Unrestricted Search

In most Practical Problems, the optimum solution is known to lie within restricted ranges of the design variables. In some cases this range is not known, and hence the search has to be made with no restrictions on the values of the variables. In restricted search methods. Fibonacci method is the first to be discussed.

Fibonacci Method

Fibonacci method is used to find the minimum of a function of one variable even if the function is not continuous. This method make use of the sequence of fibonacci numbers $\{F_n\}$ for placing the experiments. These numbers are defined as $F_0 = F_1 = 1$

$$F_n = F_{n-1} + F_{n-2} ; \quad n = 2, 3, 4, \ldots$$

Which yield the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Procedure

Let L_0 be the initial interval of uncertainty defined by $a \leq x \leq b$ and n be the total number of experiments to be conducted. Define

$$L_2^* = \frac{F_{n-2}}{F_n} L_0$$

and Place the first two experiments at Points x_1 and x_2, Which are located at a distance of ~~bot~~ ~~from~~ given by

$$x_1 = a + L_2^* \quad \text{and} \quad x_2 = b - L_2^*$$

$$x_1 = a + \frac{F_{n-2}}{F_n} L_0 \quad \text{and} \quad x_2 = b - \frac{F_{n-2}}{F_n} L_0$$

Discard part of the interval by unimodality assumption. Then there remains a smaller interval of uncertainty L_2 g. by

$$L_2 = L_0 - L_2^* = L_0\left(1 - \frac{F_{n-2}}{F_n}\right) = \frac{F_{n-1}}{F_n} L_0$$

continuing this process n times untill the ratio $\frac{L_n}{L_0}$ is approx equal to $1/F_n$.

Unimodality In functions

A unimodal function is one that has only one peak (Max.) or Valley (Min.) in a given interval

Q1. Minimize $f(x) = .65 - \left[\frac{.75}{(1+x^2)}\right] - .65x\tan^{-1}\left(\frac{1}{x}\right)$ in the interval $[0,3]$ by Fibonacci method using $n=6$.

Sol^n Here $n=6$, $L_0 = 3-0 = 3$ which gives

$$L_2^* = \frac{F_{n-2}}{F_n} L_0 = \frac{F_{6-2}}{F_6} L_0 = \frac{F_4 L_0}{F_6}$$

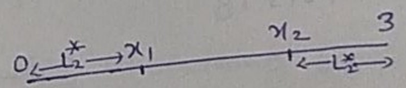
$$= \frac{F_4}{F_6} L_0$$

$\because$ Fibonacci series

1,	1,	2,	3,	5,	8,	13
F_0	F_1	F_2	F_3	F_4	F_5	F_6

$$L_2^* = \frac{5}{13}(3) = 1.153846$$

Thus the positions of the first two experiments are given by



$x_1 = 0 + L_2^* = 0 + 1.153846$, $x_2 = 3 - L_2^*$

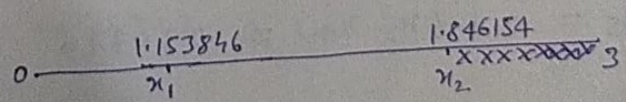
$x_1 = 1.153846$ $\quad\quad = 3 - 1.153846$

$\quad\quad x_2 = 1.846154$

$\therefore f(x_1) = -.207270$

$f(x_2) = -.115843$

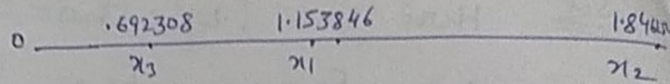
Since $f(x_1) < f(x_2)$, We can delete the interval $[x_2, 3]$



The third Experiment is placed at

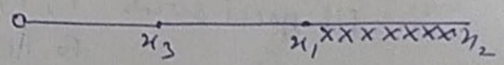
$x_3 = 0 + (x_2 - x_1)$

$x_3 = 0 + (1.846154 - 1.153846) = .692308$



Here $f(x_3) = -.291364$

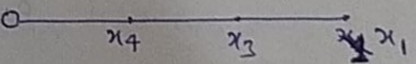
i.e $f(x_1) > f(x_3)$ i.e We delete the interval $[x_1, x_2]$



