(Please write your Enrolment No. immediately) **Enrolment No. ....**

MID TERM EXAMINATION

B.TECH PROGRAMMES (UNDER THE AEGIS OF USICT)

Third Semester, November, 2023

Paper Code: ES-201 **Subject: Computational Methods**

Time: $1^1/_2$ Hrs. **Max. Marks: 30**

Note: Attempt Q.No.1 which is compulsory and any two more questions from remaining.

Q. No.	Question	Max. Marks	CO(s)
1(a)	Write the statement of Lagrange's Mean Value Theorem and apply it to the function $f(x) = \sqrt{x}$ in [0,2].	2.5	CO1
1(b)	Find minimum value of $f(x) = x^3 - 3x - 7$, using Newton's Method taking initial approximation $x_1 = 0.5$ and $\epsilon = 0.001$.	2.5	CO1
1(c)	Prove that: $\Delta^2 y_3 = \nabla^2 y_5$.	2.5	CO2
1(d)	If $f(x) = \frac{1}{x}$, find divided differences $[a, b]$ and $[a, b, c]$.	2.5	CO2
2(a)	Find minimum value of $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2^2$, using Steepest descent method such that $\lvert f(X_{k+1}) - f(X_k)\rvert < 0.05$ taking starting point as $X_1 = \left(1, \frac{1}{2}\right)^T$.	5.0	CO1
2(b)	Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places.	5.0	CO2
3(a)	Find the minimum value of $f(x) = x^2 + 2x$ using $n = 4$ within the interval $[-3, 4]$ using Fibonacci Search method.	5.0	CO1
3(b)	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$ and Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$.	5.0	CO2
4(a)	Use Secant method to find a root correct to two decimal places, of the equation $xe^x - 2 = 0$ taking two initial approximations as $x_0 = 0.9$ and $x_1 = 1$.	5.0	CO1
4(b)	Find the polynomial of the lowest degree which assumes the values 1, 27 and 64 when x takes the values 1, 3 and 4 respectively, using Lagrange's interpolation formula and hence find $f(2)$.	5.0	CO2

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CSE-IIIA

Mid-Term 2023

CM Question Paper Solution

Q1 a) Lagrange Mean value Theorem Statement: If a function $f(x)$ is continuous on closed interval $[a,b]$ and differentiable on open interval (a,b), then there exist atleast one point $c \in (a,b)$ s.t $f'(c) = \frac{f(b)-f(a)}{b-a}$

Function $f(x) = \sqrt{x}$ is cts on $[0,2]$ and diff on open interval $(0,2)$ $\therefore$ Lagrange mean value theorem can be applied to it.

$\therefore \exists\, c \in (0,2)$ s.t. $f'(c) = \frac{f(2)-f(0)}{2-0}$

$$\frac{1}{2\sqrt{c}} = \frac{\sqrt{2}-\sqrt{0}}{2-0}$$

$$\frac{1}{2\sqrt{c}} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{c}} = \sqrt{2}$$

$$\sqrt{c} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow c = \frac{1}{2} \quad \text{(squaring)}$$

$\therefore \exists\, c = \frac{1}{2} \in (0,2)$ s.t $f'(\frac{1}{2}) = \frac{f(2)-f(0)}{2-0}$.

b) Minimize $f(x) = x^3 - 3x - 7$ using Newton's Method. $x_0 = 0.5$ & $\varepsilon = 0.001$

Iterative formula is:

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}, \quad n = 1, 2, 3, \ldots$$

$f(x) = x^3 - 3x - 7$

$\therefore f'(x) = 3x^2 - 3$

$f''(x) = 6x$

Iterative formula becomes:

$$x_{n+1} = x_n - \frac{3x_n^2 - 3}{6x_n}$$

$$= \frac{6x_n^2 - 3x_n^2 + 3}{6x_n}$$

$$= \frac{3x_n^2 + 3}{6x_n}$$

$$= \frac{x_n^2 + 1}{2x_n}.$$

$n = 1$

$$x_2 = \frac{x_1^2 + 1}{2x_1} = \frac{(0.5)^2 + 1}{2(0.5)}$$

$$= \frac{0.25 + 1}{1} = 1.25$$

$n = 2$

$$x_3 = \frac{x_2^2 + 1}{2x_2} = \frac{(1.25)^2 + 1}{2(1.25)}$$

$$= \frac{1.5625 + 1}{2.5}$$

$$= \frac{2.5625}{2.5}$$

$$= 1.025$$

$n = 3$

$$x_4 = \frac{x_3^2 + 1}{2x_3} = \frac{(1.025)^2 + 1}{2(1.025)}$$

$$= \frac{2.050625}{2.050}$$

$$= 1.00030487$$

$n = 4$

$$x_5 = \frac{x_4^2 + 1}{2x_4} = \frac{(1.0003)^2 + 1}{2(1.0003)} = \frac{2.00060009}{2.0006} \cancel{= 1.50045}$$

$$= 1.00000004$$

As $|x_5 - x_4| < 0.001$, we will stop here

Minimum value of $f(x) = x^3 - 3x - 7$ occurs at $x = 1$

& minimum value is $f(1) = 1^3 - 3(1) - 7$

$= 1 - 3 - 7$

$= 1 - 10$

$= -9$.

c) Prove: $\Delta^2 y_3 = \nabla^2 y_5$

LHS: $\Delta^2 y_3 = \Delta(\Delta y_3)$

$= \Delta(y_4 - y_3)$

$= \Delta y_4 - \Delta y_3$

$= (y_5 - y_4) - (y_4 - y_3)$

$= y_5 - 2y_4 + y_3$

RHS: $\nabla^2 y_5 = \nabla(\nabla y_5)$

$= \nabla(y_5 - y_4)$

$= \nabla y_5 - \nabla y_4$

$= (y_5 - y_4) - (y_4 - y_3)$

$= y_5 - y_4 - y_4 + y_3$

$= y_5 - 2y_4 + y_3$.

LHS = RHS

$\therefore$ Hence proved.

d) $f(x) = \frac{1}{x}$.

$$f[a,b] = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{\frac{1}{b}-\frac{1}{a}}{b-a}$$

$$= \frac{\frac{a-b}{ba}}{b-a}$$

$$= \frac{-1}{ab}.$$

$$f[a,b,c] = \frac{f[b,c]-f[a,b]}{c-a}$$

$$= \frac{\frac{-1}{bc} - \left(\frac{-1}{ab}\right)}{c-a}.$$

$$= \frac{\frac{-1}{bc} + \frac{1}{ab}}{c-a}$$

$$= \frac{\frac{-a+c}{abc}}{c-a}$$

$$= \frac{1}{abc}.$$

a) Minimize $f(x_1, x_2) = x_1^2 - x_1 x_2 + x_2^2$ using steepest Descent.

Starting pt $X_1 = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$.

$$\nabla f(x_1, x_2) = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right\rangle$$

$$= \langle 2x_1 - x_2, -x_1 + 2x_2 \rangle.$$

$$= (2x_1 - x_2)\hat{i} + (-x_1 + 2x_2)\hat{j}.$$

<u>1st Iteration X_2</u>

$$\nabla f(X_1) = \nabla f(1, 1/2)$$

$$= \langle 2(1) - \tfrac{1}{2}, -1 + 2(\tfrac{1}{2}) \rangle$$

$$= \langle 3/2, 0 \rangle.$$

$$= \frac{3}{2}\hat{i} + 0\hat{j}.$$

$$S_1 = -\nabla f(X_1) = -\langle 3/2, 0 \rangle$$

$$X_2 = X_1 + \lambda S_1$$

$$= (1, 1/2) - \lambda(3/2, 0)$$

$$= (1 - \tfrac{3}{2}\lambda, 1/2).$$

$$f(X_2) = f(1 - 3/2\lambda, 1/2)$$

$$f(X_2) = (1 - 3/2\lambda)^2 - (1 - 3/2\lambda)(\tfrac{1}{2}) + (1/2)^2$$

$$\frac{df(X_2)}{d\lambda} = 2(1 - 3/2\lambda)(-3/2) + \frac{3}{2}(\tfrac{1}{2}) + 0$$

