CHAPTER 2

SIGNALS AND WAVEFORM SYNTHESIS

2.1. INTRODUCTION

A signal may be considered to be a function of time that represents a physical variable of interest associated with a system. In electrical systems, the excitation (input) and response (output) are given in terms of currents and voltages. Mostly, these currents and voltages are function of time. In general, these functions of time are called signals, *i.e.* signals are also called as the functions.

Signals play an important role in science and technology as communication, aeronautics, bio-medical engineering and speech processing etc.

2.2. CLASSIFICATION OF SIGNALS

Signals may describe a wide variety of physical phenomena, as follows :

2.2.1. Continuous-Time and Discrete-Time Signals

To distinguish between continuous-time and discrete-time signals, we will use the symbol 't' to denote the continuous-time independent variable and 'n' or 'nT' to denote the discrete-time independent variable.

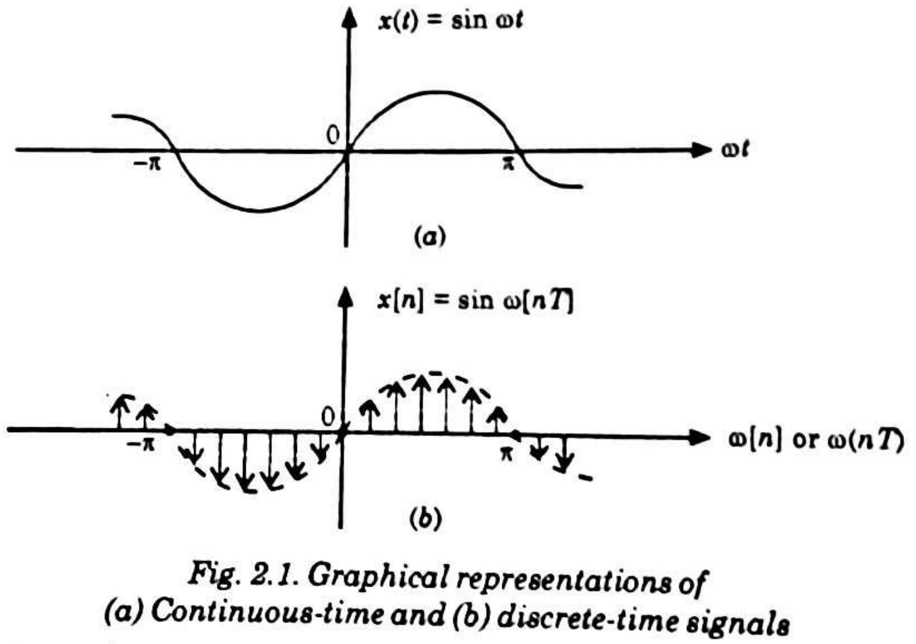

Fig. 2.1. Graphical representations of (a) Continuous-time and (b) discrete-time signals

Illustrations of a continuous-time signal $x(t)$ and a discrete-time signal $x[n]$ or $x(nT)$ are shown in figure 2.1.

2.2.2. Even and Odd Signals

Another set of useful properties of signals relates to their symmetry under time reversal.

A signal $x(t)$ or $x[n]$ is referred to as an even signal if it is identical to its time-reversed counterpart. In continuous-time a signal is *even* if

$$x(-t) = x(t)$$

while a discrete-time signal is even if

$$x[-n] = x[n]$$

Examples : (i) t^n (where n is even) or t^{2n} (where $n \in$ integer) *i.e.* $t^2, t^4, ...$

(ii) $\cos t$, $\sin^2 t$, etc.

A signal is referred to as an odd if the signal is negative of its reflection, i.e.,

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

Examples : (i) t^n (where n is odd) or t^{2n+1} (where $n \in$ integer) *i.e.*, $t, t^3, ...$

(ii) $\sin t$, etc.

Note : There are some functions (signals), which are neither even nor odd.
Examples : e^t, $t^2 + t$ etc.

Theorems :

(i) Sum of even functions = even function
(ii) Sum of odd functions = odd function
(iii) Multiplication of even and even functions = even function
(iv) Multiplication of odd and odd functions = even function
(v) Multiplication of even and odd functions = odd function
(vi) Sum of even and odd functions = Neither even nor odd.

2.2.3. Periodic and Unperiodic Signals

A signal $x(t)$ is *periodic* if and only if

$$x(t + T_0) = x(t), \qquad -\infty < t < \infty \qquad ...(1)$$

where the constant T_0 is the period of $x(t)$. The smallest value of T_0 such that equation (1) is satisifed is referred to as the Time-period. Any signal not satisfying equation (1) is called unperiodic or aperiodic.

Examples : (i) $\sin t$, $\cos t$, $\sin {}^t/_2$, $\cos 4t$ etc. are periodic signals

(ii) e^t, t^2, t etc. are unperiodic signals.

2.3. STANDARD SIGNALS OR SINGULARITY FUNCTIONS

In order to simulate any signal, some standard signals which are realisable in the laboratory environment are described in this section.

Singularity function can be obtained from one another by successive differentiation or integration.

2.3.1. Step Signal

The step signal $f_s(t)$ is defined by

$$f_s(t) = \begin{cases} 0 & ; t < 0 \\ K & ; t > 0 \end{cases} \qquad \text{(where } K \text{ is the amplitude of the step signal)}$$

The signal $f_s(t)$ is graphed in figure 2.2(a). Note that the function is undefined and discontinuous at $t = 0$.

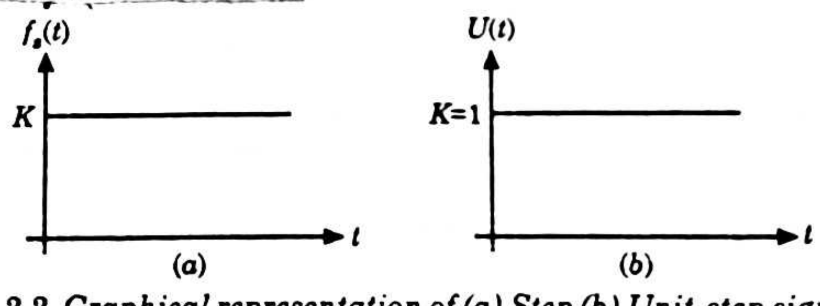

Fig. 2.2. Graphical representation of (a) Step (b) Unit-step signals.

