10 Feb 2023

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VECTORS

∇-operator

$$= \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$\phi(x,y,z) \rightarrow$ scalar

$V(x,y,z) = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$
(vector)

Gradient :- $\nabla\phi$

$$\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi(x,y,z)$$

$$= \hat{i}\frac{\partial}{\partial x}\phi + \hat{j}\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

scalar $\longrightarrow$ vector

Divergence $= \vec{\nabla}\cdot\vec{V}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$\text{div}(\vec{v}) = \frac{\partial}{\partial x}V_1 + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

$\hat{i}\cdot\hat{i} = 1$; $\hat{i}\cdot\hat{j} = 0 = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i}$ $\quad \hat{j}\cdot\hat{j} = 1 = \hat{k}\cdot\hat{k}$

Curl $(\vec{v}) = \vec{\nabla}\times\vec{V}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\times(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_1 & V_2 & V_3 \end{vmatrix}$$

NORMAL :-

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$

$$= \underbrace{\left(\hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}\right)}_{\nabla \cdot \phi} \cdot \left(\underbrace{\hat{i}\, dx + \hat{j}\, dy + \hat{k}\, dz}_{d\vec{r}}\right)$$

$\vec{x} = (x_1, x_2, x_3) = x_1\hat{i} + x_2\hat{j} + x_3\hat{k})$ | $\vec{r} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$

$$d\vec{x} = dx\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$d\phi = \vec{\nabla}\phi \cdot d\vec{r}$$

$$d\phi = |\vec{\nabla}\phi| \cdot |d\vec{r}| \cos\theta$$

Max $\theta = 0$ $\parallel^{x}$

$\theta = \pi/2$ $\perp$ $\Rightarrow \vec{\nabla}\phi =$ Normal to curve $\phi(x, y, z) = c$

unit Normal :- $\dfrac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}$

DIRECTIONAL DERIVATIVE :-

In the direction of vector D for $\vec{\nabla}\phi$ is given by

$\vec{\nabla}\phi \cdot \hat{d}$; $\hat{d} = \dfrac{\vec{d}}{|\vec{d}|}$

Q The temp at any pt in the space is given by $T = xy + yz + zx$. Determine the directional derivative of T in the dir of vector $\vec{d} = 3\hat{i} - 4\hat{k}$ at the point $(1,1,1)$.

$$\nabla . T = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)T$$
$$= \hat{i}(y+z) + \hat{j}(x+z) + \hat{k}(x+y)$$

$$\nabla T\Big|_{(1,1,1)} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$DD = \vec{\nabla} T \cdot \hat{d} = (2\hat{i} + 2\hat{j} + 2\hat{k})\left(\frac{3\hat{i} - 4\hat{k}}{5}\right)$$
$$= \frac{6}{5} - \frac{8}{5} = \frac{-2}{5}$$

Q. If the fluid $(\vec{F})$ is compressible $\vec{F} = (F_1, F_2, F_3)$ then div $(\vec{F}) = 0 = \vec{\nabla}\cdot\vec{F}$ & F is called SOLENOIDAL then $\exists$ a scalar qty $\phi(x,y,z)$ called scalar Potential s.t $\vec{F} = \vec{\nabla}\phi$

Q1. Find the value of n for which $r^n \vec{r}$ is solenoidal

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \qquad |\vec{r}| = r = \sqrt{x^2+y^2+z^2}$$
$$r^n = (x^2+y^2+z^2)^{n/2}$$

$$r^n\vec{r} = x(x^2+y^2+z^2)^{n/2}\,\hat{i} + y(x^2+y^2+z^2)^{n/2}\,\hat{j} + z(x^2+y^2+z^2)^{n/2}\,\hat{k}$$

