23-08-2022

Probability and statistics

robability and statistics → Data

statistics: Descriptive, Inferential → concerned with drawing of conclusions.

ble: parameter which takes different value.

: particular value of the variable.

ation: collection of all data.

→ Represented in the form of set.

Sample: subset of population. /subgroup of population.

Date: 27-08-2022

Arithmetic Mean.

Average → middle
- → Mean
- → Median
- → Mode.

$x_1, x_2 \dots x_n$

$$AM\ (\bar{x}) = \frac{x_1 + x_2 + \dots x_n}{n}$$

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

$$= \cancel{\frac{\sum_{k=1}^{r} f_k \cdot x_k}{\sum f_k}} = \boxed{\frac{\sum_{i=1}^{r} x_i f_i}{\sum_{i=1}^{r} f_i}} \rightarrow \text{For group data}$$

$$eg.\ \bar{x} = \frac{5\cdot 1 + 5\cdot 2 + 9\cdot 3 + 10\cdot 4 + 5\cdot 5 + 11\cdot 6}{45}$$

$$= \frac{5 + 10 + 27 + 40 + 25 + 66}{45} = 3.84.$$

eg.2:

x	70 – 75	75 – 80	80 – 85	85 – 90
f	4	4	5	6

$$\bar{x} = \frac{4 \times 72.5 + 4 \times 77.5 + \dots}{19}$$

$=$

Mid Range

$x_1 < x_2 < \dots < x_n$

$x_1 \downarrow$ Min value, $x_n \to$ Max

$$\text{Mid range} = \frac{x_1 + x_n}{2}$$

Median

Arrange in ascending order.

$$\tilde{x} = \begin{cases} x_{k+1} & n = 2k+1 \\ \dfrac{x_k + x_{k+1}}{2} & n = 2k \end{cases}$$

eg. 3 $\quad 3, 3, 5, 7, 8$

$n = 5$

$\tilde{x} = 5,$

eg. $1, 2, 5, 5, 7, 8, 8, 9$

$n = 8$

$\tilde{x} = \frac{5+7}{2}$

Method 2: (For group data).

$$\tilde{x} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

$l \to$ lower limit of median class

$N \to$ size of population.

* cumulative freq $< \frac{N}{2}$ (Median class)

$h \to$ width of median class

$f \to$ freq. of median class

$c \to$ cumulative freq. of pre-median class

Mode: (For Group data):

data point with max. freq.

x_i

$$\text{Mode} = l + \frac{h(f_m - f_1)}{\underbrace{2f_m - (f_1 + f_2)}_{(f_m - f_1)(f_m - f_2)}}$$

$l \to$ lower limit of model class

$f_m \to$ freq. of model class

$f_1 \to$ freq. of pre-model class

$f_2 \to$ freq. of post-model class

$h \to$ width of class.

S1: 7, 9, 9, 10, 10, 11, 14

S2: 7, 7, 8, 10, 11, 13, 14

$\bar{x}_1 = \frac{70}{7} = 10$

$\bar{x}_2 = \frac{70}{7} = 10.$

$e_i = (x_i - \bar{x})^2$

positive square root of the sample variance is SD.

SD → standard deviation → σ

$SD = +\sqrt{\sigma^2} = \sigma$

variance $= \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$

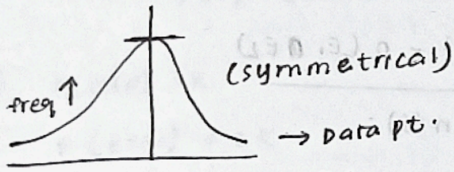
$= \frac{1}{n}\sum_{i=1}^{r} f_i(x_i - \bar{x})^2$

$\frac{1}{n}\sum (x_i^2 + \bar{x}^2 - 2x_i\bar{x})$

$= \frac{1}{n}\sum x_i^2 - 2\frac{\bar{x}}{n}\sum x_i + \frac{\bar{x}^2}{n}\sum 1$

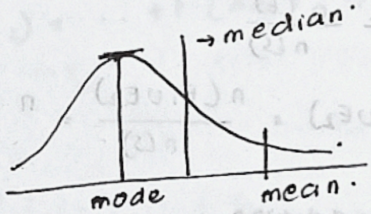
$= \frac{1}{n}\sum x_i^2 - 2\bar{x}^2 + \bar{x}^2$

$= \frac{1}{n}\sum x_i^2 - n\bar{x}^2$

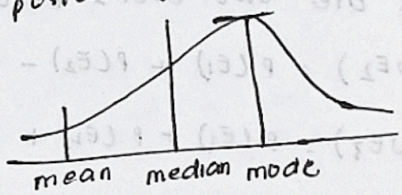


median = mode = mean

symmetrical with respect to mean.



positively ~~skute~~ iscute (to right)



Negatively ~~skute~~ iscute.

(iscute to left).

Karl-Pearson coefficient of skewness.

$\boxed{\text{skewness} = \frac{\text{Mean} - \text{mode}}{SD}}$

Iscunese → Degree of assymmetry or departure of symmetry.

Probability

Experiment:

Equally likely: Each outcome has equal prob. of occurrence. Eg. Rolling a die (⚀)

E → subset of sample space is called event.

S: sample space $= \{e_1, e_2, \dots, e_n\}$

Mutually exclusive: both events cannot occur simultaneously.

Eg. Tossing a coin.

Independent: Outcome of one event does not affect the other

Otherwise, dependent.

$$P(e_i) = \frac{\text{No. of favorable cases}}{\text{Total no. of cases}} = \frac{n(E)}{n(S)}$$

Axioms of Probability

S be a sample space and an event e, the prob. of event e is P(e) if following axiom holds $P(E) \leq 1$.

Ax. 1 $\quad 0 \leq P(E) \leq 1$

if, $P(E) = 0 \rightarrow$ impossible event

$P(E) = 1 \rightarrow$ sure event.

$A' \rightarrow$ complement of A

$P(A) + P(A') = 1$.

Ax 4: Non-negativity

$P(A) \geq 0$.

Ax 2 $\quad P(S) = 1$ (Normalization)

Ax : 3 for any two disjoint events E_1 and E_2

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$ [Additivity]

If they are not disjoint,

* disjoint → nothing common.

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E_1 \cup E_2) = \frac{n(E_1 \cup E_2)}{n(S)} = \frac{n(E_1) + n(E_2) - n(E_1 \cap E_2)}{n(S)}$$

Law of Addition

E_1 and E_2 are two events belonging to S,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2 \cap E_3)$$

* mutually exclusive → occurrence of one event stops the occurrence of another event.

