjugate matching,

$$R_r + R_L = R_g$$

$$x_A = -x_g$$

For this case

$$P_r = \frac{|V_g|^2}{2} \left[\frac{R_r}{4(R_r + R_L)^2} \right] = \frac{|V_g|^2}{8} \left[\frac{R_r}{(R_r + R_L)^2} \right] \quad \text{--- (1)}$$

$$P_L = \frac{|V_g|^2}{8} \left(\frac{R_L}{(R_r + R_L)^2} \right). \quad \text{--- (2)}$$

$$P_g = \frac{|V_g|^2}{8} \left[\frac{R_g}{(R_r + R_L)^2} \right] = \frac{|V_g|^2}{8} \left[\frac{1}{R_r + R_L} \right] = \frac{|V_g|^2}{8 R_g} \quad \text{--- (3)}$$

from (1), (2), (3)

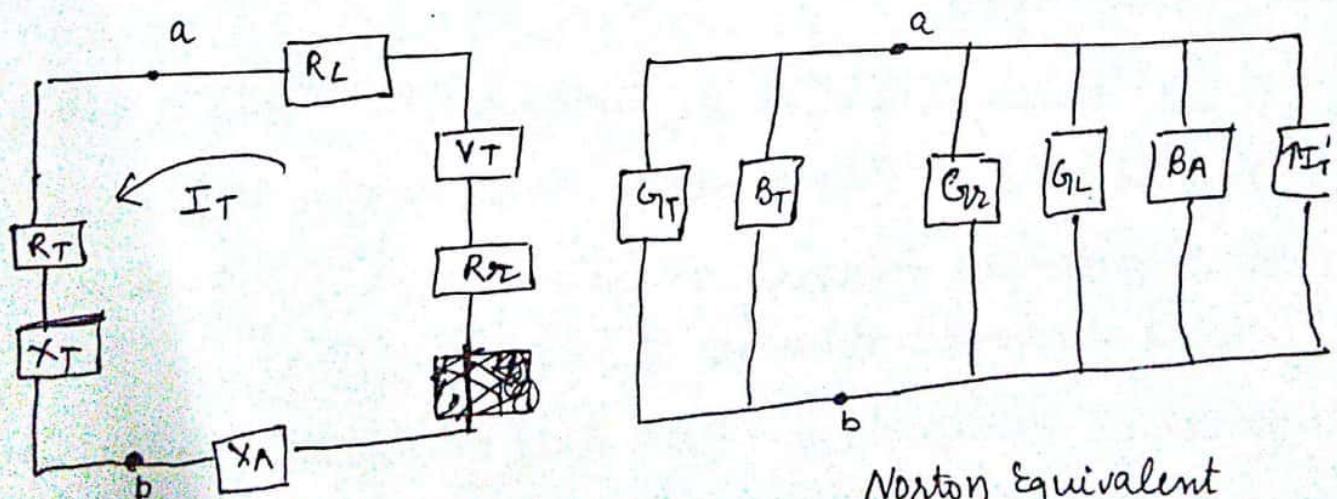
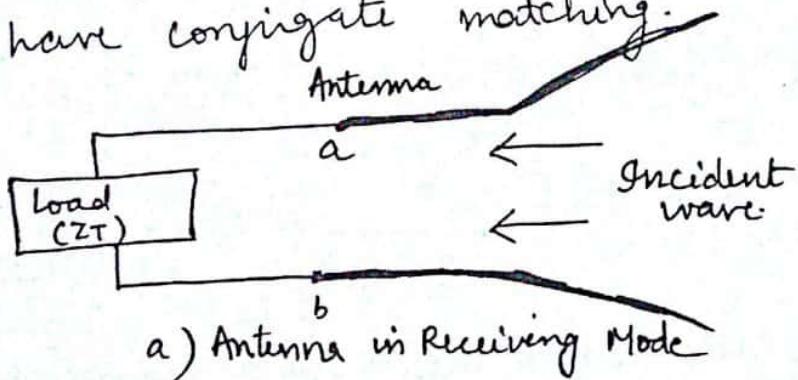
$$P_g = P_r + P_L = \frac{|V_g|^2}{8} \left[\frac{R_g}{(R_r + R_L)^2} \right] = \frac{|V_g|^2}{8} \left[\frac{R_r + R_L}{(R_r + R_L)^2} \right]$$

