Data Structure and Algorithms

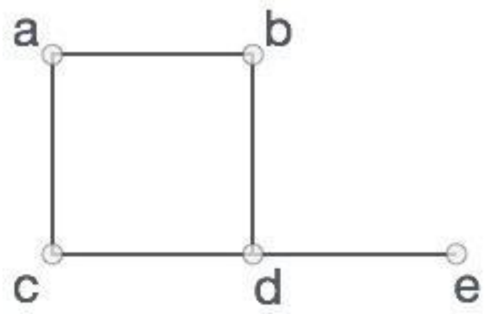
Topic :- Graphs

Graph :-

1. Definition
2. Types of graphs
3. Terminology
4. Representation
5. Uses of graph

What is Graph ?

- A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called **edges**.
- Formally, a graph is a pair of sets **(V, E)**, where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices. Take a look at the following graph −



- In the graph,

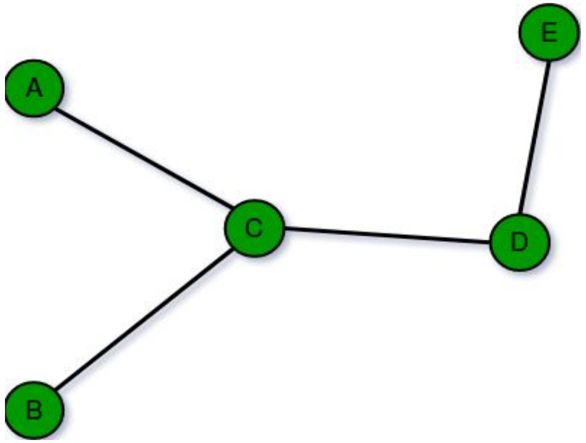
  V = {a, b, c, d, e}

  E = {ab, ac, bd, cd, de}

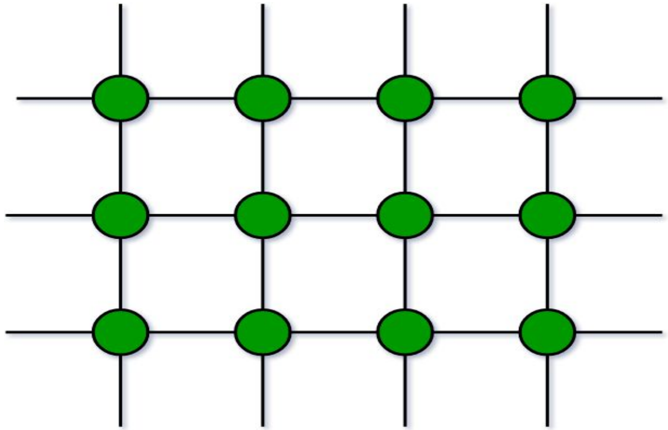
- A data structure that consists of a set of nodes (*vertices*) and a set of edges that relate the nodes to each other
- The set of edges describes relationships among the vertices .

Types of Graph

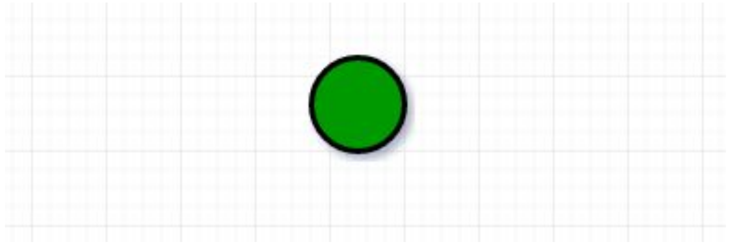
1. **Finite Graphs:** A graph is said to be finite if it has finite number of vertices and finite number of edges.



1. Finite graph



2. Infinite graph

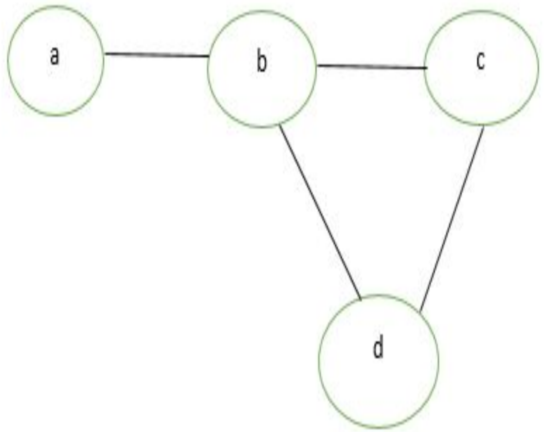


3. Trivial graph

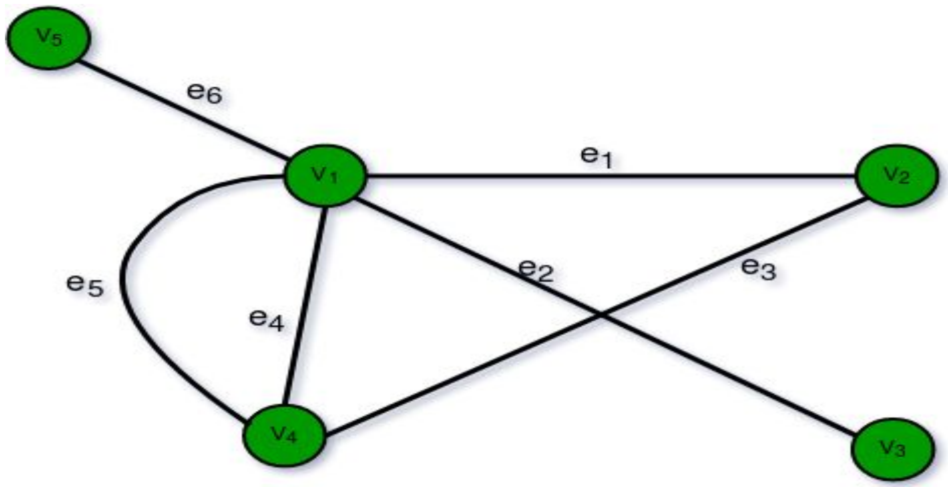
2. **Infinite Graph:** A graph is said to be infinite if it has infinite number of vertices as well as infinite number of edges.

3. **Trivial Graph:** A graph is said to be trivial if a finite graph contains only one vertex and no edge.

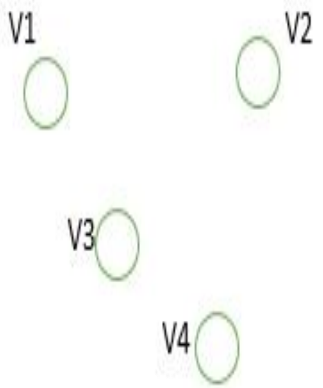
4. **Simple Graph:** A simple graph is a graph which does not contains more than one edge between the pair of vertices. A simple railway tracks connecting different cities is an example of simple graph.



4. Simple graph



5. Multi graph



6. Null graph

5. **Multi Graph:** Any graph which contain some parallel edges but doesn't contain any self-loop is called multi graph. For example A Road Map

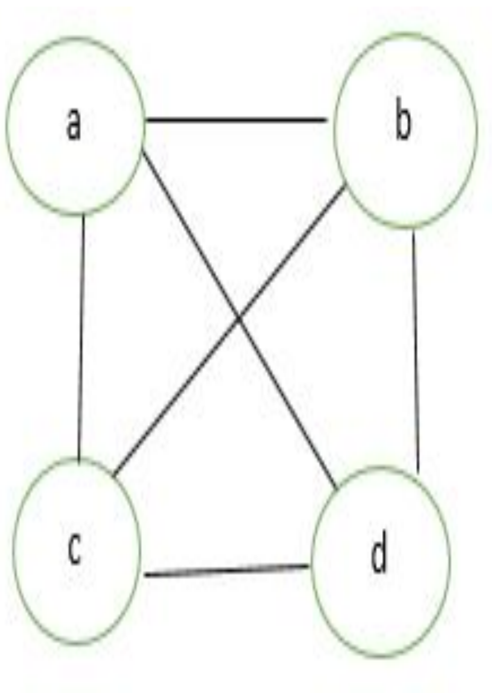
Parallel Edges: If two vertices are connected with more than one edge than such edges are called parallel edges that is many roots but one destination.

