(Please write your Enrolment No. immediately) Enrolment No. __________

MID TERM EXAMINATION

B.TECH PROGRAMMES (UNDER THE AEGIS OF USICT)

Fourth Semester, May, 2023

Paper code: BS-202 Subject: Probability, Statistics and Linear programming

Time: 1½ Hrs. Max. Marks:30

Note: Attempt Q. No. 1 which is compulsory and any two more questions from remaining.

Q1. (a) If one out of every 10 bulbs are defective. Find the mean and standard deviation for the distribution of defective bulbs in a total of 500 bulbs. **(2.5)**

(b) Find K so that $f(x, y) = K(x + y), 0 < x < 1,$ and $0 < y < 1$ is a joint probability density Function.

(c) Calculate the covariance of the following pairs of observations of the variables X and Y **(2.5)**

(1,6),(2,9),(3,6), (4,7),(5,8), (6,5),(7,12), (8,3), (9,17), (10,1). **(2.5)**

(d) If the probability of a bad reaction from a certain injection is 0.001. Find the chance that out of 1000 individuals more than two will have a bad reaction. **(2.5)**

Q2. (a) In a toy factory, machines A, B, C manufactures respectively 25%, 35% and 40% of total. Of their outputs 5,4,2 percents are respectively defective. A toy is drawn at random from the total production. What is the probability that the toy is drawn is defective? Also, the probability that it was manufactured by machine A. **(5)**

(b) X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} Kx & ; 0 \le x < 5 \\ K(10 - x); & 5 \le x < 10 \\ 0 & ; otherwise \end{cases}$$

(i) Find the value of K, (ii) Mean of X (iii) $p(5 < X \le 12)$ **(5)**

Q3. (a) In a normal distribution ,7% of the items are under 35 and 89% are under 63. Find the mean and S.D. of the distribution? Given that

$$P(0 \le z \le 0.18) = 0.07, P(0 \le z \le 1.48) = 0.43, P(0 \le z \le 1.23) = 0.39.$$ **(5)**

(b) The ages of Boys and Girls and are given in the following table. calculate the coefficient of correlation between Xand Y **(5)**

X (ages of Boys)	23	27	28	29	30
Y (ages of Girls)	18	22	23	24	25

Q4. (a) Three fairs of coins are tossed. Let X denote the number of heads on the first two coins, Let Y denote the number of tails on the last two coins. Then

(i)Find the joint distribution of X and Y **(5)**

(ii)Find the conditional distribution of Y given that X=1

(b) An electronics company manufactures resistors that have a mean resistance of 100 ohms and a standard deviation of 10 ohms. The distribution of resistance is normal. Find the probability that a random sample of n=25 resistors will have an average of fewer than 95 ohms. Given that P(Z>2.5) = 0.4938 **(5)**

From Binomial distribution.

Q.1.(a) $p = \frac{1}{10}$, $q = 1 - p = 1 - \frac{1}{10} = 9/10$, $n = 500$

mean $= n \cdot p = \frac{1}{10} \times 500 = 50$

Standard deviation $= \sqrt{npq} = \sqrt{\cancel{500} \times \frac{1}{\cancel{10}} \times \frac{9}{\cancel{10}}}$

$= 3\sqrt{5}$

$= \underline{\underline{6.70820}}$

From poisson distribution.

$\lambda = np = 50$

~~P(X=r)~~ = Standard deviation $= \sqrt{50} = \underline{\underline{7.07}}$

(mean = variance)

(b) $\int_0^1 \int_0^1 k(x+y)\, dx\, dy = 1$

$\int_0^1 k \left[\frac{x^2}{2} + xy\right]_0^1 dy = 1$

$= \int_0^1 k \left[\frac{1}{2} + y\right] dy = 1$

$k \left[\frac{1}{2} y + \frac{y^2}{2}\right]_0^1 = 1$

$k \left[\frac{1}{2} + \frac{1}{2}\right] = 1 \Rightarrow k = 1$

Q:

(c)

