* Servo Motors :

A servo motor is one in which output is some mechanical variable like position, velocity or acceleration. $

Servo system are generally automatic control systems. They work on error signals. Error signals are amplified to drive the motors used in servo systems.

Motors used in servo systems are called servo Motors. servo Motors are usually coupled to the output shaft. These motors are used to convert electrical signal applied, into angular velocity (or) movement of shaft.

* Features required in servo Motors :

1. Linear relationship between speed and electrical control system
2. servo motor operation should be stable without any oscillations (or) overshoots.
3. Inertia of rotar should be as low as possible.
4. Its response should be as fast as possible.
5. It should have linear torque-speed characteristics.
6. wide range of speed control is required.

* Types of servo Motors :

Servo motors are basically classified depending upon the nature of electric supply to be used for its operation.

* Advantages & Disadvantages of DC servo motors

- Servo motors
  - AC Servo motors
  - DC Servo motors
    - Sliding contact motors with commutators and Brushes
      - Permanent magnets
        - Cylindrical armature motors
        - Disk armature motors
        - Moving coil motors.
      - Electromagnetic field motors.
        - Armature controlled motors.
        - Field controlled motors
        - Series motor
        - Split field motors
    - Brushless (or) contactless motors with transistor commutators.
  - Special servomotors.

* Armature controlled DC servo motor :

It is a DC shunt motor designed to satisfy the requirement of servo motor. The field is excited by constant DC supply.

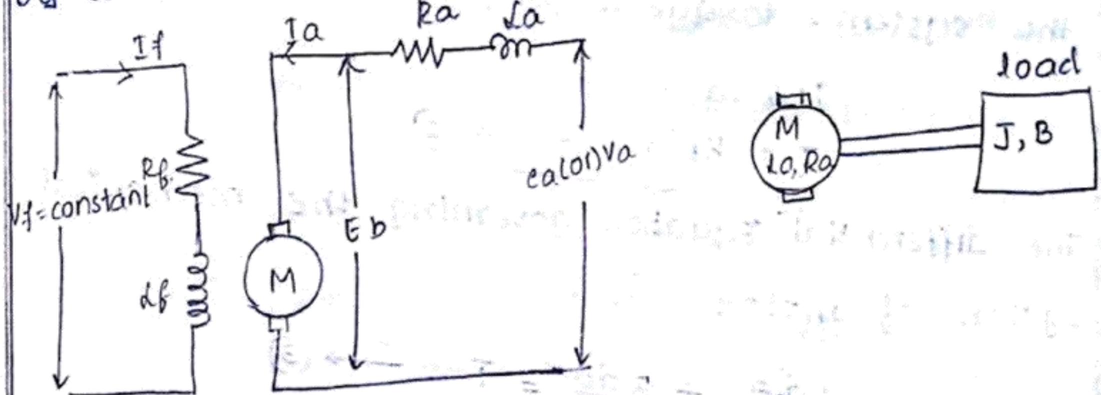
If the field current is constant, then the speed is directly proportional to Armature voltage and torque is directly proportional to Armature current.

In small motors, armature voltage is controlled by variable resistance. In large motors, in order to reduce power loss, armature voltage is controlled by Thyristors.

Transfer function of Armature controlled DC Motor:

In Armature controlled DC Motor, the desired speed is obtained by varying armature voltage.

The electrical system consists of armature and the field circuit. But for analysis purpose, only the armature circuit is considered because field is excited by a constant voltage.



Let R_a = Armature Resistance, Ω

L_a = Armature Inductance, H

I_a = Armature current, Amps

V_a = Armature voltage, V

E_b = Back emf, V

K_t = Torque constant, Newton-meter/amp

T = Torque developed by motor, N-m.

