Physics:→ Physics is the natural science that studies the matter, its constituents, its motion and behaviour through space and time.

The main goal of Physics is to understand how the universe behaves.

Thermodynamics:→ Thermodynamics is the branch of physics that deals with heat, work and temperature and their relation to energy, radiation and Physical properties of matter.

The behaviour of these quantities is governed by the four laws of thermodynamics. i.e Zeroth law, First Law, Second law, third law of thermodynics.

The Continuum Model:→

Continuum is combination of two words continuous and medium,

$\therefore$ Continuum = Continuous + medium

Definition:→ When matter is considered as continuous distribution of mass by neglecting intermolecular gaps/voids, then matter is said to be continuum.

Validity of Concept of Continuum:→ To see whether the medium is continuum or not, a number is used and this number is known as Knudsen number.

or In other words "Knudsen number is used to check whether the medium is continuum or not". It is denoted by Kn

$$Kn = \frac{\lambda}{L}$$

Where, λ = mean free path
L = characteristic dimension of the system

Kn = Knudsen number.

If $Kn << 0.01$, concept of continuum is valid other wise not valid.

Requirement of Continuum:– If we take the continuum approach of matter (solid, liquid, gas), then the properties in case of fluid such as density, viscosity, thermal conductivity, temperature etc can be expressed as continuous functions of space & time.

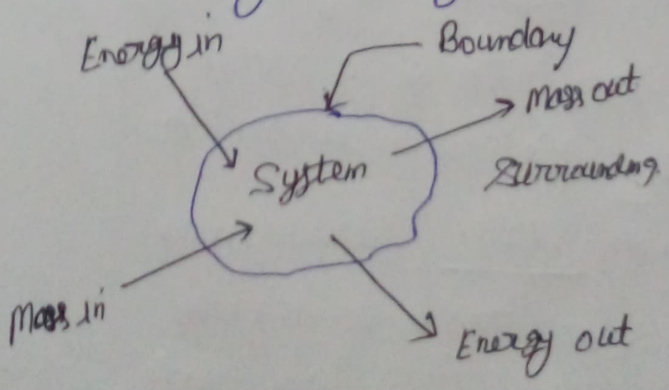
System and its Surroundings:→ Any part of the matter which is considered as seperated from its surrounding is called a system. All those things which are out side that system and influence the behaviour of the system, are known as the "Surroundings" of the system.

Example:- If a gas filled in a cylinder fitted with a piston is heated by burner, then the gas is the system, while the piston and burner are surroundings.

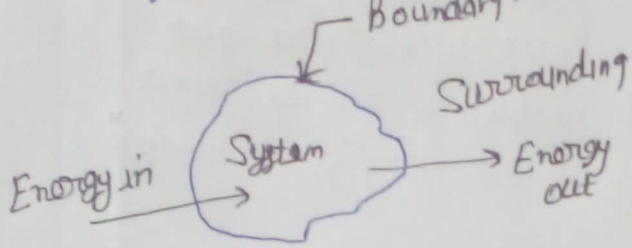
The system is seperated from the surrounding by the system boundary. The boundary may be either fixed or moving. A system and surroundings together comprise a universe.

There are three types of system-

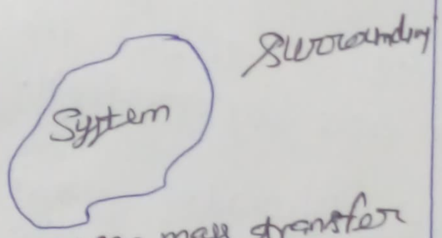
(i) open System:- The open system is one in which matter (mass) crosses the boundary of the system. There may be energy transfer also.



(2) Closed system → The closed system is one in which no mass (matter) transfer across the boundary. There may be energy transfer into or out of the system.



(3) An isolated system → An isolated system is one in which no mass and energy transfer takes place.



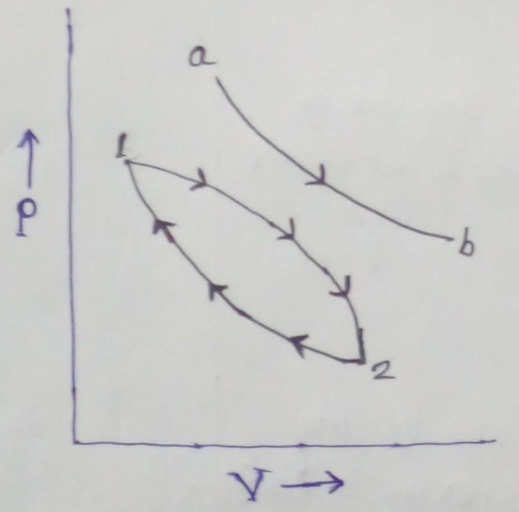
* no mass transfer
* no energy transfer.

Thermodynamic Properties, State, Processes and cycles: → Every System has certain characteristics by which its physical condition may be described e.g. Volume, temperature, Pressure etc. Such characteristics are called thermodynamic properties of the system.

When all the properties of a system have definite values, the system is said to exist at a definite state. Thermodynamic properties are the coordinates to describe the state of the system. They are state variables of the system.

Any operation in which one or more of the properties of a system changes is known as a change of state. The succession of states passed through during a change of state is called the path of change of state. When the path is completely specified, the change of state is called the process.

A thermodynamic cycle is defined as a series of state changes such that the final state is identical with the initial state (fig)



a-b A process.
1-2-1 A cycle.

Thermodynamic Equilibrium → If there is no unbalanced forces between the system and surroundings, the system is said to be in mechanical equilibrium.

If the system has no tendency to undergo a change in internal structure and also has no tendency to transfer matter from one part of the system to another, the system is said to be in chemical equilibrium.

If all parts of the system are at the same temperature which is equal to the temperature of surroundings, the system is said to be in thermal equilibrium.

When the system is under all the three types of equilibrium, the system is said to be in thermodynamic equilibrium. In this condition no change in state of system or of the surroundings occur.

Equation of State:→ The equation of state of a thermodynamic system in equilibrium is a functional relationship among the state variables of that system.

The general form of the equation of state of a gas, may be written as-

$$f(P,V,T)=0$$

It can be solved w.r.t. any one of the variables, its three equivalent forms are

$$P=P(V,T)$$
$$V=V(P,T)$$
$$T=T(P,V)$$

It implies that any two of the three variables P, V, T are enough to specify the state of a gas and to fix the value of third variable.

Zeroth Law of Thermodynamics:→ This fundamental law states that if two systems A and B are separately in thermal equilibrium with a third system C, then the systems A and B are in thermal equilibrium with each other.

A simple illustration of this law is as follows:- When A and B are two gases enclosed in vessels and C is a mercury thermometer. The zeroth law says that if there is no change in the length of mercury

~~thermometer. The zeroth law says that~~

thread when the thermometer C is placed in thermal contact of A nor when it is placed in thermal contact of B, then there will be no change if A and B are brought in thermal contact with each other.

Zeroth Law and Temperature:→ Zeroth law can be used to define temperature. It shows that all systems which are in thermal equilibrium have common property having same value for all of them. This property is temperature. So the temp of a system is the property which determines whether or not the system is in thermal equilibrium with other systems.

Temperature And Heat:→ The temperature of a body is a measure of its degree of hotness and coldness. A body appearing hotter to our sense of touch than the other is said to be at a higher temperature than the other.

If there are two bodies A and B. Let the body A is at higher temperature, and then bodies A and B are placed in contact then after some time they both acquire the same temperature which is somewhere between the two initial temperatures. This means that something from A has been transferred to B. This something is called heat. So heat is a form of energy which is transfered from one body to other because of a temp difference between them.

There is a distinction between a temperature of a body and the heat it contains. The heat that a body contains depends upon its mass as well as upon its temperature.

The sparks from a blacksmith's hammer are white hot (i.e, at a very high temperature) but they do not burn the hand since their mass is very small, and therefore they contain little heat. On the other hand, a jug of hot water, though at a much lower temp than the spark, causes a severe burn because it contains more heat.

~~First law of Thermodynamics~~

Internal Energy of a System → A system may seem to have no apparent mechanical energy but may still be capable of doing work. It is therefore said to possess 'internal energy'.

Example:- A mixture of hydrogen and oxygen does not possess '~~internal energy~~ any external kinetic or potential energy but still it can do work on explosion. This is due to the internal energy of the system.

The First Law of Thermodynamics → The first law of thermodynamics is simply the principle of conservation of energy applied to a thermodynamic system.

It was Joule's work in 1840 who established a relation between mechanical work and heat energy. His experiment proved that heat is a form of energy.

& when W amount of mechanical work is converted into heat, then amount of heat Q produced is related with mechanical work W as

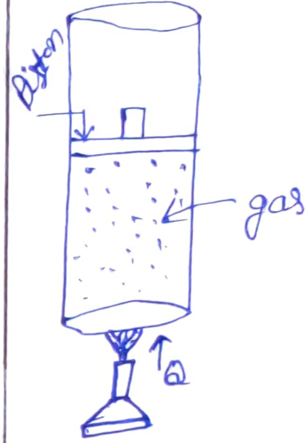
$$W \propto Q \Rightarrow W = JQ$$

where J is constant called Joule's mechanical equivalent of heat, its value is 4.18 J/calorie. Thus 4.18 J of mechanical work can produce one calorie of heat.

Consider a gas enclosed in a cylinder with non-conducting walls & piston only the bottom of the cylinder is conducting. Let gas absorbs 'Q' heat from heat source. This heat (Q) energy is used in two ways

i) To increase the internal energy of system

ii) To do work to move the piston against its weight.



If ΔU represents the increase in the internal energy of the system then

$$\boxed{Q = \Delta U + W}$$

This is the mathematic form of First Law of thermodynamics.

So, First law of thermodynamics can be stated as " If some amount of heat is given to the system, then this given amount of heat is equal to the sum of the increase in internal energy of the system & work done by the system.

In differential form this law can be written as.

$$\boxed{dQ = dU + dW}$$

Internal Energy of a System → A system may seem to have no apparent mechanical energy but may still be capable of doing work. It is therefore said to possess 'internal energy'.

Example:- A mixture of hydrogen and oxygen does not possess '~~external energy~~ any external kinetic or potential energy but still it can do work on explosion. This is due to the internal energy of the system.

The First Law of Thermodynamics → The first law of thermodynamics is simply the principle of conservation of energy applied to a thermodynamic system.

It was Joule's work in 1840 who established a relation between mechanical work and heat energy. His experiment proved that heat is a form of energy

3) When W amount of mechanical work is converted into heat, then amount of heat Q produced is related with mechanical work W as

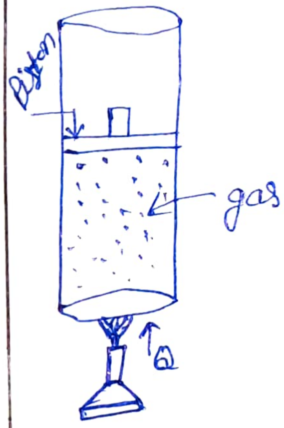
$$W \propto Q \Rightarrow W = JQ$$

Where J is constant called Joule's mechanical equivalent of heat, its value is 4.18 J/calorie. Thus 4.18 J of mechanical work can produce one calorie of heat.

Consider a gas enclosed in a cylinder with non-conducting walls & piston, only the bottom of the cylinder is conducting. Let gas absorbs 'Q' heat from heat source. This heat (Q) energy is used in two ways

i) To increase the internal energy of system

ii) To do work to move the piston against its weight.



