* Frequency Response Analysis:

(only for stable system)

[Transfer F. Model]

Revise from Std Book

LTI System.

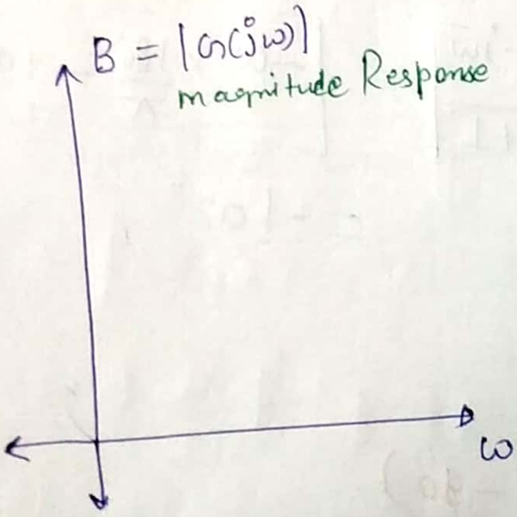
$A\sin[\omega t + \cancel{\phi}]$ → $r(t)$ → [$G(s) \rightarrow G(j\omega)$; $|G(j\omega)|$ $\angle G(j\omega)$] → $B\sin[\omega t + \phi_2]$ → $c(t)$

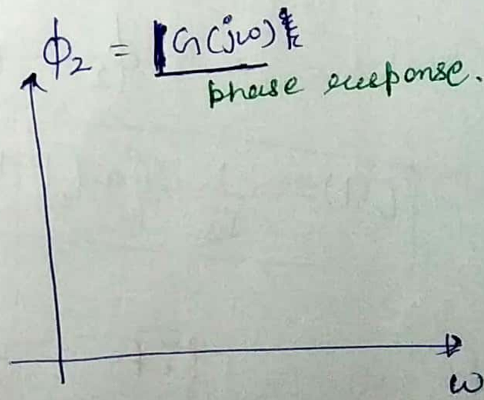
$$c(t) = r(t) * g(t)$$

$$c(t) = F^{-1}[R(j\omega) \cdot G(j\omega)]$$

α, $\int \cdot dt$, dt — Linear

$$\boxed{\begin{aligned} B &= \cancel{A} \cdot |G(j\omega)| \\ \phi_2 &= \cancel{\phi_1} + |G(j\omega)| \end{aligned}}$$





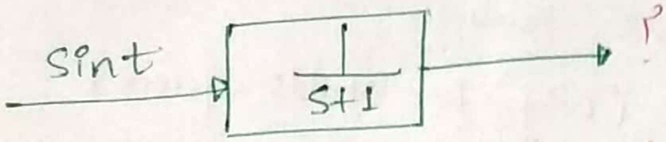
* Magnitude and phase relation between sinusoidal input, output of LTI system as a funct^n of input frequency ω varied from $0 \rightarrow \infty$ such that i/p amplitude & phase kept constt is called frequency response analysis

$$\frac{10}{s+2} \Rightarrow \boxed{\frac{10}{\sqrt{\omega^2+4}}}$$

10 → gain (constant)

$\sqrt{\omega^2+4}$ → magnitude (funct^n of ω)

Q.



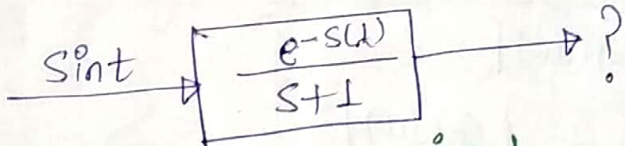
$A = 1 ; \quad \phi_1 = 0^\circ , \quad \omega = 1 .$

$$B = 1 \times \left| \frac{1}{j\omega + 1} \right|_{\omega = 1} = \frac{1}{\sqrt{2}}$$

$$\phi_2 = 0^\circ + \angle \frac{1}{j\omega + 1} = -45^\circ$$

$$\boxed{C(t) = \frac{1}{\sqrt{2}} \sin[t - 45^\circ]}$$

Q.



$$B = 1 \times \left| \frac{e^{-j\omega}}{1 + j\omega} \right|_{\omega = 1} = \frac{1}{\sqrt{2}}$$

$$\phi_2 = 0^\circ + \angle \frac{e^{-j\omega}}{j\omega + 1} = \left[-\omega \times \frac{180^\circ}{\pi} - \tan^{-1}(\omega) \right]$$

$$= -102^\circ$$

$$\boxed{C(t) = \frac{1}{\sqrt{2}} \cdot \sin(t - 102^\circ)}$$

Q.

$$T.F = \frac{S}{S+P} =$$

$x(t) = P\cos(2t - 90^\circ)$

$y(t) = 1 . \cos(2t - 60^\circ)$

$$1 = P . \left| \frac{j\omega}{j\omega + P} \right|_{\omega = 2} \qquad \sqrt{4 + P^2} = 2P$$

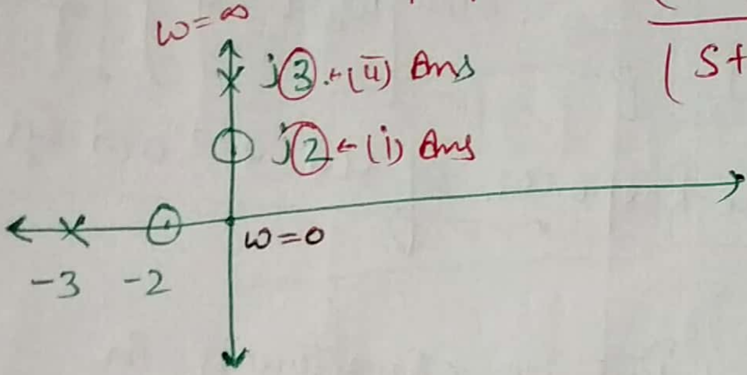
$$P = \pm 2/\sqrt{3} \Rightarrow P = +\frac{2}{\sqrt{3}}$$

$$-60^\circ = -90^\circ + \angle \frac{j\omega}{j\omega + P}$$

$$-60 = -90^\circ - \tan^{-1}\left(\frac{2}{P}\right) \Rightarrow P = \frac{2}{\sqrt{3}}$$

Q find the ω at which steady state sinusoidal response of system become 0.
find the ω at which stead. s. sinusoidal response become ∞.

$$T.F. = \frac{(s+2)(s^2+4)}{(s+3)(s^2+9)}$$



* F.R.A of Standard 2nd Order System.

T.D $\quad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\rho\omega_n s+\omega_n^2}$

F.D $\quad \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{-\omega^2+j2\rho\omega\omega_n+\omega_n^2}$

Fourier T.F
Sinusoidal system

ω → i/p freqncy

ω_n → system natural freq.

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\rho\left(\frac{\omega}{\omega_n}\right)}$$

Let $\frac{\omega}{\omega_n} = u$;
Normalized freq.

$r(t)$ = $\sin\omega t$ → $\boxed{\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1-u^2+j2\rho u}}$ → $c(t)$ = $M\sin[\omega t+\phi]$

$u \propto \omega$
$0 \to \infty$; $u \propto \omega: 0 \to \infty$

Standard 2nd order CLT.F

$$M = \frac{1}{\sqrt{(1-u^2)^2+(2\rho u)^2}}, \quad \phi = -\tan^{-1}\left[\frac{2\rho u}{1-u^2}\right]$$

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$$C(t) = \frac{1}{\sqrt{(1-u)^2 + (2\rho u)^2}} \cdot \sin\left[\omega t - \tan^{-1}\left(\frac{2\rho u}{1-u^2}\right)\right]$$

Sinusoidal or frequency Response of std 2^{nd} ord system.

$$C(t) = \left\{1 - \frac{e^{-\rho\omega_n t}}{\sqrt{1-\rho^2}} \cdot \sin\left[\omega_d t + \phi\right]\right\} \cdot u(t)$$

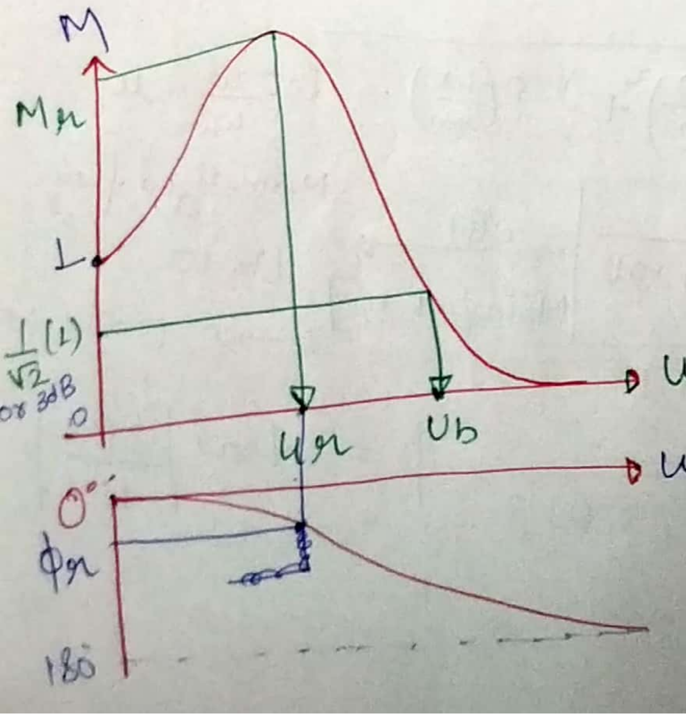
Step or time Response of std 2^{nd} ord. sys.

* Damping Conditions and Response characteristics in frequency Domain are obtained are from resonance charecterstics which are nothing but transience charact. of system.

$u = 0 \Rightarrow M = 1 \ ; \ \phi = 0^\circ$

$u = \infty \Rightarrow M = 0 \ ; \ \phi = 180^\circ$

$\omega_n, \rho \rightarrow$ system parameter



Frequency Response Parameter

1). u_r (Resonace freq.)
2). M_r (Resonance Peak)
3). ϕ_r (Resonance phase)
4). u_b (Bandwidth or 3dB BW)
5). GM
6). PM

Resonance freq

1. $U_r = \sqrt{1-2\xi^2}$

$\therefore$ $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}} = 0$

$\xi_{cri} = \frac{1}{\sqrt{2}} = 0.707$

2. $\boxed{M_r = \frac{A \times B}{2\xi\sqrt{1-\xi^2}}}$ — i/p Amp (A), correct^n factor (B)

ξ	T.D	F.D
Un-dmp.	$=0$	0
under.	<1	$<\frac{1}{\sqrt{2}}$
cri	$=1$	$=\frac{1}{\sqrt{2}}$
over	>1	$>\frac{1}{\sqrt{2}}$

$M_r \propto \frac{1}{\text{Stability}}$

good Range of ξ

$0.3 < \xi < 0.7$

$0 < \xi < \frac{1}{\sqrt{2}} \Rightarrow \infty > M_r > 1$

3. $\boxed{\phi_r = -\tan^{-1}\left[\frac{\sqrt{1-2\xi^2}}{\xi}\right]}$

4. $\boxed{\omega_b = \omega_n\sqrt{4\xi^4 - 4\xi^2 + 2 + (1-2\xi^2)}}$

$\Downarrow$

$\boxed{\omega_b \simeq \omega_n[1.85 - 1.2\xi]}$

$\omega_b \propto$ Speed, $\omega_b \propto$ Noise

There is no time parameter to tell about Noise.

5. $G_m \propto$ Stability.

6. $P_m \propto$ Stability.

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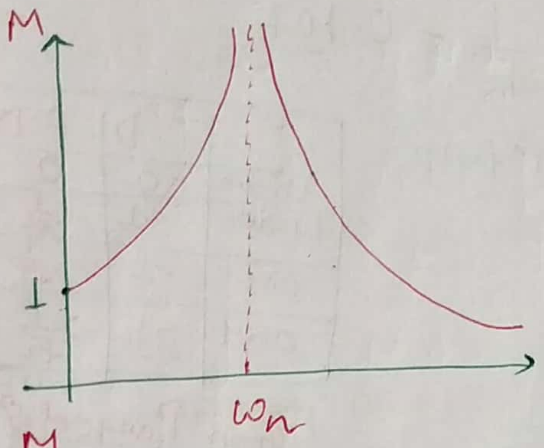
$$\boxed{t_s \propto \frac{1}{\text{Stability}}}$$

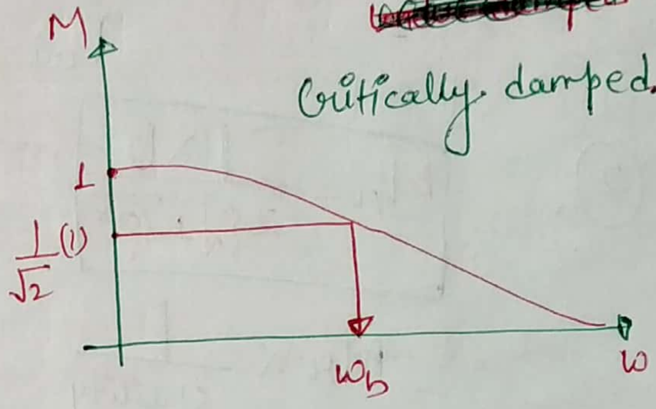
$\zeta = 0$ undamped

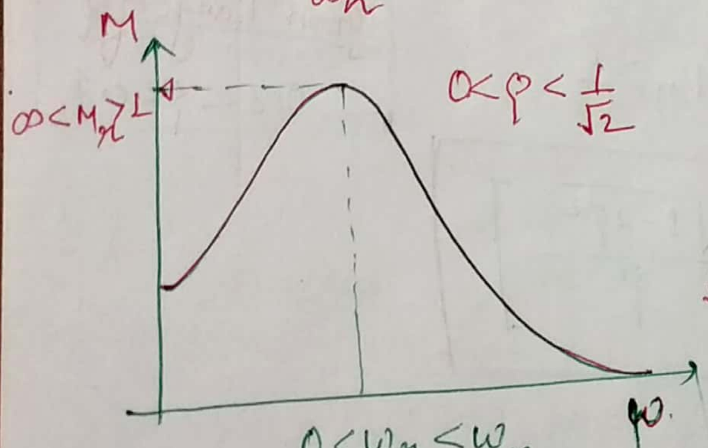


$\zeta = \frac{1}{\sqrt{2}}$

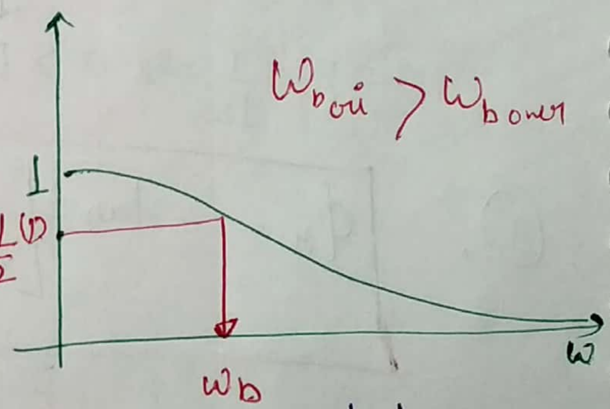
~~under damped~~

Critically damped.





$0 < \omega_r < \omega_n$



over-damped

under-damped

* Classification of System:

* Based on the location of finite zeros & poles of open loop system, a system can be classified as follow:

(I). Minimum Phase System: in which all zeros and poles are on left side of S-plane.

(II) NON-minimum Phase system / Maximum Phase System: The system in which atleast a zero or a pole is in right side of S-plane,

(III) All pass system: The system in which poles and zeros are located such that they are mirror with each other wrt to $j\omega$ axis.

* All pass system is also non-minimum phase system.

$$\frac{S-1}{(S+2).(S+3)} = \frac{S+1}{(S+2).(S+3)} \times \frac{S-1}{S+1}$$

$$\underset{NmpS}{G.H(S)} = \underset{M.P.S}{G\,H(S)} \times \underset{A.P.S}{G.H(S)}$$

$$|G.H(j\omega)| = |GH(j\omega)| \times 1$$

$$\angle GH(j\omega)_{Nmps} = \angle GH(j\omega)_{mps} + \angle GH(j\omega)_{APS}$$

* Non-Minimum phase system produces huge phase angle compare to minimum phase system due to which non-minimum phase system is less stable.

Hence C. System in practice is preferred to be Minimum phase system.

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Frequency Domain Stability Analysis

* ## Polar plot:
(polar coordinates)

$G.H(s) \Rightarrow G.H(j\omega) = x + jy = Me^{j\phi}$

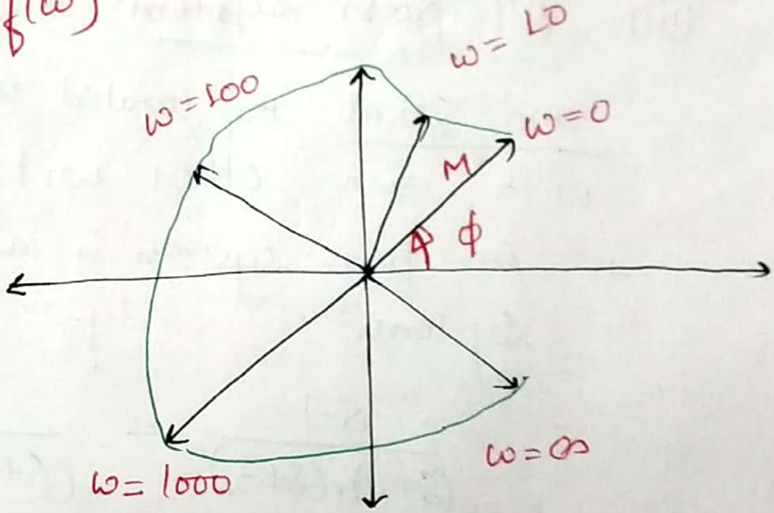
OLTF

$$M = |G.H(j\omega)|$$
$$\phi = \angle G.H(j\omega)$$

$\omega: 0 \longrightarrow \infty$

$M, \phi = f(\omega)$

ω	m	ϕ
0	–	–
:		
∞	–	–



$q(s) = 1 + G.H(s) = (0,0)$

$G.H(s) = (-1,0)$

$G.H(s) = -1$

$q(s) = 1 - G.H(s) = (0,0)$

* polar plot is the locus traced by the tip of the phasor in polar coordinate system for the values of input frequency ω varied from $0 \rightarrow \infty$ such that length of phasor is $|GH(j\omega)|$ & phase of phasor is $\underline{|G \cdot H(j\omega)}$.

* polar plot is obtained using open loop transfer function and it is open loop locus unlike Root locus which is CRL.
hence to determine -ive feedback closed loop system stability $(-1,0)$ is considered to be reference point and to determine +ive feedback stability $(1,0)$ is considered to be reference point

* Using polar plot alone stability of a system can be analysed ~~for~~ but using polar ~~polar~~ plot along with M-N circle.
(tan φ) it is possible to analyse both stablity & phase response in frequency domain.

* Though stability can be analysed using polar plot, no. of roots causing unstability can't be determined. To overcome this limitation polar ~~plot~~ is upgraded in Nyquist plot.

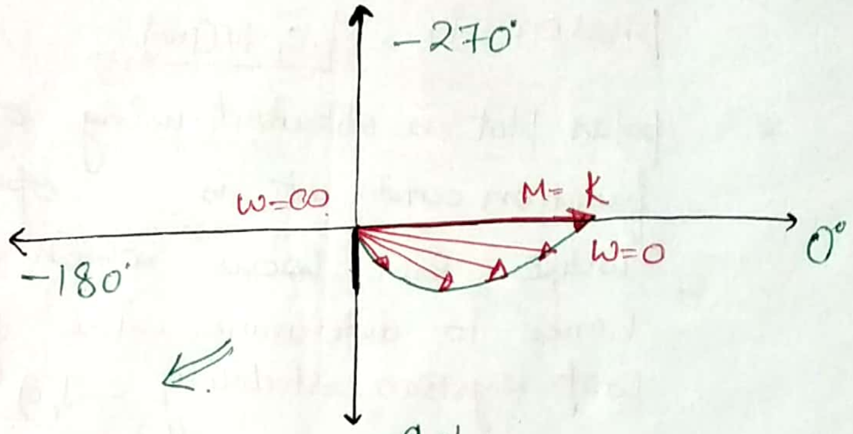
Q $G \cdot H(S) = \dfrac{K}{1+ST}$

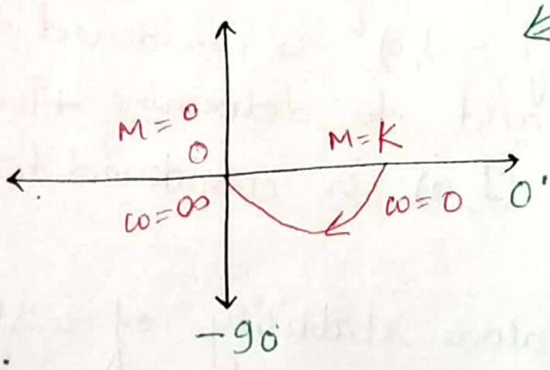
Step 1: $G\,H(S) \Rightarrow G \cdot H(j\omega)$, $M = \dfrac{K}{\sqrt{1+\omega^2 T^2}}$

$\dfrac{K}{1+j\omega T}$

$\phi = -\tan^{-1}(\omega T)$

ω	M	ϕ
0	K	0°
⋮	⋮	⋮
∞	0	-90°





$\omega : 0 \longrightarrow \infty$

$\omega : \infty \longrightarrow 0$

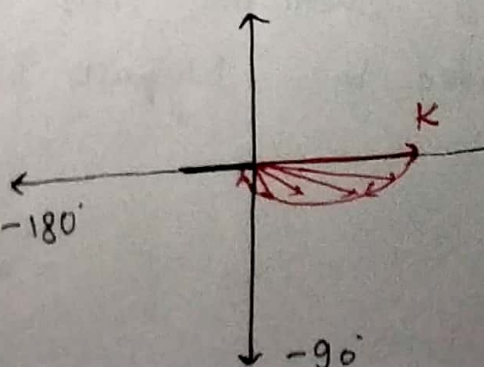
→ Direct^n will be opposite for both plot to each other

Q $G \cdot H(S) = \dfrac{K}{(1+ST_1)(1+ST_2)}$

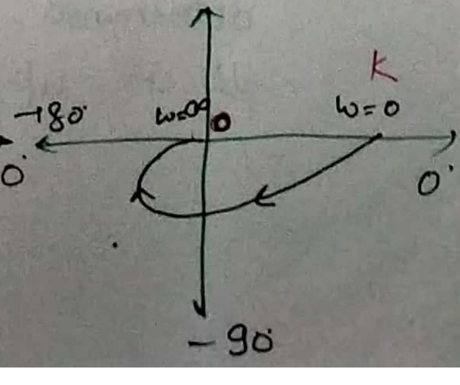
$G \cdot H(j\omega) = \dfrac{K}{(1+T_1 j\omega)(1+T_2 j\omega)}$, $M = \dfrac{K}{\sqrt{1+\omega^2 T_1^2} \cdot \sqrt{1+\omega^2 T_2^2}}$

$\phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$

ω	m	ϕ
0	K	0°
⋮	⋮	⋮
∞	0	-180°



⇒

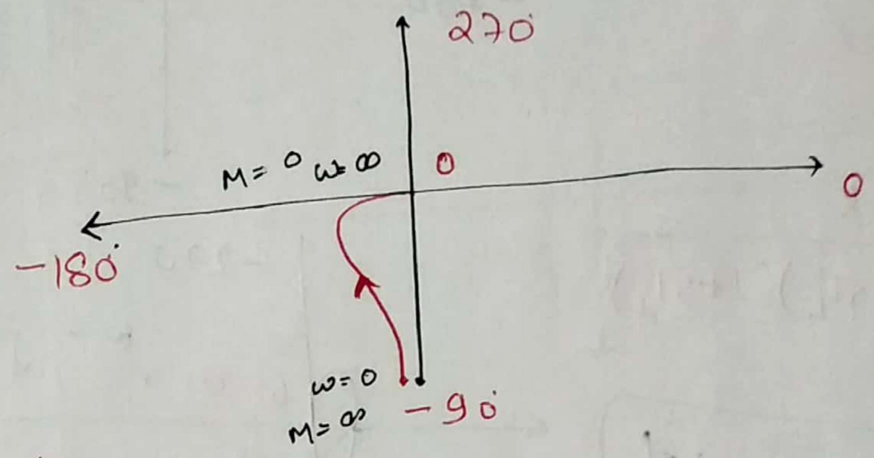


Q :

$$G \cdot H(s) = \frac{k}{s(1+sT)} \Rightarrow \frac{k}{(j\omega)(1+Tj\omega)}$$

$$M = \frac{k}{\omega\sqrt{1+\omega^2T^2}}, \quad \phi = -90^\circ - \tan^{-1}\omega T$$

