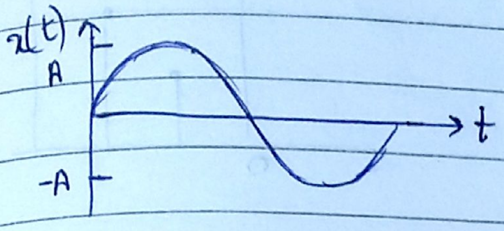
ETEE-207

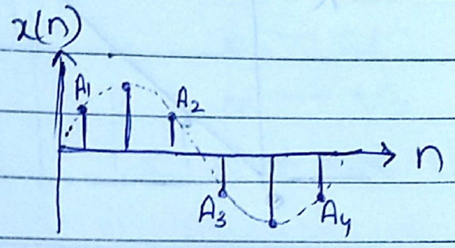
Book : K.M. Soni

UNIT-I

1. Continuous Signal : $x(t)$
2. Discrete Signal : $x(n)$



continuous



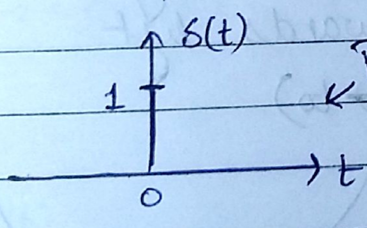
discrete

Basic Signal

1. Unit Impulse
2. Unit Step
3. Unit Ramp

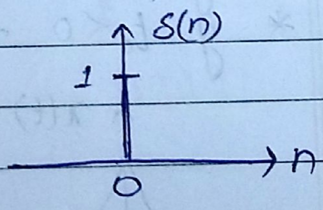
① Impulse Signal : $\delta(t)$

$\delta(t) = 1 \;;\; t = 0$
$\quad = 0 \;;\; t \neq 0$



Practical graph

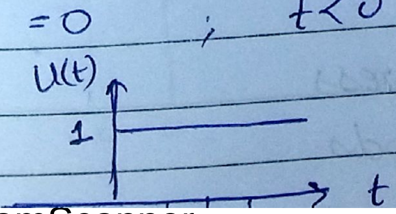
$\delta(n) = 1 \;;\; n = 0$
$\quad = 0 \;;\; n \neq 0$



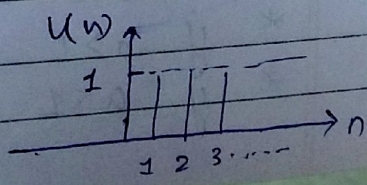
Ideal : $\delta(t) = \infty \;;\; t = 0$

② Step Signal : $u(t)$

$u(t) = 1 \;;\; t \geq 0$
$\quad = 0 \;;\; t < 0$

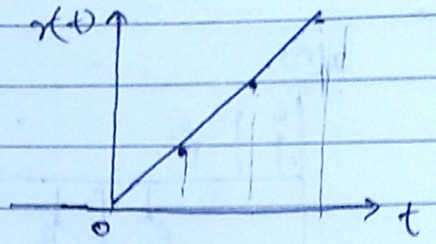


$u(n) = 1 \;;\; n \geq 0$
$\quad = 0 \;;\; n < 0$

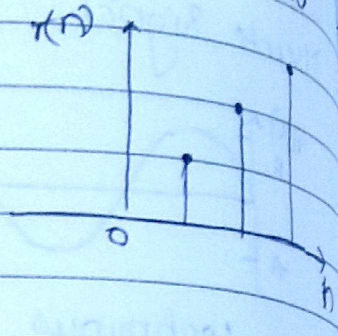


3 **Ramp signal : $r(t)$**

$r(t) = t \;;\; t \geq 0$
$\quad\;\; = 0 \;;\; t < 0$



$r(n)$ ...
$\quad = 0 \;;\; n<0$



$\boxed{t \geq 0 \equiv t = 0, 1, 2 \ldots \infty}$

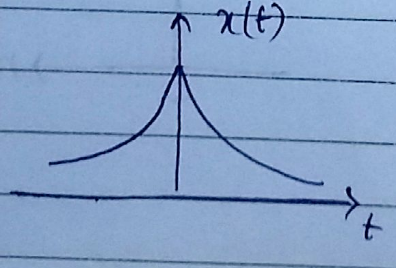
Transformation of Signal

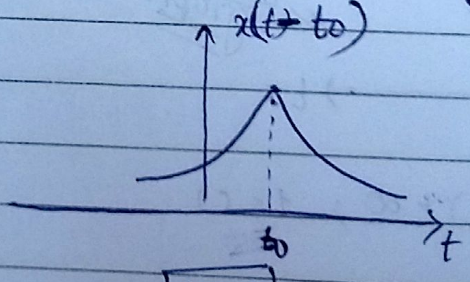
① Shifting
② Scaling
③ Folding/Reversal

order to follow

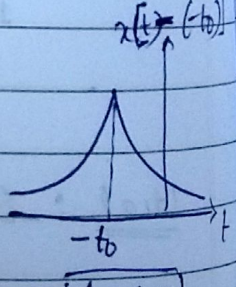
Rules for SHIFTING

* replace 't' $\xleftrightarrow{\text{with}}$ '$t - t_0$'
* if $t_0 > 0$ then signal shifts towards right
* if $t_0 < 0$ then signal shifts towards left





$\boxed{t_0 > 0}$



$\boxed{t_0 < 0}$

Rules for SCALING

* replace 't' $\xleftrightarrow{\text{with}}$ 'at'
* if $a > 1$ then signal compress
* if $a < 1$ then signal expands

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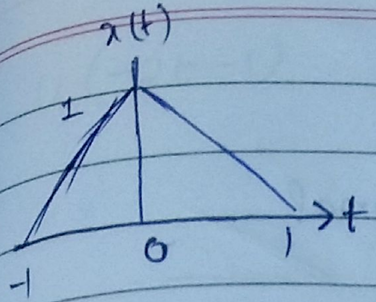
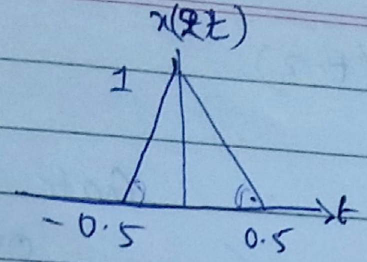
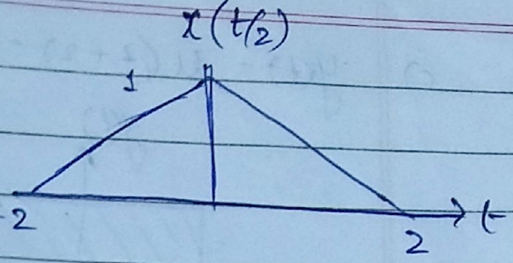






