NUMBER SYSTEMS - CODE

Weighted

1. Binary (2)
2. Octal (8)
3. Decimal (10)
4. Hex (16)
5. BCD

~~Non~~ Un-weighted

1. Gray code
2. Excess-3 Code

r [r digits, 0 to $r-1$]

→ In a number System with base r, have r different digits and they are from 0 to $r-1$.

(2) 0, 1 (2 digits)
(10) 0 to 9 (10 digits)

Conversions

Decimal to Other

(1) To convert Decimal number into any other base r, divide integer part and multiply factional part with r.

E.g. (1) $(25.625)_{10}$ to $(\ .\ )_2$

Integer part

2	25	
2	12	1 (least) (LSB)
2	6	0
2	3	0
2	1	1
	0	1 Most (MSB)

$0.625 \times 2 = 1.25 \rightarrow 1$ (MSB)
$0.25 \times 2 = 0.5 \rightarrow 0$
$0.5 \times 2 = 1.0 \rightarrow 1$ (LSB)

$(11001.101)_2$

(2) $(25.625)_{10}$ to $(\ .\ )_8$.

$$\boxed{(25)_{10} - (31)_8}$$

8	25	
8	3	1
	0	3

$0.625 \times 8 = 5.000 \rightarrow 5$

$(25.625)_{10} \rightarrow (31.5)_8$

(iii) $(29.625)_{10}$ to $(\ )_{16}$

16	29
16	1 — 13 → D
	0 — 1

$0.625 \times 16 = 10.000$ → A

$(1D)$

$\Rightarrow (1D.A)_{16}$ Ans

(iv) $(254)_{10}$ — $(\ )_{16}$.

16	254
16	15 — 14 → E
	0 → 15 → F

$\Rightarrow (FE)_{16}$ Ans

(v) $(27.4)_{10} \longrightarrow (\ )_4$

4	27
4	6 — 3
4	1 — 2
	0 — 1

$0.4 \times 4 = 1.6$ — 1
$0.6 \times 4 = 2.4$ — 2
$0.4 \times 4 = 1.6$ — 1

$\Rightarrow (123.\overline{12})$

0 A (10)
1 B (11)
2 C (12)
3 D (13)
4 E (14)
5 F (15)
6 10 (16)
7 11 (17)
8 12 (18)
9

10.
$16 + 16^0$

1) Others to Decimal Conversion

$(x_2\, x_1\, x_0 \,.\, y_1\, y_2)_r$ — $(\ )_{10}$

$r^2\ r^1\ r^0\ \ r^{-1}\ r^{-2}$ $\longrightarrow$ $x_2 \times r^2 + x_1 \times r^1 + x_0 \times r^0 + y_1 \times r^{-1} + y_2 \times r^{-2}$

→ To convert any other base into decimal multiply each digit with positional weightage then add

Eg. (1) $(10110.11)_2 \longrightarrow (\ )_{10}$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0\ \ 2^{-1}\ 2^{-2}$

10

2^0

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$\Rightarrow 16 + 4 + 2 + 0.5 + 0.25$

$\Rightarrow (22.75)_{10}$

⑪ $(37.4)_8 \longleftrightarrow (\quad)_{10}$

$(37.4)_8$

$8^1 \; 8^0 \; 8^{-1}$
$3 \; 7 \; . \; 4$

$3 \times 8^1 + 7 \times 8^0 + 8^{-1} \times 4$

$24 + 0.5 + 7$

$\cancel{24.5}$ 31.5

⑫ $(57.4)_{16} \longleftrightarrow (\quad)_{10}$

$16^1 \; 16^0 \; 16^{-1}$
$5 \; 7 \; . \; 4$

$\Rightarrow 5 \times 16^1 + 7 \times 16^0 + 16^{-1} \times 4$

$\Rightarrow 87.25$

256
256
2816

$16^2 \rightarrow 4096$

⑬ $(BAD)_{16} \longleftrightarrow (\quad)_{10}$

$16^2 \; 16^1 \; 16^0$
B A D

$11 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$

$2816 + 160 + 13$

$= (2989)_{10}$

Octal to Binary

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

$0 \longrightarrow 000$

$7 \longrightarrow 111$

Ques

$(CAFE)_{16} \longleftrightarrow (\quad)_8$

1100 1010 1111 1110

$(145376)_8$

Airthmatic operations

~~Binary~~ Binary

Sum of two binary numbers.

