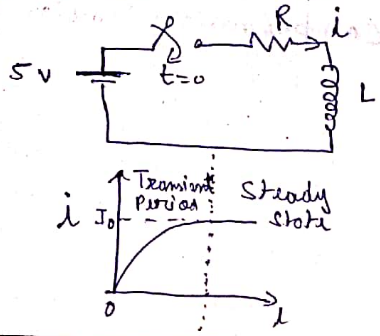
Difference between Transient & Steady State

(i) **Transient** :- Transient period is defined as a very short time period between two steady state.



- Before closing switch, current in circuit was zero, $I = 0$ (Steady state)
- After closing switch at $t = 0$, current will grow exponentially to steady value I_0 (New steady state)
- Time period between $i = 0$ to $i = I_0$ is called transient period.

- Any disturbance (disturbance or change in steady state) causes transients in electric circuit. 11
- Transients are entirely associated with the changes in stored energy in inductor and capacitors.
- Since there is no stored energy is resistors, there are no transients in pure resistive circuits.

(ii) Steady state – when transients dies out, a new steady state is achieved.

Modelling of electrical System

- Any electrical system (electrical circuit / network) can be modelled in terms of differential equation.
- System response = Solution of differential equation.
- Solution of differential equation contains Complementary function (C.F.) and Particular Integral (P.I.).
- Complementary function gives transient response.

Note :- Transient response is also called natural or force free response of a system.

- Particular Integral gives steady state response of a system. Steady state response is also called forced response of a system.

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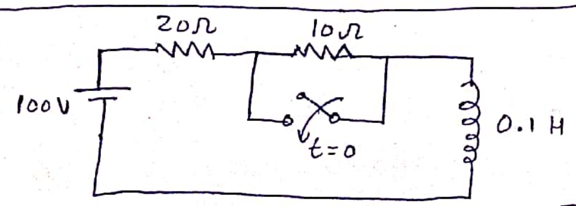
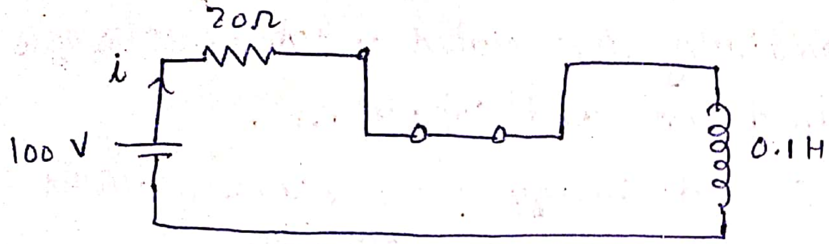


Initially circuit was in steady state before closing switch. switch is closed at $t = 0$

Find $i(t)$ supplied by 100 V battery.

Sol. When switch is closed at $t=0$; Resistor ($10\,\Omega$) is bypassed from the circuit



Apply KVL

$$100 = i(t)\,20 + 0.1\frac{d\,i(t)}{dt}$$ → Differential equation

$i(t) = C.F + P.I.$

$$1000 = 200\, i(t) + \frac{d\,i(t)}{dt}$$

$$(D + 200)\, i(t) = 1000$$

C.F ⇒ $D + 200 = 0$ ⇒ $D = -200$

$$C.F = C_1 e^{-200t}$$

$$P.I \Rightarrow e^{-200t} \int e^{200t} \cdot 1000 \, dt$$

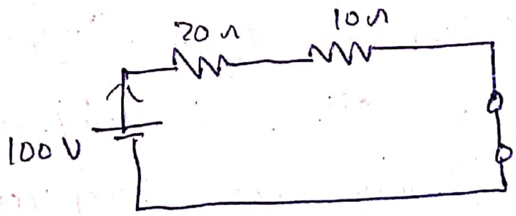
$$\Rightarrow \frac{1000 \times e^{-200t} \times e^{200t}}{200}$$ (No need for constant of integration)

$$= 5$$

$$i(t) = C_1 e^{-200t} + 5$$

$i(0^-) = i(0^+)$ → Due to presence of inductor

For $i(0^-)$ ⇒



L short circuit (under steady state)