If the value of $K = 1$ (unity), then, this step signal $f_s(t)$ is called as *unit step signal* $U(t)$, which is shown in figure 2.2(b) and defined as

$$U(t) = \begin{cases} 0 & ; t < 0 \\ 1 & ; t > 0 \end{cases}$$

It can be observed from above, that the step and unit step signals are zero whenever the argument within the parentheses, namely, t is negative, and they have magnitude K and 1 respectively when the argument is greater than zero. This helps us in defining the shifted or delayed signals as

$$f_s(t-a) = \begin{cases} 0 \ ; \ t < a \\ K \ ; t > a \end{cases}$$

And,

$$U(t-a) = \begin{cases} 0 \ ; t < a \\ 1 \ ; t > a \end{cases}$$

as shown in figure 2.3(a) and (b) respectively.

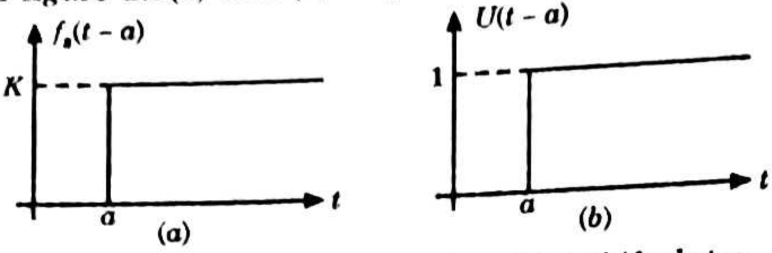

Fig. 2.3. Graphical representation of (a) Shifted step (b) Shifted unit-step signals.

Example 2.1. Express the given waveform as shown in figure 2.4 in terms of step signal.

Solution :

$$f(t) = K\,U[t-(-t_1)]$$
$$f(t) = K\,U(t+t_1)$$

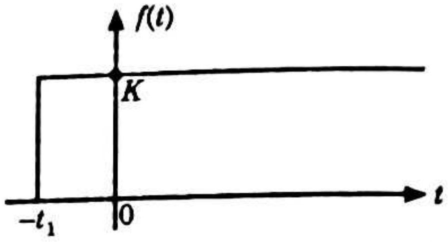

Fig. 2.4.

2.3.2. Ramp Signal

The ramp signal $f_r(t)$ is defined by

$$f_r(t) = \begin{cases} 0 \ ; t < 0 \\ Kt \ ; t \geq 0 \end{cases}$$

(where K is the slope of the ramp signal)

The signal $f_r(t)$ is graphed in figure 2.5(a). If the value of slope $K = 1$, then, this ramp signal $f_r(t)$ is called as *unit ramp signal* $r(t)$, which is shown in figure 2.5(b) and defined as

$$r(t) = \begin{cases} 0 \ ; \ t < 0 \\ t \ ; \ t \geq 0 \end{cases}$$

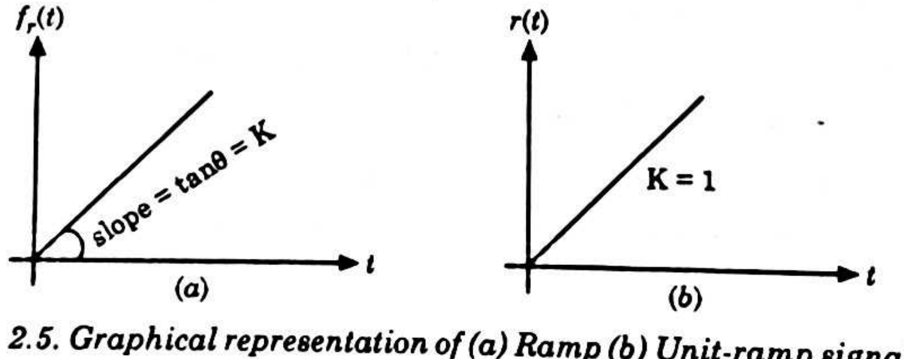

Fig. 2.5. Graphical representation of (a) Ramp (b) Unit-ramp signals.

And, shifted ramp signal as shown in figure 2.6(a) and is described as

$$f_r(t-a) = \begin{cases} 0 \qquad ; \ t < a \\ K(t-a) \ ; \ t \geq a \end{cases}$$

And, shifted unit ramp signal as shown in figure 2.6(*b*) and is described as

$$r(t-a) = \begin{cases} 0 & ; t < a \\ (t-a) & ; t \geq a \end{cases}$$

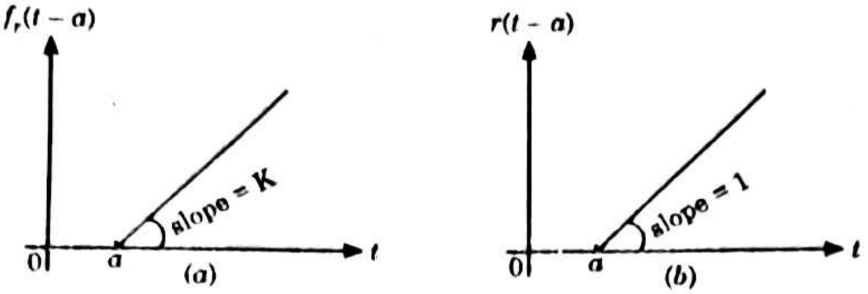

Fig. 2.6. Graphical representation of (a) Shifted ramp (b) Shifted unit-ramp signals

Note **:** Unit ramp signal;

$$r(t) = t\ U(t)$$
$$r(t-a) = (t-a)\ U(t-a)$$

And ramp signal,

$$f_r(t) = K\, r(t) = K\, t\ U(t)$$
$$f_r(t-a) = K\, r(t-a) = K\,(t-a)\ U(t-a)$$

Example 2.2. Express the given triangular waveform as shown in figure 2.7, in terms of ramp and step signals.

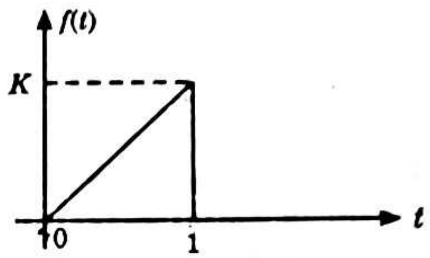

Fig. 2.7.

Solution : This triangular waveform may be generated using three signals as shown in figure 2.8(*a*), (*b*) and (*c*).

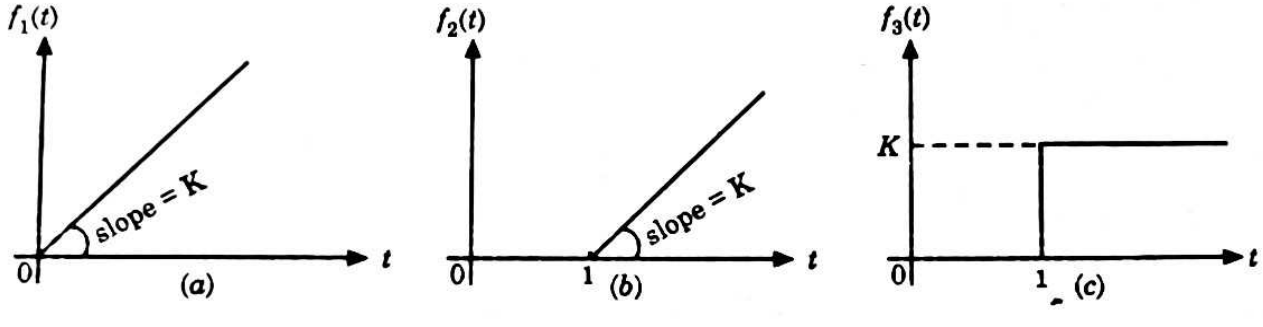

Fig. 2.8.

