Example 4.12 *A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length.*

[GGSIPU, Sept. 2013 reappear (4 marks) ; Sept. 2012 (4 marks)]

Solution. Given $\lambda = 5.82 \times 10^{-7}$ m, $\alpha = 20''$, $\mu = 1.5$

The fringe width $$\beta = \frac{\lambda}{2\mu\alpha}$$

$$\alpha = \frac{20 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

$$\beta = \frac{5.82 \times 10^{-7} \times 60 \times 60 \times 180}{2 \times 1.5 \times 20 \times \pi} = \frac{5.82 \times 6 \times 6 \times 18 \times 10^{-5}}{2 \times 1.5 \times 2 \times \pi}$$

$$= 2.0 \times 10^{-3} \text{ m} = 0.2 \text{ cm.}$$

Number of dark interference fringes (m) per cm of the wedge length *i.e.,*

$$\frac{m}{x_2 - x_1} = \frac{1}{\beta} = \frac{1}{0.2 \text{ cm}} = \frac{10}{2} \text{ per cm} = 5 \text{ fringes per cm.}$$

Example 4.13 *A glass wedge of angle 0.01 radian is illuminated by monochromatic light of wavelength 6000 Å falling normally on it. At what distances from the edge of the wedge will the 10th fringe be observed by reflected light ?*

[GGSIPU, Nov. 2006, Sept. 2005 (4 marks)]

Solution. Given that $\alpha = 0.01$ radian, $n = 10$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ m}$$

The condition for dark fringe $2t = n\lambda$

The angle of wedge $\alpha = \frac{t}{x}$ or $t = \alpha x$

$$\therefore \quad 2x\alpha = n\lambda$$

$$x = \frac{n\lambda}{2\alpha} = \frac{10 \times 6000 \times 10^{-10}}{2 \times 0.01} = 3 \text{ mm.}$$

Example 4.14 *In a Newton's ring experiment the diameters of 4th and 12th dark rings are 0.4 cm and 0.8 cm respectively. Deduce the diameter of 20th dark ring.* [GGSIPU, Dec. 2011 ; Dec. 2012 (2.5 marks)]

Solution. In Newton's ring experiment,

Given that : $n = 4$; $(m+n) = 12$, $m = 8$

$$D_n = 0.4 \text{ cm and } D_{m+n} = 0.8 \text{ cm}$$

The wavelength of sodium light using Newton's ring is

$$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR}$$

or

$$4\lambda R = \frac{D_{m+n}^2 - D_n^2}{m}$$

$$\Rightarrow \quad 4\lambda R = \frac{(0.8)^2 - (0.4)^2}{m} \quad \text{...(i)}$$

We know that the diameter of nth dark ring in presence of air is

$$D_n^2 = 4n\lambda R$$

$$\Rightarrow \quad D_{20}^2 = 20 \times (4\lambda R) \quad \text{...(ii)}$$

Putting the value of $4\lambda R$ from Eq. (*i*) in Eq. (*ii*)

$$D_{20}^2 = \frac{20 \times [(0.8)^2 - (0.4)^2]}{8} = \frac{20}{8} \times 1.2 \times 0.4 \quad \Rightarrow \quad D_{20} = 1.2 \text{ cm}$$

Problem 4.1 *Two waves of same frequency have amplitudes 1.00 and 2.00. They interference at a point, where the phase difference is 60°. What is the resultant amplitude ?* [GGSIPU, Dec. 2009 (3 marks)]

Solution. Given that $a_1 = 1.00$, $a_2 = 2.00$ and $\varphi = 60°$

We know that, the resultant amplitude

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi}$$

$$= \sqrt{1^2 + 2^2 + 2(1)(2)\cos 60°}$$

$$= \sqrt{1+4+2} = \sqrt{7} = 2.65 \text{ unit.}$$

$$= 2.50 \times 10^{-5}\ \text{m}^2$$

Problem 4.11 *A Newton ring arrangement is used with a light sources of wavelength $\lambda_1 = 6000$ Å and $\lambda_2 = 5000$ Å and it is found that the nth dark ring due to λ_1 coincide with $(n+1)$th dark ring due to λ_2. If the radius of curvature of curved surface of the lens is 90 cm, then find the diameter for the nth, dark ring for λ_1.*

[GGSIPU, Sept. 2009 (3 marks)]

Solution. Given $\lambda_1 = 6000$ Å for nth ring

$\lambda_2 = 5000$ Å for $(n+1)$th ring, $R = 90\text{ cm} = 0.9\text{ m}$

$$[\because D_n = \sqrt{4n\lambda R}]$$

and
$$(D_n)_{\lambda_1} = (D_{n+1})_{\lambda_2}$$

or
$$\sqrt{4n\lambda_1 R} = \sqrt{4(n+1)\lambda_2 R}$$

or
$$n\lambda_1 = (n+1)\lambda_2$$

$$6000 \times n = (n+1) \times 5000$$

$\Rightarrow$
$$n = 5$$

$\Rightarrow$
$$D_n = \sqrt{4n\lambda_1 R}$$

$$D_5 = \sqrt{4 \times 5 \times 6000 \times 10^{-10} \times 0.9} = 3.286 \times 10^{-3}\ \text{m}$$

Numerical Problems

4.1 Two coherent sources whose intensity ratio is 4:1 produce interference fringes, find the ratio of maximum to minimum intensity in the interference pattern. **[GGSIPU, Dec. 2015 (3 marks)]**

Hint : $$\frac{I_1}{I_2} = \frac{4}{1} = \frac{a_1^2}{a_2^2} \quad \Rightarrow \quad a_1 = 2a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(3a_2)^2}{a_2^2} = 9 : 1$$

4.2 In Young's two slits experiment, the distance between the slits is 0.2 mm and screen is at a distance 1.0 m. The third bright fringe is at a distance 7.5 mm from the central fringe. Find the wavelength of the light used.

