AREA AND AVERAGE VALUE OF A SIGNAL.

1. AREA OF SIGNAL.

$$\text{area of signal} = \int_{-\infty}^{\infty} x(t)\,dt$$

$$\text{area of signal over } (t_1, t_2) = \int_{t_1}^{t_2} x(t)\,dt$$

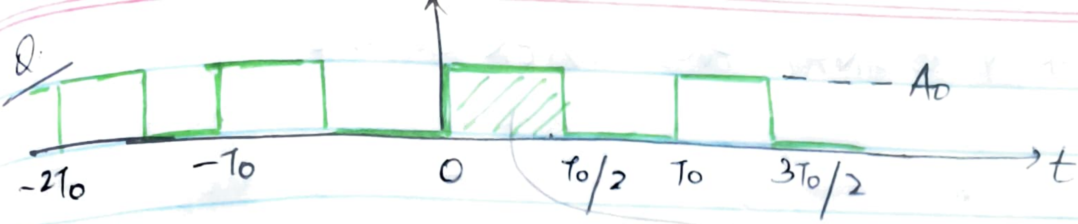
2. AVERAGE VALUE OF SIGNAL.

average value of signal

$$= \begin{cases} \dfrac{1}{T_0} \displaystyle\int_{T_0} x(t)\,dt, & \text{for periodic s/g} \\ \lim_{T_0 \to \infty} \dfrac{1}{T_0} \displaystyle\int_{-T_0/2 \,\to\, \text{fixed}}^{T_0/2 \,\to\, \text{fixed}} x(t)\,dt, & \text{for non-periodic s/g} \end{cases}$$

$$\boxed{\text{Average value of periodic s/g} = \frac{\text{area over } 'T_0'}{T_0}}$$

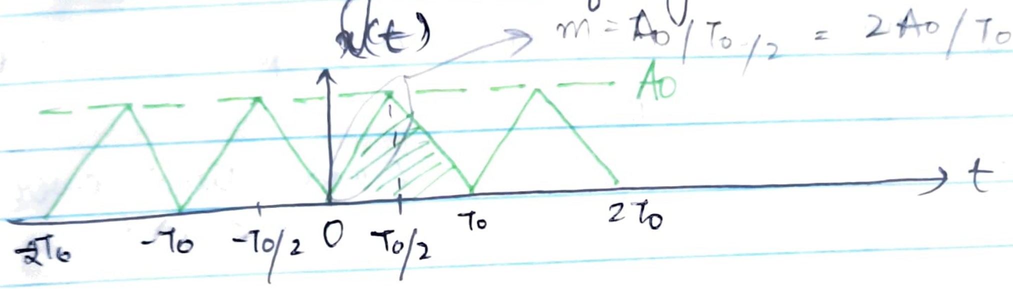
Q.



Average value of signal $= \dfrac{\text{area over } T_0}{T_0}$

$$= \frac{A_0 T_0/2}{T_0} = \frac{A_0}{2}$$

Q Calculate avg value of signal.



avg value of $x(t)$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} x(t)\,dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \left(\frac{2A_0}{T_0}t\right) dt$$

$$= \frac{A_0}{2}.$$

Alternate

avg of $x(t) = \dfrac{\text{area over } T_0}{T_0}$

$$= \frac{\frac{1}{2} \times T_0 \times A_0}{T_0} = \frac{A_0}{2}.$$

ENERGY SIGNAL AND POWER SIGNAL

ENERGY: $\boxed{E = \int_{-\infty}^{\infty} |x(t)|^2 dt}$ for both periodic & non-periodic sig.

(normalised energy $\because R = 1$)

POWER SIGNAL:

$$P = \begin{cases} \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt, & \text{for periodic signal} \\ \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt, & \text{for non-periodic sig} \end{cases}$$

(normalised power $\because R = 1$)

The amount of energy dissipated by a load resistor of 'R' Ω if a voltage source $v(t) = x(t)$ is applied across the resistor.

$$E = \int_{-\infty}^{\infty} \frac{|v(t)|^2}{R} dt$$

If $R = 1\Omega$, then

$$E = \int_{-\infty}^{\infty} |v(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |x(t)|^2 dt$$

The energy & power expressions written above are known as normalised energy & normalised power because they are calculated for 1Ω load resistance.

Property
② Total energy E = area under $|x(t)^2|$ graph
③ $P = \lim_{T\to\infty} \frac{E}{T}$.

ENERGY SIGNAL.

