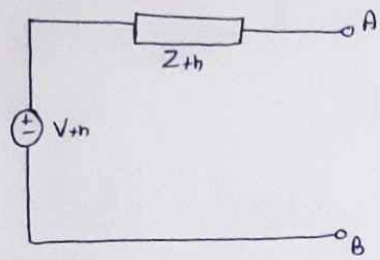
Thevenin's Theorm :-

with respect to terminal pair AB, the network N replaced by the voltage source and internal impedence of the circuit Z_{th}. The voltage source V_{th} is called Thevenin's voltage is the potential difference $(V_A - V_B)$ between the terminal A and B

Z_{th} is the internal impedence of voltage source shorted and the current source open circuited having the resistance impedence.

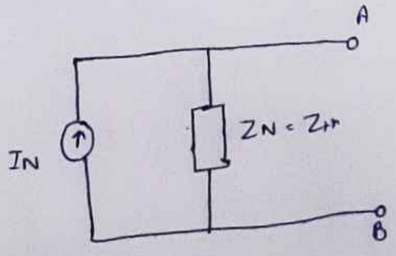
The series combination of V_{th}, Z_{th} is known as Thevenin's Theorm.



Norton's Theorm :-

with respect to the terminal AB, the Network N may be replaced with the current source parallel to the internal impedence of Z_N. The current source, I_N called as Norton's current. The current that would flow from the A to B terminal are shorted together and The current impedence is the N Network

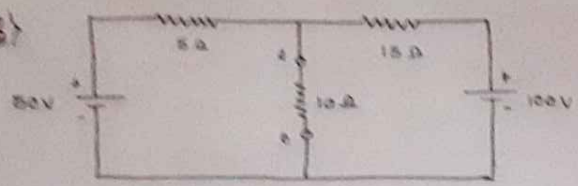
The parallel combination of I_N and Z_N is known as Norton's equivalent.



$$\boxed{I_L = \frac{V_{th}}{R_L + R_{th}}}$$

$$\boxed{I_L = \frac{I_N \cdot V_N}{Z_{th} + Z_L}}$$

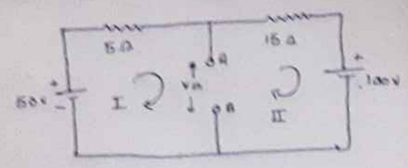
Q)



Find current across the load Resistor $R_L = 10\Omega$

Sol :- Removing the load Resistor and converting into open circuit

By the KVL



$V_{th} = -50 + 5I_0$ { loop I }

Also

$V_{th} = 100V - 15I_0$ [loop II]

$\cancel{-50 + 5I_0 = 100 + 15I_0}$

$\cancel{-50 - 100 = 15I_0 - 5I_0}$

$\cancel{10I_0 = -150}$

$\cancel{I_0 = \frac{-150}{10}}$

$\cancel{\boxed{I_0 = -15A}}$

$-50 + 5I_0 = 100V - 15I_0$

$5I_0 + 15I_0 = 100 + 50$

$20I_0 = 150$

$\boxed{I_0 = -2.5}$

$V_{th} = -50 + 5I_0 \Rightarrow -50 - 5(-2.5)$

$\cancel{V_{th} = -50} +$

$V_{th} = 50 + 12.5$

$\boxed{V_{th} = 62.5V}$

Now, R_{th} / Z_{th} will be;

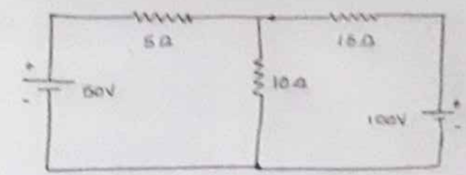
$Z_{th} = 5 \| 15 = \frac{5 \times 15}{5 + 15} \Rightarrow 3.75$

$\boxed{Z_{th} = 3.75\,\Omega}$

$I_L = \frac{V_{th}}{Z_{th} + Z_L} \Rightarrow \frac{62.5}{3.75 + 10} \Rightarrow$

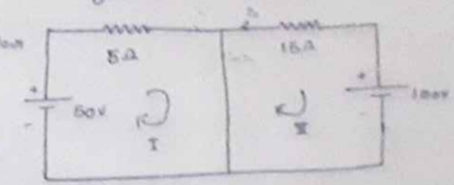
$\boxed{I_L = 4.54A}$

Q)



Here, Terminal A and B, Short circuiting the Load Resistor 10Ω

For the I_N from the Load Resistor



By the KVL

$-50 + 5I_1 = 0$

$-50 + 5I_1 = 0$

$5I_1 = 50$ —(1)

For loop II ;

$100 - 15I_2 = 0$

$15I_2 = 100$ —(2)

from eq (1) and eq(2)

$I_1 = \frac{50}{5} \Rightarrow 10$

$I_2 = \frac{100}{15} \Rightarrow 6.66$

$I_N = I_1 + I_2$

$I_N = 10 + 6.667$

$\boxed{I_N = 16.667}$

Z_{th} ; will be ; $5 \| 15 \Rightarrow \frac{5 \times 15}{5 + 15}$

$\boxed{Z_{th} = 3.75\Omega}$

Thus By the Norton's equivalent :

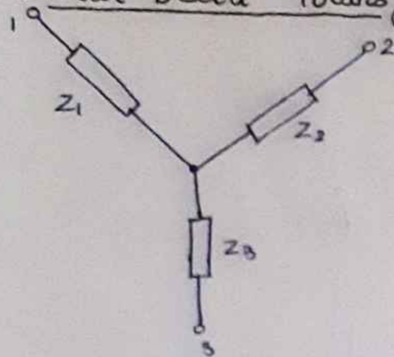
$I_L = I_N \left(\frac{Z_{th}}{Z_{th} + Z_L} \right)$

$I_L = 16.667 \left[\frac{3.75}{3.75 + 10} \right] \Rightarrow 16.667 \left[\frac{3.75}{13.75} \right]$

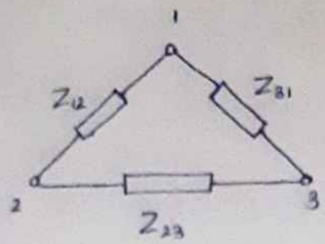
$\boxed{I_L = 4.54A}$

Network Theorems

Star Delta Transformations :-



Star Connection



Delta connection

Delta to star connection :-

$$Z_1 = \frac{Z_{12} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{23} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_3 = \frac{Z_{31} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

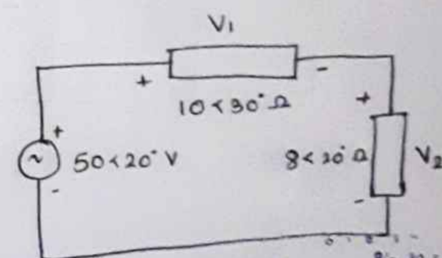
Star to Delta Connection :

$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$Z_{23} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_{31} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

Q) Find the voltage across the impedance then transform voltage source and $10 < 30° \Omega$ to an equivalent current source :



By the KVL ;

$$V_1 = 50 < 20° \times \frac{10 < 30°}{10 < 30° + 8 < 20°} \Rightarrow \frac{500 < 50°}{17.9 < 25.6°}$$

$$V_1 = 27.9 < 24.4° V$$

$$V_2 = 50 < 20° - 27.9 < 24.4 \Rightarrow 22.1 < 14.4°$$

$$I = \frac{V}{10 < 30°} \Rightarrow \frac{50 < 20°}{10 < 30} \Rightarrow 5 < -10° A$$

$$V = R_{th} \times I \Rightarrow 22.3 < 14.4$$

Rough

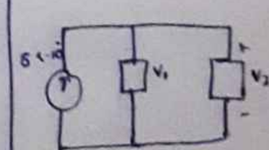
$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$10 < 30°$ $\quad \tan^{-1}(\tan 30)$

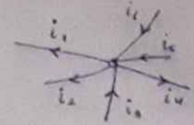
$10 < \tan^{-1} \frac{1}{\sqrt{3}}$

$1 + \sqrt{3} j$



Kirchhof's current law;

The algebraic sum of all branch currents leaving or entering a node is zero at all instants of time



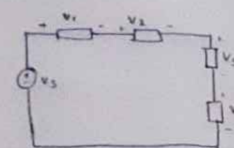
According to the statement

$$-i_1 - i_2 + i_3 - i_4 + i_5 + i_6 = 0$$

$$\boxed{i_3 + i_5 + i_6 = i_1 + i_2 + i_4}$$

Kirchhof's valtage law;

The algebraic sum of drop in all branch valtages in ~~closed~~ circuit, around a loop is zero at all instant of time

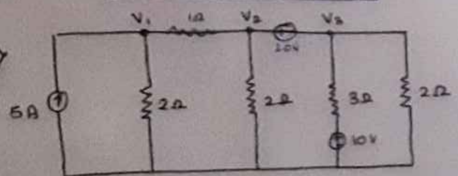


$$\Sigma v_i = 0$$

$$V_1 + V_2 + V_3 + V_4 - V_5 = 0$$

$$\boxed{V_1 + V_2 + V_3 + V_4 = V_5}$$

Q)



Calculate the current in 3Ω resistor.

$V_2 - V_3 = 20 \Rightarrow V_3 = V_2 - 20$

By the Node analysis

at node 1

$$5 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$

$$\Rightarrow 5 = \frac{V_1 + 2V_1 - 2V_2}{2}$$

$$10 = 3V_1 - 2V_2 \quad -(1)$$

~~at node 2~~

~~$\frac{V_2 - V_1}{1} + \frac{V_2}{2} = 0$~~

~~$2V_2 - 2V_1 + V_2 = 0$~~

~~$3V_2 - 2V_1 = 0$~~

~~at node 3~~

~~$\frac{V_3}{2} + \frac{V_3 - 10}{3} = 0$~~

~~$3V_3 + 2V_3 - 20 = 0$~~

~~$5V_3 = 20$~~

~~$\boxed{V_3 = 4V}$~~

at node 2 and 3

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_3 - 10}{3} + \frac{V_3}{2} = 0 \Rightarrow \frac{6V_2 - 6V_1 + 3V_2 + 2V_3 - 30 + 3V_3}{6} = 0$$

$$9V_2 - 6V_1 + 5V_3 = 0$$

$$9V_2 - 6V_1 + 5(V_2 - 20) = 0$$

$$9V_2 - 6V_1 + 5V_2 - 100 = 0$$

$$9V_2 - 6V_1 + 5V_2 = 100$$

$$14V_2 - 6V_1 = 100 \quad -(2)$$

$$3V_1 - 2V_2 = 10$$

$$-6\cancel{V_1} + 14V_2 = 100$$

$$6\cancel{V_1} - 4V_2 = 20$$

(+)

$$10V_2 = 120$$

$$\boxed{V_2 = 12}$$

$$\boxed{V_3 = -8}$$

~~$V_2 - V_3 = 20$~~

~~$\boxed{V_3 = 8}$~~

Given eq: $3V_1 - 2V_2 = 10$

$$-6V_1 + 9V_2 + 5V_3 = 20$$

$$V_2 - V_3 = 20 \Rightarrow V_3 = V_2 - 20$$

$$V_2 = 20 + V_3$$

$$-6V_1 + 9V_2 + 5(V_2 - 20) = 20$$

$$-6V_1 + 9(20 + V_3) + 5V_3 = 20$$

$$-6V_1 + 9V_2 + 5V_2 - 100 = 20$$

$$-6V_1 + 180 + 9V_3 + 5V_3 = 20$$

$$-6\cancel{V_1} + 14V_2 = 120$$

$$6\cancel{V_1} - 4V_2 = 20$$

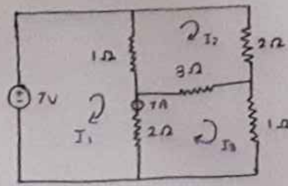
$$10V_2 = 140$$

$$\boxed{V_2 = 14}$$

$$V_3 = V_2 - 20 \Rightarrow -6$$

Super mesh Analysis :-

Q) calculate the current in 3 Ω resistor



Ans) Applying the KVL (mesh 1 and 3)

$7 = 1(I_1 - I_2) + 3(I_3 - I_2) + 1 I_3$

$7 = I_1 - I_2 + 3I_3 - 3I_2 + I_3$

$7 = I_1 - 4I_2 + 4I_3$ —(i)

In mesh 2 ;

$1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$

$I_2 - I_1 + 2I_2 + 3I_2 - 3I_3 = 0$

$-I_1 + 6I_2 - 3I_3 = 0$ —(ii)

Also from the given circuit

$I_1 - I_3 = 7$ —(iii)

$I_1 = 7 + I_3$

$\Rightarrow 7 = 7 + I_3 - 4I_2 + 4I_3$

$\cancel{7} = \cancel{7} + 5I_3 - 4I_2$

$5I_3 - 4I_2 = 0$

$-(7 + I_3) + 6I_2 - 3I_3 = 0$

$-7 - I_3 + 6I_2 - 3I_3 = 0$

$6I_2 - 4I_3 = 7$

$20I_3 - 16I_2 = 0$

$-20I_3 + 30I_2 = 35$

$14I_2 = 35$

$\boxed{I_2 = 2.5A}$

$5I_3 - 4(2.5) = 0$

$5I_3 - 10 = 0$

$\boxed{I_3 = 2A}$

$I_1 = 7 + I_3$

$I_1 = 7 + 2$

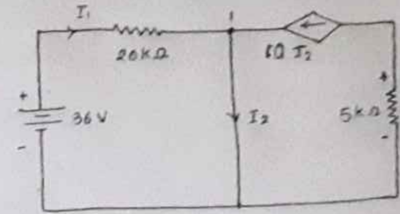
$\boxed{I_1 = 9A}$

Current from 3 Ω resistor

$I = I_2 - I_3 \Rightarrow 2.5 - 2$

$\boxed{I = 0.5A}$

Q) Calculate the V in the circuit ?



By the ohm's law ;

$I_1 = \frac{V}{R} = \frac{36}{20 \times 10^3}$

$I_1 = 1.8 \times 10^{-3}$

$\boxed{I_1 = 1.8mA}$

By the KVL ; at node 1

$I_2 = I_1 + 10 I_2$

$I_2 = 1.8 + 10 I_2$

$-9 I_2 = 1.8$

$\boxed{I_2 = -0.2mA}$

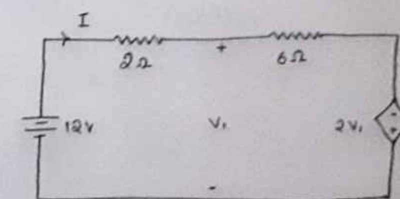
Finally the voltage at 5kΩ will be ;

$V = R I_2$

$V = 5 \times 10^3 [10 \times 0.2 \times 10^{-3}]$

$\boxed{V = 10V}$

Q) Determine the current ?



