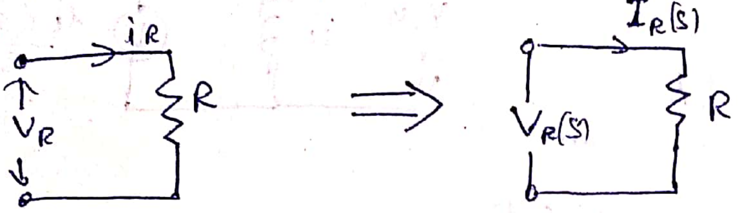
Transformed Circuit elements

1. Resistor :

$$V_R = i_R R \longrightarrow \text{Time Domain}$$

$$\mathcal{L}\left[V_R = i_R R\right] \xrightarrow{\text{S Domain}} V_R(s) = I_R(s) R$$

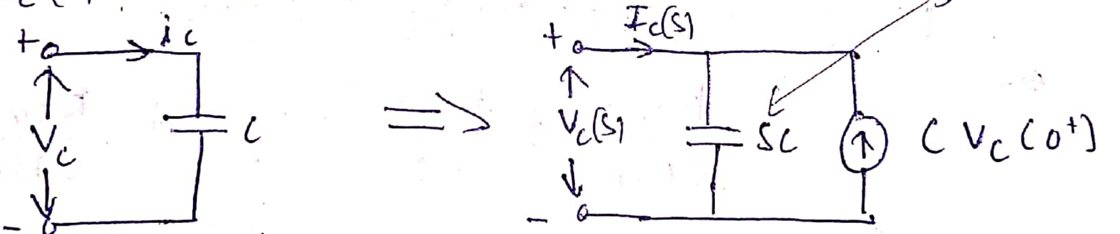


2. Capacitor :

$$i_c = C \frac{dV_c}{dt}$$

$$I_c(s) = C\left[sV_c(s) - V_c(0^+)\right]$$

$$I_c(s) = sC\, V_c(s) - C V_c(0^+)$$

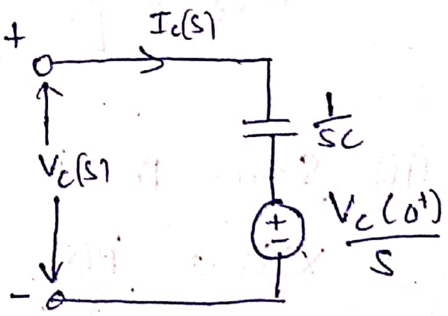


$$\frac{1}{C}\int_0^t i_c\, dt = \int_{V_c(0^+)}^{V_c} dV_c$$

$$\frac{1}{C}\int_0^t i_c\, dt = V_c - V_c(0^+)$$

$$\frac{1}{C}\,\frac{I_c(s)}{s} = V_c(s) - \frac{V_c(0^+)}{s}$$

$$V_c(s) = \frac{1}{sC} I_c(s) + \frac{V_c(0^+)}{s}$$

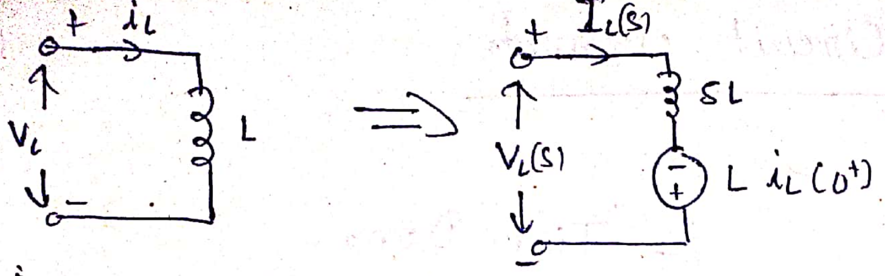


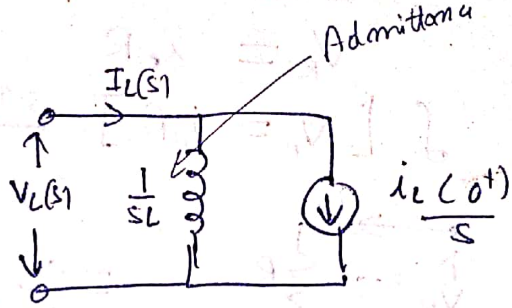
3. Inductor :

