Unit - IV

[Introduction to LASER Physics]

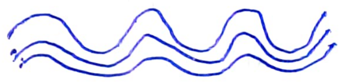
LASER :→ The full form of LASER is Light Amplification by Stimulated Emission of Radiation. In 1960, Maiman built the first laser using ruby as the active medium. Since then Laser has opened up new fields of research in optics. Laser is based on the principle of stimulated emission of radiation.

Characteristics of Laser Beam :→ Laser light has the following characteristics -

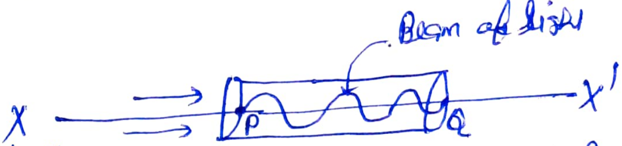
(i) highly coherent.
(ii) High directionality
(iii) Extraordinary Monochromaticity
(iv) High Intensity

Coherence :→ It describes the degree of correlation between the phases of waves at different points in a beam light.

It is of two types-

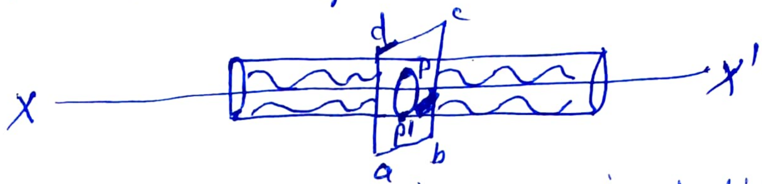


(i) **Temporal coherence :-** If the phase difference of waves crossing the two points lying along the direction of propagation of the beam is time dependent. then the beam is said to be in the temporal coherence. It is also known as Longitudinal coherence.



If the phase difference between the waves at P and Q points are constant at any instant, then beam is said to be in temporal coherence.

(ii) Spatial coherence:- If the phase difference of waves crossing the two points lying perpendicular to the direction of propagation of light beam is not time dependent but dependent on space or position, then the light beam is said to be in spatial coherence.



The beam is said to be in spatial coherence if the phase difference of the waves crossing P and P' at any instant is constant.

Einstein A and Einstein B Coefficient: → (Interaction of Radiation with Matter)

Every material is composed of atoms or molecules. they can remain in allowed energy states. An atom can move from one energy state to another ~~state~~ When it receives or releases an amount of energy equal to the energy difference between those two states.

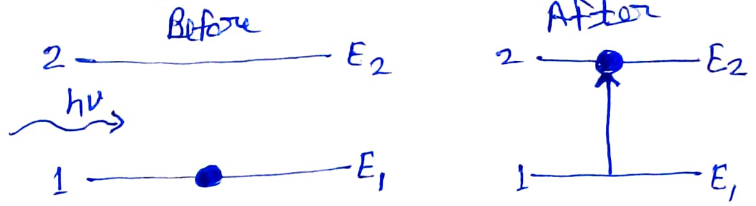
Let us consider two energy states E_1 and E_2 of an atom, $E_1 \rightarrow$ lower energy state, $E_2 \rightarrow$ excited state.

When a monochromatic radiation consisting of a stream of photons having frequency ν is incident on the ~~medium~~ material, then on the basis of interaction of radiation with atoms of material, there may occur three processes -

(i) Stimulated Absorption (ii) Spontaneous Emission

(iii) Stimulate Emission.

(i) Stimulated (Induced) Absorption or Absorption of Radiation →



When a photon having energy $h\nu$ is incident or interacts with atom in state 1, then atom is excited the excited state by absorbing the energy of photon i.e $h\nu$.

$$h\nu = E_2 - E_1 \Rightarrow \nu = \frac{(E_2 - E_1)}{h}$$

The Probability rate of transition from state 1 to 2 depends on the properties of state 1 and 2 and is proportional to energy density $u(\nu)$ of the radiation (photons) incident on atom.

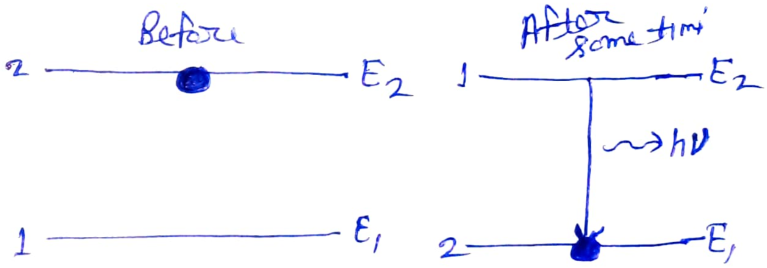
$$\boxed{P_{12} = B_{12}\, u(\nu)}$$

$u(\nu)$ → energy density of incident radiation.

B_{12} → Einstein coefficient of stimulated Absorption

P_{12} → Probability rate of transition of an atom from state 1 to 2

(ii) Spontaneous Emission →



When atom is in excited state, then after 10^{-8} sec, it comes to state 1 automatically with emitting a photon having energy $h\nu$, it is known as spontaneous emission.

$$\nu = \frac{E_2 - E_1}{h}$$

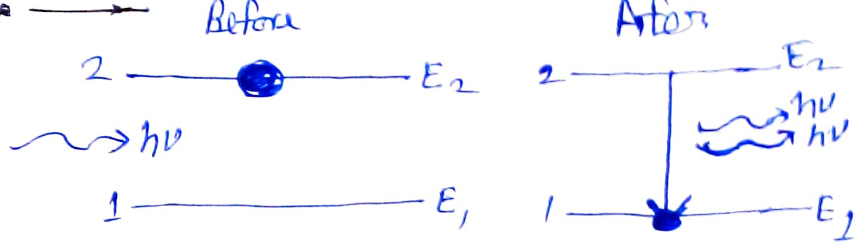
ν → freq of emitting photon.

The Probability rate of transition from state 2 to 1 may be written as

$$\boxed{(P_{21})_{\text{spontaneous}} = A_{21}}$$

A_{21} → Einstein Coefficient of spontaneous emission

(iii) Stimulated (or Induced) Emission →



When a photon having energy $h\nu$ is incident on an atom lying in excited state, then atom jumps to lower energy state with emitting an additional photon having same frequency ν. Hence two photons, one original (incident photon) and other emitted, move together. The freq, direction, phase are same of these photons, so Laser operation is based on stimulated emission of radiation.

