Unit 3 & 4 (C.S)

Control System :

Routh Stability Criterion :- (Unit - 4)

This Criterion is based on arranging the coefficients of characteristic eqn into an array called "Routh array".

$$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n = 0$$

Routh array :-

s^n	a_0	a_2	a_4	a_6	---
s^{n-1}	a_1	a_3	a_5	a_7	---
s^{n-2}	b_1	b_2	b_3	---	
s^{n-3}	c_1	c_2	c_3	---	
⋮					
s^0	a_n				

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

Statement :-

For a System to be Stable it is necessary & sufficient that each term of first column of Routh array must be +ve if $a_0 > 0$, If this condn is not met System is unstable & the no of sign changes in first column corresponds to the number of roots characteristic eqn in the right half of s-plane

Example-1 (Routh array)

$S^4 + 8S^3 + 18S^2 + 16S + 5 = 0$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4$

S^4	$a_1 = 1$	$a_3 = 18$	$a_5 = 5$
S^3	$a_2 = 8$	$a_4 = 16$	$a_6 = 0$
S^2	$\frac{8 \cdot 18 - 1 \cdot 16}{8}$	$\frac{8 \cdot 5 - 1 \cdot 0}{8}$	$\frac{8 \cdot 0 - 1 \cdot 0}{8}$
S^1			
S^0			

=

1	18	5
8	16	0
16	5	0
$\frac{16 \cdot 16 - 8 \cdot 5}{16}$	0	0
5	0	0

$5 \Downarrow a_n$

As to Routh array, it is a stable system because each term in 1st column is +ve ($a_0 > 0$)

Example-2

$3S^4 + 10S^3 + 5S^2 + 5S + 2 = 0$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4$

S_4	3	5	2	0
S_3	10	5	0	0
S_2	3.5 (1)	2	0	
S_1	-0.714 (2)	0	0	
S_1	$2 = a_n$	0	0	

This System is unstable because one term is less than 0, so it is unstable

Poles shifted on right half of S-plane

3.5 to -0.714 to 2 = 2 poles

+ (1) − (2) +

Polar Plot : (Unit - 3)

or Nyquist Plot

The Polar Plot of a Sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ Vs phase angle of $G(j\omega)$ on polar Coordinates as ω is varied from $0 \to \infty$. The Polar Plot is also called as nyquist Plot

→ Minimum Phase System :

↓

System with all zeros & Poles on the left half of S-Plane.

→ Non- minimum Phase System :

↓

System ~~with~~ in which poles are either left Side or right Side.

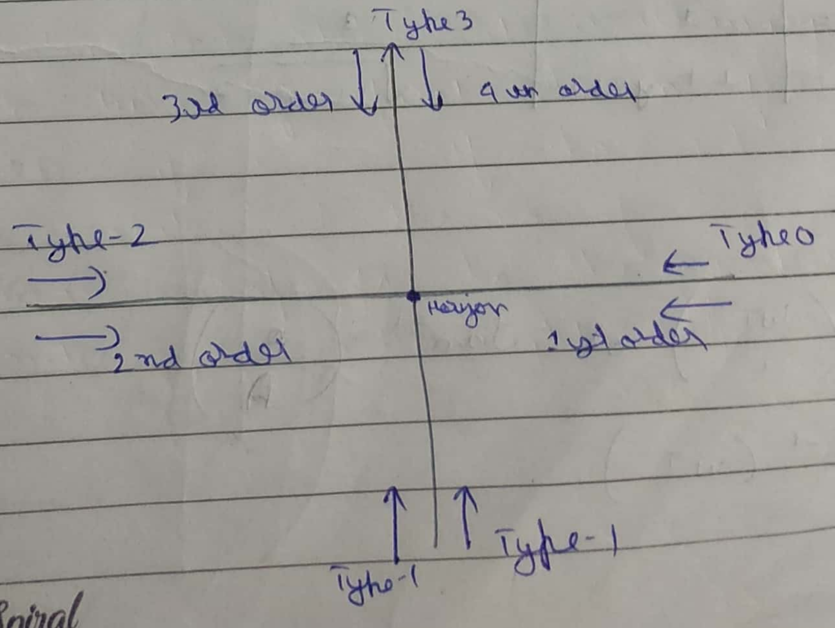
Ex : $G(s) = \dfrac{1}{S(1+KS)} = \dfrac{1}{S+KS^2}$

Type-1

order - 2

$S = 0$ (Pole at origin) , Type 1 System

order = 2 (max degree of S)



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Types of System (Polar Plot).

Note:

For Sketching the Polar plot we have to determine the type & order of the System.

Type = No of Poles in transfer function at origin

order = Highest degree of S in transfer f^n.

Example: (Type - 0 order - 1 System)

$$G(s) = \frac{1}{1+sT}$$

Type = 0
order = 1

$\Rightarrow$ Magnitude $= |G(s)| = \left|\frac{1}{1+sT}\right|$

$s = j\omega$

$$= |G(j\omega)| = \left|\frac{1}{1+j\omega T}\right| = \frac{1}{\sqrt{1+\omega^2 T^2}}$$

Phase Angle:

$$\phi = -\tan^{-1}\left(\frac{\omega T}{1}\right) \text{ or } -\tan^{-1}\left(\frac{B}{A}\right)$$

$$= -\tan^{-1}(\omega T).$$

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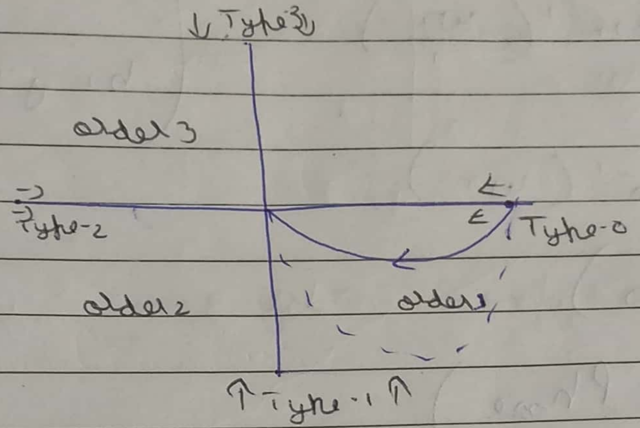
(i) $\omega = 0$ — Mag, Phase Angle

$1 \angle 0^\circ$

from $= \dfrac{1}{\sqrt{1+\omega^2 T^2}}$ from $= -\tan^{-1}(\omega T)$

(ii) $\omega = \infty$ $0 \angle -90^\circ$



(Type-1, order-2 System)

$$\boxed{G(s) = \frac{1}{s(1+sT)}}$$

Type = 1, Poles at origin is S, because $S = 0$

order = 2, $(s + s^{②} T) \rightarrow$ Max^m Power of S

$S = j\omega$

$$G(s) = G(j\omega) = \frac{1}{j\omega(1 + j\omega T)}$$

Magnitude $= |G(j\omega)|$

$$\boxed{M = \frac{1}{\omega\sqrt{1+\omega^2T^2}}}$$

Phase angle:

$$\phi = -90 - \tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$= -90 - \tan^{-1}(\omega T)$$

when, $j\omega$

a) $j\omega$ = denominator
= -90

b) $j\omega$ = numerator
= $+90$

ω (0 to ∞)

$\omega = 0$ (mag & Phase)

