Physics

Refrence → Engineering physics by Devraj Singh or SP Taneja.

Unit-1

Thermodynamics

* Branch of science that deals with the relation between heat & other forms of energy.

The str. of thermodynamics is based on 2 laws. (I st & II nd)
↳ impossible to convert all work
law of increase in entropy.

Zeroth law – A=B ; B=C ; A=C

* Temp – degree of hotness
quantity that characterises degree of heating to which a body is objected.

* heat exchange – When a hot cup of tea is placed in a surrounding then the heat from ~~cup~~ cup is transferred to the surrounding. ~~It means~~ after some time temp of 2 bodies become equal & change in temp stops. Now the 2 bodies are said to be in thermal equilibrium

→ It indicates that energy (heat) exchange takes place b/w the two bodies. Such an exchange of energy is called heat exchange.

Heat – In 18th century heat was considered to be massless & invisible substance called calorie

A/c to caloric theory every body contains an amount of calorie which depends on temp of body.

Zeroth law :- If two system are each in thermal equilibrium with a 3rd system, then they are in thermal equilibrium with each other.

It means heat is an intrinsic property.

→ later on it was concluded that heat is a form of energy

→ A/c to modern view heat is a energy that is transferred from a hot body to a cold body. eg – a hot cup of coffee at room temp, after some time the temp of cup is same as of surrounding due to heat transfer.

System – It is defined as quantity of matter of fixed mass & identity. System can interact with its surrounding in foll. 2 ways – i) transfer of heat
ii) by doing work.

open system → interact with surrounding eg earth.
closed system → no interaction. eg hot cup of tea.
isolated system

State – It is described with the macroscopic properties (P, V, T)

→ Physical variables that determine the state of a body are called thermodynamic variables.

→ 2 states of a system are considered to be diff. if the value of even one of the thermodynamic variable is diff.

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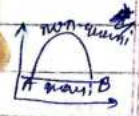
Thermodynamic equilibrium.

A system is said to be in thermal eq. if none of the thermodynamic variable determining its state changes with time.

Process → any change in the state of the system.

* Quasi-static process :- takes place at a very-slow rate. It has all its state in equilibrium.



* The process in which the system is in thermodynamic equilibrium with its sorroundings at all times.

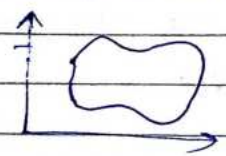
Reversible process → The process that satisfies the foll. condition

i) Process can be carried out with equal facility in 2 opp direction.

ii) in each direction the system passes through same intermediate state

irreversible process - which cann't fulfill above 2 conditions eg - diffusion of gases.

Cycle - When a system is in a given initial state passes through no. of diff. stages & finally returns to its original state.



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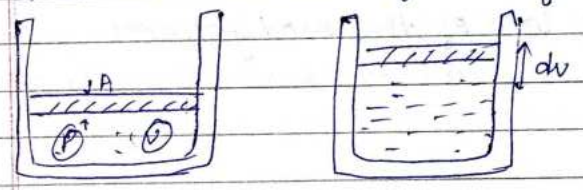
Heat reservoirs :- It is a body from (or to) which heat can be transferred without causing a change in the temp of the reservoir. eg - Atmosphere.

* high-temp reservoir - throug which we can extract heat (source)
* low-temp reservoir - (sink)

Work done - when a force acting on a body it displaces through a certain distance, the work done is $F \cdot x$.

It is the measure of transferred energy.



force exerted by the gas on the piston.

$$F = P \cdot A \quad - (1)$$

Work done by the gas.

$$dW = F \cdot dx$$

$$dW = PA\,dx$$

$$dW = P\,dV$$

$$W = \int dW = \int P\,dV \quad \int_{V_i}^{V_f} P\,dV$$

Sign conventions

i) Work done by the gas → +ve
ii) work done on the gas → −ve
iii) expansion of gases, work done → +ve
iv) compression of gases, work done → −ve

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Internal energy :- energy persists by a body due to its molecular arrangement & motion of molecules. It is very imp in calculating the thermodynamic state of the system.

⇒ For perfect / ideal gases internal energy depends only on temperature. $U = U(T)$

⇒ For non-perfect gases, vapours internal energy strongly depends on temp & weakly on pressure $U = U(T, P)$

First law of thermodynamics

→ Based on law of conservation of energy.

Heat -

Work -

Internal energy - energy stored in a system

→ It states that quantity of heat ~~by~~ supplied to the system will be equal to sum of work performed by the system & change in internal energy.

Net heat transferred = work done + Change in Internal energy.

$$\boxed{Q = W + \Delta U}$$

Application of 1st law

i) Isolated system :- doesn't interacts with surrounding.

- doesn't provide information about direction of process.
- Assumes no loss of energy
- ignores the quality of energy
- Assumes no change in system composition.

$Q = 0$
$W = 0$
$0 = 0 + \Delta U$
$\Delta U = 0$
$U_2 - U_1 = 0$

⇒ $\boxed{U_1 = U_2}$ constant ~~energy~~ internal energy no change in internal energy.

~~Signif~~

ii) Cyclic process :- $U_1 = U_2$; $U_1 - U_2 = 0$, $\Delta U = 0$

initial & final internal energies are same $\therefore \boxed{q = W}$

⇒ Net work done by the system is equal to net heat absorbed by the system over the cycle

iii) Adiabatic process : No heat is absorbed or ejected by the system. $Q = 0$

⇒ change in internal energy is equal to work done by the system in magnitude.

$0 = W + \Delta U$

$\boxed{\Delta U = -W}$

$\boxed{|W| = -\Delta U}$

iv) Isochoric process : constant volume.

⇒ Heat supplied to the system is equals to new work done by the system.

$\Delta V = 0$
$W = P\Delta V = 0$.

$\boxed{Q = \Delta U}$

v) Isothermal process : constant temp.

$Q, W, \Delta U \rightarrow$ non zero.

$$\boxed{Q = W + \Delta U}$$

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vi) Isobaric process : constant pressure.

$$W = P\Delta V$$

$$\boxed{Q = W + \Delta V}$$

vii) Isothermal expansion of an ideal gas:-

Ideal gas eq^n $PV = nRT$

$$P = \frac{nRT}{V}$$

$$\therefore dW = \frac{nRT}{V} dV$$

$$W = \int dW = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = nRT \ln \left[\frac{V_2}{V_1}\right]$$

$$\boxed{W = 2.303\, nRT \log \frac{V_2}{V_1}}$$

viii) Adiabatic expansion of an ideal gas:-

$dW = C_v dT$ $\qquad C_p - C_v = R$

$dU = nC_v dT$ — (1) $\qquad C_v = \frac{dU}{dT}$

Work done during the process $W = PdV$

$\therefore$ The process is adiabatic $\quad W = -\Delta U$

$$PdV = -nC_v dT \quad \text{— (2)}$$

Ideal gas eq^n $PV = nRT$

In differential form.

$$d(PV) = d(nRT)$$

$$W = \frac{U}{\gamma - 1}[T_1 - T_2]$$

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$$PdV + VdP = nRdT \quad \text{— (3)}$$

$$PdV = nRdT - VdP$$

$$-nC_v dT = nRdT - VdP$$

$$VdP = nRdT + nC_v dT$$

$$VdP = ndT(C_v + R)$$

$$VdP = nC_p dT \quad \text{— (4)}$$

dividing eq^n (3) & (4)

$$\frac{PdV}{VdP} = \frac{-\cancel{n}C_v dT}{\cancel{n}dTC_p}$$

$$\frac{PdV}{VdP} = \frac{-C_v}{C_p}$$

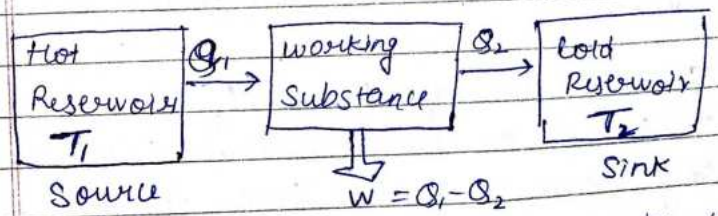
$$\cancel{\frac{PdV}{VdP} = -\gamma} \qquad \frac{VdP}{PdV} = \frac{-C_p}{C_v} = -\gamma$$

$$\boxed{\frac{dP}{P} = -\gamma \frac{dV}{V}}$$

$$\log P + \gamma \log V = \text{Constant}$$

$$\boxed{PV^\gamma = \text{constant}}$$

Heat engine:- device that converes heat energy into mechanical work through a medium (generally in the form of vapour or gas) called working substance.



Source - It is a hot reservoir which is at higher temperature T_1 & can

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extract any quantity of heat without any change in its temperature.

Working → It depends on the engine Substance which absorbs heat from the source converts it into mechanical work. and the rest of the heat is supplied to sink.

Sink → It is cold reservoir at lower temperature T_2 which can take any amount of heat by the working Substance without any change in its temperature.

Efficiency: $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$1 - \frac{Q_2}{Q_1}$ $\boxed{\eta < 1}$

efficiency can never be 100%.

Assignment Q. Explain Carnott cycle, Carnott engine & Carnott's theorem.

* Second law of thermodynamics
It based on the fact that heat can only flow from hot body to cold body by itself.

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Kelvin-planik Statement - it states that it is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of the surrounding.

Clausius statement - it is impossible for ~~self~~ self acting machine to transfer heat from colder body to hotter body without an aid of external energy.

Entropy - it is defined as degree of randomness or disorderness it is represented by 'S'

$\boxed{dS = \frac{dQ}{T}}$

solid → Gas
More ordered → less ordered
entropy changes (↑)

* Entropy & IInd law of thermodynamics
• If a process occurs in a closed system the entropy of the system increases for irreversible process and remains constant for reversible process
⇒ It never decreases.

* Entropy & IIIrd law of thermodynamics
As the temp tends to absolute zero the entropy also tends to zero & the molecules of the system are perfectly ordered.

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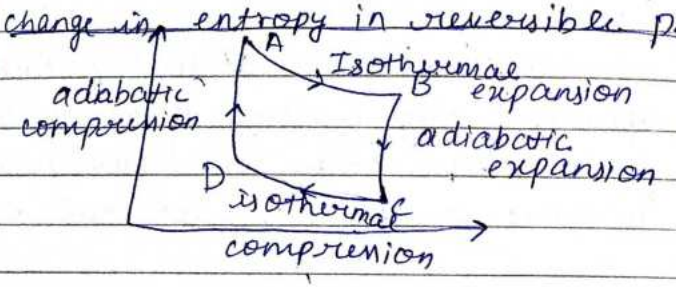
change in entropy in reversible process.

Third law of thermodynamics

→ It is impossible to atain a temperature zero Kelvin.

change in entropy in reversible process



During A to B process is isothermal
Change in entropy from A → B will be $\frac{Q_1}{T_1}$

During B to C is adiabation
change in entropy is (0).

During C to D process in isothermal compression. Change in entropy $\frac{Q_2}{T_2}$

During D to A process is adiabatic
change in entropy is zero. (0)

Total change in entropy during this process. $\frac{Q_1}{T_1} - \frac{Q_2}{T_2}$ — (1)

carnot theorem → no heat engine have more efficiency than the reversible engine operating at the same temp & the efficiency of carnot engine is independent of nature of working substance.

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In a carnott cycle $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$.

Therefore change in entropy is zero
$\boxed{\Delta S = 0}$

Change in entropy in an irreversible process :-

Consider an irreversible process such as conduction in which heat is lost by body at higher temperature T_1 & is gained by the body at lower temperature T_2, with the change in heat energy Q.

gain in entropy of colder body $= \frac{Q}{T_2}$

loss in entropy of hotter body $= \frac{Q}{T_1}$

Total change in entropy $\frac{Q}{T_2} - \frac{Q}{T_1}$

$\boxed{T_1 > T_2}$

$\Delta S = +ve$

$Q\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$

$Q\frac{(T_1 - T_2)}{T_1 T_2}$

Q. What is the heighest possible efficiency of a heat engine operating with a heat reservoir of gases at 2100°c and the cooling water is available at

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

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15°C.

$2100 + 273 = 2373$

$15 + 273 = 288$

$$\eta = 1 - \frac{288}{2373} = \cancel{87\%}.$$

$$= \frac{2085}{2373} = 87\%$$

Q. efficiency of a carnot cycle is $\frac{1}{6}$ if the temp of the sink is reduced by 60°C. The efficiency increases to $\frac{1}{3}$ find initial & final Temp b/w which the process is running.

$\frac{273}{60}$
333

Case I

$$\frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$$

Case II

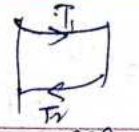
$$\frac{1}{3} = 1 - \frac{(T_2 - 333)}{T_1}$$

$$-\frac{2}{3} = -\frac{T_2}{T_1} + \frac{333}{T_1}$$

$$-\frac{2}{3} + \frac{5}{6} = \frac{333}{T_1}$$

$$\frac{1}{6} = \frac{333}{T_1} \qquad \underline{T_1 = 1998\,K}$$

$$W_{adB} = \frac{mR}{(\gamma - 1)}[T_1 - T_2]$$



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$$T_2 = \frac{5}{6} \times \cancel{1998}^{333} \Rightarrow 1665\,K$$

$$T_1 - T_2 \Rightarrow 1998 - 1665$$
$$\Rightarrow 333$$

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continuum –

matter – discrete mass

Validity of concept

$K_n = \frac{\lambda}{L}$ → mean free path. / L → ~~thermal~~ dimension of system

Knudsen number.

macroscopic approach.

$$Q_1 = T_2 \log \frac{V_3}{V_4}$$

$$Q_1 = T_1 \log \frac{V_2}{V_1}$$

carnot cycle.

$P_1V_1 = P_2V_2$

$\log \frac{V_3}{V_4} = \log \frac{V_2}{V_1}$

$R = 8.3 \times 10^7$

$127°C$
273

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Q. 1 mole of a gas expands isothermally until its volume is doubled. Calculate W.

$W = 2.303 \times 8.3 \times 10^7 \times T \log 2$

$2.303 \times 8.3 \times 10^7 \times 0.30$

$5.73 \times 10^7 \times 300$

~~17.19×10^9~~

22.92×10^9 ergs.

Q. Calculate the change in entropy when 2 gm of ice melts into water at the same temp.
latent heat of ice = 80 cal/g

→ $l = 80$ cal/g

Heat required $= ml = 2 \times 80 = 160$ cal

$T = 0°C = 273K$

$S = \frac{\Delta Q}{T} = \frac{160}{273} = 0.58$ cal/K

Unit-II

Waves & oscillation

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Motion

- Linear → car running on a straight road
- Rotational → rotation of earth around the Sun.
- Vibrational or Translational → eg - Pendulum.

* Vibrational motion - motion that moves back & forth & repeats itself in ~~equal~~ interval of time.
eg → Pendulum.

* Periodic motion - A motion which repeats itself after equal interval of time. This motion is called periodic motion.
eg → spin of earth, motion of satellite around planet.

* Oscillatory motion - A body is said to persist oscillatory motion if it moves back & forth repeatedly about the mean position.

* Simple harmonic motion - motion in which force ∝ displacement.

* equation of SHM :- Consider a block of mass m attach to a spring. When the mass is pulled in left

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$x \to$ displacement. $f \propto x$ $K \to$ spring constant

$f = -Kx$ — (1) $f \to$ restoring force

$f = m\frac{d^2x}{dt^2}$ — (2)

from (1) & (2)

$$-Kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{Kx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{Kx}{m} = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$ differential eq^n of SHM

$\omega^2 =$ constant $= \frac{K}{m}$.

angular frequency $\boxed{\omega = \sqrt{\frac{K}{m}}}$

$$2\pi f = \sqrt{\frac{K}{m}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

Time period, $\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{K}}}$

$\Rightarrow$ heavier object (m) oscillates more slowely.

Characteristics of SHM:-

(i) displacement -

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$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

general sol^n of this eq^n is -

$\boxed{x = A\sin(\omega t + \phi)}$ eq of SHM

$A \to$ Amplitude $\to$ maximum displacement

$\phi \to$ Phase diff.