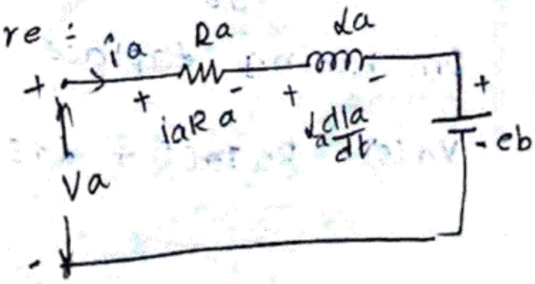
θ = Angular displacement of shaft, radians

J = Moment of Inertia of motor and load, Kg-m²/radians

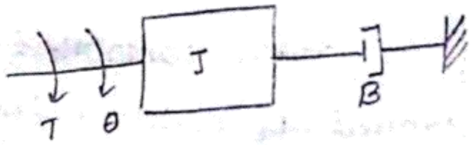
B = Frictional coefficient of motor and load, N-m/(rad/sec)

K_b = Back emf constant, V/(rad/sec)

Equivalent circuit of Armature:



Mechanical system of Motor :



By KVL , we can write

$$V_a = i_aR_a + L_a\frac{di_a}{dt} + e_b \longrightarrow ①$$

Torque of DC Motor is proportional to product of flux and current. since flux is constant in this system, torque is propositional to Ia alone.

$$T \propto i_a$$

$$\Rightarrow T = K_t i_a \longrightarrow ②$$

The differential equation governing the mechanical system of motor is ,

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \longrightarrow ③$$

Back emf of DC machine is proportional to speed of shaft i.e., angular velocity of shaft.

$$e_b \propto \frac{d\theta}{dt}$$

$$e_b = K_b\frac{d\theta}{dt} \longrightarrow ④$$

Differential Equations governing Armature control DC motor are

$$V_a = i_aR_a + L_a\frac{di_a}{dt} + e_b$$

$$T = K_t i_a$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T$$

$$e_b = K_b\frac{d\theta}{dt}$$

On taking Laplace Transform, we will get

$$V_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s) \longrightarrow ⑤$$

$$T(s) = K_t I_a(s) \longrightarrow (6)$$

$$Js^2\theta(s) + Bs\theta(s) = T(s) \longrightarrow (7)$$

$$E_b(s) = K_b s\theta(s) \longrightarrow (8)$$

Equating (6) and (7).

$$\Rightarrow K_t I_a(s) = Js^2\theta(s) + Bs\theta(s)$$

$$\Rightarrow I_a(s) = \left[\frac{Js^2 + Bs}{K_t}\right]\theta(s) \longrightarrow (9)$$

substitute (8) and (9) in (5)

$$\Rightarrow V_a(s) = (R_a + L_a s)\left[\frac{Js^2 + Bs}{K_t}\right]\theta(s) + K_b s\theta(s)$$

$$\Rightarrow V_a(s) = \theta(s)\left[\frac{(R_a + L_a s)(Js^2 + Bs) + K_t K_b s}{K_t}\right]$$

$$\Rightarrow \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + L_a s)(Js^2 + Bs) + K_t K_b s}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{R_a\left(1 + \frac{L_a s}{R_a}\right) Bs\left(1 + \frac{Js^2}{Bs}\right) + K_t K_b s}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{sR_aB\left[s\frac{L_a}{R_a} + 1\right]\left[1 + \frac{Js}{B}\right] + K_t K_b s}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_t}{sR_aB(s\tau_a + 1)(\tau_M s + 1) + K_t K_b s}$$

where $\tau_a = \frac{L_a}{R_a} \rightarrow$ Electrical Time constant

$\tau_M = J/B \rightarrow$ Mechanical Time constant

1. Construct block diagram of Armature controlled DC Motor

Sol: Differential eqns governing Armature controlled DC Motor are

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \rightarrow (1)$$

$$T = K_t i_a \rightarrow (2)$$

$$T = J\frac{d\omega}{dt} + B\omega \rightarrow (3)$$

$$e_b = K_b \omega \rightarrow (4)$$

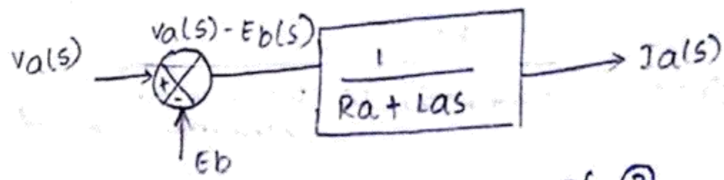
$$\omega = \frac{d\theta}{dt} \rightarrow (5)$$

