

Maxwell's Eqⁿ in time domain UNIT-I

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -j\omega H$$

Electric field int.	$\nabla \times E = -\frac{\partial B}{\partial t}$	- Faraday's electromotive force law
Mag. field int.	$\nabla \times H = J + \frac{\partial D}{\partial t}$	
E-flux density	$\nabla \cdot D = \rho_v$	- Gauss law
M. flux density	$\nabla \cdot B = 0$	

$\rho = 0$
 $\sigma = 0$
 No conducting material in free space

Unit
 $E \rightarrow$ Electric field intensity V/m
 H - Mag. " " Amp/m
 D - Electric flux density C/m^2
 B - Mag. flux density Wb/m^2

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= J + j\omega E$$

$$= E(j\sigma + j\omega \epsilon)$$

$$\nabla \times H = j\omega E$$

$J = J_c + J_o$ - Impressed current -

\downarrow
 $J = \sigma E$ - Conduction current density caused by the field

$B = \mu H$
 $D = \epsilon E$
 $J = \sigma E$
 $\epsilon = \epsilon_0 \epsilon_r$
 $\mu = \mu_0 \mu_r$

Maxwell Eqⁿ in free domain

$$\nabla \times E = -\frac{\partial \mu H}{\partial t}$$

$$\nabla \times E = -j\omega \mu H$$

$$E = E_0 e^{j\omega t}$$

Assume that x in the form of $e^{j\omega t}$
 \downarrow max electric field intensity

$$\frac{\partial E}{\partial t} = E_0 e^{j\omega t} \cdot j\omega$$

then $\left[\frac{\partial}{\partial t} = j\omega \right]$
 $\omega = 2\pi f$
 f - freq of sinusoidal variation

$$\frac{\partial^2}{\partial t^2} = (j\omega)^2$$

$$\left[\frac{\partial^2}{\partial t^2} = -\omega^2 \right]$$

Now curl of $\nabla \times E$
 $\nabla \times \nabla \times E = -j\omega \mu (\nabla \times H)$
 $-\nabla^2 E + \nabla(\nabla \cdot E) = -j\omega \mu (j\omega \epsilon E)$
 $\nabla^2 E = -(\omega)^2 \mu \epsilon E$

$$\nabla^2 E = -\omega^2 \mu \epsilon E$$

Similarly $\nabla^2 H = -\omega^2 \mu \epsilon H$

$$\mu \epsilon = \frac{1}{c^2}$$

Intrinsic Propagation
 $\gamma = \alpha + j\beta$ - Phase const.
 \downarrow
 Propagation const. Attenuation const.

$$\gamma = \sqrt{j\omega \mu (\sigma + j\omega \epsilon)}$$

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

Similarly

We know $\mu \epsilon = \mu_0 \epsilon_r \cdot \epsilon_0 \epsilon_r$
that

$$\mu_r = 1$$

$$4\pi \times 10^{-7} \text{ H/m} \quad \left(\frac{1}{36\pi} \cdot 10^{-9} \text{ f/m} \right)$$

farads/meter

$$= \frac{4\pi \times 10^{-7} \times 10^{-9}}{36\pi}$$

$$= \frac{1}{9 \times 10^{16}}$$

$$\mu \epsilon = \frac{1}{(3 \times 10^8)^2}$$

$$\mu \epsilon = \frac{1}{c^2} \quad c^2 = \frac{1}{\mu \epsilon}$$

$$c = \frac{1}{\sqrt{\mu \epsilon}} \Rightarrow 3 \times 10^8 \text{ m/sec}$$

If wave propagate through other medium then velocity then air

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

dielectric constt of the medium

A changing electric field produced mag. field and changing magnetic field induced a changing electric field in the surrounding region



Mode

TEM - Both Electric and mag. fields are purely Transverse to the direction of propagation and consequently have no z directed component:

$$E_z = 0 \quad H_z = 0$$

② TE

$$E_z = 0 \quad H_z \neq 0$$

③ TM

$$E_z \neq 0 \quad H_z = 0$$

④ HE (Hybrid) wave

$$E_z \neq 0 \quad H_z \neq 0$$

[300 MHz - 300 GHz]

[3 KHz - 300 GHz]

(1)

- (1) MW freq range is from 300 MHz - 300 GHz
 so that wavelengths (λ) \downarrow $\lambda = 1m$ \downarrow $1mm$
 may consider in microns ranges

$$1 \mu m = 10^{-6} m$$

(1 Micron)

$$1 \text{ \AA (angstrom)} = 10^{-10} m$$

- (2) means that wave also covers infrared and visible light regions.

