Goodness of Fit Test -

If f_i $(i=1,2,\ldots n)$ is a set of observed frequency and e_i $(i=1,2,\ldots n)$ is the corresponding set of expected frequencies then Karl Pearson's chi-square is given by

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(f_i - e_i)^2}{e_i}\right]$$

follows a chi-square distribution with $(n-1)$ d.f.

Decision Rule:

Accept H_0 if $\chi^2 \le \chi^2_\alpha(n-1)$ and reject H_0 if $\chi^2 > \chi^2_\alpha(n-1)$ where χ^2 is the calculated value of chi-square and $\chi^2_\alpha(n-1)$ is the tabulated value of chi-square for $(n-1)$ d.f. and level of significance α.

Q The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained.

Days :	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
No. of parts Demanded :	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depends on the day of the week. (Given: the value of chi square significance at 5, 6, 7 d.f are respectively 11.07, 12.59, 14.07 at the 5% level of significance)

Soln Here we set up the null Hypothesis, H_0 that the number of parts demanded does not depend on the day of week.

Under the null hypothesis, the expected frequencies of the spare part demanded on each of the six days would be:

$$\frac{1}{6}(1124 + 1125 + 1110 + 1120 + 1126 + 1115) = \frac{6720}{6} = 1120$$

Days	Frequency Observed (f_i)	Expected (e_i)	$(f_i - e_i)^2$	$\frac{(f_i - e_i)^2}{e_i}$
Mon.	1124	1120	16	0.014
Tues.	1125	1120	25	0.022
Wed.	1110	1120	100	0.089
Thurs	1120	1120	0	0
Fri.	1126	1120	36	0.032
Sat.	1115	1120	25	0.022
Total	6720	6720		0.179

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i} = 0.179$$

The number of degree of freedom $= 6-1 = 5$ E

The tabulated $\chi^2_{0.05}$ for 5 d.f. $= 11.07$

Since calculated value of χ^2 is less than the tabulated value, it is not significant and the null hypothesis may accepted at 5% level of significance. Hence we conclude that the number of parts demanded are same over the 6-day period.

Test for Independence (Categorical Data)

$$\chi^2 = \sum_i \frac{(O_i - e_i)^2}{e_i}$$

Where the summation extend over all 'rc' cells in the r×c contingency table. If $\chi^2 > \chi^2_\alpha$ with $\nu = (r-1)(c-1)$ degrees of freedom, reject the null hypothesis of independence at the α level of significance; otherwise fail to reject the null hypothesis.

Q Two sample polls of votes for two candidates A and B for a public office are taken, one from among the resident of rural areas. The results are given in the adjoining table. Examine whether the nature of the area is related to voting preference in this election.

Area	Votes for		Total
	A	B	
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

Soln Under the null hypothesis that the nature of the area is independent of the voting preference in the election, we get the expected frequency as follows:

$E(620) = \frac{1170 \times 1000}{2000} = 585$

$E(380) = \frac{830 \times 1000}{2000} = 415$

$E(550) = \frac{1170 \times 1000}{2000} = 585$

$E(450) = \frac{830 \times 1000}{2000} = 415$

Expected frequency

Area	Votes for A	Votes for B	Total
Rural	585	415	1000
Urban	585	415	1000
Total	1170	830	

$$\therefore \chi^2 = \sum_i \left[\frac{(O_i - e_i)^2}{e_i} \right]$$

$$= \frac{(620-585)^2}{585} + \frac{(380-415)^2}{415} + \frac{(550-585)^2}{585} + \frac{(450-415)^2}{415}$$

$$= (35)^2 \left[\frac{1}{585} + \frac{1}{415} + \frac{1}{585} + \frac{1}{415} \right]$$

$$= (1225)\left[2 \times 0.002409 + 2 \times 0.0017097\right]$$

$$= 10.089$$

Tabulated $\chi^2_{0.05}$ for $(2-1)(2-1) = 1$ d.f. is 3.841. Since calculated χ^2 is much greater than the tabulated value, it is highly significant and null hypothesis is rejected of 5% level of significance. Thus we conclude the nature of area is related to voting preference in the election.

Sign Test –

The sign test is used to test hypothesis about the median $\tilde{\mu}$.

Q Ten samples were taken from a plating bath used in an electronics manufacturing process, and the bath pH was determined. The sample pH values are 7

7.91 7.85 6.82 8.01 7.46 6.95 7.05 7.35 7.25 7.42.

Manufacturing engineering believes that pH has a median value of 7.0. Do the sample data indicate that this statement is correct? Use the sign test with $\alpha = 0.05$ to investigate this hypothesis.

