Circuit system UNIT-4

Filter Synthesis

Inside this Chapter

11.1. INTRODUCTION

Resonant circuits have already been studied in AC circuits that select relatively narrow bands of frequencies and reject others.Certain other reactive networks are available that will freely pass desired bands of frequencies while almost totally suppressing other bands of frequencies. Such reactive networks, called filters, were first discussed by G. A. Campbell and O.J. Zobel of the Bell Telephone Laboratories.

Filter networks are widely used in communication systems to separate various voice channels in carrier frequency telephone circuits. Filters also find applications in instrumentation, telemetering equipment etc., where it is necessary to transmit or attenuate a limited range of frequencies.

In this chapter, we will discuss various types of filters and complete analysis of filter circuits is carried out on the basis of certain definitions from the general field of electrical network theory, under the assumption of symmetrical network sections.

Active filters are being widely used in place of conventional (passive) filters. Inductors cannot be fabricated with high quality in *I.C.* (integrated circuit) technology, so the resistance capacitance circuit with active device replaces the conventiond *L-C* filter and provides a sharp cut-off in the attenuation band. Detailed analysis of active filters is out of the scope of this text book.

11.2. PARAMETERS OF A FILTER

The following parameters characterize a typical filter.

11.2.1. Characteristic Impedance Z_C or Z_o

The characteristic impedance of a filter must be chosen such that the filter may fit into a given line or between two types of the equipment.

11.2.2. Pass Band

Band, in which ideal filters have to pass all frequencies without reduction in magnitude are referred to as pass band.

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11.2.3. Stop Band

Band, in which ideal filters have to attenuate (or stop) frequencies are referred to as stop band.

11.2.4. Cut-off Frequency f_C

The frequency which separates the pass-band and the stop band is defined as the cut-off frequency of the filter.

11.2.5. Units of Attenuation

The attenuation of a wave filter can be expressed in decibels (dB) or Nepers or Bels.

Let V_i, I_i and P_i be the input voltage, input current and input power respectively of a filter. Similarly V_o, I_o and P_o represent output voltage, output current and output power respectively.

Then $\frac{V_o}{V_i}$ represents the voltage gain and similarly $\frac{P_o}{P_i}$ represents the power gain. In the case of filter $P_o < P_i$ so that $\frac{P_i}{P_o}$ is given by the attenuation. Various units of attenuation are given below:

(i) A decibel is defined as ten times the common logarithm of the ratio of the input power to the output power, *i.e.*,

$$\text{Attenuaton in } dB = 10 \log_{10} \frac{P_i}{P_o}$$

The decibel can also be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current), *i.e.*,

$$\text{Attenuation in dB} = 20 \log_{10} \frac{V_i}{V_o} = 20 \log_{10} \frac{I_i}{I_o}$$

(ii) A neper is defined as the natural logarithm of the ratio of input and output quantities, *i.e.*,

$$\text{Attenuation in Nepers} = \frac{1}{2} \log_e \frac{P_i}{P_o} = \log_e \frac{V_i}{V_o} = \log_e \frac{I_i}{I_o}$$

(iii) A bel is defined as the common logarithm of the ratio of input and output quantities, *i.e.*,

$$\text{Attenuation in Bels} = \log_{10} \frac{P_i}{P_o} = 2 \log_{10} \frac{V_i}{V_o} = 2 \log_{10} \frac{I_i}{I_o}.$$

From above discussion, it may be seen that

$$\text{Attenuation in dB} = 8.686 \times \text{Attenuation in Nepers}$$
$$= 10 \times \text{Attenuation in Bels}$$

11.3. CLASSIFICATION OF FILTERS

The filters may, in principle, have any number of pass bands separated by attenuation bands. However, they are classified into four common types as follows :

11.3.1. Low Pass Filters

These filters reject all frequencies above cut-off frequency, f_C. The attenuation characteristic of an ideal LP filter is shown in figure 11.1 (*a*). Thus the pass band or transmission band for the LP filter is the frequency range 0 to f_C and the stop band or attenuation band is the frequency range above f_C.

11.3.2. High Pass Filters

These filters reject all frequencies below cut-off frequency, f_C. Thus the pass band and stop band of the HP filter are the frequency range above f_C and below f_C respectively. The attenuation characteristic of a HP filter is shown in figure 11.1(b).

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11.3.3. Band Pass Filters

These filters allow transmission of freqeucnies between two designated cut off frequencies and reject all other frequencies. As shown in figure 11.1(c), a band pass filter has two cut-off frequencies and will have the pass band $f_{C2}-f_{C1}$. f_{C1} is called as the lower cut-off frequency while f_{C2} is called the upper cut-off frequency.

11.3.4. Band Stop or Band Elimination Filters

These filters pass all frequencies lying outside a certain range, while it attenuates all frequencies between the two frequencies f_{C1} and f_{C2}. The characteristic of an ideal band stop filter is shown in figure 11.1(d).

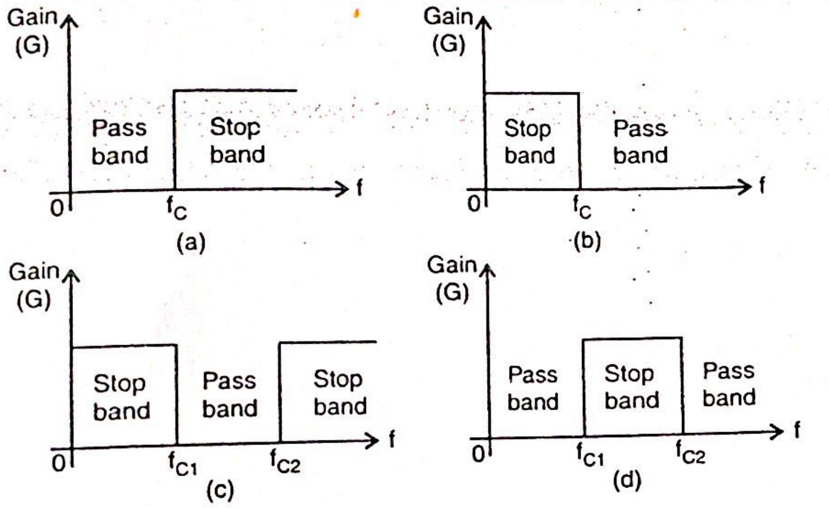


Fig. 11.1. Ideal response of (a) Low-pass (L.P.) (b) High-pass (H.P.), (c) Band-Pass (B.P.) and (d) Band stop (B.S.) filters. Here G = V_o/V_i represents the gain of the filter and attenuation is inverse of the gain of the filter

11.4. BLOCK DIAGRAM REPRESENTATION OF THE FILTERS

The block diagram of low pass and high pass filters are shown in figure 11.2 (a) and (b) respectively.

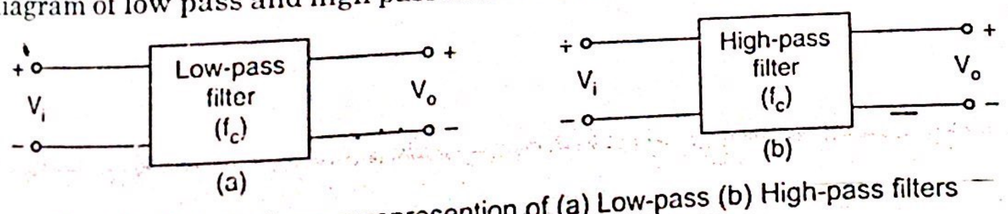
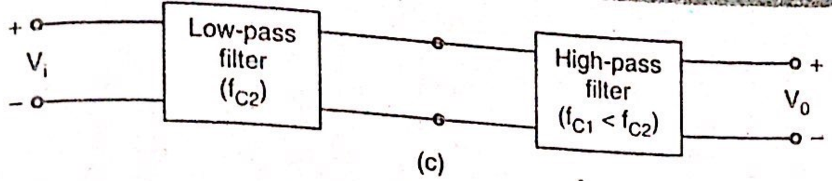


Fig. 11.2. Block diagram represention of (a) Low-pass (b) High-pass filters

A band pass filter is generally obtained by series or cascade connection of a low-pass filter and high-pass filter as shown in figure 11.3(a) where cut-off frequency f_{C1} of high-pass filter is less than the cut-off frequency f_{C2} of low pass filter. While, a band - stop filter is obtained by parallel connection of a low-pass filter and high-pass filter as shown is figure 11.3 (b), where cut-off frequency f_{C2} of high-pass filter is greater than the cut-off frequency f_{C1} of low-pass filter.



(c)

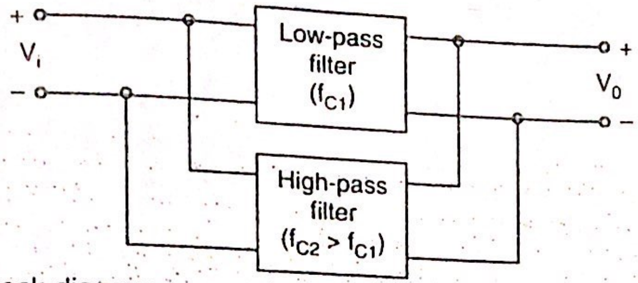


Fig. 11.3. Block-diagram representation of (a) Band-pass (b) Band-stop filters

Note :

Mathematically we can say that the band-pass filter is intersection of low-pass and high-pass filters, *i.e.*, $B.P. = (L.P.) \cap (H.P.)$, while, the band-stop filter is union of them, *i.e.*, $B.S. = (L.P.) \cup (H.P.)$.

Table 11.1 shows the main classification with their corresponding frequency ranges.

Table 11.1

Types of Filter	Pass-band	Attenuation-band
Low-pass	$0 \to f_C$	$f_C \to \infty$
High-pass	$f_C \to \infty$	$0 \to f_C$
Band-pass	$f_{C1} \to f_{C2}$	$0 \to f_{C2}, f_{C1} \to \infty$
Band - stop	$0 \to f_{C1}, f_{C2} \to \infty$	$f_{C1} \to f_{C2}$

The passive filters mentioned in article 11.3 may also be classified as :

(*i*) **Constant -*K* or Prototype Filters :** A network is said to be of the constant- K type if the series impedance Z_1 and the shunt impedance Z_2 of the network satisfy the relation

$$Z_1 Z_2 = K^2$$

where K is a real constant, independent of frequency, *i.e.*, a resistance. K is often termed as design impedance R_o of the filter. Thus

$$Z_1 Z_2 = K^2 = R_o^2$$

The constant-K type filter is also known as the prototype because other more complex networks can be derived from it.

(*ii*) ***m*-derived Filters :** In such network, $Z_1 Z_2 \neq K^2$, (where Z_1 and Z_2 defined above) but the same characteristic impedance as the corresponding constant-K networks and have much sharper attenuation characteristics.

11.5. FILTER NETWORKS

A filter ideally should have zero attenuation in the pass (transmission) band. This can only happen if the elements of the filter are dissipationless, *i.e.*, pure reactive. Filters are made of symmetrical T, or π networks. Both the T and π networks can be considered as combinations of unsymmetrical L networks as shown in figure 11.4 (*a*) and (*b*) respectively.

