Eigen Value Problem by Power Method

Power method is used to determine numerically largest eigen value and the corresponding eigen vector of a matrix A.

Step I

First we assume Initial vector

e.g. $y_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

~~Lag~~ Largest value of vector not more than 1.

Then we find out $y_1 = A y_0$, where A is given matrix

Q.1. Find the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ by using Power method.

Sol^n Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Let Initial (arbitrary) vector $y_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then

$$y_1 = A y_0 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ .5 \\ 0 \end{bmatrix}$$ {We take common max value element}

$$= K_1 Z_1$$

Now $$y_2 = A Z_1 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ .5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1+0 \\ 2+.5+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2.5 \\ 0 \end{bmatrix}$$

$$= 2.5 \begin{bmatrix} .8 \\ 1 \\ 0 \end{bmatrix} = K_2 Z_2$$

$$y_3 = A Z_2 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} .8 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .8+2+0 \\ 1.6+1+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 2.6 \\ 0 \end{bmatrix}$$

$$= 2.8 \begin{bmatrix} 1 \\ .93 \\ 0 \end{bmatrix} = K_3 Z_3$$

$$y_4 = AZ_3 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ .93 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1.86+0 \\ 2.93 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.86 \\ 2.93 \\ 0 \end{bmatrix}$$

$$= 2.93 \begin{bmatrix} .98 \\ 1 \\ 0 \end{bmatrix} = K_4 Z_4$$

$$y_5 = AZ_4 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} .98 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.98 \\ 2.96 \\ 0 \end{bmatrix} = 2.98 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= K_5 Z_5$$

$$y_6 = AZ_5 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= K_6 Z_6$$

Here $Z_5 = Z_6 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ so Process is stoped

Hence eigen ~~vector~~ value is 3 and Eigen ~~val~~ vector is $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Ans

Q.2 Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by Power method and hence find the eigen vector also.

Sol^n Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Let Initial vector $y_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore y_1 = Ay_0 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 4\begin{bmatrix} .5 \\ 1 \end{bmatrix}$

$y_1 = k_1 Z_1$

Now $y_2 = AZ_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} .5 \\ 1 \end{bmatrix} = \begin{bmatrix} .5+2 \\ 1.5+4 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 5.5 \end{bmatrix}$

$= 5.5\begin{bmatrix} .45 \\ 1 \end{bmatrix} = k_2 Z_2$

$y_3 = AZ_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} .45 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.45 \\ 5.35 \end{bmatrix} = 5.35\begin{bmatrix} .46 \\ 1 \end{bmatrix}$

$= k_3 Z_3$

$y_4 = AZ_3 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} .46 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.46 \\ 5.38 \end{bmatrix} = 5.38\begin{bmatrix} .46 \\ 1 \end{bmatrix}$

$= k_4 Z_4$ Here $Z_3 = Z_4 = \begin{bmatrix} .46 \\ 1 \end{bmatrix}$

and Eigen value $= k_4 = 5.38$

Ans

3. Find the numerically largest Eigen Value of

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$ and the corresponding Eigen vector.

Sol^n Given $$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

Let us assume Initial vector $y_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\therefore y_1 = Ay_0 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$y_1 = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25\begin{bmatrix} 1 \\ .04 \\ .08 \end{bmatrix} = K_1 Z_1$$

$$y_2 = AZ_1 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ .04 \\ .08 \end{bmatrix} = \begin{bmatrix} 25+.04+.16 = 25.2 \\ 1+.12+0 = 1.12 \\ 2+0-.32 \end{bmatrix}$$

$$= \begin{bmatrix} 25.2 \\ 1.12 \\ 1.68 \end{bmatrix} = 25.2\begin{bmatrix} 1 \\ .04 \\ .067 \end{bmatrix} = K_2 Z_2$$

$$y_3 = AZ_2 = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ .04 \\ .06 \end{bmatrix} = \begin{bmatrix} 25.17 \\ 1.12 \\ 1.76 \end{bmatrix} = 25.17\begin{bmatrix} 1 \\ .04 \\ .06 \end{bmatrix}$$

$= K_3 Z_3$ Here $Z_2 = Z_3 = \begin{bmatrix} 1 \\ .04 \\ .06 \end{bmatrix}$ Hence Process is stoped

Hence Eigen Value is 25.17 and corresponding Eigen vector is $\begin{bmatrix} 1 \\ .04 \\ .06 \end{bmatrix}$ Ans

Q.4. Find the dominant Eigen value and the corresponding Eigen vector of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Ans. Eigen value $= 4$

Eigen vector $= \begin{bmatrix} 1 \\ .5 \\ 0 \end{bmatrix}$