Conditional Probability

A and B are two events, then prob. of occurrence of A given that B has already occured is called conditional probability of A conditional to B.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A/B)\, P(B) \longrightarrow (1)$$

* intersection → both

union → either

A and B are independent, then

$$P(A/B) = P(A)$$

Subs. in eqn. (1),

$$P(A \cap B) = P(A)\, P(B)$$

$$S = \bigcup_{i=1}^{n} A_i$$

~~$P(A_i/e) \rightarrow$~~

Bayes Rule

$$P(A_i/e) = \frac{P(A_i \cap e)}{P(e)} = \frac{P(A_i)\, P(e/A_i)}{\sum_{i=1}^{n} P(A_i)\, P(e/A_i)}$$

$$e \cap \bigcup_{i=1}^{n} A_i = e \cap (A_1 \cup A_2 \cdots \cup A_n)$$

$$= (e \cap A_1) \cup (e \cap A_2) \cup \ldots (e \cap A_n)$$

$$P(A_i/e) = \frac{P(A_i \cap e)}{P(A_1)\, P(e/A_1) + P(A_2)\, P(e/A_2) + \ldots + P(A_n)\, P(e/A_n)}$$

Multiplication Rule of probability:

If E_1 and $E_2 \rightarrow$ independent

$$P(A \cap B) = P(A) \cdot P(B)$$

~~E_1 and $E_2 \rightarrow$ Dependent:~~

~~$P(A \cap B) = P(A) \cdot P(B)$~~

(1) $P(\text{odd}) = K$

$P(\text{even}) = 2K$

$E \Rightarrow$ no $< 4 = \{1, 2, 3\}$

We know,

$$P(1) + P(2) + \cdots + P(6) = 1,$$

$$K + 2K + K + 2K + K + 2K = 1$$

$$9K = 1$$

$$K = \frac{1}{9}$$

$\therefore P(\text{odd}) = \frac{1}{9}$, $P(\text{even}) = \frac{2}{9}$

$$P(E) = P(1) + P(2) + P(3)$$

$$= \frac{1}{9} + \frac{2}{9} + \frac{1}{9}$$

$$= \frac{4}{9}$$

② A and B cast each other with a pair of dice.

$S \triangleq \{(1,1), (1,2) \ldots (6,6)\} \rightarrow 36$ pairs.

A → wins if thrown 6 → (1,5) (5,1) (2,4) (4,2) (3,3)

B → wins if throw 7 → (6,1) (1,6) (5,2) (2,5) (4,3) (3,4)

$P(A) = \frac{5}{36}$ $\quad P(B) = \frac{6}{36} = \frac{1}{6}$

E → A wins

$E = A \cup A^c B^c A \cup A^c B^c A^c B^c A$

$P(E) = P(A) + P(A^c) P(B^c) P(A) + \ldots$

③ Prob. each suite is represented ~~out~~ once out of 6 cards.
(with replacement).

$SCDH\,(S^2 + C^2 + D^2 + H^2 + SC + SD + SH + CD + CH + DH)$

$S^3 CDH \qquad \frac{\lfloor 6}{\lfloor 3} \times {}^4C_1$

$S^2 C^2 DH \qquad \frac{\lfloor 6}{\lfloor 2 \lfloor 2} \times {}^4C_2$

Total no. of possible ways $= 13^6 \left({}^4C_1 \times \frac{\lfloor 6}{\lfloor 3} + {}^4C_2 \times \frac{\lfloor 6}{\lfloor 2 \lfloor 2}\right)$

Total no. of ways $= 52^6$

E → All four units are represented

$P(E) = \frac{13^6 (480 + 1080)}{52^6}$

④ A → smoke detected by device A.

B → Detected by B

$P(A) = 0.95 \qquad P(B) = 0.98$

E → smoke present & detected

$E = A \cup B$

$P(E) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.95 + 0.98 - 0.94$

$= \underline{\underline{0.99}}$

(5) $A \to 30\%$ (10% → Backtires)

$B \to 20\%$ (15% Backtires)

$C \to 50\%$ (5% Backtires)

$E_A \to$ Renting from A

$E_B \to$ Renting from B

$E_C \to$ Renting from C.

$TA \to$ car with Backtires from A.

$TB \to$ from B

$TC \to$ from C.

$P(E_A) = 0.3$, $P(E_B) = 0.2$, $P(E_C) = 0.5$.

$(P(TA) = 0.1$, $P(TB) = 0.15)$

$(P(TC) = 0.05)$

$E \to$ car rented with back tires.

$E = (E_A \cap TA) \cup (E_B \cap TB) \cup (E_C \cap TC)$

$P(E) = P(E_A \cap TA) + P(E_B \cap TB) + P(E_C \cap TC)$

$= P(E_A)\,P(TA) + P(E_B)\,P(TB) + \cdots$ (Independent events)

$= 0.3 \times 0.1 + 0.2 \times 0.15 + 0.5 \times 0.05$

$=$

(6) $X \to$ true detection of corosion

$Y \to$ false detection of corrosion

$E \to$ pipe is corrosive

$T^+ \to$ test +ve

$T^- \to$ test -ve.

$P(X) = 0.7$

$P(Y) = 0.2$

$P(E) = 0.1$

$P(X^c) = 0.3$

$P(Y^c) = 0.8$

$P(E/T^+) = ?$

$$P(E/T^+) = \frac{P(E \cap T^+)}{P(T^+)}$$

$T^+ = (X \cap E) \cup (Y \cap E^c)$

$P(T^+) = P(X \cap E) + P(Y \cap E^c) - \underbrace{P(X \cap E) \cap (Y \cap E^c)}_{}$

$\therefore P(E/T+) = \dfrac{0.1 \times 0.7}{0.25}$

$=$

$=$

$P(T+) = P(X)\,P(E) + P(Y)\,P(E^c)$

$= 0.7 \times 0.1 + 0.2 \times 0.9$

$= 0.25$

$P(E \cap T^+) = P(X)\,P(E)$

$= 0.1 \times 0.7$

Pipe
- 0.1 corrosion
  - 0.7 T+
  - 0.3 T−
- 0.9 non-corrosion
  - 0.2 T+
  - 0.8 T−

Random variable → Is a real valued function which assign a real no. to each sample point in the sample space.

→ $[a,b] \cup [c,d]$..

Continuous data:
Height, temperature, Humidity.

Discreet data:
$S: \{e_1, e_2, \dots e_n\}$

S → sample space

$X: S \to R$ (R → set of real nos)
↳ Random variable.

$S = \{e_1, e_2, \dots, e_n\}$

$X(s) = x_1$

$s \in S,\ x_1 \in R$
↳ contains result.

$R_x = \{x_1, x_2, \dots x_n\}$
- continuous → infinite
- Discreet → finite
  - → Probability mass func: i) $P(x) \geq 0$ ii) $\sum P(x) = 1$
  - → $E(x) = \sum x\,P(x)$
  - ↳ Prob-Density function: $f(x) \geq 0$, $\int_{-\infty}^{\infty} f(x)\cdot dx = 1$
  - ↳ $E(x) = \int_{-\infty}^{\infty} x f(x)\cdot dx$

* On particular sample space, more than one random variable can be defined.

Eg. $S = \{HH, HT, TH, TT\}$

X → no. of heads

$R_x = \{2, 1, 0\}$

Properties:

If X and Y are 2 rando

1) $(X+Y)(s) = X(s) + Y(s)$,

2) $(kX)(s) = k(X(s))$, k

3) $(X+k)(s) = X(s) + k$

4) $(XY)(s) = X(s) \cdot Y(s)$, provided

inde

Discreet random variable ~~→ takes~~ finite or as many countable no. of values.
Eg - No. of head obtained when tossing a coin.