$$i(0^-) = \frac{100}{30}\ A$$

$$\therefore\ i(0^+) = \frac{10}{3}\ A$$

$$i(0) = C_1 e^{-200(0)} + 5$$

$$\frac{10}{3} = C_1 + 5$$

$$C_1 = -\frac{5}{3}$$

$$\boxed{i(t) = \left(-\frac{5}{3} e^{-200t} + 5\right) A}$$ Solution

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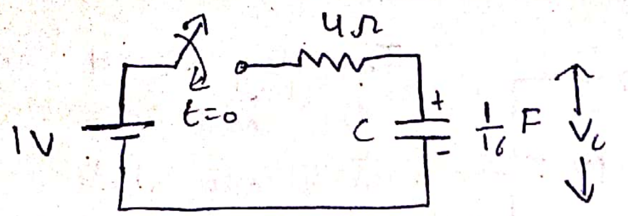
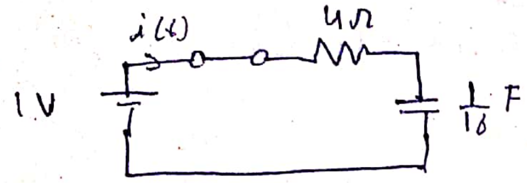
Q 

$V_c(0^-) = 9V$

Find expression of capacitor voltage $V_c(t)$

Sol. 

$i(t) = C \dfrac{dV_c}{dt}$

$\dfrac{1}{C}\int_0^t i(t)\,dt = dV_c$

$\dfrac{1}{C}\int_0^t i(t)\,dt = \int_9^{V_c} dV_c$

$V_c - 9 = \dfrac{1}{C}\int_0^t i(t)\,dt$

$V_c = \dfrac{1}{C}\int_0^t i(t)\,dt + 9$

Apply KVL

$1 = i(t)\,4 + V_c$

$1 = 4\,i(t) + 16\int_0^t i(t)\,dt + 9$

Differentiate both side

$0 = 4\dfrac{d\,i(t)}{dt} + 16\,i(t)$

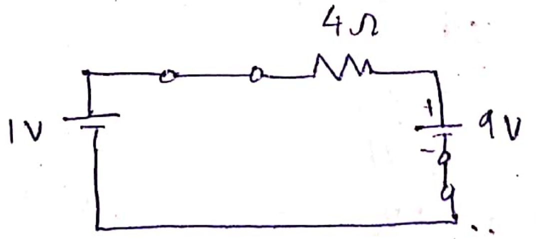
$i(t) = C.F + P.I.$

$P.I = 0$

$C.F \Rightarrow (4D + 16)\,i(t) = 0 \qquad \Rightarrow D = -4$

$i(t) = C_1 e^{-4t}$

Circuit at $t = 0^+$



$i(0^+) = \dfrac{1-9}{4} = -2$

$i(0) = C_1 e^{-4(0)}$

$-2 = C_1$

$\therefore i(t) = -2e^{-4t}$

$V_c = 16\int_0^t i(t)\,dt + 9 = 16\int_0^t -2e^{-4t}\,dt + 9$

$V_c = \dfrac{-32}{-4}\left[e^{-4t}\right]_0^t + 9 = 8[e^{-4t} - 1] + 9$

$V_c = 1 + 8e^{-4t}$

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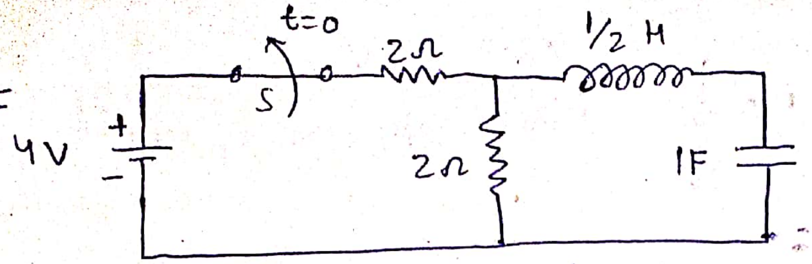
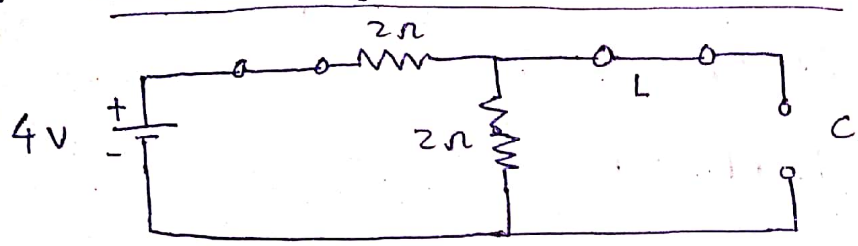


Initially Switch S was close and Circuit was in steady State.

