

→ Two port Network

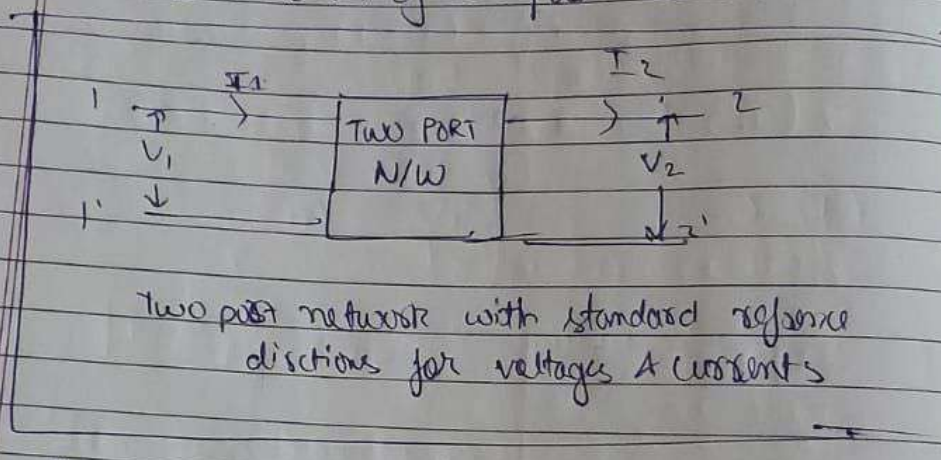
A two port network is a special case of multicode network.

Each code contains two terminals

one for entry and other for exit.

From the definition of a port the current at Entry should be  $i =$  current at exit.

→ Characterisation of 2 port network



A two port network is illustrated in given fig. by analogy with the transmission network the port given as '1-1'' is called the input port 1 & the other '2-2'' is called the output port.

The port variables are the port current and port voltage

In a general way or general manner in  $n$  port network there will be  $2n$  voltage & current variables.



$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (2)$$

From eq (1) we have,

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} \rightarrow (3)$$

$Z_{11}$  is called Input impedance with the output port is open circuited.

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} \rightarrow (4)$$

Reverse transfer impedance with the input open circuited

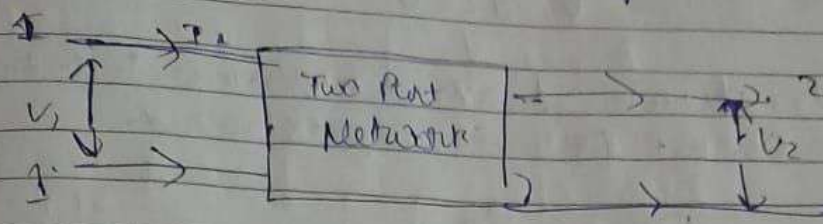
From eq (2) we have

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} \rightarrow (5)$$

$Z_{21}$  is called forward transfer impedance with the output open circuited

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

## Short circuited admittance parameters



$$(I_1, I_2) = f(V_1, V_2)$$

$$[I] = [Y][V]$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad (7)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad (8)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From eq<sup>n</sup> - (7) we have,

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad (9)$$

$Y_{11}$  is called the input admittance with the output part short circuited.

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} \quad (10)$$

$Y_{12}$  is called the reverse transfer admittance with the input part short-circuited.

From equation (8) we have

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad (11)$$



Assignment

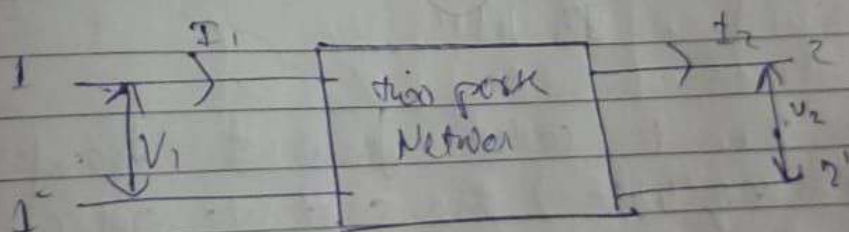
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$$\frac{(2n)1^2}{(n)1^2} \quad \text{Not } 2 \times 1$$

