11/12/12

SIGNAL AND SYSTEM.

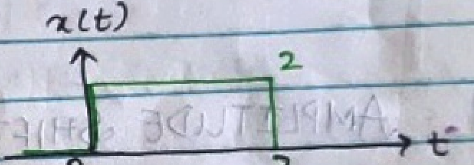
DIFFERENT OPERATIONS ON SIGNALS.

- → Shifting
- → Scaling
- → Reversal

1. SHIFTING → TIME SHIFTING / → AMPLITUDE SHIFTING.

(a) TIME SHIFTING.

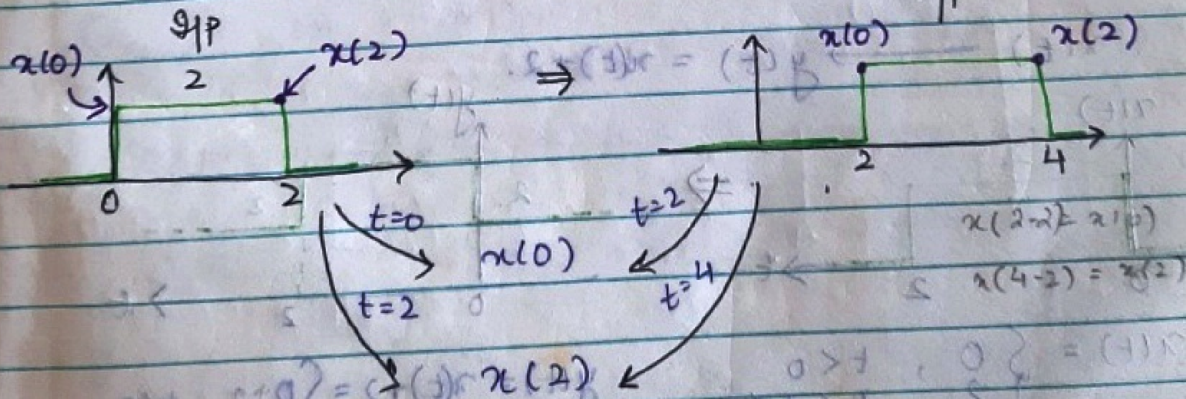
$$\boxed{x(t) \rightarrow y(t) = x(t+k)}$$



CASE 1 : when $k < 0$

eg = $k = -2$.

$x(t) \longrightarrow x(t-2)$



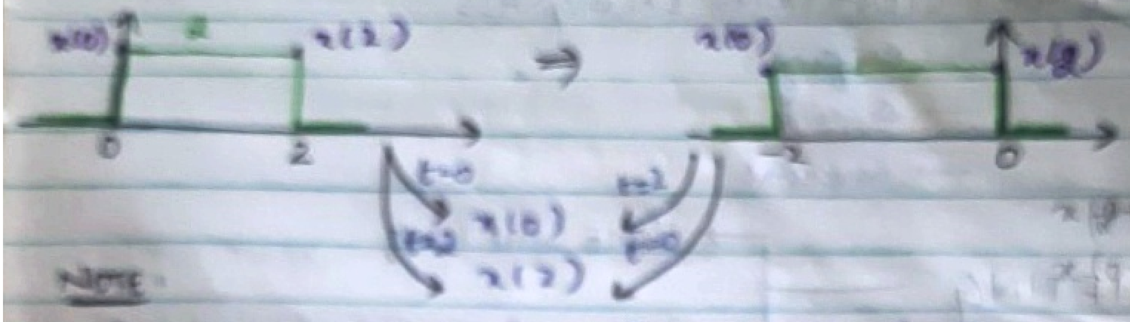
NOTE :

→ It is a case of right - shifting / time - delay
∵ here there is a time delay of 2 sec in O/P.

CASE 2: when $k > 0$

eg $k = 2$

$x(t) \longrightarrow y(t) = x(t+2)$



NOTE:

→ It is a case of time left shifting / time advance

∵ the sig is generated before 0 sec.

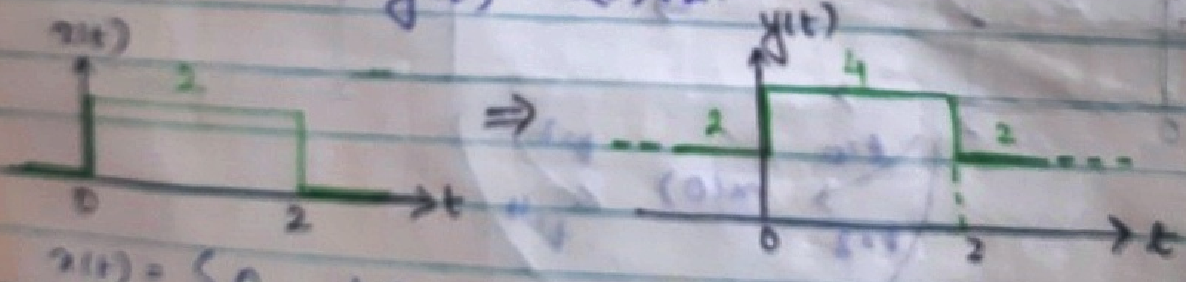
(b) AMPLITUDE SHIFTING

$$\boxed{x(t) \longrightarrow y(t) = x(t) + k}$$

CASE 1: when $k > 0$

eg $k = 2$.

