Microwave Engg. [Important Topics] from unit I + II

Unit I

1. Maxwell eq in diff | integral form.
2. Poynting Theorem
3. wave Eqn for all field Component in Rect | cylindrical waveguide.
4. TE | TM mode in Rect. waveguide] Numerical Also.
5. TE | TM mode in cylindrical waveguide]
6. Power Tr. and power losses in waveguide and excitation of waveguide.
7. Dominant and degenerative mode
8. wave impedance ZTE | ZTM.
9. Phase velocity | Group velocity | Guided wavelength (Numerical Also)
10. Comparision of different transmission with waveguide.
11. ~~Domi~~ Introduction to - Planar Tr. line
   - strip line
   - microstrip line
12. why waveguide work as HPF.
13. Difference b/w TE | TM | TEM with ex.

UNIT-II

1. why Z, Y, h, ABCD parameter ~~not~~ ~~are~~ are not used in µwave.
2. S Matrix and their properties
3. Resonant freq. for Rectangular & circular cavity Resonator
4. Short note on - Reentrant cavity Resonator | toroidal resonator
5. Components -
   - E Plane Tee
   - H Plane Tee
   - Magic Tee
   - Rat race ckt
   - Directional coupler
   - Attenuator
   - Phase shifter
   - Iris
   - Corner
   - Bend | Twist
6. Ferrite devices and its application in isolator, circulator, gyrator

UNIT- I Microwave Engg. [300 MHz – 300 GHz]

RF & Mw Engg. [3 KHz – 300 GHz]

1. Mw freq range is from 300 MHz – 300 GHz so that wave length (λ) may consider in microns ranges

$\lambda = 1m$ — $1mm$

$$\boxed{1\,\mu m = 10^{-6} m}$$ (1 Micron)

$1\,\text{Å}$ (angstrom) $= 10^{-10} m$

2. Means that μwave also cover infrared and visible light regions.

10^{18}	Exa	E
10^{15}	Peta	P
10^{12}	tera	T
10^{9}	GHz	G
10^{6}	Mega	M
10^{3}	Kilo	k
10^{2}	Hecto	h
10	deka	da

3. when freq = 300 MHz.

then $\lambda = \frac{3\times10^8}{300\times10^6}$

$$\boxed{\lambda = \frac{c}{f}}$$

$$\boxed{\lambda = 1m}$$

when 300 GHz

then $\lambda = \frac{3\times10^8}{300\times10^9}$

$\lambda = \frac{3}{300\times10} = \boxed{\frac{1}{1000} m}$ or $\frac{1\times100}{1000} = \boxed{.1\,cm}$

or $.1\times10 = \boxed{1mm}$

$\frac{1}{10}\times10$

deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ

4. IEEE freq. band

HF – 3 – 30 MHz] in AM/FM Broadcasting
VHF – 30 – 300 MHz]
UHF – 300M – 1 GHz] — GSM — 900 MHz: Tx. 890 – 915, Rx. 935 – 960 | 1800 MHz GSM: Tx 1710–1785, Rx–1805–1880
L – 1 – 2 "]
S – 2 – 4 " — Radar, Bluetooth, wifi, Mw oven.
C – 4 – 8 " — satellite
X – 8 – 12 " — Educational Purpose
KU – 12 – 18 " — DTH, VSAT, GPS.
K – 18 – 27 "] — Radar,
Ka – 27 – 40 "]
mm – 40 – 300 "] ~~Terest~~ Terrestrial Purpose.
Sub mm – > 300 — Infrared, visible light, UV, X ray, γ, Cosmic

Maxwell's Eqⁿ in time domain — UNIT-I

$\nabla \times H = j\omega\epsilon E$

$\nabla \times E = -j\omega\mu H$

Electric field int	$\nabla \times E = -\frac{\partial B}{\partial t}$	– Faraday's electromotive force law
Mag. field Int	$\nabla \times H = J + \frac{\partial D}{\partial t}$	– Ampere's circuital law
E-flux density	$\nabla \cdot D = \rho_v$	Gauss law.
M. flux density	$\nabla \cdot B = 0$	Gauss law.

$\rho = 0$, $\sigma = 0$ — No conducting material in free space

Unit

$E \rightarrow$ Electric field intensity v/m

H – Mag " " Amp/m

D – Electric flux density C/m^2

B – Mag. flux density w/m^2

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$
$$= \sigma E + j\omega\epsilon E$$
$$= E(\cancel{\sigma_0} + j\omega\epsilon)$$
$$\boxed{\nabla \times H = j\omega\epsilon E}$$

$J = J_c + J_0$ — impressed current —

$J = \sigma E \leftarrow$ Conduction current density caused by the field

(Conduction current)

$B = \mu H$

$D = \epsilon E$

$J = \sigma E$

$\epsilon = \epsilon_0\epsilon_r$ — Electric Permittivity

$\mu = \mu_0\mu_r$ — Mag. Permeability

Maxwell Eqⁿ in freq domain

Assume that Time varying fx in the form of $e^{j\omega t}$

$$\nabla \times E = -\frac{\partial}{\partial t}\mu H$$

$$\nabla \times E = -j\omega\mu H$$

Now curl of $\nabla \times E$

$$\nabla \times \nabla \times E = -j\omega\mu(\nabla \times H)$$

$$-\nabla^2 E + \nabla(\nabla \cdot E) = -j\omega\mu(j\omega\epsilon E)$$

$$\cancel{-}\nabla^2 E = \cancel{-}(j\omega)^2\mu\epsilon E$$

$$\boxed{\nabla^2 E = -\omega^2\mu\epsilon E}$$

$E = E_0 e^{j\omega t}$ ← max electric field intensity

$$\frac{\partial E}{\partial t} = E_0 e^{j\omega t} \cdot j\omega$$

then $\boxed{\frac{\partial}{\partial t} = j\omega}$

$\omega = 2\pi f$, f – freq of sinusoidal variation

$$\partial^2/\partial t^2 = (j\omega)^2$$

$$\boxed{\frac{\partial^2}{\partial t^2} = -\omega^2}$$

Similarly $\boxed{\nabla^2 H = -\omega^2\mu\epsilon H}$ → or $\nabla^2 E = \underbrace{j\omega\mu(\sigma + j\omega\epsilon)}_{\gamma^2}E$

$\mu\epsilon = \frac{1}{c^2}$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

Intrinsic Propagation

$\gamma = \alpha + j\beta$ — Phase constt

γ – Propagation constt.

