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Unit-I: Quantum Mechanics

de-broglie wavelength relation:

Dual nature of light assuming both wave and particle nature can be explained by continuing planck's relation and einstein relation of energy.

$$E = h\nu \quad \text{--- (I)}$$

$$E = mc^2 \quad \text{--- (II)}$$

Equating (I) & (II)

$$mc^2 = h\nu$$

$$mc = \frac{h\nu}{c}$$

$$\boxed{\lambda = \frac{h}{p}} \qquad \boxed{\lambda = \frac{h}{mv}}$$

de-broglie wavelength for an electron:

Consider an electron of mass 'm' under the potential diff. of V volts.

$$eV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} \Rightarrow \boxed{\lambda_e = \frac{12.28}{\sqrt{V}} \text{Å}}$$

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Calculate the λ de-broglie of a particle accelerated by a potential diff. of 30000 volts.

$$\lambda_e = \frac{12.28}{\sqrt{30000}} Å = \frac{12.28}{\sqrt{300}} = 0.071 Å$$

Calculate the λ of particle of $m = 10^{-15}$ kg moving at a speed of 2 mm/s.

$$\lambda = \frac{6.623\times10^{-34}}{10^{-15}\times2\times10^{-3}} = 3.3\times10^{-16}\ m$$

de-broglie postulated that because photon have both wave and particle characteristic, perhaps all forms of matter have both properties.

Matter Waves or de-broglie waves.

According to de-broglie a moving particle behaves sometimes as a ~~wave~~ wave and sometimes as a particle or a wave associated with moving wave, these waves are called matter waves are called de-broglie waves.

Characteristic: wavelength of matter wave can be given as

$$\lambda = \frac{h}{\sqrt{2meV}}$$

- matter waves are pilot / guiding waves.
- these waves are not em waves.
- these waves are called probability waves.

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$$V_p = \nu\lambda$$
$$= 2\pi\nu\left(\frac{\lambda}{2\pi}\right)$$

$$V_p = \frac{\omega}{K}$$

$$y = A\sin\omega\left(t - \frac{x}{v}\right) \quad \text{— (1)}$$

$$\boxed{\phi(x,t) = \omega\left(t - \frac{x}{v}\right)}$$

$$\frac{d\phi}{dt} = \omega\left(1 - \frac{1}{v}\frac{dx}{dt}\right)$$

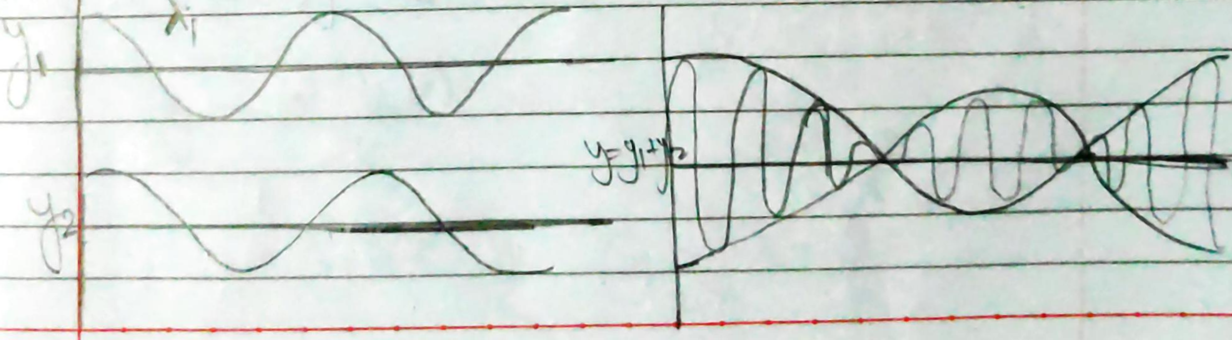
$$\frac{d\phi}{dt} = 0$$

$$1 - \frac{1}{v}\frac{dx}{dt} = 0$$

$$v = \left(\frac{dx}{dt}\right)_\phi \qquad V_p = \left(\frac{dx}{dt}\right)_\phi \quad \text{— (2)}$$

$$y = A\sin\omega\left(t - \frac{x}{V_p}\right) = A\sin\omega\left(t - \frac{xK}{\omega}\right)$$

$$\boxed{y = A\sin(\omega t - Kx)}$$



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When plane waves of slightly different wavelengths travel simantaneously along the same dirⁿ along st. line through dispersive medium.

Successive groups of waves are produced. These groups are called wave packet. Each wave group travels with velocity is called group velocity.

Let 2 harmonic wave of same amplitude but slightly different, wavelength, travelling in a dispersive medium.

$$y_1 = A\sin(\omega t - kx) \quad \text{--- (1)}$$

$$y_2 = A\sin[(\omega t - \Delta\omega)t - (k + \Delta k)x] \quad \text{--- (2)}$$

$$\boxed{y = y_1 + y_2}$$

$$y = A[\sin(\omega t - kx) + \sin[(\omega + \Delta\omega)t - (k + \Delta k)x]]$$

By using $\sin(A+B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A+B}{2}\right)$

$$y = 2A\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right]\sin\left[\left(\frac{2\omega + \Delta\omega}{2}\right)t - \left(\frac{2K + \Delta k}{2}\right)x\right]$$

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$$y = 2A\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right]\sin(\omega t - kx) \quad \text{——} (3)$$

Analytical representation of group of waves.

Resultant amplitude:

$$R = 2A\cos\left[\left(\frac{\Delta\omega}{2}\right)t - \left(\frac{\Delta k}{2}\right)x\right] \quad \text{——} (4)$$

The group velocity is the velocity with which maximum amplitude moves.

at $(x=0, t=0) \implies \boxed{R = 2A}$

$$V_g = \lim_{\Delta k \to 0} \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} \qquad V_g = \lim_{\Delta k \to 0} \frac{\Delta\omega/2}{\Delta k/2}$$

$$\boxed{V_g = \frac{d\omega}{dk}} \qquad \boxed{V_p = \frac{\omega}{k}}$$

Relation b/w group velocity and phase velocity.

$$V_p = \frac{\omega}{k} \quad \text{——} (1) \qquad V_g = \frac{d\omega}{dk} \quad \text{——} (2)$$

Substitute ω from (1) in (2)

$$V_g = \frac{d(V_p k)}{dk} \qquad k = \frac{2\pi}{\lambda}$$

$$V_g = V_p + k\frac{dV_p}{dk} \quad \text{——} (3) \qquad \lambda = \frac{2\pi}{k}$$

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$$V_g = V_p + k \frac{dV_p}{d\lambda} \cdot \frac{d\lambda}{dk} \quad \text{--- (4)}$$

$$\boxed{\frac{d\lambda}{dk} = \frac{-2\pi}{k^2}} \rightarrow \text{Put in (4)}$$

$$V_g = V_p - k\left(\frac{-2\pi}{k^2}\right)\frac{dV_p}{d\lambda}$$

$$= V_p - \frac{2\pi}{k}\left(\frac{dV_p}{d\lambda}\right)$$

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$ in dispersive medium.