$x(t) \longrightarrow y(t) = x(t) + 2$.



$$x(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$$

$$y(t) = x(t) + 2 = \begin{cases} 0+2, & t < 0 \\ 2+2, & 0 \le t \le 2 \\ 0+2, & t > 2 \end{cases}$$

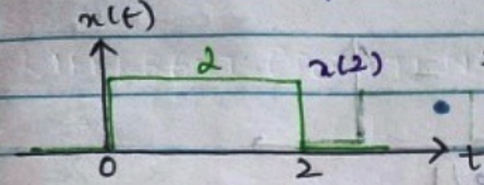
NOTE:

→ It is a case of upward shifting.

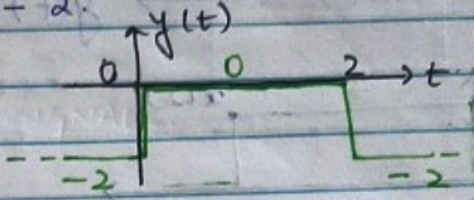
CASE 2: when $k < 0$

eg $k = -2$.

$x(t) \longrightarrow y(t) = x(t) - 2$.



$\Longrightarrow$



$$x(t) = \begin{cases} 0 & ; t<0 \\ 2 & ; 0 \le t \le 2 \\ 0 & , t>2 \end{cases} \qquad y(t) = \begin{cases} 0-2 & , t<0 \\ 2-2 & , 0 \le t \le 2 \\ 0-2 & , t>0 \end{cases}$$

NOTE:
It is a case of downward shifting.

2. SCALING → TIME SCALING (compression or expansion of sig...)
→ AMPLITUDE SCALING.

(i) TIME SCALING. (divide by 2)

$$\boxed{x(t) \longrightarrow y(t) = x(\alpha t) \quad , \alpha \neq 0}$$

$\alpha \in (-\infty, -1) \cup (1, \infty)$

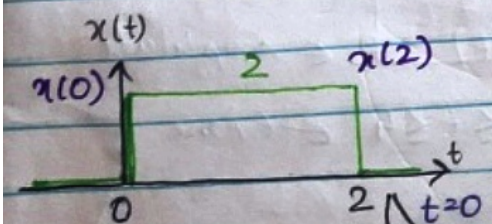
$\alpha \neq 0.1, 0.2, 0.5$ etc.

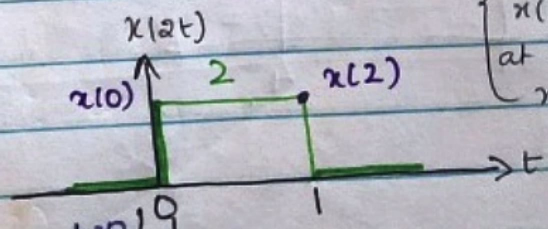
CASE 1: when $\alpha > 1$

eg: $\alpha = 2$.

$x(t) \longrightarrow y(t) = x(2t)$ — time.

Amplitu...
t=0
$x(2.0) = x($...
at $t=1$
$x(2.1) =$...
$x(2t) =$...





$x(t) = 2$
$0 \le t \le 2$

t=0 → $x(0)$ ← t=0

t=2 → $x(2)$ ← t=1.

NOTE:
→ It is a case of time - compression.
Amplitude will remain same only time will c...