ω	M	ϕ
0	∞	-90°
∞	0	-180°



$\boxed{\text{Imp}}$

$\omega : 0 \longrightarrow \infty$

$M : k \longrightarrow 0$ type = 0

[Only for strictly Proper system] $: \infty \longrightarrow 0$; type $\neq 0$

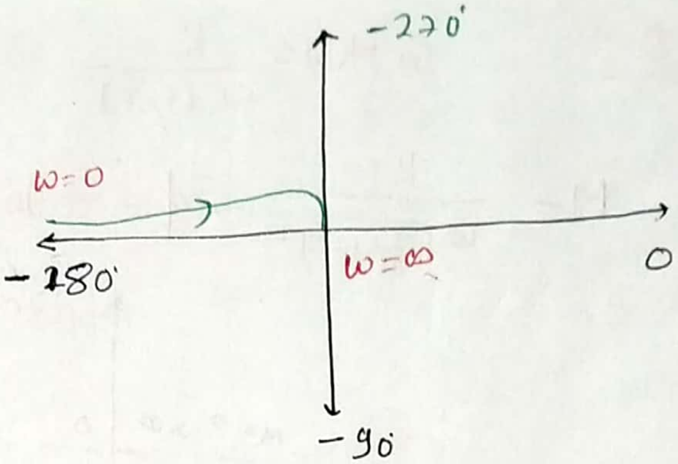
$\phi : -90^\circ \times \text{type} \longrightarrow -90^\circ \times (P-Z)$

or

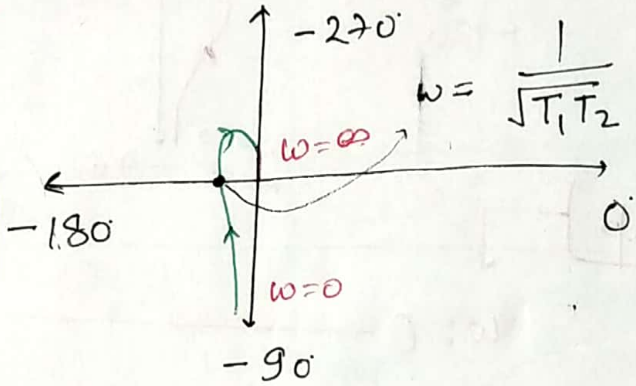
$+90^\circ \times$ (No. of zeros at origin)

[Only for Minimum phase system]

Q. $\dfrac{K}{s^2(1+sT)}$

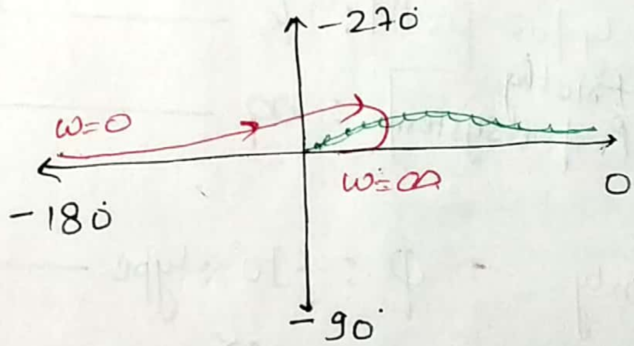


Q. $\dfrac{K}{s(1+sT_1)(1+sT_2)}$

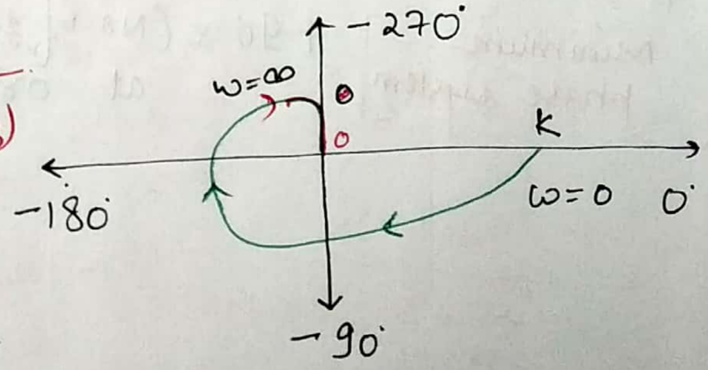


$$\boxed{\text{Gain } M = \left(\frac{K\,T_1T_2}{T_1+T_2}\right)^{-1}}$$

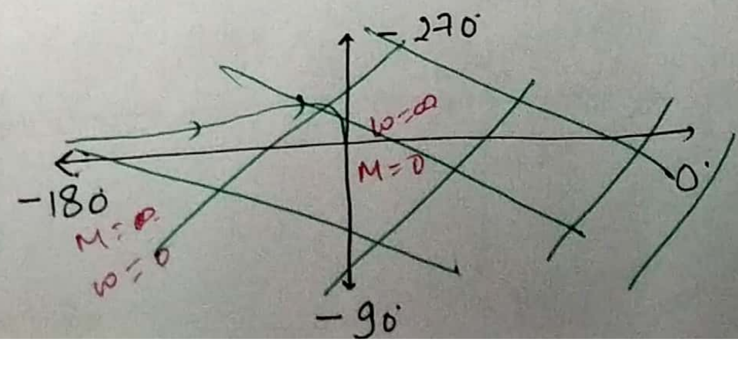
Q. $\dfrac{K}{s^2(1+sT_1)(1+sT_2)}$

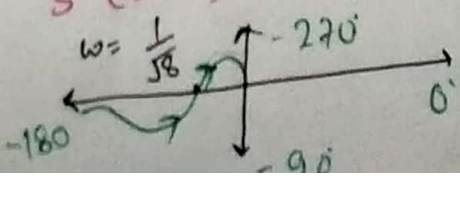


Q. $\dfrac{K}{(1+sT_1)(1+sT_2)(1+sT_3)}$

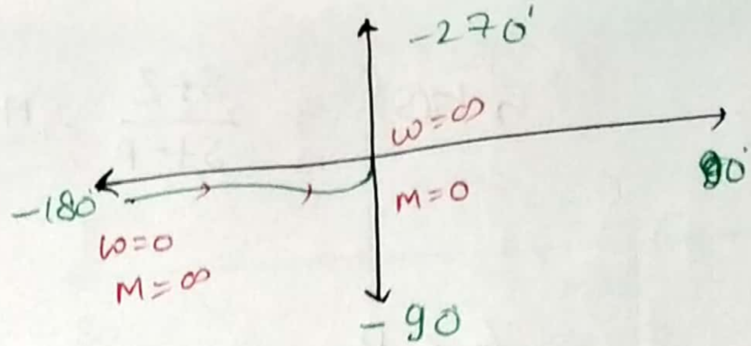


Q. $\dfrac{K(4s+1)}{s^2(s+1)(2s+1)}$ Dominant

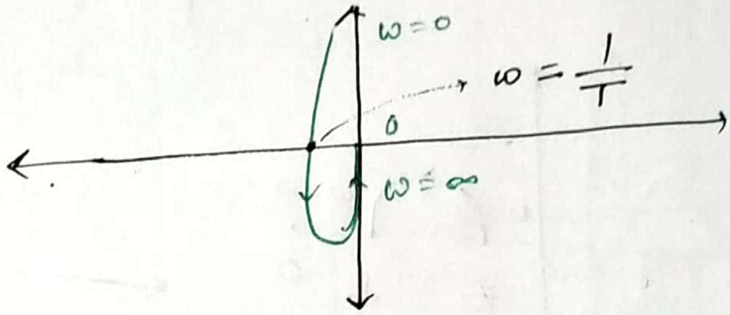




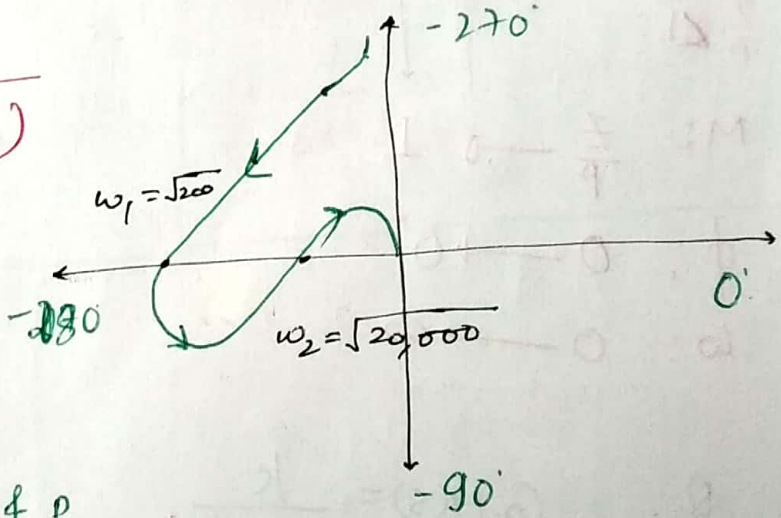
Q $\dfrac{K(1+sT)}{s^2}$



Q $\dfrac{K(1+sT)^2}{s^3}$



Q. $\dfrac{K(s+10)(s+20)}{s^3(s+100)(s+200)}$



$\omega_1 \to$ is GM of Zeros

$\omega_2 \to$ is GM of poles.

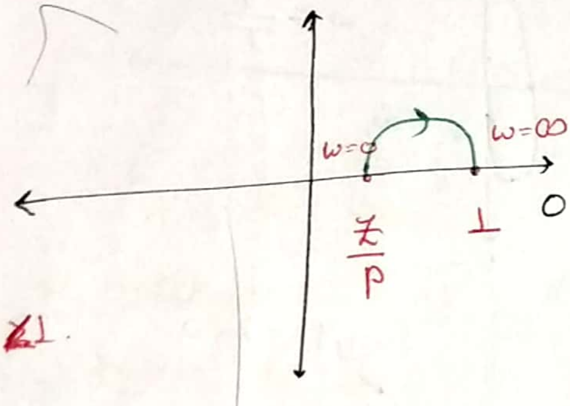
Phase time f-ine by Z & P
Only valid for Minimum phase system

$$\frac{K(4j\omega+1)}{(j\omega)^2(j\omega+1)(2j\omega+1)} = \frac{K(4j\omega+1)}{-\omega^2(-2\omega^2+j\omega+2j\omega+1)}$$

$$\frac{K(4j\omega+1)}{-\omega^2(-2\omega^2+j\omega+2j\omega+1)} = \frac{K(4j\omega+1)}{+2\omega^4+j} \qquad \omega = \frac{1}{2\sqrt{2}}$$

Q. $G\cdot H(s) = \frac{s+z}{s+p}$; $M = \sqrt{\frac{\omega^2+z^2}{\omega^2+p^2}}$

$z < p$



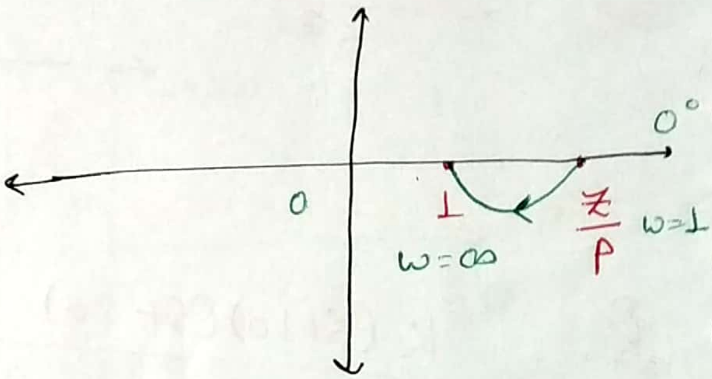
$\frac{z}{p} < 1$

M: $\frac{z}{p} \longrightarrow 1$

ϕ: $0 \longrightarrow 0^\circ$

ω: $0 \longrightarrow \infty$

$p > z$



M: $\frac{z}{p} \longrightarrow 1$; $\frac{z}{p} > 1$

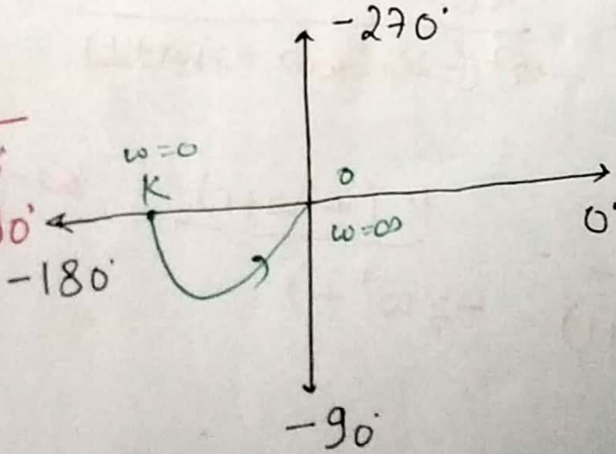
ϕ: $0^\circ \longrightarrow 0^\circ$

ω: $0 \longrightarrow \infty$

Q. $G\cdot H(s) = \frac{k}{s-1}$

M: $K \longrightarrow 0$

ω	ϕ
0	-180°
∞	-90°

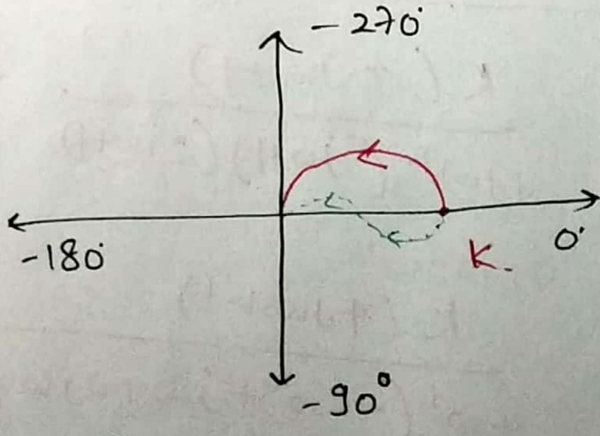


$\phi = 0 - \tan^{-1}\left(\frac{-\omega}{1}\right)$

$= -\left[-\tan(\omega) + 180^\circ\right]$

$G\cdot H(s) = \frac{k}{1-s}$

M: $K \longrightarrow 0$



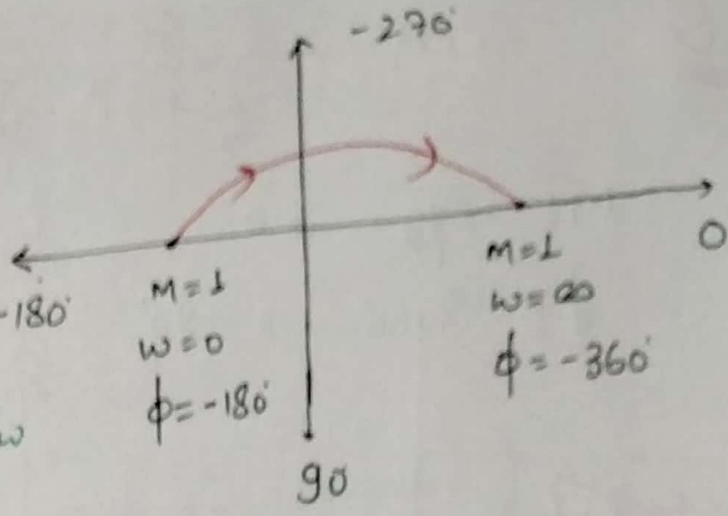
$\phi = \angle k - \angle(1-j\omega)$

$= -\left[-\tan^{-1}(\omega)\right]$

$\angle k = 0^\circ$

$\angle -k = 180^\circ$

Q. $GH(s) = \frac{S-1}{S+1}$



$\phi = \angle j\omega - 1 - \angle j\omega + 1$

$\phi = \left[\tan^{-1}\left(\frac{\omega}{-1}\right)\right] - \tan^{-1}\omega \quad -180°$

$\phi = \left[-\tan^{-1}\omega + 180°\right] - \tan^{-1}\omega$

$\phi = 180° - 2\tan^{-1}(\omega).$

$M = 1 \ \forall \omega.$

$\phi \Rightarrow 180° \longrightarrow 0°$

or

$\phi = -180° \longrightarrow -360°$

Polar plot of All pass system is circle at origin at (0, 0).

Q. $G \cdot H(S) = \frac{S+2}{(S+1)(S-1)}$

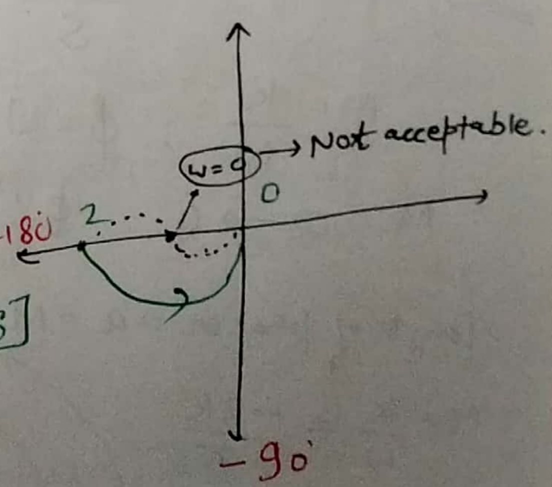
$M: 2 \longrightarrow 0$

$\phi = \angle j\omega + 2 - \angle j\omega + 1 - \angle j\omega - 1.$

$\phi = \tan^{-1}\frac{\omega}{2} - \tan^{-1}\omega - \left[\tan^{-1}(-\omega)\right]$

$\phi = \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\omega - \left[-\tan^{-1}\omega + 180°\right]$

$\phi = -180° - 2\tan^{-1}\omega + \tan^{-1}\left(\frac{\omega}{2}\right).$

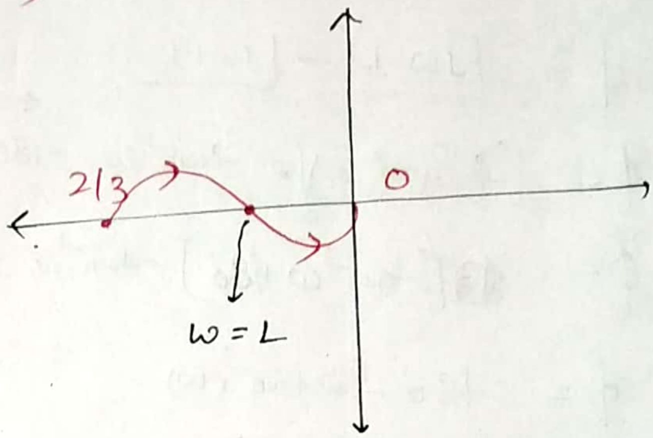


ω	ϕ
0	-180°
⋮	⋮
∞	-90°

Q. $G.H(S) = \dfrac{S+2}{(S+1)(S-3)}$

M: $-2/3 \longrightarrow 0$

ϕ: $-180^\circ \longrightarrow -90^\circ$



Check (ω) for 270°,

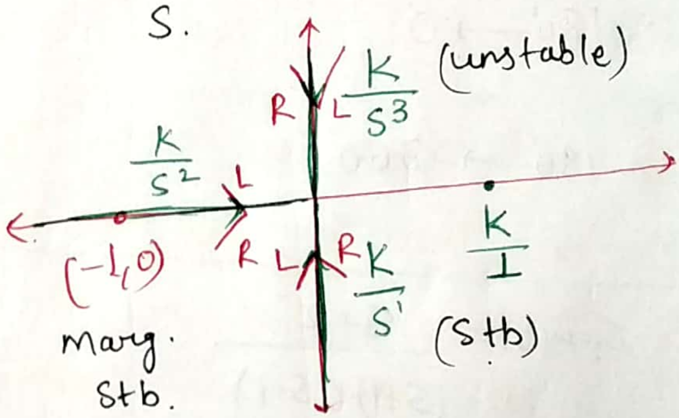
ω = 1

Type > 3
Not possible

Q. $G.H(S) = \dfrac{K}{S}$

M: $\infty \longrightarrow 0$

ϕ: $-90^\circ \longrightarrow -90^\circ$



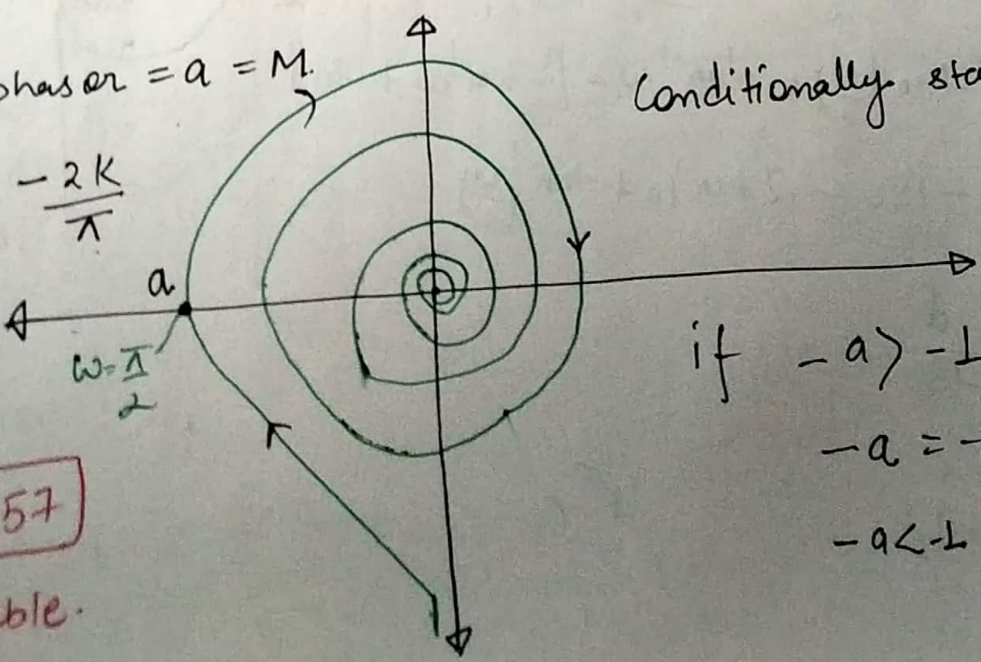
Q. $G.H(S) = \dfrac{K.e^{-S}}{S}$

$M = \dfrac{K}{\omega}$; $\phi = -90^\circ - \omega \times \dfrac{180^\circ}{\pi}$

M: $\infty \longrightarrow 0$ ϕ: $-90^\circ \longrightarrow -\infty$

length of phasor = a = M.

$M = \dfrac{K}{\omega} = \dfrac{-2K}{\pi}$



Conditionally stable.

$\dfrac{-2K}{\pi} > -1$

$\boxed{0 < K < 1.57}$

Stable.

if $-a > -1$; stb

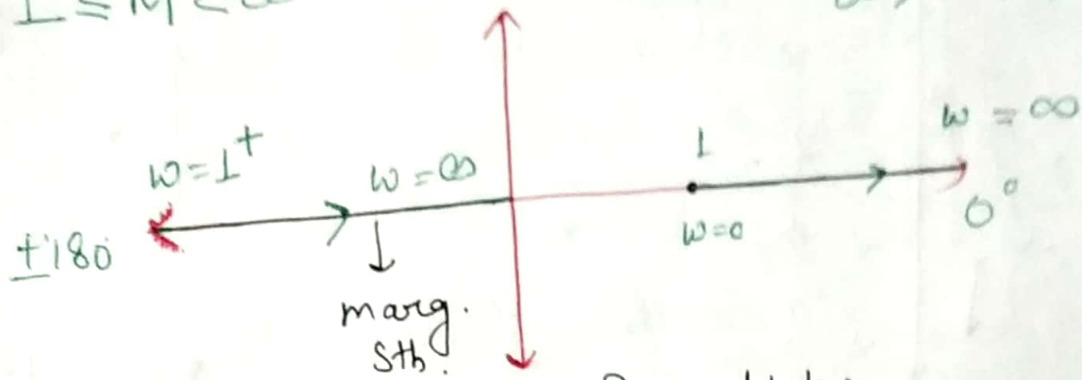
$-a = -1$ marg. stb

$-a < -1$; unstb.

Q. $G \cdot H(s) = \frac{1}{s^2+1} \Rightarrow \frac{1}{1-\omega^2}$

$0 \le \omega < 1$ $\quad\quad$ $1 < \omega < \infty$

$1 \le M < \infty$ $\quad\quad$ $\infty > M > 0$



* Stability Analysis using Polar plot:

① If (-1,0) lies to the left of polar plot then the system is stable.

② If (-1,0) lies to the right of polar plot then system is unstable.

③ If (-1,0) lies on the polar plot then system is marginally stable.

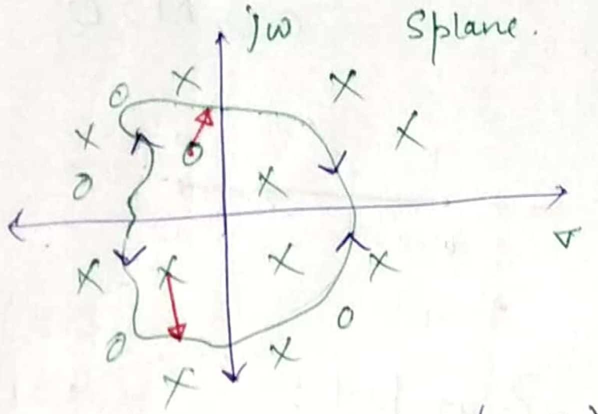
④ If the polar plot intersects -180° phase line then the system is conditionally stable.

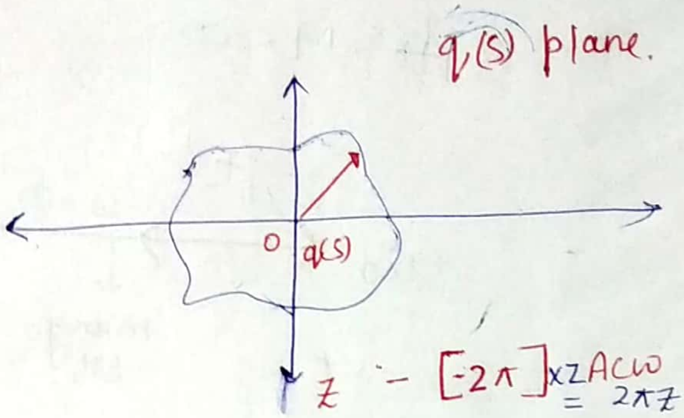
(for Non-Minimum phase system always check for ω, dominant Z, P concept is not applicable.).