For energy signal,

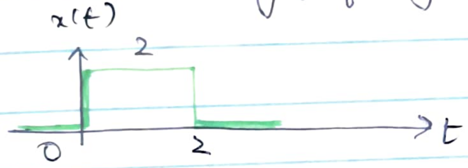
$$\boxed{E = \text{finite}, \quad \text{Power} = 0.}$$

Property
① Energy signals are absolutely integrable sig.

i.e. $\int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty.$
= finite value, or $< \infty$

if any signal is absolutely integrable its fourier tranform always exist.

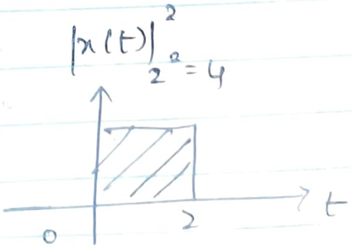
Q. Calculate energy of signal.



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{0} + \int_{0}^{2} + \int_{2}^{\infty}$$

by integration

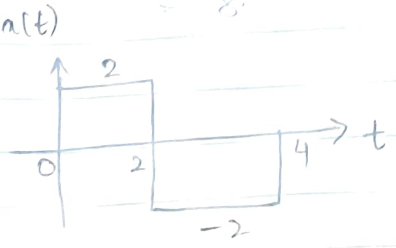
$$\int_0^2 (2)^2 \, dt = \int_0^2 4 \, dt$$
$$= 4[t]_0^2$$
$$= 4[2-0]$$
$$= 8.$$

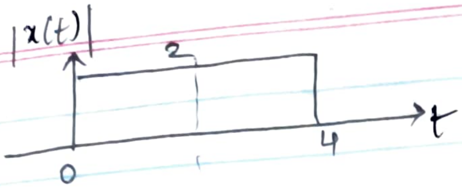


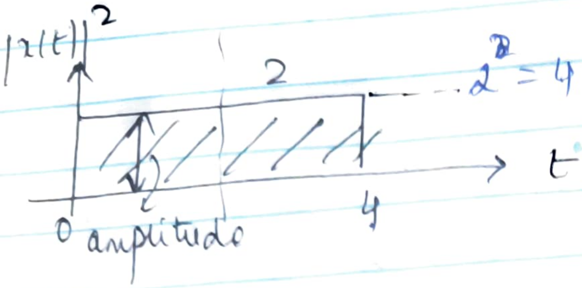
Alternate Method.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$
= area of $|x(t)|^2$ graph
= 8.

Q2.



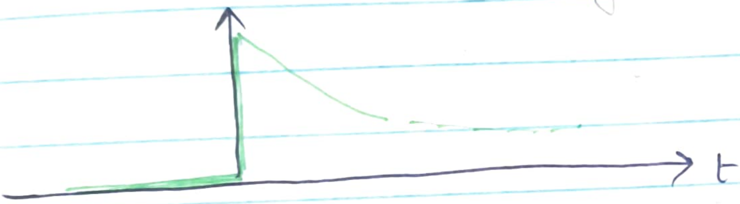




$E_{x(t)}$ = area of of $|x(t)|^2 = 16$.

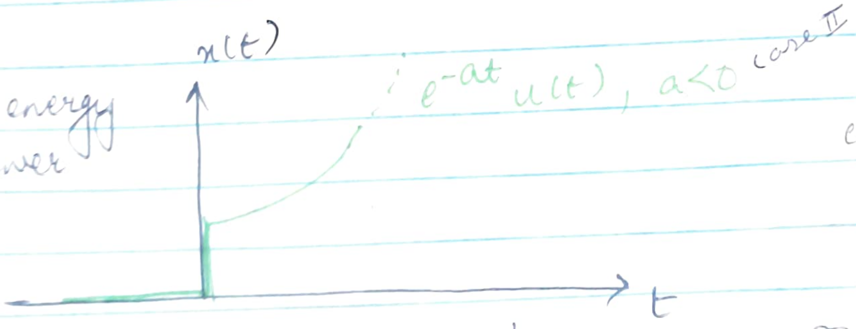
3. $x(t) = e^{-at} u(t)$ $a > 0$ case I

Infinite → nature decreasing but here it is increasing



$$\int_{-\infty}^{\infty} |e^{-at}|^2 \, dt$$

neither energy nor power



case 3 : $a = 0$ $e^0 u(t) = u(t)$ energy $= \infty$.

