UNIT-1 Microwave Engineering

Introduction to Microwave :-

- Microwaves are electromagnetic waves whose frequency range from 1 GHz to 1000 GHz. $(1\ GHz = 10^9\ Hz)$
- μwaves are so called since they are defined in terms of their wavelength in micro ranges i.e. very short.
- μwave is a signal that has a wavelength of 1 foot or less $\lambda \leq 30.5$ cm = 1 foot.
- The frequency and wavelength of a microwave are inversely proportional to each other.

i.e. $f = \frac{V_0}{\lambda}$ where f = freq.
V_0 = velocity of microwave in free space, m/sec (3×10^8 m/sec)
λ = wavelength, m

i) $f = 300$ MHz
$$\lambda = \frac{3\times10^8}{300\times10^6} = 1m$$

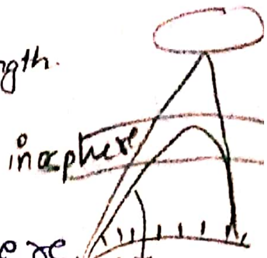
ii) $f = 300$ GHz
$$\lambda = \frac{3\times10^8}{300\times10^9} = 1\ mm$$

Characteristics of μwaves :-

1. Their wavelength is small and frequency is high.
2. They propagate in free space freely.
3. They consist of Electric and magnetic fields which are perpendicular to each other.
4. Less Attenuation in free space.
5. μwaves propagate along a straight line.
6. μwaves are reflected from good conductors.

• Advantages of μwaves :-

1. Large Bandwidth
2. High directivity of μwave radiation pattern.
3. Small Antenna size. ($\because$ Size of component $\propto \lambda$) wavelength.
4. Fading effect is low.
5. Effective for propagation through inosphere.
6. Repeaters are placed at 50 km.
7. High Reliable.



inosphere propagation

• Applications of μwaves :-

- Telecommunication → Television, Telemetry communication link for railways.
- Radars → Detect Aircrafts, Guide missiles, observe weather patterns, Air traffic Control (ATC).
- Commercials → Microwave oven, food processing industries.

• Different Types of Frequency Ranges

1) ULF (ultra low frequency) - (3Hz - 30 Hz)
2) ELF (Extra low freq.) - (30Hz - 300 Hz)
3) VF (voice freq.) - (300 Hz - 3 KHz)
4) VLF (very low freq.) - (3 KHz - 30 KHz)
5) LF (Low freq.) - (30 KHz - 300 KHz)
6) MF (Medium freq.) - (300 KHz - 3 MHz)
7) HF (High freq.) - (3MHz - 30 MHz)
8) VHF (very high freq.) - (30 MHz - 300 MHz) → T.V (Radio waves)
9) UHF (ultra high freq.) - (300 MHz - 3 GHz)
10) SHF (Super high freq.) - (3 GHz - 30 GHz)
11) EHF (Extreme high freq.) - (30 GHz - 300 GHz)

• μ-wave frequencies lie in (UHF + SHF + EHF).

• Other higher order frequencies

1) Infrared freq. – (300 GHz – 300 THz)
2) Visible light – (300 THz – 3 PHz)
3) Ultra voilet – (3 PHz – 30 PHz)
4) X-rays – (30 PHz – 300 PHz)
5) γ-rays (Gamma) – (300 PHz – 3 EHz)

PHz = Peta Hertz
THz = Tera Hertz
EHz = Exa hertz

$$\boxed{\begin{aligned} THz &= 10^{12}\ Hz \\ PHz &= 10^{15}\ Hz \\ EHz &= 10^{18}\ Hz \end{aligned}}$$

• Different Types of Microwave Bands :-

1) L-Band → (1-2 GHz) → Marine Satellites.
2) S-Band → (2-4 GHz) → Bluetooth, wi-fi, oven, weather
3) C-Band → (4-8 GHz) → Satellite communication.
4) X-Band → (8-12 GHz) → Education purpose μ-wave test bench
5) Ku-Band → (12-18 GHz) → Satellite T.V
6) K-Band → (~~27-40~~ 18-27 GHz) → RADAR, Infrared Astronomy (Defines Stars)
7) Ka-Band → (27-40 GHz) → Military Airplanes, and also for Satellite communication.