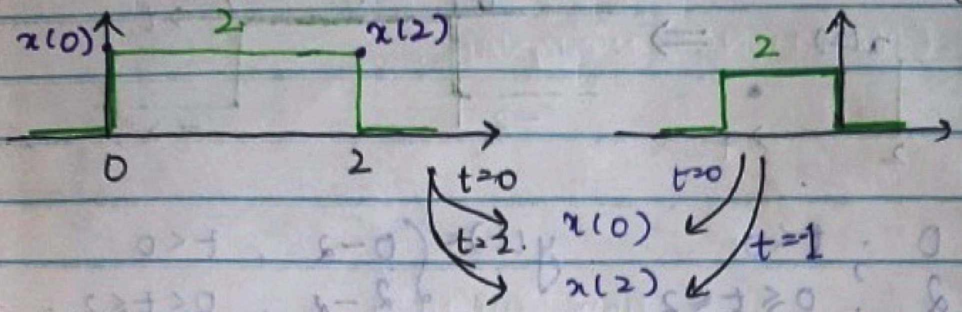
CASE 2: when $0<\alpha<1$. or $|a|<1$

eg. $\alpha = -2$. or $-1<a<1$, $a \neq 0$

$x(t) \longrightarrow y(t) = x(-2t)$ $\alpha \in (-1,0) \cup (0,1)$

$\alpha = 0.1, 0.2, 0.3$ —

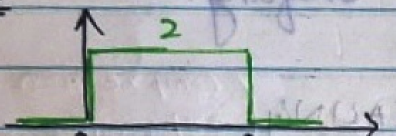
α can't be integer



① Amplitude will remain same. ② $\frac{\alpha t}{\alpha}$

ALTERNATE METHOD.

Case 1: $x(\alpha t)$



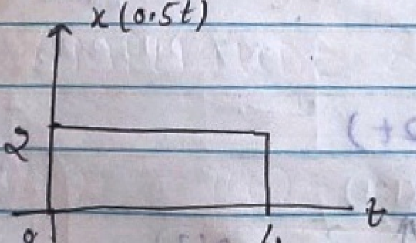
$\frac{0}{\alpha}$ $\frac{2}{\alpha}$ $\frac{\alpha t}{\alpha} = t$

$\alpha = 2$

$\longrightarrow 0$ $\frac{2}{2} = 1.$

Case 2: eg $\alpha = -0.5$

$\frac{0.5t}{0.5} = t$

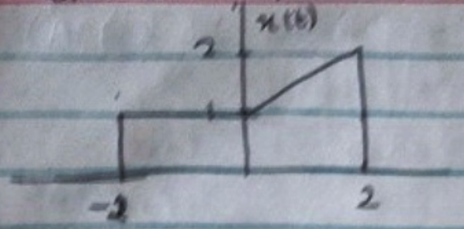


$\frac{2}{0.5} = 4$ $\frac{0}{0.5} = 0$

$x(0.5t) = 2 \leftarrow$ Amplitude

$0 \leq t \leq 4 \leftarrow$ time

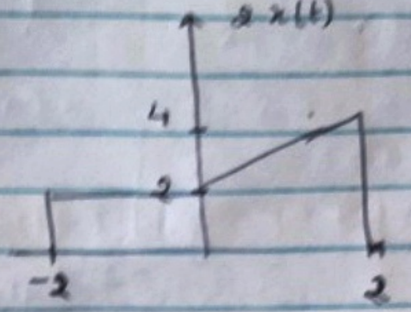
Qu on Amplitude scaling.



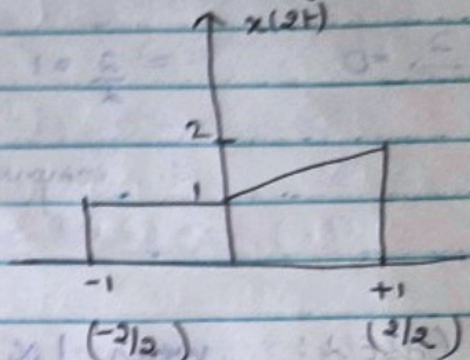
find - $2x(t)$ - Amplitude scaling

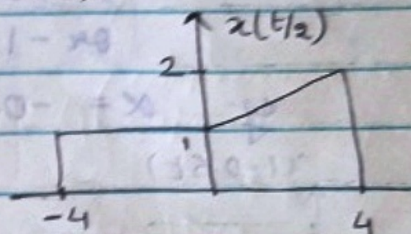
$x(2t)$ } time scaling
$x(t/2)$

Amplitude scaling.



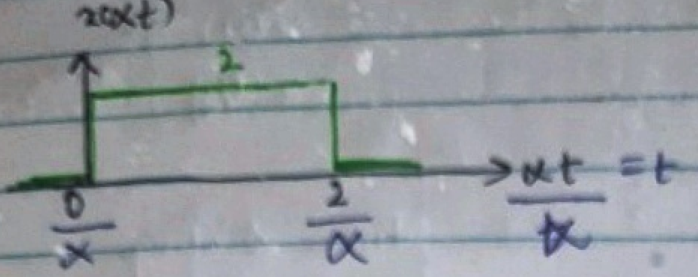
time scaling.





In case of time scaling when we multiply time then we get compressed signal whereas in case of division we get expand signal.