(i)
```
   1 0 1 1 0 1
   1 1 0 0 1 1
 -------------
 1 1 0 0 0 0 0
```

```
  1 0 1 1 0 1
+ 1 1 0 0 1 1
-------------
1 1 0 0 0 0 0
```

$1+0=0$
$1+1=10\ (2)$
$10+1=11$

Subtract

```
  1 1 0 1 1 1
- 1 0 1 1 1
-----------
  0 0 1 0 0
```

```
  1 1 0 0 1 0 1
  1 0 1 1 1
  -----------
  0 0 1 0 0
```

Multiplication

(i) 1010 × 101

$$\begin{array}{r} 1\,0\,1\,0 \\ 0\,0\,0\,0 \\ 1\,0\,1\,0 \\ \hline 1\,1\,0\,0\,1\,0 \end{array}$$

AND $\begin{cases} 1 \times 1 = 1 \\ 1 \times 0 = 0 \end{cases}$

(ii) 1111 × 1111

$$\begin{array}{r} 1\,1\,1\,1 \\ 1\,1\,1\,1 \\ 1\,1\,1\,1 \\ 1\,1\,1\,1 \\ \hline \end{array}$$

0 1

1 1 1 1

1 1 1 1

1 1 1 1

1 1 1 1

10 11 11 10 1

11 1 0 0 0 0 0 1

$(10W1Z)_2 \times (15)_{10} = (Y01011001)_2$

10W1Z × 1111

10W1Z

10W1Z

10W1Z

10W1Z

Z

↳ Z = 1

W = 1

⟹

1 0 W 1 1

1 0 W 1 1

1 0 W 1 1

1 0 W 1 1

0 0 1

4 → 100

6 − 110

101

011

100

Sum of two Octal numbers

(0 to 7)

$$\begin{array}{r} 2\,4\,6 \\ 3\,5\,1 \\ \hline 6\,1\,7 \end{array}$$

0	(0)
1	(1)
2	(2)
3	(3)
4	(4)
5	(5)
6	(6)
7	(7)
10	(8)
11	(9)

1 + 1 = 2

1 + 7 = 8 → 10

8 | 9

8 | 1 − 1

| 0 − 1

Subtraction of two Octals

(i)

$$\begin{array}{r} 5\,6\,7 \\ 4\,5\,2 \\ \hline 1\,1\,5 \end{array}$$

(ii)

8

$$\begin{array}{r} 7\,5\,4 \\ 4\,6\,5 \\ \hline 2\,6\,7 \end{array}$$

Scanned with CamScanner

89

Hexa decimal Addition

1 + 1 = 2
⋮
1 + 9 = A
1 + A = B
1 + B = C
A + A = 14 ⟶ 10 + 10 = 20 ⟹ 16 | 20, 16 | 1 − 4, 0 − 1

- Sum

```
   2  6  8  9
   3  4  5  9
  ------------
   5  A  E  2
```

16 | 18, 16 | 1 − 2, 0 − 1

Subraction:

```
   9  6  ¹⁶4  7
   5  4   7   B
  ---------------
   4  1   6   F
```

Complements

Any Number System with base r, will have two compliments

1. (r−1)'s compliments. (r−1) compliment
2. r's compliments (r) compliment

(r−1)'s compliment (r−1)'s compliment

→ To determine (r−1)'s complement, first write max^m number possible then subtract the given number

Imp

E.g. ① 10101

```
⟹  1 1 1 1 1
    1 0 1 0 1
   -----------
 → 0 1 0 1 0
```

(1's compliment of 10101)

② 7's of 543 (Octal) (0-7) 7's, 8's

```
⟹  7 7 7
    5 4 3
   -------
    2 3 4
```

(7's complement of 543)

Scanned with CamScanner

(ii) 9's compnt of (2654)

Soln

9	9	9	9
2	6	5	4
7	3	4	5

(9's compliment of 2654)

(iii) F's compnt of (2689)

F	F	F	F
2	6	8	9
D	9	7	6

(F's complement of 2689)

r's complement

→ To determine r's complement 1st find (r−1)'s compliment then add 1 to LSB. (right most)

Q (i) find 2's of Binary no. (10110)

→ 11111
10110

→ 01001
\+ 1

01010

(2's compliment of 10110)