Soln Null Hypothesis $H_0 : \tilde{\mu} = 7.0$

Alternative Hypothesis $H_1 : \tilde{\mu} \neq 7.0$

x_i	7.91	7.85	6.82	8.01	7.46	6.95	7.05	7.35	7.25	7.42
$x_i - 7.0$	0.91	0.85	−0.18	1.01	0.46	−0.05	0.05	0.35	0.25	0.42
Sign	+	+	−	+	+	−	+	+	+	+

Test Statistic –

Let R^+ denote the number of the difference $(X_i - \tilde{\mu})$ that are positive and let R^- denote the number of these differences that are negative.

Let $R = \min(R^+, R^-)$

Note :-

Alternative Hypothesis	Reject Region (Reject H_0)
$H_1 : \tilde{\mu} \neq \tilde{\mu}_0$	$r \leq r^*_\alpha$
$H_1 : \tilde{\mu} > \tilde{\mu}_0$	$r^- \leq r^*_\alpha$
$H_1 : \tilde{\mu} > \tilde{\mu}_0$	$r^+ \leq r^*_\alpha$

Now, in above example $r^+ = 8$ and $r^- = 2$ therefore $r = \min(8, 2) = 2$. The tabular value of r with $n = 10$ and $\alpha = 0.05$ is $r^*_{0.05} = 1$. Since $r \nleq r^*_\alpha$ we cannot reject the null hypothesis that the median pH is 7.0.

Wilcoxon Signed- Rank Test -

The sign makes use only of the plus and minus sign of the differences between the observation and the median $\tilde{\mu}_0$. It does not take into account the size or magnitude of these differences. Frank Wilcoxon devised a test procedure that uses both direction (sign) and magnitude.

The wilcoxon signed - rank test applies to the case of symmetric continuous distribution. Under these assumptions the mean equals the median, and we can use this procedure to test the null hypothesis $\mu = \mu_0$.

Test procedure-

Let W^+ be the sum of the positive rank and
W^- be the sum of the negative rank.

Let $W = \min(W^+, W^-)$

Alternative Hypothesis	Rejection Region
$H_1 : \mu \neq \mu_0$	$W \leq W^*_\alpha$
$H_1 : \mu > \mu_0$	$W^- \leq W^*_\alpha$
$H_1 : \mu < \mu_0$	$W^+ \leq W^*_\alpha$

Q The length of a box was measured by an inspector using a new machine. The result were as follows (in mm)

0.265 0.263 0.266 0.267 0.267 0.265 0.267 0.267

0.265 0.268 0.268 0.263

Use the Wilcoxon signed rank test to evaluate the claim the mean box length is 0.265 mm. Use $\alpha = 0.05$

Soln

x_i :	0.265	0.263	0.266	0.267	0.267	0.265	0.267	0.267	0.265	0.268
$x_i - 0.265$	0	−0.002	0.001	0.002	0.002	0	0.002	0.002	0	0.003
Absolute Difference	0	0.002	0.001	0.002	0.002	0	0.002	0.002	0	0.003

x_i :	0.268	0.263
$x_i - 0.265$	0.003	−0.002
Abs. Diff.	0.003	0.002

The Signed Rank table.

Difference $x_i - 0.265$	Absolute Difference	Rank
0	0	–
0	0	–
0	0	–
0.001	0.001	1
−0.002	0.002	−4.5
0.002	0.002	4.5
0.002	0.002	4.5
0.002	0.002	4.5
0.002	0.002	4.5
−0.002	0.002	−4.5
0.003	0.003	8.5
0.003	0.003	8.5

Note: Order the pairs by the absolute differences and assign a rank from smallest to largest absolute difference. Ignore pairs that have an absolute difference 0 and assign mean ranks when their are ties.

Here. $\frac{(2+3+4+5+6+7)}{6} = 4.5$

and $\frac{8+9}{2} = 8.5$

Null Hypothesis $H_0: \mu = 0.265$ mm

Alternative Hypothesis $H_1: \mu \neq 0.265$ mm

Test statistic -

$$W = \min(W^+, W^-)$$

Here, W^+ = the sum of the positive rank $= (1 + 4.5 + 4.5 + 4.5 + 4.5 + 8.5 + 8.5) = 36$

W^- = the sum of the absolute value of the negative rank $= (4.5 + 4.5) = 9$

Therefore, $W = \min(36, 9) = 9$

Conclusion - Since $W = 9$ is less than ~~or~~ to the critical value at $n = 12$ for $\alpha = 0.05$ is 52. i.e $W \leq W_\alpha^*$ we can reject the null hypothesis that the mean length is not equal to 0.275.