Gauss elimination method

(Solution of Linear Equations)

Consider the system of Equation

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Written In matrix form $AX = B$

1.
$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

2. Find ~~Aug~~ Augmented matrix for given system

$$c = [A : B]$$

3. Transform of augmented matrix c into upper trangular form.

4. Find equations corresponding to upper trangular matrix

5.

Solve the system of equation by Gauss Elimination method

$2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$

Sol^n Given Eq^n is

$2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

Written In matrix form

$$\underset{A}{\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}}$$

It's Augmented matrix is

$C = [A : B]$

$$= \left[\begin{array}{ccc:c} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$R_1 \Leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 9 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 2R_1$, $R_3 \Rightarrow R_3 - R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & -3 \\ 0 & -2 & 0 & -4 \end{array}\right]$$

$R_2 \Leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{array}\right]$$

$R_3 \Rightarrow 2R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & +6 \end{array}\right]$$

$\therefore \quad x + y + z = 6$

$-2y = -4$

$2z = 6$

$\boxed{z = 3}, \quad \boxed{y = 2}$

$x + 2 + 3 = 6$

$\boxed{x = 1}$

$\therefore \quad \boxed{x = 1, \; y = 2, \; z = 3}$ Ans

Q.

(3) Solve system of equation by Gauss elimination method

$2x - y + z = 10, \quad 3x + 2y + 3z = 18, \quad x + 4y + 9z = 16$

Sol^n First given eq^n written in matrix form

$$\underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}}_{B}$$

It's Augmented matrix is

$$C = [A : B] = \left[\begin{array}{ccc:c} 2 & -1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

$R_1 \Leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & -1 & 1 & 10 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 3R_1$, $R_3 \Rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -9 & -17 & -22 \end{array}\right]$$

$R_3 \Rightarrow 10R_3 - 9R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & 0 & 46 & 50 \end{array}\right]$$

$\therefore \quad x + 4y + 9z = 16$

$-10y - 24z = -30$

$46z = 50$

$z = \frac{50}{46} = \frac{25}{23}$

Solve the linear system

$$-x_1 + x_2 + 2x_3 = 2$$
$$3x_1 - x_2 + x_3 = 6$$
$$-x_1 + 3x_2 + 4x_3 = 4$$

Sol^n Given Eq^n written in matrix form

$$\underset{A}{\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}}$$

It's Augmented matrix

$$C = [A : B]$$

$$= \left[\begin{array}{ccc:c} -1 & 1 & 2 & 2 \\ 3 & -1 & 1 & 6 \\ -1 & 3 & 4 & 4 \end{array}\right]$$

$R_2 \Rightarrow R_2 + 3R_1$, $R_3 \Rightarrow R_3 - R_1$

$$\sim \left[\begin{array}{ccc:c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 2 & 2 & 2 \end{array}\right]$$

$R_3 \Rightarrow R_3 - R_2$

$$\sim \left[\begin{array}{ccc:c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 0 & -5 & -10 \end{array}\right]$$

$$\therefore \quad -x_1 + x_2 + 2x_3 = 2$$
$$2x_2 + 7x_3 = 12$$
$$-5x_3 = -10 \Rightarrow \boxed{x_3 = 2}$$
$$2x_2 + 14 = 12 \Rightarrow 2x_2 = -2$$
$$\boxed{x_2 = -1}$$

$$\therefore -x_1 + \cancel{x} - \cancel{x} = 2$$
$$\boxed{x_1 = -2} \text{ Ans}$$

Solve the following system of equation

$x+y+z=2\ ,\quad x+2y+3z=5\ ,\quad 2x+3y+4z=11$

Soln Given eqn written in matrix form

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix}}$$

It's Augmented matrix is

$$C = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 11 \end{array}\right]$$

$R_2 \Rightarrow R_2 - R_1\ ,\ R_3 \Rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & +1 & 2 & 7 \end{array}\right]$$

$R_3 \Rightarrow R_3 - R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$x+y+z=2$

$y+2z=3$

That mean, there is no solution for the given system of eqn.