[GGSIPU, Dec. 2008 (3 marks)]

Hint : $$x = \frac{n\lambda D}{2d} \quad \Rightarrow \quad \lambda = \frac{x(2d)}{nD} = 5000 \text{ Å}.$$

4.3 In Young's double slit experiment, fringes are obtained at a screen placed at some distance from the slits. If screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. Calculate the wavelength of light used. Given the distance between the slits is 10^{-3} m.

[GGSIPU, Dec. 2011 (3 marks)]

Hint : Fringe with $\beta = \frac{\lambda D}{2d}$; if $\Delta\beta$ is the change in fringe width when the screen is moved by ΔD, then

$$\Delta\beta = \frac{\lambda \Delta D}{2d} \qquad \Rightarrow \qquad \lambda = \frac{2d\Delta\beta}{\Delta D} = \frac{10^{-3} \times 3 \times 10^{-5}}{5 \times 10^{-2}} \text{ m} = 600 \text{ nm}.$$

4.4 The inclined faces of a biprism ($\mu = 1.5$) make angles of 1° with the base of the prism. The slit is 10 cm from the biprism and is illuminated by the wavelength 590 nm. Find the fringe width observed at a distance of one meter. **[GGSIPU, Nov. 2012 (3 marks)]**

Hint : $d = 2a(\mu - 1)\alpha$; $D = 10 \text{ cm} + 1\text{m} = 1.1 \text{ m}$, $\alpha = \frac{\pi}{180}$ radian,

$$\lambda = 590\text{nm} = 5.9 \times 10^{-7} \text{ m}, \quad a = 0.1 \text{ m}$$

Fringe width $\beta = \frac{\lambda D}{d} = \frac{5.9 \times 10^{-7} \times 1.1 \times 180}{2 \times 0.1 \times 0.5 \times \pi} = 0.000372 \text{ m} = 3.72 \times 10^{-4} \text{ m}.$

4.5 Newton's rings are formed between the plane surface of glass and lens. The diameter of third dark ring is 10^{-2} m. When the light of wavelength 5890×10^{-10} m is used at such an angle that the light passes through the air film at an angle of 30° to the normal. Find the radius of the lens. **[GGSIPU, Dec. 2018 (4 marks)]**

Hint : $2\mu t \cos r = n\lambda \quad \Rightarrow \frac{r^2}{R} \cos r = n\lambda \quad \Rightarrow \frac{D_3^2}{4R} \cos r = 3\lambda$

$$\Rightarrow \qquad R = \frac{D_3^2}{4 \times 3 \times \lambda} = 30° = \frac{(10^{-2})^2}{4 \times 3 \times 5890 \times 10^{-10}} = \frac{\sqrt{3}}{2} = 12.25\text{m}$$

4.6 Light of wavelength 6000 Å falls normally on a wedge shaped film of refractive index 1.4 forming fringes that are 2.0 mm apart. Find the angle of wedge in seconds. **[GGSIPU, Dec. 2013 reappear (4 marks)]**

Hint : Given $\lambda = 6000 \text{ Å} = 6.000 \times 10^{-7} \text{ m}, \mu = 1.4, \beta = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

We know that angle of wedge

$$\alpha = \frac{\lambda}{2\mu\beta} = \frac{6.0 \times 10^{-7}}{2 \times 1.4 \times 2 \times 10^{-3}} = 1.071 \times 10^{-4} \text{ radian} = 22''.$$

4.7 Interference fringes are produced by monochromatic light of wavelength 5460 Å, when a thin sheet of transparent material of thickness 6.3×10^{-4}cm is introduced in the path of one of the interfering beams, the central fringe shifts of a position occupied by 6th bright fringe. Compute refractive index of the sheet. **[GGSIPU, Sept. 2008 (4 marks)]**

Hint : $(\mu - 1)t = n\lambda$

or $\mu = \frac{n\lambda}{t} + 1 = \frac{6 \times 5460 \times 10^{-8}}{6.3 \times 10^{-4} \text{ cm}} + 1 = 1.52$

4.8 Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of refractive index, $(m) = 1.42$. What is the least thickness of the film that will disappear (*i*) dark, (*ii*) bright. **[GGSIPU, Dec. 2010 (3.5 marks)]**

Hint : Given $(\mu) = 1.42$ and $\lambda = 5893$ Å

(*i*) For the film to appear bright in reflected light at normal incidence :

$$2\mu t = (2n + 1)\lambda / 2 \quad \Rightarrow \quad t = (2n + 1)\lambda / (4\mu)$$

For least thickness, $n = 0$, $t = \lambda / (4\mu) \quad \Rightarrow \quad t = 1037.5$ Å (*ii*) For the film to appear bright in reflected light at normal incidence :

$$2\mu t = n\lambda \quad \Rightarrow \quad t = n\lambda/(2\mu)$$

For least thickness, $n = 1$, $t = \lambda/(2\mu) \Rightarrow t = 2075$ Å

4.9 Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 fringes are observed between these edges in sodium light for normal incidence, find thickness of the wire. **[GGSIPU, Sept. 2010 (2 marks)]**

Hint : Given $N = 20$, λ = wavelength of sodium = 589 nm = 5.89×10^{-7} m.