From the above representation,

$$\begin{aligned} f(t) &= f_1(t) - f_2(t) - f_3(t) \\ &= K\, r(t) - K\, r(t-1) - K\, U(t-1) \\ &= K\, t\, U(t) - K(t-1)\ U(t-1) - K\, U(t-1) \\ &= K[t\, U(t) - (t-1+1)\ U(t-1)] \\ &= K[t\, U(t) - t\, U(t-1)] \\ &= K\, t[U(t) - U(t-1)] \end{aligned}$$

2.3.3. Impulse Signal (It is also known as Dirac delta signal)

The impulse signal $f_\delta(t)$ is defined by

$$f_\delta(t) = \begin{cases} 0 & ; t \neq 0 \\ A & ; t = 0 \end{cases}$$

(where *A* is the area of the impulse signal and some times called the strength of the impulse)

2.5. DIRECT FORMULA (OR K.M. FORMULA)

If a function is a combination of various gate functions, then we can develop a formula to represent that function directly in terms of step functions. This formula can be called as Direct formula or K.M. formula. It is given by

$$f(t) = \sum_{T=-\infty}^{\infty} (A_f - A_i) U(t - T)$$

where T is the time instant at which function $f(t)$ changes its values, A_f and A_i are the final and initial values at the corresponding time instant respectively.

If the function $f(t)$ exist (define) only for $t \geq 0$, then Direct formula reduces to

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i) U(t - T)$$

To illustrate the use of the Direct formula, let us consider several examples.

Example 2.4. Synthesize the waveform as shown in figure 2.19 using standard signal. (I.P. Univ., 2001)

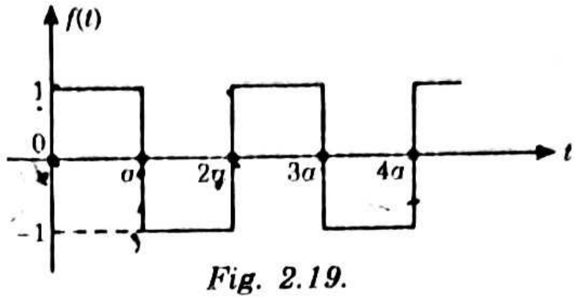

Fig. 2.19.

Solution : The function $f(t)$ can be written as the sum of the gate functions, as

$$f(t) = G_{0,\,a}(t) + (-1)\, G_{a,\,2a}(t) + G_{2a,\,3a}(t) + (-1) G_{3a,\,4a}(t) + \ldots\ldots$$

$$f(t) = 1.[U(t) - U(t - a)] + (-1)[U(t - a) - U(t - 2a)] + 1[U(t - 2a) - U(t - 3a)] + (-1)[U(t - 3a) - U(t - 4a) + \ldots\ldots$$

$$= U(t) - 2U(t - a) + 2U(t - 2a) - 2U(t - 3a) + \ldots\ldots$$

Alternative ways : (Using Direct formula)

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i) U(t - T)$$

(where T is the time at which function changes its values).

$$= (1 - 0) \cdot U(t - 0) + (-1 - 1) \cdot U(t - a) + [1 - (-1)] \cdot U(t - 2a) + (-1 - 1) \cdot U(t - 3a) + \ldots$$

$$= U(t) - 2U(t - a) + 2U(t - 2a) - 2U(t - 3a) + \ldots$$

Example 2.5. Synthesize the waveform as shown in figure 2.20 using step functions.

Solution : (*i*) Using gate functions

$$f(t) = 1 G_{0,\,2}(t) + (-2) \cdot G_{2,\,3}(t) + 2 \cdot G_{3,\,5}(t)$$

$$f(t) = 1 \cdot [U(t) - U(t - 2)] + (-2) \cdot [U(t - 2) - U(t - 3)] + 2 \cdot [U(t - 3) - U(t - 5)]$$

$$= U(t) - 3U(t - 2) + 4U(t - 3) - 2U(t - 5)$$

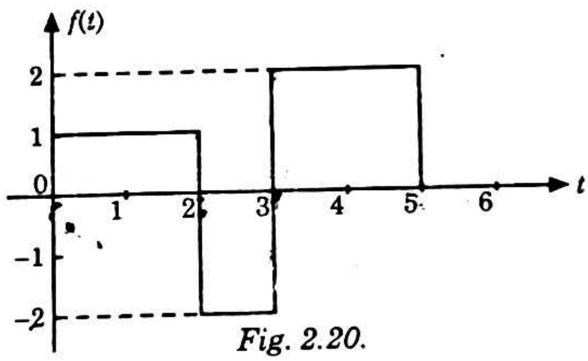

Fig. 2.20.

(*ii*) Using Direct formula :

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i) U(t - T)$$

$$= (1 - 0) \cdot U(t - 0) + (-2 - 1) \cdot U(t - 2) + [2 - (-2)] \cdot U(t - 3) + (0 - 2) \cdot U(t - 5)$$

$$= U(t) - 3U(t - 2) + 4U(t - 3) - 2U(t - 5)$$

Example 2.6. Synthesize the waveform as shown in figure 2.21.

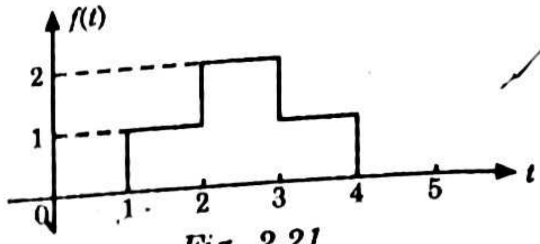

Fig. 2.21.

Solution : (*i*) Using Gate functions :

$$f(t) = G_{1,2}(t) + 2G_{2,3}(t) + G_{3,4}(t)$$
$$= 1\cdot[U(t-1) - U(t-2)] + 2\cdot[U(t-2) - U(t-3)] + 1\cdot[U(t-3) - U(t-4)]$$
$$= U(t-1) + U(t-2) - U(t-3) - U(t-4)$$

(*ii*) Using Direct formula :

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)U(t-T)$$
$$= (1-0)\cdot U(t-1) + (2-1)\cdot U(t-2) + (1-2)\cdot U(t-3) + (0-1)\cdot U(t-4)$$
$$= U(t-1) + U(t-2) - U(t-3) - U(t-4)$$

Example 2.7. Synthesize the following waveforms as shown in figure 2.22. Using gate functions.

