CONTROL SYSTEMS - UNIT-1

- System :→ means of transfer / changing a signal. It is the func. of one or more no. of independent variables.

- automatic control systems → the existing value of a quantity / condn is measured & compared with the desired value & the diff of these 2 values is used to initiate the action for reducing the diff.

CLASSIFICATION.

(1.) Linear and Non-linear:

Linear → both homogenity & superposition.

Superposition : $r_1(t) \rightarrow c_1(t)$ & $r_2(t) \rightarrow c_2(t)$

then $[r_1(t) + r_2(t)] \rightarrow [c_1(t) + c_2(t)]$

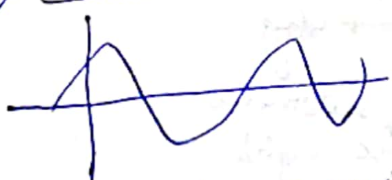
homogenity : $k r_1(t) \longrightarrow k c_1(t)$

Non-linear → non homogenous.

(2). Time Invarient & Time varying system:

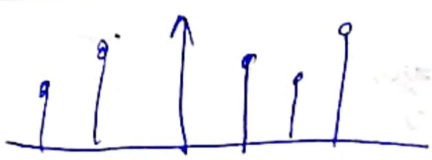
$\underbrace{\text{Time Invarient}}_{\text{Parameters are independent of time \& not varying with time.}}$ & $\underbrace{\text{Time varying system}}_{\text{vary with time}}$

(3) Continuous Time and Discrete Time System



→ defined for every instant of time.

→ func. of continuous time variable



defined at only specific instants of time.

(4) Dynamic and Static System

Dynamic: If o/p of sys is a func of time even when i/p is constant.

Static: if i/p does not change with time then o/p will also not change with time

(5) Deterministic & stochastic system

Deterministic: o/p could be exactly determined for a given i/p

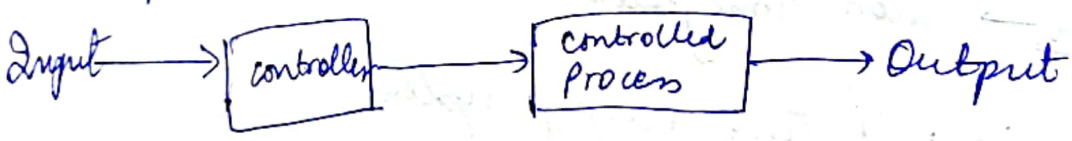
Stochastic: For a given ~~sys~~ i/p, the output is uncertain, only Probability can be predicted.

OPEN LOOP CONTROL SYSTEM:

[no provision within system for supervision of o/p & no mechanism is provided to correct system behavior for any lack of behavior of system components

→ control system without feedback or non-feedback control system.

→ control action is independent of the desired output



The components are :

1. controller
2. controlled Processes.

Advantages

→ simple, economical, less maintenance, not difficult

Disadvantages

→ inaccurate
→ not reliable
→ slow
→ optimization not possible

Eg : Automatic washing machine, automatic traffic light.

CLOSED LOOP CONTROL SYSTEM:

- → feedback control systems
- → the control action is dependent on the desired output.
- → Here, the output is compared with the reference input & error signal is produced.

Advantages:

- → more reliable
- → faster
- → optimization is possible

Disadvantages

- → expensive
- → difficult maintenance
- → complicated installation.

Eg: Air conditioner → temp regulation.

DIFFERENCE

Open loop	Closed loop
→ not reliable	→ reliable
→ easier to build	→ difficult to build
→ slow	→ fast
→ more stable	→ less stable
→ optimization not possible	→ optimization is possible.
→ not that accurate.	→ accurate

TRANSFER FUNCTION:

→ Defined as the ratio of laplace transform of the output to the laplace transform of input with all initial conditions as zero.

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{L(c(t))}{L(r(t))} \left\{ \frac{\text{output}}{\text{input}} \right\}$$

ADVANTAGES OF TRANSFER FUNC :

→ Transfer func is a mathematical model of all system components & hence of overall system & therefore it gives the gain of the system.

→ helps in study of stability of system

→ does not depend on input to system.

DISADVANTAGES OF TRANSFER FUNC :

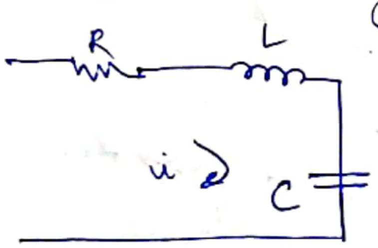
→ G(s) cannot be defined for non-linear system.

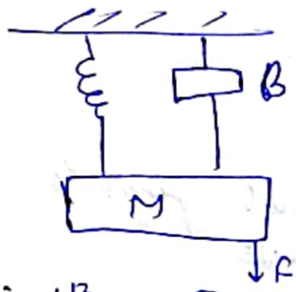
→ initial conditions loose their importance.

→ Physical structure of system cannot be determined.

ANALOGOUS SYSTEMS

→ Two diff. physical systems can be described by same mathematical model. The concept is very useful in study of complex systems.