By the KVL ;

$-12 + 2I + V_1 = 0$

$V_1 = 12 - 2I$ —(i)

and. $-12 + 2I + 6I - 2V_1 = 0$

$2I + 6I - 2V_1 = 12$ —(ii) [from eq (i)]

$2I + 6I - 2(12 - 2I) = 12$

$2I + 6I - 24 + 4I = 12$

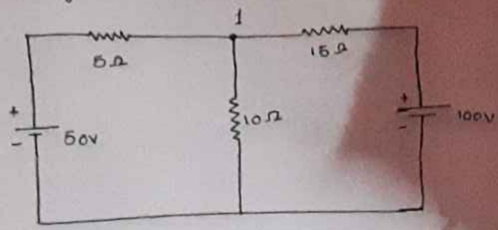
$\Rightarrow 12I = 12 + 24$

$\boxed{I = 3A}$

Q) Determine the current I through 10Ω resistance;

a) Nodal Analysis

b) Mesh Analysis



Ans)

a) Nodal Analysis :-

Apply KCL at node 1:

$$\frac{V-50}{5} + \frac{V}{10} + \frac{V-100}{15} = 0$$

$$\frac{6V - 300 + 3V + 2V - 200}{30} = 0$$

$$11V - 500 = 0$$

$$V = \frac{500}{11}$$

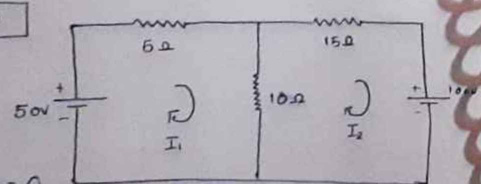
$$\boxed{V = 45.45V}$$

current from 10Ω resistor;

$$I = \frac{V}{R} = \frac{45.45}{10}$$

$$\boxed{I = 4.543A}$$

b) Mesh Analysis:



By the KVL at loop 1:

$$-50 + 5I_1 + 10(I_1 - I_2) = 0$$

$$5I_1 + 10I_1 - 10I_2 = 50$$

$$15I_1 - 10I_2 = 50$$

$$3I_1 - 2I_2 = 10 \quad \text{—(1)}$$

Now, at loop 2;

$$100 - 15I_2 - 10(I_2 - I_1) = 0 \Rightarrow 100 - 15I_2 - 10I_2 + 10I_1 = 0$$

$$10I_1 - 25I_2 = 100 \quad \text{—(2)}$$

$$2I_1 - 5I_2 = 20 \quad \text{—(2)}$$

Now, from the eq (1) and eq (2);

$$3I_1 - 2I_2 = 10$$

$$2I_1 - 5I_2 = 20$$

$$\cancel{6I_1} - 4I_2 = 20$$

$$\cancel{6I_1} - 15I_2 = 60$$

$$(-) \quad (+) \quad (-)$$

$$11I_2 = -40$$

$$\boxed{I_2 = -\frac{40}{11}}$$

$$3I_1 - 2\left(-\frac{40}{11}\right) = 10$$

$$3I_1 + \frac{80}{11} = 10 \Rightarrow 10 - \frac{80}{11}$$

$$3I_1 = \frac{110-80}{11}$$

$$3I_1 = \frac{30}{11}$$

$$\boxed{I_1 = \frac{10}{11}}$$

Current through 10Ω resistor;

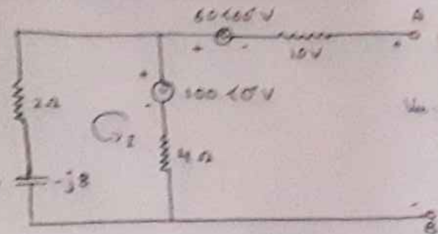
$$I = I_1 - I_2$$

$$I = \frac{10}{11} - \left(-\frac{40}{11}\right)$$

$$\boxed{I = \frac{50}{11}} \Rightarrow I = 4.545$$

$$\boxed{I = 4.545A}$$

Q) Find the thevenin's equivalent circuit at terminal AB;



For finding $V_{th} = V_{oc}$

By the KVL;

$$-100\angle 0° + 2I - 8jI + 4I = 0$$

$$6I - 8jI = 100\angle 0°$$

$$I = \frac{100\angle 0°}{6-8j} \Rightarrow \frac{100\angle 0°}{10\angle -13.8}$$

$$\boxed{I = 10\angle 13.8 A}$$

Voltage drop across $4\Omega \Rightarrow V = IR$

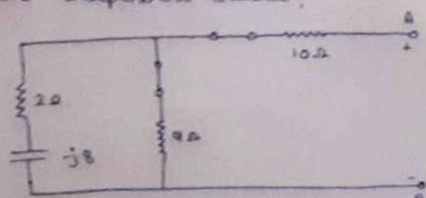
$$V_o = [10\angle 13.8]4$$

$$V_o = 40\angle 13.8 V$$

$$V_{th} = -60\angle 60° + 100\angle 0° - 40\angle 13.8$$

$$V_{th} = 96\angle -61.3° V$$

Now, short circuiting all the Independent source;

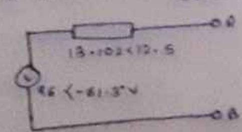


$$Z_{th} = [(2-j8) \,||\, 4] + 10$$

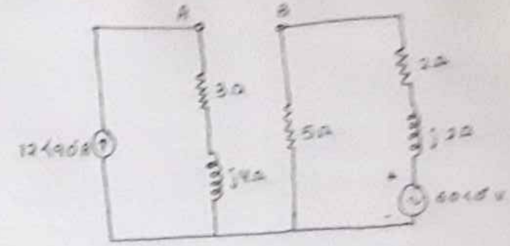
$$= \frac{(2-j8)4}{2-j8+4} + 10$$

$$\Rightarrow \frac{8-32j}{6-j8} + 10 \Rightarrow \frac{8-32j+60-80j}{6-j8}$$

$$\Rightarrow \frac{68-112j}{6-j8} \Rightarrow \frac{131.02\angle -1.31}{10\angle -13.81} \Rightarrow 13.102\angle 12.5\,\Omega$$



Q) Find the thevenin's equivalent circuit at terminals A and B?



For the V_{th};

$$V_{th} = V_A - V_B$$

For V_A;

$$V_A = 12\angle 90°[3+j4]$$

$$V_A = j12[3+j4] = 36j + 48j^2$$

$$V_A = -48 + 36j$$

$12\angle 90° = x + yj$

$x = A\cos\theta$, $y = A\sin\theta$

$12\angle 90° = 12\cos 90 + j12\sin 90$

$= 12(0) + j12$

$= j12$

$$V_B = \frac{5 \times 60\angle 0°}{(2+j2+5)} \Rightarrow \frac{300}{(7+j2)}$$

$$V_B = \frac{300 \times (7-j2)}{53} \Rightarrow 5.66(7-2j)$$

$60\angle 0° = x + yi$

$60\angle 0° = 60\cos 0° + j60\sin 0°$

$= 60 + 0$

$= 60$

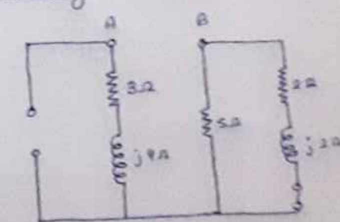
$$V_B = 46.66 - 13.334j$$

$$V_{th} = V_A - V_B \Rightarrow -48 + 36j - 46.66 + 13.33j$$

$$V_{th} = -94.66 + 49.33j$$

$$\boxed{V_{th} = 99.56\angle 151.62}$$

For calculation of Z_{th} by short circuiting the Independent source;



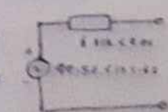
$$Z_{th} = (3+j4) + [5 \,||\, (2+j2)]$$

$$Z_{th} = 3+j4 + \left[\frac{5(2+j2)}{5+2+j2}\right]$$

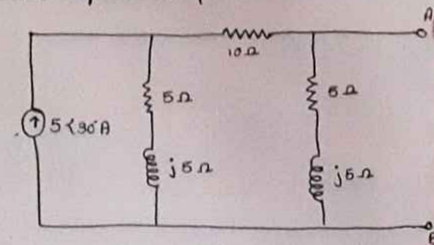
$$\Rightarrow 3+j4 + \frac{10+10j}{7+j2}$$

$$\Rightarrow \frac{3(7+j2) + j4(7+j2) + 10 + 10j}{7+j2} \Rightarrow \frac{21+6j+28j-8+10+10j}{7+2j}$$

$$Z_{th} = \frac{23+44j}{7+2j} \Rightarrow \frac{49.64\angle 3.85}{7.28\angle 9.06} \Rightarrow \boxed{6.818\angle -5.21}$$



Q) obtain the theverins and noorton's equavalent parameter



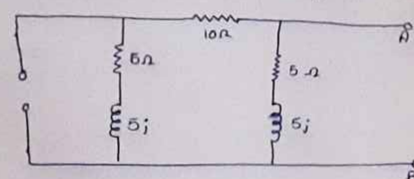
By the Thevenin's ;

$$V_{th} = \frac{(5+j5)\ (5+j5)\ 5\angle 30^\circ}{5+5+10+j5+j5}$$

$$V_{th} = \frac{(5\sqrt{2}\ \angle 45)^2\ 5\angle 30^\circ}{20+10j} \Rightarrow \frac{250\ \angle 120}{22.3\ \angle 26.51}$$

$$\boxed{V_{th} = 11.21\ \angle 97.49\ V}$$

Now, short circuiting the independent source we will get



$$Z_{in} = [(5+5j)+10]\ ||\ (5+5j)$$

$$Z_{in} = [15+5j]\ //\ (5+j5)$$

$$Z_{in} = \frac{(15+5j)(5+j5)}{15+5j+5+5j} \Rightarrow \frac{75+75j+22j-25}{20+10j}$$

$$Z_{th} = \frac{50+97j}{20+10j} \Rightarrow \frac{109.12\ \angle 1.788}{22.3\angle 22.5}$$

$$\boxed{Z_{th} = 4.89\angle -20.712}\ \Omega$$

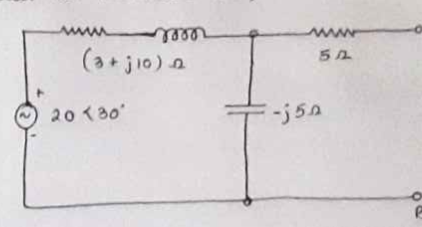
for the Norton's parameter ;

$$I_N = \frac{V_{th}}{Z_{th}} = \frac{11.21\ \angle 97.49}{4.89\ \angle -20.71}$$

$$\boxed{I_N = 2.29\ \angle 76.78\ A}$$

$$\boxed{Z_{th} = Z_n = 4.89\ \angle -20.712}$$

Q) Find the Thevenins equivalent circuit at terminals AB ;



V_{th} = Voltage across $(-j5)\ \Omega$

$$V_{th} = \frac{20\angle 30^\circ}{(3+j10-j5)} \cdot (-j5)$$

$$V_{th} = \frac{(10\sqrt{3}+10j) * (-j5)}{3+5j}$$

$20\angle 30^\circ = 20\cos 30 + 20\sin 30j$

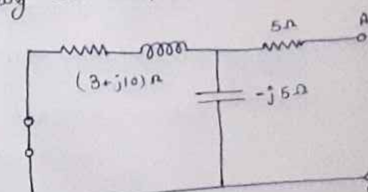
$\Rightarrow 20 \times \frac{\sqrt{3}}{2} + 20 \times \frac{1}{2}j$

$= 10\sqrt{3} + 10j$

$$V_{th} = \frac{-50\sqrt{3}j - (-1)\,50}{3+5j} \Rightarrow \frac{50-50\sqrt{3}j}{3+5j} = \frac{100\angle -60}{5.83\angle 59^\circ}$$

$$\boxed{V_{th} = 17.15\ \angle -119^\circ\ V}$$

for the calculation of Z_{th} short circuiting the Independent source ;



$$Z_{th} = [3+10j // -j5] + (5\Omega)$$

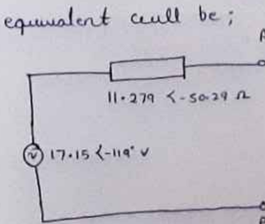
$$Z_{th} = \frac{[3+10j][-j5]}{3+10j-5j} + 5$$

$$Z_{th} = \frac{-15j-(-1)\,50}{3+5j} + 5 \Rightarrow \frac{-15j+50}{3+5j} + 5$$

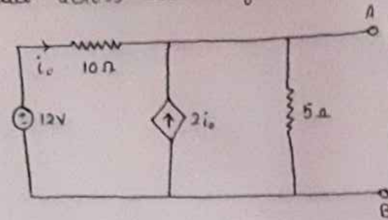
$$Z_{in} = \frac{-15j+50+15+25j}{3+5j} \Rightarrow \frac{65+10j}{3+5j} \Rightarrow \frac{65.76\angle 8.74}{5.83\ \angle 59.03}$$

$$\boxed{Z_{th} = 11.279\ \angle -50.29}$$

Thevenin equivalent will be ;



Q) Find the Norton's Equivalent circuit across terminals from AB



Applying the KVL

$-12V + 10i_o = 0$

$10i_o = 12$

$i_o = 1.2\,A$

At the node A the current will be;

$I_N = i_o + 2i_o$

$I_N = 1.2 + 2(1.2) \Rightarrow 1.2 + 2.4$

$\boxed{I_N = 3.6A}$

Voltage across the 5Ω will be:

$V_{th} = 5 \times$ (Current through 5Ω resistor)

Applying KVL;

$-12 + 10i_o + 5(i_o + 2i_o) = 0$

$25i_o = 12$

$i_o = \frac{12}{25}$

$V_{th} = 15 \times \frac{12}{25}$

$\boxed{V_{th} = 7.2V}$

$R_N = \frac{V_{th}}{I_N} = \frac{7.2}{3.6}$

$\boxed{R_N = 2\Omega}$

Superposition Theorem :-

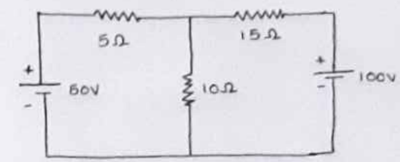
In an active linear network containg several sources:

In any branch in the newtwork equals the algebraic sum of the response of each individual source.