If ΔU represents the increase in the internal energy of the system then

$$\boxed{Q = \Delta U + W}$$

This is the mathematic form of First Law of thermodynamics.

So, First law of thermodynamics can be stated as "If some amount of heat is given to the system, then this given amount of heat is equal to the ~~so~~ sum of the increase in internal energy of the system & work done by the system.

In differential form this law can be written as.

$$\boxed{dQ = dU + dW}$$

$$dQ = dU + dW$$

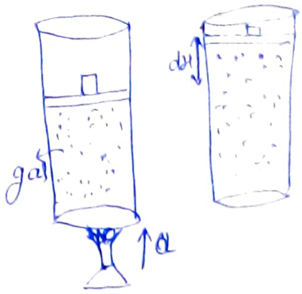
Now $dW = F \times dx$

$= (P \times A) \times dx$

$= P \times (A \times dx)$

$dW = P dV$

So putting $dW = PdV$ in above eq



$$\boxed{dQ = dU + PdV}$$

this is known as first law of thermodynamics in the form of conservation of energy.

Applications of First Law → Let us now study some special processes and the appl^n of the first law to them -

(i) Isolated System → there is no heat flow between the system and surrounding, so the work done is zero. ie $Q=0$ and $W=0$

We know Ist law of thermodynamics -

$$Q = \Delta U + W$$

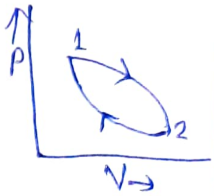
$$0 = \Delta U + 0$$

$$\Delta U = 0$$

$$U_2 - U_1 = 0 \quad \text{ie} \quad \boxed{U_2 = U_1}$$

it means internal energy of isolated system remains constant.

(ii) Cyclic Process :- In this process the initial and final states of the system are the same, so



$$U_2 = U_1 \Rightarrow \Delta U = U_2 - U_1 = 0$$

putting in Ist law

$$Q = \Delta U + W$$

$$Q = 0 + W$$

$$\boxed{Q = W}$$

It means the net heat absorbed by the system is equal to the net work done by the system

iii) Adiabatic process:- A process in which no heat is absorbed or ejected by the system is called as adiabatic process, so $Q=0$

Applying First law, $Q = \Delta U + W$

$0 = \Delta U + W$

$\boxed{\Delta U = -W}$

(iv) Isochoric Process:- The process in which the volume remains constant is known as Isochoric Process. If the volume of system remains constt, then system can do no work ~~so~~ i.e $W=0$

$Q = \Delta U + W \Rightarrow \boxed{Q = \Delta U}$ $\because W=0$

In this case the heat entered in the system stored as internal energy.

(V) Isothermal process:- A process in which temp remains constt is known as isothermal process. In isothermal process Q, W and ΔU are in general non-zero. The first law does not assume any special form ~~of~~ for an isothermal process.

(VI) Isobaric ~~pressure~~ process:- A process in which pressure remains constant is known as isobaric process. As in case of isothermal process, Q, W and ΔU are non-zero.

(vii) Isothermal expansion of an ideal gas:→

Let an ideal gas be allowed to expand at constant temp by placing the gas in good thermal contact with a heat reservoir at the same temp. Since the gas is ideal we can apply ideal gas eqn → $PV = nRT$

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV$$

if T is constt

$$W = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\boxed{W = nRT \log \frac{V_2}{V_1}}$$

this is work done by system in isothermal expansion of gas.

(viii) Adiabatic Expansion of an Ideal gas:→

(i) We have to see the relation between P and V for adiabatic process applied on ideal gas.
for adiabatic process, $dQ = 0$.
so from Ist law of thermodynamics

$$dQ = dU + dW$$
$$0 = dU + dW$$
$$dU = -dW$$

for an ideal gas, $\Delta U = nC_v\,dT$ (at constt volume)

so $\quad dW = -nC_v\,dT \qquad \because dU = -dW$

$$P\,dV = -nC_v\,dT \quad -①\qquad \because dW = P\,dV$$

We know the eqn of state for an ideal gas, $PV = nRT$
differential form of this eqn can be written as:-

$$d(PV) = d(nRT)$$
$$P\,dV + V\,dP = nR\,dT$$

from eqn ①

$$-nC_v\,dT + V\,dP = nR\,dT$$
$$V\,dP = nC_v\,dT + nR\,dT$$

$VdP = nC_v dT + nRdT$

$VdP = (C_v + R) ndT$

$VdP = C_p ndT$ — ② $\because C_p - C_v = R$

dividing eq ② by eq^n ①

$$\frac{VdP}{PdV} = -\frac{C_p n\cancel{dT}}{C_v n\cancel{dT}} = -\gamma \quad \because \frac{C_p}{C_v} = \gamma$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

Integrating above eq^n on both side

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\gamma \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\ln \frac{P_2}{P_1} = -\gamma \ln \frac{V_2}{V_1}$$

$$\ln \frac{P_2}{P_1} = \gamma \ln \left(\frac{V_2}{V_1}\right)^{-1} = \gamma \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\ln \frac{P_2}{P_1} = \ln \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\frac{P_2}{P_1} = \frac{V_1^{\gamma}}{V_2^{\gamma}} \quad \because \ln a = \ln b \Rightarrow a = b$$

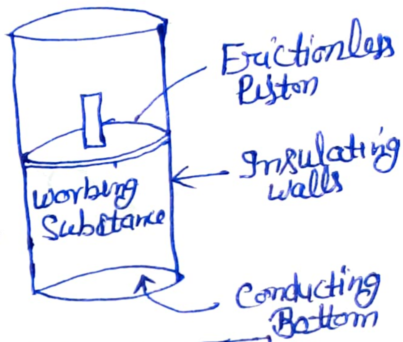
$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

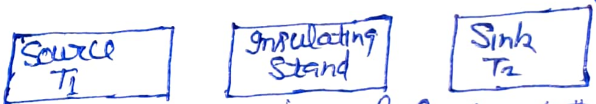
$$\Rightarrow \boxed{PV^{\gamma} = \text{Constt}}$$

Carnot's Engine (Ideal heat engine) ~~or~~ & carnot's cycle:→ A heat engine is a device which converts heat energy into mechanical work. Sadi Carnot a French engineer put ~~the~~ an idea of ~~the~~ an ideal heat engine which is a reversible heat engine. This engine is purely based upon theoretical considerations.

Construction:→ Carnot engine has the following parts-

(i) cylinder:→ It consists of non-conducting or insulating walls, with conducting Bottom, weightless and friction less piston fitted in the cylinder, ~~working substance~~ and perfect gas as a working substance.





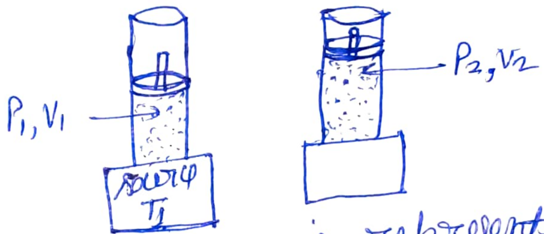
(ii) Source:→ It is a reservoir of heat ~~which~~ having a very high fixed temp T_1 and having infinite thermal capacity. Any amount of heat can be extracted from it ~~the~~ without any fall of temp.

(iii) Sink:- It is another reservoir of heat having lower fixed temperature T_2 and having infinite thermal capacity. Any amount of heat can be added to it without any rise of temperature.

(iv) Insulating stand:- It is used to make the cylinder ~~cond~~ perfectly insulator ie If cylinder is placed on insulating stand then cylinder becomes perfectly insulated ie neither heat ~~is absorbed~~ ~~nor~~ can enter nor leave the cylinder.

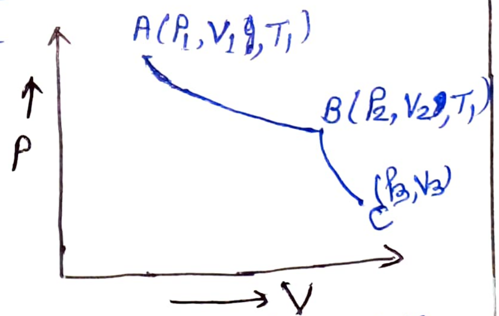
Working → The working of carnot engine can be explained in 4 operations, called a carnot cycle

(i) Isothermal expansion → Let the cylinder contains 1 mole of gas. The base of the cylinder is placed in contact with the source of temp T_1, so that gas attains temp T_1 of the source. The pressure and volume at this stage are P_1 and V_1 respectively. On ~~increasing~~ decreasing the pressure from P_1 to P_2, the gas expands from volume V_1 to V_2. As the bottom of the cylinder is perfectly conducting, heat Q_1 is absorbed at constant temp T_1 during this process



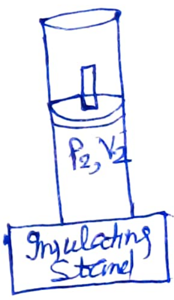
In P-V diagram, this process is represented by isotherm AB, the work done by the system during this process is

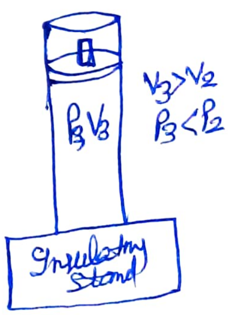
$$W_1 = RT_1 \log_e \frac{V_2}{V_1} \quad \text{—} \ (1)$$



(ii) Adiabatic Expansion →

Now the cylinder is removed from the source & placed on the insulating stand, so that base of the cylinder becomes insulating. The pressure of gas is reduced from P_2 to P_3 and gas expands further from ~~P_2 to P_3~~ V_2 to V_3. As the work is done by gas so the internal energy of gas decreases.





The temperature of gas falls to temp of sink (T_2), so the process is adiabatic. On P-V diagram, this process is represented by curve BC, work done by the gas is given by -

$$W_2 = \frac{R(T_1 - T_2)}{(\gamma - 1)} \quad -(2)$$

where $\gamma = C_P/C_V$

→ Carnot theorem

- Heat Pump (or Refrigerator) — Definition
  — Coefficient of Performance

→ Second law of Thermodynamics — Kelvin Planck Statement
  — Clausius Statement

→ Entropy — Definition
  — Formula

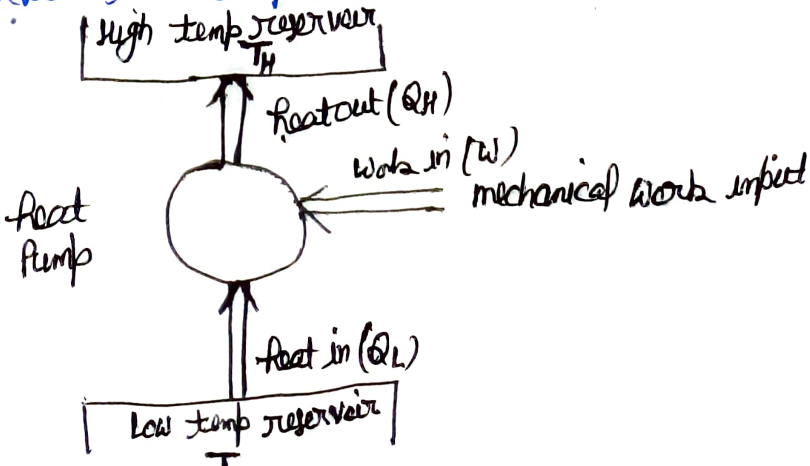
→ Third law of thermodynamics

<u>Carnot Theorem</u> :→ The main aspect of Carnot engine is that it is the most efficient engine working between the two heat reservoirs. This is summarised by Carnot's theorem as follows :-

" No heat engine working between two heat reservoirs can be more efficient than the Carnot's engine working between the same two heat reservoirs".

