(Please write your Enrolment No. immediately) Enrolment No. ____________

MID TERM EXAMINATION

B.TECH PROGRAMMES (UNDER THE AEGIS OF USICT)

2nd Semester, May, 2023

Paper Code: BS-112 **Subject: Applied Mathematics-II**

Time: 1½Hrs. **Max. Marks: 30**

Note: Attempt Q.No.1 which is compulsory and any two more questions from remaining.

1. (a) Principal Argument of $(1+i)^{20}$ (2 ½)

1. (b) Find $Re\left(e^{e^z}\right)$ (2)

1. (c) Integrate $Re(z)\,along\ the\ line\ 0\ to\ 1+2i$ (3)

1. (d) Find the residue of $f(z)=\frac{\coth z}{z-i}$ $at\ each\ of\ the\ poles.$ (2 ½)

2. (a) Find Modulus and principal argument of $z=-1-i\sqrt{3}$ and verify the result that multiplication by i is geometrically a counterclockwise rotation through $\pi/2$ by graphing z and iz and the angle of rotation. (5)

2. (b) Find all Taylor and Laurent series of $f(z)=\frac{-2z+3}{z^2-3z+2}$ in the region $1<|z|<2$. (5)

3. (a) Find $Re(\cosh z)$ and all solution of $\cosh z=1$ (5)

3. (b) (5)

$$Evaluate \oint \frac{e^z}{(z+1)^2}dz\,,along\ C$$
$$where\ C\ is\ the\ circle\ |z-1|=3$$

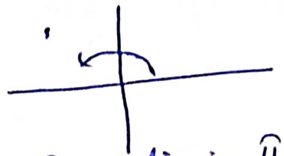
4. (a) An electrical field $f(z)=\phi(x,y)+i\psi(x,y)$ in the xy- plane, the potential function $\phi(x,y)=3x^2y-y^3$ is given. Find the stream function $\psi(x,y)$ and electric field f(z). (5)

4. (b) Find the image of the infinite strip $0<y<\frac{1}{2}$ $under\ the\ mapping\ w=\frac{1}{z}$ (5)

Q.1 (a) Principal Argument of $(1+i)^{20}$ $\qquad r = |z| = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}(y/x)$

$$z = (1+i)^{20}$$
$$= \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{20} \cdot (\sqrt{2})^{20}$$
$$= 2^{10}\left[\cos 5\pi + i\sin 5\pi\right]$$
$$= 2^{10}\left[-1 + i.0\right]$$



$$\theta = \tan^{-1}\left|\frac{0}{1}\right| = \tan^{-1} 0 = 0$$

Point lies in II quadrant

$$\theta = 0$$

Principal argument $= \pi - \theta = \pi - 0 = \underline{\underline{\pi}}$

(b) $\mathrm{Re}\left(e^{e^{z}}\right)$

$$= e^{e^{x+iy}}$$
$$= e^{e^{x}(\cos y + i\sin y)}$$
$$= e^{e^{x}\cos y + e^{x} i\sin y} \longrightarrow (1)$$

Solve (1)

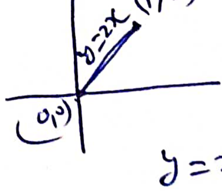
$$e^{i e^{x}\sin y} = \cos(e^{x}\sin y) + i\sin(e^{x}\sin y)$$

Putting in (1)

$$= e^{e^{x}\cos y}\left[\cos(e^{x}\sin y) + i\sin(e^{x}\sin y)\right]$$

$$\therefore \mathrm{Re}\left(e^{e^{z}}\right) = e^{e^{x}\cos y}\left[\cos(e^{x}\sin y)\right] \text{ Ans}$$

(c) $\displaystyle\int_0^{1+2i} \mathrm{Re}(z)\,dz = \int_{x=0}^{1} x\,(dx + i\,dy)$



$y = 2x$

$dy = 2dx$

$$= \int_{x=0}^{1} x(1+2i)\,dx$$
$$= (1+2i)\left(\frac{x^2}{2}\right)_0^1 = \frac{1+2i}{2} \text{ Ans}$$

2(b)

(d) $f(z) = \dfrac{\coth z}{z-i}$

Pole $z - i = 0$

$\boxed{z = i}$

∴ Residue at Pole $z = i$

$$R = \lim_{z \to i} \cancel{(z-i)} \cdot \frac{\coth z}{\cancel{(z-i)}}$$

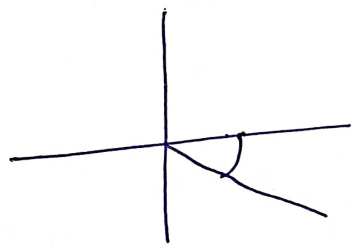
$R = \coth i$ Ans

2(a) $z = -1 - i\sqrt{3}$

∴ $\text{Mod}(z) = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

~~Tan~~ $\theta = \tan^{-1}\left|\dfrac{-\sqrt{3}}{-1}\right| = \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

∴ Principal argument $= -\dfrac{\pi}{3}$ Ans



2(b) $f(z) = \frac{-2z+3}{z^2-3z+2}$, $1<|z|<2$

$= -2z+3\left[\frac{1}{z-2} - \frac{1}{z-1}\right]$

$= (-2z+3)\left[-\frac{1}{2}(1-\frac{z}{2})^{-1}\right] - \frac{(-2z+3)}{z}\left[1-\frac{1}{z}\right]^{-1}$

