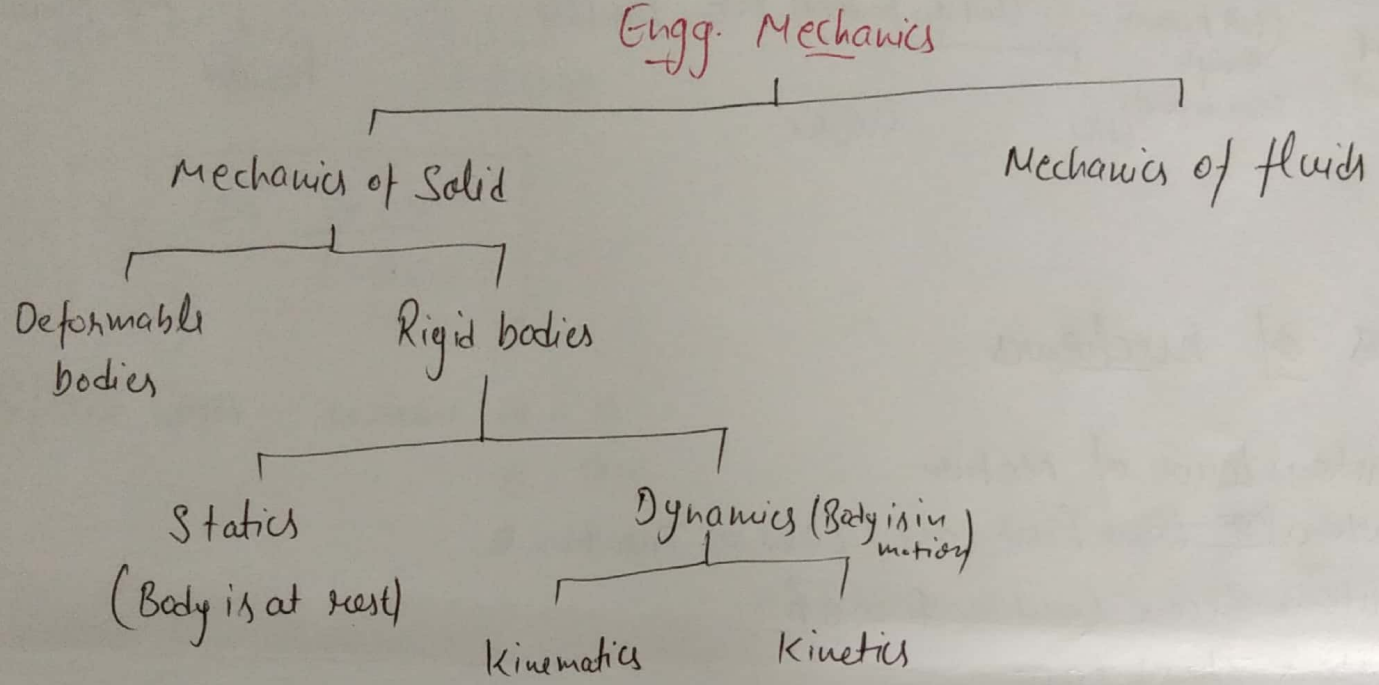
Engg. Mechanics - It is a branch of physical science which describes and predicts the state of rest or motion of bodies under the action of forces.



Basic Quantities used in mechanics

i) Matter - anything that occupies space, possesses mass and offers resistance to any external force.

ii) Particle - infinitelly small volume but has mass.

iii) Body - Made up of particles

iv) Mass - The quantity of matter possessed by body is it mass. It is measure of Inertia of the body.

v) Force - Force is an external agent which tends to change the speed or direction of a system.

vi) weight - It is a force which the system exerts due to gravitational acceleration.

System of forces

- Coplanar forces
  - Collinear (LOA along same line)
  - Concurrent (LOA passes through common point)
  - Parallel (LOA are parallel)
    - Like
    - Unlike
  - Non concurrent Non-parallel
- Non-coplanar
  - Concurrent
  - Parallel
  - Non concurrent Non Parallel

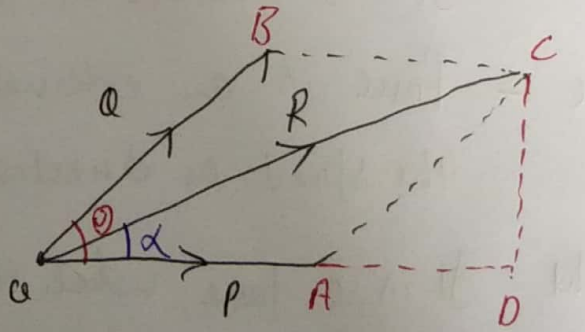
Laws of mechanics :

1) Newtons laws of Motion
   a) Newtons ~~I law~~ First Law / Law of Inertia
   b) Newtons Second Law $F \propto \frac{d\vec{p}}{dt}$
   c) Newtons Third Law

