Friction : when a body slides over another body, a force is exerted at the surface of contact by the stationary body on the moving body. This resisting force is called the force of friction. and acts in a direction opposite to the direction of motion.

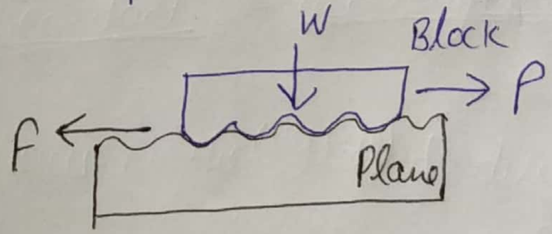
Friction is desirable as well as undesirable (a necessary evil)

UnDesirable - Power screws
bearings and gear
flow of fluids in pipes

Desirable: Friction brakes and clutches
belt and rope drives
holding and fastening devices

Dry friction - The friction between dry surfaces in contact is called dry friction. It is also known as coulomb friction.
Types - (Sliding friction & Rolling friction)

The major cause of such friction is due to minute projection (Irregularities) of the surfaces.



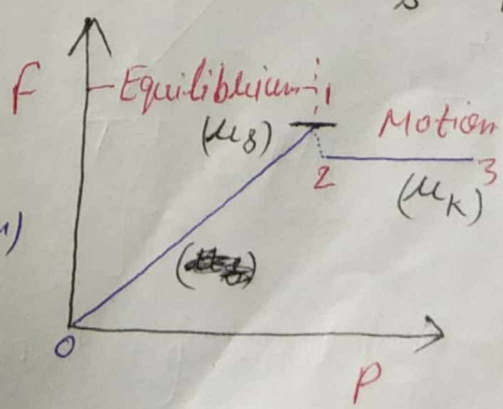
Fluid Friction -

Static and Dynamic friction -

Limiting friction - It is the maximum frictional force exerted at the time of impending motion.

$\mu_s > \mu_k$

0-1 = Zone of static friction.
2-3 - Zone of kinetic friction.
at point 1, friction is max. (Limiting friction)
1-2 - Variation is uncertain (dotted line)



Laws of dry friction

1) The ~~total~~ (Limiting) friction that can be developed is independent of the magnitude of the area of contact.

2) The ~~total~~ Limiting friction that can be developed is proportional to the normal force transmitted across surface of contact.

3) The force necessary to start the motion is greater than that necessary to maintain the motion.

Characterstics of friction

→ Friction always opposes the relative motion of the body and is tangential to the surface of contact.

→ It is a passive force, it exists as long as the tractive force acts.

→ It is a self-adjusting force

→ It is proportional to normal force.

Scanned by CamScanner

Terms related to friction

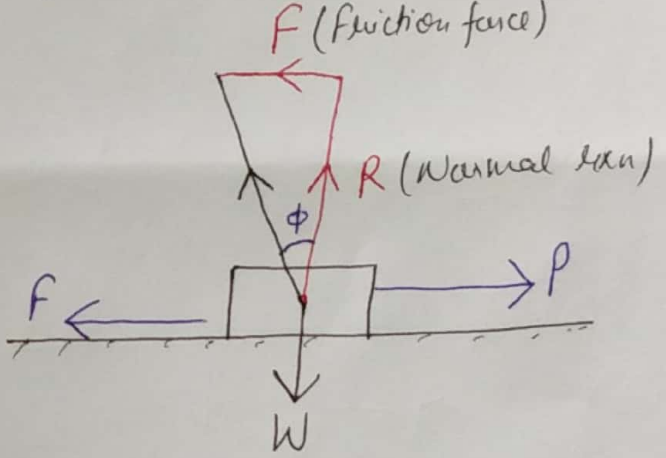
Co-efficient of friction: Ratio of force of friction to the normal reaction b/w the contact surfaces.

$$F \propto R$$

(Frictional force) (Normal reaction)

$$F = \mu R \qquad \mu_S = \frac{F_S}{R}, \quad \mu_K = \frac{F_K}{R}$$

Angle of friction- It is the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction

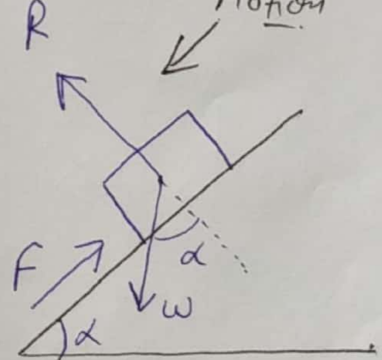


$$\tan\phi = \frac{F}{R}$$

$$\boxed{\mu = \tan\phi}$$

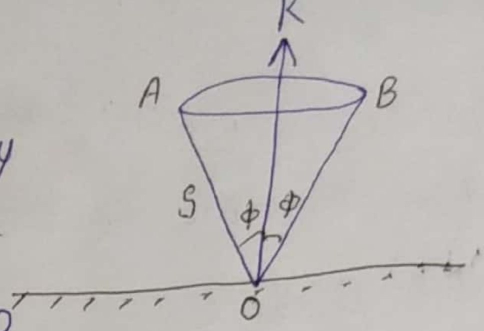
Angle of repose:-

The angle of the inclined plane at which a block resting on it is about to slide down the plane is called the angle of repose. (α)



$$R = W\cos\alpha \quad \text{-(i)}$$
$$\mu R = W\sin\alpha \quad \text{-(ii)}$$
$$\mu = \tan\alpha$$
$$\tan\phi = \tan\alpha$$
$$\boxed{\phi = \alpha}$$

Cone of friction:



The cone of friction is the imaginary cone AOB generated by revolving the static resultant about the normal OR.