Loop: An edge of a graph which join a vertex to itself is called loop or a self-loop.

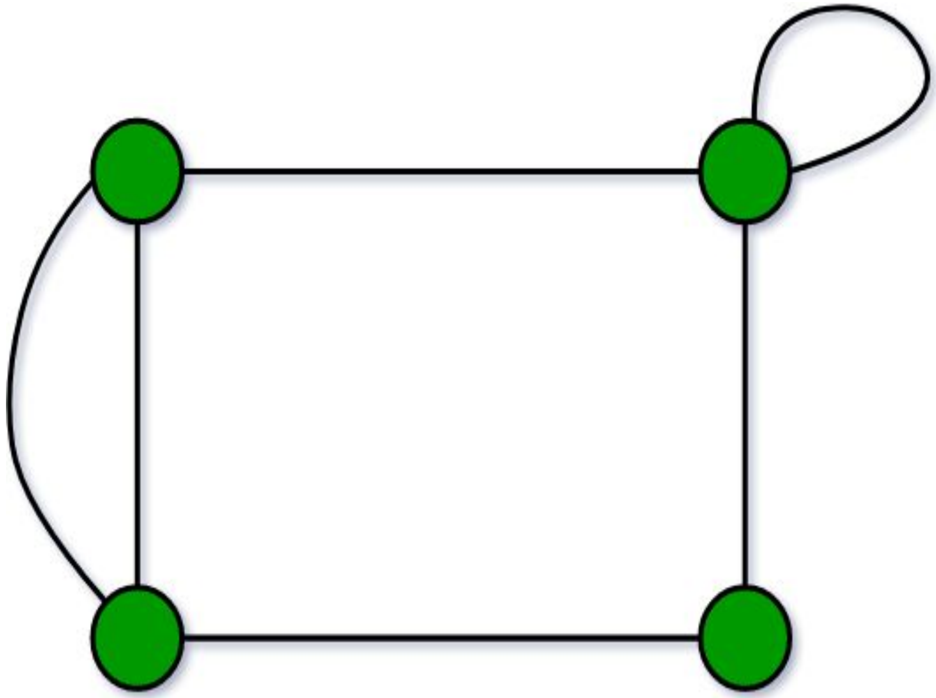
6. **Null Graph:** A graph of order n and size zero that is a graph which contain n number of vertices but do not contain any edge.

7. Complete Graph: A simple graph with n vertices is called a complete graph if the degree of each vertex is n-1, that is, one vertex is attach with n-1 edges. A complete graph is also called Full Graph.

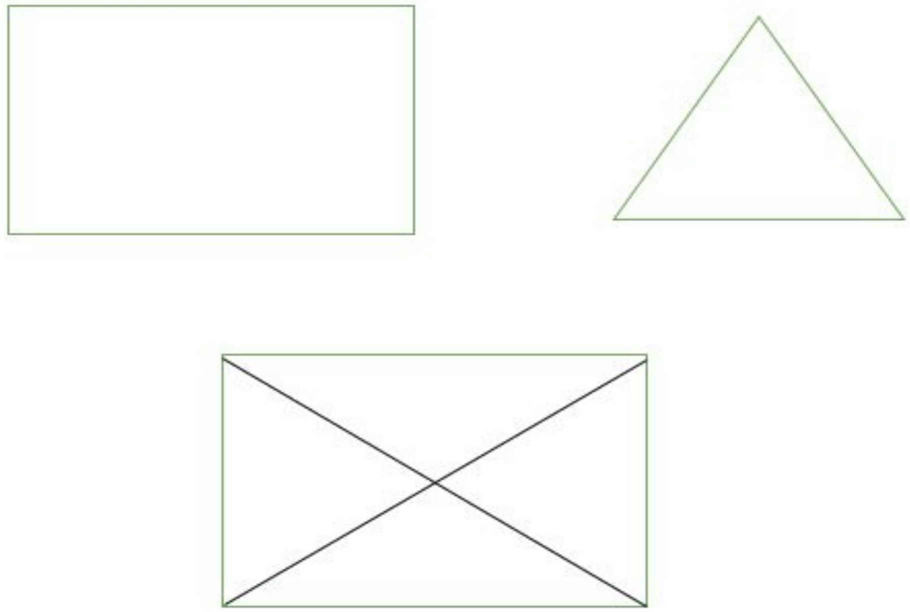
8. Pseudo Graph: A graph G with a self loop and some multiple edges is called pseudo graph.



7. Complete Graph



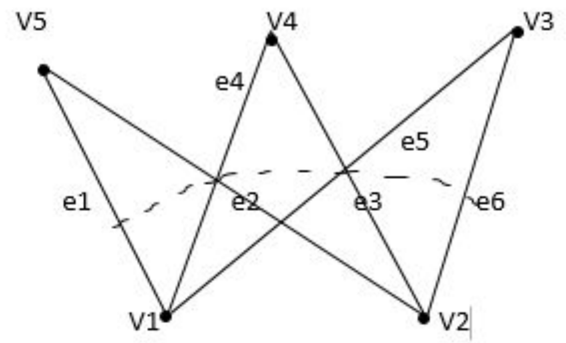
8. Pseudo Graph



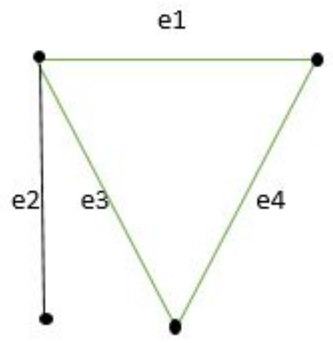
9. **Regular Graph**

9. **Regular Graph:** A simple graph is said to be regular if all vertices of a graph G are of equal degree. All complete graphs are regular but vice versa is not possible.

10. Bipartite Graph: A graph G = (V, E) is said to be bipartite graph if its vertex set V(G) can be partitioned into two non-empty disjoint subsets. V1(G) and V2(G) in such a way that each edge e of E(G) has its one end in V1(G) and other end in V2(G).
The partition V1 U V2 = V is called Bipartite of G.
Here in the figure:
V1(G)={V5, V4, V3}
V2(G)={V1, V2}



10. Bipartite Graph



11. Labelled Graph

11. Labelled Graph: If the vertices and edges of a graph are labelled with name, data or weight then it is called labelled graph. It is also called *Weighted Graph*.
12. **Connected or Disconnected Graph:** A graph G is said to be connected if for any pair of vertices (Vi, Vj) of a graph G are reachable from one another. Or a graph is said to be connected if there exist atleast one path between each and every pair of vertices in graph G, otherwise it is disconnected. A null graph with n vertices is disconnected graph consisting of n components. Each component consist of one vertex and no edge.

