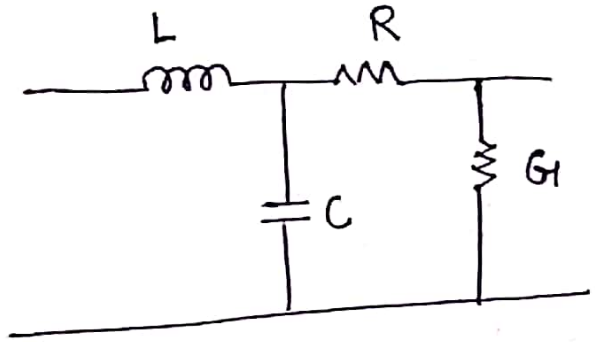
Transmission Line Equation



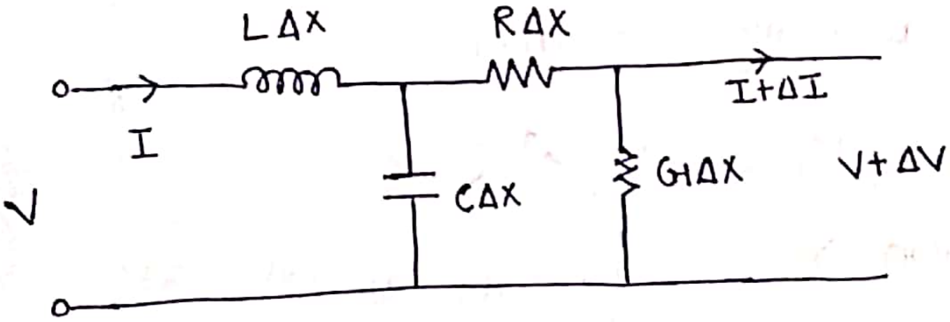
L, R, C, G are Constant parameter & its ~~unit is~~ in per Unit length

L/m → to ~~co~~ overcome it, multiply 'm' to get absolute value

R/m

C/m

G/m



$$\Delta V = -(R\Delta x + j\omega L\,\Delta x)I$$

(-ve) sign indicate that 'ΔV' is less the input 'V'

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -(R + j\omega L)I$$

$$\frac{dv}{dx} = -(R + j\omega L)I \quad \text{—} \; 1$$

similarly ΔI

$$\Delta I = -(G\Delta x + j\omega C\Delta x)V$$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = -(G + j\omega C)V$$

$$\frac{dI}{dx} = -(G + j\omega C)V \quad \text{—} \; 2$$

Double differentiate of Eq (1)

$$\frac{d^2V}{dx^2} = -(R+j\omega L)\frac{dI}{dx}$$

put the value of $\frac{dI}{dx}$

$$\frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)V$$

R, L, G and C are primary Constant

$$\frac{d^2V}{dx^2} = \gamma^2 V$$

$$V(x,t) = V^+ e^{-\gamma x} + V^- e^{+\gamma x}$$

It is a sinusodial in nature, if we want instantaneous, just multiply $e^{j\omega t}$ with these Equation

$$V(x,t) = \left(V^+ e^{-\gamma x} + V^- e^{+\gamma x}\right) e^{j\omega t}$$

(gama) $\gamma \rightarrow$ propogation Const.

$\gamma \rightarrow \alpha + j\beta$

$\alpha \rightarrow$ attenuation Constant

$\beta \rightarrow$ phase constant.

~~For Lossless Medium ($\alpha = 0$)~~

$$V(x,t) = V^+ e^{-(\alpha+j\beta)x} e^{j\omega t} + V^- e^{+(\alpha+j\beta)x} e^{j\omega t}$$

For Lossless Medium ($\alpha = 0$)

$$V(x,t) = V^+ e^{-j\beta x} e^{j\omega t} + V^- e^{j\beta x} e^{j\omega t}$$

$$V(x,t) = V^+ e^{j(\omega t - \beta x)} + V^- e^{j(\omega t + \beta x)}$$

$$\Rightarrow \underbrace{V^+ \cos(\omega t - \beta x)}_{\text{travelling in +ve direction}} + \underbrace{V^- \cos(\omega t + \beta x)}_{\text{travelling in -ve direction}}$$