(ii) Velocity -

$$v = \frac{dx}{dt} = A\cos(\omega t + \phi)\omega$$

$$\Rightarrow \boxed{A\omega\cos(\omega t + \phi)}$$

$$\boxed{v = A\omega\sqrt{1 - \sin^2(\omega t + \phi)}}$$

$$v = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)}$$

$$\boxed{v = \omega\sqrt{A^2 - x^2}}$$

$v = 0$; $x = A$ at extreme position.

$v = max$; $x = 0$ at mean position.

(iii) acceleration -

$$\boxed{a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \phi)}$$

$$a = -\omega^2 x$$

$a = 0$; $x = 0$. at mean position

$a = max$; $x = \pm A$ at extreme position.

(iv) Time period - time taken to complete 1 wave.

$\frac{d^2x}{dt^2}$ $x = A\sin(\omega t + \phi)$

replace t by $t + \frac{2\pi}{\omega}$

$x = A\sin\left(\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right)$

$A\sin(\omega t + 2\pi + \phi)$

no changes at 2π so

$\underline{x = A\sin(\omega t + \phi)}$

It means a wave take a time of $\frac{2\pi}{\omega}$ to complete itself

Time period, $T = \frac{2\pi}{\omega}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

$$\boxed{\text{frequency}, f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

5) Phase

$x = A\sin(\underbrace{\omega t + \phi}_{\text{Phase}})$

epoch :- value of phase when t is zero, (ϕ)

6) Energy :- Energy of SHM

$E = KE + PE$

$KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$

kinetic energy max ⇒ mean position

kinetic energy min extreme position

$F = -\frac{dU}{dx}$ $F = -Kx$

$\frac{dU}{dx} = Kx$

$dU = Kx\,dx$

$\int dU = Kx\,dx$

$U = \frac{Kx^2}{2} + C$

$x = 0,\ U = 0 \therefore C = 0$

$U = \frac{Kx^2}{2}$

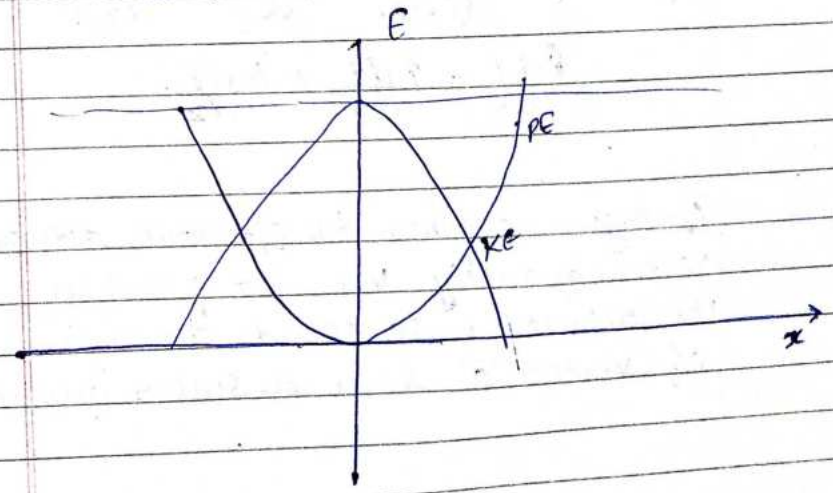
Total energy $= \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}Kx^2$

$\frac{1}{2}K(A^2 - x^2) + \frac{1}{2}Kx^2$

$\frac{1}{2}KA^2 \cancel{- \frac{1}{2}Kx^2} \cancel{+ \frac{1}{2}Kx^2}$

$TE = \frac{1}{2}KA^2$

⇒ Total energy will always be a constant.



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Electromagnetic Theory :-

(Del) ∇ operator / Nevela

$$\nabla F = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) F$$

$$\hat{i}\frac{dF}{dx} + \hat{j}\frac{dF}{dy} + \hat{k}\frac{dF}{dz}$$

when del operator acts on a scalar quantity resultant is a vector quantity.

Gradient

When del operator acts on a scalar function the resultant is a vector quantity and is called gradient of that function.

$$\text{grad}\,\phi = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right)\phi$$

$$\hat{i}\frac{d\phi}{dx} + \hat{j}\frac{d\phi}{dy} + \hat{k}\frac{d\phi}{dz}$$

Divergence - when del operator acts on vector quantity via dot product the resultant is called divergence of vector $\vec{a}$ & is a scaler quantity

no. of lines going out of surface

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$$\nabla \cdot \vec{A}$$

$$\left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) \cdot \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$\boxed{\text{div}\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

If the divergence of a vector is zero the the vector is said to be solenoid.

Curl → twist in a vector.

when del operation acts on a vector via cross product, the resultant is called curl of the vector and is a vector quantity.

$$\nabla \times \vec{A}$$

$$\left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) \times \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ A_x & A_y & A_z \end{vmatrix}$$

If the curl of a vector is zero then it is called irrotational vector.

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Q. If $\phi = x^{3/2} + y^{3/2} + z^{3/2}$ find grad ϕ.

$\cancel{\hat{i}\,\frac{3}{2}x^{1/2} + \hat{j}\,\frac{3}{2}}$

$\hat{i}\,\frac{3}{2}x^{1/2} + \hat{j}\,\frac{3}{2}y^{1/2} + \hat{k}\,\frac{3}{2}z^{1/2}$

Q. If vector $\vec{A} = 3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2\hat{k}$ find div $\vec{A}$ $= 0$.

Q. If vector $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ is the field solenoidal?
is the field irrotation?
$\text{div}\,\vec{A} = 2x + 2y + 2z$
$\text{div}\,\vec{A} = 2x + 2y + 2z$

Q. If vector $A = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$
div $\vec{A}$ at $(1, 2, 3)$

$\Rightarrow \quad 2x + 2y$
$2 + 2(2)$
$\Rightarrow \boxed{6}$

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Gauss - divergence theorem. This theorem is used to convert volume integral into surface integral.

→ It states that volume integral of divergence of $\vec{A}$ is equal to surface integral of $\vec{A}$ crossing the surface.

$$\boxed{\int_V \text{div}\,\vec{A}\,dv = \int_S \vec{A}\cdot ds}$$

Stoke's theorem - This theorem is used to convert surface integral into line integral

→ It states that surface integral of curl of $\vec{A}$ is equal to line integral of $\vec{A}$ over the close surface.

$$\cancel{\int_S \vec{A}\cdot ds} = \boxed{\int_S \text{curl}\,\vec{A}\cdot ds = \oint A\,dl}$$

Displacement Current - By Faraday's law a change in magnetic field (flux) induces an electric field, hence a changing electric field must induced magnetic field. But current is the usual source of magnetic field hence, this change in electric field is equivalent to a current named as displacement current.

→ Hence it is defined as current which comes into existence in addition to

conduction current whenever the electric field or flux changes with time

$$i_d \propto \frac{d\vec{E}\cdot ds}{dt}$$

$$\boxed{i_d = \varepsilon_0 \frac{d\vec{E}}{dt}\cdot ds} = \boxed{\varepsilon_0 \frac{d\phi}{dt}}$$

eg- between the plates of capacitor. (vacuum)

$$i = i_c + i_d$$

Hence, modified ampere's law becomes

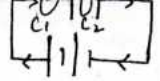
$$\oint B\cdot dl = \mu_0 i$$
$$= \mu_0 (i_c + i_d)$$

$$\boxed{\oint B\cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\phi_e}{dt}\right)}$$

Conduction current	Displacement current
• Obeys ohm's law.	doesn't obeys ohm's law.
• ~~ca~~ cann't exist in vacuum	• exists in vacuum.
• ~~flow~~ origin- flow of charge carriers in metals.	• origin - rate of change in electric flux wrt. time.
• depends on potential diff & resistance	• ~~re~~ depends on rate at which electric ~~field~~ flux is changing with time.

⇒ Inconsistency in ampere's law - only in constant flow.

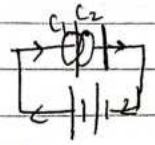
case I :



Current in loop C_1 $\oint B\cdot dl = \mu_0 i$

loop C_2 $\oint B\cdot dl = 0$

case II :



$$\int_{C_1} B\cdot dl = \mu_0 i$$
$$\int_{C_2} B\cdot dl = \mu_0 i$$

** loops are close

Properties of displacement current -

i) it exist whenever there is a change in electric flux.

ii) it is not a current, since it produces magnetic field so it is called a current.

iii) since it is called current Hence it has same dimension as of current. 'A'

iv) can exist in vacuum.

3/10 Equation of continuity :- charge is conserved

- it says that charge can neither be created nor be destroyed.
- It say's that rate of charge (current) flowing away from the enclosed volume is say as the rate of decrease of charge inside it.

* Current density to a given surface ($\vec{J}$)

$$\vec{I} = \int \vec{J}\cdot d\vec{s} \quad -(1)$$

$$I = -\frac{dq}{dt} \quad \text{(minus due to decrease)} \quad -(2)$$

$$\int \vec{J}\cdot d\vec{s} = -\frac{dq}{dt}$$

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Let ρ be the volume charge density of infinite small element of volume $d\vec{v}$

total charge $q = \rho dv$

$$\int \vec{J} \cdot d\vec{s} = -\frac{d}{dt}\int \rho dv$$

$$\int \vec{J} \cdot d\vec{s} = -\int \frac{d\rho}{dt} dv \quad -(3)$$

Using Gauss-divergence theorem in L.H.S

$$\int \vec{J} \cdot d\vec{s} = \int div \vec{J} dv$$

Therefore eq (3) becomes

$$\int div \vec{J} dv = -\int \frac{d\rho}{dt} dv$$

$$\int \left\{ div \vec{J} + \frac{d\rho}{dt} \right\} dv = 0$$

$$\therefore \boxed{div \vec{J} + \frac{d\rho}{dt} = 0}$$ This is eqn of continuity.

Maxwell's eqn in differential form

* I^{st} equation :- (Gauss law in electrostats)
It states that net electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0}\int \rho dv$$

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Using Gauss divergence theorem in L.H.S

$$\int \vec{E} \cdot d\vec{s} = \int div \vec{E} \cdot dv$$

$$\int div \vec{E} \cdot dv = \frac{1}{\varepsilon_0}\int \rho dv$$

$$\int div \vec{E} \cdot dv - \frac{1}{\varepsilon_0}\int \rho dv = 0.$$

$$\int \left(div \vec{E} - \frac{\rho}{\varepsilon_0} \right) dv = 0.$$

$$div \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$\boxed{div \vec{E} = \frac{\rho}{\varepsilon_0}}$$ Maxwell's I^{st} eqn.

* IInd equation :- (Gauss law in Magnetostats)
It states that net magnetic flux through any closed gaussian surface is always zero.

$$\phi_B = 0.$$

$$\int \vec{B} \cdot d\vec{s} = 0.$$

Using Gauss divergence theorem in L.H.S

$$\int div \vec{B} dv = 0$$

i.e $\boxed{div \vec{B} = 0}$ IInd eqn

magnetic monopole doesn't exist.

* IIIrd equation :- (Faraday's law of EMF)
It states that induced emf around a closed circuit is equal to negative time rate of change of magnetic flux linked with the circuit.

* div (curl) of any vector is zero.

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$$e = -\frac{d\phi_B}{dt}$$

$$\int \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

Using stroke's law in L.H.S

$$\int \vec{E}\cdot d\vec{l} = \int \text{curl}\, \vec{E}\cdot d\vec{s}$$

$$\int \text{curl}\, \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$\int \text{curl}\, \vec{E}\cdot d\vec{s} + \frac{d}{dt}\int \vec{B}\cdot d\vec{s} = 0.$$

$$\int \left(\text{curl}\, \vec{E} + \frac{d\vec{B}}{dt}\right) d\vec{s} = 0$$

$$\text{curl}\, \vec{E} + \frac{d\vec{B}}{dt} = 0$$

$$\text{curl}\, \vec{E} = -\frac{d\vec{B}}{dt}$$

* IVth equation :- ~~Maxwell's~~
(Modified ampere's law)

It states that line integral of magnetic field around a closed loop is equal to μ_0 times sum of conduction current & displacement current

$$\int \vec{B}\cdot d\vec{l} = \mu_0\{i_c + i_d\}.$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$$

$$i = \int \vec{J}\cdot d\vec{s} \qquad \phi_E = \int \vec{E}\cdot d\vec{s}$$

assignment

8. Explain the inconsistency in Amperes law. How this inconsistency was overcome.

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$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{s} + \mu_0\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{s}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right] d\vec{s}$$

Using stoke's theorem in L.H.S then $\oint \vec{B}\cdot dl = \int \text{curl}\, \vec{B}\cdot d\vec{s}$

$$\therefore \oint \text{curl}\, \vec{B}\cdot d\vec{s} = \mu_0 \int \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right] d\vec{s}$$

$$\int \text{curl}\, \vec{B}\cdot d\vec{s} - \mu_0 \int \left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right] d\vec{s} = 0$$

$$\int \left\{\text{curl}\, \vec{B} - \mu_0\left(\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right)\right\} d\vec{s} = 0$$

$$\therefore \text{curl}\, \vec{B} - \mu_0\left\{\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right\} = 0$$

$$\text{curl}\, \vec{B} = \mu_0\left\{\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right\}$$

$$\text{curl}\, \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Equation of continuity using 4th law.

$$\text{curl}\, \vec{B} = \mu_0\left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right]$$

taking divergence on both side.

$$\text{div}(\text{curl}\, \vec{B}) = \mu_0\, \text{div}\, \vec{J} + \mu_0\varepsilon_0\, \text{div}\, \frac{d\vec{E}}{dt}$$

$$0 = \mu_0\, \text{div}\, \vec{J} + \mu_0\varepsilon_0 \frac{d}{dt}(\text{div}\, \vec{E}).$$

$$= \mu_0\, \text{div}\, \vec{J} + \mu_0\varepsilon_0 \frac{d}{dt}\left(\frac{\rho}{\varepsilon_0}\right).$$

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$\mu_0 \, div \vec{J} + \mu_0 \frac{d\rho}{dt}$

$0 = \mu_0 \left[div \vec{J} + \frac{d\rho}{dt} \right]$

$$\boxed{div \vec{J} + \frac{d\rho}{dt} = 0}$$

~~Maxwell's equation in integral form.~~

I^{st}	$div \vec{E} = \frac{\rho}{\epsilon_0}$	$div \vec{E} = \frac{\rho}{\epsilon_0}$
IInd	$div \vec{B} = 0$	$div \vec{B} = 0$
IIIrd	$Curl \vec{E} = -\frac{d\vec{B}}{dt}$	$Curl \vec{E} = -\frac{d\vec{B}}{dt}$
IVth	$Curl \vec{B} = \mu_0 \left\{ \vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right\}$	$Curl \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right]$

$curl \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$

Maxwell's equation in integral form:-

I^{st} equation: $div \vec{E} = \frac{\rho}{\epsilon_0}$

integrating this eq^n over the volume dv

$$\int div \vec{E} \, dv = \frac{1}{\epsilon_0} \int \rho \, dv$$

Using Gauss divergence theorem in L.H.S

$$\int div \vec{E} \, dv = \int \vec{E} \cdot d\vec{S}$$

$$\therefore \boxed{\int \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \int \rho \, dv}$$

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⇒ Net outward electric flux to a surface is equal to $\frac{1}{\epsilon_0}$ times net charge enclosed within the surface.

$$\int (\vec{E} \cdot \epsilon_0) d\vec{S} = \int \rho \, dv$$

$$\int \vec{D} \cdot d\vec{S} = \int \rho \, dv$$

[$\vec{D}$ → electric displacement
$\vec{D} = \epsilon_0 \vec{E}$]

IInd eq^n :- $div \vec{B} = 0$

Integrating this equation over the volume V

$$\int div \vec{B} \, dv = 0$$

Using Gauss divergence theorem in L.H.S

$$\int div \vec{B} \, dv = \int \vec{B} \cdot d\vec{S}$$

$\boxed{\int \vec{B} \cdot d\vec{S} = 0}$ → net magnetic flux over a closed surface is zero

→ Magnetic monopole doesn't exists.

IIIrd eq^n :- $curl \vec{E} = -\frac{dB}{dt}$

Integrating this eq^n over the ds

$$\int curl \vec{E} \, ds = -\int \frac{dB}{dt} ds$$

Using stokes theorem in L.H.S

$$\boxed{\int \vec{E} \cdot d\vec{l} = -\int \frac{d\vec{B}}{dt} ds}$$

EMF around a closed path is equal to negative time rate of change of magnetic flux link with the circuit.