Relation b/w group velocity and particle velocity

$$V_g = \frac{d\omega}{dk} \quad \text{--- (1)} \qquad \omega = 2\pi\nu = \frac{2\pi E}{h}$$

$$d\omega = \frac{2\pi}{h} dE \quad \text{--- (2)}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} \qquad \boxed{\lambda = \frac{h}{p}} \rightarrow \text{de-broglie relation.}$$

$$dk = \left(\frac{2\pi}{h}\right) dp \quad \text{--- (3)}$$

$$V_g = \frac{\cancel{\left(\frac{2\pi}{h}\right)}}{\cancel{\left(\frac{2\pi}{h}\right)}} \frac{dE}{dp} \Rightarrow V_g = \frac{dE}{dp}$$

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$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$dE = \frac{p}{m}\, dp$$

$$\frac{dE}{dp} = \frac{p}{m} \quad \text{--- (5)}$$

$$V_g = \frac{p}{m} = \frac{\cancel{m} V_{particle}}{\cancel{m}}$$

$$\boxed{V_g = V_{particle}}$$

Relation b/w group, phase (velocity) & light velocity

$$V_p = \frac{\omega}{k} \quad \text{--- (1)}$$

$$\omega = 2\pi\nu = \frac{2\pi E}{h}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p \quad \text{--- (3)}$$

$$V_p = \frac{E}{p} = \frac{\cancel{m} c^2}{\cancel{m} V_g}$$

$$V_p V_g = c^2$$

Relation b/w phase velocity & group velocity for non relativistic particle.

$$\lambda = \frac{h}{mv_g} \text{ ——— } (1)$$

$$E = h\nu$$

$$E = \frac{1}{2} m v_g^2$$

$$h\nu = \frac{1}{2} m v_g^2$$

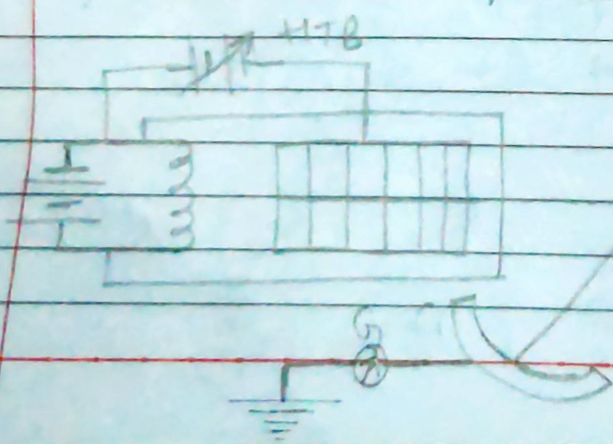
$$\nu = \frac{m v_g^2}{2h} \text{ — } (2)$$

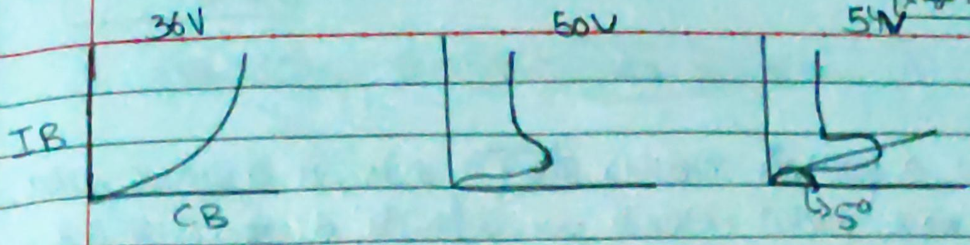
$\boxed{V_p = \nu\lambda}$ ⟶ defination of phase velocity

$$V_p = \frac{\cancel{m} v_g^{\cancel{2}}}{2\cancel{h}} \times \frac{\cancel{h}}{\cancel{m}\cancel{v_g}}$$

$$\boxed{V_p = \frac{v_g}{2}}$$

Davisson-Germer Experiment:





$2d \sin\theta = n\lambda$

$d = 0.91 A$

$\lambda = 2 \times 0.91 \times \sin 65$

$\lambda = 1.65 A^\circ$

$$\lambda = \frac{12.28}{\sqrt{V}} A^\circ = \frac{12.28}{\sqrt{54}} A^\circ = 1.66 A^\circ$$

The electron wavelength agrees with the observed wavelength, thus Davisson-Germer Experiment, directly verifies the de-broglie hypothesis of wave nature of matter particles

Heisenberg's Uncertainity Principle:

According to heisenberg uncertainity principle, it is impossible to determine the exact position and momentum of a sub-atomic particle Simultaneously.

Proof:

$$x_1 \longleftarrow \Delta x \longrightarrow x_2$$

Consider a particle moving along x-axis, in a wave-packet. When a wave packet extends over a finite distance of Δx, then two points at which the amplitude of wave particle becomes negligible, will become seperated over this distance.

The amplitude of wave packet is given by

$$R = 2A \cos\left[\left(\frac{\Delta \omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right] \quad \text{--- (1)}$$

$$0 = 2A \cos\left[\left(\frac{\Delta \omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x\right]$$

for position x_1:

$$\left(\frac{\Delta \omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x_1 = (2n+1)\pi/2 \quad \text{--- (2)}$$

for position x_2:

$$\left(\frac{\Delta \omega}{2}\right)t - \left(\frac{\Delta K}{2}\right)x_2 = (2n+3)\pi/2 \quad \text{--- (3)}$$

Subtracting (2) from (3)

$$\frac{\Delta K}{2}(x_2 - x_1) = \pi$$

$$\frac{\Delta k}{2}(\Delta x) = \pi$$

$$\Delta K = \frac{2\pi}{(\Delta x)} \quad \text{--- (4)}$$

$$\boxed{K = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p_x}$$

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~~Subtracting ④ from ⑤~~ Substituting ⑤ in ④

$$\frac{2\pi}{h}\Delta p_x = \frac{2\pi}{\Delta x}$$

$$\boxed{\Delta x . \Delta p_x = h}$$

According to fourier analysis the width of single wave Δx represents the superposition of waves, propagation constant varies in range of ΔK.

$$\boxed{\Delta K = \frac{1}{\Delta x}}$$

Heisenberg's Uncertainity Principle in terms of energy & time

Let us consider a photon of energy 'E', momentum 'p', covers the distance 'x', in time 't', with speed 'C'.

$$p = \frac{E}{c}$$

$$x = Ct$$

$$\Delta x = C\Delta t \text{ ——— } ①$$

$$\Delta p = \frac{\Delta E}{c} \text{ ——— } ②$$

$$\Delta x . \Delta p \geq \hbar \text{ ——— } ③$$

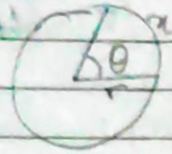
Substitute ① & ② in ③

$$\cancel{c}\Delta t . \frac{\Delta E}{\cancel{c}} \geq \hbar$$

$$\boxed{\Delta t . \Delta E \geq \hbar}$$

Heisenberg's Uncertainity Principle in terms of angular momentum and Angular depression.

