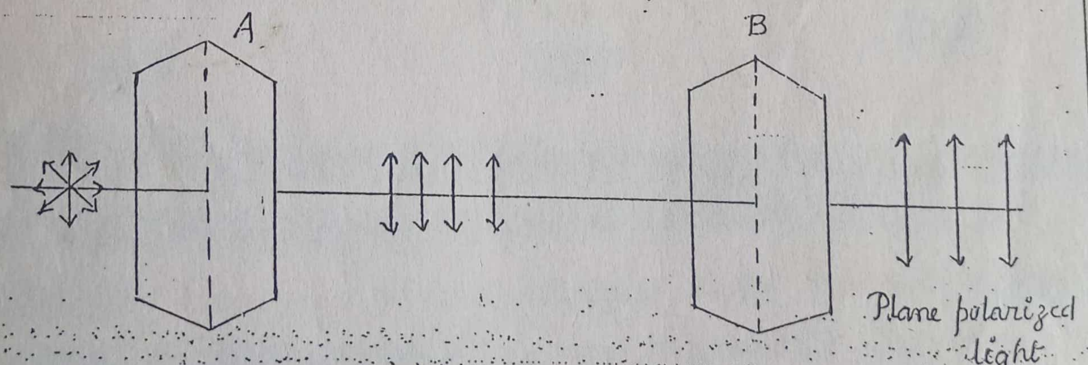
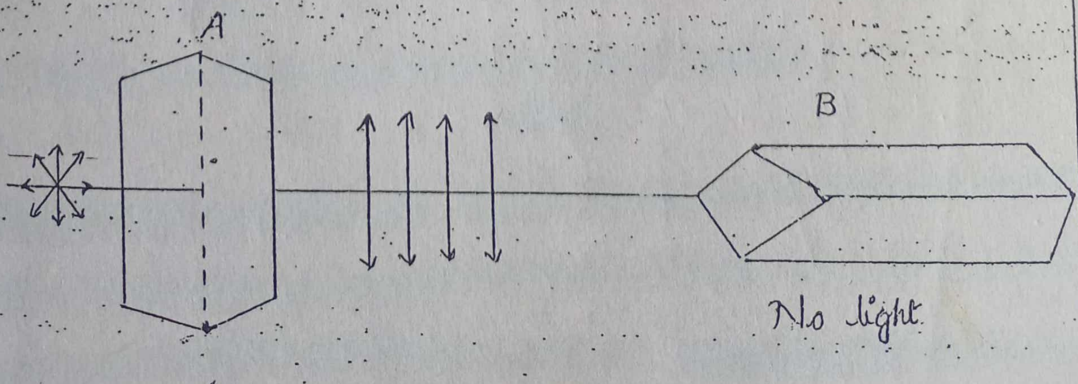
Polarisation

The phenomena of interference and diffraction establish the wave nature of light, but fail to show the exact nature of light – ie whether the light waves are longitudinal or transverse or are the vibrations linear, circular or elliptical. The phenomenon of polarisation establishes the transverse nature of light.

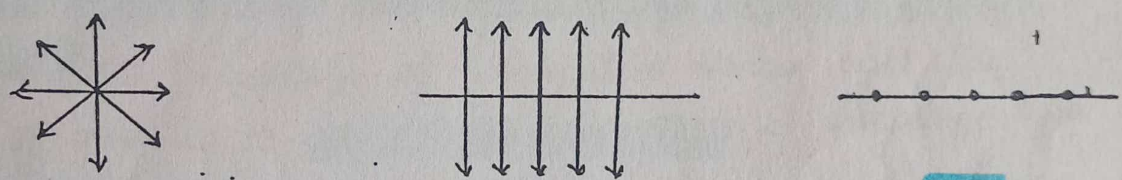
When a beam of an ordinary light from source S is allowed to fall normally on a thin plate of tourmaline crystal, cut with faces parallel to the optic axis only a part of incident light is transmitted. If the crystal A is rotated, no change in intensity of the transmitted light is observed. If this beam is allowed to pass through another similar crystal B placed with its axis parallel to A, the light is completely transmitted through B. If B is rotated gradually with respect to A, intensity goes on decreasing and is finally cut off after a rotation of 90°. Then





the crystals are said to be in crossed position. On further rotation of B, brightness gradually increases and finally attains full brightness when the axes of A and B are mutually parallel. This experiment shows that light waves are transverse in nature. It is found that after passing through the crystal, the light vibrations are confined only to a single line in a plane perpendicular to the direction of propagation of light. The light which exhibit the property of one sideness is called polarised light or the phenomenon of asymmetry of vibrations about the direction of propagation is called polarisation of light.

