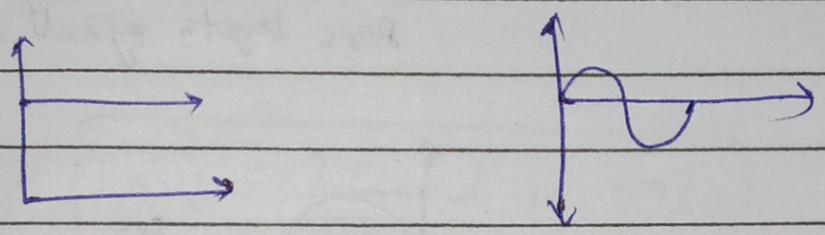
Unit II

DATE: 11/5/23
PAGE:

AC Circuit

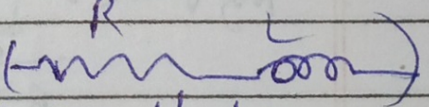
Current

- DC → R / E (battery) volt
- AC → R / L / C



50 Hz

Topics

1. Avg value / Rms value (waves ते तो sums)
2. Pure R ckt, Pure L ckt, Pure C ckt } Phasor Diagram
3. Choke Coil Circuit 
4. Resonance (series & parallel)

Ac Ckt

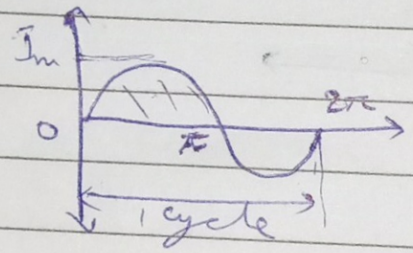
- ~~AC fundamental~~ → R-L-C
- ~~Series Circuit~~ → RL / RC / RLC

Eq^n of sinusoidal ac waveform

$i = I_m \sin\theta$; $v = V_m \sin\theta$

Avg value of sinusoidal waveform

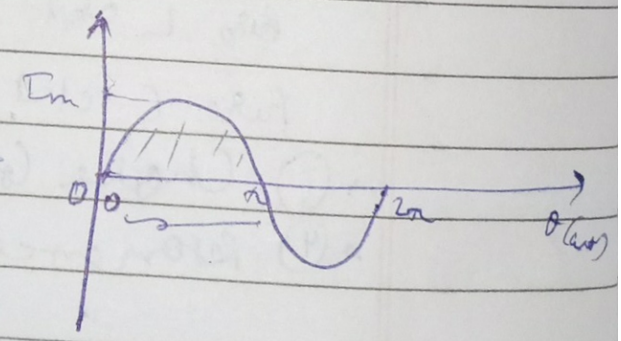
$$\text{Avg value} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}}$$



• Topic : Avg & Rms value

Avg Value

$$\text{Avg value} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}}$$



$$\therefore I_{avg} = \frac{\int_0^{\pi} i\, d\theta}{\pi} \qquad \boxed{i = I_m \sin\theta}$$

$$\Rightarrow \frac{\int_0^{\pi} I_m \sin\theta\, d\theta}{\pi}$$

$$\Rightarrow \frac{I_m}{\pi} \int_0^{\pi} \sin\theta\, d\theta$$

DATE: __/__/____
PAGE: _____

$$\Rightarrow I_{avg} = \frac{I_m}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$\Rightarrow \frac{I_m}{\pi}\left(-\cos\pi - (-\cos 0)\right)$$

$$\Rightarrow \frac{I_m}{\pi}\left(-(-1) - (-1)\right)$$

$$\Rightarrow \frac{I_m}{\pi}(1+1) = \frac{2I_m}{\pi}$$

$$\Rightarrow \boxed{I_{avg} = \frac{2I_m}{\pi} = 0.637 I_m}$$

Rms Value
(Root mean square)

$$I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2\, d\theta}{\pi}} = \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta}{\pi}}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\sin^2\theta\, d\theta}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$\Rightarrow \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta} \Rightarrow \sqrt{\frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}}$$

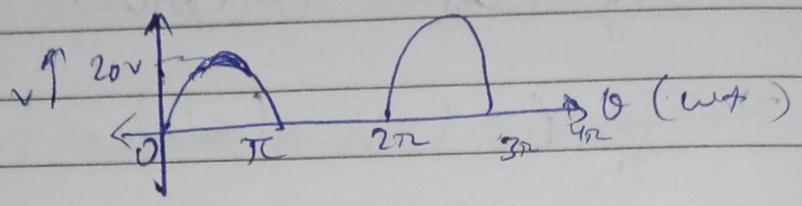
$$\Rightarrow \sqrt{\frac{I_m^2}{2\pi}(\pi - 0)} \Rightarrow \sqrt{\frac{I_m^2\pi}{2\pi}} \Rightarrow \frac{I_m}{\sqrt{2}}$$

Good Write $\Rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$

DATE : ___/___/___
PAGE : _____

Numerical

Q. Find avg value of waveform



Sol→ Given waveform is continuous.

Equation	Interval
① $v = 20\sin\theta$	$0 < \theta < \pi$
② $v = 0$	$\pi < \theta < 2\pi$

$$V_{avg} = \frac{\text{Area of ~~half~~ full Cycle}}{\text{~~base~~ length of full cycle}}$$

$$V_{avg} = \frac{\int_0^{2\pi} v\,d\theta}{2\pi}$$

$$V_{avg} = \cancel{\int_0^{2\pi} v\,\theta} \quad \frac{1}{2\pi}\left[\int_0^{\pi} 20\sin\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi}\left[20\int_0^{\pi}\sin\theta\,d\theta\right]$$

$$\Rightarrow \frac{10}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

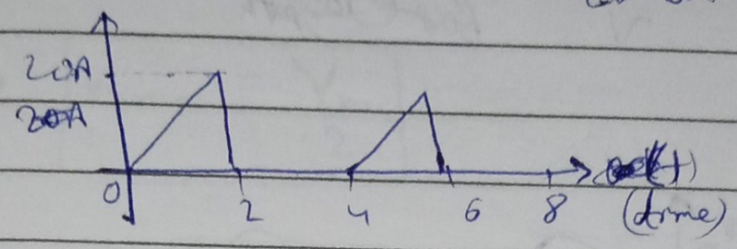
$$\Rightarrow \frac{10}{\pi}\left(-\cos\pi + \cos 0\right)$$

$$\Rightarrow \frac{10}{\pi}(1+1) \Rightarrow \frac{20}{\pi} \Rightarrow 6.366\text{ V Ans}$$

