

Statistical Physics.

①

When a system consists of a large no. of individual atoms and molecules, the study of the system has been done by applying microscopic (study of individual particles) or macroscopic (study of the system as a whole) approach. If a large no of particles are present, then we can apply the laws of probability to it. Statistical Physics consists essentially in applying the basic laws of physics to the system as a whole and combining their applications with the application of the laws of probability.

The study of statistical mechanics mainly consists of two branches.

(i) Classical statistics (ii) Quantum Statistics

Now, we shall first discuss certain physical terms -

→ Static System:- The system in which its composing particles are at rest.

Dynamic System:- The system, in which its composing particles are in constant state of motion.

Position Space:- To define the position of any particle or point in space, we need a sys-

(2)

Mechanically,

3 co-ordinates axes x, y & z . This 3-D space
is called the "Position Space."

Momentum Space:- If the particle is moving with a velocity v and momentum p in a dynamic system, then the 3 momentum components p_x , p_y & p_z will be along the three momentum space axes. Hence the momentum p , of a particle is completely specified by three momentum co-ordinates is called momentum space.

Phase Space:- Combination of position and momentum spaces is called phase space of the particle. A particle in phase space is represented by a point having six co-ordinates x, y, z, p_x, p_y, p_z .

The product of two volume (volume of position and momentum space) is called available volume in phase space and represented by τ .

\therefore Elementary volume $d\tau$ in phase space is

is given by
$$d\tau = dx dy dz dp_x dp_y dp_z$$

called phase space cell.

Classically, there is no limit on the size of the phase space cell, minimum size can be '0'.

Bur- Quantum Mechanically,

(2)

According to Heisenberg Uncertainty principle.

$$dx dp_x > \frac{\hbar}{2}, dy dp_y > \frac{\hbar}{2}, dz dp_z > \frac{\hbar}{2}$$

$$\therefore \boxed{d\tau > \frac{\hbar^3}{2}}$$

⇒ minimum size of a phase space cell is $\frac{\hbar^3}{2}$.

Microstate of a system:- The microstate of the system at a particular instant be defined when we specify as which particular cell each molecule of the system belongs at that instant.

Macrostate of a system:- A macrostate of the system, on the other hand, can be defined by just giving the no. of molecules ~~as~~ in each cell.

Hence, microstate gives more detailed information.

$$\boxed{\text{Arrangement of } n \text{ Particles in } K \text{ cells} = (K)^n}$$

Ex. Let there are 2 particles in the two cells

Macrostate.		Microstate	
cell (1)	(2)	(1)	(2)
0	2	-	a, b
2	0	a, b	-
1	1	a	b
		b	a

$$\therefore \text{Arrangement} = (2)^2 = 4$$

Classification of statistical Physics.

There are 3 kinds of particles.

(1) Classical particle

(2) Boson

Quantum
particles

(3) fermions.

→ (1) Identical particles of any spin which are so much separated that they can be distinguished from one other.

Ex. Gas molecules.

(They follow MB statistics).

→ (2) Identical particles of zero or integral spin which cannot be distinguished from one to another called Bosons or Bose particles.

Ex. Photon, phonon, γ - particles.

(They follow BE statistics)

- Do not obey Pauli Exclusion principle.

→ (3) Identical particles of half integral spin

which cannot be distinguished from one to another. (3)

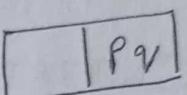
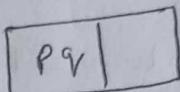
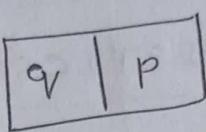
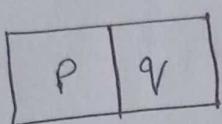
Ex. e^- , proton etc.

(They follow F-D statistics)

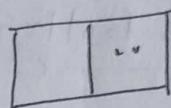
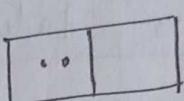
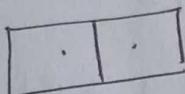
They follow Pauli - exclusion principle.

Ex. Distribution of 2 particles in 2 cells.

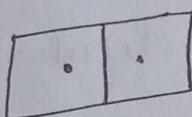
M.B.



B.E.



F.D.



Exchange Symmetry of wave function :- Let there be

N-particles described by - the wave function $\Psi(1, 2, \dots, N)$ when 1 stands for all ~~coordinates~~ co-ordinates (Position and spin) of Particle 1 and so on.

There are two kinds of wave function Ψ

(i) Symmetric wave function (Ψ_s) :-

A wave function is said to be symmetric if the interchange of any ~~one~~ pair of particles among its arguments leave the wave function unchanged.

$$P_{1,2} \Psi_s(1, 2) = +\Psi_s(2, 1) \quad (P = +1)$$

(ii) Antisymmetric wave function (Ψ_A) :-
 The wave function is said to be antisymmetric if the interchange of any pair of particles among its arguments, changes the sign of the wave function.

$$P_{1,2}, \Psi_A(1,2) = -\Psi_A(2,1) \quad (P = -1)$$

CLASSICAL STATISTICS.

Maxwell-Boltzmann statistics:-

- (1.) This statistics applies to a system of classical particles of any spin like molecules of an ideal gas.
 - (2.) The particles are distinguishable, because, they have specified position and momentum.
 - (3.) The particles do not follow any symmetric condition or restriction.
 - (4.) There is no limit on the particles present in a given state.
 - (5.) In phase space the volume of phase cell is not fixed.
 - (6.) The probability of particles to be present in a state of energy E_i , when system is at a temp T is given by
- $$f_{MB} = A e^{-E_i/kT}$$

$$f_{MB} = \frac{1}{e^{-\epsilon_i/kT}} \quad (4)$$

where f_{MB} → Distribution function

(7) The Maxwell - Boltzmann distribution is

$$n_i = \frac{g_i}{e^{-\epsilon_i/kT}}$$

where $\alpha = -\epsilon_F/kT$ $\epsilon_F \rightarrow$ fermi-energy level.