Unpolarized light : Ordinary light having vibrations along all possible planes perpendicular to the direction of propagation is said to be unpolarized. The unpolarized light is represented as in fig (a)

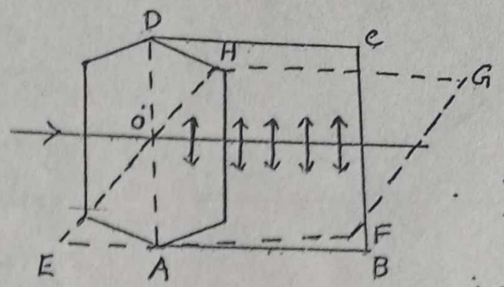


(a) (b) (c)

Plane polarized light : If the vibrations are confined along a single straight line, the light is said to be plane polarized. When the plane polarized light has got vibrations in the plane of paper, they are represented by arrows as shown in fig (b). When the vibrations lie in a direction perpendicular to the plane of paper, they are represented by dots as shown in fig (c).

Plane of polarisation : The plane containing the direction of propagation of light but containing no vibrations is called the plane of polarisation. The plane containing the direction of vibration and direction of propagation of light is called the plane of vibration.

In the figure ABCD is the plane of vibration and EFGH is the plane of polarisation



Methods to produce plane polarized light

Plane polarized light can be produced by various methods. Some of them are.

1) by reflection
2) by refraction
3) by double refraction
4) by scattering
5) by selective absorption

Plane polarisation by reflection

In 1808, Malus discovered that when a beam of unpolarized light is reflected from the surface of a transparent medium, the reflected light gets partially polarized. The percentage of polarization varies with the angle of incidence. At a particular angle of incidense the reflected beam is completely plane polarized. The angle of incidence at which maximum polarization occurs is called the angle of polarization. The polarizing angle is different for different reflecting surfaces. It is 57° for air - glass reflection, 33° for glass - air reflection and 53° for air - water reflection.

Brewster's law

Brewster discovered a simple relation between the angle of incidence at which the maximum polarization occurs and the refractive index of the medium. The tangent of the polarising angle is equal to the refractive index of the medium, which is Brewster's law. Then at

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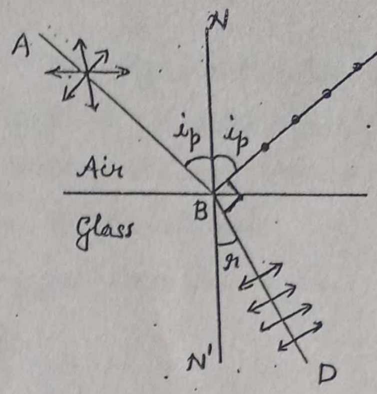
polarizing angle, the reflected and refracted rays are at right angles to each other.

Let a beam AB of an ordinary light be incident on the glass surface at the polarizing angle i_p. A part of it is reflected along BC and a part is refracted along BD. According to Brewster's law



$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p} \quad \text{--- (1)}$$

From Snell's law

$$\mu = \frac{\sin i_p}{\sin r} \quad \text{--- (2)}$$

From eqns. (1) and (2)

$$\mu = \frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$$

$$\cos i_p = \sin r$$

$$\sin(90 - i_p) = \sin r \qquad \text{or } 90 - i_p = r$$

$$\text{or } i_p + r = 90^\circ$$

From the figure

$$\angle NBC + \angle CBD + \angle DBN' = 180^\circ$$

$$\text{ie } i_p + \angle CBD + r = 180^\circ$$

$$90 + \angle CBD = 180^\circ \qquad [\text{as } i_p + r = 90^\circ]$$

$$\therefore \angle CBD = 90^\circ$$

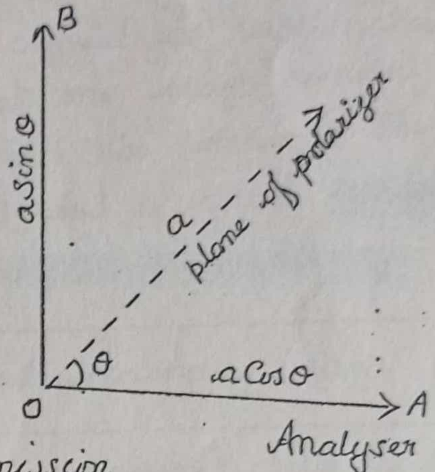
ie when the light is incident at polarizing angle, the reflected and refracted rays are mutually perpendicular to each other.

Polarization by refraction

When ordinary light gets refracted through any transparent medium, the refracted ray is partially polarized. In order to obtain completely polarized light, it is refracted through piles of plates which consists of adequate number of glass plates separated by airgaps. After multiple refraction through this arrangement the emerging light is completely polarized.