Voltage Source ⟹ short circuited [internal impedence will be 0]

Current source ⟹ open circuited [internal impedence will be infinite]

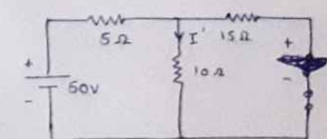
Q)



In the first situation;

$V_1 = 50V$ and $V_2 = 100V$

The V_2 independent source that will replaced by the internal resistance will be short circuited



$R_{eq} = (15 // 10) + 5$

$R_{eq} = \frac{15 \times 10}{15+10} + 5 \Rightarrow \frac{150 + 125}{25} = \frac{275}{25}$

$\boxed{R_{eq} = 11\Omega}$

$I_T = \frac{V_1}{R_{eq}} = \frac{50}{11}$

$\boxed{I_T = \frac{50}{11} A}$

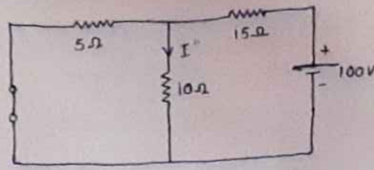
By the current division Rule

$I_x = I_T \frac{R_T}{R_x + R_T}$

$I' = \frac{50}{11} \times \frac{15}{15+10} \Rightarrow \frac{50}{11} \times \frac{15}{25} = \frac{30}{11}$

$\boxed{I' = \frac{30}{11} A}$

Now, V_1 will be short circuited and taking the $V_2 = 100V$



$R_{eq} = (5 // 10) + 15$

$R_{eq} = \frac{5 \times 10}{5 + 10} + 15 \Rightarrow \frac{50}{15} + 15$

$R_{eq} = \frac{50 + 225}{15} \Rightarrow \frac{275}{15}$

$\boxed{R_{eq} = 18.334\,\Omega}$

$I_T' = \frac{V_1}{R_{eq}} = \frac{100}{18.334} \Rightarrow \boxed{I_T' = 5.454\,A}$

Thus, $I'' = I_T' \times \frac{5}{5+10}$

$I'' = \frac{5.454 \times 5}{15}$

$\boxed{I'' = 1.818\,A}$

Hence, By super-position Theorem we will get;

$I = I' + I''$

$I = 2.72 + 1.818$

$\boxed{I = 4.545A}$

Maximum Power Transfer :-

Maximum power Transfer output is obtained from an A.C circuit when the load impedence is equal to the internal impedence of the circuit.

Any network can be converted into single voltage source with series impedence.

1) For DC circuit :-

Load resistance = Internal resistance [Thevenin's resistance]

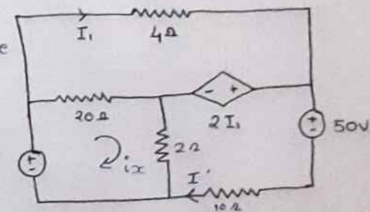
2) For AC circuit :-

=> Load Impedence = complex conjugate of Internal Impedence

=> Load Resistance = Magnitude of internal impedance

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_L}}$$

Q) Calculate current I by super position Theorem;



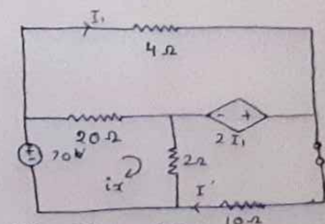
Case 1 :- When 70V source acts alone

By KVL;

$-70 + 20(i_x - I_1) - 2(i_x - I') = 0$

$20 i_x - 20 I_1 - 2 i_x + 2I' = 70$

$18 i_x - 20 I_1 + 2I' = 70$ —(i)



$-2I_1 + 2(I' - i_x) + 10I' = 0$

$12I' - 2I_1 + 2i_x = 0$ —(ii)

$4I_1 + 2I_1 + 20(I_1 - i_x) = 0$

$6I_1 + 20I_1 - 20i_x = 0$

$26I_1 - 20i_x = 0$ —(iii) $\quad I_1 = \frac{20}{26} i_x$

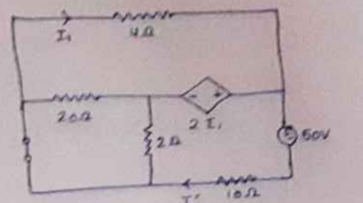
$\boxed{I_1 = \frac{10}{13} i_x}$

By putting the value of I_1 in eq (i) and (ii)

$\boxed{I' = 3.425\,A}$

Case 2: when 50V source acts alone ;

By KVL equation ;

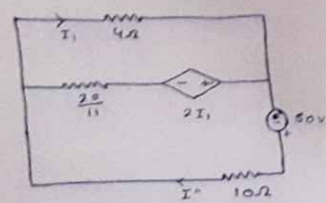


$-50 + 10I'' + \frac{20}{11}(I'' - I_1) - 2I_1 = 0$

$10I'' + \frac{20}{11}I'' - \frac{20}{11}I_1 - 2I_1 = 50$

$\frac{130}{11}I'' - \frac{42}{11}I_1 = 50$

$130I'' - 42I_1 = 550$



$4I_1 + 2I_1 + \frac{20}{11}(I_1 - I'') = 0$

$6I_1 + \frac{20}{11}I_1 - \frac{20}{11}I'' = 0$

$\frac{86}{11}I_1 - \frac{20}{11}I'' = 0$

$I_1 = \frac{10}{43}I''$

Putting I_1 into equation (i) as

$\boxed{I'' = 4.572 A}$

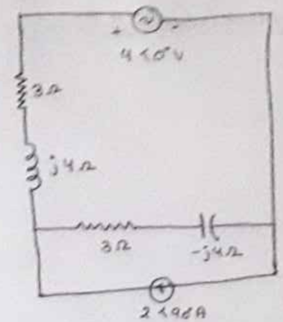
By the super-position Theorum;

$I = I' + I''$

$I = 3.425 + 4.572$

$\boxed{I = 7.997 A}$

Q) Determine the current in capacitor branch by super position theorm

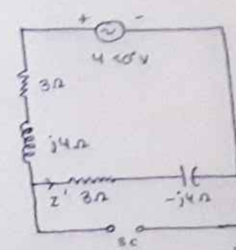


Ans Case 1 :-

when voltage source acts alone, Current source is ~~short~~ open - circuited.

$I' = \frac{4\angle 0^\circ}{(3+4j)+(3-4j)} \Rightarrow \frac{4\angle 0^\circ}{6}$

$\boxed{I' = \frac{2}{3}\angle 0^\circ A}$



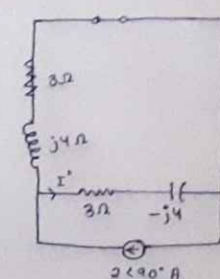
4 cos 0 + j 4 sin 0

4 +

Case 2 :- when current source acts alone Voltage source is short circuited

$I'' = 2\angle 90^\circ \left[\frac{3+4j}{(3+4j)+(3-4j)}\right]$

$I'' = 2\angle 90^\circ \left[\frac{3+4j}{6}\right] = 2j\left[\frac{3+4j}{6}\right]$



$I'' = \frac{6j - 8}{6} \Rightarrow j - \frac{4}{3} A$

$2\angle 90^\circ = 2\cos 90^\circ + j2\sin 90^\circ$

$= j2$

Hence, By the superposition Theorum;

$I = I' + I''$

$I = \frac{2}{3} - \frac{4}{3} + j \Rightarrow -\frac{2}{3} + j$

$\boxed{I = 1.2\angle 123.7^\circ A}$

Synthesis of One Port Network : [By two Elements]

Impedance and Admittance :-

Impedance is the measure of the opposition to electrical flows it defined as the voltage across an element divided by the current

Represented by Z

$$\boxed{Z = R \pm jW}$$

where, R = Resistance

W = Reactance

and + => Lagging

− => Leading

Also $\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$

Impedance Formula:

(Series)

$RL \Rightarrow Z = \sqrt{R^2 + X_L^2}$

$RC \Rightarrow Z = \sqrt{R^2 + X_C^2}$

$\cancel{LC \Rightarrow Z = X_L - X_C}$

$LC \Rightarrow Z = X_L - X_C$

$RLC \Rightarrow \sqrt{R^2 + (X_L - X_C)^2}$

(Parallel)

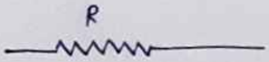
$(RL) :- \ Z = \dfrac{RX_L}{\sqrt{R^2 + X_L^2}}$

$(RC) :- \ Z = \dfrac{RX_C}{\sqrt{R^2 + X_C^2}}$

$(LC) :- \ Z = \dfrac{X_L X_C}{X_L - X_C}$

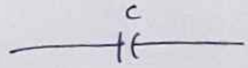
$(RLC) :- \ Z = \dfrac{RX}{\sqrt{R^2 + X^2}} \quad \left[X = \dfrac{X_L X_C}{X_L + X_C}\right]$

R



$G = \dfrac{1}{R}$ = conductance

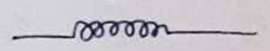
C



$X_C = \dfrac{1}{\omega C}$

$B_C = \dfrac{1}{X_C} = \omega C$

L



$X_L = \omega L$

$B_L = \dfrac{1}{X_L} = \cancel{\omega L} \ \dfrac{1}{\omega L}$

Admittance is expression of the ease which Alternating current (AC) flows through a complex circuit or system.

It is vector quantity by the two independent source

Admittance represented by Y

$$\boxed{\text{Admittance} = \frac{1}{\text{Impedance}}}$$

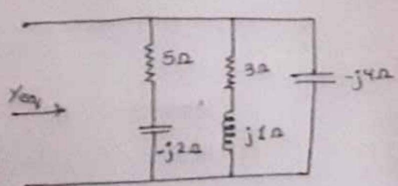
$$Y = \frac{1}{Z}$$

$$\boxed{Y = G \pm jB}$$

where, G is the conduction

B is the Susceptance

Q) Find the Y_{eq}



Ans $Y_{eq} = (5-j2) \parallel (3+j) \parallel -j4$

$$Y_{eq} = \frac{(5-j2)(3+j)(-j4)}{5-j2+3+j-j4} \Rightarrow \frac{-j4\,[15+5j-6j-(-1)2]}{8-5j}$$

~~Y_{eq} = ... 60j ... (20 + 4)~~

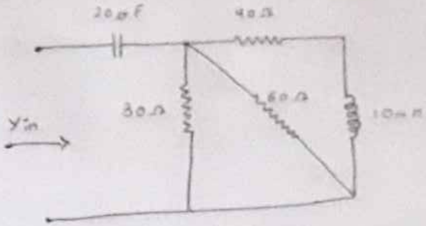
$$Y_{eq} = \frac{-j4\,[18-j]}{8-5j}$$

$$Y_{eq} = \frac{-72j + (-1)4}{8-5j} \Rightarrow \frac{-72j-4}{8-5j} \Rightarrow \frac{-(72j+4)}{64-(-1)25} \times 8+5j$$

$$Y_{eq} = -\frac{(576j + (-1)360 + 32 + 20j)}{64+25} \Rightarrow -\frac{(-328+596j)}{89}$$

$$Y_{eq} = \frac{328-596j}{89} \Rightarrow \boxed{3.685 - 6.69j}$$

Q) At $\omega = 10^3$ rad/sec, find the input admittance :-



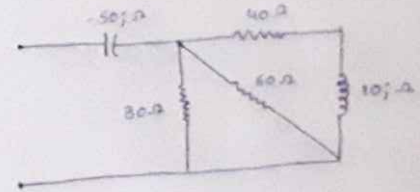
Ans

$$X_c = \frac{1}{j\omega C} \Rightarrow \frac{1}{j \times 10^3 \times 10^{-6} \times 20} \quad \xrightarrow{Y_{in}}$$

$$X_c = \frac{1}{j \times 10^{-3} \times 20}$$

$$X_c = \frac{10^{3\,2}}{2\cancel{0}j} \Rightarrow \frac{100}{2j}$$

$$\boxed{X_c = -50j}$$



$$X_L = j\omega L \Rightarrow j \times 10^3 \times 10 \times 10^{-3}$$

$$\boxed{X_L = 10j}$$

for the input Impedance ;

$$I_{in} = 30 \parallel 60 \parallel (40+10j)$$

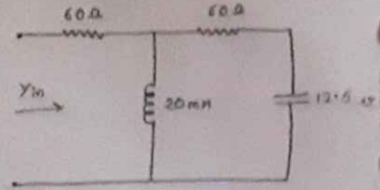
$$I_{in} = \frac{30 \times 60 \times (40+10j)}{30+60+40+10j} \Rightarrow \frac{1800\,(40+10j)}{130+10j} \Rightarrow \frac{180\,(40+10j)}{13+j}$$

$$I_{in} = \frac{7200+1800j}{13+j} \times \frac{13-j}{13-j} \Rightarrow \frac{93600 - 7200j + 23400j - (-1)1800}{169-(-1)}$$

$$I_{in} = \frac{95400+16200j}{170} \Rightarrow 561.17 + 95.26j$$

$$\boxed{I_{in} = 561.17 + 95.26j}$$

Q) At $\omega = 10^3$ rad/sec, Find the input admittance;



Ans) for the susceptance;

$B_c = j\omega C = j \times 10^3 \times 12.5 \times 10^{-6}$

$B_c = j \times 10^{-3} \times 12.5$

$\boxed{B_c = 12.5j \text{ m}\Omega}$

$B_L = \frac{1}{j\omega L} \Rightarrow \frac{1}{j \times 10^3 \times 20 \times 10^{-3}} \Rightarrow \frac{1}{20j}$

$\boxed{B_L = -\frac{1}{20}j}$

$Y_{in} = (60 + 12.5j) \,//\, (-\frac{1}{20}j)$

$Y_{in} = \frac{[60 + 12.5j] \times (-\frac{1}{20}j)}{60 + 12.5j - \frac{1}{20}j} \Rightarrow \frac{-\cancel{60}^{3} \times \frac{1}{\cancel{20}}j + -12.5 \times \frac{1}{20}(-1)}{60 + 12.45j}$

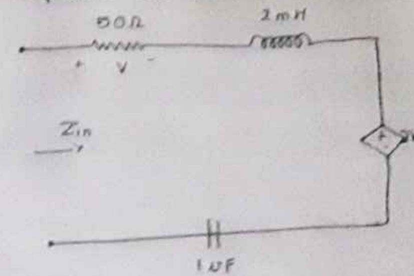
$Y_{in} = \frac{-3j + 0.625}{60 + 12.45j} \times \frac{60 - 12.45j}{60 - 12.45j}$

$y_{in} = \frac{-180j + \cancel{37.5(-1)} + \cancel{37.5} - 7.781j}{3600 - (-1)155}$

$y_{in} = \frac{-187.78j}{3755}$

$\boxed{y_{in} = -0.05j}$

Q) For the circuit; find the input impedance Zin at 10 k rad/sec



Converting into phasor form

$2mH \Rightarrow X_L = j\omega L$

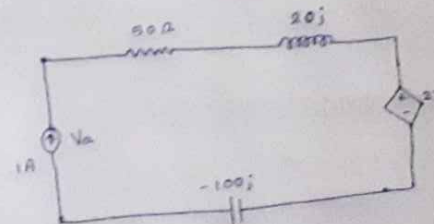
$X_L = j \times 2\cancel{0} \times 10^{-\cancel{3}} \times 10 \times 10^{\cancel{3}}$

$\boxed{X_L = 20j}$

$X_C = \frac{1}{j\omega C} \Rightarrow \frac{1}{j \times 1 \times 10^{-6} \times 10 \times 10^3}$

$X_C = \frac{1}{j \times 10^{-2}} \Rightarrow -100j$

$\boxed{X_C = -100j}$



$Z_{in} = \frac{V}{I}$

By the KVL;

$-Va + 50(1) + 20j(1) + 2 + (-100j)(1) = 0$

$50 + 20j + 2 - 100j = Va$

$Va = 52 - 80j$

Thus, the Input Impedance will be;

$Z_0 = \frac{52 - 80j}{1}$

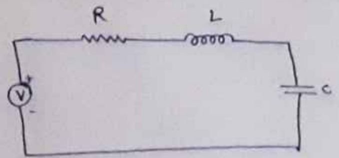
$\boxed{Z_{in} = 52 - 80j}$

Resonant Circuit :-

It describes circuit condition in which

1) Inductive and capacitive effect neutralised to each other
2) Impedence of circuit will be Resistive
3) Power factor will be unit
4) Input Voltage and Input current will be in same phase.