(ii) 2's compn of (1001.11)

→ ~~1001.11~~

1111.11
1001.11

0110.00
→ +1

0110.01

(iii) 8's compt of 2470

7+1=10

→ ~~8888~~
~~2470~~
~~6418~~
~~+1~~
~~6419~~

7	7	7	7
2	4	7	0
5	3	0	7
		+	1
5	3	1	0

(8's compliment of 2470)

(iv) 10's compt of 5370

→

9	9	9	9
5	3	7	0
4	6	2	9
		+	1
4	6	3	0

9+1=10

(10's complement of 5370)

v) 16's compt of 2359

→

F	F	F	F
− 2	3	5	9
D	C	A	6
		+	1
D	C	A	7

(16's compliment of 2359)

Codes

BCD Codes : Binary coded decimal

(i) It is a 4 bit code
(ii) It is a weighted code
(iii) Also known as 8421 code.

→ In this each decimal digit represented with 4 bit binary code

Decimal	BCD codes (8)(4)(2)(1)		Excess 3 code (+3 code)
0	0 0 0 0	+ 0011	0 0 1 1
1	0 0 0 1	+ 0011	0 1 0 0
2	0 0 1 0		0 1 0 1
3	0 0 1 1		0 1 1 0
4	0 1 0 0		0 1 1 1
5	0 1 0 1		1 0 0 0
6	0 1 1 0		1 0 0 1
7	0 1 1 1		1 0 1 0
8	1 0 0 0		1 0 1 1
9	1 0 0 1		1 1 0 0

imp In BCD there are 6 invalid code will present.

imp During addition if invalid codes is present, then 0110 is added to correct it.

$\underbrace{0001}\ \underbrace{0000}$

1010 + 0110 = 10000
1011 + 0110 = 10001
1100 + 0110 = 10010
1101 + 0110 = 10011
1100 + 0110 = 10100
1111 + 0110 → 10101

imp Invalid codes are
1010
1011
1100
1101
1110
1111
} (should not appear in BCD code)

$\underbrace{0001}_{1}\ \underbrace{0011}_{3}$

② using Nand gate to NOR

92

i) Excess 3 Code

→ 4 bit code
→ unweighted code
→ BCD code + 0011
→ ** Self complimenting code . (sum of weightage is 9)
→ only self complimenting code which is unweighted .

eg 2421 = 9
3321 = 9
4311 = 9
5211 = 9

Imp Gray Code (K-map)

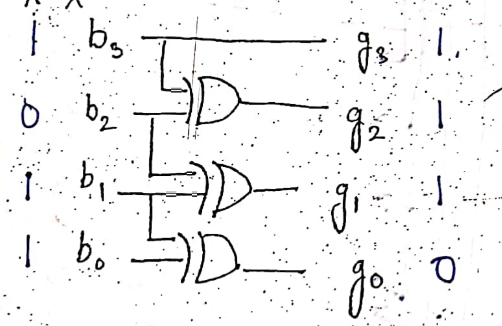
→ Unweighted code
• Sucessive number will differ by 1 bit.
• also called unit distance code / Cyclic code / reflective code
minimum error code .

Imp Binary to Gray Code

1 0 1 1 → 1 1 1 0

$b_3 \; b_2 \; b_1 \; b_0 \rightarrow g_3 \; g_2 \; g_1 \; g_0$

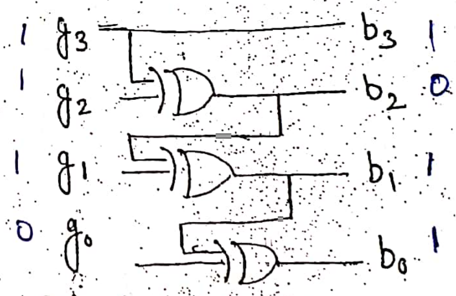
**



B → G
I/P

Gray to Binary

$g_3 \; g_2 \; g_1 \; g_0$ $b_3 \; b_2 \; b_1 \; b_0$
1 1 1 0 1011



G → B
O/P

→ ASCII — 7 bit
EBCDIC — 8 bit
Hollerath — 12 bit

Data Representation

→ Magnitude — (i) Unsigned (+ve) (mag. of value)
(ii) Signed Magnitude (−ve)

→ Complement — (i) 1's Compt
(ii) 2's compt