Let the radius be 'r', length of arc be x & the angle be θ.



$$x = \theta r$$

$$\Delta x = r\,\Delta\theta \quad \text{——} \quad (1)$$

$$L = mvr$$

$$\Delta L = \Delta p\, r \quad \text{——} \quad (2)$$

$$\Delta x \cdot \Delta p \geq \hbar \quad \text{——} \quad (3)$$

Substitute (1) & (2) in (3)

$$\cancel{r}\,\Delta\theta \cdot \frac{\Delta L}{\cancel{r}} \geq \hbar$$

$$\boxed{\Delta\theta \cdot \Delta L \geq \hbar}$$

Experimental Proof of Heisenberg's Uncertainity Principle:

Let us consider a process where an $e^{\ominus}$ is observed by a microscope, the min. distance b/w two points which can be distinguished as seperate by the microscope.

$$\Delta x = \frac{\lambda}{2\sin\theta} \quad \text{——} \quad (1)$$

Now in order that $e^{\ominus}$ may be seen through the microscope the incident photon must be scattered by the $e^{\ominus}$ into the microscope objective. During scattering the $e^{\ominus}$ recoils & suffers a change in momentum. Let the incident photon has wavelength λ' & corresponding momentum h/λ'.

It will scatter in to the microscope with wavelength λ anywhere b/w OA & OB

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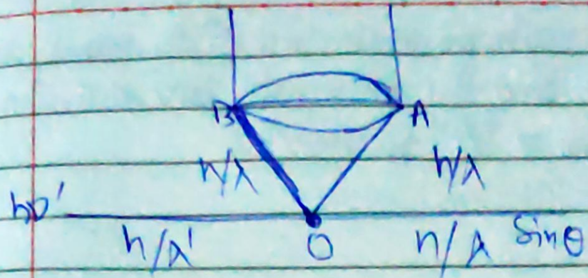

In case in enters on its momentum is given by h/λ sinθ. Hence, the lose of momentum of photon. When it goes to direction A.

$$p_A = \frac{h}{\lambda'} - \frac{h}{\lambda}\sin\theta \quad \text{——} \quad (2)$$

$$p_B = \frac{h}{\lambda'} = \left(-\frac{h}{\lambda}\sin\theta\right) \quad \text{——} \quad (3)$$

$$\Delta p = p_B - p_A = \cancel{\frac{h}{\lambda'}} + \frac{h}{\lambda}\sin\theta - \cancel{\frac{h}{\lambda'}} + \frac{h}{\lambda}\sin\theta$$

$$\Delta p = p_B - p_A = \frac{2h}{\lambda}\sin\theta \quad \text{——} \quad (4)$$

multiplying eq^n (1) and (4)

$$\boxed{\Delta x \Delta p = h}$$

Vimp Applications of Uncertainity Principle.

(1) Non-existence of e in nucleus.

According to energy momentum rel^n

$$\boxed{E^2 = p^2c^2 + m_0^2 + c^4} \quad \text{——} \quad (1)$$

$$\boxed{\Delta x \cdot \Delta p_x \geq h} \quad \text{——} \quad (2)$$

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The maximum uncertainity in position of e is same as the diameter of nucleus and the mininum uncertainity in momentum. $\Delta x = 10^{-4} m$ — (3)

$$\Delta p_x = \frac{h}{2\pi \Delta x}$$

$$\boxed{\Delta p_x = 1.055 \times 10^{-20}\ kg\,m/sec} \text{ — (4)}$$

Put Δp_x in (1)

If $e^{\ominus}$ exist in nucleus.

$$E^2 = p^2c^2 + m_0^2 c^4$$

$$E = 20\ MeV$$

Thus, if a free $e^{\ominus}$ exist in a nucleus. It must have a energy equal to 20 MeV. Maximum KE with a β particle is emitted from radioactive nuclei is of the order of 4 MeV. Therefore, it is clear that $e^{\ominus}$ can not be present within the nucleus.

Radius of Bohr's first orbit:

If Δp & Δx are uncertainity in determining in momentum and position of a particle in bohr's 1st orbit

$$\Delta x \Delta p = \hbar$$

$$\Delta p = \frac{\hbar}{\Delta x} \text{ — (1)}$$

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$$K = \frac{p^2}{2m} \quad , \quad \Delta K = \frac{(\Delta p)^2}{2m}$$

$$\Delta K = \frac{\hbar^2}{2m(\Delta x)^2} \quad \text{—— (2)}$$

Uncertainity in potential energy of sofie$^{\ominus}$:

$$\Delta U = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{\Delta x} \quad \text{—— (3)}$$

Uncertainity in total energy:

$$\Delta E = \Delta K + \Delta U$$

$$\Delta E = \frac{\hbar^2}{2m(\Delta x)^2} + \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{\Delta x}$$

The value of energy in fundamental state is minimum and uncertainity in energy will be min when.

$$\frac{d(\Delta E)}{d(\Delta x)} = 0$$

$$\Delta E = \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{(\Delta x)}$$

$$\frac{d(\Delta E)}{d(\Delta x)} = \frac{\hbar^2}{\cancel{2}m} \times \frac{-\cancel{2}}{(\Delta x)^3} - \frac{1}{4\pi\epsilon_0} Ze^2 \frac{(-1)}{(\Delta x)^2} = 0$$

$$\frac{\hbar^2}{m(\Delta x)^3} = \frac{Ze^2}{4\pi\epsilon_0(\Delta x)^2}$$

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$$\frac{\hbar^2}{m(\Delta x)^3} = \frac{Ze^2}{4\pi\epsilon_0(\Delta x)^2}$$

$$\frac{\hbar^2}{m(\Delta x)} = \frac{Ze^2}{4\pi\epsilon_0}$$

$$\boxed{\Delta x = \frac{4\pi\epsilon_0\hbar^2}{mZe^2}}$$

$$\hbar = \frac{h}{2\pi}$$

$$\Delta x = \frac{\cancel{4\pi}\epsilon_0 \times h^2}{\cancel{4}\pi^{\cancel{2}} \times mZe^2} = \frac{\epsilon_0 h^2}{\pi m Z e^2}$$

$$\boxed{\Delta x = \frac{\epsilon_0 h}{\pi m Z e^2}}$$

$\Delta x = r$

Zero Point energy of Simple Harmonic Oscillator:

Consider a particle of mass m executing simple harmonic motion with angular frequency ω. Let x be disp of particle, from its mean position & p is its momentum, so expression of total energy can be written as

$$\boxed{E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2} \quad \text{—— (1)}$$

Let a be amplitude of oscillation & sepn b/w 2 extreme position on its path is $2a$.

