Unit-2

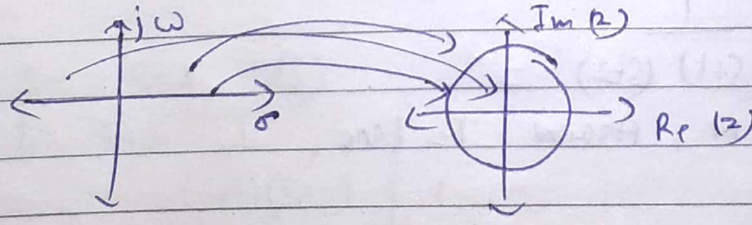
Digital Filters

IIR	FIR
eg: $h(n) = \{1, 2, 3, --\}$	eg: $h(n) = \{1, 2, 3, 4\}$
Infinite impulse response	Finite impulse response
Recursive (feedback available)	Non-recursive (no feedback available)

To design digital filter :-

1. We have to make analog filter then convert into digital.

Analog	digital
Laplace	z-transform
S-plane	z-domain



Date:
Page No.

⇒ IIR Filters

- By IIM (impulse invariance method) method.

1.) $$\frac{1}{S-P_k} = \frac{1}{1-e^{P_k T} z^{-1}} \quad \text{— (1)}$$

2.) $$\frac{S+a}{(S+a)^2+b^2} = \frac{1-e^{-at}[\cos bT]z^{-1}}{1-2e^{-at}[\cos bT]z^{-1}+e^{-2aT}z^{-2}} \quad \text{— (2)}$$

3.) $$\frac{b}{(S+a)^2+b^2} = \frac{e^{-aT}[\sin bT]}{1-2e^{-aT}[\cos bT]z^{-1}+e^{-2aT}z^{-2}} \quad \text{— (3)}$$

Q. For analog transfer function
$$H(S) = \frac{1}{(S+1)(S+2)}$$
Determine $H(z)$ using IIM, Assume $T = 1\,sec$

Sol) step = Check if $H(S)$ given or not
= Partial fraction expansion
= use formula (1), (2) or (3)

Step 1: $$H(S) = \frac{1}{(S+1)(S+2)}$$

Step 2: By using partial fraction expansion :-

Date:
Page No.

$$H(S) = \frac{A}{S+1} + \frac{B}{S+2} \quad \text{— } 2$$

$$A = H(S)\ S+1 \Big|_{S=-1}$$

$$A = \frac{1}{\cancel{(S+1)}(S+2)} \cancel{S+1} \Big|_{S=-1}$$

$$A = \frac{1}{S+2} \Big|_{S=-1}$$

$$A = \frac{1}{-1+2} = 1$$

$$\boxed{A=1}$$

$$B = S+2\ H(S)$$

$$B = \cancel{S+2}\ \frac{1}{(S+1)\cancel{(S+2)}} \Big|_{S=-2}$$

$$B = \frac{1}{-2+1}$$

$$\boxed{B=-1}$$

Date:
Page No.

Put value of A, B in eq^n (2)

$$H(s) = \frac{1}{(s+1)} - \frac{1}{(s+2)} \quad \text{— (3)}$$

Step 3:- By using IIM

$$\frac{1}{s-P_k} = \frac{1}{1-e^{P_k T} z^{-1}}$$

It can be written as:-

$$\frac{1}{s+P_k} = \frac{1}{1-e^{-P_k T} z^{-1}}$$

Applying formula on eq^n (3)

$$H(z) = \frac{1}{1-e^{-1\times1} z^{-1}} - \frac{1}{1-e^{-2\times1} z^{-1}}$$

$$H(z) = \frac{1}{1-e^{-1} z^{-1}} - \frac{1}{1-e^{-2} z^{-1}}$$

$$H(z) = \frac{(1-e^{-2} z^{-1}) - (1-e^{-1} z^{-1})}{(1-e^{-1} z^{-1})(1-e^{-2} z^{-1})}$$

$$\boxed{H(z) = \frac{0.2326 z^{-1}}{1-0.5032 z^{-1} + 0.0498 z^{-2}}}$$

(e^{-2} and e^{-1} ki value aise nikal jayegi)

Date:
Page No.

Q. For analog transfer function

$$H(s) = \frac{s+0.2}{(s+0.2)^2+9}$$

[For 5 marks ques. direct use formula.]
[For 10 marks, do partial fraction then solve]

Use IIM. Assume T= 1 sec.

Sol) Acc. to formula :-

$$\frac{s+a}{(s+a)^2+b^2} = \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}[\cos bT]z^{-1}+e^{-2aT}z^{-2}}$$

Here,

$a = 0.2$
$b = 3$

$$\frac{s+0.2}{(s+0.2)^2+3^2} = \frac{1-e^{-0.2\times1}[\cos 3\times1]z^{-1}}{1-2e^{-0.2\times1}[\cos 3\times1]z^{-1}+e^{-2\times0.2\times1}z^{-2}}$$

$$= \frac{1-(0.8187)(-0.99)z^{-1}}{1+2(0.8187)(-0.99)z^{-1}+0.6703z^{-2}}$$

$$\boxed{H(z) = \frac{1+0.8105z^{-1}}{1+1.6210z^{-1}+0.6703z^{-2}}}$$

Date:
Page No.

- By Bilinear Transformation :-

① $S = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$ or $\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

② $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$ → Digital freq.

Analog freq.

Q. Convert analog filter $H(S) = \frac{S+0.1}{(S+0.1)^2+16}$ using into IIR filter using BLT. The digital filter should have resonant freq. of $\omega_r = \frac{\pi}{2}$

Sol → Given, $H(S) = \frac{S+0.1}{(S+0.1)^2+16}$ — ①

Standard eqⁿ :-
$\frac{S+a}{(S+a)^2+\Omega_c^2}$

By formula,

$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$

$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right)$

$4 = \frac{2}{T}\tan\left(\frac{\pi/2}{2}\right)$

$\boxed{T = 0.5\ sec}$

Date:
Page No.