Continuous R.V → variable which takes infinite no. of values in an interval.
Eg. weight of group of individuals.

$P(x = x_1) = P(x_1) = 1 (s \in S, X(s) = x_1)$

S, X, Rx

x	x_1	x_2	. . .	x_n
$p(x)$	$P(x_1)$	$P(x_2)$	. . .	$P(x_n)$

probability distribution table.

Note:
$P(x)$ = probability mass function.
For continuous random variable:
Rx → will be an interval and union of interval.
$f(x)$ → probability density function (pdf) → for continuous random variable

$\Sigma P(x_i) = 1$

Eg.

x	1	3	4
$P(x)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{5}$

$R_x = \{1, 3, 4\}$

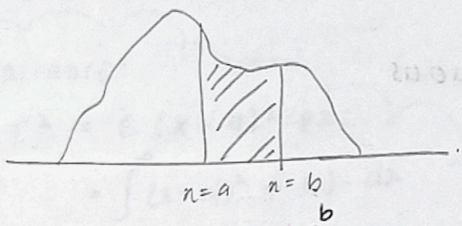
$P(1) = \frac{1}{3}$

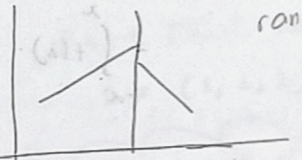
$P(3) = \frac{1}{4}$

$P(4) = \frac{2}{5}$

But, $\frac{1}{3} + \frac{1}{4} + \frac{2}{5} \neq 1$

∴ Not probability distribution.





→ piece-wise continuity.

$P(a \le x \le b) = \int_a^b f(x) \cdot dx,$

$f(x) \ge 0 \ \forall x \in R$

$$\boxed{\int_{-\infty}^{\infty} f(x) \cdot dx = 1}$$

For continuous prob. distribution, probabity of single value is always zero.
$P(x = a) = 0$.
~~Discreet R.V.~~

* CDF (Cumulative ~~Descriptive~~ distribution function): Represented by $F(x)$

For C.R.V,
Cumulative D.F:
$F_x(x) = P(X \le x) = \int_{-\infty}^{x} f(x) \cdot dx.$

$F(x) = P(X \le x) = \sum_{x_i \le x} P(x_i)$

$P(7.5) = P(x \le 7) = P(x < 7.5)$

→ Discreet R.V.

Expectation (Mean or Average):

→ Represented by μ.

$x \to$ random variable.

$E(x) = \mu = \sum x_i \cdot p(x_i)$

$= x_1 p(x_1) + x_2 p(x_2) + \cdots x_n p(x_n)$

→ If sample space is finite, $R_x \to$ finite.

Same with infinite (∞)

Eg.

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$E(x) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4}$

$= \underline{\underline{1}}$

Cumulative Distribution Function

$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i) \to$ Discreet

$= \int_{-\infty}^{x} f(x) \cdot dx \to$ continuous

properties:

1) $\lim_{x \to -\infty} F(x) = 0$

2) $\lim_{x \to \infty} F(x) = 1$

$P(a \leq x \leq b) = F(b) - F(a)$

$\boxed{\frac{d}{dx} F(x) = f(x)}$ → Relation b/w distribution func. and density func.

$\lim_{x \to \infty} F(x) = 1$

$\lim_{x \to -\infty} F(x) = 0$.

Expection (Mean).

$E(x) = \mu = \sum_{i=1}^{n} x_i p(x_i) \to$ Discreet

$= \int_{-\infty}^{\infty} x f(x) \cdot dx \to$ Continuous.

$E\{g(x_i)\} = \sum_{i=1}^{n} g(x_i)\, p(x_i)$

variance $(\sigma^2) = \sum_{i=1}^{n} (x_i - \mu)^2 \cancel{p(x_i)}\, p(x_i)$

$= (x_i - \mu)^2 p(x_i) + \dots + (x_n - \mu)^2 p(x_n)$

$= \sum_{i=1}^{n} (x_i^2 - 2x_i\mu + \mu^2)\, p(x_i)$

$= \sum_{i=1}^{n} x_i^2 p(x_i) - 2\mu \underbrace{\sum_{i=1}^{n} x_i p(x_i)}_{\mu} + \mu^2 \underbrace{\sum p(x_i)}_{1}$

$= \sum_{i=1}^{n} x_i^2 p(x_i) - 2\mu^2 + \mu^2$

$= \sum x_i^2 p(x_i) - \mu^2$

$\boxed{\sigma^2 = E(x^2) - (E(x))^2} = E(x - \bar{x})^2 \rightarrow$ D.R.V

If $h(t)$ is any polynomial logarithmic or exponential or any cont. func.

$h(x)(s) = h(x(s))$

If $E(x) = \mu = \sum x_i\, p(x_i)$

$E(h(x)) = \sum h(x_i)\, p(x_i)$

Note

$h(x) = x^2 + 5x + 2$

$h(x)(s)$

$(hx)(HH)$

$y = x^2 = \{0, 1, 4\}$

$5x = (0, 5, 10)$

$2 = (2, 2, 2)$

variance:

$\sigma^2 = E(x-\mu)^2 p x_i$

$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$= E(x^2) - (E(x))^2$

for C.R.V.

$\sigma^2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx - \left[\int_{-\infty}^{\infty} x\, p(x)\, dx\right]^2$

If var $x = \sigma_x^2$ and $a, b \in R$

$\mathrm{var}(ax + b) = \mathrm{var}(ax) = a^2\, \mathrm{var}(x)$

Theorem:

$\mathrm{var}(ax + R) = a^2\, \mathrm{var}(x)$

proof: 1) $\mathrm{var}(x + R) = \mathrm{var}\, x$.

If $E(x) = \mu_x$

$E(x + R) = \mu_{x+R} = \sum (x + R)\, p(x_i)$

$= \sum x_i\, p(x_i) + \sum R\, p(x_i)$

$= E(x) + R$

$= \mu_x + R$

$\mathrm{var}(x + R) = \sum (x_i + R - \mu_x + R)^2\, p(x_i)$

$= \sum (x_i + \cancel{R} - \mu_x - \cancel{R})^2\, p(x_i)$

$= \sum (x_i - \mu_x)^2\, p(x_i)$

$= \sigma_x^2$ //

11) $Var(kx) = k^2 \, Var\, x$

$var(kx) = \Sigma (kx_i - \mu_{kx})^2 P(x_i)$

$= k \Sigma x_i P(x_i)$

$= kE(x) = k\mu_x$

$= \Sigma (kx_i - k\mu_x)^2 P(x_i)$

$= k^2 \Sigma (x_i - \mu_x)^2 P(x_i)$

$= k^2 \, var\, (x)$

1)

$R_x = \{-3, -1, 2, 5\}$

X	-3	-1	2	5
P(x)	$\frac{2k-3}{10}$	$\frac{k-2}{10}$	$\frac{k-1}{10}$	$\frac{k+1}{10}$