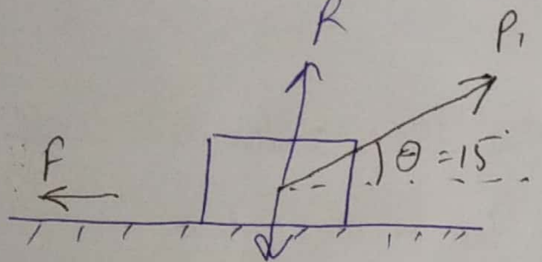
For the motion to occur the resultant R will lie on the surface of the cone.

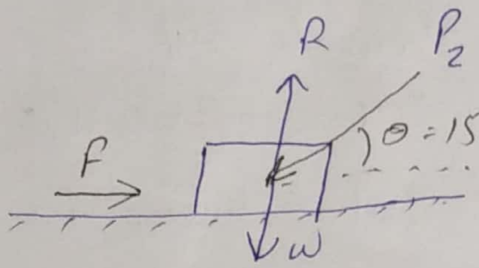
when the friction force is less than the limiting friction, the total reaction would lies within the cone.

(This aspect forms the working principle for self-locking mechanisms.)

Q A wooden block of weight 50N rests on a horizontal plane. Determine the force required to just (a) pull it (b) push it $\mu = 0.4$. Comment on the result. (3)

Sol.





$\Sigma F_x = 0 \quad F = P_1 \cos 15$

$\Sigma F_y = 0 \quad R = W - P_1 \sin\theta$

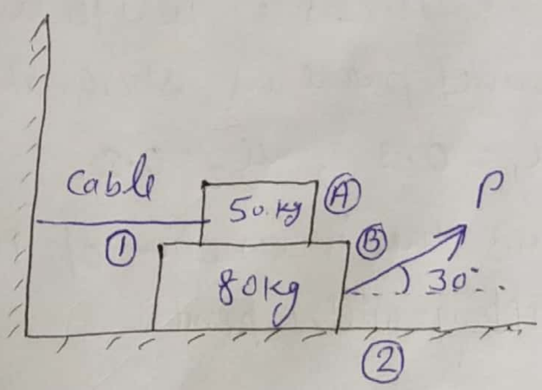
$P_1 = 18.7\ N$

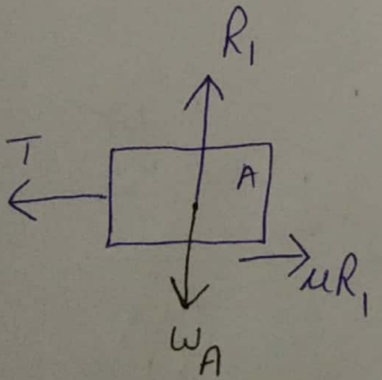
$P = 23.17\ N$

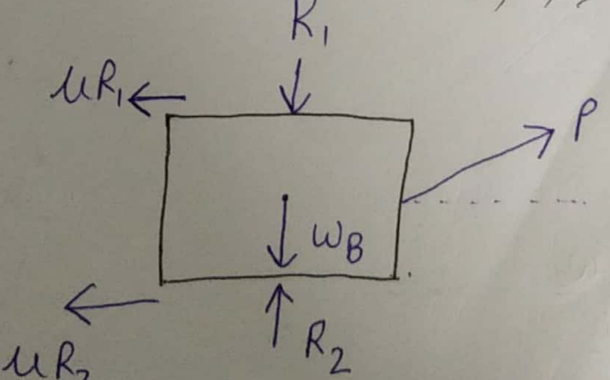
It is easier to pull the block than push it

Q Two blocks A and B weighing 50kg and 80kg resp. are in equilibrium as shown in fig. Calculate the force P required to move the lower block B and tension in the cable. ($\mu = 0.3$)

Sol.







$\boxed{P = 521.4N}$

Q A 7m long ladder rests against a vertical wall, with which it makes an angle of 45° with the floor. If a man, whose weight is one half of that of ladder climbs it, at what distance along the ladder will he be, when the ladder is about to slip?

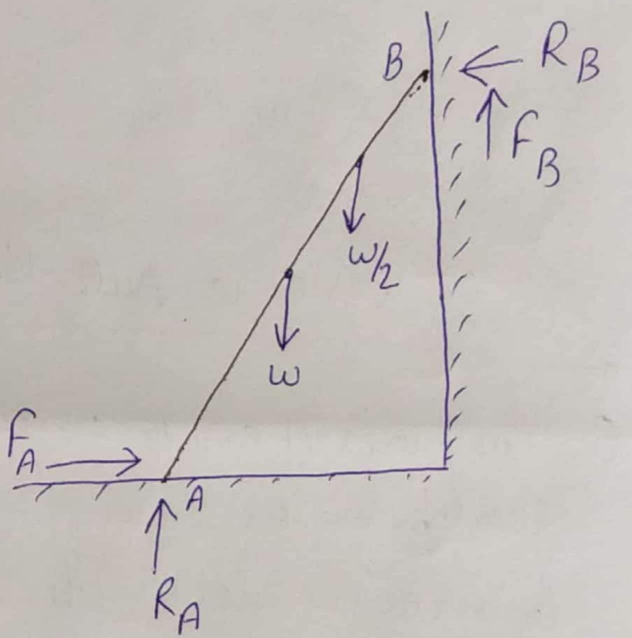
The Coefficients of friction b/w the ladder and wall is 1/3 and that between the ladder and the floor is 1/2.

Ans

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M = 0$



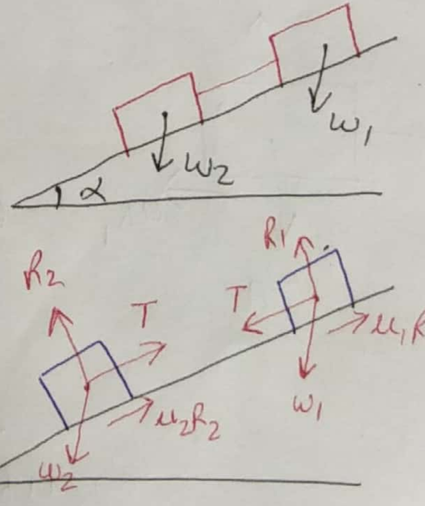
$l = 5m$

Q Two blocks of weigh $W_1 = 50N$ and $W_2 = 50N$ rest on a rough inclined plane as shown in fig.