Charateristic Impedance

$$\frac{dv}{dx} = -(R+j\omega L)\, I$$

$$\frac{d\left(V^+ e^{-\gamma x} + V^- e^{+\gamma x}\right)}{dx} = -(R+j\omega L)\left(I^+ e^{-\gamma x} + I^- e^{+\gamma x}\right)$$

$$V^+ e^{-\gamma x}(-\gamma) + V^- e^{+\gamma x}(\gamma) = -(R+j\omega L)\left(I^+ e^{-\gamma x} + I^- e^{+\gamma x}\right)$$

taking part of travelling Forward

$$V^+ e^{-\gamma x}(-\gamma) = -(R+j\omega L)\left(I^+ e^{-\gamma x}\right)$$

$$\frac{V^+}{I^+} = \frac{R+j\omega L}{\gamma}$$

(where $\gamma^2 = (R+j\omega L)(G+j\omega C)$

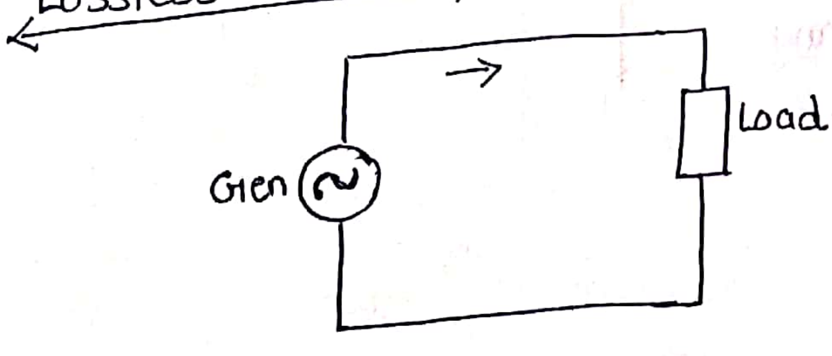
$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$)

$$\frac{V^+}{I^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\boxed{\frac{V^+}{I^+} = Z_0}$$

$Z_0 \rightarrow$ It is the Ratio of (+ve) travelling voltage wave to current wave at any point on the line.

Lossless Tx Line



(4)

A wave or voltage is travelling over a line, their loss should be zero.

$$\boxed{R=0,\ G=0}$$

Now find propogation constant (γ) & characteristic Impedance

→ propogation constant (γ)

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

put $R=G=0$

$$\gamma = \sqrt{j^2\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}} \quad \text{— (1)}$$

propogation constant (γ) is nothing but $\alpha + j\beta$

$\cancel{\alpha + j\beta = j\omega\sqrt{LC}}$

~~compare it~~

$$\boxed{\gamma = \alpha + j\beta} \quad \text{— (2)}$$

Compare these 2 Equation we get

$\alpha = 0$

$$\boxed{\beta = \omega\sqrt{LC}}$$

α → attenuation constant and it is zero, that means no any attunation is happeing.

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

put $R=G=0$

$$\boxed{Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}}$$

$\beta = \omega\sqrt{LC}$

$$\frac{2\pi}{\lambda} = 2\pi f\sqrt{LC}$$

$$\boxed{f\lambda = \frac{1}{\sqrt{LC}}}$$

$$\boxed{V_p = \frac{1}{\sqrt{LC}}}$$

V_p → phase velocity

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Distortion Less Transmission Line

A Transmission Line is said to be Distortion Less if
attenuation constant (α) → independent on Frequency
phase constant (β) → Linearly dependent on Frequency