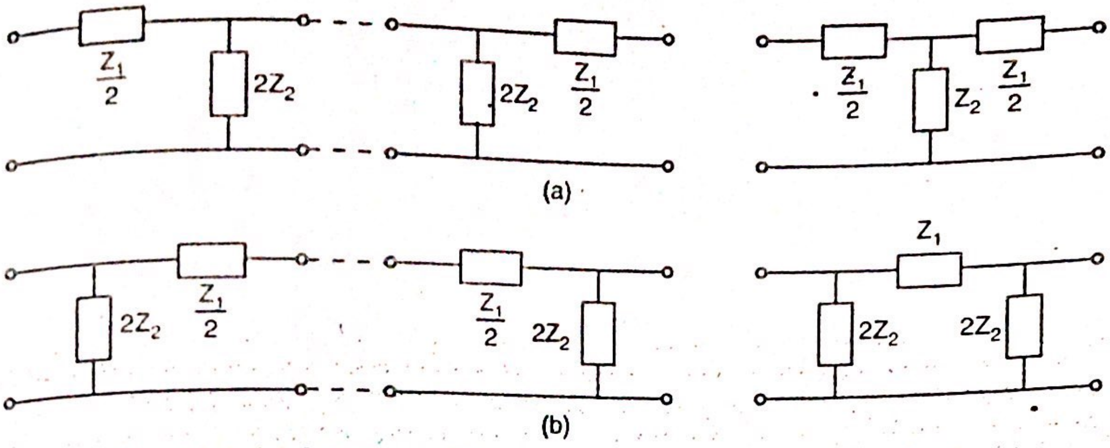


Fig. 11.4. (a) Symmetrical T-network and (b) Symmetrical p-network as derived from unsymmetrical two L-networks

The ladder structure is one of the commonest froms of filter network. A cascade connection of several symmetrical T and π networks constitutes a ladder network. The common forms of the ladder networks are shown in figure 11.5 (a) and (b). It can be observed that both networks are identical except at the ends.

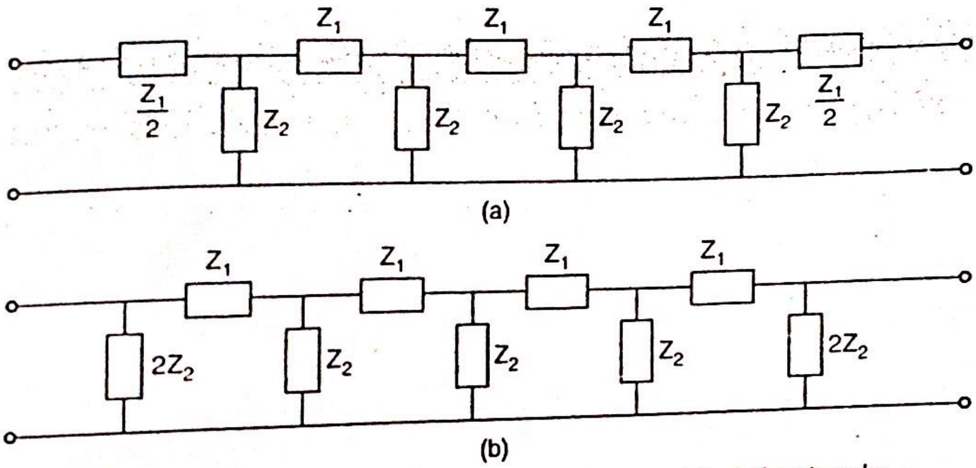


Fig. 11.5. Ladder network (a) formed by symmetrical T-networks (b) formed by symmetrical p-networks

11.6. CHARACTERISTICS OF FILTER NETWORKS

The study of the characteristics of any filter requires the calculation of its characteristic impedance Z_o. Propagation constant γ, attenuation α, phase shift β.

11.6.1. Characteristic Impedance (Z_o)

As discussed in chapter 8, (Article 8.14) if the image impedances at input port and output port are equal to each other, the image impedance is then called the characteristic or the iterative impedance, Z_o.

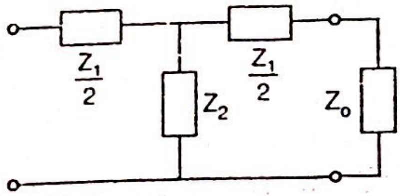


Fig. 11.6. Symmetrical T-network

(A) For T-network: Consider a symmetrical T-network, as shown in figure 11.6, is terminated in Z_o, its input impedance will also be Z_o. The value of input impedance for the T-network can be determined as

$$Z_{in} = Z_o = \frac{Z_1}{2} + \left[Z_2 \| \left(\frac{Z_1}{2} + Z_o \right) \right]$$

$$Z_o = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_o\right)}{Z_2 + \frac{Z_1}{2} + Z_o} = \frac{Z_1}{2} + \frac{(Z_1 Z_2 + 2Z_2 Z_o)}{Z_1 + 2Z_2 + 2Z_o}$$

or

$$= \frac{Z_1^2 + 2Z_1 Z_2 + 2Z_1 Z_o + 2Z_1 Z_2 + 4Z_2 Z_o}{2(Z_1 + 2Z_2 + 2Z_o)}$$

or $$2Z_o(Z_1 + 2Z_2 + 2Z_o) = Z_1^2 + 4Z_1 Z_2 + 2Z_1 Z_o + 4Z_2 Z_o$$

$$4Z_o^2 = Z_1^2 + 4Z_1 Z_2$$

$$Z_o^2 = \frac{Z_1^2}{4} + Z_1 Z_2$$

or

The characteristic impedance of a symmetrical T-network is

$$Z_{oT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad \text{...(1)}$$

Z_{oT} can also be expressed in terms of open-circuit impedance Z_{oc} and short- circuit impedance Z_{sc} of the T- network. From figure 11.6, the open circuit impedance

$$Z_{oc} = \frac{Z_1}{2} + Z_2 = \frac{1}{2}(Z_1 + 2Z_2)$$

and short-circuit impedance

$$Z_{sc} = \frac{Z_1}{2} + \left[Z_2 || \frac{Z_1}{2}\right] = \frac{Z_1}{2} + \frac{Z_2 \cdot \frac{Z_1}{2}}{Z_2 + \frac{Z_1}{2}} = \frac{Z_1^2 + 4Z_1 Z_2}{2(Z_1 + 2Z_2)}$$

Therefore, $$Z_{oc} \times Z_{sc} = \frac{Z_1^2}{4} + Z_1 Z_2 = Z_{oT}^2 \qquad \text{...(2)}$$

or $$Z_{oT} = \sqrt{Z_{oc} Z_{sc}}$$

(B) For π-network : Consider a symmetrical π-network as shown in figure 11.7. When the network is terminated in Z_o at output port, its input impedance is given by

$$Z_{in} = \left[\{Z_o || 2Z_2\} + Z_1\right] || (2Z_2)$$

$$= \frac{\left(\frac{Z_o \cdot 2Z_2}{Z_o + 2Z_2} + Z_1\right) \cdot 2Z_2}{\frac{Z_o \cdot Z_2}{Z_o + 2Z_2} + Z_1 + 2Z_2}$$

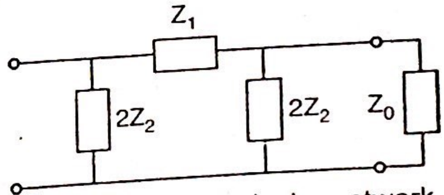


Fig. 11.7. Symmetrical p-network

By definition of characteristic impedance, $Z_{in} = Z_o$

$$Z_o = \frac{(2Z_o Z_2 + Z_1 Z_o + 2Z_1 Z_2) \cdot 2Z_2}{2Z_o Z_2 + Z_1 Z_o + 2Z_1 Z_2 + 2Z_o Z_2 + 4Z_2^2}$$

or $$2Z_o^2 Z_2 + Z_o^2 Z_1 + 2 \cdot Z_o \cdot Z_1 Z_2 + 2Z_o^2 Z_2 + 4Z_o Z_2^2 = 4Z_o Z_2^2 + 2Z_o Z_1 Z_2 + 4Z_1 Z_2^2$$

or $$4Z_o^2 Z_2 + Z_o^2 Z_1 = 4Z_1 Z_2^2$$

$$Z_o^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

The characteristic impedance of a symmetrical π - network is

$$Z_{o\pi} = \frac{2Z_1 Z_2}{\sqrt{Z_1^2 + 4Z_1 Z_2}} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} \quad ...(3)$$

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and $$Z_{o\pi} = \frac{Z_1 Z_2}{Z_{oT}} \quad \text{[From equation (1)]} \quad ...(4)$$

$Z_{o\pi}$ can also be expressed in terms of Z_{oc} and Z_{sc} of the π-network shown in figure 11.7.

$$Z_{oc} = (2\,Z_2 + Z_1) \,||\, (2Z_2)$$

$$Z_{oc} = \frac{(2Z_2 + Z_1)\cdot 2Z_2}{2Z_2 + Z_1 + 2Z_2} = \frac{(2Z_2 + Z_1)\cdot 2Z_2}{Z_1 + 4Z_2}$$

and $$Z_{sc} = (2\,Z_2) \,||\, Z_1 = \frac{2Z_1 Z_2}{2Z_2 + Z_1}$$

Therefore, $Z_{oc} \cdot Z_{sc} = \dfrac{4Z_1 Z_2^2}{Z_1 + 4Z_2} = Z_{o\pi}^2$ [from equation (3)]

or $$Z_{o\pi} = \sqrt{Z_{oc} \cdot Z_{sc}} \quad ...(5)$$

It can be seen from Equation (4) that

$$Z_{oT} \cdot Z_{o\pi} = Z_1 \cdot Z_2$$

11.6.2. Propagation Constant (γ)

By definition the propagation constant γ of the network is given by

$$\gamma = \log_e \frac{I_1}{I_2}$$

(A) For *T*-network : Taking the mesh equation for the second mesh, we get

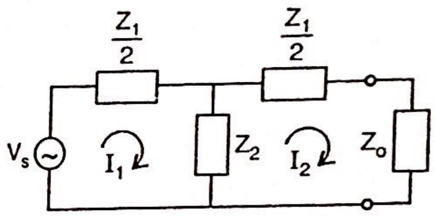

Fig. 11.8. Symmetrical T-network terminated in Z_o for finding propagation constant

$$(I_2 - I_1)\, Z_2 + I_2\left(\frac{Z_1}{2} + Z_o\right) = 0$$

$$I_1\, Z_2 = I_2\left(\frac{Z_1}{2} + Z_o + Z_2\right)$$

or $$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_o + Z_2}{Z_2} = e^{\gamma}$$

