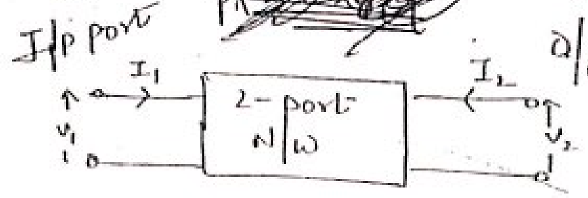
New Notes

CBS III [redacted]

Two-port N/W

[redacted]



[redacted]

If I entering 1 terminal of a pair is equal & opposite to the I leaving other terminal of the pair ~~then~~ this type of terminal pair is called as a "port".

2-port N/W - represented by 4 variables.
2 voltages (V_1, V_2) & 2 Current (I_1, I_2)

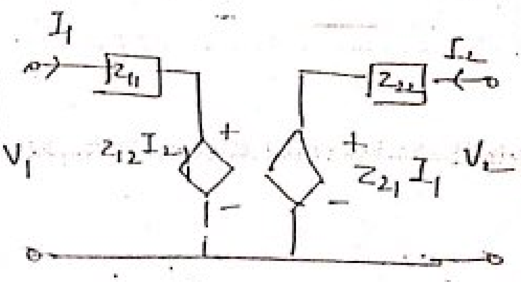
Relationship of 2 port Variables:

no. of possible combi generated by 4 variables taken 2 at a time = 6. so total 6 combi are there with which we can analyse any type of 2-port N/W.

(1) open circuit Impedance parameters or Z-para.

$(V_1\ V_2) = f(I_1\ I_2)$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$V_1 = Z_{11}I_1 + Z_{12}I_2$
$V_2 = Z_{21}I_1 + Z_{22}I_2$

$I_2 = 0$

$V_1 = V_1$

$I_1 = ?$ $V_2 = ?$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ i/p driving point impedance (2)

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ frwd transfer impedance

$I_1 = 0$

$V_2 = V_2$ $V_1 = ?$

$I_2 = ?$ $I_1 = 0$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ reverse transfer impedance

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$ o/p driving pt imp.

(2) <u>Short ckt Admittance or Y-par.</u>

$(I_1, I_2) = f(V_1, V_2)$

$[I] = [Y][V]$

$$\boxed{Y = Z^{-1}}$$

$$\therefore Y_{ij} \neq \frac{1}{Z_{ij}}$$

③ Transmission (T) or chain or ABCD para. ②

$$(V_1, I_1) = f(V_2, -I_2)$$

3

T-para is used in the analysis of transmission line. O/p port → receiving end
& I/p port → sending end.

2

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

④ Inverse T parameters

$$(V_2\ I_2) = f(V_1\ -I_1).$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}.$$

⑤ Hybrid parameters : (h-para.

widely used in electronic ckts, esp in constructing models for transistors.

$$-(V_1, I_2) = f(I_1\ V_2).$$

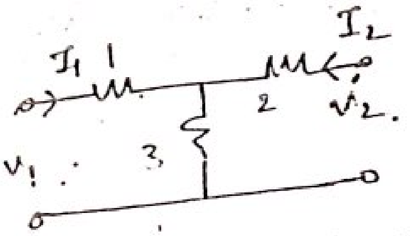
⑥ Inverse hybrid / g-para:

$$[g] = [h^{-1}]$$

$$(I_1\ V_2) = f(V_1\ I_2)$$

Calc. (1) Z, Y, T, T' -parameters.

Q 1



loop eq: $V_1 = 1I_1 + 3(I_1+I_2) = 4I_1 + 3I_2$

$V_2 = 2I_2 + 3(I_1+I_2) = 3I_1 + 5I_2$

(1) Z-para $(V_1, V_2) = f(I_1\ I_2)$

(a) $I_2 = 0$

$V_1 = 4I_1$

$V_2 = 3I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{V_1}{\frac{V_1}{4}} = 4\Omega$

$Z_{21} = \frac{V_2}{I_1} = 3$

(b) $I_1 = 0$

$V_1 = 3I_2$ & $V_2 = 5I_2$.

$Z_{12} = 3$ & $Z_{22} = 5$

(2) Y-par $(I_1, I_2) = f(V_1\ V_2)$.

a) $V_2 = 0$.

$3I_1 = -5I_2$.

$V_1 = 4I_1 + 3I_2$.

$V_1 = 4I_1 + 3\cdot\left(-\frac{3}{5}\right)I_1$

$V_1 = \left(\frac{20-9}{5}\right)I_1$

$V_1 = \frac{11}{5}I_1$

$\therefore Y_{11} = \frac{5}{11}\mho$

$V_1 = 4\left(\frac{-5}{3}\right)I_2 + 3I_2$

$V_1 = \frac{(9-20)}{3}I_2$

$V_1 = \frac{-11}{3}I_2$

$\therefore Y_{21} = \frac{-3}{11}\mho$

(3)

$\underline{V_1 = 0}$

$Y_{12} = -\frac{3}{11}\,\mho$ $\quad Y_{22} = \frac{4}{11}\,\mho$.

3

(3) <u>T-parameters</u>:

$(V_1, I_1) = f(V_2, -I_2)$

(a) $I_2 = 0$

$V_1 = 4 I_1$

$V_2 = 3 I_1$

$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = \frac{4}{3}$

$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} = \frac{1}{3}\,\mho$

(b) $V_2 = 0$.

$3 I_1 = -5 I_2 \qquad V_1 = 4\left(\frac{-5}{3} I_2\right) + 3 I_2 = \frac{-11}{3} I_2$

$B = \frac{V_1}{-I_2} = \frac{11}{3}\,\Omega$

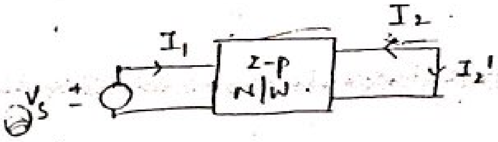
$D = \frac{I_1}{-I_2} = \frac{5}{3}$.

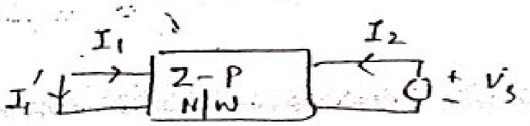
Condition for Reciprocity :-

2-port N/W is reciprocal, if the ratio of excitation to response is invariant to an interchange of their positions. 6

N/W having R, L, C → reciprocal

" " dependent sources → non-reciprocal.





$V_1 = V_s$, $I_1 = I_1$,
$V_2 = 0$ $\quad I_2 = I_2'$

$V_1 = 0$ $\quad I_1 = I_1'$
$V_2 = V_s$ $\quad I_2 = I_2$.

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \quad \text{or} \quad I_2' = I_1'$$

(1) In terms of Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (a)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (b)$$

$V_1 = V_s$, $\quad I_1 = I_1$
$V_2 = 0$ $\quad I_2 = -I_2'$

$$V_s = Z_{11} I_1 - Z_{12} I_2' \quad -(1)$$
$$0 = Z_{21} I_1 - Z_{22} I_2' \quad -(2)$$

from (2) $\quad I_1 = \dfrac{Z_{22} I_2'}{Z_{21}}$

$$V_s = \left(\frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12} \right) I_2'$$

$$I_2' = \frac{V_s Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}} \quad -③$$

||ly on putting in a & b.

$V_2 = V_s \quad I_2 = I_2$

$V_1 = 0 \quad I_1 = -I_1'$

we get

$$I_1' = \frac{V_s Z_{12}}{Z_{11} Z_{22} - Z_{12} Z_{21}} \quad -④$$

On comparing ③ & ④

$$\boxed{Z_{21} = Z_{12}}$$

2) In terms of Y-parameters

$Y_{21} = Y_{12}$.

3) T- parameters

$V_1 = A V_2 + B(-I_2)$

$I_1 = C V_2 + D(-I_2)$.

$V_1 = V_s \quad I_1 = I_1$

$V_2 = 0 \quad I_2 = -I_2'$

$V_s = B I_2' \Rightarrow I_2' = \frac{V_s}{B} \quad -①$.

$I_1 = D I_2'$

$V_2 = V_s \quad I_2' = I_2$

$V_1 = 0 \quad I_1 = -I_1'$

$0 = AV_s + B(-I_2) \Rightarrow I_2 = \frac{A V_s}{B}$

$-I_1' = CV_s + B(-I_2)$

$-I_1' = CV_s - \frac{DA}{B}V_s$

$I_1' = \frac{(-BC + DA)V_s}{B}$ — (2)

Comparing (1) & (2)

$$\boxed{\begin{array}{l} AD - BC = 1 \\ \text{or } \Delta T = 1 \end{array}}$$

(4) In terms of T' parameters.

$A'D' - B'C' = 1 \quad \text{or} \quad \Delta T' = 1$

(5) h-parameters

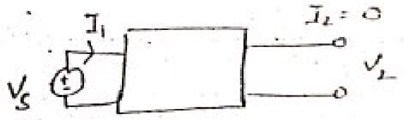
$h_{12} = -h_{21}$

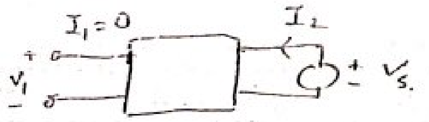
6) g-parameters

$g_{12} = -g_{21}$

Condition for Symmetry :-

A 2-port N/W is symmetrical if the ports can be interchanged without changing the port voltages & currents.





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$V_1 = V_s \quad V_2 = V_2$

$I_1 = I_1 \quad I_2 = 0$

$V_1 = V_1 \quad I_1 = 0$

$V_2 = V_s \quad I_2 = I_2$

$$\boxed{\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}}$$

(1) In terms of Z-parameters :

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

~~$V_1 = V_s$~~

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = Z_{11}$$

$$\left.\frac{V_s}{I_2}\right|_{I_1=0} = Z_{22}$$

(2) Y-para.

$Y_{11} = Y_{22}$

(3) T-parameters

$A = D$

T' - parameters

$A' = D'$ (1)

(5) h - para

$h_{11} h_{22} - h_{12} h_{21} = 1$

or $\Delta h = 1$

6) g - para

$\Delta g = 1$ or $g_{11} g_{22} - g_{12} g_{21} = 1$

Relationships b/w parameter Sets:

① Z - parameters in terms of other

(a) Z in terms of Y - par.