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IVth eq^n $\text{curl}\,\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}\right]$

$$\text{curl}\,\frac{\vec{B}}{\mu_0} = J + \frac{d}{dt}(\varepsilon_0 \vec{E}) = J + \frac{d}{dt}\vec{D}$$

$$\left[\text{Curl}\,\vec{H} = \vec{J} + \frac{d}{dt}\vec{D}\right]$$

$\boxed{\vec{B} = \mu_0 H}$ Magnetic field intensity

integrating over surface ds

$$\int \text{curl}\,\vec{H}\,d\vec{s} = \int\left(J + \frac{d}{dt}\vec{D}\right)d\vec{s}$$

Using Stoki's theorem in L.H.S

$$\boxed{\underbrace{\int \vec{H}\cdot d\vec{l}}_{\text{MMF Magnetic motive force}} = \int\left(\vec{J} + \frac{d\vec{D}}{dt}\right)d\vec{s}}$$

mmf around a closed path is equal to sum of conduction current and displacement current through the surface.

Poynting vector ($\vec{S}$)

Rate of flow of energy per unit area in a plane electromagnetic wave can be described by a vector called the Poynting vector.

SI unit = Watt/m^2

$$S = \frac{1}{\mu_0}[\vec{E}\times\vec{B}] = \vec{E}\times\vec{H}$$

$$= EH\sin\theta$$

$$\boxed{\vec{S} = EH} \quad (\theta = 90^\circ)$$

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10/10/23 Poynting theorem:

→ It states that work done on the charge by an electromagnetic force is equal to decrease in energy stored in the field less than the energy which flows out through the surface

OR

→ Rate of decrease of electromagnetic energy in a certain volume 'V' is equal to work done by the field forces per unit time + the power transferred into electromagnetic field.

→ This theorem is also known as work energy theorem in electromagnetic field.

$$\underbrace{-\frac{d}{dt}\int\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv}_{\text{rate of decrease in electromagnetic energy}} = \underbrace{\int \vec{E}\cdot\vec{J}\,dv}_{\text{rate of doing work}} + \underbrace{\int(\vec{E}\times\vec{H})\cdot d\vec{s}}_{\text{power flux}}$$

Physical interpretation:-

(i) $-\frac{d}{dt}\int\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv$. it represents the rate of change (decrease) of electromagnetic energy in a volume V.

(ii) $\int \vec{E}\cdot\vec{J}\,dv$ it represents the rate of doing work (energy transfer into electromagnetic field)

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(iii) $(\vec{E}\times\vec{H})d\vec{s}$ → ~~po~~ it represent amount of electromagnetic energy crossing the closed surface per second per unit area.

EMW in free space

Maxwell's eqⁿ

$$\left.\begin{aligned} &\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0} \\ &\text{div}\,\vec{B} = 0 \\ &\text{Curl}\,\vec{E} = -\frac{d\vec{B}}{dt} \\ &\cancel{\text{Curl}\,\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\right.} \quad \text{Curl}\,\vec{H} = \vec{J} + \frac{\partial\vec{D}}{dt} \end{aligned}\right\} \text{①}$$

Condition of free space.

$q=0$; $\rho=0$; $i=0$; $\vec{J}=0$

Therefore maxwell's eqⁿ in free space become

$$\left.\begin{aligned} &\text{div}\,\vec{E} = 0 \quad -(a) \\ &\text{div}\,\vec{B} = 0 \quad -(b) \\ &\text{Curl}\,\vec{E} = -\frac{d\vec{B}}{dt} \quad -(c) \\ &\text{Curl}\,\vec{H} = \frac{\partial\vec{D}}{dt} \quad -(d) \end{aligned}\right\} -\text{②} \qquad \text{Curl}\,\vec{B} = \mu_0\varepsilon_0\frac{d\vec{E}}{dt}$$

→ Taking curl of eqⁿ (2)(c)

$$\text{curl}(\text{Curl}\,\vec{E}) = -\frac{\partial}{\partial t}(\text{Curl}\,\vec{B})$$

$$\text{grad}(\text{div}\,\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{d\vec{E}}{dt}\right)$$

$$0 - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

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$$\text{grad}(\text{div}\,\vec{E}) = 0$$

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{dt^2}}$$

Wave eqⁿ for electric field in freespace

Taking curl of eqⁿ 2(d)

$$\cancel{\text{curl}(\text{curl}\,\vec{B}) = \mu_0 J}$$

$$\boxed{\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{dt^2}}$$

Wave eqⁿ for magnetic field in freespace

Therefore, general eqⁿ for electromagnetic wave in free space.

$$\nabla^2 u = \mu_0\varepsilon_0\frac{\partial^2 u}{\partial t^2}$$

$$\nabla^2 u - \mu_0\varepsilon_0\frac{\partial^2 u}{\partial t^2} = 0 \quad -\text{③}$$

We already have a wave eqⁿ for a wave propogating with velocity v.

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad -\text{④}$$

Comparing eqⁿ ③ & ④.

$$\frac{1}{v^2} = \mu_0\varepsilon_0$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

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$$= \sqrt{\frac{4\pi}{(4\pi\epsilon_0)\mu_0}}$$

$$= \sqrt{9\times10^9\times10^{-7}}$$

$$= \sqrt{9\times10^{16}}$$

$$V = 3\times10^8 \ m/s.$$

$$\boxed{V = c}$$

~~Wave~~ Speed of wave propagating in a free space is equal to the speed of light in vacuum.

Transverse nature of EMW.

$$\nabla^2\vec{E} - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{dt^2} = 0$$

$$\nabla^2\vec{B} - \mu_0\epsilon_0\frac{d^2\vec{B}}{dt^2} = 0.$$

general solⁿ of these two equation

$$\vec{E} = E_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$
$$\vec{B} = B_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

$E_0 \rightarrow$ complex amplitude of electric field.

$$\vec{k} = k\hat{n}$$
$$= \frac{2\pi}{\lambda}\hat{n}$$
$$\vec{k} = \frac{2\pi\nu}{c}\hat{n}$$
$$\vec{k} = \frac{\omega}{c}\hat{n}$$

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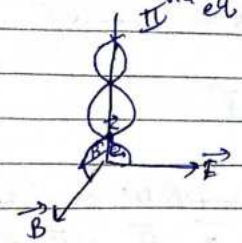
I^st eqⁿ

$$\text{div}\,\vec{E} = 0$$
$$\nabla\cdot\vec{E} = 0$$
$$\Rightarrow i(\vec{k}\cdot\vec{E}) = 0$$
$$\vec{k}\cdot\vec{E} = 0$$
$$\Rightarrow \vec{k}\perp\vec{E}$$

IInd eq



$$\text{div}\,\vec{B} = 0.$$
$$\nabla\cdot\vec{B} = 0$$
$$i(\vec{k}\cdot\vec{B}) = 0$$
$$\vec{k}\cdot\vec{B} = 0$$
$$\Rightarrow \vec{k}\perp\vec{B}$$

Hence they are transverse in nature.

Orthogonal nature of EMW.

general solⁿ of eqⁿ

$$\vec{E} = E_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$
$$\vec{B} = B_0 e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

IIIrd eq.

$$\text{curl}\,\vec{E} = -\frac{d\vec{B}}{dt}$$

$$\nabla\times\vec{E} = -\frac{d}{dt}(B_0 e^{ikr - i\omega t})$$

$$i(\vec{k}\times\vec{E}) = i\omega \underbrace{B_0 e^{ikr - i\omega t}}_{B}$$

$$\cancel{i}(\vec{k}\times\vec{E}) = \cancel{i}\omega\vec{B}$$
$$\vec{k}\times\vec{E} = \omega\vec{B}$$

$\Rightarrow$ as we already have $\vec{k}\perp\vec{B}$ & $\vec{k}\perp\vec{E}$,
i.e $\vec{E}\perp\vec{B}$

IVth eq.

$$\text{Curl}\,B = \mu_0\epsilon_0\frac{d\vec{E}}{dt}$$

$$\nabla\times\vec{B} = -\frac{d}{dt}(\mu_0\epsilon_0 E_0 e^{ikr - i\omega t})$$

$$\cancel{i}(\vec{k}\times\vec{B}) = \mu_0\epsilon_0\cancel{i}\omega \underbrace{E_0 e^{ikr - i\omega t}}_{E}$$

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$\vec{K} \times \vec{B} = \mu_0 \epsilon_0 \omega \vec{E}$

$\Rightarrow$ So $\vec{B} \perp \vec{E}$

~~That means EMW in free~~
That means $\vec{K}$, $\vec{B}$ & $\vec{E}$ are mutually perpendicular to each other.

$\rightarrow$ Hence, EMW in free space is orthogonal in nature

$(Z) = \frac{E}{H}$

Impedence / wave impedence / intrinsic impedence

$\omega \vec{B} = \vec{K} \times \vec{E}$

$\omega \vec{B} = K(\hat{n} \times \vec{E})$

$\vec{B} = \frac{K}{\omega}(\hat{n} \times \vec{E}) = \frac{1}{c}(\hat{n} \times \vec{E})$

$\mu_0 H = \frac{1}{c}(\hat{n} \times \vec{E})$

$\vec{H} = \frac{1}{c\mu_0}(\hat{n} \times \vec{E})$

$\vec{H} = \sqrt{\frac{\mu_0 \epsilon_0}{\mu_0}}(\hat{n} \times \vec{E})$

$\vec{H} = \sqrt{\frac{\epsilon_0}{\mu_0}}(\hat{n} \times \vec{E})$

$\frac{\vec{E}}{\vec{H}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\hat{n}}$

In terms of magnitude.

$\left|\frac{E}{H}\right| = \sqrt{\frac{\mu_0}{\epsilon_0}} \left|\frac{1}{\hat{n}}\right| = \sqrt{\frac{\mu_0}{\epsilon_0}}$

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$Z = \boxed{\left|\frac{\vec{E}}{H}\right| = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega}$ since this ratio is real & +ve it means -

$\Rightarrow$ E & H both are in the same phase

Ratio of electrostatic & magnatostatic energy density :-

$U_E = \frac{1}{2}\epsilon_0 E^2$ $\quad U_B = \frac{1}{2}\mu_0 H^2$.

Ratio $= \frac{\epsilon_0}{\mu_0}\left(\frac{E}{H}\right)^2 = \frac{\epsilon_0}{\mu_0} \times \frac{\mu_0}{\epsilon_0} = 1$

$\Rightarrow$ $\boxed{U_E = U_B}$ electrostatic energy density is equal to magnatostatic energy density

Total energy

$U = U_E + U_M = 2U_E = 2U_M$

$\boxed{U = \epsilon_0 E^2}$ $\quad \boxed{U = \mu_0 H^2}$

time average of energy density.

$\langle U \rangle = \epsilon_0 \langle E^2 \rangle$

$\epsilon_0 \langle (E_0 e^{ikr - i\omega t})^2 \rangle$ $\quad \left\{ e^{ikr - i\omega t} \text{ avg value } \Rightarrow 1/2 \right\}$

$\boxed{\langle U \rangle = \epsilon_0 \frac{E_0^2}{2}}$

$\boxed{\langle U \rangle = \epsilon_0 E_{rms}^2}$ $\quad E_{rms} = \frac{E_0}{\sqrt{2}}$

OR

$\boxed{\langle U \rangle = \mu_0 B_{rms}^2}$

$\vec{a}\times\vec{b}\times\vec{c}$
$= (a\cdot c)b - (a\cdot b)c$

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* Poynting vector -

$$\vec{H} = \frac{1}{\mu_0 c}(\hat{n}\times\vec{E})$$

$$\vec{S} = \vec{E}\times\vec{H}$$

$$= \vec{E}\times\frac{1}{\mu_0 c}(\hat{n}\times\vec{E})$$

$$= \frac{1}{\mu_0 c}\{\vec{E}\times(\hat{n}\times\vec{E})\}$$

$$\frac{1}{\mu_0 c}\{(E\cdot E)\hat{n} - (\vec{E}\cdot\hat{n})\vec{E}\}$$

$$\frac{1}{\mu_0 c}\{E^2\hat{n} - 0\}$$

$$\vec{S} = \frac{1}{\mu_0 c}E^2\hat{n}$$

$$= \frac{1}{\mu_0}\frac{1}{\sqrt{\mu_0\epsilon_0}}E^2\hat{n}$$

$$= \sqrt{\frac{\epsilon_0}{\mu_0}}E^2\hat{n}$$

Momentum
$$\vec{P} = \frac{\vec{S}}{c^2} = \frac{EH}{c^2}$$

$$\vec{S} = \frac{E^2}{Z_0}\hat{n}$$

angular momentum
$$L = \frac{1}{c^2}\int\vec{r}\times(\vec{E}\times\vec{H})\,dv$$

Time average of Poynting vector -

$$\langle\vec{S}\rangle = \frac{\langle E^2\rangle}{Z_0}\hat{n}$$

$$\langle\vec{S}\rangle = \frac{1}{Z_0}\hat{n}\langle E^2\rangle$$

$$\langle\vec{S}\rangle = \frac{1}{Z_0}\hat{n}\frac{E_0^2}{2}$$

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$$\langle\vec{S}\rangle = \frac{1}{Z_0}\hat{n}E_{rms}^2$$

$$\frac{\langle S\rangle}{\langle U\rangle} = \frac{\frac{1}{Z_0}\hat{n}E_{rms}^2}{\epsilon_0 E^2} = \frac{1}{Z_0\epsilon_0}\hat{n}$$

$$\frac{1}{\sqrt{\frac{\mu_0}{\epsilon_0}}\times\epsilon_0}\hat{n}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}}\hat{n}$$

$$\boxed{\frac{\langle S\rangle}{\langle U\rangle} = c\hat{n}}$$

$$\langle S\rangle = \langle U\rangle c\hat{n}$$

* EMW in non-conducting medium.

$$\oint \text{div}\,\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \text{div}\,\vec{B} = 0$$

$$\oint \text{curl}\,\vec{E} = -\frac{d\vec{B}}{dt}$$

$$\text{Curl}\,\vec{B} = \mu_0\left[\vec{J} + \epsilon_0\frac{d\vec{E}}{dt}\right]$$

$q = 0$
$\rho = 0$
$i = 0$
$J = 0$.
μ, ϵ
$\mu = \mu_0\mu_r$
$\epsilon = \epsilon_0\epsilon_r$

Therefore maxwell's eq^n in non-cond. below.

$$\left.\begin{aligned} \text{div}\,\vec{E} &= 0 \\ \text{div}\,\vec{B} &= 0 \\ \text{Curl}\,\vec{E} &= -\frac{d\vec{B}}{dt} \\ \text{Curl}\,\vec{B} &= \cancel{\mu_0\epsilon_0\frac{d\vec{E}}{dt}}\ \mu\epsilon\frac{d\vec{E}}{dt} \end{aligned}\right\} \text{②}$$

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Taking curl of eqn 2 (c)

$$\text{curl}\,(\text{curl}\,\vec{E}) = -\frac{d}{dt}(\text{curl}\,\vec{B})$$

$$-\nabla^2 E = -\frac{d}{dt}\left(\mu\varepsilon\frac{d\vec{E}}{dt}\right)$$

$$\nabla^2 E - (\mu\varepsilon)\frac{d^2\vec{E}}{dt^2} = 0$$

Wave eqn of electric field in non conducting medium.

Similarly $\nabla^2 B - \mu\varepsilon\frac{d^2\vec{B}}{dt^2} = 0$

Wave eqn of magnetic field in non-conducting medium.

Therefore general eqn of EMW in non-cond medium.

$$\nabla^2 v - \mu\varepsilon\frac{d^2 v}{dt^2} = 0$$

but we already have a wave eqn for a wave propagating with velocity v

$$\nabla^2 v - \frac{1}{v^2}\frac{\partial^2 v}{\partial t^2} = 0$$

Comparing these 2

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$V = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_r\varepsilon_0}}$$

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$$\boxed{v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}}$$

Speed of EMW in ~~(non-conducting)~~ dielectric is less than the speed of free space.

