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- *-ve, +ve Feed back*

Control Systems

UNIT - I

Introduction :

- System : A system is a combination of different physical components or an arrangement of different physical components which act together as an entire unit to achieve certain objective. Every physical object is a system.

Example : A class room, lamp, kite etc.

- Control System : In a system, when the output quantity is controlled by varying the input, then the system is called control system.
  - Output quantity is called controlled variable (or) response and input quantity is called command signal or excitation.
  - The word 'control' refers to command, regulate or direct.

Example : 1. In a classroom, the professor is delivering lecture, the combination becomes a control system.

2. A lamp with ON-OFF switch combination becomes a control system.

- Classification of control systems :
  1. Natural control system
  2. Man-made control system
  3. combinational control system
  4. Time varying systems.

Time varying system :

The systems in which parameters of the system are varying with time.

It is not dependent on whether input and output are functions of time or not.

5. Time Invariant control system :

The systems in which inputs and outputs are functions of time but the parameters are independent of time and constant.

6. Linear control system :

A control system is said to be linear, if it satisfies

a) principle of superposition which says that the response to several inputs can be obtained by considering one input at a time and then algebrically adding individual results.

b) The differential equation describing the system is linear having its coefficients as constant.

c) Practically, the output varies linearly with the input.

7. Non-Linear control system :

A control system is said to be non-linear if it :

a) does not satisfies the principle of superposition.

b) the differential equation describing the system is non-linear

c) If the output does not vary linearly with input.

8. Continuous-time control system :

The system in which all system variables are function of continuous time variable t.

9. Discrete time control system :

The system in which all system variables are discrete with respect to time.

10. Deterministic control system :

A control system is said to be deterministic when its response to input as well as behaviour to external disturbances is predictable and repeatable.

11. Stochastic control System :

A control system is said to be stochastic in nature if its response is unpredictable.

12. Lumped parameter control system :

If the system can be described by ordinary differential equation, then it is called lumped parameter control system.

13. Distributed parameter control system :

If the system is described by partial differential equation, then it is called Distributed parameter control system

14. Single input - single output system (SISO) :

A system having only one input and one output is called single input single output system.

15. Multiple Input Multiple output system (MIMO):

A system having multiple inputs and outputs. is called MIMO system

16. Open Loop control system :

Any physical system which does not automatically correct the variation in its output is called open loop control system.

17. Closed Loop control system :

Control systems in which the output has an effect upon the input quantity in such a manner as to maintain that the desired value are called closed Loop control systems

- A control system is an interconnection of physical components to provide a desired function, involving some kind of controlling action in it.

Plant :

The position of the system which is to be controlled or regulated is called Plant (or) Process.

Controller :

The element of the system itself or external to the system which controls the plant is called controller.

Input :

It is an applied signal or an excitation signal applied to the control system from an external energy source in order to produce a specified output.

Output :

It is the particular signal of interest or the actual response obtained from the control system when input is applied to it.

Disturbances :

It is a signal which tends to adversely effect the output. If such a disturbance is generated within the system, it is called Internal disturbance and if it is generated outside the system acting as an extra input to the system. then it is called external Disturbances.

* Open Loop control System :

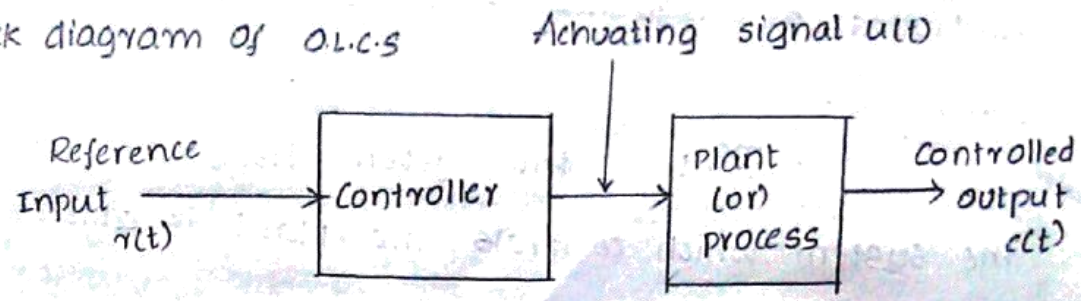
A system in which control action is independent of output is called open loop control system.