Let t be the thickness of the wire and l, the length of the wedge as shown in Fig. 4.46.

The angle of wedge $\theta = t/l$,

Fringe width in air wedge $\beta = \lambda/2\theta = \lambda l/2t$

If N fringes are seen, that $l = N\beta$;

$\therefore \quad \beta = \lambda N\beta/2t$

$\Rightarrow \quad t = N\lambda/2 = 5.89 \times 10^{-6}$ m.

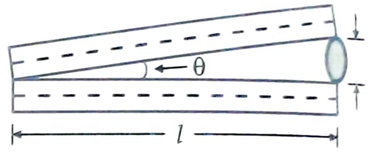


Fig. 4.46

4.10 An interference pattern is first obtained using a bi-prism set up. When a thin sheet of glass ($\mu = 1.5$) o 5 mm thickness is introduced in the path of one of interfering rays, the central fringe is shifted to position normally occupied by the fifth fringe. Calculate the wavelength of light used.

Hint : $\lambda = \dfrac{(\mu - 1)t}{n} = \dfrac{(1.5 - 1) \times (5 \times 10^{-6})}{5}$ $5 \times 10^{-7}\text{ m} = 500$ nm.

4.11 In Newton's ring experiment the diameters of the 4th and 12th bright rings are 0.4 cm and 0.7 cm respectively. Deduce the diameter of 20th bright ring.

[GGSIPU, Dec. 2011 (3 marks), Dec. 2013 (4.5 marks

Hint : $$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR} \qquad ...(i)$$

and $$D_n^2 = 4n\lambda R \qquad ...(ii)$$

From Eq. (*i*) $4\lambda R = \dfrac{D_{m+n}^2 - D_n^2}{m}$, then $D_n^2 = \dfrac{n}{m}(D_{m+n}^2 - D_n^2)$

$$D_{20}^2 = \frac{20}{8} \times [(0.7)^2 - (0.4)^2] = \frac{20}{8} \times 1.1 \times 0.3 = 0.91 \text{ cm.}$$

4.12 In a Newton's ring experiment the diameter of the 13th ring was found to be 0.590 cm and that of th 3rd ring was 0.336 cm. If the focal length of the plano-convex lens is 50 cm, calculate the wavelengt of light used. **[GGSIPU, Sept. 2008 (2 marks**

Hint : $$\left(\frac{1}{f}\right) = (\mu - 1)\left(\frac{1}{R}\right) \quad \Rightarrow \quad R = (\mu - 1) f$$

$$\therefore \quad R = 0.5 \times 50 = 25 \text{ cm.}$$

$$\therefore \quad \lambda = \frac{D_{m+n}^2 - D_n^2}{4mR} = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 25} = 22520 \text{ Å}$$

Note wavelength of light is too much being wrong data.

4.13 Newton's rings are formed by a light of wavelength 4000 Å.

(i) Between the 3rd and 6th bright fringe, what is the change in thickness of the air film ?

(ii) If the radius of curvature of the curved surface is 5.0 cm, what is the radius of 3rd bright fringe ?

[GGSIPU, Dec. 2019 (4 marks)]

Hint : (i) $2t = \frac{r_n^2}{R} \quad \Rightarrow \quad t_n = (2n+1)\frac{\lambda}{4}$

$$t_3 = 7 \times 10^{-7}\,\text{m}, \qquad t_6 = 13 \times 10^{-7}\,\text{m}$$

Spacing $\Delta t = t_3 \sim t_6 = 60\ \mu\text{m}$

(ii) $r_n = \sqrt{\frac{(2n+1)\lambda R}{2}} = 2.65 \times 10^{-4}$ cm

4.14 A drop of liquid of volume 0.2 cm^3 is dropped on the surface of the tank water of area 1 m^2. The drop spreads uniformly over the whole surface. White light is incident normally on the surface. The spectrum contains one dark band whose centre gas wavelength 5500 Å in air. Find the refractive index of the liquid.

[GGSIPU, Dec. 2019 (2.5 marks)]

Hint : $t = \frac{0.2\ \text{cm}}{100 \times 100} = 2 \times 10^{-5}\text{cm}$

$2\mu t \cos r = n\lambda$ and $\mu = \frac{n\lambda}{2t} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1} = \mathbf{1.375}$

4.15 Two wavelengths of light λ_1 and λ_2 are sent through a Young's double slit experiment set up simultaneously. What must be true concerning λ_1 and λ_2 if the third order λ_1 bright fringe is to coincide with the fourth order λ_2 fringe ?

[GGSIPU, Dec. 2016 (3 marks)]

Hint : $\beta_1 = \beta_2 \quad \Rightarrow \quad \frac{3\lambda_1 D}{2d} = \frac{4\lambda_2 D}{2d} \quad \Rightarrow \quad \lambda_2 = \frac{3}{4}\lambda_1$

4.16 Newton's rings are obtained by source emitting light of wavelength $\lambda_1 = 6000$ Å and $\lambda_2 = 5000$ Å. It is found that the n^{th} dark ring due to λ_1 coincides with $(n-1)^{th}$ dark ring due to λ_2. If the radius of curvature of convex surface is 90 cm, calculate the diameter of n^{th} dark ring of λ_1.