EMW in conducting medium

$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$

$\vec{J} = \sigma\vec{E}$ (conductivity)

$\text{div}\,\vec{B} = 0$

$\text{Curl}\,\vec{E} = -\frac{d\vec{B}}{dt}$

Condition for conducting medium

$\text{Curl}\,\vec{H} = \vec{J} + \frac{\partial D}{\partial t}$

$\vec{J} = \sigma\vec{E}$

$q = 0,\ \rho = 0$

therefore maxwell eqn in conducting medium becomes -

$\text{div}\,\vec{E} = 0$

$\text{div}\,\vec{B} = 0$

$\text{Curl}\,\vec{E} = -\frac{d\vec{B}}{dt}$

$\text{Curl}\,\vec{B} = \mu\left[\sigma\vec{E} + \varepsilon\frac{d\vec{E}}{dt}\right]$

⇒ The amplitude of electromagnetic waves keeps on antanuated as the wave travels in conducting medium.

Skin depth :- δ

distance at which the amplitude of EMW reduces to $1/e$ (37%) of its initial value

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$\omega \rightarrow$ angular frequency $\rightarrow 2\pi f$.

$\mu = \mu_0\mu_r$.

$\sigma \rightarrow$ conductivity.

SI unit - m

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Numericals -

Q EMW intensity recieved on the surface of earth from the sun is found to be 1.33 k Watt/m² find the amplitude of electric field vector & magnetic field vector recieved on the earth surface.

→ $\vec{S} = \cancel{1.33}\ 1330\ W/m^2$

$\vec{S} = \vec{E} \times \vec{H} = EH = 1330$ — (1)

$\frac{E}{H} = Z = 377$ — (2)

$377 H^2 = 1330$

$H^2 = \frac{1330}{377}$

$H^2 = 3.52$

$H = 1.876$ A/m

$E = 707.31$ W/m

$E_0 = 707.31 \times \sqrt{2} = 1000.29403$

$H_0 = 2.64$ A/m

Q If the earth recieves ~~2 cal/min~~ 2 cal $min^{-1} cm^{-2}$ solar energy. Find E_0 & B_0

→ $\frac{2 \times 4.2}{60 \times 10^4}\ J s^{-1} m^{-2} = W/m^2$

$S = 1.4 \times 10^3 = 1400 = EH$ — (1)

$\frac{E}{H} = Z = 377$ — (2)

$377 H^2 = 1400$

$H^2 = 3.71$

$H = 1.92$ A/m

$E = 726.15$ W/m

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$E_0 = 726.15\sqrt{2}$

$H_0 = 1.92\sqrt{2}$

Q: Calculate the poynting vector at the surface of sun given that energy radiated per second 3.8×10^{26} J/s and the radius of the sun 0.7×10^9 m.

→ Area $= 4\pi r^2$

$4 \times 3.14 \times 0.49 \times 10^{18}$

$6.1544 \times 10^{18}\ m^2$

$S = \frac{3.8 \times 10^{26}}{6.1544 \times 10^{18}}$

$= 0.617 \times 10^8$

$\boxed{6.17 \times 10^7\ J/s/m^2}$

Q: When the ~~amplitude~~ magnitude of Magnetic field in a plane wave is 2 A/m. Calculate the magnitude of electric field.

→ $\left|\frac{E}{H}\right| = 377 \times 2$

$= 754$ W/m

Determine the magnitude of Electric field when $\sigma = 0$, $\mu_r = 1$, $\epsilon = 4\epsilon_0$

$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{377}{\sqrt{4}} = \frac{377}{2}$

$\Rightarrow 188.5\ \Omega$.

Q Find the depth of penetration of EMW in copper. $\nu = 60\ Hz$ $\sigma \rightarrow 5.8 \times 10^7$ mho m^{-1}

$\mu_r = 1$ $\epsilon_r = 1$

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$$S = \sqrt{\frac{2}{\mu\sigma\omega}} = \sqrt{\frac{2}{\mu_0\mu_r\sigma 2\pi f}}$$

$\omega = 2\pi f$

$$= \sqrt{\frac{1}{4\pi\times10^{-7}\times1\times5.8\times10^{7}\times\pi\times60}}$$

$$= \sqrt{\frac{1}{13724.5}} = 8.5\times10^{-3}\ m$$

$$8.5\ mm$$

$\nu = 100\ Hz$

$$= \frac{1}{4\pi\times5.8\times\pi\times10^{2}} = \sqrt{\frac{1}{22874.2}}$$

$$4.3\times10^{-5}$$

$$= \frac{1}{151.2} = 6.6\times10^{-3}\ m$$

$$6.6\ mm$$

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Unit – IV

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Laser

(Light amplification by stimulated emission of radiation)

* Characteristics of Laser –

i) divergence – laser has very low divergence as compared to ordinary source.

ii) intensity – laser has a very high intensity.

iii) Coherence – laser is highly coherent such that two independent laser can give rise the phenomenon of interference.

iv) Highly directional

* Absorption of Radiation – An atom has quantised energy state, initially an atom is in ground state when energy is given to this atom its electron jumps to higher energy state by absorbing the photon/radiation. This process is c/d absorption of radiation.

→ The frequency of absorbed radiation is

$$E_2 - E_1 = h\nu$$

$$\boxed{\nu = \frac{E_2 - E_1}{h}}$$

→ Probable rate of transition from 1→2

$$\boxed{(P_{12}) = B_{12}\, u(\nu)}$$

$u(\nu)$
↳ energy density
↳ energy per unit volume.

B_{12} → Einstien's coefficient for absorption process

→ Absorption is an induced process.

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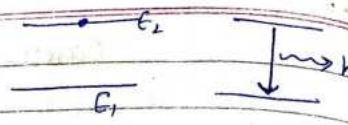
Spontaneous emission -



→ An atom in excited state remains for only about 10^{-8} seconds. it then by its own jumps to lower energy state by emitting radiation. This process is called spontaneous emission of radiation.

→ Probable rate of transition from 2→1

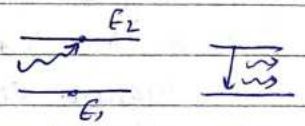
$$\boxed{(P_{21})_{spn} = A_{21}}$$

A_{21} → Einstein's coefficient for spontaneous process.

→ Spontaneous emission isn't induced one.

$$\boxed{\frac{E_2 - E_1}{h} = \nu}$$

Stimulated emission -



→ If an atom is in excited state then an incident photon of correct energy may cause the atom to jump to lower energy state by emitting an additional photon of same frequency. This phenomenon is cld stimulated emission.

→ Probable rate of transition from 2→1

$$\boxed{(P_{21})_{sti} = B_{21}\, u(\nu)}$$

B_{21} → Einstein's coefficient for Stimulated process.

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→ Total probability of emission

$$P_{21} = (P_{21})_{spn} + (P_{21})_{sti}$$

$$\boxed{P_{21} = A_{21} + B_{21}\, u(\nu)}$$

Relation b/w einstien's A & B coefficient -

→ consider an assembly of atom in thermal equilibrium at temp T. with frequency ν and energy density $u(\nu)$.

→ Let N_1 N_2 be the population of states 1 & 2 respectively at any instant

→ no. of atom in state 1 that can absorb a photon and give rise to absorption process $= N_1 P_{12}$

→ no. of atom in state 2 that can cause emission process $= N_2 (P_{21})_{total}$

→ at equilibrium

rate of absorption = rate of emission.

$$N_1 P_{12} = N_2 (P_{21})_{total}.$$

$$N_1 B_{12}\, u(\nu) = N_2 (A_{21} + B_{21}\, u(\nu))$$

$$N_1 B_{12}\, u(\nu) = N_2 A_{21} + N_2 B_{21}\, u(\nu)$$

$$N_1 B_{12}\, u(\nu) - N_2 B_{21}\, u(\nu) = N_2 A_{21}$$

$$u(\nu)\left[N_1 B_{12} - N_2 B_{21}\right] = N_2 A_{21}$$

$$\boxed{u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}}$$

$$\boxed{u(\nu) = \frac{N_2 A_{21}}{B_{21}\left(\frac{N_1 B_{12}}{B_{21}} - N_2\right)}}$$

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$$u(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{N_1}{N_2}\frac{B_{12}}{B_{21}} - 1}\right] \quad -(1)$$

⇒ A/c to Boltzmann distribution law

$$N_1 = N_0 e^{-E_1/KT}$$

$$N_2 = N_0 e^{-E_2/KT}$$

$$\frac{N_1}{N_2} = \frac{e^{-E_1/KT}}{e^{-E_2/KT}} = e^{-\frac{E_1}{KT}+\frac{E_2}{KT}}$$

$$= e^{\frac{1}{KT}[E_2 - E_1]}$$

$$= e^{\frac{h\nu}{KT}} \quad -(2)$$

putting (2) in (1).

$$u(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{e^{h\nu/KT}\frac{B_{12}}{B_{21}} - 1}\right] \quad -(2)$$

Comparing this with planck's radiation formula.

$$u(\nu) = \frac{8\pi h \nu^3}{c^3}\frac{1}{e^{h\nu/KT} - 1}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \left\{\frac{8\pi h}{c^3} \rightarrow \text{constant}\right\}$$

$$\frac{A_{21}}{B_{21}} \propto \nu^3$$

$$\boxed{A_{21} \propto B_{21}\nu^3}$$

Spontanous emission always dominates over stimulated emission.

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$$\frac{B_{12}}{B_{21}} = 1$$

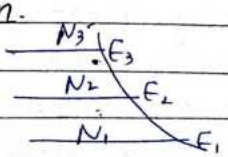
$$\boxed{B_{12} = B_{21}}$$

→ Stimulated emission.

→ Absorption

* Population ~~inv~~ inversion.

→ A condition ~~when~~ in which higher energy states are highly populated as compared to lower energy state. is called population inversion.



→ In normal condition.

$$N_1 > N_2 > N_3.$$

population inversion

$$N_1 < N_2 < N_3$$

→ population inversion is achieved by pumping

* pumping - Method of supplying energy to the atoms which are present in lower energy state so that they can jump to higher energy state to achieve population inversion.

→ electrical pumping →

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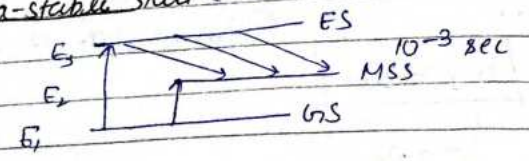
Meta-stable state (MSS)

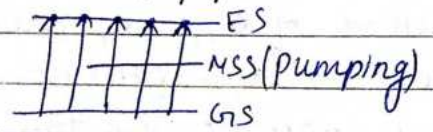


→ an atom remain in the excited state for very short time (10^{-8} sec). Therefore, even if we supply energy continuously to the atom to transfer them from ground state to excited state they immediately come down to ground state. So in this way population inversion can never be achieved.

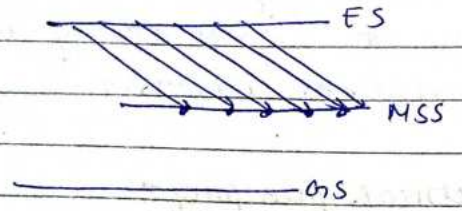
→ In order to achieve population inversion we must have an energy state which has a longer life time such an energy state is called meta stable state
lifetime → 10^{-3} sec

Process of population inversion :-

Step 1:



Step 2:



Spontaneous emission.

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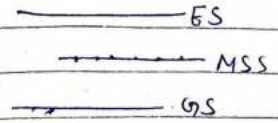
Step 3:



population inversion achieved.

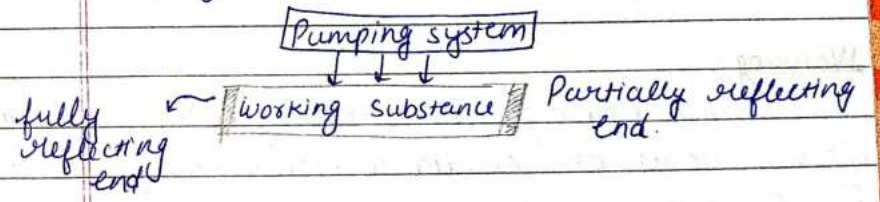
Principle of LASER -

1) Pumping -
2) Population inversion.
3) stimulated emission should dominate over spontaneous emission.

$$(P_{21})_{stimulated} > (P_{21})_{spontaneous}$$

Components of LASER -

① Working substance - Having metal stable state
② Resonant cavity
③ Pumping system



He-Ne Laser :- Ruby laser

4 energy level in He-Ne

Main drawback of Ruby laser is that output beam is not continuous though very intense for the continuous laser beam, gas lasers are used.
Main advantage of gas laser are exceptionally high monochromaticity, most pure spectrum and high stasicity of frequency. Power of gas laser is moderate but inferior to that of crystal lasers.

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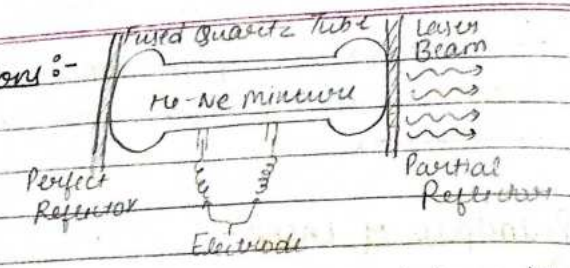
Construction :-



Gas laser consist of fused quartz tube with diameter about 1.5cm and 80cm long. This tube is filled with a mixture of Neon under a pressure of 0.1mmHg and He under a pressure of 1mm Hg. There is a majority of He atoms and minority of Ne atoms (3:1). At the end of the tube, there is a perfect reflector while at the other end there is a partial reflector. Active materials is excited by means of high frequency generator.

• Quartz is used because it absorbs the photon released by state E offers spontaneous emission.

Working :-

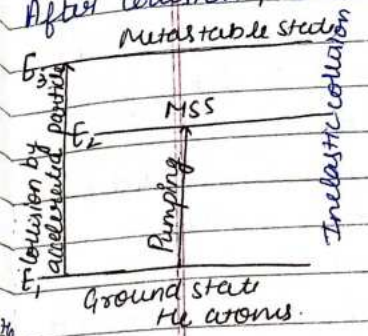
When a discharge passes through the gas mixture, helium atoms are excited to higher energy levels E_2 & E_3 through collisions with accelerated particles. This is known as pumping.

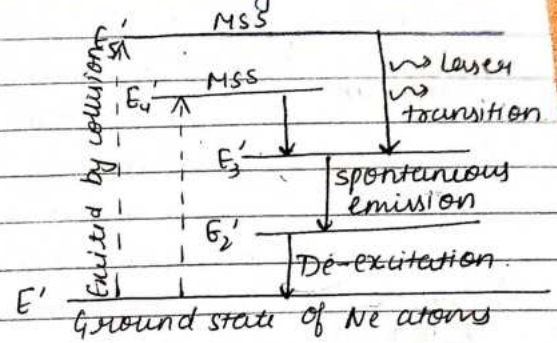
• E_2 & E_3 are metastable state from which there are no allowed transitions.

Excited He atoms, then collide inelastically (loss of energy) with neon atoms still in ground state and transfer energy to them.

Advantage of this collision process is that fairly light Neon atoms can easily jumped to energy state E_3', E_4', E_5'

After collision, He atoms return to ground state





Higher Ne states E_4' & E_5' are metastable states and have longer life time than E_3'

Here a population inversion takes place b/w E_5', E_4' and E_3'

When an excited Ne atom passes from metastable state E_5' & E_4' to state E_3', it emits a photon. This photon travels through the gas mixture. If the photon is moving parallel to the axis of ~~fuse~~ tube, it is reflected back and forth by the mirror ends until it stimulates an excited Ne atom. Thus it causes a fresh photon in phase with stimulating photon.

This stimulated transition is LASER transition.

This process continues till a beam of coherent radiation builds up in the tube. When the beam become sufficiently intense, a portion of it escapes through the partially silvered end

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Characteristics of He-Ne laser

1) Type: It is four energy level (3 in Ne & 1 in He)
2) Active medium: It uses a mixture of He & Ne gases as active medium.
3) Pumping method: Electric discharge method is used for pumping action for achieving population inversion.
4) Optical Resonator: A pair of plane mirrors facing each other is used as an optical resonator.
5) frequency: frequency of ~~operating~~ output beam is 4.7×10^{14} Hz
6) wavelength: wavelength of output laser beam is 6328 Å
7) Nature: Nature of output beam is continuous waves.
8) power output: Power output of laser is 0.5 – 50 mW

Advantages of He-Ne lasers:-

- It is more monochromatic & more directional than solid state lasers.
- High stability of frequency.
- No cooling is required
- It is less expensive.