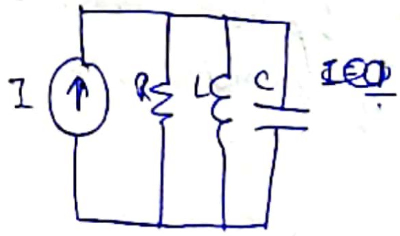


$$V_i = R\frac{dq}{dt} + L\frac{d^2q}{dt} + \frac{q}{C}$$

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

F-V Analogy

F	V
B	R
M	L
1/K	C



$$I = \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} + C\frac{d^2\phi}{dt}$$

F	I
M	C
B	1/R
K	1/L

For F-V

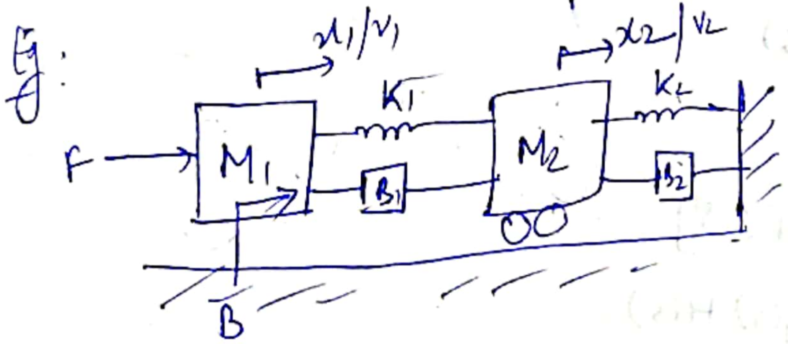
- elements having same velocity = are in series
- each node (mass) = closed loop in elec.
- no of meshes in elec = no. of masses in mech.
- element connected b/w 2 masses in mech is represented as a common ele b/w two meshes in elec.

For F-I :

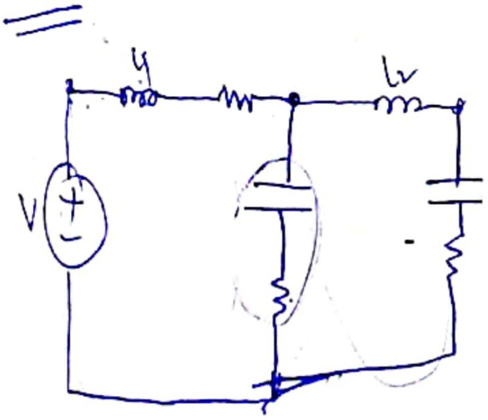
- In mech, the elements in parallel will have same force. Similarly, in elec, ele in parallel have same voltage.
- each node (mass) in mech corresponds to a node in elec.
- no. of nodes in elec = no. of nodes in ~~elec~~ mech.
- ele connected b/w 2 nodes in mech is represented as common ele b/w nodes in electrical system ~~(in parallel)~~

↳ Element taken in parallel.

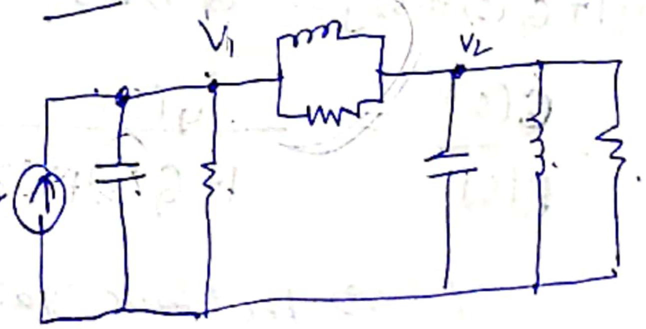
Eg.:



F-V



F-I

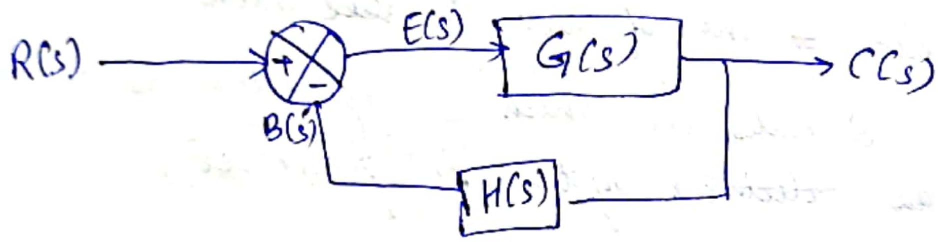


BLOCK DIAGRAM:

single line diagram that represents flow of system variables from one block to another. & each block represents a single component or a group of component which is characterised as transfer fun.

(A) How to draw?

1. Write diff. eqn
2. Take Laplace
3. Find expression ~~fron~~ for I(s)



$$E(s) = R(s) - B(s)$$
$$= R(s) - C(s).H(s)$$

$$C(s) = G(s).E(s)$$
$$C(s) = G(s)[R(s) - C(s)H.(s)]$$
$$C(s) = G(s)R(s) - C(s)G(s)H(s)$$
$$C(s)[1 + G(s)H(s)] = G(s)R(s)$$
$$\frac{G(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

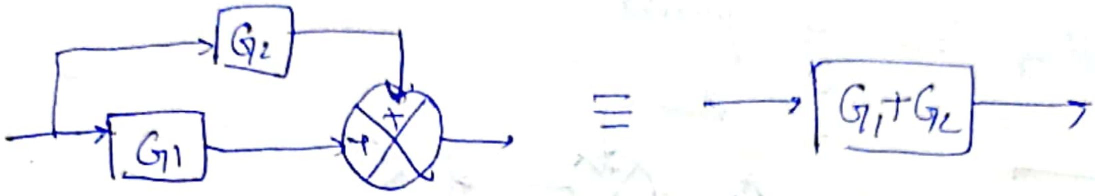
(−ve feedback gain)

$$\text{+ve feedback gain} = \frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$$

