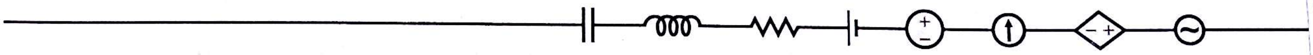
CHAPTER

10

Analysis of Special Signal Waveforms



10.1 INTRODUCTION

Circuits frequently experience response of the sources having complicated waveforms such as rectangular, pulse, square wave, ramp type etc. in addition to step d.c., exponential or sinusoidal (the circuit responses to these signals have been discussed amply in the other parts of this book). The signals may be recurrent type or non-recurrent type and mostly involve the passive circuits. This chapter deals with analysis of these basic types of signals, termed as *special signals*, which are reasonably common.

10.2 BASIC TYPES OF SPECIAL SIGNALS

Shifted Unit Step Function

This function remains at zero value till $t = a$ and at $t = a$, it abruptly changes from 0 to unity.

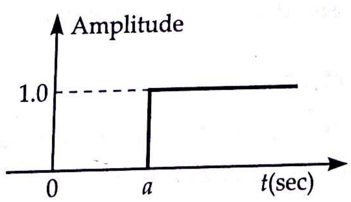


Fig. 10.1 Shifted Unit Step function $u(t-a)$.

It is expressed as

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases}$$

and is shown in Fig. 10.1 [the quantity $(t-a)$ is called the *argument* of the function].

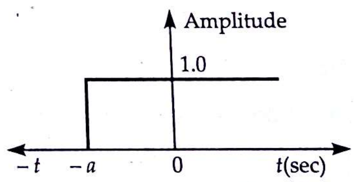


Fig. 10.2 Shifted Unit step function $u(t+a)$.

Figure 10.2 shows another shifted unit step function which remains at zero value as long as $(t+a)$ *i.e.*, the argument is less than 0. The function abruptly changes to unity value at time $t = -a$.

Obviously,

$$u(t+a) = \begin{cases} 0, & t < -a \\ 1, & t \geq -a \end{cases}$$

Similarly, two other forms of shifted unit step functions are shown in Figs. 10.3 and 10.4. The function in Fig. 10.3 is $u(a-t)$ and remains zero so long as $(a-t)$, the argument, is negative *i.e.*,

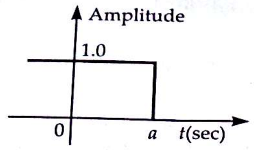


Fig. 10.3 Shifted Unit Step function $u(a-t)$.

$(a-t)<0$ or $t>a$. It abruptly rises to unity value for positive value of argument $(a-t)$ *i.e.*, when $t<a$.

Obviously, $u(a-t)=\begin{cases}0, & t>a\\1, & t\le a\end{cases}$

Figure 10.4 represents another form of shifted unit step function which remains of zero value as long as the argument $(-a-t)$ is negative *i.e.*, till $(-t-a)<0$ or,

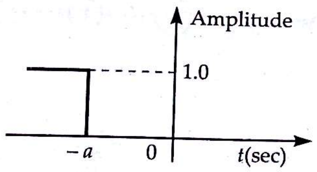


Fig. 10.4 Shifted Unit Step function $u(-t-a)$.

$t>-a$. The value of the function abruptly becomes unity at $t=-a$ and remains so for $t<-a$.

It is represented as

$$u(-t-a)=\begin{cases}0, & t>-a\\1, & t\le -a\end{cases}$$

Ramp Function

When a time variant current or voltage increases linearly with time, it is known as *ramp function* or simply an ordinary *ramp* (Fig. 10.5).

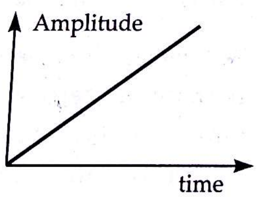


Fig. 10.5 Ramp function

To give a practical example of a ramp function, let us consider a unit step voltage applied to an inductor of L Henry (Fig. 10.6).

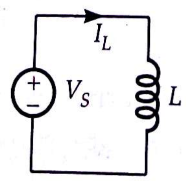


Fig. 10.6 Example to show ramp function.

Obviously, $I_L=\frac{1}{L}\int_{-\infty}^{t} V_S\, dt$

where $V_S=u(t)=1.0$

Assuming zero initial condition, $i_L(0+)=0$.

$$\therefore \qquad I_L=\frac{t}{L}.$$

This response current constitutes the ramp function where the inductor current is seen to vary linearly with time (Fig. 10.6(*a*)).

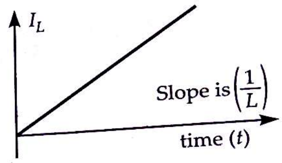


Fig. 10.6 (*a*) Inductor current *vs* time.

Similarly, if a unit step current is applied to a capacitor C, the capacitor voltage would be

$$V_C=\frac{1}{C}\int_{-\infty}^{t} i_C\, dt=\frac{1}{C}\int_{-\infty}^{t} u(t)\, dt.$$

This, with zero initial condition gives [*i.e.*, V_C at $0^+=0$]

$$V_C=\frac{t}{C}.$$

This voltage response constitutes the ramp function where the capacitor voltage is seen to vary linearly with time. Thus, the response of unit step function of voltage to inductor and of current to capacitor gives us the ramp function of response *i.e.*, the inductor current and capacitor voltage. To clarify further, it can be said that the *integration of unit step function results in unit ramp function* (Fig. 10.7).

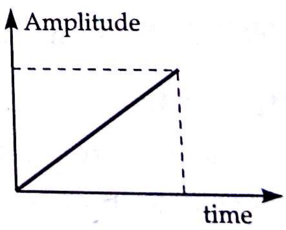


Fig. 10.7 Unit Ramp function.

Impulse Function

When a current i_L flows through an inductor, the voltage induced is given by

$$v_L=L\frac{di_L}{dt} \quad (v_L \text{ is directed to oppose } i_L)$$

Assuming $i_L = u(t)$, the step function, $\frac{di_L}{dt} = 0$.

Thus the derivative $\frac{di_L}{dt}$ is zero for all time except $t = 0$ *i.e.*, at the moment when stepping occurs. At $t = 0$, the slope would become infinite as $\frac{di_L}{dt}$ is infinite at $t = 0$. Thus the response of step current at the inductor makes an impulse [*i.e.*, the function is infinite at $t = 0$ and zero at $t > 0$] (Fig. 10.8). Similarly, application of unit step voltage to a capacitor C results in impulse current given by

$$i_C = C\frac{dv_C}{dt}$$

$$\left[\text{obviously, } \frac{dv_C}{dt} = 0 \text{ at } t > 0 \text{ and infinity at } t = 0\right]$$

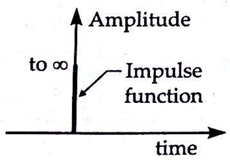


Fig. 10.8 Impulse function.

The above examples clearly show that :

Differentiation of step function results in impulse function.

An unit impulse function is denoted by $\delta(t - t_1)$, where t_1 is the instant of time at which the pulse occurs. Obviously, if the pulse occurs at origin of the axis, $t_1 = 0$ leading to the unit impulse function to be $\delta(t)$. Further, as the differentiation of step function results to impulse function, hence we can say that :

The integration of unit impulse function would result in a unit step function.

i.e., $$\int_{-\infty}^{t} \delta(t - t_1)\,dt \begin{cases} = 0 & \text{for } t < t_1 \\ = 1 & \text{for } t \geq t_1 \end{cases}$$

and $$\frac{d}{dt}u(t - t_1) = \delta(t - t_1) = \begin{cases} 0 & \text{for } t \neq t_1 \\ = \infty & \text{for } t = t_1. \end{cases}$$

[Refer to Fig. 10.9]

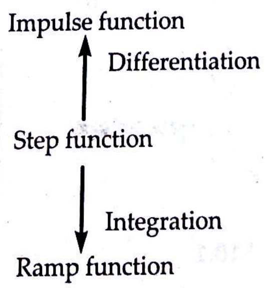


Fig. 10.9

Unit Doublet Function

Differentiating a unit impulse function with respect to t, we find

$$\frac{d}{dt}\delta(t - t_1)\begin{cases} = 0, & t \neq t_1 \\ = +\infty \text{ and } -\infty, & t = t_1. \end{cases}$$

The function $\frac{d}{dt}\delta(t - t_1)$ is called *unit doublet function*. Fig. 10.10 represents such a function. For academic interest, it may be noted that starting from unit step function, successive integrations result in new functions (as discussed above and summarised below in Fig. 10.11).