(From definition)

or $$\frac{Z_1}{2} + Z_o + Z_2 = Z_2\, e^{\gamma}$$

$$Z_o = Z_2(e^{\gamma} - 1) - \frac{Z_1}{2} \quad ...(6)$$

From equation (1), the characteristic impedance of a symmetrical *T*-network is given by

$$Z_{oT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \quad ...(1)$$

Squarring equation (6) and (1), and subtracting equation (1) from equation (6), we get

$$Z_2^2\,(e^{\gamma}-1)^2 + \frac{Z_1^2}{4} - Z_1 Z_2 (e^{\gamma} - 1) - \frac{Z_1^2}{4} - Z_1\, Z_2 = 0$$

$$Z_2^2\,(e^{\gamma} - 1)^2 - Z_1\, Z_2\,(e^{\gamma} - 1 + 1) = 0$$

or $$Z_2^2 (e^{\gamma} - 1)^2 - Z_1\, Z_2\, e^{\gamma} = 0$$

$$Z_2(e^{\gamma} - 1)^2 - Z_1\, e^{\gamma} = 0$$

$$(e^\gamma - 1)^2 = \frac{Z_1 e^\gamma}{Z_2}$$

$$e^{2\gamma} + 1 - 2e^\gamma = \frac{Z_1}{Z_2 e^{-\gamma}}$$

$$(e^\gamma + e^{-\gamma} - 2) = \frac{Z_1}{Z_2}$$

Dividing both the side by 2, we have

$$\frac{e^\gamma + e^{-\gamma}}{2} = 1 + \frac{Z_1'}{Z_2}$$

$$\cos h\gamma = 1 + \frac{Z_1}{2Z_2} \quad ...(7)$$

$$\sin h\gamma = \sqrt{\cos h^2\gamma - 1} = \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{Z_1}{2Z_2}\right)^2 + \frac{Z_1}{Z_2}} = \frac{1}{Z_2}\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \frac{Z_{oT}}{Z_2} \quad ...(8) \text{ [From equation (1)]}$$

Dividing equation (8) by equation (7), we get

$$\tan h\gamma = \frac{Z_{oT}}{Z_2\left(1+\frac{Z_1}{2Z_2}\right)} = \frac{Z_{oT}}{Z_2 + \frac{Z_1}{2}}$$

But
$$Z_2 + \frac{Z_1}{2} = Z_{oc}$$

Also from equation (2),

$$Z_{oT} = \sqrt{Z_{oc} \cdot Z_{sc}}$$

Therefore,
$$\tan h\gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} \quad ...(9)$$

Also
$$\sin h\frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cos h\gamma - 1)} = \sqrt{\frac{1}{2}\left(1+\frac{Z_1}{2Z_2}-1\right)} = \sqrt{\frac{Z_1}{4Z_2}} \quad ...(10)$$

(B) For π-**network** : The propagation constant of a symmetrical π-network is the same as that for a symmetrical T-network, *i.e.*

$$\cos h\gamma = 1 + \frac{Z_1}{2Z_2}$$

11.6.3. Classification of Pass-band and Stop-band

Idealy it is desired that a filter network transmits or passes a desired frequency band without loss, whereas it should attenuate or completely stop all undesired frequencies. The propagation constant $\gamma = \alpha + j\beta$, being a function of frequency, the pass band, stop band and the cut-off point *i.e.* the point of separation between the two bands, can be identified.

For symmetrical T or π-network, the expression for propagation constant γ in terms of hyperbolic function is given by

$$\sin h\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

If Z_1 and Z_2 are both pure imaginary values,their ratio, and hence $\frac{Z_1}{4Z_2}$, will be a pure real number.

Since Z_1 and Z_2 may be anywhere in the range from $-j\infty$ to $+j\infty$, $\frac{Z_1}{4Z_2}$ may also have any real value between the infinite limits. Then $\sin h\gamma/2 = \sqrt{Z_1/4Z_2}$ will also have infinite limits, but may be either real or imaginary depending upon whether $\frac{Z_1}{4Z_2}$ is positive or negative.

As discussed earlier, the propagation constant is a complex function $\gamma = \alpha + j\beta$, the real part of the complex propagation constant α, is a measure of the change in magnitude of the current or voltage in the network, known as the attenuation constant. β is a measure of the difference in phase between the inut and output currents or voltages, known as phase shift constant. Therefore α and β take on different values depending upon the range of $\frac{Z_1}{4Z_2}$. We have

$$\sin h\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$$

or

$$\sin h\left(\frac{\alpha + j\beta}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

Expanding, we get

$$\sin h\,\frac{\alpha}{2}\cdot\cos\frac{\beta}{2} + j\cos h\frac{\alpha}{2}\cdot\sin\frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \qquad \text{...(11)}$$

Case I : If Z_1 and Z_2 are the same type of reactances, then $\frac{Z_1}{4Z_2}$ is real and positive equal to say p.

The imaginary part of the equation (11) must be zero, *i.e.*,

$$\cos h\frac{\alpha}{2}\cdot\sin\frac{\beta}{2} = 0 \qquad \text{...(12.}a\text{)}$$

and

$$\sin h\frac{\alpha}{2}\cdot\cos\frac{\beta}{2} = p \qquad \text{...(12.}b\text{)}$$

α and β must satisfy both the above equations.

Equation (12.a) can be satisfied if

$$\frac{\beta}{2} = 0, \quad \text{or } n\pi \quad \text{where } n = 0,\ 1,\ 2,\ \ldots\ ;$$

then

$$\cos\frac{\beta}{2} = 1 \text{ and } \sin h\,\frac{\alpha}{2} = p = \sqrt{\frac{Z_1}{4Z_2}}$$

That p should be always positive implies that

$$\frac{Z_1}{4Z_2} > 0 \text{ and } \alpha = 2\sin h^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

Since $\alpha \neq 0$, it indicates that the attenuation exists.

Case II : If Z_1 and Z_2 are the opposite type of reactances, then $\frac{Z_1}{4Z_2}$ is real and negative, making $\sqrt{\frac{Z_1}{4Z_2}}$ imaginary and equal to say jp.

The real part of the equation (11) must be zero.

$$\sin h\frac{\alpha}{2}\cos\frac{\beta}{2} = 0 \qquad \text{...(12.c)}$$

and $$\cos h\frac{\alpha}{2}\sin\frac{\beta}{2} = p \qquad \text{...(12.d)}$$

Both the above equations must be satisfied simultaneously by α and β. Equation (12.c) may be satisfied when $\alpha = 0$, or when $\beta = \pi$.

(A) $\alpha = 0$, therefore, $\sin h\frac{\alpha}{2} = 0$. And from equation (12.d),

$\sin\frac{\beta}{2} = p = \sqrt{\frac{Z_1}{4Z_2}}$. But $\frac{Z_1}{4Z_2}$ must be negative, therefore,

$$-1 \le \frac{Z_1}{4Z_2} \le 0$$

$$\therefore \quad \beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

(B) $\beta = \pi$, therefore, $\cos\frac{\beta}{2} = 0$. And from equation (12.d) $\sin\frac{\beta}{2} = \pm 1$.

$$\cos h\frac{\alpha}{2} = p = \sqrt{\frac{Z_1}{4Z_2}}.$$

Since $\cos h\frac{\alpha}{2} \ge 1$, this solution is valid for negative $\frac{Z_1}{4Z_2}$, and having magnitude greater than or equal to unity.

$$-\alpha \le \frac{Z_1}{4Z_2} \le -1$$

This represents a stop or attenuation band, since $\alpha \ne 0$. The phase angle is π and the attenuation constant is represented by

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$$

11.6.4. Cut-off Frequency

As defined earlier, the frequency at which the network changes from a pass-band to stop-band and vice-verse is called the cut-off frequency. The cut-off frequencies can be determined using.

$$-1 < \frac{Z_1}{4Z_2} < 0$$

$$\therefore \quad \frac{Z_1}{4Z_2} = 0 \quad \text{or} \quad Z_1 = 0$$

and $$\frac{Z_1}{4Z_2} = -1 \quad \text{or} \quad Z_1 + 4Z_2 = 0$$

Where Z_1 and Z_2 are opposite type of reactances. Since Z_1 and Z_2 may have a number of different values, as L and C elements, or as parallel and series combinations, a variety of types are possible.

11.7. SUMMARY OF RELATIONS FOR FILTER NETWORKS

For symmetrical T and π-networks we now summarize the different relationships derived in last Article 11.6, which will be used frequently for the analysis of various filters in forthcoming Articles:

1. **Characteristic Impedance :**

$$Z_{oT} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} \quad \text{... for } T\text{-network}$$

$$Z_{o\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} \quad \text{... for } \pi \text{ - network}$$

$$Z_{oT} \cdot Z_{o\pi} = Z_1 Z_2 = K^2 = R_o^2$$

2. **Propagation Constant :** Propagation constand γ is the same for both T and π-networks and is given by

$$\cos h\gamma = 1+\frac{Z_1}{2Z_2}$$

3. **Attenuation Constant :**

$$\alpha = 0 \quad \text{..... in pass-band.}$$

$$\alpha = 2\cos h^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} \quad \text{... in stop-band}$$

4. **Phase Constant :** $\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|}$ ... in pass - band

$\beta = n\pi$; $n = 0, 1, 2, \ldots\ldots$ in stop - band

5. **Cut - off Frequency :** $Z_1 = 0$ and $Z_1 + 4Z_2 = 0$

11.8. CONSTANT-K LOW PASS FILTERS

A network, either T or π, is said to be of the constant-K type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1 Z_2 = K^2$$

where Z_1 and Z_2 are impedances in the T and π networks as shown in figure 11.9.(*a*) and (*b*) respectively.

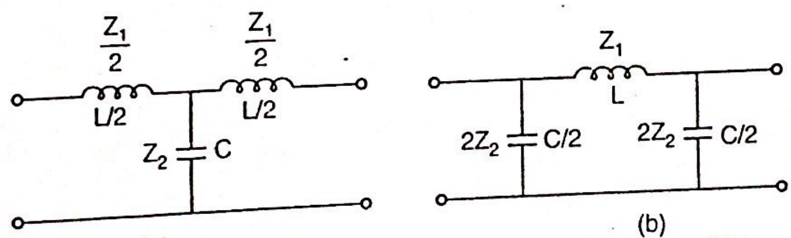

Fig. 11.9. General configurations of constant - K low pass filter as (a) T-network (b) p-network

For both T and π-network filters

If series impedance $Z_1 = j\omega L$ and Shunt impedance $Z_2 = \frac{1}{j\omega C}$, then

$$Z_1 Z_2 = \frac{L}{C} = K^2 \quad \text{(which is independent of frequency)}$$

Since $Z_1 Z_2 = K^2 = R_o^2$ for constant-K filter, therefore, the design impedance or the load resistance R_o is given by

$$R_o = \sqrt{\frac{L}{C}}$$

(i) **Cut-off Frequency :** The cut-off frequencies are given by

and $Z_1 + 4Z_2 = 0$, or $j\omega_C L = 0$ or $\omega_C = 0$ or $f_C = 0$

$j\omega_C C$

or
$$\omega_C = \frac{2}{\sqrt{LC}}$$

or
$$f_C = \frac{1}{\pi\sqrt{LC}}$$

Thus, the passband of constant-K low pass filter extends from 0 to $\frac{1}{\pi\sqrt{LC}}$.