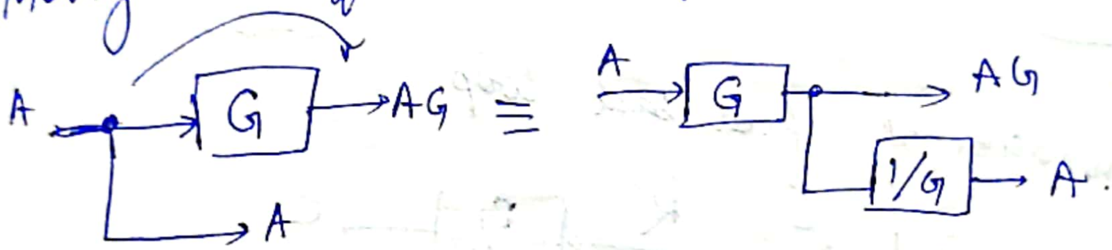
BLOCK DIAGRAM REDUCTION RULES:

1) combining 2 blocks in cascade / series:

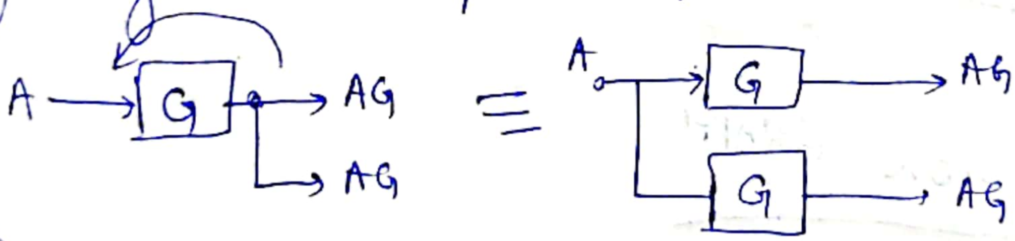
2) combining blocks in parallel



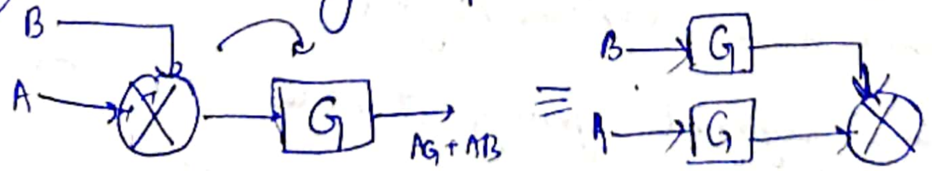
3) Moving take off pt ahead of block



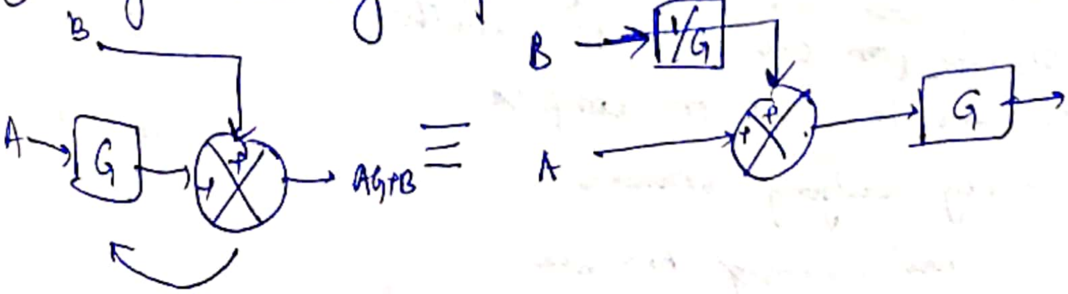
4) Moving take-off point before block.



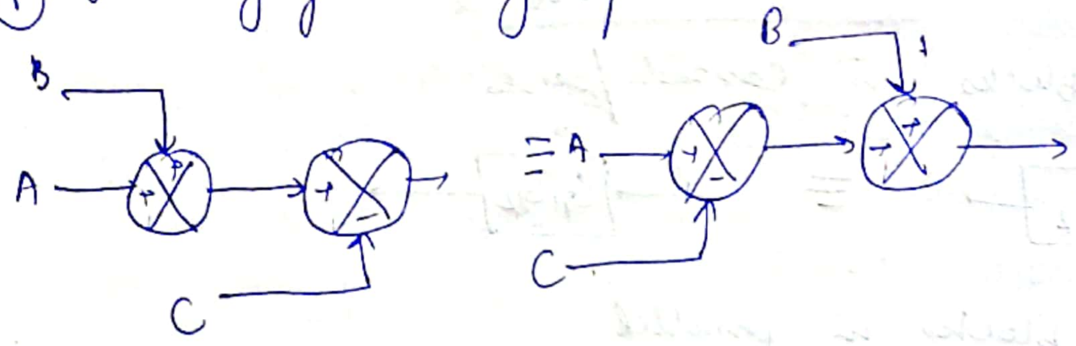
5) Moving summing point ahead of block.