$[I] = [Y][V]$

$[V] = [Z][I]$

$\therefore [Z] = [Y]^{-1}$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{11} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$Z_{11} = \frac{Y_{22}}{\Delta Y}$ $Z_{12} = \frac{-Y_{11}}{\Delta Y}$ $Z_{21} = \frac{-Y_{21}}{\Delta Y}$

$Z_{22} = \underline{Y_{11}}$

(b) Z in terms of T-para.

$$V_1 = A V_2 + B(-I_2) \quad -①$$
$$I_1 = C V_2 + D(-I_2) \quad -②$$
$$\Downarrow$$
$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

$$\boxed{Z_{21} = \frac{1}{C} \; ; \; Z_{22} = \frac{D}{C}}$$

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from ①

$$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2$$

$$\boxed{Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C}}$$

c) in T'-para.

$$Z_{21} = \frac{1}{C'} \, , \; Z_{22} = \frac{D'}{C'}$$

$$Z_{11} = \frac{A'}{C'} \; \& \; Z_{12} = \frac{\Delta T'}{C'}$$

Scanned with CamScanner

h- parameters :

$$V_1 = h_{11} I_1 + h_{12} V_2 .$$

$$I_2 = h_{21} I_1 + h_{22} V_2 .$$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} . I_1$$

$$\Rightarrow Z_{21} = \frac{-h_{21}}{h_{22}} \quad \& \quad Z_{22} = \frac{1}{h_{22}} .$$

$$V_1 = h_{11} I_1 + h_{12}\left(\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1\right).$$

$$V_1 = \frac{h_{22} h_{11} - h_{21} h_{12}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 .$$

So

$$\Rightarrow Z_{11} = \frac{\Delta h}{h_{22}} \quad \& \quad Z_{12} = \frac{h_{12}}{h_{22}} .$$

(e) g-parameters :

$$Z_{11} = \frac{1}{g_{11}} \; ; \; Z_{12} = -\frac{-g_{12}}{g_{11}} \; ; \; Z_{21} = \frac{-g_{21}}{g_{11}}$$

$$Z_{22} = \frac{\Delta g}{g_{11}}$$

Interconnection of two-Port N/W.

2-port N/w can be connected as series, parallel, cascade, series-parallel & parallel-series connection.

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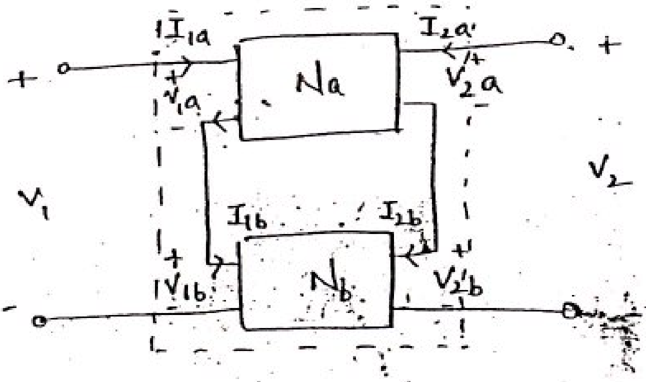
For each configuration certain set of parameters may be more useful than others to describe the n/w.

① Series Connection :



Na

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

Nb

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22b} \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

Series Connection

$I_1 = I_{1a} = I_{1b}$, $I_2 = I_{2a} = I_{2b}$.

$V_1 = V_{1a} + V_{1b}$. $V_2 = V_{2a} + V_{2b}$.

$$V_1 = V_{1a} + V_{1b} = Z_{11a} I_{1a} + Z_{12a} I_{2a} + Z_{11b} I_{1b} + Z_{12b} I_{2b}.$$

$$V_1 = (Z_{11a} + Z_{11b}) I_1 + (Z_{12a} + Z_{12b})$$

||ly $V_2 = V_{2a} + V_{2b}$

$$= (Z_{21a} + Z_{21b}) I_1 + (Z_{22a} + Z_{22b}) I_2$$

$Z_{11} = Z_{11a} + Z_{11b}$.

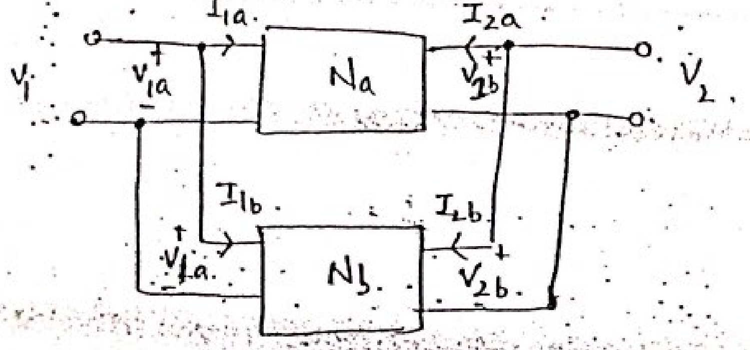
$Z_{12} = Z_{12a} + Z_{12b}$.

$Z_{21} = Z_{21a} + Z_{22b}$.

$Z_{22} = Z_{22a} + Z_{22b}$.

so $[Z] = [Z_a] + [Z_b]$

(2) Parallel connection:



N_a

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

N_b

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{21b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

for || connection →

$$V_1 = V_{1a} = V_{1b} \qquad V_2 = V_{2a} = V_{2b}$$

$$I_1 = I_{1a} + I_{1b} \text{ — (1)} \qquad I_2 = I_{2a} + I_{2b} \text{ — (2)}$$

In (1).

$$I_1 = Y_{11a} V_{1a} + Y_{12a} V_{2a} + Y_{11b} V_{1b} + Y_{12b} V_{2b}$$

$$I_1 = (Y_{11a} + Y_{11b}) V_{1} + (Y_{12a} + Y_{12b}) V_2$$

Illy $I_2 = (Y_{21a} + Y_{21b}) V_1 + (Y_{22a} + Y_{22b}) V_2$.

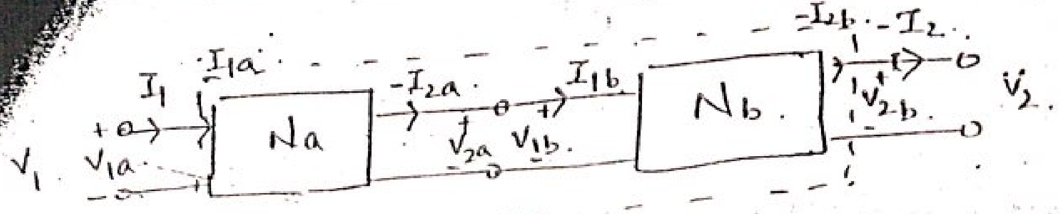
So $[Y] = [Y_a] + [Y_b]$

$$Y_{11} = Y_{11a} + Y_{11b} \qquad Y_{22} = Y_{22a} + Y_{22b}$$

$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

Cascade Connection: (Tandem)



⟹ This is the simplest possible interconnection 2-N/w r cascaded when O/P of Ist N/w is the I/P of the 2nd N/w.

Na

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

Nb

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

In Cascade

$V_1 = V_{1a}$; $V_{2a} = V_{1b}$.

$V_2 = V_{2b}$.

$I_1 = I_{1a}$ $\quad -I_{2a} = I_{1b}$ $\quad I_{2b} = I_2$.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} =$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$ 9

So $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$A = A_a A_b + B_a C_b$. $C = C_a A_b + D_a C_b$.

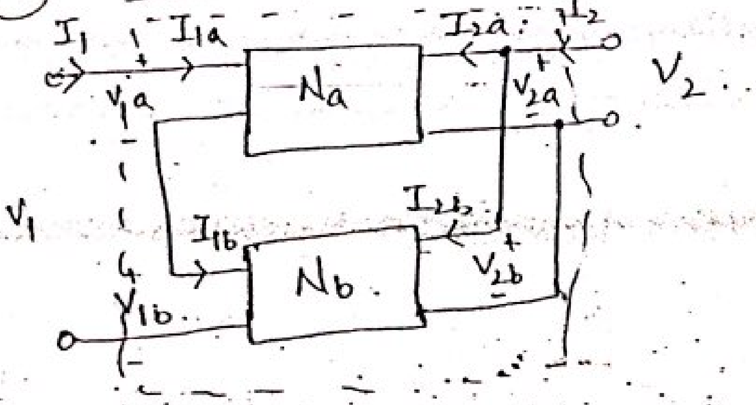
$B = A_a B_b + B_a D_b$. $D = C_a B_b + D_a D_b$.

$$[T] = [T_a].[T_b]$$

In terms of T'-parameters

$$[T'] = [T_b'].[T_a']..$$

(4) Series-parallel.



Series - parallel

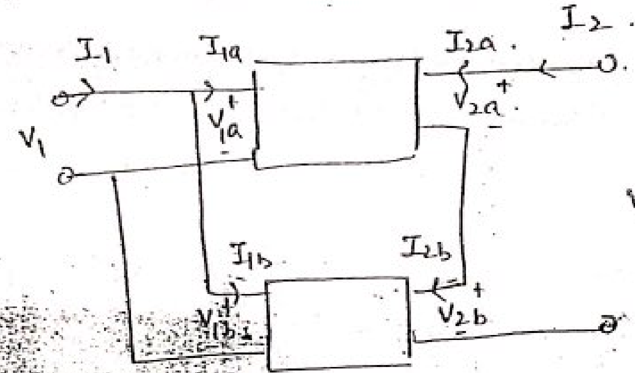
$V_1 = V_{1a} + V_{1b}$. $\quad V_2 = V_{2a} = V_{2b}$

$I_1 = I_{1a} = I_{1b}$. $\quad I_2 = I_{2a} + I_{2b}$.

here

$$[h] = [h_a] + [h_b].$$

(5) Parallel - Series



$V_1 = V_{1a} = V_{1b}$. $\quad I_1 = I_{1a} + I_{1b}$.

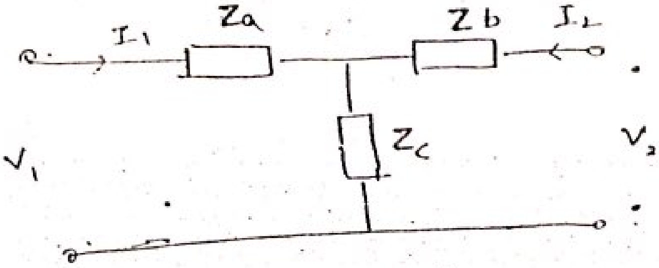
$V_2 = V_{2a} + V_{2b} \quad I_2 = I_{2a} = I_{2b}$.

here

$$[g] = [g_a] + [g_b]$$

Ques:

(1) For the T-n/w shown find Z-parameters. (4)



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$$V_1 = Z_a I_1 + Z_c (I_1 + I_2)$$
$$= (Z_a + Z_c) I_1 + Z_c I_2$$
$$V_2 = Z_b I_2 + Z_c (I_1 + I_2)$$
$$= Z_c I_1 + (Z_b + Z_c) I_2 .$$

$$Z = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix}$$

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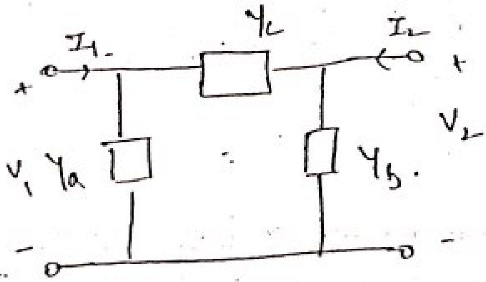
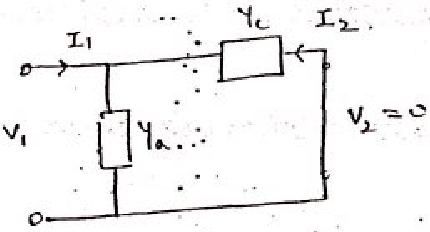
(2)



at ~~$V_2 = 0$~~.