Good Write

DATE: ___/___/___
PAGE: _____

Q. 2 Find avg value of waveform (sawtooth)



$y_2 - y_1 = m(x_2 - x_1)$

$m = \frac{20}{2} \Rightarrow 10$

Sol: Given waveform is continuous

Equation	Interval
① $i = mt = 10t$	$0 < t < 2$
② $i = 0$	$2 < t < 4$

$$I_{avg} = \frac{\int_0^4 i\,dt}{4}$$

$$\Rightarrow \frac{1}{4}\int_0^4 10t\,dt$$

$$\Rightarrow \frac{10}{4}\left[\frac{t^2}{2}\right]_0^4 \Rightarrow \frac{160}{8} \Rightarrow \frac{20}{2}$$

$$\Rightarrow \frac{1}{4}\left[\int_0^2 10t\,dt + \int_2^4 0\,dt\right]$$

$$\Rightarrow \frac{1}{4}\left[\frac{10}{2}[t^2]_2^2\right] \Rightarrow \frac{40}{8} \Rightarrow 5A$$

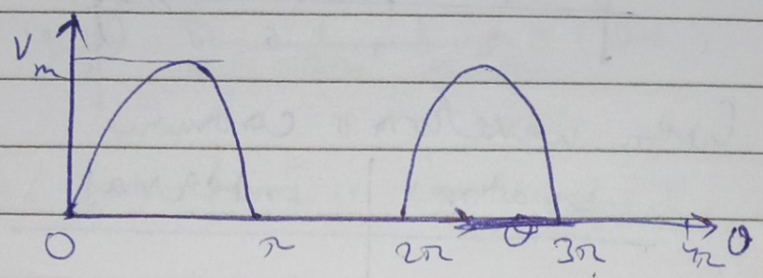
$$\boxed{I_{avg} = 5A}$$ Ans

Good Write

DATE: ___/___/___
PAGE: _____

Q. $V_{rms} = \sqrt{\dfrac{\text{Area of full cycle}}{\text{Base length}}}$

Q.



Sol. $v = V_m \sin\theta$ — $0 < \theta < \pi$

$v = 0$ — $\pi < \theta < 2\pi$

$$V_{rms} = \sqrt{\frac{\int_0^{2\pi} v^2 d\theta}{2\pi}}$$

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} v^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} (V_m \sin\theta)^2 d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$\Rightarrow \frac{V_m^2}{2\pi}\int_0^{\pi} (\sin^2\theta)\, d\theta$$

$$\Rightarrow \frac{V_m^2}{4\pi}\int_0^{\pi} (1 - \cos 2\theta)\, d\theta$$

$$\Rightarrow \frac{V_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$\Rightarrow \frac{V_m^2}{4\pi} \times \pi$$

Good Write

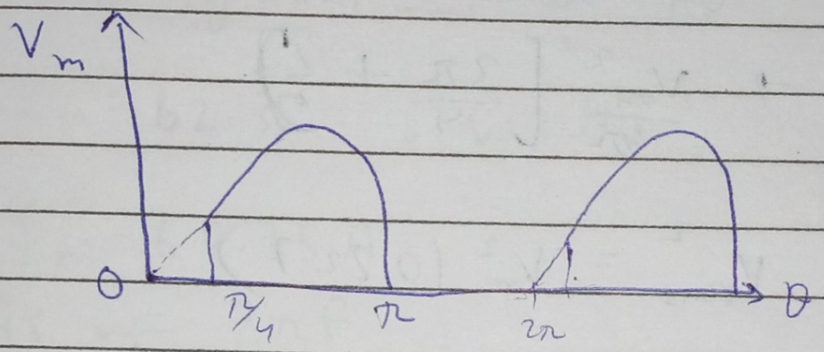
DATE: ___/___/___
PAGE: _____

$$\Rightarrow V_{rms}^2 = \frac{V_m^2}{4}$$

$$\Rightarrow \boxed{V_{rms} = \frac{V_m}{2}}$$

$$\boxed{V_{rms} = 0.5\, V_m}$$ Ans

Q.



Soln

Equation	Interval
$V = 0$	$0 < \theta < \pi/4$
$V = V_m \sin\theta$	$\pi/4 < \theta < \pi$
$V = 0$	$\pi < \theta < 2\pi$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2\, d\theta}$$

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi/4} 0\, d\theta + \int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{V_m^2}{4\pi}\int_{\pi/4}^{\pi} (1 - \cos 2\theta)\, d\theta$$

Good Write

DATE: ___/___/___
PAGE: _____

$$= \frac{V_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_{\pi/4}^{\pi}$$

$$= \frac{V_m^2}{4\pi}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin \pi/2}{2}\right)\right]$$

$$= \frac{V_m^2}{4\pi}\left[\pi - 0 - \pi/4 + 1/2\right]$$

$$\Rightarrow \frac{V_m^2}{4\pi}\left[\frac{3\pi}{4} + \frac{1}{2}\right]$$

$$\Rightarrow V_{rms}^2 = V_m^2 (0.227)$$

$$\Rightarrow \boxed{V_{rms} = V_m (0.476)}$$ Ans

• Algebra of Phasors

* Some Important Formulae

① $\omega = 2\pi f = \frac{2\pi}{T}$

② Rms value $= I_{rms} = \frac{I_m}{\sqrt{2}}$, $V_{rms} = \frac{V_m}{\sqrt{2}}$

③ Avg value $= I_{avg} = 0.637\, I_m$

④ Form factor $= \frac{I_{rms}}{I_{avg}}$

DATE: ___/___/___
PAGE: _____

Q The equation of an AC is $i = 62.35 \sin 323t$
Determine: (1) Max value (2) frequency
(3) Rms value (4) avg value
(5) form factor

Sol → Given: $i = 62.35 \sin 323t$.

