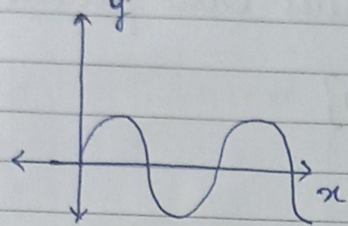
CIRCUITS AND SYSTEM

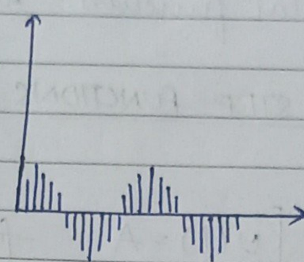
Signals: A signal is an electrical or electromagnetic current that is used for carrying data from one device or network to another.

For eg:

$f(t) = \sin \omega t$



Continuous signal



Discrete signal.

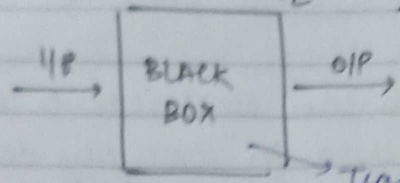
CLASSIFICATION OF SIGNAL

** Note: when we quantize the discrete signal with respect to amplitude it becomes digital signal

* CONTINUOUS AND DISCRETE
  - A signal is said to be continuous when it is defined for all instants of time.
  - A signal is said to be discrete when it is defined at only discrete instance of time.

* DETERMINISTIC AND NON DETERMINISTIC
* EVEN AND ODD SIGNALS
* ENERGY AND POWER SIGNALS
* REAL AND IMAGINARY SIGNALS

$$h(t) = \frac{O/P}{I/P}$$



Transfer function / impulse function

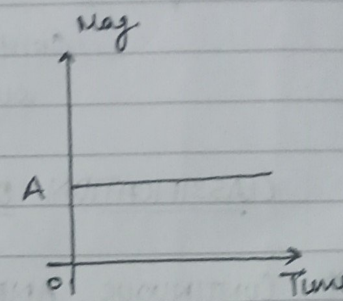
$u(t)$, $r(t)$, $\delta(t)$, $y(t)$ } Singular function

★ singular function: which gives basic functions

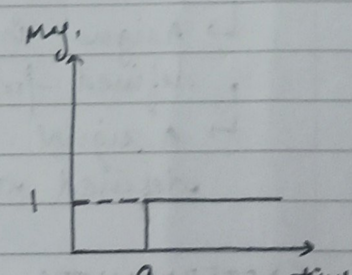
>> UNIT STEP FUNCTIONS [$u(t)$]

STEP FUNCTION: $u(t) = A$ for $t \geq 0$; 0 otherwise

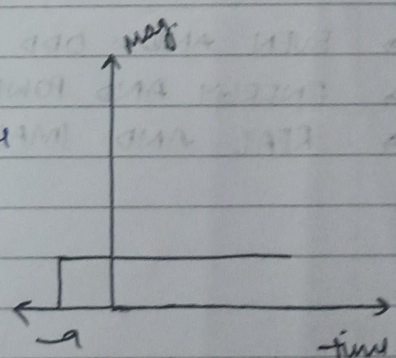
UNIT STEP FUNC.: $u(t) = 1$ for $t \geq 0$; 0 otherwise



$u(t-a) = 1$ for $t \geq a$; 0 otherwise

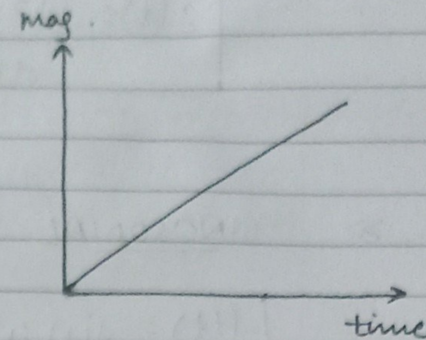


$u(t+a) = 1$ for $t \geq a$.; 0 otherwise



» UNIT RAMP FUNCTION

$$r(t) = t \qquad t \geq 0$$
$$= 0 \qquad \text{otherwise}$$



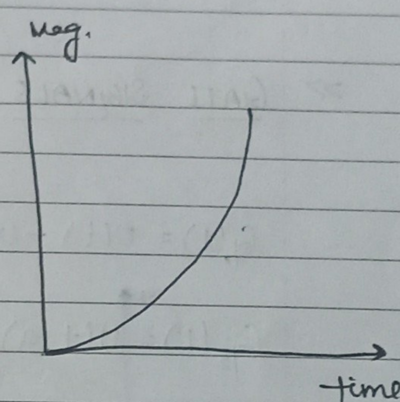
$$\boxed{r(t) = t\,u(t)}$$

$$r(t) = \int u(t)\,dt = \int 1\,dt = t$$

$$\boxed{u(t) = \frac{d\,r(t)}{dt}}$$

» UNIT PARABOLA FUNCTION

$$p(t) = \frac{t^2}{2} \quad \text{for } t \geq 0$$
$$0 \quad \text{otherwise}$$

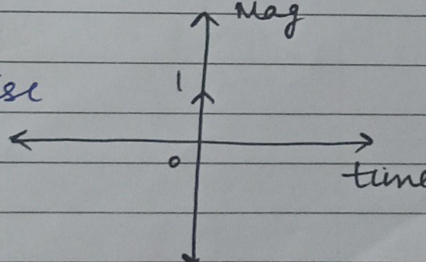


$$p(t) = \int_0^t r(t)\,dt$$
$$= \int t\,dt = \frac{t^2}{2}$$

$$\boxed{p(t) = \frac{t^2}{2}\,u(t)} \qquad \boxed{r(t) = \frac{d}{dt}\,p(t)}$$

UNIT IMPULSE FUNCTION

$$\delta(t) = 1 \qquad t = 0$$
$$0 \qquad \text{otherwise}$$

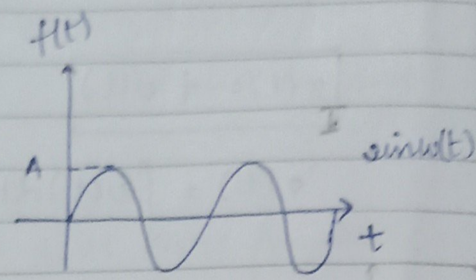


$$\boxed{\delta(t) = \frac{d\,u(t)}{dt}}$$

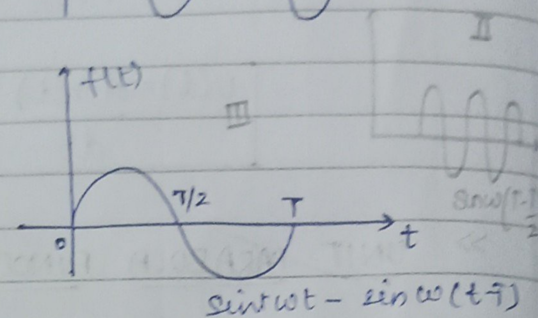
$$\boxed{\delta(t) = \frac{d^2 r(t)}{dt^2} = \frac{d^3 p(t)}{dt^3}}$$

>> SINOSODIAL SIGNALS

$\boxed{f(t) = \sin \omega t}$ $t \geq 0$



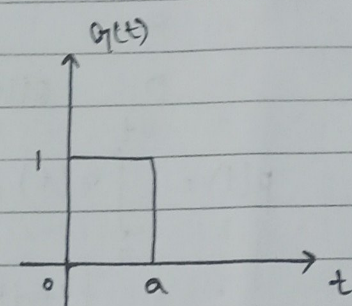
$\boxed{f(t) = \sin\omega t - \sin\omega(t-T)}$

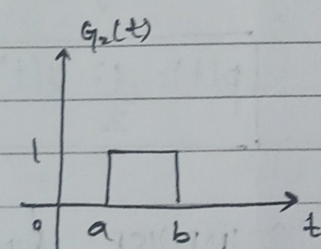


>> GATE SIGNALS.

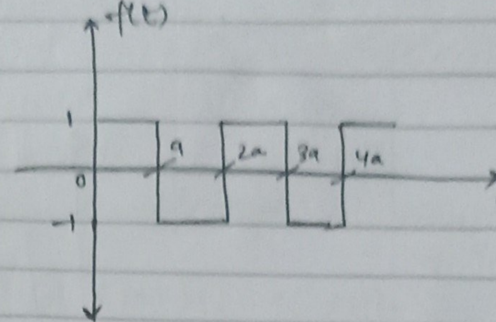
$G_1(t) = u(t) - u(t-a)$

$G_2(t) = u(t-a) - u(t-b)$





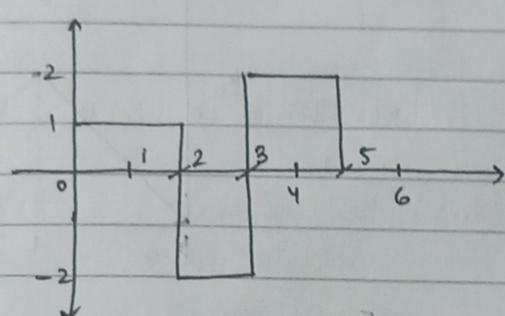
Ques Synthesis the wave form shown in the figure.



Ans $f(t) = 1[u(t) - u(t-a)]$
$\quad - 1[u(t-a) - u(t-2a)]$
$\quad + 1[u(t-2a) - u(t-3a)] \dots$

$= u(t) - 2u(t-a) + 2u(t-2a)$
$\quad - 2u(t-3a) + \dots$

Ques Synthesis the wave form shown in the given figure

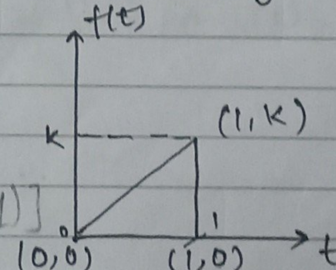


$f(t) = 1[u(t) - u(t-2)] - 2[u(t-2) - u(t-3)] + 2[u(t-3) - u(t-5)]$

$= u(t) - u(t-2) - 2u(t-2) + 2u(t-3) + 2u(t-3) - 2u(t-5)$

$f(t) = u(t) - 3u(t-2) + 4u(t-3) \cancel{+} - 2u(t-3)$

Ques Synthesis the wave form shown in the given figure.



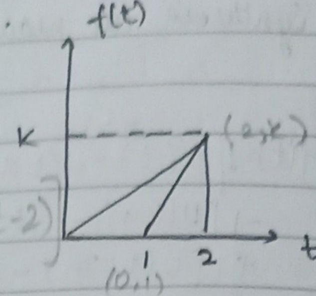
$(y - y_1) = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$f(t) - 0 = \dfrac{k-0}{1-0}(t-0)\,[u(t) - u(t-1)]$

$f(t) = kt\,[u(t) - u(t-1)]$

Q Synthesis of given wave from.

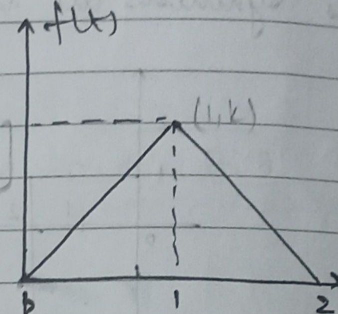
$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$



$f(t)-0 = \frac{k-0}{2-1}(t-1)[u(t-1)-u(t-2)]$

$f(t) = k(t-1)[u(t-1)-u(t-2)]$
$= (kt-k)[u(t-1)-u(t-2)]$

Q Synthesis of given form.