$P(x_i) = 1$

$\frac{2k-3}{10} + \frac{k-2}{10} + \frac{k-1}{10} + \frac{k+1}{10} = 1$

$\therefore k = 3$

x	-3	-1	2	5
P(x)	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{4}{10}$

$\cancel{E(x) = (x-\mu)^2 P(x)}$ σ^2

~~(or)~~

$E(x) = \Sigma x_i \, P(x_i)$

$= -3 \cdot \frac{3}{10} - 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 5 \cdot \frac{4}{10}$

$= \frac{14}{10}$

$Var(x) = \Sigma (x_i - \mu)^2 P(x_i)$

$= (-3 - \frac{14}{10})^2 \cdot \frac{3}{10} + (-1 - \frac{14}{10})^2 \cdot \frac{1}{10} + (2 - \frac{14}{10})^2 \cdot \frac{2}{10} + (5 - \frac{14}{10})^2 \cdot \frac{4}{10}$

$= \frac{136}{10}$

(or)

$Var(x) = E(x^2) - (E(x))^2$

$E(x^2) = \Sigma x_i^2 P(x_i)$

$= 9 \cdot \frac{3}{10} + 1 \cdot \frac{1}{10} + 4 \cdot \frac{2}{10} + 25 \cdot \frac{4}{10}$

$= \frac{136}{10}$

$\sigma^2 = \frac{136}{10} - (14/10)^2 = \frac{1164}{100}$

Q. 8000 tickets sold → Rs. 5 each

prize → TV of 12,000

X : gain in Rs

$R_x = \{-10, \ 11{,}990\}$

x	-10	$11{,}990$
$P(x)$	$\frac{7998}{8000}$	$\frac{7999 \times 1}{8000 C_2} = \frac{2}{8000}$

$$E(x) = -10 \cdot \frac{7998}{8000} + 11{,}990 \cdot \frac{2}{8000}$$

$$= -\underline{1}$$

Conclusion: It is a loss of Rs. 1 ($\because$ -ve sign)

Q. Item of 5000 Rupees

prob. of theft = 0.01

$X \to$ gain in Rs.

$R_x = \{N, \ N-5000\}$

x	N	$N-5000$
$P(x)$	0.99	0.01

$$E(x) = 1000 = N \times 0.99 + 0.01\,(N-5000)$$

$$= N - 50$$

$$N > 8N - 1050$$

Q. Coin → tossed 2 times

1 Head → 1 Dollar

3 Head → 3 Dollars.

$R_x = \{-3, 1, 2\}$

x	-3	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$S = \{HH, \ HT, \ TH, \ TT\}$

$x = \begin{cases} 1 & \text{one head} \\ 2 & \text{two heads} \\ -3 & \text{0 head.} \end{cases}$

$$E(x) = -\frac{3}{4} + \frac{1}{2} + \frac{2}{4} = \frac{1}{4}\ \$$$

$= \cancel{\frac{1}{4}}$ (fayorable to players)

~~= 0 (fair)~~

$$var = \sigma^2 = \cancel{(E(x^2)) - (E(x))^2}$$

$$= \cancel{-\frac{3}{4} - \left(\frac{1}{4}\right)^2}$$

$$= \cancel{\frac{-13}{16}}$$

(or)

$$var = E(x-4)^2\, P(x)$$

$$= \frac{59}{16}$$

Q. $X \to$ cont. random variable

$$f(x) = \begin{cases} \frac{x}{16} + k & 0 \le x \le 3 \\ 0 & \text{Else where} \end{cases}$$

Find K:

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1 \qquad \int_0^3 \left(\frac{x}{16} + k\right) \cdot dx = 1$$

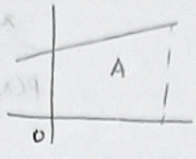
If $f(x)$ is pd f,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

$$\int_{0}^{3} \left(\frac{x}{6} + k\right) \cdot dx = 1$$

$$k = \frac{1}{5}$$

II Method:



$$A = \frac{\left(k + k + \frac{1}{2}\right) \times 3}{2} = 1$$

$$P(1 \leq x \leq 2) = \int_{1}^{2} f(x) \cdot dx$$

$$= \int_{1}^{2} \frac{x}{6} + \frac{1}{12} \cdot dx = \frac{1}{3}$$

Q. $f(x) = \begin{cases} \frac{20,000}{x^3} & x > 100 \\ 0 & \text{elsewhere} \end{cases}$

$x \to$ life in hrs. of an electronic device.

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$= \int_{100}^{\infty} x \cdot \frac{20,000}{x^3} \cdot dx$$

$$= 200 \text{ hours}$$

$$E(x) = \int_{100} x \cdot f(x) \cdot dx = \int_{100}^{\infty} x \cdot \frac{20000}{x^3} \cdot dx$$

$$= \int_{100}^{\infty} \frac{20,000}{x^2} \cdot dx = 20,000 \int_{100}^{\infty} x^{-2} \cdot dx$$

$$= 20,000 \left(-\frac{1}{x}\right)_{100}^{\infty} = 200 \text{ hours}$$

Note. x and y are two random variables defined on same sample spaces.

$R_x = \{x_1, x_2 \dots x_n\}$

$R_y = \{y_1, y_2, \dots y_n\}$

m may be equal to n.

Joint probability are joint distribution function defined as

$n = R_x \times R_y$

$$\boxed{f(x_i, y_i) = P(\{s \in S) : x(s) = x_i, y(s) = y_j\})}$$

Note: $\sum_i \sum_j f(x_i, y_j) = 1$

$y = \begin{cases} 0 & \text{even} \\ 1 & \text{odd} \end{cases}$

$x \to$ no. appearing on face

$R_x = \{1, 2, 3, 4, 5, 6\}$

$R_y = \{0, 1\}$

$R_x \cdot R_y = \{(1,0), (1,1), (3,0), (2,1) \dots\}$

$p(2, 1) = \frac{1}{12}$

$\Sigma f(x, y) = 1 = \sum_j \sum_i f(x_i, y_j)$

the prob. of

For any given region A in xy plane, $P[(x,y) \in A] =$

$\sum\sum_{x,y \in A} f(x, y)$.

X and Y are said to be independent:

$P(X = x, Y = y) = P(X = x) \cdot P(Y = y)$.

X \ Y	y_1	y_2	...	y_n	
x_1	$p(x_1, y_1)$	-	-	$p(x_1, y_n)$	$h(x_1)$
x_2					$h(x_2)$
$\vdots$					
x_n	$p(x_n, y_1)$	-	-	$p(x_n, y_n)$	$h(x_n)$
	$k(y_1)$	$k(y_2)$		$k(y_n)$	1

Date: 08-09-2022

$S_{X,Y}$ $\quad R_X = \{x_1, \dots x_n\}$

$R_Y = \{y_1, \dots y_n\}$

General expression of $h(x)$:

$h(x_i) = \sum_j^m f(x_i, y_j)$, for $i = 1, 2, \dots n$

$K(y_j) = \sum_i^n f(x_i, y_j)$, for $j = 1, 2, \dots n$

For marginal probability.

Marginal probability,

$h(x) = \int_{-\infty}^{\infty} f(x, y) \cdot dy$

$K(y) = \int_{-\infty}^{\infty} f(x, y) \cdot dx$

If it fulfills the condition of pdf,

$f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y) \cdot dx\, dy = 1$.