If signal is having infinite value at any instant of time, then signal is neither energy nor power

Area of $x(t) = \int_{-\infty}^{\infty} x(t) \, dt$

$= \int^{\infty} e^{-at} \, dt$

$$= \frac{\left[e^{-at}\right]_0^{\infty}}{-a}$$

$$= \frac{0-1}{-a} = \frac{1}{a}$$

* Energy is finite for infinite converging graph

Energy of $x(t)$ = area of $|x(t)|^2$

$$= \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{0}^{\infty} \left(e^{-at}\right)^2 dt$$

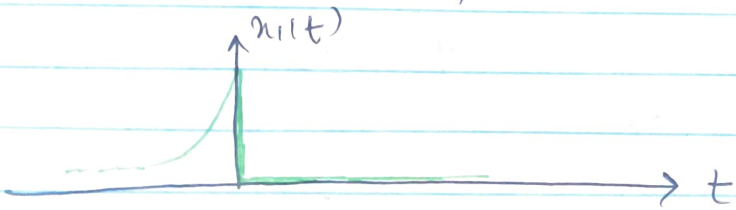
$$= \int_{0}^{\infty} e^{-2at} dt$$

$$= \frac{\left[e^{-2at}\right]_0^{\infty}}{-2a}$$

$$= \frac{0-1}{-2a} = \frac{1}{2a}$$

4. $x_1(t) = x(-t)$

$= e^{at} u(-t)$, $a > 0$



area = $\frac{1}{a}$, energy = $\frac{1}{2a}$

* Hence no effect of time reversal on ~~engy~~ energy signal

5. $x(t) = e^{-a|t|}$, $a > 0$

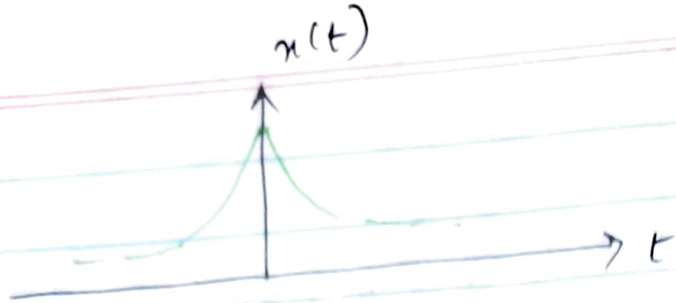
$$= \begin{cases} e^{at}, & t < 0 \\ e^{-at}, & t > 0 \end{cases}$$

$$|t| = \begin{cases} -t, & t<0 \\ t, & t>0 \end{cases}$$

$= e^{at} u(-t) + e^{-at} u(t)$

Effect of Amplitude Scaling: $x(t) \to E$

eg $2x(t) = |2|^2 E = 4 \cdot E$.



$$\text{area} = \frac{1}{a} + \frac{1}{a} = \frac{2}{a}$$

$$\text{energy} = \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}.$$

Q. $x(t) \longrightarrow$ 'E'

$x(2t) \longrightarrow$?

a) $E/2$ b) $E/4$ c) $2E$ d) $4E$

Ans Energy of $x(2t)$

$$E' = \int_{-\infty}^{\infty} |x(2t)|^2 \, dt$$

let $2t = k$, $dt = dk/2$

$$E' = \int_{-\infty}^{\infty} |x(k)|^2 \left(\frac{dk}{2}\right)$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} |x(k)|^2 \, dk \right.$$

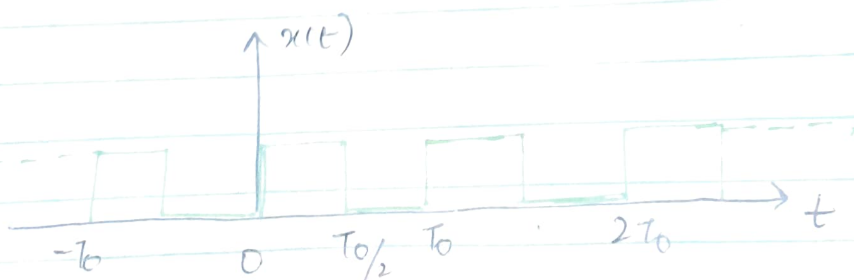
$$= \frac{E}{2}$$

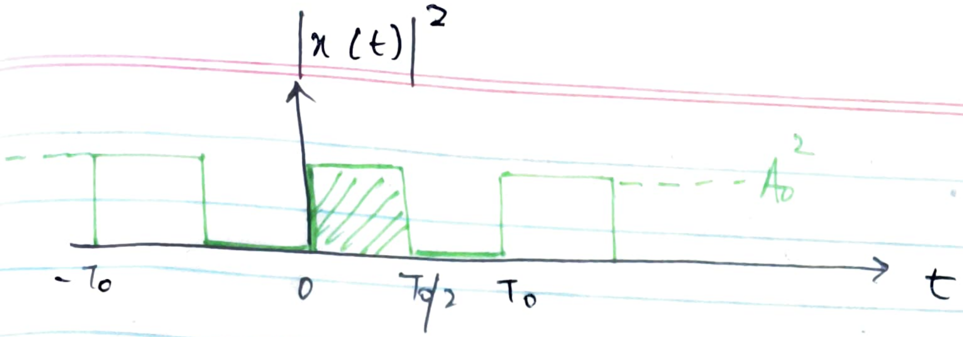
Time Scaling effect on energy of signal is

$x(at) = \frac{E}{|a|}$ $a \neq 0$

★ TR, T. Shifiting, AR has no eff... on energy of signal

Q.