$$V_L = L \frac{di_L}{dt}$$

$$V_L(s) = L\left[s I_L(s) - I_L(0^+)\right]$$

$$V_L(s) = sL\, I_L(s) - L I_L(0^+)$$





$$\int_{i_L(0^+)}^{i_L} di_L = \frac{1}{L}\int_0^t V_L \, dt$$

$$i_L - i_L(0^+) = \frac{1}{L}\int_0^t V_L \, dt$$

$$I_L(s) = \frac{1}{L}\frac{V_L(s)}{s} + \frac{i_L(0^+)}{s}$$

Partial Fraction Expansion

(i) Simple Zeros

$$X(s) = \frac{p(s)}{q(s)} = \frac{p(s)}{(s+s_1)(s+s_2)\cdots(s+s_m)}$$

$$= \frac{K_{s1}}{s+s_1} + \frac{K_{s2}}{s+s_2} + \cdots + \frac{K_{sm}}{s+s_m}$$

$$K_{sj} = \left[(s+s_j)\frac{p(s)}{q(s)}\right]_{s=-s_j}$$

(ii) Same Multiple Zeros

$$X(s) = \frac{p(s)}{q(s)} = \frac{p(s)}{(s+s_1)(s+s_2)\cdots(s+s_i)^r(s+s_m)}$$

$$X_s = \frac{K_{s1}}{s+s_1} + \frac{K_{s2}}{s+s_2} + \cdots + \frac{K_{sm}}{(s+s_m)} + \frac{A_{i1}}{s+s_i} + \frac{A_{i2}}{(s+s_i)^2} + \cdots \frac{A_{ir}}{(s+s_i)^r}$$

where $A_{ir} = \left[(s+s_i)^r \frac{p(s)}{q(s)}\right]_{s=-s_i}$

$$A_{i(r-1)} = \frac{d}{ds}\left[(s+s_i)^r \frac{P(s)}{Q(s)}\right]_{s=-s_i}$$

$$A_{i(r-2)} = \frac{1}{2!}\frac{d^2}{ds^2}\left[(s+s_i)^r \frac{P(s)}{Q(s)}\right]_{s=-s_i}$$

$$A_{i1} = \frac{1}{(r-1)!}\frac{d^{(r-1)}}{ds^{(r-1)}}\left[(s+s_i)^r \frac{P(s)}{Q(s)}\right]_{s=-s_i}$$

Q. Find Inverse Laplace transform

(i) $X(s) = \dfrac{5s+3}{(s+1)(s+2)(s+3)}$

Sol

$$\frac{5s+3}{(s+1)(s+2)(s+3)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)} + \frac{k_3}{(s+3)}$$

$$k_1 = \left[\cancel{(s+1)}\frac{5s+3}{\cancel{(s+1)}(s+2)(s+3)}\right]_{s=-1} = \frac{-5+3}{(-1+2)(-1+3)} = -1$$

$$k_2 = \left[\cancel{(s+2)} \times \frac{5s+3}{(s+1)\cancel{(s+2)}(s+3)}\right]_{s=-2} = \frac{-10+3}{(-2+1)(-2+3)} = 7$$

$$k_3 = \left[\cancel{(s+3)} \times \frac{5s+3}{(s+1)(s+2)\cancel{(s+3)}}\right]_{s=-3} = \frac{-15+3}{(-3+1)(-3+2)} = -6$$

$$X(s) = \frac{-1}{(s+1)} + \frac{7}{(s+2)} - \frac{6}{(s+3)}$$

$$x(t) = -e^{-t} + 7e^{-2t} - 6e^{-3t}$$

(ii) $X(s) = \dfrac{1}{s(s+1)^3(s+2)}$

$$X(s) = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{A_1}{(s+1)^3} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)}$$

$$k_1 = \frac{1}{2}, \quad k_2 = \frac{1}{(-2)(-2+1)^3} = \frac{1}{2}$$

$$A_1 = \left[\cancel{(s+1)^3}\frac{1}{s\cancel{(s+1)^3}(s+2)}\right]_{s=-1} = \frac{1}{(-1)(-1+2)} = -1$$

