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(SUMMARY + IMPORTANT QUESTIONS)

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SYLLABUS (From Academic Session 2021-22)

Applied Physics-I [BS-105]

Marking Scheme:

(a) *Teacher Continuous Evaluation: 25 marks*

(b) *Term End Theory Examination : 75 marks*

UNIT I

Introduction to Thermodynamics: Fundamental Ideas of Thermodynamics, The Continuum Model, The concept of a "System", "State", "Equilibrium", "Process". Equations of state, Heat, Zeroth Law of Thermodynamics, Work, first and second laws of thermodynamics, entropy. **[8 Hrs.]**

UNIT II

Waves and Oscillations: Wave motion, simple harmonic motion, wave equation, superposition principle.

Introduction to Electromagnetic Theory: Maxwell's equations, work done by the electromagnetic field, Poynting's theorem, Momentum, Angular momentum in electromagnetic fields, Electromagnetic waves: the wave equation, plane electromagnetic waves, energy carried by electromagnetic waves. **[8 Hrs.]**

UNIT III

Interference: Interference by division of wave front (Young's double slit experiment, Fresnel's biprism), interference by division of amplitude (thin films, Newton's rings, Michelson's interferometer), Coherence and coherent sources.

Diffraction: Fraunhofer and Fresnel diffraction; Fraunhofer diffraction for Single slit, double slit, and N-slit (diffraction grating), Fraunhofer diffraction from a circular aperture, resolving power and dispersive power of a grating, Rayleigh criterion, resolving power of optical instruments.

Polarization: Introduction to polarization, Brewster's law, Malu's law, Nicol prism, double refraction, quarter-wave and half-wave plates, optical activity, specific rotation, Laurent half shade polarimeter. **[12 Hrs.]**

UNIT IV

Theory of relativity: The Michelson-Morley Experiment and the speed of light; Absolute and Inertial frames of reference, Galilean transformations, the postulates of the special theory of relativity, Lorentz transformations, time dilation, length contraction, velocity addition, mass energy equivalence Invariance of Maxwell's equations under Lorentz Transformation.

Introduction to Laser Physics: Introduction, coherence, Einstein A and B coefficients, population inversion, basic principle and operation of a laser, the He-Ne laser and the Ruby laser. **[12 Hrs.]**

FIRST SEMESTER
APPLIED PHYSICS-I [BS 105]
FROM ACADEMIC SESSION 2021-22

UNIT-I : INTRODUCTION TO THERMODYNAMICS

Introduction to Thermodynamics: Fundamental Ideas of Thermodynamic, The Continuum Model, The Concept of a "System", "State", "Equilibrium", "Process". Equations of state, Heat. Zeroth Law of Thermodynamics, Work, first and second laws of thermodynamics, entropy

[8 Hrs]

1. Two bodies are in the thermal equilibrium with each other if they have a common temperature.

2. The zeroth law of thermodynamics states that if bodies A and B are separately in thermal equilibrium with a third body C, then A and B are in thermal equilibrium with each other.

3. Heat exchange is a form of energy transfer that takes places as a consequence of temperature difference.

4. Thermodynamic work is a path dependent quantity.

5. The general form of first law of thermodynamics is

$Q = U + W$; where Q is the heat added to a system, U is the change in internal energy and W is the work done by the system.

6. Heat engines are cyclic devices that extract heat from a high-temperature reservoir and convert part of it into work.

7. The thermal efficiency η of a heat engine is defined as the ratio of the net work done to the heat absorbed per cycle.

$$\eta = \frac{\text{work out put}}{\text{heat absorbed}} = \frac{W}{Q_H}$$

8. The efficiency of Carnot engine is

$\eta = 1 - \frac{T_L}{T_H}$; No heat engine operating between T_H and T_L can have an efficiency greater than η (Carnot).

9. The second law of thermodynamics places limits on the conversion of heat into work. The complete conversion of heat into work is not possible.

10. Entropy measures the disorder of a system at the molecular level.

11. The entropy changes dS during a reversible process is defined by

$dS = \frac{dQ}{T}$; where dQ is the heat exchanged and T is the temperature.

12. Entropy is not a conserved quantity.

13. Irreversible processes drive an isolated system toward equilibrium while increasing its entropy. The equilibrium state corresponds to a state of maximum entropy.

14. In terms of entropy, the second law of thermodynamics may be stated as follows: "The entropy of an isolated system never decreases."

15. The third law of thermodynamics state that "it is impossible to attain absolute zero temperature".

Example 1. An engineer claims his engine to develop 5 H.P. On testing the engine consumes 0.44 kg of fuel per hour having a calorific value 10,000 K cal/kg. The maximum temperature recorded in the cycle is 1400°C and the minimum is 350°C. Find whether the engineer is justified in the claim.

Solution: The maximum possible efficiency between the specified limits of T_1 and T_2 is

$$\eta = \frac{T_1 - T_2}{T_1}$$

here

$$T_1 = 1400 + 273 = 1673 \text{ K}$$

$$T_2 = 350 + 273 = 623 \text{ K}$$

$$\therefore \quad \eta_{max} = \frac{1673 - 623}{1673} \times 100 = 62.8\%$$

Efficiency of the engine developed by the engineer.

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

Q_1 = heat consumed in one hour

$$= 0.44 \times 10000 = 4400 \text{ kcal.} = \text{heat input.}$$

Power developed = 5 HP

$$\therefore \text{ Equivalent heat output } = \frac{5 \times 746 \times 3600}{4200} \text{ k cal}$$

Hence

$$\eta = \frac{5 \times 746 \times 3600}{4200 \times 4400} \times 100 = \mathbf{72.66\%}.$$

Since the efficiency of the engine is more than the maximum possible efficiency of Carnot cycle, hence the claim is not justified.

Example 2. An engine is designed on Carnot cycle to operate between 500 K and 300 K. Assuming that the engine actually produces 1.56 kJ mechanical energy per kilo calorie of heat absorbed, compare the actual efficiency with theoretical maximum efficiency.

Solution: As the engine works on Carnot cycle its efficiency will be

$$\eta = \frac{500 - 300}{500} = 50\%$$

$$\text{Actual efficiency} = \frac{\text{Energy output}}{\text{Energy input}} = \frac{1.57 \times 1000}{4.184 \times 1000} = 0.375 = 37.5\%$$

The actual efficiency is about 75% of the maximum efficiency.

Example 3. A steam engine operating between a boiler temperature 493 K and a condenser temperature of 308 K delivers 8 H.P. If its efficiency is 30% of that are Carnot engine operating between these temperature limits how many calories are absorbed each second by the boiler? How many calories are exhausted to the condenser per second?

Solution: Efficiency of Carnot engine working between these limits

$$\eta = 1 - \frac{308}{493} = 0.375 \text{ or } 37.5\%$$

$\therefore$ Actual efficiency $= 0.30 \times \eta = 0.30 \times 37.5$

$= 0.113$ or 11.3%

We know Efficiency $= \dfrac{\text{Output work}}{\text{Input heat}}$

$\therefore$ Heat input = Output work / Efficiency

$$= \frac{8 \times 746}{0.113} W = \frac{8 \times 747}{0.113 \times 4.184}$$

$$= 12.6 \text{ k cal/s}$$

The energy rejected to the condenser can be determined by using energy conservation law i.e. Input energy = Output work + Rejected energy

$\therefore$ Rejected energy = Input energy – Output work

= Input energy (1 – efficiency)

= 12.6 (1 – 0.113) = **11.3 k cal/s.**

Example 4. How many kilograms of water at 0°C can a freezer with a performance coefficient 5 make into ice cubes at 0°C with a work input of 3.6 MJ. Take latent heat of ice as 80 cal/g.

Solution: Let m kg be the mass of the water converted in to ice. Heat required to be taken out for freezing m kg of water

$$= m \times 80 \times 4.184 \times 10^3 = 334.72 \times 10^3 \text{ J}$$

The coefficient of performance,

= Heat removed/work input

$$\therefore \quad 5 = \frac{m \times 334.72 \times 10^3}{3.6 \times 10^6} = 92.98 \times 10^{-3} \text{ m}$$

or m = **53.8 kg.**

Example 5. The efficiency of an otto cycle is 50% and γ is 1.5. Calculate the compression ratio.

Solution: The efficiency of an otto cycle

$$\eta = \left[1 - \frac{}{\gamma -}\right] \times 100\%$$

$$\therefore \quad 50 = \left[1 - \frac{1}{r^{1.5-1}}\right] \times 100$$

or $$\frac{1}{2} = 1 - \frac{1}{r^{1.5-1}}$$

or $$\frac{1}{r^{0.5}} = \frac{1}{2} \text{ or } r^{0.5} = 2$$

$$\therefore \quad r = 2^{(1/0.5)} \text{ or } r = (2)^2 = 4$$

Hence compression ratio = **4.**

Example 6. A gas engine working on the otto cycle has cylinder diameter of 15 cm and a stroke of 22 cm. If the clearance volume is 1311 cm³ find the air standard efficiency of the engine. Given γ as 1.4.

Solution: Swept volume

$$\pi r^2 l = \pi \times \frac{15}{2} \times \frac{15}{2} \times 22 = 3887.72 \text{ cm}^2$$

Total volume of the cylinder = Swept volume + Clearance volume

$$= 3887.72 + 1311 = 5198.72 \text{ cm}^3$$

$$\text{Compression ratio} = \frac{\text{Total volume}}{\text{Clearance volume}} = \frac{5198.72}{1311} = 3.966.$$

∴ Air standard efficiency

$$\eta = 1-\left(\frac{1}{3.966}\right)^{1.4-1} = 1-\left(\frac{1}{3.966}\right)^{0.4}$$

$$= 1 - 0.576 = 0.4236$$

Hence efficiency is **42.36%.**

Example 7. What is the thermal efficiency and the compression ratio of an engine working on Otto cycle. The measured suction temperature was 100°C and the temperature at the end of the compression was 300°C. The value of γ is 1.41.

Solution:

$$T_1 = \text{Suction temperature}$$

$$= 100 + 273 = 373 \text{ K}$$

$$T_2 = \text{Temperature at the end of compression}$$

$$= 300 + 273 = 573 \text{ K}$$

∴
$$\eta = 1-\frac{T_1}{T_2} = 1-\frac{373}{573} = 1 - 0.65 = 0.35$$

or
$$\eta = 35\%$$

Now we know efficiency of an otto engine is

$$\eta = 1-\left(\frac{1}{r}\right)^{\gamma-1} \quad \therefore 0.35 = 1-\left(\frac{1}{r}\right)^{1.41-1}$$

or
$$0.35 = 1-\left(\frac{1}{r}\right)^{0.41} \text{ or } 0.65 = \left(\frac{1}{r}\right)^{0.41}$$

or
$$\frac{1}{r} = (0.65)^{1/0.41} \text{ or } \frac{1}{r} = (0.65)^{2.439}$$

or
$$r = \mathbf{2.86.}$$

Example 8. Calculate the percentage increase in the efficiency of an engine if its compression ratio is increased from 6 to 8. Take γ = 1.4

Solution: Let η_1 and η_2 be the efficiency when compression ratio is 6 and 8 respectively.

∴
$$\eta_1 = 1-\left[\frac{1}{6}\right]^{1.4-1} = 1-\left[\frac{1}{6}\right]^{0.4}$$

and
$$\eta_2 = 1-\left[\frac{1}{8}\right]^{1.4-1} = 1-\left[\frac{1}{8}\right]^{0.4}$$

Percentage increase in efficiency

$$= \frac{\eta_2 - \eta_1}{\eta_1} \times 100 = \frac{1 - \left[\frac{1}{8}\right]^{0.4} - 1 + \left[\frac{1}{6}\right]^{0.4}}{1 - \left[\frac{1}{6}\right]^{0.4}} \times 100$$

$$= \frac{-\left[\frac{1}{8}\right]^{0.4} + \left[\frac{1}{6}\right]^{0.4}}{1 - \left[\frac{1}{6}\right]^{0.4}} \times 100 = \frac{-0.435275 + 0.488359}{1 - 0.4884} \times 100 = \frac{0.05308}{0.5116} \times 100$$

$$= 0.10375 \times 100 = \mathbf{10.38\%}.$$

Example 9. Calculate the efficiency of an engine having compression ratio 13.8 and the expansion ratio and working on diesel cycle. Given $\gamma = 1.4$

Solution:

$$\rho = 13.8;\ r = 6$$

$$k = \frac{\rho}{r} = \frac{13.8}{6} = 2.3$$

Since for a diesel engine

$$\eta = 1 - \left[\frac{1}{\rho}\right]^{\gamma - 1} \frac{(k^{\gamma} - 1)}{\gamma(k - 1)}$$

$$\eta = 1 - \left[\frac{1}{13.8}\right]^{1.4 - 1} \frac{(2.3^{1.4} - 1)}{1.4(2.3 - 1)}$$

$$= 1 - \left[\frac{1}{13.8}\right]^{0.4} \frac{(2.3^{1.4} - 1)}{1.4 \times 1.3}$$

$$= 1 - 0.34998 \times \frac{(3.209 - 1)}{1.82}$$

$$= 1 - \frac{0.34998 \times 2.209}{1.82}$$

$$= 1 - 0.42578 = 0.5752$$

or

$$\eta = \mathbf{57.25\%}.$$

Example 10. Find the efficiency of a diesel engine having a compression ratio of 15 and cutoff taking place at 6% of the stroke. Take $\gamma = 1.4$.

Solution: Let the clearance volume = V

Since the compression ratio is 15

$$\frac{\text{Total volume}}{\text{Clearance volume}} = 15$$

or Total volume = $15 \times V$

Swept volume = $15V - V = 14V$

Hence value of 6% of stroke

$$= 6\% \text{ of swept volume}$$

$$= \frac{6}{100} \times 14V = 0.84\ V$$

Cut off volume

$$= \text{clearance volume} + 6\% \text{ of swept volume}$$

$$= V + 0.84\,V = 1.84\,V$$

Now air standard efficiency

$$\eta = 1 - \frac{1}{\gamma}\frac{1}{(\rho^{\gamma-1})}\frac{(k^{\gamma}-1)}{(k-1)}, \text{ here } \rho = 15$$

$$k = \frac{V_3}{V_2} = \frac{\text{Cut off volume}}{\text{Clearance volume}} = \frac{1.84\,V}{V} = 1.84$$

Substituting the volume of r, k and γ we have

$$\eta = 1 - \frac{1}{1.4} \times \frac{1}{(15)^{1.4-1}} \frac{1.84^{1.4}-1}{1.84-1}$$

$$= 1 - \frac{1}{1.4} \times \frac{1}{(15)^{0.4}} \times \frac{(2.348-1)}{0.84}$$

$$= 1 - \frac{1}{1.4} \times \frac{1}{2.954} \times \frac{1.348}{0.84}$$

$$= 1 - 0.388 = 0.6119$$

Hence efficiency is **61.2%**.

Example 11. 50 gram of water at 0°C is mixed with an equal mass of water at 80°C. Calculate resultant increase in entropy.

Solution:

$$m_1 = 50 \text{ g}, T_1 = 273$$

$$m_2 = 50 \text{ g}, T_2 = 353$$

If final temperature of the mixture is T

$$m_1 C (T - T_1) = m_2 C (T_2 - T_1)$$

or $$50 \times 1 \times (T - 273) = 50 \times 1 \times (353 - T)$$

or $T = 313$ K change in entropy of 50 g of water while its temperature increases to 313 from 273

at

$$= \frac{\delta H}{T} = mC \int_{T_1}^{T} \frac{dT}{T}$$

$$= 50 \times 1 \times \log_e \frac{313}{273}$$

$$= 50 \times 2.3026 \log_{10} \frac{313}{273}$$

$$= +6.837 \text{ cal / K (gain in entropy)}$$

Similarly change in entropy of 50 g of water when its temperature falls from 353 K to 313 K

$$= \frac{\delta H}{T} = mC \int_{T_1}^{T} \frac{dT}{T}$$

$$= 50 \times 1 \times \log_e \frac{313}{353}$$

$$= 50 \times 2.3026 \times \log_{10} \frac{313}{353}$$

$$= -6.013 \text{ cal/K} \qquad \text{(loss of entropy)}$$

∴ The total gain in entropy = 6.837 – 6.013 = **0.824 cal/K.**

Example 12. A quantity of heat ΔQ is transferred from a large heat reservoir at temperature T_1 to another large heat reservoir at temperature T_2 ($T_1 > T_2$) required for spontaneous transfer. The heat reservoirs are of infinite capacity so that there is no observable change in their temperature. Show that the entropy of the entire system has increased.

Solution: As reservoir is at constant temperature T_1 and gives heat ΔQ, its entropy change is

$$\Delta\phi_1 = \frac{\Delta Q_1}{T_1} = -\frac{\Delta Q}{T_1}$$

The reservoir at constant temperature T_2 receives heat ΔQ, so its entropy change is

$$\Delta\phi_2 = \frac{\Delta Q_2}{T_2} = \frac{\Delta Q}{T_2}$$

The total entropy change

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 = -\frac{\Delta Q}{T_1} + \frac{\Delta Q}{T_2}$$

$$= \Delta Q \frac{T_1 - T_2}{T_1 T_2}$$

Since $T_1 > T_2$, $\Delta\phi$ is positive

Example 13. In a specific heat experiment 200 g of lead at 200°C is mixed with 400 g of water at 20°C. Find the difference in entropy of the system at the end from its value before mixing, C_p for lead = 145 J/kg-K.

Solution: Let the final temperature be T

Heat lost by lead = (0.200 kg) × (145) (373 – T)

Heat gained by water = (0.400 kg) (4200) (T–293)

Heat lost = Heat Gained

$$(0.2 \times 145)(373 - T) = (0.4 \times 4200)(T - 293)$$

or

$$T = 294.36 \text{ K}$$

If ϕ_1 is the entropy of a substance at temperature T_1 and ϕ_2 at higher temperature T_2,

The change is entropy

$$\Delta\phi = \int_{S_1}^{S_2} d\phi = \phi_2 - \phi_1 = \int_{T_1}^{T_2} \frac{dQ}{T}$$

$$= \int_{T_1}^{T_2} \frac{mCdT}{T} = mC \int_{T_1}^{T_2} \frac{dT}{T}$$

or

$$\Delta\phi = mC\,(\log_e T_2/T_1)$$

where C is the specific heat capacity of the substance. Now if $\Delta\phi_1$ is the change in entropy of lead and $\Delta\phi_2$ that of water, we have for the change in entropy of the system $\Delta\phi = \Delta\phi_1 = \Delta\phi_2$. From above relation we have

$$\Delta\phi_1 = m_1 C_1\left(\frac{T_1}{T}\right) = -0.2 \times 145 \times \log_e\left(\frac{/}{294.36}\right) = -6.866$$

$$\Delta\phi_2 = m_2 C_2\left(\frac{T}{T_2}\right) = -0.4 \times 4200 \times \log_e\left(\frac{294.36}{293}\right) = 7.78$$

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 = -6.866 + 7.78$$
$$= \mathbf{0.914\ J/K.}$$

OBJECTIVE TYPE QUESTIONS

1. The conduction of heat from hot body to a cold body is an example of

(a) reversible process **(b) irreversible process** (c) none of these

2. In the Carnot engine when heat is taken from the source its temperature

(a) remains constant (b) does not remain constant

(c) decreases (d) increases

3. The net gain in entropy of the working substance in a Carnot cycle is

(a) zero (b) –ve

(c) +ve (d) may be – ve or + ve

4. For the efficiency of the Carnot cycle to be maximum.

(a) the temperature of the source should be infinity

(b) the temperature of sink should be infinity

(c) the temperature of the source should be zero

5. Which of the following is correct

(a) $\frac{T_1}{H_2} + \frac{T_2}{H_1} = 0$ **(b)** $\frac{\mathbf{H_1}}{\mathbf{T_1}} = \frac{\mathbf{H_2}}{\mathbf{T_1}}$

(c) $H_1 T_1 = H_2 T_2$ (d) $H_1 T_1 + H_2 T_2 = 0$

6. The efficiency of an otto cycle increases as

(a) compression ratio increases

(b) compression ratio decreases

(c) none of above

7. A diesel cycle works at

(a) constant volume

(b) constant pressure

(c) constant temperature

8. The fuel cut off for increasing efficiency in a diesel engine should be

(a) delayed (b) advanced

UNIT II : WAVES AND OSCILLATIONS

(Simple Harmonic Motion)

Waves and Oscillations: Wave motion, simple harmonic motion, wave equation, superposition principle. Introduction to Electromagnetic Theory: Maxwell's equations. work done by the electromagnetic field, Poynting's theorem, Momentum, Angular momentum in electromagnetic fields, Electromagnetic waves: the wave equation, plane electromagnetic waves, energy carried by electromagnetic waves **[8 Hrs]**

Displacement equation of a particle in SHM

$$\boxed{y = a \sin (\omega t + \phi)}$$

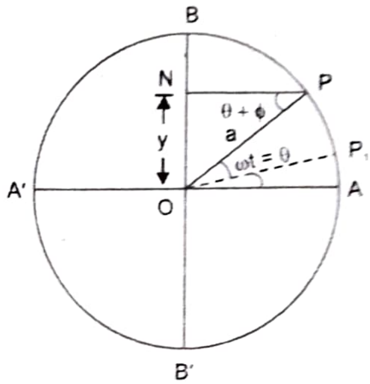
Angular velocity ω

Initial phase ϕ

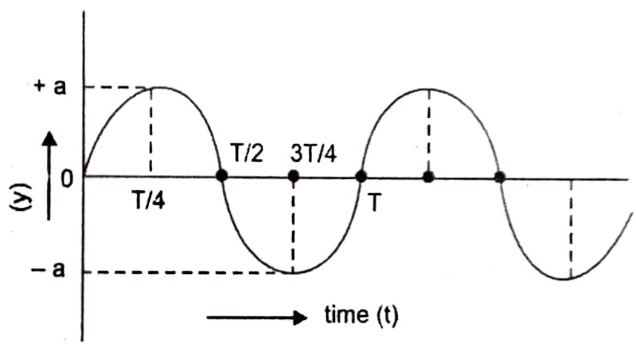
Amplitude $= a$

Time period $T = \dfrac{2\pi}{\omega}$

Frequency $n = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$



displacement vs time graph of $y = a \sin \omega t$



Velocity expression for a particle in SHM

$$u = a\omega \cos \omega t$$

as a function of time t

$$u = \omega\sqrt{(a^2 - y^2)}$$

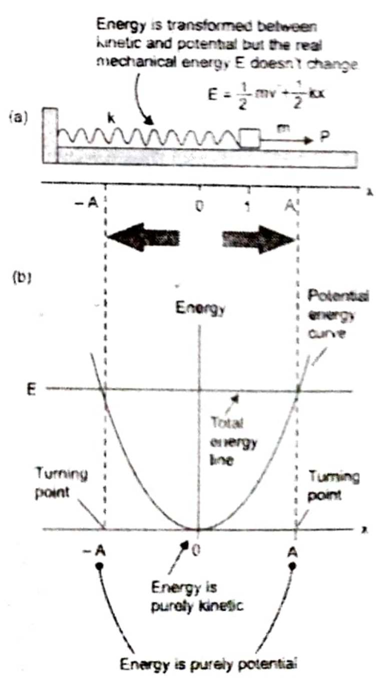
as a function of displacement y

1. if $y = 0$ then Max velocity u_{max}

$$= \omega\sqrt{(a^2 - 0)} = a\omega$$

2. if $y = \text{a}$ then Min Velocity u_{min}

$$= \omega\sqrt{(a^2 - a^2)} = 0$$



Simple Harmonic Motion

$$U = \frac{1}{2}kx^2$$

$$K = \frac{1}{2}mv^2$$

$$E = K + U$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At $x = \pm A$, $E = U = \frac{1}{2}kA^2$

At $x = 0$, $E = K = \frac{1}{2}mv_{max}^{\;2}$

E = constant, so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^{\;2}$

$$V_{max} = A\sqrt{\frac{k}{m}} = \omega A,$$

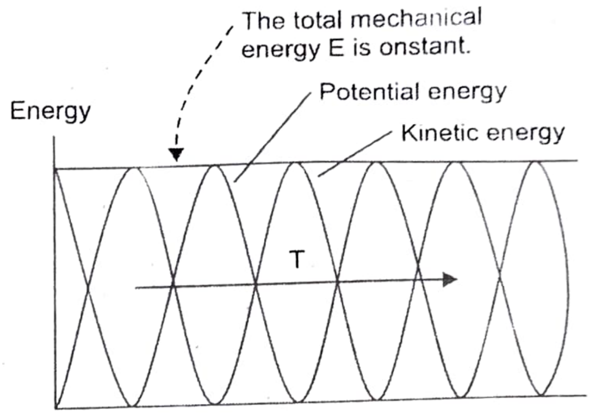
so, $w = \sqrt{\frac{k}{m}}$

$$\omega = \sqrt{\frac{k}{m}};\; f = \frac{1}{2\pi}\sqrt{\frac{k}{m}};\; T = 2\pi\sqrt{\frac{m}{k}}$$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \omega\sqrt{A^2 - x^2}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^{\;2} \rightarrow A = \sqrt{x_0^2 + \frac{mv_0^2}{k}} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$



Example of Simple Harmonic Motion-

(i) **Simple Pendulum-**

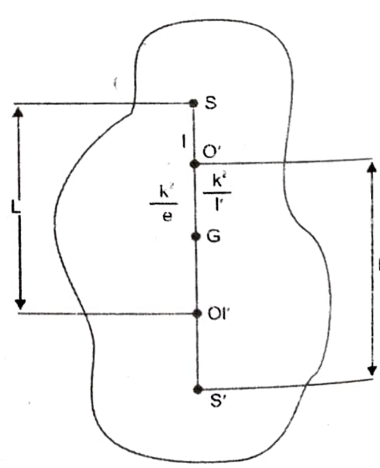
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

(ii) **Compound pendulum**

$$T = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{\frac{k^2}{l} + l}}{g} = 2\pi\sqrt{\frac{L}{g}}$$

where, $L = \frac{k^2}{l} + l$

k-radius of gynational Interchanging ability of centres of suspension and oscillation-



$$T' = 2\pi\sqrt{\frac{k^2}{l'} + l'}$$

But $\frac{k^2}{l} = l'$

$$T' = 2\pi\sqrt{\frac{l + l'}{g}} = T$$

$$\boxed{T' = T}$$

Maximum and Minimum Time period-

$T = \alpha$ or maximum if $l = o$ or α Ignoring $l = \alpha$, the time period of pendulum is maximum when it length is zero, i.e, when the axis of suspension passes through it centre of gravity (CG).