$Y_{12} =$

at $V_2 = 0$



$$\left.\begin{aligned} V_1 &= \frac{(I_1 + I_2)}{Y_a} \\ V_1 &= \frac{-I_2}{Y_c} \end{aligned}\right\} (1)$$

at $V_1 = 0$ $\quad V_2 = \dfrac{(I_1 + I_2)}{Y_b}$ & $V_2 = \dfrac{-I_1}{Y_c} \longrightarrow (2)$

from ①

$$I_2 = (-Y_c)\,V_1$$

$$V_1 = \left[I_1 + V_1(-Y_c)\right]\frac{1}{Y_a}$$

⑪

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$$(Y_a + Y_c)\,V_1 = I_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_a + Y_c$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_c$$

from ② &

$V_1 = 0$

$$V_2 = (I_1 + I_2)\frac{1}{Y_b}$$

$$V_2 = -\frac{I_1}{Y_c}$$

2) $Y_{12} = \dfrac{I_1}{V_2} = -Y_c$

$$V_2 = (-Y_c V_2 + I_2)\frac{1}{Y_b}$$

$$Y = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix}$$

$$Y_b V_2 + Y_c V_2 = I_2$$

$$(Y_b + Y_c)\,V_2 = I_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_b + Y_c$$

Open ckt & short ckt impedences

Open ckt impedences → measurement at i/p port while o/p → is open

$$Z_{1o} = \frac{V_1}{I_1}\Big|_{I_2=0} \equiv Z_{11}$$

$$Z_{2o} = \frac{V_2}{I_2}\Big|_{I_1=0} \equiv Z_{22}.$$

Short ckt impedences →

$$Z_{1s} = \frac{V_1}{I_1}\Big|_{V_2=0} \equiv \frac{1}{Y_{11}}$$

$$Z_{2s} = \frac{V_2}{I_2}\Big|_{V_1=0} \equiv \frac{1}{Y_{22}}.$$

In terms of T-parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2.$$

$$Z_{1o} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{A}{C}$$

$$Z_{2o} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{D}{C}.$$

$$Z_{1s} = \frac{V_1}{I_1}\Big|_{V_2=0} = \frac{B}{D}$$

$$Z_{2s} = \frac{V_2}{I_2}\Big|_{V_1=0} = \frac{B}{A}$$

T-para. in terms of O.C & S.C. Imped

$$A = \pm\sqrt{\frac{Z_{1o}}{Z_{2o} - Z_{2s}}}$$

$$B = \pm Z_{2s}\sqrt{\frac{Z_{1o}}{Z_{2o} - Z_{2s}}}$$

$$C = \pm\sqrt{\frac{1}{Z_{1o}(Z_{2o} - Z_{2s})}}$$

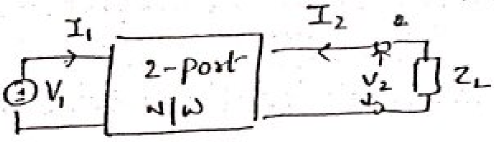
$$D = \pm\frac{Z_{2o}}{\sqrt{Z_{1o}(Z_{2o} - Z_{2s})}}$$

Input impedance

If load imp. Z_L is connected to O/P port

then $Z_{ip} = \frac{V_1}{I_1}$ is I/P Z.

$\rightarrow$ 12



① Z_{ip} in terms of Z

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -I_2 Z_L$$

$$-I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$$

$$I_2 = \frac{-Z_{21}}{Z_{22} + Z_L} I_1$$

$$V_1 = Z_{11} I_1 + Z_{12} \left[\frac{-Z_{21}}{Z_{22} + Z_L}\right] I_1$$

$$Z_{ip} = \frac{V_1}{I_1} = \frac{Z_{11} Z_{22} - Z_{21} Z_{12} + Z_{11} Z_L}{Z_{22} + Z_L}$$

② in terms of T-para.

$$Z_{ip} = \frac{V_1}{I_1} = \frac{A Z_L + B}{C Z_L + D}.$$

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Imp in terms of h-parameters:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$V_2 = -I_2 Z_L$$

$$V_2 = -Z_L(h_{21} I_1 + h_{22} V_2) \Rightarrow$$

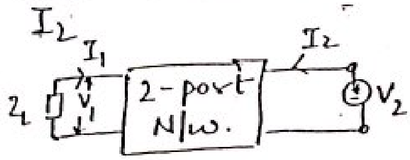
$$V_2 = \frac{-h_{21} Z_L}{1 + h_{22} Z_L} I_1$$

$$\therefore \quad V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21} Z_L}{1 + h_{22} Z_L}\right] I_1$$

$$Z_{ip} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12} h_{21} Z_L}{1 + h_{22} Z_L}$$

O/p Impedance

If a load Imp. Z_L is connected to I/p port then $Z_{op} = \frac{V_2}{I_2}$ is termed as o/p Imp.



① O/p Imp. in terms of Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_1 = -I_1 Z_L$$

$$I_1 = -\left(\frac{Z_{11} I_1 + Z_{12} I_2}{Z_L}\right)$$

$$I_1 = -\frac{Z_{12}}{Z_{11} + Z_L} I_2$$

$$V_2 = Z_{21}\left[-\frac{Z_{12}}{Z_{11}+Z_L}\right] I_2 + Z_{22} I_2 \quad 13$$

$$Z_{op} = \frac{V_2}{I_2} = \frac{Z_{11} Z_{22} - Z_{21} Z_{12} + Z_{22} Z_L}{Z_{11} + Z_L}$$

(2) in terms of T-par.

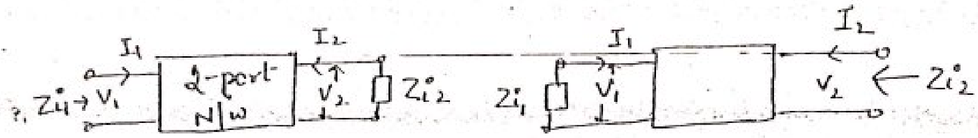
$$Z_{op} = \frac{D Z_L + B}{C Z_L + A}$$

(3) h-par

$$Z_{op} = \frac{h_{11} + Z_L}{h_{11} h_{22} - h_{12} h_{21} + h_{22} Z_L}$$

Image Impedances :

In a 2-port N/w, if the impedance at I/p port with impedance Z_{i2} connected across O/p port be Z_{i1} & the imp. at O/p port with imp Z_{i1} connected across I/p port be Z_{i2}, then Z_{i1} & Z_{i2} r image impedance.



For symmetrical n/w

$Z_{i1} = Z_{i2}$ & is called characteristic or iterative imp. Z_c.

$Z_{i1} = \frac{V_1}{I_1}$ (driving point imp. at I/p port)

$Z_{i2} = \frac{V_2}{I_2}$ (" " " O/p ").

Image Imp. in terms of I/p & O/p Imp.

$Z_{i1} = Z_{ip}$ & $Z_{i2} = Z_{op}$

in terms of T-parameters

$Z_{i1} = Z_{ip}$

$Z_{i1} = \frac{A Z_{i2} + B}{C Z_{i2} + D}$ — ①

$Z_{i2} = Z_{op} = \frac{D Z_{i1} + B}{\ }$ ②

Solving ① & ②.

$Z_{i1} = \sqrt{\frac{AB}{CD}}$

&

$Z_{i2} = \sqrt{\frac{BD}{AC}}$

In terms of o/p ckt & s.c. imp:

$$\Pi \quad Z_{i_1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{A}{C}\cdot\frac{B}{D}} = \sqrt{Z_{10}\cdot Z_{1s}}$$

$$Z_{i_2} = \sqrt{\frac{BD}{AC}} = \sqrt{\frac{D}{C}\cdot\frac{B}{A}} = \sqrt{Z_{20}\cdot Z_{2s}}$$

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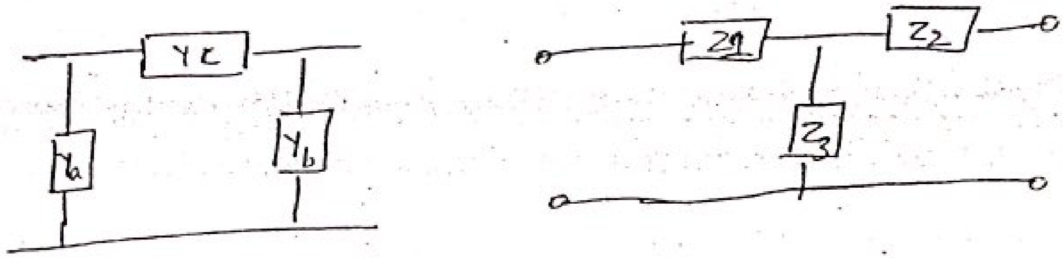
Π to T transformation.



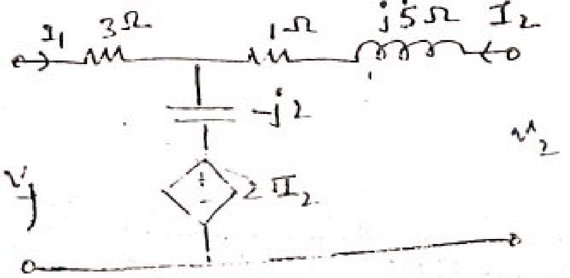
Π to T

$$Z_1 = \frac{\frac{1}{Y_a}\cdot\frac{1}{Y_c}}{\frac{1}{Y_a}+\frac{1}{Y_b}+\frac{1}{Y_c}} = \frac{\frac{1}{Y_aY_c}}{\frac{Y_bY_c + Y_cY_a + Y_aY_b}{Y_aY_bY_c}}$$

$$= \frac{Y_b}{Y_aY_b + Y_bY_c + Y_cY_a}.$$

T to Π

Ques 1 Calc. z-par. of the N/w shown & find out whether its reciprocal or symmetrical or none



$V_1 = 3I_1 - j2(I_1 + I_2) + 2I_2$

$V_1 = (3 - j2)I_1 + 2I_2(1 - j)$

$V_2 = I_2 + j5I_2 - j2(I_1 + I_2) + 2I_2$

$V_2 = (3 + 3j)I_2 - j2I_1$

$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (3 - j2)\Omega$

$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -j2\ \Omega$

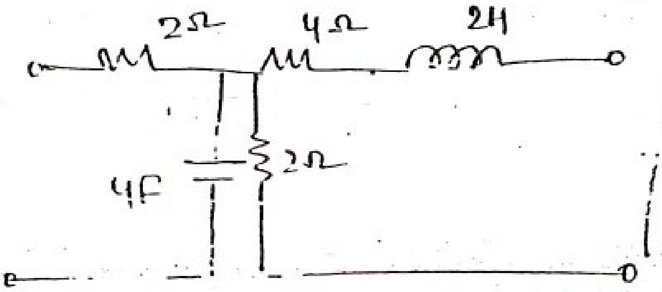
$z_{12} = (2 - j2)\ \Omega$

$z_{22} = (3 + 3j)\ \Omega$

Since $z_{12} \neq z_{21}$ so not reciprocal

$z_{11} \neq z_{22}$ so not symm.