α – Attenuation constt

$\boxed{\nabla^2 E = \gamma^2 E}$ or $\nabla^2\psi = \gamma^2\psi$

Similarly $\boxed{\nabla^2 H = \gamma^2 H}$

we know that $\mu\epsilon = \mu_0 \mu_r \cdot \epsilon_0 \epsilon_r$ $\qquad \begin{rcases} \mu_r = \\ \epsilon_r = \end{rcases} 1$

$\mu_0 = 4\pi\times10^{-7}\ H/m \qquad \epsilon_0 = \frac{1}{36\pi}\cdot 10^{-9}\ f/m$ faradsmeter or $8.854\times10^{-12}\ F/m$

$$= \frac{\cancel{4\pi}\times10^{-7}\times10^{-9}}{36\pi\ 9\cancel{\pi}}$$

$$= \frac{1}{9\cancel{\pi}\times10^{16}}$$

$$\mu\epsilon = \frac{1}{(3\cancel{\pi}\times10^{8})^2}$$

$$\mu\epsilon = \frac{1}{c^2} \qquad c^2 = \frac{1}{\mu\epsilon}$$

$$\text{or} \quad c = \frac{1}{\sqrt{\mu\epsilon}} \Rightarrow \boxed{3\times10^{8}\ m/sec}$$

If wave propagate through other medium then velocity decreases then air

$$d = \frac{c}{f(\sqrt{\epsilon_r})}$$

ϵ_r — dielectric constt of the medium

A changing electric field produced mag. field and changing magnetic field induced a changing electric field in the surrounding region

Mode

TEM – Both Electric and mag. fields are purely Transverse to the direction of propagation and consequently have no Z directed component:

$E_z = 0 \qquad H_z = 0$

(2) TE

$E_z = 0 \qquad H_z \neq 0$

(3) TM

$E_z \neq 0 \qquad H_z = 0$

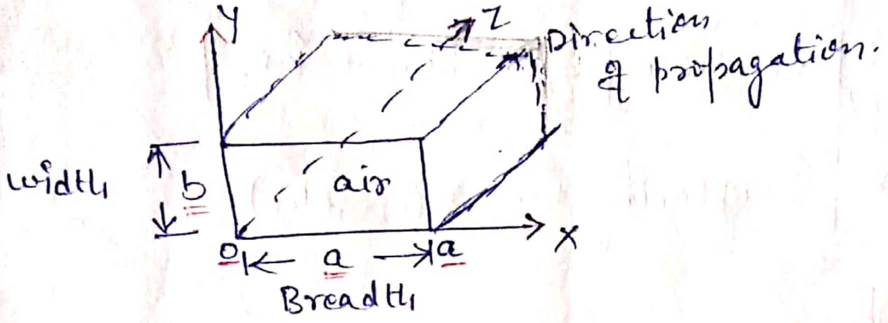
(4) HE (Hybrid) wave

$E_z \neq 0 \qquad H_z \neq 0$

(General solu) of Rectangular w/G

Propagation of waves in Rectangular waveguides | TM Modes in R/w guide.

1. Consider a Rectangular w/G in rectangular coordinate system with its breadth along x axis, width along y axis and wave propagate along z direction.
2. wave Guide is filled with air as dielectric.



⇒ wave Eq^n for TE and TM wave are given as

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \quad \text{for TE wave} \quad (E_z = 0)$$

$$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \quad \text{for TM wave} \quad (H_z = 0)$$

Expanding $\nabla^2 E_z$ in Rectangular Coordinate system

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \epsilon E_z$$

Taking direction inside of w/G. so distance ↓es.

$$\therefore \frac{\partial}{\partial z} = -\gamma \qquad \frac{\partial^2}{\partial z^2} = \gamma^2$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \epsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \underbrace{(\gamma^2 + \omega^2 \mu \epsilon)}_{h^2} E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{for TM wave}$$

Similarly $\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$ for TE wave

By solving these differential Eq^{n}s we get solu. for E_z, H_z.

using maxwell's eqn. it is possible to find the various comp. Now along x and y directions $[E_x, H_x, E_y, H_y]$.

from maxwell's Eqn

$$\sigma E + jw\epsilon E$$
$$\sigma \to 0$$

$$\nabla \times H = jw\epsilon E$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = jw\epsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

Replacing by $-\gamma$ (operator)

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = jw\epsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

Equating coefficient of i, j, k

$$\frac{\partial H_z}{\partial y} + \gamma H_y = jw\epsilon E_x \quad — (1)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -jw\epsilon E_y \quad — (2)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = jw\epsilon E_z \quad — (3)$$

Similarly

$$\nabla \times E = -jw\mu H$$

Replace $\partial/\partial z = -\gamma$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -jw\mu\left[iH_x + jH_y + kH_z\right]$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -jw\mu H_x \quad — (4)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -jw\mu H_z \quad — (6)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = +jw\mu H_y \quad — (5)$$

Now

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \quad (\text{from } \nabla \times E \text{ II}^{nd} \text{ eq}^n) \text{ from Eq. 5}$$

find out H_y.

$$H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu} E_x$$

Put this value (in $\nabla \times H$ 1st eq^n) in Eq. 1

$$\frac{\partial H_z}{\partial y} + \frac{\gamma \cdot \partial E_z}{j\omega\mu\, \partial x} + \frac{\gamma^2 E_x}{j\omega\mu} = j\omega\epsilon E_x$$

Multiply by $j\omega\mu$ we get

$$\frac{j\omega\mu\, \partial H_z}{\partial y} + \frac{\gamma \cdot \cancel{j\omega\mu}}{\cancel{j\omega\mu}}\frac{\partial E_z}{\partial x} + \frac{\gamma^2 \cdot \cancel{j\omega\mu}\, E_x}{\cancel{j\omega\mu}} = j\omega\epsilon E_x \cdot j\omega\mu$$

$$j\omega\mu \frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x} + \gamma^2 E_x = -\omega^2\mu\epsilon E_x$$

$$'' \quad '' \quad = \cancel{E_x} - \omega^2\mu\epsilon E_x - \gamma^2 E_x$$

$$'' \quad '' \quad = E_x\big(-\underbrace{(\gamma^2 + \omega^2\mu\epsilon)}_{-h^2}\big)$$

$$E_x\left(-(\gamma^2 + \omega^2\mu\epsilon)\right) = j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x} \cancel{+ \gamma^2 E_x}$$

$$E_x(-h^2) = j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x}$$

$$\boxed{E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}}$$

① From Eq 5 find out H_y and put in eq^n ① for E_x.
② From Eq 4 find out H_x and put in Eq ② for E_y.
③ From Eq 2 find out E_y and put in Eq 4 for H_x.
④ From Eq 1 find E_x and put in Eq 5 for H_y.

or $\cancel{E_x} = \cancel{-\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}$

or

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

General relationship for field component within W/G.

from Eq 4th H_x and put in eq^n ②

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

from eq. 2, E_y Put in eq ④

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$$

from eq. 1 E_x Put in eq ⑤

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Propagation of TEM waves.

