Statistical mechanics

It is a theortical tool or theoritical physicist. The Branch of physics which considers how the overall behaviour of a System of many particles is related to the properties of the particles themselves is known as statistical mechanics.

Distribution laws:

Every matter (solid, liquid, gas) is assembly of a number of microscopic particles. The actual motion or interactions of individual particles in an assembly cannot be described. But after considering the distribution of individual particles in different possible states and with the help of distribution laws the microscopic properties of an assembly can be explained in terms of individual particles.

macroscopic properties

There are three distribution laws

1. Maxwell Boltzman (M-B) distribution law

1. This law can be apply to a System having identical, distinguishable particles of any spin.

Eg: molecules of gas

2. So the particles are distinguishable because they have specified position and momentum.
3. The particles don't follow restrictions because they have 'n' spin.

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4. If there are 2 particles 'P' and 'Q' then they can arrange in two states in four ways.

P	Q		Q	P		PQ				PQ
A	B		A	B		A	B		A	B

5. The mathematical form of maxwell-Boltzman distribution law,

$$n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}},$$

n_i = number of particles in ith state
g_i = no. of cells in ith state
ε_i - Energy of ith state
α - Constant

$\beta = \frac{1}{KT}$, K = Boltzmann Constant
T = temperature

* Bose Einstein (B-E) distribution law

1. This law is applied to a system having identical, indistinguishable particles of integral spin.
Eg: Photon.

2. They have symmetric wave function.

$$\hat{P}\psi(1,2) = +\psi(2,1), \quad \hat{P} - \text{operator}$$

3. These particles don't follow Pauli's Exclusion law.

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4. mathematical form of B-E distribution law is

$$n_i = \frac{g_i}{(e^{\alpha+\beta\epsilon_i}-1)}$$

eg: Radiation in cavity

⇒ Application:

It is used to derive plank's Black Body radiation formula.

3. Fermi Dirac (F-D) distribution law

1. This law can be applied to a system having identical indistinguishable particles of half integral spin.

Eg: Electron

2. They have anti-symmetric wave function. $\hat{P}\psi(1,2) = -\psi$
3. These particles follow pauli's exclusion law.
4. mathematical form.

$$n_i = \frac{g_i}{(e^{\alpha+\beta\epsilon_i}+1)}$$

Applications:

using this law we can explain the properties of metals like electrical conductivity, thermal conductivity, thermoionic emission, photoelectrical effect, specific heat of metals.

Bosons

- These are identical particles having zero or integral spin. They can not be distinguish from one another. They don't obey Pauli's exclusion principle.
- These particles are also known as Bose particles.
- These particles are governed by Bose Einstein distribution law.

Example: photon (spin 1), phonon (spin zero), meson (spin zero)

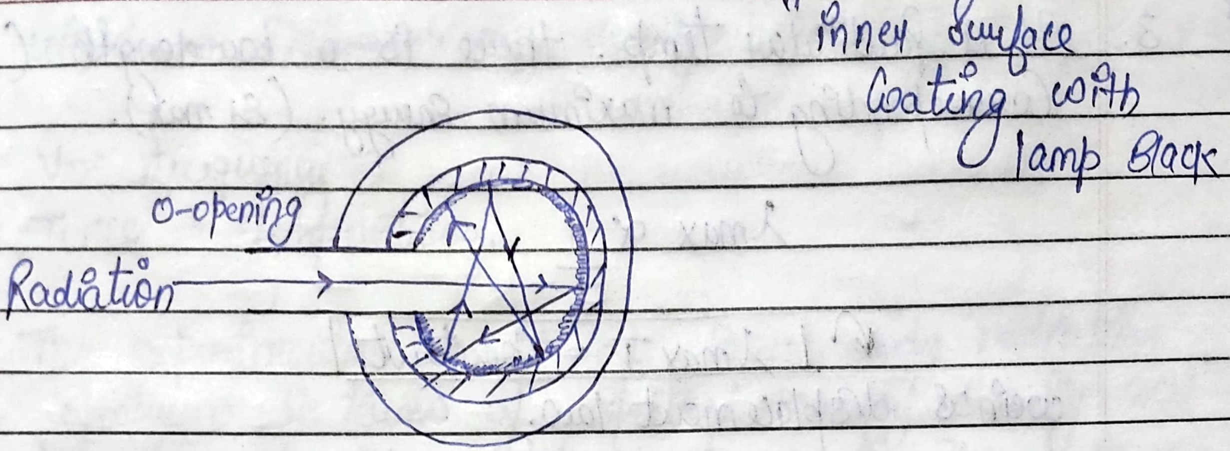
Fermions

- These are identical particles having odd half integral spin.
- They can not be distinguish from one another.
- They follow Pauli Exclusion principle.
- These particles are also known as fermi particles.
- They are governed by Fermi Dirac distribution law.