$n_i \rightarrow$ no. of particles in ϵ_i states.

$g_i \rightarrow$ Sublevels.

(8) The energy of M-B system, such an ideal gas (monoatomic) may be zero at absolute zero.

Molecular Energies of an Ideal Gas. — (5)
we now apply M-B statistics to find the distribution of energies among the gas molecules.

Distribution function for M-B statistics is given by

$$n_i = \frac{g_i}{e^{(E_i - E_F)/kT}} \quad \text{where } \zeta = \frac{-E_F}{kT} \quad \text{--- (1)}$$

where n_i = No. of particles in E_i states

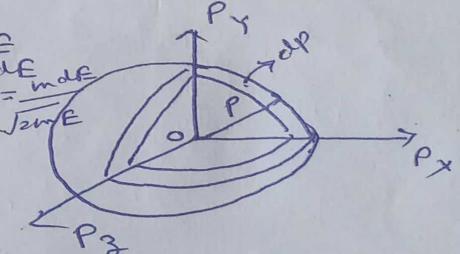
g_i = No. of sublevels.

Consider an ideal gas containing N molecules.
Let energies are continuous, then no. of molecules having energies $E \rightarrow E + dE$ is given by

$$n(E) dE = g(E) e^{-\zeta} e^{-E/kT} \quad \text{--- (2)}$$

Let a molecule of energy E has a momentum.

$\therefore p = \sqrt{2mE} \Rightarrow p^2 = \frac{2mE}{dp/dp} = \frac{2mE}{dm/dE} \cdot \frac{dp}{dp} = \frac{mdE}{\sqrt{2mE}}$
No. of momentum state available \propto volume of spherical shell in momentum space radius p and thickness dp .



$$g(p) dp = \text{volume} = 4\pi p^2 dp = B p^2 dp. \quad \text{where } B = 4\pi$$

each momentum magnitude p corresponds to a single energy E

$$g(E) dE = g(p) dp = B p^2 dp = B \cdot 2mE \cdot \frac{m dE}{\sqrt{2mE}}$$

$$g(E) dE = B \sqrt{2m}^{3/2} E^{1/2} dE \quad \text{--- (3)}$$

Applies to system of bosons (Photon, Phonon etc)

from equ. ⑪

$$n(E) dE = B \sqrt{2} m^{3/2} E^{\gamma_2} e^{-\epsilon} e^{-E/kT}$$

$$= C E^{\gamma_2} e^{-E/kT} dE$$

$$\text{where } C = B e^{-\epsilon} \sqrt{2} m^{3/2} = \text{Constant.}$$

after solving using normalized condition.

we have

$$n(E) dE = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{E} e^{-E/kT} dE$$

Total Energy

$$E = \int_0^\infty E n(E) dE$$

$$= \frac{2\pi N}{(\pi kT)^{3/2}} \int_0^\infty E^{3/2} e^{-E/kT} dE$$

$$= \frac{2\pi N}{(\pi kT)^{3/2}} \cdot \frac{3}{4} (kT)^2 \sqrt{\pi kT}$$

$$\cdot \underbrace{\int_0^\infty E^{3/2} e^{-E/kT} dE}_{= \frac{3}{4} (kT)^2 \sqrt{\pi kT}}$$

$$= \frac{3}{2} N kT$$

Average energy of an ideal gas = $\frac{E}{N}$

$$\boxed{\bar{E} = \frac{3}{2} kT}$$

Bose-Einstein Statistics.

- (1) Applies to system of bosons (Photon, Phonon etc)
- (2) Particles are identical but indistinguishable.
- (3) Spin is either '0' or an integral like $1, 2, 3, \dots$
- (4) No limit on the particle present in a given state i.e. Pauli exclusion principle does not hold for bosons.
- (5) Volume of ~~the~~ Phase cell $\simeq h^3$
- (6) They have symmetric wave functions.
- (7) The probability of particles to be present in a state of energy E_i when system is at a temp' T is given by

$$f_{BE} = \frac{1}{e^{E_i/kT} - 1}$$

f_{BE} = average no. of particles in each of the states of that energy

- (8) The distribution law is

$$n_i = \frac{g_i}{e^{E_i/kT} - 1}$$

n_i = no. of particles

g_i = sublevels

k = Boltzmann's constant.

- (9) The energy may be zero at absolute zero.

Spectrum i.e., it contain quantum

Fermi-Dirac Statistical:-

- (1) Applied to a system of fermions (e, p etc).
- (2) Identical and indistinguishable.
- (3) Odd half integral value of spin like $\frac{1}{2}, \frac{3}{2} \dots$.
- (4) only one particle in a given quantum state is allowed. (Obey Pauli's exclusion principle)
- (5) Volume of phase cell is π^3
- (6) Having antisymmetric wave function
- (7)
$$f_{FD} = \frac{1}{e^{(E_i/kT)} + 1}$$

distribution function

- (8) Distribution law is

$$n_i = \frac{g_i}{e^{(E_i/kT)} + 1}$$

- (9) The energy at absolute zero cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

Spectrum i.e., it contain radiation of all the frequencies

Variation of Intensity of Black-body Radiation :-

- At a given temp^r, the energy is not uniformly distributed in the radiation.

Spectrum of a hot body.

- first intensity of radiations increases with increase in wavelength and at a particular wavelength its value is maximum and then decreases with increase in wavelength.