The Probability of stimulated transition ~~from~~ from state 2 to 1 may be written as –

$$(P_{21})_{\text{stimulated}} = B_{21}\, u(\nu)$$

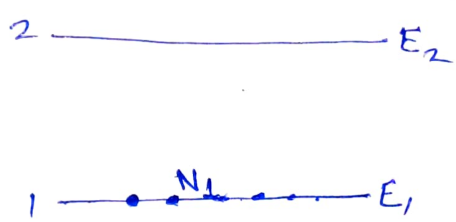
B_{21} ↳ Einstein's coefficient of stimulated emission.

The total Probability of emission transition from state $2 \to 1$ is sum of spontaneous and stimulated emission

$$P_{21} = (P_{21})_{\text{spontaneous}} + (P_{21})_{\text{stimulated}}$$

$$\boxed{P_{21} = A_{21} + B_{21}\, u(\nu)}$$

Relation Between Einstein's A and B coefficients or Relation Between Spontaneous and Stimulated Probabilities →



Let N_1 is the number of atoms in state 1, then the number of atoms in state 1 that absorbs photons and rise to state 2 is

$$N_1 P_{12} = N_1 B_{12}\, u(\nu) \quad \text{—①}$$

Similarly, Let N_2 is the number of atoms in state 2 that drop 2 to 1, due to emission (spontaneous + stimulated) is

$$N_2 P_{21} = N_2 [A_{21} + B_{21}\, u(\nu) \quad \text{—②}$$

In equilibrium, Absorption = emission

$$N_1 P_{12} = \cancel{N_1 B_{12} u(\nu)} \; N_2 P_{21}$$

$$N_1 B_{12} u(\nu) = N_2 [A_{21} + u(\nu) B_{21}]$$

$$N_1 B_{12} u(\nu) = N_2 A_{21} + N_2 B_{21} u(\nu)$$

$$N_1 B_{12} u(\nu) - N_2 B_{21} u(\nu) = N_2 A_{21}$$

$$u(\nu) [N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$u(\nu) = \frac{\cancel{N_2} A_{21}}{\cancel{N_2} B_{21} \left[\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right]} = \frac{A_{21}}{B_{21}} \frac{1}{\left[\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right]}$$

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{[e^{h\nu/kT} - 1]}$$

$$B_{12} = B_{21}$$

$$\frac{N_1}{N_2} = e^{h\nu/kT}$$

Comparing with Planck's Radiation Law

$$N_1 = N_0 e^{-E_1/kT}$$

$$N_2 = N_0 e^{-E_2/kT}$$

N_0 = total no. of atoms in ground state

$$\frac{N_1}{N_2} = \frac{\cancel{N_0}}{\cancel{N_0}} \frac{e^{-E_1/kT}}{e^{-E_2/kT}}$$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT}$$

$$\frac{N_1}{N_2} = e^{h\nu/kT}$$

A/c to Einstein $B_{12} = B_{21}$

$$u(\nu) = \frac{8\pi h\nu^3}{c^3 [e^{h\nu/kT} - 1]}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

$$\boxed{\frac{A_{21}}{B_{21}} \propto \nu^3}$$

It means the ratio between the spontaneous emission and stimulated emission is directly proportional to the ν^3.

Basic Principle and operation of LASER:-

(i) Population Inversion :- This is the necessary condition for Laser action. Under ordinary conditions of thermal equilibrium, the number of atoms in higher energy state 2 is ~~considerably~~ considerably smaller than the number in lower energy state ($N_1 > N_2$). In this ~~case~~ condition there is very little stimulated emission compared with absorption. An incident photon is more likely to be absorbed than to cause emission.

If, however, by some means a large number of atoms are available in higher energy state, stimulated emission is promoted.

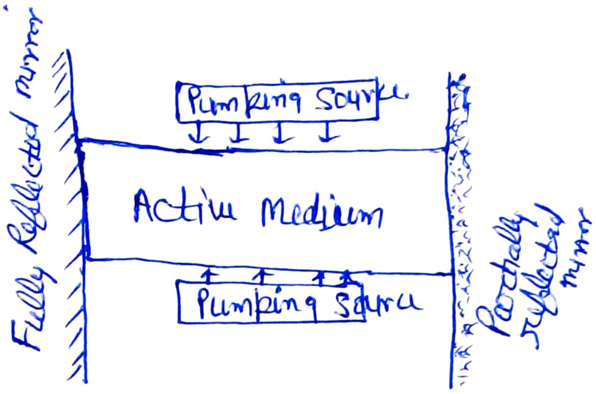
The Situation, in which the number of atoms ~~are~~ ~~more than~~ in higher energy states are ~~more~~ more than that of Lower energy State ($N_2 > N_1$), is known as Population Inversion.

(ii) Pumping :- The process or method used to achieve population inversion, is known as Pumping of atoms. There are various types of pumping process like-

(i) optical Pumping - It is used in Ruby Laser.
(ii) Electric Discharge Method - It is used in He-Ne Laser.
(iii) Inelastic Atom-Atom collision
(iv) Direct conversion
(v) chemical conversion.

(iii) Main Components of Laser :→

Laser consists of three main components.–



(i) Pumping Source or Energy Source :→ It is used to achieve population inversion. In Ruby Laser, Optical Pumping method is used to achieve population inversion. In He-Ne Laser, electric discharge method is used to achieve population inversion.

(ii) The Active medium → It may be solid, liquid, gases. These are the materials which are used in Laser like– He + Ne is used in He-Ne Laser, $Al_2O_3 + Cr_2O_3$ is used in Ruby Laser.

(iii) Optical Resonator :→ It consists of two ~~set~~ mirrors facing each other. One is fully reflecting mirror and other is partially reflecting mirrors. The active ~~medium~~ medium or material is filled in cavity. This cavity is used to make stimulated emission possible. This naturally increases the intensity of Laser beam.

. Ruby Laser (A Pulsed Laser)

The Ruby Laser : This is the first laser developed in 1960, and is a solid-state laser. It consists of a pink ruby cylindrical rod whose ends are optically flat and parallel (Fig. 5). One end is fully silvered and the other is only partially silvered. Upon the rod is wound a coiled flash lamp filled with xenon gas.