$$A_2 = \left[\frac{d}{ds}\left[\cancel{(s+1)^3}\frac{1}{s\cancel{(s+1)^3}(s+2)}\right]\right]_{s=-1} = \left[\frac{-(2s+2)}{s^2(s+2)^2}\right]_{s=-1}$$

$$= 0$$

$$A_3 = \frac{1}{2!}\frac{d^2}{ds^2}\left[\cancel{(s+1)^3}\frac{1}{s\cancel{(s+1)^3}(s+2)}\right]_{s=-1} = \frac{1}{2}\frac{d}{ds}\left[\frac{-(2s+2)}{s^2(s+2)^2}\right]_{s=-1}$$

$$= \frac{1}{2}\left[\frac{-s^2(s+2)^2(2) + (2s+2)[4s^3+8s+12s^2]}{s^4(s+2)^4}\right]_{s=-1}$$

$$= \frac{1}{2}\left[\frac{-(1)^2(1)^2(2)+0}{(-1)^4(1)^4}\right] = \frac{1}{2}(-2) = -1$$

$$X(s) = \frac{1}{2}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{(s+2)} - \frac{1}{(s+1)^3} + \frac{0}{(s+1)^2} - \frac{1}{(s+1)}$$

$$x(t) = \frac{1}{2} + \frac{1}{2}e^{-2t} - \frac{1}{2}t^2e^{-t} - e^{-t}$$

(iii) $X(s) = \dfrac{1}{s^2+4s+8}$

Sol. $$X(s) = \frac{1}{(s+2+2j)(s+2-2j)} = \frac{k_1}{(s+2+2j)} + \frac{k_2}{(s+2-2j)}$$

$$k_1 = \left[\frac{\cancel{(s+2+2j)}\,1}{\cancel{(s+2+2j)}(s+2-2j)}\right]_{s=-2-2j}$$

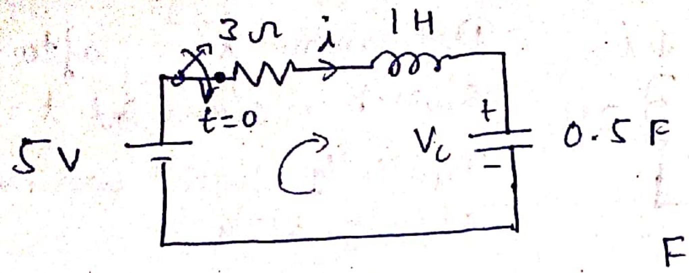
$$= \frac{1}{(\cancel{-2}-2j\cancel{+2}-2j)} = \frac{1}{-4j} = \frac{j}{4}$$

$$k_2 = -\frac{j}{4}$$

$$X(s) = \frac{j}{4}\frac{1}{(s+2+2j)} - \frac{j}{4}\frac{1}{(s+2-2j)}$$

$$x(t) = \frac{j}{4}e^{(-2-2j)t} - \frac{j}{4}e^{(2+2j)t} = \frac{1}{2}e^{-2t}\sin 2t$$

Q



If initial conditions are $V_c(0^-) = 2V$ & $i_L(0^-) = 2A$
Find $i(t)$ after switch is closed at $t=0$

(26)

Sol Apply KVL

$$5 - 3i - 1\frac{di}{dt} - \left[2\int_0^t i\,dt + V_c(0^+)\right] = 0$$

Laplace both sides

$$\frac{5}{s} - 3\,I(s) - [s\,I(s) - i_L(0^+)] - \left[2\frac{I(s)}{s} + \frac{V_c(0^+)}{s}\right] = 0$$

since $i_L(0^+) = i_L(0^-) = 2A$ &
$V_c(0^+) = V_c(0^-) = 2V$

$$\therefore \frac{5}{s} - 3\,I(s) - s\,I(s) + 2 - 2\frac{I(s)}{s} - \frac{2}{s} = 0$$