(or)

Any physical system which does not automatically

correct the variations in its output is called Open loop control system. [O.L.C.S]

Block diagram of O.L.C.S



Reference Input r(t) is applied to the controller which generates actuating signal u(t) required to control the process which is to be controlled.

Process is giving out the necessary desired controlled output c(t)

Advantages :

- simple to design
- Economical
- Easier to construct
- maintainance is very easy
- Generally they are stable.

Disadvantages :

- They are inaccurate and unreliable because accuracy of such systems totally depends on accurrate pre-caliberation. of the controller.
- Open loop systems gives inaccurrate results if there are variations in external environments ie, such systems cannot sense environmental changes.
- Open loop systems cannot sense internal

disturbances in the system after controller stage.

- To maintain quality and accuracy re-calibration of the controller is necessary from time to time.

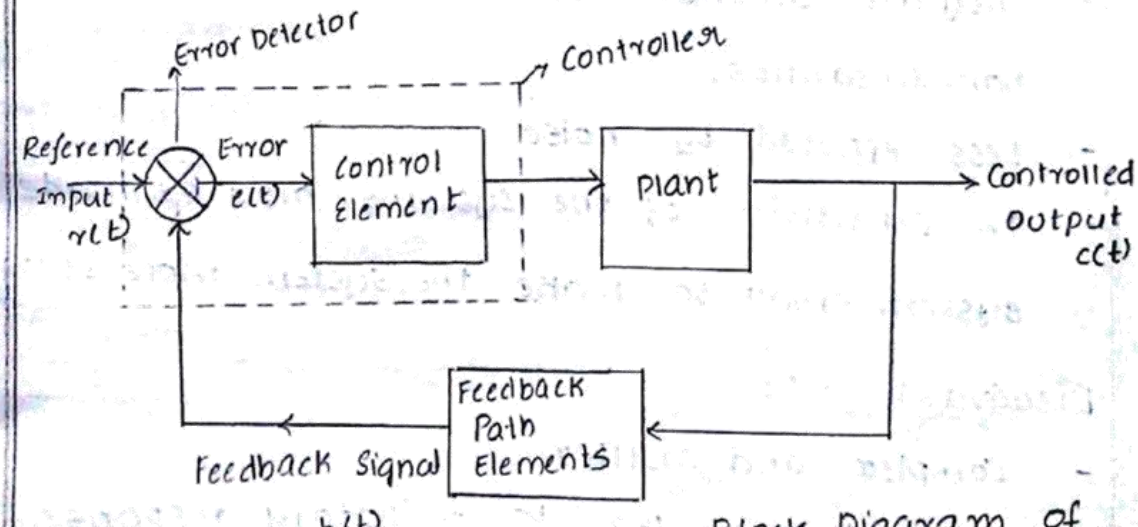
Examples : Sprinkler used to water a lawn, Traffic Light controller, Automatic door opening and closing system, Toaster system, stepper Motor positioning system

* Closed Loop Control System :

A system in which controlling Action or Input is depending on the output (or) changes in the output is called closed loop system.

To have dependence of Input on output, such system uses feedback property

- Feedback is the property of a system by which it permits the output to be compared with reference input to generate error signal based on which appropriate controlling action can be decided.



Block Diagram of closed Loop Control System

The part of the output which is feedback signal b(t) is fed back to the input (reference). It is then compared with reference input giving error signal

$\therefore e(t) = r(t) \pm b(t)$

If the feedback sign is positive, then the systems are called positive feedback systems and vice versa.

This error signal is then modified by controller and decides proportional manipulated signal for the process to be controlled. This manipulation is such that the error will approach zero.

This system signal then activates the actual system and produces an output. As output is controlled one, hence called controlled output c(t)

Advantages :

- Accurrate
- They are accurrate even in the presence of non-linearities.
- Less effected by noise
- The sensitivity of the systems may be made system small to make the system more stable

Disadvantages :

- Complex and costlier
- Feedback may lead to oscillatory response
- Feedback reduces overall again
- Stability is the major problem in closed loop system.

Examples : Traffic Light controller, Home heating system, voltage stabiliser, ship stabilisation systems, manual speed control systems, Missile launching systems etc.