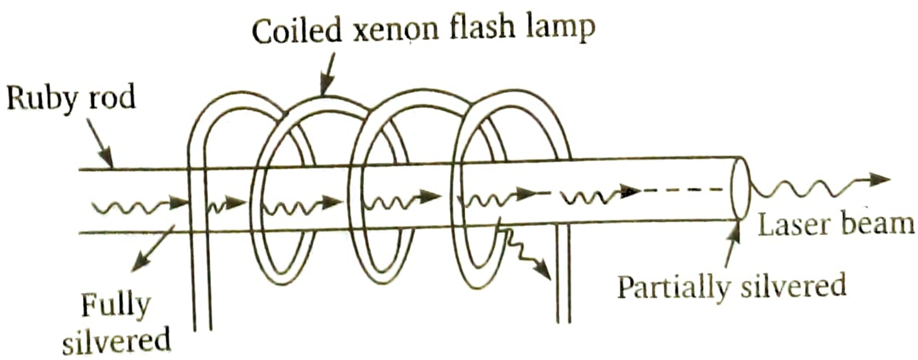


Fig. 5

Working : The ruby rod is a crystal of aluminium oxide (Al_2O_3) doped with 0.05% chromium oxide (Cr_2O_3), so that some of the Al^{+++} ions are replaced by Cr^{+++} ions. These "impurity" chromium ions give pink colour to the ruby and give rise to the laser action.

In Fig. 6 is shown a simplified version of the energy-level diagram of chromium ion. It consists of an upper short-lived energy level (rather energy band) E_3 above its ground-state energy level E_1, the energy difference $E_3 - E_1$ corresponding to a wavelength of about 5500Å. There is an intermediate excited-state level E_2 which is metastable* having a life-time of 3×10^{-3} sec (about 10^5 times greater than the life-time of E_3 which is $=10^{-8}$ sec).

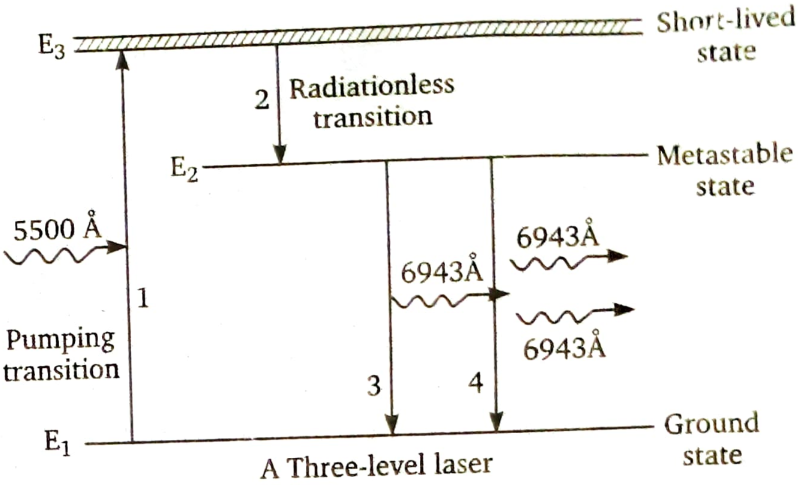


Fig. 6

Normally, most of the chromium ions are in the ground state E_1. When a flash of light (which lasts only for about a milli-second) falls upon the ruby rod, the 5500–Å radiation photons are absorbed by the chromium ions which are "pumped" (raised) to the excited state E_3. The transition 1 is the (optical) pumping transition. The excited ions give up, by collision, part of their energy to the crystal lattice and decay to the "metastable" state E_2. The corresponding transition 2 is thus a radiationless transition. Since the state E_2 has a much longer life-time, the number of ions in this state goes on increasing while, due to pumping, the number in the ground state E_1 goes on decreasing. Thus, population inversion is established between the metastable (excited state E_2) and the ground state E_1.

When an (excited) ion passes *spontaneously* from the metastable state to the ground state (transition 3), it emits a photon of wavelength 6943Å. This photon travels through the ruby rod and, if it is moving parallel to the axis of the crystal, is reflected back and forth by the silvered ends until it stimulates an excited ion and causes it to emit a fresh photon *in phase with the stimulating photon*. This "stimulated" transition 4 is the laser transition. (The photons emitted spontaneously which do not move axially escape through the sides of the crystal). The process is repeated again and again because the photons repeatedly move along the crystal, being reflected from its ends. The photons thus multiply. When the photon-beam becomes sufficiently intense, part of it emerges through the partially-silvered end of the crystal.

*A metastable state is relatively long-lived state whose life-time may be 10^{-3} sec, or more, instead of the usual life-time of 10^{-8} sec.

There is a drawback in the three-level laser such as ruby. The laser requires high pumping power because the laser transition terminates at the ground state and more than one-half of the ground-state atoms must be pumped up to the higher state to achieve population inversion. Moreover, ions which happen to be in their ground state absorbs the 6943-Å photons from the beam as it builds up.

The ruby laser is a "pulsed" laser. The active medium (Cr^{+++} ions) is excited in pulses, and it emits laser light in pulses. While the Xenon pulse is of several millisecond duration; the laser pulse is much shorter, less than a millisecond duration. It means enhanced instantaneous power.

. Helium-Neon Laser

Ans. Helium-Neon Laser : It is a four-level laser in which the population inversion is achieved by electric discharge. A mixture of about 7 parts of helium and 1 part of neon is contained in a glass tube at a pressure of about 1 mm of mercury (Fig. 7). At both ends of the tube are fitted optically plane and parallel mirrors, one of them being only partially silvered. The spacing of the mirrors is equal to an integral number of half-wavelengths of the laser light. An electric discharge is produced in the gas-mixture by electrodes connected to a high-frequency electric source.

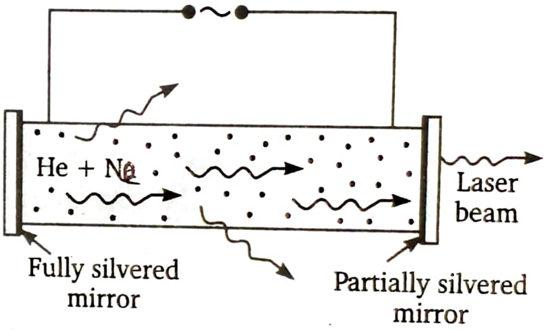
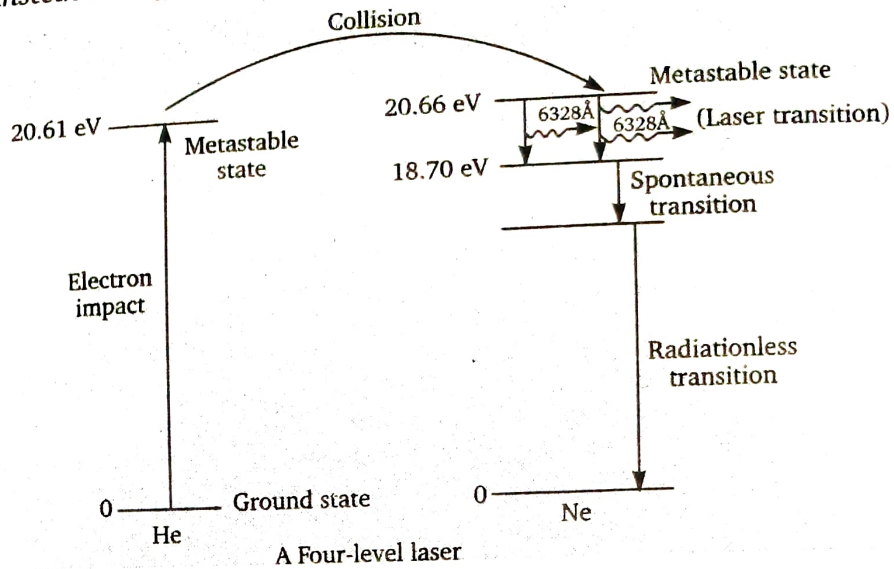