$f(t)-0) = \frac{k-0}{1-0}(t-0)[u(t)-u(t-1)]$

$f(t) = kt[u(t)-u(t-1)]$ — (1)

$f(t)-k) = \frac{0-k}{2-1}[t-1)$

$f(t)-k = \frac{-k}{1}(t-1) \Rightarrow f(t) = -kt$

$= -kt[u(t-1)-u(t-2)]$

$f(t) = kt[u(t)-u(t-1)] - kt[u(t-1)-u(t-2)]$

SIGNALS:

A signal is a physical quantity that varies with time, space or any other independent variable. If the signal depends upon two independent variables then the signal is known as 2-D signal. Multidimensional signals depends on many variables

CLASSIFICATIONS OF SIGNAL

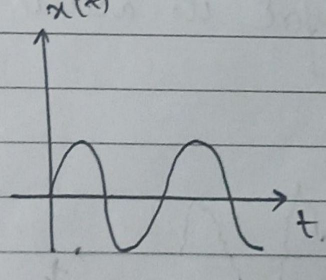
Signals are classiffied

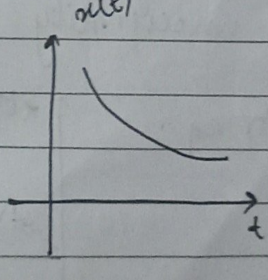
1. Continuous and Discrete Time Signal
2. Analog and Digital Signal
3. Deterministic and Random Signal
4. Periodic and non-periodic signal
5. Even and odd signal
6. Energy and Power Signal
7. Causal and Non-causal Signal.

CONTINUOUS AND DISCRETE TIME SIGNAL

* Continuous Time Signal (CTS) are defined for all values of time and is represented by $x(t)$. It is also called as an analog signal.

For eg.

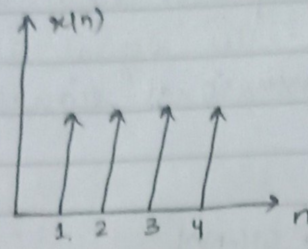




Normalisation: simplifying with the function it is respected to

Discrete Time Signal (DTS) are defined at discrete instant of time and it is represent by $x(n)$

For eg:



The discrete time signals are continuous in amplitude and discrete in terme.

DIGITAL SIGNAL.

The signals which are discrete in time and quantised in amplitude.

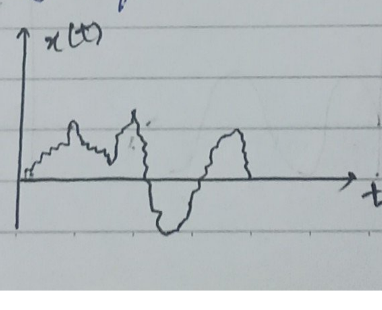
DETERMINISTIC AND RANDOM SIGNAL.

* Deterministic signal is a signal that exhibits no uncertainity of value at any instant of time. Its instantaneous value can be predicted by given mathematical eq.

For eq. $A \sin \omega t$.

* Random Signal is a signal that is characterised by uncertinity before its actual occurence

For eq.



PERIODIC AND NON-PERIODIC SIGNALS

* ~~Per~~ A continuous time signal will be a periodic signal if it satisfies the given equation

$$\boxed{x(t+T) = x(t) \quad \forall t} \quad \text{—— (1)}$$

* A DTS time signal will be a periodic signal if it ~~satf~~ satisfies the given equation

$$\boxed{x(n+N) = x(n) \quad \forall n} \quad \text{—— (2)}$$

here in eq. (1) the T is known as fundamental period of time.

Fundamental Period.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

Q Find the fundamental time period of the following CTS.

(i) je^{j5t} (ii) $\sin 50\pi t$ (iii) $20\cos(10\pi t + \frac{\pi}{6})$

(i) je^{j5t} =

$$T = \frac{2\pi}{5} =$$

(iii) $20\cos(10\pi t + \frac{\pi}{6})$

$$T = \frac{2\pi}{10\pi} = \frac{1}{5}$$

(ii) $\sin 50\pi t$

$$T = \frac{2\pi}{50\pi} = \frac{1}{25}$$

Find the fundamental period T if they are periodic

(i) $4\cos 5\pi t$ (ii) $\sin\pi + u(t)$ (iii) $e^{-|t|}$

Periodic aperiodic aperiodic

(i) $4\cos 5\pi t$ • $T = \frac{2\pi}{5\pi} = \frac{2}{5}$ sec.

NOTE :

The sum of 2 periodic signal $x_1(T)$ & $x_2(T)$ with periods T_1 & T_2 may and maynot be periodic depending upon the relation b/w T_1 and T_2. If the sum to be periodic then the ratio of the periods T_1 & T_2 must be a rational no. or ratio of two integer. Otherwise sum is not periodic.

Ques. Find the given signal is periodic or not.

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{10} = \frac{\pi}{5}$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4} = \frac{\pi}{2}.$$

$$\frac{T_1}{T_2} = \frac{\cancel{\pi}/5}{\cancel{\pi}/2} = \frac{2}{5}$$

$\therefore$ $x(t)$ is a periodic signal.

Ques $\cos 60\pi t + \sin 50\pi t$ is periodic or not.

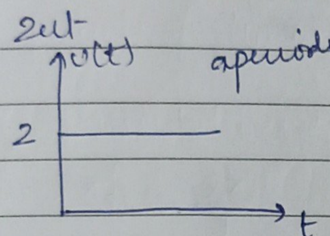
$$T_1 = \frac{2\pi}{\omega} = \frac{2\pi}{60\pi} = \frac{1}{30}$$

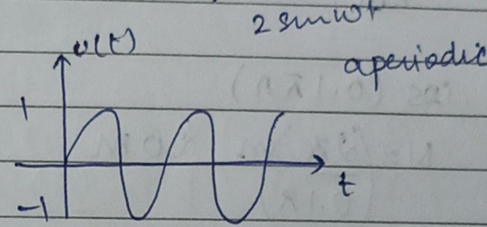
$$T_2 = \frac{2\pi}{\omega} = \frac{2\pi}{50\pi} = \frac{1}{25}.$$

$$\frac{T_1}{T_2} = \frac{1/30}{1/25} = \frac{\cancel{25}\,5}{\cancel{30}\,6} = \frac{5}{6}.$$

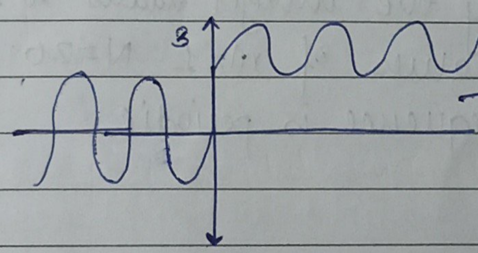
$\therefore$ It is a periodic signal.

Ques: $2u(t) + 2\sin 2t$





⇓



→ aperiodic.

Ques $u(t) - \frac{1}{2}$

A-periodic as its unit step signal.

Ques $x(t) = \sin^2 t$

$$\sin^2 t = \frac{1}{2} - \frac{\cos 2t}{2}.$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$$

CONDITION OF DISCRETE TIME SIGNAL TO BE PERIODIC.

condition

$$x(n+N) = x(n) \quad \forall \, n$$

$$\omega N = 2\pi m \rightarrow \text{to make integer}$$

$$\boxed{N = \left(\frac{2\pi}{\omega}\right)m}$$ where m is an ~~not~~ integer

Ques find whether the following are periodic or not.

(i) $\cos(0.1\pi n)$

$$N = \left(\frac{2\pi}{0.1\pi}\right)m = 20m$$

$\because$ N takes only +ve integer values so for minimum values of $m=1$ $N=20$ and the sequence is periodic.

(ii) $x(n) = e^{j6\pi n}$

$$N = \left(\frac{2\pi}{6\pi}\right)m = \frac{1}{3}m$$

for $m=3$, $N=1$

So the given signal is periodic

(iii) $x(n) = \sin\left(\frac{6\pi n}{7} + 1\right)$

$N = \frac{2\pi \cdot 7}{6\pi} m = \frac{7}{3} m.$

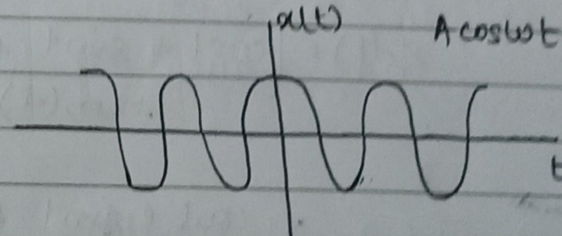
for $m = 3$, $N = 7$.
So the given signal is periodic.

SYMMETRIC (EVEN) AND ANTI SYMMETRIC (ODD) SIGNAL

☆ Symmetric (even): A CTS signal is said to be even if

$$\boxed{x(-t) = x(t) \quad \forall\ t.}$$

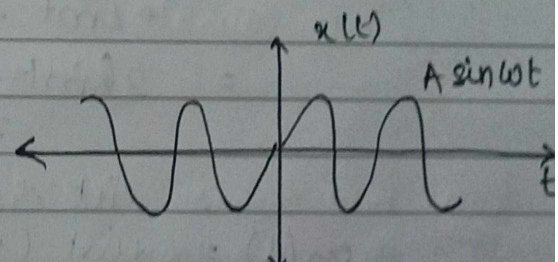
For eg:
A cos ωt



☆ Anti symmetric (odd): A CTS signal is said to be odd if

$$\boxed{x(-t) = -x(t) \quad \forall\ t}$$

For eg:
A sin ωt



A signal can be represented as x(t)

$$x(t) = x_e(t) - x_o(t) \quad — ①$$

After replacing t with -t eq. becomes

$$\left.\begin{aligned} x(-t) &= x_e(-t) - x_o(-t) \\ x(-t) &= x_e(t) - x_o(-t) \end{aligned}\right\} ②$$

solving ① & ② we get

$$\boxed{\begin{aligned} x_e(t) &= [x(t) + x(-t)]/2 \\ x_o(t) &= \left[\frac{x(t) - x(-t)}{2}\right] \end{aligned}}$$

Ques Find the even and odd function component of the following signal.

(i) $x(t) = \cos(t) + \sin(t) + \cos t \sin t$

$x_e(t) = [\cos t + \sin t + \cos t \sin t + \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)]/2$

$= \cos t + \cancel{\sin t} + \cancel{\cos t \sin t} + \cos t - \cancel{\sin t} - \cancel{\cos t \sin t}$

$x_e(t) = \cos t$

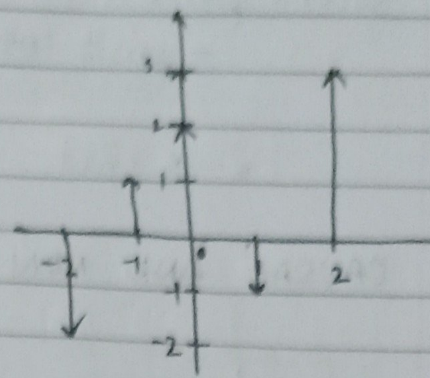
$x_o(t) = [\cancel{\cos t} + \sin t + \cos t \sin t \cancel{\cos t} - \cancel{\cos t} + \sin t + \cos t \sin t]/2$

$= \frac{2(\sin t + \cos t \sin t)}{2}$

$= \sin t + \cos t \sin t$

$x_o(t) = \sin t\,(1 + \cos t).$

Q $x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$.