Power supplied by generator during conjugate matching

$$P_S = \frac{1}{2} V_g I_g^* = \frac{1}{2} V_g \left[\frac{V_g^*}{2(R_s + R_L)} \right] = \frac{1}{4} V_g^2 \left[\frac{1}{R_s + R_L} \right]$$

(W)

Power that is provided by generator, half is dissipated as heat in internal resistance (R_g) of generator and other half is delivered to antenna, this happens when we have conjugate matching.



Thevenin Equivalent

In receiving mode under ~~under~~ conjugate matching

$$R_R + R_L = R_T \text{ and } X_A = -X_T$$

Power Delivered to R_T , R_r and R_L is

$$P_T = \frac{|V_T|^2}{8} \left[\frac{R_T}{(R_r + R_L)^2} \right] = \frac{|V_T|^2}{8} \left(\frac{1}{R_r + R_L} \right) = \frac{|V_T|^2}{8 R_T} - (4)$$

$$P_r = \frac{|V_T|^2}{2} \left[\frac{R_r}{4(R_r + R_L)^2} \right] = \frac{|V_T|^2}{8} \left[\frac{R_r}{(R_r + R_L)^2} \right] - (5)$$

$$P_L = \frac{|V_T|^2}{8} \left[\frac{R_L}{(R_r + R_L)^2} \right] - (6)$$

while induced (collected or captured) is

$$P_c = \frac{1}{2} V_T I_T^* = \frac{1}{2} V_T \left[\frac{V_T^*}{2(R_r + R_L)} \right] = \frac{|V_T|^2}{4} \left(\frac{1}{R_r + R_L} \right)$$

Eq (5) Power P_r delivered to R_r is scattered or re-radiated Power.

Total Power Collected or Captured (P_c) half is delivered to load R_T (Eq (4)) and half is scattered or re-radiated through R_r (Eq (5)) and dissipated as heat through R_L (Eq (6)).

If losses are zero ($R_L = 0$), half of captured power is delivered to load and other half is scattered.

(in order to deliver half the power to load you must scatter half)

Aperture Efficiency - (ϵ_{ap}) \rightarrow ratio of max. effective area to physical area.

Effective Area or Effective Aperture or Capture Area

Transmitting antenna transmits EM waves and receiving antenna receives a fraction of same.

So effective area or aperture over which antenna extracts EM energy from travelling electromagnetic waves.

Defined as ratio of Power received at antenna load terminal to Poynting vector (or power density) in watts/m² of incident wave.

$$\text{Effective area or Effective aperture or } = \frac{\text{Power Received}}{\text{Poynting vector of incident wave}}$$

$$A_e = \frac{W}{P} \equiv A$$

$$W = PA$$

W = Power received, in watts.

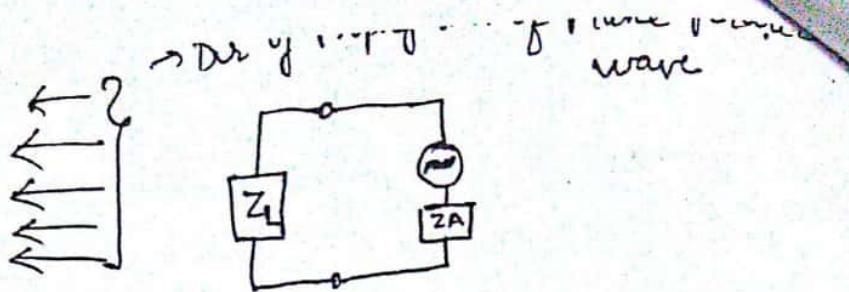
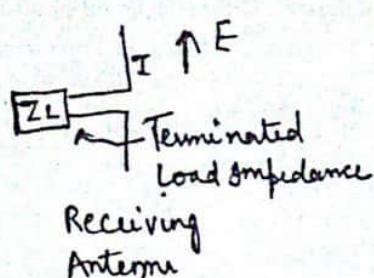
P = Poynting vector of incident Plane wave.