Apply Gauss Elimination with the concept of Partial Pivoting to solve

$$2x - 7y - 10z = -17, \quad 5x + y + 3z = 14,$$
$$x + 10y + 9z = 7.$$

Sol^n Here ~~coefficient~~ higher coefficient of x is 5, so given eq^n written as

$$5x + y + 3z = 14 \longrightarrow (i) \quad \text{Pivotal eq}^n$$
$$2x - 7y - 10z = -17 \longrightarrow (ii)$$
$$x + 10y + 9z = 7 \longrightarrow (iii)$$

Step 1 Eliminating x from eq^n (ii) and Eq^n (iii) using eq^n (i)

$$2\,[5x + y + 3z = 14]$$
$$5\,[2x - 7y - 10z = -17]$$

$$\cancel{10x} + 2y + 6z = 28$$
$$\cancel{10x} - 35y - 50z = -85$$
$$- \quad + \quad + \quad +$$
$$37y + 56z = 113 \longrightarrow (iv)$$

Again

$$5x + y + 3z = 14$$
$$5\,[x + 10y + 9z = 7]$$

$$\cancel{5x} + y + 3z = 14$$
$$\cancel{5x} + 50y + 45z = 35$$
$$- \quad - \quad - \quad -$$
$$-45y - 42z = 21 \longrightarrow (v)$$

From Eq^n (iv) and (v)

$$37y + 56z = 113 \longrightarrow (iv)$$
$$-45y - 42z = 21 \longrightarrow (v)$$
$$45\,[37y + 56z = 113]$$
$$37\,[-45y - 42z = 21]$$

$$\boxed{z = 4}$$

Putting iv

$$37y + 56(4) = 113$$
$$\boxed{y = -3}$$

Putting in (i)

$$\boxed{x = 1} \quad \text{Ans}$$

Gauss Elimination Method by Concept of Pivoting

Q1. Solve

$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16$

Sol. First we select Pivote equation (i.e coefficient of x is max)

$2x + y + z = 10 \longrightarrow (i)$

$3x + 2y + 3z = 18 \longrightarrow (ii)$ Pivote Equation

$x + 4y + 9z = 16 \longrightarrow (iii)$

Step-I

$2x + y + z = 10 \longrightarrow (i)$

$\frac{2}{3}[3x + 2y + 3z = 18] \longrightarrow (ii)$

$\cancel{2x} + y + z = 10$

$\cancel{2x} + \frac{4}{3}y + 2z = 12$

$-\quad\ -\quad\ -\quad\ -$

$(1 - \frac{4}{3})y - z = -2$

$-\frac{1}{3}y - z = -2$

$\frac{1}{3}y + z = 2 \longrightarrow (iv)$

Step-II

Now $x + 4y + 9z = 16 \longrightarrow (iii)$

$\frac{1}{3}[3x + 2y + 3z = 18] \longrightarrow (ii)$

$\cancel{x} + 4y + 9z = 16$

$\cancel{x} + \frac{2}{3}y + z = 6$

$-\quad\ -\quad\ -\quad\ -$

$(4 - \frac{2}{3})y + 8z = 10$

$\frac{10}{3}y + 8z = 10 \longrightarrow (v)$

Now

$$3x + 2y + 3z = 18 \longrightarrow (ii)$$

$$\frac{1}{3}y + z = 2 \longrightarrow (iv)$$

$$\frac{10}{3}y + 8z = 10 \longrightarrow (v)$$

Again we choose Pivoting Equation for y

i.e

$$3x + 2y + 3z = 18$$

$$\frac{1}{3}y + z = 2$$

$$\frac{10}{3}y + 8z = 10 \quad \text{Pivoting Equation}$$

Step (iii)