For TEM wave

$E_z = 0 \quad H_z = 0$

Put these value in Eqn. E_x, E_y, H_x, H_y vanish and hence a TEM wave cannot exist inside a waveguide.

Propagation of TM waves in Rectangular W/G.

For TM ~~mag.~~ wave

$H_z = 0 \quad E_z \neq 0$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2}{\partial y^2} E_z + \frac{\partial^2}{\partial z^2} E_z = -\omega^2 \mu \epsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2}{\partial y^2} E_z + \underbrace{(\gamma^2 + \omega^2 \mu \epsilon)}_{h^2} E_z = 0$$

$$h^2 E_z = 0$$

Let $E_z = XY$ — Y → a fx. of y, X → a fx of x

Since X and Y are independent variable. so

$$Y \cdot \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

sum of these is a constt.

Divide by XY we get

$$\underbrace{\frac{1}{X} \frac{d^2 X}{dx^2}}_{-B^2} + \underbrace{\frac{1}{Y} \frac{d^2 Y}{dy^2}}_{-A^2} + h^2 = 0 \quad (A)$$

Let →

we use separation of variable method to solve differential Eqn.

$h^2 = A^2 + B^2$

$\frac{1}{X}\frac{d^2X}{dx^2}$, $\frac{1}{Y}\frac{d^2Y}{dy^2}$ — Both are ordinary 2nd order diff Eqn.

$X = C_1 \cos Bx + C_2 \sin Bx$

$Y = C_3 \cos Ay + C_4 \sin Ay$

Solu. of these Eqn.

$$\boxed{E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]}$$

C_1, C_2, C_3, C_4 are constt and can evaluated by Boundary condn.

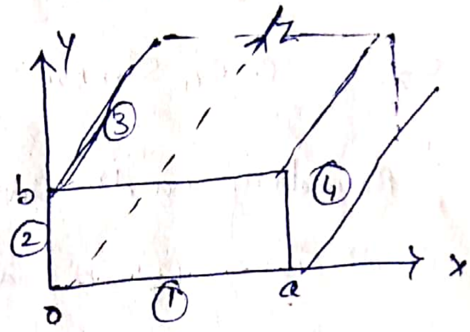
Boundary Condⁿ for [TM] wave

Rect.

① Since entire surface of the w/G acts as a short ckt or Ground for electric field, $E_z = 0$ all along the boundary walls of the w/G.

② Since there are 4 walls there fore four boundary condⁿ

③ Always tangential component should be zero, for $E_z = 0$



Ist B. Condⁿ (Bottom wall)

$E_z = 0$ all along the bottom wall

$E_z = 0$ at $y = 0$ $x \to 0$ to a

2nd B. Condⁿ (Left side wall)

$E_z = 0$ at $x = 0$ $y \to 0$ to b

3rd B. Condⁿ (Top wall)

$E_z = 0$ at $y = b$ $x \to 0$ to a

4th B. Condⁿ (Right side wall)

$E_z = 0$ at $x = a$ $y \to 0$ to b.

Now substitute Ist B. Condⁿ. in E_z Eqⁿ.

$$E_z = [C_1 \cos B_x + C_2 \sin B_x][C_3 \cos Ay + C_4 \sin Ay] \quad \text{— ①}$$

$$0 = [C_1 \cos B_x + C_2 \sin B_x][C_3 \underbrace{\cos(0)}_{1} + C_4 \sin(0)]$$

$$C_1 \cos B_x + C_2 \sin B_x \neq 0$$

$$C_3 = 0$$

Now Eq. ① becomes

$$E_z = [C_1 \cos B_x + C_2 \sin B_x][C_4 \sin Ay] \quad \text{— ②}$$

Now 2nd Boun. condⁿ

$E_z = 0$ at $x = 0$ $y = 0$ to b.

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_4 \sin Ay] \quad \text{— Eq. (2)}$$

$$0 = \{C_1 \cos(0) + C_2 \sin(0)\}[C_4 \sin Ay]$$

$$0 = C_1 C_4 \sin Ay$$

$\sin Ay \neq 0$ so that $C_4 \neq 0$ also only $\boxed{C_1 = 0}$

Now eqn. (2) becomes

$$E_z = [C_2 \sin Bx][C_4 \sin Ay]$$

or

$$E_z = C_2 C_4 \sin Bx \sin Ay \quad \text{— (3)}$$

By 3rd Boundary condn substitute in Eq. (3)

Condn. is $E_z = 0$ at $y = b$, $x \to 0$ to a

$$0 = C_2 C_4 \sin Bx \sin Ab$$

$\sin Bx \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$ } otherwise there would be no solution.

$\therefore \sin n\pi = 0$

$\sin Ab = 0 \leftarrow \sin n\pi$

$Ab =$ a multiple of π

$Ab = n\pi$ ↳ a constt. $[n = 0, 1, 2 \ldots]$

then $Ab = n\pi$

$$\boxed{A = \frac{n\pi}{b}}$$

By 4th B. condn. in eqn. (3)

$\Rightarrow E_z = 0$ at $x = a$, $y \to 0$ to b

$$E_z = 0 = C_2 C_4 \sin Bx \sin Ay$$

$$0 = C_2 C_4 \sin Ba \sin Ay$$

$\sin Ay \neq 0$ $\quad C_2 \neq 0$

$\sin Ba = 0$ $\quad C_4 \neq 0$

$$Ba = m\pi$$

$$\boxed{B = \frac{m\pi}{a}}$$

Now Eq^n.