Comparing with $i = I_m \sin \omega t$

$I_m = 62.35$; $\omega = 323$

(1) $I_m = 62.35\,A$

(2) As $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi} = \frac{323}{2\pi} = \frac{323}{2 \times 3.14} = 51.41\ Hz \text{ Ans}$$

(3) Rms value $= \frac{I_m}{\sqrt{2}} = \frac{62.35}{\sqrt{2}} = 44.1\,A$ Ans

(4) Avg value $\Rightarrow I_{avg} = 0.637\, I_m$
$= (62.35)(0.637)$
$= 39.7\,A$ Ans

(5) form factor $= \frac{I_{rms}}{I_{avg}} = \frac{44.1}{39.7} = 1.11$ Ans

DATE: ___/___/___
PAGE: _____

- Algebra of Phasor

① Length of Phasor ⇒ $V = \frac{V_m}{\sqrt{2}}$; $I = \frac{I_m}{\sqrt{2}}$

② Rectangular form / Cartesian form ⇒
$x + iy = x + jy$

③ Polar form
$r \angle \theta$

* Rectangular form

$z_1 = x_1 + jy_1$; $z_2 = x_2 + jy_2$

① $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$ — (Addition)

② $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$ — (Subtraction)

* Polar form

① $z_1 = r_1 \angle \theta_1$ & $z_2 = r_2 \angle \theta_2$

$(z_1 \cdot z_2) = (r_1 \angle \theta_1) \cdot (r_2 \angle \theta_2)$
$= (r_1 \cdot r_2) \angle (\theta_1 + \theta_2)$
— (Multiplication)

② $\frac{z_1}{z_2} = \left(\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2}\right) = \left(\frac{r_1}{r_2}\right) \angle (\theta_1 - \theta_2)$
— (Division

Good Write

DATE: __/__/____
PAGE: _____

Q $v_1 = (42.43 \angle 0^\circ)$, $v_2 = (28.28 \angle 60^\circ)$
Calculate $(v_1 + v_2)$ in polar form.

Sol→ $v_1 = 42.43 \angle 0^\circ$; $v_2 = 28.28 \angle 60^\circ$

$(v_1 + v_2) = (42.43 \angle 0^\circ) + (28.28 \angle 60^\circ)$

Polar

$= Rec\ (42.43, 0) + Rec(28.28, 60)$

$= [42.43 + 0j] + \cancel{28.28} \cdot (14.14 + j(24.49))$

$v_1 + v_2 = 56.57 + 24.49j$ → Rectangle

$(v_1 + v_2) = Pol\ (56.57, 24.49)$

Ans $v_1 + v_2 = 61.64 \angle 23.41$ → Polar form

• Phasor Calculations

* Phasor Numericals

① $i = \frac{I_m}{\sqrt{2}}$, $V = \frac{V_m}{\sqrt{2}}$

② Cartesian (Rec) form : $z = x + jy$

③ Polar form : $z = r(\cos\theta + i\sin\theta)$
$= r \angle \theta$

DATE: __/__/____
PAGE: _____

Q. Find resultant of following voltages

$v_1 = 25 \sin \omega t$; $v_2 = 10 \sin\left(\omega t + \frac{\pi}{6}\right)$

$v_3 = 30 \cos \omega t$; $v_4 = 20 \sin\left(\omega t - \frac{\pi}{4}\right)$

Sol→ $v_1 = 25 \sin \omega t$

$v_2 = 10 \sin(\omega t + \pi/6)$

$\boxed{\cos\theta = \sin\left(\frac{\pi}{2} + \theta\right)}$

$v_3 = 30 \sin(\omega t + \pi/2)$

$v_4 = 20 \sin(\omega t - \pi/4)$

∴ Converting standard sine waveform;

$\bar{v}_1 = \frac{v_m}{\sqrt{2}} \angle \theta_1 = \frac{25}{\sqrt{2}} \angle 0 \Rightarrow (17.68 \angle 0)$

$\bar{v}_2 = \frac{10}{\sqrt{2}} \angle \theta_2 = (7.07 \angle 30)$

$\bar{v}_3 = \frac{30}{\sqrt{2}} \angle \theta_3 = (21.21 \angle 90)$

$\bar{v}_4 = \frac{20}{\sqrt{2}} \angle \theta_4 = (14.14 \angle -45)$

For resultant voltage;

$v = \bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4$

Good Write

DATE: ___/___/___
PAGE: _____

$= (17.68\angle 0) + (7.07\angle 30) + (21.21\angle 90) + (14.14\angle -45)$

Add^n & Sub^n → Rectangular form

multiplication & Division → Polar $\left(\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)\right)$

→ Rec $(17.68, 0)$

$\Rightarrow (17.68 + j(0)) + (6.12 + j(3.54)) + (0 + j(21.21)) + (10 - j(10))$

$\Rightarrow \bar{v} = (17.68 + 6.12 + 0 + 10) + j(0 + 3.54 + 21.21 - 10)$

$\Rightarrow \bar{v} = 33.80 + j(14.75)$

Converting into polar form

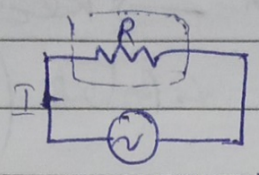
Pol $(33.80, 14.75) = 36.88\angle 23.58$ Ans

$(36.88\angle 23.58)$ v

Good Write

• Pure Circuit

* Resistive Circuit



$V = V_m \sin \omega t$

$$\boxed{\omega = 2\pi f}$$

frequency for DC = 0 Hz

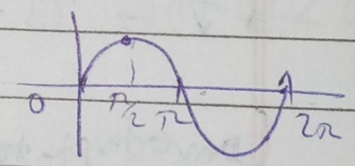
" " AC (in India) = 50 Hz

Acc to Ohm's law

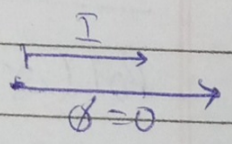
$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$$

For max ; $\sin \omega t = 1$

$$i = \frac{V_m}{R}$$



$$* \boxed{I_m = \frac{V_m}{R}}$$



(Phase changing at π & 2π.)