$$s^2 I(s) + 3s\,I(s) + 2\,I(s) = 2s + 3$$

$$I(s) = \frac{2s+3}{s^2+3s+2} = \frac{2s+3}{(s+1)(s+2)}$$

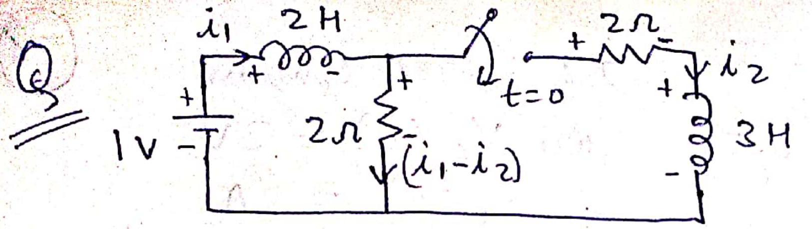
$$I(s) = \frac{k_1}{s+1} + \frac{k_2}{s+2}$$

$$k_1 = \frac{2(-1)+3}{(-1+2)} = 1 \quad ; \quad k_2 = \frac{+2(-2)+3}{(-2+1)} = 1$$

$$I(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

$i(t)$ = inverse of $I(s)$
$= e^{-t} + e^{-2t}$

Q



Determine i_2, after the switch is closed at $t=0$. Initially circuit was in steady state

Sol

$$i_1(0^-) = 0.5\ A \quad , \quad i_2(0^-) = 0$$

KVL is left side loop, (after $t=0$)

$$1 - 2\frac{di_1}{dt} - 2(i_1 - i_2) = 0 \quad \Rightarrow \quad 2\frac{di_1}{dt} + 2i_1 - 2i_2 = 1$$

Laplace both sides

$$2[sI_1(s) - i_1(0^+)] + 2I_1(s) - 2I_2(s) = \frac{1}{s}$$

$$2sI_1(s) - 1 + 2I_1(s) - 2I_2(s) = \frac{1}{s}$$

$$I_1(s) = \frac{2sI_2(s) + s + 1}{2s^2 + 2s} \rightarrow \text{(I)}$$

KVL in Right side loop

$$2(i_1 - i_2) - 2i_2 - 3\frac{di_2}{dt} = 0$$

Laplace both side

$$2I_1(s) - 2I_2(s) - 2I_2(s) - 3[sI_2(s) - i_2(0^+)] = 0$$

$$2I_1(s) - 4I_2(s) - 3sI_2(s) = 0$$

$$2\left[\frac{2sI_2(s) + s + 1}{2s^2 + 2s}\right] - 4I_2(s) - 3sI_2(s) = 0 \quad \text{(using I)}$$

$$-3s^3 I_2(s) - 7s^2 I_2(s) - 2sI_2(s) + s + 1 = 0$$

$$I_2(s) = \frac{s+1}{3s^3 + 7s^2 + 2s} = \frac{1}{3}\frac{(s+1)}{s(s+\frac{1}{3})(s+2)}$$

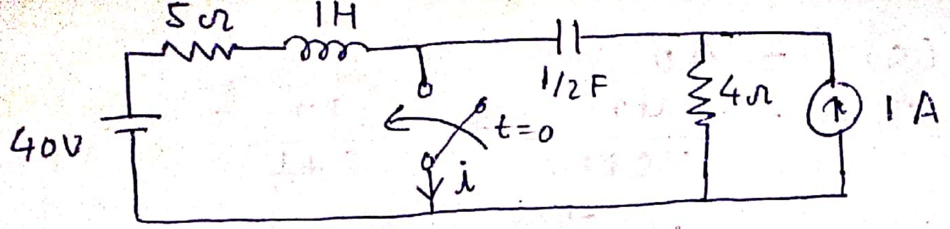
$$I_2(s) = \frac{1}{3}\left[\frac{K_1}{s} + \frac{K_2}{s+\frac{1}{3}} + \frac{K_3}{s+2}\right]$$

$$K_1 = \frac{3}{2} \quad ; \quad K_2 = -\frac{6}{5} \quad ; \quad K_3 = -\frac{3}{10}$$

$$I_2(s) = \frac{1}{3}\left[\frac{3}{2}\cdot\frac{1}{s} - \frac{6}{5}\frac{1}{s+\frac{1}{3}} - \frac{3}{10}\frac{1}{s+2}\right]$$

$$i_2(t) = \frac{1}{2} - \frac{2}{5}e^{-\frac{1}{3}t} - \frac{1}{10}e^{-2t}$$ Solution

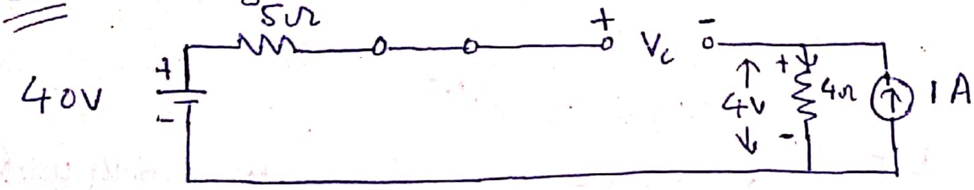
Q



Initially Circuit was in steady State Switch was closed at $t=0$. (27)

Find $i(t)$ (current through switch)

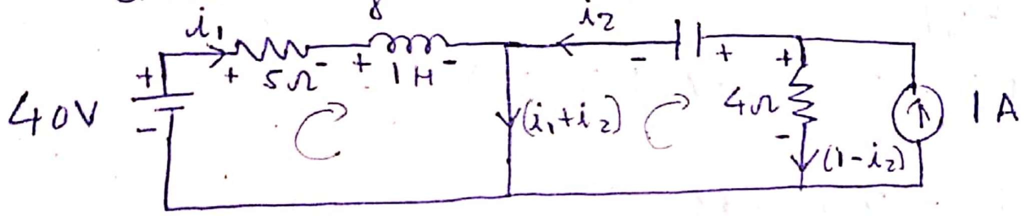
Sol. Before closing switch, Circuit was in steady State.



Circuit before closing switch (steady state)

$i_L(0^-) = 0$; $V_c(0^-) = 36\ V$

Circuit after closing switch



$i_1(0^+) = i_L(0^-)$
$= 0$
$V_c(0^+) = V_c(0^-) = -36V$
$= -36\ V$

KVL in left side loop

$$40 - 5i_1 - 1\frac{di_1}{dt} = 0$$

Laplace both sides

$$\frac{40}{s} - 5I_1(s) - [sI_1(s) - i_1(0^+)] = 0$$

$$\frac{40}{s} - 5I_1(s) - sI_1(s) = 0$$

$$I_1(s) = \frac{40}{s^2+5s} = \frac{40}{s(s+5)}$$

$$= \frac{8}{s} - \frac{8}{s+5}$$

$$i_1(t) = 8 - 8e^{-5t}$$

KVL in right side loop.

$$2\int_0^t i_2\,dt + V_c(0^+) - (1-i_2)4 = 0$$

$$2\int_0^t i_2\,dt - 36 = 4 - 4i_2$$

Laplace both sides

$$2\frac{I_2(s)}{s} - \frac{36}{s} = \frac{4}{s} - 4I_2(s)$$

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$$(4s+2)\, I_2(s) = 40$$

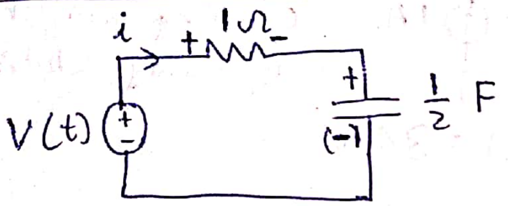
$$I_2(s) = \frac{40}{4s+2} = \frac{10}{s+\frac{1}{2}}$$

$$i_2(t) = 10\, e^{-1/2 t}$$

Current through switch, $i = i_1 + i_2$

$$i(t) = 8 - 8e^{-5t} + 10\, e^{-1/2 t}$$

Q



$V(t) = 2e^{-0.5t}$

$V_c(0^+) = 0$ (Capacitor initially uncharged)