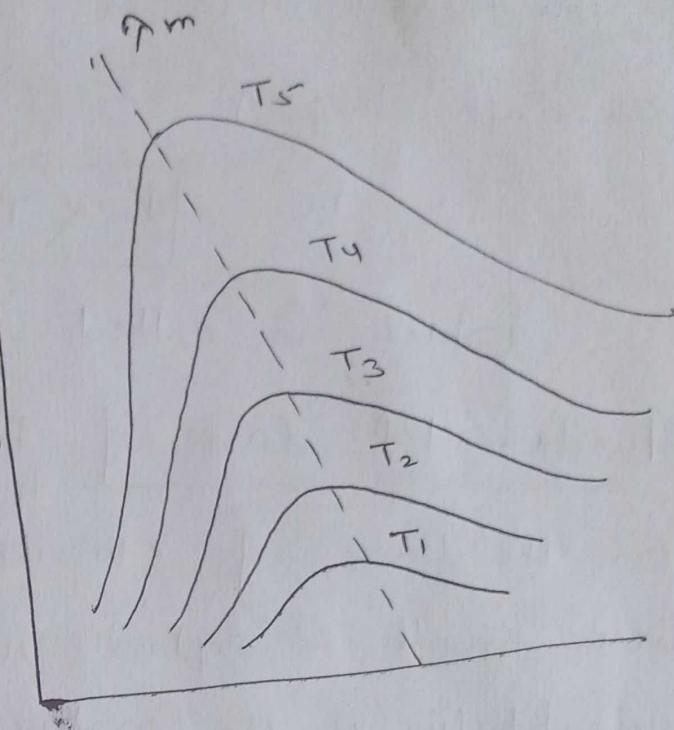
- With increase in temp^r, λ_m decreases such.

-that- $\lambda_m T = \text{Constant}$ or $[\lambda_m T = 0.2896 \text{ cm K}]$

This is Wien's displacement law.

where λ_m - wavelength at which maximum emission of energy takes place.

- The area under each curve represents the total energy emitted by the body at a particular temp^r for the range of the wavelength Considered.



This area increase with increase of temp^r. It is found that area is $\propto T^4$. (fourth power of absolute temp^r)

i.e. $E \propto T^4$.

which is called Stefan's Law.

Fundamental laws of Black body Radiation.

On the basis of classical theory, several attempts were made to explain qualitatively and quantitatively the nature of the black-body spectrum.

Stefan's - Boltzmann law:- The energy of radiation is \propto the fourth power the absolute temp^r T of the black body

$$E \propto T^4$$

$$E = \sigma T^4$$

$$\text{where } \sigma \rightarrow \text{stefan's Constant} = 5.67 \times 10^{-8} \text{ J/m}^2 \text{ sec}^{-4} \text{ K}^4$$

Stefan's law does not throw any light on the energy distribution among different wavelengths.

Wein's displacement law:- In 1983, Wein showed that the wavelength at the maximum energy emission is inversely proportional to the temperature.

i.e. $\lambda_{\text{max}} T = \text{constant} = 0.2896 \text{ K cm}$

$$\text{also } E_{\lambda} T^{-5} = \text{a constant}$$

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Where E_{λ} is the emissive power (quantity of radiation emitted per sq cm per sec. at a given temperature).

The amount of energy contained in spectral region λ and $\lambda + d\lambda$ is given by

$$\begin{aligned} v &= \frac{c}{\lambda} \\ \lambda &= \frac{c}{v} \\ d\lambda &= -\frac{c}{v^2} dv \end{aligned}$$

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$$

$$E_{\nu} dv = \frac{8\pi h\nu^3 - h\nu/kT}{c^3} e^{-h\nu/kT} dv$$

It was found that the above equation gives very good fitting of experimental curve in the low wavelength region (or high frequency) but did not produce the results at high wavelength (or low frequency).

Rayleigh - Jeans Contribution

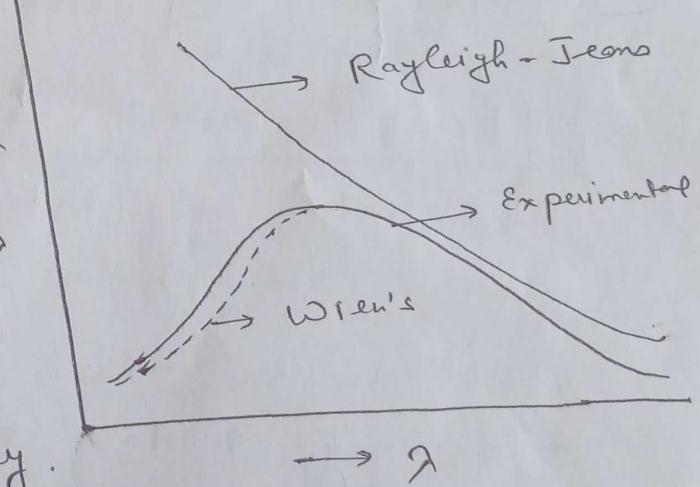
According to this it E_{λ} was assumed that a black body radiator.

emits radiation continuously.

According to this the energy density in wavelengths range λ and $\lambda + d\lambda$ is given by

$$E_{\lambda} d\lambda = \frac{8\pi}{\lambda^4} K T d\lambda$$

$$\text{or } E_{\nu} dv = \frac{8\pi v^2}{c^3} K T dv$$



$$E_\lambda \propto \frac{1}{\lambda^4}$$

$$\lambda \rightarrow 0 \\ E_\lambda \rightarrow \infty$$

$\Rightarrow E_\lambda$ Continuously increases with decreasing λ .

\Rightarrow This law agree with ~~exp~~ experimental results at low frequency (high wavelength) but does not fit the curve at low wavelength (high frequency).