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

taking 'R' and 'G' common

$$\gamma = \sqrt{R\left(1+j\omega\frac{L}{R}\right)G\left(1+j\omega\frac{C}{G}\right)}$$

$$\Rightarrow \sqrt{RG\left(1+j\omega\frac{C}{G}\right)^2}$$

$$\gamma \Rightarrow \sqrt{RG}\left(1+j\omega\frac{C}{G}\right)$$

we know that

$$\gamma = \alpha + j\beta$$

$$\alpha + j\beta = \sqrt{RG}\left(1+j\omega\frac{C}{G}\right)$$

By Comparing

$$\alpha = \sqrt{RG}$$

$$\beta = \sqrt{RG}\,\omega\frac{C}{G}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\Rightarrow \sqrt{\frac{R\left(1+j\omega\frac{L}{R}\right)}{G\left(1+j\omega\frac{C}{G}\right)}}$$

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$ → Distortion Less condition

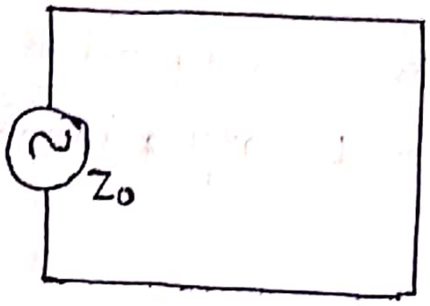
$$Z_0 = \sqrt{\frac{R}{G}}$$

In another way

$$Z_0 = \sqrt{\frac{j\omega L\left(1+\frac{R}{j\omega L}\right)}{j\omega C\left(1+\frac{G}{j\omega C}\right)}}$$

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} \Rightarrow \sqrt{\frac{L}{C}}$$

Input Impedance of Short Circuit



In short circuit, the o/p terminal is short instead of Load that means no any Load is connected. so $\boxed{Z_L = 0}$

The Formula of input impedance is

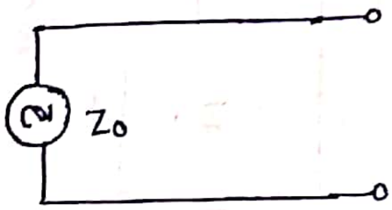
$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right] \quad \text{—— (1)}$$

put $Z_L = 0$ in Equation 1

$$Z_{in} = \cancel{Z_0} \left[\frac{0 + jZ_0 \tan\beta l}{\cancel{Z_0} + 0} \right]$$

$$\boxed{Z_{in} \Rightarrow jZ_0 \tan\beta l}$$

Input impedance of Open Circuit



$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$= \frac{Z_0 \times \cancel{Z_L} \left(1 + j\frac{Z_0}{Z_L} \tan\beta l\right)}{\cancel{Z_L} \left(\frac{Z_0}{Z_L} + j\tan\beta l\right)}$$

~~Zin/ 4/Zo~~

When $Z_L = \infty$, we know that any Denominator is ∞ then that value ze become zero

$$Z_{in} = \frac{Z_0}{j\tan\beta l} \Rightarrow \boxed{Z_{in} = -j Z_0 \cot\beta l}$$

~~Zin / Z / jZL cot~~

~~Zin =~~

Relationship b/w voltage standing wave Ratio (VSWR) and Reflection co-efficient (K)

$$|V_{max}| = |V_i| + |V_R| \quad \text{--- (1)}$$

$$|V_{min}| = |V_i| - |V_R| \quad \text{--- (2)}$$

where $V_i \rightarrow$ incident voltage

$V_R \rightarrow$ Reflected voltage

For Current standing wave Ratio

$$S = \frac{I_{max}}{I_{min}}$$

Similarly for voltage

$$VSWR = S = \frac{|V_{max}|}{|V_{min}|} \quad \text{--- (3)}$$

put Eq (1) & (2) in Eq (3)

$$VSWR = S = \frac{|V_i| + |V_R|}{|V_i| - |V_R|}$$

$$S = \frac{\cancel{|V_i|}\left(1 + \frac{|V_R|}{|V_i|}\right)}{\cancel{|V_i|}\left(1 - \frac{|V_R|}{|V_i|}\right)}$$

$$S = \frac{\left(1 + \frac{|V_R|}{|V_i|}\right)}{\left(1 - \frac{|V_i|}{|V_R|}\right)} \quad \text{--- (4)}$$