ALTERNATE METHOD:



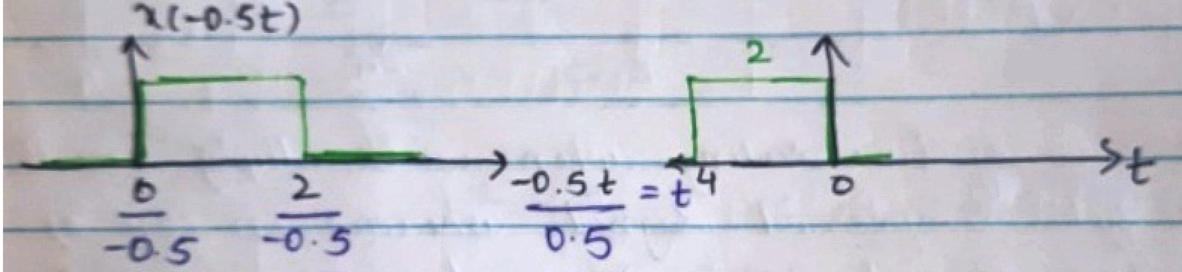
$\alpha = 2$, $\rightarrow 0$ $\quad = \frac{2}{2} = 1$

∴ compression

CASE 2: when $|\alpha| < 1$

or $-1 < \alpha < 1$, $\alpha \neq 0$

eg. $\alpha = -0.5$



$= 0 \qquad = -4$

NOTE: → It is a case of time expansion

(II) AMPLITUDE ~~SHIFTING~~ SCALING.

$$\boxed{x(t) \longrightarrow y(t) = \alpha\, x(t) \;,\; \alpha \neq 0}$$

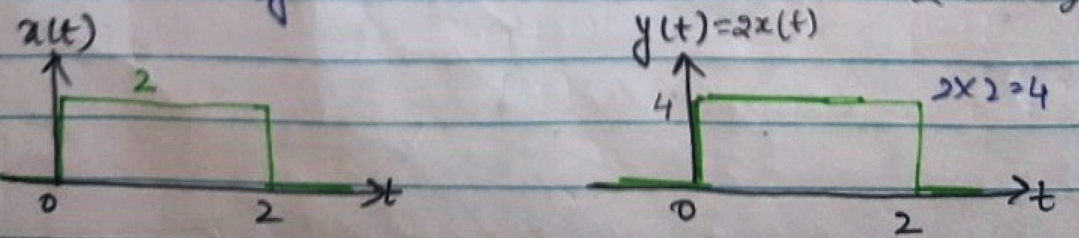
$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$

CASE 1: when $\alpha > 0$ — $\alpha > 0$ — $|\alpha| > 1$

$\alpha \in (-\infty, -1) \cup (1, \ldots$

α is integer

eg $\alpha = 2$.



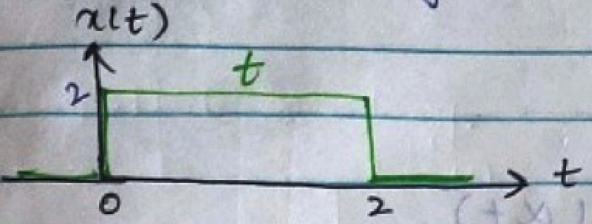
NOTE: It is a case of ~~amplificati~~ amplification.
time will not change only amplitude will change

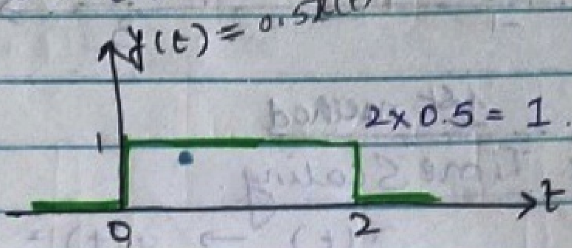
$\beta \in (-1,0) \cup (0,1)$
$\alpha = 0.1,\ 0.7$

CASE 2: when $|\alpha| < 1$

eg $\alpha = 0.5$

$x(t) \longrightarrow y(t) = 0.5\, x(t)$





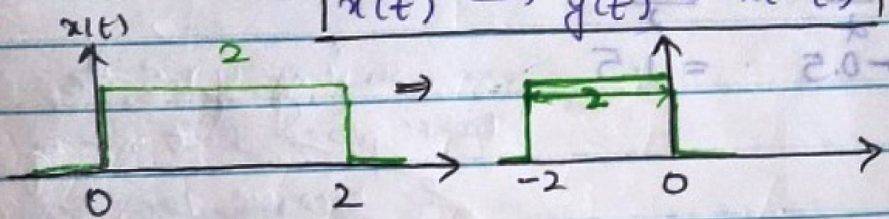
NOTE: → It is a case of attenuation.

3. REVERSAL → TIME REVERSAL
→ AMPLITUDE REVERSAL.

It is a special case of scaling with $\alpha = -1$.

a) TIME-REVERSAL.