$\mu_1 = 0.3$, $\mu_2 = 0.2$

Find the inclination of the plane for which slipping will impend.

Sol.

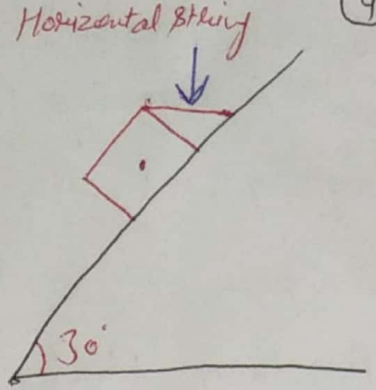


$$\tan\alpha = \frac{\mu_1 W_1 + \mu_2 W_2}{W_1 + W_2}$$

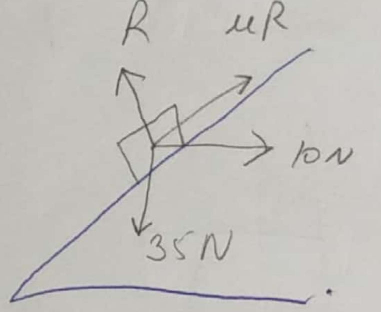
$\boxed{\alpha = 14°}$

Scanned by CamScanner

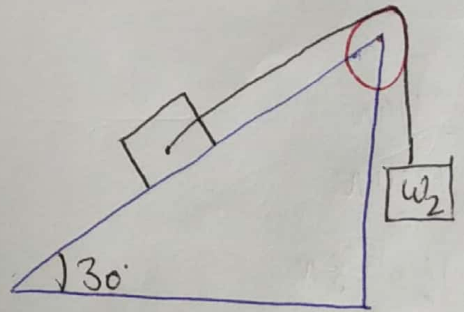
Q The block is tied up by a horizontal string which has a tension of 10N. If the block weighs 35 N, determine
i) friction force on the block
ii) normal rxn of the inclined plane
iii) μ.



Sol: $f = \mu R = 8.84N$
$R = 35.31N$
$\mu = 0.25$

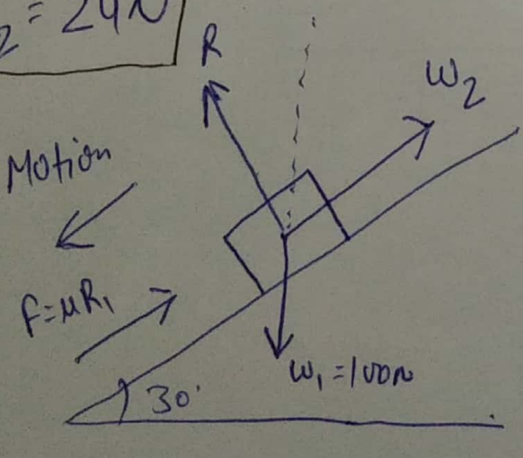


Q A block of weight $W_1 = 100N$ rests on an inclined plane and another weight W_2 is attached to the first weight through a string as shown at fig no. If the μ b/w the block and plane is 0.3, determine the max. and min values of W_2 so that eq. can exist.

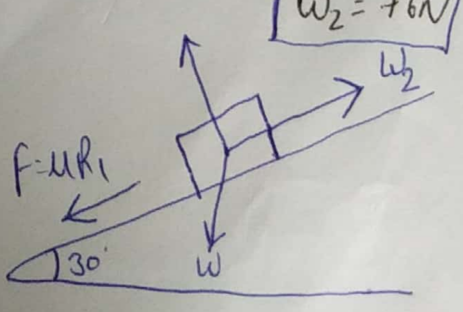


Sol.

$\boxed{W_2 = 24N}$



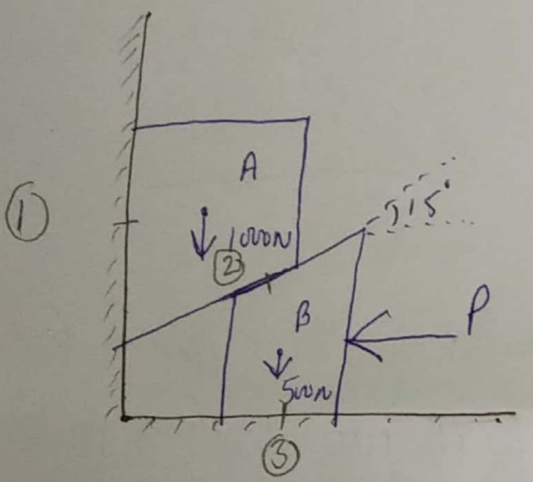
$\boxed{W_2 = 76N}$

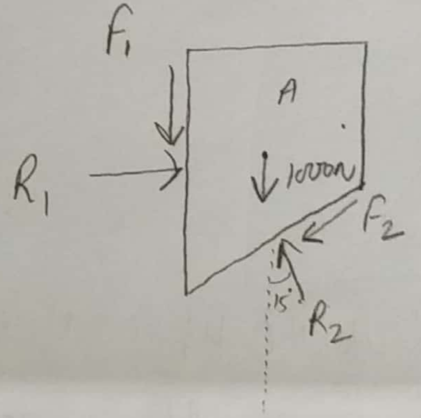


Q (wedge Problem) 6.9

a) Block A weighing 1000 N is to be raised by means ~~by~~ of a 15° wedge B weighing 500N. Assuming $\mu = 0.2$ for all contact. determine what minimum horizontal force P should be applied to raise the block.

b) Assuming that there is no friction between the block A and the vertical surface and wedge is of negligible weight, what is the minimum value of 'μ' required for the wedge to be self-locking?