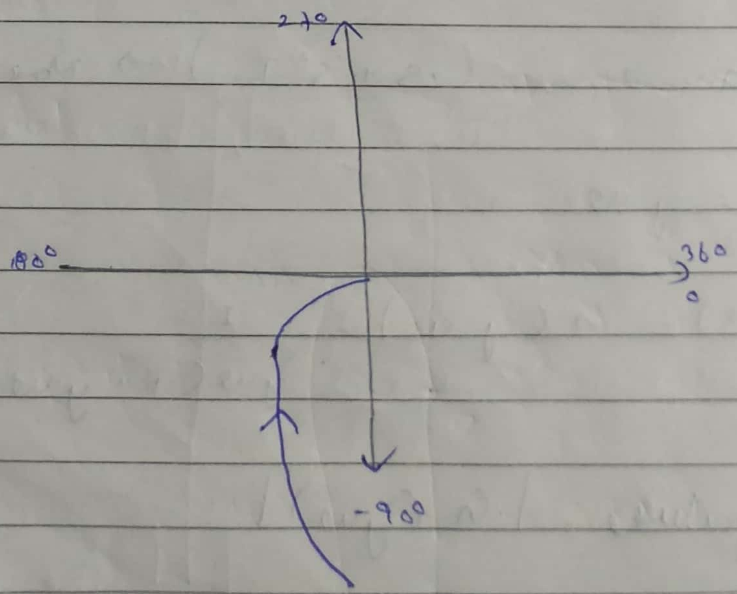
$= \infty \angle -90^\circ$

$\tan 0^\circ = 0$

$\omega = \infty$

$= 0 \angle 180^\circ$

Plot



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(Type-0, order-2 System)

$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$

Type-0 (Plot starting)

order = s^2, So = 2 (Plot ending)

$s = j\omega$

$$\boxed{G(s) = G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}}$$

Magnitude

$$|G(j\omega)| = \frac{1}{\sqrt{(1+\cancel{j}(\omega T_1)^2)(1+(\omega T_2)^2}}$$

Phase Angle :

$$\phi = -\tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

Here, ω = 0 to ∞

$\omega = 0$ (Magnitude & Phase angle)

$1 \angle 0^\circ$

$\omega = \infty$

$\tan(\infty) = 90^\circ$

$0 \angle -180^\circ$

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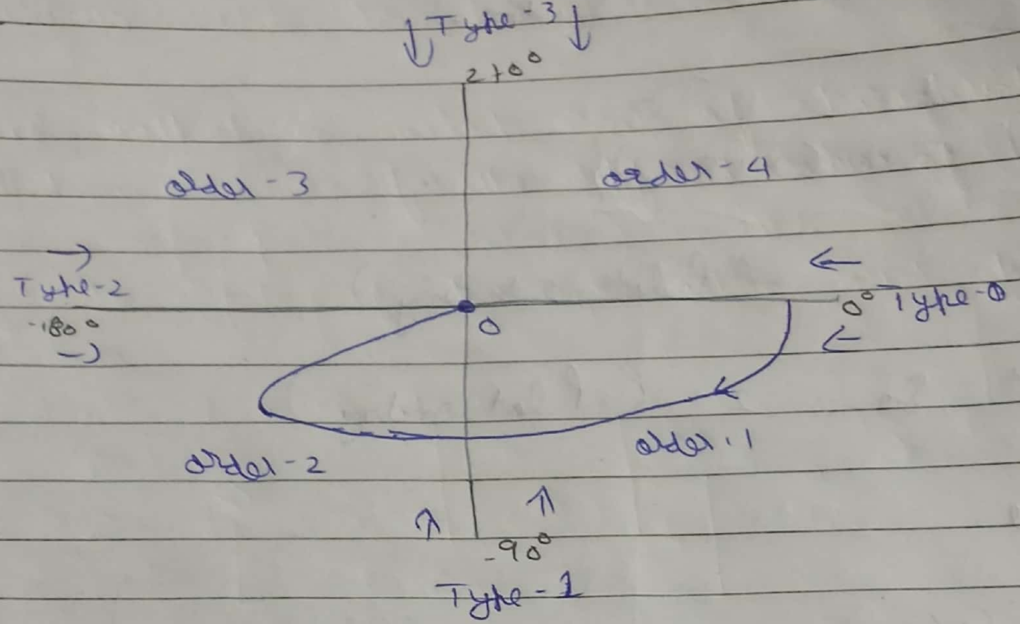
Plot:



Example of (Type-1, order-3 System)

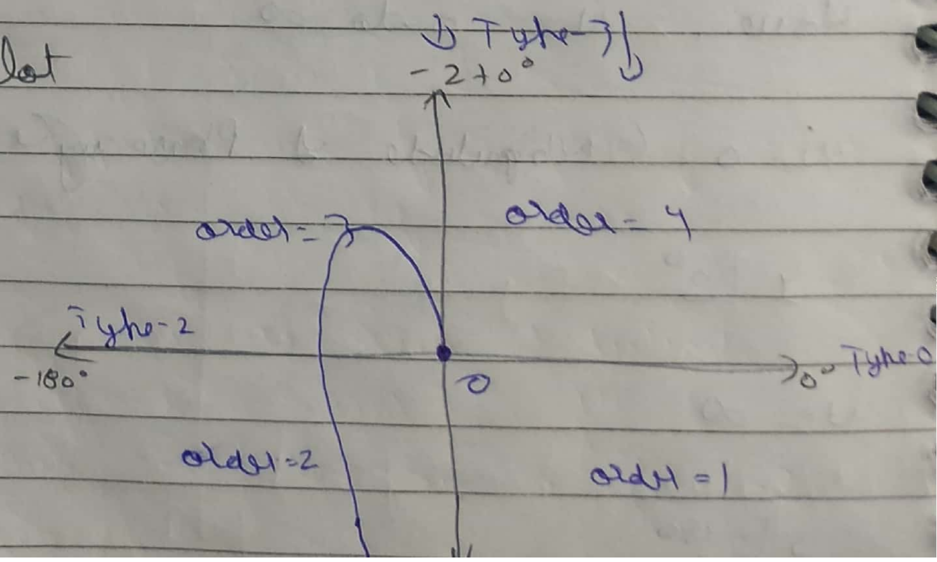
$$G(s) = \frac{1}{s(sT_1+1)(1+sT_2)}$$

$\omega = 0$ (mag & Phase)

$\infty \angle -90^\circ$

$\omega = \infty$ (mag & Phase)

$0 \angle -270$ Plot



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1) Nyquist Criterion : (Extension of Polar Plot)
or Nyquist Plot Stability :

→ It is basically an extended form of Polar Plot because here ω varies from (-∞ to ∞)

Characteristic eqn:

$$D(s) = 1 + G(s)\,H(s)$$

A feedback system (Close loop System) is Stable if and only if there is no zeroes of D(s) in the right half of S-Plane i.e (Z = 0)

$$(N = -P)$$

→ $D(s) = 1 + G(s)\,H(s)$

$$\boxed{G(s)\,H(s) = D(s) - 1}$$

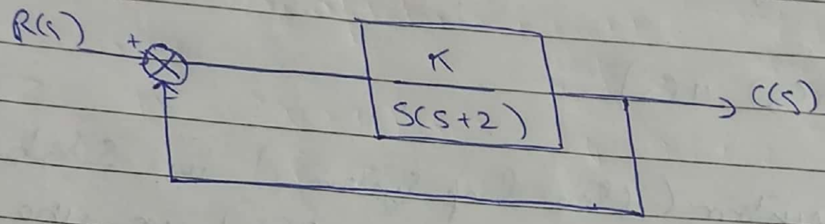
Also

Zeroes of the D(s) are the roots of characteristic eqn.

