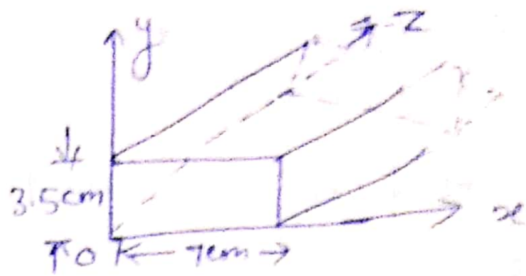
Ex 4-1-1 TE_{10} in Rect | W/G

Q: Rectangular W/G of inside dimension 7 × 3.5 cm operates in the dominant TE_{10} mode.



Calculate

1. cut off frequency
2. Phase velocity of the wave in the guide at a freq. of 3.5 GHz.
3. Guided wavelength at the same freq.

Soln :

$f_c = \frac{c}{\lambda_c} \Rightarrow \frac{c}{2a}$

1. $f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 7 \times 10^{-2}} = \boxed{2.14 \text{ GHz}}$

2. $v_p = \frac{c}{\sqrt{1-(f_c/f)^2}} = \frac{3 \times 10^8}{\sqrt{1-(2.14/3.5)^2}} = \boxed{3.78 \times 10^8 \text{ m/s}}$

3. $\lambda_g = \frac{\lambda_0}{\sqrt{1-(f_c/f)^2}} = \frac{3 \times 10^8/(3.5 \times 10^9)}{\sqrt{1-(2.14/3.5)^2}} = \boxed{10.8 \text{ cm}}$

$\lambda_0 \longrightarrow$

$\frac{\lambda_0}{\lambda_c} = \frac{c}{f} \times \frac{f_c}{c}$

$\boxed{\frac{\lambda_0}{\lambda_c} = \frac{f_c}{f}}$

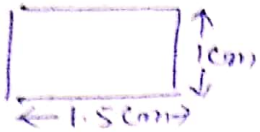
1. Hint $\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$

$\lambda_{c_{m,n}} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$

Q: Show that for a TE_{10} mode a freq. of 8GHz will pass through a waveguide of dimension a = 1.5cm b = 1cm if a dielectric with $\epsilon_r = 4$ is inserted in the guide?

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Solu:



Since a = 1.5cm

Cut off wavelength of TE_{10} mode m = 1, n = 0

$\lambda_c = 2a = 3cm$.

Hence $f_c = \frac{3\times10^{10}}{3} = \boxed{10\ GHz}$.

$$f_c = \boxed{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

As is evident a freq of 8 GHz which is less than 10GHz (cut off freq.) will not ordinarily pass through the guide.

But when a dielectric is inserted inside the w/G this freq of 8 GHz will readily pass through the same guide.

Normal λ for 8 GHz $= \frac{3\times10^{10}}{8\times10^{9}} = 3.75$ Cm in air

we know that

$$\boxed{\lambda_{dielectric} = \frac{\lambda(\text{freespace})}{\sqrt{\epsilon_r}}}$$

when the medium changes wavelength also changes. wavelength of an electromagnetic wave in a dielectric medium is shorter than in free space

$$\lambda = \frac{c}{f\sqrt{\epsilon_r}}$$

$$= \frac{3.75}{\sqrt{4}}$$

λ with dielectric $\lambda_{die.} = 1.87$ Cm

$\lambda_{die.}$ is less than 3cm and hence 8 GHz freq will pass through the same guide.

given Data	
f = 8 GHz	$\lambda_{c_{10}} = 2\cdot a = 2\times1.5 = \boxed{3cm}$
a = 1.5cm	and $\lambda_0 = \frac{3\times10^{10}}{8\times10^{9}} = \boxed{3.75cm}$
b = 1 cm	Condition 1 $\boxed{\lambda_c > \lambda_0}$ Not satisfied
$\epsilon_r = 4$	Now $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{10}}{3} = 10GHz$
Mode = TE_{10}	Condition 2 $\boxed{f > f_c}$ Not satisfied

Now dielectric is inserted with $\epsilon_r = 4$

$$\lambda_{die} = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{3.75cm}{\sqrt{4}}$$

$$\lambda_{die} = \frac{3.75}{2} = \boxed{1.87cm}$$

Condition 1 $\boxed{\lambda_c > \lambda_0 \text{ or } \lambda_{die}}$ satisfied

$\boxed{3 > 1.87}$

For a w/G with dimension $a = 2.286$ cm and $b = 1.016$ cm

Part ① Find the cut off freq. of TE_{01}, TE_{20}, TE_{12} and TE_{21} modes.