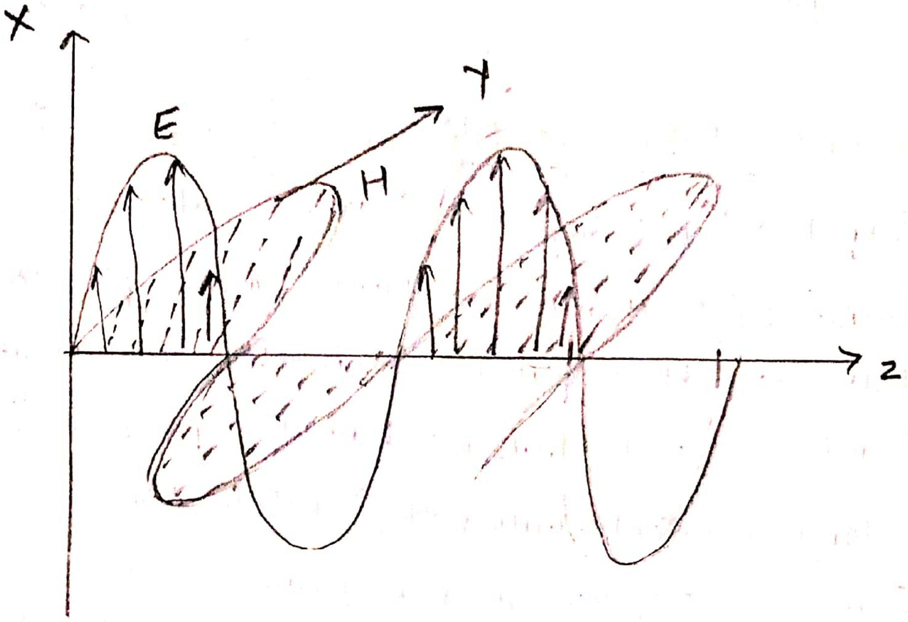
How Does Microwaves or EM waves Propagates ???

→ EM or μwaves propagate similar to the propagation of water waves on a pond after a stone has been thrown into it, but the difference is that water waves are longitudinal (oscillations in the direction of propagation) and EM waves are transverse (oscillations perpendicular to the direction of propagation).

→ The direction of propagation of the electric field and the magnetic field are mutually perpendicular in EM waves.

- These electric fields and magnetic fields vary with f
Therefore these variations of electric and magnetic fields are described by the set of laws and equations which are called MAXWELL's Equations.

- Maxwell's Equations describe how electric and magnetic fields are generated by charges, currents and changes of the fields.



Maxwell's Equations :-

- For Time-varying fields

→ In differential form

① $\nabla \times H = \frac{\partial D}{\partial t} + J$

② $\nabla \times E = -\frac{\partial B}{\partial t}$

③ $\nabla \cdot D = \rho$

④ $\nabla \cdot B = 0$

, where J = Electric or Conduction Current density.
H = magnetic field strength
D = Electric flux density
B = Magnetic flux density
ρ = charge density
$\frac{\partial D}{\partial t}$ = Displacement electric current density.
E = Electric field
$\frac{\partial B}{\partial t}$ = Time-derivative of Magnetic Flux Density.

$\stackrel{+}{n^o}$ In Integral form, Maxwell Equations are defined by some laws :- (2)

① **Ampere's Law** → It states that the magnetomotive force around a closed path is equal to the sum of the displacement current and conduction current through any surface enclosed by the path.

i.e. $$\boxed{\oint H\cdot dl = \oint\left(J+\frac{\partial D}{\partial t}\right)\cdot ds}$$ $\int(\nabla\times H)\cdot ds$

I^{st} Eq^n of Maxwell

② **Faraday's Law** → It states that the electromotive force around a closed path is equal to the negative of time derivative of magnetic flux flowing through any surface enclosed by the path.

i.e. $$\boxed{\oint E\cdot dl = -\frac{\partial}{\partial t}\oint B\cdot ds}$$

II^{nd} Eq^n of Maxwell

③ **Gauss's Law** (Electric fields) → It states that the total electric displacement flux passing through a closed surface is equal to the total charge enclosed.

III^{rd} Eq^n of Maxwell $$\boxed{\oint D\cdot ds = \oint \rho_v\, dv}$$

where, ρ_v = volume charge density.