$$\frac{df(X_2)}{d\lambda} = 0$$

$$-3\left(1-\frac{3}{2}\lambda\right) + \frac{3}{4} = 0$$

$$\frac{1}{4} = 1-\frac{3}{2}\lambda$$

$$\frac{1}{4} = \frac{2-3\lambda}{2}$$

$$\frac{1}{2} = 2-3\lambda$$

$$1 = 4-6\lambda$$

$$6\lambda = 3$$

$$\lambda = 1/2$$

$$\therefore \quad X_2 = \left(1-3/2(1/2), 1/2\right)$$

$$= \left(1/4, 1/2\right)$$

$$f(X_1) = f(1, 1/2) = (1)^2 - (1)\left(\frac{1}{2}\right) + \left(1/2\right)^2$$

$$= 1-\frac{1}{2}+\frac{1}{4} = \frac{4-2+1}{4} = 3/4$$

$$f(X_2) = f(1/4, 1/2) = (1/4)^2 - (1/4)(1/2) + (1/2)^2$$

$$= \frac{1}{16} - \frac{1}{8} + \frac{1}{4}$$

$$= \frac{1-2+4}{16} = 3/16$$

$$|f(X_2)-f(X_1)| = \left|\frac{3}{16} - 3/4\right| = \left|-\frac{9}{16}\right| = 0.5625$$
$$\not< 0.05$$

$\therefore$ Continue

$\therefore X_3 = \left(\frac{1}{4}, \frac{1}{2} - \frac{3}{4}\left(\frac{1}{2}\right)\right)$

$= \left(\frac{1}{4}, \frac{1}{8}\right)$

$f(X_3) = f\left(\frac{1}{4}, \frac{1}{8}\right) = \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{8}\right)^2$

$= \frac{1}{16} - \frac{1}{32} + \frac{1}{64}$

$= \frac{4-2+1}{64}$

$= \frac{3}{64}$

$|f(X_3) - f(X_2)| = \left|\frac{3}{64} - \frac{3}{16}\right|$

$= \frac{9}{64}$

$= 0.140625 \not< 0.05$

$\therefore$ Continue.

<u>3rd iteration X_4</u>

$\nabla f(X_3) = \nabla f\left(\frac{1}{4}, \frac{1}{8}\right)$

$= \left\langle 2\left(\frac{1}{4}\right) - \frac{1}{8}, -\frac{1}{4} + 2\left(\frac{1}{8}\right)\right\rangle$

$= \left\langle \frac{3}{8}, 0\right\rangle$

$S_3 = -\nabla f(X_3) = -\left\langle \frac{3}{8}, 0\right\rangle$

$X_4 = X_3 + \lambda S_3$

$= \left(\frac{1}{4}, \frac{1}{8}\right) - \lambda\left(\frac{3}{8}, 0\right)$

$= \left(\frac{1}{4} - \frac{3}{8}\lambda, \frac{1}{8}\right).$

2^{nd} Iteration X_3

$$\nabla f(X_2) = \nabla f(1/4, 1/2)$$

$$= \langle 2(1/4) - 1/2 \,,\, -1/4 + 2(1/2) \rangle$$

$$= \langle 0 \,,\, 3/4 \rangle$$

$$S_2 = -\nabla f(X_2)$$

$$= -\langle 0, 3/4 \rangle$$

$$X_3 = X_2 + \lambda S_2$$

$$= \left(\tfrac{1}{4}, 1/2\right) - \lambda\left(0, 3/4\right)$$

$$= \left(1/4 \,,\, \tfrac{1}{2} - \tfrac{3}{4}\lambda\right).$$

$$f(X_3) = f\left(\tfrac{1}{4}, \tfrac{1}{2} - \tfrac{3}{4}\lambda\right)$$

$$= \left(\tfrac{1}{4}\right)^2 - \left(\tfrac{1}{4}\right)\left(\tfrac{1}{2} - \tfrac{3}{4}\lambda\right) + \left(\tfrac{1}{2} - \tfrac{3}{4}\lambda\right)^2$$