2) Newtons Law of gravitation - $F = \frac{G m_1 m_2}{r^2}$ $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

3) Parallelogram Law of forces - If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal passing their point of intersection represented the resultant both in magnitude and direction.

$$\vec{R} = \vec{P} + \vec{Q}$$



$$R = OC = \sqrt{OD^2 + CD^2} = \sqrt{(OA + AD)^2 + CD^2}$$

$$= \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\tan\alpha = \frac{CD}{AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\boxed{\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}}$$

Special Cases - when $\theta = 0$

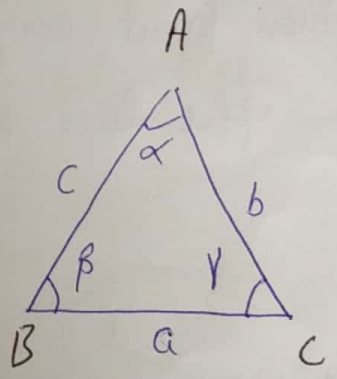
$\theta = 90^\circ$

$\theta = 180^\circ$

4) Sine Law And Cosine Law

Sine Law

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



Cosine Law

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

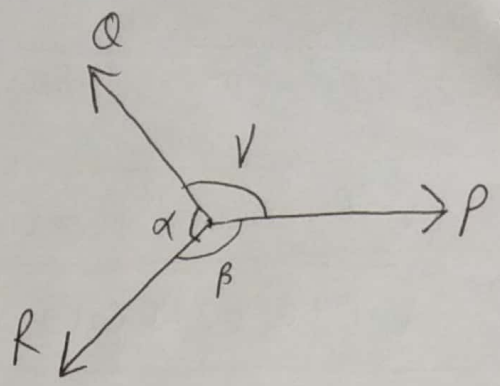
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

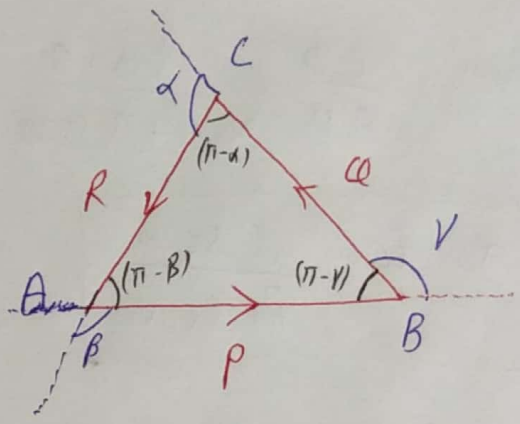
5) Lami's Theorem - If three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

Applying sine rule

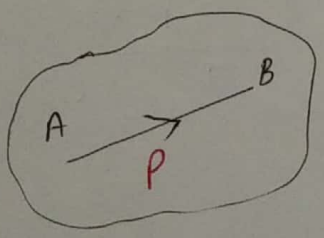
$$\frac{AB}{\sin(\pi-\alpha)} = \frac{BC}{\sin(\pi-\beta)} = \frac{CA}{\sin(\pi-\gamma)}$$

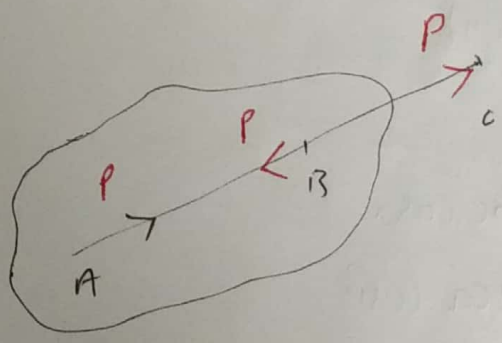
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

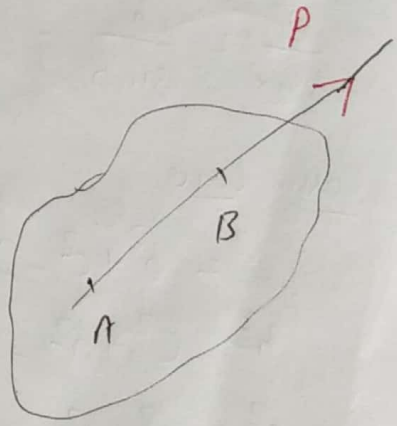




6) Principle of Transmissibility - It states that " the external effect on a rigid body remain unchanged when a force, acting at a given point on the body, is applied to another point lying on the line of action of force.







Equilibrium: when the two or more forces act on a body in such a way that the body remains in a state of rest or of uniform motion, then the system of forces is said to be in equilibrium.

Resultant - when a body is acted upon by a system of forces, then vectorial sum of all the force is known as resultant.