So, Carnot's theorem shows the limitation on the conversion of heat into work.

<u>Heat Pump</u> (Refrigerator) :→ It is a device which is used to transfer heat from low temp reservoir to high temp reservoir. It is just reverse of heat engine. It requires work input because the transfer of heat from low temp reservoir to high temp reservoir is not possible without using external agency (Work)



Coefficient of Performance (K_r) → It is the ratio of the heat absorbed from low temp reservoir to the work done on the system (or on refrigerant)

$$K_r = \frac{Q_L}{W}$$

$$\boxed{K_r = \frac{Q_L}{(Q_H - Q_L)}} \quad \because Q_H = Q_L + W$$

If we assume the cycle is reverse carnot cycle then

$$\boxed{K_r = \frac{T_L}{T_H - T_L}}$$

Second law of Thermodynamics → Second law of thermodynamics regarding the operation of tells us about the two classical statements, these statements are known as Kelvin - Planck statement and Clausius statement.

Kelvin-Planck Statement: → After analysing the operation of heat engine Kelvin & Planck concluded that -

It is impossible to construct a heat engine that will operate in a cycle and which will receive given amount of heat from a high temp reservoir and does an equal amount of work. The only alternative is that some amount of heat must be transferred from working fluid to the low temp reservoir. So work can be done by the transfer of heat only if there are two temp levels involved.

Clausius Statement → After analysing the operation of heat-Pump clausius concluded that -

It is impossible to construct a device which can transfer the heat from cold body to hot body without an input of work.

Exp → heat flow from hot coffee to surrounding is possible but not from surrounding to coffee.

Entropy → It is measure of disorder or randomness in a system. An increase in disorder is equivalent to ~~the~~ an increase in entropy. It is denoted by S.

The relation b/w change in entropy (ds), heat absorbed or rejected dQ & temp T of the system is given by

$$\boxed{ds = \frac{dQ}{T}}$$ Reversible process

ds → infinitesimal change in entropy
dQ → infinitesimal heat absorbed or rejected

The total entropy change in reversible process will be obtained by integrating the above eq^n.

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

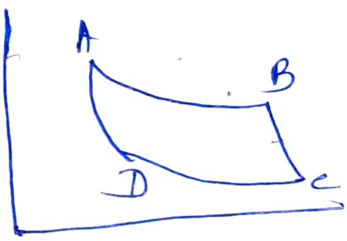
Entropy is function of only ~~state~~ not path, like internal energy

for isothermal process

$$\Delta S = \frac{1}{T}\int_1^2 dQ$$

$$\boxed{\Delta S = \frac{Q}{T}}$$ isothermal process.

Proof that $\Delta S \geq 0$ i.e. $\Delta S = 0$ for reversible process and $\Delta S > 0$ for irreversible process →



change in Entropy in reversible cycle →

During AB, $\Delta S_1 = \frac{Q}{T}$ so entropy change is zero ~~ΔS_3 & $\Delta S_4 = 0$~~

During BC & DA, $Q = 0$,

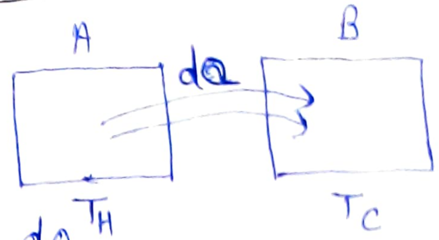
During CD, $\Delta S_2 = -\frac{Q}{T}$

So total change $= \Delta S_1 + \Delta S_2$ ~~$+ \Delta S_3 + \Delta S_4$~~

$= \frac{Q}{T} - Q/T = 0$

$\boxed{\Delta S = 0}$ for reversible

Entropy Increases in all irreversible processes →



$$(\Delta S_1)_A = -\frac{dQ}{T_H}$$

$$(\Delta S_2)_B = \frac{dQ}{T_C}$$

$$\Delta S = \Delta S_1 + \Delta S_2$$

$$= -\frac{dQ}{T_H} + \frac{dQ}{T_C}$$

$$\Delta S = \frac{dQ}{T_C} - \frac{dQ}{T_H}$$

$$T_H > T_C$$

$$\boxed{\text{So, } \Delta S > 0}$$

So entropy increases in irreversible process.

~~Irreversible process are natural process.~~

All natural process in universe are irreversible. there is a continuous increase in entropy. Entropy is not conserved. In this respect, entropy differs from energy.

Irreversible processes are isothermal expansion of gas, Free expansion of gas.

Third law of Thermodynamics (Nernst theorem) →

According to this law "the entropy of a perfectly crystalline substance is zero at absolute zero temp (0K)."

$$S_{0K} = 0 \text{ (for perfect crystal)}$$

If for a substance $S_{0K} \neq 0$, then substance will not be perfectly crystalline. This is also known as Nernst theorem.

Third law of thermodynamics is some times known as the principle of unattainability of absolute zero. It is as follows

It is impossible to attain a temp of 0K.

No area is enclosed by the P-V diagram.

Example 1.2 *If in an isothermal expansion the volume of 1 g mole of a gas at 27° C is doubled, calculate the workdone in the process.* $(R = 8.3\ J\ mol^{-1}\ K^{-1})$.

Solution. Given $T = 27°\text{C} = 300\ \text{K},\ R = 8.3\ \text{J mol}^{-1}\text{K}^{-1},\ V_2 = 2V_1$

The workdone in isothermal process is

$$W = 2.3026\ RT \log_{10} \frac{V_2}{V_1}$$

$$= 2.3026 \times 8.3 \times 300 \log_{10} \frac{2V}{V}$$

$$= 2.3026 \times 8.3 \times 300 \times 0.3010 = 1725.8\ \text{J}.$$

Example 1.3 *A definite mass of a perfect gas is compressed adiabatically to half of its original volume. Determine the resultant pressure if the initial pressure was 1 atmosphere.* [$\gamma = 1.4$ *and* $2^{14} = 2.64$]

Solution. Given $P_1 = 1$ atmosphere, $V_2 = \frac{V_1}{2}$, $\gamma = 1.4$ and $2^{1.4} = 2.64$

For an adiabatic change

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\therefore \quad P_2 = P_1 \left[\frac{V_1}{V_2}\right]^{\gamma} = 1 \times \left[\frac{V_1}{V_1/2}\right]^{1.4} = 1 \times (2)^{1.4} = 2.64 \text{ atmosphere.}$$

Wave motion— Wave motion is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their mean positions. The disturbance is handed over from one particle to the next.

For Exp— When a stone is dropped into pond containing water, waves are produced at the point where the stone strikes the water. The waves travel outward, the particles of the water vibrate only up and down about their mean position. Water particles do not travel along with the wave.

In other words wave motion refers to the transfer of energy from one point to the another point of the medium. Transfer of different kinds of energy like sound, light take place in the form of wave motion.

Characteristics of wave motion:→

(i) Wave motion is a disturbance created in the medium by the repeated periodic motion of the particles of the medium.

(ii) Only wave travels forward in the medium whereas the particles of the medium vibrate about their mean position.

(iii) The particles ahead starts vibrating a little later than the particle just preceding it.

(iii) The velocity of the wave is different from the velocity with which the particles of the medium are vibrating about their mean positions. The velocity of the particle is maximum at the mean position and zero at the ~~mean~~ extreme positions of the particle.

There are two types of wave motions.

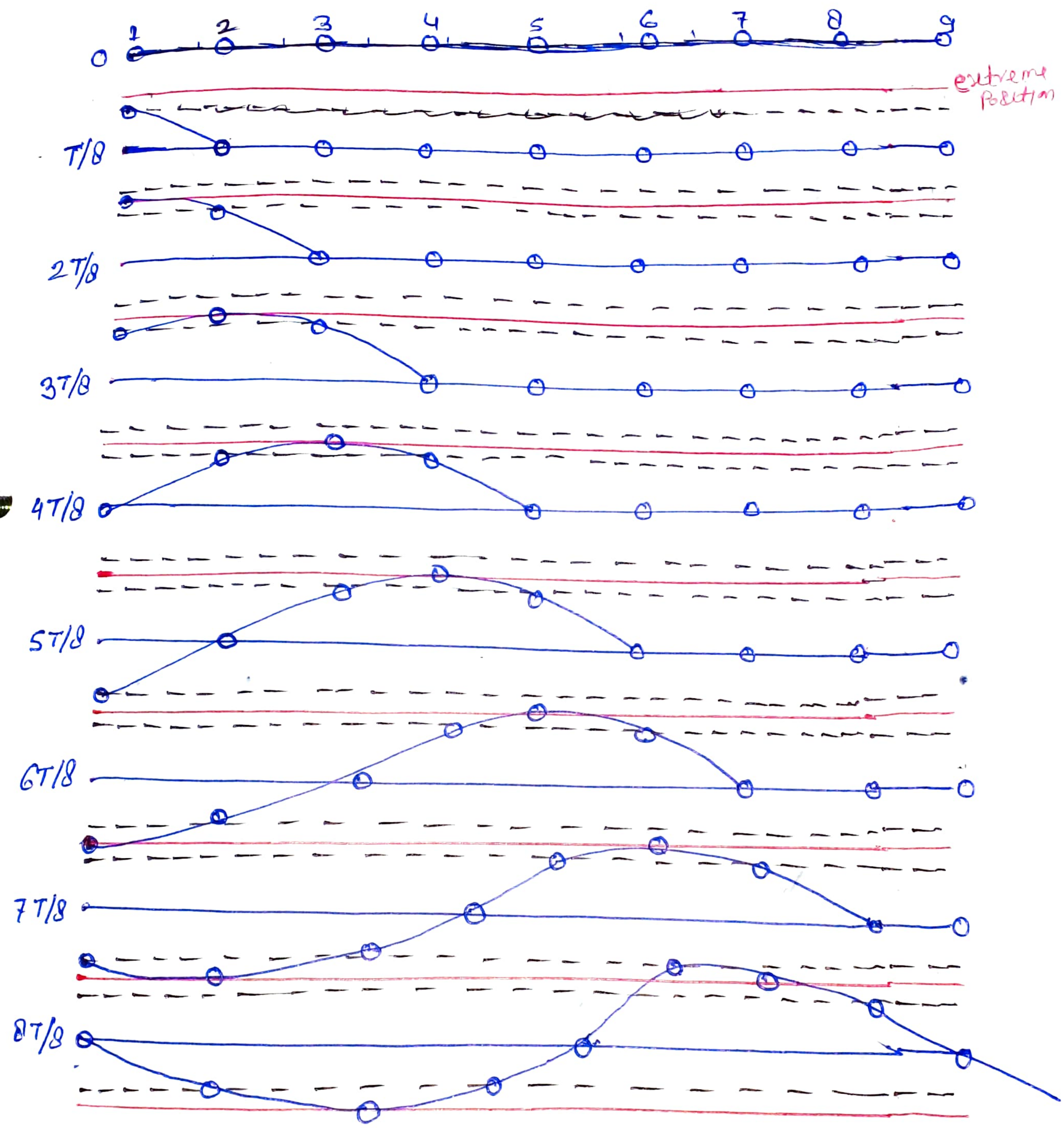
(i) Transverse wave motion - Example - light waves.