Example: Electron, proton and neutron.

Black Body

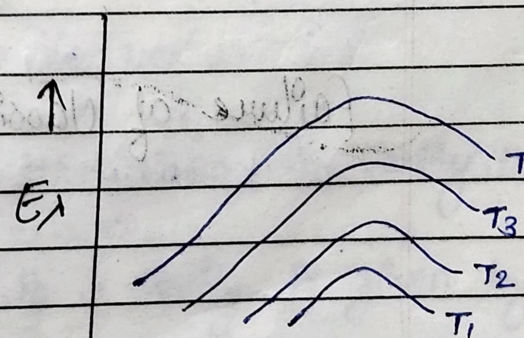
Ideal or perfect Black body is one which can absorb all the radiation incident on it and can emit all the radiation on heating it. But it is impossible to design such type of ideal Black body.
Ferry Design a Black body which can absorb 97% of radiation incident on it. and can emit 97% radiation on heating it.



→ Construction of Fery's Black Body:

It Consists of double walled metallic Sphere. The inner Surface of this Sphere is Coated with material lamp Black. This Sphere has a Small opening-O. when the radiation is incident on this double-walled metallic Sphere through opening-O then this radiation Suffers multiple reflections & 97% of this radiation is absorbed. So this double walled metallic Sphere works as a Black Body.

$E\lambda$ = Energy emitted per Second
λ = wavelength
T = Temperature



$T4 > T3 > T2 > T_1$

* The Black Body radiation Spectrum is Shown in graph. These are the following Conclusions from the graph.

1. The Energy emitted per Second from the black body increases on increases the temp. of black body.
2. There is a non-uniform distribution of energy in black body.

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3. At a particular temp. there is a wavelength (λmax) Corresponding to maximum Energy. ($E_\lambda max$).

$$\lambda max \propto \frac{1}{T}$$

$$\boxed{\lambda max\, T = Constant}$$

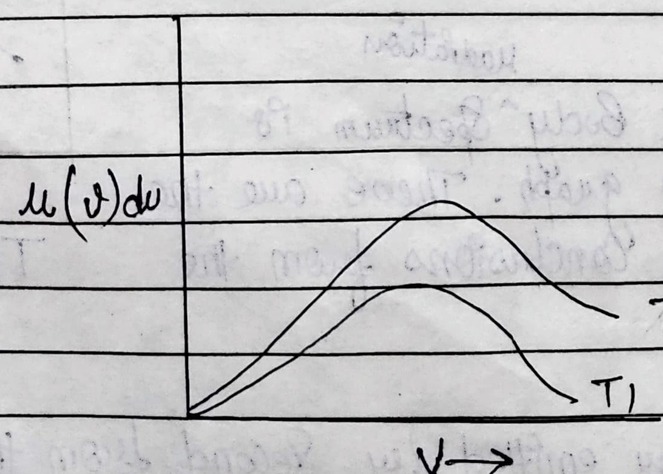
wein's displacement law.

4. The area under each Curve Shows the energy emitted per Second of the black body. It is experimentally found that the energy emitted per Second is directly proportional to fourth power of T i.e $E_\lambda \propto T^4$

$$\boxed{E_\lambda = \sigma T^4}$$

This is Stefan's law.

Failure of classical theory to explain Black body radiation Spectrum



$T_1 < T_2$

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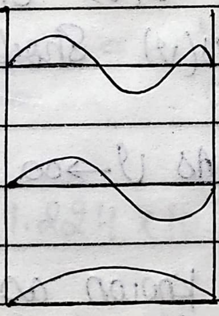
$u(\nu)d\nu \rightarrow$ Spectral energy density between frequency ν and $\nu + d\nu$

$\nu \rightarrow$ frequency

$T_1, T_2 \rightarrow$ temperatures.