Fig. 7

The electrons from the discharge collide with and "pump" (excite) the He and Ne atoms to *metastable* states 20.61 and 20.66eV respectively above their ground states (Fig. 8). Some of the excited He atoms transfer their energy to ground-state Ne atoms by collisions, with the 0.05 eV of additional energy being provided by the kinetic energy of atoms*. Thus, He atoms help in achieving a population inversion in the Ne atoms.

When an excited Ne atom passes *spontaneously* from the metastable state at 20.66 eV to state at 18.70 eV, it emits a 6328–Å photon. This photon travels through the gas-mixture, and if it is moving parallel to the axis of the tube, is reflected back and forth by the mirror-ends until it stimulates an excited Ne atom and causes it to emit a fresh 6328–Å photon in phase with the stimulating

*The advantage of this collision process is that the lighter He atoms can be easily pumped up to their excited states; the much heavier Ne atoms could not be raised efficiently without them.



A Four-level laser

Fig. 8

photon. This stimulated transition from 20.66-eV level to 18.70-eV level is the laser transition. This process is continued and a beam of coherent radiation builds up in the tube. When this beam becomes sufficiently intense, a portion of it escapes through the partially-silvered end.

From the 18.70-eV level the Ne atom passes down spontaneously to a lower metastable state emitting incoherent light, and finally to the ground state through collision with the tube walls. The final transition is thus radiationless.

Obviously, the Ne atom in its ground state cannot absorb the 6328-Å photons from the laser beam, as happens in the three-level ruby laser. Also, because the electron impacts that excite the He and Ne atoms occur all the time, unlike the pulsed excitation from the xenon flash lamp in the ruby laser, the He-Ne laser operates continuously.

Further, since the laser transition does not terminate at the ground state, the power needed for excitation is less than that in a three-level laser.

Properties of a Laser Beam

Properties of a Laser Beam : The laser beam has certain characteristic properties which are not present in beams derived from other light sources :

(i) The laser beam is *completely* spatially *coherent*, with the waves all exactly in phase with one another. An interference pattern can be obtained not merely by placing two slits in a laser beam but also by using beams from separate lasers.

(ii) The laser light is almost perfectly monochromatic, i.e. highly temporally coherent.

(iii) The laser rays are almost *perfectly parallel*. Hence a laser beam is very narrow and can travel to long distances without spreading. It can be brought to an extremely sharp focus.

(iv) The laser beam is *extremely intense*. It can vaporise even the hardest metal. Because of its high energy density and directional property, a laser beam can produce temperatures of the order of $10^4 C$ at a focussed point.

Important Applications of Laser

Applications of Lasers : The laser beam being narrow, intense, parallel, monochromatic and highly coherent is finding increasing applications in various fields :

(i) In the technical and industrial field, the laser beam is used for cutting fabric for clothing on one hand and steel sheets on the other. It can drill extremely fine holes in paper clips, single human hair and hard materials including teeth and diamond. Extremely thin wires used in cables are drawn through the diamond hole. Metallic rods can be melted and joined by means of a laser beam (laser welding). The surfaces of engine crank-shafts and the cylinder walls are hardened through heat-treatment by laser. The laser beam is used to vaporise unwanted material during the manufacture of electronic circuits on semiconductor chips.

(ii) In the medical field, the laser beam is used in delicate surgery like cornea grafting. Using laser beam, the surgical operation is completed in a much shorter time. It is also used in the treatment of kidney stone, cancer, tumour and in cutting and sealing the small blood vessels in brain operation.

(iii) During war-time, lasers are used to detect and destroy enemy missiles. Now, laser-rifles, laser-pistols and laser bombs are also being made which can be aimed at the enemy in the night. In space, laser has been used to control rockets and satellites and in directional radio-communication like fiber-optic telephony.

(iv) Laser is very useful in science and research. It has been used to perform Michelson-Morley experiment which is the building stone of the Einstein's theory of relativity. It can be used to determine the temperature of plasma and the density of electron. Laser-torch is used to see objects at long distances.

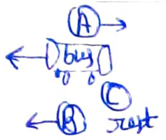
(v) Laser is used in holography and nonlinear optics.

(vi) Since laser rays are very much parallel, so they are used for communications and measuring long distances. The distance between earth and moon has been measured by laser rays to an accuracy of 15 cm.

(vii) Laser rays have proved to be useful in detecting nuclear explosions and earthquakes, in vaporising solid fuel of rockets, in the study of the surface of distant planets and satellites.

(viii) Laser beams have also been used in the "inertial confinement" of plasma.

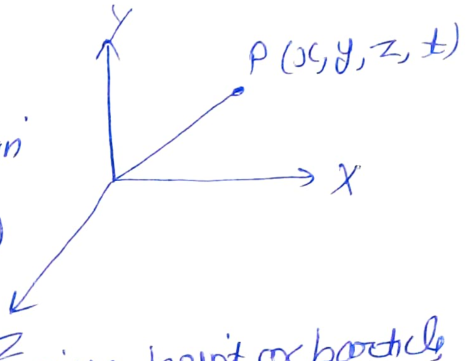
*Exp-



Unit-IV
Special Theory of Relativity

Frame of Reference:- Frame of reference is a coordinate system relative to which the position and motion of a body may be described.

*Exp- The Simple frame of reference is the Cartesian coordinate system in which every point of space may be described by 3 numbers (x, y, z) or three coordinates of that point.