* Argument principle

(phase angle) — Cauchy.

$q(s = \sigma + j\omega) = u + jv$





$Z - [-2\pi] \times Z\ ACW = 2\pi Z$

$P + [-2\pi] \times P\ CW = -2\pi P$

$q(s) = \dfrac{(s+Z_1)\ldots}{(s+P_1)\ldots}$

$q(s) = \angle s+Z_1 - \angle s+P_1$

S-plane contour is in C.W direction.
$N = P - Z$; ACW   or $N = Z - P$; CW

No. of Rotation

S-plane contour is in A.C.W direct^n
$N = Z - P$; ACW   or $N = P - Z$; CW

* If the closed contour in S-plane that doesn't pass through any pole of a function q(S) encircles Z no. of zeros and P no. of poles of the function in clockwise direction then the corresponding q(S) contour encircles the origin (P−Z) no. of time in anticlockwise direction, or (Z−P) no. of times in clockwise direction.

* A point is said to encircled by a closed contour if it is lies inside contour.
Not-encircled if it lies outside.

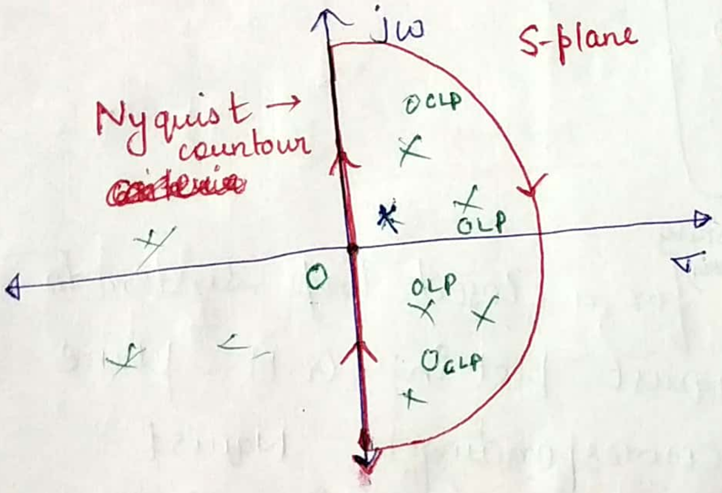
* Encirclements can't be defined wrt point through which contour passes.

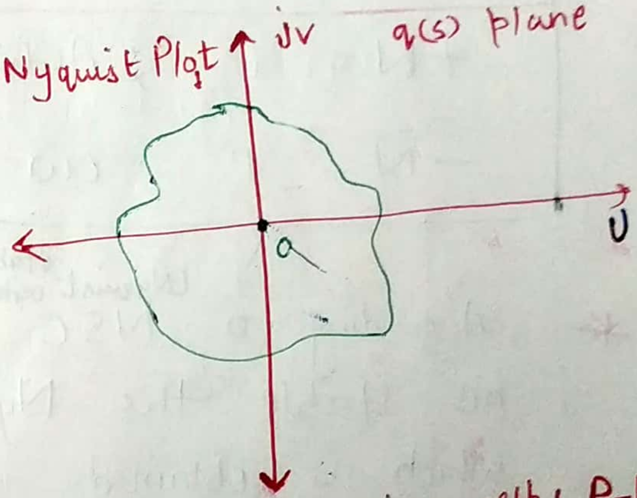
* Nyquist Stability Criteria :

CLP

$$q(s) = \frac{(s+Z_1)}{(s+P_1)}$$

OLP





S-plane contour is in CW direction

$N = P - Z$; ACW

$N = Z - P$; CW

No. of poles causing unstb : P-N

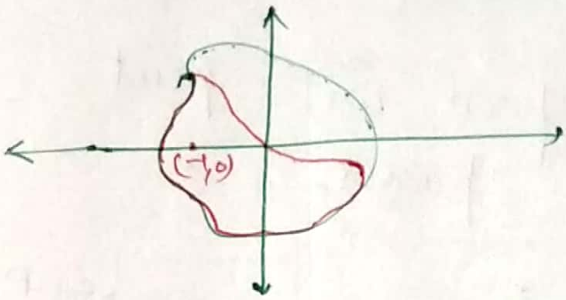
if $N = P$ stable

if $N \neq P$ unstable

if N = undefined ⇒ Marg. Stb

∵ Nyquist ~~phases~~ passes through reference point.

$$G \cdot H(S) = \frac{(S+Z_1) \longrightarrow OLZ}{(S+P_1) \longrightarrow OLP}$$



reference point = (-1, 0)

$G \cdot H(S) = (-1, 0)$

S-plane contour is in CW direction.
$+N = P$; ACW
$-N = P$; CW

* According to NSC (Nyquist stability criteria) for a closed loop system to be stable the Nyquist plot in G.H plane which is obtained corresponding to Nquist contour in S-plane that doesn't pass through any pole of the function G(S).H(S) must encircle the reference point (-1, 0) in anticlockwise direction as many times as the no. of open loop poles in right side of S-plane.

* if the NP (Nyquist Plot) incircles (-1, 0) in clockwise direction then no. of encirclements are considered to be -ive.

NOTE: Above statement is valid wrt clockwise direction of Nyquist contour. In case anticlockwise direction of Nyquist countour, N is -ive in case

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of ACW encirclements & N is +ive for clockwise encirclements.

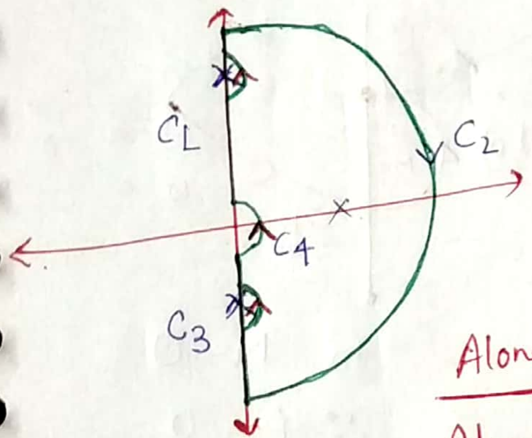
* Nyquist Contour :

$$G.H(s) = \frac{1}{s+2} \; ; \; P=0$$

$$= \frac{1}{(s+2)(s-3)} \; ; \; P=1$$

$$= \frac{1}{s(s+2)(s-3)} \; ; \; P=1.$$

$$= \frac{1}{s^3(s+2)(s-3)(s^2+1)} \; ; \; P=1$$



Along C_1 : $s = j\omega \; ; \; \omega : 0 \to \infty$

Along C_3 : $s = j\omega \; ; \; \omega : -\infty \to 0$

Along C_2 : $s = \lim_{R\to\infty} R.e^{j\theta} \; ; \; \theta, +90^\circ \text{ to } -90^\circ$

Along C_4 : $s = \lim_{r\to 0} r.e^{j\theta} + j.0 \quad \theta : -90 \text{ to } 90^\circ$

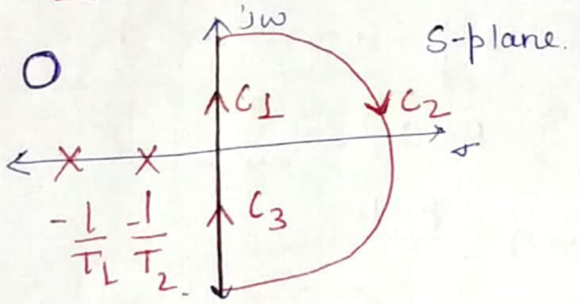
Along C_5 : $s = \lim_{r\to 0} r.e^{j\theta} + j.1 \quad \theta : -90^\circ \text{ to } 90^\circ$

Along C_6 : $s = \lim_{r\to 0} r.e^{j\theta} - j.1 \quad \theta : -90^\circ \text{ to } 90^\circ$

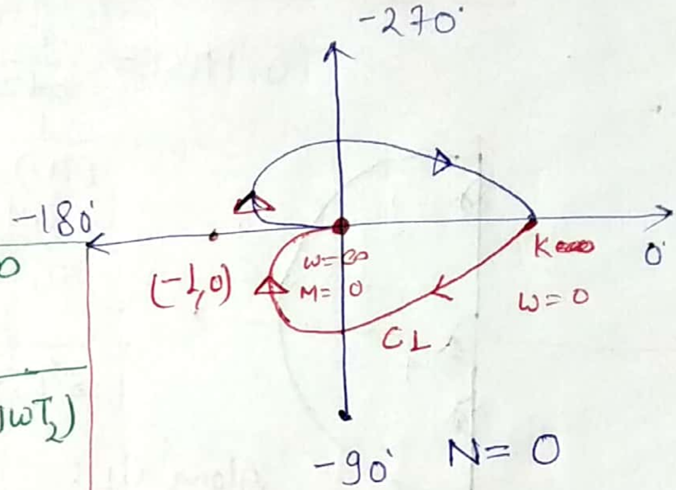
Q $G.H(S) = \dfrac{K}{(1+ST_1)(1+ST_2)}$

Nyquist Contour

$P = 0$



G.H(s) plane



$N = 0$

$\therefore P = 0$

$\therefore N = P$

Stable

Along C_1: $S = j\omega$; $\omega : 0 \to \infty$

$$\therefore GH(j\omega) = \frac{K}{(1+j\omega T_1).(1+j\omega T_2)}$$

$$M = \frac{K}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$$

$$\phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

ω	M	φ
0	K	0°
⋮	⋮	⋮
∞	0	-180°

Along C_2: $S = \underset{R\to\infty}{Lt}\ R.e^{j\theta}$; $\theta : +90°$ to $-90°$

$$G.H(s) = \underset{R\to\infty}{Lt} \frac{K}{(1+Re^{j\theta_1})(1+Re^{j\theta_2})}$$

$$M = \underset{R\to\infty}{Lt} \frac{K}{R^2 T_1 T_2} = 0$$

$$\phi = -2\theta$$

$\therefore \phi$ $\underline{|-180° \text{ to } +180°}$

Along C_3:

$S = -j\omega$; $\omega : \infty \to 0$

$$G.H(j\omega) = \frac{K}{(1-j\omega T_1)(1-j\omega T_2)}$$

$$M = \frac{K}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$$

$$\phi = +\tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_2)$$

ω	M	φ
∞	0	180°
⋮	⋮	⋮
0	K	0°

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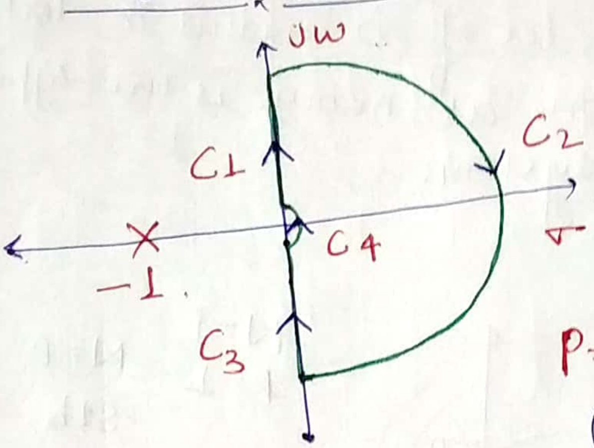
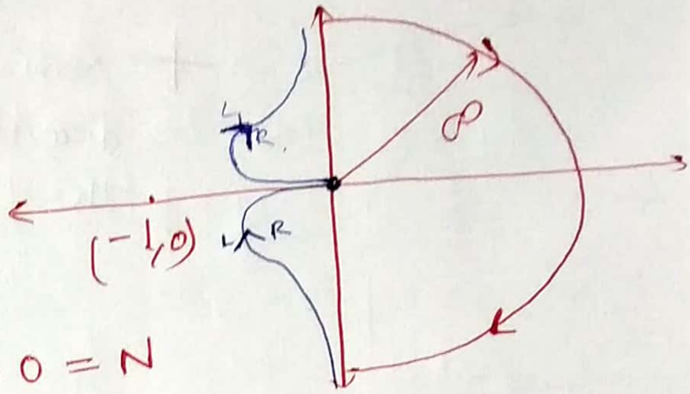
Q. $G.H(s) = \frac{K}{s(s+1)}$

Nyquist contour





$P = 0 = N$

(Stable)

Along C4:

$s = \underset{r \to 0}{Lt}\ r.e^{j\theta}$; $\theta : -90^\circ$ to 90°

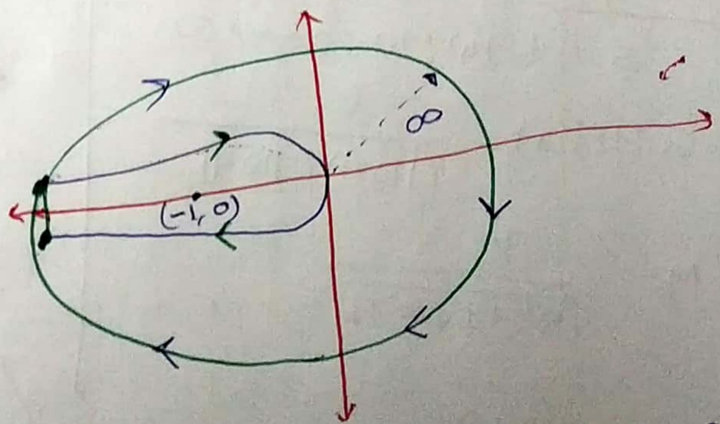
$G.H(s) = \underset{r \to 0}{Lt} \frac{K}{r.e^{j\theta}[1+\cancel{re^{j\theta}}]}$

$M = \underset{r \to 0}{Lt} \frac{K}{r} = \infty$

$\phi = -\theta$

$\therefore\ \infty \angle +90^\circ$ to -90°

Q $G.H(s) = \frac{K}{s^2(s+1)}$



$N = -2$

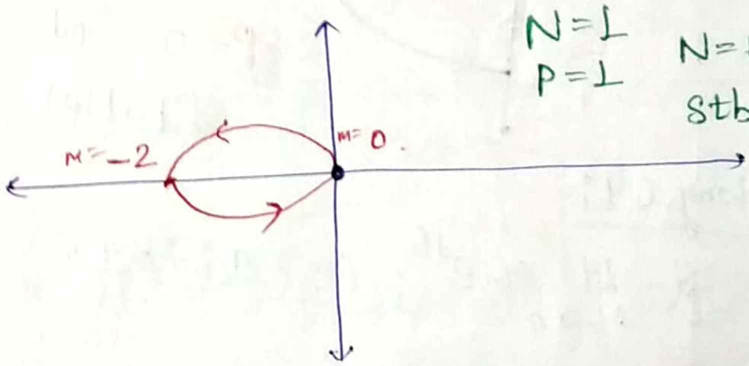
$N \neq P$ (unstable)

No. of Roots Causing Unstability: $P - N = 0 - (-2) = 2$

* Nyquist plot = polar plot + mirror image of polar plot with respect to real axis with opposite dirⁿ.

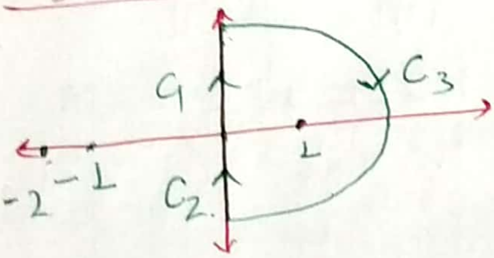
+ Semicircles of ∞ radius in clockwise direction as many as the type of the system.

Q. $\dfrac{S+2}{(S+1)(S-1)}$



N=1
P=1
N=P
Stb

Nyquist Contour



Along C1:

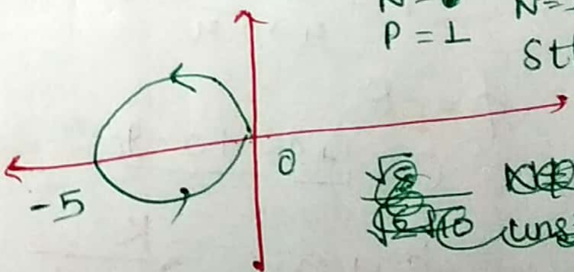
$S = j\omega : \omega : 0 \longrightarrow \infty$

$$G.H(j\omega) = \frac{k}{(j\omega+1)(j\omega-1)}$$

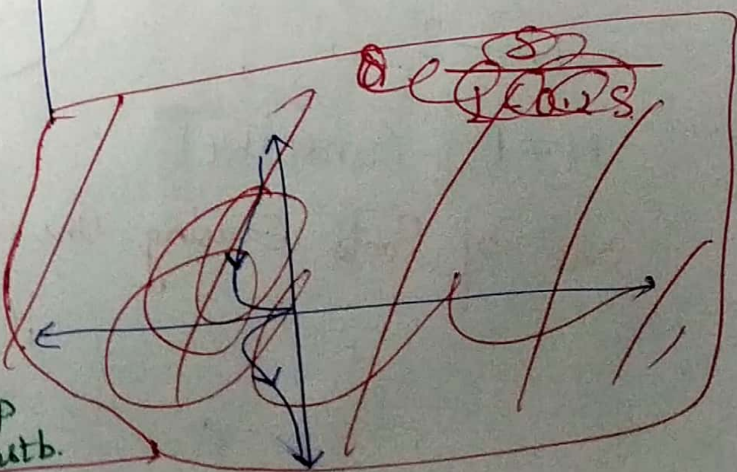
$$M = \frac{k}{\sqrt{\omega^2+1}\sqrt{\omega^2+1}}$$

Q. $\dfrac{S}{1-0.2S}$

N=1
P=1
N=P
Stb

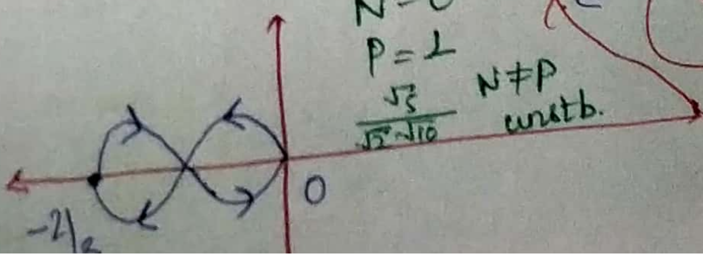


~~unstb.~~

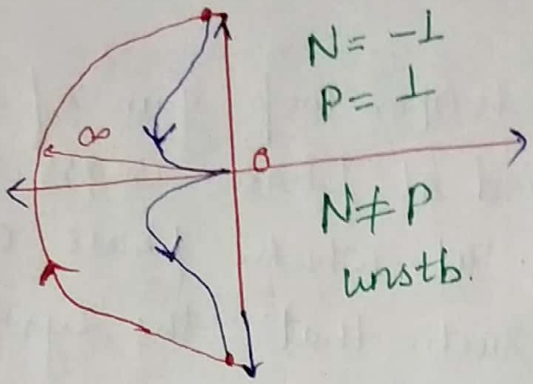


Q. $\dfrac{S+2}{(S+1)(S-3)}$

N=0
P=1
$\frac{\sqrt{5}}{\sqrt{2}\cdot\sqrt{10}}$
N≠P
unstb.



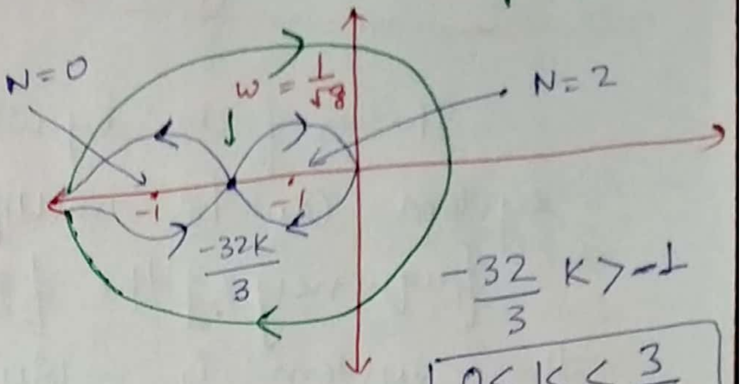
Q. $\dfrac{K}{S(S-1)}$



$N = -1$
$P = 1$
$N \neq P$
unstb.

Q. $\dfrac{K(4S+1)}{S^2(S+1)(2S+1)}$

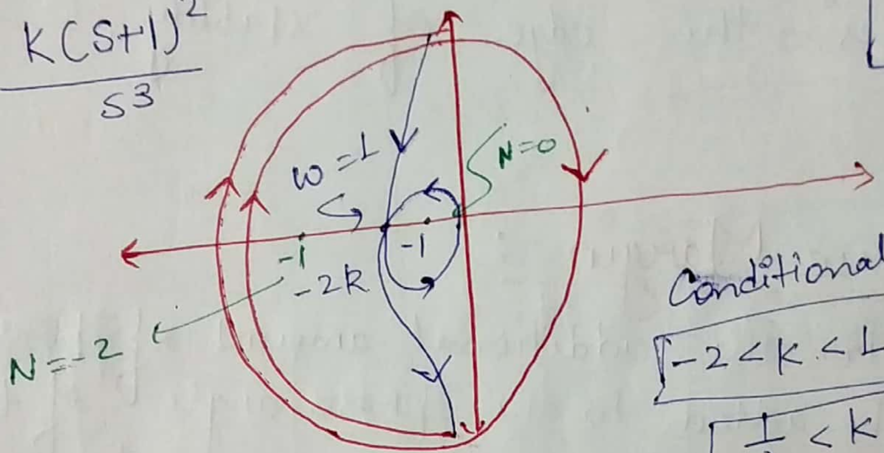
$P = 0$



$-\dfrac{32}{3}K > -1$

$\boxed{0 < K < \dfrac{3}{32}}$

Q. $\dfrac{K(S+1)^2}{S^3}$



Conditionally stable

$\boxed{-2 < K < 1}$

$\boxed{\dfrac{1}{2} < K < \infty}$

---

* Gain Margin & Phase Margin:

$0 < \omega < \infty$

Stb

Marg. Stb

$M = (\ ) \times [GM] \Rightarrow M = 1$

$\phi = 180^\circ$ $\quad \phi = -180^\circ$

$\omega_{pc} \rightarrow$ phase cross over freq.

Stb

$1 = M$ $\quad \omega: 0 \xrightarrow{\omega_{gc}} \infty$

$[PM] + (\ ) = \phi$ $\quad m: \infty \xrightarrow{1} 0$

$\Rightarrow 180$

ω_{gc} = Gain Cross over frequency.

Gain M → Non linear approach

ϕ.margin → linear appro

hence PM is preff.

* Gain Margin :

It is factor by which open loop gain of a system can be multiplied at phase cross over frequency, the freq. at which phase of the system is $-180°$ such that the system reaches to the edge of stablity.

* Phase Margin :

It is the additional amount of phase lag to be added to the phase angle of the system at gain cross-over frequency, the frequency at which ~~phase~~ magnitude is 1 such that the system reaches to the edge of stability.

* Sign of the margin is used for stability analysis
* And amount of margin is used for relative stability analysis.

* $G.H(s) = \frac{1}{sT}$

$\omega : 0 \longrightarrow \infty$. $m : \infty \longrightarrow 0$.

$\phi = -90°$ ω_{pc} X $G_m = X$

$m = \frac{1}{\omega T}$, $\omega_{gc} = \frac{1}{T}$, $\phi_{at\ \omega_{gc}=1} = -90°$

So, $PM = -90°$

★ $G.H(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

$\omega : 0 \longrightarrow \infty$

$m : \infty \longrightarrow 0$

$\phi : -90^\circ \longrightarrow -180^\circ$

$\omega_{pc} = ✓$

$G_m = ✓$

ω_{gc} ✓ PM ✓

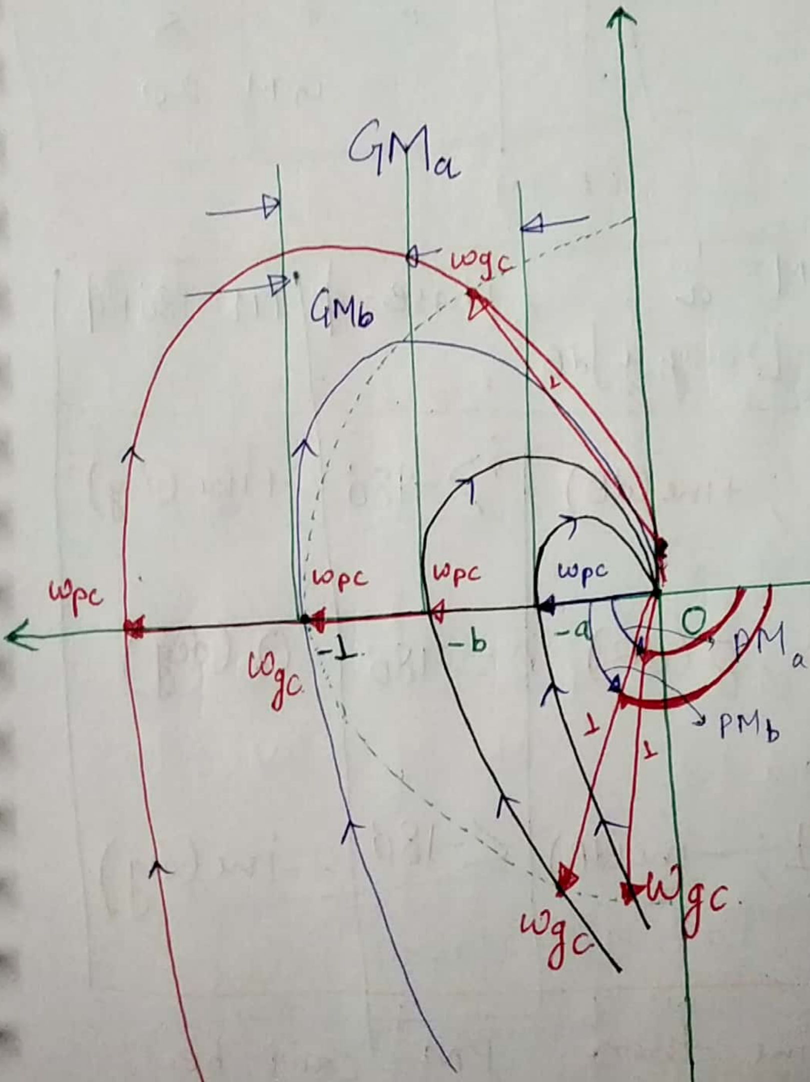
Q. $G.H(s) = \dfrac{K}{s(T_1 s+1)(1+sT_2)}$

$a = |G.H(j\omega)|_{\omega_{pc}}$

$a \times \left[\frac{1}{a}\right] = 1$

$\Rightarrow G.M = \frac{1}{a}$

$= [0 - 20\log_{10} a]$ dB



GM ∝ Stb

PM ∝ Stb

$\angle G.H(j\omega) = -180^\circ + PM$

$\therefore PM = 180^\circ + \angle G.H(j\omega)\big|_{\omega_{gc}}$

$\omega_{gc} < \omega_{pc} \Rightarrow$ Stable

$\omega_{gc} = \omega_{pc} \Rightarrow$ Marg. stb

$\omega_{gc} > \omega_{pc} \Rightarrow$ Unstb.