The experimental result of Black Body radiation Spectrum in terms of Spectral Energy density and frequency at different temperatures is shown in graph.
But the question arises why does the Black body radiation Spectrum has the shape as in figure. (graph).
It was first explained by the Rayleigh – Jean's law. This law is based on classical theory.



$L = \frac{n\lambda}{2}$

Standing wave

Acc. to Rayleigh – Jean's the radiation in cavity is a Series of electromagnetic waves.
The no. of Standing waves in a cavity is given by

$$\boxed{G(\nu)d\nu = \frac{8\pi \nu^2 d\nu}{c^3}} \quad - (1)$$

Now we will find out the average energy per Standing electromagnetic wave. Rayleigh – Jean's assumed that the Standing electromagnetic wave behaves as a 1 dimensional harmonic oscillator. Because the electromagnetic wave is produced by the oscillating electrons. According to the classical theory the average Energy per oscillator or the Average energy per standing electromagnetic wave is given by

$$\bar{E} = KT \quad - (2)$$

$$\boxed{u(\nu)d\nu = \frac{8\pi \nu^2}{c^3} KT d\nu} \quad - (3) \quad \text{as } \boxed{u(\nu)d\nu = G(\nu)\bar{E}\, d\nu}$$

This is the Rayleigh - Jean's formula

It is clear from eqn (3) as ν increases towards ultra-violet region $u(\nu)$ should increase as ν^2 and if $\nu \to \infty$, then $u(\nu)$ should be tends to ∞ (infinity).

But experimentally As $\nu \to \infty$ then $u(\nu) \to 0$.

This discrepency is known as ultraviolet catastrophe. and hence classical physics is failed to explain the black body radiation spectrum.

Que.1 Calculate the de-Broglie wavelength associated with electron having 10 KeV energy.

$m_e = 9.1 \times 10^{-31}$ Kg

$h = 6.6 \times 10^{-34}$ joule-second.

Soln: $\lambda = \frac{h}{mv}$, $\lambda = \frac{h}{\sqrt{2eVm}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-34} \times 10^{4}}}$

$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 10^{3}}}$, $\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{18.2 \times 10^{-27}}}$

$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{1.82 \times 10^{-28}}} = \frac{6.6 \times 10^{-34}}{1.34 \times 10^{-28}} = \frac{4.9 \times 10^{-6}}{\sqrt{1.6 \times 10^{-19}}}$

$\lambda = \frac{4.9 \times 10^{-6}}{\sqrt{0.16 \times 10^{-20}}} = \frac{4.9 \times 10^{-6}}{0.4 \times 10^{-10}} = 12.25 \times 10^{+4}$

$= 1.22 \times 10^{5}$

$$\boxed{\lambda = 1.224 \times 10^{-11} m}$$

Que. what is the de-broglie wavelength of an electron which has been accelerated from rest through a potential difference of 100 V.

Soln: $\lambda = \frac{h}{\sqrt{2eVm}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 100 \times 9.1 \times 10^{-31}}}$

$$\boxed{\lambda = 1.22 A^{\circ}}$$

Que.3 The position & momentum of 1 KeV electron are simultaneously determine of its position is located within 1A°. what is the % of uncertainity of its momentum.

Soln: $\Delta x \Delta p = h/2\pi$

$10^{-10}\ \Delta p = \dfrac{6.6\times10^{-34}}{2\times3.14}$, $\boxed{\Delta p = 1.05\times10^{-24}}$

$K.E = \dfrac{p^2}{2m}$, $p = \sqrt{2mE}$, $\boxed{p = 1.71\times10^{-23}}$

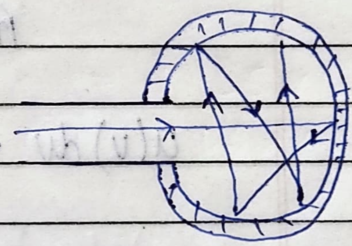
$\% = \dfrac{\Delta p}{p}\times100 = \dfrac{1.05\times10^{-24}}{1.71\times10^{-23}}\times100$

$\boxed{\% = 6.1\ \%}$

Bose Einstein applied to explain Black Body radiation.

Black Body radiation Spectrum Couldn't be explain using Rayleigh - Jean's law which is based on classical mechanics. But Planck's Quantum theory which is based on Quantum mechanics Can explain the Black Body radiation Spectrum.