On taking Laplace Transform of (1)

$$\Rightarrow V_a(s) = I_a(s) R_a + L_a s I_a(s) + E_b(s)$$

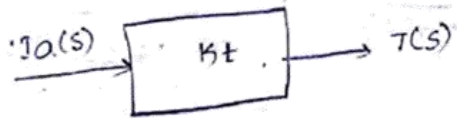
$$\Rightarrow V_a(s) = I_a(s)[R_a + L_a s] + E_b(s)$$

$$\Rightarrow I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + L_a s}$$



On taking Laplace Transform of (2)

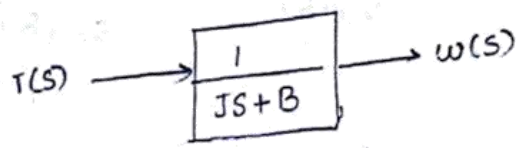
$$T(s) = K_t I_a(s)$$



On taking Laplace transform of (3)

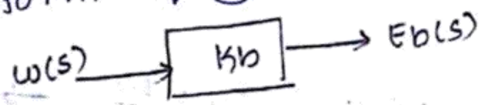
$$T(s) = Js\omega(s) + B\omega(s)$$

$$\Rightarrow \omega(s) = \frac{T(s)}{Js + B}$$



On taking Laplace transform of (4)

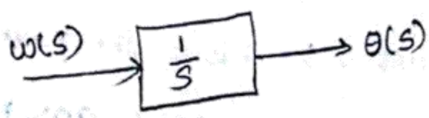
$$E_b(s) = K_b \omega(s)$$

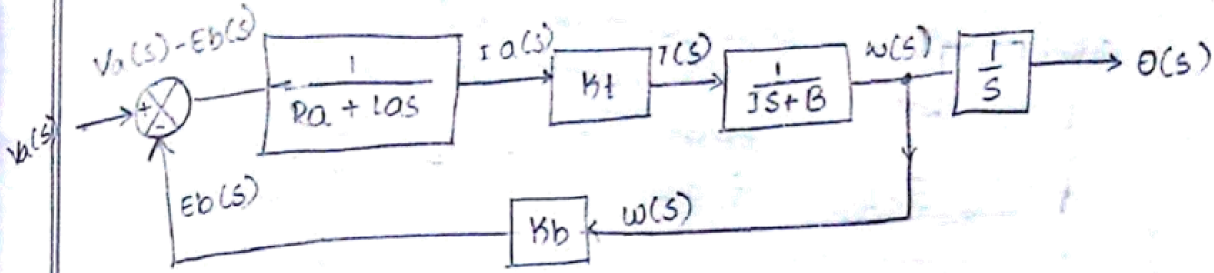


On taking Laplace transform of (5)

$$\Rightarrow \omega(s) = s\theta(s)$$

$$\Rightarrow \theta(s) = \frac{\omega(s)}{s}$$





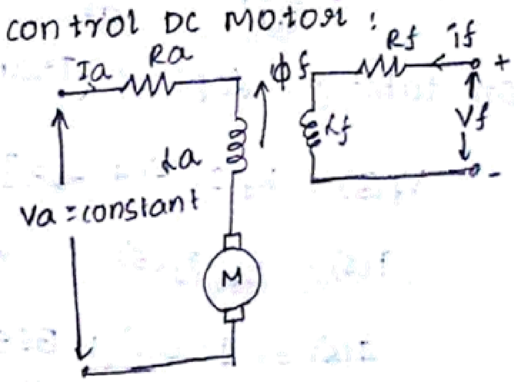
* Field control DC servo motor :

It is a DC shunt motor designed to satisfy the requirement of servo motor. In this the armature is supplied with a constant current or voltage.

When armature voltage is constant, the torque is directly proportional to field flux.

The torque of the motor is controlled by controlling field current.