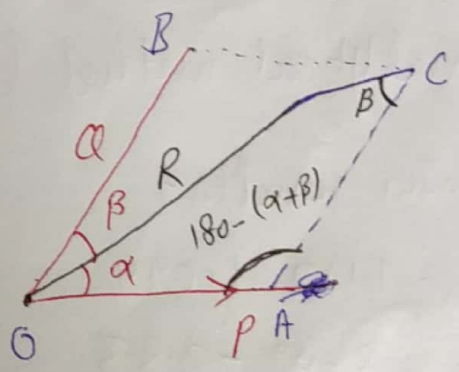
Equilibrant - The single force may apply to the body to bring it in equilibrium state is called equilibrant.

Composition and resolution of forces - The process of determination of resultant is known as composition of forces.

When a single force acting on a particle is replaced by two or more forces in different direction, the process is called resolution of forces.

Q Find the magnitude of two forces such that if they act at right angles, their resultant is $\sqrt{10}$ KN and when they act at an angle of 60°, their resultant is $\sqrt{13}$ KN.

Resolution of forces:



$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin\{180-(\alpha+\beta)\}}$$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

$$\boxed{P = \frac{R\sin\beta}{\sin\alpha+\beta}} \qquad \boxed{Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)}}$$

Special cases

when $\alpha + \beta = 90^\circ$

$\beta = 90 - \alpha$

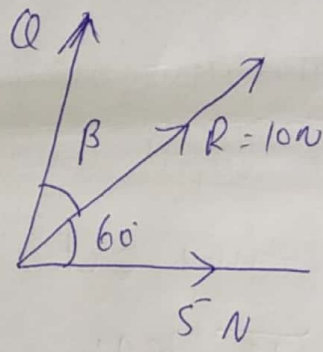
$$P = \frac{R \sin(90-\alpha)}{\sin 90} = R\cos\alpha \Rightarrow \boxed{P = R\cos\alpha}$$

$$Q = \frac{R\sin\alpha}{\sin 90} \Rightarrow \boxed{Q = R\sin\alpha}$$

Q The resultant of two forces is 10N and it is inclined at 60° to one of the forces whose magnitude is 5N. Determine the magnitude and direction of the other force.

Sol $Q = 8.66N$

$\beta = 30^\circ$



Resultant of Coplanar - Concurrent forces.

1) Analytical method (Principle of resolved parts)