$\mu_{g(x,y)} = E(g(x,y)) = \sum_y \sum_x g(x_i, y_j) f(x_i, y_j)$

eg. $g(X, Y) = XY$

$\mu_{xy} = E(XY) = \sum_i \sum_j x_i y_j f(x_i, y_j)$

In case 2 random variables are involved.

co-variance $= \text{cov}(x, y) = \sum_i \sum_j (x_i - \mu_x)(x_j - \mu_y) f(x_i, y_j)$

$= E[(x - \mu_x)(y - \mu_y)]$

$\text{cov}(x, y) = E(XY) - E(X) \cdot E(Y)$

If x and y are independent, then

$cov(x, y) = 0$

$\boxed{\Rightarrow E(XY) = E(X) \cdot E(Y)}$ → Independence of x and y

Conditional Distribution

$f(Y/X = x) = \frac{f(x,y)}{h(x)}$ → density function, $h(x)$ → Marginal Probability.

$f(X/Y = y) = \frac{f(x,y)}{k(y)}$

$Z = ax + by$

$\sigma_z^2 = \sigma^2_{ax+by} = E\{Z - \mu_z\}^2$

$= E\{ax + by - (a\mu_x + b\mu_y)\}$

$= E\{a(x - \mu_x) + b(y - \mu_y)\}$

$\boxed{var(ax+by) = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\ cov}$

Note:

$var(ax + by) = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\,\sigma_{xy}$

Q. $S = \{(1,1) \ldots (2,1) \ldots (5,5)\}$

X, Y, Z → random variables d

$X = \begin{cases} 0 & \text{if first no. is even} \\ 1 & \text{if first no. is od} \end{cases}$

$Y = \begin{cases} 1 & \text{if second no. is o} \\ 0 & \text{if " " " even (10)} \end{cases}$

$Z = X + Y$

Find Distribution of X, Y and Z, joint distribution, independence of x and y.

Ans: $R_X = \{0, 1\}$ $R_Y = \{0, 1\}$

x	0	1
$p(x)$	$\frac{10}{25}$	$\frac{15}{25}$

y	0	1
$p(x)$	$\frac{10}{25}$	$\frac{15}{25}$

$E(x) = \mu_x = \frac{15}{25}$

$\mu_y = 0 \cdot \frac{10}{25} + 1 \cdot \frac{15}{25}$

$= \frac{15}{25}$

$\sigma_x^2 = E(x^2) - (E(x))^2$

$= \frac{15}{25} - \left(\frac{15}{25}\right)^2$

$= \frac{6}{25}$

$\sigma_y^2 = E(y^2) - (E(y))^2$

$= \frac{15}{25} - \left(\frac{15}{25}\right)^2$

$= \frac{6}{25}$

If X and Y are independent, the
$Cov(X,Y) = 0$

$$\boxed{\Rightarrow E(XY) = E(X) \cdot E(Y)} \longrightarrow \text{Indep}$$

Conditional Distribution

$$f(Y/X=x) = \frac{f(x,y)}{h(x)} \quad \begin{matrix}\rightarrow \text{Density fu} \\ \rightarrow \text{Marginal}\end{matrix}$$

$$f(X/Y=y) = \frac{f(x,y)}{k(y)}$$

$$Z = ax + by$$

$$\sigma_Z^2 = \sigma^2_{ax+by} = E\{Z - \mu_Z\}^2$$

$$= E\{ax + by - (a\mu_x + b\mu_y)\}^2$$

$$= E\{a(x-\mu_x) + b(y-\mu_y)\}^2$$

$$\boxed{var(ax+by) = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\, cov(X,Y)}$$

Q. $S = \{(1,1) \dots (2,1) \dots (5,5)\} \rightarrow 25$ pairs.

$X, Y, Z \rightarrow$ random variables defined as

$$X = \begin{cases} 0 & \text{if first no. is even. } (10) \\ 1 & \text{if first no. is odd } (15) \end{cases}$$

$$Y = \begin{cases} 1 & \text{if second no is odd } (15) \\ 0 & \text{if " " " even } (10) \end{cases}$$

$Z = X+Y$

Find Distribution of X, Y and Z, joint distribution, independence of x and y.

Ans: $R_x = \{0, 1\}$ $\quad R_y = \{0, 1\}$

x	0	1
$p(x)$	$\frac{10}{25}$	$\frac{15}{25}$

y	0	1
$p(x)$	$\frac{10}{25}$	$\frac{15}{25}$

$$E(x) = \mu_x = \frac{15}{25}$$

$$\mu_y = 0 \cdot \frac{10}{25} + 1 \cdot \frac{15}{25} = \frac{15}{25}$$

$$\sigma_x^2 = E(x^2) - (E(x))^2$$

$$= \frac{15}{25} - \left(\frac{15}{25}\right)^2$$

$$= \frac{6}{25}$$

$$\sigma_y^2 = E(y^2) - (E(y))^2$$

$$= \frac{15}{25} - \left(\frac{15}{25}\right)^2$$

$$= \frac{6}{25}$$

Joint distribution:

Y / X	0	1	h(x)
0	$\frac{4}{25}$	$\frac{6}{25}$	$\frac{10}{25}$
1	$\frac{6}{25}$	$\frac{9}{25}$	$\frac{15}{25}$
E(y)	$\frac{10}{25}$	$\frac{15}{25}$	1

$E(XY) = E(X) \cdot E(Y)$

$p(x,y) = p(x) \cdot p(Y) \quad \forall x, y$

$\boxed{E(X,Y) = \sum_i \sum_j x_i y_j f(x_i, y_j)}$

$= \sum_j \{ (x_i = 0 \cdot y_j f(0, y_j) + (x_i = 1 \; y_j f(1, y_j)) \}$

$= (x_2 = 1)(Y_2 = 1) f(1,1)$

$= \frac{9}{25} = E(x) E(Y)$

∴ The random variables are independent.

$Z = X + Y$

$R_Z = \{0, 1, 2\}$

$\{(0,0), (0,1), (1,0), (1,1)\}$

$Z = 0 \Rightarrow x = 0$ and $y = 0 = \{(0,0)\}$

$Z = 1 = \{(0,1) \; (1,0)\}$

$Z = 2 = \{(1,1)\}$

z	0	1	2
P(z)	$\frac{4}{25}$	$\frac{12}{25}$	$\frac{9}{25}$

$P(Z=0) = P(X=0) + P(Y=0)$

$P(Z=1) = P((0,1), (1,0))$

$= P(0,1) + P(1,0)$

$= \frac{12}{25}$

$E(Z) = \mu_Z = \frac{12}{25} + \frac{18}{25}$

$= \frac{30}{25} = \mu_x + \mu_y$

$\sigma^2 = \frac{40}{25} - \frac{30^2}{25^2} = \frac{12}{25}$

Q2) $S = \{1, \dots 6\}$

$X \to$ twice the no. appearing

$Y \Rightarrow \begin{cases} 1 & \text{No. is odd} \\ 3 & \text{No. is even.} \end{cases}$