Now Fourth Experiment is placed at

$x_4 = 0 + (x_1 - x_3) = 0 + (1.153846 - .692308)$

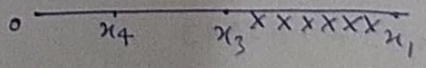
$x_4 = .461538$



$f(x_4) = -.309811$

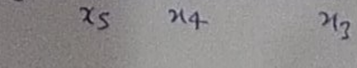
Here $f(x_4) < f(x_3)$

so deleted interval is $[x_3, x_1]$



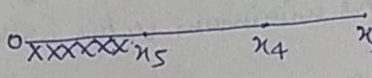
Fifth experiment is placed at

$x_5 = 0 + (x_3 - x_4) = .230770$



$f(x_5) = -.263678$

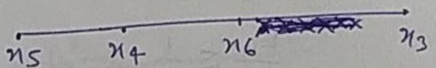
i.e $f(x_5) > f(x_4)$, we delete the interval $[0, x_5]$



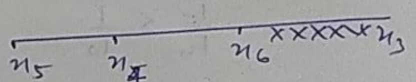
Now sixth experiment is placed at

$$x_6 = x_3 + (x_5 - x_4)$$

$$= .692308 + .230770 - .461538 = .461540$$



$f(x_6) = -.309810$



i.e $f(x_6) > f(x_4)$

delete the interval $[x_6, x_3]$

We obtain the final Interval of uncertainty $L_6 = [x_5, x_6]$

$L_6 = [.230770, .461540]$

The Ratio of the final to the initial interval of uncertainty is

$$\frac{L_6}{L_0} = \frac{.461540 - .230770}{3} = .076923$$

This value can be compared with the value $\frac{1}{F_6}$

$$= \frac{1}{13} = .076923 \text{. As}$$

Q.2. Find the minimum of $f(x) = x^2 + 2x$ within the interval $(-3, 4)$ using the fibonacci method, obtain the minimum value within the 5% of exact value.

Sol^n

To find n, we have

$$\frac{\text{Length of final of interval of uncertainty}}{2 \times \text{Length of initial interval of uncertainty}} \leq \frac{5}{100}$$

$$\frac{L_n}{2L_0} \leq \frac{5}{100} = \frac{1}{20}$$

$$\frac{L_n}{L_0} \leq \frac{1}{10}$$

$$\frac{1}{F_n} \leq \frac{1}{10}$$

$$F_n \geq 10$$

$\because$ Fibonacci series

1, 1, 2, 3, 5, 8, 13

F_0 F_1 F_2 F_3 F_4 F_5 F_6

$13 \geq 10$ i.e. for $\underline{\underline{n=6}}$

Note $\because$

$$\frac{L_n}{L_0} = \frac{F_n - F_{j-1}}{F_n} \qquad j = n$$

$$L_n = \frac{F_{n-(j-1)}}{F_n} L_0$$

$$\frac{L_n}{L_0} = \frac{F_{n-(j-1)}}{F_n} \qquad j = 0, 1, 2 \ldots n$$

$$= \frac{F_{n-n+1}}{F_n} \qquad \boxed{j = n}$$

$$\frac{L_n}{L_0} = \frac{F_1}{F_n} \qquad \because F_1 = 1$$

$$\boxed{\frac{L_n}{L_0} = \frac{1}{F_n}}$$

Here $f(x) = x^2 + 2x$, $[-3, 4]$

$n = 6$ $\quad$ $L_0 = 4+3 = 7$

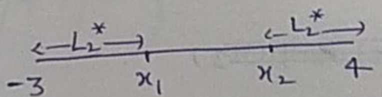
$$\therefore L_2^* = \frac{F_{n-2}}{F_n} L_0$$

$$= \frac{F_{6-2}}{F_6}(7) = \frac{F_4}{F_6}(7)$$

$$= \frac{5}{13} \times 7 \qquad \because \; \begin{matrix} 1, & 1, & 2, & 3, & 5, & 8, & 13 \\ F_0 & F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \end{matrix}$$

$$L_2^* = 2.6923$$

Thus the Position of the first two experiments are given by



$x_1 = -3 + L_2^* = -3 + 2.6923$

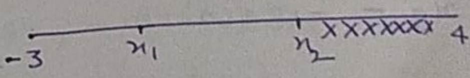
$x_1 = -.3076$

$x_2 = 4 - L_2^* = 4 - 2.6923 = 1.3077$

$f(x_1) = -.52072$

$f(x_2) = 4.3255$

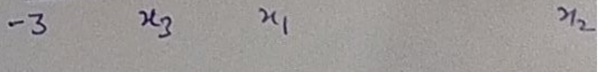
Here $f(x_1) < f(x_2)$, We can delete Interval $(x_2, 4)$