The pass-band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as shown in figure 11.10. The cut-off frequency at the intersection of the curves Z_1 and $-4Z_2$ is indicated as f_C. All the frequencies above f_C lie in a stop or attenuation band. Thus, the network is called a low-pass-filter.

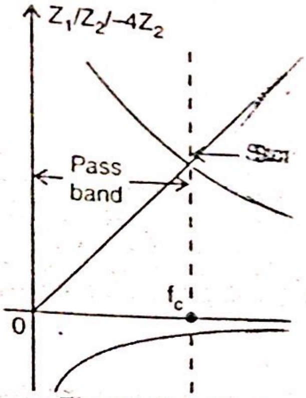

Fig. 11.10. Variatio... $4Z_2$ with fre...

(ii) **Attenuation Constant :** In pass-band, $\alpha = 0$

and in stop-band,
$$\alpha = 2\cos h^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\cos h^{-1}\sqrt{\left|\frac{j\omega L}{4/j\omega C}\right|}$$
$$= 2\cos h^{-1}\sqrt{\frac{\omega^2 LC}{4}} = 2\cos h^{-1}\left(\frac{f}{f_C}\right)$$

(iii) **Phase constant :** In stop-band, $\beta = \pi$

and in pass-band,
$$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\sin^{-1}\left(\frac{f}{f_C}\right)$$

The variation of attenuation constant α and phase constant β with frequency is shown in figure 11.11.

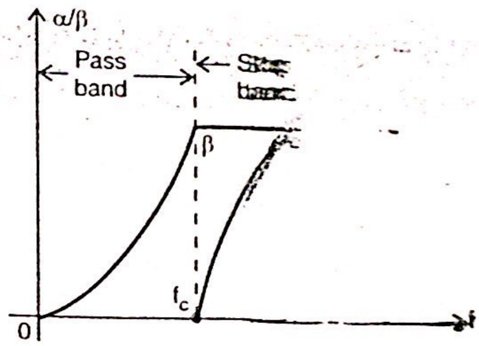

Fig. 11.11. Variation of attenuation constant a, and phase constant b with frequency

(iv) **Characteristic Impedance :** For T-network filter

$$Z_{oT} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

$$Z_{oT} = \sqrt{\frac{L}{C}}\cdot\sqrt{1 - \left\{\frac{f}{f_C}\right\}^2} = R_o\sqrt{1 - \left(\frac{f}{f_C}\right)^2}$$

In the pass-band, $f < f_C$, so that Z_{oT} is real,

In the stop-band, $f > f_C$, so that Z_{oT} is imaginary,

and if $f = f_C$, $Z_{oT} = 0$.

For π-network filter

$$Z_{o\pi} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \sqrt{\frac{\frac{L}{C}}{1 - \frac{\omega^2 LC}{4}}} = \frac{\sqrt{\frac{L}{C}}}{\sqrt{1 - \left(\frac{f}{f_C}\right)^2}}$$

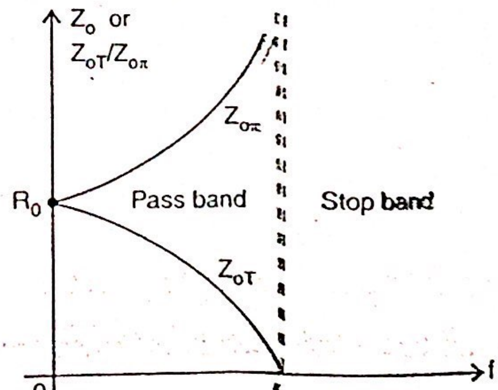

Fig. 11.12. Variation of characteristic impedance Z_o with frequency for constant K, T and p-networks Low pass filter

$$\therefore \quad Z_{o\pi} = \frac{R_o}{\sqrt{1-\left(\frac{f}{f_C}\right)^2}}$$

In the pass-band, $f < f_C$, so that $Z_{o\pi}$ is real,

In the stop-band, $f > f_C$, so that $Z_{o\pi}$ is imaginary, and if $f = f_C$, $Z_{o\pi} \to \infty$

It can be verified that, $Z_{oT} Z_{o\pi} = R_o^2$.

Variation of characteristic impedances Z_{oT} and $Z_{o\pi}$ with frequency is shown in figure 11.12.

v) Filter Component Values :

We know that $\quad Z_1 Z_2 = K^2$

$$= R_o^2 = \frac{L}{C}$$

or
$$K^2 = \frac{L}{C} \quad \text{...(A)}$$

and
$$f_C = \frac{1}{\pi\sqrt{LC}}$$

or
$$\pi^2 f_C^2 LC = 1 \quad \text{...(B)}$$

From equations (A) and (B) we have

Series inductance ;

$$L = \frac{K}{\pi f_C} \quad \text{and Shunt Capacitance } C = \frac{1}{\pi f_C K}$$

EXAMPLE 11.1 **(A) Design a low pass filter (both π and T-networks) having a cut-off frequency of 1 kHz to operate with a terminated load resistance of 200Ω.**

(B) Find the frequency at which this filter offers attenuation of 19.1dB.

Solution : (A) Given $\quad R_o = K = \sqrt{\frac{L}{C}} = 200\ \Omega$ and $f_C = 1000$ Hz

We know that, $\quad L = \frac{K}{\pi f_C} = \frac{200}{3.141 \times 1000} = 63.66$ mH

and $\quad C = \frac{1}{\pi f_C K} = \frac{1}{3.141 \times 1000 \times 200} = 1.59\ \mu$F

The π and T-networks of this filter are shown in figure 11.13 (a) and (b) respectively.

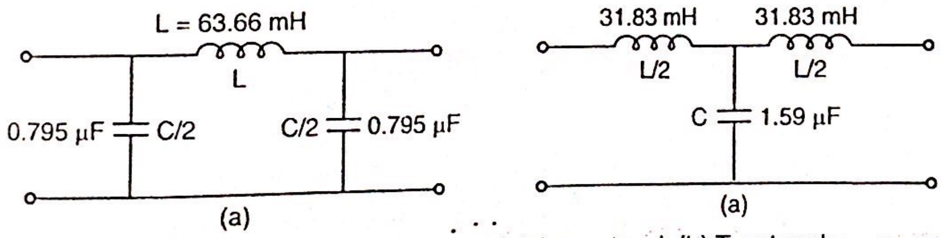

Fig. 11.13. Filter of example 11.1 (a) p-network (b) T-network

(B) Since attenuation in $dB = 8.686 \times$ attenuation in nepers

$$\therefore \text{ attenuation in nepers} = \frac{19.1}{8.686} = 2.2 \text{ nepers}$$

For Low pass filter ; $\quad \alpha = 2\cosh^{-1}\left(\frac{f}{f_C}\right)$

or $$2.2 = 2\cosh^{-1}\left(\frac{f}{1}\right)$$

or $$f = \cosh(1.1) = 1.67 \text{ kHz}.$$

11.9. CONSTANT-K HIGH PASS FILTERS

Constant-K high filter can be obtained by changing the positions of series and shunt impedances of the network shown in figure 11.9. The general configurations of constant-K high pass filter is shown in figure 11.14, where $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$.

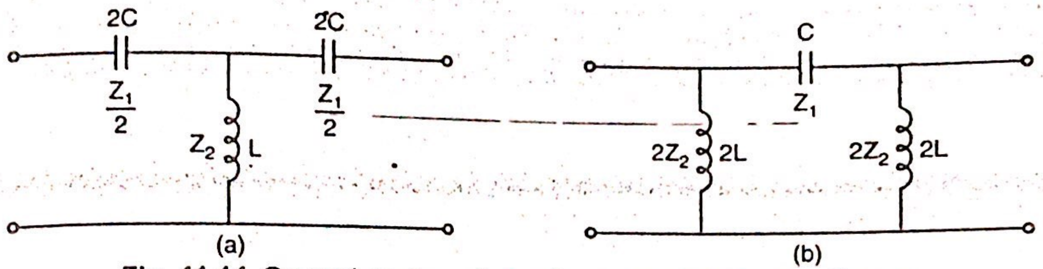

Fig. 11.14. General configurations of constant-K high pass filter as (a) T-network (b) p-network

Again it can be observed that the product of Z_1 and Z_2 is independant of frequency, and the filter design obtained will be of the constant-K type.

Thus, $$Z_1 Z_2 = \frac{1}{j\omega C}\cdot j\omega L = \frac{L}{C} = K^2$$

since, $Z_1 Z_2 = K^2 = R_o^{\,2}$ for constant-K filter, therefore, the design impedance or the load resistance

$$R_o = \sqrt{\frac{L}{C}}$$

(i) Cut-off Frequency : The cut-off frequencies are given by

$$Z_1 = 0 \quad \text{or} \quad \frac{1}{j\omega_C C} = 0 \text{ or } \omega_C \text{ and hence } f_C \to \infty$$

and $$Z_1 + 4Z_2 = 0 \quad \text{or} \quad \frac{1}{j\omega_C C} + 4.\, j\omega_C L = 0$$

or $$\omega_C = \frac{1}{2\sqrt{LC}} \quad \text{or} \quad f_C = \frac{1}{4\pi\sqrt{LC}}$$

Thus the pass band of constant-K high pass filter extends from $\frac{1}{4\pi\sqrt{LC}}$ to ∞.

The pass - band can be determined graphically. The reactances Z_1 and Z_2 will vary with frequency as shown in Figure 11.15. The cut-off frequency at the intersection of the curves Z_1 and $-4Z_2$ is indicated as f_C. The filter transmits all frequencies between f_C and ∞.

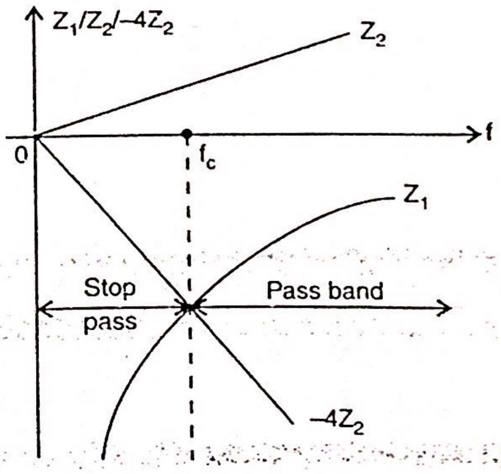

Fig. 11.15. Variation of Z_1 and $-4Z_2$ with frequency

(ii) Attenuation Constant : In pass-band, $\alpha = 0$ and in stop-band,

$$\alpha = 2\cosh^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\cosh^{-1}\sqrt{\frac{1}{4\omega^2 LC}}.$$

$$= 2\cos h^{-1}\left(\frac{f_C}{f}\right)$$

(iii) **Phase constant :** In stop-band, $\beta = \pi$ and in pass-band,

$$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_2}\right|} = 2\sin^{-1}\sqrt{\frac{1}{4\omega^2 LC}}$$

$$= 2\sin^{-1}\left(\frac{f_C}{f}\right)$$

The variation of attenuation constant α, and phase constant β with frequency is shown in figure 11.16.