$$\frac{1}{3}y + z = 2 \longrightarrow (iv)$$

$$\frac{1}{10}\left[\frac{10}{3}y + 8z = 10\right] \longrightarrow (v)$$

$$\cancel{\frac{1}{3}y} + z = 2$$

$$\cancel{\frac{1}{3}y} + \frac{8}{10}z = 1$$

$$- \qquad - \qquad -$$

$$z - \frac{8}{10}z = 1$$

$$z\left(\frac{10-8}{10}\right) = 1$$

$$z\left(\frac{2}{10}\right) = 1$$

$$\boxed{z = 5}$$

Putting in (iv)

$$\frac{1}{3}y + 5 = 2$$

$$\frac{1}{3}y = -3$$

$$y = -9$$

$$\therefore 3x + 2(-9) + 3(5) = 18$$

$$3x - 18 + 15 = 18$$

$$3x = 21 \Rightarrow x = 7$$

$$\therefore \boxed{x = 7,\ y = -9,\ z = 5} \quad \text{Ans}$$

Complete Pivoting (Gauss Elimination Method by Complete Pivoting)

Q. Solve $x+4y-z=-5$, $x+y-6z=-12$, $3x-y-z=4$ by Gauss Elimination method and concept of complete pivoting.

Sol^n First given Equation written in matrix form

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

Its Augmented matrix is

$[A : B]$

$$C = \left[\begin{array}{ccc:c} 1 & 4 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ 3 & -1 & -1 & 4 \end{array}\right]$$

As we know that In complete pivoting we search bigest number in A matrix and It should be on First place so

$C_2 \Longleftrightarrow C_1$ [because 4 is Largest in A matrix]

$$\sim \left[\begin{array}{ccc:c} 4 & 1 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ -1 & 3 & -1 & 4 \end{array}\right]$$

Now we convert It in upper tringular form matrix

so $R_2 \Rightarrow 4R_2 - R_1$, $R_3 \Rightarrow 4R_3 + R_1$

$$\sim \left[\begin{array}{ccc:c} 4 & 1 & -1 & -5 \\ 0 & 3 & -23 & -43 \\ 0 & 13 & -5 & 11 \end{array}\right]$$

$$\begin{bmatrix} 4 & 1 & -1 & : & -5 \\ 0 & 3 & -23 & : & -43 \\ 0 & 13 & -5 & : & 11 \end{bmatrix}$$

$R_3 = 3R_3 - 13R_2$

$$\sim \begin{bmatrix} 4 & 1 & -1 & : & -5 \\ 0 & 3 & -23 & : & -43 \\ 0 & 0 & 284 & : & 592 \end{bmatrix}$$

$\therefore \quad AX = B$

$$4x + y - z = -5$$

$$3y - 23z = -43$$

$$284z = 592$$

$$z = 2.0845$$

$$y = \cancel{6.5945} \; 1.6478$$

$$1.6478$$

$$\boxed{x = -1.1408} \text{ Ans}$$

Inverse of a system of Linear Equations Using Gauss-Jordan Method :—

Q Apply Gauss-Jordan method to solve the equations

$x+y+z=9, \quad 2x-3y+4z=13, \quad 3x+4y+5z=40$

Sol^n Given eq^n $x+y+z=9$, $2x-3y+4z=13$; $3x+4y+5z=40$

Written in Matrix form.

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}}$$

$$\boxed{AX = B}$$

It's Augmented matrix $C = [A : B]$

$$C = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right]$$

(Now our aim is to convert I ~~incl~~ unit matrix)

$R_2 \Rightarrow R_2 - 2R_1$, $R_3 \Rightarrow R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right]$$

$R_2 \Leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & -5 & 2 & -5 \end{array}\right]$$

$R_3 \Rightarrow R_3 + 5R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & 13 \\ 0 & 0 & 12 & 60 \end{array}\right]$$

$R_3 \Rightarrow \frac{1}{12} R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & +1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \Rightarrow R_1 - R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -1 & -4 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 2R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & -1 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \Rightarrow R_1 + R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$\therefore AX = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$\boxed{x = 1,\ y = 3,\ z = 5}$ Ans

Q2. Apply Gauss Jordon method to solve the equation

~~x+y+z=9, 2x−3y+4~~ $x + 3y + z = 10$, $x - 2y - z = -6$

$2x + y + 2z = 10$, $2x + y + 2 = 10$,

$x = 1,\ y = 2,\ z = 3$

To Find the solution of the Equations using Gauss-Elimination method.