The third Experiment is Placed at

$x_3 = -3 + (x_2 - x_1) = -3 + [1.3077 + .3076]$

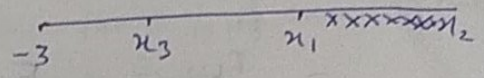
$x_3 = -1.3846$



$f(x_3) = -.8520$

Since $f(x_1) > f(x_3)$.

Delete the interval $[x_1, x_2]$

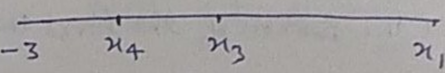


Now Fourth Experiment is placed at

$x_4 = -3 + [x_1 - x_3]$

$= -3 + (-.3076) + 1.3846 = -1.923$

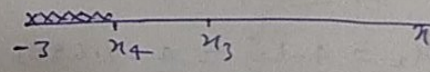
$f(x_4) = -.14807$



Here $\cancel{f(x_4) < f(x_3)}$

$f(x_4) > f(x_3)$

i.e Deleted Interval is $[-3, x_4]$



Now fifth Experiment is placed at

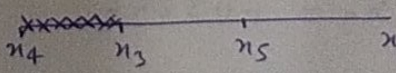
$x_5 = x_1 + (x_4 - x_3)$

$= -.3076 + [-1.923 + 1.3846]$

$x_5 = -.8462$

$f(x_5) = -.976384 \approx -.97634$

$f(x_3) > f(x_5)$ i.e Delete Interval is $[x_4, x_3]$

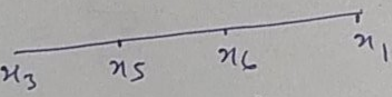


Now sixth experiment is placed at

$x_6 = x_1 + (x_3 - x_5)$

$= -.3076 + (-1.3846 + .846)$

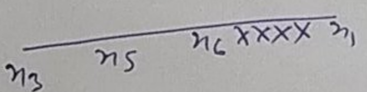
$x_6 = -.8462$



$f(x_6) = -.97634$

Here $f(x_6) > f(x_5)$

~~i.e. f(x5)~~ ~~So we delete the interval [x6, x1]~~



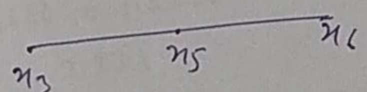
~~So we consider interval [x3, x1]~~

$\cancel{L_5 = x_1 - x_3 = -.3076 + 1.3846} = 1.077$

$\cancel{\frac{L_5}{L_0} = \frac{1.077}{7} = .1538}$

$\cancel{\frac{1}{F_5} = \frac{1}{8} = .13}$

Deleted Interval is $[x_6, x_1]$



So we obtain the final

Interval of uncertainty $L_6 = [x_3, x_6]$

$= [-1.384, -.8462]$ Ans

The Ratio of final and Initial Interval of uncertainty is

$$\frac{L_6}{L_0} = \frac{-.8462 + 1.384}{4-(-3)} = \frac{.5386}{7} = .07694$$

This value can be compaired with value $\frac{1}{F_6} = \frac{1}{13} = .07694$

① Minimize the function $f(x) = x^2 + \frac{54}{x}$ in the region $[0, 5]$, by taking $n = \cancel{8}\ 3$