By BLT formula,

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$H(s) = H(z)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

Putting s in eqn (1) :-

$$H(z) = \frac{\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right) + 0.1\right]}{\left[\frac{2}{T}\left(\frac{z-1}{z+1}\right) + 0.1\right]^2 + 16}$$

$$H(z) = \frac{\left[\frac{2}{0.5}\left(\frac{z-1}{z+1}\right) + 0.1\right]}{\left[\frac{2}{0.5}\left(\frac{z-1}{z+1}\right) + 0.1\right]^2 + 16}$$

$$H(z) = \frac{\frac{(4z-4) + 0.1(z+1)}{(z+1)}}{\frac{(4z-4+0.1z+0.1)^2 + 16(z+1)^2}{(z+1)^2}}$$

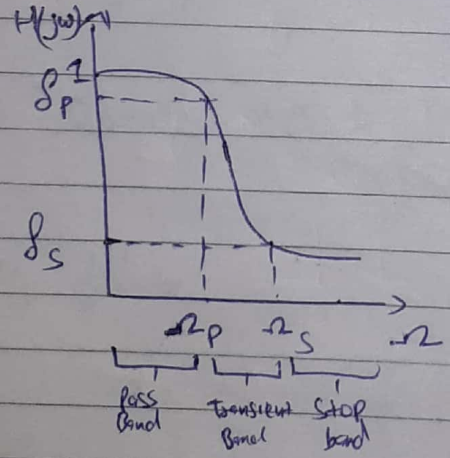
$$H(z) = \frac{4.1z^2 + 0.2z - 3.9}{z^2\,32.81 + 0.02z + 31.21}$$

$$H(z) = \frac{\cancel{z^2}\left[4.1 + 0.2z^{-1} + 3.9z^{-2}\right]}{\cancel{z^2}\left[32.81 + 0.02z^{-1} + 31.21z^{-2}\right]}$$

$$\boxed{H(z) = \frac{4.1 + 0.2z^{-1} + 3.9z^{-2}}{32.81 + 0.02z^{-1} + 31.21z^{-2}}}$$

=> Design of Butterworth Filter :-

- The desired low pass filter response



$\delta_p = \delta_1 = A_p =$ Attenuation in Passband

$\delta_s = \delta_2 = A_s =$ " " Stop band

$\Omega_p = \Omega_1 =$ Pass band edge frequency

$\Omega_s = \Omega_2 =$ Stopband " "

Date:
Page No.

→ Design steps:-

Step ① :- Calc. of analog filter's edge freq. (Ω) :-

a) For IIM :-

$$\Omega = \frac{\omega}{T}$$

b) For BLT :-

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

Step ② :- Calc. of order of filter (N)

$$N = \frac{1}{2}\frac{\log\left[\dfrac{\left(\frac{1}{\delta_s^2}-1\right)}{\left(\frac{1}{\delta_p^2}-1\right)}\right]}{\log\left[\dfrac{\Omega_s}{\Omega_p}\right]}$$

For linear

If parameters are in decibel [dB]:

$$N = \frac{1}{2}\frac{\log\left[\dfrac{10^{0.1\,\delta_s(dB)}-1}{10^{0.1\,\delta_p(dB)}-1}\right]}{\log\left[\dfrac{\Omega_s}{\Omega_p}\right]}$$

Date:
Page No.

Step 3: Calc. of $-3dB$ cut off freq. (Ω_c)

$$\Omega_c = \frac{\Omega_p}{\left[\frac{1}{\delta_p^2} - 1\right]^{\frac{1}{2N}}}$$

linear

If parameter in dB :

$$\Omega_c = \frac{\Omega_p}{\left[10^{0.1\delta_p} - 1\right]^{\frac{1}{2N}}}$$

Step 4: Calc. of Analog Transfer func. $H(S)$:-

(a) If N = even number $= 2, 4, 6 \dots$

$$H(S) = \prod_{k=1}^{N/2} \frac{B_K \Omega_c^2}{S^2 + b_K \Omega_c S + C_K \Omega_c^2}$$

sign of multiply

$\rightarrow b_k = 2\sin\left[\frac{(2K-1)\pi}{2N}\right]$ & $C_K = 1$

B_K

$$A = \prod_{K=1}^{N/2} B_K$$

B_K will be 1 everytime

Date:
Page No.

(b) If N is odd = 3, 5, 7, —

$$H(s) = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\left(\frac{N-1}{2}\right)} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right] \text{ \& } c_k = 1$$

$\rightarrow B_k$

$$A = \prod_{k=1}^{\frac{N-1}{2}} B_k$$

B_k is 1 everytime.

Step (5) :- Calc. of Digital transfer func. $H(z)$:-

(a) For IIM :-

Same 3 formula of IIM of converting $H(s) \rightarrow H(z)$

(b) For BLT

Same formula of converting $H(s) \rightarrow H(z)$

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

Date:
Page No.

Q. Design digital butterworth filter that satisfies following constraint using BLT. Assume T=1 sec.

$$\begin{bmatrix} 0.9 \le \left|H(e^{j\omega})\right| \le 1 \to A & 0 \le \omega \le \frac{\pi}{2} \\ \left|H(e^{j\omega})\right| \le 0.2 & \frac{3\pi}{4} \le \omega \le \pi \end{bmatrix}$$

Sol^n :- Given :-

$\delta_p = 0.9$; $\omega_p = \frac{\pi}{2}$

$\delta_s = 0.2$; $\omega_s = \frac{3\pi}{4}$

$A = 1$ $T = 1$ sec.

Step I :- $\Omega = \frac{2}{T} \tan\left(\frac{\omega}{2}\right)$

$$\Omega_p = \frac{2}{1} \tan\left(\frac{\pi/2}{2}\right) = 2 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{1} \tan\left(\frac{3\pi/4}{2}\right) = 4.828 \text{ rad/sec}$$

Step 2 :-

$$N = \frac{\frac{1}{2}\log\left[\frac{\frac{1}{0.2^2} - 1}{\frac{1}{0.9^2} - 1}\right]}{\log\left[\frac{4.828}{2}\right]} = 2.626$$

$N = 3$ (round off) (If 2.1 then also 3)

Date:
Page No.