$G_m = \dfrac{\text{Critical gain}}{\text{Given gain}}$

GM ∝ Stb.

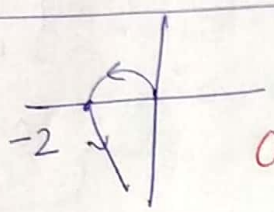
①. Gain = 5 at ω_{pc}

$$Gm = \frac{1}{5}$$

②. Gain 5dB at ω_{pc}

$$G.M = -5dB.$$

③.



$$G.M = \frac{1}{2}$$

④. Stb K_{cri}

$0 < K < 100$

find G.M for K=5

$$G.M = \frac{\text{Critical gain}}{\text{given gain}}$$

$$\Rightarrow G.M = \frac{100}{5}$$

$$G.M = 20$$

*	Gain = a	$G.M = \frac{1}{a}$ $= -[20\log_{10} a]$ dB	Phase = ϕ	PM = $180° + \phi$
Stable	< 1	> 1 ; +ive (dB)	$> -180°$	+ive (deg)
Marg. stb	$= 1$	$= 1$; 0 (dB)	$= -180°$	0 (deg)
Unstb.	> 1	< 1 ; -ive (dB)	$< -180°$	-ive (deg)

If G.M is +ive then PM can't be -ive

$\cancel{GM_1 > GM_2}$

$\cancel{PM_1 < PM_2}$

With Transportation delay ~~cause~~ ω_{pc} changes hence GM changes.

NOTE:

Due to transportⁿ delay both GM & PM decreases.

Imp

$\xi \propto \frac{1}{\sqrt{K}}$; $Gm = \frac{1}{m}$, $m \propto K$

$Gm \propto \frac{1}{K}$ $\therefore$ $\boxed{GM \propto \xi^2}$

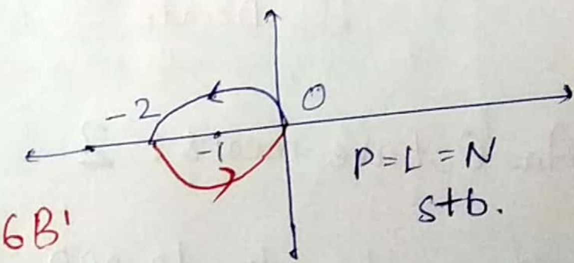
$$\boxed{P.M = \tan^{-1}\left[\frac{2\xi}{\sqrt{\sqrt{4\xi^4+1}-2\xi^2}}\right] \approx 100\xi}$$ (± 5° error)

$$\boxed{PM \propto \xi \; ; \; 0^\circ < PM < 80^\circ}$$

Q

$$G.H(S) = \frac{S+2}{(S+1)(S-1)}$$

$q(s) = s^2 + s + 1 = 0$



$\cancel{GM} - 6B'$

NOTE:

The concept of GM, PM is applicable only for minimum phase system.

* Bode plot

(H.w Bode)

- Magnitude plot $[M_{dB} \times \log_{10}\omega]$ (Y: M_{dB}, X: $\log_{10}\omega$)
- phase plot $[\phi_{deg} \times \log_{10}\omega]$

ω : $1_{23456789}10 \; \ldots \; 100 \; \ldots \; 1000 \; \ldots \; 10^4 \ldots 10^5 \ldots 10^6$

$X = \log_{10}\omega$: $0 \ldots 1 \ldots 2 \ldots 3$

$\omega \notin [0, \infty)$: $\omega \in (0, \omega)$

$x_1 = \log_{10}\omega_1$, $x_2 = \log_{10}\omega_2$. $x_2 - x_1 = \log_{10}\frac{\omega_2}{\omega_1}$.

if $\boxed{\omega_2 = 10\,\omega_1}$ Decade then $x_2 - x_1 = 1$

$\Rightarrow \boxed{x_2 = x_1 + 1}$

An Octave means : 2

from 10 to 10,000

No. of decades : 3

from 2 to 2048

No. of octave : 10

from $\cdot 1$ to $16000 \Rightarrow \frac{16000}{\cdot 1} = 1,60,000$

No of decades = ~~10~~ 4

No of octaves = 4.

$\boxed{\begin{matrix} 16 \times 10^4 \\ 2^4 \times 10^4 \end{matrix}}$

* Advantages of Bode plot:

1. using Bode plot both response analysis and stability analysis can be performed in frequency domain.
2. Response analysis of a system having huge bandwidth is made easy by considering ω on the $\log_{10}(\ )$ scale.
3. Response analysis of higher order system is made easy by considering magnitude in decibal (dB).
4. Analysis and design of controllers & compensators is effective using Bode plot.

* Magnitude Plot [$M_{dB} \times \log_{10}\omega$]

consider OLTF of minimum phase system in time constant form as: [∵ GM PM valid for MPS] minimum phase sys.

$$G.H(s) = \frac{K(T_1 s+1)\ldots}{s^r (T_2 s+1)\ldots}$$

① $\frac{K}{s^r} \Rightarrow \frac{K}{(j\omega)^r}$

$$M = \frac{K}{\omega^r} \Rightarrow M = 20\log\frac{K}{\omega^r}\ \text{dB}$$

$$\boxed{M = +20r\log_{10}\omega + 20\log_{10}K}\ \text{dB}$$

$$y = m\,x + c$$

① Point: $\left[\underset{\omega_1}{1}, \underset{y_1}{20\log_{10}K}\right]$; $\left[\underset{\omega_2}{10}, \underset{y_2}{-20r + 20\log_{10}K}\right]$

$\left[\underset{x_1}{0}, \underset{y_1}{20\log_{10}K}\right]$; $\left[\underset{x_2}{1}, \underset{y_2}{-20r + 20\log_{10}K}\right]$

$\text{Slope}_1 = \frac{y_2 - y_1}{x_2 - x_1}$, $\text{Slope}_2 = \frac{y_2 - y_1}{\omega_2/\omega_1}$

w/o Semilog Graph paper

$\text{Slope}_1 = \frac{-20r\,dB}{1}$

with Semilog graph paper.

$\frac{-20r\,dB}{10} = \frac{-20r\,dB}{\text{decade}}$.

$\boxed{\frac{20dB}{\text{decade}} = \frac{6dB}{\text{Octave}}}$

$\left[\underset{\omega_2}{2}, \underset{y_2}{-6r + 20\log_{10}K}\right]$

$\text{Slope}_2 = -\frac{6r\,dB}{2}$

$= -\frac{6r\,dB}{\text{Octave}}$.

$+90^\circ \rightarrow +\frac{20dB}{\text{decade}}$

$-90^\circ \rightarrow -\frac{20dB}{\text{decade}}$.

② . $\frac{1}{Ts+1} \Rightarrow \frac{1}{Tj\omega+1}$

$M = \frac{1}{\sqrt{T^2\omega^2+1}} \Rightarrow M_{dB} = -\frac{20}{2}\log_{10}(T^2\omega^2+1)$

$M_{dB} = -10\log_{10}(T^2\omega^2+1)\ dB.$

Actual Bode plot

* Approximati (or) Assymptode Bode plot:

Case – 1: $\omega^2 T^2 << 1 \Rightarrow \omega < \omega_c$

$M = 0\,dB$

$\therefore$ Slope $= \dfrac{0dB}{decade}$

Case – 2: $\omega^2 T^2 >> 1. \Rightarrow \omega > \omega_c$

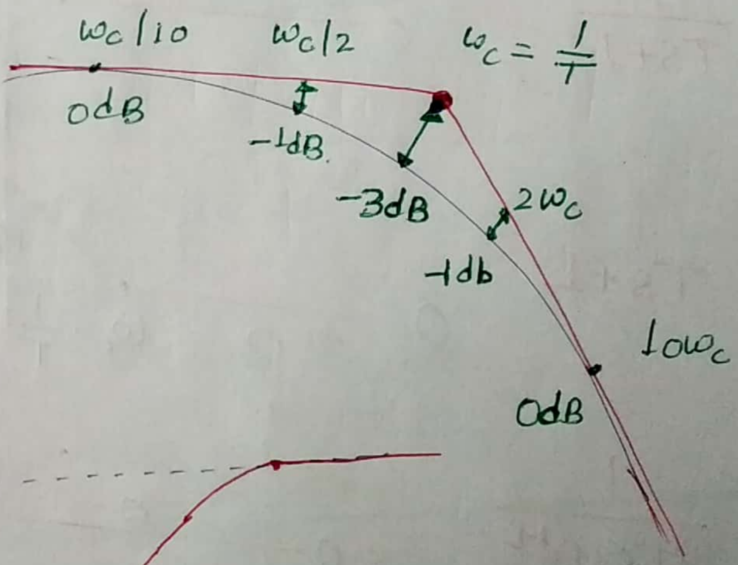
$M = -10\log(\omega^2 T^2)$

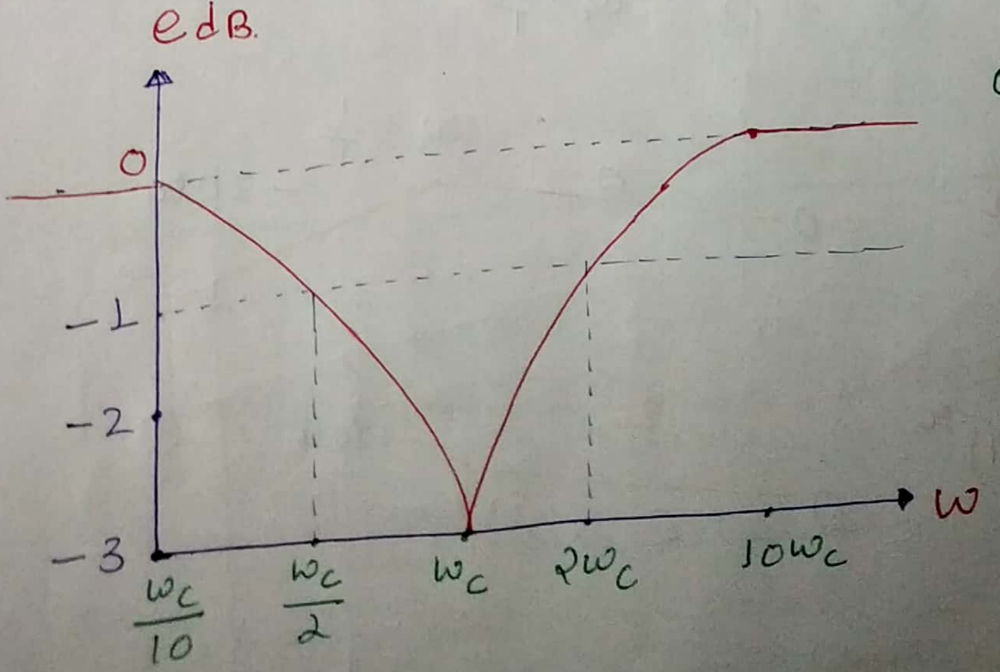
$M_{dB} = -20\log_{10}\omega - 20\log_{10}T$

Slope $= -20dB = -\dfrac{20dB}{decade} = \dfrac{-6dB}{Octave}.$

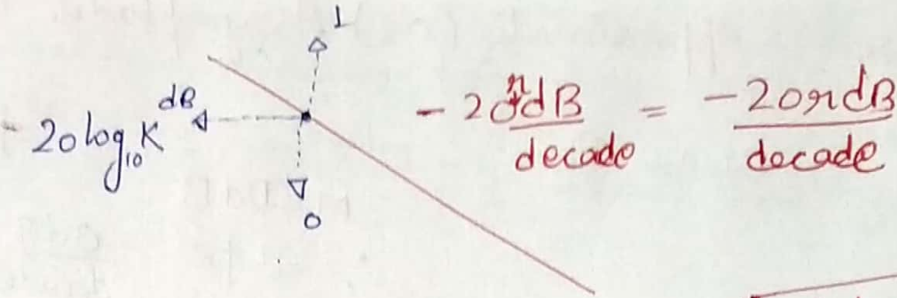
$$\boxed{\begin{array}{l} \omega^2 T^2 << 1 \\ \omega T < 1 \\ \omega < \frac{1}{T} = \omega_c \\ \omega_c = \frac{1}{T};\ \text{Corner freq:} \end{array}}$$

error = Actual – Approximate.



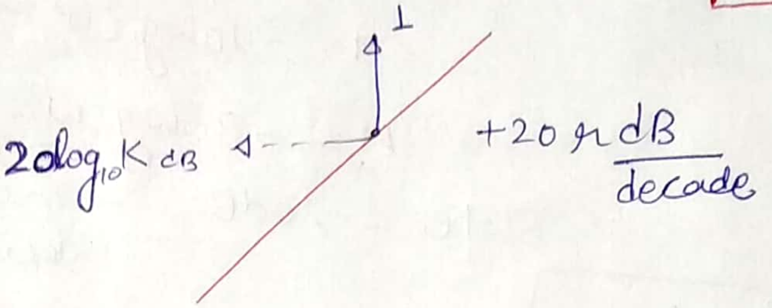


1. $\dfrac{K}{s^r}$

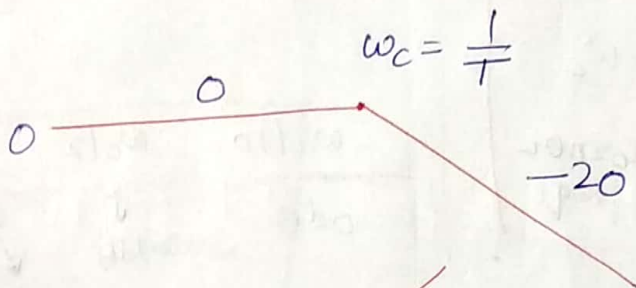


$\boxed{\text{finite pole zero } \neq 0, \infty}$

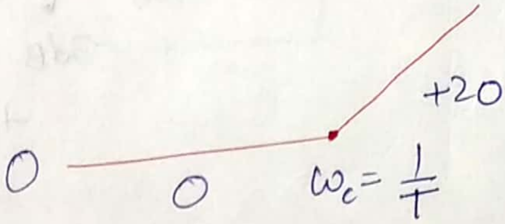
2. $\dfrac{K.s^r}{\cancel{0}}$



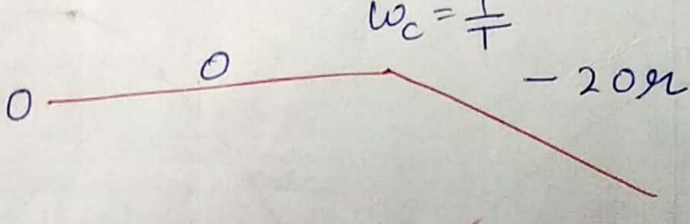
3. $\dfrac{1}{Ts+1}$



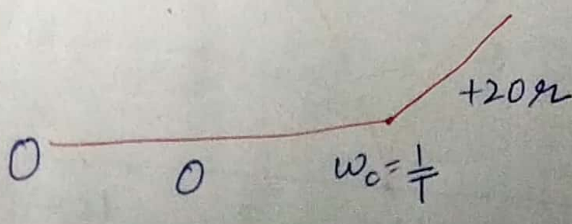
4. $Ts+1$



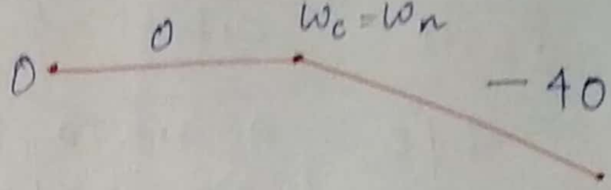
5. $\dfrac{1}{(Ts+1)^r}$



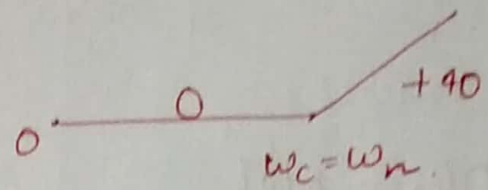
6. $(Ts+1)^r$



(7). $\dfrac{1}{\left(\dfrac{s}{\omega_n}\right)^2 + 2\xi\dfrac{s}{\omega_n} + 1}$



(8). $\left(\dfrac{s}{\omega_n}\right)^2 + 2\xi\dfrac{s}{\omega_n} + 1$



$\xi < 1$

$\xi^2 <<< 1$

$\dfrac{1}{1-u^2+j2\xi u}$

$M = \dfrac{1}{\sqrt{(1-u)^2+(2\xi u)^2}}$ $\qquad M = -10\log_{10}\left[(1-u^2)^2 + \cancel{(2\xi u)^2}_{\,0}\right] dB$

Case ①: $u^2 << 1 \Rightarrow \boxed{\omega < \omega_n}$

$m = 0\ dB$

slope $= 0$

Case ②: $u^2 >> 1$ $\qquad \boxed{\omega > \omega_n}$

$m = -40\log u$

$m = (-40\log_{10}\omega + 40\log_{10}\omega_n)\ dB$

Slope $= -40$.

Q $\quad G.H(s) = \dfrac{100}{s^2(s+10)}$

step①: $G.H(s) = \dfrac{10}{s^2(\cdot 1s+1)}$ (time constt form)

Magnitude plot:

① $\dfrac{k}{s^n} = \dfrac{10}{s^2}$ $\qquad$ point $= [1, 20dB]$

slope $= -40dB/decade$

② $\frac{1}{TS+1} = \frac{1}{0.1S+1}$; $\omega_c = \frac{1}{T} = 10$

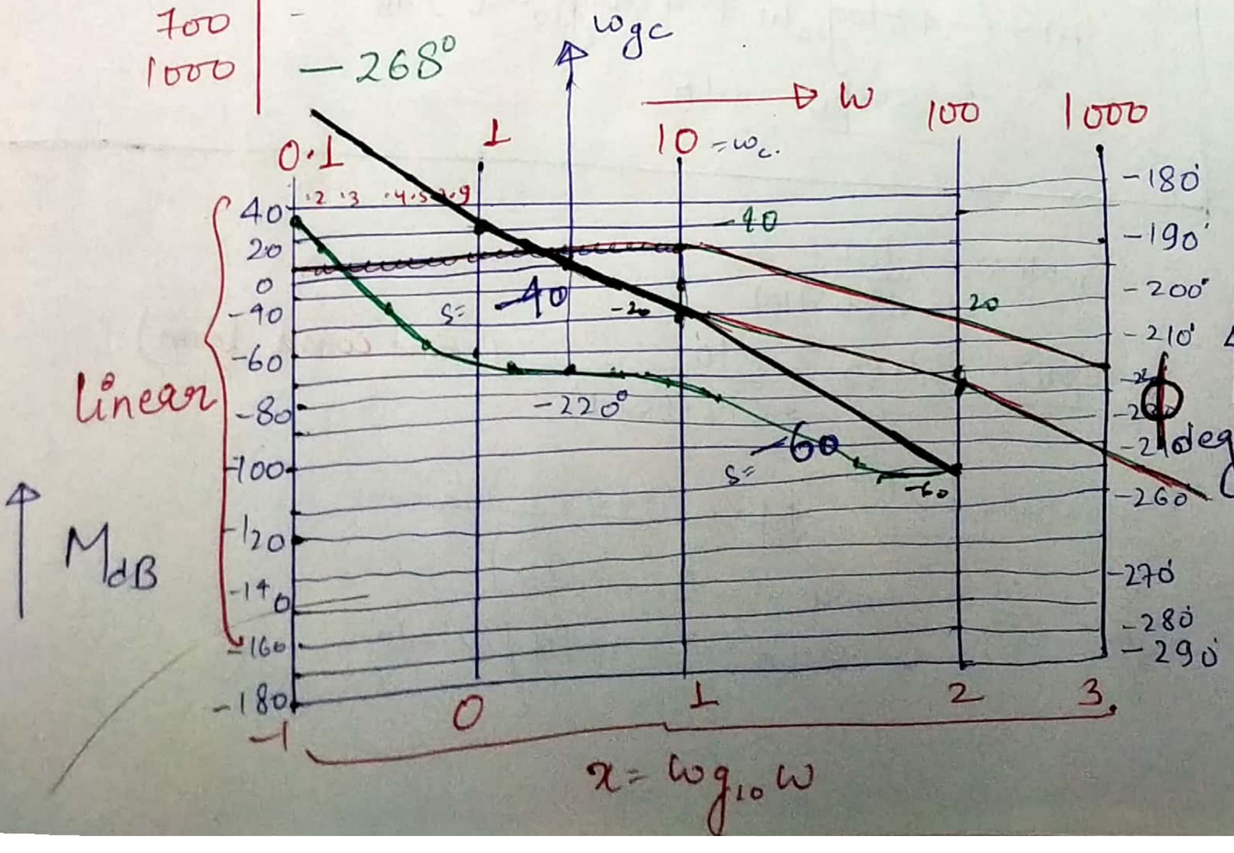
Slope = 0, $\omega < 10$

= -20, $\omega > 10$

Phase plot

ω	$\phi = -180° - \tan^{-1}(0.1\omega)$
0.1	-181°
0.2	-
0.5	-
0.7	-
1	-
2	-
5	-
7	-
10	-225°
20	-
50	-
70	-
100	-
200	-
500	-
700	-
1000	-268°

$\boxed{\text{① } \frac{K}{s^r} = \frac{10}{s^2} \text{ reference line. } \therefore \text{ this existed from } \omega = 0^+}$



Slop: $\frac{-20dB}{decade}$ $\longrightarrow$ $\frac{-20dB}{dec} \times (P-Z)$

(or)

$\frac{+20dB}{decade} \times$ No. of zeros at origin

ϕ: $-90^\circ \times$ type $\longrightarrow$ $-90^\circ (P-Z)$

or

$+90^\circ \times$ No. of zeros at origin

$G.M = -[X]_{dB} = -\infty dB$

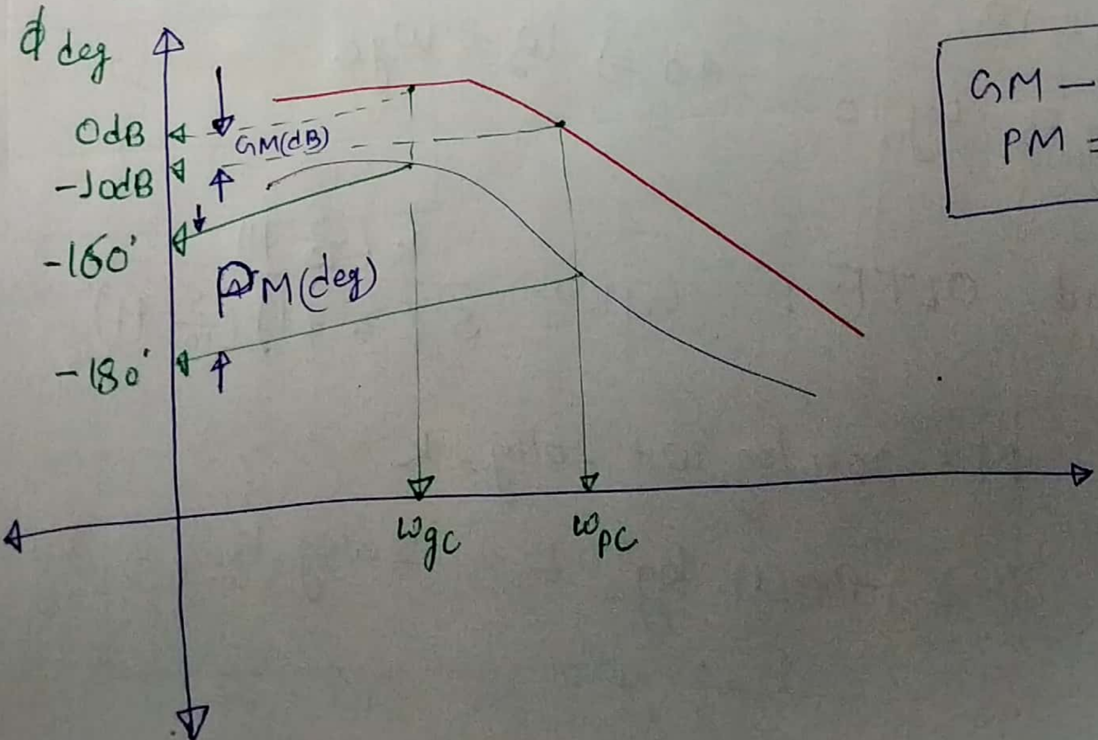
$\omega_{pc} = X$

$P.M = 180^\circ + [-220^\circ]_{\omega gc}$

$= -38^\circ$

[unstable] (-ive PM).