Root locus : (16 November) : (11:20 P.M)

Graphical method in which roots of characteristic eqn are plotted in s-plane for the different values of parameter

Locus of the roots of char eqn when varied from 0 to ∞ is root locus.



Char eqn:

$$1 + G(s)H(s) = 0$$

$$G(s) = \frac{K}{S(S+2)} \quad , \quad H(s) = 1$$

$$1 + \frac{K}{S(S+2)} \cdot 1 = 0$$

$$S^2 + 2S + K = 0$$, So

Roots are:

$$S_1 = -1 + \sqrt{1-K} \quad , \quad S_2 = -1 - \sqrt{1-K}$$

Location of the roots are:

1) $0 < K < 1$, roots are real & distinct.

2) $K = 0$, the roots are $S_1 = 0$, $S_2 = -2$

3) $K = 1$, both roots are real loop poles
4) $K > 1$, roots are complex conjugate with real part $= -1$

→ overall transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

char eqⁿ:

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

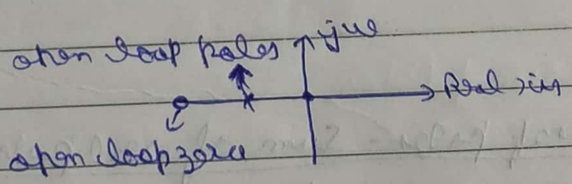
$$|G(s)H(s)| = -1$$

$$\boxed{\angle G(s)H(s) = \pm 180(2K+1)}$$

$K = 0, 1, 2, 3 \ldots$

→ Construction Rules:

1) Root locus is always symmetrical about real axis.



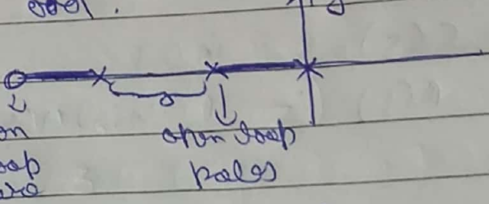
2) Root locus always starts from open loop poles & terminates on either finite open loop zero or infinity.

$P > Z$ (branch terminate at ∞)
$P < Z$ (branch will start at ∞)

3) No of branches terminating on at infinity $= P - Z$

No of poles ↓ No of zeroes ↓

4) A Pt on the real axis on the locus if the no of open loop (poles + zeroes) on the real axis to the right half of s-plane is odd.



5) The angle of asymptotes :

If $P > Z$ $\phi_A = \frac{(2q+1)180°}{P-Z}$, $q = 0, 1, 2, \ldots P-Z-1$.

If $P < Z$ $\phi_A = \frac{(2q+1)180°}{Z-P}$, $q = 0, 1, 2, \ldots Z-P-1$.

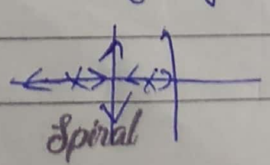
Note a) If branch is terminating at ∞, it implies a zero is there at ∞

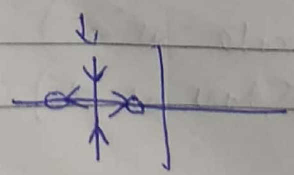
If branch is starting at ∞, it implies a pole at ∞.

6) Centroid : The asymptotes meet the real axis at Centroid

$$\text{Centroid} = \frac{\text{Sum of real parts of poles} - \text{Sum of real parts of zeroes}}{\text{No of poles} - \text{No of zeroes}}$$

7) Break away & Break in point is solved by : $\frac{dK}{ds} = 0$





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8) Angle of departure from open loop given by ϕ_p

$$\phi_p = \pm 180^\circ (2q+1) + \phi \quad , q = 0, 1, 2 ---$$

where $\phi = \theta_2 - (\theta_1 + \theta_3 + \theta_4)$ {Angle by zero (Sum) - (Sum of Angle by poles)}

9) Angle of arrival from open loop zero

$$\phi_z = \pm 180^\circ (2q+1) - \phi \; , \boxed{q = 0, 1, 2 ---}$$

$$\phi = \theta_2 - (\theta_1 + \theta_3 + \theta_4)$$

10) Intersection of root locus branches with the imaginary axis can by determined by use of routh criterion.

Note :

When we don't have any pole or zero with img part then we don't have to take angle of departure & angle of arrival.

Spiral

$-a + \frac{521}{3}$ $\quad -a + \frac{4.58}{7}$

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Example to Find Root locus.

$$G(s)\,H(s) = \frac{K}{S(S+5)(S+10)}$$

→ Numerator given = Zeroes
→ Denominator " " = Poles

Step-1 Identify loci

No of Poles $P = 3$
" " Zeroes $Z = 0$ } Loci = Max (P, Z) $= 3$

Position of Poles $P = 0, -5, -10$

Step-2 Identify No of Asymptotes: (N.O.A)

$= P - Z$

$= 3 - 0 = 3$

Step-3 Centroid of Asymptotes:

$$\sigma_c = \frac{\Sigma \text{Real } P - \Sigma \text{Real } Z}{P - Z}$$

(σ = Sigma)

$$= \frac{(0-5-10) - 0}{3} = -5$$

Step-4 Angle of asymptotes:

$$\theta = \frac{(2q+1)}{P-Z}\,180^\circ, \quad q = 0, 1, 2 \;(\text{N.O.A} = 3)$$

when $q = 0$	$q = 1$	$q = 2$
$= 60^\circ$	$= 180^\circ$	$= 300^\circ$

Spiral

Step-5 Calculation of Break away pt

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Char eq ($1 + G(s)\ H(s) = 0$)

$$1 + \frac{K}{s(s+5)(s+10)} = 0$$

$$s^3 + 15s^2 + 50s + K = 0$$

$$K = -(s^3 + 15s^2 + 50s)$$

Hence $\frac{dK}{ds} = 0$

$$\frac{dK}{ds} = -(3s^2 + 30s + 50) = 0$$

$S = -2.11$ ✓, $S = -7.88$ ×

Step 6 Intersection of Img axis

char eqn ($1 + G(s)\ H(s) = 0$)

$$s^3 + 15s^2 + 50s + K = 0$$

Routh array

s^3	1	50
s^2	15	K
s^1	$\frac{750-K}{15}$	0
s^0	K	

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Dt.:
Pg.:

for Stability

$K > 0$, $\frac{750 - K}{15} > 0$

$750 > K_{max}$

Aux eqⁿ

$A(s) = 0$

$K = 750$

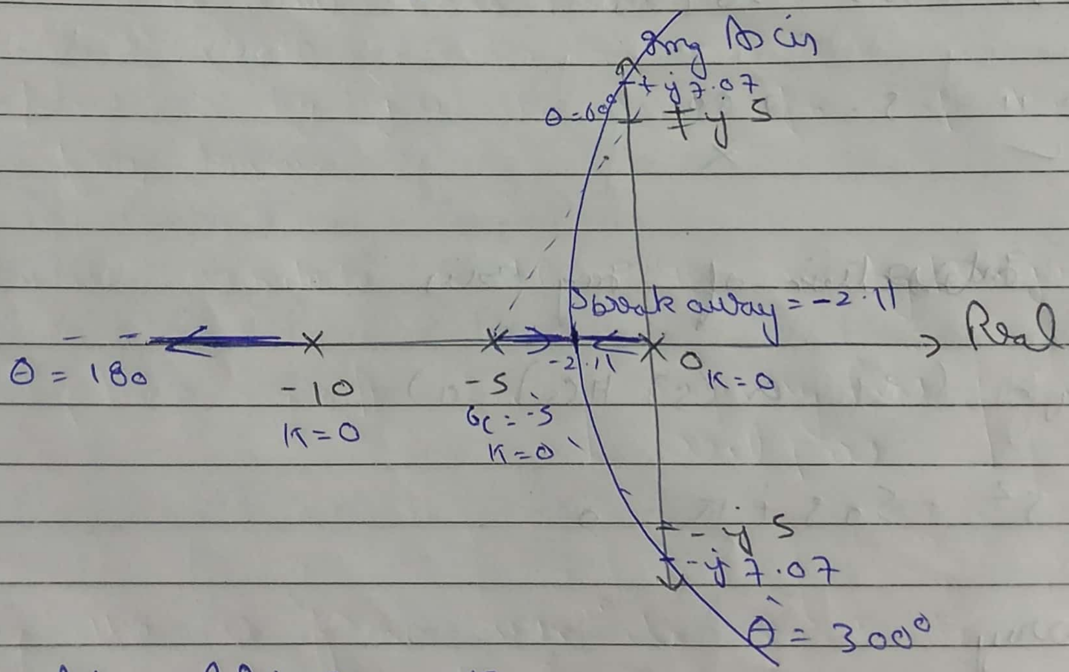
$15s^2 + K = 0$

$= 15s^2 + 750 = 0$

$s = \pm j\, 7.07$

Step 7 Angle of departure

we don't have any pole or zero with img part



Here At all poles, K = 0

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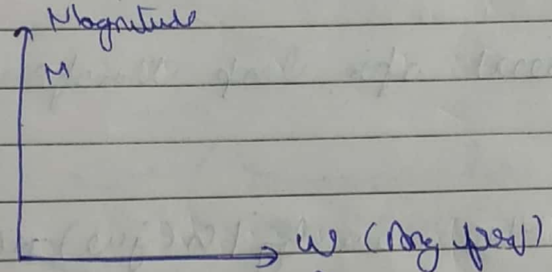
Bode Plot:

Graphical representation of the transfer function for determining the stability of the control system.

Bode Plot consist of:

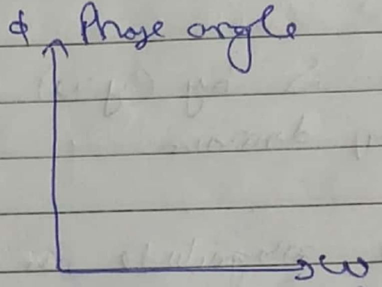
- Magnitude Plot. a)
- Phase Plot b)

a) mag Plot



$M = 20 \log |G(j\omega)|$

b) Phase Plot



$\phi = \angle G(j\omega)$

Basic Bode Plots:

Ideal System - Unity feedback Syst

a) Constant, K

b) Integral term $\frac{K}{j\omega}$ or $\frac{K}{(j\omega)^n}$

c) Derivative term $K(j\omega)$ or $K(j\omega)^n$

d) 1st order term in denominator $\left[\frac{1}{1+j\omega}\right.$ or $\frac{1}{1+(j\omega)^n}$

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(e) 1st order term in numerator:

$$(1+j\omega) \text{ or } (1+j\omega)^n$$

f) Quadratic term in denominator $= \dfrac{1}{1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$

g) Quadratic term in numerator $= \dfrac{1}{1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$

Procedure to draw Bode Plots:

1) Replace S by $(j\omega)$ to convert open loop transfer fn into freq domain.

2) Calculate magnitude in dB, $M|_{dB} = 20 \log |G(j\omega)\, H(j\omega)|$

3) Find phase angle $\phi = \tan^{-1}\left[\dfrac{\text{imaginary part}}{\text{real part}}\right]$

4) Vary the ω from $\min^m$ to $\max^m$ to find M & ϕ & draw magnitude & Phase Plot.

1) Constant = K

$G(s) = K$

Let $S = j\omega$

$\boxed{G(j\omega) = K}$ →

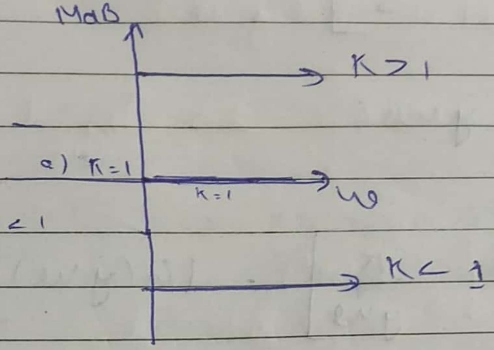
Magnitude Plot:

$M_{dB} = 20 \log |G(j\omega)|$

$= 20 \log_{10} |K|$

b) When $K > 1$, +ve value

c) When $K < 1$, -ve value or < 1



Magnitude

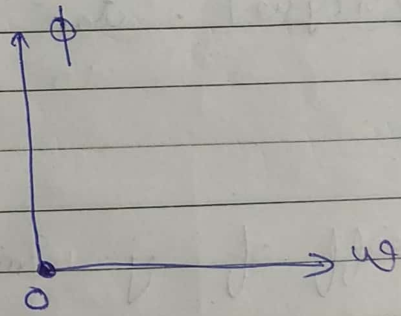
Phase Plot

$\phi = \tan^{-1}\left(\frac{\text{Imaginary Part}}{\text{real Part}}\right)$

Img Part = 0
real Part = K

$\phi = \tan^{-1}\left(\frac{0}{K}\right) = 0$

$\phi = 0$



2) Bode Plot for = Integral factor

$\frac{K}{S}$ or $\frac{K}{S^n}$

(i) $\boxed{G(s) = \frac{K}{S}}$

Let $S = j\omega$

$\boxed{G(j\omega) = \frac{K}{j\omega}} = |G(j\omega)| = \frac{K}{\omega}$

Gain dB or $Mag_{dB} = 20 \log_{10} |G(j\omega)|$

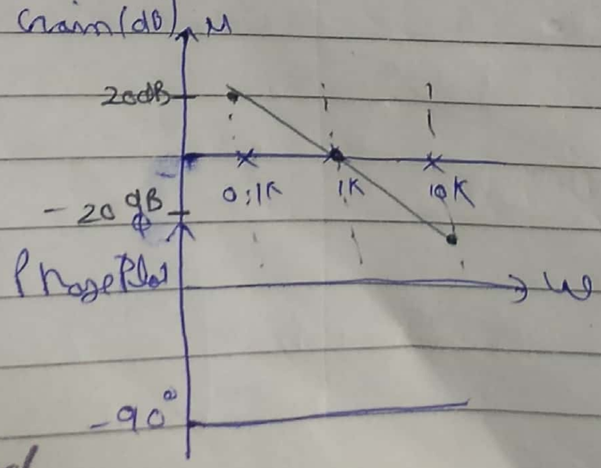
$= 20 \log \frac{K}{\omega}$

$\phi = \angle G(j\omega) = \tan^{-1}\left(\frac{Img}{Real}\right) = \tan^{-1}\left(\frac{K/\omega}{0}\right) = \tan^{-1}(\infty)$

$\boxed{\phi = -90^\circ}$

Here Coeff of j = Imaginary Part

[W = is in denominator
So $\phi = -90^\circ$]



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y $\omega = 0.1K$ for M_{dB}

$20\log\left(\frac{K}{0.1K}\right) = 20\log\left(\frac{1}{0.1}\right) = 20\log_{10}(10)$

$\boxed{\omega = 20\,dB \text{ (for } = 0.1)}$ 1st Value

y $\omega = K$ for M_{dB}

$20\log_{10}\left(\frac{K}{K}\right) = 20\log_{10}(1) = 20\times 0 = 0\,dB$

$\boxed{\omega = 0\,dB}$ 2nd Value

y $\omega = 10K$ for M_{dB}

$20\log_{10}\left(\frac{K}{10K}\right) = 20\log\left(\frac{1}{10}\right) = -20$

$\boxed{\omega = -20\,dB}$ 3rd Value

(b)

$\boxed{\text{For } G(s) = \frac{K}{s^n}}$

We will put values of ω as $= 0.1K^{1/n}$, $M = 20n\,dB$

$= K^{1/n}$, $M = 0\,dB$

$= 10K^{1/n}$, $M = -20n\,dB$

Spiral.