At $t=0$, switch S was opened. Find the expression for inductor current.

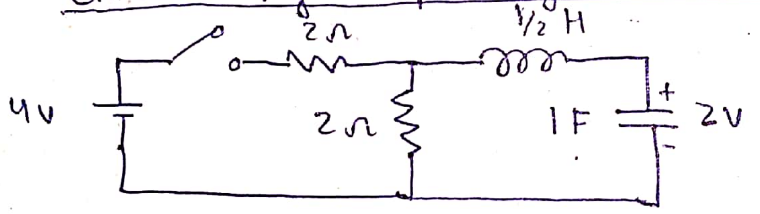
Sol. Circuit before opening switch S.



Voltage across Capacitor (under steady state, S closed) $= \left(\frac{4}{2+2}\right)\times 2 = 2V$

Inductor current (under steady state, S closed) $= 0$

Circuit After opening switch S.



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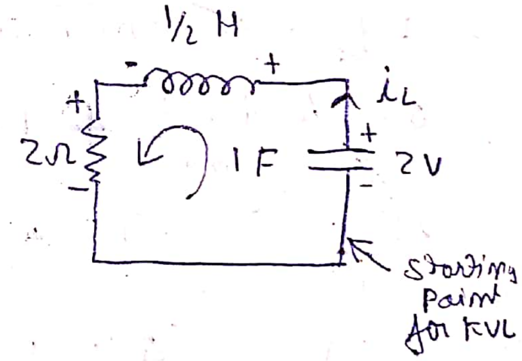

KVL in above circuit

$$V_c - L\frac{d\,i_L(t)}{dt} - i_L(t)\,2 = 0$$

Also $i_L(t) = C\frac{dV_c}{dt}$

$$\frac{1}{C}\int_0^t i_L(t)\,dt = \int_2^{V_c} dV_c$$

$$\frac{1}{C}\int_0^t i_L(t)\,dt = V_c - 2$$

Put V_c in KVL eq.

$$\frac{1}{1}\int_0^t i_L(t)\,dt + 2 = \frac{1}{2}\frac{d\,i_L(t)}{dt} + 2\,i_L(t)$$

Differentiate both side

$$i_L(t) = \frac{1}{2}\frac{d^2 i_L(t)}{dt^2} + 2\frac{d\,i_L(t)}{dt}$$

$$\frac{d^2 i_L(t)}{dt^2} + 4\frac{di_L(t)}{dt} - 2 i_L(t) = 0$$

$i_L(t) =$ C.F

CF $\Rightarrow$ $D^2 + 4D - 2 = 0$

$$D = \frac{-4 \pm \sqrt{16 - 4(-2)}}{2} = -4.449, 0.449.$$

$$i_L(t) = C_1 e^{-4.449t} + C_2 e^{0.449t}$$

Initial condition after opening switch at $t=0$

(i) $i_L(0^+) = 0$

(ii) Since voltage across capacitor cannot change instantaneously & inductor current is zero, therefore, voltage across inductor should be 2V (It should follow KVL)

$V_L = L\frac{di_L}{dt}$ $\Rightarrow$ $2 = \frac{1}{2}\frac{di_L}{dt}$ @ $t=0^+$

$\therefore \frac{di_L}{dt} = 4$ at $t = 0^+$ $\Rightarrow$ $\frac{di_L(0^+)}{dt} = 4$

$i_L(0) = C_1 + C_2$ → from initial condition 1

$0 = C_1 + C_2$

$C_1 = -C_2$

$$\frac{di_L(0^+)}{dt} = C_1 \times (-4.449)\, e^{-4.449t} + C_2 \times (0.449)\, e^{0.449t}$$

$4 = -4.449 C_1 + 0.449 C_2$

$C_2 = 0.816$, $C_1 = -0.816$

$$\therefore \boxed{i_L(t) = -0.816\, e^{-4.449t} + 0.816\, e^{0.449t}}$$ Solution.