Applications of He-Ne laser:-

- Used to perform interference & diffraction pattern
- Used in opthalmology
- Used to produce 3D pictures.
- Used in optical communication w/o for moderate distance.

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FOR - Frame of Ref

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RELATIVITY

Frame of reference - it is defined as ~~system~~ of a coordinate system relative to which position and motion of a body may be described.

* Inertial FOR - Those FOR in which law of inertia is valid

→ Inertial frame of reference are always unaccelerated frame.

→ eg - a person sitting in a car.
a person in lift.

* Non-inertial FOR - Those FOR in which law of inertia is not valid.

~~eg~~ Such frames are accelerated frame.

→ eg - Earth
Merry-go round.

M.M experiment → stationary frame.

Aim - To prove the existence of ether by calculating the relative velocity between earth & ether.

Apparatus - Michelson's interferometer.

$\boxed{l = 11m}$

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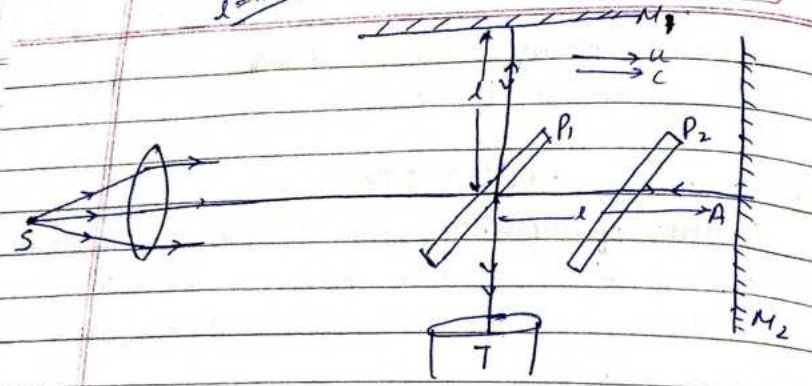
$l = 11m$



- function of plate P_2 (compensating plate)
  - beam reflected towards M_1 crosses the thickness of the plate P_1 thrice. whereas the transmitted beam which is going toward M_2 do so only once. (3:1)
  - To make their optical path through glass and air equal an additional plate P_2 which is of the same material and same thickness as that of P_1 is introduced b/w plate P_1 & mirror M_2. Hence it is known as compensating plate.

Conclusion: existence of hypothetical medium was disapproved. It means the motion of earth through ether cannot be detected experimentally.

→ The relative velocity b/w earth and ether is zero.

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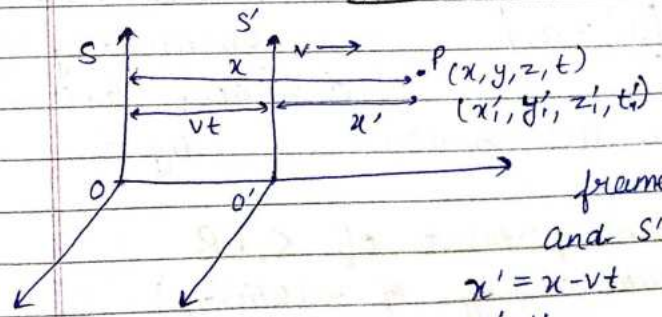
(Explanation of negative result) -

1) Ether drag hypothesis -
when the earth crosses the ether present in the space then the earth will drag it with its own speed. Hence relative velocity b/w two is zero.
2) Constancy of speed of light hypothesis -
3) Length contraction hypothesis

Significance of negative result -

1) Velocity of light is constant in all directions
2) Presence of ether in entire space is undetectable it means all the attempts to make ether a universal frame is meaningless.
3) A new theory having different concept of space, time and mass is required because gallilion transformation fail to give the correct result.

Gallilion transformation for space and time



frame S is at rest and S' in motion.

$x' = x - vt$
$y' = y$
$z' = z$
$t' = t$

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general eqn

$x' = x - vt \qquad t' = t$

Gallelion eqn for velocity

velocity of frame

$x' = x - vt \qquad t' = t$

diffrentiating wrt t.

$\frac{dx'}{dt} = \frac{dx}{dt} - v \qquad \frac{dt'}{dt} = 1$

$dt' = dt$

$\Rightarrow \boxed{\frac{dx'}{dt'} = \frac{dx}{dt} - v}$

$u' = \frac{dx'}{dt'} \rightarrow$ velocity of P from O'

$\boxed{u' = u - v}$

$u = \frac{dx}{dt} \rightarrow$ velocity of P from O

drawback only valid for small velocities

Gallelion transformation for acceleration

$\rightarrow$

$u' = u - v$

diffrentiating wrt t

$\frac{du'}{dt} = \frac{du}{dt} - 0 \qquad$ {velocity is constant}

$\Rightarrow \frac{du'}{dt'} = \frac{du}{dt}$

a' = acceleration observed by O'

$\Rightarrow \boxed{a' = a}$

a = acc. observed by O

$\Rightarrow$ Same acceleration

Einstien's postulate of S.T.R
(Special theory of relativity)

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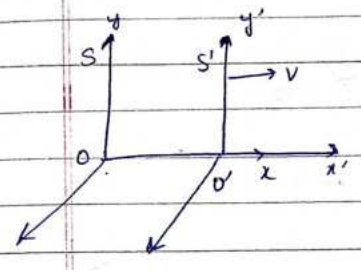
1) All ~~Inertia~~ physical laws are same in every inertial FOR. which are moving with constant velocity relative to each other

2) Speed of light in vacuum is same in every inertial frame

Lorentz - transformation eqn



S' in motion with uniform velocity v wrt S along +ve x-axis

$x' \propto x - vt \qquad$ K is independent of x & t.

$x' = K(x - vt)$ — (1)

Inverse relation can be written as -

$x = K(x' + vt')$ — (2)

Substitute the value of x' from (1) in (2)

$x = K[K(x - vt) + vt']$

$\frac{x}{K} = K(x - vt) + vt'$

$vt' = \frac{x}{K} - K(x - vt)$

$\cancel{t' = \frac{1}{v}\left[\frac{x - K^2(x^2 - vt)}{K}\right]}$

$t' = \frac{1}{v}\left[\frac{x}{K} - K(x - vt)\right] = \frac{x}{Kv} - \frac{Kx}{v} + Kt$

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$$t' = kt + \frac{x}{v}\left[\frac{1}{k} - k\right]$$

$$t' = kt + \frac{kx}{v}\left[\frac{1}{k^2} - 1\right] = \cancel{kt + \frac{kx}{v}\left[\frac{1-k^2}{k^2}\right]}$$

$$t' = kt - \frac{kx}{v}\left[1 - \frac{1}{k^2}\right] \quad \text{--- (3)}$$

A/c to (2) second postulate speed of light is constant

$$\left.\begin{aligned} x &= ct \\ x' &= ct' \end{aligned}\right] \text{--- (4)}$$

Substitute (4) in eqn (1) & (2)

$$ct' = k(ct - vt)$$

$$ct' = kt(c - v) \quad \text{--- a}$$

$$ct = k(ct' + vt')$$

$$ct = kt'(c + v)$$

Multiplying these ~~two~~ eqn.

$$c^2 \cancel{tt'} = k^2 \cancel{tt'} (c^2 - v^2)$$

$$c^2 = k^2(c^2 - v^2)$$

$$\Rightarrow k^2 = \frac{c^2}{c^2 - v^2} = \frac{c^2}{c^2\left[1 - \frac{v^2}{c^2}\right]} = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\boxed{k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

k is independent of x & t.

Rewriting eqn (3).

$$t' = kt - \frac{kx}{v}\left[1 - \frac{1}{k^2}\right]$$

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$$\cancel{\frac{1}{k^2} = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1 - \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = \frac{-\frac{v^2}{c^2}}{\frac{v^2}{c^2}\left(\frac{c^2}{v^2} - 1\right)} = \frac{1}{c^2}}$$

$$1 - \frac{1}{k^2} = 1 - \left(1 - \frac{v^2}{c^2}\right) = \frac{v^2}{c^2}$$

$$t' = kt - \frac{kx}{v}\left[\frac{v^2}{c^2}\right]$$

$$t' = kt - \frac{kxv}{c^2}$$

$$\boxed{t' = k\left[t - \frac{xv}{c^2}\right]}$$

Hence, lorentz transformation eqn becomes.

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad \boxed{\begin{aligned} y' &= y \\ z' &= z \end{aligned}}$$

$$\boxed{t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Inverse lorentz transformation eqn.

$$\boxed{x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}} \quad \boxed{\begin{aligned} y &= y' \\ z &= z' \end{aligned}}$$

$$\boxed{t = \frac{\left(t' + \frac{x'v}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

If the speed of the frame is very smaller than speed of light $v \ll c$

$K = 1$

$x' = x - vt$

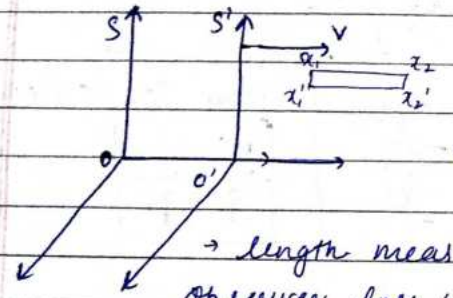
$y' = y$

$z' = z$

$t' = t$

Application of Lorentz transformation eqn

① Length contraction -



rod in S'

- for observer O rod is in motion
- while for observer O' rod is at rest

→ length measured by the observer for which the rod is at rest is called proper length (L_0)

→ length of the rod for observer O

$L = x_2 - x_1$ — ①

for observer O'

$L_0 = x_2' - x_1'$ — ②

from lorentz transformation eqn

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

③

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

putting in ②

$$L_0 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} (x_2 - \cancel{vt} - x_1 + \cancel{vt})$$

$$L_0 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} (x_2 - x_1)$$

$$\boxed{L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{L = L_0 \sqrt{1 - \frac{v^2}{c^2}}} \qquad \boxed{L_0 > L}$$

(a part of L_0 is L)

Case I:	Case II:
$v = c$	$v = 0$
$L = 0$	$L = L_0$
	frame at rest
	length measured by both observer is same. (proper length)

Assignment

Q. Show that the circle $x^2 + y^2 = a^2$ in frame S appears to be an ellipse in frame S' which is moving with velocity v wrt S.

→ $x^2 + y^2 = a^2$

Q. Calculate velocity of rod when its length appears $\frac{3}{4}^{th}$ of its proper length.

→ $L = \frac{3}{4} L_0$.

$$\frac{3L_0}{4} = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{9}{16} = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\frac{9}{16} = \frac{7}{16}$$

$$\cancel{v^2 = \frac{7}{16} \times 9 \times 10^{16}}$$

$$\frac{v}{c} = \frac{\sqrt{7}}{4}$$

$$\boxed{v = \frac{\sqrt{7}}{4} c}$$

Q. Calculate the velocity of rod when its length appear half of its proper length

$$L = \frac{L_0}{2}$$

$$\frac{L_0}{2} = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{4} = \frac{3}{4}$$

$$\frac{v}{c} = \frac{\sqrt{3}}{2}$$

$$\boxed{v = \frac{\sqrt{3}}{2} c} \quad \text{or } 0.86c$$

Q. If a rod travels with a speed 0.8c along its length calculate the % of contraction.

Q. Calculate the length contraction & orientation of a rod of length 5 m in a FOR which is moving with velocity 0.6c in a direction making an angle 30° with the rod.

Ans. (1) $v = 0.8c$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{L}{L_0} = \sqrt{1-\frac{0.64c^2}{c^2}} = \sqrt{0.36}$$

$$L = 0.6 L_0$$

% of contraction

$$\frac{\Delta L}{L} \times 100$$

$$\Rightarrow \frac{L_0 - L}{L_0} \times 100 \qquad \Rightarrow \frac{L_0 - 0.6L_0}{L_0} \times 100$$

$$\frac{0.4L_0}{L_0} \times 100 = 40\%$$

(2) $L_0 = 5 \quad v = 0.6c \quad \theta = 30°$

$$L_x = \frac{L_0 \cos 30°}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5\cos 30°}{\sqrt{1-\frac{0.36c^2}{c^2}}} = \frac{5\cos 30°}{\sqrt{0.64}}$$

$$= \frac{5 \times \sqrt{3}}{2 \times 0.8} = 5.31$$

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$$L_y = \frac{L_0 \sin 30^\circ}{0.8} = \frac{5 \times \frac{1}{2}}{0.8} = \frac{25}{8}$$

$$= 3.12$$

$$L = \sqrt{L_x^2 + L_y^2}$$

$$= \sqrt{(5.3)^2 + (3.12)^2}$$

$$= \sqrt{(28.19) + (9.73)}$$

$$= \sqrt{37.92} = \underline{6.15\ m}$$

(2) $L_0 = 5\ m \quad v = 0.6c \quad \theta = 30^\circ$

$$L_x = L_0 \cos\theta = 5 \cos 30 \sqrt{1 - \frac{v^2}{c^2}} = 5 \times \frac{\sqrt{3}}{2} \sqrt{1 - \frac{v^2}{c^2}}$$

$$5 \times \frac{\sqrt{3}}{2} \times 0.4 = 2\sqrt{3}$$

$$L_y = L_0 \sin\theta \sqrt{1 - \frac{v^2}{c^2}} = 5 \sin 30^\circ \sqrt{1 - \frac{v^2}{c^2}} = \frac{5}{2} \times 0.4$$

$$= 2$$

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{12 + 4}$$

$$L = \sqrt{16} = 4m$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = 60^\circ}$$

$$\theta = 30^\circ \qquad \tan\theta = 0.72$$

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(2) Variation of mass with velocity.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad m_0 \rightarrow \text{rest mass}$$

$$v << c$$

$$\Rightarrow m = m_0$$

Q. Find the speed that a particle must have so that its mass becomes double the rest mass

$$m = 2m_0$$

$$2m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$4\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$4 - \frac{4v^2}{c^2} = 1$$

$$\frac{4v^2}{c^2} = 3$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\boxed{v = \frac{\sqrt{3}}{2} c = 0.86c}$$

Q find the speed that a particle must have so that its mass become 5 times the rest mass.

$\rightarrow$ $m = 5m_0$

$$5m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \left(1 - \frac{v^2}{c^2} = \frac{1}{25}\right)$$

$$\frac{v^2}{c^2} = \frac{24}{25}$$

$$\boxed{v = \frac{\sqrt{24}}{5} c}$$

Q. Find the velocity that an e^- must be given so that its momentum is 10 times its rest mass times the speed of light.

→ $mv = 10 m_0 c$ What is energy at this speed.

$$m = \frac{10 m_0 c}{v}$$

$$\frac{10 m_0 c}{v} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{100 c^2}{v^2} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)}$$

$$\frac{100 c^2}{v^2} - \cancel{\frac{100 c^2}{v^2} \times \frac{v^2}{c^2}} = 1$$

$$100\left(\frac{c^2}{v^2} - 1\right) = 1$$

$$\frac{c^2}{v^2} = \frac{1}{100} + 1$$

$$\frac{v^2}{c^2} = \frac{100}{101}$$

$$v = \frac{10}{\sqrt{101}} c$$

$$E = mc^2$$

$$= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \times c^2$$

$$m_0 = 9.1 \times 10^{-31}$$

$$\frac{9.1 \times 10^{-31}}{0.09} \times 9 \times 10^{16}$$

$$\frac{}{9.1 \times 10^{-13}} \quad 8.2 \times 10^{-13}$$

Find the mass & velocity of an e^- having total energy 2 MeV

$$2 \text{ MeV} = 2 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}.$$

$$E = 3.2 \times 10^{-13} \text{ J}.$$

$$E = mc^2$$

$$m = \frac{3.2 \times 10^{-13}}{9 \times 10^{16}}$$

$$m = 0.35 \times 10^{-29}$$

$$m = 3.5 \times 10^{-30} \text{ kg}$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$(3.5 \times 10^{-30})^2 \left(1-\frac{v^2}{c^2}\right) = 9.1 \times 10^{-31}$$

galat hai

$$1-\frac{v^2}{c^2} = \frac{9.1 \times 10^{-31}}{11.2 \times 10^{-60}}$$

$$1-\frac{v^2}{c^2} = 0.8125 \times 10^{29} \times 9 \times 10^{-31}$$

$$\frac{v^2}{c^2} = 7.39 \times 10^{-2} - 1$$

$$\frac{v^2}{c^2} = 6.39 \times 10^{-2}$$

$$v^2 = 6.39 \times 10^{-2} \times 9 \times 10^{16}$$

$$v^2 = 57.51 \times 10^{14}$$

$$v = 7.5 \times 10^7 \text{ m/s}.$$

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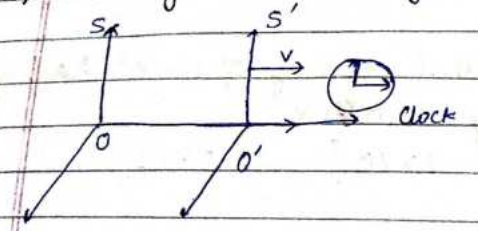
★ Time Dylation - (moving clock appears to go slow)



for observer O clock is in motion & for O' clock is at rest.