(ii) Longitudinal wave motion - Example - Sound waves

<u>Transverse wave motion</u> → In this wave motion, the particles of the medium vibrate at right angles to the direction of propagation of the wave.

Let us consider 9 particles to understand the propagation of transverse wave in the medium. The particles are vibrating about their mean positions up and down and the wave is travelling from left to right. The disturbance takes T/8 seconds to travel from one particle to the next.

3



—— → extreme position
—— → mean position
- - - - → just before ~~mean~~ mean position

(i) At $t=0$ all the particles are at their mean position.

(ii) After T/8 seconds, particle 1 travel a certain distance ~~and~~ upward and the disturbance reaches particle 2.

~~(iii) After 2T/8 seconds, particle 1 has completed 3/8 of its vibration and disturbance has reached particle 4.~~

(4)

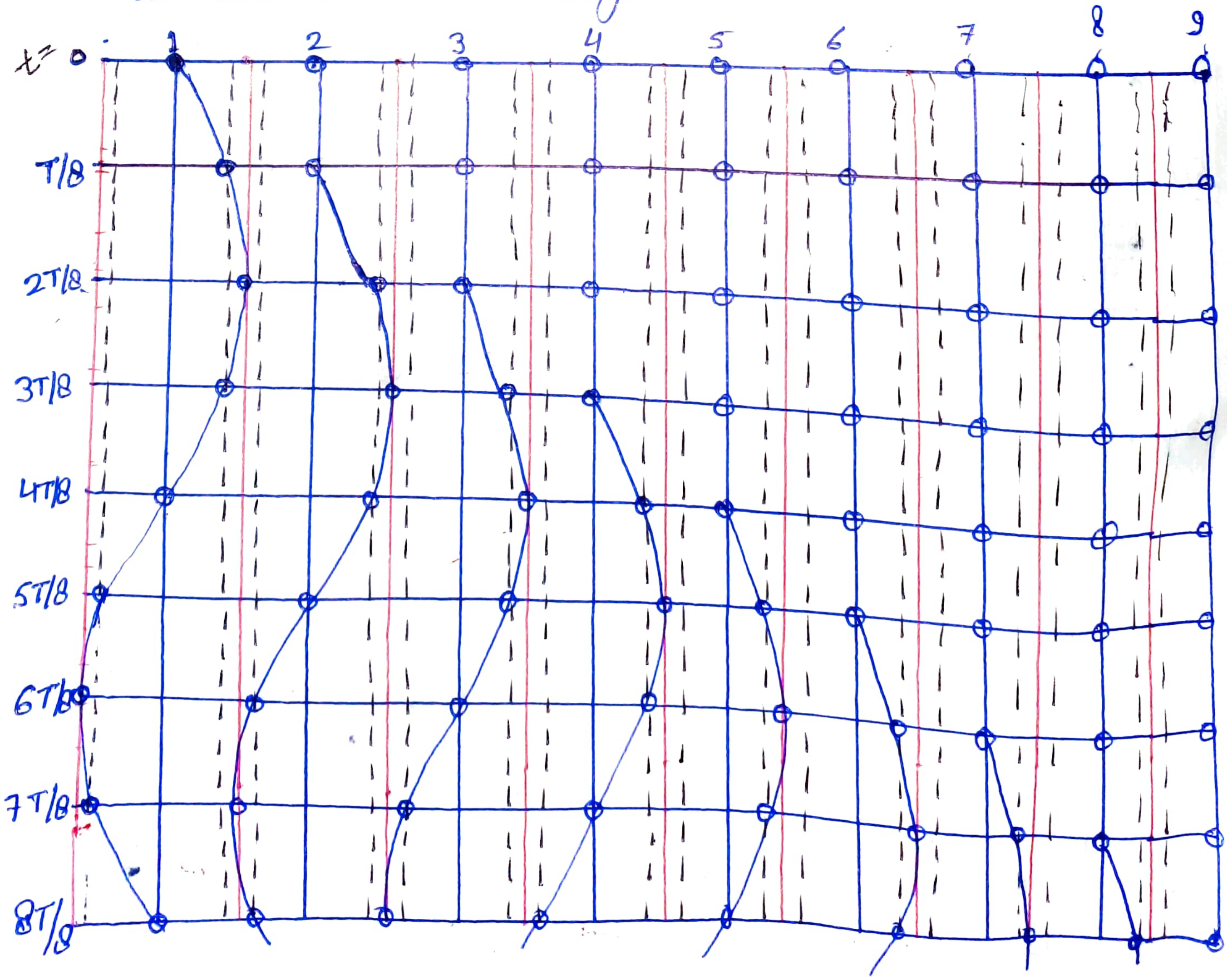
(iii) After 2T/8 seconds, particle (1) has reached its extreme position and the disturbance has reached particle (3).

(iv) After 3T/8 seconds particle (1) has completed 3/8 of its vibration and the disturbance has reached particle (4)

In this way after T seconds. particle (1) completes its one vibration.

Longitudinal wave motion:- In this type of wave motion, the particles of medium vibrate along the direction of propagation of the wave.

Let us consider nine particles of the medium to understand the longitudinal wave motion.



The wave travels from left to right and particles vibrate about their mean position. It is clear from the graph (figure), After T/8 seconds, the particle 1 goes to right and completes 1/8 of its vibration. The disturbance reaches particle 2, after 2T/8 the particle 1 goes to the extreme right position and completes 1/4 of its vibration and particle 2 completes 1/8 of its vibration, similarly the positions of other particles change with time.

After one complete time period, particle 1 complete its 1 cycle. The wave have reached particle 9. Note that—

* particles 1, 5 and 9 are at their mean position
* particles 1 and 3 are close to the particle 2, this is the position of condensation.
* Similarly particles 9 and 8 are close to particle 7, this is also the position of condensation.
* particle 4 and 6 are far away from particle 5, this is the position of rarefaction.

Hence in longitudinal wave motion condensation and rarefactions are alternatively formed.

[Simple Harmonic Motion]

Periodic Motion or Harmonic motion:→ A motion which repeats itself after equal interval of time is called periodic motion or harmonic motion.

Example:- spin of earth, the motion of satellite around a planet etc.

Oscillatory Motion or Vibratory motion:→ If a body moves back and forth repeatedly about the mean position, then body is said to oscillatory or vibratory motion.

Example- The pendulum of clock swings back and forth has oscillatory motion, motion of simple pendulum etc.

Simple harmonic motion→ It is defined as the motion of an oscillatory particle which is acted upon by a restoring force which is directly proportional to the displacement but opposite to it in direction.

The characteristics of simple harmonic motion are as follows-

(i) The motion is periodic.
(ii) The motion is along a straight line about the mean or equilibrium position.
(iii) The acceleration is proportional to displacement
(iv) Acceleration is directed towards the mean or equilibrium position.

Simple Harmonic Motion and Harmonic Oscillator:→

A system executing Simple harmonic motion is known as Simple Harmonic oscillator.

Let us consider a particle having mass m, executing simple harmonic motion. If the displacement of the particle at any instant t be x, then its acceleration will be $\frac{d^2x}{dt^2}$

According to the definition of Simple harmonic motion,

Restoring force $\propto$ -displacement

$$\frac{md^2x}{dt^2} \propto -x$$

$$\frac{md^2x}{dt^2} = -Kx \quad , \quad \text{where } K \text{ is constant}$$

-ve sign shows that the force on the particle is directed opposite to x increasing.

$$\frac{md^2x}{dt^2} + Kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0$$

putting $\omega^2 = \frac{K}{M}$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad - (1)$$

Equation (1) is known as differential equation of motion for a simple harmonic oscillator

The solution of eq (1) is given by

$$x = A \sin(\omega t + \phi) \quad - (2)$$

where A is amplitude of wave

ω is angular frequency of wave

$$x = A \sin(\omega t + \phi) \quad - (2)$$

Characteristics of SHM:-

(i) Displacement:- The displacement of harmonic oscillator is -

$$x = A \sin(\omega t + \phi)$$

Where A is the maximum displacement, which is known as amplitude of oscillation.

(2) Velocity - We know the displacement,

$$x = A \sin(\omega t + \phi)$$

differentiating wrt t -

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

This is the velocity of a body which executes SHM.

(3) Acceleration - We know the velocity of a body which executes SHM-

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

Differentiating wrt t -

$$\text{acceleration, } a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$$

$$\boxed{a = -\omega^2 x}$$

It shows that acceleration is proportional and opposite to displacement.

(4) Time period:→ We know,

$$x = A\sin(\omega t + \phi)$$

Let t is increased by $\frac{2\pi}{\omega}$

$$x = A\sin\left\{\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right\}$$

$$x = A\sin(\omega t + 2\pi + \phi)$$

$$x = A\sin(\omega t + \phi)$$

This shows that displacement repeats itself after a time $2\pi/\omega$, so $\frac{2\pi}{\omega}$ is known as periodic time.

$$T = \frac{2\pi}{\omega}$$

but we have assumed earlier $\omega^2 = \frac{K}{m}$

so $$T = \frac{2\pi}{\sqrt{\frac{K}{m}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

(5) Frequency:- The number of oscillations made in 1 sec is called as frequency

$$\boxed{\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

(6) Phase:- Angle $(\omega t + \phi)$ is called the phase of oscillation.

(7) Epoch:- Value of phase at $t = 0$ is called the epoch, so in our case ϕ is epoch.

Energy of harmonic oscillator:→ In general, a harmonic oscillator has potential as well as kinetic energy. The potential energy is due to the displacement from mean position and kinetic energy is due to its velocity.

At any instant the total energy of the oscillator will be sum of ~~its~~ these two energies.

Total energy = (P.E + K.E)

Potential energy→ The displacement of harmonic oscillator at any instant t is given by

$$x = A\sin(\omega t + \phi) \quad \text{— } (1)$$

its velocity, $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$ — (2)

its acceleration, $a = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$

$$a = \frac{d^2x}{dt^2} = -\omega^2 x \quad (3) \qquad \because x = A\sin(\omega t + \phi)$$

~~We know the restoring force on the harmonic oscillator is directly proportional to displacement but opposite to it in direction~~

$\cancel{F \propto -x}$

$\cancel{F = -Kx}$

from Newton's law, $F = ma$

$\cancel{ma = -Kx}$

~~putting the value of a from~~ eq (3)

$\cancel{-m\omega^2 x = -Kx}$

from Newton's law

$$F = ma$$

putting the value of a from eq 3

$$F = -m\omega^2 x$$

but $\omega^2 = \frac{K}{m}$ (we have assumed earlier)

$$F = -m\frac{K}{m}x$$

$$F = -Kx$$

If the oscillator is displaced through dx distance, then the work done on the oscillator is -

$$dW = \text{force} \cdot dx$$

$$dW = Kx\,dx \qquad [\because \text{ neglecting the } -\text{ve sign}]$$

If oscillator is displaced from $x=0$ to $x=x$ position, then work done on the oscillator

$$W = K\int_0^x x\,dx$$

$$W = \frac{1}{2}Kx^2$$

This work done will be stored as potential energy on the oscillator,

So, $W = U$

$$\boxed{U = \frac{1}{2}Kx^2} \quad \leftarrow (4)$$

This is the potential energy of harmonic oscillator.