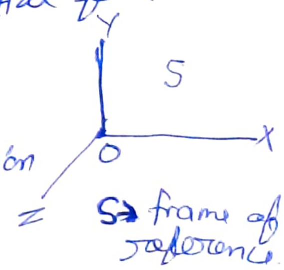
If $\vec{r}$ is the position vector of Point P relative to origin O of a cartesian frame of reference, then $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$

In order to specify the given point or particle exact time of knowing the position of the given point or particle we must assign the more coordinate t, the time coordinate. Such a reference frame is called space-time reference frame. It consists of 4 coordinates (x, y, z, t)

Frame of reference are of two types.

(i) **Inertial Frame of Reference** ⇒ A frame of reference in which Newton's Laws of motion holds good is known as inertial frame of reference.

It means a frame of refrence which is either at rest or moving with uniform velocity is inertial frame of reference.



From Newton's 2^n law of motion

$F = ma$

$0 = ma$ if $F = 0$

$a = 0$

$\boxed{v = \text{Constant}}$

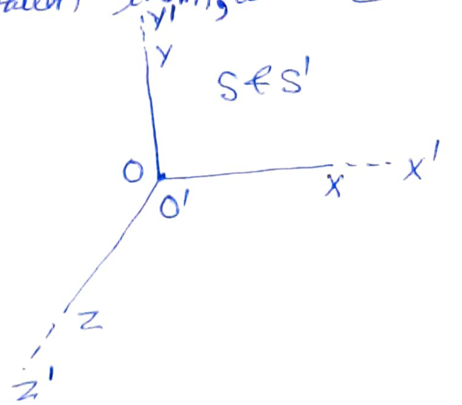
Thus inertial frame of reference is also known as unaccelerated or non-accelerated frame of reference.

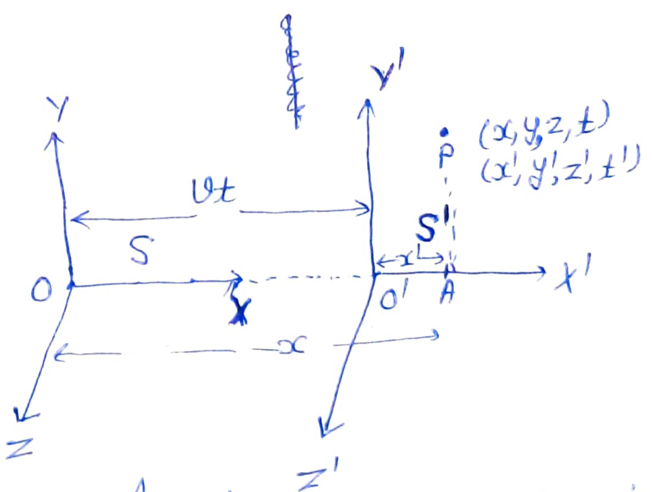
(ii) Non-Inertial frame of reference :→ A frame of reference in which Newton's laws of motion are not valid, is known as Non-inertial frame of reference.

A non-inertial frame of reference is an accelerated frame of reference.

For Exp:→ When a traveller travels in a bus and bus driver applies force on brake of bus then bus is deaccelerated, but traveller also move forward but no force is applied here on traveller, so without applying force on traveller, traveller move forward, so here Newton' Law is not valid in this frame of reference (bus).

Galilean Transformation:→ The set of equations which relate space-time coordinates of an event w.r.t two inertial frames of references having relative motion between them, are called Galilean transformation equations.





Let us consider 2 inertial frame of references S & S' having cartesian coordinates axes as x, y, z & x', y', z' and origin O and O' respectively. At $t=0$ both the frames are at rest so that their origin O and O' coincides with each other.

Now let frame S' start moving with constant velocity v along +ve direction of X-axis. Let an event occurs at point P at any instant of time. The coordinates of point P wrt to observer O in frame S are (x, y, z, t). The coordinates of point P wrt observer O' in frame S' are (x', y', z', t')

From figure, it is clear that

$$OA = OO' + O'A$$

$$x = vt + x'$$

$$x' = x - vt \quad \text{--- (1)}$$

S' is moving in x direction only, so

$$y' = y \quad \text{--- (2)}$$

$$z' = z \quad \text{--- (3)}$$

its time is considered to be absolute in nature i.e. time becomes same in all inertial frames of references. so $t' = t$ --- (4)

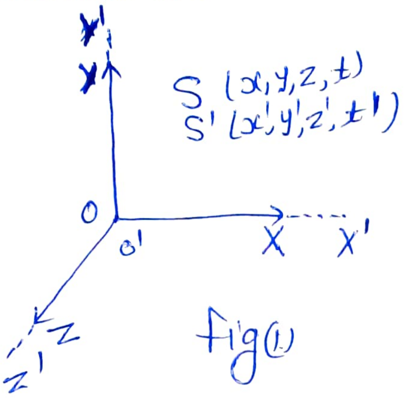
$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$ — Galilean transformation equations

Postulates for Special Theory of Relativity → In 1905, Einstein published his special theory of relativity which is based upon the two postulates.—

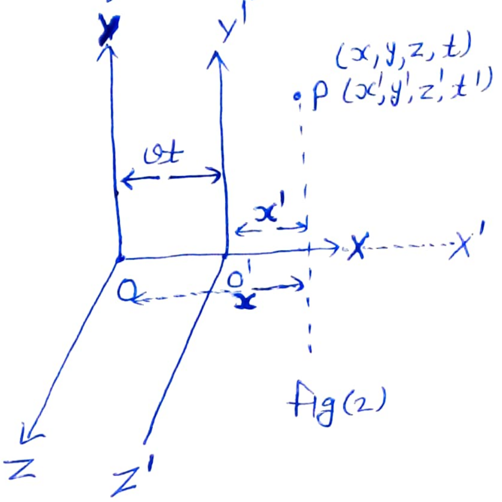
Postulate I ⇒ The laws of physics have the same form in all inertial frames of references moving with constant velocity relative to one another. This is known as principle of relativity.

Postulate II ⇒ The speed of light in free space is the same in all inertial frames of references. This is the principle of constancy of speed of light. This postulate comes from the result of Michelson-Morley Experiment.