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot \underbrace{e^{-\gamma z}}_{\text{Propagation along z dir}^n} \cdot \underbrace{e^{+j\omega t}}_{\text{Sinusoidal variation w.r.t t.}}$$

Let $C = C_2 C_4$

$$E_z = C \sin\left(m\frac{\pi}{a}\right)x \sin\left(n\frac{\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

Since E_z is known E_x, E_y, H_x, H_y are given by the Eq^n.

Now from General relationship Eq^{n}s.

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\underbrace{\frac{\partial H_z}{\partial y}}_{=0}$$ For TM wave $H_z = 0$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left[C \sin\left(m\frac{\pi}{a}\right)x \sin\left(n\frac{\pi}{b}\right)y\, e^{j\omega t - \gamma z}\right].$$

$$\boxed{E_x = \frac{-\gamma}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}}$$

Now

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\underbrace{\frac{\partial H_z}{\partial x}}_{=0}$$

$E_x = -$
$E_y = +$
$H_x = +$
$H_y = -$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}\left[C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}\right]$$

$$\boxed{E_y = \frac{-\gamma}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}}$$

Now

$$H_x = \frac{-\gamma}{h^2}\underbrace{\frac{\partial H_z}{\partial x}}_{0} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_x = \frac{j\omega\epsilon}{h^2}\frac{\partial}{\partial y}\left[C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$\boxed{H_x = \frac{j\omega\epsilon}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}.}$$

Now

$$H_y = \boxed{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}} \xrightarrow{} 0 \;\; * \;\; \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{j\omega\epsilon}{h^2}\frac{\partial}{\partial x}\left[C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t-\gamma z}\right]$$

$$\boxed{H_y = -\frac{j\omega\epsilon}{h^2}\, C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \,\sin\left(\frac{n\pi}{b}\right)y\; e^{j\omega t-\gamma z}}$$

TM Modes in Rectangular waveguides.

(1) Depending upon the values of m and n we have various modes in TM waves as TM_{mn}.

(2) m indicates the no. of half wave variation of field across wider dimension a.

(3) n indicates no. of half wave variations of field across narrow dimension b. TM_{mn}

(i) TM_{00} mode : $m=0$ and $n=0$

If $m=n=0$ then E_x, E_y, H_x, H_y all vanish and hence TM_{00} mode does not exist.

(ii) ~~TM~~ TM_{01} : $m=0$ and $n=\phi$

All field components vanish and hence mode does not exist

(iii) TM_{10} mode : $m=1$ and $n=0$

All field components vanish hence mode does not exist.

(iv) TM_{11} mode : $m=1$ and $n=1$

Now E_x, E_y, H_x, H_y components exist and TM_{11} mode exist and for all higher values of m and n ~~the~~ ie all higher modes do exist.

Cut off frequency of wave guide (W/G as high pass filter)

we know that

$$h^2 = \gamma^2 + \omega^2\mu\epsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

or $$\gamma^2 + \omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$\gamma = \alpha + j\beta$

At lower frequencies:

Ist Cond^n $\omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

Now → γ becomes real and (+ve) and equal to the attenuation constant α.

→ wave is completely attenuated and there is no phase change.

→ Hence wave cannot propagate.

2nd Cond^n At higher frequencies: $\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

(1) γ becomes imaginary there will be phase change β and hence wave propagates.

[illegible] cond^n (2) At ~~this~~ the transition γ becomes 0 and propagation just starts.

(3) Frequency at which γ just becomes 0 is defined as cutoff frequency or threshold frequency) (f_c).

At $\boxed{f = f_c}$ then $\gamma = 0$ or $\omega = 2\pi f = 2\pi f_c = \omega_c$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

Cond^n (3)

$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\epsilon$

then $\gamma = 0$

So that propagation just starts.

$$\omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2 = \frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$= \frac{c}{2\pi}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \cdot (\pi)^2}$$

$u_p = \frac{1}{\sqrt{\mu\epsilon}}$ is phase velocity in an unbounded dielectric

$$= \frac{c\cdot\cancel{\pi}}{2\cancel{\pi}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$\mu = \mu_0 \mu_r \leftarrow 1$ for air

$\mu = \mu_0$

or $\epsilon = \epsilon_0 \epsilon_r \leftarrow 1$ for air

$\epsilon = \epsilon_0$

$$\boxed{f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Now cutoff wavelength λ_c

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = \frac{2}{\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}} \qquad \sqrt{\frac{m^2b^2 + n^2a^2}{a^2b^2}}$$

or

$$\boxed{\lambda_c = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}}$$

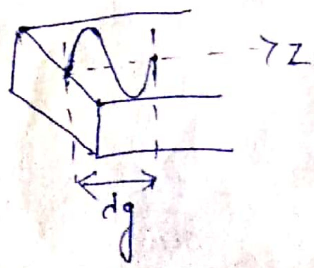
All wavelength greater than λ_c are attenuated and those less then λ_c are allowed to propagate inside the waveguide. //

Guide wavelength, Group velocity, Phase velocity.

Guide wavelength (λ_g): Distance travelled by the wave in order to undergo a phase shift of 2π radians.

λ_g relates with phase constant by relation

$$\boxed{\lambda_g = \frac{2\pi}{\beta}}$$



* wavelength in w/G is different from wavelength in free space.

λ_g is related to free space wave length λ_0 and cutoff wavelength λ_c as

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

$$\boxed{\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}}$$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

If $\boxed{\lambda_0 << \lambda_c}$ denominator terms ≈ 1 and $\lambda_g = \lambda_0$.

As $\boxed{\lambda_0 >> \lambda_c}$ then λ_g is imaginary which is nothing but no propagation in the w/G.