Kinetic energy of harmonic oscillator:-

we know the formulae of K.E.

$$K.E = \frac{1}{2} m v^2$$

In case of harmonic oscillator, the displacement of harmonic oscillator at time t is given by

$$x = A \sin(\omega t + \phi) \quad - (5)$$

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

$$= A\omega \sqrt{(1 - \sin^2(\omega t + \phi))} \quad \because \sin^2\theta + \cos^2\theta = 1$$

$$= A\omega \sqrt{\{1 - \sin^2(\omega t + \phi)\}}$$

$$= \omega \sqrt{\{A^2 - A^2 \sin^2(\omega t + \phi)}$$

from eq (5).

$$v = \omega \sqrt{\{A^2 - x^2\}}$$

putting this value of v in K.E. formula-

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\boxed{K.E = \frac{1}{2} m\omega^2 (A^2 - x^2)} \quad - (6)$$

~~This is the K.E. of harmonic~~ oscillator

~~Total energy of harmonic oscillator,~~

$$\cancel{= K.E. + P.E.}$$

~~from eq (4) & (6)~~

$$\cancel{= \frac{1}{2} m\omega^2 (A^2 - x^2) + \frac{1}{2} K x^2}$$

$$K.E = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

but $\omega^2 = K/m, \Rightarrow K = m\omega^2$

$$\boxed{K.E = \frac{1}{2} K (A^2 - x^2)}$$

This is K.E. of harmonic oscillator.

Total energy of harmonic oscillator (E)

$$= K.E. + P.E.$$

$$= \frac{1}{2}K(A^2-x^2) + \frac{1}{2}Kx^2$$

$$\boxed{E = \frac{1}{2}KA^2}$$

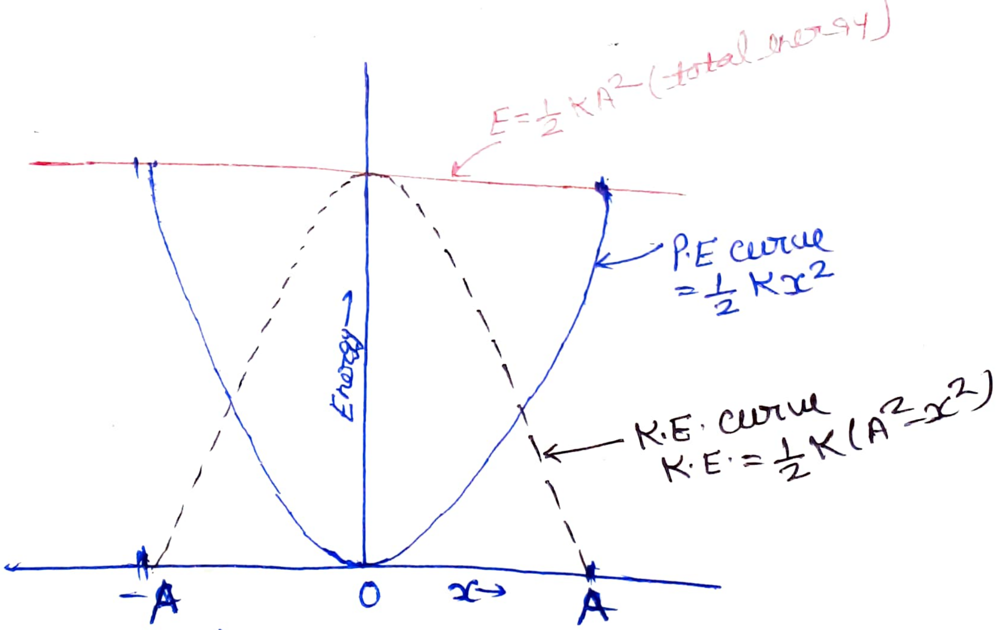
So the total energy is constant.

Now we will plot a common graph for K.E, P.E. and total energy versus displacement.

We have $K.E. = \frac{1}{2}K(A^2-x^2)$

$P.E. = \frac{1}{2}Kx^2$

total energy, $E = \frac{1}{2}KA^2$



Conclusion :→ (i) The curve for total energy E (constant) is the horizontal line. The particle cannot go beyond these points, where this line intersects the potential energy curve, because U can never be larger than E. These points are called turning points of the motion, and corresponds to the maximum displacement.

(ii) At $x = \pm A$, the total energy is wholly potential, but K.E. is zero.

(iii) At $x = 0$ ie in equlibrium position, the total energy is wholly kinetic, but P.E. is zero.

(IV) Between $x = 0$ to $x = \pm A$, the total energy is partly kinetic and partly potential but their sum is always constant ie $\frac{1}{2}KA^2$

Average values of Kinetic & Potential Energies :-

Average kinetic energy of oscillator for a period T is given by

$$K.E._{av} = \frac{\int_0^T K.E dt}{\int_0^T dT} = \frac{\int_0^T \frac{1}{2} m v^2 dt}{\int_0^T dt}$$

$$K.E._{av} = \frac{1}{T} \cdot \frac{1}{2} m \int_0^T v^2 dt$$

we know for a harmonic oscillator displacement, $x = A \sin \omega t$, if $\phi = 0$

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

putting this value of v in above eq^n.

$$K.E._{av} = \frac{m}{2T} \int_0^T A^2 \omega^2 \cos^2 \omega t \, dt$$

$$K.E._{av} = \frac{m A^2 \omega^2}{2T} \int_0^T \cos^2 \omega t \, dt$$

$\cancel{\cos 2\omega t = 1 - 2\sin^2 \omega t}$

$\cancel{\sin^2 \omega t =}$

$$K.E._{av} = \frac{m A^2 \omega^2}{2T} \int_0^T \frac{(1 + \cos 2\omega t)}{2} dt$$

$\because \cos 2\omega t = 2\cos^2 \omega t - 1$

$\cos^2 \omega t = \frac{(1 + \cos 2\omega t)}{2}$

$$K.E_{av} = \frac{m A^2 \omega^2}{2T}\frac{1}{2}\int_0^T (1+\cos 2\omega t)\,dt$$

$$K.E_{av} = \frac{m A^2 \omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{m A^2 \omega^2}{4T}\left[T + \frac{\sin 2\omega T}{2\omega}\right]$$

$$= \frac{m A^2 \omega^2}{4T}\left[T + \frac{\sin 2\times\frac{2\pi}{\cancel{T}}\times\cancel{T}}{2\times 2\pi/T}\right] \quad \because \omega = \frac{2\pi}{T}$$

$$= \frac{m A^2 \omega^2}{4T}\left[T + \frac{\sin 4\pi}{4\pi/T}\right]$$

$$\because \sin n\pi = 0,$$

$$= \frac{m A^2 \omega^2}{4T}\left[T + 0\right]$$

$$= \frac{1}{4\cancel{T}} m A^2 \omega^2 \cancel{T} = \frac{1}{4} m A^2 \omega^2$$

$$K.E_{av} = \frac{1}{4} m \omega^2 A^2$$

$$\boxed{K.E_{av} = \frac{1}{4} K A^2} \quad \because \omega^2 = \frac{K}{m}$$

~~Potential energy:~~

Average value of Potential energy:

The average value of potential energy for a period T is given by,

$$U_{av} = \int_0^T U\,dt \Big/ \int_0^T dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2} K x^2\,dt = \frac{1}{2T}\int_0^T A^2 \sin^2 \omega t\,dt \quad \because x = A\sin\omega t$$

on solving, we get

$$\boxed{U_{av} = \frac{1}{4} K A^2}$$

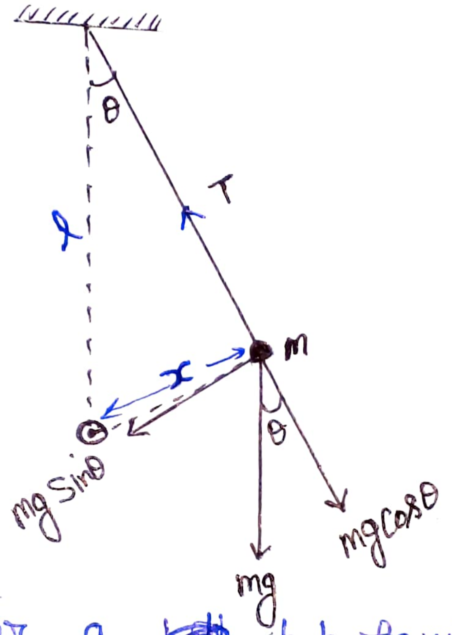
So Average K.E. is equal to average P.E. i.e

$$\boxed{U_{av} = K.E_{av}}$$

Example of Simple harmonic Motion ⇒

Simple Pendulum → A simple pendulum is an ideal body which consists of a point mass suspended by weightless, inextensible cord from a rigid support.

In reallity, a small metallic ~~bob~~ bob suspended by a cotton thread is used as simple pendulum.



Let us consider a ~~bob~~ bob having mass m which is suspended by a thread having length l from a rigid support. In the equilibrium position, the bob hangs vertically below the point of suspension. When the bob is displaced slightly from its equilibrium position, then a restoring force will act towards the equilibrium position, In this case the restoring force

$$F_{restoring} = -mg\sin\theta$$

where θ – angular displacement of the pendulum from its equilibrium position –

$$F_{restoring} = -mg\sin\theta$$

If angle θ is sufficiently small, then

$$\sin\theta \simeq \theta$$

$$F_{Restoring} = -mg\theta$$

from figure $\theta = \frac{x}{l}$

$$F_{restoring} = -\frac{mgx}{l} \quad -(1)$$

from Newton's second law

$$F = ma = m\frac{d^2x}{dt^2} \quad -(2)$$

from eq (1) & (2)

$$m\frac{d^2x}{dt^2} = -\frac{mgx}{l}$$

$$m\frac{d^2x}{dt^2} + \frac{mg}{l}x = 0$$

$$\frac{d^2x}{dt^2} + \frac{\cancel{m}g}{l\cancel{m}}x = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{g}{l}x = 0}$$

This is the differential eq^n of Simple pendulum

Time period→ we know the differential eq^n of simple pendulum,

$$\frac{d^2x}{dt^2} + \frac{g}{l}x = 0$$

$$\frac{d^2x}{dt^2} = -\frac{g}{l}x \quad -(3)$$

the Acceleration of SHM is given by

$$a = -\omega^2 x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad -(4)$$

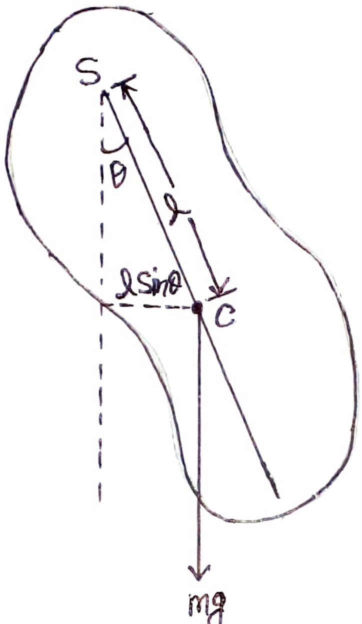
Comparing eq (3) & (4)

$$\omega^2 = \frac{g}{l} \Rightarrow \frac{4\pi^2}{T^2} = \frac{g}{l}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Example of Simple harmonic motion →

Compound Pendulum → Any rigid body which is so mounted ~~that~~ it can swing in vertical plane about a horizontal axis passing through it is known as physical or compound Pendulum.