Resonance describes the Energy transformation b/w Inductor and capacitor at a some frequency i.e Resonance frequency



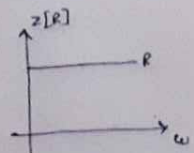
According to the impedance

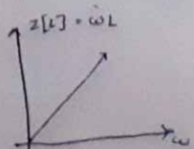
$V = V_R + j(V_L - V_C)$

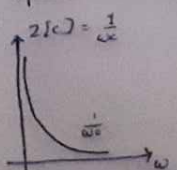
$Z = R + j(X_L - X_C)$

By Resonance condition

$Im[V] = 0$	$Im[z] = 0$
$V_L - V_C = 0$	$X_L - X_C = 0$
$V_L = V_C \quad [V = IR]$	$\boxed{X_L = X_C}$
$IX_L = IX_C$	
$\boxed{X_L = X_C}$	







for the Resonance frequency;

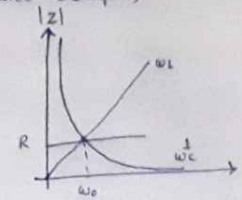
$\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0^2 = \frac{1}{LC} \Rightarrow \frac{1}{\sqrt{LC}}$

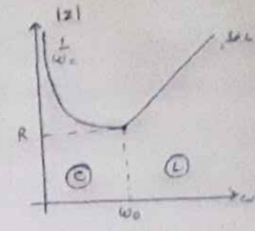
$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ rad/sec

$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$

Resonance Graph;



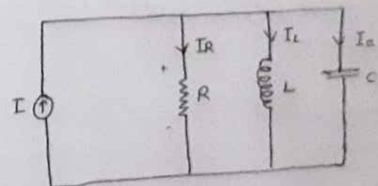
$\Rightarrow$



ω	R/L/C	Power factor
$\omega < \omega_0$	C	Leading
$\omega = \omega_0$	R	1
$\omega > \omega_0$	L	Lagging

Note: Series RLC Resonant Circuit also referred as acceptance circuit because it accepts maximum current in Resonance

Parallel RLC Resonance Circuit :-



$I = I_R + j(I_C - I_L)$

$Y = \frac{1}{R} + j\omega c + \frac{1}{j\omega L}$

$\boxed{Y = \frac{1}{R} + j\left[\omega c - \frac{1}{\omega L}\right]}$

Condition of Resonance;

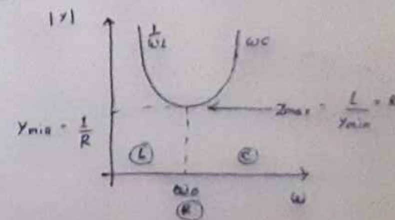
$Im[I] = 0$ or $Im[Y] = 0$

$\omega c - \frac{1}{\omega L} = 0$

$\omega c = \frac{1}{\omega L}$

$\omega^2 = \frac{1}{Lc}$

$\boxed{\omega = \frac{1}{\sqrt{Lc}}}$ rad/sec



ω	R/L/C	P/F
$\omega < \omega_0$	L	Lagging
$\omega = \omega_0$	R	1
$\omega > \omega_0$	C	Leading

$|Y| = \sqrt{\frac{1}{R^2} + \left(\omega c - \frac{1}{\omega L}\right)^2}$

Filter Synthesis :-

Filter network are widely used in communication systems to seperate various voice channels in carrier frequency telephone circuits.

It is necessary to transmit a limited range of frequency.

Active filters are widely used in place of passive filters.

Inductors cannot be fabricated with high Quality IC.

Resistance capacitance circuit with device ~~active~~ active replace the conventional LC filter.

Parameter of a Filter :-

Parameter charaterize a typical filter.

1) Characteristic Impedance Z_c or Z_0 :-

characteristic Impedance of a filter must be choosen

∴ the filter fit ⟶ given line and b/w two ~~eqe~~ Types

$\downarrow$

$\boxed{\text{equipment}}$

2) Pass Band :-

Band in which → ideal filters have to pass all frequencies without any reduction in magnitude.

3) Stop Band :-

Band in which ⇒ ideal filters have to stop the frequencies

4) Cut-off frequency :-

The frequency which seperates the Pass Band and Stop Band.

5) Unit of Attenaution :-

The attenaution of wave filters can be expressed as decibel or Nerps or Bels.

let V_i, I_i and P_i are reffered as input voltage, input current and input power respectively.

Similarly, V_o, I_o, P_o referred as output Voltage, output current and output power respectively.

Then $\frac{V_o}{V_i}$ will represent the voltage gain and $\frac{P_o}{P_i}$ represents as the Power gain. If $P_i > P_o$ so that $\frac{P_i}{P_o}$ is given by attenuation.

Classification of filters :-

The filter have any number of pass bands that is seperated by the attenuation bands.

1) Low pass filters :-

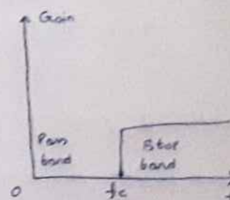
Filters reject the all frequencies which is above the cut-off frequency.

The pass band for low pass filter range from 0 to f_c and

The stop band for low pass filter range from above f_c

pass band ⟹ transmission band

stop band ⟹ attenuation band

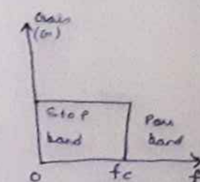


2) High Pass Filter :-

Filter reject the all frequencies which is below the cut-off frequency.

The pass band of the High pass filter is above the f_c

The stop band of the High pass filter is below the f_c



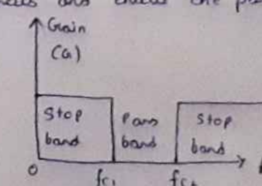
3) Band Pass Filter :-

This filter allows the transmission of all frequencies b/w two designated cut-off frequencies but rejects all the other frequency.

A band pass filter has two cut-off frequencies and show the pass band $f_{c_1} - f_{c_2}$

f_{c_1} = lower cut off frequency

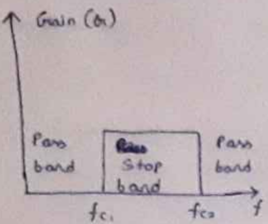
f_{c_2} = upper cut off frequency



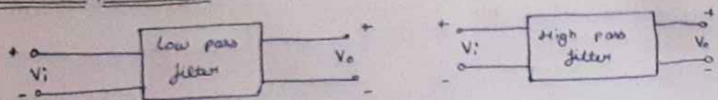
4) **Band Stop or Band Elimination :-**

This filters pass all frequencies lying outside the range.

It attenuate all frequencies between the two frequencies f_{c_1} and f_{c_2}



Block Representation :-



Band Pass filter ⟶ series connection of Low pass filter and High pass filter

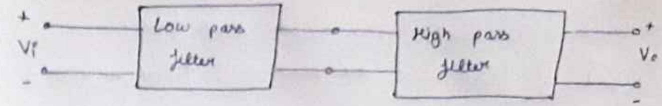
cut-off frequency of high pass filter f_{c_1} is less than the cut-off frequency of low pass filter f_{c_2}

$$\boxed{f_{c_1} < f_{c_2}}$$

Band stop filter ⟶ parallel connection of Low pass filter and High pass filter

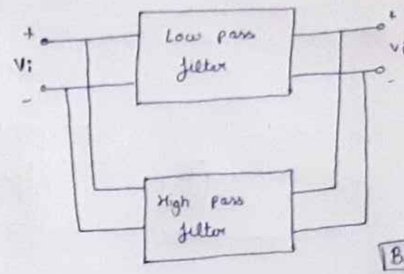
cut-off frequency of high pass filter f_{c_1} is greater than the cut-off frequency of low pass filter

$$\boxed{f_{c_1} > f_{c_2}}$$



Band pass filter

$$\boxed{BP = L.P \cap H.P}$$



$$\boxed{BS = L.P \cup H.P}$$

Band stop filter

Passive Filters

It is classified as :

1) Constant-K or Prototype Filters :-

A network is said to be constant-k type If the series impedance Z_1 and the shunt impedance Z_2 satisfy the relation

$$Z_1 \cdot Z_2 = k^2$$

$k \Rightarrow$ real constant {independent of frequency}
i.e Resistance

$$\Rightarrow Z_1 \cdot Z_2 = k^2 = R_0^2$$

The constant-k type is known as the prototype because other more complex network can be derive from it.

2) m-derived Filters :- In this filter $Z_1 \cdot Z_2 \neq k^2$ as the corresponding constant k and have same characteristic impedance much sharper attenuation characteristic.

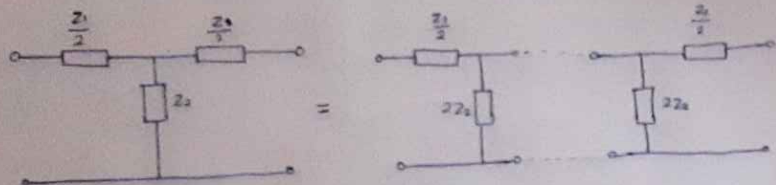
Filter Networks :-

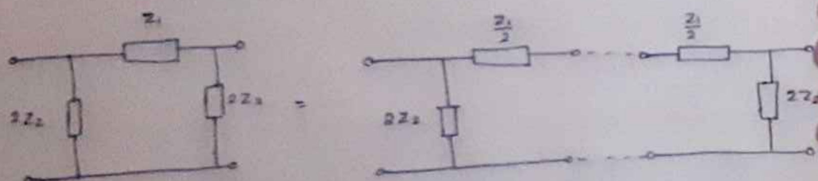
Ideal Filter have zero Attenuation in the pass band.

when the element of filter are dissipation less [Pure reactive]

Filter made symmetrical ⟶ T and π networks

Both T and π network can be considered as combinations of unsymmetrical T or π networks.

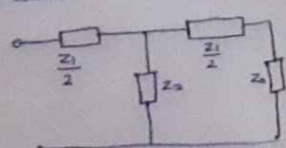




Characteristics of filter Network :-

1) $\boxed{\text{Characteristic Impedance } (Z_0)}$ -

when image impedance at input port and output port are equal to each other. The image ~~imp~~ impedance is called the characteristic Impedance Z_0.



a) $Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$ —— (T network)

b) $Z_{0\pi} = \sqrt{\dfrac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}}$ —— (π network)

$Z_{0T}\, Z_{0\pi} = Z_1 Z_2 = k^2 = R_0^2$

2) Propogation Constant :-

Propogation constant γ is the same for both T and π - networks given by;

$$\boxed{\cosh \gamma = 1 + \frac{Z_1}{2Z_2}}$$

3) Attenuation Constant :-

$\alpha = 0$ —— {pass band}

$\alpha = 2\cosh^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}$ [Stop band]

4) Phase constant :-

$\beta = 2\sin^{-1} \sqrt{\left|\frac{Z_1}{4Z_2}\right|}$ [pass band]

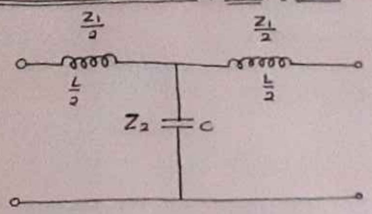
$\beta = n\pi \;;\; n = 0, 1, 2 \ldots$ [stop band]

5) Cutt-off frequency :-

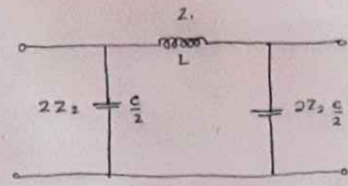
$Z_1 = 0$

and $Z_1 + 4Z_2 = 0$

Constant k low pass filter :-



T-network



Π- network

For T and π network ;

series Impedance $Z_1 = j\omega L$ and shunt impedance $Z_2 = \frac{1}{j\omega C}$, then

$$Z_1 Z_2 = k^2 = R_0^2$$

$$j\omega L \times \frac{1}{j\omega C} = R_0^2$$

$$R_0^2 = \frac{L}{C}$$

$$\boxed{R_0 = \sqrt{\frac{L}{C}}}$$

i) Cutt - off frequency :-

The cutt-off frequency are given by ;

$Z_1 = 0$ and $j\omega_c L = 0$, $\omega_c = 0, f_c = 0$

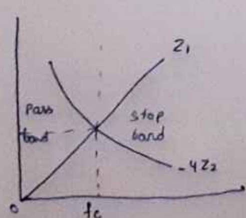
By the cutt - off frequency $Z_1 + 4Z_2 = 0$

$$j\omega_c L + 4 \times \frac{1}{j\omega_c C} = 0 \Rightarrow j\omega_c L = -\frac{4}{j\omega_c C}$$

$$\omega_c^2 = -\frac{4}{LC}$$

$$\boxed{\omega_c = \frac{-2}{\sqrt{LC}}}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$



~~The constant k pass band~~ The passband of constant k low pass filter extends from 0 to $\frac{1}{\pi\sqrt{LC}}$

ii) Attenuation Constant :- [Pass Band]

If the pass Band $\alpha = 0$

$Z_1 = j\omega_c L$

Stop Band ; $\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

$Z_2 = \frac{1}{j\omega_c C}$

$$\alpha \quad 0 = 2\cosh^{-1}\sqrt{\frac{j\omega_c L}{4 \times \frac{1}{j\omega_c C}}} \Rightarrow 2\cosh^{-1}\sqrt{\frac{\omega_c^2 LC}{4}}$$

$$0 = 2\cosh^{-1}\frac{\omega_c\sqrt{LC}}{2} \Rightarrow \boxed{2\cosh^{-1}\left(\frac{f}{f_c}\right)}$$

$\omega = 2\pi f$

iii) Phase constantant :- [Stop Band]

In stop band $\beta = \pi$

$Z_1 = j\omega_c L$

and pass band : $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$

$Z_2 = \frac{1}{j\omega_c C}$

$$\boxed{\beta = 2\sin^{-1}\left[\frac{f}{f_c}\right]}$$

$$\left[\begin{array}{l} Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} \\ Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{\left(1 + \frac{Z_1}{4Z_2}\right)}} \end{array}\right]$$

iv) Characteristic Impedance :-

$Z_1 = j\omega_c L$ and $Z_2 = \frac{1}{j\omega_c C}$

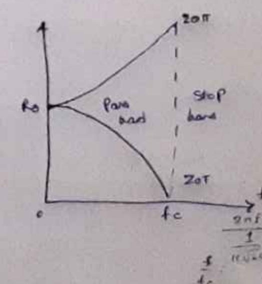
$$Z_{0T} = \sqrt{j\omega_c L \times \frac{1}{j\omega_c C}\left[1 + \frac{j\omega_c L}{4/j\omega_c C}\right]}$$

$$Z_{0T} = \sqrt{\frac{L}{C}\left[1 + \frac{(-1)\omega_c^2 LC}{4}\right]} \Rightarrow \sqrt{\frac{L}{C}\left[1 - \frac{\omega_c^2 LC}{4}\right]} = \sqrt{\frac{L}{C}}\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$\boxed{Z_{0T} = R_0\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{\left(1 + \frac{Z_1}{4Z_2}\right)}} \Rightarrow \sqrt{\frac{j\omega_c L \times \frac{1}{j\omega_c C}}{\left(1 + \frac{j\omega_c L}{4/j\omega_c C}\right)}} \Rightarrow \frac{\sqrt{\frac{L}{C}}}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}}$$