\Rightarrow Black-body spectrum could not be explained on the basis of classical concept. This discrepancy in classical calculations and observed fact was removed by Planck on the basis of Quantum theory. Bose-Einstein statistics was applied to Black-body radiation spectrum.

Application of Bose-Einstein statistics

Planck's Radiation Law:-

Radiator or photon gas is considered as Bose-Einstein gas. Each photon has energy $h\nu$.

Let dn be - the no. of photons distributed in frequency ν to $(\nu + d\nu)$

$$dn = n(\nu) d\nu \\ = g(\nu) f(\nu) d\nu \quad \text{--- (i)}$$

$$\therefore g(\nu) = \frac{8\pi h\nu^2}{c^3} d\nu \quad \text{--- (ii)}$$

$$f(\nu) = \frac{1}{e^{[E/kT]} - 1} d\nu \quad \therefore \lambda = 0 \\ E = h\nu \quad \text{--- (iii)}$$

$$f(v) = \frac{1}{e^{\frac{hv}{kT}} - 1} \quad \text{--- (14)}$$

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putting these values in equ. (1)

$$dn = \frac{8\pi v^2}{c^3} \frac{1}{e^{\frac{hv}{kT}} - 1} dv$$

Let dE be the energy distributed in frequency interval v and $(v+dv)$

$$\begin{aligned} dE &= E dn \\ &= hv dn \end{aligned}$$

$$dE = \frac{8\pi hv^3}{c^3} \frac{1}{(e^{\frac{hv}{kT}} - 1)} dv \quad \text{--- (V)}$$

Equation (V) is known as Planck's radiation formula for the spectral density of black body spectrum, which agrees with experimental curve.

for low frequency (or large wavelength) =

$$\begin{aligned} e^{\frac{hv}{kT}} - 1 &= 1 + \frac{hv}{kT} - \dots - \cancel{1} \\ &= \frac{hv}{kT}. \end{aligned}$$

\therefore equ. (V) becomes

$$dE = \frac{8\pi \frac{hv^3}{c^3}}{1 + \frac{hv}{kT}} dv$$

$$\boxed{dE = \frac{8\pi kT v^2}{c^3} dv} \quad \text{--- (VI)}$$

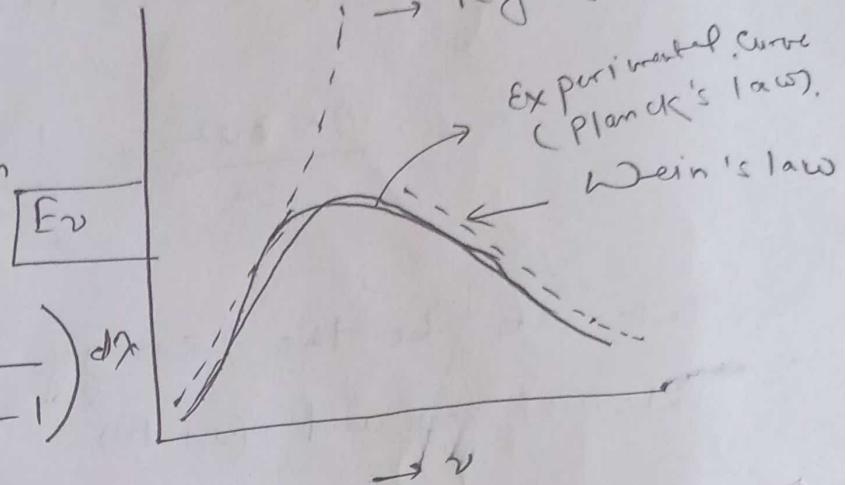
FREE ELECTRON THEORY

which is Rayleigh-Jeans law. This law is good for low frequencies and high wavelength.

$$\therefore \nu = \frac{c}{\lambda}$$

Equ. (1) can be written as

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \left(\frac{1}{e^{hc/\lambda KT} - 1} \right) d\lambda$$



for low wavelength $\lambda T \rightarrow \text{small}$

$e^{hc/\lambda KT}$ becomes large $\gg 1$

$\Rightarrow 1$ can be neglected

$$\therefore E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT}}$$

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda}$$

$$\text{or } \boxed{E_\nu dv = \frac{8\pi h\nu^3}{c^3} e^{-\frac{hv}{KT}} dv}$$

which is Wien's law, which valid at low wavelength (or high frequency)

\Rightarrow Rayleigh-Jeans and Wien's laws are unable to explain the complete energy distribution curve, while Planck's law is explain it completely, which is based on B-E statistics.

FREE ELECTRON THEORY :-

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This theory is applicable to all solids, both metals and non-metals. It explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids. It has been divided into two

(i) Classical free electron theory :- Given by

Drude and Lorentz in 1900. According to this theory, metal contains free e^- s which are responsible for the electrical conductivity and e^- s obey the laws of classical mechanics. (Applied M-B statistics)

(ii) Quantum free electron theory :- Given by

Sommerfeld in 1928. According to this e^- s move with a constant potential. This theory obeys quantum laws (applied f-D. statistics).

Assumptions (OR) Salient features

- In metals there are a large no. of free e^- s moving freely in all possible directions.
- These free e^- s behave like gas molecules in a container obey the laws of Kinetic theory of gases i.e. they are free to move in random direction and collide

with either +ve ions fixed to the lattice or other free e^- s. The collision is elastic i.e there is no loss of energy. Hence the electric conduction is due to free e^- s.

- In the absence of field the energy associated with each e^- at a temp^r T is given by $\frac{3}{2}kT$

$$\therefore \frac{3}{2}kT = \frac{1}{2}mv_{th}^2 \text{, where } v_{th} \rightarrow \text{Thermal velocity.}$$

- When electric field is applied to a metal, the free e^- s are accelerated in direction opposite to the direction of applied electric field with a velocity called drift velocity v_d .