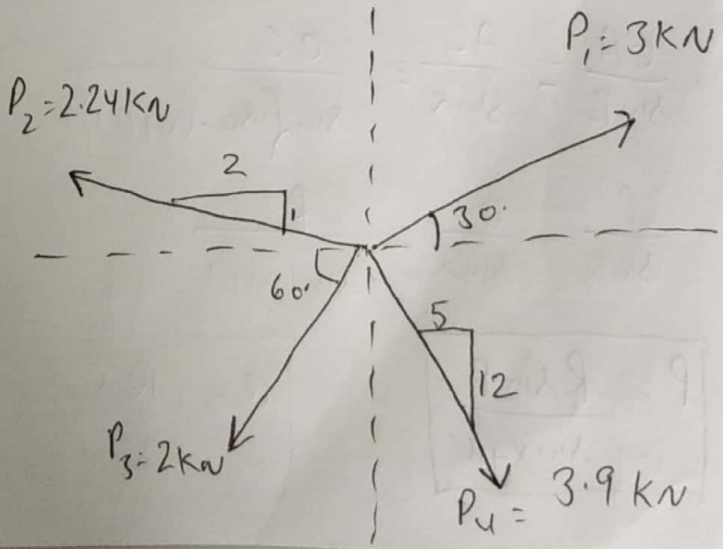
Determine resultant

$\sum F_x = \cancel{1.097}\ 1.094$

$\sum F_y = \cancel{8.007}\ -2.83$

$R = \sqrt{F_x^2 + F_y^2} = 3.0341 kN$

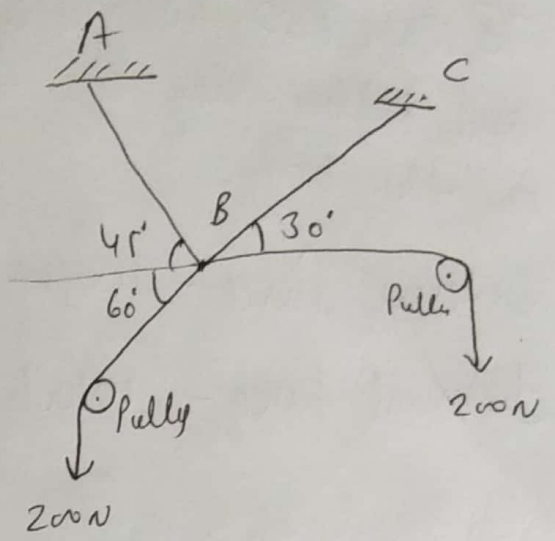
$\alpha = 68.86^\circ$



Q. Calculate the tensile force in cables AB and BC.

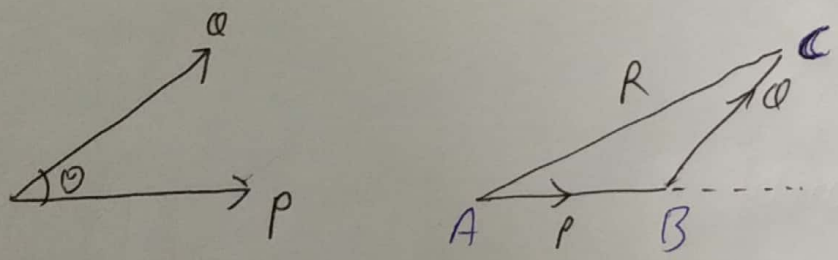
$T_A = 206.21\,N$

$T_C = 53.45\,N$



Graphical Method:

(a) Triangular Law of forces:- "If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, their resultant is represented by the closing side of the triangle taken in opposite order.



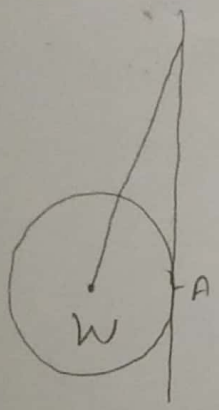
(b) Polygon's Law- If a no. of forces acting on a particle simultaneously are represented in magnitude and direction by the sides of a polygon taken in order then the resultant force can be represented in magnitude & direction by the closing side of the polygon taken in opposite Order.

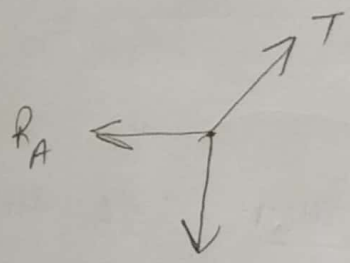
Free body diagram :-

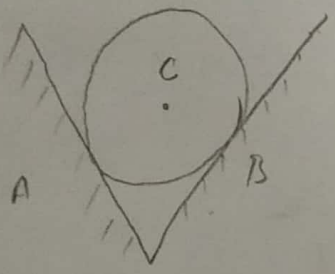
To draw FBD, we have to remove all the supports (like wall, floor) and replace them by the reactions which these supports exert on the body.

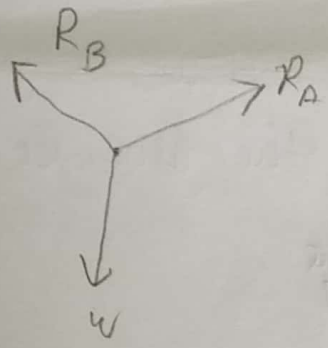
External force - weight, external force etc.

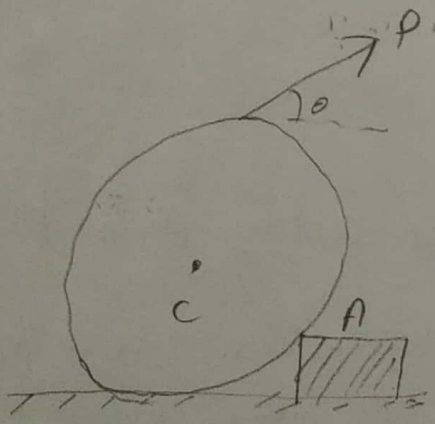
Internal force - which hold particles of a body.

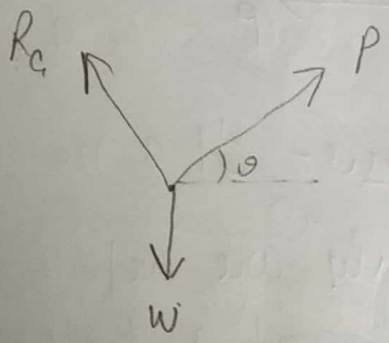




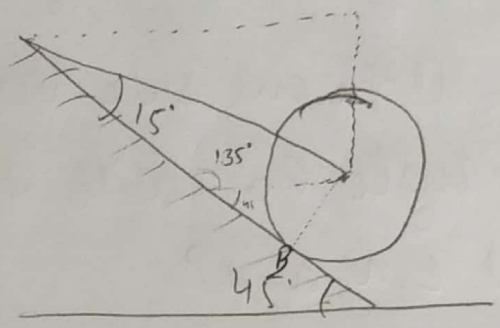




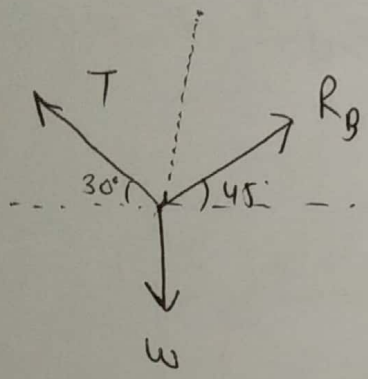




Q. A roller of weight 500 N rests on a smooth inclined plane and is kept free from rolling down by a string as shown in fig. Work out tension in the string and rxn. at the point of contact.