VECTORS

Scaler - It is a quantity which has magnitude only.

Eg. - mass, length, temp etc.

Vector - It is a quantity which has both magnitude and direction.

Eg. Force, velocity, etc.

Scalar Product - A.B = $|A||B|\text{Cos }\theta = AB\text{Cos}\theta$

Vector Product - $A \times B = |A||B|\text{Sin }\theta = AB\text{Sin}\theta$

Gradient of a Scaler - $\phi(x, y, z)$

$$\text{Grad } \phi = \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

Divergence of a vector- $\vec{A}$ $\left\{\left[\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}\right]\right\}$

$$\vec{\nabla}.\vec{A} = \frac{\partial Ax}{\partial x} + \frac{\partial Ay}{\partial y} + \frac{\partial Az}{\partial k}$$

curl of a vector- $\vec{A}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ax & Ay & Az \end{vmatrix}$$

$$\vec{\nabla} \times \vec{A} = \hat{i}\left(\frac{dAz}{dy} - \frac{\partial Ay}{\partial z}\right) - \hat{j}\left(\frac{\partial Az}{\partial x} - \frac{\partial Ax}{\partial z}\right) + \hat{k}\left(\frac{\partial Ay}{\partial x} - \frac{\partial Ax}{\partial y}\right)$$

Divergence Theorem-

$$\int_v (\vec{\nabla}.\vec{A})\, dv = \int_s \vec{A}.\vec{ds}$$

Stoke's Theorem-

$$\int_s (\vec{\nabla} \times \vec{A})\, ds = \oint_l \vec{A}.\vec{dl}$$

Some Useful Vectors Relations-

If we take U and V as scalar junct and $\vec{A}$ and $\vec{B}$ as Vector junctions then-

(1) $\vec{A}.\vec{B} \times \vec{C} = \vec{B}.\vec{C} \times \vec{A} = \vec{C}.\vec{A} \times \vec{B}$

(2) $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$

(3) $\vec{\nabla}(U + V) = \vec{\nabla}U + \vec{\nabla}V$

(4) $\vec{\nabla} \cdot (\vec{A} + \vec{B}) = \vec{\nabla} \cdot \vec{A} + \vec{\nabla} \cdot \vec{B}$

(5) $\vec{\nabla} \times (\vec{A} + \vec{B}) = \vec{\nabla} \times \vec{A} + \vec{\nabla} \times \vec{B}$

(6) $\vec{\nabla}(UV) = U\vec{\nabla}V + V\vec{\nabla}U$

(7) $\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$

(8) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

(9) $\vec{\nabla} \times \vec{\nabla}V = 0$

(10) $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$

1. Maxwell's Equations-

(I) First Equation- Gauss's Law in Electrostatics

Internal Form-

$$\int \vec{E}.\overline{ds} = q/\epsilon \qquad \text{Differential Form-}$$

$$\vec{\nabla}.\vec{E} = q/\epsilon$$

Physical Significance- It states that the electric flux through any closed surface is $\frac{1}{E}$ times the total charge enclosed by the surface.

(II) Second Equation- Gauss's Law in Magnetostatics

Internal Form-

$$\int \vec{B}.\overline{ds} = 0 \qquad \text{Differential Form-}$$

$$\vec{\nabla}.\vec{B} = 0$$

Physical Significance- The net magnetic flux through any closed Gaussian Surface is always zero.

(III) Third Equation- Faraday's Law-

Integral Form-

$$\oint \vec{E}.\overline{dl} = -\frac{\int \partial \vec{B}}{\partial t}\overline{ds} \qquad \text{Differential Form-}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Physical Significance- An electric flux can be produced magnetic flux.

(IV) Fourth Equation- Modified Ampere's Law

Integral Form-

$$\oint \vec{B}.dl = \mu \iint \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) \overline{ds}$$

Differential Form-

$$\vec{\nabla} \times \vec{B} = \mu \left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

Physical Significance- Time varying electric field produces magnetic field.

2. Displacement Current- It is defined as rate of change of electric displacement field $(\vec{D} = \varepsilon \vec{E})$

$$id = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

The current through a capacitor is called displacement current and it flows only when voltage applied to it is changing.

3. Continuity Equation-

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

It states that current flowing out of a given volume must be equal to rate of decreases of charge within the volume.

4. Poynting Theorem -

$$\int_v (\vec{E}.\vec{J})dv = \frac{-\partial}{\partial t}\int_v\left(\frac{\mu H^2}{2}+\frac{\varepsilon E^2}{2}\right)dv - \oint_s(\vec{E}\times\vec{H})ds$$

It states that the work done on the charge by an electromagnetic field is equal to the decreases in energy stored in the field, less than the energy which flowed out through the surface.

5. Poynting vector- $\vec{S}=\vec{E}\times\vec{H}$

It is the rate of flow of energy per unit area in a plane electromagnetic wave.

Unit - watt/m^2

6. Plane Electromagnetic wave in Free Space-

(i) Maxwell's Equations are-

$$\vec{\nabla}.\vec{E} = 0$$

$$\vec{\nabla}.\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

(ii) Wave equations are-

$$\nabla^2\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\nabla^2\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}=0$$

(iii) Velocity is-

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}}$$

$$v \approx 3\times10^8\frac{\text{m}}{\text{s}}$$

(iv) Solution of plane electromagnetic waves are-

$$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{i\vec{k}.\vec{r}-i\omega t}$$

$$\vec{B}(\vec{r},t) = \vec{B}_0\, e^{i\vec{k}.\vec{r}-i\omega t}$$

(v) Transverse Nature of the wave-

$$\vec{\nabla}.\vec{E}=0 = i(\vec{k}.\vec{E})$$

$$\vec{\nabla}.\vec{B}=0 = i(\vec{k}.\vec{B})$$

$\Rightarrow \quad \vec{k}\perp\vec{E}$ and $\vec{k}\perp\vec{B}$

(vi) Orthogonal Nature of the wave-

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = i\left(\vec{k}\times\vec{E}\right)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = i\left(\vec{k} \times \vec{B}\right)$$

$$\Rightarrow \qquad \vec{k} \perp \vec{E} \perp \vec{B}$$

(vii) Intrinsic Impedance-

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$Z_0 = 376.72 \text{ ohms}$$

(viii) Poynting Vector-

$$\vec{S} = \vec{E} \times \vec{H} = \frac{E^2}{z_0}\hat{n}$$

where, $\vec{k} = k\hat{n} \quad K = \frac{2\pi}{\lambda}$ k - Propagation vector

$$< \vec{s} > = \frac{E_0^2}{2Z_0}\hat{n} = \frac{E_{rms}}{Z_0}\hat{n}$$

(ix) Energy Density-

$$\mu_E = \frac{1}{2}\epsilon_0 E^2$$

$$\mu_M = \frac{1}{2}\mu_0 H^2$$

$$\mu = \mu_E + \mu_M$$

But $$\frac{U_E}{U_M} = 1$$

$$\therefore \quad U = 2\,U_E = \epsilon_0 E^2$$

$$<U> = \frac{1}{2}\epsilon_0 E_0^2 = E_0 E_{rms}^2$$

(x) Momentum-

$$\vec{p} = \frac{\vec{S}}{C^2} = \frac{(E \times \vec{H})}{C^2} = \frac{EH}{C^2}$$

(xi) Angular Momentum-

$$\vec{L}_{EM} = \frac{1}{C^2}\int \vec{r} \times (\vec{E} \times \vec{H})dv$$

SHORT QUESTION ANSWER

Q.1. What do you mean by (i) solenoidal and (ii) irrotational ?

Ans. (i) Solenoidal: A vector whose divergence is zero is called solenoidal.

(ii) Irrotational: A vector whose curl is zero is called irrotational.

Q.2. What is Lamellar field ?

Ans. Lamellar field: A vector field which can be expressed as the gradient of a scalar field is known as lamellar (or laminar) vector field.

Electric field is an example of a lamellar field, since

$$E = -\nabla V$$

where V is the electrical potential, which is a scalar function.

The word lamellar (or laminar) means that the field can be divide into laminas or layers over which the value of the scalar function, whose gradient gives the vector field E remains constant.

Q.3. "The gradient of a scalar quantity is a vector". Explain

Ans. The gradient of a scalar field point in the direction of maximum change of the scalar function and the magnitude of the gradient of a scalar field gives the slope (i.e., rate of change) along this maximum direction. Therefore, this is a vector quantity.

Mathematically, if ϕ is a scalar field, then gradient of ϕ is

$$\nabla\phi = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

so from expression, it is clear that it is vector quantity.

Q.4. Why does not one see the portion other than visible one of electromagnetic spectrum?

Ans. The retina of the eye is sensitive only to colours in the visible region i.e., wavelength lying between 00Å and 7800Å. This region corresponds to visible part of the spectrum.

Q.5. An electromagnetic wave carries momentum. What is signifies?

Ans. An electromagnetic interaction between two electric charges means an exchange of energy and momentum between the charges.

Q.6. A plane monochromatic wave, travelling in a homogeneous medium, meets a denser medium. What are the changes in (i) amplitude (ii) frequency (iii) speed of propagation and (iv) phase of reflected and transmitted waves in comparison with the corresponding properties of the incident wave?

Ans. (i) Amplitude decreases for both cases.

(ii) Frequency remains same for both cases.

(iii) Speed same for reflected wave and lens for transmitted waves in comparison to reflected waves.

(iv) Phase change by π for reflected wave. No phase change for transmitted wave.

UNIT–III: INTERFERENCE, DIFFRACTION, POLARIZATION

Interference: Interference by division of wave front (Young's double slit experiment, Fresnel's biprism), interference by division of amplitude (thin films, Newton's rings, Michelson's interferometer), Coherence and coherent sources

Diffraction: Fraunhofer and Fresnel diffraction: Fraunhofer diffraction for Single slit, double slit, and N-slit (diffraction grating), Fraunhofer diffraction from a circular aperture, resolving power and dispersive power of a grating, Rayleigh criterion, resolving power of optical instruments

Polarization: Introduction to polarization, Brewster's law, Malu's law, Nicol prism, double refraction, quarter-wave and half-wave plates, optical activity, specific rotation, Laurent half shade polarimeter.

1. Interference: The phenomenon of modification of intensity due to two coherent waves of same frequency in the region of superposition is called interference.

2. Coherent Sources: Two sources emitting waves of same frequency and constant initial phase difference are called coherent sources. It can be produced by a single source either by division of wavefront as in Fresnel's biprism or by division of amplitude as in thin films.

3. Conditions for Maxima and Minima:

For maxima:

Phase difference = $2n\pi$ Path difference = $n\lambda$; $I_{max} = (a_1 + a_2)^2$

For minima:

Phase difference = $(2n + 1)\pi$; Path difference = $(2n + 1)\dfrac{\lambda}{2}$;

$I_{min} = (a_1 - a_2)^2$

4. Double slit Interference and Fresnel's Biprism experiment: If $2d$ is the separation between coherent sources and D is distance of screen from the sources,

Then the position of maxima can be represented by the following relation:

$$y_n = \frac{n\lambda D}{2d}$$

The position of minima is given by:

$$y_n = \frac{(2n+1)\lambda d}{4d}$$

Fringe width:

$$\beta = \frac{\lambda D}{2d}$$

5. Interference in thin film:

(i) Reflected System:

For bright fringe:

$$2\,\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

For dark fringe:

$$2\mu t \cos r = n\lambda$$

6. Wedge Shaped film of wedge angle θ:

Reflected system:

Condition of Maxima:

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

Condition for Minima:

$$2\mu t = n\lambda$$

Fringe Width:

$$\beta = \frac{\lambda}{2\mu\theta}$$

7. Newton's Rings: It is formed due to reflection of light rays from upper and lower surfaces of a film enclosed between a plane glass plate and planoconvex lens. Each ring is locus of points of equal thickness and is a circle-

(i) Reflected System

Radius of bright ring:

$$r = \frac{\sqrt{(2n-1)\lambda R}}{2}$$

Radius of dark ring:

$$r = \sqrt{n\lambda R}$$

Centre of Newton's rings is dark.

(ii) **Transmitted System:**

Radius of bright ring

$$r = \sqrt{n\lambda R}$$

Radius of dark ring:

$$r = \frac{\sqrt{(2n-1)\lambda R}}{2}$$

Centre of Newton's rings is bright

8. Wavelength of Sodium light is given by:

$$\lambda = \frac{D^2_{n+m} - D_n^2}{4mR} \text{(for air } \mu = 1)$$

where, R is the radius of curvature of the plano-convex lens.

and D_{n+m} and D_n are the diameters of the $(n+m)^{th}$ and n^{th} dark rings respectively.

9. Refractive index of liquid is given by:

$$\mu = \frac{\left[D^2_{n+m} - D_n^2\right]_{air}}{\left[D^2_{n+m} - D_n^2\right]_{liquid}}$$

10 (a) Newton's rings disappears when the planoconvex lens is replaced by a plane mirror.

(b) Seperation between diameters of Newton's rings decreases with increases in order.

11. Michelson's interferometer

(a) Wavelength of light:

$$\lambda = \frac{2d}{n}$$

(b) small difference in two wave lengths from the same source is given by:

$$\Delta\lambda = \lambda_1 - \lambda_2 = —$$

(c) Refractive index of thin film:

$$\mu = \left(\frac{n\lambda}{2b} + 1\right)$$

where b is the length of the tube.

DIFFRACTION

1. The phenomenon of bending of light around the corners of an object or obstacle is called diffraction. The necessary condition for observing diffraction is that the size of the object should be comparable to the wavelength of light used.

2. Diffraction can be divided into two type

(i) Fresnel diffraction (source and screen are place at finite distance)

(ii) Fraunhofer diffraction (source and screen are placed at infinite distance)

3. Fresnel type of diffraction due to Straight edge.

$$x = \sqrt{\frac{\lambda b(a+b)}{\lambda}}\sqrt{2n \pm 1}$$

$$\lambda = \frac{a^{(x_{m+n}{}^2 - x_n{}^2)}}{2b(a+b)m}$$

4. Fraunhofer diffraction at single slit

- *Resultant intensity*

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

- *Principle maximum*

$$\alpha = 0$$

$$\alpha = \frac{\pi e \sin\theta}{\lambda}$$

$$\sin\theta = 0$$

$$\theta = 0$$

$$I \propto A^2 \propto R^2$$

- *Condition for minima*

$$e \sin\theta = \pm n\lambda$$

where $n = 1, 2, 3, \ldots$

$n \neq 0$ as $n = 0$ or $\theta = 0$ corresponds to principal maximum

- *Direction of secondary maxima*

$$e \sin\theta = \pm\frac{(2n+1)\lambda}{2}$$

- for central maxima $I = I_0$

- for first secondary maxima $I_1 = \dfrac{I_0}{22}$

- for second secondary maxima $I_2 = \dfrac{I_0}{62}$

- for third secondary maxima $I_3 = \frac{I_0}{120}$
- Thus the relation intensities of principal first second, ... maxima are

$$1, \frac{4}{9\pi^2}, \frac{4}{25\pi^2}, \frac{4}{49\pi^2}, \ldots$$

- Width of central maxima $= \frac{2f\lambda}{e}$

5. Fraunhofer diffraction at Double slits

(a) *Resultant Intensity*

$$I = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

where $\frac{\sin^2\alpha}{\alpha^2}$ – diffraction pattern due to each list

$\cos^2\beta$ – Interference pattern due to two waves of same Amplitude A

(b) *According to the interference terms, $\cos^2\beta$, we have for maxima*

$$\beta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda$$

$$n = 0, 1, 2, \ldots$$

minima

$$(e+d)\sin\theta = \pm(2n+1)\frac{\lambda}{2}$$

$$n = 0, 1, 2, \ldots$$

(c) The factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ gives a principal maxima in a direction $\theta = 0$

- The position of maxima

$$\sin\alpha = 0 \text{ but } \alpha \neq 0$$

$$\sin\theta = \pm\left(\frac{m\lambda}{a}\right) \quad m = 1, 2, 3, \ldots$$

- Position of secondary maxima are

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} + \ldots$$

6. Plane diffraction Grating

This consist of large number of slits place side by side. $(e+d)$ is called grating element $e \rightarrow$ width of slit, $d \rightarrow$ width of opaque part

- *Resultant intensity*

$$I = R^2$$

$$= \frac{A^2\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

- *Principal maxima*

$$\beta = \pm n\pi$$

where

$$n = 0, 1, 2, 3, \ldots$$

$$(e+d)\sin\theta = \pm n\pi$$

- *Secondary minima*

$$\sin N\beta = 0$$
$$N\beta = \pm m\pi$$
$$N(e+d)\sin\theta = \pm m\lambda$$
$$n = 1, 2, 3, \ldots (N-1)$$

- *Secondary maxima*

$$r = \frac{A^2 \sin^2\alpha}{\alpha^2} \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

- $\dfrac{\text{Intensity of secondary maxima}}{\text{Intesnity of principal maxima}} = \dfrac{I'}{I} = \dfrac{1}{1+(N^2-1)\sin^2\beta}$

- Angular half width of principal maxima

$$d\theta = \frac{\lambda}{N(e+d)\cos\theta}$$

- Condition for absent spectra

$$\frac{e+d}{e} = \frac{n}{m}$$

- Maximum number of orders in a diffraction grating is

$$n = \frac{(e+b)}{\lambda}$$

7. **Dispersive power of grating** is defined as the change the angle of diffraction corresponding to the unit change in the wavelength of light and given as

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

8. **Resolving power of grating:** The resolving power of grating is defined to show two spectral lines of wavelength very close and separate

$$\frac{\lambda}{d\lambda} = nN$$

; n is the order

; N is the number of lines on the grating

9. **Resolving power of prism**

$$\frac{\lambda}{d\lambda} = \frac{t\,d\mu}{d\lambda}$$

POLARISATION

1. The phenomenon, in which the vibration of the light wave are restricted to particular direction in a plane is called polarisation.

2. Polarisation is a typical wave characteristics shown only by transverse wave. Longitudinal waves cannot be polarised.

3. Light from an ordinary source is said to be unpolarised. The vibration of light vector are allowed in all possible directions perpendicular to the direction of propagation. If the vibration of light vectors are restricted to a particular direction or plane then it is said to be polarised.

4. The plane in which the vibration take place i.e., the plane containing the direction of vibration and the direction of propagation, is called **plane of vibration**.

5. The plane at right angles to the plane of vibration in the light wave is called **plane of polarisation**.

6. Brewster's law: According to this law the angle between the reflected and the refracted ray is 90°. It states that the tangent of the polarising angle is equal to the refractive index of the refracting medium $\tan i_p = \mu$

7. The Malus law states that intensity of light emerging from the analyzer is proportional to the square of the consine of the angle between the polarised and the analyser.

$$I = I_0 \cos^2\theta$$

8. Double refraction: The phenomenon of splitting the incident unpolarised light ray into two refracted ray i.e., ordinary and extra ordinary ray is called double refraction.

9. NICOL prism is an optical device made from calcite and is used in many instruments for producing and analysing plane polarised light. Its action is based on the phenomenon of double refraction.

10. NICOL prism allows O-ray to be absorbed, while e-ray is transmitted through it. Thus it produce a plane polarised light.

11. Quarter Wave Plate: The thickness of plate 't' in calcite crystal.

$$t = \frac{\lambda}{\mu(\mu_o - \mu_e)}$$

and in case of quartz

$$t = \frac{\lambda}{\mu(\mu_e - \mu_o)}$$

12. Half wave plate

The thickness of the calcite plate

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

for quartz

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

13. General equation of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

(i) if $\delta = 0, y = \frac{b}{a}x$, straight line; (ii) if $\delta = , y = \frac{-b}{a}x$, straight line

(iii) if $\delta = \frac{\pi}{2}, a = b, x^2 + y^2 = a^2$, circle (iv) if $\delta = \frac{\pi}{2}, a \neq b, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ellipse

14. Circularly polarised light can be produced by passing a plane polarised light normally through a quarter wave plate whose optic axis is set at an angle of 45° to the plane of vibration of the incident plane polarised light.

15. Elliptically polarised light can be produced from a plane polarised beam by passing it through other than 0°, 45° or 90° to the plane of vibration of the incident light.

16. The phenomena of rotation of the plane of polarisation of the light when it passes through certain crystal is called optical rotation. The substance exhibiting this phenomena are called **optically active substance.**

17. Specific Rotation

$$S = \frac{10\theta}{lC}$$

This is determined by an instrument called Laurent's half shade polarimeter. Here θ is the angle of rotation, l is the length of the tube and C is the concentration of the solution.

SHORT ANSWER QUESTIONS

Q.1. What is difference between fringes obtained by Fresnel's biprism and those obtained by Newton's rings ?

Ans. (i) The biprism fringes are straight and equally spaced whereas the fringes in Newton's rings are circular and not equally spaced.

(ii) In biprism fringes are obtained by division of wavelength whereas in Newton's rings, they are obtained by division of amplitudes.

(iii) In biprism, fringes are non-localised while in Newton's rings they are localised.

Q.2. What are Newton's rings? Why the central ring is dark when observed in reflected light?

Ans. When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, on air film is formed between the lower surface of the lens and the upper surface of the plate. If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings with their centre dark concentric rings with their centre dark is formed. Since these rings were discovered by Newton, so these are called Newton's rings. The central ring is dark when observed in reflected light because effective path difference $\Delta = 2t + \frac{\lambda}{2}$. At the point of contact $t = 0$, then $\Delta = \frac{\lambda}{2}$. This is the condition for minimum intensity. Hence the central ring is dark.

Q.3. Why do the Newton's rings get closer as the order of the rings increases?

Ans. Because radius of dark ring $D_n \propto \sqrt{n}$, while for bright ring $D_n \propto \sqrt{(2n+1)}$ i.e., square root of odd natural numbers.

Q.4. In Newton's ring experiment, predict, what will happen?

(a) If sodium lamp is replaced by white light source in Newton's rings?

(b) If a few drops of a transparent liquid introduced between the lens and plate?

Ans. (a) Few coloured fringes will be observed near the centre.

(b) The diameter of fringes is contracted.

Q.5. What happen to the ring system if a plane polished mirror is used instead of a glass plate in Newton's arrangement?

Ans. If a plane polished mirror is used instead of a glass plate in Newton's ring arrangement interference pattern produced due to reflected and transmitted light will be superimposed and we get the two patterns, which are complimentary their superposition. Hence we get uniform illumination.

Q.6. The interference fringes produced in the Newton's ring experiment are real or virtual? Justify.

Q.7. What is the cause of diffraction?

Ans. Diffraction occurs due to interference of secondary wavelets between different portions of wavefront allowed to pass a small aperture or obstacle.

Q.8. What two main changes in diffraction pattern of single slit will you observe when the monochromatic sources of light is replaced by a source of white light.

Ans. When the monochromatic source is replaced by a source of white light, the diffraction pattern show following changes:

(i) In each diffraction order, the diffracted image of the slit gets dispersed into component colours of white light. As fringe width is directly proportional to wavelengths, so the red fringe with higher wavelength is wider than the violet fringe with smaller wavelength.

(ii) In higher spectra, the dispersion is more and it causes overlapping of different colours.

Q.9. Why is diffraction of sound waves more evident in our daily life than of light wave?

Or

Interference and diffraction, which is more common and why.

Ans. To obtain a well defined diffraction pattern, the size of the obstacle or aperture should be of the same order as the wavelength. The wavelength of sound is comparable to the size of most of the obstacle or aperture, we come across in daily life. The diffraction of sound is therefore, a common experience. But wavelength of light is very small as compared to the size of the obstacles or aperture, we come across in daily life. We, therefore, do not observe diffraction of light as an everyday phenomenon.

Q.10. What is the significanceq of diffraction phenomenon on the nature of light?

Ans. The diffraction phenomenon is a very strong and convincing proof in favour of Huygen's hypothesis of the undulatory nature of light.

Q.11. A single slit diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?

Ans. Diffraction fringes becomes narrower and crowded together.

Q.12. What is the effect of increasing the number of lines per cm of the grating on the diffraction gating?

Or

What are the advantages of increasing the number of rulings in a grating?

Ans. On increasing the number of lines per cm, decreases the grating element (a + b). As a result the angle of diffraction θ increases for a given order. This results in a less number of spectra separated by large angles or an increased dispersive power.

Q.13. What are uniaxial and biaxial crystals?

Ans. The crystals having one direction (optic axis) along which the two refracted rays travel with the same velocity are called as uniaxial crystals. In biaxial crystals, there are two optic axes.

Q.14. What is difference between 'ordinary' and 'extraordinary' rays?

Ans. While passing through a double refracting crystal, the ray which obeys the laws of refraction in the ordinary ray while the other, which does not obey the laws of refraction, is extraordinary ray.

Q.15. How does the polarisation of light afford a convincing evidence of transverse nature of light?

Ans. If light waves were not transverse in nature then the vibrations would have passed through the tourmaline crystals with their optic axes perpendicular to each other i.e., in crossed position.

Q.16. A calcite crystal is placed over a dot on a piece of paper and rotated. What will you observe through the crystal?