→ Two port network parameters -

- (1) Open circuit impedance or  $Z$  parameter
- (2) Short circuit admittance ( $Y$ ) parameter
- (3) Transmission parameter ( $A, B, C, D$ )
- (4) Inverse transmission parameter ( $A', B', C', D'$ )
- (5) Hybrid Parameter ( $h$ )
- (6) Inverse Hybrid Parameter ( $g$ )

→ Open circuit impedance ( $Z$ ) parameter



Characteristic eq:

$$(V_1, V_2) = f(I_1, I_2)$$

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_2 = \frac{V_1}{-I_2} \quad \text{or } V_2 = 0 \quad \rightarrow (16)$$

∴ derive transfer impedance with the output per circuit.

from eq (14) we have

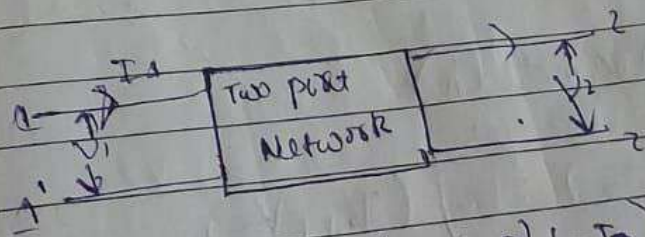
$$Z = \frac{V_1}{I_1} \quad \text{or } V_2 = 0 \quad \rightarrow (17)$$

Reverse transfer impedance with the output open circuit.

$$Y = \frac{I_1}{V_1} \quad \text{or } V_2 = 0 \quad \rightarrow (18)$$

Output short circuit

★ Inverse transmission parameters ( $A', B', C', D'$ )



$$V_2 = A'V_1 + B'(-I_1) \quad \rightarrow (19)$$

$$I_2 = C'V_1 + D'(-I_1) \quad \rightarrow (20)$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$



$$Y_{21} = \frac{I_2}{V_1} \Big|_{I_2=0} \rightarrow (12)$$

$Y_{21}$ : forward transadmittance with the output port is short circuited.

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} \rightarrow (13)$$

$Y_{22}$ : Output admittance with the help of input port short circuit.

$$V_1 = AV_2 + B(-I_2) \rightarrow (13)$$

$$V_2 = CV_1 + D(-I_1) \rightarrow (14)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

From eq<sup>n</sup> (13) we have

$$A = \frac{V_1}{V_2} \Big|_{I_2=0} \rightarrow (15)$$

A: reverse voltage gain with the output port open circuited.



$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (25)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (26)$$

From eq. (25) we have

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} \rightarrow (27)$$

Input impedance with output short circuit

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0} \rightarrow (28)$$

Reverse voltage gain with input open circuit

From eq. (26) we have,

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$

(Forward current gain with output short circuit)

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} \rightarrow (30)$$

(Forward current impedance with open circuit)

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Inverse Hybrid parameters (g)

$$(V_2, I_2) = f(I_1, V_1)$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_1 \end{bmatrix}$$

$$V_2 = g_{11} I_1 + g_{12} V_1 \longrightarrow (31)$$

$$I_2 = g_{21} I_1 + g_{22} V_1 \longrightarrow (32)$$

For eq<sup>n</sup> (31) we have,

$$g_{11} = \left. \frac{V_2}{I_1} \right|_{V_1=0} \longrightarrow (33)$$

$$g_{12} = \left. \frac{V_2}{V_1} \right|_{I_1=0} \longrightarrow (34)$$

For eq<sup>n</sup> (32) we have

$$g_{21} = \left. \frac{I_2}{I_1} \right|_{V_1=0} \longrightarrow (35)$$

$$g_{22} = \left. \frac{I_2}{V_1} \right|_{I_1=0} \longrightarrow (36)$$



From eq<sup>n</sup> (19) we have,

$$A' = \left. \frac{V_2}{V_1} \right|_{I_2=0} \rightarrow (21)$$

Forward voltage ratio

$$B' = \left. \frac{V_2}{-I_1} \right|_{V_1=0} \rightarrow (22)$$

Forward transfer impedance.