X	Y	$(X-\bar{X})$	$Y-\bar{Y}$	$(X-\bar{X}).(Y-\bar{Y})$
1	6	-4.5	-1.4	6.3
2	9	-3.5	1.6	-5.6
3	6	-2.5	-1.4	$+3.5$
4	7	-1.5	-0.4	$+0.6$
5	8	-0.5	0.6	-0.3
6	5	0.5	-2.4	-1.2
7	12	1.5	4.6	6.9
8	3	2.5	-4.4	-11.0
9	17	3.5	9.6	33.6
10	1	4.5	-6.4	-28.8

$\Sigma X = 55$ $\Sigma Y = 74$ $\Sigma X-\bar{X} = 0$ $\Sigma(X-\bar{X})(Y-\bar{Y}) = 4$

$$\bar{Y} = \frac{74}{10} = 7.4$$

$$\bar{X} = \frac{55}{10} = 5.5$$

$$\text{cov}(x, y) = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{n} =$$

$$= \frac{4}{10} = \underline{0.4}$$

(d) $p = 0.001$ $n = 1000$

$$\lambda = np = 1000 \times 0.001 = 0.1$$

$$P[r>2] = 1 - P[r \leq 2]$$

$$= 1 - [P(r=0) + P(r=1) + P(r=2)]$$

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \qquad = 1 - [0.904 + 0.0904 + 0.004524]$$

$$= \underline{0.0011}$$

Q. 2. (a) Let E be event that the toys is defective.

and $E_1 \equiv$ produced by machines A

$E_2 \equiv$ " " " B

$E_3 =$ " " " C

$P(E_1) = .25$ $P(E_2) = 0.35$, $P(E_3) = 0.40$

$P(E/E_1) = 0.05$ $P(E/E_3) = 0.02$

$P(E/E_2) = 0.04$

By Total probability

$$P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)$$

$$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$

$$= 0.0345$$

By Bays theorem :.

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E)} = \frac{0.25 \times 0.05}{0.0345}$$

$$= \frac{.0125}{.0345}$$

$$= \frac{125}{345} = \frac{25}{69}$$

$$= \underline{0.362}$$

Question (b) Here $f(x) = \begin{cases} Kx & ; 0 \le x \le 5 \\ K(10-x) & ; 5 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$

$\because \int_{-\infty}^{\infty} f(x)\,dx = 1$

or $\int_0^5 K(x)\,dx + \int_5^{10} K(10-x)\,dx = 1$

$K\left[\frac{x^2}{2}\right]_0^5 + K\left[10x - \frac{x^2}{2}\right]_5^{10} = 1$

$K\left[\frac{25}{2}\right] + K\left[(100-50) - \left(50 - \frac{25}{2}\right)\right] = 1$

$\frac{25K}{2} + K\left[50 - \frac{75}{2}\right] = 1$

$\frac{25K}{2} + K\left[\frac{25}{2}\right] = 1$

or $50K = 2$ or $K = \frac{2}{50} = 1/25$

$= .04$

(ii) mean of the distribution.

$= \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^5 x \cdot f(x)\,dx + \int_5^{10} x f(x)\,dx$

$= \int_0^5 x \cdot Kx\,dx + \int_5^{10} K(10-x) \cdot x\,dx$

$= K\left[\frac{x^3}{3}\right]_0^5 + K\left[\frac{10x^2}{2} - \frac{x^3}{3}\right]_5^{10}$ $750 - 250$

$K\left[\frac{250}{6} + \frac{500}{6}\right]$

$= K\left[\frac{125}{3}\right] + K\left[\left(\frac{1000}{2} - \frac{1000}{3}\right) - \left(\frac{250}{2} - \frac{125}{3}\right)\right]$

$\frac{K \cdot 750}{6}$

$= K\left(\frac{125}{3}\right) + K\left[\left(\frac{1000}{6}\right) - \left(\frac{500}{6}\right)\right] = K\left[\frac{125}{3}\right] + \left(\frac{500}{6}\right)$