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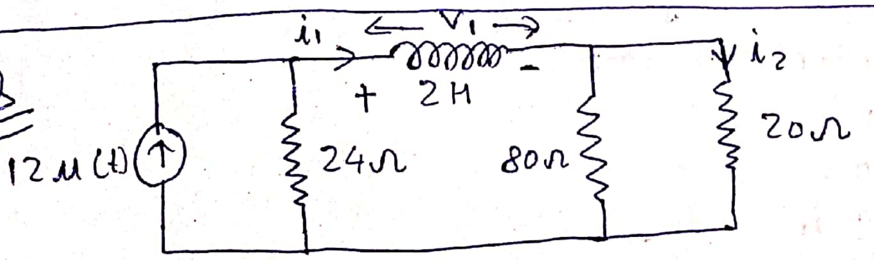


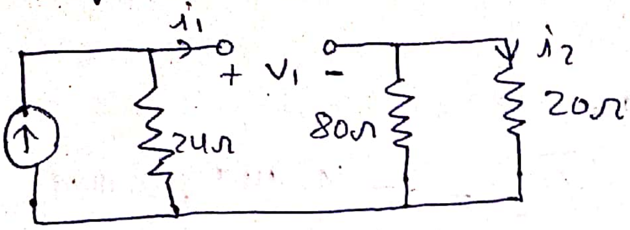
Find i_1, i_2, V_1 at
(i) $t = 0^+$
(ii) $t = 0^-$
(iii) $t = \infty$; (IV) $t = 50$ ms

Sol. (ii) Since excitation source is step signal, it has zero value for $t < 0$.

$\therefore$ $i_1(0^-) = 0$; $i_2(0^-) = 0$; $V_1(0^-) = 0$.

(i) $t = 0^+$ (initial stage)

Eq. circuit at $t = 0^+$



$i_1(0^+) = 0$

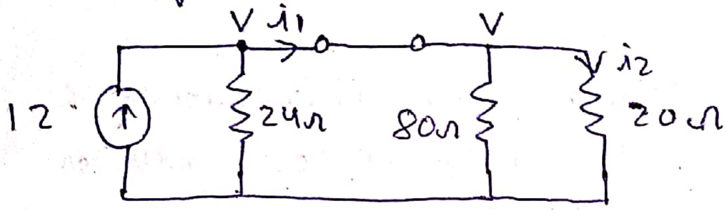
$i_2(0^+) = 0$

$V_1(0^+) =$ Voltage across 24Ω

$= 24 \times 12 = 288V$

(ii) $t = \infty$ (Final or steady state)

Eq. circuit at $t = \infty$



Apply KCL at the node

$$12 = \frac{V}{24} + \frac{V}{80} + \frac{V}{20} = 0.0416V + 0.0125V + 0.05V = 0.1041V$$

$V = 115.2 \text{ V}$

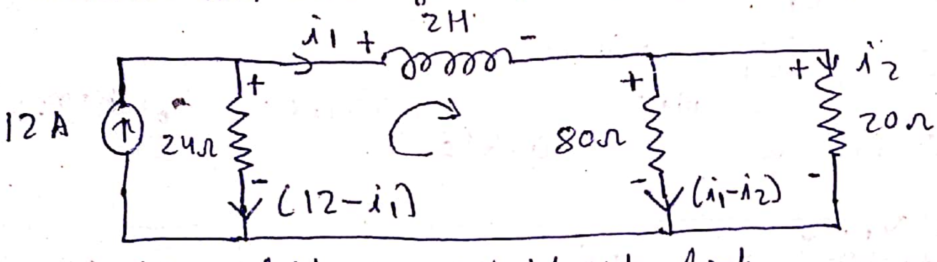
$i_1 = 12 - \frac{115.2}{24} = 7.2 \text{ A}$

$i_2 = \frac{115.2}{20} = 5.76$

$V_1 = 0$

$\therefore i_1(\infty) = 7.2 \text{ A}$; $i_2(\infty) = 5.76 \text{ A}$; $V_1(\infty) = 0$

(iv) General expression for $i_1(t)$, $i_2(t)$, $V_1(t)$



KVL in ~~left side~~ right side loop

$80(i_1 - i_2) = 20 i_2$

$80 i_1 = 100 i_2$

$i_2 = \frac{80}{100} i_1$

KVL in left side loop

$$(12 - i_1)24 - 2\frac{di_1}{dt} - 80(i_1 - i_2) = 0$$

$$288 - 24i_1 - 2\frac{di_1}{dt} - 80\left(i_1 - \frac{80}{100} i_1\right) = 0$$

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$288 = 40 i_1 + 2\frac{di_1}{dt} \Rightarrow \frac{di}{dt} + 20 i = 144$