④ **Gauss's Law** (Magnetic fields) → It states that the net magnetic flux passing through any closed surface is zero.

$$\boxed{\oint B\cdot ds = 0}$$

IV^{th} Eq^n of Maxwell

• Maxwell's Equation for free space. ($\sigma = 0$, $\rho = 0$) $\Downarrow$ Conditions for free space.

① $\nabla \times H = \frac{\partial D}{\partial t}$

② $\nabla \times E = \frac{-\partial B}{\partial t}$

③ $\nabla \cdot D = 0$

④ $\nabla \cdot B = 0$

• WAVE EQUATIONS OR Helmholtz's Equations

→ For the propagation of waves, all field vectors should vary w.r.t time and it requires harmonics in nature to propagate the wave forward.

→ Electric field (E) should contain harmonics for wave propagation,

So, $E = E_0 e^{j\omega t}$ ---- ①

where E_0 = Maximum value of electric field intensity
$\omega = 2\pi f$ (angular frequency of harmonics variations)

Differentiating eqn ①;

$$\frac{\partial E}{\partial t} = \underbrace{E_0 \cdot e^{j\omega t}}_{E} \cdot j\omega = E \cdot j\omega$$

$$\frac{\partial E}{\partial t} = j\omega E \quad — ② \quad \left[\because \frac{\partial}{\partial t} = j\omega\right]$$

Again differentiation;

$$\frac{\partial^2 E}{\partial t^2} = E_0 e^{j\omega t} \cdot (j\omega)^2$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 e^{j\omega t} = -\omega^2 \cdot E \quad — ③ \quad \left[\because \frac{\partial^2}{\partial t^2} = -\omega^2\right]$$

Now let us consider a medium which does not contain any free charges and is also non-conducting, for example air or free space, [∴ σ (conductivity) = 0] (4)

→ From maxwell's 1st equation :-

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \frac{\partial}{\partial t}(\varepsilon E)$$

For free space ($\sigma = 0$)

$$\therefore \quad \nabla \times H = 0 + \frac{\partial E}{\partial t} \cdot \varepsilon$$

from eq^n ② $\left(\frac{\partial}{\partial t} = j\omega\right)$

So, $$\nabla \times H = j\omega \varepsilon E \quad — ④$$

. From Maxwell's 2nd equation,

$$\nabla \times E = \frac{-\partial B}{\partial t} = \frac{-\partial(\mu H)}{\partial t} = -j\omega\mu H \quad \left(\because \frac{\partial}{\partial t} = j\omega\right) \text{ from eq}^n ②$$

$$\nabla \times E = -j\omega\mu H \quad — ⑤$$

Now, taking curl of $\nabla \times E$, we get

$$\nabla \times (\nabla \times E) = \nabla \times (-j\omega\mu H) \quad — (\text{from eq}^n\ 5)$$

From vector analysis

$$\nabla(\nabla \cdot E) - \nabla^2 E = -j\omega\mu(\nabla \times H)$$

$$= -j\omega\mu(j\omega\varepsilon E) \quad — (\text{from eq}^n\ 4)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = \omega^2 \mu \varepsilon E \quad — ⑥$$

Now, maxwell's third equation

$$\nabla \cdot D = \rho = 0 \quad (\text{for free space})$$

$$\nabla \cdot \varepsilon E = 0$$

$$\varepsilon(\nabla \cdot E) = 0$$

But $\varepsilon = \varepsilon_0 \cdot \varepsilon_r \neq 0$

∴ neglecting ε

So, $\nabla \cdot E = 0$

Now from eq^n ⑥, we will get

$$-\nabla^2 E = \omega^2 \mu \varepsilon E \Rightarrow \nabla^2 E = -\omega^2 \mu \varepsilon E \quad — ⑦$$

Similarly, for magnetic field intensity,

$$\nabla^2 H = -\omega^2 \mu \varepsilon H \quad - \quad (8)$$

Eqⁿ (7) & (8) are known as wave equation or Helmholtz eqⁿ which are the wave equation to propagate in free space.