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max^m uncertainity in position is given by

$$\boxed{\Delta x_{max} = 2a}$$

and minimum uncertainity in momentum according to HUP:

$$\Delta p_{min} = \frac{\hbar}{\Delta x} = \frac{\hbar}{2a} \quad \text{—— (3)}$$

The momentum of oscillation cannot be smaller than (3).

$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \quad \text{—— (1)}$$

$$E = \frac{\hbar^2}{8ma^2} + \frac{1}{2} m\omega^2 a^2 \quad \text{—— (4)}$$

Eq^n (4) expresses the total energy as the function of amplitude. Let $a = \alpha$ be the amplitude of oscillator when its total energy remains its minimum value. For that the following conditions must be satisfied.

(i) $\left(\frac{dE}{da}\right)_{a=\alpha} = 0$ (ii) $\left(\frac{d^2E}{da^2}\right)_{a=\alpha} > 0$

$$\frac{dE}{da} = \frac{\hbar^2}{\cancel{8}_4 m} \frac{(-\cancel{2})}{a^3} + \frac{1}{\cancel{2}} m\omega^2 \times \cancel{2}a$$

$$= -\frac{\hbar^2}{4ma^3} + m\omega^2 a = 0$$

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$$\frac{\hbar^2}{4ma^3} = m\omega^3 a$$

$$\frac{\hbar^2}{4m^2\omega^2} = a^4$$

$$a^2 = \alpha^2 = \frac{\hbar}{2m\omega} \quad \text{--- (5)}$$

Substitute (5) in (4):

We will get lowest possible value of total energy:

$$E_{min} = \frac{\hbar^2}{8ma^2} + \frac{1}{2}m\omega^2 \times \frac{\hbar}{2m\omega}$$

$$= \frac{\hbar^2}{\cancel{4}8\cancel{m}}\frac{\cancel{2}\cancel{m}\omega}{(\cancel{\hbar})} + \frac{\omega\hbar}{4}$$

$$= \frac{\hbar\omega}{4} + \frac{\omega\hbar}{4}$$

$$E_{min} = \frac{\omega\hbar}{2} \quad \text{--- (6)}$$

$$\omega = 2\pi\nu$$

$$E_{min} = \frac{\cancel{2}\pi\nu\hbar}{\cancel{2}}$$

$$E_{min} = \pi\hbar\nu$$

$$= \cancel{\pi}\frac{h}{2\cancel{\pi}}\nu$$

$$E_{min} = \frac{h\nu}{2} \quad \text{--- (7)}$$

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$$\boxed{E = \left(n + \frac{1}{2}\right) h\nu}$$ $n \rightarrow$ discrete energy levels

for $n = 0$, system has minimum energy & is called zero point energy.

Wave function :

$$\psi = a + ib$$
$$\psi^* = a - ib$$
$$|\psi|^2 = |\psi\psi^*| = a^2 + b^2$$

Probability density $= \int_{-\infty}^{\infty} |\psi|^2 dx$

Normalization condition

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

orthogonality cond^n

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 0$$

Properties of wave function :

(I) ψ must be single valued, finite and continuous for all values of x

(II) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$, must be finite and continuous for all values of x, y and z except where V is infinite.

(III) ψ must be normalised.

Physical significance of wave function - ψ cannot be interpreted in terms of an experiment. The probability of experimentally finding the body described by the wave function ψ at the point (x, y, z) at time t is proportional to the value of $|\psi|^2$, larger the value of $|\psi|^2$ means

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Strong probability of ~~process~~ presence of particle. While lesser values of $|\psi|^2$ mean the less probability of finding of particle.

<u>Operators</u>: Let us assume that wave f^n ψ is specified in x diversion.

$$\psi = Ae^{-i(\omega t - kx)} \quad \text{--- (1)}$$

$$\omega = 2\pi\nu = 2\pi \frac{E}{h} \quad , k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p$$

$$\psi = Ae^{-i \frac{2\pi}{h}(Et - px)}$$

$$\boxed{\psi = Ae^{-i/\hbar (Et - px)}} \quad \text{--- (2)}$$

Diffrentiate eq^n (2) w.r.t time

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} Ae^{-i/\hbar (Et - px)}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi$$

$$E\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$\boxed{E\psi = -i\hbar \frac{\partial \psi}{\partial t}}$$

Energy $\quad i\hbar \frac{\partial}{\partial t}$

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differentiate eq^n 2 w.r.t x :

$$\frac{\partial \psi}{\partial x} = \frac{-i}{\hbar}(-p)\,Ae^{-i/\hbar(Et-px)}$$

$$= \frac{ip}{\hbar}Ae^{-i/\hbar(Et-px)}$$

$$\frac{\partial \psi}{\partial x} = \frac{ip}{\hbar}\psi$$

$$p\psi = \frac{\hbar}{i}\frac{\partial \psi}{\partial x}$$

$$\boxed{\text{momentum Operator} \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x}}$$

Schrodinger's wave equation:

Time dependent schrodinger wave equation :-

from free particle wave eqn

$$\psi = Ae^{-i/\hbar(Et-px)} \text{ ——— } (1)$$

$$E\psi = i\hbar\frac{\partial \psi}{\partial t} \qquad p\psi = -i\hbar\frac{\partial \psi}{\partial x} = \frac{\hbar}{i}\frac{\partial \psi}{\partial x}$$

total energy of system can be defined as $E = K.E + PE$

$$E = \frac{p^2}{2m} + V \text{ ——— } (2)$$

$$E\psi = \frac{p^2\psi}{2m} + V\psi \text{ ——— } (3)$$

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$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi} \quad (4)$$

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boxed{\nabla^2 \phi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}} \quad (5)$$

$$E\psi = \left(\frac{-\hbar^2}{2m} \nabla^2 + V\right) \psi$$

$$\boxed{E\psi = H\psi}$$ $H \rightarrow$ Hamiltonian operator.

