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Unit-3

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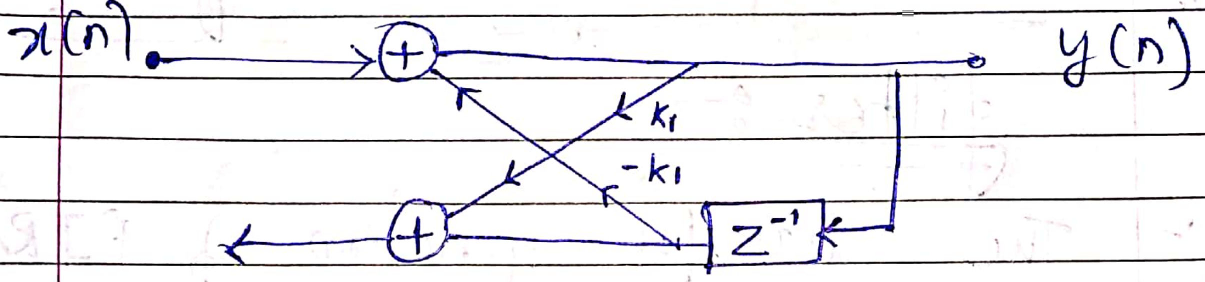
Lattice Realization :-

- **For IIR :-**

Step 1 :- find values of k

Step 2 :- $A_{m-1}(z) = \dfrac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$

All pole system :-



First stage

Stability :- K < 1

- **For FIR :-**

Step 1 :- find k values

Step 2 :- $A_{m-1}(z) = \dfrac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$

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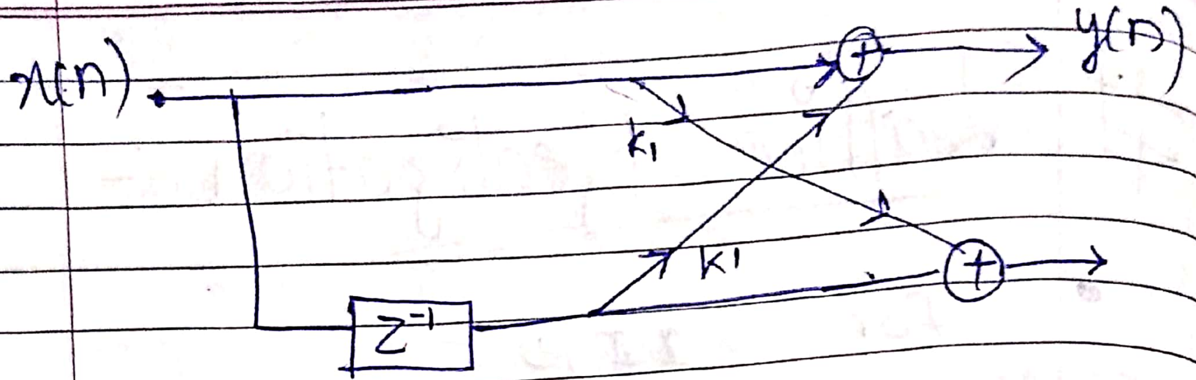

first stage

for stability $k < 1$

Lattice Realization of FIR filter :-

Ex:- The transfer function of FIR filter

max power

$$H(z) = 1 + \frac{3}{4} z^{-1} + \frac{1}{2} z^{-2} + \frac{1}{4} z^{-3}$$

find the lattice coeff. and show lattice realization.

A (i) To find k_3

$$H(z) = A_3(z) = 1 + \frac{3}{4} z^{-1} + \frac{1}{2} z^{-2} + \boxed{\frac{1}{4}} z^{-3}$$

$\rightarrow k_3$

[As it is]

$$\boxed{k_3 = 0.25}$$

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(ii) To find k_2

$$A_{m-1}(z) = \frac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$$

$$A_{3-1}(z) = \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2}$$

$$\boxed{B_3(z) = z^{-3} A_3(z^{-1})}$$

$$= z^{-3}\left[1 + \frac{3}{4}z + \frac{1}{2}z^2 + \frac{1}{4}z^3\right]$$

$$\boxed{B_3(z) = z^{-3} + \frac{3}{4}z^{-2} + \frac{1}{2}z^{-1} + \frac{1}{4}}$$

$$A_2(z) = \frac{1 + \frac{3}{4}z^{-1} + \frac{1}{2}z^{-2} + \cancel{\frac{1}{4}z^{-3}} - \cancel{0.25z^{-3}} - \frac{3z^{-2}}{16} - \frac{1}{8}z^{-1} - \frac{1}{16}}{1 - \frac{1}{16}}$$

$\frac{3}{4} - \frac{1}{8}$

$\frac{6}{8} - \frac{1}{8}$

$$= \frac{\frac{15}{16} + \frac{5z^{-1}}{8} + \left(\frac{8z^{-2} - 3z^{-2}}{16}\right)}{\frac{15}{16}} \Rightarrow \frac{\frac{15}{16} + \frac{5}{8}z^{-1} + \frac{5z^{-2}}{16}}{\frac{15}{16}}$$

$$A_2(z) = \frac{15 + 10z^{-1} + 5z^{-2}}{15} \Rightarrow 1 + \frac{2}{3}z^{-1} + \frac{1}{3}z^{-2}$$

for $A_2 = 1 + \frac{2}{3}z^{-1} + \frac{1}{3}z^{-2}$

$$\boxed{k_2 = \frac{1}{3}}$$

(iii) To find k_1

$$A_{m-1}(z) = \frac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$$

$$A_1(z) = \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2}$$

$$B_2(z) = z^{-2} A_2(z^{-1})$$

$$= z^{-2}\left[1 + \frac{2}{3}z + \frac{1}{3}z^2\right]$$

$$B_2(z) = z^{-2} + \frac{2}{3}z^{-1} + \frac{1}{3}$$

$$A_1(z) = \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2}$$

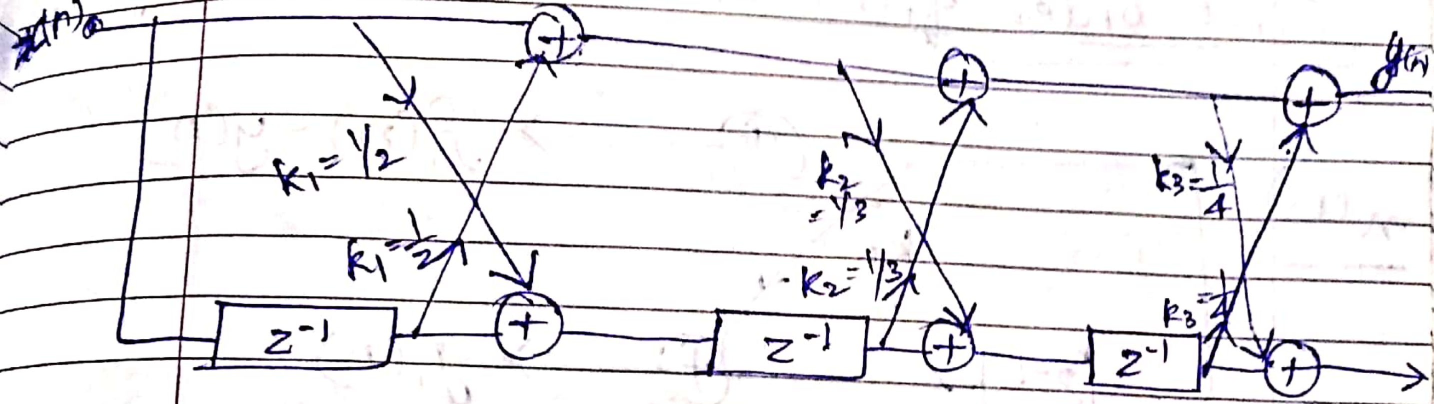
$$= \frac{1 + \frac{2}{3}z^{-1} + \cancel{\frac{1}{3}z^{-2}} - \cancel{\frac{1}{3}z^{-2}} - \frac{2}{9}z^{-1} - \frac{1}{9}}{1 - \frac{1}{9}}$$