Malus law

In 1809, Malus found experimentally that when a beam of completely plane polarized light is incident on an analyzer, the intensity of the emergent ray varies as the square of the cosine of the angle between the planes of transmission of the analyzer and polarizer. This law is known as Malus law.



Let θ be the angle between the planes of the polarizer and analyzer and 'a' be the amplitude of the incident plane polarized light emerging from the polarizer. The plane polarized light of amplitude 'a' may be resolved into two components along and perpendicular to the planes of transmission of the analyser.

$$OA = a\cos\theta \quad \text{and} \quad OB = a\sin\theta$$

The perpendicular component is eliminated in the analyser while the parallel component is transmitted through it. ∴ The intensity of light emerging from the analyser is

$$I = (a\cos\theta)^2 = a^2\cos^2\theta = I_0\cos^2\theta$$

where $I_0 = a^2$ is the intensity of the completely plane polarized light.

Double refraction

When a ray of ordinary unpolarized light is passed through calcite or quartz crystal, it is split up into two refracted rays. One of the refracted rays follows the ordinary laws of refraction and hence is called ordinary ray. The other ray does not follow the ordinary laws of refraction and is called extraordinary ray. If an object is viewed through such a crystal,

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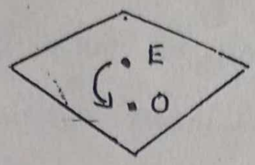
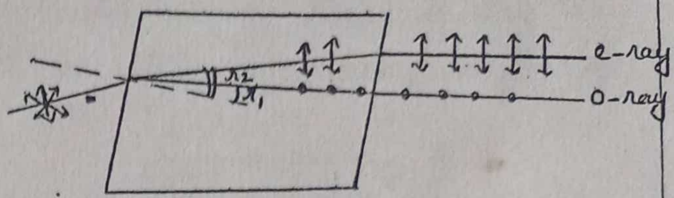
two images of the object are observed. One corresponds to ordinary or o-ray and other is extraordinary or e-ray. This phenomenon is called double refraction.





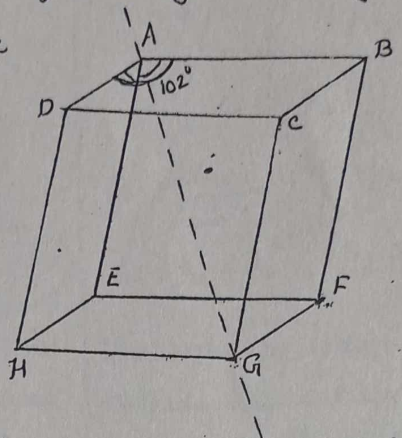
An ink dot is marked on a white paper and calcite crystal is placed over it. Two images of the ink dot are observed. Now if the crystal is rotated about a vertical axis, it is observed that one image remains fixed and the other image rotates with the rotation of the crystal. The fixed image is normal and is called ordinary image while other image is called extra-ordinary image.

	Ordinary rays	Extraordinary rays
1.	O-rays obeys the laws of refraction	E-rays does not obeys the laws of refraction
2.	O-rays have vibrations perpendicular to the principal section of the crystal.	E-rays have vibrations parallel to the principal section of the crystal
3	The refractive index of o-ray, $\mu_o = \frac{\sin i}{\sin r}$ is constant	The refractive index of e-ray $\mu_e = \frac{\sin i}{\sin r}$ varies with the angle of incidence i.
4	The velocity of o-ray is constant in all directions	The velocity of e-ray varies with the direction
5	The wavefront of o-rays emanating from a point source in a crystal will be spheroid	The wavefront of e-rays emanating from a point source in a crystal will be ellipsoid

$\mu = \frac{c}{v}$

Geometry of calcite crystal

The calcite crystal, also known as Iceland spar is chemically calcium carbonate. Each of the six faces of this crystal is a parallelogram having angles of 78° and 102° nearly. At the diametrically two opposite corners A and G, the angles of the three faces meeting there are all obtuse. The corners A and G are called blunt corners. At the rest of the six corners, one angle is obtuse and two are acute. The line passing through the opposite blunt corners and making equal angles with the three faces which meet there is the direction of the optics axis. Optic axis is a direction and not a particular line. A ray passing through the optic axis does not break up into ordinary and extraordinary rays, ie their velocities remain the same along the optic axis.



A plane containing the optic axis of the crystal and perpendicular to the two opposite refracting faces is called the principal section of the crystal.