then $K = \frac{|V_R|}{|V_i|}$

$$S = \frac{(1+|K|)}{(1-|K|)}$$

$$S(1-|K|) = 1+|K|$$

$$S - S|K| = 1+|K|$$

$$S-1 = S|K|+|K|$$

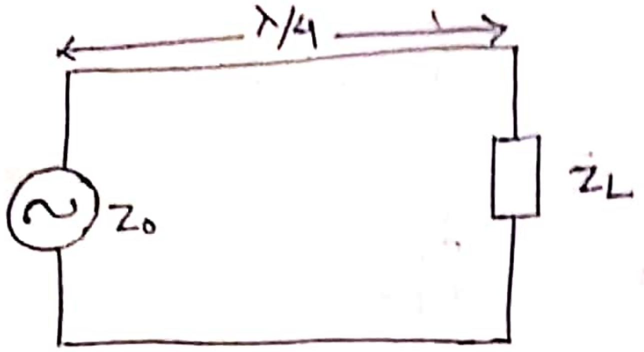
$$S-1 = |K|(S+1)$$

$$\boxed{|K| = \frac{S-1}{S+1}} \quad \text{--- (5)}$$

Equation 5th shows the Relationship b/w VSWR & K

Lines of Different Length

① $\lambda/4$



As we know that Input Impedance (Z_{in})

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right)$$

where $\beta = \frac{2\pi}{\lambda}$ & $l = \frac{\lambda}{4}$

$$\beta \times l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \Rightarrow \frac{\pi}{2}$$

$$Z_{in} = Z_0 \frac{\cancel{\tan\beta l} \left(\frac{Z_L}{\tan\beta l} + jZ_0 \right)}{\cancel{\tan\beta l} \left(\frac{Z_0}{\tan\beta l} + jZ_L \right)}$$

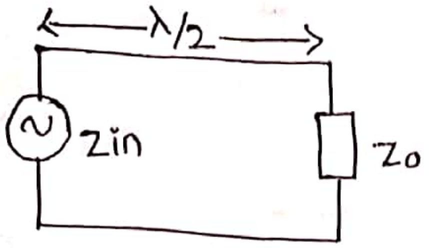
put $\beta l = \frac{\pi}{2}$

$$\Rightarrow Z_0 \frac{\left(\frac{Z_L}{\tan\pi/2} + jZ_0 \right)}{\left(\frac{Z_0}{\tan\pi/2} + jZ_L \right)} \qquad (\tan\pi/2 = \infty)$$

$$Z_{in} \Rightarrow Z_0 \frac{(jZ_0)}{(jZ_L)} \Rightarrow \frac{Z_0^2}{Z_L}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

2) Half wave Transmission Line ($\lambda/2$)



$$\boxed{Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right)}$$

$$\beta = \frac{2\pi}{\lambda}, \quad l = \frac{\lambda}{2}$$

$$\beta \times l = \frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} = \pi$$

$$\boxed{\beta l = \pi}$$

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\pi}{Z_0 + jZ_L \tan\pi} \right) \qquad (\tan\pi = 0)$$

$$Z_{in} = \cancel{Z_0} \left(\frac{Z_L + 0}{\cancel{Z_0} + 0} \right)$$

$$\boxed{Z_{in} = Z_L}$$

Smith chart

what is smith chart

① Smith chart is a graphical method to solve transmission Line problem.

② It is graph b/w Resistance Component (R/z_0) and Reactance Component $(\pm jx/z_0)$

③ where R is normalized Resistance

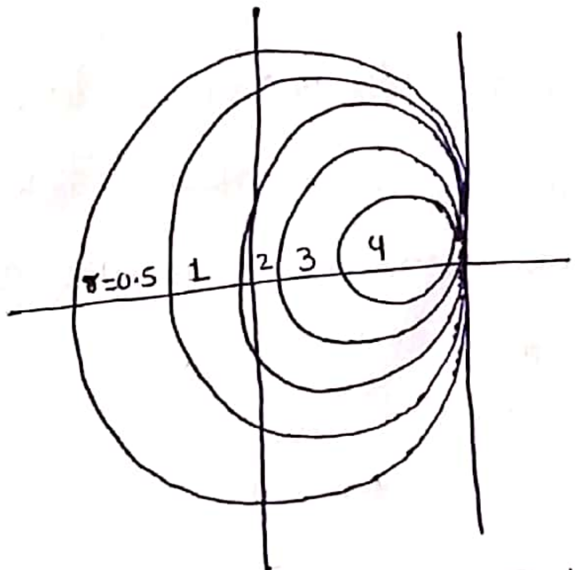
④ z_0 is characteristic impedance and x is normalized Reactance.