1) Use Gauss elimination method to solve the following system of Eqn

$2x+y+z=10$, $3x+2y+3z=18$, $x+4y+9z=16$.

Solⁿ First written In matrix form

$$\underset{A}{\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}}$$

It's Augmented Matrix $C = [A : B]$

$$C = \left[\begin{array}{ccc:c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$$

Convert it in upper triangular form

$R_1 \Leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$

$R_2 \Rightarrow R_2 - 3R_1$, $R_3 \Rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{array}\right]$$

$R_3 \Rightarrow R_3 - \frac{7}{10} R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & 0 & -\frac{1}{5} & -1 \end{array}\right]$$

Now $AX = B$

$x + 4y + 9z = 16$

$-10y - 24z = -30$

$-\frac{z}{5} = -1$

$\boxed{z = 5}$

$-10y = -30 + 24 \times 5$

$-10y = -30 + 120 = 90$

$\boxed{y = -9}$

$\therefore x - 36 + 45 = 16$

$\boxed{x = 16 - 9 = 7}$

$\boxed{x = 7, \ y = -9, \ z = 5}$ Ans

Solution of Simultaneous Algebraic Equation By Crouts - Method

This method is based on the fact that every square matrix A can be expressed as the product of a lower ~~tria~~ triangular matrix and an upper triangular matrix, provided all the principal minors of A are non singular. i.e. if $A = [a_{ij}]$, then $a_{11} \neq 0$,
$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$

Consider the following ~~system~~ equations:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

These equations are writen in the matrix form as.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{B} \longrightarrow (1)$$

Now let $A = LU$

Where L is Lower triangular matrix and U is upper triangular matrix.

$$\therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (2)$$

Multiplying the matrices on R.H.S, we get

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix}$$

Equating corresponding elements from both sides, we get

$l_{11} = a_{11}$, $\quad l_{11}U_{12} = a_{12}$ $\quad l_{11}U_{13} = a_{13}$,

$l_{21} = a_{21}$ $\quad l_{21}U_{12} + l_{22} = a_{22}$ $\quad l_{21}U_{13} + l_{22}U_{23} = a_{23}$

$l_{31} = a_{31}$, $\quad l_{31}U_{12} + l_{32} = a_{32}$ $\quad l_{31}U_{13} + l_{32}U_{23} + l_{33} = a_{33}$

We solve these equations in the following order.

step 1. solve equation in I column

step 2. solve equation in I Row

step 3. Solve equation in II Column

step 4. solve equation in II row

step 5. solve equation in III Column

step 6. solve equation in III row.

$AX = B$ $\because A = LU$.

Putting LU For A in ①,

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \longrightarrow ③$$

Put

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \longrightarrow ④$$

in ③ we get

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \longrightarrow ⑤$$

on solving eqn ⑤, we get the value of u, v, w, we substitute the value of u, v, w in ④, on solving ④ we get the values of x_1, x_2, x_3.

1) Solve by Crouts method.