$$\frac{df(X_3)}{d\lambda} = 0 - \left(\tfrac{1}{4}\right)\left(-3/4\right) + 2\left(\tfrac{1}{2} - 3/4\,\lambda\right)\left(-3/4\right)$$

$$= \frac{3}{16} - \frac{3}{2}\left(\frac{2 - 3\lambda}{4}\right)$$

$$\frac{df(X_3)}{d\lambda} = 0$$

$$\frac{3}{16} - \frac{3(2-3\lambda)}{8} = 0$$

$$\frac{1}{2} = 2 - 3\lambda$$

$$1 = 4 - 6\lambda$$

$$6\lambda = 3 \Rightarrow \lambda = 1/2.$$

$$f(X_4) = \left(\frac{1}{4} - \frac{3}{8}\lambda\right)^2 - \left(\frac{1}{4} - \frac{3}{8}\lambda\right)\left(\frac{1}{8}\right) + \left(\frac{1}{8}\right)^2$$

$$\frac{df(X_4)}{d\lambda} = 2\left(\frac{1}{4} - \frac{3}{8}\lambda\right)\left(-\frac{3}{8}\right) + \frac{3}{8}\left(\frac{1}{8}\right) + 0$$

$$\frac{df(X_4)}{d\lambda} = 0$$

$$-\frac{3}{4}\left(\frac{2-3\lambda}{8}\right) + \frac{3}{64} = 0$$

$$\frac{1}{2} = 2 - 3\lambda$$

$$1 = 4 - 6\lambda$$

$$6\lambda = 3$$

$$\lambda = \frac{1}{2}$$

$$\therefore X_4 = \left(\frac{1}{4} - \frac{3}{16}, \frac{1}{8}\right)$$

$$= \left(\frac{1}{16}, \frac{1}{8}\right)$$

$$f(X_4) = \left(\frac{1}{16}\right)^2 - \left(\frac{1}{16}\right)\left(\frac{1}{8}\right) + \left(\frac{1}{8}\right)^2$$

$$= \frac{1}{256} - \frac{1}{128} + \frac{1}{64}$$

$$= \frac{1-2+4}{256}$$

$$= \frac{3}{256}$$

$$|f(X_4) - f(X_3)| = \left|\frac{3}{256} - \frac{3}{64}\right|$$

$$= \left|\frac{9}{256}\right| = 0.0351562 < 0.05$$

so we will stop here.

Minimum value of ~~f(x)~~ $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2$ occurs at $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/16 \\ 1/8 \end{pmatrix}$ & min. value is $f(1/16, 1/8) = \frac{3}{256} \approx 0.01172$.

Q2 b) Compute Integral $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places using Romberg's Method.

→ lower limit $a = 0$
Upper limit $b = 1$
Integrand $y = f(x) = \frac{1}{1+x^2}$.

Compute I_1 (Integral using Trapezoidal rule with width $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$).

$x_0 = a = 0$, $x_1 = a+h = 0+0.5 = 0.5$, $x_2 = a+2h = 0+2(0.5) = 1$.

$y_0 = f(x_0) = f(0) = \frac{1}{1+0^2} = 1$
$y_1 = f(x_1) = f(0.5) = \frac{1}{1+(0.5)^2} = 0.8$
$y_2 = f(x_2) = f(1) = \frac{1}{1+1^2} = 0.5$

$\therefore I_1 = \frac{h}{2}(y_0 + y_2 + 2y_1)$
$= \frac{0.5}{2}(1 + 0.5 + 2(0.8)) = 0.775$.

Now compute I_2 (Integrand using trapezoidal rule with width $h = \frac{b-a}{4} = 0.25$).

$x_0 = a = 0$, $x_1 = a+h = 0.25$, $x_2 = a+2h = 0.5$,
$x_3 = a+3h = 0.75$, $x_4 = a+4h = 1$.