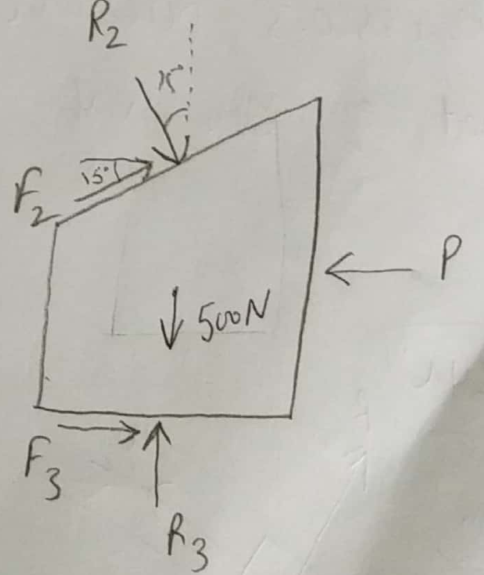
when upper block moving up, the lower block moving right to left.

$$\boxed{P = 871N}$$

$R_1 = 549N$

$R_2 = 1214N$

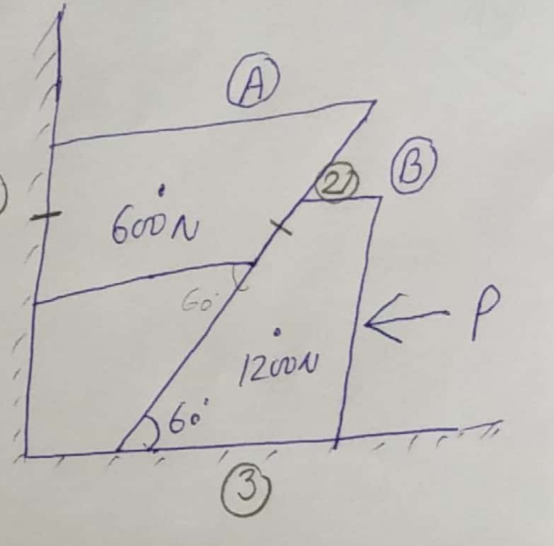
$R_3 = 1609.8N$



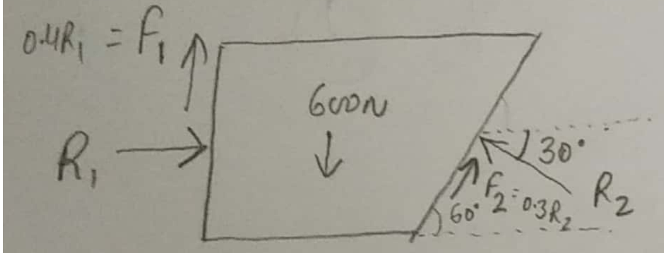
Q A system comprises two blocks A and B that against a wall and a floor as shown in fig. The weights of blocks are 600N and 1200N resp. Make calculations for the minimum horizontal force P that needs to be applied to keep the blocks in equilibrium. Assume the following values for μ : 0.2 for floor

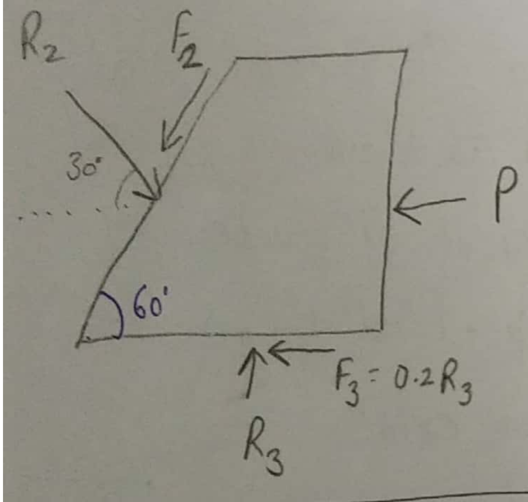
0.3 for blocks

0.4 for wall.



Sol.





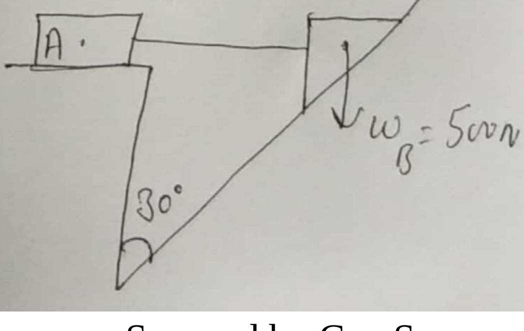
$R_2 = 573.4N$

$R_3 = 1635.67N$

$\boxed{P = 83.42N}$

Q Two blocks are connected by a horizontal link AB as shown in fig. What is the smallest weight W_A of block A for which equilibrium can exist. Assume $\mu = 0.4$ for block A and floor.

$\phi = 20°$ for block B and plane



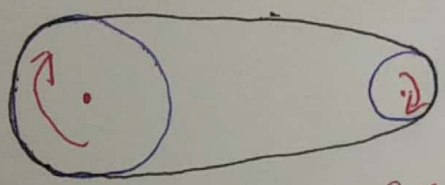
Ans (1050N)

Belt drive :

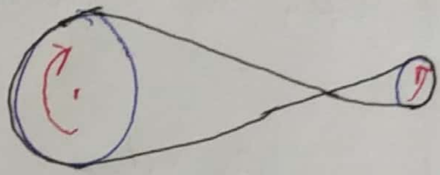
- Used for power transmission. (other method - ropes, chain, gear, clutch, shaft)
- It is a non positive drives (∵ of slip)

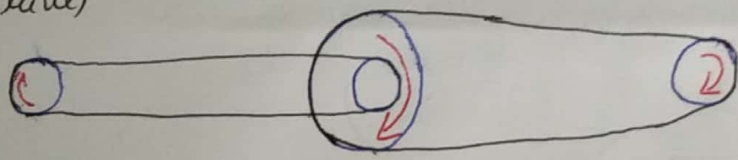
Types of belt drive

i) open belt drive - rotation of driver and follower is in same direction.
ii) ~~closed~~ Crossed belt drive - " " " in opposite direction
iii) Compound belt drive - more than two pulleys are used.