A = Effective or capture area, m².

Let receiving antenna be placed in field of plane polarised travelling wave having effective area

A and receiving antenna (dipole) is terminated

at load impedance $Z_L = R_L + jX_L$



a) Receiving antenna in field of Plane polarised wave

If I be terminal current, received Power

$$W = I^2 \text{rms} R_L \quad - \quad (1)$$

R_L = Load Resistance (Ω)

I_{rms} = Terminal rms current

$$A = \frac{W}{P} = \frac{I^2 \text{rms} R_L}{P}$$

Antenna Extracts energy from incident EM waves, delivers the same to terminated load impedance Z_L and power flowing per square metre or Poynting vector P W/m^2 .

Acc. to Thvenin's Theorem.

V = Equivalent Thvenin's Voltage

Z_A = Equivalent Thvenin's Impedance

Voltage V is induced by passing EM waves which produces current I_{rms} through terminal load impedance Z_L

$$I_{\text{rms}} = \frac{\text{Equivalent Voltage}}{\text{Equivalent Impedance}} \Rightarrow I_{\text{rms}} = \frac{V}{Z_L + Z_A} \text{ Amp.} \quad - (2)$$

$Z_A = R_A + jX_A$ = Complex Antenna Impedance

$R_A = R_r + R_L = R_r$ if $R_L = 0$ is assumed

= Radiation Resistance + loss resistance

$$I_{\text{rms}} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)} \Rightarrow (3)$$

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L+R_A)^2 + (X_L+X_A)^2}} - \textcircled{3}$$

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L+R_r+R_e)^2 + (X_L+X_A)^2}} - \textcircled{4}$$

X_L = load reactance (-z)

X_A = Antenna reactance (-z)

Power Received by terminal load impedance is given by
eq①, Put Eq ③ in ④ ①

$$W = I_{rms}^2 \cdot R_L$$

$$W = \frac{V^2 R_L}{(R_L+R_A)^2 + (X_L+X_A)^2} - \textcircled{5}$$

Power delivered by antenna at terminating load impedance Z_L

$$\text{Effective Aperture}(A_e) = \frac{\text{Power received } (W) \text{ m}^2 \text{ or } \lambda^2}{\text{Poynting Vector } (P)}$$

from Eq ⑤

$$A_e = \frac{V^2 R_L}{[(R_L+R_A)^2 + (X_L+X_A)^2] P} \text{ m}^2 \text{ or } \lambda^2$$

$$A_e = \boxed{\frac{V^2 R_L}{[(R_L+R_A)^2 + (X_L+X_A)^2] P} \lambda^2} - \textcircled{6}$$

$$A_e = \frac{V^2 R_L}{[(R_L+R_r+R_e)^2 + (X_L+X_A)^2] P} \lambda^2 - \textcircled{7}$$

The induced Voltage is max. when antenna is oriented for max. response and antenna, incident wave both have same polarization.

Acc. to Maximum Power transfer theorem, Max. Power will be transferred from antenna to antenna terminating load.

$$X_L = -X_A \quad - \textcircled{4}$$

$$R_L = R_A = R_r + R_e, \quad - \textcircled{5}$$

$$R_L = R_r \text{ if } R_L = 0$$

Max. Power Received in Antenna terminating load impedance Z_L can be obtained if Eq \textcircled{4} substituted in Eq \textcircled{5}

$$W_{\max} = \frac{V^2 R_L}{4 R_L^2} = \frac{V^2}{4 R_L} = \frac{V^2}{4 R_r}$$

$$W_{\max} = \frac{V^2}{4 R_r} \quad \boxed{\therefore R_L = R_r}$$

This is max. power received in antenna terminating load impedance Z_L under condition of max. power transfer and without antenna loss and effective aperture is known as max. effective Aperture.