unit circle of polar plot is mapped in Bode plot

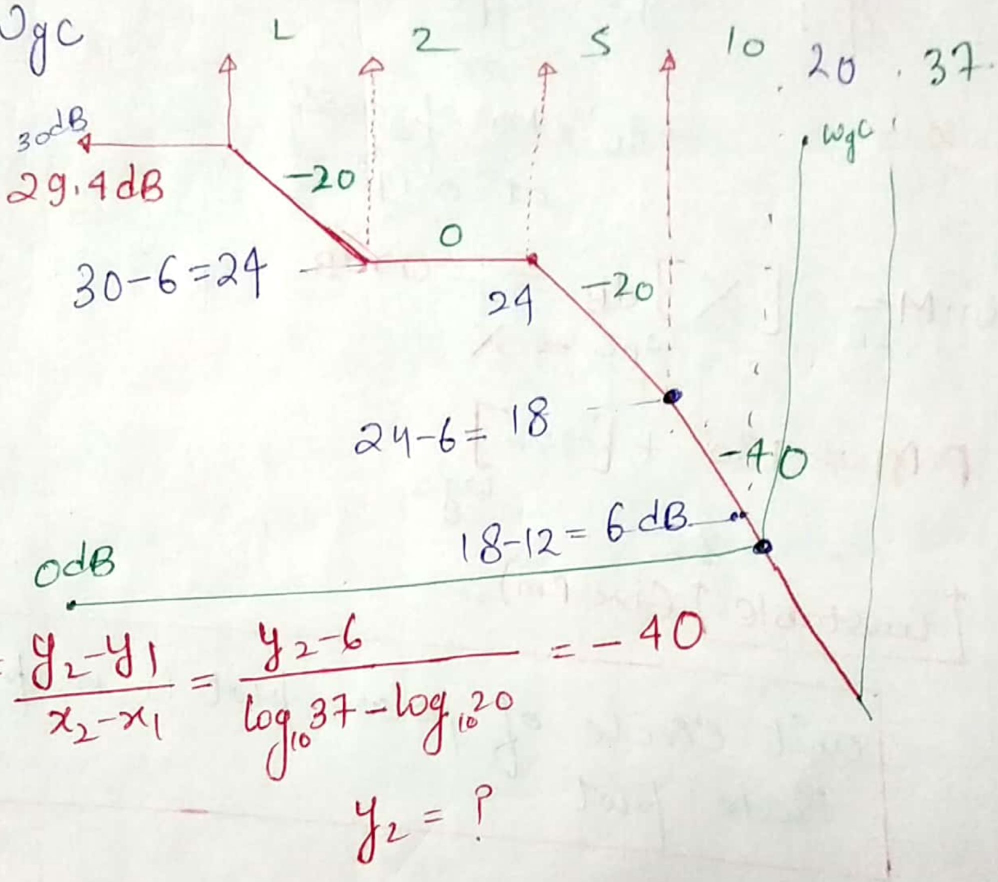


GM — 10
PM = 20

Q $G.H(S) = \dfrac{20}{S(1+0.1S)(1+0.2S)}$

1). Magnitude plot
2). $|G.H(j20)| = ?$
3). $|G.H(j37)| = ?$
4). ω_{gc}

Ans:



$$slope = \frac{y_2-y_1}{x_2-x_1} = \frac{y_2-6}{\log_{10}37-\log_{10}20} = -40$$

$y_2 = ?$

$$\frac{0-18}{\log_{10}\omega_2 - \log_{10}10} = -40 \Rightarrow \omega_2 = \omega_{gc}.$$

find OLTF : $G.H(S) = \dfrac{k(\frac{S}{2}+1)}{S^1(\frac{S}{5}+1)(\frac{S}{10}+1)}$

$M = -20n\log_{10}\omega + 20\log_{10}K.$

$30 = -20(1).\log_{10}0.1 + 20\log_{10}K.$

$K = ?$

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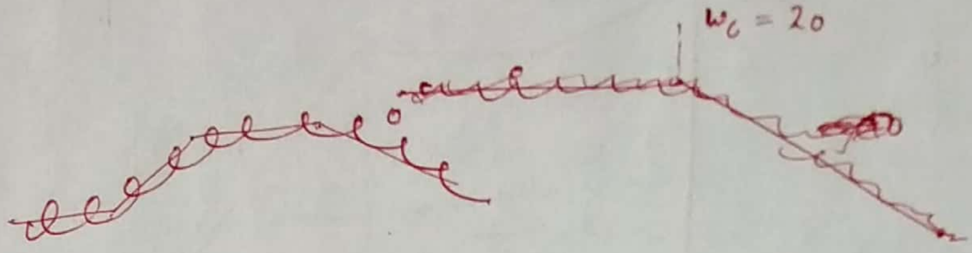
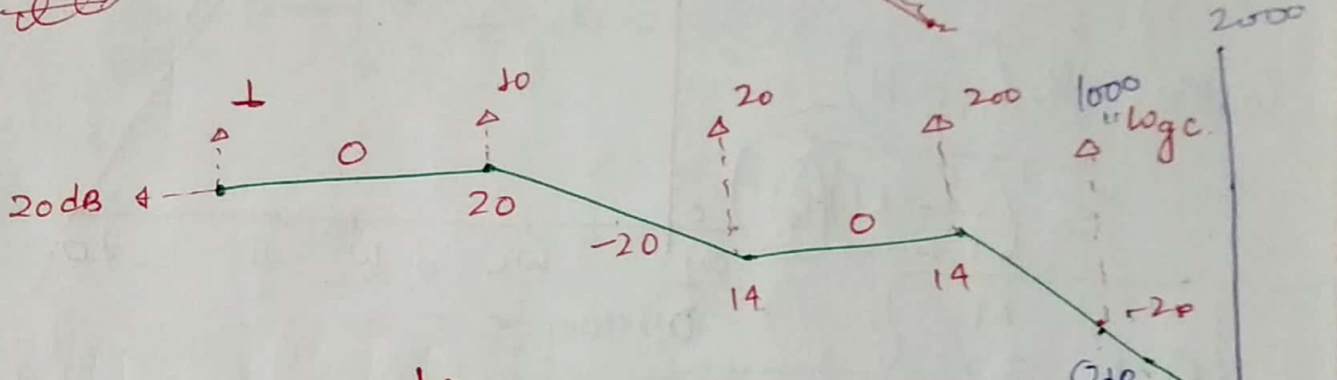
Q. $$G\cdot H(s) = \frac{10^3 (s+20)}{s^2 + 210s + 2000}$$



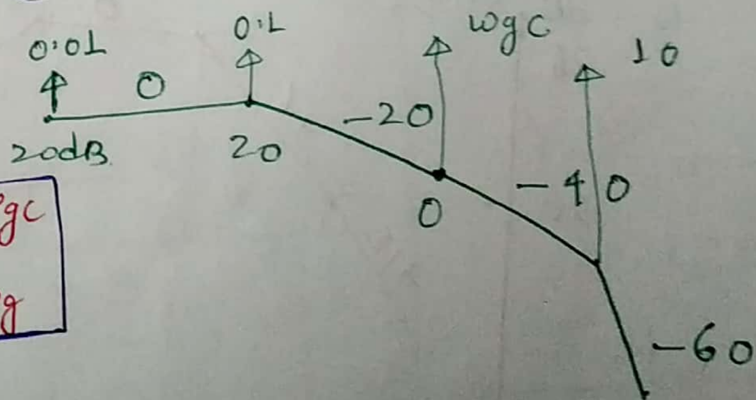


Q. $$G\cdot H(s) = \frac{10}{(s+0.1)(s+1)(s+10)}$$

PM = ?

$$PM = 180^\circ + \left[-\tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)\right]_{\omega=\omega_{gc}}$$

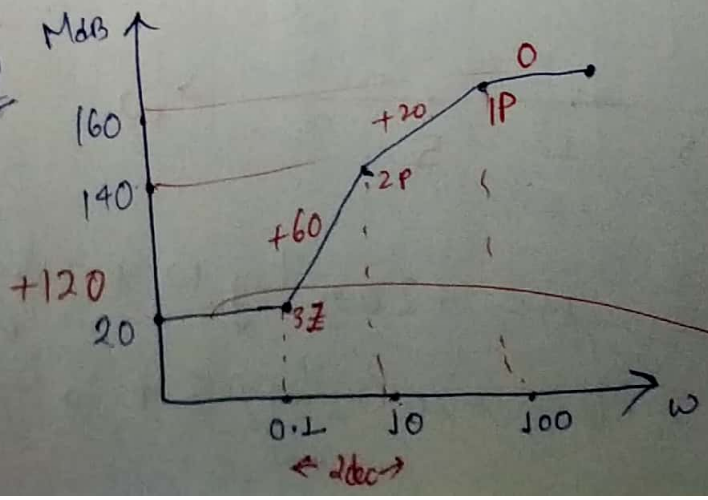
$$G\cdot H(s) = \frac{10}{(10s+1)(s+1)(0.1s+1)}$$



$$PM = 180^\circ + \left[-\tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{1}\right)\right]_{\omega=\omega_{gc}}$$

$$PM = 180^\circ + [\phi]_{\omega=\omega_g}$$

Q.



$$\frac{k\left(\frac{s}{0.10}+1\right)^3}{s^0\left(\frac{s}{10}+1\right)^2\left(\frac{s}{100}+1\right)^1}$$

200

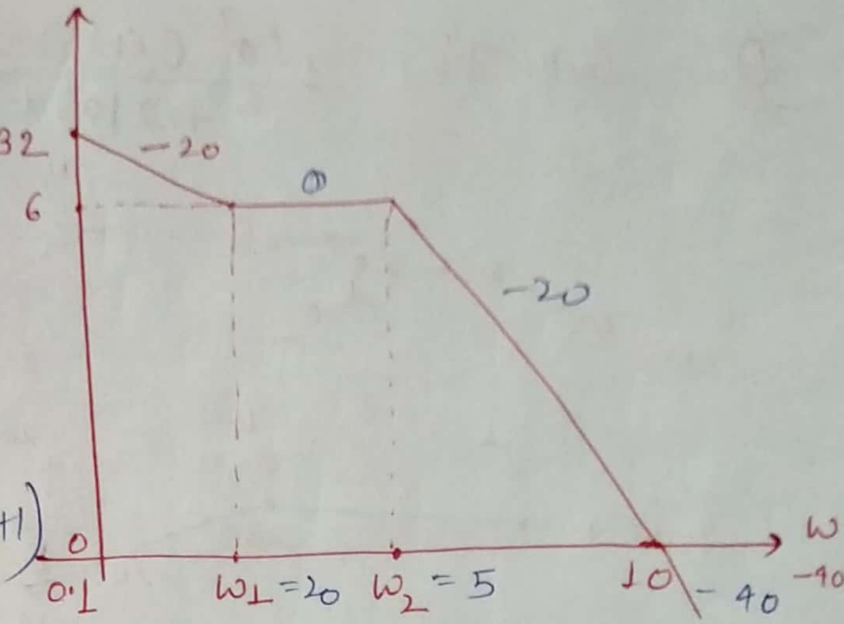
$20 = 20\log_{10} k$

∵ slope = 0.

8.

$-26 = 20 - 6$

1 octave 1



$$GH(S) = \frac{K\left(\frac{S}{\omega_1}+1\right)}{S\left(\frac{S}{\omega_2}+1\right)\left(\frac{S}{10}+1\right)}$$

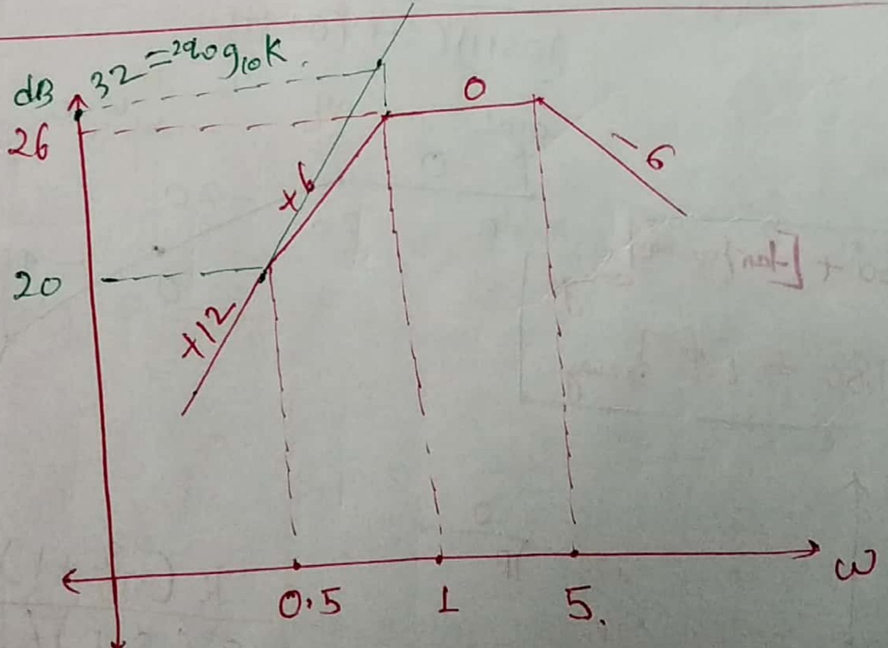
ω	M
0.1	32
1	$12 = 20\log_{10}K$
	$k = 4$

$0.1 \times 10 \times 2 =$

or $32 = -20(1)\log_{10}0.1 + 20\log_{10}K$

$$\frac{32-6}{\log_{10}0.1 - \log_{10}\omega_1} = -20 \Rightarrow \log_{10}\omega_1 = ?$$

8



$$\frac{Ks^2}{\left(\frac{S}{0.5}+1\right)\left(\frac{S}{1}+1\right)\left(\frac{S}{5}+1\right)}$$

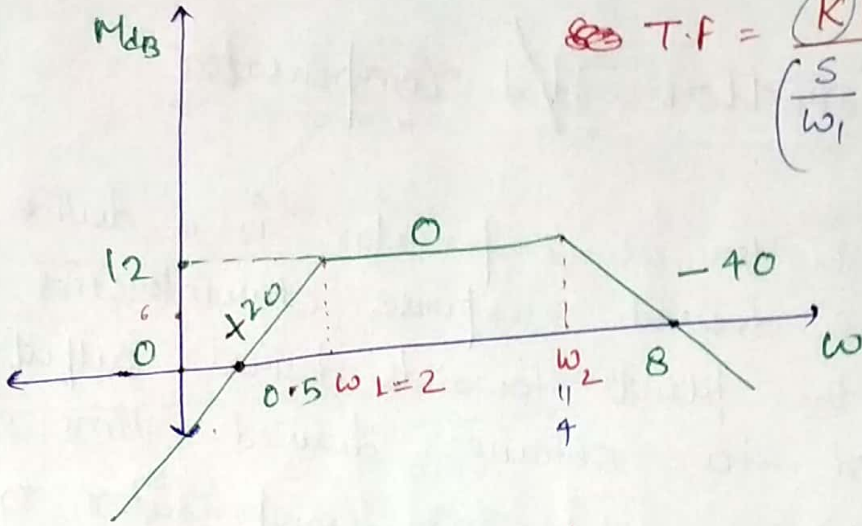
$M = +20n\log_{10}\omega + 20\log_{10}K$

$20 = +20(2)\log_{10}0.5 + 20\log_{10}K.$

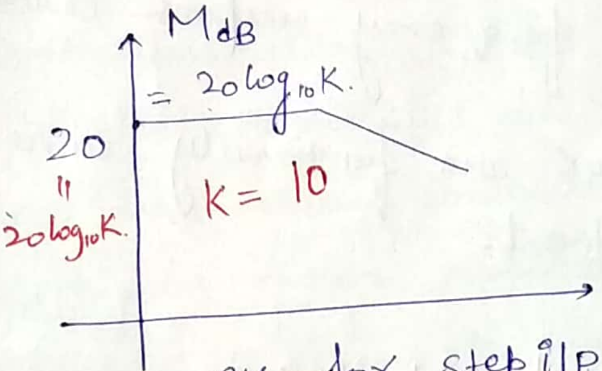
Q

Doubt

$$T.F = \frac{K s^2}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)^2}$$



Q



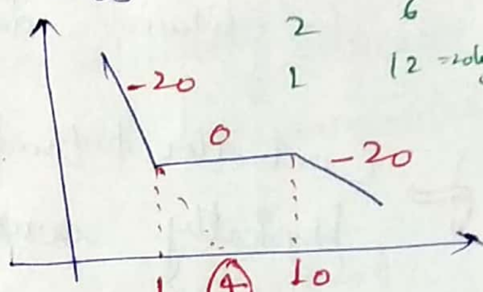
$K = 10$

e_{ss} for step i/p

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} = \frac{1}{11}$$

Q

ω	M_{dB}
4	0
2	6
1	12 = $20\log K$

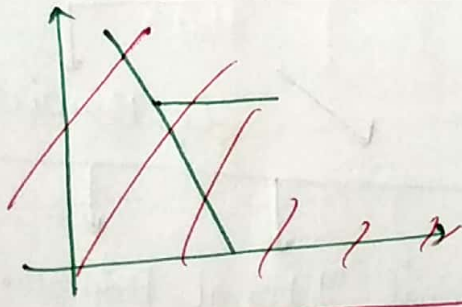


4.

e_{ss} for ramp i/p

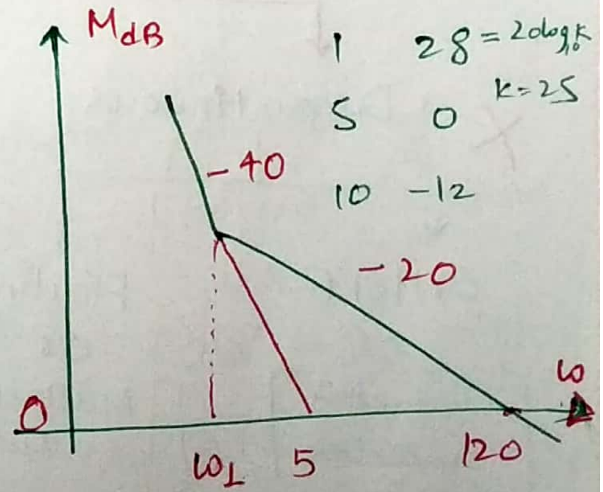
$$e_{ss} = \frac{1}{K_v} = \frac{1}{K} = \frac{1}{4}.$$

Q.



Q.

1	28 = $20\log_{10}K$
	K = 25
5	0
10	-12



e_{ss} for parabolic i/p

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K} = \frac{1}{25}$$

1. b	11. c	21. a
2. a	12. d	22. a
3. b	13. d	23. c
4. d	14. a	24. 3
5. b	15. b	25. $\frac{1}{\sqrt{2}}$
6. b	16. b	26. 45°
7. d	17. b	27. 10.5°
8 a	18. b	28. -18.5 dB
9 a	19. a	29. 8.970 Hz
10. a	20. b	

30 84.5° ~~31~~ 32 33 K=2

34 K = 60

35 $P_1 = 1$

36 16|109

32. $\frac{8\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{20}+1\right)\left(\frac{s}{50}+1\right)}$

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Ch-6

* Controller A/d Compensator

A controller or compensator is a device used to obtaine desired response characterstics i.e to drive the plant towards desired output. Controller is used to obtaine desired time response characterstics where as Compensator is used to obtaine desired frequency response characterstics.

Controller & Compensator are functionally same but physically ~~same~~ different.

Types of Controllers

- Discountinuous
  - ON/OFF or [Two state controller]
  - Floating or [Multistate Controller]
- Continuous
  - Proportional
  - Integral
  - Derivative
  - Composite controller
    - P+D
    - P+I
    - P+I+D

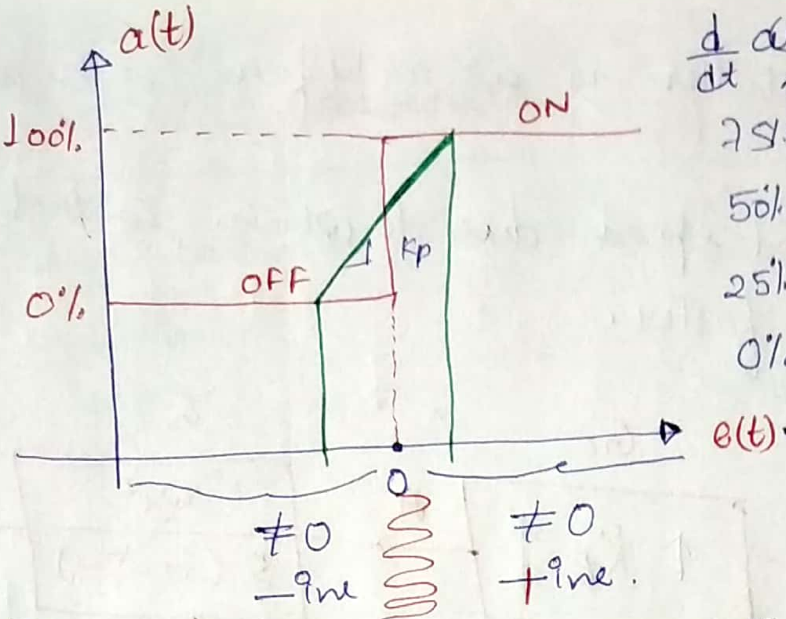
ON/OFF

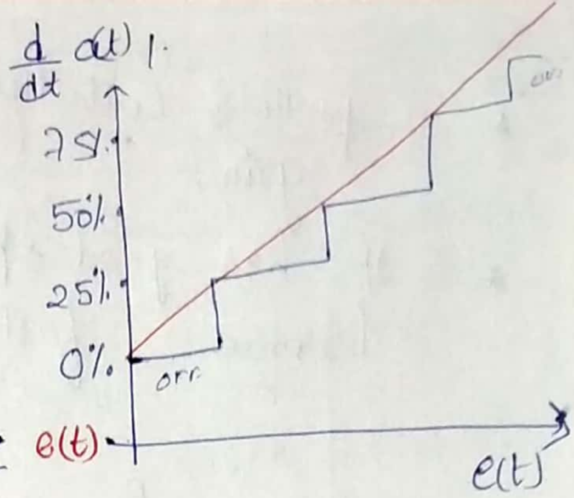
$$a(t) = \begin{cases} 100\%\ (\text{ON}) \\ 0\%\ (\text{OFF}) \end{cases} \quad e(t) \neq 0$$

$$= \text{un-defined}; \quad e(t) = 0$$

$$u(t) = \begin{cases} 1 \\ 0 \end{cases} \quad t \neq 0$$

$$= \text{undefined} \quad t = 0$$





* This ~~~~ discontinuous controller introduces oscillations in plant due to its abrupt characteristics due to which stability decreases hence discontinuous controllers are not recommended for higher order applications.

* Their applications are limited to domestic control system like first order system.

* Proportional Controller :

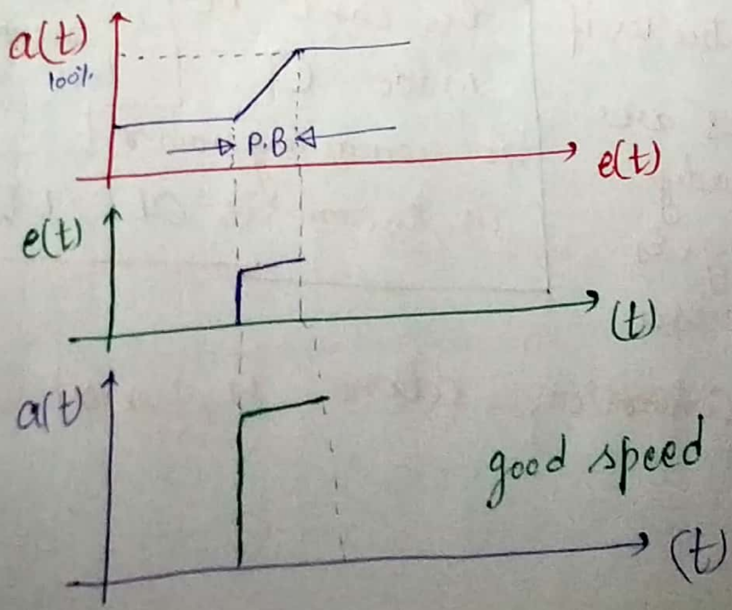
Proportional Controller is a natural extension of ON/OFF controller.