Soln :

1 Cut off freq. of TE modes

$$\left[f_{c_{mn}} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\right] \quad \text{or} \quad \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For TE_{01} mode

$$f_{c_{01}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{0\times\pi}{2.286}\right)^2 + \left(\frac{1\times\pi}{1.016}\right)^2} = 14.764 \text{ GHz}.$$

For TE_{20}

$$f_{c_{20}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{0\times\pi}{1.016}\right)^2} = 13.123 \text{ GHz}.$$

For TE_{12}

$$f_{c_{12}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{2\pi}{1.016}\right)^2} = 30.248 \text{ GHz}.$$

For TE_{21}

$$f_{c_{21}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 19.753 \text{ GHz}.$$

Part ② cut off freq.

For TM_{11}, TM_{12} and TM_{21} modes.

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For TM_{11} mode

$$f_{c_{11}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 16.156 \text{ GHz}.$$

For TM_{12} mode

$$f_{c_{12}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{2\pi}{1.016}\right)^2} = 30.248 \text{ GHz}.$$

For TM_{21} mode

$$f_{c_{21}} = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 19.753 \text{ GHz}.$$

Q: A hollow rectangular w/G has dimensions a = 4cm, b = 2cm. Calculate the amount of attenuation if the frequency is 3GHz.

$b = 2cm$

Soln:

$\lambda_c = 2a$ $f = 3GHz$.

$\lambda_c = 2 \times 4 = 8cm$

Condn.
$\lambda_c > \lambda_o$
$f > f_c$

and $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{10}}{8} = 3.75GHz$

Bcz signal freq. is less than fc
f<fc So that signal not propagate through w/G but will get attenuated.

So that

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$= \left[\left(\frac{\pi}{0.04}\right)^2 + 0 - (2\pi\times3\times10^9)^2\, 4\pi\times10^{-7} \times \frac{1}{36\pi}\times10^{-9}\right]^{1/2}$$

$\uparrow$
8.854×10^{-12}

$\alpha = 15\pi$ Np

$\alpha = 47.13$ nepers/m

$\boxed{\alpha = 409 dB/ms.}$

$1Np = 20\log_{10} e\, dB$ (2.7182)
$1Np = 8.6859 dB.$

$1dB = \frac{1}{20\log_{10} e} dB$

$= 0.11513 Np$

$dB = 10\log\frac{P_1}{P_2}$

$dBm = 10\log\frac{P}{mW}$ (Absolute Power level)

* If dBm then dBmW is acquired bcz of mW reference
* A value can be in a form of dBW if it is referred to 1 watt.

$dB = 10\log P_1/P_2$

$dB = 20\log(V_1/V_2)$

Bcz P is proportional to V^2

$dB = 10\log(V_1/V_2)^2$

$= 20\log\left(\frac{V_1}{V_2}\right)$

A Rect. W/G has dimension 2.5 x 5 cms. Determine
1) Guide wavelength 2) Phase Constant β
3) Phase velocity v_p
at a wavelength of 4.5 cms for the dominant mode TE_{10}.

Soln: For TE_{10} mode

$$\lambda_c = 2 \cdot a = 2 \times 2.5 = 5\,cm$$

$$\lambda_0 = 4.5\ cms.$$

Now we know that

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{4.5}{\sqrt{1-\left(\frac{4.5}{5}\right)^2}} = \frac{4.5}{0.573} = 7.803\,cm \quad 10.32\,cm$$

$$\boxed{\lambda_g = \cancel{7.803}\ 10.32}\ cm. \quad \boxed{10.32\ cm}$$

$$v_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{3\times10^{10}}{\sqrt{1-\left(\frac{4.5}{5}\right)^2}} = \frac{3\times10^{10}}{\cancel{0.573}\ 0.4359} = \cancel{5.22\times10^{10}\,cm/sec}$$

wrong in book $v_p = \cancel{5.22\times10^{10}\,cm/sec}$ $\boxed{6.88\times10^{8}}$ m/s

$$\beta = \frac{1}{c}\sqrt{\omega^2-\omega_c^2} \qquad f_0 = \frac{c}{\lambda_0}, \quad \omega = 2\pi f \text{ or } \omega = 2\pi \frac{c}{\lambda_0}$$

or $f = \frac{\omega}{2\pi}$

$$= \frac{1}{c}\sqrt{\left(\frac{2\pi c}{\lambda_0}\right)^2-\left(\frac{2\pi c}{\lambda_c}\right)^2} \qquad f_c = \frac{c}{\lambda_c} \quad \omega_c = 2\pi f_c \quad \omega_c = 2\pi\frac{c}{\lambda_c}$$

$$\beta = \frac{2\pi \cancel{c}}{\cancel{c}}\sqrt{\left(\frac{1}{\lambda_0}\right)^2-\left(\frac{1}{\lambda_c}\right)^2}$$

$$\boxed{\beta = \frac{2\pi}{\lambda_0\lambda_c}\sqrt{(\lambda_c)^2-(\lambda_0)^2}} \text{ or } \boxed{\lambda_g = \frac{2\pi}{\beta}}$$

$v_g = \cancel{1/3\times10^{7}}$

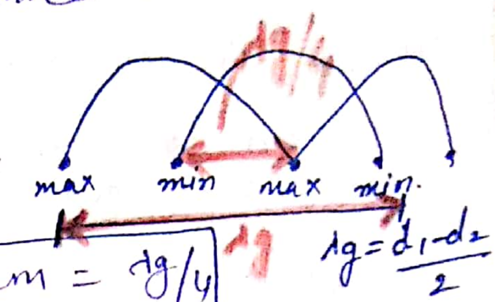
$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{10.32} = 0.6088 \text{ radians.}$$

$$\beta = \frac{2\pi}{4.5\times10}\sqrt{(5)^2-(4.5)^2}$$

$$\beta = \frac{2\pi}{4.5\times10}\sqrt{25-20.25}$$

$\Rightarrow \beta = \cancel{1.246\ radians}$ $\cancel{6.6}$ 0.6088 radians

TE_{10} mode is propagated in a rectangular w/g of dim. $a=6$ cm, $b=4$ cm. By means of travelling detector distance b/w a maximum and minimum is found to be 4.55 cms. Find the freq. of the wave.



Soln:

$a = 6$ cm $\quad b = 4$ cm

Distance b/w maxi. and minima $= 4.55$ cm $= \lambda g/4$

for TE_{10} mode $\boxed{\lambda_c = 2a} = 2\times 6 = \boxed{12\text{ cm}}$

$$\lambda g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}, \quad \frac{\lambda g}{4} = 4.55\text{ cm}$$

$\boxed{\lambda g = 18.2\text{ cm}}$ $\quad \boxed{\lambda_c = 2a} = 12$ cm

$$18.2 = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}} \Rightarrow \lambda_0 = 18.2 \cdot \sqrt{1-(\lambda_0/\lambda_c)^2}$$

S.B.S

$$331.24\left[1-\left(\frac{\lambda_0}{\lambda_c}\right)^2\right] = \lambda_0^2$$

$$1-\left(\frac{\lambda_0}{\lambda_c}\right)^2 = \frac{\lambda_0^2}{331.24} \Rightarrow 1 = \frac{\lambda_0^2}{144} + \frac{\lambda_0^2}{331.24}$$

$$1 = \lambda_0^2\left[6.944\times10^{-3} + 3.01895\times10^{-3}\right]$$

$\cancel{\lambda g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$, $\lambda_0 = \frac{c}{f} = \frac{3\times10^{10}}{3\times10^{9}} = 10$ cms.

$$\lambda_0^2 = \frac{1}{9.363\times10^{-3}} = 100.36$$

$\boxed{\lambda_0 = 10\text{ cm}}$

$\lambda_0 = \frac{c}{f}$ and $\boxed{f = \frac{c}{\lambda_0}} = \frac{3\times10^{10}}{10} = 3\times10^{9}$ Hz