* Differences between Open and Closed systems :

Open Loop	Closed Loop
1. Output has no effect on input	1. Change in output effect the input
2. Easy to design	2. Complex to design
3. No Feedback is present	3. Feedback is present.
4. Error detection is not possible	4. Error Detection is possible
5. These systems are stable	5. These are unstable in certain conditions.
6. High sensitivity to disturbances	6. Low sensitivity.
7. They are inaccurate	7. They are accurrate
8. These systems are cheaper	8. Costlier.
9. Easy to maintain	9. Maintainance is difficult.
10. Cannot sense the changes	10. These systems can sense the changes.
11. Gain Increases.	11. Overall gain decreases

Feedback :

The process of taking a fraction of output signal and feeding it back to the input circuit is known as feedback.

There are two types of Feedback

1. Positive Feedback
2. Negative Feedback

1. Positive Feedback :

If the feedback voltage (or) current is in phase with the input signal, then it is known as positive feedback.

2. Negative Feedback :

If the feedback voltage (or) current is 180° out of phase with the input signal and thus decreases the amplifier input, then it is called Negative Feedback Amplifier

* Comparision of Positive and Negative Feedback :

Positive Feedback	Negative Feedback.
1. In positive feedback, the feedback signal is in phase with the input signal.	1. In negative Feedback, the feedback signal is out of phase with the the input signal.
2. It is also called as Regenerative Feedback	2. It is also called as Degenerative Feedback.

	Positive Feedback	Negative Feedback.
3.	It increases the net input to amplifier $V_i = V_s + V_f$	3. It reduces net input to amplifier $V_i = V_s - V_f$
4.	It increases gain of the amplifier	4. It reduces gain of the amplifier.
5.	It increases noise and distortion.	5. It reduces the noise and distortion.
6.	It decreases stability of amplifier gain	6. It improves the stability of amplifier gain
7.	It is used in oscillator circuits	7. It is used in amplifiers.

* Mathematical Models :

Transfer function is one of the ways to model a system mathematically when a physical system is analysed, mathematical model is developed by writing differential equations with the help of various physical laws governing the systems.

Most of the control systems, consists of electrical and mechanical equipment. The transfer function is very helpful concept in modeling the interconnection of sub-systems by forming a block diagram.

- Transfer function of Linear Time Invariant system [LTI] is defined as ratio of Laplace Transform of ~~Input~~ output

to laplace transform of input under the assumption that initial conditions are zero

Steps involved in obtaining Transfer function are as follows :

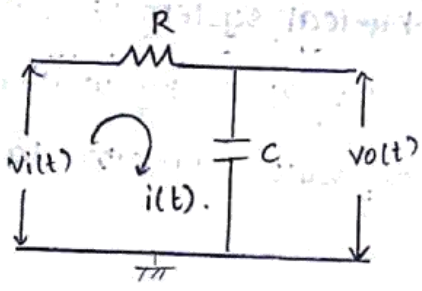
1. Write differential equations governing the system.
2. Find laplace transform i.e., replace the term d/dt by s and $\int dt$ by $\frac{1}{s}$
3. Find ratio of transformed output to input variables.

Impulse Response :

Impulse Response is defined as output when the input is a unit impulse function.

So laplace transform of Impulse Response is its transfer function with all its initial conditions assumed to be zero.

1.



$$V_i(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$$

$$\Rightarrow \quad V_o(t) = \frac{1}{C}\int i(t)\,dt.$$

$$V_i(s) = I(s)R + \frac{I(s)}{Cs}.$$

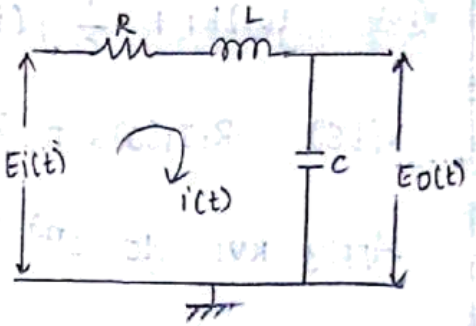
$$V_i(s) = I(s)\left[R + \frac{1}{Cs}\right].$$

$$V_o(s) = \frac{I(s)}{Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{I(s)}{Cs}}{I(s)\left[\frac{RCs+1}{Cs}\right]}$$

$$= \frac{1}{1+RCs}$$

2. $E_i(t) = i(t)R + L\cdot\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt$