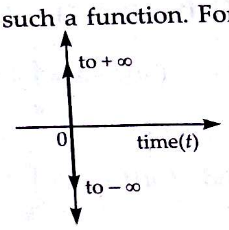


Fig. 10.10 Unit doublet function.

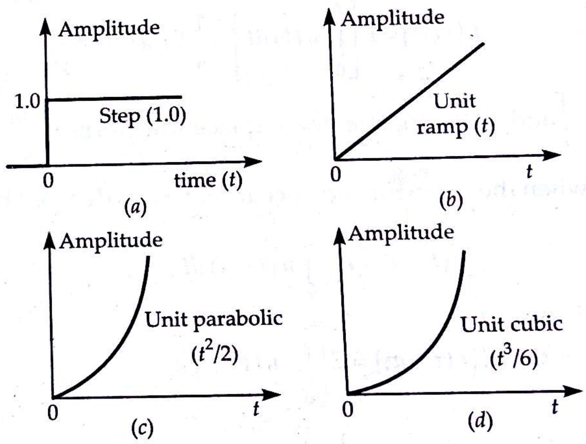


Fig. 10.11 Successive integration of unit step function ; (on integration of unit step function of Fig. (*a*) results to unit ramp of Fig. (*b*) which on further integration leads to unit parabolic of Fig. (*c*) and then to unit cubic function of Fig. (*d*))

It may be recalled here that :

Differentiation of unit step function gives unit impulse function which on further differentiation gives unit doublet function (Fig. 10.12).

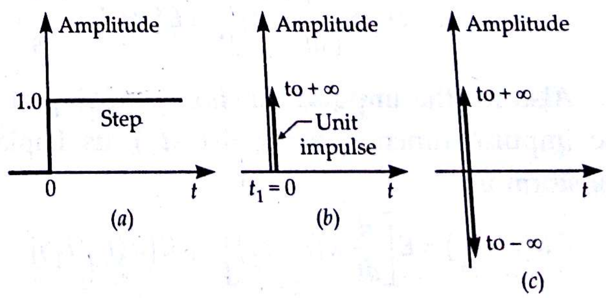


Fig. 10.12 Differentiation of Fig. (*a*) gives Fig. (*b*) *i.e.*, unit impulse function which on further differentiation gives unit doublet function (Fig. (*c*))

CHAPTER 10

10.3 LAPLACE TRANSFORMATION OF SPECIAL SIGNAL WAVEFORMS

Unit Step Function

$$£\,u(t) = \int_0^\infty 1.e^{-st}\,dt = \left[\frac{-e^{-st}}{s}\right]_0^\infty = \frac{1}{s}.$$

Shifted Unit Step Function

$$£\,u(t-a) = \int_a^\infty 1.e^{-st}dt = \left[\frac{-e^{-st}}{s}\right]_a^\infty = \left(\frac{1}{s}\right)e^{-as}$$

and $£\,u(t+a) = \int_{-a}^\infty 1.e^{-st}dt = \left[\frac{-e^{-st}}{s}\right]_{-a}^\infty = \left(\frac{1}{s}\right)e^{as}.$

Ramp Function

$$£[r(t)] = £\left[\int_0^t u(t)\,dt\right] = \frac{1}{s}£[\,u(t)] = \frac{1}{s^2},$$

[and with slope m, the Laplace transform is $\frac{m}{s^2}$]

when the ramp function occurs at $t = a$, is denoted by

$$r(t-a)\ \text{i.e.,}\ \int_0^t u(t-a)\,dt\,;$$

$$\therefore\quad £[r(t-a)] = £\left[\int_0^t u(t-a)\,dt\right];$$

$$= \frac{1}{s}£\,u(t-a)$$

$$= \frac{1}{s}.\frac{1}{s}.e^{-as} = \frac{e^{-as}}{s^2}.$$

[with slope m, the ramp function becomes $m\frac{e^{-as}}{s^2}$ in s-domain].

Impulse Function

$$£\,\delta(t) = £\left[\frac{d}{dt}u(t)\right] = s\,£[u(t)] = s.\frac{1}{s} = 1.$$

Also for the impulse function occurring at t_1, the impulse function being $\delta(t-t_1)$, its Laplace transform is

$$£\,\delta(t-t_1) = £\left[\frac{d}{dt}u(t-t_1)\right] = s\,£[u(t-t_1)]$$

$$= s.\frac{e^{-t_1 s}}{s} = e^{-t_1 s}.$$

Unit Doublet Function

Let the function occurs at t_1. The function is then represented by $\delta'(t-t_1)$.

Obviously,

$$\delta'(t-t_1) = \frac{d}{dt}\delta(t-t_1)$$

$$£[\delta'(t-t_1)] = £\left[\frac{d}{dt}\delta(t-t_1)\right]$$

$$= s£\,\delta(t-t_1) = se^{-t_1 s}.$$

Obviously, $\delta'(t) = s$ and the Laplace transform of a doublet function with slope m is

$$£[m\delta'(t-t_1)] = m£[\delta'(t-t_1)] = mse^{-t_1 s}.$$

The following table shows the Laplace transform of unit functions.

Table 10.1 *Laplace Transform of Unit Functions*

Function	Laplace transform occurring at $t = 0$	Laplace transform occurring at $t = a$
$u(t)$ [*unit step function*]	$\frac{1}{s}$	$\frac{1}{s}e^{-as}$
$r(t)$ [*ramp function*]	$\frac{1}{s^2}$	$\frac{e^{-as}}{s^2}$
$\delta(t)$ [*impulse function*]	1	$e^{-t_1 s}$ [$a = t_1$ here]
$\delta'(t)$ [*doublet function*]	s	se^{-as}
$p(t)$ [*unit parabolic function*]	$\frac{1}{s^3}$	$\frac{1}{s^3}e^{-as}$

EXAMPLE 10.1 *A series R-L circuit has $R = 2\,\Omega$ and $L = 1$ H. A pulse voltage of magnitude 4 V and width θ is applied at $t = 0$. Assuming initial current through the inductor to be zero, find $i(t)$.*

SOLUTION. A pulse voltage of magnitude 4 V, width θ is shown in Fig. E10.1 and can be represented by $[u(t) - u(t-\theta)]\,4$. At $t = 0$, this impulse being applied in the R-L circuit, the KVL yields

$$Ri(t) + L\frac{di(t)}{dt} = 4\,[u(t) - u(t-\theta)].$$

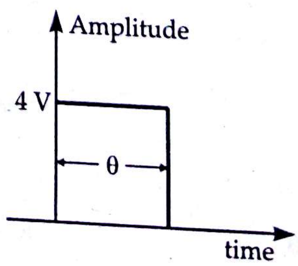


Fig. E10.1

Laplace transformation of this equation yields

$$RI(s) + L[sI(s) - i(0+)] = \frac{4}{s}[1 - e^{-\theta s}].$$

However, $i(0+) = 0$. Thus finally,

$$RI(s) + Ls\,I(s) = \frac{4}{s}(1 - e^{-\theta s})$$

or,
$$I(s) = \frac{4}{s}\frac{1-e^{-\theta s}}{R+Ls} = \frac{4}{s}\frac{1-e^{-\theta s}}{2+s}$$
$$= \frac{4}{s(s+2)}\cdot(1 - e^{-\theta s}).$$

However, $\frac{4}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$

where $K_1 = \left.\frac{4}{s+2}\right|_{s=0} = 2;$

$K_2 = \left.\frac{4}{s}\right|_{s=-2} = -2$

i.e., $\frac{4}{s(s+2)} = \frac{2}{s} - \frac{2}{s+2}.$

Thus $I(s) = \left[\frac{2}{s} - \frac{2}{s+2}\right][1 - e^{-\theta s}]$

$$= \frac{2}{s} - \frac{2e^{-\theta s}}{s} - \frac{2}{s+2} + \frac{2e^{-\theta s}}{s+2};$$

$$\therefore \quad \boldsymbol{i(t) = 2[u(t) - u(t)e^{-2t} - u(t-\theta) + u(t-\theta)e^{-2(t-\theta)}].}$$

Example 10.2 *In a R-L series circuit assume $R = 10\,\Omega$ while $L = 2$ H. Assuming the initial current through the inductor to be zero, find i (t) following the application of an unit impulse voltage at t = 0.*

Solution. The unit impulse voltage being represented by $\delta(t)$ in time domain, application of KVL in the *R-L* circuit yields

$$L\frac{di(t)}{dt} + R\,i(t) = \delta(t).$$

By Laplace transformation of the above equation,

$$L[sI(s) - i(0+)] + RI(s) = 1.$$

However, $i(0+) = 0$

$\therefore \quad sL\,I(s) + RI(s) = 1$

or, $$I(s) = \frac{1}{R+sL} = \frac{1}{L\left(s+\frac{R}{L}\right)} = \frac{1}{2(s+5)}.$$

$\therefore$ Inverse Laplace transformation yields

$$\boldsymbol{i(t) = \frac{1}{2}e^{-5t}}.$$

Example 10.3 *Develop the equation of the function shown in Fig. E10.2 in terms of ramp function.*

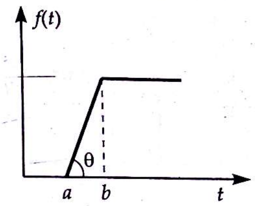


Fig. E10.2

Solution. The function of Fig. E10.2 is graphically segregated into following functions (Ref. Fig. E10.3).