→ Time measured by the observer for which the clock is at rest is c/d. proper time

→ Time interval between two 'ticks' as measured by observer O.

$$t = t_2 - t_1 \quad -(1)$$

Similarly,

→ Time measured by observer O' b/w two ticks proper time ← $t_0 = t_2' - t_1'$ -(2)

from inverse lorentz equation.

$$t = \frac{t' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t_1 = \frac{t_1' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2 = \frac{t_2' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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$$t_2 - t_1 = \frac{\cancel{\frac{x'v}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} \left[t_2' - t_1'\right]$$

$$\boxed{t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{t_0 = t\sqrt{1 - \frac{v^2}{c^2}}} \qquad t > t_0$$

★→ Time Dylation is a real effect or experimetal verification of time dylation.

Cosmic rays + air molecule ⟶ μ mesons.
$v_{fastest} = 0.998c$

μ mesons have very short life 2×10^{-6} sec. or 2 μsec.

Distance covered by μ mesons in the time

$$d = vt$$
$$= 0.998(3 \times 10^8)(2 \times 10^{-6})$$
$$\Rightarrow 5.988 \times 10^2 \, m$$
$$598 m \approx 600 m.$$

→ Now the question exist if μ meson decays after travelling a distance of 600m then how they reach on the surface of the earth (6 km – 20 km)

→ The answer to this problem is time dilation.

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t = \frac{2\times10^{-6}}{\sqrt{1-0.998}}$$

$$\frac{2\times10^{-6}}{0.02\times10^{-2}}$$

$$t = 3.1\times10^{-5}\ sec$$

$$\cancel{3.1\times10^{-4}}$$

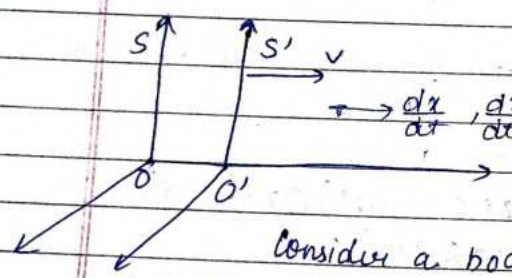
Hence, distance travelled by μ mesons in this time

$$(0.998\times3\times10^{8})(3.17\times10^{-5}).$$

$$\Rightarrow 9.49\times10^{3}\ m.$$

$$\boxed{d \Rightarrow 10\ Kms.}$$

<u>Addition & transformation of velocity</u>



frame S is at rest & S' is in motion with constant velocity v.

$\rightarrow \frac{dx}{dt}, \frac{dx'}{dt'}$

Consider a body moves certain distance at a certain time

- Velocity of body for observer O $\Rightarrow \frac{dx}{dt} = u$

$$O' \Rightarrow \frac{dx'}{dt'} = u'$$

from inverse lorentz eq^n

$$x = K(x'+vt')$$

$$t = K(t' + x'v/c^2)$$

diff wrt dt'

$$\cancel{\frac{dx}{dt'} = K\frac{dx'}{dt'} + v = K(u'+v)}$$

$$\cancel{dx = K(u'+v)dt' = Kdt'(u'+v)}$$

$$\cancel{\frac{dt'}{dt'}} \qquad dx = K(dx'+vdt')$$

$$dt = K(dt' + \frac{v}{c^2}dx')$$

dividing these 2 eq^n

$$\frac{dx}{dt} = \frac{dx'+vdt'}{dt'+\frac{v}{c^2}dx'}$$

$$= \frac{\frac{dx'}{dt'}+v}{1+\frac{v}{c^2}\frac{dx'}{dt'}}$$

$$u = \frac{u'+v}{1+\frac{vu'}{c^2}}$$

$$\boxed{u = \frac{u'+v}{1+\frac{vu'}{c^2}}}$$

special case :-

<u>case 1</u> - if we consider a photon moving with speed c in frame S' & the frame S' is moving with speed v.

→ $u' = c \qquad v = v.$

$$u = \frac{c+v}{1+\frac{v}{c}} = \frac{\cancel{c+v}}{\frac{\cancel{c+v}}{c}} = c$$

$$\boxed{u = c}$$

If we add any velocity to the velocity of light the result will be velocity of light.

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case II consider a photon moving with speed c in frame S' ~~to~~ frame S' is moving with speed c

$$u = \frac{c+c}{1+\frac{c^2}{c^2}} = \frac{2c}{2} = c$$

$\boxed{u=c}$

If we add velocity of light with velocity of light the result is velocity of light

⇒ Velocity of light is maximum attainable velocity.

Einstien mass energy relation

a particle of mass m of velocity v force f is appered to the particle.

If force displaces the particle through a small distance dx, then work done is stored in the form of

$$dW = dE_K = F \cdot dx.$$

$$F = \frac{dp}{dt}$$

$$= \frac{d}{dt}(mv)$$

$$dW = \frac{d}{dt}(mv)dx$$

$$= \frac{dx}{dt} d(mv)$$

$$= v\left[m dv + v.dm\right]$$

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$$\boxed{dE_K = mv dv + v^2 dm} \quad -(1)$$

we already have

Mass - velocity relation $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\cancel{m_0^2 c^2} =$$

$$m^2 = \frac{m_0^2}{1-\frac{v^2}{c^2}}$$

$$m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

differentiating this equation

$$0 = 2m dm c^2 - (2m dm v^2 - 2v dv m^2)$$

$$2m\left[c^2 dm - v^2 dm - m v dv\right]$$

$$0 \frac{1}{2m} = c^2 dm - v^2 dm - mv dv$$

$$c^2 dm = \frac{1}{dm} + v^2 dm + mv dv$$

putting in (1)

$$\boxed{dE_K = c^2 dm}$$

$$E_K = \int dE_K = \int_{m_0}^{m} c^2 dm = c^2(m-m_0)$$

$$\boxed{E_K = c^2(m-m_0)}$$ Total kinetic energy.

$$E_K = mc^2 - m_0c^2$$

rest mass energy

→ This is relativistic ~~rate~~ relation for kinetic energy of particle.

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Total energy is the sum of rest mass energy and kinetic energy.

E = rest mass energy + K.E.

$m_0c^2 + mc^2 - m_0c^2$

$$\boxed{E = mc^2}$$

Relativistic relation between Energy & momentum $\left(E = \frac{p^2}{2m}\right)$

Einstein mass energy relation

$E = mc^2$

$$E = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}c^2 \qquad P = mv, \quad v = P/m.$$

$$E = \frac{m_0}{\sqrt{1-\frac{p^2}{m^2c^2}}}c^2$$

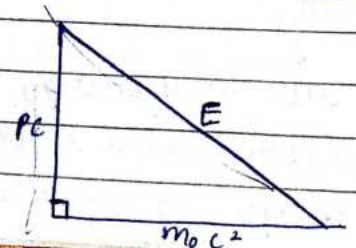
→ multiply & divide by c^2.

$$E = \frac{m_0c^2}{\sqrt{1-\frac{p^2c^2}{m^2c^4}}} = \frac{m_0c^2}{\sqrt{1-\frac{p^2c^2}{E^2}}} = \frac{m_0c^2}{\frac{\sqrt{E^2-p^2c^2}}{E}}$$

$$\cancel{E^2 = \frac{m_0c^2}{\sqrt{E^2-p^2c^2}}} \qquad 1 = \frac{m_0c^2}{\sqrt{E^2-p^2c^2}}$$

$$\cancel{E^4 = \frac{m_0^2c^4}{E^2-p^2c^2}} \qquad E^2 - p^2c^2 = m_0^2c^4$$

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$



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Q. A rocket ship is 50 m long when it is on flight its length appears to be 49.5m to an observer on ground. Find speed of rocket.

$$L_0 = L_0\sqrt{1-\frac{v^2}{c^2}} \qquad L_0 = 50m, \quad L = 49.5m$$

$$\frac{49.5}{50} = \sqrt{1-\frac{v^2}{c^2}}$$

$$(0.99)^2 = 1-\frac{v^2}{c^2}$$

$$1 - 0.00891 = \frac{v^2}{c^2}$$

$$0.99c^2 = v^2$$

$$v = 0.094c$$

$v = 0.141c$

$v =$

Q. An e^- has an initial speed of 1.4×10^8 m/s how much additional energy must be imported to it for its speed to double.

→ $u = 1.4\times10^8$

$E = mc^2 \qquad v = \frac{u'+v}{1+\frac{vu'}{c^2}}$

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{9.1\times10^{-31}\times 9\times10^{16}}{\sqrt{1-\left(\frac{1.4\times10^8}{3\times10^8}\right)}}$$

$$\frac{9.1\times9}{0.88}\times10^{-15}$$

$\Rightarrow 93.06\times10^{-15}$ J

$\rightarrow 9.3\times10^{-14}$ J

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$$E' = mc^2 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \times c^2$$

$$\Rightarrow \frac{= 9.1\times10^{-31} \times 9\times10^{16}}{\sqrt{1-0.86}}$$

$$= \frac{9.1\times9 \times 10^{-15}}{0.37}$$

$$221.35 \times 10^{-15}$$

$$E' = 2.21\times10^{-13}\ J$$

$\Delta E \rightarrow$ additional energy supplied.

Q A clock measures the proper time with ~~with~~ what velocity it should travel relative so that it appears to go slow 30 second in a day

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$t = 86400.$

$t = 86400 - 30$

$t_0 = 86370$

~~86400~~

$86400 = 86370$

$$\sqrt{1-\frac{v^2}{c^2}} = 0.99$$

$$1-\frac{v^2}{c^2} = 0.98$$

$$\frac{v^2}{c^2} = 0.02$$

$v = 0.02c$

$$v^2 = 0.02c^2$$

$$v = 0.14c$$

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Q A clock keeps correct time, with what speed should it be moved relative to an observer so that it may appear to loose 4 minutes in 24 hrs.

$\rightarrow$ $24\times60 = 1440$ minutes. $= t$

$t_0 = 1440 - 4 = 1436$ minutes $= t_0$.

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1436}{1440} \qquad v = 0.07c$$

$$\sqrt{1-\frac{v^2}{c^2}} = 0.99$$

$t > t_0$
$L_0 > L$
$m > m_0$

$t = 1440$ minutes.

$t_0 = 1440 - 4$
$= 1436$

$6\times3600\times24$

$\frac{86400}{3600}$ 24

$$\frac{1440}{1436} = \sqrt{1-\frac{v^2}{c^2}}$$

$$= 1-v^2$$

$$(1.002)^2 = 1-\frac{v^2}{c^2}$$

$$1.004 = 1-\frac{v^2}{c^2}$$

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1436}{1440}$$

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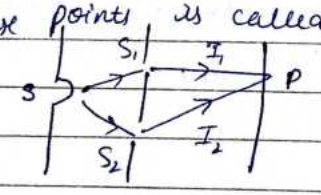
Unit-3

29/11 Interference

The modification in intensity of light due to two waves of same frequency and constant phase difference (coherent) b/w them in the region of superposition is called interference.

constructive interference at certain point wave superimpose in such a way that resulting intensity is more than the sum of intensity due to individual wave. Interference at these points is called constructive interference.



- at certain point wave superimpose in such a way that resulting intensity is less than the sum of intensities due to individual wave. Interference at these points is called destructive interference.

path difference : $\Delta = S_2P - S_1P$

phase diff: $\frac{2\pi}{\lambda} \times$ path diff. $= \frac{2\pi}{\lambda}(S_2P - S_1P)$

Principle of Superposition – It states that resultant displacement of a particle acted upon by two or more waves simultaneously is algebric sum of displacement due to individual waves.

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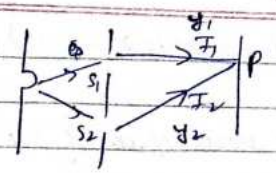
Resultant



$y = y_1 + y_2$.

phase diff.

$y_1 = a_1 \sin\omega t$ $\quad y_2 = a_2 \sin(\omega t + \delta)$

$y = y_1 + y_2$

$y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$

$y = a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$

$y = \sin\omega t (a_1 + a_2 \cos\delta) + a_2 \cos\omega t \sin\delta$ – ①

Put $a_1 + a_2 \cos\delta = A\cos\phi$ – (a)

$a_2 \sin\delta = A\sin\phi$ – (b)

$y = A\sin\omega t \cos\phi + A\sin\phi \cos\omega t$

$\boxed{y = A\sin(\omega t + \phi)}$

Signification :- Resultant wave is also simple harmonic

Squaring and adding (a) & (b)

$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$\Rightarrow I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

dividing (b) by (a)

$\tan\phi = \frac{a_2 \sin\delta}{a_1 + a_2\cos\delta}$

⇒ (Condition for maximum) :-

I max when

$\delta = 0, 2\pi, 4\pi, 6\pi \ldots$

(even multiple of π) phase diff ⇒ $2n\pi$ $(n = 0, 1, 2, \ldots)$

path diff ⇒ $\frac{\lambda}{2\pi} \times$ phase diff.

$\frac{\lambda}{2\pi} \times 2n\pi =$

$I_{max} = (a_1 + a_2)^2$

$\frac{2n\pi}{2}$ $(n = 0, 1, 2 \ldots)$

even multiple of half wavelength.

Condition for minima.

$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$\cos\delta = -1$ $\qquad I_{min} = (a_1 - a_2)^2$

$\delta = \pi, 3\pi, 5\pi \ldots$

phase diff: $\delta = (2n+1)\pi$ $\{n = 0, 1, 2 \ldots\}$

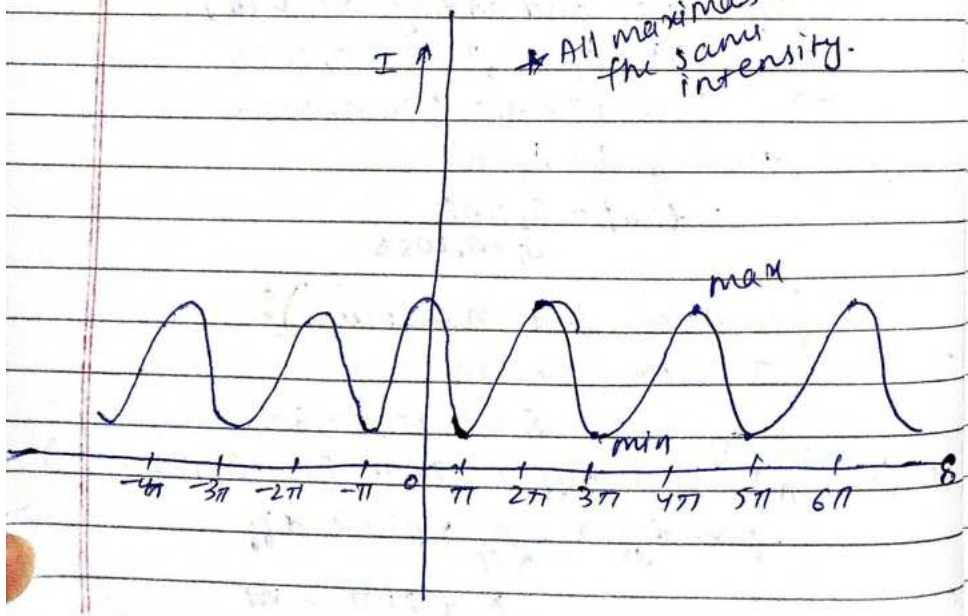
odd multiple of π.

path diff: $\frac{\lambda}{2\pi} \times (2n+1)\pi$

$(2n+1)\frac{\lambda}{2}$ $\{n = 0, 1, 2 \ldots\}$

odd multiple of half wavelength.