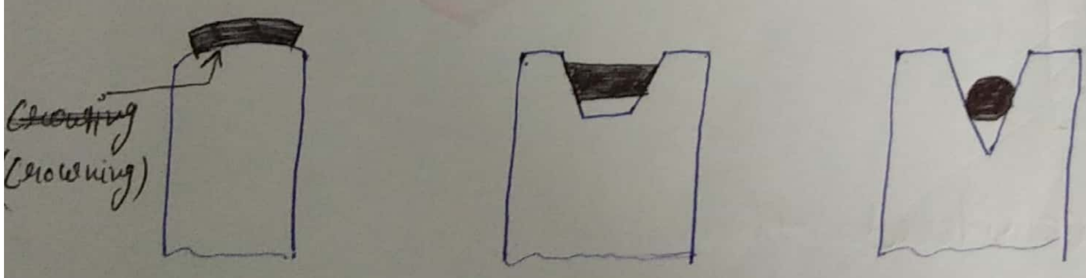


(open belt drive)


(Cross belt drive)


(Compound belt drive)

Belt material and its types



Flat belt V-belt Circular belt

Belt material - Rubber, leather, fabric, balata

Velocity ratio - It is the ratio of speed of driven pulley to that of driving pulley.

d_1, d_2 = dia of driver and driven pulleys

ω_1, ω_2 = angular velocities of driver and driven pulleys

N_1, N_2 = rotational speeds of driver and driven pulleys.

Assuming belt is inelastic and there is sufficient friction to prevent any slip, the pulleys will have the same linear speed.

$$\omega_1 \times \frac{d_1}{2} = \omega_2 \times \frac{d_2}{2}$$

$$\frac{\omega_2}{\omega_1} = \frac{d_1}{d_2} \quad \text{or} \quad \frac{2\pi N_2}{2\pi N_1} = \frac{d_1}{d_2}$$

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

If thickness of belt (t) is taken into account,

$$\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

If slip is also considered, then

$$\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1 - \frac{S}{100}\right) \qquad S = S_1 + S_2 + 0.01 S_1 S_2$$

S = Total effective slip

S_1 = % slip b/w driver & belt

S_2 = % slip b/w belt & follower

Length of belt

⇒ open drive → $l = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$

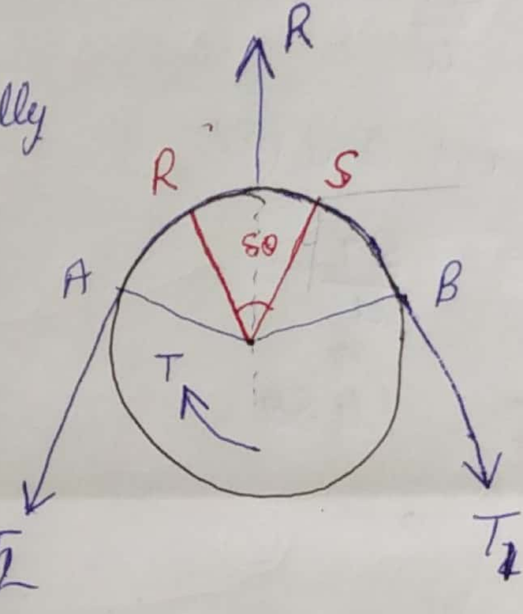
⇒ Crossed belt drive → $l = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$

where r_1, r_2 radius of pulleys

x = center distance centers of the two pulleys

Ratio of Tensions

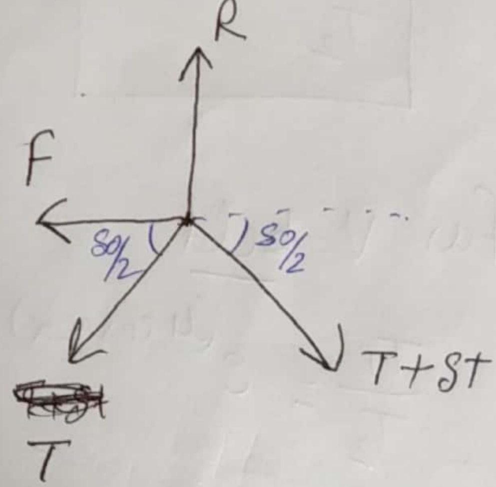
of driven pully



Consider the impending motion to be clockwise. $(T_1 > T_2)$

The angle subtended at the center of the pully by the position of belt in contact with it is called the angle of contact or the angle of lap (θ)

Consider equilibrium of forces in the radial direction. (vertical)



$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

For small value of $\delta\theta$, $\sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2}$

$$R = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$= T\,\delta\theta \quad -(i) \qquad \left[\text{neglect } \delta T \frac{\delta\theta}{2}\right]$$

Consider equilibrium of forces in tangential (horizontal) direction.

$$\mu R + T\cos\frac{\delta\theta}{2} = (T+\delta T)\cos\frac{\delta\theta}{2}$$

For small values of $\delta\theta$; $\cos\frac{\delta\theta}{2} \to 1$

$$\mu R = (T+\delta T) - T$$

$$\mu R = \delta T$$

$$R = \frac{\delta T}{\mu} \quad -(ii)$$

From (i) and (ii)

$$T\delta\theta = \frac{\delta T}{\mu}$$

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \mu\,\delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