$= \frac{0.4 \times 750}{6}$

$= 5$

$P[5<X<12]$

(c) $F(x) = \int_5^{12} f(x)\,dx$

$= \int_5^{10} f(x)\,dx + \int_{10}^{12} f(x)\,dx$

$= \int_5^{10} k(10-x)\,dx + 0$

$= k\left(10x - \frac{x^2}{2}\right)_5^{10}$

$= k\left[(100-50) - \left(50 - \frac{25}{2}\right)\right]$

$= k\left[50 - \frac{25}{2}\right] \quad = k\left[\frac{25}{2}\right]$

$= 0.04 \times \frac{25}{2} = .50$

$= \underline{0.5}$ $\underline{\text{Ans.}}$

Ques: 3(a) Let $X \sim N(\mu, \sigma^2)$, then we have

$P[X < 63] = 0.89$, $P[X < 35] = 0.07$

$P[\mu < X < 63] = 0.89 - 0.5 = 0.39$

$P[35 < X < \mu] = 0.5 - 0.07 = 0.43$

$\therefore P\left[0 < \frac{X-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right] = 0.39$ and

~~P[0 < X -~~ $P\left[\frac{35-\mu}{\sigma} < \frac{X-\mu}{\sigma} < 0\right] = 0.43$

$\Rightarrow P\left[0 < Z < \frac{63-\mu}{\sigma}\right] = 0.39$, $P\left[0 < Z < \frac{\mu-35}{\sigma}\right] = 0.43$

$\frac{63-\mu}{\sigma} = 1.23$, $\frac{\mu-35}{\sigma} = 1.48$

$\Rightarrow \frac{28}{\sigma} = 2.71 \Rightarrow \sigma = \frac{28}{2.71} = 10.33$

$\mu = 35 + 1.48 \times \sigma$

$= 35 + 1.48 \times 10.33 = 35 + 15.3$

$= \underline{\underline{50.3}}$

Q-4-1

Q- 3-(b)

x	y	$x \cdot y$ ($\cancel{x \cdot y}$)	x^2 ($\cancel{x^2}$)	y^2
23	18	414	529	324
27	22	594	729	484
28	23	644	784	529
29	24	696	841	576
30	25	750	900	625

$\Sigma xy = 3098$ $\Sigma x^2 = 3783$ $\Sigma y^2 = 2538$

$$\rho(x,y) = \frac{\text{cov}(x,y)}{\sigma_x \cdot \sigma_y}$$

$\cancel{\bar{x} = \frac{137}{5}} =$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \times \Sigma y^2}} = \frac{3098}{\sqrt{3783 \times 2538}}$$

$$= \frac{3098}{3098.58} = \frac{\cancel{1.009}}{}$$

$$\simeq 0.9981$$

~~r = 1.0055~~

Q-4-(a) Sample space. $\{$ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT $\}$

	HHH	HHT	HTH	HTT	THH	TTT	TTH
X	2	2	1	1	1	0	0
Y	0	1	1	2	0	2	1

	THT
X	1
Y	1

(a)

Y \ X	0	1	2	$f_Y(y)$
0	0	1/8	1/8	1/4
1	1/8	2/8	1/8	1/2
2	1/8	1/8	0	1/4
$f_X(x)$	2/8	4/8	2/8	1

(ii) $P[Y|X=1] = \frac{f(x,y)}{f_X(x)} = \cancel{\frac{f(1,y)}{f_X(1)} = 2 \cdot f(1,y)}$

$y=0$

$= \cancel{2 \times \frac{1}{4}} = \frac{1/8}{1/2} = 1/4$

$y=1$ $P(Y|X=1) = 2/8 / 1/2 = 1/2$

$y=2$ $P(Y|X=1) = \frac{1/8}{1/2} = 1/4$

Q 4(b) $\mu = 100$ ohms. (mean of population)

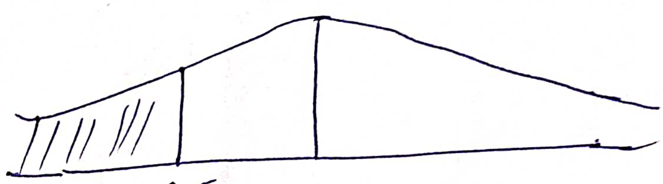
σ 10 ohms.

$n = 25$

$\bar{x} = 95$ (sample mean)

by C.L.T theorem.

$$Z_n = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} = \frac{95-100}{10/\sqrt{25}} = \frac{-5}{10/5} = -2.5$$



$P\{Z < -2.5\} = (P\ Z > 2.5)$ (by the properperty of inequality)

$= .5 - P(0 < Z < 2.5)$ $[P(0 < Z < \infty) - P(0 < Z < 2.5)]$

$= .5 - 0.4938$

$= .0062$