Two Important kinds of graphs

- Directed
- Undirected

1. A **directed** graph, or **digraph**, is a graph in which the edges are ordered pairs
    - $(v, w) \neq (w, v)$
2. An **undirected** graph is a graph in which the edges are unordered pairs
    - (v, w) == (w, v)

Directed vs. Undirected Graphs

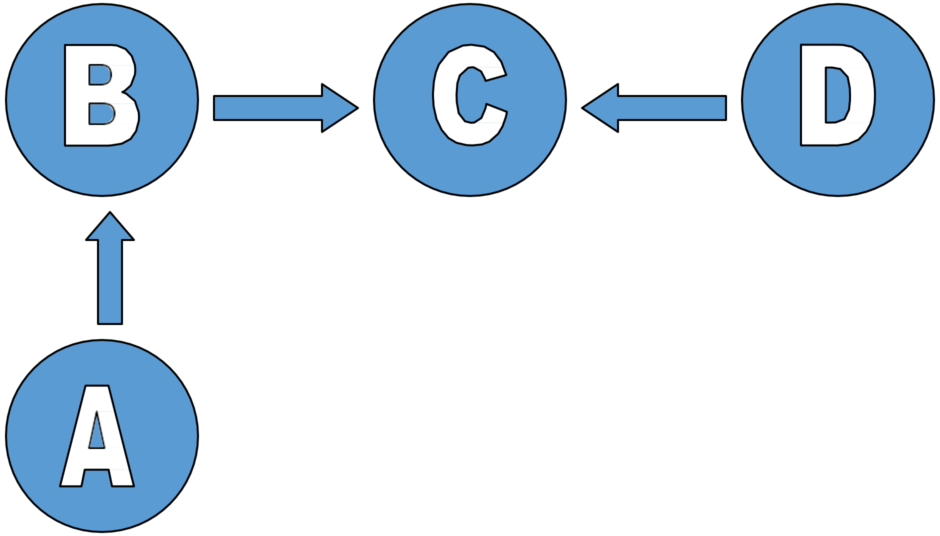
- Undirected edge has no orientation (no arrow head)
- Directed edge has an orientation (has an arrow head)
- Undirected graph – all edges are undirected
- Directed graph – all edges are directed

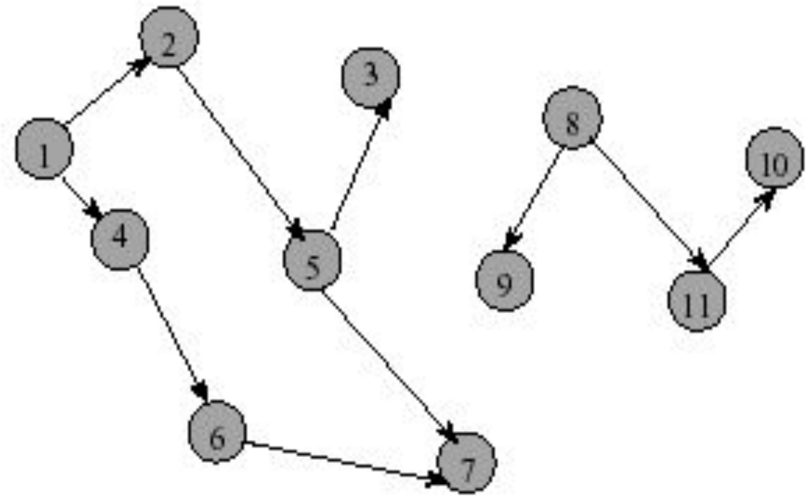
undirected edge

directed edge

Introduction: Directed Graphs

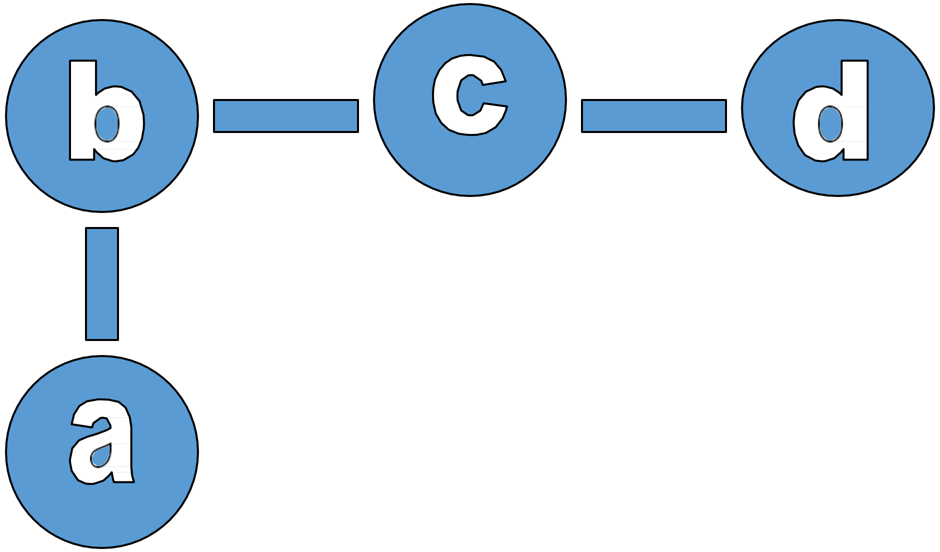
- In a directed graph, the edges are arrows.
- Directed graphs show the flow from one node to another and not vise versa.

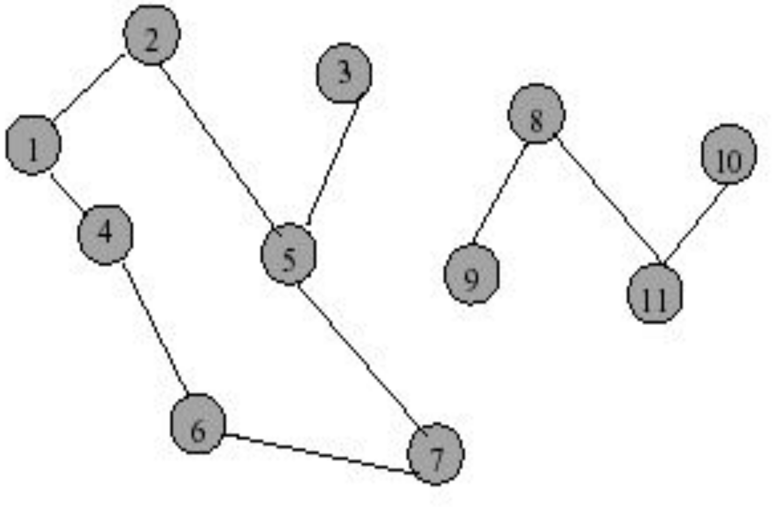




Introduction: Undirected Graphs

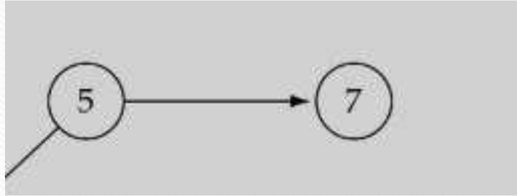
- In a Undirected graph, the edges are lines.
- UnDirected graphs show a relationship between two nodes.





Graph terminology

- Adjacent nodes: two nodes are adjacent if they are connected by an edge

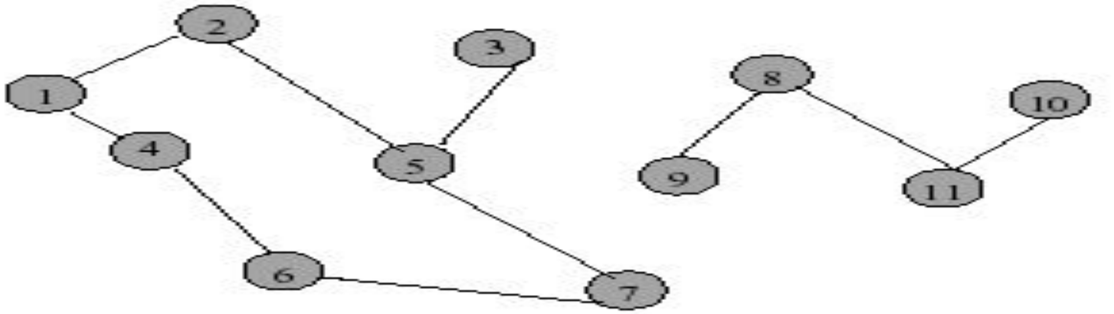


5 is adjacent to 7
7 is adjacent from

- Path: a sequence of vertices th at connect two nodes in a graph
- A simple path is a path in which all vertices, except possibly in the first and last, are different.
- Complete graph: a graph in which every vertex is directly connected to every other vertex

Terminology

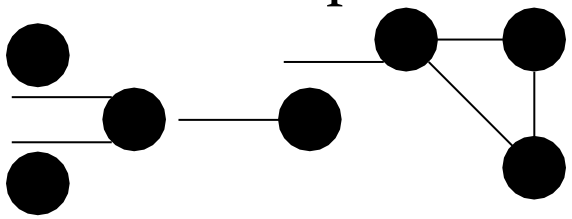
- A cycle is a simple path with the same start and end vertex.
- The **degree** of vertex *i* is the no. of edges incident on vertex *i*.