Compression

$t_0 \to at$

$t_0 \to 2t$

$\Rightarrow t = t_0/2$

Expansion

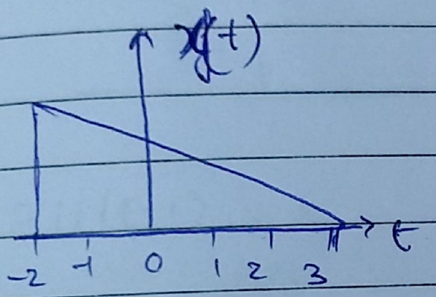
$t_0 \to at$

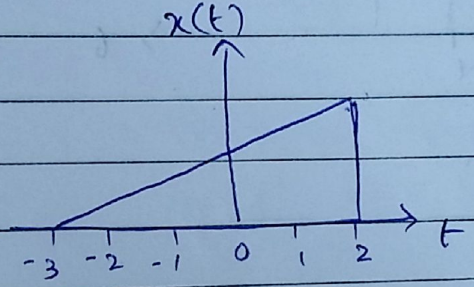
$t_0 \to t/2$

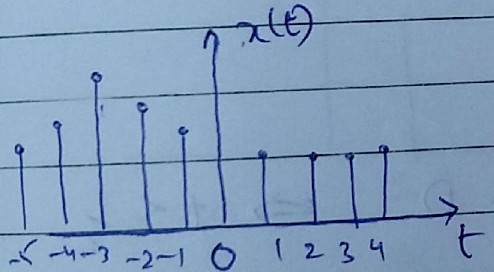
$\Rightarrow t = 2t_0$

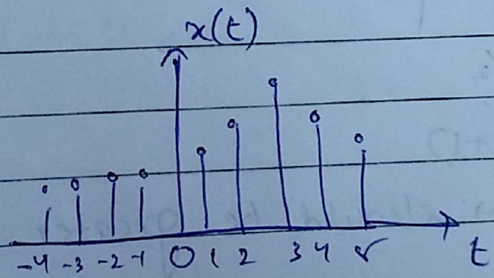
Rules for FOLDING

* Replace 't' $\xleftrightarrow{\text{with}}$ '-t'

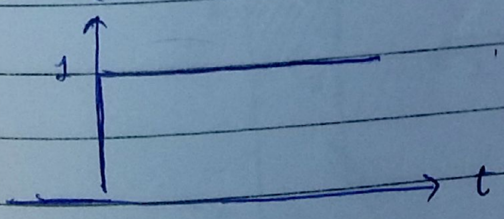




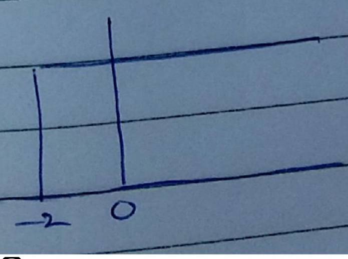




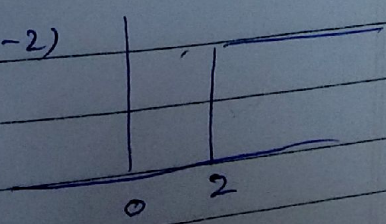
Q u(t)



a) u(t+2)



b) u(t-2)



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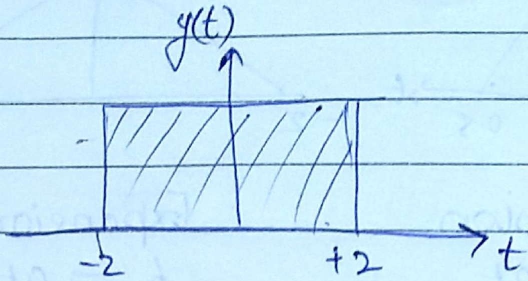
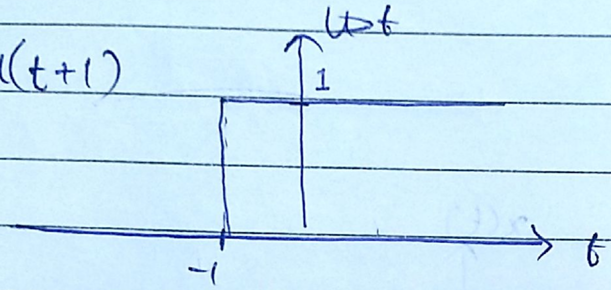
c) $y(t) = u(t+2) - u(t-2)$



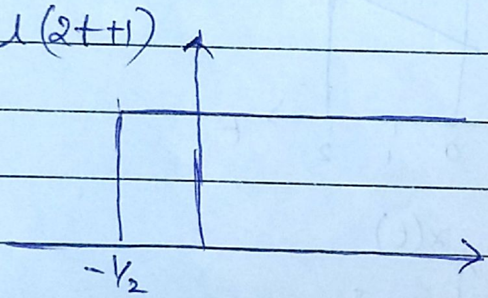
Gate Signal
or
Rectangular signal

Q $u(-2t+1)$

$u(t+1)$



$u(2t+1)$

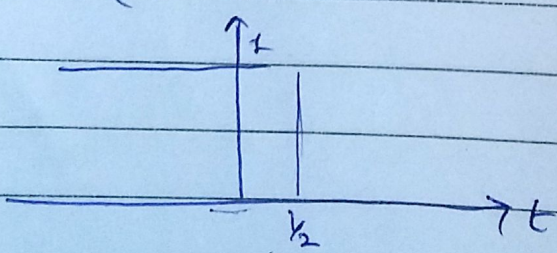


$u(-2t+1)$

↳ should be greater than 0 $\Rightarrow t < \frac{1}{2}$

check ☆

$u(-2t+1)$



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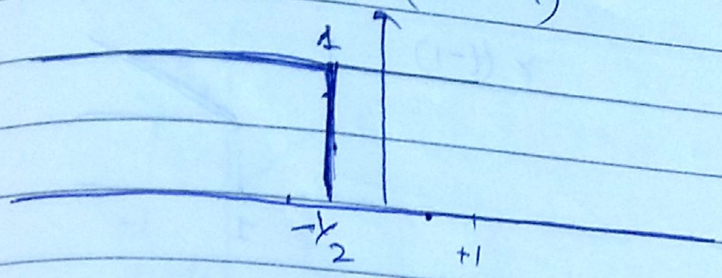
Page