5) Filter component Value :-

We know, $Z_1 Z_2 = k^2$

$\Rightarrow R_o^2 = \frac{L}{C}$ or $k^2 = \frac{L}{C}$

also. $f_c = \frac{1}{\pi\sqrt{LC}}$

$\pi^2 f_c^2 LC = 1$

Therefore,

Series Inductance $\Rightarrow \boxed{L = \frac{k}{\pi f_c}}$ $\boxed{\text{Shunt capcitance} \Rightarrow \frac{1}{\pi f_c k}}$

Q) Design a low pass filter (both π and T networks) having a cut-off frequency = 1kHz
load resistance = 200 Ω
b) Find the frequency at which this filter offers attenuation of 19.1 dB.

a) Given : $f_c = 1000\,Hz$ and $R_o = 200\Omega$

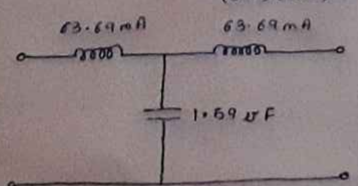
Thus ;

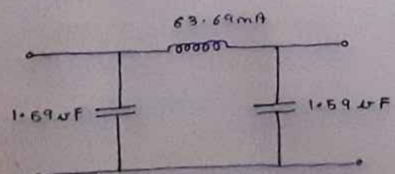
Series Inductance ; $L = \frac{k}{\pi f_c}$

$L = \frac{200}{(3.14)(1000)} \Rightarrow 0.0636 \Rightarrow 63.69\ mA$

Shunt capacitance ; $C = \frac{1}{\pi f_c k}$

$C = \frac{1}{(3.14)(1000)(200} \Rightarrow 1.59\ \mu F$





b) Attenuation is dB = 8.686 × attenuation in nepres

19.1 = 8.686 × attenuation in nepres

attenuation in nepores = $\frac{19.1}{8.686} \Rightarrow 2.192$

attenuation in nepres = 2.2

for low pass filters ;

$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$

$\frac{2.2}{2} = \cosh^{-1}\left(\frac{f}{1}\right)$

$\cosh(1.1) = f$

$\boxed{f = 1.67\,KHz}$

Constant K- High pass filter :- $K = R_o = \sqrt{\frac{L}{C}}$

1) Cutt-off frequency :-

$\boxed{\omega_c = \frac{1}{2\sqrt{LC}}}$ and $\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$

2) Attenuation Constant :-

In pass Band $\alpha = 0$ and stop band $\boxed{\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)}$

3) phase constant :-

In stop - Band , $\beta = \pi$ and pass band $\boxed{\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)}$

4) Characteristic Impedance :-

$\boxed{Z_{oT} = R_o\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ and $\boxed{Z_{o\pi} = \frac{R_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$

5) Filter component value :

Shunt Inductance $\boxed{L = \frac{k}{4\pi f_c}}$ and Series capacitance. $\boxed{C = \frac{1}{4\pi f_c k}}$

Q) Design a high pass filter (both π and T) having a cut-off frequency of 2 kHz with load resistance 300 Ω.

Ans) $f_c = 2000\,Hz$ and $R_o = k = 300\,\Omega$

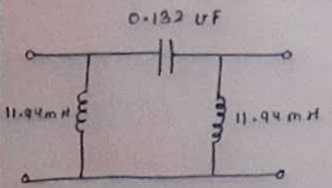
We know that;

$$L = \frac{k}{4\pi f_c} = \frac{300}{4 \times 3.14 \times 2000} = 0.0119$$

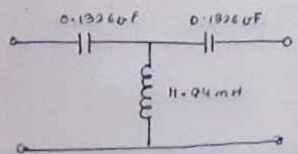
$$\boxed{L = 11.94\ mH}$$

$$C = \frac{1}{4\pi k f_c} \Rightarrow \frac{1}{4 \times 3.14 \times 300 \times 2000} \Rightarrow 0.1326\ \mu F$$

Thus, the T and π network will be;



π Network



T-Network

Q) Cut-off frequency of 8 kHz and characteristic impedance of 600 Ω. Find the component value of π network constant k high pass filter and its characteristic impedance. ~~and~~
phase constant at $f = 12\,kHz$
attenuation at $f = 0.8\,kHz$

Ans) Given $f_c = 8000\,Hz$ and $R_o = k = \sqrt{\frac{L}{C}} = 600\,\Omega$

Now,

Shunt Inductance : $L = \frac{k}{4\pi f_c} \Rightarrow \frac{600}{4 \times 3.14 \times 8000} \Rightarrow 5.971 \times 10^{-3}\,H$

$$\boxed{L = 5.971\ mH}$$

Series capacitance : $C = \frac{1}{4\pi f_c k}$

$$C = \frac{1}{4 \times 3.14 \times 8000 \times 600} \Rightarrow 1.658 \times 10^{-8}$$

$$\boxed{C = 0.01658\ \mu F}$$

The value of components will be :

$2L = 11.94\ mH$

$C = 0.0166\ \mu F$

Characteristic Impedance ; at $f = 12\,kHz$

$$Z_{o\pi} = \frac{R_o}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \Rightarrow \frac{600}{\sqrt{1 - \left(\frac{8}{12}\right)^2}}$$

$\boxed{Z_{o\pi} = 805\ \Omega}$ at 12 kHz

Characteristic Impedance ; at $f = 0.8\,kHz$

$$Z_{o\pi} = \frac{600}{\sqrt{1 - \left(\frac{8}{0.8}\right)^2}} \Rightarrow \boxed{Z_{o\pi} = 603\ \Omega} \text{ at } 0.8\,kHz$$

Phase constant at $f = 12000\,Hz$

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)$$

$$\beta = 2\sin^{-1}\left(\frac{8000}{12000}\right) \Rightarrow 2\sin^{-1}\left(\frac{2}{3}\right)$$

$$\boxed{\beta = 83.6^\circ}$$

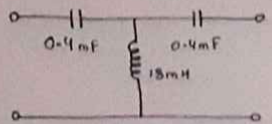
Attenuation constant at $f = 800\,Hz$ (0.8 kHz)

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)$$

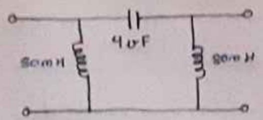
$$\alpha = 2\cosh^{-1}\left(\frac{8000}{800}\right) \Rightarrow 2\cosh^{-1}(10)$$

$$\boxed{\alpha = 5.99\ nepers}$$

Q> Determine the cut-off frequency for high pass filter :



T network



π network

Ans> For the T Network;

$2C = 0.4 \mu F \Rightarrow C = 0.2 \mu F$

$L = 18 mH$

$k = \sqrt{\frac{L}{C}} = \sqrt{\frac{18 \times 10^{-3}}{0.2 \times 10^{-6}}} = 300 \Omega$

$\boxed{k = 300 \Omega}$

Cut-off frequency;

$f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4 \times 3.14} \times \frac{1}{\sqrt{18 \times 0.2 \times 10^{-9}}}$

$f_c = 1326.96 \text{ Hz}$

$\boxed{f_c = 1326 \text{ Hz}}$

For the π network;

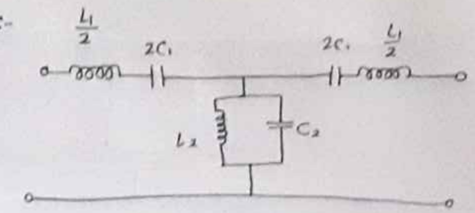
$2L = 80 \times 10^{-3}$ $\quad C = 4 \mu F$

$L = 40 \times 10^{-3} H$ $\quad C = 4 \times 10^{-6} F$

Cut-off frequency $\quad f_c = \frac{1}{4\pi\sqrt{LC}} \Rightarrow \frac{1}{4 \times 3.14 \times \sqrt{40 \times 10^{-3} \times 4 \times 10^{-6}}}$

$\boxed{f_c = 199.04 \text{ Hz}}$

Constant k Band Pass Filter :-



$R_o = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$

i> Cut-off Frequencies :-

$Z_1 + 4Z_2 = 0$

$Z_1 = -4Z_2$

The cut-off frequency will be :

$\boxed{f_o = \sqrt{f_{c1} f_{c2}}}$

ii> Filter component Values :-

$\boxed{C_1 = \frac{f_{c2} - f_{c1}}{4\pi R_o f_{c1} f_{c2}}}$ $\quad \boxed{L_1 = \frac{R_o}{\pi(f_{c2} - f_{c1})}}$

$\boxed{L_2 = \frac{(f_{c2} - f_{c1}) R_o}{4\pi f_{c1} f_{c2}}}$ $\quad \boxed{C_2 = \frac{1}{\pi R_o (f_{c2} - f_{c1})}}$

Q> Design a constant k band pass filter with cut-off frequencies of 3kHz and 7.5 kHz and characteristic impedance $R_o = 900 \Omega$

Ans Given : $R_o = 900 \Omega$, $f_{c1} = 3 \text{ kHz}$ and $f_{c2} = 7.5 \text{ kHz}$

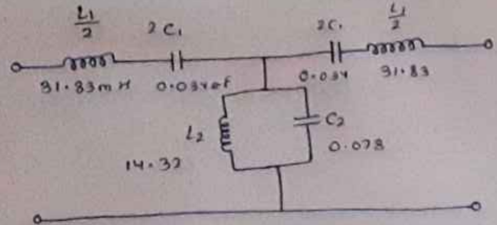
$L_1 = \frac{R_o}{\pi(f_{c2} - f_{c1})} \Rightarrow \frac{900}{3.14(7.5 - 3)} \Rightarrow 63.66 mH$

$C_1 = \frac{f_{c2} - f_{c1}}{4\pi R_o f_{c1} f_{c2}} \Rightarrow \frac{4.5}{3.14 \times 4 \times 900 \times 7.5 \times 3} \Rightarrow 0.017 \mu F$

$L_2 = \frac{(f_{c2} - f_{c1}) R_o}{4\pi f_{c1} \cdot f_{c2}} \Rightarrow \frac{4.5 \times 900}{4 \times 3.14 \times 7.5 \times 3} \Rightarrow 14.33 mH$

$C_2 = \frac{1}{\pi R_o (f_{c2} - f_{c1})} \Rightarrow \frac{1}{3.14 \times 900 \times 84.5} \Rightarrow 0.078 \mu F$

Hence, $\frac{L_1}{2} = 31.83 mH$ and $2C_1 \Rightarrow 0.034 \mu F$



Limitations of Passive Filters :-

Passive Filter have may limitations are as follows:-

1) The use of inductor in filter element is not desirable at low frequencies (< 1 KHz)

In these frequencies pure practical inductors of reasonable & tends to $\boxed{\text{costly}}$ $\boxed{\text{Bulky}}$ $\boxed{\text{large}}$ $\boxed{\text{heavy}}$.

2) High range of Q factor is not possible,

3) need of an external amplifier to provide suitable gain.

4) Cascade many sections to give a composite filter

Cascading different sections of filter → $\boxed{\text{buffer}}$ and $\boxed{\text{isolation amplifier}}$ is required to prevent the load of circuit.

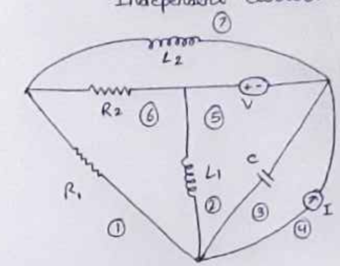
due to above limitation the passive filter became obsolete.

Graph Theory :-

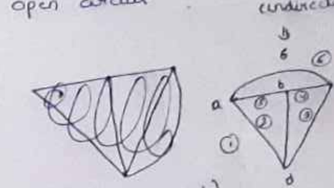
Graph is obtain from arbitrary circuit replacing with open circuit and short circuit

Passive Elements and Independent Voltage source → short circuit

Independent current source → open circuit .



→

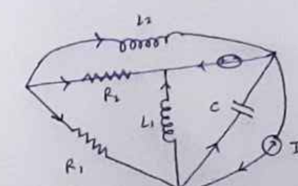


Nodes → 4 (a, b, c, d)

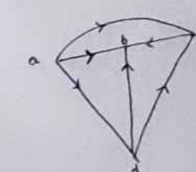
Branches / edges = 6 (1, 2, 3, 4, 5, 6)

Graph is defined as collection nodes and Branches or Vertexes and edges.

Directed Graph :-



⟹



a) Rank of Graph = n - 1 [n = no of nodes]

b) Degree of node :-
[no of incoming branches across any nodes]

$$\left[\begin{array}{l} D[a] = 0 \\ D[b] = 2 \\ D[c] = 2 \\ D[d] = 1 \end{array} \Rightarrow 2+2+1 \Rightarrow 5\right]$$

c) Complete Graph :-
only one line segment for every one node pair

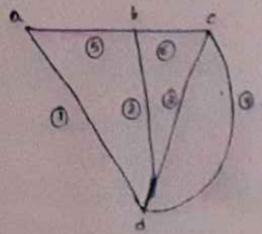
Node pair = ab, bc, cd, bd, ad, ac

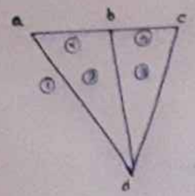
No of Branches [complete Graph] $\Rightarrow {}^{n}C_2 \Rightarrow \frac{n!}{2!\,(n-2)!}$

$$\boxed{\text{No of Branches} = \frac{n(n-1)}{2}}$$

Connected Graph

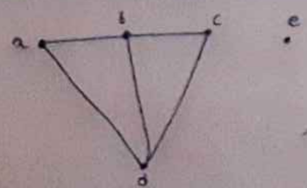
In connected all node pair contains atleast one branch Should be connected





Not Connected Graph

In unconnected Graph all node pair contains atleast one branch Should be connected but there will be one isolated node



Note :-

All complete graph are connected graph but vice versa are not True

$\boxed{\text{Number of node pair voltage} = \frac{n(n-1)}{2}}$

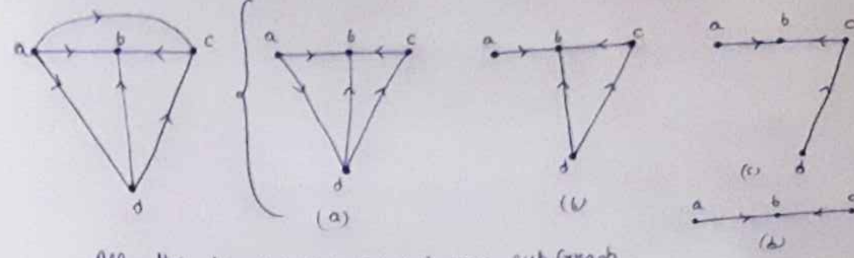
for both complete and incomplete Graph

Sub Graph

It contains less number of branch from other branch.