∴ Instantaneous Power ⇒ $P = V \times i$

$\Rightarrow P = V_m \sin \omega t \times I_m \sin \omega t$

$\Rightarrow P = V_m I_m \sin^2 \omega t$

$\Rightarrow V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$

Good Write

DATE: ___/___/___
PAGE: _____

$$\Rightarrow \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}}$$

$\therefore$ Power Consumed $\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} P \, d\omega t$

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi}\frac{V_m I_m \, d\omega t}{2} - \int_0^{2\pi}\frac{V_m I_m}{2}\cos 2\omega t \cdot d\omega t\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_m I_m}{2}\int_0^{2\pi} d\omega t - \frac{V_m I_m}{2}\int_0^{2\pi}\cos 2\omega t \, d\omega t\right]$$

$$\Rightarrow \frac{V_m I_m}{4\pi}\left[(\omega t)_0^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)_0^{2\pi}\right]$$

$$\Rightarrow \frac{V_m I_m}{4\pi}\left[(2\pi - 0) - 0\right] = \frac{V_m I_m}{2}$$

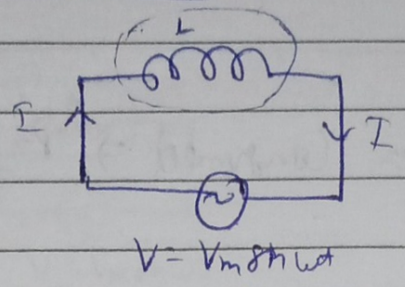
$$\Rightarrow \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow \boxed{P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}}$$

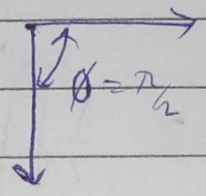
Good Write

DATE: ___/___/___
PAGE: _____

* Pure Inductive Circuit

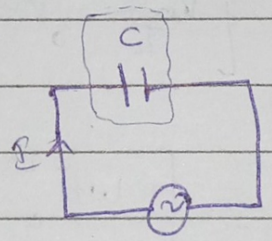


$i = I_m \sin(\omega t - \pi/2)$



Current lagging behind voltage by 90°

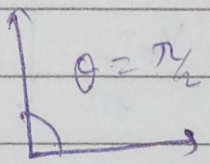
* Pure Capacitive Circuit



$I = I_m \sin(\omega t + \pi/2)$

Current leads voltage by $\pi/2$.

$V = V_m \sin \omega t$



* Pure L circuit

- Opposition to current (z):

$$I_m = \frac{V_m}{\omega L}$$

$$\therefore \frac{V_m}{I_m} = \omega L$$

Good Write

DATE: __/__/____
PAGE: _____

$$\Rightarrow \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \omega L \Rightarrow \frac{V}{I} = \omega L$$

$$\Rightarrow Z = \omega L$$

$$\Rightarrow \boxed{Z = 2\pi f L}$$

$$\boxed{X_L = 2\pi f L} = \text{Inductive Reactance}$$

* Pure C Circuit

- Opposition to current (Z)

$$\frac{I_m}{V_m} = \omega C$$

$$\frac{V_m}{I_m} = 1/\omega C$$

$$\frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = 1/\omega C$$

$$\frac{V}{I} = \frac{1}{\omega C}$$

$$\Rightarrow Z = \frac{1}{\omega C}$$

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

Capacitive Reactance

Q. A 318 uF capacitor is connected across a 230 V, 50Hz supply. Determine
i) Capacitive Reactance
ii) rms value of current
iii) Eqn for voltage & current

DATE: ___/___/___
PAGE: _____

Sol:- Given ① $C = 318 = 318 \times 10^{-6} F$
② $V = 230 V$

① Capacitive reactance

$$X_C = \frac{1}{2\pi f C}$$

$$\Rightarrow \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}}$$

$$\boxed{X_C = 10 \Omega}$$

② Rms value of current

$$I = \frac{V}{Z} = \frac{V}{X_C} = \frac{230}{10} = 23A$$

③ $V = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = \sqrt{2} V \Rightarrow \sqrt{2} \times 230 = 325.27 V$

$I = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2} I = \sqrt{2} \times 23 = 32.53 A$

④ Eqn for voltage

$v = V_m \sin \omega t = 325.27 \sin (2\pi \times 50) t$

$$\boxed{v = 325.27 \sin 314 t}$$

Eqn for current

$i = I_m \sin(\omega t + \pi/2)$

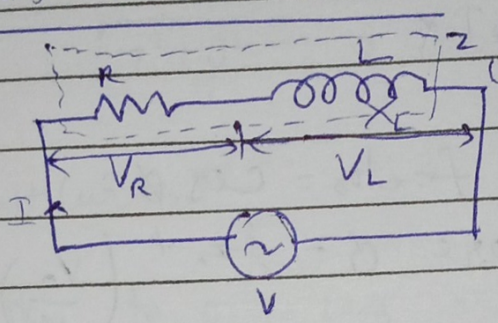
$$\boxed{i = 32.53 \sin (314 t + \pi/2)}$$

DATE: __/__/____
PAGE: _____

* AC Series Circuit

1. R-L circuit
2. R-C circuit
3. R-L-C circuit.

→ Series R-L circuit



Z (Impedance) (Poore box ka opposition to current)