$i_1(t) = C_1 e^{-20t} + e^{-20t}\int 144\, e^{20t}\, dt$

$i_1(t) = C_1 e^{-20t} + 7.2$

$i_1(0) = C_1 + 7.2$ → initial condition

$\therefore C_1 = -7.2$

$i_1(t) = -7.2 e^{-20t} + 7.2 = 7.2(1 - e^{-20t})$

$i_2(t) = 0.8\, i_1(t) = 5.76(1 - e^{-20t})$

$V_1(t) = L\frac{di_1}{dt}$

$= 2\left[\frac{d}{dt}[7.2(1 - e^{-20t})]\right] = 2(-7.2)(-20)\, e^{-20t}$

$V_1(t) = 288\, e^{-20t}$

$i_1(0.05) = 4.55A$; $i_2(0.05) = 3.64A$; $V_1(0.05) = 105.94$

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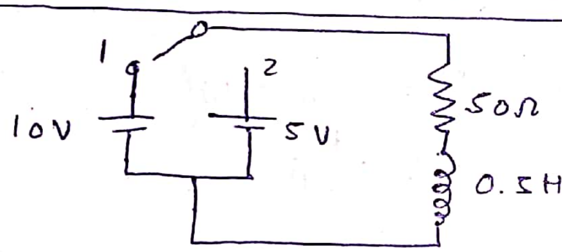
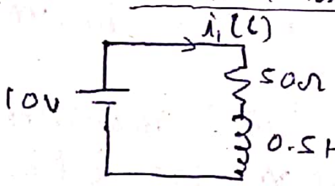


At $t = 0$ switch is closed at position 1
After $t = 0.5ms$ switch is moved to position 2.
Find $i(t)$ in both cases.

Sol.

Switch at position 1



$10 = i_1 50 + 0.5\frac{di_1}{dt}$

$20 = 100 i_1 + \frac{di_1}{dt}$

$i_1(t) = C_1 e^{-\frac{R}{L}t}$ + steady state current

$i_1(t) = C_1 e^{-100t} + 0.2$

$i_1(0) = C_1 + 0.2$

$0 = C_1 + 0.2$

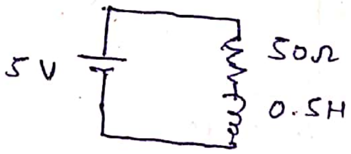
$C_1 = -0.2$

$i_1(t) = -0.2 e^{-100t} + 0.2$

$i_1(0.0005) = -0.2 e^{-100(0.0005)} + 0.2$

$i_1(0.0005) = 0.00975$

Switch at position 2



$5 = i_2 50 + 0.5\frac{di_2}{dt}$

$10 = 100 i_2 + \frac{di_2}{dt}$

$i_2(t) = C_2 e^{-\frac{R}{L}(t-t')}$ + steady state current

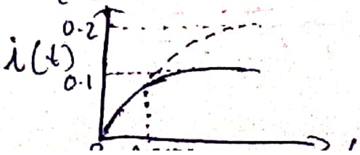
$i_2(t) = C_2 e^{-100(t-0.0005)} + 0.1$

$i_2(0.0005) = C_2 + 0.1$

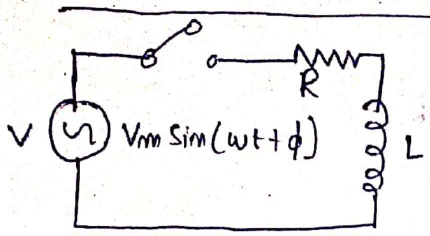
$0.00975 = C_2 + 0.1$

$C_2 = \cancel{0.009} - 0.0902$

$i_2(t) = -0.0902\, e^{-100(t-0.0005)} + 0.1$



Transient response with Sinusoidal excitation



$V = V_m \sin(\omega t + \phi)$ where ϕ varies from 0 to 2π depending on the switching instant.