$i = \sqrt{-1}$

$$\frac{\lambda_0 \cdot \lambda_c}{\sqrt{-\lambda_0^2}} = \frac{\lambda_0 \cdot \lambda_c}{i\lambda_0^2}$$

Phase velocity (v_p): It is defined as the rate at which the wave changes its phase in terms of guide wave length.

$$v_p = \frac{\lambda_g}{\text{unit time}} = \lambda_g \times f = \cancel{2\pi f \cdot \lambda_g} \quad \cancel{2\pi f}$$

$$\left[\lambda_g = \frac{2\pi}{\beta}\right]$$

$$\frac{2\pi \times f}{\beta} = \frac{\omega}{\beta}$$

$$\boxed{v_p = \frac{\omega}{\beta}}$$

Group velocity (v_g): Rate at which wave propagate through the w/G.

$$v_g = \frac{d\omega}{d\beta} \qquad \beta = \sqrt{\mu\epsilon(\omega^2 - \omega_c^2)}$$

Now differenting β w.r.t ω we get

$$\frac{d\beta}{d\omega} = \frac{1}{2\sqrt{(\mu\epsilon)(\omega-\omega_c)^2}} \cdot 2\omega\mu\epsilon$$

$$\frac{\cancel{2\pi} f_c}{\cancel{2\pi} f}$$

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\epsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\mu\epsilon}}{\sqrt{1-(f_c/f)^2}}$$

Divide by ω

$$\frac{1}{\sqrt{\mu\epsilon}} = c$$

or

$$\frac{d\omega}{d\beta} = \frac{\sqrt{1-(f_c/f)^2}}{\sqrt{\mu\epsilon}} = c \cdot \sqrt{1-(f_c/f)^2}$$

or $$\boxed{v_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Product of v_p and v_g.

$$v_p \cdot v_g = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \cdot c\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}$$

$$\boxed{v_p v_g = c^2}$$

Propagation of TE waves in Rectangular w/Guide

For TE Mode $\quad E_z = 0 \quad H_z \neq 0$

Z Component of magnetic field H must exist in order to have energy transmission in the guide.

wave eqn for TE Mode

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \epsilon H_z$$

$$\left[\frac{\partial^2}{\partial z^2} = \gamma^2\right]$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + \omega^2 \mu \epsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \underbrace{(\gamma^2 + \omega^2 \mu \epsilon)}_{h^2} H_z = 0 \quad \text{--- (1)}$$

This is a partial differential equation whose solution can be assumed.

Assume a solu $H_z = XY$ → Y: A pure fx of y only; X is a pure fx of x only

Put $H_z = XY$ in eqn (1)

$$Y \cdot \frac{d^2 X}{dx^2} + X \cdot \frac{d^2 Y}{dy^2} + h^2 XY = 0$$

Dividing throughout by XY we get

$$\frac{\cancel{Y} \cdot}{XY}\frac{d^2 X}{dx^2} + \frac{\cancel{X}}{XY}\frac{d^2 Y}{dy^2} + h^2 \frac{\cancel{XY}}{\cancel{XY}} = 0$$

$$\underbrace{\frac{1}{X} \cdot \frac{d^2 X}{dx^2}}_{-B^2} + \underbrace{\frac{1}{Y} \frac{d^2 Y}{dy^2}}_{-A^2} + h^2 = 0$$

Equating these terms to a constant

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -B^2, \quad \frac{1}{Y} \frac{d^2 Y}{dy^2} = -A^2$$

$$-B^2 - A^2 + h^2 = 0$$

$$\boxed{h^2 = A^2 + B^2}$$

Solving for x and y by separation of variable method

$$X = C_1 \cos Bx + C_2 \sin Bx$$

$$Y = C_3 \cos Ay + C_4 \sin Ay$$

Therefore

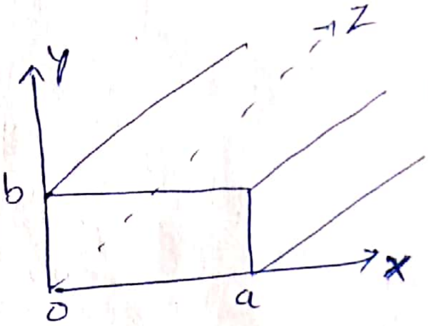
$$\boxed{H_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]} \quad \text{--- (2)}$$

for constants C_1, C_2, C_3, C_4 applying Boundary condn.

we have four Boundary Condn.

Here Tangent component of electric field E_z is zero on the conducting surface.

$E_z = 0$ but we have component along x and y direction.



$E_x = 0$ all along bottom and top wall of the w/G.

$E_y = 0$ all along left and right walls of the w/G.

Ist B. Condn. (Bottom wall)

$E_x = 0$ at $y = 0$ $x \rightarrow 0$ to a

2nd B. Condn

$E_x = 0$ at $y = b$ $x \rightarrow 0$ to a (top wall)

3rd B. Condn

$E_y = 0$ at $x = 0$ $y \rightarrow 0$ to b (Left side wall)

4th B. Condn

$E_y = 0$ at $x = a$ $y \rightarrow 0$ to b (right side wall)

Before Substituting Ist B.C in Eq. (2) find and write down Eqn of E_x & E_y.

We know that

$$E_x = \cancel{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$ 0

In TE $\begin{bmatrix} E_z = 0 \\ H_z \neq 0 \end{bmatrix}$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$ Put the value of H_z and der. w.r.t y.

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}[C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$E_x = \frac{-j\omega\mu}{h^2}[C_1 \cos Bx + C_2 \sin Bx][-AC_3 \sin Ay + AC_4 \cos Ay]$$ E.

Now Apply 1st Bound. Condn.

i.e $[E_x = 0$ at $y = 0$ $x \to 0$ to $a]$

$$0 = \frac{-j\omega\mu}{h^2}[C_1 \cos Bx + C_2 \sin Bx][-AC_3 \cancel{\sin}(0) + AC_4 \cos(0)]$$

$$= \frac{-j\omega\mu}{h^2}[C_1 \cos Bx + C_2 \sin Bx][AC_4]$$

$\boxed{C_4 = 0}$

Put $\boxed{C_4 = 0}$ in Eqn. (2)

Revised H_z

ie

$$H_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$H_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay] \quad — (3)$$ 0

We know that

$$E_y = \cancel{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y}} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$ 0