* Transfer function of Field control DC Motor :



Let,

R_f = Field Resistance, Ω

L_f = Field Inductance, H

I_f = Field current, Amps

V_f = Field voltage, v

T = Torque developed by the motor, N-m

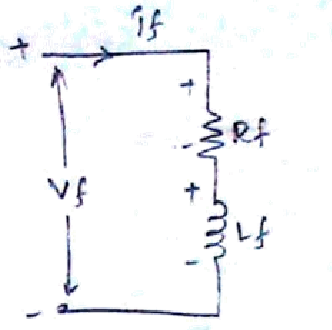
K_{tf} = Torque constant, N-m/Amp

K = Moment of Inertia of motor and load, kg-m²/rad

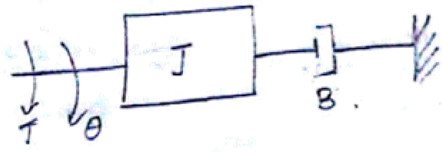
B = frictional coefficient of motor and load, $\frac{N\text{-}m}{rad/sec}$

The Electrical system consists of Armature and field circuit, but for analysis purpose only field circuit is considered because armature is excited by a constant voltage.

The mechanical system consists of the rotating part of motor and the load connected to shaft of the motor.



Equivalent circuit of field



Mechanical system of the motor

Apply KVL to the circuit

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \longrightarrow (1)$$

$$T = K_{tf} I_f \longrightarrow (2)$$

Equation of mechanical system

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \longrightarrow (3)$$

On taking, Laplace Transform we will get.

$$V_f(s) = R_f I_f(s) + L_f s I_f(s) \longrightarrow (4)$$

$$T(s) = K_{tf} I_f(s) \longrightarrow (5)$$

$$T(s) = Js^2\theta(s) + Bs\theta(s) \longrightarrow (6)$$

Equating (5) and (6)

$$K_{tf} I_f(s) = Js^2\theta(s) + Bs\,\theta(s).$$

$$I_f(s) = \theta(s)\left[\frac{Bs + Js^2}{K_{tf}}\right]$$

sub in (4)

$$\Rightarrow \quad V_f(s) = I_f(s)\left[R_f + L_f s\right].$$

$$V_f(s) = \theta(s)\left[\frac{Bs + Js^2}{K_{tf}}\right]\left[R_f + L_f s\right]$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{(Bs + Js^2)(R_f + L_f s)}$$

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{Bs\left[1+\frac{Js}{B}\right] R_f\left(1+\frac{L_f s}{R_f}\right)}$$

$$\Rightarrow \frac{\theta(s)}{V_f(s)} = \frac{K_{m}}{s(1+T_m s)(1+T_f s)}$$

where

$T_f = \frac{L_f}{R_f}$ = Electrical Time constant

$T_m = \frac{J}{B}$ = Mechanical Time constant

$K_m = \frac{K_{tf}}{R_f B}$ = motor gain constant.

1. Block Diagram of Field control DC Motor :

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \longrightarrow (1)$$

$$T = K_{tf} I_f \longrightarrow (2)$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \longrightarrow (3) \quad , \quad \omega = \frac{d\theta}{dt} \longrightarrow (4)$$

Taking Laplace transform of (1)

$$V_f(s) = R_f I_f(s) + L_f s I_f(s)$$

$$V_f(s) = I_f(s)[R_f + L_f s].$$

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s}$$

$$V_f(s) \longrightarrow \boxed{\frac{1}{R_f + L_f s}} \longrightarrow I_f(s)$$

Taking Laplace transform of (2)

$$T(s) = K_{tf} I_f(s)$$

$$I_f(s) \longrightarrow \boxed{K_{tf}} \longrightarrow T(s)$$

Taking Laplace transform of (3)

$$T = J\frac{d\omega}{dt} + B\omega$$

$$\Rightarrow T(s) = Js\,\omega(s) + B\omega(s)$$

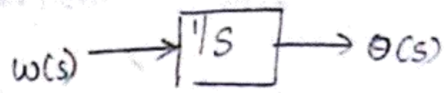
$$\Rightarrow T(s) = \omega(s)[Js + B].$$