Ans. One dot rotating about the other.

Q.17. What do you mean by optic axis?

Ans. Line passing through any one of the blunt corners and making equal angles with three faces called optic axis. It is the direction along which o- and e-rays travel with same velocity.

UNIT–IV: THEORY OF RELATIVITY

Theory of relativity: The Michelson-Morley Experiment and the speed of light; Absolute and Inertial frames of reference, Galilean transofrmations, the postulates of the special theory of relativity, Lorentz transformations, time dilation, length contraction, velocity addition, mass energy equivalence. Invariance of Maxwell's equations under Lorentz Transformation.

Introduction to Laser Physics: Introduction, coherence, Einstein A and B coefficients, population inversion, basic principle and operation of laser, the He-Ne laser and the Ruby laser.

INTRODUCTION TO LASER

1. Frame of Reference-

(i) Inertial Frames: The system in which the law of inertia holds good are called inertial frames of reference.

(ii) Non-Inertial Frames: The system in which the law of inertial does not hold good. This frame is an accelerated motion with respect to an inertial frame.

2. Galilean Transformations-

(i) $x' = x - v; \quad y' = y$

$z' = z; \quad t' = t$

(ii) $x' = x - v_x t; \quad y' = y - v_y t$

$z' = z - v_z t; \quad t' = t$

(iii) $u_x' = u_x - v_x; \quad u_y' = u_y - v_y$

$u_z' = u_z - v_z; \quad \vec{u}' = \vec{u} - \vec{v}$

(iv) $a_x' = a_x; \quad a_y' = a_y \quad a_z' = a_z$

3. Michelson-Morely Experiment- This experiment gave the evidence for the non-existence of the ether.

(i) The effective path difference $= \dfrac{2lv^2}{c^2}$

(ii) Number of fringes $= \dfrac{2lv^2}{c^2\lambda}$

4. Einstein's Postulates-

(i) Law of physics have identical form in all frames of references

(ii) The velocity of light in free space is constant

5. Lorentz Transformation- Invariance of speed of light in all inertial frames implies that Galilean transformation equations are not suitable. Therefore, another transformation, named Lorentz transformation was introduced. The Lorentz transformation equation are-

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \left(\frac{vx}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\left(t - \frac{vx}{c^2}\right)$$

where, $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

6. Inverse Lorentz Transformation equations-

$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma(x'+vt')$$

$$y' = y$$

$$z' = z$$

$$t = \frac{t'+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\left(t'+\frac{vx'}{c^2}\right)$$

7. Length Contraction-

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

where L_0 is the length of the rod in rest frame, S and L is the length of the rod in moving frame, S'.

8. Time Dilation-

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta t$$

$\Delta t'$ is the time interval measured in the moving frame S' and Δt is the time interval measured in the rest frame, S.

9. Simultaneity- Two events that are simultaneous in one inertial frame of reference are in general not simultaneous in another reference frame moving with respect to first.

10. Addition to Velocities-

(i) $$u_x' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}; \qquad u_y' = \frac{u_y}{\gamma\left(1-\frac{u_x v}{c^2}\right)};$$

$$u_z' = \frac{u_z}{\gamma\left(1-\frac{u_x v}{c^2}\right)}$$

(ii) $$u_x = \frac{u'_x + v}{1+\frac{u'_x v}{c^2}}; \qquad u_y = \frac{u'_y}{\gamma\left(1+\frac{u'_x v}{c^2}\right)};$$

$$u_z = \frac{u'_z}{\gamma\left(1+\frac{ux'v}{c^2}\right)}$$

11. Variation of mass with velocity- If m_0 is the rest mass of a particle then its mass m at velocity v is

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Thus the mass of the particle increases with velocity

12. Equivalence of mass and energy-

(i) Kinetic Energy = $mc^2 - m_0c^2$

(ii) Rest Energy = m_0c^2

(iii) Total Energy = mc^2

13. Relativistic Energy-momentum relation-

$$E^2 = p^2c^2 + m_0^2c^4$$

14. For Massless particle-

$$E = pc \text{ (it travels with the speed of light)}$$

LASER

1. The word LASER stands for **light Amplification by stimulated emission of radiation.**

2. The interaction of radiation with matter leads to absorption and emission of radiation.

3. **Stimulated absorption:** When an atom in the ground level interact with a photon of energy $h\nu$, it absorb the photon and goes in to an excited state. This is called stimulated absorption of radiation.

4. **Spontaneous emission:** The excited atom after a times period of 10^{-8} sec (life time of an atom) comes back to the ground state with the emission of photon. This process is called spontaneous emission.

5. **Stimulated emission:** In stimulated emission, the process is forced by the incident photon stream and the excited atom come back to the ground state with the emission of photon having phase, energy frequency and state of polarisation exactly same as that of the incident photon.

6. **Population inversion:** If the number of atoms in the higher energy state is more than the number of atoms in the lower energy state, the state is said to be in population inversion.

$N_2 = N_1 \exp(-(E_2 - E_1)/kT)$ N_2 = Number of atoms in excited state

E_2 = Energy of excited state N_1 = Number of atoms in ground state

E_1 = Energy of ground state then for population inversion $N_2 >> N_1$

7. **Pumping:** To achieve population inversion, the external energy is supplied to excite the atoms to the material and the process is called laser pumping.

8. The ratio of the Einstein's coefficients A_{21} and B_{21} is given by the relation

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

or $$\frac{A_{21}}{B_{21}} \propto \nu^3$$

Thus, the probability of spontaneous emission increases rapidly with energy difference between the two states.

9. **Essential components of laser:** (1) Active medium (2) Pumping (3) Optical Resonator

10. **Optical resonator:** This consists of two reflecting surfaces one mirror is partially reflecting and other is fully reflection.

11. Some of the significant characteristics of laser are (i) directionality (ii) coherence (iii) monochromaticity (iv) High intensity.

12. **Coherence:** is a term used to define the existence of a definite phase relationship between different parts of a travelling wave. Coherence are of two types (i) spatial coherence (ii) temporal coherence.

13. **Coherence time:** The average time interval during which the field remains sinusoidal i.e., a definite phase,. is called coherence time τ_c.

$$l_c = \frac{2\pi}{\Delta\omega}$$

14. **Coherence length:** The distance travelled by the wave train during time τ_c is called coherence length.

$$l_c = c\tau_c$$

15. Lasers can be classified as solid-state lasers, gaseous lasers, dye lasers, semiconductor laser and so-on.

Solid state laser - Ruby laser; Gas laser - He - Ne, laser

16. **Ruby laser:** In ruby laser, the active medium is Al_2O_3 doped with chromium ions (0.05%). It is a three level, solid-state pulsed laser with a wavelength of $\lambda = 6943\text{Å}$

17. **He - Ne laser:** In He - Ne laser, the active medium is a mixture of He - Ne in the ratio of 10 :1. A a four level gas laser operating in a continuous wave mode with $\lambda = 6328$ Å

18. Because of different characteristics of laser, they found applications in space communication medicine, material research, atomic physics and optical phenomena.

Example 1. A particle executes a S.H.M. of period 10 seconds and amplitude of 1.5 metre. Calculate its maximum acceleration and velocity.

Solution: If ω is the angular velocity, the time to complete one revolution with this speed i.e., to cover 2π radians will be equal to the time period T.

Thus $$T = \frac{2\pi}{\omega}$$

or $$\omega = \frac{2\pi}{T} = \frac{2\pi}{10} = 0.628 \text{ rad./s}$$

Now we know that linear velocity,

$$v = \omega\sqrt{a^2 - y^2}$$

and will be maximum only when $y \to 0$ i.e., at the mean position. Thus substituting values of ω (= 0.628) and y (= 0) and a (= 1.5), we have

$$v_{max} = 0.628\sqrt{(1.5)^2 - (0)^2} = 0.942 \text{ m/s}.$$

Similarly we know that the acceleration of a body executing S.H.M. is given by $\omega^2 y$ and will be maximum only when y is maximum, ($= a$). Acceleration $= \omega^2 y$

$\therefore$ Maximum acceleration $= (0.628)^2 \times 1.5 = \mathbf{0.59\ m/s^2}$

Example 2. A body executing S.H.M. has its velocity 16 cm/s when passing through its centre mean position. If it goes 1 cm either side of mean position, calculate its time period.

Solution: We know the velocity of body executing S.H.M. is given by

$$v = \omega\sqrt{a^2 - y^2}$$

Now when at mean position $y = 0$ and $v = 16$ cm/s and $a = 2$ cm

$$\therefore \quad 16 = \sqrt{(2)^2 - (0)^2}$$

or $$\omega = \frac{16}{8} = 8 \text{ rad./s.}$$

Since $$T = \frac{2\pi}{\omega}$$

$$\therefore \quad T = \frac{3 \times 3.14}{8} = \mathbf{0.785\ second.}$$

Example 3. A hydrogen atom has a mass of 1.68×10^{-27} kg. When attached to a certain massive molecule it oscillates as a classical oscillator with a frequency of 10^{14} cycles per second and with an amplitude of 10^{-10} m. Calculate the force acting on the hydrogen atom.

Solution: The hydrogen atom executes S.H.M. of amplitude, $a = 10^{-10}$ m.

The angular velocity $\omega = 2\pi n = 2\pi \times 10^{14}$ rad./s

The acceleration of the atom $= \omega^2 y$ or $\omega^2 a$

$\therefore$ The force on the hydrogen atom

$$= m\,\omega^2 a = \text{mass} \times \text{acceleration}$$

$$= 1.68 \times 10^{-27} \times (2\pi \times 10^{14})^2 \times 10^{-10}$$

$$= 1.68 \times 4 \times \pi^2 \times 10^{-9} = \mathbf{66.3 \times 10^{-9}\ Newton.}$$

Example 4. A body executing S.H.M describes 120 vibrations per minute and has a velocity of 5 m/s. What is the length of its path? What is the velocity? When it is half way between its mean position and an extremity of its path?

Solution: Time period of the body $= \frac{1}{120}$ min. $= \frac{60}{120} = 0.5$ s

Velocity at mean position (when $y = 0$) $= 5$ m/s

Since $v = \omega\sqrt{a^2 - y^2}$ at the mean position its value will be $a\,\omega$

$$\therefore \quad a\,\omega = 5$$

or $$a.\frac{2\pi}{T} = 5 \qquad (\because \omega = 2\pi/T)$$

When $$a = \frac{5 \times T}{2\pi} = \frac{5 \times 0.5}{2 \times 3.14} = 0.398 \text{ m.}$$

Now total path length will be double the amplitude.

Hence length of path = 2 × 0.398 = 0.788 m.

Velocity of the body executing S.H.M. at any point is given by

$$v = \omega\sqrt{a^2 - y^2}$$

When the body is half way between its mean position ad an extremity of its path, $y = a/2$ or 0.197. Substituting the values of y, a and ω, we have

$$v = \frac{2\pi}{T}\sqrt{(0.398)^2 - \left(\frac{0.398}{2}\right)^2}$$

$$= \frac{2\pi}{T} \times 0.398\sqrt{1 - \left(\frac{1}{2}\right)^2}$$

or $$v = \frac{2 \times 3.14}{0.5} \times 0.398 \times \frac{\sqrt{3}}{4}$$

$$= 2 \times 3.14 \times 0.398\sqrt{3} = \mathbf{4.329\ m/s.}$$

Thus the length of path of the body is 0.788 m and its velocity when it is half way between its mean position and an extremity of its path is 4.287 m/s.

Example 5. A body executes S.H.M. such that its velocity at the mean position is 1 m/s and acceleration at one of the extremity is 1.57 m/s². Calculate the time period of vibration.

Solution: Velocity at any point of a body executing S.H.M. is given by

$$v = \omega\sqrt{(a^2 - y^2)}$$

at mean position $v = 1$ m/s and $y = 0$

$$\therefore \quad 1 = \omega a \quad \text{...(1)}$$

Acceleration at any point of a body executing S.H.M. is given by:

$$\text{Acceleration} = \omega^2 y$$

$$\text{At extremity } y = a$$

$$\text{and Acceleration} = 1.57$$

$$\therefore \quad 1.57 = \omega^2 a \quad \text{...(2)}$$

Dividing (2) by (1), we have $\omega = 1.57$

Now $$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.57} = \mathbf{4\ seconds}$$

SHORT ANSWER QUESTIONS

Q.1. What are the inertial frames of reference? Give an example of a non-inertial frame.

Ans. Inertial frames of reference are those in which bodies obey Newton's law of inertia and other laws Newtonian Mechanics.

Non-inertial frames of reference are those in which the bodies do not obey Newton's laws. Example of non-inertial frame is man inside the lift. When lift is moving up with a uniform acceleration, the man feels more weight than his real weight and feels less weight when lift is moving down with constant acceleration.

Q.2. Discuss the negative result of Michelson-Morley experiment and conclusion drawn thereform.

Ans. The theoretical calculations show that if there is a relative velocity between the ether and the earth, then a fringe shift of 0.4 would occur. Michelson-Morley experimental setup is sufficient sensitive of measure this fringe shift. But experimentally no shift to fringe was detected.

Then this was concluded that

(i) the relative velocity between earth and either is zero.

(ii) either does not exist at all.

Q.3. If photons have a speed in one reference frame, can they be found at rest in any other frame?

Ans. No, when a body moves with a speed less than c in one reference frame, we can find another reference frame in which it is at rest. But when the body moves with speed c in one frame of reference, it will move with the same speed c in all reference frame.

Q.4. Why we do not observe time dilation in day to day phenomenon?

Ans. We do not observe or experience time dilation in every day (day to day) life phenomenon because we do not travel anywhere near fast enough for that to happen.

Q.5. State whether a laser is an amplifier or an oscillator?

Ans. The mirror in a resonator provide positive feedback to the photons amplified in the active medium. In analogy with electronic oscillator (in which a part of the output voltage is feedback to the input in paper phase i.e., positive feedback) the laser is an amplifier.

Q.6. A What is the wavelength of laser light from: (i) Ruby laser and (ii) He-Ne laser ? How are the two lasers different?

Ans. (i) Ruby Laser – 694.3 nm (ii) He-Ne laser 632.8 nm.

Ruby laser is a three level laser and has pulsed output, whereas He-Ne laser is a four level laser and has continuous output.

Q.7. What is an active medium?

Ans. A medium in which population inversion is achieved for laser action is called an active medium.

Q.8. What is an optical resonator?

Ans. It is a pair of reflecting surfaces (mirrors); of which one is being a perfect reflector and the other being a partial reflector. It is used for amplification of photons thereby producing an intense and highly coherent output.

Q.9. Why is it necessary to use a narrow tube in a He-Ne laser?

Ans. A narrow discharge tube is necessary for the rapid deexciation of atoms by collision with the walls. With a tube of large diameter, the probability of collisions of atoms with the walls decreases and less atoms are available at the ground level for further excitation. This can cease the laser action in due course.

UNIT II

Waves and Oscillations: Wave motion, simple harmonic motion, wave equation, superposition principle.

Introduction to Electromagnetic Theory: Maxwell's equations, work done by the electromagnetic field, Poynting's theorem, Momentum, Angular momentum in electromagnetic fields, Electromagnetic waves: the wave equation, plane electromagnetic waves, energy carried by electromagnetic waves. **[8 Hrs.]**

Q.1.
(a) Find the gradient of a vector –

$$A = (x^2 - xy + z)\,\hat{i} + (x^3 - xz + x)\,\hat{j} + (y^2 - y + z)\,\hat{k}$$

Ans.
$$\vec{A} = (x^2 - xy + z)\,\hat{i} + (x^3 - z + x)\,\hat{j} + (y^2 - y + z)\,\hat{k}$$

Taking A as scalar.
Gradient

$$\vec{A} = \vec{\nabla}A = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)[(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)]$$

$$= \frac{\partial}{\partial x}\left[(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)\right]\hat{i} +$$

$$\frac{\partial}{\partial y}\left[(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)\right]\hat{j} +$$

$$\frac{\partial}{\partial z}\left[(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)\right]\hat{k}$$

$$= [2x - x + 3x^2 - x + 1 + 0]\hat{i} + [-x + 0 + 2y - 1]\hat{j} +$$

$$[1 - x + 1]\hat{k}$$

$$= [3x^2 + 1]\hat{i} + [2y - x - 1]\hat{j} + [2 - x]\hat{k}$$

At point (2, 1, 1)

$$\vec{\nabla}A = [3\times4 + 1]\hat{i} + [2\times1 - 2 - 1]\hat{j} + [2 - 2]\hat{k}$$

$$\boxed{\vec{\nabla}A = 13\hat{i} - \hat{j}}$$

Q.1 (b) Write the expression of gradient in spherical coordinates. (2)
Ans. In spherical co-ordinate gradient is given by following–

$$x = r\sin\theta\cos\phi;\ y = r\sin\theta\sin\phi;\ z = r\cos\theta$$

$$\vec{\nabla}f = \hat{r}\frac{\partial f}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial f}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}$$

Q.1 (c) Give the expression of Gauss's law for magnetic field in both differential and integral form. (2)
Ans. Gauss's law for magnetic field in differential form is–

$$\vec{\nabla}.\vec{B} = 0$$

Gauss's law for magnetic field in integral form is–

$$\int \vec{B}.d\vec{S} = 0$$

Q.1 (d) Define the divergence of a vector and its significance (2)
Ans. The divergence of a vector field at any point is defined as the amount of flux diverging through the surface enclosing unit volume.

Physical significance of divergence $\vec{A}$ – If $\vec{A}$ represents the velocity of a moving fluid at any point P, then div $\vec{A}$ gives the rate at which the fluid is diverging per unit volume from point P.

If div $\vec{A} > 0$ at any point P, then either the fluid is expanding or, its density at P is decreasing with time or, the point P is source of the fluid. If div A < 0, then either fluid is contracting or its density is increasing at P or, the point is a sink.

If div $\vec{A} = 0$, then flux of $\vec{A}$ entering any element of space is exactly balanced by the flux leaving it. Then there is no source or sink in the field nor its density is changing.

Q. ... An electric field in a region is given by $\vec{E} = 3\hat{i} + 4\hat{j} - 5\hat{x}$. Calculate the electric flux through the source. $\vec{S} = 2.0 \times 10^{-5}\ m^2$ (2.5)

Ans.
$$\phi_E = \vec{E}.\vec{S} = (3\hat{i} + 4\hat{j} - 5\hat{k}).(2.0\times10^{-5}\hat{k})$$
$$= 0 + 0 - 10 \times 10^{-5} = 1.0 \times 10^{-4}\ \text{Vm}.$$
$$\boxed{\phi_E = 1.0\times10^{-4}\ \text{Vm}}$$

Q. ... Discuss the continuity equation. Distinguish between conduction current and displacement current. (4)

Ans. An equation, which expresses the equality of incoming and outgoing charges in a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density J and the charge density ρ are related at each point through a differential equation. This relation is based on the fact that *electric charge can neither be created nor be destroyed and rate of increase* of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.

$$\therefore \qquad I = \int_S \mathbf{J}.d\mathbf{S} \qquad ...(1)$$

Again, considering charge leaving a volume V per second

$$I = -\frac{\partial}{\partial t}\int_V \rho dV \qquad ...(2)$$

The negative sign comes here because the current is positive when the net charge is from the outside of V to within.

Since, we are dealing with a fixed volume V, hence we may write,

$$-\frac{\partial}{\partial t}\int_V \rho dV = -\int_V \frac{\partial\rho}{\partial t}dV \qquad ...(3)$$

Again, using divergence theorem, we can write

$$\int_S J.d\mathbf{S} = -\int_V (\nabla.J)dV \qquad ...(4)$$

Therefore, from Eqs. (2),(3) and (4), we have

$$I = \int_V (\nabla.J)dV = -\int_V \frac{\partial\rho}{\partial t}dV \qquad ...(5)$$

$$\therefore \qquad \int_V (\nabla.J)dV = -\int_V \frac{\partial\rho}{\partial t}dV$$

or
$$\int\left\{(\nabla.J) + \frac{\partial\rho}{\partial t}\right\}dV = 0$$

This integral must be zero for any arbitrary volume. It is only possible when integral is zero, i.e.,

$$\nabla.J + \frac{\partial\rho}{\partial t} = 0$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial\rho}{\partial t} = 0$ and hence for steady current, we have

$$\nabla.J = 0$$

Difference between conduction current and displacement current is given below:

S. No.	Conduction current	Displacement current
1.	Actually flows through in conductivity medium and obeys Ohm's law.	Set up in a dielectric medium due to changing electric field across the dielectric, which leads to variation of induced displacement of charge.
2.	$J_c = \sigma E, I = \frac{V}{R}$	$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t}$
3.	For constant E, $J_c \neq 0$	For constant E, $J_d = 0$

Q. (c) If the earth receives 20 cal/min/sq. cm. solar energy, what are the amplitudes of electric and magnetic fields of radiation. (4)

Ans. Poynting vector is–

$$\vec{S} = \vec{E} \times \vec{H} = EH \sin 90° = EH$$

$$\text{Solar energy} = 20 \text{ Cal/min/cm}^2$$

$$= \frac{20 \times 4.18 \times 10^4}{60} Jm^{-2}s^{-1}$$

$$\therefore EH = \frac{20 \times 4.18 \times 10^4}{60} = 14000$$

But $$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$$

$\therefore$ $$EH \times \frac{E}{H} = 14000 \times 377$$

$$E^2 = \sqrt{14000 \times 377} = \sqrt{5278000} = 2297.4 \frac{V}{m}$$

and $$H = \frac{E}{377} = \frac{2297.4}{377} = 6.094 \approx 6.1 \frac{A}{m}$$

Amplitudes of electric and magnetic fields of radiation are-

$$E_0 = E\sqrt{2} = 2297.4\sqrt{2} = 3249.01 \text{V/m}$$

$$H_0 = H\sqrt{2} = 6.1\sqrt{2} = 8.63 \frac{A}{m}$$

Q. (a) A 2 KW laser beam is concentrated by a lens into cross-sectional area about 10^{-6} cm². Find the poynting vector. (2)

Ans. Poynting vector is $\frac{\text{Power}}{\text{Area}}$

$\therefore$ $$\vec{S} = \frac{P}{\text{Area}}$$

Given $$P = 2\text{kw} = 2 \times 10^3 \text{ w}$$

$$\text{Area} = 10^{-6} \text{ cm}^2 = 10^{-10} \text{ m}^2$$

$\therefore$ $$\vec{S} = \frac{2 \times 10^3}{10^{-10}} = 2 \times 10^{13} \frac{w}{m^2}$$

Q. If the average distance between the sun and the earth is 1.5×10¹¹ m and the power radiated by the sun is 3.8 × 10²⁶ watt, show that the average Solar energy incident on earth is 2 cal/cm²- min. (2)

Ans. Poynting vector $= \dfrac{\text{Power}}{\text{Area}}$

$$\text{Area} = 4\pi r^2$$

where, r = distance between sun and earth

$$\bar{S} = \frac{P}{4\pi r^2}$$

where, $P = 3.8\times10^{26}$ watt (given) $r = 1.5\times10^{11}$ m (given)

$$\bar{S} = \frac{3.8\times10^{26}}{4\times3.14\times(1.5\times10^{11})^2}\,\frac{\text{watt}}{m^2}$$

$$= \frac{3.8\times10^{26}\times60}{4\times3.14\times(1.5\times10^{11})^2\times4.2\times10^4}\,\frac{\text{Cal}}{\text{cm}^2\text{-min}} = 2\text{ cal /cm}^2\text{-min}$$

As, 1 Cal = 4.2 J and
1 watt = 1J/E

Q. Find constant a, b and c so that
$V = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$ is irrotational. (2.5)

Ans. For irrotational vector-

$$\bar{\nabla}\times\bar{A} = 0$$

$$\bar{\nabla}\times\bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

Here,

$$A_x = x + 2y + az$$
$$A_y = bx - 3y - z$$
$$A_z = 4x + cy + 2z$$

Solving for $\bar{\nabla}\times\bar{A}$

$$\bar{\nabla}\times\bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(4x + cy + 2z) - \frac{\partial}{\partial z}(bx - 3y - z)\right] +$$

$$\hat{j}\left[\frac{\partial}{\partial z}(x + 2y + az) - \frac{\partial}{\partial x}(4x + cy + 2z)\right] +$$

$$\hat{k}\left[\frac{\partial}{\partial x}(bx - 3y - z) - \frac{\partial}{\partial y}(x + 2y + az)\right]$$

$$= \hat{i}(c + 1) + \hat{j}(a - 4) + \hat{k}(b - 2) = 0$$

$$\therefore \quad \boxed{\begin{array}{l} c = 1 \\ a = 4 \\ b = 2 \end{array}}$$

Q. (c) Prove that vector $\vec{A} = 3Y^2Z^2\hat{i} + 3X^2Z^2\hat{j} + 3X^2Y^2\hat{k}$ is solenoidal. **(2)**

Ans. For solenoidal vector-

$$\vec{\nabla}.\vec{A} = 0$$

$$\vec{\nabla}.\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\left(3Y^2Z^2\hat{i} + 3X^2Z^2\hat{j} + 3X^2Y^2\hat{k}\right)$$

$$= 0 + 0 + 0$$

$$\boxed{\vec{\nabla}.\vec{A} = 0}$$

Q. (d) Show that in free space the direction of flow of electromagnetic energy is along the direction of wave propagation. **(4)**

Ans. The Poynthing vector (i.e., energy flow per unit area per unit time) for a plane electromagnetic wave is given by

$$\vec{S} = \vec{E} \times \vec{H} = \vec{E} \times \frac{\hat{n} \times \vec{E}}{\mu_0 c}$$

$$= \frac{1}{\mu_0 c}\vec{E} \times (\hat{n} \times \vec{E})$$

$$= \frac{1}{\mu_0 c}E^2\hat{n} \qquad [\vec{E}\cdot\hat{n} = 0; E \text{ being} \perp \text{to } \hat{n}]$$

$$= \frac{E^2}{Z_0}\hat{n}$$

For a plane electromagnetic wave of angular frequency ω, the average value of S over a complete cycle is given by

$$\langle S \rangle = \frac{1}{Z_0}\langle E^2 \rangle \hat{n} = \frac{1}{Z_0}\left\langle \left(E_0 e^{ik.r - i\omega t}\right)^2 \right\rangle_{real} \hat{n}$$

$$= \frac{1}{Z_0}E_0^2 \langle \cos^2(k.r - \omega t) \rangle \hat{n}$$

$$= \frac{1}{Z_0}\frac{E_0^2}{2}\hat{n} \qquad \left[\text{Average value of } \cos^2(k.r - \omega t) = \frac{1}{2}\right]$$

$$= \frac{1}{Z_0}E_{rms}^2\hat{n} \qquad \left[\because E_{rms} = E_0/\sqrt{2}\right]$$

It is obvious that the direction of Poynting vector is along the direction of propagation of electromagnetic wave. This means that the flow of energy in a plane electromagnetic wave in free space is along the direction of wave.