Construction

→ smith chart Consist of two type of Circles

① The Constant R- Circles

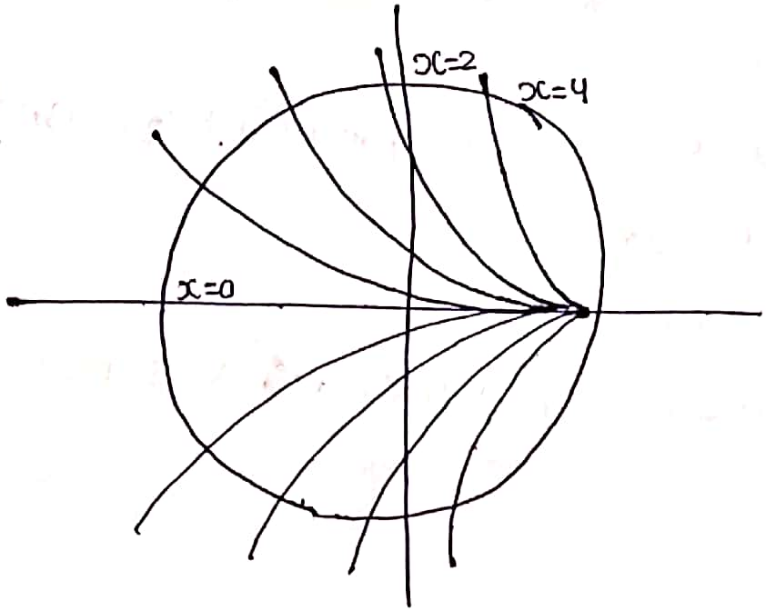
→ Smith chart basically an impedance chart Containing two set of lines.



→ All lines are tangent to Each other at Right hand & the value of 'R' is decreasing from Left to Right.

② The constant X- Circle

→ There is another set of lines called Constant Reactance lines.



→ The Lines in the Upper half Represent (+Ve) Reactance while other in the lower Represent (-Ve) Reactance.

→ The Complete Smith chart is obtain by the Super--position of the two sets of X- Circle & R- Circle.

Application

1. Calculate Admittance on any transmission line
2. Calculate Impedance on any transmission line
3. Calculate the Length of a transmission line to provide Capacitance & Inductance Reactance.

Losses in transmission Line

① Copper losses

→ This type of loss is in the form of heat being dissipated ~~in the~~ by line Conductor.

→ This type of Losses appear due to Existance of Resistance in the Conductor

→ It Expressed as I^2R

② Dielectric loss

→ In transmission Lines, air acts as dielectric medium b/w Conductor & Earth but in Co-axial Cable, the dielectric medium may be channel Compound.

→ Dielectric ~~medium~~ losses are also in the form of heat.

→ Dielectric loss $\propto$ voltage across dielectric

→ Dielectric loss $\propto \frac{1}{Z}$ (in Co-axial Cable)

③ Radiation loss

→ Radiation loss arise when the line act as an antenna

→ Radiation Loss $\propto$ distance b/w 2 Conductor

→ Radiation losses $\propto F^2$ → frequency

(4) Reflection Loss

→ It is also known as Mismatch loss

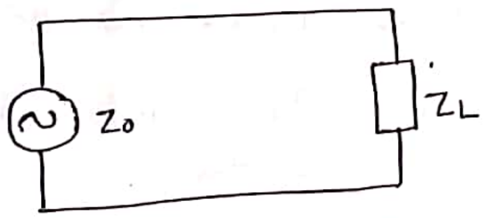
→ When signal is flowing from source to Receiver End & at Receiving End it start Reflect back to source.