$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$= \text{area of } |x(t)|^2$

$= \text{no. of rectangles} \times \text{area of one rectangle}$

$$= \infty \times \frac{A_0^2 T_0}{2}$$

$E \Rightarrow \infty.$

NOTE: The periodic s/g are not energy signals because there energy content is ∞. Periodic signals are power signals but vice-versa is not true.
Best category of signal is POWER SIGNAL.

TR, T.shifting, T.scaling
AR, Phase shifting cause
No effect on power signal

★ avg. power = (RMS)

~~only~~ amplitude scaling changes the value of power for signal
$x(t) \rightarrow P$
$kx(t) \rightarrow k^2 P$

POWER SIGNAL

for power signal,

$$\boxed{P = \text{finite}, \quad E = \infty}$$

$$\boxed{E = \lim_{T \to \infty} P \times T.}$$

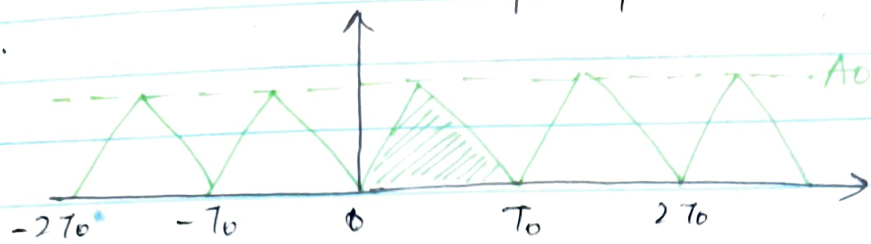
Condition for a periodic signal to be a power signal.

$\longrightarrow \int_{T_0} |x(t)| \, dt < \infty.$

→ Signal should be absolutely integrable over 'T_0'.

$$x(t) = |x(t)|$$

Q1.



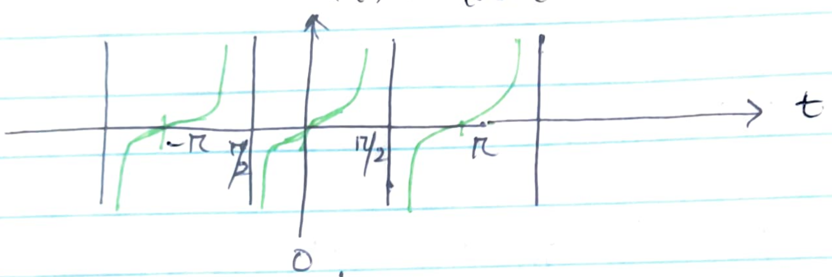
This s/g is power s/g because it is absolutely integrable.

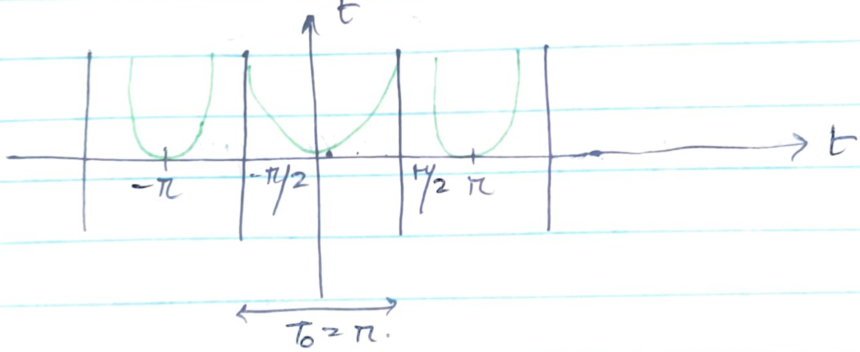
$$\int_{T_0} |x(t)|\, dt = \frac{A_0 T_0}{2} < \infty.$$

↳ power signal.

$$x(t) = \tan t$$

Q2.





$$\int_{T_0} |x(t)|\, dt = \infty$$

neither energy nor power s/g.

Q3. Calculate power of signal.