$Z = X + Y$

Ans: $R_X = \{2, 4, 6, 8, 10, 12\}$

$R_Y = \{1, 3\}$

x	2	4	6	8	10	12
P(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

y	1	3
P(y)	$\frac{1}{2}$	$\frac{1}{2}$

$\mu_x = 7$

$\sigma_x^2 = 11.7$

$\mu_y = 2$

$\sigma_y^2 = 1$

Joint Distribution:

Y \ X	2	4	6	8	10	12	h(x)
1	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{3}{6}$
3	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{3}{6}$
k(y)	$\frac{1}{6}$	. . .				$\frac{1}{6}$	1

$f(2,3) = P(x=2) \cdot P(y=3)$

$0 \neq \frac{1}{12}$ (Not independent)

$z(1) = x(1) + y(1)$
$= 2+1$
$= 3$

$z(3) = 6+1 = 7$

$z(2) = 4+3 = 7$

Q. $S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

$X = \begin{cases} 0 & \text{if first toss head} \\ 1 & \text{if first toss tail} \end{cases}$

Y : total no. of heads.

cov (x, y)

$R_x = \{0, 1\}$ $\qquad R_Y = \{0, 1, 2, 3\}$

X \ Y	y_1 0	y_2 1	y_3 2	y_4 3	
$x_1 = 0$	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{4}{8}$
$x_2 = 1$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0	$\frac{4}{8}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	1

x	0	1
P(x)	$\frac{1}{2}$	$\frac{1}{2}$

y	0	1	2	3
P(y)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\mu_x = \frac{1}{2} = E(x)$

$\mu_y = \frac{12}{8} = \frac{3}{2} = E(y)$

$Cov(X,Y) = E(XY) - E(X)E(Y)$

$E(XY) = \sum_{j=1}\sum_{i=1} x_i y_j f(x_i y_j)$

$= \cancel{x_1 y_1} f(x_1 y_1) + \cancel{x_1 y_2} f(x_1 y_2) + \cancel{x_1 y_3} f(x_1 y_3) + \cancel{x_1 y_4}$

$f(x_1 y_4) + \cancel{x_2 y_1} f(x_2 y_1) + x_2 y_2 f(x_2 y_2) + x_2 y_7 + (x_2 y_3)$

$+ \cancel{x_2 y_4} f(x_2 y_4)$

$= 1 \cdot 1 \cdot \frac{2}{8} + 1 \cdot 2 \cdot \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

$Cov(x,y) = \frac{1}{2} - \frac{1}{2} \cdot \frac{3}{2} = -\frac{1}{4}$

$f(y/x=1) = \frac{f(1,4)}{h(x=1)} = \frac{f(1,4)}{1/2} = 2f(1/4)$

$f(0/1) = \frac{1}{4}$ $\quad f(2/1) = \frac{1}{4}$

$f(1/1) = \frac{1}{2}$ $\quad f(3/1) = 0$

\# Joint Density function for variable x where x is unit temp-change and y is unit proportional change that a certain unique particle 1

$$f(x,y) = \begin{cases} 10xy^2 & 0 \leq x < y < 1 \\ 0 & \text{else where} \end{cases}$$

a) find the marginal densities $h(x)$ and $k(y)$ and the conditional densities $f(y/x)$

b) find the prob. that the spectrum shifts more than $\frac{1}{2}$ of the total given the temp. is ↑ed by 0.25 unit.

Ans: $h(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_x^1 10xy^2\,dy$

$= \frac{10x}{3}(y^3)_x^1$

$= \frac{10x}{3}(1-x^3)$

$k(y) = \int_{-\infty}^{\infty} f(x,y) \cdot dx = \int_0^y 10xy^2\,dx$

$= \frac{10y^2}{2} \cdot (x^2)_0^y$

$= \underline{5y^4}$

$f(y/x=x) = \frac{f(x,y)}{h(x)}$

$= \frac{10xy^2}{\frac{10}{3}(x-x^4)} = \frac{3y^2}{1-x^3}$ $0<x<4<1$

$x = 0.25$

$y > \frac{1}{2}$

ii) $P(y > 1/2 / x = 0.25) = \int_{1/2}^{1} \frac{3y^2}{1-0.25^3} \cdot dy$

$= \frac{3\times 64}{63} \left(\frac{y^3}{3}\right)_{\frac{1}{2}}^{1} = \frac{3\times 64}{63\times 3} \cdot \frac{7}{8}$

Q. x and y are two random variables having joint distribution

$f(x,y) = \begin{cases} 6xy^2 & 0 \le x, y \le 1 \\ 0 & \text{else where} \end{cases}$

Ans:

$P(A \cap B) = P(A)\,P(B)$ $E(XY) = E(X).E(Y)$

$f(x,y) = f_x(x)\, f_y(y)$

$f_x(x) = \int_0^1 6xy^2\, dy = 2x$ Question is to prove in dependence

$f_y(y) = \int_0^1 6xy^2\, dx = 3y^2$

$f_x(x)\, f_y(y) = 2x\, 3y^2 = 6xy^2 = f(x,y)$

$E(x) = \int_0^1 x\, 2 \quad = \frac{2}{3}$

Chebyshev's Inequality

let x be a random variable, $\mu_x = E(x) = \mu$, $SD = \sigma$, $k > 0$ the prob. that x lies in the interval $(\mu - k\sigma, \mu + k\sigma)$ is atleast $\left(1 - \frac{1}{k^2}\right)$.

$P(\mu - k\sigma \le X \le \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

$P(|x-\mu| \le k\sigma) \geq k^2$

$$E(x) = \int_{0}^{\infty} x f(x) \cdot dx$$

$f(x) \to$ p.d.f

$$= \int_{0}^{a} x f(x) \cdot dx + \int_{a}^{\infty} x f(x)\, dx$$

$$E(x) \geq \int_{a}^{\infty} x f(x)\, dx \geq a \int_{a}^{\infty} f(x) \cdot dx \to a P(x \geq a)$$

$$P(x \geq a) \leq \frac{E(x)}{a}$$

$$P((x-\mu)^2 \geq k^2) \leq \frac{E(x-\mu)^2}{k^2} \qquad Y = (x-\mu)^2$$

$$P(|x-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$$

Q) $\sigma = 0.002$ mm

Ans: X: diameter of cylindrical part in mm

$$P(|x-\mu| \leq 0.006) = ?$$

$$k\sigma = 0.006$$

$$k = 3$$

$$P(|x-\mu| \leq 0.006) \geq 1 - \frac{1}{3^2} = \frac{8}{9}$$

Moments and Moment Generating functions

If $g(x) = x^r$, $r = 0, 1, 2, \ldots$, rth moment of the random var. x about the origin $\mu_r' = E(x^r) = \begin{cases} \sum x^r p(x) \;\forall x \\ \int_{-\infty}^{\infty} x^r f(x) \cdot dx \end{cases}$

discrete random var.

cont. random var.

The rth moment of the r.v x about mean will be denoted by $\mu_r = E(x-\mu)^r = \begin{cases} \sum (x_i - \mu)^r p(x_i) \\ \int_{-\infty}^{\infty} (x-\mu)^r f(x) \cdot dx \end{cases}$

The method to obtain moments other than definitions, we have moment generating functions.