Given $r^n\vec{r}$ is solenoidal

$$\text{div}\,(r^n\vec{r}) = 0$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left[x(x^2+y^2+z^2)^{n/2}\hat{i} + y(x^2+y^2+z^2)^{n/2}\hat{j} + z(x^2+y^2+z^2)^{n/2}\hat{k}\right] = 0$$

$$(x^2+y^2+z^2)^{n/2} + \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cdot(2x^2) + (x^2+y^2+z^2)^{n/2}$$
$$+ \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cdot(2y^2) + (x^2+y^2+z^2)^{\frac{n}{2}} + \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2z^2)$$

$$(x^2+y^2+z^2)^{\frac{n}{2}-1}\left(3x^2\cancel{3y^2}\cancel{3z^2}+n(x^2+y^2+z^2)\right)=0$$

$$(x^2+y^2+z^2)^{\frac{n}{2}-1}\left(3x^2+3y^2+3z^2+n(x^2+y^2+z^2)\right)=0$$

$$(x^2+y^2+z^2)^{\frac{n}{2}-1}\underbrace{(x^2+y^2+z^2)}_{\text{non zero}}(3+n)=0$$

$$\boxed{n=-3}$$

If $\vec{\nabla}\times\vec{V}=0$ (curl $\vec{V}=0$) ; $\vec{V}$ is irrotational then $\exists$ a scalar ϕ st $\vec{V}=\vec{\nabla}\phi$.

Q. Show that the vector $\vec{v}=2xyz\hat{i}+(x^2z+2y)\hat{j}+x^2y\hat{k}$ is irrotational.
& find scalar pot u.

$$\vec{v}=2xyz\hat{i}+(x^2z+2y)\hat{j}+x^2y\hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z+2y & x^2y \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial}{\partial y}(x^2y)-\frac{\partial}{\partial z}(x^2z+2y)\right)-\hat{j}\left(\frac{\partial}{\partial x}(x^2y)-\frac{\partial}{\partial z}(2xyz)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}(x^2z+2y)-\frac{\partial}{\partial y}(2xyz)\right)$$

$$=\hat{i}(x^2-x^2)-\hat{j}(2xy-2xy)+\hat{k}(2zx-2zx)$$

$$=0$$

To find $u(x,y,z)$

$$d\vec{u} = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$= \left(\hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}\right)\left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$= \vec{\nabla}u \cdot d\vec{r}$$

because $\text{curl}\,\vec{v} = 0$ $\therefore \exists\ u(x,y,z)$ called scalar pot s.t $\vec{v} = \vec{\nabla}u$

$$d\vec{u} = \vec{v}\cdot d\vec{r}$$

$$= \left[2xyz\hat{i} + (x^2z+2y)\hat{j} + x^2y\,\hat{k}\right]\cdot\left[\hat{i}dx + \hat{j}dy + \hat{k}dz\right]$$

$$= 2xyz\,dx + (x^2z+2y)\,dy + x^2y\,dz$$

$$d\vec{u} = yz(2x\,dx) + x^2z\,dy + 2y\,dy + x^2y\,dz$$

$$\cancel{\int d\vec{u} = yz\int 2x\,dx + x^2z\int dy + \int 2y\,dy + x^2y\int dz}$$

$$= yz\,d(x^2) + x^2z\,dy + d(y^2) + x^2y\,dz$$

$$= z\left[y\,d(x^2) + x^2dy\right] + d(y^2) + x^2y\,dz$$

$$= z\left[d(x^2y)\right] + x^2y\,dz + d(y^2)$$

$$du = d(x^2yz) + d(y^2)$$

$$du = d\left[x^2yz + y^2\right]$$

$$u = x^2yz + y^2 + c$$

Q. $\vec{F} = (y^2 + 2xz^2)\hat{i} + (2xy - z)\hat{j} + (2x^2z - y + 2z)\hat{k}$
irrotational & scalar potential

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2+2xz^2 & 2xy-z & 2x^2z-y+2z \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial}{\partial y}(2x^2z-y+2z) - \frac{\partial}{\partial z}(2xy-z)\right)$$
$$-\hat{j}\left(\frac{\partial}{\partial x}(2x^2z-y+2z) - \frac{\partial}{\partial z}(y^2+2xz^2)\right)$$
$$+\hat{k}\left(\frac{\partial}{\partial x}(2xy-z) - \frac{\partial}{\partial y}(y^2+2xz^2)\right)$$

$$= \hat{i}(-1+1) - \hat{j}(4xz - 4zx) + \hat{k}(2y - 2y)$$

$$= 0$$

$$\vec{du} = \vec{v} \cdot \vec{dr}$$
$$= (y^2+2xz^2)dx + (2xy-z)dy + (2x^2z-y+2z)dz$$
$$= y^2dx + z^2(2x\,dx) + (2y)(dy)x - z\,dy + x^2(2z\,dz) - y\,dz + (2z)dz$$

$$= y^2dx + z^2d(x^2) + d(y^2)x - z\,dy + x^2d(z^2) - y\,dz + d(z^2)$$

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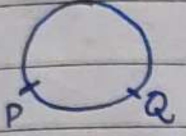
INTEGRATION