Q. Obtain Z-parameters of the N/w.



$Z = R = Ls = \frac{1}{Cs}$

$Y = \frac{1}{R} = \frac{1}{Ls} = Cs$

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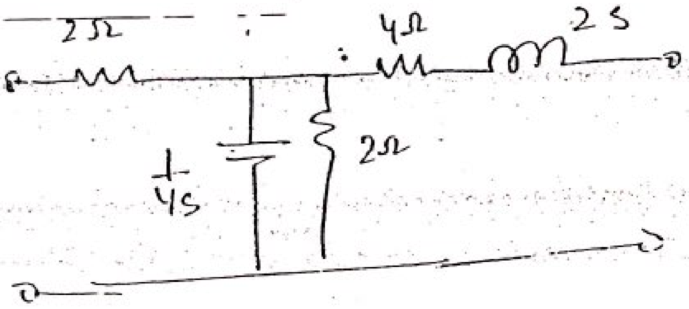
Converting into s-domain



$$\frac{1}{4s} \parallel 2\Omega = \frac{2}{1+8s}.$$

$Z_a = 2\Omega$ $\quad Z_b = 4+2s.$

$Z_c = \frac{2}{1+8s}.$

So $Z_{11} = Z_a + Z_c = 2 + \frac{2}{1+8s} = \frac{4+16s}{1+8s}.$

$Z_{12} = Z_{21} = Z_c = \frac{2}{1+8s}$

$Z_{22} = Z_b + Z_c = 4+2s + \frac{2}{1+8s} = \frac{6+34s+16s^2}{1+8s}$

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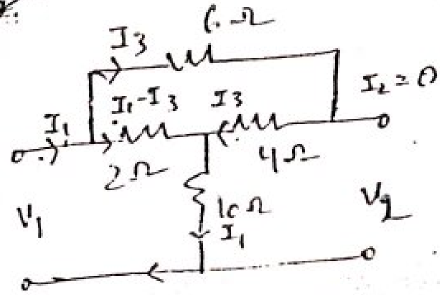
1. obtain the open circuit param. of the N/w.



$\underline{I_2 = 0}$.

$V_1 = 2(I_1 - I_3) + 10 I_1$

$V_1 = 12 I_1 - 2 I_3$.

$V_2 = 4 I_3 + 10 I_1$.

$I_3 = I_1 \times \frac{2}{2+6+4}$

$= \frac{2 I_1}{12} = \frac{I_1}{6}$.

$\therefore V_1 = 2\left(I_1 - \frac{I_1}{6}\right) + 10 I_1$

$= \frac{10}{6} I_1 + 10 I_1$

$= \frac{70}{6} I_1 = \frac{35}{3} I_1$.

$V_2 = 4 \cdot \frac{I_1}{6} + 10 I_1 = \frac{32}{3} I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{35}{3}\,\Omega$

$Z_{21} = \frac{32}{3}\,\Omega$.

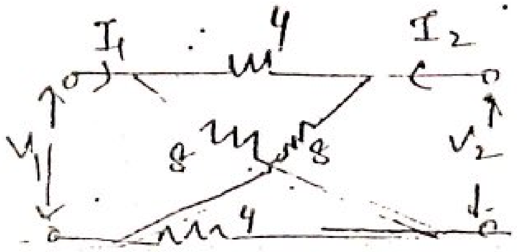
||ly for $\underline{I_1 = 0}$

$V_1 = 2 I_4 + 10 I_2$

$V_2 = 4(I_2 - I_4) + 10 I_2 =$

$Z_{12} = \frac{32}{3}\,\Omega$; $Z_{22} = \frac{38}{3}\,\Omega$.

Q1 for the lattice N/w find z-parameters



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Case 1 $I_2 = 0$

$$V_1 = 4\frac{I_1}{2} + 8\frac{I_1}{2} = 6I_1$$

$$V_2 = 8\frac{I_1}{2} - 4\frac{I_1}{2} = 2I_1$$

Case 2 $I_1 = 0$

$$V_1 = 8\frac{I_2}{2} - 4\frac{I_2}{2} = 2I_1$$

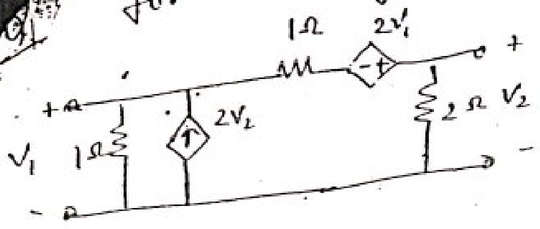
$$V_2 = 6I_2$$

$$z_{11} = z_{22} = 6\,\Omega$$

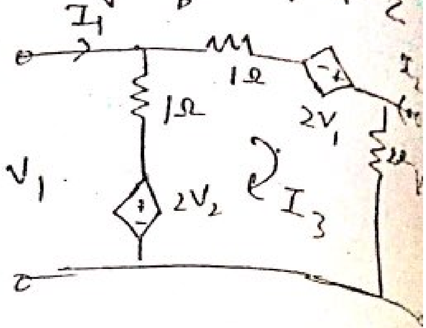
$$z_{12} = z_{21} = 2\,\Omega.$$

Questions:

Q1 for the given N/w find Y & Z



⇒



Sol^n

$$\to V_1 = 1(I_1 - I_3) + 2V_2$$

$$V_1 = I_1 - I_3 + 2V_2$$

$$I_3 = I_1 + 2V_2 - V_1$$

$$\to 1.I_3 - 2V_1 + 2(I_2 + I_3) - 2V_2 + 1(I_3$$

$$4I_3 + 2I_2 - I_1 = 2V_1 + 2V_2 \quad \text{—(1)}$$

$$\to V_2 = 2(I_2 + I_3) \quad \text{—(2)}$$

Putting I_3 in (1)

$$2V_1 + 2V_2 = 4(I_1 + 2V_2 - V_1) + 2I_2 - I_1$$

$$0 = 3I_1 + 6V_2 - 6V_1 + 2I_2$$

$$6(V_1 - V_2) = 3I_1 + 2I_2 \leftarrow \quad (3)$$

I_3 in (2) &

$$V_2 = 2I_2 + 2(I_1 + 2V_2 - V_1)$$

$$0 = 2I_2 + 2I_1 + \cancel{5}3V_2 - 2V_1$$

$$2V_1 - 3V_2 = 2(I_2 + I_1) \leftarrow \quad (4)$$

(3) - (4)

$$4V_1 - 3V_2 = I_1 \longleftrightarrow (5)$$

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so $I_1 = 4V_1 - 3V_2$ — (a)

Putting (5) in (4)

$2V_1 - 3V_2 + 2 = 2(4V_1 - 3V_2) + 2I_2$

$I_2 = -\frac{6}{2}V_1 + \frac{3}{2}V_2$ — (b) 17

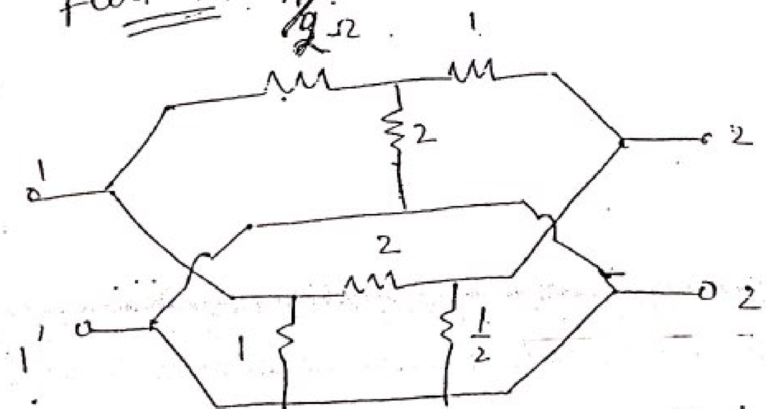
$= -3V_1 + \frac{3}{2}V_2$

Using (a) & (b)

$$Y = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix}$$

$$Z = [Y]^{-1} = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}$$

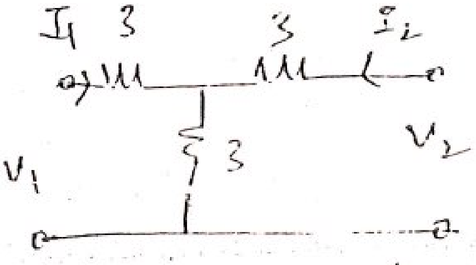
Ques find Y-par.



$$Y_A = [Z_A]^{-1} = \begin{bmatrix} \frac{1}{2}+2 & 2 \\ 2 & 2+1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{6}{7} & -\frac{4}{7} \\ -\frac{4}{7} & \frac{5}{7} \end{bmatrix}$$

$$Y_B = \begin{bmatrix} \frac{1}{1}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$ $Y = Y_A + Y_B$

Q. For the given N/w find image para.



$A = 2 \quad \frac{V_1}{V_2}$

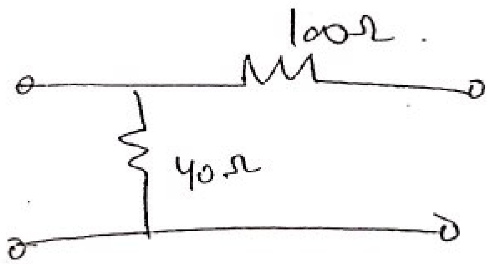
$B = 9\Omega, \quad \frac{V_1}{I_2}$

$C = \frac{1}{3} ℧ \quad \frac{I_1}{V_2}$

$D = 2 \quad \frac{I_1}{I_2}$

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{2\times 9}{\frac{1}{3}\times 2}} = \sqrt{27} = 5.2\,\Omega$$

Q



h-par.

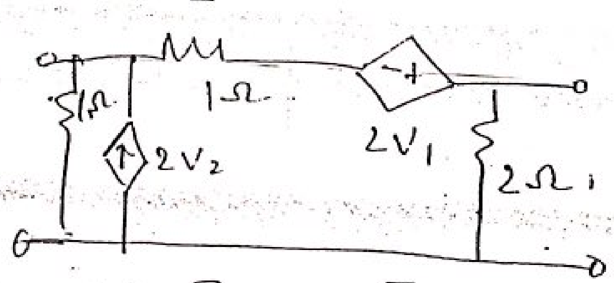
$h_{11} = \frac{200}{7}$

$h_{21} = \frac{-2}{7}$

$h_{12} = \frac{2}{7}$

$h_{22} = \frac{1}{140}$

Q



Y & Z - parameters

$$Y = \begin{bmatrix} 4 & -3 \\ -3 & \frac{3}{2} \end{bmatrix} \quad Z = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix}$$

Y-parameters.