Spanning Sub Graph :-

It contains all the ~~real~~ ~~loran~~ nodes of original Graph



All the fig (a) (b) (c) (d) are sub Graph but (d) is ~~spanning~~ not spanning sub Graph.

Trees & Co-Tree

Tree is connected sub Graph which contains all nodes without forming any closed loop

Tree is also a spanning sub Graph. without forming closed loop.

Tree Branch $\Longrightarrow$ Twigs $(n-1)$ [Present Tree Branches]

Absent Tree Branch $\Longrightarrow$ Links / chords $= l$

$\boxed{\text{Total no of Branches in Graph} = \text{Twigs} + \text{Link}}$

$$B_T = n-1+l$$

$$\boxed{l = B_T - n + 1}$$

Rank of Tree $= n-1$

For complete Graph :-

$\boxed{\text{no of Tree} = n^{(n-2)}}$

Spanning Tree :-

$\boxed{\text{no of spanning Tree} = n^{(n-2)}}$

→ there should no closed loop

→ complete Graph

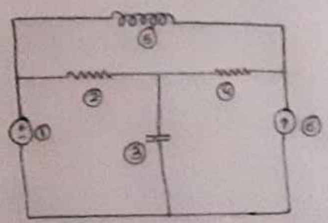
$$\left.\begin{array}{l}\text{Tree Branch} \Rightarrow \text{Twigs} \\ \text{Co Tree Branch} \Rightarrow \text{Links}\end{array}\right\} \text{no closed Loop}$$

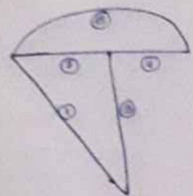
But co Tree ~~Bra~~ consist closed loop.

~~Note~~

no of Tree Branch in complete Tree = no of co Tree Branch in complete Tree = $n^{(n-2)}$

Q) Find the number of chords ;





n of nodes = 4

no of Branches = 5

Twigs = $n-1 = 4-1 = 3$

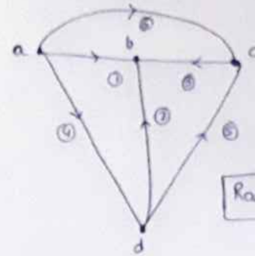
no of chords / link = $B - n + 1$

$= 5 - 4 + 1$

$\Rightarrow \underline{\underline{2}}$

Incidence Matrix :- [complete Incidence Matrix]

Incidence Matrix describe the information regarding incoming and outgoing Branches across any graph.



$\boxed{\text{Rank}\,[IM] = n-1} = 4-1 \Rightarrow 3$

Note:-

$\boxed{\text{Rank}\,[IM] = \text{Rank}\,[\text{Graph}] = \text{Rank}\,[\text{Tree}] = n-1}$

Incidence Matrix = $[A]_{n \times B}$ $\left\{\begin{array}{l}\text{outgoing} = 1 \\ \text{incoming} = -1\end{array}\right\}$

n = row = no of node

B = column = no of Branch

$$A = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & 1 & 0 & 0 & 1 & 0 & 1 \\ b & -1 & -1 & -1 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & -1 & -1 \\ d & 0 & 1 & 0 & -1 & 1 & 0 \\ \hline & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 0 & 0 & 0 & 0 & 0 & 0 \end{array}_{4 \times 6}$$

$\boxed{\text{The Algebraic sum of column} = 0}$ {for all column Matrix}

If $n = B$

$[A]_{n \times B}$ = square Matrix

$\boxed{|A| = 0}$ {Singular Matrix}

Reduced Incidence Matrix :-

$[R]_{(n-1) \times B}$

$$[R]_{3 \times 6} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline a & 1 & 0 & 0 & 1 & 0 & 1 \\ b & -1 & -1 & -1 & 0 & 0 & 0 \\ c & 0 & 0 & 1 & 0 & -1 & -1 \end{array}$$

$\boxed{\begin{array}{l}\text{If The Algebraic Sum of column is not 0} \\ \text{Then Reduced Incidence Matrix}\end{array}}$

$$\boxed{\text{No of trees} = \det[R \cdot R^T]}$$

Q) Find the no of possible tree by Reduce Incidence Matrix :-

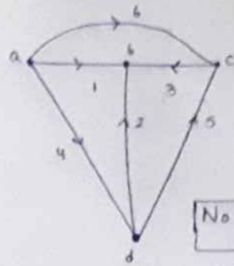
$$[R] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$$[R^T] = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R \cdot R^T = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\text{Det}\,|R \cdot R^T| = 3(6-1) - (-1)[-3-1] + (-1)[1-(-2)]$$
$$= 15 + [-4] - [3]$$
$$= 15 - 4 - 3$$
$$\Rightarrow 15 - 7$$
$$\Rightarrow \underline{\underline{8}}$$

Tie-Set Matrix / Fundamental loop Matrix



Tie-set Matrix is a link dependent Matrix

$$\boxed{\text{Rank}[\text{Tie-Set}] = B - n + 1}$$

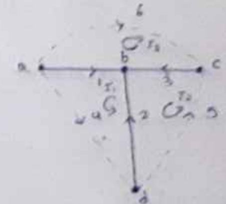
$$[T]_{l \times B}$$

$$\boxed{\text{No of fundamental loop current} = l = B - n + 1}$$

Let link $l = I_1, I_2, I_3$

$B = 6$

$n = 4$ link $= 6 - 4 + 1 \Rightarrow 3$



$$A = \begin{array}{c|c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} & \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}_{3 \times 6} \\ & \begin{array}{cccccc} j_1 & j_2 & j_3 & j_4 & j_5 & j_6 \end{array} \end{array}$$

fundamental loop current depends on direction of link

Branch Current $= j_1, j_2, j_3, j_4, j_5, j_6$

In tie set matrix we determine Branch current in terms of loop current.

$J_1 = -I_1 - I_3$ $\quad J_4 = I_1$

$J_2 = I_1 - I_2$ $\quad J_5 = I_2$

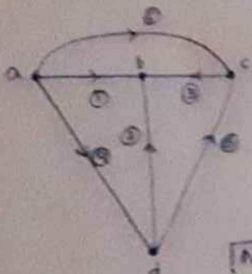
$J_3 = I_2 - I_3$ $\quad J_6 = I_3$

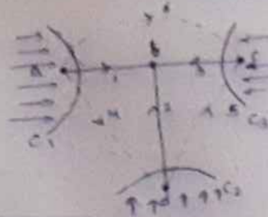
$$\boxed{\text{No of KVL} = \text{No of links} = \text{No of Fundamental loop current} = B - n + 1}$$

$$\left.\begin{aligned} I_1 &= -V_1 + V_2 + V_4 \\ I_2 &= -V_2 + V_3 + V_5 \\ I_3 &= -V_1 - V_3 + V_6 \end{aligned}\right\} \text{KVL equation}$$

Cut - Set Matrix :-

It is twigs dependent Matrix :-





$\boxed{\text{No of cut set} = T = n-1}$

$\boxed{\text{Rank [cut set]} = n-1}$

$\boxed{\text{No of cut-set Matrix} = n^{(n-2)}}$

→ for complete Graph

$T = 4-1 = 3\ [C_1, C_2, C_3]$

one cut set can only contain one Twigs

order of cut set Matrix = $[CM]_{T \times B}$

$$[C]_{3 \times 6} = \begin{array}{c} \\ C_1 \\ C_3 \\ C_3 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 0 & 1 & 0 \end{bmatrix} \\ \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \end{array}$$

In cut set Matrix we determine branch voltage in terms of induced voltage.

$$\begin{cases} E_1 = I_1 + I_2 + I_3 \\ E_2 = I_3 - I_5 - I_6 \\ E_3 = I_2 - I_3 + I_5 \end{cases}$$

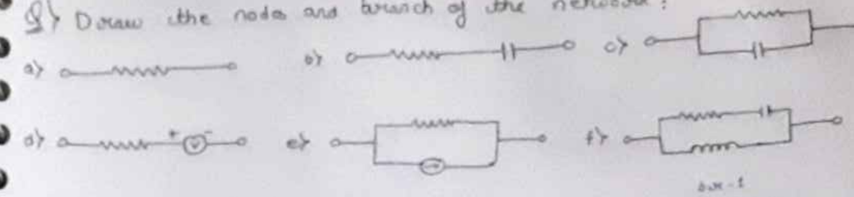
$V_1 = e_1 \qquad V_4 = e_1$

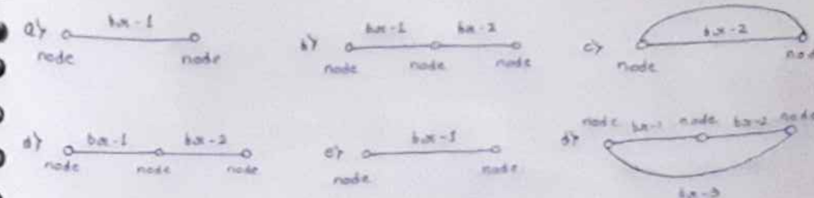
$V_2 = e_4 \qquad V_5 = e_2 - e_3$

$V_3 = -e_2 + e_3 \qquad V_6 = e_1 - e_3$

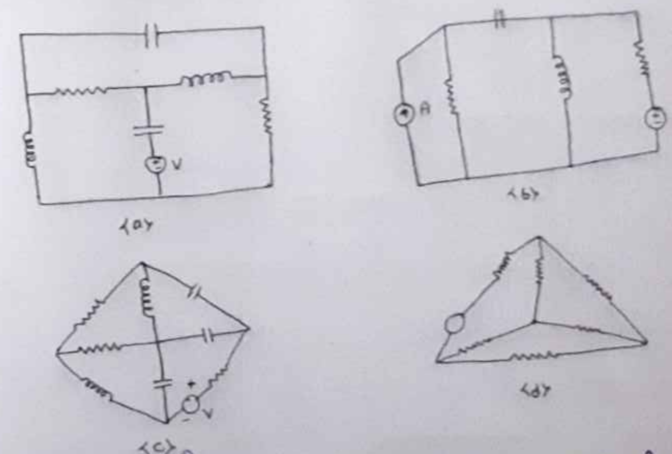
$\boxed{\text{No of KCL equation} = \text{No of Twigs} = n-1}$

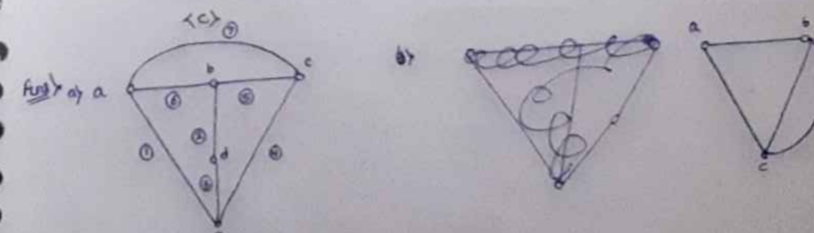
Q) Draw the node and branch of the network :



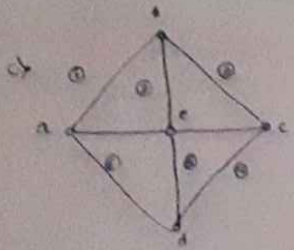


Q Draw the graph of the Network ;

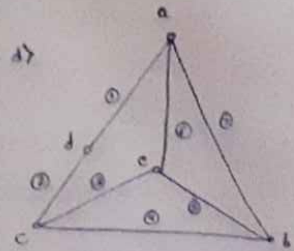




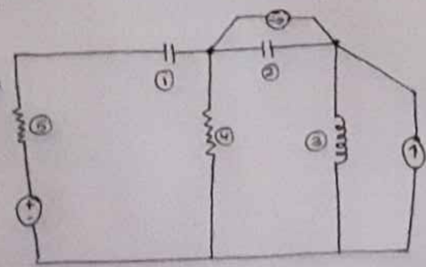
c)



d)



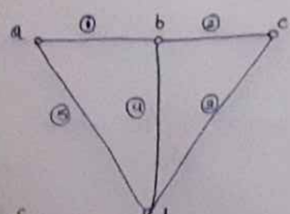
Q) For the Given Network;
Determine all the tree and co-tree for the graph of network.
Consider the tree formed by the branches (1, 2, 5)
write A_c, B_f and Q_f.



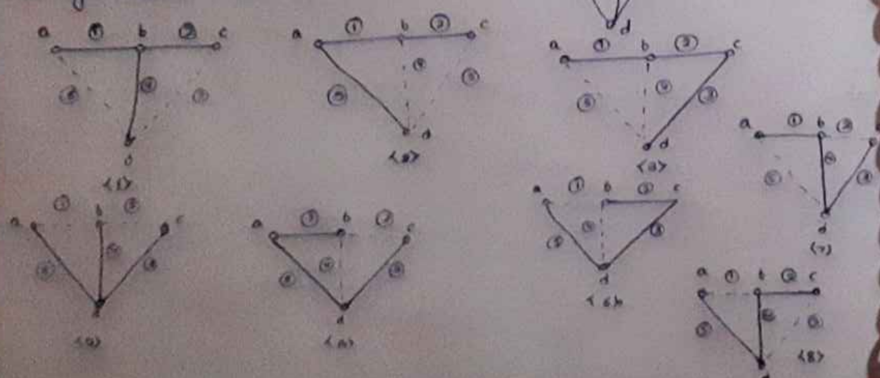
The graph of the given network
Keeping the voltage short-circuited and current sources are open-circuited.

no of twigs = $n-1 = 4-1 = 3$

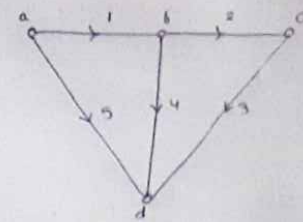
no of links = $b-n+1 = 5-4+1 = 2$



No of Possible Trees will be;



Tree (Twigs)	Links
(1, 2, 4)	(5, 3)
(2, 1, 5)	(4, 3)
(1, 2, 3)	(5, 4)
(4, 5, 3)	(1, 2)
(1, 5, 3)	(4, 2)
(2, 3, 5)	(1, 4)
(2, 4, 5)	(1, 3)
(1, 4, 3)	(5, 2)



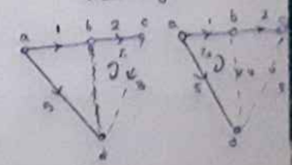
The complete incidence Matrix :-

$$[A] = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

Reduced Incidence Matrix :-

$$[A_R] = A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow \begin{array}{c} \begin{array}{ccc|cc} 1 & 2 & 5 & 3 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \end{array}\right] \end{array}$$

This is for A (1, 2, 5)



Now for Tie - Set Matrix :-

no of Tie - Set = $b-n+1 = 5-4+1$
$= 2$

Tie - set (3, 4), Branch = 1 2 3 4 5

$$[B_f] = \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

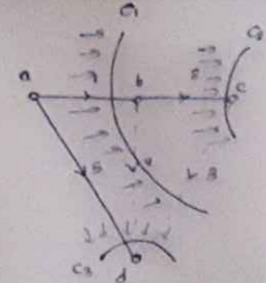
for the Branch (1, 2, 5) :

$$[B_f] = \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc|cc} 1 & 2 & 5 & 3 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 1 \end{array}\right] \end{array}$$

for cut-set Matrix :-

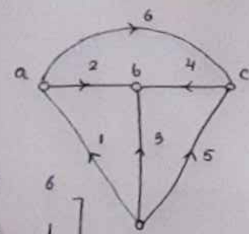
no of cut-set = n-1 = 4-1 = 3

Cut Set = 1 , cut Set = 2 , cut Set = 5



$$[Q] = \begin{array}{c} \\ 1 \\ 2 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc|cc} 1 & 2 & 5 & 3 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \end{array}$$

Q) For the given graph; Consider the tree branch (2, 3, 4) Determine A, B_f and Q_f ?



