Putting $n=2$ in quadrature formula and taking curve through $(x_0,y_0),(x_1,y_1)$ and (x_2,y_2) as Parabola i.e a Polynomial of second order so that difference of order higher than second vanish.

Simpson's One-Third Rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\Big[(y_0+y_n)+4(y_1+y_3+y_5+y_7+\cdots+y_{n-1}) + 2(y_2+y_4+y_6+y_8+\cdots+y_{n-2})\Big]$$

Which is known as Simpson's one-third rule.

Note: While using Simpson one-third formula, the given interval of integration must be divided into an even number of sub-intervals.

Simpson's Three-Eight Rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\Big[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\cdots+y_{n-2}+y_{n-1}) + 2(y_3+y_6+y_9+\cdots+y_{n-3})\Big]$$

Which is known as Simpson Three-Eight Rule

Note: While using Simpson's three-eight formula, the given interval of integration must be divided into sub intervals whose number n is a multiple of 3.

Note: Putting $n=3$ in quadrature formula and taking the curve through as Polynomial of third order so differences above the third order vanish.

Q. The table below gives the velocity v of a speeding car at time t seconds. Approximate the distance travelled by car in 12 seconds.

t (in sec)	0	2	4	6	8	10	12
v (in m/sec)	4	6	16	34	60	94	136

Solⁿ We know that velocity $v = \frac{ds}{dt}$

$\therefore \; ds = v\,dt$

$s = \int v\,dt$, here $h = 2$.

Total distance s travelled in 12 seconds using Simpson's one third rule is

$$s = \int_0^{12} v\,dt$$

$$= \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots) + 2(y_2+y_4+\cdots)\right]$$

$$= \frac{2}{3}\left[(4+136) + 4(6+34+94) + 2(16+60)\right]$$

$$= \frac{2}{3}\left[140+536+152\right] = 552 \text{ Metres.}$$

Q.2 Calculate $\int_1^2 \frac{dx}{x}$,

(i) Simpson's rule with $n=2$ (ii) Simpson's Rule with $n=4$

(i) $n=2$

$h = \frac{2-1}{2} = \frac{1}{2} = .5$

x :	1	1.5	2
$f(x) = y$:	1	.666	.5
	y_0	y_1	y_2

According to Simpson one third Rule

$$S = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots) + 2(y_2+y_4+\cdots)\right]$$

$$= \frac{.5}{3}\left[(1+.5) + 4(\overset{.666}{\cancel{\quad}}) + 0\right] = .694$$

(ii) $\int_1^2 \frac{dx}{x}$, Simpson Rule with $h=4$

Here $h = \frac{2-1}{4} = \frac{1}{4}$

x	1	5/4	6/4	7/4	8/4
y	1	$\frac{4}{5} = .8$	$\frac{4}{6} = .666$	$\frac{4}{7} = .5714$	$\frac{4}{8}$ .5
	y_0	y_1	y_2	y_3	y_n

$$\int_1^2 \frac{dx}{x} = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots)\right]$$

$$= \frac{1}{4\times3}\left[(1+.5) + 4(.8+.5714) + 2(.666)\right] = \underline{\underline{.693}} \text{ Ans}$$

Q. A river is 80 ~~meter~~ metres wide. The depth 'd' in metres at a distance x metres from one bank is given by the following table. Calculates the area of cross section of the river, using Simpson's One-third Rule

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_n

Here $h = 10$

By simpson's one Third Rule

$$\text{Area of cross-section} = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{10}{3}\left[(0+3) + 4(4+9+15+8) + 2(7+12+\cancel{14})\right]$$

$$= \frac{10}{3}\left[3+144+66\right] = \underline{\underline{710}}$$

Q. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$, using simpson's ~~Three Eight~~ one Third rule

taking $h = \frac{1}{6}$

Sol^n

x	0	$1/6$	$\frac{2}{6}$	$3/6$
y	1	$\frac{36}{37}$	$\frac{36}{45}$	

Q. Evaluate $\int_0^1 \frac{dx}{1+x^2}$, by using simpson's three Eight Rule

taking $h = \frac{1}{6}$

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$3/6$	$4/6$	$5/6$	$6/6=1$
y	1	$\frac{36}{37}$	$\frac{36}{40}$	$\frac{36}{45}$	$\frac{36}{52}$	$\frac{36}{61}$	$\frac{36}{72}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_n

By Simpson Three Eight Rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\cdots) + 2(y_3+y_6+\cdots)\right]$$

$$= \frac{3\times\frac{1}{6}}{8}\left[1+\frac{36}{72} + 3\left(\frac{36}{37}+\frac{36}{40}+\frac{36}{52}+\frac{36}{61}\right) + 2\left(\frac{36}{45}\right)\right]$$

$$= \frac{3}{48}\left[1.5 + 3(.9729+.9+.6923+.5901) + 1.6\right] = \underline{\underline{.7853\ \text{Ans}}}$$

Q. Approximate $I = \int_0^6 \frac{dx}{1+x^2}$

(i) Simpson's $\frac{1}{3}$ Rule

(ii) Simpson's $\frac{3}{8}$ Rule

And compare the result with the exact value of Integral.