(i) $x(t) = A_0 \sin w_0 t$

(ii) $x_1(t) = x(t - t_1)$ shifting

$= A_0 \sin [w_0 (t - t_1)]$

(iii) $x_2(t) = x(2t) = A_0 \sin 2w_0 t$ (scaling)

(iv) $x_3(t) = A_0 \sin (w_0 t + \theta)$

$x(t) = A_0 \sin w_0 t$

$P = \frac{A_0^2}{2}$

* Av Power of step signal $= A_0^2/2$
* Av Power of DC signal $= A_0$
* Av Power of sinusoidal fn $A_0 \sin \omega t = A_0^2/2$

1) $$P = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt$$

$$= \frac{1}{T_0}\int_0^{T_0} A_0^2 \sin^2 \omega_0 t \, dt$$

$$= \frac{A_0^2}{T_0}\int_0^{T_0} \frac{1-\cos 2\omega_0 t}{2} dt$$

$$P = \frac{A_0^2}{2T_0}\int_0^{T_0} [1-\cos 2\omega_0 t] dt$$

$$= \frac{A_0^2}{2T_0}\left\{ [t]_0^{T_0} - \left[\frac{\sin 2\omega_0 t}{2\omega_0}\right]_0^{T_0} \right\}$$

$$= \frac{A_0^2}{2T_0}\left[\cancel{T_0} - \frac{\underbrace{\sin 2\omega_0 T_0}_{=0} - \underbrace{\sin 0}_{=0}}{2\omega_0} \right]$$

$$= \frac{A_0^2}{2}.$$

$\omega = 2\pi f$

$\frac{\omega_0}{2\pi} = \frac{1}{T_0}.$

$\boxed{\omega_0 T_0 = 2\pi}$

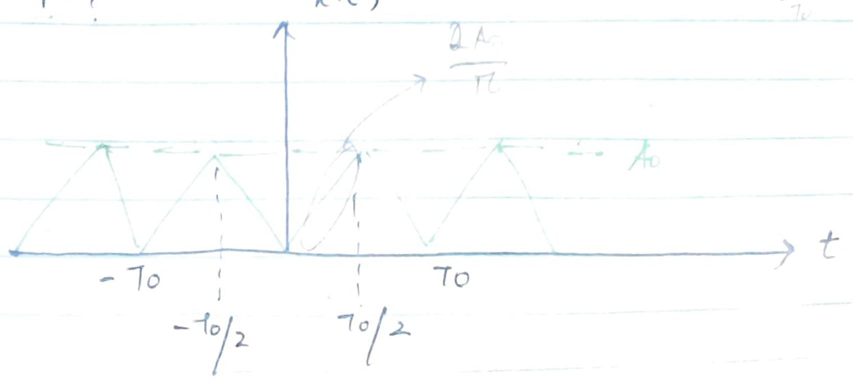
Power is also known as mean square value

$$\boxed{P = RMS^2}$$

Power of a signal is unaffected by

1. time shifting
2. time scaling
3. change in fundamental freq. or time period of a signal.
4. change in phase of a signal

Q. $P = ?$



$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt$$

$$= 2 \times \frac{1}{T_0}\int_{0}^{T_0/2} |x(t)|^2 \, dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} \left(\frac{2A_0}{T_0} t\right)^2 dt$$

$$P = \frac{2}{T_0} \times \frac{4A_0^2}{T_0^2}\int_{0}^{T_0/2} t^2 \, dt$$

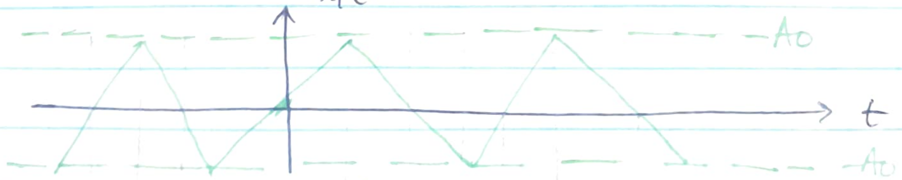
$$= \frac{8A_0^2}{T_0^3}\left[\frac{t^3}{3}\right]_0^{T_0/2}$$

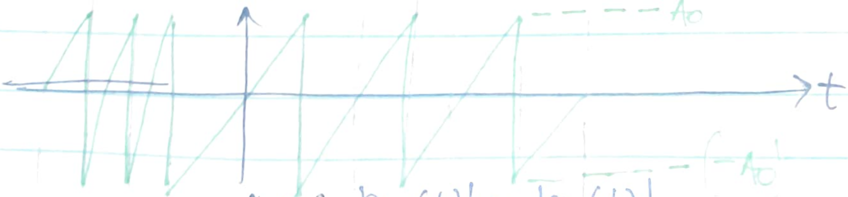
$$= \frac{8A_0^2}{3T_0^3}\left[\left(\frac{T_0}{2}\right)^3 - 0\right]$$

$$= \frac{\cancel{8} A_0^2}{3 \cancel{T_0^3}}\left[\frac{\cancel{T_0^3}}{\cancel{8}}\right]$$