$$\boxed{A_1(z) = \frac{\frac{8}{9} + \frac{4}{9}z^{-1}}{8/9}}$$

$$A_1(z) = \frac{\frac{8}{9} + \frac{4}{9}z^{-1}}{\frac{8}{9}} \Rightarrow \boxed{1 + \frac{1}{2}z^{-1}}$$

$$\boxed{k_1 = 1/2}$$

* **Lattice Realization :-**



Lattice Structure :-

F.I.R filter differencep^n :-

* **First Order filter :-**

$$y(n) = x(n) + a_1(1)x(n-1)$$

$k_1 = a_1(1)$ is the filter coeff.

* **Second order filter :-**

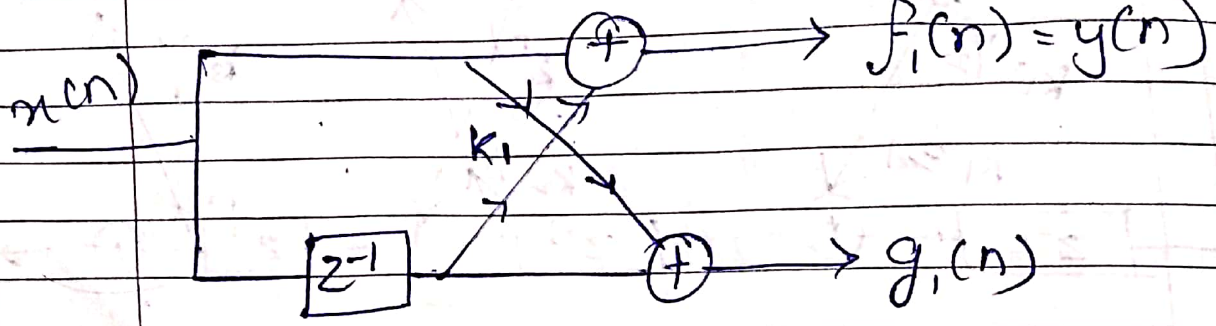
$$y(n) = x(n) + a_2(1)x(n-1) + a_2(2)x(n-2)$$

Here $k_1 = \dfrac{a_2(1)}{1 + a_2(2)}$

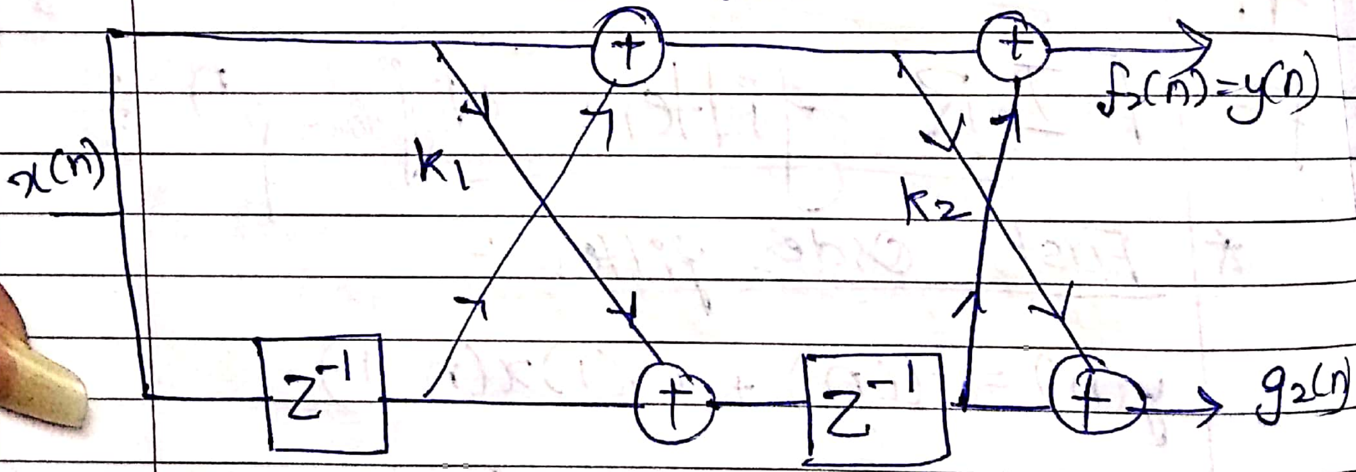
$k_2 = a_2(2)$

} filter coefficients

· **First order filter structure** ~~second order filter~~



Second Order filter structure :-

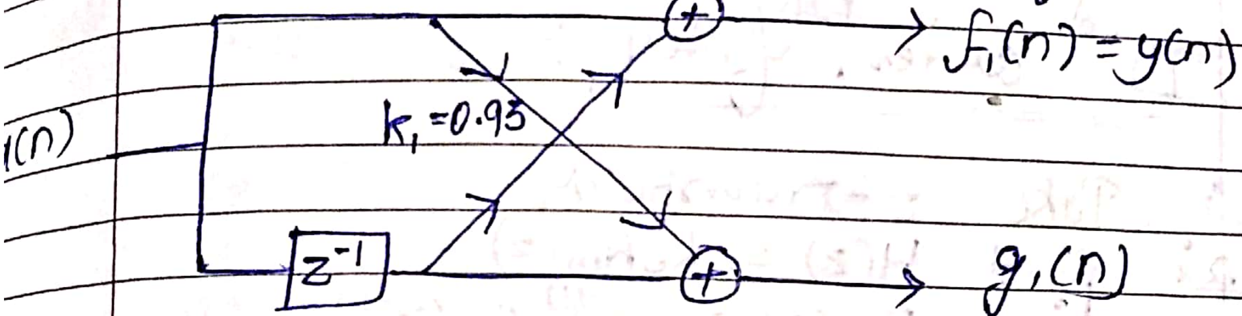


Q.1 Implement the first order filter represented by eqⁿ

$y(n) = x(n) + 0.93x(n-1)$ using lattice structure.

Sol:- $y(n) = x(n) + 0.93\,x(n-1)$

Here, $k_1 = 0.93$ is the filter coeff.



Q2 Implement the following 2nd order FIR filter

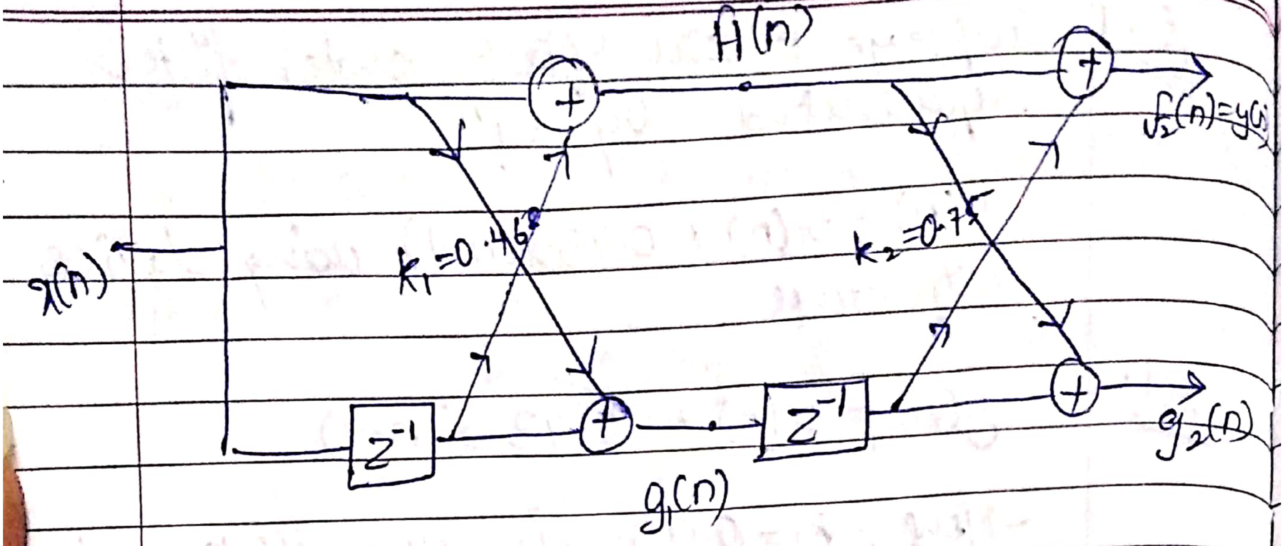
$y(n) = x(n) + 0.82\,x(n-1) + 0.75\,x(n-2)$

Sol:- $k_2 = a_2(2) = 0.75$

$k_1 = a_2(1) = \dfrac{a_2(1)}{1 + a_2(2)}$

$k_1 = \dfrac{0.82}{1 + 0.75} = \dfrac{0.82}{1.75}$

$\boxed{k_1 = 0.468}$



* IIIrd order FIR filter :-

Step 1 :- Take z-transform
Step 2 :- find $H(z) = k_0 A_m(z)$
Step 3 :- find $B_m(z) = z^{-m} A_m(z^{-1})$

Step 4 :- find

$$A_{m-1}(z) = \frac{A_m(z) - k_m B_m(z)}{1-k_m^2}$$

Step 5 :- Repeat until $k_1 = ?$
$k_2 = ?$ & $k_3 = ?$

Q. Implement = ?

$$y(n) = 2x(n) + \frac{13}{12}x(n-1) + \frac{5}{4}x(n-2) + \frac{2}{3}x(n-3)$$

Sol:- Take z transform :-

$$Y(z) = 2X(z) + \frac{13}{12}z^{-1}X(z) + \frac{5}{4}z^{-2}X(z) + \frac{2}{3}z^{-3}X(z)$$

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$$Y(z) = X(z)\left[2 + \frac{13}{12} z^{-1} + \frac{5}{4} z^{-2} + \frac{2}{3} z^{-3}\right]$$

$$\frac{Y(z)}{X(z)} = 2\left[1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}\right]$$

→ first term must be unity

we know that

step 2 :- $H(z) = k_0 A_3(z)$

$$\boxed{k_0 = 2}$$

$$\boxed{A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}}$$

$$\boxed{k_3 = \frac{1}{3}}$$

Step 3 :- $B_3(z) = z^{-3} A_3(z^{-1})$ → Write in Reverse order

$$B_3(z) = z^{-3}\left[1 + \frac{13}{24} z + \frac{5}{8} z^2 + \frac{1}{3} z^3\right]$$

$$\boxed{B_3(z) = \frac{1}{3} + \frac{5}{8} z^{-1} + \frac{13}{24} z^{-2} + z^{-3}}$$

Step 4 :- $A_2(z) = \dfrac{A_3(z) - k_3 B_3(z)}{1 - k_3^2}$

$\frac{5}{8} \cdot \frac{13}{72}$ $\frac{45-13}{72} \Rightarrow \frac{32 \cdot 16 \cdot 4}{72 \cdot 36 \cdot 9}$

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$$= \frac{1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3} - \frac{1}{9} - \frac{5}{24} z^{-1} - \frac{13}{72} z^{-2} - \cancel{\frac{1}{3} z^{-3}}}{1 - \frac{1}{9}}$$

$$= \frac{\frac{8}{9} + \frac{8}{24} z^{-1} + \frac{4}{9} z^{-2}}{\frac{8}{9}}$$

$$\boxed{A_2(z) = 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2}} \quad \therefore \boxed{K_2 = \frac{1}{2}}$$

Step 5:- $\therefore$ $A_1(z) = ?$ $K_1 = ?$

$$\boxed{A_1(z) = 1 + \frac{1}{4} z^{-1}} \quad \boxed{K_1 = \frac{1}{4}}$$

Step 6:- $B_1(z) = z^{-1}\left(1 + \frac{1}{4} z\right)$

$$\boxed{B_1(z) = z^{-1} + \frac{1}{4}}$$

$\therefore$ we get

$$\left\{ \begin{array}{l} K_1 = 1/4 \\ K_2 = 1/2 \\ K_3 = 1/3 \end{array} \right\}$$

Step 8

$x(n)$

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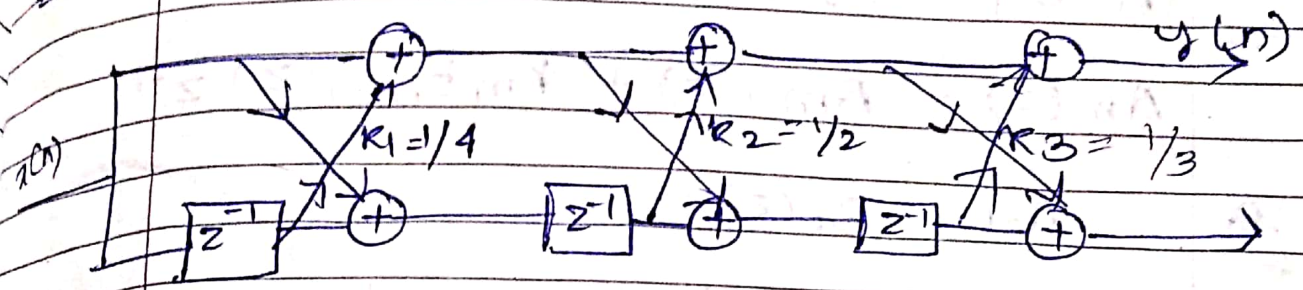
Step 8 :- Structure :-

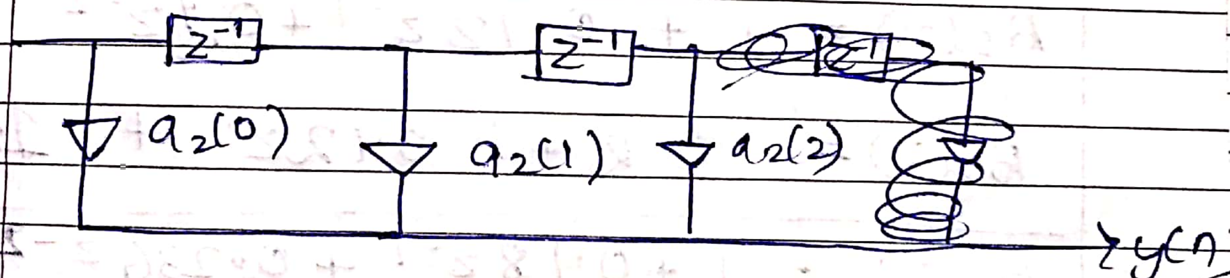


Lattice & ladder (IIR)

$\boxed{H(z) = A_m(z)}$

$= a_m(0) + a_m(1)z^{-1} + a_m(2)z^{-2} \dots a_M(M)z^{-M}$

if m = 2 $\quad a_2(0) + a_2(1)z^{-1} + a_2(2)z^{-2}$



* Forward eqn ⇒ $A_2(z) = 1 + 3/8\, z^{-1} + \frac{1}{2} z^{-2}$
* Backward eqn ⇒ $B_2(z) = \frac{1}{2} + \frac{3}{8} z^{-1} + 1\, z^{-2}$
* for zero order ~~m = 0~~ $A_0(z) = B_0(z) = 1$

$$\boxed{A_m(z) = A_{m-1}(z) + K_m z^{-1} B_{m-1}(z)}$$

① $k_1 = 0.1 \quad k_2 = 0.2 \quad k_3 = 0.3$

$$A_m(z) = A_{m-1}(z) + k_m B_{m-1}(z) z^{-1}$$

A: $A_0(z) = B_0(z) = 1$

$$A_1(z) = A_0(z) + k_1 B_0(z) z^{-1}$$
$$= 1 + 0.1(1) z^{-1}$$
$$= 1 + 0.1 z^{-1}$$

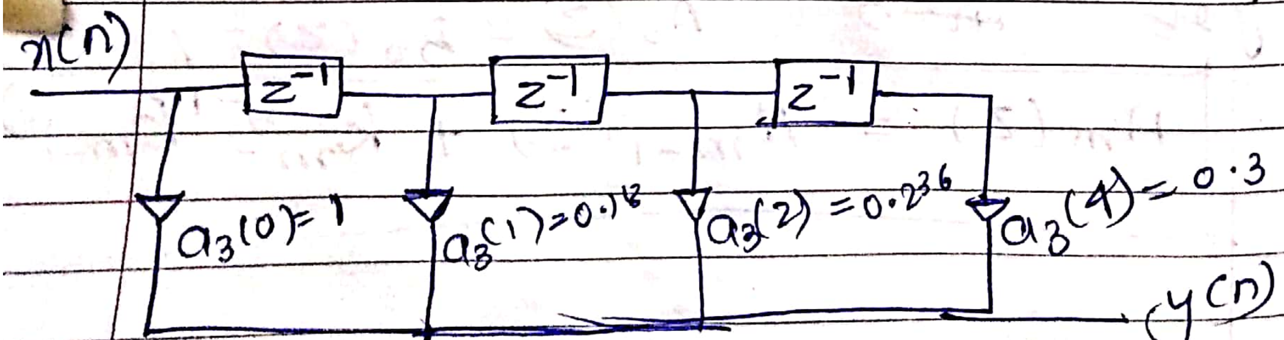
$$B_1(z) = 0.1 + 1 z^{-1}$$ [Interchanging coeffs.]

$$A_2(z) = A_1(z) + k_2 B_1(z) z^{-1}$$
$$= 1 + 0.1 z^{-1} + 0.2 z^{-1} (0.1 + 1 z^{-1})$$
$$= 1 + 0.1 z^{-1} + 0.02 z^{-1} + 0.2 z^{-2}$$
$$A_2(z) = 1 + 0.12 z^{-1} + 0.2 z^{-2}$$

$$B_2(z) = 0.2 + 0.12 z^{-1} + 1 z^{-2}$$

$$\boxed{A_3(z) = 1 + 0.18 z^{-1} + 0.236 z^{-2} + 0.3 z^{-3}}$$

* Structure :-



$a_3(0) + a_3(1) z^{-1} + a_3(2) z^{-2} + \ldots$

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6 $H(z) = \dfrac{0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}}{1 - 0.0010z^{-1} + 0.6546z^{-2} - 0.0715z^{-3}}$

Draw & explain lattice ladder structure

A:

$B_M(z) = 0.2759 + 0.5121z^{-1} + 0.5121z^{-2} + 0.2759z^{-3}$

$A_M(z) = 1 - 0.0010z^{-1} + 0.6546z^{-2} - 0.0715z^{-3}$

highest power = 3 $\Rightarrow a_3(1) = -0.0010$; $a_3(2) = 0.6546$

$\boxed{a_3(3) = -0.0715 = k_3}$

* for m=3 i=1

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1 - k_m^2}$$

$a_{3-1}(1) = a_3(1) - a_3(3)a_3(2) / 1 - (k_3)^2$

$$= \frac{(-0.001) - [(-0.0715)(0.6546)]}{1 - (-0.0715)^2}$$

$= (0.000001 - 0.047) / 0.995$

$\boxed{a_2(1) = -0.047}$

* (i=2) (m=3)

$a_2(2) = a_3(2) - a_3(3)\,a_3(1) / 1 - k_3^2$

$\boxed{a_2(2) = 0.658 = k_2}$

* for m=2 i=1

$a_1(1) = a_2(1) - a_2(2)\,a_2(1) / 1 - k_2^2$

$= -0.0161 / 0.567$

$\boxed{a_1(1) = 0.02835 = k_1}$

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$$B(z) = 0.2759 + 0.5121 z^{-1} + 0.5121 z^{-2} + 0.2759 z^{-3}$$

$b_0 = 0.2759$ | $b_2 = 0.5121$

$b_1 = 0.5121$ | $b_3 = \beta_3 = 0.2759$

$$\boxed{\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)}$$

$$\beta_2 = b_2 - \sum_{2+1}^{3} \beta_m a_m(m-i) = b_2 - \beta_3 a_3(1)$$

$$= 0.5121 - [0.276(-0.001)] = \boxed{0.5124}$$

$$\beta_1 = b_1 - [\beta_2 a_2(1) + \beta_3 a_3(2)]$$

$$= 0.5121 - [0.5124(-0.047) + (0.276)(0.65)]$$

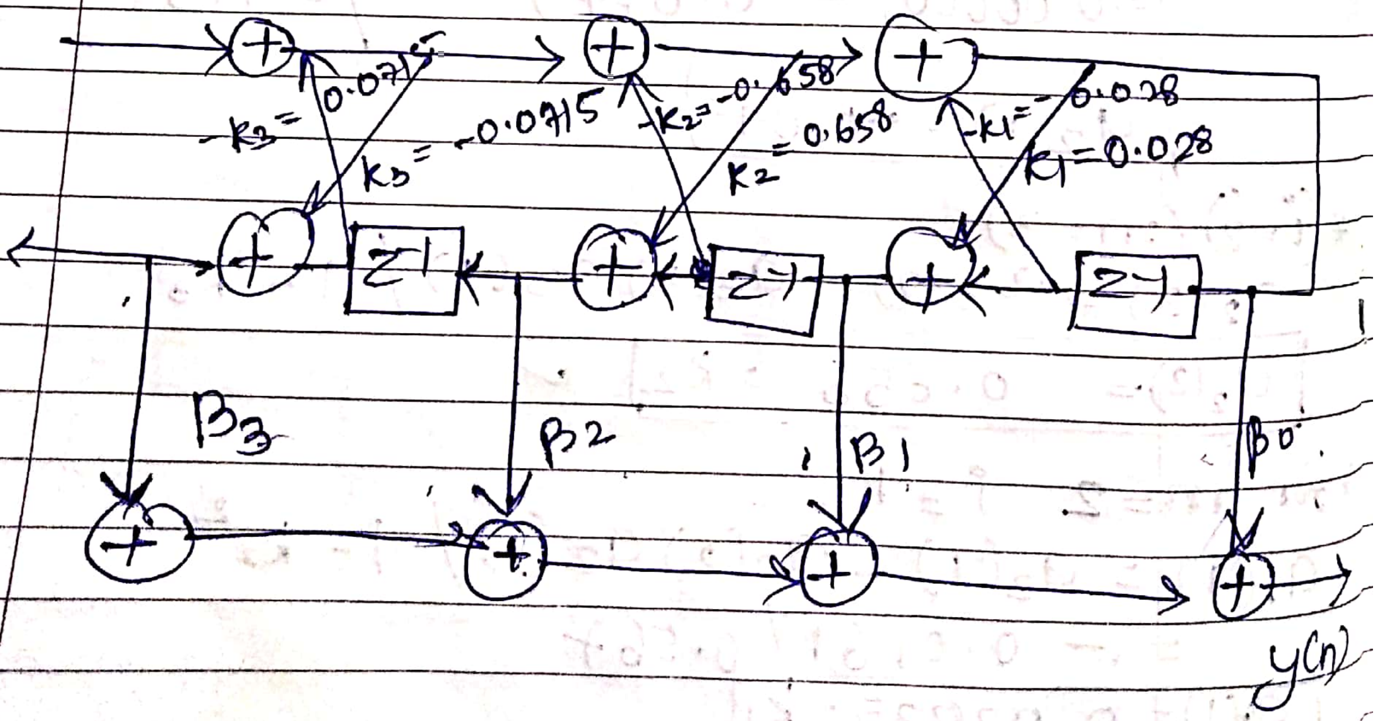
$$= 0.5121 - 0.157 = \boxed{0.355}$$

$$\beta_0 = b_0 - [\beta_1 a_1(1) + \beta_2 a_2(2) + \beta_3 a_3(3)]$$

$$= 0.2759 - [0.355(0.028) + 0.512(0.66) + (0.2759)(-0.0715)]$$

$\beta_0 =$ ✓

Lattice - ladder structure :-



* Ladder structure for IIR filter

Ex $$H(z) = \frac{2z^{-2} + 3z^{-1} + 1}{z^{-2} + z^{-1} + 1}$$

Routh array Table

$$\frac{3-2}{1}$$

$$\begin{array}{c|ccc}
z^{-2} & 2 & 3 & 1 \\
z^{-2} & 1 & 1 & 1 \\
z^{-1} & 1 & -1 & 0 \\
z^{-1} & 2 & 1 & \\
1 & -3/2 & 0 & \\
1 & 1 & 0 &
\end{array}$$

$$\alpha_0 = \frac{2}{1} = 2$$

$$\beta_1 = \frac{1}{1} = 1$$

$$\alpha_1 = \frac{1}{2} = 1/2$$

$$\beta_2 = \frac{2}{-3/2} = -4/3$$

$$\alpha_2 = \frac{-3/2}{1} = -3/2$$

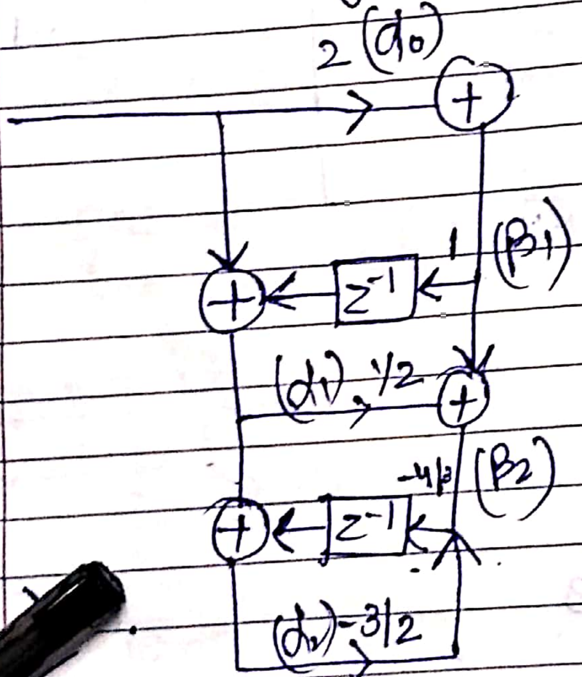
Standard eq^n for $H(z)$

$$H(z) = \alpha_0 + \cfrac{1}{\beta_1 z^{-1} + \cfrac{1}{\alpha_1 + \cfrac{1}{\beta_2 z^{-2} + \cdots \cfrac{1}{\alpha_m}}}}$$

Put all coeff. in H(z)

$$H(z) = 2 + \cfrac{1}{z^{-1} + \cfrac{1}{\frac{1}{2} + \cfrac{1}{-\frac{4}{3}z^{-2} + \cfrac{1}{-\frac{3}{2}}}}}$$

ladder form Realization



$$H(z) = \frac{0.129 + 0.38687z^{-1} + 0.3869z^{-2} + 0.129z^{-3}}{1 - 0.2971z^{-2} + 0.3564z^{-2} - 0.0276z^{-3}}$$