Acc. to Planck's Quantum theory radiation Consist of a no. of tiny particles these particles are known as Photons. Photons have discrete energies not Continuous energies.



The no. of Standing electromagnetic wave per unit volume in the Cavity is given by

$$G(\nu)\,d\nu = \frac{8\pi\nu^2 \, d\nu}{c^3} \quad \text{— (1)}$$

The Spectral energy density between the freq. range ν and $\nu + d\nu$ is given by

$$u(\nu)\,d\nu = G(\nu).\bar{E}\,d\nu$$
$$= G(\nu).\bar{E}\,d\nu \quad \text{— (2)}$$

$\bar{E}$ = Average energy per Standing wave

Acc. to Quantum mechanical theory the avg. energy per Standing wave is given by

$$\bar{E} = \frac{h\nu}{(e^{h\nu/KT} - 1)}$$

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$$u(\nu)\,d\nu = \frac{8\pi\nu^2 h\nu}{c^3\,(e^{h\nu/KT}-1)}\cdot d\nu$$

$$\boxed{u(\nu)\,d\nu = \frac{8\pi h\nu^3\,d\nu}{c^3\,(e^{h\nu/KT}-1)}} \quad -(3)$$

This is planck's Black Body formula. which agrees the experimental result of Black Body radiation spectrum at all frequencies.

(i) # Plack's radiation formula at low frequency.

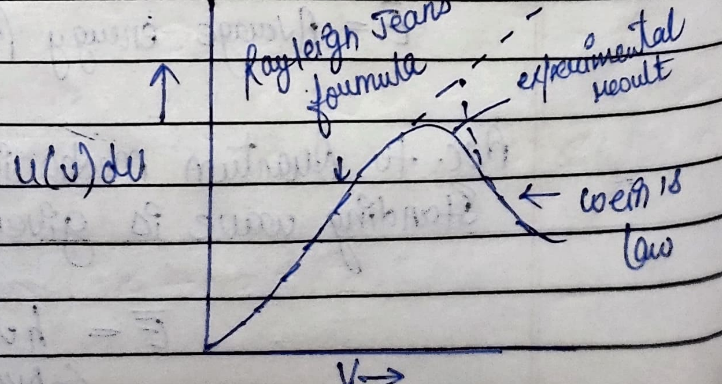
⇓

At low frequency $(h\nu \ll KT)$

$$u(\nu)\,d\nu = \frac{8\pi h\nu^3\cdot d\nu}{c^3\left[1+\frac{h\nu}{KT}-1\right]}$$

$$\boxed{u(\nu)\,d\nu = \frac{8\pi\nu^2 KT\cdot d\nu}{c^3}}$$

This is ~~planck's~~ Rayleigh-Jean's ~~Radiation~~ formula which is derived from planck's Radiation formula at low frequency it means Rayleigh-Jeans formula is valid at low frequencies.



At low frequency → Rayleigh Jean's formula
At high frequency → wein's law
All over graph → Planck's Black Body radiation formula

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(ii) # Planck's radiation formula at high frequency ($h\nu >> KT$) or short wavelength.

Planck's Radiation formula in λ form.

putting $\nu = \frac{c}{\lambda}$

$$d\nu = \frac{c}{\lambda^2} \cdot d\lambda$$

$$u(\lambda)\,d\lambda = \frac{8\pi h\, c^3/\lambda^3}{c^3\left(e^{hc/\lambda KT} - 1\right)} \cdot \left(\frac{-c}{\lambda^2}\right) d\lambda$$

$$u(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda KT} - 1\right)} \cdot d\lambda$$

$\frac{du(\lambda)}{d\lambda} = 0$ and put $\lambda = \lambda_{max}$

$$\boxed{\lambda_{max}\, T = \text{Constant}}$$

This is wein's formula

molecular energies in an ideal gas (Application of maxwell - Boltzmann distribution law).

let us Consider an ideal gas having 'N' no. of molecules here we will apply maxwell Boltzmann distrib- law to find the distribution of energies among the molecules in this ideal gas.