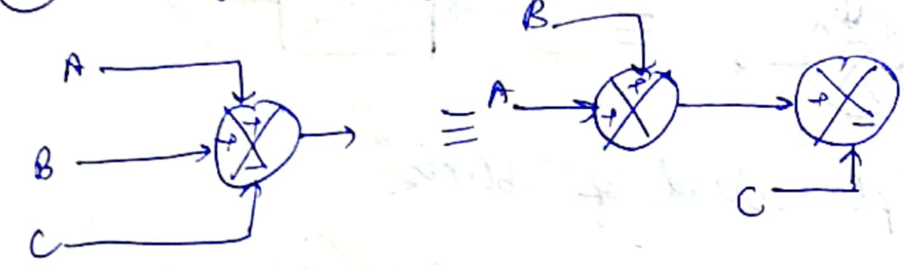
6) Moving summing point before block



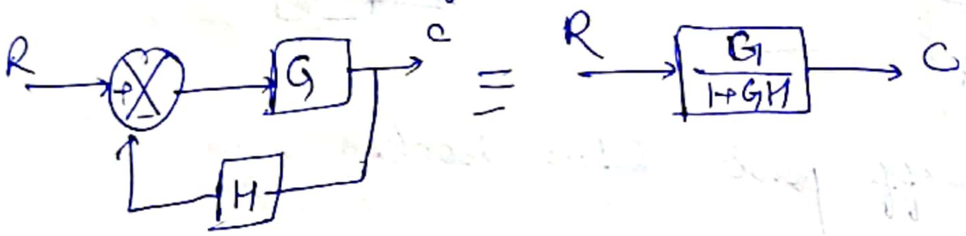
(7) Interchanging summing point.



(8) Splitting summing points.



(9) Elimination of feed back loop.



SIGNAL FLOW GRAPH

graphical representation.

- Node(s) point representing a variable / signal
- Branch - directed line segment joining 2 nodes. The arrow on branch indicates signal flow.
- Transmittance → The gain acquired by the signal when it travels from one node to another.
  → It can be real or complex.
- input node (source) → only outgoing branches
  output node (sink) → only incoming branches

- Path : traversal of connected branches in the direction of arrows. The path should not cross a node more than once.

(i) open path (ii) closed path (starts & end at same node)

- Forward path → the path from i/p to o/p
- Forward path gain → combination of transmittances in forward path.
- Individual loop → start & end at same node without crossing a node more than once.
- loop gain → product of gains of a loop
- Non-touching loop → if the loops don't have common node.

MASON's GAIN FORMULA:

$$T = \frac{C}{R} = \frac{\sum P_i \Delta_i}{\Delta}$$

where P_i = gain of i^{th} path

(alternate signs) $\Delta = 1-$ (sum of individual loop gains) + (sum of gain products of 2 non touching loops) + ...

$\Delta_i = 1 -$ (~~....~~ loop gain which are not touching to i^{th} part of path ($1-$ (that part of Δ not common with i^{th} part)

Step 1: Write all forward paths gain

Step 2: Write all Loop gains. (start & end node of loop same)

Step 3: Find non touching loops..

PROPERTIES OF SIGNAL FLOW GRAPH

→ applicable to linear time-invariant system

→ signal flow only along direction of arrows

→ value of variable at each node is equal to algebraic sum of all signals entering at that node.

→ signal flow graph is not unique property of system

CONSTRUCTION OF SFG :

Eg: $y_2 = t_{21} y_1 + t_{23} y_3$

to from

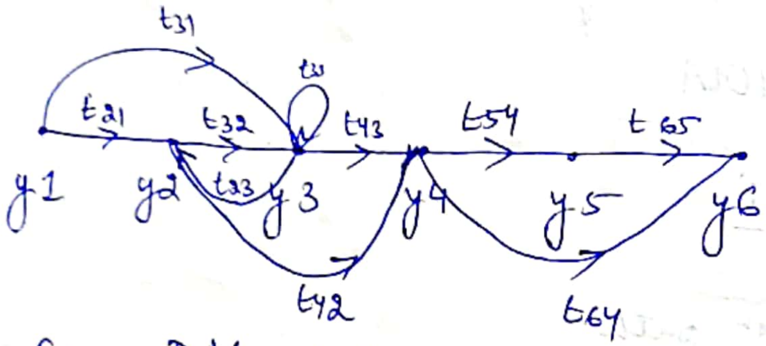
$y_3 = t_{32} y_2 + t_{33} y_3 + t_{31} y_1$

$y_4 = t_{43} y_3 + t_{42} y_2$

$y_5 = t_{54} y_4$

$y_6 = t_{65} y_5 + t_{64} y_4$

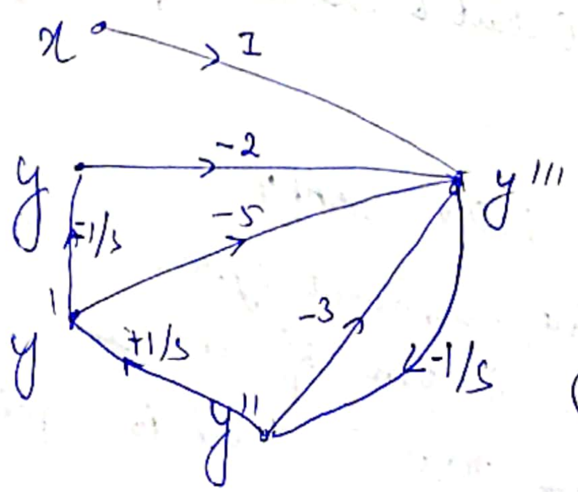
From eqns



* From Differential eqn.

$$y''' + 3y'' + 5y' + 2y = x$$

$$y''' = x - 3y'' - 5y' - 2y$$



Reverse sign of

① $\begin{cases} y''' \text{ to } y'' \\ y'' \text{ to } y' \end{cases}$

② $\begin{cases} y'' \text{ to } y' \\ y' \text{ to } y \end{cases}$

DIFFERENCE BTW BLOCK DIAGRAM & SFG.