For V-belt

$$\frac{T_1}{T_2} = e^{(\mu\,\mathrm{cosec}\,\alpha)\theta}$$

θ = angle of lap

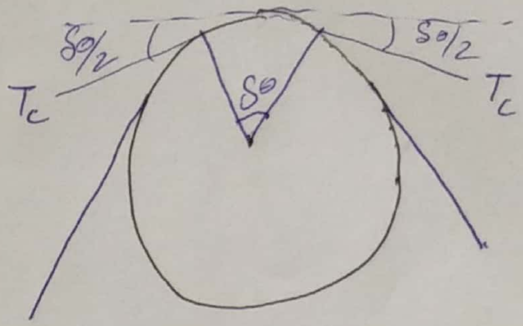
2α = angle of groove.

Centrifugal Tension : Due to the mass and speed, the belt is subjected to centrifugal force that acts radially outwards. This reduces [the normal rxn and hence frictional resistance.

Let r = radius of pulley

V = speed of belt

m = mass per meter length of belt



Then

length of elemental segment $= r\,\delta\theta$

mass of elemental segment $= m.r\,\delta\theta$

Centrifugal ~~Tension~~ Force $(F_c) = \dfrac{m v^2}{r} = \dfrac{m r \delta\theta V^2}{r} = m\,\delta\theta\, V^2$

This centrifugal force is counter balanced by tensions at the ends A and B of the elemental segment.

Considering equilibrium of forces in vertical direction.

$$2T_c \sin\frac{\delta\theta}{2} = m\,\delta\theta\, V^2$$

$$\sin\frac{\delta\theta}{2} \to \frac{\delta\theta}{2} \quad [\text{as } \frac{\delta\theta}{2} \text{ is very small})$$

$$2T_c \frac{\delta\theta}{2} = m\,\delta\theta\, V^2$$

$$\boxed{T_c = mV^2}$$

Tension on tight side ~~Side~~ $= T_1 + T_c$

" " slack " $= T_2 + T_c$

Power Transmission -

T_1 = Tension on tight side

T_2 = Tension on slack side

V = Velocity of the belt

Then, effective turning force $= T_1 - T_2$

~~Work done $(T_1 - T_2)V$ Nm/s~~

Power $= (T_1 - T_2)V$ watts

Second method

Turning moment acting on the pulley $= (T_1 - T_2)r$

work done per second $= (T_1 - T_2)r\omega$

$= (T_1 - T_2)V$

Condition for transmission of max. power

$$P = (T_1 - T_2)V$$

$$= T_1\left[1 - \frac{T_2}{T_1}\right]V = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)V$$

$$= (T - T_c)KV \qquad \left[\begin{array}{l} T = \text{max. permissible tension} = T_1 + T_c \\ \left(K = 1 - \frac{1}{e^{\mu\theta}}\right) \end{array}\right]$$

$$= (T - mv^2)KV$$

$$= (TV - mV^3)K$$

$$\frac{dP}{dV} = T - 3mV^2 = 0$$

$$T = 3mV^2 = 3T_c$$

$$\boxed{V = \sqrt{\frac{T}{3m}}}$$

$$\boxed{T_c = \frac{T}{3}}$$

Centrifugal Tension

$$\boxed{T_c = mV^2}$$

[m = mass per meter length of belt

V = speed of the belt.]

Total tension on tight side $= T_1 + T_c$

" " " slack " $= T_2 + T_c$

Initial tension (T_0)

During motion and power transmission,

The tight side of the belt stretches until the tension increases from T_0 to T_1.

The corresponding increase in length of belt on the tight side is $= \alpha (T_1 - T_0)$ —(i)

where α = coeffient of belt length per unit force.

The resulting decrease in length of belt on slack side $= \alpha (T_0 - T_2)$ —(ii)

∵ The belt is inelastic, then length of belt remains unchanged

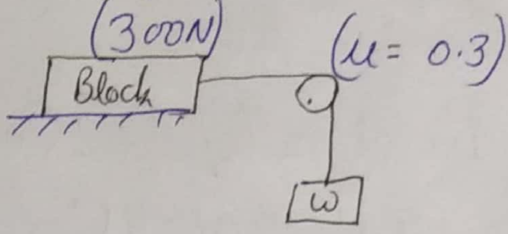
$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2)$$

$$\boxed{T_0 = \frac{T_1 + T_2}{2}}$$

If centrifugal tension is taken into account.

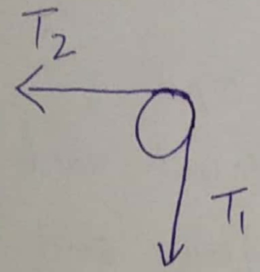
$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

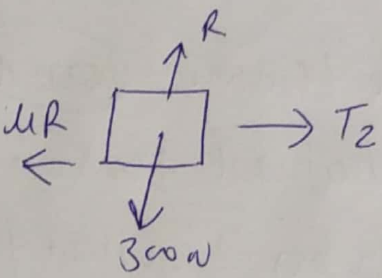
Q Determine the minimum value of weight W required to cause motion of the block on surface as shown in fig. Take ($\mu = 0.6$)



Angle of lap = 90°

Sol:





$T_1 = W$

$\frac{T_1}{T_2} = e^{\mu \theta}$, 1.60

$T_2 = \mu R$

$R = 300N$

$T_2 = 180N$

$T_1 = 288N$

Lifting machine: A machine may be defined as a device that receives energy in some available form and uses it for doing a particular useful work.

Some basic machines are

i) Lever ii) Pulley iii) Inclined plane

iv) Screw v) Wedge vi) wheel and axel

The machine which are used to lift heavy loads are called lifting machines.