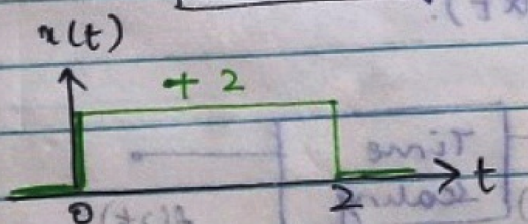
$$\boxed{x(t) \longrightarrow y(t) = x(-t)}$$

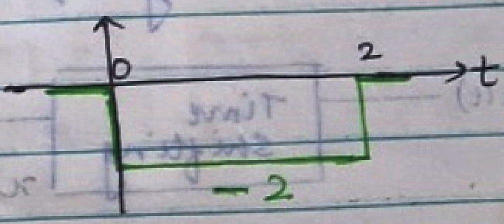


NOTE: signal folding will take place about y-axis.

b) AMPLITUDE-REVERSAL.

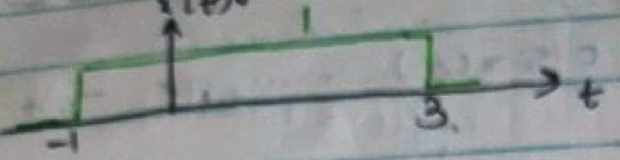
$$\boxed{x(t) \longrightarrow y(t) = -x(t)}$$





NOTE: Signal folding will take place about x-axis.

Q. Draw sig, $y(t) = x(2t+3)$



1st method

Ans Time Scaling

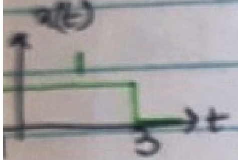
$x(t) \rightarrow y(t) = x(\alpha t)$.

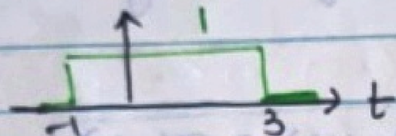
Time shifting

$x(t) \rightarrow y(t) = x(t+k)$.

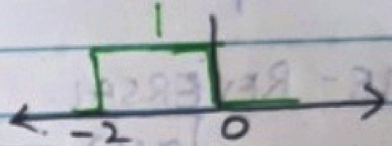
$y(\frac{t}{2}) = x(2t+\frac{3}{2}) = x[3(t+1.5)]$.

$x(t)$ → [Time scaling] → $x(2t)$ → [Time shifting] →





$\frac{-1}{2} = -0.5$ $\quad \frac{3}{2} = 1.5$



shifting of 1.5 sec takes place but not of 3 because the scaling parameter is seperated.

2nd method

Time shifting

$x(t) \rightarrow y(t) = x(t+k)$

Time scaling

$x(t) \rightarrow y(t) = x(\alpha t)$.

$x(t)$ → [Time shifting] → $x(t+1.5)$ → [Time scaling] →

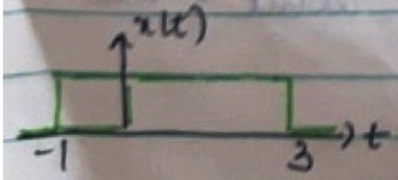
$f(t)$ ✗ $\quad$ because ques is $(2t+3)$ not $(2t+1.5)$ $\quad f(2t)$ $\quad x(2t+1.5)$ ✗

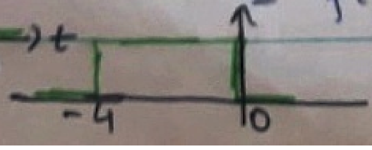
$= x(t+3)$

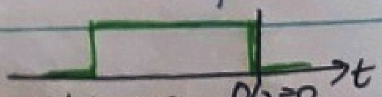
$= f(t)$

$= f(2t)$

$= f(2t+3)$



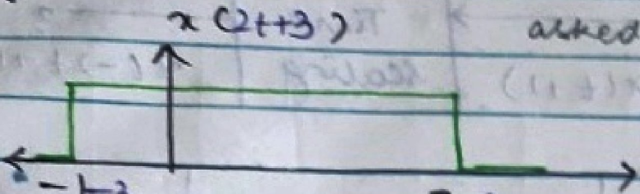




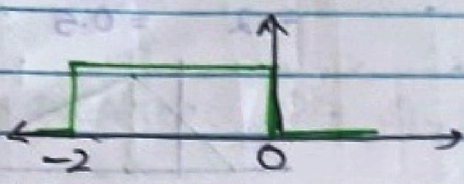
$-4/2 = -2 \quad 0/2 = 0$

SHORTCUT → This shortcut is applicable for ques in which s/g is given & we are asked to draw the waveform.