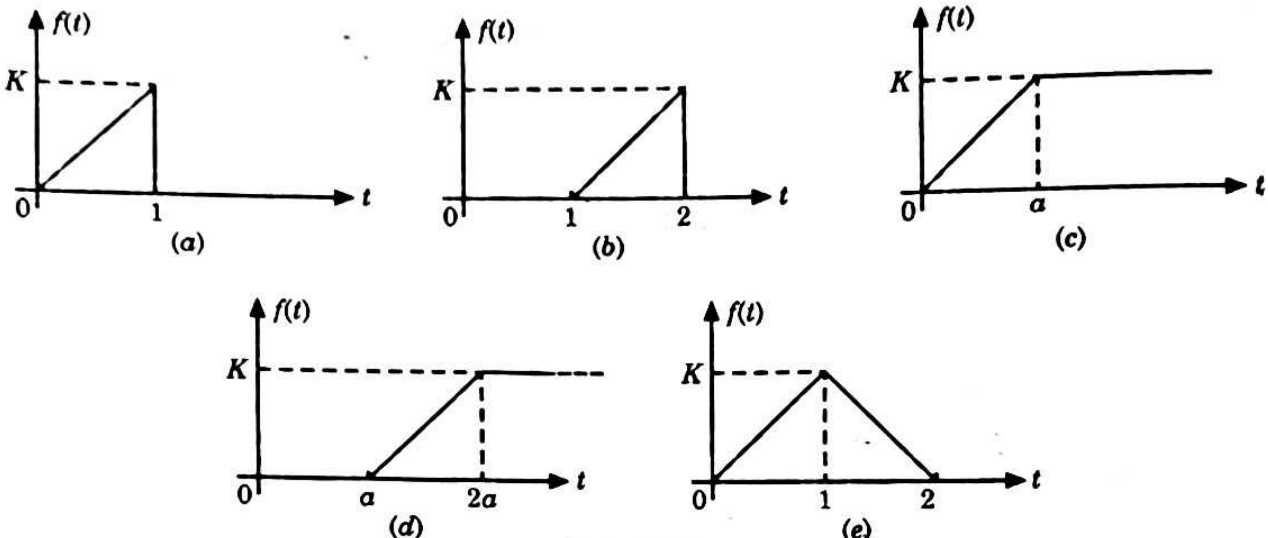

Fig. 2.22.

Solution :

(*a*) $f(t) = Kt\cdot[U(t) - U(t-1)]$

(*b*) $f(t) = K(t-1)\cdot[U(t-1) - U(t-2)]$ (since $K(t-1)$ is the expression of the line)

(*c*) This function $f(t)$ can be represented as the sum of two functions, one is from 0 to a and other is a to infinity. As,

$$f(t) = \frac{K}{a}\cdot t[U(t) - U(t-a)] + KU(t-a)$$
$$= \frac{K}{a} tU(t) - \frac{K}{a}(t-a)U(t-a)$$

(*d*) Similar to part (*c*)

$$f(t) = \frac{K}{a}(t-a)[U(t-a) - U(t-2a)] + KU(t-2a)$$
$$= \frac{K}{a}(t-a)U(t-a) - \frac{K}{a}(t-2a)U(t-2a)$$

(*e*) Given waveform can break into two functions. One is from 0 to 1 and other from 1 to 2. As,

$$f(t) = Kt\cdot[U(t) - U(t-1)] - K(t-2)\cdot[U(t-1) - U(t-2)]$$
$$= Kt\,U(t) - 2\,Kt\,U(t-1) + 2K\,U(t-1) + Kt\,U(t-2) - 2K\,U(t-2)$$
$$= Kt\,U(t) - 2K(t-1)\,U(t-1) + K(t-2)\,U(t-2)$$

Example 2.8. Synthesize the given waveform as shown in figure 2.23.

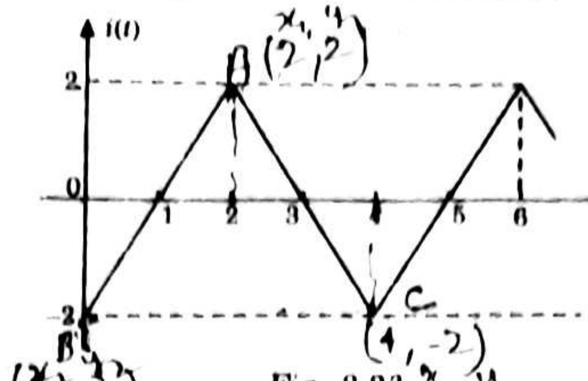

Fig. 2.23.

Solution : Using gate functions, we can represent as,

$$i(t) = (2t-2)[U(t) - U(t-2)] + (-2t+6)[U(t-2) - U(t-4)] + (2t-10)[U(t-4) - U(t-6)] + \ldots$$

$$= (2t-2)U(t) + (-2t+2-2t+6)\,U(t-2) + (2t-6+2t-10)U(t-4) + \ldots$$

$$= (2t-2)\,U(t) + (-4t+8)\,U(t-2) + (4t-16)\,U(t-4) + \ldots$$

$$= 2(t-1)\,U(t) - 4(t-2)\,U(t-2) + 4(t-4)\,U(t-4) + \ldots$$

Example 2.9. Express the given waveform as shown in figure 2.24 using standard signal.

Solution : The function $v(t)$ can be written as the sum of a ramp and a step functions, as

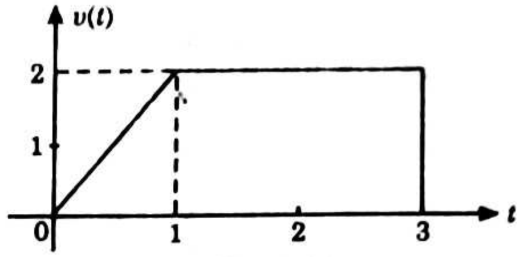

Fig. 2.24.

$$v(t) = 2[r(t) - r(t-1)] - 2U(t-3)$$

Putting

$$r(t) = t\,U(t)$$

and $r(t-1) = (t-1)\,U(t-1)$

$$v(t) = 2t\,U(t) - 2(t-1)\,U(t-1) - 2U(t-3)$$

Alternative way : (using gate functions) :

$$v(t) = 2t \cdot G_{0,1}(t) + 2 \cdot G_{1,3}(t)$$
$$= 2t[U(t) - U(t-1)] + 2 \cdot [U(t-1) - U(t-3)]$$
$$= 2t\,U(t) - 2t\,U(t-1) + 2U(t-1) - 2U(t-3)$$
$$= 2t\,U(t) - 2(t-1)\,U(t-1) - 2U(t-3)$$

Example 2.10. Express the waveform shown in figure 2.25 in terms of delayed functions.