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3) Bode Plot for = Derivative factor.

$G(s) = K(s)$ or $K s^n$

Let $S = j\omega$

$G(j\omega) = K(j\omega)$

Gain dB = MdB = $20 \log_{10} |G(j\omega)|$

$= 20 \log_{10} (K\omega)$

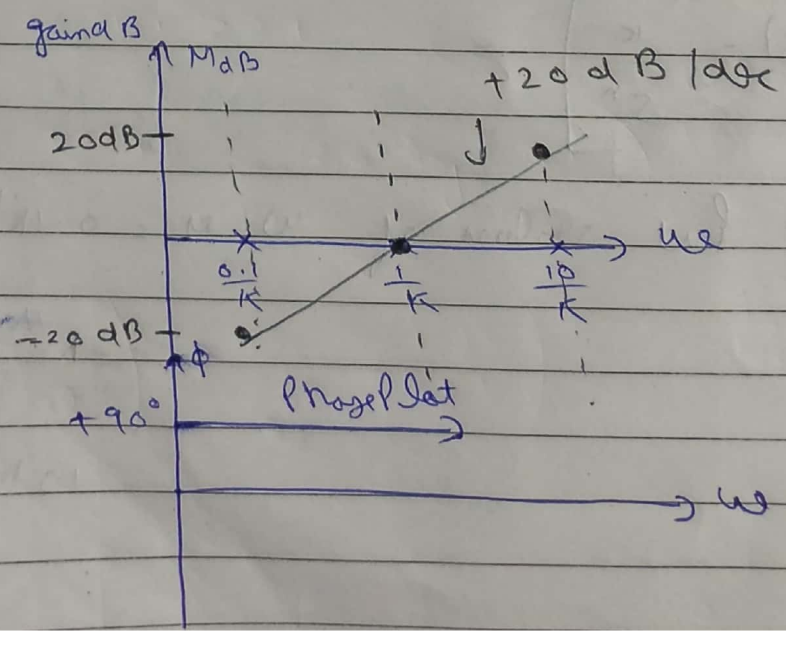
Note:- When we remove mode then it got Saved and Img Part gets deleted

$\phi = \tan^{-1}\left(\frac{\text{Img Part}}{\text{real Part}}\right) = \tan^{-1}\left(\frac{K\omega}{0}\right)$

Imaginary Part = $K\omega$

(ω = numerator So $\phi = 90°$)

$= \tan^{-1}(\infty) = 90°$



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$M = 20 \log_{10} K(\omega)$

(i) $\omega = \frac{0.1}{K}$

$= 20 \log_{10} (10^{-1}) = -20$ dB

(ii) $\omega = \frac{1}{K}$

$= 20 \log_{10} (1) = 0$

(iii) $\omega = \frac{10}{K}$

$= 20 \log_{10} (10) = 20$

Note:

For $G(s) = K s^n$

$M_{dB} = 20 \log (K^{1/n} \omega)^n = 20n \log (K^{1/n} \omega)$

Value of $\omega = \frac{0.1}{K^{1/n}}, \frac{1}{K^{1/n}}, \frac{10}{K^{1/n}}$

Spiral

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4) Bode Plot: 1st order in denominator

$$G(s) = \frac{1}{1+sT}$$

Let $s = j\omega$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$|G(j\omega)| = \left|\frac{1}{1+j\omega T}\right| = \frac{1}{\sqrt{1+\omega^2T^2}}$$

Gain dB = MdB

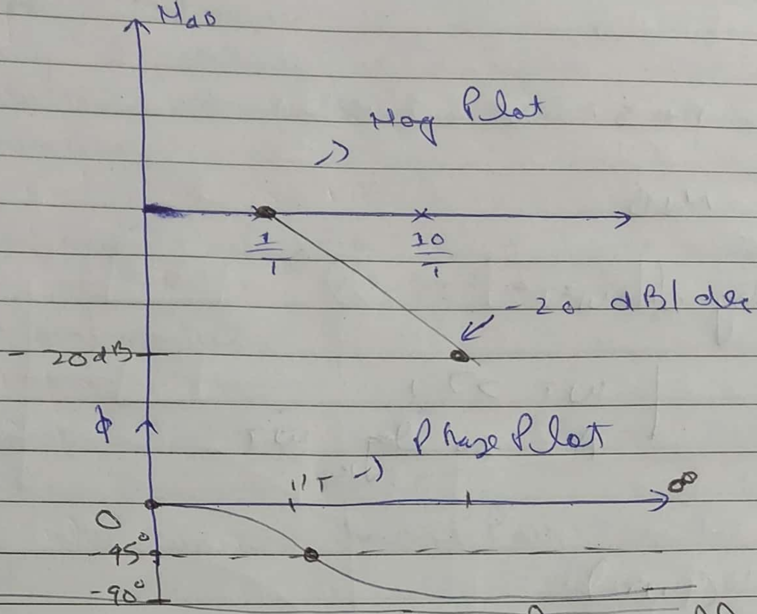
$= 20 \log |G(j\omega)|$

$$= 20 \log_{10} \frac{1}{\sqrt{1+\omega^2T^2}}$$

$\omega T << 1$	$\omega T >> 1$
So	So
$[M dB = 0]$	$(M dB = -20 \log (\omega T))$

$$\phi = \tan^{-1}\left(\frac{Img}{Real}\right) = \tan^{-1}\left(\frac{\omega T}{1}\right) = -\tan^{-1}(\omega T)$$

Telco Dt.: Pg.:



(i) For $\omega = \frac{1}{T}$, MdB:

$MdB = -20\log(\omega T)$

$= -20\log_{10}\left(\frac{1}{\cancel{T}} \times \cancel{T}\right)$

$= 0$

(ii) $\omega = \frac{10}{T}$, MdB

$MdB = -20\log\left(\frac{10}{\cancel{T}} \times \cancel{T}\right)$

$= -20\ dB$

Phase Plot

$\phi = -\tan^{-1}(\omega T)$

(i) $\omega = 0$

$\phi = -\tan^{-1}(0) = 0$

(ii) $\omega = 1/T$

$\phi = -\tan^{-1}\left(\frac{1}{T} \times T\right) = -45^\circ$

(iii) $\omega = \infty$

$\phi = -\tan^{-1}(\infty) = -90^\circ$

Spiral

telco Dt.: Pg.:

Note:

$G(s) = 1 + sT$

Gain dB = Mod B

$A = 20 \log \sqrt{1 + \omega^2 T^2}$

$\omega T << 1$ | $\omega T >> 1$

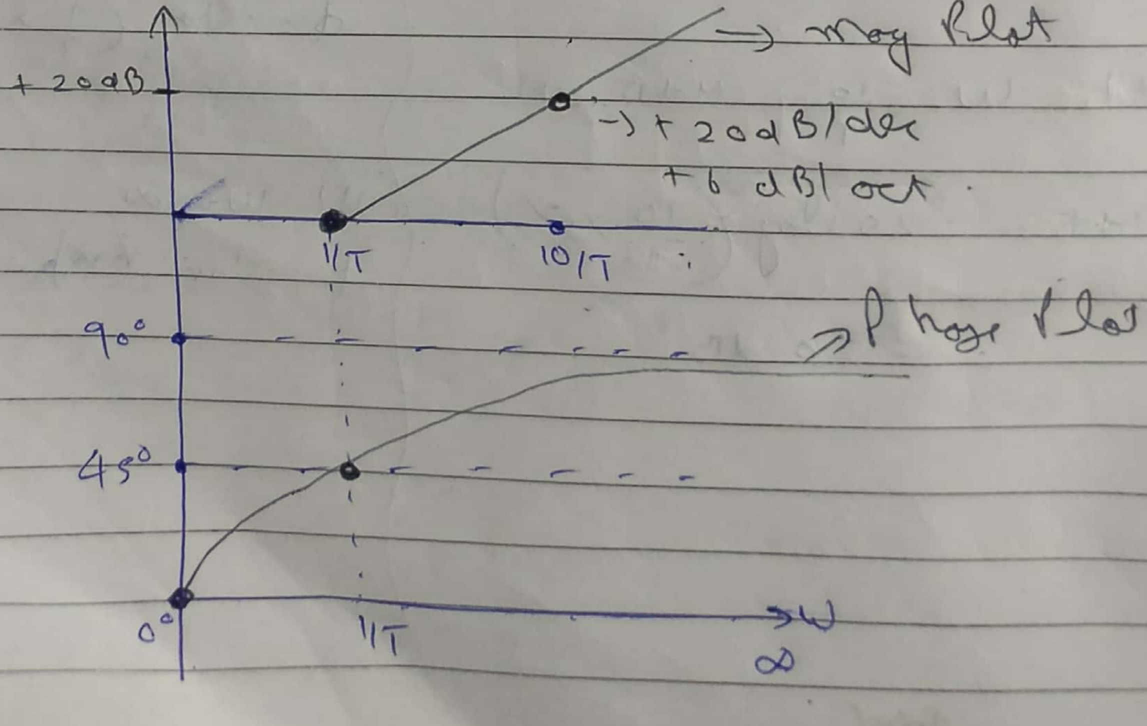
$A = 0$ | $A = 20 \log_{10} \omega T$

$\phi = \tan^{-1}(\omega T)$

Take Values

$\omega = 1/T$, $\frac{10}{T} = \omega$ (magnitude Plot)

$\omega = 0°, 1/T, \infty$ (Phase Plot)



Spiral

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Compensation techniques:-

For accuracy & stability of a System we connect a circuit between error detector & plant this is known as compensation.

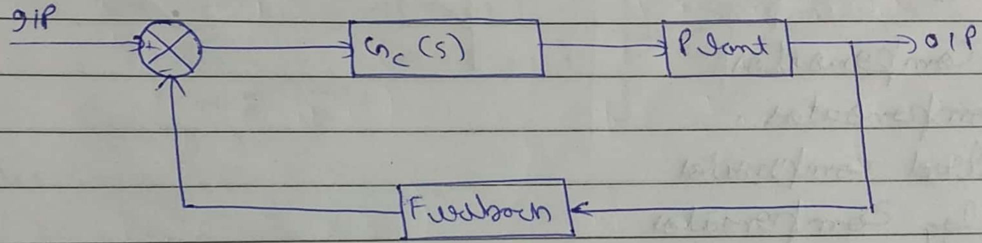
→ System Performance is given by → Time response
→ Freq response

Types of compensation:-

1) Series Compensation:-

↓

Compensating N/W in the forward Path.

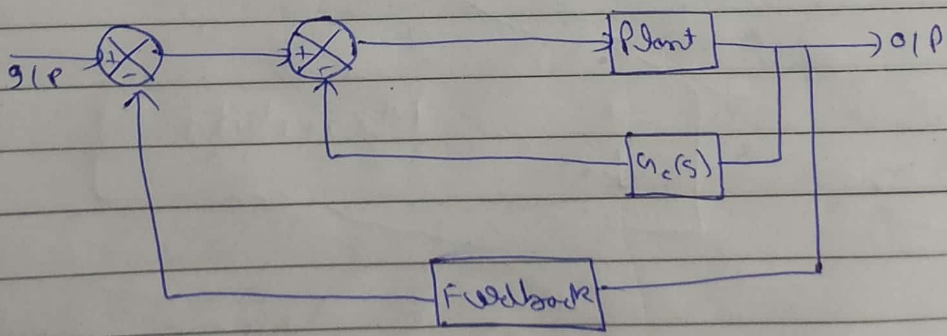


$G_c(s)$ = Compensator.

2) Feedback Compensator.

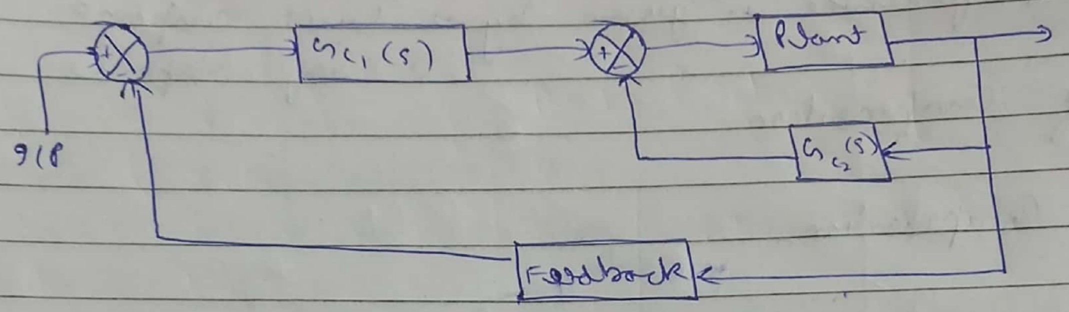
↓

Compensator is inserted in feedback path.



3) Lead Compensation:

Comb^n of other two compensators.



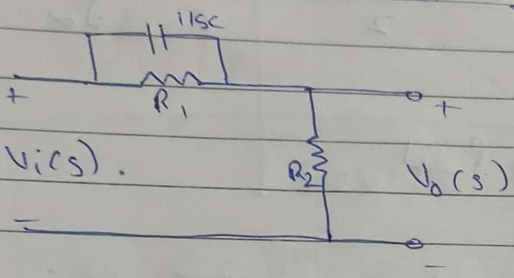
Types of Compensator:

1) Lead Compensator
2) Lag Compensator
3) Lag-lead compensator
4) Lead-lag compensator

Lead Compensator

↓

Electrical NW which produces Sinusoidal O/P having phase lead when a Sinusoidal I/P is applied



$\alpha = \frac{Z}{P}$

$P = \frac{Z}{\alpha}$

$$G_c(s) = \frac{S+Z}{S+P} = \frac{S+Z}{S+\frac{Z}{\alpha}}$$

$$= \frac{S+\frac{1}{Z}}{S+\frac{1}{\alpha Z}} = \frac{\alpha(SZ+1)}{(\alpha S Z+1)} \rightarrow (A)$$

$\frac{1}{Z} \rightarrow \omega_{c1}$, $\frac{1}{\alpha Z} \rightarrow \omega_{c2}$