$y_0 = f(x_0) = f(0) = 1$

$y_1 = f(x_1) = f(0.25) = \frac{1}{1+(0.25)^2} = 0.9412$

$y_2 = f(x_2) = f(0.5) = 0.8$

$y_3 = f(x_3) = f(0.75) = \frac{1}{1+(0.75)^2} = 0.64$

$y_4 = f(x_4) = f(1) = \frac{1}{1+1^2} = 0.5.$

$I_2 = \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$

$= \frac{0.25}{2}[1 + 0.5 + 2(0.9412 + 0.8 + 0.64)]$

$= 0.125(1.5 + 4.7624)$

$= 0.125(6.2624)$

$= 0.7828.$

Compute I_3: (Integral value using trapezoidal rule with subinterval width $h = \frac{b-a}{8}$) $= \cancel{0}\ \frac{1}{8} = 0.125$)

$x_0 = a = 0$, $x_1 = 0 + h = 0.125$, $x_2 = 0 + 2h = 0.25$;

$x_3 = 0.375$, $x_4 = 0.5$, $x_5 = 0.625$, $x_6 = 0.75$,

$x_7 = 0.875$, $x_8 = 1.$

$y_0 = f(x_0) = f(0) = 1$

$y_1 = f(x_1) = f(0.125) = \frac{1}{1+(0.125)^2} = 0.9846$

$y_2 = f(x_2) = f(0.25) = 0.9412$

$y_3 = f(x_3) = f(0.375) = \frac{1}{1+(0.375)^2} = 0.8767$

$y_4 = f(x_4) = f(0.5) = 0.8$

$y_5 = f(x_5) = f(0.625) = \frac{1}{1+(0.625)^2} = 0.7191$

$y_6 = f(x_6) = f(0.75) = 0.64$

$y_7 = f(x_7) = f(0.875) = \frac{1}{1+(0.875)^2} = 0.5664$

$y_8 = f(x_8) = f(1) = 0.5$

$I_3 = \frac{h}{2}[y_0 + y_8 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)]$

$= \frac{0.125}{2}[1 + 0.5 + 2(0.9846 + 0.9412 + 0.9412 + 0.8767 + 0.8 + 0.7191 + 0.64 + 0.5664)$.

$= 0.7848$.

Now compute I_1', I_2'.

$I_1' = I_2 + \frac{1}{3}(I_2 - I_1)$

$= 0.7828 + \frac{1}{3}(0.7828 - 0.775)$

$= 0.7854$

$I_2' = I_3 + \frac{1}{3}(I_3 - I_2)$

$= 0.7848 + \frac{1}{3}(0.7848 - 0.7828)$

$= 0.7855$

Now compute I_1''

$I_1'' = I_2' + \frac{1}{3}(I_2' - I_1')$

$= 0.7855 + \frac{1}{3}(0.7855 - 0.7854)$

$= 0.7855$

As $I_1'' = I_2' = 0.7855$ we will stop here.

$\therefore$ Ans $\int_0^1 \frac{1}{1+x^2}dx = 0.7855$

a) Minimum value of $f(x) = x^2 + 2x$ using $n = 4$, within interval $[-3, 4]$ using fibonacci Search Method.

→ Fibonacci sequence is:

$$\underset{F_0}{1},\ \underset{F_1}{1},\ \underset{F_2}{2},\ \underset{F_3}{3},\ \underset{F_4}{5}.$$

lower bound of interval $a = -3$

Upper bound of interval $b = 4$.

$L = b - a = 4 - (-3) = 7$

$$L_2^* = \frac{F_{n-2}}{F_n} L$$

$$= \frac{F_{4-2}}{F_4} \times L$$

$$= \frac{F_2}{F_4} \times L$$

$$= \frac{2}{5} \times 7$$

$$= \frac{14}{5}$$

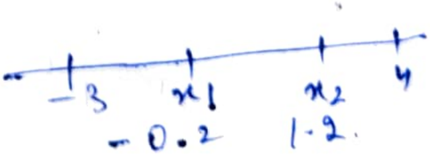
$$= 2.8$$

$x_1 = a + L_2^* = -3 + 2.8 = -0.2$

$x_2 = b - L_2^* = 4 - 2.8 = 1.2$

$f(x_1) = f(-0.2) = (-0.2)^2 + 2(0.2) = 0.04 - 0.4 = -0.36$

$f(x_2) = f(1.2) = (1.2)^2 + 2(1.2)$
$= 1.44 + 2.4$
$= 3.84$



As $f(x_1) < f(x_2)$, discard $(1.2, 4)$.