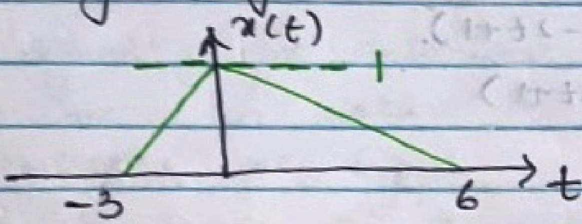
$y(t) = x(2t+3)$



$\frac{-1-3}{2} = -2$ $\qquad$ $\frac{3-3}{2} = 0.$ $\qquad$ $\frac{\cancel{2}t+\cancel{3}-\cancel{3}}{\cancel{2}} = t$

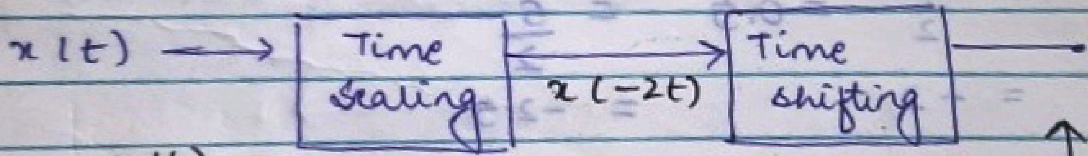


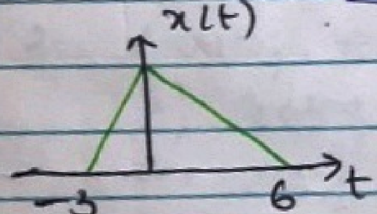
Q. Draw signal, $y(t) = x(-2t+1)$.

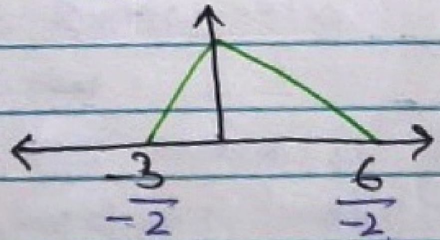


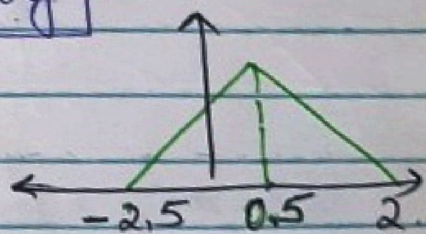
Ans → 1st method.

$y(t) = x(-2t+1) = x\left[-2\left(t-0.5\right)\right]$

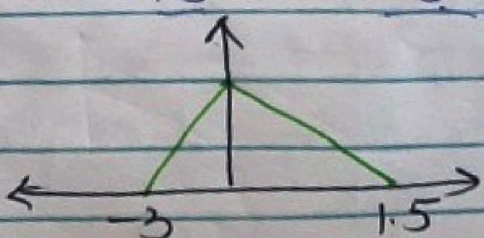






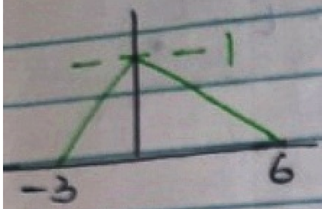


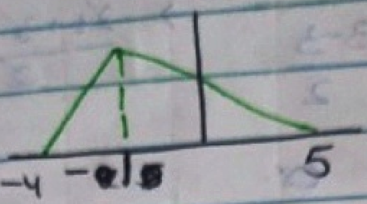
$= 1.5$ $\qquad$ $= -3$

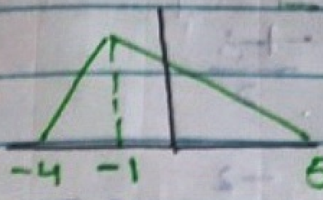


2nd method

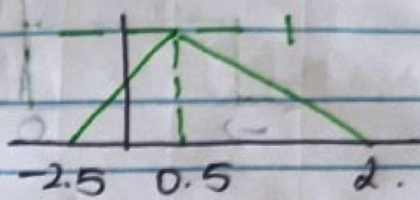
$x(t) \rightarrow$ Time shifting $\rightarrow x(t+1) \rightarrow$ Time scaling $\rightarrow x(-2t+1)$







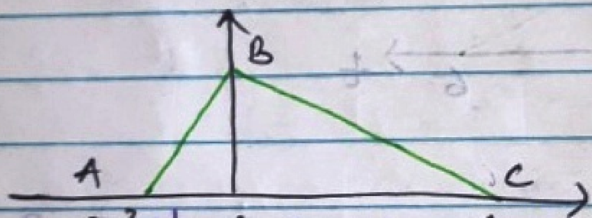
$\frac{-4}{-2} = 2 \qquad \frac{-1}{-2} = 0.5 \qquad \frac{5}{-2} = -2.5$



SHORTCUT.