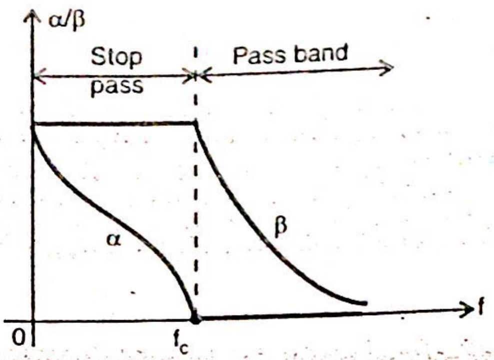

Fig. 11.16. Variation of attenuation constant a, and phase constant b with frequency

(iv) **Characteristic Impedance :** For *T*-network filter

$$Z_{oT} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1-\frac{1}{4\omega^2 LC}\right)}$$

$$\therefore \quad Z_{oT} = R_o\sqrt{1-\left(\frac{f_C}{f}\right)^2}$$

In the pass-band, $f > f_C$, so that Z_{oT} is real,

In the stop-band, $f < f_C$, so that Z_{oT} is imaginary,

and if $f = f_C$, $Z_{oT} = 0$

For π-network filter

$$Z_{o\pi} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{\frac{L}{C}}{1-\frac{1}{4\omega^2 LC}}} = \frac{\sqrt{\frac{L}{C}}}{\sqrt{1-\left(\frac{f_C}{f}\right)^2}}$$

$$\therefore \quad Z_{o\pi} = \frac{R_o}{\sqrt{1-\left(\frac{f_C}{f}\right)^2}}$$

In the pass-band, $f > f_C$, so that $Z_{o\pi}$ is real,

In the stop-band $f < f_C$, so that $Z_{o\pi}$ is imaginary,

and if $f = f_C$, $Z_{o\pi} \to \infty$

It can be verified that, $Z_{oT} \cdot Z_{o\pi} = R_o^2$

Variation of characteristic impedances Z_{oT} and $Z_{o\pi}$ with frequency is shown in figure 11.17.

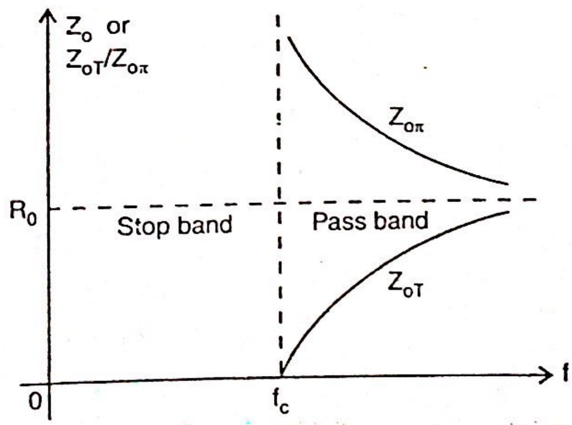

Fig. 11.17. Variation of characteristic impedance Z_o with frequency for constant-K, T and p-networks high pass filter

(v) **Filter Component Values :**

We know that $\quad Z_1 Z_2 = K^2 = R_o^2 = \frac{L}{C}$

or $\quad K^2 = \frac{L}{C} \quad \ldots(A)$

and $\quad f_C = \frac{1}{4\pi\sqrt{LC}}$

or $\quad 16\,\pi^2 f_C^2\, LC = 1 \quad \ldots(B)$

From equations (A) and (B), we have

Shunt Inductance ; $L = \frac{K}{4\pi f_C}$ and series Capacitance ; $C = \frac{1}{4\pi f_C K}$

EXAMPLE 11.2 **Design a high pass filter (both π and T- networks) having a cut-off frequency of 2 kHz with a load resistance of 300 Ω.**

Solution : Given $R_o = K = \sqrt{\frac{L}{C}} = 300\ \Omega$ and $f_C = 2000\ Hz$

We know that, $$L = \frac{K}{4\pi f_C} = \frac{300}{4\times 3.141\times 2000} = 11.93\ mH$$

and $$C = \frac{1}{4\pi K f_C} = \frac{1}{4\times 3.141\times 300\times 2000} = 0.133\ \mu F$$

The π and T- networks of this filter are shown in figure 11.18 (a) and (b) respectively.

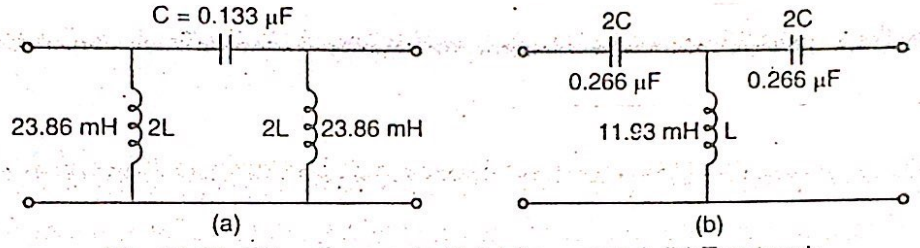

Fig. 11.18. Filter of example 11.2 (a) p -network (b) T-network

EXAMPLE 11.3 **Find the component values of π - network constant-K high-pass filter having cut-off frequency of 8 kHz and nominal charactristic impedance of 600 Ω. Hence, find its characteristic impedance and phase constant at f = 12 kHz and attenuation at f = 0.8 kHz.**

Solution : Given $R_o = K = \sqrt{\frac{L}{C}} = 600\ \Omega$ and $f_C = 8000\ Hz$

We know that,

$$L = \frac{K}{4\pi f_C} = \frac{600}{4\times 3.141\times 8000} = 5.97\ mH$$

$$C = \frac{1}{4\pi K f_C} = \frac{1}{4\times 3.141\times 600\times 8000} = 0.0166\ \mu F$$

Therefore, the values of components in π-network are

$$2L = 11.94\text{ mH}$$
$$C = 0.0166\ \mu\text{F}$$

Characteristic impedance ; at f = 12000 Hz

$$Z_{o\pi} = \frac{R_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{600}{\sqrt{1-\left(\frac{8000}{12000}\right)^2}} = 805\ \Omega$$

Phase constant; at f = 12000 Hz

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right) = 2\sin^{-1}\left(\frac{8000}{12000}\right) = 83.6° = 1.46\text{ rad}$$

Attenuation, at f = 800 Hz

$$\alpha = 2\cos h^{-1}\left(\frac{f_c}{f}\right) = 2\cos h^{-1}\left(\frac{8000}{800}\right) = 5.99\text{ nepers}$$

EXAMPLE 11.4 **Determine the nominal characteristic impedance or load resistance and the cut-off frequency for the low pass filter shown in figures 11.19 (*a*) and (*b*).**

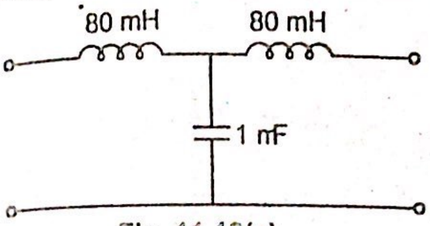

Fig. 11.19(a).

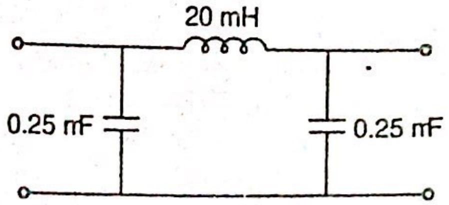

Fig. 11.19(b).

Solution : (*a*) For the *T*-network

Given $$\frac{L}{2} = 80\ mH \text{ and } C = 1\ \mu F$$

$$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{80\times2\times10^{-3}}{1\times10^{-6}}} = 400\ \Omega$$

Cut-off frequency ; $$f_c = \frac{K}{L\pi} = \frac{400}{160\times10^{-3}\times3.141} = 795.5\text{Hz}$$

or $$f_c = \frac{1}{\pi KC} = \frac{1}{3.141\times400\times1\times10^{-6}} = 795.5\text{ Hz}$$

(*b*) For the π-network

Given $$\frac{C}{2} = 0.25\ \mu F,\quad L = 20m\text{H}$$

$$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{20\times10^{-3}}{0.5\times10^{-6}}} = 200\Omega$$

Cut-off frequency ; $$f_c = \frac{K}{L\pi} = \frac{200}{20\times10^{-3}\times3.141} = 3183\text{ Hz}$$

or $$f_c = \frac{1}{\pi\ Kc} = \frac{1}{3.141\times200\times0.5\times10^{-6}} = 3183\text{ Hz}$$

EXAMPLE 11.5 **Determine the cut-off frequency for the high pass filter shown in figures 11.20 (*a*) and (*b*).**

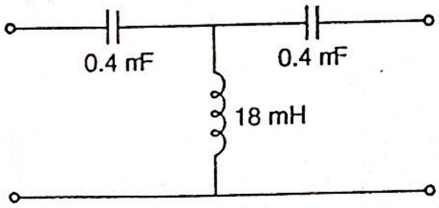

Fig. 11.20(a).

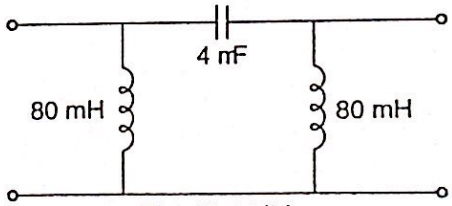

Fig. 11.20(b).

Solution : (*a*) For the *T*-network

Given $$2C = 0.4\ \mu F \quad\text{and}\quad L = 18\ mH$$

$$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{18\times10^{-3}}{0.2\times10^{-6}}} = 300\ \Omega$$

Cut-off frequency ; $$f_C = \frac{1}{4\pi\sqrt{LC}} = \frac{K}{4\pi\ L} = \frac{300}{4\times3.141\times18\times10^{-3}} = 1326\text{ Hz}$$

or $$f_C = \frac{1}{4\pi KC} = \frac{1}{4\times3.141\times300\times0.2\times10^{-6}} = 1326\text{Hz}$$

(*b*) For the π-network

Given $\quad 2L = 80$ mH and $C = 4\mu F$

Cut-off frequency ; $\quad f_C = \dfrac{1}{4\pi\sqrt{LC}} = \dfrac{1}{4 \times 3.141 \times \sqrt{40 \times 10^{-3} \times 4 \times 10^{-6}}} = \mathbf{198.94\ Hz}$

11.10. CONSTANT-*K* BAND PASS FILTERS

A general configuration of the constant- *K* band pass filter has been shown in figure 11.21. As discussed earlier, a band pass filter can be thought of as a series connection of a low pass filter and a high pass filter, in which the cut-off frequency f_{C1} of the high pass filter is less than the cut-off frequency f_{C2} of the low pass filter. Thus the overlap allow only a band of frequencies to pass.