Denotion: $M_x(t)$ $\quad t \to$ parameter.

$$M_x(t) = E(e^{xt}) = \begin{cases} \sum e^{xt} \cdot p(x) \to \text{Discreet r.v} \\ \int_{-\infty}^{\infty} e^{xt} f(x)\, dx \to \text{cont. r.v} \end{cases}$$

The rth moment about (0,0) is obtained by diff. mgf w.r.t t at $t=0$.

M.G.F of x about arbitrary point a, $M_a(t) = E(e^{(x-a)t})$.

$$M_a(t) = E(\underbrace{e^{-at}}_{\text{constant}} e^{xt})$$

$$= e^{-at} E(e^{xt})$$

$$= e^{-at} M_x(t)$$

Sum of 2 random variables,

$$M_{(x+y)}(t) = E(e^{(x+y)t})$$

$$= E(e^{xt} \cdot e^{yt})$$

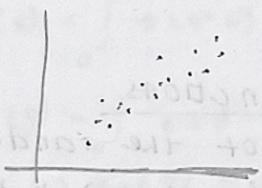
$$= E(e^{xt}) E(e^{yt}) \text{ if } x \text{ and } y \text{ are independent.}$$

$$= M_x(t) \cdot M_y(t)$$

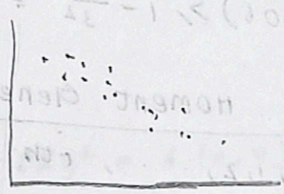
$M_{ax}(t) = E(e^{axt}) = E(e^{(at)x})$

$$= M_x(at)$$

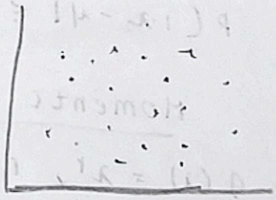
Correlation



+vely correlated



-vely correlated



No correlation.

Karl Pearson coefficient correlation,

$$r(x,y) = r_{xy} = r = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$= \frac{E(x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y}$$

$$= \frac{E(XY) - E(X)E(Y)}{(E(X^2) - (E(X))^2)^{\frac{1}{2}} (E(Y^2) - (E(Y))^2)^{\frac{1}{2}}}$$

$$= \frac{\frac{1}{n}\Sigma XY - \frac{\Sigma X}{n} \cdot \frac{\Sigma Y}{n}}{\left\{\frac{1}{n}\Sigma x^2 - \left(\frac{\Sigma x}{n}\right)^2\right\}^{\frac{1}{2}} \left\{\frac{1}{n}\Sigma y^2 - \left(\frac{\Sigma y}{n}\right)^2\right\}^{\frac{1}{2}}}$$

shido si
shido!

$$r = \frac{\Sigma xy - \frac{\Sigma x \cdot \Sigma y}{n}}{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)^{\frac{1}{2}}\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)^{\frac{1}{2}}}$$

x ma	y	x^2	y^2	xy
5	2	25	4	10
11	4	121	16	44
15	6	225	36	90
19	8	361	64	152
24	10	526	100	240
28	12	784	144	336
33	14	1089	196	462
$\Sigma x = 135$	$\Sigma y = 56$	$\Sigma x^2 = 3181$	$\Sigma y^2 = 56$	$\Sigma xy = 1334$

$n = 7$

$\text{cov}(x, y) = 36.285$

$r_{xy} = 0.9988$

$$E\left(\frac{x - \mu_x}{\sigma_x} \pm \frac{y - \mu_y}{\sigma_y}\right)^2 \geq 0$$

$$\frac{E(x - \mu_x)^2}{\sigma_x^2} + \frac{E(y - \mu_y)^2}{\sigma_y^2} \pm \frac{2E(x - \mu_x)(y - \mu_y)}{\sigma_x \sigma_y} \geq 0$$

$$1 + 1 \pm 2\frac{\text{cov}(x, y)}{\sigma_x \sigma_y} \geq 0$$

$$2 \pm 2 \cdot r_{xy} \geq 0 \quad \text{if} \quad r = 1$$

$$= -1$$

$$-1 \leq r_{xy} \leq 1$$

Perfectly correlated.

Curve fitting (Line)

$$y = a + bx$$

$$E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y' - y_i)^2 = \sum_{i=1}^{n} (a + bx_i - y_i)^2$$

$$\frac{\partial E}{\partial a} = 0 = \frac{\partial E}{\partial b}$$

$$\frac{\partial E}{\partial a} = \sum 2(a + bx_i - y_i) = 0$$

$$\frac{\partial E}{\partial b} = \sum 2x_i(a + bx_i - y_i) = 0$$

$$\sum (a + bx_i - y_i) = 0$$

$$\sum x_i (a + bx_i - y_i) = 0$$

$$\left.\begin{aligned} \sum y_i &= \sum a + \sum b x_i \\ \sum x_i y_i &= \sum a x_i + \sum b x_i^2 \end{aligned}\right\} \text{Normal eqn}$$

$$\sum y_i = na + b\sum x_i$$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2$$

Let (x_i, y_i) be a bivariate data assuming one of them independent, say x and other dependent on first, say y (which may be random in nature). We predict the value of y by fitting a curve to data. If the curve is line then it called line of regression and there said to be a linear relation.

If the curve is fit for y on x, it is called regression curve of y on x. And similarly for x too.

Line of regression always passes through $\bar{x}, \bar{y}$.

Note:

$$r_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\sum xy - \frac{\sum x \cdot \sum y}{n}}{\sigma_x \sigma_y}$$

$$\sum x_i y_i = a\sum x_i + b\sum x_i^2 \longrightarrow (1)$$

Divide by n to (1),

$$\frac{\sum x_i y_i}{n} = a\bar{x} + b\frac{\sum x_i^2}{n} \longrightarrow (2)$$

$$\frac{\Sigma xy}{n} - \bar{x}\bar{y} = b\left[\cancel{-\bar{x}^2} + \sigma_x^2 + \cancel{\bar{x}^2}\right]$$

$$\Rightarrow b = \frac{\left(\frac{\Sigma xy}{n} - \bar{x}\bar{y}\right)}{\sigma_x^2} \frac{\sigma y}{\sigma y} = \frac{\sigma y}{\sigma x} \cdot r_{xy}.$$

$$y - \bar{y} = b(x - \bar{x}),$$

$$y - \bar{y} = \frac{\sigma y\, r}{\sigma x}(x - \bar{x}) \longrightarrow y \text{ on } x.$$

$$x - \bar{x} = \frac{\sigma x}{\sigma y} r(y - \bar{y}) \longrightarrow x \text{ on } y$$

Q. x (current), y (voltage)

$\bar{x} = 19.285$, $\bar{y} = 8$

$\sigma_x^2 = 82.49$, $\sigma y^2 = 16$

$\sigma x = 9.08$, $\sigma y = 4$

$\text{cov}(x \cdot y) = 36.285$, $r_{xy} = 0.9988$

<u>y on x</u> (voltage dependent on current).