10 ¹⁸	Era - E
10 ¹⁵	Peta - P
10 ¹²	tera - T
10 ⁹	Giga - G
10 ⁶	mega - M
10 ³	kilo - K
10 ²	Hecto - h
10	deka - da

- (3) when = 300 MHz.

then $\lambda = \frac{3 \times 10^8}{300 \times 10^6}$

$$\lambda = \frac{c}{f}$$

$$\lambda = 1m$$

when 300 GHz

then $\lambda = \frac{3 \times 10^8}{300 \times 10^9}$

$$\lambda = \frac{3}{300 \times 10^9} = \frac{1}{1000} m \text{ or } \frac{1 \times 10^6}{1000} = .1 \text{ cm}$$

or $.1 \times 10 = 1mm$

$$\frac{1 \times 10}{10}$$

- (4) IEEE freq. band

HF - 3 - 30 MHz
 VHF - 30 - 300 MHz
 UHF - 300 - 1 GHz

L - 1 - 2 "

S - 2 - 4 "

C - 4 - 8 " — Satellite

X - 8 - 12 " — Educational Purpose

KU - 12 - 18 " — DTH, VSAT, GPS.

K - 18 - 27 "

Ka - 27 - 40 "

min - 40 - 300 "

in AM/FM Broadcasting
 GSM — 900 MHz TR 880-915 MHz RX 925-960 MHz GSM RX 1805-1880 MHz
 Radar, Bluetooth, wifi, microwave
 Terrestrial Purpose
 visible light, UV, x ray, γ , Cosmic

10 ⁻¹	deci - d
10 ⁻²	centi - c
10 ⁻³	milli - m
10 ⁻⁶	micro - μ

Advantages of MW

① B.W ↑

Bcz freq. range of information channel will be small % of carrier freq.

B.W for speech — 4 KHz

music — 10-15 KHz.

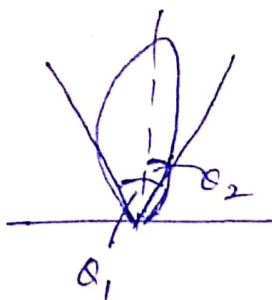
TV — 5-7 MHz.

Ex. ^{consider} speech signal is 4 KHz Bandwidth

when carrier

$$\begin{array}{l} \downarrow 300 \text{ MHz} \\ \text{then} \quad \frac{75}{300 \times 10^6} = \frac{10^3}{4 \times 10^8} = \frac{75000}{\text{channels}} \\ \text{no. of channels.} \end{array} \quad \begin{array}{l} 3 \text{ GHz} \\ \downarrow \times 10^6 \\ \frac{3 \times 10^6}{4 \times 10^8} = 7.5 \text{ Lakhs channels.} \end{array}$$

② As freq. ↑ Directivity ↑ Beamwidth ↓



For Diameter of an antenna

We know that

$$B = \frac{140}{D/\lambda} \quad \begin{array}{l} \text{Beam} \\ \text{in degree} \end{array} \quad \begin{array}{l} \text{Diameter} \\ \text{of an} \\ \text{antenna} \\ \text{in cm} \end{array} \quad \begin{array}{l} \text{wavelength} \\ \text{in cm.} \end{array}$$

$$D = \frac{140 \times \lambda}{B}$$

Consider 1° so that

$$D = \frac{140 \times \lambda}{B}$$

At 30 GHz.

$$D = \frac{140 \times \lambda}{B}$$

$$D = \underline{140 \text{ cm}}$$

when 300 MHz.
 $D = 140 \text{ m.}$

③ Fading Effect and Reliability

- Due to variation in the medium, Fading is more effective at L. freq.
- Due to LOS propagation and high freq. there is less fading effect.

Power Requirement is less at Rx/Tr.

Application - 1. Industrial

2. Domestic - MW oven 2.45 GHz ($\lambda = 12\text{cm}$)

3. Military

4. Radar

5. Comm.

6. Mobile Comm.

7. Sat. Comm.

$$J = \sigma E$$

For wave Eqn.

Maxwell's Eqn.