→ From eqⁿ (7) & (8) :-

if we consider, EF → x direction
M.F → y direction
then, Propagation direction → z. direction

As, $E \perp H \perp$ propagation direction
So, we can modify eqⁿ (7) & (8) in unidirection
i.e.

$$\nabla^2 E_x = -\omega^2 \mu \varepsilon E_x$$

$$\nabla^2 E_y = -\omega^2 \mu \varepsilon E_y$$

$$\nabla E_z = -\omega^2 \mu \varepsilon E_z$$

Similarly,

$$\nabla^2 H_x = -\omega^2 \mu \varepsilon H_x$$

$$\nabla^2 H_y = -\omega^2 \mu \varepsilon H_y$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z$$

But, In General, wave equations can be written as,

$$\boxed{\nabla^2 E = -\omega^2 \mu \varepsilon E}$$

and

$$\boxed{\nabla^2 H = -\omega^2 \mu \varepsilon H}$$

Replacing $-\omega^2$ by $\frac{\partial^2}{\partial t^2}$ in above equations,

$$\nabla^2 E = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}, \quad \nabla^2 H = \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

where $\mu = \mu_0 \mu_r$, $\varepsilon = \varepsilon_0 \varepsilon_r$

$\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = \frac{1}{36\pi} \cdot 10^{-9}$ F/m $(\varepsilon_r = 1, \mu_r = 1$ for air$)$

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$$\therefore \mu \varepsilon = 4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}$$

$$= \frac{1}{9\times 10^{16}} = \frac{1}{(3\times 10^{8})^2} = \frac{1}{c^2}$$

where c is velocity of light.

So,
$$\boxed{\begin{aligned} \nabla^2 E &= \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \\ \nabla^2 H &= \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2} \end{aligned}}$$

⇒ These equations represent wave propagation through free space with a velocity equal to that of light.

• **Significance of Wave Equations** :- By the wave equations, we can easily understand that if source is time harmonic then the effect is a harmonic propagation in the medium.

• **Types of Waves**

① TEM (Transverse ElectroMagnetic Waves)

→ TEM waves are those whose electric and magnetic field lines are transverse to the direction of propagation.

ie. $E_z = 0$, $H_z = 0$

② TE (Transverse Electric Waves)

→ An EM wave in which the electric fields are purely transverse to the direction of propagation

ie. $E_z = 0$, $H_z \neq 0$

③ TM (Transverse Magnetic waves)

→ An EM wave in which only magnetic fields are purely transverse to the direction of propagation.

ie. $E_z \neq 0$, $H_z = 0$

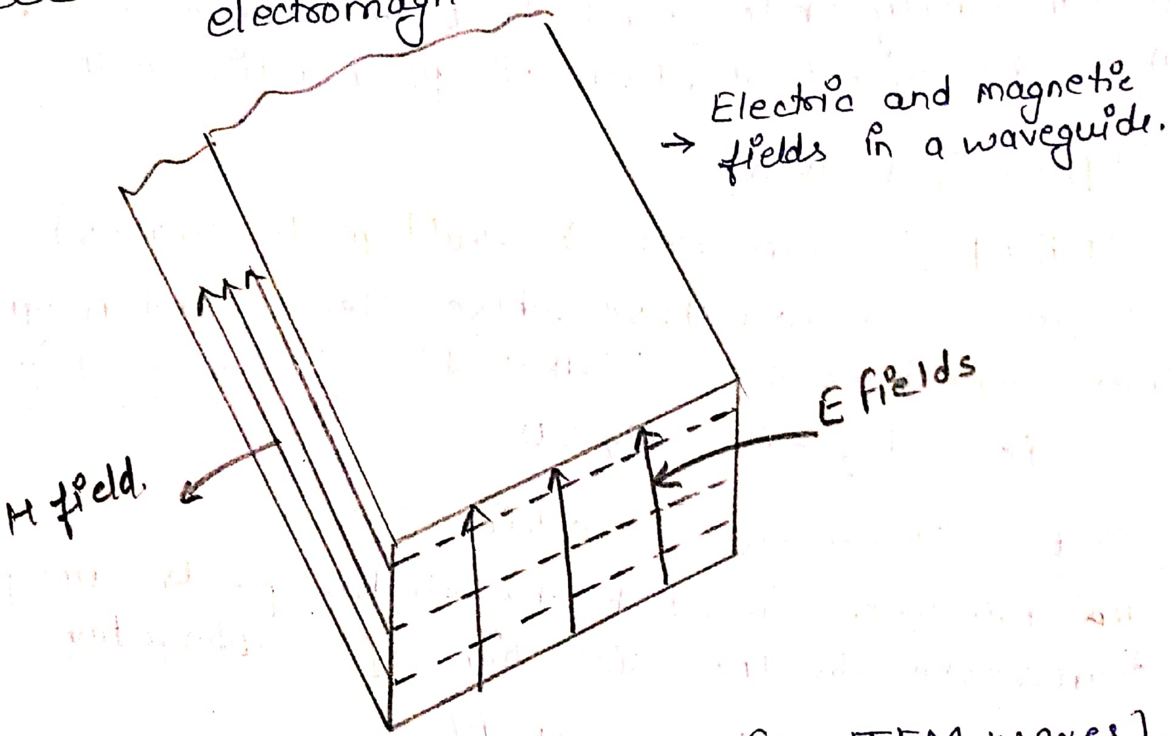
• Poynting Theorem ⇒ This theorem states that the cross product of electric field vector ($\vec{E}$) and magnetic field vector ($\vec{H}$) at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point.