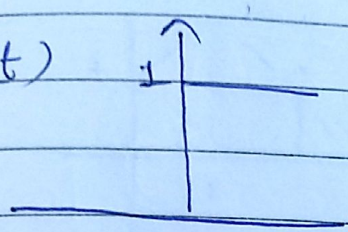
Q. $u(-2t-1)$

$u(-2t-1)$

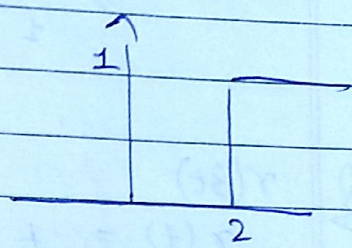


Q. $x(t) = u(t) - 2u(t-1) + u(t-2)$

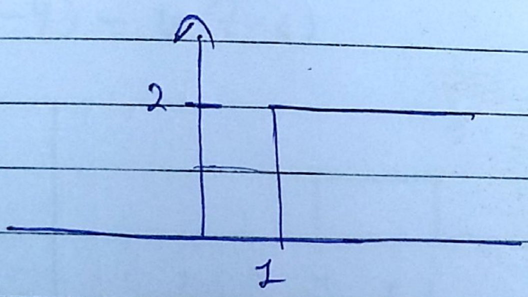
$u(t)$



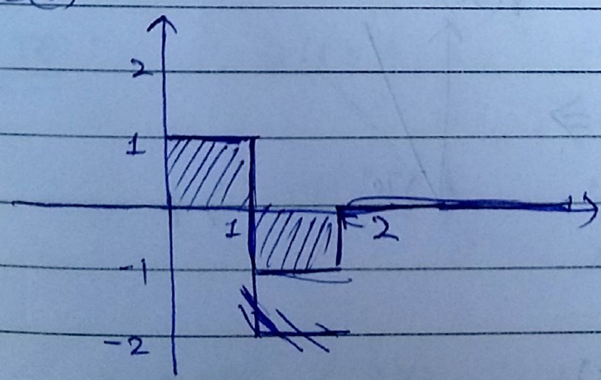
$u(t-2)$



$2u(t-1)$



$\therefore x(t)$



Square signal

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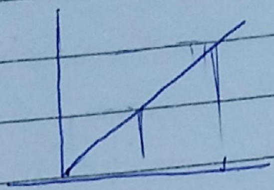
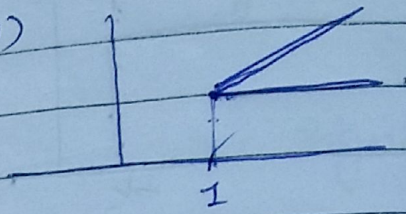
$t-1$ $\quad$ $t-1 \geq 0$

Q. $x(t) = r(t) - r(t-1)$

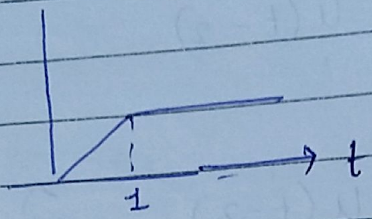
$r(t)$



$r(t-1)$



$x(t)$



Q. $r(3t)$

$r(t) = t \quad t \geq 0$

$ 0 \quad t < 0$

$\Rightarrow r(3t) = 3t \quad 3t \geq 0$

$ 0 \quad 3t < 0$

$= 3t \quad t \geq 0$

$ 0 \quad t < 0$

$\tan\theta = m$

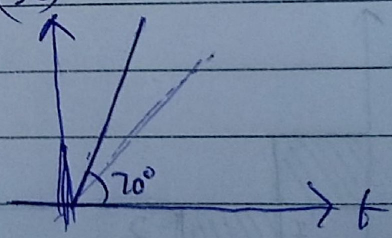
$= 3$

$\theta = \tan^{-1}(3)$

$= 70^\circ$

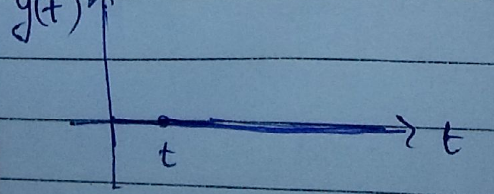
$\Rightarrow$

$r(3t)$



8/8/17

$y(t) = u(t-1) - u(t-1)$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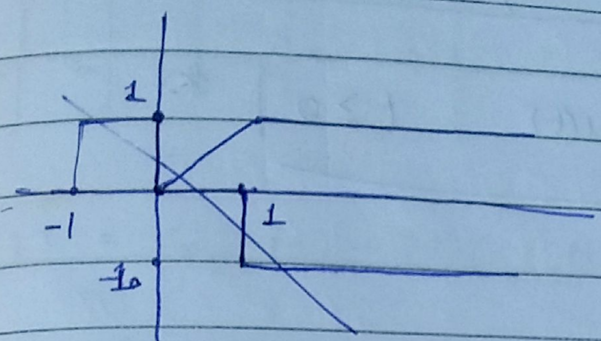
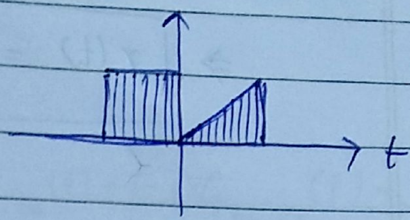
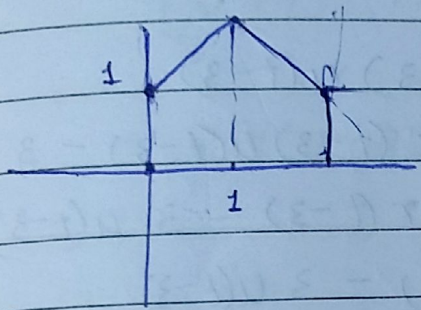
Q $u(t+1) - u(t) + r(t) - r(t-1) - u(t-1)$

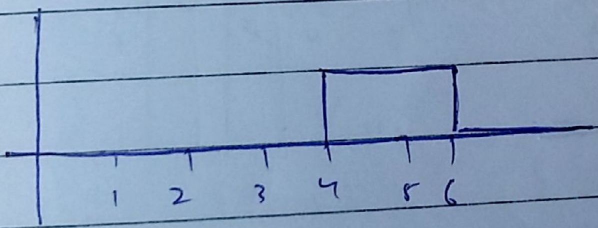




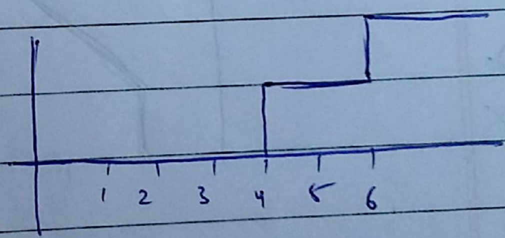
Q $u(t) + r(t) - 2\,r(t-1) + r(t-2) - u(t-2)$



Q $u(t-4) - u(t-6)$



Q $u(t-4) + u(t-6)$



<u>NOTE</u> :

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\Rightarrow \frac{d\,r(t)}{dt} = \frac{d(t)}{dt} \; ; \; t \geq 0$$

$$0 \; ; \; t < 0$$

$; \; t > 0$

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$$\Rightarrow \boxed{\frac{d\,r(t)}{dt} = \begin{cases} u(t) ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}} \; *$$

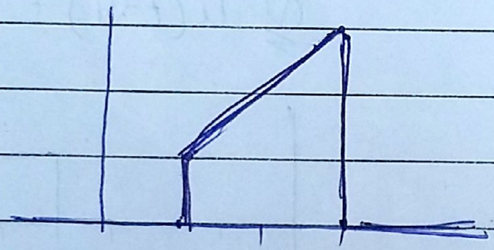
$$\Rightarrow \boxed{r(t) = t \cdot u(t) \quad ; \quad t \geq 0} \; *$$

Q. $f(t) = t\,[u(t-1) - u(t-3)]$

$$\Rightarrow f(t) = t \cdot u(t-1) - t \cdot u(t-3)$$
$$= (t-1+1)\,u(t-1) - (t-3+3)\,u(t-3)$$
$$= (t-1)\,u(t-1) + u(t-1) - (t-3)\,u(t-3) - 3\,u(t-3)$$
$$= r(t-1) + u(t-1) - r(t-3) - 3\,u(t-3)$$
$$= u(t-1) + r(t-1) - r(t-3) - 3\,u(t-3)$$

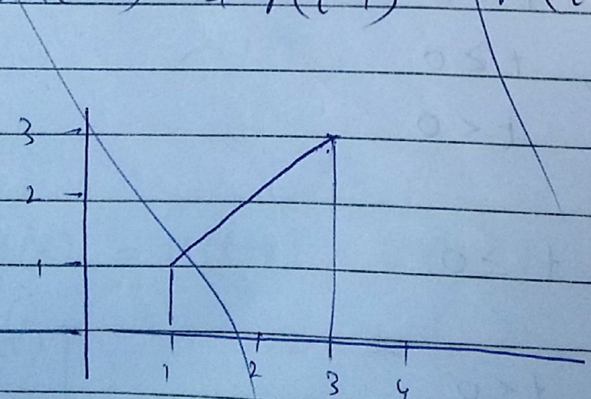
Check

$u(t)$.