$n = -2, -1, 0, 1, 2$

$x_e(n) = \dfrac{x(n) + x(-n)}{2}$

$x_o(n) = \dfrac{x(n) - x(-n)}{2}$

for even component

$n = -2, 0, 2$

$x_e = \dfrac{x(-2) + x(2)}{2}$

$= \dfrac{-2+3}{2} = \dfrac{1}{2}$

$x_e(0) = \dfrac{x(0) + x(0)}{2}$

$= \dfrac{2+2}{2} = 2.$

$x_e(2) = \dfrac{x(2) + x(-2)}{2}$

$= \dfrac{3-2}{2} = \dfrac{1}{2}$

for odd component

$n = -1, 1.$

$x_o(-1) = \dfrac{x(-1) - x(1)}{2}$

$= \dfrac{-1+1}{2} = 0$

$x(1) = \dfrac{x(1) - x(-1)}{2}$

$= \dfrac{-1-1}{2} = -1$

Ques2(t) $\sin t + 2\sin t + 2\sin^2 t \cos t$

$x_e(t) = \left[\cancel{\sin t} + 2\cancel{\sin t} + 2\sin^2 t \cos t + (\cancel{-\sin t}) - \cancel{2\sin t} + 2\sin^2 t \cos t\right]/2$

$= 2\sin^2 t \cos t$

$x_o(t) = [\sin t + 2\sin t + 2\sin^2 t\cos t + \sin t + 2\sin t - \sin^2 t\cos t]/2$

$= 3\sin t$

CAUSAL AND NON-CAUSAL SIGNAL

causal.
$x(t) = 0$ for $t < 0$

non causal
$x(t) = 0$ for otherwise.

For a continuous time signal $x(t)$ is said to be causal if follows the above equation

For a DTS $x(t)$ is said to be causal if follows.

causal $x(n) = 0$ for $n < 0$

non causal
otherwise.

Find which following signal are causal or non-causal.

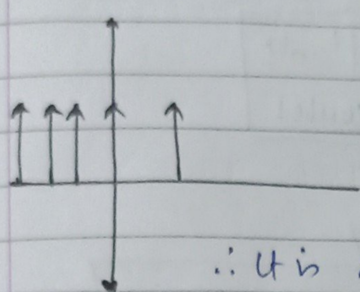
$x(t) = e^{at}.u(t)$.

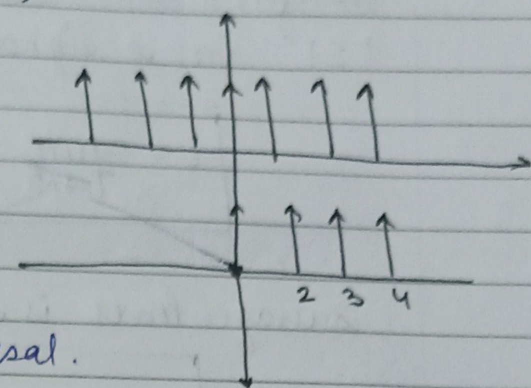
causal. because all the values for $t<0$ is 0.

- $e^{-2t} u(-t)$

non causal because all the values for $t<0$ is not 0.

$x(n) = u(n+3) - u(n-2)$





∴ It is non causal.

$x(n) = \left(\frac{1}{2}\right)^n u(n+2)$

It is non causal because all the values for $t<0$ is not zero.

ENERGY AND POWER SIGNAL

→ A signal maybe represented as a voltage or a current. If we apply a voltage V accross a register R. The current passing through it $i(t)$. Then instantaneous power is given

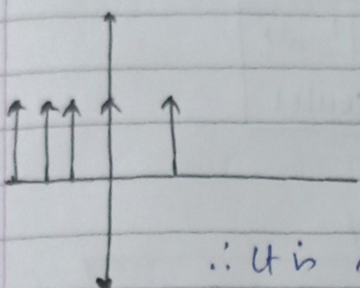
$$P_i = v(t) \cdot i(t)$$
$$= \frac{V^2(t)}{R} = i^2(t) R$$

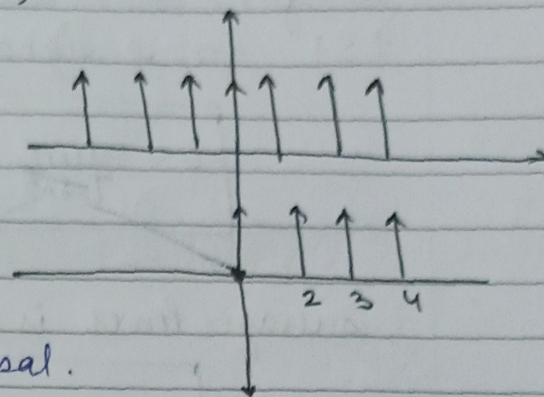
$$P_i = v^2(t) = i(t) \qquad \text{for } R=1$$

- $e^{-2t}u(-t)$

non causal because all the values for $t<0$ is not 0.

$x(n) = u(n+3) - u(n-2)$





$\therefore$ it is non causal.

$x(n) = \left(\frac{1}{2}\right)^n u(n+2)$

It is non causal because all the values for $t<0$ is not zero.

ENERGY AND POWER SIGNAL

→ A signal maybe represented as a voltage or a current. If we apply a voltage V across a register R. The current passing through it $i(t)$. Then instantaneous power is given

$$P_i = v(t) \cdot i(t)$$

$$= \frac{V^2(t)}{R} = i^2(t)R$$

$P_i = v^2(t) = i(t)$ for $R=1$

If voltage or current is represented by signal $x(t)$ then

$$\boxed{P_i = x^2(t)}$$

Therefore, the total energy over a time interval $t \le T$ for a signal (CTS) is given by

$$\boxed{E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt \quad \text{Joules}}$$

average Power is given by

$$\boxed{P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \quad \text{watt}}$$

For discrete time signal. total energy is given by

$$\boxed{E = \lim_{N \to \infty} \int_{-N}^{N} |x(n)|^2 \, dn}$$

average power is given by

$$\boxed{P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}$$

→ A signal $x(t)$ is called an energy signal if energy satisfy $\boxed{0 < E < \infty}$ along with this condition $P = 0$ has to be there

→ A signal $x(t)$ is called a power signal if the power satisfied this condition. For a power signal

$E = \infty$

Note: The square root of P is known as RMS value of the signal.

Determine the RMS value and Power of the signal.

$x(t) = A \cos(\Omega_0 t + \theta)$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cos^2(\Omega_0 T + \theta)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T}^{T} \frac{1 + \cos 2(\Omega_0 T + \theta)}{2}\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} [1 + \cos 2(\Omega_0 T + \theta)]\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} 1\, dt + \underbrace{\lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} \cos 2(\Omega_0 T + \theta)\, dt}_{0}$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} [T]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{A^2\, 2T}{4T_2} = \lim_{T\to\infty} \frac{A^2}{2} = \frac{A^2}{2}$$

Rms value of $P = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$

) $x_1(t) = 5\cos(40t + \frac{\pi}{3})$

$P = \frac{25}{2}$ Rms value $= \frac{5}{\sqrt{2}}$

$E = \infty$

Note: The square root of P is known as RMS value of the signal.

Determine the RMS value and Power of the signal.

$x(t) = A \cos(\Omega_0 t + \theta)$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cos^2(\Omega_0 T + \theta)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T}^{T} \frac{1 + \cos 2(\Omega_0 T + \theta)}{2}\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} [1 + \cos 2(\Omega_0 T + \theta)]\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} 1\, dt + \underbrace{\lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} \cos 2(\Omega_0 T + \theta)\, dt}_{0}$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} [T]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{A^2 \, 2T}{4T_2} = \lim_{T\to\infty} \frac{A^2}{2} = \frac{A^2}{2}$$

Rms value of $P = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$

) $x_1(t) = 5\cos\left(40t + \frac{\pi}{3}\right)$

$P = \frac{25}{2}$ Rms value $= \frac{5}{\sqrt{2}}$

(iii) $x_2(t) = 10\sin(25t + \pi/4) + 16\cos(30t + \pi/2)$

$$P = \frac{10^2}{2} + \frac{16^2}{2}$$

(iv) $x_3(t) = 10\cos 5t \cos 10t$

$\cos(A+B) + \cos(A-B)$

$= 5(\cos 15t + \cos 5t)$

$= 5\cos 15t + 5\cos 5t$

$= \frac{25}{2} + \frac{25}{2} = 25$

rms value = 5

SYSTEM

\# ↳ Basic operations on signals or transformation of dependent / independent variables.

1. Time Shifting 2. Time Reversal 3. Time Scaling
4. ~~Ap~~ Amplitude Scaling 5. Signal addition
6. Signal multiplication

TIME SHIFTING

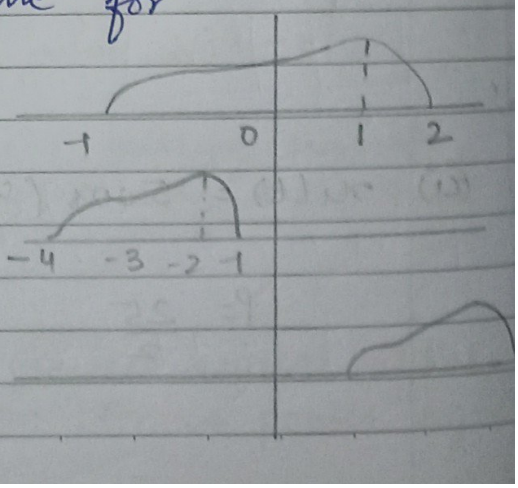
↳ Time Shifting of $x(t)$ may delay or advance the signal in time for

CTS

$$\boxed{y(t) = x(t-T)}$$

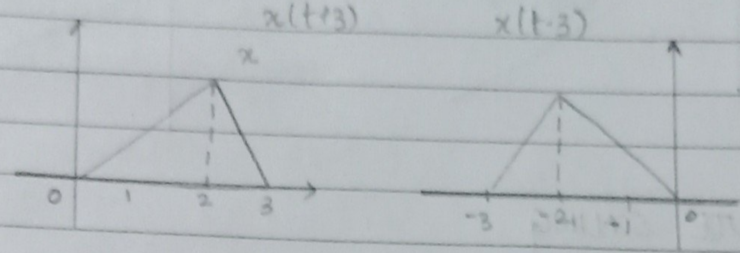
DTS

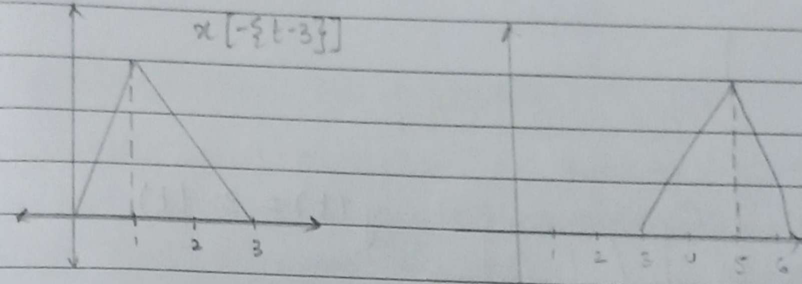
$$\boxed{y(n) = x\cancel{y}(n-N)}$$



TIME REVERSAL

↳ It can be obtained by folding the signal about $t=0$.



