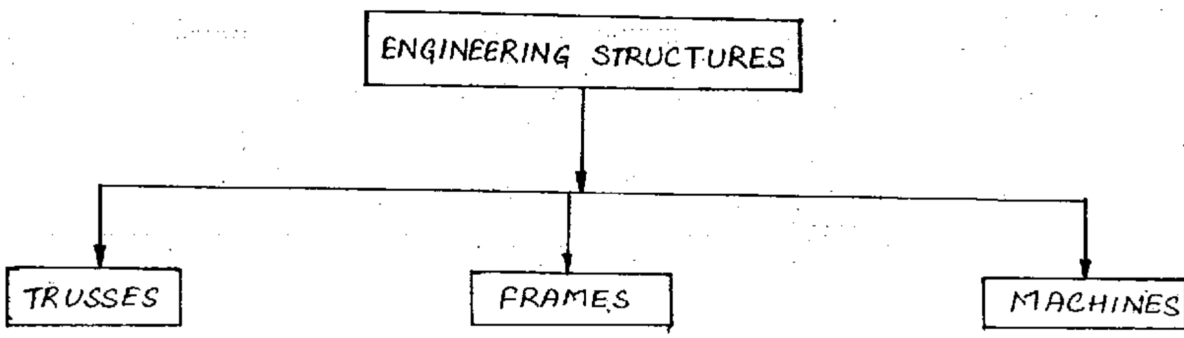
ENGINEERING STRUCTURES

These are defined as any system of connected members built to support or transfer forces acting on them and to safely withstand these forces.

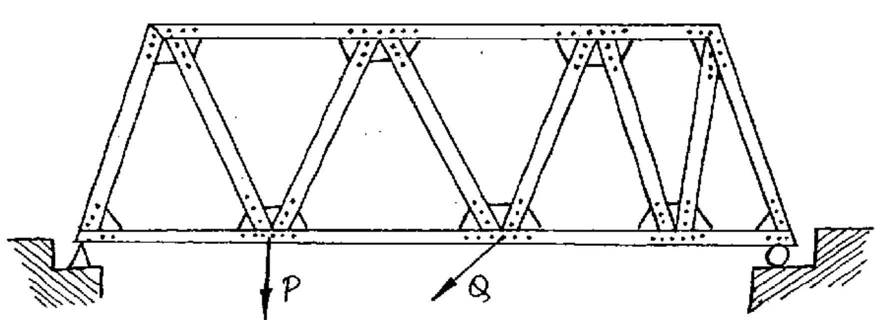


1) TRUSS → System of uniform bars or members joined together at their ends by rivetting or welding and constructed to support loads.

a) Members of a truss are straight members.

b) Loads are applied only at joints.

c) Every truss member is a two force system.

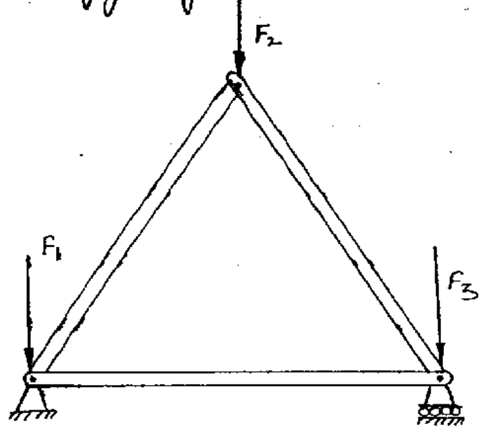


2) FRAME ⟶ Structure consisting of several bars or members pinned together and in which one or more than one of its members is subjected to more than two forces.

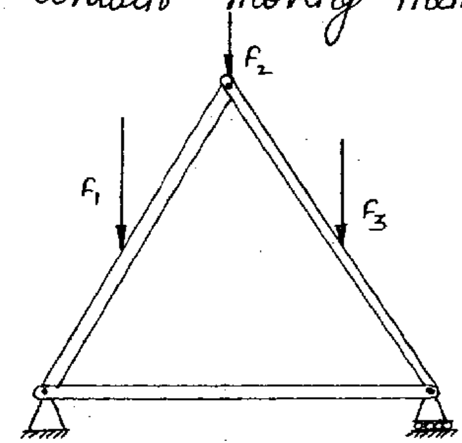
a) Designed to support loads

b) Stationary structures.

3) MACHINE ⟶ Structures designed to transmit and modify forces and they contain moving members.