$y(t) = x(-2t+1).$

$x(-2t+1)$



$\cancel{(-2t+1)-1} = t$

$\frac{-3-1}{-2} \qquad \frac{0-1}{-2} \qquad \frac{6-1}{-2}$

$= \frac{-4}{2} \qquad = 0.5 \qquad = \frac{5}{-2}$

$= -2 \qquad\qquad\qquad = -2.5$

II. Long Answer Type Questions

1. A triangular pulse signal $x(t)$ is shown in Figure 1.77(a). Sketch the following signals. (a) $x(4t)$; (b) $x(4t+3)$; (c) $x(-3t+2)$; (d) $x(\frac{t}{3}+2)$; (e) $x(3t-2)$; (f) $x(4t+3)+x(2t)$.

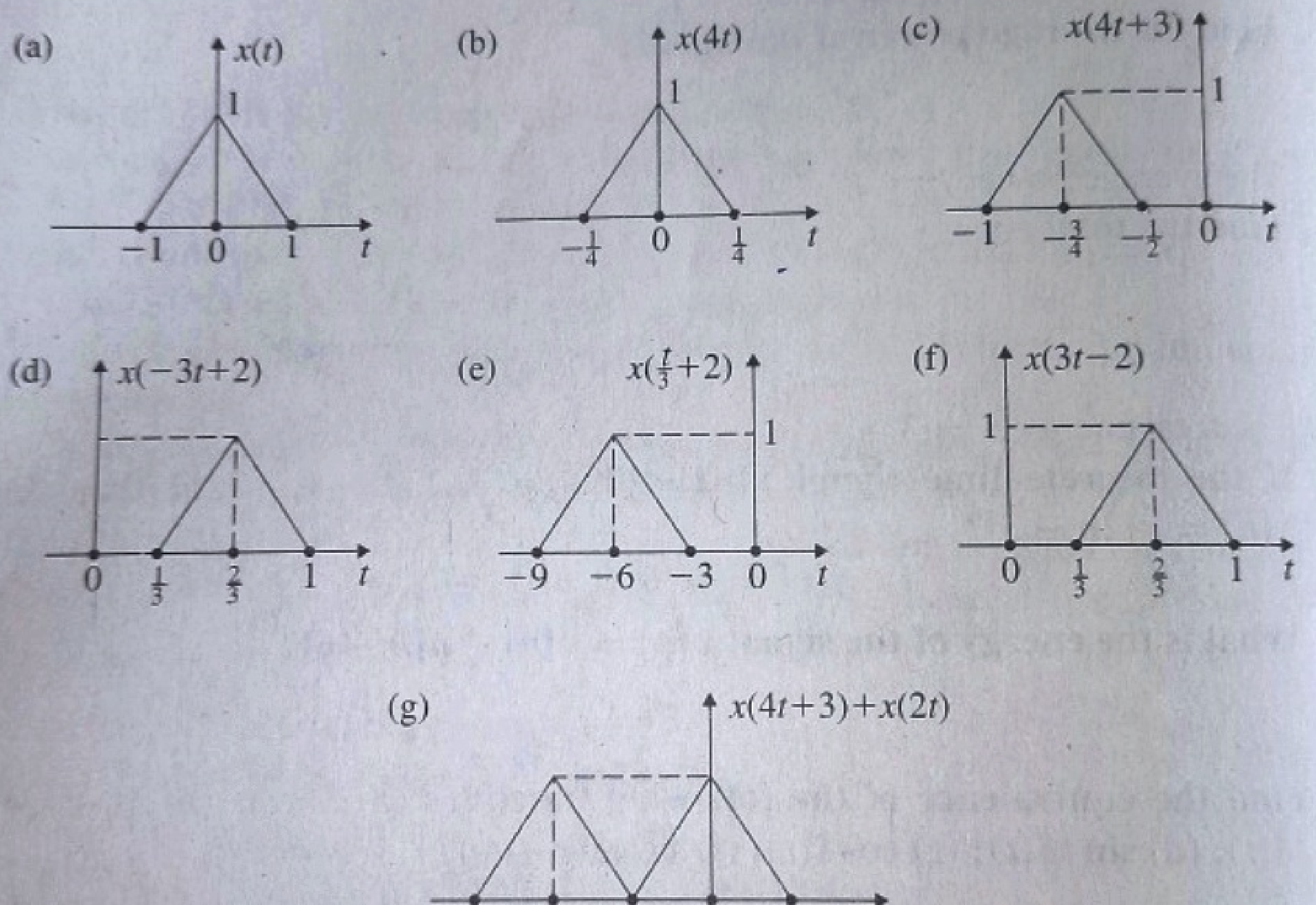


Figure 1.77 Operations of CT signals.