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

Sol^n. The equation are written in matrix form

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} \longrightarrow (1)$$

$$AX = B$$

And $A = LU$

$$\therefore \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \longrightarrow (2)$$

$$\begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$\therefore\ l_{11} = 2$, $l_{11}U_{12} = 3$, $l_{11}U_{13} = 1$

$l_{21} = 1$, $l_{21}U_{12} + l_{22} = 2$, $l_{21}U_{13} + l_{22}U_{23} = 3$

$l_{31} = 3$, $l_{31}U_{12} + l_{32} = 1$, $l_{31}U_{13} + l_{32}U_{23} + l_{33} = 2$

$\because\ l_{11} = 2\ \therefore\ U_{12} = 3/2$ $\quad\therefore$ And $U_{13} = 1/2$

$l_{21} = 1\ \therefore\ 1 \times \frac{3}{2} + l_{22} = 2 \Rightarrow l_{22} = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2}$, $U_{23} = 5$

$l_{31} = 3\ \therefore\ 3 \times \frac{3}{2} + l_{32} = 1 \Rightarrow l_{32} = 1 - \frac{9}{2} = -\frac{7}{2}$, $l_{33} = 18$

On substituting these value of L and U in (2)

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 3 & -7/2 & 18 \end{bmatrix} \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

Putting $\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \longrightarrow (3)$

$$\therefore \begin{bmatrix} 2 & 0 & 0 \\ 1 & \frac{1}{2} & 0 \\ 3 & -\frac{7}{2} & 18 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

21 16 37 27 10

$\therefore \quad 2y_1 = 9 \Rightarrow y_1 = \frac{9}{2}$

$y_1 + \frac{1}{2}y_2 = 6 \Rightarrow \frac{9}{2} + \frac{1}{2}y_2 = 6 \Rightarrow \frac{9}{2} - 6 = -\frac{1}{2}y_2$

$\frac{9-6}{2} = -\frac{1}{2}y_2$

$\boxed{y_2 = 3}$

$3y_1 - \frac{7}{2}y_2 + 18y_3 = 8$

$3 \times \frac{9}{2} - \frac{7}{2} \times 3 - 8 = -18y_3 \Rightarrow \frac{27}{2} - \frac{21}{2} - 8 = -18y_3$

$\frac{27-21-16}{2} = -18y_3$

$-\frac{10}{2} = -18y_3 \Rightarrow y_3 = \frac{5}{18}$

$\therefore \quad y_1 = \frac{9}{2}, y_2 = 3, y_3 = \frac{5}{18}$

Putting the value of y_1, y_2, y_3 in (3)

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9/2 \\ 3 \\ 5/18 \end{bmatrix}$$

$$\therefore \quad x + \frac{3}{2}y + \frac{1}{2}z = 9/2$$

$$y + 5z = 3$$

$$\boxed{z = 5/18}$$

$$\therefore y = 3 - 5z = 3 - \frac{25}{18} = \frac{54-25}{18} = \frac{29}{18}$$

$$\therefore x + \frac{3}{2} \times \frac{29}{18} + \frac{1}{2} \times \frac{5}{18} = \frac{9}{2}$$

$$x = \frac{9}{2} - \frac{29}{12} - \frac{5}{36} = \frac{162 - 87 - 5}{36} = \frac{80}{36} = \frac{35}{18}$$

$$\therefore \boxed{x = \frac{35}{18}, y = \frac{5}{18}, z = \frac{5}{18}}$$ Ans

Q2. Solve by crouts method.

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

Sol^n. Given eq^n are written in matrix form.

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

i.e $\quad AX = B$.

Put $\quad A = LU$

$$\therefore \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix} \longrightarrow (1)$$

$$\begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 5 \\ 3 & 2 & -3 \end{bmatrix}$$

Doolitt

step

$\therefore\ l_{11} = 1$, $l_{11}u_{12} = 1 \Rightarrow u_{12} = 1$; $l_{11}u_{13} = -1 \Rightarrow u_{13} = -1$

$l_{21} = 2$; $l_{21}u_{12} + l_{22} = 3$; $l_{21}u_{13} + l_{22}u_{23} = 5$

$2\times 1 + l_{22} = 3 \Rightarrow \boxed{l_{22} = 1}$ $\qquad 2\times(-1) + 1\,u_{23} = 5 \Rightarrow u_{23} = 7$