A force acts on a particle and moves it $\overrightarrow{AB}$

$\vec{F} = F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k}$

work done $= \int_A^B \vec{F} \cdot \vec{dr}$



$= \oint \vec{F} \cdot \vec{dr}$

Closed path

$\int \vec{F} \cdot dr = 0$ then $\vec{F}$ is conservative (no work is done)

$\vec{F}$ is irrotational

i.e $\vec{\nabla} \times \vec{F} = 0$

$\exists$ a scalar $\phi(x, y, z)$ st $\vec{F} = \vec{\nabla}\phi$

Q1 Show that integral $\int_{(1,2)}^{(3,4)} (xy^2 + y^3)dx + (x^2y + 3xy^2)dy$ independent of the path taken, hence find scalar pot.

Ans) It means curl $\vec{F} = 0$

$$\vec{F} = (xy^2 + y^3)\hat{i} + (x^2y + 3xy^2)\hat{j}$$

curl F =

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 + y^3 & x^2y + 3xy^2 & 0 \end{vmatrix}$$

$$\hat{i}\left(-\frac{\partial}{\partial z}(x^2y + 3xy^2)\right) - \hat{j}\left(\frac{\partial}{\partial z}(xy^2 + y^3)\right) = 0$$

$\therefore$ Independent of path taken.

$\exists$ Scalar $\phi(x, y, z)$ st $\vec{F} = \vec{\nabla}\phi$

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$$\cancel{I = \int_{(1,2)}^{(3,4)} (xy^2 + y^3)\,dx + (x^2y + 3xy^2)\,dy}$$

$$\cancel{I = \int_{(1,2)}^{(3,4)} (xy^2dx) + y^3dx + x^2ydy + 3xy^2dy}$$

$$\cancel{= \int_{(1,2)}^{(3,4)}} \quad \& \quad \vec{F} = \vec{\nabla}\phi$$

$$d\phi = \vec{\nabla}\phi \cdot \vec{dr}$$

$$d\phi = \vec{F} \cdot \vec{dr}$$

$$= ((xy^2 + y^3)\hat{i} + (x^2y + 3xy^2)\hat{j}) \cdot (\hat{i}dx + \hat{j}dy)$$

$$= (xy^2 + y^3)dx + (x^2y + 3xy^2)dy$$

$$= (xy^2 dx) + y^3dx + x^2ydy + 3xy^2dy$$

$$= d\left(\frac{x^2}{2}\right)y^2 + y^3dx + d\left(\frac{y^2}{2}\right)x^2 + d\left(\frac{y^3}{3}\right)3x$$

$$= \cancel{y^2\left(d\left(\frac{x^2}{2}\right) + y\,dx\right) + x^2\left(d\left(\frac{y^2}{2}\right)x + d\left(\frac{y^3}{3}\right)3x\right)}$$

$$= d\left(\frac{x^2}{2}\right)y^2 + d\left(\frac{y^3}{\cancel{3}}\right)\cancel{3}x + y^3dx + d\left(\frac{y^2}{2}\right)x^2$$

x^3y^3
$3x^2y^3$
$+ x^3 3y^2$

$$= d\left(\frac{x^2}{2}\right)y^2 + d(y^3)x + y^3dx + d\left(\frac{y^2}{2}\right)x^2$$

$$d\phi = d\left(\frac{x^2y^2}{2}\right) + d(xy)^3$$

$$\phi = \int_{(1,2)}^{(3,4)} d\left(\frac{x^2y^2}{2} + xy^3\right)$$

$$= \frac{(3)^2(4)^2}{2} + (3)(4)^3 - \cancel{(3^2)4}\ \frac{(1^2)(2)^2}{2} - (1)(2)^3$$