Lorentz Transformation Equations → The set of equations which relate the space-time coordinates of two frame of reference having relative motion between them.



fig(1)



fig(2)

Let S and S' are two inertial frames of reference at $t=0$, they coincide with each other (fig 1). The origin of S and S' are O and O', the coordinate axis of S and S' are (x, y, z, t) and (x', y', z', t') respectively. When S' start to move with constant velocity v relative to S and v is comparable with c, then the relation b/w x & x', y & y' & z & z' may be written as -

$$x' = \gamma(x - vt) \quad \text{--- (1)},$$

$$y' = y$$
$$z' = z$$
because S' is moving in +X direction only not in Y and Z directions.

eq (1) may be written as by replacing $v \to -v$

$$x = \gamma(x' + vt') \quad \text{--- (2)}$$

∵ Acc to I^st postulate, The equation of Physics or Laws of Physics must have the same form in both S and S'.

putting the value of x' from equation (1) into eq (2) -

$$x = \gamma\left[\{\gamma(x - vt) + vt'\}\right]$$

$$\frac{x}{\gamma} = \gamma x - \gamma vt + vt'$$

$$vt' = \frac{x}{\gamma} - \gamma x + \gamma vt$$

$$t' = \frac{x}{\gamma v} - \frac{\gamma x}{v} + \gamma t$$

$$t' = \gamma t - \frac{\gamma x}{v}\left(1 - \frac{1}{\gamma^2}\right) \quad \text{--- (3)}$$

Similarly, we can write for t by replacing $v \to -v$

$$t = \gamma t' + \frac{\gamma}{v}x'\left(1 - \frac{1}{\gamma^2}\right) \quad \text{--- (4)}$$

γ can be evaluated from the II postulate.

Suppose, a light signal is given at time $t = 0$, $t' = 0$, that is when O and O' coincides or when S and S' coincides. The signal travels with a speed c which is same for both the frames (II Postulate). Its position as seen from S and S' after some time is given by,

$$x = ct$$
$$x' = ct'$$

putting $x = ct$ and $x' = ct'$ in eq (1), we get

$$ct' = \gamma(ct - vt)$$
$$ct' = \gamma t(c - v) \quad \text{—(5)}$$

putting $x = ct$ and $x' = ct'$ in eq (2), we get

$$ct = \gamma t'(c + v) \quad \text{—(6)}$$

Solving eq (5) & (6), we get.

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

putting the value of γ in eq (1) and (3), we get

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \text{—(A)}$$

$$t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \text{—(B)}$$

These A, B, C, D are Lorentz Transformation equations

S' is not moving in y and z-direction, so

$$y' = y \quad \text{— (C)}$$

$$z' = z \quad \text{— D}$$

other form of Lorentz transformation equations can be written by replacing $v \to -v$ in eq A, B, C, D - [Inverse Lorentz transformation equation]

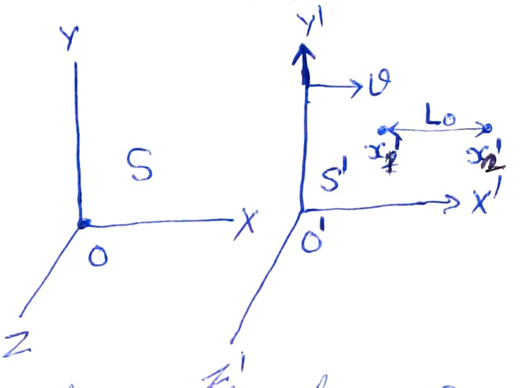
$$\boxed{x = \frac{x' + vt'}{\sqrt{(1 - v^2/c^2)}}, \; y' = y, \; z' = z, \; t = \frac{t' + x'v/c^2}{\sqrt{(1 - v^2/c^2)}}}$$

Length Contraction →

Relative motion affects measurement of length. A body moving with velocity v relative to an observer appears to ~~be~~ the observer to be contracted in length in the direction of motion by a factor $\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, whereas its dimensions perpendicular to the direction of motion are unaffected.

$$\boxed{L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

Derivation →



Let us consider two frames of reference S and S'. S is in rest position while S' is moving with velocity v. Let a rod having L_0 is placed in S'. Suppose x_1' and x_2' be the coordinates of ~~a~~ the ends ~~of~~. The length of the rod as observed from S' is,

$$L_0 = x_2' - x_1' \quad -(1)$$

Now suppose another observer measures the length of the rod from a stationary frame S, relative to which the rod (or S') is moving with velocity v. If the coordinates of the ends of the rod as x_1 and x_2 at time t, the rod appears to him having length,

$$L = x_2 - x_1 \quad -(2)$$

From the Lorentz transformation, $x_2' = \frac{x_2 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, $x_1' = \frac{x_1 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$

putting these values of x_2' and x_1' in eq (1),

$$L_0 = \frac{x_2 - vt}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} - \frac{(x_1 - vt)}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{x_2 - \cancel{vt} - x_1 + \cancel{vt}}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{x_2 - x_1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$L_0 = \frac{x_2 - x_1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \quad \because L = (x_2 - x_1)$$

$$L_0 = \frac{L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$\boxed{L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

If $v = c$

$L = 0$, Hence if rod moves with speed c (speed of light) then it appears like a point to an stationary observer.

L_0 is known as Proper Length.

→ The Length L_0 measured in the frame of reference in which the rod is at rest.

Time Dialation: Time intervals are also affected by the relative motion. A clock moving with velocity v with respect to an observer appears him to have slowed down by a factor, than when at rest with respect to him.

$$\boxed{t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}}$$

Derivation: Suppose a clock is placed at the point x' in the moving frame S'. An observer in S' finds that the clock gives two ticks at times t_1' and t_2'. The time interval b/w the ticks judged from S' is

$$t_0 = t_2' - t_1' \quad \text{—(1)}$$

Now Another observer measures the time interval b/w two ticks from a stationary frame of reference S, relative to which the clock (or S') is moving with velocity v. If he records the ticks at t_1 and t_2, the time interval appears to him as

$$t = t_2 - t_1 \quad \text{—(2)}$$

From Lorentz transformation eqn –

$$t_2 = \frac{t_2' + x'v/c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}, \quad t_1 = \frac{t_1' + x'v/c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

putting the values of t_2 and t_1 in eq ②

$$t = t_2 - t_1$$

$$t = \frac{t_2' + \frac{x'v}{c^2}}{\sqrt{(1-\frac{v^2}{c^2})}} - \frac{(t_1' + x'v/c^2)}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$t = \frac{t_2' + \cancel{\frac{x'v}{c^2}} - t_1' - \cancel{x'v/c^2}}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{(1-\frac{v^2}{c^2})}}$$

$$\boxed{t = \frac{t_0}{\sqrt{(1-\frac{v^2}{c^2})}}}$$

It shows that the moving clock appears to be slowed down to a stationary observer. This is known as time dialation.

t_0 is known as Proper time interval. ↳ It is the time interval of clock seen from a frame to which the clock is attached.