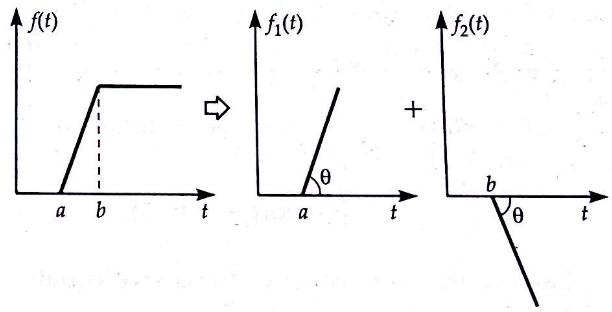


Fig. E10.3

Here, $f(t) = f_1(t) + f_2(t)$
$$= K[r(t-a)] + \{-K[r(t-b)]\}$$
$$= Kr(t-a) - Kr(t-b)$$
$$= K[r(t-a) - r(t-b)].$$

Thus, finally $f(t) = K[r(t-a) - r(t-b)]$

when $\quad K = \tan\theta.$

Example 10.4 *Express the function shown in Fig. E10.4 as a product of two step functions.*

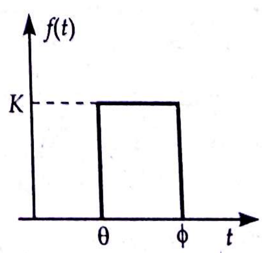


Fig. E10.4

Solution. Graphically, the given pulse function is the product of two step functions as shown in Fig. E10.5.

CHAPTER 10

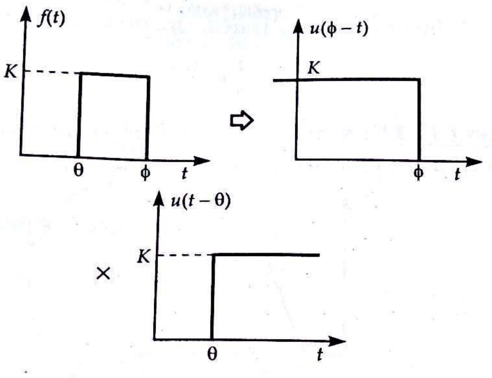


Fig. E10.5

Obviously, with reference to Fig. E10.5,

$$f(t) = Ku\,(t-\theta)\,.\,u\,(\phi - t).$$

Example 10.5 *A unit ramp voltage $r(t-3)$ is applied to a series R-L circuit [$R = 5\,\Omega$; $L = 2$ H] at $t = 0$. Find its response with respect to current. [Assume zero initial condition]*

Solution. With application of the ramp voltage at $t = 0$, the KVL yields

$$L\frac{di(t)}{dt} + Ri(t) = r(t-3).$$

Laplace transformation of the above equation yields

$$L[sI(s) - i(0+)] + RI(s) = \frac{1}{s^2}e^{-3s}.$$

However, due to zero initial condition, $i(0+) = 0$.

Thus we obtain,

$$sLI(s) + RI(s) = \frac{1}{s^2}e^{-3s};$$

however, $L = 2$ H, $R = 5\,\Omega$;

This gives, $(2s+5)\,I(s) = \dfrac{1}{s^2}e^{-3s}.$

$$\therefore \quad I(s) = \frac{e^{-3s}}{s^2(2s+5)} = \frac{e^{-3s}}{2s^2(s+2.5)}.$$

However,

$$\frac{1}{s^2(s+2.5)} = \frac{K_1}{s} + \frac{K_2}{s^2} + \frac{K_3}{s+2.5} \quad \text{...}(a)$$

$$\text{Hence } K_3 = \frac{1}{s^2(s+2.5)}\cdot(s+2.5)\bigg|_{s=-2.5} = 0.16$$

$$K_2 = \frac{1}{s^2(s+2.5)}\cdot s^2\bigg|_{s=0} = 0.4$$

$\therefore$ Let us now multiply equation (a) by s^2. This gives

$$\frac{1}{s+2.5} = K_1 s + K_2 + \frac{K_3 s^2}{s+2.5}. \quad \text{...}(b)$$

Differentiating (b) with respect to (a), we get

$$-\frac{1}{(s+2.5)^2} = K_1 + K_3\,\frac{(s+2.5)(2s) - s^2}{(s+2.5)^2}. \quad \text{...(c)}$$

Putting $s = 0$ in (c),

$$K_1 = -\frac{1}{(2.5)^2} = -0.16$$

$$\text{Hence,} \quad I(s) = \left[\frac{1}{s}(-0.16) + \frac{0.4}{s^2} + \frac{0.16}{s+2.5}\right]\frac{e^{-3s}}{2}.$$

i.e.,

$$\boldsymbol{i(t) = \frac{1}{2}[-0.16\,u(t-3) + 0.4\,r(t-3) + 0.16\,e^{-2.5(t-3)}\,u(t-3)]\ \text{A}.}$$

Example 10.6 *A unit step voltage $u(t-2)$ is applied to a series L-R circuit at $t = 0$ when, $L = 1$ H, $R = 2\,\Omega$. Find i (t) assuming zero initial condition.*

Solution. With application of KVL in the series LR circuit with unit step voltage of $u(t-2)$ being applied at $t = 0$, we get

$$L\frac{di(t)}{dt} + Ri(t) = u(t-2).$$

Laplace transformation gives,

$$L[sI(s) - i(0+)] + RI(s) = \frac{e^{-2s}}{s}.$$

With zero initial condition, $i(0+) = 0$. Thus,

$$I(s)[sL + R] = \frac{e^{-2s}}{s}$$

$$\text{or,} \quad I(s)[s+2] = \frac{e^{-2s}}{s} \quad [\because\ R = 2\Omega,\ L = 1\,\text{H}]$$

$$\text{or,} \quad I(s) = \frac{1}{s(s+2)}e^{-2s}.$$

$$\text{Obviously,} \quad \frac{1}{s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$\text{where,} \quad K_1 = \frac{1}{s(s+2)}\cdot s\bigg|_{s=0} = 0.5$$

$$K_2 = \frac{1}{s(s+2)}\cdot(s+2)\bigg|_{s=-2} = -0.5$$

$$\text{Then,} \quad I(s) = \left[\frac{0.5}{s} - \frac{0.5}{s+2}\right]e^{-2s}.$$

Inverse Laplace transformation gives,

$$i(t) = 0.5\,u(t-2) - 0.5\,e^{-2(t-2)}\,u(t-2)$$

i.e., $$\mathbf{i(t) = 0.5\,u(t-2)\,[1 - e^{-2(t-2)}]}.$$

EXAMPLE 10.7 *A unit impulse function of voltage is applied at t = 0 to a series RC circuit. Find $i(t)$ assuming the initial charge stored in the capacitor to be zero. Assume $R = 5\,\Omega$, $C = 2$ F.*

SOLUTION. Applying KVL at the series *RC* circuit, we can write

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \delta(t)$$

[∵ applied voltage is a unit impulse function],

or, $$Ri(t) + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt = \delta(t).$$

Laplace transformation yields

$$RI(s) + \frac{1}{C}\cdot\frac{q(0+)}{s} + \frac{1}{C}\frac{I(s)}{s} = 1.$$

However, as the initial charge stored in C is zero, hence $q(0+) = 0$. Thus, we get

$$RI(s) + \frac{1}{Cs}I(s) = 1$$

or, $$I(s) = \frac{1}{R + \frac{1}{Cs}}$$

$$= \frac{1}{R}\frac{s}{s + \frac{1}{RC}} = \frac{1}{5}\frac{s}{s + \frac{1}{10}}$$

$$= 0.2\frac{s}{s + 0.1}$$

$$= 0.2\left[1 - \frac{0.1}{s + 0.1}\right].$$

Inverse Laplace transformation yields

$$\mathbf{i(t) = 0.2\,[\delta(t) - 0.1\,e^{-0.1t}]\ A.}$$

EXAMPLE 10.8 *In the above problem assume the excitation voltage to be a shifted impulse function $\delta(t-a)$. Assuming zero initial condition, find $i(t)$.*