Switch is closed at $t = 0^+$

$$Ri + L\frac{di}{dt} = V_m \sin(\omega t + \phi)$$

$$i(t) = C.F. + P.I$$

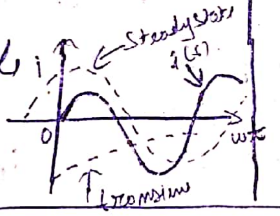
$$C.F = C_1 e^{-\frac{R}{L}t}$$ → Transient (Dies out after some time)

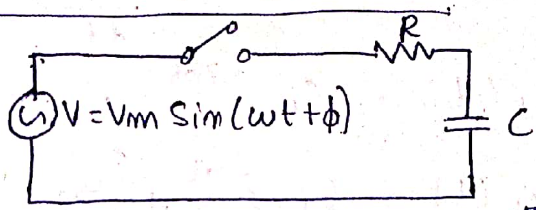
P.I = Steady State current

$$P.I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t + \phi - \theta)$$

where θ is p.f angle

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$





Switch is closed at $t = 0^+$ (t=0)

$$iR + \frac{1}{C}\int_0^t i\,dt + V_c(0^+) = V_m \sin(\omega t + \phi)$$

Differentiate both the sides

$$R\frac{di}{dt} + \frac{1}{C}i = \omega V_m \cos(\omega t + \phi)$$

$$i(t) = C.F + P.I.$$

$$C.F = C_2 e^{-\frac{t}{RC}}$$ → Transient (Dies out after some time)

P.I = Steady State Current

$$P.I = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi + \theta)$$

where θ is p.f angle

$$\theta = \tan^{-1}\frac{1}{\omega C R}$$

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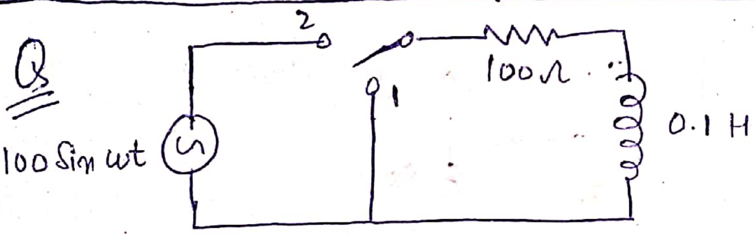
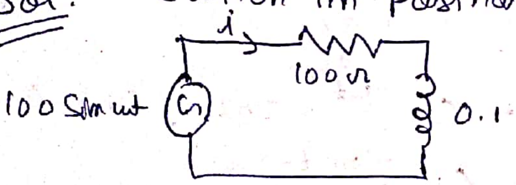


System frequency is 50Hz

Initially Switch was in position 1 and steady state in inductor was 1 A.
At $t=0$ switch is moved to 2.
Find expression for $i(t)$

Sol. Switch in position 2



KVL in the loop.

$$100i + 0.1\frac{di}{dt} = 100 \sin \omega t$$

$$1000i + \frac{di}{dt} = 1000 \sin \omega t$$

$$i(t) = C.F + P.I.$$

$$C.F = C_1 e^{-1000t}$$

$$P.I = \frac{100}{\sqrt{(100)^2 + (2\pi(50)\,0.1)^2}} \sin\left(\omega t - \tan^{-1}\frac{2\pi(50)(0.1)}{100}\right)$$

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$$P.I. = \frac{100}{104.8} \sin(\omega t - 17.43^\circ)$$

$$= 0.954 \sin(\omega t - 17.43^\circ)$$

$$i(t) = C_1 e^{-1000t} + 0.954 \sin(\omega t - 17.43^\circ)$$

$i(0) = 1A$ (Initial Condition)

$$1 = C_1 + 0.954 \sin(-17.43^\circ)$$

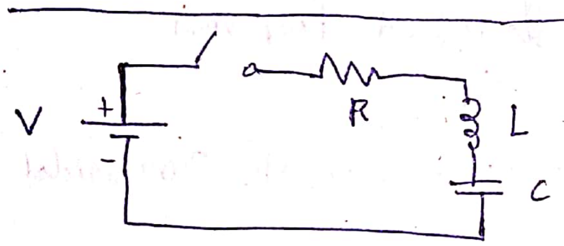
$$1 = C_1 + (-0.285)$$

$$C_1 = \cancel{0.714}\ 1.285$$

$$i(t) = 1.285 e^{-1000t} + 0.954 \sin(\omega t - 17.43^\circ)$$

$$\boxed{i(t) = 1.285\, e^{-1000t} + 0.954 \sin(\omega t - 17.43^\circ).}$$ Solution.