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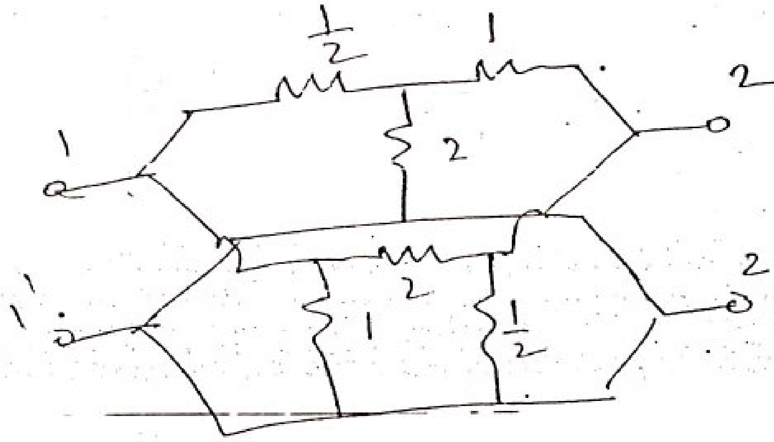
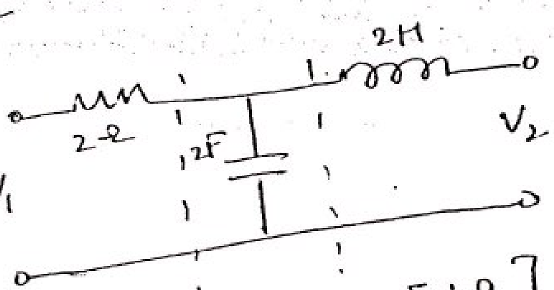


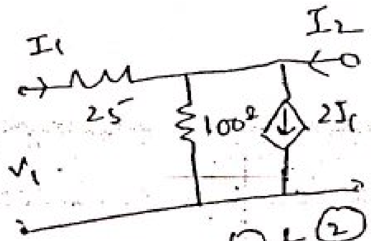
Q Determine T-parameters.



$$T_a = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad T_b = \begin{bmatrix} 1 & 0 \\ 2s & 1 \end{bmatrix} \quad T_c = \begin{bmatrix} 1 & 2s \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 4s+1 & 2(4s^2+s+1) \\ 2s & 4s^2+1 \end{bmatrix}$$

Q Find Y-parameters



$$I_1 + I_2 = \frac{V_2}{100} + 2I_1$$

$$I_1 = -\frac{V_2}{100} + I_2 \quad -(1)$$

Also $I_1 = \dfrac{V_1 - V_2}{25}$ $-(2)$ $= \frac{1}{25}V_1$ [illegible] V_2 $-(3)$

from (1) & (2)

$$\frac{V_1 - V_2}{25} = -\frac{V_2}{100} + I_2$$

$$I_2 = \frac{1}{25}V_1 - \frac{3}{100}V_2$$

$y_{21} = y_{11} = \frac{1}{25}$ $\quad y_{12} = -\frac{1}{25}$

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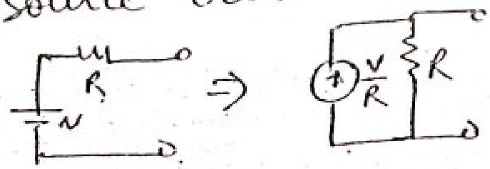
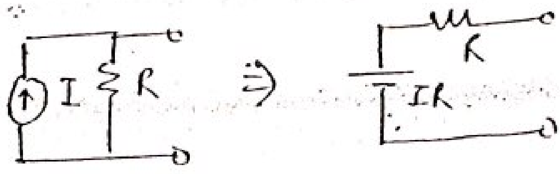
N/w Theorems

→ electric ckt / N/w

→ Star - Delta transformation

→ Source trans.

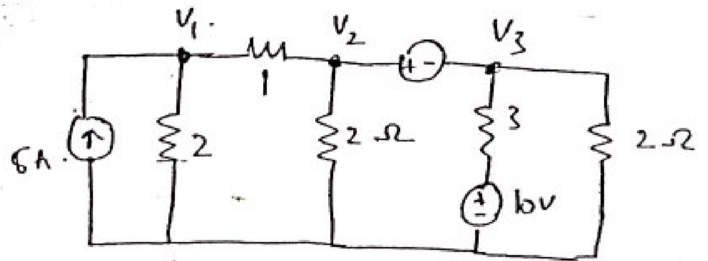




→ Kirchoff's Law → KCL & KVL

→ **Super node Analysis**

↳ If a branch has only a V.S. then the adjacents nodes r considered as 1 node & KCL is applied as usual.



At node 1

$$5 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$

$$3V_1 - 2V_2 = 10$$

At super node (i.e. 2 & 3)

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_3 - 10}{3} + \frac{V_3}{2} = 0$$

$1 - 6V_1 + 9V_2 + 5V_3 = 20$ ..

$\rightarrow V_2 - V_3 = 20$.

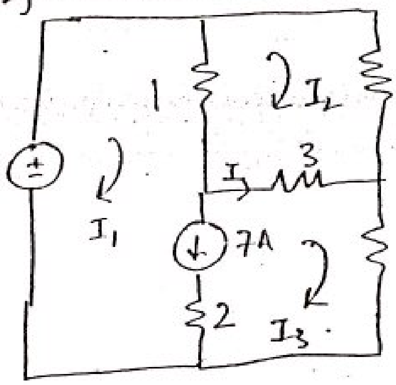
$V_3 = 12V$.

$I = \frac{V_3 - 10}{3} = \frac{2}{3} A$

Super-mesh Technique

If a branch in the ckt has a C.S. then ~~nox~~ applying KVL is difficult so we may apply super-mesh tech., in which a super-mesh is constituted by 2 adja loops that have a common C.S.

Q find I



Super mesh (mesh 1 & 3)

$$7 = 1(I_1 - I_2) + (I_3 - I$$
$$+ 1 \cdot I_3$$

$$I_1 - 4I_2 + 4I_3 =$$

mesh 2

$$1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \quad \text{—(2)}$$

$$I_1 - I_3 = 7 \quad \text{—(3)} \Rightarrow I_1 = 7 + I_3$$

solving eq (1), (2) & (3)

$$I_1 = 9\,A, \quad I_2 = 2.5\,A \;\&\; I_3 = 2\,A$$

$$I = I_2 - I_3 = .5\,A$$

Superposition theorem :

In an active linear n/w containing several sources, the overall response in any branch in the n/w equals the algebraic sum of the responses of each individual source considered separately with all other sources made inoperative.

Appl^n → ① any linear ckt having time varying or time-invariant elements.

② → ckt ~~any~~ analysis when large no. of independent sources are present.

Limitations → ① not applicable to ckt containing dependent sources only. & also to non-linear elements like diode etc

② not useful for ckt having less than 2 independent sources.

Reciprocity thrm

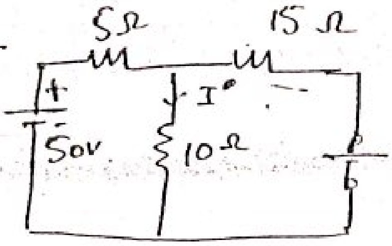
ratio of excitation to response remains invariant on changing the point of appl^n of both.

Appl^n → to linear, time-invariant N/w of passive N/w elements :

This thrm provides bilateral property of N/w, Limitations :

1. N/A to dependent source.
2. N/A to any time varying element.
3. N/w has non-linear elements.

Ques 2. Find I by Superposition theorm.

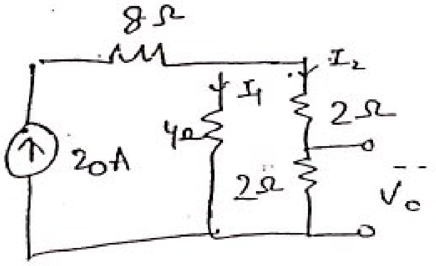


$I' = \frac{30}{11}$

$I'' = \frac{20}{11}$

$I = \frac{80}{11} = 4.5\ A$

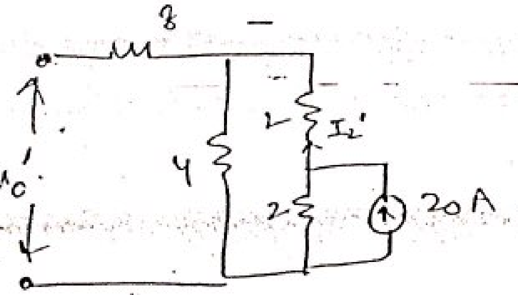
Q- 3



$I_2 = 20 \times \frac{4}{4+2+2} = 10\ A$

$V_o = 2 \times 10 = 20V$

→ verify Reciprocity theorm



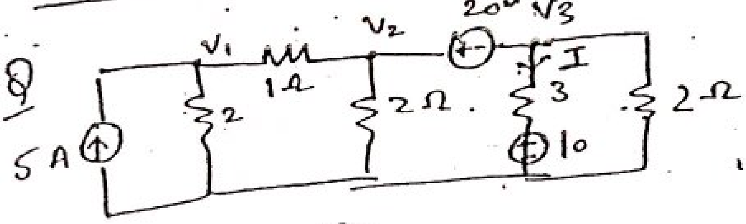
$I_2' = \frac{2 \times 20}{2+(4+2)}$

$= 5\ A$

$V_o' = 5 \times 4 = 20V$

Thevenin's Theorem : find I.

Q



node 1

$5 = \frac{V_1}{2} + \frac{V_1 - V_2}{1} \Rightarrow V_1$

$3V_1 - 2V_2 = 10$ — (1)

node 2+3

$\frac{V_2 - V_1}{1} + \frac{V_2}{2} + \frac{V_3 - 10}{3} + \frac{V_3}{2} = 0$

$\Rightarrow -6V_1 + 9V_2 + 5V_3 = 20$ — (2)

$V_2 - V_3 = 20$ — (3)

from (1) & (3)

$3V_1 - 2V_3 = 50$

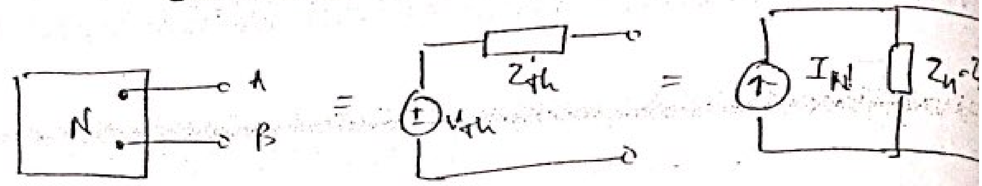
$\Rightarrow V_1 = \frac{50 + 2V_3}{3}$

Thevenin's theorem :

With respect to terminal pair AB, the n/w N may be replaced by a volt. source V_{th} in series with an int. imp. Z_{th}. The

V_{th} = Thev. volt. pot. diff. b/w A & B.