$$= \frac{9\times\cancel{16}^{8}}{\cancel{2}} + 9(64) - \frac{(4)}{2} - (8)$$

$$= 72 + 576 - 10 = 72 + 566 = 254\cancel{8}$$

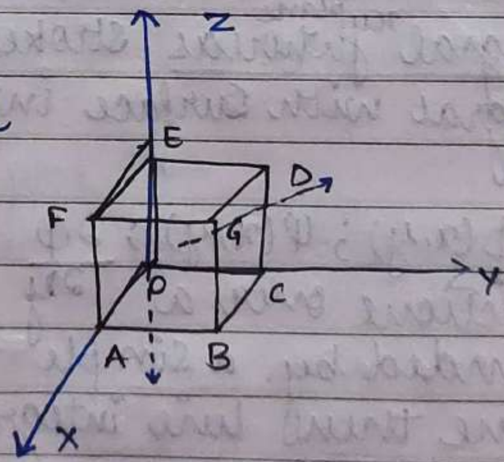
Surface Integral :-

$\vec{F}$ on surface S

$S = \iint \vec{F}\cdot\hat{\eta}\, ds$ $\quad \hat{\eta}$ is unit normal to the S.

$$\hat{\eta} = \frac{\vec{\nabla} S}{|\vec{\nabla} S|}$$

** Q. Find surface integral where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ & S :- cube bounded by $x=0, x=1; y=0; y=1; z=0, z=1$



	Face	Eqt of surface	dS	$\hat{n}$
1	OABC	$z=0$	$dx\,dy$	$-\hat{k}$
2	BGCD	$y=\cancel{0}1$	$dx\,dz$	$\hat{j}$
3	GDEF	$z=1$	$dx\,dy$	$\hat{k}$
4	FAOE	$y=0$	$dx\,dz$	$-\hat{j}$
5	ABGF	$x=1$	$dy\,dz$	$\hat{i}$
6	OCDE	$x=0$	$dy\,dz$	$-\hat{i}$

$$\iint \vec{F}\cdot\hat{\eta}\, ds = \left[\iint_{OABC} + \iint_{BGDC} + \iint_{GDEF} + \iint_{ABGF} + \iint_{OCDE} + \iint_{AOEF}\right] \vec{F}\cdot\hat{\eta}\, ds$$

$$= \iint_{z=0} \cancelto{0}{(-y^2\hat{j})\cdot -\hat{k}}\ dx\,dy + \iint_{x=0} (4xz\hat{i} - \hat{j} + z\hat{k})\cdot\hat{j}\ dx\,dz$$

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$$+\int_{x=0}^{1}\int_{y=0}^{1}(4x\hat{i}-y^2\hat{j}+y\hat{k})\cdot\hat{k}\,dxdy+\int_{y=0}^{1}\int_{z=0}^{1}(4z\hat{i}-y^2\hat{j}+yz\hat{k})\cdot\hat{i}\,dydz$$

$$+\iint(-y^2\hat{j}+\cancel{yz\hat{k}}^{\,0})\cdot-\hat{i}\,dydz+\iint(4xz\cancel{\hat{i}}^{\,0})\cdot-\hat{j}\,dxdz$$

$$=\int_0^1\int_0^1 -1\,dxdz+\iint y\,dxdy+\iint 4z\,dydz$$

$$=-xz\Big|_0^1\Big|_0^1+\frac{xy^2}{2}\Big|_0^1\Big|_0^1+\frac{4z^2}{2}\cdot y\Big|_0^1\Big|_0^1$$

$$=-1+\frac{1}{2}+2(1)=\cancel{2\text{ Ans}}$$

$$=\frac{5}{2}-1=\frac{3}{2}\text{ Ans.}$$

GREEN'S THEOREM

★★ Imp: green's theorem relates line integral with double integral (xy plane) whereas stokes theorem relate line integral with Surface integral in xyz plane.