9/8/17

Q. $u(t-1) + r(t-1) - r(t-3) - 3\,u(t-3)$



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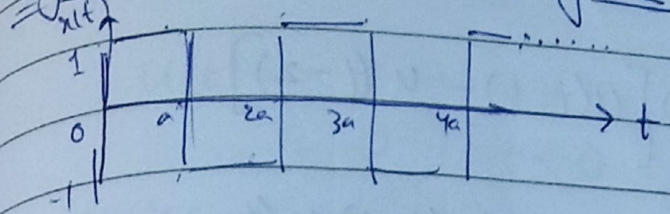
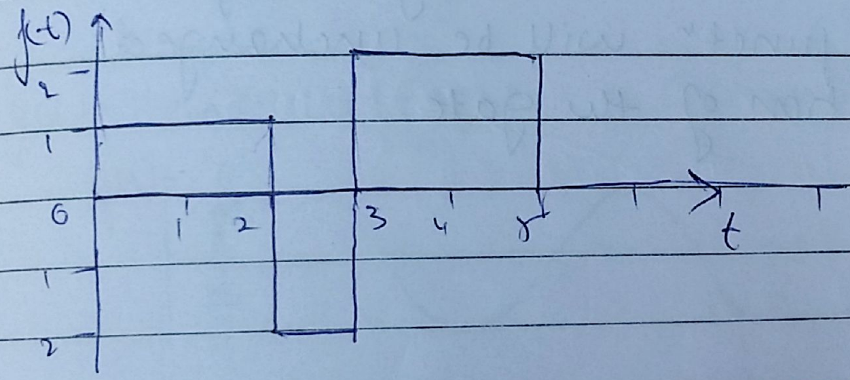
Synthesize the Waveform



$$x(t) = G_{0,a}(t) + (-1)G_{a,2a}(t) + G_{2a,3a}(t) + (-1)G_{3a,4a}(t) + \dots$$

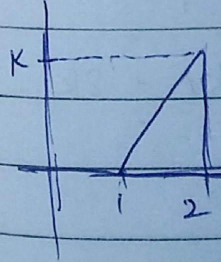
$$= [u(t) - u(t-a)] - [u(t-a) - u(t-2a)] + [u(t-2a) - u(t-3a)] - [u(t-3a) - u(t-4a)] \dots$$

$$= u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + 2u(t-4a) \dots$$



$$x(t) = u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)$$

$$y(t) = G_{0,2}(t) - 2G_{23}(t) + 2G_{35}(t)$$



$$(y - y_1) = m(x - x_1)$$

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$(y(t) - 0) = \frac{k - 0}{2 - 1}(t - 1)$$

$$\boxed{y(t) = k(t-1)} \quad (1)$$

$f_2(t) = [u(t-1) - u(t-2)]$ —— ②

① × ② ⇒ $f(t) = k(t-1)[u(t-1) - u(t-2)]$

$= k(t-1)u(t-1) - k(t-1)u(t-2)$

$= k\,r(t-1) - k(t-1+1-1)u(t-2)$

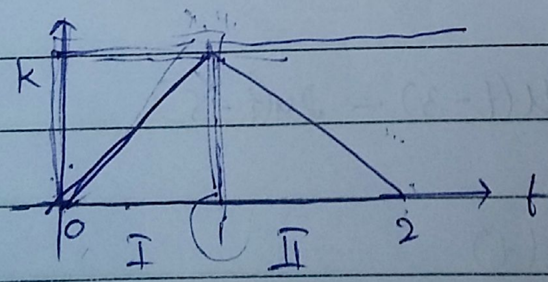
$= k\,r(t-1) - k\,r(t-2) - k\,u(t-2)$

1

Property of Gate funct^n

If any funct^n (signal) multiplied by a gate funct^n (or signal) then that funct^n will have zero value outside the duration of the Gate and the value of the funct^n will be unchanged within the duration of the gate

Q



$f_1(t) - 0 = \left(\frac{k-0}{1-0}\right)(t-0)$

$\Rightarrow f_1(t) = kt$

$f_2(t) - k = \left(\frac{0-k}{2-1}\right)(t-1)$

$f_2(t) = -kt + 2k$

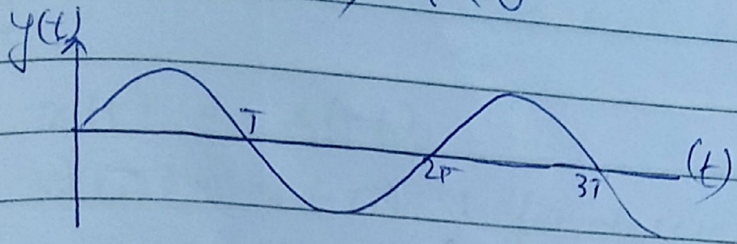
Date 10/8/17

Page

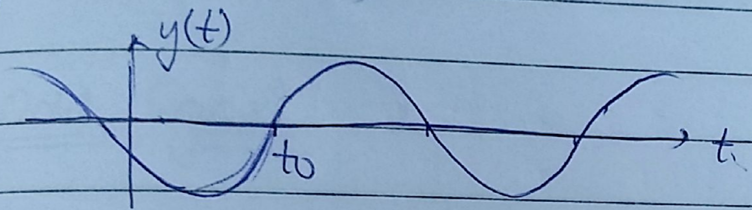
Q. $y(t) = A \sin \omega t \cdot u(t)$

$u(t) = 1 \; ; \; t \geq 0$

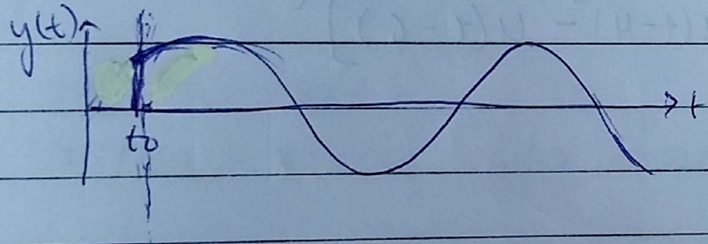
$ 0 \; ; \; t < 0$



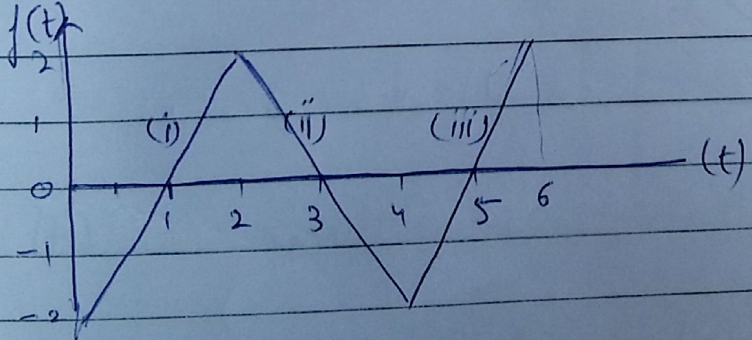
Q. $y(t) = \sin(t - t_0)$



Q. $y(t) = \sin t \cdot u(t - t_0)$



Q.