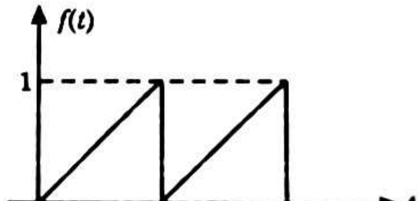

Fig. 2.25.

Solution : (Using gate functions)

$$f(t) = t \cdot G_{0,1}(t) + (t-1)\,G_{1,2}(t)$$
$$= t[U(t) - U(t-1)] + (t-1)[U(t-1) - U(t-2)]$$
$$= t\,U(t) - U(t-1) - (t-1)\,U(t-2)$$
$$= t\,U(t) - U(t-1) - (t-2)\,U(t-2) - U(t-2)$$
$$= r(t) - U(t-1) - r(t-2) - U(t-2)$$

Example 2.11. Synthesize the given waveform as shown in figure 2.26 using step and ramp signals.

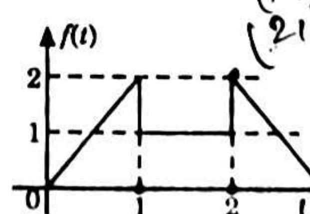

Fig. 2.26.

$m = \frac{0-2}{3-2}$

$m = -2$

$y - 0 = (-2)(x-3)$

$y = -2x + 6$

$\therefore f(t) = -2t + 6$

Solution : (Using gate functions)

$$f(t) = 2t\,G_{0,1}(t) + G_{1,2}(t) + (-2t+6)\,G_{2,3}(t)$$
$$= 2t[U(t) - U(t-1)] + [U(t-1) - U(t-2)] + (-2t+6)[U(t-2) - U(t-3)]$$
$$= 2t \cdot U(t) - (2t-1)\,U(t-1) - (2t-5)\,U(t-2) + (2t-6)\,U(t-3)$$

Example 2.18. Sketch the following signals :

(*i*) $U(t-5) - U(t-8)$

(*ii*) $U(t-6) + U(t-4)$

(*iii*) $t^2\,[U(t-1) - U(t-3)]$

(*iv*) $(t-4)\,[U(t-1) - U(t-4)]$

Solution:

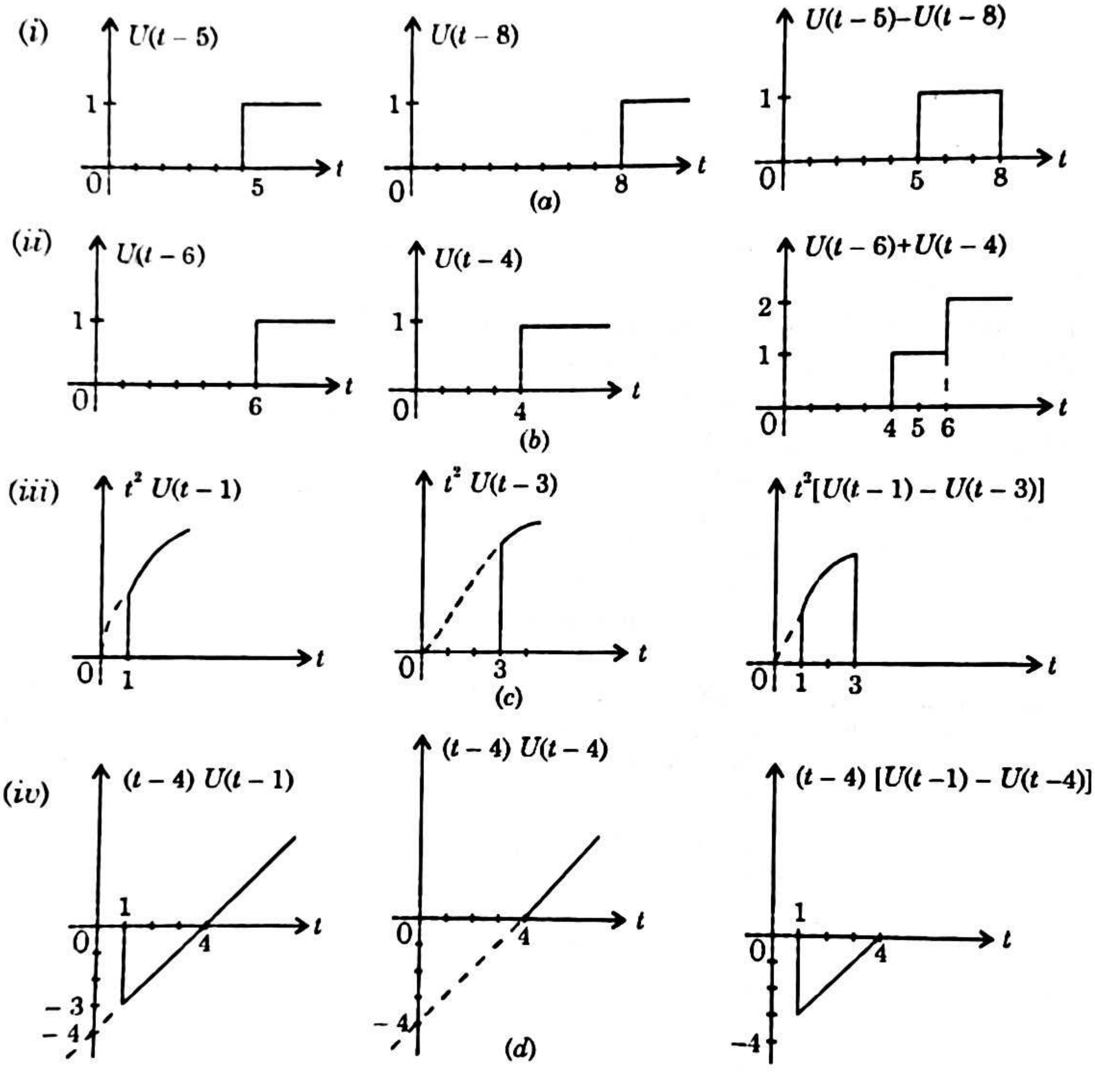

Fig. 2.33.