New interval is $[-3, 1.2]$

length of New interval $L_2 = L - L_2^*$

$= 7 - 2.8$

$= 4.2$.

As $K = 2 \neq 4 = n$, ∴ therefore continue & take $K = 3$.

$L_3^* = \frac{F_{n-3}}{F_4} \times L$

$= \frac{F_{4-3}}{F_4} \times L$

$= \frac{F_1}{F_4} \times L$

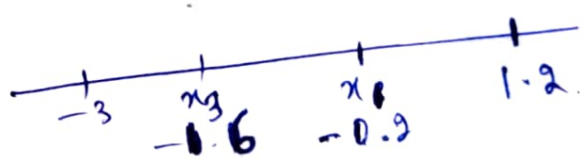
$= \frac{1}{5} \times 7$

$= 1.4$.

$x_3 = -3 + L_3^*$

$= -3 + 1.4$

$= -1.6$



$x_4 = 1.2 - 1.4$

$= -0.2 = x_1$.

$3.20 - 2.56 = 0.64$

$f(x_3) = f(-1.6) = (-1.6)^2 + 2(-1.6)$

$= 2.56 \cancel{+} - 3.2$

$= \cancel{5.76}\ -0.64$

As $f(x_1) \cancel{>} f(x_3)$ ∴ therefore discard $\cancel{[-3, -1.6]}$ $[-0.2, 1.2]$

New interval is $\cancel{[x_3, 1.2]} = \cancel{[-1.6, 1.2]}$ $[-3, -0.2]$

length of new interval $= L_2 - \cancel{(-1.6+3)}\ (1.2+0.2)$

$= 4.2 - 1.4$

$= 2.8$

As $K = 3 \neq 4 = n$, therefore, continue & take $K = 4$.

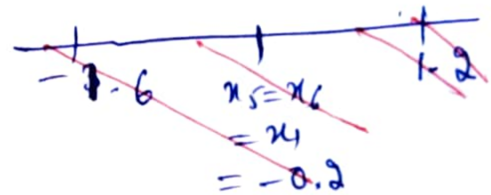
$$L_4^* = \frac{F_{n-4}}{F_n} \times L$$

$$= \frac{F_{4-4}}{F_4} \times L$$

$$= \frac{F_0}{F_4} \times L$$

$$= \frac{1}{5} \times 7$$

$$= 1.4$$



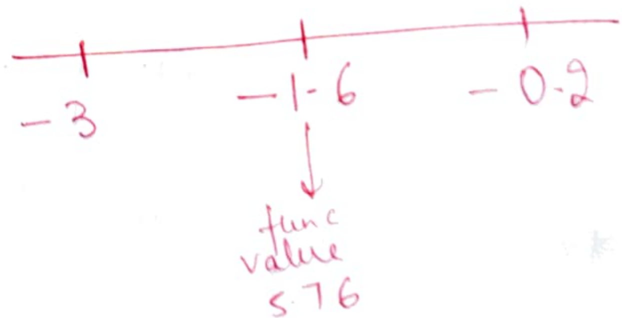
$x_5 = -3.0 + 1.4 = -1.6 = x_3$

$x_6 = -0.2 - 1.4 = -1.6$.

$f(-1.6) = -0.64$

$\cancel{f(-0.2) = 0.44}$

$\cancel{f(1.2) = 3.84}$.



Q3 b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using simpson's $\frac{1}{3}$ rule. taking $h = 1/4$.

$a = 0$

$b = 1$

Integrand $= y = f(x) = \frac{1}{1+x^2}$

Width of subinterval $h = 1/4$.

$x_0 = a = 0$, $x_1 = a+h = 1/4$, $x_2 = a+2h = 1/2$, $x_3 = a+3h = 3/4$,

$x_4 = a+4h = 1$.