$l_{31} = 3$; $l_{31}u_{12} + l_{32} = 2.$; $l_{31}u_{13} + l_{32}u_{23} + l_{33} = -3$

$3\times 1 + l_{32} = 2 \Rightarrow l_{32} = -1$ $\qquad 3\times -1 + (-1)(7) + l_{33} = -3$

$\cancel{-3} + 7 + \cancel{3} = +l_{33} \Rightarrow l_{33} = 7$

Putting these value in (1)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} \longrightarrow (2)$$

Put $$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \longrightarrow (3)$$

Putting in (2)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 7 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

$\therefore\ \boxed{u = 2}$, $2u + v = -3$; $3u - v + 7w = 6$

$4 + v = -3$ $\qquad \cancel{6} + 7 + 7w = \cancel{6}$

$\boxed{v = -7}$ $\qquad \boxed{w = -1}$

Putting the value of u, v and w in (3)

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}$$

$\therefore\ x_1 + x_2 - x_3 = 2 \longrightarrow (i)$ $\quad x_2 + 7x_3 = -7 \longrightarrow (ii)$ and $\boxed{x_3 = -1}$

$\boxed{x_2 = -7 + 7 = 0}$

$x_1 + 0 + 1 = 2$

$\boxed{x_1 = 1}$, $\boxed{x_2 = 0}$, $\boxed{x_3 = -1}$ A

Doolittle Method or Doolittle LU Decomposition Method

step · I Given equation written in matrix form

$$AX = B$$

step - II Decompose A As LU

i.e $LUX = B$ Here $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$

Step III Put $UX = Y$

$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$

Step IV $LY = B$

Q1 Solve $x+y+z=5$, $x+2y+2z=6$, $x+2y+3z=8$ by Doolittle method.

Sol^n Given $x+y+z=5 \longrightarrow (1)$

$x+2y+2z=6 \longrightarrow (2)$

$x+2y+3z=8 \longrightarrow (3)$

Written in matrix method

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}}_{B}$$

Now Let $A = LU$

In Doolittle method we put $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$

And $U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae+g & af+h \\ bd & be+cg & bf+ch+i \end{bmatrix}$$

$d=1$, $e=1$, $f=1$

$ad=1$ $\quad$ $\boxed{1=a}$

$ae+g=2$

$1+g=2$

$g=1$

$af+h=2$

$(1)(1)+h=2$

$\boxed{h=1}$

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$$bd = 1 \qquad be + cg = 2 \qquad bf + ch + i = 3$$

$$b(1) = 1 \qquad (1)(1) + c(1) = 2 \qquad (1)(1) + (1)(1) + i = 3$$

$$\boxed{b = 1} \qquad c = 2 - 1 \qquad i = 3 - 2 = 1$$

$$\boxed{c = 1}$$

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\because \quad LUX = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix} \longrightarrow \text{(I)}$$

Let

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \longrightarrow (2)$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_1 + y_2 \\ y_1 + y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$$y_1 = 5, \quad y_1 + y_2 = 6 \implies y_2 = 6 - 5 = 1$$

$$y_1 + y_2 + y_3 = 8$$

$$5 + 1 + y_3 = 8 \implies \boxed{y_3 = 2}$$

Putting this value in ~~①~~ ②

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

From ②

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$$x + y + z = 5$$
$$y + z = 1$$
$$z = 2$$
$$y = 1 - 2 = -1$$

$$x - 1 + 2 = 5$$
$$x = 4$$

$\therefore$ $x = 4, \; y = -1, \; z = 2$ Ans

Q2. ~~[illegible]~~

Find LU decomposition using Doolittle's method of matrix
$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Let $A = LU$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

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$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae+g & af+h \\ bd & be+cg & bf+ch+i \end{bmatrix}$$