From eq<sup>n</sup> (20) we have,

$$C' = \left. \frac{I_2}{V_1} \right|_{I_1=0} \rightarrow (23)$$

Forward transfer admittance

$$D' = \left. \frac{I_2}{-I_1} \right|_{V_1=0} \rightarrow (24)$$

Forward current gain

### (5) Hybrid Parameters (h)

$$(V_1, I_2) = f(I_1, V_2)$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The original representation for  $z$  parameters is -

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \rightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \rightarrow (2)$$

From eq (1) & (2)

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \rightarrow (3)$$

$$Z_{11} = \frac{V_1}{I_1} \quad I_2 = 0$$

$$Z_{12} = \frac{V_1}{I_2} \quad I_1 = 0 \quad \rightarrow (4)$$

$$Z_{21} = \frac{V_2}{I_1} \quad I_2 = 0 \quad \rightarrow (5)$$

$$Z_{22} = \frac{V_2}{I_2} \quad I_1 = 0 \quad \rightarrow (6)$$

In order to determine the open circuit impedance parameters or  $z$  parameters we open the input coil and excite output coil with a known voltage source  $V_s$  so that  $V_2 = V_s$  &  $I_1 = 0$ .

We determine  $I_2$  &  $V_1$  to obtain  $Z_{12}$  &  $Z_{22}$ .

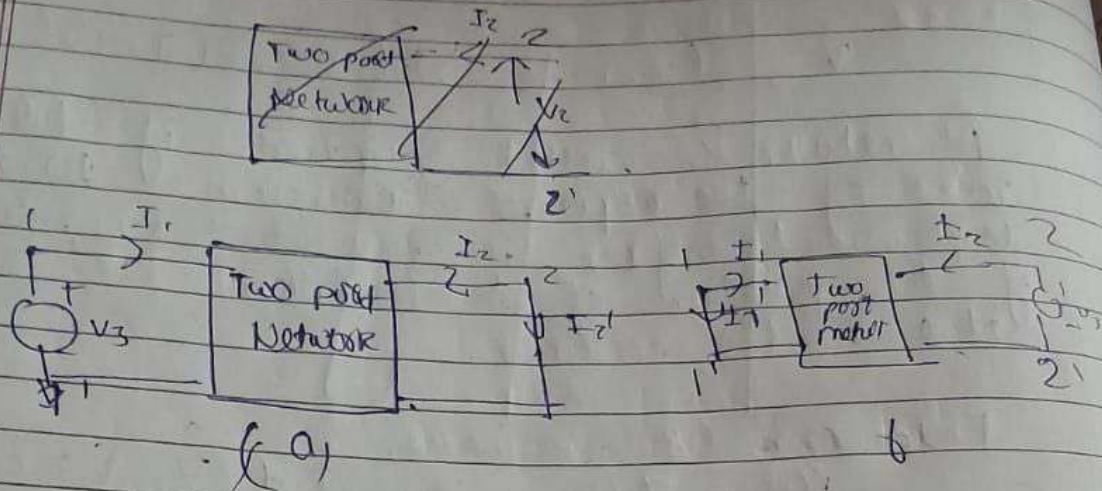
Then the output coil is open circuited and the input coil is excited with the same voltage source  $V_s$ .

The circuit is analysed to determine  $I_1$ ,  $V_2$  so as to obtain  $Z_{11}$  &  $Z_{21}$ .