$\Rightarrow E_i(s) = RI(s) + LsI(s) + \frac{1}{C}\frac{I(s)}{s}$

$$E_i(s) = I(s)\left[R + Ls + \frac{1}{Cs}\right]$$

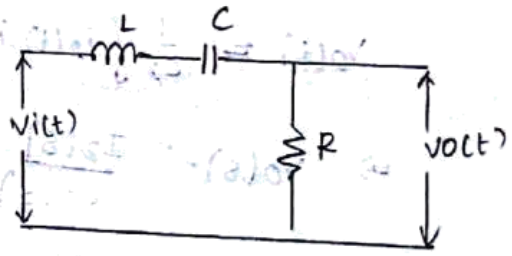
$$E_o(t) = \frac{1}{C}\int i(t)\cdot dt$$

$$E_o(s) = \frac{I(s)}{Cs}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{I(s)}{Cs}}{I(s)\left[\frac{RCs+s^2LC+1}{Cs}\right]}$$

$$= \frac{1}{s^2LC + RCs + 1}$$

3.



$$V_i(t) = L\frac{di}{dt} + \frac{1}{C}\int i(t)dt + i(t)R$$

$\Rightarrow V_i(s) = LI(s)s + \frac{1}{C}\frac{I(s)}{s} + RI(s)$

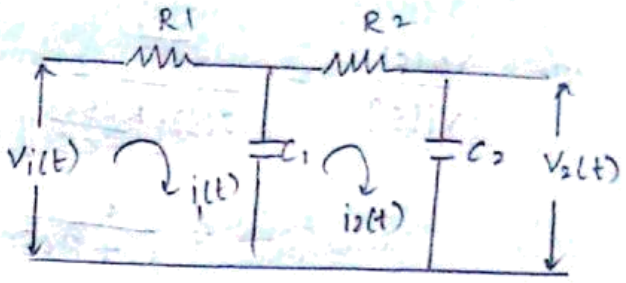
$$V_i(s) = I(s)\left[Ls + \frac{1}{Cs} + R\right]$$

$$V_o(t) = i(t)R$$

$$V_o(s) = RI(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{RI(s)}{I(s)\left[\frac{s^2LC+RCs+1}{Cs}\right]} = \frac{sRC}{s^2LC+RCs+1}$$

4.



$$V_i(t) = i_1(t)R_1 + \frac{1}{C_1}\int (i_1(t) - i_2(t))\,dt$$

$$V_i(s) = R_1 I_1(s)\$ + \frac{I_1(s)}{C_1 s} - \frac{I_2(s)}{C_1 s} \quad \text{— (1)}$$

Apply KVL to 2nd loop.

$$\frac{1}{C_1}\int (i_2(t) - i_1(t))\,dt + R_2 i_2(t) + \frac{1}{C_2}\int i_2(t)\,dt = 0$$

$$\frac{I_2(s)}{C_1 s} - \frac{I_1(s)}{C_1 s} + R_2 I_2(s)\$ + \frac{I_2(s)}{C_2 s} = 0 \quad \text{— (2)}$$

$$-\frac{I_1(s)}{C_1(s)} + I_2(s)\left[R_2\$ + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] = 0. \quad \text{— (2)}$$

$$V_i(s) = I_1(s)\left[R_1\$ + \frac{1}{C_1 s}\right] - \frac{I_2(s)}{C_1 s} \quad \text{— (1)}$$

$$V_0(t) = \frac{1}{C_2}\int i_2(t)\,dt = \frac{I_2(s)}{C_2 s}$$

$$\Rightarrow V_0(s) = \frac{I_2(s)}{C_2(s)} \quad \text{— (3)}.$$

$$(2) \Rightarrow I_1(s) = I_2(s)\left[R_2\$ + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right] C_1(s)$$

subs. in eqn — (1)

$$V_i(s) = I_2(s)\left[R_2\$ + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right]\left[R_1\$ + \frac{1}{C_1 s}\right] - \frac{I_2(s)}{C_1 s}.$$

$$V_i(s) = I_2(s)\left[R_1 + R_1R_2C_1s + \frac{C_1}{C_2}R_1 + \frac{1}{C_1s} + \frac{1}{C_2s} + \frac{1}{C_1s} + R_2\right]$$

$$V_o(s) = \frac{\frac{I_2(s)}{C_2s}}{I_2(s)\left[R_1 + R_1R_2C_1s + \frac{C_1R_1}{C_2} + \frac{1}{C_1s} + \frac{1}{C_2s} + \frac{1}{C_1s} + R_2\right]}$$