Block diag

1. applicable to linear Time invarient system only
2. each element represented by block.
3. summing pt & take off pts are separate.
4. Self loop does not exist
5. time consuming method.
6. Block diag is reqd at each and every step.
7. Transfer func of element is shown inside the corresponding block.
8. feedback path is present.

Signal flow graph:

1. applicable to LTI systems
2. each variable is represented by a node.
3. summing & take off points are absent
4. self loop can exist
5. require less time by using Mason gain formula.
6. At each step, it is not necessary to draw SFG
7. Transfer func is shown along the branches connecting the nodes.
8. feedback loops are used.

CONSTRUCTION OF SFG from BLOCK DIAG:

Rules: (CLR) R 1 2 3 C

1. All variables, summing pts & take off points are represented with nodes.

2. If summing pt is placed before a take off point in the direction of signal flow, then represent summing pt & take off pt by a single node.

3. If suming pt is placed after a take off point in the direction of signal flow, then represent summing pt & take off pt by separate nodes connected by a branch having transmittance unity.

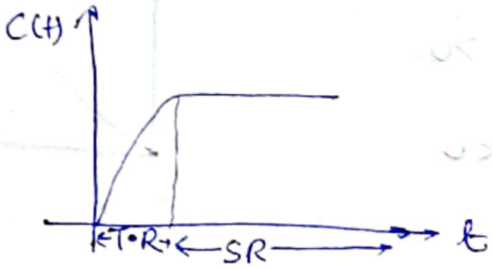
ST → single pt.

CONTROL SYSTEMS - UNIT-2

TIME DOMAIN ANALYSIS

→ variation of o/p of system with time

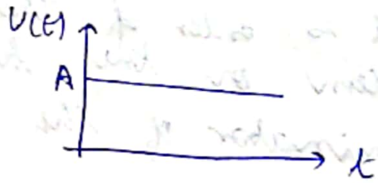
* Transcient Response → that part of response varying with time.
* Steady state Response → that part of total response after transcient has died.



TEST SIGNALS

1) Step signal (displacement function)

$$U(t) = \begin{cases} A & , t \geq 0 \\ 0 & t < 0 \end{cases}$$



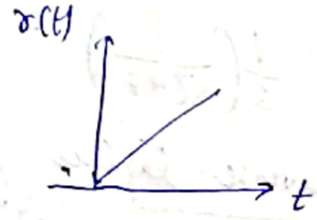
When $A = 1$ → unit step

$$L[U(t)] = \frac{1}{S} \quad \text{(unit step signal)}$$

$$R(S) = \frac{1}{S}$$

2) Ramp function.

$$r(t) = \begin{cases} Kt & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Where K = slope for line

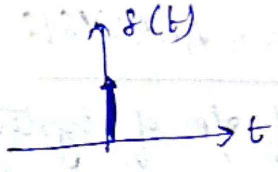
for $k > 0$ → positive slope, $k < 0$ → negative slope; downwards.

$$L[r(t)] = \frac{K}{S^2}$$

for unit ramp $= \frac{1}{S^2}$

3) Impulse signal

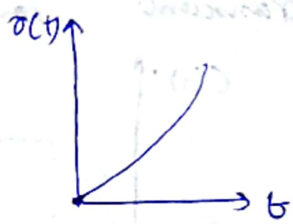
$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t\neq 0 \end{cases}$$



$L[\delta(t)] = 1$

4) Parabolic signal

$$r(t) = \begin{cases} \frac{Kt^2}{2} & , t>0 \\ 0 & t<0 \end{cases}$$



$L[r(t)] = \frac{K}{s^3}$

ORDER OF THE SYSTEM:

- defined as order of the differential eqⁿ representing the system or the highest power of 's' in the denominator of the transfer function.

RESPONSE OF FIRST ORDER SYSTEM WITH UNIT STEP I/P.

$$\frac{C(s)}{R(s)} = \frac{1}{s\tau+1} \Rightarrow C(s) = \frac{1}{s\tau+1} R(s)$$

$$\Rightarrow C(s) = \frac{1}{s}\left(\frac{1}{s\tau+1}\right) = \frac{1}{s} - \frac{\tau}{\tau s+1}$$

⇒ Inverse laplace gives:

$$\boxed{C(t) = 1 - e^{-t/\tau}}$$

When $t=\tau$,

$$C(t) = 1-e^{-1} = 0.632 = 63.2\%$$

τ = time constant

= time reqd for signal to attain 63.2% of final or steady state value.

= how fast system reaches final value.