SOLUTION. Here, application of KVL yields

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt = \delta(t-a).$$

Laplace transformation yields

$$RI(s) + \frac{1}{C}\frac{q(0+)}{s} + \frac{1}{C}\frac{I(s)}{s} = e^{-as}.$$

With zero initial condition, we get

$$RI(s) + \frac{1}{Cs}I(s) = e^{-as}$$

$$\therefore\quad I(s) = \frac{e^{-as}}{\left(R + \frac{1}{Cs}\right)}$$

$$= \frac{e^{-as}}{5 + \frac{1}{2s}} = \frac{2s\,e^{-as}}{10s + 1} = 2e^{-as}\left[\frac{s}{10s+1}\right]$$

$$= e^{-as}\frac{(2\times 0.1)s}{s + 0.1} = 0.2\,e^{-as}\frac{s}{s+0.1}$$

$$= 0.2\,e^{-as}\left[1 - \frac{0.1}{s + 0.1}\right].$$

∴ Inverse of Laplace transformation yields

$$i(t) = 0.2\,[\delta(t-a) - \delta(t-a)\,e^{-0.1(t-a)} \times 0.1],$$

i.e., $$\mathbf{i(t) = 0.2\delta(t-a) - 0.02\delta(t-a)\,e^{-0.1(t-a)}\ A.}$$

EXAMPLE 10.9 *A unit step voltage $2u(t-\theta)$ is applied in a series RC circuit with $R = 2\,\Omega$, $C = 1$ F. Assuming zero initial condition, find $i(t)$.*

SOLUTION. Here, KVL in *RC* series circuit yields

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 2u(t-\theta)$$

which, after Laplace transformation with zero initial condition gives

$$I(s)\left[R + \frac{1}{Cs}\right] = 2\,\frac{1}{s}\,e^{-\theta s}$$

i.e., $$I(s) = \frac{2e^{-\theta s}}{s\left(R + \frac{1}{Cs}\right)} = \frac{2e^{-\theta s}}{s\left(2 + \frac{1}{s}\right)} = \frac{2e^{-\theta s}}{2s+1}$$

$$= \frac{e^{-\theta s}}{s + 0.5}.$$

Inverse Laplace transformation yields

$$\mathbf{i(t) = u(t-\theta)\,e^{-0.5(t-\theta)}\ A.}$$

EXAMPLE 10.10 *A ramp voltage $2r(t-2)$ is applied in a series RC circuit at $t = 0$ where $R = 3\,\Omega$, $C = 1$ F. Assuming zero initial conditions, find $i(t)$.*

SOLUTION. With ramp voltage $2r(t-2)$ being applied in series *RC* circuit,

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 2r(t-2).$$

CHAPTER 10

Laplace transformation provides

$$RI(s)+\frac{1}{C}\frac{q(0+)}{3}+\frac{1}{C}\frac{I(s)}{s}=2\frac{1}{s^2}e^{-2s}$$

However, the initial condition being zero, $q(0+)=0$.

$\therefore$ We get,

$$RI(s)+\frac{1}{C}\frac{I(s)}{s}=2\frac{1}{s^2}e^{-2s}$$

or,
$$I(s)\left[R+\frac{1}{Cs}\right]=\frac{2e^{-2s}}{s^2}$$

or,
$$I(s)=\frac{2e^{-2s}}{s^2\left(R+\frac{1}{Cs}\right)}=\frac{2e^{-2s}}{s^2\left(3+\frac{1}{s}\right)}=2\frac{e^{-2s}}{s(3s+1)}$$

$$=\frac{2}{3}\frac{e^{-2s}}{s(s+0.33)}.$$

However,
$$\frac{1}{s(s+0.33)}=\frac{K_1}{s}+\frac{K_2}{s+0.33}$$

where,
$$K_1=\frac{1}{s(s+0.33)}\cdot s\Big|_{s=0}=3$$

$$K_2=\frac{1}{s(s+0.33)}(s+0.33)\Big|_{s=-0.33}=-3$$

$$\therefore\quad I(s)=\frac{2}{3}\left[\frac{3}{s}-\frac{3}{s+0.33}\right]e^{-2s}.$$

Using inverse of Laplace transformation,

$$i(t)=0.67\,[3u(t-2)-3u(t-2)\,e^{-0.33(t-2)}]\text{ A.}$$

Thus the expression of the current becomes

$$\boldsymbol{i(t)=2.01\,u(t-2)-2.01\,u(t-2)\,e^{-0.33(t-2)}\text{ A.}}$$

EXAMPLE 10.11 *Synthesize the given gate function (Fig. E10.6) using addition and multiplication of two unit step functions.*

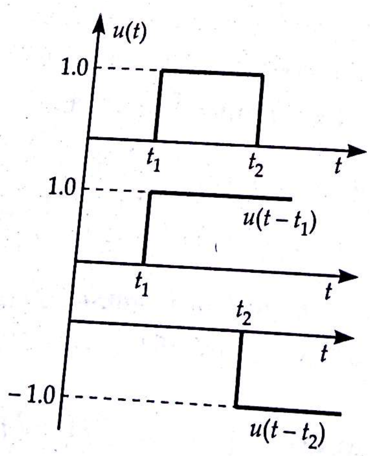


Fig. E10.6

SOLUTION. Figure E10.6 represents a gate function $g(t)$. The waveform analysis reveals that this function can be represented by the sum of two unit step functions as shown in Fig. E10.6.

i.e.,
$$g(t)=u(t-t_1)+[-u(t-t_2)]$$
$$=u(t-t_1)-u(t-t_2).$$

Also, using the concept of waveform analysis, the gate function can also be represented by the product of two unit step functions as shown in Fig. E10.6(*a*).

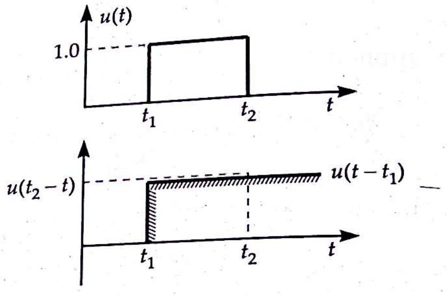


Fig. E10.6 (*a*)

Here, $\boldsymbol{g(t)=u(t-t_1)\,.\,u(t_2-t)}$.

EXAMPLE 10.12 *A function f(t) is shown in Fig. E10.7. Write its equation in terms of step functions.*

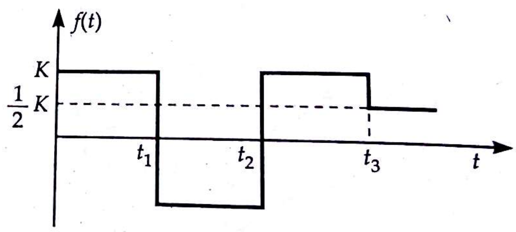


Fig. E10.7

SOLUTION. The waveform analysis shows that the given waveform is a combination of a number of step functions as shown below (Fig. E10.8).

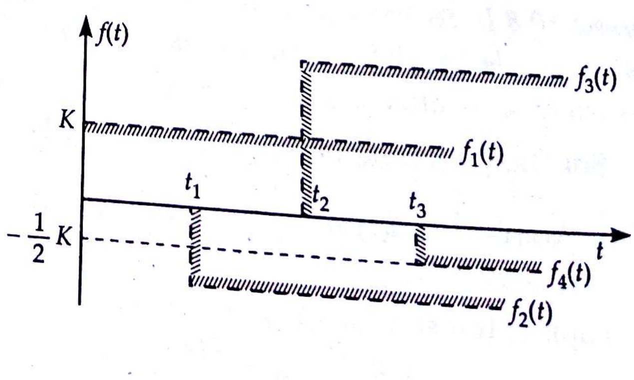


Fig. E10.8

Obviously,

$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

where $f_1(t) = Ku(t)$

$f_2(t) = -2Ku(t - t_1)$

$f_3(t) = 2Ku(t - t_2)$

$f_4(t) = -\frac{1}{2}Ku(t - t_3);$

i.e., the equation of the given waveform is

$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

or, $$f(t) = K\left[u(t) - 2u(t - t_1) + 2u(t - t_2) - \frac{1}{2}u(t - t_3)\right].$$

EXAMPLE 10.13 *Obtain the waveform of the figure shown in Fig. E10.9.*

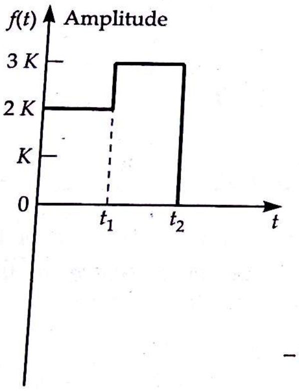


Fig. E10.9

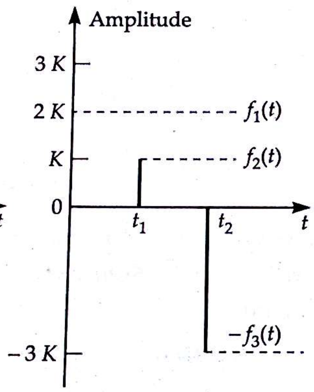


Fig. E10.9 (*a*)

SOLUTION. Utilising the concept of waveform analysis, the given waveform is shown in Fig. E10.9(*a*).