P.B = proportional Band

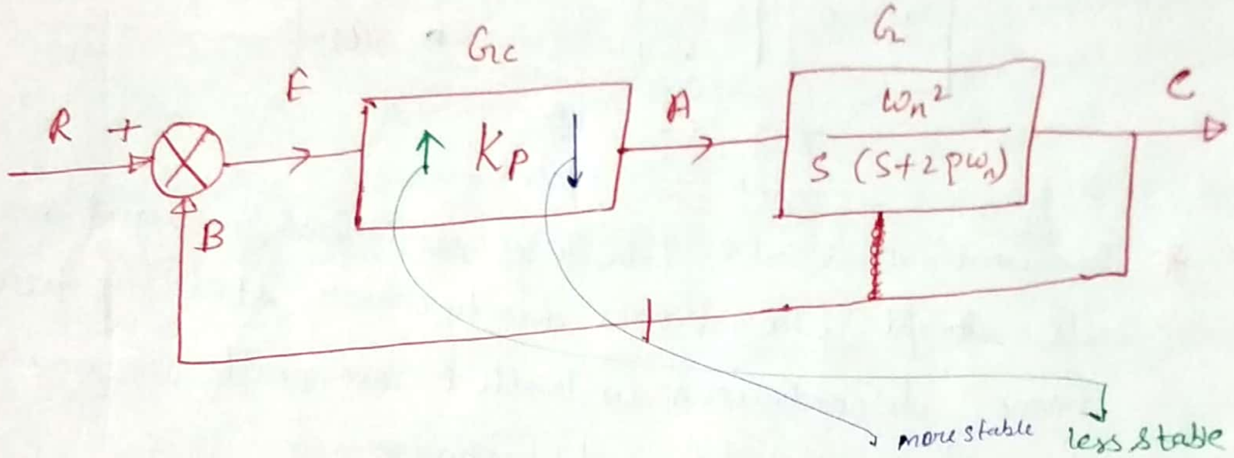
$a(t) \propto e(t)$

$a(t) = K_p \, e(t)$

$G_c(S) = \frac{A(S)}{E(S)} = K_p$



* Proportional Controller is a amplifier with adjustable gain.
* It has good speed due to which control system become faster.



stable

W/O Controller

$q(s) = S^2 + 2\rho\omega_n S + \omega_n^2 = 0$

Natural freq. $= \omega_n$

Damping Ratio $\rho = \frac{x}{2\omega_n}$

type = 1, $e_{ss} = \frac{1}{K_v} = \frac{2\rho}{\omega_n}$

$R(s) = \frac{1}{S^2}$

With Controller

$q(s) = S^2 + 2\rho\omega_n S + (\sqrt{K_p}\omega_n)^2 = 0$

Natural freq $= \sqrt{K_p} \cdot \omega_n$

Damping Ratio $\rho = \frac{x}{2\sqrt{K_p}\omega_n}$

type = 1

$R(S) = \frac{1}{S^2} \Rightarrow e_{ss} = \frac{1}{K_v} = \frac{2\rho}{K_p\omega_n}$

more accurate

less accurate

e_{ss} can't be made 0

so remaining error in known as OFFSET error

* If using proportional controller if transience characteristics are improved then steady state characteristics gets affect & vice-versa.

Hence proportional controller alone is not recommended.

* Integral Controller :

$$G.H(S) = \frac{1}{S\,T} = \frac{\left(\frac{1}{T}\right)}{S}$$

$\frac{1}{T}$ = Gain constt

T → time constt

Integral Controller is the natural extension of multistate Controller.

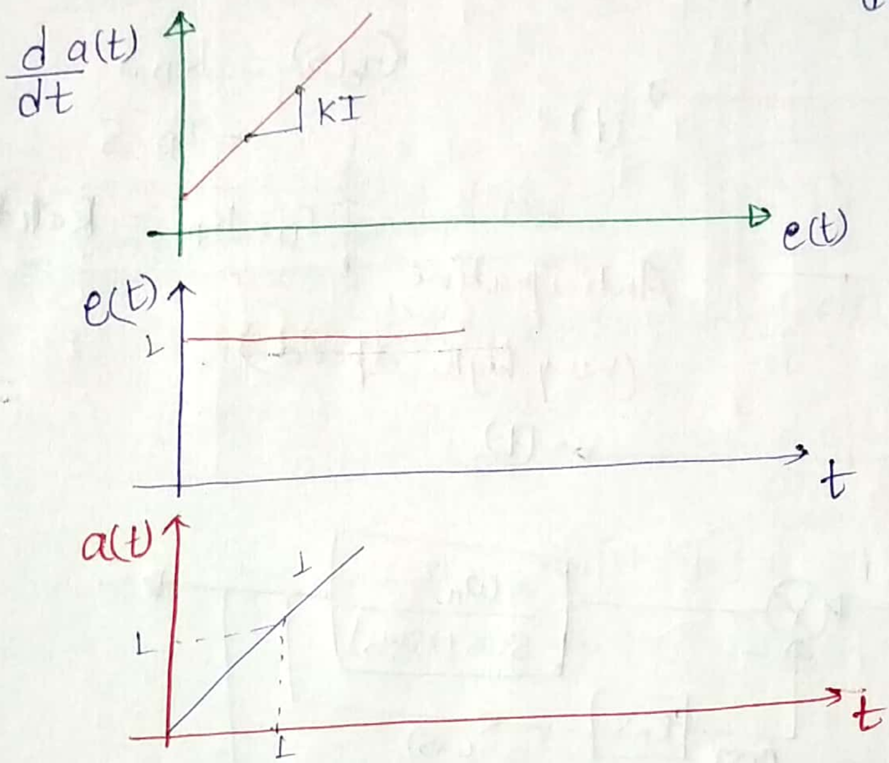
$$\frac{d}{dt} a(t) \propto e(t)$$

$$a(t) \propto \int_0^t e(t)\,dt$$

$$a(t) = K_I \int_0^t e(t)\,dt$$

$$G_c(S) = \frac{A(S)}{E(S)} = \frac{K_I}{S} = \frac{1}{T_n S}$$

$$\boxed{T_n = \frac{1}{K_I}} \text{ Reset time}$$



(Plant) with Controller

Type = 2

$$R(S) = \frac{1}{S^2} \Rightarrow e_{ss} = 0$$

$$q(S) = S^3 + 2\xi\omega_n S^2 + K_I\,\omega_n^2 = 0$$

NOT Stable

$\boxed{bc < ad}$ [Necessary condition for stability doesn't satisfied] as $0.S^1$

* Integral controller is sluggish due to which speed of response decreases.

* Integral controller decreases the steady state error to zero but it also decreases the stability. Hence Integral Controller alone is not Used.

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* Derivative Controller (OR) Rate (or) tachometer Controller :

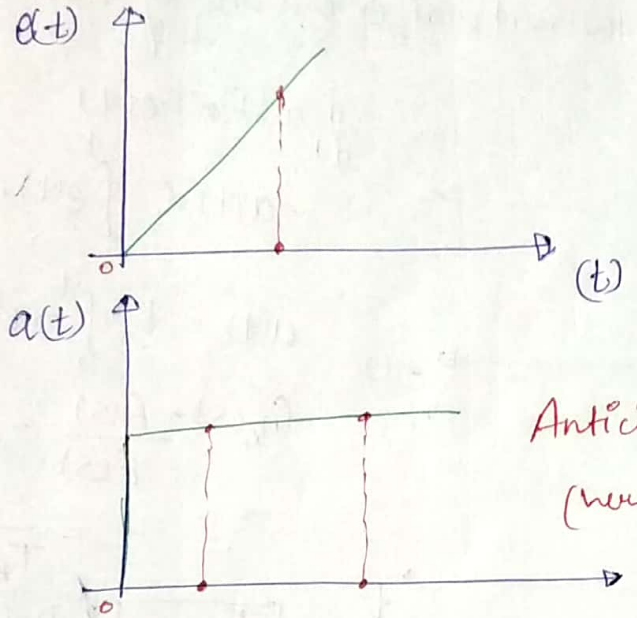
$a(t) \propto \frac{d}{dt} e(t)$ c(s)

$a(t) = K_D \cdot \frac{d}{dt} e(t)$ c(s)

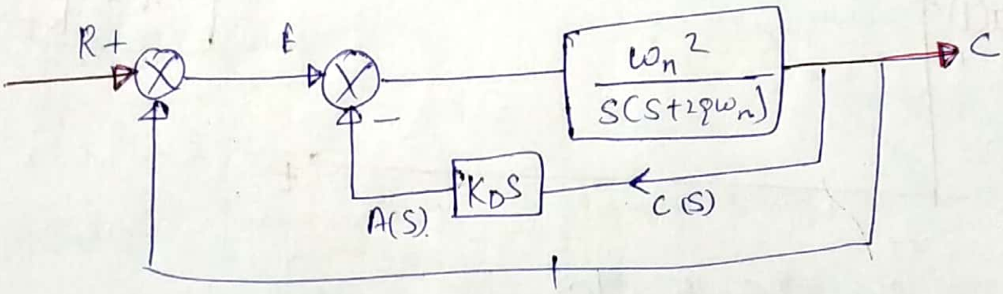
$G_c(s) = K_D S$

$= T_D S$

$\boxed{T_D = K_D = \text{Rate time}}$



Anticipative,
(very high speed)



With Controller

$$G \cdot H(S) = \frac{\omega_n^2}{S\left[S + 2\rho\omega_n + K_D\omega_n^2\right]}$$

type = 1

$$R(S) = \frac{1}{S^2} \Rightarrow e_{ss} = \frac{1}{K_v} = \frac{2\rho}{\omega_n} + K_D$$

$$q(s) = s^2 + 2\omega_n s\left[\rho + \frac{K_D\omega_n}{2}\right] + \omega_n^2 = 0$$

more stability ↑, Speed ↑, Accuracy ↓

* Derivative Controller is anticipative due to which speed of control system becomes very high.
* Derivative Controller increases stability but it also steady state error hence derivative controller alone is not used.

$\boxed{\text{P-D Controller}}$

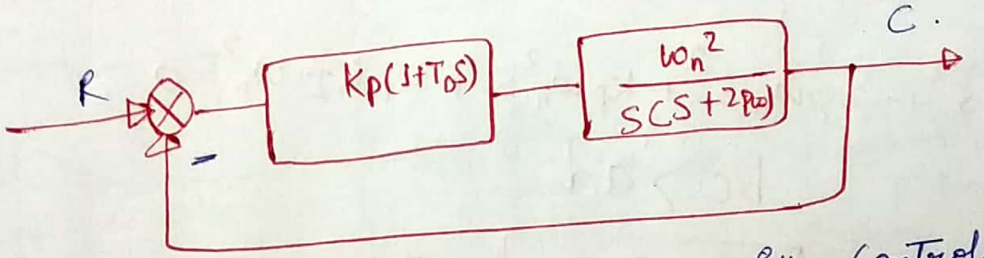
* **Composite Controllers:**

$$a(t) \propto \left[e(t) + K_D \cdot \frac{d}{dt} e(t) \right]$$

$$a(t) = K_p \left[e(t) + K_D \cdot \frac{d}{dt} e(t) \right]$$

$$G(s) = \underset{\text{constt}}{K_p}[1 + \overset{\text{Varied}}{K_D}]$$

$$= K_p [1 + T_D S]$$



w/o Controller

with Controller

$$G.H(s) = \frac{K_p \cdot \omega_n^2 (1 + K_D S)}{S(S + 2\rho\omega_n)}$$

Speed ↑

Stability ↑

$e_{ss} \downarrow (\neq 0)$

$$e_{ss} = \frac{1}{K_v} = \frac{2\rho}{K_p \omega_n} \quad ; \text{type} = 1$$

constt

$$q(s) = s^2 + 2\omega_n s \left[\rho + \frac{\overset{\text{varied}}{K_p K_D \omega_n}}{2}\right] + \underset{\text{constt}}{K_p \omega_n^2} = 0$$

P+D controller is recommended to improve transient response characteristics of a system having good steady state response characteristics.

P+I :

$$a(t) \propto \left[e(t) + k_I \int_0^t e(t)\,dt \right]$$

$$a(t) = k_P \left[e(t) + k_I \int_0^t e(t)\,dt \right]$$

$$G_c(s) = \underset{\text{constt}}{K_P} \left[1 + \frac{k_I}{s} \right]$$

$$= k_P \left[1 + \frac{1}{T_i s} \right]$$

$$\underset{PI}{G_c(s)} = \underbrace{\frac{K_P}{s}}_{I} \times \underbrace{[s + k_I]}_{PD}$$

speed ↓ stb ↓ e_{ss} ↓

Speed ↑ stb ↑ constant

with controller

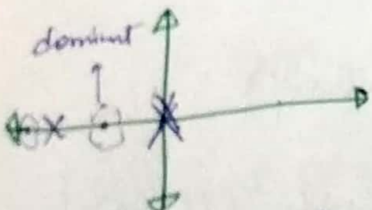
type = 2.

$R(s) = \frac{1}{s^2} \Rightarrow e_{ss} = 0$

$$q(s) = s^3 + 2\xi\omega_n s^2 + K_P \omega_n^2 s + K_P K_I \cdot \omega_n^2 = 0$$

$$bc > ad$$

$$\boxed{2\xi\omega_n > K_I}$$ Stable.



PI controller is recommended to improve steady state characteristics of a system having good transient characteristics.

PI	PD
Speed ↑, Stability ↑	e_{ss} ↓ ✓ $e_{ss} = 0$ ✗
M_P ↓ $M_P = 0$ ✗	

P + I + D : $a(t) \propto \left[e(t) + K_I \int_0^t e(t)\,dt + K_D \cdot \frac{d\,e(t)}{dt} \right]$

$$a(t) = K_P \left[e(t) + K_I \int_0^t e(t)\,dt + K_D \cdot \frac{d\,e(t)}{dt} \right]$$

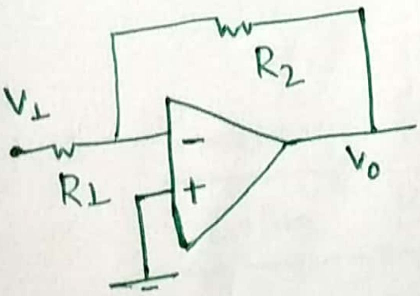
$$G_c(S) = K_P \left[1 + \frac{K_I}{S} + K_D S \right]$$

$$= K_P \left[1 + \frac{1}{T_{i} S} + T_D S \right]$$

$$\underbrace{G_c(S)}_{PID} = \underbrace{\frac{K_P}{S}(S+Z_1)}_{PI}\underbrace{(S+Z_2)}_{PD}$$

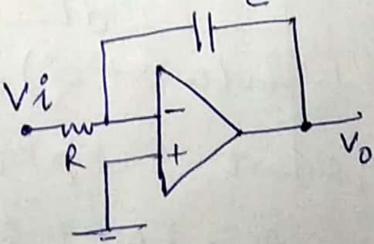
Using PID controller which is nothing but cascade combination of PD and PI controllers, both transient and steady state characteristics of system can be improved.

P :



$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} = -K_P$$

I :

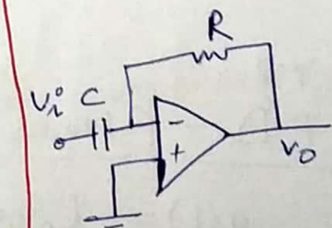


$$\frac{V_o}{V_i} = -\frac{1}{SRC}$$

$RC \rightarrow T_i$

$$\boxed{\frac{V_o}{V_i} = -\frac{K_I}{S}}$$

D :

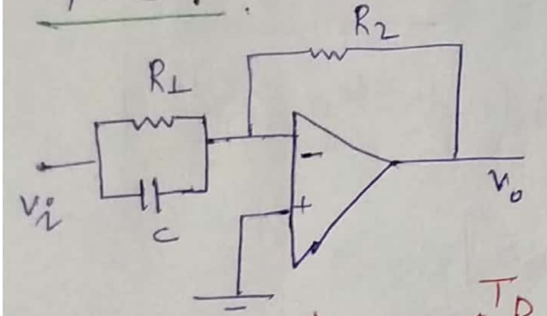


$$\frac{V_o}{V_i} = -SRC$$

$RC \rightarrow T_D$

$$\boxed{\frac{V_o}{V_i} = -K_D S}$$

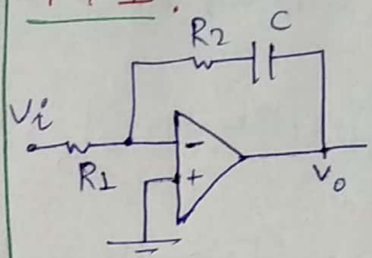
P+D:



$$\frac{V_o}{V_i} = -\underbrace{\frac{R_2}{R_1}}_{k_p}\left[1 + s\underbrace{R_1 C}_{T_D}\right]$$

$$\boxed{\frac{V_o}{V_i} = -k_p\left[1 + K_D s\right]}$$

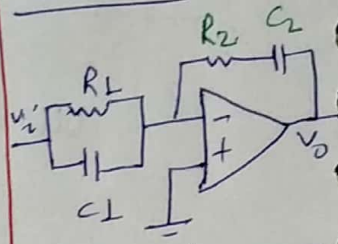
P+I:



$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\left[1 + \frac{1}{sR_2C}\right]$$

$$\boxed{\frac{V_o}{V_i} = -k_p\left[1 + \frac{k_I}{s}\right]}$$

P+I+D:



$$\frac{V_o}{V_i} = -\underbrace{\frac{R_2}{R_1}}_{k_p}\left[1 + \frac{1}{s\underbrace{R_2 C_2}_{T_{\text{r}}}} + s\underbrace{R_1C_1}_{T_D}\right]$$

$$\boxed{\frac{V_o}{V_i} = -K_p\left[1 + \frac{k_I}{s} + k_D s\right]}$$

T.D F.D

PD → phase lead , PI → Phase lag , PID → phase lead lag.

* **P+D** (phase lead).

$$a(t) = k_p\, e(t) + k_p . K_D \frac{d}{dt} e(t)$$

$$e(t) = 1 . \sin(\omega t + 0°)$$

$$a(t) = k_p \sin\omega t + k_p k_D \omega \cos\omega t$$

$$a(t) = A\sin[\omega t + \phi]$$

$$\phi \rightarrow \tan^{-1}[\underset{T_D}{k_D}\,\omega]$$

$$A = \sqrt{k_p^2 + k_p^2 k_D^2 + 0}$$

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* Compensator:

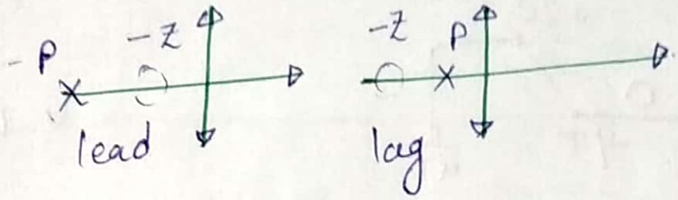
$$G_c(s) = \frac{s+z}{s+p}$$

1). Phase lead $(s+z_1)$

2). Phase lag $\frac{1}{(s+p)}$

3). Phase lead-lag or lag-lead.

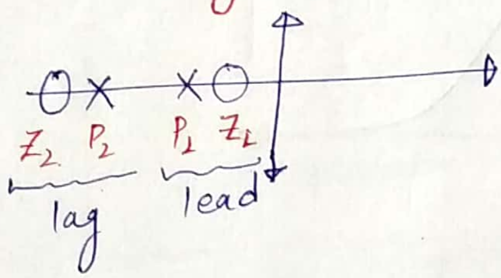
$$\frac{z}{p} = \alpha < 1 \rightarrow \text{lead}$$
$$> 1 \rightarrow \text{Lag}$$



③. Phase lead-lag or lag-lead.

$$G_c = \underbrace{\left(\frac{s+z_1}{s+p_1}\right)}_{\text{lead}} \underbrace{\left(\frac{s+z_2}{s+p_2}\right)}_{\text{lag}}$$

$$\frac{z_1}{p_1} < 1 \; ; \; \frac{z_2}{p_2} > 1$$



* frequency Response characteristics of compensator using Bode plot:

$$G_c(s) = \left(\frac{s+z_1}{s+p}\right)$$

$$= \left(\frac{z}{p}\right)\left[\frac{\frac{1}{z_1}s+1}{\frac{1}{p}s+1}\right]$$

$\frac{z}{p} \to \alpha$, $\frac{1}{z_1} \to T$, $\frac{1}{p} \to \alpha T$

$$G_c(s) = \frac{\alpha\,[Ts+1]}{\alpha Ts+1} \approx \frac{Ts+1}{\alpha Ts+1}$$

$$\omega_{cz} = \frac{1}{T} = z$$

$$\omega_{cp} = \frac{1}{\alpha T} = P$$

$$M = \sqrt{\frac{\omega^2 T^2+1}{\alpha^2\omega^2 T^2+1}}$$

$$\phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\alpha\omega T_2)$$

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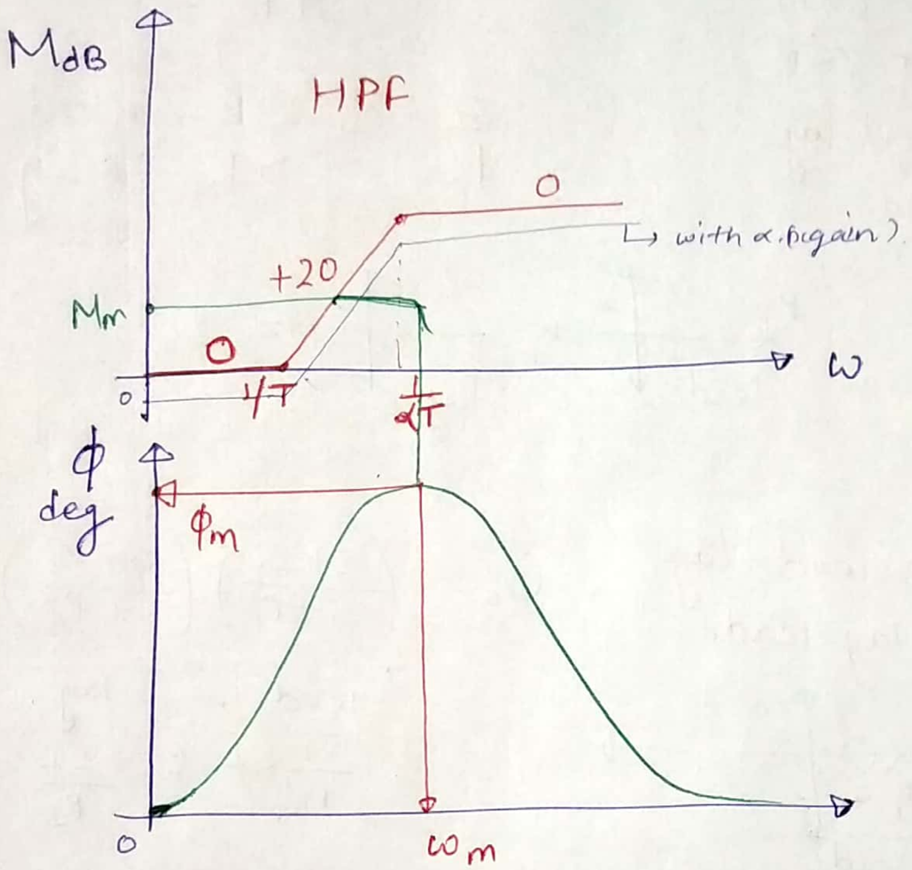
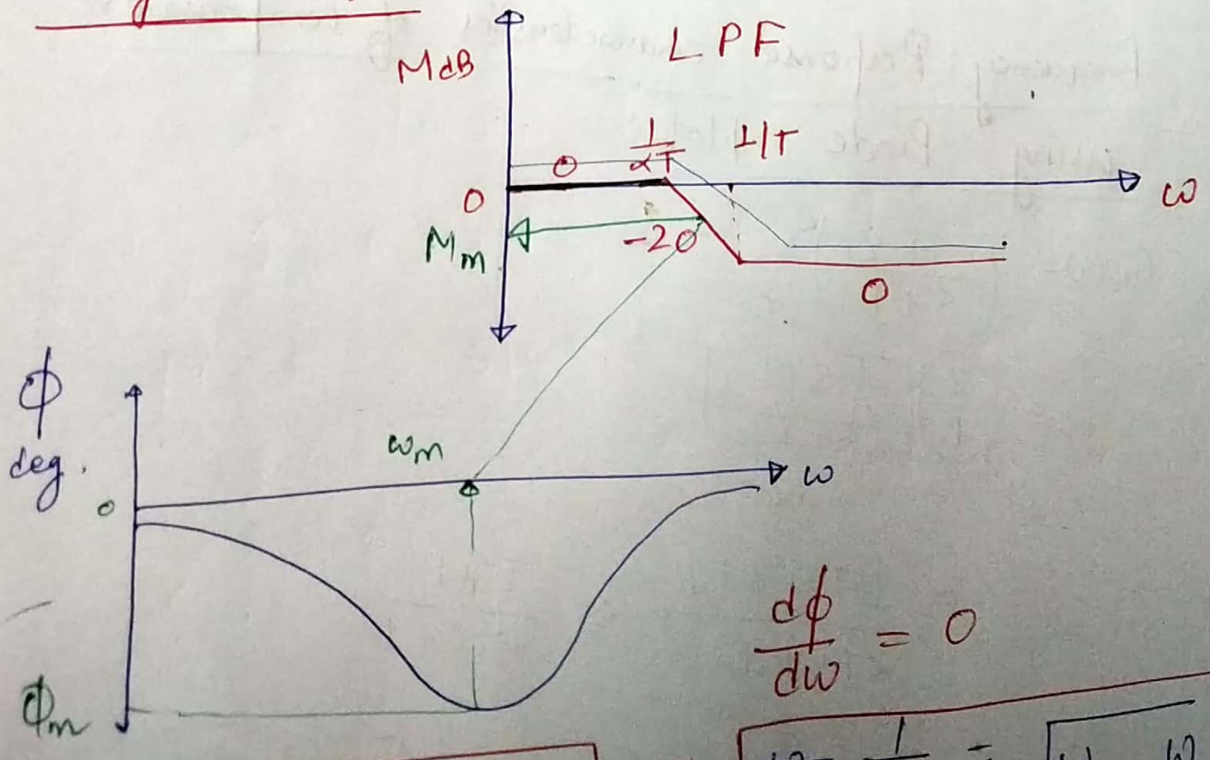
* lead ($\alpha < 1$): $W_{cz} < W_{cp}$