2-port parameters			
Name	Functions	Express	Equations
Z-parameter	$V_1, V_2$	$I_1, I_2$	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Y-parameter	$I_1, I_2$	$V_1, V_2$	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
A, B, C, D parameter	$V_1, I_1$	$V_2, I_2$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
A', B', C', D' parameters	$V_2, I_2$	$V_1, I_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$
h-parameter	$V_1, I_2$	$I_1, V_2$	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
g-parameter	$V_2, I_1$	$I_2, V_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_1 \end{bmatrix}$

For determining reciprocity we consider the following network.



As in fig(a) we apply short circuit in the output side and apply voltage source  $V_s$  in Port. 1

Then,

$$V_s = V_1 \quad - (10)$$

$$V_2 = 0 \quad - (11)$$

$$I_2 = -I_2' \quad - (12)$$

Applying these 3 equations (10) (11) (12) on equation 4.2 we have

$$V_s = Z_{11}I_1 - Z_{12}I_2' \rightarrow (13)$$

$$0 = Z_{21}I_1 - Z_{22}I_2'$$



$$V_s = (Z_{11} + Z_{22}) I_1 + Z_{12} I_2 \quad \text{--- (13)}$$

From equation (13) we have,

$$I_1 = \frac{V_s + Z_{12} I_2}{Z_{11} + Z_{22}} \quad \text{--- (14)}$$

From eqn (14) we have,

$$I_1 = \frac{Z_{22} I_2'}{Z_{11} + Z_{22}} \quad \text{--- (15)}$$

From eqn (15) we have

$$\frac{V_s + Z_{12} I_2'}{Z_{11} + Z_{22}} = \frac{Z_{22} I_2'}{Z_{11} + Z_{22}} \quad \text{--- (16)}$$

$$V_s Z_{21} + Z_{12} Z_{21} I_2' = Z_{11} Z_{22} I_2'$$

$$(Z_{11} Z_{22} - Z_{12} Z_{21}) I_2' = V_s Z_{21}$$

$$I_2' = \frac{V_s Z_{21}}{(Z_{11} Z_{22} - Z_{12} Z_{21})} \quad \text{--- (17)}$$

we apply short circuit on part 2  
A voltage source on part 2

$$V_2 = V_s \quad \text{--- (18)}$$

$$V_1 = 0 \quad \text{--- (19)}$$

$$I_1 = -I_1' \quad \text{--- (20)}$$

Eq<sup>n</sup> (1) can be written as

$$V_1 = (Z_{11} - Z_{12})I_1 + Z_{11}(I_1 + I_2) \quad (7)$$

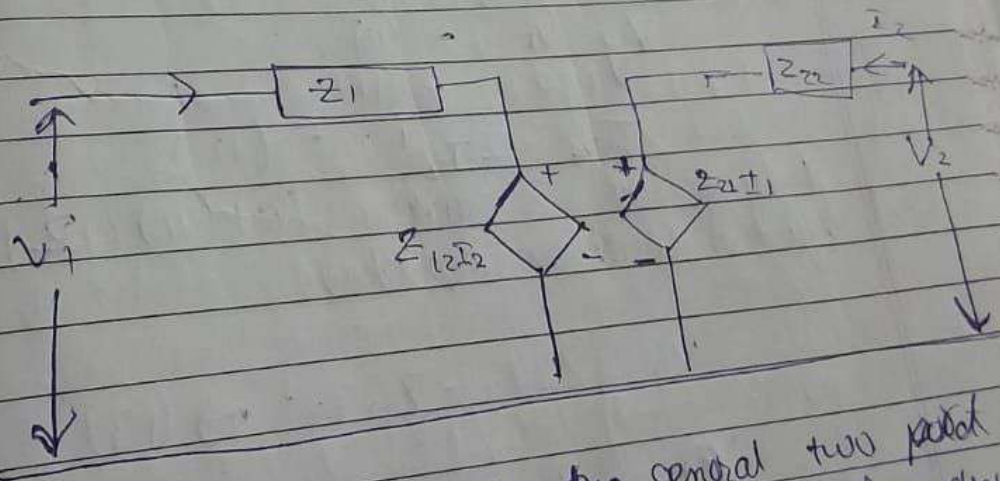
$$V_2 = (Z_{21} - Z_{22})I_1 + Z_{22}(I_1 + I_2) \quad (8)$$

A network is called reciprocal if the ratio of the response transform to the excitation transform is invariant to interchange of the positions of the excitation and response in the network.