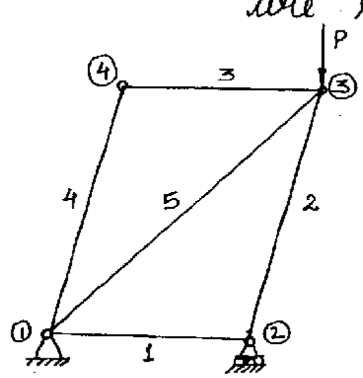
TRUSS

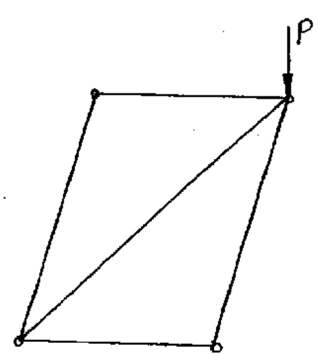


FRAME

RIGID OR PERFECT TRUSS

Rigid ⟶ Non-collapsible if all external supports are removed.



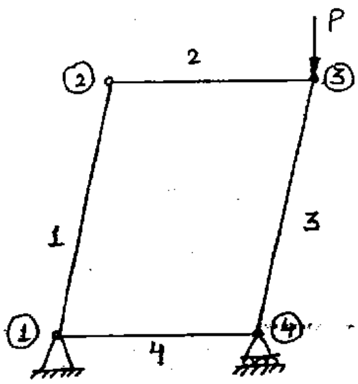


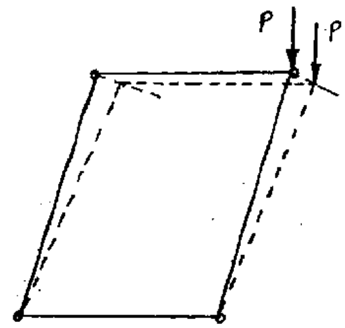
$m = 5$

$j = 4$

$\boxed{m + 3 = 2j}$

NON-COLLAPSIBLE





$m=4$
$j=4$
$\boxed{m+3<2j}$

COLLAPSIBLE

MATHEMATICAL CONDITION FOR RIGID OR PERFECT TRUSS

For a truss to be rigid or perfect, the relationship between its number of members and number of joints is

$$\boxed{m+3=2j}$$

$m \longrightarrow$ Number of members.
$j \longrightarrow$ Number of joints

1) If $m+3>2j$

Truss contains more members than required to be rigid and hence it is over rigid and statically indeterminate.

2) If $m+3<2j$

Truss contains less members than required to be just rigid and hence it is collapsible or under rigid.

STATICALLY DETERMINATE $\longrightarrow$ A truss is statically determinate if the equations of equilibrium (static) alone are sufficie to determine the axial forces in the members without the need of considering their deformations.

BASIC ASSUMPTIONS FOR THE PERFECT TRUSS.

1) Joints of a simple truss are assumed to be pin connections and frictionless. Therefore, joints cannot resist moments.

2) Loads on the truss are applied at the joints only.

3) Members are straight two force members with the forces acting collinear with the centre line of the members.

4) Weights of the members are negligibly small unless otherwise mentioned.

5) Truss is statically determinate.

TRUSS : DETERMINATION OF AXIAL FORCES IN THE MEMBERS.

1) Method of Joints

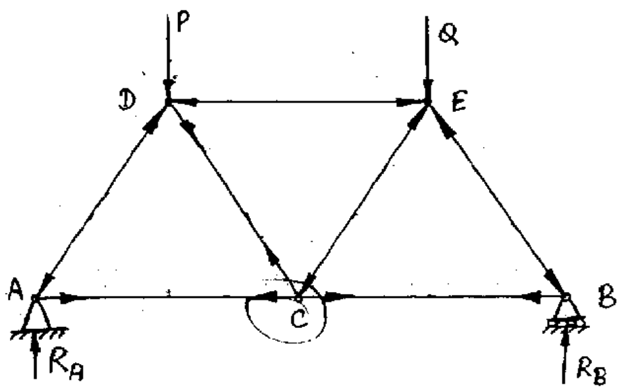
2) Method of Sections

3) Graphical Method.

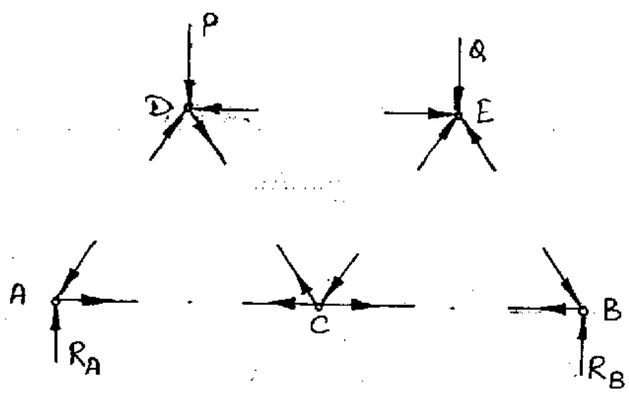
METHOD OF JOINTS

Consider a free-body diagram of the entire truss and compute the support reactions using equations of equilibrium.