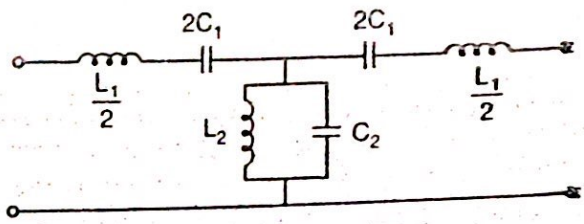

Fig. 11.21. General configuration of constant-K band pass filter

For series arm, $\quad \dfrac{Z_1}{2} = \dfrac{j\omega L_1}{2} + \dfrac{1}{j\omega \cdot 2C_1}$

or $\quad Z_1 = j\omega L_1 + \dfrac{1}{j\omega C_1} = j\left(\dfrac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$

For shunt arm, $\quad Z_2 = j\omega L_2 \Bigg\| \dfrac{1}{j\omega C_2} = \dfrac{j\omega L_2}{1 - \omega^2 L_2 C_2}$

Therefore, $\quad Z_1 \cdot Z_2 = \dfrac{L_2(1 - \omega^2 L_1 C_1)}{C_1(1 - \omega^2 L_2 C_2)}$

For the condition of equal resonant frequencies, the resonant frequency is related as

For series arm, $\quad \omega_o^2 L_1 C_1 = 1$

For shunt arm, $\quad \omega_o^2 L_2 C_2 = 1$

Therefore, $\quad L_1 C_1 = L_2 C_2 = \dfrac{1}{\omega_o^2}$

Hence, $\quad Z_1 Z_2 = \dfrac{L_2}{C_1} = \dfrac{L_1}{C_2} = R_o^2 \qquad$ (since $L_1 C_1 = L_2 C_2$)

or $\quad R_o = \sqrt{\dfrac{L_2}{C_1}} = \sqrt{\dfrac{L_1}{C_2}}$

(*i*) Cut-off Frequencies : We know that, the cut-off frequencies are given by

$$Z_1 + 4Z_2 = 0 \quad \text{or} \quad Z_1 = -4Z_2$$

Multiplying both the sides by Z_1, we have

$$Z_1^2 = -4Z_1 Z_2 = -4 R_o^2$$

Therefore $\quad Z_1 = \pm j2 R_o$

This equation defines two cut-off frequencies :

At cut-off frequency, f_{C1}, $Z_1 = -j2 R_o$

At cut-off frequency, f_{C2}, $Z_1 = +j2R_o$

Therefore, the impedance Z_1 of the series arm at f_{C1} is negative of the impedance Z_1 at f_{C2}. *i.e.*,

$$j\left[\omega L_1 - \frac{1}{\omega C_1}\right]_{f_{C1}} = -j\left[\omega L_1 - \frac{1}{\omega C_1}\right]_{f_{C2}}$$

or $$\omega_1 L_1 - \frac{1}{\omega_1 C_1} = -\left(\omega_2 L_2 - \frac{1}{\omega_2 C_1}\right)$$

or. $$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = -\left(\frac{\omega_2^2 L_1 C_1 - 1}{\omega_2 C_1}\right)$$

$$1 - \omega_1^2 L_1 C_1 = \frac{\omega_1}{\omega_2}(\omega_2^2 L_1 C_1 - 1)$$

Since $$L_1 C_1 = \frac{1}{\omega_0^2} \text{ and } \omega_o = 2\pi f_o \text{, we have}$$

$$1 - \left(\frac{f_{C1}}{f_o}\right)^2 = \frac{f_{C1}}{f_{C2}}\left[\left(\frac{f_{C2}}{f_o}\right)^2 - 1\right] \qquad (\text{Since } \omega_1 = 2\pi f_{C1} \text{ and } \omega_2 = 2\pi f_{C2})$$

or $$f_o^2 - f_{C1}^2 = \frac{f_{C1}}{f_{C2}}(f_{C2}^2 - f_o^2)$$

$$f_{C2}(f_o^2 - f_{C1}^2) = f_{C1}(f_{C2}^2 - f_o^2)$$

Rearranging $$f_o^2 (f_{C1} + f_{C2}) = f_{C1} f_{C2} (f_{C1} + f_{C2})$$

or $$f_o^2 = f_{C1} f_{C2}$$

Therefore, $$f_o = \sqrt{f_{C1} f_{C2}}$$

Hence, the resonant frequency (or the centre frequency), of the band pass filter is equal to the geometric mean of the two cut-off frequencies.

***(ii)* Filter Component Values :** The values of circuit components can be found in terms of R_o or K (design impedance or load resistance or nominal characteristic impedance) and cut-off frequencies f_{C1} and f_{C2}.

At cut-off frequency f_{C1}, we have

$$Z_1 = -j\,2\,R_o$$

$$\left(j\omega L_1 + \frac{1}{j\omega C_1}\right) = -j2\,R_o$$

$$\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1} = -2\,R_o$$

or $$1 - \left(\frac{\omega_1}{\omega_o}\right)^2 = 2R_o\,\omega_1\,C_1 \qquad (\text{since } \omega_o^2 = \frac{1}{L_1 C_1})$$

or $$1 - \left(\frac{f_{C1}}{f_o}\right)^2 = 4\pi\,R_o f_{C1}\,C_1$$

Since $$f_o^2 = f_{C1} \cdot f_{C2} \text{, we have}$$

$$1 - \frac{f_{C1}^2}{f_{C1} f_{C2}} = 4\pi\,R_o f_{C1}\,C_1$$

or $$f_{C2} - f_{C1} = 4\pi\,R_o f_{C1} f_{C2}\,C_1$$

Therfore, $$C_1 = \frac{f_{C2} - f_{C1}}{4\pi\,R_o f_{C1} f_{C2}}$$

As we know that, $L_1 C_1 = \dfrac{1}{\omega_o^2}$ or $L_1 = \dfrac{1}{4\pi^2 f_o^2 C_1} = \dfrac{1}{4\pi^2 f_{C1} f_{C2} C_1}$

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or

$$L_1 = \frac{R_o}{\pi(f_{C2} - f_{C1})}$$

The other components can then be found easily as follows :

$$R_o = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$$

or

$$R_o^2 = \frac{L_2}{C_1} = \frac{L_1}{C_2}$$

so

$$L_2 = R_o^2 C_1 \quad \text{or } L_2 = \frac{(f_{C2} - f_{C1})R_o}{4\pi f_{C1} f_{C2}}$$

and

$$C_2 = \frac{L_1}{R_o^2} \quad \text{or} \quad C_2 = \frac{1}{\pi R_o (f_{C2} - f_{C1})}$$

EXAMPLE 11.6 Design a constant-K band pass filter with cut off frequencies of 3 kHz and 7.5 kHz and nominal characteristic impedance or R_o = 900 Ω.

Solution : Given $R_o = 900\Omega$, $f_{C1} = 3$ kHz and $f_{C2} = 7.5$ kHz

Using the results of band pass filter, we have

$$L_1 = \frac{R_o}{\pi(f_{C2} - f_{C1})} = \frac{900}{3.141(7.5-3)\times 10^3} = 63.66\ mH$$

$$C_1 = \frac{f_{C2} - f_{C1}}{4\pi R_o f_{C1} f_{C2}} = \frac{(7.5-3)\times 10^3}{4\times 3.141\times 900\times 3\times 7.5\times 10^6} = 0.017\mu F$$

$$L_2 = \frac{(f_{C2} - f_{C1}) R_o}{4\pi f_{C1} f_{C2}} = \frac{(7.5-3)\times 10^3\times 900}{4\times 3.141\times 3\times 7.5\times 10^6} = 14.32\ mH$$

$$C_2 = \frac{1}{\pi R_o (f_{C2} - f_{C1})}$$

$$= \frac{1}{3.141\times 900\times (7.5-3)\times 10^3} = 0.078\ \mu F$$

Hence, $\frac{L_1}{2} = 31.83\ mH$ and $2C_1 = 0.034\mu F$

The configuration of constant- K band pass filter is shown in figure 11.22.

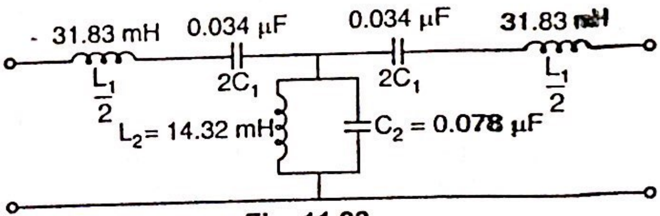

Fig. 11.22.

11.11. CONSTANT-K BAND STOP FILTERS

A general configuration of the constant-K band stop filter has been shown in figure 11.23. As discussed earlier, a band stop filter can be thought of as a parallel connection of a low pass filter and a high pass filter, in which the cut-off frequency f_{c2} of high pass filter is greater than the cut-off frequency f_{C1} of low pass filter.

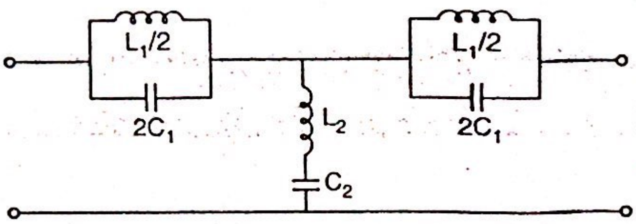

Fig. 11.23. General Configuration of constant - K band stop filter

For series arm,

$$\frac{Z_1}{2} = \frac{j\omega L_1}{2} \Bigg\| \frac{1}{j\omega \cdot 2C_1}$$

$$= \frac{\frac{j\omega L_1}{2} \cdot \frac{1}{j\omega \cdot 2C_1}}{\frac{j\omega L_1}{2} + \frac{1}{j\omega \cdot 2C_1}}$$

$$\frac{Z_1}{2} = \frac{j\omega L_1}{2(1-\omega^2 L_1 C_1)}$$

or

$$Z_1 = \frac{j\omega L_1}{1-\omega^2 L_1 C_1}$$

or

For shunt arm, $$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2} = \frac{-\omega^2 L_2 C_2 + 1}{j\omega C_2}$$

or $$Z_2 = \frac{(1-\omega^2 L_2 C_2)}{j\omega C_2} = j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right)$$

Therefore, $$Z_1 Z_2 = \frac{L_1(1-\omega^2 L_2 C_2)}{C_2(1-\omega^2 L_1 C_1)}$$

For the condition of equal resonant frequencies, the resonant frequency is related as

For series arm, $\omega_o^2 L_1 C_1 = 1$

For shunt arm, $\omega_o^2 L_2 C_2 = 1$

Therefore, $$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_o^2}$$

Hence, $$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = R_o^2 \qquad \text{(Since } L_1 C_1 = L_2 C_2\text{)}$$

or $$R_o = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}}$$

(*i*) Cut-off Frequencies : We know that, the cut-off frequencies are given by

$$Z_1 + 4Z_2 = 0 \quad \text{or} \quad Z_1 = -4Z_2$$

Multiplying both the sides by Z_1, we have

$$Z_1^2 = -4Z_1 Z_2 = -4R_o^2$$

Therefore, $$Z_1 = \pm j2R_o$$

This equation defines two same cut-off frequencies as that obtained for band pass filter. Thus, the impedance Z_1 of the series arm at frequency f_{C1} is negative of the impedance Z_1 at frequency f_{C2}.