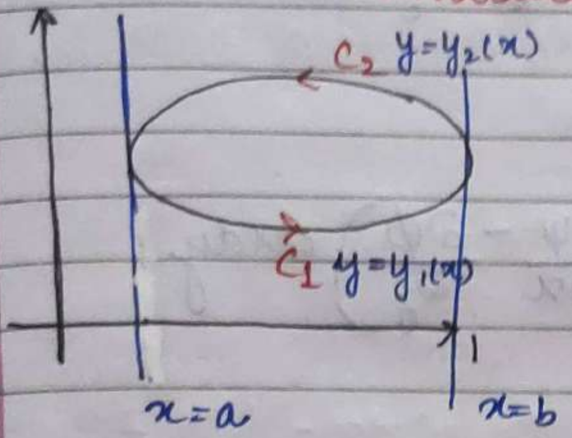
If $\phi(x,y)$; $\psi(x,y)$; $\frac{\partial\phi}{\partial y}$; $\frac{\partial\psi}{\partial x}$ are continuous functions over a region R bounded by a simple closed curve C in xy plane then line integral over C

$$\int_C(\phi\,dx+\psi\,dy)=\iint_R\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$$

$$\int_C\vec{F}\cdot d\vec{r}=\iint_R \text{curl}\,\vec{F}\cdot\hat{k}\,d\vec{R}\qquad(dR=dxdy)$$

Proof:-

anticlockwise



C_1 : $a \le x \le b$; $y = y_1(x)$

C_2 : $b \le x \le a$; $y = y_2(x)$

$$\iint_R \frac{\partial \phi}{\partial y}\,dx\,dy = \iint_R \frac{\partial \phi}{\partial y}\,dx\,dy = \int_{x=a}^{x=b}\int_{y_1(x)}^{y_2(x)} \frac{\partial \Phi}{\partial y}\,dx\,dy + \int_{x=b}^{x=a}\int_{y=y_1(x)}^{y_2(x)} \frac{\partial \Phi}{\partial y}\,dx\,dy$$

$$\iint_R \frac{\partial \phi}{\partial y}\,dx\,dy = \int_{x=a}^{b} \phi(x,y)\Big|_{y_1(x)}^{y_2(x)} dx = \int_{x=a}^{b} \phi(x,y_2(x)) - \phi(x,y_1(x))\,dx$$

$$= \int_{x=a}^{x=b} \Phi(x,y_2(x))\,dx - \int_{x=a}^{b} \Phi(x,y_1(x))\,dx$$

$$= \underbrace{-\int_{x=b}^{a} \phi(x,y_2(x))\,dx}_{C_2} - \int_{x=a}^{b} \underbrace{\phi(x,y_1(x))\,dx}_{C_1}$$

$$= -\left[\int_{C_2} \phi(x,y)\,dx + \int \phi(x,y)\,dx\right]$$

$$= -\left[\left(\int_{C_2} + \int_{C_1}\right) \phi(x,y)\,dx\right] = -\int_C \phi(x,y)\,dx \quad \text{—(i)}$$

Similarly, $\iint_R \frac{\partial \psi}{\partial x} dx dy = \int_c \psi(x,y) dy$ —(ii)

(ii) + (i)

$$\int_C (\phi dx + \psi dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx dy$$

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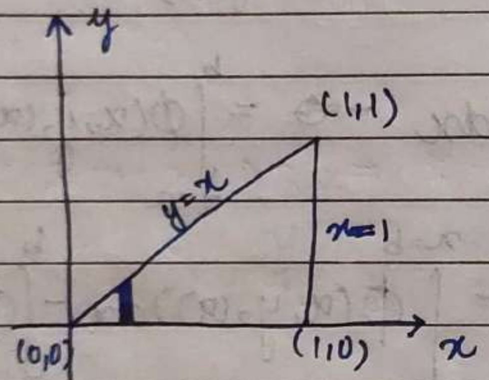
Q. $\int_C x^2 y \, dx + x^2 dy$; C = Δ with vertices (0,0), (1,0) & (1,1)



$\phi(x,y) = x^2 y$

$\psi(x,y) = x^2$

$$\int_C \phi dx + \psi dy$$

$$= \int_{x=0}^{1} \int_{y=0}^{x} (2x - x^2) dx dy$$

$$= \int_{x=0}^{1} (2x - x^2) dx \int_{y=0}^{x} dy$$

$\cancel{x^2} - \frac{x^3}{3}$

$$= \int_{x=0}^{1} (2x^2 - x^3) dx = \left.\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^1$$