$\therefore\ d=8,\quad e=-6,\quad f=2$

$ad=-6$
$a\cdot 8=-6$
$a=-3/4$

$ae+g=7$
$(-\frac{3}{\cancel{4}_2})(-\cancel{6}_3)+g=7$
$\frac{9}{2}+g=7$
$g=7-\frac{9}{2}=\frac{5}{2}$

$af+h=-4$
$(-\frac{3}{\cancel{4}})(\cancel{2})+h=-4$
$h=-4+\frac{3}{2}=-\frac{5}{2}$

$bd=2$
$b(8)=2$
$b=1/4$

$be+cg=-4$
$\frac{1}{4}(-6)+c(\frac{5}{2})=-4$
$-\frac{3}{2}+\frac{5}{2}c=-4$
$\frac{5}{2}c=-4+\frac{3}{2}=-\frac{5}{2}$
$c=-1$

$\frac{3+10}{6}=\frac{13}{6}$
$\frac{3-13}{6}$
$\frac{10-13}{6}=\frac{5}{6}$

$bf+ch+i=3$
$\frac{1}{4}(2)+(-1)(-5/3)+i=3$
$\frac{1}{2}+\frac{5}{2}+i=3$
$i=3-\frac{1}{2}-\frac{5}{2}=\cancel{\frac{18-3-10}{6}=\frac{5}{6}}$
$i=0$

$$\therefore L=\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3/4 & 1 & 0 \\ \frac{1}{4} & -1 & 1 \end{bmatrix}$$

$$U=\begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 8 & -6 & 2 \\ 0 & 5/2 & -5/2 \\ 0 & 0 & 0 \end{bmatrix}$$

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Solution of Linear Algebraic Equation by Cholesky Method

Working Rule

First Given set of eqⁿ $AX = B$

Now Let $A = LL^T$

where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

It's $L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$

$$A = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} \\ l_{31} l_{11} & l_{31} l_{21} + l_{32} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

Note We solve it Row wise

Solve the system of equation by Cholesky Method

$x+2y+3z=5$, $2x+8y+22z=6$, $3x+22y+82z=-10$

Sol^n

Given set of equation written in matrix form

$AX=B$

$$\underbrace{\begin{bmatrix}1 & 2 & 3\\ 2 & 8 & 22\\ 3 & 22 & 82\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x\\ y\\ z\end{bmatrix}}_{X}=\underbrace{\begin{bmatrix}5\\ 6\\ -10\end{bmatrix}}_{B}$$

Now Let $A = LL^T$

Where $L = \begin{bmatrix} l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}$

$$\therefore \begin{bmatrix}1 & 2 & 3\\ 2 & 8 & 22\\ 3 & 22 & 82\end{bmatrix}=\begin{bmatrix} l_{11} & 0 & 0\\ l_{21} & l_{22} & 0\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\begin{bmatrix} l_{11} & l_{21} & l_{31}\\ 0 & l_{22} & l_{32}\\ 0 & 0 & l_{33}\end{bmatrix}$$

$$=\begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31}\\ l_{21}l_{11} & l_{21}^2+l_{22}^2 & l_{21}l_{31}+l_{22}l_{32}\\ l_{31}l_{11} & l_{31}l_{21}+l_{32}l_{22} & l_{31}^2+l_{32}^2+l_{33}^2\end{bmatrix}$$

$\because 1 = l_{11}^2 \Rightarrow \boxed{l_{11}=1}$, $l_{11}l_{21}=2 \Rightarrow \boxed{l_{21}=2}$, $l_{11}l_{31}=3$, $\boxed{l_{31}=3}$

$l_{21}l_{11}=2$, $l_{21}^2+l_{22}^2=8$

$4+l_{22}^2=8 \Rightarrow l_{22}^2=8-4=4 \Rightarrow \boxed{l_{22}=2}$

$l_{21}l_{31}+l_{22}l_{32}=22$

$2\times3+2\times l_{32}=22 \Rightarrow 6+2l_{32}=22 \Rightarrow 2l_{32}=16 \Rightarrow l_{32}=8$