Step 3:

$$\Omega_c = \frac{2}{\left[\frac{1}{0.9^2}-1\right]^{\frac{1}{2\times 3}}}$$

$$\boxed{\Omega_c = 2.5467}$$

Step 4: Since N is odd

$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right]$

$$H(s) = \frac{B_0\,\Omega_c}{s + c_0\,\Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k\,\Omega_c^2}{s^2 + b_k\,\Omega_c\,s + c_k\,\Omega_c^2}$$

Put N = 3

So, k will be from 1 to 1 only

$$b_1 = 2\sin\left[(2\times1-1)\frac{\pi}{2\times3}\right]$$

$$\boxed{b_1 = 1}$$

$\boxed{c_1 = 1}$ (always)

$\boxed{c_0 = 1}$ (If $c_1 = 1$ then $c_0 = 1$)

B_k

$$A = \prod_{k=1}^{\frac{N-1}{2}} B_k$$

Date:
Page No.

$$1 = \prod_{k=1}^{\frac{2+1}{2}} B_k$$

$$1 = \prod_{k=1}^{1} B_k$$

(b_k ki value nikalo
baaki sabki value 1 hi aati hai)

$$\boxed{B_1 = 1}$$

$$\boxed{B_0 = 1}$$

Putting all values in eqⁿ :-

$$H(S) = \frac{1 \times 2.5467}{s + 1 \times 2.5467} \prod_{k=1}^{\frac{3-1}{2}} \frac{1 \times (2.5467)^2}{s^2 + 1 \times 2.5467 s + 1 (2.5467)^2}$$

$$\boxed{H(s) = \left(\frac{2.5467}{s + 2.5467}\right)\left(\frac{6.4857}{s^2 + 2.5467 s + 6.4857}\right)}$$

Step-5: $s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$

$$= \frac{2}{1}\left(\frac{z-1}{z+1}\right)$$

$$H(S) = H(z)\Big|_{s = 2\left(\frac{z-1}{z+1}\right)}$$

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Date:
Page No.

Answer.

$$H(z) = \left(\frac{2.5467}{\left\{2\left(\frac{z-1}{z+1}\right)\right\} + 2.5467} \right) \left(\frac{6.4857}{\left\{2\left(\frac{z-1}{z+1}\right)\right\}^2 + 2.5467\left\{2\left(\frac{z-1}{z+1}\right)\right\} + 6.4857} \right)$$

You can simply further or leave it here.

Q. Design butterworth filter low pass IIR for following specification:

passband ripple :- ≤ 3.01 dB
Passband edge freq. : 0.15 Hz
Stopband attenuation : ≥ 13.97 dB
Stopband edge freq. : 0.375 Hz
Sampling rate : 1 Hz

Use IIM :-

Sol→

$\delta_p(dB) = 3.01$ dB
$F_p = 0.15$ Hz
$\delta_s(dB) = 13.97$ dB
$F_s = 0.375$ Hz
$F = 1$ Hz
$T = \frac{1}{F} = \frac{1}{1}$ sec

(sometimes in ques- directly ω_s & ω_p is given)

$\omega_p = 2\pi f_p$
$= 2\pi \frac{F_p}{F}$
$= 2\pi \frac{0.15}{1}$

$\boxed{\omega_p = 0.3\pi \text{ rad/sample}}$

$\omega_s = 2\pi f_s$
$= 2\pi \frac{F_s}{F} = 2\pi \frac{0.375}{1}$

$\boxed{\omega_s = 0.75\pi \text{ rad/sample}}$

0.942

$\frac{1}{\pi}$ 0.25

0.999986
0.99996546 5

Date:
Page No.

Step 1: $\Omega_s = \frac{\omega_s}{T} = \frac{0.75\pi}{1} = 0.75\pi$ rad/sec

$\Omega_p = \frac{\omega_p}{T} = \frac{0.3\pi}{1} = 0.3\pi$ rad/sec

Step 2:

$$N = \frac{1}{2} \frac{\log\left[\frac{10^{0.1\times 13.97}-1}{10^{0.1\times 3.01}-1}\right]}{\log\left[\frac{0.75\pi}{0.3\pi}\right]}$$

$N = 1.75$

$\boxed{N = 2}$ (round off)

Step 3:

$$\Omega_c = \frac{0.3\pi}{\left[10^{0.1\times 3.01}-1\right]^{\frac{1}{2\times 2}}}$$

$\boxed{\Omega_c = 1.0645 \text{ rad/sec}}$ (Mera $\Omega_c = 0.94$ aayi thi)

Step 4: $N = 2$ (even)

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2\times 1}\right]$$

$$B_k \Rightarrow A = \prod_{k=1}^{N/2} B_k \quad (k = 1 \text{ only})$$

Date:
Page No.

$b_1 = 2\sin\left[\left(2\times 1 - 1\right)\frac{\pi}{2\times 2}\right]$ & $C_1 = 1$

$$\boxed{b_1 = 1.414}$$

$B_1 = 1$ (Come always 1)

$$H(S) = \prod_{k=1}^{\frac{2}{2}} \frac{B_1 \Omega_c^2}{s^2 + b_1 \Omega_c S + C_1 \Omega_c^2}$$

$$= \frac{1 \times (1.0645)^2}{s^2 + (1.414)\, 1.0645 \times S + 1\,(1.0645)^2}$$

$$\boxed{H(S) = \frac{1.1332}{s^2 + 1.5054\,S + 1.1332}}$$

Simplifying :- to put into formula :-

$$H(S) = \frac{1.1332}{s^2 + 0.7527 \times 2S + 0.7527^2 - 0.7527^2 + 1.1332}$$

$$H(S) = \frac{1.1332}{(S + 0.7527)^2 + 0.5666}$$

$$H(S) = \frac{1.1332}{0.7527} \times \frac{0.7527}{(S+0.7527)^2 + 0.7527^2}$$

$$\boxed{H(S) = 1.5055 \times \frac{0.7527}{(S+0.7527)^2 + 0.7527^2}}$$

Date:
Page No.

Step 5 :-

$$H(z) = H(s) \Big|$$

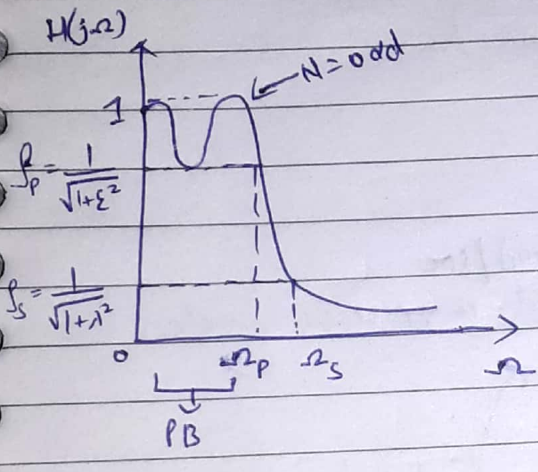
$$\frac{b}{(s+a)^2 + b^2} = \frac{e^{-aT} [\sin bT] z^{-1}}{1 - 2e^{-aT} [\cos bT] z^{-1} + e^{-2aT} z^{-2}}$$