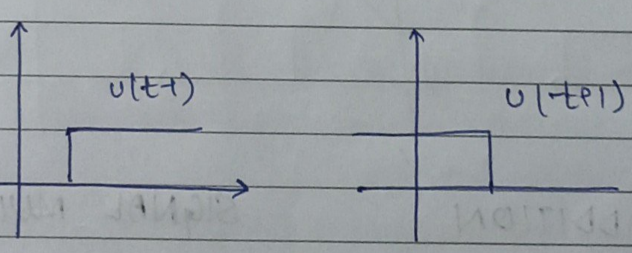


Ques. Sketch the following signal

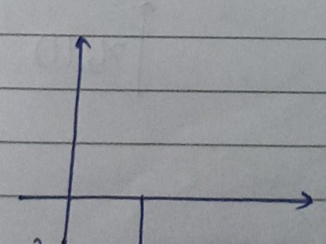
(i) $u(-t+1)$ (ii) $-2u(t-1)$ (iii) $3r(t-1)$ (iv) $-2r(t)$

(v) $r(-t+2)$ (vi) $r(t+3)$

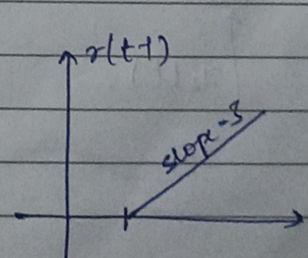
(i) $u(-t+1) = u[-(t-1)]$



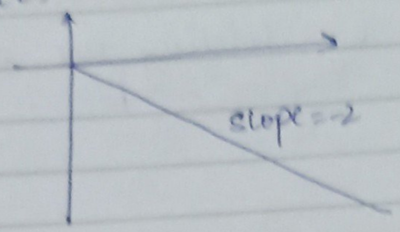
(ii) $-2u(t-1)$



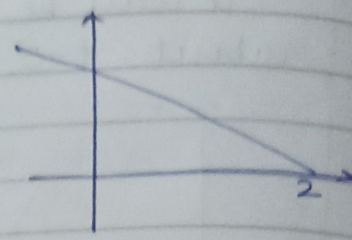
(iii) $3r(t-1)$



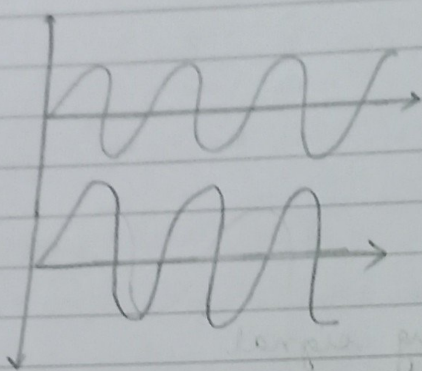
(iv) $-2x(t)$



(v) $x(-t+2) = x[-(t-2)]$



AMPLITUTE SCALING



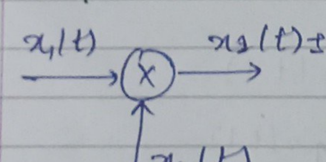
$y(t) = 3x(t)$

TIME SCALING

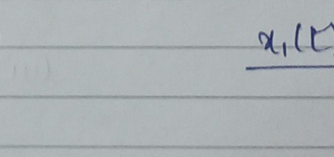
$y(t) = \underbrace{x(2t)}_{\text{value divided}}$

$y(t) = \underbrace{x(t/2)}_{\text{value multiplied.}}$

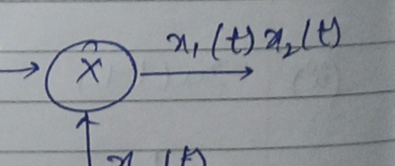
$y(t) = x(t)$



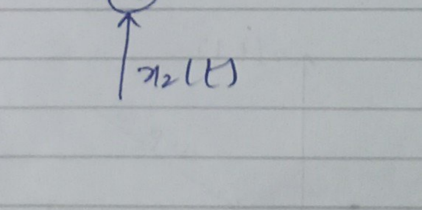
$y(t) = x(2t)$



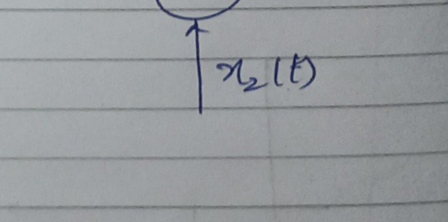
$y(t) = x(t/2)$



SIGNAL ADDITION

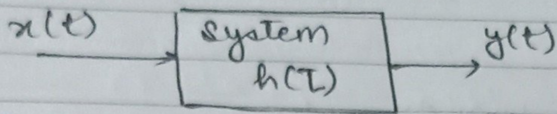


SIGNAL MULTIPLICATION



SYSTEM

A system is defined as a physical device that performs an operation on input signal and produces an output signal



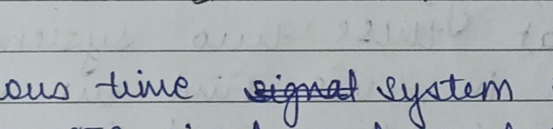
τ → time

CLASSIFICATION OF SYSTEM

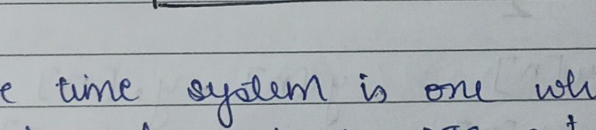
1. Continuous and Discrete Time system
2. Lumped and distributed parameter system
3. Static and Dynamic system.
4. Causal and Non-Causal System
5. Time Variant and Invariant System
6. Stable and Unstable system.
7. Linear and non-linear system.

CONTINUOUS AND DISCRETE TIME SIGNAL

* A continuous time ~~signal~~ system is one which operates on CTS signal and produces CTS output.



* A discrete time system is one which operates on DTS signal and produces DTS output.



LUMPED AND DISTRIBUTED TIME SIGNAL.

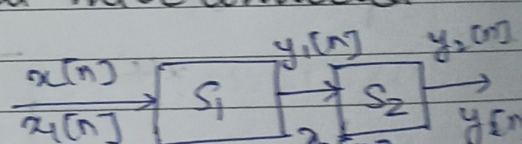
* In lumped parameter system each and every component is lumped at one point. These system are described by ordinary differential equation.

parameter system

* ~~Distributive time signal are defined along and it is defined by partial differential~~

* Distributive parameter system are defined along the whole length of the line like transmission line and it is defined by the partial differential equation.

Q Consider two systems with the following input output relationship.
System1 (S_1): $y_1[n] = 2x_1[n] + 4x_1[n-1]$
System2 (S_2): $y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$.
Suppose that these two systems are connected in series. Find the input output relationship for the overall inter connected system if



a. S_2 follows S_1
b. S_1 follows S_2

$x[n] = x_1[n]$

$x_2[n] = y_1[n]$

$y[n] = y_2[n]$

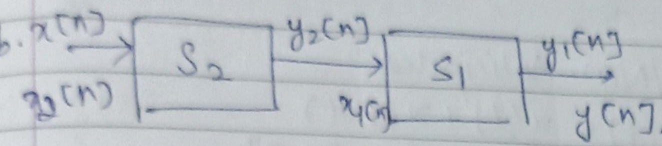
$= x_2[n-2] + \frac{1}{2}x_2[n-3]$

$= y_1[n-2] + \frac{1}{2}y_1[n-3]$

$$= 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}\left\{2x_1[n-3] + 4x_1[n-4]\right\}$$
$$= 2x_1[n-2] + 4x_1[n-3] + x_1[n-3] + 2x_1[n-4]$$

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

b.



$x[n] = x_2[n]$ $\qquad$ $y[n] = y_1[n]$

$x_1[n] = y_2[n]$ $\qquad$ $= 2x[n] + 4x_1[n-1]$

$\qquad\qquad\qquad\qquad = 2y_2[n] + 4y_2[n-1]$.

$$2\left[x_2[n-2] + \frac{1}{2}x_2[n-3]\right] + 4\left[x_2[n-3] + \frac{1}{2}x_2[n-4]\right]$$

$$= 2x_2[n-2] + x_2[n-3] + 4x_2[n-3] + 2x_2[n-4]$$

$$y[n] = 2x_2[n-2] + x_2[n-3] + 4x_2(n-3] + 2x_2(n-4]$$
$$y[n] = 2x_2(n-2] + 5x_2[n-3] + 2x_2[n-4]$$

STATIC AND DYNAMIC SYSTEMS

Memory less $\qquad\qquad$ with memory.

→ A system is called static or memory less if its output at any instant depends on the input at that instance but not on past or future values otherwise the system is dynamic or with memory.

Static : $y(t) = 2x(t)$ $\qquad$ $y[n] = nx[n]$

Dynamic : $y[n] = x[n-1]$ $\qquad$ $y(t) = \frac{dx(t)}{dt}$

CAUSAL AND NON-CAUSAL

* A system which depends on present and past input but not on future are causal systems otherwise the system is non-causal.

Eg Causal / Non-Anticipated

$$y(t) = x(t) + x(t-1)$$
$$y[n] = 5[n] + x[n-3]$$

Non-causal

$$y[n] = 3x[n] + x[n-5] + x[n+2]$$

Check wether the systems are causal or not

(i) $y[n] = x[n] + \frac{1}{x[n-1]}$

$n=0$

$y[0] = x[0] + \frac{1}{x[-1]}$ → Depends one present & past inputs

∴ Causal System

(ii) $y(t) = x^2(t) + x(t-2)$

$t=0$

$y(0) = x^2(0) + x(-2)$

Depends on present and past input

∴ Causal System

(iii) $y(t) = x(t-2) + x(2-t)$

$t=0$

$y(0) = x(-2) + x(2)$

∴ Depends on future input also

∴ Non Causal System.

(iv) $y(t) = \int_{-\infty}^{2t} x(t)\,dt$

$y(t) = z(2t) - z(-\infty)$

$t=0$

$y(0) = z(0) - z(-\infty)$

$y(1) = z(2) - z(-\infty)$ $\therefore$ Non-causal.

(v) $y[n] = x[n]^2$

$n=2$

$y[2] = x[2]^2 = x[4]$ $\because$ Depends on future input

$\therefore$ Non-causal

NON LINEAR AND LINEAR SYSTEM

↳ A system is said to be linear system if that holds the principle of superposition i.e additivity and scaling otherwise it is non-linear system

(i) Response to i/p : $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$. additivity

Response to i/p : $ax(t) \rightarrow ay(t)$ scaling.

$$\boxed{ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)}$$

Ques Check wether the following systems are linear or not

(i) $y(t) = t^2 x(t-1)$

$y_1(t) = t^2 x_1(t-1)$ — ①

$y_2(t) = t^2 x_2(t-1)$ — ②

Consider

$x_3(t-1) = ax_1(t-1) + bx_2(t-1)$

Now apply in the system

$y_3(t) = t^2 x_3(t-1)$

$= t^2[ax_1(t-1) + bx_2(t-1)]$

$= at^2 x_1(t-1) + bt^2 x_2(t-1)$

$= ay_1(t) + by_2(t)$ —— (3)

(3) $= a(1) + b(2)$

$\therefore$ linear system

(ii) $y[n] = x^2[n]$

$y_1[n] = x_1^2[n]$ —— (1)

$y_1[n] = x_2^2[n]$ —— (2)

consider,

$x_3[n] = ax_1[n] + bx_2[n]$

Now apply in the system

$y_3[n] = x_3^2[n]$

$y_3[n] = (ax_1[n] + bx_2[n])^2$

$= a^2x_1^2[n] + b^2x_2^2[n] + 2abx_1[n]$

$+ b[x_2]n$

$\neq ay_1[n] + by_2[n]$

$\therefore$ Non-linear

Q $\frac{dy(t)}{dt} + 3ty(t) = t^2 x(t)$

if $x_1(t)$ is ~~generated~~ considered to generate the o/p of $y_1(t)$

$\frac{dy_1(t)}{dt} + 3ty_1(t) = t^2 x_1(t)$ —— (1).