$\boxed{f = 3\text{ GHz}}$

Q: Dimensions of a guide are 2.5 × 1 cms. operating freq. is 8.6 GHz. find the (a) Possible modes. (b) cut off freq. (c) Guide wavelength

Solu: for TE modes propagation

$a = 2.5$ cm $f = 8.6$ GHz.
$b = 1$ cm

$$\lambda_0 \text{ or } \boxed{\lambda = \frac{c}{f}} = \frac{3\times10^{10}}{8.6\times10^9} = \frac{30}{8.6} = \boxed{3.488 \text{ cm}}$$

Condn for wave to propagate $\lambda_c > \lambda_0$

Part (a)

for TE_{01}
$$\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} = \frac{2\times2.5\times1}{\sqrt{0+a^2}} = \frac{5}{a} = \frac{5}{2.5} = \boxed{2 \text{ cm}}$$

Since $\lambda_c \not> \lambda_0$ So TE_{01} does not propagate.

For TE_{10} $\lambda_c = 2a = 2\times2.5 = \boxed{5 \text{ cm}}$

$\lambda_c > \lambda_0$ So TE_{10} propagate.

for TE_{11} or TM_{11} (same) λ_c
$$\lambda_c = \frac{2ab}{\sqrt{a^2+b^2}} = \frac{2\times2.5\times1}{\sqrt{(2.5)^2+(1)^2}} = \frac{5}{\sqrt{6.25+1}} = \boxed{1.856 \text{ cm}}$$

$\lambda_c < \lambda_0$ So TE_{11} & TM_{11} donot propagate

Part (b)

Now cut off freq. of propagating mode ie TE_{10} $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{10}}{5} = \boxed{6 \text{ GHz}}$.

Part (c)

Now
$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{3.488}{\sqrt{1-(3.488/5)^2}} = \frac{3.488}{\sqrt{1-0.486}} = \frac{3.488}{\sqrt{0.513}}$$

$$\lambda_g = \frac{3.488}{0.7164} = \boxed{4.868 \text{ cms.}}$$

mode is propagated in R/w/g of dim a = 6 b = 4 by means of Travelling detector distance b/w minima and maxima is 4.55 cm · find freq.

$\lambda_g = (18.2\ cm)$

Soln $a = 6\ cm \quad b = 4\ cm$

$$\lambda_g/4 = 4.55\ cm$$

$$\lambda_g = 4 \times 4.55\ cm$$

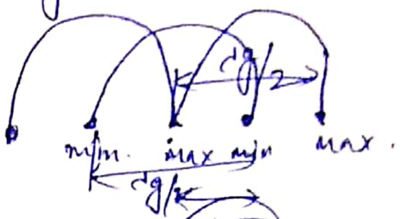
$$\boxed{\lambda_g = 18.2\ cm}$$

$$\lambda_c = 2a = 6 \times 2 = 12\ cm$$

$$2\lambda_g = 2(d_1 - d_2)$$

$$(\underset{d_1}{12.00} - \underset{d_2}{8.00})$$

$$\lambda_g = 4$$



$\lambda_g/4$

$$\lambda_g = \boxed{\frac{d_1 - d_2}{2}}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

18.2

$$\lambda_0 = 18.2 \cdot \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

S.B.S. $\lambda_0^2 = (18.2)^2 \cdot \cancel{\sqrt{\ }}\, 1 - \left(\frac{\lambda_0}{12}\right)^2$

$$\lambda_0^2 = 331.24 \cdot \left[1 - \frac{\lambda_0^2}{144} \right]$$

$$\lambda_0^2 = 331.24 \left[\frac{144 - \lambda_0^2}{144} \right]$$

$$\lambda_0^2 = \frac{331.24 \times \cancel{144}}{\cancel{144}} - \frac{331.24 \cdot \lambda_0^2}{144}$$

$$= 331.24 - 2.30\, \lambda_0^2$$

$$\lambda_0^2 + 2.30\, \lambda_0^2 = 331.24$$

$$\lambda_0^2 (1 + 2.30) = 331.24$$

$$\lambda_0^2 = \frac{331.24}{3.30} = 100.37$$

$$\boxed{\lambda_0 = 10\ cm}$$