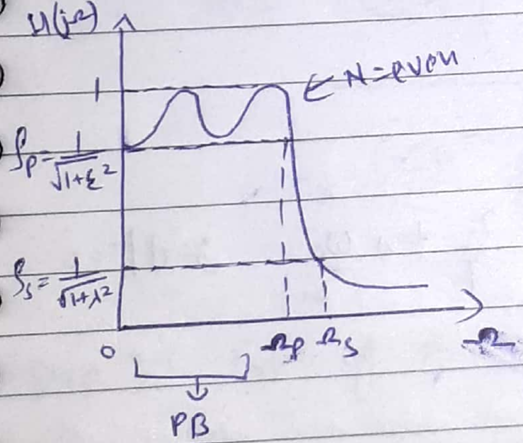
$$H(z) = 1.5055 \times \frac{e^{-0.7527 \times 1} [\sin(0.7527 \times 1)] z^{-1}}{1 - 2e^{-0.7527 \times 1} [\cos(0.7527 \times 1)] z^{-1} + e^{-2(0.7527 \times 1)} z^{-2}}$$

$$\boxed{H(z) = \frac{0.4848 z}{z^2 - 0.6877 z + 0.2219}}$$

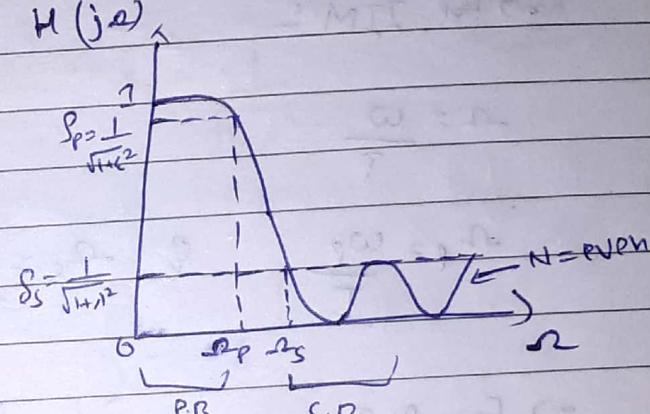
=> Design of Chebyshev Filter :-

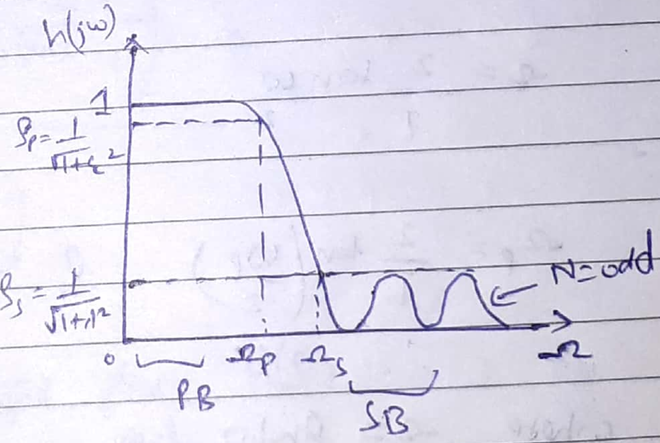
Type - I





Type - II (Inverse Chebyshev)





δ_p = P.B attenuation

δ_s = S.B "

Ω_p = P.B edge freq.

Ω_s = S.B " "

Date:
Page No.

⇒ Design steps :-

Step ① :- Calc. of analog filter edge freq (Ω)

→ For IIM :-

$$\Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{\omega_p}{T} \quad \& \quad \Omega_s = \frac{\omega_s}{T} \quad \text{rad/sec}$$

→ For BLT :-

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) \quad \& \quad \Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2} \quad \text{rad/sec}$$

where Ω = Analog freq.
ω = Digital freq.
ω_p = Pass band edge freq digital in rad/sample
ω_s = Stop band " " " " " "

Date:
Page No.

Step 2: Calc. of order of filter (N)

$$N = \frac{\cosh^{-1}\left(\frac{\lambda}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\delta_p = \frac{1}{\sqrt{1+\varepsilon^2}}$$

Linear

$$\delta_s = \frac{1}{\sqrt{1+\lambda^2}}$$

when parameters in dB :-

$$N = \frac{\cosh^{-1}\left(\frac{10^{0.1\delta_s(dB)}-1}{10^{0.1\delta_p(dB)}-1}\right)^{\frac{1}{2}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Step 3: Calc. of Analog transfer func. $H(S)$:-

1.) Axis - Minor and Major axis (a & b)

$$a = \Omega_p\left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] \quad \& \quad b = \Omega_p\left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right]$$

where $\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2}+1}$ & $\varepsilon = \sqrt{10^{0.1\delta_p(dB)}-1}$

Date:
Page No.

2. Poles of chebyshev filter:

$S_k = a\cos\phi_k + jb\sin\phi_k$; $k = 1, 2, \ldots N$

where $\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi$; $k = 1, 2, \ldots N$

3. To find denominator of $H(s)$

$H_{Den}(s) = (S - P_k)$ OR $H_{Den}(S) = (S - S_k)$

4. To find numerator of $H(s)$

(i) For $N =$ odd
put $S = 0$ in denominator polynomial
$H_{NUM}(S) = (0 - P_k)$ OR $H_{NUM}(S) = (0 - S_k)$

(ii) For $N =$ even
put $S = 0$ in the denominator polynomial and divide it by $\sqrt{1-\varepsilon^2}$

$H_{NUM}(S) = \frac{(0 - P_k)}{\sqrt{1-\varepsilon^2}}$ OR $H_{NUM}(S) = \frac{(0 - S_k)}{\sqrt{1+\varepsilon^2}}$

~~total~~

Date:
Page No.

Step 4: Calc. of $H(z)$

→ For IIM:

Use same 3 formula

→ For BLT

use same formula

Q. Design a chebhshev low pass filter whose Transfer function.

$$0.8 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \qquad 0.6\pi \le \omega \le \pi$$

Using BLT

Sol→ Given:

$\delta_p = 0.8$

$\delta_s = 0.2$

$\omega_p = 0.2\pi$

$\omega_s = 0.6\pi$

$\delta_p = \dfrac{1}{\sqrt{1+\varepsilon^2}} = 0.8$ $\boxed{\varepsilon = 0.75}$

$\delta_s = \dfrac{1}{\sqrt{1+\lambda^2}} = 0.2$ $\boxed{\lambda = 4.89}$

$\frac{2.56}{2.12}$ $\quad 2^{0.97}$ $\quad 1.52$ $\quad 23$

Date: 4.