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$$l_{31}^2 + l_{32}^2 + l_{33}^2 = 82$$

$$9 + 64 + l_{33}^2 = 82$$

$$l_{33}^2 = 82 - 73 = 9 \Rightarrow \boxed{l_{33} = 3}$$

$\therefore$ $Ax = B$

$LL^T x = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix} \longrightarrow (1)$$

Put
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \longrightarrow (2)$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

$y_1 = 5$,

$2y_1 + 2y_2 = 6$, $\quad 3y_1 + 8y_2 + 3y_3 = -10$

$10 + 2y_2 = 6$, $\quad 3\times 5 - 16 + 3y_3 = -10$

$2y_2 = -4$ $\quad 15 - 16 + 3y_3 = -10$

$\boxed{y_2 = -2}$ $\quad 3y_3 = -9$, $\boxed{y_3 = -3}$

Putting in (2)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$$

$x + 2y + 3z = 5$

$2y + 8z = -2$, $\quad 2y - 8 = -2$

$3z = -3$, $\boxed{z - 1}$ $\quad \boxed{y = 3}$ $\quad x + 6 - 9 = 5$ $\quad \boxed{x = 2}$

Solve the foll
method $x + y + 3z =$
Written In
Sol

Solve the following system of equation by Cholesky's Method

$$x + y + 3z = 6, \quad x + 5y + 5z = 20, \quad 3x + 5y + 19z = 106$$

Sol
Written In matrix form

$$\underset{A}{\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 5 \\ 3 & 5 & 19 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 6 \\ 20 \\ 106 \end{bmatrix}}$$

Let $A = LL^T$

Where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$, $L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$

$$\therefore A = \begin{bmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} \\ l_{31} l_{11} & l_{31} l_{21} + l_{32} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 5 \\ 3 & 5 & 19 \end{bmatrix}$$

$\therefore l_{11}^2 = 1 \Rightarrow l_{11} = 1$, $\quad l_{11} l_{21} = 1 \Rightarrow \boxed{l_{21} = 1}$, $\quad l_{11} l_{31} = 3$ $\boxed{l_{31} = 3}$

$l_{21}^2 + l_{22}^2 = 5$

$1 + l_{22}^2 = 5 \Rightarrow \boxed{l_{22} = 2}$, $\quad l_{21} l_{31} + l_{22} l_{32} = 5$

$(1)(3) + (2) l_{32} = 5$

$2 l_{32} = 2 \Rightarrow \boxed{l_{32} = 1}$

$l_{31}^2 + l_{32}^2 + l_{33}^2 = 19$

$9 + 1 + l_{33}^2 = 19 \Rightarrow l_{33}^2 = 9 \Rightarrow \boxed{l_{33} = 3}$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 1 & 3 \end{bmatrix}, \quad L^T = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$\because \quad AX = B$

$LL^T X = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 106 \end{bmatrix}$$

Let
$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_2 \end{bmatrix} \longrightarrow (2)$$

Putting in (1)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 106 \end{bmatrix}$$

$y_1 = 6$

$y_1 + 2y_2 = 20 \Rightarrow 2y_2 = 14 \Rightarrow \boxed{y_2 = 7}$

$3y_1 + y_2 + 3y_3 = 106$

$18 + 7 + 3y_3 = 106$

$3y_3 = 106 - 18 - 7 = 106 - 25$

$3y_3 = 81$

$\boxed{y_3 = 27}$

Putting in (2)

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 27 \end{bmatrix}$$

$\therefore x + y + 3z = 6$

$2y + z = 7 \quad \therefore 2y = 7 - 9 = -2 \Rightarrow \boxed{y = -1}$

$3z = 27 \quad x = 6 + 1 - 27 = -20$

$\boxed{z = 9}$

$\boxed{x = -20}$ Ans