Q. (i) If $\vec{A} = 2x\hat{i} + 2y\hat{j} + 3z\hat{k}$. Find Curl $\vec{A}$ (2)

Ans.
$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x & 2y & 3z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}3z - \frac{\partial}{\partial z}2y\right) - \hat{j}\left(\frac{\partial}{\partial x}3z - \frac{\partial}{\partial z}2x\right) + \hat{k}\left(\frac{\partial}{\partial x}2y - \frac{\partial}{\partial y}2x\right)$$

$$= 0 + 0 + 0$$

$$\boxed{\vec{\nabla} \times \vec{A} = 0}$$

Q. (ii) A charge of 1500μC is distributed over a very large sheet having surface area of 300m². Calculate the electric field at a distance of 25 cm. (2.5)

Ans.
$$\sigma = \frac{1500\mu c}{300\, m^2}$$

$$= \frac{1500 \times 10^{-6}}{300} = 5 \times 10^{-6} \frac{c}{m^2}$$

$$\boxed{\sigma = 5 \times 10^{-6} \frac{c}{m^2}}$$

Consider a vector field $\bar{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Is this field solenoidal? (2)

Ans. Solving for $\vec{\nabla}.\vec{A}$

$$\vec{\nabla}.\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\left(x^2\hat{i} + y^2\hat{j} + z^2\hat{k}\right)$$

$$= 2x + 2y + 2z \neq 0$$

Q. State Ampere's Law. Discuss the inconsistency in Ampere's Law

Ans. According to Ampere's circuital law, the line integral of the magnetic field along any closed loop C is proportional to the current I passing through the closed-loop, i.e.

$$\oint B.dl = \mu_0 I$$

In 1864, Maxwell showed that Eq. (1) is logically inconsistent. To prove the inconsistency, we consider a parallel plates capacitor being charged by a battery as shown in Fig.1. (a). As the charging continues, a current I flows through the connecting wire which of course changes with time. This current produces a magnetic field around the capacitor. Consider two planar loops C_1 and C_2, C_1 just left of the capacitor and C_2 between the capacitor plates, with their planes parallel to these plates.

Now the current I flows across the area bounded by loop C_1 because connecting wire pass through it. Hence from Ampere's law, we have

$$\oint B.dl = \mu_0 I$$

But the area bounded by C_2 lies in the region between the capacitor plates, so no current flows across it.

$$\oint B.dl = 0 \qquad ...(3)$$

Imagine the loops C_1 and C_2 to be infinitesimally close to each other, as shown in Fig. 1.

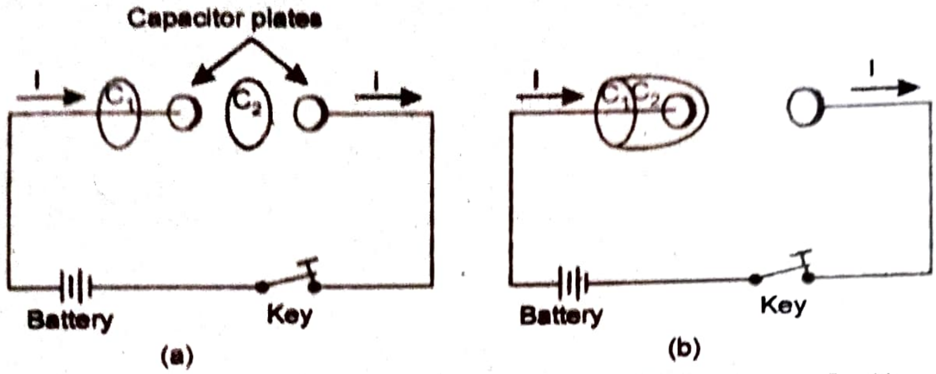


Fig. 1. A parallel plate capacitor being charged by a battery.

Then we must have $$\oint B.dl = \oint B.dl \qquad ...(4)$$

This result is inconsistent with Eqs. (3) and (4) so a need for modifying Ampere's law was felt by Maxwell.

Maxwell's Modification of Ampere's Law: Displacement Current: To modify Ampere's law, Maxwell followed a symmetry consideration. By Faraday's law, a changing magnetic field induces an electric field, hence a changing electric field must induce magnetic field. As currents are the usual sources of magnetic fields, a changing electric field must be associated with a current. Maxwell called this current as the 'displacement current' to distinguish it from the usual current caused by the drift of electrons.

Displacement current is that current, which comes into existence, in addition to the conduction current, whenever the electric field and hence the electric flux changes with time.

To maintain the dimensional consistency, the displacement current is given by the form.

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

where ϕ_E = electric field × area = ES, is the electric flux across the loop.

∴ Total current across the closed loop.

$$= I_c + I_d = I_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

Hence the modified form of Ampere's law is

$$\oint B.dl = \mu_0\left[I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right] \qquad ...(5)$$

Unlike the conduction current, the displacement current exists whenever the electric field and hence the electric flux is changing with time. Thus according to Maxwell, the source of magnetic field is not just conduction electric current due to flowing charges, but also the time varying electric field. Hence the total current I is the sum of the conduction current I_c and displacement current I_d, *i.e.*,

$$I = I_c + I_d = I_c + \varepsilon_0 \frac{d\phi_E}{dt} \qquad ...(6)$$

Q. Define poynting vector. State and prove poynting theorem. (7)

Ans. The electromagnetic waves carry energy when they propagate and there is an energy density associated with both electric and magnetic field. The amount of energy flowing through unit area, perpendicular to the direction of energy propagation per unit time, *i.e.* the rate of energy transport per unit area is called poynting vector. It is also called instaneous energy flux density and is represented by. $\vec{S} = \vec{E} \times \vec{H}$

Poynting theorem:

When em wave propagates through space from source to receiver there exist a simple and direct relationship between power transferred and amplitude of electric and magnetic field strength. The relationship may be obtained through maxwell equation.

from Maxwell IV^{th} eqn.

$$\vec{\nabla} \times \vec{B} = \mu\left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right]$$

But, aa

$$\vec{B} = \mu \vec{H} \Rightarrow \frac{\vec{B}}{\mu} = \vec{H}$$

and

$$\epsilon \vec{E} = \vec{D}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon \frac{d\vec{E}}{dt}$$

$$\vec{J} = \vec{\nabla} \times \vec{H} - \varepsilon \frac{\partial \vec{E}}{\partial t}$$

This is equation of current density, when multiplied by E, this will result in a relation between the quantities which have the dimension of power.

$$\vec{E}.\vec{J} = \vec{E}.(\vec{\nabla} \times \vec{H}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$$

from vector identity-

$$\vec{\nabla}.(\vec{E} \times \vec{H}) = \vec{H}.(\vec{\nabla} \times \vec{E}) - \vec{E}.(\vec{\nabla} \times \vec{H})$$

$$\vec{E}.(\vec{\nabla} \times \vec{H}) = \vec{H}(\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\therefore \quad \vec{E}.\vec{J} = \vec{H}(\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) - \epsilon \vec{E}.\frac{\partial \vec{E}}{\partial t}$$

From Maxwell's III^{nd} equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

we have,

$$= \vec{E}.\vec{J} = -\mu\left(\vec{H}.\frac{\partial \vec{H}}{\partial t}\right) - \in\left(\vec{E}.\frac{\partial \vec{E}}{\partial t}\right) - \vec{\nabla}.(\vec{E}\times\vec{H})$$

$$\vec{H}.\frac{\partial \vec{H}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}H^2$$

$$\vec{E}.\frac{\partial \vec{E}}{\partial} = \frac{1}{2}\frac{\partial}{\partial t}E^2.$$

$$\vec{E}.\vec{J} = \frac{-\mu}{2}\frac{\partial}{\partial t}H^2 - \frac{\in}{2}\frac{\partial}{\partial t}E^2 - \vec{\nabla}.(\vec{E}\times\vec{H})$$

Integrating over volume, v –

$$\int_v \vec{E}.\vec{J}\,dv = \frac{-\partial}{\partial t}\int_v\left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right)dv - \int_v \vec{\nabla}.(\vec{E}\times\vec{H})dv$$

Using divergence theorem-

$$\int_v \vec{\nabla}.(\vec{E}\times\vec{H})dv = \oint_s (\vec{E}\times\vec{H}).ds$$

$$\boxed{\oint_s (\vec{E}\times\vec{H}).ds = -\frac{\partial}{\partial t}\int_s\left(\frac{\mu}{2}H^2 + \frac{\varepsilon}{2}E^2\right).dv - \int_v \vec{E}.\vec{J}\,dv}$$

or

$$\frac{-\partial}{\partial t}\left\{\left(\frac{\mu}{2}H^2 + \frac{\in E^2}{2}\right)\right\}dv = \int_s\left(\vec{E}\times\vec{H}\right)\overline{ds} + \int_v \vec{E}.\vec{J}\,dv$$

i.e. Rate of energy flow = rate of decrease of stored energy + total instantaneous ohmic power dissipated within the volume.

This is called the "Poynting theorem" and

$\vec{S} = \vec{E}\times\vec{H}$ is called "poynting vector".

$-\int E.J\ dv$ – Rate of energy transferred into em field through the motion of free charge in Volume V.

$-\frac{\partial}{\partial t}\int_v\left(\frac{\mu}{2}H^2 + \frac{\in}{2}E^2\right)dv$ – Rate of decrease of em energy stored in volume V.

$\oint_s\left(\vec{E}\times\vec{H}\right).ds$ – Amount *of em* energy crossing the closed surface per second.

Physical meaning of equation- The time rate of change of em energy with certain volume plus time rate of energy flowing out through the boundary surface is equal to power transferred into em field.

Q.) If the magnitude of $\vec{H}$ in a plane wave is 1.0 Amp/m. Find the magnitude of $\bar{E}$ for a plane wave in free space. **(2)**

Ans. In free space

$$Z = \left|\frac{E_0}{H_0}\right| = \mu_0 c = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.720 \text{ ohm}$$

Given

$$H = 1\frac{\text{Amp}}{\text{m}}$$

$\therefore$

$$E = ZH_0$$

$$\boxed{E = 376.72\frac{\text{V}}{\text{m}}}$$

Q.) Deduce the equation for propagation of electromagnetic wave in free space and obtain an expression for the velocity. Show that electric and magnetic field vectors are normal to each other and to the direction of propagation of waves. **(8)**

Ans. Maxwell's equations are:

$$\text{div } D = \nabla . D = \rho \quad \text{...(a)}$$

$$\text{div } B = \nabla . B = 0 \quad \text{...(b)}$$

$$\text{curl } E = -\frac{\partial B}{\partial t} \quad \text{.....(c)}$$

and

$$\text{curl } H = J + \frac{\partial D}{\partial t} \quad \text{...(d)}$$

Free space is characterised by,

$$\rho = 0, \ \sigma = 0, \ \mu = \mu_0 \text{ and } \varepsilon = \varepsilon_0 \quad \text{...(2)}$$

Therefore, Maxwell's Eqs. (1) reduce to

$$\text{div } E = 0 \quad \text{...3(a)}$$

$$\text{div } H = 0 \quad \text{...3(b)}$$

$$\text{curl } E = -\mu_0 \frac{\partial H}{\partial t} \quad \text{...3(c)}$$

and

$$\text{curl } H = \varepsilon_0 \frac{\partial E}{\partial t} \quad \text{...3(d)}$$

Taking curl of *Eq.* 3. (c), we get

$$\text{curl curl } E = -\mu_0 \frac{\partial}{\partial t}(\text{curl H})$$

Substituting curl H from *Eq.* 3. (d) we get

$$\text{curl curl } E = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial E}{\partial t}\right)$$

i.e.,

$$\text{curl curl } E = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{...(4)}$$

Now, curl curl E = grad div $\dot{E} - \nabla^2 E = -\nabla^2 E$ [$\because$ div $E = 0$ from *Eq.* 3 (a)]

Making substitution *Eq.* (4), becomes

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad ...(5)$$

Similarly, on taking curl of *Eq.* 3. (d) and putting the value of curl *E* from *Eq.* 3. (c) we have

$$\text{curl curl } H = \varepsilon_0 \frac{\partial}{\partial t}\left(-\mu_0 \frac{\partial H}{\partial t}\right)$$

$$= -\mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} \qquad ...(6)$$

Again using identity curl curl H = grad div $H - \nabla^2 H$ and noting that div $H = 0$ from *Eq.* 3 (b), we obtain

$$\text{curl curl } H = -\nabla^2 H$$

Making this substitution in *Eq.* (6), we get

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \qquad ...(7)$$

Equations (5) and (7) represent wave equation governing electric and magnetic fields (*E* and *H*) in free space. It may be noted that these equations may be obtained by using *Eq.* (2) in *Eqs.*(5) and (7) are vector equations of identical form, which means that each of six components of *E* and *H* separately satisfies the same scalar wave equation of the form

$$\nabla^2 \phi - \mu_0 \varepsilon_0 \frac{\partial^2 \phi}{\partial t^2} = 0 \qquad ...(8)$$

where ϕ is a scalar and cannot stand for one of the components of *E* and *H*. It is obvious that *Eq.* (8) resembles with general wave equation

$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} \qquad ...(9)$$

where v is the velocity of wave.

Comparing *Eqs.* (8) and (9), we see that the field vectors *E* and *H* are propagated in free space as waves at a speed equal to

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad ...(10)$$

$$= \sqrt{\frac{4\pi}{\mu_0 4\pi\varepsilon_0}} = \sqrt{\frac{4\pi}{4\pi \times 10^{-7}} \times 9 \times 10^9}$$

$$= 3 \times 10^8 \text{ m/s}$$

$$= c = \text{speed of light in vaccum.}$$

Therefore, it is reasonable to write c the speed of light in place of $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, so *Eqs.* (5) and (7) take the form

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \qquad ...(11)$$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0 \qquad ...(12)$$

and $$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \qquad ...(13)$$

The plane wave solution of above equations in well known form many be written as

$$E(r,t) = E_0 e^{ik.r - i\omega t} \qquad ....(14)$$

$$H(r,t) = H_0 e^{ik,r - i\omega t} \qquad ....(15)$$

$$\phi(r,t) = \phi_0 e^{ik,r - i\omega t} \qquad ....(16)$$

where E_0, H_0 and ϕ_0 are complex amplitudes which are constant in space and time while k is a wave propagation vector denoted as

$$k = k\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\upsilon}{c}\hat{n} = \frac{\omega}{c}\hat{n} \qquad ....(17)$$

Here $\hat{n}$ is a unit vector in the direction of wave propagation. Now in order to apply the condition $\nabla.E = 0$ and $\nabla.H = 0$. Let us first find $\nabla.E$ and $\nabla.H$.

$$\nabla.E = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).E_0 e^{ik,r - i\omega t}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).[(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})e^{i(k_x x + k_y y + k_z z) - i\omega t}]$$

$$[\because \hat{k}.\hat{r} = (\hat{i}k_x + \hat{j}k_y + \hat{k}k_z).(\hat{i}x + \hat{j}y + \hat{k}z) = k_x x + k_y y + k_z z]$$

$$\therefore \quad \nabla.E = (E_{0x}\hat{i}\, k_x + E_{0y}\,\hat{i}\, k_y + E_{0z}\,\hat{i}\, k_z)\, e^{ik.r - i\omega t}$$

$$= \hat{i}(k_x E_{0x} + k_y E_{0y} + k_z E_{0z})e^{ik.r - i\omega t}$$

$$= \hat{i}(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z).(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})e^{ik.r - i\omega t}$$

$$= \hat{i}k.E_0 e^{ik.r - i\omega t} = \hat{i}k.E$$

Similarly, $$\nabla.H = \hat{i}k.H$$

Thus the requirement $\nabla.E = 0$ and $\nabla.H = 0$ demands that

$$k.E = 0 \text{ and } k.H = 0 \qquad ...(18)$$

This means that electromagnetic field vectors E and H are both perpendicular to the direction of propagation vector k. This implies that electromagnetic waves are transverse in character.

Further restrictions are provided by curl. From Eqs. 3. (c) and 3. (d).

$$\text{curl } E = -\mu_0\frac{\partial H}{\partial t} \text{ and curl } H = \varepsilon_0\frac{\partial E}{\partial t}$$

Using Eqs. (14), (15) and (17), above equation yields

$$ik.E = -\mu_0.(-i\omega H) \quad \text{or} \quad k.E = \mu_0\omega H \qquad ...(19)$$

and $$ik.H = \varepsilon_0.(-i\omega E) \quad \text{or} \quad k.H = -\varepsilon_0\omega E \qquad ...(20)$$

From Eq. (17), it is obvious that field vector H is perpendicular to both k and E and according to Eq. (18), E perpendicular to both k and H. This simply means that field vectors E and H are mutually perpendicular and they are also perpendicular to the direction of propagation of wave.

Q. For position vector $\vec{r}$ prove that $\nabla r^n = nr^{n-1}\hat{r}$. **(2)**

Ans. Solving for ∇r^n

Where,
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\nabla r^n = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot(x^2 + y^2 + z^2)^{n/2}$$

as
$$r = (x^2 + y^2 + z^2)^{1/2}$$

$$\vec{\nabla} r^n = \left[\frac{n}{2}(x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2x\right]\hat{i} + \left[\frac{n}{2}(x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2y\right]\hat{j} + \left[\frac{n}{2}(x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2z\right]\hat{k}$$

$$= n\left(x^2 + y^2 + z^2\right)^{\frac{n}{2}-1}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$= nr^{n-2}\vec{r}$$

$$= nr^{n-2}\,\hat{r}\cdot r \qquad \left(\text{as } \hat{r} = \frac{\vec{r}}{r}\right)$$

$$\vec{\nabla} r^n = nr^{n-1}\hat{n} \quad \text{proved}$$

Q. A conducting sphere of radius 5cm has an unknown charge. If the electric field 10 cm from the centre of the sphere is $1.5 \times 10^3 \frac{\text{N}}{\text{C}}$ and points inwards, what is the charge on the sphere? **(2.5)**

Ans. For conducting sphere at a point outside the charged sphere is given by

$$E = \frac{q}{4\pi r^2 \epsilon_0 \epsilon_r}$$

Given:

Radius of sphere $= a = 5$ cm ; $E = 1.5 \times 10^3 \frac{N}{C}$; $r = 10$ cm $= 10^{-1}$ m $= 0.1$ m

For air $\epsilon_r = 1$

and $\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{F}}{\text{m}}$

$$\therefore \quad 1.5 \times 10^3 = \frac{q}{4 \times 3.14 \times (10^{-1})^2 \times 1 \times 8.854 \times 10^{-12}}$$

$$q = 1.5 \times 10^3 \times 4 \times 3.14 \times 10^{-2} \times 8.854 \times 10^{-12} = 166.80 \times 10^{-11}\ \text{C}$$

$$\boxed{q = 0.1668 \times 10^{-8}\ \text{C}}$$

Q. Show that curl of gradient of a scalar function is always zero. (2)

Ans. Let the scalar function be $\phi(x, y, z)$

Solving for $\vec{\nabla} \times \vec{\nabla}\phi$

$$\vec{\nabla} \times \vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \hat{i}\left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial z \partial y}\right) + \hat{j}\left(\frac{\partial^2 \phi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial x \partial z}\right) + \hat{k}\left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x}\right)$$

$$= 0 \qquad \left[\because \frac{\partial^2 \phi}{\partial y \partial z} = \frac{\partial^2 \phi}{\partial z \partial y} \text{ and so on.}\right]$$

UNIT III

Interference: Interference by division of wave front (Young's double slit experiment, Fresnel's biprism), interference by division of amplitude (thin films, Newton's rings, Michelson's interferometer), Coherence and coherent sources.

Diffraction: Fraunhofer and Fresnel diffraction; Fraunhofer diffraction for Single slit, double slit, and N-slit (diffraction grating), Fraunhofer diffraction from a circular aperture, resolving power and dispersive power of a grating, Rayleigh criterion, resolving power of optical instruments.

Polarization: Introduction to polarization, Brewster's law, Malu's law, Nicol prism, double refraction, quarter-wave and half-wave plates, optical activity, specific rotation, Laurent half shade polarimeter. [12 Hrs.]

Q. If the diameter of n^{th} dark ring in a Newton's rings arrangement changes from 1.2 cm to 1.0 cm, when air is replaced by a transparent liquid find the refractive index of liquid. (2)

Ans. Given $D_n = 1.2$ cm; $D_n' = 1.0$ cm;

$$D_n'^2 = \frac{4n\lambda R}{\mu}$$

and the diameter of n^{th} ring in air is

$$D_n^2 = 4n\lambda R$$

$$\mu = \frac{D_n^2}{D_n'^2} = \frac{(1.2)^2}{(1.0)^2} = 1.44$$

Q. How would you obtain a sustained interference pattern with good contrast? (2.5)

Ans. (*i*) The two sources should continuously emit waves of the same wavelength or frequency; otherwise maxima, minima will change with time.

(*ii*) The two sets of wave trains from the two sources should either have the same phase or a constant difference in phase *i.e.*, the two sources must be coherent.

(*iii*) The amplitude of two waves must be equal or nearly equal. In this case the cancellation of amplitude will near zero. While the addition will make the amplitude double and the intensity four times. Thus maxima & minima will be seen distinctly.

Q. Distinguish between Fresnel and Fraunhofer class of diffraction. (2.5)

Ans.

Fresnel diffraction	Fraunhofer diffraction
1. The source of the screen or both are at fine distance from the diffracting element.	1. The distance of the source and screen from the diffracting element is infinite.
2. No lenses are used to make the rays parallel or convergent.	2. A convex lens is used to make the light from the source parallel before it falls on the aperture another convex lens is used to focus the light after diffraction on the screen.
3. The incident wavefront is not plane but is either cylindrical or spherical.	3. The incident waveport is plane and the secondary waves originating from the exposed part of wavefront are in the same phase at every point in the plane of the aperture.

Q. Why is diffraction of sound waves more evident in our daily life than that of light wave? (2.5)

Ans. The wavelength of sound wave is much greater than that of light waves and is comparable with the size of the obstacle, which is essential condition for diffraction. This is why deffraction of sound is more evident in daily life than light waves.

Q. Calculate the minimum thickness of a calcite plate that would convert plane polarized light into circularly polarized light. The principal refractive indices for the ordinary and extraordinary rays are 1.658 and 1.486 respectively at wavelength 5890 Å.

Ans. The plane polarized light shall be converted to circularly polarized light if the plate thickness introduced with a path difference which is an odd multiple of $\lambda/4$. Thus the thickness t of the calcite with plate is given as

$$t = \frac{\lambda(2n-1)}{(\mu_0 - \mu_e)4}$$

$(n = 1)$ for minimum thickness, so $t = 8.50 \times 10^{-5}$ cm

Q. Show the geometry of calcite crystal by a neat diagram. (2.5)

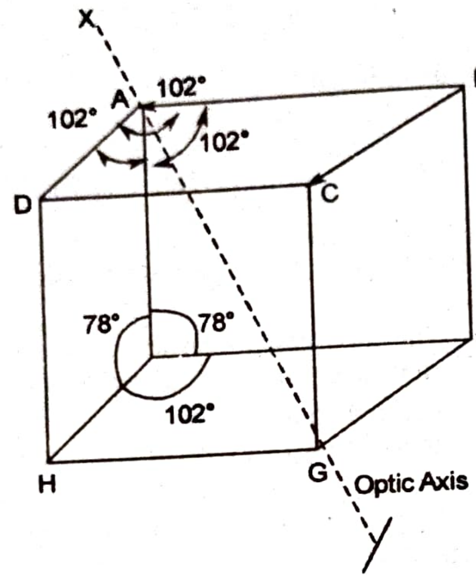
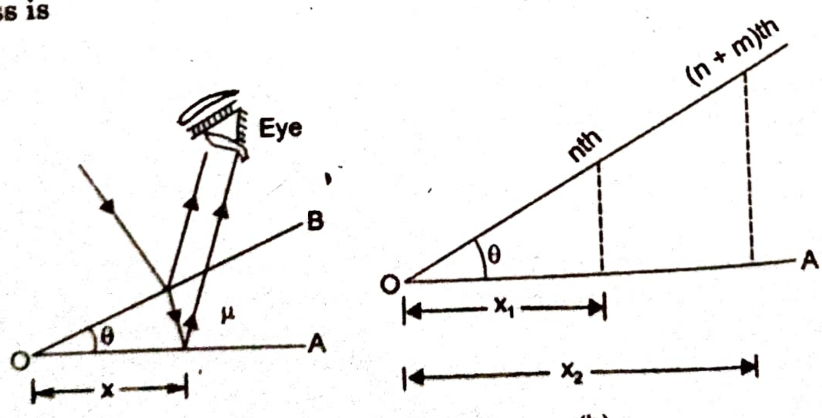


Fig. 1

Q. Discuss the formation of interference fringes due to wedge shaped thin film seen by normally reflected sodium light and obtain an expression for fringe width. (6)

Ans. If t is the thickness of the film at a distance x from the edge, the path difference between the two reflected rays producing interference will be $2\mu t \cos r \pm \lambda/2$. The condition for brightness is



$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

Fig. (2)

or
$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots$$

And condition for *darkness* is
$$2\mu t \cos r = n\lambda \qquad n = 0, 1, 2, 3, \ldots$$

The film appears *bright* when 't' the thickness of film satisfies

$$2\mu t \cos r = (2n+1)\lambda/2$$

or
$$t = \frac{(2n+1)\lambda}{4\mu \cos r}$$

or
$$t = \frac{\lambda}{4\mu \cos r}, \frac{3\lambda}{4\mu \cos r}, \frac{5\lambda}{4\mu \cos r}, \ldots \text{etc.} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

The film will appear *dark* if "t" satisfies the condition

$$2\mu t \cos r = n\lambda$$

or
$$t = \frac{n\lambda}{2\mu \cos r}$$

and
$$t = 0, \frac{\lambda}{2\mu \cos r}, \frac{2\lambda}{2\mu \cos r}, \ldots \text{etc.} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

It is observed that in the direction of increasing thickness of the film there will be alternate bright and dark fringes parallel to the edge of the film.