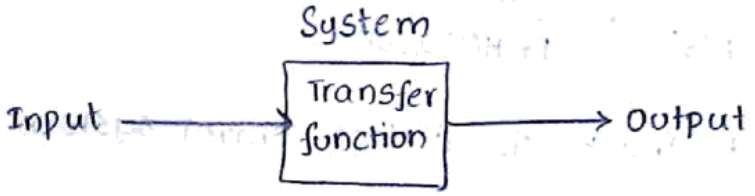
$$\text{(1)} \Rightarrow V_i(s) = R_1I(s) + \frac{I(s)}{C_1(s)} -$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{C_2(s)\left[R_1 + R_1R_2C_1s + \frac{C_1}{C_2}R_1 + \frac{1}{C_2s} + R_2\right]}$$

$$= \frac{1}{R_1C_2s + R_1R_2C_1C_2s^2 + C_1R_2R_1s + 1 + R_2C_2s}$$

* Transfer Functions of Open loop and closed loop systems :

Open loop system :

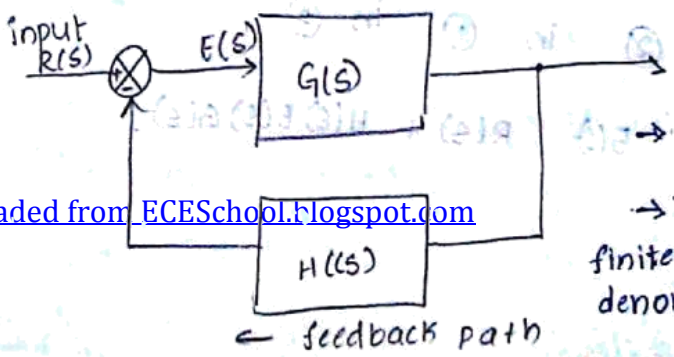


closed loop System :

They can be divided into 2 categories based on nature of feedback signal.

1. Negative feedback closed loop control system (Degenerative feedback)
2. Positive feedback closed loop control system (Regenerative feedback)

1' Negative feedback closed loop system :



→ The transfer function of -ve feedback system is generally less than 1.

→ The closed loop T.F is a finite value as long as denominator is >0

G represents the gain of all the blocks in forward path and H represents the gain of all the blocks in feedback path

From the figure $C(S) = G(S)\,E(S)$ — ①

But $E(S) = R(S) - H(S)\,C(S)$ — ②

Subs. ① in ②

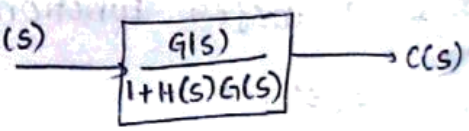
$\Rightarrow E(S) = R(S) - H(S)G(S)E(S)$

$\Rightarrow R(S) = E(S)[1 + H(S)G(S)]$

$\Rightarrow E(S) = \dfrac{R(S)}{1 + H(S)G(S)}$ — ③

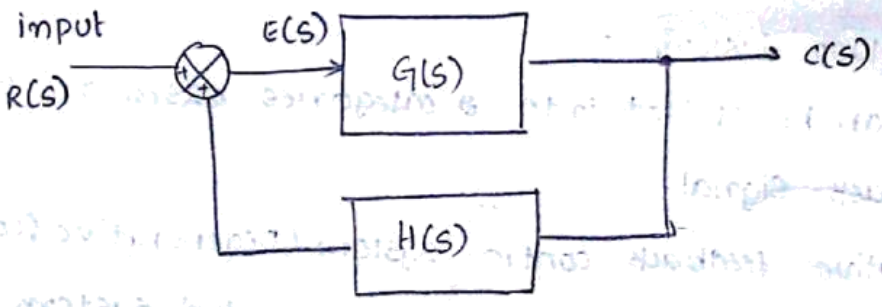
Subst ③ in ①

$\Rightarrow C(S) = G(S) \cdot \dfrac{R(S)}{1 + H(S)G(S)}$



$\Rightarrow \boxed{\dfrac{C(S)}{R(S)} = \dfrac{G(S)}{1 + H(S)G(S)}}$

2. Positive Feedback closed loop control system :



From the figure $C(S) = E(S)G(S)$ — ①

But $E(S) = R(S) + H(S)\,C(S)$ — ②

Subs. ② in ① in ②

$\Rightarrow E(S) = R(S) + H(S)\,E(S)\,G(S)$.