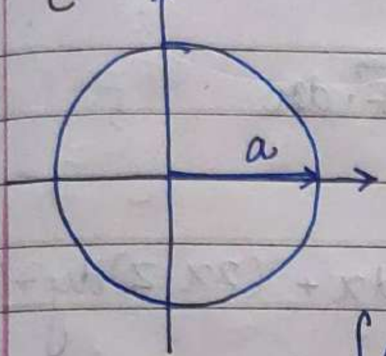
$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12} \text{ Ans.}$$

$\phi\, dx + \psi\, dy$

Q2. $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\,\hat{j}$

$$\int_C \vec{F}\cdot\vec{dr} \;;\; C = x^2+y^2=a^2 \qquad \vec{dr} = dx\,\hat{i} + dy\,\hat{j}$$

A2



$$\vec{F}\cdot\vec{dr} = \sin y\, dx + x(1+\cos y)\, dy$$

$$\phi(x,y) = \sin y$$
$$\psi(x,y) = x(1+\cos y)$$

$$\int_C (\phi\, dx + \psi\, dy) = \iint_R (\cancel{\cos}\ 1+\cos y - \cos y)\, dx\,dy$$

$$= \iint_R dx\,dy = \pi a^2$$

Stokes Theorem

Relates line integral with double integral.

$$\int_C \vec{F}\cdot\vec{dr} = \iint_S \operatorname{curl}(\vec{F})\cdot\hat{\eta}\, ds$$

$\hat{\eta}$ = unit normal external to the surface element s

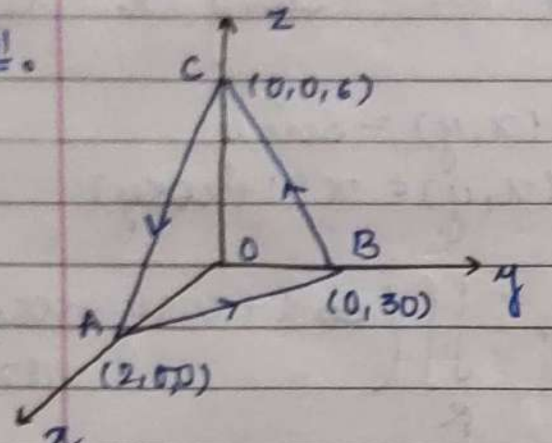
$$\hat{\eta} = \frac{\vec{\nabla} s}{|\vec{\nabla} s|}$$

Divergence Theorem

$$\iint_S \vec{F}\cdot ds = \iiint_V \operatorname{div}(F)\, dv \qquad dv = dx\,dy\,dz$$

Q. verify strokes theorem for
$\vec{F} = (x+y)\hat{i} + (2x-z)\hat{j} + (y+z)\hat{k}$
over the surface of a Δ lamina w vertices
$(2,0,0)$; $(0,3,0)$; $(0,0,6)$

A.



$$LHS = \int_C \vec{F}\cdot d\vec{r}$$

$$= \int_C (x+y)\,dx + (2x-z)\,dy + (y+z)\,dz$$

$C = AB + BC + CA$

$$= \int_{AB} + \int_{BC} + \int_{CA} \left[(x+y)\,dx + (2x-z)\,dy + (y+z)\,dz \right]$$

C along AB $\Rightarrow$ $z = 0$; $\frac{x}{2} + \frac{y}{3} = 1$

$dz = 0$; $y = 3 - \frac{3}{2}x = \frac{6-3x}{2}$

$dy = -\frac{3}{2}\,dx$

$2 \le x \le 0$

$$\int_{x=2}^{0} \left[x + \frac{6-3x}{2}\right] dx + (2x-0)\left(\frac{-3}{2}\,dx\right) + 0$$

$$= \int_{x=2}^{0} \left(x + 3 - \frac{3x}{2}\right) dx + \left(-3x\,dx\right)$$

$$= \int_{x=2}^{0} \left(x + 3 - \frac{3x}{2} - 3x\right) dx$$

$$= \left[\frac{x^2}{2} + 3x - \frac{3x^2}{4} - \frac{3x^2}{2}\right]_2^0 = \left[-2 - \cancel{6} + 3 + \cancel{6}\right] = 1$$