Further, for small angle of incidence, $\cos r = 1$ because angle r will also be small. For small angle θ, the thickness $t = x\theta$, thus the condition for darkness reduces to

$$2\mu x\theta = n\lambda$$

If x_1 is the distance of the nth dark band from the edge of the wedge and x_2 that of the $(n+m)$th dark band (Fig. 2(*b*)) then

$$x_1 = \frac{n\lambda}{2\mu\theta} \quad \text{and} \quad x_2 = \frac{(n+m)\lambda}{2\mu\theta}$$

$$x_2 - x_1 = \frac{(n+m)\lambda}{2\mu\theta} - \frac{n\lambda}{2\mu\theta} = \frac{m\lambda}{2\mu\theta}$$

Here $(x_2 - x_1)$ is the distance corresponding to m fringes. The fringe wdth

$$\beta = \frac{x_2 - x_1}{m} = \frac{\lambda}{2\mu\theta}$$

Q. In a Newton's ring experiment the diameter of 4th and 12th dark rings are 0.400 cm and 0.700 cm. Deduce the diameter of 20th dark ring. (4)

Ans. The diameter of nth dark ring is given by

$$d_n^2 = \frac{4n\lambda R}{\mu}$$

or
$$d_n^2 = 4n\lambda R \qquad (\mu = 1) \text{ for air}$$

Further we can write

$$d_{n+m}^2 - dn^2 = 4m\lambda R$$

$$n = 4, n+m = 12 \quad \text{thus } m = 8$$

$$d_{12} = d_{n+m} = 0.700 \text{ cm} \quad \text{and } d_4 = d_n = 0.400 \text{ cm}$$

$$4\lambda R = \frac{(0.7)^2 - (0.4)^2}{8} = \frac{0.33}{8}$$

$$d_{20}^2 = 20 \times \frac{0.33}{8}$$

$$d_{20} = 0.907 \text{ cm}$$

Q. Discuss analytically the intensity distribution in fraunhofer diffraction at a single slit. (6)

Ans. The resultant intensity for single slit is

given by $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$, we have

(a) *For Principal (central) maximum,* $\alpha = 0$.

$\therefore$
$$I = I_0 \quad ...(1)$$

(b) For the *first secondary maximum,* $\alpha = \frac{3\pi}{2}$

$\therefore$
$$I = I_0\left[\frac{\sin 3\pi/2}{3\pi/2}\right]^2 = I_0\left[\frac{-1}{3\pi/2}\right]^2$$

or
$$I = \frac{4I_0}{9\pi^2} = \frac{I_0}{22} \quad ...(2)$$

(c) For the *second secondary maximum,* $\alpha = \frac{5\pi}{2}$

$\therefore$
$$I = I_0\left[\frac{\sin 5\pi/2}{5\pi/2}\right]^2 = I_0\left[\frac{1}{5\pi/2}\right]^2$$

or
$$I = \frac{4I_0}{25\pi^2} = \frac{I_0}{61} \quad ...(3)$$

and so on.

If the intensity of central maximum is taken us unity, the value of second, third and fourth will be approximately equal to

$$\left(\frac{2}{3\pi}\right)^2 : \left(\frac{2}{5\pi}\right)^2 : \left(\frac{2}{7\pi}\right)^2 = \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \cdots$$

This means that in Fig. which depicts the *intensity distribution* in the Fraunhofer diffraction of a slit, the first secondary maximum on either side of the principal maxiumim have only 4% the height of the principal maximum. The principal maximum occurs at $\alpha = 0$ and from this the intensity diminishes to zero at $\alpha = \pm\pi$, then passes through several secondary maxima at $\alpha = \pm 1.42\pi, \pm 2.46\pi$ etc. and minima at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ etc. The width of the principal maximum is inversely proportional to the width of the slit and a very large amount of light falls in the principal maximum.

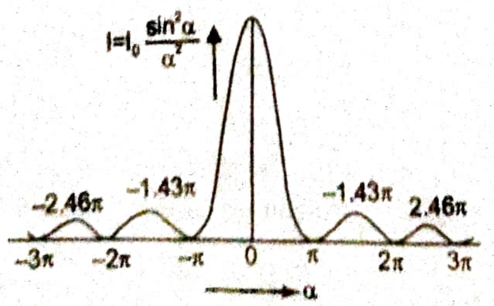


Fig. 3. Variation of Intensity as a function of a in Fraunhofer diffraction

Q. What is Rayleigh's criterion of resolution. **(2)**

Sol. According to Rayleigh crieterion, two close wavelengths can be identified as two if the principal maximum of one falls on the first minimum of the other.

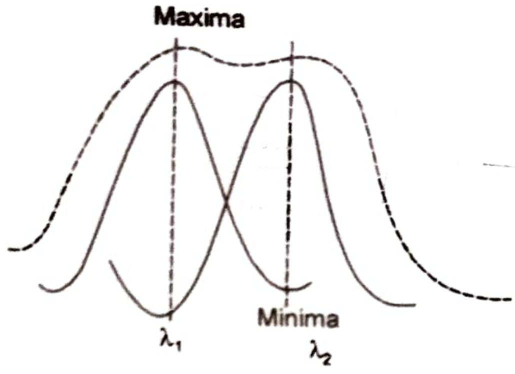


Fig. 4

Q. (c) In a diffraction grating the width of opacities and transparencies are in the ratio 1:2. Find the absent spectra. **(2)**

Sol. For absent spectra

$$\frac{(a+b)}{a} = \frac{n}{m}$$

a is the width of transparencies

b is the width of opacities

It is given that

$$b = 2a$$

$$\frac{a+2a}{a} = \frac{n}{m}$$

$$\frac{3}{a} = \frac{n}{m}$$

$$n = 3m$$

Therefore 3rd, 6th or 9th spectra are missing.

Q. Describe Laurent's half shade polarimeter. How can it be used to find the specific rotation of an optically active substance. **(6)**

Ans. Laurent's Half-Shade Polarimeter: It consists of two Nicol prisms N_1 (polarizer) and N_2 (analyser) capable of rotation about a common axis. A glass tube T, containing the optically active substance and closed at the ends by cover slips and metal covers, is mounted between the two Nicols as shown in Fig.

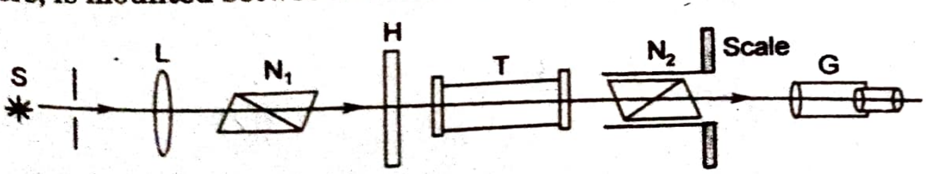


Fig.5. Laurent's half-shade polarimeter

Working of the half shade device: Half shade device (Laurent's plate) consists of a semi-circular plate of quartz, cut with faces parallel to the optic axis, cemented together with another semi-circular plate of glass so as to form a composite circular plate as shown in Fig. 6(a). The thickness of the quartz plate is such that it introduces a path difference of $\lambda/2$ (or a phase difference of π) between the ordinary and extra-ordinary rays in transmission normally through it. Thus it is simply a half wave plate, the thickness of the glass plate is so adjusted that it ransmits and absorbs the same amount of light as the quartz plate.

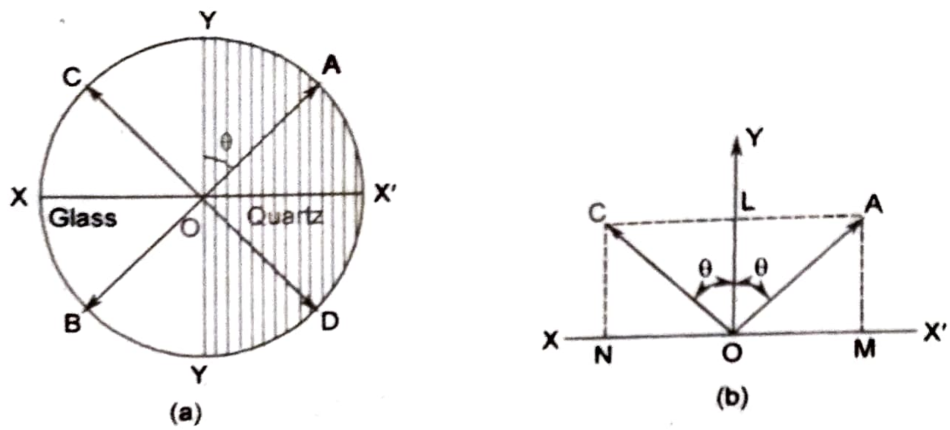


Fig.6. Principle of Laurent's half-shade device

Let the principal plane of the polarizing Nicol be parallel to AB *i.e.*, the vibration plane of the light incident on glass quartz combination is parallel to AB inclined at an angle θ to the optic axis YY' of the quartz half. On passing through the glass half, the vibrations of light will remain in the same plane *i.e.*, AB while a change occurs in the quartz half. Inside the quartz half, the light is divided into two components—one E component parallel to the optic axis (YY') of the quartz and the other O-component perpendicular to it *i.e.*, along XX'. The speed of O-component is greater than that of E-component within quartz crystal and hence on emergence the O-component will gain a phase of π over the E-component. Due to this phase difference of π, the direction of the O-component is reversed *i.e.*, if the initial position of O-component is represented by OM Fig. 6 (*b*), its final position must be represented by ON (advance in phase of π). Therefore, if the components of initial incident vibrations along OA (in quartz) are along OL and OM, the emergent wave will be the resultant of the vibrations along OL and ON. The resultant of OL and ON is obviously OC which makes the same angle θ (with y-axis) as the incident vibrations along AB do but on the other side *i.e.*, $\angle AOL = \angle COL$. *Thus the effect of quartz plate is to rotate the plane of polarization by an angle 2θ.*

Hence there are two plane polarized beams-one emerging from glass half with vibrations in the plane OA while other emerging from quartz half with vibrations in the plane OC. If the principal plane of the analyser Nicol is parallel to AOB, light from the glass portion will pass unobstructed while that from the quartz portion will be partly obstructed by the analysing Nicol and hence *glass half will be brighter than the quartz half*. If, however, the principal plane of the analysing Nicol is parallel to OC, light from the quartz portion will pass unobstructed while that through the glass portion will

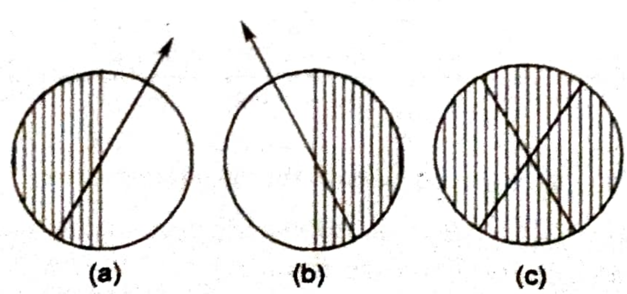


Fig. 7. Different positions of analysing Nicol

be partly obstructed by the analysing Nicol and hence *quartz half will be brighter than the glass half.* But when the principal plane of the analysing nicol is parallel to OL (Y axis), it is equally inclined to the two plane polarized beams, one from the glass half and the other from the quartz half; the amplitude of light incident on the analyser N_2 from both the halves will be qual and hence the *field of view will be equally bright* (or strictly speaking, equally dark). The appearance of the circular plate in three position is shown in Fig. 7 (*a*), (*b*) and (*c*) respectively.

Determination of specific rotation of sugar: In order to find the specific rotation of an optically active substance the angle of rotation is determined for a number of solutions of different concentrations. Then a graph is plotted between concentration C and the angle of rotation Q which comes out to be a straight line Fig. 8 From graph the ratio θ/C is determined and the specific rotation of sugar is calculated from the relation

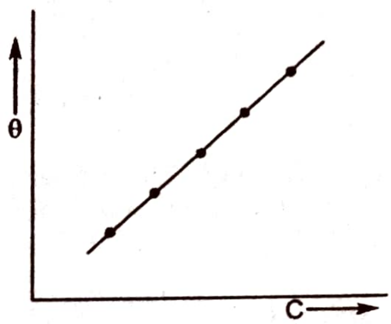


Fig.8. Plot of concentration *vs* angle of optical rotation

$$S = \frac{10\theta}{lc}$$

l = length of tube in cm

θ = rotation in degrees

C = concentration of sol. in gm/cc

Q. (?) Calculate the minimum thickness of a quarter wave plate for the light of wavelength 5893 Å. Given **(4)**

$$\mu_0 = 1.57 \text{ and } \mu_e = 1.526$$

Sol.

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)} = \frac{5899Å}{4(1.5701.526)}$$

$$t = 3.35\ \mu m$$

Q. Explain the formation of interference fringes by means of fresnel's biprism when a monochromatic source of light is used and deserve the expression for fringe width. How will you measure a wavelength of light using bi-prism method. **(8)**

Ans: Biprism is a combination of two prism with their bases joined and their two faces making an angle of about 179° and other were each of 30′.

Biprism is used to get two virtual coherent sources S_1 and S_2. The point C on the screen is equidistant from S_1 and S_2 hence the two waves reaching C reinforce each other and the point C will be the centre of a bright fringe.

For any point of the screen to be at the centre of bright fringe $\frac{D}{d}x = n\lambda$

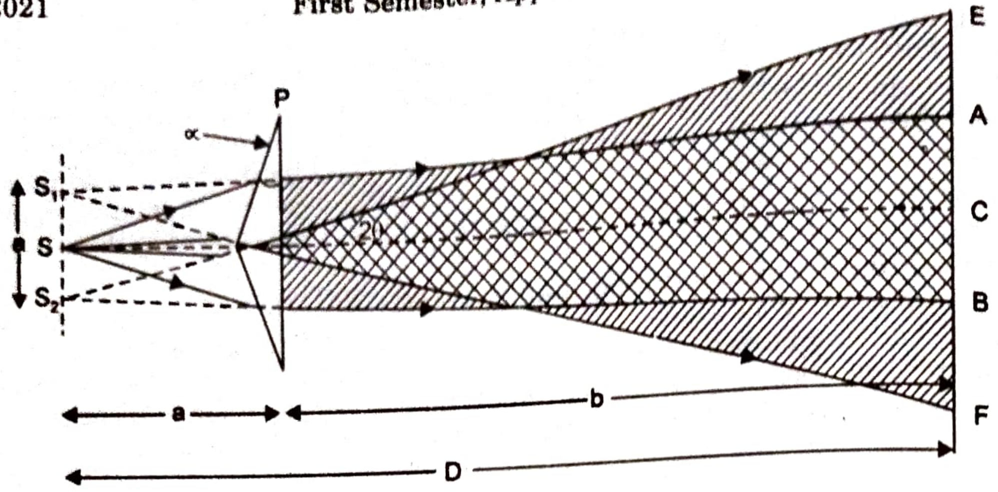


Fig. 14

$$\frac{D}{d}\lambda = \frac{x}{n} = \beta \text{ the fringe width}$$

Determination of wavelength (λ) of light : Fresnel's bi-prism can be used to determine the wavelength of a given source of monochromatic light. The measurement of fringe width in a bi-prism enables one to determine wavelength of light if D and d are known.

as
$$\beta = \frac{\lambda D}{d}$$

$$\lambda = \frac{d}{D}\beta = \frac{d}{(a+b)}\beta$$

where
$$D = (a+b)$$

Main difficulty is in to find out the distance between two virtual source S_1 and S_2. There are two methods to find the distance between two virtual source.

Determination of *d*. The value of *d* can be measured by the displacement method. A convex lens of suitable focal lenght is placed in between the eye-piece and the bi-prism without disturbing their positions. The position of the lens is adjusted so that the images of sources S_1 and S_2 are in the field of view of eye-piece. Let the position of the lens be L_1 (Fig. 15). Measure the distance between the images of S_1 and S_2 as seen through the eye-piece. Let it be x. The magnification in this case is

$$\frac{x}{d} = \frac{v}{u}$$

Now move the lens towards the eye-piece and set it to some other position L_2 so that the images of S_1 and S_2 are seen in the eye-piece again. Measure the distance between the two images in this case too. Let it by y. Thus the magnification is

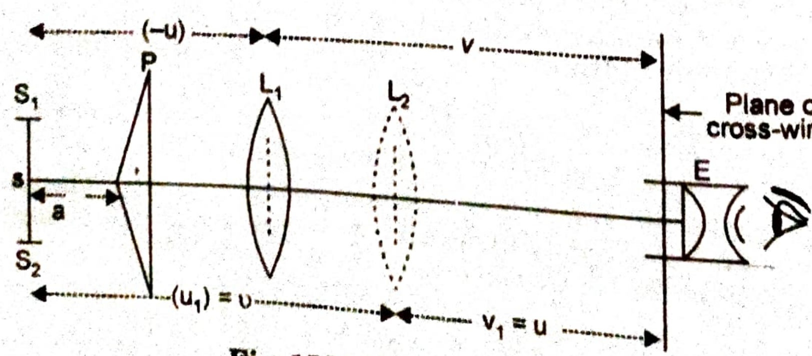


Fig. 15 Method of finding *d*

where u and v are the distances of the object and image respectively from the lens,

$$\therefore \quad \frac{x}{d}\times\frac{y}{d} = \frac{v}{u}\frac{u}{v} = 1$$

or

$$xy = d^2$$

$$d = \sqrt{xy}$$

Thus substituting the value of β, D and d, the wave-length λ can be calculated from

$$\lambda = \frac{d}{D}\beta$$

The value of d can be found by making use of the fact that for a prism of very small angle, the deviation θ of the ray is given by $\theta = (\mu - 1)\alpha$, where μ is the refractive index of the material of the prism and α its refracting angle. Both μ and α can be measured and thus θ found out. Also

$$d = 2\theta \times a$$

$$\therefore \quad \lambda = \frac{d}{D}\beta = \frac{2\theta\times a}{a+b}\beta$$

or

$$\lambda = \frac{2a(\mu-1)\alpha}{a+b}\beta.$$

(b) Thickness Measurement of Thin Transparent Sheet: Bi-prism can be used to determine the thickness of mica sheet or thin sheet of transparent material. Let S_1 and S_2 be the two coherent monochromatic source of light which are producing interference fringes on the screen so that C is

Q.... Newton's rings are formed between a plane surface of glass and lens. The diameter of third dark ring is 10^{-2}m. When a light of wavelength 5890×10^{-10}m is used a such an angle that the light passes through the air film at an angle of 30° to the normal. Find the radius of lens. (4.5)

Ans:

$$\lambda = 5890 \times 10^{-10}\text{ m}$$

$$d = 10^{-2}\text{ m}$$

$$\mu = 1$$

$$r = 30°$$

$$n = 3$$

for dark ring

$$2\mu t\cos r = n\lambda$$

and

$$\frac{dn^2}{4R} = 2t$$

So we have

$$\frac{\mu dn^2}{4R}\cos r = n\lambda$$

$$R = \frac{\mu dn^2\cos r}{4n\lambda}$$

$$= \frac{1\times10^{-2}\times10^{-2}\times\cos 30}{4\times3\times5890\times10^{-10}}$$

$$R = 12.23\text{ m}$$

Q. Why two independent sources cannot produce observable interference pattern. **(2.5)**

Ans: If-two independent sources of light of same wavelength are placed side by side, no interference fringes or effect are observed because the light waves from one source are emitted independently of those from the other source. The emissions from the two independently do not maintain constant phase relationship with each other over time as the light from an ordinary source undergo random phases change in time interval less than a nonosecond.

Q. What is the effect of increasing the angle of Biprism on the fringes? Explain **(2.5)**

Ans: If the angle $\propto$ of the biprism be increased, the distance 2d between the virtual sources would increase because 2d= 2a (μ-1)$\propto$. This, in turn, would reduce the fringe width $\left(\beta = \frac{D\lambda}{2d}\right)$.

The fringes will not be separately visible and may disappear ultimately

Q. Why central ring is dark instead of bright some times in reflected system. Give appropriate reason. **(2.5)**

Ans: This is because a dust particle comes between the two surface at the point of contanct. This dust particle introduce of path difference of the order of $\pm\lambda/_2$.

Hence the central rings appears dark instead of bright in reflected system

Q. In a differaction grating how are spectral lines affected when ruling are mase closer. **(2.5)**

Ans: When spectal lines are made closer the value of grating element, $(a + b)$ decreases. As a result angle of diffraction θ increases for a given order. This results in an increased dispersive power.

Q. A transparent plate is given. How will you distinguish whether the given plate is a quater wave plate or a half wave plate or a simple glass plate. **(2.5)**

Ans: For the recognition of the given plate, we can take two Nicol prisms N_1 and N_2 placed parallel to each other. Now the given plate is inserted between the two Nicol prisms. and unpolarized light in allowed to incident of N_1. Light transmitted through N_1 will be plane polarized light.

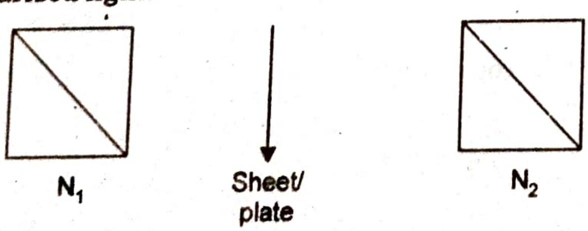


Fig. 13

(*a*) it there is no light after N_2, then plate is half wave plate.

(*b*) If on rotating N_2, there is variation in light intensity but it in not zero in any position, then plate is quarter wave plate

(*c*) if on rotating N_2, there is variation in intensity from zero to maximum value, the plate is glass plate.

Q. In a plane transmission grating the angle of diffraction for second order maxima for wavelength 5×10^{-5} cm is 30°. Calculate the number of lines in one centimeter of the grating surface (3.5)

Ans:

$$\lambda = 5\times10^{-5}\ cm$$
$$\theta = 30°$$
$$N = ?$$
$$n = 2$$
$$(e+d)\sin\theta = n\lambda$$

$$(e+d) = \frac{n\lambda}{\sin\theta} = \frac{2\times5\times10^{-5}}{\sin 30°} = 2.0\times10^{-4}\ cm$$

The number lines are

$$N = \frac{1}{(e+d)} = \frac{1}{2.0\times10^{-4}} = 5000\ \text{lines/cm.}$$

Q. What do you mean by double refraction What are ordinary and extra ordinary rays?

Ans: A beam of ordinary light when incident on a calcite or quartz crystal splits up into two polorized referacted rays in place of the usual one as in class. This phenomenon is called double refraction and crystal having this property are said to be doubly refracting. The two refracted rays are called ordinary rays (o-ray) and extra ordinary ray (e-ray).

The ordinary ray obeys the law of refraction where as the extra ordinary ray does not obey laws of refraction

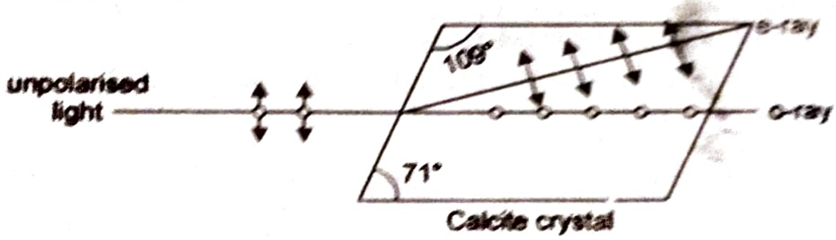


Fig. 16

Q. A 20 cm long tube containing sugar solution is placed between crossed nicols and illuminated by light of wavelength 6×10^{-5} cm. If specific rotation is 60° and optical rolation produced is 12° what is the strength of the solut on. (3)

Ans: Specific rotation

$$S = \frac{\theta}{l\times c}$$
$$S = 60°,\ l = 20\ cm,\ \theta = 12°,\ \lambda = 6\times10^{-5}\ cm$$

$$C = \frac{\theta}{l\times S} = \frac{12}{2\times60} = \frac{1}{10}\ gm/cc$$

It is 10% of the sugar i.e., 1 gm of sugar dissolved in 10cc of water.

Q. ... Each slit has width of 0.15 *mm* and distance between their centre is 0.75 *mm* what are missing orders?

Ans.

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\frac{0.75}{0.15} = \frac{n}{m}$$

$$\Rightarrow \quad \boxed{n = 5m}$$

5th, 10th, 15th... order will be missing.

