# GRIEBACH NORMAL FORM.



1 $S \to a \mid AB$

$A \to a \mid BC$

$B \to b$

$C \to b$

No null production

No Unit production

No useless Symbol

Already it is in CNF.

<u>GNF:</u>

$S \to A_1$

$A \to A_2$

$B \to A_3$

$C \to A_4$

After Assigning grammar becomes

$A_1 \to a \mid A_1 A_2$

$A_2 \to a \mid A_3 A_4$

$A_3 \to b$

$A_4 \to b$

$$\left.\begin{array}{l} A_1 \to a \\ A_2 \to a \\ A_3 \to b \\ A_4 \to b \end{array}\right\} \text{Already in GNF.}$$

$A_1 \to A_2 A_3 \Rightarrow i<j \quad 1<2$

$A_2 \to A_3 A_4$

$$\boxed{\begin{array}{l} A_2 \to A_3 A_4 \\ A_2 \to bA_4 \end{array}}$$

$$\boxed{\begin{array}{l} A_1 \to A_2 A_3 \\ A_1 \to bA_4 A_3 \end{array}}$$

$A_1 \to bA_4A_3 \mid a$

$A_2 \to bA_4 \mid a$

$A_3 \to b$

$A_4 \to b$

Final Grammar:

$S \to bCB \mid a$

$A \to bC \mid a$

$B \to b$

$C \to b$

(2) $S \to AB$

$A \to BC$

$B \to d$

$C \to e$

No null production

No Unit production

No useless symbol

CNF: Already grammar is in CNF.

GNF: $S \to A_1$

$A \to A_2$

$B \to A_3$

$C \to A_4$

Now the grammar

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_4$

$A_3 \to d$

$A_4 \to e$

$\left.\begin{array}{l} A_3 \to d \\ A_4 \to e \end{array}\right\}$ Already in GNF

$A_1 \to A_2 A_3$ $\quad 1 < 2$

$A_2 \to A_3 A_4$ $\quad 2 < 3$

$A_2 \rightarrow d A_4$

$A_1 \rightarrow d A_4 A_3.$

$\therefore$ soln is

| | | |
|---|---|---|
| $A_1 \rightarrow d A_4 A_3$ | Final $\xrightarrow{}$ Grammar | $S \rightarrow d C B$ |
| $A_2 \rightarrow d A_4$ | | $A \rightarrow d C$ |
| $A_3 \rightarrow d$ | | $B \rightarrow d$ |
| $A_4 \rightarrow e$ | | $C \rightarrow e$ |

## Elimination of Left Recursion

$A \rightarrow A\alpha \mid \beta$

Left Recursion is eliminated by

$A \rightarrow \beta A' \mid \beta$

$A' \rightarrow \alpha A' \mid \alpha$

1. $S \rightarrow AA \mid 0$

   $A \rightarrow SS \mid 1$

No null production
No Unit production
No useless Symbols

Already is in CNF

$S \rightarrow A_1$

$A \rightarrow A_2$

Now grammar becomes

$A_1 \rightarrow A_2 A_2 \mid 0$

$A_2 \rightarrow A_1 A_1 \mid 1$

$\left.\begin{array}{l} A_1 \rightarrow 0 \\ A_2 \rightarrow 1 \end{array}\right\}$ Already in GNF

$A_1 \rightarrow A_2 A_2 \qquad 1 < 2$

$A_2 \rightarrow A_1 A_1 \qquad 2 > 1 \qquad \times$ Fails.

Eliminate left recursion.

$A_2 \rightarrow A_1 A_1 \mid$

$A_2 \rightarrow A_2 A_2 A_1 \mid 0 A_1 \mid 1$

Eliminate left recursion.

$A_2 \rightarrow \underset{A}{A_2}\ \underset{\alpha}{A_2 A_1} \mid \underset{\beta}{0 A_1} \mid \underset{\beta}{1}$

$A_2 \rightarrow 0 A_1 A_2' \mid 0 A_1 \mid 1 A_2' \mid 1$

$A_2' \rightarrow A_2 A_1 A_2' \mid A_2 A_1$

$A_2' \rightarrow 0 A_1 A_2' A_1 A_2' \mid 0 A_1 A_1 A_2' \mid 1 A_2' A_1 A_2' \mid 1 \mid$

$0 A_1 A_2' A_1 \mid 0 A_1 A_1 \mid 1 A_2' A_1 \mid 1 A_1 \mid 1 A_1 A_2'$

$A_1 \rightarrow 0A_1A_2'A_2 \mid 0A_1A_2 \mid 1A_2'A_2 \mid 1A_2 \mid 0$

Let $A_2' \rightarrow B$

Final Grammar is

$S \rightarrow 0SBA \mid 0SA \mid 1BA \mid 1A \mid 0$

$A \rightarrow 0SB \mid 0S \mid 1B \mid 1$

$B \rightarrow 0SBSB \mid 0SBS \mid 0SSB \mid 0SS \mid 1BSB \mid$

$1BS \mid 1SB \mid 1S$

**Pumping Lemma:**