Intensity distribution curve :-



special case : when the amplitude of two waves are equal.

$a_1 = a_2 = a$

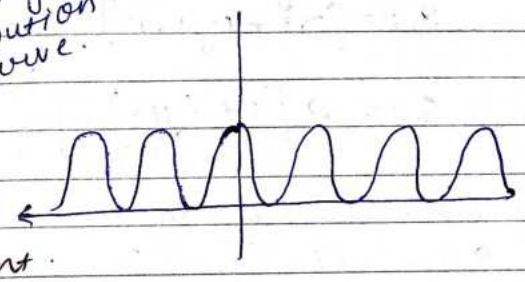
$I_{max} = \cancel{2a_1^2 + 2a^2\cos\delta}$ $(a+a)^2$

$\cancel{2a}$ $\Rightarrow 4a^2$

$I_{min} = (a-a)^2 = 0$

(perfectly dark).

intensity distribution curve.

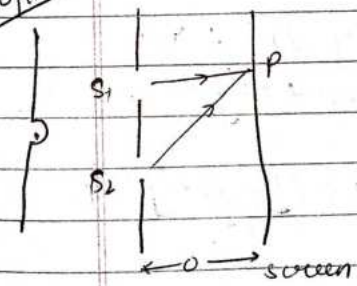


Assignment

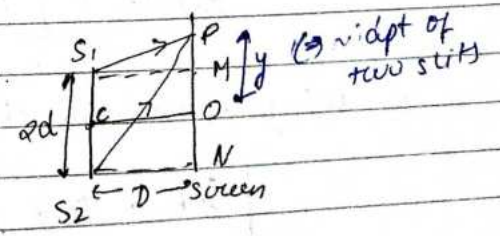
Q. Calculate average intensity hence show that Energy is conserved during interference.

Q. Explain YDSE (theory). YDSE → yourself.

Solⁿ



Analytical treatment for YDSE



~~O → point.~~

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- Light rays reaching from S_1 & S_2 at point O will have zero path diff.
- O is a point of ~~central~~ maxima.

$PM = y - d \qquad PN = y + d.$

ΔS_2PN

$$PN^2 + D^2 = S_2P^2$$

$$S_2P^2 = y^2 + d^2 + 2yd + D^2$$

$$S_2P = \sqrt{y^2 + d^2 + 2yd + D^2}$$

$$S_2P = [D^2 + (y+d)^2]^{1/2}.$$

$$S_2P = D\left[1 + \frac{(y+d)^2}{D^2}\right]^{1/2}$$

Using binomial theorem.

$$S_2P = D\left[1 + \frac{(y+d)^2}{2D^2}\right].$$

$$\boxed{S_2P = D + \frac{(y+d)^2}{2D}.} \quad \text{— (1)}$$

ΔS_1PM

$$S_1P^2 = (y-d)^2 + D^2.$$

$$S_1P = [(y-d)^2 + D^2]^{1/2}.$$

$$S_1P = D\left[1 + \frac{(y-d)^2}{D^2}\right]^{1/2}.$$

using binomial theorem.

$$\boxed{S_1P = D + \frac{(y-d)^2}{2D}} \quad \text{— (2)}$$

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Path difference at P

$$\Delta = S_2P - S_1P$$

$$\cancel{D} + \frac{(y+d)^2}{2D} - \cancel{D} - \frac{(y-d)^2}{2D}$$

$$\frac{1}{2D}[(y+d)^2 - (y-d)^2]$$

$$= \frac{1}{2D}[\cancel{y^2} + \cancel{d^2} + 2yd - \cancel{y^2} - \cancel{d^2} + 2yd]$$

$$= \frac{4yd}{2D} = \boxed{\frac{2yd}{D}.}$$

$$\boxed{\delta = \frac{2\pi}{\lambda}\left(\frac{2yd}{D}\right)}$$

for maxima:

$$\frac{2yd}{D} = \frac{2n\lambda}{2} \qquad (n = 0, 1, 2)$$

$$y = \frac{n\lambda D}{2d}$$

distance of nth bright fringe ~~of~~ from O

for minima: (dark fringe)

$$\frac{2yd}{D} = (2n+1)\frac{\lambda}{2} \qquad (n = 0, 1, 2 \ldots)$$

$$y = \frac{(2n+1)}{2d}\frac{\lambda D}{2} = \frac{(2n+1)\lambda D}{4D}.$$

distance of nth dark fringe from O.

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Fringe width - distance b/w two consecutive dark fringe or bright fringe. is k/w fringe width 'β'

for bright fringe -

$y_n = \frac{n\lambda D}{2d}$ $\quad y_{n+1} = \frac{(n+1)\lambda D}{2d}$.

$(n=0,1,2)$.

$\beta = y_{n+1} - y_n$

$\frac{\lambda D}{2d}[\cancel{n}+1-\cancel{n}]$

$$\boxed{\beta = \frac{\lambda D}{2d}}$$

$2d \rightarrow$ distance b/w slits.

for good interference pattern $\beta \uparrow$.

for dark fringe -

$y_n = \frac{(2n+1)\lambda D}{4d}$ $\quad y_{n+1} = \frac{(2n+3)\lambda D}{4d}$.

$\beta = \frac{\lambda D}{4d}(\cancel{2n}+3-\cancel{2n}-1)$

$= \frac{2\lambda D}{4d} =$

$$\boxed{\beta = \frac{\lambda D}{2d}}$$

* Condition necessary for sustained (permanent) interference -

- Two sources must be coherent. (They can derived from same source, constant phase diff.)

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- Amplitude of two waves must be same.
- Fringe width should be as large as possible.

→ Distance between source and screen (D) ~~is as~~ Should be as large as possible

→ Distance b/w two slits (2d) should be as small as possible.

- Two waves must be propagating along the same direction.
- Both sources should be narrow.

Coherent Source :-

→ Two sources are said to be coherent if they have same frequency and constant phase difference between them.

→

Two independent sources can never be ~~coherent, why~~?

Sustained interference can never be obtained with two independent sources of light, such as two bulb or two candles. It is due to the fact that any two independent beam of light are always incoherent

Every source of light is made up of large no. of atoms which are responsible for producing waves. Each atom consist of electrons moving in certain definite orbits around +vely charged nucleus. Energy of the e⁻ depend on its orbit. It is maximum in the outermost orbit and minimum in innermost orbit when an atom get

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sufficient energy by any mean, its e⁻ jumps from inner orbit to outer orbit. This state of atom is called excited state and lasts only for about 10^{-8} s. After this interval of time the e⁻ falls back to the inner orbit. During this process the atom radiates light. Out of large no. of atoms, a no. of them emits light at an instant. At the next instant other atoms do so and so on. As all atoms emit light pulses randomely independent of each other, the initial phase of light wave from a source varies with time in a random way.

Hence if we have two independent light sources, the phase of wave emitted from them will change randomely independent of each other.

Thus independent sources cannot maintain a constant phase difference between them & hence ~~coherent~~ they can't be coherent.

5/12 Classification of interference. modification in intensity.

1) Division of amplitude -

2) Division of wavefront.

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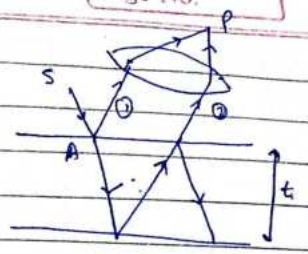
1) Division of amplitude -

Two virtual sources are obtained from the same source by partial reflection & transmission.



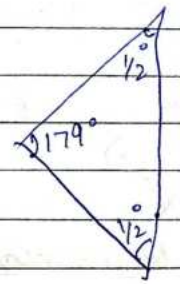
eg - Newton's Ring & thin films.

2) Division of wavefront -

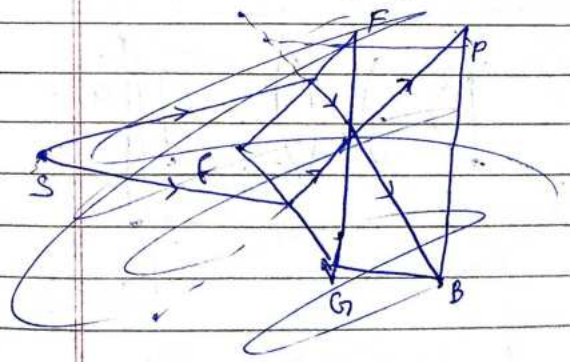
If we introduce lens, mirror or glass in the path of the light ray then 2 virtual sources will be formed.

eg - YDSE and fresnel's biprism.

Fresnel's Biprism



Interference by fresnel's biprism.



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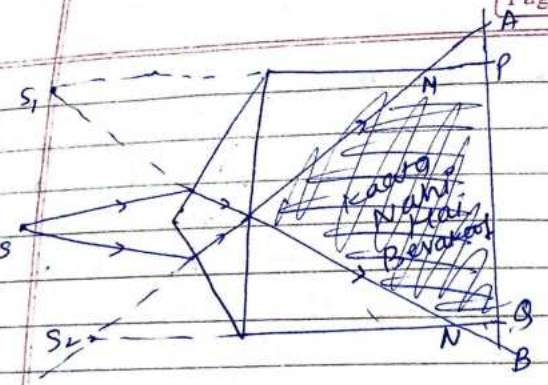


Condition for maxima :-

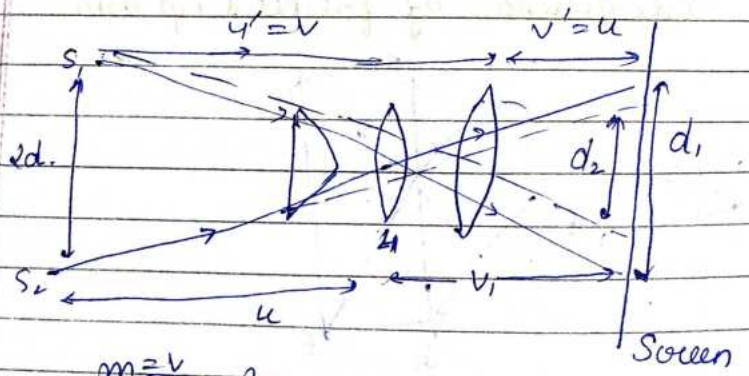
$$y = \frac{nD\lambda}{2d} \qquad n = 0, 1, 2$$

minima $\quad y = \frac{(2n-1)D\lambda}{4d} \qquad n = 1, 2, 3$

fringe width $\quad \beta = \frac{D\lambda}{2d}$.

(Division of wavefront)

Measurement of 2d by displacement method-



$$m = \frac{v}{u}, \quad m = \frac{d_1}{2d} \Rightarrow \frac{v}{u} = \frac{d_1}{2d} \quad (1)$$

$$m' = \frac{v'}{u'} = \frac{u}{v} = \frac{d_2}{2d} \quad (2)$$

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multiplying (1) × (2).

$$\frac{v}{u} \times \frac{u}{v} = \frac{d_1 d_2}{2d^2}$$

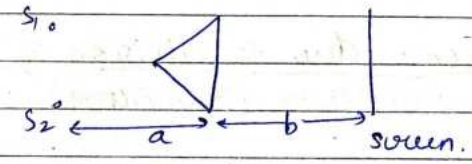
$$2d^2 = d_1 d_2$$

$$d = \sqrt{\frac{d_1 d_2}{2}}$$

$$\boxed{2d = \sqrt{d_1 d_2}}$$

- Deviation method.

$$\boxed{2d = 2a(\mu - 1)\alpha}$$



$D = a + b$.

$\alpha = \frac{1}{2}$ (small angle)

Q- Interference fringes are obtained with a glass biprism of base angle 2° each. ($\mu = 1.5$) If the distance b/w slit & biprism is 10cm. Calculate the fringe width obtained on the screen at a distance of 90cm from biprism.

① $\lambda = 5890$ Å

$\lambda =$

$\alpha = 2° = 2 \times \frac{\pi}{180} = 0.34$ rad.

$\beta = \frac{D\lambda}{2d}$.

2d

$D = 10 + 90 = 100$ cm $= 1$ m.

$$\beta = \frac{1 \times 5890 \times 10^{-10}}{2 \times 0.1 (0.5) 0.34}$$

$$\frac{5890 \times 10^{-10}}{34 \times 10^{-3}}$$

$$\Rightarrow 173.2 \times 10^{-7}$$

1.6×10^{-6}

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In a biprism exp with sodium lamp the eye piece is placed at a distance of 100cm from the source. If the distance b/w 2 virtual source is 2cm. Find β.

→ $D = 100cm$ ~~= 100~~ $\lambda = 5890 \mathring{A}$

$2d = 2$

$$\beta = \frac{5890 \times 10^{-8} \times 10^{2}}{2}$$

$$= 2.995 \times 10^{-6}$$

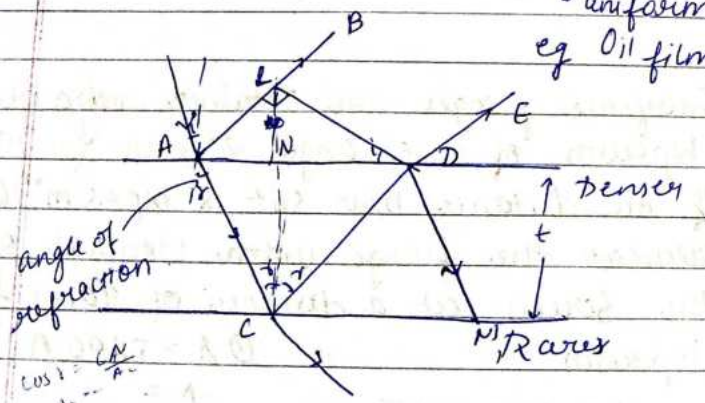
$$\Rightarrow 2.995 \times 10^{-3} \text{ cm}.$$

Interference due to division of Amplitude

(i) due to reflected light (Thin film)

↳ uniform thickness

eg Oil film on H_2O



$\cos r = \frac{CN}{AC}$

Path diff b/w AB & DE

Δ = Path (ACD) in film − Path AL in air

$\mu(AC+CD)$ − $\mu(AL)$

in film in air

$$\Delta = \mu(AC+CD) - AL \quad -(1)$$

from ΔACN $\quad AC = \frac{CN}{\cos r} = \frac{t}{\cos r}$

$CD = \frac{t}{\cos r}$ $\quad AL = AN \sin i$

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$(AN+ND)\sin i$

$AN = CN \tan r$

$AL = (t\tan r + t\tan r)\sin i$

$= 2d \tan r \sin i$

from Snell's law $\frac{\sin i}{\sin r} = \mu$

$2t \tan r \sin r \mu$

$= 2\mu t \frac{\sin^2 r}{\cos r}$

Put eq in (1)

$$\Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$\frac{\mu 2t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = 2\mu t \cos r \qquad \boxed{\mu = 2\mu t \cos r}$$

Path diff.

Stock's treatment - when light is reflected from denser to rarer medium a phase difference of π or a path diff of $\frac{\lambda}{2}$ takes place.

phase change of $x \rightarrow A$

$O \rightarrow C$

$O \rightarrow D$ $\quad DM \rightarrow$ reflected ray

Total phase change $= \pi$

$$\boxed{\Delta_{ref} = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

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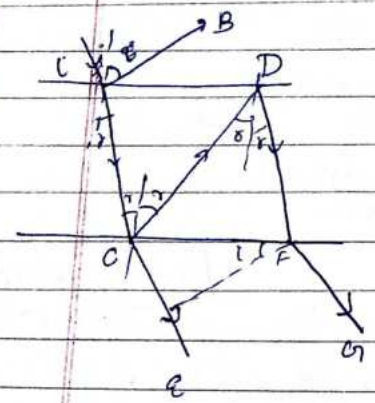
Condition for Maxima → $2\mu t\cos r \pm \frac{\lambda}{2} = \frac{2n\lambda}{2}$ $n = 0, 1, 2$

$$\boxed{2\mu t\cos r = (2n \mp 1)\frac{\lambda}{2}}$$

for minima – $\boxed{2\mu t\cos r = 2n\frac{\lambda}{2}}$
even multiple of half wavelength.