1.1. NETWORK AND CIRCUIT

An electric network is any possible interconnection of electric circuit elements, while an electric circuit is a closed energised network. (An electric circuit is characterised by currents (I) in the elements and voltages (V) across them). Therefore, all the circuits are networks, while a network is not necessarily a circuit, *e.g.* T-network. Circuit elements may be :

(*i*) Active or passive
(*ii*) Unilateral or bilateral
(*iii*) Linear or non-linear
(*iv*) Lumped or distributed

1.2. SYSTEM

A system is a combination and interconnection of several components to perform a desired task. Systems may be interconnected with each other to form other systems. In such cases the component systems are referred to as subsystems.

1.3. SYSTEM MODELING CONCEPTS FOR 'LINEAR TIME INVARIANT SYSTEM'*

1.3.1. Some Terminology

We need to be able to construct appropriate system models that adequately represent the interaction of signals and systems and the relationship of causes and effects for that system. Usually, we refer to certain causes of interest as inputs and certain effects of interest as outputs. For example, in the accelerator of an automobile, the input of interest could be the applied force and the output could be a speed of automobile.

1.3.2. Representation of Systems

It is convenient to visualize a system as shown in figure 1.1. On the left hand side of the box we represent the inputs (excitations, causes) as a series of arrows labeled $x_1(t)$, $x_2(t)$ ....., $x_m(t)$. The output (responses, effects), $y_1(t)$, $y_2(t)$, ..., $y_p(t)$, are represented as arrows of the right hand side of the box.

Such systems are referred to as two-port or single input, single output (SISO) systems since they have one input port and one output port.

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* The terms "linear" and "time invariant" will be given precise mathematical definitions shortly. Although it is possible to analyse systems that are time varying, it is more difficult than for the time invariant systems. Because of this, and because many systems are time invariant, we restrict our attention to this category.

SISO System

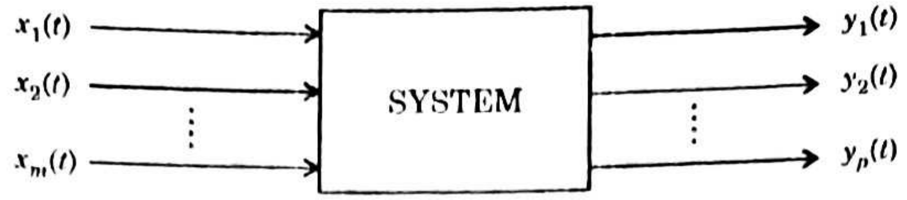

Fig. 1.1. Block diagram representation of a system

1.3.3. Interconnections of Systems

Many real systems are built as interconnections of several subsystems. One example is an audio system, which involves the interconnection of a radio receiver, compact disc (CD) player, or tape deck with an amplifier and one or more speaker. By viewing such a system as an interconnection of its components, we can use our understanding of the component systems and of how they are interconnected in order to analyse the operation and behaviour of the overall system. There are various types of interconnection as shown in figure 1.2.

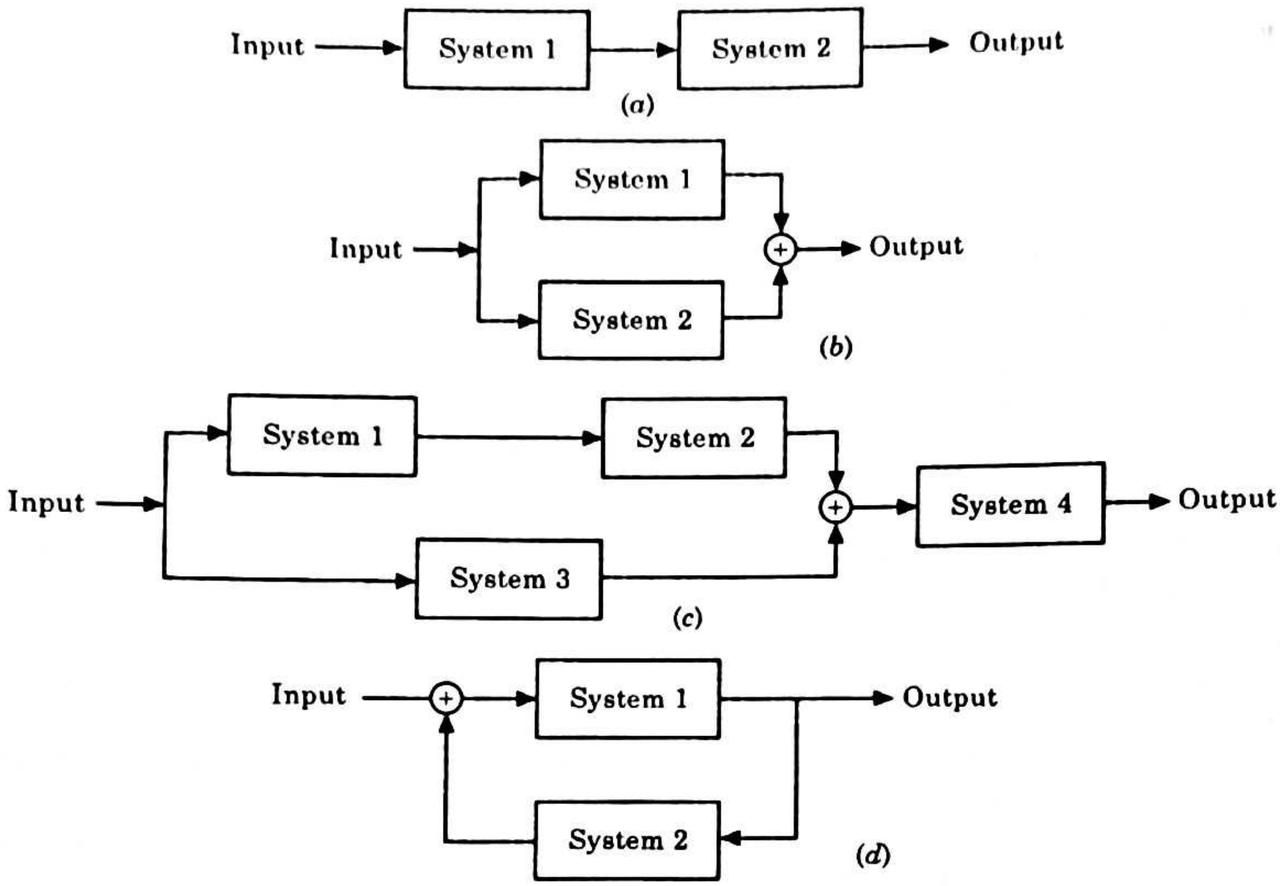

Fig. 1.2. Interconnection of systems : (a) Series (Cascade connection), (b) Parallel connection, (c) Series parallel connection, (d) Feedback interconnection.