Z_{th} ∃ int. imp. of N/w N as seen from terminals A & B. with all sources set to



$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

Norton's theorem :

With respect to terminal pair AB, the N/w N may be replaced with a I.S (↑) . I_N in || with

I_N = I from A to B. on S.C. A & B.

$Z_N = Z_{Th}$.

$I_L = V_{th}$ $\quad I_L = I_N \cdot \dfrac{Z_N}{Z_L + Z_N}.$

Case I Ckt having only Independent Sources :

set all sources at 0 values

i.e. V.S → shorted circuited

& C.S → open circuited } leaving behind their internal resis. imp.

Case II independent + dependent :

calc. V_{th} & I_N then

$$Z_{th} = Z_N = \frac{V_{th}}{I_N}$$

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Case III Only dependent Sources:

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Apply voltage V at AB terminal.
Current I flow due to this V.

Now $Z_N = \frac{V}{I}$

Appl^n (1) replacement of a large, complicated n/w by simple equivalent.

(2) mptt.

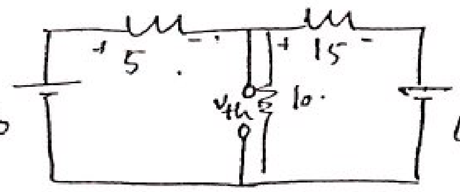
Limitations:

(1) N/A to unilateral elements.

(2) N/A to non linear elements.

(3) :

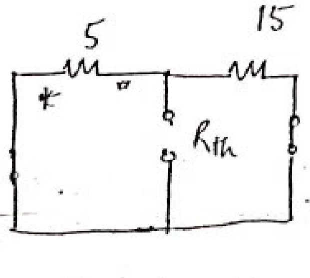
Q



$V_{th} = 50 - 5I_0 = 15I_0 + 100. = 15(-2.5) + 100 = 62.5$

$I_0 = \frac{50-100}{20} = -2.5A.$

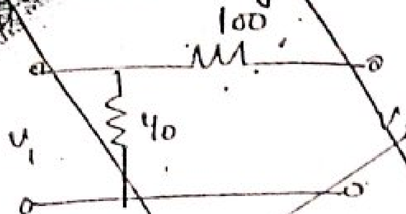
R_{th}



$R_{th} = 5 \parallel 15 = \frac{75}{20} = 3.75$

$I = \frac{V_{th}}{R_{th}+10} = \frac{62.5}{3.75+10} = \frac{62.50}{13.75} = 4.548A$

Find hybrid p. of the n/w.



$h_{11} = \frac{200}{7}\,\Omega$ $h_{21} = \frac{-2}{7} = \frac{I_2}{I_1}\Big|_{V_2=0}$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{2}{7}$

$h_{22} = \frac{I_2}{V_2} = \frac{1}{140}\,\mho$

Millman's theorem:

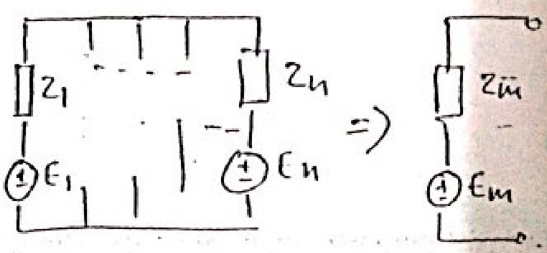
Can be used for V.S or C.S.

(1) for V.S.:

If for n voltage sources having series imp. operating in parallel, then by Millman's theorem, we can determine E_m, the equivalent V.S & Z_m the series imp.

$$E_m = \frac{\sum_{i=1}^{n} E_i Y_i}{\sum Y_i}$$

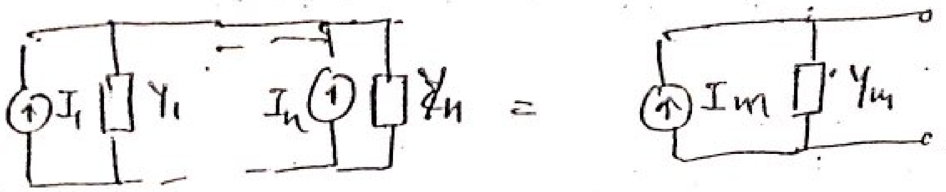
$$Z_m = \frac{1}{\sum_{i=1}^{n} Y_i}$$



Proof V.S. to C.S transformation.

$$I_i = \frac{E_i}{Z_i} = E_i Y_i$$

$$Y_i = \frac{1}{Z_i}$$



$$I = \sum I_i = \sum E_i Y_i$$

$$Y_m = \sum Y_i$$

$$E_m = \frac{I}{Y_m} = \frac{\sum E_i Y_i}{\sum Y_i}$$

||ly for C.S

n no. of C.S with || imp. connected in series

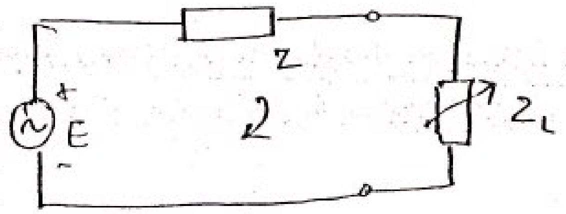
the $I_m = \frac{E}{Z_m} = \frac{\sum I_i Z_i}{\sum Z_i}$

Appl^n ~~(I) applies only to independent~~

M.P.T.T.

used for analysing Comm. circuits where there is transfer of max. power from source to the load & not highest η.

Max. Power O/P is obtained from an A.C. ckt when the load imp. = the complex conjugate of the int. imp. of the ckt as seen from terminals of the load.



→ MPTT aims at finding Z_L such that ~~the~~ power dissipated in it is max.

$$I = \frac{E}{(Z+Z_L)^2}$$

$$P = |I|^2 R_L$$

$$Z = R+jX \quad \& \quad Z_L = R_L + jX_L$$

$$P = \frac{E^2}{(R+R_L)^2 + (X+X_L)^2} \cdot R_L$$

for max. Power

$$\frac{\partial P}{\partial X_L} = 0 \qquad \frac{\partial P}{\partial X_L} = \frac{0 - E^2 \cdot R_L \cdot 2(X+X_L)}{\left[(R+R_L)^2 + (X+X_L)^2\right]^2}$$

$$E^2 \cdot R_L \cdot 2(X+X_L) = 0$$

$$\Rightarrow X + X_L = 0 \Rightarrow X = -X_L$$

Putting $X_L = -X$.

$$P = \frac{E^2 R_L}{(R+R_L)^2}$$

$$\frac{\partial P}{\partial R_L} = 0$$

$$\frac{\partial P}{\partial R_L} = \frac{(R+R_L)^2 \cdot E^2 \cdot 1 - E^2 R_L \cdot 2 \cdot (R+R_L)}{\left[(R+R_L)^2\right]^2}$$

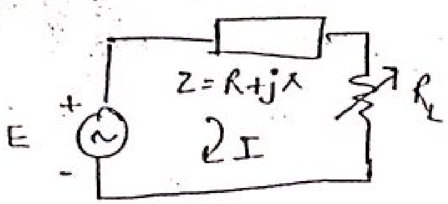
$$E^2(R+R_L) - 2E^2 R_L = 0$$

$$R_L = R.$$

$$\therefore \quad Z_L = Z^*$$

$$P_m = \frac{E^2}{4R_L} = \frac{(E/2)^2}{R_L}.$$

(B)



In this case.

$$R_L = \sqrt{R^2+X^2} = |Z|$$

Various Cases

(1) D.C. ckt

$R_L = R$ (Thev.'s resistance)

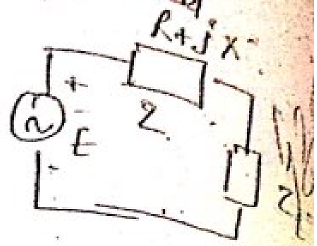
(2) for A.C. ckt

(a) $Z_L = Z^*$

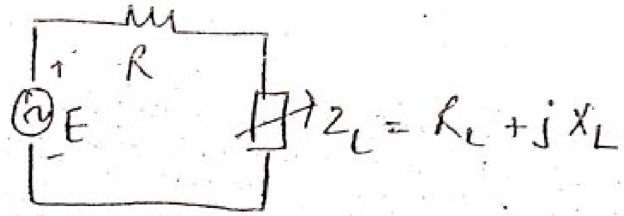
(b) $R_L = |Z| = \sqrt{R^2+X^2}$

for AC ckt when Z_L (R_L) X_L = fixed

$$R_L^2 = R^2 + (X + X_L)^2$$



(4)

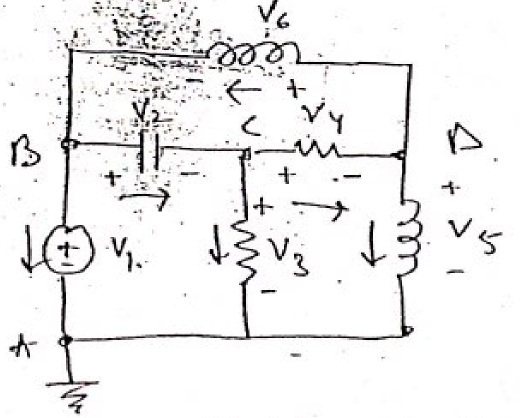


$$R_L^2 = R^2 + X_L^2$$

Tellegen's theorem :

→ Completely independent of nature of ckt elements.

→ based on KVL & KCL.



If the ckt having nodes & b branches with branch voltage be V_K the " current be i_K

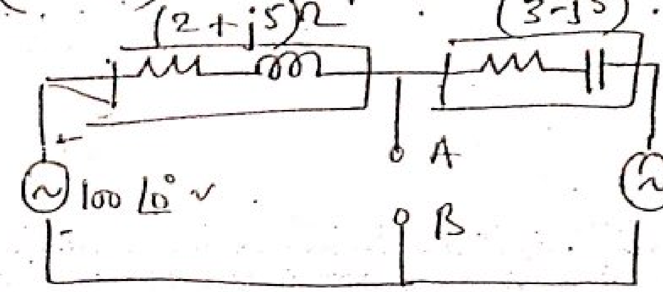
$K \to 1$ to b.