Q. ... In Young's double slits experiment the distance between the slits is 0.2 mm and screen is at distance of 1.0*m*. The Third bright fringe is at a distance of 7.5 mm from the central fringe. Find the wavelength of light used.

Ans.

$$y = \frac{n\lambda D}{2d}$$

$$7.5\times10^{-3}_{m} = \frac{30+\lambda\times1}{0.2\times10^{-3}m}$$

$$\lambda = \frac{7.5\times10^{-3}\times0.2\times10^{-3}}{3} = 5000A°.$$

Q. ... Discuss the phenomenon of interference of light in thin film and obtain the conditions of maxima and minima. Show that interference patterns in reflected and transmitted lights are complimentary.

Ans. Interference due to Reflected Light: Let us consider a transparent film GHH'G' (Fig.1) of thickness *t* and refractive index μ. A ray *AB* incidents on the upper surface of the film is partly reflected along *BR* and partly refracted along *BC*. At *C*, part of it is internally reflected along *CD* and finally emerges out DR_1, parallel to BR. This will continue further in the same way.

Aim: To get path difference between two reflected rays.

Draw a normal *DE* on BR and other normal *BF* on *CD*.

One also produces *DC* in backward direction which meets at P on BQ line

In Fig.1

$$\angle ABN = i = \text{angle of incidence}$$
$$\angle QBC = r = \text{angle of refraction.}$$

From geometry of Fig.1.

and

The optical path difference between two reflected rays (*BR* and DR_1) given by

$$\Delta = \text{Path } (BC + CD) \text{ in film- Path BE in air}$$
$$= \mu(BC + CD) - BE \quad ...(1)$$

We know that,

$$\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{FD/BD} = \frac{BE}{FD}.$$

or

$$BE = \mu(FD) \quad ...(2)$$

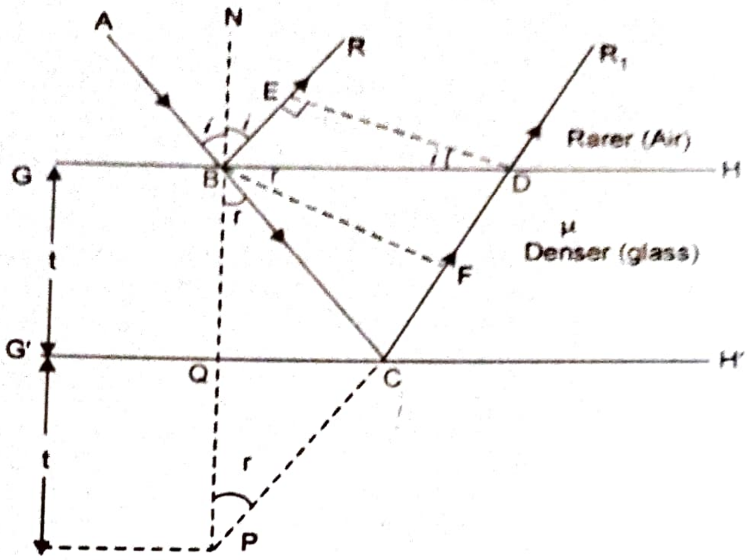


Fig. 1. Path difference created by a thin film (reflected case)

From eqn. (1) and (2)

$$\Delta = \mu(BC + CD - FD)$$
$$= \mu(BC + CF + FD - FD) = \mu(PC\text{-}CF)$$
$$= \mu(PF) \; [\because PC = BC] \quad ...(3)$$

From ΔBPF,

$$\cos r = \frac{PF}{BP}$$

or

$$PF = BP \cos r$$
$$= 2t \cos r \quad ...(4)$$

Substituting eqn. (3) in eqn. (4)

$$\Delta = \mu \times 2t \cos r$$
$$\Delta = 2\mu t \cos r \quad ...(5)$$

It should be remembered that a ray reflected at a surface backed by denser medium suffers an abrupt phase of π, which is equivalent to a path difference of $\frac{\lambda}{2}$

Thus the effective path difference between the two reflected rays = $2 \mu t \cos r \pm \frac{\lambda}{2}$

Condition for bright bands in thin film for reflected light.

The path difference D = $n\lambda$, n = 0, 1, 2, 3, 4,....

[Constructive interference]

Then

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

or

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \quad ...(A)$$ [Film will apper bright]

Condition for dark bands in thin film for reflected light.

If the path difference

$$\Delta = (2n \pm 1)\frac{\lambda}{2}, n = 0, 1, 2, 3, ...$$

[Destructive interference]

then $$2\mu t\cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

or $$2\mu t \cos r = n\lambda \quad \text{...(B)[Film will appear dark]}$$

Interference due to Transmitted Light:

Let us consider a transparent parallel film *GHH' G'G* (Fig.2) of thickness *t* and refractive index (μ). A ray *AB* incidents on the upper surface. This ray *AB* refracted as *BC*. The ray *BC* is partly internally reflected as CD and partly transmitted as *CT*. The ray *CD* also partly internally reflected as *DE* and finally emerges as transmitted ray ET_1, parallel to *CT*. This will continue further in the same way.

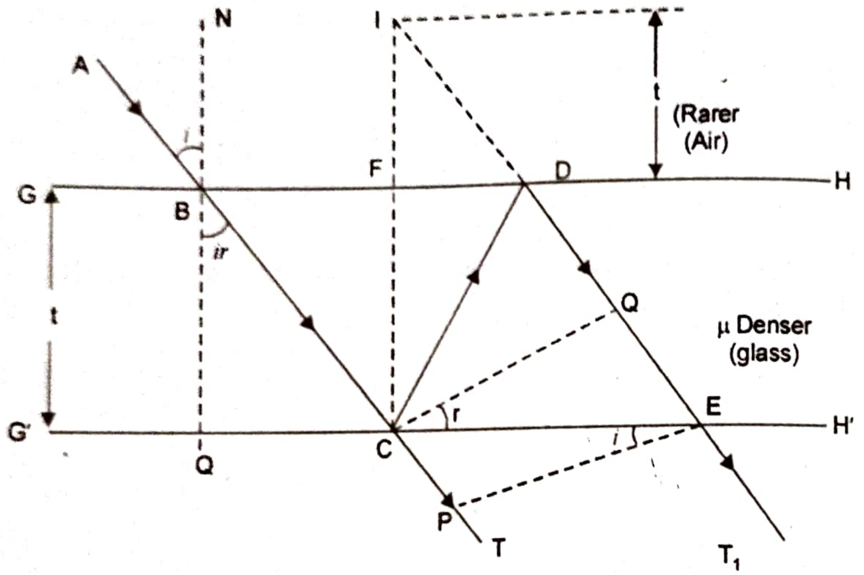


Fig. 2. Path difference created by a thin film (transmitted case)

AIM: To get path difference between two transmitted rays.

We also produce *ED* in backward direction which meets produced *CF* at *I*.

Draw a normal *EP* to *CT* and other normal *CQ* on *DE*, one also produces *ED* in backward direction, which meets at *I on CF* and angle of incidence $\angle ABN = i$ and angle of refraction $\angle CBN' = r$. From the geometry of Fig. 2.

$$\angle ECQ = r, \angle PEC = \text{i and}$$
$$\angle CIQ = r.$$

The effective path difference

$$\Delta = \mu(CD + DE) - CP$$

Also, $$\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE} \quad \text{...(6)}$$

$$CP = \mu(QE) \quad \text{...(7)}$$

From equation (6) and (7)

$$\begin{aligned} \Delta &= \mu(CD + DQ + QE) - \text{QE}\,(\mu) \\ &= \mu(CD + DQ) \\ &= \mu(ID + DQ) \qquad [\because \text{CD} = \text{ID}] \\ &= \mu(IQ) \\ &= \mu(2t\cos r) \end{aligned}$$

$$\Rightarrow \quad \Delta = 2\mu t \cos r \quad \text{...(8)}$$

Hence it should be remembered that inside the film, reflection at differents points takes place at the surface backed by rarer medium (air), thus no abrupt change of π in this case.

$$\Delta = 2\mu t \cos r = n\lambda \quad \text{...(A) (Film will appear bright)}$$

Condition for dark bands in transmitted light (Destructive interference)

$$\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

...(B) (Film will appear dark)

These condition (A) and (B) are just reverse by reflected light.

Hence the interference pattern in Reflected and transmitted light are complimentary.

Q. (c) Using Sodium light ($\lambda = 5893$ A°) interference fringes are formed in a thin air wedge. when viewed normally 10 fringes are obtained in distance of 1cm. Calculate the angle of wedge.

Ans. Fringe width is

$$\beta = \frac{x}{m} = \frac{\lambda}{2\mu\theta}$$

x = distance from edge
μ = no of fringes
μ = refractive index of medium ($\mu = 1$ for air)
λ = wavelength

$$\therefore \quad \beta = \frac{1}{10} = \frac{.5893 \times 10^{-8}\, cm}{2 \times 1 \times \theta}$$

$$\theta = 5 \times 5893 \times 10^{-8} \text{ radian} = 2.9465 \times 10^{-4} \text{ radian}$$

Q. (c) Discuss the phenomenon of Fraunhofer's diffraction at a single slit and show that the intensities of successive maximam are nearly in the ratio of (7)

$I_0 : I_1 : I_2 : I_3 = 1 : 4/9\pi^2 ; 4/25\pi^2\ 4/49\pi^2$

Ans. Fraunhoffer diffrection at single slit.

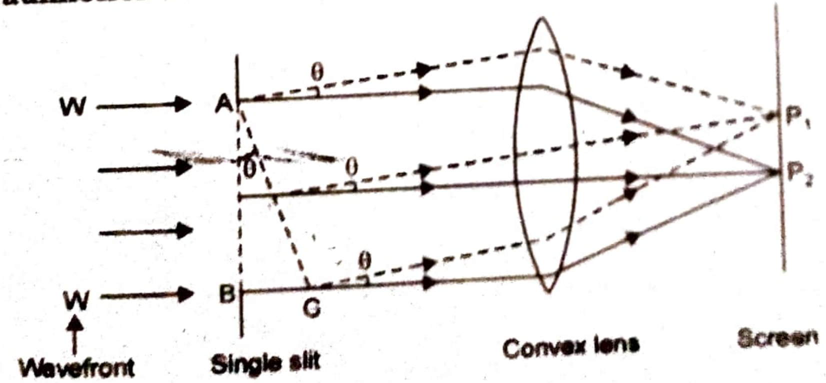


Fig. 3.

Diffracted light be focussed by means of a convex lens. According to Huygen-Fresnel Theory every point of the wavefront in the plane of slit is a source of secondary wavelents The intensity at P_1 depends on the path difference between secondary wave originating from the corresponding points of the wavefront.

The path difference between secondary wavelets from A and B in direction θ.

$$BC = AB \sin\theta = e \sin\theta$$

$$\text{Phase diff} = \frac{2\pi}{\lambda} \times e \sin\theta$$

Let width of slit is divided into n equal parts then phase difference. between any two consecutive waves from these parts is

$$= \frac{1}{n}(\text{total phase})$$

$$= \frac{1}{n}\left[\frac{2\pi}{\lambda} e \sin\theta\right]$$

$$= d(\text{say}) \quad ...(1)$$

Using the method of vector addition, the resultant amplitude

$$R = \frac{a \sin\frac{nd}{2}}{\sin\frac{d}{2}} = \frac{a \sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

$$= \frac{a \sin\alpha}{\sin\frac{\alpha}{n}} \quad \left(\alpha = \frac{\pi e \sin\theta}{\lambda}\right)$$

$$= \frac{a \sin\alpha}{\frac{\alpha}{n}} \quad \left(\because \frac{\alpha}{n} \text{ is very small}\right)$$

$$= na \frac{\sin\alpha}{\alpha}$$

$$= A \frac{\sin\alpha}{\alpha} \quad ...(2) \{ na\text{- finite term}$$

Intensity

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad ...(3)$$

Principal maximum

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

R would be maximum when $\alpha = 0$

$$\therefore \quad \alpha = \frac{\pi e \sin\theta}{\lambda} = 0$$

$$\sin\theta = 0$$

$$\theta = 0$$

Means maxima is formed by those secondary wavelets which travel normally to the slit *i.e.* (position for principal maximum)

Minimum Intensity Position:

From (3)

$$I = 0 \text{ when}$$

$$\text{Sin}\,\alpha = 0$$

the values of α which satisfy this eqn. are

$$\alpha = \pm, \pi, \pm 2\pi, \pm 3\pi...., \pm m\pi$$

or $$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$e \sin\theta = \pm m\lambda$$

$$m = 1, 2, 3.....$$

$$m \neq 0, \text{ i.e. for principal maximum}$$

Secondary maxima:

Can be calculated with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of I w.r.t α and equating to zero. We have

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or $$A^2 \frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

either $$\sin\alpha = 0 \text{ or } (\alpha\cos\alpha - \sin\alpha) = 0$$

Since $\sin\alpha = 0$ gives the value of minima therefore position of maxima are given by root of eqn.

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\tan\alpha = \alpha \qquad ...(4)$$

Values of α satisfying the above eqn. are obtained graphically by plotting the curves

$$y = \alpha$$

and $$y = \tan\alpha \text{ on the graph}$$

The points of intersection of two curves gives the values of α which satisfy eqn. (4)

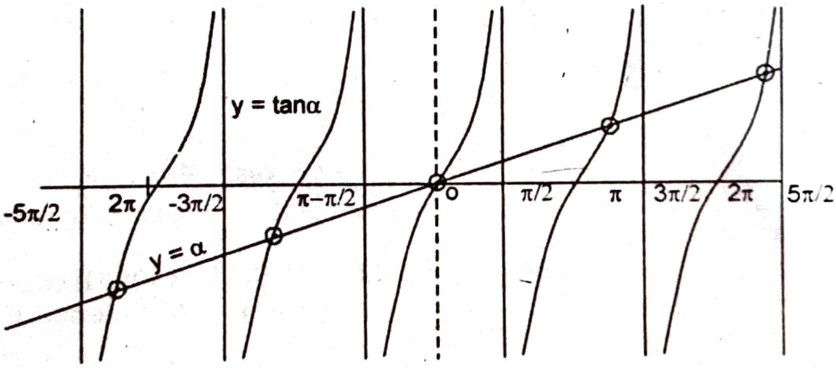


Fig. 4.

Point of intersections are

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$$

or $$\alpha = 0, \pm 1.430\pi, \pm 2.462\pi, \pm 3.471\pi$$

$$\alpha = 0 \text{ gives principal maximum}$$

Substituing value of α in eqn. (3) we get

$$I_0 = A^2 \qquad \text{[Principal maximum]}$$

$$I_1 = A^2\left[\frac{\sin\left(\frac{3\pi}{2}\right)^2}{\left(\frac{3\pi}{2}\right)}\right]^2$$

$$I_1 = \frac{A^2}{22} \text{ (Ist maxima)}$$

$$I_2 = A^2\left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)}\right]^2$$

$$I_2 = \frac{A^2}{62} \text{ (2nd maxima)}$$

$$I_3 = \frac{A^2}{121} \text{ (3rd maxima)}$$

$$I_0 : I_1 : I_2 : I_3 ....$$

$$1 : \frac{1}{22} : \frac{1}{62} : \frac{1}{121} ....$$

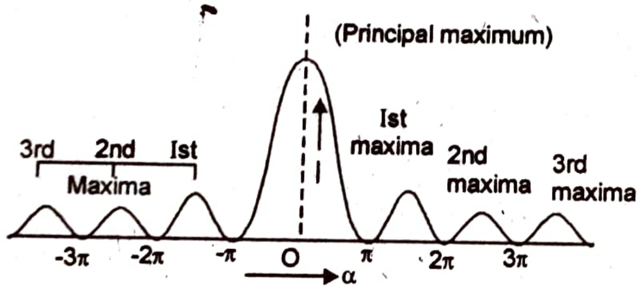


Fig. 5.

also $$= 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$$

from expressions of I_0, I_1, I_2, I_3 it is clear that most of the incident light is concentrated at principal maximum.

Q.) A plane transmission grating of length 6m has 5000 liner/cm find the resolving power of grating and smallest wavelength difference that can be resolved for light of wavelength 5000A°. **(3)**

Ans. The resolving power of the grating = n N = order of spectrum × no. of lines on the grating

Length of grating = 6 cm

No. of lines per con on the grating = 5000

Total number of lines on the grating = 5000 × 6 = 30,000

Resolving power $$\frac{\lambda}{d\lambda} = n\lambda = 2 \times 30{,}000$$

$$= 60{,}000$$

The smallest wavelength difference $d\lambda$ that can be resolved is a given as

$$\frac{\lambda}{d\lambda} = nN$$

$$d\lambda = \frac{\lambda}{Nn}$$

$$d\lambda = \frac{5000 \times 10^{-8}}{60{,}000} = 8.33\text{A}^\circ$$

Q. Two coherent sources whose intensity ratio is 4:1 produce interference fringes, find the ratio of maximum to minimum intensity in the interference pattern. (4)

Ans.

$$I_{max} = (a_1 + a_2)^2$$

$$I_{min} = (a_1 - a_2)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

Given
$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1} = 2$$

$$\frac{a_1}{a_2} = \frac{2}{1} \text{ or } a_1 = 2a_2$$

$\therefore$
$$\frac{I_{max}}{I_{min}} = \frac{(2a_2 + a_2)^2}{(2a_2 - a_2)^2} = \frac{9}{1}$$

$$\boxed{I_{max} : I_{min} = 9 : 1}$$

Q. A slit is located 'at infinity' in front of lens of focal length 1 m and is illuminated on either side of the central maximum of the diffraction pattern observed in the focal plane of the lens are separated by 6 mm. What is the width of the slit? (3)

Ans. Width of central maximum

$$y = \frac{f\lambda}{a}$$

where
y = linear half width of the central maximum
f = focal length
λ = wavelength of light
a = slit width

given
y = 6 mm
f = 1 m
λ = 600 nm

$\therefore$
$$6\text{mm} = \frac{1m \times 600nm}{a}$$

$\Rightarrow$
$$a = \frac{1m \times 600 \times 10^{-9} m}{6 \times 10^{-3}}$$

$$a = 0.1\text{m}$$

Q. The axes of a polarizer and analyzer are oriented at 30° to each other.
(1) If un-polarized light of intensity I_0 is incident on them, what is the intensity of the transmitted light.

(2) Polarized light of intensity I_0 is incident on this polarizer analyzer system. If the amplitude of the light makes an angle of 30° with the axis of the polarizer, what is the intensity of the transmitted light? **(3)**

Ans. Given that I_0 is the unpolariged light incident on polarizer then after passing through it I_1 is the intensity of polarizer and I_2 is that of analyzer.

(1) The intensity of light after passing through polarizer

$$\boxed{I_1 = \frac{I_0}{2}} \quad ...(i)$$

(2) If polarized light incident on polarizer at an angle of 30°

$$I_1 = I_0 \cos^2(30°)$$

$$I_1 = I_0 \frac{3}{4}$$

Q. ...) Explain the terms temporal and spatial coherence in the context of the interfernce phenomenon. Explain why interference due to division of amplitude is observed in thin films. (1+1+1)

Ans. Temporal Coherence: If the phase difference between the two field is constant during the period normally covered by observation, the wave has temporal coherence.

Spatial Coherence: If two fields, at two different points on a wave front of a given electromagnetic wave, have constant phase difference over any time, they possess spatial coherence.

When a thin film of oil spreads on the surface of water is exposed to white light beautiful colours are seen. These phenomena can be explain on the basis of interference between light reflected from the upper and lower surface of a film. Which is the interference due to division of amplitudé in which the incident amplitude is alomost equally divided into two parts either by reflection of refraction. These divided parts reunite to produce interference pattern.

Q.... Illustrate with a neat scientific, well-labeled diagram the formation of fringes due to a Fresnel's Bi-prism. (1.5)

Ans.

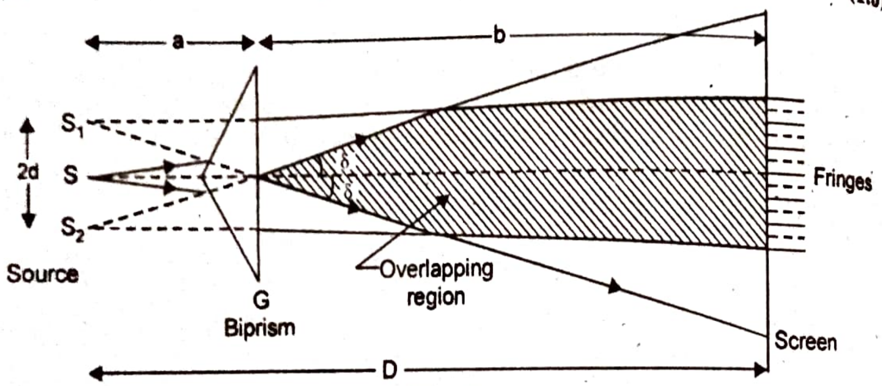


Fig. 2.

Q... (c) Illustrate with a neat scientific, well labeled diagram the necessity of an extended sources to observe fringes in a thin film. (1.5)

Ans.

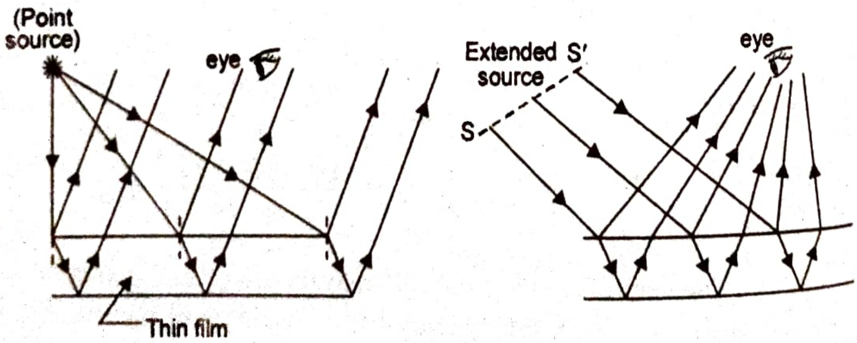


Fig. 3.

Q... (...) Derive the relation for path difference and subsequently the width of a single band for a wedge shaped film. (4)

Ans. If t is the thickness of film at a distance x from the edge the path difference between the two reflected rays producing interference will be $2\mu t \cos r \pm \lambda/2$ the condition for brightness is

$$2\mu t \cos r - \lambda/2 = n\lambda$$

or

$$2\mu t \cos r = (2n+1)\lambda/2$$

and condition for darkness

$$2\mu t \cos r = 2n\lambda$$

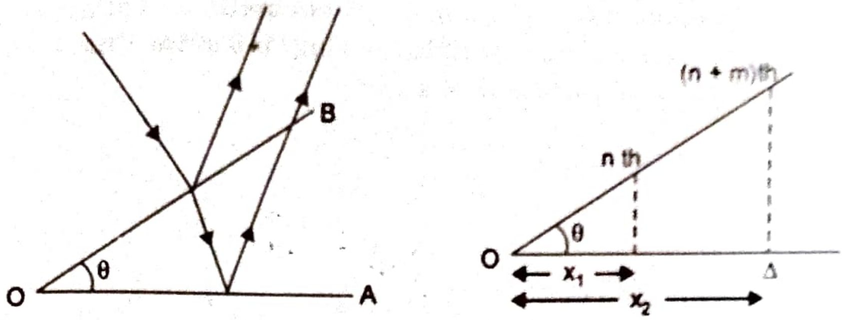


Fig. 4.

The film appear bright when 't' the thickness of film satisfies.

$$2\mu t \cos r = (2n+1)\lambda/2$$

or
$$t = \frac{(2n+1)\lambda}{2\mu \cos r}$$

The film appear dark when 't' satisfies the condition.

$$2\mu t \cos r = n\lambda$$

or
$$t = \frac{n\lambda}{2\mu \cos r}$$

for small angle of incidence $\cos r = 1$ and $t = x\theta$

So condition for darkness reduces to

$$2\mu x\theta = n\lambda$$

if x_1 is the distance of the nth dark band from the edge of wedge x_2 that of the $(n+m)$th dark band then

$$x_1 = \frac{n\lambda}{2\mu\theta} \text{ and } x_2 = \frac{(n+m)\lambda}{2\mu\theta}$$

$$x_2 - x_1 = \frac{(n+m)\lambda}{2\mu\theta} - \frac{n\lambda}{2\mu\theta} = \frac{m\lambda}{2\mu\theta}$$

$$\beta = \frac{x_2 - x_1}{m} = \frac{\lambda}{2\mu\theta} \text{ so}$$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Q. [illegible] An interference pattern is first obtained using a bi-prism set-up. When a thin sheet of glass ($\mu = 1.5$) of 5 μm thickness is introduced in the path of one of the interfering rays, the central fringes is shifted to a position normally occupied by the fifth fringes. Calculate the wavelength of light used. (2)

Ans.
$$t = \frac{n\lambda}{\mu - 1}$$

where t = μm, $n = 5$, $\mu = 1.5$; $\lambda = ?$

$$\lambda = \frac{(\mu-1)t}{n} = \frac{(1.5-1)\times(5\times10^{-6})}{5}$$

$$= 5 \times 10^{-7}\text{m or } 500\text{nm}$$

Q. [illegible] What are localized and non-localized fringes. **(2.5)**

Ans. Localize fringes are those fringes which are observed only over a particular surface. Eg–Newton's rings.

Non-localized fringes are those fringes which exists everywhere in the region of space and is not restricted to small region Eg–Fringes of Youngs's double slit experiment.