From eqn (8) we have,

$$V_2 = (Z_{21} - Z_{22})I_1 + (Z_{22} - Z_{12})I_2 + Z_{12}(I_1 + I_2) \quad (9)$$

The equivalent circuit representation for eqn (1) & (2).



[Two generator equivalent of network in terms of parameters] the general two port open circuit & impedance



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$$0 = -Z_{11}I_1' + Z_{12}I_2 \rightarrow (21)$$

$$V_s = Z_{21}I_1' + Z_{22}I_2 \rightarrow (22)$$

From eq (21) we have,

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1' \rightarrow (23)$$

From eq (22) we have

$$I_2 = \frac{V_s + Z_{21}I_1'}{Z_{22}} \rightarrow (24)$$

From eq (23) & (24) we have

$$\frac{Z_{11}}{Z_{12}} I_1' = \frac{V_s + Z_{21}I_1'}{Z_{22}}$$

$$\text{On } Z_{11}Z_{22}I_1' = Z_{12}V_s + Z_{12}Z_{21}I_1'$$

$$\text{On } (Z_{11}Z_{22} - Z_{12}Z_{21})I_1' = V_s Z_{12}$$

$$I_1' = \frac{V_s \cdot Z_{12}}{(Z_{11}Z_{22} - Z_{12}Z_{21})} \rightarrow (25)$$

5) Condition for reciprocity of (Y) parameter.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_2 = 0 \quad (\text{Short circuited})$$

$$V_1 = V_2$$

$$I_1 = I_2$$

$$Y_{12} = Y_{21} \quad \leftarrow \text{Reciprocity}$$

$$Y_{11} = Y_{22} \quad \leftarrow \text{Symmetry}$$

Self Reciprocity

Condition for reciprocity A symmetry of (A B C D)

Notation

$$V_1, V_2 = f(I_1, I_2) \quad \leftarrow Z$$

$$I_1, I_2 = f(V_1, V_2) \quad \leftarrow Y$$

$$V_1, I_1 = f(V_2, I_2) \quad \leftarrow A, B, C, D$$

$$V_2, I_2 = f(V_1, I_1) \quad \leftarrow A', B', C', D'$$

$$V_1, I_2 = f(I_1, V_2) \quad \leftarrow h$$

$$V_2, I_1 = f(I_2, V_1) \quad \leftarrow g$$



eg of prob 10

$$V_1 = AV_2 - BI_2$$

$$V_1 = (V_2 - BI_2) \rightarrow (1)$$

For the network to be reciprocal we use the principle of reciprocity. we apply a voltage source  $V_S$  at ascending end with the terminal and shorted.

So,  $V_1 = V_S$  — (3)

$V_2 = 0$  — (5)

$I_2 = -I_2' - (5)$

$$V_0 = BI_2'$$

$$\begin{vmatrix} I_1' \\ I_2' \\ V_S \end{vmatrix} = \frac{1}{B} \quad (6)$$

Now we interchange the positions of the shorted terminals. So,

$$V_1 = 0 \quad \text{--- (7)}$$

$$V_2 = V_S \quad \text{--- (8)}$$

$$I_1 = -I_1' \quad \text{--- (9)}$$

eg 7, 8, 9 apply in (1) & (2)

$$0 = AV_S - BI_2 \rightarrow (10)$$

$$-I_1' = (V_S - 0)I_2 \rightarrow (11)$$

Comparing  $I_1$  &  $I_2$  for reciprocity theorem condition

$$Z_{12} = Z_{21}$$

Similarly for condition of symmetry we apply voltage source  $V_s$  at port 1 with port 2 open circuited and find  $\frac{V_s}{I_1}$