i.e. $\boxed{P = \vec{E} \times \vec{H}}$

where P = poynting vector

⇒ The direction of P is perpendicular to E and H.

WAVEGUIDES :- It is a hollow pipe which guides the electromagnetic wave.



• It supports only TE and TM waves (No TEM waves).

• When the wave propagates in the waveguide, it can have infinite type of field patterns which are called "Modes."

• They are used in transmission lines at μwave freq.

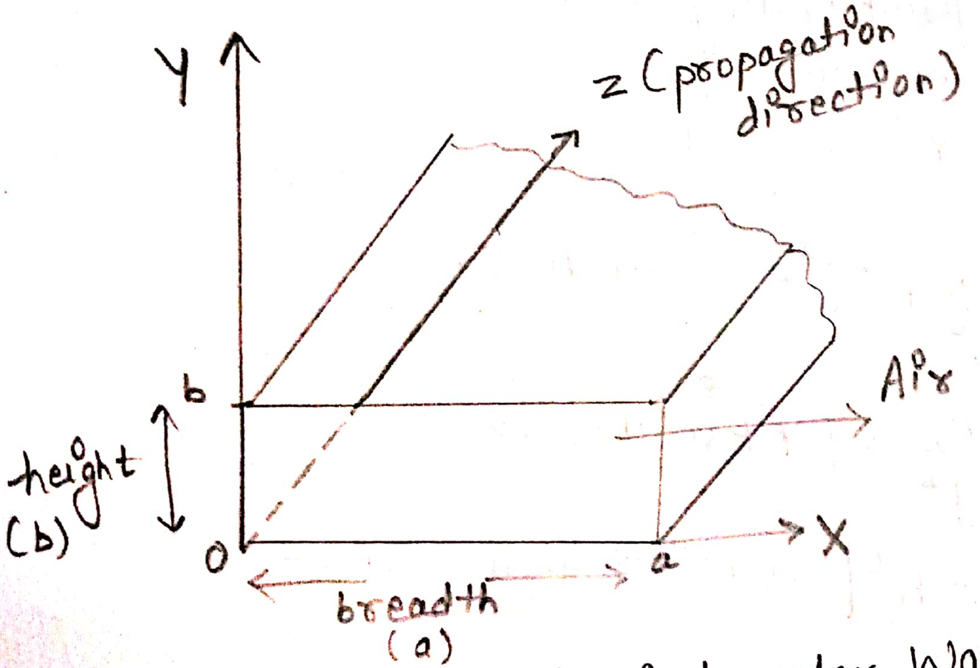
• They can be used as high pass filter radiators and antenna feed elements.

2. Waveguide consist of a hollow metallic tube of rectangular or circular shape used to give an EM wave by successive reflections from the inner walls of the tube. ⑥

- No power loss. Their radiation and dielectric loss is also negligible as waveguide are normally air ~~filled~~ filled.

· RECTANGULAR WAVEGUIDE →

→ Rectangular waveguides are one of the earliest type of transmission lines. They are used in many applications such as isolators, detectors, attenuators, couplers and slotted lines etc.



Propagation of waves in Rectangular Waveguide :-

As we know that propagation direction is perpendicular to the cross product of electric field and magnetic field.