$= -\frac{1}{2}(-2z+3)\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\cdots\right] - \frac{(3-2z)}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\cdots\right]$ Ans

3(a) $\because$ $\cosh z = \cos iz$

$= \cos i(x+iy)$

$= \cos(ix-y)$

$= \cos ix \cos y + \sin ix \sin y$

$= \cosh x \cos y + i \sinh x \sin y$

$\therefore Re(\cosh z) = \cosh x \cos y$

Given

$\cosh z) = 1$ ①

we have to find solution

we know that

$e^z = 1$

$\Rightarrow z = 2k\pi i$ $k \in \mathbb{Z}$.

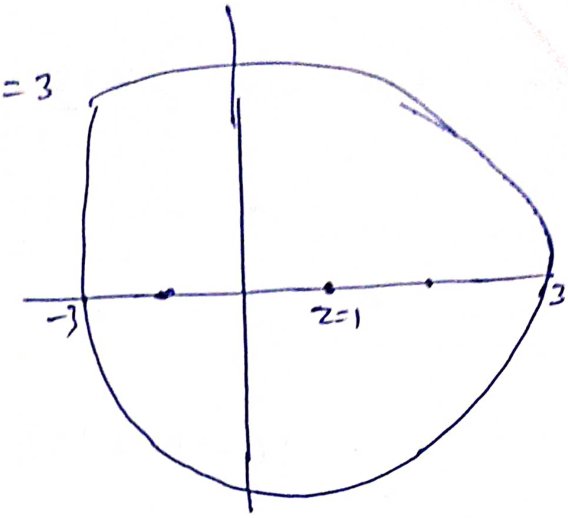
from ①

$\frac{e^z + e^{-z}}{2} = 1$

$e^z + e^{-z} = 2$

$\Rightarrow z = 2k\pi i$ $k \in \mathbb{Z}$.

3(b) $\int_c \frac{e^z}{(z+1)^2} dz$, where c, $|z-1| = 3$



Pole $z = -1$ (order 2)

i.e Pole lies inside c.

$$R = \frac{1}{\lfloor 2-1} \frac{d}{dz} \cdot \cancel{(z+1)^2} \cdot \frac{e^z}{\cancel{(z+1)^2}} \Bigg|_{z=-1}$$

$$= \frac{1}{1} \cdot (e^z)_{z=-1} = e^{-1}$$

$$= \frac{1}{e} \quad \text{Ans}$$

So $\int_c \frac{e^z}{(z+1)^2} dz = 2\pi i \left[\frac{1}{e}\right] = 2\pi i e^{-1}$ Ans

4(a) $f(z) = \phi + i\psi = u + iv$

Here $u = 3x^2y - y^3$ $\qquad \frac{\partial u}{\partial x} = 6xy$; $\frac{\partial u}{\partial y} = 3x^2 - 3y^2$

$\therefore dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$

$= -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$

$dv = -(3x^2 - 3y^2) dx + 6xy$

$v = -\int (3x^2 - 3y^2) dx + 0$

$v = -\frac{\cancel{3}x^3}{\cancel{3}} + 3xy^2 + c$

$$\boxed{v = -x^3 + 3xy^2 + c}$$

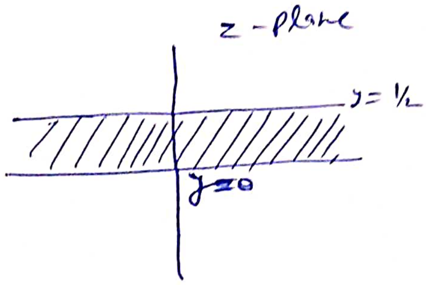
4(b) $W = \frac{1}{z}$

$$z = \frac{1}{w} = \frac{1}{u+iv} \times \frac{u-iv}{u-iv} = \frac{u-iv}{u^2+v^2}$$

$$x+iy = \frac{u-iv}{u^2+v^2}$$

$$x = \frac{u}{u^2+v^2}, \quad y = \frac{-v}{u^2+v^2}$$

Given $0 < y < 1/2$



$$y > 0$$

$$\frac{-v}{u^2+v^2} > 0 \quad \therefore -v > 0$$

$$v < 0$$

And $y < \frac{1}{2}$

$$\frac{-v}{u^2+v^2} < \frac{1}{2}$$

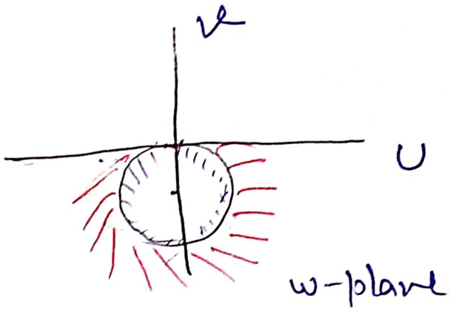
$$-2v < u^2+v^2$$

$$0 < u^2+v^2+2v$$

$$0 < u^2+v^2+2v+1-1$$

$$1 < u^2+(v+1)^2$$

$$u^2+(v+1)^2 > 1$$



Centre (0,1)
radius = 1