along BC $\int_{BC} \vec{F} \cdot \vec{dr}$

$x=0 \Rightarrow dx=0$; $3 \leq y \leq 0$; $0 \leq z \leq 6$

Eqt of line BC $= \frac{y}{3} + \frac{z}{6} = 1$

$$y = \left(1 - \frac{z}{6}\right)3 = 3 - \frac{3z}{6}$$

$$= \frac{6-z}{2} \Rightarrow dy = \frac{-dz}{2}$$

$$\int_{BC} \vec{F} \cdot \vec{dr} = \int_{z=0}^{6} -z\left(\frac{-dz}{2}\right) + \left(\frac{6-z}{2} + z\right)dz \quad \text{—(ii)}$$

along CA :- $y=0 \Rightarrow dy=0$

$0 \leq x \leq 2$; $6 \leq z \leq 0$

$$\frac{x}{2} + \frac{z}{6} = 1 \Rightarrow z = \left(1 - \frac{x}{2}\right)6 = 6 - 3x$$

$$dz = -3dx$$

$$\int_{CA} \vec{F} \cdot \vec{dr} = \int_{x=0}^{x=2} x\,dx + (6-3x)(-3dx) = (-16)$$

(ii) $$\int_{BC} \vec{F} \cdot \vec{dr} = \int_{z=0}^{6} \left(\frac{z}{2} + 3 - \frac{z}{2} + z\right) dz$$

$$= 36$$

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RHS :- $\iint_S \text{curl}(\vec{F})\hat{\eta}\, ds$

$$\text{curl}\,\vec{F} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & 2x-z & y+z \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial}{\partial y}(y+z) - \frac{\partial}{\partial z}(2x-z)\right) - \hat{j}\left(\frac{\partial}{\partial x}(y+z) - \frac{\partial}{\partial z}(x+y)\right) + \hat{k}\left(\frac{\partial}{\partial x}(2x-z) - \frac{\partial}{\partial y}(x+y)\right)$$

$$= \hat{i}\,(2) + \hat{k}$$

$$S : \frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1 \qquad \hat{\eta} = \frac{\vec{\nabla}S}{|\vec{\nabla}S|}$$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{6}\right)$$

$$\vec{\nabla}S = \frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{6}\hat{k}$$

$$\sqrt{\frac{1}{4} + \frac{1}{9} + \frac{1}{36}} = \sqrt{\frac{9+4+1}{36}} = \sqrt{\frac{14}{36}} = \frac{\sqrt{14}}{6}$$

$$\hat{\eta} = \frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}$$

$$\iint \left(2\hat{i} + \hat{k}\right)\cdot\left(\frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}\right)\frac{ds}{\hat{\eta}\cdot\hat{k}} = \iint \left(\frac{7}{\sqrt{14}}\right)\frac{dx\,dy}{\hat{\eta}\cdot\hat{k}}$$

$ds = \frac{dx\,dy}{\hat{n}\cdot\hat{k}}$ $\quad ds = \frac{dy\,dz}{\hat{n}\cdot\hat{i}}$ $\quad ds = \frac{dx\,dz}{\hat{n}\cdot\hat{j}}$

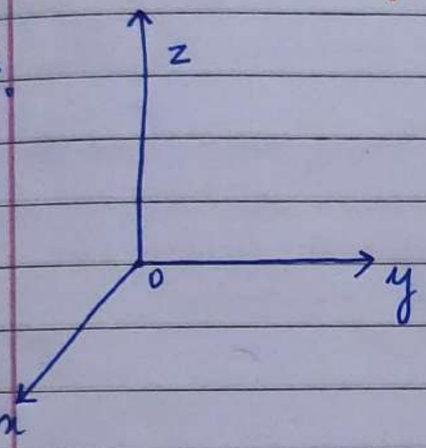
$= \frac{7}{\cancel{\sqrt{14}}} \times \frac{1}{\cancel{2}} \times \cancel{2} \times 3$

$= \frac{7}{\cancel{\sqrt{14}}} \times 3 = 21.$

Q. verify stokes theorem $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xz + z^2)\hat{k}$

S:- Surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xoy plane.

Ans.



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INSPECTION METHOD.

(1). $x\,dy + y\,dx = d(xy)$

(2). $\dfrac{y\,dx - x\,dy}{y^2} = d\left(\dfrac{x}{y}\right)$

(3). $a\,dx + b\,dy = d(ax+by)$

(4) $x\,dx = d\left(\dfrac{x^2}{2}\right)$