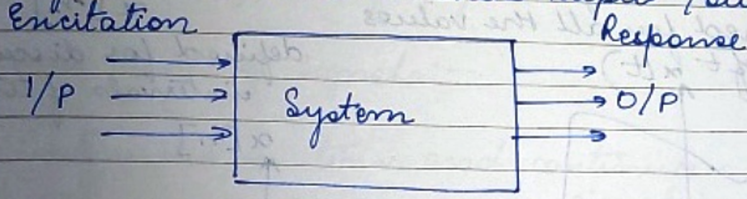
Signal :- Any time varying physical phenomenon that is intended to convey information.
E.g:- human voice, voltage on telephone lines, electrical signal, etc.
Noise is a signal which is unwanted.
Signal is a function of time. $f(t)$

System :- It is a device which operates on signal according to its characteristics.
System has one or more input/output.



E.g:- communication system

Signals may be one dimensional or multi dimensional.

when function depends only on one variable
E.g:- speech

when function depends on more than one variable.
E.g image

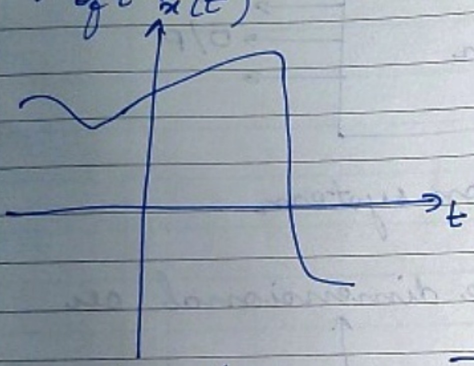
Classification of signals :-

1. Continuous and discrete time signals
2. Continuous and discrete time valued signals
3. Periodic and non periodic signals
4. Even and Odd signals
5. Deterministic and Random signal
6. Energy and Power signal.

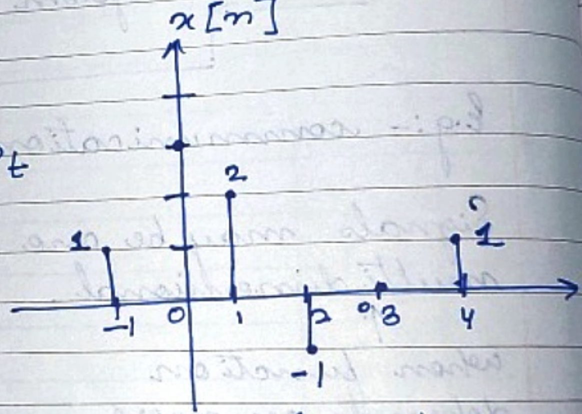
Signal is a physical quantity that varies with time, space or other independent variable or variables.

One dimensional signal e.g:- V, I

Signal
- Continuous signal $x(t)$
  defined for all the values of t
- Discrete time signal $x[n]$
  defined for discrete intervals of time



Signal



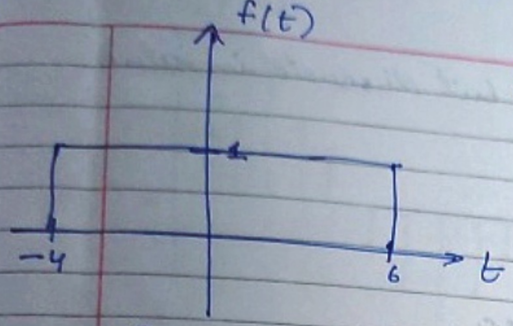
Signal and Sequence

$$x[n] = \{\overset{-1}{+1}\ \underset{\uparrow 0}{3}, \underset{1}{2}, \underset{2}{-1}, \underset{3}{0}, \underset{4}{1}\}$$

Difference between discrete and digital signal.

discrete – time is discrete, amplitude is continuous

digital – both time and amplitude is discrete.

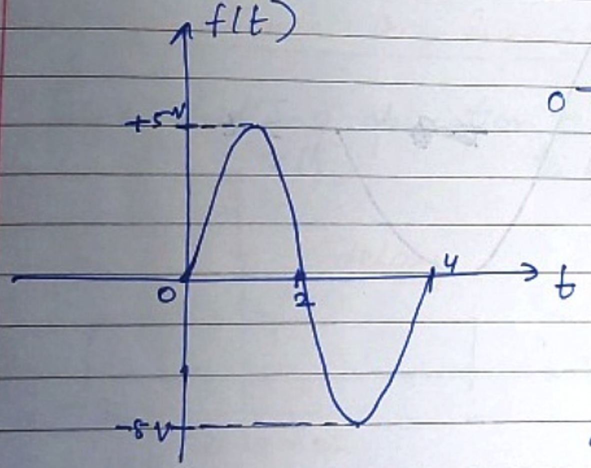


$$f(t) = \begin{cases} 1 & -4 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

System :- interconnection of diff physical components to convert one form of signal to another.

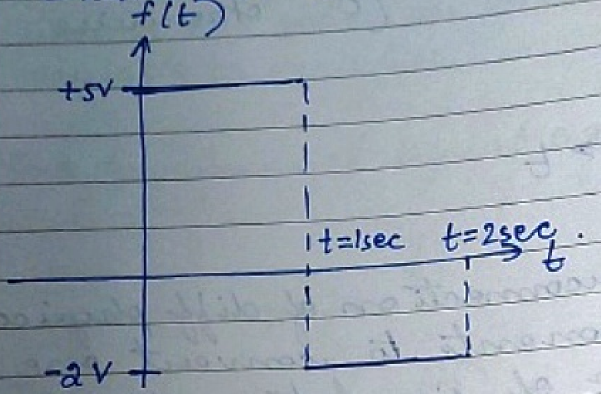
Basic types of signals:-

(1) Continuous in time and continuous in value signals:-

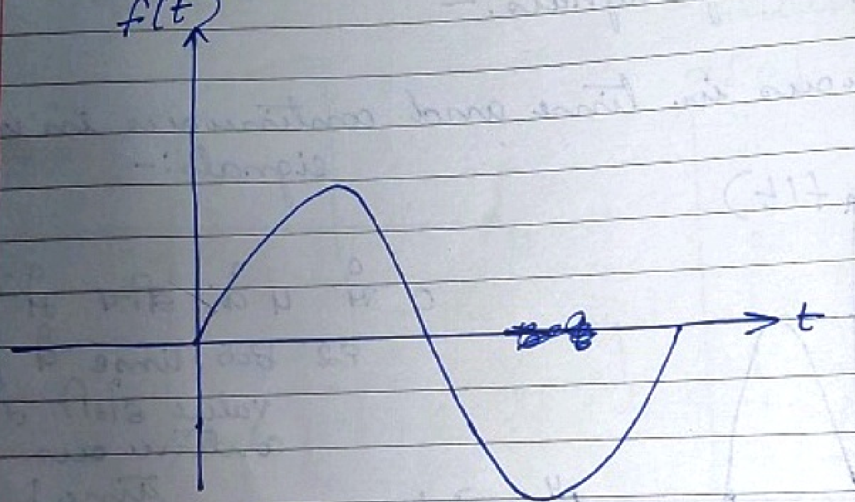


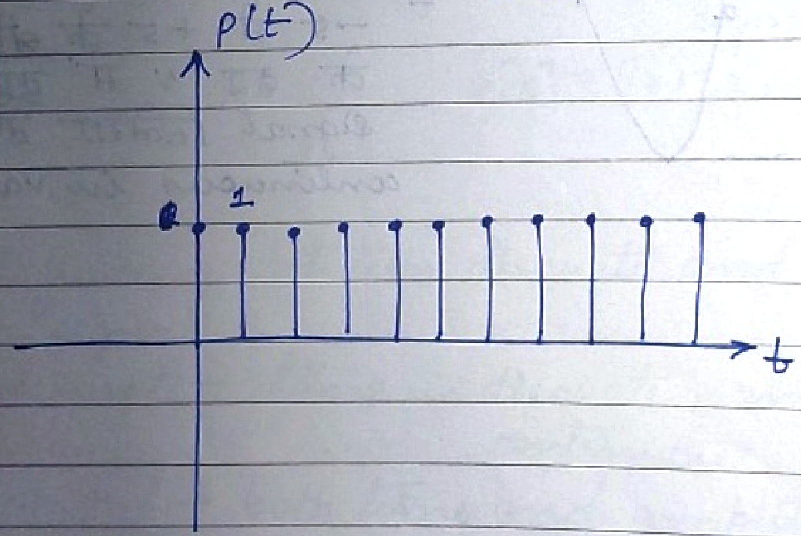
0 से 4 के बीच में हर एक time पे कोई value होगी तो continuous in time।
−5 से +5 के बीच के हर V में हमें signal मिलेगा तो continuous in value.

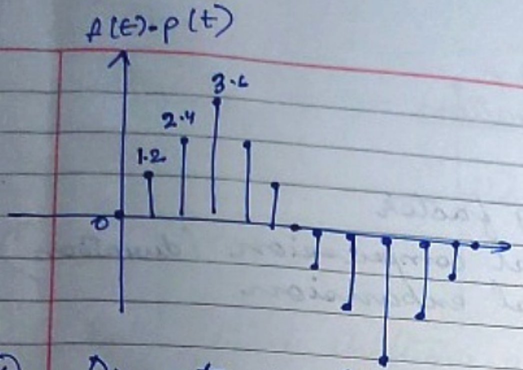
(2) Continuous in time but discrete in value.

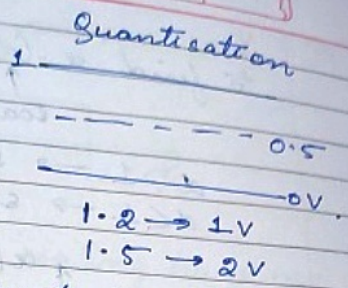


(3) Continuous in value but discrete in time

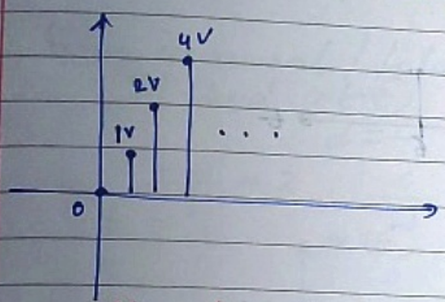






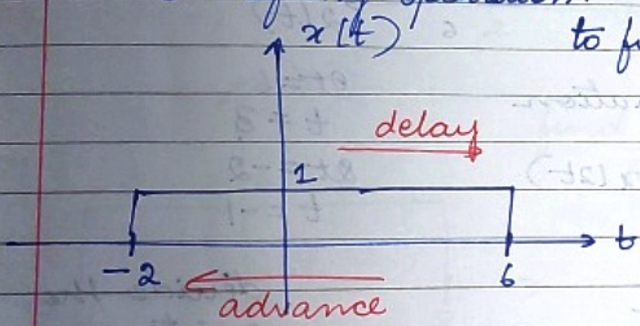


(4) Discrete in time and discrete in value:-



Operations on Signals

(1) Time shifting operation:-



to find $x(t \pm t_0)$

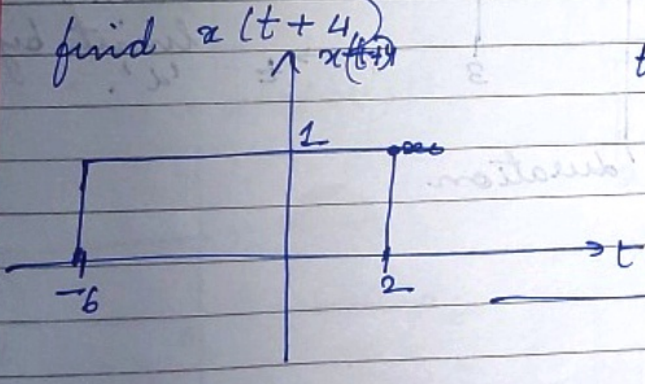
$\downarrow$

any constant

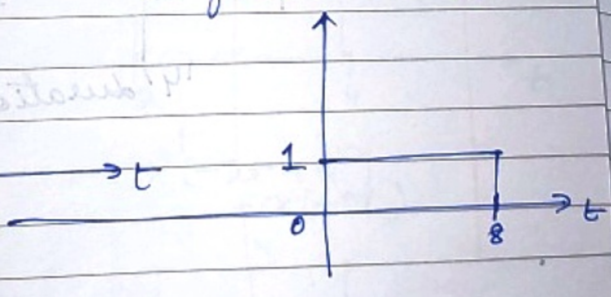
(−) → delay signal

(+) → advance signal

find $x(t+4)$



to find $x(t-2)$.



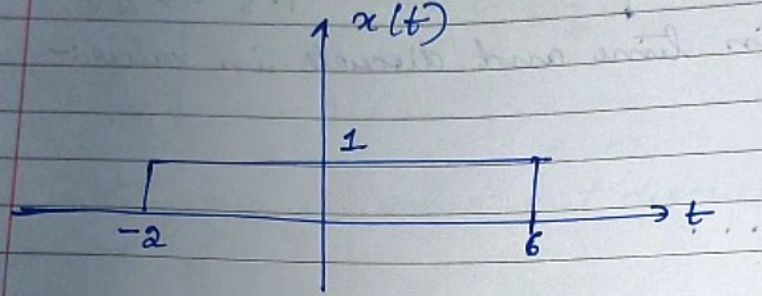
② Time scaling operation:-

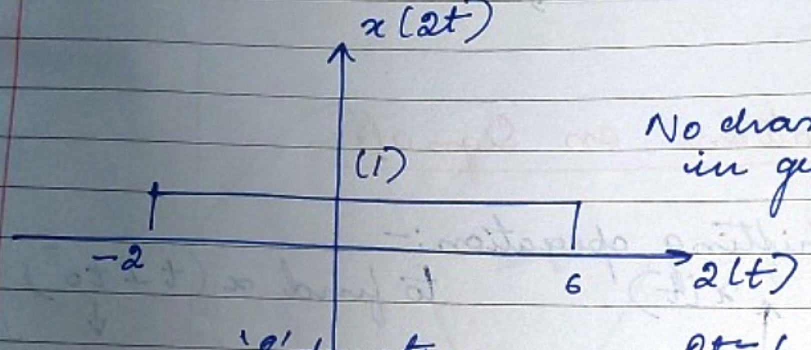
to find $x(\alpha t)$

↑ scaling factor

$\alpha > 1 \rightarrow$ signal compression (duration)

$\alpha < 1 \rightarrow$ signal expansion





No change in graph

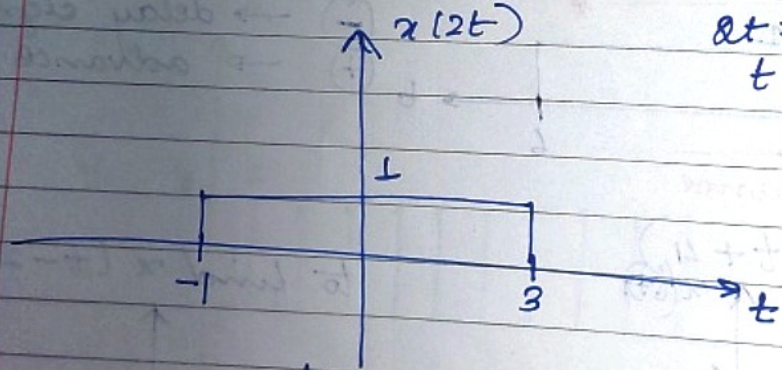
'8' duration

$2t = 6$

$t = 3$

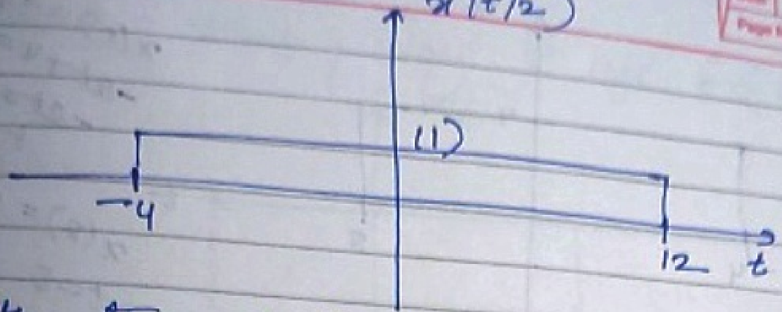
$2t = -2$

$t = -1$



divide the existing limits by 'α'.

'4' duration



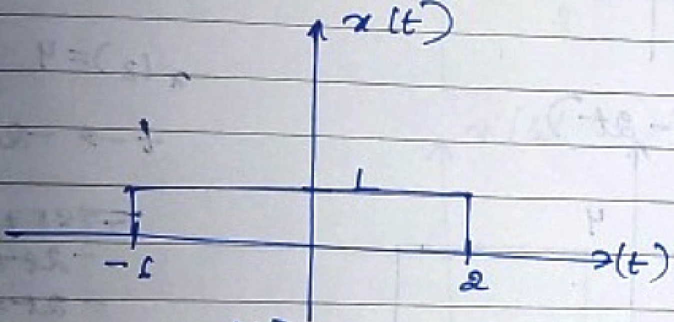
Both time shifting and scaling have no effect on amplitude.

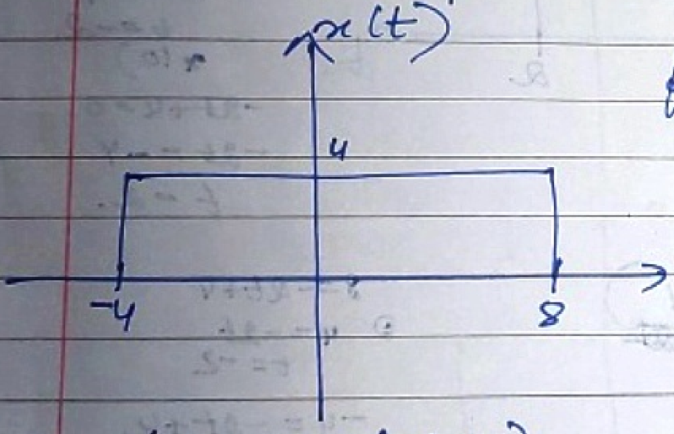
(3) Time reversal / folding operation:-

To find $x(-t)$.

$\alpha = -1$

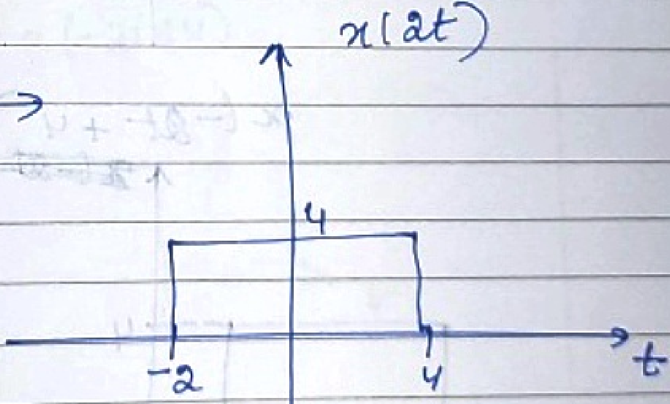
$\frac{-2}{-1} = 2 \qquad \frac{6}{-1} = -6$

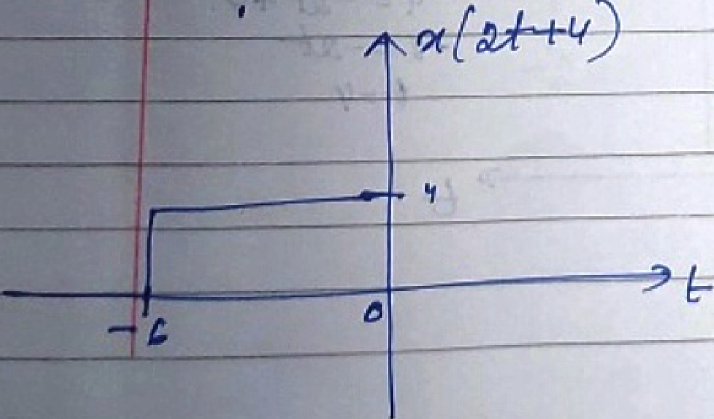


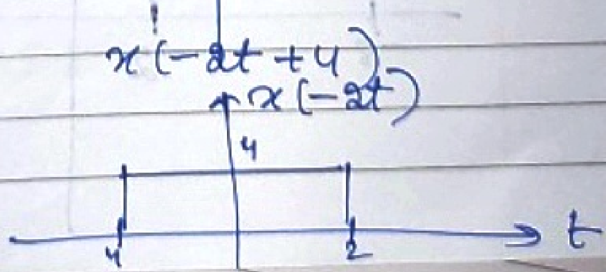


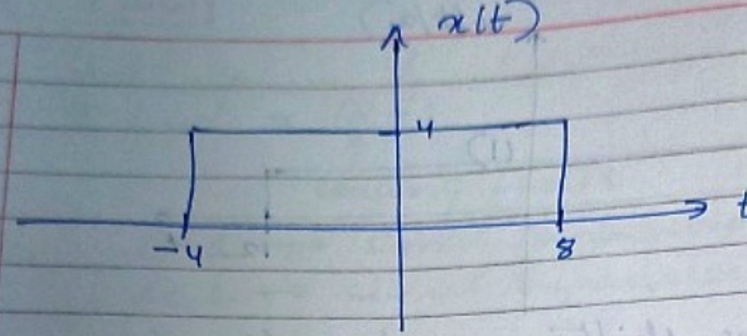
find $x(2t+4)$

$\alpha = 2$



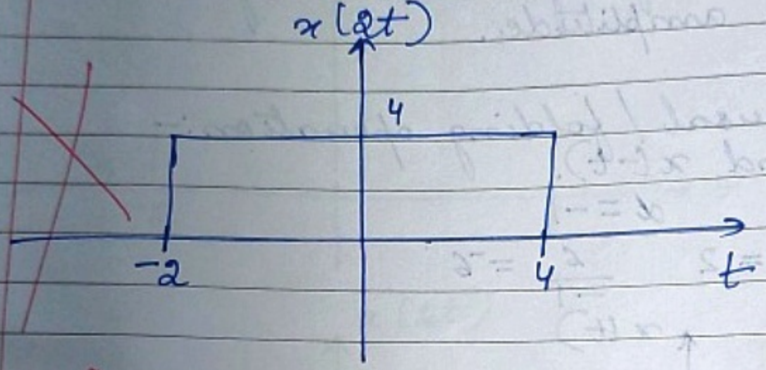




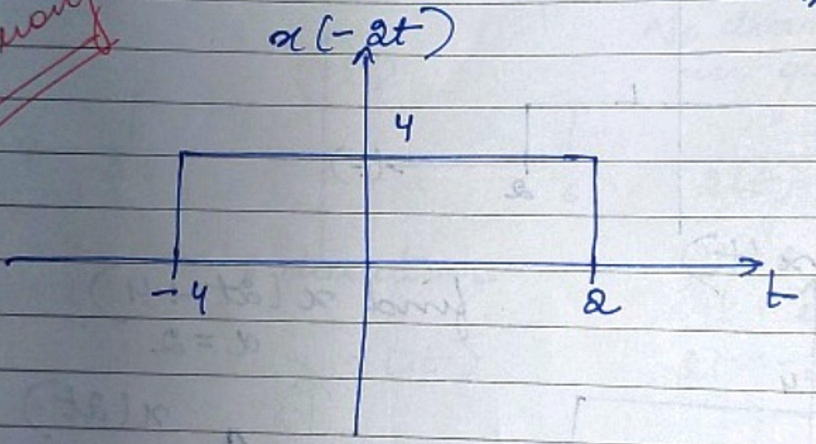


$x(0) = -4$
$-2t+4 =$
$-2t =$
t

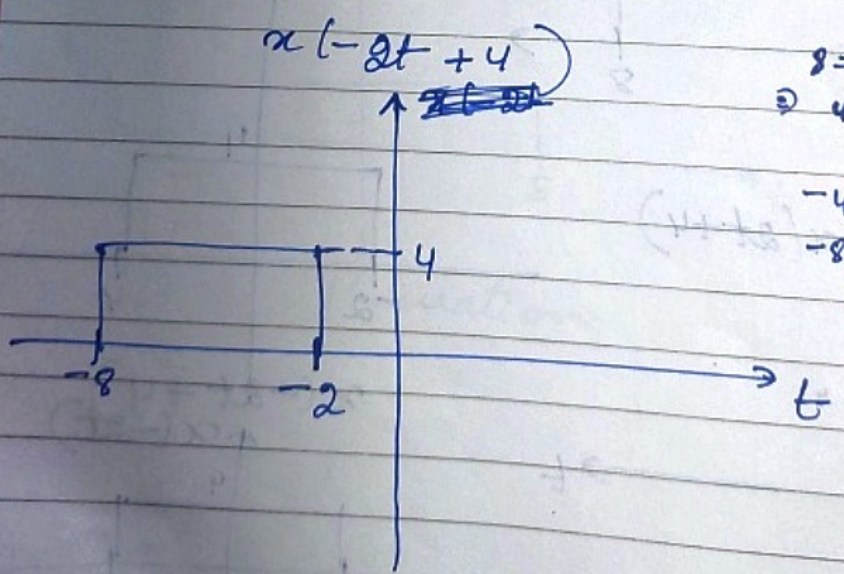
$x(8) = 4$
$t = 8$



Wrong



$x(8) = 4$
$t \rightarrow -$
-2×8
-2
-2
x
$-2t+$
$-2t =$
t

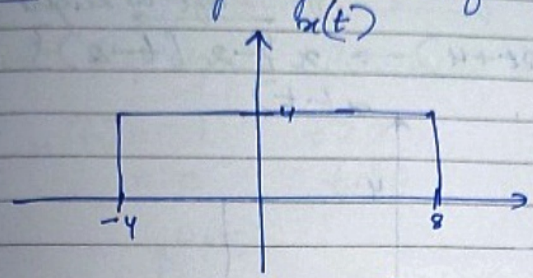


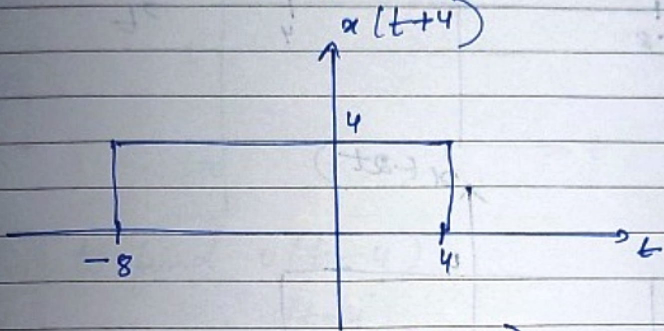
$8 = -2t+4$
$\Rightarrow 4 = -2t$
$t = -2$

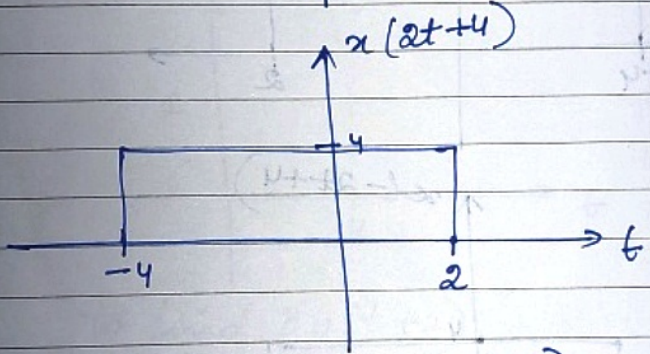
$-4 = -2t+$
$-8 = -2t$
$t = 4$

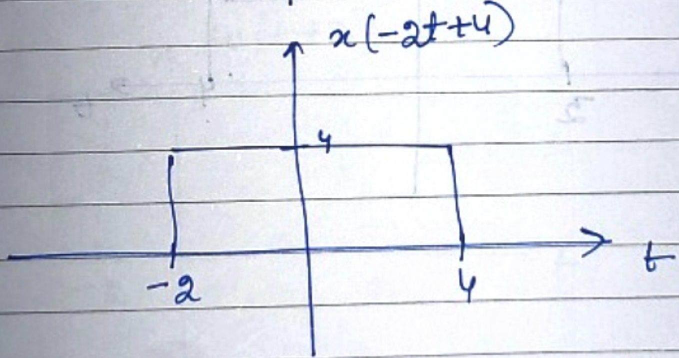
144 studen

Method:- Moving right to left $x(-2t+4)$



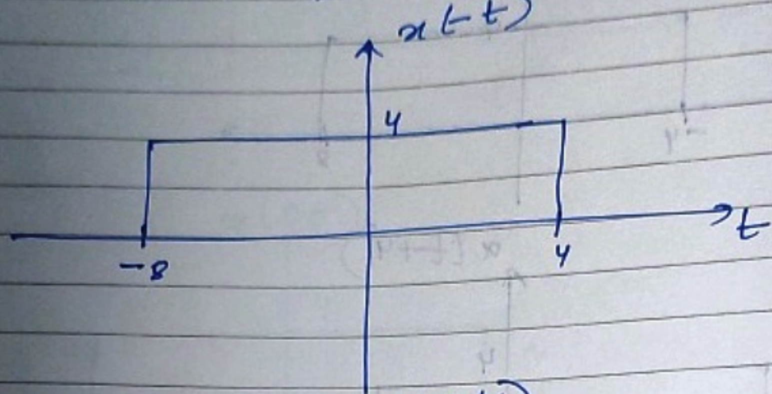


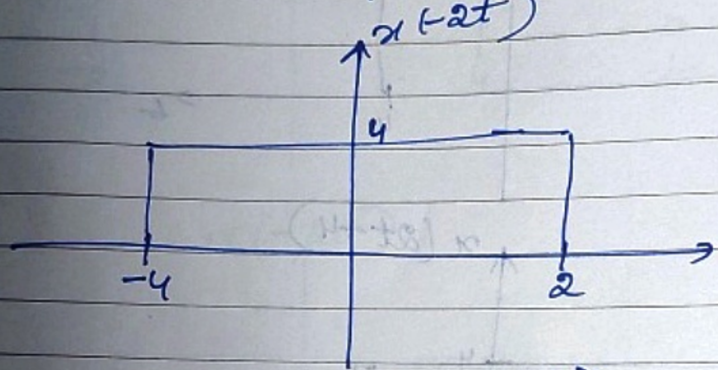


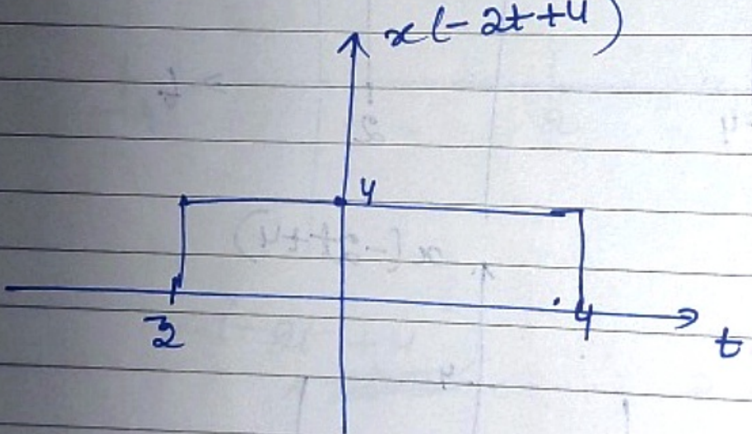


Method 2:- left to right

$$x(-2t+4) \xrightarrow{\text{left to right}} x\{-2(t-2)\}$$



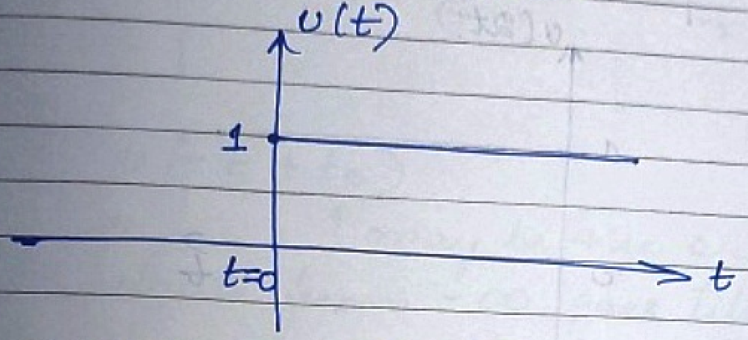




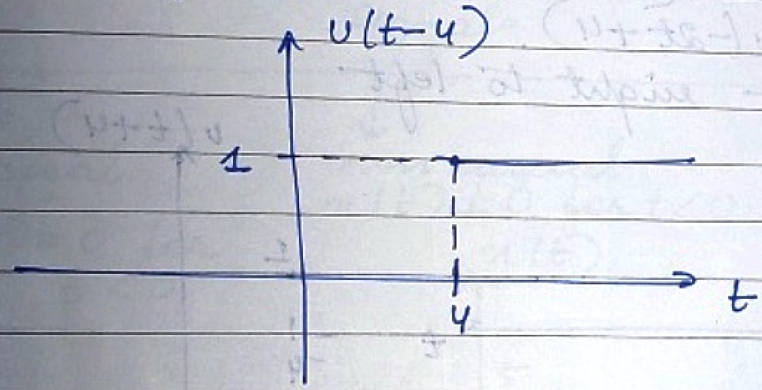
Elementary Signals

① Unit Step signal (Heaviside step function)