1. $V_R = IR$
2. $V_L = IX_L$
3. $V = IZ$

4. Power factor = $\cos\phi$ → Phase difference

$\tan^{-1}\left(\frac{X_L}{R}\right) = \phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$ = Phase angle

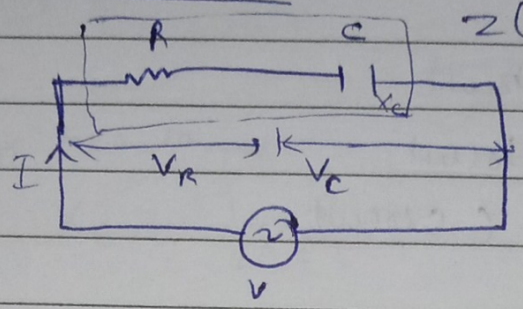
5. $Z = \sqrt{R^2 + X_L^2}$

6. $\bar{Z} = R + jX_L \Rightarrow Z\angle\phi$

$\bar{Z}$ → Means in form of complex no.

DATE: __/__/____
PAGE: _____

* Series R-C Circuit



① $V_R = IR$

② $V_C = IX_C$

③ $V = IZ$

④ Power factor $= \cos\phi$

where $\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \tan^{-1}\left(\frac{X_C}{R}\right)$

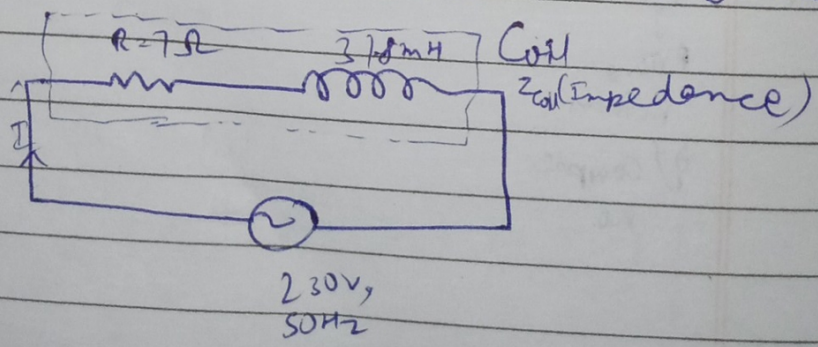
⑤ Impedence : $Z = \sqrt{R^2 + X_C^2}$

$$\bar{Z} = R - jX_C = Z\angle -\phi$$

Q. A coil have a resistance of 7Ω and inductance of 31.8 mH is connected to 230V, 50Hz supply. Calculate.

① Circuit current ② Phase angle
③ Power factor ④ Power consumed

Soln



DATE: __/__/____
PAGE: _____

Inductive Reactance $= X_L = 2\pi f L$

$= 2\pi (50)\ 31.8 \times 10^{-3}$

$= 10\Omega$

$\boxed{X_L = 10\Omega}$

Impedance of coil

$Z = \sqrt{R^2 + X_L^2}$

$\sqrt{7^2 + 10^2} \Rightarrow 12.21\Omega$

① Circuit current

Acc to Ohm's Law, $V = IZ$

$I = \frac{V}{Z_{coil}} = \frac{230}{12.21} = 18.84A$ Ans

② Phase angle $= \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{7}\right)$

$\phi = 55°$ Ans

③ Power factor $= \cos\phi$

$= \cos(55)$

$= 0.574$ (lagging)

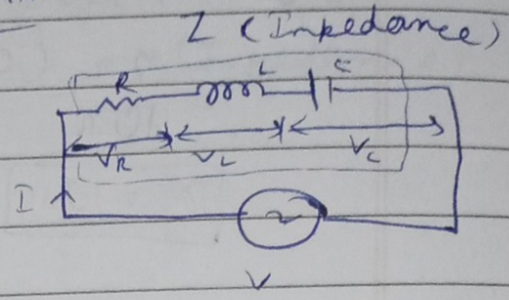
④ Power Consumed $\Rightarrow P = VI\cos\phi$

$= 230 \times 18.84 \times 0.574$

$= 2487.26\ W$ Ans

DATE: ___/___/___
PAGE: _____

• Series R-L-C Circuit



According to ohm's law

$V = IZ$

$V_R = IR$; $V_L = IX_L$

$V_C = IX_C$

ϕ = Phase angle $= \tan^{-1}\left(\frac{V_L - V_C}{R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

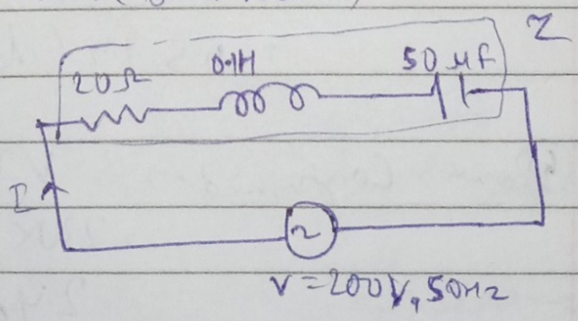
Circuit Impedance:

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = R + j(X_L - X_C)$ ---- Rectangular form

$Z = Z\angle\phi$ ---- Polar form

Q For given circuit determine; (1) Circuit Impedance (2) Circuit current (3) Power factor (4) Active Power (Total Power)



Sol: $V = 200\,V$, $f = 50\,Hz$, $R = 20\,\Omega$

$L = 0.1\,H$; $C = 50\,\mu F = 50 \times 10^{-6}\,F$

$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \, 50 \times 50 \times 10^{-6}} \Rightarrow 63.66\,\Omega$

Good Write

NOTES

Net Reactance $\Rightarrow X = X_C - X_L$ (Becoz Reactance Resistance ko torch tre hotahai)

$\Rightarrow X = 32.24\,\Omega$

~~Impedance $\Rightarrow Z = \sqrt{R}$~~

① Impedance $\Rightarrow Z = R - j(X_C - X_L)$

$Z = (20 - j\,32.24)$ --- Rectangular form.