- Electron velocities in a metal obey Maxwell-Boltzmann distribution law.

Success of Classical free e^- theory :-

- It verifies Ohm's law.
- It explains electrical and thermal conductivities of metals.
- It derives Wiedemann - Franz law. **
- It explain optical properties of metals.

** The ratio of the thermal conductivity of the material and the electrical conductivity of the material is directly relative to the temperature.

$$\left[\frac{K}{\sigma} = LT \right]$$

where K = electrical conductivity

$K \rightarrow$ Thermal " constant called Lorentz no. = 2.44×10^{-8} work

Limitations or drawbacks.

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- It fails to explain the electric conductivity of semiconductor and insulators.
- It fails to explain the temperature variation of electrical conductivity at low temperature. (i.e. Wiedemann-Franz law is not applicable at low temp.)
- It fails to explain the concept of specific heat of metals.
- The phenomenon like photo-electric effect, Compton effect and black body radiation could not be explained by classical free e^- theory.
- It fails to explain temperature dependence of paramagnetic susceptibility and ferromagnetism.

Quantum free Electron Theory:-

Assumptions. Valence e^- s move freely in a constant potential within the boundaries of metal and is prevented from escaping the metal at the boundaries (High potential). Hence the electron is trapped in a potential well.

- The distribution of e^- s in various allowed energy levels occurs as per Pauli's Exclusion Principle.

- The particles behaves just a particle in a box
- Hence the energy values of free e^- s are quantized.
$$E_n = \frac{n^2 h^2}{8mL^2}$$
 where $n = 1, 2, 3, \dots$
- The distribution of energy among the free e^- s is according Fermi-Dirac statistics.
- The attraction between the free e^- s and lattice ions and the repulsion between e^- s themselves are ignored.
- To find the possible energy values of e^- Schrodinger time independent wave equations applied.

Merits of quantum free electron theory:-

- Successfully explains the electrical and thermal conductivity of metals.
- Can explain the thermionic phenomenon.
- Can explain temperature dependence of conductivity of metals.
- Can explain the specific heat of metals
- Can explain magnetic susceptibility of metals.
- Can explain photo electric effect, Compton effect and black-body radiation, etc.

- Demerits:-
1. Unable to explain the metallic properties exhibited by only certain crystals (14)
 2. Unable to explain, why the atomic arrays in metallic crystals should prefer certain structures only
 3. It fails to distinguish between metal, semiconductor and insulator.
 4. It fails to explain the true value of Hall coefficient.
 5. According to this theory, only two e^- s are present in the fermi level and they are responsible for conduction which is not true.

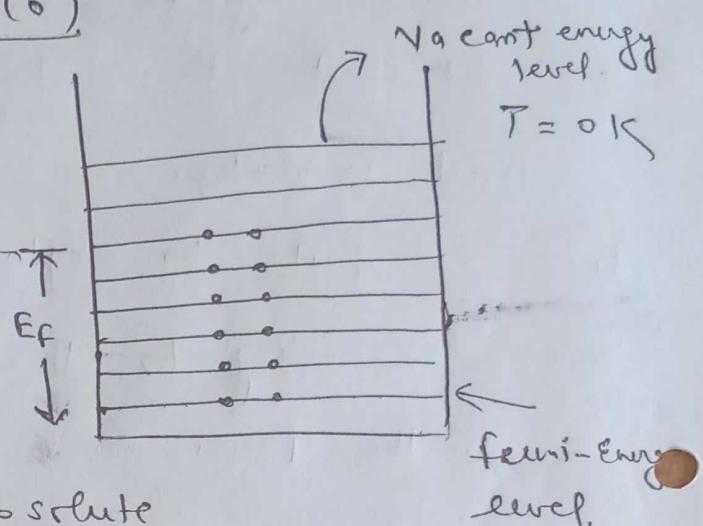
Fermi Energy, Fermi Level and Fermi Factor:-

We know that for a metal containing N atoms, there will be N no. of energy levels in each band. According to Pauli's exclusion principle, each energy level can accommodate a maximum of two e^- s, one with spin up ($+1/2$) and other with spin down ($-1/2$). As temp' decrease, the energy of fermions also decrease, but at $T=0\text{ K}$, the energy of all fermions is not zero. It means that the fermions will go on occupying the lower and lower energy states as the system is cooled.

Thus at 0 K , starting from the ground state all the energy states are filled till all the fermions are accommodated.

The energy value upto which the energy states are filled at 0K and above which all the energy states are empty is known as Fermi-Energy and denoted by $E_F(0)$.

→ Top most energy level ↑
occupied by e's at E.
 0K is called Fermi-energy level.



→ At temp^r above absolute zero, the e's get thermally excited and move up to higher energy levels. As a result there will be many vacant energy level below as well as above Fermi-energy level. Under thermal equilibrium, the distribution of e's among various energy levels is given by function $f(E)$, called Fermi factor and gives the probability of occupation of a given energy level under thermal equilibrium is given by

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

(15)

FERMI ENERGY:- Consider an e^- gas having n free e^- s in a conductor of volume V . Total energy of e^- be E . This energy is distributed among all the N electrons.