Velocity Addition Theorem :→ Very high velocities can not be added directly as in classical mechanics. Very high velocities which are comparable with speed of light, should be added using Lorentz transformation equations.

Let two frames of reference S and S'. S' is moving with uniform velocity v relative to S (S is in rest). Let a body move a distance dx in a time interval dt in S frame. Then the velocity of the body measured by an observer in S is

$$u = \frac{dx}{dt} \quad -①$$

To an observer in S', both the distance and the time interval will be appear different, dx' and dt', for an observer in S'

$$u' = \frac{dx'}{dt'} \quad \text{--- (2)}$$

From the Lorentz transformation equations -

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}, \quad t' = \frac{t - xv/c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Differentiating, we get

$$dx' = \frac{dx - v\,dt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}, \quad dt' = \frac{dt - dx\frac{v}{c^2}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

putting these values in eq (2) -

$$u' = \frac{(dx - v\,dt)/\cancel{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}{\left(dt - dx\frac{v}{c^2}\right)/\cancel{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}$$

$$u' = \frac{(dx - v\,dt)}{\left(dt - \frac{v}{c^2}dx\right)} = \frac{\cancel{dt}\left(\frac{dx}{dt} - v\right)}{\cancel{dt}\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

from eq (1), $\frac{dx}{dt} = u$

$$\boxed{u' = \frac{(u - v)}{\left(1 - \frac{uv}{c^2}\right)}}$$

- This is relativistic addition of velocities u and v.

If we consider $u = c$, i.e. a ray of light is emitted in the S frame along the x-axis, then observer in S' frame will measure the velocity ⇒

$$u' = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{\cancel{(c-v)}\,c}{\cancel{(c-v)}} = c$$

$$\boxed{u' = c}$$

So $u = c$, $u' = c$ ⟹ It means that the velocity of light in free space is same in all inertial frames of reference.

Mass-Energy Equivalence :→

According to Einstein's special theory of relativity, The variation of mass m with velocity v is given by-

$$m = \frac{m_0}{\sqrt{(1-\frac{v^2}{c^2})}} \quad \text{—} ①,$$

v - velocity of mass of body
m_0 - rest mass of body.
c - speed of light.

This variation of mass m, with velocity v, has modified our ideas about energy. Let us consider a particle having mass m. If a force F is applied on this particle, then the force F may be written as-

$$F = \frac{dp}{dt}, \quad p \to \text{momentum}$$

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Here m and v are variables

This force F will produce K.E in particle, If particle is displaced through distance ds on applying force F, then

$$dK = F\,ds$$

$$= \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)ds$$

$$= m\frac{dv}{dt}ds + v\frac{dm}{dt}ds$$

$$= m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

$$dK = mv\,dv + v^2 dm \quad \text{—} ② \quad \because \frac{ds}{dt} = v$$

from eq①, $m = \dfrac{m_0}{\sqrt{(1-\frac{v^2}{c^2})}} = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v}{c^2}\right)dv$$

$$= \frac{m_0}{c^2}\frac{v\,dv}{(1-\frac{v^2}{c^2})^{3/2}}$$

$$dm = \frac{m_0\,v\,dv}{c^2\,(1-\frac{v^2}{c^2})^{3/2}}$$

$$dm = \frac{1}{c^2} \frac{m_o}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}\left(1-\frac{v^2}{c^2}\right)} v\,dv$$

$$= \frac{m\,v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)} \quad \because \; m = \frac{m_o}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$= \frac{m\,v\,dv}{\cancel{c^2}\frac{(c^2-v^2)}{\cancel{c^2}}} = \frac{m\,v\,dv}{(c^2-v^2)}$$

$$m\,v\,dv = (c^2-v^2)\,dm$$

putting this value in eq^n (2)

$$dK = (c^2-\cancel{v^2})\,dm + \cancel{v^2}\,dm$$

$$dK = c^2\,dm$$

Suppose the body has mass m_o when at rest and a mass m when accelerated to a velocity v, The K.E acquired is then,

$$K = \int dK = \int_{m_o}^{m} c^2\,dm$$

$$K = c^2(m-m_o)c^2$$

$$\boxed{K.E = c^2(m-m_o)}$$

The total energy = rest mass energy + K.E.

$$= \cancel{m_o}c^2 + c^2(m-\cancel{m_o})$$

$$E = mc^2$$

$$\boxed{E = mc^2}$$

This is the mass- energy relation.

Verification :- The mass and Energy relation has been verified in a number of phenomena. like - Nuclear phenomena, - Compton effect.

In nuclear phenomena, The explanation of mass defect and release of huge amount of energy in nuclear fission is based on mass- energy relation. It gives the strong support to the relation.

Invariance of Maxwell's Equations under Lorentz Transformations

In special theory of relativity one distinguish between proper charge ~~and~~ density ρ_0, measured when the charge under consideration is at rest relative to the observer and the "nonproper" or "relativistic" or "Lorentz contracted" charge density defined as –

$$\rho = \frac{\rho_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}}$$

, $u \rightarrow$ speed of charge

then Maxwell' equations can be written as –

$$\boxed{\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \frac{\rho_0}{\sqrt{\left(1-\frac{u^2}{c^2}\right)}}, \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \vec{J}+\frac{\partial\vec{D}}{\partial t} \end{aligned}}$$

Michelson - Morley Experiment :→

❋ 1.4 Michelson-Morley Experiment: Search for Ether

Ans. Michelson-Morley Experiment : According to the wave theory of light, a light source sets up a disturbance travelling in all directions through a hypothetical medium called 'ether' which fills all space and penetrates all matter. The assumption of ether, however, created a problem. Does ether remain stationary in space when material bodies (including earth) move in it, or is it dragged along with the moving bodies? Bradley's observation of the aberration of light from stars had indicated that *the ether must be stationary in space*. It means that if a material body, say earth, moves in space*, there is a *relative* motion between the body and the ether. A number of experiments were performed to detect a relative motion between the earth and the ether. The most famous among them is the one performed by Michelson and Morely in 1887 using the Michelson interferometer.

A simplified plan of the experiment is shown in Fig. 3. A beam of light from a source S falls upon a half-silvered glass plate P placed at 45° to the beam and is divided into two beams 1 and 2. The beams 1 and 2 travelling at right angles to each other, fall normally on mirrors M_1 and M_2 which reflect them back to P. The two beams returned to P are directed towards a telescope T in which interference fringes are observed.