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{-h^2}\frac{\partial}{\partial x}[C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay]$$

$$E_y = \frac{j\omega\mu}{h^2}[-BC_1 \sin Bx + BC_2 \cos Bx][C_3 \cos Ay]$$

Now applying 3^{rd} B. Cond^n

ie $E_y = 0$ at $x = 0$ $y = 0$ to b

$$E_y = \frac{j\omega\mu}{h^2}\left[-BC_1 \sin Bx + BC_2 \cos Bx\right]\left[C_3 \cos Ay\right]$$

$$0 = \frac{j\omega\mu}{h^2}\left[-BC_1 \sin(0) + BC_2 \cos(0)\right]\left[C_3 \cos Ay\right]$$

$$0 = \frac{j\omega\mu}{h^2}\left[0 + BC_2\right]\left[C_3 \cos Ay\right]$$

$$\boxed{C_2 = 0}$$

Substitute $\boxed{C_2 = 0}$ in eqn. (3)

$$H_2 = \left[C_1 \cos Bx + C_2 \sin Bx\right]\left(C_3 \cos Ay\right)$$

$$H_z = \left[C_1 \cos Bx\right]\left[C_3 \cos Ay\right]$$

$$H_z = C_1 C_3 \cos Bx \cos Ay \quad \text{--- (4)}$$

we know that

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[C_1 C_3 \cos Bx \cos Ay\right]$$

$$E_x = -\frac{j\omega\mu}{h^2}\left[-C_1 C_3 A \cos Bx \sin Ay\right]$$

$$E_x = \frac{j\omega\mu}{h^2}\left[C_1 C_3 A \cos Bx \sin Ay\right]$$

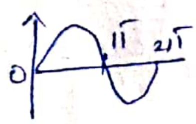
Now applying 2^{nd} B. Cond^n

ie $E_x = 0$ at $y = b$ $x \to 0$ to a

$$0 = \frac{j\omega\mu}{h^2}\left[C_1 C_3 A \cos Bx \sin Ab\right]$$

$\sin Ab = 0$ $\quad Ab = n\pi$

$$\boxed{A = \frac{n\pi}{b}}$$



$Ab =$ a multiple of π

$n = 1, 2, 3 \ldots$

we know that

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

0

$$E_y = \frac{j\omega\mu}{-h^2}\frac{\partial}{\partial x}\left[C_1C_3 \cos Bx \cos Ay\right]$$

$$E_y = \frac{j\omega\mu}{-h^2}\left[-C_1C_3 B \sin Bx \cos Ay\right]$$

$$E_y = -\frac{j\omega\mu}{-h^2}\left(C_1C_3 B \sin Bx \cos Ay\right)$$

Now applying 4th B. Condn

i.e $E_y = 0$ at $x = a$ $y \to 0$ to b

$$0 = \frac{-j\omega\mu}{-h^2} C_1C_3 B (\sin Ba) \cos Ay$$

$$\sin Ba = 0$$

$$Ba = m\pi$$

$$\boxed{B = \frac{m\pi}{a}}$$

Now ~~final~~ H_z becomes from Eqn (4)

$$H_z = C_1 C_3 \cos Bx \cos Ay$$

$$H_z = (C_1C_3) \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

or

$$\boxed{H_z = C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}}$$

Note: In TM wave E_z has Sin Sin components

In TE wave H_z has Cos Cos components.

TE continue

Now field components E_x, E_y, H_x, H_y are written as.

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$ $\swarrow 0$ for TE wave

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}\right]$$

$$\boxed{E_x = +\frac{j\omega\mu}{h^2}C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$ $\swarrow 0$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}\right]$$

$$\boxed{E_y = -\frac{j\omega\mu}{h^2}C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$$ $\swarrow 0$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left[C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}\right]$$

$$\boxed{H_x = \frac{+\gamma}{h^2}C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}}$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}$$ $\swarrow 0$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}\left[C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}\right]$$

$$\boxed{H_y = \frac{+\gamma}{h^2}C\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x\sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}}$$

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TE Modes in Rectangular waveguides.

TE_{mn} – General mode and various mode are depends upon m & n values..

(a) TE_{00} mode $n=0$, $m=0$

$E_x, E_y, H_x, H_y = 0$

All field Components vanish, It cannot exist.

(b) TE_{01} mode $n=1$, $m=0$

E_x = exist
E_y = Not exist
H_x = "
H_y = exist

(c) TE_{10} mode $n=0$, $m=1$

TE_{10} mode exists.

$\boxed{\lambda_c = 2a}$ when mode is dominant

$E_x = 0$
$E_y = 1$
$H_x = 1$
$H_y = 0$

(d) TE_{11} mode $n=1$, $m=1$

TE_{11} mode exist and even higher modes.

Dominant mode – That mode for which the cut off wavelength (λ_c) assumes a maximum value.

Bcz TE_{01}, TE_{10}, TE_{11} mode All exist so that with the help of cut off wavelength find out, which one is dominant mode.

we know that

$$\boxed{\lambda_{cmn} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}}$$

for TE_{01} mode $\lambda_{c01} = \frac{2ab}{\sqrt{0+a^2}} = 2b$

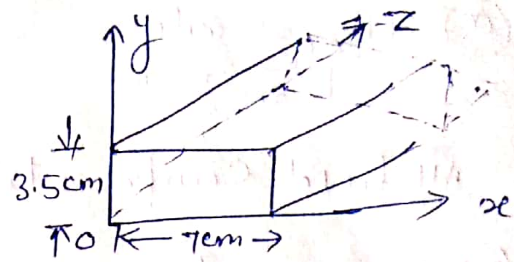
for TE_{10} mode $\boxed{\lambda_{c10}} = \frac{2ab}{\sqrt{b^2+0}} = \boxed{2a}$

for TE_{11} mode $\lambda_{c11} = \frac{2ab}{\sqrt{a^2+b^2}}$

→ λ_{c10} has the max value since a is the larger dimension. Hence TE_{10} mode is the dominant mode in Rect. W/G.

Ex 4.1.1 TE_{10} in Rect. W/G

Q: Rectangular W/G of inside dimension 7 × 3.5 cm operates in the dominant TE_{10} mode.



Calculate

1. cutoff frequency
2. Phase velocity of the wave in the guide at a freq. of 3.5 GHz.
3. Guided wavelength at the same freq.