Determination of support reactions may not be necessary in case of a cantilever type of truss.

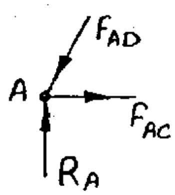


If in the solution the magnitude of a force comes out to be negative the assumed direction of the force in the member is simply reversed.

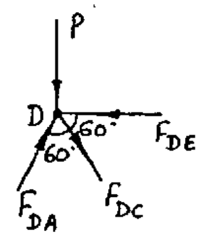


Now choose a joint and consider its free-body diagram. The forces acting on the joint represent a system of concurrent forces in equilibrium. Hence only two equations of equilibrium can be written for each joint and can be solved to find unkown forces.

Hence, start from a joint where not more than two unknown forces append.



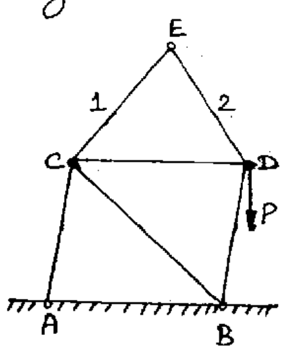
JOINT A

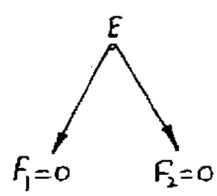


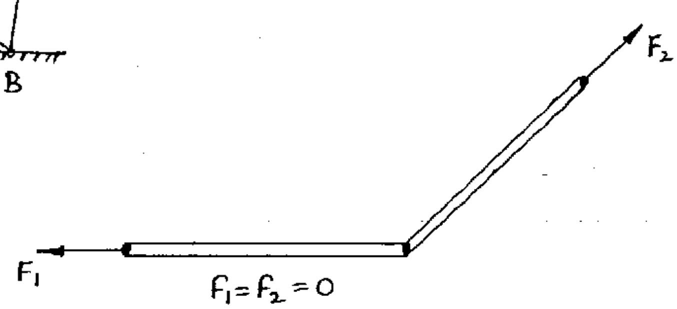
JOINT D

SPECIAL CONDITIONS

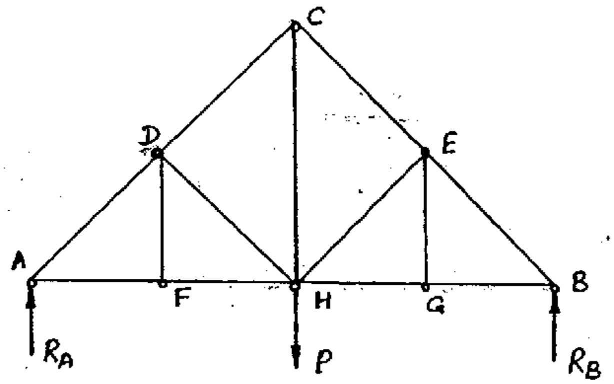
1) When two members meeting at a joint are not collinear and there is no external force acting at the joint, then the forces in both the members are zero.

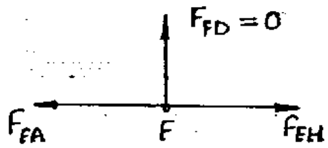


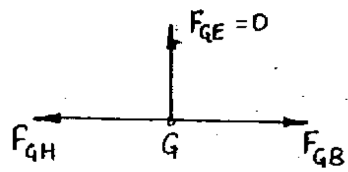


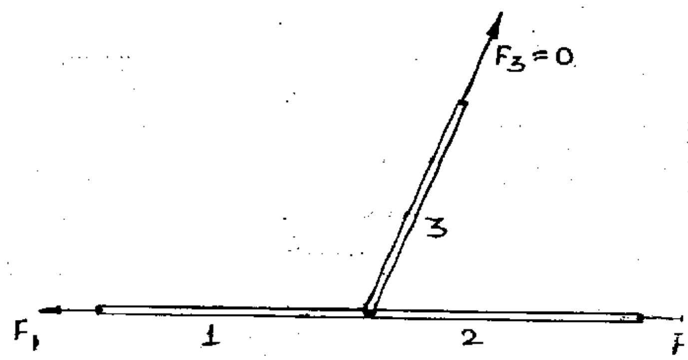


2) When there are three members meeting at a joint, of which two are collinear and the third be at an angle and if there is no load at the joint the force in the third member is zero.