EF dir^n × MF dir^n = Propagation direction

Consider; (X axis) (Y axis) = (Z-axis)

- For Helmholtz equation; TE and TM waves are represented by :-

z as propagation direction
$$\begin{cases} \nabla^2 H_z = -\omega^2 \mu \varepsilon H_z & \text{— TE mode } (E_z = 0) \\ \nabla^2 E_z = -\omega^2 \mu \varepsilon E_z & \text{— TM mode } (H_z = 0) \end{cases}$$

we know, $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ (Rectangular co-ordinate)

Expanding $\nabla^2 E_z$ in rectangular coordinate :

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z$$

As wave is propagating in z-dir.. So there will be a propagation constant γ.

So $\frac{\partial^2}{\partial z^2} = \gamma^2$ — (1)

- $$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

- $\boxed{\gamma^2 + \omega^2 \mu \varepsilon = h^2}$ → consider as a constant.

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad (\text{for TM}) \text{ — (2)}$$

- Similarly, $\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0$ (for TE) — (3)

- By solving eqn (2) and (3), we get E_z, H_z
- By using Maxwell's eqn, we can easily solve the other components also [ie. E_x, H_x, E_y, H_y].

From Maxwell's 1^{st} equation ; (1)

$$\nabla \times H = j\omega\varepsilon E$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

$\left(\frac{\partial}{\partial z} = -\gamma \text{ constant operator}\right)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

After expanding ;

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \quad -(4)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y \quad -(5)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad -(6)$$

Similarly, for $\nabla \times E = -j\omega\mu H$

we will get.

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \quad -(7)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = +j\omega\mu H_y \quad -(8)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad -(9)$$

Now find eqn for H_y from eqn (8) and then put the value of H_y in eqn (4).

From eqn 8, $H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x$

put it into eqn (4)

$$\frac{\partial H_z}{\partial y} - \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu} E_x = j\omega\varepsilon E_x$$

$$E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}$$

Multiplied by $j\omega\mu$;

$$E_x\left[-\omega^2\mu\varepsilon - \gamma^2\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

$$E_x\left[-(\underbrace{\gamma^2 + \omega^2\mu\varepsilon)}_{h^2\text{(Constant)}}\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y}$$

Dividing by $(-h^2)$;

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad \text{— (10)}$$

Similarly,

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \quad \text{— (11)}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \quad \text{— (12)}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \quad \text{— (13)}$$

There are the field components equations within rectangular waveguide.

- <u>Propagation of TEM modes</u> → For TEM mode; $E_z = 0$, $H_z = 0$ substituting in above equations (10), (11), (12), (13) all the field components vanished and Hence "TEM" mode in rectangular waveguide doesn't exist.

Propagation of TM waves in rectangular waveguide :-

- If propagation is in 'z' direction,

For TM wave ; $H_z = 0$, $E_z \neq 0$

From propagation wave equation 2 in RWG;

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{—} (1)$$

To solve different field components E_x, E_y, H_x, H_y, we have to use Separation of variables method.

Let us consider, $\boxed{E_z = X \cdot Y}$ → pure y components.

↓ pure x components

- where X and Y are the independent variables ;

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 (X \cdot Y)}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2}$$

and

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 (X \cdot Y)}{\partial y^2} = X \frac{\partial^2 Y}{\partial y^2}$$

- From eq^n (1), $Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + h^2 (XY) = 0$

Dividing by XY,

$$\underbrace{\frac{1}{X} \frac{\partial^2 X}{\partial x^2}}_{\text{Pure x funct}^n} + \underbrace{\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}}_{\text{Pure y funct}^n} + h^2 = 0 \quad \text{—} (2)$$

⇓

These two functions are constants.

Let

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -B^2$$

$$\frac{\partial^2 X}{\partial x^2} = -B^2 X \quad \text{—} (3)$$

Let

$$\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -A^2$$

$$\frac{\partial^2 Y}{\partial y^2} = -A^2 Y \quad \text{—} (4)$$

→ From eqⁿ ②; substitute ③ and ④ eqⁿ;

$$-B^2 - A^2 + h^2 = 0$$

$$\boxed{h^2 = A^2 + B^2} \quad - ⑤$$

• Eqⁿ ③ and ④ are partial differential eqⁿ. So solving as 2nd order differential eqⁿ;