Time independent schrodinger wave equation:

time independent $\quad \psi = Ae^{-i/\hbar\,(Et - px)} \quad (1)$

$$= Ae^{-iEt/\hbar} \cdot e^{ipx/\hbar}$$

$$= Ae^{ipx/\hbar} \cdot e^{-iEt/\hbar}$$

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$$(\psi_0 = Ae^{-ipx/\hbar})$$

$$\psi = \psi_0 e^{-iEt/\hbar} \quad \text{---} \ (2)$$

differentiate eq^n (2) w.r.t time:

$$\frac{\partial \psi}{\partial t} = \psi_0 \left(\frac{-iE}{\hbar}\right) e^{-iEt/\hbar}$$

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi_0 e^{-iEt/\hbar} \quad \text{---} \ (a)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} e^{-iEt/\hbar} \quad \text{---} \ (b)$$

$\Big] \text{---} \ (3)$

from time dependent schrodinger's wave eq^n:

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$i\hbar \left(\frac{-iE}{\hbar}\right) \psi_0 = e^{-iEt/\hbar} = \frac{-\hbar^2}{2m} \left(\frac{\partial^2 \psi_0}{\partial x^2} e^{-iEt/\hbar}\right) + V\psi$$

$$i\cancel{\hbar} \left(\frac{-iE}{\cancel{\hbar}}\right) \psi_0 \cancel{e^{-iEt/\hbar}} = \frac{-\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} \cancel{e^{-iEt/\hbar}}\right) + V\psi_0 \cancel{e^{-iEt/\hbar}}$$

$$E\psi_0 = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\frac{-\hbar}{2m} \frac{\partial^2 \psi_0}{\partial x^2} = (E - V)\psi_0$$

$$\frac{\partial^2 \psi_0}{\partial x^2} = -\frac{2m}{\hbar} (E - V)\psi_0$$

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$$\frac{\partial^2 \psi_0}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi_0 = 0$$

Applications of schrodinger wave eqn:

Supportive Application:

free particle:

When a particle is not subjected to any external force. So that it moves in a region in which its potential energy is constant is said to be free particle. Such particle has definite values of energy and momentum, but position is uncertain.

Suppose a particle of mass m is moving along x axis is optional. From time independent schrodinger wave eqn

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

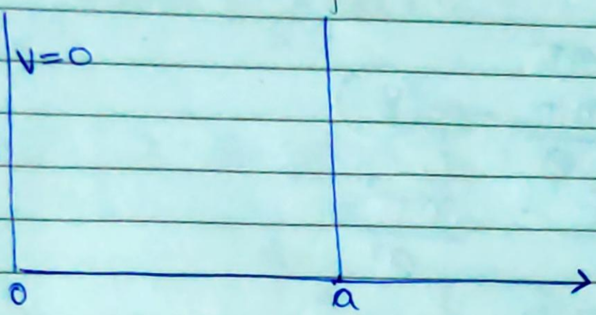
$$\frac{8\pi^2 mE}{h^2} = k^2$$

$$\boxed{E = \frac{k^2 h^2}{8\pi^2 m}}$$

In, ~~this case~~ state energy is not quantised.

Hence a particle not bound in a system it does not have quantised energy states.

Particle in a box or infinite square well potential.



$$\text{(1)} \left[\begin{array}{l} V=0 \quad 0<x<a \\ x\leq 0 \\ x\geq a \end{array} \right.$$

Let us consider a particle of mass m is restricted to move along x-axis with wave function ψ

From time independent schrodinger's wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 m}{h^2}(E-V)\psi=0 \quad \text{———} (2)$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 mE}{h^2}\psi=0 \quad \text{———} (3)$$

$$\frac{8\pi^2 mE}{h^2}=K^2 \quad \text{———} (4)$$

$$\frac{\partial^2\psi}{\partial x^2}+K^2\psi=0 \quad \text{———} (5)$$

$$\psi=A\sin kx+B\cos kx \quad \text{———} (6)$$

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at $x=0$, $\psi=0 \Rightarrow B=0$

$$\psi = A\sin kx \quad \text{--- (7)}$$

at $x=a$, $\psi=0$

$$\sin ka = 0$$

$$ka = n\pi$$

$$k = \frac{n\pi}{a} \quad \text{--- (8)}$$

Substituting (8) in (9)

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$$

$$\boxed{E = \frac{n^2h^2}{8ma^2}} \quad ; \; (n = 1, 2, 3 \ldots)$$

From eq^n (9) it is clear that inside the potential well particle cannot have arbitary energy but can have discrete energy levels corresponding to n.

Each permitted energy level is called eigen value and the wave function corresponding to each eigen value is called eigen function. To find the eigen function we will apply the normalization condition.

$$\int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$$\int_0^a \left|A\sin\frac{n\pi x}{a}\right|^2 dx = 1$$

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$$A^2 \int_0^a \sin^2 \frac{n\pi x}{a}\, dx = 1$$

$$\frac{A^2}{2} \int_0^a \sin^2 \frac{n\pi x}{a}\, dx = 1$$

$$\frac{A^2}{2} \int_0^a \left(1 - \cos 2\frac{n\pi x}{a}\right) dx = 1 \qquad \cos 2x = 1 - 2\sin^2 x$$

$$\frac{A^2}{2} \left[\frac{x - \sin \frac{2n\pi x}{a}}{\frac{2n\pi}{a}} \right]_0^a = 1$$

$$\frac{A^2}{2} \left(a - \frac{a}{2n\pi} \sin 2\frac{\cancel{a}\pi\cancel{x}}{\cancel{a}} \right) = 1$$

$$\frac{A^2}{2} a = 1$$

$$A = \sqrt{\frac{2}{a}}$$

$$\boxed{\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}}$$

Q Calculate the ground state energy of an $e^{\ominus}$ confined to a box of 1 $\mathring{A}$ wide width.