Page No.

Step 1: By BLT

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\Omega_p = \frac{2}{1}\tan\left(\frac{0.2\pi}{2}\right) \qquad \text{(Assume } T = 1\text{ sec)}$$

$$\boxed{\Omega_p = 0.65 \text{ rad/sec}}$$

$$\Omega_s = \frac{2}{2}\tan\left(\frac{0.6\pi}{2}\right)$$

$$\boxed{\Omega_s = 2.75 \text{ rad/sec}}$$

Step 2:-

$$N = \frac{\cosh^{-1}\left(\frac{\lambda}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\cosh^{-1}\left(\frac{4.89}{0.75}\right)}{\cosh^{-1}\left(\frac{2.75}{0.65}\right)}$$

$N = 1.20$ (round off)

$$\boxed{N = 2}$$

Step 3: 1) Calc. of axis (a & b):-

$$a = \Omega_p\left[\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] \quad \& \quad b = \Omega_p\left[\frac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right]$$

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$= (0.75)^{-1} + \sqrt{(0.75)^{-2} + 1} = 3$$

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Date:
Page No.

$\boxed{\mu = 3}$

$a = 0.65\left[\frac{3^{\frac{1}{2}} - 3^{-\frac{1}{2}}}{2}\right]$ & $b = 0.65\left[\frac{3^{\frac{1}{2}} + 3^{-\frac{1}{2}}}{2}\right]$

$\boxed{a = 0.375}$ $\boxed{b = 0.750}$

2.) $S_k = a\cos\phi_k + jb\sin\phi_k$, $K = 1, 2, \ldots N$

where $\phi_k = \frac{\pi}{2} + \left(\frac{2K-1}{2N}\right)\pi$, $K = 1, 2 \ldots N$

Since $N = 2$
so $K = 1, 2$

For $K = 1$

$$\phi_1 = \frac{\pi}{2} + \left(\frac{2\times 1 - 1}{2\times 2}\right)\pi$$

$$\phi_1 = \frac{3\pi}{4}$$

For $K = 2$

$$\phi_2 = \frac{\pi}{2} + \left(\frac{2\times 2 - 1}{2\times 2}\right)\pi$$

$$\phi_2 = \frac{5\pi}{4}$$

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Date:
Page No.

$S_1 = a\cos\phi_1 + jb\sin\phi_1$

$= 0.375\cos\left(\frac{3\pi}{4}\right) + j(0.750)\sin\left(\frac{3\pi}{4}\right)$

$\boxed{S_1 = -0.265 + j0.530}$

$S_2 = 0.375\cos\left(\frac{5\pi}{4}\right) + j(0.750)\sin\left(\frac{5\pi}{4}\right)$

$\boxed{S_2 = -0.265 - j0.530}$

3.) $H_{DEN}(S) = [S-(-0.265+j0.530)][S-(-0.265-j0.530)]$

4) Numerator of $H(S)$

$N = 2$ (even)

So, $S=0$

$H_{Num}(S) = \left.\frac{0-S_k}{\sqrt{1+\varepsilon^2}}\right|_{S=0}$

$H_{Num}(S) = \frac{[S-(-0.265+j0.530)][S-(-0.265-j0.530)]}{\sqrt{1+\varepsilon^2}}$

Date:
Page No.

$$H_{Num}(s) = \frac{(0+0.265-j0.530)(0+0.265+j0.530)}{\sqrt{1+0.75^2}}$$

$$\boxed{H_{Num}(s) = 0.2809}$$

$\therefore$

$$H(s) = \frac{0.2809}{(s+0.265-j0.530)(s+0.265+j0.530)}$$

$$\boxed{H(s) = \frac{0.2809}{s^2+0.53s+0.351}}$$

Step 4 :-

By BLT:

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{0.2809}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 0.53\left(\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + 0.351}$$

$$H(z) = \frac{0.2809(1+z^{-1})^2}{4(1-z^{-1})^2 + 1.06(1-z^{-1})(1+z^{-1}) + 0.351(1+z^{-1})^2}$$

$$\boxed{H(z) = \frac{0.052+0.1038z^{-1}+0.052z^{-2}}{1-1.349z^{-1}+0.609z^{-2}}}$$

FIR FILTER

- Window
- Sampling

⇒ Design steps using window technique

Step 1: The desired freq. respone $\left(H_d\left(e^{j\omega}\right)\right)$

$$H_d\left(e^{j\omega}\right) = \begin{cases} e^{-\omega\left(\frac{N-1}{2}\right)} & \text{for } \omega < |\omega_c| \\ 0 & \text{otherwise} \end{cases}$$

If $H_d e^{(j\omega)}$ not given in ques then find by putting value of N (which must be given in ques.).

Step 2: The desired impulse respone coefficients using Inverse fourier transform $(h_d(n))$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right) e^{j\omega n}\, d\omega$$

Value of n is 0 to $N-1$

Step 3:- The impulse respone / filter coefficients of FIR filter using window function:

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Date:
Page No.

a) Rectangular window

$$w_R(n) = \begin{cases} 1 & \text{for } 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$$

b) Hamming window

$$w_{Hn}(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{M-1}\right), \quad \left(\frac{M-1}{2}\right) \ge n \ge -\left(\frac{M-1}{2}\right)$$

c) Hanning window

$$w_{Hn}(n) = 0.5 + 0.5 \cos\left(\frac{2\pi n}{M-1}\right), \quad \frac{M-1}{2} \ge n \ge -\left(\frac{M-1}{2}\right)$$

d) Blackman window

$$w_{Bm}(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{M-1}\right) + 0.08 \cos\left(\frac{4\pi n}{M-1}\right),$$

$$\frac{M-1}{2} \ge n \ge -\left(\frac{M-1}{2}\right)$$

e) Bartlett window (Triangular window)

$$w_B(n) = 1 - \frac{2|n|}{M-1}; \quad \left(\frac{M-1}{2}\right) \ge n \ge -\left(\frac{M-1}{2}\right)$$

$$\Rightarrow h[n] = h_d[n] \cdot w[n]$$

Date:
Page No.

If asked in ques:-

Step 4: Frequency response of designed LPF

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n}$$

Q. Design digital LPF FIR filter using window method

$$H(e^{j\omega}) = \begin{cases} e^{-j2\omega} & ; -\frac{\pi}{4} \le |\omega| \le \frac{\pi}{4} \\ 0 & ; \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Use rectangular window of length 5.