Ans from the Given Graph ;

Incidence Matrix for complete Graph;

$$A_c = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Reduced Incidence Matrix eliminating any Row;

$$A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right] \end{array}$$

for the branch 2 3 4

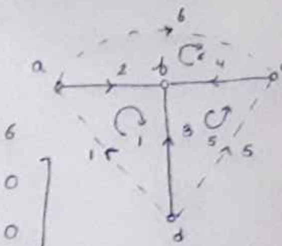
$$A = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right] \end{array}$$

For the Tie-Set Matrix $[B_f]$:-

~~no of Tie set = external~~ no of Tie set = B - n + 1

=> 6 - 4 + 1 => 3

Tie set will be 1, 5, 6



$$B_f = \begin{array}{c} \\ 1 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{array}\right] \end{array}$$

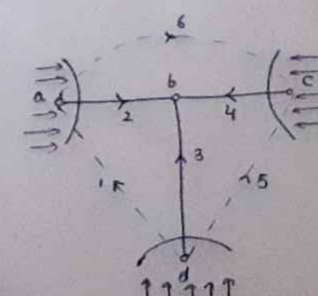
for the branch 2 3 4 will be

$$B_f = \begin{array}{c} \\ 1 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

For the Cut Set Matrix $[Q_f]$:-

no of cut set = n-1 = ~~6~~ 4-1 => 3

cut set ~~and~~ branch or Twigs will be 2 4 3

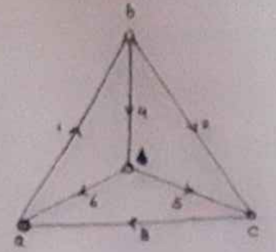


$$Q_f = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} -1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right] \end{array}$$

for the branch (2, 3, 4)

$$Q_f = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right] \end{array}$$

Q) Consider the graph shown;
Select a tree with twigs (4, 5, 6) for the tree write down the tie set matrix.

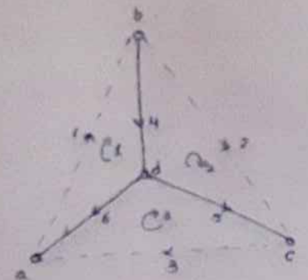


For the tie set Matrix

no of tie set = $B - n + 1 \Rightarrow 6 - 4 + 1$
$\Rightarrow 3$

links = (1, 3, 2)

$$B_f = \begin{array}{c} \\ 1 \\ 3 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$



for the twigs of (4, 5, 6) the Matrix will be;

$$B_f = \begin{array}{c} \\ 1 \\ 3 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc|ccc} 4 & 5 & 6 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Relation between A and B_f :-

For a graph, The two matrices A and B_f are orthogonal to each other

We know:-
$A = [A_t : A_l]$
$B_f = [B_{ft} : B_{fl}]$
$Q_f = [Q_{ft} : Q_{fl}]$

$$A B_f^T = 0$$

$$[A_t : A_l] \begin{bmatrix} B_{ft}^T \\ \vdots \\ U \end{bmatrix} = 0$$

$$A_t B_{ft}^T + A_l = 0$$

Then; $\boxed{B_{ft}^T = -A_l \cdot A_t^{-1}}$ or $\boxed{B_{ft} = [-A_l \cdot A_t^{-1}]^T}$

Relation between A and Q_f :-

for a graph KCL equations obtained by using incidence and cut set matrices;

$$A I_b = 0$$
$$Q_f I_b = 0$$

where, $I_b = \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix}$ branch matrix of $b \times 1$

Now: $[A_t : A_l] \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$

$$A_t I_t + A_l I_l = 0 \Rightarrow A_t I_t = -A_l I_l$$

$$I_t = -A_l I_l A_t^{-1} \text{ —— (i)}$$

Also, $[Q_{ft} : Q_{fl}] \begin{bmatrix} I_t \\ \vdots \\ I_l \end{bmatrix} = 0$

$$Q_{ft} I_t + Q_{fl} I_l = 0$$
$$Q_{ft} I_t = -Q_{fl} I_l \Rightarrow I_t = -Q_{fl} I_l Q_{ft}^{-1} \text{ —— (ii)}$$

from eq (i) and eq (ii)
$-Q_{fl} \cancel{I_l} Q_{ft}^{-1} = -A_l \cancel{I_l} A_t^{-1}$

$$\boxed{Q_{fl} = A_t^{-1} A_l}$$

Relation between B_f and Q_f :-

We know; $B_{fl}^{'} = -A_{b1} A_{t1}^{'}$ ——(1)

$Q_{fl} = \bullet A_t^{'} A_l$ ——(2)

from the above eq (1) and (2)

$$\boxed{-B_{fl}^{T} = Q_{fl}}$$

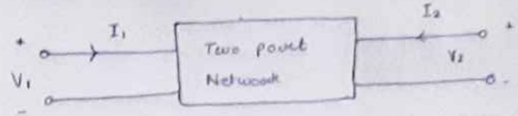
or

$$\boxed{B_{ft} = -Q_{fl}^{'}}$$

Two port Networks :-

Current entering or incoming from one terminal → $\boxed{\text{equal and opposite}}$ → Current leaving the other terminal of the port

This is called a port.



Two port Network consists four variables i.e Two voltages (V_1, V_2) and Two current (I_1, I_2) which are available for measurements and relevant for the analysis of two port Network.

Open Circuit Impedance (z) Parameters :-

Two port voltages in terms of two port current

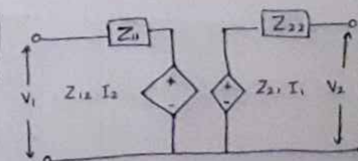
$$[V_1 \; V_2] = f[I_1 \; I_2]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = Z[I]$$

Here, Z is the open parameter circuit impedance matrix and Z_{ij} are the open circuit impedance parameter

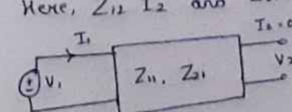
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} I_1 + Z_{12} I_2 \\ Z_{21} I_1 + Z_{22} I_2 \end{bmatrix}$$

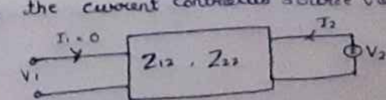


Thus, $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Here, $Z_{12} I_2$ and $Z_{21} I_1$ is the current controlled source voltage





Case 1 :- Outport port open circuited

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Case 2 :- Input port open circuited

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Short circuit Admittance (Y) parameter :-

Two port Current in terms of Two port voltage

$$(I_1\ I_2) = f\,(V_1, V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

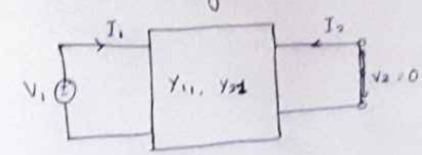
$$[I] = [Y]\,[V]$$

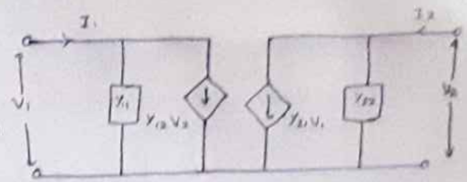
Here, Y is the Short circuit Admittance matrix and Y_{ij} are the short circuit Admittance parameter.

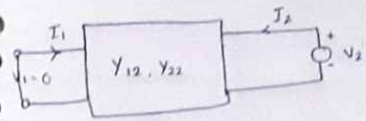
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11}V_1 + Y_{12}V_2 \\ Y_{21}V_1 + Y_{22}V_2 \end{bmatrix}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Determination of Y parameters:-







Case 1 :- when output short circuited

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Case 2 :- when Input port short circuited

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Transmission (T) or Chain or ABCD Parameters

One port variable in terms of other port variable :

$$(V_1, I_1) = f(V_2, I_2)$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

T parameter are used in the analysis of power transmission line

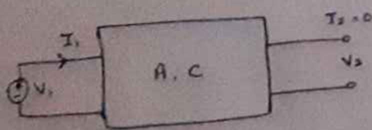
Input port $\longrightarrow$ Sending ends

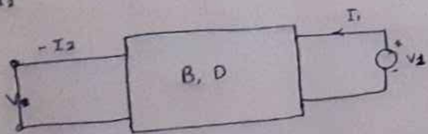
output port $\longrightarrow$ Receiving ends

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} AV_2 + BI_2 \\ CV_2 + DI_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$





Case 1 :- Output port open circuited ;

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

Case 2 :- Output port short circuited ;

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

Inverse Transmission (T) Parameter :-

output port variable in terms of input port variable

$$(V_2, I_2) = f(V_1, -I_1)$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

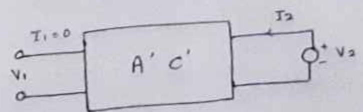
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A'V_1 + (-B'I_1) \\ C'V_1 + (-D'I_1) \end{bmatrix}$$

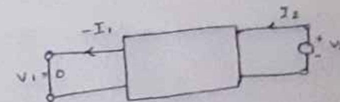
where, A' B' C' D' is called as an inverse Transmission parameter (T')

$$V_2 = A'V_1 + B'(-I_1)$$

$$I_2 = C'V_1 + D'(-I_1)$$

~~Case 1~~





Case 1: Input port open circuited

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad B' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

Case 2 :- Input port short circuited

$$C' = \left.\frac{V_2}{-I_1}\right|_{V_1=0} \qquad D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$

Hybrid (h) Parameters :-

The Hybrid (h) parameter are wide usage in electronic circuits for modeling transistors.

Voltage input port and current output port in terms of voltage output port and current input port.

$$(V_1, I_2) = f(V_2, I_1)$$

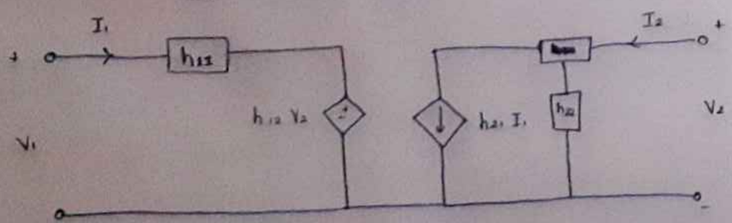
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} V_2 + h_{12} I_1 \\ h_{21} V_2 + h_{22} I_1 \end{bmatrix}$$

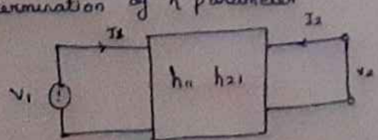
The matrix equation define h - parameter and h - parameters matrix known as hybrid matrix.

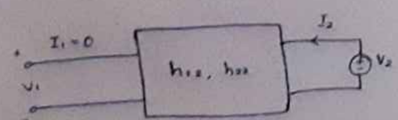
$V_1 = h_{11} V_2 + h_{12} I_1$ or $h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} V_2 + h_{22} I_1$ $h_{21} I_1 + h_{22} V_2$



Determination of h parameter





Case 1 :- output port short circuited

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$

Case 2 :- input port open circuited

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$

Inverse hybrid (g) parameter

Hybrid parameter and inverse hybrid parameter are dual of each other.

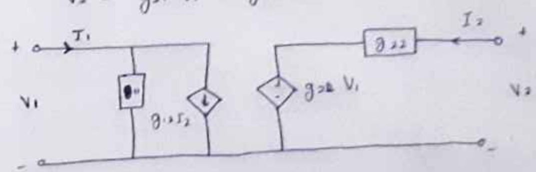
$$[g] = [h]^{-1}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

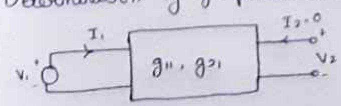
$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} V_1 + g_{12} I_2 \\ g_{21} V_1 + g_{22} I_2 \end{bmatrix}$$

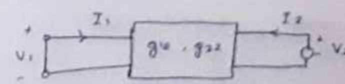
$I_1 = g_{11} V_1 + g_{12} I_2$

$V_2 = g_{21} V_1 + g_{22} I_2$



Determination of g - parameter :-





Case 1 :- output port open circuited

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$

$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$

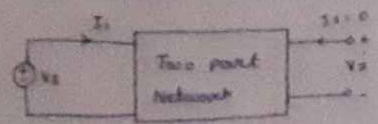
Case 2 :- input port short circuited

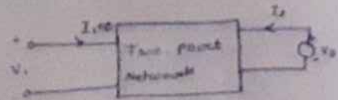
$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$

$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$

Condition for Symmetry :-

Two port networks are said to be symmetrically if port can interchanged without changing the voltages and current port.