Sol^n Here $n = 3$, $L_0 = 5 - 0 = 5$

Which Gives $L_2^* = \frac{F_{n-2}}{F_n} L_0 = \frac{F_{3-2}}{F_3} L_0$

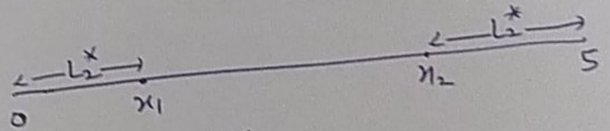
$= \frac{F_1}{F_3} L_0$

As we know that Fibonacci series is

1,	1,	2,	3,	5,	8, ...
F_0	F_1	F_2	F_3	F_4	F_5

$\therefore L_2^* = \frac{1}{3}(5) = 1.6666$

Thus the position of the first Two experiment are given by



$\therefore x_1 = 0 + L_2^* = 0 + 1.6666 = 1.6666$

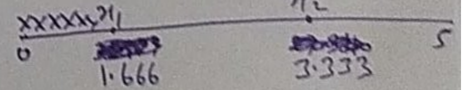
$x_2 = 5 - L_2^* = 5 - 1.6666 = 3.333$

$\therefore f(x_1) = f(1.6666) = (1.6666)^2 + \frac{54}{1.6666} = 35.17749$

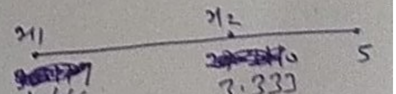
$f(x_2) = f(3.333) = (3.333)^2 + \frac{54}{3.3333} = 27.3110$

Here $f(x_1) > f(x_2)$

So we can delete the interval $(0, x_1)$



Now third experiment is placed at



$x_3 = 5 + (x_1 - x_2) =$

$= 5 + 1.666 - 3.333 = 3.333$

i.e $x_3 = 3.333$ (Hence $x_2 = x_3$)

Process is stoped

So final Interval of Uncertainty $L_3 = [x_2, x_3]$

$$L_3 = [1.6666, 3.333]$$

The Ratio of final to the initial interval of uncertainty is

$$\frac{L_3}{L_0} = \frac{3.333 - 1.666}{5} = \frac{1.66667}{5} = .33334 \ \underline{\underline{Ans}}$$

This value can be compared with value $\frac{1}{F_3}$

$$= \frac{1}{3} = .3333 \ \underline{\underline{Ans}}$$

Golden Section Method

The Golden section method is same as the Fibonacci method except that in the Fibonacci method the total number of experiments to be conducted has to be specified before beginning the calculation, where as this is not required in the golden section method.

Golden ratio is denoted by $\gamma\ (\simeq 1.618)$

$\because \quad F_n = F_{n-1} + F_{n-2}$ divid by F_{n-1}

$$\frac{F_n}{F_{n-1}} = \frac{F_{n-1}}{F_{n-1}} + \frac{F_{n-2}}{F_{n-1}}$$

$$\frac{F_n}{F_{n-1}} = 1 + \frac{F_{n-2}}{F_{n-1}} \longrightarrow (1)$$

$$\gamma = \lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \lim_{n\to\infty} \frac{F_{n-1}}{F_{n-2}} = \lim_{n\to\infty} \frac{F_{n-2}}{F_{n-3}}$$

From (1)

$$\gamma = 1 + \frac{1}{\gamma}$$

$$\gamma^2 - \gamma - 1 = 0$$

$$\gamma = \frac{1 \pm \sqrt{5}}{2}$$

$$\boxed{\gamma = 1.618 \text{ and } \frac{1}{\gamma} = .618}$$

Q.1. Minimize $f(x) = 4x^3 + x^2 - 7x + 14$ within the interval $[0, 1]$ using Golden section method.

Sol^n Here $f(x) = 4x^3 + x^2 - 7x + 14$,

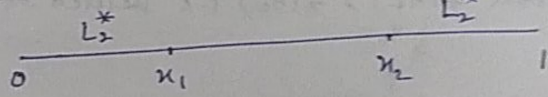
$L_0 = 1 - 0 = 1$

In Golden section method.