$$\Rightarrow \omega(s) = \frac{T(s)}{Js + B}$$

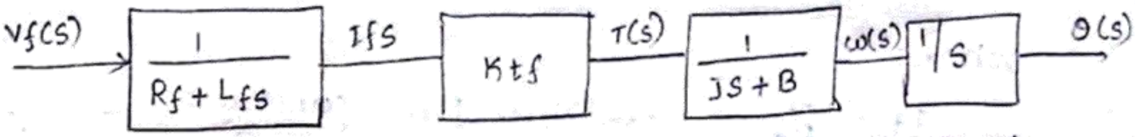
$$T(s) \longrightarrow \boxed{\frac{1}{Js + B}} \longrightarrow \omega(s)$$

Taking Laplace transform of 4

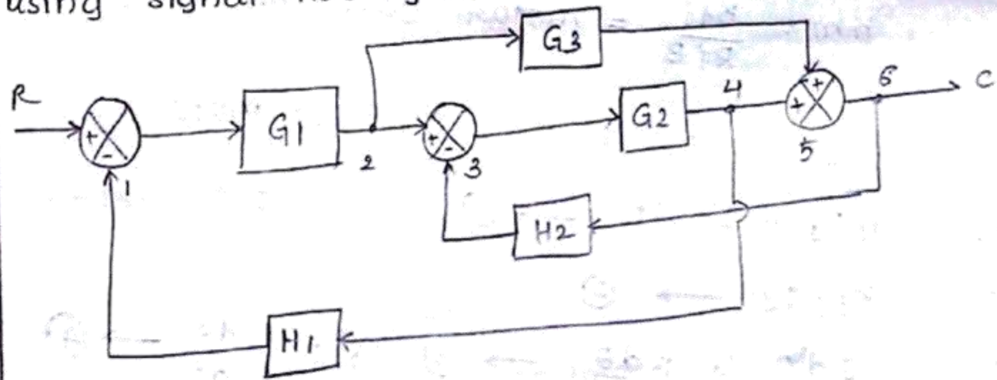
$$\omega(s) = s\,\theta(s)$$



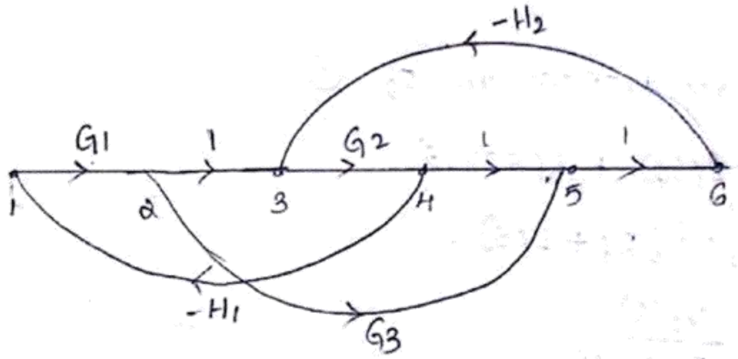
$$\Rightarrow \quad \theta(s) = \omega(s)/s$$



* Find transfer function for the following block diagram using signal flow graph method.



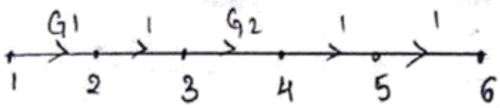
Sol:-

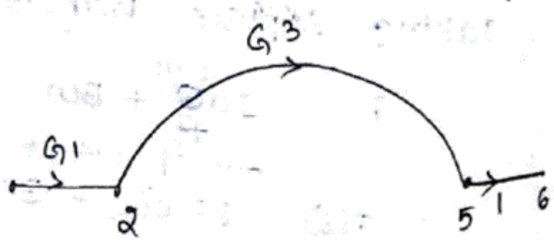


1. Finding forward path gains.

No. of forward paths = 2

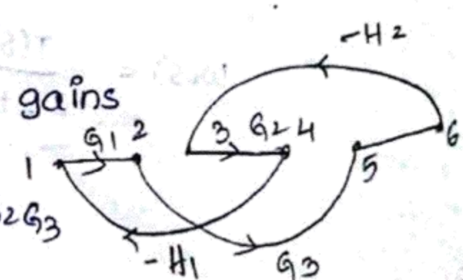
Let the forward paths be P_1 & P_2





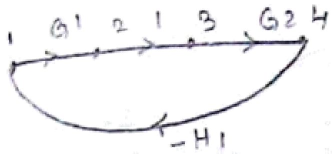
$P_1 = G_1G_2$ $\qquad$ $P_2 = G_1G_3$.

