MYNAMEIS

→ (ϕ) Null set / Empty set / Void set :- A set which contains no. elements at all is called null set.

eg. $A = \{x : x^2 + 4 = 0,\ x \text{ is real no.}\}$

→ Singleton set → a set which has only one element is called a singleton.

→ Subset → If A and B are sets such that every element of A is also an element of B then A is said to be a subset of B.

→ Super set → If A is subset of B then B is called superset of A.

$A \subseteq B$ $\quad A = \{1, 3, 4\}$

$B = \{2, 1, 3, 4, 6\}$

→ Proper subset → Any subset A is said to be a proper subset of another subset B. If A is a subset of B but there is atleast one element of B which doesnot belong to A

eg. $A = \{1, 5\}$ $\quad A \subseteq B$

$B = \{1, 5, 6\}$ $\quad$ but $A = B$.

→ Equal set - Two set A & B are said to be equal if and only if every element of A is element of B also every element of B is element of A. i.e.

i.e. $A \subseteq B$ and $B \subseteq A$

then $A = B$.

→ (*) Universal - a universal set is a set which contains all the elements of the other set including its own element (U)

eg. $A = \{$ even no. $\}$
$B = \{$ odd no. $\}$
$U = A \cup B = \{$ set of natural no. $\}$

→ Power set - the collection of all subsets of a set A is called the power set of A and it is denoted by P(A)

P(A)

let $A = \{1, 2\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$2^n = 2^2 = 4.$

$B = \{1, 2, 3\}$

→ Operations of Sets
↳ union
↳ Intersection
↳ difference

1. Union of sets - a union of 2 sets A and B is the set which consists of all those elements which are either in A or B. And it is written as

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

let $A = \{1, 2\}$
$B = \{3, 2, 4\}$
$A \cup B = \{1, 2, 3, 4\}$

Properties of union sets
1. Commutative $A \cup B = B \cup A$
2. Associative

Date: / /
Page No.

2. <u>Intersection of set</u> - The intersection of sets A and B is the set of all those elements which belongs to both A & B. And it is represented as $(A \cap B)$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A = \{1, 2\}$

$B = \{3, 2, 4\}$

$A \cap B = \{2\}$

3. <u>Difference of set</u> : The difference of the set A & B in this order is the set of element which belongs to A but not to B.

$A - B = \{x : x \in A \text{ but } x \notin B\}$

$A = \{1, 2\}$

$B = \{3, 2, 4\}$

$A - B = \{1\}$

<u>Complement of a Set</u>

Let U be the universal set and A is a subset of U then the complement of A is the set of all elements of U which are not the element of A and it is represented as $\bar{A}$ or A' or A^c

$A' = \{x : x \in U \text{ and } x \notin A\}$ or $A' = U - A$

Let $U = \{1, 2, 3, 4, 5\}$

$A = \{2, 3, 4\}$

$A' = \{1, 5\}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

n of sets A
ments which
represented

of the set A
element which

A is a
of A is the
are not the
ted as $\bar{A}$ or A'
or A^c

$' = U - A$

$- n(A \cap B)$
)

If $n(A) = 10$,
$n(B) = 20$
$n(A \cap B) = 10$
find $n(A \cup B) = ?$
$n(A \cup B) = 20$

Cartesian of set

Given the 2 non empty set $P \times Q$. The cartesian product $P \times Q$ is the set of all ordered pairs of element from P and Q.
i.e. $P \times Q = \{(p, q) : p \in P, q \in Q\}$.

If $A : \{a, b\}$
$B : \{c, d\}$
$A \times B = \{(a, b)(a, d)(b, c)(b, d)\}$.

Let $U = \{a, b, c, d, e\}$, $A = \{a, b, d\}$ and
$B = \{b, d, e\}$.

Find

1. $A \cup B = \{a, b, d, e\}$
2. $A \cap B = \{b, d\}$.
3. $B - A = \{e\}$
4. $A^c \cap B = \{e\}$
5. $B^c \cap A^c = \{c\}$.

$\{c, e\}$
$\{a, c\}$
$\{c, d\}$
A

Q If $U = \{1, 2, 3, 4, 5, 6\}$
$A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
And the bit string for the set A and B use bit string to find A' and union and intersection of sets A and B.

Date : / /
Page No.

$A = \{1, 2, 3, 4\} = A = 111100$

$B = \{3, 4, 5, 6\} = B = 001111$

$A \cup B = \{1111111\}$

$A \cap B = \{001100\}$

$A' = \{000011\}$

$U = \{1, 2, 3, 4, 5, 6\}$

let $A = \{1, 2\}$, $B = \{2, 4, 6\}$.

Calculate $A \cup B$

$B \cap C$

$A^c \cap B^c$

$A^c \cup B^c$

Relation

$A = \{$Calcutta, Patna, Lucknow, chennai$\}$

$B = \{$West bengal, Bihar, Uttar pradesh, Tamil nadu$\}$

Relation 'is a capital'

$R = \{$(Calcutta, west bengal), (Patna, Bihar), (Lucknow, uttar pradesh), (chennai, Tamil nadu)$\}$;

$P(A \times B) = |A \times B| = 16 = 2^{16}$

Defination - let A and B be two sets. A relation from A to B is a subset of a cartesian product $(A \times B)$. Suppose R is a relation from A to B then R is a set of ordered pairs (a, b) where $a \in A$ & $b \in B$. Every such ordered pair is written as aRb and read as a is related to b by 'R'

Ex. let

Find a

$A \times B =$

$R =$

$A =$

$B =$

Ex. let A

List th

(a) $a \in A$

(b) $a \in A$

(a) A

(b) R

(2). If A

D =

defin

rel

S =

Fin

R

$R \cup S$

$R \cap S$

$R - S$

7:12

6.50 7. 8:30

Ex: let $A = \{1, 2, 3\}$
$B = \{4, 5\}$
Find a relation set which follows '<' relation.
$A \times B = \{(1,4), (1,5) (2,4) (2,5) (3,4) (3,5)\}$
$a < b$.
$R = \{(1,4) (1,5) (2,4) (2,5) (3,4) (3,5)\}$

A = { Printf(), ~~cout~~ :: out, Print()}
B = { C, C++, python }
relation is 'output'

Ex: let $A = \{2, 3, 4\}$ and $B = \{3, 4, 5\}$
List the elements of each relation R defined below
(a) $a \in A$ is related to $b \in B$, that is aRb if $a < b$
(b) $a \in A$ is related to $b \in B$, that is aRb if a and b are both odd numbers

(a) $R = \{(2,3) (2,4) (2,5) (3,4) (3,5) (4,5)\}$

(b) $R = \{(3,3) (3,5)\}$.