Sol->

Step 1: Given in ques:-

$$H(e^{j\omega}) = \begin{cases} e^{-j2\omega} & -\frac{\pi}{4} \le |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

Rectangular window

$M = N = 5$

so $n = 0, 1, 2, 3, 4$

Step 2:-

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j2\omega}\, e^{j\omega n}\, d\omega$$

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$= \sin \frac{2.46}{2\pi}$ $\frac{}{6.28}$ $-j2\omega + j\omega n$

6.785 $- \sin \pi \frac{\pi}{4}$ $j\omega(n-2)$ $j\omega n - j\omega 2$

Date:
Page No.

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{j(n-2)\omega} \, d\omega \quad —— (1)$$

$$= \frac{1}{2\pi} \left[\frac{e^{j(n-2)\omega}}{j(n-2)} \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j(n-2)\frac{\pi}{4}} - e^{-j(n-2)\pi/4}}{j(n-2)} \right]$$

$$\boxed{h_d(n) = \frac{\sin (n-2)\frac{\pi}{4}}{\pi(n-2)} \quad \text{where } n \neq 2} \quad —— (2)$$

Put $n=0$

$$h_d(0) = \frac{\sin (0-\pi)\frac{\pi}{4}}{\pi(0-2)} = 0.16$$

Put $n=1$

$$h_d(1) = \frac{\sin (1-\pi)\frac{\pi}{4}}{\pi(1-2)} = 0.225$$

Put $n=2$

from eqn (1)

$$h_d(2) = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\pi/4} e^{j(2-2)\omega} \, d\omega$$

[For $n=2$ we can't put in eqn (2) so we put in eqn (1)]

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6.094

Date:
Page No.

$$= \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\pi/4} e^{0} \, d\omega = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\pi/4} 1 \cdot d\omega = \frac{1}{2\pi} \Big[\omega \Big]_{-\frac{\pi}{4}}^{\pi/4}$$

$$= \frac{1}{2\pi} \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] = 0.25$$

Put $n=3$

$$h_d(3) = \frac{\sin(3-2)\frac{\pi}{4}}{\pi(3-2)} = 0.225$$

Put $n=4$

$$h_d(4) = \frac{\sin(4-2)\frac{\pi}{4}}{\pi(4-2)} = 0.16$$

Step 3:- Rectangular window:-

$$\omega_R(n) = \begin{cases} 1 & ; \; 0 \le n \le 4 \\ 0 & ; \; \text{otherwise} \end{cases}$$

$n=0$, $\omega(0) = 1$

$n=1$, $\omega(1) = 1$

$n=2$, $\omega(2) = 1$

$n=3$, $\omega(3) = 1$

$n=4$, $\omega(4) = 1$

Date:
Page No.

$$h[n] = h_d[n] \cdot \omega[n]$$

$n=0$, $h[0] = 0.16 \cdot 1$

$h[0] = 0.16$

$n=1$ $h[1] = 0.225 \times 1$

$h[1] = 0.225$

$n=2$ $h[2] = 0.25 \times 1$

$h[2] = 0.25$ Filter coefficients.

$n=3$ $h[3] = 0.225 \times 1$

$h[3] = 0.225$

$n=4$ $h[4] = 0.16 \times 1$

$h[4] = 0.16$

If asked in ques:-

Step 4:-

$$H(e^{j\omega}) = \sum_{n=0}^{4} h(n)\, e^{-j\omega n}$$

$$= h(0)e^{-j\omega \times 0} + h(1)e^{-j\omega} + h(2)e^{-j2\omega} + h(3)e^{-j3\omega} + h(4)e^{-j4\omega}$$

$$\boxed{H(e^{j\omega}) = (0.16 \times 1) + (0.225 \times e^{-j\omega}) + (0.25 \times e^{-j2\omega}) + (0.225 \times e^{-j3\omega}) + (0.16 \times e^{-j4\omega})}$$

It can be left here or simplify.

Date:
Page No.

⇒ Design steps frequency sampling.

Step 1: Calc. of desired freq response.
Length of filter (M) = given

Cut off freq. (ω_c) = given (rad/ sample)

$$H_d(\omega) = \begin{cases} e^{-j\omega\left(\frac{M-1}{2}\right)} & \text{for } 0 \le \omega \le \omega_c \\ 0 & \text{for } \omega_c \le \omega \le \pi \end{cases}$$

Step 2: Calc. of $H(K)$

- $\omega = \frac{2\pi K}{M}$, $K = 0, 1, 2, \ldots M-1$

- $H(k) = H_d(\omega)\Big|_{\omega = \frac{2\pi K}{M}}$

- Modify range of K.

Step 3: Calc. of $h(n)$

$$h(n) = \frac{1}{M}\left\{ H(0) + 2\sum_{k=1}^{P} Re\left[H(k)\, e^{j2\pi\frac{nK}{M}}\right]\right\}$$

Here $P = \begin{cases} \frac{M-1}{2} & \text{if } M \text{ is odd} \\ \frac{M}{2} - 1 & \text{if } M \text{ is even} \end{cases}$

Date:
Page No.

Q. Determine impulse response $h(n)$ of filter having desired freq. response.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{(M-1)\omega}{2}} & \text{for } 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le |\omega| < \pi \end{cases}$$

$M = 7$, Use freq sampling approach.

Sol→ Step 1:-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\left(\frac{7-1}{2}\right)\omega} & \text{for } 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le |\omega| < \pi \end{cases}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & \text{for } 0 \le |\omega| \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le |\omega| < \pi \end{cases} \quad -(1)$$

Step 2:-

(i) $\omega = \frac{2\pi K}{M}$, $K = 0, 1, 2, M-1$

$\omega = \frac{2\pi K}{7}$, $K = 0, 1, 2, 3, 4, 5, 6$

(ii) $H(K) = H_d(\omega)\Big|_{\omega = \frac{2\pi K}{7}}$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\left(\frac{2\pi K}{7}\right)} & \text{For } 0 \le \frac{2\pi K}{7} \le \frac{\pi}{2} \\ 0 & \text{For } \frac{\pi}{2} \le \left|\frac{2\pi K}{7}\right| < \pi \end{cases}$$

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Date:
Page No.

$$H_d(e^{j\omega k}) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le |k| \le \frac{7}{4} \\ 0 & \text{for } \frac{7}{4} \le |k| \le \frac{7}{2} \end{cases}$$

(iii) Modification of K. $\boxed{\begin{array}{l} \frac{7}{4} = 1.75 \text{ round off} \to 2 \\ \frac{7}{2} = 3.5 \text{ round off} \to 4 \end{array}}$

$$H(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le |k| \le 2 \\ 0 & \text{for } 2 \le |k| \le 4 \end{cases} \quad \text{--- (2)}$$

Step 3: Calc. of h(n)

$$h(n) = \frac{1}{M}\left\{ h(0) + 2\sum_{k=1}^{P} \mathrm{Re}\left[H(k)\, e^{j2\pi \frac{nk}{M}}\right] \right\}$$

Here $P = \frac{M-1}{2}$ bcz $M = 7$ (odd)

$$P = \frac{7-1}{2} = 3$$

From eq^n (2)

$$H(k) = e^{-j\frac{6\pi k}{7}}$$

If $k=0 \Rightarrow H(0) = e^{-j\frac{6\pi 0}{7}} = 1$

$$\boxed{H(0) = 1}$$

Date:
(from 2) Page No.