[GGSIPU, Sept. 2009 (3 marks)]

Hint : $D_n = 4n\lambda_1 R = 4(n+1)\lambda_2 R \quad \Rightarrow \quad n\lambda_1 = (n+1)\lambda_2 \quad \Rightarrow \quad n = 5$

and then $D_n = \sqrt{4n\lambda_1 R} = 3.26$ nm.

4.17 Michelson interferometer experiment is performed with a source which consists of two wavelength 4882 Å and 4886 Å. Through what distance does the mirror have to be moved between two position of disappearance of fringes ?

Hint : $(\Delta\lambda) = \frac{\lambda^2}{2(x_2 - x_1)} = \frac{\lambda_1\lambda_2}{2(x_2 - x_1)}$

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{(4882 + 4886)\,\text{Å}}{2} = 4884\,\text{Å} = 4884 \times 10^{-10}\,\text{m}$$

and $\Delta\lambda = (4886 - 4882)\,\text{Å} = 4 \times 10^{-10}$ meter.

Then $l = \frac{\lambda^2}{2\Delta\lambda} = \frac{(4884 \times 4884) \times 10^{-10} \times 10^{-10}}{2 \times 4 \times 10^{-10}} = 3.519 \times 10^{-4}\,\text{m} = 0.352$ mm.

4.18 In a Michelson interferometer 100 fringes cross the field of view when the movable mirror is displaced by 0.022948 mm. Calculate the wavelength of monochromatic light. **[GGSIPU, Sept. 2009 (3 marks)]**

Hint : $\lambda = \dfrac{2l}{m} = \dfrac{2 \times 0.022948 \times 10^{-3}}{100}\ \text{m} = 4.459 \times 10^{-7}\ \text{m} = 4459\ \text{Å}.$

4.19 The initial and final readings of a Michelson's interferometer screw are 10.7347 mm and 10.7051 mm respectively when 100 fringes pass through the field of view. Calculate the wavelength of light used.

Hint : $x_2 - x_1 = l = n\dfrac{\lambda}{2}$ or $\lambda = \dfrac{2l}{n} = \dfrac{2[10.7347 - 10.7051] \times 10^{-3}}{100} = 5920\ \text{Å}.$

4.20 When a thin glass plate ($\mu = 1.5$) is introduced in one of arms of Michelson interferometer using light of wavelengths 5890 Å, there is a shift of 10 fringes. Calculate the thickness of the plate.

Hint : $t = \dfrac{n\lambda}{2(\mu - 1)} = \dfrac{10 \times 5890 \times 10^{-10}}{2 \times 0.5} = 5.89 \times 10^{-6}\ \text{m}.$

4.21 In a Michelson interferometer 200 fringes cover the field of view when the movable mirror is displaced through 0.0589 mm. Calculate the wavelength of monochromatic light used.

Hint : $2(x_2 - x_1) = n\lambda$

$$\Rightarrow \quad \lambda = \frac{2(x_2 - x_1)}{n}$$

$$= \frac{2 \times 5.89 \times 10^{-5}}{200} = 5.89 \times 10^{-7}\ \text{m} = 5890\ \text{Å}.$$

4.22 Sodium light ($\lambda = 5893$ Å) is used first in a Fresnel's Bi-prism set up. A total of 60 fringes are observed in the field of view of the eye-piece. Calculate the number of fringes that would be observed in the same field of view if the sodium light is replaced by mercury vapour lamp ($\lambda = 5460$ Å).

[GGSIPU, Dec. 2016 (4 marks)]

Hint : $\beta_S = \dfrac{D\lambda_S}{d},\ \beta_M = \dfrac{D\lambda_M}{d}$

$$\frac{\beta_S}{\beta_M} = \frac{\lambda_S}{\lambda_M} = \frac{5893}{5460}$$

Given $60\beta_S = n\beta_M$ $\quad \Rightarrow \quad n = 60 \times \dfrac{\beta_S}{\beta_M} = 60 \times \dfrac{5893}{5460} = 65.$

Example 5.5 *How many orders will be visible if the wavelength of an incident radiation is* 5000 Å *and number of lines on the grating is* 2620 *per inch ?* **[GGSIPU, Dec. 2013 reappear (2 marks)]**

Solution. Given $\lambda = 5000\ \text{Å} = 5.0 \times 10^{-7}$ cm,

$$N = 2620 \text{ LPI, then grating element } (a+b) = \frac{2.54}{2620} \text{ cm,}$$

We know grating formula $(a+b)\sin\theta = n\lambda$ (for highest order $\theta = 90°$), then $(a+b) = n\lambda$

or
$$n = \frac{(a+b)}{\lambda}$$

$$= \frac{2.54}{2620} \times \frac{1}{5.0 \times 10^{-5}} = 19.38 = 19$$

which is ...

Example 5.7 A parallel beam of light is made incident on a plane transmission diffraction grating of 15000 lines per inch and angle of 2nd order diffraction is found to be 45°. Calculate the wavelength of light used.