$L_2^* = \frac{1}{\gamma^2} L_0$, where γ is Golden ratio

$\gamma = 1.618$, And $\frac{1}{\gamma} = .618$

Thus the position of the first two experiments are given by



$L_2^* = \frac{1}{(1.618)^2} \times 1 = .3819$

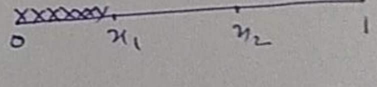
$x_1 = 0 + L_2^* = 0 + .3819 = .3819$

$x_2 = 1 - L_2^* = 1 - .3819 = .6181$

$f(x_1) = 4(.3819)^3 + (.3819)^2 - 7(.3819) + 14 = 11.6953$

$f(x_2) = 4(.6181)^3 + (.6181)^2 - 7(.6181) + 14 = 10.9999$

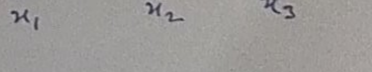
Here $f(x_1) > f(x_2)$, we can delete the Interval $[0, x_1]$



The Third Experiment is Placed at

$x_3 = 1 + (x_1 - x_2)$

$= 1 + .3819 - .6181 = .7638$

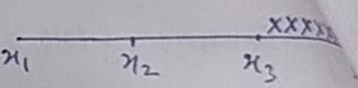


$f(x_3) = 4(.7638)^3 + (.7638)^2 - 7(.7638) + 14$

$= 11.0192$

Here $f(x_3) > f(x_2)$,

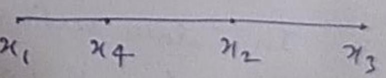
So we delete the Interval $[x_3, 1]$



Now Fourth Experiment is placed at

$x_4 = x_1 + (x_3 - x_2)$

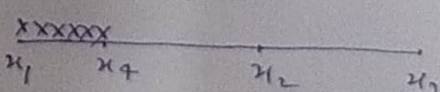
$= .3819 + .7638 - .6181 = .5278$



$f(x_4) = 4(.5278)^3 + (.5278)^2 - 7(.5278) + 14$

$= 11.1721$

Here $f(x_4) > f(x_2)$ i.e Deleted Interval is $[x_1, x_4]$



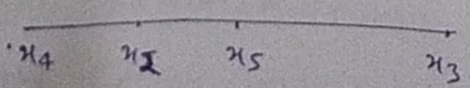
Now fifth ~~E~~ Experiment is placed at

$x_5 = x_3 + (x_4 - x_2)$

$= .7638 + .5278 - .6181$

$x_5 = .6736$

$f(x_5) = 4(\cancel{.7636}\ .6736)^3 + (.6736)^2 - 7(.6736) + 14 = 10.9611$



Newton's Method

$$x_{i+1} = x_i - [J_i]^{-1} \nabla f_i$$

Where

$$J_1 = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

And ∇f_1 is called gradient of f_1

and find as

$$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1}$$. where X_1 is given

How to find out X_2

$$X_2 = X_1 - [J_1]^{-1} \nabla f_1$$

$$X_3 = X_2 - [J_2]^{-1} \nabla f_2$$

if $X_2 = X_3$ Then Process will stoped. or If ∇f_1 is zero $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

then Process will stoped.

Q1 Minimize $f_1(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ taking the starting point as $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ by Newton method.

Now we J_1^{-1}

Solⁿ By Newton method, we find X_2 as

$\because X_{i+1} = X_i - [J_i]^{-1} \nabla f_i$

$X_2 = X_1 - [J_1]^{-1} \nabla f_1 \longrightarrow (1)$

Here $J_1 = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$

And ∇f_1 (gradient of f_1) $= \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1}$

$\therefore \because f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$

$\frac{\partial f}{\partial x_1} = 1 + 4x_1 + 2x_2$

$\frac{\partial^2 f}{\partial x_1^2} = 4$ And $\frac{\partial^2 f}{\partial x_1 \partial x_2} = 2$

$\frac{\partial f}{\partial x_2} = 0 - 1 + 0 + 2x_1 + 2x_2$

$\frac{\partial^2 f}{\partial x_2^2} = 2$

$J_1 = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$.