Let us consider a body having mass m and distance l between the centre of mass (C) and suspension point (S). If at some instant body starts to oscillate, then a restoring torque will act on the body towards equilibrium position i.e

the restoring torque $(\tau) = -mg\,l\sin\theta$

[$\because \tau =$ Force . ⊥ distance]

If angle θ is small,

$$\tau = -mgl\theta \quad - ①$$

We know the relation between torque, moment of inertia (I) of a body

$$\tau = I\frac{d^2\theta}{dt^2} \quad - ②$$

From eq① & ②

$$I\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$I\frac{d^2\theta}{dt^2} + mgl\theta = 0$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0}$$

This is differential eqⁿ of compound pendulum.

Time period - $\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$ - ③

In simple Harmonic motion angular accⁿ -

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \quad -④$$

Comparing eq ③ & ④

$$\omega^2 = \frac{mgl}{I}$$

$$\frac{4\pi^2}{T^2} = \frac{mgl}{I}$$

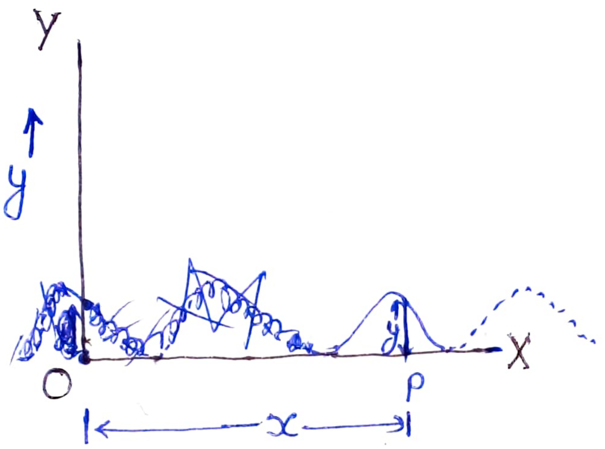
$$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

In terms of k, l and g the time period -

$$\boxed{T = 2\pi\sqrt{\frac{\left(\frac{k^2}{l}+l\right)}{g}}}$$

k → constant or known as radius of gyration.

Wave equation:→ Let us consider a number of particles forming a wave along the X-axis. Suppose the particle at the origin O acts as the source of disturbance which travels as a transverse wave in the positive direction of X-axis with velocity v. As the wave proceeds, each successive particle is set in motion.



Suppose At time $t=0$, the particle is in equilibrium or mean position position at point O, then after a time interval t or at any instant t, the displacement of particle at origin O is given by

$$y = f(t) \quad , \quad f(t) - \text{any function of time}$$

If the wave is propagating with velocity v, then ~~it will take to reach at poi~~
It will take $\frac{x}{v}$ seconds to reach at point P, so the particle at point P will start moving $\frac{x}{v}$ second later then the particle at O, So the displacement y of the particle at point P ~~at~~ at any instant t will be the same as the displacement at the origin $\frac{x}{v}$ second ~~later~~ earlier, i.e at $(t - x/v)$

hence the displacement of particle at point P is given by

$$y = f\left(t - \frac{x}{v}\right)$$, v is constant so,

~~Since this is~~, we may write it,

$$y = f(vt - x) \quad — (1)$$

This is the equation of a wave of any shape travelling along the positive direction of X-axis. The function f determines the exact shape of wave.

Similarly the wave propagating in negative X-direction may be written as.

$$y = f(vt + x) \quad — (2)$$

So the function $f(vt \pm x)$ shows the ~~travelling~~ travelling wave.

From eq (1) & (2) the general equation for wave can be written as-

$$y = f_1(vt - x) + f_2(vt + x) \quad — (3)$$

To get the general differential equation of wave, we differentiate eq (3) twice partially w.r.t. t–

$$\frac{\partial y}{\partial t} = v f_1'(vt - x) + v f_2'(vt + x)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 f_1''(vt - x) + v^2 f_2''(vt + x)$$
$$= v^2 [f_1''(vt - x) + f_2''(vt + x)] \quad — (4)$$

f_1'', f_2'' are second differentials of f_1 & f_2

Again differentiating eq (3) twice partially wrt x,

$$\frac{\partial^2 y}{\partial x^2} = f_1''(vt-x) + f_2''(vt+x) \quad -(5)$$

from eq (4) & (5) —

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

This is general differential equation of single wave.

Superposition Principle:→ This principle states "When two (or more) waves travel simultaneously in an elastic medium, the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements of that particle corresponding to separate waves at that instant."

This is seen in daily life. When we listen to an orchestra, we receive a complex sound due to the superposition of many sound waves of different characteristics produced by different musical instruments. Still we can recognize the separate sounds of different instruments.

Mathematical form of Superposition principle:→

Suppose y_1 is the displacement of a point x at time t due to a wave described by a function $y_1(x,t)$ and y_2 be the displacement of the same point x at the same point t due to another wave described by $y_2(x,t)$, then the resultant displacement may be written as -

$$\boxed{y(x,t) = y_1(x,t) + y_2(x,t)}$$

The superposition principle produces various phenomena like - Interference, diffraction, beats etc.

Maxwell's Equations:→ There are four Maxwell's equations in electricity & magnetism. Maxwell's equations are based on Basic laws of electricity & magnetism. Maxwell's equations are in two forms i.e. Differential form and Integral form.

Maxwell's equations in Differential form→

(i) $\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$ where $\vec{D}$- electric displacement vector or electric flux density.
ρ- Volume charge density.

This is differential form of Gauss's law of electrostatics.

(ii) $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$
Where $\vec{B}$- magnetic induction or magnetic field.
This is differential form of Gauss's law in Magnetostatics.

(iii) $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$, where $\vec{E}$- electric field intensity
This is Faraday's law in electromagnetic induction.

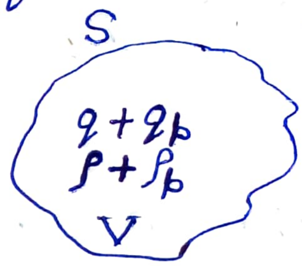
(iv) $\boxed{\vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t}}$
Where, $\vec{H}$- magnetic field intensity
$\vec{J}$- current density
$\vec{D}$- electric displacement vector or electric flux density.

This is modified form of ~~Farade~~ Ampere's circuital law in differential form.

Derivation of Maxwell's equations in Differential form :-

(i) Derivation of Ist maxwell's eq. in Differential form i.e $\vec{\nabla}\cdot\vec{D} = \rho$

Let us consider a closed surface S. There is free charge q and Polarisation charge q_p enclosed in that surface S. Correspondingly there will be volume charge densities ρ and ρ_p of free charge and Polarisation charges.



Recall Gauss's law for electrostatics -

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

where q - free charge.

If there is polarisation charge q_p and free charge q in surface S, then Gauss's law is -

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}(q + q_p) \quad — (1)$$

If ρ and ρ_p are free charge density & polarisation charge density, then we may write -

$$q = \int_V \rho\, dv, \quad q_p = \int_V \rho_p\, dv$$

then eq (1) will be -

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\left(\int_V \rho\, dv + \rho_p\, dv\right)$$

$$\oint_S \varepsilon_0\vec{E}\cdot d\vec{s} = \left(\int_V \rho\, dv + \rho_p\, dv\right)$$

$$\oint_S \varepsilon_0\vec{E}\cdot d\vec{s} = \int_V (\rho + \rho_p)\, dv$$

$$\oint_S \varepsilon_0 \vec{E}\cdot d\vec{s} = \int_V (\rho + \rho_p)\, dV$$

$$\oint_S \varepsilon_0 \vec{E}\cdot d\vec{s} = \int_V (\rho - \vec{\nabla}\cdot\vec{P})\, dV$$, where $\rho_p = \vec{\nabla}\cdot\vec{P}$, $\vec{P}$ – polarisation vector.

$$\oint_S \varepsilon_0 \vec{E}\cdot d\vec{s} = \int_V \rho\, dV - \int_V \vec{\nabla}\cdot\vec{P}\, dV \quad -②$$

Using Gauss divergence theorem to change surface integral into volume integral

$$\int_S \vec{A}\cdot d\vec{s} = \int_V \vec{\nabla}\cdot\vec{A}\, dV$$ – Gauss divergence Theorem

Applying Gauss div theorem on LHS in eq②

$$\int_V \vec{\nabla}\cdot \varepsilon_0\vec{E}\, dV = \int_V \rho\, dV - \int \vec{\nabla}\cdot\vec{P}\, dV$$

$$\int_V \vec{\nabla}\cdot \varepsilon_0\vec{E}\, dV + \int_V \vec{\nabla}\cdot\vec{P}\, dV = \int_V \rho\, dV$$

$$\int_V \vec{\nabla}\cdot (\varepsilon_0\vec{E} + P)\, dV = \int_V \rho\, dV$$

$$\int_V \vec{\nabla}\cdot\vec{D}\, dV = \int_V \rho\, dV \qquad \because \varepsilon_0\vec{E} + \vec{P} = \vec{D}$$

$$\int_V (\vec{\nabla}\cdot\vec{D} - \rho)\, dV = 0$$

so,

$$(\vec{\nabla}\cdot\vec{D} - \rho) = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

This is Ist maxwell eq. in Differential form.

Derivation of IInd maxwell's equation:→ $\vec{\nabla}\cdot\vec{B}=0$

Gauss's law for magnetostatics is

$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

Using Gauss div theorem to change surface integral into volume integral.

$$\int_V (\vec{\nabla}\cdot\vec{B})\, dV = 0 \qquad \because \int_S \vec{A}\cdot d\vec{S} = \int_V (\vec{\nabla}\cdot\vec{A})\, dV$$

So,

$$\boxed{\vec{\nabla}\cdot\vec{B}=0}$$

This is differential form of IInd maxwell's equation.

Derivation of IIIrd Maxwell's equation in Differential form→ $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

According to the Faraday's law of electro-magnetic induction, The induced emf (ε) is equal to the negative rate of change of magnetic flux (ϕ_B)

i.e.

$$\varepsilon = -\frac{d\phi_B}{dt} \quad -①$$

We know, magnetic flux $\phi_B = \oint \vec{B}\cdot d\vec{S}$

and emf (ε) $= \int_C \vec{E}\cdot d\vec{l}$

So eq ① becomes–

$$\int \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{S}$$

$$\int \vec{E}\cdot d\vec{l} = -\oint_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}$$

$$\int \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} \quad -(2)$$

Using Stoke's theorem to change line integral into surface integral.

$$\int \vec{A}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{A})\cdot d\vec{s} \quad \text{- Stoke's theorem}$$

Applying Stoke's theorem on LHS of eq(2)

$$\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

Taking multiple of $\int_S d\vec{s}$ on both side-

$$\boxed{(\vec{\nabla}\times\vec{E}) = -\frac{\partial \vec{B}}{\partial t}}$$ - This is III^{rd} maxwell's eq.