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{ — Faraday's } B = \mu H$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \text{ — Ampere's } D = \epsilon E$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\mu = \mu_0 \mu_r$$

$$\nabla \cdot D = \rho_v \quad \nabla \cdot B = 0 \quad \left. \vphantom{\begin{matrix} \nabla \cdot D = \rho_v \\ \nabla \cdot B = 0 \end{matrix}} \right\} \text{ Gauss}$$

$$\nabla \times E = -j\omega \mu H$$

$$\nabla \times H = j\omega \epsilon E$$

EMT wave follow exponential form then

$$E = E_0 e^{j\omega t}$$

$$\frac{\partial E}{\partial t} = j\omega \cdot E_0 e^{j\omega t}$$

$$\frac{\partial E}{\partial t} = j\omega \cdot E$$

$$\text{or } \frac{\partial^2 E}{\partial t^2} = (j\omega)^2 \cdot E$$

$$\frac{\partial^2}{\partial t^2} = j\omega$$

$$\frac{\partial^2}{\partial t^2} = (j\omega)^2 = -\omega^2$$

Ques no. 1 (a) write Maxwell's Equation in Differential and in integral form.

Ans. Maxwell's Equations:

- $\oint_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$ Gauss's law electric.
- $\oint_S \vec{B} \cdot d\vec{A} = 0$ Gauss's law magnetism
- $\int \vec{E} \cdot d\vec{s} = -\frac{d\phi_s}{dt}$ Faraday's law
- $\int \vec{B} \cdot d\vec{s} = \mu_0 I + \frac{d\phi_E}{dt}$ Ampere-Maxwell law

→ In differential form:

- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}_i = -\vec{M}_d - \vec{M}_i$
- $\nabla \times \vec{H} = \vec{J}_i + \vec{J}_c + \frac{\partial \vec{D}}{\partial t} = \vec{J}_i + \vec{J}_c + \vec{J}_d$
- $\nabla \cdot \vec{D} = \rho_{ev}$
- $\nabla \cdot \vec{B} = \rho_{mv}$
- $\vec{M}_d = \frac{\partial \vec{B}}{\partial t}, \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

Here,

\vec{E} = Electric field intensity [V/m]

\vec{B} = Magnetic flux density [weber/m² = Vs/m² = Tesla]

\vec{M}_i = Impressed (source) magnetic current density [V/m²]

\vec{M}_d = magnetic displacement current density [V/m²]

\vec{H} = Magnetic field intensity [A/m]

\vec{J}_i = Impressed (source) electric current density [A/m²]

\vec{D} = Electric flux density or electric displacement $[C/m^2]$

\vec{J}_c = Electric conduction current density $[A/m^2]$

\vec{J}_d = Electric displacement current density $[A/m^2]$

ρ_{ev} = Electric charge volume density $[C/m^3]$

ρ_{mv} = Magnetic charge volume density $[weber/m^3]$.

→ In integral form:

• $\iint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \iiint_V \rho dv$ (Gauss's law)

• $\iint_A \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)

• $\int_C \vec{E} \cdot d\vec{l} = - \frac{d\phi_B}{dt} = - \frac{d}{dt} \iint_A \vec{B} \cdot d\vec{A}$ (Faraday's law)

• $\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\phi_E}{dt} = \mu_0 \iint_A \left[\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right] \cdot d\vec{A}$
(Ampere's Law).

Ques no. 1(b) write different modes in Rectangular waveguide.

Ans. There are three type of waveguide mode in Rectangular waveguide.

1) Transverse Electric (TE) mode: This waveguide mode is dependent upon the transverse electric waves also sometimes called H waves, characterised by the fact that the electric vector (E) being always perpendicular to the direction of propagation.

$E_z = 0$ $H_z \neq 0$ Exist in waveguide.

1) Transverse Electric (TE) mode: This waveguide mode is dependent upon the transverse electric waves also sometimes called H waves, characterised by the fact that the electric vector (E) being always perpendicular to the direction of propagation.

$$E_z = 0$$

$$H_z \neq 0$$

Exist in waveguide.

2) Transverse Magnetic (TM) Mode: Transverse magnetic waves also called (E) waves are characterised by the fact that the magnetic vector (H) is always perpendicular to the direction of propagation.

$H_z = 0$ $E_z \neq 0$ Exist in waveguide.

3) Transverse electromagnetic (TEM) Mode: The TEM waves can not be propagated within a waveguide, but is included for completeness. It is the mode that is commonly used within coaxial and open wire feeders.

The TEM wave is characterised by the fact that both the electric vector (E) and the magnetic vector (H) are perpendicular to the direction of propagation.

$E_z = 0$ $H_z = 0$ Not exist in waveguide.