END TERM EXAMINATION

SECOND SEMESTER [B.TECH] JULY 2023

Paper Code: BS-112	**Subject: Applied Mathematics-II**
Time: 3 Hours	**Maximum Marks: 75**

Note: Attempt five questions in all including Q.No.1 which is compulsory. Select one question from each unit.

Q1. (a) Find all the point at which the following mapping is not Conformal $w = \frac{z+\frac{1}{2}}{4z^2+2}$. (2.5)

(b) Split the real and imaginary part of i^i. (2.5)

(c) Find the Laplace transform of t^2e^{-2t}. (2.5)

(d) Using half range sine series of function $f(x) = 1$ for $0 < x < \pi$, prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2.5)

(e) Taylor series expansion of $\frac{1}{z-2}$ in $|z| < 1$ is……. (2.5)

(f) Classify the type of PDE: $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} = 0$, whether it is parabolic, ellipticor hyperbolic? (2.5)

UNIT-I

Q 2. (a) Find all the values of $(-1+\sqrt{3}i)^{\frac{3}{2}}$. (7)

(b) Verify that the function $u(x,y) = x^3y - xy^3$ is harmonic and find the harmonic conjugate of $u(x,y)$ to express the function $f(z) = u + iv$ as an analytic function. (8)

Q 3. (a) Evaluate the integral $\oint_C \frac{e^{2z}}{(z-1)(z-2)}dz$ along the curve C, where C is a circle $|z| = 3$. (7)

(b) Integrate the function $f(z) = \bar{z}$ along the curve C, where C is the square with vertices $z = 0, 2, 2i, 2+2i$ (8)

UNIT-II

Q 4. (a) Find the bilinear transformation or Mobious transformation which maps $1, i, 1$ of the z – plane onto $1, i, -1$ of the w – plane respectively. Also find the fixed points or invariant points. (8)

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(b) *Find the singular points, type of singularities and the corresponding residues of the function* $f(z) = \frac{1}{(z^2-1)^2}$. **(7)**

Q 5.(a) *Sketch and graph the given region:* $|z| \le \frac{1}{2}, -\frac{\pi}{8} < Arg(z) < \frac{\pi}{8}$,

and its image under the given mapping: $w = z^2$. **(7.5)**

(b) *Prove that the integral* $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx = \frac{\pi}{3}$. **(7.5)**

UNIT-III

Q 6. (a) *Using Laplace transform solve the ordinary differential equation*

$y'''(t) + 2y''(t) - y'(t) - 2y = 0$, *with conditions* $y(0) = y'(0) = 0$, *and* $y''(0) = 6$. **(8)**

(b) *Find the inverse Laplace transform of* $\log\left(\frac{s+1}{s-1}\right)$. **(7)**

Q 7.(a) *Find the Fourier series to represent the function* $f(x) = x^2$ *in the interval* $(-\pi, \pi)$. **(7.5)**

(b) *Find the Fourier transform of* $f(x) = \begin{cases} 1 & for \quad |x| < 1 \\ 0 & for \quad |x| > 1 \end{cases}$

Hence evaluate $\int_0^{\infty} \frac{\sin x}{x}$. **(7.5)**

UNIT-IV

Q 8. (a) *Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero, assuming that the initial temprature is*

$$f(x) = \begin{cases} x & for \quad 0 < x < \frac{L}{2} \\ L - x & for \quad \frac{L}{2} < x < L \end{cases}$$ **(8)**

(b) *A tightly stretched string with fixed end points* $x = 0$ *and* $x = l$, *is initially in a position given by* $y = \sin^3\left(\frac{\pi x}{l}\right)$. *If it is released from rest from this position, find the displacement* $y(x,t)$. **(7)**

Q 9. (a) **Solve the Laplace equation** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, *subject to the conditions*

$u(0,y) = u(l,y) = u(x,0) = 0$ *and* $u(x,a) = \sin\frac{n\pi x}{l}$. **(8)**

(b) *Solve the equation* $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, *with boundary condition s* $u(x,0) = 3\sin(n\pi x)$,

$u(0,t) = 0$ *and* $u(1,t) = 0$, *where* $0 < x < 1, t > 0$. **(7)**

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