Obviously, $f(t) = f_1(t) + f_2(t) + f_3(t)$

where $f_1(t) = 2Ku(t)$; $f_2(t) = Ku(t - t_1)$

$f_3(t) = -3Ku(t - t_2)$.

∴ The wave equation is given by

$$f(t) = 2Ku(t) + Ku(t - t_1) - 3Ku(t - t_2).$$

EXAMPLE 10.14 *A wave equation is given by*

$$f(t) = 6u(t - t_1) + 2u(t - t_2) - 10u(t - t_3)$$

Draw the waveform.

SOLUTION. Let

$$f_1(t) = 6u(t - t_1)$$

$$f_2(t) = 2u(t - t_2)$$

$$f_3(t) = 10u(t - t_3)$$

As $f(t) = f_1(t) + f_2(t) + f_3(t)$, the waveforms are drawn in Fig. E10.10.

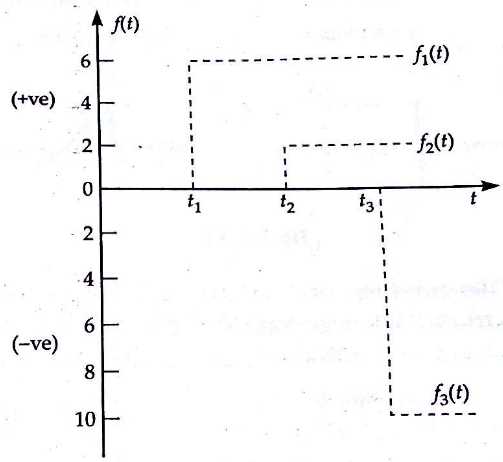


Fig. E10.10

The resultant waveform is shown by hatched line in Fig. E10.11.

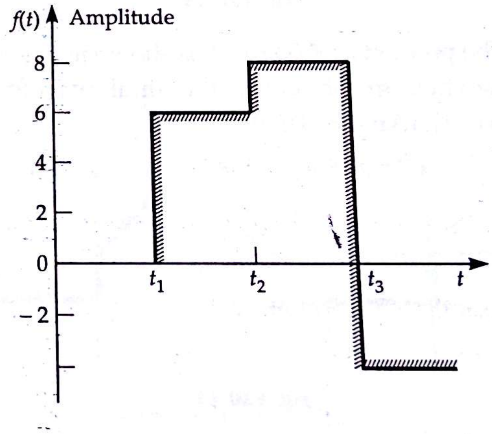


Fig. E10.11

CHAPTER 10

EXAMPLE 10.15 *A ramp function, having slope "m" starts at $t = t_x$ sec. Obtain its equation.*

SOLUTION. As the unit ramp function is given by $r(t)[= tu(t)]$, hence here, m being the slope, the function being starting at $t = t_x$, we can write, the ramp function $= m(t - t_x)\,u(t - t_x)$.

EXAMPLE 10.16 *A function is given by $f(t) = (t_1 - t)$. Between the interval $-t_2 \le t \le t_2$, sketch the waveforms of*

(*i*) $f(t)\,u(t)$

(*ii*) $f(t)\,u(t - t_1)$

(*iii*) $f(t)\,u(-t)$.

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Solution. The product of $f(t)\,u(t)$ is shown below (Fig. E10.12). The hatched graph in Fig. 10.12 shows $f(t)\,u(t)$ within $-t_2 \le t \le t_2$ **(Ans. of (i)).**

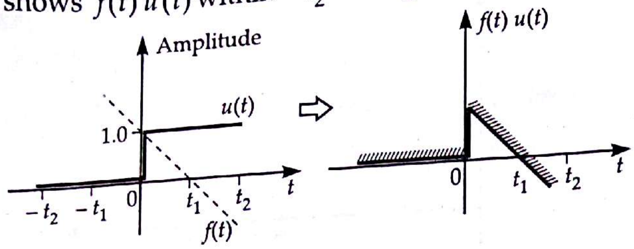


Fig. E10.12

The product of $f(t)\,u(t-t_1)$ is shown in Fig. E10.13, the hatched graph is the final waveform of $f(t)\,u(t-t_1)$ within $-t_2 \le t \le t_2$. **(Ans. of (ii))**

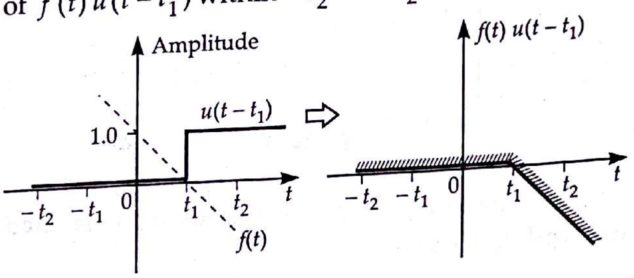


Fig. E10.13

The product of $f(t)\,u(-t)$ is shown in Fig. E10.14, the hatched graph being the final waveform of $f(t)\,u(-t)$. **(Ans. of (iii))**

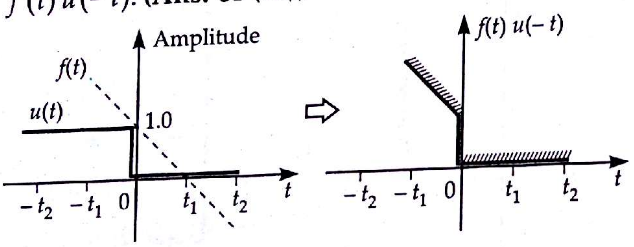


Fig. E10.14

Example 10.17 *Find the value of* $M = \int_{-\infty}^{+\infty} 10\delta(t)\,dt.$

Solution. It is obvious that $\delta(t)$ is zero everywhere except at $t=0$. Thus the limits of integrations can be put as $0(-)$ to $0(+)$ *i.e.*, just before and after zero instead of $-\infty$ to $+\infty$.

$$\therefore \quad M = \int_{0(-)}^{0(+)} 10\,\delta(t)\,dt = 10\int_{0(-)}^{0(+)} \delta(t)\,dt.$$

However, $\int_{0(-)}^{0(+)} \delta(t)\,dt$ is simply the area of the unit impulse $\delta(t)$.

$$\therefore \quad M = 10\,(1) = \mathbf{10}.$$

Example 10.18 *Find the value of*

$$f(t) = \int_{-\infty}^{t} 4tu(t-1)\,\delta(t-2)\,dt.$$

Solution. It is evident that the integral is a function of t. It is also evident that the integrand is zero for all values of $t<2$. And for $t<2$,

$$f(t) = \int_{-\infty}^{t} 0\,.\,dt = 0.$$

However, for $t>2$,

$$f(t) = \int_{-\infty}^{t} 4t\,u(t-1)\,\delta(t-2)\,dt$$

$$= \int_{2(-)}^{2(+)} 4\,(2)(1)\;\delta\,(t-2)\,dt$$

$$= 8\int_{2(-)}^{2(+)} \delta(t-2)\,dt$$

$$= 8\times 1 = 8 \text{ for } t>2.$$

Example 10.19 *An exponential waveform is given by* $v(t) = V_0\,e^{-at}$. *Sketch the derivative waveform of this function.*

Solution. $v(t) = V_0\,e^{-at}$

$$\therefore \quad \frac{d\,v(t)}{dt} = -a\,V_0\,e^{-at}.$$

The required sketch is shown in Fig. E10.15.