Now we find out J_1^{-1}

$$J_1^{-1} = \frac{1}{8-4}\begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$$

$$J_1^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

And $$\nabla f_1 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1}$$

$$= \begin{bmatrix} 1 + 4x_1 + 2x_2 \\ -1 + 2x_1 + 2x_2 \end{bmatrix}_{\begin{pmatrix} 0 \to x_1 \\ 0 \to x_2 \end{pmatrix}}$$

$$\nabla f_1 = \begin{bmatrix} 1 + 0 + 0 \\ -1 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore X_2 = X_1 - [J_1]^{-1} \nabla f_1$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ -\frac{1}{2} - 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix}$$

To see ~~whether~~ whether or not X_2 is the optimum point we evaluate

g_2 i.e ∇f_2

$$\nabla f_2 = \begin{bmatrix} \frac{\partial f_2}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_2}$$

$$= \begin{bmatrix} 1+4x_1+2x_2 \\ -1+2x_1+2x_2 \end{bmatrix} \left(\begin{matrix} -1 \rightarrow x_1 \\ 3/2 \rightarrow x_2 \end{matrix} \right.$$

$$= \begin{bmatrix} 1-4+2\times 3/2 \\ -1+2(-1)+2(3/2) \end{bmatrix}$$

$$\nabla f_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~As ∇f_2 is~~:

As $\nabla f_2 = 0$, X_2 is the optimum point.

Ans

~~$X_3 = X_2 - [J_1]^{-1} \nabla f_2$~~

Now we calculate $X_3 = X_2 - [J_2]^{-1} \nabla f_2 \quad \because \nabla f_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$X_3 = X_2 - 0$

$X_3 = X_2$, we stoped the process.

Steepest Descent Method

In this method we start from an initial trial point x_i and iteratively move along the steepest descent directions until the optimum point is found. The steepest descent method can be summarized by following steps.

Step I. Start with arbitrary Initial Point X_1 (Given)

Step-2. Find the search direction S_i

$$S_i = -\nabla f_i \quad (\text{Gradient})$$

i.e. $S_1 = -\nabla f(x_1)$

Here $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{x_1}$

Step-3 Determine the optimal step ~~leng~~ length λ_i^*

We minimize the $f(x_1 + \lambda_1^* S_1)$

then $\frac{df}{d\lambda_i^*} = 0 \Rightarrow$ then find λ_i^*

Step-4 Find X_2

As $X_2 = X_1 + \lambda_i^* S_1$

find $\nabla f_2 = \left\{ \begin{matrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{matrix} \right\}_{X_2} =$

(If This is equal to initial X_1 then Process stoped)

otherwise Next iteration

Q1 Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the Point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ by using Steepest descent Method.

Sol^n

Iteration first $\qquad X_2 = X_1 + \lambda_i^* S_1$

$$X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Now search direction S_1

$$S_1 = -\nabla f_1$$

Here $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_1}$

$$\nabla f = \begin{bmatrix} 1 + 4x_1 + 2x_2 \\ -1 + 2x_1 + 2x_2 \end{bmatrix}_{\binom{0}{0}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Determine step length λ_1 And we minimize

$f(X_i + \lambda_i^* S_i)$

$$f\left[\binom{0}{0} + \lambda_i^* \binom{-1}{1}\right] = f\begin{bmatrix} -\lambda_i^* \\ \lambda_i^* \end{bmatrix} \begin{matrix} \to x_1 \\ \to x_2 \end{matrix}$$

$$f[-\lambda_i^* - \lambda_i^* + 2(-\lambda_i^*)^2 + 2(-\lambda_i^* \lambda_i^*) + (\lambda_i^*)^2$$

$$f[-2\lambda_i^* + \cancel{2\lambda_i^{*2}} - \cancel{2\lambda_i^{*2}} + \lambda_i^{*2}] = f(-2\lambda_i^* + \lambda_i^{*2})$$

Now $\frac{df}{d\lambda_i^*} = -2 + 2\lambda_i^* = 0$

$$\therefore \boxed{\lambda_i^* = 1}$$

We obtain X_2

$\therefore X_2 = X_1 + \lambda_1^* S_1$

$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\therefore \nabla f_2 = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}_{X_2} = \begin{bmatrix} 1 + 4x_1 + 2x_2 \\ -1 + 2x_1 + 2x_2 \end{bmatrix}_{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}$

$= \begin{pmatrix} 1 - 4 + 2 \\ -1 - 2 + 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

X_2 is not optimum

Iteration 2

$S_2 = -\nabla f_2$

$= -\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$\therefore f(X_2 + \lambda_2 S_2) = f\left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$