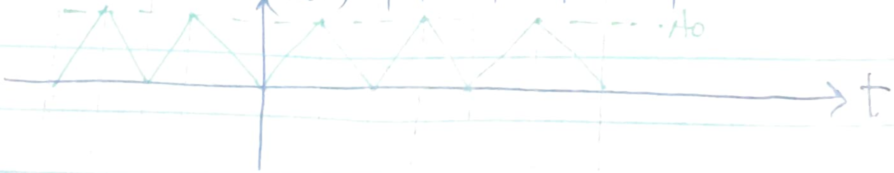
$$\boxed{P = \frac{A_0^2}{3}}$$

Q.





$x_3(t) = |x_1(t)| = |x_2(t)|$



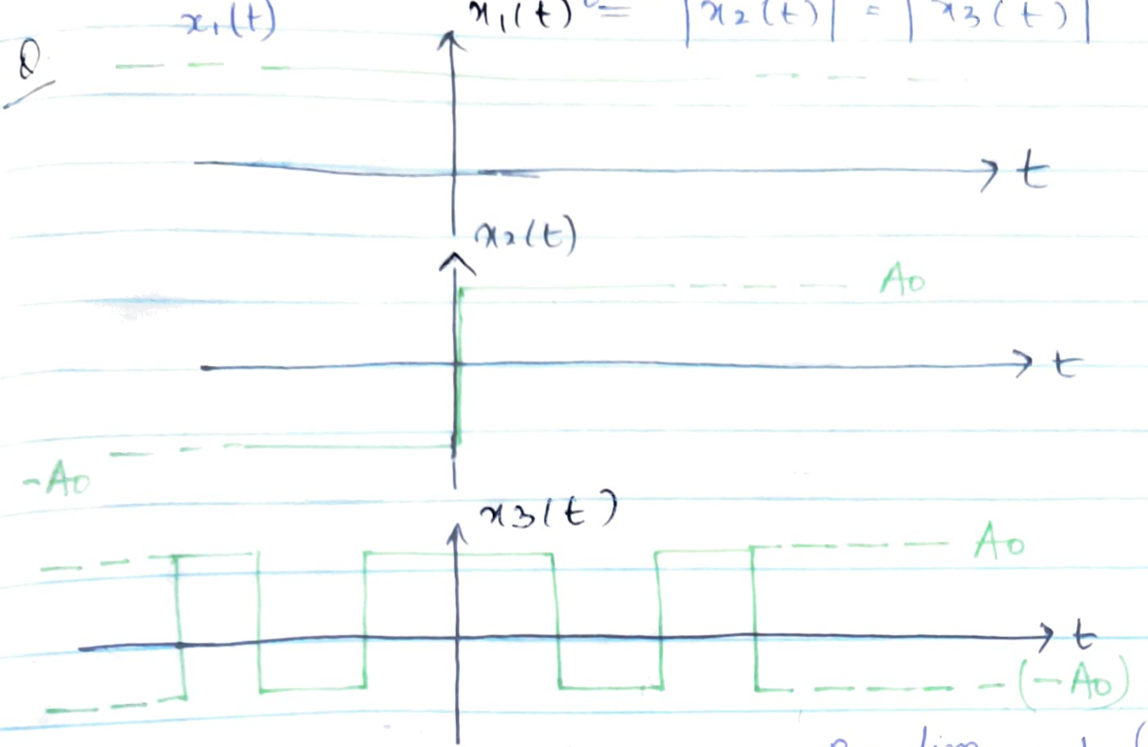
$$P = \frac{A_0^2}{3}$$

$x_3(t) = |x_1(t)|$
$= |x_2(t)|$

Signals having same modulus value contain equal amount of power.

$x_1(t) = |x_2(t)| = |x_3(t)|$

Q.