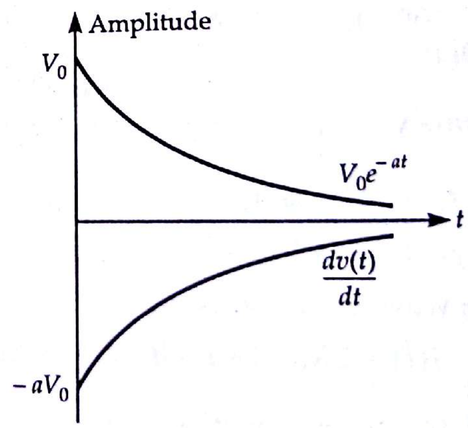


Fig. E10.15

Example 10.20 *A function is given by*

$$f(t) = F_0\,e^{-at}\,u(t)$$

Find its derivative and sketch the waveform.

Solution. $f(t) = F_0\,e^{-at}\,u(t)$

$$\therefore \quad \frac{df(t)}{dt} = F_0\,[e^{-at}\,\delta(t) - ae^{-at}\,u(t)]$$

$$= \mathbf{F_0\,[\delta(t) - ae^{-at}u(t)]}.$$

It may be noted that the impulse function is only valid at $t=0$. Thus at $t=0$, $e^{-at}\,\delta(t)$ becomes $\delta(t)$ only.

Figure E10.16 represents the final waveform of $\frac{df(t)}{dt}$.

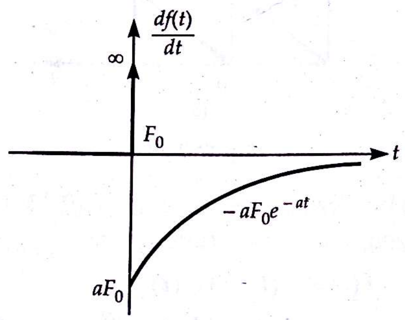


Fig. E10.16

Example 10.21 *Two positive straight line functions are given by $f_1(t) = \alpha_1\, t\, u(t)$ and $f_2(t) = \alpha_2(t-t_1)\,u(t-t_1)$. Add these two functions and find the waveform.*

Solution. Given that

$$f_1(t) = \alpha_1\, t\, u(t);$$
$$f_2(t) = \alpha_2\,(t-t_1)\,u(t-t_1)$$

[where α_1 and α_2 are the slopes of the respective straight line functions].

$$\mathbf{f(t) = f_1(t) + f_2(t)}$$
$$\mathbf{= \alpha_1 t\, u(t) + \alpha_2\,(t-t_1)\,u(t-t_1)}$$

[Ref. Fig. E10.17]

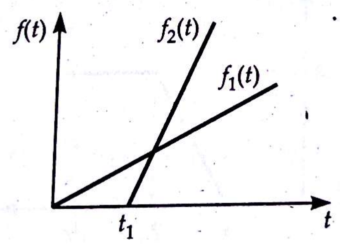


Fig. E10.17

The waveform of $f(t)$ is shown in Fig. E10.18.

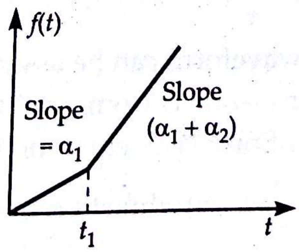


Fig. E10.18

Example 10.22 *A triangular pulse waveform is shown in Fig. E10.19. By waveform analysis, draw the component functions.*

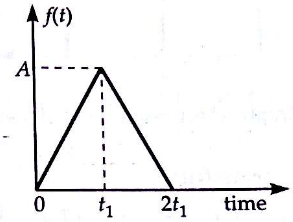
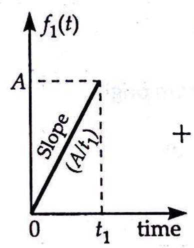
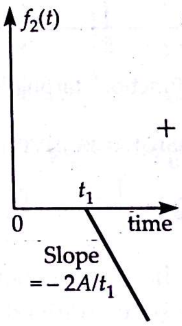


Fig. E10.19

Solution. To obtain the components of the triangular pulse, from time 0 to t_1, we draw a ramp function with slope (A/t_1) and is denoted by $f_1(t)$ (Ref. Fig. E10.20). We then add another ramp with a slope $(-2A/t_1)$, delayed by t time and denoted by $f_2(t)$ (Fig. E10.21). Since the function is zero at all time intervals greater then $2t$, we add another ramp with a +ve slope of (A/t_1) at $t=2t$. It is denoted by $f_3(t)$ [Ref. Fig. E10.22].



+



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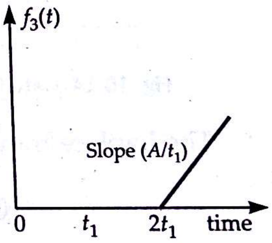

Fig. E10.20 **Fig. E10.21** **Fig. E10.22**

Obviously,

$$f(t) = f_1(t) + f_2(t) + f_3(t).$$

$$\mathbf{= \frac{A}{t_1}\,tu(t) - \frac{2A}{t_1}\,(t-t_1)\,u(t-t_1)}$$
$$\mathbf{+ \frac{A}{t_1}\,(t-2t_1)\,u(t-2t_1)}.$$

10.4 GATE FUNCTION

A rectangular pulse of unit height (*i.e.*, having amplitude of one unit), initiating at $t=a$ and ending at $t=b$ with duration T is defined as a gate function. It is expressed in terms of unit step function and can be represented by (also see Fig. 10.13)

$$G(t) = u(t-a) - u(t-b)$$
$$= u(t-a) - u(t-a-T)$$

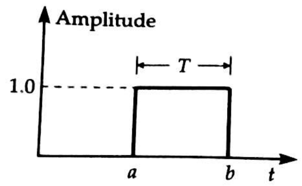

Fig. 10.13 Graphical representation of a gate function.

In Laplace transform,

$$G(s) = e^{-as} \times \frac{1}{s} - e^{-(a+T)s} \times \frac{1}{s}$$

$$= \frac{e^{-as}}{s} [1 - e^{-T.s}]$$

When the gate function starts from zero, *i.e.*, from origin (Fig. 10.14)

$$G(t) = u(t) - u(t-a)$$

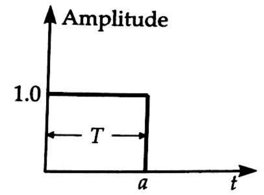

Fig. 10.14 Gate function starting from origin.

The Laplace transform is given by

$$G(s) = \frac{1}{s}(1 - e^{-Ts})$$

This function is called gate function because the rectangular pulse can be visualised as a "gate" just opening at $t = a$ and closing at $t = b$. As the gate opens at $t = a$, its value becomes 1.0 (amplitude) and as it closes at $t = b$, its value becomes 0.

If any function is multiplied by a gate function then that function will have zero value outside the interval of the gate function while the value of the function remains unchanged within the duration of the gate function.

EXAMPLE 10.23. *Synthesize the following waveforms using gate function.*

SOLUTION. (*a*) Using gate function, we can synthesize the waveform in Fig. E10.23(*a*) as

$$f(t) = 2t.G(t)$$

[∵ the equation of the given function is $\left(\frac{2}{1} \times t + 0\right)$ in terms of $(mx + c)$]

or, $$f(t) = 2t\,[u(t) - u(t-1)].$$

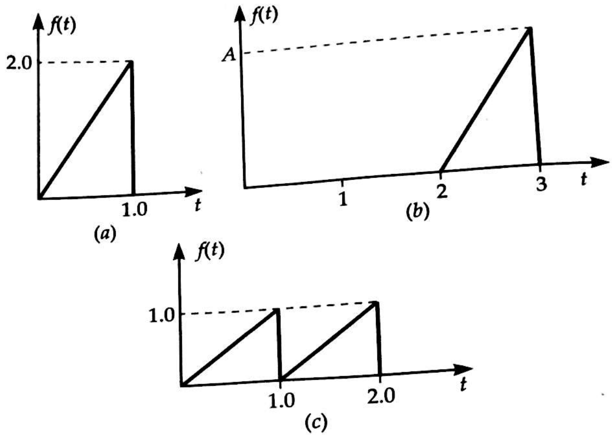

Fig. E10.23

(*b*) The waveform in Fig. E10.23(*b*) can be synthesized, using gate function as

$$f(t) = A(t-2)\,G(t)$$

$$= A(t-2)\,[u(t-2) - u(t-3)].$$

(*c*) For the waveform in Fig. E10.23(*c*), we can use gate function as

$$f(t) = 1.t\,G(t)_{0-1} + 1.(t-1)\,G(t)_{1-2}$$

$$= t[u(t) - u(t-1)] + (t-1)[u(t-1) - u(t-2)]$$

$$= t\,u(t) - t\,u(t-1) + tu(t-1) - u(t-1)$$

$$- tu(t-2) + u(t-2)$$

$$\therefore \quad f(t) = t\,u(t) - u(t-1) - (t-1)\,u(t-2).$$

EXAMPLE 10.24 *Synthesize the following waveforms using gate function :*

SOLUTION. (*a*) Using gate function, the waveform in Fig. E10.24(*a*) can be synthesized as