Find the expression for $i(t)$

Sol. Apply KVL

$$V(t) = 1\, i + 2\int_0^t i\, dt + V_c(0^+)$$

$$2e^{-0.5t} = i + 2\int_0^t i\, dt$$

Laplace both side

$$2 \cdot \frac{1}{(s+0.5)} = I(s) + 2\frac{I(s)}{s}$$

$$I(s) = \frac{2s}{(s+0.5)(s+2)} = \frac{k_1}{(s+0.5)} + \frac{k_2}{(s+2)}$$

$$k_1 = \frac{2(-0.5)}{(-0.5+2)} = \frac{-1}{1.5} = \frac{-2}{3}$$

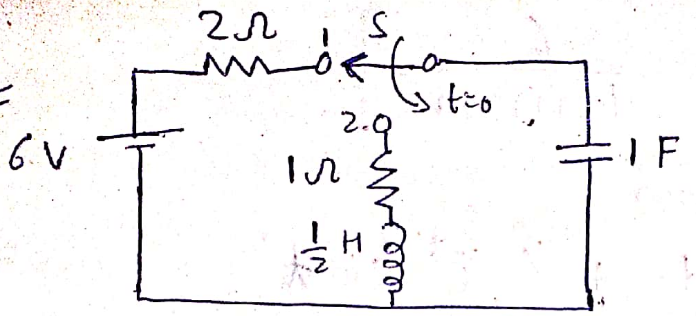
$$k_2 = \frac{2(-2)}{(-2+0.5)} = \frac{8}{3}$$

$$I(s) = -\frac{2}{3}\frac{1}{(s+0.5)} + \frac{8}{3}\frac{1}{(s+2)}$$

Inverse Laplace transform

$$i(t) = -\frac{2}{3}e^{-0.5t} + \frac{8}{3}e^{-2t}$$

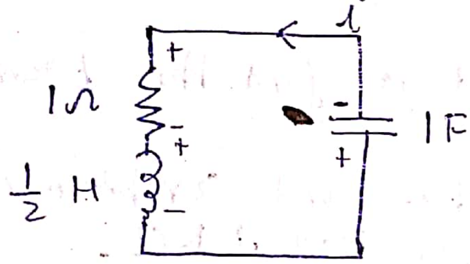
Q



Initially switch was at position 1 and circuit was in steady state. Switch is moved to position 2 at $t=0$. Find voltage across capacitor.

Sol. When switch was at Position 1, voltage across capacitor in steady state was 6V. (i.e $V_c(0^-) = 6V$)

Circuit when switch is in Position 2.



Apply KVL

$$1i + \frac{1}{2}\frac{di}{dt} + \left[\frac{1}{1}\int_0^t i\,dt + (-6)\right] = 0$$

Laplace transform both sides.

$$I(s) + \frac{1}{2}[sI(s) - i(0^+)] + \frac{I(s)}{s} - \frac{6}{s} = 0$$

$$I(s) + \frac{1}{2}[sI(s) - 0] + \frac{I(s)}{s} - \frac{6}{s} = 0$$

$$I(s) = \frac{12}{s^2+2s+2}$$

$$V_c + i(1) + \frac{1}{2}\frac{di}{dt} = 0$$

Laplace both sides

$$V_c(s) + I(s) + \frac{1}{2}[sI(s) - 0] = 0$$

$$V_c(s) = -I(s) - \frac{1}{2}sI(s)$$

$$V_c(s) = -\left[I(s)\left(1 + \frac{s}{2}\right)\right]$$

$$V_c(s) = -I(s)\left(\frac{s+2}{2}\right)$$

$$V_c(s) = -\frac{12}{s^2+2s+2}\left(\frac{s+2}{2}\right)$$

$$= -\frac{6(s+2)}{s^2+2s+2}$$

25) $V_C(s)_{out} = -\left[\frac{6(s+1)}{(s+1)^2+1} + \frac{6}{(s+1)^2+1}\right]$

Inverse Laplace transform

$V_C(t) = -\left[6\,e^{-t}\cos t + 6\,e^{-t}\sin t\right]$

Graph Theory

- Any electrical network can be solved by applying KVL and KCL.
- When we apply KVL /or KCL in any electric network, we get network equations.
- Network equations can be solved to find the branch voltages & currents.
- But when network is very complex and complicated, then it become very difficult to apply KVL & KCL.
- Such complicated and complex networks can be solved easily using graph Theory.
- Graph Theory is used in Computer Aided Electric network analysis.
- Application of Graph Theory :-

(i) It is used in Power System Analysis.

(ii) It can be used as a tool to construct software programs related to electric network analysis & simulation.

- Graph Theory is very generalized approach to solve (or analysis) any complex electric network.
- Graph Theory use network topology i.e. only the geometrical pattern of a network is considered. The various standard matrix are formed (or constructed) for a given network topology. Then matrix manipulation (or calculations) are used to form network equations and hence find the solution of a given network.