Types of crystals

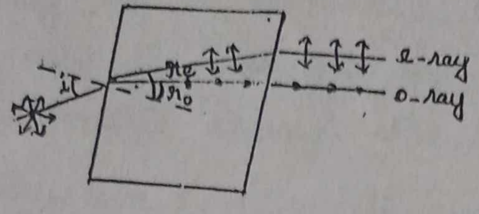
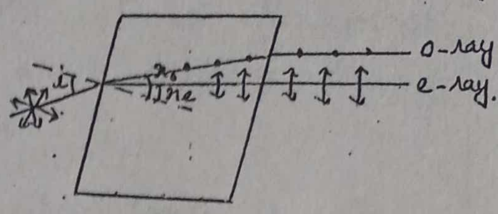
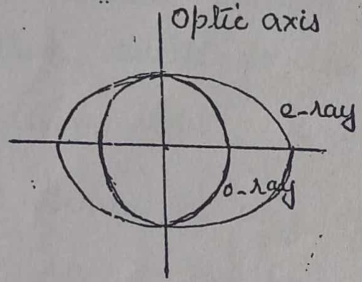
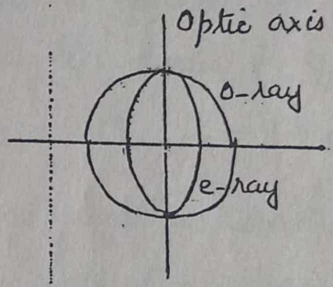
Depending upon the number of optic axis a crystal can have, crystals can be classified into two.

Uniaxial crystal – Crystals with one optic axis.

eg: calcite, quartz, tourmaline

Biaxial crystal – Crystals with two optic axes

eg: borax, mica, topaz

	Negative crystal	Positive crystal
		
	$r_o < r_e$.	$r_o > r_e$
	$\mu_o = \frac{\sin i}{\sin r_o}$ & $\mu_e = \frac{\sin i}{\sin r_e}$	$\mu_o = \frac{\sin i}{\sin r_o}$ & $\mu_e = \frac{\sin i}{\sin r_e}$
	As $r_o < r_e$, $\mu_o > \mu_e \Rightarrow v_o < v_e$	As $r_e < r_o$, $\mu_e > \mu_o$
3	The wavefront of o-rays lies inside the extraordinary wave surface as the velocity of o-ray is less than e-ray. 	The ordinary wave surface lies outside the extraordinary wave surface as the velocity of e-ray is less than o-ray. 
4	The velocity of o-ray is constant in all directions	The velocity of o-ray is constant in all direction
5	The velocity of e-ray varies with the direction. It is maximum in a direction perpendicular to the direction of optic axis	The velocity of e-ray varies with the direction. It has a minimum value in a direction perpendicular to the optic axis

Nicol prism

In 1826, William Nicol invented a convenient optical device for producing and analysing plane polarized light known as Nicol prism.

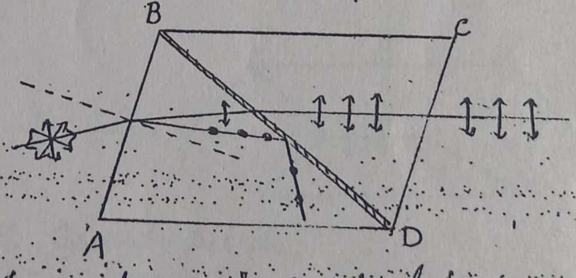
Principle

When ordinary ray of light is passed through a calcite crystal, it is broken up into two rays: ordinary and extraordinary rays. If by some means one of the two rays is eliminated, the ray emerging through the crystal will be plane polarized. In Nicol prism, ordinary ray is eliminated by total internal reflection so that only the extraordinary ray which is plane polarized is transmitted through the prism.

Construction : A calcite crystal whose length is 3 times its breadth is taken. The end faces of the crystal are cut in such a way that they makes angles of 68° and 112° in the principal section instead of 71° and 109°. The crystal is then cut into two pieces from one blunt corner to the orther along a plane perpendicular to the principal section. The cut faces are polished and cemented together with a thin layer of transparent substance called Canada Balsam. It is optically more denser than calcite for the e-ray and less denser for o-ray.