[GGSIPU, Dec. 2015 reappear (4.5 marks)]

Solution. Given : $N = 15000$ lines/inch $= \frac{15000}{2.54}$ lines/cm,

$$n = 2, \quad \theta = 45°, \quad \lambda = ?$$

We know the grating formula,

$$(a+b)\sin\theta = n\lambda$$

or

$$\lambda = \frac{(a+b)\sin\theta}{n} \qquad ...(i)$$

$$(a+b) = \frac{1}{N} = \frac{2.54}{15000} \text{ cm} \qquad ...(ii)$$

Putting Eq. (*ii*) in Eq. (*i*), we get

$$\lambda = \frac{2.54 \sin 45°}{15000 \times 2} = 5.987 \times 10^{-5} = 5987 \text{ Å}$$

Example 5.8 A plane transmission grating has 15000 lines per inch. What is the highest order of the spectra which can be observed for wavelength 6000 Å? If opaque spaces are exactly two times the transparent spaces, which order of spectra will be absent ?

[GGSIPU, Dec. 2015 reappear (3 marks)]

Solution. $N = 15000$ lines/inch

$$(a+b) = \frac{2.54}{15000} \text{ cm} ; \quad \lambda = 6000 \text{ Å} = 6.000 \times 10^{-5} \text{ cm}$$

We know the grating formula

$$(a+b)\sin\theta = n\lambda$$

For highest order, $\sin\theta = 1$

$$n = \frac{(a+b)}{\lambda} = \frac{2.54 \times 10^5}{15000 \times 6} = 2.8 \cong 3 \text{ (approximately)}$$

Hence the third order is highest order visible.

For just resolved the $(\theta_n + d\theta)$ corresponds the direction of first secondary minimum after nth primary maximum at P_1 of wavelength λ. So we introduce extra path difference, so the extra path difference $= \frac{\lambda}{N}$.

where N = number of lines on grating surface

So, $$(a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N} \qquad ...(5.62)$$

From Eqs. (5.61) and (5.62),

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$\boxed{\frac{\lambda}{d\lambda} = nN} \qquad ...(5.63)$$

Equation (5.63) is nothing but the expression for resolving power for grating. From Eq. (5.63), it is clear that *resolving power is directly proportional to order of spectrum and number of lines in grating surface.*

For central maxima $n = 0$, hence resolving power is zero. As the dispersive power of a grating is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \qquad ...(5.64)$$

Therefore the resolving power of a diffraction grating may be expressed as

$$\frac{\lambda}{d\lambda} = Nn = N(a+b)\cos\theta\frac{d\theta}{d\lambda} \qquad ...(5.65)$$

Resolving power of grating = Total aperture × dispersive power. ...(5.66)

Example 5.10 *What is the least separation between wavelengths that can be resolved near 640 nm in the second order, using diffraction grating that is 5 cm wide and ruled with 32 lines per millimetre.*

[GGSIPU, Oct. 2013 (2 marks)]

Solution. Given $\lambda = 640$ nm, $n = 2$, $N = 32 \times 50 = 1600$, $d\lambda = ?$

We know resolving power of grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

$$d\lambda = \frac{\lambda}{nN}$$

$$= \frac{640 \times 10^{-9}\text{ m}}{2 \times 1600} = \frac{6400}{3200} \times 10^{-10}\text{ m}$$

$$= 2 \times 10^{-10}\text{ m} = 2\text{ Å}$$

$$\because \quad a+b = 5.0 \times 10^{-5}\text{m}$$

$$a = 2.5 \times 10^{-5}\text{m}$$

Then $$a = b = 2.5 \times 10^{-5}\text{m}$$

(*i*) *Angle of diffraction*

We know that the grating equation

$$(a+b)\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{(a+b)} = \frac{3\times 6\times 10^{-5}}{5\times 10^{-3}}$$

$$\theta = \sin^{-1}(0.036) = 2.06^\circ$$

(*ii*) *The condition for absent spectra*

$$\frac{a+b}{a} = \frac{m}{n}$$

if $$a = b$$

then $$m = 2n$$

i.e., second, fourth, ... order spectra will be absent.

Problem 5.7 *A plane transmission grating having 6000 lines per cm used to obtain a spectrum of light from a sodium light in the second order. Find the angular separation between the two sodium lines* ($\lambda_1 = 5890$ Å *and* $\lambda_2 = 5896$ Å).

[GGSIPU, Dec. 2017 (5.5 marks)]

Solution. For diffraction grating,

$$(a+b) = \frac{1}{6000}\text{ cm} = \frac{1}{6000\times 100}\text{ m}, \quad \lambda_1 = 5.890\times 10^{-7}\text{ m}, \quad \lambda_2 = 5.896\times 10^{-7}\text{ m}$$

$(\theta_2 - \theta_1)$ = angular separation between two spectral lines = ?

$\because$ Condition for maxima,

$$(a+b)\sin\theta = n\lambda$$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\sin\theta_1 = \frac{n\lambda_1}{(a+b)}$$

$$\Rightarrow \quad \theta_1 = \sin^{-1}\left[\frac{2\times 5.890\times 10^{-7}\times 6000\times 100}{1}\right] = 44^\circ 59'$$

and $$(a+b)\sin\theta_2 = n\lambda_2$$

$$\sin\theta_2 = \frac{n\lambda_2}{(a+b)}$$

$$\Rightarrow \quad \theta_2 = \sin^{-1}\left[\frac{2\times 5.896\times 10^{-7}\times 6000\times 100}{1}\right] = 44^\circ 61'$$

Hence $\theta_2 - \theta_1 = 2'$.