$$f_i(t) - (-2) = \left(\frac{2-(-2)}{2-0}\right)(t-0)$$

$$f_i(t) + 2 = 2t$$

$$\boxed{f_i(t) = 2t - 2}$$

Page

$$f_{ii}(t) - 2 = \left(\frac{-2-2}{4-2}\right)(t-2)$$

$$f_{ii}(t) - 2 = -2(t-2)$$

$$\boxed{f_{ii}(t) = -2t + 6}$$

$$f_{iii}(t) - (-2) = \left(\frac{2-(-2)}{6-4}\right)(t-4)$$

$$f_{iii}(t) + 2 = 2(t-4)$$

$$\boxed{f_{iii}(t) = 2t - 10}$$

$$y(t) = (2t-2)\, G_{0,2}(t) + (-2t+6)\, G_{2,4}(t) + (2t-10)\, G_{4,6}(t)$$

$$= (2t-2)[u(t) - u(t-2)] + (-2t+6)[u(t-2) - u(t-4)]$$
$$+ (2t-10)[u(t-4) - u(t-6)]$$

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FREEMIND

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Page

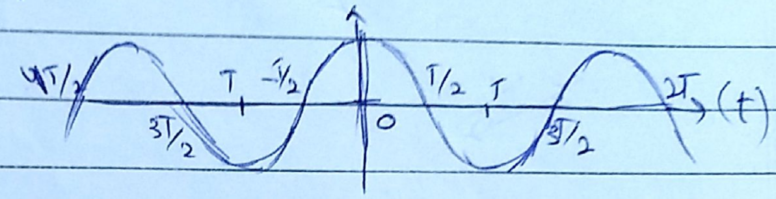
Even Signals / Symmetric Signal

A signal is said to be even signal if inversion of time axis does not change the amplitude

$x(t) = x(-t)$

$x(n) = x(-n)$: for descrete domain

Eg: cosine wave



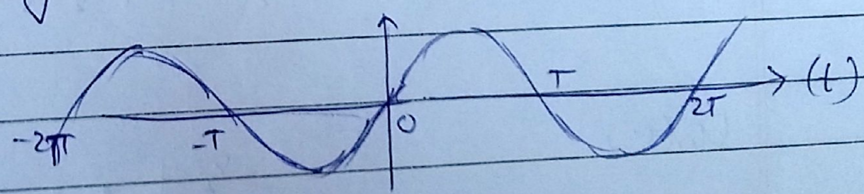
Odd Signal

A signal is said to be odd signal if the inversion of time axis ~~change~~ also inverse amplitude of the signal

$x(t) = -x(-t)$

$x(n) = -x(-n)$: for descrete domain

Eg: sine wave



$x(t) = x_e(t) + x_o(t) = E[x(t)] + O[x(t)]$

$x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$x_o(t) = \dfrac{x(t) - x(-t)}{2}$

} Similarly for descrete domain

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Q. Determine the even & odd part of the unit step signal.

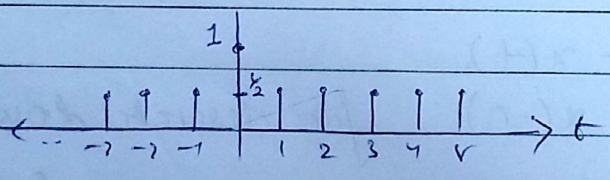
$$u(n) = \begin{cases} 1 \,; & n \geq 0 \\ 0 \,; & n < 0 \end{cases}$$

$$u_e(n) = \frac{u(n) + u(-n)}{2} \qquad u_o(n) = \frac{u(n) - u(-n)}{2}$$

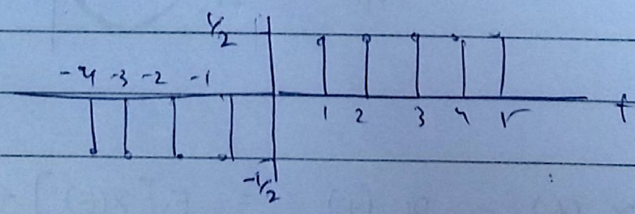
$$\cancel{= \frac{1}{2}} \qquad \qquad \cancel{= \frac{1}{2}}$$

n	...	-2	-1	0	1	2	----
u(n)	0	0	0	1	1	1	1
u(-n)	1	1	1	1	0	0	0
$u_e(n)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$u_o(n)$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

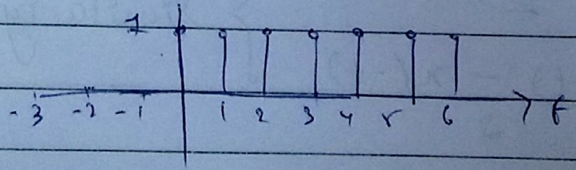
$u_e(n) =$



$u_o(n)$



$u(n) = u_e(n) + u_o(n)$



Date
Page

Energy Signals

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dx \; ; \qquad 0 < E_x < \infty \text{ and Power}(P_x) = 0$$

$$E_x = \sum_{n=-\infty}^{\infty} |x(t)|^2$$

Power Signals

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2 \; ;$$

- $0 < P_x < \infty$
- $E_x = \infty$

① $$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \; ; \quad -1 < \alpha < 1$$

② $$\sum_{n=0}^{P} \alpha^n = \frac{1-\alpha^{P+1}}{1-\alpha} \; ; \quad \alpha \neq 1$$

③ $$\sum_{\text{lower limit}}^{\text{upper (UL) limit}} \text{constant} = \text{constant}\,[UL - LL + 1]$$