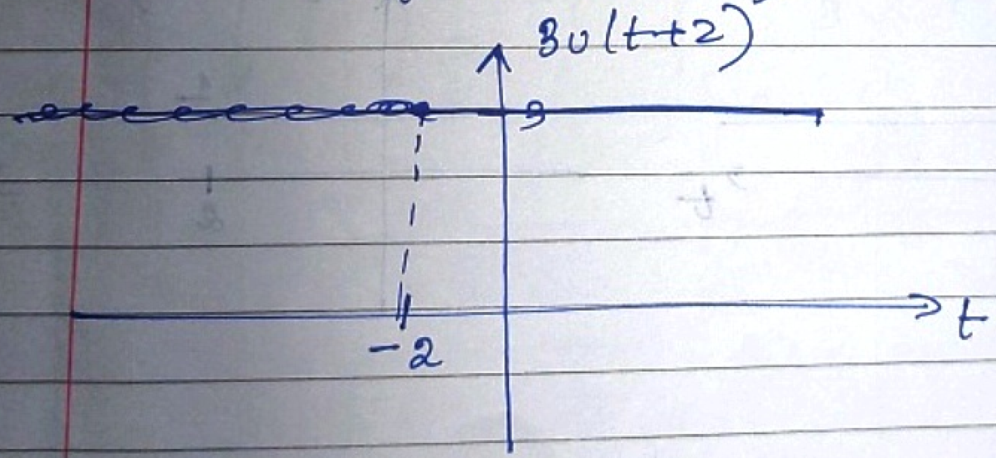
$$u(t) = 1 \quad t > 0$$
$$= 0 \quad t < 0$$



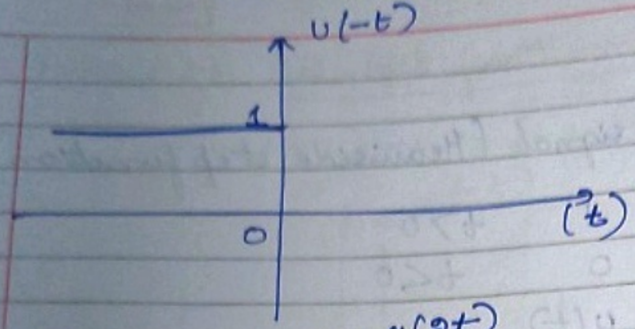
to find $u(t-4)$

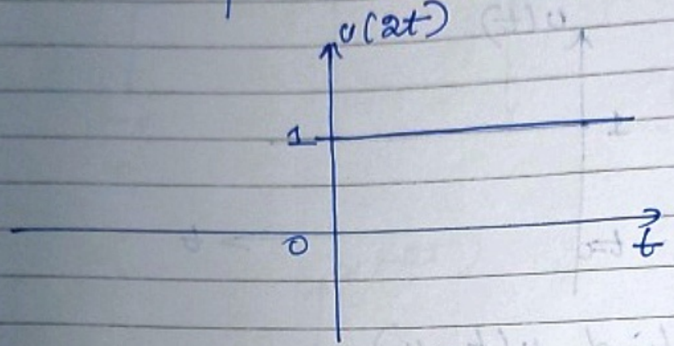


to find $3u(t+2)$



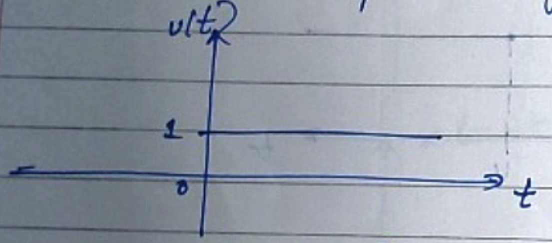
$u(-t)$

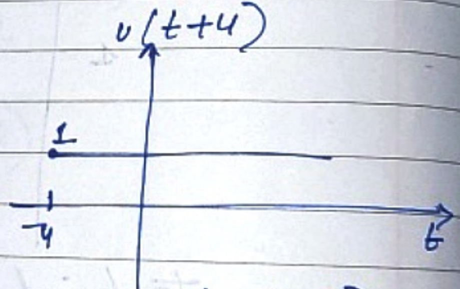


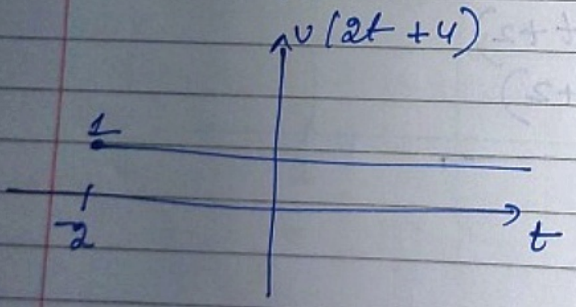


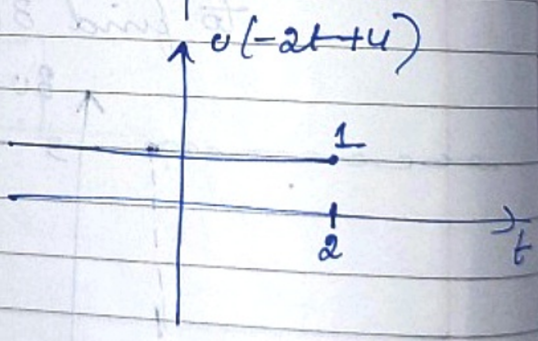
To draw $u(-2t+4)$.

Method 1:- right to left

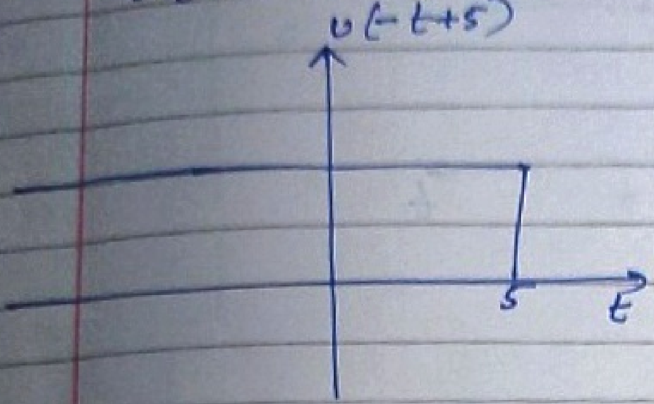




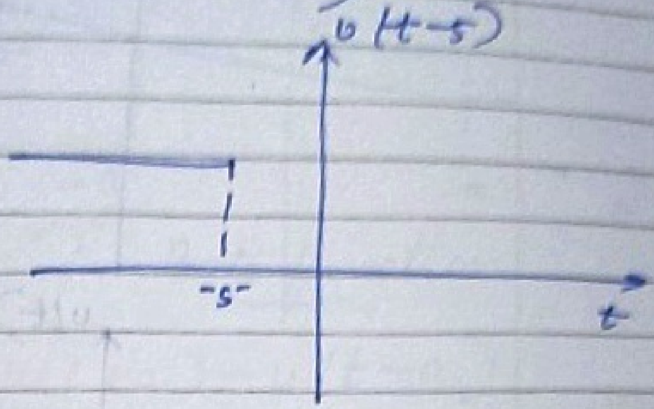




$u(-t+5)$



$u(-t-5)$



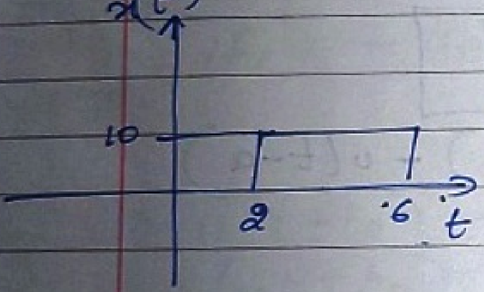
$u(-t+t_0)$

↑ may be +ve or -ve

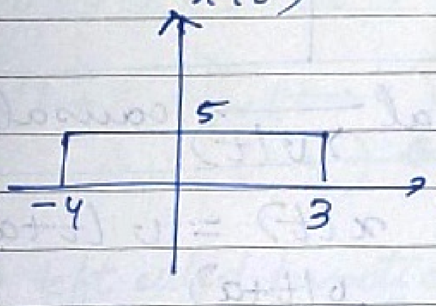
starts from $-\infty$ goes till t_0 (including sign)

Signals

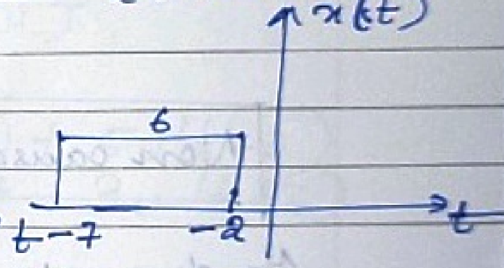
- causal
  $x(t)=0$ for $t<0$



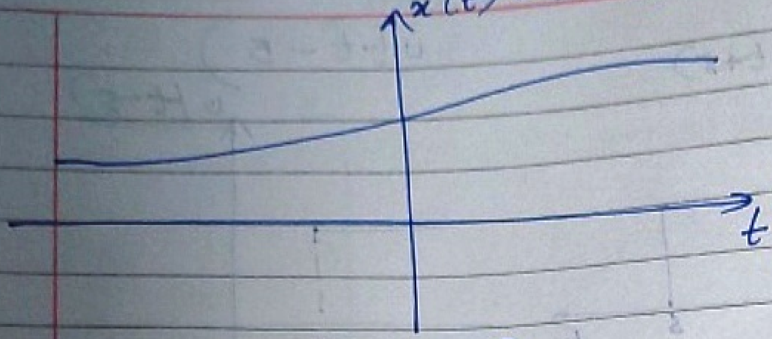
- non casual
  $x(t)\neq 0$ for $t<0$

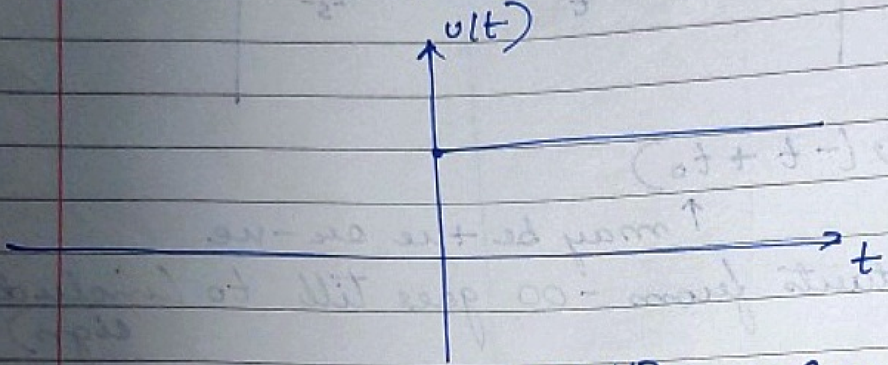


- anticausal
  $x(t)=0 \quad t>0$

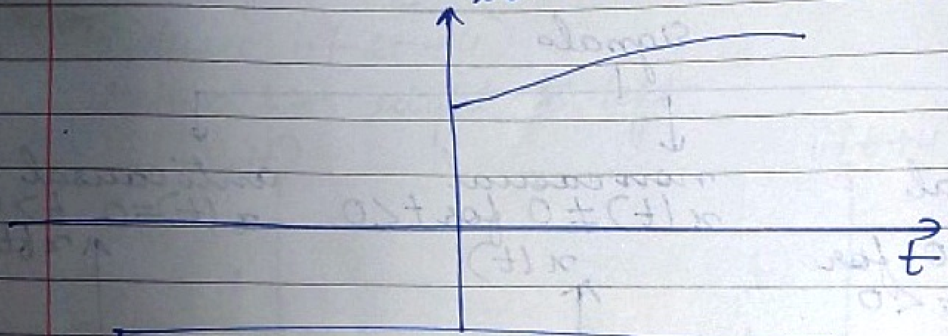


$x(t) \rightarrow$ Non causal



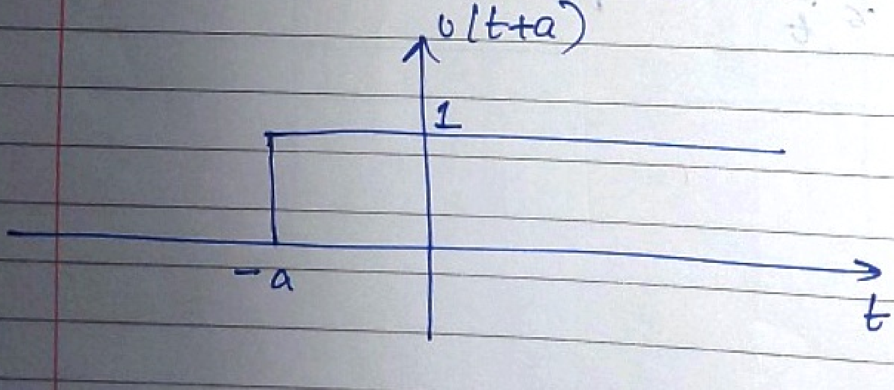


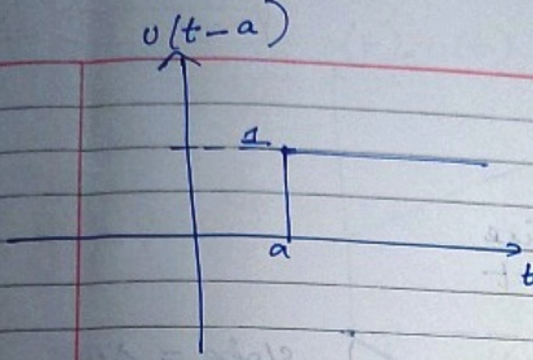
$x(t) \cdot u(t) \rightarrow$ Causal

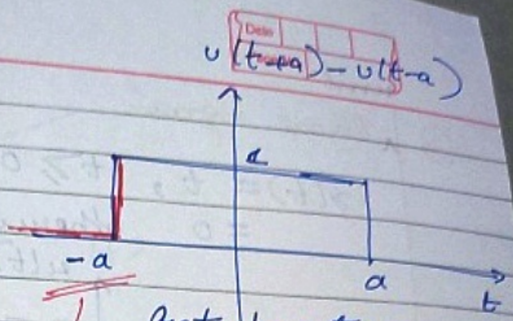


$\boxed{\text{Non causal} \xrightarrow[\times u(t)]{} \text{causal}}$

To draw:- $x(t) = u(t+a) - u(t-a)$

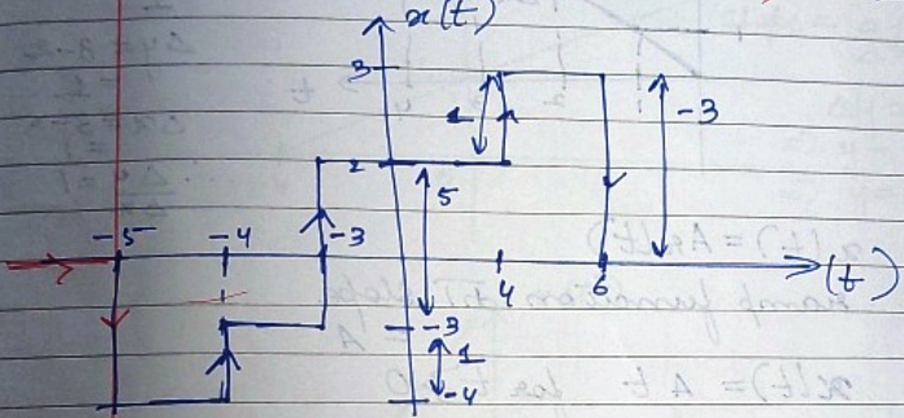






Gate function

$u(t+a) - u(t-a)$



$\cancel{x(t) = -4u(t+5) - 3u(t+4)}$

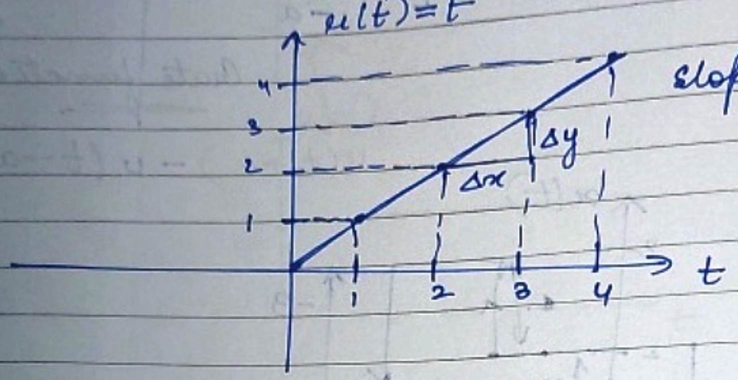
$$x(t) = -4u(t+5) + u(t+4) + 5u(t+3) + u(t-4) - 3u(t-6)$$

$u(t \pm t_0) \rightarrow$ left sided function

$u(+t \pm t_0) \rightarrow$ right sided function unit

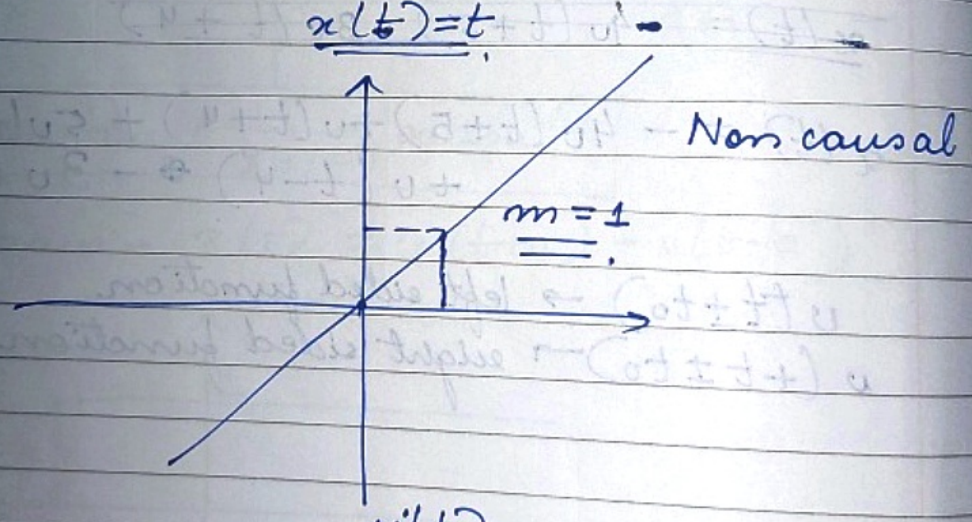
② Ramp Signal

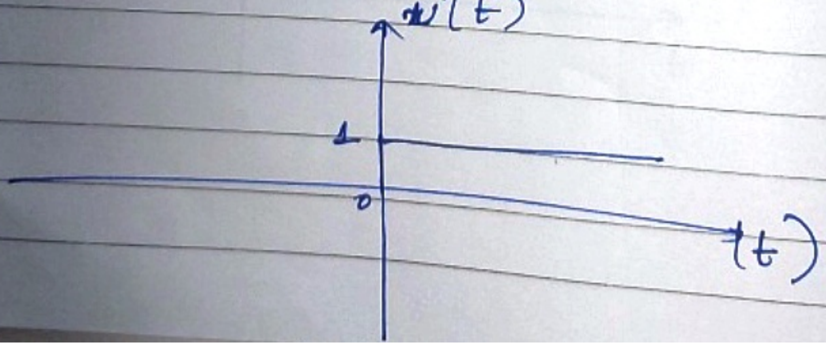
$x(t) = t, \quad t \geq 0$
$\quad = 0 \quad$ otherwise

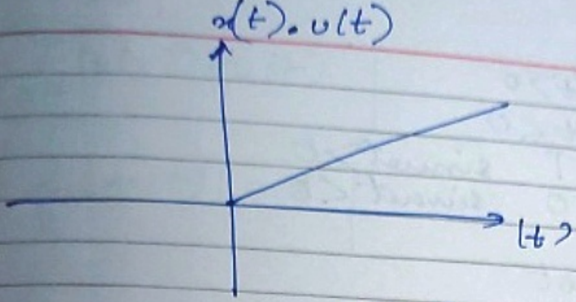


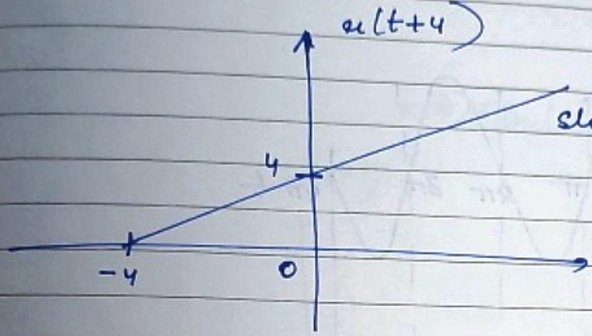
slope $= \frac{\Delta y}{\Delta x}$
$= 1$
$\Delta y = 3-2 = 1$
$\Delta x = 3-2 = 1$
$\therefore \frac{\Delta y}{\Delta x} = 1$

$x(t) = A r(t)$
ramp function का slope $= A$
$x(t) = At$ for $t \geq 0$







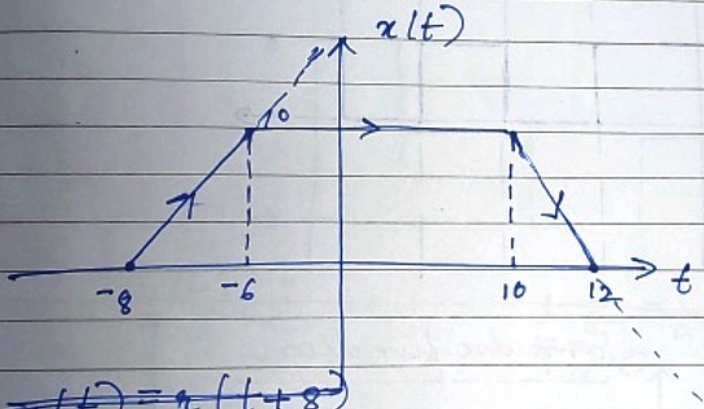


slope $= \frac{\Delta y}{\Delta x} = 1.$

$\Delta y = \Delta x$

$\Rightarrow y - 0 = 0 - (-4)$

$\Rightarrow y = 4$



$\frac{10}{2}$

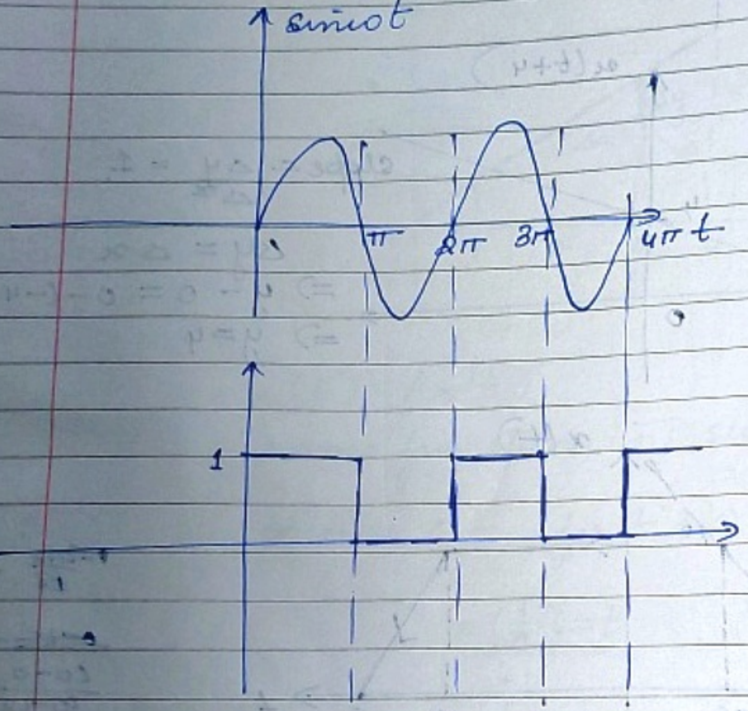
$\frac{10-0}{10-12} = \frac{10}{-2}$

$\cancel{x(t) = r(t+8)}$

$\cancel{x(t) = 5r(t+8) + u(t+6) - u(t-10) - 5r(t-10)}$

$x(t) = 5r(t+8) - 5r(t+6) - 5r(t-10) + 5r(t-12)$

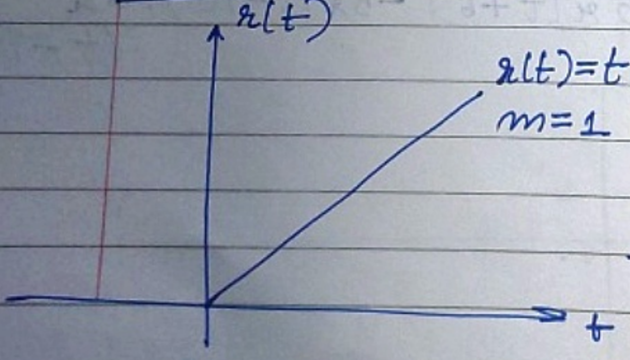
$u(t) = 1 \quad t > 0$
$\quad\;\; = 0 \quad t < 0$
$u(\sin\omega t) = 1 \quad \sin\omega t > 0$
$\quad\quad\quad\quad\;\; 0 \quad \sin\omega t < 0$



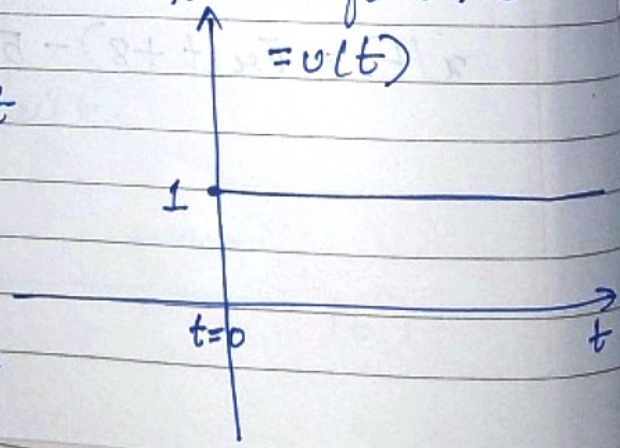
Impulse function

$r(t) = t \quad t \geq 0$
$\quad\;\; 0 \quad t < 0$

$\boxed{r(t) = t\,u(t)}$

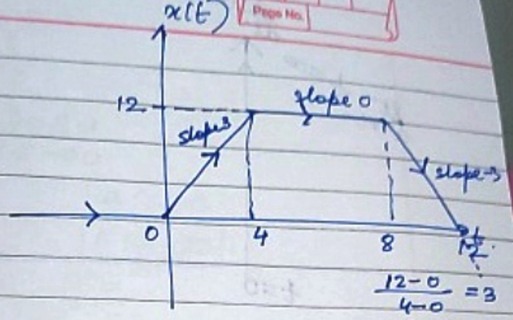


$r'(t) = 1$ for $t > 0$
$= u(t)$



$$\boxed{\frac{d}{dt}(r(t)) = u(t)}$$

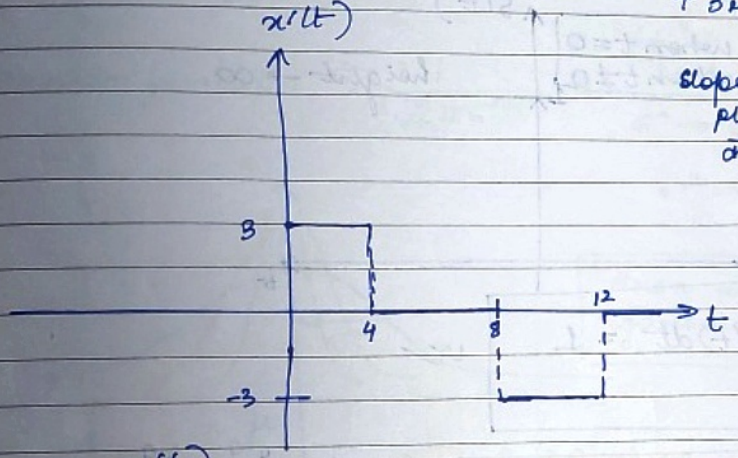
$$\boxed{\frac{d}{dt}[A\,r(t)] = A}$$



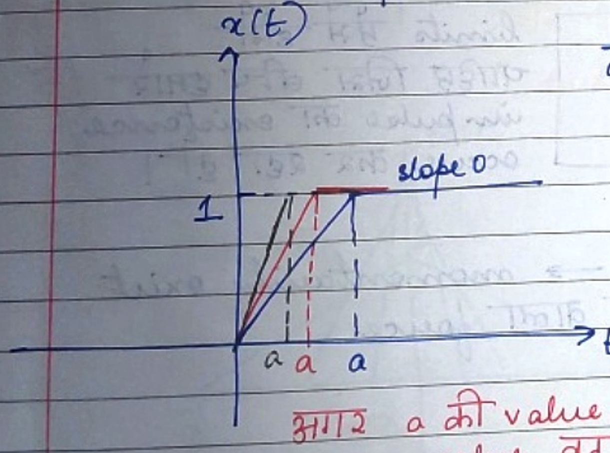
$\frac{12-0}{4-0} = 3$

$\frac{0-12}{12-8} = \frac{-12}{4}$

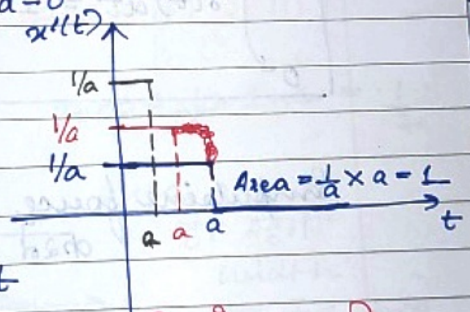
$x(t) = 3r(t) - 3r(t-4) - 3r(t-8) + 3r(t-12)$



slope को plot कर दो।



$\frac{1-0}{a-0} = \frac{1}{a}$ slope

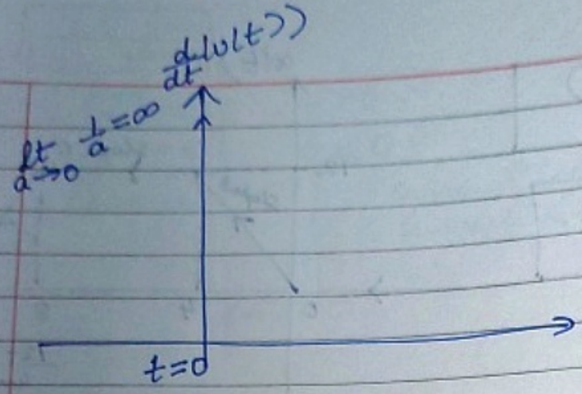


$\text{Area} = \frac{1}{a} \times a = 1$

अगर a की value कम करदी तो 1/a की value बढ़ जाएगी।

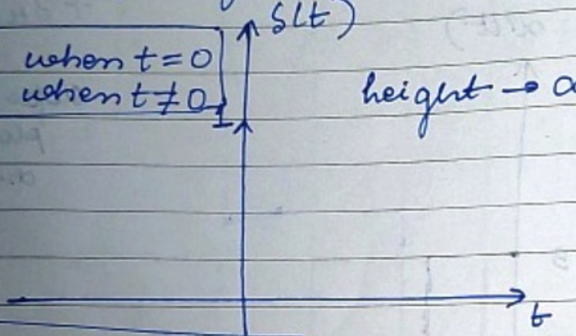
$x(t)$ becomes $\delta(t)$ when $a \to 0$

$\frac{1}{a} \to \infty$



unit impulse function / direct delta function

$$\boxed{\begin{aligned} \delta(t) &= \infty \quad \text{when } t=0 \\ &= 0 \quad \text{when } t \neq 0 \end{aligned}}$$



height $\to \infty$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = 1}$$ ~~ix~~

$$\boxed{\int_{0^-}^{0^+} \delta(t)\,dt = 1}$$

limits ऐसे होने चाहिए जिस बीच हमारे impulse का existence occur कर रहा हो ।

Impulsive force → momentarily exist करने वाला force

$\boxed{F = ma}$

$$a = \frac{v-u}{t} = -\frac{20\hat{a}_x}{t \to 0}$$

A $\xrightarrow{10\,ms^{-1}}$ $\quad$ $\xleftarrow{10\,ms^{-1}}$ B

$a \approx \infty$

$\boxed{F \approx \infty}$

$10\hat{a}_x \quad -10\hat{a}_x$

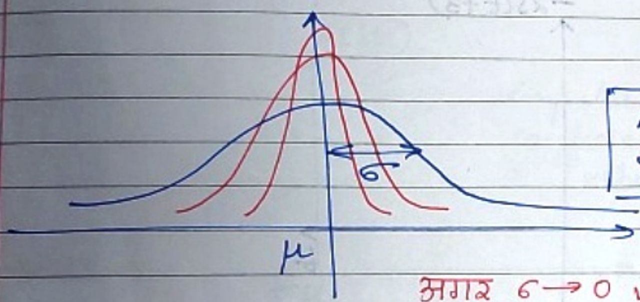
$v-u = -10\hat{a}_x - 10\hat{a}_x$

$= -20\hat{a}_x$

Gaussian function $\Rightarrow \dfrac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(t-\mu)^2}{2\sigma^2}}$ mean