For sodium light, $\mu_o = 1.66$, $\mu_{CB} = 1.55$, $\mu_e = 1.49$

Action : When a ray of unpolarized light is incident on face AB of the prism, it splits into two refracted rays - o and e-rays. Both are plane polarized. The o-ray going from calcite to canada balsam travels from optically denser to a rarer medium. If θ is the critical angle, then



$$\theta = \sin^{-1}\left(\frac{\mu_{CB}}{\mu_o}\right)$$

$$= \sin^{-1}(0.933) = 69°$$

If the angle of incidence at canada balsam surface for o-ray is greater than critical angle, then o-ray ~~is~~ totally internally reflected and is finally absorbed by the side AD which is blackened. The e-ray travels from an optically rarer medium to a denser

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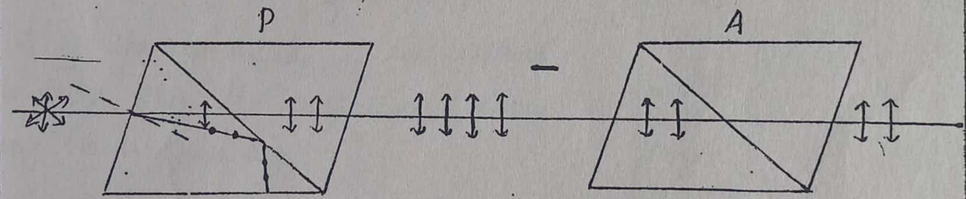
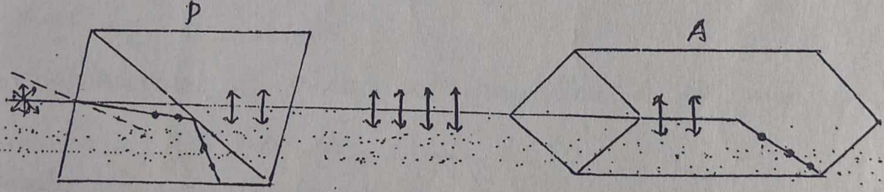
medium. ∴ it is not affected by the calcite - canada balsam surface and is transmitted. Thus by Nicol prism we are able to get a single beam of plane polarized light. Thus Nicol prism can be used as a polariser.

Nicol as an analyser

Consider two Nicols arranged co-axially one after another. When a beam of unpolarized light is incident on the first prism P, the emergent beam is plane polarized with it's vibrations in the principal section of prism P. This prism is called polarizer. The beam emerging from the polarizer falls on the second Nicol prism A called the analyser. When the principal section of prism A is parallel to that of P, the ray is completely transmitted and the intensity is maximum. In this position, the two Nicols are said to be parallel Nicols.



(a) parallel Nicols



(b) Crossed Nicols

When the prism A is rotated, the intensity of emergent beam decreases. When the principal section of A is at right angles to that of P, then no light emerges from A. In this position e-ray has no vibrations in the principal section of A and therefore it acts as ordinary ray inside the prism A and it totally reflected at the calcite - canada balsam surface and no light is transmitted. Here P produces plane polarized light and A detect it.

$\delta = 0$ $\delta = \pi/2$ $\delta = \pi$ $\delta = \pi/2$, $\theta = 45°$

Retardation plates

Retardation plates are plates cut from a doubly refracting crystal so as to produce a definite value of path difference or phase difference between e-ray and o-ray. The plate is cut with its face parallel to the optic axis. There are two types of retardation plates - Quarter wave plate and Half wave plate.

Quarter wave plate [QWP]

A plate of uniaxial doubly refracting crystal cut with optic axis parallel to the refracting faces and capable of producing a path difference of $\lambda/4$ or phase difference of $\pi/2$ between the ordinary and extra-ordinary waves is called a quarter wave plate.

When a plane polarized beam of light is incident normally on such a plate, the vibrations in the beam breakes into two, one along the optic axis and the other perpendicular to it. These two components travel in the same direction along the normal to the faces but with different velocities. If 't' is the thickness of the crystal and μ_o and μ_e be the refractive indices of o and e beams respectively, then the path difference introduced between o and e-rays is

$$\Delta = (\mu_o \sim \mu_e) t$$

For quarter wave plate, the path difference between the two emergent rays must be equal to $\lambda/4$

$$\therefore (\mu_o \sim \mu_e) t = \frac{\lambda}{4}$$

$$\text{or} \quad \boxed{t = \frac{\lambda}{4(\mu_o \sim \mu_e)}}$$

For negative uniaxial crystal like calcite, $\mu_o > \mu_e$, then

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

For positive crystal like quartz, $\mu_e > \mu_o$

$$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

Quarter wave plates are widely used for the production and detection of circularly and elliptically polarized light.

2. Half wave plate [HWP]

A plate of doubly refracting uniaxial crystal cut with its optic axis parallel to the refracting faces and capable of producing a path difference of $\lambda/2$ or phase difference of π between the o and e-rays is called a half wave plate.