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Q. $x(t) = e^{-2t}\, u(t)$

$$|x(t)|^2 = e^{-4t}\, u(t)$$

$$\therefore E_x = \int_{-\infty}^{\infty} e^{-4t}\, u(t)\, dt$$

$$= \int_{0}^{\infty} e^{-4t}\, dt$$

$$= \left| \frac{e^{-4t}}{-4} \right|_{0}^{\infty}$$

$$= \frac{1}{4}$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{-4t}\, u(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-4t}}{-4} \right]_{0}^{T}$$

$$= 0$$

Q. $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

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Date
Page

$|x(n)|^2 = \left(\frac{1}{2}\right)^{2n} u(n)$

$= \left(\frac{1}{4}\right)^{n} u(n)$

$E_x = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n)$

$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$

$= \frac{1}{1-1/4} = \frac{4}{3}$

$P_x = \lim_{n \to \infty}$

Q $\cancel{x(n) = u(n)}$ ~~Find u(n) is energy signal or power signal~~

$\cancel{E_x = \int_{-\infty}^{\infty} u(n)\, dn}$

$= \int_{0}^{\infty} u(n)\, dn$

Date
Page

Q. $x(n) = u(n)$

$$P_{x} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} (1)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} [1+1+1 \ldots (1)_n] \rightarrow N+1 \text{ times as N starts with 0}$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= \lim_{N\to\infty} \frac{1+1/N}{2+1/N}$$

$$= \frac{1}{2}$$

Step signal is one of the type of Power signal

Q. $x(t) = A \cos t$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$|x(t)|^2 = |A \cos t|^2$$

$$= A^2 \cos^2 t$$

$$= A^2 \left(\frac{1+\cos 2t}{2}\right)$$

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Date ______

Page ______

$$E_x = \int_{-\infty}^{\infty} \frac{A^2}{2}(1+\cos 2t)\,dt$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty}(1+\cos 2t)\,dt$$

$$= \frac{A^2}{2}\left| t + \frac{\sin 2t}{2}\right|_{-\infty}^{\infty}$$

$$= \infty$$

$$P_x = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{A^2}{2}(1+\cos 2t)\,dt$$

$$= \frac{A^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T}(1+\cos 2t)\,dt$$

$$= \frac{A^2}{2}\lim_{T\to\infty}\frac{1}{T}\left|t + \frac{\sin 2t}{2}\right|_{-T}^{T}$$

$$= \frac{A^2}{2}\lim_{T\to\infty}\frac{1}{T}\left|\left(T + \frac{\sin 2T}{2}\right) - \left(-T - \frac{\sin 2T}{2}\right)\right|$$

$$= A^2$$

Q $x(t) = e^{j(2t+\pi/4)}$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$= \sqrt{\cos^2\theta + \sin^2\theta}$$

$$= 1$$

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$$E_x' = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} 1^2 \, dt$$

$$= |t|_{-\infty}^{\infty}$$

$$= \infty$$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} 1 \, dt$$

$$= 1$$

1/8/17

Laplace Transform

Converge

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt} \rightarrow \text{Bi-lateral LT}$$

s: complex Independent Variable
s: Laplace operator

$$\boxed{X(s) = \int_{0}^{\infty} x(t) e^{-st} \, dt} \rightarrow \text{Unilateral LT}$$

$$s = \sigma + j\omega$$

σ → real part

$j\omega$ → complex frequency

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FREEMIND
Date
Page

Inverse LT

$$X(\sigma+j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma+j\omega)t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t) \cdot e^{-\sigma t} \cdot e^{-j\omega t}\, dt$$

Change of integral

$$x(t) \cdot e^{-\sigma t} = \int_{-\infty}^{\infty} X(\sigma+j\omega)\, e^{j\omega t}\, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\sigma+j\omega)\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\sigma+j\omega) \cdot e^{(\sigma+j\omega)t}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x(S)\, e^{st}\, d\omega$$

Change of variable is peformed by replacing $S = \sigma + j\omega$

Now diff. s wrt ω

$$\frac{ds}{d\omega} = j$$

$$\Rightarrow d\omega = \frac{dS}{j}$$

$$\therefore x(t) = \frac{1}{2\pi j} \int_{\sigma-\infty j}^{\sigma+\infty j} x(S)\, e^{st}\, ds$$

$\frac{\text{Numerator}}{\text{Denominator}} = \text{Poles} \rightarrow \times$

(ckt system)

$|x(t) \cdot e^{-\sigma t}| < \infty$

$\Rightarrow$ L.T is convergent (gives finite value)

Q $x(t) = e^{at} u(t)$, find ROC (region of convergence) and L.T.

$$X(s) = \int_{-\infty}^{\infty} e^{at} u(t) e^{-st} dt$$

$$= \int_{0}^{\infty} e^{(a-s)t} dt$$

$$= \left| \frac{e^{(a-s)t}}{a-s} \right|_0^{\infty}$$

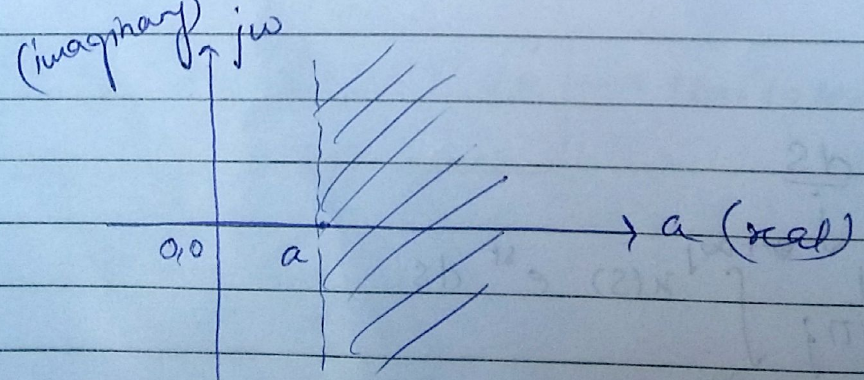
$$= \left| \frac{e^{-(s-a)t}}{-(s-a)} \right|_0^{\infty}$$

$$= \frac{1}{s-a}$$

$\therefore$ The funct^n will converge or give a real value when $s-a > 0$

$\Rightarrow s > a$

Check



FREEMIND
Date
Page

$x(t) = e^{-at} u(t)$

$$x(s) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} dt$$

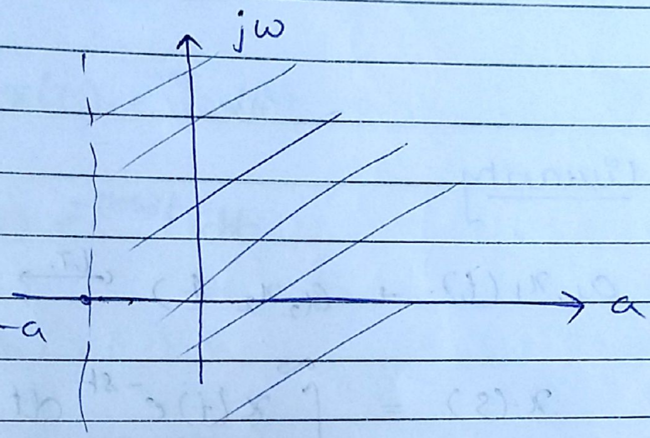
$$= \int_{0}^{\infty} e^{-(s+a)t} dt$$

$$= \left| \frac{e^{-(s+a)t}}{-(s+a)} \right|_{0}^{\infty}$$

$$= \frac{1}{s+a}$$

$\therefore$ $s+a > 0$

$s > -a$



V. Imp.

$$\boxed{e^{\mp at} u(t) \Rightarrow \frac{1}{s \mp a}}$$

1/8/17

Properties of Laplace Transform

② Time Swifting

$$x(t - t_0) \xleftrightarrow{L.T.} e^{-st_0} x(s)$$

$$\boxed{x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt}$$

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$$= \int_{-\infty}^{\infty} x(t-t_0)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-s(\tau+t_0)}\, dt$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-s\tau} e^{-st_0}\, d\tau$$