Basic Definitions:

⇒ Mechanical Advantage (MA) $= \frac{W}{P} = \frac{\text{weight lifted}}{\text{Effort applied}}$

⇒ Velocity Ratio (VR) $= \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{y}{x}$

⇒ Input work - $P \cdot y$

⇒ Output work - $W \cdot x$

⇒ Efficiency of machine -

$$\eta = \frac{\text{Useful work done by the machine}}{\text{work expended on the machine}}$$

$$= \frac{\text{Output of the machine}}{\text{Input of the machine}}$$

$$= \frac{Wx}{Py} \Rightarrow \frac{W}{P} \times \frac{1}{\frac{y}{x}}$$

$$\boxed{\eta = \frac{MA}{VR}}$$

[In a simple machine, a small force when applied through a large distance overcomes a large force through a small distance.]

Reversible and Irreversible machine (Self locking)

Load fall
(Pulley)

Load does not fall.
(Screw Jack)

In an irreversible machine, some work done is lost due to friction

Friction work = Input - Output = $P.y - Wx$

On removal of effort, the load will not fall if the friction work is more than the output of machine.

$$(Py - Wx) > Wx$$

$$Py > 2Wx$$

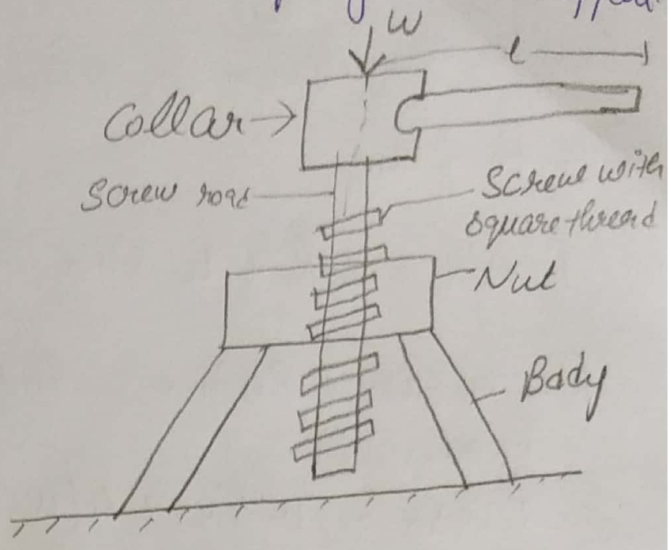
$$\frac{Wx}{Py} < \frac{1}{2}$$

$$\eta < \frac{1}{2}$$

$$\eta < 50\%$$

$\therefore$ For irreversibility or self-locking of a machine the $\eta < 50\%$.

Screw Jack - It is a simple machine used for lifting heavy loads, through short distances, with the help of small effort.



when one rotation is given to the handle,

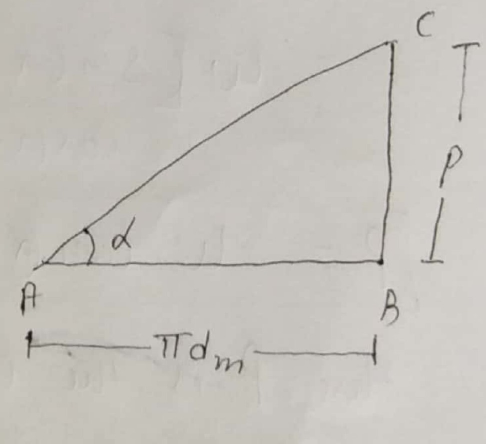
distance moved by effort = $2\pi l$

distance through which load is lifted is pitch of the thread (p)

$$V.R. = \frac{2\pi l}{p}$$

The working principle of a screw jack is the same as that of the inclined plane.

This fig shows development of one complete turn of a screw thread. Distance AB will be equal to the circumference and distance BC is equal to the pitch of the screw.



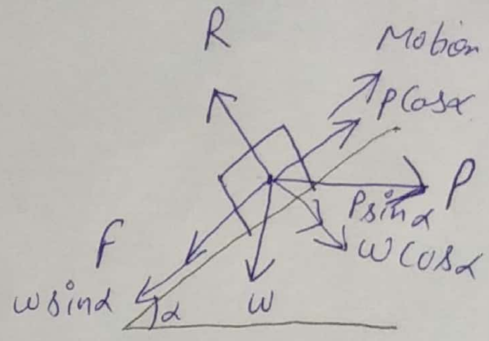
$$\tan\alpha = \frac{p}{\pi d_m}$$

[α : helix angle
d_m : mean dia. of thread]

Effort required to lift the load:

Consider equilibrium condition:

along the plane



$$P\cos\alpha = \mu R + W\sin\alpha \quad -(i)$$

$$R = W\cos\alpha + P\sin\alpha \quad -(ii)$$

From (i) & (ii)

$$\therefore P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha)$$

$$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$P = \frac{W(\sin\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\sin\alpha}$$

But $\because \mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

$$P = \frac{W[\sin\alpha\cos\phi + \mu\sin\phi\cos\alpha]}{\cos\alpha\cos\phi - \sin\phi\sin\alpha}$$

$$= \frac{W[\sin(\alpha+\phi)]}{\cos(\alpha+\phi)}$$

$$\boxed{P = W\tan(\alpha+\phi)}$$

Here P is the effort applied at the mean radius of screw jack. But in practice, the effort is applied at the end of the handle of the jack.