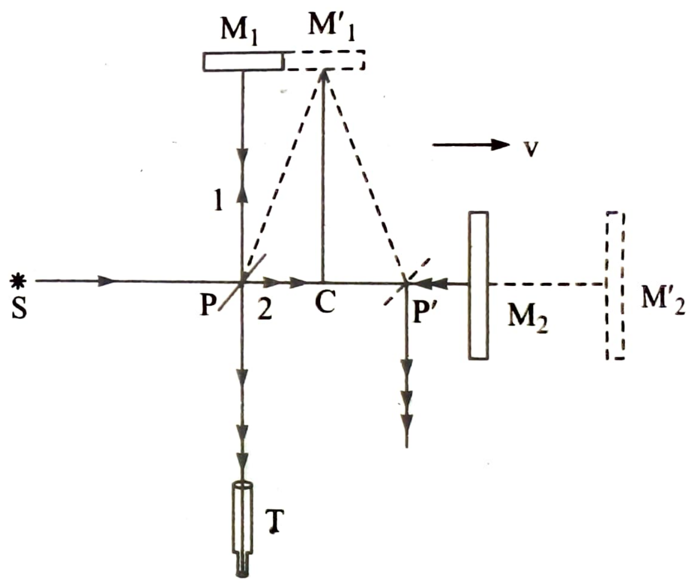


Fig. 3

Let the mirrors M_1 and M_2 be at the same distance l from the plate P. Then, if the apparatus were at rest in ether, the two beams would take the same time to return to P. But, actually the earth, and hence the apparatus, is moving in space through the ether with a velocity v (say). Suppose this motion is in the direction of the initial beam of light. Then,

* The earth is moving round the sun with a velociy of 3×10^4 m/s which is one ten-thousandth of the velocity of light in free space.

if the initial beam strikes the plate P in the position shown, the paths of the two beams and the positions of their reflections from the mirrors will be as shown by the dotted lines. The time taken by the two beams on their journeys are *not* equal.

Let c be the velocity of light through the ether. The beam 2 moving towards M_2 has a velocity $(c - v)$ *relative to the apparatus* on the outgoing trip, and $(c + v)$ on the return trip. If t_2 be the total time taken by this beam to go from P to M_2 and back, then

$$t_2 = \frac{l}{c - v} + \frac{l}{c + v} = \frac{2lc}{c^2 - v^2} = \frac{2l}{c}\left(\frac{1}{1 - \frac{v^2}{c^2}}\right). \quad \text{...(i)}$$

The beam 1 moving *transversely* with respect to the apparatus retains its velocity c throughout. Let it take a time t' to go from P to strike M_1, travelling a distance ct'. In the same time, the mirror M_1 advances a distance vt'. Thus, in the right-angled triangle PM_1M_1', we have

$$(PM_1')^2 = (PM_1)^2 + (M_1M_1')^2$$

But $PM_1 = l$, $M_1M_1' = vt'$ and $PM_1' = ct'$

$$\therefore \quad (ct')^2 = l^2 + (vt')^2$$

or

$$t' = \frac{l}{(c^2 - v^2)^{1/2}} = \frac{l}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If t_1 be the total time taken by the beam to travel the whole path $PM_1'P'$, then

$$t_1 = 2t' = \frac{2l}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{...(ii)}$$

Hence the difference between the times of travel of the two beams is, from eq. (i) and (ii), given by

$$t_2 - t_1 = \frac{2l}{c}\left[\frac{1}{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

$$= \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-1} - \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

Using binomial expansion and dropping terms higher than the second order, we have

$$t_2 - t_1 = \frac{2l}{c}\left[\left(1+\frac{v^2}{c^2}\right) - \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\right]$$

$$= \frac{2l}{c}\left[\frac{1}{2}\frac{v^2}{c^2}\right] = \frac{lv^2}{c^3}$$

The path difference δ between the beams corresponding to a time difference $(t_2 - t_1)$ is

$$\delta = c\,(t_2 - t_1) = \frac{lv^2}{c^2}$$

If the interferometer is suddenly brought to rest (v is made zero), then the path difference δ would become zero. We know that if the path difference between two interfering waves changes by λ, there is a shift of one fringe across the cross-wires in the field of view. Thus, if ΔN is the number of fringes which shift when the interferometer is stopped, then

$$\Delta N = \frac{\delta}{\lambda} = \frac{lv^2}{c^2\lambda}$$

In the actual experiment, the whole apparatus, which was placed on a block of stone floated on mercury, was rotated through 90°. This introduced a path difference of the same amount in the opposite direction. Hence a shift of $\frac{2lv^2}{c^2\lambda}$ was expected.

To have an observable shift, Michelson and Morley increased the effective value of l upto 11 meters by reflecting the light back and fourth several times. Then, using values : $l = 11$ meter, $v = 3 \times 10^4$ meter/sec, $c = 3 \times 10^8$ meter/sec and $\lambda = 5.5 \times 10^{-7}$ meter (for visible light), the expected shift is

$$\Delta N = \frac{2lv^2}{c^2\lambda} = \frac{2 \times 11 \times (3 \times 10^4)^2}{(3 \times 10^8)^2 \times 5.5 \times 10^{-7}} = 0.4,$$

or a shift of four-tenths a fringe.

Michelson and Morley were extremely surprised to see that there was *no* shift in the fringes when the interferometer was rotated through 90°. They repeated the experiment during various times of the day and various seasons of the year but no shift was observed. Trouton and Noble, in 1902, performed an electromagnetic experiment for the same purpose but with no positive result. Thus, ***the motion of the earth through the ether could not be experimentally detected.***

Explanation of the Negative Result : Three separate explanations were given to the negative result of the Michelson-Morley experiment :

1. Ether-drag Hypothesis : The moving earth *completely drags* the ether with it so that there is not relative motion betwen the two and hence the question of shift does not arise. But this explanation was not accepted for two reasons : (i) It goes against the observed aberration of light from stars. (ii) Fizeau had experimentally shown that a moving body could drag the 'light waves' only *partially*. Furthermore, this partial dragging of light waves was explained by the electromagnetic theory, without introducing the ether-drag hypothesis.

2. Fitzgerald-Lorentz Contraction Hypothesis : Fitzgerald and Lorentz independently put an adhoc hypothesis that all material bodies moving through the ether are contracted in the direction of motion by a factor $\sqrt{1-(v^2/c^2)}$. It is easily seen that such a contraction in the interferometer arm would equalize the times t_1 and t_2 and no fringe-shift would be expected.This explanation also, being purely adhoc could not be accepted. Further, Rayleigh worked out that such a contraction is expected to produce double refraction which was, however, never observed.