$P = RMS^2$

$= A_0^2$

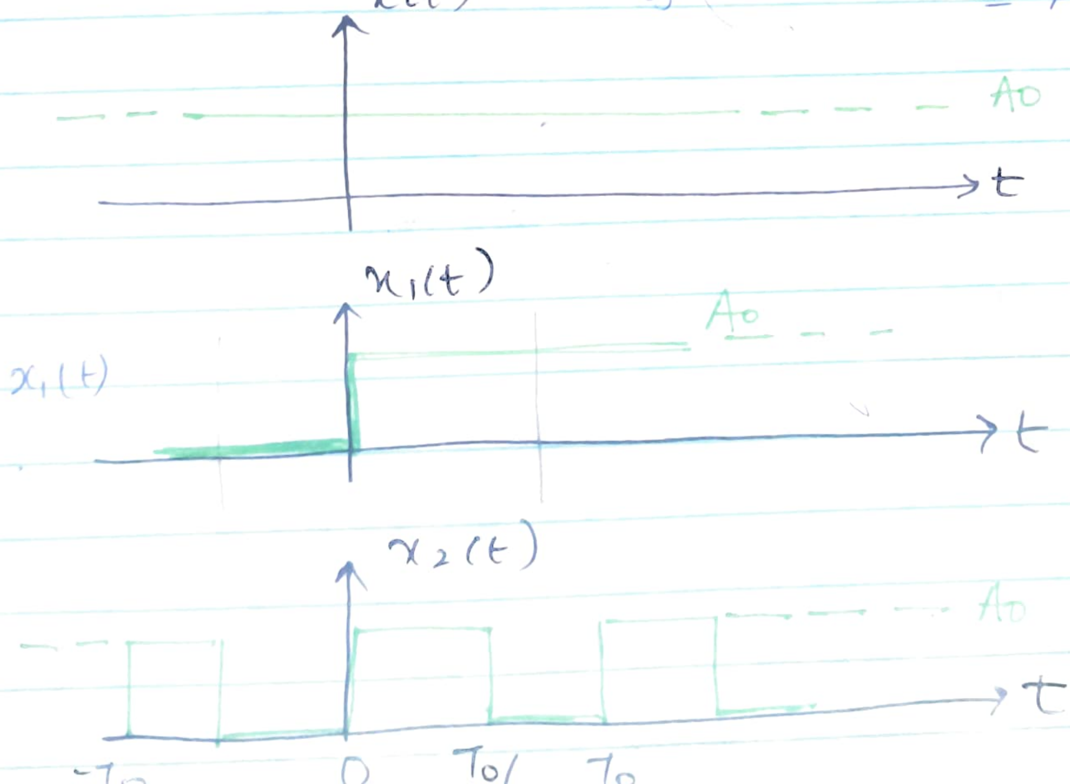
$$P = \lim_{T\to\infty} = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A_0^2 \, dt$$

power of $x_1(t)$

$= A_0^2$

Q.



Avg of $x(t) = A_0$

Power of $x(t) = A_0^2$

	Power	Average
$x(t)$	A_0^2	A_0
$x_1(t)$	$\frac{A_0^2}{2}$	$\frac{A_0}{2}$
$x_2(t)$	$\frac{A_0^2}{2}$	$\frac{A_0}{2}$.

Power of $x_1(t)$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt$$

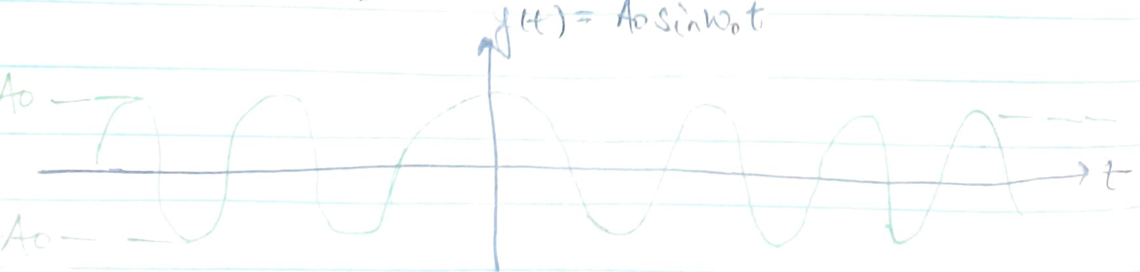
$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{0}^{T_0/2} A_0^2 \, dt$$

$$= \lim_{T_0 \to \infty} \frac{1}{\cancel{T_0}} A_0^2 \cdot \frac{\cancel{T_0}}{2}.$$