2. Sketch the following CT functions. (a) $x(t) = 8u(5-t)$; (b) $x(t) = 3\delta(t+2)$; (c) $x(t) = \text{ramp}(t+1)$; (d) $x(t) = 5\text{rect}\frac{(t+1)}{4}$; (e) $x(t) = -\text{tri}\frac{t-1}{4}$; (f) $x(t) =$

$u(t) - u(t-5)$; (g) $x(t) = u(t) - u(t+5)$; (h) $x(t) = -\mathrm{ramp}(t)u(t-3)$; (i) $x(t) = u(t)(t+\frac{1}{3})\mathrm{ramp}(\frac{1}{3}-t)$; (j) $x(t) = \mathrm{rect}(t+2) - \mathrm{rect}(t-2)$.

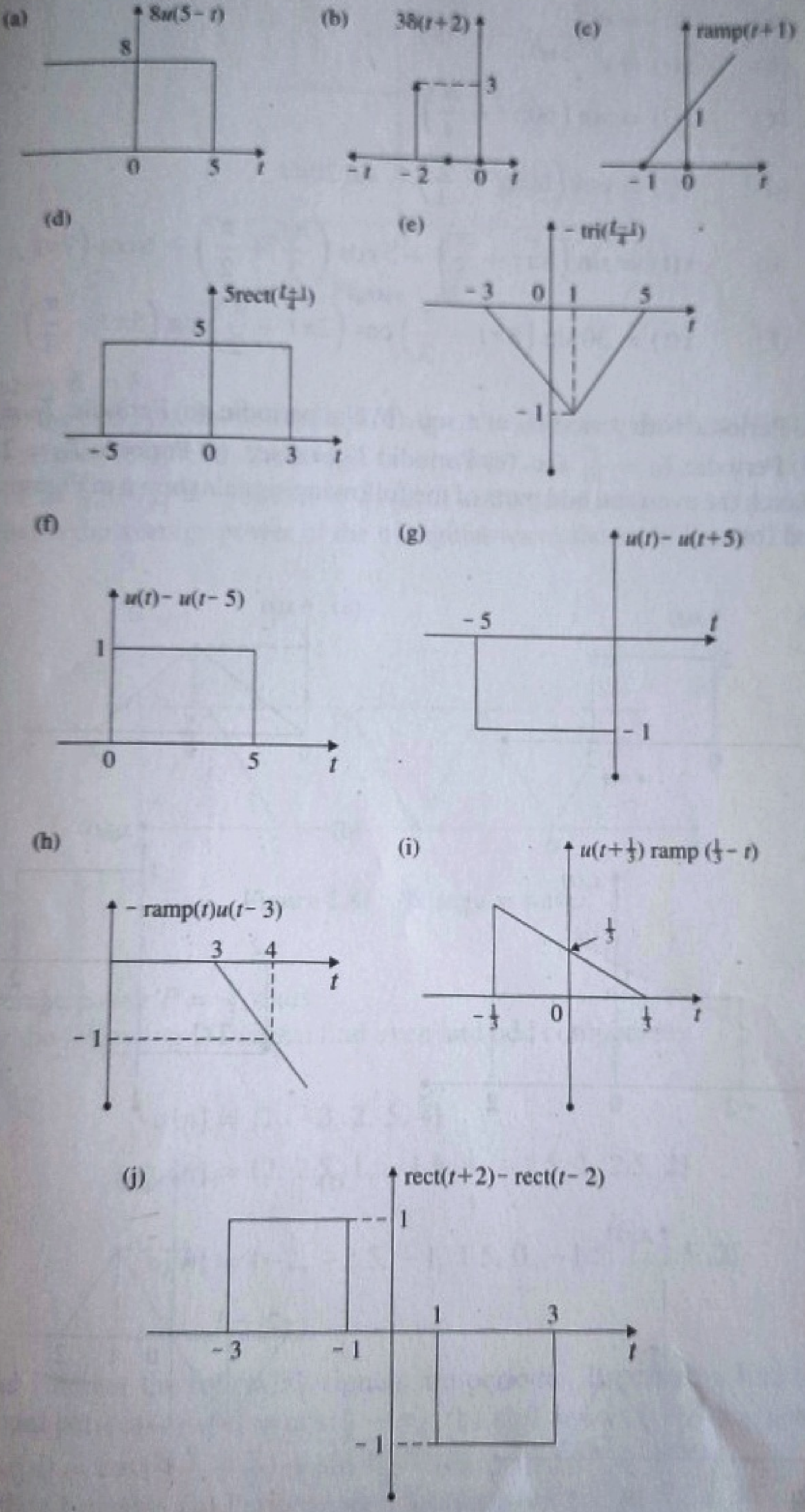


Figure 1.78 Operations of CT signals.