$$= e^{-st_0} X(s)$$

5. **Linearity**

$$a_1 x_1(t) + a_2 x_2(t) \xrightarrow{L.T.} a_1 X_1(s) + a_2 X_2(s)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} [a_1 x_1(t) + a_2 x_2(t)]\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} a_1 x_1(t)\, e^{-st}\, dt + \int_{-\infty}^{\infty} a_2 x_2(t)\, e^{-st}\, dt$$

$$= a_1 x_1(t) + a_2 x_2(t)$$

Date
Page

③ Frequency or S-Domain shifting

$$e^{s_0 t}\, x(t) \xleftrightarrow{LT} x(s-s_0)$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

$$= \int_{-\infty}^{\infty} e^{+s_0 t}\, x(t) \cdot e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-(s-s_0)t}\, dt$$

$$= x(s-s_0)$$

$$= \int_{-\infty}^{\infty} x(\cancel{t})\, e^{-st} \cdot e^{-st_0}\, d\tau$$

$$= e^{-st_0}\, x(s)$$

$$\boxed{e^{\pm s_0 t}\, x(t) \xleftrightarrow{LT} x(s \mp s_0)t}$$

④ Time Scaling

$$x(at) \xleftrightarrow{LT} \frac{1}{a} x\left(\frac{s}{a}\right)$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

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$$= \int_{-\infty}^{\infty} x(at)\, e^{-st}\, dt$$

Put $at = \tau$

$$\boxed{t = \tau / a}$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-s(\tau/a)}\, d\tau$$

$$= \frac{1}{a} X\left(\frac{s}{a}\right)$$

⑤ <u>Diff in time-domain</u>

$$\frac{d}{dt}[x(t)] \xleftrightarrow{LT} s\cancel{X}(s)$$

$$x(t) = \frac{1}{2\pi s} \int_{-\infty}^{\infty} X(s)\, e^{st}\, dt$$

Diff both sides wrt 't'

$$\frac{d}{dt}[x(t)] = \frac{1}{2\pi s} \int_{-\infty}^{\infty} X(s)\, e^{st}\, (s)\, ds$$

$$\frac{d}{dt}[x(t)] = sX(s)$$

$$\boxed{\frac{d^2}{dt}[x(t)] = s^2 X(s)}$$

⑥ <u>Diff in S-Domain</u>

$$-t\, x(t) \xleftrightarrow{LT} \frac{d}{ds} X(s)$$

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$$x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

Diff both sides wrt s

$$\frac{d}{ds} x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} (-t) dt$$

$$\frac{dx(s)}{ds} = -t x(t)$$

$$t^2 x(t) \xleftrightarrow{LT} \frac{d^2}{ds} x(s)$$

$$\vdots$$

$$\boxed{(-t)^n x(t) \xleftrightarrow{LT} \frac{d^n}{ds} x(s)} \leftarrow \text{General form}$$

Convolution

Both the signals should be in time domain

1) Convolution in Time Domain

$$x_1(t) * x_2(t) \xleftrightarrow{LT} x_1(s)\ x_2(s)$$

S: independent complex variable
: laplace operator

$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\ x_2(t-\tau)\ d\tau}$$

$$x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

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$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, e^{-st}\, dt\, d\tau$$

Change the order of integration

$$= \int_{-\infty}^{\infty} x_1(\tau) \underbrace{\int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-st}\, dt}\, d\tau$$

→ using time shifting prop.! ~~graph~~

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-s\tau}\, x_2(s)\, d\tau$$

$$= x_2(s) \int_{-\infty}^{\infty} x_1(\tau)\, e^{-s\tau}\, d\tau$$

$$= x_1(s) \cdot x_2(s)$$

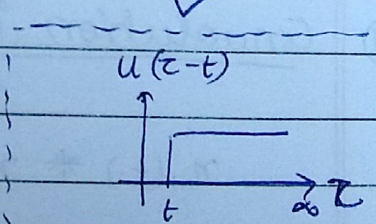
8/17

(8) Integration in Time Domain

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{LT} \frac{x(s)}{s}$$

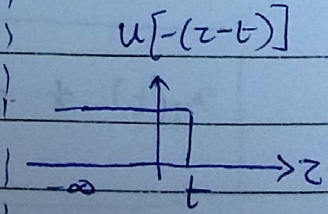
$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau}$$

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau$$



Check ↳

$$u(t-\tau) = 1 \; ; \; t-\tau \ge 0$$
$$= 0 \; ; \; t-\tau < 0$$
$$\Rightarrow u(t-\tau) = 1 \; ; \; t \ge \tau$$
$$= 0 \; ; \; t < \tau$$



$$= \int_{-\infty}^{t} x(\tau) \times 1 \, d\tau$$

$$= \int_{-\infty}^{t} x(\tau) \, d\tau$$

$$\therefore LT\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = LT\left[x(t) * u(t)\right]$$

$$= LT[x(t)] \; LT[u(t)]$$

$$= \frac{x(S)}{S}$$

⑨ <u>Periodic Function</u>

$$F(S) = \frac{F_1(S)}{1 - e^{-ST}} \; ; \quad T \rightarrow \text{Time Period}$$

⑩ <u>Initial Value Theorem</u>

$$LT\left[\lim_{t \to 0} x(t)\right] = \lim_{s \to \infty} S\, X(S)$$

⑪ <u>Final Value Theorem</u>

$$LT\left[\lim_{t \to \infty} x(t)\right] = \lim_{s \to 0} S \cdot x(S)$$

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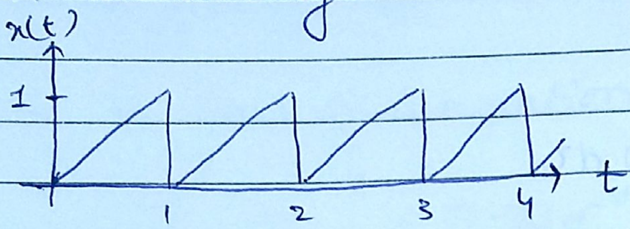
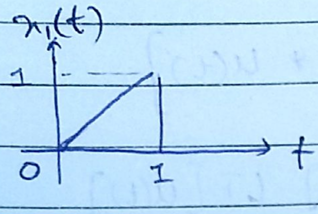
- $\delta(t) \xrightarrow{LT} 1$
- $u(t) \xrightarrow{LT} 1/s$
- $r(t) \xrightarrow{LT} 1/s^2$

Q. Find the L.T. of the saw tooth wave chain

a)