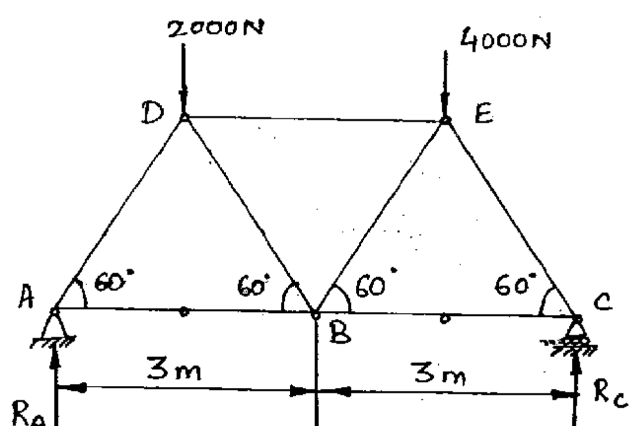








EXAMPLE :



SOL :

Taking moment about A

$$\Sigma M_A = 0$$

$$-2000(1.5) - 4000(4.5) + R_C(6) = 0$$

$$-3000 - 18000 + R_C(6) = 0$$

$$6R_C = 21000$$

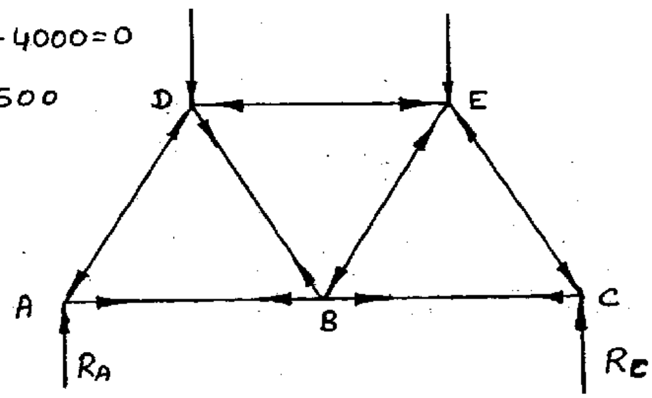
$$\boxed{R_C = 3500\text{N}}$$

$\Sigma F_y = 0$

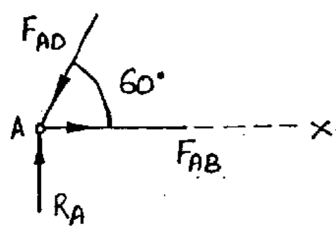
$R_A + R_C - 2000 - 4000 = 0$

$R_A = 6000 - 3500$

$\boxed{R_A = 2500 N}$



JOINT A :



$\Sigma F_x = 0$

$$- F_{AD} \cos 60^\circ + F_{AB} = 0$$

$$F_{AD} = \frac{F_{AB}}{\cos 60^\circ} \longrightarrow (1)$$

$\Sigma F_y = 0$

$$R_A - F_{AD} \sin 60^\circ = 0$$

$$R_A = 2500 N$$

$$\therefore \quad F_{AD} = \frac{R_A}{\sin 60^\circ} = \frac{2500}{0.866}$$

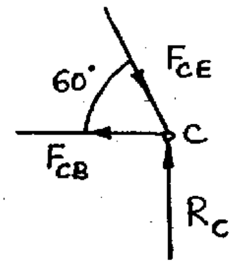
$$F_{AD} = 2886.7$$

$$\boxed{F_{AD} = 2887 N}$$

$$F_{AB} = 2887 \cos 60^\circ = 2887 \times 0.5$$

$$\boxed{F_{AB} = 1443.5 N}$$

JOINT C :



$\Sigma F_x = 0$

$$F_{CE}\cos 60^\circ - F_{CB} = 0$$

$\Sigma F_y = 0$

$$R_C - F_{CE}\sin 60^\circ = 0$$

$$F_{CE} = \frac{R_C}{\sin 60^\circ} = \frac{3500}{\sin 60^\circ}$$

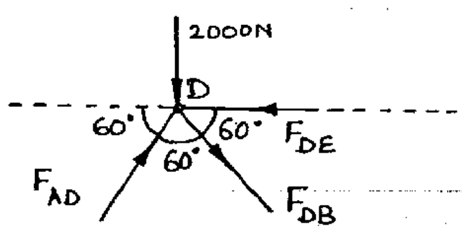
$$F_{CE} = 4041.4$$

$$\boxed{F_{CE} = 4041\,N}$$

$$F_{CB} = F_{CE}\cos 60^\circ$$
$$= 4041\cos 60^\circ$$

$$\boxed{F_{CB} = 2020.5\,N}$$

JOINT D :