A. $B(z) = 0.129 + 0.38687z^{-1} + 0.3869z^{-2} + 0.129z^{-3}$

$A(z) = 1 - 0.2971z^{-2} + 0.3564z^{-2} - 0.0276z^{-3}$

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$a_3(1) = -0.2971 \quad a_3(2) = 0.3564 \quad \underbrace{a_3(3) = -0.0276 = k_3}$

* for $m=3 \quad i=1$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1-k_m^2}$$

$$a_{3-1}(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1-(-0.0276)^2}$$

$$\boxed{a_2(1) = \frac{-0.2971 - (-0.0276)(0.3564)}{0.99923}}$$

$$\boxed{a_2(1) = -0.28748}$$

for $i = 2$

$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1-k_3^2}$$

$$k_2 = \boxed{a_2(2) = 0.36418}$$

* for $m = 2, \quad i=1$

$$a_{2-1}(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1-k_2^2}$$

$$\boxed{k_1 = a_1(1) = -0.2106}$$

Now, $B(z) = 0.129 + 0.38687 z^{-1} + 0.3869 z^{-2} + 0.129 z^{-3}$

$b_0 = 0.129$
$b_1 = 0.38687$
$b_2 = 0.3869$
$b_3 = \beta_3 = 0.129$ $\quad [M = 3]$

[Highest power = 3 = β_3]

Imp. $\boxed{\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)}$

$$\beta_0 = b_0 - \sum_{0+1}^{M=3} \beta_m a_m(m-i)$$

$$= b_0 - [\beta_1 a_1(1) + \beta_2 a_2(2) + \beta_3 a_3(3)]$$

$$= 0.129 - [-0.21\beta_1 + 0.37\beta_2 + (-0.03\beta_3)] \quad (1)$$

$$\beta_2 = b_2 - \sum_{m=3=2+1}^{3} \beta_m a_m(m-i)$$

129
3
387

$$= b_2 - [\beta_3 a_3(1)]$$

$$= 0.3869 - [0.129(-0.30)]$$

$$= 0.3869 + [0.0387]$$

$$\boxed{\beta_2 = 0.4256} \quad (2)$$

$$\boxed{\begin{matrix} k_1 & k_2 & k_3 \\ \beta_0 & \beta_1 & \beta_2 \end{matrix}}$$ $\cancel{\beta_3}$ not used in structure.

$$\beta_1 = b_1 - [\beta_2 a_2(1) + \beta_3 a_3(2)]$$

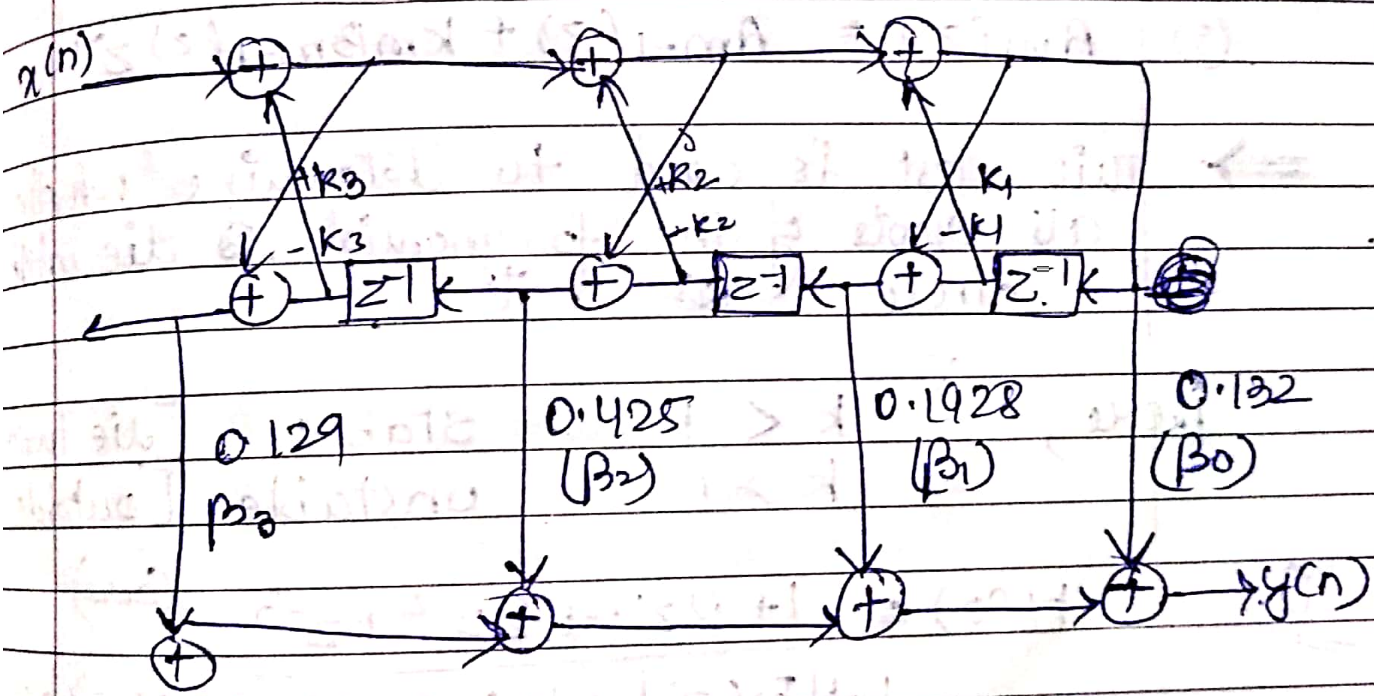
$$\beta_1 = 0.39 - [(0.42)(-0.29) + (0.129)(0.36)]$$

$$\beta_1 = 0.1928 \quad \text{--- (3)}$$

Put (2) (3) in (1),

$$\boxed{\beta_0 = 0.132}$$

lattice-ladder structure :-



Schur- Cohn Stability test :-

① $a_{m-1}(k) = \dfrac{a_m(k) - a_m(m)\,a_m(m-k)}{1 - (a_m(m))^2}$

② $A_{m-1}(z) = \dfrac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$

③ $A_m(z) = A_{m-1}(z) + k_m B_{m-1}(z) z^{-1}$

⟹ This test is used to determine whether all roots of the polynomial lie within circle or not

here, $k < 1$ stable [lie inside]
$k > 1$ unstable [outside]

Q. $H(z) = \dfrac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + 1.58z^{-1} + 1.638z^{-2} + 1.556z^{-3} + 0.4z^{-4}}$ → (zeros) → (poles)

A. On the basis of highest value of z

$a_4(0) = 1$
$a_4(1) = 1.58$
$a_4(2) = 1.638$
$a_4(3) = 1.556$
$\boxed{a_4(4) = 0.4 = k_4}$