(2). If $A = \{x, y, z\}$, $B = \{x, y, z\}$, $C = \{x, y\}$ and $D = \{y, z\}$. R is a relation from A to B defined by $R = \{(x, x), (x, y), (y, z)\}$ and S is a relation from C to D defined by $S = \{(x, y) (y, z)\}$.
$U = A \times B = \{$
$R' = U - R$
Find R', $R \cup S$, $R \cap S$ and $R - S$
$R' = \{$
$R \cup S = \{(x,x), (x,y) (y,z)\} (y,z)\}$
$R \cap S = \{(x,y)\}$
$R - S = \{(x,x), (y,z)\}$

Date: / /
Page No.

$U = A \times B$
$= \{(x, X), (x, Y)\ (x, Z)\ (y, X)\ (y, Y)\ (y, Z)\ (z, X)\ (z, Y)\ (z, Z)$

$R = \{(x, X), (x, Y), (y, Z)$

$R' = U - R$
$= \{(x, Z)\ (y, X)\ (y, Y)\ (z, X)\ (z, Y), (z, Z)$

Inverse relation

Let 'R' be any relation from set A to set B. The inverse of R denoted by 'R^{-1}' is the relation from B to A which consists of those ordered pairs which when reversed belong to 'R' i.e. $R^{-1} = \{(b, a) : (a, b) \in R\}$

Ex. Let $A = \{2, 3, 5\}$ and $B = \{6, 8, 10\}$ defined relation R from A to B as follows.
For all $(x, y) \in A \times B$, $(x, y) \in R \Leftrightarrow x/y$ (x divides y)
Write R and R^{-1}.

$A \times B = \{(2, 6), (2, 8)\ (2, 10)\ (3, 6)\ (3, 8)\ (3, 10)\ (5, 6)\ (5, 8)\ (5, 10)$

$R = \{(2, 6)\ (2, 8)\ (2, 10)\ (3, 6)\ (5, 10)\}$

$R^{-1} = \{(6, 2)\ (8, 2)\ (10, 2)\ (6, 3)\ (10, 5)\}$

Identity relation

A relation 'R' in a set 'A' is said to be identity relation generally denoted by I_A. if $I_A = \{(x, x) : x \in A\}$

Ex. let $A = \{1, 2, 3\}$ then $I_A = \{(1,1)\ (2,2)\ (3,3)\}$ is identity relation in set A.

Properties of Relations

1) Reflexive relation - a relation 'R' on a set 'A' is reflexive if aRa. for every $a \in A$ i.e. if $(a, a) \in R$, for every $a \in A$. This implies that element a of A is related to itself.

ex. if $R_1 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\}$ be a relation on $A = \{1, 2, 3\}$ then R_1 is reflexive relation: Since for every $a \in A$, $(a,a) \in R$

2) Irreflexive relation - a relation 'R' on a set 'A' is irreflexive if for every $a \in A$, $(a, a) \notin R$. In other words there is no $a \in A$ such that aRa. $R_1 = \{(1,2)\ (2,3)\ (3,1)\}$ is an irreflexive relation on $A = \{1, 2, 3\}$

3) Non-reflexive relation → a relation 'R' on a set 'A' is non-reflexive if R is neither reflexive nor Irreflexive.

Let $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2)\ (3,2), (3,3)\}$

Date : / /
Page No.:

4) Symmetric Relation :- a relation 'R' on a set 'A' is symmetric if whenever $(a,b) \in R$ then $(b,a) \in R$. i.e. $aRb \Rightarrow bRa$

$R = \{(1,1)(1,2)(2,1)(3,2)\}, (2,3)\}$

5) Asymmetric relation :- a relation R on a set A is asymmetric if whenever $(a,b) \in R$ then $(b,a) \notin R$ i.e $aRb \Rightarrow bRa$

$R = \{(1,2)(3,2)\}$

6) Antisymmetric relation :- a relation R on a set A is antisymmetric if aRb and $bRa \Rightarrow a = b$ $a, b \in A$

$R_1 = \{(1,1)(1,2)(3,2)\}$

$[(1,1)(1,2)]$

7) Transitive relation :- a relation R on a set A is transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$.

Ex. $\{(1,1)(1,2)\}$

8) Equivalence relation :- a relation R on a is a equivalence relation on A if it has the following properties :-

(i) $(a,a) \in R$ for all $a \in A$ (Reflexive)

(ii) $(a,b) \in R$ then $(b,a) \in R$ (Symmetric)

(iii) $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ (transitive

Q Let R be the relation in the natural number defined by "$(x-y)$ is divisible by 5" prove that R is an equivalence relation

$(1,2) \in R$, $(2,1) \in R$

$1, 2 \in$

Date: / /

Page No.

1. <u>Reflexive</u>

$xRx \Rightarrow x - x$ is divisible by 5

0 is divisible

R is a reflexive relation

2. <u>Symmetric</u>

xRy the yRx

$xRy \Rightarrow x - y$ divisible by 5

$yRx \Rightarrow y - x$ divisible by 5

$\forall x, y \in N$.

R is symmetric relation

3. <u>Equivalence</u>

$xRy \Rightarrow x - y$ is divisible by 5

$yRx \Rightarrow y - x$ is divisible by 5

then $xRz = x - z$ will also be divisible

<u>Equivalence class</u>

If R is equivalence relation on a set 'S' and let $x \in S$ then the set of all those element of R that are related to x is called the equivalence class of x under R and is denoted by $[x]$

$[y / y \in S$ and $yRx]$.

ex. $R = \{(1,2)(2,1)(1,1)(2,2)(3,3)(4,4)\}$

on $S = \{1, 2, 3, 4\}$

has the following equivalence class.

$[1] = \{2, 1\}$

$[2] = \{1, 2\}$

$[3] = \{3\}$

$[4] = \{4\}$

Date : / /
Page No.

Q If R is an equivalence relation in a set A then prove that R^{-1} is also an equivalence relation.

Reflexive closure of a binary relation 'R' on a set A is the smallest reflexive relation of the set A that contains 'R'. Reflexive closure is usually denoted by $R_r^r = R \cup \{(a,a) : a \in A\}$

Q Let R is a relation on the set $\{0,1,2,3\}$ containing the ordered pair $\{(0,1)\ (1,1)\ (1,2)\ (2,0)\ (2,2)$ and $(3,0)\}$ Find the reflexive closure of R

$R_r^r = \{(0,0)\ (1,1), (2,2), (3,3), (0,1)\ (1,2)\ (2,0)\ (3,0)\}$

Symmetric closure of a binary relation 'R' on a set A is smallest symmetric relation on a set A that contains R and it is denoted by $R_r^s = R \cup \{(b,a) / (a,b) \in R\}$

Date: / /
Page No.