$\sigma \rightarrow$ standard deviation

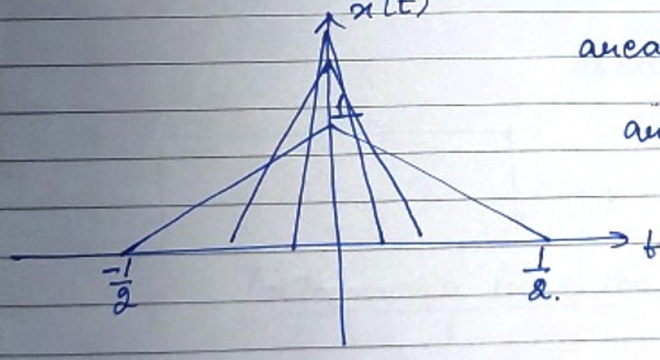
$\sigma^2 \rightarrow$ variance



Area under gaussian curve $= 1$

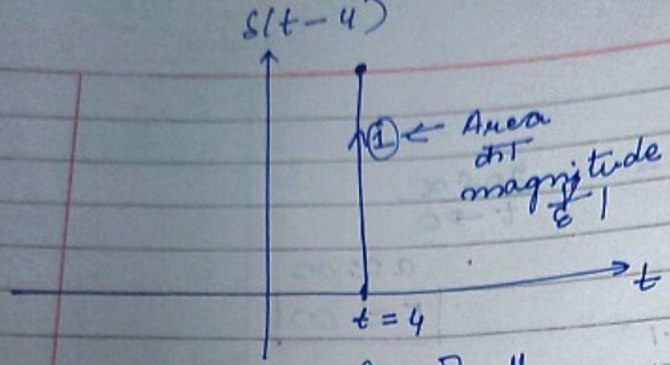
अगर $\sigma \rightarrow 0$ value function की ∞

but area 1 ही रहेगा।



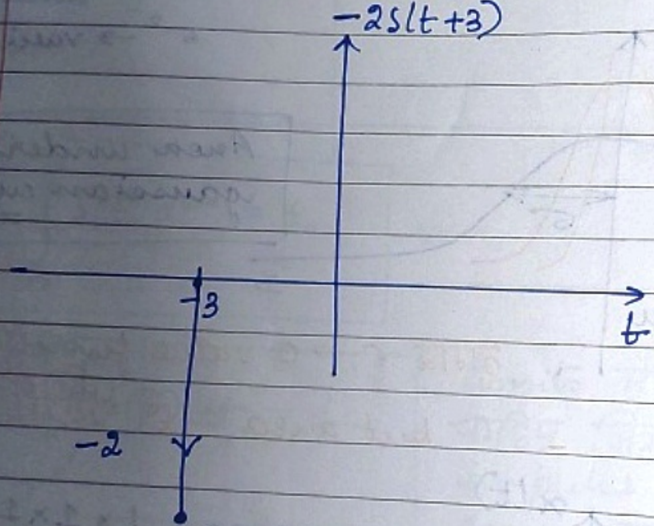
area $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

area same ही रहेगी width ~~कम~~ कम करने पर।



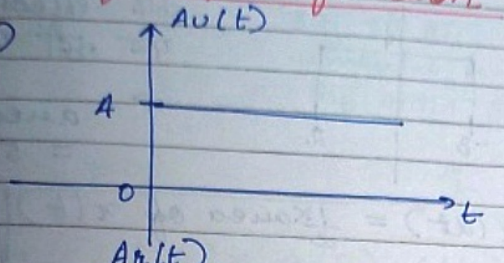
दो cheezein देखनी है — impulse कहाँ exist करेगा।
impulse के area का magnitude

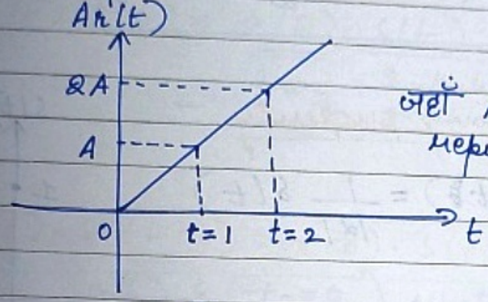
$-2\delta(\underline{t+3})$
↓
यहाँ exist करेगा।



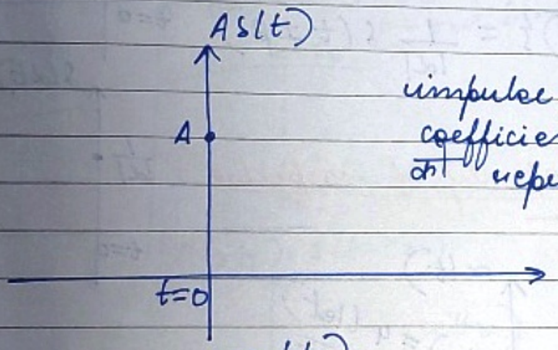
Properties of Impulse function

$Au(t)$

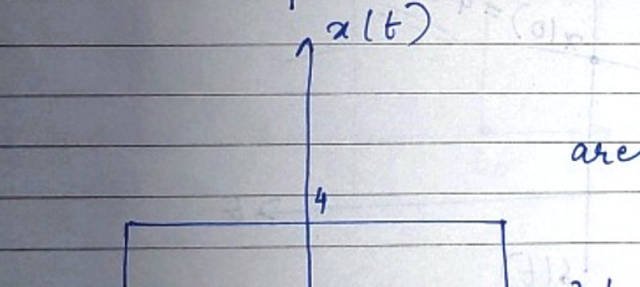




जहाँ A slope को represent करता है।

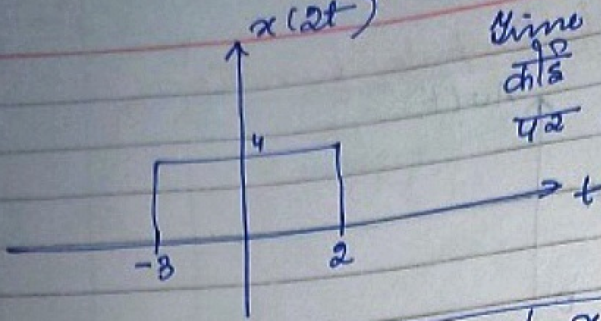


impulse function का coefficient उसके area को represent करता है।



$$\text{area} = 10 \times 4 = 40$$

Performing time scaling.



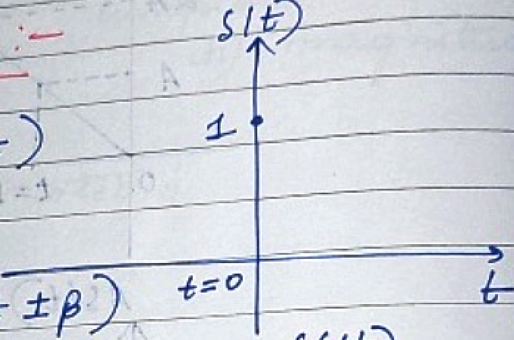
Time scaling का कोई effect amplitude पर नहीं पड़ता।

area $= 5 \times 4 = 20$

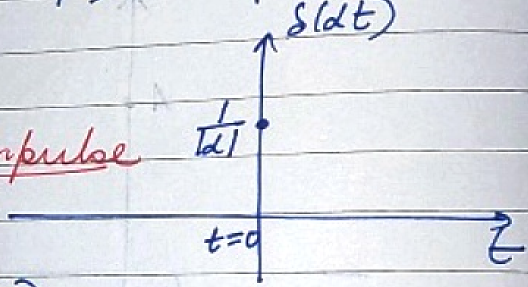
$$\boxed{\text{area of } x(\alpha t) = \frac{1}{\alpha} \times \text{area of } x(t)}$$

① Time scaling property:-

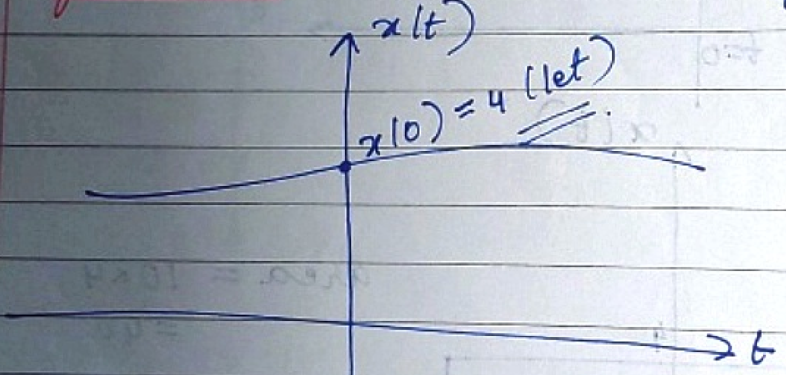
$$\delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t)$$

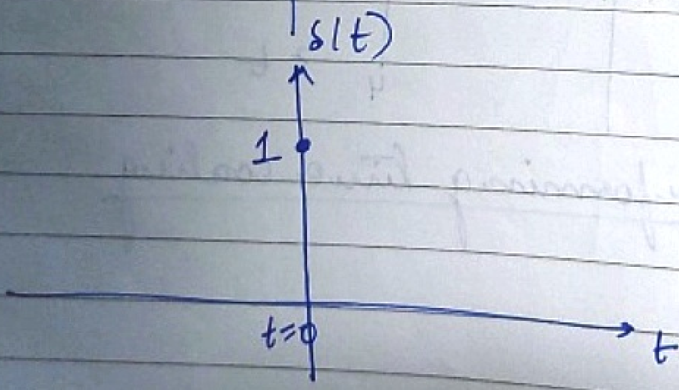


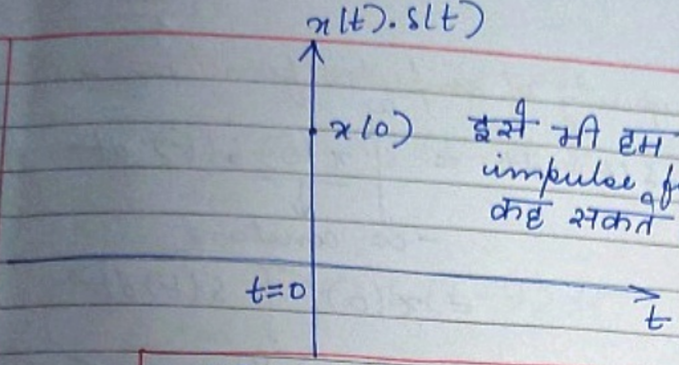
$$\delta\{\alpha(t \pm \beta)\} = \frac{1}{|\alpha|}\,\delta(t \pm \beta)$$



② Product property of impulse function:-







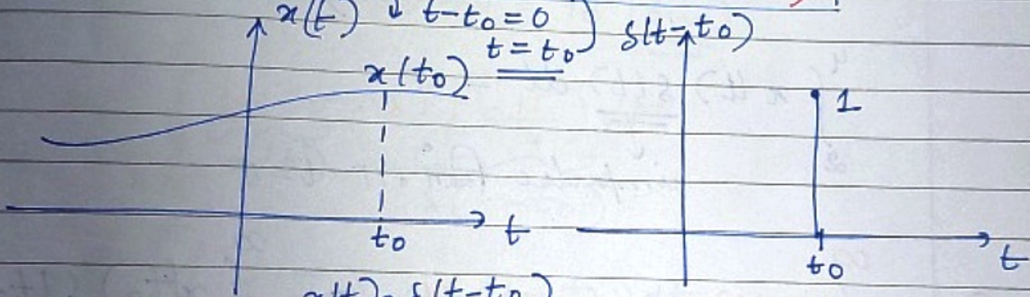
इससे भी हम impulse function कह सकते हैं जिसका area $x(0)$ है।

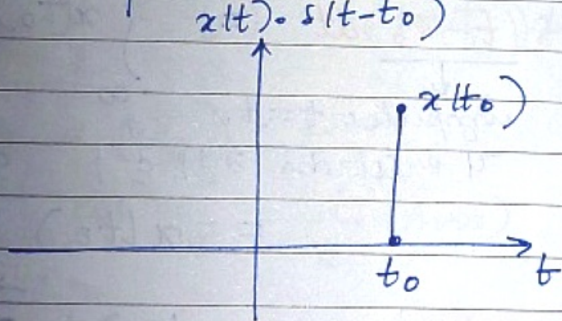
$$\boxed{x(t)\cdot\delta(t) = \underbrace{x(0)}_{\text{Area}}\cdot\underbrace{\delta(t)}_{\text{Impulse function existing at } t=0}}$$

$$\boxed{x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)}$$

$t-t_0=0$

$t=t_0$





Date | Page No

③ Shifting property of impulse function:-

$$\int_{-\infty}^{\infty} x(t)\cdot \delta(t)\,dt = \int_{-\infty}^{\infty} \underbrace{x(0)}_{\text{constant}} \delta(t)\,dt$$

$$= x(0)\underbrace{\int_{-\infty}^{\infty} \delta(t)\,dt}_{\text{area under impulse} = 1}$$

$$= x(0)$$

$$\int_{0^-}^{0^+} x(t)\cdot \delta(t)\,dt = \int_{0^-}^{0^+} x(0)\,\delta(t)\,dt$$

$$= x(0)$$

$$\int_{2}^{4} x(t)\,\underbrace{\delta(t)}_{\text{impulse मिलेगा } t=0}\,dt = 0$$

④ $$\int_{-\infty}^{\infty} x(t)\cdot \underbrace{\delta(t-t_0)}_{\text{impulse } t=t_0 \text{ पे exist कर रहा है।}}\,dt = \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt$$

$$= x(t_0)\underbrace{\int_{-\infty}^{\infty} \delta(t-t_0)\,dt}_{\text{impulse का area} = 1}$$

$$= x(t_0)$$

$$\boxed{\int_{t_0^-}^{t_0^+} x(t)\cdot \delta(t-t_0)\,dt = x(t_0)}$$

Evaluate the value of :-

$$I = \int_{-\infty}^{\infty} (t^2+4)\,\delta(t-3)\,dt \qquad x(t) = t^2+4$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-3)\,dt$$

$$= \int_{-\infty}^{\infty} x(3)\,\delta(t-3)\,dt$$

$$= x(3) \int_{-\infty}^{\infty} \delta(t-3)\,dt = 1.$$

$$= x(3) = 3^2+4 = 13 \ (\underline{\underline{\text{Ans}}})$$

$$I = \int_{-\pi}^{\pi} \underbrace{\cos^2 t}_{x(t)} \cdot \underbrace{\delta\left(t-\frac{\pi}{4}\right)}_{\delta(t-t_0)} dt$$

$$= \int_{-\pi}^{\pi} x\left(\frac{\pi}{4}\right) \cdot \delta\left(t-\frac{\pi}{4}\right) dt$$

$$= x\left(\frac{\pi}{4}\right) \cdot 1$$

$$= \cos^2 \frac{\pi}{4} = \frac{1}{2} \ (\text{Ans})$$

$$I = \int_{-3}^{4} (t^3+5)\,\delta(t-6)\,dt$$

impulse existing at 6

$$= 0$$

$$I = \int_{-6}^{5} (t-2)\,\delta(2t-4)\,dt$$

$t = 2$

$$= \int_{-6}^{5} x(t)\,\delta(2t-4)\,dt$$ ✗ Wrong

$$= x(2)\int_{-6}^{5} \delta(2t-4)\,dt$$

$$= 0$$

$$\delta(2t-4) = \delta(2(t-2))$$
$$= \frac{1}{2} \times \delta(t-2)$$

$$\int_{-6}^{5} (t-2) \times \frac{1}{2} \times \delta(t-2)$$

$$= (t-2)\Big|_{t=2} \times \frac{1}{2} \times 1$$

$$= 0 \text{ (Ans)}$$

① $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$

$\delta(\alpha(t \pm \beta)) = \frac{1}{|\alpha|}\delta(t \pm \beta)$

② $x(t)\cdot\delta(t) = x(0)\,\delta(t)$

$x(t)\cdot\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

③ $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

Extension of scaling property of impulse function

$$I = \int_{-\infty}^{\infty} x(t) \cdot \delta\{g(t)\}\, dt$$

$\delta\{g(t)\} = t^2 - 16$

$\delta\{g(t)\} = 0$ if $g(t)$ is nowhere 0

$\delta\{g(t)\} = \frac{\delta(t-t_0)}{|g'(t_0)|}$ if 'g' has a real root at $t = t_0$

If g has more than one real roots at $t_0, t_1, \ldots t_i$ then $\delta\{g(t)\} = \sum_i \frac{\delta(t-t_i)}{|g'(t_i)|}$

$$= \frac{\delta(t-t_0)}{|g'(t_0)|} + \frac{\delta(t-t_1)}{|g'(t_1)|} + \frac{\delta(t-t_2)}{|g'(t_2)|} + \ldots$$

Evaluate the integral

$$I = \int_{-10}^{+10} (t^2+10)\, \delta(t^2-16)\, dt$$

$\delta\{g(t)\} = \delta(t^2-16)$

$g(t) = t^2 - 16 \quad t = \pm 4 \qquad g'(t) = 2t$

$$\delta\{g(t)\} = \frac{\delta(t+4)}{8} + \frac{\delta(t-4)}{8}$$

$$= \frac{1}{8}\left[\delta(t+4) + \delta(t-4)\right]$$

$$\int_{-10}^{10} (t^2+10)\, \frac{1}{8}\left[\delta(t+4) + \delta(t-4)\right]$$

$$\frac{1}{8}\left[\int_{-10}^{10}(t^2+10)\,\delta(t+4) + \int_{-10}^{10}(t^2+10)\,\delta(t-4)\right]$$

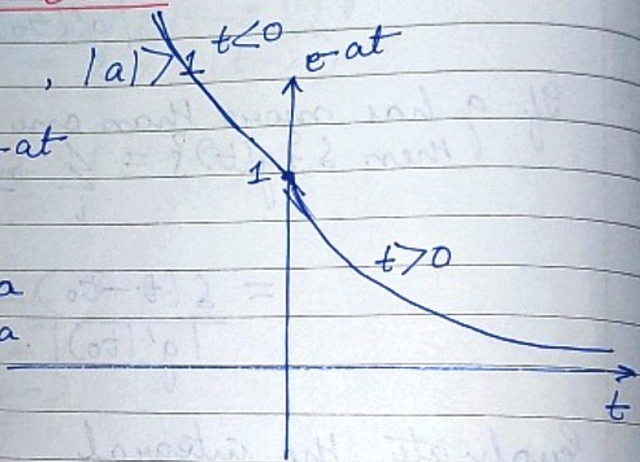
$$= \frac{1}{8}\left[(-4)^2+10 + (4)^2+10\right] = \frac{1}{8}\left[20+32\right]$$

$$= \frac{1}{8} \times 52 = 6.5$$

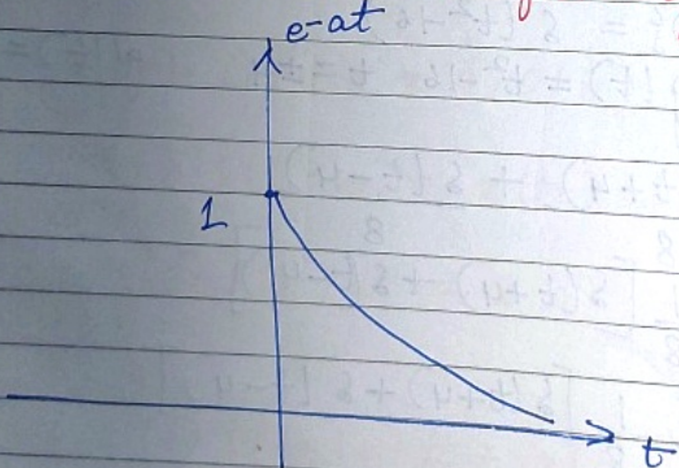
Exponential functions :-

$x(t) = e^{-at}$, $|a| > 1$

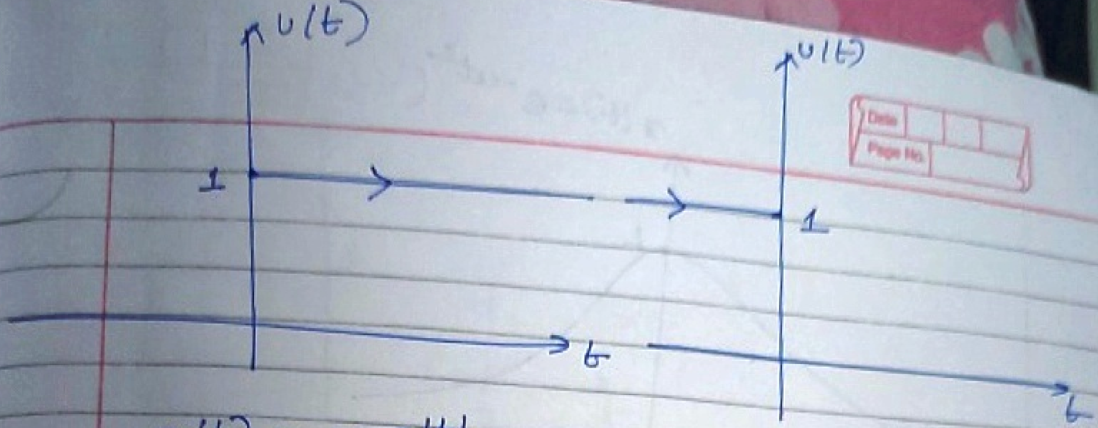
t	$x(t) = e^{-at}$
0	1
1	e^{-a}
2	e^{-2a}
3	e^{-3a}
.	.
-1	e^{a}
-2	e^{2a}



$x(t) = e^{-at}u(t)$ function becomes causal



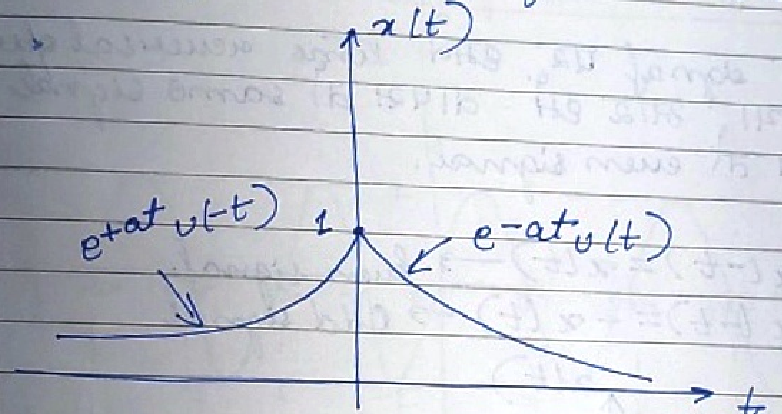
~444 students



$x(t) = e^{-a|t|}$

$= e^{-at}u(t) + e^{at}u(-t)$

when $t>0$ from 0 to ∞ | when $t<0$ from $-\infty$ to 0



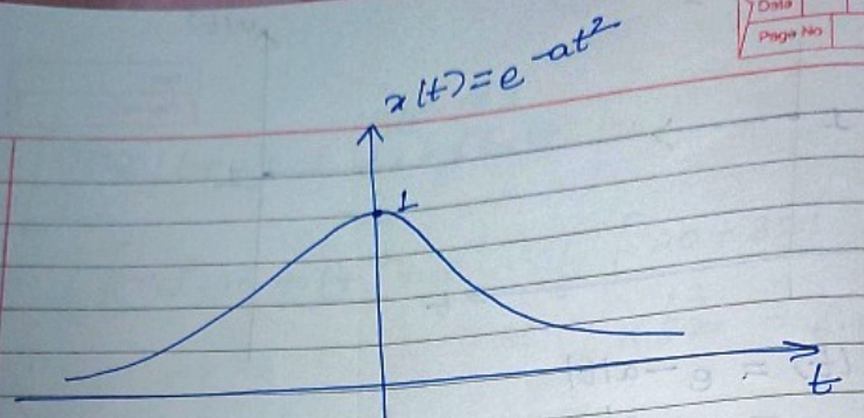
$x(t) = e^{-at^2} \longrightarrow$ Gaussian function

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

t	e^{-at^2}
0	1
+1	e^{-a}
-1	e^{-a}

$t^2 \longrightarrow$ always positive

overall function दोनों तरफ से decrease होगा।

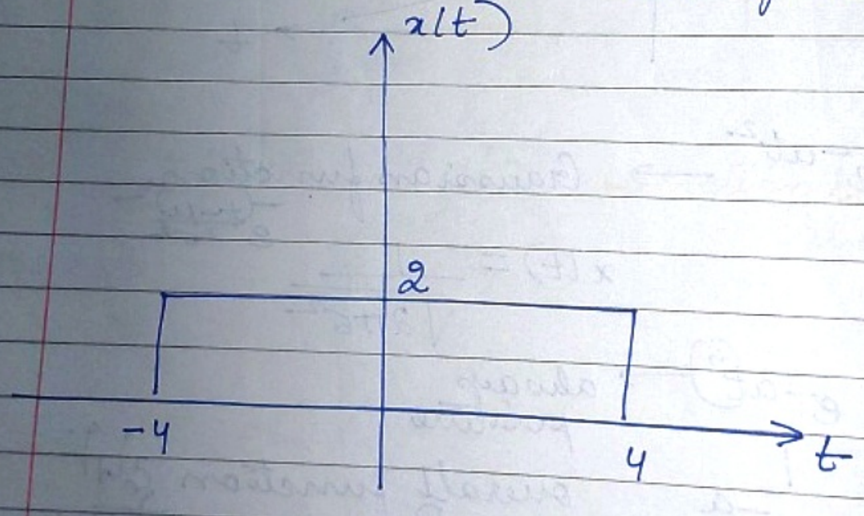


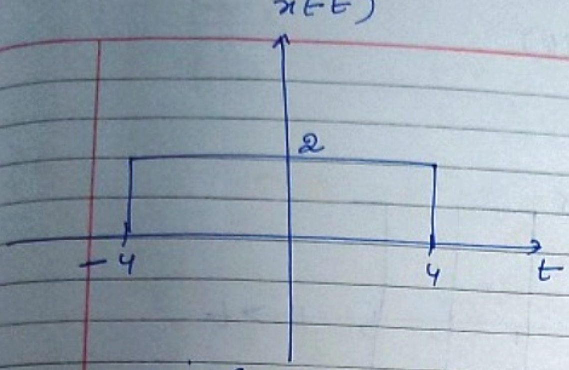
Classification of Signals :-

① Even and odd signals :-

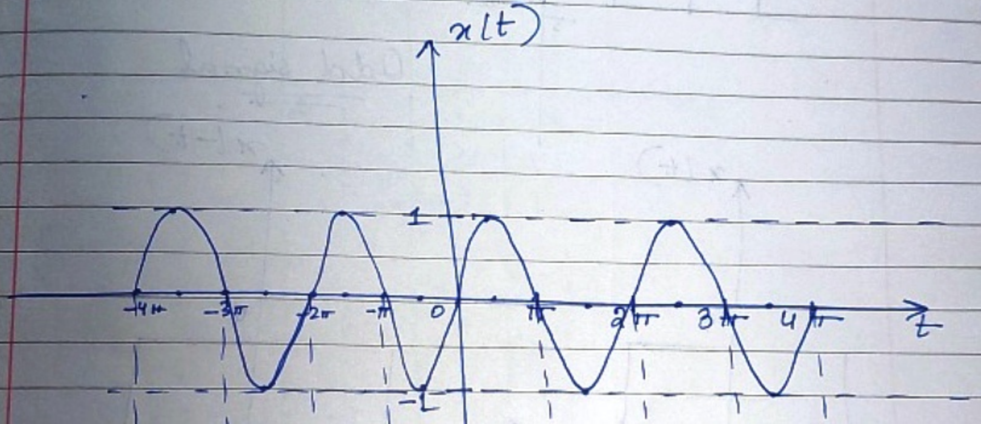
किसी signal पर हमने time reversal operation लगाया, और हमें वापस वो same signal मिला तो even signal.

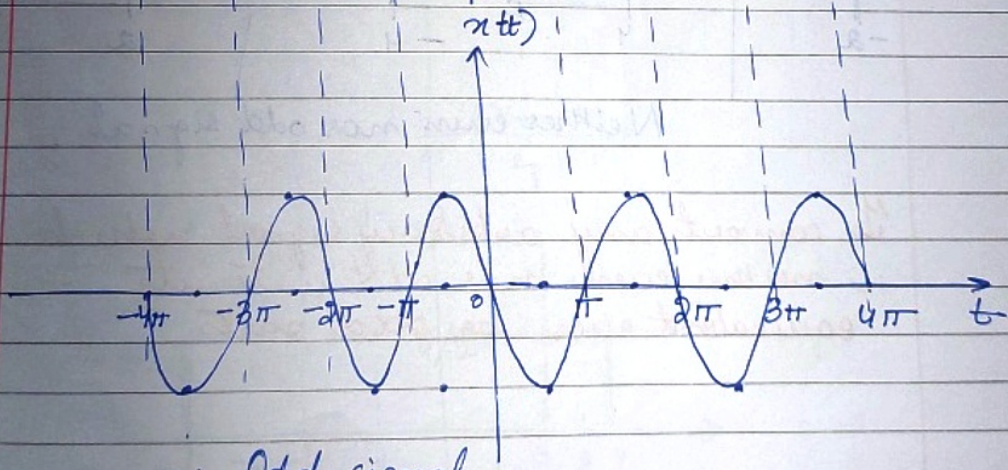
If $x(-t)=x(t) \rightarrow$ Even signal
$x(-t)=-x(t) \rightarrow$ Odd signal



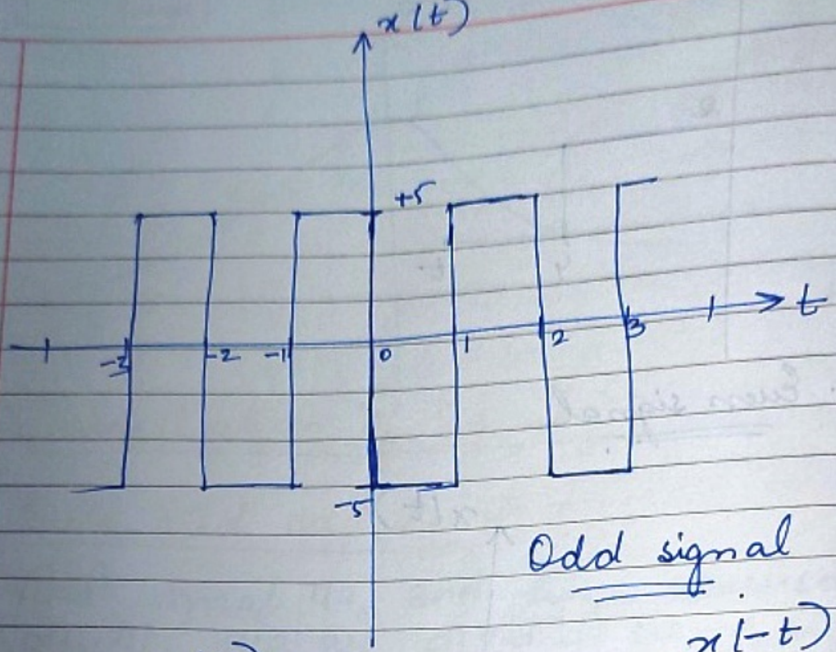


∴ Even signal

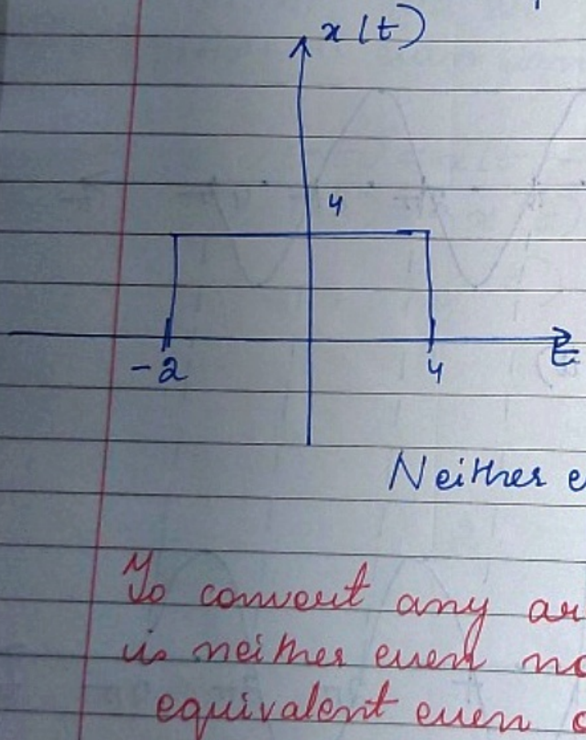


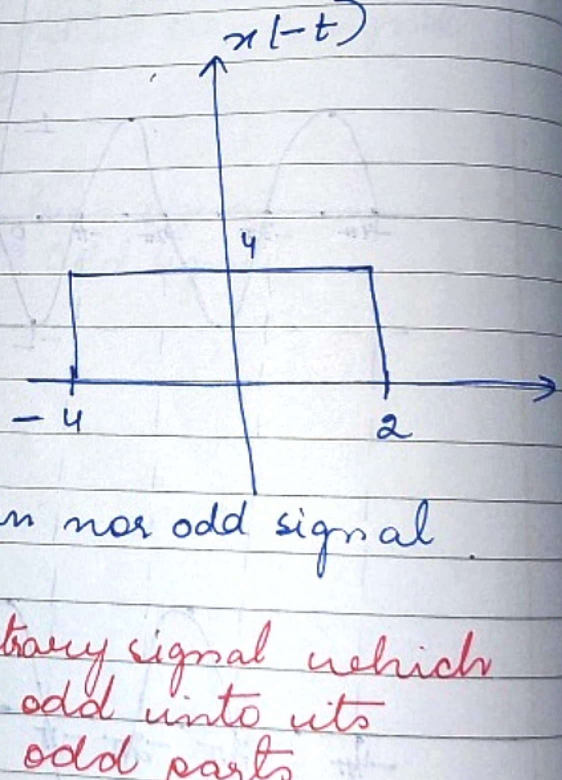


∴ Odd signal



Odd signal.