It is not 3 k2 ~~not of~~ k value is 2

$$H(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} Re\left[H(k)\, e^{\frac{j2\pi nk}{7}}\right]\right\}$$

$$h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} Re\left[e^{\frac{-j6\pi k}{7}}\, e^{\frac{j2\pi nk}{7}}\right]\right\}$$

$$= \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} Re\left[e^{\frac{-j2\pi k}{7}(3-n)}\right]\right\}$$

From euler's theorem :-

$$e^{i\theta} = \cos\theta + j\sin\theta$$

$$Re\left[e^{i\theta}\right] = \cos\theta$$

Here in ques. $\theta = \frac{2\pi k}{7}(3-n)$

$$\boxed{h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} \cos\left[\frac{2\pi k}{7}(3-n)\right]\right\} \quad ; \quad n = 0,1,2,3,4,5,6}$$

Answer

(If you want, then put value of n in find all values of h(n))

Date:
Page No.

⇒ Realization/ Structure of IIR system :-

- → Direct - form - I
- → Direct - form - II
- → Cascade Realization
- → Parallel "
- → Ladder structure

All zero system

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots}{\underbrace{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots}_{\text{All pole system}}}$$

Q. Determine direct form - I and direct form - II realization for system,

$$y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + 3\, x(n) + 36\, x(n-1) + 0.6\, x(n-2)$$

Sol:-

Sol-> Taking z transform:-

$$\boxed{\begin{aligned} y(n-k) &\xleftrightarrow{z} z^{-k}\, y(z) \\ x(n-k) &\xleftrightarrow{z} z^{-k}\, x(z) \end{aligned}}$$

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Date:
Page No.

$$Y(z) = -0.1z^{-1}Y(z) + 0.2z^{-2}Y(z) + 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

$$Y(z) + 0.1z^{-1}Y(z) - 0.2z^{-2}Y(z) = 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

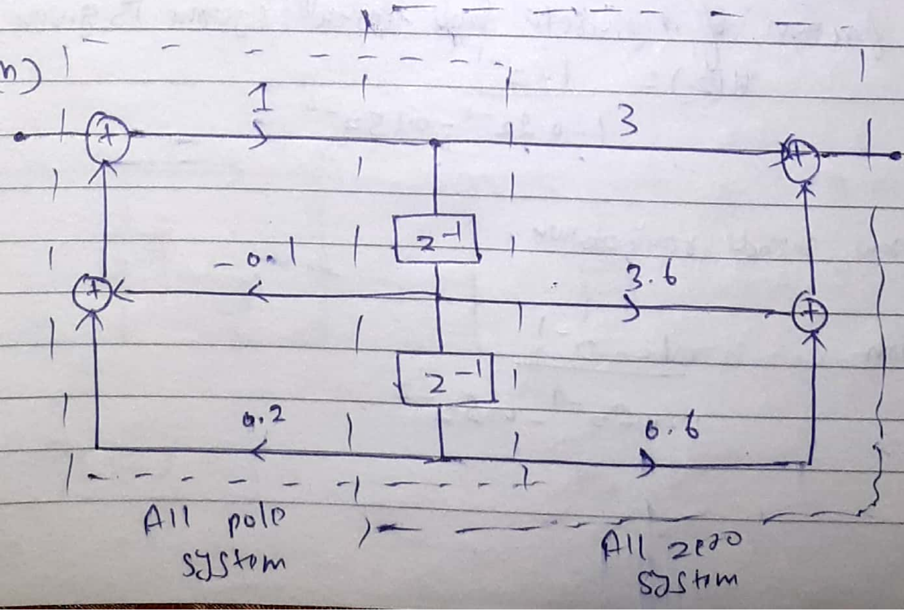
$$Y(z)\left[1 + 0.1z^{-1} - 0.2z^{-2}\right] = X(z)\left[3 + 3.6z^{-1} + 0.6z^{-2}\right]$$

$$\underbrace{\frac{Y(z)}{X(z)}}_{H(z)\ \text{(transfer func)}} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

[In ques we have to draw Direct form I first but it is easy to find direct form-I from direct form II. So first find direct form-II.]

