$$= -17.72 \times 10^{-9}\,C = -1.77 \times 10^{-8}\,C$$

Problem 3.4 *If the average distance between the sun and earth is* 1.5×10^{11} *m and power radiated by sun is* 3.8×10^{26} *W, show that the average solar energy incident on the earth's surface is 2 cal/cm*^{2} *min.*

[GGSIPU, Feb. 2009 (2 marks)]

Solution. As total power (P) is radiated uniformly, energy flux per unit area per second at a distance r (the distance between the sun and earth) is given by

1 ca = 4.125

1 J = 1/4.1

$$S = \frac{\text{Energy}}{A \times t} = \frac{P}{A} = \frac{\cancel{\text{Energy}}}{\text{Area}} = \frac{\text{Joule}}{\text{sec}}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = E\,H \sin 90^\circ$$

$$= \frac{P_0}{4\pi r^2} = \frac{3.8 \times 10^{26}\ \text{W}}{4 \times \pi \times (1.5 \times 10^{11}\ \text{m})^2} = 1344.656\ \text{W/m}^2$$

So, the average solar energy per minute is given as

$$= \frac{1344.656 \times 60}{4.18 \times 10^4}\ \text{cal/cm}^2\ \text{min}$$

$$= 1.9\ \text{cal/ cm}^2\ \text{min} = 2\ \text{cal/ cm}^2\ \text{min.}$$

Problem 3.7 *Assuming that all the energy from a 1000 W lamp is radiated uniformly, calculate the average values of intensities of electric and magnetic fields of radiations at a distance of 2 m from the lamp.*

Or

Considering that all the energy from a 1000 W lamp is radiated uniformly, calculate the average of intensity of electric field of radiation at a distance of 2 m, away from the lamp. **[GGSIPU, Feb. 2014 (4 marks)]**

Solution. If the total power P_0 is radiated uniformly in all directions, then the power or energy flux per unit area per second at a distance r from the point source (*i.e.*, lamp) is

$$S_{av} = \frac{P_0}{4\pi r^2} = \frac{1000}{4\pi(2)^2} \text{ W/m}^2$$

From the definition of Poynting vector.

$$|\mathbf{S}| = (\mathbf{E} \times \mathbf{H}) = EH \sin 90^\circ \qquad (\because E \text{ and } H \text{ are } \perp \text{ to each other})$$

$$EH = \frac{1000}{16\pi} (\Omega)^{-1} \qquad \text{...}(i)$$

But

$$Z = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 376.72\ \Omega \qquad \text{...}(ii)$$

Multiplying Eqs. (*i*) and (*ii*), we get

$$EH . \frac{E}{H} = \frac{376.72 \times 1000}{16\pi} \quad \text{or} \quad E = \sqrt{\frac{376.72 \times 1000}{16\pi}} = 86.59 \text{ V/m}$$

From Eq. (*i*),

$$H = \frac{1000}{16\pi E} = \frac{1000}{16 \times 3.14 \times 86.59} = 0.23 \text{ A/m}$$

Problem 3.9 *A plane electromagnetic wave travelling in the positive z-direction in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 3$ has an electric field intensity $E = 6$ V/m. Find (i) speed of the em waves in the given medium (ii) impedance of the medium.*

[GGSIPU, Feb. 2005 (2 marks), May 2017 (2.5 marks)]

Solution. Given $\mu_r = 1,\ \varepsilon_r = 3,\ E_0 = 6$ V/m

but $\varepsilon = \varepsilon_r \varepsilon_0$ and $\mu = \mu_r \mu_0$, where ε and μ are the permittivity and permeability of the medium and ε_0 and μ_0 corresponding constant for free space.

(*i*) The speed of the electromagnetic waves in given medium

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \times \varepsilon_r}} = \frac{3\times 10^8}{\sqrt{1\times 3}} = \sqrt{3}\times 10^8 = 1.732\times 10^8 \text{ m/s}$$

(*ii*) The impedance Z of the medium is given by

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \times \mu_0}{\varepsilon_r \times \varepsilon_0}}$$

where $\mu_0 = 4\pi\times 10^{-7}$ H/m and $\varepsilon_0 = 8.86\times 10^{-12}$ F/m, we get

$$Z = \sqrt{\frac{4\pi\times 10^{-7}\times 1}{8.86\times 10^{-12}\times 3}} = 2.17\times 10^2\ \Omega$$

3.4 A plane electromagnetic wave travelling in +Z-direction in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 3$, has peak electric field intensity $E_0 = 6$ V/m. Find :

(i) Speed of the wave

(ii) Independence of the medium

(iii) Peak magnetic field intensity. **[GGSIPU, Feb. 2011 (3 marks) ; Feb. 2011 (4 marks-reappear)]**

Hint : Go through Problem 3.9 at page 161 for parts (i) and (ii)

(iii) Peak magnetic field intensity

$$H_0 = \frac{E_0}{Z} = \frac{6}{217.6} = 2.76 \times 10^{-2} \text{ A / m.}$$

$\sqrt{\mu\sigma\omega} \quad \sqrt{\mu\,\sigma(2\pi\nu)}$

3.9 A 2 kW laser beam is concentrated by a lens into cross-sectional area about 10^{-6}cm^{-2}. Find the Poynting vector. **[GGSIPU, May 2016 (2 marks)]**

Hint : $\vec{S} = \dfrac{p}{A} = \dfrac{2\times10^3}{10^{-6}\times10^{-4}} = 2\times10^{10}\ \text{kW/m}^2.$

3.10 Calculate the magnitude of Poynting vector at the surface of the sun. Given that power radiated by the sun $= 3.8\times10^{26}$ watt and radius of sun $= 7\times10^8$ m. **[GGSIPU, Feb. 2011 ; Feb. 2013 (3 marks)]**

Hint : Poynting vector at the surface of the sun,

Power radiated per unit area $= |s| = (3.8\times10)^{26}/(4\pi\times7\times7\times10^{16}) = 4.87\times10^7\ \text{Jm}^{-2}\text{s}^{-1}.$

3.11 In a plane e.m.-wave in free space, the electric field oscillates at a frequency of 2×10^{14} Hz and amplitude 5 V/m. Find (*i*) the wavelength of the wave and (*ii*) the amplitude of oscillating magnetic field. **[GGSIPU, Feb. 2012 (2 marks)]**

Hint : Wavelength of the wave $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{14}} = 1.5 \times 10^{-6}$ m $= 1.5$ μm ;

Amplitude of oscillating magnetic field $B_0 = \frac{E_0}{c} = \frac{5}{3 \times 10^8} = 1.67 \times 10^{-8}$ T.

3.15 Calculate the penetration depth for 2 MHz e.m. wave through copper. Given : $\sigma = 5.8 \times 10^7$ S/m, $\mu = 4\pi \times 10^{-7}$.

[GGSIPU, June 2015 Reappear (3 marks); May, 2007 (1.5 marks) ; Feb. 2010 (3 marks), May 2019 (2.5 marks)]

Hint : $$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = \sqrt{\frac{2}{\mu\sigma(2\pi\nu)}}$$

$$= 46.7\ \mu\text{m}$$

3.16 Find the skin depth at a frequency 1.6 MHz in aluminium where $\sigma = 38.2$ MS/m and $\mu = 1$.

[GGSIPU, Feb 2008 (2 marks)]

Hint : $$\delta = \sqrt{\left(\frac{2}{\mu\sigma\omega}\right)} = \sqrt{\frac{2}{\mu\sigma(2\pi\nu)}}$$

$$= 7.21 \times 10^{-8}\text{m}.$$