Problem 5.8 *Deduce the missing order for double slits Fraunhofer diffraction pattern, if the slit widths 0.16 mm and they are 0.8 mm apart.*

[GGSIPU, Sept. 2011 (2 marks)]

Solution. Given that

$$a = 0.16 \text{ mm} \quad ; \quad b = 0.8 \text{ mm}$$

If a be the slit width and b the separation between slits ; the condition of missing order spectra is given by

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\frac{0.16+0.8}{0.16} = \frac{n}{m}$$

$$\frac{0.96}{0.16} = \frac{n}{m}$$

$$n = 6\,m = 6,\ 12,\ 18,\ \ldots\ (m = 1, 2, 3, \ldots)$$

Thus 6th, 12th, 18th, ... orders will be missing.

Problem 5.9 *How many lines per cm are there in a grating which gives an angle of diffraction of 30° in first*

Problem 6.14 *A 20 cm long tube containing sugar solution is placed between crossed Nicols and illuminated with light of wavelength 6×10^{-5} cm. If the specific rotation is 60°/dm/gm/cm^3 and optical rotation produced is 12°, determine the strength of the solution.*

[GGSIPU, Sept. 2011 (2 marks) ; Jan 2015 (3 marks)]

Solution. The specific rotation S of a solution is given by

$$[S]_T^{\lambda} = \frac{\theta}{l \times C}$$

Here, $\theta = 12°$, $l = 2.0$ dm and $S = 60/\text{dm/gm/cm}^3$

$$\therefore \qquad C = \frac{12}{2.0 \times 60} = 0.1 \text{ gm/cc} = 10\%.$$

4×0.161

Problem 6.5 *A plane polarised light is incident on a quartz plate cut parallel to the axis. Calculate the least thickness of the plate for which the o- and e-rays combine to form plane polarised light.*
Assume that $\mu_e = 1.5533$ and $\mu_0 = 1.5442$ and $\lambda = 5.4 \times 10^{-5}$ cm **[GGSIPU, Dec. 2015 (2 marks)]**

Solution. In this case the quartz plate must act as half wave plate. Thus if t be the required thickness then we have

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

or

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

Putting the given values, we get

$$t = \frac{5.4 \times 10^{-5}\ \text{cm}}{2(1.5533 - 1.5442)}$$

$$= \frac{5.4 \times 10^{-5}\ \text{cm}}{2 \times 0.0091} = 3 \times 10^{-3}\ \text{cm.}$$

Example 6.11 *A 5% solution of cane sugar placed in a tube of length 40 cm, causes the optical rotation of 20°. How much length of 10% solution of the same substance will causes 35° rotation ?*

[GGSIPU, Sept. 2012 (3 marks)]

Solution. Given : $C_1 = 5\% = 0.05$; $l_1 = 40$ cm ; $\theta_1 = 20°$

$C_2 = 10\% = 0.1$; $l_2 = ?$; $\theta_2 = 35°$

We know the specific rotation $[S]_T^\lambda = \dfrac{\theta}{l \times C}$

According to problem,

$$S = \frac{\theta_1}{l_1 \times C_1} = \frac{\theta_2}{l_2 \times C_2}$$

or $$l_2 = \left(\frac{\theta_2}{\theta_1}\right) \times \left(\frac{C_1}{C_2}\right) \times l_1 = \frac{35}{20} \times \frac{0.05}{0.10} \times 40 \text{ cm} = 35 \text{ cm}.$$

6.11 Two Nicols are oriented with their planes making an angle of 60°. What percentage of incident unpolarised light will pass through the system ?

[GGSIPU, Sept. 2004 (5 marks), Sept. 2008, Oct. 2013 (3 marks) ; Dec. 2017 (3 marks)]

Hint : If unpolarised light incidents on a polariser, the intensity of light transmitted through the polariser.

$$\frac{I_0}{2} = I_1 \quad \text{and} \quad I = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 60° = 0.125\, I_0 = 12.5\% \text{ of } I_0.$$

6.12 A polariser and an analyser are oriented so that maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of transmitted light reduced when the analyser is rotated through 22.5° ? **[GGSIPU, Dec. 2008 (4 marks)]**

Hint : According to Malus law, intensity of transmitted light through analyser is given by

$$I = I_0 \cos^2 \theta = I_0 \cos^2 22.5 = 0.85\, I_0$$

Thus its maximum intensity is reduced to 85% of the maximum.

6.13 What will be the state of polarisation of the emerging light when

(*i*) A beam of circularly polarised light is passed through a quarter wave plate (QWP) ;

[GGSIPU, Sept. 2012 (2 marks)]

(*ii*) A beam of plane polarised light is passed through a quarter wave plate ;

(*iii*) A beam of elliptically polarised light is passed through quarter wave plate ; and

(*iv*) A beam of plane polarised light is passed through a QWP such that the plane polarised light falling on QWP makes an angle of 45° with optic axis ?

Hint : (*i*) Plane/linearly polarised light. (*ii*) Circularly or elliptically or plane polarised light.

(*iii*) Plane/linearly polarised light. (*iv*) Circularly polarised light.