$x_2(t) \rightarrow y_2(t)$

$\frac{dy_2(t)}{dt} + 3t\,y_2(t) = t^2 x_2(t)$ —— (2).

Multiply eq(1) with a and eq(2) with b

$$a\frac{dy_1(t)}{dt} + 3at\, y_1(t) + b\frac{dy_2(t)}{dt} + 3bt\, y_2(t) = at^2 x_1(t) + bt^2 x_2(t) \quad \text{—③}$$

$$\frac{d}{dt}\{\underbrace{ay_1(t) + by_2(t)}_{\text{weighted sum of o/p}} + 3t\} + 3t\{ay_1(t) + by_2(t)\} = t^2\{\underbrace{ax_1(t) + bx_2(t)}_{\text{weighted sum of i/p}}\}$$

∴ linear.

$$\frac{dy(t)}{dt} + 2y(t) = x^2(t)$$

If $x_1(t)$ is considered ~~together~~ generate the o/p of $y_1(t)$

$$\frac{dy_1(t)}{dt} + 2y_1(t) = x_1^2(t) \quad \text{—①}$$

$x_2(t) \rightarrow y_2(t)$

$$\frac{dy_2(t)}{dt} + 2y_2(t) = x_2^2(t) \quad \text{—②}$$

Multiplying ① by a and ② by b and adding we get.

$$a\frac{dy_1(t)}{dt} + 2ay_1(t) + b\frac{dy_2(t)}{dt} + 2by_2(t) = ax_1^2(t) + bx_2^2(t)$$

$$\frac{d}{dt}\{ay_1(t) + by_2(t)\} + 2(ay_1(t) + by_2(t)) = ax_1^2(t) + bx_2^2(t)$$

∵ weighted sum of input is not ~~not~~ equal.

∴ It is non-linear

TIME VARIANT AND INVARIANT SYSTEM

A system is time invariant or fixed its input output relationship doesn't change with time otherwise it is said to be time variant.

whether the given system is time variant or invariant

1) $y(t) = t\,x(t)$

O/P delayed i/p
$$y(t,T) = t\,x(t-T) \quad —①$$

delayed output
$$y(t-T) = (t-T)\,x(t-T) \quad —②$$

$$y(t,T) \neq y(t-T)$$
$\therefore$ System is time variant.

2) $y(t) = x(t)\cos 40\pi t$

O/P due to delayed i/p
$$y(t,T) = x(t-T)\cos 40\pi t \quad —①$$

delyed O/P
$$y(t-T) = x(t-T)\cos 40\pi(t-T)$$

$$y(t,T) \neq y(t-T)$$

$\therefore$ Time variant.

(iii) $y(t) = x(t^2)$

o/p due to delayed i/p

$$y(t,T) = x(t^2 - T)$$

delayed o/p

$$y(t-T) = x((t-T)^2)$$

$$y(t,T) \neq y(t-T)$$

(iv) $y(t) = e^{x(t)}$

o/p due to delayed i/p

$$y(t,T) = e^{x(t-T)}$$

delayed o/p

$$y(t-T) = e^{x(t-T)}$$

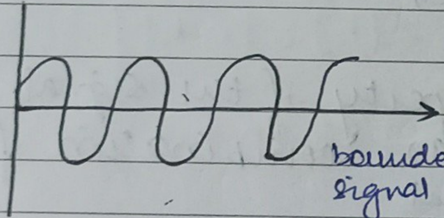
$$\because y(t,T) = y(t-T)$$

$\therefore$ Time invariant

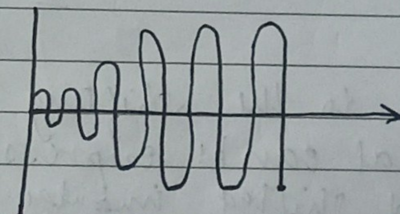
STABLE AND UNSTABLE SYSTEM

BIBO

↳ Bounded input bounded output. $0 \leq |M_x| < \infty$



bounded signal



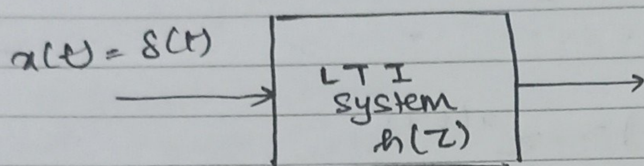
unbounded signal

LINEAR TIME INVARIENT SYSTEM (LTI)

↳ A system that innibits two basic property namely linearity and time invaient is known as LTI system. There are two major reasons behind the use of LTI system.

- Mathematical analyses becomes easier.
- Maximum no. of systems can be approx. with the properties of linearity and timinvariat.

CONTINUOUS TIME LTI SYSTEM.



$$y(t) = T[x(t)]$$

or

$$y(t) = T[\delta(t)]$$

The LTI systems are always considered with respect to the impulse responses that means the output will be the function of impulse response.

$$\boxed{x(t) = \delta(t)}.$$

$$y(t) = T[x(t)] = T[\delta(t)]$$
$$\simeq h(t)$$

Acc. to the shifting property of the signal, any signal can be expressed as combination of weighted and shifted impulse signal.

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,dt$$

Then the impulse response is

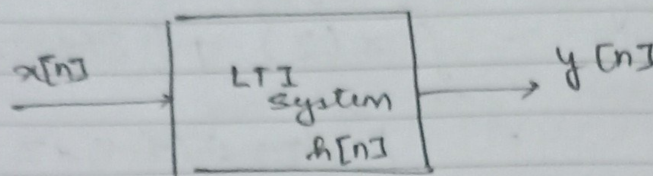
$$y(t) = T[x(t)]$$
$$= \int_{-\infty}^{\infty} x(\tau) \cdot T[\delta(t-\tau)] d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\boxed{\begin{array}{l}\text{convolation Integral} \\ y(t) = x(t) * h(t)\end{array}}$$ → when we multiply 2 signal

Calculate the output $y(t)$ of the system to the input $x(t) = u(t)$ if the impulse response of the system is given by $h(t) = e^{-at} u(t)$ for $a \geq 0$.

$$x(t) = u(t).$$
$$h(t) = e^{-at} u(t) \; ; \quad a > 0.$$
$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} u(t)\, e^{-a(t-\tau)}\, u(t-\tau)\, d\tau$$
$$= \int_{0}^{\infty} e^{-a(t-\tau)}\, u(t-x)\, d\tau$$
$$= e^{-at} \int_{0}^{\infty} e^{a\tau}\, u(t-\tau)\, d\tau$$
$$= e^{-at} \int_{0}^{t} e^{a\tau}\, d\tau$$
$$= e^{-at} \left. \frac{e^{a\tau}}{a} \right|_{0}^{t}$$
$$= \frac{e^{-at}}{a}\left[e^{at} - e^{0}\right] = \frac{e^{-at}}{a}\left[e^{at} - 1\right]$$

DISCRETE TIME LTI SYSTEM



Input to the system is impulse i.e

$$x[n] = \delta[n]$$

correspondingly o/p can be written as

$$y[n] = T[x[n]]$$
$$= T[\delta[n]]$$

Acc. to the shifting property of the signal any signal can be represented as weighted and shifted impulse signal.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$y[n] = T[x[n]]$$
$$= T\left[\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Ques Calculate the step response of discrete time LTI system with impulse response $h[n] = a^n u[n]$

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$$

$x[n] = u[n]$ → step response.

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, a^{n-k} u[n-k] \quad \left[\because \sum_{-\infty}^{\infty} x[k]\,\delta[n-k]\right]$$

step function negative nahi exist

$$= \sum_{k=0}^{\infty} a^{n-k} u[n-k] = a^n \sum_{k=0}^{\infty} a^{-k} u[n-k]$$

graph

$$= a^n \sum_{k=0}^{n} a^{-k} = a^n \left\{ \frac{1 - a^{-(n+1)}}{1 - a^{-1}} \right\}$$

$$\boxed{y[n] = \frac{a^{n+1} - 1}{a - 1}}$$

PROPERTIES OF LTI

1. **Commutative**

$$y(t) = x(t) * h(t)$$
$$= h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, dt = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, dt$$

2. **Distributive**

$$y(t) = x(t) * [h_1(t) + h_2(t)]$$
$$= x(t) * h_1(t) + x(t) * h_2(t)$$

3. **Associativity**

$$y(t) = [x(t) * h_1(t)] * h_2(t) = [x(t) * h_2(t)] * h_1(t)$$

$$x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t)$$

$$x(t) \rightarrow \boxed{h_2(t)} \rightarrow \boxed{h_1(t)} \rightarrow y(t)$$

4. Causility

CTS LTI system $y(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau \quad t \leq 0$

DTS LTI system $y[n] = \sum_{k=\infty}^{n} x(k)\, h(n-k) \quad n \leq 0.$

A causal LTI system is non-anticipatory and doesn't produce an output before an input is applied.

5. Stability

↳ If for a given system every bounded input produces a bounded output when the system is stable for a LTI system to be stable, the impulse response must absolutely integrable or sumable.

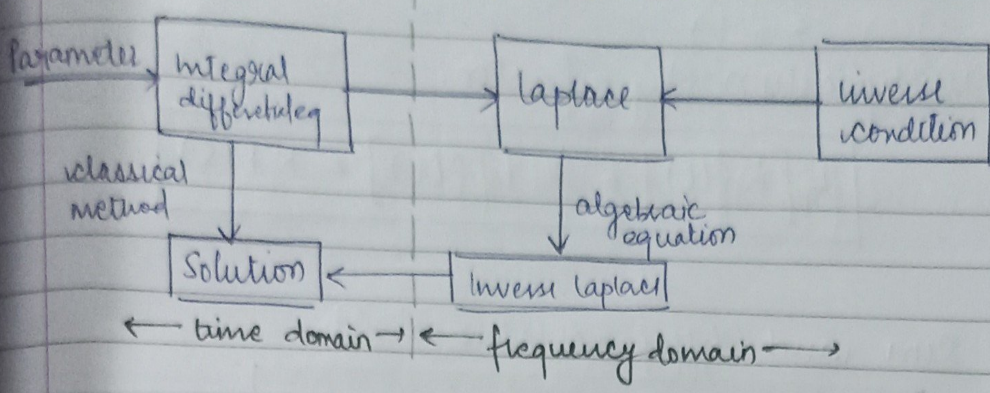
CTS $y(t) = \int_{-\infty}^{\infty} h(\tau)\, d\tau < \infty$

DTS $y[n] = \sum_{\tau=-\infty}^{\infty} |h(\tau)| < \infty$

6. With or without memory LTI

↳ An LTI system is called static or memory less system if its output at any time depends only on present input otherwise system is dynamic system.