If 't' is the thickness of half wave plate, μ_o and μ_e be the refractive indices for o and e-rays respectively, then the path difference introduced by the half wave plate between o and e-rays is

$$\Delta = (\mu_o \sim \mu_e) t$$

For half wave plate, the path difference is equal to $\lambda/2$.

$$\Delta = (\mu_o \sim \mu_e)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_o \sim \mu_e)}$$

For negative crystal like calcite, $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$

For positive crystal like quartz, $t = \dfrac{\lambda}{2(\mu_e - \mu_o)}$

Half wave plates are used in polarimeter for the construction of half shade device.

Production and Detection of circularly & elliptically

Plane polarized light by selective absorption

Some doubly refracting crystals possess the property of selective absorption. Such crystals not only produce two beams polarized in directions right angles to each other but also absorb one of the polarized component much more strongly than the other. In a crystal of proper thickness; one of the component is completely removed by absorption, whereas the other is transmitted. The crystals exhibiting this property are said to be dichroic and the phenomenon is known as dichroism.

Dic

Dic

One example of a dichroic crystal is tourmaline. A beam of unpolarized light incident normally on a 1mm thick crystal of tourmaline splits into ordinary and extraordinary beams both being plane polarized. The ordinary beam is completely absorbed in the crystal while extraordinary beam is partly absorbed and emerges from the crystal. Thus plane polarized light with vibrations in the plane of incidence is produced.

Optical Activity

When a beam of plane polarized light propagates through certain substances or crystals the plane of vibration (or plane of polarisation) of the emergent beam is not the same as that of the incident polarized beam but has been rotated through a certain angle. This phenomenon of rotation of the plane of vibration (or plane of polarization) is called rotatory polarization and this property of crystals and other substances is called optical activity and substances which show this property are called optically active.

Specific Rotation

The optical activity of a substance is measured by its specific rotation. The specific rotation of an optically active substance at a given temperature for a given wavelength of light is defined as the rotation (in degrees) produced by a path of one decimetre length in a substance of unit density.

If θ is the rotation produced by l decimeter length of an optically active substance, concentration is c gm/cc, then the specific rotation at a given temperature t for a given wavelength of light λ is expressed by

$$S_\lambda^t = \frac{\theta}{l \times c}$$

If l is in cm, then $S_\lambda^t = \frac{100\theta}{l \times c}$

The unit of specific rotation is deg. $(\text{decimetre})^{-1} (\text{gm/cc})^{-1}$.

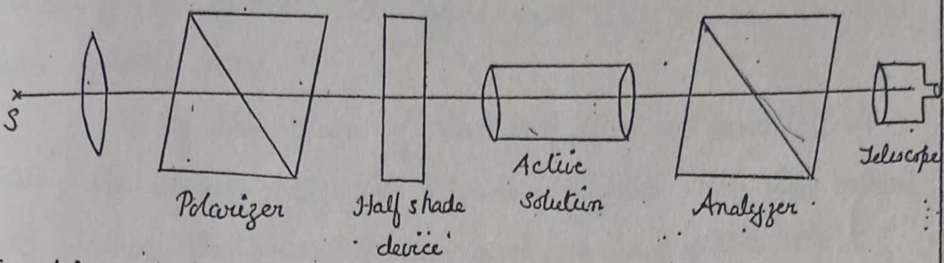
Polarimeters

A device designed for the accurate measurement of the angle of rotation of the plane of polarization of a plane polarized light by an optically active medium is said ~~to~~ be polarimeter. By measuring this angle, specific rotation can be determined. There are two types of polarimeters – a) Laurent's half shade polarimeter

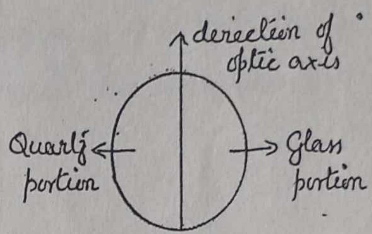
b) Biquartz polarimeter

Laurent's Half shade polarimeter

Laurent's half shade polarimeter consists of two Nicol prisms – one acts as a polarizer and the other as analyzer. The two are capable to rotate about a common axis. The glass tube containing optically active solution is placed between the two Nicols. A half shade device is placed between the polarizer and the glass tube. A Galilean telescope with graduated scale is used to view the emergent light and also to determine the angle of rotation.