From eq (1) & (2)

$$V_1 = V_s \rightarrow (26)$$

$$I_2 = 0 \rightarrow (27)$$

Applying equation 26 & 27 on eq 1

$$V_s = Z_{11} I_1 \rightarrow (28)$$

$$V_2 = Z_{21} I_1$$

$$\frac{V_s}{I_1} = Z_{11} \rightarrow (29)$$

Now we apply  $V_s$  at port 2 & port (1) is open circuited.

$$I_1 = 0 \rightarrow (30)$$

$$V_2 = V_s \rightarrow (31)$$

$$V_s = Z_{22} I_2$$

$$\frac{V_s}{I_2} = Z_{22} \rightarrow (32)$$

Condition of symmetry

$$Z_{11} = Z_{22}$$





Khushi

Missed voice call

8:14 Pm

unit 3 k

8:05 pm

notes bhjde apne

8:05 pm

32°C  
Haze

Search

Symmetry

The definition of symmetry for 3 parameter case  
 as  $Z_{11} = Z_{22}$  (10)

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{AV_1 - B I_2}{C V_1 - D I_2} = \frac{A}{C} \rightarrow (12)$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

## Interpolation and Approximation

### UNIT - IV

Numerical solution of ordinary Differential Equations: Picard's method, Runge-Kutta's methods, Predictor-corrector methods: Euler's method.

Numerical Solution of Partial Differential equations: Parabolic equations: Implementation to be done in C/C++

#### Textbook(s):

David R. Kincaid, "Numerical Mathematics"



from eq<sup>n</sup> (10) we have  
$$I_2 = \frac{AV_s}{R_s} \rightarrow (12)$$

from eq (11) we have,

$$DI_2 = (V_s + I_1')$$

$$I_2 = \frac{V_s + I_1'}{D} \rightarrow (13)$$

Equating eq 13 & 12

$$\frac{AV_s}{R_s} = \frac{V_s + I_1'}{D}$$

$$\text{or } ADV_s = R_s(V_s + I_1')$$

$$\text{or } V_s(AD - R_s) = R_s I_1'$$

$$\left[ \frac{I_1'}{V_s} = \frac{AD - R_s}{R_s} \right] \rightarrow (14)$$

As per the definition of reciprocity, the LHS of (8) & (14) should be identical

$$AD - R_s = 1$$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1 \rightarrow (15)$$

Reciprocity



$$F = \{q_1\}$$

$$q_0 = \{q_0\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\delta = Q \times \Sigma \rightarrow Q$$

Good Write

string should end with 11.

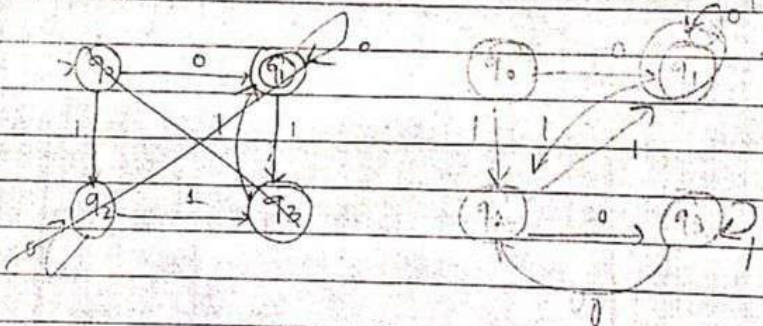
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Q The set of all strings which when interpreted as binary integer is a multiple of 3.

$q_0, q_1, q_2$

$q_0: 0, 1, 2$



Q The set of all strings which when interpreted as binary integer is a multiple of 5.