(*) Smaller the τ, faster is the system response

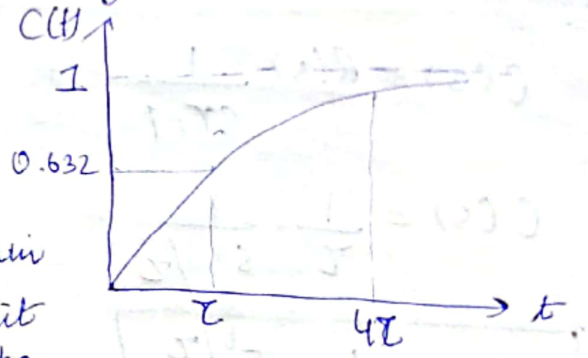
$\tau = 63.2\%$

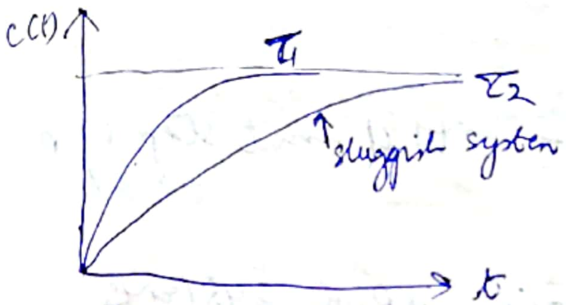
$2\tau = 86.4\%$

$4\tau = 98\%$

Settling time

(When actual o/p reaches within 2% of the desired output, it is said that steady state has reached).



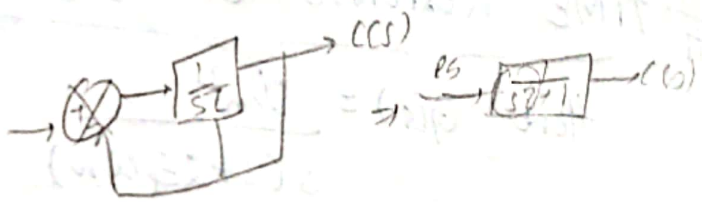


$\tau_1 < \tau_2$

Error $e(t) = r(t) - c(t)$

$= 1 - (1 - e^{-t/\tau})$

$= e^{-t/\tau}$



RESPONSE OF FIRST ORDER SYSTEM WITH UNIT RAMP

$R(s) = \frac{1}{s^2}$

$C(s) = \frac{1}{s^2}\left(\frac{1}{s\tau+1}\right) \Rightarrow C(s) = \frac{A}{\tau s+1} + \frac{Bs+C}{s^2}$

$= \frac{\tau^2}{\tau s+1} + \frac{1-\tau s}{s^2}$

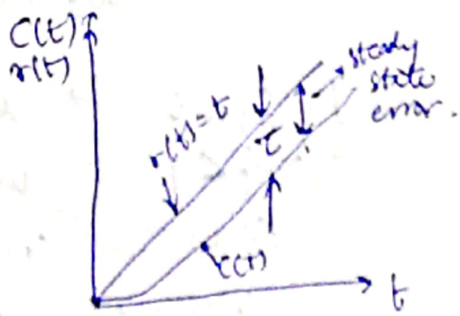
$\Rightarrow C(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau}{s+1/\tau}$

$\Rightarrow \boxed{c(t) = t - \tau(1) + \tau e^{-t/\tau}}$

error signal, $e(t) = r(t) - c(t)$

$= t - [t - \tau + \tau e^{-t/\tau}]$

$= \tau(1 - e^{-t/\tau})$



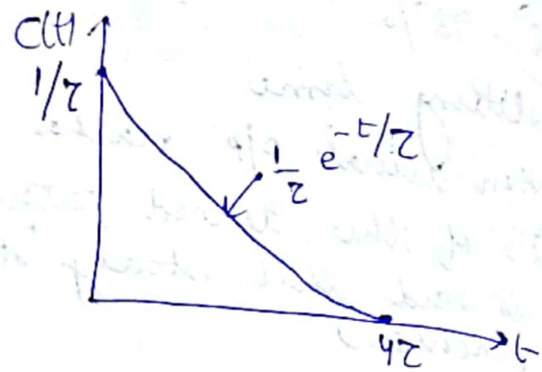
(*) less τ, more speed.

RESPONSE OF UNIT IMPULSE FUNTION (FIRST ORDER)

$$C(s) = R(s)\frac{1}{s\tau+1} = 1 \cdot \frac{1}{\tau s+1}$$

$$C(s) = \frac{1}{\tau} \cdot \frac{1}{s+1/\tau}$$

$$\boxed{C(t) = \frac{1}{\tau} e^{-t/\tau}}$$



For LTI systems,

$$\frac{d}{dt} C(t) \text{ of unit ramp} = C(t) \text{ of unit step i/p.}$$

TIME RESPONSE OF SECOND ORDER SYSTEMS

Here $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

So, generalized transfer func we get is:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Where $\omega_n \to$ undamped natural freq

$\xi \to$ damping factor

$\omega_d \to$ freq of damped oscillations

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

if $\xi = 0 \to$ undamped system

$0<\xi<1 \to$ under damped

$\xi = 1 \to$ critically damped

$\xi > 1 \to$ overdamped system

Scanned with CamScanner

Characteristic eq^n:

$$s^2 + 2\xi_p \omega_n s + \omega_n^2 = 0$$

$$s_1, s_2 = \frac{-2\xi_p\omega_n \pm \sqrt{(2\xi_p\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= -\xi_p\omega_n \pm \omega_n\sqrt{\xi_p^2 - 1}$$