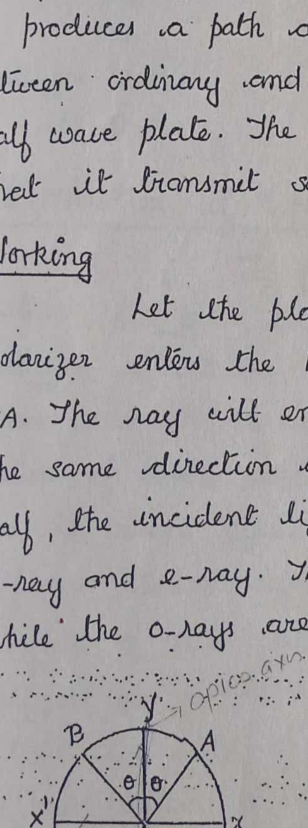
Function of half shade device



The half shade device consists of two semi-circular plates, one made of glass and other of quartz. They are cemented together as shown in the figure. The thickness of the quartz half is taken such that it produces a path difference of $\lambda/2$ or phase difference of π between ordinary and extraordinary rays. ie it acts as a half wave plate. The thickness of the glass plate is such that it transmit same amount of light as quartz plate.

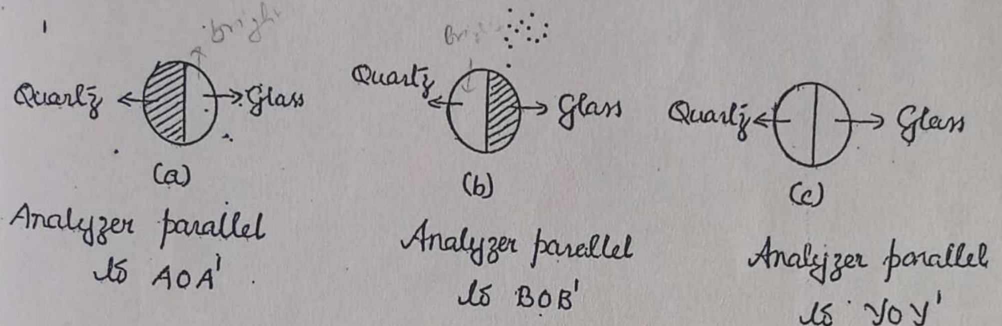
Working

Let the plane polarised light emerging from the polarizer enters the half shade device along the direction OA. The ray will emerge out from the glass plate along the same direction of vibration, ie along OA. In the quartz half, the incident light splits up into two components – o-ray and e-ray. The e-rays are parallel to optic axis (YY') while the o-rays are perpendicular to it (XX'). Within the quartz half they travel with unequal speed and gain a phase difference of π or path difference of $\lambda/2$ on emergence from quartz plate. Thus the emergent light will have vibrations along OX' (o-ray) and OY (e-ray). The two combine to form linear resultant vibration along the direction OB. If the principal plane of the analyser



is parallel to AOA', the light from glass half will pass unobstructed while that through quartz half is partially obstructed. The glass part will appear brighter than quartz half.

If the plane of the analyser is parallel to BOB', the quartz half will appear brighter than the glass half. When the plane of the analyzer is parallel to YOY', both the quartz and glass half are equally inclined to the principal plane of analyzer and hence two components make the two halves equally bright.



Determination of specific rotation of sugar solution

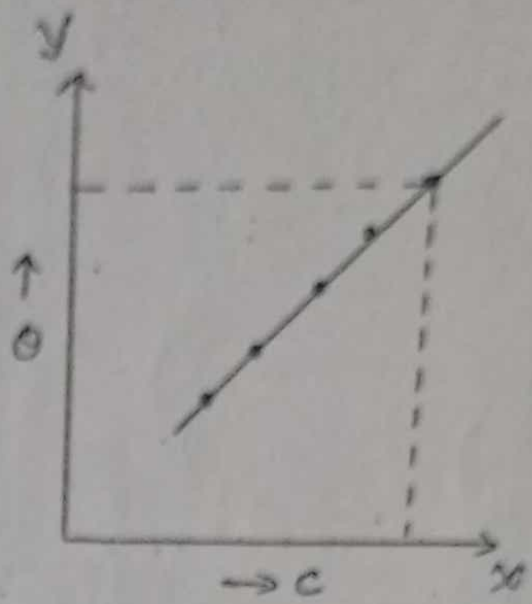
To find the specific rotation of sugar solution, the glass tube is filled with clean water and analyzer is set in the position of equal brightness of two halves and the vernier reading is noted. Now the sugar solution of known concentration is filled in the tube and tube is placed again in same place. Due to the rotation of plane polarized light by the optically active solution, the field of view will not be equally bright. Then the analyzer has to be rotated and brought to a position so that the field of view again appear equally bright. The new position of vernier on the circular scale is noted. The difference of two readings give the angle of rotation θ produced by the solution. A graph is plotted between concentration c and angle of rotation θ and the ratio θ/c is determined.

Then specific rotation is determined by the formula

$$S_{\lambda}^{t} = \frac{\theta}{l \times c}$$

where l is the length of the tube in decimeter

$c \rightarrow$ conc. in gm/cc.