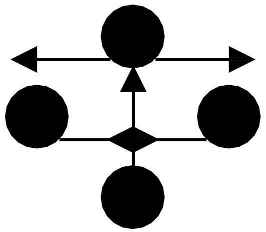
e.g., degree(2) = 2, degree(5) = 3, degree(3) = 1

Terminology

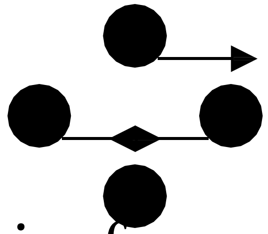
Undirected graphs are *connected* if there is a path between any two vertices



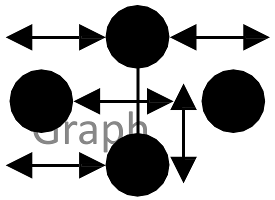
Directed graphs are *strongly connected* if there is a path from any one vertex to any other



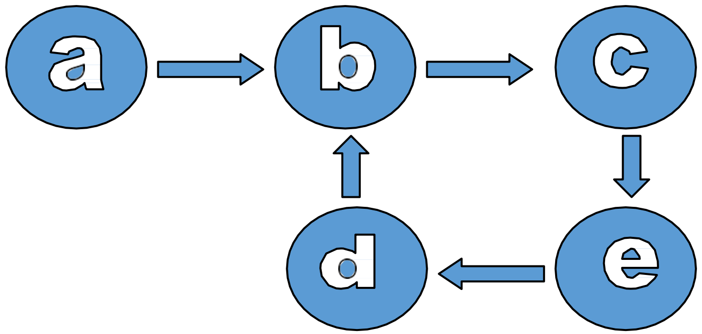
Directed graphs are *weakly connected* if there is a path between any two vertices, *ignoring direction*



A *complete* graph has an edge between every pair of vertices



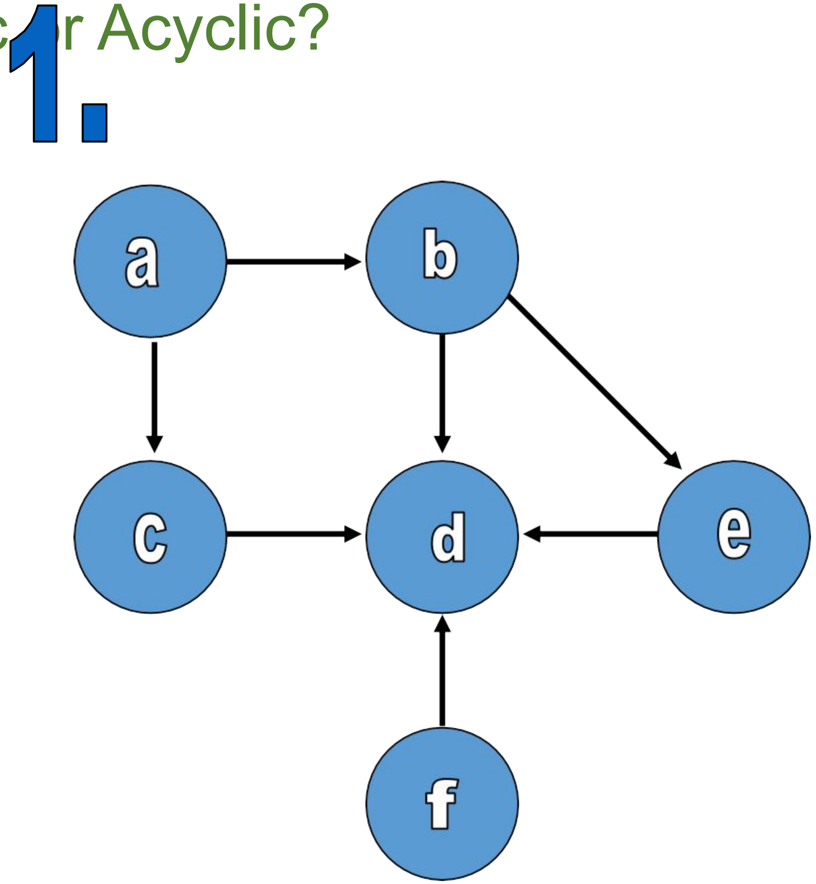
Terminology

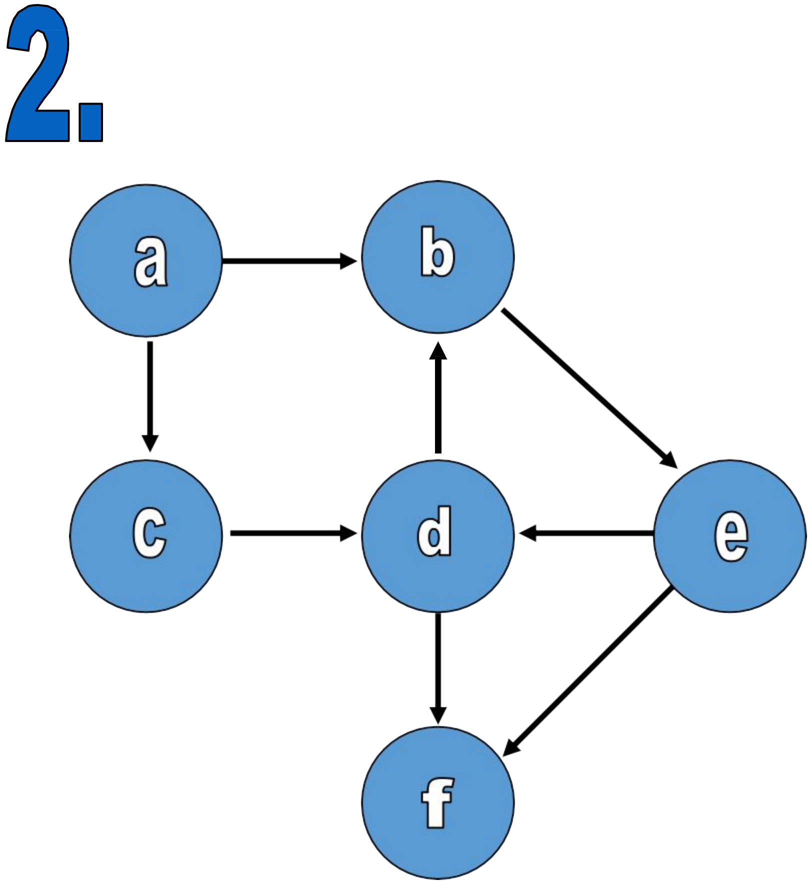


- An **acyclic path** is a path which does not follow a sequence.
- A **cyclic path** is a path such that
  - There are at least two vertices on the path
  - $w_1 = w_n$ (path starts and ends at same vertex)
  - And also maintains the sequence

Test Your Knowledge

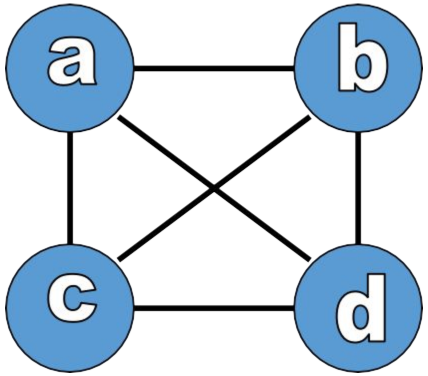
Cyclic or Acyclic?