LAPLACE TRANSFORM (continuous)



laplace transform is used to convert integro into simpler algebraic equation. it then becomes easier to manipulate the algebraic equation by simple algebraic rule to obtain the expression in suitable form.

laplace transform of any function can be represented as

$$\boxed{L = [f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-st} dt} \qquad s = \sigma + j\omega$$

inverse laplace

$$\boxed{L^{-1}\left[F(s)\right] = \frac{1}{2\pi j} \int_{\sigma - \infty j}^{\sigma + \infty j} F(s)\, e^{+st} ds}$$

Ques find the laplace of unit step signal.

$$F(s) = \int_{0}^{\infty} u(t) e^{-st} dt = \int_{0}^{\infty} e^{-st} dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = \left[\frac{e^{\infty} - e^{0}}{-s}\right] = \frac{1}{s}$$

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PROPERTIES OF LAPLACE.

* Multiplication by a factor.

$$\boxed{L[k f(t)] = k\, L[f(t)] = k F(s)}$$

* Sum and difference

$$\boxed{L[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)}$$

* differentiation with respect to t.

$$\frac{d f(t)}{dt} = s F(s) - f(0^+)$$

$$\frac{d^n f(t)}{dt^n} = s^n F(s) - s^{n-1} f(0^+) - s^{n-2} f(0^+) \ldots$$

* Integration by T

$$\boxed{\int_0^t f(t)\, dt = \frac{F(s)}{s} + \frac{f(0^+)}{s}}$$

* Differentiation w.r.t s (frequency differentiation)

$$\boxed{L[t f(t)] = -\frac{d F(s)}{ds}}$$

* Integration by s (frequency Integration)

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} f(x)\, ds$$

* Shifting theorem

$$\boxed{\begin{array}{l} L[f(t-a)] = e^{-as} F(s) \\ L[e^{at} f(t)] = F(s-a) \end{array}}$$

time shifting

frequency shifting

* Initial value theorem

$$\boxed{f(0^+) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)}$$

$$\boxed{f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)}$$

Q find the laplace transform of the following function.

(1) $f(t) = k\,u(t-a)$

$$L[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^\infty k\,u(t-a)\, e^{-st}\, dt$$

$$= \int_a^\infty k\,u(t)\, e^{-st}\, dt$$

$$= k \int_a^\infty 1\, e^{-st}\, dt = k \left[\frac{e^{-st}}{s}\right]_a^\infty$$

$$= \frac{k e^{-as}}{s}$$

$$f(t) = k\, r(t)$$
$$= k\, t\, u(t).$$

$$L\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st}\, dt.$$

$$= \int_0^\infty k\, t\, u(t)\, e^{-st}\, dt$$

$$= k \int_0^\infty t\, u(t)\, e^{-st}\, dt$$

$$= k \left[t \int_0^\infty e^{-st}\, dt - \int_0^\infty \frac{dt}{dt} \int_0 e^{-st}\, dt \right]$$

$$= k \left[t \int_0^\infty e^{-st}\, dt - \int_0^\infty \frac{e^{-st}}{-s}\, dt \right]$$

$$= k \left[t \left[\frac{e^{-st}}{-s} \right]_0^\infty - \left[\frac{e^{-st}}{s^2} \right]_0^\infty \right]$$

$$= k \left[0 \left[\frac{-1}{s} \right]_0^\infty - \left[\frac{-1}{s^2} \right] \right]$$

$$\cancel{= k \left[\frac{t}{s} + \frac{1}{s^2} \right] = k \left[\frac{st+1}{s^2} \right]}$$

$$= k \left[\frac{1}{s^2} \right]$$

(iii) $f(t) = k\,r(t-a)$.

$r(t-a) = (t-a)\,u(t-a)$

$$L\left[f(t)\right] = \int_0^\infty k\,r(t-a)\,e^{-st}\,dt$$

$$= \int_a^\infty k\,r(t)\,e^{-st}\,dt$$

$$= \int_a^\infty \cdot$$

$$L\{f(t)\} = \int_0^\infty k\,(t-a)\,u(t-a)\,e^{-st}\,dt$$

$$= \int_a^\infty k\,(t-a)\,e^{-st}\,dt.$$

$$= k\left[\int_a^\infty (t-a)\,e^{-st}\,dt\right]$$

$$= k\left[(t-a)\int_a^\infty e^{-st}\,dt - \int_a^\infty \frac{d(t-a)}{dt}\int e^{-st}\,dt\right]$$

$$= k\left\{\left[(t-a)\frac{e^{-st}}{-s}\right]_a^\infty - \left[\frac{e^{-st}}{s^2}\right]_a^\infty\right\}$$

$$= k\left[-(a-a)\frac{e^{-as}}{-s} - \left[\frac{e^{-as}}{s^2}\right]\right.$$

(ii) $f(t) = \delta(t)$ - impulse function
$= 1.$

$= e^{-st}\Big|_{t=0} = 1$

(iii) $f(t) = Ke^{-at}$

$$L[f(t)] = \int_0^\infty k e^{-at} \cdot e^{-st} \, dt$$

$$= \int_0^\infty k \, e^{(a-s)t} \, dt$$

$$= k \left[\frac{e^{(a-s)t}}{-(a+s)} \right]_0^\infty$$

$$= k \left[\frac{e^{\infty} - e^{0}}{-(a+s)} \right]$$

$$= \frac{k}{a+s}$$

(iv) $f(t) = \sin \omega t$

$$L[f(t)] = \int_0^\infty f(t) \, e^{-st} \, dt$$

$$= \int_0^\infty \sin\omega t \, e^{-st} \, dt.$$

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$$= \int_0^\infty \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right] e^{-st}\, dt.$$

$$= \frac{1}{2j} \int_0^\infty e^{j\omega t} \cdot e^{-st} - e^{-j\omega t} \cdot e^{-st}\, dt$$

$$= \frac{1}{2j}\left[\int_0^\infty e^{(j\omega - s)t}\, dt - \int_0^\infty e^{(-j\omega - s)t}\, dt\right]$$

$$= \frac{1}{2j}\left[\left.\frac{e^{j\omega - st}}{j\omega - s}\right|_0^\infty - \left.\frac{e^{-j\omega - st}}{(-j\omega - s)}\right|_0^\infty\right]$$

$$= \frac{1}{2j}\left[\frac{e^{(j\omega - s)\infty} - e^0}{j\omega - s} - \frac{e^\infty - e^0}{-j\omega - s}\right].$$

$$= \frac{1}{2j}\left(\frac{-1}{j\omega - s} + \frac{-1}{j\omega + s}\right)$$

$$= \cancel{\frac{1}{2j}\left(\frac{j\omega + s - j\omega + s.}{(j\omega - s)(j\omega + s)}\right)} = \cancel{\frac{s}{j(j\omega - s)(j\omega + s)}}$$

$$= \frac{1}{2j} \frac{-j\omega - \cancel{s} - j\omega + \cancel{s}}{(j\omega)^2 - s^2} = \frac{-\omega}{(j\omega)^2 - s^2}$$

$$= \frac{\omega}{s^2 - (j\omega)^2} = \frac{\omega}{s^2 + \omega^2}$$

1) $f(t) = \cos \omega t$

$$L[f(t)] = \int_0^\infty \cos\omega t \, e^{-st} \, dt$$

$$= \int_0^\infty \frac{e^{j\omega t} + e^{j\omega t}}{2} e^{st} dt$$

$$= \frac{1}{2} \int_0^\infty (e^{j\omega t} + e^{j\omega t}) e^{-st} \, dt$$

$$= \frac{1}{2} \int_0^\infty e^{j\omega t - st} + e^{j\omega t - st} \, dt$$

$$= \frac{1}{2} \left[\int_0^\infty e^{(j\omega - s)t} dt + \int_0^\infty e^{-(j\omega + s)t} dt \right]$$

$$= \frac{1}{2} \left[\frac{e^{(j\omega - s)t}}{j\omega - s} \Big|_0^\infty + \frac{e^{-(j\omega + s)t}}{-(j\omega + s)} \Big|_0^\infty \right]$$

$$= \frac{1}{2} \left[\frac{e^{\infty} - e^{0}}{j\omega - s} + \frac{e^{\infty} - e^{0}}{-j\omega - s} \right]$$

$$= \frac{1}{2} \left[\frac{-1}{j\omega - s} + \frac{1}{j\omega + s} \right]$$

$$= \frac{1}{2} \left[\frac{\cancel{j\omega} + s + \cancel{j\omega} - s}{(j\omega)^2 - s^2} \right] = \frac{-s}{-\omega^2 - s^2} = \frac{s}{s^2 + \omega^2}$$

(vi) $f(t) = e^{-at}\sin\omega t$

$$= e^{-at}\left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right)$$

$$L[f(t)] = \frac{1}{2j}\int_0^\infty \left(e^{-at} e^{j\omega t} - e^{-at} e^{-j\omega t}\right) e^{-st}\,dt$$

$$= \frac{1}{2j}\int_0^\infty e^{-at+j\omega t-st} - e^{-at-j\omega t-st}\,dt$$

$$= \frac{1}{2j}\int_0^\infty e^{(-a+j\omega-s)t}\,dt - \int_0^\infty e^{(-a-j\omega-s)t}\,dt$$

$$= \frac{1}{2j}\int_0^\infty \frac{e^{(-a+j\omega-s)t}}{-a+j\omega-s} - \int_0^\infty \frac{e^{(-a-j\omega-s)t}}{-a-j\omega-s}$$

$$= \frac{1}{2j}\left[\frac{e^{\infty}-e^{0}}{-a+j\omega-s} - \frac{e^{\infty}-e^{0}}{-a-j\omega-s}\right]$$

$$= \frac{1}{2j}\left[\frac{-1}{-a+j\omega-s} - \frac{1}{a+j\omega+s}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{a}-j\omega-\cancel{s}+a-j\omega+s}{(-a+j\omega-s)(a+s+j\omega)}\right]$$

$$= \frac{1}{\cancel{2j}}\left[\frac{-\cancel{2}j\omega}{(j\omega-(a+s))(j\omega+(a+s))}\right] = \frac{-\omega}{(j\omega)^2-(a+s)^2}$$

$$= \frac{\omega}{(a+s)^2-(j\omega)^2} = \frac{\omega}{(a+s)^2+\omega^2}$$

(ii) Obtain the laplace transform of periodic waveform defined by a function

$$i(t) = (2t-2)[u(t) - u(t-2)] + [-2t+6)[u(t-2) - u(t-4)]$$

Sol.

$$= 2tu(t) - 2u(t) - 2tu(t-2) + 2u(t-2) + 2tu(t-4)$$
$$+ -6u(t-4) - 2tu(t-2) + 6u(t-2)$$

$$= 2tu(t) - 2u(t) - 4tu(t-2) + 8u(t-2) + 2tu(t-4)$$
$$-6u(t-4)$$

$$I(s) = L[i(t)] = \frac{2}{s^2} - \frac{2}{s} - \frac{4e^{-2s}}{s^2} + \frac{2e^{-4s}}{s^2} - \frac{6e^{-4s}}{s}$$
$$+ \frac{8e^{-2s}}{s}$$