Sol^n

(i) Simpson $\frac{1}{3}$ Rule

Divide the Interval (0, 6) into six equal Part

$h = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
$y=f(x)$	1	.5	.2	.1	.0588	.03846	.0270
	y_0	y_1	y_2	y_3	y_4	y_5	y_n

By Simpson $\frac{1}{3}$ Rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots) + 2(y_2+y_4+\cdots)\right]$$

$$= \frac{1}{3}\left[(1+.0270) + 4(.5+.1+.03846) + 2(.2+.0588)\right]$$

$$= 1.3661 \quad \text{Ans}$$

By Simpson 3/8 Rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\cdots) + 2(y_3+y_6+\cdots)\right]$$

$$= \frac{3}{8}\left[(1+.0270) + 3(.5+.2+.0588+.03846) + 2(.1)\right]$$

$$= 1.3570 \quad \text{Ans}$$

By Rules of Integration, the exact value is

$$I = \int_0^6 \frac{dx}{1+x^2}$$

$$= \left[\tan^{-1} x\right]_0^6$$

$$= \tan^{-1} 6 - \tan^{-1} 0$$

$$= \tan^{-1} 6 - 0 = \tan^{-1} 6 = \underline{\underline{1.4056}}$$

Q. Evaluate $\int_0^1 e^{-x^2}\,dx$, using Simpson one-third rule.

Sol^n

Divide the Interval (0,1) into 10 Parts each of width.

$$h = \frac{1-0}{10} = .1$$

x	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	10
x^2	0	.01	.04	.09	.16	.25	.36	.49	.64	.81	1
e^{-x^2}	1	.9900	.9608	.9139	.8521	.7788	.6977	.6126	.5273	.4448	.3679
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_n

By Simpson's one third rule.

$$\int_0^1 e^{-x^2}\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{.1}{3}\left[1+.3679+4(.9900+.9139+.7788+.6126+.4448) + 2(.9608+.8521+.6977+.5273)\right]$$

$$= \underline{\underline{.7468}} \text{ Ans}$$

Q. A curve passes through even points (1, 2), (1.5, 2.4), (2.0, 2.7), (2.5, 2.8), (3, 3), (3.5, 2.6), and (4, 2.1), obtain the area bounded by the curve, the x-axis and the ordinates at x = 1 and x = 4. Also find the volume of solid of revolution obtained by revolving this area about the axis of x.

Solⁿ

Given

x	1	1.5	2.0	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Here $h = .5$

$$\text{Area} = \int_a^b y\,dx$$

$$= \int_1^4 y\,dx$$

$$= \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots) + 2(y_2+y_4+y\right]$$

$$= \frac{.5}{3}\left[(2+2.1)+4(2.4+2.8+2.6)+2(2.7+3)\right]$$

$$= \frac{.5}{3}\left[4.1+31.2+11.4\right]$$

$$= 7.783 \text{ sq.units} \quad \text{Ans}$$

Now

$$\text{volume} = \int_1^4 \pi y^2\,dx$$

$$= \pi\int_1^4 y^2\,dx \quad (\text{using } \tfrac{1}{3} \text{ Rule})$$

$$= \pi \times \frac{h}{3}\left[(y_0^2+y_n^2) + 4(2.4^2+2.8^2+2.6^2) + 2(2.7^2+3^2)\right]$$

$$= \frac{\pi}{3}\times .5\left[(4+4.41)+4(5.76+7.84+6.76)+2(7.29+9)\right]$$

$$= 64.13 \text{ cubic units}$$

Q. A Train is moving at the speed of 30 m/sec. Suddenly breaks are applied. The speed of train per second after t seconds is given by

Time (t) :	0	5	10	15	20	25	30	35	40	45
Speed (v) :	30	24	19	16	13	11	10	8	7	5

Apply Simpson's three Eight rule to determine the distance moved by train in 45 seconds.