Sol.

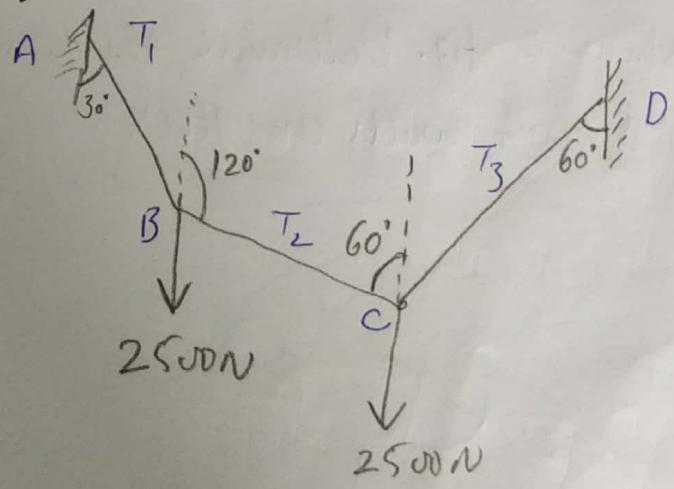


$$R_B = \frac{W \sin 120}{\sin 105} = 448.24\ N$$

$$T = \frac{W \sin 135}{\sin 105} = 365.94\ N$$

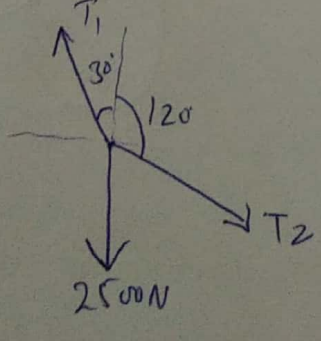
Q. Two equal loads of 2500 N are supported by a flexible string ABCD at points B and C. Find tensions in the portions AB, BC and CD of the string. (AK Tayal 34)

Sol:

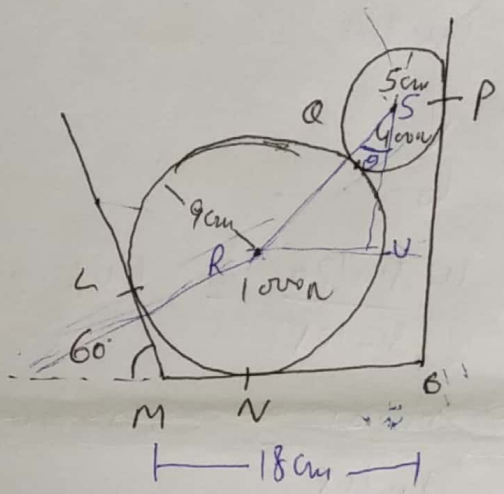


$T_1 = 4330\ N$

$T_2 = T_3 = 2500\ N$



Q Two cylinders A and B rest in a horizontal channel as shown in fig. The cylinder A has a weight of 1000 N and radius of 9.0 cm. The cylinder B has a weight of 400 N and radius of 5.0 cm. The channel is 18 cm wide at the bottom with one side vertical. The other side is inclined at an angle 60° with the horizontal. Find the reactions at point L, N and P.



Sol.

$\theta = 33.86°$

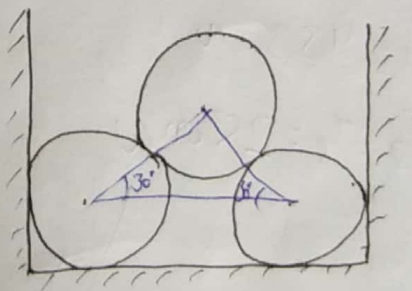
$R_Q = 481.9\ N$

$R_P = 268.5\ N$

$R_L = 310\ N$

$R_N = 1248.2\ N$

Q (2017) Three identical tubes of weights 8 kN each are placed as shown in fig. Determine the forces exerted by the tubes on the smooth walls and floor.



Q A uniform wheel of 60.0 cm diameter & weighing 1000 N rests against a rectangular block 15 cm high lying on a horizontal plane as shown in fig. It is to be pulled over this block by a horizontal force P applied to the end of a string wound round the circumference of the wheel. Find the force P when the wheel is just about to roll over the blocks.