Let P_h = horizontal force applied at the end of handle.

l = length of handle.

$$P_h \times l = P \times \frac{d_m}{2}$$

$$= W \tan(\alpha+\phi) \frac{d_m}{2}$$

$$\boxed{P_h = \frac{W d_m}{2l} \tan(\alpha+\phi)}$$

$$P = W \tan(\alpha+\phi)$$

In the absence of friction $\phi = 0$ [Ideal condition]

$$P_0 = W \tan\alpha$$

$$\eta = \text{Efficiency} = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{W \tan\alpha}{W \tan \alpha+\phi} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}}$$

To find max. efficiency, differenciate η with respect to α

$$\frac{d\eta}{d\alpha} = \frac{d}{d\alpha}\left[\frac{\tan\alpha}{\tan(\alpha+\phi)}\right] = 0$$

$$\frac{\sec^2\alpha \tan(\alpha+\phi) - \sec^2(\alpha+\phi)\tan\alpha}{\tan^2(\alpha+\phi)} = 0$$

$$\sec^2\alpha \tan(\alpha+\phi) = \sec^2(\alpha+\phi)\tan\alpha$$

$$\frac{1}{\cos^2\alpha} \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = \frac{1}{\cos^2(\alpha+\phi)} \times \frac{\sin\alpha}{\cos\alpha}$$

$$2\cdot\sin(\alpha+\phi)\cos(\alpha+\phi) = 2\cdot\sin\alpha\cos\alpha$$

$$\sin 2(\alpha+\phi) = \sin 2\alpha = \sin(\pi - 2\alpha)$$

$$2(\alpha+\phi) = \pi - 2\alpha$$

$$\alpha = \frac{\pi}{4} - \frac{\phi}{2} = 45^\circ - \frac{\phi}{2}$$

$$\boxed{\alpha = 45^\circ - \frac{\phi}{2}}$$

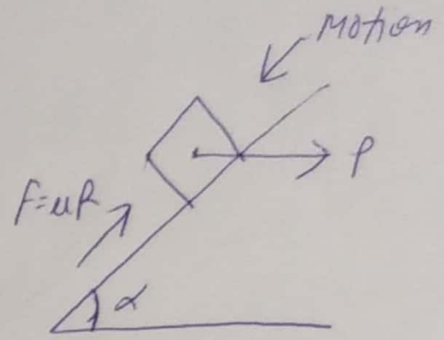
$$\eta_{max} = \frac{\tan\left[45 - \frac{\phi}{2}\right]}{\tan\left[45 - \frac{\phi}{2} + \phi\right]} = \frac{\dfrac{1-\tan\frac{\phi}{2}}{1+\tan\phi/2}}{\dfrac{1+\tan\phi/2}{1-\tan\phi/2}} = \left(\frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}}\right)^2$$

$$= \left[\frac{\cos\frac{\phi}{2} - \sin\phi/2}{\cos\frac{\phi}{2} + \sin\frac{\phi}{2}}\right]^2 = \frac{1 - 2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}{1 + 2\cos\frac{\phi}{2}\sin\frac{\phi}{2}} = \frac{1 - \sin\phi}{1+\sin\phi}$$

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$

Lowering of load:

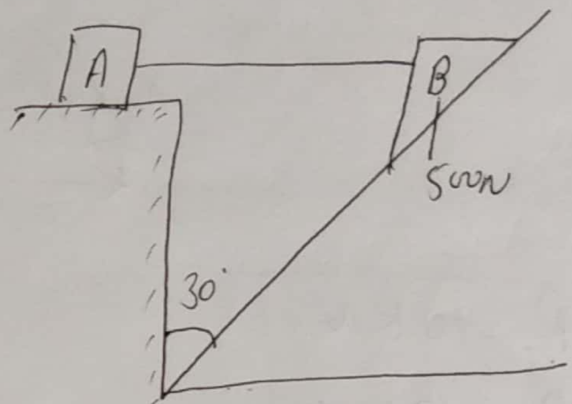
$$P = w \tan(\alpha - \phi)$$



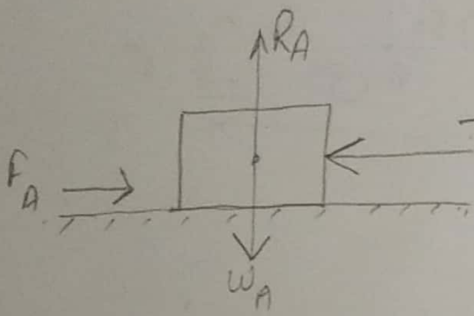
Scanned by CamScanner

6.6 (Friction)

Q Two blocks are connected by a horizontal link AB and rest on two planes as shown in fig. what the smallest weight W_A of the block A for which equilibrium can exist? Assume μ for the block A and horizontal surface to be 0.4 and the angle of friction for the block B on the inclined plane is $\phi = 20^\circ$. [1050N]



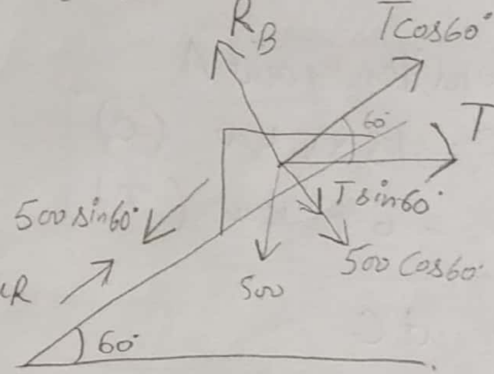
Ans



$T = F_A$ (i)

$R_A = W_A$ (ii)

$\mu = \tan 20^\circ$
$= 0.3639$



$R_B = T\sin 60 + 500\cos 60^\circ$ — (iii)

$T\cos 60^\circ + \mu R_B = 500\sin 60^\circ$ — (iv)

$T\cos 60^\circ + \mu[T\sin 60 + 500\cos 60^\circ] = 500\sin 60^\circ$

$T[\cos 60^\circ + \mu\sin 60^\circ] = 500\sin 60 - \mu\, 500\cos 60^\circ$

$T = 419.549N$

From (i)

$T = F_A = 419.549 = \mu R_A$

$R_A = 1048\,N$

$W_A = R_A = 1048\,N$