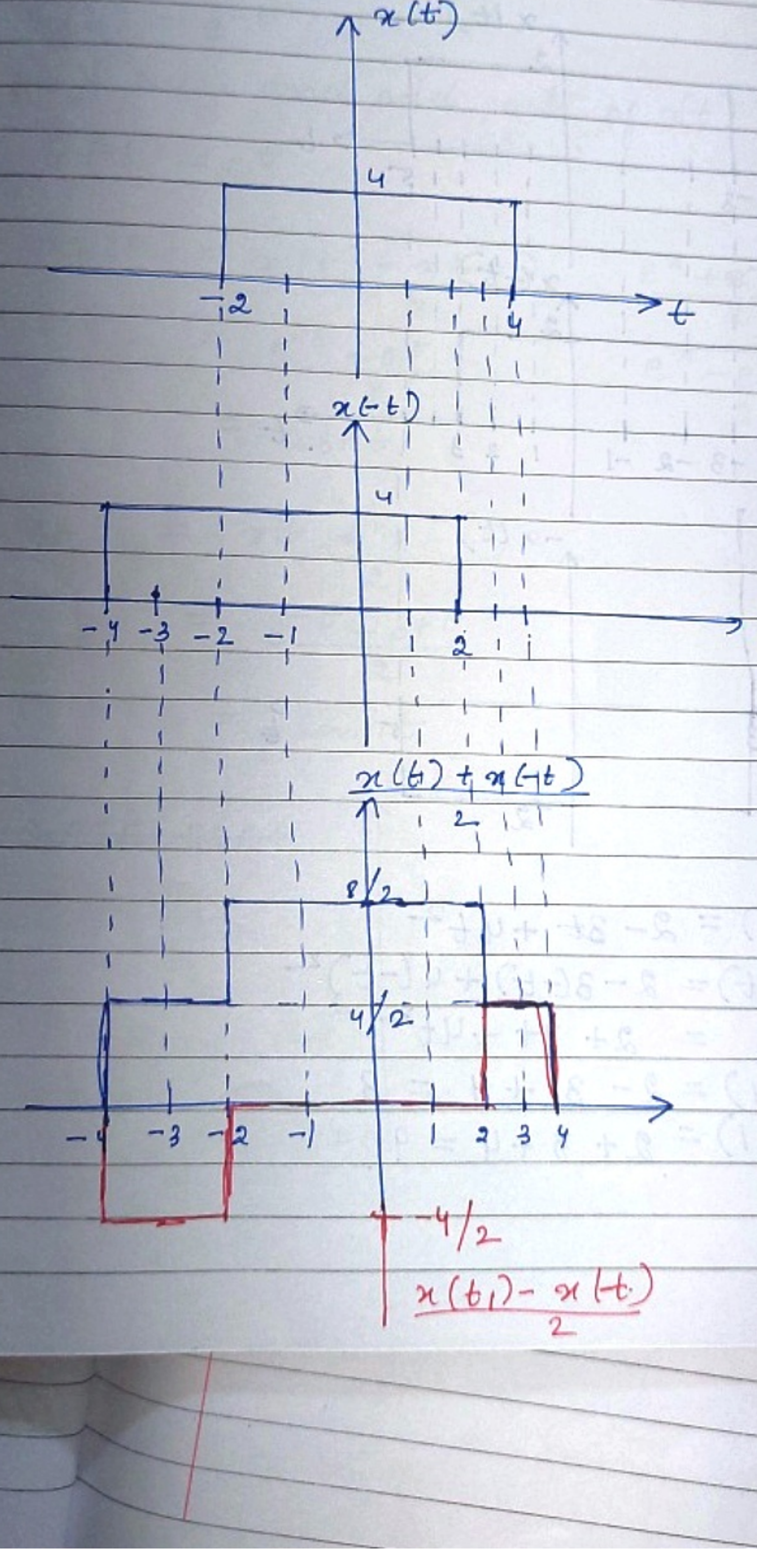


Neither even nor odd signal.

To convert any arbitrary signal which is neither even nor odd into its equivalent even or odd parts.

$$x_e(t) = \frac{x(t)+x(-t)}{\textcircled{2}} \rightarrow \text{magnitude की division}$$

$$x_o t = \frac{x(t)-x(-t)}{2}$$

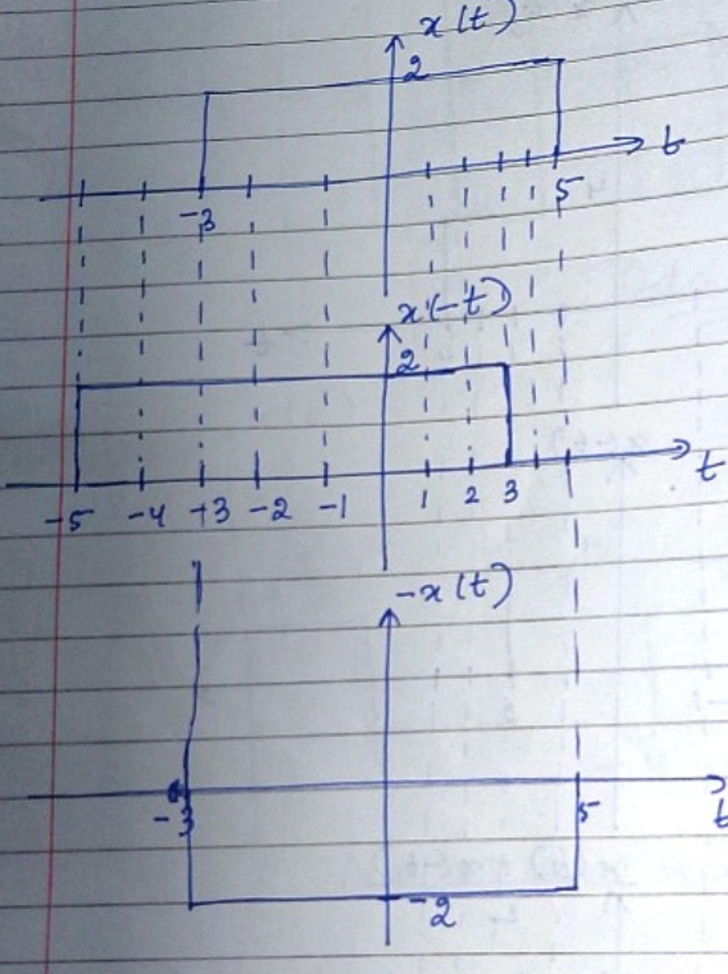


~~Sinusoidal~~

Even and odd signals:-

If $x(-t) = x(t) \rightarrow$ even

$x(-t) = -x(t) \rightarrow$ odd



$x(t) = 2 - 3t + 4t^2$

$x(-t) = 2 - 3(-t) + 4(-t)^2$

$= 2 + 3t + 4t^2$

$x(1) = 2 - 3 + 4 = 3$

$x(1) = 2 + 3 + 4 = 9$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$x(t) = e^{-t}$

find even and odd parts of $x(t)$.

$x(-t) = e^{t}$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad \frac{e^{x} + e^{-x}}{2} = \cosh x$$

$$= \frac{e^{-t} + e^{t}}{2} \qquad \frac{e^{x} - e^{-x}}{2} = \sinh x$$

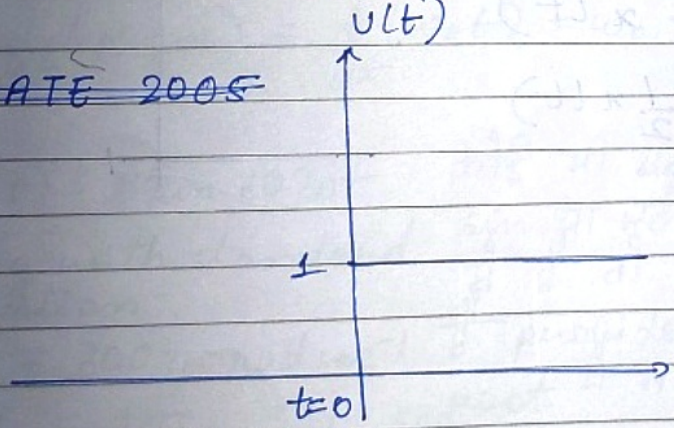
$$= \cosh t$$

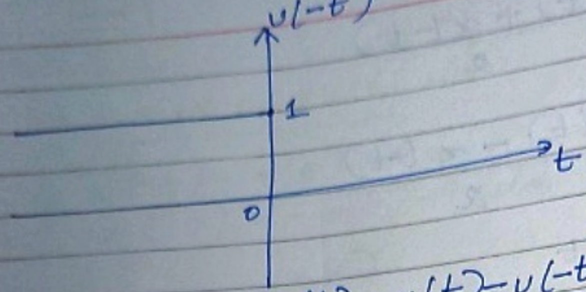
$$x_o t = \frac{x(t) - x(-t)}{2}$$

$$= \frac{e^{-t} - e^{+t}}{2}$$

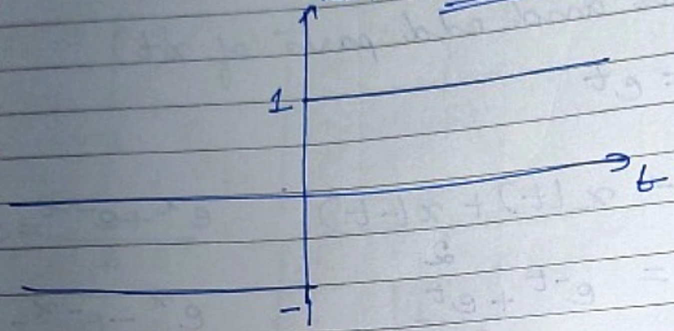
$$= -\sinh t$$

~~# GATE 2005~~



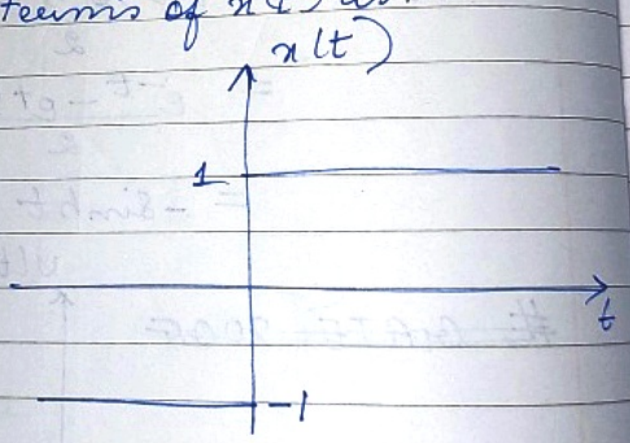


$x(t) = u(t) - u(-t) \Rightarrow \text{sgn}(t)$



GATE 2005

Even and odd parts of $u(t)$ can be represented in terms of $x(t)$ as:-



(a) $\frac{1}{2}, \frac{1}{2}x(t)$

(b) $-\frac{1}{2}, \frac{1}{2}x(t)$

(c) $-\frac{1}{2}, -\frac{1}{2}x(t)$

(d) $\frac{1}{2}, -\frac{1}{2}x(t)$

Periodic and aperiodic signals:-

A signal is said to be periodic if it satisfies following two properties:-

1. It must exist for $-\infty \leq t \leq \infty$
2. It must repeat itself after some constant amount of time 'T' which is called as.

T = Fundamental Time period $= \frac{2\pi}{\omega_0}$

ω_0 = fundamental frequency
rad sec^{-1}

$T = \frac{1}{f}$ → Hertz $= \frac{1}{sec}$

① Sinusoidal signals:-

General representation:-

$$x(t) = \underbrace{A}_{\text{Amplitude}} \sin(\underbrace{\omega t}_{\text{phase angle}} + \underbrace{\phi}_{\text{phase shift}})$$

'+' → advance
'-' → delay

$$\frac{d}{dt}(\text{phase}) = \frac{d}{dt}(\omega_0 t) = \omega_0 \rightarrow \text{frequency}$$

$x(t) = 4\sin 300\pi t$

Comparing with standard equation.

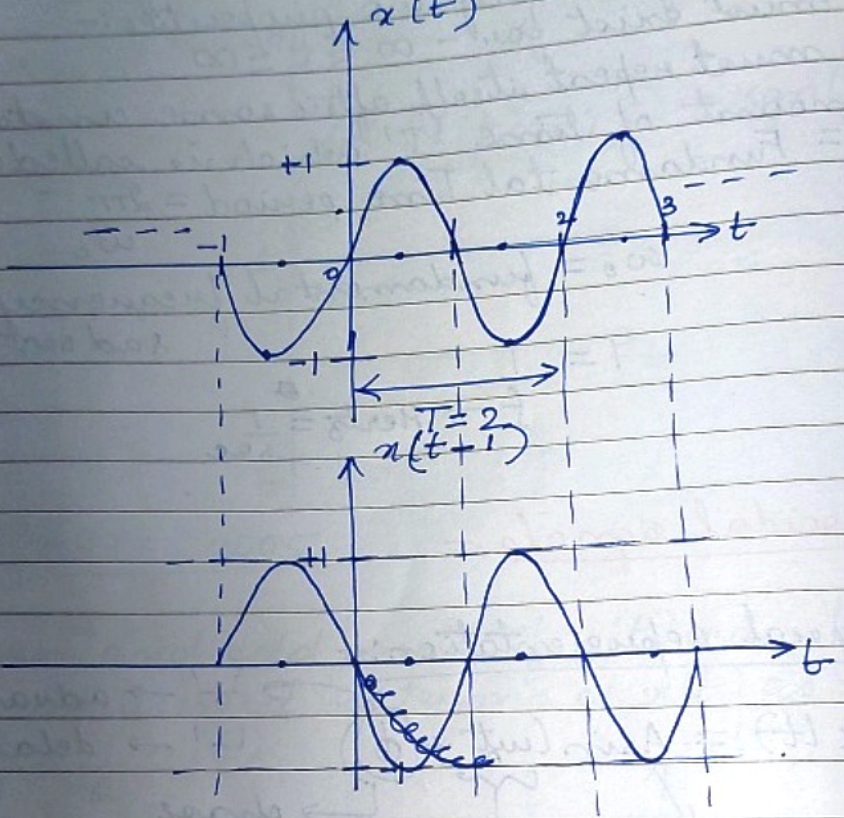
$\omega_0 = 300\pi \text{ rad sec}^{-1}$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{300\pi} = \frac{1}{150} \text{ sec}$$

कोई भी signal एक sin या एक cos के form में है तो वो periodic है provided t under root में ना हो।

$$x(t) = 5\cos\left(20\pi t + \frac{\pi}{4}\right)$$

Shifting का कोई भी effect periodicity पर नहीं होगा।



$\omega_0 = 20\pi$

$\phi = \frac{\pi}{4}$

$A = 5$

$T = \frac{2\pi}{\omega} = \frac{1}{10}$ sec $= 0.1$ sec.

Combination of periodic signals:-

will be periodic if ratio of individual time periods is a rational number.

$$x(t) = \underbrace{4\cos 2\pi t}_{T_1} + \underbrace{3\sin 6t}_{T_2} + \underbrace{5\cos 60\pi t}_{T_3}$$

Not periodic

$\frac{T_2}{T_1}$, $\frac{T_3}{T_2}$, $\frac{T_1}{T_3}$ $\Rightarrow$ Rational Number

Then period of resultant signal

$T =$ LCM of $[T_1, T_2, T_3]$

or frequency

$\omega_0 =$ GCD/HCF of $[\omega_{01}, \omega_{02}, \omega_{03}]$

Rational no. :- In the form of ratio of integers $\frac{p}{q}$ or if ratio is converted into decimal then it must be terminating or repeating decimals.

$\frac{T_1}{T_2} = \frac{10}{3} = 3.333$ (repeating) $\therefore$ Rational

$\frac{T_1}{T_2} = \frac{10}{4} = 2.5 \rightarrow$ terminating decimal

$\frac{T_1}{T_2} = \pi = \frac{22}{7} = 3.142\,78\ldots \rightarrow$ non terminating $\therefore$ irrational

$x(t \pm T) = x(t)$
$\downarrow$
time period

$x(t \pm kT) = x(t)$
$\downarrow$
$\boxed{k \in I}$

$x(t) = A\sin(\omega t + \phi)$

$x(t) = \underbrace{4\cos t}_{T_1} + \underbrace{3\sin 2\pi t}_{T_2} + \underbrace{2\sin 3\pi t}_{T_3}$

अपने आप में तीनों periodic है।

$\frac{T_2}{T_1} \quad \frac{T_3}{T_2} \quad \frac{T_1}{T_3}$

LCM of rational numbers $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

HCF of rational numbers $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

LCM of $\left(\frac{1}{2}, \frac{3}{6}, \frac{5}{4}\right)$
$= \frac{15}{2} = 7.5$

$\omega_{01} = 1 \qquad T_1 = 2\pi$
$\omega_{02} = 2\pi \qquad T_2 = 1$
$\omega_{03} = 3\pi \qquad T_3 = \frac{2}{3}$

$\frac{T_2}{T_1} = \frac{1}{2\pi} \qquad \frac{T_3}{T_2} = \frac{3}{2}. \qquad \frac{T_1}{T_3} = 3\pi$

irrational $\qquad$ irration

$x(t)$ is aperiodic

signal combination में periodicity check करनी है तो या तो same frequency में π हो या किसी में भी π ना हो।

$x(t) = 4\cos\pi t + 3\sin 2\pi t + 2\sin 3\pi t$

$\omega_1 = \pi \qquad \omega_2 = 2\pi \qquad \omega_3 = 3\pi$

$T_1 = 2 \qquad T_2 = 1 \qquad T_3 = \frac{2}{3}$

T of resultant signal $= \text{LCM}(T_1, T_2, T_3)$

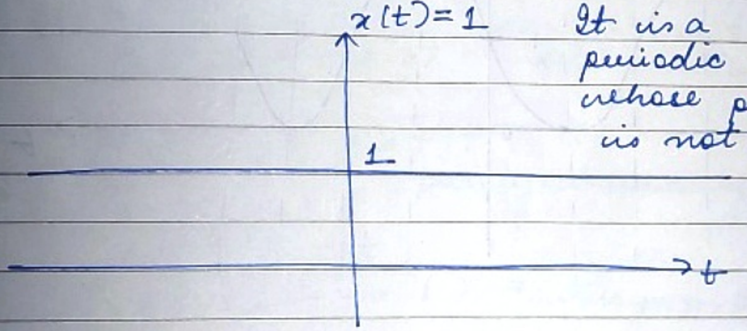
$= \frac{2}{1} = 2$

ω of resultant signal $= \text{HCF}(\omega_1, \omega_2, \omega_3)$

$= \pi$

constant signal

$x(t) = 1$

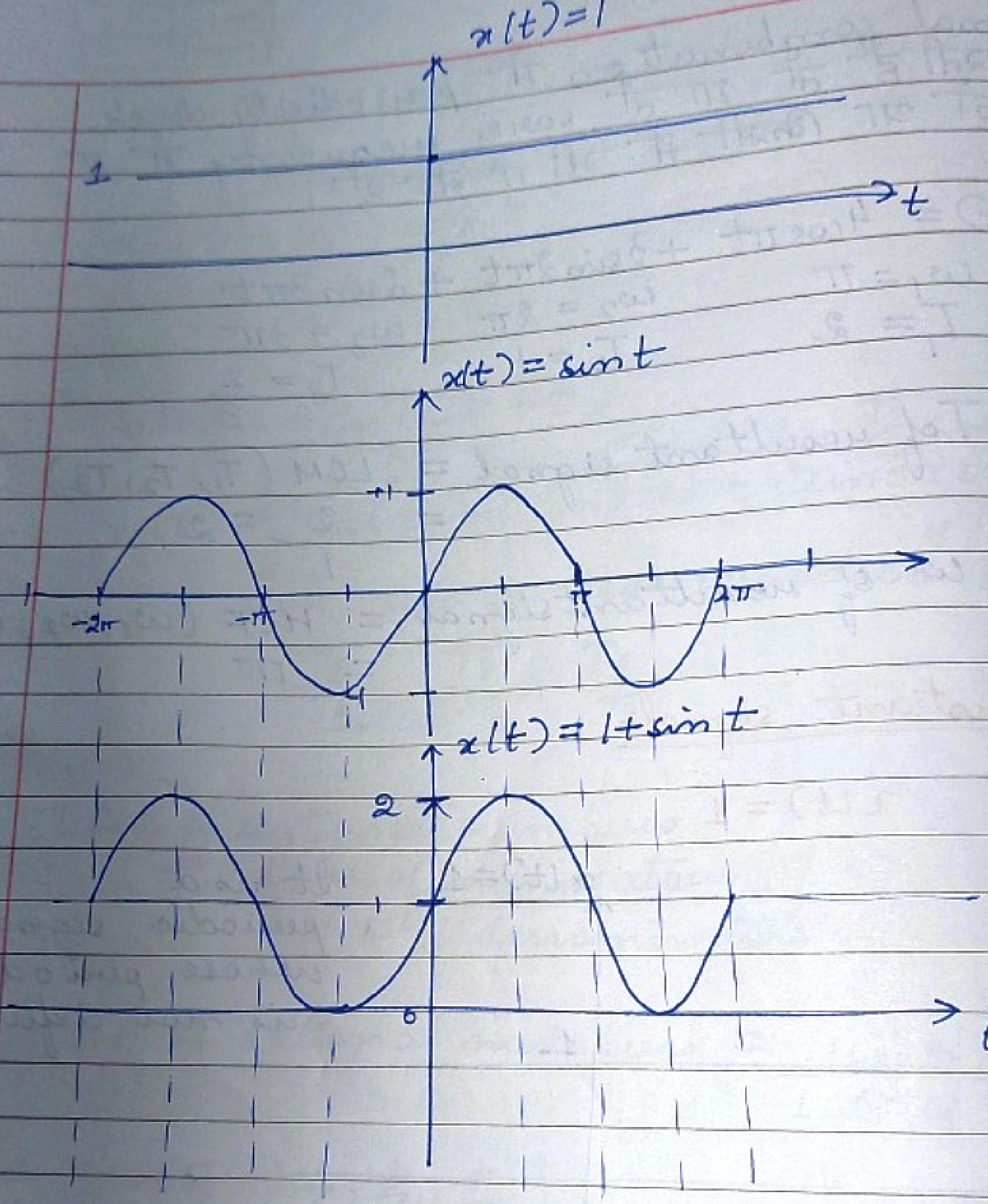


It is a periodic signal whose period is not defined

$x(t) = 1 + \sin t$ is always periodic.

$\omega_0 = 1 \quad T = \frac{2\pi}{1} = 2\pi$ sec

never consider dc signal to determine the periodicity of any signal.



$x(t) = 4 + \cos^2 4\pi t$

find x(t) is periodic.

If yes then find frequenc

and time period.

$\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \frac{1+\cos 2\theta}{2} = \cos^2\theta$

$\cos^2 4\pi t = \frac{1+\cos 8\pi t}{2}$

$x(t) = 4 + \frac{1}{2} + \frac{\cos 8\pi t}{2}$

$$\boxed{x(t) = \frac{9}{2} + \frac{1}{2}\cos 8\pi t}$$

$\omega_0 = 8\pi$ r/s

$T = \frac{2\pi}{8\pi} = \frac{1}{4} = 0.25$ sec.

Even and odd

If $x(t)$ is real or purely imaginary

If $x(-t) = x(t) \rightarrow$ even / symmetric

$x(-t) = -x(t) \rightarrow$ odd / antisymmetric

$x(t) = jt = 0 + jt$
purely imaginary

$x(-t) = -jt$

$x(-t) = -x(t) \rightarrow$ antisymmetric

Even $\rightarrow$ E

Odd $\rightarrow$ O

$O \pm O = O$

$E \pm E = E$

$O \pm E$ = neither even nor odd

$E \times E = E$

$O \times O = E$

$E \times O = O$

$O \times E = O$

$\frac{E}{E} = E$ $\quad \frac{O}{E} = O$

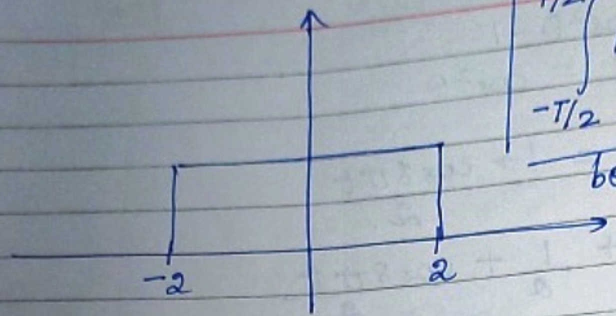
$\frac{O}{O} = E$ $\quad \int E = O$

$\frac{E}{O} = O$ $\quad \int O = E$

$\frac{d}{dt} E = O$

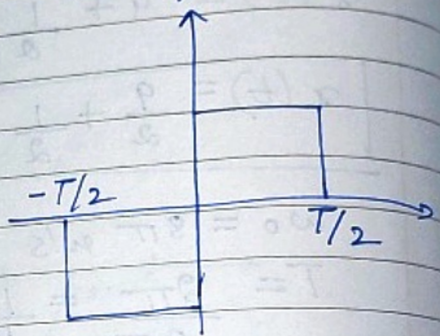
$\frac{d}{dt} O = E$

students



$$\int_{-T/2}^{T/2} E = 2 \times \int_{0}^{T/2} E$$

because symmetrical



$$\int_{-T/2}^{T/2} \underset{\text{odd signal}}{O} = 0$$

If $x(t)$ is complex

$$x(t) = a \pm jb$$

for even or symmetric

$$x(t) = x^{*}(-t)$$

$\downarrow$ conjugate

$$3 + j4 \rightarrow 3 - j4$$

for odd or antisymmetric

$$x(t) = -x^{*}(-t)$$

Conjugate symmetric part of $x(t)$

$$x_e(t) = \frac{x(t) + x^{*}(-t)}{2}$$

Conjugate antisymmetric part of $x(t)$

$$x_o(t) = \frac{x(t) - x^{*}(-t)}{2}$$

$x(t) = 5 - jt$

$x(-t) = 5 + jt$

$x^*(-t) = 5 - jt$

Here $x(t) = x^*(-t)$

$\therefore$ $x(t)$ is conjugate symmetric signal

$$x_e(t) = \frac{x(t) + x^*(-t)}{2}$$

$$= \frac{5 - jt + 5 - jt}{2}$$

$$= 5 - jt$$

$$x_o(t) = \frac{x(t) - x^*(-t)}{2}$$

$$= \frac{5 - jt - 5 + jt}{2} = 0.$$

Q $x(t) =$ Even of $[\sin 4\pi t\, u(t)]$

Is $x(t)$ periodic? If yes find period.