A $$E = \frac{1 \times (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2} = \frac{43.90 \times 10^{-68}}{72.8 \times 10^{-51}}$$

$$= 0.60 \times 10^{-17} = 6.06 \times 10^{-18}$$

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Q. Find the value of normalization const. A for the func

$\Psi = Axe^{-x^2/2}$

Soln $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$

$\int_{-\infty}^{\infty} A^2 x^2 e^{-x^2} dx = 1$

$2\int_{0}^{\infty} A^2 x^2 e^{-x^2} dx = 1$

$2A^2 \int_{0}^{\infty} x^2 e^{-x^2} dx = 1$

$\cancel{2} A^2 \times \frac{\sqrt{\pi}}{\cancel{4}2} = 1$

$A^2 = \frac{2}{\sqrt{\pi}} \Rightarrow A = \frac{\sqrt{2}}{\pi^{3/4}}$

Q. Find the value of A and find the probability that the particle be found from 0 to $\pi/4$

$\Psi = A\cos^2 x \quad -\pi/2 < x < \pi/2, \quad 0 < x < \pi/4$

$\cancel{\int_{-\infty}^{\infty} |\Psi|^2 dx = 1}$

$\cancel{A^2 \int_{-\infty}^{\infty} \cos^4 x \, dx = 1}$

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$$\int_{-\pi/2}^{\pi/2} (A\cos^2 x)^2\, dx = 1 \longrightarrow A^2 \int_{-\pi/2}^{\pi/2} \cos^4 x\, dx = 1$$

$$\int \cos^4 x = \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + C$$

$$A^2\left(\frac{3\pi}{16} \times 2\right) \Rightarrow A^2\left(\frac{3\pi}{8}\right) = 1$$

$$\boxed{\cos 2x = 2\cos^2 x - 1}$$

$$A = \sqrt{\frac{2\sqrt{2}}{3\pi}} \Rightarrow A = \sqrt{\frac{8}{3\pi}}$$

$$\int \cos^4 x \longrightarrow \int (\cos^2 x)^2\, dx$$

$$\int_0^{\pi/4} \frac{8}{3\pi} \cos^4 x\, dx \Rightarrow \frac{8}{3\pi}\left[\frac{3\pi}{32} + 0 + \frac{1}{4} - 0\right]$$

$$\Rightarrow \frac{8}{3\pi}\left(\frac{3\pi}{32} + \frac{1}{4}\right)$$

$$\frac{1}{4} + \frac{8}{3\pi} \longrightarrow \boxed{\frac{1}{4} + \frac{8}{3\pi}} \rightarrow \boxed{\frac{3\pi + 8}{12\pi}}$$

Q Calculate the ground state energy of an $e^{\ominus}$ confined in a box of 1Å width.

$$E = \frac{n^2 h^2}{8 m a^2}$$

$a = 1Å =$ width

$$E = \frac{(1)(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-20}} = 0.6030 \times 10^{-17}$$

$$= \boxed{6.03 \times 10^{-18}\ J}$$

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Ques An $e^{\ominus}$ is in a box of 0.1 Å. Find its ~~potential~~ permitted energy. (n=1,2,3)

$$E = \frac{n^2h^2}{8ma^2}$$

$$E = \frac{(1)(6.626\times10^{-34})^2}{8\times9.1\times10^{-31}\times10^{-22}}$$

$$E = \frac{0.603\times10^{-15}}{1.6\times10^{-19}}$$

$E = 0.37\times10^{4}$ eV
$= 3.786\times10^{3}$ eV

$n=1$ 3.786×10^{3} eV
$n=2$ 1.5×10^{4} eV
$n=3$ 3.39×10^{4} eV

Ques A particle limited to x-axis has a wave funⁿ, $\psi = ax$ b/w $x=0$ and 1. Find the probability that particle can be found b/w (i) 0.45 to 0.55
(ii) exponential value of particle posⁿ

expⁿ value $\langle x\rangle = \int_0^1 x|\psi|^2 dx$

$A^2\int x^2 dx = 1 = A^2\left[\frac{x^3}{3}\right]_0^1 = 1$, $A^2 = 3$

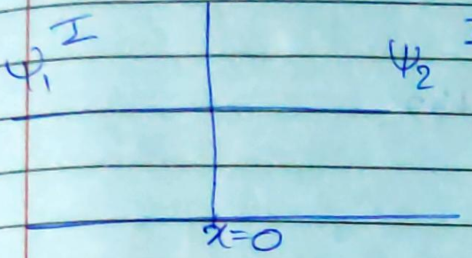
$P = \int_{0.45}^{0.55} (3)(x)dx$ $3\left[\frac{x^2}{3}\right]_{0.45}^{0.55}$ $= 0.166375 - 0.91125$

$= 0.744$

~~$\langle x\rangle = \frac{x^2}{4} \to \frac{3}{4} \to 0.75$~~

Potential Step Barrier

Assume that the potentials are constant in time and that are also constant for prescribed region of space. The potential function of a potential step is defined by.



$R+T=1$

$E > V_0 \qquad V=0 \qquad x<0$

$E < V_0 \qquad V=V_0 \qquad x>0$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 m}{h^2}(E-V)\psi=0$$

I $$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 mE}{h^2}\psi_1=0 \quad \text{—— (2)}$$

II $$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 mE}{h^2}\psi_2(E-V_0)=0 \quad \text{—— (3)}$$

$$K_1^2=\frac{8\pi^2 mE}{h^2}, \quad K_2^2=\frac{8\pi^2 m(E-V_0)}{h^2} \quad \text{—— (4)}$$

$$\frac{\partial^2\psi_1}{\partial x^2}+K_1^2\psi_1=0 \quad \text{—— (5)}$$

$$\frac{\partial^2\psi_2}{\partial x^2}+k_2^2\psi_2=0 \quad \text{—— (6)}$$

$$\psi_1=Ae^{K_1 i x}+Be^{-K i x} \quad \text{—— (7)}$$

$$\psi_2=Ce^{iK_2 x}+De^{-iK_2 x} \quad \text{—— (8)}$$

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$D=0$ [No reflection in medium 2]

$\psi_2 = Ce^{k_2 i x}$ —— (9)

First boundary condⁿ

$x=0 \quad \psi_1 = \psi_2$

$Ae^{k_1 i x} + Be^{-k_1 i x} = Ce^{ik_2 x}$

$\boxed{A+B=C}$ —— (10)

$x=0, \quad \frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x}$

$Ae^{ik_1 x}(ik_1) + Be^{-ik_1 x}(-ik_2) = Ce^{ik_2 x} \quad V^{ik_2}$

$(A-B)\,K_1 = C(K_2)$

$A-B = \frac{K_2}{K_1} C$ —— (11)

Adding ~~Subtracting~~ (10) to ~~from~~ (11)

$2A = \left(\frac{K_2}{K_1} + 1\right) C$

$\cancel{C = \frac{2A}{}}$

$\boxed{C = \left(\frac{2K_1}{K_1 + K_2}\right) A}$ —— (12)

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Subtracting (10) from (11)

$$2B = \left(1 - \frac{K_2}{K_1}\right) C$$

$$2B = \left(\frac{K_1 - K_2}{K_1}\right) C$$

$$2B = \left(\frac{K_1 - K_2}{K_1}\right)\left(\frac{2AK_1}{K_1 + K_2}\right)$$

$$B = \left(\frac{K_1 - K_2}{K_1 + K_2}\right) A \text{ —— } (13)$$

In eqn (12) & (13) B and C represent amplitude of reflected and transmitted beams respectively in terms of A.