2. Finding individual loop gains

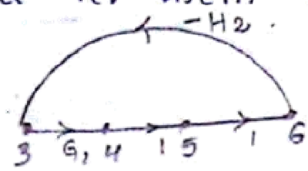


$P_{13} = H_1H_2G_1G_2G_3$

There are 2 loops and let them be P_{11} & P_{12}



$P_{11} = -H_1G_1G_2$



$P_{12} = -H_2G_2$

3. Finding gain product of 2 non-touching loops.

There are no non-touching loops.

4. Finding Δ and Δ_K.

$$\Delta = 1 - (P_{11} + P_{12}) - P_{13}$$

$$= 1 + H_1G_1G_2 + H_2G_2 - G_1G_2G_3H_1H_2.$$

$$\Delta_K = 1 - [0]$$

Since there is no part of graph non-touching two forward paths

$$\Rightarrow \Delta_1 = 1, \Delta_2 = 1.$$

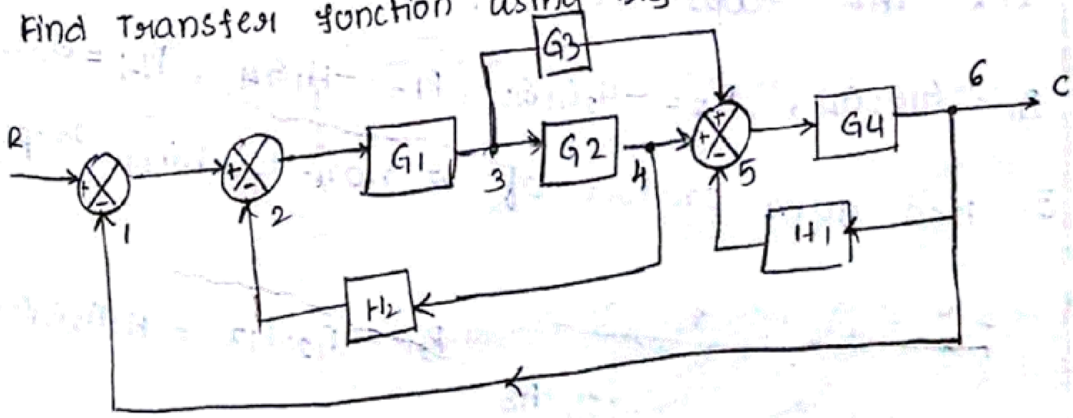
5. Find transfer function.

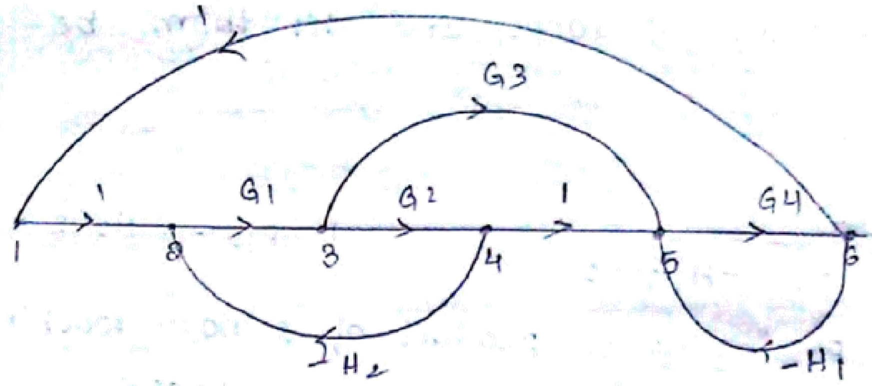
$$T = \frac{1}{\Delta} \sum P_K \Delta_K$$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$T = \frac{G_1G_2 + G_1G_3}{1 + H_1G_1G_2 + H_2G_2 - G_1G_2G_3H_1H_2}$$