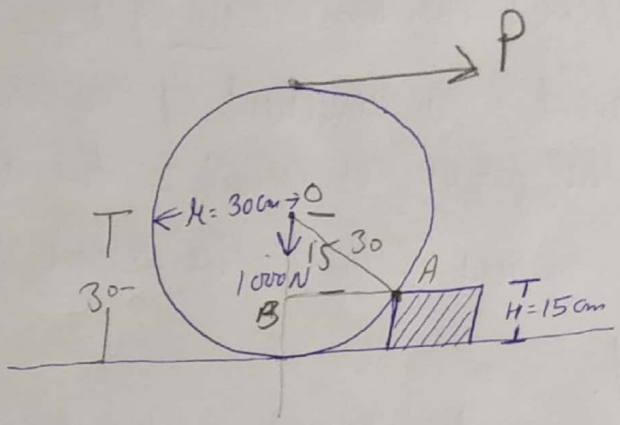
Sol.

$AB = \sqrt{30^2 - 15^2}$

$= \sqrt{}$

$P \times 45 = 1000 \times 25.98$

$P = 577.35 N$

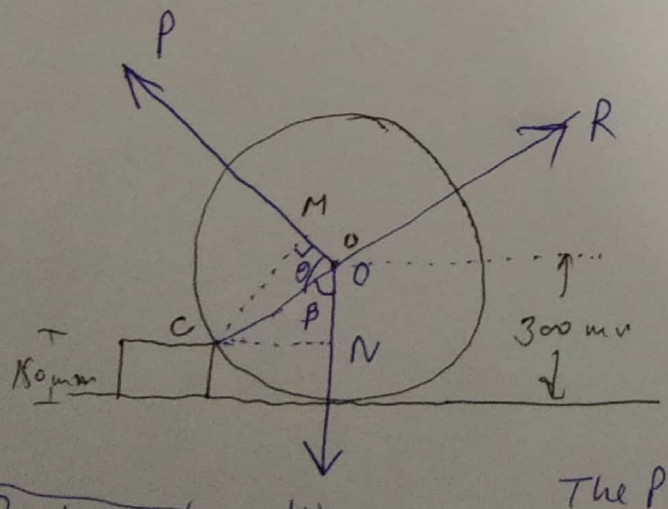


Q A uniform wheel weighing 20 kN and of 600 mm dia rests against 150 mm thick rigid blocks as shown in fig.

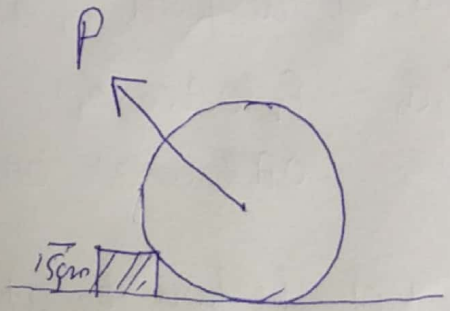
Determine a) least pull through the centre of wheel to just turn the wheel over the corner of the blocks.

b) the rexn. of the blocks.

Sol.



$R = 10 kN$



Taking moment about C

$P \times CM - W \times CN = 0$

$P \times OC \sin\theta - W \times 259.81$

$P = \frac{17.32}{\sin\theta}$

The P will be minimum when $\sin\theta$ is max.

$P_{min} = 17.32 kN$

Moment of a force : Moment of a force about a point is defined as the turning or rotational effect of a force about that point.

It is measured by product of force and the perpendicular distance of the point from line of action of the force.

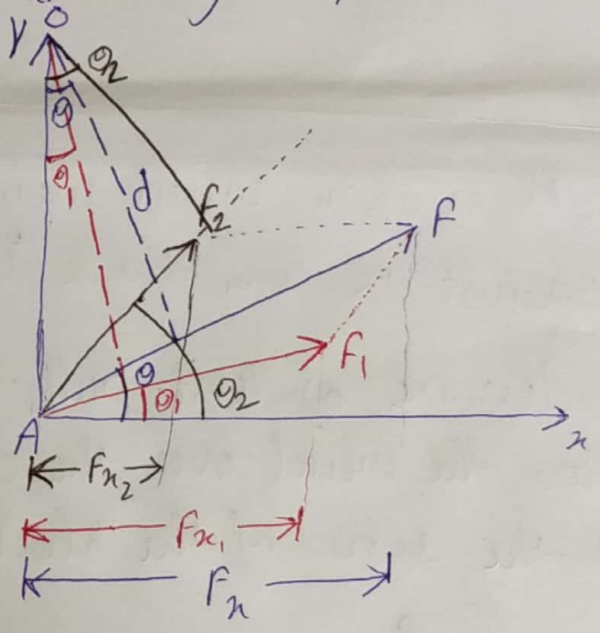
Unit = N-m

Varignon's Theorem : Law of moments

"Moment of a resultant of two forces, about a point lying in the plane of the forces, is equal to the algebraic sum of moments of these two forces about the same point."

Proof:

Consider a force F acting at a point A and having component F_1 and F_2.