Reflectance: $\dfrac{\text{magnitude of reflected current}}{\text{magnitude of incident current}}$

Transmittance: $\dfrac{\text{Magnitude of transmitted current}}{\text{magnitude of incident current}}$

(I) $E > V_0$

then K_2 is real and probability density for Region II is defined as

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$$[J_x]_1 = \frac{\hbar}{2m}\left[\psi_1^* \frac{\partial \psi_1}{\partial x} - \psi_1 \frac{\partial \psi_1^*}{\partial x}\right]$$

$$\psi_1 = Ae^{K_1 i x} + Be^{-iK_1 x}$$

$$\psi_1^* = A^* e^{K_1 x(-i)} + B^* e^{iK_1 x}$$

$$\frac{\partial \psi_1}{\partial x} = Ae^{K_1 i x}(K_1 i) - (Be^{-iK_1 x})$$

$$\frac{\partial \psi_2}{\partial x} = A^* e^{-iK_1 x}(-iK_1) + B^* e^{iK_1 x}(iK_1)$$

$$[J_x]_I = \frac{K_1 \hbar}{m}\left[AA^* - BB^*\right]$$

$$[J_x]_I = \frac{K_1 \hbar}{m}\left[|A|^2 - |B|^2\right] \quad \text{--- (14)}$$

In eq^n (14), current in region one is equal to diff b/w 2 terms where first one is proportional to $K_1|A|^2$, represent the incident beam travelling from left to right whereas other one is proportional to $K_1|B|^2$ represent reflected beam from right to left,

$\therefore$ probability current in incident beam $= \frac{K_1 \hbar}{m}|A|^2$

" " " reflected beam $= \frac{K_1 \hbar}{m}|B|^2$

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Probability current density in transmitted wave:

$$[J_x]_{II} = \frac{\hbar}{2im}\left[\psi_2^* \frac{\partial \psi_2}{\partial x} - \psi_2 \frac{\partial \psi_2^*}{\partial x}\right]$$

$$[J_x]_{II} = \frac{\hbar}{2im}\left[\left\{C^* e^{-ik_2 x}(ik_2) C e^{ik_2 x/\hbar}\right\} - \left\{C e^{ik_2 x}(-ik_2) C^* e^{-ik_2 x}\right\}\right]$$

$$= \frac{\hbar k_2}{2m}\left[CC^* + CC^*\right] = \frac{\hbar k_2}{m}(CC^*) = \frac{k_2 \hbar}{m}|C|^2 \quad (15)$$

$$R = \frac{[J_x]_{I(R)}}{[J_x]_{I(i)}} = \frac{|B|^2 \frac{k_1 \hbar}{m}}{|A|^2 \frac{k_1 \hbar}{m}} = \left|\frac{B}{A}\right|^2 = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$T = \frac{[J_x]_{II(T)}}{[J_x]_{I(i)}} = \frac{|C|^2 \frac{k_2 \hbar}{m}}{|A|^2 \frac{k_1 \hbar}{m}} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1} = \left(\frac{2k_1}{k_1 + k_2}\right)^2 \cdot \frac{k_2}{k_1}$$

$$= \frac{4k_1 k_2}{(k_1 + k_2)^2}$$

$$R + T = \frac{(k_1 - k_2)^2 + 4k_1 k_2}{(k_1 + k_2)^2} = \frac{(k_1 + k_2)^2}{(k_1 + k_2)^2} = 1$$

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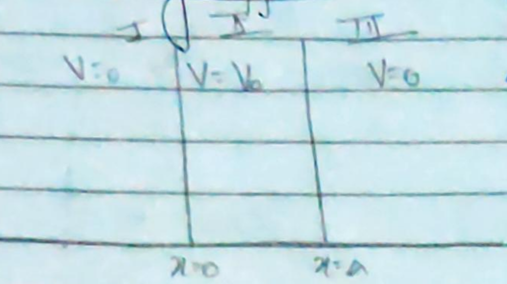
Tunneling Effect :



Let us consider the following regions where potentials are defined.

If a particle having energy less than V_0, $(E < V_0)$ approaches this barrier from the left from first region, classically the particle will always be reflected and hence will not penetrate the barrier.
The particle has some probability of penetrating to region III, the probability of penetration being greater if $(V_0 - E)$ and a are smaller.

Write three schrodinger equations for three regions

I $\quad \frac{d^2\psi_1}{dx^2} + \frac{2mE\psi_1}{\hbar^2} = 0 \quad \Rightarrow \quad \frac{d^2\psi_1}{dx^2} + K^2\psi_1 = 0$ — (I)

II $\quad \frac{d^2\psi_2}{dx^2} + \frac{2m(E-V_0)\psi_2}{\hbar^2} = 0 \quad \Rightarrow \quad \frac{d^2\psi_2}{dx^2} - \beta^2\psi_2 = 0$ — (II)

III $\quad \frac{d^2\psi_3}{dx^3} + \frac{2mE\psi_2}{\hbar^2} \quad \Rightarrow \quad \frac{d^2\psi_3}{dx^2} + K^2\psi_3$ — (III)

$$K = \sqrt{\frac{2mE}{\hbar^2}}, \quad \beta = \sqrt{\frac{2m(V_0 - E)}{\hbar}}$$

$$\psi_1 = Ae^{ikx} + Be^{-ikx} \qquad \psi_2 = Ce^{\beta x} + De^{-\beta x}$$

$$\psi_3 = Ge^{ikx} + He^{-ikx}$$

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$= Ge^{ikx}$ [H=0 ; because no waves travel back from infinity in III region]

Applying boundary conditions to particle.