This phenomenon is known as Reflection loss

→ This happen when air & glass are interfere together, this occur in OFC.

→ In Decibel (dB) $= 20 \log_{10} \left| \dfrac{Z_1 + Z_2}{2\sqrt{Z_1 \times Z_2}} \right|$

Impedance Matching

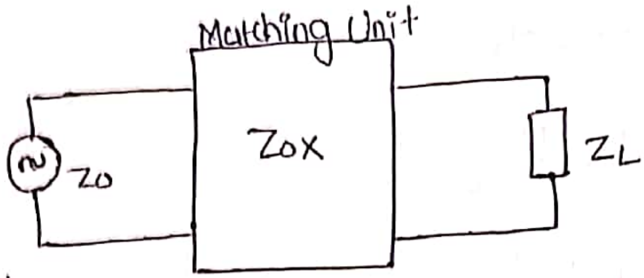


→ Impedance matching is, if $Z_0 = Z_L$ then maximum power deliver from source to Load. (100% power deliver)

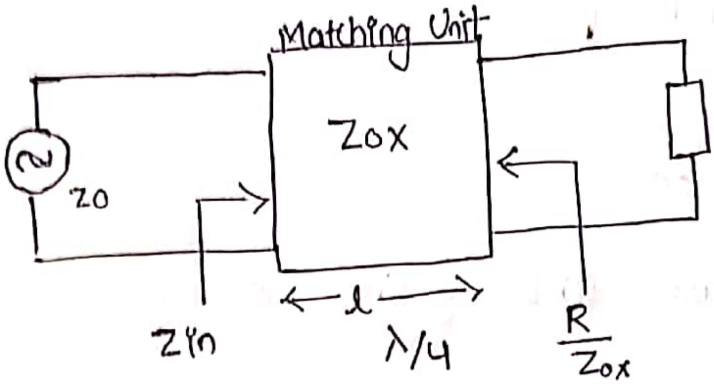
→ But if $Z_0 \neq Z_L$ then Γ power will be Reflect back. (60% Receive and 40% Reflect back)

→ For Impedance matching, put Maching Unit b/w them

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$



1) Resistive Load



Normalise Impedace

$\frac{R}{Zox}$

$$Zin = \frac{(Zox)^2}{R} = Zo$$

$$\boxed{Zox = \sqrt{R\,Zo} = Z_L}$$

* Standing wave Ratio

When the transmission line is not Correctly torminated, the traving EM wave is Reflected back. The interference of incident & Reflected wave give standing wave of Current and voltage along the line.

It is the Ratio of Maximum to minimum, Current or Voltage on the line.

SWR = standing wave Ratio

$$= \left|\frac{V_{max}}{V_{min}}\right|$$

S = give ~~stay~~ amount of mismatch b/w Inci. & Ref.

$S > 1 \rightarrow$ Mismatch

$S = 1 \rightarrow$ totally match.

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

Input Impedance

There are two cases ① Lossy line ② Loss less line

Lossy Line

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tan h \beta l}{Z_0 + Z_L \tanh \beta l} \right]$$

$\rightarrow$ For short Circuit, $Z_L = 0$

$$Z_{in} = \cancel{Z_0} \left[\frac{0 + Z_0 \tanh \beta l}{\cancel{Z_0} + 0} \right] \Rightarrow \boxed{Z_0 \tanh \beta l}$$