1.4. BASIC SYSTEM PROPERTIES

1.4.1. Continuous-Time and Discrete-Time Systems

A continuous-time system is a system in which continuous-time input signals* are applied and result in continuous-time output signals. Such a system as shown in figure 1.3 (*a*), where $x(t)$ is the continuous-time input and $y(t)$ is the continuous-time output.

On the other hand, the discrete-time system is a system that transform discrete time inputs into discrete time outputs-will be shown as in figure 1.3(*b*), where $x[n]$ is the discrete-time input and $y[n]$ is the discrete-time output.

* Signals are discussed in next chapter *i.e.*, chapter 2.

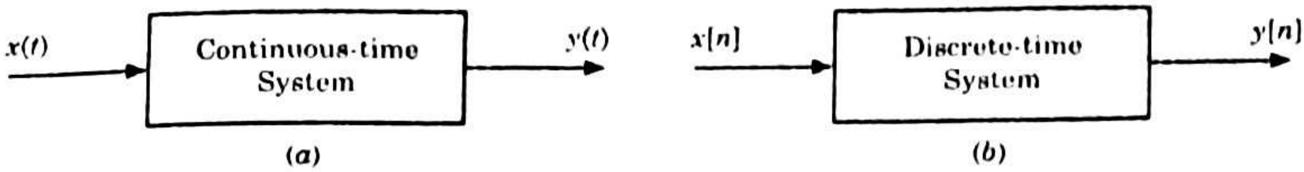

Fig. 1.3(a) Continuous-time system, (b) Discrete-time system.

Examples of continuous-time systems are electric circuits composed of resistors, capacitors and inductors that are driven by continuous-time sources. Continuous-time systems are described by "*Differential equations*", which can easily be solved by "*Laplace Transformation*".

An example of discrete-time systems is a simple model for the balance in a bank account from month-to-month. Discrete-time systems are described by "*difference equations*", which can easily be solved by "*z-Transformation*".

1.4.2. Linear and Non-Linear Systems

A system is said to be linear system, if that holds the principle of superposition *i.e.*, additivity and homogeneity (or scaling). Otherwise it is said to be non-linear. Mathematically a system is linear, if :

I. The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$

II. The response to $ax_1(t)$ is $ay_1(t)$, where a is any constant.

where $y_1(t)$ and $y_2(t)$ be the responses (outputs) of a continuous time system to inputs $x_1(t)$ and $x_2(t)$ respectively.

The above two properties defining a linear system can be combined into a single statement :

Continuous-time system : $ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)$,

Discrete-time system : $ax_1[n] + bx_2[n] \longrightarrow ay_1[n] + by_2[n]$.

Here a and b are any constant.

An example of a linear system is $y(t) = mx(t)$ and for non-linear systems are

$$y(t) = mx(t) + C,$$

and $$y(t) = x^2(t) \quad \text{etc.}$$

1.4.3. Time-Invariant and Time-Varying Systems

A system is time-invariant, or fixed, if its input-output relationship does not change with time. Otherwise, it is said to be time varying.

Specifically, a system is time invariant if a time shift in the input signal results in an identical time shift in the output signal. That is, if $y(t)$ is the output of a continuous-time-invariant system when $x(t)$ is the input, then $y(t - t_0)$ is the output when $x(t - t_0)$ is applied. In discrete-time with $y[n]$ the output corresponding to the input $x[n]$, a time-invariant system will have $y[n - n_0]$ as the output when $x[n - n_0]$ is the input.

An example of a time invariant system is $y(t) = \sin[x(t)]$ and for time varying system is $y(t) = x(2t)$.

1.4.4. Instantaneous and Dynamic Systems

A system for which the output is a function of the input at the present time only is said to be instantaneous (or memoryless, or zero memory). On the other hand, A dynamic system is one whose output depends on past or future values of the input or past output in addition to the present input.

For example, the system specified by the relationship

Continuous-time system : $y(t) = 3x(t) - x^2(t)$

Discrete-time system : $y[n] = 3x[n] - x^2[n]$

is instantaneous or memory less system, as the value of $y(t)$ or $y[n]$ at any particular time t_0 or n_0 depends only on the value of $x(t)$ or $x[n]$ at that time. Similarly a resistor is a instantaneous system as

$$v(t) = R \cdot i(t)$$

while a capacitor is a dynamic system as

$$v(t) = \frac{1}{C}\int_0^t i\,dt + v_c(0^-)$$

1.4.5. Causal and Non-Causal Systems

A system is said to be causal or non-anticipative if the output of the system at any time depends only on values of the input at the present time and in the past, *i.e.*, the system output does not anticipate future values of the input.

For example,

$$y(t) = x(t) + x(t-1) \text{ or } y[n] = x[n] + x[n-1] \text{ is}$$

causal, while

$$y(t) = x(t) + x(t+1) \text{ or } y[n] = y[n] + y[n+1] \text{ is}$$

non-causal systems.

1.4.6. Invertibility and Inverse Systems

A system is said to be invertible if distinct inputs lead to distinct outputs. If a system is invertible, then an inverse system exists that, when cascaded with the original system, yields an output $\omega(t)$ or $\omega[n]$ equal to the input $x(t)$ or $x[n]$ to the first system, as illustrated in figure 1.4(*a*).

An example of an invertible continuous-time system is illustrated in figure 1.4(*b*).

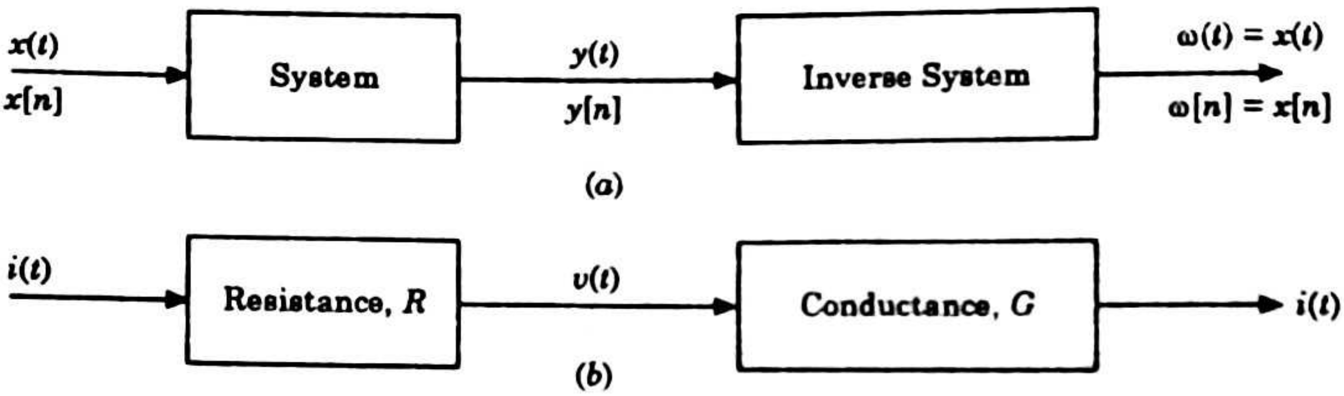

Fig. 1.4 (a) A general invertible system (b) an example of invertible system.