Terminology

- A directed graph that has *no* cyclic paths is called a
  DAG (a Directed Acyclic Graph).
- An undirected graph that has an edge between every pair of vertices is called a **complete** graph.

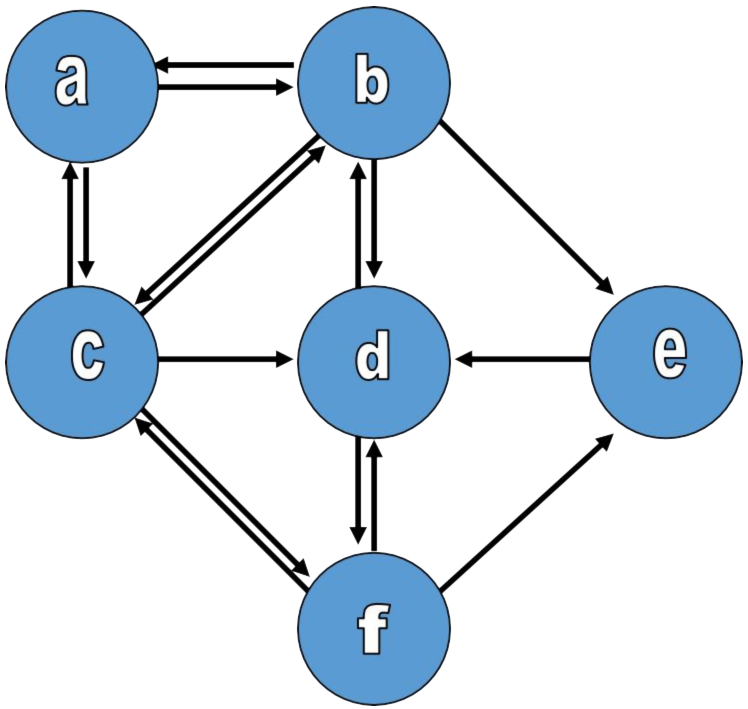


Note: A directed graph can also be a complete graph; in that case, there must be an edge from every vertex to every other vertex.

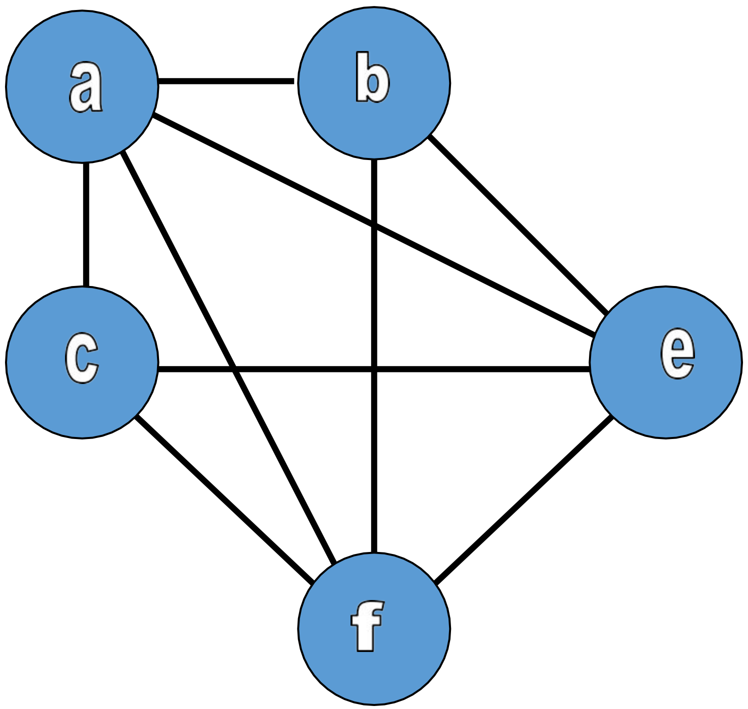
Test Your Knowledge

Complete, or “Acomplete” (Not Complete)

1.



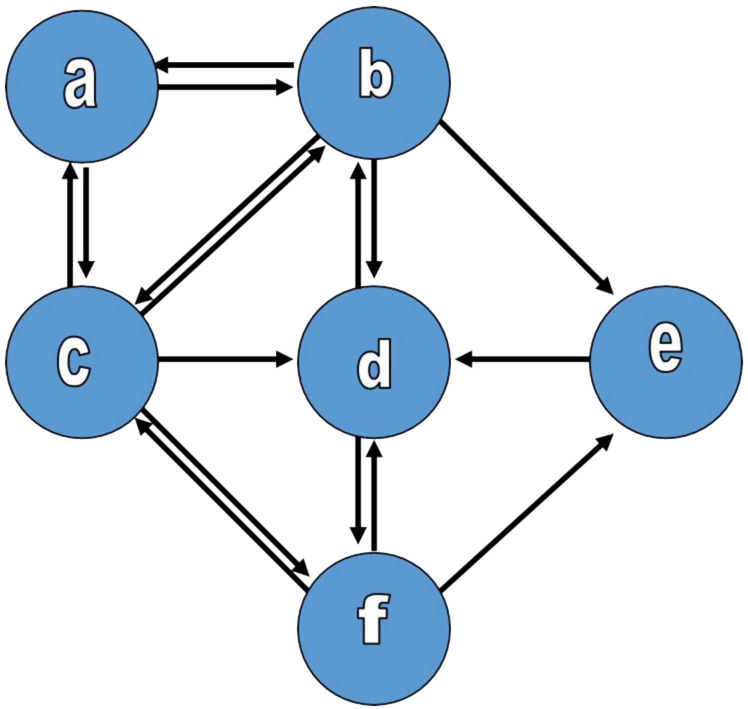
2.



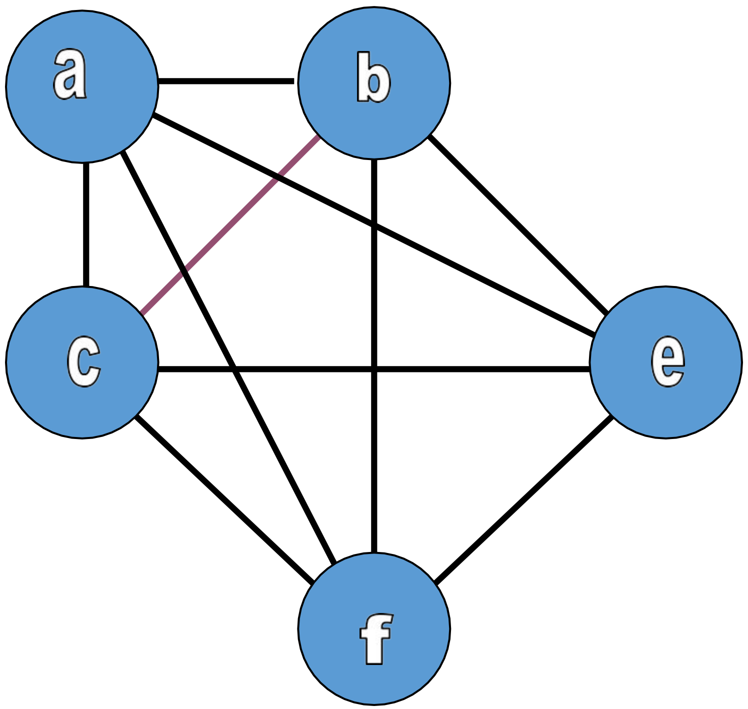
Test Your Knowledge

Complete, or “Acomplete” (Not Complete)

1.



2.