Polarization

1. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is a maximum. Through what angle must either sheet be turned so that the intensity becomes one half the initial value ?

$$I = I_0 \cos^2\theta$$

$$\frac{I_0}{2} = I_0 \cos^2\theta \qquad \cos^2\theta = 1/2$$

$$\cos\theta = 1/\sqrt{2} \qquad \theta = \cos^{-1}(0.707) = 45^\circ$$

2. At what angle the light should be incident on a glass plate ($\mu = 1.5697$) to get a plane polarized light by reflection.

$$\mu = \tan i_p$$

$$i_p = \tan^{-1}(1.5697) = 57^\circ 30'$$

3. A beam of linearly polarized light is changed into circularly polarized light by passing it through a thin sheet of crystal 0.003 cm thick. Calculate the difference of refractive indices of two rays, assuming it to be of minimum thickness and the wavelength is 6000 Å.

$$t = \frac{\lambda}{4(\mu_o \sim \mu_e)}$$

$$\mu_o \sim \mu_e = \frac{\lambda}{4 \times t} = \frac{6000 \times 10^{-8}}{4 \times 0.003}$$

$$= \frac{6000 \times 10^{-8}}{0.012} = 0.005$$

4. Calculate the thickness of the Quartz half wave plate for sodium light of wavelength 5893 Å, given that the indices of refraction of Quartz of the ordinary and extraordinary rays are 1.5442 and 1.5533 respectively.

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

$$= \frac{5893 \times 10^{-8}}{2 \times (1.5533 - 1.5442)}$$

$$= \frac{5893 \times 10^{-8}}{2 \times 0.0091} = 0.0032 \text{ cm}$$

5. Calculate the thickness of half wave plate for sodium light ($\lambda = 5893$ Å), given $\mu_o = 1.54$ and ratio of velocity of ordinary and extra-ordinary components is 1.007. Is the crystal positive?

$$\frac{v_o}{v_e} = \frac{\mu_e}{\mu_o} = 1.007$$

$$\mu_e = 1.007 \times 1.54 = 1.551$$

$$t = \frac{\lambda}{2(\mu_e - \mu_o)} = \frac{5893 \times 10^{-10}}{2(1.551 - 1.54)} = 0.267 \times 10^{-4} \text{ m}$$

As $\mu_e > \mu_o$, the crystal is positive.

6. A sugar solution in a tube of length 20 cm. produces optical rotation of 30°. The solution is then diluted to one third of its previous concentration. Find the optical rotation produced by 30 cm. long tube containing diluted solution.

$$S = \frac{\theta}{l \times c}$$

In 1st case, $l = 20$ cm., let $c = c$, $\theta = 30^\circ$

$$S = \frac{30^\circ}{0.2 \times c} \quad \text{--- (1)}$$

In 2nd case, $l = 30$ cm, let $c = \frac{c}{3}$, $\theta = ?$

$$S = \frac{\theta}{0.3 \times c/3} \quad \text{--- (2)}$$

Equating (1) and (2)

$$\frac{30}{0.2 \times c} = \frac{\theta}{0.3 \times c}$$

$\theta = 15^\circ$

7. Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.

$$S = \frac{\theta}{l \times c}$$

$\theta = 13.2^\circ$, $l = 20\text{ cm} = 2\text{ dm}$, $c = 10\% = 10/100$

$$S = \frac{13.2 \times 100}{2 \times 10} = 66^\circ$$

8. A solution of dextrose [specific rotation 52.5°] causes a rotation of 12° in a column 10 cms. long. Find the concentration of the solution.

$$S = \frac{\theta}{l \times c}$$

$$c = \frac{\theta}{S \times l} = \frac{12}{1 \times 52.5^\circ} = 0.228\text{ gm/cc}$$

9. On introducing a polaristeter tube 25 cm. long and containing sugar-solution of unknown strength, it is found that the plane of polarization is rotated through 10°. Find the strength of the sugar solution in g/cm³. Given the specific rotation of sugar solution is 60° per decimeter per unit concentration.

$$c = \frac{\theta}{S \times l} = \frac{10}{2.5 \times 60} = 6.67\%$$

A 200 mm long tube containing 48 cm³ of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution.

$$S = \frac{\theta}{l \times c}$$

$S = 66^\circ \quad \theta = 11^\circ, \; l = 200\,mm = 2\,dm$

$$c = \frac{x\,gm}{48\,cm^3}$$

$$66 = \frac{11}{2 \times x/48}$$

$$x = \frac{11 \times 48}{2 \times 66} = 4 \text{ gms.}$$