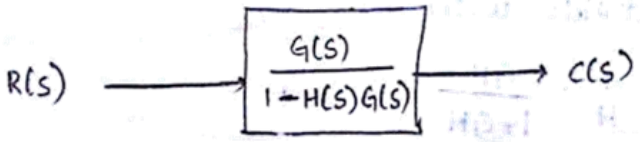
$$\Rightarrow R(s) = E(s)\left[1 - H(s)G(s)\right]$$

$$\Rightarrow E(s) = \frac{R(s)}{1 - H(s)G(s)} \quad \text{---} (3)$$

Subs. (3) in (1)

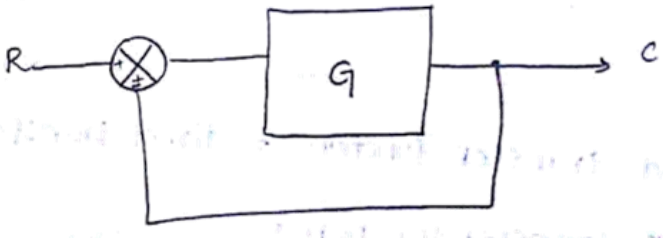
$$\Rightarrow C(s) = G(s) \cdot \frac{R(s)}{1 - H(s)G(s)}$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - H(s)G(s)}}$$



- The transfer function tends to very large value as GH approaches 1. It is generally used in oscillators

* Unity Feedback Systems :



Unity feedback systems are those in which the feedback factor H is 1.

For Negative Feedback

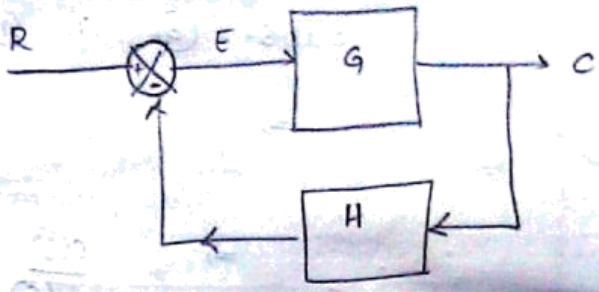
$$\boxed{\frac{C(s)}{R(s)} = \frac{G}{1+G}}$$

For positive feedback

$$\boxed{\frac{C(s)}{R(s)} = \frac{G}{1-G}}$$

* Conversion to a unity feedback systems :

Any closed loop system can be converted to an equivalent unity feedback system.



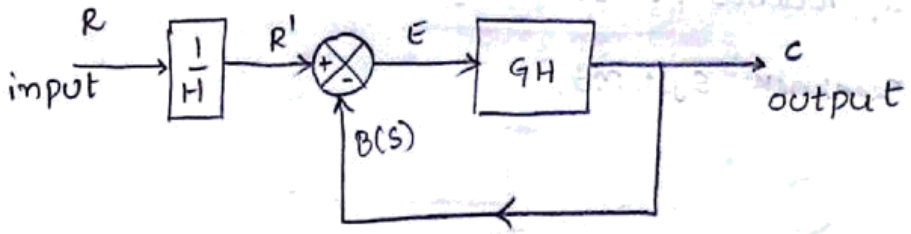
For Negative Feedback system

$$TF = \frac{G}{1+GH}$$

Multiply and divide with H

$$TF = \frac{1}{H} \cdot \frac{GH}{1+GH}$$

$$TF = \frac{1}{H} \cdot \frac{G'}{1+G'}, \text{ where } G' = GH$$



G(S) = forward transfer function = direct transfer function

H(S) = feedback transfer function

G(S) H(S) = Loop transfer function (or) open loop transfer function.

$\frac{C(S)}{R(S)}$ = closed loop transfer function = control ratio

$\frac{E(S)}{R(S)}$ = Actuating signal ratio (or) Error ratio

$\frac{B(S)}{R(S)}$ = Primary feedback ratio.

* Analysis of Mechanical Systems :

1. Mechanical systems whether translational (or) rotational, obey the basic law that the sum of