Ans



$x_1(t) = r(t) - r(t-1) - u(t-1)$

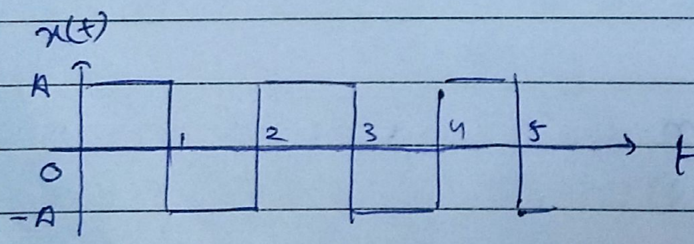
$LT[x_1(t)]$

$$\Rightarrow x_1(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$$

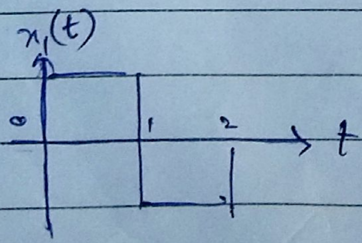
$$= \frac{1}{s^2}(1 - e^{-s} - se^{-s})$$

$$\therefore X(s) = \frac{1 - e^{-s} - se^{-s}}{s^2(1-e^{-s})} \quad ; \text{ here } T = 1$$

b)



Ans



$x_1(t) = AU(t) - 2AU(t-1) + AU(t-2)$

$$\therefore LT[x_1(t)] \Rightarrow x_1(s) = \frac{A}{s} - \frac{2e^{-s}A}{s} + \frac{Ae^{-2s}}{s}$$

Date
Page

$$\therefore x(s) = \frac{A}{s} \frac{(1-2e^{-s}+e^{-2s})}{(1-e^{-2s})}$$

$$= A$$

Classification of System

Time variant & Time Invariant System

A continuous time system is time invariant if the time shift in the input signal, results in corresponding time shift in the output signal

$$y(t) = F[x(t)]$$

$y(t)$ is response for $x(t)$, if $x(t)$ is delayed by time 't_1' then output $y(t)$ will also be delayed by the same time.

$$\left.\begin{aligned} y(n-k) &= F[x(n-k)] \longrightarrow \text{discrete} \\ y(t-t_1) &= F[x(t-t_1)] \longrightarrow \text{continuous} \end{aligned}\right\} \text{Time invariant}$$

Q $y(t) = \sin x(t)$

Let us determine the output of the system for delayed input function i.e. $x(t-t_1)$

$$y(t, t_1) = F[x(t-t_1)]$$
$$= \sin[x(t-t_1)] \longrightarrow (1)$$

Now delay the output in $y(t)$ by 't_1'

$$y(t-t_1) = \sin[x(t-t_1)] \longrightarrow (2)$$

$$\boxed{y(t, t_1) = y(t-t_1)} \quad \therefore \text{Time invariant}$$

Q) $y(t) = t \cdot x(t)$

Let us determine the output of the system for delayed input function

$$y(t, t_1) = t \quad x(t-t_1) \quad \text{——} \ (1)$$

Now delay the output by t_1

$$y(t-t_1) = (t-t_1) \cdot x(t-t_1) \quad \text{——} \ (2)$$

$$\therefore \boxed{y(t, t_1) \neq y(t-t_1)} \quad \therefore \text{Time variant}$$

Causal System

A system is causal if the output at any time 't_0' or 'n_0' depends only on the value of the input at the present time and in the past time i.e for $t \leq t_0$ or $n \leq n_0$

$y(n) = n \cdot x(n)$

Consider the output $y(n)$ at +ve time (n_0)

$$y(n)\Big|_{n=n_0} = y(n_0) = n_0 \cdot x(n_0)$$

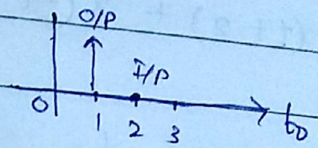
Consider the output $y(n)$ at -ve time i.e. $(-n_0)$

$$y(n)\Big|_{n=-n_0} = y(-n_0) = (-n_0) \cdot x(-n_0)$$

The given system is causal system as in ~~a~~ both the cases mentioned above, the ~~outp~~ present output depends on present input

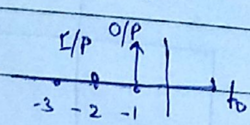
$y(t) = x(2t)$

$y(t)\big|_{t=t_0} = y(t_0) = x(2t_0)$



Past value

$y(t)\big|_{t=-t_0} = y(-t_0) = x(2(-t_0))$

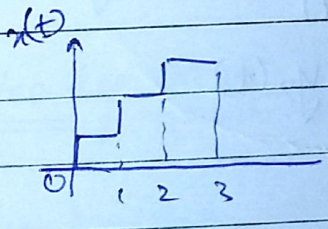


Future Value

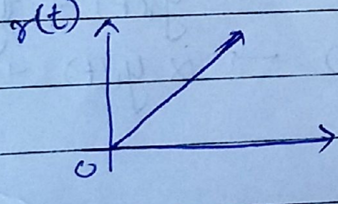
∴ The system is not a causal system
In first case the present output depends upon the future input but in second case the present output depends upon past input.

Bounded & Unbounded Signal (Stable and Unstable)

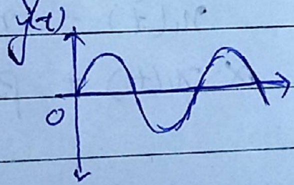
Bounded : If the amplitude of the signal is constant
Unbounded : If the amplitude of the signal inc.s



Unbounded



Unbounded



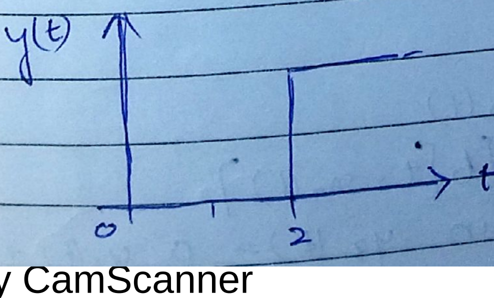
Bounded

Q $y(t) = x(t-2)$

$x(t) = u(t)$

$\therefore x(t-2) = u(t-2)$

$y(t) = u(t-2)$



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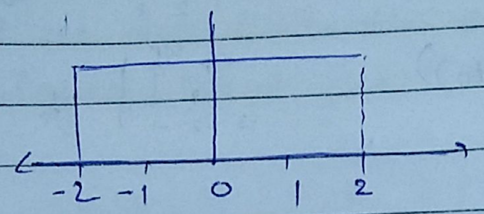
Q. $x(t+2) + x(-t+2) = y(t)$

$-t+2 \geq 0$

$t \leq 2$



Bounded

Linear & Non linear System

A system is said to be linear if it satisfy the condition of superposition principle

Superposition principle simply implies that the response resulting from several input signals can be computed as the sum of the responses resulting from each input signal acting along

$$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$$

$$\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$$

Q. $y(t) = t \cdot x(t)$

$$x_1(t) \longrightarrow y_1(t) = t\, x_1(t)$$

$$x_2(t) \longrightarrow y_2(t) = t \cdot x_2(t)$$

$$x_3(t) \longrightarrow y_3(t) = t \cdot x_3(t)$$

$x_3(t)$ is a linear combination of $x_1(t)$ & $x_2(t)$

$$x_3(t) = a\, x_1(t) + b\, x_2(t)$$

*

$\therefore$ $y_3(t) = a\, y_1(t) + b\, y_2(t)$

$= a[t \cdot x_1(t)] + b[t \cdot x_2(t)]$

* If the system is linear then $y_3(t) = a\, y_1(t) + b\, y_2(t)$

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