→ In terms of Z :

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$\boxed{Z_{11} = Z_{22}}$$

→ In terms of Y :

$$Y_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Y_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$\boxed{Y_{11} = Y_{22}}$$

→ In terms of T parameter :

$$A = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad D = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$\boxed{A = D}$$

→ In terms of T' parameter

$$A' = \frac{V_2}{I_2}\Big|_{I_1=0} \qquad D' = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$\boxed{A' = D'}$$

→ In terms of h parameter

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = 1 \quad \left\{ \begin{matrix} h_{11}h_{22} - h_{12}h_{21} = 1 \\ \Delta h = 1 \end{matrix} \right\}$$

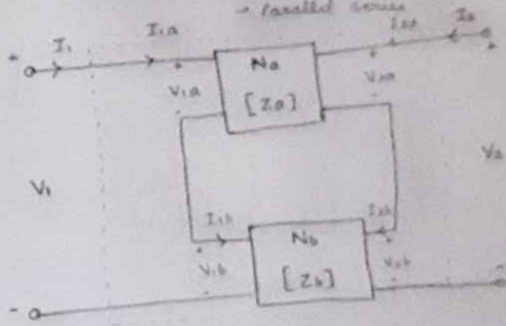
→ In terms of g parameter

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = 1 \quad \left\{ \begin{matrix} g_{11}g_{22} - g_{12}g_{21} = 1 \\ \Delta g = 1 \end{matrix} \right\}$$

Interconnection of Two port Network :-

→ Parallel → Series
→ Cascade
→ Series Parallel
→ Parallel series

i) $\boxed{\text{Series Connection}}$:-



$$(V_{1a}\ V_{2a}) = f(I_{1a}\ I_{2a})$$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

Similarly for Network N_b ;

$$(V_{1b}\ V_{2b}) = f(I_{1b}\ I_{2b})$$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Thus ;

$$I_1 = I_{1a} = I_{1b}$$
$$I_2 = I_{2a} = I_{2b}$$
$$V_1 = V_{1a} + V_{1b}$$
$$V_2 = V_{2a} + V_{2b}$$

We know that ;

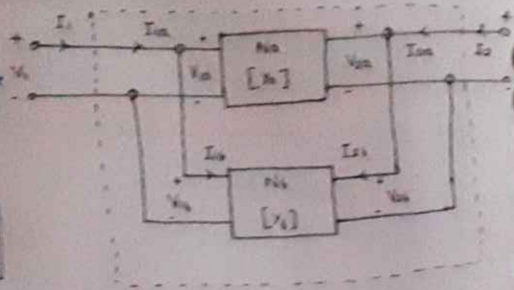
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus we get :

$$Z_{11} = Z_{11a} + Z_{11b}$$
$$Z_{12} = Z_{12a} + Z_{12b}$$
$$Z_{21} = Z_{21a} + Z_{21b}$$
$$Z_{22} = Z_{22a} + Z_{22b}$$

$$\boxed{[Z] = [Z_a] + [Z_b]}$$

2) **Parallel Connection** :-



Current port in terms of voltage port

$(I_1, I_2) = f(V_1, V_2)$

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

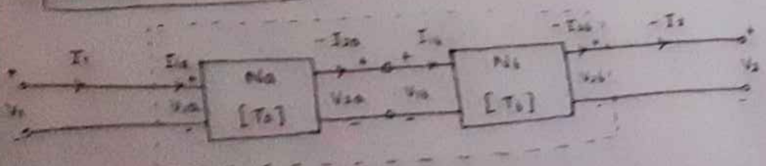
Similarly for Network N_b ;

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

By the Y parameter matrix in the parallel connection

$$\boxed{[Y] = [Y_a] + [Y_b]}$$

3) **Cascade Connection** :-



Input current and voltage in terms output current and voltage

$(V_1, I_1) = f(V_2, -I_2)$

for Network N_a ;

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

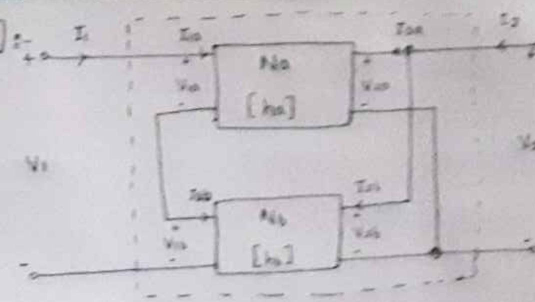
for Network N_b ;

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

By the T parameter matrix in cascade connection

$$\boxed{[T] = [T_a][T_b]}$$

4) **Series - Parallel Connection** :-



$(V_1, I_2) = f(I_1, V_2)$

for Network N_a ;

$$\begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix}$$

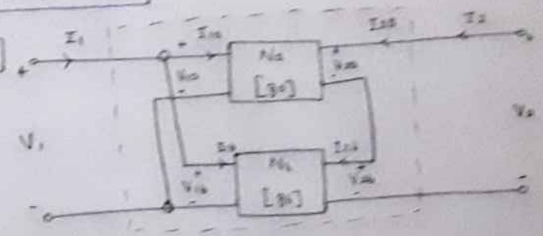
for Network N_b ;

$$\begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

In matrix from the h parameter of series parallel connection

$$\boxed{[h] = [h_a] + [h_b]}$$

5) **Parallel Series Connection** :-



$(I_1, V_2) = f(V_1, I_2)$

for Network N_a ;

$$\begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} g_{11a} & g_{12a} \\ g_{21a} & g_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix}$$

for Network N_b ;

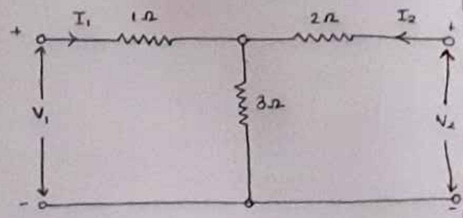
$$\begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} g_{11b} & g_{12b} \\ g_{21b} & g_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

In matrix from the g parameter of parallel series connection

$$\boxed{[g] = [g_a] + [g_b]}$$

Q7 For the Network shown :

Calculate : i) Z ii) Y iii) T iv) T' v) h and vi) g parameter



By the loop equation;

Applying the KVL :

$V_1 = I_1 + 3(I_1 + I_2)$

or, $V_1 = 4I_1 + 3I_2$ —— (1)

$V_2 = 2I_2 + 3(I_1 + I_2)$

or, $V_2 = 3I_1 + 5I_2$ —— (2)

i) Z parameter will be :

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Rightarrow \frac{4I_1}{I_1} = 4\Omega$

$Z_{11} = 4\Omega$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Rightarrow \frac{3I_1}{I_1} = 3\Omega$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Rightarrow \frac{3I_2}{I_2} \Rightarrow 3\Omega$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Rightarrow \frac{5I_2}{I_2} \Rightarrow 5\Omega$

$\boxed{Z_{11} = 4\Omega}$ $\boxed{Z_{21} = 3\Omega}$ $\boxed{Z_{12} = 3\Omega}$ $\boxed{Z_{22} = 5\Omega}$

ii) Y parameter will be :

from the eq(1) and eq(2) converting into terms of current

$V_1 = 4I_1 + 3I_2$

$4I_1 = V_1 - 3I_2$

$I_1 = \frac{V_1}{4} - \frac{3}{4}I_2$ —— (3)

$V_2 = 3I_1 + 5I_2$

$5I_2 = V_2 - 3I_1$

$I_2 = \frac{V_2}{5} - \frac{3}{5}I_1$ —— (4)

Case 1 :- when $V_2 = 0$ from eq(1) and eq(2)

$V_2 = 3I_1 + 5I_2$

$3I_1 = -5I_2$

$V_1 = 4I_1 - \frac{3}{5}I_1 \Rightarrow \frac{11}{5}I_1$

$V_1 = \frac{11}{5}I_1$ and $V_1 = -\frac{11}{3}I_2$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \Rightarrow \frac{-\frac{5}{\cancel{3}}\cancel{I_2}}{-\frac{11}{\cancel{3}}\cancel{I_2}} \Rightarrow \frac{5}{11}\,\mho$

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \Rightarrow \frac{-\frac{3}{\cancel{5}}I_1}{\frac{11}{\cancel{5}}I_1} \Rightarrow -\frac{3}{11}\,\mho$

Case 2 :- when $V_1 = 0$ from eq(1) and eq(2)

$4I_1 = -3I_2$

$V_2 = -\frac{11}{3}I_1$ and $V_2 = \frac{11}{4}I_2$

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \Rightarrow \frac{-\frac{3}{\cancel{4}}\cancel{I_2}}{\frac{11}{\cancel{4}}\cancel{I_2}} = -\frac{3}{11}\,\mho$

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \Rightarrow \frac{-\frac{4}{\cancel{3}}I_1}{-\frac{11}{\cancel{3}}I_1} \Rightarrow \frac{4}{11}\,\mho$

$\boxed{Y_{11} = \frac{5}{11}\,\mho}$ $\boxed{Y_{12} = -\frac{3}{11}\,\mho}$ $\boxed{Y_{21} = \frac{-3}{11}\,\mho}$ $\boxed{Y_{22} = \frac{4}{11}\,\mho}$

iii) <u>T parameters</u> :-

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

Case 1 :- when $I_2 = 0$ from eq(1) and eq(2)

$$V_1 = 4I_1$$
$$V_2 = 3I_1$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \Rightarrow \frac{4I_1}{3I_1} \Rightarrow \frac{4}{3}$$

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} \Rightarrow \frac{I_1}{3I_1} = \frac{1}{3}\ \mho$$

Case 2 :- when $V_2 = 0$ from eq(1) and eq(2)

$$3I_1 = -5I_2$$
$$V_1 = -\frac{11}{3}I_2$$

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0} \Rightarrow \frac{+\frac{11}{3}\cancel{I_2}}{+\cancel{I_2}} \Rightarrow \frac{11}{3}\ \Omega$$

$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0} = \frac{+\frac{5}{3}\cancel{I_2}}{+\cancel{I_2}} \Rightarrow \frac{5}{3}$$

iv) <u>T' parameters</u> :-

$$(V_2, I_2) = f(V_1, I_1)$$

Case 1 :- when $I_1 = 0$; $V_1 = 3I_2$, $V_2 = 5I_2$

$$A' = \frac{V_2}{V_1}\bigg|_{I_1=0} = \frac{5I_2}{3I_2} \Rightarrow \frac{5}{3}, \quad B' = \frac{I_2}{V_1}\bigg|_{I_1=0} = \frac{I_2}{3I_2} = \frac{1}{3}$$

Case 2 :- when $V_1 = 0$; $4I_1 = -3I_2$, $V_2 = -\frac{11}{3}I_1$

$$C' = \frac{V_2}{I_1}\bigg|_{V_1=0} \Rightarrow \frac{+\frac{11}{3}\cancel{I_1}}{+\cancel{I_1}} \Rightarrow +\frac{11}{3}\ \Omega, \quad D' = \frac{I_2}{I_1}\bigg|_{V_1=0} \Rightarrow \frac{+\frac{4}{3}\cancel{I_1}}{+\cancel{I_1}} = +\frac{4}{3}$$

iv) <u>h- parameter</u> :-

$$(V_1, I_2) = f(I_1, V_2)$$

Case 1 :- when $V_2 = 0$;

$$3I_1 = -5I_2$$
$$V_1 = \frac{11}{5}I_1$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{\frac{11}{5}I_1}{I_1} \Rightarrow \frac{11}{5}\ \Omega \qquad h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{-\frac{3}{5}\cancel{I_1}}{\cancel{I_1}} \Rightarrow -\frac{3}{5}$$

Case 2 :- when $I_1 = 0$;

$$V_1 = 3I_2$$
$$V_2 = 5I_2$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = \frac{3I_2}{5I_2} \Rightarrow \frac{3}{5} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = \frac{I_2}{5I_2} \Rightarrow \frac{1}{5}\ \mho$$

v) <u>g parameter</u> :-

$$(I_1, V_2) = f(V_1, I_2)$$

Case 1 when $I_2 = 0$;

$$V_1 = 4I_1, \quad V_2 = 3I_1$$

$$g_{11} = \frac{I_1}{V_1}\bigg|_{I_2=0} = \frac{I_1}{4I_1} = \frac{1}{4}\ \mho \ ; \quad g_{12} = \frac{V_2}{V_1}\bigg|_{I_2=0} = \frac{3I_1}{4I_1} = \frac{3}{4}$$

Case 2 :- when $V_1 = 0$:-

$$4I_1 = -3I_2$$
$$V_2 = \frac{11}{4}I_2$$

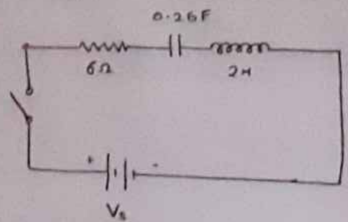
$$g_{12} = \frac{I_1}{I_2}\bigg|_{V_1=0} = \frac{-\frac{3}{4}\cancel{I_2}}{\cancel{I_2}} = -\frac{3}{4}, \quad g_{22} = \frac{V_2}{I_2}\bigg|_{V_1=0} = \frac{\frac{11}{4}\cancel{I_2}}{\cancel{I_2}} \Rightarrow \frac{11}{4}\ \Omega$$

Q1) Considering the RLC series circuit:

$V_s = 2V$, $R = 6\Omega$, $L = 2H$ and $C = 0.25F$

To Determine: $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i}{dt}(0^+)$ and $i(t)$

Ans At $t = 0$. By the KVL:



$$-V_s + V_R + V_C + V_L = 0$$

$$V_R + V_C + V_L = V_s$$

By the laws we know:

$$V = IR, \quad V_C = \frac{1}{C}\int I\,dt \quad \text{and} \quad V_L = L\frac{d}{dt}I$$

Thus, we get

$$i(t)R + \frac{1}{C}\int i(t)\,dt + L\frac{d}{dt}i(t) = V_s$$

$$6i(t) + 4\int i(t)\,dt + 2\frac{d}{dt}i(t) = 2 \text{ ——— (1)}$$

Multiplying with the $\frac{d}{dt}$ to eq(1), we get

$$6\frac{d}{dt}i(t) + 4i(t) + 2\frac{d^2i(t)}{dt^2} = 0$$

Assuming the $\frac{d}{dt}i(t)$ as P and $\frac{d^2i(t)}{dt^2}$ as P^2

$$6P + 4 + 2P^2 = 0$$

$$P^2 + 3P + 2 = 0$$

$$P(P+2) + 1(P+2) = 0$$

$$(P+1)(P+2) = 0$$

$$P = -1 \text{ and } -2$$

The general solution will be: $i(t) = k_1e^{-t} + k_2e^{-2t}$

At $t = 0$,

$$i(0^+) = 0$$

Inductor does not allow the change in current and act as a open circuit

$i(0^-)$ at $t = 0$

$$i(0) = k_1e^{-0} + k_2e^{-2(0)} \Rightarrow k_1 + k_2$$

$$k_1 + k_2 = 0$$

$$i(t) = k_1e^{-t} + k_2e^{-2t}$$

$$\frac{di(t)}{dt} \Rightarrow -k_1e^{-t} + (-2)k_2e^{-2t} \Rightarrow -k_1e^{-t} - 2k_2e^{-2t}$$

$$\frac{d}{dt}i(0^+) \Rightarrow -k_1 - 2k_2$$

Since $\frac{di(0^+)}{dt} = \frac{V_s}{L} \Rightarrow \frac{2}{2} = 1$

$$\boxed{\frac{di(0^+)}{dt} = 1\ \text{Amp/sec}}$$

Now, we know

$$\frac{d^2i(0^+)}{dt^2} + 3\frac{di(0^+)}{dt} + 2i(0^+) = 0$$

$$\frac{d^2i(0^+)}{dt^2} + 3(1) + 2(0) = 0$$

$$\boxed{\frac{d^2i(0^+)}{dt^2} = -3A/\text{sec}^2}$$

The complete solution will be solved by $k_1 + k_2 = 0$ and $-k_1 - 2k_2 = 1$

$$\cancel{k_1} + k_2 = 0$$

$$-\cancel{k_1} - 2k_2 = 1$$

$$\boxed{k_2 = -1} \text{ and } \boxed{k_1 = 1}$$

Thus, $\boxed{i(t) = e^{-t} - e^{-2t}}$