Q. ... Newton's rings are observed normally in reflected light of wavelength 5893 A° the diameter of 10th dark rings is 0.005m. Find the radius of curvature of the lens and thickness of air film (3)

Ans.

$$D_n^2 = 4n\lambda R$$

$$D_{10} = 0.005$$

$$n = 10^{th}$$

$$\lambda = 5893\text{Å}$$

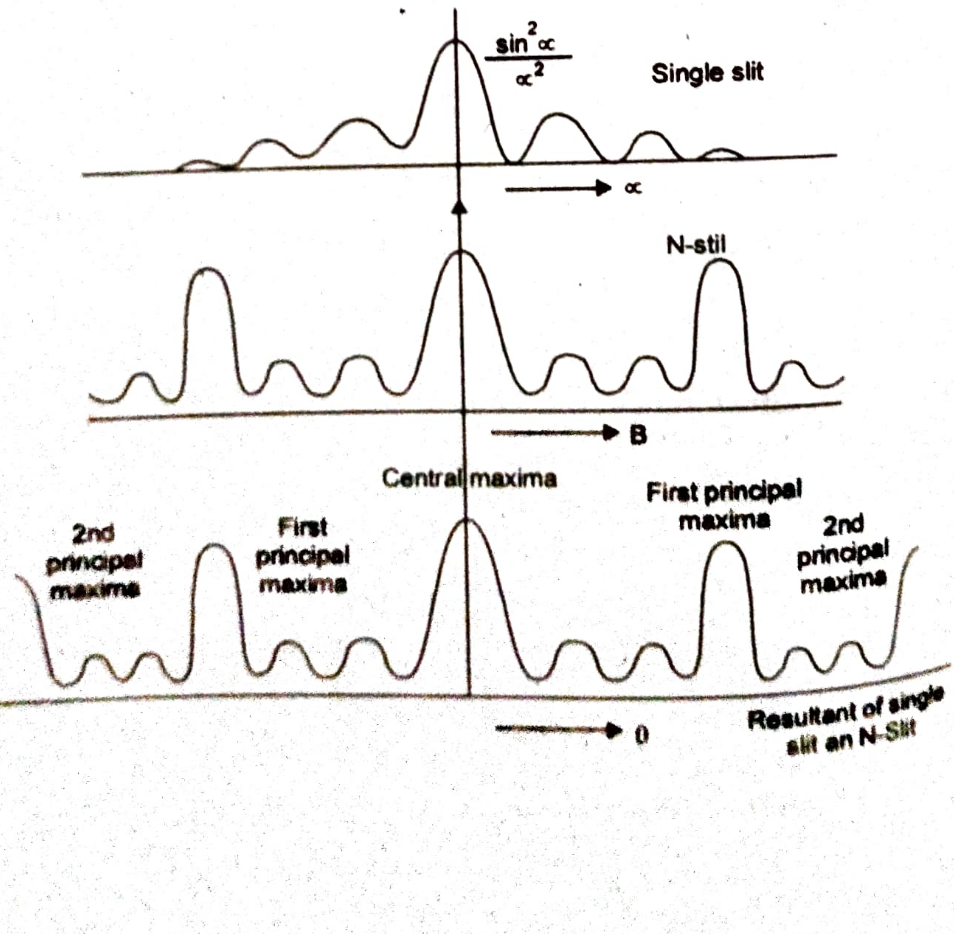
$$R = \frac{D_{10}^2}{4n\lambda} = 1.061 \text{ m}$$

$$t = \frac{D_{10}^2}{8R}$$

$$= 2.94 \times 10^{-6} \text{m.}$$

Q. ... Show that the intensity pattern due to N slits is the product of two terms the diffraction pattern due to a single slit and the interference pattern due to N slits. (5)

Ans.



Q. Transparency and opacity ratio in a grating having 5000 line in one cm. is 1:2 which orders of the maxima will be missing the diffraction grating? (2)

Ans.

$$\frac{a+b}{a} = \frac{n}{m} \quad \text{here } b = 2a$$

$$\frac{3a}{a} = \frac{n}{m}$$

$$n = m$$

for m = 1, 2, 3.... so 3th, 6th, 9th order will be missing

n = 3.6, 9

UNIT-II

Q. Explain the superposition of polarized light Hence, differentiale between plane polarized, circularly polarized and elliptically polarized lights. (2+3)

Ans. When two plane polarised light waves superimposed the resultant wave rotate under certain condition.

(a) Circularly Polarised light: If the magnitude of the resultant wave remains constant and direction varies regularly so that the resultant vector traces a circle, the light is said to be circularly polarised

(b) Elliptically polarised: If the magnitude and direction of the resultant both vary and resultant vector traces an ellipse then light is said to be elliptically polarised

(c) Plane polarised light: If the vibration of light are confined only to one direction and perpendicular to the direction of propagation of light, it is called plane polarized light.

Q. Differentiate between uniaxial and biaxial crystals. (1.5+1.5)

Ans. Uniaxial: Uniaxial crystal has only one direction (optic axis) along which refracted ray travel with the same velocity *e.g.* calcute, tourmaline.

Biaxial: Biaxial crystal has two such direction (two optic axis) along which the velocities of refracled rays are same eg topaz, mica,, canesugar.

Q. Illustrate with a series of neat scientific well labeled diagrams the formation of a Nicol prism from a double refracting crystal. (2)

Ans.

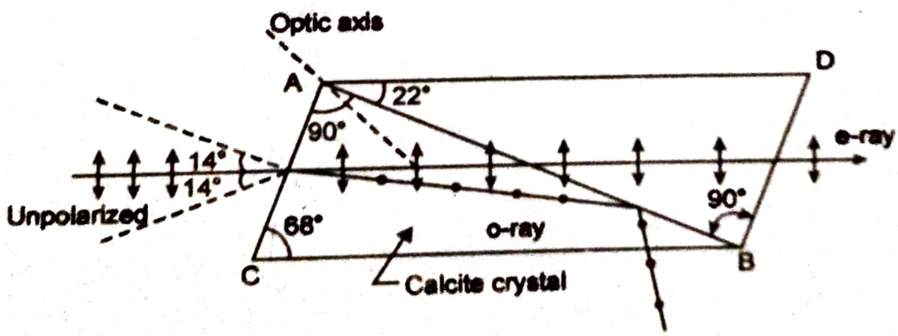


Fig. 5.

Q. A plane polarized light is incident on a quartz plate is cut parallel to the axis. Calculate the least thickness of the plate for which the o-and e-ray recombine to form a plane polarized light. Assume that μ_e = 1.5533; μ_o = 1.5442 and λ = 5.4×10^{-5} cm. (2)

Ans. Here the Quatz plate must act as half wave plate.

$$t = \frac{\lambda}{2[\mu_e - \mu_o]} = \frac{\lambda}{2(1.5533 - 1.5442)}$$

$$t = 0.0029 \text{ cm.}$$

Q. Give the nature of fringes in (i) YDSE (ii) Newton's Rings (iii) Wedge shaped film. (iv) Fresnel's Biprism. (2)

Ans. (i) **YDSE:**

$$\text{Fringe width } \beta = \left(\frac{\lambda D}{d}\right)$$

Fringes are striaght parallel and equidistant, central fringe is bright.

(ii) **Fresnel's Biprism:**

$$\text{Fringe width } \beta = \left(\frac{\lambda D}{d}\right)$$

Fringes are striaght parallel and equidistant, central fringe is bright.

(iii) **Wedge Shaped Film:**

$$\text{Frige width } \beta = \left(\frac{\lambda}{2\mu\,\alpha}\right)$$

Fringes are straight, parallel and equidistant, the apex is dark.

(iv) Newton's Rings:

$$\text{Diameter; } D^2 = \sqrt{4\,n\lambda R} \qquad \text{(for dark ring)}$$

Fringes are circular and get closes and closer as the order of fringe increases. So fringes are not equidistant.

Q. Prove that at Brewster's angle the reflected and refracted light rays are perpendicular to each other. (5)

Ans. Sir David Brewster in 1811 found that there was a simple relation between the angle of polarization and the refractive index μ of the medium which refracted the light. This relationship is $\mu = \tan i$. That is refractive index of the material of the refracting medium equals the tangent of the angle of polarization. This is called Brewster's law. A direct dedution of this law is that when light is incident at the polarizing angle the reflected rays are at right angles to the refracted rays.

Let XY be the surface of separation of a transparent medium, say glass, PQ the incident beam, QR the reflected beam QS the refracted beam. Angles i and angle r are the angles of incidence and refraction respectively.

From Brewster's Law

$$\mu = \tan i = \frac{\sin i}{\cos i} \qquad \ldots(1)$$

From Snell's Law

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad \frac{\sin i}{\cos i} = \frac{\sin i}{\sin r}$$

$$\text{or} \quad \frac{\sin i}{\sin(90 - i)} = \frac{\sin i}{\sin r}$$

$$\therefore \quad 90^\circ - i = r$$

or $$i + r = 90^\circ$$

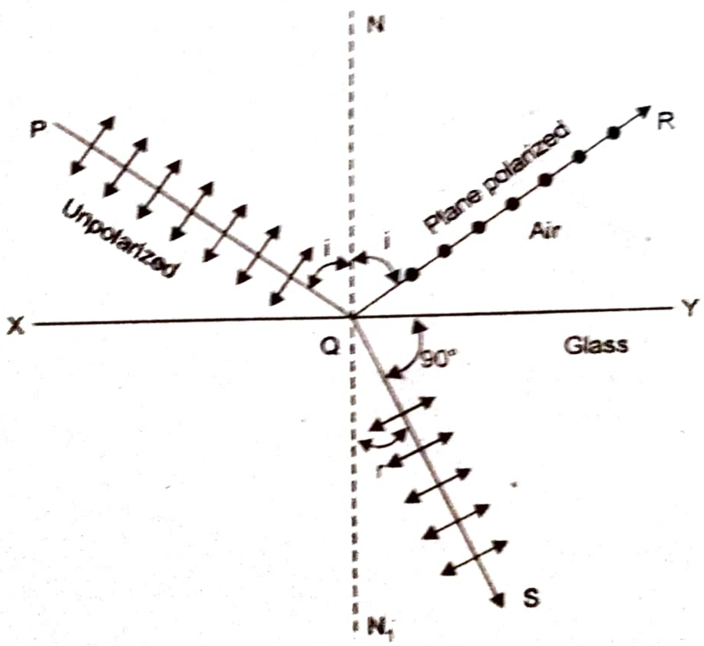
As $$\angle NQN_1 = p, \text{ we have}$$

$$i + \angle RQS + r = p$$

or $$\angle RQS = p - (i - r) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

i.e., at polarizing angle, the reflected ray is at right angles of the refracted ray.

This law is obeyed even when light is reflected at the surface of a rarer medium. Thus if light is incident on a glass plate at the polarizing angle *i.e.* 57.5°, the beams reflected from both, the upper and lower, surfaces will be plane-polarized. This given us a very convenient method for obtaining plane-polarized light. But this method is very efficient.



Q. (c) Polarized light of intensity I_0 is incident on polarizer-analyzer system. If the amplitude of light makes an angle of 30° with the axis of polarizer what is the intensity of the transmitted light? (3)

Ans. $$I = I_0 \cos^2 \theta$$

I_0 = Intensity of incident radiation

$$\theta = 30^\circ$$

$$I = I_0 \cos^2 (30^\circ)$$

$$\boxed{I = I_0 \frac{3}{4}}$$

Q. Two wavelength of light λ_2 are sent through a young's Double slit expreimental set up simultaneously. What must be true concerning λ_1 and λ_2 if the third order bright fringe is to coincide with the fourth-order λ_2 fringe. (2.5)

Ans. Path difference $= n\lambda$

$$n_1\lambda_1 = n_2\lambda_2$$

$$3\lambda_1 = 4\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

Q. A slit is located at infinity in front of a lens of focal length in 1m and is illumenated normally with light of wavelength. The first minima on either side of the central maximum of the diffraction pattern observed in the focal plane of the lens are separated by 7 mm what is the width of the slit. (2.5)

Ans.

$$\frac{x}{f} = \frac{\lambda}{d}$$

$$d = \frac{\lambda f}{x} = \frac{\overset{100\,mm}{\cancel{7}00\,mm} \times 1m}{\cancel{7}m}$$

$$= 100\times10^{-3} m \times 1 \times 10^{-3}$$

$$\frac{1\cancel{00}\,mm}{10\cancel{00}} = .1\text{ mm} \qquad ...(1)$$

Q. The velocity of light in water is 2.2×10^8m/s. What is the polarising angle of incidence for water surface. (2.5)

Ans. $\mu = \tan ip$

$$\mu = \frac{\mu_{water}}{\mu_{air}} = \frac{v_{air}}{v_{water}} = \tan ip = \frac{3\times10^8}{2.2\times10^8} = 1.36$$

$$ip = \tan^{-1}(1.36)$$

$$= 53.7°$$

Q. Derive an expression for intensity distribution due to N-slits. (4

Ans. Consider N slits illuminated by a source 'S' the width of each slits is e and the width of opaque part is d.

$(e + d)$ – grating element

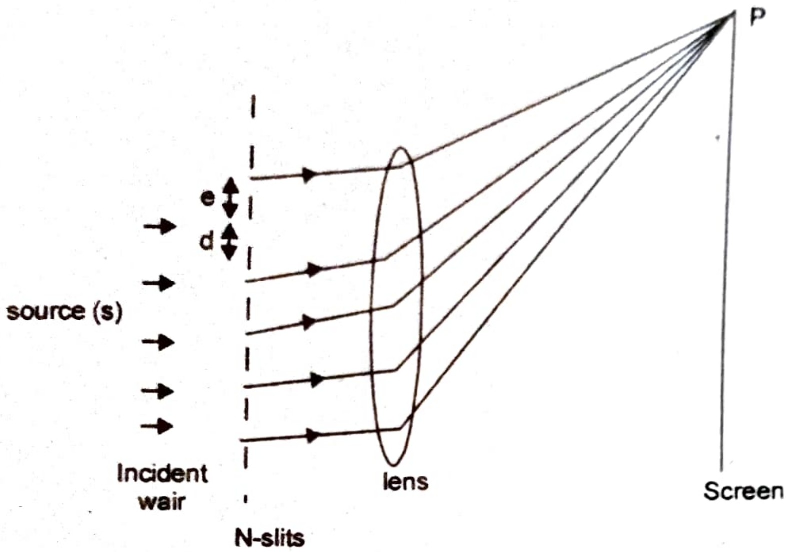
here BC = path difference

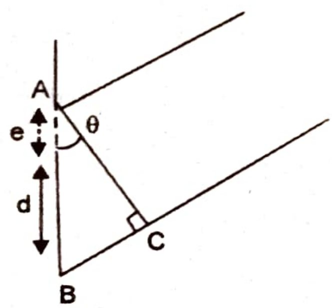
Consider ΔABC

$$\sin\theta = \frac{BC}{AB}$$

$$BC = AB\sin\theta$$

$$\therefore \quad BC = (e+d)\sin\theta$$





$$\Rightarrow \qquad \Delta = (e+d)\sin 0$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\Delta$$

or,
$$= \frac{2\pi}{\lambda}\cdot(e+d)\sin\theta$$

$$\text{Phase diff} = 2\beta \qquad \text{As } \beta = (e+d)\sin\theta$$

Vector addition formula

$$R = \frac{a\sin(n\delta/2)}{\sin\delta/2}$$

here
$$a = \frac{A\sin\alpha}{\alpha}$$
$$n = N$$
$$\delta = 2\beta.$$

$$\Rightarrow \qquad R = \frac{A\sin\alpha}{\alpha}\cdot\frac{\sin(N\beta)}{\sin\beta}$$

$$\Rightarrow \quad \text{(Intensity)} \quad \boxed{I = R^2 = \frac{A^2\sin\alpha}{\alpha^2}\frac{\sin^2(N\beta)}{\sin^2\beta}}$$

(i) **Condition for principal Maxima**

$$\text{Sin}\,\beta = 0$$

$$\Rightarrow \quad \beta = \pm n\pi$$

Applying limits

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta}$$

$$\Rightarrow \quad \lim_{\beta \to \pm n\pi} \frac{\partial/\partial\beta(\sin N\beta)}{\partial/\partial\beta(\sin\beta)}$$

$$\Rightarrow \quad \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

$$\boxed{I = \frac{A^2 \sin^2\alpha}{\alpha^2}.[N]^2}$$

(*ii*) **Condition for Minima** $\quad \sin N\beta = 0$

$$\Rightarrow \quad N\beta = \pm m\pi$$

$$\Rightarrow \quad I = \left[\frac{A\sin\alpha}{\alpha}\right]^2$$

Q. Define absent spectra. If the width of opaque space of grating is doubled the width of transparent space, which order of spectrum will be absent. (2)

Ans. Condition for Principal maxima in grating is

$(a + b)\sin\theta = n\lambda$ ...(*i*)

Condition for minima in a single slit is

$$a\sin\theta = m\lambda \quad ...(ii)$$

If both conditions are simultaneously satisfied, a particular maximum of order n will be absent in grating spectrum these are known as absent spectra or missing order spectra

from (*i*) and (*ii*)

$$\boxed{\frac{(a+b)}{a} = \frac{n}{m}}$$

As

$$\frac{a+b}{a} = \frac{n}{m}$$

here

$$b = 2a$$

$$\frac{3a}{a} = \frac{n}{m}$$

$$n = 3m$$

for

$$m = 1, 2, 3, 4$$

$$n = 3, 6, 9, 12, \ldots$$

So 3rd, 6th, 9th & 12th order spectrum will be missing.

Q. Define dispersive power and resolving power. Calculate the minimum number of lines in grating which will first resolve the lines of wavelenghts 5890 Å and 5896 Å in the second order. (5)

Ans. Dispersive Power of grating

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in the wavelength between the two spectral lines. It can also be defined as the diffraction in the angle of diffraction per unit change in wavelength. The diffraction of the nth order principal maximum for a wavelength λ is given by the equation,

$$(a + b) \sin \theta = n\lambda \qquad \text{(i)}$$

Differentiating this equation with respect to θ and λ (a + b) is constant and n is constant in a given order)

$$(a + b) \cos \theta \, d\theta = n \, d\lambda$$

Or,
$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

Or,
$$\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta} \qquad \text{(ii)}$$

In equation (ii) $d\theta / d\lambda$ is the dispersive power, n is the order of the spectrum, N' is the number of lines per cm of the grating surface and θ is the angle of diffraction for the n th order principal maximum of wavelength λ.

We know that resolving power

R.P = $\lambda/d\lambda$ nN

and dispersive power D.p. $d\theta / D\lambda = n/(a+b) \cos \theta_n$

Therefore, $\lambda / d\lambda = nN = N(a+b) \cos \theta_n \, n/(a+b) \cos \theta_n$

$$\lambda/d\lambda = Ax \, d\theta / d\lambda$$

Resolving power = total aperture of telescope objective x dispersive power.

The resolving power of a grating can be increased by

(i) Increasing the number of lines on the grating N.

(ii) Increasing the sides of spectrum n.

(iii) Increasing the total width of grating 'W', for which one has to make use of whole aperture of telescopes objective.

As $R.P = \frac{\lambda}{d\lambda}$ where λ is mean of two wavelength of two wavelength of $d\lambda$ is difference of two wavelength

$$\lambda_1 = 5890 \text{ A}^\circ$$
$$\lambda_2 = 5896 \text{ A}^\circ$$

so,
$$\frac{5890 + 5896/2}{5893 - 5890} = \frac{5893}{6} = 982$$

Q . . .) What particular spectra of plane transmission grating would be absent if the width of the transparencies and opacities of the grating are equal? (2.5)

Ans. $\frac{a+b}{a} = \frac{n}{m}$, where a – is slit width (transparencies)

b – width of opacities

here $a = b$

So $\frac{2a}{a} = \frac{n}{m}$

$n = 2m$

So 2nd, 4th, 6th, 8th order would be absent

Q. Two nicols are oriented with their principle planes making an angle of 60°. What percentage of incident unpolarized light will pass through the system. **(1)**

Ans. If unpolarised light incidents on a polariser the intensity of light transmitted through the polariser

$$\frac{I_0}{2} = I$$

and

$$I = I_1 \cos^2\theta = \frac{I_0}{2} \cos^2 60° = 0.125\, I_0$$

$$= 12.5\% \text{ of } I_0$$

Q. Sodium light (λ = 5893 A) is used in a fresnel's Bi-prism set up. A total of 60 fringes are observed in the field of view of the eye-piece calculate the number of fringes that would be observed in the same field of view if the sodium light is replaced by a mercurry vapour lamp with (λ = 5461A°) **(2)**

Ans.

$$n_1\lambda_1 = n_2\lambda_2$$

$$\frac{60 \times 5893}{5461} = n_2 = 64$$

Q. List five differences between interference and diffraction fringes. **(2.5)**

Ans.

Interference	Diffraction
1. The interference occurs between two separate wave fronts originating from coherent sources.	1. Diffraction occurs between the secondary wavelets originating from the exposed part of same wave front.
2. In an interference pattern the pringes are are normally of the same width.	2. In diffraction pattern the fringes never of the same width and go on decreasing in width as we move away from the edge of the shadow.
3. In an interference pattern the regions of minimum intensity are perfectly dark. (zero intensity)	3. In diffraction pattern the intensity of minima is never perfectly dark.
4. In an interfence pattern all the bright fringes (maxima) are of uniform intensity.	4. The intensity of the central maxima is maximum and decreases on either side as the order of maxima increases.
5. There is requirement of two coherent sources.	5. Only one source of light is required

Q. What is meant by a diffraction grating. How is it useful for the determination on of wavelength of a monochromatics source of light? What is the advantages of increasing the number of lines on the grating? What is the fundamental difference in the spectra obtained by a prism to that of a diffraction grating for a white light source.

Ans. (1) An arrangement consisting of a large number of equidistant parallel rectangular slits of equal width separated by equal obaque portion is known as a diffraction

(2) By using the diffraction grating formula

$$(e + d) \sin \theta = n\lambda$$

and observing the diffraction pattern the wavelength of light can be determined

where $(e + d)$ = grating elment

n – order of spectrum

λ – wavelength of light

(3) If we increase the numer of lines of the grating, the principal maxima will become intense and sharp, while secondary maxima become weaker.

(4) The fundamental difference in the spectra obtained by prism & grating is in the sequence of colour

In prism the sequence of colours is VIBGYOR

In grating the sequence of colours is ROYGBIV

Q. In a Fraunhofer diffraction pattern experiment using two slits, the third, sixth and ninth interference maxima are found to be missing if the slit width is 0.05×10^{-3}m, Calculate the inter-slit separation. **(2)**

Ans. $$\frac{a+b}{a} = \frac{n}{m}$$

if $$a = b \quad \frac{n}{m} = 2$$

if $$2a = b \; n = 3\,m$$ third six ninth spectram are mission

$$a = 0.05 \times 10^{-3}\ \text{m}$$

$$b = 0.1 \times 10^{-3}\ \text{m}$$

Q. Determine the minimum number of lines in a grating that are just able to resolve the sodium lines of wavelength 5890A & 5896 Å in the first order spectrum. **(2)**

$$\frac{\lambda}{d\lambda} = nN$$

for n = 1

λ = 5893 (mean of 5890 & 5896 A°)

$d\lambda$ = 6 A° (difference of 5890 & 5896 A°)

N = 986

Ans. $$\frac{5890 + 5896/2}{5893 - 5890} = \frac{5893}{6} = 982$$

Q. A diffraction pattern is observed using a beam of red light. What happens if the red light is replaced by the blue light. **(2)**

Ans. Diffraction formula for grating.

$$(e + d) \sin \theta = n\lambda$$

if λ_{red} is replaced by λ_{blue}

as $\lambda_{red} > \lambda_{blue}$

therefore the number of maxima formed on the screan increases

Q.3. Light is incident normally on a grating 0.5cm wide with 2500 lines. Find the angles of diffraction for the principle maxima of the two sodium lines in the first order spectrum, $\lambda_1 = 5890A°$, $\lambda_2 = 5896A°$ **(5.5)**

Ans. For Grating

$$(a + b) \sin \theta = n\lambda$$

$$\sin \theta = \frac{\lambda}{a+b} \text{ as } n = 1$$

$$(a + b) = \frac{0.5}{2500} = \frac{5}{25000} = 2 \times 10^{-4} \text{cm}$$

Angle of diffraction for 5890 A° line

$$(a + b) \sin \theta = \lambda$$

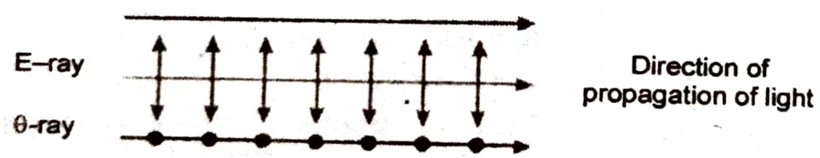
$$\theta_1 = 17.13°$$

Angle of diffraction for 5896 Å line

$$\theta_2 = 17.15°$$

Q.4. What do you mean by polarisation of light? Describe the construction of Nicol prism and show how it can be used as a poloriser and analyser. **(3.5)**

Ans. If the vibrations of light are confined only to one direction in a plane perpendicular to the direction of propagation of light, such light is called plane polarized or linearly polarized or linearly polarized.



Construction of Nicol Prism: Nicol prism is based on the phenomenon of double refraction. When light is passed through a doubly refracting crystal just as iceland spar (calcite), it is broken up into two refracted rays: *(i)* the ordinary ray with its vibrations perpendicular to the principal section of the crystal and *(ii)* the extra-ordinary ray with its vibrations parallel to the principal section. Both these rays are plane polarized but have their vibrations at right angles to each other. In order to get a beam of plane polarized light, one of these rays should be got rid off. The Nicol prism is so made that one of the rays i.e. O-ray is eliminated by total internal reflection so that only E-ray is transmitted through the prism. Hence plane polarized light is obtained with vibrations in the principal section of the crystal.