$\Sigma F_x = 0$

$$F_{DB}\cos 60^\circ + F_{AD}\cos 60^\circ - F_{DE} = 0$$

$\Sigma F_y = 0$

$$F_{AD}\sin 60^\circ - F_{DB}\sin 60^\circ - 2000 = 0$$

$F_{AD} = 2887N$

$2887 \times 0{\cdot}866 - F_{DB} \times 0{\cdot}866 - 2000 = 0$

$-F_{DB} \times 0{\cdot}866 = 2000 - 2887 \times 0{\cdot}866$

$-F_{DB} \times 0{\cdot}866 = -500{\cdot}142$

$F_{DB} = 577{\cdot}5$

$\boxed{F_{DB} = 577N}$

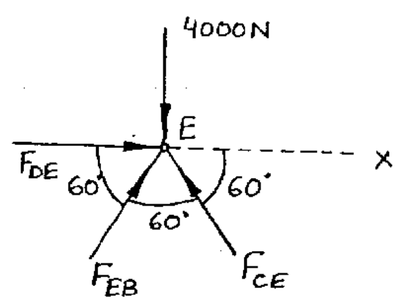
$F_{DE} = 577 \cos 60^\circ + 2887 \cos 60^\circ$

$= 577 \times 0{\cdot}5 + 2887 \times 0{\cdot}5$

$= 288{\cdot}5 + 1443{\cdot}5$

$\boxed{F_{DE} = 1732N}$

JOINT E



$\Sigma F_x = 0$

$F_{DE} + F_{EB} \cos 60^\circ - F_{CE} \cos 60^\circ = 0$

$1732 + F_{EB} \times 0{\cdot}5 - 4041 \times 0{\cdot}5 = 0$

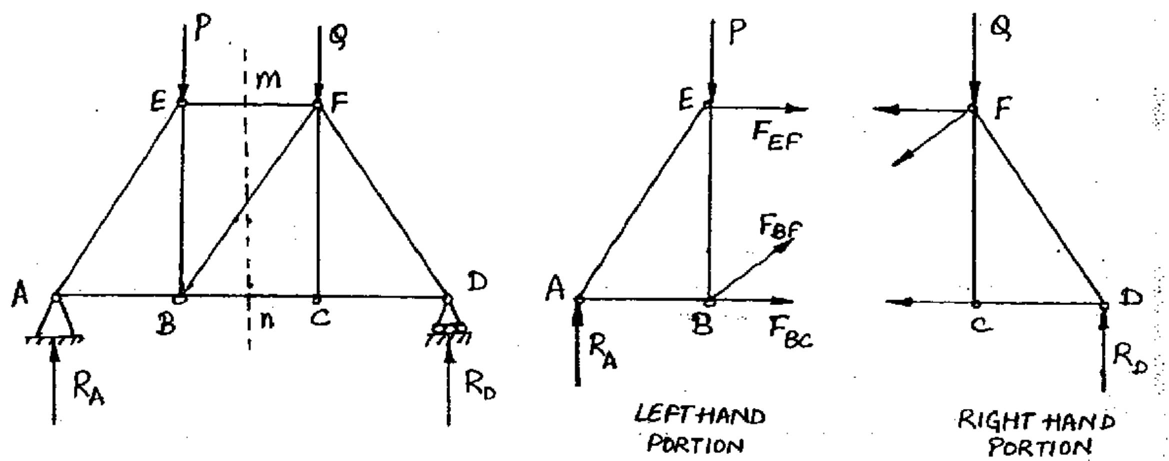
$$F_{EB} = \frac{4041 \times 0{\cdot}5 - 1732}{0{\cdot}5}$$

$$= \frac{2020{\cdot}5 - 1732}{0{\cdot}5}$$

$\boxed{F_{EB} = 577N}$

METHOD OF SECTIONS.

In this, the equilibrium of a portion of a truss is considered which is obtained by cutting the truss by some imaginary portion.



mn ⟶ Cutting line

mn cuts the members EF, BF, BC and the internal forces in these members become external forces acting on two portions of the truss.

Equilibrium of entire truss ⟶ Every part of truss will be in equilibrium.

Hence, three equations of equilibrium can be used,

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

POINTS TO BE REMEMBERED.

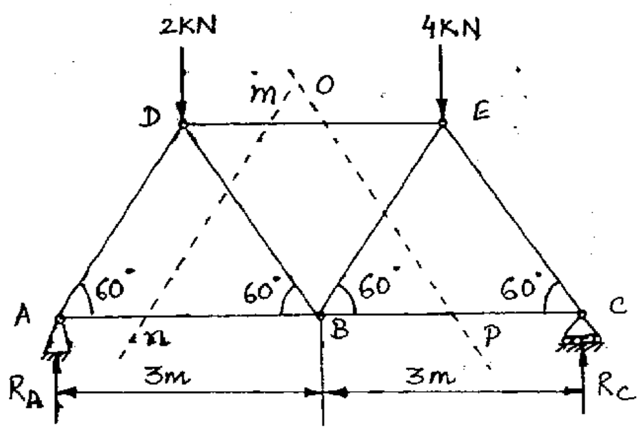
1) Section should be passed through members and not through the joints.
2) Section should divide the truss into two clearly separate and unconnected portions.