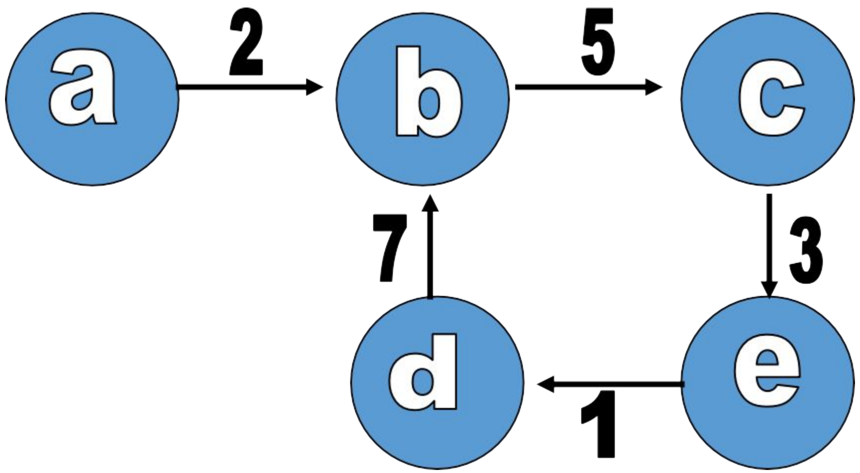


Terminology

- An undirected graph is **connected** if a path exists from every vertex to every other vertex
- A directed graph is **strongly connected** if a path exists from every vertex to every other vertex
- A directed graph is **weakly connected** if a path exists from every vertex to every other vertex, disregarding the direction of the edge

Terminology

- A graph is known as a **weighted graph** if a weight or metric is associated with each edge.



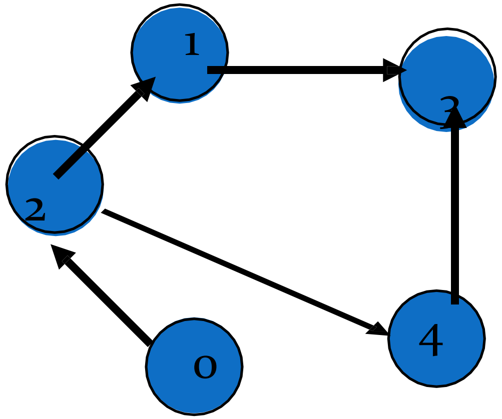
Representation of Graph

Graph Representation

- For graphs to be computationally useful, they have to be conveniently represented in programs
- There are two computer representations of graphs:
  - Adjacency matrix representation
  - Adjacency lists representation

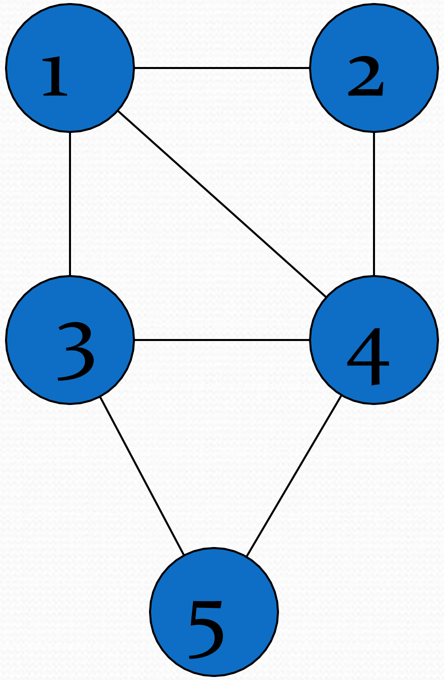
Adjacency Matrix

- A square grid of boolean values
- If the graph contains N vertices, then the grid contains N rows and N columns
- For two vertices numbered I and J, the element at row I and column J is true if there is an edge from I to J, otherwise false



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{pmatrix}
\text{false} & \text{false} & \text{true} & \text{false} & \text{false} \\
\text{false} & \text{false} & \text{false} & \text{true} & \text{false} \\
\text{false} & \text{true} & \text{false} & \text{false} & \text{true} \\
\text{false} & \text{false} & \text{false} & \text{false} & \text{false} \\
\text{false} & \text{false} & \text{false} & \text{true} & \text{false}
\end{pmatrix}
\end{array}
$$

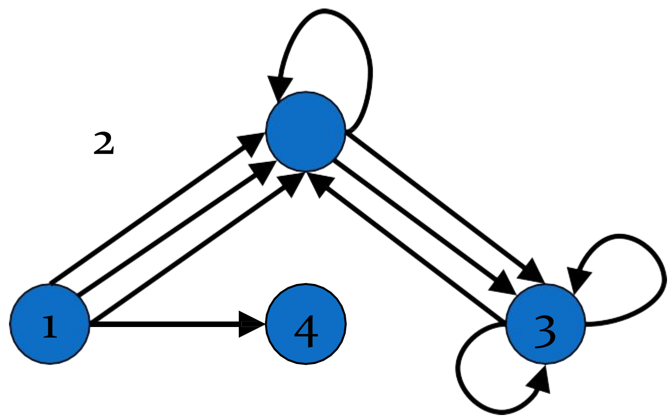
Adjacency Matrix



	1	2	3	4	5
1	0	1	1	1	0
2	1	0	0	1	0
3	1	0	0	1	1
4	1	1	1	0	1
5	0	0	1	1	0

Adjacency Matrix

Directed Multigraphs



A:

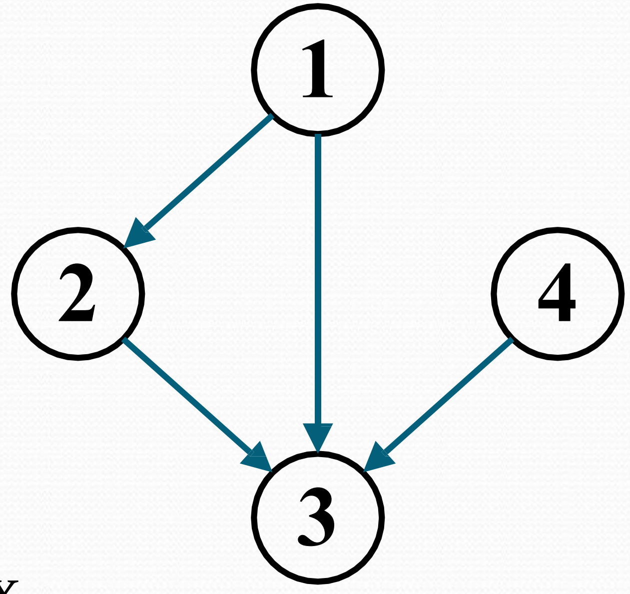
$$\begin{pmatrix} 0 & 3 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency Lists Representation

- A graph of n nodes is represented by a one-dimensional array L of linked lists, where
  - L[i] is the linked list containing all the nodes adjacent from node i.
  - The nodes in the list L[i] are in no particular order

Graphs: Adjacency List

- Adjacency list: for each vertex $v \in V$, store a list of vertices adjacent to v
- Example:
  - Adj[1] = {2,3}
  - Adj[2] = {3}
  - Adj[3] = {}
  - Adj[4] = {3}
- Variation: can also keep a list of edges coming *into* vertex



Graphs: Adjacency List

- How much storage is required?
  - The *degree* of a vertex v = # incident edges
    - Directed graphs have in-degree, out-degree
  - For directed graphs, # of items in adjacency lists is

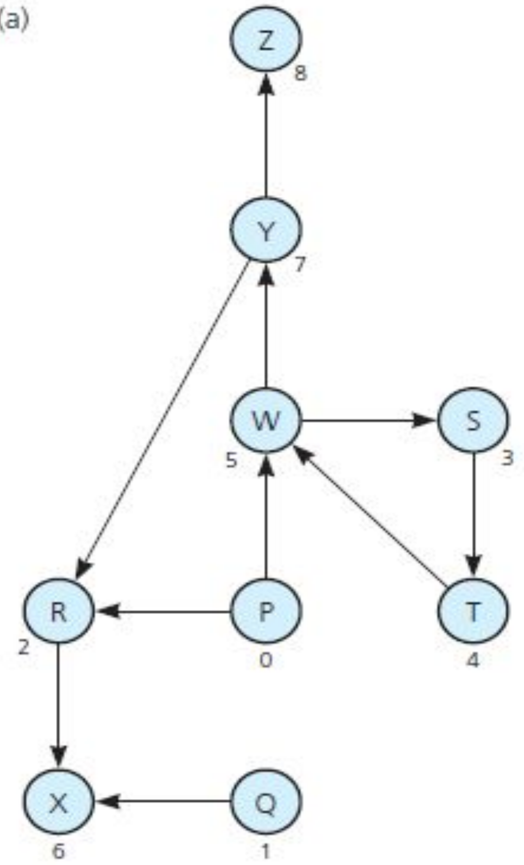
    $$\Sigma \text{ out-degree}(v) = |E|$$