Transient response in RLC circuit with DC excitation



Switch is closed at $t=0$

Apply KVL.

$$V = i(R) + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_c(0^+)$$

Differentiate both sides

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C} i$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0$$

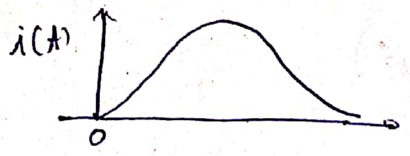
$$i(t) = C_1 e^{P_1 t} + C_2 e^{P_2 t}$$

$$P_1, P_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \alpha \pm \beta$$

$$\alpha = -\frac{R}{2L}, \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case I : If p_1 & p_2 are real & unequal. $\left[\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}\right]$

$$i(t) = e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right)$$



Over Damped Response

Case II If $p_1 = p_2$ $(\beta = 0)$ $\left[\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}\right]$

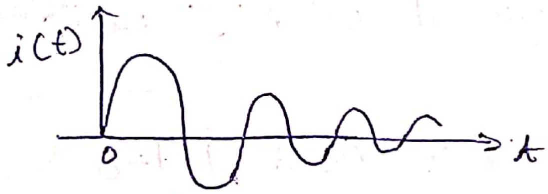
$$\therefore\ i(t) = e^{\alpha t}\left(C_1 + C_2 t\right)$$

Critical Damped Response

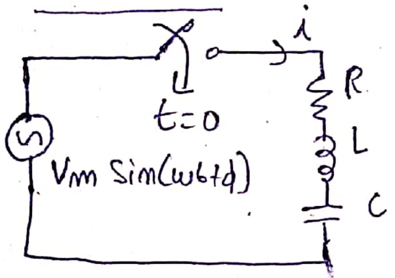
Case III If p_1 & p_2 are Complex Conjugate $\left[\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}\right]$

$$i(t) = e^{\alpha t}\left[C_1 \cos(\beta t) + C_2 \sin(\beta t)\right]$$

Underdamped Response



Transient Response in Series RLC Circuit with Sinusoidal Excitation.



$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + V_c(0^+) = V_m \sin(\omega t+\phi)$$

Differentiate both sides

$$R\frac{di}{dt} + L\frac{d^2 i}{dt} + \frac{1}{C} i = \omega V_m \cos(\omega t+\phi)$$

$i(t) = C.F + P.I$

C.F gives transient response and it is same as in case of RLC with DC excitation (Above previous Case).

P.I gives steady state response

$$P.I = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \sin\left[\omega t + \phi + \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)\right]$$

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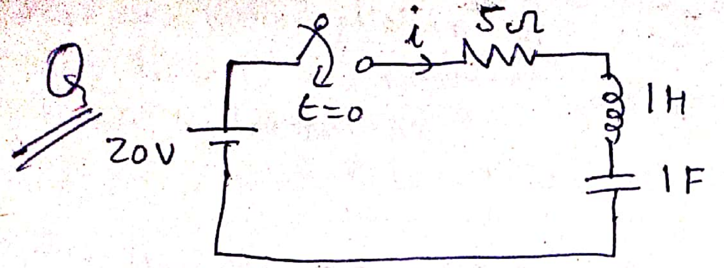


Find $i(t)$

Initially there is no energy stored in the system.

Sol. Apply KVL

$$20 = 5i + 1\frac{di}{dt} + \frac{1}{1}\int_0^t i\,dt$$

Differentiate both sides

$$0 = 5\frac{di}{dt} + \frac{d^2i}{dt^2} + i \Rightarrow \frac{d^2i}{dt^2} + 5\frac{di}{dt} + i = 0$$

$i = C.F + P.I.$

$P.I. = 0$

$C.F \Rightarrow \quad D^2 + 5D + 1 = 0$

$$D = \frac{-5 \pm \sqrt{25-4}}{2} = -0.21, -4.74$$

$$\therefore i(t) = C_1 e^{-0.21t} + C_2 e^{-4.74t}$$

Initial conditions $i(0^+) = 0$

$V_C(0^+) = 0$ (Behaviour of capacitor)

$V_R(0^+) = 0$ (No current at $t = 0^+$)