$37.94 \angle -58.19$ ... Polar

$Z = 37.94$, $\phi = -58.19^\circ$

② Circuit Current

$$I = \frac{V}{Z} = \frac{200 \angle 0}{37.94 \angle -58.19}$$

$$= \left(\frac{200}{37.94}\right) \angle (0 + 58.19)$$

$$\boxed{I = 5.27 \angle 58.19}$$

③ Power factor $= \cos\phi = \cos(-58.19)^\circ$

$= 0.527$ (lagging)

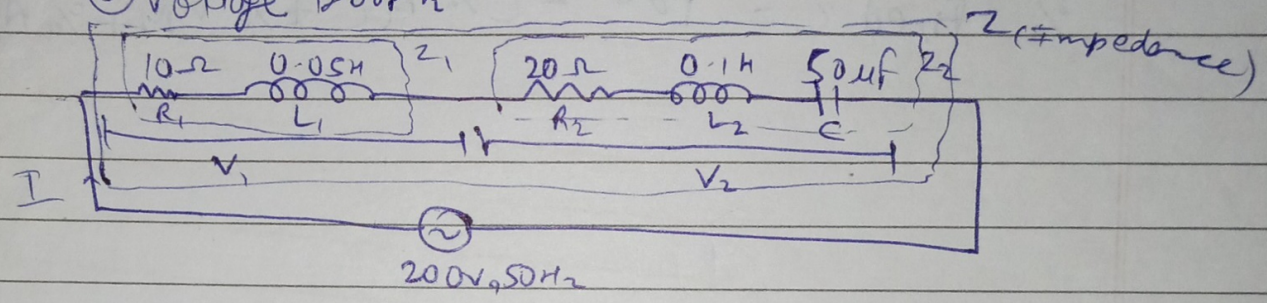
④ Active Power $\Rightarrow P = VI\cos\phi$

$\Rightarrow 200(5.27)(0.527)$

$\Rightarrow \boxed{P = 555.40\,W}$

Topic → Series Circuit

Q → for given circuit, Determine
① ~~Dro~~ Circuit current
② Voltage Drop V_1
③ voltage Drop V_2



Sol → Taking applied voltage as reference;

$\bar{V} = 200 \angle 0$; $R = 10 + 20 = 30\,\Omega$

$L_1 = 0.05\,H$

$X_{L_1} = 2\pi f L_1 = 2\pi 50 \times 0.05 = 15.71\,\Omega$

$L_2 = 0.1\,H$; $X_{L_2} = 2\pi(50)(0.1) = 31.42\,\Omega$

$X_L = X_{L_1} + X_{L_2} = 15.7 + 31.42 = 47.13\,\Omega$

$C = 50\,\mu f = 50 \times 10^{-6}\,F$

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi\, 50 \times 50 \times 10^{-6}}$$

$X_c = 63.66\,\Omega$

Net reactance $\Rightarrow X = X_C - X_L = 63.66 - 47.13$
$= 16.53\,\Omega$

Total Impedance of circuit;

$\bar{Z} = (30 - j(16.53))\,\Omega$ ___ Rectangular form

$\bar{Z} = (34.25 \angle -28.85)\,\Omega$ --- Polar form

Acc to ohm's Law

(1) Circuit current $= \bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{(200 \angle 0)}{(34.25 \angle -28.85)}$

$\Rightarrow \left(\dfrac{200}{34.25}\right) \angle (0 + 28.85)$

$\boxed{\bar{I} \Rightarrow (5.84 \angle 28.85)}$ Ans

(2) $Z_1 = R_1 + jX_{L_1} = 10 + j(15.71) = (18.62 \angle 57.52)\,\Omega$

& (3) $Z_2 = R_2 + j(X_{L_2} - X_C)$

$Z_2 = R_2 - j(X_C - X_L)$

$Z_2 = 20 - j(63.66 - 31.42) = (20 - j(32.24))\,\Omega$

$Z_2 = (37.94 \angle -58.19)$

For voltage drops

$V_1 = IZ_1 = (5.84 \angle 28.85)(18.62 \angle 57.52)$

$\Rightarrow (5.84 \times 18.62) \angle (28.85 + 57.52)$

$\Rightarrow (108.74) \angle (86.37)$ V Ans

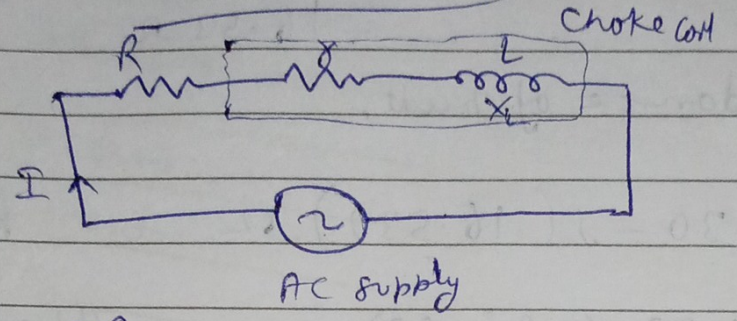
$V_2 = IZ_2 = ((5.84) \angle (28.85))(37.94 \angle -58.19)$

$\Rightarrow (221.57 \angle -29.34)$ V Ans

furana Tubelight mein got choke coil.