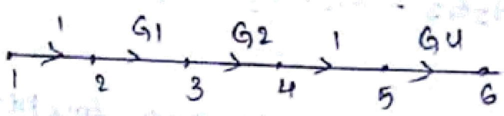
2 Find Transfer function using signal flow graph method



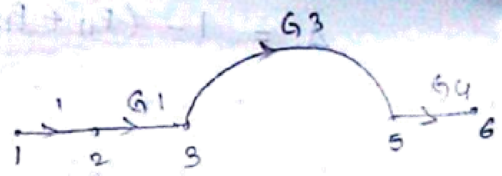


1. Find forward path gains.

No. of forward paths $K = 2$ & let them be P_{11} & P_{12}

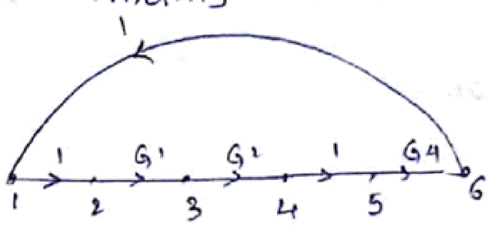


$P_{11} = G_1G_2G_4$

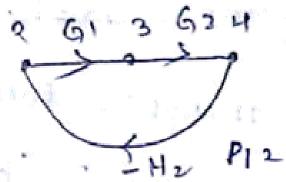


$P_{12} = G_1G_3G_4$.

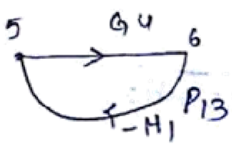
2. Finding individual loop gains.



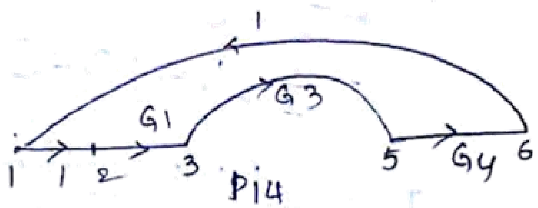
P_{11}



P_{12}



P_{13}

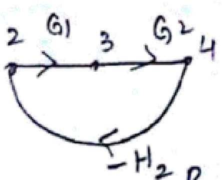


P_{14}

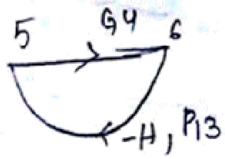
Let the loops be $P_{11}, P_{12}, P_{13}, P_{14}$.

$P_{11} = G_1G_2G_4$, $P_{12} = -H_2G_1G_2$, $P_{13} = -H_1G_4$, $P_{14} = G_1G_3G_4$

3. Find gain product of 2 non-touching loops.



P_{12}



P_{13}

$P_{21} = P_{12} \cdot P_{13} = H_1H_2G_1G_2G_4$.

4. Finding Δ & Δ_K.

$$\Delta = 1 - (P_{11} + P_{12} + P_{13} + P_{14}) + (P_{21})$$

$$= 1 - (G_1G_2G_4 - H_2G_1G_2 + H_1G_4 + G_1G_3G_4) + H_1H_2G_1G_2G_4.$$

$$= 1 - G_1G_2G_4 + H_2G_1G_2 + H_1G_4 - G_1G_3G_4 + H_1H_2G_1G_2G_4.$$

$$\therefore \Delta_1 = 1 - [0] = 1$$

$$\Delta_2 = 1 - [0] = 1$$

5 Δ. Find Transfer function

$$T = \frac{1}{\Delta}[\Sigma P_K \Delta_K]$$

$$T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$T = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_2G_4 + H_2G_1G_2 + H_1G_4 - G_1G_3G_4 + H_1H_2G_1G_2G_4}$$