6.14 Calculate the thickness of (*i*) quarter wave plate and (*ii*) half wave plate, given $\lambda = 5000\,\text{Å}, \mu = 1.544$, $\mu_e = 1.553$. **[GGSIPU, Dec. 2019 (2.5 marks)]**

Hint : (*i*) QWP $t = \dfrac{\lambda}{4(\mu_e - \mu_0)}$

$$t = \frac{5000 \times 10^{-10}}{4 \times (1.553 - 1.544)} = \frac{5000 \times 10^{-10}}{4 \times 0.009}$$

$$= \frac{50}{36} \times 10^{-5}\,\text{m} = 1.39 \times 10^{-5}\,\text{m} = 0.139\,\mu\text{m}$$

(*ii*) HWP $t = \dfrac{\lambda}{2(\mu_e - \mu_0)} = 0.278\,\mu\text{m}$

6.15 Calculate the thickness of (*i*) a quarter wave plate and (*ii*) a half wave plate given that $\mu_e = 1.553$, $\mu_o = 1.544$ and $\lambda = 6328$ Å. **[GGSIPU, Dec. 2018 (2 marks)]**

Hint : (*i*) QWP $t = \dfrac{\lambda}{4(\mu_e - \mu_0)} = \dfrac{6328 \times 10^{-10}}{4 \times (1.553 - 1.544)} = 0.176\,\mu\text{m}$

(*ii*) HWP $t = \dfrac{\lambda}{2(\mu_e - \mu_0)} = \dfrac{6328 \times 10^{-10}}{2 \times (1.553 - 1.544)} = 0.352\,\mu\text{m}$

6.4 Light travelling in water ($\mu = 1.33$) is incident on a plate of glass ($\mu = 1.53$). At what angle of incidence will the light be fully polarised ?

[GGSIPU, Dec. 2009 (4 marks)]

Hint : ${}_a\mu_\omega = 1.33$ and ${}_a\mu_g = 1.53$ then ${}_\omega\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_\omega} = \dfrac{1.53}{1.33} = 1.15$

$${}_\omega\mu_g = \tan i_p \qquad \Rightarrow \qquad i_p = \tan^{-1}(1.15) = 49°\,12'.$$

6.5 What will be the Brewster's angle for a glass slab ($\mu = 1.5$) immersed in water ?

[GGSIPU, Dec. 2007 (4 marks)]

Hint : $\mu = \tan i_p \;\; i_p = d\tan^{-1} 1.5 = 56.7°.$

6.6 What will be the Brewster's angle for a glass slab ($\mu = 1.5$) immersed in water ($\mu = 1.33$) ?

[GGSIPU, Sept., 2010 ; Sept 2012 (2 marks)]

Hint : ${}_a\mu_g = 1.5,\; {}_a\mu_\omega = 1.33$ then ${}_w\mu_g = \dfrac{1.5}{1.33} = 1.128$

then ${}_w\mu_g = \tan i_p \qquad \Rightarrow i_p = \tan^{-1}({}_w\mu_g) = \tan^{-1}(1.128) = 48.5°.$

6.7 What is the polarising angle for glass whose refractive index for light used in 1.6827 ?

[GGSIPU, Sept. 2011 Reappear (2 marks)]

Hint : Using Brewster's law, we know that

$$\tan i_p = \mu \qquad \Rightarrow \qquad i_p = \tan^{-1}(1.6827) = 59°.$$

6.8 Refractive index of water is 1.33. Calculate the angle of polarisation of light reflected from the surface of the pond.

[GGSIPU, Nov. 2012 (2 marks)]

Hint : $\mu = \tan i_p \;\Rightarrow\; i_p = \tan^{-1}(1.33) = 53.06°.$

6.9 If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.

Hint : $\dfrac{I_e}{I_o} = ?,$ where I_e and I_o are the intensity of e-ray and o-ray respectively.

$$\frac{I_e}{I_o} = \frac{\cos^2\theta}{\sin^2\theta} = \frac{3}{1}.$$

6.10 Determine the state of polarization (SOP) of the following set of equations as :

$\vec{E} = E_0\,\hat{x}\cos(kz - \omega t)$ and $\vec{E} = E_0\,\hat{y}\cos(kz - \omega t + \phi)$ when (a) $\phi = \dfrac{\pi}{2}$ and (b) $\phi = \pi$.

[GGSIPU, Dec. 2018 (3 marks)]

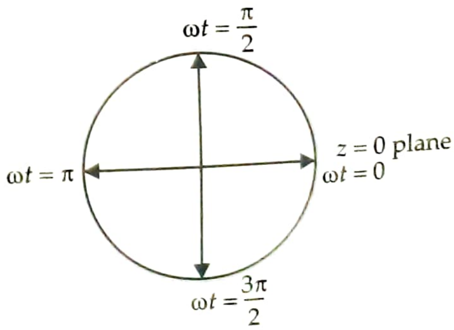
Hint : Case (a)

When $\phi = \dfrac{\pi}{2}$, $\vec{E} = E_0$ and consider a plane $z = 0$.

at time $\omega t = 0 \qquad \vec{E} = E_x\hat{x}$ and $\vec{E} = 0\hat{y}$

at time $\omega t = \dfrac{\pi}{2} \qquad \vec{E} = 0\hat{x}$ and $\vec{E} = E_0\hat{y}$

at time $\omega t = \dfrac{3\pi}{2} \qquad \vec{E} = 0\hat{x}$ and $\vec{E} = -E_0\hat{y}$



Case (b). When $\phi = \pi$, contribution of both component aligned in same direction irrespective SOP will be linearly polarized life.

Numerical Problems

6.1 The velocity of light in water is 2.2×10^8 ms^{-1}. What is the polarizing angle of incidence for water surface for water surface ? (Given the speed of light in free space = 3×10^8 ms^{-1}).