Since $k_4 < 1$ ∴ stable

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$k_3 = a_3(3)$ $\quad (m = 4 \quad k = 3)$

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)\, a_m(m-k)}{1 - k_m^2}$$

$$a_3(3) = \frac{a_4(3) - a_4(4)\, a_4(1)}{1 - k_4^2}$$

$$= \frac{1.556 - (0.4)(1.58)}{1 - (0.4)^2}$$

$$\boxed{a_3(3) = 1.1 = k_3}$$

$k_3 > 1$ [unstable]

$k_2 = a_2(2)$ $\quad (m = 3 \quad k = 2)$

$$\boxed{a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - k_3^2}} \quad \text{—— (1)}$$

To find $a_3(2)$

$$a_3(2) = \frac{a_4(2) - a_4(4)\, a_4(2)}{1 - k_4^2} \boxed{= 1.17}$$

To find $a_3(1)$

$$a_3(1) = \frac{a_4(1) - a_4(4)\, a_4(3)}{1 - k_4^2} \boxed{= 1.14}$$

Put these values in (1)

$$k_2 = a_2(2) = \frac{1.17 - (1.1)(1.14)}{1 - (1.1)^2} \boxed{= 0.4}$$ $k_2 < 1$ stable.

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$m=2 \quad k=1$

$$\boxed{k_1 = a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1-k_2^2}} \quad \text{—— (2)}$$

To find $a_2(1)$

apply formula and get

$$\boxed{a_2(1) = 0.7}$$

Put $a_2(1)$ in (2),

$$k_1 = a_1(1) = \frac{0.7 - (0.7)(0.4)}{1-(0.4)^2}$$

$\boxed{k_1 = 0.5} < 1 \quad \therefore$ stable.

$$\left\{ \begin{array}{lll} k_1 & = 0.5 < 1 & \text{stable} \\ k_2 & = 0.4 < 1 & \text{stable} \\ k_3 & = 1.1 \nless 1 & \text{[unstable]} \\ k_4 & = 0.4 < 1 & \text{stable} \end{array} \right\}$$

Q2 $\quad H(z) = \dfrac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$

As this eq^n is not in the standard form.

Taking z^3 common from N & D :-

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$$H(z) = \frac{\cancel{z^3}\left[\frac{0.28}{z} + \frac{0.319}{z^2} + \frac{0.04}{z^3}\right]}{\cancel{z^3}\left[0.5 + \frac{0.3}{z} + \frac{0.17}{z^2} - \frac{0.2}{z^3}\right]}$$

$$= \frac{0.28z^{-1} + 0.319z^{-2} + 0.04z^{-3}}{0.5 + 0.3z^{-1} + 0.17z^{-2} - 0.2z^{-3}}$$

This term should be unity in standard form

So, $$\boxed{H(z) = \frac{0.28z^{-1} + 0.319z^{-2} + 0.04z^{-3}}{1 + \frac{3}{5}z^{-1} + \frac{0.17}{0.5} - \frac{2}{5}z^{-3}}}$$

Discrete Hilbert Transform

Fourier Series & Fourier transform are used to convert time domain sequence to frequency domain sequence.

time $\longleftrightarrow$ fourier.

$x(t) \xleftrightarrow{\text{Exponential fourier series}} C_n$

$x(t) \xleftrightarrow{\text{Fourier Transf.}} X(\omega)$

$x(t) \xleftrightarrow{\text{Hilbert Transf.}} \hat{x}(t)$

* Hilbert transform of a signal $x(t)$ is defined as the transform in which phase angle of all components of the signal is shifted by $\pm 90°$.

* HT of $x(t)$ is $\hat{x}(t)$ and represented by

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\alpha}^{\alpha} \frac{x(k)}{t-k}\, dk}$$

* Properties :

① Signal $x(t)$ & its hilbert transform is having
- → same amplitude spectrum
- → same autocorrelation function
- → same ESD

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② $x(t)$ & $\hat{x}(t)$ are orthogonal

③ If fourier transform exists then hilbert transform also exists for energy & power signals.

Lavinson Durbin Algorithm:-

It is a computationally efficient algorithm for solving the prediction coefficients.

This algorithm exploits the special symmetry in the autocorrelation matrix.

$$P = \begin{bmatrix} \gamma_{xx}(0) & \gamma_{xx}^{*}(1) & \cdots & \gamma_{xx}(p-1) \\ \gamma_{xx}(1) & \gamma_{xx}(0) & \cdots & \gamma_{xx}(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{xx}(p-1) & \gamma_{xx}(p-2) & \cdots & \gamma_{xx}(0) \end{bmatrix}$$

The key to the Lavinson Durbin method of solution, that exploits the Toeplitz property of matrix, is to proceed recursively, beginning with a predictor of order m and then to increase the order recursively using the lower-order solutions to obtain the solution to the next-higher order.

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Thus, the solution to the first-order predictor is

$$\boxed{a_1(1) = \frac{-\gamma_{xx}(1)}{\gamma_{xx}(0)}} \quad — (1)$$

and the resulting MMSE is

$$\boxed{E_1^f = \gamma_{xx}(0) + a_1(1)\gamma_{xx}(-1) = \gamma_{xx}(0)[1-|a_1(1)|^2]} \quad — (2)$$

$a_1(1) = k_1$ [I^{st} reflection coeff of lattice filter]

now find $a_2(1)$ & $a_2(2)$ [II^{nd} order predictor] and express solutions in terms of $a_1(1)$,

The two equations are :-

$$a_2(1)\gamma_{xx}(0) + a_2(2)\gamma_{xx}^*(1) = -\gamma_{xx}(1)$$
$$a_2(1)\gamma_{xx}(1) + a_2(2)\gamma_{xx}(0) = -\gamma_{xx}(2)$$

Use (1), eliminate $\gamma_{xx}(1)$,

$$a_2(2) = -\frac{\gamma_{xx}(2) + a_1(1)\gamma_{xx}(1)}{\gamma_{xx}(0)[1-|a_1(1)|^2]} = -\frac{\gamma_{xx}(2)+a_1(1)\gamma_{xx}(1)}{E_1^f}$$

$$a_2(1) = a_1(1) + a_2(2)a_1^*(1)$$ [second order predictor]

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again, note that $a_2(2) = k_2$ [IInd reflection coeff]

Proceeding in this manner, we can express the coeff of the m^{th} order predictor in terms of coeff of $(m-1)$ order predictor.