3) Section should cut only three members since only three unknowns can be determined from three equations of equilibrium.

4) When using moment equation, the moment can be taken about any convenient point which may or may not lie on the section under consideration.

EXAMPLES

Q1.



SOL:

Consider entire truss as a free body.

Taking moment about A

$\Sigma M_A = 0$

$$R_C(6) - 2(1.5) - 4(4.5) = 0$$

$$6R_C = 3 + 18$$

$$\boxed{R_C = 3.5 \text{ KN}}$$

$\Sigma F_y = 0$

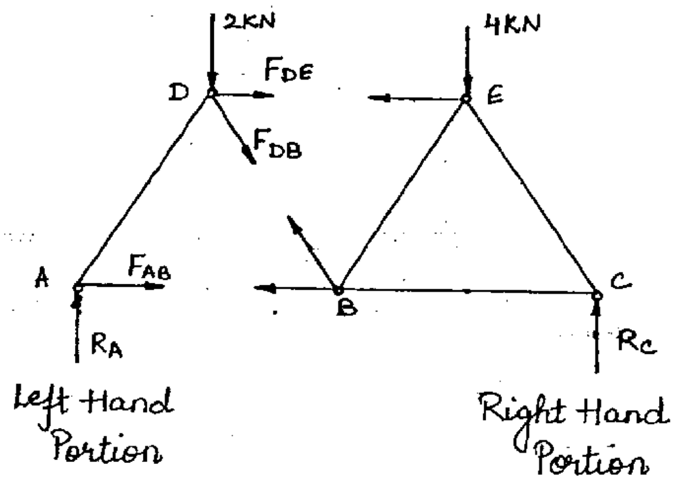
$$R_A + R_C - 2 - 4 = 0$$

$$R_A = 6 - 3.5$$

$$\boxed{R_A = 2.5 \text{ KN}}$$

There can be more than one way to pass a section (mn or op).

Taking mn as the cutting line. The two portions of the truss are as follows:



Considering the equilibrium of left hand portion

Taking moment about B.

$\Sigma M_B = 0$

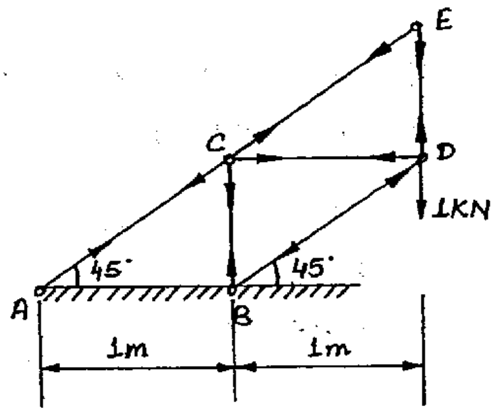
$$2000(3\sin 30^\circ) - R_A(3) + F_{DE}(3\cos 30^\circ) = 0$$

$$2000 \times 3 \times 0.5 - 3R_A + F_{DE} \times 3 \times 0.866 = 0$$

$$F_{DE} = \frac{-3000 + 7500}{3 \times 0.866}$$

$$\boxed{F_{DE} = 1732.1\,N}$$

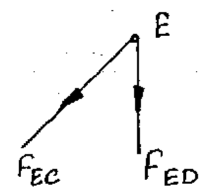
Q2.



SOL: Considering the equilibrium of various joints.

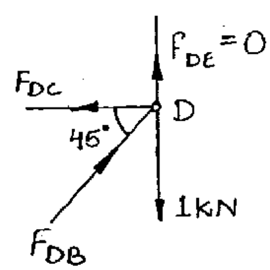
JOINT E

F_{EC} and F_{ED} are non-collinear and no external force is applied at the joint hence



$$\boxed{F_{EC} = F_{ED} = 0}$$

JOINT D



$\Sigma F_x = 0$

$$F_{DB}\cos 45° - F_{DC} = 0$$

$$F_{DB}\left(\frac{1}{\sqrt{2}}\right) - F_{DC} = 0$$

$$F_{DC} = \frac{F_{DB}}{\sqrt{2}}$$

$\Sigma F_y = 0$

$$F_{DB}\sin 45° + F_{DE} - 1 = 0$$

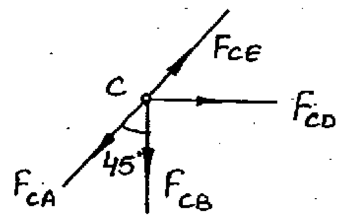
$$F_{DB}\left(\frac{1}{\sqrt{2}}\right) + 0 = 1$$