$\therefore V_L(0^+) = 20\,V$ (To satisfy KVL)

$$V_L(0^+) = L\frac{di(0^+)}{dt} = 20V$$

$\therefore i(0^+) = C_1 + C_2$

$0 = C_1 + C_2$

$C_1 = -C_2$

$$\frac{di(0^+)}{dt} = -0.21C_1 - 4.74C_2$$

$$20 = -0.21(-C_2) - 4.74C_2$$

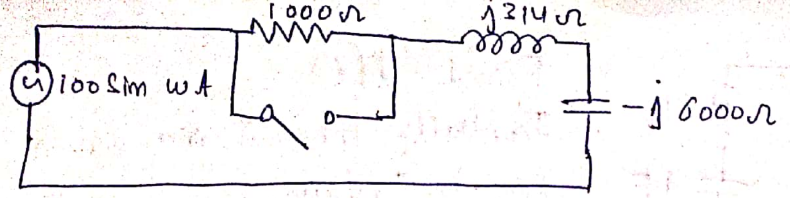
$$20 = 0.21C_2 - 4.74C_2$$

$C_2 = -4.36$

$C_1 = 4.36$

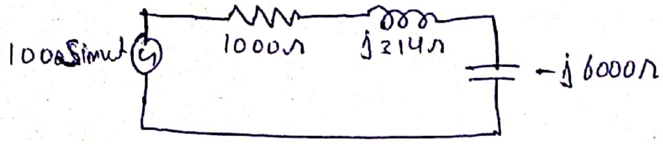
$$\therefore i(t) = 4.36e^{-0.21t} - 4.36e^{-4.74t}$$

Q



Initially switch was open. Switch was close at $t=0$. Find current in circuit.

Sol switch in open position

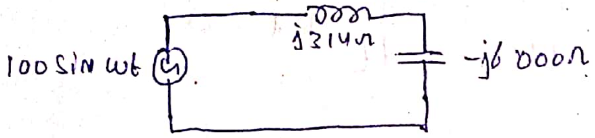


$$\text{steady state current} = \frac{100 \sin\left(\omega t + \tan^{-1}\left(\frac{X_C - X_L}{R}\right)\right)}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$i(\infty) = \frac{100}{\sqrt{(1000)^2 + (5686)^2}} \sin\left(\omega t + \tan^{-1}\left[\frac{5686}{1000}\right]\right)$$

$$i(\infty) = 0.0173 \sin(\omega t + 80^\circ)$$

switch in close position (Resistor is bypassed)



KVL in loop:

$$100 \sin \omega t = 1 \frac{di}{dt} + \frac{1}{5.3 \times 10^{-7}} \int i\, dt + V_c(0^+)$$

$$\omega 100 \cos \omega t = \frac{d^2 i}{dt^2} + \frac{1}{5.3 \times 10^{-7}} i$$

$$i(t) = C.F + P.I$$

$$C.F \Rightarrow \left(D^2 + \frac{1}{5.3 \times 10^{-7}}\right) = 0 \quad , D = \pm j1373.6$$

$$C.F = C_1 \cos(1373.6 t) + C_2 \sin(1373.6 t)$$

$$P.I = \text{Steady State Current} = \frac{100}{5686} \sin(\omega t + 90^\circ)$$

$$P.I = 0.0176 \sin(\omega t + 90^\circ) = 0.0176 \cos \omega t$$

$$i(0^+) = 0.0173 \left[\sin(\omega(0) + 80^\circ)\right] = 0.0173 \times \sin 80^\circ = 0.0170$$

$$V_c = 0.0173 \angle 80^\circ \times 6000 \angle -90^\circ \rightarrow \text{switch in open condition}$$

$$= 103.8 \angle -10 = 103.8 \sin(\omega t - 10^\circ).$$

$$V_c(0^+) = 103.8 \sin(\omega(0) - 10^\circ) = -18.02$$

Voltage across inductor at $t = 0^+$ = − (voltage across capacitor at $t = 0^+$)

$$1 \cdot \frac{di(0^+)}{dt} = 18.02$$

$$18.02 = -C_1 (1373.5) \sin(1373.5 (0)) + C_2 (1373.5) \cos(1373.5 (0)) - 0.176\omega \sin(\omega(0))$$

$$18.02 = C_2 (1373.5)$$

$$C_2 = 0.0131$$

$$i(0^+) = C_1 + 0.0176$$

$$0.0170 = C_1 + 0.0176$$

$$C_1 = -0.0006$$

$$\therefore i(t) = -0.0006 \cos(1373.5 t) + 0.0131 \sin(1373.5 t) + 0.0176 \sin(\omega t + 90^\circ)$$