Q1. What are de-Broglie waves? State de-Broglie wave hypothesis. Give experiment verification of it.

Ans. All particles in motion have a wave character characteristic, Particles such as electron, proton, neutron have wave associated in motion with them. These waves are matter waves or de-Broglie waves.

de-Broglie Hypothesis:

All material particles have a wave character in motion. Matter has particles as well as wave nature. The corresponding wavelength $\lambda = \frac{h}{p}$ $h \rightarrow$ Plank's const. $p \rightarrow$ momentum of particle.

This relation is applicable for microscopic only.

Davisson and Germer Exp.

This was the exp proof of matter waves. If material particles have wave character associated, then they expected to show the phenomena like interference, diffraction & polarization.

Experimental arrangement and working

In this exp., the e^- beam shows diffraction pattern, Electrons are emitted by thermionic emission and are accelerated by applying potential diff. The electrons are reflected from crystal along diff. direction & scattered of e^- are ~~called~~ caught in faraday cylinder (cylinder capable of rotating on circular scale to recieve e^- in all directions).

Observation

The current is plotted against an angle b/w the incident beam. Distinct, min and max. are observed.

Result

The most prominent of plot of $\theta = 50°$ & moving electrons. The e^- are diffracted by the crystal verifies the existence of electron wave.

$\lambda = \frac{12.28}{\sqrt{V}} = \text{Å}$ · The K.E of e (54eV) is small compared to rest energy (0.51MeV) of emitted e^-,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times54\times1.6\times10^{-19}}} = \frac{6.63\times10^{-34}}{4.0\times10^{-24}} = \boxed{1.66\times10^{-10}\,m = 0.166\,nm}$$

Thus, the results of Davisson & Germer Exp. completely confirm the de-Broglie's concept of matter wave.

Physics

Uni-1 Ass.

Q2. What are the characteristics of matter waves? Explain phase & group velocities.

Ans. Characteristics of matter waves:-

1. Matter waves are not electromagnetic waves and can be associated with any particle whether charged or uncharged.
2. Velocity of matter wave is greater than the vel. of electromagnetic waves.
3. Matter wave's wavelength associated with a slow moving particle is greater than the wavelength associated with fast moving particle.
4. The wave nature of matter introduces an uncertainty in the position of particle because a wave cannot be exactly at this or that point.

Phase Velocity- It is the velocity with which a particular phase of the wave propagates in medium.

$$v_p = \frac{\omega}{k} = \frac{c^2}{v}$$

Group Velocity- Waves can be in a group and such groups are called wave packets, so the velocity with which a wave packet moves forward in the medium is called group velocity.

$$v_g = \frac{d\omega}{dk} = \frac{\omega_1 - \omega_2}{k_1 - k_2}$$

Q3. The de-broglie matter waves travel with a wave velocity > speed of light. Does this violate Einstein's special theory of relativity? Justify.

Ans. We know that fact that in quantum mechanics, particles behave as waves, so, all matters behave like a wave,
Let us assume the phase vel. to be v_p.
From de Broglie hypothesis, we can write it as,

$$v_p = \frac{\omega}{k}$$

$$\Rightarrow v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p} \quad \text{--- (i)}$$

Now, we need to relate phase vel. with the relation of energy and momentum.
Therefore, we can write eq. (i) as,

$$v_p = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v} > c$$

Now, as the particle speed $v<c$ for any particle which has mass, the phase velocity of matter waves will exceed,
It means $v_p > c$
So, we can see that the vel. of matter wave is more than that of light.

Q4. a) Why is the wave nature of matter not more apparent in our daily life?

Ans. de-Broglie wavelength associated with a body of mass m, moving with vel. v is given by $\lambda = \frac{h}{mv}$. Since, the mass of the object, ~~is~~ not the de-Broglie wavelength associated with it is quite small hence it is not visible.

That's why, wave nature is not apparent.

Q5. State Heisenberg's Uncertainty Principle. By applying this, explain non-existence of e^- in atomic nucleus.

Ans. It states that, it is impossible to measure precisely and simultaneously both the members of pairs of certain canonically conjugate variables that describe the behaviour of an atomic system.

Thus, the uncertainty in momentum Δp_x, and uncertainty in position Δx in direction of x of a particle are related by

$$\Delta p_x \cdot \Delta x \geq \hbar \equiv \frac{h}{2\pi}$$

Non-existence of electrons in the nucleus –

- Electrons have a -ve charge and if they can exist in the nucleus, the +ve charges will neutralize them. The radius of a nuclei is 10^{-15} m when Heisenberg's uncertainty principle is applied and if e^- were in the nucleus, the highest uncertainty inside its position would have been 10^{-15} m.

- We know that the size of an atomic nucleus is in the range of 10^{-15} m.
- ~~The un~~ Mass of $e^- = 9.1 \times 10^{-31}$ Kg.