Moment of the force F about O,

$$F \cdot d = F\, OA \cos\theta = OA(F\cos\theta)$$
$$= OA\, F_x \quad - (i)$$

Moment of the force F_1 about O,

$$F_1 d_1 = F_1 (OA \cos\theta_1)$$
$$= OA (F_1 \cos\theta_1) = OA\, F_{x_1} - (ii)$$

Moment of the force F_2 about O,

$$F_2 d_2 = F_2 (OA \cos\theta_2) = OA(F_2 \cos\theta_2)$$
$$= OA\, F_{x_2} \quad - \quad (iii)$$

Adding (ii) and (iii)

$$F_1 d_1 + F_2 d_2 = OA(F_{x_1} + F_{x_2})$$

$$= OA(F_x)$$

(∵ The sum of x-components of the forces F_1 & F_2 = x component of the resultant F)

$$\boxed{F_1 d_1 + F_2 d_2 = F d}$$

Equations of equilibrium:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

Q(42) Three cylinders are piled up in a rectangular channel as shown in fig. Determine the reaction between the cylinder A and the vertical wall of the channel.

Sol:

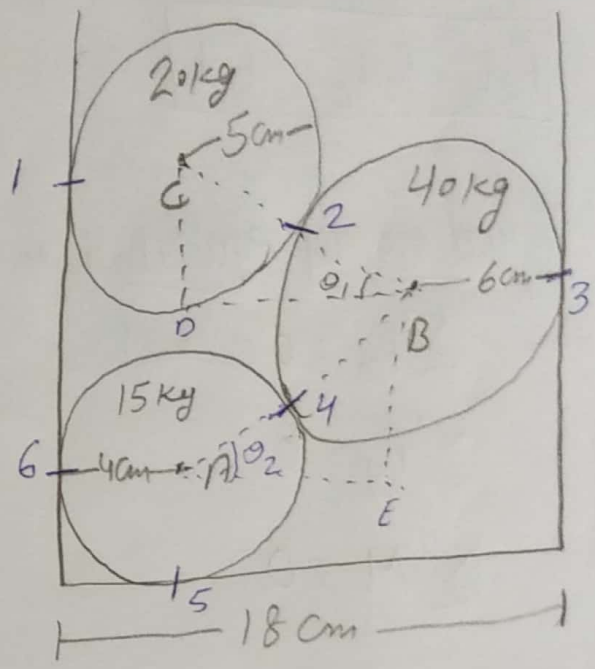
$$\cos\theta_1 = \frac{BD}{BC} = \frac{18-5-6}{6+5} = \frac{7}{11}$$

$\cos\theta_1 = 0.636$, $\theta_1 = 50.5°$.