Determine the initial value $f(0^+)$ if

$$f(S) = \frac{2(S+1)}{S^2+2S+5}$$

$$= \lim_{s \to \infty} s\left(\frac{2(s+1)}{s^2+2s+5}\right)$$

$$= \lim_{s \to \infty} \frac{2s^2+2s}{s^2+2s+5} = \lim_{s \to \infty} \frac{\cancel{s^2}\left[2+2/s\right]}{\cancel{s^2}\left[1+\frac{2}{s}+\frac{5}{s^2}\right]}.$$

$$= \frac{2+0}{1+0+0} = 2$$

Q For the current $i(t) = 5u(t) - 3e^{-2t}$, find $i(s)$ at determine the initial and final value also

$$I(s) = \frac{5}{s} - \frac{3}{2+s}$$

$$I(s) = \frac{10 + 5s - 3s}{s(2+s)} = \frac{10 + 2s}{s(s+2)}$$

$$f(0^+) = \lim_{s \to \infty} s\left(\frac{10+2s}{s(s+2)}\right)$$

$$= \lim_{s \to \infty} \frac{s(10/s + 2)}{s(1 + s/2)} = \underline{2}$$

$$f(\infty^+) = \lim_{s \to 0} s\left(\frac{10+2s}{s(s+2)}\right) = \frac{10}{2} = 5$$

» Z-Transform (Discrete).

$$\boxed{H(z) = \sum_{n=0}^{\infty} \underbrace{h(n)}_{\text{impulse response}} z^{-n}}$$

Types of z transform
- Unilateral
- bilateral

$x[n] = \delta[n] = 1 \longrightarrow$ system $h[n]$ $\longrightarrow y[n]$

$= h[n] * x[n] = h[n]$

Z-transform is used to convert discrete time domain signal into frequency domain signal.

$$\boxed{\begin{aligned} \frac{1}{1-r} &= 1 + r + r^2 + r^3 + \dots \infty \\ \frac{1}{1+r} &= 1 - r + r^2 - r^3 + \dots \infty \end{aligned}}$$

Q. Find the Z transform of the unit impulse sequence defined by $x[nT] = 0;\ n \neq 0$
$1;\ n = 0$

$$x(z) = \sum_{n=0}^{\infty} x[nT] z^{-n} = 1 z^{0} = 1$$

Q. Find the Z-transform of the unit step sample sequence defined by sample values, $f(nT) = 0;\ n<0$
$= 1;\ n \geq 0$

Sol.
$$F(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}$$

$$= f(0) z^{-0} + f(1) z^{-1} + f(2) z^{-2} + \dots$$

$$= 1 + z^{-1} + z^{-2} + \dots + \infty$$

$$= \frac{1}{(1-z^{-1})}$$

Q. Find the Z-transform of the unit ramp sequence defined by $f(nT) = nTU(nT)$

$$F(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}$$

$$= Tz^{-1} + 2Tz^{-2} + 3Tz^{-3} + \dots$$

$$= Tz^{-1}\left[1 + 2z^{-1} + 3Tz^{-2} + \dots\right.$$

$$= Tz^{-1}\left[1 + z^{-1} + z^{-2} + z^{-3} + \dots\right.$$
$$+ z^{-1} + z^{-2} + z^{-3} + \dots$$
$$\left. z^{-2} + 2z^{-3} + \dots \right]$$

$$= Tz^{-1}\left[\frac{1}{1-z^{-1}} + \frac{z^{-1}}{1-z^{-1}} + \frac{z^{-2}}{1-z^{-1}} + \dots\right]$$

$$= \frac{Tz^{-1}}{1-z^{-1}} \left[1 + z^{-1} + z^{-2} + \dots\right] = \frac{Tz^{-1}}{(1-z^{-1})^2}$$

Ques ~~Define~~ Determine the z-transform of $a^n \sin\left(\frac{\pi n}{2}\right)$

$$F(z) = \sum_{n=0}^{\infty} a^n \sin\left(\frac{n\pi}{2}\right) z^{-n}$$

$$= 0 + a \sin\frac{\pi}{2} z^{-1} + a^2 \sin \pi z^{-2} + a^3 \sin\frac{3\pi}{2} z^{-3} + \dots$$

$$= \cancel{a} + az^{-1} - a^3 z^{-3} + \dots$$

$$= az^{-1}\left[1 - a^2z^{-2} + a^4z^{-4} - \dots\right]$$

$$= az^{-1} \left[\frac{1}{1 + a^2 z^{-2}}\right]$$

Ques $f(nT) = \left[3(2)^n - 4(3)^n\right] u(n)$

$$F(z) = \sum_{n=0}^{\infty} \left[3(2)^n - 4(3)^n\right] z^{-n}$$

$$= 3\sum_{n=0}^{\infty} 2^n z^{-n} - 4\sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\left[\because \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}\right]$$

$$= \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}$$

$$= \frac{3 - 9z^{-1} - 4 + 8z^{-1}}{(1-2z^{-1})(1-3z^{-1})} = \frac{-z^{-1} - 1}{(1-2z^{-1})(1-3z^{-1})}$$

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↳ Z-transform is the extension of ~~fourier~~ DTFT (Discrete time fourier transform)

↳ DTFT can only be applied to stable system but z transform can be applied to both stable and unstable system

TYPES OF ~~...~~ Z-TRANSFORM

↳ UNILATERAL

$$\boxed{X(n) = \sum_{n=0}^{\infty} x(n) z^{-n}}$$

↳ BILATERAL

$$\boxed{X(n) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

REGION OF CONVERGENCE

⇒ Region of convergence of Z-transform. ~~convergence~~

↳ It is the region where Z-transform convergence.

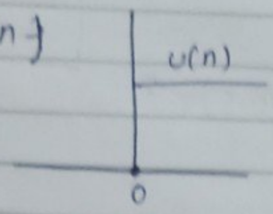
⇒ Properties.

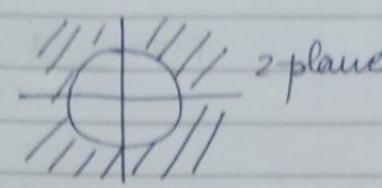
1. ROC of any function consists of a circle in Z-plane centered about the origin. It is indicated as circles containing unit circle.

2. The ROC should not contain any poles i.e $X(Z)$ should be having finite values.

3. The ROC of right-sided sequence is outside the circle.

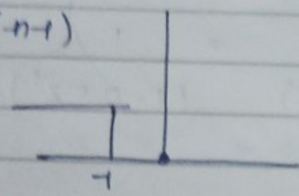
Eg. $\alpha^n u(n)$

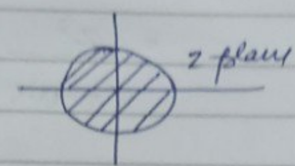




4. The ROC of left sided sequence is inside the circle.

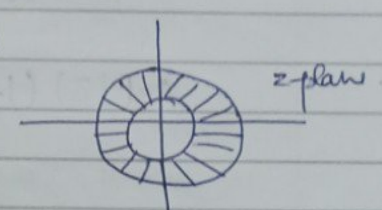
Eg. $\alpha^n u(-n-1)$





5. The ROC of two sided sequence is concentric circle / ring.

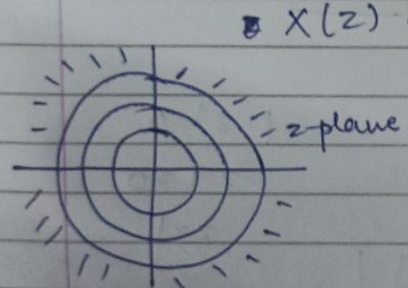
Eg $x(n) = \alpha^n u(n) - 2^n u(-n-1)$



6. If $x(n)$ is rational and right-sided then ROC is the z-plane outside the outermost pole.

Eg: $x(n) = \alpha^n u(n) - \beta^n u(n) + c^n u(n)$

$$X(z) = \frac{1}{1-\alpha z^{-1}} - \frac{1}{1-\beta z^{-1}} + \frac{1}{1-Cz^{-1}}$$



If z-transform of $x(n)$ is rational and left sided sequence then ROC is the z-plane inside the innermost pole.

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INVERSE Z-TRANSFORM

$$\boxed{\frac{1}{1-\alpha z^{-1}} = \alpha^n u(n)}$$

$$F(z) = \frac{1}{1-(1.5)z^{-1}+(0.5)z^{-2}}.$$

$$= \frac{z^2}{z^2-1.5z+0.5}$$

$$= \frac{z^2}{(z-0.5)(z-1)} = \frac{1}{(1-0.5z^{-1})(1-z^{-1})}$$

$$F(z) = \frac{A}{1-0.5z^{-1}} + \frac{B}{(1-z^{-1})}$$

$$A = F(z)(1-0.5z^{-1})\Big|_{z=0.5}$$

$$= \frac{1}{(1-0.5z^{-1})(1-z^{-1})} \times (1-0.5z^{-1})\Big|_{z=0.5}$$

$$= \frac{1}{1-2} = -1 \qquad \boxed{A=-1}$$

$$B = F(z)\cdot(1-z^{-1})\Big|_{z=1}$$

$$1-z^{-1}=0$$
$$z^{-1}=0$$
$$z=1$$

$$= \frac{1}{(1-z^{-1})(1-0.5z^{-1})} (1-z^{-1})\Big|_{z=1}$$

$$B = \frac{1}{1 - 1/2} = \frac{1/2^{-1}}{2} \qquad \boxed{B = 2}$$

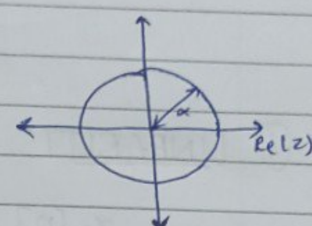
$$F(z) = \frac{2}{1 - z^{-1}} - \frac{1}{1 - 0.5z^{-1}}$$

$$F(n) = 2(1)^n u(n) - (0.5)^n u(n).$$

Q $x(n) = \alpha^n u(n).$

$$x(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$

$$x(z) = \frac{1}{1 - \alpha z^{-1}}$$



Q Determine the z-transform of the signal $x(n) = -\alpha^n u[-n-1]$

$$= -\sum_{n=0}^{\infty} \alpha^n u[-n-1] z^{-1}$$

$$= -\sum_{n=-\infty}^{-1} \alpha^n z^{-n} = -\sum_{n=1}^{\infty} \alpha^{-n} z^n$$

$$= -\sum_{n=1}^{\infty} (\alpha^{-1} z)^n$$

$$= -\left[\alpha^{-1} z + \alpha^{-2} z^2 + \alpha^{-3} z^3 + \dots\right]$$

$$= -\alpha^{-1} z \left[1 + \alpha^{-1} z + \alpha^{-2} z^2 + \dots\right]$$

$$x(z) = \frac{\alpha - z}{1 - \alpha^{-1} z}$$

Q $x[n] = \alpha^n u[n] + \beta^n u[-n-1]$

$$= \frac{1}{1-\alpha z^{-1}} + \frac{\beta^{-1} z}{1-\beta^{-1} z}$$

ROC $|z| > \alpha$

ROC. $|z| < \beta$

$\alpha < |z| < \beta$

① LINEARITY

$x_1[n] \xleftrightarrow{z\text{-transform}} X_1(z)$

$x_2[n] \xleftrightarrow{z\text{-transform}} X_2(z)$

$a x_1[n] + b x_2[n] \longleftrightarrow a X_1[z] + b X_2(z)$

② MULTIPLICATION BY A CONSTANT

if $x[n] \rightarrow X[n]$

$a x[n] \rightarrow a X[n]$

③ TIME DELAY

if $x[n] \xleftrightarrow{z} X[n]$

then $x[n-k] \longleftrightarrow z^{k} X(z)$

④ TIME ADVANCE

if $x[n] \xleftrightarrow{z} X[n]$

$x[n+k] \longleftrightarrow z^{k} X(z)$

⑤ Scality in z-domain

If $x[n] \xleftrightarrow{z} X[z]$

z_0

⑥ Time Reversal

If $x[n] \xleftrightarrow{z} X[z]$

$x[-n] \longleftrightarrow X(z^{-1})$

Ex. $f[n] = \left(\frac{1}{5}\right)^n u[n] + \left(\frac{3}{4}\right)^n u[n-4] - 2u[n-8]$