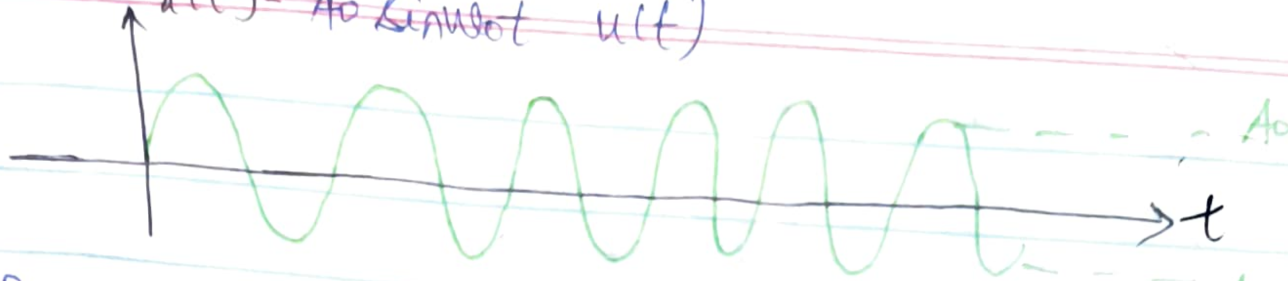
$$= \frac{A_0^2}{2}.$$

Q. $x(t) = A_0 \sin w_0 t \cdot u(t)$

$P = ?$



$x(t) = A_0 \sin \omega_0 t \; u(t)$



$$P_{x(t)} = \frac{P_{y(t)}}{2} = \frac{A_0^2/2}{2} = \frac{A_0^2}{4}$$

Q. $x(t) = A_0 e^{j\omega_0 t}$

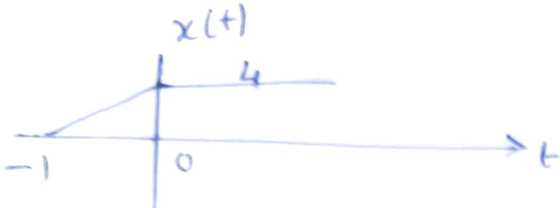
$P = ?$

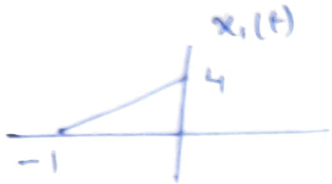
$|x(t)| = A_0$

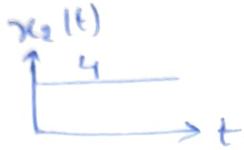
$P = A_0^2$

ORTHOGONAL SIGNALS

Cal. av. Power for foll. signal







$x_1(t)$ finite duration signal hence it is energy signal so its $P =$

$x_2(t)$ is step signal $P = \frac{A_0^2}{2} = \frac{4^2}{2} = 8$

av Power $= 0 + 8 = 8$ Ans

Q $\quad x(t) = \underbrace{5\cos(10t+\phi)}_{x_a(t)} + \underbrace{10\sin(5t+\phi)}_{x_b(t)}$

$x_b(t) = 10\sin(5t+\phi) \rightarrow \frac{A_0^2}{2} = \frac{10^2}{2} = 50$

$A_0 \sin w_0 t = A_0^2/2$

phase shift $A_0 \sin(w_0 t + \phi) = \frac{A_0^2}{2}$

so $A_0 = 10$

$x_a(t) = A_0 \sin(w_0 t + \pi/2 - \phi) = \frac{5^2}{2} = 12.5$

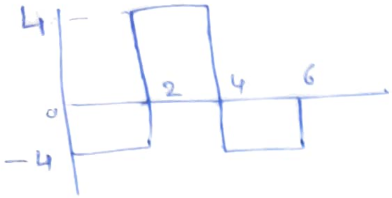
$P = x_a(t) + x_b(t) = 50 + 12.5 = 62.5$ J.

★ E is finite for infinite converging signal   T.R. having No effect on E)

★ Energy. $x(t) \to E \quad x(-t) \to E$

★ $x_1(t) = E_1 \quad x_2(t) = E_2 \quad x_3(t) = x_1(t) + x_2(t) = E_1 + E_2.$

Solved prob on Energy of CTS



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{0} 0\,dt + \int_0^2 (4)^2 dt + \int_2^4 4^2 dt + \int_4^6 4^2 dt + \int_6^{\infty} 0\,dt$$

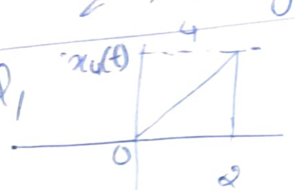
$$= 0 + 16(t)_0^2 + (16)(t)_2^4 + 16(t)_4^6 + 0$$

$$= 0 + 16\times 2 + 16\times 2 + 16\times 2 + 0$$

$E =$ ~~32×3~~ 96 J (finite energy signal)

& Average power = 0.

Q1



$$E = \int_{-\infty}^{t} |x_4(t)|^2 dt$$

$$E = \int_{-\infty}^{0} 0\,dt + \int_0^2 |x_4(t)|^2 + \int_0^{\infty} 0\,dt$$

$y = mx + c \to 0$

$x_4(t) = 2t$

$$E = \int_0^2 |2t|^2 t$$

$$= \frac{4}{3}(t^3)_0^2 = 32/3\,J.$$

Q

$$E = \int_0^2 4\,dt$$