- By Heisenberg's principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$v \geq \frac{h}{4\pi \Delta x \cdot m}$$

$$= \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 10^{-15} \times 9.1 \times 10^{-31}}$$

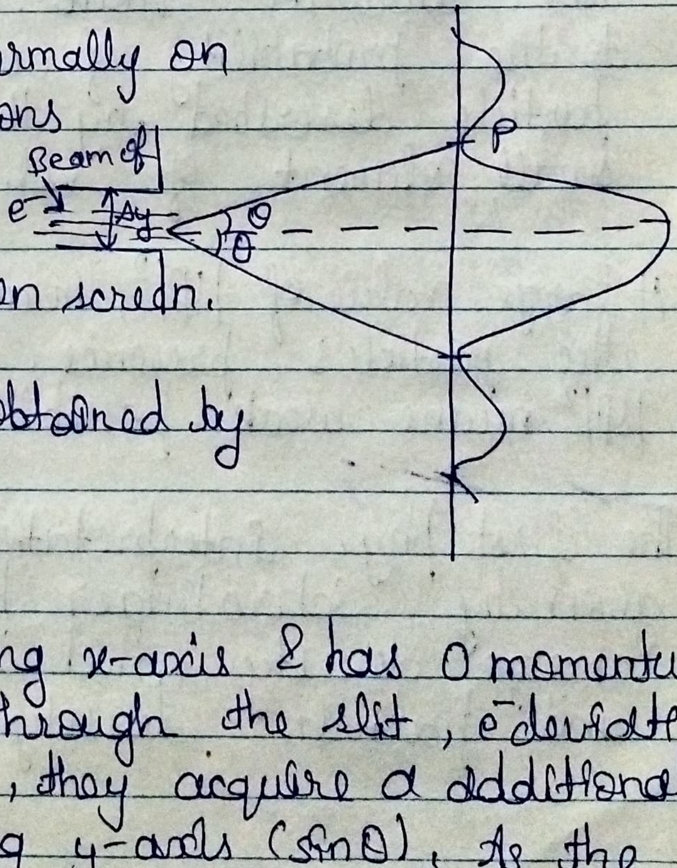
$$= 5.79 \times 10^{10} \text{ m/s}$$

- As a result, the vel. uncertainty will be 5.79×10^{10} m/s which is far more than the speed of light, which is not possible.
- In the atom, no e^- or particle has an energy greater than 4 MeV. As a result, e^- do not exist within the nucleus.

Q6. Give an experimental illustration of Heisenberg Uncertainty principle.

Ans. Diffraction of e^- beam at a single slit –

A parallel beam of e^-'s fall normally on a single slit of width Δy. Electrons get diffracted after passing through the slit & a diffraction pattern is obtained on screen.



The first minima of pattern is obtained by

$$d \sin\theta = n\lambda$$

$$\Delta y \sin\theta = \lambda$$

$$\therefore \Delta y = \lambda / \sin\theta \quad \text{— (i)}$$

Initially, the e^- are moving along x-axis & has 0 momentum along y-axis. After passing through the slit, e^- deviate from their original path, they acquire a additional component of momentum along y-axis ($\sin\theta$). As the e^- may be anywhere within the pattern from angle $-\theta$ to $+\theta$. So, the uncertainty in y-component of momentum of the electron –

$$\Delta p_y = P\sin\theta - (-P\sin\theta)$$
$$= 2P\sin\theta$$
$$= 2\frac{h}{\lambda}\sin\theta \quad \text{— (ii)}$$

$$\therefore \Delta y \cdot \Delta p_y = \frac{\lambda}{\sin\theta} \cdot \frac{2h}{\lambda} \cdot \sin\theta$$
$$= 2h$$
$$\Rightarrow \Delta y \cdot \Delta p_y \geq \hbar$$

i.e. the product of uncertainty in determining the position & momentum simultaneously is always > $\hbar$, which is Heisenberg's uncertainty principle.

Q. a) What is the physical significance of wave func?

Ans. The phy. significance is that the square of its absolute value, $|\psi|^2$, at a pt. is proportional to the probability of experimentally finding the particle described by the wave func. in a small element of vol. $d\tau$ $(dx\,dy\,dz)$ at that pt.

A large value of $|\psi|^2$, means strong possibility of the particle's presence, whereas small value of $|\psi|^2$ means weak possibility of its presence.

The first phy. interpretation of wave func. ψ was given by Schrodinger himself in terms of charge density which is a product of charge and particle density, $|\psi|^2$.

b) State the properties of a wave function.

Ans. 1. It must be finite everywhere because if ψ is infinite then means infinity possibility of finding the particle which is impossible.

2. It must be single valued bcoz if ψ has more than one value means more than 1 probability of finding the particle, which is impossible.

3. It must be continuous and have a continuous first derivative everywhere.

4. It must be normalized.

c) What is Zero-Point Energy (particle in a box).

Ans. The quantum state with lowest ($n=1$) called ground state while state with higher n ($=2,3---$) are excited states.

- For a particle in 1-D box of length 'l', the energy is given by $E_n = \frac{n^2\pi^2\hbar^2}{2ml^2}$

This is the energy of ground state as well as the min. energy of particle or we can say even when temp. of box is reduced, the total energy will still be E, and not be 0, so, it is zero point energy of particle.

d) What are Eigen values & Eigen function?