$$\therefore (A_e)_{\max} = \frac{\text{Max. Recd. Power}}{\text{Power density of incident wave}} = \frac{V^2}{4 R_r P}$$

$$(A_e)_{\max} = \frac{V^2}{4 P R_r} \text{ } \text{J}^2 \text{ or m}^2$$

Ratio of effective area and max. effective area is effectiveness ratio, denoted by α

$$\alpha = \frac{A_e}{(A_e)_{\max}}$$

$$\alpha = \text{lies b/w 0 and 1}$$

Relation b/w Max. Aperture and Gain vs Directivity

Directivity of Receiving antennas are directly proportional to max. effective Aperture.

Let there be two antennas A and B whose directivities and max. effective apertures are denoted by D_a, D_b and $(A_{ea})_{max}$ and $(A_{eb})_{max}$ respectively.

$$D_a \propto (A_{ea})_{max} \quad \frac{D_a}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}} - ①$$

$$D_b \propto (A_{eb})_{max}$$

Gain and directivity wrt isotropic source or antenna is $G_D = K D$, $G_D = \text{Gain of Transmitting or Receiving antenna}$

$K = \text{Efficiency factor}$

$D = \text{Directivity}$

If losses of Efficiency factor K and mismatch are included, K can be replaced by effectiveness ratio α .

$$\boxed{G_D = \alpha D} - ②$$

Assume G_{Da}, α_a, D_a is gain, effectiveness ratio & directivity of antenna A and G_{Db}, α_b, D_b are corresponding quantity of Antenna B.

$$G_{Da} = \alpha_a D_a \quad \text{for Ant. A}$$

$$G_{Db} = \alpha_b D_b \quad \text{for .. B}$$

$$\frac{G_{Da}}{G_{Db}} = \frac{\alpha_a D_a}{\alpha_b D_b} = \frac{\alpha_a (A_{ea})_{max}}{\alpha_b (A_{eb})_{max}} - ③$$

$$\alpha_a = \frac{A_{ea}}{(A_{ea})_{max}}$$

or

$$A_{ea} = \alpha_a (A_{ea})_{max} - ④$$

$$A_{eb} = \alpha_b (A_{eb})_{max} - ⑤$$

from eq (3)

$$\frac{G_{10a}}{G_{10b}} = \frac{A_{ea}}{A_{eb}}$$

A_{ea} ; A_{eb} \rightarrow effective aperture of antenna A & B.

Let A is isotropic antenna with $D_a = 1$

$$\frac{D_a}{D_b} = \frac{1}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}$$

$$(A_{ea})_{max} = \frac{(A_{eb})_{max}}{D_b} \quad - (6)$$

$$D_b = \frac{(A_{eb})_{max}}{(A_{ea})_{max}} \quad - (7)$$

Directivity of antenna is ratio of max. effective aperture to max. effective aperture of an isotropic antenna.

Let $(A_{ea})_{max} = \frac{\frac{3\lambda^2}{8\pi}}{\frac{3/2}{\lambda^2}} = \frac{\lambda^2}{4\pi}$

$$D_b = \frac{4\pi}{\lambda^2} (A_{eb})_{max}$$

$$D = \frac{4\pi}{\lambda^2} (A_e)_{max}$$

$$D = \frac{4\pi}{\lambda^2} A_{em}$$

← Relation b/n
Directivity or max.
effective aperture
of an antenna

effective length

Effective length of an antenna represents effectiveness of an antenna as radiator or collector of EM energy.