∴

$x_0 = 0$	$x_1 = 1/4$	$x_2 = 1/2$	$x_3 = 3/4$	$x_4 = 1$
$y_0 = \frac{1}{1+0^2} = 1$	$y_1 = \frac{1}{1+(1/4)^2} = 0.941176$	$y_2 = \frac{1}{1+(1/2)^2} = 0.8$	$y_3 = \frac{1}{1+(3/4)^2} = 0.64$	$y_4 = \frac{1}{1+1^2} = 0.5$

$$\int_0^1 \frac{dx}{1+x^2} \approx \frac{h}{3}\left[y_0 + y_4 + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{1}{4(3)}\left[1 + 0.5 + 4(0.941176 + 0.64) + 2(0.8)\right]$$

$$= \frac{1}{12}\left[1.5 + 4(1.581176) + 1.6\right]$$

$$= 0.785392.$$

$\int_0 \frac{dx}{1+x^2}$ using simpson 3/8 rule with $h = 1/6$.

$x_0 = 0$	$x_1 = 1/6$	$x_2 = 2/6$	$x_3 = 3/6$	$x_4 = 4/6$	$x_5 = 5/6$	$x_6 = 6/6 = 1$
$y_0 = \frac{1}{1+0^2} = 1$	$y_1 = \frac{1}{1+(1/6)^2} = 0.97297$	$y_2 = \frac{1}{1+(2/6)^2} = 0.9$	$y_3 = 0.8$	$y_4 = \frac{1}{1+(4/6)^2} = 0.692307$	$y_5 = \frac{1}{1+(5/6)^2} = 0.590164$	$y_6 = 0.5$

$$\int_0 \frac{dx}{1+x^2} \approx \frac{3h}{8}\left[y_0 + y_6 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$$= \frac{3}{8}\left(\frac{1}{6}\right)\left[1 + 0.5 + 3(0.97297 + 0.9 + 0.692307 + 0.590164) + 2(0.8)\right]$$

$$= \frac{3}{48}\left[1.5 + 9.466323 + 1.6\right]$$

$$= 0.785395187$$

Q4 a) find root of $f(x) = xe^x - 2$ using secant method correct to two decimal places using $x_0 = 0.9$ & $x_1 = 1$.

$f(x_0) = f(0.9) = (0.9)e^{0.9} - 2 = 0.2136428$

$f(x_1) = f(1) = 1e^1 - 2 = 0.718281828$

Secant Method formula is:

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) \quad ; n = 1, 2, 3, \ldots$$

$\underline{n=1}$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_1)$$

$$= 1 - \frac{(1 - 0.9)}{(0.718281828 - 0.2136428)} (0.718281828)$$

$$= 0.85766$$

$$f(x_2) = f(0.85766)$$

$$= (0.85766)\, e^{0.85766} - 2$$

$$= 0.02205126$$

$\underline{n=2}$

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_2)$$

$$= 0.85766 - \frac{0.85766 - 1}{0.02205126 - 0.718281828} (0.02205126)$$

$$= 0.8531517 5733$$

As $|x_2 - x_3| < 0.01$,

$\therefore$ Ans is $\boxed{0.85}$ correct to two decimal places.

$f(x) = 0$

Q4 b)

	x_0	x_1	x_2
x	1	3	4
y	y_0 1	y_1 27	y_2 64

Lagrange polynomial is:

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-3)(x-4)}{(1-3)(1-4)}(1) + \frac{(x-1)(x-4)}{(3-1)(3-4)}(27) + \frac{(x-1)(x-3)}{(4-1)(4-3)}(64)$$

$$= \frac{x^2-7x+12}{(-2)(-3)} + \frac{x^2-5x+4}{(2)(-1)}(27) + \frac{x^2-4x+3}{(3)(1)}(64).$$

$$= \frac{x^2-7x+12}{6} - \frac{27x^2-135x+108}{2} + \frac{64x^2-256x+192}{3}$$

$$= \frac{x^2-7x+12 - 81x^2+405x-324+128x^2-512x+384}{6}$$

$$= \frac{48x^2-114x+72}{6} = 8x^2-19x+12.$$

$$y = f(2) = 8(2)^2-19(2)+12 = 32-38+12 = 6$$

384
12
396
324
72

119
405
114