The mathematical form of maxwell Boltzmann law is given

$$n_i = \frac{g_i}{e^{\alpha + \beta \varepsilon_i}}$$

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Here we are considering that the energy in ideal gas is in continuous form. So the mathematical form of M-B law can be written as

$$n(\varepsilon)\,d\varepsilon = \frac{g(\varepsilon)\,d\varepsilon}{e^{\alpha+\beta\varepsilon}} = \frac{g(\varepsilon)\,d\varepsilon}{e^{\alpha}.e^{\beta\varepsilon}}$$

where, $n(\varepsilon)\,d\varepsilon$ = no. of molecules having energy between ε and $(\varepsilon+d\varepsilon)$

$g(\varepsilon)\,d\varepsilon$ = no. of states having energy between ε and $(\varepsilon+d\varepsilon)$

α = Constant

β = Constant, $\beta = 1/KT$

K = Boltzmann Constant

T = temperature

$$n(\varepsilon)\,d\varepsilon = g(\varepsilon)\,e^{-\alpha}\,e^{-\beta\varepsilon}\,d\varepsilon$$

$\beta = 1/KT$

$$\boxed{n(\varepsilon)\,d\varepsilon = g(\varepsilon)\,e^{-\alpha}\,e^{-\varepsilon/KT}\,d\varepsilon} \quad \text{--- (1)}$$

$$g(p)\,dp \propto 4\pi p^2.dp$$

Our next step is to find the value of $g(\varepsilon)\,d\varepsilon$ in terms of momentum.

A molecule having energy ε as a momentum P. whose magnitude can be written as

$$p = \sqrt{2m\varepsilon} \qquad (\because \varepsilon = p^2/2m)$$

$$= \sqrt{p^2x + p^2y + p^2z}$$

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P_x, P_y and P_z represents different States of motion of molecules. let us Consider a momentum space whose Coordinate axis are P_x, P_y and P_z.
The no. of States,

$$g(p)\,dp$$

having momentum between p and $(p+dp)$ is proportional to the volume of Spherical Cell in momentum Space having radius P and thickness dp.

$$g(p)\,dp \propto 4\pi p^2 dp$$

$$g(p)\,dp = \underline{A\,4\pi}\,p^2 dp$$

$\Downarrow$

B – Constant

$$\boxed{g(p)\,dp = Bp^2 dp}$$

Put in this eqn.

$$\varepsilon = p^2/2m$$

$$p^2 = \varepsilon 2m$$

$$2p\,dp = 2m.d\varepsilon \qquad \boxed{dp = \frac{m\,d\varepsilon}{p}}$$

$$g(p)\,dp = Bp^2 \frac{m\,d\varepsilon}{p}, \quad = Bp\,m\,d\varepsilon$$

$$g(p)dp = B\sqrt{2m\varepsilon}\; m\,d\varepsilon \qquad \because p = \sqrt{2m\varepsilon}$$

$$g(p)dp = B\sqrt{2}\, m^{3/2} \varepsilon^{1/2} d\varepsilon$$

$$g(p)\,dp \simeq g(\varepsilon)\,d\varepsilon$$

$$\boxed{g\varepsilon(d\varepsilon) = B\sqrt{2}\, m^{3/2}\, \varepsilon^{1/2} d\varepsilon} \quad -(2)$$

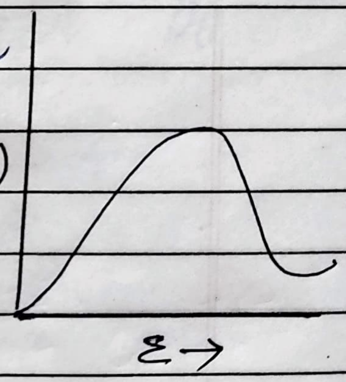
Put eqn ② in eqn ①

$$n(\varepsilon)\,d\varepsilon = \underbrace{B\sqrt{2}\, m^{3/2}\, e^{-\alpha}}_{C}\, e^{-\varepsilon/KT}\, \varepsilon^{1/2}\, d\varepsilon$$

$$\boxed{n(\varepsilon)\,d\varepsilon = C\,\varepsilon^{1/2}\, e^{-\varepsilon/KT}\, d\varepsilon} \quad \text{—— ③}$$

Normalised condn: $\int_0^\infty n(\varepsilon)\,d\varepsilon = N$

using normalisation condition in ideal gas having N no. of molecules.