Soln :

$$f_c = \frac{c}{\lambda_c} \Rightarrow \frac{c}{2a}$$

1. $$f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times 7\times10^{-2}} = \boxed{2.14\ GHz}$$

2. $$v_p = \frac{c}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^8}{\sqrt{1-(2.14/3.5)^2}} = \boxed{3.78\times10^8\ m/s.}$$

3. $$\lambda_g = \frac{\lambda_0}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^8/(3.5\times10^9)}{\sqrt{1-(2.14/3.5)^2}} = \boxed{10.8\ cm}$$

λ_0 →

1. Hint $$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^{1/2}}$$

$$\lambda_{c_{m,n}} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

$$\frac{\lambda_0}{\lambda_c} = \frac{c}{f}\times\frac{f_c}{c}$$

$$\boxed{\frac{\lambda_0}{\lambda_c} = \frac{f_c}{f}}$$

Wave Impedance / char wave Impedance :

$$Z_z = \frac{\text{strength of E-field in one transverse dir}}{\text{strength of M-field in another one}^{r} \text{ dir}}$$

$$Z_z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$\Rightarrow \underset{H_z=0}{Z_{TM}} = \frac{E_x}{H_y} = \frac{\cancel{\frac{-\gamma}{h^2}\frac{\partial}{\partial x}E_z} - \cancel{\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}H_z}}{\cancel{\frac{-\gamma}{h^2}\frac{\partial}{\partial y}H_z} - \cancel{\frac{j\omega\epsilon}{h^2}\frac{\partial}{\partial x}E_z}} = \frac{\cancel{+}\gamma}{\cancel{+}j\omega\epsilon} = \frac{j\beta}{j\omega\epsilon} = \frac{\beta}{\omega\epsilon}$$

As we know that $\beta = \sqrt{\mu\epsilon(\omega^2-\omega_c^2)}$

$$= \frac{\sqrt{\mu\epsilon(\omega^2-\omega_c^2)}}{\omega\epsilon} = \frac{\sqrt{\mu\epsilon}\cdot\cancel{\omega}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}{\cancel{\omega}\epsilon} = \sqrt{\frac{\mu}{\epsilon}}\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

or $Z_{TM} = \eta\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$\eta \quad \sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}} = 120\pi$$

$377\,\Omega$

Intrinsic impedance

the impedance of wave everywhere in space

or $\boxed{Z_{TM} = \eta\cdot\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$$\boxed{\frac{f_c}{f} = \frac{1}{\lambda_c}\times\frac{\lambda_0}{1}}$$

Since λ_0 is always less then λ_c for wave propagation

$$\boxed{Z_{TM} < \eta}$$

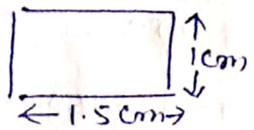
$$\underset{E_z=0}{Z_{TE}} = \frac{E_x}{H_y} = \frac{\cancel{\frac{-\gamma}{h^2}\frac{\partial}{\partial x}E_z} + \frac{j\omega\mu}{\cancel{h^2}}\cancel{\frac{\partial}{\partial y}}H_z}{\frac{+\gamma}{\cancel{h^2}}\cancel{\frac{\partial}{\partial y}}H_z - \cancel{\frac{j\omega\epsilon}{h^2}\frac{\partial}{\partial x}E_z}} = \frac{\cancel{j}\omega\mu}{\cancel{j}\beta} = \frac{\omega\cdot\mu}{\sqrt{\mu\epsilon(\omega^2-\omega_c^2)}} = \frac{\sqrt{\mu}\,\cancel{\omega}}{\sqrt{\epsilon}\,\cancel{\omega}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{\eta}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \boxed{\frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Q: Show that for a TE_{10} mode a freq. of 8GHz will pass through a waveguide of dimension $a = 1.5cm$ $b = 1cm$ if a dielectric with $\epsilon_r = 4$ is inserted in the guide?

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Soln :



Since $a = 1.5cm$

Cut off wavelength of TE_{10} mode $m=1$, $n=0$

$$\lambda_c = 2a = 3cm.$$

Hence $f_c = \frac{3\times10^{10}}{3} = \boxed{10\,GHz}$

$$f_c = \boxed{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

As is evident a freq. of 8 GHz which is less than 10GHz (cut off freq.) will not ordinarily pass through the guide.

But when a dielectric is inserted inside the w/G this freq of 8 GHz will readily pass through the same guide.

Normal λ for 8GHz $= \frac{3\times10^{10}}{8\times10^{9}} = 3.75$ Cm in air

We know that

$$\boxed{\lambda_{dielectric} = \frac{\lambda(\text{freespace})}{\sqrt{\epsilon_r}}}$$

when the medium changes wavelength also changes. wavelength of an electromagnetic wave in a dielectric medium is shorter than in free space

$$\lambda = \frac{c}{f\sqrt{\epsilon_r}}$$

$$= \frac{3.75}{\sqrt{4}}$$

λ with dielectric $\lambda_{die.} = 1.87$ Cm

$\lambda_{die.}$ is less than 3cm and hence 8 GHz freq will pass through the same guide.

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given Data

$f = 8\,GHz$

$a = 1.5cm$

$b = 1cm$

$\epsilon_r = 4$

Mode $= TE_{10}$

$\lambda_{c_{10}} = 2\cdot a = 2\times1.5 = \boxed{3cm}$

and $\lambda_0 = \frac{3\times10^{10}}{8\times10^{9}} = \boxed{3.75cm}$

Condition 1 $\boxed{\lambda_c > \lambda_0}$ Not satisfied

Now $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{10}}{3} = 10GHz$

Condition 2 $\boxed{f > f_c}$ Not satisfied

Now dielectric is inserted with $\epsilon_r = 4$

$$\lambda_{die} = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{3.75cm}{\sqrt{4}}$$

$$\lambda_{die} = \frac{3.75}{2} = \boxed{1.87cm}$$

Condition 1 $\boxed{\lambda_c > \lambda_0 \text{ or } \lambda_{die}}$ satisfied

$\boxed{3 > 1.87}$

~~Hence $f_{c\,die} = \frac{3\times10^{10}}{1.87}$ [illegible] $= 16$ GHz~~

~~[illegible] $= 16$ GHz~~

~~$f > f_c \rightarrow$ 8 GHz > 16 GHz satisfied.~~

Q: For a W/G with dimension $a = 2.286$ cm and $b = 1.016$ cm Part ① Find the cutoff freq. of TE_{01}, TE_{20}, TE_{12} and TE_{21} modes.