Tellegen's thom states that

$$\sum_{K=1}^{b} V_K i_K = 0$$

This implies that the sum of instt power delivered to all branches of a circuit is

Q1 Using Thevenin's theorm, find power in $(4+j6)\,\Omega$ imp. connected across terminals AB



for $V_{th} = 100 - \text{drop in } (2+j5)$
$= \text{drop in } (3-j5) + 50\angle 90°$

$$= 100\angle 0° - (2+j5)\left[\frac{100\angle 0° - 50\angle 90°}{(2+j5)+(3-j5)}\right]$$

$$= 100\angle 0° - (2+j5)\left(\frac{100 - j50}{5}\right)$$

$$= 100\angle 0° - [40 + j100 - j20 + 50]$$

$$= 100 - j80 = 80.62\angle -82.87° \text{ V}$$

$Z_{th} \Rightarrow (2+j5) \parallel (3-j5)$

$$\Rightarrow \frac{(2+j5)(3-j5)}{(2+j5)+(3-j5)} = \frac{31+j5}{5} = (6.2+j1)\,\Omega$$

$$I_L = \frac{V_{th}}{Z_{th}+Z_L} = \frac{80.62\angle -82.87}{(6.2+j1)+(4+j6)} = \frac{-2.99 - 5.78j}{}$$

$$= \frac{80.62\angle -82.87}{12.371\angle 34.46} = 6.52\angle -117.33° \text{ A}$$

Power $= I_L^2 \times R_L = (6.52)^2 \times 4 = 170.04\,W$

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(11)

2 Properties :

(1) N_1 & N_2 → 2 diff ckts having same graph with same reference directions assigned to the branches in the 2 ckt

Let for N_1 → v_{1k} & i_{1k} ∃
N_2 → v_{2k} & i_{2k}

All V's & I's satisfy Kirchoff's laws

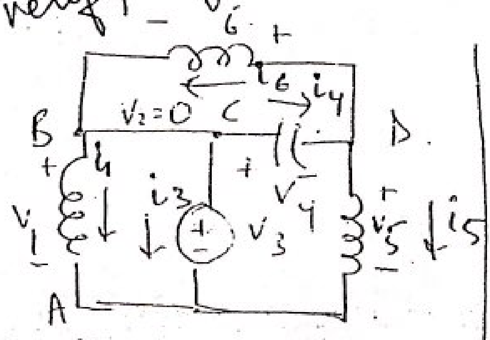
So by Tellegen's thrm

$$\sum_{k=1}^{b} v_{1k} \cdot i_{2k} = 0 \quad \& \quad \sum_{k=1}^{b} v_{2k} \cdot i_{1k} = 0$$

(2) If t_1 & t_2 are 2 diff instt of obs. it still follows that

$$\sum_{k=1}^{b} v_k(t_1) \cdot i_k(t_2) = 0$$

Q verify Tellegen's T



$V_1 = 5V$, $V_4 = 2V$
$V_2 = 0$ & $i_1 = 2A$ $i_3 = 1A$

Soln ABCA $V_1 = V_3 = 5$

ACDA

$V_5 = V_3 - V_4 = 5-2$

BCDB

$-V_4 - V_6 = 0$

$\therefore V_6 = -V_4 = -2$

at Node B $i_6 = i_1 + i_2 = 2A$

C $i_2 = i_3 + i_4 \Rightarrow i_4 = -i_3 = -1A$

D $i_5 = i_4 + i_6 = -3A$

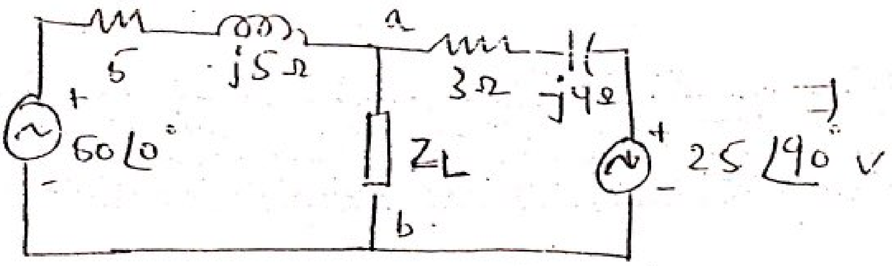
$25\angle 90^\circ = 25\cos 90^\circ + j\,25\sin 90^\circ$

$= 0 + j25$

$\therefore (5\cdot 2) + 0\cdot 0 + 5\cdot 1 + 2\cdot(-1) + 3\cdot(-3)$
$+ (-2)\cdot(2) = 0$

MPTT

Q



Z_L is varying in R_L & X_L

find Z_L for Max. power using millman theorem.

& also Calc. max. power

Soln

$E_1 = 50\angle 0^\circ = 50 \quad Y_1 = \frac{1}{Z_1} = \frac{1}{5+j5} = 0.1 - 0.1j$

$Z_1 = (5 + j5)\,\Omega$

$E_2 = 25\angle 90^\circ = 25\cos 90^\circ + j\,25\sin 90^\circ$

$= +j25\text{ V}$

$Z_2 = 3 - j4\,\Omega$

$Y_2 = \frac{1}{Z_2} = \frac{1}{3-j4} \times \frac{3+j4}{3+j4} = \frac{3+j4}{9-j^2 16} = \frac{3+j4}{25}$

$E_m = \frac{E_1Y_1 + E_2Y_2}{Y_1 + Y_2} = \frac{1-j2}{0.22+j0.06} = 9.807\angle -78.65^\circ\text{V}$

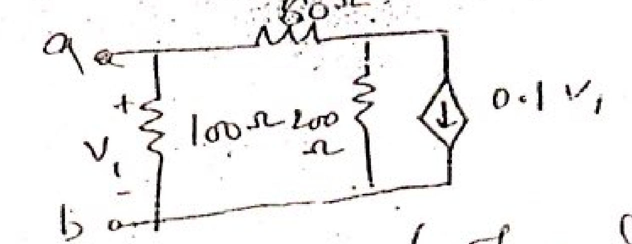
$Z_m = \frac{1}{Y_1+Y_2} = \frac{1}{0.22+j0.06} = 4.385\angle -15.25^\circ$

$= 4.23 - j1.15$

$Z_L = Z_m^* = 4.385\angle 15.25^\circ = 4.23 + j1.15$

$P_m = \frac{E_m^2}{4R_L} = \frac{(9.807)^2}{4\times 4.23} = 5.68\text{ W}$

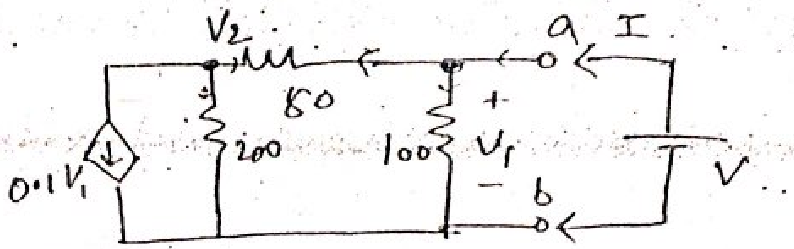
Q. Find the Norton's equivalent of the n/w.



$\frac{V_2}{200} + \frac{V_2 - V_1}{50} + 0.1V_1 = 0$

$I_{sc} = I_N = 0$ (why?)

$R_N \rightarrow$



Now $R_N = \frac{V}{I}$

$I = \frac{V_1}{100} + \frac{V_1 - V_2}{50} \Rightarrow 100 I = 3V_1 - 2V_2$

$\frac{V_1 - V_2}{50} = \frac{V_2}{200} + 0.1V_1 \Rightarrow V_2 = \frac{-16}{5} V_1$ (2)

also $V = V_1$ (3)

from (1) & (2)

$100 I = 3V_1 - 2\left(\frac{-16}{5} V_1\right)$

$R_N = \frac{V_1}{I} = \frac{500}{47} = 10.63$

Ques: Determine I in Capacitor by super position theorem.

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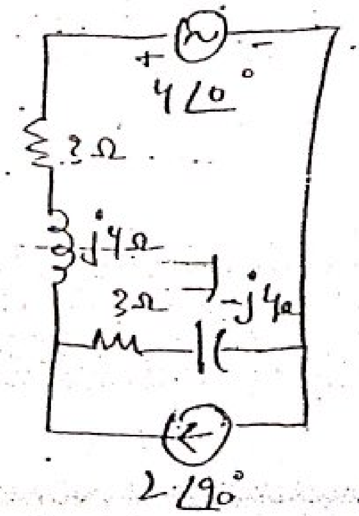


① V.S → $4\angle 0°$

$$I' = \frac{4\angle 0°}{(3+j4)+(3-j4)} = \frac{4\angle 0}{6} = \frac{2}{3}\angle 0° = \frac{2}{3}A$$

② C.S → $2\angle 90°$

$$I'' = 2\angle 90° \times \frac{3+j4}{(3+j4)+(3-j4)}$$

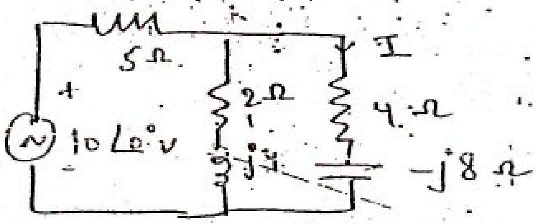
$$= 2j \times \frac{3+j4}{6} = \frac{-4+j3}{3} = \left(-\frac{4}{3}+j\right) A$$

$$I = I' + I''$$

$$= \frac{2}{3} - \frac{4}{3} + j = -\frac{2}{3}+j = 1.2\angle 123.7° A$$

Verify reciprocity theorem 10

5y



Case I

$$Z_{eff} = \left[(4-j8) \,||\, (2+j4)\right] + 5$$

$$= \frac{(4-j8)(2+j4)}{(4-j8)+(2+j4)} + 5$$

$$= \frac{8+j16-j16+32}{6-j4} + 5$$

$$= \frac{40}{6-j4} + 5 = \frac{70-j20}{6-j4}$$

$$= \frac{72.8 \angle -15.94}{7.21 \angle -33.69}$$

$$= 10.1 \angle 17.75$$

$$I = \frac{10 \angle 0^\circ}{10.1 \angle 17.75^\circ} \cdot \frac{(2+j4)}{(2+j4)+(4-j8)}$$

$$= 0.614 \angle 79.3^\circ A$$

Case II

$$Z'_{eff} = [5 \,||\, (2+j4)] + (4-j8) \quad \Omega$$

$$= \frac{10+j20}{7+j4} + (4-j8) = \frac{70-j20}{7+j4}$$

$$\Rightarrow \frac{72.8\angle -15.9^\circ}{8.06\angle 29.74} = 9.03\angle -45.64$$

$$\underline{\underline{I'}} = \frac{10\angle 0^\circ}{9.03\angle -45.64} \cdot \frac{2+j4}{5+(2+j4)}$$

$$= \underline{\underline{0.614\angle 79.3^\circ \text{ A}}}$$

N/w Synthesis

Elements of Realizability theory:

(1) → $T(s) = \frac{R(s)}{E(s)}$ $\frac{\text{Response}}{\text{Excitation}}$

(1) Eg (S) (j)

(1) Causality & Stability

↓

response of a n/w must be = 0 for $t < 0$.

for ~~n/w to be~~ Stable → following 3 condtⁿ must be satisfied.

(1) $T(s)$ can't have poles in righ half of s-pla

(2) $T(s)$ Can't have multiple poles in the ($j\omega$) axis

(3) degree of $N(s)$ of $T(s)$ can't exceed the degree of $D(s)$ by more than unity

$$T(s) = \frac{N(s)}{D(s)}$$

(2) Hurwitz Polynomial

It is the $D(s)$ polynomial of the n/w funcⁿ satisfying certain condtⁿ.