$$\int_0^\infty n(\varepsilon)\,d\varepsilon = N$$

$$C\int_0^\infty \varepsilon^{1/2}\, e^{-\varepsilon/KT}\, d\varepsilon = N \quad , \; a = 1/KT$$

$$C \frac{1}{2 \times 1/KT}\sqrt{\frac{\pi}{1/KT}} = N \qquad \because \left[\int_0^\infty x^{1/2} e^{-ax}\,dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}\right]$$

$$\frac{C}{2} KT \sqrt{\pi}\sqrt{KT} = N$$

$$C(KT)^{3/2}\sqrt{\pi} = 2N \rightarrow C = \frac{2N}{(KT)^{3/2}}\frac{1}{\sqrt{\pi}}$$

$$C = \frac{2N}{(KT)^{3/2}}\frac{\pi}{\sqrt{\pi}\times\pi} \;,\; \frac{2\pi N}{(KT)^{3/2}\pi^{3/2}}$$

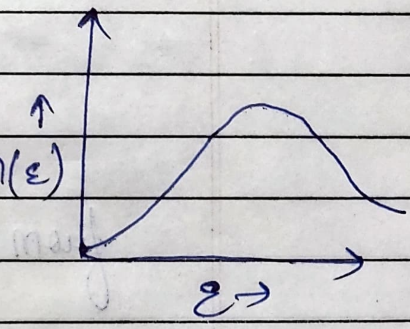
$$\boxed{C = \frac{2\pi N}{(\pi KT)^{3/2}}}$$ → put this eqn in eqn ③

$$\boxed{n_{\varepsilon} d(\varepsilon) = \frac{2N\pi}{(\pi KT)^{3/2}} \varepsilon^{1/2} e^{-\varepsilon/KT} \, d\varepsilon} \quad — ④$$

This is maxwell-Boltzmann distribution law for ideal gas.

∴ Total Energies of molecule,

$$\varepsilon = \int_0^{\infty} \varepsilon \, n(\varepsilon) \, d\varepsilon$$



$$E = \frac{2N\pi}{(\pi KT)^{3/2}} \int_0^{\infty} \varepsilon^{3/2} e^{-\varepsilon/KT} \, d\varepsilon$$

$\sqrt{\frac{\pi}{a}}$

$$\boxed{\varepsilon = \frac{3}{2} NKT}$$

Average Energy: $(\bar{\varepsilon}) = \frac{E}{N} = \frac{3/2 \, \cancel{N} KT}{\cancel{N}}$

$$\boxed{\bar{\varepsilon} = \frac{3}{2} KT}$$

molecular speed in ideal gas →

we know, The M-B energy distribution law in ideal gas.

$$n(\varepsilon)d\varepsilon = \frac{2N\pi}{(\pi KT)^{3/2}} \varepsilon^{1/2} e^{-\varepsilon/KT} d\varepsilon \quad -(5)$$

for ideal gas, $\varepsilon = \frac{1}{2}mv^2$, $\boxed{\text{i.e. } \varepsilon = 0}$

$$\varepsilon = \frac{1}{2}mv^2 \quad , \quad d\varepsilon = \frac{1}{2} m\, 2v\, dv$$

$$d\varepsilon = mv\, dv$$

from eqn (5)

$$n(v)\,dv = \frac{2N\pi}{(\pi KT)^{3/2}} \sqrt{\frac{1}{2}mv^2}\; e^{-\frac{mv^2}{2KT}}\; mv\,dv$$

$$\boxed{n(v)\,dv = \frac{2N\pi}{(\pi KT)^{3/2}} \sqrt{\frac{1}{2}}\; m^{3/2} e^{-mv^2/KT}\, v^2 dv} - (6)$$

This is M-B velocity distribution law for ideal gas.

(i) $V_{rms} = \sqrt{3KT/m}$

(ii) $V_{average} = \sqrt{8KT/m\pi}$, $\sqrt{\frac{8KT}{\pi m}}$

(iii) $V_{most\ probable} = \sqrt{\frac{2KT}{m}}$

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(i) Rms velocity

$$\varepsilon = \frac{3}{2}KT, \quad \frac{1}{\cancel{2}}mv^2 = \frac{3}{\cancel{2}}KT, \quad v^2 = \frac{3KT}{m}$$