Soln:

1 Cut off freq. of TE modes

$$\left[f_{c_{mn}} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \right] \quad \text{or} \quad \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For TE_{01} mode

$$f_{c_{01}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{0 \times \pi}{2.286}\right)^2 + \left(\frac{1 \times \pi}{1.016}\right)^2} = 14.764 \text{ GHz}.$$

For TE_{20}

$$f_{c_{20}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{0 \times \pi}{1.016}\right)^2} = 13.123 \text{ GHz}.$$

For TE_{12}

$$f_{c_{12}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{2\pi}{1.016}\right)^2} = 30.248 \text{ GHz}.$$

For TE_{21}

$$f_{c_{21}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 19.753 \text{ GHz}.$$

Part ② cut off freq. For TM_{11}, TM_{12} and TM_{21} modes.

$$f_c = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For TM_{11} mode

$$f_{c_{11}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 16.156 \text{ GHz}.$$

For TM_{12} mode

$$f_{c_{12}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{\pi}{2.286}\right)^2 + \left(\frac{2\pi}{1.016}\right)^2} = 30.248 \text{ GHz}.$$

For TM_{21} mode

$$f_{c_{21}} = \frac{3 \times 10^{10}}{2\pi} \sqrt{\left(\frac{2\pi}{2.286}\right)^2 + \left(\frac{\pi}{1.016}\right)^2} = 19.753 \text{ GHz}.$$

Q: A hollow rectangular w/G has dimensions $a = 4cm$, $b = 2cm$. Calculate the amount of attenuation if the frequency of signal is 3GHz.

Soln:

$\lambda_c = 2a$ $f = 3GHz$.

$\lambda_c = 2 \times 4 = 8cm$

Condn.
$\lambda_c > \lambda_0$
$f > f_c$

and $f_c = \frac{c}{\lambda_c} = \frac{3 \times 10^{10}}{8} = 3.75GHz$

Bcz signal freq. is less than fc
$f < f_c$ So that signal not propagate through w/G but will get attenuated.

So that

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$= \left[\left(\frac{\pi}{0.04}\right)^2 + 0 - (2\pi \times 3 \times 10^9)^2 \, 4\pi \times 10^{-7} \times \frac{1 \times 10^{-9}}{36\pi}\right]^{1/2}$$

8.854×10^{-12}

$\alpha = 15\pi$ Np

$\alpha = 47.13$ nepers/m

$\boxed{\alpha = 409 dB/ms}$

$1Np = 20\log_{10} e \, dB$ (e = 2.7182)

$1Np = 8.6859 dB$.

$1dB = \frac{1}{20\log_{10} e} dB$

$= 0.11513 Np$

$dB = 10 \log \frac{P_1}{P_2}$

$dBm = 10 \log \frac{P}{mW}$
(Absolute Powerlevel)

* If dBm then dBmW is acquired bcz of 1mW reference
* A value can be in a form of dBW if it is referred to 1 watt.

$dB = 10 \log P_1/P_2$

$dB = 20 \log (V_1/V_2)$

Bcz P is proportional to V^2

$dB = 10 \log (V_1/V_2)^2$

$= 20 \log \left(\frac{V_1}{V_2}\right)$

Relationship b/w $\lambda_0, \lambda_g, \lambda_c$

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

or

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

guided wave length (λ_g)

⇒ Distance travelled by the wave to undergo a phase shift of 2π radians.

$$\boxed{\lambda_g = \frac{2\pi}{\beta}}$$

β → Phase constt

Phase velocity

V_p → The rate at which the wave changes its phase in terms of guide wave length.

$$V_p = \frac{\lambda_g}{\text{unit time}} = \lambda_g \cdot f$$

$$V_p = \frac{2\pi \cdot f}{\beta} = \frac{\omega}{\beta}$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

Group velocity (V_g) : Rate at which the wave propagates through w/G.

$$V_g = \frac{d\omega}{d\beta} \qquad \beta = \sqrt{\mu\epsilon(\omega^2 - \omega_c^2)}$$

Now differentiating β w.r.t ω we get

$$\frac{d\beta}{d\omega} = \frac{1}{2\sqrt{(\mu\epsilon)(\omega^2-\omega_c^2)}} \cdot 2\omega\mu\epsilon$$

Divide by ω

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\epsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\mu\epsilon}}{\sqrt{1-(f_c/f)^2}}$$

$$\frac{d\omega}{d\beta} = \frac{\sqrt{1-(f_c/f)^2}}{\sqrt{\mu\epsilon}} = c\cdot\sqrt{1-f_c/f}$$

$$V_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

Product of V_p and V_g.

$$V_p \cdot V_g = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \cdot c\cdot\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\boxed{V_p \cdot V_g = c^2}$$

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Last class concerned.

$\uparrow R = \frac{V}{I}\uparrow\downarrow$

$f > f_c$

$\boxed{\lambda_0 < \lambda_c}$

1. $Z_{TM} = \eta \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$

2. $Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$\frac{4(2)^2}{2}$

$\sqrt{1-4}$

$\eta \cdot \sqrt{-3}$

3. $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

4. $\lambda_g = \frac{2\pi}{\beta}$

5 $\to \beta \to \sqrt{\mu\epsilon(\omega^2-\omega_c^2)}$

6. $v_p = \frac{\lambda_g}{\text{unit time}} = \lambda_g \cdot f \to \frac{2\pi f}{\beta}$

$\boxed{v_p = \frac{\omega}{\beta}} \leftarrow \cancel{\omega}\sqrt{\mu\epsilon}\left(\cancel{\omega^2}\, 1-\left(\frac{\omega_c}{\omega}\right)^2\right)$

$\boxed{v_p = \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}}$

$\boxed{v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$

$\boxed{v_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$

$Z_{TM} = \eta \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$

$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$\boxed{\lambda_g = \frac{2\pi}{\beta}} \leftarrow \cancel{\omega}\sqrt{\mu\epsilon}\left(\omega^2-\omega_c^2\right)\; 1-\left(\frac{\omega_c}{\omega}\right)^2$

$v_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \leftarrow \lambda_g \cdot f = \frac{2\pi f}{\beta} = \frac{\omega}{\beta}$

$v_g = c \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$

$v_g = \frac{d\omega}{d\beta}$

$\frac{d\beta}{d\omega} = \frac{1}{\cancel{2}} \frac{\mu\epsilon \cdot \cancel{2}\omega}{\sqrt{\mu\epsilon(\omega^2-\omega_c^2)}}$

$= \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\left(1-\left(\frac{\omega_c}{\omega}\right)^2\right)}} \cdot \sqrt{\mu\epsilon}$

$\frac{d\omega}{d\beta} = \frac{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}{\sqrt{\mu\epsilon}} = c \cdot \sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$

$\boxed{v_p \cdot v_g = c^2}$