$S = j\omega$

$$G_c(j\omega) = \frac{\alpha(j\omega Z+1)}{(j\omega\alpha Z+1)} = \frac{(j\omega+1)}{(j\omega\alpha Z+1)}$$

α = small

$$\phi = \angle G_c(j\omega) = \tan^{-1}(\omega Z) - \tan^{-1}(\omega\alpha Z)$$

$$\tan(A) - \tan(B)$$

$$\tan\phi = \frac{\omega Z - \omega\alpha Z}{1+\omega^2 Z^2\alpha} \quad - (1)$$

$$\left.\frac{d\phi}{\partial\omega}\right|_{\omega=\omega_m} = 0$$

↓

max^m compen^n cond^n

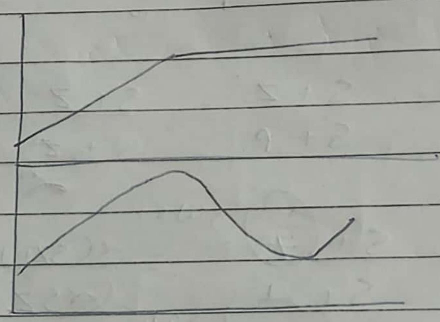
$$\omega_m = \sqrt{\omega_{c1}\cdot\omega_{c2}}$$

$$\omega_m = \sqrt{\frac{1}{Z}\cdot\frac{1}{\alpha Z}} = \frac{1}{Z\sqrt{\alpha}}$$

Putting $\omega = \omega_m$ in (1)

$$= \frac{\frac{1}{\tau\sqrt{\alpha}}\cancel{\tau} - \frac{1}{\tau\sqrt{\alpha}}\alpha\cancel{\tau}}{1 + \frac{1}{\cancel{\tau^2}\alpha}\cancel{\tau^2}\cancel{\alpha}} = \frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{2} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

Bode Plot



$$\phi = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)$$

$$\sin\phi = \frac{1-\alpha}{1+\alpha}$$

$$\alpha = \frac{1-\sin\phi}{1+\sin\phi}$$

Magnitude :

$$|G_c(j\omega)| = \frac{\sqrt{1+\omega^2\tau^2}}{\sqrt{1+\omega^2\alpha^2\tau^2}} \qquad \omega = \omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

$$= \frac{\sqrt{1+\frac{1}{\cancel{\tau^2}\alpha}\cancel{\tau^2}}}{\sqrt{1+\frac{1}{\cancel{\tau^2}\cancel{\alpha}}\alpha^{\cancel{2}}\cancel{\tau^2}}} = \frac{\sqrt{1+\frac{1}{\alpha}}}{\sqrt{1+\alpha}}$$

$$= \frac{\sqrt{\frac{\cancel{\alpha+1}}{\alpha}}}{\cancel{\sqrt{1+\alpha}}} = \boxed{\frac{1}{\sqrt{\alpha}} = \text{Magnitude}}$$

→ s-domain transfer function

telco Dt.: Pg.:

$$\frac{V_o}{V_i} = \frac{R_2}{R_2 + \frac{R_1 \cdot \frac{1}{sC}}{R_1 + \frac{1}{sC}}}$$

→ Electrical NW Transfer function

$$= \frac{R_2}{R_2 + \frac{R_1}{R_1 sC + 1}}$$

$$= \frac{R_2(R_1 sC + 1)}{R_2 R_1 sC + R_2 + R_1}$$

$$= \frac{R_2 R_1 sC + R_2}{(R_1 + R_2) + R_2 R_1 sC}$$

$$= \frac{R_2(R_1 sC + 1)}{R_1 + R_2\left(1 + \frac{R_1 R_2 sC}{R_1 + R_2}\right)}$$

$$= \frac{R_2}{R_1 + R_2}\left(\frac{R_1 sC + 1}{1 + \frac{R_1 R_2}{R_1 + R_2} sC}\right) \quad \text{--- } A'$$

Comparing A & A'

$$\boxed{\alpha = \frac{R_2}{R_1 + R_2}}$$

$$\boxed{Z = R_1 C}$$

Spiral

telco Dt.:
Pg.:

Effects of Lead Compensator :-

1) Stability increases
2) Bandwidth increases
3) Rise time decreases
4) Transient ~~time~~ response is improved

→ Pros

5) Noise enters into the System at high freq.

→ Cons

Note :-

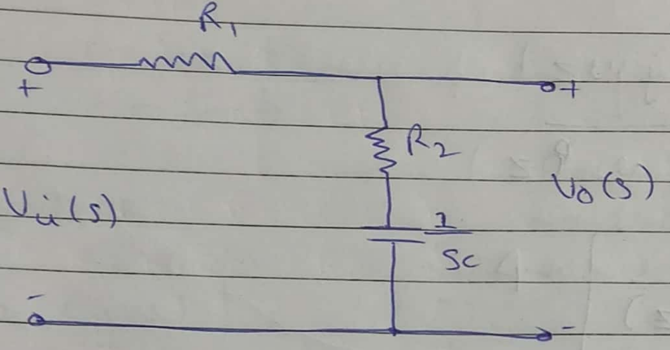
Here Phase is +ve

Telco Dt.: Pg.:

Lag Compensator

i) Electrical NW which produces a sinusoidal o/p having phase lag when sinusoidal i/p is given

Phase = -ve



$$G_c(s) = \frac{S+Z}{S+P}$$

poles

$P = \frac{Z}{\beta}$ zeroes

$\beta = \frac{Z}{P} > 1$

$$= \frac{S + Z}{S + \frac{Z}{\beta}}$$

we have to take $Z = 1/Z$ all the time

$$G_c(s) = \frac{S + \frac{1}{Z} \rightarrow \omega_{c1}}{S + \frac{1}{\beta Z} \rightarrow \omega_{c2}}$$

$\angle G_c(s) = -ve$, $\frac{P}{Z} < 1 \Rightarrow \frac{Z}{P} > 1$

$\phi = \tan^{-1}(Z) - \tan^{-1}(\beta Z)$

$S = j\omega$

Spiral

$$G_c(j\omega) = \frac{j\omega + \frac{1}{Z}}{j\omega + \frac{1}{\beta Z}}$$

$$= \frac{\beta(1 + j\omega Z)}{(1 + j\omega\beta Z)}$$

$$\boxed{G_c(j\omega) = \frac{1 + j\omega Z}{(1 + j\omega\beta Z)}} - A$$

Now

$$\left.\frac{d\phi}{d\omega}\right|_{\omega = \omega_m} = 0 \quad \text{max Comp}^n \text{ Cond}^n$$

$$\omega_m = \sqrt{\omega_{c1} \cdot \omega_{c2}} = \sqrt{\frac{1}{Z} \cdot \frac{1}{\beta Z}} = \frac{1}{Z\sqrt{\beta}}$$

$$\phi = \tan^{-1}(\omega Z) - \tan^{-1}(\omega\beta Z)$$

$$\omega = \omega_m = \frac{1}{Z\sqrt{\beta}}$$

$$\tan\phi = \left.\frac{\omega Z - \omega\beta Z}{1 + \omega^2 Z^2 \beta}\right|_{\omega = \omega_m = 1/Z\sqrt{\beta}}$$

$$\tan\phi = \frac{\frac{1}{\sqrt{\beta}} - \sqrt{\beta}}{1 + \frac{1}{\cancel{Z^2}\beta^2}\cancel{Z^2}\cancel{\beta}} = \frac{1 - \beta}{2\sqrt{\beta}}$$