\therefore Fermi distribution function

$$n_i = \frac{g_i}{e^{(E_i - E_F)/k_B T} + 1} \quad \text{--- (1)}$$

This equation can be written as

$$n(E) dE = \frac{g(E) dE}{e^{(E - E_F)/k_B T} + 1} \quad \text{--- (2)}$$

$n(E) dE$ — No. of e^- s. within energy range $E \rightarrow E + dE$

The no. of cells lying within the range P to $P + dP$

$$g(P) dP h^3 = g_s (4\pi P^2) dP V \quad \text{--- (3)}$$

$g_s \rightarrow$ spin degeneracy factor, for $e^- (s) = \frac{1}{2}$

$$\text{given by } g_s = (2s+1) \Rightarrow g_s = 2$$

$$\therefore g(P) dP = 2 \times \frac{4\pi P^2 dP}{h^3} V$$

$$g_{p1} dP = 2 \cdot \frac{4\pi V}{h^3} P \cdot P dP \quad \text{--- (4)}$$

$$\therefore E = P^2/2m \Rightarrow P^2 = 2mE \Rightarrow P = \sqrt{2mE}$$

$$2P dP = 2m dE$$

$$P dP = m dE$$

\therefore The no. of states in the energy range between E and $E + dE$ will be given by

$$g(E) dE = 2 \cdot \frac{4\pi V}{h^3} (2mE)^{1/2} m dE$$

$$g(E) dE = \frac{8\sqrt{2}\pi h^3 m^{3/2}}{V} E^{1/2} dE$$

$g(E) \rightarrow$ density states function.

$$\therefore dn(E) = n(E) dE = \frac{8\sqrt{2}\pi m^{3/2} V}{h^3} \cdot \frac{\sqrt{E} dE}{e^{(E-E_F)/k_T} + 1}$$

$$dn(E) = n(E) dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \cdot \frac{\sqrt{E} dE}{e^{(E-E_F)/k_T} + 1}$$

\therefore Fermi Energy = Energy at $T=0K$, denoted by $E_F(0)$

\therefore At $T=0K$, Total no. of fermions = Total no. of energy states.

occupied by the fermions from 0 to $E_F(0)$ as each energy state can accommodate only one fermion

\therefore The value of E_F at $T=0K$ is $E_F(0)$ and can be found as

$$N = \int_0^{E_F(0)} dn(E)$$

$$N = \int_0^{E_F(0)} \frac{8\sqrt{2}\pi m^{3/2}}{h^3} (E)^{3/2} dE$$

(from equ. v)

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_F(0)} (E)^{3/2} dE$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \cdot \left[\frac{2}{3} (E)^{3/2} \right]_0^{E_F(0)}$$

$$= \frac{8\pi (2)^{3/2} V}{h^3} \cdot (E_F(0))^{3/2}$$

$$E_F(0) = \frac{h^2}{2m} \left[\frac{3N}{8\pi V} \right]^{2/3}$$

$$\begin{aligned} &\because \text{At } T=0K \\ &\frac{1}{e^{(E-E_F)/k_T} + 1} \\ &= \frac{1}{e^{0/0} + 1} = \frac{1}{e^0 + 0} = \frac{1}{1+0} = 1 \end{aligned}$$

- (vi)

Equation (vi) gives the energy of the highest level occupied at $T = 0K$. (16)

$$\therefore \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} = \frac{3}{2} N [E_F(0)]^{-3/2}$$

put in eqn. (v)

$$n(E) dE = dn(E) = \frac{3}{2} N [E_F(0)]^{-3/2} \frac{\sqrt{E} dE}{e^{(E-E_F)/kT} + 1}$$

at $T = 0K$, $E \leq E_F(0)$

$$n(E) dE = \frac{3}{2} N [E_F(0)]^{-3/2} (E)^{1/2} dE$$

$$\begin{aligned} e^{-\infty} &= 1 \\ -\infty &= 1 \\ 0 &= 1 \\ &= 1 \end{aligned}$$

$$\therefore \text{Average Energy} = \overline{E} = \frac{\int_0^{E_F(0)} E n(E) dE}{\int_0^{E_F(0)} n(E) dE}$$

$$= \frac{\int_0^{E_F(0)} \frac{3}{2} N [E_F(0)]^{-3/2} [E]^{3/2} dE}{\int_0^{E_F(0)} \frac{3}{2} N [E_F(0)]^{-3/2} [E]^{1/2} dE}$$

$$\therefore \overline{E} = \frac{\frac{2}{5} [E_F(0)]^{5/2}}{\frac{2}{3} [E_F(0)]^{3/2}}$$

$$\boxed{\overline{E} = \frac{3}{5} E_F(0)}$$

\Rightarrow The average K.E. of a mobile e^- in a conductor is $\frac{3}{5}$ times of the fermi-energy.

Zero Point Energy of Electron Gas i.e. (ie Energy at 0K)

$$\begin{aligned} U_0 &= \int_0^{E_F(0)} E n(E) dE \\ &= \frac{3}{2} N [E_F(0)]^{-3/2} \int_0^{E_F(0)} [E]^{3/2} dE \\ &= \frac{3}{2} N [E_F(0)]^{-3/2} \left[\frac{2}{5} E^{5/2} \right]_0^{E_F(0)} \\ &= \frac{3}{5} N [E_F(0)]^{-3/2} [E_F(0)]^{5/2} \end{aligned}$$

$$\boxed{U_0 = \frac{3}{5} N E_F(0)}$$

Specific Heats of Solids

Blackbody radiation is not only the phenomenon, which couldn't be explained by classical theory. This required quantum statistical mechanics. One more is - the internal energy of solid with varies with temperature.

Specific heat capacity :- The amount of heat required to raise the temp^r of 1kg of substance by 1K

unit - $J \text{ kg}^{-1} \text{ K}^{-1}$

Let us consider - the molar specific heat of a solid at constant volume, C_V . (This is the energy that must be added to 1 kmol of the solid, whose volume is held fixed, to raise its temperature by 1K.)

The internal energy of a solid resides in the vibrations of its constituent particles which may

be atoms, ions, or molecules. here we refer as atoms. we may represent each atom by 3D harmonic oscillator and vibrations are resolved into 3 axes have energy $\frac{3}{2} kT$.