When $\xi_p = 0$,

$$s_1, s_2 = \pm j\omega_n \quad \text{(purely imaginary)}$$

When $\xi_p = 1$,

$$s_1, s_2 = -\omega_n \quad \text{(purely real)}$$

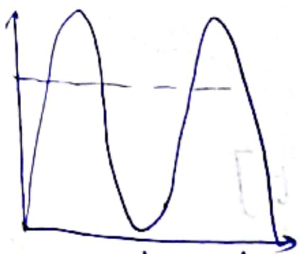
When $0 < \xi_p < 1$,

$$s_1, s_2 = -\xi_p\omega_n \pm j\omega_n\sqrt{1-\xi_p^2} \quad \text{(complex conjugates)}$$

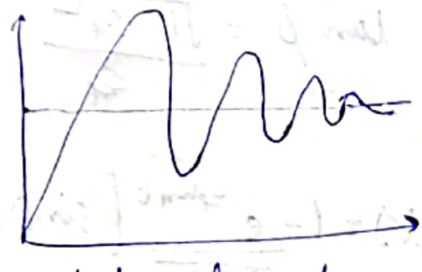
$$= -\xi_p\omega_n \pm j\omega_d$$

When $\xi_p > 1$,

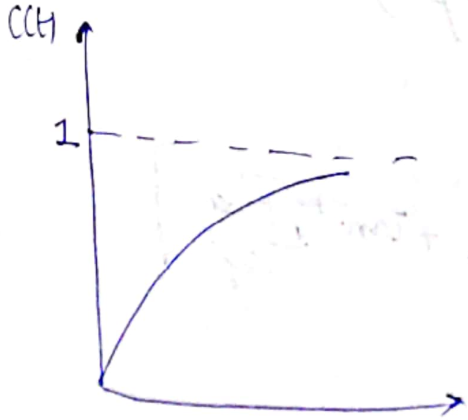
$$s_1, s_2 = -\xi_p\omega_n \pm \omega_n\sqrt{\xi_p^2 - 1}$$



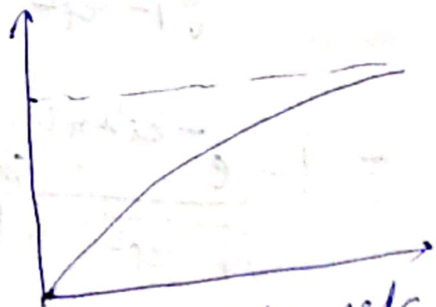
Undamped



Under-damped.



Critically damped



over damped

RESPONSE OF 2ND ORDER SYSTEM WITH UNIT STEP I/P

$$\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

On simplification,

$$C(s) = \frac{1}{s} - \frac{s + \xi\omega_n + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}$$

$$\Rightarrow c(t) = 1 - \left[e^{-\xi\omega_n t} \cos\omega_d t + \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi \sin\omega_d t\right]$$

Put $\sqrt{1-\xi^2} = \sin\phi$

$\therefore \cos\phi = \xi$

$\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$

$$\therefore c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\phi\cos\omega_d t + \cos\phi \sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left((\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Case 1) For undamped case, $\xi = 0$

$$c(t) = 1 - \sin\left(\omega_n t + \frac{\pi}{2}\right)$$

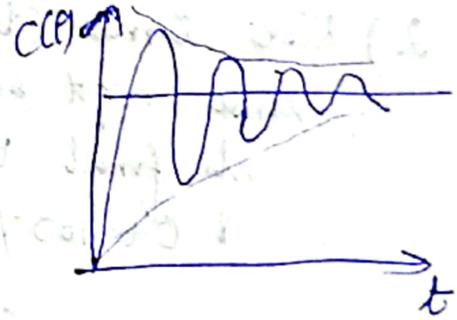
$$\boxed{c(t) = 1 - \cos\omega_n t}$$

Case 2: For underdamped system, $0 < \xi < 1$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

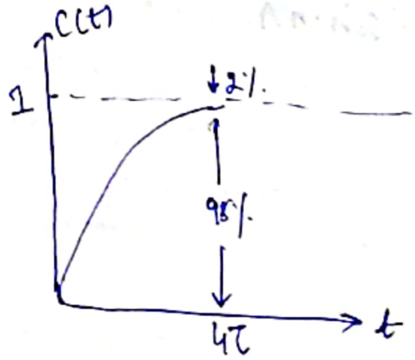


Case 3: Critically damped system, $\xi = 1$

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

In this oscillations disappear.

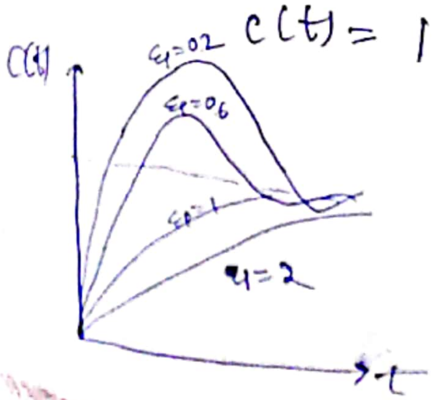
Damping ratio, $\xi = \frac{\text{Actual damping}}{\text{Critical damping}} = \frac{\xi\omega_n}{\omega_n}$



time constant $= \frac{1}{\xi\omega_n}$

Case 4: Overdamped system, $\xi > 1$

$$c(t) = 1 + \frac{e^{-\omega_n t}}{2\sqrt{\xi^2-1}}\left[\frac{e^{\xi+\sqrt{\xi^2-1}}}{\xi+\sqrt{\xi^2-1}} - \frac{e^{\xi-\sqrt{\xi^2-1}}}{\xi-\sqrt{\xi^2-1}}\right]$$



$$\tau_1 = \frac{1}{\omega_n(\xi+\sqrt{\xi^2-1})}$$

$$\tau_2 = \frac{1}{\omega_n(\xi-\sqrt{\xi^2-1})}$$

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TIME - DOMAIN SPECIFICATIONS

- desired performance characteristics of control systems are specified in-Terms of time domain specification.