$$\boxed{F_{DB} = \sqrt{2}\ KN}$$

Now, $F_{DC} = \frac{\sqrt{2}}{\sqrt{2}}$

$$\boxed{F_{DC} = 1KN}$$

JOINT C



$\Sigma F_x = 0$

$$-F_{CA}\sin 45^\circ + F_{CE}\cos 46^\circ + F_{CD} = 0$$

$$-\frac{F_{CA}}{\sqrt{2}} + 0 \times \frac{1}{\sqrt{2}} + F_{CD} = 0$$

$$F_{CD} = \frac{F_{CA}}{\sqrt{2}}$$

$$F_{CA} = \sqrt{2} \times 1$$

$$\boxed{F_{CA} = \sqrt{2}\ KN}$$

$\Sigma F_y = 0$

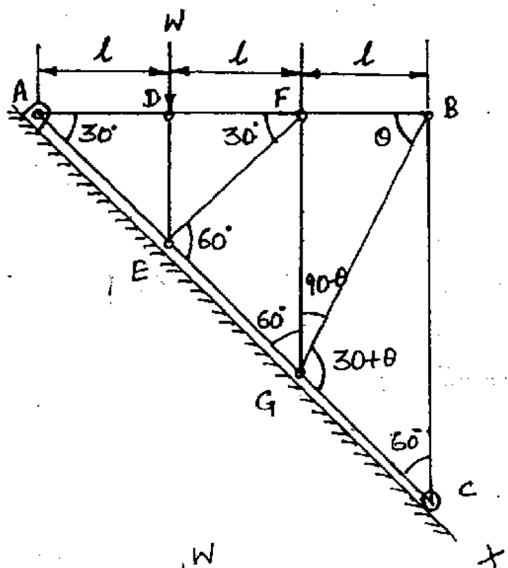
$$-F_{CA}\cos 45^\circ + F_{CE}\sin 45^\circ - F_{CB} = 0$$

$$-\frac{\sqrt{2}}{\sqrt{2}} + 0 \times \frac{1}{\sqrt{2}} = F_{CB}$$

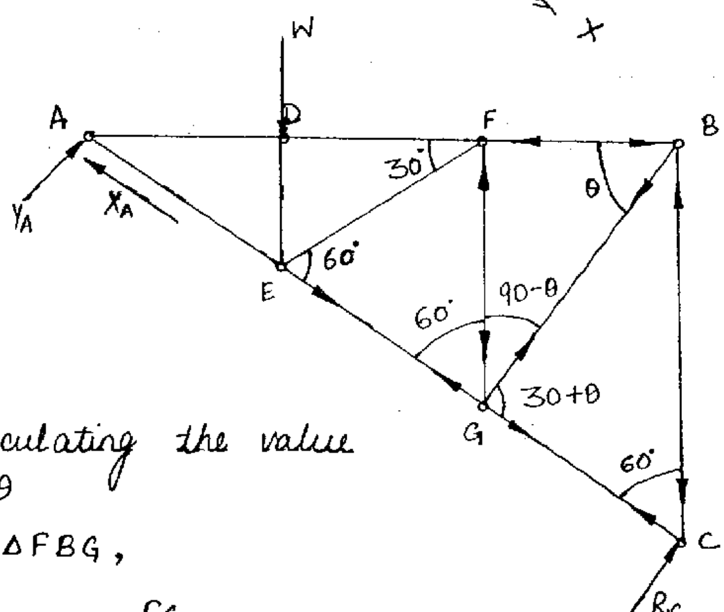
$$\boxed{F_{CB} = -1\ KN}$$

Reverse the direction of the force in the member CB.

Q3. Find the axial force in the members BC, BG, BF, GC, GF and GE of the truss.



SOL :



Calculating the value of θ

ΔFBG,

$$\frac{FG}{FB} = \tan\theta$$

$$\tan\theta = \frac{FG}{l}.$$

ΔAFG, $$\tan 60^\circ = \frac{AF}{FG}$$

$$FG = \frac{AF}{\tan 60^\circ} = \frac{2l}{\sqrt{3}}$$

$$FG = \frac{2l}{\sqrt{3}} \implies \tan\theta = \frac{2l}{\sqrt{3}\,l}$$

$$\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$\theta = 49.10^\circ$$

Consider the entire truss as a free-body.