Foreg. Let $R = \{(0,1), (1,1), (1,2) (2,0)(2,2)(3,0)\}$
$R^s = \{(0,1), (1,0) (1,1)(1,2)(2,1) (2,0) (0,2)(2,2) (3,0)(0,3)\}$

Transitive closure of a binary relation

R on a set 'A' is the smallest transitive relation on a set A that contains R. It is denoted by $R_t^t = R \cup \{(a,c) \mid (a,b) \in R \wedge (b,c) \in R\}$

Foreg. Let $A = \{0, 1, 2, 3\}$
$R = \{(1,1) (2,3) (3,1)\}$
$R_t^t = \{(1,1) (2,3) (3,1) (2,1)\}$
a, b

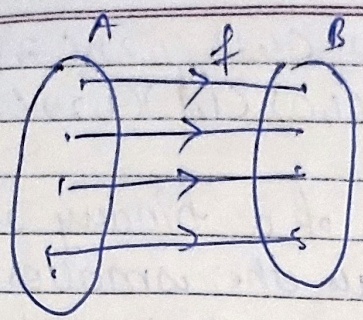
Q. Let R be the relation in the natural $N = \{1,2,\ldots\}$ defined by the open sentence '$2x + y = 10$'. Let,
$R = \{(x,y), x \in N, y \in N, 2x + y = 10\}$.
Find.
1. Domain of R
2. Range of R
3. R^{-1}

$R = \{(1,8), (2,6) (3,4), (4,2)\}$

$2x$
$2 + y = 1$
$y = 10 - 2x$

1. $\{1, 2, 3, 4\}$
2. Range $= \{8, 6, 4, 2\}$
3. $R^{-1} = \{(8,1), (6,2) (4,3) (2,4)\}$

Function: Let A & B be two non-empty sets. A function f from a to b. is a set of ordered pair $f \subseteq A \times B$ with the property that each element 'x' in 'A' there is a unique element 'y' in 'B' such that $(x,y) \in f$ and it is represented as $f: A \longrightarrow B$



Domain the set of all possible values which qualify as input to a function is known as the domain of the function.

Range the set of all output of a function is known as the range of the function

Types of function

1. One two one function → a function from a to b One two one or injective if for all elements x_1, x_2 in 'A' such that $f(x_1) = f(x_2)$ implies $x_1 = x_2$ each.

2. Many to one functions a function f from a to b is said to be many to one function if and only if two or more elements of a have same image in b

3. Into function → a function f from a to b is called into function if and only if there exist atleast one element in 'B' which is not the ~~element~~ image of any element in 'A'

3 (50)

3

4. Onto function → a function f from A → B is onto or surjective if every element of B is the image of some element in A.

(5) Bijective function → A function ~~f from~~ from A → B is said to be bijective if f is both injective or surjective is both one-one or onto.

Composition of function

Let f from A to B and g from B to C be two functions. Then the composition of 'f' ~~and~~ and 'g' denoted by ~~fog~~ gof is defined as the function gof: ~~A→~~ A → C given by $gof(x) = g(f(x))$.

→ Inverse function

A function 'f' from x to y is defined to be invertible if there exist a function $g: \cancel{x} \to \cancel{y}$ such that $gof = I_x$ & $fog = I_y$. The function g is called the inverse of f and is denoted by f^{-1}. Thus if f is invertible then 'f' must be (one-one) and onto and vice versa.

Q Let f be a mapping from R to R such that $f(x) = 2x+3$. Show that f is invertible

$f(x_1) = f(x_2)$ if $x_1 = x_2$

$2x_1 + \cancel{3} = 2x_2 + \cancel{3}$

$\cancel{2}x_1 = \cancel{2}x_2$

$\boxed{x_1 = x_2}$

$y = f(x) = 2x+3$

$y = 2x+3$

$x = \frac{y-3}{2}$

$f(x) = y$ $f(x) = \cancel{2}x \frac{y-3}{\cancel{2}} + 3$

$f(x) = y-3+3$

Date: / /
Page No.

Permutation Function

A ~~bij~~ bijection from a set 'A' to itself is called permutation of 'A'

$$f : A \to A.$$

$$p = \begin{pmatrix} a_1, & a_2, & a_3 & - & - & - \\ p(a_1), & p(a_2), & p(a_3) & \cdots & & \end{pmatrix}$$

$A = \{1, 2, 3\}$, $|3| = 6$.

$$p = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = I$$

$$p = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = I.$$

Identity permutation.

let $A = \{1, 2, 3\}$

$f : A \to A$

$$I = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$p_4 = \{(1,3)(2,1)(3,2)\}$

$$P_2 \times P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Yes sir

Date: / /
Page No.

$$p_1 \cdot p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

Cyclic permutation

let $b_1, b_2 \ldots b_r$ be r distinct of the set $A = \{a_1, a_2 \ldots a_n\}$. The permutation $p: A \to A$ defined by

$$p(b_1) = b_2$$
$$p(b_2) = b_3$$
$$\vdots$$
$$p(b_{r-1}) = b_r.$$
$$p(x) = x \text{ if } x \in A, \; x \notin \{b_1, b_2 \ldots b_r\}$$

is called a cyclic permutation of length r. or simply a cycle of length r.

→ let $A = \{1, 2, 3, 4, 5\}$ the cycle $(1, 3, 5)$ denotes the permutation. ?

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$$

→ let $A = \{1, 2, 3, 4, 5, 6\}$ compute $(4, 1, 3, 5) \circ (5, 6, 3)$

$$P = \begin{pmatrix} 4 & 1 & 3 & 5 \\ 2 & 3 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 1 & 3 & 5 \end{pmatrix}$$

Date : / /
Page No.

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 4 & 6 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 4 & 1 & 6 & 5 \end{pmatrix}$$

Disjoint

If $p_1 = (a_1, a_2, \ldots a_r)$ and $p_2 = (b_1, b_2, \ldots b_s)$ are disjoint cycle of A, then $p_1 \circ p_2 = p_2 \cdot p_1$

Note :-
two cycles of set A are said to disjoint if no. element of A appears in both cycles.

Q Let $A = \{1, 2, 3, 4, 5, 6\}$. Prove ~~that~~ the cycle $(1, 2, 5)$ and $(3, 4, 6)$ are disjoint whereas the cycle $(1, 2, 5)$ and $(2, 4, 6)$ are not.

$$p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 3 & 4 & 1 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 4 & 6 & 5 & 3 \end{pmatrix}$$

$$p_1 \cdot p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 6 & 1 & 3 \end{pmatrix}$$

$$p_2 \cdot p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 6 & 1 & 3 \end{pmatrix}$$

as $p_1 \cdot p_2 = p_2 \cdot p_1$

Date: / /
Page No.

Transposition - A cycle of length 2 is called a transposition.
Every cycle can be written as a product of transposition

Note :- 1. $(b_1, b_2 \ldots b_r) = (b_1, b_r) \cdot (b_1, b_{r-1}) \ldots (b_1, b_2)$
2. If $p = (a_i, a_j)$ is a transposition of A then, $p \cdot p = I_n$, the identity permutation of A

Example :- $(1, 2, 3, 4, 5) = (1,5) \cdot (1,4) \cdot (1,3) \cdot (1,2)$

Q Write the permutation.

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$$

as a product of Trasposition.