$\sin\theta_1 = 0.771$

$$\cos\theta_2 = \frac{AE}{AB} = \frac{18-4-6}{4+6} = \frac{8}{10}$$

$\cos\theta_2 = 0.8$, $\theta_2 = 36.87°$, $\sin\theta_2 = 0.6$

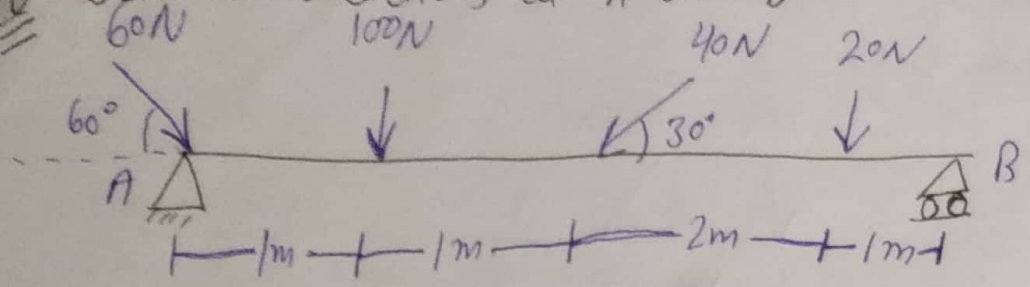


$R_2 = 254.5\,N$

$R_4 = 980.8\,N$

$R_6 = 784.6\,N$

Q. Determine reactions at A and B.



$R_b = 44N$

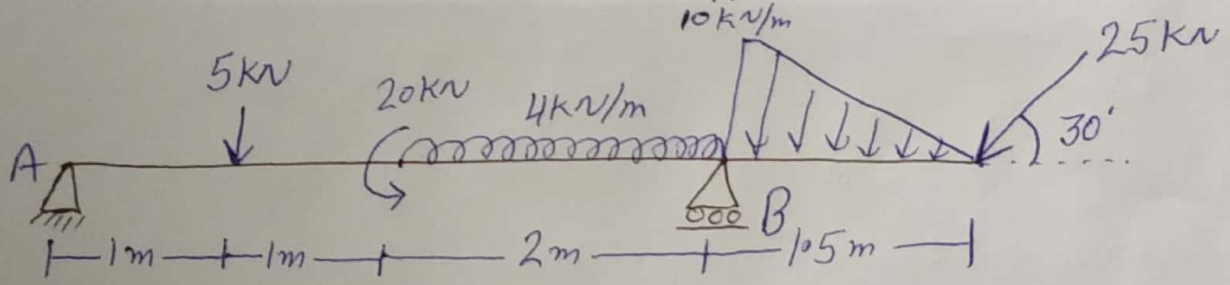
$R_{ah} = 4.64N$

$R_{av} = 147.96N$

$R_a = 148N$

$\theta_A = 88.2°$

Q A beam has been loaded and supported as shown in fig Determine the reaction at the supports point A and B.



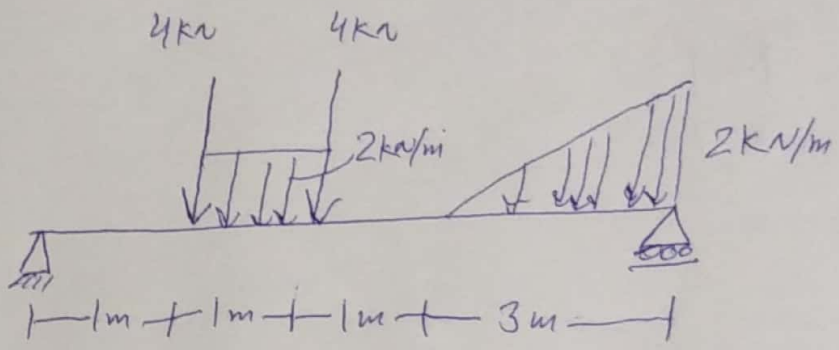
Sol.

$R_{ah} = 21.65\ KN$

$R_{av} = 5.125\ KN$

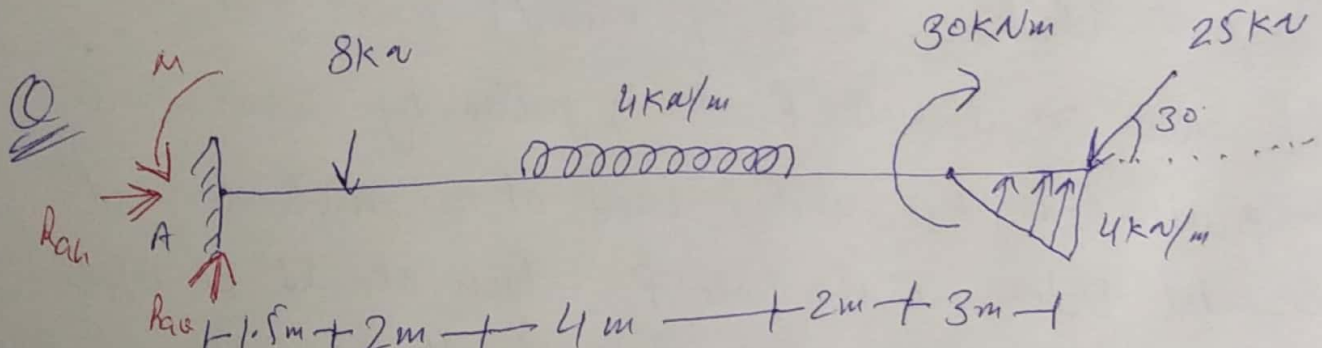
$R_b = 27.875N$

© Determine the reaction R_A and R_B for the beam supported and loaded as shown in fig



$R_A = 8$ kN

$R_B = 5$ kN

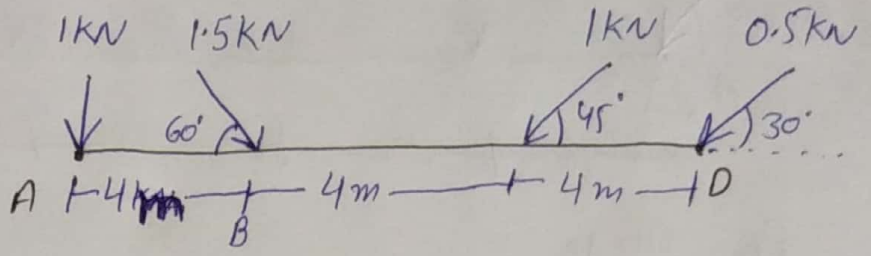


$R_{ah} = 21.65$ kN

$R_{av} = 30.5$ kN

$M = 217.25$ kN m

Q A horizontal beam AD of length 12m is acted upon by a set of forces as shown in fig



Sol.

$\sum f_x = -0.39\ kN$

$\sum f_y = -3.256\ kN$

$R = 3.279\ kN$

$\alpha = 83.17°$

$x_A = 4.25\ m$