Lattice filter designing :-

Consider an all-pole system

$$H(z) = \frac{1}{1 + \left(\sum_{k=1}^{N} a_N(k) z^{-k}\right)} \Rightarrow \frac{1}{A_N(z)}$$

the difference eq^n for this IIR system is

$$\boxed{y(n) = -\sum_{k=1}^{N} a_N(k) y(n-k) + x(n)}$$

Or

$$\boxed{x(n) = y(n) + \sum_{k=1}^{N} a_N(k) y(n-k)}$$

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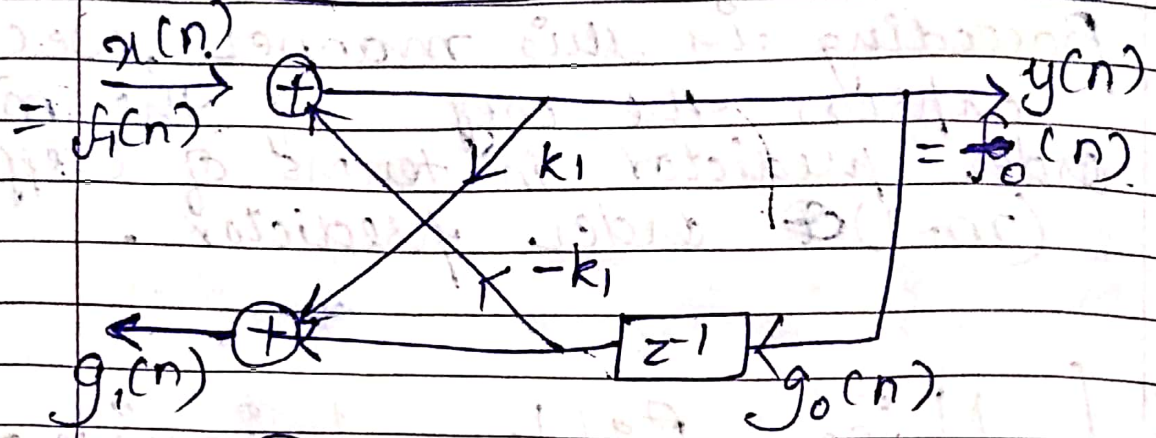
for $N=1$ $\qquad$ $k=1$

$$\boxed{x(n) = y(n) + a_1(1)\, y(n-1)} \quad \text{(a)}$$



(1) Single stage all pole lattice filter

Here ~~$x(n) = g f_1(n)$~~

$$\boxed{x(n) = f_1(n)}$$

$$y(n) = f_0(n) = g_0(n)$$

$$y(n) = f_1(n) - k_1 g_0(n-1)$$

$$y(n) = x(n) - k_1 y(n-1) \quad \text{(1)}$$

from (1),

$$x(n) = y(n) + k_1 y(n-1) \quad \text{(2)}$$

$$g_1(n) = k_1 f_0(n) + g_0(n-1)$$

$$\boxed{g_1(n) = k_1 y(n) + y(n-1)} \quad \text{(3)}$$

Comparing (a) & (2)

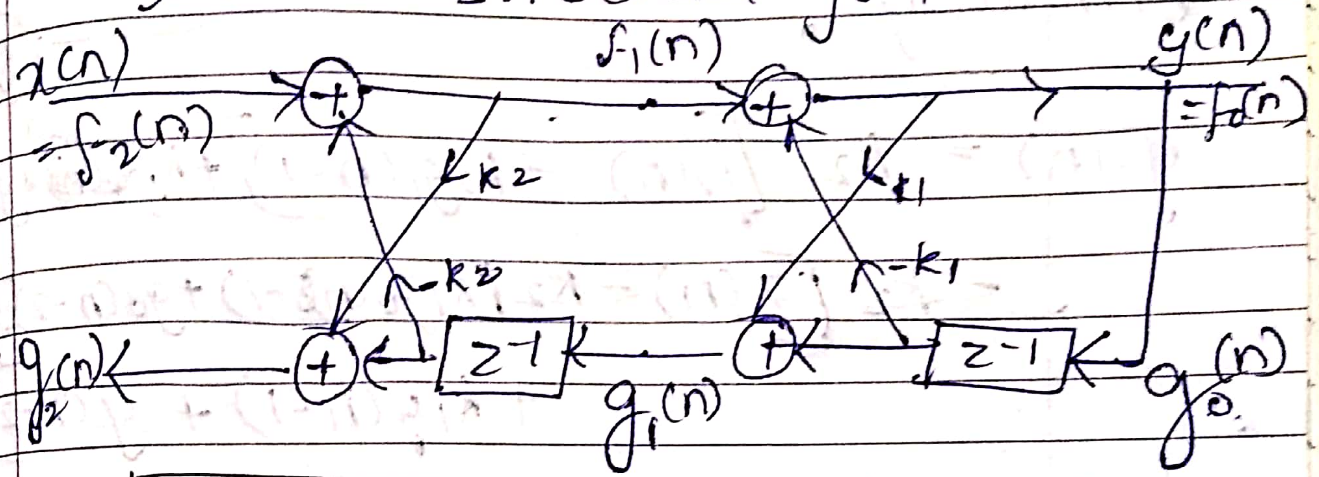
$$\boxed{a_1(1) = k_1}$$

for $N=2$ $\quad k=1$ $\quad$ [using formula]

$$x(n) = y(n) + a_2(1)\, y(n-1) + a_2(2)\, y(n-2)$$

also $\boxed{x(n) = f_2(n)}$ —— (4)

lattice structure for $N=2$



$\boxed{y(n) = f_0(n) = g_0(n)}$ —— (5)

$$f_2(n) = x(n)$$

$$f_1(n) = f_2(n) - k_2 g_1(n-1)$$

$$f_0(n) = f_1(n) - k_1 g_0(n-1)$$

$$g_2(n) = k_2 f_1(n) + g_1(n-1)$$

$$g_1(n) = k_1 f_0(n) + g_0(n-1)$$

In eq (5)

$$y(n) = f_0(n) = f_1(n) - k_1 g_0(n-1)$$

$$= f_2(n) - k_2 g_1(n-1) - k_1 g_0(n-1)$$

$$= x(n) - k_2 [k_1 f_0(n-1) + g_0(n-2) - k_1 g_0(n-1)$$

$$y(n) = x(n) - k_2 k_1 f_0(n-1) - k_2 g_0(n-2) - k_1 g_0(n-1)$$

now using 5

$$y(n) = x(n) - k_2 k_1 y(n-1) - k_2 y(n-2) - k_1 y(n-1)$$

$$\boxed{y(n) = x(n) - k_1 y(n-1)[1+k_2] - k_2 y(n-2)} \quad (6)$$

similarly,

$$g_2(n) = k_2 [f_2(n) - k_2 g_1(n-1)] + k_1 f_0(n-1) + g_0(n-2)$$

$$= k_2 [x(n) - k_2 [k_1 f_0(n-1) + g_0(n-2)]] + k_1 y(n-1) + y(n-2)$$

$$= k_2 [x(n) - k_2 k_1 y(n-1) - k_2 y(n-2)] + k_1 y(n-1) + y(n-2)$$

$$= k_2 x(n) - k_2^2 k_1 y(n-1) - k_2^2 y(n-2) + k_1 y(n-1) + y(n-2)$$

$$\boxed{g_2(n) = k_2 y(n) + k_1 (1+k_2) y(n-1) + y(n-2)} \quad (7)$$

Comparing eq (4) & (6)

$$a_2(0) = 1$$

$$a_2(1) = k_1 (1+k_2)$$

$$a_2(2) = k_2$$