i.e.,

$$\left[\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right]_{f=f_{C1}} = -\left[\frac{j\omega L_1}{1-\omega^2 L_1 C_1}\right]_{f=f_{C2}}$$

$$(j\omega_1 L_1)(1-\omega_2^2 L_1 C_1) = -(j\omega_2 L_1)(1-\omega_1^2 L_1 C_1)$$

$$\frac{\omega_1}{\omega_2}(1-\omega_2^2 L_1 C_1) = -(1-\omega_1^2 L_1 C_1)$$

or $$1-\omega_1^2 L_1 C_1 = \frac{\omega_1}{\omega_2}(\omega_2^2 L_1 C_1 - 1)$$

Since $$L_1 C_1 = \frac{1}{\omega_o^2} \text{ and } \omega_o = 2\pi f_o, \text{ we have}$$

$$1-\left(\frac{\omega_1}{\omega_o}\right)^2 = \frac{\omega_1}{\omega_2}\left[\left(\frac{\omega_2}{\omega_o}\right)^2 - 1\right]$$

or
$$1-\left(\frac{f_{C1}}{f_o}\right)^2 = \frac{f_{C1}}{f_{C2}}\left[\left(\frac{f_{C2}}{f_o}\right)^2 - 1\right]$$

Simplifying exactly in the same manner as in previous section, *i.e.* in case of band pass filter, we get

$$f_o = \sqrt{f_{C1} f_{C2}}$$

Hence, the resonant frequency (or the centre frequency) of the band stop filter is also equal to the geometric mean of the two cut-off frequencies.

(ii) **Filter Component Values :** As in previous section, the values of circuit components can be found in terms of R_o and cut-off frequencies f_{C1} and f_{C2}.

At cut-off frequency f_{C1}, we have

$$Z_1 = +j\,2\,R_o$$

$$\frac{j\omega_1 L_1}{1-\omega_1^2 L_1 C_1} = j\,2\,R_o$$

Since $L_1\,C_1 = \frac{1}{\omega_o^2}$, we get

$$\frac{\omega_1 L_1}{1-\left(\frac{\omega_1}{\omega_o}\right)^2} = 2R_o$$

or
$$1-\left(\frac{f_{C1}}{f_0}\right)^2 = \frac{\omega_1 L_1}{2R_0}$$

Since $f_o = \sqrt{f_{C1}\cdot f_{C2}}$, we have

$$1-\left(\frac{f_{C1}}{\sqrt{f_{C1}f_{C2}}}\right)^2 = \frac{\omega_1 L_1}{2R_o}$$

$$1-\frac{f_{C1}}{f_{C2}} = \frac{\omega_1 L_1}{2R_o}$$

or
$$f_{C2}-f_{C1} = \frac{\omega_1 L_1 f_{C2}}{2R_o} = \frac{2\pi f_{C1} L_1 f_{C2}}{2R_o}$$

Therefore,
$$L_1 = \frac{R_o(f_{C2}-f_{C1})}{\pi f_{C1} f_{C2}}$$

since
$$L_1\,C_1 = \frac{1}{\omega_o^2} \quad \text{or} \quad C_1 = \frac{1}{\omega_o^2 L_1} = \frac{1}{4\pi^2 f_o^2 L_1}$$

or
$$C_1 = \frac{1}{4\pi R_o(f_{C2}-f_{C1})}$$

The other components can then be found easily as follows :

$$R_o^2 = \frac{L_2}{C_1} = \frac{L_1}{C_2}$$

or
$$L_2 = R_o^2\,C_1 \quad \text{or} \quad L_2 = \frac{R_o}{4\pi(f_{C2}-f_{C1})}$$

and
$$C_2 = \frac{L_1}{R_o^2}$$

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$$\text{or} \qquad C_2 = \frac{f_{C2} - f_{C1}}{\pi R_o f_{C1} f_{C2}}$$

EXAMPLE 11.7 **Design a constant–K band stop filter with cut - off frequencies of 3 kHz and 7.5 kHz and nominal characteristic impedance of $R_o = 900\ \Omega$.**

Solution : Given $R_o = 900\Omega$, $f_{C1} = 3000\text{Hz}$, $f_{C2} = 7500\text{Hz}$

Using the results of band stop filter, we have

$$L_1 = \frac{R_o(f_{C2} - f_{C1})}{\pi f_{C1} \cdot f_{C2}} = \frac{900\,(7.5-3)\times 10^3}{3.141\times 3\times 7.5\times 10^6} = 57.3\text{ mH}$$

$$C_1 = \frac{1}{4\pi R_o(f_{C2} - f_{C1})}$$

$$= \frac{1}{4\times 3.141\times 900\times (7.5-3)\times 10^3} = 0.0196\mu\text{F}$$

$$L_2 = \frac{R_o}{4\pi (f_{C2} - f_{C1})} = 15.9\text{ mH}$$

$$C_2 = \frac{f_{C2} - f_{C1}}{\pi R_o f_{C1} f_{C2}} = 0.0707\ \mu\text{F}$$

Hence, $\frac{L_1}{2} = 28.6\text{ mH}$ and $2C_1 = 0.0392\ \mu\text{F}$

The configuration of constant-K band stop filter is shown in figure 11.24.

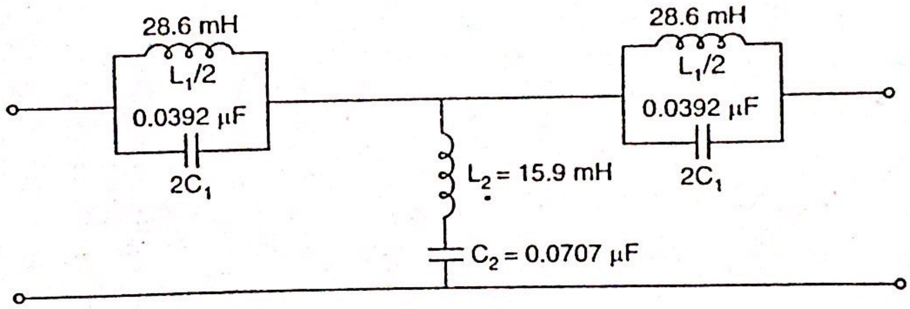

Fig. 11.24.

11.12. LIMITATIONS OF PASSIVE FILTERS

The passive filters have many limitations some of these are as follows :

(*i*) The use of inductor as a filter element is not desirable especially at low frequencies (< 1 kHz) as at these frequencies practical inductors of reasonable Q tend to become large, heavy (bulky) and costly.

(*ii*) Necessity to cascade many sections so as to give a composite filter, while cascading different sections of filter a buffer or isolation amplifier is required to prevent loading of the circuit.

(*iii*) There is a need for an external amplifier to provide suitable gain.

(*iv*) High range of Q-factor (quality factor) is not possible.

Because of above limitations the passive filters are becoming obsolete. The network with operational amplifiers are called active network. These networks have the advantages like small and compact size, high reliability, high Q-factor etc. Unfortunately it is not possible to build inductors as integrated circuits (ICs). Therefore, active filter consists of resistors capacitors and operational amplifiers (OP-AMPs).

11.13. ACTIVE FILTERS

In the previous articles, we studied design of passive filters where the inductors are the integral parts of all types of filters. The inductor creates some of the problems and because of the other advantages of active filters, passive filters remain the good academic exercise rather than a practical utility. Also we had seen that the design of passive filters (constant-K and m-derived filters) is based on the concept of image or iterative impedance, while the design of active filter is based on the filter transfer function which satisfies certain specifications. Active filters are a class of frequency selective circuits in which resistances, capacitances and operational amplifiers *i.e.*, (active elements) are used. We can list some of the advantages of active filters :

(*i*) They eliminate the need for inductors which are large, heavy (bulky), costly, non-linear (some-times), generate stray magnetic fields and may dissipate considerable power.

(*ii*) Due to absence of inductors, the active filters are readily compatible with ICs.

(*iii*) Reduction in power consumption.

(*iv*) They provide gain and excellent isolation properties, *i.e.*, high input and low output impedances.

(*v*) Due to excellent isolation property, active filters can be designed and tuned independently with minimum interaction.

(*vi*) High range of Q-factor (quality factor) is possible.

11.13.1. First Order Active Low Pass Filter

Figure 11.25 shows the circuit of an active low-pass filter. Since the input impedance of an operational amplifier is very high, therefore, the input current is almost zero and hence the currents through R_1 and R_2 are nearly equal.

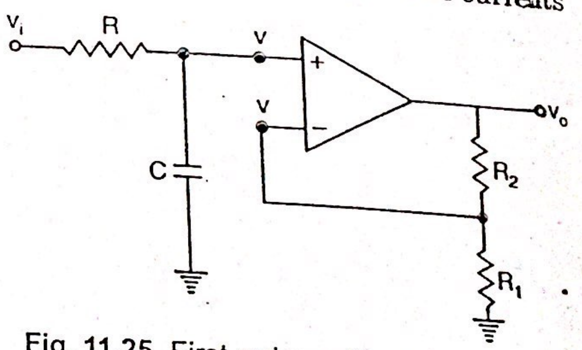

Fig. 11.25. First order active low-pass filter

Now, $$\frac{v_0 - v}{R_2} = \frac{v - 0}{R_1}$$

or $$v = \frac{v_0}{1 + \frac{R_2}{R_1}} \qquad ...(1)$$

And $$\frac{v_i - v}{R} = \frac{v}{1/sC}$$

or $$v = \frac{v_i}{1 + RCs} \qquad ...(2)$$

From equations (1) and (2),

$$\frac{v_0}{1 + \frac{R_2}{R_1}} = \frac{v_i}{1 + RCs}$$

or $$\frac{v_0}{v_i} = \frac{1 + R_2/R_1}{1 + RCs} \qquad ...(3)$$

From equation (1) the gain of the OP-AMP circuit is

$$A_0 = \frac{v_0}{v} = 1 + \frac{R_2}{R_1} \qquad ...(4)$$

Now, from equation (3) and (4),

$$H(s) = \frac{v_0}{v_i} = \frac{A_0}{1 + RCs} = \frac{A_0}{1 + (s/\omega_0)}$$

which is the transfer function of first order low pass filter, where $\omega_0 = 1/RC$ represents the cut-off frequency.

Hence for given cut-off frequency ω_0 and assumed value of C, the value of R can be evaluated. Similarly, for arbitrary gain A_0 and some assumed value of R_1, the value of R_2 can be obtained and hence first order active low-pass filter can be designed.