(i) At $x=0$, we have

$$\psi_1 = \psi_2$$

$$\left[Ae^{ikx} + Be^{-ikx} = Ce^{\beta x} + De^{-\beta x}\right]_{x=0}$$

$$A + B = C + D \quad \text{—— (4)}$$

Also,

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx}$$

$$\left[ikAe^{ikx} - ikBe^{-ikx} = \beta Ce^{\beta x} - \beta De^{-\beta x}\right]_{x=0}$$

$$A - B = \frac{\beta}{ik}(C - D) \quad \text{—— (5)}$$

Adding (4) & (5)

$$A+B+A-B = C+D+\frac{\beta}{ik}(C-D)$$

$$A = \frac{1}{2}\left[\left(1+\frac{\beta}{ik}\right)C + \left(1-\frac{\beta}{ik}\right)D\right] \text{—— (6)}$$

Subtract (4) & (5)

$$A+B-A+B = C+D-\left[\frac{\beta}{ik}(C-D)\right]$$

$$B = \frac{1}{2}\left[\left(1-\frac{\beta}{ik}\right)C + \left(1+\frac{\beta}{ik}\right)D\right] \text{—— (7)}$$

(ii) at $x=a$, we have

$$\psi_2 = \psi_3$$

$$\left[Ce^{\beta x} + De^{-\beta x} = Ge^{ikx}\right]_{x=0}$$

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$$Ce^{\beta a} + De^{-\beta a} = Ge^{ika} \quad \text{--- } (8)$$

also $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

$$\left[C\beta e^{\beta x} - D\beta e^{-\beta x} = Gike^{ikx}\right]_{x=a}$$

$$C\beta e^{\beta a} - D\beta e^{-\beta a} = Gike^{ika}$$

$$Ce^{\beta a} - De^{-\beta a} = \frac{ik}{\beta} Ge^{ika} \quad \text{--- } (9)$$

Add (8) & (9)

$$Ce^{\beta a} + \cancel{De^{-\beta a}} + Ce^{\beta a} - \cancel{De^{-\beta a}} = Ge^{ika} + \frac{ik}{\beta} Ge^{ika}$$

$$2Ce^{\beta a} = Ge^{ika}\left(1 + \frac{ik}{\beta}\right)$$

$$C = \frac{1}{2}\left(1 + \frac{ik}{\beta}\right) e^{-\beta a} Ge^{ika} \quad \text{--- } (10)$$

Subtract (8) & (9)

$$Ce^{\beta a} + De^{-\beta a} - Ce^{\beta a} + De^{-\beta a} = Ge^{ika} - \frac{ik}{\beta} Ge^{ika}$$

$$2De^{-\beta a} = Ge^{ika}\left(1 - \frac{ik}{\beta}\right)$$

$$D = \frac{1}{2}\left(1 + \frac{ik}{\beta}\right) e^{\beta a} Ge^{ika} \quad \text{--- } (11)$$

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Substituting values of (10) & (11) in (6)

$$A = \frac{1}{4}\left(1+\frac{ik}{\beta}\right)Ge^{(ik-\beta)a}\left(1+\frac{\beta}{ik}\right) + \frac{1}{4}\left(1-\frac{ik}{\beta}\right)\left(1-\frac{\beta}{ik}\right)Ge^{(ik+\beta)a}$$

$$= \left[\left(\frac{e^{\beta a}+e^{-\beta a}}{2}\right) - \frac{1}{2}\left(\frac{\beta}{ik}+\frac{ik}{\beta}\right)\left(\frac{e^{\beta a}-e^{-\beta a}}{2}\right)\right]Ge^{ika}$$

$$= \left[\cosh\beta a + \frac{i}{2}\left(\frac{\beta}{k}-\frac{k}{\beta}\right)\sinh\beta a\right]Ge^{ika}$$

$$T = \frac{GG^*}{AA^*} = \left|\frac{G}{A}\right|^2$$

$$\frac{A}{G} = \cosh\beta a + \frac{i}{2}\left(\frac{\beta}{k}-\frac{k}{\beta}\right)\sinh\beta a\, e^{ika}$$

$$\left|\frac{A}{G}\right|^2 = \left(\frac{A}{G}\right)\left(\frac{A}{G}\right)^*$$

$$\left|\frac{A}{G}\right|^2 = \cosh^2\beta a + \frac{1}{4}\left(\frac{\beta}{k}-\frac{k}{\beta}\right)^2\sinh^2\beta a \qquad \left(\because \cosh^2 x - \sinh^2 x = 1\right)$$

$$= 1 + \left[1+\frac{1}{4}\left(\frac{\beta}{k}-\frac{k}{\beta}\right)^2\right]\sinh^2\beta a = 1 + \frac{1}{4}\left[\left(\frac{\beta}{k}-\frac{k}{\beta}\right)^2+4\right]\sinh^2\beta a$$

$$\frac{\beta}{k} - \frac{k}{\beta} = \sqrt{\frac{V_0-E}{E}} - \sqrt{\frac{E}{V_0-E}} = \frac{V_0-E-E}{\sqrt{E(V_0-E)}} = \frac{V_0-2E}{\sqrt{E(V_0-E)}}$$

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$$\left(\frac{\beta}{K}-\frac{K}{\beta}\right)^2+4=\frac{(V_0-2E)^2}{E(V_0-E)}+4=\frac{V_0^2+4E^2-4EV_0+4EV_0-4E^2}{E(V_0-E)}$$

$$=\frac{V_0^2}{E(V_0-E)}$$

$$\frac{1}{T}=1+\frac{1}{4}\left[\frac{V_0^2}{E(V_0-E)}\right]\sinh^2\beta a$$

$$T=\frac{1}{1+\frac{V_0^2}{4E(V_0-E)}\sinh^2\beta a} \quad \text{--- (12)}$$

If $e^{\beta a}>>1$

$$\sinh^2\beta a=\left(\frac{e^{\beta a}-e^{-\beta a}}{2}\right)=\frac{e^{2\beta a}}{4} \quad \left(\begin{array}{l}e^{\beta a}\text{ is very large}\\ e^{-\beta a}\text{ is very small}\end{array}\right)$$

$$\frac{1}{T}=1+\frac{V_0^2}{4E(V_0-E)}\frac{e^{2\beta a}}{4}=1+\frac{V_0^2}{16E(V_0-E)}e^{2\beta a}$$

Here 1 is very small in comparison to $e^{\beta a}$, then.

$$\frac{1}{T}=\frac{V_0^2}{16E(V_0-E)}e^{2\beta a}$$

$$T=\left[\frac{16E(V_0-E)}{V_0^2}\right]e^{-2\beta a}$$

$$\boxed{T=\frac{16E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\beta a}}$$

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$$T = \frac{16E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{\frac{-2\sqrt{2m(V_0 - E)}\,a}{\hbar}}$$

$$T \approx e^{-2\beta a} \quad \text{———} \quad (13)$$

If a particle with energy E incidents on a thin energy barrier of height greater than E, there is a finite probability of the particle penetrating the barrier. This phenomenon is called tunnel effect.

Anju
6/5/22