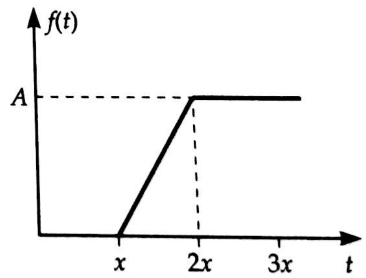

Fig. E10.24 (*a*)

$$f(t) = \frac{A}{x}(t-x)\,G(t)_{x-2x} + AG(t)_{2x-\infty}$$

[∵ the given waveform can be assumed to be sum of two functions—one is from x to $2x$ and the other is from $2x$ to infinity. The equation of the function from x to $2x$ is $\frac{A}{x}(t-x)$ while that from $2x$ to infinity is A]

$$\therefore\ f(t) = \frac{A}{x}(t-x)[u(t-x) - u(t-2x)] + Au(t-2x)$$

$$= \frac{A}{x}(t-x)\,u(t-x) - \frac{A}{x}(t-x)\,u(t-2x) + Au(t-2x)$$

$$= \frac{A}{x}(t-x)\,u(t-x) - Au(t-2x)\left[\frac{1}{x}(t-x)+1\right]$$

$$= \frac{A}{x}(t-x)\,u(t-x) - \frac{Au}{x}(t-2x).t$$

$$= \frac{A}{x}(t-x)u(t-x) - \frac{A}{x}\,.\,tu(t-2x).$$

(*b*) We synthesize the given triangular wave in Fig. E10.24(*b*) as sum of two functions, one from 0 to x and another from x to $2x$.

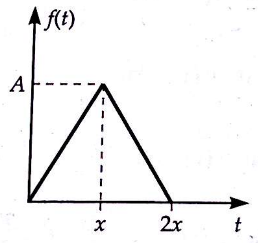

Fig. E10.24 (*b*)

$$f(t) = \frac{A}{x}.t.G(t)_{0-x} + \frac{-A}{x}(t-2x)\,G(t)_{x-2x}$$

$$= \frac{A}{x}.t[u(t) - u(t-x)] - \frac{A}{x}(t-2x)[u(t-x) - u(t-2x)]$$

$$= \frac{A}{x}.tu(t) - \frac{A}{x}.tu(t-x) - \frac{A}{x}(t-2x)u(t-x) + \frac{A}{x}(t-2x)\,u(t-2x)$$

$$= \frac{A}{x}t\,u(t) - \frac{A}{x}u(t-x)[t+t-2x] + \frac{A}{x}(t-2x)\,u(t-2x)$$

$$= \frac{A}{x}t\,u(t) - \frac{A}{x}u(t-x)2\,(t-x) + \frac{A}{x}(t-2x)\,u(t-2x)$$

$$= \frac{A}{x}\,tu(t) - \frac{2A}{x}(t-x)\,u(t-x) + \frac{A}{x}(t-2x)\,u(t-2x)$$

EXAMPLE 10.25 *Synthesize the following waveforms using gate function :*

SOLUTION. (*a*) The given waveform [Fig. E10.25(*a*)] can be synthesized as

$$f(t) = 1\times G(t)_{0-x} + 1\times G(t)_{2x-3x}$$

$$= 1[u(t) - u(t-x)] + 1[u(t-2x) - u(t-3x)]$$

$$= u(t) - u(t-x) + u(t-2x) - u(t-3x).$$

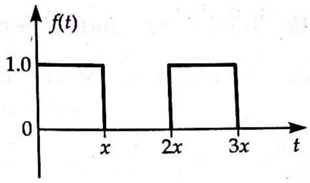

Fig. E10.25 (*a*)

(*b*) The given waveform [Fig. E10.25(*b*)] can be synthesized in terms of gate function as :

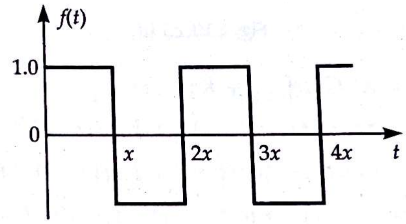

Fig. E10.25 (*b*)

$$f(t) = 1\times G(t)_{0-x} + (-1)\,G(t)_{x-2x} + 1\times G(t)_{2x-3x} + (-1)\,G(t)_{3x-4x+\ldots\ldots}$$

$$= 1[u(t) - u(t-x)] - 1[u(t-x) - u(t-2x)] + 1[u(t-2x) - u(t-3x)] - 1[u(t-3x) - u(t-4x)] + \ldots\ldots$$

$$= u(t) - 2u(t-x) + 2u(t-2x) - 2u(t-3x) + \ldots$$

(*c*) Using gate function, the waveform in Fig. E10.25(*c*) can be synthesized as

$$f(t) = (At-A)[G(t)_{0-2}] + (-At+3A)[G(t)_{2-4}]$$

$$= (At-A)[u(t) - u(t-2)] + (-At+3A)[u(t-2) - u(t-4)]$$

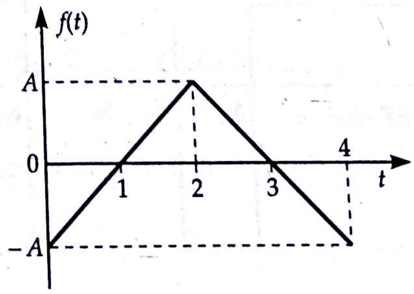

Fig. E10.25 (*c*)

$$= At\,u(t) - Au(t) - Atu(t-2) + Au(t-2)$$
$$- At\,u(t-2) + 3Au(t-2)$$
$$+ At\,u(t-4) - 3Au(t-4)$$
$$= (At - A)\,u(t) - (2At - 4A)\,u(t-2)$$
$$+ (At - 3A)\,u(t-4)$$
$$= A(t-1)\,u(t) - 2A(t-2)\,u(t-2)$$
$$+ A(t-3)\,u(t-4)$$

or, $\boldsymbol{f(t) = A[(t-1)\,u(t) - 2(t-2)u(t-2) + (t-3)\,u(t-4)]}$

(*d*) Using gate function, for Fig. E10.25(*d*), we can write

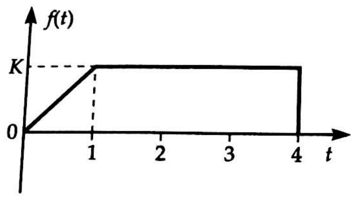

Fig. E10.25 (*d*)

$$f(t) = Kt\,G(t)_{0-1} + K.G(t)_{1-4}$$
$$= Kt[u(t) - u(t-1)] + K[u(t-1) - u(t-4)]$$
$$= Kt\,u(t) - Ktu(t-1) + K\,u(t-1) - Ku(t-4)$$
$$= Kt\,u(t) - Ku(t-1)[t-1] - Ku(t-4)$$
$$\boldsymbol{= Kt\,u(t) - K(t-1)\,u(t-1) - Ku(t-4).}$$

EXAMPLE 10.26 *Synthesize the following waveforms in terms of gate functions.*

SOLUTION. (*a*) Using gate function, the waveform in Fig. E10.26(*a*) can be synthesized as

$$f(t) = 1 \times G(t)_{0-3x} + (-2) \times G(t)_{3x-4x} + 2 \times G(t)_{4x-6x} + (-2)\,G(t)_{6x-7x}$$
$$= 1[u(t) - u(t-3x)] - 2\,[u(t-3x) - u(t-4x)]$$
$$+ 2\,[u(t-4x) - u(t-6x)]$$
$$- 2\,[u(t-6x) - u(t-7x)]$$