[GGSIPU, Dec. 2016 (2.5 marks)]

Hint : $\mu_{water} = \frac{c}{v} = \frac{3 \times 10^8}{2.2 \times 10^8} = 1.36 \Rightarrow \mu = \tan i_p \Rightarrow i_p = \tan^{-1}\mu = \tan^{-1}(1.36) = 53.74°.$

6.2 Calculate the Brewster angle for ethyl alcohol ($\mu = 1.46$). **[GGSIPU, Dec. 2013 reappear (2 marks)]**

Hint : $\mu = \tan i_p$ or $i_p = \tan^{-1}(\mu) \Rightarrow i_p = \tan^{-1}(1.46) = 56°.$

6.3 At what angle the light should be incident on a glass plate ($\mu = 1.5697$) to get a plane polarised light by reflection ? **[GGSIPU, Sept. 2012 (2 marks)]**

Hint : $\mu = \tan i_p$ Here $\mu = 1.5697$

Then $i_p = \tan^{-1}(1.5697) = 57.5°.$

6.23 A sugar solution in a tube of length 20 cm produces optical rotation of 13°. The solution is the diluted to one-third of its previous concentration. Find optical rotation produced lay 30 cm long tub containing the dilute solution.

[GGSIPU, Dec. 2013 (4 marks

Hint : Given $l_1 = 20$ cm, $\theta_1 = 13°$, $C_1 = C$ and $l_2 = 30$ cm, $\theta_2 = ?$ $C_2 = \frac{1}{3}C$

Now, as per question $\quad s = \dfrac{\theta_1}{l_1 \times C_1} = \dfrac{\theta_2}{l_2 \times C_2}$

$$\theta_2 = \frac{\theta_1 \times l_2 \times C_2}{l_1 \times C_1} = \frac{13° \times 30 \times C}{20 \times C \times 3} = 6.5°$$

Hint : For quarter wave plate

$$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

So $$(\mu_e - \mu_0) = \frac{\lambda}{4t} = \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-4}} = \text{[illegible]} \times 10^{-3} = 5.0 \times 10^{-4}.$$

6.17 Calculate the minimum thickness of a quarter wave plate for the light of wavelength 5893 Å. Given $\mu_0 = 1.57$ and $\mu_e = 1.526$. **[GGSIPU, Sept. 09, 10 reappear, 12 reappear, 2013 reappear (2 marks)]**

Hint : $$t = \frac{\lambda}{4(\mu_0 - \mu_e)} = 3.35\ \mu\text{m}.$$

6.18 Calculate the minimum thickness of a calcite plate which would convert plane polarised light into circularly polarised light. The principal refractive indices for the ordinary and extraordinary rays are 1.658 and 1.486 respectively at wavelength 5890 Å.

[GGSIPU, Nov. 2012 reappear ; Dec. 2011 (3.5 marks)]

Hint : This will be QWP and formula for thickness is

$$t = \frac{\lambda}{4(\mu_2 - \mu_a)} = \frac{5890 \times 10^{-8}}{4 \times 0.172} = 0.86\ \mu\text{m}.$$

6.19 Linearly polarised light is changed into circularly polarised light after passing through a slice of the crystal 2.5×10^{-5} m thick. Find the wavelength of light used, if the difference in refractive indices for ordinary and extraordinary rays is 0.005. **[GGSIPU, Nov. 2012 (2 marks)]**

Hint : Given $t = 2.5 \times 10^{-5}$ m and $(\mu_0 - \mu_e) = 0.005$.

This plate is QWP, then

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)} \quad \text{or} \quad \lambda = 4t(\mu_0 - \mu_e)$$

$$\Rightarrow \quad \lambda = 4 \times 2.5 \times 10^{-5} \times 0.005 = 5000 \times 10^{-10}\ \text{m} = 5000\ \text{Å}$$

6.20 Calculate the thickness of half wave plate for sodium light ($\lambda = 5893$ Å), if $\mu_0 = 1.54$ and ratio of velocity of ordinary and extraordinary waves is 1.007. Is this crystal is positive or negative ? **[GGSIPU, Dec. 2010 (3.5 marks)]**

Hint : Given $\lambda = 5893$ Å, $\mu_0 = 1.54$, $v_e / v_0 = 1.007$

We know that $$\frac{\mu_0}{\mu_e} = \frac{v_e}{v_0} \quad \Rightarrow \quad \mu_e = 1.54 \times 1.007 = 1.551$$

$\because\ \mu_e > \mu_0$, so crystal is positive, then $$t = \frac{\lambda}{2(\mu_e - \mu_0)} = 2.733 \times 10^{-5}\ \text{m}.$$

6.21 The plane of polarization gets rotated through 23.4° as light travels through an 18 cm long column of 20% sugar solution. Determine the specific rotation of solution. **[GGSIPU, Dec. 2016 (2.5 marks)]**

Hint : $$[s]_T^{\lambda} = \frac{\theta}{l \times C} = \frac{23.4 \times 100}{1.8 \times 20} = 65°/\text{dm}/\text{gm}/\text{cm}^3.$$

6.22 A 10 cm long tube contains 10% sugar solution and produces an optical rotation of 13.2°. Calculate the specific rotation. **[GGSIPU, Dec. 2009 (4 marks)]**

Hint : $$[S]_T^{\lambda} = \frac{\theta}{l \times C} = \frac{13.2 \times 10}{1 \times 1} = 132°/\text{dm}/\text{gm}/\text{cm}^3.$$