Derivation of IV^{th} Maxwell's equation in Differential form:- $\vec{\nabla}\times\vec{H} = \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

Recall that Ampere's circuital law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i$$

$$\oint \frac{\vec{B}}{\mu_0}\cdot d\vec{l} = i \quad -(1)$$

We know $\vec{H} = \frac{\vec{B}}{\mu_0}$ and $i = \oint \vec{J}\cdot d\vec{s}$

Where $\vec{H}$ - magnetic field intensity

$\vec{J}$ - current density.

then eq(1) becomes-

$$\oint \vec{H}\cdot d\vec{l} = \oint_S \vec{J}\cdot d\vec{s}$$

$$\oint \vec{H}\cdot d\vec{l} = \oint \vec{J}\cdot d\vec{s} \quad - (2)$$

Using stoke's theorem to change line integral into surface integral-

$$\int \vec{A}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{A})\, d\vec{s} \quad \text{- Stoke's theorem}$$

Applying stoke's theorem in eq (2), we get

$$\oint (\vec{\nabla}\times\vec{H})\, d\vec{s} = \oint \vec{J}\cdot d\vec{s}$$

Taking multiple of $\oint d\vec{s}$ on both side

$$\vec{\nabla}\times\vec{H} = \vec{J} \quad - (3)$$

This is ampere's law in differential form.

Now to get Maxwell's IVth eq^n in Differential form, Taking divergence of eq (3), we get -

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} \quad \text{OR} \quad \text{Div}(\text{curl}\vec{H}) = \text{div}\vec{J} \quad (4)$$

[$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0$, because $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ where $\vec{A}$ is any vector]

So, eq (4) reduces to -

$$\vec{\nabla}\cdot\vec{J} = 0 \quad - (5)$$

Recall that the continuity eq^n.

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} \quad - (6)$$

putting the value of $\vec{\nabla}\cdot\vec{J}$ from eq (6) into eq (5), we get -

$$-\frac{\partial \rho}{\partial t} = 0 \quad \text{OR} \quad \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = 0$$

It means $\rho =$ Constt

If ρ is constant then equal charge will flow at every time $\Rightarrow$ steady current will flow. Hence eq 3 or Ampere's circuital law is valid only for steady current.

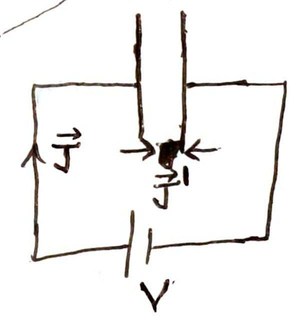
If there is not steady current, then there is need to modify eq 3 or Ampere's circuital law.

So if there is steady current and changing current as well, then eq 3 or Ampere's law is modified as-

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}' \quad - 7$$

where $\vec{J}$ is conduction current or steady current

$\vec{J}'$ is changing current due to some region, Capacitor



Taking divergence of eq 7 on both side-

$$\underbrace{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H})}_{0} = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}'$$

0, because $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$, $\vec{A}$ - Vector

$$\vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}' = 0$$

$$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J}' = 0$$

Recall, continuity eq^n

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

putting the value of $\vec{\nabla}\cdot\vec{J}$ in above eq^n, we get

$$-\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J}'$$

$$\vec{\nabla}\cdot\vec{J}' = \frac{\partial \rho}{\partial t}$$

Recall maxwell's Ist eq^n.

$$\vec{\nabla}\cdot\vec{D} = \rho \Rightarrow \rho = \vec{\nabla}\cdot\vec{D}$$

putting the value of ρ in above eq^n.

$$\vec{\nabla}\cdot\vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$$

$$\vec{\nabla}\cdot\vec{J}' = \vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t}$$

So comparing on both side, we get

$$J' = \frac{\partial \vec{D}}{\partial t}$$

So from eq(7), the modified Ampere's law is

$$\vec{\nabla}\times\vec{H} = J + J'$$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

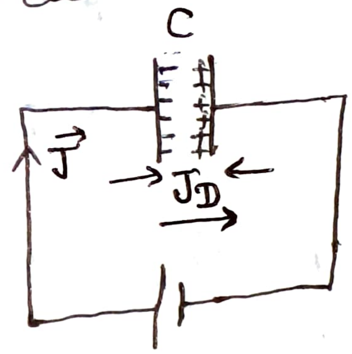
This is known as modified Ampere's law in Differential form or IVth maxwell's eq in Differential form. The term $\frac{\partial \vec{D}}{\partial t}$ is known as displacement current and is denoted by J_D.

Physical Significance of Displacement Current Density (J_D) :→ $J_D = \frac{\partial \vec{D}}{\partial t}$

$$J_D = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \because \vec{D} = \varepsilon \vec{E}$$

$$J_D = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

The changing electric field produces current, this current is known as displacement current (J_D). It is equally effective as conduction current ($\vec{J}$) density. This displacement current density (J_D) is responsible for the production of magnetic field in empty space, where the conduction current is zero.



Maxwell's equations in Integral form and Their Physical Significance :-

Maxwell's Ist equation In Integral form -

Recall the Ist Maxwell's equation in differential form i.e. $\vec{\nabla}\cdot\vec{D} = \rho$

Taking volume integral of this equation

$$\int_V (\vec{\nabla}\cdot\vec{D})\,dv = \int_V \rho\,dv$$

Using Gauss div theorem to change volume integral into surface integral in LHS

$$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\,dv \qquad \left[\because \int_V (\vec{\nabla}\cdot\vec{A})\,dv = \int \vec{A}\cdot d\vec{S}\right]$$

$$\boxed{\oint_S \vec{D}\cdot d\vec{S} = q}$$ - This is the integral form of Maxwell's Ist equation

Physical significance - The flux of electric displacement vector through a closed surface is equal to the net charge enclosed in that surface.

Maxwell's IInd equation in Integral form →

Recall Maxwell's IInd equation in Differential form i.e. $\vec{\nabla}\cdot\vec{B} = 0$

Taking volume integral.

$$\int_V (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

Using Gauss div theorem to change volume integral into surface integral -

$$\oint_S \vec{B}\cdot d\vec{S} = 0 \;, \qquad \left[\because \int_V (\vec{\nabla}\cdot\vec{A})\,dv = \int_S \vec{A}\cdot d\vec{S}\right]$$

Physical Significance → The flux of magnetic induction ($\vec{B}$) through a closed surface is equal to zero.

Maxwell's IIIrd equation in Integral form →

Recall the Maxwell's eq in Differential form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Taking surface integral on both side

$$\oint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Using stokes theorem to change surface integral into line integral on LHS.

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s} \qquad \left[\because \int_V (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} = \int \vec{A} \cdot d\vec{l} \right]$$

$$\cancel{\boxed{emf = -\frac{\partial \phi_B}{\partial t}}}$$ ~~This is~~

~~Where emf → induced electromotive force~~

~~ϕ_B → magnetic flux.~~

→ This is integral form of Maxwell's IIIrd equation.

Physical Significance: → The line integral of electric field around a closed path is equal to the negative rate of change of magnetic flux (ϕ_B), where $\phi_B = \oint \vec{B} \cdot d\vec{s}$

Maxwell's IVth equation in Integral form

Recall Maxwell's IVth equation in Differential form-

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking surface integral on both side-

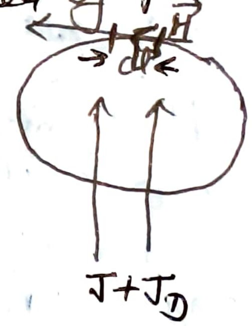
$$\oint_S (\vec{\nabla} \times \vec{H}) ds = \int_S \left(J + \frac{\partial D}{\partial t} \right) ds$$

Using Stokes theorem to change surface integral into line integral-

$$\boxed{\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}}$$

This is integral form of Maxwell's IVth eq^n.

Physical Significance → The line integral of magnetic field intensity ($\vec{H}$) around a closed path is equal to the conduction plus displacement current through any surface bounded by path.

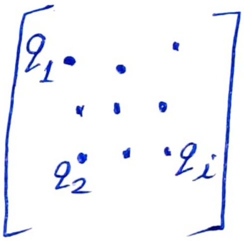


Work done by electromagnetic field:→ To find work done by electromagnetic field, we will first get work done by electric field after that we will get work done by magnetic field and then we will add both work done.

Work done by electric field or The energy of a continuous charge distribution:→ Suppose we have a number of charges $q_1, q_2, q_3, q_4 \cdots q_i$. We want to assemble an entire collection of ~~point~~ these point charges, so to assemble these charges, we have to do some work, so work required to assemble these charges is given by

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(r_i) \quad \text{— (1)}$$

where $V(r_i)$ - potential



If the charges are continuously distributed then we have to replace summation into integral. then eq (1) becomes -

$$W = \frac{1}{2} \int \rho V d\tau \quad \text{— (2)} \qquad \because q = \int \rho d\tau$$

where, ρ - volume charge density
V - electrical potential
$d\tau$ - volume element

From Ist Maxwell equation,

$$\vec{\nabla} \cdot \vec{D} = \rho$$

putting, $\vec{D} = \epsilon_0 \vec{E}$

So, $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

$$\rho = \epsilon_0 (\vec{\nabla} \cdot \vec{E})$$

putting the value of ρ in eq (2)

$$W = \frac{\epsilon_0}{2} \int (\vec{\nabla} \cdot \vec{E}) V d\tau$$

$$W = \frac{\varepsilon_0}{2}\int_V (\vec{\nabla}\cdot\vec{E})V\,d\tau \quad -③$$

Using the product rule of vector calculus.

$$\vec{\nabla}\cdot(f\vec{A}) = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot\vec{\nabla}f$$

$$(\vec{\nabla}\cdot\vec{A})f = \vec{\nabla}\cdot(f\vec{A}) - \vec{A}\cdot\vec{\nabla}f$$

~~putting f = V~~

$$\cancel{(\vec{\nabla}\cdot\vec{A})V = \vec{\nabla}\cdot(V\vec{A}) - \vec{A}\cdot\vec{\nabla}V}$$

using this property of vector in eq ③

$$W = \frac{\varepsilon_0}{2}\Big[\underbrace{\int_V \vec{\nabla}\cdot(V\vec{E})\,d\tau}_{\mathrm{I}} - \underbrace{\int_V \vec{E}\cdot(\vec{\nabla}V)\,d\tau}_{\mathrm{II}}\Big]$$

using Gauss divergence theorem to change volume integral into surface integral in I term and putting $\vec{E} = -\vec{\nabla}V$ in II term

$$W = \frac{\varepsilon_0}{2}\left[\oint_S V\vec{E}\cdot d\vec{s} + \int_V \vec{E}\cdot\vec{E}\,d\tau\right]$$

$$W = \frac{\varepsilon_0}{2}\Big[\underbrace{\int_V E^2\,d\tau}_{\mathrm{I}} + \underbrace{\oint_S V\vec{E}\cdot d\vec{s}}_{\mathrm{II}}\Big]$$

on increasing the volume, the second term becomes negligible in comparison to I term, So,

$$\boxed{W = \frac{\varepsilon_0}{2}\int_V E^2\,d\tau}$$

This is the expression for work done by electric field or energy ~~density~~ of a continuous charge distribution.

Work done by electromagnetic field = work done by electric field + work done by magnetic field

Work done by electric field $= \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau$

Work done by magnetic field $= \frac{1}{2\mu_0} \int_V B^2 d\tau$

So ~~total~~ work done by electromagnetic field

$$\boxed{= \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau + \frac{1}{2\mu_0}\int_V B^2 d\tau}$$

This is the required expression for work done by electromagnetic field.