Solⁿ. If s metres is the distance covered in t seconds, Thus

$$v = \frac{ds}{dt}$$

$$\therefore \frac{ds}{dt} = v$$

on Integrating under Limit t=0 to 45

$$[s]_0^{45} = \int_0^{45} v\, dt$$

Since the number of sub interval is 9 (a multiple of 3), hence we use Simpson Three Eight rule

$$\therefore \int_0^{45} v\, dt = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8) + 2(y_3 + y_6 +) \right.$$

$$= \frac{3 \times 5}{8}\left[(30+5) + 3(24+19+13+11+8+7) + 2(16+10)\right]$$

$$= \frac{15}{8}\left[35 + 246 + 52\right] = 624.375 \text{ metres}$$

Q. Estimate the length of the arc of the curve $3y = x^3$ from $(0,0)$ to $(1, 1/3)$ using Simpson's 1/3 Rule by taking $h = .125$

Sol Given curve $3y = x^3$

$$\cancel{3}\frac{dy}{dx} = \cancel{3}x^2 \Rightarrow \frac{dy}{dx} = x^2$$

$\therefore$ Length $s = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+x^4}\,dx = \int_0^1 f(x)\,dx$

Where $f(x) = \sqrt{1+x^4}$

x	0	.125	.250	.375	.5	.625	.75	.875	1
$f(x)$	1	1.0001	1.001	1.0098	1.0307	1.0735	1.1473	1.2594	1.4142
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_7

By Simpson one third rule

$$s = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+y_7+) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{.125}{3}\left[1+1.4142+4(1.0001+1.0098+1.0735+1.2594) + 2(1.001+1.0307+1.1473)\right]$$

$S. = 1.089$ Ans

x :

y : 1 ~~1.0221~~ 1.01049

Q. Evaluate $\int_0^1 \frac{dx}{1+x}$ by dividing the interval of integration into 8 equal parts. Hence find $\log_e 2$ approximately.

Solⁿ Here $h = \frac{1-0}{8} = \frac{1}{8}$

Here $y = \frac{1}{1+x}$

x :	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	1
y :	1	$8/9$	$\frac{4}{5}$	$\frac{8}{11}$	$\frac{2}{3}$	$\frac{8}{13}$	$\frac{4}{7}$	$\frac{8}{15}$	$\frac{1}{2}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_n

Here Sub Interval = 8 ·(Even)

i.e ~~divided into~~.

So We Use Simpson's $\frac{1}{3}$ Rule

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+y_6+y_8+\cdots) + 4(y_1+y_3+y_5+\cdots)\right]$$

$$\int_0^1 \frac{dx}{1+x} = \frac{1}{8\cdot 3}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{4}{5}+\frac{2}{3}+\frac{4}{7}\right) + 4\left(\frac{8}{9}+\frac{8}{11}+\frac{8}{13}+\frac{8}{15}\right)\right]$$

$$= \cancel{\tfrac{1}{24}}\ [\cdot 693154] \text{ Ans}$$

Again $\int_0^1 \frac{dx}{1+x} = \left[\log_e(1+x)\right]_0^1$

$= \log(2) - \log 1 = \log_e 2 = \cdot 6931147$ Ans

Q Compute $I = \int_0^{1/2} \frac{x}{\sin x}\,dx$, using Simpson's $\frac{1}{3}$ Rule

Let $h = \frac{1}{4}$

x :	0	$1/4$	$\frac{1}{2}$
y :	1	~~1.04291~~ 1.01049	1.04291

By simpson $\frac{1}{3}$ Rule

$$\int_0^{1/2} \frac{x}{\sin x}\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+\cdots)\right]$$

$$= \frac{1}{12}\left[(1+1.04291)+\cancel{2(0)}+4(1.01049)\right]$$

$$= .5070725$$

Ans

Again Case-2

If we take $h = 1/8$

x :	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$
y :	1	1.00261	1.01049	1.02383	1.04291
	y_0	y_1	y_2	y_3	y_n

By simpson 1/3 Rule

$$\int_0^{1/2} \frac{x}{\sin x}\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+\cdots)\right]$$

$$= \frac{1}{24}\left[(1+1.04291)+2(1.01049)+4(1.00261+1.02383)\right]$$

$$= .5070688 \text{ Ans}$$

Q. Compute $\int_0^{\pi/2} \sin x\, dx$, Using Simpson's three Eight rule of numerical Integration.

To Apply Simpson three Eight rule, the Interval of Integration $(0, \frac{\pi}{2})$ must be divided into a (multiple of 3).

Let us divide $(0, \frac{\pi}{2})$ into 9 sub intervals each of width $\frac{\pi}{18}$.

x :	0	$\pi/18$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$	$\frac{7\pi}{18}$	$\frac{8\pi}{18}$
y :	0	.1736	.3420	.5000	.6428	.7660	.8660	.9397	.9848
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

x :	$9\pi/18$
y :	1
	y_n

By Simpson 3/8 Formula

$$\int_0^{\pi/2} \sin x\, dx = \frac{3h}{8}\left[(y_0+y_n) + 2(y_3+y_6+\cdots) + 3(y_1+y_2+y_4+y_5+y_7+y_8\cdots)\right]$$

$$= \frac{3}{8}\cdot\frac{\pi}{18}\left[(0+1) + 2(.5+.8660) + 3(.3420+.6428+.7660+.9397 + .9848)\right]$$

$$= .999988 \simeq 1 \text{ Ans}$$