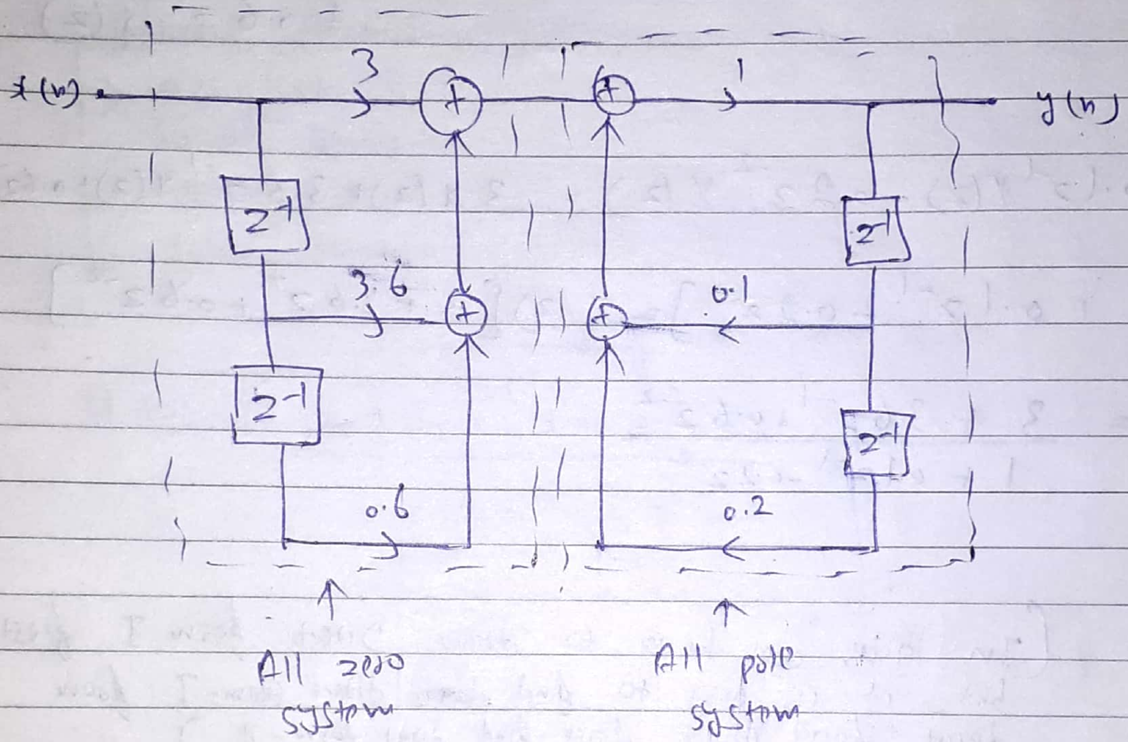
$\Rightarrow$ Direct - Form II

In pole system sign of coeff. of z^{-1} and z^{-2} are changed
z^{-1} are written on basis of max power of z^{-1}. Here it is 2. so 2 z^{-1}.



Date:
Page No.

=> Direct - form I



=> Cascade Realization :-

Q. Transfer function of discrete time causal system is given by

$$H(z) = \frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$$

Draw cascade realization.

Sol-> Given $\frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$

Date:
Page No.

$$\text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0.2 \pm \sqrt{(0.2)^2 + 4 \times 1 \times 0.15}}{2 \times 1}$$

$= 0.5, -0.3$ (It can be found in calci by solving eqn)

Thus roots, $(1 - 0.5z^{-1})(1 + 0.3z^{-1})$

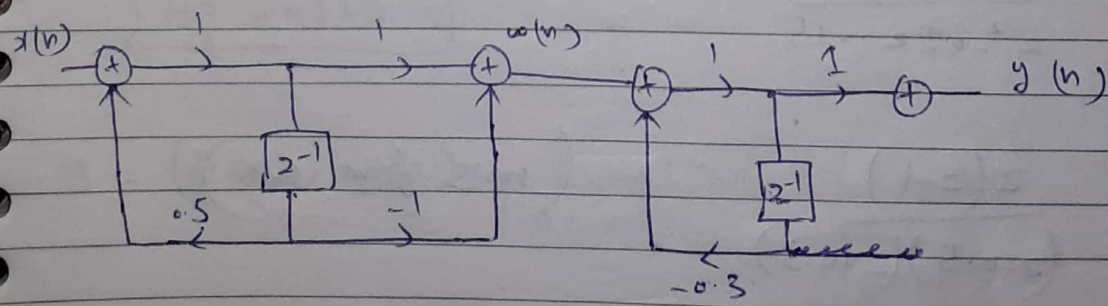
$$H(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.3z^{-1})}$$

~~H(z)~~ Assume, $H_1(z) = \dfrac{1 - z^{-1}}{1 - 0.5z^{-1}}$

$$H_2(z) = \frac{1}{(1 + 0.3z^{-1})}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(z) = \left(\frac{1 - z^{-1}}{(1 - 0.5z^{-1})}\right)\left(\frac{1}{1 + 0.3z^{-1}}\right)$$



$H_1(z)$ $H_2(z)$

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Date:
Page No.

→ Parallel Realization :-

Q. The transfer function of discrete time causal system is given by

$$H(z) = \frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$$

Draw parallel structure.

Sol → Given $H(z) = \frac{1 - z^{-1}}{1 - 0.2z^{-1} - 0.15z^{-2}}$

Converting into +ve power :-

$$H(z) = \frac{z^2(1 - z^{-1})}{z^2(1 - 0.2z^{-1} - 0.15z^{-2})}$$

$$H(z) = \frac{z^2 - z}{z^2 - 0.2z - 0.15}$$

$$H(z) = \frac{z(z-1)}{z^2 - 0.2z - 0.15}$$

$$H(z) = \frac{z(z-1)}{(z-0.5)(z+0.3)}$$ (roots from calci)

Date:
Page No.

we have to do partial fraction expansion but for that Degree in N < Degree in D but here both has same degree.
So,

$$\frac{H(z)}{z} = \frac{z-1}{(z-0.5)(z+0.3)} \quad \text{— (1)}$$

Taking partial fraction :-

$$\frac{H(z)}{z} = \frac{A}{z-0.5} + \frac{B}{z+0.3} \quad \text{— (2)}$$

$$A = (z-0.5)\frac{H(z)}{z}\Bigg|_{z=0.5}$$

This is bcz
$z-0.5=0$
$z=0.5$

From (1):

$$A = \cancel{(z-0.5)}\frac{z-1}{\cancel{(z-0.5)}(z+0.3)}$$

$$A = \frac{0.5-1}{0.5+0.3}$$

$$\boxed{A = -0.625}$$

$$B = (z+0.3)\frac{H(z)}{z}\Bigg|_{z=-0.3}$$

This is bcz
$z+0.3=0$
$z=-0.3$

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Date:
Page No.

$$B = \cancel{(z+0.3)} \frac{z-1}{(z-0.5)\cancel{(z+0.3)}} \Bigg|_{z=-0.3}$$

$$B = \frac{-0.3-1}{-0.3-0.5}$$

$$\boxed{B = 1.625}$$

Put A & B in (2):

$$\frac{H(z)}{z} = \frac{-0.625}{z-0.5} + \frac{1.625}{z+0.3} \quad \text{(8)}$$

$$H(z) = \frac{-0.625z}{z-0.5} + \frac{1.625z}{z+0.3}$$

Converting into -ve power.

$$H(z) = \frac{-0.625\cancel{z}}{\cancel{z}(1-0.5z^{-1})} + \frac{1.625\cancel{z}}{\cancel{z}(1+0.3z^{-1})}$$

$$H(z) = \frac{-0.625}{1-0.5z^{-1}} + \frac{1.625}{1+0.3z^{-1}} \quad - \quad (3)$$

Assume $H_1(z) + H_2(z)$

Date:
Page No.

Using Direct form II :-