    For undirected graphs, # items in adjacency lists is

    $$\Sigma \text{ degree}(v) = 2\,|E|$$
- So: Adjacency lists take O(V+E) storage

Implementing Graphs

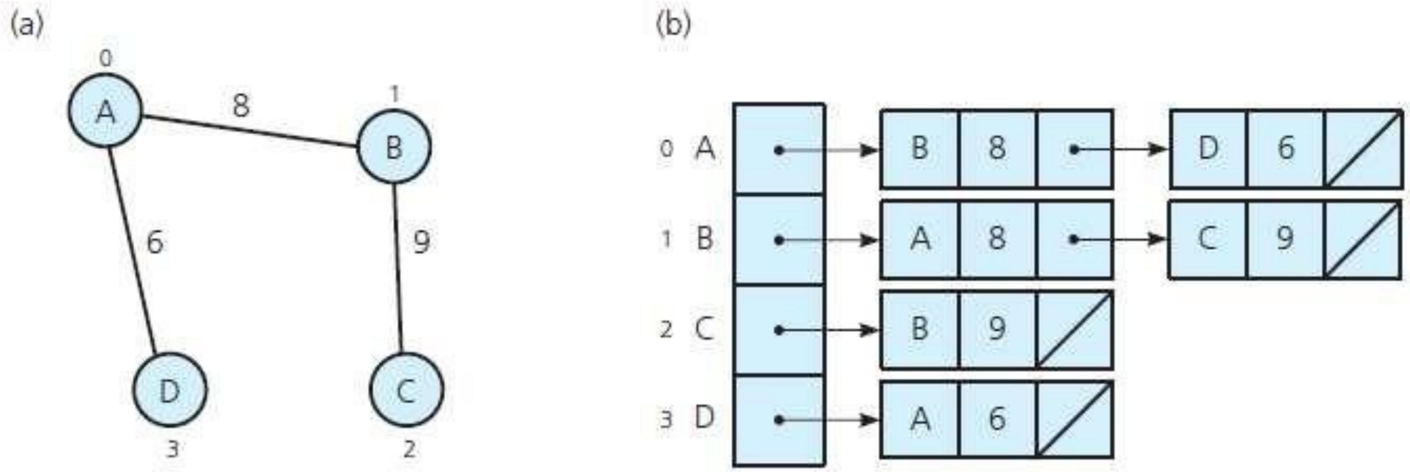
(a)



(b)

		0	1	2	3	4	5	6	7	8
		P	Q	R	S	T	W	X	Y	Z
0	P	0	0	1	0	0	1	0	0	0
1	Q	0	0	0	0	0	0	1	0	0
2	R	0	0	0	0	0	0	1	0	0
3	S	0	0	0	0	1	0	0	0	0
4	T	0	0	0	0	0	1	0	0	0
5	W	0	0	0	1	0	0	0	1	0
6	X	0	0	0	0	0	0	0	0	0
7	Y	0	0	1	0	0	0	0	0	1
8	Z	0	0	0	0	0	0	0	0	0

Implementing Graphs



- (a) A weighted undirected graph and
  (b) its adjacency list

Uses For Graphs

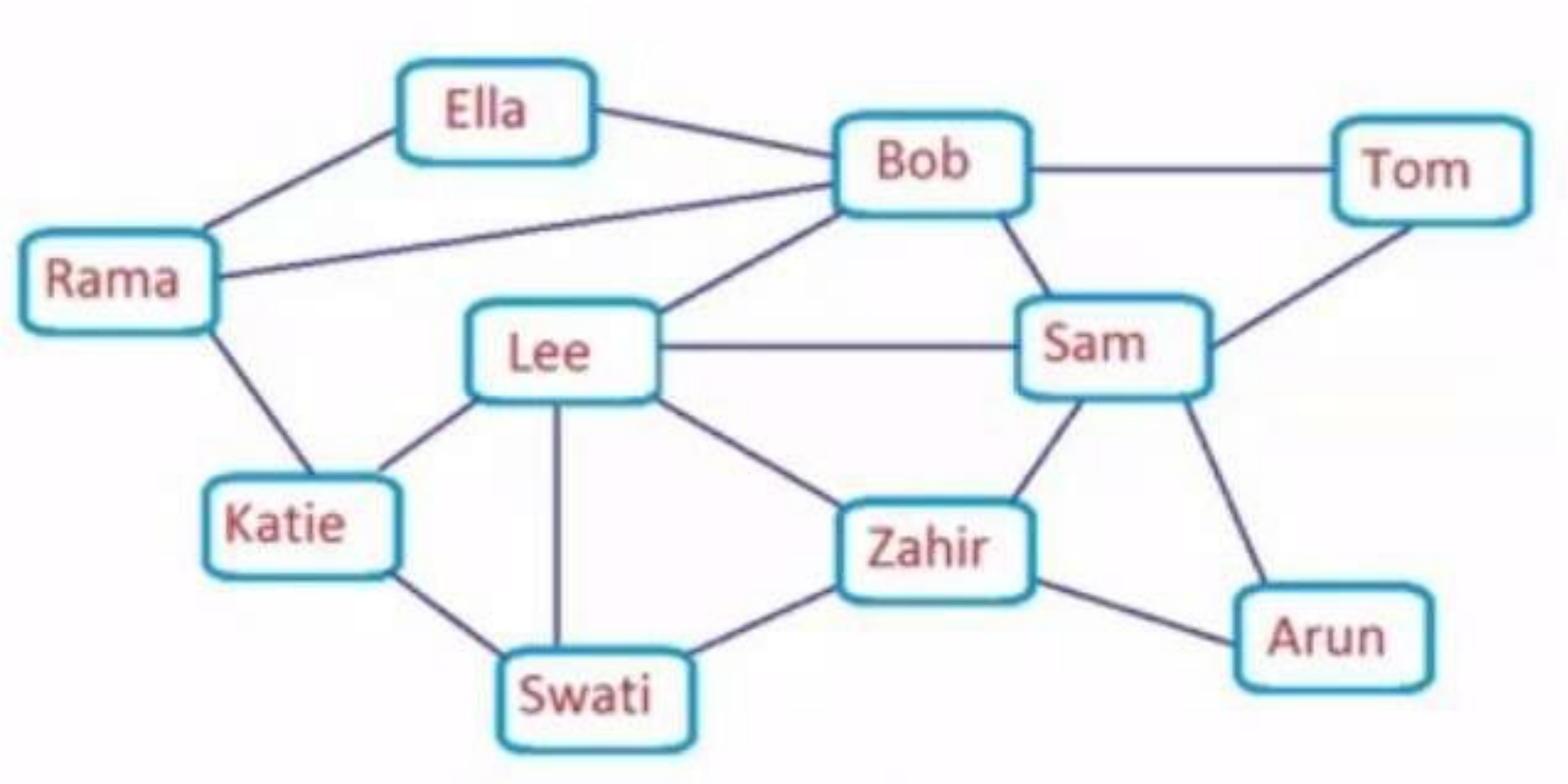
Uses for Graphs

- **Computer network:** The set of vertices V represents the set of computers in the network. There is an edge (u, v) if and only if there is a direct communication link between the computers corresponding to u and v.