$x(t) = \sin 4\pi t\, u(t)$

$x(-t) = \sin 4\pi(-t)\, u(-t)$

$= -\sin 4\pi t\, u(-t)$

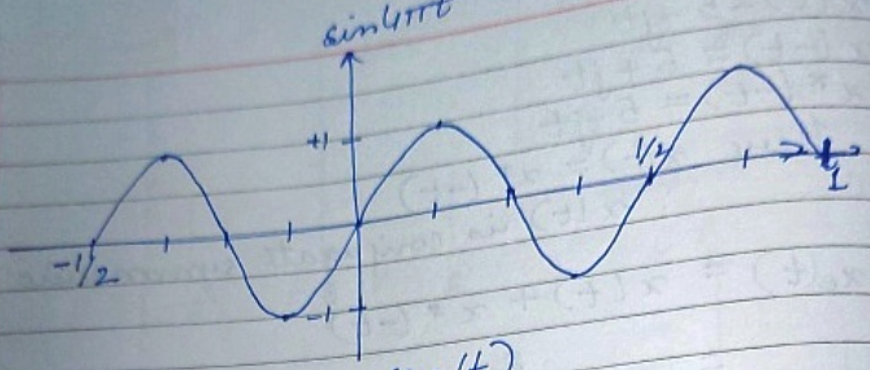
$$x_e(t) = \frac{\sin 4\pi t\, u(t) - \sin 4\pi t\, u(-t)}{2}.$$

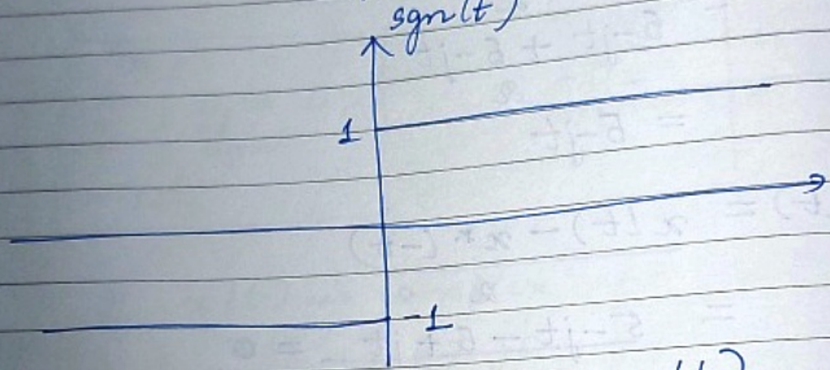
$$= \sin 4\pi t\, \frac{[u(t) - u(-t)]}{2}.$$

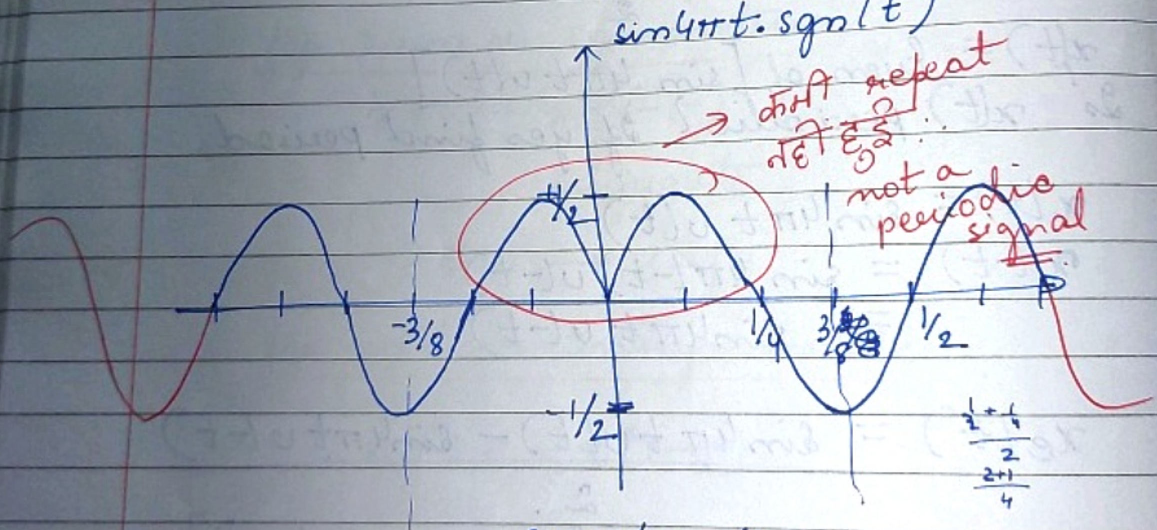
$$x_1(t) = \sin 4t\, \frac{1}{2}[u(t) - u(-t)]$$

$$= \sin 4t\, \frac{1}{2}\, sgn(t)$$

$T = \frac{2\pi}{4\pi}$

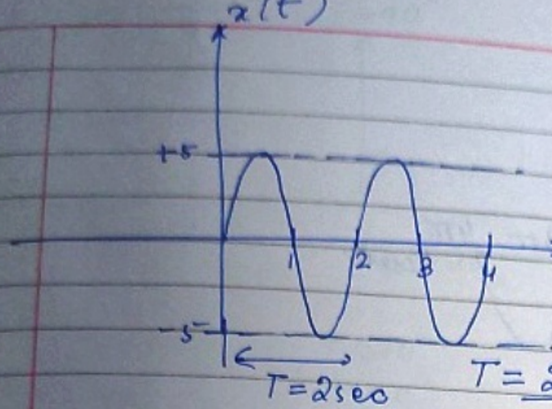






even function

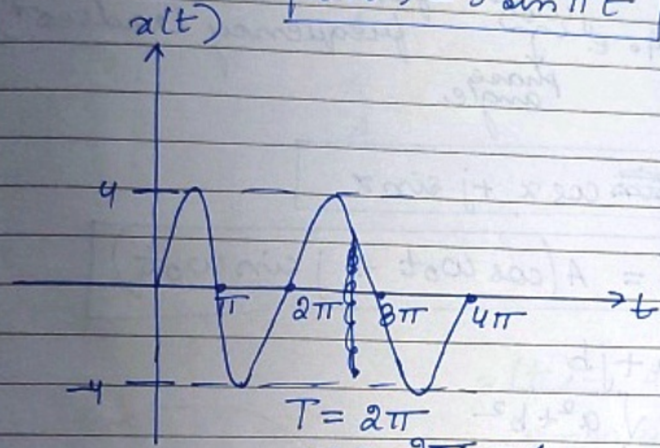
$\cancel{\frac{3}{8}+\frac{3}{8}=\frac{6}{8}=\frac{3}{4}} \to T$



$$T = \frac{2\pi}{\omega}$$

$$\Rightarrow \boxed{\omega = \pi}$$

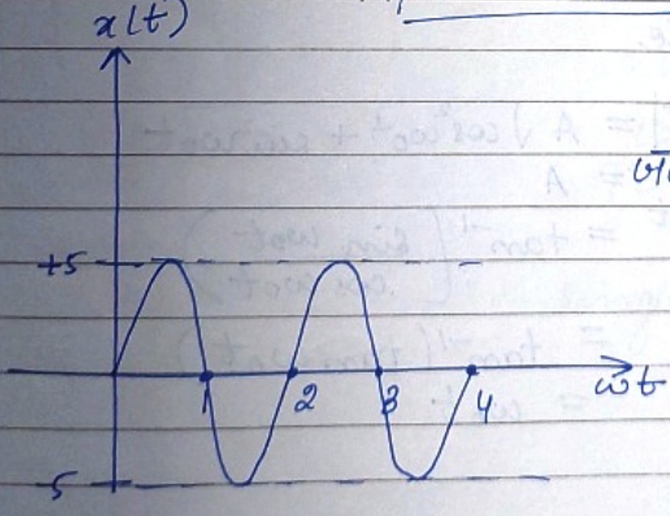
$$\therefore \boxed{x(t) = 5 \sin \pi t}$$



$$T = 2\pi$$

$$\omega = \frac{2\pi}{2\pi} = 1$$

$$\therefore \boxed{A = 4 \sin t}$$



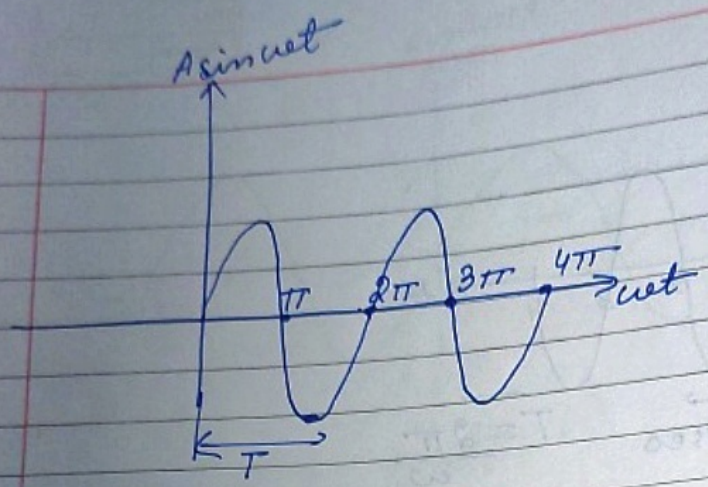
जब भी $\omega t \in I$

then $x(t) = 0$

$$\therefore \boxed{x(t) = A \sin \pi \omega t}$$

35

4 students



Complex Exponentials:-

$x(t) = A_0\, e^{j\omega_0 t}$ — ω_0: fundamental frequency (rad sec^{-1}); $\omega_0 t$: phase angle

$$\boxed{e^{jx} = \cos x + j \sin x}$$

$$\boxed{Ae^{j\omega_0 t} = A(\cos \omega_0 t + j \sin \omega_0 t)}$$

$x = a + jb$

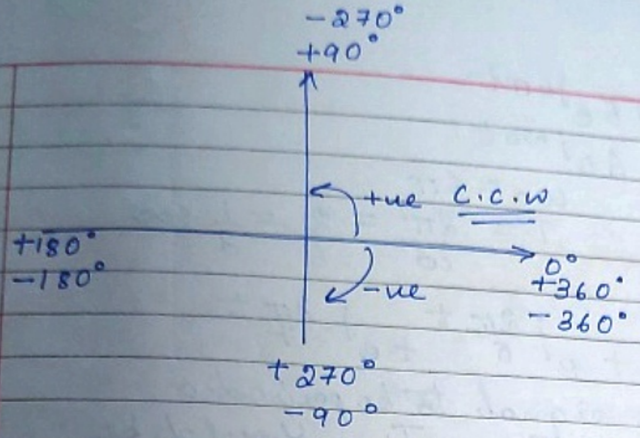
$|x| = \sqrt{a^2 + b^2}$

$\angle x = \tan^{-1}\left(\frac{b}{a}\right)$

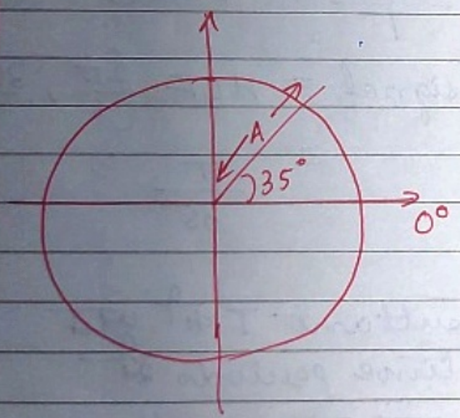
↑ phase angle

$$|A e^{j\omega_0 t}| = A\sqrt{\cos^2 \omega_0 t + \sin^2 \omega_0 t} = A$$

$$\angle A e^{j\omega_0 t} = \tan^{-1}\left(\frac{\sin \omega_0 t}{\cos \omega_0 t}\right) = \tan^{-1}(\tan \omega_0 t) = \omega_0 t$$



ω_0	$\lvert Ae^{j\omega_0 t}\rvert$	$\angle Ae^{j\omega_0 t} = \omega_0 t$
0	A	
⋮	A	
	A	
	⋮	
∞	A	



$x(t) = Ae^{j3\pi t}$

Comparing with standard representation $Ae^{j\omega_0 t}$

$x(t) = Ae^{j(\omega_0 t + \phi)}$

$\omega_0 = 3\pi$

$T = \frac{2\pi}{3\pi} = \frac{2}{3}$ sec.

single representation always periodic.

$$x(t) = 1 + 5e^{j6\pi t}$$
$$Ae^{j\omega_0 t}$$
$$\omega_0 = 6\pi$$
$$T = \frac{2\pi}{\omega} = \frac{2}{6} = \frac{1}{3} \text{ sec.}$$

$$x(t) = 4 + e^{j\frac{2\pi}{5}t} + e^{j\frac{3\pi}{7}t}$$

For the signal to be periodic $\frac{T_2}{T_1}$ $\frac{T_3}{T_2}$ $\frac{T_1}{T_3}$ should be rational.

either ω all in terms of π or none

$$T_1 = \frac{2\pi \times 5}{2\pi} = 5$$

$$T_2 = \frac{2\pi}{3\pi}\,7 = \frac{14}{3}$$

$\therefore$ This signal is periodic.

$$\text{resultant } T = LCM(T_1, T_2)$$
$$= LCM\left(5, \frac{14}{3}\right)$$
$$= \frac{5 \times 14}{1} = 70 \text{ sec}$$

$$\omega \text{ of resultant signal} = HCF\left(\frac{2\pi}{5}, \frac{3\pi}{7}\right)$$
$$= \frac{\pi}{35}$$

Trick:- हम अपने resultant T को जब individual time periods से divide करेंगे हमें हमेशा integer value मिलेगा।

$$\frac{T}{T_1} = \frac{70}{5} = 14$$ oscillations complete करेगा signal 1

$$\frac{T}{T_2} = \frac{\cancel{70}^{5} \times 3}{\cancel{14}_{1}} = 15$$ oscillations complete करेगा signal 2.

Complex Numbers

Rectangular form

Addition & Subtraction

$x = a + jb$

$|x| = \sqrt{a^2+b^2} = r$

$\angle x = \tan^{-1}\left(\frac{b}{a}\right) = \theta$

Polar form

Multiply / Divide / Sq root

$x = re^{j\theta}$ or $r\angle\theta$

$= r(\cos\theta + j\sin\theta)$

$= \underbrace{r\cos\theta}_{a} + \underbrace{rj\sin\theta}_{b}$

① $x = a + jb$

$|x| = \sqrt{a^2+b^2}$

$\angle x = \tan^{-1}\left(\frac{b}{a}\right)$

② $x = a$ (real)

$|x| = a$

$\angle x = \tan^{-1}\left(\frac{0}{a}\right) = 0$

③ $x = jb$ (purely imaginary)

$|x| = b.$

$\angle x = \tan^{-1}\left(\frac{b}{0}\right) = 90^\circ$

④ $x = -a + jb$

$|x| = \sqrt{a^2+b^2}$

$\angle x = 180^\circ - \tan^{-1}\left(\frac{b}{a}\right)$

$x = -(a - jb)$

$= (-1)(a - jb)$

$= j^2(a - jb)$

दो complex nos multiply होते हैं तो phase angle add होगा।

$x = j$ $x = j^2$

$\angle x = 90^\circ$ $= j \cdot j$

$\angle x = 90^\circ + 90^\circ$

$= 180^\circ$

$180^\circ + \tan^{-1}\left(\frac{-b}{a}\right)$

$180^\circ - \tan^{-1}\left(\frac{b}{a}\right)$

$x = 3 + j4$ find $\sqrt{x}$

$r = \sqrt{3^2 + 4^2} = 5$

$x = 5 \angle \tan^{-1}\left(\frac{4}{3}\right)$

$\sqrt{x} = \sqrt{5} \angle \frac{\tan^{-1}\left(\frac{4}{3}\right)}{2}$

phase angle divide हो गया 2 से।

$x = \frac{(4+j5)(3+j6)}{(8+j4)(9+j3)} = \frac{(x_1 \angle \theta_1)(x_2 \angle \theta_2)}{(x_3 \angle \theta_3)(x_4 \angle \theta_4)}$

$= \frac{x_1 x_2}{x_3 x_4} \angle (\theta_1 + \theta_2) - (\theta_3 + \theta_4)$

अगर cube करना है तो

magnitude का cube और phase angle को multiply by 3

Energy and Power Signals

Energy of signal $x(t)$ is given as :-

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

If $0 < E_x < \infty$

then $x(t)$ is said to be an energy signal

Power of $x(t)$ is given as:-

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Normally preferred

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt$$

symmetric limit नहीं है तो generally not preferred.

If $0 < P_x < \infty$ then $x(t)$ is said to be a power signal.

IMP

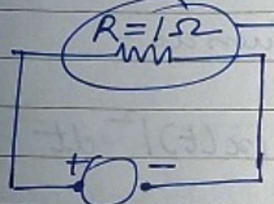
* If energy of a signal is finite then the power of a signal will be zero.

* If the power of a signal will be finite then energy will be infinite.

Any aperiodic signal can be assumed to be periodic with $T = \infty$.

Energy = power × time

$= v(t) \times i(t) \cdot \text{Time}$



→ इस resistance में जितना power आएगा उसे बोलते हैं normalized power.

← signal apply किया voltage signal या current signal जिसे check करना है कि power signal है या energy signal

$$P = I_{rms}^2 R = I_{rms}^2 \text{ (if current signal)}$$
$$= V_{rms}^2 R = V_{rms}^2 \text{ (if voltage signal)}$$

$R = 1\Omega$

$$\boxed{P = (RMS)^2}$$

then the
ll be zero.
be finite
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nal)

$$I_{rms} = \sqrt{\frac{1}{T}\int i(t)^2 \, dt}$$

$$\text{Power} = \frac{1}{T}\int_{-T/2}^{T/2} (i(t))^2 \, dt$$

इतना sufficient है अगर signal periodic है।

Incase signal periodic नहीं है तो $T \to \infty$

$$\text{Power} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |i(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |i(t)|^2 \, dt$$

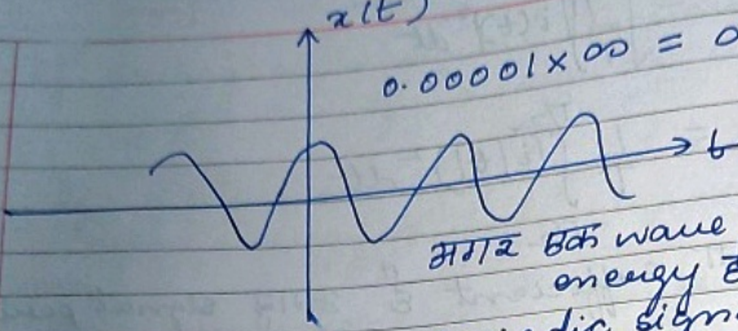
Energy = Power × time

$$\text{Energy} = \lim_{T\to\infty} T \cdot \frac{1}{T}\int_{-T/2}^{T/2} |i(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} |i(t)|^2 \, dt$$

$$E_x = \int_{-\infty}^{\infty} |i(t)|^2 \, dt$$

Assume all the periodic signals are power signals hence cannot be energy signals.



$0.00001 \times \infty = \infty$

अगर एक wave में इतनी energy है $\rightarrow 0.00001$
periodic signals में infinite no of oscillations
so energy $\rightarrow \infty$

$\boxed{\text{It is not necessary that all power signals must be periodic.}}$

$x(t) = e^{-4t} u(t) \rightarrow$ signal not periodic

signal exists from $0 \rightarrow \infty$ "$u(t)$"

"causal signal"

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} |e^{-4t} u(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |e^{-4t} u(t)|^2 dt$$

$$= \int_{0}^{\infty} (e^{-4t})^2 dt \qquad \boxed{u(t)^2 = 1}$$

$$= \int_{0}^{\infty} e^{-8t} dt$$

$$= \left. \frac{e^{-8t}}{-8} \right|_0^{\infty} = 0 - \frac{e^0}{-8} = \frac{1}{8} J$$

energy finite आहे $\therefore$ $x(t) = e^{-4t} u(t)$ is an energy signal

$$\text{Energy} = \lim_{T \to \infty} P \cdot T$$

$$P = \lim_{T \to \infty} \frac{1}{T} \cdot E = 0$$

finite

$$x(t) = [4e^{-2t} + 3e^{-4t}]\, u(t)$$

$\lim_{T \to \infty}$ × cause periodic नाही है u(t) × ही रहा so $0 \to \infty$ lt

$$E_x = \int_0^\infty |4e^{-2t} + 3e^{-4t}|^2 dt$$

$$= \int_0^\infty |16e^{-4t} + 9e^{-8t} + 24e^{-6t}|\, dt$$

$$= \int_0^\infty |16e^{-4t}|\, dt + \int_0^\infty |9e^{-8t}|\, dt + \int_0^\infty |24e^{-6t}|\, dt$$

$$= \cancel{16}^{4}\frac{e^{-4t}}{-\cancel{4}} + 9\frac{e^{-8t}}{-8} + \cancel{24}^{4}\frac{e^{-6t}}{-\cancel{6}} \Big|_0^\infty$$

$$= 0 + 4 + \frac{9}{8} + 4$$

$$= 16 + \frac{9}{8} = \frac{128 + 9}{8} = \frac{137}{8}$$

$$0 < E_x < \infty$$

$\therefore$ $x(t)$ is an energy signal.

$$\boxed{Power = (RMS)^2}$$

$x(t) = u(t)$

$x(t) = A \sin \omega t$

$RMS = \frac{A}{\sqrt{2}}$

$P_x = (RMS)^2 = \frac{A^2}{2}$

For any sinusoidal signal

$x(t) = A \sin t$

$\omega = \cancel{2\pi} 1$

$T = 2\pi$

$P = \lim_{T \to \infty}$

cause periodic

$$\frac{1}{T} \int_{-T/2}^{T/2} |A \sin t|^2 \, dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} A^2 \sin^2 t \, dt$$

$$= \frac{A^2}{2\pi} \int_{-\pi}^{\pi} \sin^2 t \, dt$$

$$= \frac{A^2}{2\pi} \int_{-\pi}^{\pi} \left(\frac{1 - \cos 2t}{2} \right) dt$$

$$= \frac{1}{2} \frac{A^2}{2\pi} \left[\int_{-\pi}^{\pi} dt - \int_{-\pi}^{\pi} \cos 2t \, dt \right]$$

0

$\omega = 2$

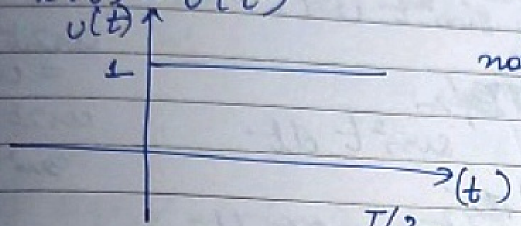
$T = \frac{2\pi}{2} = \pi$

$\pi - (-\pi) = 2\pi$

$$\frac{A^2}{4\pi} \times t \Big|_{-\pi}^{\pi}$$

$$= \frac{A^2}{\cancel{2}\cancel{4}\cancel{\pi}} \times \cancel{2}\cancel{\pi} = \frac{A^2}{2}.$$

* $x(t) = u(t)$



not periodic.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \, t \Big|_{0}^{T/2}$$

$$= \lim_{T\to\infty} \left(\frac{1}{\cancel{T}}\right)\left(\cancel{T}/2\right)$$

$$= \frac{1}{2} \text{ Watt}$$

$$0 < P_x < \infty$$

$\therefore$ $u(t)$ is a power signal.

$$\text{Energy} = \lim_{T\to\infty} \text{power} \times \underset{\infty}{\underset{\downarrow}{\text{time}}}$$

$$= \infty$$

$\omega = 1$
$T = 2\pi$

$x(t) = \sin t \, u(t)$ not periodic

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\sin t \, u(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} \sin^2 t \, u^2(t) \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} \sin^2 t \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} \frac{1-\cos 2t}{2} \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\frac{1}{2} \left(\int_{0}^{T/2} dt - \int_{0}^{T/2} \cos 2t \, dt \right)$$

$\omega = 2$
$T = 2\pi$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(T/2 - \frac{\sin 2t}{2} \Big|_0^{T/2} \right)$$

$$= \lim_{T\to\infty} \frac{1}{4} - \frac{\sin \cancel{2} T/\cancel{2}}{2 \cdot 2T}$$

$$= \frac{1}{4} - \lim_{T\to\infty} \frac{\sin T}{4T}$$

$$= \frac{1}{4}\left(1 - \lim_{T\to\infty} \frac{\sin T}{T} \right)$$

$$= \frac{1}{4}$$

$\therefore$ $x(t)$ is a power signal

$\cos 2t = \cos^2 t - \sin^2 t$

$\cos 2t = 1 - 2\sin^2 t$

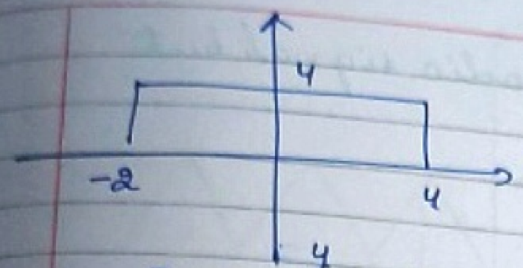
$\sin^2 t = 1 - \cos 2t$

$\lim_{x\to\infty} \frac{\sin x}{x}$

$\frac{-1}{\infty} \le \frac{\sin x}{\infty} \le \frac{1}{\infty}$

$0^- \le \frac{\sin x}{x} \le$

$= 0$



$$E_x = \int_{-2}^{4} (4^2)\, dt = 16 \int_{-2}^{4} dt$$
$$= 16\, (t)_{-2}^{4}$$
$$= 96\, J$$

जो signal infinite duration के लिए exist करते रहते हैं और dec function हैं तो वो energy signal है।

अगर finite duration का signal है और उसका magnitude भी finite है तो energy signal.

* If $x(t)$ is existing for infinite duration and is dec in nature, i.e. $\lim_{T \to \infty} f(t) = 0$

  then $f(t)$ will be an energy signal.

* If $x(t)$ exists for finite duration and value of $x(t)$ is finite at all points then $x(t)$ be an energy signal.

* All periodic signals are power signals but converse is not true.

Power signal*

If $x(t)$ is not a periodic signal but follows :-

$$\lim_{T \to \infty} f(t) \neq 0$$

$$\lim_{T \to \infty} f(t) \neq \infty$$

Signals	Power
$A \sin \omega t$ or $A \cos \omega t$	$\frac{A^2}{2}$
$A u(t)$	$\frac{A}{2}$
$A e^{j\omega_0 t}$	A^2
A	A^2

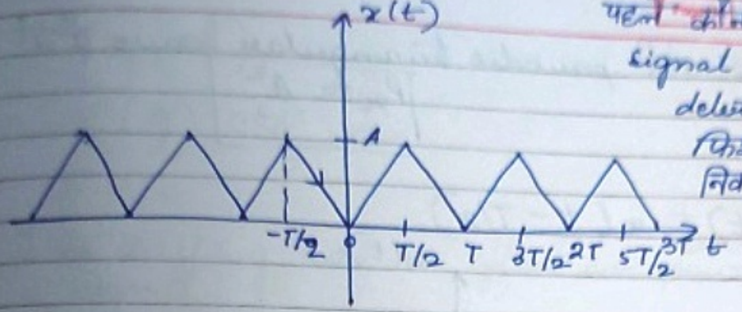
Energy and Power Signals

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

real signal है तो | | का कोई काम नहीं है but complex signal में | | काम आएगा।

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

पहले कौन सा signal है वो determine करें फिर values निकालें।



Periodic signal $\longrightarrow$ Power signal

$$x(t) = -\frac{2A}{T}(t) \quad ---- \quad -T/2 < t < 0$$

$$\frac{2A}{T}(t) \quad ---- \quad 0 < t < T/2$$

$$x_{rms}(t) = \cancel{\frac{4A^2(t^2)}{T^2}} \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt}$$

$$P_x = |x_{rms}(t)|^2$$

$$\therefore P_x = \frac{1}{T}\left[\int_{-T/2}^{0} \left(-\frac{2A}{T}t\right)^2 dt + \int_{0}^{T/2}\left(\frac{2A}{T}t\right)^2 dt\right]$$

$$= \frac{1}{T}\left[\frac{4A^2}{T^2}\left(\int_{-T/2}^{0} t^2 dt + \int_{0}^{T/2} t^2 dt\right)\right]$$

$$= \frac{1}{T}\left[\frac{4A^2}{T^2}\left(\frac{t^3}{3}\Big|_{-T/2}^{0} + \frac{t^3}{3}\Big|_{0}^{T/2}\right)\right]$$

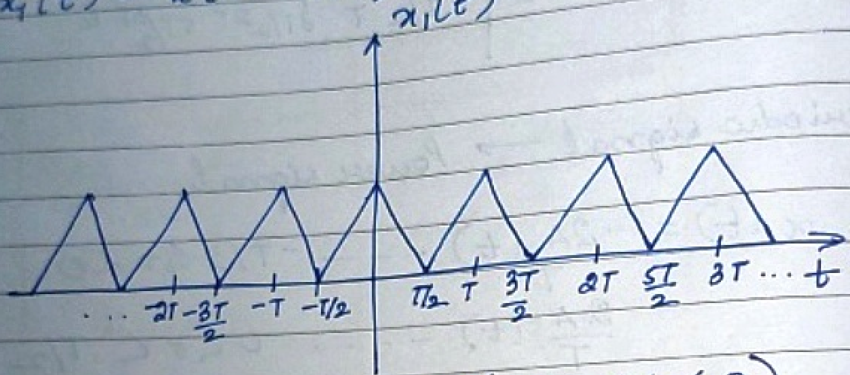
$$= \frac{1}{T}\left[\frac{4A^2}{T^2}\left(-\frac{(-T/2)^3}{3} + \frac{(T/2)^3}{3}\right)\right]$$

$$= \frac{1}{\cancel{T}}\left[\frac{\cancel{4}A^2}{\cancel{T^2}} \quad \frac{\cancel{2} \times \cancel{T^3}}{3 \times \cancel{8}\,\cancel{4}}\right] = \frac{A^2}{3}$$

अगर कोई periodic triangular wave है तो

$$\boxed{P_x = \frac{A^2}{3}}$$

$x_1(t) = x(t - T/2)$



$$x_1(t) = \frac{2A}{T}(t) + A \quad (-T/2 < t < 0)$$

$$-\frac{2A}{T}(t) + A \quad (0 < t < T/2)$$

$$P_x = \frac{1}{T}\left[\int_{-T/2}^{0}\left(\frac{2A}{T}(t) + A\right)^2 dt + \int_{0}^{T/2}\left(-\frac{2A}{T}(t) + A\right)^2 dt\right]$$

even function

$$= \frac{1}{T} \times 2 \times \int_0^{T/2}\left(\frac{2A}{T}(t) - A\right)^2 dt$$

$$= \frac{1}{T} \times 2 \times \left[\int_0^{T/2} \frac{4A^2t^2}{T^2}\, dt - \int_0^{T/2} \frac{4A^2 t}{T}\, dt + \int_0^{T/2} A^2\, dt\right]$$

$\left(\frac{T}{2}\right)^3 \times \frac{1}{3}$

$\frac{T^3}{24}$

$$= \frac{1}{T} \times 2 \times \left[\frac{4A^2}{T^2}\, \frac{t^3}{3}\Big|_0^{T/2} - \frac{4A^2}{T}\, \frac{t^2}{2}\Big|_0^{T/2} + A^2\, t\Big|_0^{T/2}\right]$$

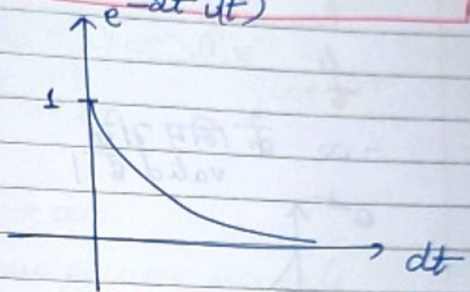
$$= \frac{1}{\cancel{T}} \times 2\left[\frac{\cancel{4}A^2}{\cancel{T^2}}\, \frac{\cancel{T^3}}{\cancel{24}_6} - \frac{\cancel{4}A^2}{\cancel{T}} \times \frac{\cancel{T^2}}{\cancel{8}_2} + \frac{A^2 \cancel{T}}{2}\right]$$

$$A^2\left(\frac{1}{3}\right)$$

$$= A^2\left(\frac{\cancel{T}}{3}\right) = \frac{A^2}{3}$$

$\boxed{\text{Time shifting has no effect on power or energy of a signal.}}$

$$x(t) = e^{-2t}u(t)$$



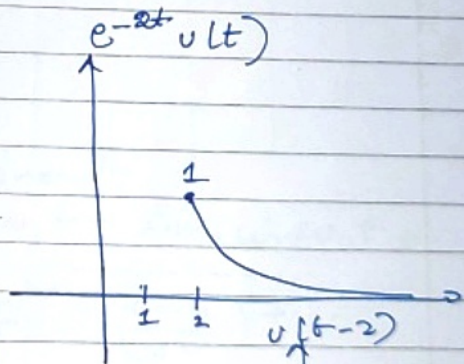
$$E_x = \int_0^\infty (e^{-2t})^2\,dt$$

$$= \int_0^\infty e^{-4t}\,dt$$

$$= -\frac{e^{-4t}}{4}\Big|_0^\infty$$

Energy signal

$$= -\left(0 - \frac{1}{4}\right) = \frac{1}{4}.$$

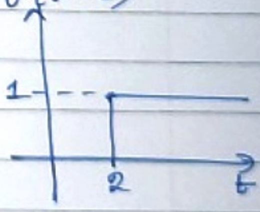


$$x_1(t) = x(t-2)$$

$$= e^{-2(t-2)}u(t-2)$$

$$= e^{-2t+4}u(t-2)$$

$$= e^4 \cdot e^{-2t}u(t-2)$$



$$E x_1(t) = \int_2^\infty \left(e^4 \cdot e^{-2t}u(t-2)\right)^2 dt$$

$$= \int_2^\infty \left(e^4 \cdot e^{-2t}\right)^2 dt$$

$$= e^8 \int_2^\infty e^{-4t}\,dt = e^8\,\frac{e^{-4t}}{-4}\Big|_2^\infty$$

$= \frac{-e^{8}}{4}\left(0 - \frac{1}{e^{8}}\right)$

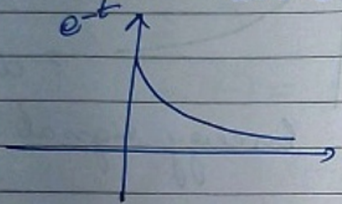
$= \frac{1}{4}$ (same)

Signals

Bounded

$\lim_{t\to\infty} x(t) < \infty$

$-\infty$ के लिए भी valid है।

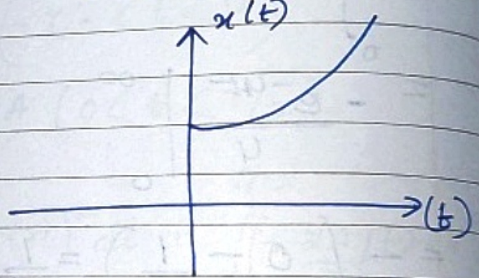


Unbounded

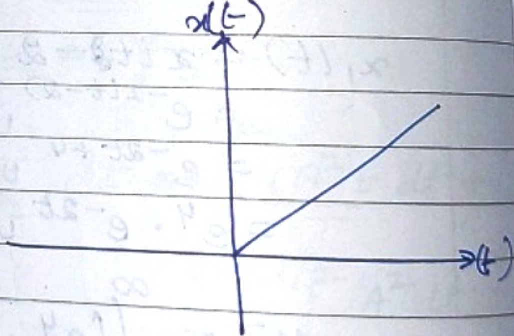
$\lim_{t\to\infty} x(t) = \infty$

$\lim_{t\to\infty} x(t) = \infty$

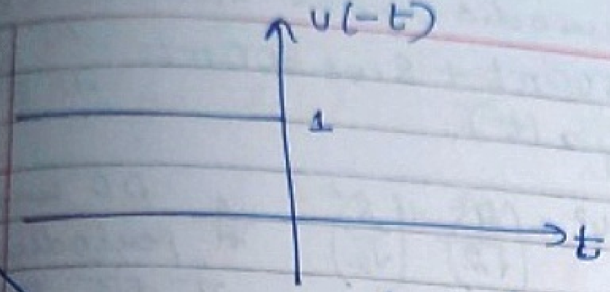
$x(t) = e^{2t}u(t)$

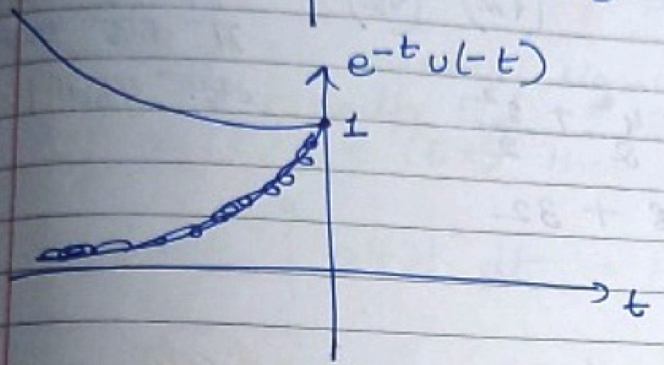


$x(t) = t \cdot u(t) = r(t)$



$x(t) = e^{-t} u(-t)$





$t \to -\infty \quad x(t) \to \infty$

Neither energy nor power signals.