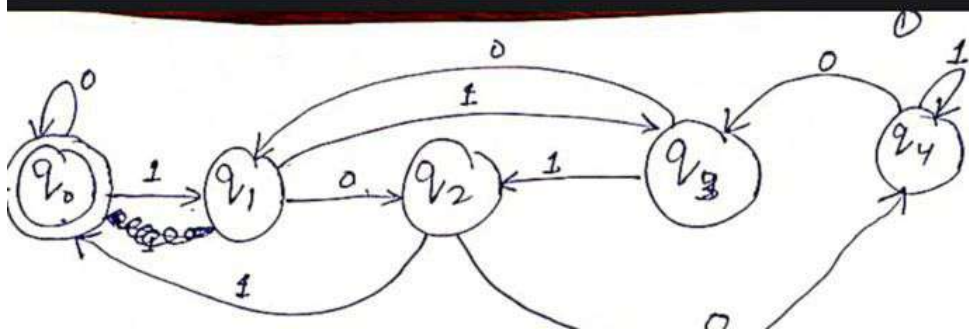
$$n \div 5 = 0 \rightarrow q_0 \quad 0 \rightarrow 0, 5 \rightarrow 101, 10 \rightarrow 1010, 15 \rightarrow 1111$$

$$n \div 5 = 1 \rightarrow q_1 \quad 1 \rightarrow 1, 6 \rightarrow 110, 11 \rightarrow 1011, 16 \rightarrow 10000$$

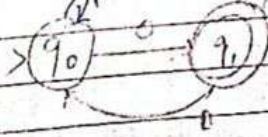
$$n \div 5 = 2 \rightarrow q_2 \quad 2 \rightarrow 10$$

$$n \div 5 = 3 \rightarrow q_3 \quad 3 \rightarrow 11$$

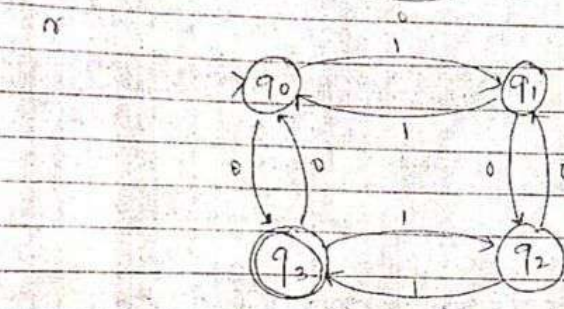
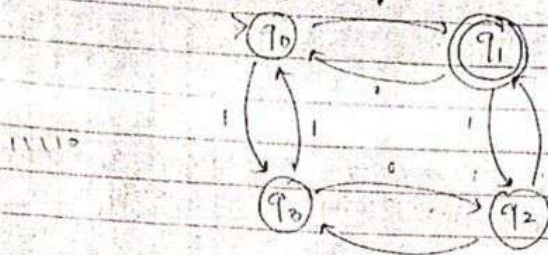
$$n \div 5 = 4 \rightarrow q_4 \quad 4 \rightarrow 100$$



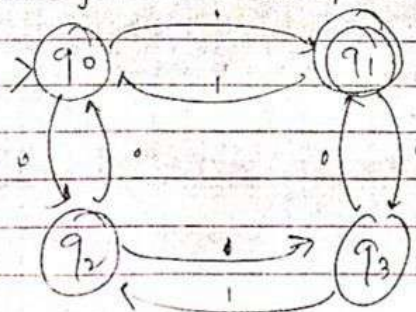




Ques Draw a DPA for all strings over  $\{0,1\}$  consisting odd no. of zeros & even no. of '1's.



Ques Draw a DPA for all strings over  $\{0,1\}$  consisting even no. of '0' & odd no. of '1'.



010  
111

$M = \langle Q, F, q_0, \delta, \epsilon \rangle$

$\Sigma = \{0,1\}$

$F = \{q_1\}$

$q_0 = \{q_0\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$\delta = Q \times \Sigma \rightarrow Q$

Good Write

string should end with 11.

0111011