Taking moment about A

$$\Sigma M_A = 0$$

$$-W(AD) + R_c(AC) = 0$$

$$-W\ell + R_c\left(\frac{6\ell}{\sqrt{3}}\right) = 0$$

$$\left[\; AB/AC = \cos 30^\circ \qquad AC = \frac{AB}{\cos 30^\circ} \qquad AC = \frac{3\ell \times 2}{\sqrt{3}} \right.$$

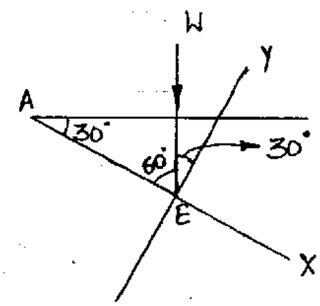
$$R_c \, \frac{6\ell}{\sqrt{3}} = W\ell$$

$$R_c = \frac{W\ell\sqrt{3}}{6\ell}$$

$$\boxed{R_C = \frac{W}{2\sqrt{3}}}$$

$\Sigma F_x = 0$

[Along AC]



$$W\sin 30^\circ - X_A = 0$$

$$\boxed{X_A = \frac{W}{2}}$$

$\Sigma F_y = 0$

$$Y_A + R_c - W\cos 30^\circ = 0$$

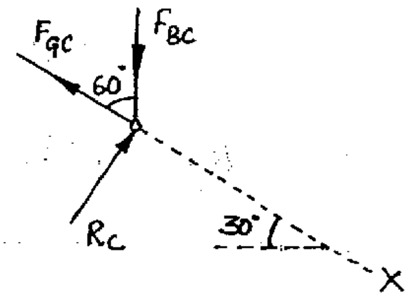
$$Y_A = \frac{W\sqrt{3}}{2} - \frac{W}{2\sqrt{3}}$$

$$= \frac{W}{2}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$$

$$= \frac{W}{2} \times \frac{2}{\sqrt{3}}$$

$$\boxed{Y_A = \frac{W}{\sqrt{3}}}$$

JOINT C



$\Sigma F_x = 0$

$$F_{BC}\cos 60^\circ - F_{GC} = 0$$

$\Sigma F_y = 0$

$$-F_{BC}\sin 60^\circ + R_C = 0$$

$$F_{BC} = \frac{R_C}{\sin 60^\circ}$$

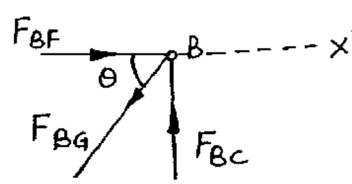
$$= \frac{W}{2\sqrt{3}} \times \frac{2}{\sqrt{3}}$$

$$\boxed{F_{BC} = \frac{W}{3}}$$

$$F_{GC} = \frac{W}{3} \times \frac{1}{2}$$

$$\boxed{F_{GC} = \frac{W}{6}}$$

JOINT B



$\Sigma F_x = 0$

$$F_{BF} = F_{BG}\cos\theta$$

$$F_{BF} = F_{BG}\cos 49.10^\circ$$

$\Sigma F_y = 0$

$$F_{BC} - F_{BG}\sin 49.10^\circ = 0$$

$$F_{BG} = \frac{W}{3}\left(\frac{1}{0.756}\right)$$

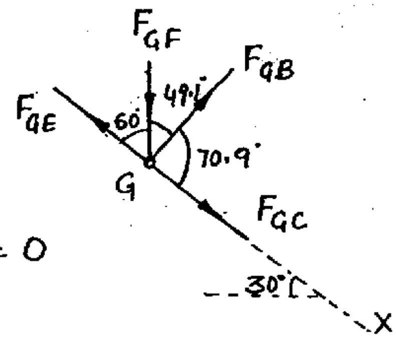
$\sin 49.10^\circ = 0.756$

$\cos 49.10^\circ = 0.655$

$$\boxed{F_{BG} = 0.441W}$$

$$F_{BF} = 0.441 \times W \times 0.655$$

$$\boxed{F_{BF} = 0.289W}$$



JOINT G

$\Sigma F_x = 0$

$$-F_{GE} + F_{GC} + F_{GF}\cos 60^\circ + F_{GB}\cos 70.9^\circ = 0$$

$\Sigma F_y = 0$

$$-F_{GF}\sin 60^\circ + F_{GB}\sin 70.9 = 0$$

$$F_{GF} = \frac{F_{GB}\sin 70.9^\circ}{\sin 60^\circ}$$

$$= \frac{0.441W \times 0.944}{0.866}$$

$$\boxed{F_{GF} = 0.48W}$$

$$F_{GE} = \frac{W}{6} + 0.48W \times 0.5 + 0.441W \times 0.327$$

$$= 0.166W + 0.24W + 0.144207W$$

$$\boxed{F_{GE} = 0.55W}$$