$v = a + bi$

$b = r_{xy}\frac{\sigma y}{\sigma x}$

$= 0.9988\left(\frac{4}{9.08}\right)$

$= 0.4399$

$a = -0.4838$

$b = 0.4399$

$(y - 8) = 0.4399(x - 19.285)$

$y = 0.4399x - (0.4399 \times 19.285) + 8$

$y = 0.4399x + 8.48$

$m_1 = r\frac{\sigma y}{\sigma x}$, $m_2 = \cancel{\frac{\sigma x}{\sigma y r}}\ \frac{\sigma y}{\sigma x r}$

$$\tan^{-1}\left|\frac{\left(r\frac{\sigma x}{\sigma y} - \frac{\sigma y}{\sigma x r}\right)}{1 + \frac{\sigma y^2}{\sigma x^2}}\right| = \tan^{-1}\frac{|1 - r^2|}{|r|} \cdot \frac{\sigma x^2 \cdot \sigma y}{\sigma x^2 + \sigma y^2\ \cancel{\sigma x}}$$

$r = \pm 1$, $\tan\theta = 0$

$r = 0$, $\tan\theta = \pi/2$ ($\perp$ lines)

Bernoulli Trial

{success, failure} → only two outcomes.
(p)   (1−p)

Binomial Experiment

→ Binomial trial is conducted n times, (n → fixed), n Bernoulli trials with prob. of success p.

B (n, p). → n Bernoulli trials with prob. of success p.

X → no. of success

$R_X = \{0, 1, 2, \dots, n\}$

Probability of r-success :

$= {}^nC_r p^r (1-p)^{n-r}$

x	0	1	2	...	n
$p(x)$	q^n	${}^nC_1 p q^{n-r}$			p^n

$\Sigma p(x) = q^n + {}^nC_1 p q^{n-r} + \dots + {}^nC_n p^n$

$= (p+q)^n = \underline{\underline{1}}$

$X_1 = \{0, 1\}$, $X_2 = \{0, 1\}$

$Y = X_1 + X_2 = \{0, 1, 2\}$

$X = X_1 + X_2 + \dots + X_n.$

$E(X) = E(X_1 + X_2 + \dots + X_n)$

$= E(X_1) + E(X_2) + \dots + E(X_n) = p + p + \dots + p = \underline{\underline{np}}$

x	0	1
$p(x)$	q	p

$E(X_i) = 0 \cdot q + 1 \cdot p = p.$

$var(x_i) = E(x^2) - (E(x))^2$

$= 0^2 q + 1^2 p - p^2$

$= p - p^2$

$= p(1-p)$

$= \underline{\underline{p.q}}$

$var X = var(X_1 + X_2 + \dots + X_n)$

$= var X_1 + var X_2 + \dots var X_n$

$= \underline{\underline{npq}}$

M.G.f

$$M_x(t) = E(e^{xt}) = \sum_{r=0}^{n} e^{xt}\, {}^nC_r\, p^r q^{n-r}$$

{ x and r same : no. of success (both)

$$= \sum_{r=0}^{n} e^{rt}\, {}^nC_r\, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} {}^nC_r\, (pe^t)^r q^{n-r}$$

~~$= (pe^t)$~~

$$M_x(t) = (pe^t + q)^n$$

$\mu = E(x)$

$$\mu_1 = \frac{d}{dt} M_x(t)\Big|_{t=0} = \frac{d}{dt}(e^t p + q)^n\Big|_{t=0}$$

$$= ne^t.p\,(e^t p + q)^{n-1}\Big|_{t=0}$$

$$= np. = E(x).$$

$$E(x^2) = \frac{d^2}{dt^2} M_x(t)\Big|_{t=0} = npe^t(pe^t + q)^{n-1} + n(n-1)p^2 e^{2t}(pe^t + q)^{n-2}\Big|_{t=}$$

$$= np + (n^2 - n)p^2$$

$$E(x^r) = \frac{d^r}{dt^r} M_x(t)\Big|_{t=0}$$

$$\sigma_x^2 = E(x^2) - (E(x))^2 = np + (n^2 - n)p^2 - n^2p^2$$

$$= np + \cancel{n^2p^2} - np^2 - \cancel{n^2p^2}$$

$$= np(1-p)$$

$$= npq$$

Note : If X and Y are independent binomial random variables with parameters (n,p) and (m,p), then

$$M_{x+y}(t) = E(e^{(x+y)t})$$

$$= E(e^{xt} e^{yt})$$

$$= E(e^{xt})\, E(e^{yt})$$

$$= M_x(t)\, M_y(t)$$

$M_X(t) = (pe^t + q)^n$

$X \to B(n, p)$

$M_Y(t) = (pe^t + q)^m$

$\therefore M_{X+Y}(t) = (pe^t+q)^n (pe^t+q)^m$

$= (pe^t+q)^{m+n}$

$\therefore X+Y = B(n+m, p)$

* Consider an exp. with similar properties as those of binomial with an exception that trials are to be continued till desired success

k^{th} success $\to$ x^{th} trial.

$x \to$ no. of success

$P(X = k-1) = {}^{x-1}C_{k-1}\, p^{k-1} q^{x-1} \sqrt{k-1}$

$P(k^{th}$ success in x^{th} trial$) = {}^{x-1}C_{k-1}\, p^k q^{x-1}$

Mgf

$M_X(t) = p^k (1-qe^t)^{-k}$

$E(X) = \frac{kq}{p}$

$\sigma^2(X) = \frac{kq}{p^2}$

Q. $P(D) = 0.01$ (independent of each other).

pkt. of $10. = n.$

$P(G) = 0.99$

X : no. of defective disks in one pkt.

$E \to$ pkt. is defective

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - P(X=0) - P(X=1)$

$= 1 - (0.99)^{10} - {}^{10}C_1 (0.99)^{10}(0.01)$

$= 1 - 0.995$

$= 0.005$

$= 0.5\%$ are to return

b) Out of 3 pkts, exactly one is returned.

Y: pkt. being defective out of bunch of 3.

$P(Y=1) = {}^3C_1 (0.005)^1 (0.995)^2$

=

=

Q) prob. patient recovers = 0.4.

15 patients in total = n.

i) Atleast 10 survives.

ii) from 3 - 8 survives

iii) exactly 5 survives.

$P(\text{success}) = 0.4$
$P(\text{failure}) = 0.6$

$B(15, 0.4)$

$X \rightarrow$ No. of patients recovered out of 15.

i) $P(X \geq 10) = P(X=10) + P(X=11) + P(X=12) + \dots + P(X=15)$

=

ii) $P(3 \leq X \leq 8) = P(X=3) + P(X=4) + \dots + P(X=8)$

=

iii) $P(X=5) = {}^{15}C_5 (0.4)^5 (0.6)^{10}$

=

Q) $P(WA) = 0.55$

$P(A) = {}^5C_3 (W_A)^3 (L_A)^2 \times W_A$ → Prob. A wins in 6 ~~ma~~ sets match.

$= {}^5C_3 (0.55)^4 (0.45)^2$

= 0.1853

=

b) X : No. of matches to be played for A to win.

$R_X = \{4, 5, 6, 7\}$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$= (0.55)^4 +$