$p \cdot p =$

$(1, 3, 6) \cdot (2, 4, 5) \cdot (7, 8)$

$(1, 6) \cdot (1, 3) \cdot (2, 5) \cdot (2, 4) \cdot (7 8)$

Even and odd permutation

A permutation of finite set is called even if it can be written as product of an even no. of transposition and it is called odd if it can be written as a product of an odd no. of transposition.

Remark :- 1. The product of 2 even permutation is even
2. The product of 2 odd permutation is even
3. The product of an even and odd permutation is odd.

Date: / /
Page No.

Q1 $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ is even or odd permutation?

$p = (1, 2, 4, 7) \cdot (3, 5, 6)$.

$= (1, 7)(1, 4)(1, 2)(3, 6)(3, 5)$

= odd transposition.

Q2. $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 2 & 1 & 6 & 5 & 8 & 7 & 3 \end{pmatrix}$

$(1, 4, 6, 8, 3) \cdot \cancel{(2, 5, 7)}$

$(1, 3), (1, 8)(1, 6)(1, 4), \cancel{(2, 7)(2, 5)}$

Even permutation

Q Let $A = \{a, b, c, d, e, f, g\}$
Compute the product
(a) $(a, f, g) \cdot (b, c, d, e)$
(b) $(f, g) \cdot (b, c, f) \cdot (a, b, c)$

(a) $\cancel{(a, g), (a, f) \cdot (b, e)(b, d)(b, c)}$
(b) $\cancel{(f, g)(b, f)(b, c)(a, c)(a, b)}$

$\begin{pmatrix} a & b & c & d & e & f & g \\ f & b & c & d & e & g & a \end{pmatrix}$

$\begin{pmatrix} a & b & c & d & e & f & g \\ a & c & d & e & b & f & g \end{pmatrix}$

$\begin{pmatrix} a & b & c & d & e & f & g \\ f & c & d & e & b & g & a \end{pmatrix}$

Date : / /
Page No.

Tautology :- A statement that is true for all possible values of its propositional variable is called tautology. (True for all)

Contradiction :- A statement that can be either true or false depending on the truth value of its propositional variable is called contingency.

Logically Equivalent → the compound proposition p and q are called logically equivalent if

* $p \leftrightarrow q$ is tautology
* the notation $p \equiv q$

is denote that p And q are logically equivalent.

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

not or

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	T

So, it is logically equivalence

Algebra of operations :→

Date : / /
Page No.

(1) Idempotent laws :-

$p \vee p = p \qquad p \wedge p = p$

(2) Associative law :-

$(p \vee q) \vee r = p \vee (q \vee r) \qquad (p \wedge q) \wedge r = p \wedge (q \wedge r)$

(3) Commutative law :-

$p \vee q = q \vee p \qquad p \wedge q = q \wedge p$

(4) Distributive law :-

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee q)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \vee \wedge r)$

(5) Identity laws :-

$p \vee F = p \qquad p \wedge T = p$

$p \vee T = T \qquad p \wedge F = F$

(6) Complement laws :-

$p \vee \sim p = T \qquad p \wedge \sim p = F$

$\sim T = F \qquad \sim p = T$

(7) Involution laws :-

$\sim(\sim p) = p$

(8) De - morgan's law :-

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim(p \wedge q) = \sim p \vee \sim q$

Important Results :-

(1) $p \leftrightarrow q = \overline{(\sim p \vee q)}$

(2) $p \leftrightarrow q = (\sim q \rightarrow \sim p)$

(3) $p \leftrightarrow q$

(4) $\sim(p$

(5) $\sim(p$

Ques. → Show

Ques → Show

Solⁿ

p
T
T
F
F

Solⁿ

p
T
F
F

Non

↳ Disj

↳ Conj

→ A

nor

elem

(3) $p \leftrightarrow q = ((p \to q) \wedge (p \to q))$

(4) $\sim(p \to q) = (p \wedge \sim q)$

(5) $\sim(p \leftrightarrow q) = ((p \wedge \sim q) \vee (q \wedge \sim p))$

Ques. → Show that $p \wedge (q \wedge \sim p)$ is a contradiction

Ques → Show that $p \to (p \vee q)$ is a tautology

Solⁿ

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	~~T~~ F	F
F	F	T	F	F

So, it is contradiction.

Solⁿ

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

So, it is tautology

Normal form →

↳ Disjunctive normal form

↳ Conjunctive normal form

→ A logical expression is said to be disjunctive normal form if it is the ~~sum~~ sum of elementary product

eg → $p \vee (q \wedge \sim q)$

↓ OR.

Date: / /
Page No.

Predicates

The statement "x is greater than 3" by P(x) where P denotes the predicates "is greater than 3" and x is a variable. The statement P(x) is also said to be the value of the proposition function P at x. Once a value has been assigned to the variable x the statement p(x) become a proposition and has a truth value.

Logic in Proof

Argument - An argument is the process of defining a conclusion
TF → F from a set of proposition

Rule of Inference

1. Modus Ponens

$$\begin{array}{l} \overset{T}{p} \to \overset{T}{q} \\ \underline{p \to T} \\ \therefore q_T \end{array}$$

$$\left.\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{array}\right\} \text{premises or hypothesis}$$

$\therefore C \to$ conclusion

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Date : / /
Page No.

2. Hypothetical Syllogism

$p \to q$ T
$q \to r$ T
$\therefore p \to r$ T
T T

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ tautology

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

3. Modus Tollens

$p \to q$ T
$\sim q$ T
$\therefore \sim p$ T

$[(p \to q) \wedge \sim q] \to \sim p$

p	q	$\sim p$	$\sim q$	$p \to q$	
T	T	F	F	T	
T	F	F	T	F	
F	T	T	F	T	
F	F	T	T	T	✓

4. Addition

p
$\therefore p \vee q$

$p \to (p \vee q)$

Disjunctive Syllogism

$p \vee q$
$\sim q$
$\therefore p$

$[(p \vee q) \wedge \sim q] \to p$

Date : / /
Page No.

Q Show that 't' is a valid argument from the premises $p \to q$, $q \to r$, $r \to s$, $\sim s$ and $p \vee t$

~~R & RES t o P → q q → r r → s ~s p v t~~

$p \to q$, $q \to r$.

$p \to r$, $r \to s$.

& $p \to s$.

$\sim s$

$\sim p$ $\sim p$ $p \vee t$

$\sim p$

t

Q Proof the validity of the following argument

→ If I get the job and workhard then I will get promoted to a stock. If I get promoted then I will be happy. I will not happy. ∴ Either will not get the job or I will not work hard.

I get the job → p.

workhard = q

i will get promoted = r

~~promoted~~ & I will be happy = s.

I will not be happy = ~s

$p \wedge q \to r$

$q \to r$ $r \to s$

$p \to r$ $\sim s$

$r \to s$.

$p \to s$.

$\sim s$.

$\sim p$

$\therefore \sim p \vee \sim q$

$p \wedge q \to r$ premise

$r \to s$ premise

$(p \wedge q) \to s$ hypothesis

$\sim s$ premise

$\sim (p \wedge q)$ by US Modus Tollens

$\therefore \sim p \vee \sim q$

$\frac{a}{b}$

Date: / /
Page No.

$\sqrt{2}$ is rational, Let us assume

$\sqrt{2} = \frac{a}{b}$ a and b are only multiple of

$\sqrt{2}b = a$

$\cancel{(\sqrt{2}b)^2 = a^2}$

$\cancel{4b^2 = a^2}$

$2b^2 = a^2 \rightarrow a^2$ is even $\Rightarrow a$ is even

Let $a = 2k$

$2b^2 = (2k)^2$

$2b^2 = 4k^2$

$b^2 = 2k^2 \rightarrow b^2$ is even $\Rightarrow b$ is even

$\sqrt{2} = \frac{a}{b}$ Here a and b are multiple of 2

Since our assumption is contradict

Hence $\sqrt{2}$ is irrational.

Rule of Inference	Tautological form	Name
1) a. $\frac{p}{\therefore p \vee q}$ b. $\frac{q}{\therefore p \vee q}$	a. $p \rightarrow (p \vee q)$ · b. $q \rightarrow (p \vee q)$	Addition
2) a. $\frac{p \wedge q}{\therefore p}$ b. $\frac{p \wedge q}{q}$	a. $(p \wedge q) \rightarrow p$ b. $(p \wedge q) \rightarrow q$	Simplifica
3) $p \rightarrow q$, p, $\therefore q$	$[(p \rightarrow q) \wedge p] \rightarrow q$	Modus ponens
4) p, q, $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunct
5) $p \rightarrow q$, $\sim q$, $\therefore \sim p$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$	Modus tollens
6) $p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$.	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothe Syllogis

7. $p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge \sim p] \rightarrow q$	Disjunction syllogism
8. $p \rightarrow q \wedge (r \rightarrow s)$ $p \vee r$ $\therefore q \vee s$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) \rightarrow (q \vee s)$	constructive dilemma
9. $p \rightarrow q \wedge (r \rightarrow s)$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s) \rightarrow (\sim p \vee \sim r)$	Destructive dilemma

Principle of Mathematical Induction

→ Let $S(n)$ be a statement that involves positive integer $n = 1, 2, 3 \ldots \infty$ the $S(n)$ is true for all positive integer and provided that

1. $S(1)$ is true
2. $S(K+1)$ is true whenever $S(K)$ is true

Q Show that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \geq 1$$

by mathematical induction.

Sol^n For $n = 1$

$$1^2 = \frac{1(1+1)(2 \times 1 + 1)}{6} = \frac{1 \times 2 \times 3}{6} = 1$$

$$\frac{6}{6} = 1$$

Let us assume that $S(n)$ is true at $n = K$

i.e. $S(K) = 1^2 + 2^2 + 3^2 + \cdots K^2 = \frac{K(K+1)(2K+1)}{6}$

Now, we will show that $S(n)$ is true for $n = K+1$

i.e. $S(K+1) = 1^2 + 2^2 + \cdots (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

Date : / /
Page No.

$$1^2 + 2^2 + \cdots K^2 + (K+1)^2$$

$$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$$

$$= \frac{(K+1)}{6}\left[K(2K+1) + 6(K+1)\right]$$

$$= \frac{(K+1)}{6}\left[2K^2 + K + 6K + 6\right]$$

$$= \frac{(K+1)}{6}\left(2K^2 + 7K + 6\right)$$

$$= \frac{(K+1)}{6}\left(2K^2 + 4K + 3K + 6\right)$$

$$= \frac{(K+1)}{6}\left[2K(K+2) + 3(K+2)\right]$$

$$= \frac{(K+1)}{6}(K+2)(2K+3)$$

Q Show that

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

$n = 1$

$$\frac{1}{2} = \frac{1}{2}$$

$$S(K) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} \cdots + \frac{1}{K(K+1)} = \frac{K}{(K+1)}$$

$$S(K+1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} \cdots + \frac{1}{(K+1)(K+2)} = \frac{K+}{(K+2}$$

$$= \frac{K}{(K+1)}$$

Date: / /
Page No.

Principle of complete Induction/ Strong Induction.

→ For a given statement $S(n)$ involving a natural no. 'n' is true for all positive integer $n \geq n_0$ if

(1) $S(1)$ is true

(2) $S(K+1)$ is true whenever $S(K)$ is true for all $1 < n \leq K$

Q Prove that any integer $n \geq 2$ is either a prime or a product of prime.

Soln $S(n)$: is an integer which is either a prime no. or a product of prime n.

for $n = 2$

$S(2) = 2$

let $n = 3$

$S(3) = 3$

$S(4) = 4 = 2 \times 2$

$S(5) = 5$

$S(6) = 2 \times 3$.

So, $S(K)$ is true for $2 < n \leq K$.

$S(K+1)$ Now let $n = K+1$

1. If $K+1$ is a prime no.

2. If $K+1$ is not a prime no.

$K+1 = u \cdot v$.

u is lie b/w $2 < u \leq K$

v is lie b/w $2 < v \leq K$.

So, $S(K+1) = S(u) \cdot S(v)$.

$S(K+1)$ is true

Sol $S(16)$
$= (4 \times 4)$
16
4×4
15
3×5

Date: / /
Page No.

Unit - 2.

Characteristic root method.

Q Solve the recurrence relation

$a_n = a_{n+1} + 6a_{n-2}$ $\quad a_0 = 1, a_1 = 1$

$a_n - a_{n-1} - 6a_{n-2} = 0.$

let $a_n = x^n$

characteristic equation $\leftarrow x^n - x^{n-1} - 6x^{n-2} = 0$

divide by x^{n-2}

$x^2 - x^1 - 6 = 0.$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x-3)(x+2) = 0$

characteristic root $\leftarrow$ $x = 3, -2$

If we have distinct root

$a_n = b_1 x^n + b_2 x^n + \cdots + b_m x^n$

If we have repeated root

$a_n = (b_1 + nb_2 + n^2 b_3 + \cdots + n^{m-1} b_m) x^n$

$a_n = b_1 (3)^n + b_2 (-2)^n$

$a_0 = b_1 + b_2$

$1 = b_1 + b_2$ ✓

$a_1 = 3b_1 - 2b_2$

$1 = 3b_1 - 2b_2$ ✓

$3b_1 - 2b_2 = 1$

$2b_1 + 2b_2 = 2 \times 2$

$5b_1 = 3$

$b_1 = 3/5$

$b_1 + b_2 = 1$

$b_2 = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

$a_n = \frac{3}{5}(3)^n + \frac{2}{5}(-2)^n$

Date: / /
Page No.