Ans. Eigen values: In infinite potential well, the particle cannot have an arbitary energy, but can have only certain discrete energy corresponding to $n=1,2,3,...$ Each energy level is Eigen value.

Eigen functions: The corresponding wave func ψ to each eigen values are eigen functions.

The normalised wave function for 1D:

$$\varphi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x.$$

where L is the length of the box.

Q8. a) Write postulates of quantum mechanics.

Ans. 1. The superposition principle is valid for functions representing phy. states as

$$\psi = \sum_i C_1\varphi_1 + C_2\varphi_2 + C_3\varphi_3 + \cdots C_i\varphi_i$$

where $c \rightarrow$ expansion co-efficient.

2. The measurements of an observable can provide the values (λ) given by the eq.

$$\beta\varphi = \lambda\psi$$

This is eigen value eq. & the values 'λ' are eigen values.

3. There is a wave func. associated with every phy. state of the system. The wave func. is a func. of all position coordinate & time and contains info. abt. the properties of the system.

b) Derive Time Independent and Time dependent Schrodinger wave equation.

Ans. Time dependent Wave Eq.:

$$\frac{\partial\psi}{\partial t} = i\omega\,\psi$$

$$= -i\,(2\pi\nu)\varphi$$

$$= -2\pi\nu i\psi$$

$$= -\frac{2\pi E i}{h}\psi \times \frac{i}{i}$$

$$= \frac{E}{i\hbar}\psi \Rightarrow E\psi = i\hbar\frac{\partial\varphi}{\partial t}$$

Subs. this value in this eq.

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\nabla^2 \psi = -\frac{2m}{\hbar^2}\left(i\hbar \frac{\partial \psi}{\partial t} - V\psi\right)$$

$$-\frac{\hbar}{2m}\nabla^2 \psi = i\hbar \frac{\partial \psi}{\partial t} - V\psi$$

$$\boxed{\left(-\frac{\hbar}{2m}\nabla^2 + V\right)\psi = i\hbar \frac{\partial \psi}{\partial t}}$$

Time- Independent Wave Eq.

Let $\psi(\vec{r},t)$ be wave displacement for de-Broglie wave at any location $\vec{r}$ at time t.

The diff. eq. of wave motion in 3-D in accordance with Maxwell's wave eq. can be written as:

$$\nabla^2 \psi = \frac{1}{u^2}\frac{\partial^2 \psi}{\partial t^2}$$

or
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2}\cdot\frac{\partial^2 \psi}{\partial t^2} \quad \text{—(i)}$$

[$u \rightarrow$ wave vel.]

The sol. of eq.(i) $\psi(\vec{r},t) = \psi_0 e^{-i\omega t}$

where ψ_0 is amplitude at pt. considered, it is a func. of position of '$\vec{r}$' & not of time 't'.

$$\frac{\partial \psi}{\partial t} = -i\omega\psi(r,t)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi(\vec{r},t)$$

From eq.(i), $\frac{\partial^2 \psi}{\partial x^2} + \cancel{\frac{\partial^2 \psi}{\partial y^2}} + \cancel{\frac{\partial^2 \psi}{\partial z^2}} = \frac{-\omega^2}{u^2}\psi(\vec{r},t)$

$\because \omega = 2\pi\nu = \frac{2\pi u}{\lambda}$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

or $$\nabla^2\psi = \frac{-4\pi^2}{\lambda^2}\psi$$

$$\left[\lambda = \frac{h}{mv}\right]$$

$$\nabla^2\psi + \frac{4\pi m^2v^2}{h^2}\psi = 0$$

If E & V are the total & potential energies of particle respectively, then

$$K.E = E - V$$

$$\frac{1}{2}mv^2 = E - V$$

$$m^2v^2 = 2m(E-V)$$

$$\therefore \nabla^2\psi + \frac{4\pi^2}{h^2} \cdot 2m(E-V)\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{8m\pi^2}{h^2}(E-V)\psi = 0}$$

or

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Q12. a) Explain principle and working of SEM.

Ans. Scanning Electron Microscope (SEM): is an instrument that produces a largely magnified image by using e^- instead of light to form an image.

Working

Principle: Electrons are liberated from a field emission source & accelerated in a high electrical field gradient, within the high vacuum column, Schrondinger Time Independent wave eq.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E-V]\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar}[E - \frac{1}{2}Kx^2]\psi = 0$$

solving this eq. under the condition

$$\psi^*\psi \longrightarrow 0 \quad at \quad x \rightarrow \infty$$

we get,

$$\boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega}$$

- These energy levels are equally spaced also.

$$E_1 = \frac{3}{2}\hbar\omega = \frac{3}{2}h\nu$$

$$E_2 = \frac{5}{2}\hbar\omega = \frac{5}{2}h\nu$$

$$E_3 = \frac{7}{2}\hbar\omega = \frac{7}{2}h\nu$$

$$\boxed{E_{n+1} - E_n = \hbar\omega = h\nu}$$