$\text{Power} = (RMS)^2$

$x(t) = A \sin \omega t$

$x_{rms}(t) = \frac{A}{\sqrt{2}}$

$P_x = \frac{A^2}{2}$

$\xrightarrow{\text{DC component}}$

$x(t) = A + A_{m_1} \sin \omega_1 t + A_{m_2} \sin \omega_2 t + \ldots$

$x_{rms}(t) = \sqrt{A^2 + \left(\frac{A_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{A_{m2}}{\sqrt{2}}\right)^2 + \ldots}$

$P_x = A^2 + \left(\frac{A_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{A_{m2}}{\sqrt{2}}\right)^2$

periodic signal सारे ω में न हो
या तो ना हो।

$x(t) = 4 + 4\sin 500\pi t + 8\cos 300\pi t$

find power of $x(t)$.

$x_{rms}(t) = \sqrt{4^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{8}{\sqrt{2}}\right)^2}$

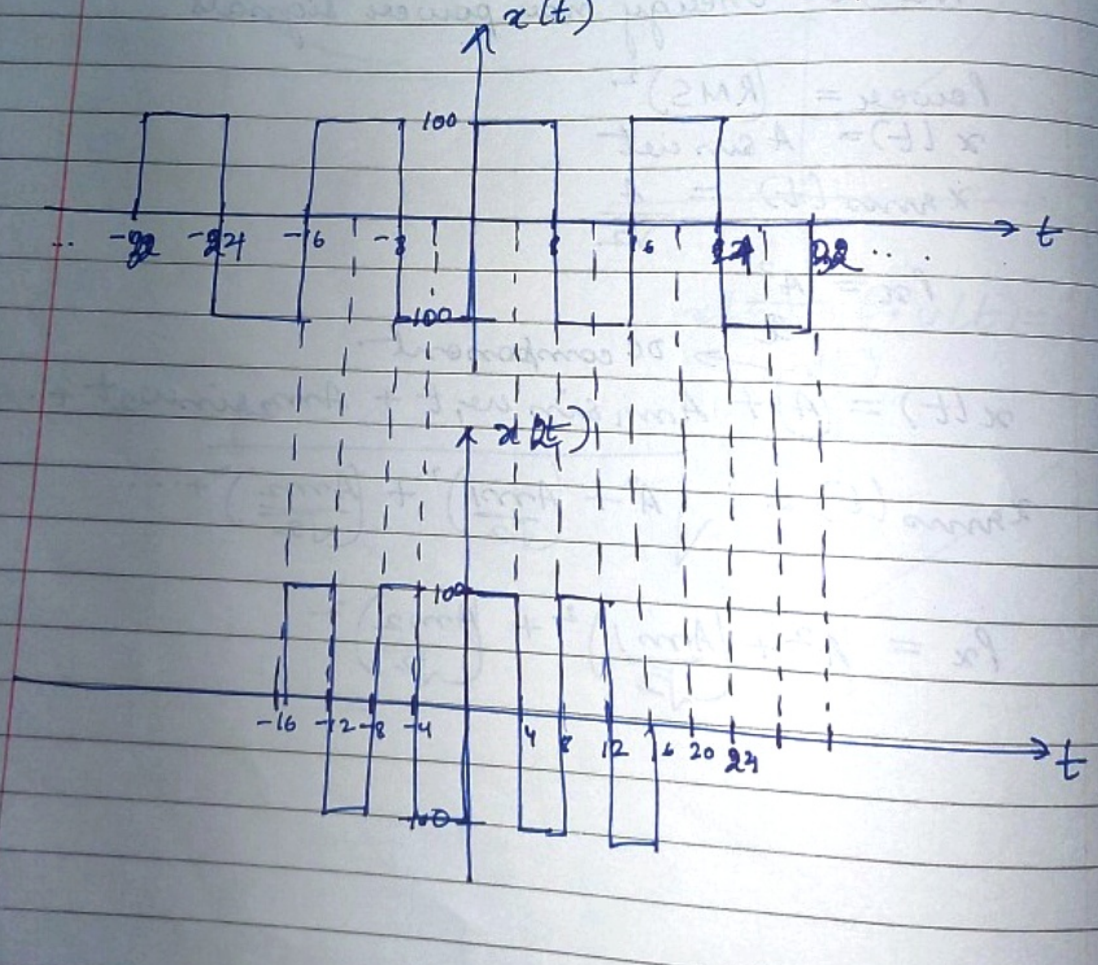
DC component से periodicity में कोई फर्क नहीं पड़ता।

$P_x = 4^2 + \frac{4^2}{2} + \frac{8^2}{2}$

$= 16 + 8 + 32$

$= 56$

Effect of time scaling on periodicity and period:-



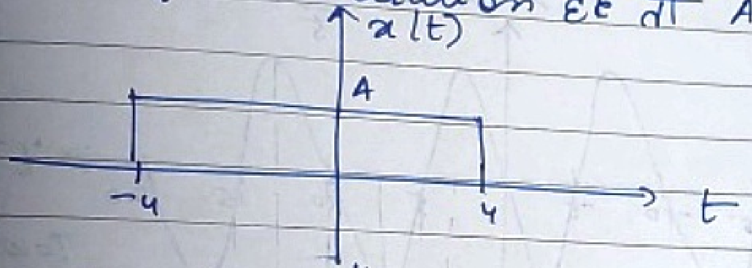
If for $x(t)$, $T = T_1$
for $x(\alpha t)$, $T = \frac{T_1}{\alpha}$

(2) Effect of Time scaling on energy of a signal:-

Consider an energy signal $x(t)$ existing for $-4 < t < 4$ with $E_x = 100J$
If $x_1(t) = x(5t)$ then $E_{x_1(t)} = ?$

$$E_x = \int_{-4}^{4} |x(t)|^2 dt = 100J.$$

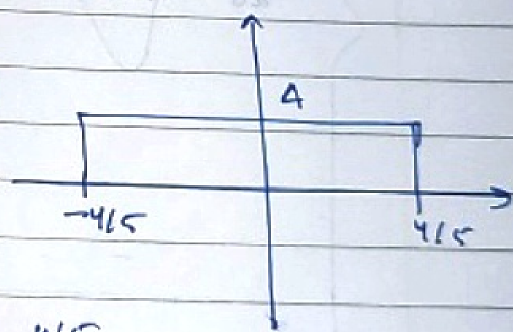
अगर finite duration है तो A भी finite



$$\Rightarrow A^2 \; t\Big|_{-4}^{4} = 100$$

$$\Rightarrow A^2 (8) = 100$$

$$\Rightarrow A^2 = \frac{100}{8}$$



$$E_{x_1} = \int_{-4/5}^{4/5} |x(t)|^2 dt = A^2 \int_{-4/5}^{4/5} dt$$

$$= A^2 \left(\frac{8}{5}\right)$$

$$= \frac{100}{8} \times \frac{8}{5} = 20J.$$

IMP

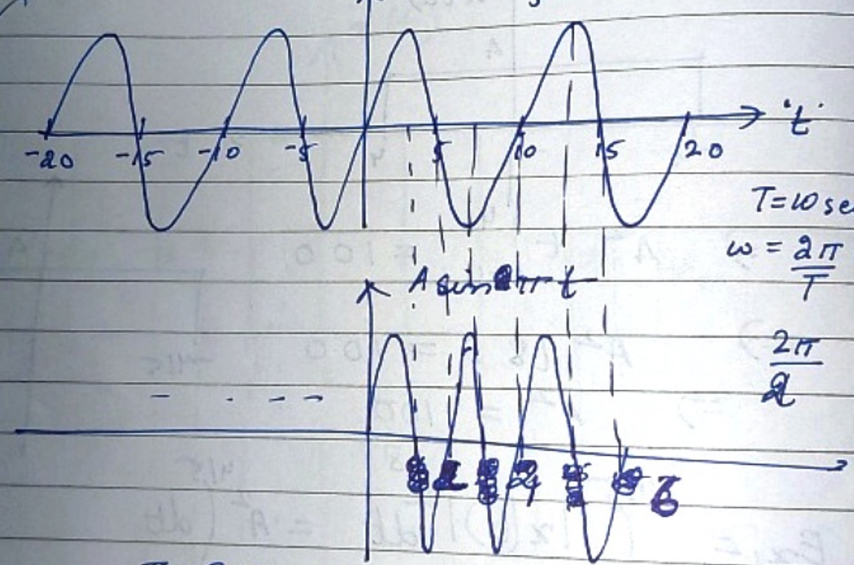
Conclusion:-

$$\boxed{\begin{aligned} \text{If } E_{x(t)} &= E \\ E_{x(\alpha t)} &= \frac{E}{\alpha} \end{aligned}}$$

(3) Effect of time scaling on power of a signal.

$x(t)$ is a power signal with $T = 10\,sec$ and
$P_x = 100W$
$x_1(t) = x(5t)$ find $P_{x_1 t}$?

$x(t)$ $A \sin \frac{\pi}{5} t$



$T = 10\,sec$

$\omega = \frac{2\pi}{T} = \frac{\pi}{5}$

$\frac{2\pi}{2}$

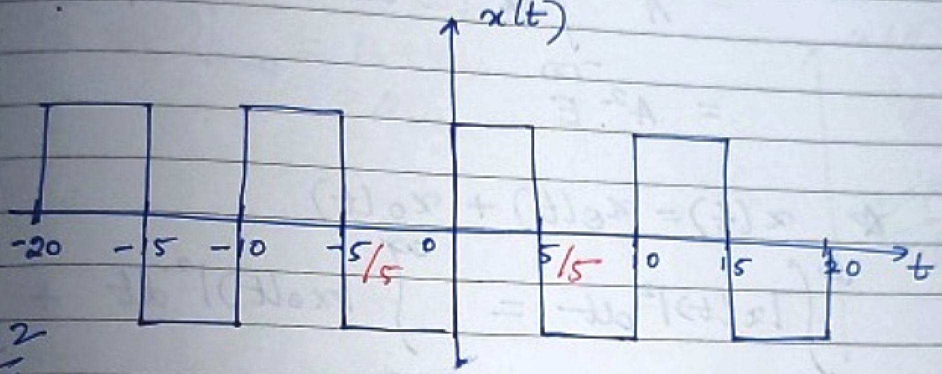
$T = \frac{10}{5} = 2\,sec$

$$P_{x}t = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \sin^2 \frac{\pi}{5} t \, dt = 100W$$

$$= A^2 \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{1 - \cos \frac{2\pi}{5} t}{2} \, dt$$

$$\Rightarrow \frac{A^2}{2} \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(1 - \cos \frac{2\pi}{5} t \right) dt = 100$$

$$\Rightarrow \frac{A^2}{2} \lim_{T \to \infty} \frac{1}{T} \left(t \Big|_{-T/2}^{T/2} - \right.$$



$T = \frac{10^2}{8}$

$$P_x = \frac{1}{10} \int_{-5}^{5} A^2 dt = 100\,W$$

$$\frac{A^2}{10}(10) = 100$$

$$A = 10$$

$$P_{x_1} = \frac{1}{2} \int_{-1}^{1} A^2 dt$$

$$= \frac{1}{2}(100)(2)$$

$$= 100\,W.$$

Time scaling का कोई effect power पे नह

IMP

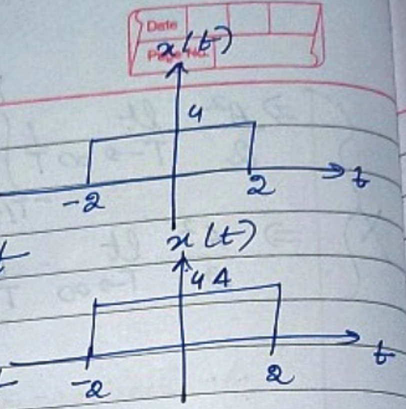
$E_x(t) = E$

$E_{Ax}(t) = ?$

$$E_{Ax}(t) = \int_{-\infty}^{\infty} A^2 x^2(t)\, dt$$

$$= A^2 \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= A^2 E$$



☆ $x(t) = x_e(t) + x_o(t)$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x_e(t)|^2 dt + \int_{-\infty}^{\infty} |x_o(t)|^2 dt$$

$$x^2(t) = x_e^2(t) + x_o^2(t) + 2x_e(t)x_o(t)$$

$$\boxed{\int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} x_e^2(t)\, dt + \int_{-\infty}^{\infty} x_o^2(t)\, dt} + \cancel{\int_{-\infty}^{\infty} 2x_e(t)x_o(t)}^{\,0}$$

even × odd = odd func.

$$\int_{-\infty}^{\infty} \text{odd}\, dt = 0$$

Discrete Time Signals

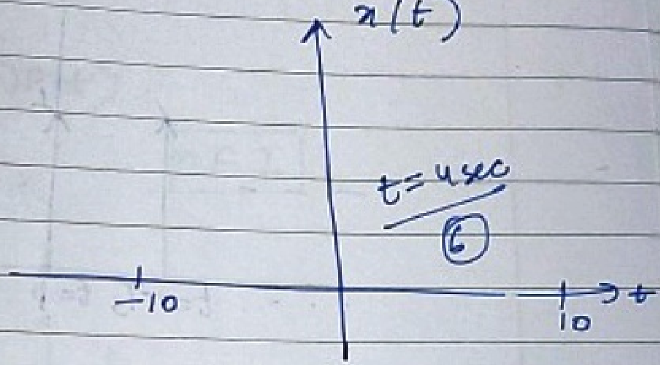
Signals

Deterministic | Stochastic / Random

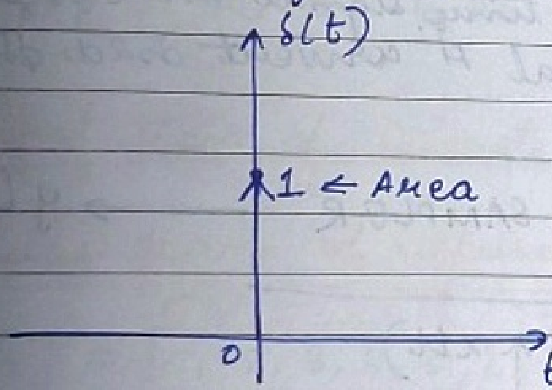
$x(t) = t^2+4;\ t>0$

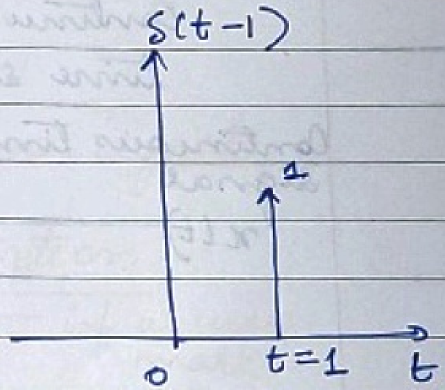
$x(t)\big|_{t=4} = 16+4$

$= 20$

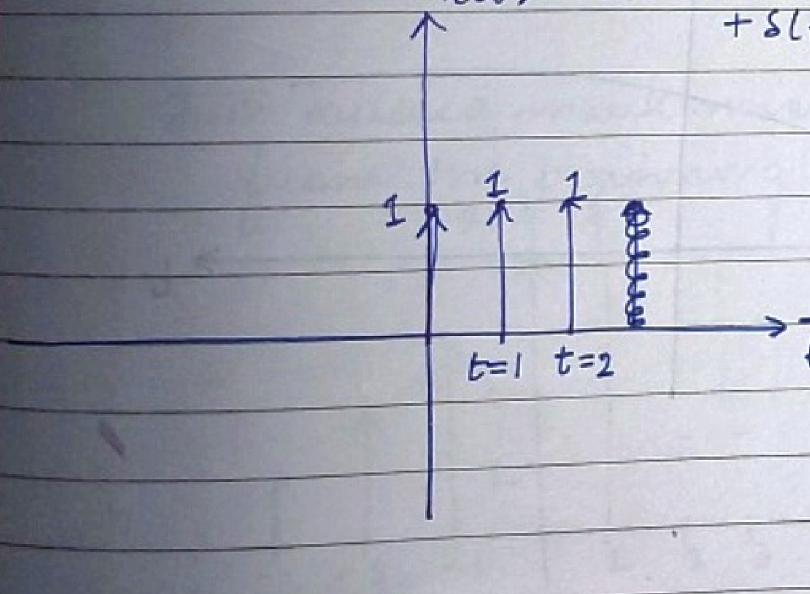


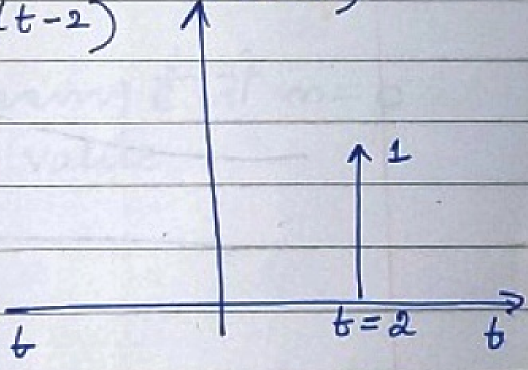
Impulse function





$x(t) = \delta(t) + \delta(t-1) + \delta(t-2)$



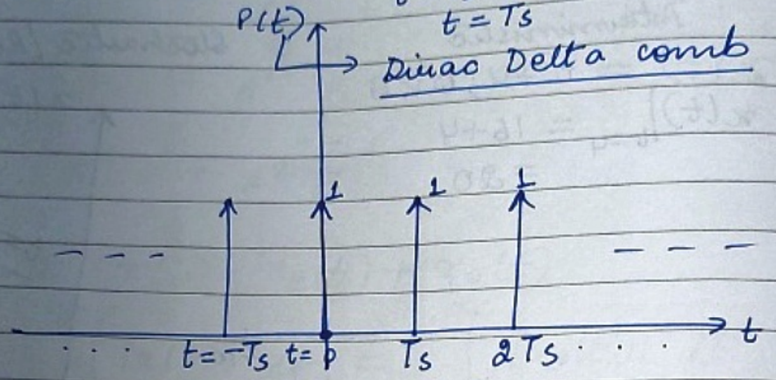


$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$

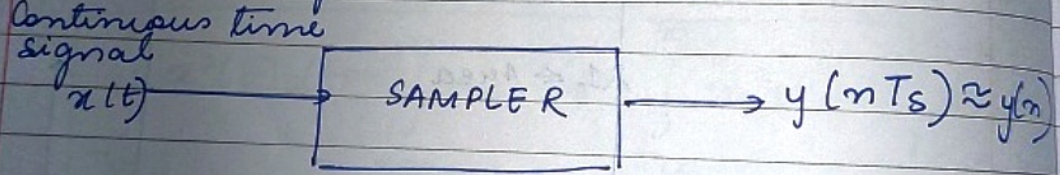
$$p(t) = \dots \delta(t+2T_s) + \delta(t+T_s) + \delta(t) + \delta(t-T_s) + \delta(t+2T_s) + \dots$$

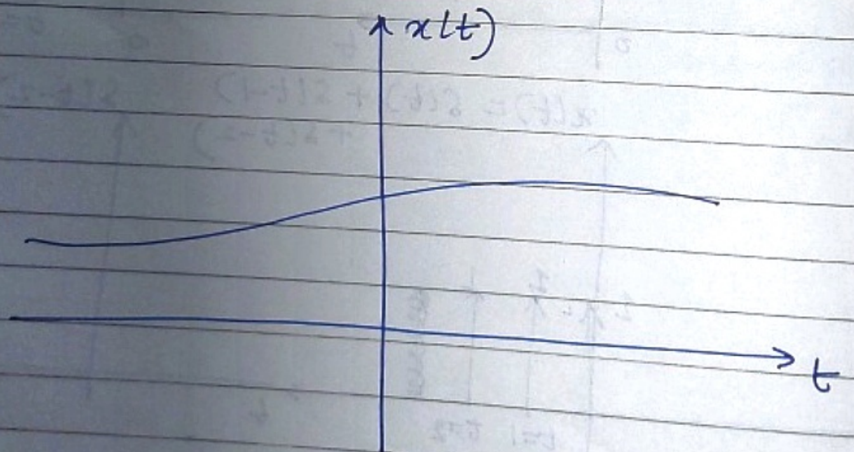
$t - T_s = 0$

$t = T_s$



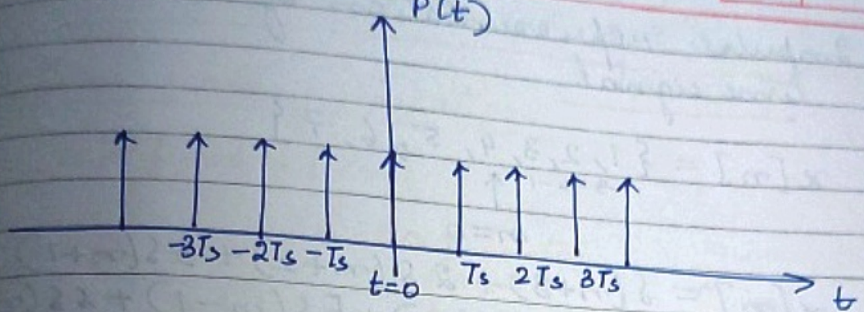
यही एक function है जो किसी भी continuous time signal को discrete time signal में convert करके देगा।





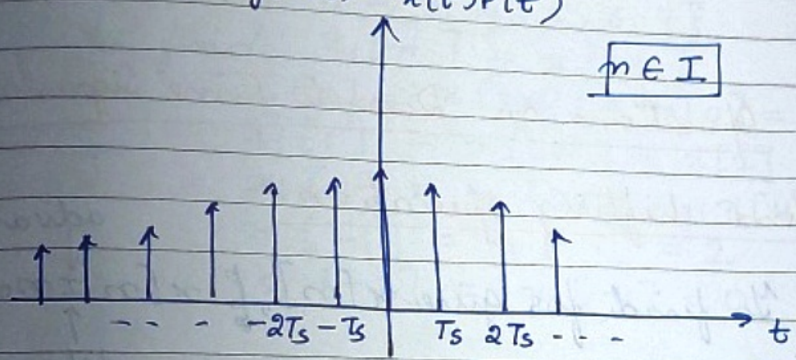
$+\delta(t)$

$\approx y(n)$



$y(nT_s) = x(t)P(t)$

$\boxed{n \in I}$



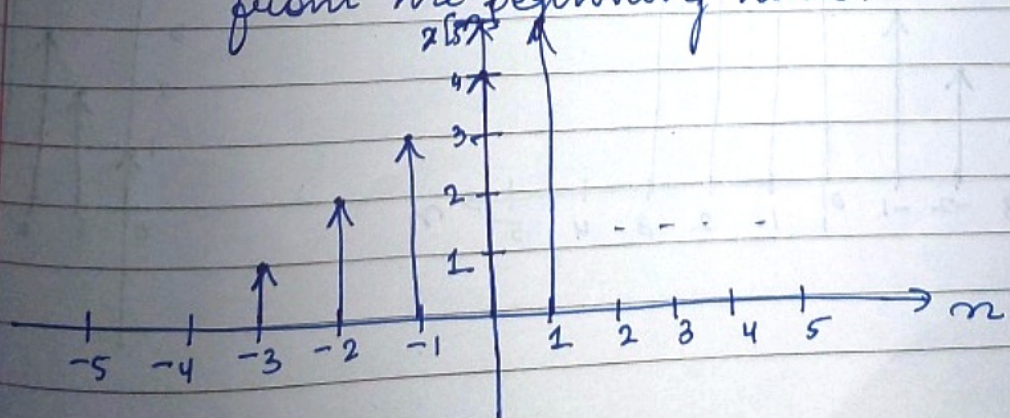
Representation of Discrete Time signal

① Tabular form of representation:-

n=0 n=1 n=2 . . . — if arrow mark missing

$$x[n] = \{\underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\substack{\uparrow \\ n=0}}{4}, 5, 6, 7\}$$

अगर arrow mark missing है तो n=0 from the beginning value.



② Impulse representation of discrete time signal.

$$x[n] = \{\underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\uparrow}{4}, 5, 6, 7\}$$

$n=0$

$$x[n] = \delta(n+3) + 2\delta(n+2) + 3\delta(n+1) + 4\delta(n) + 5\delta(n-1) + 6\delta(n-2) + 7\delta(n-3)$$

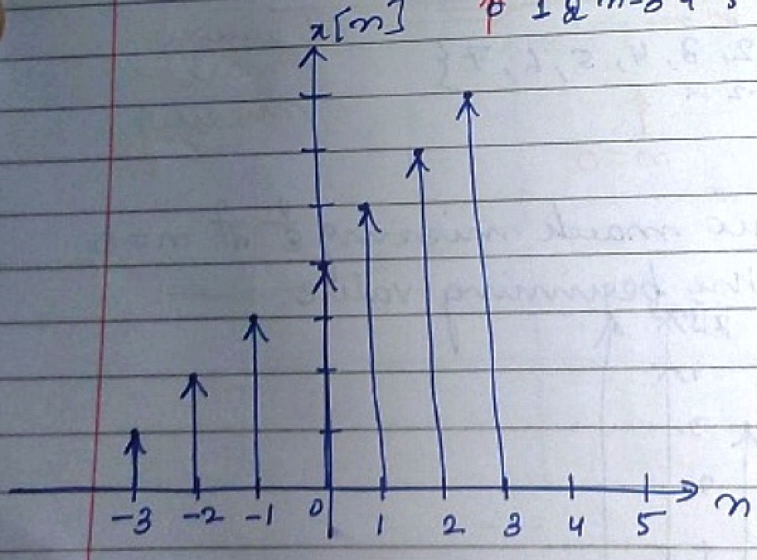
Operations on Discrete Time Signals

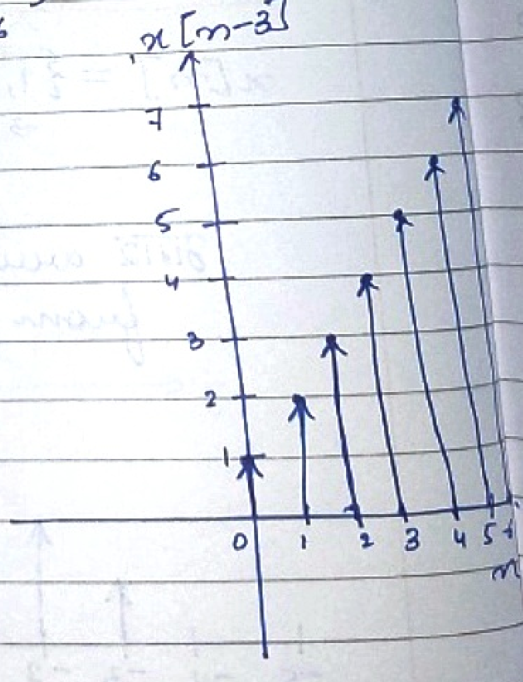
① Time shifting operation:-

To find for give $x[n]$, $x[n \pm n_0]$ — advance ($+$), delay ($-$)

$$x[n] = \{\underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\substack{0\\ \uparrow}}{4}, \underset{1}{5}, \underset{2}{6}, \underset{3}{7}\}$$ both $n, n_0 \in I$

$$x[n-3] = \{\underset{\substack{\uparrow\\ 0}}{1}, \underset{1}{2}, \underset{2}{3}, \underset{n=3}{4}, \underset{4}{5}, \underset{5}{6}, \underset{6}{7}\}$$





$x[n+2] = \{1, 2, 3, \overset{-2}{4}, \overset{-1}{5}, \underset{\uparrow}{\overset{0}{6}}, \overset{1}{7}\}$

② Time scaling operation

to find $x[\underset{\substack{\downarrow \\ \text{scaling factor}}}{\alpha} n]$

$\alpha > 1$ decimation in time

$\alpha < 1$ interpolation

$x[n] = \{1, 2, 3, \underset{\uparrow}{4}, 5, 6, 7\}$

to find $x_1[n] = x[2n]$

$x_1[0] = x[2\times0] = x[0] = 4$

$x_1[1] = x[2\times1] = x[2] = 6$

$x[2] = x[2\times2] = x[4] = 0$

$x[-1] = x[-2] = 2$

$x[-2] = x[-4] = 0.$

$x_1[n] = \{2, \underset{\uparrow}{4}, 6\}$

compress

$x[n] = \{\underset{-4}{2}, \underset{-3}{5}, \underset{-2}{8}, \underset{-1}{7}, \underset{\substack{\uparrow \\ n=0}}{9}, \underset{1}{12}, \underset{2}{14}, \underset{3}{18}, \underset{4}{20}, \underset{5}{27}\}$

$x_1[n] = x[2n]$

$= \{2, 8, \underset{\uparrow}{9}, 14, 20\}$

$x_2[n] = x[3n]$

$= \{5, \underset{\uparrow}{9}, 18 \cancel{27}\}$

values के बीच में 0 append करनी।

Interpolation (i.e when $\alpha < 1$)

$$x[n] = \{\underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\uparrow 0}{4}, \underset{1}{5}, \underset{2}{6}, \underset{3}{7}\}$$

$$x_1[n] = x\left[\frac{n}{2}\right] = \{1,0,2,0,3,0,4,0,5,0,6,0,7\}$$

$$x_1[0] = x\left[\frac{0}{2}\right] = 4$$

$$x_1[1] = x\left[\frac{1}{2}\right] = 0$$

$$x_1[2] = x\left[\frac{2}{2}\right] = 5$$

$$\boxed{x_1[n] = x\left[\frac{n}{\alpha}\right]}$$

($\alpha-1$) zeroes will be appended between each two values of $x[n]$

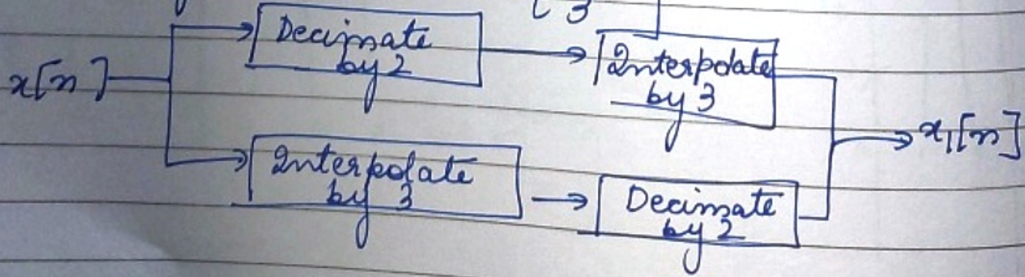
$$x[n] = \{5, \underset{\uparrow}{9}, 3, 12, 14, 18\}$$

$$x\left[\frac{n}{3}\right] = \{1\ 0\ 0\ 2\ 0\ 0\ 3\ 0\ 0\ 4\ 0\ 0\ 5\ 0\ 0\ 6\ 0\ 0\ 7\}$$

$$x\left[\frac{n}{3}\right] = \{5,0,0,9,0,0,3,0,0,12,0,0,14,0,0,18\}$$

$$x[n] = \{3, \underset{\uparrow}{5}, 6, 8, 9\}$$

To find $x_1[n] = x\left[\frac{2}{3}n\right]$



$$\{\underset{-1}{3}, \underset{\underset{0}{\uparrow}}{5}, \underset{1}{6}, \underset{2}{8}, \underset{3}{9}\}$$

$$x[2n] = \{\underset{\uparrow}{5}\ 8\}$$

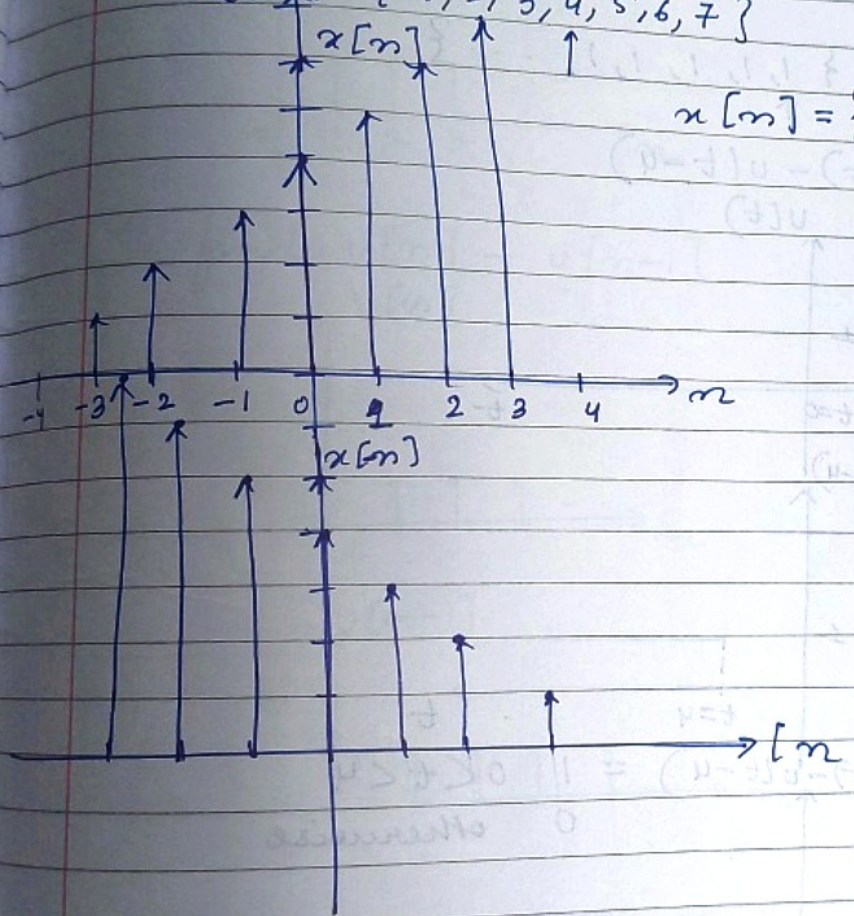
$$x\left[\frac{2n}{3}\right] = \{\underset{\uparrow}{5}, 0, 0, 8\}$$