$= f \begin{bmatrix} -1 + \lambda_2 \\ 1 + \lambda_2 \end{bmatrix} \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \end{matrix}$

$\therefore$ Given function $f[(-1+\lambda_2) - (1+\lambda_2) + 2(-1+\lambda_2)^2 + 2(-1+\lambda_2)(1+\lambda_2) + (1+\lambda_2)^2]$

$f[-2 + 2(1 + \lambda_2^2 - 2\lambda_2) + 2(-1 - \lambda_2 + \lambda_2 + \lambda_2^2) + (1 + \lambda_2^2 + 2\lambda_2)$

$f[5\lambda_2^2 - 2\lambda_2 - 1]$

So $\frac{df}{d\lambda_2} = 0 \Rightarrow 10\lambda_2 - \lambda_2 = 0 \Rightarrow \lambda_2 = 1/5$

$X_3 = X_2 + \lambda_2 S_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{1}{5}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -.8 \\ 1.2 \end{bmatrix}$

$\therefore \nabla f_3 = \begin{Bmatrix} \partial f / \partial x_1 \\ \frac{\partial f}{\partial x_2} \end{Bmatrix}_{X_3} = \qquad = \begin{bmatrix} .2 \\ -.2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

We proceed to the next iteration

~~tion Step 3~~

Iteration 3

$$S_3 = -\nabla f_3 = -\begin{bmatrix} +.2 \\ -.2 \end{bmatrix} = \begin{bmatrix} -.2 \\ .2 \end{bmatrix}$$

$$F(X_3 + \lambda_3 S_3) = F\left[\begin{pmatrix} -.8 \\ .2 \end{pmatrix} + \lambda_3 \begin{pmatrix} -.2 \\ .2 \end{pmatrix}\right]$$

$$= f\begin{bmatrix} -.8 - .2\lambda_3 \\ .2 + .2\lambda_3 \end{bmatrix} \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \end{matrix}$$

Putting in function

$$f\left[.8 - .2\lambda_3 - (.2 + .2\lambda_3) + 2(-.8 - .2\lambda_3)^2 + 2(-.8 - .2\lambda_3)(.2 + .2\lambda_3) + (.2 + .2\lambda_3)^2\right].$$

After solving

$$f\left[.04\lambda_3^2 - .08\lambda_3 - 1.2\right]$$

$$\therefore \frac{df}{d\lambda_3} = 0 \Rightarrow$$

we get $\lambda_3 = 1$

$$\therefore X_4 = X_3 + \lambda_3 S_3 = \begin{pmatrix} -.8 \\ 1.2 \end{pmatrix} + \begin{pmatrix} -.2 \\ .2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1.4 \end{pmatrix}$$ is not optimum

~~$\nabla f_4 = \begin{pmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \end{pmatrix}_{X_4} =$~~

~~Iteration Step 4~~ ~~$S_4 =$~~ $\because$ $S_4 = -\nabla f_4 = \begin{Bmatrix} 1 \\ -1.4 \end{Bmatrix} \neq \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

X_4 is not optimum and hence we have to proceed to the next iteration $S_4 = -\nabla f_4 = \begin{Bmatrix} 1 \\ -1.4 \end{Bmatrix}$

As $F(X_4 + \lambda_4 S_4) = f\left[\begin{pmatrix} -1 \\ 1.4 \end{pmatrix} + \lambda_4 \begin{pmatrix} 1 \\ -1.4 \end{pmatrix}\right] = f\left[-1 + \lambda_4,\ 1.4 - \lambda_4(1.4)\right]$

Note When ∇f_3 .., ∇f_4 are same Process will stop.

$$X_5 = X_4 + \lambda_4^*(S_4) = \begin{Bmatrix} -1 \\ 1.4 \end{Bmatrix} + \lambda_4^* \begin{pmatrix} 1 \\ -1.4 \end{pmatrix}$$

$$X_5 = \begin{Bmatrix} -1 \\ 1.5 \end{Bmatrix}$$

$\therefore x_5 \approx x_4$

optimal solⁿ is $x_4 = \begin{pmatrix} -1 \\ 1.4 \end{pmatrix}$ and

$\nabla f_5 = \begin{bmatrix} -.12 \\ -.2 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \end{bmatrix}$