* Lag ($\alpha > 1$):



$$\frac{d\phi}{d\omega} = 0$$

$$\boxed{\omega_m = \sqrt{Z \cdot P}} \Rightarrow \boxed{\omega_m = \frac{1}{T\sqrt{\alpha}} = \sqrt{\omega_{cz} \cdot \omega_{cp}}}$$

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$$\boxed{\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}}$$

$$\boxed{\sin\phi_m = \frac{1-\alpha}{1+\alpha}}$$

$$M_m = \frac{1}{\sqrt{\alpha}}$$

$$\boxed{\cos\phi_m = \frac{2\sqrt{\alpha}}{1+\alpha}}$$

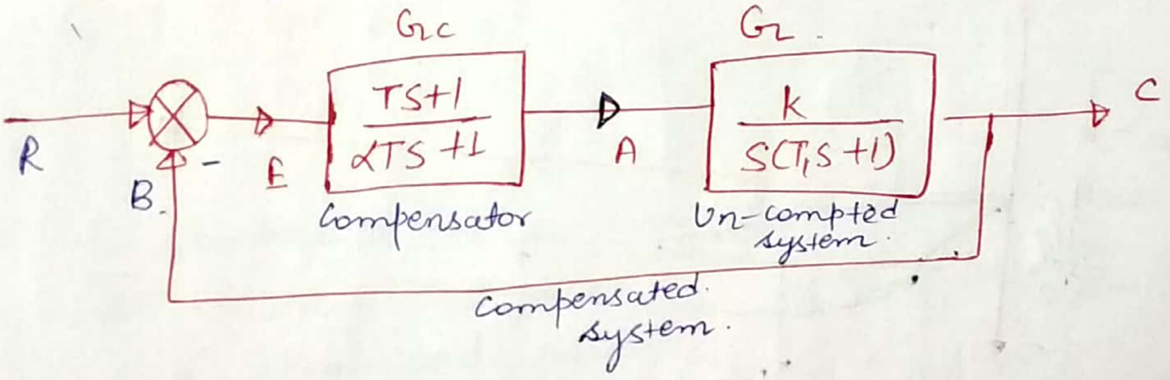
$\phi_m = 30^\circ$

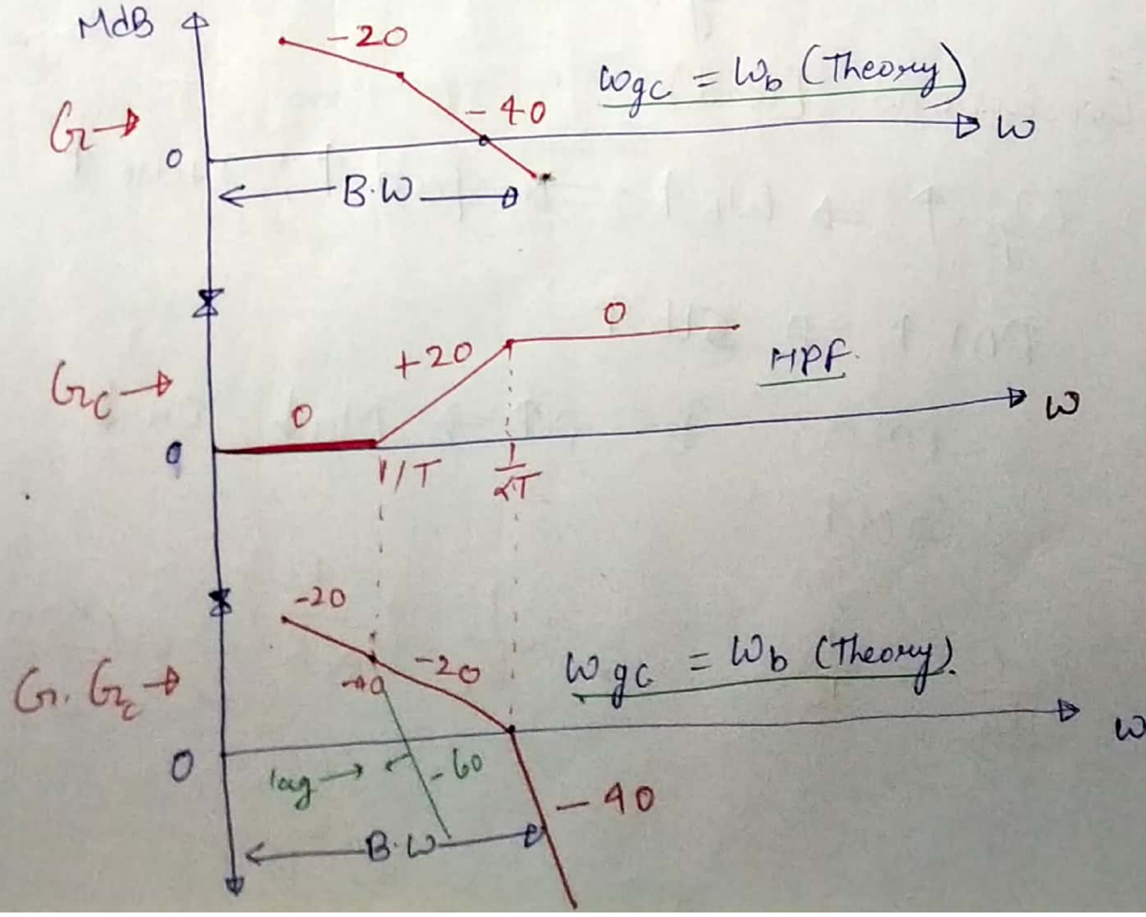
$\omega_m = 10\ r/s$

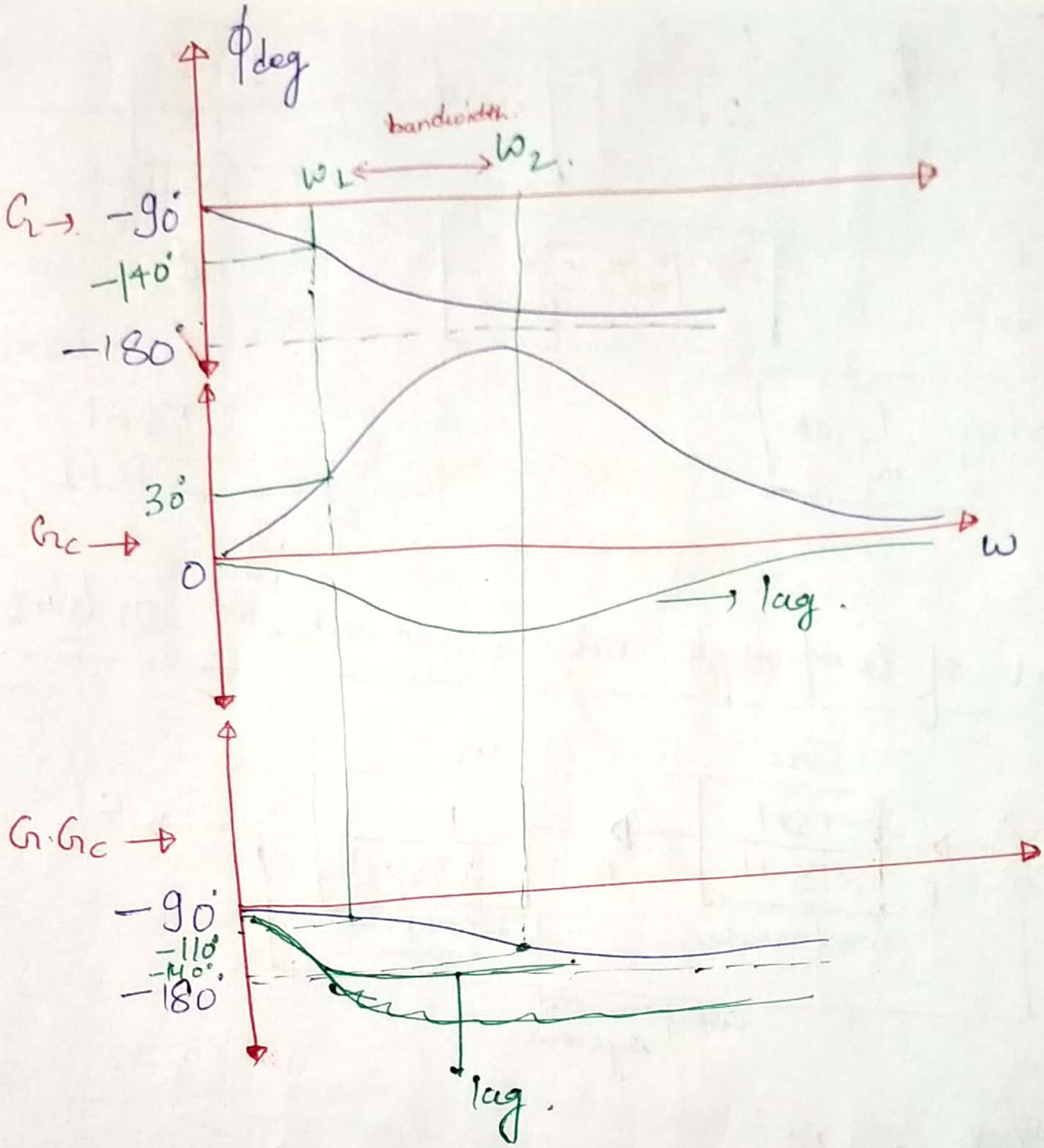
$$\frac{Ts+1}{\alpha Ts+1}$$

$$\boxed{M_m = 10\log_{10}\frac{1}{\alpha}\ dB}$$

*. Effect of Compensator on standard 2nd Order:







* Conclusions: (Lead)

$\omega_{gc} \uparrow \Rightarrow \omega_b \uparrow \Longrightarrow$ speed $\uparrow$; Noise $\uparrow$

$PM \uparrow \Rightarrow$ Stb $\uparrow$

$PM \propto \xi \Rightarrow \xi \uparrow \Rightarrow M_r \downarrow , \omega_r \downarrow$

$GM \uparrow$

$\frac{1}{K_v} = \frac{1}{K}$, $\frac{1}{K_v} = \frac{1}{\alpha K}$ $e_{ss} \uparrow$

* lead compensator is recommended to improve ~~noise and accuracy~~ ~~chara~~ speed and stability characteristics of system having good accuracy & noise characteristics.

Lag:

$\omega_{gc} \downarrow \Rightarrow \omega_b \downarrow \Rightarrow$ speed $\downarrow$; Noise $\downarrow$

$PM \downarrow \Rightarrow Stb \downarrow$

$PM \propto \xi \downarrow \Rightarrow \quad M_r \uparrow \quad \omega_r \uparrow$

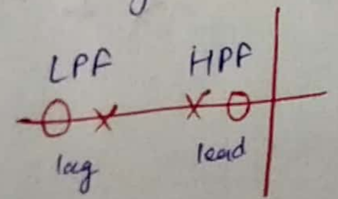
$GM \downarrow$

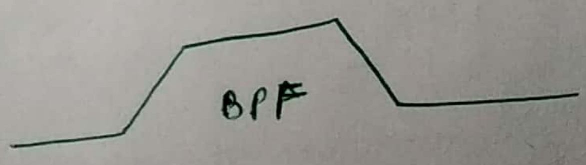
$(e_{ss} \downarrow)$

* lag compensator is recommended to improve noise and accuracy characteristics of a system having good speed and stability.

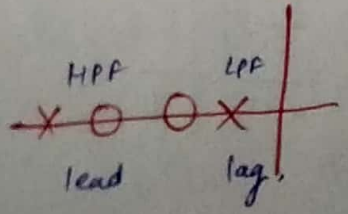
* lead-lag compensator which is cascade combination of lead + lag compensator is recommended to improve complete characteristics of a system.

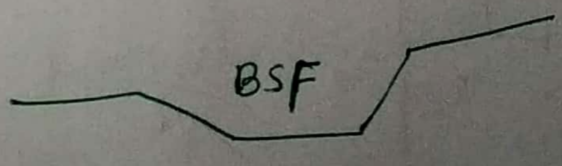
BPF





BSF





lead $\rightarrow$ HPF

lag $\rightarrow$ LPF

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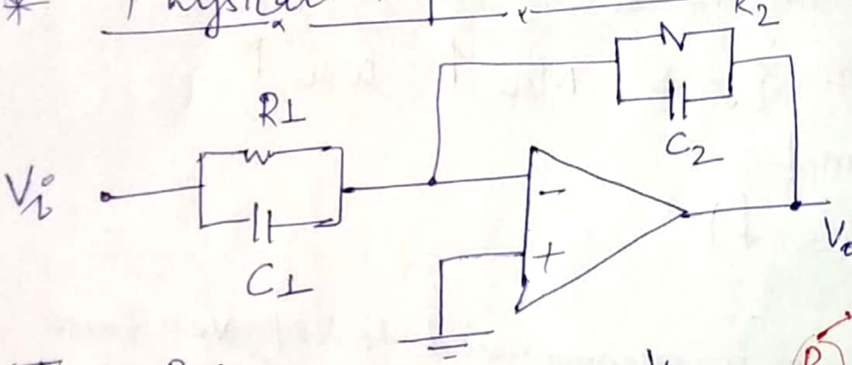
NOTE :

If single stage controller | Compensator fails to obtained desired characteristics then we implement multi-stage compensator | controller.

* Physical implementation



$T_z = R_1 C_1$.

$T_p = R_2 C_2$.

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\frac{[sR_1C_1+1]}{[sR_2C_2+1]}$$

α.

$R_1C_1 > R_2C_2$: Lead

$R_2C_2 > R_1C_1$; lag.

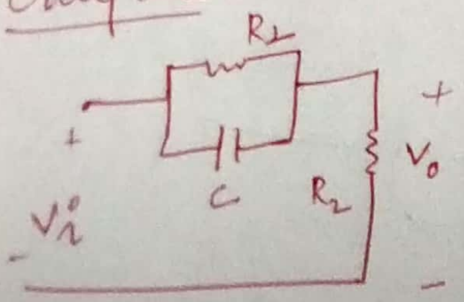
$$\frac{V_o}{V_i} \approx \frac{Ts+1}{\alpha Ts+1}$$

$T = R_1C_1$; $\alpha T = R_2C_2$.

$\alpha = \frac{R_2C_2}{R_1C_1}$ $\quad \alpha > 1$ lead

$\alpha < 1$ lag.

Only lead:



$$\frac{V_o}{V_i} = (\quad) = \frac{Ts+1}{\alpha Ts+1}.$$

$T = R_1 C$

$$\alpha = \frac{R_2}{R_1+R_2} < 1.$$

* State Variable Analysis :

$\underline{\text{Transfer F}} = f\,[\overset{x}{i/p},\ \overset{y}{o/p}]$

$\underline{\text{State Model}} = f[\underset{\text{present}}{\underset{\downarrow}{i/p}},\ \underset{\text{future}}{\underset{\downarrow}{o/p}},\ \underset{\text{past}}{\underset{\downarrow}{\text{state Variable}}}]$

State description of dynamic system is describing the future behaviour of system based on present and past behaviour.

* Input Variables describes present behaviour.
* O/p Varibles " future " .
* State " " past " .

* <u>State Variable</u>: These are the minimum set of variables such that knowledge of these variables along with input can completely describe the behaviour of system.

* Minimum no. of state Variables = Order.

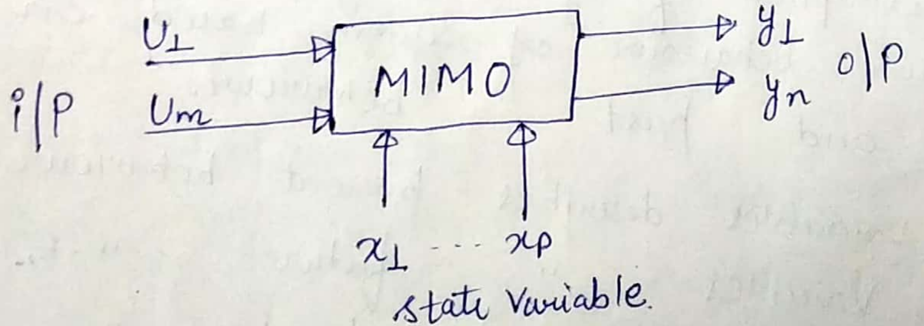
$$\frac{d^n y(t)}{dt^n} \longleftrightarrow s^n Y(s) - s^{n-1} y(0) - s^{n-2} y(0) \cdots$$

* Output and its derivates of system can be considered as state variables.

* Current through inductor, voltage across capacitors are state Variables.

* Output of an integrator is a state Variable.

* State Model:



$f(t) \rightarrow (\sin \omega_0 t, \cos n\omega_0 t)$

$$f(t) = a_0 \cos 0t + a_1 \cos \omega t + a_2 \cos 2\omega t + \cdots + b_0 \sin 0t + \cdots$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

Homogenous Steady Eqt^n

$$\boxed{\dot{x}(t) = \underset{P\times P}{A}\,\underset{P\times 1}{x(t)}} + \underset{P\times M}{B}.\underset{m\times 1}{U(t)} \quad ; \quad \text{state equations.}$$

↓ Sol^n ↓ Z[IR]

$$y(t) = \underset{n\times p}{C}.\underset{p\times 1}{x(t)} + \underset{n\times m}{D}.\underset{m\times 1}{U(t)} \quad ; \quad \text{o/p equations.}$$

1. $U(t)$ = i/p vector

$$= \begin{bmatrix} U_1 \\ \vdots \\ U_m \end{bmatrix}_{m\times 1}$$

2. $y(t)$ = o/p vector

$$= \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}_{n\times 1}.$$

3. $x(t)$ = State vector

$$= \begin{bmatrix} x_1 \\ \vdots \\ x_p \end{bmatrix}_{p\times 1}.$$

4. $\dot{x}(t) = \frac{d}{dt}x(t)$

$$= \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_p \end{bmatrix}$$

A, B, C, D ⟶ cofficient matrices

A ⟶ System matrix

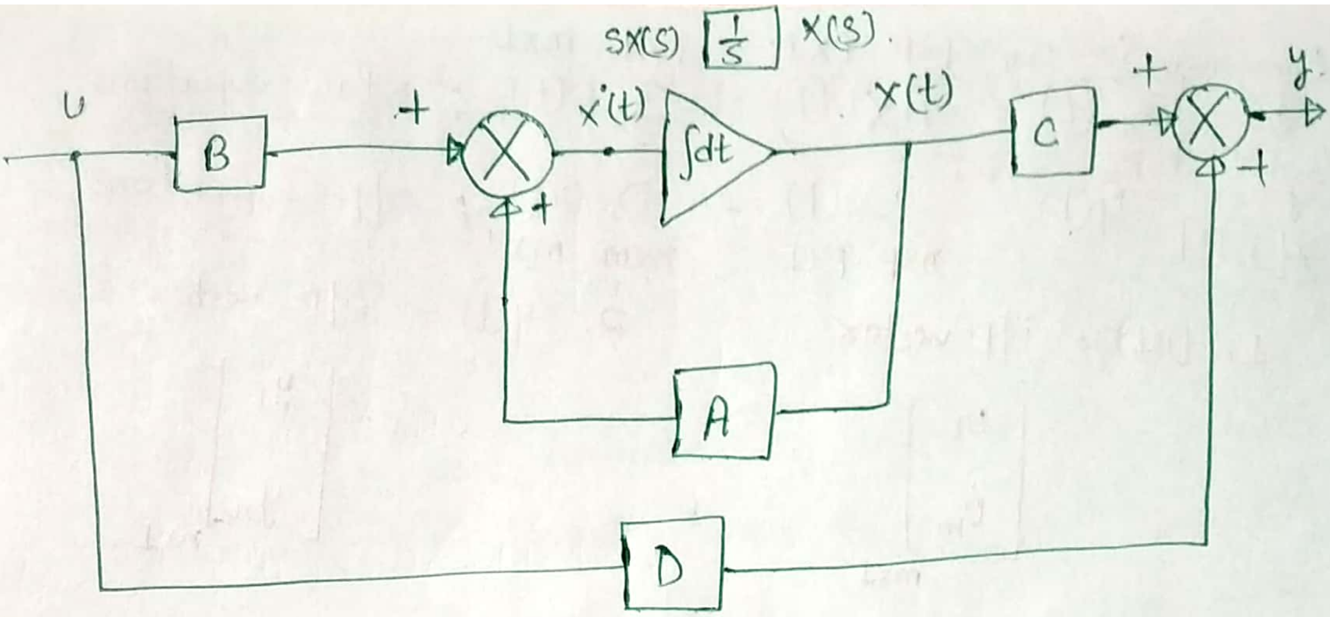
B ⟶ Input matrix

C ⟶ Output matrix

D ⟶ Transfer ~~function~~ matrix.

* State Diagram:

State diagram is graphical representation of state model which is either block diagram or signal flow graph.



* Only that block diagram or signal flow graph becomes state diagram which has atleast one integrator.

* <u>Differences b/w Transfer F. and State model based Analysis</u> :-

1. Transfer function model is applicable only for LTI system, whereas state model is applicable for any system,

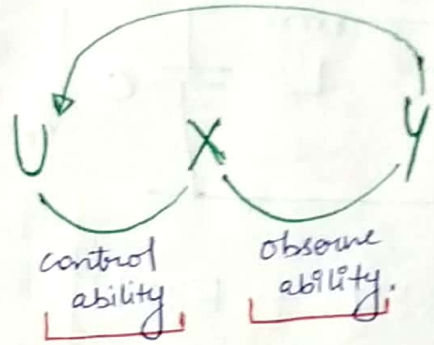
2. Transfer function model is valid both in time and frequency domain. But State model valid only in time domain.

3. Initial Conditions can't be considered in case of transfer function model hence Z(IR) can't be obtained but it can be obtained using state model.

4. Analysis of multivariable system is difficult using transfer-function model but it is easy using state model.

⑤. Transfer function model of system is unique but state model is not unique because of state variable is random.

⑥. Control ability and observe ability property of a system can be determine using state model but can't be determined using transfer function model.



⑦. It is easy to obtaine transfer function model and also easy to analyse system characteristics hence transfer function model is popular than state model.

* State Model

- physical Variable Method.
- Phase Variable Method.
  - Direct Method.
- Canonical variable method.
  - Series — Repeated eigen value
  - Parallel. — Distinct eigen value.

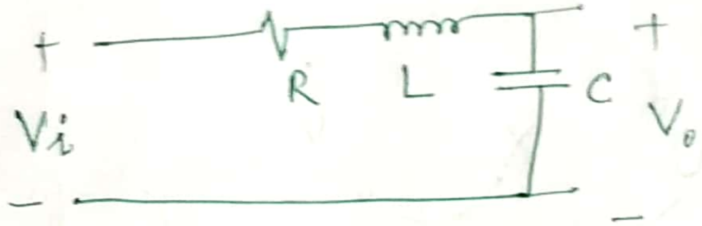
State Diagram.

* Physical Variable Method:

Physical Variables are those Variables that can be measured.

Eg: Voltage, current, displacement, velocity acceleration.

Q. obtain state model.



$$x^{\cdot}(t) = A.x(t) + B.U(t)$$

$$y(t) = C.x(t) + D.U(t).$$

$U = V_i$ $\quad\quad x = \begin{bmatrix} i_L \\ V_c \end{bmatrix}$

$y = V_o$ $\quad\quad x^{\cdot} = \begin{bmatrix} \dot{i}_L \\ \dot{V}_c \end{bmatrix}$

$$-V_i + Ri_L + L.\dot{i}_L + V_c = 0$$

$$\therefore \quad \dot{i}_L = -\frac{R}{L}.i_L - \frac{V_c}{L} + \frac{V_i}{L}$$

$$V_c = \frac{1}{C}\int \dot{i}_L.dt$$

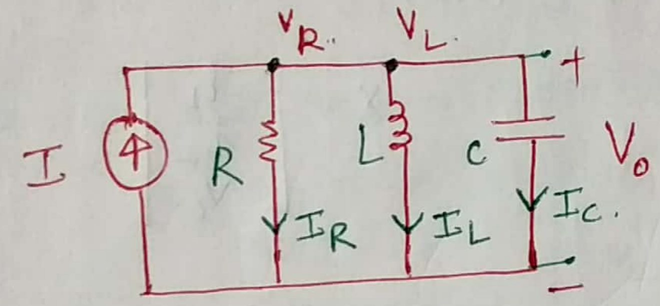
$$\therefore \boxed{\dot{V}_c = \frac{1}{C}.i_L}$$

$$\therefore \boxed{V_o = V_c}$$

$$\dot{X} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{matrix} i_L \\ X \\ v_C \end{matrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} U = V_i$$

$$V_o = Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{matrix} i_L \\ X \\ v_C \end{matrix} + \begin{bmatrix} 0 \end{bmatrix} U = V_i$$

Q.