The atom in a solid contains Avogadro's no. No. 1 kmol of a solid contains Avogadro's no. No.

of atoms.

Its total internal energy E at temp^r

$$\text{W} = 3kT = 3N_0kT = 3RT$$

Where $R = N_0k = 8.31 \times 10^3 \text{ J/Kmol.K}$

$$= 1.99 \text{ kJ/mol} \cdot \text{K}$$

$R \rightarrow$ universal gas constant-

The specific heat at constant volume is given by

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{\partial}{\partial T} (3RT)$$

$$\boxed{C_V = 3R = 5.97 \text{ kJ/mol} \cdot \text{K}}$$

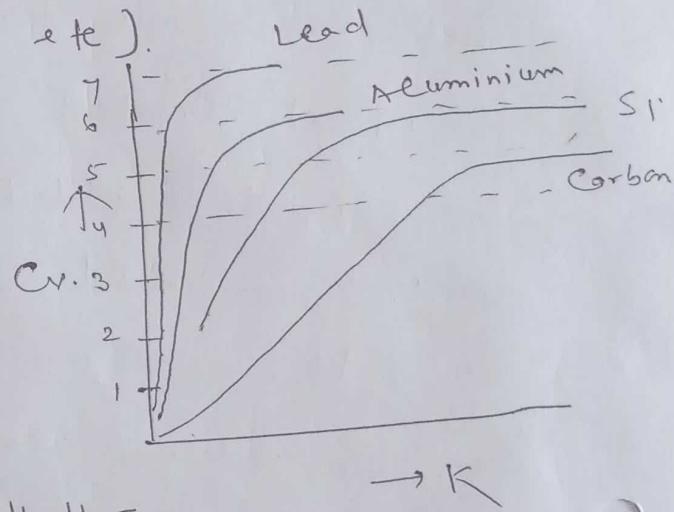
— (1)

This is known as Dulong - Petit law

However, this law fails for many elements (like boron, beryllium and carbon etc.).

The specific heats of all solids drop sharply at low temp^r and $\rightarrow 0.$ at 0K

(fig)



\Rightarrow Something is wrong with the analysis leading up to eqn. (1)

(fig)

In 1907 Einstein discerned that the basic flaw in the derivation of eqn. (1) lies in the figure of kT for the average energy per oscillation in a solid.

Average energy per oscillator is

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

not kT .

Internal energy of solid

(18)

$$U = 3N_0 \bar{E} = \frac{3N_0 \hbar v}{e^{\hbar v / kT} - 1}$$

and its molar specific heat is

$$C_V = \left(\frac{\partial U}{\partial T} \right)_{V, P} = 3R \left(\frac{\hbar v}{kT} \right)^2 \frac{e^{\hbar v / kT}}{(e^{\hbar v / kT} - 1)^2} \quad (11)$$

\Rightarrow this approach on the right track.

$C_V \rightarrow 0$

At high temp $\frac{\hbar v}{kT} \Rightarrow \hbar v \ll kT$

$$e^{\hbar v / kT} \approx 1 + \frac{\hbar v}{kT}$$

$$\begin{aligned} C_V &= 3R \left(\frac{\hbar v}{kT} \right)^2 \cdot \frac{1 + \frac{\hbar v}{kT}}{\left(\frac{\hbar v}{kT} \right)^2} \\ &= 3R \left[1 + \frac{\hbar v}{kT} \right] \approx 3R \end{aligned}$$

$$C_V \approx 3R$$

\Rightarrow classical physics holds.

from eqn. (11) As temp decrease, C_V is also decrease.

—x—

Comparison of the three Distribution Laws:-

$$\therefore f(E_i) = \frac{n_i}{g_i}$$

$f(E_i)$ is known as "occupation index" of a state of energy E_i . It represents the average no. of particles in each of the states of that energy.

$$f_{MB} = \frac{1}{e^{-E_i/kT}}$$

$$f_{BE} = \frac{1}{e^{-E_i/kT} - 1}$$

$$f_{FD} = \frac{1}{e^{-E_i/kT} + 1}$$

$$\frac{g_i}{n_i} = e^{-E_i/kT} \quad \text{for MB}$$

$$\frac{g_i}{n_i} = \frac{e^{-E_i/kT}}{e^{-E_i/kT} - 1} \quad \text{for BE}$$

$$\frac{g_i}{n_i} = \frac{e^{-E_i/kT}}{e^{-E_i/kT} + 1} \quad \text{for FD}$$

$$\text{If } \frac{g_i}{n_i} \gg 1 \text{ then } \frac{g_i}{n_i} = \frac{g_i}{n_i} + 1 \approx \frac{g_i}{n_i} - 1$$

In this limit both BE and FD are identical with MB. This limit occurs when temp is not too low and pressure (or density) is not too high.