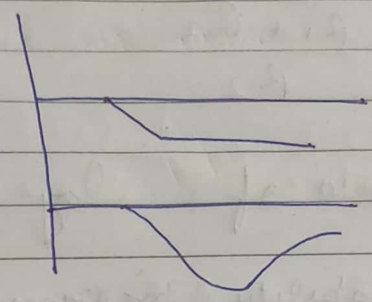
Dt.:
Pg.:

$$\phi = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)$$

$$\sin\phi = \frac{1-\beta}{1+\beta}$$

$$\beta = \frac{1-\sin\phi}{1+\sin\phi}$$

Bode Plot



→ Magnitude :

$$|G_c(j\omega)| = \left.\frac{\sqrt{1+\omega^2 Z^2}}{\sqrt{1+\omega^2\beta^2 Z^2}}\right|_{\omega=\omega_m=\frac{1}{Z\sqrt{\beta}}}$$

$$= \frac{\sqrt{1+\frac{1}{\cancel{Z^2}\beta}\times\cancel{Z^2}}}{\sqrt{1+\frac{1}{\cancel{Z^2}\beta}\times\beta^{\cancel{2}}\times\cancel{Z^2}}} = \frac{1}{\sqrt{\beta}}$$

→ Analyzing Electrical N/W

$$\frac{V_o}{V_i} = \frac{R_2+\frac{1}{SC}}{R_1+R_2+\frac{1}{SC}}$$

$$= \boxed{\frac{1+R_2SC}{S(R_1+R_2)C+1}} - A'$$

Comparing A & A'

telco Dt.:
Pg.:

$Z = R_2 C$

$$\beta = \frac{R_1 + R_2}{R_2}$$

→ Effects of lag- Compensator

1) Stability increases.
2) Band width increases.
3) Rise time increases.

4) Transient response become poor.
5) Steady state response increases.

→ It can eliminate noise entering at high freq.

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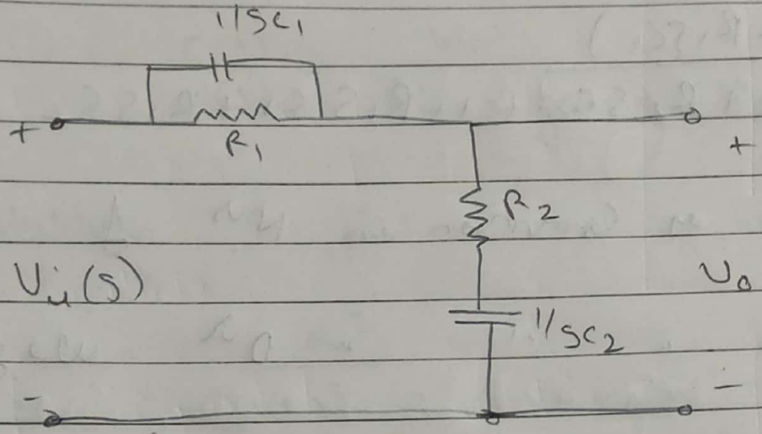
Dt.:
Pg.:

Lag- lead Compensator :

Electrical NW which produces phase lag at one freq region & phase lead at other freq region.



$$G_c(s) = \left(\frac{S + \frac{1}{Z_1}}{S + \frac{1}{\alpha Z_1}} \right) \left(\frac{S + \frac{1}{Z_2}}{S + \frac{1}{\beta Z_2}} \right)$$

Lead Section

$\alpha < 1$

Lag section

$\beta > 1$

$$= \alpha \left(\frac{1 + S Z_1}{1 + S \alpha Z_1} \right) \cdot \beta \left(\frac{1 + S Z_2}{1 + S \beta Z_2} \right)$$

$$G_c(s) = \frac{(1 + S Z_1)}{(1 + S \alpha Z_1)} \cdot \frac{(1 + S Z_2)}{(1 + S \beta Z_2)} \quad A$$

Analyzing electrical NW

$$\frac{V_o}{V_i} = \frac{R_2 + \frac{1}{SC_2}}{R_2 + \frac{1}{SC_2} + \frac{R_1}{1 + R_1 S C_1}}$$

Spiral

Dt.:
Pg.:

$$= \frac{R_2 S C_2 + 1}{R_2 S C_2 + 1 + \frac{R_1 S C_2}{1 + R_1 S C_1}}$$

$$= \frac{(R_2 S C_2 + 1)(1 + R_1 S C_1)}{R_1 R_2 S^2 C_1 C_2 + R_2 S C_2 + 1 + R_1 S C_1 + R_1 S C_2}$$

Taking $R_2 C_2$ & $R_1 C_1$ as common in Nr &

$R_1 R_2 \; C_1 C_2$ " " " Dr we get

$$= \left[\frac{\left(S + \frac{1}{R_2 C_2}\right)\left(S + \frac{1}{R_1 C_1}\right)}{S^2 + S\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}} \right] A'$$

Comparing A & A' : we get

$Z_1 = R_1 C_1$

$Z_2 = R_2 C_2$

$\alpha \beta = 1$

Spiral

Dt.:
Pg.:

Frequency Domain Specⁿ. (Unit - 3) Pg - 177 Book

Mag & Phase of 2nd order System

Freq Response is the steady state output of a System due to the sinusoidal I/P

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Substitute $s = j\omega$

$$= \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2}$$

$(j)^2 = -1$

$$= \frac{\omega_n^2}{-\omega^2 + 2j\omega_n\omega + \omega_n}$$

$$= \frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2}\left(1 - \frac{\omega^2}{\omega_n^2} + \frac{2j\zeta\omega}{\omega_n}\right)}$$

Let $\boxed{\frac{\omega}{\omega_n} = x}$

$$T(s) = T(j\omega) = \frac{1}{1 - x^2 + 2j\zeta x}$$

$$= \frac{1}{(1-x^2) + j2\zeta x}$$

Magnitude

$$|M(j\omega)| = |T(j\omega)| = \left| \frac{1}{(1-x)^2 + j2\zeta x} \right|$$

Spiral

telco Dt.: Pg.:

$$= \frac{1}{\sqrt{(1-x^2)^2 + (2 \zeta x)^2}}$$

Phase:

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{2\zeta x}{\sqrt{1-x^2}}\right)$$

→ F.D.S

Resonant Peak (M_r): Maxm value of magnitude

Mag of M_r gives Info" about rel stability of System

Large val of M_r = Undesirable Transient resp

Resonant freq (ω_r): Freq at which magnitude has max value.

If ω_r = large, Time resp is fast

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

Also

$$\boxed{x_r = \frac{\omega_r}{\omega_n}}$$

normalized resonant freq

Spiral

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Note:

Magnitude = $\frac{1}{\sqrt{(1-x^2)^2 + (2\zeta x)^2}}$

Resonant Peak, M_r

We know $x = \frac{\omega_r}{\omega_n}$

$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$ Resonant Peak

→ Bandwidth : (From zero freq value, $\omega = 0$)
↓
Band of freq^n lying btw −3 dB Points

→ Cut-off freq : Freq at which mag is 3 dB below its zero freq value is cut off freq (ω_b)

→ Cut-off rate : It is the slope of log mag curve near the cut off freq

telco Dt.: Pg.:

→ Classification of System based on Stability :

1) Absolutely Stable System (A.S.S)
2) Unstable System.
3) Conditionally Stable System.

a) A.S.S :

For all the variation of Parameter we get the finite output

2) U.S

For all the variation of Parameter we get infinite o/p.

3) C.S.S

For a Specific Period of Condition the system is Stable after that it get Unstable