

Difference equation specifies actual operations that are performed by digital system on input data in time domain.

Advantages of representing digital filter in Block dia

- easy to analyse I/O relationship
- easy to determine hardware requirements.
- easy to develop block dia from transfer function.

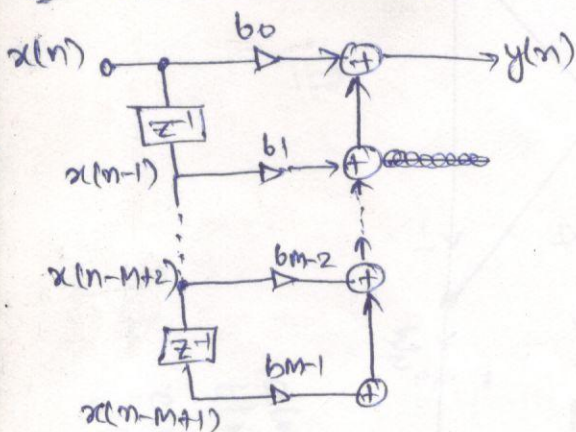
FIR filter structures

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_{M-1} x(n-M+1)$$

length of filter = M = no. of coeff.
order = M-1.

I. Direct form



D.F. Structure of $(M-1)^{th}$ order FIR filter requires.

- M-1 memory locations to store M-1 previous I/P samples.
- M memory locations to store M coefficients (b).
- M multipliers
- M-1 additions.

III. Polyphase structure

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k} = \sum_{k=0}^{M-1} h(k) z^{-k}$$

$$= \sum_{k=0}^{M-1} h(2k) z^{-2k} + \sum_{k=0}^{M-1} h(2k+1) z^{-(2k+1)}$$

→ separating odd & even indexes.

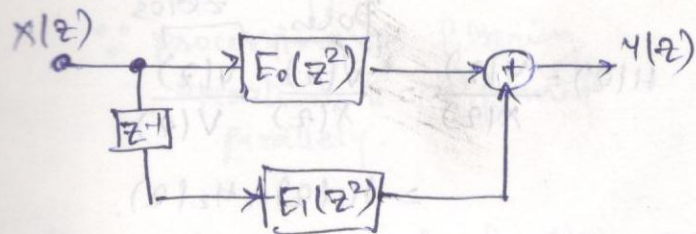
$$= \sum_{k=0}^{M-1} h(2k) z^{-2k} + z^{-1} \sum_{k=0}^{M-1} h(2k+1) z^{-(2k)}$$

Let $E_0(z) = \sum_{k=0}^{M-1} h(2k) z^{-k}$, $E_1(z) = \sum_{k=0}^{M-1} h(2k+1) z^{-k}$

$$\Rightarrow H(z) = E_0(z^2) + z^{-1} E_1(z^2)$$

Ex. For length = 7.

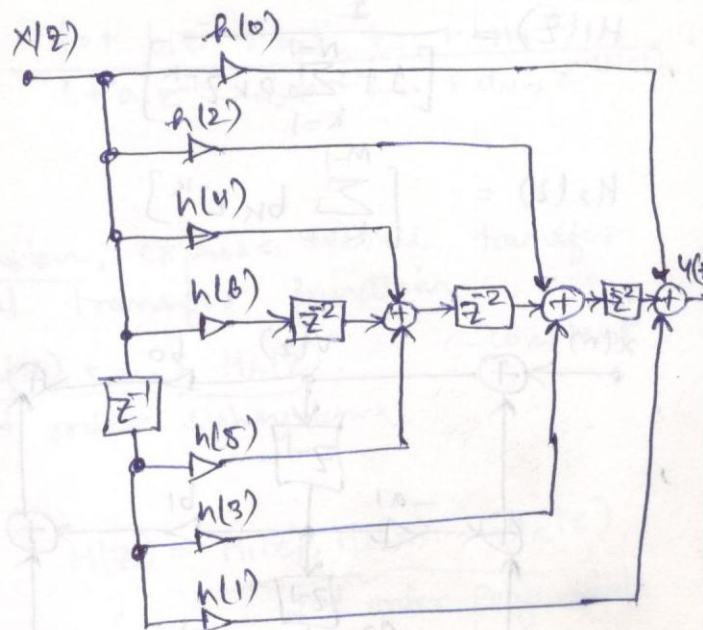
$$H(z) = [a(0) + a(2)z^{-2} + a(4)z^{-4} + a(6)z^{-6}] + z^{-1} [a(1) + a(3)z^{-2} + a(5)z^{-4}]$$



Polyphase struct. of FIR filter.

IIR filter structures.

$$\sum_{k=0}^{M-1} z^{-k}$$



For M=7.

II

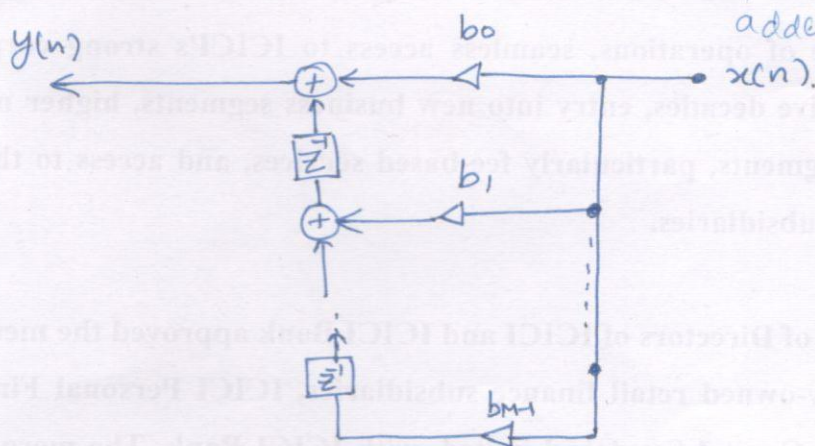
Transposed structure
(equivalent to direct form)

→ interchange o/p & i/p nodes

FIR

→ Reverse all paths.

→ Replace pick-off nodes by adders & vice versa.



IV

Cascade form structure.

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \dots H_K(z)$$

$$H_k(z) = b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}$$

$$\text{also } H_k(z) = \frac{Y_k(z)}{X_k(z)} \Rightarrow Y_k(z) = b_{k0}X_k(z) + b_{k1}z^{-1}X_k(z) + b_{k2}z^{-2}X_k(z)$$

$$\Rightarrow y_k(n) = b_{k0}x_k(n) + b_{k1}x_k(n-1) + b_{k2}x_k(n-2)$$

Let $H(z) = 1 + \frac{6}{5}z^{-1} + \frac{7}{5}z^{-2} + \frac{26}{25}z^{-3} + \frac{1}{5}z^{-4}$

M=5
factorizing $\Rightarrow H(z) = \left[1 + \frac{1}{5}z^{-1} + z^{-2}\right] \left[1 + z^{-1} + \frac{1}{5}z^{-2}\right]$
 $= H_1(z) \cdot H_2(z)$

Cascade Realization:

