Introduction to Control Systems

V SEMESTER

EEC-307

Department of Electrical and Electronics Engineering, BVCOE New Delhi

Course Outcomes

CO1: Ability to define, understand various terms related to control system and evaluation of transfer Function

CO2: Ability to apply knowledge of various types of signals in time response of systems

CO3: Ability to analyse frequency response of systems

CO4: Ability to design compensators and controllers

UNIT-I
METHOD TO FIND TRANSFER FUCTION

- Control system definition
- Finding the transfer function using block reduction technique
- Mason's gain formula
- different types of control system

What is control system ?

A control system manages, commands, directs, or regulates the behaviour of other devices or systems using control loops. It can range from a single home heating controller using a thermostat controlling a domestic boiler to large Industrial control systems which are used for controlling processes or machines.

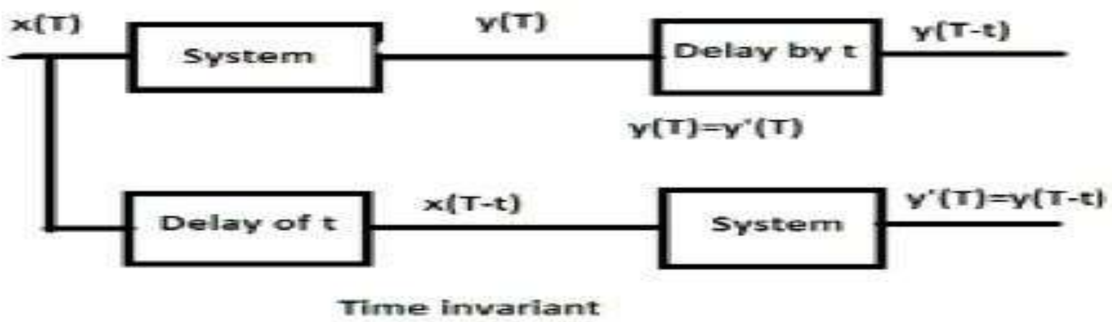
Types of system in control system:-

1.Linear and non linear system

A system is known as linear if and only if it posses both homogeneity and superposition properties.

A system is said to be non linear if it does not satisfy the superposition and homogeneity properties.

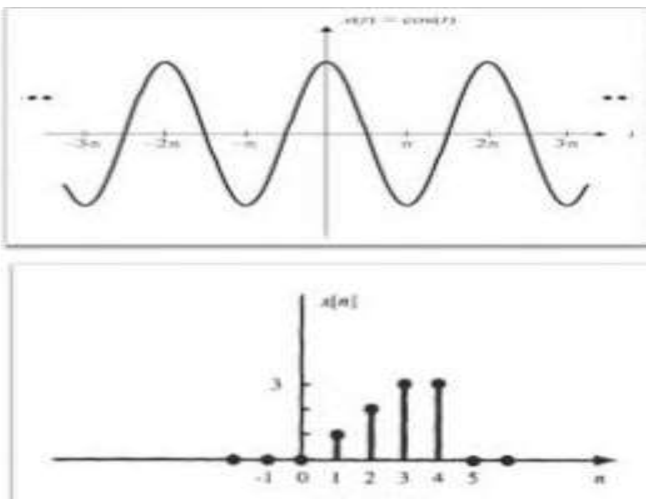
2.Time invariant and time varying system



3.Continuous time and discrete time system

The system in which the variables are the function of continuous time variable ‘t’ is called continuous time system.

The system in which the variables are defined at only instant of time is called discrete time system.



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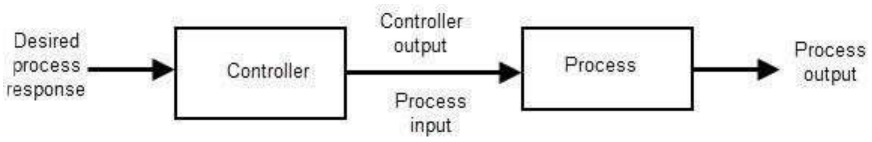
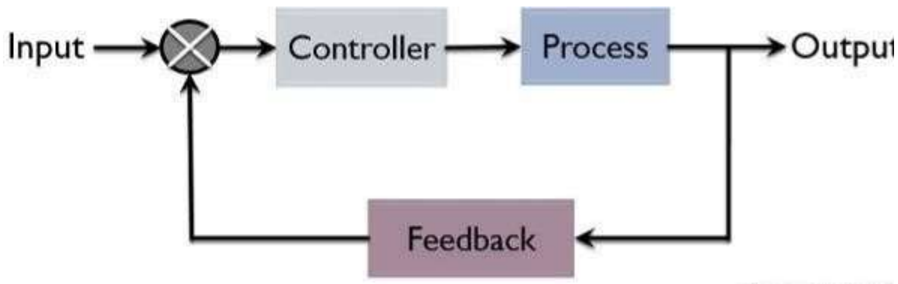
Open loop system

The open loop control system is also known as control system without feedback or non feedback control systems. In open loop systems the control action is independent of the desired output.



Closed loop system

Closed loop control systems are also known as feedback control systems. In closed loop control system the control action is dependent on the desired output.



What is transfer function?

The transfer function is defined as the ratio of Laplace transform of the output to the Laplace transform of the input with all initial conditions are zero.

$$G(s)=C(s)/R(s)$$

where $C(s)$ is output function and,

$R(s)$ is input function

- What is the block diagram?

A block diagram may represent a single component or a group of component ,but each block is completely characterized by a transfer function. The transfer function is an expression which relates output to input in s domain. Transfer function does not give any information about the internal structure of the system.

CONTROL SYSTEM

- A control system manages, commands, directs, or regulates the behavior of other devices or systems using control loops. It can range from a single home heating controller using a thermostat controlling a domestic boiler to large Industrial control systems which are used for controlling processes or machines.
- For continuously modulated control, a feedback controller is used to automatically control a process or operation. The control system compares the value or status of the process variable (PV) being controlled with the desired value or set point (SP), and applies the difference as a control signal to bring the process variable output of the plant to the same value as the set point.

Open loop and Closed loop system

- here are two common classes of control action: open loop and closed loop. In an open-loop control system, the control action from the controller is independent of the process variable. An example of this is a central heating boiler controlled only by a timer. The control action is the switching on or off of the boiler. The process variable is the building temperature. This controller operates the heating system for a constant time regardless of the temperature of the building.
- In a closed-loop control system, the control action from the controller is dependent on the desired and actual process variable. In the case of the boiler analogy, this would utilize a thermostat to monitor the building temperature, and feed back a signal to ensure the controller output maintains the building temperature close to that set on the thermostat. A closed loop controller has a feedback loop which ensures the controller exerts a control action to control a process variable at the same value as the set point. For this reason, closed-loop controllers are also called feedback controllers

Feedback control systems

- In the case of linear feedback systems, a control loop including sensors, control algorithms, and actuators is arranged in an attempt to regulate a variable at a set point (SP). An everyday example is the cruise control on a road vehicle; where external influences such as hills would cause speed changes, and the driver has the ability to alter the desired set speed. The PID algorithm in the controller restores the actual speed to the desired speed in the optimum way, with minimal delay or overshoot, by controlling the power output of the vehicle's engine.
- Control systems that include some sensing of the results they are trying to achieve are making use of feedback and can adapt to varying circumstances to some extent. Open-loop control systems do not make use of feedback, and run only in pre-arranged ways.

Logic control

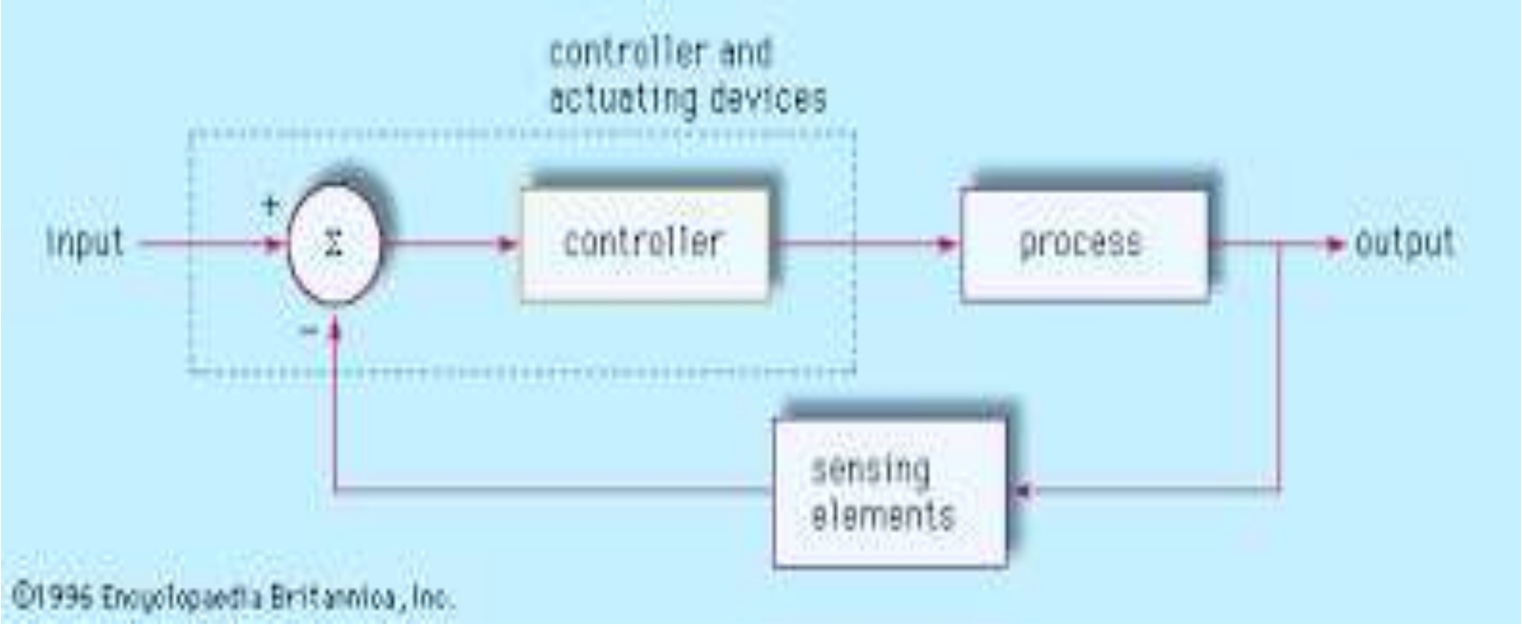
- Logic control systems for industrial and commercial machinery were historically implemented by interconnected electrical relays and cam timers using ladder logic. Today, most such systems are constructed with microcontrollers or more specialized programmable logic controllers (PLCs). The notation of ladder logic is still in use as a programming method for PLCs.
- Logic controllers may respond to switches and sensors, and can cause the machinery to start and stop various operations through the use of actuators. Logic controllers are used to sequence mechanical operations in many applications. Examples include elevators, washing machines and other systems with interrelated operations. An automatic sequential control system may trigger a series of mechanical actuators in the correct sequence to perform a task. For example, various electric and pneumatic transducers may fold and glue a cardboard box, fill it with product and then seal it in an automatic packaging machine.
- PLC software can be written in many different ways – ladder diagrams, SFC (sequential function charts) or statement lists.

On–off control

- On–off control uses a feedback controller that switches abruptly between two states. A simple bi-metallic domestic thermostat can be described as an on-off controller. When the temperature in the room (PV) goes below the user setting (SP), the heater is switched on. Another example is a pressure switch on an air compressor. When the pressure (PV) drops below the set point (SP) the compressor is powered. Refrigerators and vacuum pumps contain similar mechanisms. Simple on–off control systems like these can be cheap and effective.

Components of Control System

- A feedback control system consists of five basic components: (1) input, (2) process being controlled, (3) output, (4) sensing elements, and (5) controller and actuating devices.



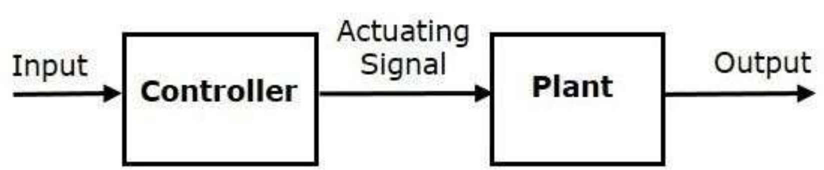
CLASSIFICATION AND TYPES OF CONTROL SYSTEMS

CLASSIFICATION OF CONTROL SYSTEMS

- Open loop control system
- Close loop control system

OPEN LOOP CONTROL SYSTEM

- Open Loop and Closed Loop Control Systems
- Control Systems can be classified as open loop control systems and closed loop control systems based on the **feedback path**.
- In **open loop control systems**, output is not fed-back to the input. So, the control action is independent of the desired output.
- The foll … agram of the open loo…



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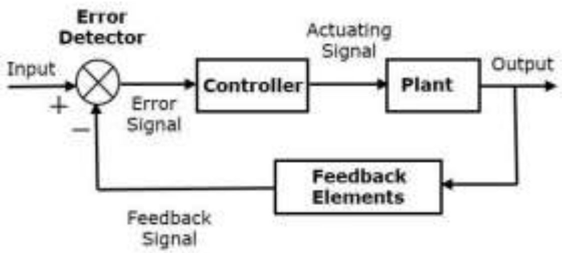
CLOSE LOOP CONTROL SYSTEM

- In **closed loop control systems**, output is fed back to the input. So, the control action is dependent on the desired output.
- The following figure shows the block diagram of negative feedback closed loop control system.
- The error detector produces an error signal, which is the difference between the input and the feedback signal. This feedback signal is obtained from the block (feedback elements) by considering the output of the overall system as an input to this block. Instead of the direct input, the error signal is applied as an input to a controller.
- So, the controller produces an actuating signal which controls the plant. In this combination, the output of the control system is adjusted automatically till we get the desired response.



LOOP AND CLOSED LOOP CONTROL SYSTEMS

OPEN LOOP CONTROL SYSTEMS

•	Control action is independent of the desired output.
•	Feedback path is not present.
•	These are also called as **non-feedback control systems**.
•	Easy to design.
•	These are economical.
•	Inaccurate.

CLOSE LOOP CONTROL SYSTEMS

- Control action is dependent of the desired output.
- Feedback path is present.
- These are also called as **feedback control systems**.
- Difficult to design.
- These are costlier.
- Accurate.

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TYPES OF CONTROL SYSTEMS

LINEAR CONTROL SYSTEM

- In order to understand the **linear control system**, we should first understand the principle of superposition. The principle of superposition theorem includes two the important properties and they are explained below:
- Homogeneity: A system is said to be homogeneous, if we multiply input with some constant A then the output will also be multiplied by the same value of constant (i.e. A).
- Additivity: Suppose we have a system S and we are giving the input to this system as a_1 for the first time and we are getting the output as b_1 corresponding to input a_1. On the second time we are giving input a_2 and correspond to this we are getting the output as b_2.

- Now suppose this time we are giving input as a summation of the previous inputs (i.e. $a_1 + a_2$) and corresponding to this input suppose we are getting the output as ($b_1 + b_2$) then we can say that system S is following the property of additivity. Now we are able to define the **linear control systems** as those **types of control systems** which follow the principle of homogeneity and additivity.
- **Examples of Linear Control System**
- Consider a purely resistive network with a constant DC source. This circuit follows the principle of homogeneity and additivity. All the undesired effects are neglected and assuming ideal behavior of each element in the network, we say that we will get linear voltage and current characteristic. This is the example of a **linear control system**.

- **Examples of Non-linear System**
- A well-known example of a non-linear system is a magnetization curve or no load curve of a DC machine.We will discuss briefly no-load curve of DC machines here: No load curve gives us the relationship between the air gap flux and the field winding mmf. It is very clear from the curve given below that in the beginning, there is a linear relationship between winding mmf and the air gap flux but after this, saturation has come which shows the nonlinear behavior of the curve or characteristics of the **nonlinear control system**.

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ANALOG OR CONTINUOUS SYSTEM

- In these **types of control systems,** we have a continuous signal as the input to the system. These signals are the continuous function of time. We may have various sources of continuous input signal like sinusoidal type signal input source, square type of signal input source; the signal may be in the form of continuous triangle etc.

DIGITAL OR DISCRETE SYSTEM

- In these types of control systems, we have a discrete signal (or signal may be in the form of pulse) as the input to the system. These signals have a discrete interval of time. We can convert various sources of continuous input signal like sinusoidal type signal input source, square type of signal input source etc into a discrete form using the switch.
- Now there are various advantages of discrete or digital system over the analog system and these advantages are written below:
- Digital systems can handle nonlinear control systems more effectively than the analog type of systems.

- Power requirement in case of a discrete or digital system is less as compared to analog systems.
- Digital system has a higher rate of accuracy and can perform various complex computations easily as compared to analog systems.
- Reliability of the digital system is more as compared to an analog system. They also have a small and compact size.
- Digital system works on the logical operations which increases their accuracy many times.
- Losses in case of discrete systems are less as compared to analog systems in general.

SINGLE INPUT AND SINGLE OUTPUT SYSTEM

- These are also known as SISO type of system. In this, the system has single input for a single output. Various example of this kind of system may include temperature control, position control system, etc.

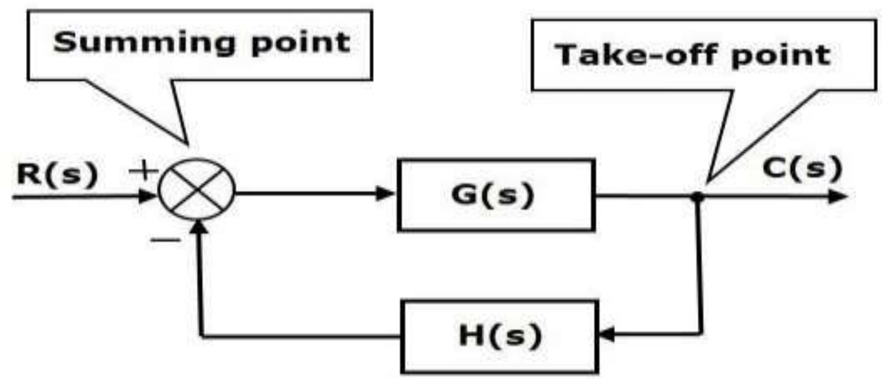
MULTIPLE INPUT AND MULTIPLE OUTPUT SYSTEM

- These are also known as MIMO type of system. In this, the system has multiple outputs for multiple inputs. Various example of this kind of system may include PLC type system etc.

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a **block,** the **summing point** and the **take-off point**. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals.

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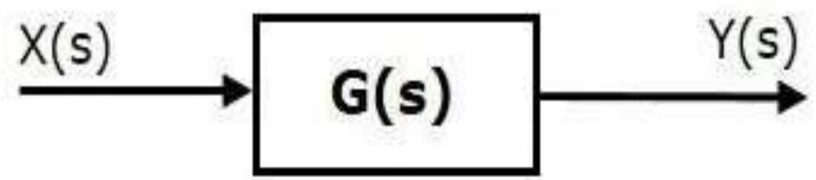
Block

The transfer function of a component is represented by a block.
Block has single input and single output.
The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).



Transfer Function $G(s)=Y(s)/X(s)$

$$\Rightarrow Y(s)=G(s)X(s)$$

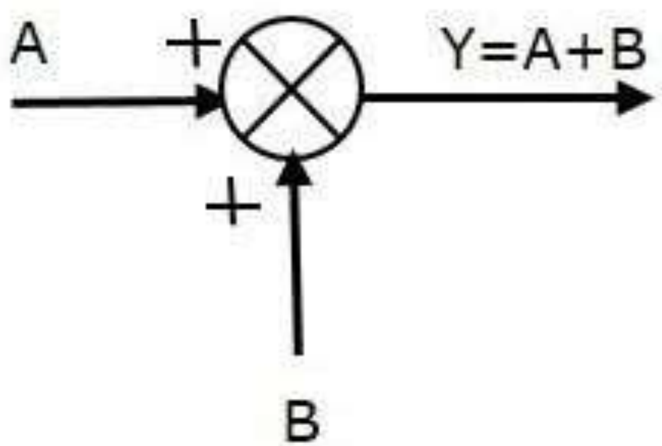
Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

i.e. Y = A + B.



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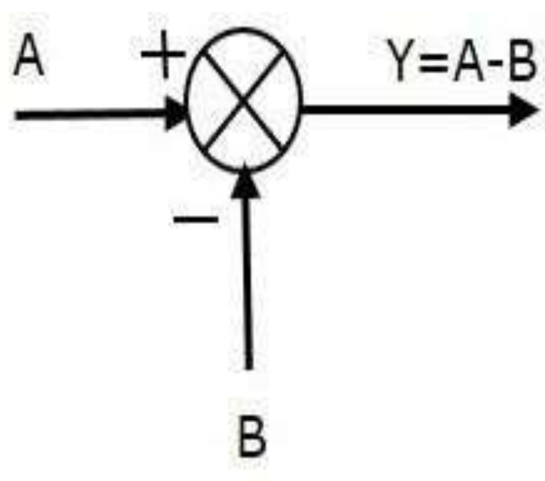
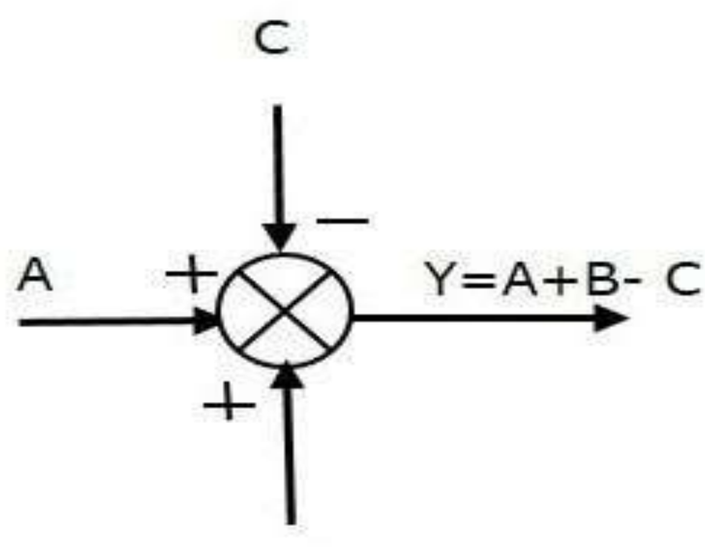
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

Y = A + (-B) = A - B.



The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as



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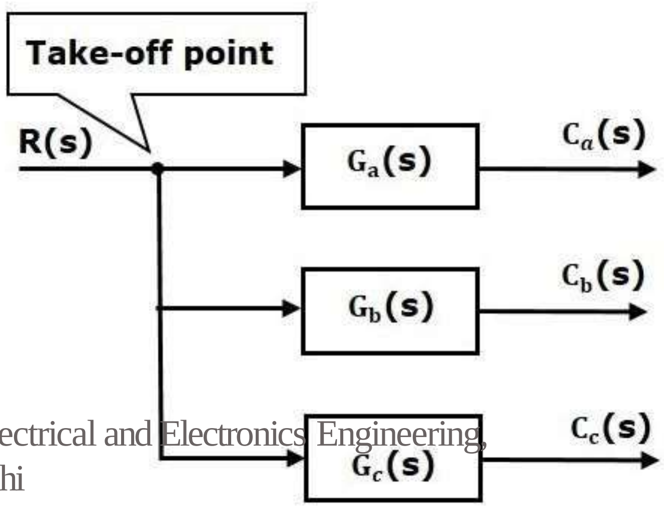
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



Typical Block Diagram Forms

Cascaded Form / Series Form:

–Components or sub-systems of a system are connected in series each having its own transfer function

–Overall transfer function is product of individual transfer functions

U(s) Y(s)

Transfer Function: $G(s)=Y(s)/U(s)=G1(s)G2(s)G3(s)$

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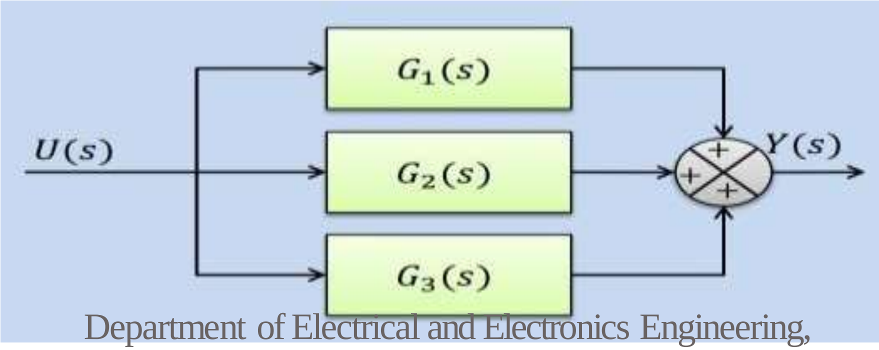
Typical Block Diagram Forms

Parallel Form:

–Components or sub-systems of a system are connected in parallel

–Overall transfer function is sum of individual transfer functions



Transfer Function:
$G(s)=Y(s)/U(s)=G1(s)+G2(s)+G3(s)$

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Typical Block Diagram Forms

Feedback Form:

–One component is present in the feedback loop of another component

Transfer Function:

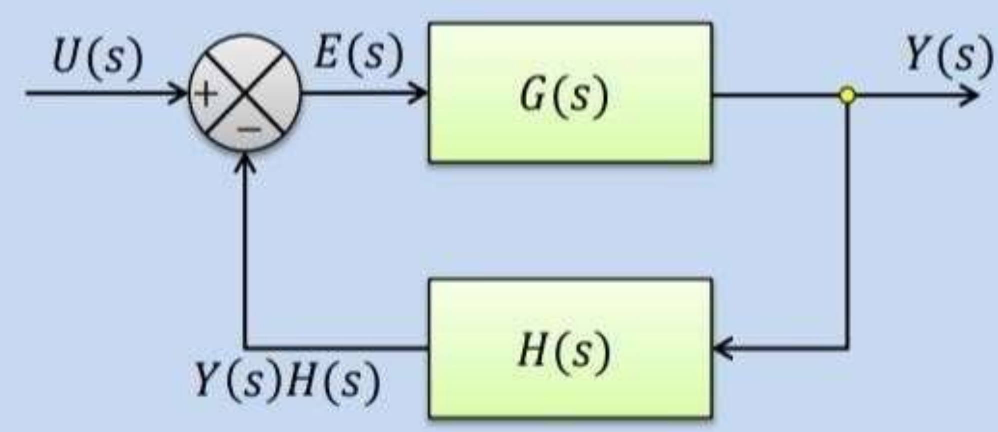
$$Y(s) = G(s)E(s)$$

$$Y(s) = G(s)[U(s) - Y(s)H(s)]$$

$$Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

$$Y(s)[1 + G(s)H(s)] = G(s)U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



Negative Feedback Loop

Block Diagram of a System

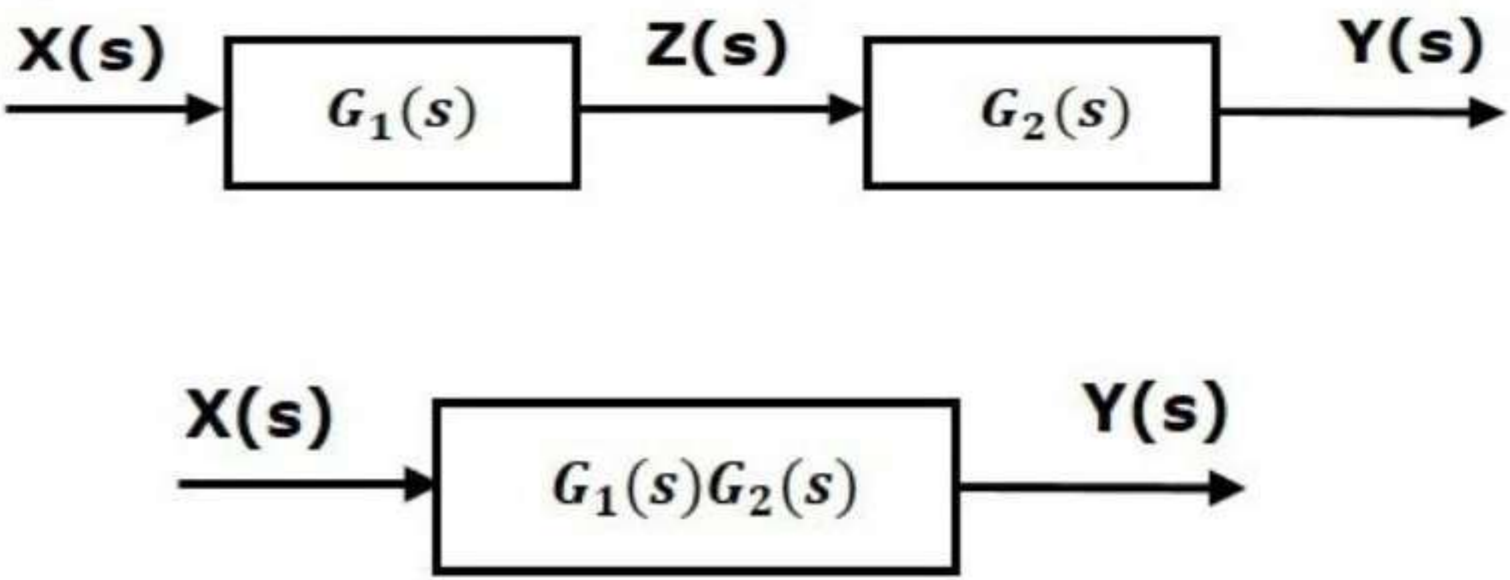
- It is a short hand pictorial representation of the system which depicts
  - –Each functional component or sub-system and
  - –Flow of signals from one sub-system to another
- Components of a block diagram:
  - –Blocks to represent components
  - –Arrows to indicate direction of signal flow
  - –Summing points to show merging signals
  - –Take off points to indicate branching of signals

Block Diagram Reduction

- Block diagram reduction refers to simplification of block diagrams of complex systems through certain rearrangements
- Simplification enables easy calculation of the overall transfer function of the system
- Simplification is done using certain rules called the ‘rules of block diagram algebra’
- All these rules are derived by simply algebraic manipulations of the equations representing the blocks

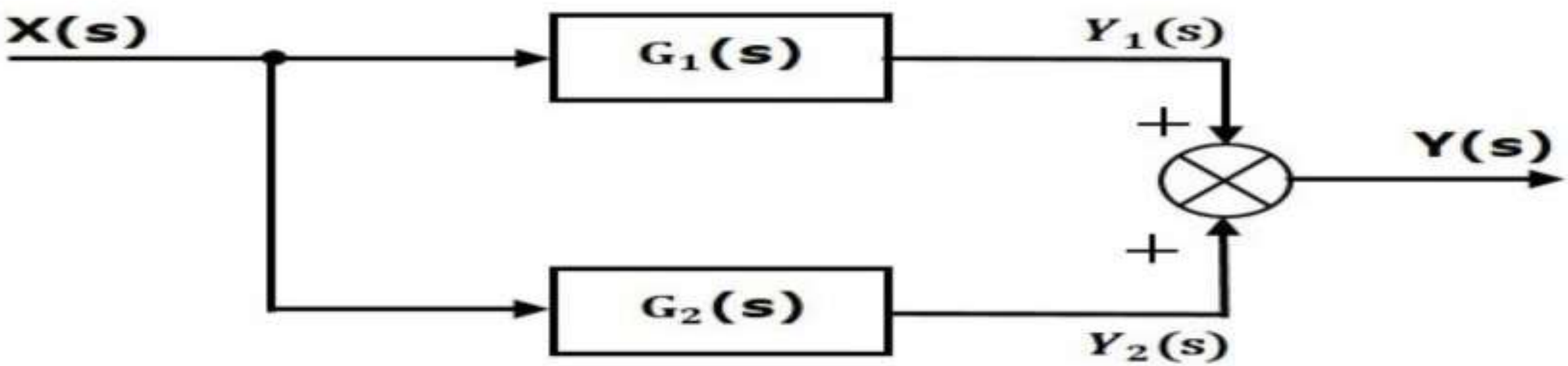
Rules of Block Diagram Algebra

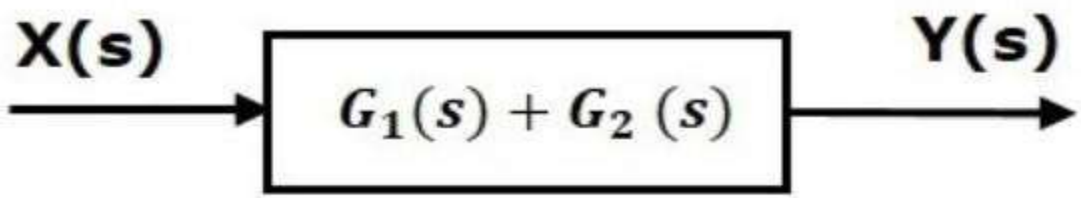
1.Combining blocks in cascade



Rules of Block Diagram Algebra

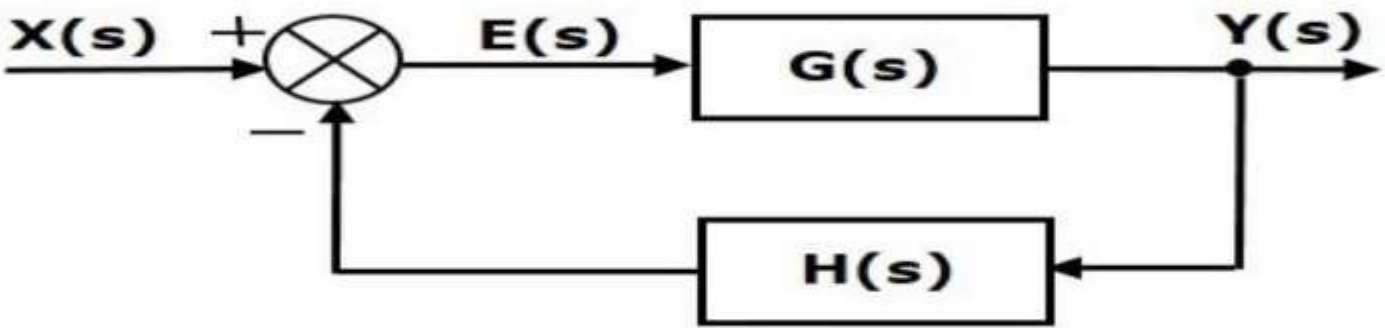
2.Combining blocks in parallel

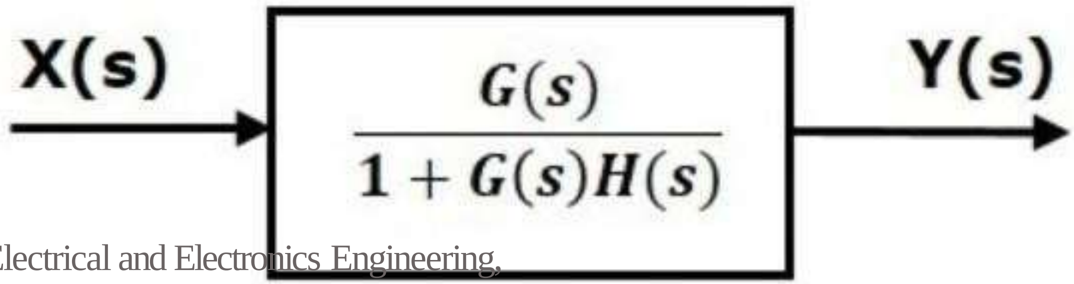




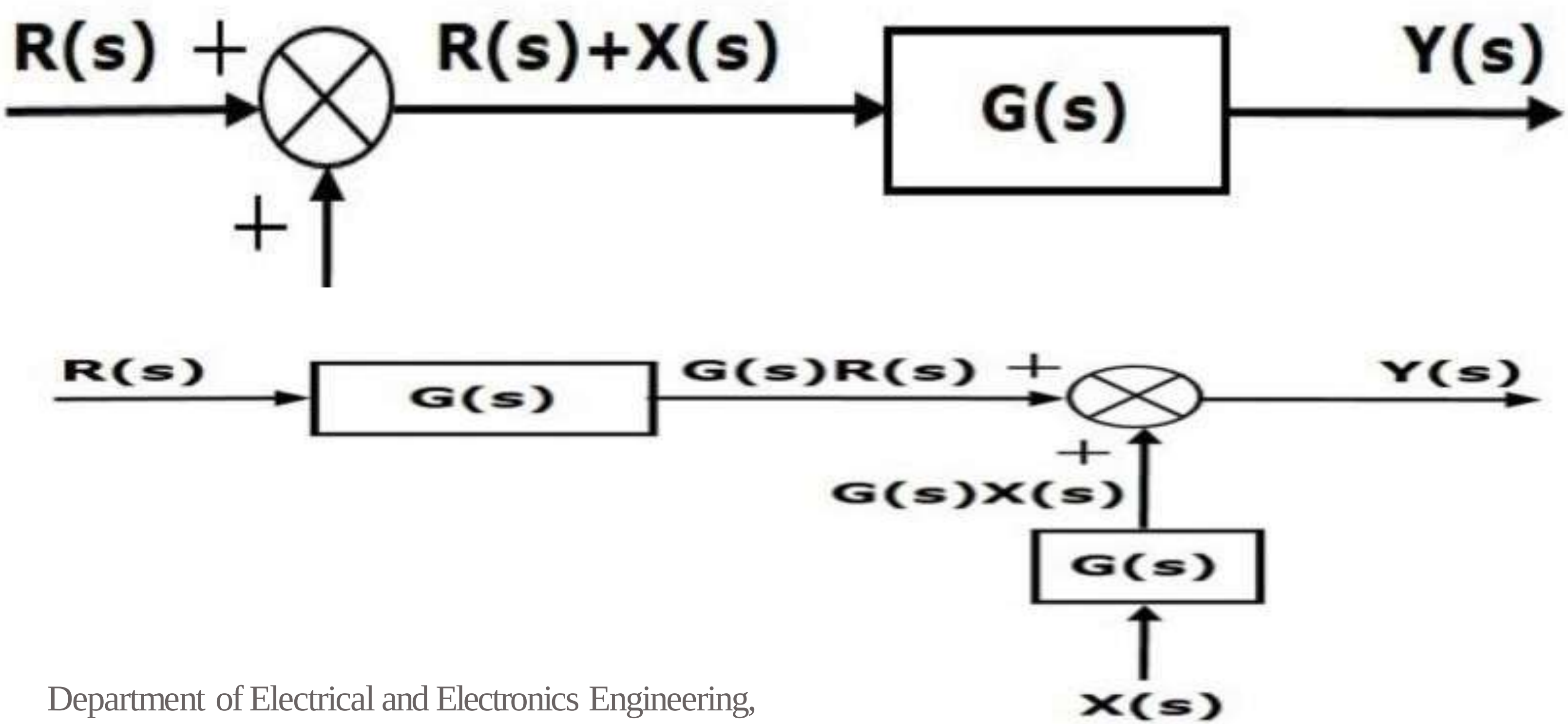
Rules of Block Diagram Algebra

3.Eliminating a feedback loop

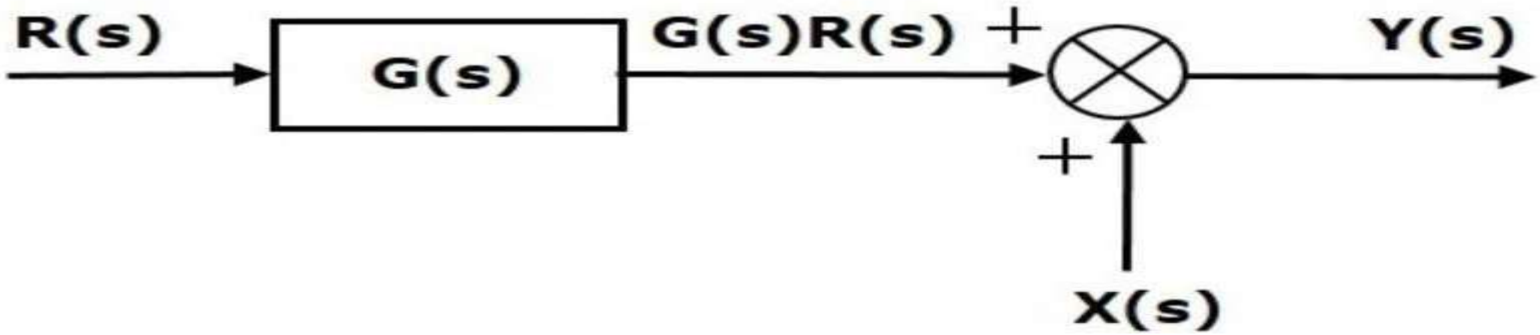


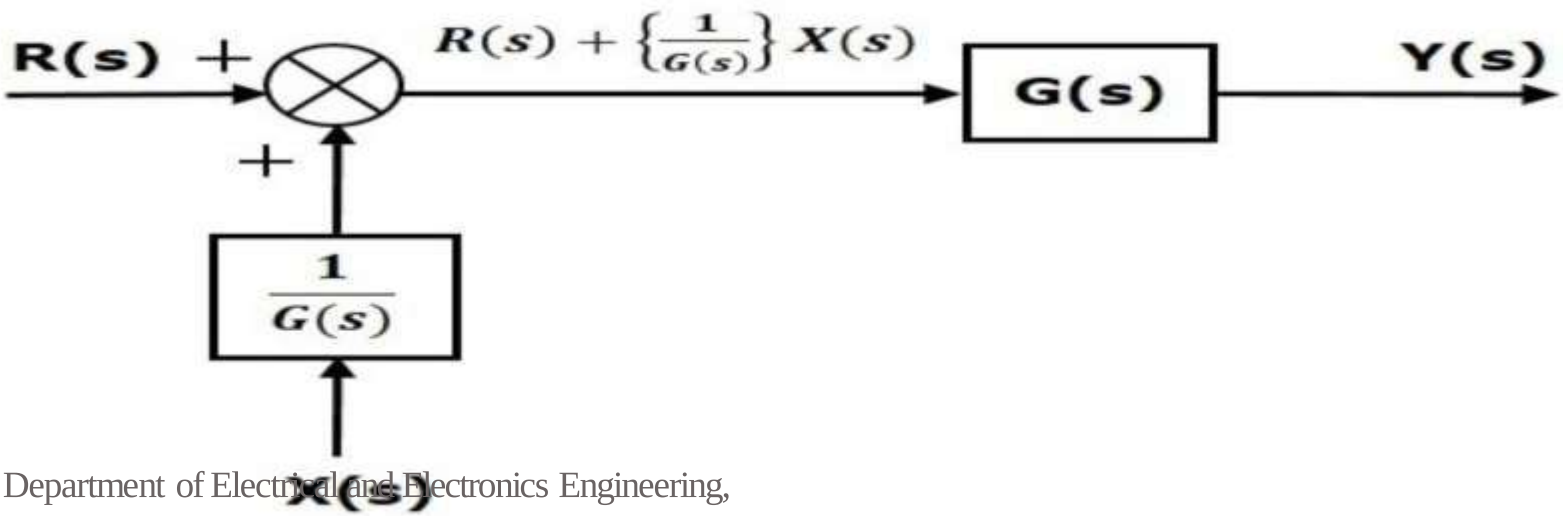


4.Moving a summing point after a block



5.Moving a summing point ahead of a block





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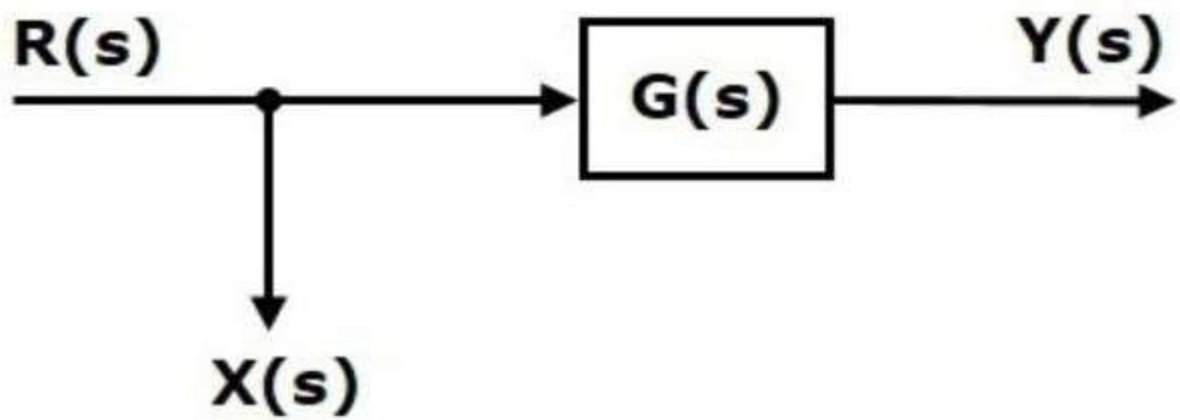
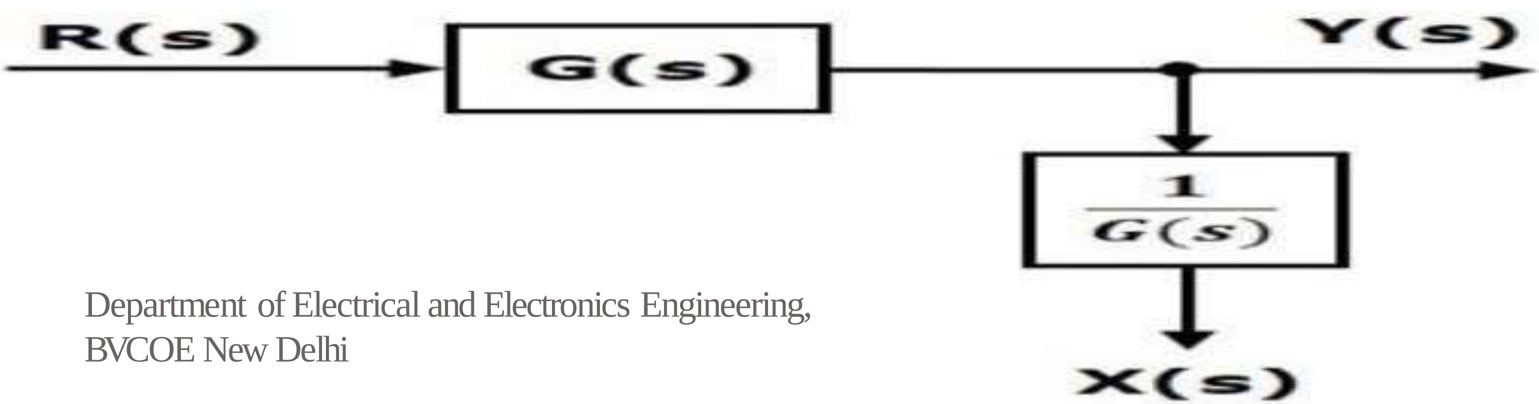
Rules of Block Diagram Algebra

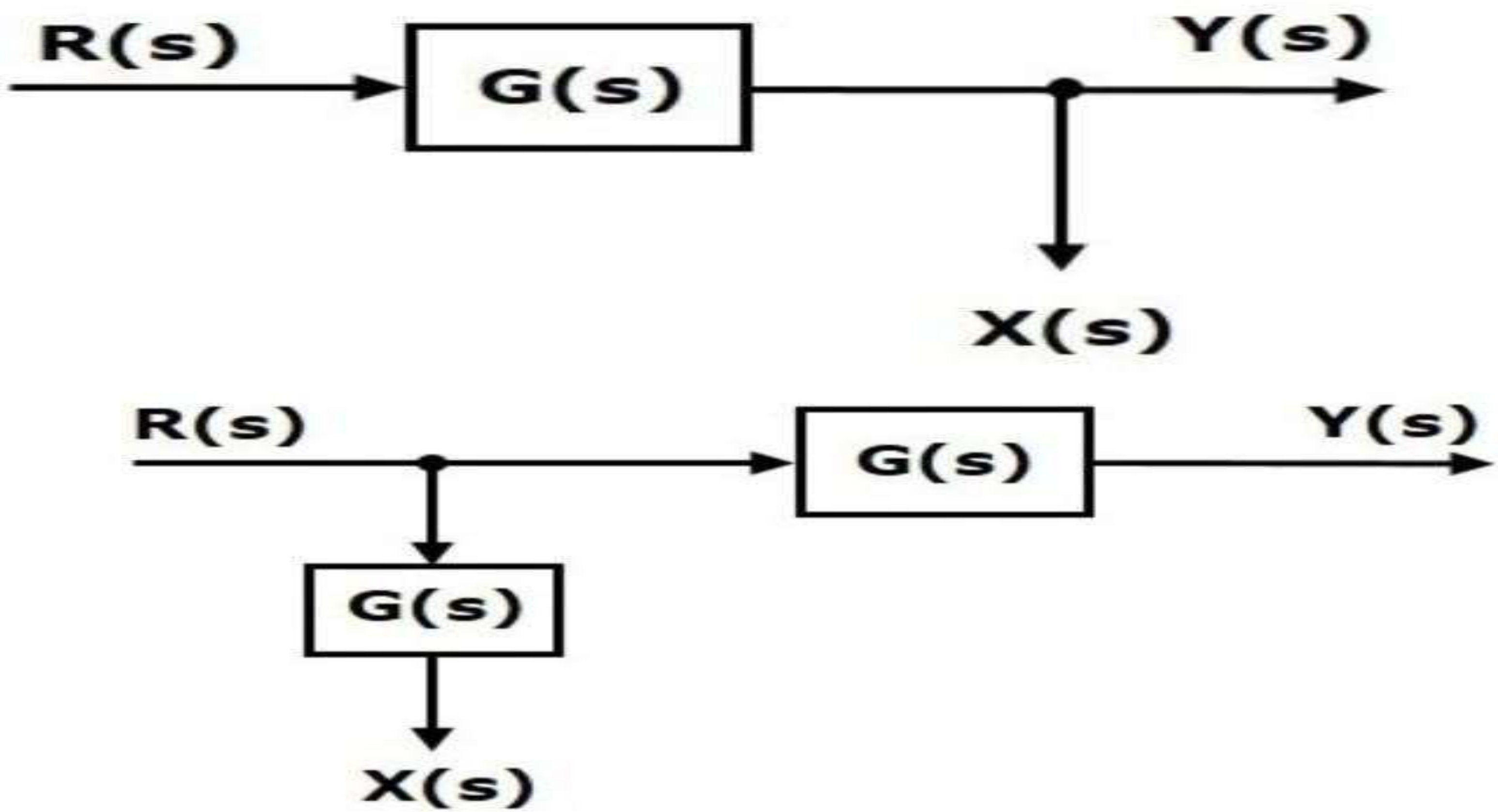
6.Moving a take-off point after a block





Rules of Block Diagram Algebra

7.Moving a take-off point ahead of a block



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Mason's Gain Formula

- **Mason's gain formula (MGF)** is a method for finding the transfer function of a linear signal-flow graph (SFG).
- The formula was derived by Samuel Jefferson Mason, whom it is also named after.
- MGF is an alternate method to finding the transfer function algebraically by labeling each signal, writing down the equation for how that signal depends on other signals, and then solving the multiple equations for the output signal in terms of the input signal.

Mason's Gain Formula

- MGF provides a step by step method to obtain the transfer function from a SFG. Often, MGF can be determined by inspection of the SFG.

- The method can easily handle SFGs with many variables and loops including loops with inner loops.

- MGF comes up often in the context of control systems and digital filters because control systems and digital filters are often represented by SFGs.

Formula

- *Mason's gain formula is*

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

- Where,
- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between R(s)R(s) and C(s)C(s)
- $\mathbf{P_i}$ is the i^{th} forward path gain

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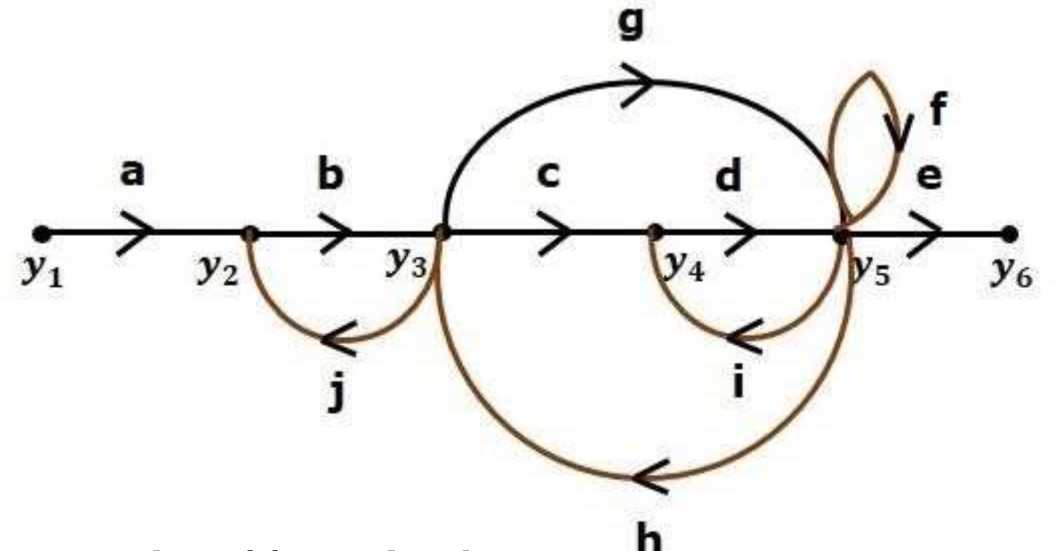
Formula

- Δ= 1− (*Sum of all individual loop gain*) + (*Sum of gain products of all possible two non touching loops*) – (*Sum of gain products of all possible three non touching loops*) +…..

- Δ_i *is obtained from Δ by removing the loops which are touching the* i^{th} *forward path.*

Definitions

- **Path**: a continuous set of branches traversed in the direction that they indicate.
- **Forward path**: A path from an input node to an output node in which no node is touched more than once.
- **Loop**: A path that originates and ends on the same node in which no node is touched more than once.
- **Path gain**: the product of the gains of all the branches in the path.
- **Loop gain**: the product of the gains of all the branches in the loop.



Number of forward paths, **N = 2**.

First forward path is - $\mathbf{y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6}$

First forward path gain, $\mathbf{p_1 = abcde}$

Second forward path is - $\mathbf{y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6}$

Second forward path gain, $\mathbf{p_2 = abge}$

Number of individual loops, **L = 5.**

Loops are - $\mathbf{y_2 \rightarrow y_3 \rightarrow y_2}$ **,** $\mathbf{y_3 \rightarrow y_5 \rightarrow y_3}$ **,** $\mathbf{y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_5}$ **,** $\mathbf{y_4 \rightarrow y_5 \rightarrow y_4}$ **and** $\mathbf{y_4 \rightarrow y_5}$

Loop gains are - **l1=bj , l2=gh , l3=cdh , l4=di and l5=fl**

Number of two **non-touching loops = 2.**

First non-touching loops pair is - $\mathbf{y_2 \rightarrow y_3 \rightarrow y_2, y_4 \rightarrow y_5 \rightarrow y_4}$**.**

Gain product of first non-touching loops pair, **l1l4=bjdi**

Second non-touching loops pair is - $\mathbf{y_2 \rightarrow y_3 \rightarrow y_2, y_5 \rightarrow y_5}$**.**

Gain product of second non-touching loops pair is - **l1l5=bjf**

Δ=1−(sum of all individual loop gains) + (sum of gain products of all possible two non-touching loops) −(sum of gain products of all possible three non-touching loops)+...

Substitute the values in the above equation,

$\Delta=1-(bj+gh+cdh+di+f)+(bjdi+bjf)-(0)$

$\Delta=1-(bj+gh+cdh+di+f)+bjdi+bjf$

There is no loop which is non-touching to the first forward path.

So,

$\mathbf{\Delta_1=1,}$

Similarly, $\mathbf{\Delta_2=1}$.

Since, no loop which is non-touching to the second forward path.

Substitute, **N = 2** in Mason's gain formula,

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

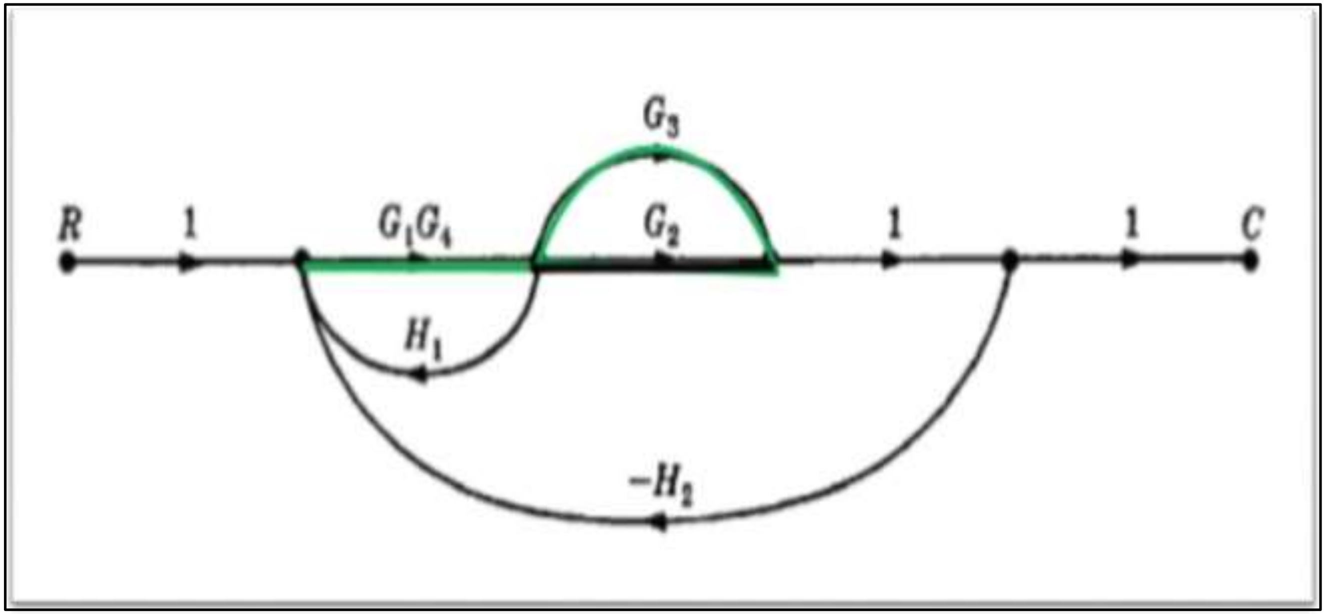
$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

EXAMPLE-1

Let us consider the following signal flow graph for finding transfer function.

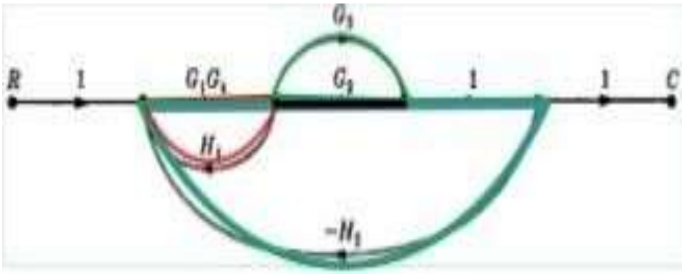


Number of forward paths, **N = 2**.

First forward path gain, $\mathbf{p_1=G_1G_4G_2}$

Second forward path gain, $\mathbf{p_2=G_1G_4G_3}$

Number of individual loops, **L = 3**



Loops are:-

$L_1 = \mathbf{G_1G_4H_1}$
$L_2 = \mathbf{G_1G_4G_2H_2}$
$L_3 = \mathbf{G_1G_4G_3H_2}$

There are **No non-touching loops**

Δ=1−(sum of all individual loop gains) + (sum of gain products of all possible two non-touching loops) −(sum of gain products of all possible three non-touching loops)+...

Substitute the values in the above equation,

$\Delta = 1 - (L_1 + L_2 + L_3)$

$\Delta = 1 - (\mathbf{G_1G_4H_1 + G_1G_4G_2H_2 + G_1G_4G_3H_2})$

There is no loop which is non-touching to the first forward path.

So,

$\boldsymbol{\Delta_1=1}$

Similarly,

$\boldsymbol{\Delta_2=1}$.

Since, no loop which is non-touching to the second forward path.

Substitute, **N = 2** in Mason's gain formula,

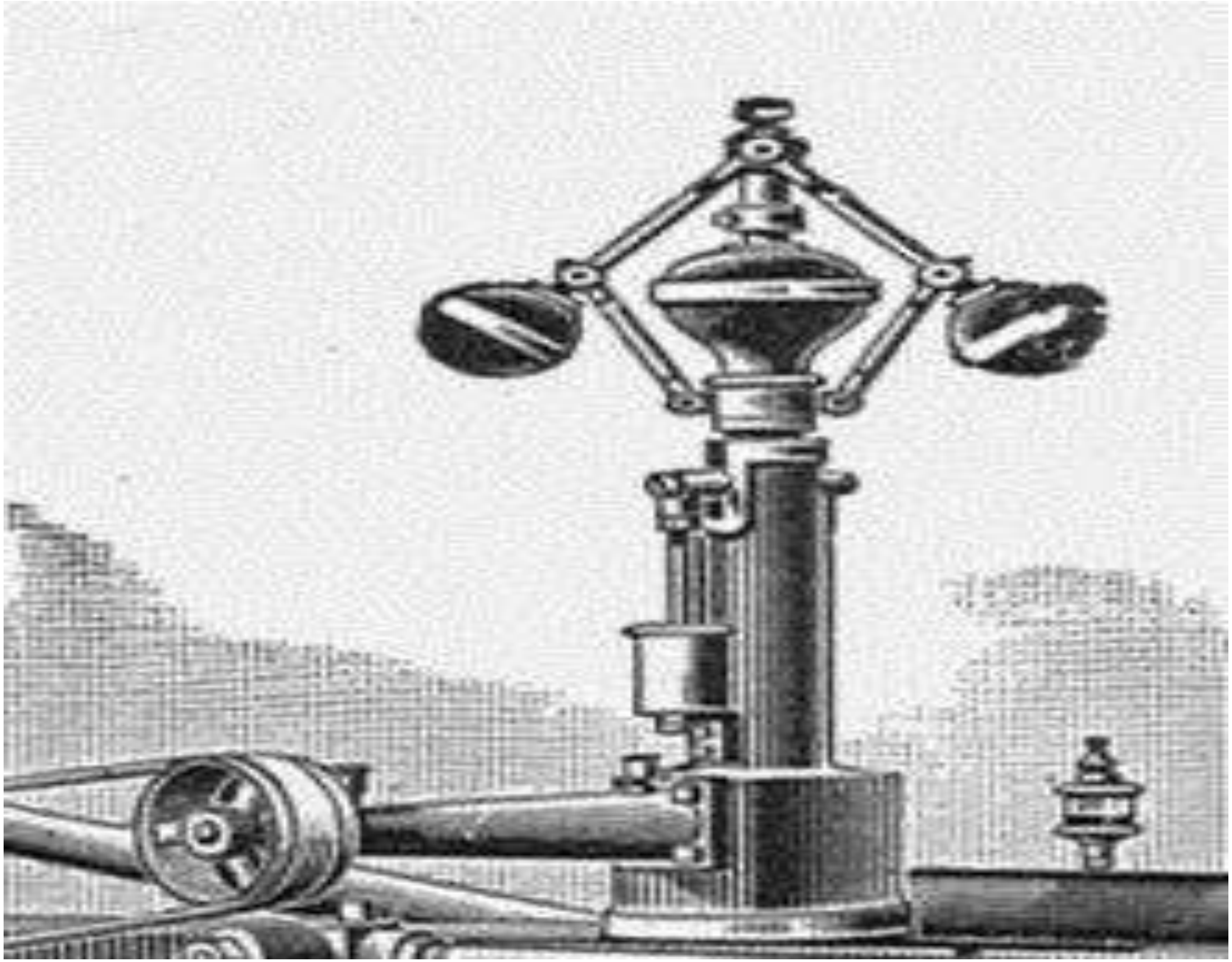
$$\boxed{T=\frac{C(s)}{R(s)}=\frac{\Sigma_{i=1}^{2}P_i\Delta_i}{\Delta}}$$

$$\frac{C}{R}=\frac{P_1\Delta_1+P_2\Delta_2}{\Delta}$$

$$=\frac{G_1G_2G_4+G_1G_3G_4}{1-G_1G_4H_1+G_1G_2G_4H_2+G_1G_3G_4H_2}$$

Transfer function of SFG

$$\boxed{\frac{C}{R}=\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_2G_4H_2+G_1G_3G_4H_2}}$$



CONTROL SYSTEM

- A control system manages, commands, directs, or regulates the behavior of other devices or systems using control loops. It can range from a single home heating controller using a thermostat controlling a domestic boiler to large Industrial control systems which are used for controlling processes or machines.
- For continuously modulated control, a feedback controller is used to automatically control a process or operation. The control system compares the value or status of the process variable (PV) being controlled with the desired value or set point (SP), and applies the difference as a control signal to bring the process variable output of the plant to the same value as the set point.

COMPONENT OF CONTROL SYSTEM

- They are of two types:-

1)Transitional Mechanical system

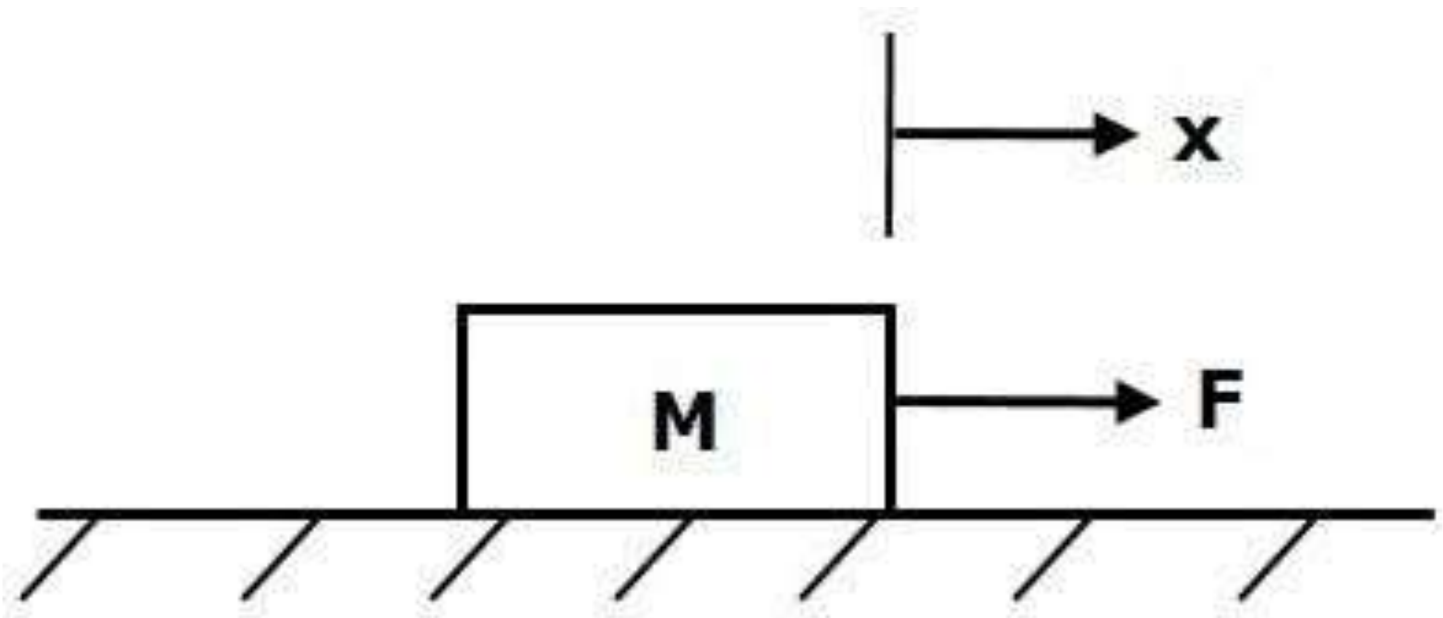
2)Rotational mechanical system

Modeling of Translational Mechanical Systems

- Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.
- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

MASS

- Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.


X
M
F

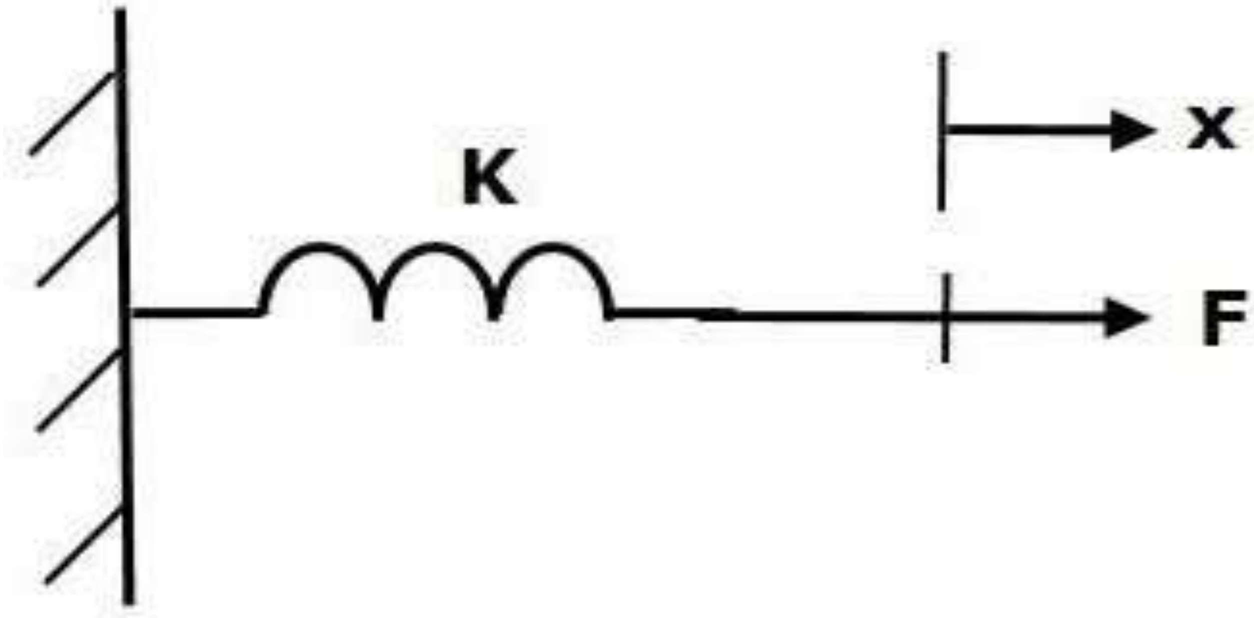
- Fmaa

Fm=Ma=Md2xdt2
F=Fm=Md2xdt2
Where,

- **F** is the applied force
- **F_m** is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Spring

- Spring is an element, which stores **potential energy**. If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.



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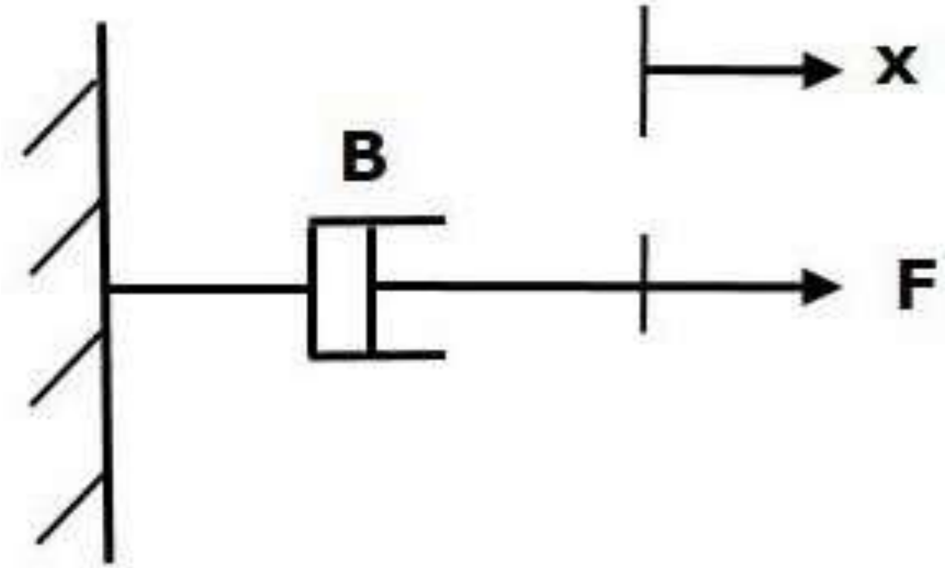
- FaxFax
- ⇒Fk=Kx⇒Fk=Kx
- F=Fk=Kx
-

Where,
- **F** is the applied force
- $\mathbf{F_k}$ is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement

Dashpot

- If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.


x
B
F

- FbavFbav
- ⇒Fb=Bv=Bdxdt⇒Fb=Bv=Bdxdt
- F=Fb=Bdxdt
-

  Where,
- $\mathbf{F_b}$ is the opposing force due to friction of dashpot
- **B** is the frictional coefficient
- **v** is velocity
- **x** is displacement
-

Modeling of Rotational Mechanical Systems

- Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.
- If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

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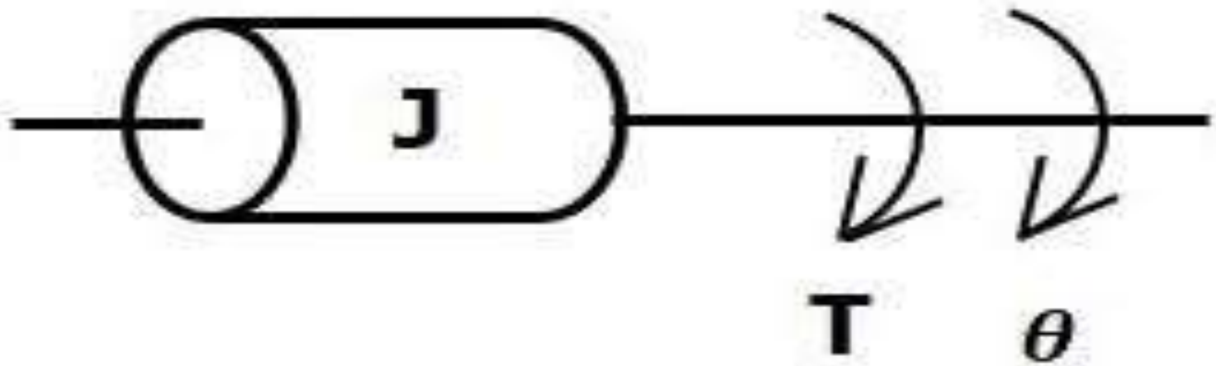
Moment of Inertia

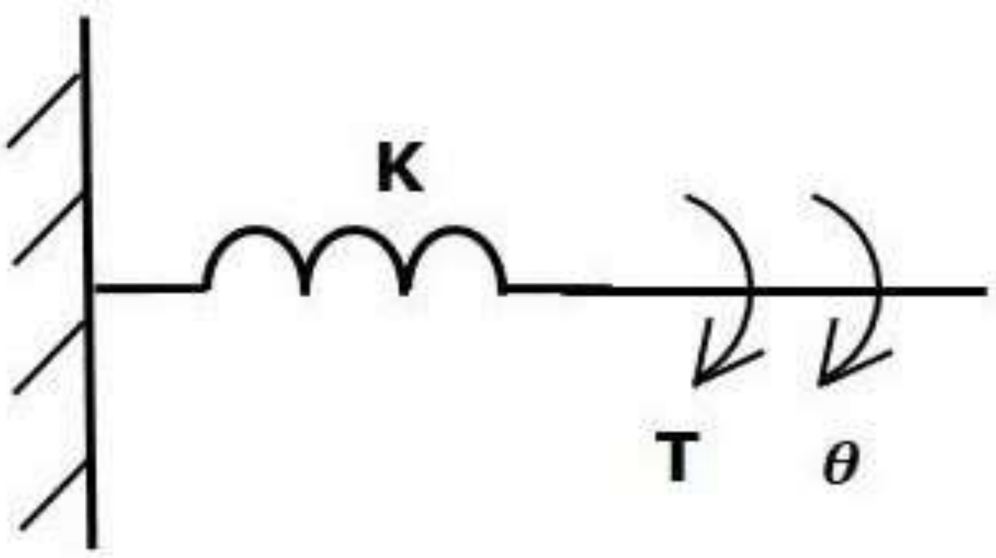
- In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.
- If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.


J
T
θ

- TjaαTjaα
- ⇒Tj=Jα=Jd2θdt2⇒Tj=Jα=Jd2θdt2
- T=Tj=Jd2θdt2T=Tj=Jd2θdt2
- Where,
- **T** is the applied torque
- **T_j** is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration
- **θ** is angular displacement

Torsional Spring

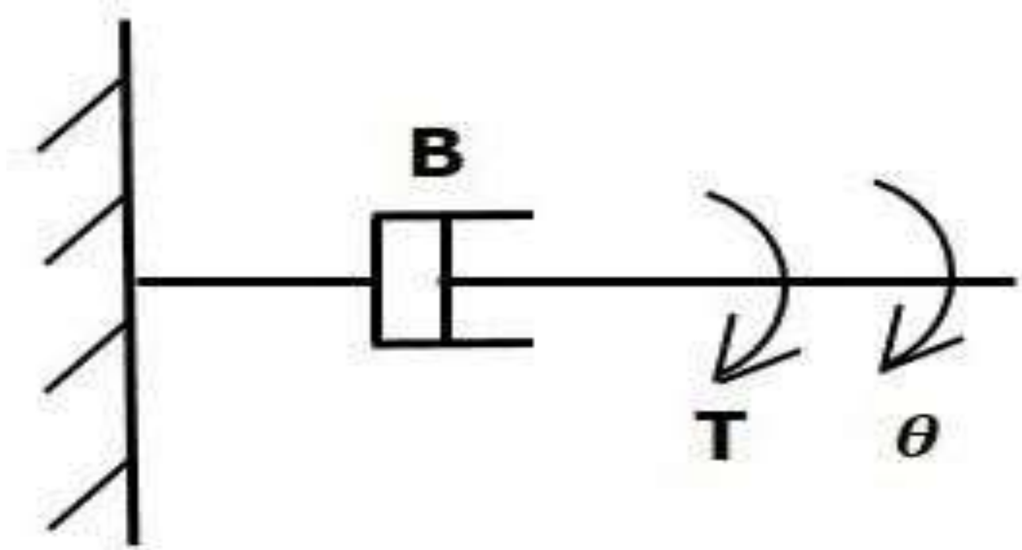
- In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.
- If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.


K
T
θ

- TkaθTkaθ
- ⇒Tk=Kθ⇒Tk=Kθ
- T=Tk=KθT=Tk=Kθ
- Where,
- **T** is the applied torque
- **T_k** is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

Dashpot

- If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.


B
T
θ

- TbaωTbaω
- ⇒Tb=Bω=Bdθdt⇒Tb=Bω=Bdθdt
- T=Tb=BdθdtT=Tb=Bdθdt
- Where,
- $\mathbf{T_b}$ is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement

SERVOMOTORS

- Servomotors are also called control motors. These motors are used in feedback control systems as output actuators.
- The principle of the Servomotor is similar to that of the other electromagnetic motor, but the construction and the operation are different.
- Their power rating varies from a fraction of a watt to a few hundred watts.
- The rotor inertia of the motors is low and have a high speed of response. The rotor of the Motor has the long length and smaller diameter.
- They operate at very low speed and sometimes even at the zero speed.
- The servo motor is widely used in radar and computers, robot, machine tool, tracking and guidance systems, processing controlling, etc.
- They have larger size than that of conventional motors of similar power rating

Classification of servomotors

They are classified as AC and DC Servo Motor. The AC servomotor is further divided into two types.

1.Two phase ac servomotor

2.Three Phase AC Servo Motor

DC Servo Motor

- DC Servo Motors are separately excited DC motor or permanent magnet DC motors.
- The figure (a) shows the connection of Separately Excited DC Servo motor and the figure (b) shows the armature MMF and the excitation field MMF in quadrature in a DC machine.
- This provides a fast torque response because torque and flux are decoupled. Therefore, a small change in the armature voltage or current brings a significant shift in the position or speed of the rotor.
- Most of the high power servo motors are mainly DC.
- The speed torque characteristics are also shown below.

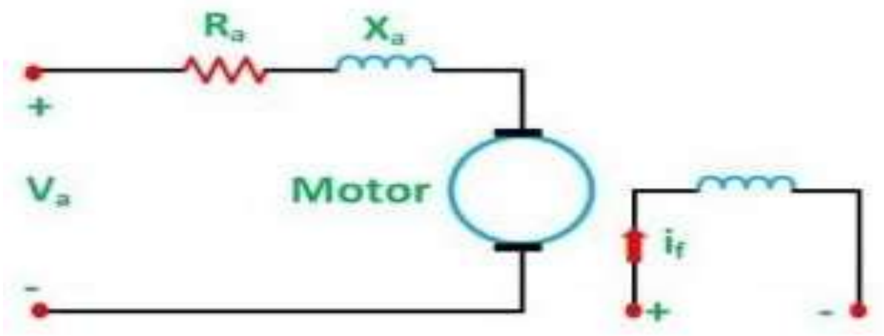


Figure a

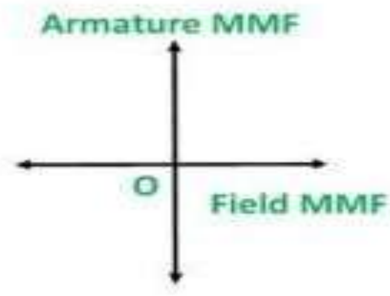
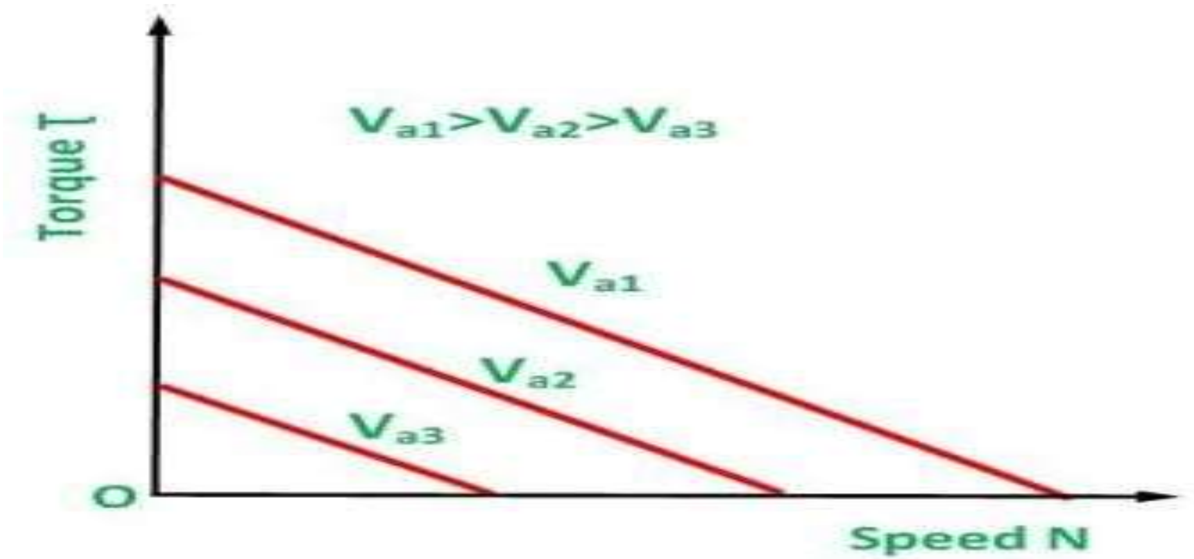


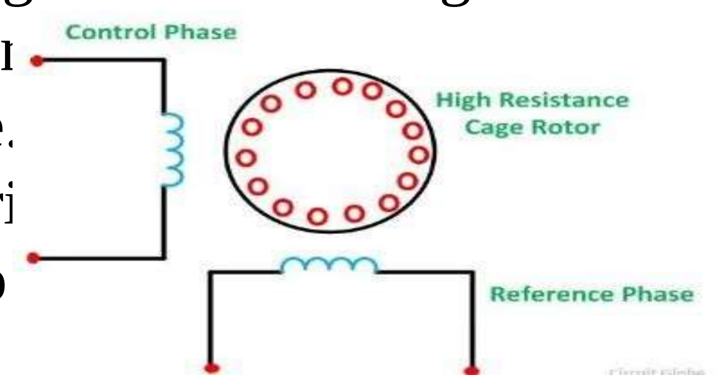
Figure b



A.C. SERVOMOTORS

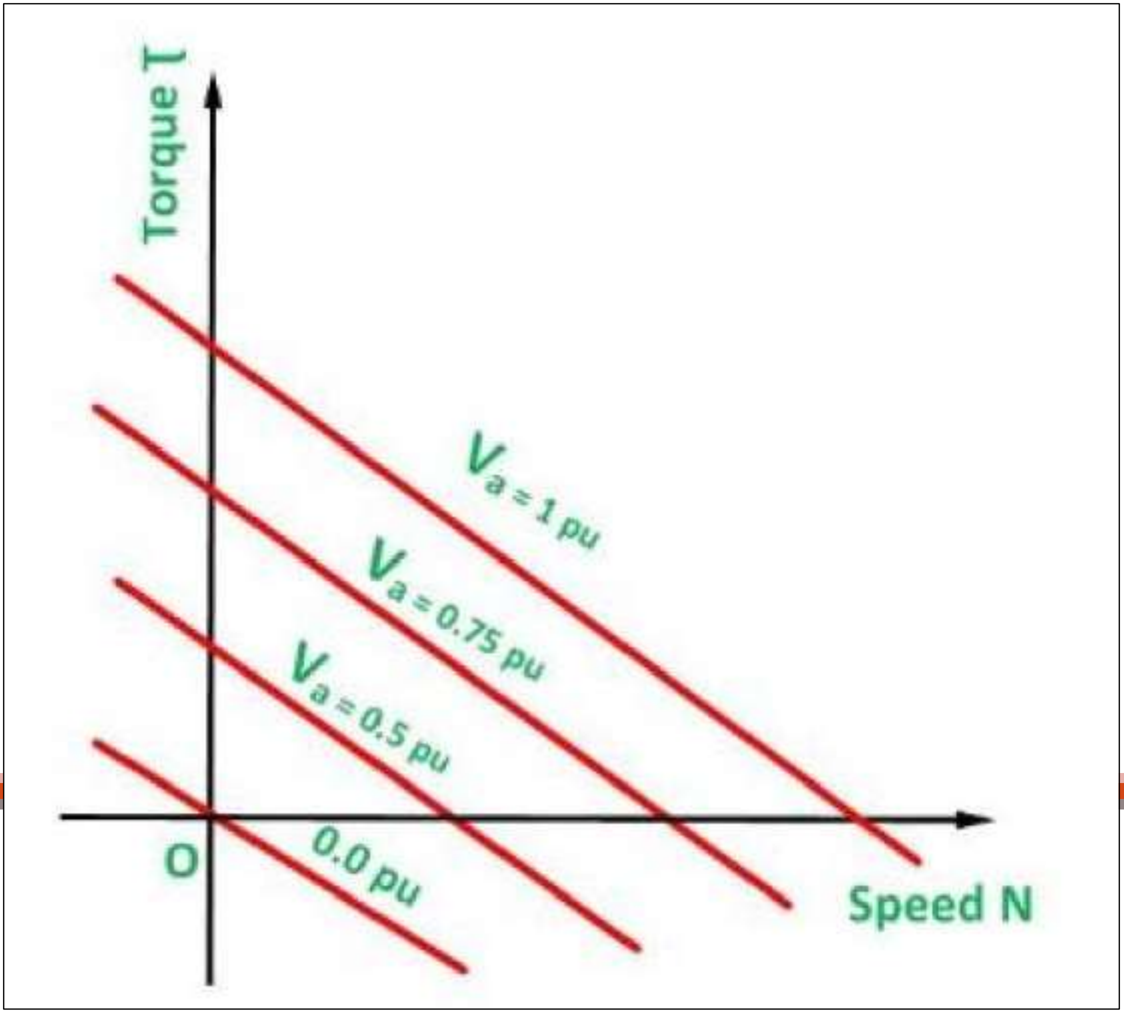
- At present most of the ac servomotors are of two phase squirrel cage induction type for lower power application. Recently,three phase squirrel cage induction motors have been modified for application in high power servo systems.
- *TWO-PHASE AC SERVOMOTOR:*

The stator has two distributed windings which are displaced from eacg other by 90 electrical degrees.one winding called One winding is known as a Reference oı lied from a constant voltage source. Control Phase, and it is provided with a vari diagram of the two Phase AC Servo mo



The control phase is usually supplied from a servo amplifier. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltage. By reversing the phase difference from leading to lagging or vice versa, the direction of the rotation of the rotor can be reversed.

The torque speed characteristic of the two phase AC servomotor is shown in the figure



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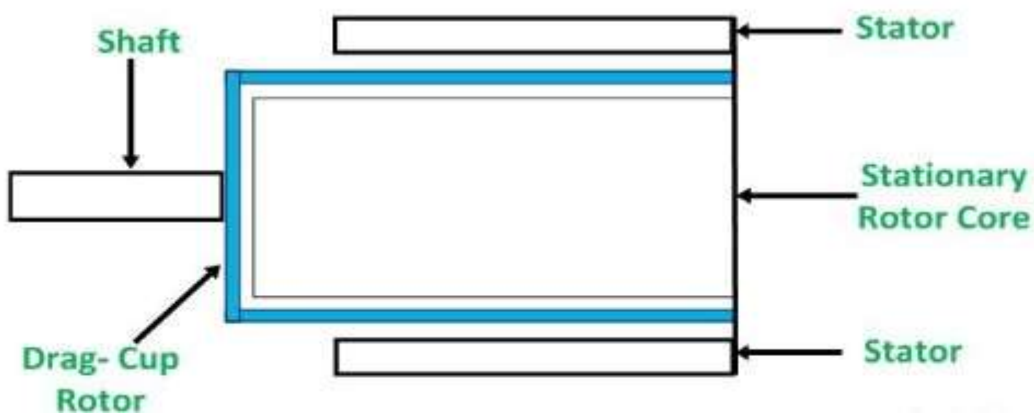
- The negative slope represents a high rotor resistance and provides the motor with positive damping for better stability. The curve is linear for almost various control voltages. The response of the motor to a light control signal is improved by reducing the weight and inertia of the motor in a design known as the Drag Cup Servo motor as shown in the figure. The rotor of the Drag cup servo motor is made of a thin cup of the

nonmagnet iron core is
placed in t arrangement
completes the motor is
made of th which results
in the high



THREE PHASE AC SERVOMOTORS

- Three phase induction motors with the voltage control are used as a Servo motor for the applications in the high power Servo systems. A three phase squirrel cage induction motor is a highly nonlinear coupled circuit device. It is used as a linear decoupled machine by using a control method known as a Vector Control or Field Oriented Control.
- The current in this type of machine is controlled in such a way that the torque and flux are decoupled.The decoupling results in high speed and high torque response.

Intoduction

- An electric motor is an electrical machine that converts electrical energy into mechanical energy.
- Most electric motors operate through the interaction between the motor's magnetic field and electric current in a wire winding to generate force in the form of rotation of a shaft.
- Electric motors may be classified by considerations such as power source type, internal construction, application and type of motion output. In addition to AC versus DC types, motors may be brushed or brushless, may be of various phase
- An electric motor is generally designed for continuous rotation, or for linear movement over a significant distance compared to its size.

Introduction

- A stepper motor (or step motor) is a brushless, synchronous electric motor that can divide a full rotation into a large number of steps.

- The motor's position can be controlled precisely, without any feedback mechanism.

- Stepper motors are similar to switched reluctance motors (which are very large stepping motors with a reduced pole count, and generally are closed-loop commutated.)

Fundamentals of Operation

- ✓ Stepper motors operate differently from DC brush motors, which rotate when voltage is applied to their terminals. Stepper motors, on the other hand, effectively have multiple "toothed" electromagnets arranged around a central gear-shaped piece of iron.
- ✓ The electromagnets are energized by an external control circuit, such as a microcontroller. To make the motor shaft turn, first one electromagnet is given power, which makes the gear's teeth magnetically attracted to the electromagnet's teeth.
- ✓ When the gear's teeth are thus aligned to the first electromagnet, they are slightly offset from the next electromagnet. So when the next electromagnet is turned on and the first is turned off, the gear rotates slightly to align with the next one, and from there the process is repeated.
- ✓ Each of those slight rotations is called a "step," with an integer number of steps making a full rotation. In that way, the motor can be turned by a precise angle.

Stepper motor characteristics

1. Stepper motors are constant power devices.

2. As motor speed increases, torque decreases.

3.The torque curve may be extended by using current limiting drivers and increasing the driving voltage.

4. Steppers exhibit more vibration than other motor types, as the discrete step tends to snap the rotor from one position to another.

5.This vibration can become very bad at some speeds and can cause the motor to lose torque.

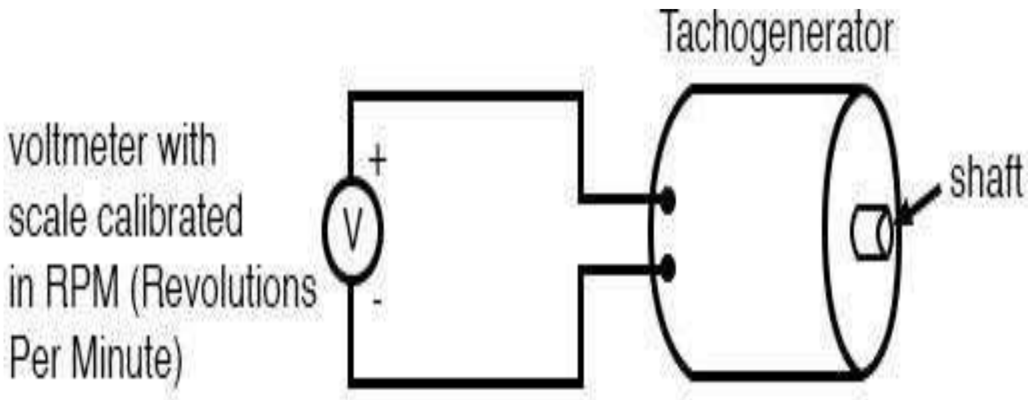
Stepper motor characteristics

6. The effect can be mitigated by accelerating quickly through the problem speeds range, physically damping the system, or using a micro-stepping driver.

7. Motors with a greater number of phases also exhibit smoother operation than those with fewer phases.

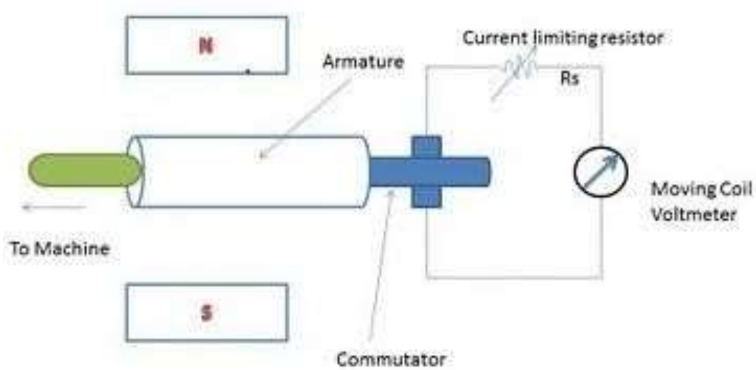
What is a Tachogenerator ?

- A tachogenerator, or tachometer generator, is an electromechanical device used to accurately measure the speed of engines and motors. In fact, a tachogenerator can measure the speed and direction of all manner of rotational devices once connected and will output a voltage proportional to the rotation around its own shaft. In other words, a tachogenerator can convert mechanical energy into electrical energy. A tachogenerator is commonly used to handle voltages between 0 and 10 volts and is classed as a precision instrument. We supply solid shaft, blind shaft and hollow shaft types of tachogenerator.


Tachogenerator
voltmeter with
scale calibrated
in RPM (Revolutions
Per Minute)
+
V
-
shaft

Design of a Techogenerator

- The majority of modern tacho generators are permanent magnet types. These devices use a rotating armature, one end of which is attached to a machine shaft, to measure rotational speed. The armature rotates within a fixed magnetic field, so that its rotation induces electromotive force (voltage) proportional to the shaft's speed. The armature contacts are connected to a voltmeter circuit, which converts the voltage into a speed value.



- Drag cup tacho generators are a less common type which use an aluminum cup rotating within a wound electromagnetic stator; the cup is attached to a shaft. Alternating current is supplied to one winding of the stator, generating eddy currents around the cup. The rotation of the cup induces a proportional voltage in the other stator winding.

Types of Techogenerator

- **Solid Shaft Tachogenerator**

Solid shaft tachogenerators are connected to a shaft which, in turn, is connected to an external rotational device. It is the rotation of the external device which turns the tachogenerator shaft and thus generating a very specific range of voltages in accordance with the speed and direction of the shaft. Tachogenerators can indicate rotational direction due to the fact that if a tachogenerator shaft is reversed, the output voltage polarity will change. The solid-shaft tachogenerator is best suited to high load applications

Hollow Shaft Tachogenerator :

A Hollow-shaft tachogenerator differs internally from solid shaft tachogenerators in that they are designed with four magnetic poles rather than two. This design allows for the tachogenerator to work with lower voltage loads. An example of the use of a DC hollow shaft tachogenerator is to sense the speed of an elevator. The DC tachogenerator is installed in the hoisting equipment on the traction sheave that drives the cables. It enables the accurate control of cable speed to make sure that the lift stops at the right floor and does so smoothly.

Uses & Applications of Tachogenerator

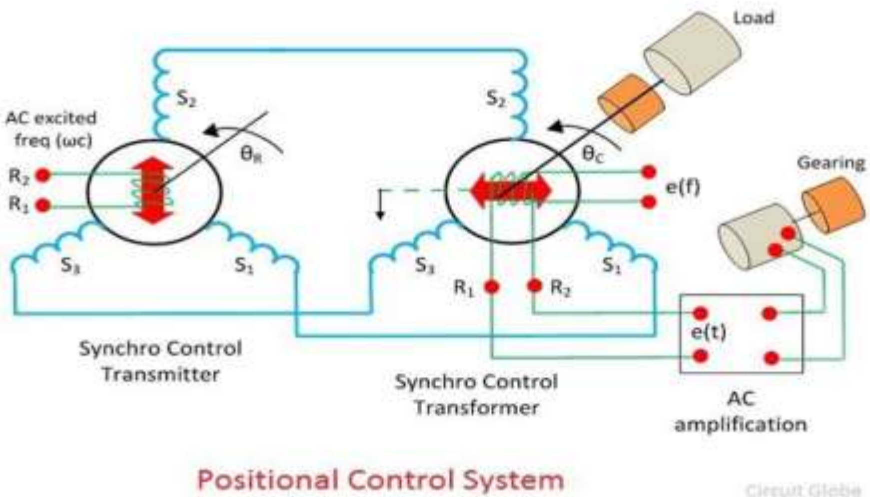
- A tachogenerator can be found in various kinds of machine tool and other equipment where the measurement of speed and direction is essential.

- These include electric motors, conveyor belts, engines, fans and mixers. Consumer products where you would expect to find a tachogenerator would be in a washing machine or a food processor.

- A tachogenerator is also used in voltage control and current limiting circuits to protect equipment from overheating or to protect other components within an electromechanical device.

SYNCHROS

- **Definition:** The Synchro is a type of transducer which transforms the angular position of the shaft into an electric signal. It is used as an error detector and as a rotary position sensor. The error occurs in the system because of the misalignment of the shaft. The transmitter and the control transformer are the two main parts of the synchro.



- **The synchro system is of two types. They are**

1. Control Type Synchro.
2. Torque Transmission Type Synchro.

- **Torque Transmission Type Synchros**

- This type of synchros has small output torque, and hence they are used for running the very light load like a pointer. The control type Synchro is used for driving the large loads.

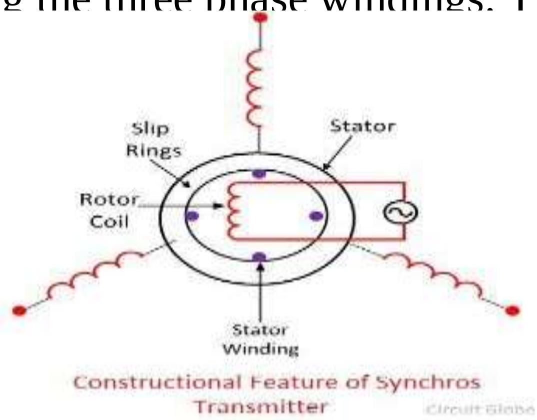
- **Control Type Synchros System**

- The controls synchros is used for error detection in positional control systems. Their systems consist two units. They are

1. Synchro Transmitter
2. Synchro receiver

- The synchro always works with these two parts.

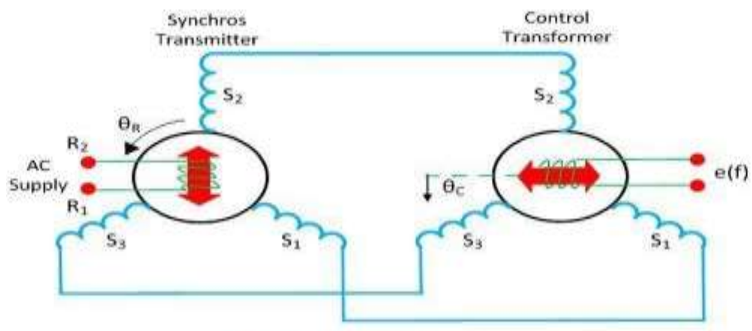
- **Synchros Transmitter –** Their construction is similar to the three phase alternator. The stator of the synchros is made of steel for reducing the iron losses. The stator is slotted for housing the three phase windings. The axis of the stator winding is kept 120° apart from e



- The AC voltage is applie $v_r = \sqrt{2}Sin\omega_c t$ the transmitter and it is expressed as

- Where Vr – r.ms. value of rotor voltage
- ωc – carrier frequency

- Consider the voltage is applied to the rotor of the transmitter as shown in the figure,



Syncho Error Dectector

-

- Let Vs1, Vs2, Vs3 be the r windings S1, S2, and S3 respectively.

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 120°)$$

$$V_{s2n} = kV_r Sin\omega_c t cos\theta_R$$

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 240°)$$

- The three terminals of the stator windings are

$$V_{s1s2} = V_{s1n} - V_{s2n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 240^\circ) Sin\omega_c t$$

$$V_{s3s2} = V_{s2n} - V_{s3n}$$

$$V_{s1s2} = \sqrt{3}kV_r Sin(\theta_R + 120^\circ) Sin\omega_c t$$

$$V_{s3s1} = V_{s3n} - V_{s1n}$$

$$V_{s3s1} = kV_r Sin\omega_c t Sin\theta_R$$

The variation in the stator terminal axis concerning the rotor is given by,

$$e(t) = k'V_r Cos(90^\circ - \theta_R + \theta_C) Sin\omega_c t$$

$$e(t) = k'V_r Sin(\theta_R - \theta_C) Sin\omega_c t$$

When the rotor angle becomes zero, the maximum current is produced in the stator windings S2. The zero position of the rotor is used as a reference for determining the rotor angular position.

- Consider the position of the rotor and the transmitter is changing in the same direction. An angle θR deflects the rotor of the transmitter and that of the control transformer is kept θC. The total angular separation between the rotors is Φ = (90º – θR + θC)

- The rotor terminal voltage of the Synchro transformer is given as

$$e(t) = k'V_r Cos(90^\circ - \theta_R + \theta_C) Sin\omega_c t$$

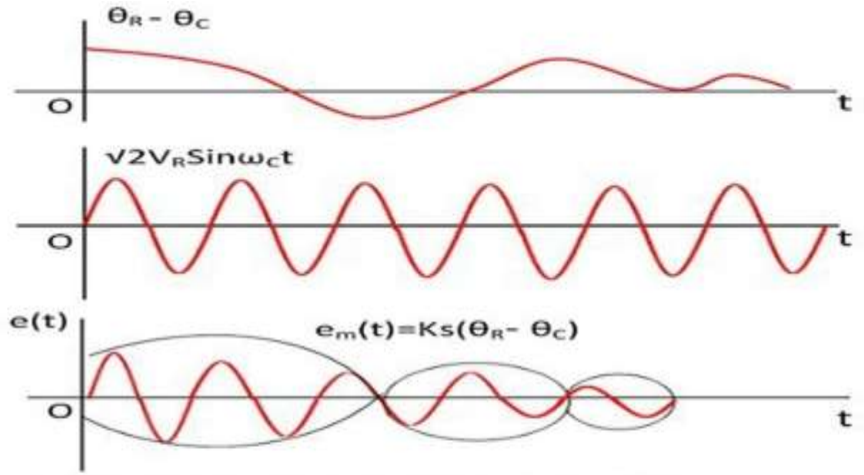
$$e(t) = k'V_r Sin(\theta_R - \theta_C) Sin\omega_c t$$

- The small angular displacement between their rotor position is given as
- Sin (θR – θC) = (θR – θC)

- On substituting the value of angular displacement

$$e(t) = k'V_r Cos\emptyset Sin\omega_c t$$

- The voltage equation shown above is equal to the shaft position of the rotors of control transformer and transmitter.
- The error signal is applied to the differential amplifier which gives input to the servo motor. The gear of the servo motor rotates the rotor of the control transformer.



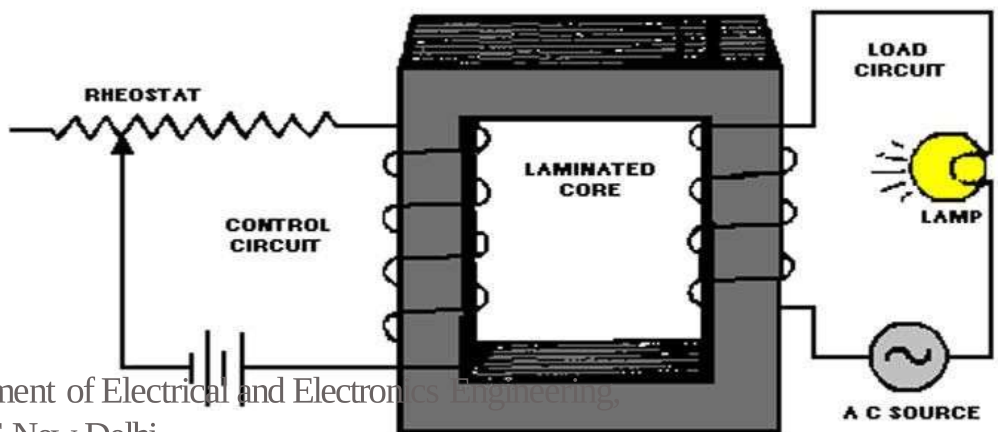
Waveform of Synchro Error Detector

Introduction

- A magnetic amplifier is an electromagnetic device which is used to amplify electrical signals.
- It works on the principle of non-linearity of transformer's core and its saturation.
- It is placed in series with a power supply output for the purpose of regulation.

Construction and Working

- It consists of two windings—AC winding and controlled winding.
- A small DC current is passed through the controlled winding.
- This alters the saturation level of the core, leading to high passage of current through the AC winding.



- Hence, the AC winding is able to control large AC currents.
- An inductive element is used as a controlled switch.
- The coil is wound on a core having square B-H characteristics which gives the core two modes of operation.
- When the core is unsaturated, there is high impedance that supports a high voltage with negligible current.
- When the core is saturated, there is nearly zero impedance. Therefore, current flows with little to no voltage drop.

Factors Affecting Amplification

- Core magnetization cross-section area
- Magnetizing force
- Core magnetic path length
- Flux density
- Magnetic amp coil turns

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Advantages and Disadvantages

Advantages:

- Can withstand neutron radiation
- Does not require warm-up time
- No wear due to absence of moving parts
- Can withstand momentary overloads better than solid state devices.

Disadvantages:

- Bulky
- Frequency response is limited to 1/10th of excitation frequency.

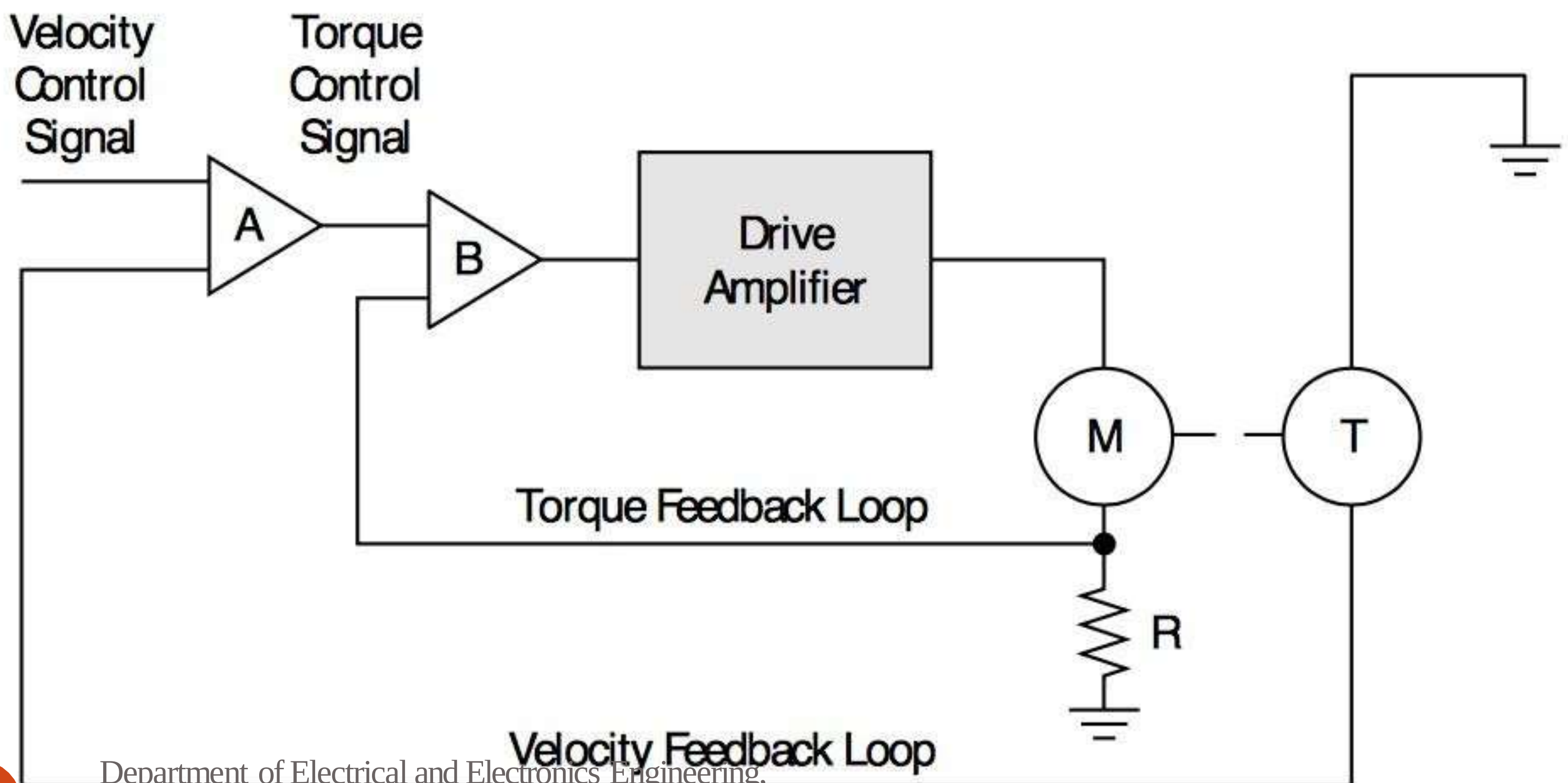
Applications

- In arc welders
- As switching component in switched mode power supplies
- Measuring high voltages
- In radio communication
- Stage lighting
- Speed regulation in paper, steel, and other assembly lines

Introduction

- A servo amplifier also known as, servo amplifier is a special electronic ampl
used to power elec
servomechanisms.
- A servo amplifier moni
the feedback signal from
servomechanism
continually adjusts
deviation from expe
behaviour.
- It basically takes an i
signal from a controller
amplifies that signal whi
then sent to the motor.

Block Diagram of a Basic Servo Amplifier

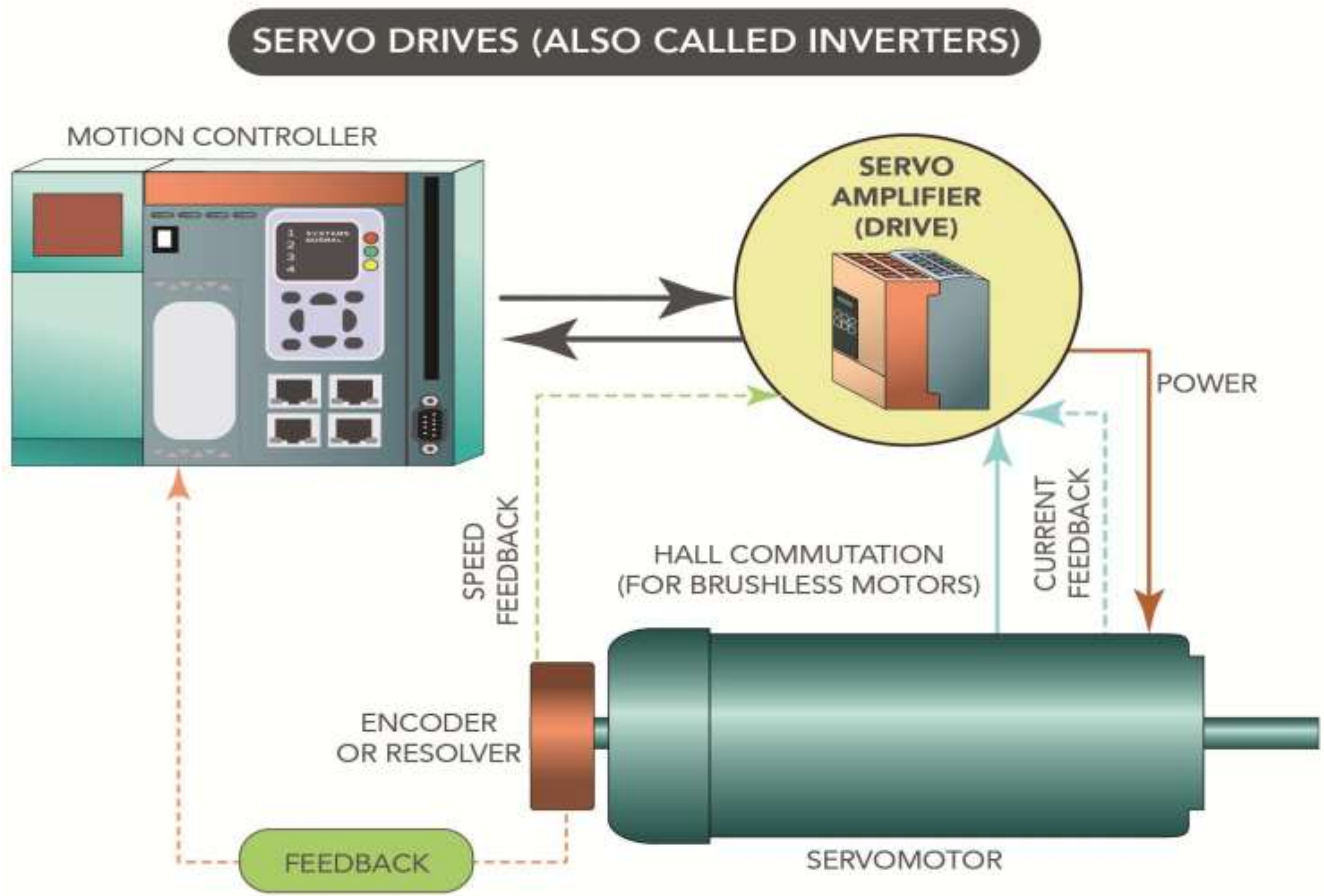


Functions

A servo amplifier receives a command signal from a control system, amplifies the signal, and transmits electric current to a servo motor in order to produce motion proportional to the command signal. Typically, the command signal represents a desired velocity, but can also represent a desired torque or position.

A sensor attached to the servo motor reports the motor's actual status back to the servo amplifier. The servo amplifier then compares the actual motor status with the commanded motor status. It then alters the voltage, frequency or pulse width to the motor so as to correct for any deviation from the commanded status.

In a properly configured control system, the servo motor rotates at a velocity that very closely approximates the velocity signal being received by the servo amplifier from the control system. Several parameters, such as stiffness (proportional gain), damping (derivative gain), and feedback gain, can be adjusted to achieve this desired performance. The process of adjusting these parameters is called **performance tuning**.


SERVO DRIVES (ALSO CALLED INVERTERS)
MOTION CONTROLLER
SERVO AMPLIFIER (DRIVE)
POWER
SPEED FEEDBACK
HALL COMMUTATION (FOR BRUSHLESS MOTORS)
CURRENT FEEDBACK
ENCODER OR RESOLVER
FEEDBACK
SERVOMOTOR

Types

DIGITAL

- They have a microprocessor which analyses the incoming signals.
- The repetitive tasks performed by a processor allows a digital drive to be quickly self-adjusting.
- They consume high energy.
- Highly efficient and performance.

ANALOG

- It controls velocity through electrical inputs of ±10V adjusted using a potentiometer.
- It is not self adjusting.
- Analog drives consume less energy than digital drives.
- Relatively low performance.

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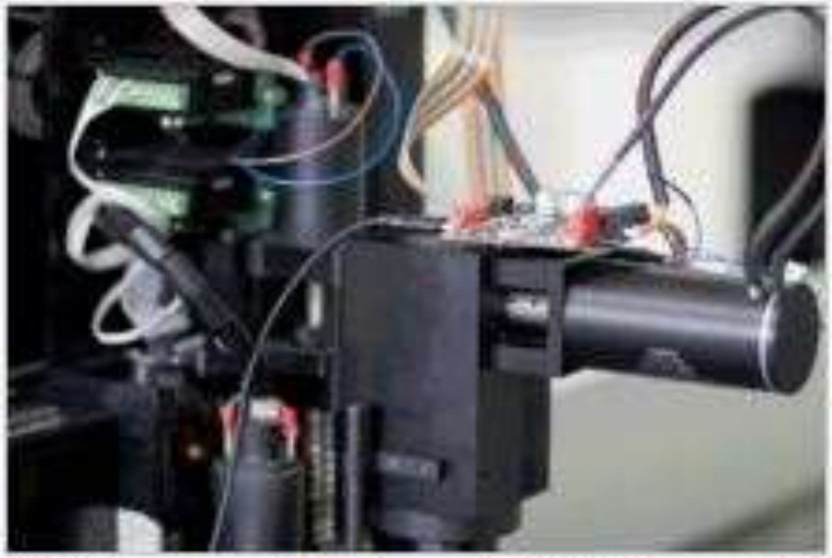
Applications

- Servo systems can be used in CNC machining, factory automation, and robotics, among other uses. Their main advantage over traditional DC or AC motors is the addition of motor feedback. This feedback can be used to detect unwanted motion, or to ensure the accuracy of the commanded motion.
- Servos, in constant speed changing use, have a better life cycle than typical AC wound motors.
- Servo amplifiers can also act as a brake by shunting off generated electricity from the motor itself.



OEM servo drive from INGENIA installed on CNC router machine controlling a Faulhaber motor

UNIT-II
TIME DOMAIN ANALYSIS

- Time domain analysis
- Time domain specification
- Error signal

CONTENTS

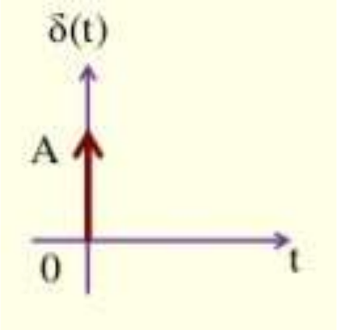
STANDARD TEST SIGNALS

- Impulse function

  Impulse function imitate sudden shock characteristic of actual input signal.

$$u(t) = \delta(t) = \begin{cases} A, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

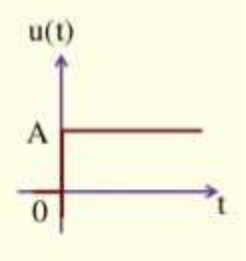
$$\mathrm{U}(s) = A$$



- Step function

  Step function can be described as sudden applicartion of input signal to the system.

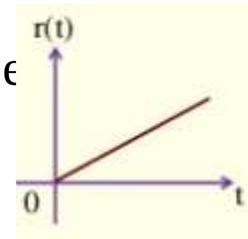
$$u(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad U(s) = \frac{A}{s}$$



❑ Ramp function

Ramp function start from origin and increases or de time.

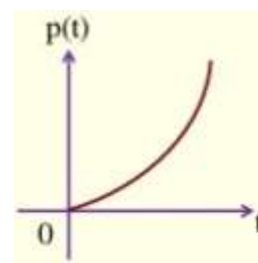
$$r(t) = \begin{cases} At, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad U(s) = \frac{A}{s^2}$$



❑ Parabolic function

Parabolic function imitates constant acceleration characteristic of input signal.

$$p(t) = \begin{cases} \frac{At^2}{2}, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad P(s) = \frac{A}{s^3}$$



Time Response of Control Systems

- Time resposnse of dynamic system is response to an input expressed as a function of time.

- Time Response of any system has 2 components:
  1. Transient response
  2. Steady State response

$$C(t)=c_{tr}(t)+c_{ss}(t)$$

- When response of a system is changed from rest or equilibrium, it takes some time to settle.

- Transient response is the response of a system from rest or equilibrium to steady state.

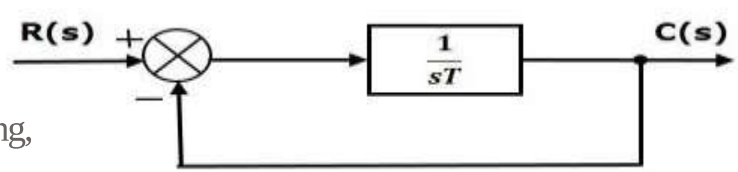
- The response of system after transient response is called steady state response.

- Transient response depends on system pole only and not on type of input.

- The steady state response depends on system dynamics, system type and system quantity.

Transient Response of First order System

- Consider the following block diagram of the closed loop control system. Here, an open loop transfer function, 1sT1sT is connected with a unity negative feedback.
- We know that the transfer function of the closed loop control system has unity negative feedback as,
- C(s)R(s)=G(s)1+G(s)C(s)R(s)=G(s)1+G(s)
- Substitute, G(s)=1sTG(s)=1sT in the above equation.
- C(s)R(s)=1sT1+1sT=1sT+1C(s)R(s)=1sT1+1sT=1sT+1
- The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the **first order system.**



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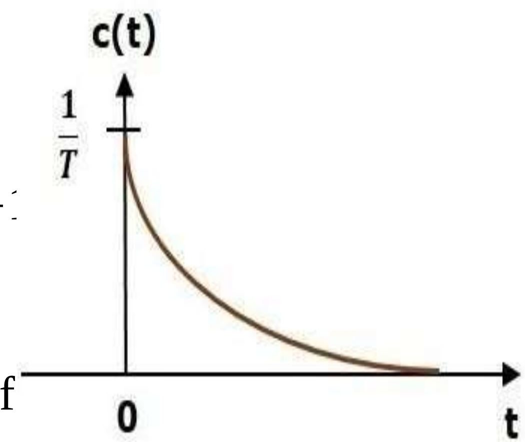
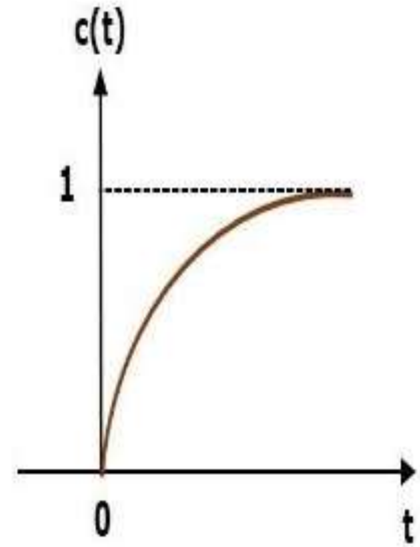
First Order Transient Response of Impulse Signal

- Consider the **unit impulse signal** as an input to the first order system.
- So, r(t)=δ(t)r(t)=δ(t)
- Apply Laplace transform on both the sides.
- R(s)=1R(s)=1
- Consider the equation, C(s)=(1sT+1)R(s)C(s)=(1sT+1
- Substitute, R(s)=1R(s)=1 in the above equation.
- C(s)=(1sT+1)(1)=1sT+1C(s)=(1sT+1)(1)=1sT+1
- Rearrange the above equation in one of the standard f transforms.



- C(s)=1T(s+1T)⇒C(s)=1T(1s+1T)C(s)=1T(s+1T)⇒C(s)=1T(1s+1T)
- Apply inverse Laplace transform on both sides.
- c(t)=1Te(−tT)u(t)c(t)=1Te(−tT)u(t)
- The unit impulse response is shown in the following figure.
- The **unit impulse response**, c(t) is an exponential decaying signal for positive values of 't' and it is zero for negative values of 't'.

First Order Time Response of Step Signal

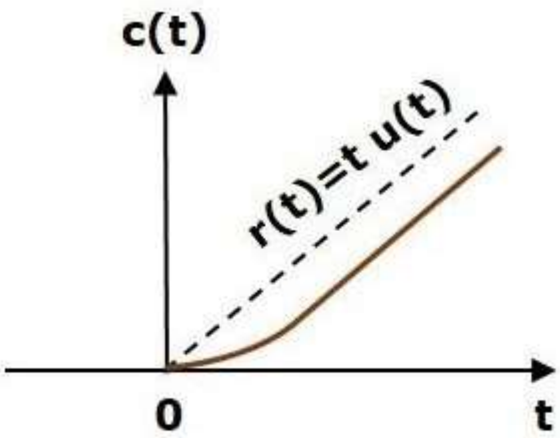


- Consider the **unit step signal** as an input to first order system.
- So, r(t)=u(t)r(t)=u(t)
- Apply Laplace transform on both the sides.
- R(s)=1sR(s)=1s
- Consider the equation, C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)
- Substitute, R(s)=1sR(s)=1s in the above equation.
- C(s)=(1sT+1)(1s)=1s(sT+1)C(s)=(1sT+1)(1s)=1s(sT+1)
- Do partial fractions of C(s).
- C(s)=1s(sT+1)=As+BsT+1C(s)=1s(sT+1)=As+BsT+1
- ⇒1s(sT+1)=A(sT+1)+Bss(sT+1)⇒1s(sT+1)=A(sT+1)+Bss(sT+1)

- On both the sides, the denominator term is the same. So, they will get cancelled by each other. Hence, equate the numerator terms.
- 1=A(sT+1)+Bs1=A(sT+1)+Bs
- By equating the constant terms on both the sides, you will get A = 1.
- Substitute, A = 1 and equate the coefficient of the **s** terms on both the sides.
- 0=T+B⇒B=−T0=T+B⇒B=−T
- Substitute, A = 1 and B = −T in partial fraction expansion of C(s)C(s).
- C(s)=1s−TsT+1=1s−TT(s+1T)C(s)=1s−TsT+1=1s−TT(s+1T)
- ⇒C(s)=1s−1s+1T⇒C(s)=1s−1s+1T
- Apply inverse Laplace transform on both the sides.
- c(t)=(1−e−(tT))u(t)c(t)=(1−e−(tT))u(t)
- The **unit step response**, c(t) has both the transient and the steady state terms.
- The transient term in the unit step response is -
- ctr(t)=−e−(tT)u(t)ctr(t)=−e−(tT)u(t)
- The steady state term in the unit step response is -
- css(t)=u(t)css(t)=u(t)
- The following figure shows the unit step response.
-

First Order Transient Response of Ramp Signal

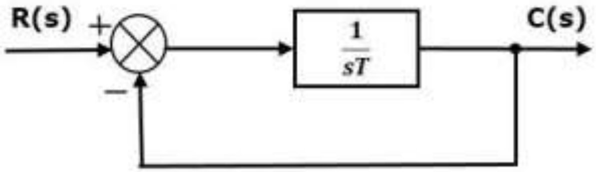
- Consider the unit ramp signal as an input to the first order system.
- So,r(t)=tu(t)So,r(t)=tu(t)
- Apply Laplace transform on both the sides.
- R(s)=1s2R(s)=1s2
- Consider the equation, C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)
- Substitute, R(s)=1s2R(s)=1s2 in the above equation.
- C(s)=(1sT+1)(1s2)=1s2(sT+1)C(s)=(1sT+1)(1s2)=1s2(sT+1)
- Do partial fractions of C(s)C(s).
- C(s)=1s2(sT+1)=As2+Bs+CsT+1C(s)=1s2(sT+1)=As2+Bs+CsT+1
- ⇒1s2(sT+1)=A(sT+1)+Bs(sT+1)+Cs2s2(sT+1)⇒1s2(sT+1)=A(sT+1)+Bs(sT+1)+Cs2s2(sT+1)


c(t)
r(t)=t u(t)
0
t

- On both the sides, the denominator term is the same. So, they will get cancelled by each other. Hence, equate the numerator terms.
- $1=A(sT+1)+Bs(sT+1)+Cs21=A(sT+1)+Bs(sT+1)+Cs2$
- By equating the constant terms on both the sides, you will get $A=1$.
- Substitute, $A=1$ and equate the coefficient of the s terms on both the sides.
- $0=T+B\Rightarrow B=-T0=T+B\Rightarrow B=-T$
- Similarly, substitute $B=-T$ and equate the coefficient of s2s2 terms on both the sides. You will get $C=T2C=T2$.

- Substitute A = 1, B = −T and C=T2C=T2 in the partial fraction expansion of C(s)C(s).
- C(s)=1s2−Ts+T2sT+1=1s2−Ts+T2T(s+1T)C(s)=1s2−Ts+T2sT+1=1s2−Ts+T2T(s+1T)
- ⇒C(s)=1s2−Ts+Ts+1T⇒C(s)=1s2−Ts+Ts+1T
- Apply inverse Laplace transform on both the sides.
- c(t)=(t−T+Te−(tT))u(t)c(t)=(t−T+Te−(tT))u(t)
- The unit ramp response, c(t) has both the transient and the steady state terms.
- The transient term in the unit ramp response is -
- ctr(t)=Te−(tT)u(t)ctr(t)=Te−(tT)u(t)
- The steady state term in the unit ramp response is -
- css(t)=(t−T)u(t)css(t)=(t−T)u(t)
- The following figure shows the unit ramp response.

- Consider the following block diagram of the closed loop control system. Here, an open loop transfer function, 1/sT is connected with a unity negative feedback.



- We know that the transfer function of the closed loop control system has unity negative feedback as,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1 + \frac{1}{sT}} = \frac{1}{sT + 1}$$

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- The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the **first order system**.

We can re-write the above equation as,

$$C(s) = \left(\frac{1}{sT + 1}\right) R(s)$$

Where,

C(s) is the Laplace transform of the output signal c(t),

R(s) is the Laplace transform of the input signal r(t), and

T is the time constant.

Step Response of First Order System

- Consider the **unit step signal** as an input to first order system.

So, $r(t) = u(t)$

Apply Laplace transform on both the sides.

$$R(s) = \frac{1}{s}$$

Consider the equation, $C(s) = \left(\frac{1}{sT+1}\right) R(s)$ and substitute R(s) = 1/s

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s}\right) = \frac{1}{s\,(sT+1)}$$

$$C(s) = \frac{1}{s\,(sT+1)} = \frac{A}{s} + \frac{B}{sT+1}$$

$$\Rightarrow \frac{1}{s\,(sT+1)} = \frac{A\,(sT+1) + Bs}{s\,(sT+1)}$$

- On both the sides, the denominator term is the same. So, they will get cancelled by each other. Hence, equate the numerator terms.

$$1 = A(sT + 1) + Bs$$

$$0 = T + B \Rightarrow B = -T$$

$$C(s) = \frac{1}{s} - \frac{T}{sT+1} = \frac{1}{s} - \frac{T}{T\left(s + \frac{1}{T}\right)}$$

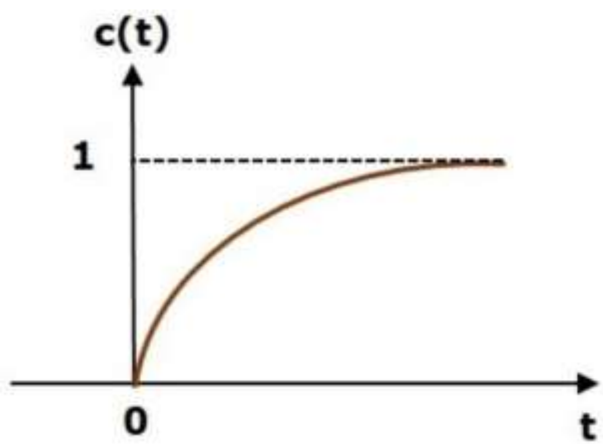
$$\Rightarrow C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

Apply inverse Laplace transform on both the sides.

$$c(t) = \left(1 - e^{-\left(\frac{t}{T}\right)}\right) u(t)$$

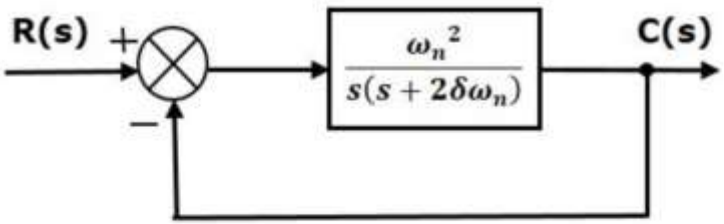
The **unit step response**, c(t) has both the transient and the steady state terms.

The transient term in th $c_{tr}(t) = -e^{-\left(\frac{t}{T}\right)}u(t)$ esponse is –

The following fig 

response.

Transient Response Of Second Order System With Unit Step Input



If we consider a unit step function as the input of the system, then the output equation of the system can be rewritten as

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$Now,\ r(t) = 1\ or\ R(s) = \frac{1}{s}$$

$$\therefore C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$= \frac{1}{s} \cdot \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

$$Putting,\ \omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$\begin{aligned}
&= \frac{1}{s} - \frac{s + 2s\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)} \\
&= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \\
&= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}
\end{aligned}$$

Taking inverse Laplace transform of the above equation, we get

$$\begin{aligned}
&\mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right] \\
&= \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\right] - \mathcal{L}^{-1}\left[\frac{\zeta\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right] \\
&\therefore c(t) = 1 - e^{-\zeta\omega_n t}\cdot\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d}\cdot e^{-\zeta\omega_n t}\cdot\sin\omega_d t
\end{aligned}$$

$$\because \mathcal{L}^{-1}\left[\frac{1}{s}\right] = 1,\ \mathcal{L}^{-1}\left[\frac{s+\alpha}{(s+\alpha)^2+\omega^2}\right] = e^{-\alpha t}\cos\omega t,$$
$$\mathcal{L}^{-1}\left[\frac{\omega}{(s+\alpha)^2+\omega^2}\right] = e^{-\alpha t}\sin\omega t$$

The above expression of output c(t) can be rewritten as

$$c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\cdot\sin\omega_d t\right)$$
$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\cos\omega_d t + \zeta\cdot\sin\omega_d t\right)$$
$$\left[Say,\ \zeta = \cos\phi,\ hence,\ \sqrt{1-\zeta^2} = \sin\phi\right]$$
$$\therefore c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$
$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)$$

INTRODUCTION

- **Time response = Transient response + Steady state response**
- Transient response $\rightarrow$ Due to system poles only
- Steady state response $\rightarrow$ Due to both input and system poles
- $\Rightarrow$ The nature of transient response of a linear control system is revealed by any of the standard test signals – impulse, unit step, ramp, parabola – **as this nature is dependent upon system poles only not on the type of the input.**

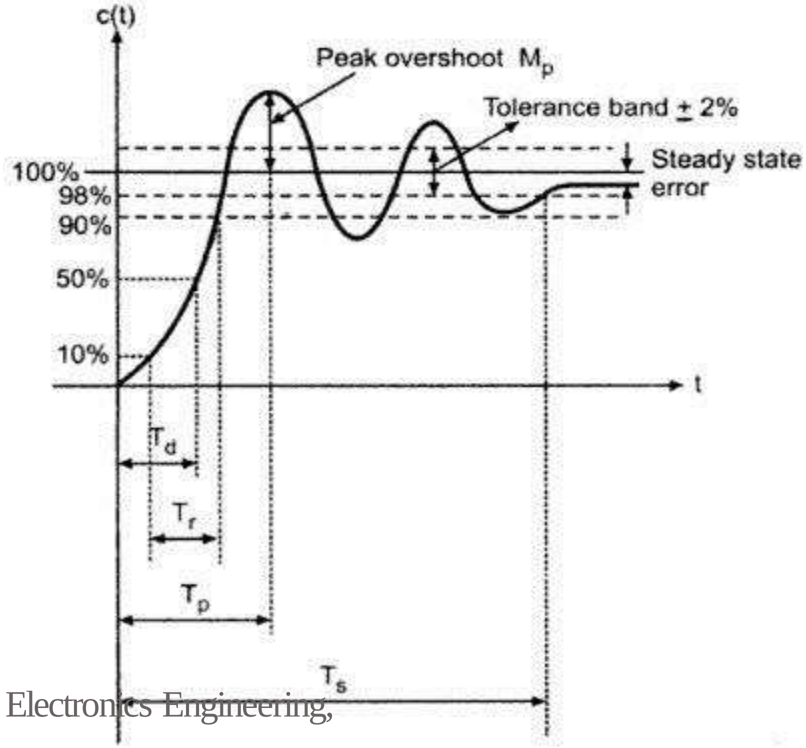
⇒ So therefore it is sufficient to analysis the transient response to one of the standard test signals; a step is generally used.

⇒ **Steady state response depends upon both the system and the type of input.** So the easiest input is generally a step since it requires only maintaining the output at a constant value once the transient is over. (Tracking of ramp and parabola inputs are difficult)

⇒ ***It may be noted that all performance specifications are meaningless unless the system is absolutely stable.***

underdamped system)

⇒ The transient response of a practical system often exhibits damped oscillations before reaching steady state.



Time Response Specifications

<u>Delay Time</u> :- The time taken by the response to reach 0 to 50 % of its steady state value.

<u>Rise Time</u> :- The time taken by the response to reach 0 to 100 % of its steady state value.

Underdamped system → 0 to 100 %

Critically damped system → 5 % to 95 %

Overdamped system → 10 % to 90 %

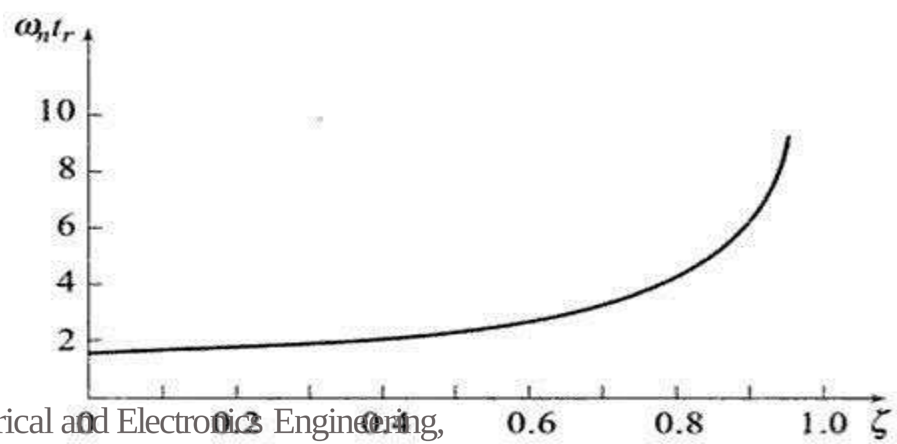


Fig. 6.10 Normalized rise time versus damping ratio

<u>Peak Time</u> :- Time required for the response to reach the first peak of the overshoot

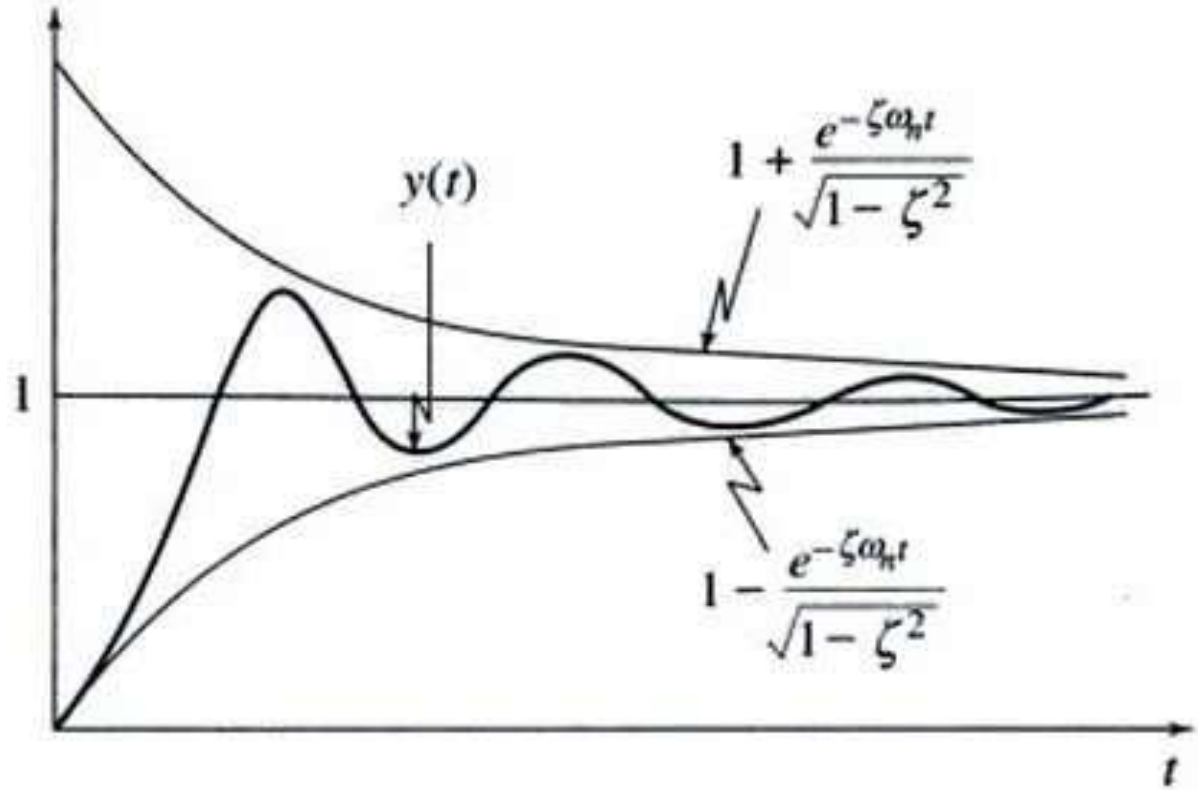
$$\frac{dC(t)}{dt} = 0$$

<u>Peak Overshoot</u> :- It is the peak value of response curve measured from unity. If the steady state value differs from unity, then it is common to use *percent peak overshoot*.

The peak overshoot is a monotonically decreasing function of damping

It is therefore an ***excellent measure of system damping***

Settling Time :- The time taken by the response to settle to its final value within its specified tolerance band.



IMPORTANT REMARKS

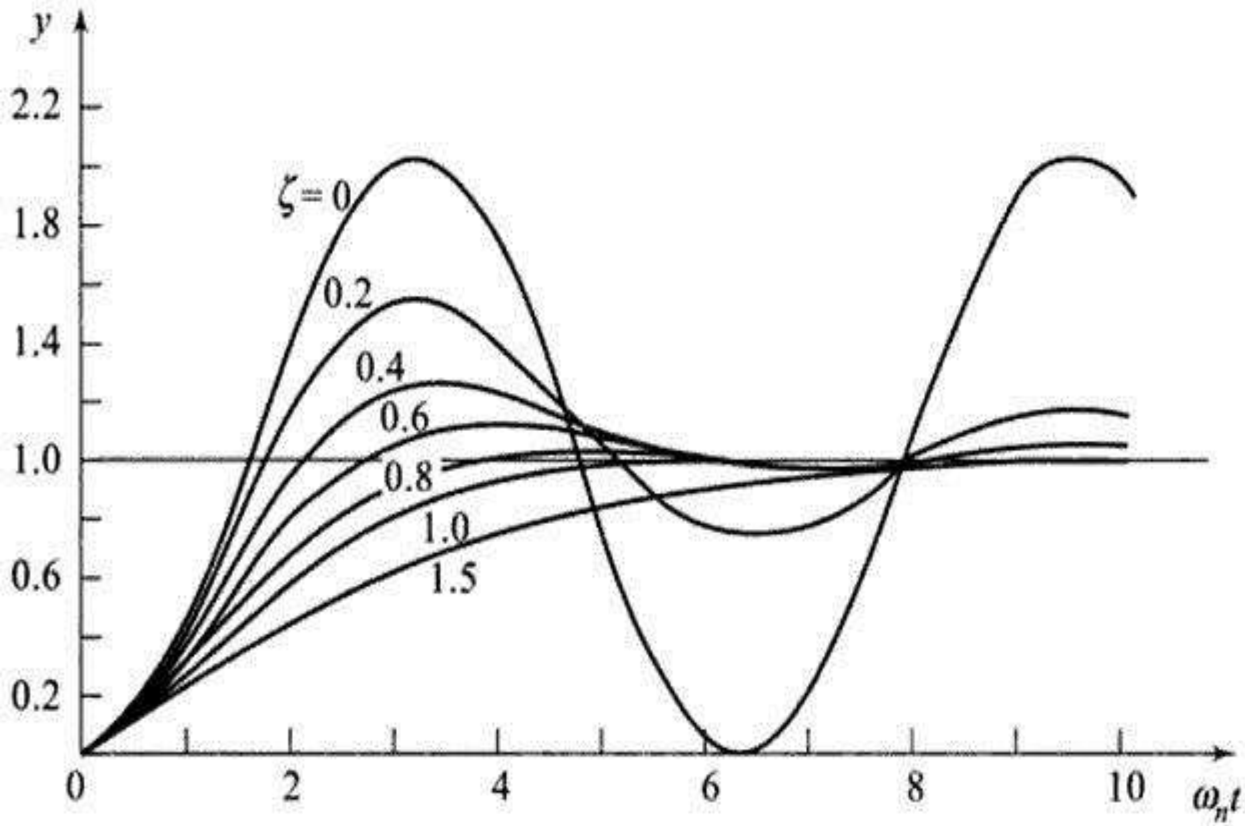


Fig. 6.9 Unit-step response curves of standard second-order system

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- **As $\xi \uparrow$ and $0 < \xi < 1$ (Underdamped response)**

1. % Peak Overshoot decreases. (Relative Stability increases)
2. Rise time, peak time increases. (Fastness of response decreases)
3. **But settling time decreases. (Affects both stability and Fastness)**

- **As $\xi \uparrow$ and $\xi > 1$ (Overdamped response)**

4. % Peak Overshoot does not exist.
5. Rise time, delay time, **settling time increases. (Sluggishness increases)**

INTRODUCTION

Time-domain specifications (TDS) include the lower and/or upper bounds of the quantities of the **time** response.

Such as-:

1. Peak time
2. Delay Time
3. Rise time
4. Peak overshoot
5. Settling time

Peak Time

It is the time required for the response to reach the maximum or Peak value of the response

$$t_P = \frac{\pi}{\omega_n \sqrt{1-\delta^2}}$$

Delay time td

It is the time required for the response to reach 50% of the steady state value for the first time

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Peak overshoot (M)

It is defined as the difference between the peak value of the response and the steady state value. It is usually expressed in percent of the steady state value

$$M_p = 100\ e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}\ \%$$

Rise time tr:

It is the time required for the response to reach 100% of the steady state value for under damped systems. However, for over damped systems, it is taken as the time required for the response to rise from 10% to 90% of the steady state value

$$t_r = \frac{\pi - \phi}{\omega_n \sqrt{1-\delta^2}} = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\delta^2}}{\delta}}{\omega_n \sqrt{1-\delta^2}}$$

Settling time (ts)

It is the time required for the response to reach and remain within a specified tolerance limits (usually ± 2% or ± 5%) around the steady state value.

$$t_s \simeq \frac{e^{-\delta \omega_n t}}{\sqrt{1-\delta^2}}$$

- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss}. We can find steady state error using the final value theorem as follows.

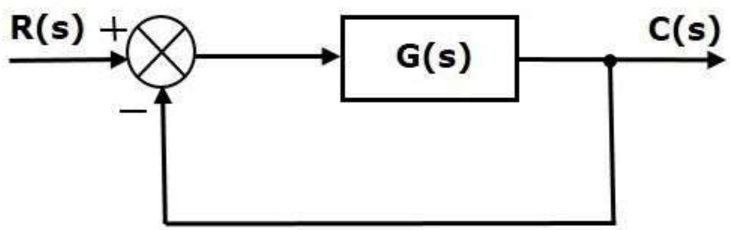
$$e_{ss} = \lim_{t \to \infty} = \lim_{s \to 0} sE(s)$$

Where,

- E(s) is the Laplace transform of the error signal, e(t).
- Let us discuss how to find steady state errors for unity feedback and non-unity feedback control systems one by one.

Steady State Errors for Unity Feedback Systems

Consider the following block diagram of closed loop control egative feedback.



Where,

R(s) is the Laplace transform of the reference Input signal r(t).

C(s) is the Laplace transform of the output signal c(t).

We know the transfer function of the unity negative feedback closed loop control system as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1+G(s)}$$

The output of the summing point is –

$$E(s) = R(s) - C(s)$$

Substitute C(s) value in the above equation.

$$E(s) = R(s) - \frac{R(s)G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

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Substitute E(s) value in the steady state error formula

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

The following table shows the steady state errors and the

e step,

u

Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p=\lim_{s\to 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v=\lim_{s\to 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a=\lim_{s\to 0} s^2G(s)$

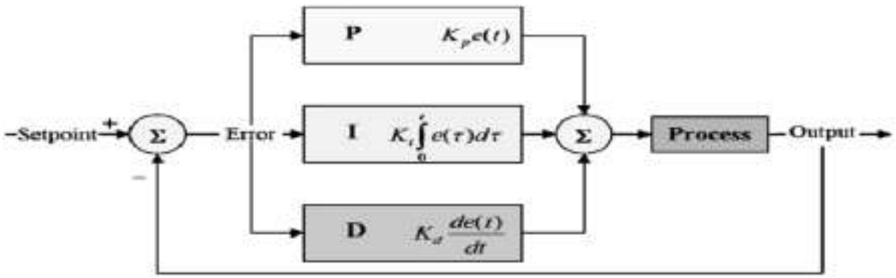
- Where, Kp, Kv and Ka are position error constant, velocity error constant and acceleration error constant respectively.

- Note – If any of the above input signals has the amplitude other than unity, then multiply corresponding steady state error with that amplitude.

- Note – We can’t define the steady state error for the unit impulse signal because, it exists only at origin. So, we can’t compare the impulse response with the unit impulse input as t denotes infinity.

What is a PID Controller?

- A **PID controller** is an instrument used in industrial control applications to regulate temperature, flow, pressure, speed and other process variables. PID (proportional integral derivative) controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controller.



- PID control is a well-established way of driving a system towards a target position or level. It's a practically ubiquitous as a means of controlling temperature and finds application in myriad chemical and scientific processes as well as automation. PID control uses closed-loop control feedback to keep the actual output from a process as close to the target or setpoint output as possible.

Working of pid controller

- The working principle behind a PID controller is that the proportional, integral and derivative terms must be individually adjusted or "tuned." Based on the difference between these values a correction factor is calculated and applied to the input. For example, if an oven is cooler than required, the heat will be increased. Here are the three steps:

-**Proportional tuning** involves correcting a target proportional to the difference. Thus, the target value is never achieved because as the difference approaches zero, so too does the applied correction.

-**Integral tuning** attempts to remedy this by effectively cumulating the error result from the "P" action to increase the correction factor. For example, if the oven remained below temperature, “I” would act to increase the head delivered. However, rather than stop heating when the target is reached, "I" attempts to drive the cumulative error to zero, resulting in an overshoot.

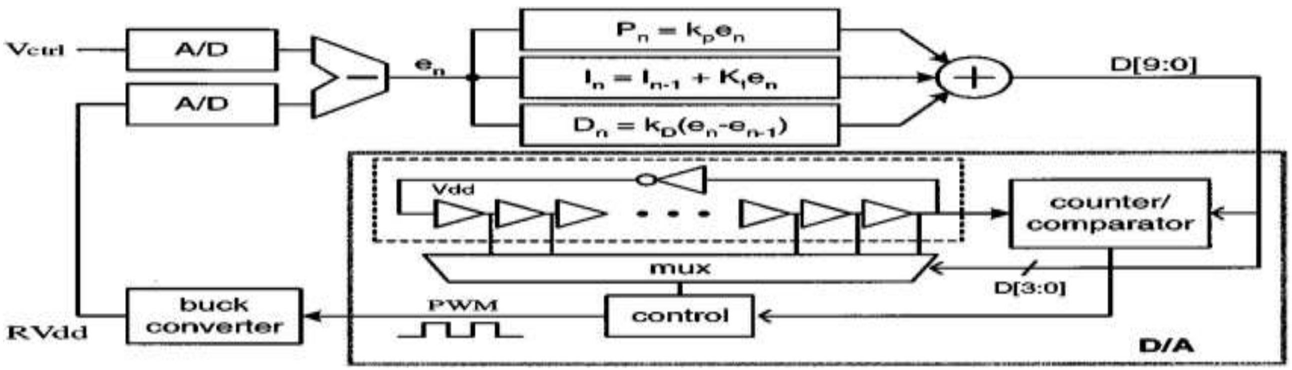
-**Derivative tuning** attempts to minimize this overshoot by slowing the correction factor applied as the target is approached.

What is a PID Temperature Controller?

- A PID temperature controller, as its name implies, is an instrument used to control temperature, mainly without extensive operator involvement. A PID controller in a temperature control system will accept a temperature sensor such as a thermocouple or RD as input and compare the actual temperature to the desired control temperature or setpoint. It will then provide an output to a control element.

What is a Digital PID Controller?

- A digital PID controller reads the sensor signal, normally from a thermocouple or RTD and connects the measurement to engineering units, such as degree Fahrenheit or Celsius, that are then displayed in a digital format.



History of PID Controller

- The first evolution of the PID controller was developed in 1911 by Elmer Sperry. However, it wasn't until 1933 that the Taylor Instrumental Company (TIC) introduced the first pneumatic controller with a fully tunable proportional controller. A few years later, control engineers went eliminate the steady state error found in proportional controllers by resetting the point to some artificial value as long as the error wasn’t zero. This resetting “integrated” the error and became known as the proportional-Integral controller. Then, in 1940, TIC developed the first PID pneumatic controller with a derivative action, which reduced overshooting issues. However, it wasn’t until 1942, when Ziegler and Nichols tuning rules were introduced that engineers were able to find and set the appropriate parameters of PID controllers. By the mid-1950’s, automatic PID controllers were widely adopted for industrial use.

CONTENTS

INTRODUCTION

- What is a PI controller?
- In this controller the output power equals to the sum of proportion and integration coefficients. The actuating signal in the time domain is given by:

$$e_a(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau$$

Here the constants Kp and Ki proportional and integral gains known as control parameters.

- The higher the proportion coefficient, the less the output power at the same control error. The higher the integration coefficient, the slower the accumulated integration coefficient.
- PI control provides zero control error and is insensitive to interference of measurement channel.

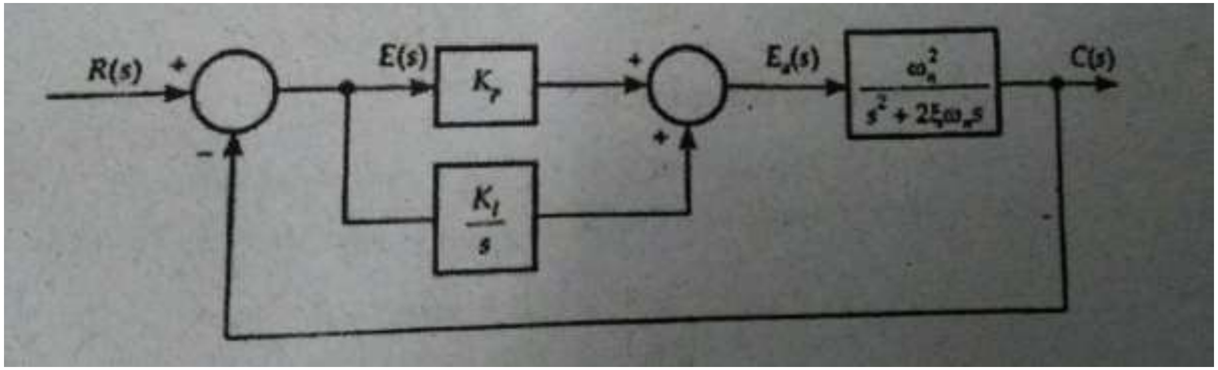
- By taking Laplace transform of both sides of error constant equation, we get:

$$E_a(s) = K_p E(s) + K_I \frac{E(s)}{s}$$

$$E_a(s) = \left(K_p + \frac{K_I}{s}\right) E(s)$$

Block diagram

The block diagram of a PI control system is given below:



Advantages/properties of pi controller

- Output power is proportional to the integral of the input signal.
- As the integrator is involved, it increases the type of the system.
- As the type of the system increases, it reduces steady state error and hence improves accuracy.
- Integral action enables PI controllers to eliminate offset, a major weakness of a P-only controller. Thus, PI controllers provide a balance of complexity and capability that makes them by far the most widely used algorithm in process control applications.
- As the type of the system is increased, there is some negative impact over the stability of the system.

DISADVANTAGES OF PI CONTROLLER

- The PI control disadvantage is slow reaction to disturbances.
- Hence we cannot use PI controller for slow moving process variables.

APPLICATIONS OF PI CONTROLLER

Following are the applications where PI controller is used:

1. Liquid Flow Control
2. Steam Pressure Control
3. Heat Exchanger Temperature Control
4. Majority of speed control applications use PI control action, e.g., speed control of motors, generators, turbines.

UNIT-III
FREQUENCY DOMAIN DOMAIN ANALYSIS

- Nyquist plot
- Stability
- Bode plot

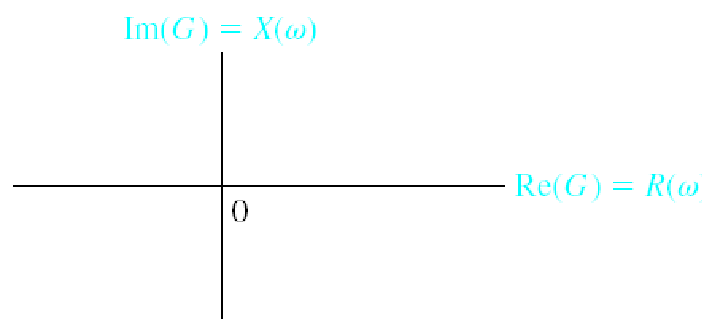
INTRODUCTION

i) Polar plot is a plot which can be drawn between magnitude and phase. Here, the magnitudes are represented by normal values only.

The polar form of **G(jω)H(jω)** is

$$\underline{\mathbf{G(j\omega)H(j\omega)=|G(j\omega)H(j\omega)|\angle G(j\omega)H(j\omega)}}$$

ii) The **Polar plot** is a plot, which can be drawn between the magnitude and the phase angle of G(jω)H(jω) by varying ω from zero to ∞.



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ADVANTAGES OF POLAR PLOTS

- It depicts the frequency response characteristics over the entire frequency range in a single plot.(there is single graph to represent both phase and magnitude , unlike in Bode plot).
- Much easier to determine both and .
- Here we will have to work with open loop transfer function G(s)H(s) (and not with closed loop transfer function and unlike Bode plot we need not required to convert G(s)H(s) to the time constant form).

EXAMPLES TO UNDERSTAND POLAR PLOTS

- **Polar Plot of a 1st Order Pole (1/(s+a)):**

given transfer function: G(s)H(s) = 10/(s+2)

Step 1 : The first step would be convert this transfer function to the frequency domain.This can be done by converting 's' by 'jω'

G(jω)H(jω) = 10+0i/(jω+2)

Step 2 : We now find the magnitude and phase

$$|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{\omega^2+4}}$$

$$\angle G(j\omega)H(j\omega) = \frac{\tan^{-1}\frac{0}{10}}{\tan^{-1}\frac{\omega}{2}} = \frac{0}{\tan^{-1}\frac{\omega}{2}}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2}$$

Step 3 : Vary 'ω' from 0 to ∞

Now instead of taking different values of ω, we simply take two extreme values of ω i.e ω = 0 and ω = ∞

At $\omega = 0$, $|G(j\omega)H(j\omega)| = 10/\sqrt{4} = 5$

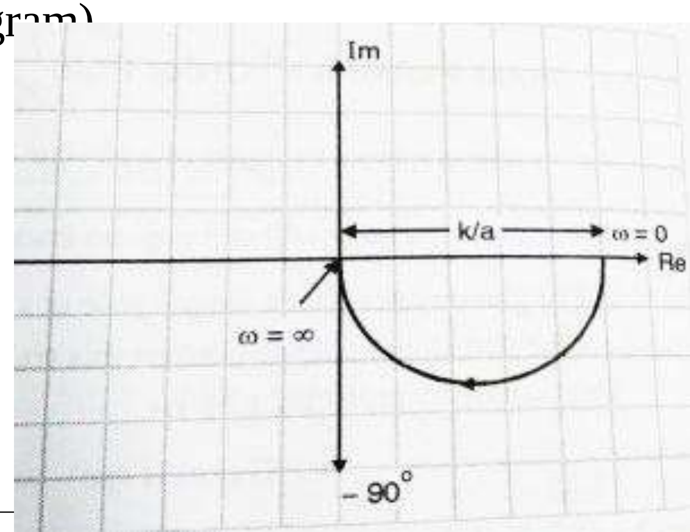
$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2} = 0°$

At $\omega = \infty$, $|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{4+\infty^2}}\ 0$

$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\infty}{2} = -90°$

Now these two points are sufficient to draw the polar plot. At ω = 0 since the magnitude is +5 and angle is 0,we draw it on the right side horizontal axis. At ω = ∞ , the magnitude is 0 while angle is -90 , hence we draw it as dot(zero magnitude) on the -90 axis. The direction is represented by an arrow(refer below diagram)

We can now generalize that the polar plot of a 1st order pole (k/s+p) will always have a shape shown above.



Effect of adding more Simple Poles:

Then we have, $G(s)H(s) = 10/(s+2)(s+4)$

The given system has two poles i.e s = -2 and s = -4 ,

Step 1 : The first step would be convert this transfer function to the frequency domain.This can be done by converting 's' by 'jω'

$$G(j\omega)H(j\omega) = \frac{10+0i}{(j\omega+2)(j\omega+4)}$$

We don't need to convert this to time constant form.

Step 2 : We now find the magnitude and phase

$$|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{(\omega^2+4)}\sqrt{\omega^2+16}}$$

$$\angle G(j\omega)H(j\omega) = \frac{\tan^{-1}\frac{0}{10}}{(\tan^{-1}\frac{\omega}{2}+\tan^{-1}\frac{\omega}{4})} = \frac{0}{\tan^{-1}\frac{\omega}{2}+\tan^{-1}\frac{\omega}{4}}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$$

Step 3 : Vary 'ω' from 0 to ∞

Now instead of taking different values of ω, we simply take two extreme values of ω i.e $\omega = 0$ and $\omega = \infty$

At $\omega = 0$, $|G(j\omega)H(j\omega)| = \dfrac{10}{\sqrt{4}\sqrt{16}} = 1.25$

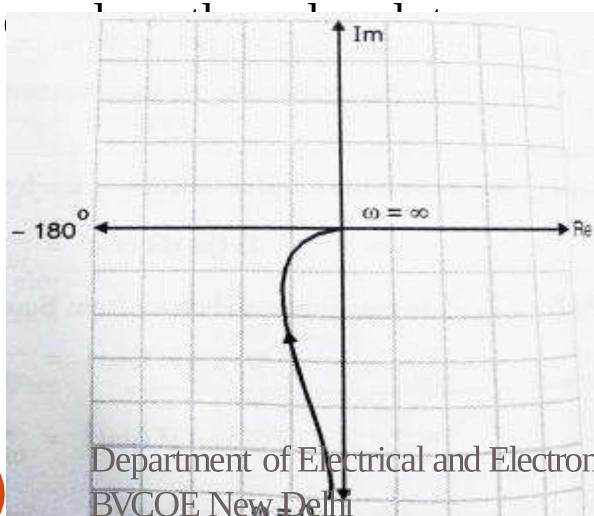
$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4} = 0°$

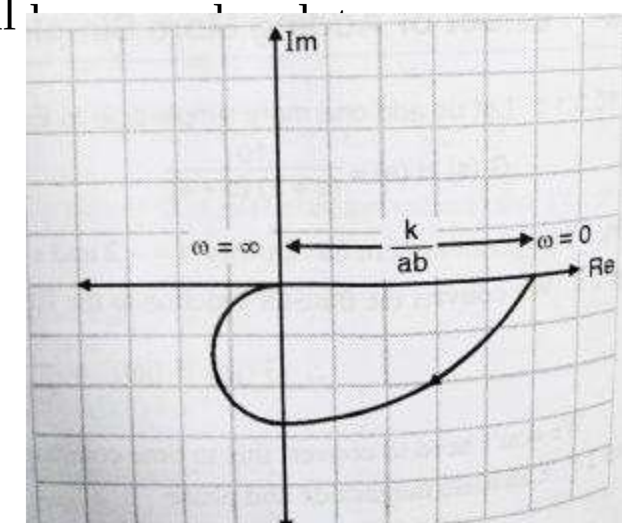
At $\omega = \infty$, $|G(j\omega)H(j\omega)| = \dfrac{10}{\sqrt{4+\infty^2}\sqrt{16+\infty^2}} = 0$

$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{\infty}{4} = -90° \; -90° = -180°$

From above two point now we

Thus, we can generalize that $G(j\omega)H(j\omega) = k/(s+a)(s+b)$ will





Effect of adding pole at origin:

Now we have, $G(s)H(s) = 10/s(s+2)$

The given system has two poles i.e s = -2 and s = 0 ,

Step 1 : The first step would be convert this transfer function to the frequency domain.This can be done by converting 's' by 'jω'

$$G(j\omega)H(j\omega) = \frac{10+0i}{j\omega(j\omega+2)}$$

We don't need to convert this to time constant form.

Step 2 : We now find the magnitude and phase

$$|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{(\omega^2+4)}\sqrt{\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = \frac{\tan^{-1}\frac{0}{10}}{(\tan^{-1}\frac{\omega}{2}+\tan^{-1}\frac{\omega}{0})} = \frac{0}{\tan^{-1}\frac{\omega}{2}+90^\circ}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\omega}{2} - 90^\circ$$

Step 3 : Vary ‘ω’ from 0 to ∞

Now instead of taking different values of ω, we simply take two extreme values of ω i.e ω = 0 and ω = ∞

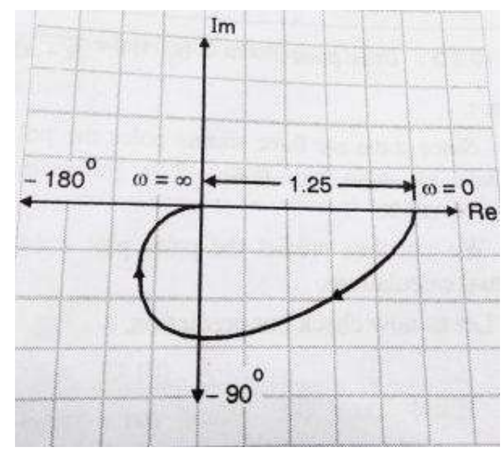
At $\omega = 0$, $|G(j\omega)H(j\omega)| = \frac{10}{0\sqrt{16}} = \infty$

$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{0}{2} - 90^\circ = -90^\circ$

At $\omega = \infty$, $|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{16+\infty^2}\infty} \quad 0$

$\angle G(j\omega)H(j\omega) = -\tan^{-1}\frac{\infty}{2} - 90^\circ = -90^\circ - 90^\circ = -180^\circ$

From above two point now we can draw the polar plot,



We can note here that the at ‘ω’ = 0 , magnitude is ∞ , we draw line parallel to the -90 axis assuming it will touch -90 axis at ∞.

We note that adding a pole at the origin:

- Shifts the polar plot by -90 degree at ω = 0 and
- Magnitude at ω = 0 becomes infinity

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- **What will happen if two poles present at the origin?**
- As expected the polar plot will get shifted by -180 degree (since two poles present at origin)
- **So we have seen effect of addition of poles to the system, so what will be the effect of addition of zeros?**
- Result of addition of zeros is exactly opposite to that of poles. A pole adds -90 degree to the plot, while a zero adds +90 degree to the polar plot.

- Polar plots are simple method to check the stability of the system. They are however not the preferred choice when the system has poles on the right half plane.

- The **inverse polar plot** is the mirror image of polar plot with respect to real axis.
- The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$on polar coordinates as 'ω' varied from zero to infinity.
- The inverse polar plot of $G(j\omega)$ is a graph of $1/G(j\omega)$ as a function of ω.

PROCEDURE

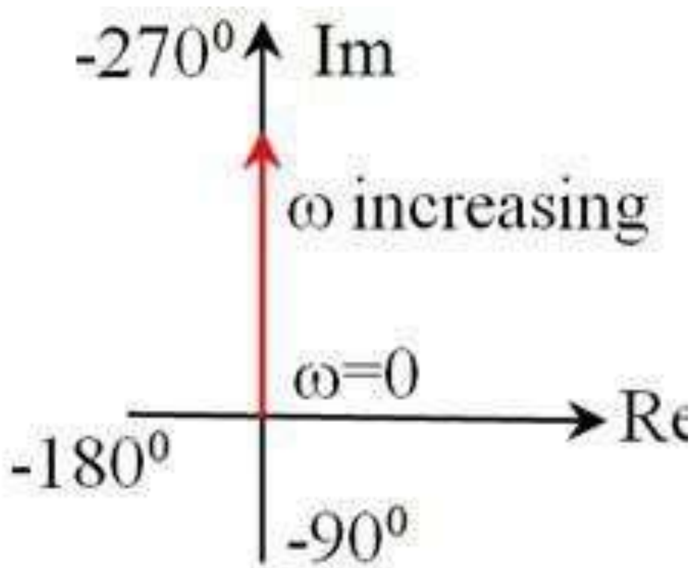
- Find the inverse transfer function.
- Substitute, $s=j\omega$ in the transfer function.
- Write the expressions for magnitude and the phase of $1/G(j\omega)$
- Find the starting magnitude of $1/G(j\omega)$ by substituting $\omega=0$. So, the polar plot starts with $\lim_{\omega \to 0}|1/G(j\omega)|$.
- Find the ending magnitude of $1/G(j\omega)$ by substituting $\omega=\infty$. So, the polar plot ends with this magnitude $\lim_{\omega \to \infty}|1/G(j\omega)|$.

- ADVANTAGE

It depicts the frequency response characteristics of a system over the entire frequency range in a single plot

- DISADVANTAGE

The plot does not indicate the contributions of a each individual factor of the transfer function.



Inverse polar Plot of $1/j\omega$

The inverse polar plot of $G(j\omega)$ is a graph of $1/G(j\omega)$ as a function of ω.

Ex: if $G(j\omega) = 1/j\omega$ then $1/G(j\omega) = j\omega$

Definitions:

- The response of a system can be partitioned into transient response and the steady state response. We can find the transient response by using Fourier integrals. The steady state response of a system for an input sinusoidal signal is known as the **frequency response**.

- The frequency domain specifications are **resonant peak, resonant frequency and bandwidth**.

- The resonant frequency wr is the frequency at which the peak resonance Mr , for the first time occurs.

- It is the range of frequencies over which, the magnitude of T(jω) drops to 70.7% from its zero frequency value.

- **Resonant frequency:**

The transfer function of a second order closed loop system is given as:

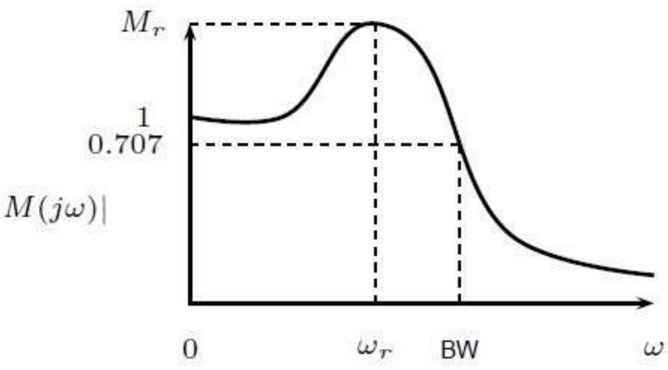
$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} \qquad \_\_1$$

substituting s=jw in the above equation we get :

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n(j\omega) + \omega_n^2}$$

which can be written as:

$$T(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\left(\frac{2\delta\omega}{\omega_n}\right)}$$



Substituting w/wn as u we can write the equation as:

$$T(j\omega) = \frac{1}{(1 - u^2) + j(2\delta u)} \qquad \_\_2$$

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Thus the magnitude of T(jw) or M can be written as:

$$|T(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\delta u)^2}} \qquad \text{\_\_3}$$

The phase angle can be written as:

$$\angle T(j\omega) = -tan^{-1}\left(\frac{2\delta u}{1-u^2}\right) \qquad \text{\_\_4}$$

At w = w_r, T reaches its peak value M

Thus differentiating M with respect to u we get dM/du. Equating dM/du to 0 we get :

$$0 = -\frac{1}{2}\left[(1-u_r^2)^2 + (2\delta u_r)^2\right]^{-\frac{3}{2}}\left[4u_r(u_r^2 - 1 + 2\delta^2)\right]$$

Which gives:

$$4u_r(u_r^2 - 1 + 2\delta^2) = 0$$

Thus:

$$u_r = \sqrt{1 - 2\delta^2} \qquad \text{\_\_5}$$

After substituting the value of u we get :

$$\frac{\omega_r}{\omega_n} = \sqrt{1 - 2\delta^2}$$

Thus resonant frequency w_r is given by the following expression:

$$\omega_r = \omega_n \sqrt{1 - 2\delta^2} \qquad \text{\_\_6}$$

- **Bandwidth:**

From its definition ,for 3db frequencies ,at $w=w_b$, M=0.707=1/√2 . Substituting the value in equation 3 we get :

$$2 = (1 - u_b^2)^2 + (2\delta)^2 u_b^2$$

Solving for the value of u_b we get :

$$u_b^2 = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)} \qquad \text{\_\_7}$$

Substituting the value of $u_b = w_b/w_n$ we get:

$$\frac{\omega_b^2}{\omega_n^2} = 1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}$$

8

Thus we get the bandwidth w_b as:

$$\omega_b = \omega_n \sqrt{1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}}$$

Frequency Domain Analysis

- Introduction
- Advantages
- Stability of closed loop system can be estimated
- Transfer function of complicated systems can be determined experimentally by frequency tests
- Effects of noise disturbance and parameter variations are relatively easy to visualize
- Analysis can be extended to certain nonlinear control systems

Frequency Domain Specifications

- Resonant Peak : Maximum value of the closed loop transfer function.
- Resonant Frequency : Frequency at which resonant peak occurs.
- Bandwidth : Range of frequencies for which the system normalized gain is more than 3db.
- Cut-off rate : It is the slop of the log-magnitude curve near the cut-off frequency.
- Gain Margin : The value of gain to be added to system in order to bring the system to the verge of instability.
- Phase Margin : Additional phase lag to be added at the gain cross over frequency in order to bring the system to verge of instability.

Resonant Peak

It is the peak (maximum) value of the magnitude of T(jω). It is denoted by M_r .

At u=ur, the Magnitude of T(jω) is –

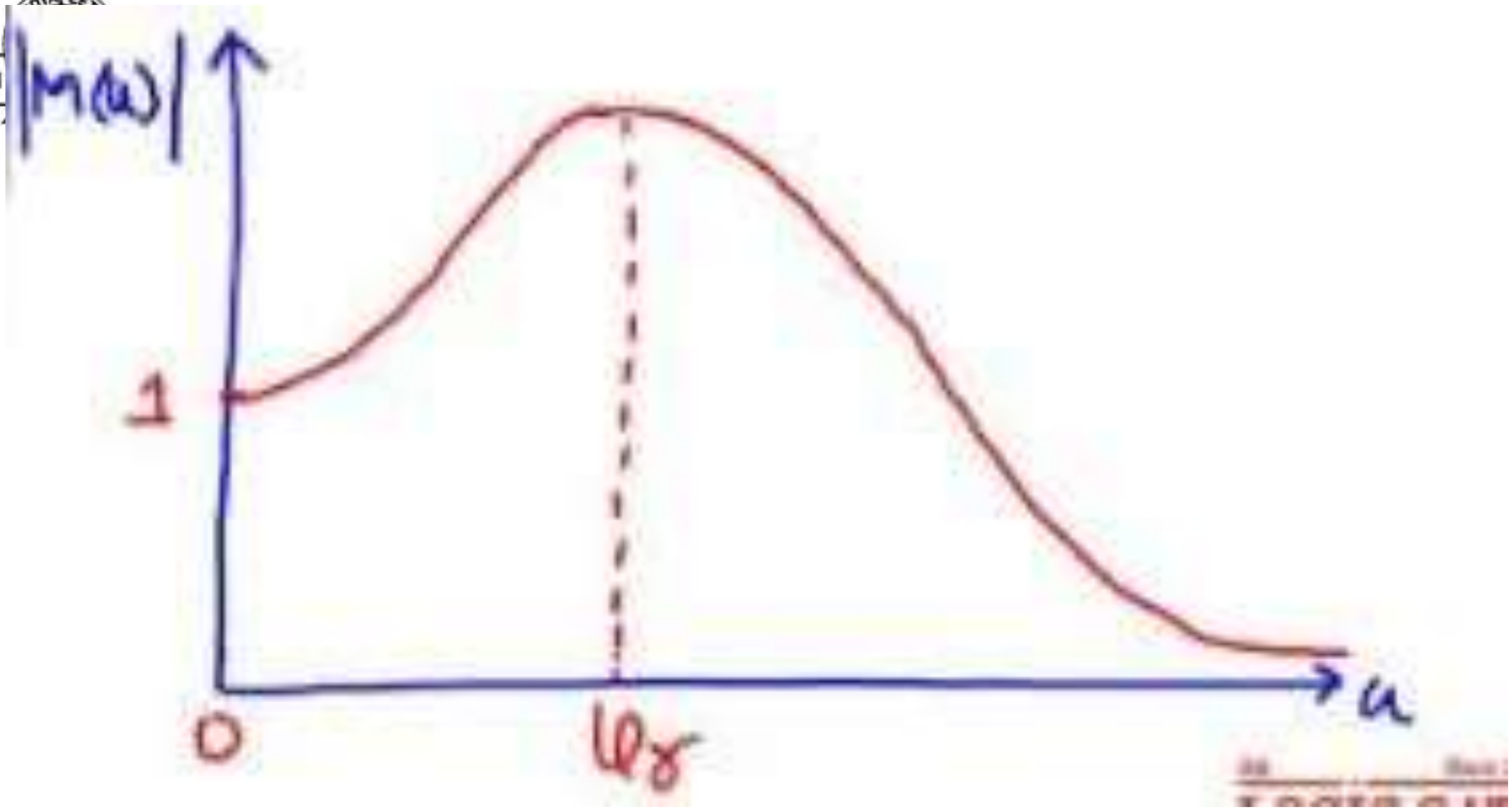
$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2 + (2\delta u_r)^2}}$$

Substitute, $u_r = \sqrt{1-2\delta^2}$ and $1-u_r^2 = 2\delta^2$ in the above equation.

$$M_r = \frac{1}{\sqrt{(2\delta^2)^2 + (2\delta\sqrt{1-2\delta^2})^2}}$$

$$\Rightarrow M_r = \frac{1}{2\delta\sqrt{1-\delta^2}}$$

- Resonant peak in frequency response corresponds to the peak overshoot in the time domain transient response for certain values of damping ratio δ. So, the resonant peak and peak overshoot are correlated to each other.



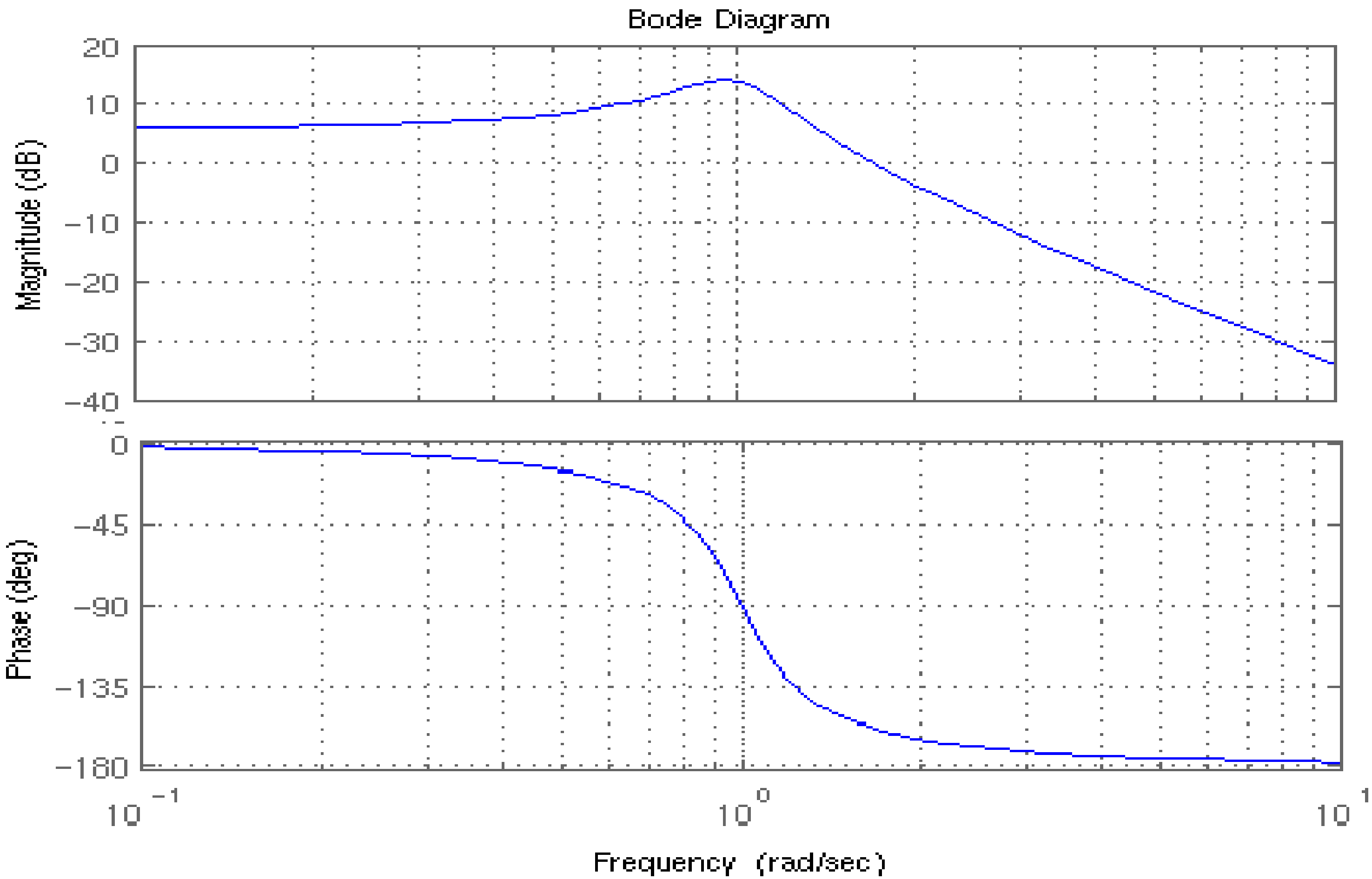
where, $u = \omega / \omega_n$

$u_r = \omega_r / \omega_n$

M=magnitude

LOGARITHMIC PLOTS(BODE PLOTS)

- A Bode Plot is a useful tool that shows the gain and phase response of a given LTI system for different frequencies. Bode Plots are generally used with the Fourier Transform of a given system.
- The frequency of the bode plots are plotted against a logarithmic frequency axis. Every tickmark on the frequency axis represents a power of 10 times the previous value. For instance, on a standard Bode plot, the values of the markers go from (0.1, 1, 10, 100, 1000, ...) Because each tickmark is a power of 10, they are referred to as a decade.
- The bode Magnitude plot measures the system Input/Output ratio in special units called decibels. The Bode phase plot measures the phase shift in degrees



Bode Diagram
Magnitude (dB)
20
10
0
−10
−20
−30
−40
Phase (deg)
0
−45
−90
−135
−180
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

INTRODUCTION:-

Bode plot introduced by H.W. Bode was first used in the study of feedback amplifiers. It is one of the popular graphical methods used for determining the stability of the system when the system is subjected to sinusoidal input. The stability of the closed-loop system is determined based on the frequency response of the loop transfer function of the system, ie, G(s)H(s). The gain or magnitude and phase angle of the system can be easily represented as a function of frequency using Bode plot. It is also a very useful graphical tool in analysing and designing of linear control systems.

The Bode plot consists of two plots:

(i)Magnitude plot: To plot the logarithmic magnitude or gain of the loop transfer function in dB versus frequency w i.e, M =20 log G(jw)H(w)in dB versus w.

(ii)Phase plot: To plot the phase angle of the loop transfer function versus frequency w i.e., o= <G(jw)H(jw) versus w.

In Bode plot, both the magnitude and phase plots are plotted against the frequency in the logarithmic scale. In addition, the magnitude of the system is plotted in dBs (decibels).

Hence, the Bode plot is also called logarithmic plot. Since the magnitude and phase plots of a system are sketched based on the asymptotic properties instead of detailed plotting, the Bode plot is also called asymptotic plots.

REASONS FOR USING LOGARITHMIC SCALE:-

The reasons for plotting the magnitude and phase angle plots of the Bode plot in a logarithm mic scale are:

(i)In higher order systems, magnitudes of the individual subsystem have to be multi plied to get the magnitude of the higher order system that is a tedious process. But if we use the logarithmic scale, the multiplication part can be replaced by the addition that makes the process of determining the magnitude of the higher order system easier.

(ii)In addition, the variation of frequency in a large scale is easier if we use a logarithmic rather than the ordinary scale.

ADVANTAGES OF BODE PLOTS:-

Advantages of using Bode plot in plotting the frequency response of a system are:-

(i) Expansion of frequency range of a system is simple.
(ii)Experimental determination of transfer function of a system is simpler if the frequency response of the system is shown using Bode plot.
(iii) Usage of asymptotic straight lines for approximating the frequency response of the system
(iv) Bode plot can be plotted for complicated systems.
(v)Relative stability of the closed-loop system can be analysed by plotting the frequency response of the loop transfer function of the system using Bode plot.
(vi)Frequency domain specifications such as gain crossover frequency, phase crossover frequency gain margin and phase margin can easily be determined.
(vii) The variation of frequency domain specifications can easily be viewed when a con troller is added to the existing system.
(viii) The system gain K can be designed based on the required gain and phase margins.
(ix)Polar plot and Nyquist plot can be constructed based on the data obtained from Bode plot.

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DISADVANTAGES OF BODE PLOTS:-

The disadvantages of using Bode plot in plotting the frequency response of a system are:-

(i) Using Bode plot, it is possible only to determine the absolute and relative stability of the minimum phase system.
(ii) Corrections are to be made in the obtained plot to meet the desired frequency plot.

Determination of Frequency Domain Specifications from Bode Plot:-
The different frequency domain specifications that can easily be determined using Bode plot are gain margin, phase margin, gain crossover frequency and phase crossover frequency

The plot shown below Indicates the determination of the above said frequency domain specifications.

PROCEDURE FOR SKETCHING BODE PLOTS:-

Here are the steps involved in sketching the approximate Bode magnitude plots:-

Bode Magnitude Plots

Step 1 :- Factor the transfer function into pole-zero form.

Step 2 :- Find the frequency response from the Transfer function.

Step 3 :- Use logarithms to separate the frequency response into a sum of decibel terms

Step 4 :- Use omega = 0 to find the starting magnitude.

Step 5 :- The locations of every pole and every zero are called break points. At a zero breakpoint, the slope of the line increases by 20dB/Decade. At a pole, the slope of the line decreases by 20dB/Decade.

Step 6 :- At a zero breakpoint, the value of the actual graph differs from the value of the straight-line graph by 3dB. A zero is +3dB over the straight line, and a pole is -3dB below the straight line.

Step 7 :- Sketch the actual bode plot as a smooth-curve that follows the straight lines of the previous point, and travels through the breakpoints.

Here are the steps to drawing the Bode phase plots:-

Bode Phase Plots

Step 1 :- If A is positive, start your graph (with zero slope) at 0 degrees. If A is negative, start your graph with zero slope at 180 degrees (or -180 degrees, they are the same thing).

Step 2 :- For every zero, slope the line up at 45 degrees per decade when, (1 decade before the break frequency). Multiple zeros means the slope is steeper.

Step 3 :- For every pole, slope the line down at 45 degrees per decade when, (1 decade before the break frequency). Multiple poles means the slope is steeper.

Step 4 :- Flatten the slope again when the phase has changed by 90 degrees (for a zero) or -90 degrees (for a pole) (or larger values, for multiple poles or multiple zeros.

GAIN MARGINS:-

The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB. We can usually read the gain margin directly from the Bode plot

PHASE MARGINS:-

The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees. We can usually read the phase margin directly from the Bode plot

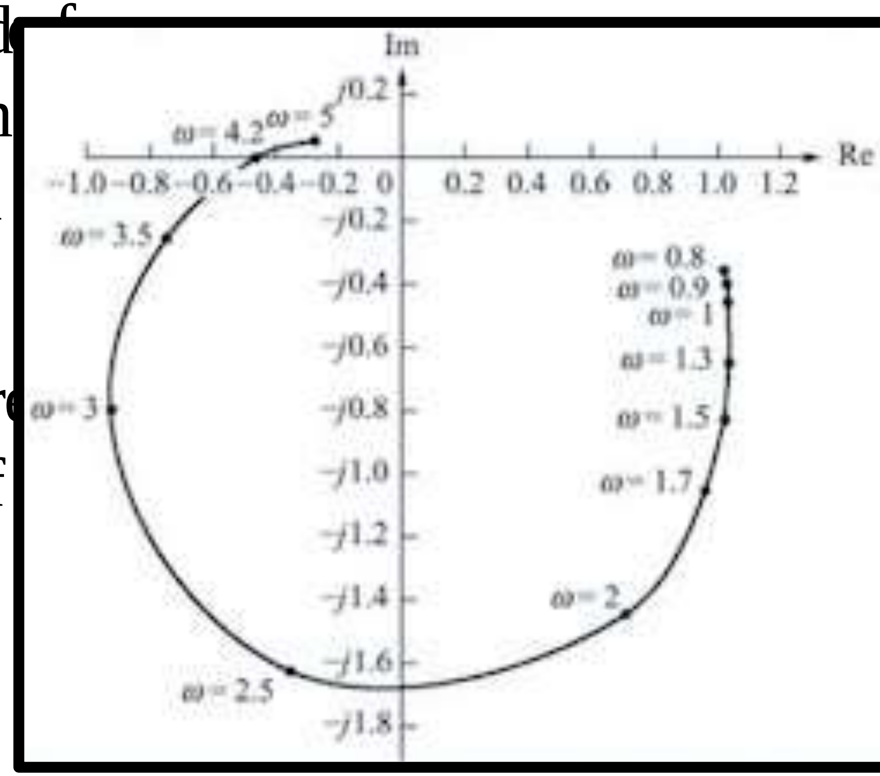
- **Close loop frequency response :**

- 1) It approximately predicts the time response of the System.
- 2) The time response are converted into time domain specifications.
- 3) After design time domain specification is converted into frequency domain

Close loop frequency response from open loop frequency response :

- The closed loop magnitud
now be obtained by findin
point of Nyquist plot with

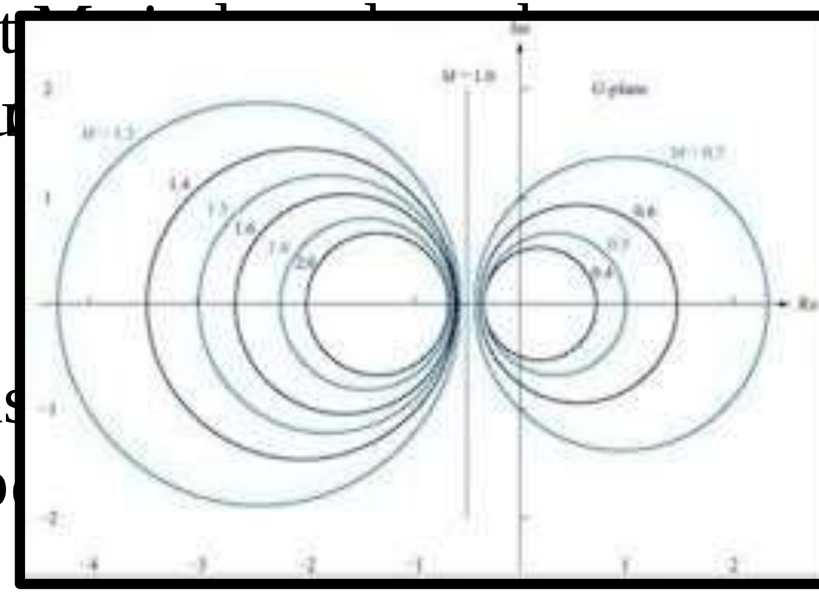
- While closed loop phase r
finding the intersection of
plot on N circle.



Relation between close loop and open loop responses :

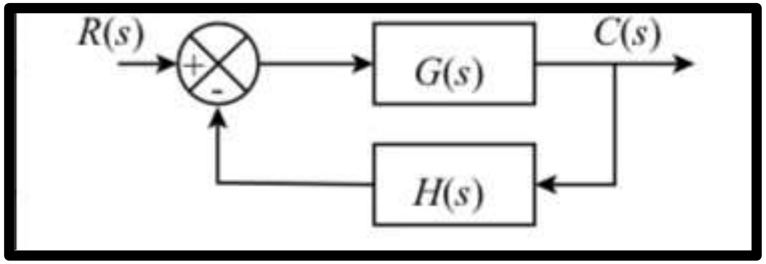
- The circle are called constant locus of closed loop magnitu unity feedback system.

- If the polar frequency respons G(s), is plotted and superimp circle.

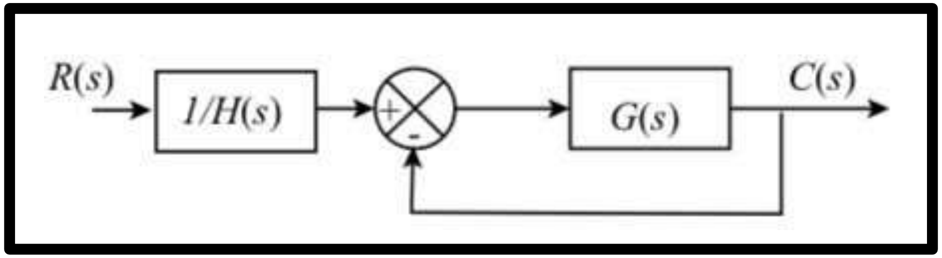


Closed loop frequency response :

- Consider the following non unity feedback system.



- The frequency response can be then obtained using
- The additive feature of bode plot.



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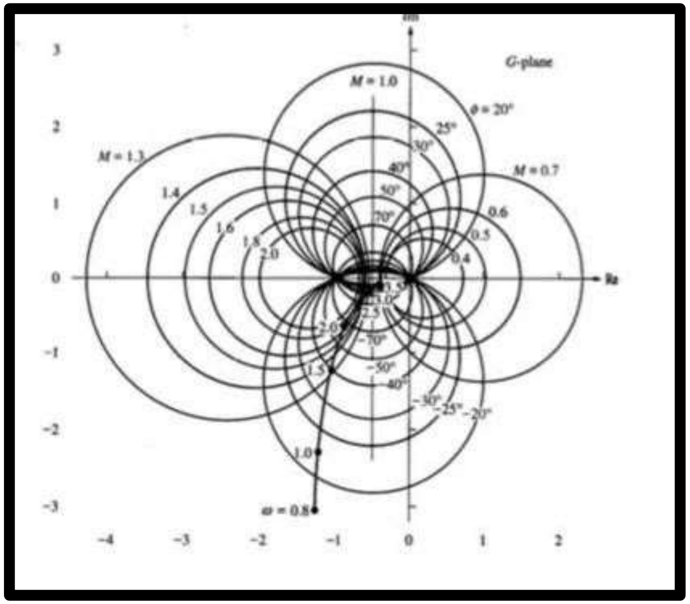
Closed loop response from open loop response :

- Using the constant magnitude and phase circle and the polar plot of the open loop to obtain the closed loop frequency response.

$$G(s)=\frac{50}{s(s+3)(s+6)}$$



FREQUENCY DOMAIN ANALYSIS

- The frequency domain analysis is generally done by using a sinusoidal input signal. When a sinusoidal input signal is given to a linear time-invariant system, the output response consists of transient and steady-state parts, whereas when the transient part dies down as $\rightarrow \infty$, only the steady-state part remains. The frequency response is the steady-state response of a system to a sinusoidal input signal.
- The frequency domain analysis is a method of obtaining steady output.

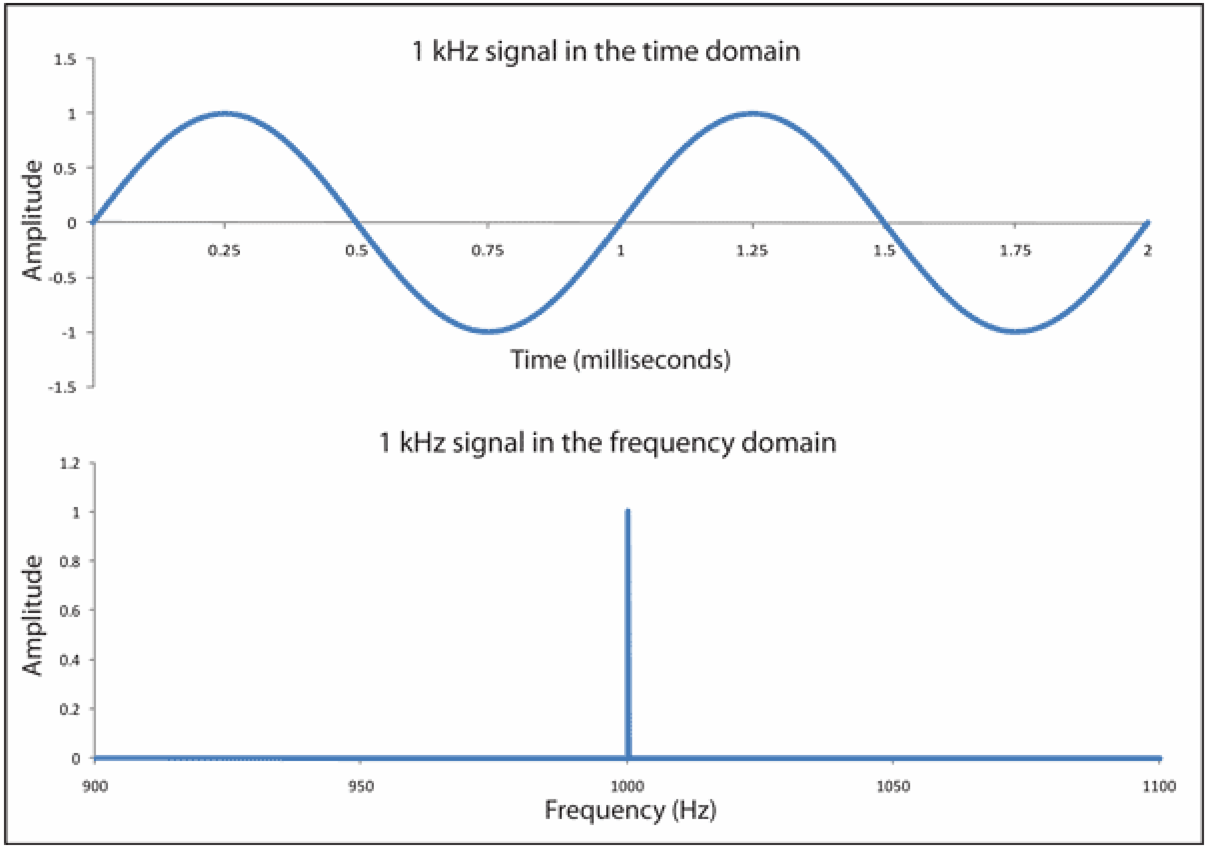


Fig1:Frequency domain analysis of a sinusoidal wave.

LIMITATIONS

- Since, frequency domain analysis is a method that is used to analyse sinusoidal wave in order to generate a steady output but due to changes in reference input it will cause unavoidable errors known as **‘steady state error’.**
- Frequency domain analysis cannot ensure the analysis of any other wave other than the sinusoidal wave such as square, spike waves.

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STEADY STATE ERRORS

Changes in the reference input will cause unavoidable errors during transient periods and may also cause steady-state errors. Imperfections in the system components, such as static friction, backlash, and amplifier drift, as well as aging or deterioration, will cause errors at steady state. Thus ,in frequency domain analysis the reference input is changing hence steady state error appears.

ANALYSIS OF SINUSOIDAL WAVES

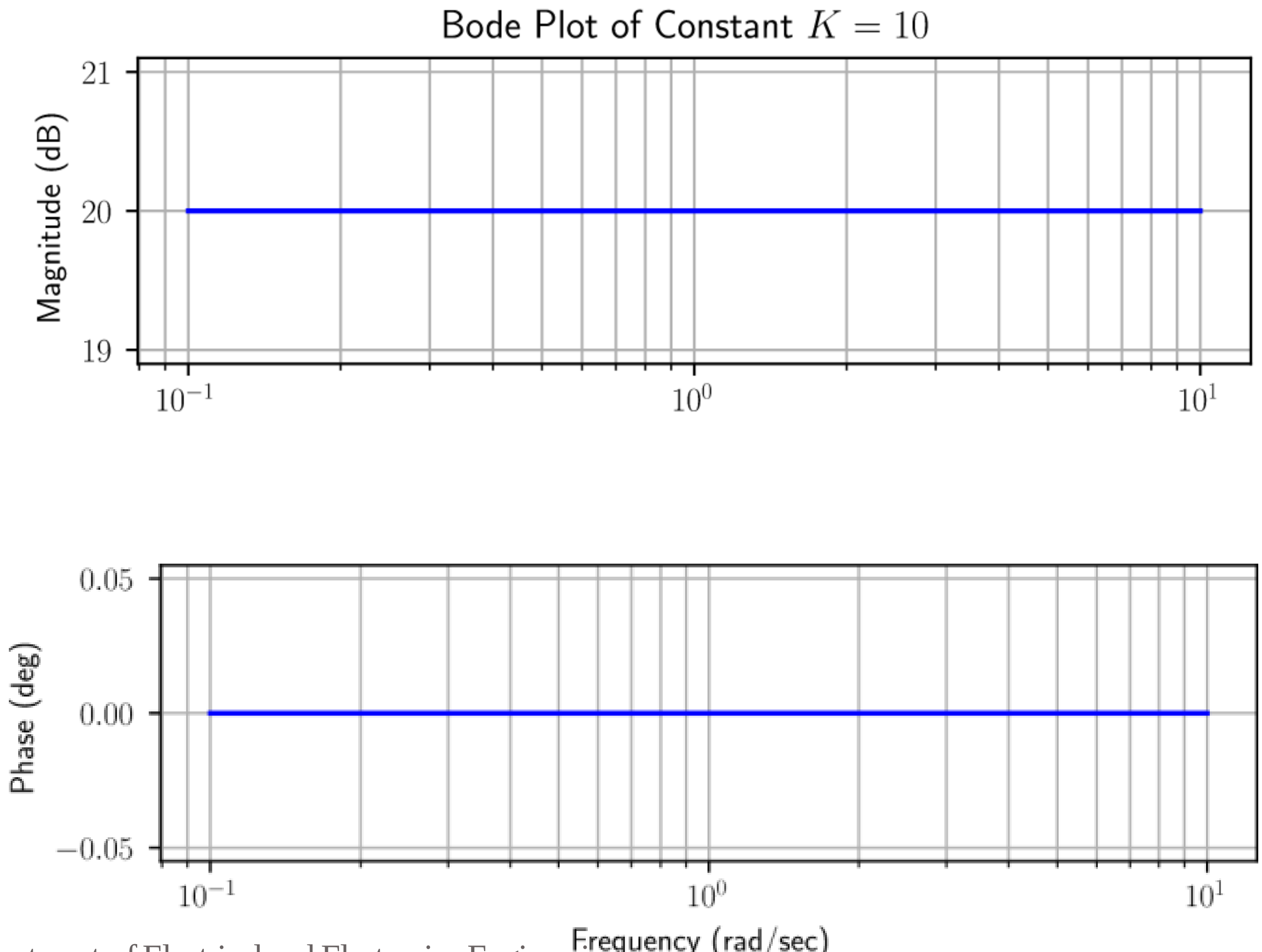
- Sinusoidal signals have a distinct frequency.
- Since ,an arbitrary signal does not have a unique frequency.
- So ,in order to maintain a unique frequency different transformations methods such as ‘**fourier series**’ and **‘fourier transform’** are used.
- Hence ,the analysis of arbitrary signals is complicated and time inefficient.

Minimum Phase System

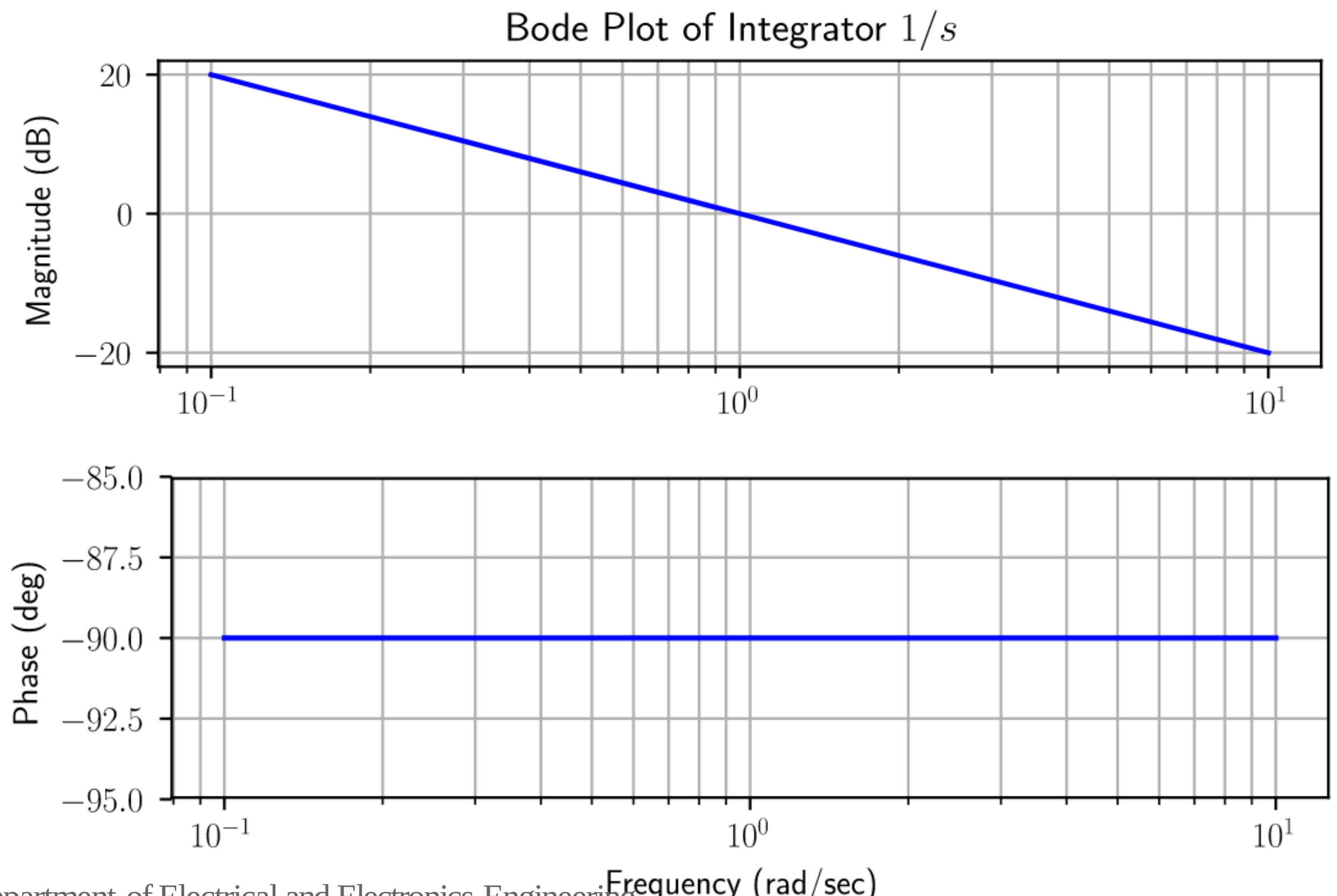
- A transfer function G(s) is minimum phase if both G(s) and 1/G(s) are causal and stable.
- Roughly speaking it means that the system does not have zeros or poles on the right-half plane. Moreover, it does not have delay.
- Bode discovered that the phase can be uniquely derived from the slope of the magnitude for minimum-phase system. (**Bode's Relation)**

Basic Factor	Mag Slope(Low Freq)	Phase(Low Freq)	Mag Slope(High Freq)	Phase (High Freq)
K	0	0	0	0
Sn	20N	90N	20N	90N
1/(τs+1)	0	0	-20	-90
1/((s/ωn)2+2ζ(s/ωn)+1	0	0	-40	-180

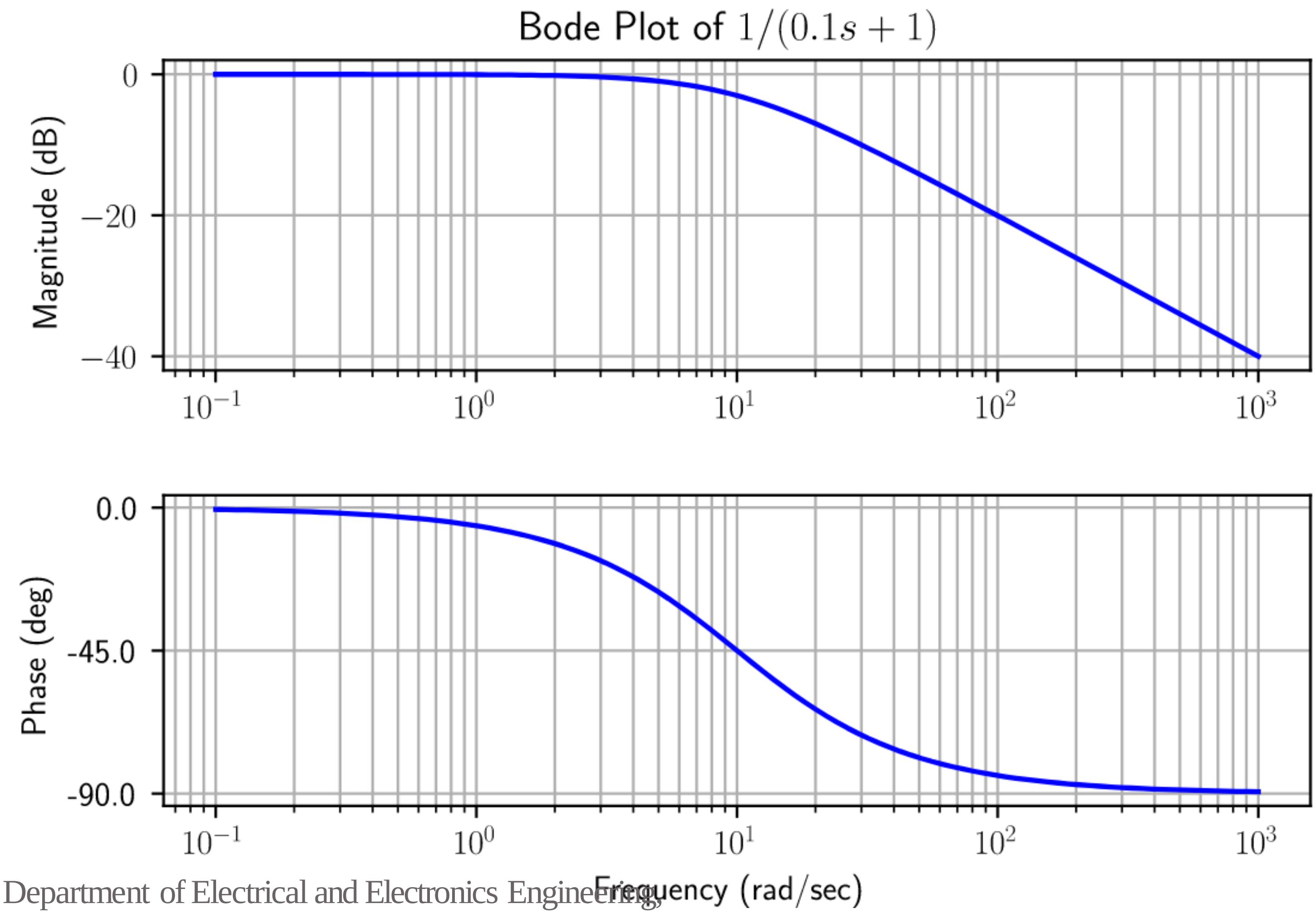
1. Constant K


Bode Plot of Constant $K = 10$
Magnitude (dB)
21
20
19
10^{-1}
10^{0}
10^{1}
Phase (deg)
0.05
0.00
−0.05
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

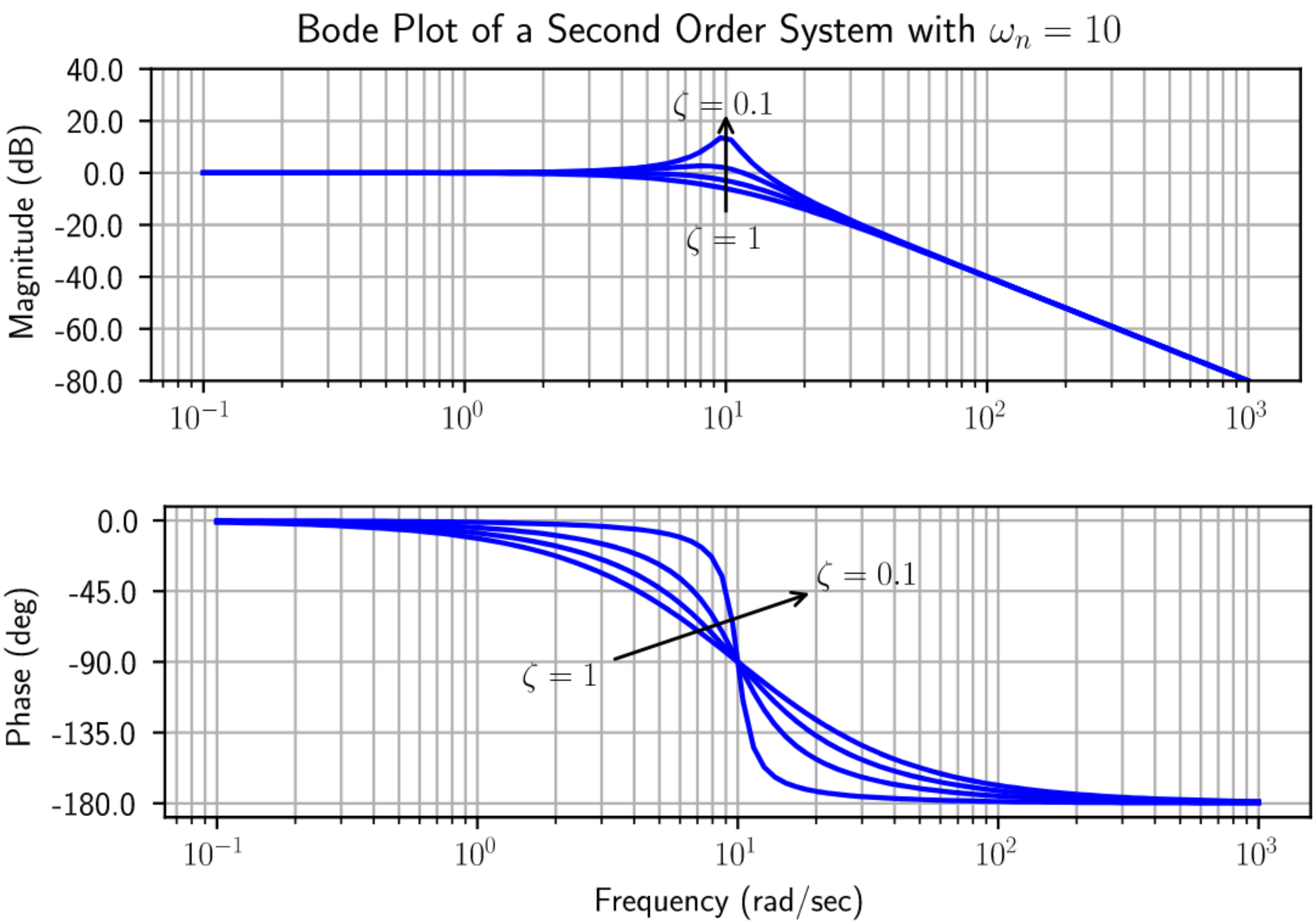
2. Integrator 1/s



3. First Order 1/(τs+1)



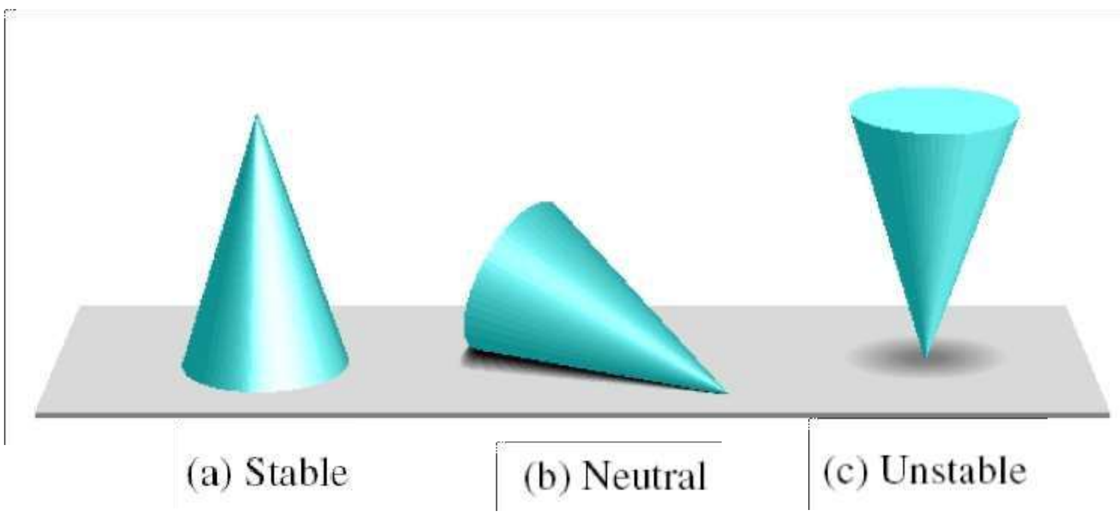
4. Second Order $1/((s/\omega_n)^2+2\zeta(s/\omega_n)+1)$



UNIT-IV
ROOT LOCUS ANALYSIS

- **Root locus**
- **Compensation technique**

Physical Meaning of Stability



Stability

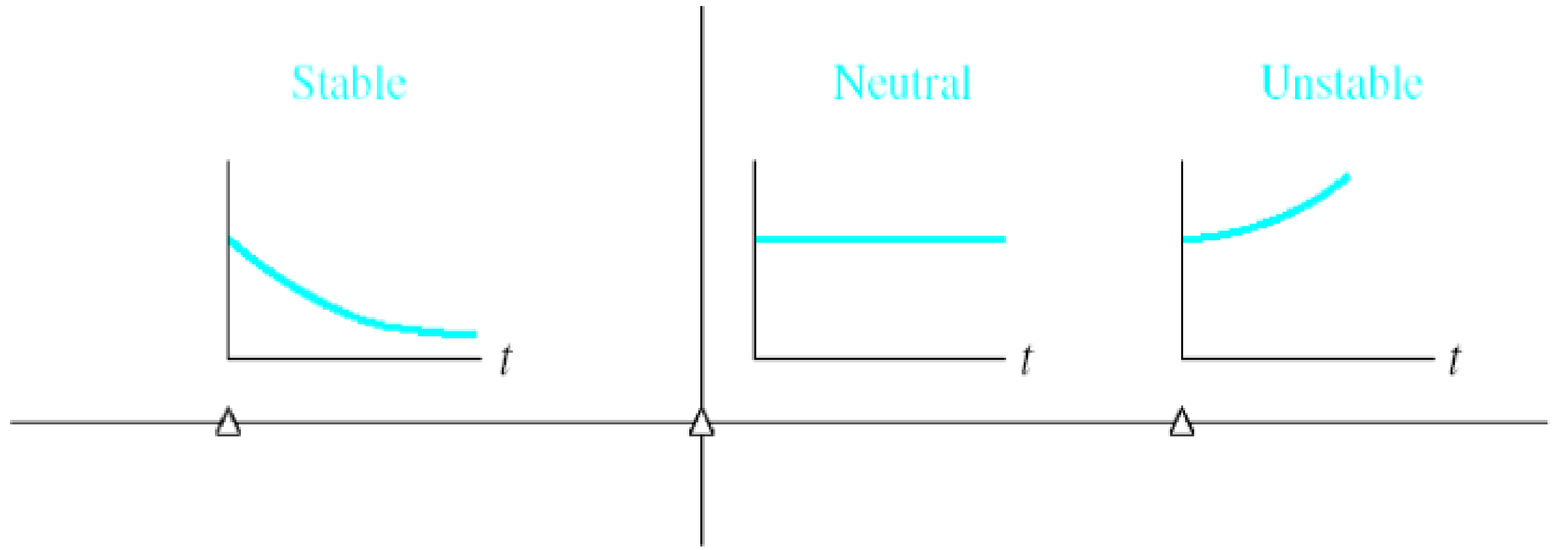
Stability of any system is a very important characteristic of any system.

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A **stable system** produces a bounded output for a given bounded input.

In Control System STABILITY cab be judged by observing the time response curve. (Which is basically depends on location of poles)

Generally for a stable system oscillations must die out as early as possible or steady state should be reached fast. (in time response curve)

Time Response Curve



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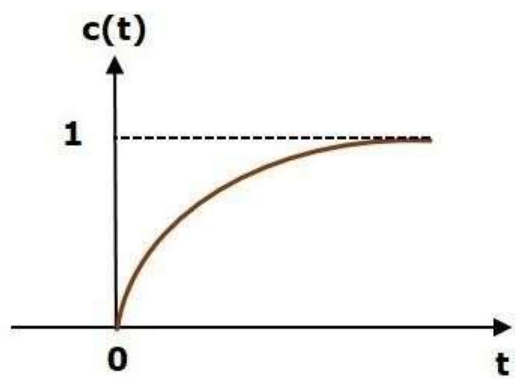
Absolute Stability

If the system is stable for all the range of system component values, then it is known as the **absolutely stable system**. The open loop control system is absolutely stable if all the poles of the open loop transfer function present in left half ... arly, the closed loop control system is a ... all the poles of the closed loop transfe ... n the left half of the ‘s’ plane.



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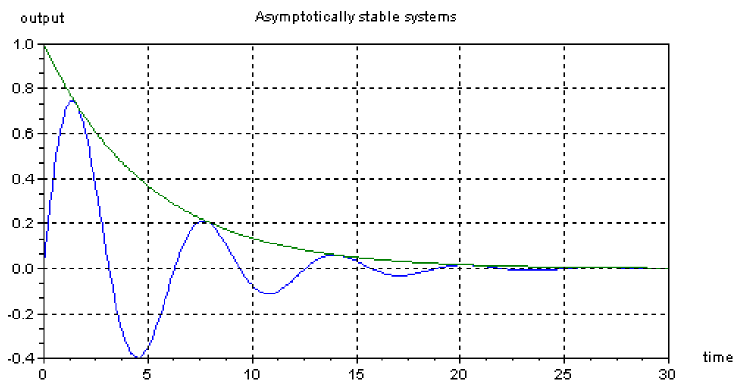
Asymptotic Stability

An LTI system is asymptotically stable if its impulse response g(t) satisfies the condition

$$\int_0^{\infty} |g(t)|dt < \infty$$



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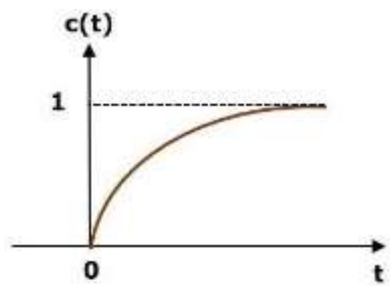
What is Stability?

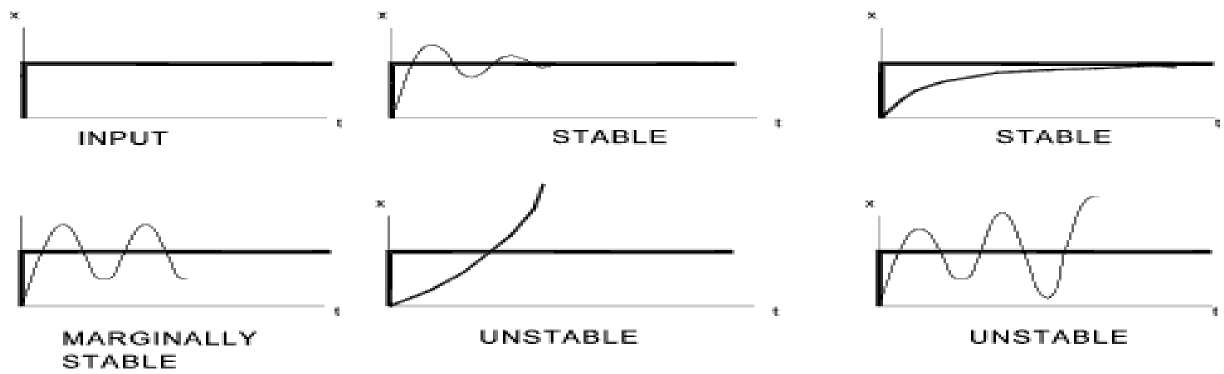
- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A **stable system** produces a bounded output for a given bounded input.
- The following figure shows the response of a stable system.



- This is the response of first order control system for unit step input. This response has the values between 0 and 1. So, it is bounded output. We know that the unit step signal has the value of one for all positive values of **t** including zero. So, it is bounded input. Therefore, the first order control system is stable since both the input and the output are bounded.

Types of Stability

- We can classify the systems based on stability as follows.
  1) Absolutely stable system
  2) Asymptotic stable system
  2) Conditionally stable system
  3) Marginally stable system
- This presentation explains conditional and marginal stability.



Conditional Stability

- If the system is stable for a certain range of system component values, then it is known as **conditionally stable system**.

- A conditional stability has no guarantee that a system will be all time stable. Two case arises here is:

1. Necessary but not sufficient conditions. Example- Routh-Hurwitz Criteria
2. Sufficient but not necessary conditions. Example- if a system is BIBO stable for a particular gain but unstable for any other gain value.

Routh-Hurwitz Stability Criterion

- The characteristic equation of the nth order continuous system can be write as:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

- The stability criterion is applied using a Routh table which is defined as;

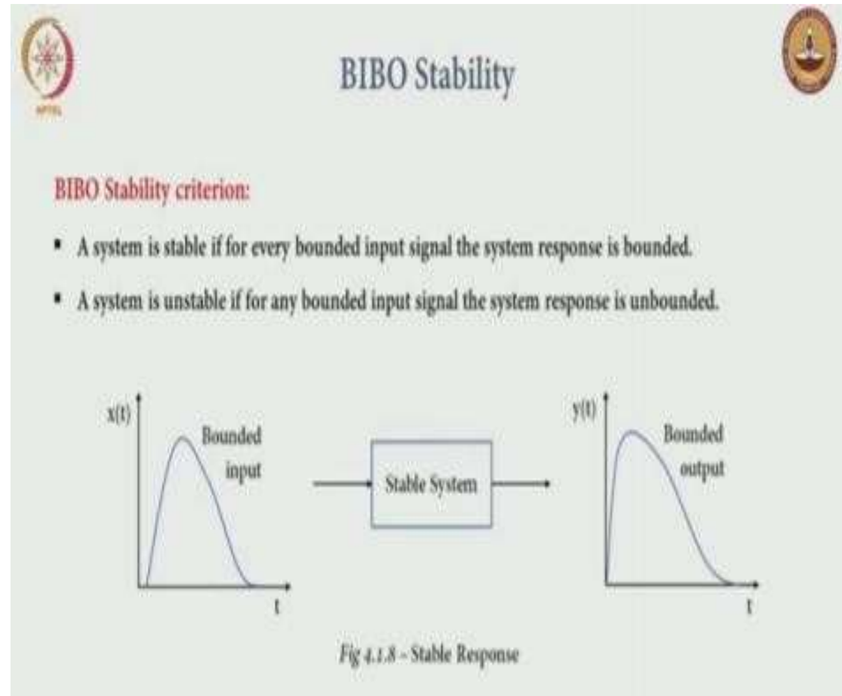
$$\begin{array}{c|cccc}
s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\
\cdot & b_1 & b_2 & b_3 & \cdots \\
\cdot & c_1 & c_2 & c_3 & \cdots \\
\cdot & \cdots & \cdots & \cdots & \cdots
\end{array}$$

- Where $a_n, a_{n-1}, \ldots, a_0$ re coefficients of the characteristic equation.

$$b_1 \equiv \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad b_2 \equiv \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 \equiv \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad c_2 \equiv \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$$

{taken from:3}

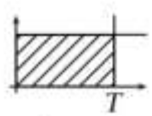


Marginal Stability

- If the system is stable by producing an output signal with constant amplitude and constant frequency of oscillations for bounded input, then it is known as **marginally stable system**. The open loop control system is marginally stable if any two poles of the open loop transfer function is present on the imaginary axis. Similarly, the closed loop control system is marginally stable if any two poles of the closed loop transfer function is present on the imaginary axis.
- An LTI system is marginally stable if it is not asymptotically stable, but there nevertheless exist numbers A, B < ∞ such that,
- Examples of marginal sta

$$\int_0^T |g(t)|dt < A + BT \qquad \text{for all } T$$

1. Integrator:

$g(t) = H(t) =$  $\int_0^T |g(t)|\,dt = T$

$G(s) = 1/s \implies j\omega\text{-axis pole at } s = 0$

2. Undamped spring-mass system:

$$g(t) = \cos(3t) \quad \Longrightarrow \int_0^T |g(t)|\,dt \le \int_0^T 1\,dt = T$$

$$G(s) = s/(s^2+9) \Longrightarrow j\omega\text{-axis poles at } s = \pm 3j$$

3. Delay line with lossless reflections:

$$g(t) = \sum_{k=0}^{\infty} \delta(t-k), \quad \Longrightarrow \int_0^T |g(t)|\,dt \le T+1$$

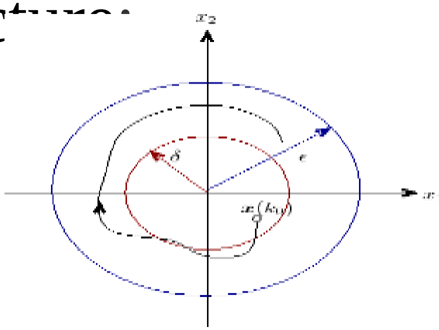
4. Something which cannot arise as the impulse response of any ODE:

($g(t) \to 0$, but system is *not* asymptotically stable)

$$g(t) = \frac{1}{t+1} \quad \Rightarrow \int_0^T |g(t)|\,dt = \log(T+1) \quad \le T$$

Asymptotic stability is generally pronounced in non linear system, where we *guess* the stability in reference to some equilibrium point of the system which we define in the trajectory we plot in phase plane.

Lets say we have plotted x1 vs x2 where them describes the dynamic state of system. He said, there exist a circle of red circumference, within which if any trajectories emanates *x(0)*, which never leave blue circle at any time and also returns back to the origin. This in his sense, was a asymptotically stable system. See the picture:



INTRODUCTION

- It was discovered that all coefficients of the characteristic polynomial must have the same sign and non-zero if all the roots are in the left-hand plane
- These requirements are necessary but not sufficient. If the above requirements are not met, it is known that the system is unstable. But, if the requirements are met, we still must investigate the system further to determine the stability of the system.

CRITERION

- The Routh-Hurwitz criterion states that the number of roots of q(s) with positive real parts is equal to the number of changes in sign of the first column of the Routh array.

Characteristic equation, q(s) $\longrightarrow$ $a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdot + a_1 s + a_0 = 0$

Routh array $\longrightarrow$

$$
\begin{array}{c|ccc}
s^n & a_n & a_{n-2} & a_{n-4} \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \\
s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} \\
s^{n-3} & c_{n-1} & c_{n-3} & c_{n-5} \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
s^0 & h_{n-1} & &
\end{array}
$$

$$b_{n-1} = \frac{(a_{n-1})(a_{n-2}) - a_n(a_{n-3})}{a_{n-1}} = \frac{-1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

$$b_{n-3} = \frac{-1}{a_{n-1}}\begin{vmatrix} a_{n-2} & a_{n-4} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

$$c_{n-1} = \frac{-1}{b_{n-1}}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}$$

Case One: No element in the first column is zero.

Example 6.1 Second-order system

The Characteristic polynomial of a second-order system is:

$$q(s) = a_2 \cdot s^2 + a_1 \cdot s + a_0$$

The Routh array is written as:

$$\begin{array}{c|cc} s^2 & a_2 & a_0 \\ s^1 & a_1 & 0 \\ s^0 & b_1 & 0 \end{array}$$

where:

$$b_1 = \frac{a_1 \cdot a_0 - (0) \cdot a_2}{a_1} = a_0$$

Therefore the requirement for a stable second-order system is simply that al coefficients be positive or all the coefficients be negative.

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Case Two: Zeros in the first column while some elements of the row containing a zero in the first column are nonzero.

If only one element in the array is zero, it may be replaced with a small positive number ε that is allowed to approach zero after completing the array.

$$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

The Routh array is then:

$$\begin{array}{c|ccc}
s^5 & 1 & 2 & 11 \\
s^4 & 2 & 4 & 10 \\
s^3 & b_1 & 6 & 0 \\
s^2 & c_1 & 10 & 0 \\
s^1 & d_1 & 0 & 0 \\
s^0 & 10 & 0 & 0
\end{array}$$

where:

$$b_1 = \frac{2 \cdot 2 - 1 \cdot 4}{2} = 0 \rightarrow \varepsilon \qquad c_1 = \frac{4\varepsilon - 2 \cdot 6}{\varepsilon} = \frac{-12}{\varepsilon} \qquad d_1 = \frac{6 \cdot c_1 - 10\varepsilon}{c_1} \rightarrow 6$$

There are two sign changes in the first column due to the large negative number calculated for c1. Thus, the system is unstable because two roots lie in the right half of the plane.

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other elements of the row containing the zero are also zero.

This case occurs when the polynomial q(s) has zeros located symmetrically about the origin of the s-plane, such as $(s+\sigma)(s-\sigma)$ or $(s+j\omega)(s-j\omega)$. This case is solved using the auxiliary polynomial, U(s), which is located in the row above the row containing the zero entry in the Routh array.

$$q(s) = s^3 + 2\cdot s^2 + 4s + K$$

Routh array:

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 2 & K \\ s^1 & \frac{8-K}{2} & 0 \\ s^0 & K & 0 \end{array}$$

For a stable system we require that $0 < K < 8$

For the marginally stable case, K=8, the s^1 row of the Routh array contains al zeros. The auxiliary plynomial comes from the s^2 row.

$$U(s) = 2s^2 + Ks^0 = 2\cdot s^2 + 8 = 2\left(s^2 + 4\right) = 2(s + j\cdot 2)(s - j\cdot 2)$$

It can be proven that U(s) is a factor of the characteristic polynomial:

$$\frac{q(s)}{U(s)} = \frac{s + 2}{2}$$

Thus, when K=8, the factors of the characteristic polynomial are:

$$q(s) = (s + 2)(s + j\cdot 2)(s - j\cdot 2)$$

Case Four: Repeated roots of the characteristic equation on the jw-axis.

- With simple roots on the jw-axis, the system will have a marginally stable behavior. This is not the case if the roots are repeated. Repeated roots on the jw-axis will cause the system to be unstable. Unfortunately, the routh-array will fail to reveal this instability.

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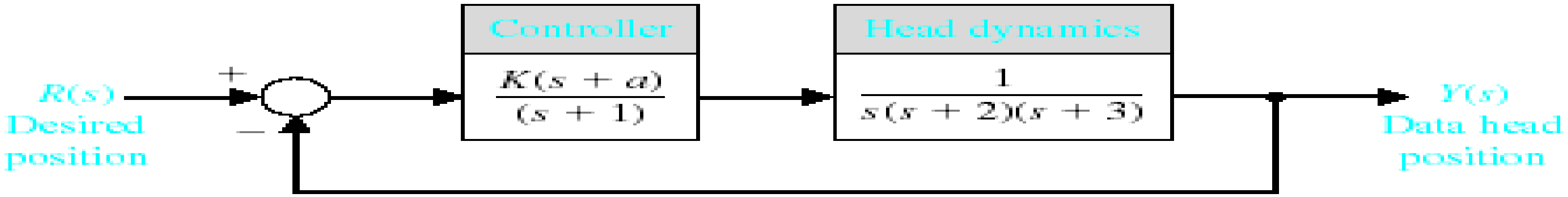
Example : Welding control



Using block diagram reduction we find that: $q(s) = s^4 + 6s^3 + 11s^2 + (K+6)s + Ka$

The Routh array is then:

$$\begin{array}{c|ccc} s^4 & 1 & 11 & Ka \\ s^3 & 6 & (K+6) & \\ s^2 & b_3 & Ka & \\ s^1 & c_3 & & \\ s^0 & Ka & & \end{array}$$

where: $b_3 = \frac{60 - K}{6}$ and $c_3 = \frac{b_3(K+6) - 6 \cdot Ka}{b_3}$

For the system to be stable both b_3 and c_3 must be positive.

Using these equations a relationship can be determined for K a

ROOT LOCUS TECHNIQUE

- The **root locus technique in control system** was first introduced in the year 1948 by Evans. Any physical system is represented by a transfer function in the form of
G(S)=k (numerator of s/denominator of s)

- We can find poles and zeros from G(s). The location of poles and zeros are crucial keeping view stability, relative stability, transient response and error analysis.

- In **root locus technique** we will evaluate the position of the roots, their locus of movement and associated information. These information will be used to comment upon the system performance.

- Now there **are two ways** of determining the value of K, each way is described below:-

  1)Magnitude criteria

  2)Root locus plot

- **1)Magnitude criteria**:-At any points on the root locus we can apply magnitude criteria as, **|G(S)H(S)|=1**, Using this formula we can calculate the value of K at any desired point.

2)Root locus plot:- The value of K at any s on the root locus is given by

K=product of all vector length drawn from poles of g(s)h(s) to s / product of all vector length drawn from zeros of g(s)h(s) to s

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ADVANTAGES OF LOOT LOCCUS TECHNIQUE

- Root locus technique in control system is easy to implement as compared to other methods.
- With the help of root locus we can easily predict the performance of the whole system.
- Root locus provides the better way to indicate the parameters.

APPLICATION OF ROOT LOCUS TECHNIQUE

- In control theory , we use some methods to analize the stability of the system , in the particular case of the root locus method , we analize a system in the form of a transfer function.

- The root locus method allows to modify our poles (the values for the denominator of our transfer funcion that are equal to zero) , and the zeroes (the same values to zero but for the denominator of the transfer function)

- The root locus method allows to determine a gain for the transfer function in a open loop system (usually a SISO system).

- Root Locus analysis is useful for you to check the places where the poles of your linear system will be located on when you close its feedback loop with a gain factor. When you change the gain, the poles change their place through the loci.

Basic diagram for compensation technique

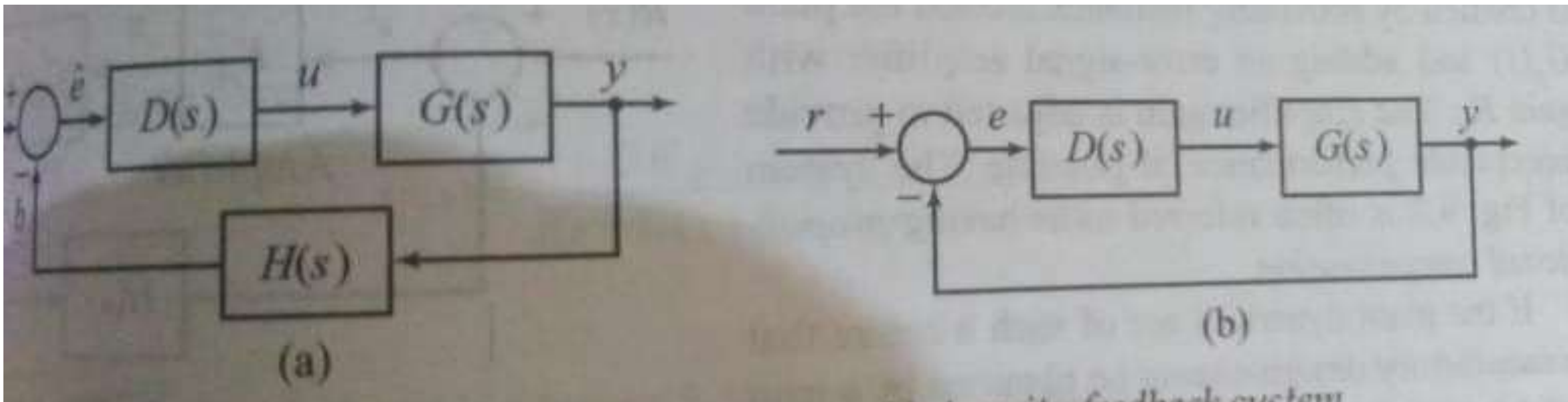
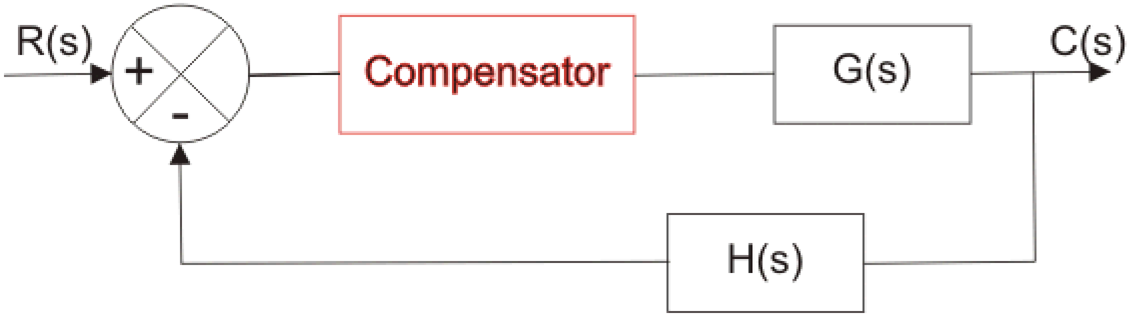


Fig. 9.1 (a) A non-unity feedback system (b) A unity feedback system

1. Series compensation

- When a compensating network is inserted in the forward path, this is called series or cascade compensation.



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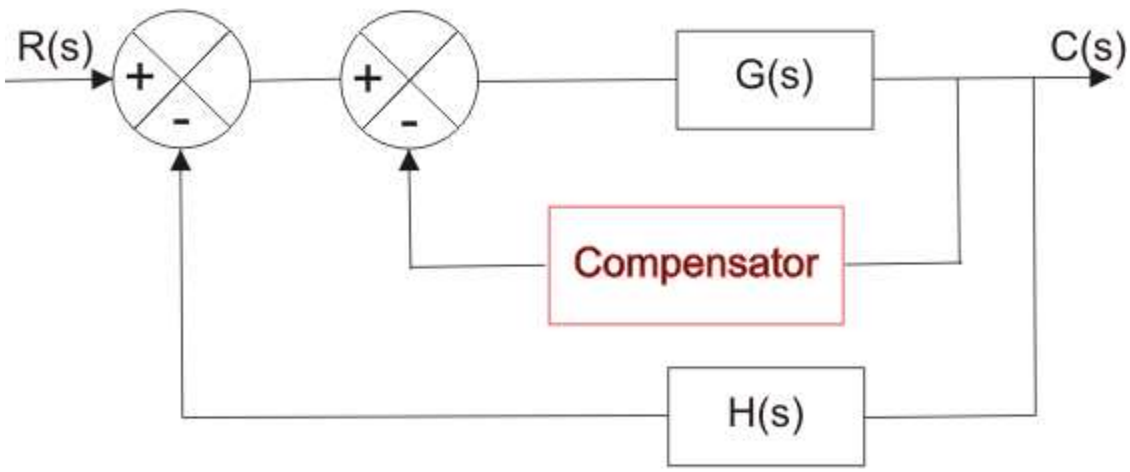
2. Feedback compensation

- When a compensator is inserted in the feedback path, this is called feedback compensation.



3. Load compensation

- A combination of series and feedback compensation is called load compensation or combined cascade and feedback compensation.

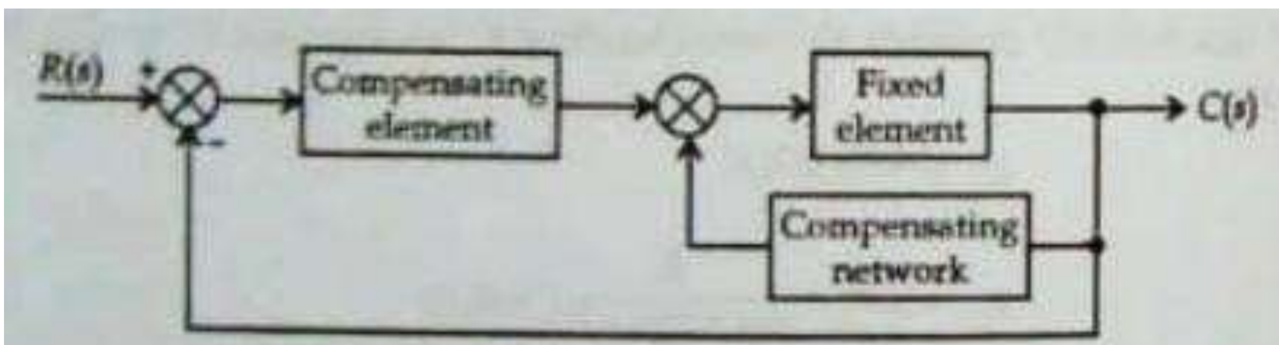


Fig. 6.2(c) Load compensation

NEED FOR COMPENSATION

- In order to obtain the desired performance of the system, we use compensating networks. Compensating networks are applied to the system in the form of feed forward path gain adjustment.
- Compensate a unstable system to make it stable.
- A compensating network is used to minimize overshoot.
- These compensating networks increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system.
- Compensating networks also introduces poles and zeros in the system thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system change.

Necessary of Compensation

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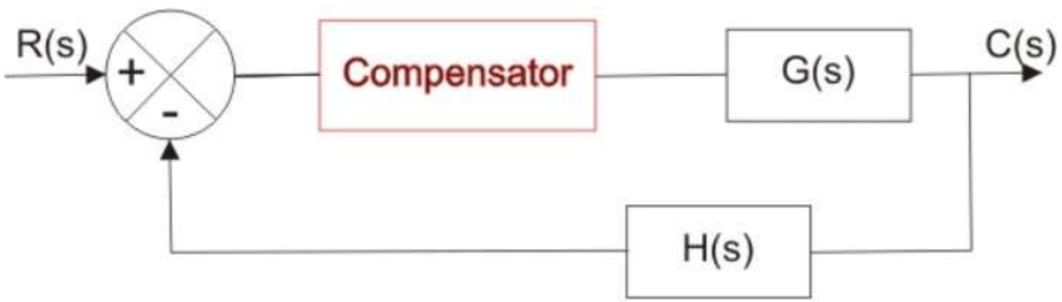
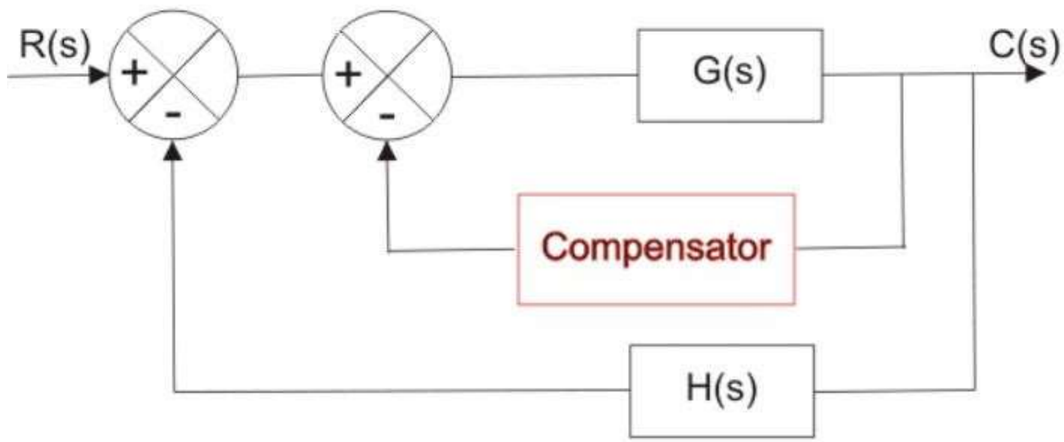
Methods of Compensation

1. Connecting compensating circuit between error detector and plants known as **series compensation**.



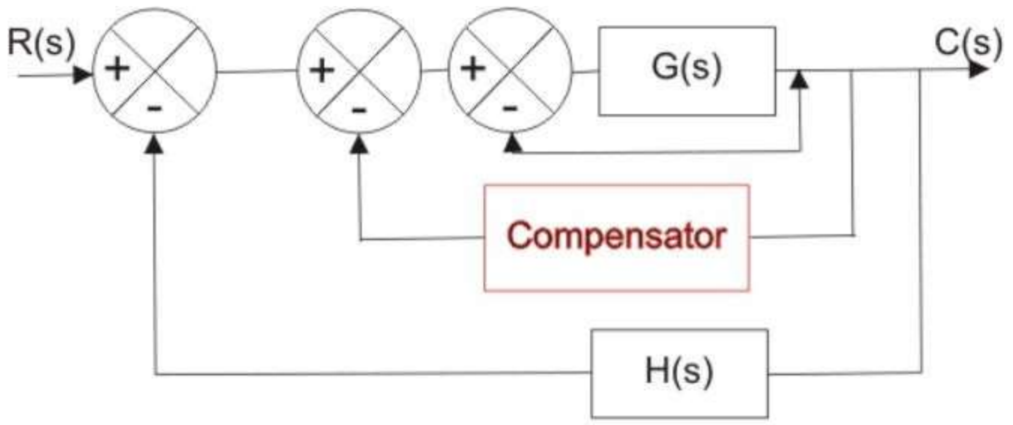
Series Compensator

2. When a compensator used in a feedback manner called **feedback compensation**.



Feedback Compensator

3. A combination of series and feedback compensator is called **load compensation**.



Load Compensator

Compensating Network

A compensating network is one which makes some adjustments in order to make up for deficiencies in the system. Compensating devices are may be in the form of electrical, mechanical, hydraulic etc. Most electrical compensator are RC filter. The simplest network used for compensator are known as lead, lag network.

Types of Compensation

Series Compensation or Cascade Compensation

Feedback compensation or Parallel compensation

Series-feedback compensation

Feed forward compensation

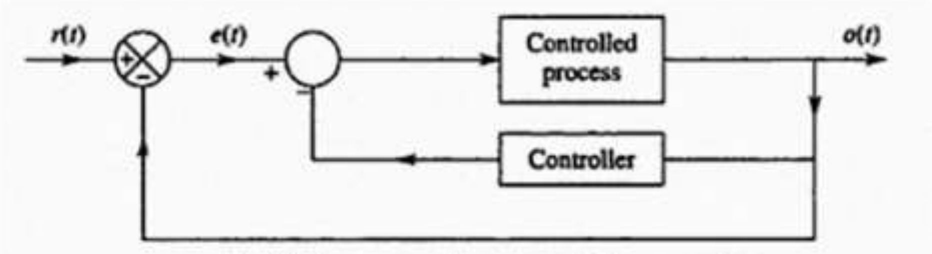
Lead Compensator

Lag Compensator

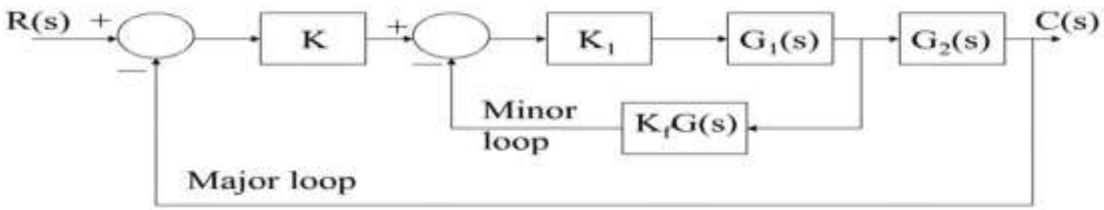
Lag-Lead Compensator

Feedback compensation or Parallel compensation

This is the system where the controller is placed in the sensor feedback path as shown in fig.



Feedback compensation or parallel compensation.



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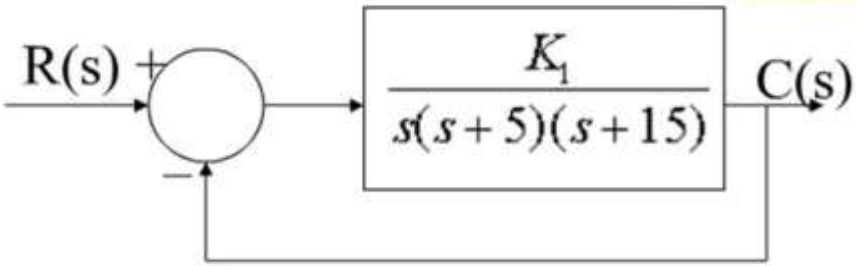
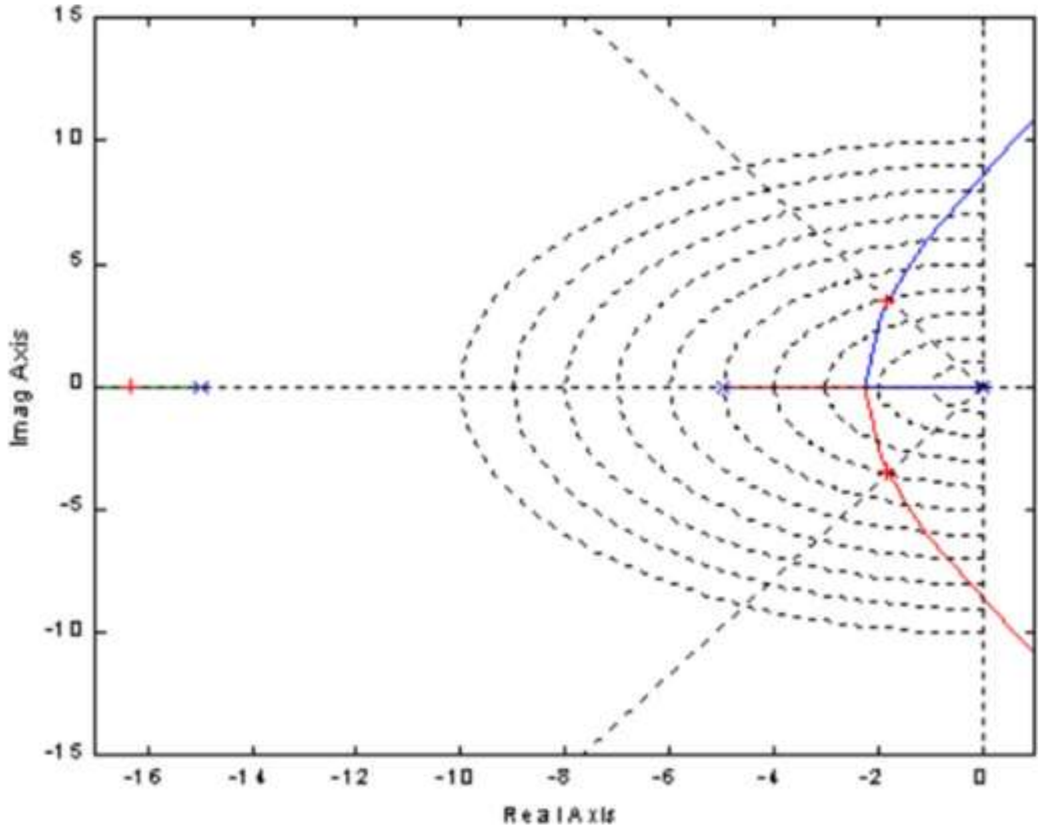
- More complicated than for cascade compensation
- Yielding faster responses
- Designing faster responses into portion of a control loop in order to provide isolation
- Being used in case where noise problems preclude the use of cascade compensation
- Not requiring additional amplification

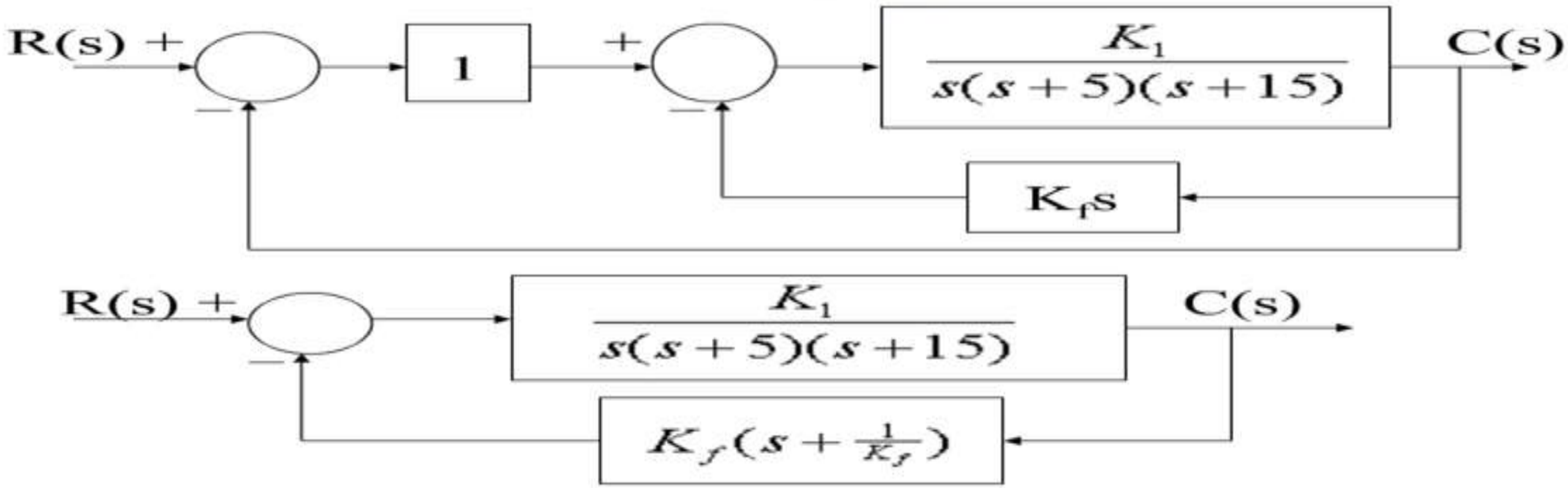
Example :



Design rate feedback compensation to reduce the settling time by a factor of 4 while continuing to operate the system with 20% overshoot.

First, checking the values for the uncompensated system: Plotting the root locus and determining the 0.456 line intersection with the root locus gives the dominant poles at $-1.8\pm j3.5$. The settling time is $4/1.8=2.22$sec and should be reduced to $2.22/4=0.555$sec


Imag Axis
Real Axis



To achieve a fourfold decrease in settling time, the real part of the pole must be increased by a factor of 4 to 4(-1.8)=-7.2. The imaginary part is then, w_d=-7.2 tan(180^0-62.87^0)=14

Using the compensated dominant pole position of -7.2+j14, the sum of the angles from the system's poles to be -277^0, then the angle condition of the root locus requires the contribution from zero to be 97^0. The zero location is calculated as

$$\frac{14}{7.2 - z_c} = \tan(180^0 - 97^0) \Rightarrow z_c = 5.48$$

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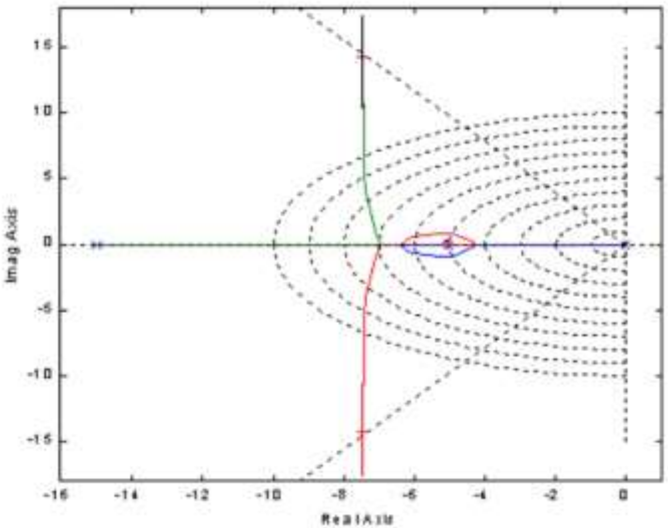
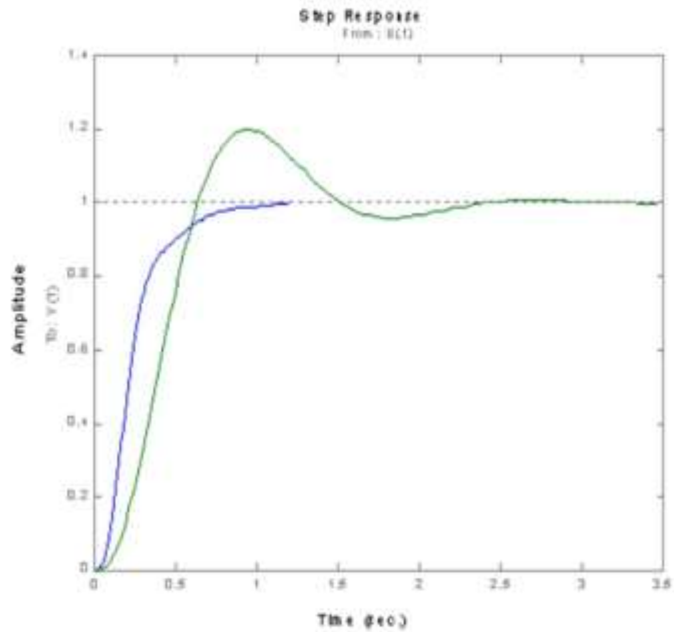


$$s_{1,2} = -7.2 \pm j14$$

$$s_3 = -5.1$$

$$K = 259$$



Since the third pole is not far away than the dominant poles it cannot be neglected. Its' effect is seen in the step response

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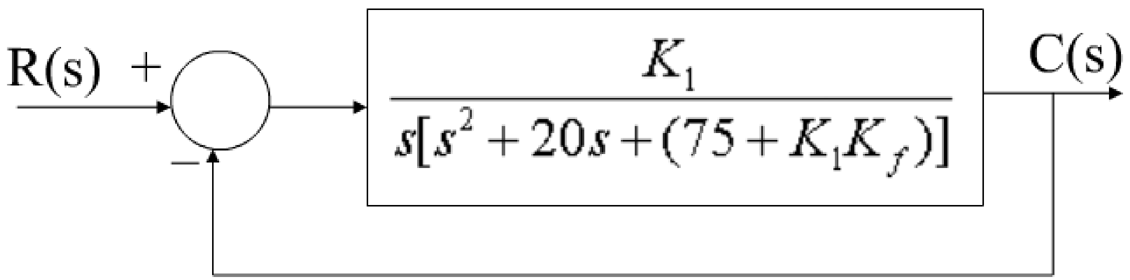
The gain at the desired point is found to be 259, which is $K_1 K_f$. Since K_f is the reciprocal of the compensator zero (1/5.48), then K_1=1419.3 In order to evaluate the steady-state error characteristics, redraw the block diagram as follows

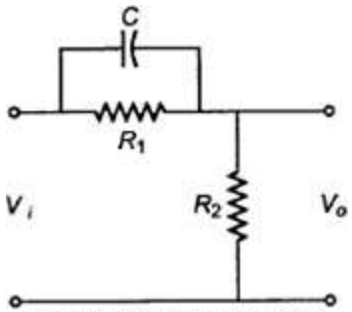


$$K_v = \frac{K_1}{75 + K_1 K_f} = 4.25$$

Compensation

- Modification of system dynamics, to improve characteristics of the open-loop plant so that it can safely be used with feedback control. For exemple to improve stability, uncouple a system, to a 'lead/lag compensator, phase compensator. In case of Lead and Lag compensator which are often designed to satisfy phase and gain margins, these bode plot of the system itself changes after the insertion of compensator, bode plot indirectly represents system dynamics.
- There are three types of compensators

1. Lead Compensators
2. Lag Compensators
3. Lead-Lag Compensators



Phase lead compensation

Lag Compensator

The Lag Compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied.

Here, the capacitor is in series with the resistor R2R2 and the output is measured across this combination.

The transfer function of this lag compensator is –

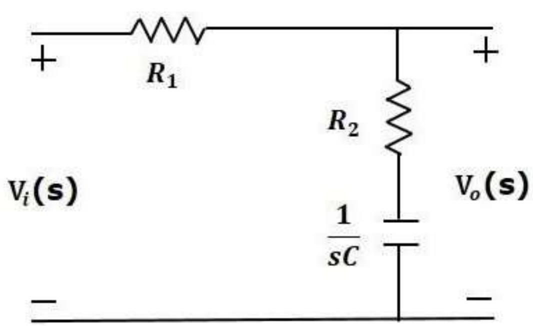
Lead Compensation

- Generally Lead compensators are represented by following transfer function

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1}, \quad (0 < \alpha < 1)$$

- or

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \quad (0 < \alpha < 1)$$



From the above equation, α is always greater than one.

Lead Compensator

The lead compensator is an electrical network which produces a sinusoidal output having phase lead when a sinusoidal input is applied.

Here, the capacitor is parallel to the resistor R1R1 and the output is measured across resistor $R_2.

The transfer function of this lead compensator is –

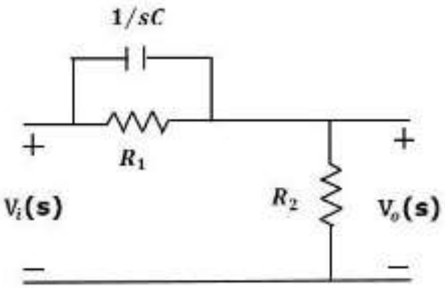
Lead Compensation

- Generally Lead compensators are represented by following transfer function

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1}, \quad (0 < \alpha < 1)$$

- or

$$G_c(s) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \quad (0 < \alpha < 1)$$



From the transfer function, we can conclude that the lead compensator has pole at s=−1βs=−1β and zero at s=−1βτs=−1βτ.

Lead-Lag Compensators

Lag-Lead compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region. It is a combination of both the lag and the lead compensators.

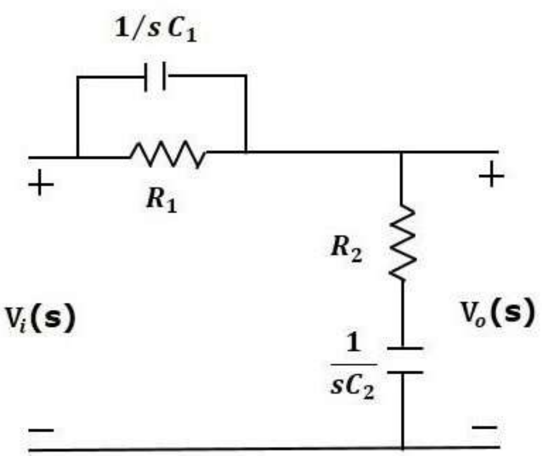
This circuit looks like both the compensators are cascaded. So, the transfer function of this circuit will be the product of transfer functions of the lead and the lag compensators.

Lag-Lead Compensation

- Lag-Lead compensators are represented by following transfer function

$$G_c(s) = K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right), \quad (\gamma > 1 \ and \ \beta > 1)$$

- Where K_c belongs to lead portion of the compensator.



- Lead–lag compensators influence disciplines as varied as robotics, satellite control, automobile diagnostics, LCD displays and laser frequency stabilization. They are an important building block in analog control systems, and can also be used in digital control.
- Given the control plant, desired specifications can be achieved using compensators. I, D, PI, PD, and PID, are optimizing controllers which are used to improve system parameters (such as reducing steady state error, reducing resonant peak, improving system response by reducing rise time). All these operations can be done by compensators as well, used in cascade compensation technique.

- Ideal Derivative Compensation
- PID controller design
- Feedback Compensation
- Physical Realization of Compensation

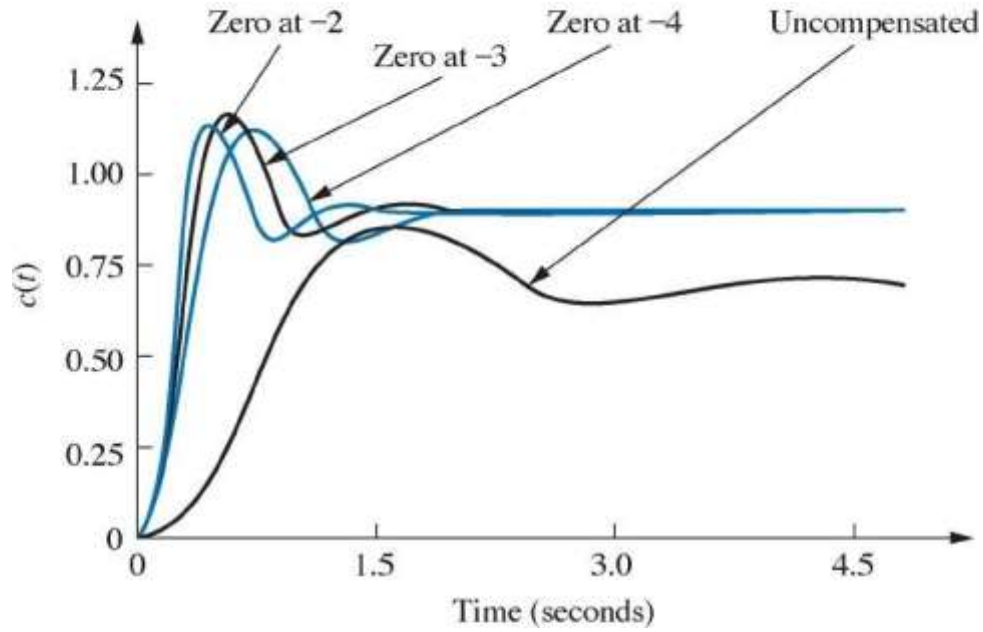
Generally, we want to speed up the transient response (decrease T_s and T_p). If we are lucky then a system's desired transient response lies on its RL. However, if no point on the RL corresponds to the desired transient response then we must compensate the system. A **derivative compensator** modifies the RL to go through the desired point.

A derivative compensator adds a zero to the forward path.

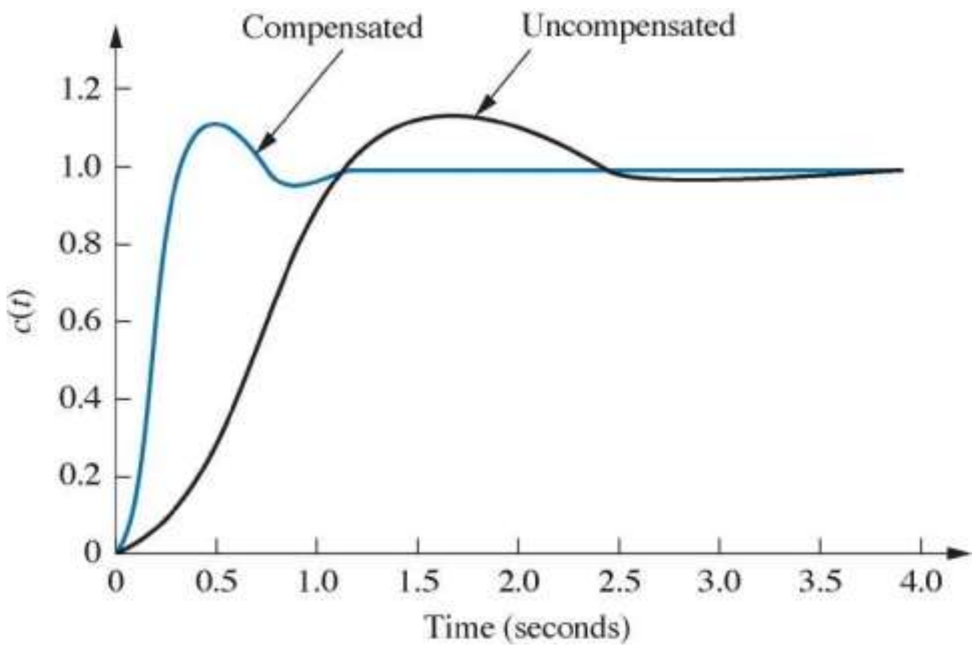
$$G_c(s) = s + z_c$$

Notice that this transfer function is the sum of a differentiator and a pure gain. Thus, we refer to its use as **PD control** (**p**roportional + **d**erivative).

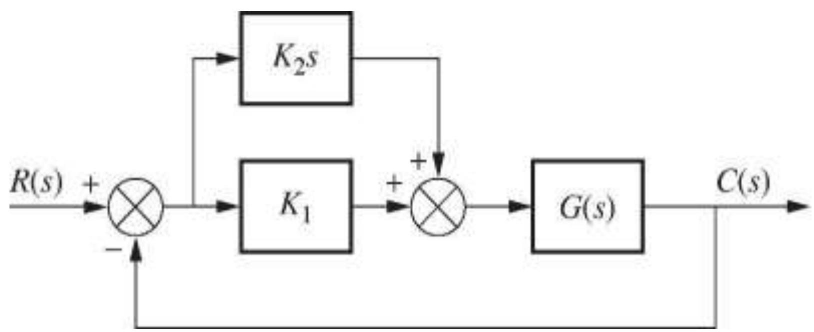
As the zero is moved we get changes in T_s and T_p . In this case, when the zero is moved to -2 we get the fastest response. All the while, we are maintaining $\%OS$.



We show how to best place the zero by example...


Compensated
Uncompensated
1.2
1.0
0.8
0.6
0.4
0.2
0
c(t)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Time (seconds)

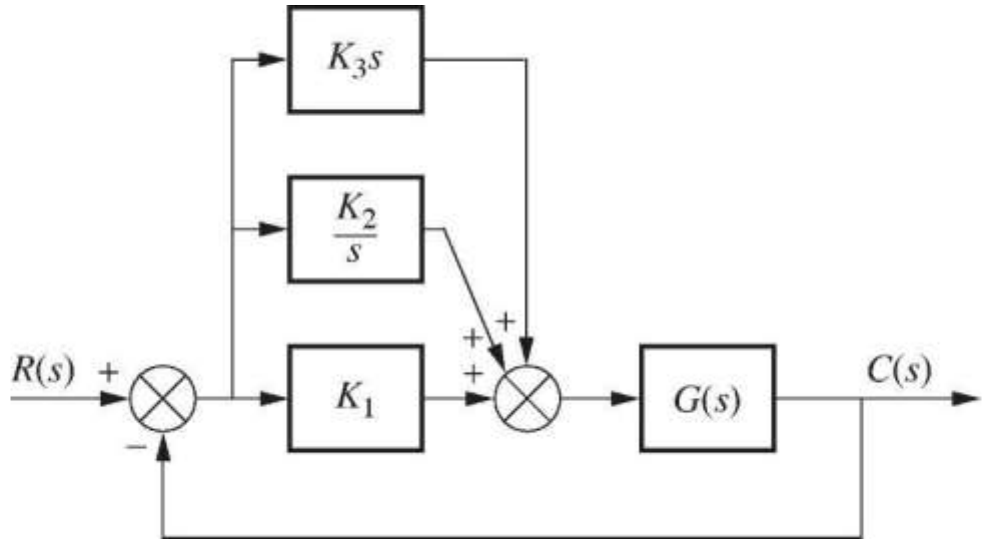
A PD controller can be implemented in a similar manner to the PI controller by placing the proportional and derivative compensators in parallel:



A PID controller utilizes PI and PD control together to address both steady-state error and transient response. There are two ways to proceed:

- Design for transient response, then design for steady-state eror
  - Con: May slightly decrease response speed when designing for steady-state error.
- Design for steady-state eror, then design for transient response
  - Con: May increase (or possibly decrease) steady state error when designing for transient response.

We choose to design for transient response first.



The transfer function for a PID controller is as follows:

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

Notice that this function has two zeros and one pole. The location of one zero will come from the transient response design, the other zero will come from the steady-state error design.

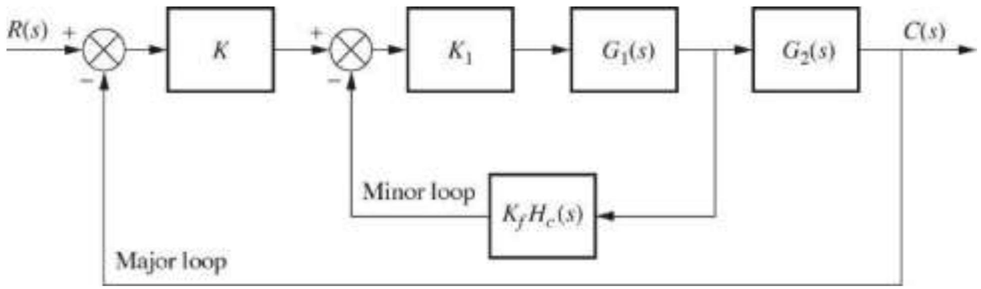
TABLE 9.10 Active realization of controllers and compensators, using an operational amplifier (table continues)

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\dfrac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\dfrac{R_2}{R_1}$
Integration	R	C	$-\dfrac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$
PI controller	R_1	R_2 C	$-\dfrac{R_2}{R_1}\dfrac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	C R_1	R_2	$-R_2C\left(s+\dfrac{1}{R_1C}\right)$
PID controller	C_1 R_1	R_2 C_2	$-\left[\left(\dfrac{R_2}{R_1}+\dfrac{C_1}{C_2}\right)+R_2C_1s+\dfrac{\frac{1}{R_1C_2}}{s}\right]$

We can implement lag and lead compensators with both op-amps and with passive circuits (see text).

Feedback Compensation

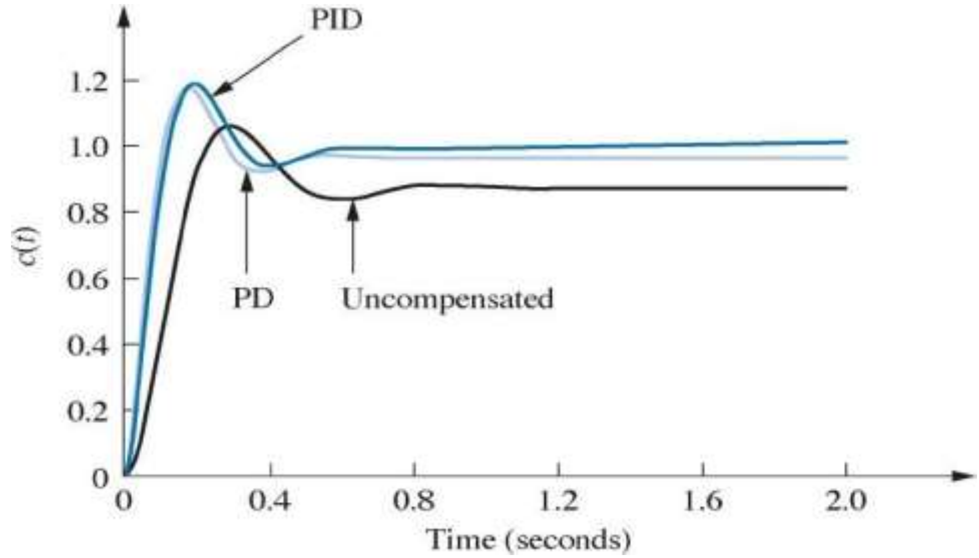
We have focussed on the addition of compensators in the forward path. It is also possible to add compensators in the feedback path:



Feedback compensators can yield faster responses than cascade compensators. They also tend to require less amplification since the compensator's input comes from the high-power output of the system, rather than from the low-lower actuating signal. Reduced amplification is preferred in noisy systems where we want to avoid amplifiying the noise.

Design techniques for feedback compensators are related to the design techniques for cascade compensators, but we will not study them in this course.

he system's step response shows both the improvement in speed and in reduction of steady-state error:



Since the second-order approximation is no longer valid, it is important to simulate the response to verify that requirements are met. In this case the desired reduction in T_p of 2/3 was not achieved (uncompensated: 0.297, PID-compensated 0.214). If this is deemed significant, we could re-design the PD component, for a greater than 2/3 reduction in T_p . Alternately we could move the PI component's zero further from the origin to yield a faster response.