

Exercise

Reduce the following quadratic forms to canonical forms or to sum of squares by linear transformation. Write also the rank, index and signature.

- 1) $2x^2 + 2y^2 + 3z^2 + 2xy - 4yz - 4xz$
- 2) $12x_1^2 + 4x_2^2 + 5x_3^2 - 4x_2x_3 + 6x_1x_3 - 6x_1x_2$
- 3) $2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx$
- 4) $x^2 + 4y^2 + z^2 + 4xy + 6yz + 2zx$

Solution

- 1) $2y_1^2 + \frac{3}{2}y_2^2 + \frac{1}{3}y_3^2$; Rank = 3, index = 3, sig. = 3
- 2) $12y_1^2 + \frac{13}{4}y_2^2 + \frac{49}{13}y_3^2$; Rank = 3, index = 3, sig. = 3
- 3) $2y_1^2 + y_2^2 - \frac{5}{2}y_3^2$; Rank = 3, index 2, sig. = 1
- 4) $y_1^2 + 2y_2^2 - \frac{1}{2}y_3^2$; Rank = 3, index = 2, Sig. = 1