Poynting Theorem:→ The work required to assemble a static continuous charge distribution is given by

$$W_e = \frac{\varepsilon_0}{2}\int_V E^2 d\tau, \quad \text{where } E\text{- electric field}$$
$d\tau$ - volume element

Similarly the work required to get currents going (against emf) is given by

$$W_m = \frac{1}{2\mu_0}\int B^2 d\tau, \quad B\text{- magnetic field}$$

hence total energy stored in electromagnetic field may be written as.

$$W = U_{em} = W_e + W_m$$

$$U_{em} = W = \frac{\varepsilon_0}{2}\int E^2 d\tau + \frac{1}{2\mu_0} B^2 d\tau \quad \text{— (1)}$$

Suppose we have some charge and current configuration, which at time t produces electric field $\vec{E}$ and magnetic field $\vec{B}$. In the next time dt, the charges move around a bit, then according to Lorentz force law the force experienced by charge q will be

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

and the work done on this charge q to move distance dl -

$$dW = q(\vec{E}+\vec{v}\times\vec{B})\cdot d\vec{l} \qquad [\because W = \vec{F}\cdot d\vec{s}]$$

$$dl = \vec{v}dt$$

$$dW = q(\vec{E}+\vec{v}\times\vec{B})\cdot\vec{v}\,dt$$

$$dW = q(\vec{E}+(\vec{v}\times\vec{B}))\cdot\vec{v}dt$$

$$dW = q\vec{E}\cdot v\,dt + (\vec{v}\times\vec{B})\cdot\vec{v}dt \qquad (\vec{v}\times\vec{B})\cdot\vec{v} = 0$$

from vector property — (1)

$$dW = q\vec{E}\cdot v\,dt$$

$$dW = q\vec{E}\cdot\vec{v}\,dt \quad \text{—} ①$$

putting $q = \rho\, d\tau$, $\rho \to$ volume charge density
$d\tau$ – volume element

$$dW = \rho\, d\tau\, \vec{E}\cdot\vec{v}\,dt$$

putting $\rho v = J$

$$dW = \vec{E}\cdot\vec{J}\,d\tau\,dt$$

$$\frac{dW}{dt} = (\vec{E}\cdot\vec{J})\,d\tau$$

So the rate at which work is done on all the charges in a volume V is

$$\frac{dW}{dt} = \int_V (\vec{E}\cdot\vec{J})\,d\tau \quad \text{—} ②$$

It is clear from above equation that $\vec{E}\cdot\vec{J}$ is the work done per unit volume per unit time or the power delivered per unit volume. Now we will express this quantity $\vec{E}\cdot\vec{J}$ in terms of electric and magnetic fields using Maxwell's equations.

We know IIIrd Maxwell's equation & IVth Maxwell equation

$$(\vec{\nabla}\times\vec{E}) = -\frac{\partial\vec{B}}{\partial t} \quad \text{—} ③$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \quad \text{—} ④$$

Taking scalar product of eq ③ with $\vec{H}$ and eq ④ with $\vec{E}$,

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} \quad \text{—} ④$$

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} \quad \text{—} ⑤$$

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} \quad -④$$

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} \quad -⑤$$

Using vector identity -

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})$$

Now putting the value of $\vec{H}\cdot(\vec{\nabla}\times\vec{E})$ and $\vec{E}\cdot(\vec{\nabla}\times\vec{H})$ from eq ④ & ⑤

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\left[\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right] - \vec{E}\cdot\vec{J}$$

putting $\vec{H} = \frac{\vec{B}}{\mu_0}$ and $\vec{D} = \varepsilon_0\vec{E}$

$$\vec{\nabla}\cdot\frac{(\vec{E}\times\vec{B})}{\mu_0} = -\left[\frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} + \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}\right] - \vec{E}\cdot\vec{J}$$

$$\vec{E}\cdot\vec{J} = -\left[\frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} + \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}\right] - \vec{\nabla}\cdot\frac{(\vec{E}\times\vec{B})}{\mu_0}$$

putting $\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(E^2)$

$$\vec{E}\cdot\vec{J} = -\left[\frac{1}{\mu_0}\frac{1}{2}\frac{\partial}{\partial t}(B^2) + \varepsilon_0\frac{1}{2}\frac{\partial}{\partial t}(E^2)\right] - \vec{\nabla}\cdot\frac{(\vec{E}\times\vec{B})}{\mu_0}$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) - \vec{\nabla}\cdot\frac{(\vec{E}\times\vec{B})}{\mu_0}$$

Putting this value of $\vec{E}\cdot\vec{J}$ in eq ②

$$\frac{dW}{dt} = -\int_V \frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)d\tau - \frac{1}{\mu_0}\int_V \vec{\nabla}\cdot(\vec{E}\times\vec{B})\,d\tau$$

$$\frac{dW}{dt} = -\int_V \frac{\partial}{\partial t}\frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)d\tau - \int_V \frac{\vec{\nabla}\cdot(\vec{E}\times\vec{B})}{\mu_0}d\tau$$

$$\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)d\tau - \int_V \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})\,d\tau$$

Using Gauss divergence theorem to change volume integral into surface integral in second term

$$\boxed{\frac{dW}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)d\tau - \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\cdot d\vec{s}}$$

This is Poynting theorem. It is also known as Work-energy theorem of electrodynamics.

Other form of Poynting theorem is

$$\boxed{\frac{dW}{dt} = -\frac{dU_{em}}{dt} - \frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})\cdot d\vec{a}} \qquad [\because d\vec{s}\rightarrow d\vec{a})$$

Where U_{em} - energy of electromagnetic field

Poynting theorem says that The work done on the charges by electromagnetic force is equal to the decrease in energy stored in field less the energy that flowed out through the surface.

We can also write Poynting theorem in the following way-

$$\boxed{\frac{dW}{dt} = -\frac{dU_{em}}{dt} - \oint \vec{S}\cdot d\vec{a}}$$

Where $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$

$\vec{S}$ is known as Poynting vector.

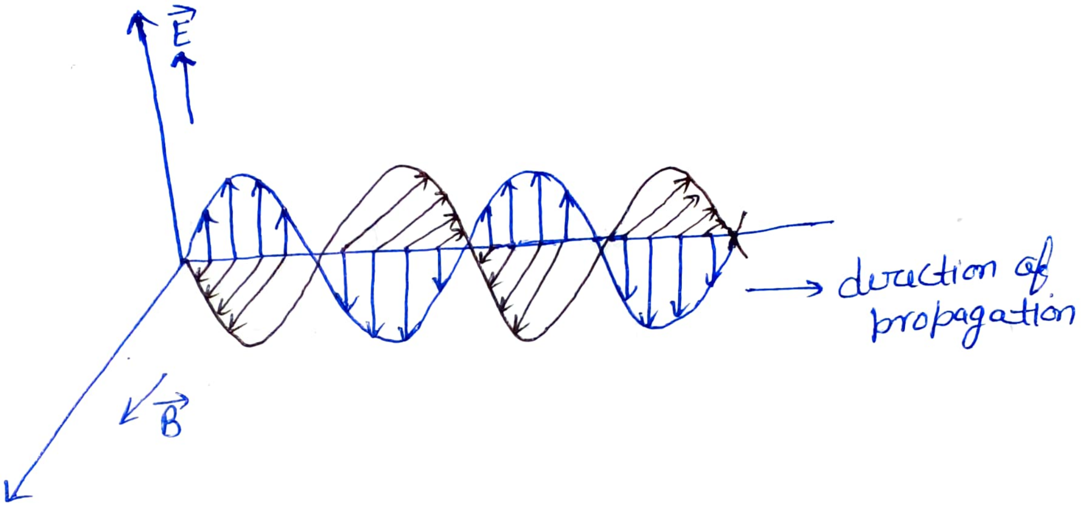
Physical significance of Poynting vector → $\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})}$

The energy per unit time per unit area transported by the fields is known as Poynting vector.

Electromagnetic wave equation in Free space:→

Electromagnetic wave:- The wave in which the electric vectors, magnetic vectors and direction of propagation are mutually perpendicular to each other, is called as electro magnetic wave.

Electromagnetic wave is represented as-



Electromagnetic wave is a plane wave because it travels only in single direction and do not depend on the other coordinates or directions. So this electromagnetic wave is also known as Plane electromagnetic wave.

Now we will derive the wave equation for electromagnetic waves using maxwell's equations in free space-

Recall maxwell's equations in differential form.

(i) $\vec{\nabla} \cdot \vec{D} = \rho$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

maxwell's equations in free space may be written as

In free space, $\rho = 0$ and also conductivity$(\sigma) = 0$

we know $J = \sigma \vec{E}$

so $J = 0$

$\vec{D} = \varepsilon_0 \vec{E}$,

so putting $\rho = 0$, $J = 0$, $D = \varepsilon_0 \vec{E}$ in maxwell's equations

(i) $\vec{\nabla} \cdot \varepsilon_0 \vec{E} = 0 \Rightarrow \vec{\nabla} \cdot \vec{E} = 0$ —— ①(a)

(ii) $\vec{\nabla} \cdot \vec{B} = 0$ —— ①(b)

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ —— ①(c)

(iv) $\vec{\nabla} \times \vec{H} = 0 + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ — ①(d)

Taking curl of eq ①(c), we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

putting $\vec{B} = \mu_0 \vec{H}$ in R.H.S.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \mu_0 \vec{H})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H})$$

putting the value of $\vec{\nabla} \times \vec{H}$ from eq①(d) into the above equation.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

We know the vector property-

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}$$

Using this vector property on LHS in above equation

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

putting $\vec{\nabla}.\vec{E} = 0$ [First maxwell's equation in free space]

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad - ②$$

Similarly using maxwell's equations,
we get the above equation in terms of $\vec{H}$

$$\boxed{\nabla^2 \vec{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0} \quad - ③$$

Equation ② & ③ are electromagnetic wave equations in free space in terms of electric field & magnetic field.

Proof of electromagnetic wave is light wave or em wave in free space has velocity, equal to the velocity of light

Suppose there is a common function ϕ, which satisfies eq ② & ③, so we can write eq ② or ③

$$\nabla^2\phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = 0 \quad -④$$

We know the general wave equation

$$\nabla^2\phi - \frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2} = 0, \quad -⑤$$

where v – velocity of wave

Comparing eq ④ & ⑤, we get

$$\frac{1}{v^2} = \mu_0 \epsilon_0$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

v is the velocity of electromagnetic wave in free space.

putting the value of μ_0 & ϵ_0,

$\mu_0 = 4\pi \times 10^{-7}$ weber/Ampere-meter

$\epsilon_0 = 8.54 \times 10^{-12}$ unit

$$\boxed{v = 3 \times 10^8 \, m/sec} = c \text{ (speed of light)}$$

It proves that the velocity of em wave in free space is equal to the velocity of light. It means em wave is light wave.

Energy Carried by electromagnetic wave-

Recall the energy contained in electromagnetic ~~magnetic~~ field-

$$W = U_{em} = \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau + \frac{1}{2\mu_0}\int_V B^2 d\tau$$

But for whole volume V, we can write above equation

$$U_{em} = \frac{1}{2}\varepsilon_0 E^2 V + \frac{1}{2\mu_0} B^2 V$$

$$\frac{U_{em}}{V} = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}.$$

$$\text{Energy density}(u) = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

$$\because u = \frac{U_{em}}{V}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

But for free space the contribution of electric & magnetic fields in em wave is equal, so putting-

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2$$

$$\boxed{u = \varepsilon_0 E^2}$$

This is expression for energy density in em wave.