$\sum_{0}^{\infty} \frac{3}{4}^n u(n-4) z^{-n}$ $\cancel{\frac{1}{1-(1/5)z^{-1}}}$ $+ \left[\frac{1}{1-(1/5)z^{-1}} + \frac{(3/4)^4 z^{-4}}{1-(3/4)z^{-1}}\right.$

Σ

$= \cancel{\left(\frac{1}{5}\right)^n} + 2\frac{z^{-8}}{1-z^{-1}}\Big]$

$= \frac{1}{1-(1/5)z^{-1}} + \frac{(3/4)^4 z^{-4}}{1-\left(\frac{3}{4}\right)z^{-1}} + \frac{2z^{-8}}{1-z^{-1}}$

RO

$\left\lvert\frac{1}{5}z^{-1}\right\rvert < 1$	$\left\lvert\frac{3}{4}z^{-1}\right\rvert < 1$	$\lvert z^{-1}\rvert < 1$
$\lvert z^{-1}\rvert < 5$	$z^{-1} < \frac{4}{3}$	$z > 1$
$z > 1/5$	$z > \frac{3}{4}$	

$\frac{1}{2}$

$\therefore z > 1$

Note: when less than inside condition, greater than outside.

$$f(nT) = e^{-anT} \; ; \quad a > 0, n \geq 0$$

$$F(z) = \sum_{n=0}^{\infty} e^{-anT} z^{-n}.$$

$$= \sum_{n=0}^{\infty} e^{-atn} z^{-n}$$

$$= \sum_{n=0}^{\infty} (e^{-at} z^{-1})^{n}.$$

$$= \frac{1}{1-(e^{-at})z^{-1}}$$

ROC

$$|e^{-at} z^{-1}| < 1$$

$$z^{-1} < \frac{1}{e^{-at}}$$

$$\boxed{z > e^{-at}}$$

INITIAL VALUE THEOREM

(continuous time)

$$x[n] \overset{z}{\longleftrightarrow} X(z)$$

$$x[0] \rightarrow \lim_{n \to 0} x[n] = \lim_{z \to \infty} X(z)$$

FINAL VALUE THEOREM

$$x[\infty] = \lim_{n \to \infty} x(n] = \lim_{z \to 1} (z-1)X(z)$$

DIFFERENCIATION IN Z DOMAIN.

$$n x[n] \longleftrightarrow -z \frac{d X(z)}{dz}$$

Q Find $x[\infty]$ if $X(z)$ is given by $\frac{z+2}{(z-0.8)^2}$

$$x[\infty] = \lim_{z\to 1} \frac{(z-1)(z+2)}{(z-0.8)^2}$$

$$= \underline{0}$$

(ii) $\frac{(z+1)}{3(z-1)(z+0.9)}$

$$x[\infty] = \lim_{z\to 1} \frac{(z+1)}{3(\cancel{z-1})(z+0.9)} \cancel{(z-1)} = \frac{z+1}{3(z+0.9)}.$$

$$= \frac{2}{3(1.9)} = 0.3508$$

> ## INVERSE Z-TRANSFORM

The process of $x[n]$ from its z-transform is called the inverse z-transform.

There are 4 methods to find the inverse z-transform

* long division method
* partial fraction method
* residue method
* convolution method.

LONG DIVISION METHOD

↳ If ROC is causal equation is in highest order
If ROC is non-causal equation is in lowest order

Determine the inverse z-transform of $x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$

a) if $x(n)$ is causal
b) if $x(n)$ is non causal.

$$
\begin{array}{r|l}
 & 1 + 4z^{-1} + 7z^{-2} + 10z^{-3}. \\
1-2z^{-1}+z^{-2} & 1+2z^{-1} \\
 & 1-2z^{-1}+z^{-2} \\
 & - \quad + \quad - \\ \hline
 & 4z^{-1} - z^{-2} \\
 & 4z^{-1} - 8z^{-2} + 4z^{-3} \\
 & - \quad + \quad - \\ \hline
 & 7z^{-2} - 4z^{-3} \\
 & 7z^{-2} - 14z^{-3} + 7z^{-4} \\
 & - \quad + \quad - \\ \hline
 & 10z^{-3} - 7z^{-4}
\end{array}
$$

$x(n) = \{\underset{\uparrow}{1}, 4, 7, 10, 13, 16, \ldots\}$

z^{-2}

6. $z^{-2} - 2z^{-1} + 1 \overline{) \, 2z^{-1} + 1 \,} (2z + 5z^2 + 8z^3$

$2z^{-1} - 4 + 2z$

$5 - 2z$

$5 - 10z + 5z^2$

$8z - 5z^2$

$$x(n) = \{ \ldots, 11, 8, 5, 2, \underset{\uparrow}{0} \}$$

PARTIAL FRACTION EXPANSION METHOD

$$\frac{a_1 z + a_2 z^2 + a_3 z^2 + \ldots}{b_1 z + b_2 z^2 + b_3 z^3 + \ldots} = \frac{k_1}{(z-a_1)} + \frac{k_2}{(z-a_2)} + \frac{k_3}{(z-a_3)}.$$

Types of roots

① $\frac{k}{z-a}$

② $\frac{k}{z-a+jb}$, $\frac{k^*}{z-a-jb}$

③ $\frac{k}{(z-a)^r}$.

Q. Find the inverse z-transform of $X(z) = \frac{5z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$

ROC → causal.

$$X(z) = \frac{5z^{-1}}{(1-2z^{-1})(1-3z^{-1})} = \frac{5z}{(z-2)(z-3)}$$

$$= \frac{A}{(z-2)} + \frac{B}{(z-3)}$$

$$A = \cancel{(z-2)} \cdot \frac{5}{\cancel{(z-2)}(z-3)}\Bigg|_{z=2} = \frac{5}{2-3} = -5$$

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$$B = (z-3)\frac{5}{(z-2)(z-3)}\Bigg|_{z=3} = 5$$

$$\boxed{x(n) = 5(2)^n u(n) + 5(3)^n u(n)}$$

Q $\frac{1}{1-1.5z^{-1}+0.5z^{-2}}$ $|z|>1$

$$\frac{z^2}{1-1.5z+0.5} = \frac{z^2}{(z-1)(z-0.5)}$$

$$\frac{F(z)}{z} = \frac{A}{(z-0.5)} + \frac{B}{(z-1)}$$

$$A = \cancel{z-0.5} \times \frac{z}{\cancel{(z-0.5)}(z-1)}\Bigg|_{z=0.5} = \frac{0.5}{-0.5} = -1$$

$$B = \frac{\cancel{(z-1)} \cdot z}{(z-0.5)\cancel{(z-1)}}\Bigg|_{z=1} = \frac{1}{0.5} = 2$$

$$F(s) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$

$$= \frac{z}{1-z^{-1}} - \frac{1}{1-0.5z^{-1}} \qquad \left[\alpha^n u[n] \xrightarrow{z} \frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}\right.$$

$$F(n) = 2(1)^n u[n] - (0.5)^n u(n)$$

Note : if $|z| < 1$

$$\therefore -2(1)^n u(-n-1) + (0.5)^n u(-n-1)$$

Q. $F(z) = \dfrac{z^4}{(z-1)(z+1)^3}$

$$\frac{F(z)}{z} = \frac{z^3}{(z-1)(z+1)^3} = \frac{A}{(z-1)} + \frac{B}{(z+1)} + \frac{C}{(z+1)^2} + \frac{D}{(z+1)^3}$$

$$A = \cancel{(z-1)} \frac{(z^3)}{\cancel{(z-1)}(z+1)^3}\Bigg|_{z=1}$$

$$= \frac{1}{8}$$

$$\cancel{B}, D = \cancel{(z+1)^3} \frac{z^3}{(z-1)\cancel{(z+1)^3}}\Bigg|_{z=-1} = \frac{-1}{-2} = \frac{1}{2}$$

$$C = \frac{1}{\lfloor 3-2} \frac{d^{3-2}}{dz^{(3-2)}} \frac{z^3}{(z-1)(z+1)^3} = \frac{d}{dz} \frac{z^3 \cancel{(z+1)^3}}{(z-1)\cancel{(z+1)^3}}$$

$$= \frac{d}{dz}\frac{z^3}{z-1} = \frac{3z^2(z-1) - z^3(1)}{(z-1)^2} = \frac{3z^3 - 3z^2 - z^3}{(z-1)^2}$$

$$= \frac{2z^3 - 3z^2}{(z-1)^2}\Bigg|_{z=-1}$$

$$= \frac{-2-3}{(-2)^2} = \frac{-5}{4}$$

$$B = \frac{1}{\lfloor 3-1} \frac{d^2}{dz^2} \frac{(z+1)^2 z^3}{(z-1)(z+1)^3} \Big|_{z=-1}$$

$$= \frac{1}{2} \frac{d^2}{dz^2} \frac{z^3}{z-1}$$

$$= \frac{1}{2} \frac{d}{dz} \frac{3z^3 - 3z^2}{(z-1)^2}$$

$$= \frac{1}{2} \cancel{\frac{(6z^2-6z)(z-1)^2 - (2z^3-3z^2)}{(z-1)^4}} \Big|_{-1}$$

$$= \frac{1}{2} \cancel{\frac{(3+6)(-2)^2 - (-1-3)}{(-2)^4}}$$

$$= \frac{1}{2} \cancel{\frac{(9)(4) - (-4)}{16}} = \frac{1}{2} \times \frac{5+4}{16} = \frac{9}{32}$$

$$= \frac{1}{2} \frac{(6z^2-6z)(z-1)^2 - 2(z-1)(2z^3-3z^2)}{(z-1)^4} \Big|$$

$$= \frac{1}{2} \frac{(6+6)(-2)^2 - 2(-2)(-2-3)}{(-2)^4}$$

$$= \frac{1}{2} \frac{(12)(4) + 4(-5)}{16} = \frac{12-5}{8} = \frac{7}{8}$$

$$= \frac{1/8}{1-z^{-1}} + \frac{7/8}{1+z^{-1}} - \frac{5/4 z^{-1}}{(1+z^{-1})^2} + \frac{1/2 z^{-2}}{(1+z^{-1})^3}$$

$$\boxed{n\alpha^n u[n] = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}}$$

$$= \frac{1}{8}(1)^n u(n) + \frac{7}{8}(-1)^n u(n) - \frac{5}{4} n(-1)^n u(n)$$

$$+ \frac{1}{2}(n-1)(-1)^n u(n)$$