③ Time reversal operation

find $x[-n]$

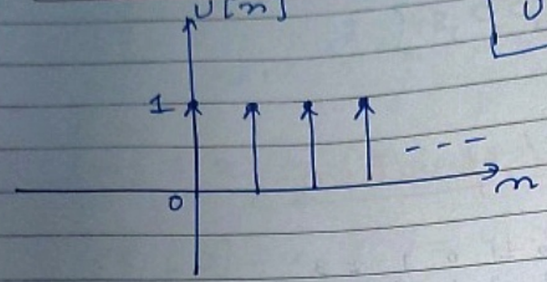
$$x[n] = \{\underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\underset{\uparrow}{0}}{4}, \underset{1}{5}, \underset{2}{6}, \underset{3}{7}\}$$

$$x[n] = \{7, 6, 5, 4, 3, 2, 1\}$$



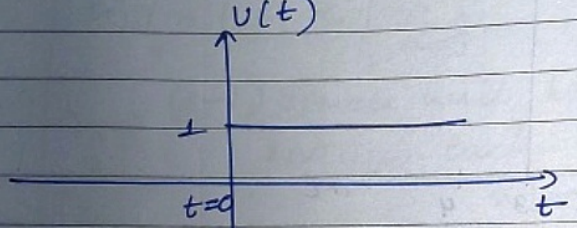
① Unit Step sequence :-

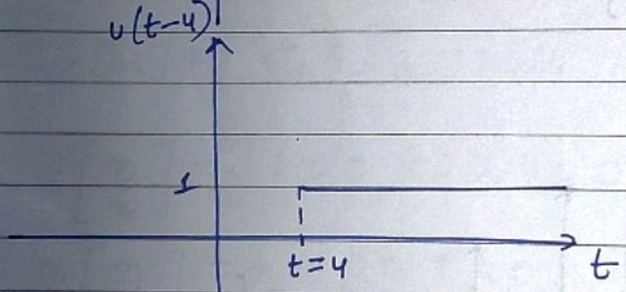
$$\boxed{u[n] = 1 \;, n \geq 0} \\ = 0 \;, n < 0$$



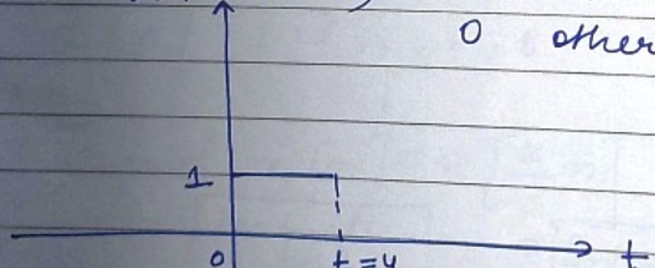
$$u[n] = \{ \underset{\uparrow}{1}, 1, 1, 1, 1, \ldots \}$$

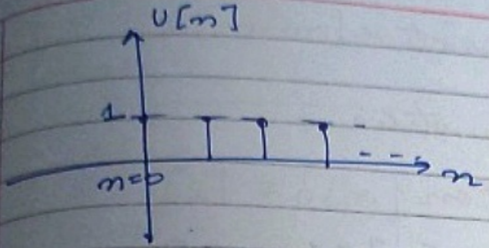
$u(t) - u(t-4)$

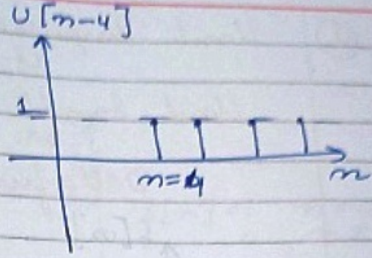


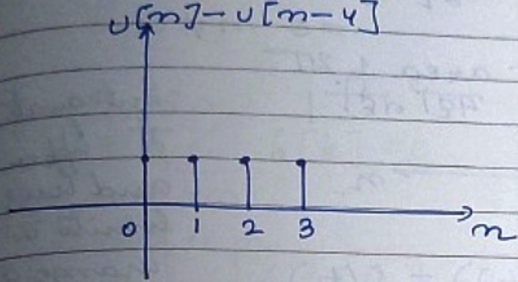


$$u(t) - u(t-4) = 1 \quad 0 < t < 4$$
$$\qquad\qquad\qquad 0 \quad \text{otherwise}$$

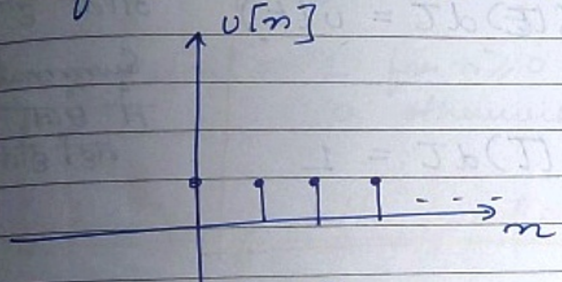


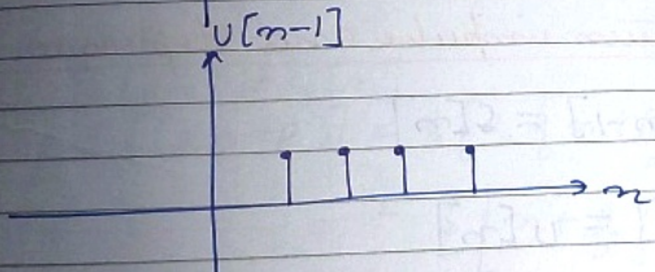




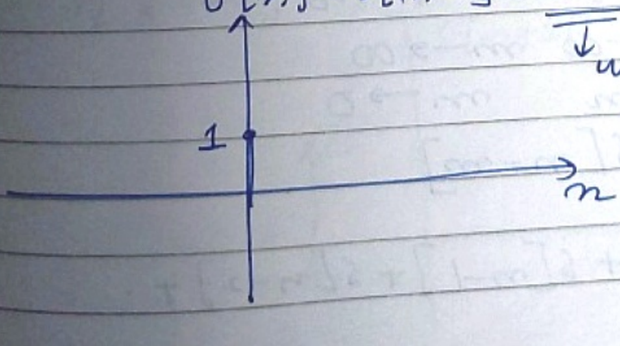


To find $u[n] - u[n-1]$



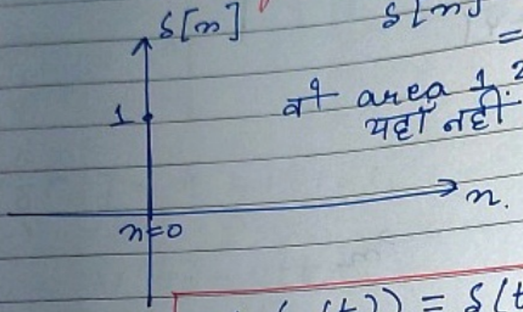


$u[n] - u[n-1] = \underline{\delta[n]}$ ↓ unit impulse function



② Unit Impulse function
or
Unit Sample function
or
Delta function

$$\delta[n] = 1 \quad n = 0$$
$$= 0 \quad n \neq 0$$



वो area 1 था यहाँ नहीं।

$$\boxed{\begin{aligned} &\frac{d}{dt}(u(t)) = \delta(t) \\ &\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t) \\ &\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1 \end{aligned}}$$

Integuation में upper and lower limits को change करने में (-) बाहर आता है।

Summation में ऐसा नहीं होता है।

Relation between impulse and step sequence:-

① $u[n] - u[n-1] = \delta[n]$

② $\sum_{k=-\infty}^{n} \delta[k] = u[n]$

Let $-k+n = m \Rightarrow k = n-m$

When $k \to -\infty \quad m \to \infty$

$k \to n \quad m \to 0$

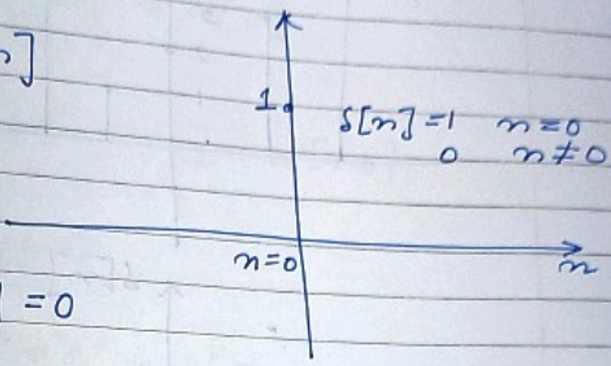
$$\delta = \sum_{m=0}^{\infty} \delta[n-m]$$

$$= \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

$$= u[n]$$

$$\boxed{\sum_{m=0}^{\infty} \delta[n-m] = u[n]}$$

$$\sum_{k=-\infty}^{n} \delta[k] = u[n]$$



$\delta[n] = 1 \quad n=0$
$\qquad\;\; 0 \quad n \neq 0$

for $n<0$ n

$$\sum_{k=-\infty} \delta[k] = 0$$

for $n \geq 0$ n

$$\sum_{k=-\infty} \delta[k] = 1$$

$u[n] = 1$ for $n \geq 0$
$\qquad\;\; 0$ otherwise

Properties of delta function

$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$

cause α area को affect करता है।

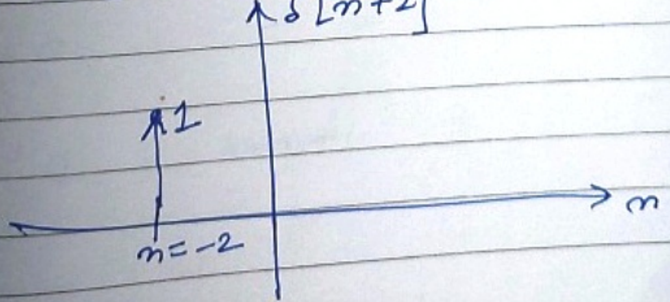
① Time scaling

$$\delta[\alpha n] \neq \delta[n]$$

$$\delta[(\alpha n+\beta)] = \delta\left[\alpha\left\{n+\frac{\beta}{\alpha}\right\}\right]$$

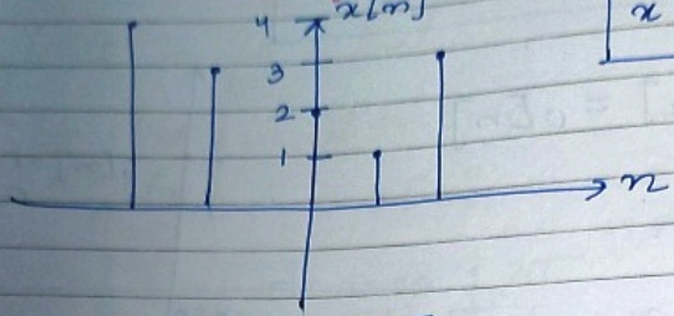
$$= \delta\left[n+\frac{\beta}{\alpha}\right]$$

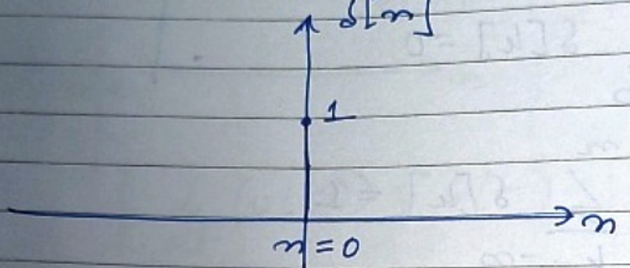
$$\delta[2n+4] = \delta[2(n+2)] = \delta[n+2]$$

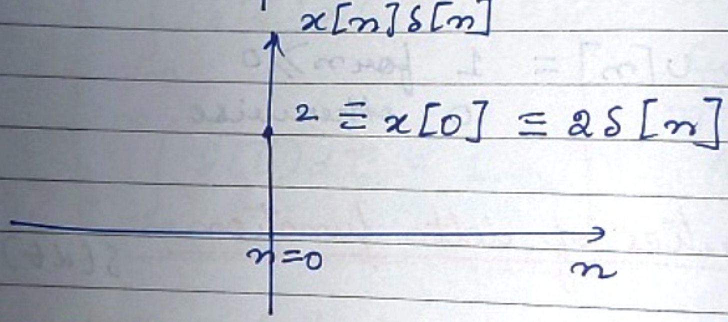


② Product property of delta function :-

$$\boxed{x(t)\cdot\delta(t) = x(0)\cdot\delta(t)}$$







$$\boxed{x[n]\cdot\delta[n] = x[0]\cdot\delta[n]}$$

$$\boxed{x[n]\cdot\delta[n-n_0] = x[n_0]\cdot\delta[n-n_0]}$$

(t)

$\boxed{x(0)\cdot\delta(t)}$

② Shifting property of $\delta[n]$

$$\sum_{n=-\infty}^{\infty} x[n]\,\delta[n] = x[0]$$

$$\sum_{n=-\infty}^{\infty} x[n]\,\delta[n-n_0] = x[n_0]$$

Some imp results:-

① $\displaystyle\sum_{n=0}^{\infty} a^n = a^0 + a^1 + a^2 + \dots$

$\displaystyle = \cancel{\frac{1-(a^{\infty}-1)}{a}}\ \frac{1}{1-a}$ given $|a|<1$

② $\displaystyle\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$

③ $\displaystyle\sum_{n=0}^{N} 1^n = N+1$

④ $\displaystyle\sum_{n=0}^{N} n = \frac{N(N+1)}{2}$

⑤ $\displaystyle\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$

⑥ $\displaystyle\sum_{n=0}^{N} n^3 = \left[\frac{N(N+1)}{2}\right]^2$

⑦ $\displaystyle\sum_{n=0}^{b} 1 = b-a+1$

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Even and Odd Sequences

$$x[-n] = x[n] \quad \text{(even sequence)}$$
$$x[-n] = -x[n] \quad \text{(odd sequence)}$$

$$x[n] = \{\overset{-2}{2}, \overset{-1}{3}, \underset{\uparrow\, n=0}{1}, \overset{1}{3}, \overset{2}{2}\} \rightarrow \text{even sequence}$$

$$x[n] = \{-2 \quad -3 \quad \underset{\uparrow\, n=0}{0} \quad 3 \quad 2\}$$

$$x[-n] = \{2 \quad 3 \quad 0 \quad -3 \quad -2\}$$
odd sequence

Any discrete time sequence can never be an odd sequence if its having any other digit other than 0 at origin.

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Find even and odd parts of $x[n] = \{1 \ \underset{\uparrow}{2} \ 3 \ 4\}$

$$x[n] = \{\overset{-1}{1} \quad \underset{\uparrow}{\overset{0}{2}} \quad \overset{1}{3} \quad \overset{2}{4}\}$$

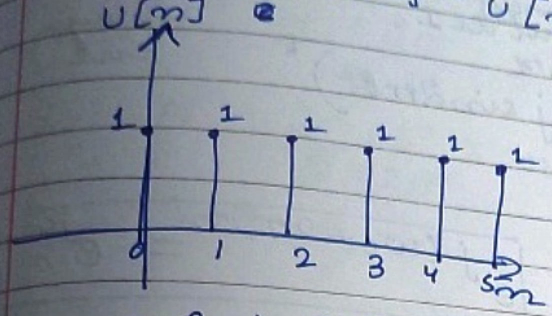
$$x[-n] = \{\overset{-2}{4} \quad \overset{-1}{3} \quad \underset{\uparrow}{\overset{0}{2}} \quad \overset{1}{1}\}$$

$$x_e[n] = \{4 \quad 4 \quad 4 \quad 4 \quad 4\}/2$$
$$= \{2 \quad 2 \quad \underset{\uparrow}{2} \quad 2 \quad 2\}$$

$$x_o[n] = \{-4 \quad -2 \quad 0 \quad 2 \quad 4\}/2$$
$$= \{-2 \quad -1 \quad \underset{\uparrow}{0} \quad 1 \quad 2\}$$

Find even and odd parts of :-

$$x[n] = u[n] - u[n-5]$$



$$x[n] = \{\underset{\uparrow}{\overset{0}{1}} \ \overset{1}{1} \ \overset{2}{1} \ \overset{3}{1} \ \overset{4}{1}\}$$

$$x[-n] = \{\overset{-4}{1} \ \overset{-3}{1} \ \overset{-2}{1} \ \overset{-1}{1} \ \underset{\uparrow}{\overset{0}{1}}\}$$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$= \left\{\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \underset{\uparrow}{1} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}\right\}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$$= \left\{-\frac{1}{2} \ -\frac{1}{2} \ -\frac{1}{2} \ -\frac{1}{2} \ \underset{\uparrow}{0} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}\right\}$$

Periodic and Aperiodic sequences

$$x[n \pm N] = x[n]$$

$e^{j\omega_0 t}$ — may or may not be an integer

$e^{j\omega_0 n}$ — always an integer

$e^{j(\omega_0+2\pi)t}$ $\qquad$ $e^{j(\omega_0+2\pi)n}$

$e^{j\omega_0 t} \cdot e^{j2\pi t}$ $\qquad$ $e^{j\omega_0 n} \cdot e^{j2\pi n}$

may or may not be 1. $\qquad$ always one

$e^{jx} = \cos x + j\sin x$

$(e^{j2\pi})^t = (\cos 2\pi + j\sin 2\pi)^t$

$= (1)^t$

$= 1$

$$\boxed{e^{j(\omega_0+2\pi)n} = e^{j\omega_0 n}}$$

Discrete time complex exponential at frequence ω_0 is identical to that of exponential at $(\omega_0 \pm 2n\pi)$

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n}$$

$$\cancel{e^{j\omega_0 n}} \cdot e^{j\omega_0 N} = \cancel{e^{j\omega_0 n}}$$

$$e^{j\omega_0 N} = 1$$

$$e^{j\omega_0 N} = e^{j2\pi m}$$

$$\omega_0 N = 2\pi m$$

$$\boxed{N = \left(\frac{2\pi}{\omega_0}\right) m}$$

smallest integer which makes N an integer.

$x[n] = 5 \sin \frac{3\pi}{5} n$

$A \sin \omega_0 n$

$\omega_0 = \frac{3\pi}{5}$

$N = \frac{2\cancel{\pi} \times 5}{3 \times \cancel{\pi}} \times m$

$N = \frac{10}{3} m$

$m = 3$

$\boxed{N = 10}$

<u>Periodic Signals</u>

$x[n \pm N] = x[n]$

$e^{j\omega_0 (n+N)} = e^{j\omega_0}$

$\boxed{N = \left(\frac{2\pi}{\omega_0}\right) m}$

smallest integer which makes overall quantity as integer.

$x[n] = \cos \frac{3\pi}{4} n.$

$\omega_0 = \frac{3\pi}{4}$ $\quad N = \frac{2\cancel{\pi} \times 4 \times m}{3\cancel{\pi}}$ $\quad \boxed{N = 8}$ at $m = 3$

$= \frac{8}{3} \times m \quad \boxed{m = 3}$

$x[n] = e^{j5\pi n}$

$\omega_0 = 5\pi$

$N = \frac{2\cancel{\pi}}{5\cancel{\pi}} \times m$

$= \boxed{m = 5}$

$\boxed{N = 2}$

$x[n] = \sin 3n$

$\sin \omega_0 n$

$\omega_0 = 3$

$N = \frac{2\pi}{\omega} \times m$

$= \frac{2\pi}{3} \times m$ $\longrightarrow$ हम m को कोई integer नहीं ले सकते

$\therefore x[n]$ is not periodic

अगर individual signals periodic है तो overall signal भी periodic होगा।

$x[n] = A\sin \frac{3\pi}{5} n + 2\cos \frac{5\pi}{4} n + 3\sin \frac{7\pi}{2} n$

हर ω_0 में π है तो periodic है।

LCM of individual time periods.

$\omega_{01} = \frac{3\pi}{5}$ $\qquad \omega_{02} = \frac{5\pi}{4}$ $\qquad \omega_{03} = \frac{7\pi}{2}$

$N_1 = \frac{2\cancel{\pi}}{3\cancel{\pi}} \times 5m = \frac{10}{3}$ $\qquad N_2 = \frac{2\cancel{\pi}}{5\cancel{\pi}} \times 4m$ $\qquad N_3 = \frac{2\pi}{7\pi} \times 2m$

$N_1 = 10$ at $m = 3$ $\qquad = \frac{8}{5}$ $N_2 = 8$ at $m = 5$ $\qquad = \frac{4}{7}$ $N_3 = 4$ at $m = 7$

LCM of $\left(\frac{10}{3}, \frac{8}{5}, \frac{4}{7}\right) = \frac{40}{1}$

$x[n] = \cos^2 \frac{\pi}{4} n$

$= \frac{1 + \cos \frac{2\pi}{4} n}{2}$

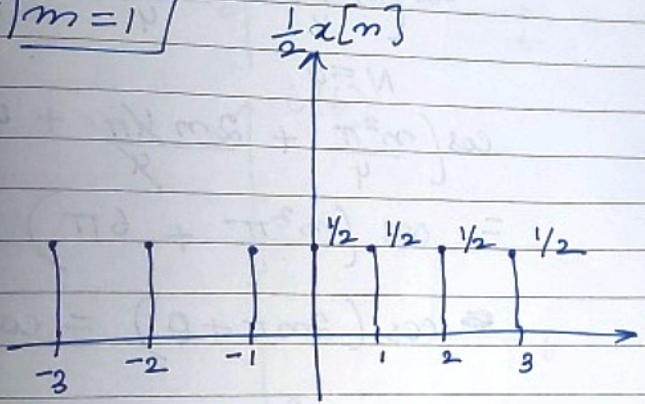
$= \frac{1}{2} + \frac{1}{2} \cos \frac{2\pi}{4} n$

Time period के लिए DC signals को neglect कर देते हैं।

$\omega_0 = \frac{2\pi}{4}$

$T = \frac{\cancel{2\pi} \times 4}{\cancel{2\pi}} \times m$

$\boxed{N=4}$ at $\boxed{m=1}$



$x[n] = 1 + e^{j\frac{3\pi}{5} n} + e^{j\frac{5\pi}{4} n}$

$\omega_{01} = \frac{3\pi}{5}$ $\quad \omega_{02} = \frac{5\pi}{4}$

$N_1 = \frac{2\cancel{\pi} \times 5 \times m}{3\cancel{\pi}}$ $\quad N_2 = \frac{2\pi \times 4 \times n}{5\pi}$

$= \frac{10}{3} \times m$ $\quad N_2 = \frac{8}{5} \times m$

$\boxed{N_1 = 10 \text{ at } m=3}$ $\quad \boxed{N_2 = 8 \text{ at } m=5}$

LCM of $N_1, N_2 = 40$.

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$x[n] = e^{j\left(\frac{3}{5}n + \frac{\pi}{4}\right)}$

$= e^{j\frac{3n}{5}} \cdot e^{j\frac{\pi}{4}}$

≠ has to have a π term

$\therefore$ Not periodic

$e^{i\theta}$

$= \cos\theta + i\sin\theta$

$\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

$\frac{1}{\sqrt{2}}(1+i)$

$x[n] = \cos\frac{n^2}{4}\pi$

$x[n+N] = \cos\frac{(n+N)^2}{4}\pi$

$= \cos\left(\frac{n^2\pi}{4} + \frac{2nN\pi}{4} + \frac{N^2\pi}{4}\right)$

$N = 4.$

$\cos\left(\frac{n^2\pi}{4} + \frac{2n\cancel{4}\pi}{\cancel{4}} + \frac{4^{\cancel{2}}\pi}{\cancel{4}}\right)$

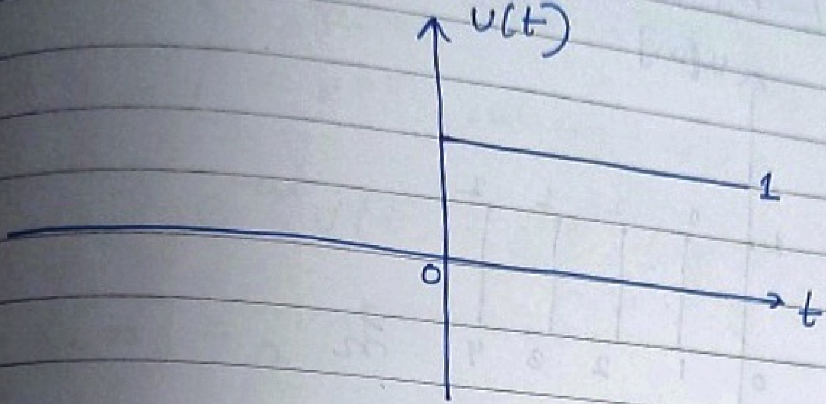
$= \cos\left(\frac{n^2\pi}{4} + 6\pi\right)$

$\cos(2n\pi + \theta) = \cos\theta$

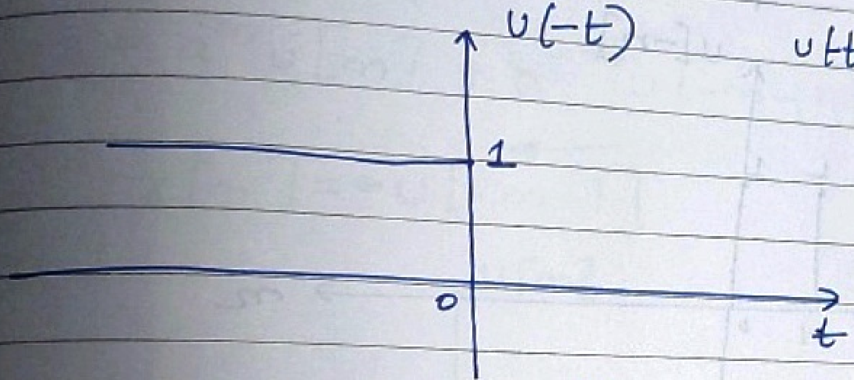
$= \cos\frac{n^2\pi}{4}$

$\therefore \boxed{x[n+N] = x[n] \text{ at } N=4}$

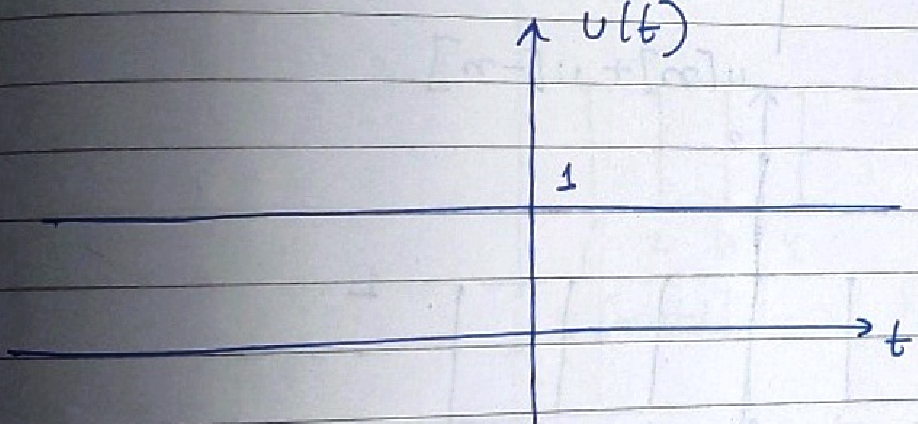
$u(t) + u(-t) =$



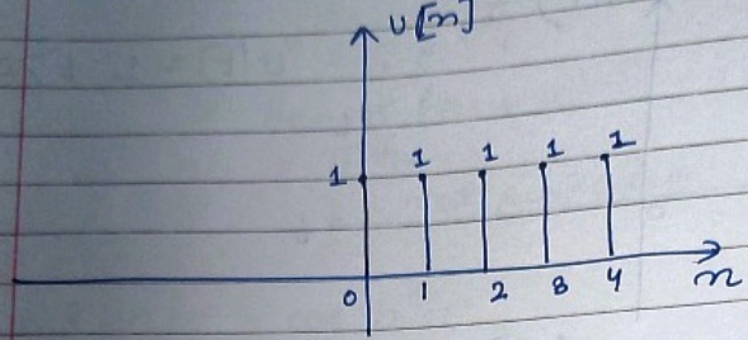
$u(t) = 1 \quad t > 0$

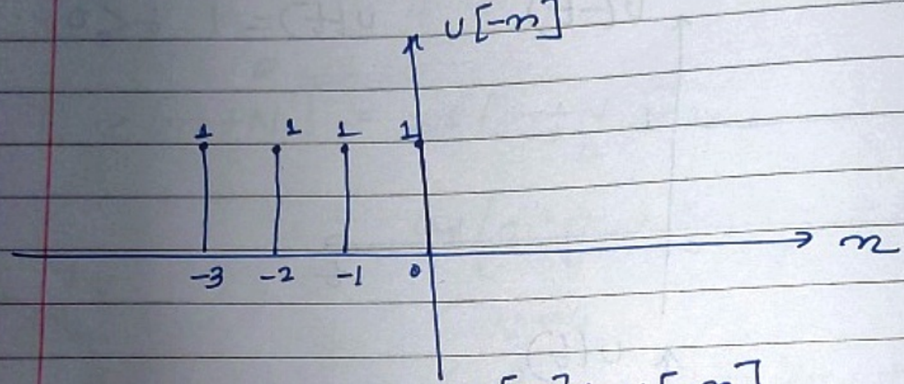


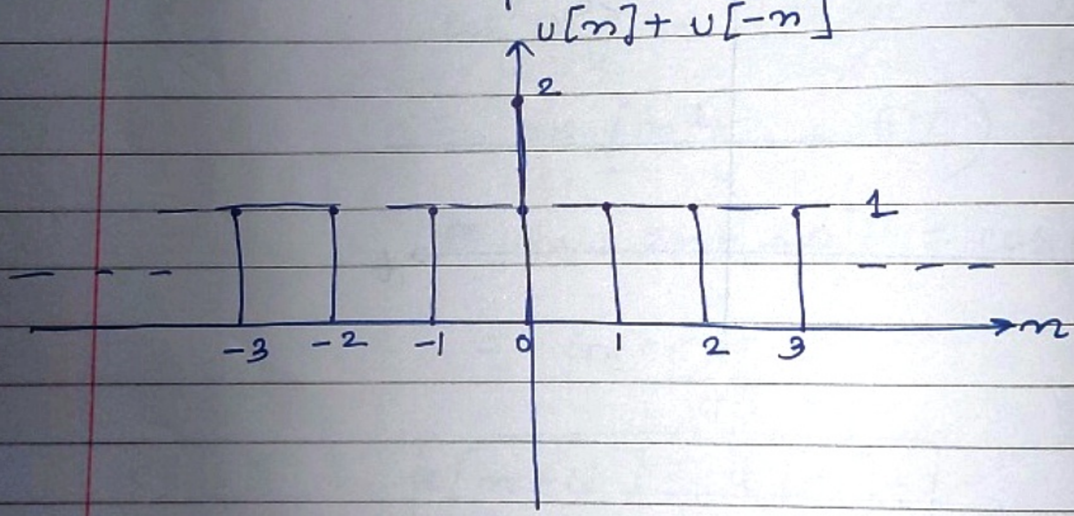
$u(-t) = 1 \quad t < 0$



$u[n] + u[-n] \neq 1$







$$\boxed{u[n] + u[-n] = 1 + \delta[n]}$$

$$x(t) = e^{-a|t|}$$

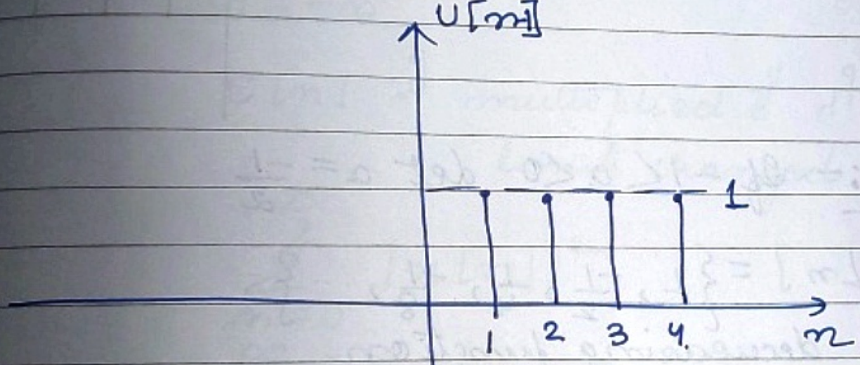
$$= \begin{cases} e^{-at} & \text{when } t>0 \\ e^{-a(-t)} = e^{at} & \text{when } t<0 \end{cases}$$

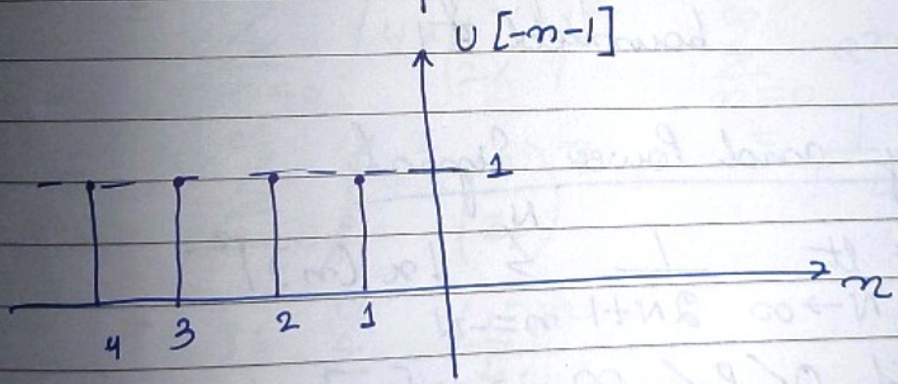
$$= e^{-at}u(t) + e^{at}u(-t)$$

$$x[n] = a^{|n|}$$

$$= a^{n}u[n] + a^{-n}u[-n-1]$$

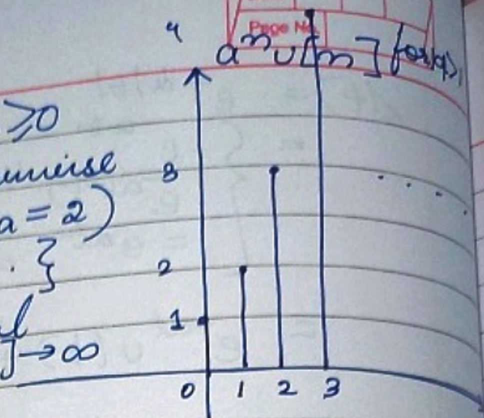
$$x[n] = u[-n-1]$$



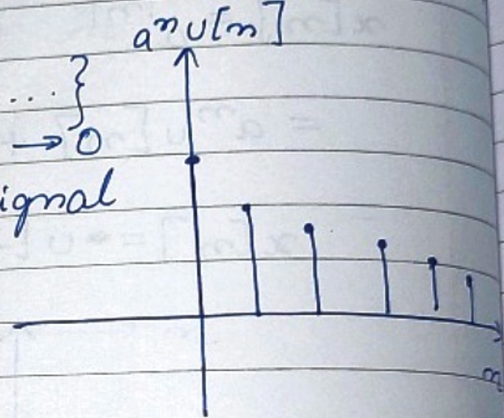


Draw :- $x[n] = a^n, \; n \geq 0$
$= 0$ otherwise

Case-1 :- If $|a| > 1$ (let $a = 2$)
$x[n] = \{1, 2, 4, 8, \dots\}$
unbounded signal
at $t \to \infty$ $x[n] \to \infty$



Case 2 :- If $|a| < 1$
$x[n] = \{1, \frac{1}{2}, \frac{1}{4}, \dots\}$
at $t \to \infty$ $x[n] \to 0$
bounded signal



Case 3 :- If $-1 < a < 0$ let $a = \frac{-1}{2}$

$x[n] = \{1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \dots\}$
decreasing function
bounded $\frac{y}{\varepsilon}$ 1

Energy and Power Signal

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

if $0 < P < \infty \to x[n]$ is a power signal.