11.13.2. First Order Active High Pass Filter

Figure 11.26 shows the circuit of an active high-pass filter. In this case also

$$v = \frac{v_0}{1+\frac{R_2}{R_1}} \quad \ldots(1)$$

while $$\frac{v_i - v}{1/sC} = \frac{v}{R}$$

or $$v = \frac{v_i}{1+\frac{1}{RCs}} \quad \ldots(2)$$

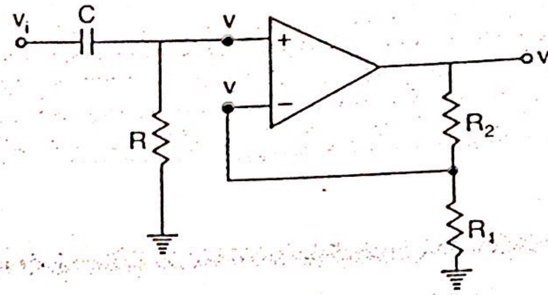

Fig. 12.26. First order active high-pass filter

From equations (1) and (2),

$$\frac{v_0}{v_i} = \frac{1+R_2/R_1}{1+1/RCs} \quad \ldots(3)$$

or $$H(s) = \frac{v_0}{v_i} = \frac{A_0}{1+1/RCs} = \frac{A_0}{1+\frac{1}{s/\omega_0}} = \frac{s/\omega_0}{1+\left(\frac{s}{\omega_0}\right)}$$

which is the transfer function of first order high-pass filter.

Alternatively : A high-pass first order filter is obtained from the low-pass first order filter by applying the transformation

$$\left.\frac{s}{\omega_0}\right|_{\text{Low pass filter}} \longrightarrow \left.\frac{1}{(s/\omega_0)}\right|_{\text{High pass filter}}$$

Thus, interchanging R and C in figure 11.25, results in a first order high pass filter of figure 11.26.

11.13.3. Active Band Pass Filter

The band-pass filter can be obtained by cascading low-pass and high pass filters as shown in figure 11.27.

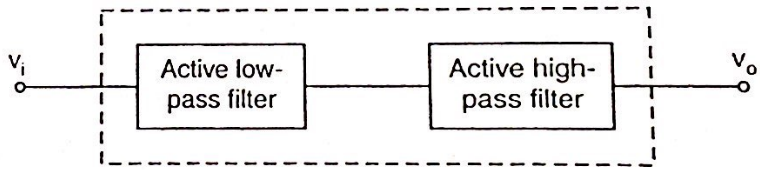

Fig. 11.27. Block diagram of an active band-pass filter

If the frequency responses of the active low-pass and high-pass filters are as shown in figure 11.28(a) and (b) respectively and $\omega_1 > \omega_2$, then the frequency response of band-pass filter is the overall frequency response of the cascaded circuit as shown in figure 11.28(c).

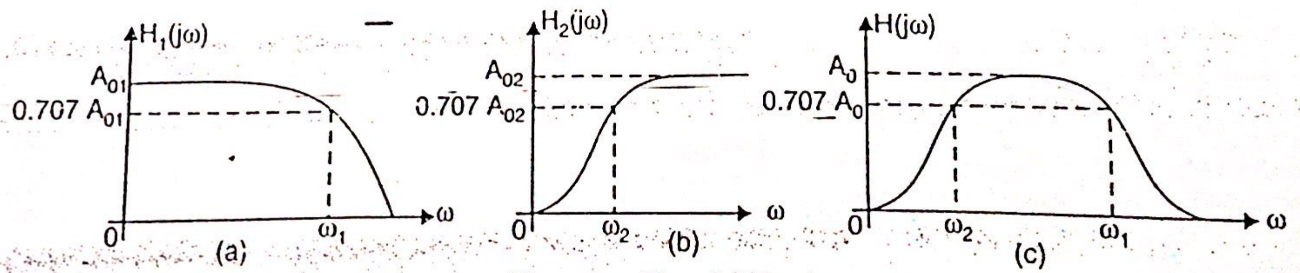

Fig. 11.28. (a) The frequency response of low-pass filter, (b) The frequency response of high-pass filter and (c) The frequency response of the cascaded circuit (band-pass filter)

11.13.4. Active Band Stop Filter

. The band-stop filter can be obtained by parallel combination of low-pass and high-pass filters as shown in figure 11.29.

If the frequency responses of the active low-pass and high-pass filters are as shown in figures 11.30(*a*) and (*b*) respectively, and $\omega_1 < \omega_2$, then figure 11.30(*c*) represents the frequency response of band-stop filter.

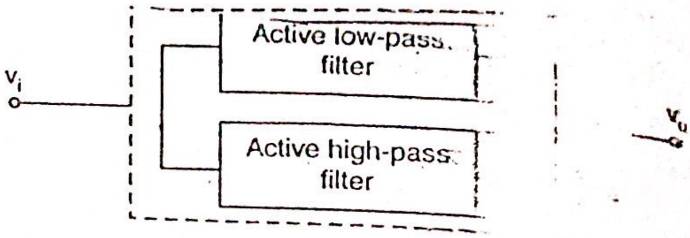

Fig. 11.29. Block diagram of a band-stop filter

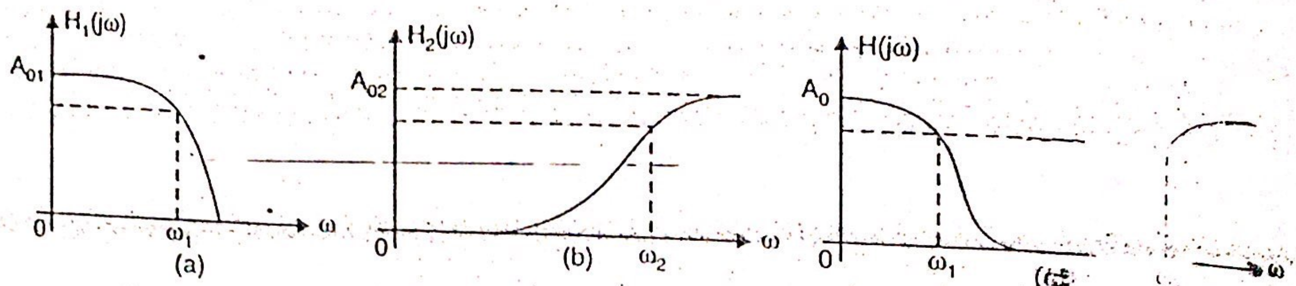

Fig. 11.30. (a) The frequency response of low-pass filter, (b) The frequency response of high-pass filter and (c) The frequency response of the parallel circuit (band-stop).

EXAMPLE 11.11 **A *T*-section low pass filter has series inductance 80 mH and shunt capacitance 0.022 μF. Determine the cut-off frequency and nominal design impedance. Also design an equivalent π-section.**

Solution : Given, $\frac{L}{2} = 80$ mH

and $C = 0.022\ \mu F$

Hence, $L = 160$ mH

Cut-off frequency ; $f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{(160\times10^{-3})(0.022\times10^{-6})}}$

or $f_c = 5.37$ kHz.

Design (or characteristic) impedance ;

$$R_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{160\times10^{-3}}{0.022\times10^{-6}}} = 2.697\ k\Omega.$$

And equivalent π-section is shown in figure 11.31.

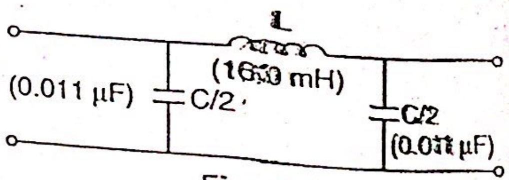

Fig. 11.31.

EXAMPLE 11.12 **A series LCR type band stop filter has $R = 1.5$ kΩ, $L = 140$ mH and $C = 300$ pf. Find the**

(*i*) Resonant frequency and

(*ii*) Bandwidth. Also find the cut-off frequencies.

(*U.P.T.U., May 2006*)

Solution : Given, $R = 1.5$ kΩ, $L = 140$ mH, $C = 300$ pf.

(*i*) Resonant frequency ; $f_r = \frac{1}{2\pi\sqrt{LC}} = 24.57$ kHz.

(ii) Bandwidth ; $BW = \dfrac{R}{2\pi L} = 1.706$ kHz.

Cut-off frequency ; $f_{c_1, c_2} = f_r \pm \dfrac{BW}{2} = 24.57 \pm 0.853$

or $f_{c_1} = 23.72$ kHz

and $f_{c_2} = 25.42$ kHz

EXAMPLE 11.13 Design a *T*-section constant K-high pass filter having cutoff frequency of 10 kHz and design impedance of 600 Ω. Find its characteristic impedance and phase constant at 25 kHz. *(U.P.T.U., 2006)*

Solution : $f_c = 10$ kHz, $R_0 = 600\ \Omega$, $R_0 = \sqrt{\dfrac{L}{C}}$ and $f_c = \dfrac{1}{4\pi\sqrt{LC}}$

From the above equations,

$$L = \frac{R_0}{4\pi f_C} \text{ and } C = \frac{1}{4\pi f_c R_0}$$

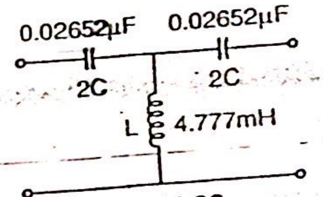

Fig. 11.32.

$$\therefore \quad L = \frac{600}{4\pi \times 10 \times 10^3} = 4.777 \text{ mH}$$

and
$$C = \frac{1}{4\pi f_c R_0} = \frac{1}{4\pi \times 10 \times 10^3 \times 600} = 0.01326\ \mu\text{F}$$

Therefore, the T-section constant K-high pass filter as shown in figure 11.32.

T-section HPF:

At $f = 25$ KHz,

$$Z_{0T} = R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 600 \sqrt{1 - \left(\frac{10}{25}\right)^2} = 545\ \Omega$$

Phase constant,
$$\beta = 2 \sin^{-1}\left(\frac{f_c}{f}\right) = 2 \sin^{-1}\left(\frac{10}{25}\right) = 47.2°$$
$$\beta = 0.82 \text{ rad.}$$

EXERCISES

11.1. Define all the parameters of a filter.

11.2. Define all the types of filters and also draw their characteristics and block-diagrams.

11.3. Define all the characteristics of filter networks.

11.4. Derive all the characteristics of constant K (*a*) low-pass, (*b*) high-pass, (*c*) band-pass, (*d*) band-stop filters.

PROBLEMS

11.1. Design a low pass filter (both *T* and π-networks) having a cut-off frequency of 2 kHz to operate with a terminated load resistance of 500 Ω.

11.2. Design a high-pass filter having a cut-off frequency of 1 kHz with a load resistance of 600 Ω.

11.3. (*a*) Design constant-*K* (both *T* and π-networks) high-pass filter having cut-off frequency $f_C = 10$ kHz and nominal characteristic impedance $R_o = 600\ \Omega$. (*b*) Find characteristic impedance and phase constant of the filter at 25 kHz. (*c*) Calculate the attenuation of the filter at 5 kHz.

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