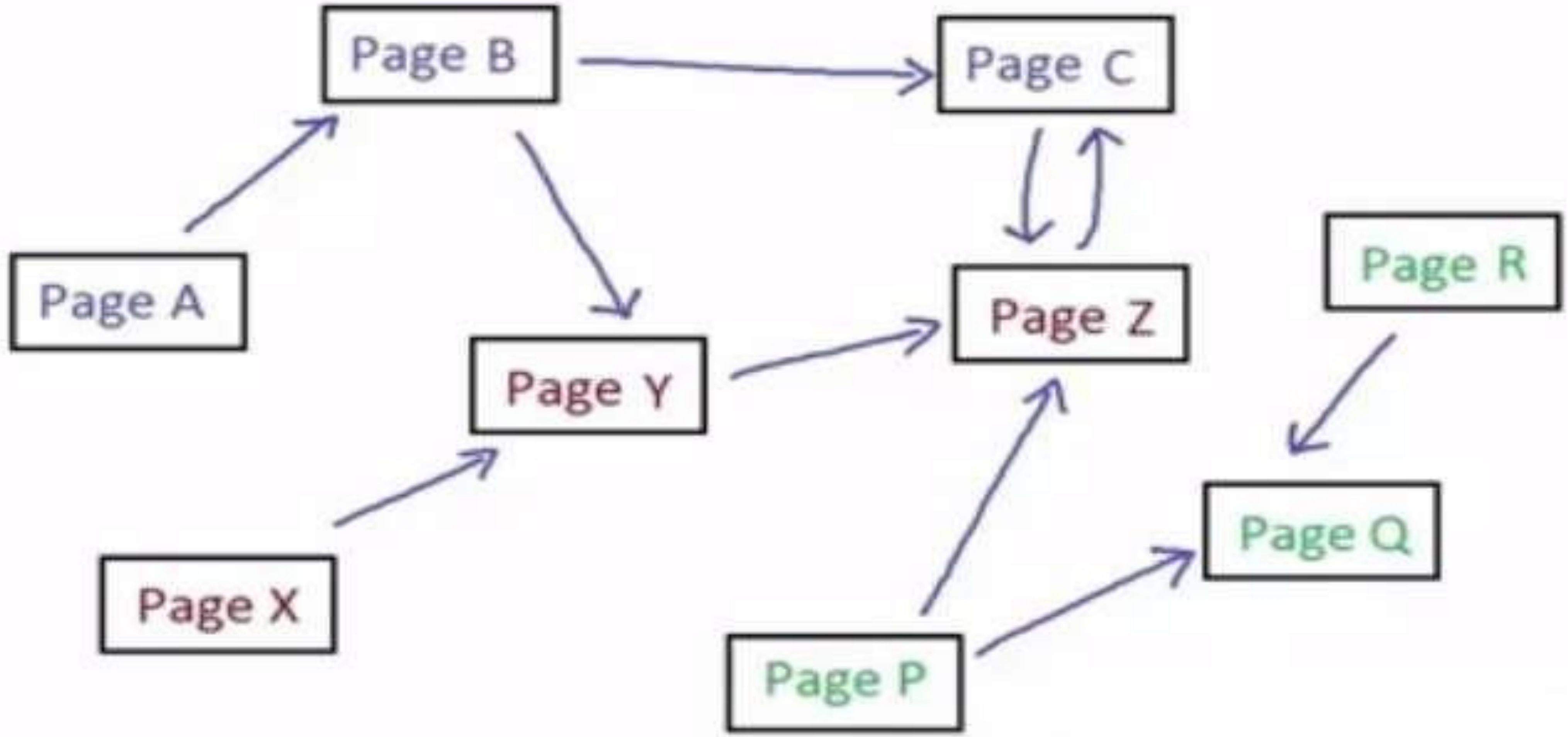
Uses for Graphs

- **Two-Player Game Tree:** All of the possibilities in a board game like chess can be represented in a graph. Each vertex stands for one possible board position. (For chess, this is a very big graph!)

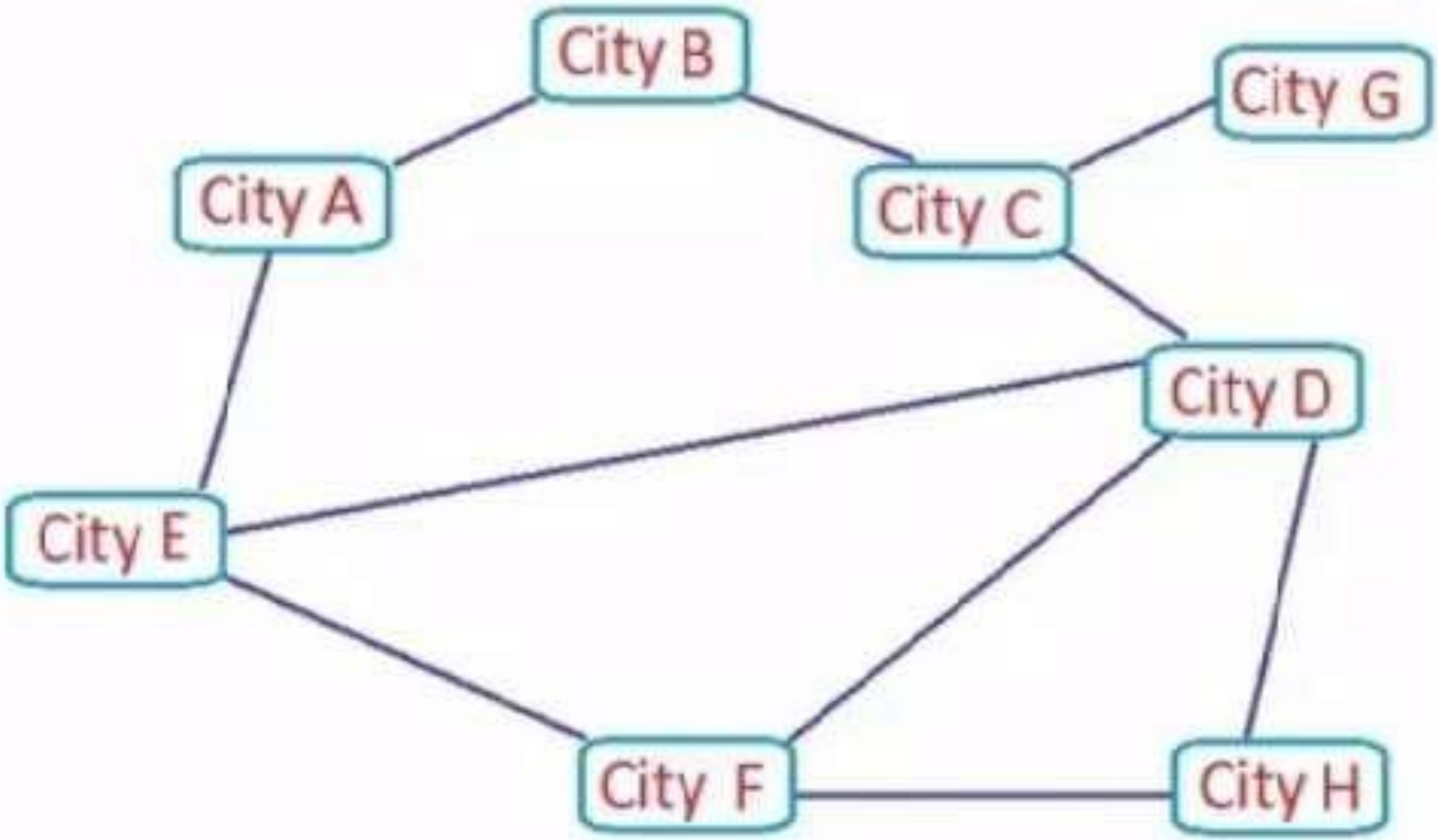
Social Media (Facebook)



Website
s



Intercity Road Network



Thank you