$U = I, \quad Y = V_o$

$X = \begin{bmatrix} I_L \\ V_C \end{bmatrix}, \quad \dot{X} = \begin{bmatrix} \dot{I_L} \\ \dot{V_C} \end{bmatrix}$

$$I = i_R + i_L + i_C$$

$$I = \frac{V_C}{R} + i_L + C\dot{V_C}$$

$$\therefore \dot{V_C} = -\frac{i_L}{C} - \frac{V_C}{R \cdot C} + \frac{I}{C}$$

$$\therefore V_L = V_C = L\dot{i_L} \Rightarrow \dot{i_L} = \frac{V_C}{L}$$

$$\therefore V_o = (1) \cdot V_C$$

$$\begin{matrix} \dot{i_L} \\ \dot{V_C} \end{matrix} \dot{X} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{matrix} i_L \\ X \\ V_C \end{matrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} U = I$$

$$V_o = Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{matrix} i_L \\ X \\ V_C \end{matrix} + \begin{bmatrix} 0 \end{bmatrix} U = I$$

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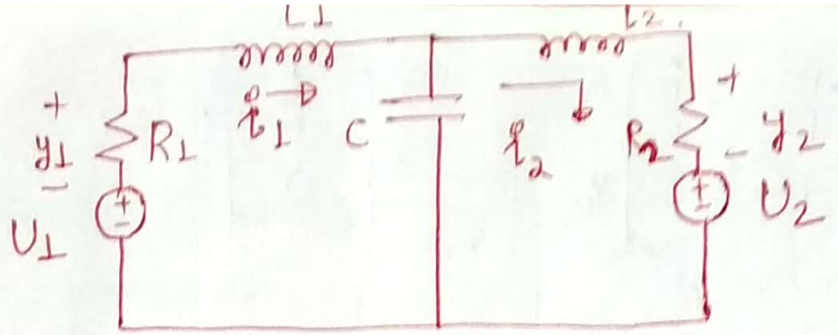
Q



$$\dot{x}(t) = A.x(t) + B.U(t).$$

$$y(t) = C.x(t) + D.U(t).$$

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad X = \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{V}_c \end{bmatrix}$$

$$-U_1 + R_1 i_1 + L_1 \dot{i}_1 + V_c = 0$$

$$\therefore \quad \dot{i}_1 = -\frac{R_1}{L_1}.i_1 - \frac{V_c}{L_1} + \frac{U_1}{L_1}$$

$$-V_c + L_2.\dot{i}_2 + R_2.i_2 + U_2 = 0$$

$$\therefore \quad \dot{i}_2 = -\frac{R_2}{L_2}.i_2 + \frac{V_c}{L_2} - \frac{U_2}{L_2}$$

$$V_c = \frac{1}{C}\int (i_1 - i_2).dt$$

$$\therefore \quad \dot{V}_c = \frac{1}{C}i_1 - \frac{1}{C}i_2.$$

$$\dot{x} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} X + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} \\ 0 & 0 \end{bmatrix} U$$

$$\therefore \quad y_1 = -R_1 i_1$$

$$y_2 = R_2 i_L.$$

$$y = \begin{bmatrix} -R_1 & 0 & 0 \\ 0 & R_2 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} U$$

Phase Variable:

* Phase variables are those variables that are obtained from one of the system variable and its derivates. Usually the variable choosen is the o/p of the system.

$$\frac{d^3 y(t)}{dt^3} + 5\cdot\frac{d^2 y(t)}{dt^2} + 6\cdot\frac{dy(t)}{dt} + \underbrace{3\cdot y(t)}_{o/p} = \underbrace{U(t)}_{i/p}$$

No. of state var. = order = 3.

Let $x_1 = y$ | $\dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y}$ | $\dot{x}_2 = \ddot{y} = x_3$

$x_3 = \ddot{y}$ | $\dot{x}_3 = \dddot{y} = -3x_1 - 6x_2 - 5x_3 + U$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -6 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

(Controlable form)

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \end{bmatrix} U$$

* State model obtained using phase variable method will either in controllable form or in observable form.

Controlable form (CF)		(OF) Observable form
A^T	=	A
B^T	=	C
C^T	=	B
D^T	=	D

(Observable form :)

$$\dot{x} = \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & -6 \\ 0 & 1 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x + [0] U$$

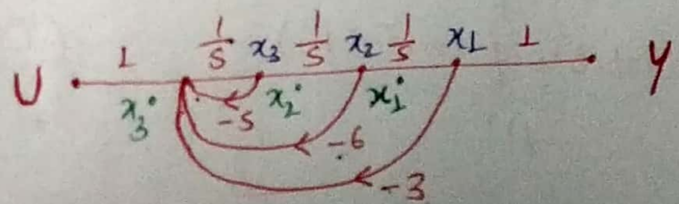
* Direct Method of ~~state~~ ~~ooo~~ decomposition of Transfer function into state Model.

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 5s^2 + 6s + 3} \times \frac{s^{-3}}{s^{-3}}$$

$$= \frac{s^{-3}(1-0) + 0}{1 - [-5s^{-1} - 6s^{-2} - 3s^{-3}] + 0}$$

No. of nodes = 3 + order

i/P   O/P   Reference.



$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -3x_1 - 6x_2 - 5x_3 + U$

$y = x_1$.

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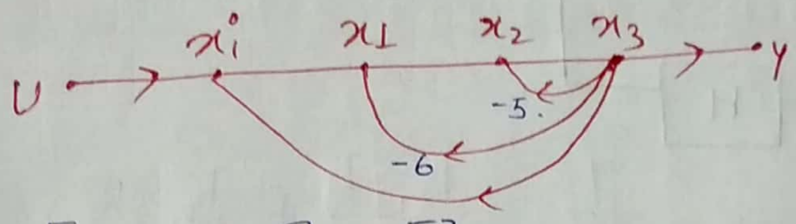
$$\dot{X} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -3 & -6 & -5\end{bmatrix}x + \begin{bmatrix}0\\0\\1\end{bmatrix}U$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}x + [0]U$$

> if all loops departs from same points then observable & if arrive at single path then controlable.



$\dot{x}_1 = -3x_3 + U$

$\dot{x}_3 = x_2 - 5x_3$

$\dot{x}_2 = x_1 - 6x_3$

$y = x_3$

$$\dot{X} = \begin{bmatrix}0 & 0 & -3\\ 1 & 0 & -6\\ 0 & 1 & -5\end{bmatrix}x + \begin{bmatrix}1\\0\\0\end{bmatrix}U$$

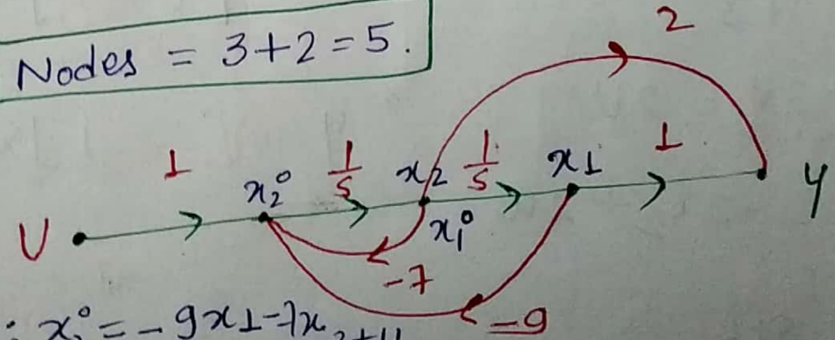
$$y = \begin{bmatrix}0 & 0 & 1\end{bmatrix}x + [0]U$$

$\boxed{\text{Observable form}}$

Q. $T.F = \dfrac{2s+1}{s^2+7s+9}$; obtain SD from by Direct Method thus obtain State Model.

$$\frac{2s+1}{s^2+7s+9}\cdot\frac{s^{-2}}{s^{-2}} = \frac{2s^{-1}(1-0)+s^{-2}(1-0)}{1-[-7s^{-1}+9s^{-2}]}$$

$\boxed{\text{Nodes} = 3+2 = 5.}$



$\dot{x}_1 = x_2$; $\dot{x}_2 = -9x_1 - 7x_2 + U$

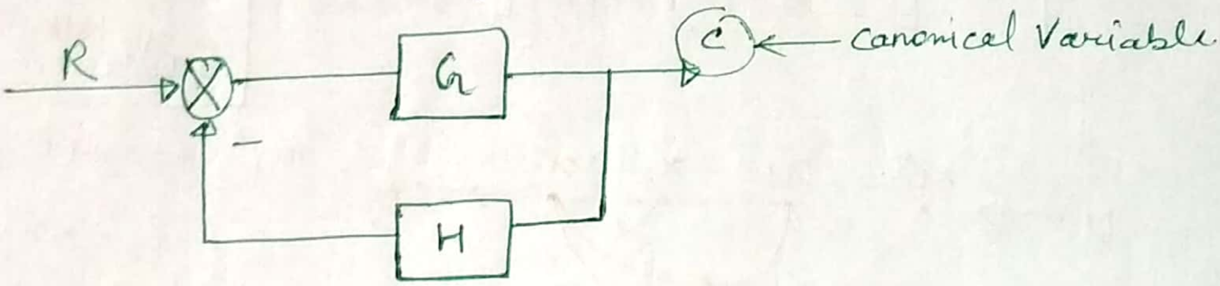
$y = x_1 + 2x_2$

$$\dot{X} = \begin{bmatrix}0 & 1\\ -9 & -7\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}U$$

$$y = \begin{bmatrix}1 & 2\end{bmatrix}x + [0]U$$

* Canonical Variable Method:

Canonical variable is a o/p of canonical system also called closed loop or feedback system.



$$\frac{Y}{U} = \frac{S+1}{(S+2)(S+3)} \qquad \frac{Y}{U} = \frac{k_1 = -1}{S+2} + \frac{k_2 = 2}{S+3}$$

$$\frac{Y}{U} = \frac{X_1}{U_1} + \frac{X_2}{U_2}$$

$$\boxed{\begin{matrix} U = U_1 = U_2 \\ Y = X_1 + X_2 \end{matrix}}$$

$$\frac{X_1}{U} = -\frac{1}{S+2}$$

$$SX_1 + 2X_1 = U \quad [ILT]$$

$$\dot{x}_1 = -2x_1 - u$$

$$\frac{X_2}{U} = \frac{2}{S+3}$$

$$SX_2 + 3X_2 = 2U$$

$$\dot{x}_2 = -3x_2 + 2u$$

eigen value

$$\dot{X} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} X + \begin{bmatrix} -1 \\ 2 \end{bmatrix} U$$

$$Y = [1 \quad 1] X + [0] U$$

$$\frac{Y}{U} = \frac{S+1}{(S+2)(S+3)}$$

State model obtained using canonical variable method ~~during~~ will be in diagonal form in case of distinct eigen values & will be in triangular form in case of one or more repeated eigen values.

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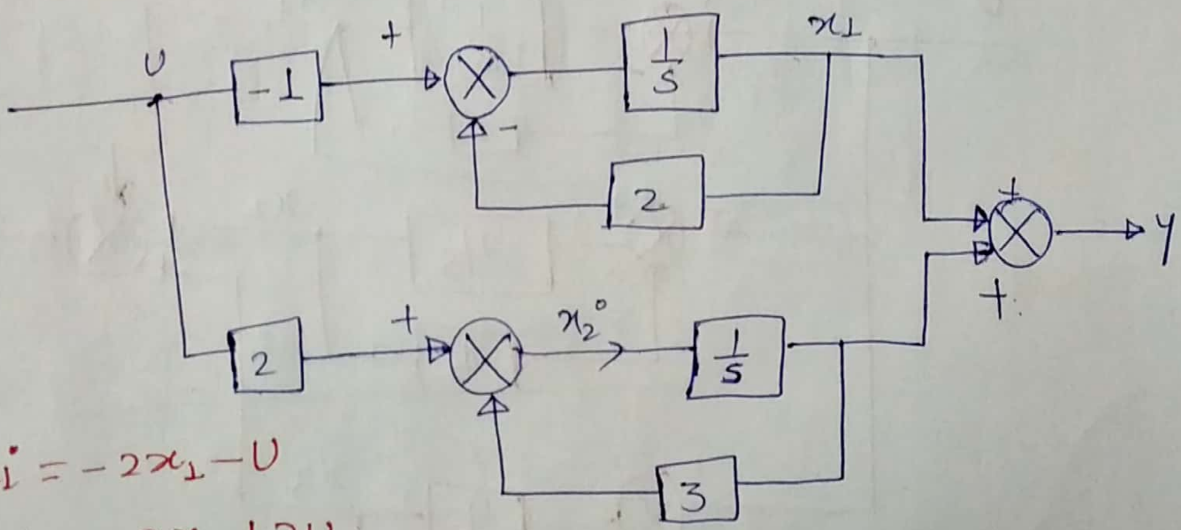
* Parallel method of decomposing the T.F into State Diagram:-

Q. $\frac{y}{u} = \frac{s+1}{(s+2)(s+3)}$; $\frac{y}{u} = -\frac{1}{s+2} + \frac{2}{s+3}$.

$$y = \underbrace{\underbrace{\frac{\frac{1}{s}}{1+\left(\frac{1}{s}\right)(2)}}_{\text{Canonical}}\underbrace{(-1u)}_{i/p}}_{x_1} + \underbrace{\underbrace{\frac{\frac{1}{s}}{1+\frac{1}{s}(3)}}\;\underbrace{2(u)}_{i/p}}_{x_2}$$



$\dot{x_1} = -2x_1 - u$

$\dot{x_2} = -3x_2 + 2u$

$y = -3x_2 + 2u$

$$\dot{X} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 2 \end{bmatrix} x + [0] u$$

* Obtain State ~~method~~ model by state diagram using series method:

(~~Diagonal form~~)

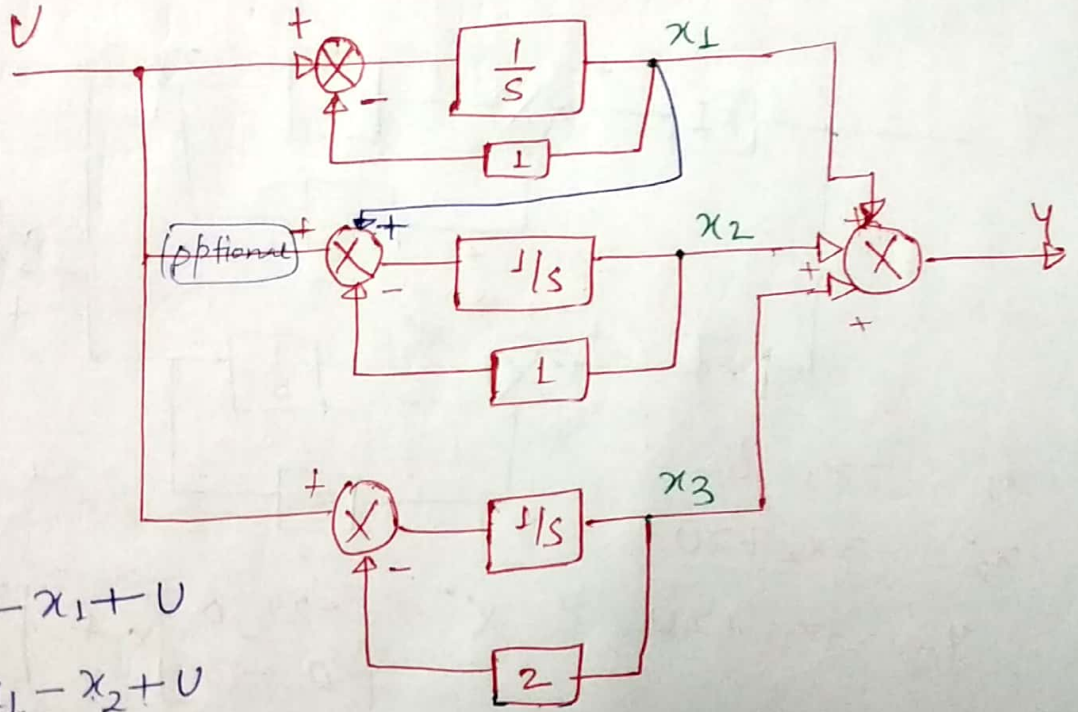
(Triangular form)

Q. $$\frac{y}{U} = \frac{2s^2+6s+5}{s^3+4s^2+5s+2}$$

$$\frac{y}{U} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+2)}$$

$$\frac{y}{U} = \frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+2)}$$

$$y = \left[\frac{1/s}{1+\frac{1}{s}(1)} + \left[\frac{1/s}{1+\frac{1}{s}(1)}\right]^2 + \frac{\frac{1}{s}}{1+\frac{1}{s}(2)} \right]$$



$\dot{x}_1 = -x_1 + U$

$\dot{x}_2 = x_1 - x_2 + U$

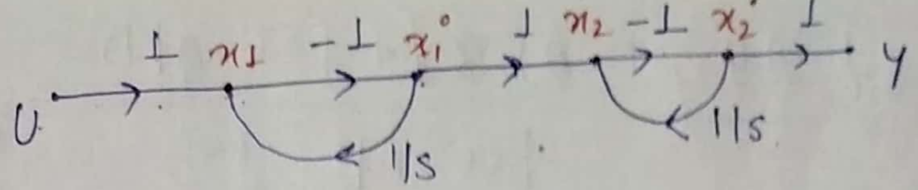
$\dot{x}_3 = -2x_3 + U$

$y = x_1 + x_2 + x_3.$

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} U$$

Q



$x_1^\circ = -x_1$, $x_2^\circ = -x_2$

$x_1 = x_1 + U$ $\qquad$ $x_2 = x_1^\circ + x_2 = -x_1 - U + x_2$

$x_1^\circ = -x_1 - U$, $\quad x_2^\circ = +x_1 - x_2 + U$

$$y = x_2^\circ = x_1 - x_2 + U.$$

$$\therefore \quad x^\circ = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \end{bmatrix} U$$

* **State Model $\longrightarrow$ T.F :-**

$x^\circ = Ax + BU$; $y = Cx + DU$

L.T $\qquad\qquad\qquad\qquad$ L.T

$sX(s) - x(0) = A.X(s) + B.U(s)$ $\qquad$ $Y(s) = C.X(s) + D.U(s)$

$x(0) \rightarrow 0$

$X(s)[s - A] = B.U(s)$ $\qquad$ $= C[(sI - A)^{-1}B].U(s) + D.U(s)$

$X(s) = [sI - A]^{-1} B.U(s)$ $\qquad$ $\dfrac{Y(s)}{U(s)} = C[(sI-A)^{-1}.B] + D$

$$\frac{Y}{U} = C\left[(sI - A)^{-1}.B\right] + D$$

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$$\frac{Y}{U} = \frac{C[\,Adj(SI-A).B\,] + |SI-A|D}{|SI-A|} = \frac{G(S)}{1+G(S).H(S)}$$

$$= \sum_{k=1}^{n} \frac{P_k \Delta_k}{\Delta}$$

Standard Charteristics Eqtn:

$$q(S) = |SI-A| = 1 \pm G(S).H(S) = \Delta(S) = 0$$

$$\boxed{\text{Roots of } q(S) = \text{Eigen Values} = CLP}$$

R.H crit-

* Stability analysis using state model is same as time domain stability analysis using transfer function model.

* Response:

$$\boxed{X^{\cdot} = AX} + BU$$

$$\underbrace{Y}_{\text{find}} = C\underbrace{X}_{\text{required}} + D\underbrace{U}_{\text{Given}}$$

* Solution of State Equation:-

$$x^{\cdot}(t) = A.\,x(t) + B.\,U(t)$$

$$S.\,X(S) - X(0) = A.\,X(S) + B.\,U(S)$$

$$X(S)[SI-A] = x(0) + B.\,U(S).$$

$$X(S) = (SI-A)^{-1}.\,x(0) + (SI-A)^{-1}.\,B.\,U(S)$$

$$\Rightarrow x(t) = L^{-1}[(SI-A)^{-1}].\,x(0) + L^{-1}[(SI-A)^{-1}.\,B\,U(S)]$$

$$\boxed{x(t) = e^{At}.\,x(0)} + e^{At} * B.\,U(t)$$

ZIR.

$$\Rightarrow \quad x(t) = e^{At} \cdot x(0) + \int_0^t e^{Ak} \cdot B\, U(t-k)\, dk.$$

$$\boxed{\phi(t) = e^{At} = L^{-1}[(SI-A)^{-1}]}$$; state transition matrix.

(governs responses by initial conditions)

$[SI-A]^{-1}$; state resolvent matrix.

ZIR : $X(t) = e^{At} \cdot x(0)$

$X(0) = e^{[0]} \cdot x(0)$

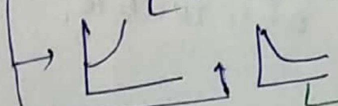
$X(0) = I\, x(0)$

$$\boxed{X(0) = x(0)}$$

1. $\phi(0) = I$
2. $\phi^{-1}(t) = \phi(-t)$
3. $\left[\frac{d}{dt}\phi(t)\right]_{t=0} = A$

$e^{At} = [\quad]$ $X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$ZIR = \begin{bmatrix} e^{\lambda t} \\ e^{-\lambda t} \end{bmatrix}$



↳ Assymptotic stability.

state transition matrix

* $A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \Rightarrow \phi(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$

* $A = \begin{bmatrix} \lambda_1 & 0 \\ 1 & \lambda_1 \end{bmatrix} \Rightarrow \phi(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ t\cdot e^{\lambda_1 t} & e^{\lambda_1 t} \end{bmatrix}$

* Response Analysis Using state Model is same as time response analysis using transfer function model.

$X(t) = ZIR + ZSR$

$X(t) = e^{A(t)} \cdot x(0) + e^{A(t)} \cdot B\, U(S)$

$y(t) = [C]\, X(t) + [D]\, U(S)$

$Z[SR] \rightarrow$ Forced Response

$Z[IR] \rightarrow$ Free Response

* Contrallability & Observalibility:

→ A system is said to be controlable or state controlable if it is possible to transform existing state of the system to any desired state by applying the input within finite time.

*** Observalilibity:** A system is said to be observable or o/p observable if it is possible to observe all the state variables in o/p within finite time.

↳ If the system is stable then steady state is guarenteed but desired characterstics are not guarenteed, it can be guarenteed if the system is controlable and observable.

$$\dot{X} = AX + BU \quad ; \quad Y = CX + DU$$

$Q_c = [B \;\; AB \;\; A^2B \dots]$

$|Q_c| \neq 0 \Rightarrow$ Controllable

$1 \le r \le n$ $\quad = 0 \Rightarrow$ Un-controllable ⇓ $(n-r)$ [No. of states causing un-controlability]

Order $= n$

rank $= r$

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}$$

$|Q_o| \neq 0 \Rightarrow$ observable

$= 0 \Rightarrow$ un-observable ⇓ $(n-r)$

No. of states causing un-observability

* if any two state becomes equal then controlability & observality fails

* if $x_1 = x_2$ or if $A = I$ then system is neither contralable nor observable.

* Only that system can be shown controlable & observable in which pole zero calcilation doesn't takes place. But vice-versa need not be true.

Q. $T.F = \frac{S+1}{(S+1)(S+2)} = \frac{S+1}{S^2+3S+2}$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + [0] u$$

$$Q_c = \begin{array}{cc} B & AB \\ \left[\begin{array}{c} 0 \\ 1 \end{array}\right. & \left.\begin{array}{c} 1 \\ -3 \end{array}\right] \end{array} \qquad Q_o = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{array}{c} C \\ CA \end{array}$$

$|Q_c| \neq 0$ ✓ controlable

$|Q_o| = 0.$ un-observable

Q.

controllable form

$$\left.\begin{aligned} \dot{X} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -6 & -5 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U \\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X + [0].U \end{aligned}\right.$$

$\} = 0$

$\neq 0$ for contrialibity

$$\dot{X} = \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & -6 \\ 0 & 1 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} .U$$

$$y = [\underbrace{0 \quad 0}_{=0} \quad \underbrace{1}_{\neq 0}] X + [0].U$$

→ Observable form

for observility

Diagonal form

$$\dot{X} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} X + \begin{bmatrix} -1 \\ 2 \end{bmatrix} U \quad \} \neq 0$$

$\boxed{\text{Controllable}}$

$$y = [\underbrace{1 \quad 1}_{\neq 0}] X + [0] U$$

$\boxed{\text{Observable}}$

$\boxed{\text{Controllable.}}$

triangular form

$$\dot{X} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} U \quad \} \text{ Don't care}$$

$\neq 0$

$$y = [\underbrace{1}_{\neq 0} \quad \underbrace{1}_{\text{Don't Care}}] X + [1] U$$

$\boxed{\text{Observable}}$

Q.

$$\dot{X} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} U$$

$\alpha \rightarrow$ Don't care

$\beta \neq 0$

$\gamma \neq 0$ $\boxed{\text{controllable}}$

$$y = \begin{bmatrix} a & b & c \end{bmatrix} x + [0] U$$

$a \neq 0$ $\quad$ ~~b≠0~~ Don't care $\quad$ $c \neq 0$

$\boxed{\text{observable}}$

6th ch.

1 → b
2 → d
3 → c
4 → c
5 → b
6 → b
7 → d
8. → b
9. → a
10. → c
11. → a.
12. → a.
13 → c.
14 → d.
15 → c
16. → a
17. → c
18. → a.
19. → c
20. → a.
21. → 30°
22. 50 n/s.

Sir: $\boxed{8527020784}$

7th ch.

1 → a
2 → d
3 → d
4 → a
5 → a.
6. → c
7. → c
8. → a.
9. → c
10. → b
11. → a.
12. → a
13. → d.
14. → a.
15. → d
16 → a
17 → a.
18 $K_1 = 1$, $K_2 = -1$
19 b.