Interference in Thin films in Transmitted System



Δ b/w CE & FG → wherever Same source originate → by partial reflected & transmitted.

Δ b/w CE & FG | Δ-path diff
Δ = Path (CDF) in film
– Path (CL) air.

$\mu(CD + DF) - CL$ } solve as like previous one.

$$\boxed{\Delta = 2\mu t\cos r}$$

We will not consider reflection as we are not studying AB $\Delta = 0$
Effective path diff (Δ_{eff}) = $\boxed{2\mu t\cos r}$

for maxima $\boxed{2\mu t\cos r = 2n\lambda/2}$

for minima $\boxed{2\mu t\cos r = (2n-1)\frac{\lambda}{2}}$

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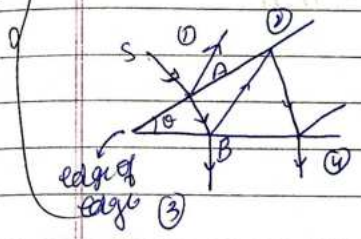
Conditions are complementary, reflected.

colours in Thin film [B blue, G green, O orange
V, Y, R]

when white source. satisfies maxima condn.

★ Phase always change in reflected system.

not in syllabus / Wedge Shaped film –



A film of non uniform thickness θ = angle of edge.

$$\boxed{\Delta = 2\mu t\cos(r+\theta)}$$

$$\boxed{\Delta = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}}$$

Q. A parallel beam of light $\lambda = 5890\,Å$ is incident on a thin glass plate ($\mu = 1.5$) such that angle of refraction is 60°. Calculate the smallest thickness of the glass plate which will appear dark by reflection.

$2\mu t\cos r = n\lambda$

$2 \times 1.5 \times t \times \frac{1}{2} = 5890 \times 10^{-10}$

$t = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times 0.5} \times 100 \times 10^{-2}$

$$\boxed{t = 3.9 \times 10^{-7}\,m}$$

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VV. Imp

A parallel beam of sodium light strikes on film of oil $\mu = 1.46$ when viewed at an angle of 30° from normal. 8th dark band in sum - find thickness of film.

$2\mu t\cos r = n\lambda$ $\qquad i = 30°$

$2\times1.46\times\cos r\times t = n\lambda$ $\qquad \mu = \frac{\sin i}{\sin r}$

$t = \frac{8\times5890\times10^{-10}\times2\times100}{2\times1.46\times\sqrt{3}}$

$= 1.8\,\mu m$

$\cos r = \sqrt{1-\sin^2 r}$

$\cos r = \sqrt{1-\frac{1}{4}}$

$\cos r = \frac{\sqrt{3}}{2}$

Q. White light falls normally on a film of soapy water with thickness 5×10^{-5} cm and $\mu = 1.33$ which λ in visible region will be reflected more strongly.

$2\mu t = n\lambda$ $\qquad 1\,\mu m = 10^{8}$ Å

$2\times1.33\times t = n\times3\times10$

$t = \frac{1\times3\times10^{-5}\times10^{8}}{2\times1.35}$

$t = \frac{3}{2}\times\frac{10^{3}}{133}\times100$

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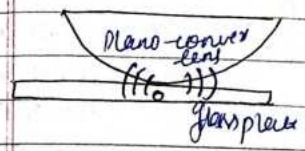
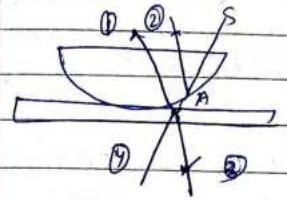
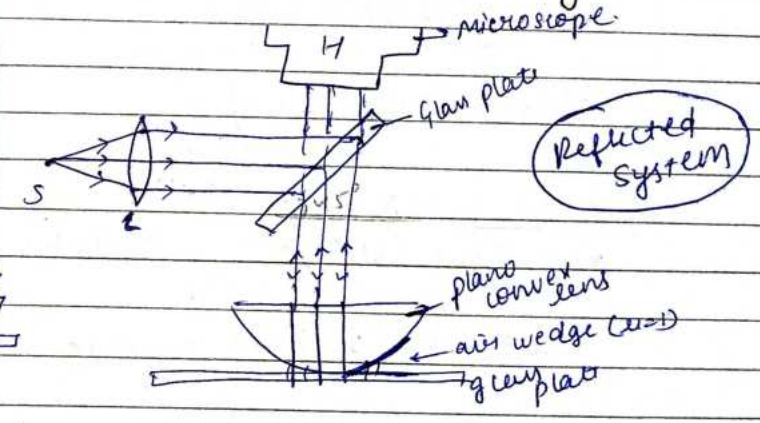
Newton's Ring

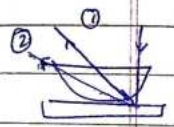


O → centre → point of contact of plano-convex lens & glass plate.



(5) ~~experimental~~ Experimental arrangement.





Path diff. b/w the ray ① & ②

$\Delta = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}$

For normal incidence and smaller angle of wedge.

$i = 0$, $r = 0$

θ is small

$\cos\theta = 1$

Path diff.

$\boxed{\Delta = 2\mu t \pm \frac{\lambda}{2}}$

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At the point O, $t=0$

$\boxed{\Delta = \pm\frac{\lambda}{2}}$ → This is the condition for minimum intesity. Hence centre of Newton's ring is dark.

max $\Delta = \frac{2n\lambda}{2}$ n, 0, 1

min $\Delta = \frac{(2n-1)\lambda}{2}$ n = 1, 2

⇒ Thats why the centre of newton's ring is dark in reflecting system

Condition for maxima.-

$2\mu t \pm \frac{\lambda}{2} = \frac{2n\lambda}{2}$

$$\boxed{2\mu t = (2n \mp 1)\frac{\lambda}{2}}$$

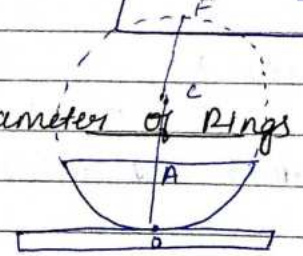
condition for minima

$2\mu t \pm \frac{\lambda}{2} = \frac{(2n\pm1)\lambda}{2}$ ~~n = 1, 2~~

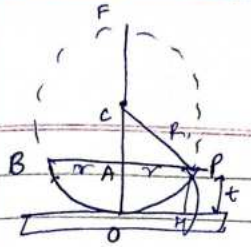
$2\mu t = \frac{(2n \pm 1 \mp 1)\lambda}{2} = \frac{2n\lambda}{2}$

$$\boxed{2\mu t = \frac{2n\lambda}{2}}$$

(b) Diameter of Rings



OC = R
OF = 2R
CP = R
PH = t
OA = t
OH = r
AP = r



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from the property of a circle.

$AB \times AP = OA \times AF$

$r \times r = t \times (2R - t)$

$r^2 = t(2R - t)$

$r^2 = 2Rt - t^2$.

neglecting t^2 (very small)

$r^2 = 2Rt$.

$$\boxed{t = \frac{r^2}{2R}}$$

for bright rings-

$2\mu t = (2n \pm 1)\frac{\lambda}{2}$.

$2\mu\left(\frac{r^2}{2R}\right) = (2n\pm1)\frac{\lambda}{2}$

$2 \times 2\mu r^2 = 2 \times (2n \pm 1)\frac{\lambda}{2} \times R$

$r^2 = \frac{1}{2\mu} R(2n \pm 1)\lambda$

$\boxed{D = 2r}$

$$\boxed{D^2 = \frac{2(2n\pm1)\lambda R}{\mu}}$$

$D^2 =$

$D^2 \propto (2n \pm 1)$

$$\boxed{D \propto \sqrt{2n \pm 1}}$$

for air wedge.

$$\boxed{D^2 = 2(2n\pm1)\lambda R}$$

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dark fringe –

$$2\mu t = n\lambda$$

$$2\mu\left(\frac{r^2}{2R}\right) = n\lambda$$

$$r^2 = \frac{nR\lambda}{\mu}$$

$$D^2 = \frac{4n\lambda R}{\mu}$$

$$D \propto \sqrt{n}$$

for air wedge.

$$\boxed{D^2 = 4n\lambda R}$$

(c) Determination of wavelength of light.

for dark ring.

Diameter of nth dark ring $\quad D_n^2 = 4n\lambda R$ —(1)

Diameter of $(n+p)^{th}$ dark ring $\quad$ (2) – $D_{(n+p)}^2 = 4(n+p)\lambda R$

(2) – (1)

$$D_{(n+p)}^2 - D_n^2 = 4p\lambda R.$$

$$\boxed{\lambda = \frac{D_{(n+p)}^2 - D_n^2}{4pR}}$$

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(d) Determination of Refractive index of a liquid.

in the absence of liquid.

$$\left[D_{(n+p)}^2 - D_n^2\right]_{air} = 4p\lambda R. \quad -(1)$$

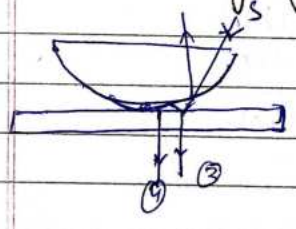
When liquid is applied on the glass plate

$$\left[D_{n+p}^2 - D_n^2\right]_{liquid} = \frac{4p\lambda R}{\mu} \quad -(2)$$

(1) ÷ (2)

$$\boxed{\frac{\left[D_{(n+p)}^2 - D_n^2\right]_{air}}{\left[D_{(n+p)}^2 - D_n^2\right]_{liquid}} = \mu}$$

(e) Newton's Ring by transmitted light. (3)(4)



path diff.

$$\Delta = 2\mu t\cos(r+\theta).$$

For normal incidence and smaller angle of wedge.

$i = 0 \quad r = 0$

θ is small

$\cos\theta = 1$

$$\Delta = 2\mu t$$

for maxima

$$\boxed{2\mu t = \frac{2n\lambda}{2}}$$

for minima

$$\boxed{2\mu t = \frac{(2n-1)\lambda}{2}}$$

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$t = \frac{r^2}{2R}$

$2\mu t = 2n\frac{\lambda}{2}$

$\frac{2\mu r^2}{2R} = \frac{2n\lambda}{2}$

$r^2 = \frac{nR\lambda}{\mu}$

$4r^2 = \frac{4nR\lambda}{\mu}$

$D^2_{bright} = \frac{4nR\lambda}{\mu}$

$2\mu t = (2n-1)\frac{\lambda}{2}$

$\frac{2\mu r^2}{2R} = (2n-1)\frac{\lambda}{2}$

$r^2 = \frac{(2n-1)\lambda R}{2\mu}$

$D^2_{dark} = \frac{2(2n-1)\lambda R}{\mu}$

Numericals

Q. Newton's ring are observed normally in reflected light of wavelength 5900Å. Diameter of 10th dark ring is 0.50 cm. Find radius of curvature of lens & thickness of film.

→ $\lambda = 5900 \times 10^{-10}$ m $D_{10}^2 = 4n\lambda R$.

$40\lambda R$.

$D = 2r$.

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$0.5 \times 10^{-2} = 40 \times 5900 \times 10^{-10} \times R$.

$R = \frac{0.5 \times 10^{-2} \times 0.5 \times 10^{-2}}{40 \times 5900 \times 10^{-10}}$

$\frac{0.25}{236000 \times 10^{-10}} \times 10^{-4}$

$\frac{25}{236} \times \frac{10^{-4}}{10^{-5}}$

0.105×10

$R = 1.06$ m

$D^2 = 4r^2$

$t = \frac{r^2}{2R} = \frac{0.25 \times}{2 \times 106}$

$\frac{0.25}{212}$

0.0011.

$t = \frac{r^2}{2R} = \frac{0.25 \times 0.25}{2 \times 106}$ 0.25, 0.5, 2

$\Rightarrow \frac{0.0625}{212}$

$= 2.9 \times 10^{-4}$ cm

2.9×10^{-6} m

$2.9\ \mu m$

Q. In NR experiment diameter of 5th ring is 0.3cm and d of 25th ring is 0.8cm. R = 100cm. Find λ.

$\lambda = \frac{0.64 - 0.09}{4 \times 20 \times 100}$

$\frac{0.55}{8 \times 10^3}$

$\frac{0.06 \times 10^{-3}}{}$ cm

$\Rightarrow 6 \times 10^{-5}$ cm

6875Å

6875×10^{-8} cm

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Q In NR exp diameter of 4th & 12th dark ring are 0.4cm & 0.7cm. Find the ~~20~~ D of 20th dark ring.

$\cancel{D_4^2 = 0.16 = 16\lambda R}$

$\cancel{D_{12}^2 = 0.49 = 48\lambda R}$

$(D_{n+p}^2 - D_n^2) = 4p\lambda R.$ $\quad p=8.$

$(0.49 - 0.16) = 32\lambda R$ — (1)

$(D_{20}^2 - D_4^2) = 64\lambda R$ — (2)

$0.33 \times 2 = D_{20}^2 - 0.16$

$D_{20}^2 = 0.66 + 0.16$

$D_{20}^2 = 0.82$

$D_{20} = 0.905$ cm

Q diameter of 10th bright ring in NR exp changes from ~~the point 4 to~~ 1.4 to 1.27cm when the liquid in introduced b/w lens & frame. Find μ.

→ $\cancel{(0.13)^2 = 0.0169}$

$\mu = \sqrt{\frac{1.96}{1.6129}}$

$\mu = 1.215$

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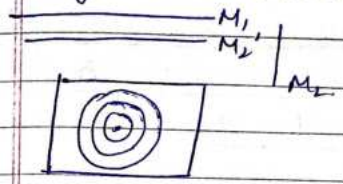
Michelson's interferometer → from relativity: constⁿ & working.

Types of fringes

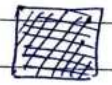
① circular fringe — when M_1 & M_2 are at right angle to each other.



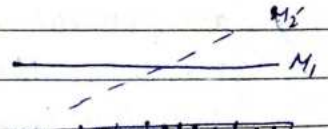
When M_1 & M_2' coincide

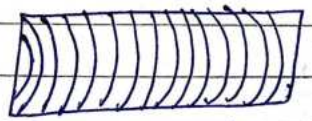
$t = 0$

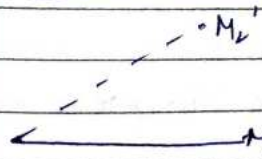
$\Delta = \frac{\lambda}{2}$ dark fringes

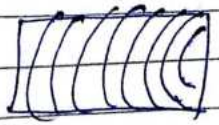


② localised fringes — when M_1 & M_2 are not at right angle to each other. (inclined).









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- Types of fringe in Michelson Interferometer depend upon the inclination of M_1 & M_2. Let M_2' be the image of M_2 formed by the reflection at half silvered surface of plate P so that $OM_2 = OM_2'$. The interference pattern may be regarded due to the light reflected from M_1 & M_2' (real) (virtual). Thus the arrangement is equal to an air film enclosed b/w reflecting surfaces M_1 & M_2'.

Circular fringes - Path difference b/w two beams produced by the reflecting surfaces M_1 & M_2' is equal to twice the thickness of the film M_1M_2'. This path difference can be varied by moving M_1 backward or forward parallel to itself. If we use monochromatic light, system of bright & dark fringes will be produced. Shape of the film depend upon the inclination of M_1 & M_2.

- If M_1 & M_2 are exactly at right angles to each other, the reflecting surfaces M_1 & M_2' are parallel and hence air film b/w M_1 & M_2' is of constant thickness so that we get circular fringes of equal inclination or Haidinger's fringes in the field of view of telescope. When distance b/w M_1 & M_2 or b/w M_1 & M_2' is decreased the circular fringe shrink & a ring disappear each time when it decrease by λ.

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Because the condⁿ of minima is
$2t\cos r = n\lambda$
at centre $\cos r = 1$ & if n changes by unity, t must be changes by $\lambda/2$.

Path diff b/w the rays reaching the ~~edge~~ eye is
$2t\cos\theta$.

An additional path diff. of $\lambda/2$ is introduced b/w two rays as the ray after reflection from M_2 suffers reflection at the silvered surface of glass plate

$$\boxed{\Delta = 2t\cos\theta + \frac{\lambda}{2}}$$