Q. Plot of F-D distribution function at 0K.

8

$$\therefore f_{FD} = \frac{1}{e^{\frac{E_i - E_F}{kT}} + 1} = \frac{1}{e^{(E_i - E_F)/kT}}$$

$$\therefore \alpha = e^{-E_F/kT}$$

at $T = 0K$. $e^{\left(\frac{E_i - E_F}{kT}\right)} = e^{-\infty} = 0$

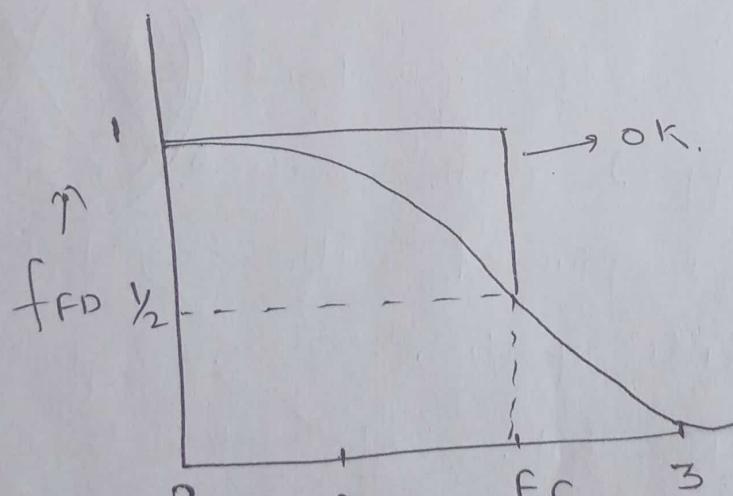
$$\therefore f_{FD} = \frac{1}{0+1} = 1 \quad \text{for } E_i < E_F$$

$$e^{\left(\frac{E_i - E_F}{kT}\right)} = e^{\infty} = \infty \quad \text{for } E_i > E_F$$

$$f_{FD} = \frac{1}{\infty+1} = \frac{1}{\infty} = 0$$

for $E_i = E_F$

$$f_{FD} = \frac{1}{e^0+1} = \frac{1}{1+1} = \frac{1}{2} \quad (\text{at all temp})$$



\Rightarrow at $T=0$, all energy states from $E_i=0$ to E_F are occupied because $f_{FD}=1$ while all states above E_F are vacant
 \Rightarrow As temp rises, some of states just below E_F become vacant, while some just above E_F are occupied

(*) Birth and Death of stars

- Like living things, star eventually reach the end of its life several billion years after its life starts.
- How the star dies, depends upon the type, mass and size of the star.
- The most massive stars die by exploding as supernovae.

Birth of a star

(#) Stars begin very small, mere particles in vast clouds of dust and gas.



To form active stars, these nebulae remain cold for ages



They get stirred up when the disturbance occurs in the form of a streaking comet or the shockwave from a distant supernova



As the resulting force moves through the cloud, particles collide and begin to form clumps.



Individually, a clump attains more mass and therefore a stronger gravitational pull,

attracting even more particles from the surrounding cloud.

↓
As more matter falls into the clump, its center grows denser and hotter.

↓
Over the course of a million years, the clump grows into a small, dense body called a protostar.

↓
When the protostar becomes hot enough (7 million Kelvin), its hydrogen atoms begin to fuse, producing helium and an outflow of energy in the process.

↓
This atomic reaction is just like nuclear fusion. The outward push of its fusion energy is still weaker than the inward pull of gravity at this point in the star's life.

↓
Material continues to flow into the protostar, providing increased mass and heat.

↓
Finally after millions of years, some of these stars reach the tipping point.

↓
If enough mass (0.1 solar mass) collapses into the protostar, a bipolar flow occurs.

Two massive gas jet erupt from the protostar and blast the remaining gas and dust clear away from its surface



At this point, the young star stabilizes and it reaches the point where its output exceeds its intake.



The outward pressure from hydrogen fusion now counteracts gravity's inward pull.



It is now a main sequence star and will remain so until it burns through all its fuel.

Death of a star

Star of the size of the sun



It takes roughly 50 million years to reach main sequence and maintains that level for approx 10 billion years.



When the core runs out of hydrogen fuel, it will contract under the weight of gravity.



As the core contracts, it heats up.

This heats the upper layers, causing them to expand.

As the outer layers expand, the radius of the star will increase and it will become a red giant.

The radius of the red giant sun will be just beyond earth's orbit.

At some point after this, the core will become hot enough to cause the helium to fuse into carbon.

When the helium fuel runs out, the core will expand and cool.

The upper layers will expand and eject materials that will collect around the dying star to form a planetary nebula.

Finally the core will cool into a white dwarf and then eventually into a black dwarf.

This entire process will take billion years.

Stars more massive than the Sun



When the core runs out of hydrogen, these stars fuse helium into carbon just like the sun.



After the helium is gone, their mass is enough to fuse carbon into heavier elements such as oxygen, neon, silicon, magnesium, sulfur & iron.



Once the core has turned into iron, it can no longer burn



The star collapses by its own gravity and the iron core heats up



The core becomes so tightly packed that protons and electrons merge to form neutrons.



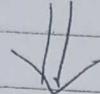
In less than a second, the iron core, which is about the size of the earth, shrinks to a neutron core with a radius of about 10 Km.



The outer layers of the star fall inwards on the neutron core, thereby crushing it further.



The core heats to billions of degrees and explodes (supernova), thereby releasing large amounts of energy and materials into space.



The shockwave from the supernova can initiate star formation in other interstellar clouds.



The remains of the core can form a neutron star or a black hole depending upon the mass of the original star.

At $E_i = E_F \Rightarrow$ The average no. of particles per quantum state is exactly $\frac{1}{2}$ ie., the prob. of finding an e^- with energy equal to the fermi energy in a metal is $\frac{1}{2}$ at any temperature.

Black body Radiation

A Black body is defined as one which absorbs all the incident radiations falling on it. When such a body heated, it emits radiations which we call black body radiation. The notion of blackbody is an idealized one and no such body exists in nature.

Lampblack and Platinum black behave close to the perfect black body for all practical purpose.

- A cavity is made out of a hollow container of any material (iron or copper) with a narrow opening and painted with lampblack in the inside portion. When any radiation falls on this hole, it enters the cavity, gets reflected by the wall of the cavity and gets absorbed. Now if we heat the container at various temperatures, it will emit radiation of all the frequencies (or wavelengths). So, the emitted radiation from a black body is a continuous