Q $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ $a_0 = 0, a_3 = 3$
$a_5 = 10$

let $a_n = x^n$

$x^n - 3x^{n-1} + 3x^{n-2} - x^{n-3} = 0.$

divide by x^{n-3}

$x^3 - 3x^2 + 3x - 1 = 0.$

at $x = 1$

$1 - 3 + 3 - 1 = 0$

$(x-1)$ root

$(x-1)\overline{)x^3 - 3x^2 + 3x - 1}\ (x^2 - 2x + 1$

$x^3 - x^2$

$-2x^2 + 3x$

$-2x^2 + 2x$

$x - 1$

$x - 1$

x

$x^2 - 2x + 1$

$x^2 - x - x + 1$

$x(x-1) - 1(x+1)$

$(x-1)(x-1) \Rightarrow x = 1, 1, 1$

$(x-1)^3$

$x = 1, 1, 1$

$b_1 = 0, b_2 = -\frac{1}{2}$

$b_3 = \frac{1}{2}$

$a_n = b_1(1)^n + b_2(1)^n + b_3(1)^3$

$a_0 \Rightarrow b_1 + b_2 + b_3 = 0.$

$a_3 \Rightarrow b_1 + b_2 + b_3 = 3$

$a_5 \Rightarrow b_1 + b_2 + b_3 = 10.$

$a_n = (b_1 + nb_2 + n^2 b_3)$

$a_0 \Rightarrow b_1 = 0.$

$a_3 \Rightarrow b_1 + 3b_2 + 9b_3 \Rightarrow 3$

$a_5 \Rightarrow b_1 + 5b_2 + 25b_3 \Rightarrow 10.$ $b_1 = 0$

$5b_2 + 25b_3 = 10.$

$3b_2 + 9b_3 = 3$

$b_2 + 5b_3 = 2$

$b_2 + 3b_3 = 1$

$2b_3 = 1$

$b_3 = \frac{1}{2}$

$b_2 + 5b_3 = 2$

$b_2 + \frac{5}{2} = 2$

$b_2 = 2 - \frac{5}{2}$

$= -\frac{1}{2}$

Date: / /
Page No.

Non-Homogeneous Recurrence relation

$$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_n a_0 = f(n)$$

General Sol^n

$$a_n = a_n^{(h)} + a_n^{(p)}$$

Case1 when $f(n) = a$ (constant)

Q Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$ $a_0 = 1, a_1 = -1$

$$a_{n+2} - 5a_{n+1} + 6a_n = 0$$

Characteristic eq^n $x^2 - 5x + 6 = 0$

x^n

$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-3)(x-2) = 0$$
$$x = 3, 2$$
$$a_n = b_1 (2)^n + b_2 (3)^n$$

let $a_n = A$

$$A - 5A + 6A = 2$$
$$2A = 2$$
$$A = 1$$

General sol^n

$$a_n = a_n^{(h)} + a_n^{(p)}$$
$$= b_1(2)^n + b_2(3)^n + 1$$
$$a_0 = b_1 + b_2 + 1 \quad \text{—— (1)}$$
$$b_1 + b_2 = 0$$
$$a_1 = 2b_1 + 3b_2 + 1 \Rightarrow -1$$
$$2b_1 + 3b_2 = -2 \quad \text{—— (2)}$$

$$b_1 + b_2 = 0 \times 2$$
$$2b_1 + 3b_2 = -2 \times 1$$

Date : / /
Page No.

Case when $\cancel{2}b_1 + 2b_2 = 0$

$$\underline{2b_1 + 3b_2 = -2}$$

$$-b_2 = 2$$

$$b_2 = -2$$

$$b_1 + b_2 = 0$$

$$b_1 = 2$$

Case 2. When $f(x)$ is a polynomial $\underline{y_{n+2} - y_{n+1} - 2y_n = 0}$

Q2 $y_{n+2} - y_{n+1} - 2y_n = n^2$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2$

$(x-2)(x-1)$

$f(n) = n^2$

$y_n = A_0 + A_1 n + A_2 n^2$ (Second degree equation)

$$[A_0 + A_1(n+2) + A_2(n+2)^2] - [A_0 + A_1(n+1) + A_2(n+1)^2] - 2[A_0 + A_1 n + A_2 n^2]$$

$$[A_0 + A_1 n + 2A_1 + A_2 n^2 + 4A_2 + 4nA_2] - [2A_0 + 2A_1 n + 2A_2 n^2] - [A_0 + A_1 n + A_1 + n^2 A_2 + A_2 + 2nA_2]$$

$$[\cancel{A_0} + \cancel{A_1 n} + 2A_1 + \cancel{A_2 n^2} + 4A_2 + 4nA_2] - 2A_0 - 2A_1 n - 2A_2 n^2 - [\cancel{A_0} - \cancel{A_1 n} - A_1 - \cancel{n^2 A_2} - A_2 - 2nA_2]$$

$$A_1 + 3A_2 + 2nA_2 - 2A_0 - 2A_1 n - 2A_2 n^2 = n^2$$

$$(-2A_0 + A_1 + 3A_2) + (-2A_1 + 2A_2)n + (-2A_2)n^2 = n^2$$

$$-2A_0 + A_1 + 3A_2 = 0.$$

$$-2A_1 + 2A_2 = 0.$$

$$-2A_2 = 1. \Rightarrow A_2 = -1/2$$

$-2A_1 - 1 = 0$

$A_0 = 1 + 3 = 3.$

$-2A_0 + \frac{-2A_0 - 1}{2}$

$-2A_0 - \frac{1}{2} - \frac{3}{2} = 0.$

$-2A_0 - 2 = 0$

$A_0 = -1$

$-2A_1 = 1$

$A_1 = -1/2$

$b_1(-1)^n + b_2(2)^n.$

$a_n^{(p)} = -1 - \frac{1}{2}n - \frac{1}{2}n^2$

$a_n = a_n^{(h)} + a_n^{(p)}$

$a_n = b_1(-1)^n + b_2(2)^n - 1 - \frac{1}{2}n - \frac{1}{2}n^2$

Q $a_{n+2} - 2a_{n+1} + a_n = 3n+5$

$y_n = A_0 + A_1 n$:

$[A_0 + A_1(n+2)] - 2[A_0 + A_1(n+1)] + [A_0 + A_1 n] = 3n+5$

$\cancel{A_0} + \cancel{A_1 n} + \cancel{2A_1} - \cancel{2A_0} - \cancel{2A_1 n} - \cancel{2A_1} + \cancel{A_0} + \cancel{A_1 n} = 3n+5$

$[A_0 + A_1 n + 2A_1] - 2[A_0 + A_1 n + A_1] + [A_0 + A_1 n]$

$\cancel{A_0} + \cancel{A_1 n} + 2A_1 - \cancel{2A_0} - \cancel{2A_1 n} - 2A_1 + \cancel{A_0} + \cancel{A_1 n} = 3n+5$

Date : / /
Page No.

Generating Function :-

→ The generating function of a sequence $a_1, a_2 \ldots a_n$ of a real no. of infinite series.

$$G(x) = a_0 + a_1 x + a_2 x^2 + \ldots a^k x^k + \ldots$$
$$= \sum_{k=0}^{\infty} a^k x^k$$

Special Generating functions :-

(1) The generating function of a sequence 1, 1, 1 is

$$G(x) = 1 + x + x^2 + \ldots = \sum_{k=0}^{\infty} x^n$$

$$\boxed{G(x) = \frac{1}{1-x} \quad \text{(closed form)}}$$

(2) The generating function of a sequence 1, 2, 3, 4 ... is

$$G(x) = 1 + 2x + 3x^2 + 4x^3 + \ldots = \sum_{k=0}^{\infty} (k+1)x^k$$

$$\boxed{G(x) = \frac{1}{(1-x)^2} \quad |x| < 1}$$

(3) The generating function of a sequence 0, 1, 2, 3 ... is

$$G(x) = 0 + 1x^2 + 2x^2 + 3x^3 + \ldots = \sum_{k=0}^{\infty} kx^k$$

$$\boxed{G(x) = \frac{x}{(1-x)^2}}$$

$2 \times (2)^n$
$= 4^n$

(4) The generating function of the sequence $1, a, a^2, a^3 \ldots$ is

$$G(x) = 1 + ax + a^2x^2 + a^3x^3 \ldots$$
$$= \sum_{k=0}^{\infty} a^k x^k$$

$$G(x) = \frac{1}{1-ax} \quad |ax| < 1$$

Date: / /
Page No.

Q Solve the recurrence relation by using generating form

$$a_n = 3a_{n-1} + 2 \quad , a_0 = 1.$$

Multiply each eqn by x^n and summing from 1 to ∞

$$\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} 3a_{n-1} x^n + \sum_{n=1}^{\infty} 2x^n$$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$G(x) = a_0 + \sum_{n=1}^{\infty} a_n x^n$$

$$G(x) - a_0 = x \sum_{n=1}^{\infty} 3a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} 2x^n$$

$$G(x) - a_0 = 3x \sum_{n=0}^{\infty} a_n x^n + \sum_{n=1}^{\infty} 2x^n$$

$$G(x) - a_0 = 3x\, G(x) + 2 \sum_{n=1}^{\infty} x^n$$

$$G(x) - 3x\, G(x) = a_0 + 2 \sum_{n=1}^{\infty} x^n$$

$$\sum_{n=0}^{\infty} x^n = 1 + \sum_{n=1}^{\infty} x^n$$

$$G(x)[1-3x] = a_0 + 2 \sum_{n=1}^{\infty} x^n$$

$$\left[\sum_{n=p}^{\infty} x^n = \sum_{n=0}^{\infty} - 1\right]$$

$$G(x)[1-3x] = a_0 + 2\left[\frac{1}{1-x} - 1\right]$$

$$\sum_{n=1}^{\infty} x^n = \left[\frac{1}{1-x} - 1\right]$$

$$G(x) = \frac{a_0}{(1-3x)} + \frac{2}{(1-3x)}\left[\frac{1-1+x}{1-x}\right]$$

$$G(x) = \frac{1}{1-3x} + \frac{2x}{(1-3x)(1-x)}$$

$$G(x) = \frac{1}{1-3x} + \frac{2x}{(1-3x)(1-x)}$$

$$G(x) = \frac{2}{1-3x} - \frac{1}{(1-x)}$$

$$\sum_{n=0}^{\infty} a_n x^n = 2\sum_{n=0}^{\infty} 3^n x^n - \sum_{n=0}^{\infty} x^n$$

$$a_n = 2 \cdot 3^n - 1$$

Date : / /
Page No.

Divide and Conquer algorithum

Divide and conquer algorithum divide a problem into one or more instances of the same problem of smaller size and conquer the problem by using the solution of the smaller problem to find a sol^n of the original problem.

It is represented as.

$$f(n) = a f(n/b) + g(n)$$

Master theorem - Let f be an increasing function that satisfies the recurrence relation

$$f(n) = a f\left(\frac{n}{b}\right) + c n^d$$

whenever $n = b^k$ where k is a positive integer $a \geq 1$, b is an integer greater than 1. and c and d are real number with c positive and d : non-negative. then

$$f(n) = \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

Date: / /
Page No.

Expression of a Boolean function as a Canonical flow

1. From a Given truth table:

Q Find the sum of minterms and product of maxterms

Input			Output
X	Y	Z	F.
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Sol

~~Sometimes~~ Sum of minterm.

$F(x, y, z) = x'y'z + x'y'z + x'yz' + xy'z$

Product of maxterm,

$F(x, y, z) = (x + y' + z') \cdot (x' + y' + z) \cdot (x' + y' + z')$

Q 2 - Find the sum of disjunctive normal form and product of conjunctive normal form

Date : / /
Page No.

x y z	f(x, y, z).	
1 1 1	1	$x \cdot y \cdot z$
1 1 0	0	$x + y + z'$
1 0 1	0	$x + y' + z$
1 0 0	1	$x y' z'$
0 1 1	0	$x' + y + z$
0 1 0	0	$x' + y + z'$
0 0 1	0	$x' + y' + z$
0 0 0	1	$x' \cdot y' \cdot z'$

Sum of disjunctive $= x \cdot y \cdot z + x \cdot y' \cdot z' + x' \cdot y' \cdot z'$

Sum of conjunctive $= (x + y + z') \cdot (x + y' + z) \cdot (x' + y + z) \cdot (x' + y + z') \cdot (x' + y' + z)$

→ 2. ~~Alg~~ ~~Algb~~ Algebric Method

1. To obtain the minterms expression, first write the expression as a sum of product and then introduce the missing variable in each term by applying the theorem $a + a' = 1$

2. To find the maxterm expression, factor the fun^n to obtain a product of sums, introduce the missing variable in each sum term by using $a \cdot a' = 0$ and then factor again to obtain the max term.

Q Express the boolean fun^n $f(x, y, z) = x + y'z$ in a sum of minterm.

Date: / /
Page No.

$x = x(y + y')$
$= xy + xy'$ (by using dist. law)
$= xy(z + z') + xy'(z + z')$
$= xyz + xyz' + xy'z + xy'z'$

In second term
$yz = y'z(x + x')$
$xy'z + x'y'z$

$f(x, y, z) = xyz + xyz' + xy'z + xy'z' + xy'z + x'y'z$
$= xyz + xyz' + xy'z + xyz' + x'y'z$

Q Express the boolean function $f(a,b,c) = ab + a'c$ as a product of maxterm.

$f(a,b,c) = ab + a'c$
$= (ab + a') \cdot (ab + c)$

$a + (b \cdot c) = (a+b) \cdot (a+c)$

$= (a + a') \cdot (b + a') \cdot (a + c)(b + c)$
$= (b + a')(a + c) \cdot (b + c)$

$(b + a') = (b + a' + c \cdot c')$
$= (b + a' + c) \cdot (b + a' + c')$

$(a + c) = a + b \cdot b' + c$
$= (a + b + c)(a + b' + c)$

$(b + c) = a \cdot a' + b + c$
$= (a + b + c) \cdot (a' + b + c)$

$f(a,b,c) = (b + a' + c)(b + a' + c') \cdot (a + b + c) \cdot (a + b' + c)(a + b + c)(a' + b + c)$
$= (a' + b + c)(a + b + c)(b + a' + c')(a + b' + c)$

Date : / /
Page No.

Simplification of boolean expression by algebric Method.

Q Simplify $f(x,y) = xy' + xy + x'y$

$= x(y'+y) + x'y$

$= x + x'y$

$= (x+x')(x+y)$

$= (x+y)$

Q Simplify the following boolean expression using boolean algebra.

1. $(AB'C' + AB'C + ABC + ABC')(A+B)$

d. $[AB'(C'+C) + AB(C+C')](A+B)$

$(AB' + AB)(A+B)$

$A(B'+B)$

$A \cdot (A+B)$

$A \cdot A + A \cdot B$

$A + AB = A(B+1)$

$= A$

2. $P + P'QR' + (Q+R)'$

$P + P'QR' + Q'R'$

$P + R'(P'Q +$

2. $P + (P'QR') + (Q+R)'$

$P + P'QR' + Q'R'$

$P + R'(P'Q + Q')$

Date: / /
Page No.

Partially Ordered set (POSET)

A relation R on a set S is called a partial order if it is reflexive, antisymmetric and transitive. That is

1. aRa for $a \in S$ (reflexive)
2. aRb ~~for~~ and $bRa \Rightarrow a = b$ (antisymmetric)
3. aRb and $bRc \Rightarrow aRc$ (transitive)

A set S together with a partial order relation R is called a partially ordered set or a poset. It is denoted by $(S, \preceq)$

Q Show that the relation $\geq$ is a partial order on the set of integer Z.

A.

1. Reflexive
   $a \geq a$, $\geq$ is reflexive

2. Antisymmetric
   $a \geq b$ and $b \geq a \Rightarrow a = b$, $\geq$ is antisym

3. Transitive
   $a \geq b$ and $b \geq c \Rightarrow a \geq c$, $\geq$ is a transiti

Hence $(Z, \geq)$ is a poset

Q Show that the set Z^+ of all positive integer under divisibility relation forms a poset

Date: / /
Page No.

1. Reflexive

$\forall a \mid 1$ every integer is divisible by itself

$\therefore$ $\mid$ is reflexive

2. Antisymmetric

$\frac{a}{b} = c$ & $\frac{b}{a} = c$

then $a = b$

$\therefore$ $\mid$ is antisymmetric

3. transitive

$\frac{a}{b} =$ & $\frac{b}{c} =$ $\quad a = m \quad \frac{a}{b} = m$

then $a/c = e \Rightarrow a/c \quad a = bm$

$\therefore$ $\mid$ is transitive.

$\frac{b}{c} = n$

$b = nc$

$\therefore$ $(Z, \mid)$ is a poset

$\frac{a}{b} = m, \quad \frac{b}{c} = n \Rightarrow a/c$

$a = bm = , b = cn$

$\frac{a}{m} = cn$

$a = \frac{cmn}{c} = mn$

$a/b = m$

$b/c = n$

$\frac{a}{m} = cn$

$a = cmn$

$\frac{a}{b} \quad \frac{b}{c}$

$a/b, \; b/c$

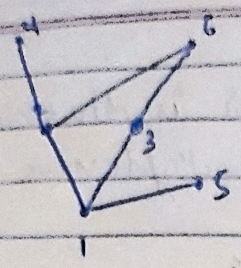
$\Rightarrow a/c$

$\frac{cmn}{c} = mn$

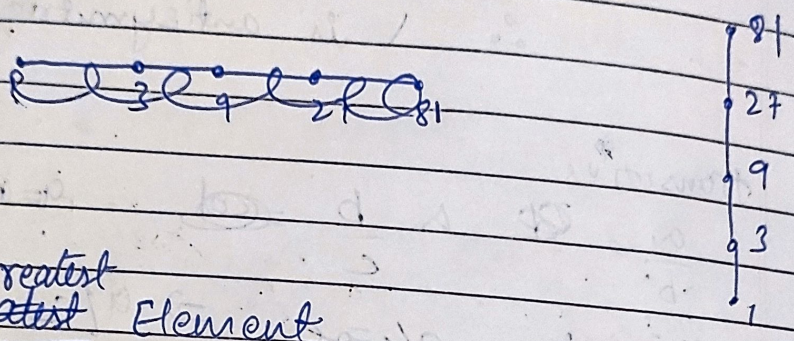
Hasse Diagram -

Q let $X = \{1, 2, 3, 4, 5, 6\}$, then $\mid$ is a partial order relation on X. Draw the hasse diagram of $(X, \mid)$

Date: / /
Page No.



Q Let $A = \{1, 3, 9, 27, 81\}$. draw the hass diagram of the poset $(A, /)$



Greatest ~~Greatest~~ Element

An element is called the greatest element if it is greater than every other element in the poset. i.e. 'a' is the greatest element of the poset $(S, \leq)$ if $b \leq a$ $\forall\, b \in S$.

Least element

An ——— least ——— less than ———
——— least element ———
if $a \leq b$ $\forall\, b \in S$

Q Find the least & greatest element in the poset $(Z^+, /)$

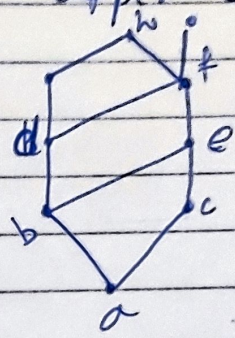
→ least element ~~to~~ of poset is 1. because 1 divide all integer
Greatest element there is no no. which divide all poset

Date: / /
Page No.

→ Lower Bound - An element that is lower than or equal to all the element in a subset 'A' of the poset $(S, \leq)$ is called a lower bound of 'A'.

→ Greatest lower bound - The element 'y' is called greatest lower bound

Q. Find the lower and upper bounds of the subset $\{a, b, c\}$ $\{i, h\}$ and $\{a, c, d, f\}$ in the poset with hasse diagram shown in fig. Also find the greatest lower bound and least upper bound.



① Lower bound

$\{a\ b\ c\} = a$

$\{i, h\} = f, e, c, a$

$\{a, c, d, f\} = a$

upper bound.

$\{a\ b\ c\} = d, e\ f\ h\ i$

$\{a, c, d, f\} = h, i.$

$\{i, h\} = \phi$

greatest lower bou

$\{i\ h\} = f$

$\{a, b, c\} = a$

$\{a, c, d, f\} = a$

least upper bound

$\{a, b, c\} = e$

$\{a, c, d, f\} = \phi$

$\{i, h\} = \phi$