A polynomial $P(s)$ is hurwitz if

(1) $P(s)$ is real when s is real

(2) roots of $P(s)$ have real parts which are 0 or negative

Properties of hurwitz

$P(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0$

(1) All coefficients a_i must be +ve.

⇒ none of of coeff. b/w highest order term in s & the lowest order term may be 0. unless the polynomial is even or odd.

(2) Both odd & even parts of a hurwitz polynomial $P(s)$ have roots on $j\omega$ axis only.

even → $M(s)$

odd → $N(s)$

so $P(s) = M(s) + N(s)$.

then $M(s)$ & $N(s)$ have roots on $j\omega$ axis only.

(3) If $P(s)$ → either even or odd, all its roots are on $j\omega$ axis.

(4) Continued fraction expansion of the ratio odd to even or vice a versa. yields all +ve quotient terms.

As $\psi(s) = \frac{N(s)}{M(s)}$ or $\frac{M(s)}{N(s)}$.

$$\psi(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\cdots}}}$$

(5) If $P(s) \to$ hurwitz poly.

& $W(s) \to$ multiplicative factor

Then $P_1(s) = P(s) \cdot W(s)$. is also hurwitz if $W(s)$ is hurwitz polynomial.

(6) If $P(s) \to$ only odd or even then Cont. fraction expansion is not possb.

So $P(s) \to$ hurwitz if $\frac{P(s)}{P'(s)}$ gives a continued fr. ex.

Q. Check whether the given polynomial $P(s) = s^4 + s^3 + 5s^2 + 3s + 4$ is hurwitz or n

Sol^n (1) all coeff of $P(s)$ are +ve. so $P(s) \to$ real for s real

(2) even & odd

$$M(s) = s^4 + 5s^2 + 4$$

$$N(s) = s^3 + 3s.$$

$$\psi(s) = \frac{M(s)}{N(s)} = s^3+3s \overline{\big) s^4 + 5s^2 + 4} \big(s$$

$$\frac{s^4 + 3s^2}{2s^2 + 4 \big) s^3 + 3s \big(\frac{1}{2}}$$

$$\frac{s^3 + 2s}{s \big) 2s^2 + 4}$$

$$\frac{2s^2}{4}$$

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Q. $P(s) = s^4 + s^3 + 2s^2 + 4s + 1$

Sol^n

(i) is satisfied.

(ii) $M(s) = s^4 + 2s^2 + 1$

$N(s) = s^3 + 4s$

$\psi(s) = \dfrac{M(s)}{N(s)}$

$s^3+4s \,)\, s^4 + 2s^2 + 1 \,(\, s$

$s^4 + 4s^2$

$-2s^2+1 \,)\, s^3+4s \,(\, -\frac{1}{2}s$

$s^3 - \frac{s}{2}$

$\frac{9}{2}s \,)\, -2s^2+1 \,(\, -\frac{2}{9} \cdot 2s = -\frac{4}{9}s$

$-2s^2$

$1 \,)\, \frac{9}{2}s \,(\, \frac{9}{2}s$

$\frac{9}{2}s$

$\times$

not hurwitz.
bcz of -ve quotient terms

Q. Find the range of values of a so that $P(s) = s^4 + s^3 + as^2 + 2s + 3$ is hurwitz.

Sol^n

Cond^n (1) All a_i must be +ve so $a > 0$.

(2) $M(s) / N(s)$

$a > 2$ & $a > 3.5$

so $a > 3.5$

Positive Real func^n:

$T(s) = \frac{N(s)}{D(s)} \rightarrow$ P.r. if foll^n condit^n are satisfied

(i) $T(s)$ is real for s real.

(2) $D(s)$ is Hurwitz polynomial.

(3) $T(s)$ may have poles on the $j\omega$ axis.

(4) real part of $T(s)$ is greater than or equal to 0 ~~for~~ real part of $s \geq 0$.

$$Re[T(s)] \geq 0 \quad Re\, s \geq 0$$

$$T(s) = \frac{N(s)}{D(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}.$$

$$T(s) = \frac{M_1 + N_1}{M_2 + N_2} \cdot \frac{M_2 - N_2}{M_2 - N_2}$$

$$= \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2} \cdot \frac{N_1 M_2 - M_1 N_2}{M_2^2 - N_2^2}.$$

$$Ev[T(s)] = \frac{M_1 M_2 - N_1 N_2}{M_2^2 - N_2^2}$$

$$odd[T(s)] = \frac{N_1 M_2 - M_1 N_2}{M_2^2 - N_2^2}$$

If $s = j\omega$.

even part → real

odd part → imag.

$$Re[T(j\omega)] = Ev[T(s)]\Big|_{s=j\omega}.$$

$$j\,Im[T(j\omega)] = odd[T(s)]\Big|_{s=j\omega}.$$

So To prove this condt.

$$Re[T(j\omega)] \geq 0 \text{ for all } \omega.$$

or $M_1(j\omega) \cdot M_2(j\omega) - N_1(j\omega) N_2(j\omega) \geq 0$

Since Denominator is always +ve.

Properties:

(1) $T(s) \rightarrow$ p.r then $\frac{1}{T(s)} \rightarrow$ p.r.

if $Z(s) \rightarrow$ p.r

then $Y(s) \rightarrow$ also p.r.

(2) Sum of p.r $f^n \rightarrow$ p.r.

(3) Poles & zeros of a p.r.f can't be in the right half of s-plane.

in addition only simple poles with real +ve residues can exist on $j\omega$ axis

(4) poles & zeros of a p.r.f are real occur in conjugate pair.

(5) highest power of $N(s)$ & $D(s)$ may differ atmost by unity.

- ⟹ prohibits multiple poles or zeros at ∞.

(6) Lowest power of $D(s)$ & $N(s)$ may differ by atmost unity.

⟹ prohibits multiple poles or z at $s=0$.

- Necessary & sufficient condtn for $T(s)$ to be p.r

(1) $D(s)$ must be hurwitz.

(2) this condtn is checked only when the poles of $T(s)$ are on the $j\omega$ axis otherwise not.

& is tested by making partial fr expansion of $T(s)$ & checking whether the residues of the poles on $j\omega$-axis are +ve & real.

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Cond^n 3 → $Re[T(j\omega)] \geq 0$ for all ω. $6\frac{3}{8}$ ✓

$M_1(j\omega) M_2(j\omega) - N_1(j\omega) N_2(j\omega) \geq 0.$

Q. show them $f(s) = \dfrac{(s+2)(s+4)}{(s+1)(s+3)}$ is p.r. $= \dfrac{s^2+6s+8}{s^2+4s+3}$

Sol^n. (i) D(s) → hurwitz.

(ii) → doesn't exist.

(iii) $M_1 = s^2+8 \quad N_1 = 6s$

$M_2 = s^2+3 \quad N_2 = 4s.$

$M_1 M_2 - N_1 N_2 \geq 0$

$(s^2+8)(s^2+3) - (6s)(4s) \geq 0$

$s^4 - 13s^2 + 24 \geq 0.$

$A(\omega^2) = \omega^4 + 13\omega^2 + 24 \geq 0$ for all ω.

$\dfrac{-3s}{s^2+1} + \dfrac{5}{s}\Big|_{s=j} \quad \dfrac{-3j}{j^2+1} + \dfrac{5}{j}$

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Q $F(s) = \dfrac{s^5 + s^2 + 3s + 5}{s^2 + 6s + 8}$.

Sol^n

Cond^n 1 : $M(s) = s^2 + 8 \quad N(s) = 6(s)$.

$D(s) \rightarrow$ hurwitz.

(2) $\rightarrow$ no poles on $j\omega$.

(3) $M_1 = s^2 + 5 \quad M_2 = s^2 + 8$.

$N_1 = s^3 + 3s \quad N_2 = 6s$

$A(\omega^2) = M_1 M_2 - N_1 N_2 \geq 0$.

$(s^2 + 5)(s^2 + 8) - (s^3 + 3s)(6s) \geq 0$.

$-5s^4 - 5s^2 + 40 \geq 0$.

$-5\omega^4 + 5\omega^2 + 40 \geq 0 \quad (s = j\omega)$.

Since all coeff = not +ve.

So we use sturm's test.

Put $\omega^2 \Rightarrow x$

$A_0(x) = -5x^2 + 5x + 40$.

$A_1(x) = \dfrac{dA_0 x}{dx} = -10x + 5$

$\dfrac{A_0(x)}{A_1(x)} = (K_1 x - K_0) - \dfrac{A_2(x)}{A_1(x)} =$

18 Determine whether the func^n is P.r. or not

Q. 7^4 $Z(s) = \dfrac{2s^2+5}{s(s^2+1)}$

(1) $D(s) = s^3+s$ then $D'(s) = 3s^2+1$

$D(s) \rightarrow$ hurwitz.

(2) $Z(s) \rightarrow$ poles $\Rightarrow s = \pm j1$

$s(s^2+1)$

$s=0$

$s^2+1=0$

$s^2=-1$

$s=\sqrt{-1}$

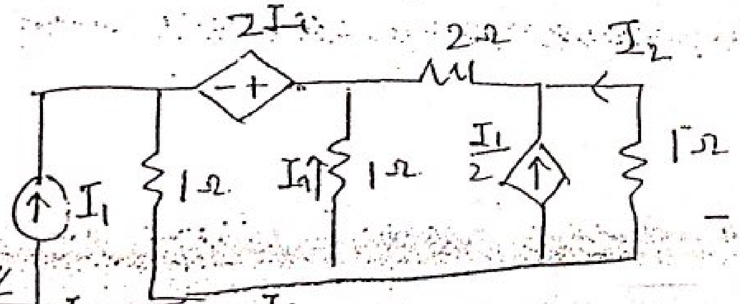
$$Z(s) = \frac{-3s}{s^2+1} + \frac{5}{s}$$

$$k\Big|_{s^2=-1} = \frac{2s^2+5}{s(s^2+1)}(s^2+1)\Big|_{s^2=-1} = \frac{-2+5}{-1} = -3$$

Since residue of the poles at $s = \pm j1$ is -ve.

So its not p.r.f.

Q find α.



$I_1 =$

$$-10x+5\,\overline{)\,-5x^2+5x+40\,}\left(\frac{x}{2}-\frac{1}{4}\right.$$
$$-5x^2+\frac{5}{2}x$$

$$\frac{5}{2}x+40$$
$$\frac{5}{2}x+\frac{5}{4}$$

$$\frac{165}{4}$$

$$\frac{A_0(x)}{A_1(x)}=\left(\frac{x}{2}-\frac{1}{4}\right)+\frac{165/4}{-10x+5}$$

$$A_2x=-165/4$$

	A_0	A_1	A_2	No. of sign change
$x=0$	+	+	−	$S_0=1$
$x=\infty$	−	−	−	$S_\infty=0$

$$S_\infty \sim S_0 = 1$$

$\therefore$ $A(\omega^2)\not> 0$ for all ω.

hence given func^n is not a p.r.f

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