1) Delay time (t_d)

- time taken by response to reach 50% of final value in first time. ~~in~~ f

2) Rise time. (t_r):

time reqd by response to rise from 10% to 90% of its final value. for overdamped system & 0 to 100%. for underdamped systems.

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left[(\omega_n\sqrt{1-\xi^2})t + \phi\right]$$

At $t = t_r$, $C(t) = 1$

$$\Rightarrow \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left[(\omega_n\sqrt{1-\xi^2})t_r + \phi\right] = 0$$

$$\Rightarrow \sin\left[(\omega_n\sqrt{1-\xi^2})t_r + \phi\right] = 0 = \sin n\pi$$

Put $n = 1$

$$(\omega_n\sqrt{1-\xi^2})t_r + \phi = \pi$$

$$\Rightarrow t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}}$$

3) Peak time (t_p): time reqd for the response to the reach the first peak of the time response or first peak overshoot.

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n\sqrt{1-\xi^2}\, t_p + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

For Max, $\frac{d}{dt}C(t_p) = 0$

$$\rightarrow t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$$

For first peak put $n=1$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

The first minimum undershoot occurs at $n=2$

$$t_{min} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

4) Maximum Overshoot / Peak overshoot (M_p)
– normalized difference b/w the peak of time response & steady output.

$$\text{Maximum \% overshoot} = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100$$

Put $t = t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ in $C(t)$, You will get:

$$C(t) = 1 - \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi+\phi)$$

$$= 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\phi$$

$$C(t)_{max} = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \quad (\text{by putting } \sin\phi = \sqrt{1-\xi^2})$$

$$M_p = C(t)_{max} - 1$$

$$= \frac{e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}}$$

$$\% M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \times 100$$

5) Settling time (t_s):

time reqd for response to reach & stay within the specified range (2 to 5 %) of its final value.

$$t_s = 4\tau = \frac{4}{\xi \omega_n}$$

6) Steady state error (e_{ss}):

difference b/w actual output & desired o/p as time 't' tends to ∞.

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

$$\boxed{t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n \sqrt{1-\xi^2}}}$$

$$\boxed{t_s = \frac{4}{\xi \omega_n}}$$

$$\boxed{t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}}$$

$$\boxed{M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \times 100}$$

✪ Type number of Control system:

→ specified for loop transfer function $G(s)H(s)$

→ no. of poles of loop transfer function lying at the origin decides the type no. of system.

poles → zeroes of D^r

$$G(s)\,H(s) = \frac{K\,(s+z_1)(s+z_2)\ldots}{s^N\,(s+P_1)(s+P_2)\ldots}$$

N = no. of poles at origin

Static error constants :

→ When control sys. is excited with standard input signal, the steady state error may be zero, constant or ∞.

→ The value depends on type number & input signal :

Types :

1) Positional error constant $K_p = \lim_{s \to 0} G(s)\,H(s)$

2) Velocity error constant $K_v = \lim_{s \to 0} s\,G(s)\,H(s)$

3) Acceleration error constant $K_a = \lim_{s \to 0} s^2\,G(s)\,H(s)$

where K_p, K_v, K_a → static error constants.

Steady state error when i/p is unit step signal :

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

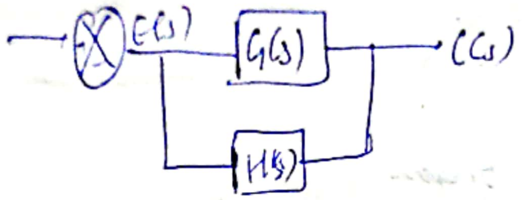
where $E(s) = \frac{R(s)}{1+G(s)H(s)}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{1+G(s)\,H(s)} = \frac{1}{1+\left(\lim_{s \to 0} G(s)\,H(s)\right)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$

Steady state error :

Value of error signal when $t \to \infty$. It is measure of system accuracy. These errors arise from nature of inputs, type of system & from non-linearity of system components.



$$E(s) = R(s) - C(s).H(s)$$
$$= R(s) - [E(s)\,G(s)]\,H(s)$$

$$E(s)[1 + G(s)H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$e(t) = L^{-1}[E(s)]$$
$$= L^{-1}\left[\frac{R(s)}{1+G(s)\,H(s)}\right]$$

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

$$= e(\infty)$$
$$= \lim_{s \to 0} s.E(s)$$ (using final value theorem)

$$\boxed{e_{ss} = \lim_{s \to 0} s\,E(s)}$$

Limitations of time-domain analysis -

- don't give info on steady state error when i/ps are other than standard: step, ramp, ~~parabolic~~ parabolic.
- does not give precise error value
- applicable to stable system
- does not provide variation of error with time which may be needed for design purpose