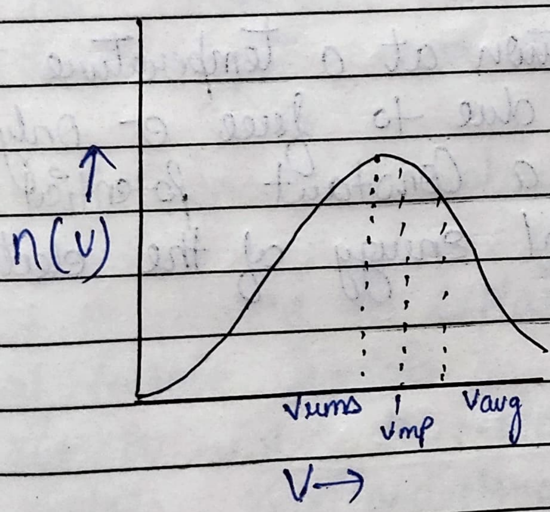
$$\boxed{V_{rms} = \frac{3KT}{m}}$$

(ii) average Velocity: $V = \int_0^\infty v\,n(v)\,dv$

$$V_{avg} = \frac{V}{N}, \quad \boxed{V_{avg} = \sqrt{\frac{8KT}{\pi m}}}$$

(iii) most probable Speed: $n(v) =$, $\frac{dn(v)}{dv} = 0$

$$\boxed{V = V_{most\ prob.} = \sqrt{\frac{2KT}{m}}}$$



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gup.

Free Electron theory

Free e⁻ theory is of 2 types :-

1. Classical free e⁻ theory
2. Quantum free e⁻ theory

1. Classical free e⁻ theory:

(Proposed by Drude and Lorentz)

It is based on the following assumptions :

1. In metals there are a large no. of free electrons moving freely in all possible directions.
2. These free electrons behaves as a gas molecules in a container which obey the laws of kinetic theory of gases.
3. Average energy per electron at a temperature T is $\frac{3}{2}KT$.
4. Electric conduction is due to free e⁻ only.
5. The free e⁻ move in a constant potential field and hence the potential energy of the electrons is constant.

Limitations :

1. It fails to explain the concept of specific heat of metals.
2. It fails to explain the electrical conductivity of semiconductor and insulators.
3. It fails to explain the mean free path of the electrons.
4. It fails to explain the phenomenon like photoelectric effect, Black body radiation spectrum and Compton effect.

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2. Quantum free e⁻ theory: This theory is proposed by Sommerfeld.

This theory is based on the following assumptions:

1. Valence e⁻ moves freely in a constant potential within the boundaries of metal.
2. The distribution of electrons in various allowed energy level is according to Pauli's Exclusion principle.
3. The attraction b/w the free e⁻ and ions and the repulsion b/w the e⁻ are ~~to be load~~ ignored.
4. The Energy values of free electrons are quantized.
5. The distribution of energy among the free e⁻ is according to Fermi-Dirac Statistics.
6. The possible Energy values can be calculated by Schrodinger wave Equation.

Limitations / Drawbacks / Demerits:

1. It is unable to explain the material metallic properties of certain crystals.
2. It is unable to explain the difference among the metals, Semiconductors & insulators.
3. Unable to explain the atomic arrangement in the metallic crystals.
4. Unable to explain the +ve value of Hall Coefficient.

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Electronic Specific Heat

* Specific Heat is defined as the amount of Energy required to raise the temp of 1Kg Substance by 1K.

Acc. to Classical Statistics of (the law of equipartition Energy). The average Energy per electron is

$$\bar{\varepsilon} = \frac{3}{2}KT$$

if N is the total no. of electrons, Then, total Energy

$$E = \bar{\varepsilon}N = \frac{3}{2}KTN$$

$$E = \frac{3}{2}NKT$$

$$(C_v)_{electronic} = \frac{dE}{dt} = \frac{3}{2}NK = \frac{3}{2}R \quad -(1)$$

$$\boxed{(C_v)_{electronic} = 3/2\,R}$$, where $R = NK$

Eqn no. (1) is the Electronic Specific heat acc. to the Classical Statistics which don't agree with the experimental result.

Experimentally $(C_v)_{elect.} \propto T$

which is explained by Quantum free Electron theory.

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Quantum

Acc. to free e⁻ theory note "not every e⁻ gains energy" from the supplied heat but only those electrons in the levels within energy range KT of the fermi energy absorb and get excited thermally to higher states.