1.4.7. Stability

Stability is another important system property. A stable system is one that will remain at rest unless excited by an external source and will return to rest if all the excitations are removed. The necessary condition for the continuous-time system to be stable is that the roots of the characteristic equation have negative real parts (or lie in the left-half of s-plane), while for the discrete-time system, the roots of the characteristic equation are inside the unit circle as shown in figure 1.5(*a*) and (*b*) respectively.

2.6. LINEAR TIME INVARIANT (LTI) SYSTEMS

A number of basic system properties has introduced and discussed in previous article 1.4. Two of these, linearity and time invariance, play an important role in signal and system analysis for two major reasons. First, many physical systems possess these properties and thus can be modeled as LTI systems. And second, as we know that the LTI systems possess the super-position property as a consequence, if we can represent the input or excitation or cause to an LTI system in terms of a linear combination of a set of basic signals, we can then use super-position to calculate the output or response or effect of the system in terms of its response to these basic signals.

2.6.1. Analysis of LTI Systems

If we take the product of unit impulse signal $\delta(t)$ and any other signal $x(t)$, then this product will provide the signal $x(t)$ existing only at $t = 0$, since $\delta(t)$ exists only at $t = 0$. Mathematically,

$$\int_{-\infty}^{\infty} x(t).\delta(t)\,dt = x(t)\big|_{t=0} = x(0)$$

Where $x(0)$ is the value of the signal $x(t)$ at $t = 0$.

The above equation is known as *shifting property* of the impulse signal because the impulse shifts the value of $x(t)$ at $t = 0$.

The shifting may also be done at any instant say at $t = t_0$, if we define the impulse signal at the instant t_0, Mathematically,

$$\int_{-\infty}^{\infty} x(t).\delta(t - t_0)dt = x\,(t_0)$$

Therefore, the shifted unit impulse signal $\delta(t - t_0)$ shifts the value of $x\,(t)$ at $t = t_0$.

From above discussion, any signal can be expressed as linear combination of shifted impulse signals. Using linearity property, the response of the system to any input can be evaluated in terms of linear combination of shifted impulse signals. Thus an LTI system can be characterized by its impulse response. Such a representation, referred to as the convolution integral in the continuous-time case and the convolution sum in discrete-time. Here we will discuss only continuous-time case.

The convolution of $x(t)$ and $h(t)$ is denoted by a special notation [by putting a star between $x(t)$ and $h(t)$] as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

2.6.2. Properties of LTI Systems

As we have seen that the characteristics of an LTI system are completely determined by its impulse response. It is important to note that this property hold in general only for LTI systems.

A. LTI Systems with and without Memory

As specified in article 1.4.4, a system is instantaneous or memoryless or

Note: The unit impulse response of a non-linear system doesnot completely characterize the behaviour of the system.

zero memory if its output at any time depends only on the value of the input at the same time otherwise dynamic *i.e.*, a dynamic system or system with memory is one whose output depends on past or future values of the input or past output in addition to the present input.

A memoryless continuous-time LTI system has the form

$$y(t) = K\, x(t) \text{ (where } K \text{ is any constant)}$$

Its impulse response

$$h(t) = K\,\delta(t)$$

Note that if $K = 1$, then this system becomes identity system, with output equal to the input and with unit impulse response equal to the unit impulse.

or $$h(t) = 0 \text{ for } t \neq 0.$$

If a continuous-time LTI system has an impulse response $h(t)$ that is not identically zero for $t \neq 0$, then the system is dynamic or has memory.

B. Causality of LTI Systems

As specified in article 1.4.5, a system is causal or non-anticipative if the output of the system at any time depends only on values of the input at the present time and in the past otherwise non-causal *i.e.* a non-causal system is the system whose output depends (or anticipates) future values of the input. Specifically, a continuous-time LTI system to be causal, $y(t)$ must not depend $x(\tau)$ for $\tau > t$. This results

$$h(t) = 0 \qquad \text{for} \qquad t < 0$$

i.e. the impulse response of a causal LTI system must be zero before the impulse occurs. More generally, causality for an LTI system is equivalent to the condition of initial rest. And the system output (using convolution integral) is

$$y(t) = \int_{0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

Note : The causality is a property of systems, it is common terminology to refer to a signal as being causal if it is zero for $t < 0$. Causality of an LTI system is equivalent to its impulse response being a causal signal.

C. Invertibility of LTI Systems

As discussed in article 1.4.6, a system is invertible only if an inverse system exists that, when cascaded with the original system, yields an output equal to the input to the first system. Here, if an LTI system is invertible, then it has an LTI inverse. If we have a system with impulse response $h(t)$. The inverse system with impulse response $g(t)$, must satisfy

$$h(t) * g(t) = \delta(t)$$

D. Stability of LTI Systems

An LTI system is said to stable if the impulse response is absolutely integrable, *i.e.*, if

$$\int_{-\infty}^{\infty} |\, h(\tau)\,|\, d\tau < \infty$$

More general, an LTI system is said to be stable if the impulse response approaches zero as $t \to \infty$.

Note: All instantaneous (memoryless) systems are causal, since the output responds only to the present value of input.

E. Commutative Property

The output of an LTI system of impulse response $h(t)$ to input $x(t)$ is equal to the output of system of impulse response $x(t)$ to input $h(t)$, *i.e.*,

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

F. Distributive Property

The output of an LTI system of impulse response $[h_1(t) + h_2(t)]$ to input $x(t)$ is equal to the sum of the output of system with impulse response $h_1(t)$ to input $x(t)$ and system with impulse response $h_2(t)$ to input $x(t)$, *i.e.*,

$$y(t) = x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t).$$

The distributive property is useful when two or more systems are connected in parallel.

Also, as a consequence of both commutative and distributive properties, we have

$$y(t) = [x_1(t) + x_2(t)] * h(t) = x_1(t) * h(t) + x_2(t) * h(t)$$

which simply state that the response of an LTI system to the sum of two inputs must equal the sum of the responses to these signals individually.

G. Associative Property

The output of an LTI system of impulse response $[h_1(t) * h_2(t)]$ to input $x(t)$ is equal to the output of the system with impulse response $h_2(t)$ to input $[x(t) * h_1(t)]$, *i.e.*,

$$y(t) = x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

The associative property is useful when two or more systems are connected in series or in cascade.

Also, as a consequence of this property, we can say that the overall system response doesnot depend upon the order of the systems in the cascade.