] for $|a|>1$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If $0 < E < \infty$ then $x[n]$ is Energy signal.

$$x[n] = 4 \times \left(\frac{1}{2}\right)^n u[n]$$

decreasing signal
bounded है तो
energy signal.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$u[n]$ से multiplied है तो
limits changed

$$\sum_{n=0}^{\infty} |x[n]|^2$$

$$= \sum_{n=0}^{\infty} \left|4 \times \left(\frac{1}{2}\right)^n\right|^2$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{when } |a|<1$$

$$= \sum_{n=0}^{\infty} 4^2 \times \left(\frac{1}{4}\right)^n$$

$$= 16 \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 16 \frac{1}{1-\frac{1}{4}} = 16 \frac{1}{\frac{3}{4}}$$

$$= \boxed{\frac{64}{3} \text{ J}}$$

$x[n] = 4\cos\frac{3\pi n}{5}$

periodic signal ∴

power signal

$P_x = \frac{A^2}{2} = \frac{4^2}{2} = 8$

$x[n] = 3e^{j\frac{2\pi n}{5}}$

π है तो periodic signal.

$P_x = A^2 = 3^2 = 9$

$x[n] = u[n]$

bounded signal तो नहीं है

$\left(t \to \infty \text{ or } -\infty \quad x[n] \to 0 \right)$

तो energy signal पकका नहीं है

power signal check करेंगे।

$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} (1+1+\cdots (N+1) \text{ times})$

$= \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$

$= \lim_{N\to\infty} \frac{\cancel{N}\left(1+\frac{1}{N}\right)}{\cancel{N}\left(2+\frac{1}{N}\right)} = \frac{1}{2}$,

power finite है

तो power signal.

Systems

I/P or Excitation $x(t)$ or $x[n]$ → System → O/P or Response $y(t)$ or $y[n]$

Impulse response :- Response for the system for impulse input is called impulse response.

$x(t) = \delta(t)$ → [ ] → $y(t) = h(t)$ impulse response

$x[n] = \delta[n]$ → [ ] → $y[n] = h[n]$

Static and Dynamic functions :-

Static (memory less) and Dynamic (system with memory)

$x(t)$ → [ ] → $y(t) = x[t^2]$

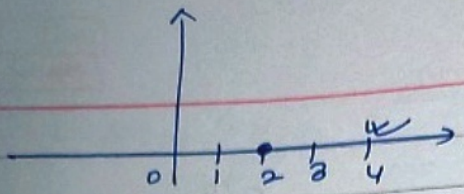
static :- If present output depends only on present input.

$y(t) = x[t^2]$

$y(1) = x(1)$

$y(2) = x(4)$ → present output depends on future input

$\therefore$ dynamic System



$y(t) = x^2(t) = [x(t)]^2$

$y(0) = [x(0)]^2$ present output
$y(-1) = [x(-1)]^2$ depends only
on present input
$y[2] = [x(-2)]^2$ $\therefore$ static system.

(2) Causal, Non causal and Anti causal systems

If present output depends on present + past inputs / present inputs	If present output depends on present + future or present + past + future or past + future I/Ps	special case → depends only on future inputs

$y(t) = x^2[t]$
output depends only on present input
$\therefore$ causal.

$\boxed{\text{static systems will always be causal.}}$

$y[n] = x[n^2]$
$y[0] = x[0]$ present + future
$y[1] = x[1]$ input
$y[-1] = x[1]$ $\therefore$ non causal system.

$y(t) = x(t^2+1)$

$y(0) = x(1) \rightarrow$ depends only on future input

$\therefore$ anti causal inputs

All anticausal systems are non causal systems but all non causal systems are not anti causal systems.

All static systems are causal systems but all causal systems are not static systems.

Causal	Non causal	Anticausal systems
$h(t)=0 \quad t<0$	$h(t)\neq 0 \quad t<0$	$h(t)=0 \quad t>0$
$h[n]=0 \quad t<0$	$h[n]\neq 0 \quad n<0$	$h[n]=0 \quad n>0$
exists for positive t	exists for both negative and positive t	exists for only negative t

(3) Time variant and invariant Systems

Invariant systems:-

If time shift in input results in identical time shift in output without disturbing the nature of output.

To check time invariancy :-

① find $y(t-t_0)$ .. delayed response

② find $T[x(t-t_0)] = y(t, t_0)$ -- response of the system for delayed input

T → symbol of response

If $y(t-t_0) = y(t,t_0)$
then system is called time invariant.

$$y(t) = x(t^2)$$

delayed response :-

$$y(t-t_0) = x[(t-t_0)^2] \quad \text{— ①}$$

Response of system for delayed input :-

$$y(t) = x(t^2 - t_0) \quad \text{— ②}$$

$$y(t-t_0) = y(t,t_0)$$

∴ System is time variant

$$y(t) = t\,x(t)$$

$$y(t-t_0) = (t-t_0)\,x(t-t_0) \quad \text{— ①} \rightarrow \text{delayed response}$$

Response for delayed input.

$$y(t,t_0) = t\,x(t-t_0) \quad \text{— ②}$$

$$y(t-t_0) \neq y(t,t_0)$$

∴ system is time variant.

$y(t) = x^2(t)$

$y(t-t_0) = x^2(t-t_0)$ — ①

$y(t, t_0) = x^2(t-t_0)$ — ②

$\therefore y(t-t_0) = y(t, t_0)$

$\therefore$ System is time invariant.

For a system to be time invariant :-

① There must not be any scaling in $x(t)$ or $y(t)$.

② Coefficients must not be function of time.

③ Any extra term except $x(t)$ or $y(t)$ must be zero or constant.

~~y(t)~~ $y[n] = x[3n]$

$y[n-n_0] = x[3(n-n_0)]$

$= x[3n-3n_0]$ — ①

$y[n] = x[3n-n_0]$ — ②

not equal

$\therefore$ time variant.

यहाँ scaling thi

Date
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$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

यहाँ ~~पे~~ system past + present input पे depended है तो dynamic + causal system

यहाँ यह आ जाता तो non causal हो जाता।

जहाँ t है वहाँ $t - t_0$

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau \quad \text{--- (1)}$$

$$y(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \quad \text{--- (2)}$$

~~यह~~ relation को same वैसे का वैसे लिखना है और input में t_0 से shift करना है।

Let $\tau - t_0 = \lambda$

$$\frac{d\tau}{d\tau} - \frac{dt_0}{d\tau} = \frac{d\lambda}{d\tau}$$

dt_0 → constant

$\tau \to -\infty$

$\lambda \to -\infty$ $\Rightarrow$ $d\lambda = d\tau$

$\tau \to t$

$\lambda \to t - t_0$

$$= \int_{-\infty}^{t-t_0} x(\lambda)\, d\lambda \quad \text{--- (11)}$$

Since λ and τ are dummy variables

$\therefore$ System is time invariant.

Linear and Non linear systems

Linear → follows principle of superposition

homogeneity

$x(t) \rightarrow y(t)$

$\alpha x(t) \rightarrow \alpha y(t)$

additivity

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_1(t)+x_2(t) \rightarrow y_1(t)+y_2(t)$

If $\boxed{\alpha x_1(t)+\beta x_2(t) = \alpha y_1(t)+\beta y_2(t)}$

then system is linear

For a system to be linear:-

1. Graph b/w O/P and I/P must be throughout a straight line passing through origin without having saturation or dead time.

2. If the system is represented in the form of differential equation then the equation must be linear.

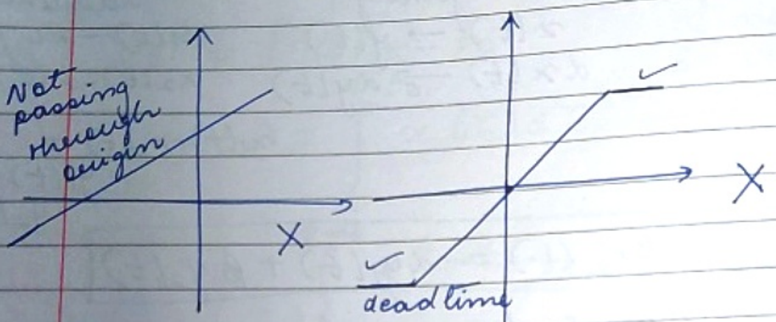
$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + a_{n-2} \frac{d^{n-2} x(t)}{dt^{n-2}} + \dots + a_0 x(t) = y(t)$$

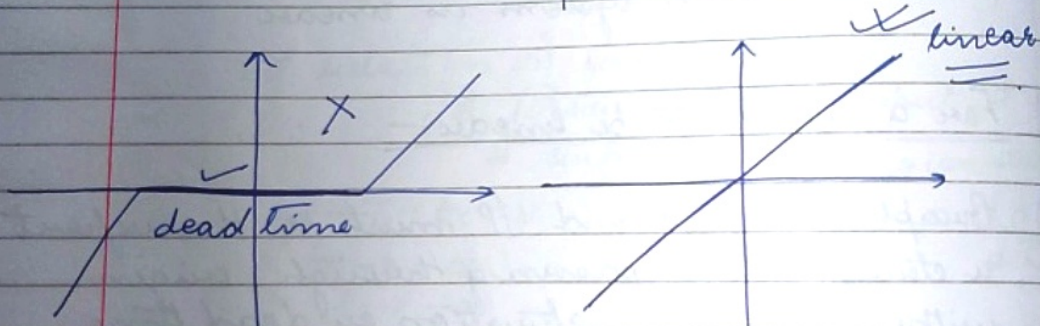
highest order derivative का degree 1.

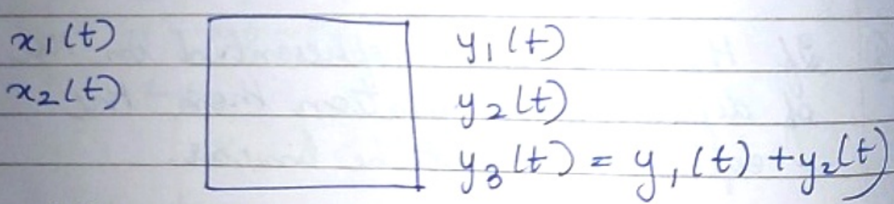
③ System must follow zero I/P zero O/P criteria.

$x(t) \longrightarrow y(t)$

$\underset{0}{\underbrace{0 \cdot x(t)}} \longrightarrow \underset{0}{\underbrace{0 \cdot y(t)}}$







$x_3'(t) = x_1(t) + x_2(t)$ $\qquad y_3'(t)$

$$\boxed{y_3'(t) = y_3(t)}$$

o O/P

linear

$\cdot y_2(t)$

$y(t) = t \cdot x(t)$

$y_1(t) = t \cdot x_1(t)$

$y_2(t) = t \cdot x_2(t)$

$y_3(t) = y_1(t) + y_2(t)$

$= t(x_1(t) + x_2(t))$

$x_3'(t) = \cancel{t}\, x_1(t) + x_2(t)$

$y_3''(t) = t \cdot x_3''(t)$

$= t\,[x_1(t) + x_2(t)]$

$\boxed{y_3'(t) = y_3(t)}$

additivity is followed

$\alpha \rightarrow$ Real/ Imaginary/ complex.

$x(t) \longrightarrow y(t)$

$\alpha x(t) \longrightarrow \alpha y(t)$

$y(t) = t\, x(t)$

$\alpha\, x(t)$

$y'(t) = t \alpha\, x(t)$

$= \alpha(t\, x(t))$ ✓

$\alpha \cdot y(t) = \alpha t\, x(t)$ ✓

homogeneous

$\therefore$ system is linear.

$y(t) = t \cdot x(t)$

linear

$y(t) = x(t^3)$

Additivity

$x_1(t) \longrightarrow x_1(t^3)$

$x_2(t) \longrightarrow x_2(t^3)$

$y_3(t) = x_1(t^3) + x_2(t^3)$

$x_3'(t) = x_1(t) + x_2(t)$

$x_3'(t^3) \longrightarrow t^3$ से replace करदो।

$y_3'(t) = x_1(t^3) + x_2(t^3)$

Q

$y(t) = x(t^3)$

$\alpha x(t) \longrightarrow \alpha x(t^3)$

If I/P is $\alpha x(t)$

then O/P is

$y'(t) = \alpha x(t^3)$

$\therefore$ System is linear.

$y(t) = x[\sin t]$

$y_1(t) = x_1[\sin t]$

$y_2(t) = x_2[\sin t]$

$y_3(t) \quad \alpha y_1(t) + \beta y_2(t) = \alpha x_1[\sin t] + \beta x_2[\sin t]$

~~$y_3'(t) = x_3'[\sin t]$~~

let $x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

~~$y_3'(t) = x$~~

$y_3'(t) = x_3'[\sin t]$
$= \alpha x_1[\sin t] + \beta x_2[\sin t]$

$\alpha x_1(t) + \beta x_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$ ✓

* $y(t) = x[e^t]$

$y_1(t) = x_1[e^t]$
$y_2(t) = x_2[e^t]$

$y_3 = \alpha y_1(t) + \beta y_2(t) = \alpha x_1[e^t] + \beta x_2[e^t]$

$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$
$y_3'(t) = x_3'(e^t)$
$= \alpha x_1(e^t) + \beta x_2(e^t)$

* $\left.\begin{array}{l} y(t) = x[t^3] \\ y(t) = x[e^t] \\ y(t) = x[\sin t] \end{array}\right\}$ All are linear

$y(t) = \sin t \cdot x(t)$
$y_1(t) = \sin t \cdot x_1(t)$
$y_2(t) = \sin t \cdot x_2(t)$
$y_3(t) = y_1(t) + y_2(t)$
$= \sin t\,[x_1(t) + x_2(t)]$

Let $x_3'(t) = x_1(t) + x_2(t)$
$y_3'(t) = \sin t \cdot x_3'(t)$
$= \sin t\,(x_1(t) + x_2(t))$

$\therefore$ ~~[illegible]~~ additivity followed.

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$
$$= \alpha \sin t\, x_1(t) + \beta \sin t\, x_2(t)$$
$$= \sin t(\alpha x_1(t) + \beta x_2(t))$$

Let $x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$$y_3'(t) = \sin t\ x_3'(t)$$
$$= \sin t\ (\alpha x_1(t) + \beta x_2(t))$$

$\therefore$ linear

$$y(t) = 4x(t)$$

$$y_1(t) = 4x_1(t)$$
$$y_2(t) = 4x_2(t)$$
$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$
$$= 4(\alpha x_1(t) + \beta x_2(t))$$

Let $x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$$y_3'(t) = 4x_3'(t)$$
$$= 4(\alpha x_1(t) + \beta x_2(t))$$

$\therefore$ linear

Differential and integrating operators are linear.

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

$$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau$$

$$y_2(t) = \int_{-\infty}^{t} x_2(\tau)\,d\tau$$

$$y_3(t) = \alpha\, y_1(t) + \beta\, y_2(t)$$

$$= \alpha \int_{-\infty}^{t} x_1(\tau)\,d\tau + \beta \int_{-\infty}^{t} x_2(\tau)\,d\tau \quad \text{--- (1)}$$

$$y_3'(t) = \int_{-\infty}^{t} x_3'(\tau)\,d\tau$$

$$x_3'(\tau) = \alpha\, x_1(\tau) + \beta\, x_2(\tau)$$

$$= \int_{-\infty}^{t} \left(\alpha\, x_1(\tau) \cancel{d\tau} + \beta\, x_2(\tau)\right) d\tau$$

$$= \int_{-\infty}^{t} \alpha\, x_1(\tau)\,d\tau + \int_{-\infty}^{t} \beta\, x_2(\tau)\,d\tau \quad \text{--- (2)}$$

$\therefore$ linear

$$\left.\begin{aligned} y(t) &= \int_{-\infty}^{t} x(\tau)\,d\tau \\ y(t) &= \frac{d}{dt} x(t) \end{aligned}\right\} \text{linear}$$

$$y(t) = \text{even}[x(t)]$$

$$y(t) = \frac{x(t) + x(-t)}{2}.$$

$$y(t) = \underbrace{\frac{1}{2}x(t)}_{\text{linear}} + \underbrace{\frac{1}{2}x(-t)}_{\text{linear}}$$

$$y_1(t) = \frac{1}{2}x_1(t) + \frac{1}{2}x_1(-t)$$

$$y_2(t) = \frac{1}{2}x_2(t) + \frac{1}{2}x_2(-t)$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$

$$= \alpha\left[\frac{1}{2}x_1(t) + \frac{1}{2}x_1(-t)\right] + \beta\left[\frac{1}{2}x_2(t) + \frac{1}{2}x_2(-t)\right]$$

$$= \frac{1}{2}\left[\alpha x_1(t) + \alpha x_1(-t)\right] + \frac{1}{2}\left[\beta x_2(t) + \beta x_2(-t)\right]$$

$$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$$

$$y_3'(t) = \frac{1}{2}x_3'(t) + \frac{1}{2}x_3'(-t)$$

$$= \frac{1}{2}(\alpha x_1(t) + \beta x_2(t)) + \frac{1}{2}(\alpha x_1(-t) + \beta x_2(-t))$$

$$= \frac{1}{2}(\alpha x_1(t) + \alpha x_1(-t)) + \frac{1}{2}\left[\beta x_2(t) + \beta x_2(-t)\right]$$

linear

$$y(t) = x(t-2) + x(t+2)$$

$$y_1(t) = x_1(t-2) + x_1(t+2)$$
$$y_2(t) = x_2(t-2) + x_2(t+2)$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$
$$= \alpha x_1(t-2) + \alpha x_1(t+2) + \beta x_2(t-2) + \beta x_2(t+2)$$

$$\boxed{x_3'(t) = \alpha x_1(t) + \beta x_2(t)}$$

$$y_3'(t) = x_3'(t-2) + x_3'(t+2)$$
$$= \alpha x_1(t-2) + \beta x_2(t-2) + \alpha x_1(t+2) + \beta x_2(t+2)$$

<u>linear</u>

$$\boxed{y(t) = a\,x(t+t_0) + b\,x(t-t_0)}$$ <u>linear</u>

$$y(t) = \begin{cases} x(t-4) & \text{for } t>0 \\ x(t+4) & \text{for } t<0. \end{cases}$$

$$y(t) = x(t-4)\,u(t) + x(t+4)\,u(-t)$$

<u>linear</u>

Non linear

$y(t) = \cos(x[t])$ or $\sin(x[t])$

$y_1(t) = \cos(x_1(t))$

$y_2(t) = \cos(x_2(t))$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$

$= \alpha \cos(x_1[t]) + \beta \cos(x_2[t])$ — ①

$x_3'(t) = \alpha x_1[t] + \beta x_2[t]$

$y_3'(t) = \cos(x_3'(t))$

$= \cos(\alpha x_1[t] + \beta x_2[t])$ — ②.

not equal

∴ not linear

$y(t) = x^2(t)$

$y_1(t) = x_1^2(t)$

$y_2(t) = x_2^2(t)$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$

$= \alpha x_1^2(t) + \beta x_2^2(t)$

$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$y_3'(t) = x_3'^2(t)$

$= (\alpha x_1(t) + \beta x_2(t))^2$

$y_3(t) \neq y_3'(t)$

∴ Not linear

$$y(t) = 4t + 8x(t)$$
$$y(t) = 2 + x(t)$$

} Zero I/P Zero O/P criteria is not followed.

$$y(t) = x^*(t)$$
$$= [x(t)]^*$$

$$y_1(t) = [x_1(t)]^*$$
$$y_2(t) = [x_2(t)]^*$$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t)$$
$$= \alpha [x_1(t)]^* + \beta [x_2(t)]^* \quad \text{--- (1)}$$

$$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$$
$$y_3'(t) = [x_3'(t)]^*$$
$$= [\alpha x_1(t) + \beta x_2(t)]^* \quad \text{--- (2)}$$

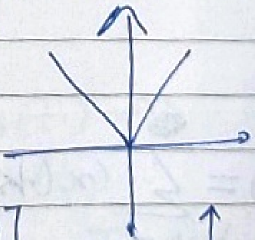
$$y_3(t) \neq y_3'(t)$$

$\therefore$ non linear.

$$\left\{ \begin{array}{l} y(t) = \text{real}(x[t]) \\ y(t) = \text{img}(x[t]) \end{array} \right. \qquad \text{real}(x[t]) = \frac{x(t) + x^*(t)}{2}, \quad \text{img}(x[t]) = \frac{x(t) - x^*(t)}{2}$$

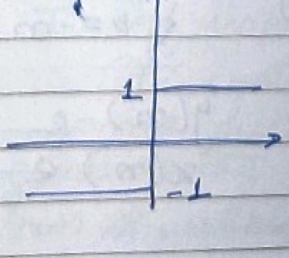
System non linear

$$y(t) = |x(t)|$$



Non linear

$$y(t) = \text{sgn}[x(t)]$$



$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 4y(t) = x(t)$$

linear + invariant

$$\frac{d^2y(t)}{dt} + t\frac{dy(t)}{dt} + 4y(t) = x(t)$$

time variant

linear
सारे coeff या तो constant या तो independent variable.

$$\frac{d^2y(t)}{dt} + y(t)\frac{dy(t)}{dt} + 4y(t) = x(t)$$

non linear

time dependent function

time variant

Invertible and non invertible systems

→ if distinct input produces distinct output

→ if input can be determined by observing the O/P.
like one-to-one mapping

→ if it is possible to design an inverse system.

$y(t) = 4x(t)$

$y(t) = x^2(t)$

Non invertible.

$x(t)$	$x^2(t)$
0	0
−1	1
1	1
+2	4
−2	4

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$y(n) \longleftrightarrow Y(z)$

$x(n) \longleftrightarrow X(z)$

$$\sum_{k=-\infty}^{n} x(k) \longleftrightarrow \frac{1}{1-z^{-1}} X(z)$$

$y(n) \rightarrow Y(z)$

$y(n-1) \rightarrow z^{-1} Y(z)$

$$y'(n) = y(n) - y(n-1)$$
$$Y'(z) = Y(z) - z^{-1} Y(z)$$
$$= (1 - z^{-1}) Y(z)$$

$y(t) = |x(t)|$ Non invertible

$x(t)$	$y(t) = \|x(t)\|$
0	0
−1	1
1	1
j	1
−j	1

Test signals :—

$u(t)$ $\quad \delta(t)$ $\quad 2u(t)$
$-u(t)$ $\quad -\delta(t)$ $\quad 2\delta(t)$ } Continuous

$u[n]$ $\quad \delta[n]$ $\quad 2u[n]$
$-u[n]$ $\quad -\delta[n]$ $\quad 2\delta[n]$ } Discrete

$y(t) = \sin t \, x(t)$

For $x(t) = \delta(t)$

$$y(t) = \sin t \, \delta(\underbrace{t}_{0})$$
$$= \sin 0 \, \delta(t) = 0$$

$$x(t)\,\delta(\underbrace{t}_{0}) = x(0)\,\delta(t)$$

For $x(t) = 4\delta(t)$

$$y(t) = \sin t \, 4\delta(t)$$
$$= 4 \sin 0 \, \delta(t)$$
$$= 0$$

दो diff input के लिए system same output generate कर रहा तो non invertible system.

$y[n] = x(n)\,x(n-1)$ System non invertible

For I/P $x(n) = u(n)$

$y(n) = u(n)\,u(n-1)$

I/P $x(n) = -u(n)$

$y(n) = -u(n) - u(n-1)$

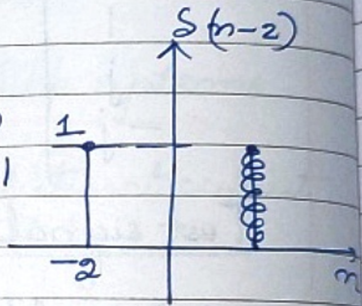
$= u(n)\,u(n-1)$

$$y(n) = \begin{cases} x(n+2) & n \geq 0 \\ x(n) & n < -1 \end{cases}$$

For $x(n) = \delta(n)$

$$y(n) = \begin{cases} \delta(n+2) & n \geq 0 \\ \delta(n) & n < -1 \end{cases}$$

0 0



For $x(n) = 2\delta(n)$

$$y(n) = \begin{cases} 2\delta(n+2) & n \geq 0 \\ 2\delta(n) & n < -1 \end{cases}$$

0 0

∴ System is non invertible.

Stable and unstable systems

Bounded input – Bounded output criteria

BIBO Stability

The output of the system must be bounded for bounded input.

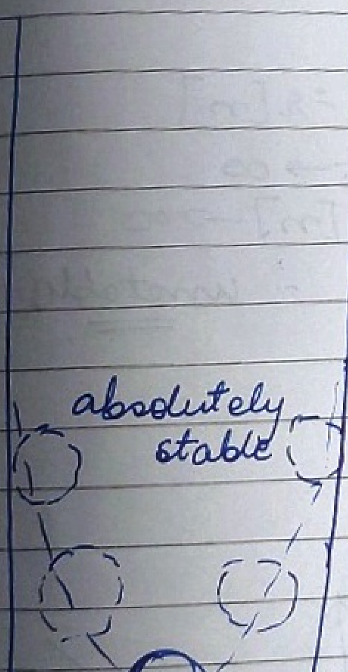
For $0 \leq |x(t)| < \infty$

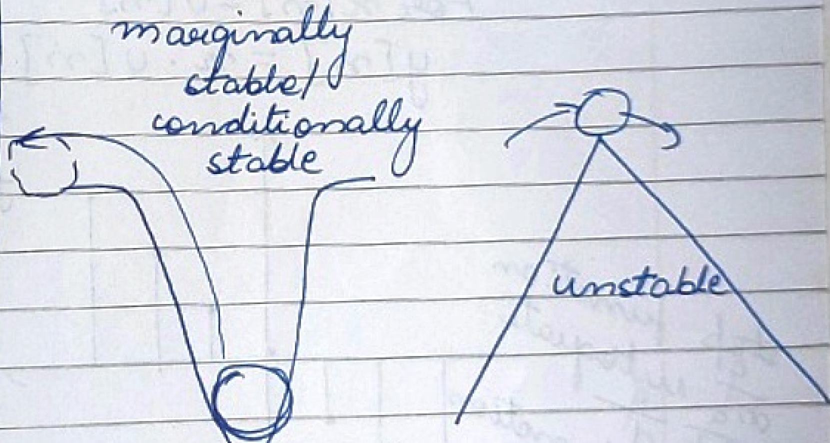
$0 \leq |y(t)| < \infty$

BIBO implies impulse response must tend to 0 as 't' tends to ∞

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty \quad \text{absolutely integrable}$$

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty \quad \text{absolutely summable.}$$





Test signals

$x(t) = u(t)$

$\sin t$

$\cos t$

$y(t) = x^2(t)$

For $x(t) = u(t)$

$y(t) = (u(t))^2 = u(t)$

bounded input के लिए output bounded ∴ system is stable.

$y(t) = \sin(t)\, x(t)$

For i/p $x(t) = u(t)$

$|y(t)| = |\sin(t)| \cdot \underbrace{|x(t)|}_{1}$

$\in [-1, 1]$

∴ system is stable.

$y[n] = n\, x[n]$

For $x[n] = u[n]$

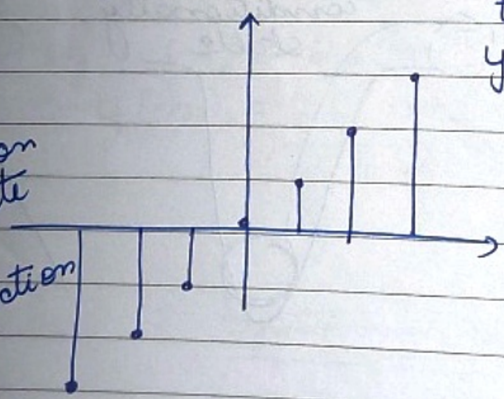
$y[n] = n \cdot u[n] = r[n]$

$t \to \infty$

$y[n] \to \infty$

∴ unstable.

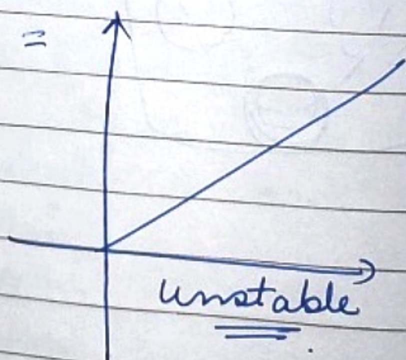
step function को integrate करते तो ramp function आता।



$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

For $x(\tau) = u(t)$.

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau =$$



unstable.

$$y(t) = \int_{-\infty}^{t} x(t) \sin 2t \, dt$$

$$x(t) = \sin 2t$$

$$\int_{-\infty}^{t} \sin^2 2t \, dt = \frac{1}{2}\int_{-\infty}^{t} 1 - \cos 4t \, dt$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= \frac{1}{2}\left[\underbrace{\int_{-\infty}^{t} dt}_{\infty} - \int_{-\infty}^{t} \cos 4t \, dt\right]$$

$\therefore$ unstable