So fraction no. of e⁻ which are participated in this process is given by

$$\frac{KT}{E_F}$$ ∴ Ef - Fermi Energy

$$E = \frac{3}{2} NKT \times \frac{KT}{E_F}$$

(Quantum mechanically)

$$E = \frac{3}{2} \frac{NK^2T^2}{E_F}$$

$$(C_v)_{electronic} = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2} NK^2 \cdot \frac{2T}{E_F}$$

$$(C_v)_{elect.} = \frac{3NK^2}{E_F} T = aT$$

$\frac{3NK^2}{E_F} \rightarrow a$

$$(C_v)_{ele} = aT$$

$$\boxed{(C_v)_{electronic} \propto T}$$

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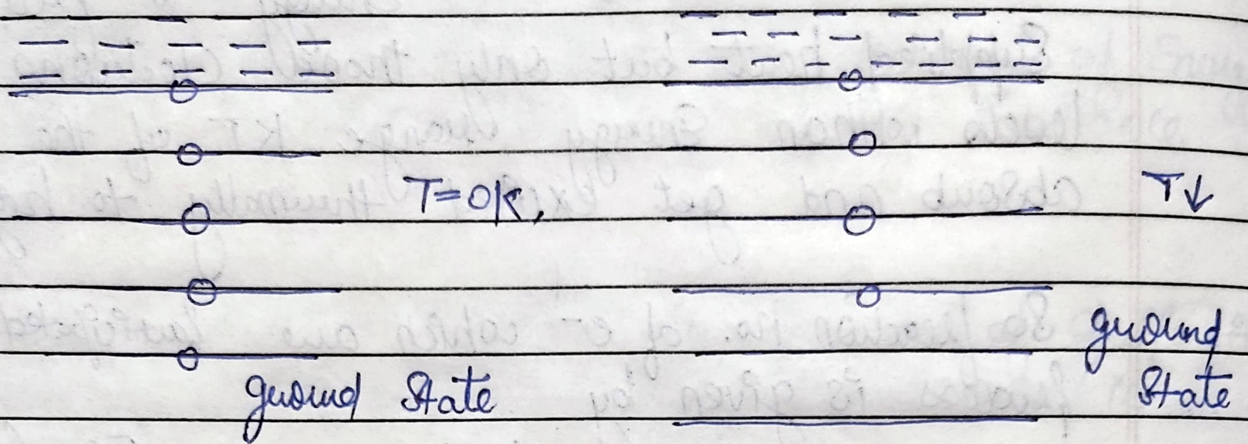
Fermi Energy



As the temp. of the System decreases the Energy of fermions also decreases but at T=0K the Energy of all fermions is not zero. It means the fermions will go occupying the lower & lower Energy States as the System is cooled ~~it means~~. At T=0K the System has the minimum possible Energy and the lowest Energy States are filled with fermions.

But acc to pauli Exclusion principle, only one fermion can occupy one Energy level. Hence ~~only~~ there will be only 1 fermion in the ground State having zero Energy.

Thus at 0K Starting from the ground State ~~of the~~ all the Starting States are filled with 1 fermion each till all the N fermions are accomodated so all Energy States upto A Certain level are Completely filled and all the Energy States under a Certain level are Completely empty.

This Certain level is fermi level and the Energy Corresponding to the fermi level is known as fermi Energy.

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The number of fermions having Energy ε and $(\varepsilon + d\varepsilon)$ is given by

$$n(\varepsilon)d\varepsilon = \frac{8\sqrt{2}\,\pi V m^{3/2}\,\varepsilon^{1/2}\,d\varepsilon}{h^3}$$

$$n = \int_0^{\varepsilon_f} n(\varepsilon)\,d\varepsilon$$

$$\boxed{\varepsilon_f(0) = \frac{h^2}{2m}\left(\frac{3n}{8\pi V}\right)^{2/3}} \rightarrow \text{fermi Energy}$$

Avg. energy $(\bar{\varepsilon}) = \dfrac{\int_0^{\varepsilon_f} \varepsilon\, n(\varepsilon)\,d\varepsilon}{\int_0^{\varepsilon_f} n(\varepsilon)\,d\varepsilon}$

$$\boxed{\bar{\varepsilon} = \frac{3}{5}\varepsilon_f(0)} \rightarrow \text{Average Energy}$$