$\rightarrow$ For Open Circuit, $Z_L = \infty$

$$Z_{in} = Z_0 \times \frac{\cancel{Z_L}}{\cancel{Z_L}} \left[\frac{1 + \frac{Z_0}{Z_L} \tan \alpha \beta l}{\frac{Z_0}{Z_L} + \tanh \beta l} \right]$$

$$\Rightarrow Z_0 \left[\frac{1 + 0}{0 + \tanh \beta l} \right] \Rightarrow \frac{Z_0}{\tanh \beta l}$$

$$\boxed{Z_{in} = Z_0 \coth \beta l}$$

② Loss Less Line

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$

→ For short Circuit, $Z_L = 0$

$$Z_{in} = Z_0 \left[\frac{0 + jZ_0 \tan \beta l}{Z_0 + 0}\right]$$

$$\boxed{Z_{in} = jZ_0 \tan \beta l}$$

→ For Open Circuit, $Z_L = \infty$

$$Z_{in} = Z_0 \times \cancel{Z_L} \left[\frac{1 + j\frac{Z_0}{Z_L} \tan \beta l}{\cancel{Z_L}\left(\frac{Z_0}{Z_L} + j \tan \beta l\right)}\right]$$

$$\Rightarrow Z_0 \left[\frac{1 + 0}{0 + j \tan \beta l}\right]$$

$$\Rightarrow \frac{Z_0}{j \tan \beta l} \Rightarrow -jZ_0 \cot \beta l$$

Loading of transmission Line

Loading is a process in which inductance is adding Artificially increase in order to get distortion less Condition.

How to increase

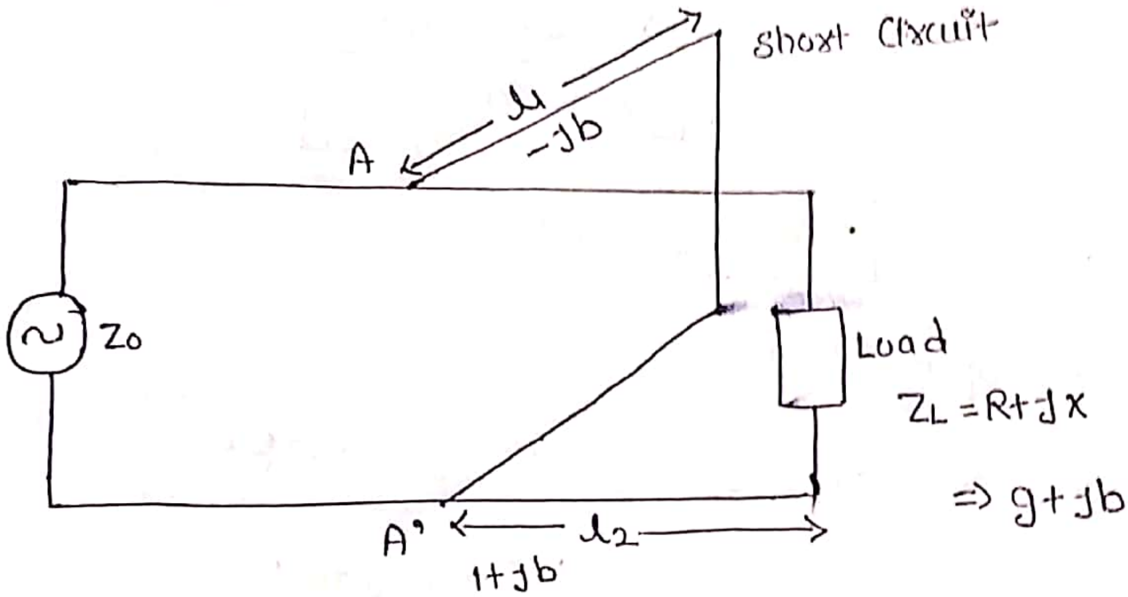
→ By installation of Co-axial cable in series with the line

→ By adding induction in lumped form at specific location.

Single stub Matching

→ Stub Matching is another way of impedance matching, In IM, The Matching Unit is used for matching but they are only for Resistive Load, not Complex.

→ Stub Matching is used for Resistance as well as Complex.



→ Input wave flowing from source, At point A & A', Some will flow toward Z_L & some will flow toward Short Ckt.

→ These are Reflect back from both, at A & A', & at this point ~~Reflection~~ if the Reflection is out of phase & Equal ~~to~~ in amplitude, both the Reflection is Cancled.

→ At A and A', Reflection is zero and it is only possible when impedance is matching.