DATE: __/__/____
PAGE: _____

- Choke Coil Concept



Coil mtbl yehan inductor

① $V_R = IR$

② $V_{coil} = I\, Z_{coil}$

③ $Z_{coil} = \sqrt{r^2 + X_L^2}$

④ $X_L = 2\pi f L$

for whole circuit

⑤ $Z = \cancel{\sqrt{R^2 + (r + X_L)^2}} = \sqrt{(R + r)^2 + X_L^2}$

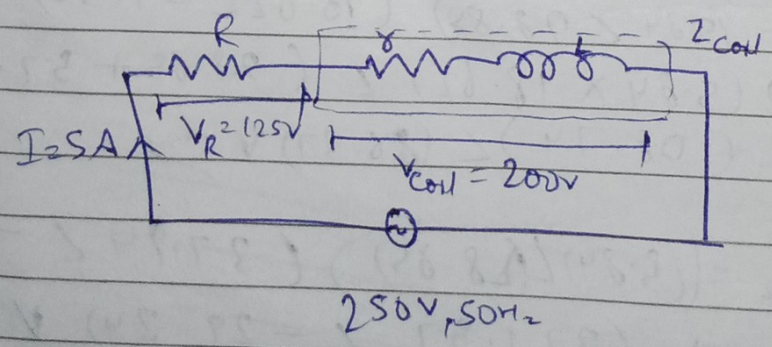
⑥ Power factor $= \cos\phi = \dfrac{r}{Z_{coil}}$

Q. A current of 5A flows through a non-inductive resistance in series with a choking coil supplied at 250V, 50 Hz. If voltage across resistance is 125V & across coil is 200 V, Calculate:

① Impedance, reactance & resistance of coil.

② Power absorbed by coil

③ Power factor.



DATE: __/__/____
PAGE: _____

Sol→ Acc to Ohm's law

$V_R = IR$

$R = \frac{V_R}{I} = \frac{125}{5} \Rightarrow 25\Omega$

Impedance of Coil;

$Z_{coil} = \frac{V_{coil}}{I} \Rightarrow \frac{200}{5} \Rightarrow 40\Omega$

Total Impedance;

$Z = \frac{V}{I} = \frac{250}{5} = 50\Omega$

$Z_{coil} = \sqrt{r^2 + X_L^2}$

$Z_{coil}^2 = r^2 + X_L^2$

$\boxed{1600 = r^2 + X_L^2}$ — (1)

Impedance for a circuit

$Z = \sqrt{(R+r)^2 + X_L^2}$

$Z^2 = (R+r)^2 + X_L^2$

$2500 = (25+r)^2 + X_L^2$ — (2)

$2500 = 625 + 50r + r^2 + X_L^2$

$2500 - 625 = 50r + 1600$

$\frac{2500 - 1600 - 625}{50} = r$

$\boxed{r = 5.5\Omega}$

$\boxed{X_L = 39.62\Omega}$

Power factor $= \frac{r}{Z_{coil}} = \frac{5.5}{40} = 0.137$ (lagging)

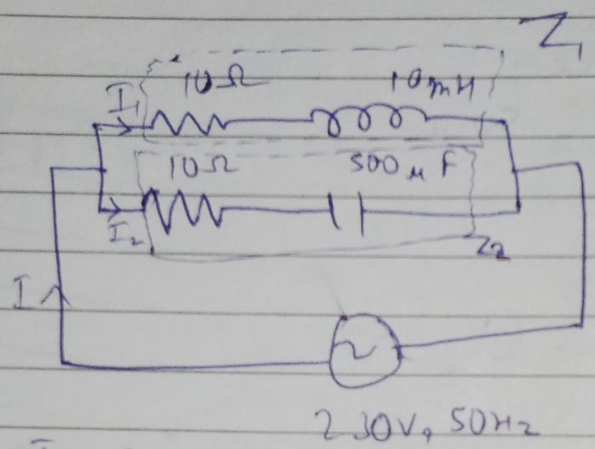
Power absorbed $= I^2 r = 5^2 \times 5.5 = 137.5$ w Ans

Good Write

DATE: __/__/____
PAGE: _____

AC Parallel Circuit

Q. From given circuit, Determine ① Total Impedence
② Branch current I_1 & I_2
③ Power factor & each branch power factor
④ Power consumed



Sol→ As; $\bar{V} = 230\angle 0$

$L = 10mH = 10\times10^{-3} H$

$X_L = 2\pi f L = 2\pi\, 50\times10\times10^{-3} \Rightarrow \boxed{3.14\Omega}$

$C = 500\mu F = 500\times10^{-6} F$

$$\boxed{X_C = \frac{1}{2\pi(50)(500\times10^{-6})} = 6.37\,\Omega}$$

$Z_1 = R_1 + jX_L = 10 + j3.14 = (10.48\angle 17.43)$

$Z_2 = R_2 - jX_C = 10 - j6.37 = (11.86\angle -32.5)$

$Z_1 \| Z_2$ $\quad \bar{Z}_{Total} = \dfrac{\bar{Z}_1 \cdot \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$

$$= \frac{(10.48\angle 17.43)(11.86\angle -32.5)}{(10 + j3.14) + (10 - j6.37)}$$

DATE: __/__/____
PAGE: _____

$$\Rightarrow \frac{(10.48 \times 11.86) \angle (17.43 - 32.5)}{(10+10) + j(3.14 - 6.37)}$$

$$\Rightarrow \frac{124.29 \angle (-15.07)}{(20 - j3.23)}$$

$$\Rightarrow \frac{(124.29 \angle -15.07)}{(20.26 \angle -9.17)}$$

$$\Rightarrow \frac{124.29}{20.26} \angle (-15.07 + 9.17)$$

$$\boxed{Z_T \Rightarrow (6.13) \angle (-5.9) \ \Omega}$$

Total Current;

$$I_{Total} = \frac{V}{Z_T} = \frac{230 \angle 0}{(6.13) \angle (-5.9)}$$

$$\Rightarrow \frac{230}{6.13} \angle (0 + 5.9)$$

$$I_T = \boxed{37.52 \angle 5.9} \text{ Ans}$$

Branch Current; $I_1 = \frac{V}{Z_1} = \frac{230 \angle 0}{10.48 \angle 17.43}$

$$\Rightarrow \left(\frac{230}{10.48}\right) \angle (0 - 17.43)$$

$$\Rightarrow \boxed{21.95 \angle (-17.43)} \text{ Ans}$$

$$I_2 = \frac{V}{Z_2} = \frac{230 \angle 0}{11.86 \angle -32.5} \Rightarrow \frac{230}{11.86} \angle 32.5$$