One of the most common forms of the Nicol Prism is made by taking a crystal about three times as long as it is wide. The two end faces AB and CD of the crystal are cut down so as to reduce the angle from 71° to 68°. The resulting crystal is then cut into two parts along the plane A_1C_1 passing through the blunt corners and perpendicular to both the principal section and the end faces, so that A_1C_1 makes an angle of 90° with ends C_1D and A_1B as shown in Fig. The two cut faces are ground and polished optically flat. They are then cemented by a layer of Canada balsam which is a clear transparent substance whose index of refraction lies midway between the index of O-ray and E-ray. For example, for sodium light, $\lambda = 5893$ Å

$$\mu_0 = 1.65837$$
$$\mu_{CB} = 1.55$$
$$\mu_E = 1.48641$$

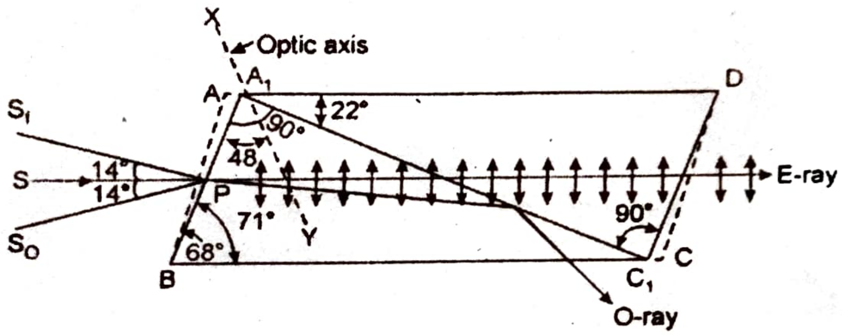


Fig. 2.

Nicol prism can be used both as a polarizer and an analyser. When two Nicols are arranged coaxially, as shown in Fig. 2. then the first Nicol prism P which produces plane polarized light is termed as the polarizer while the second Nicol A which analyses the polarized light is termed as analyser. Such a combination of two Nicols is called a polariscope.

When the principal sections of the two Nicols are parallel, as shown in Fig. 3(a), the emergent E-ray from P has its vibrations parallel to the principal sections of both polarizer and analyser. Hence the refracted beam inside A behaves as E beam and is therefore freely transmitted. This position and the other position corresponding to the angle of 180° between the two principal sections is known as parallel Nicols. The intensity of emergent light in these settings is maximum, although it is only about 40% of the incident unpolarized beam intensity-50% of incident light being totally reflected in the polarizer and rest 10% reflected from prism surfaces and absorbed in balsam.

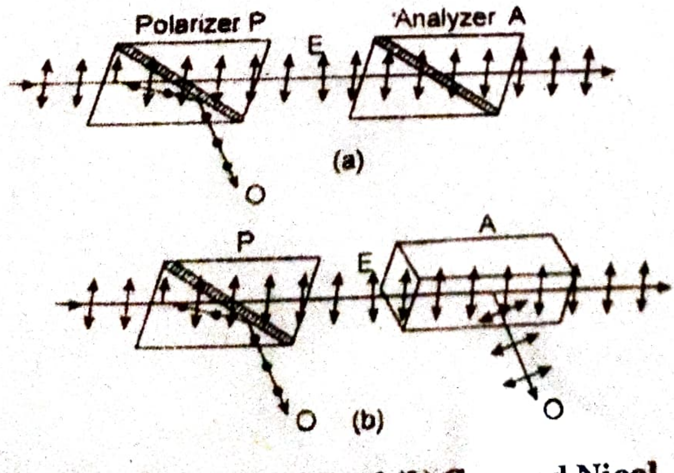


Fig. 3. (*a*) Parallel Nicol (*b*) Crossed Nicol

UNIT IV

Theory of relativity: The Michelson-Morley Experiment and the speed of light; Absolute and Inertial frames of reference, Galilean transformations, the postulates of the special theory of relativity, Lorentz transformations, time dilation, length contraction, velocity addition, mass energy equivalence. Invariance of Maxwell's equations under Lorentz Transformation.

Introduction to Laser Physics: Introduction, coherence, Einstein A and B coefficients, population inversion, basic principle and operation of a laser, the He-Ne laser and the Ruby laser. **[12 Hrs.]**

Q. What is the function of compensating plate in Michelson's Interferometer. (2)

Ans. The ray coming from the sources and entering into the telescope after being reflected from mirror M_1 along (Normal direction) transverse the plate three time while that reflected from Mirror M_2 (along horizontal direction) traseverse it only once. Hence, the optical paths of two interfering rays are not equal. They are made the same by placing a compensating plate of the same thickness and material.

Q. Explain the principle of laser action. 4

Ans. In thermal equilibrium the population of atoms in different states is governed by Maxwell-Boltzman law and the rate of transition of atoms by absorption of radiation is equal to the rate of transition by total emission of radiation. In the presence of incident radiation the equilibrium is disturbed and the ratio of emission and absorption rate is given by

$$\frac{\text{Rate of emission}}{\text{Rate of absorption}} = \frac{R_{21}}{R_{12}} = \frac{A_{21} + B_{21}E(v)}{B_{12}E(v)}\frac{N_2}{N_1} \quad ...(1)$$

But

$$B_{21} = B_{12}$$

$$\therefore \quad \frac{R_{21}}{R_{12}} = \left[1 + \frac{A_{21}}{B_{21}E(v)}\right]\frac{N_2}{N_1} \quad ...(2)$$

For laser action two conditions must be satisfied, (*i*) *the emission rate must be larger than the absorption rate and (ii) the probability of spontaneous emission which produces incoherent radiation must be much smaller than the probability for stimulated radiation i.e.,* $A_{21} << B_{21} E(v)$.

A higher probability of stimulated emission can be achieved by the following methods.

(*a*) The spectral energy density of incident radiation must be made very large *i.e.*, $E(v)$ must be made very large. To increase energy density, the emitted radiation is made to reflect coherently again and again between two parallel mirrors in a cavity containing active medium.

(*b*) A_{21}/B_{21} must be made very small. That is to minimize the value of A_{21}/B_{21} metastable states of higher energy is chosen for stimulated transition because tansitions from metastable states by spontaneous emission are not allowed.

Q. Give the main conclusion of Michelson Morley's experiment. How the negative results obtained from the experiment were interpreted. (2+3)

Ans. In Michelson Morley's experiment the total fringe shift calculated is given by the expression.

or
$$\delta n = \frac{2d}{\lambda}\frac{v^2}{c^2} \quad ...(3)$$

The orbital velocity of the earth is about 3×10^6 cm/sec then $v^2/c^2 \sim 10^{-8}$ at least at some time of the year. The arrangement should be sensitive so as to detect effects of the order of v^2/c^2 *i.e.* one part in 10^8. The expected shift for $d = 11$ m (by repeated reflections) and $\lambda = 6 \times 10^{-5}$ cm is

$$\delta n = \frac{2 \times 1100 \times (3 \times 10^6)^2}{6 \times 10^{-5} \times (3 \times 10^{10})^2} = 0.37$$

or
$$\delta n = 0.4$$

The experimental arrangement was capable of measuring one-hundredth of a fringe and as such the above shift was capable of accurate measurability. However, no effect was observed. The experiment was repeated after an other six months to eliminate the unbseeming possibility that the earth might be at rest relative to ether at the time of the experiment. The effect should have been observed at least at one of the two occassions. But experiment gave null effect on both occassions.

Conclusions or significance of null result: The negative results of Mirchelson-Morley experiment as the following conclusion:

(*i*) Ether has no observable properties. Therefore, there is no such things as absolute space or fixed fundamental frame of reference with respect to which absolute motion of the bodies can be termined. Hence absolute motion is meaningless.

(*ii*) The velocity of light is same in all directions and does not depend upon the motion of the source or the observer or both.

Q. Define time dilation and show that time dilation is a real effect. (5)

Ans.

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}}$$

where

t_0 – proper time
t – apperant time

The time interval appears to lengthened by a factor $\frac{t_0}{\sqrt{1-v^2/c^2}}$ which is observed by the observer is known as time delation.

Experimental verification of Time-dilation: Time-dilation has been best verified in experiments on a nuclear particle, called meson. There are two types of mesons. The π^+ meson decay (break into fragments) in such a way that in every 1.8×10^{-8} s, half of them die out *i.e.*, their flux decreases to 2^{-1} in every 1.8×10^{-8} s, half of them die out *i.e.*, their flux decreases to 2^{-1} in every 1.8×10^{-8} s.

Now, in an experiment, in the laboratory, π^+ mesons were produced with speed 0.99c and their flux was measured at two places separated by 30 m. The laboratory time interval Δt for travelling this distance was given by

$$\Delta t = \frac{30\text{ m}}{0.99\text{ c}} \cong \frac{30}{3 \times 10^8\text{ m/s}} = 10 \times 10^{-8}\text{ s.}$$

This is about 5.6 times of 1.8×10^{-8} s. Hence, the flux of π^+ mesons should decrease to $2^{-5.6}$ or less than 2% of the original flux in travelling 30 meters. But the actual flux at the second place was nearly 60% of that at the first place.

This discrepancy is explained by computing the *proper time* (Δt) given by relation

$$\Delta t = \Delta t\left(1-\frac{v^2}{c^2}\right) = 10 \times 10^{-8}\ [1-(0.99)^2]^{1/2}$$

$$= 1.4 \times 10^{-8}\text{ s.}$$

This is 0.78 times of 1.8×10^{-8} s. Hence, in this time the flux shoudl fall to $2^{-0.78}$ (nearly 60%) of the original flux. This is exactly what is observed.

It amounts to this: In laboratory measurements the elapsed time for 30 m travels is 10×10^{-8} s. while the π^+ mesons themselves measure the time as 1.4×10^{-8} s only.

Q. Population inversion cannot be achieved in two-level laser. Justify. (2.5)

Ans. Two level laser is not possible because in order to achieve the population inversion condition a metastable is required where the lifetime of atom is very large in order to achieve large population. Without metastable state there is no lasing action and metastable state is not present in two level laser.

Q. What is the role of H_e in H_e-N_e laser? (2.5)

Ans: Neon atoms are much heavier than helium atom. so it is not possible to excite neon atom efficiently by means of collision with electrons generated from electric discharge in the mixture of neon and helium. At the same time, the possibility of transfer of energy of an accelerated electron through the collision with helium atom is maximum because thay are light in weight. There are metastable states of helium atom that are identical with two energy states of neon atom. So Helium atom act as carrier of excitation.

Q. What is relativistic energy? Prove that the relation $E^2 - p^2c^2 = m_0^2c^4$ where p is the relativistic momentum. (6)

The relativistic energy expression include both rest mass energy and the kinetic energy of motion

$$E = \text{Kintetic energy} + \text{rest mass energy}$$

$$c^2(m - m_0) + m_0c_2$$

$$\therefore \quad \boxed{E = mc^2}$$

where

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \quad E = mc^2 = \frac{m_0c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

and

$$p = mv = \frac{m_o v}{\sqrt{1 - v^2/c^2}}$$

$$E^2 = \frac{m_o^2c^4}{\left(1 - \frac{v^2}{c^2}\right)}$$

and

$$p^2 = \frac{m_o^2v^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$p^2c^2 = \frac{m_o^2v^2c^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\text{subtracting } p^2c^2 = \frac{m_o^2c^4 - m_0^2v^2c^2}{\left[1 - \frac{v^2}{c^2}\right]}$$

$$= \frac{m_o^2c^4\left[1 - \frac{v^2}{c^2}\right]}{\left[1 - v^2/c^2\right]}$$

$$\boxed{E^2 - p^2c^2 = m_0^2c^4} \text{ Hence Proved.}$$

Q. Discuss Einstein's Coefficients. Derive relation between them. (5)

Ans. When an atom interact with photon of energy $h\nu$, then three process takes place (1) Absoption, (2) Spontaneous emission, (3) Stimulated emission. The absorption process depends on the energy density of radiation at the frequency ω; this energy density is denoted by $u(\omega)$ and is defined such that $u(\omega)\,d\omega$ represents the radiation energy per unit volume within the frequency interval ω and $\omega + d\omega$. The rate of absorption is proportional to N_1 and also to $u(\omega)$. Thus, the number of absorption per unit time per unit volume can be written as

$$N_1 B_{12} \mu(\omega)$$

where B_{12} is the coefficient of proportionality and is a characteristic of the energy levels.

Now, let us consider the reverse process of emission of radiation at a frequency ω when the atom de-excites from the level E_2 to E_1. Atoms can de-excited to lower level through either spontaneous of stimulated emission.

In spontaneous emission, the probability per unit time of the atom making downward transition is independent of the energy density of the radiation field and depends only on the level involved in the transition, if we represent the coefficient of proportionately by A_{21}, then N_2A_{21} is the rate of spontaneous emission (per unit volume) to the lower level.

In the case of stimulated emissions, the rate of transition to the lower energy is directly proportional to the number of atoms in the upper energy level and the energy density of the radiation at the frequency ω.

Thus, the rate of stimulated emission is given by,

$$N_2 B_{21} u(\omega)$$

Here B_{21} represents the corresponding proportionality constant. The constant A_{21}, B_{12} and B_{21} are called **Einsteins coefficients.**

But number of upward transition is equal to the number of downwards transition.

i.e.
$$N_1 B_{12} u(\omega) = N_2 A_{21} + N_2 B_{21} u(\omega)$$

$$u(\omega) = \frac{A_{21}}{\frac{N_1}{N_2} B_{12} - B_{21}}$$

From Boltzman's law we have.

$$\frac{N_1}{N_2} = \exp\left[\frac{E_2 - E_1}{K_B T}\right] = Exp\left[\frac{h\omega}{K_B T}\right]$$

where K_B is the Boltzman constant

$$u(\omega) = \frac{A_{21}}{B_{12}\exp\left(\frac{h\omega}{K_B T}\right) - B_{21}} \quad ...(1)$$

Now, according to Plank's Law the energy density of radiation

$$u(\omega) = \frac{h\omega^2 n_0^3}{\pi^2 c^3} \times \frac{1}{\exp(hw / K_B T) - 1} \quad ...(2)$$

where n_0 is the refractive index of the medium. Comparing the above two equations, we have

$$B_{12} = B_{21} = B \text{ (say)}$$

and
$$\frac{A_{21}}{B_{21}} = \frac{h\omega^3 n_0^3}{\pi^2 c^3}$$

However, at the thermal equilibrium, the ratio of the number of spontaneous to stimulated emission is

$$\frac{A_{21}}{B_{21}\mu(\omega)} = \exp\left(\frac{\hbar\omega}{K_B T}\right) - 1$$

Q. Compute the speed of a particle of which its mass will become 8 times of its rest mass **(3)**

Ans:

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

given $m = 8m_0$

$$\therefore \quad 8\,m_0 = \frac{m_o}{\sqrt{1-v^2/c^2}}$$

$$v = 0.99c \text{ or } 2.99\times10^8 \text{ m/s}$$

Q. At what speed will an object of length 100 cm be measured as 50 cm an observe at rest. **(2.5)**

Ans.

$$l = lo\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

$$\frac{l_o}{2} = l_o\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{4} = \left[1-\frac{v^2}{c^2}\right]$$

$$v^2/c^2 = 3/4$$

$$v/c = \sqrt{\frac{3}{2}}$$

$$\therefore \quad \boxed{v = \frac{\sqrt{3}}{2}c} = 0.866c$$

Q. The total energy of the particle is exactly twice its rest energy. Calculate the velocity of the particle. **(2.5)**

Ans.

$$mc^2 = 2m.c^2$$

Where

$$m = 2m_0$$

$$m = \frac{m_o}{\sqrt{1-v^2/c^2}}$$

$$\therefore \quad \frac{m_o}{\sqrt{1-v^2/c^2}} = 2m_0$$

$$\frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4} \quad v = 0.886c$$

Q. What do you understand by time ditation? What is proper time and establish a relation for proper time establish a relation for proper time. (3.5)

Ans: Time delation: A moving clock runs slower when moving with velocitiy v ≈ c and this effect of relativity is called time dilation. Suppose an object from system S' sends a light signal from (x', 0,0) at time t'_1 and then at time t'_2. We will like to find how the interval (t'_2-t'_1) appears to an observer in system S. if t_1 and t_2 are the corresponding times in system S then from inverse Lorentz transformation, we get,

$$t_1 = \frac{t'_1 + \frac{v}{c^2}x'}{\sqrt{1-v^2/c^2}}$$

and

$$t_2 = \frac{t'2 + \frac{v}{c^2}x'}{\sqrt{1-v^2/c^2}}$$

Therefore

$$t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1-v^2/c^2}} \quad ...(5)$$

assuming $t_2 - t_1 = \tau$ and t'_2-$t'_1 = \tau$

then

$$\tau = \tau / \sqrt{1-v^2/c^2} \quad ...(6)$$

Consider the time read by clock moving with a given observer as the proper time, because it is the time shown by the clock w.r.t the point at which both events occured remains at rest. We see from equation that the proper time of the moving object is always less than corresponding interval in the rest frame, It implies that a moving clock runs slower when moving with velocity v ≈ c and this effect of relativity is called time dilation.

Q. How much energy must be given to an electron to accelerate it 0.98 C

Ans.

$$KE = (m - m_0)c^2$$

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \frac{m_0}{\sqrt{1-\left[\frac{0.98C}{c}\right]^2}} = 5.02m_0$$

$$\begin{aligned} K.E &= [5.02m_0 - m_0]\,c^2 \\ &= 4.02\, m_0c^2 \\ &= 4.02 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \\ &= 3.296 \times 10^{-13}\, J \end{aligned}$$

Q. Explain the construction & working of a He-Ne laser. What are the merits of laser. (4)

Ans. He-Ne laser is a four level laser, which was first successfully built by Ali Javan, W. Bennett and D. Herriott in 1961.

Construction:

The He-Ne laser consists of a mixture of He and Ne in a ratio of about 10 : 1, placed inside a long narrow discharge tube. The pressure inside the tube is about 1 mm of Hg. The gas system is enclosed between a pair of plane mirrors or a pain of concave mirrior so that a resonator system is formed. One of the mirror is of very high reflectivity while the other is partially transparent so that energy may be coupled out of the system. This shown in fig. (a).

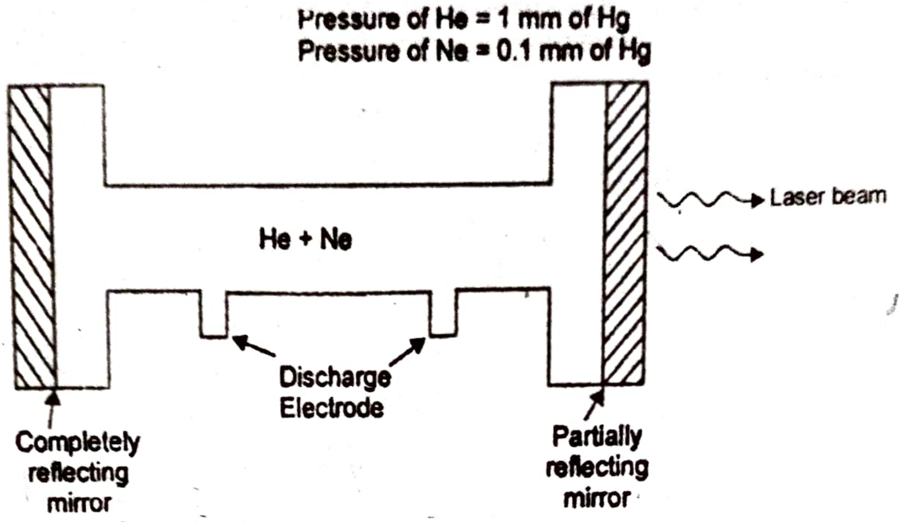


Fig. (a)

Working:

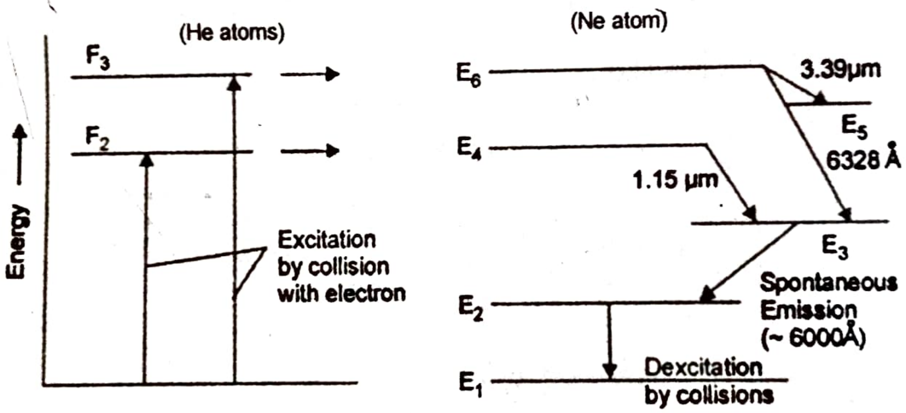


Fig. (b)

When an electric discharge is passed through the gas, the electron travelling down the tube collide with the He-Ne atom and excite them to level marked F_2 and F_3. These levels are metastable, *i.e.,* He atoms excite to this state stay there for sufficiently long time before losing energy through collisions. Through this collision, the Ne atoms are excited to the levels marked E_4 and E_6 which have nearly the same energy as levels F_2 and F_3 of He. The population in these levels are more than E_3 and E_5. Thus, a state of population inversion triggers lasing action. The Ne atoms then drop down from the lower laser level to the level E_2 through spontaneous emission. From the level E_2, the Ne atoms are brought to the ground state through collisions with the wall. The transition from E_6 to E_5, E_4 to E_3 and E_6 to E_3 result in emission of radiation, 3.39 μm, 1.15 um and 6328 Å, respectively. The transition corresponding to 3.39 μm and 1.15 μm are not visible but 6328 Å corresponds to the red light of He-Ne laser. The pressure of two gases must be so chosen that the condition of population inversion is not quenched. Thus, the condition must be such that there is an efficient transfer of energy from He to Ne atoms.

Merits of laser:

Laser is highly coherent, intense, monochromatic & directional beam.

Q. The light output of a typical laser is 10.6 μm. What is the energy difference between the enrgy levels of the exciled slate and the meta stable state. What is the energy of the photon emitted. What is the frequency associated with photon. If two moles of photons are emitted per second. What is the power of the laser output.

Ans. (i) $\lambda = 10.6\ \mu m$

10.6×10^{-6} m

$$E = hv = \frac{h \times c}{\lambda} = \frac{19.89 \times 10^{-26}}{10.6 \times 10^{-6}} = 1.87 \times 10^{-20}\ J$$

$$\Delta E = \frac{20 \times 1.6 \times 10^{-19} \times 10^{6}}{50 \times 10^{-3} \times 1.6 \times 10^{-19}} \begin{bmatrix} 1.87 \times 10^{-20} \\ \times 6.25 \times 10^{18}\ ev \end{bmatrix}$$

(ii) ∴

$$\Delta E = 1.8 \times 10^{-20} \times 6.25 \times 10^{18}\ ev$$

$$= 1.172\ ev$$

(iii)

$$V = \frac{c}{\lambda} = \frac{3 \times 10^{8}}{10.6 \times 10^{-6}}\ V = .28 \times 10^{14}\ Hz$$

(iv) 1 mol photons $= 6.023 \times 10^{-23}$

$$E = \frac{\frac{hc}{\lambda} 6.626 \times 10^{-34} \times 6.023 \times 10^{+23} \times 3 \times 10^{8}}{10.6\ \mu m}$$

$$\frac{119}{10.6} = 11.29 \times 10^{-5}$$

$$11.29 \times 10^{-5}\ J \times 2$$

$$= 22.58 \times 10^{-5}$$

$$V = 22.58\ KW$$

UNIT-III

Q. (a) Length of a moving spaceship is found to be 80% of its actual length to an observer at rest. Calculate the speed of the spaceship. (2)

Ans.

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

$$80\% = l\sqrt{1 - \frac{v^2}{c^2}}$$

let

$$l = 100$$

$$\frac{80}{100} = \sqrt{1 - \frac{-v^2}{c^2}}$$

$$v = 0.6\ c$$

Q. The ratio of the proper life to the mean life of moving fundamental particle is 1/5 calculate the speed of the fundamental particle. (2)

Ans.

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{\Delta t}{\Delta t'} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\left(\frac{1}{5}\right) = \sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{25} = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{25}$$

$$= \frac{25-1}{25}$$

$$\frac{v}{c^2} = \frac{24}{25}$$

$$v = .97c$$

Q. An electron has an initial speed of 1.4 × 10⁸m/s. How much additional energy must be imparted in it for its speed to double. (4)

Ans.

$$\Delta E = E_2 - E_1 = m_0c^2\left[\frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_1^2}{c^2}}}\right]$$

$$\Delta E = [9.1\times10^{-31}]\times[3\times10^8]^2\left[\frac{1}{\sqrt{1-\frac{1-(2.8\times10^8)^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{(1.4\times10^8)^2}{c^2}}}\right]$$

$$= [9.1\times10^{-31}]\times[3\times10^8]^2\left[\frac{1}{(0.37)} - \frac{1}{(0.88)}\right]J \quad \left\{\begin{array}{c}\text{as}\\ 1\,MeV = 1.6\times10^{-13}J\end{array}\right\}$$

$$= \frac{[9.1\times10^{-31}][3\times10^8]^2\times0.51}{1.6\times10^{-13}} = 0.26 \text{ meV}$$

Q.6. (a) Write down the posulates of special theory of relativity. (2.5)

Ans. Einstein's special theory of relativity has two postulates

(i) In two inertial system of reference in uniform translatory motion relative to each other all laws of nature are identically the same

(ii) The velocity of light in vacuum is a constant and independent not only of the direction of propagation but also of the relative velocity between the source and the observer.

Q. (b) Define Population Inversion. (2)

Ans. Population inversion is the non-equilibrium condition of a material where the number of atoms in the upper energy level exceeds the number of atoms in lower energy level. It is achieved by pumping energy from external sources.