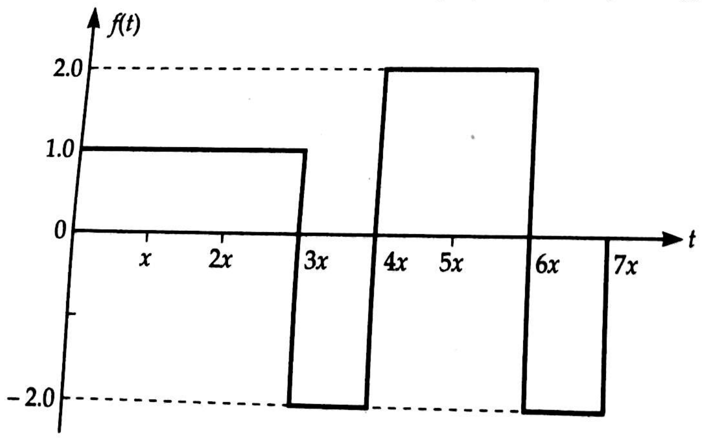

Fig. E10.26 (*a*)

$$= u(t) - u(t-3x) - 2u(t-3x) + 2u(t-4x)$$
$$+ 2u(t-4x) - 2u(t-6x) - 2u(t-6x) + 2u(t-7x)$$
$$\boldsymbol{= u(t) - 3u(t-3x) + 4u(t-4x) - 4u(t-6x) + 2u(t-7x).}$$

(*b*) For the waveform in Fig. E10.26(*b*), we can use gate function to get the following result :

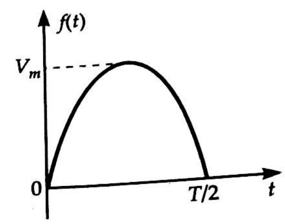

Fig. E10.26 (*b*)

$$v(t) = V_m\,\sin\omega t\,.\,G(t)$$
$$= V_m\,\sin\omega t\left[u(t) - u\left(t - \frac{T}{2}\right)\right]$$
$$= V_m\left[\sin\omega t\,.\,u(t) - \sin\omega t\,.\,u\left(t - \frac{T}{2}\right)\right]$$
$$= V_m\left[\sin\omega t.u(t) - \sin\frac{2\pi}{T}\left(t - \frac{T}{2} + \frac{T}{2}\right).u\left(t - \frac{T}{2}\right)\right]$$
$$= V_m\left[\sin\omega t\,.\,u(t) - \sin\left\{\frac{2\pi}{T}\left(t - \frac{T}{2}\right) + \pi\right\}u\left(t - \frac{T}{2}\right)\right]$$
$$\boldsymbol{= V_m\left[\sin\omega t\,.\,u(t) + \sin\left\{\frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right\}u\left(t - \frac{T}{2}\right)\right]}$$

(*c*) For the waveform shown in Fig. E10.26(*c*), we write :

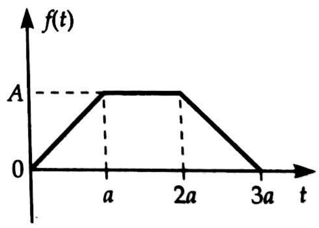

Fig. E10.26 (*c*)

$$f(t) = \frac{A}{a}.t.G(t)_{0-a} + A.G(t)_{a-2a} + \left[\left(-\frac{A}{a}\right)t + 3a\right]G(t)_{2a-3a}$$
$$= \frac{A}{a}t[u(t) - u(t-a)] + A[u(t-a) - u(t-2a)]$$
$$-\left(\frac{A}{a}.t - 3a\right)[u(t-2a) - u(t-3a)]$$

$$= \frac{A}{a}.t\,u(t) - \frac{A}{a}.t\,u(t-a) + Au(t-a) - Au(t-2a)$$
$$-\frac{A}{a}.tu(t-2a) + 3au(t-2a)$$
$$+\frac{A}{a}.t\,u(t-3a) - 3au(t-3a)$$

$$= \frac{A}{a}.t.u(t) - Au(t-a)\left[\frac{1}{a}.t - 1\right]$$
$$-u(t-2a)\left[A + \frac{A}{a}.t - 3a\right]$$
$$+u(t-3a)\left[\frac{A}{a}.t - 3a\right]$$

or, $$\boldsymbol{f(t) = \frac{A}{a}.t.u(t) - \frac{A}{a}(t-a)\,u(t-a)}$$
$$\boldsymbol{-\frac{A}{a}\left(a + t - \frac{3a^2}{A}\right)u(t-2a)}$$
$$\boldsymbol{+\frac{1}{a}(At - 3a^2)\,u(t-3a).}$$

(*d*) For the diagram shown in Fig. E10.26(*d*) use of gate function is helpful in solving the synthesization problem of the given waveform.

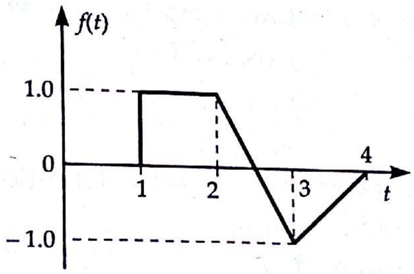

Fig. E10.26 (*d*)

Here,

$$f(t) = 1 \times G(t)_{1-2} + \{(-1)\,t + 5\}\,G(t)_{2-3}$$
$$+(t-4)\,G(t)_{3-4}$$

$$= [u(t-1) - u(t-2)] + (-2t+5)[u(t-2)$$
$$-u(t-3)] + (t-4)[u(t-3) - u(t-4)]$$

$$= u(t-1) - u(t-2) - 2tu(t-2) + 2tu(t-3)$$
$$+5u(t-2) - 5u(t-3) + tu(t-3)$$
$$-tu(t-4) - 4u(t-3) + 4u(t-4)$$

$$= u(t-1) - t(t-2)(2t-4) + u(t-3)(3t-9)$$
$$-u(t-4).(t-4)$$

or, $$\boldsymbol{f(t) = u(t-1) - 2(t-2)\,u(t-2) + 3(t-3)\,u(t-3)}$$
$$\boldsymbol{-(t-4)\,u(t-4).}$$

EXAMPLE 10.27 *Synthesize the following waveforms in terms of gate function :*

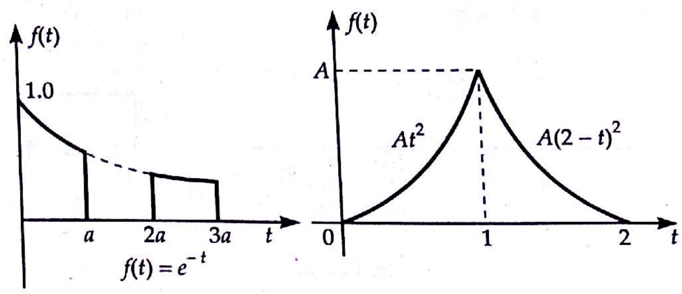

Fig. E10.27 (*a*) (*b*)

SOLUTION. (*a*) The gate function can be utilised as follows for the waveform as shown in Fig. E10.27(*a*)

$$f(t) = e^{-t}.G(t)_{0-a} + e^{-t}.G(t)_{2a-3a}$$
$$= e^{-t}[u(t) - u(t-a)] + e^{-t}[u(t-2a)$$
$$-u(t-3a)]$$
$$\boldsymbol{= e^{-t}[u(t) - u(t-a) + u(t-2a) - u(t-3a)].}$$

(*b*) For the waveform shown in Fig. E10.27(*b*), we find that

$$f(t) = At^2.G(t)_{0-1} + A(2-t)^2\,G(t)_{1-2}$$
$$= At^2.[u(t) - u(t-1)] + A(2-t)^2[u(t-1)$$
$$-u(t-2)]$$
$$= At^2[u(t) - u(t-1)] + A(4 - 4t + t^2)$$
$$[u(t-1) - u(t-2)]$$
$$= At^2[u(t) - u(t-1)] + 4A\,u(t-1)$$
$$-4Au(t-2) - 4At\,u(t-1) + 4At\,u(t-2)$$
$$+At^2\,u(t-1) - At^2\,u(t-2)$$
$$= At^2\,u(t) - Au(t-1)\,t^2 + Au(t-1).4$$
$$-Au(t-1).4t + Au(t-2).4t - Au(t-2).t^2$$
$$-Au(t-2).4 + At^2\,u(t-1)$$
$$= At^2\,u(t) - Au(t-1)[t^2 - 4 + 4t - t^2]$$
$$+Au(t-2)[4t - t^2 - 4]$$
$$= A[t^2\,u(t) - 4u(t-1).(t-1) - u(t-2)(2-t)^2]$$
$$\boldsymbol{= A\,[t^2\,u(t) - 4(t-1)\,u(t-1) - (2-t)^2\,u(t-2)].}$$

10.5 ADDITIONAL EXAMPLES

EXAMPLE 10.28 *A function is given by* $X(t) = (t_1 - t)$. *Sketch the waveform between* $-t_2 \le t \le t_2$ *of the following :*

(*i*) $X(t) + u(t - t_1)$ (*ii*) $X(t - t_2).u(t)$
(*iii*) $X(t - t_2)\,u(t + t_1)$.