$$\Rightarrow \boxed{I_2 = 19.39 \angle 32.5} \text{ A}$$

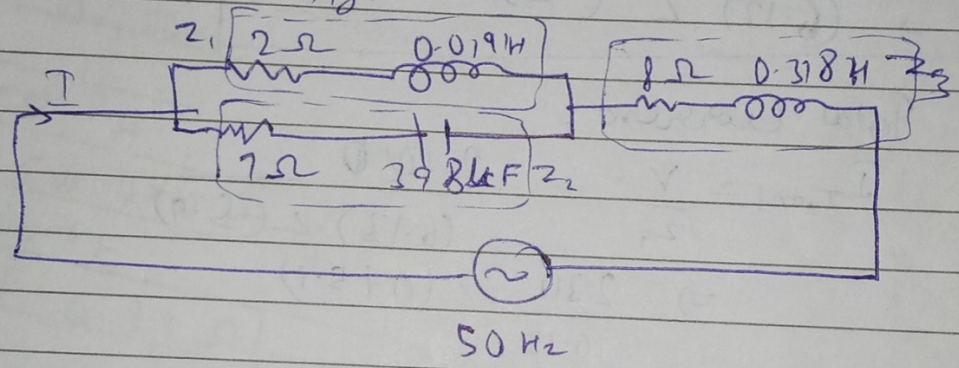
$Pf_1 = \cos\phi_1 = \cos(17.43)$
$= 0.954$ (lag)

$Pf_2 = \cos\phi_2 = \cos(32.5) = 0.849$ (lead)

Total $Pf = \cos\phi_T = \cos(5.9) = 0.995$ (lead) Ans

Power consumed $= P_1 = I_1^2 R = (21.95)^2 \times 10 = 4.818$ kW Ans
$P_2 = I_2^2 R = (19.39)^2 \times 10 = 3.76$ kW Ans

Q. Determine the equivalent impedance of circuit shown in fig.



Sol→ $L_1 = 0.0191$ H ; $X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.0191$
$= 6\ \Omega$

$C = 398 \times 10^{-6}$ F

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1 \times 10^6}{2\pi 50 \times 396} = 80\ \Omega$

$L_2 = 0.318$ H
$X_{L_2} = 2\pi f L_2 = 10\ \Omega$

$Z_1 = R_1 + jX_{L_1} = (2 + j6)\ \Omega = (6.32 \angle 71.57)\ \Omega$

$Z_2 = R_2 - jX_C = (7 - j8)\ \Omega = (10.63 \angle -48.81)\ \Omega$

$Z_3 = R_3 + jX_{L_2} = (8 + j10)\ \Omega = (12.81 \angle 51.34)\ \Omega$

Good Write

DATE: __/__/____
PAGE: _____

Total Impedance

$$Z_{eq} = (Z_1 \| Z_2) + Z_3$$

$$\left(\frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3\right)$$

Ans $\Rightarrow \dfrac{(6.32 \angle 71.57)(10.63 \angle -48.81)}{(2+j6)+(7-j8)} + 8 + j10$

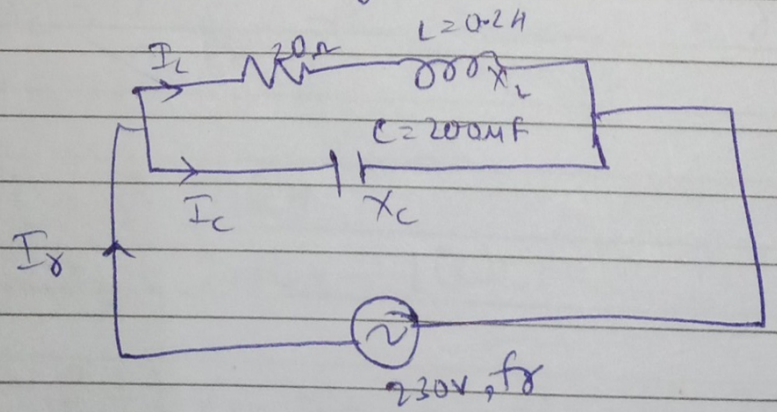
$$\left(\frac{(6.32 \times 10.63) \angle (71.57 - 48.81)}{9 - 2j}\right) + (8 + j10)$$

DATE: / /
PAGE:

• Topic → Resonance Circuit

Parameters	Series Circuit	Parallel Circuit
① Impedance	Min. ($Z = R$)	Max ($Z = L/CR$)
② Current	Max ($I = V/R$)	Min ($I = \frac{V}{Z}$)
Imp. ③ Resonating frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
④ Q (quality) factor	$= \frac{V_L \text{ or } V_C}{V}$	$= \frac{I_L \text{ or } I_C}{I}$

Q. An inductive coil of resistance 20Ω & Inductance 0.2H is connected in parallel with 200μf capacitor with variable freq. 230V supply. Find resonant freq. at which total current taken from supply is in phase with supply voltage. Also find value of current.



Resonant frequency;

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Good Write

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 200 \times 10^{-6}} - \frac{400}{(0.2)^2}}$$

$$= \frac{1}{2\pi}\sqrt{25000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{15000} \Rightarrow 19.49 \text{ Hz Ans}$$

Impedance $Z_r = \frac{L}{CR} = \frac{0.2}{200 \times 10^{-6} \times 20} = 50\,\Omega$

Current at resonance

$$I_r = \frac{V}{Z_r} = \frac{230}{50} = 4.6\text{ A Ans}$$

Resonance frequency
that frequency at which $X_L = X_C$.
Another condition is resonance.

Q factor = $\frac{V \text{ developed across } L \text{ or } C}{\text{Applied voltage}}$

Power factor
$\cos\phi = \frac{Z}{R}$
$\cos\phi_0 = \frac{Z_0}{R}$ $\cos\phi = 1$

$$= \frac{I_r X_L}{I_r R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{\text{Resonating frequency}}{\text{Bandwidth}}$$

→ used in design CKT
→ doesn't depend on frequency