Effective length indicates how an antenna is effective in transmitting or receiving EM wave energy

For Receiving Antenna, effective length is ratio of induced voltage at terminal of receiving antenna under open circuited condition to incident electric field intensity (or strength) E

$$\text{Effective length} = \frac{\text{open circuited Voltage}}{\text{Incident field strength (electric)}}$$

$$l_e = \frac{V}{E} \text{ m or wavelength}$$

$$A_e = \frac{V^2 R_L}{[(R_A + R_L)^2 + (X_A + X_L)^2] P}$$

$$V^2 = A_e \frac{[(R_A + R_L)^2 + (X_A + X_L)^2] P}{R_L}$$

$$V = \sqrt{\frac{A_e [(R_A + R_L)^2 + (X_A + X_L)^2] E^2}{Z_R}}$$

$$P = \frac{E^2}{Z}$$

$$l_e = \frac{V}{E} = \sqrt{\frac{A_e [(R_A + R_L)^2 + (X_A + X_L)^2]}{Z_R L}}$$

Cond'n for max. effective aperture when

$$X_A = -X_L$$

$$R_A = R_R + R_L = R_L$$

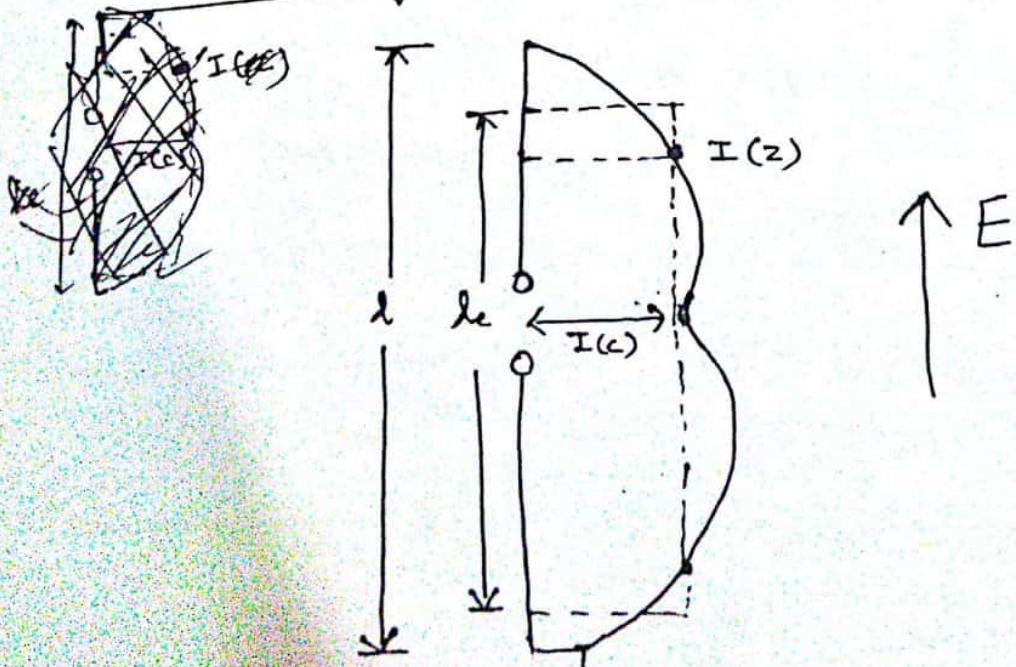
$$R_A = R_R = R_L \text{ if } R_L = 0$$

$$l_e = \frac{(A_e)m \cancel{R_L}^2}{\sqrt{Z_R L}}$$

$$l_e = 2 \frac{\sqrt{(A_e)m R_R}}{\sqrt{Z}} \quad m \neq 1$$

$$(A_e)_{max} = \frac{l_e^2 Z}{4 R_R}$$

For transmitting Antenna



For transmitting Antenna, Effective length is that length of an equivalent linear antenna that has same current $I(c)$ at all point along its length and that radiates the same field Intensity E as actual antenna.

$I(c)$ = Current at terminals of actual antenna

$I(z)$ = Current at any point z of antenna

l_e = Effective length

l = Actual length.

$$I(c) l_e = \int_{-l/2}^{+l/2} I(z) dz$$

$$l_e = \frac{1}{I(c)} \int_{-l/2}^{+l/2} I(z) dz$$

$$l_e = \frac{2}{I(c)} \int_0^{l/2} I(z) dz$$