$$X = C_1 \cos Bx + C_2 \sin Bx \quad - ⑥$$

$$Y = C_3 \cos Ay + C_4 \sin Ay \quad - ⑦$$

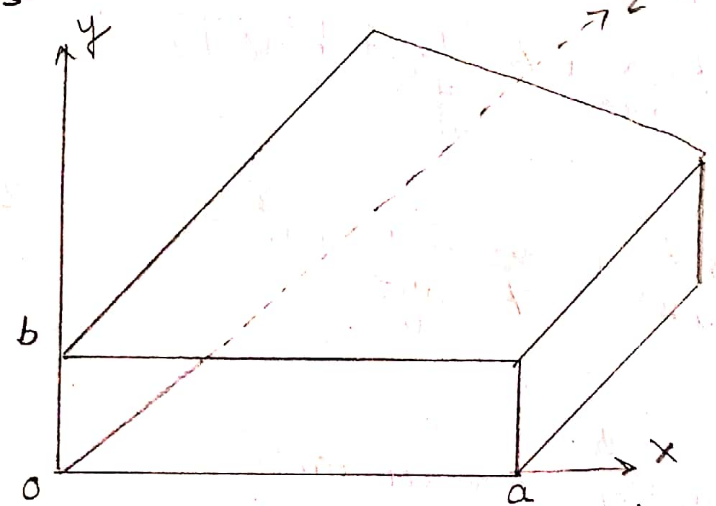
where C_1, C_2, C_3, C_4 are constants.

• Get the solution of $E_z = X \cdot Y$ By putting eqⁿ 6 and 7.

$$\therefore \; E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad - ⑧$$

• Boundary condition exist in RWG as at the surface RWG behaves as short circuit and $E_z = 0$, all along the wall of RWG. So for 4 boundaries there will be 4 boundary conditions.

→ z (Propagation direction)



• Walls are perfectly conducting sheets So, All along the boundary wall $E_z = 0$

1st Boundary Condition : (Bottom wall) ie. $y = 0$, $x \to 0$ to a

2nd Boundary Condition : (Left side wall) ie. $x = 0$, $y \to 0$ to b

3rd Boundary Condition : (Top plane wall) ie. $y = b$, $x \to 0$ to a

4th Boundary Condition : (Right side wall) ie. $x = 0$, $y \to 0$ to b

from 8^{th} eq^n; after applying 1^{st} B.C ; (9)

$E_z = 0$ at $y=0$ and $x \to 0$ to a

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$$0 = (C_1 \cos Bx + C_2 \sin Bx) \cdot C_3$$

As ; $C_1 \cos Bx + C_2 \sin Bx \neq 0$ [$\because x$ is varying]

So, $C_3 = 0$ — (9a)

So eq^n 8^{th} reduces to ;

$$\boxed{E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay)} \quad \text{— (9)}$$

- Similarly after applying 2^{nd} B.C and condition comes from 1^{st} B.C ie. eq^n (9a) in eq^n 9, we will get

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay)$$

$$0 = C_1 (C_4 \sin Ay) \qquad [x=0, y \to 0 \text{ to } b]$$

As $C_4 \sin Ay \neq 0$ or $C_4 \neq 0$ (as y is varying)

So, $C_1 = 0$ — (9b)

So, eq^n (9) reduced to ;

$$\boxed{E_z = (C_2 \sin Bx)(C_4 \sin Ay)} \quad \text{— (10)}$$

- After applying 3^{rd} B.C and (9b) in eq^n 10

$$E_z = (C_2 \sin Bx)(C_4 \sin Ay) \qquad [y=b, x \to 0 \text{ to } a]$$

$$0 = \underbrace{(C_2 \sin Bx)}_{\substack{x \text{ is varying} \\ \text{can't be zero}}} \underbrace{(C_4 \sin Ab)}_{\text{constant}}$$

So, $0 = C_4 \sin Ab$; $\{ \sin Ab = 0$ where Ab is multiple of π

$Ab = n\pi \}$

So, $\sin Ab = \sin n\pi = 0$

$$Ab = n\pi$$

$$\boxed{A = \frac{n\pi}{b}} \quad \because \text{ where } n = 0, 1, 2 \dots \rightarrow (9c)$$

- After applying 4th B.C;

$$0 = C_2 \sin Ba \cdot \underbrace{C_4 \sin Ay}_{\text{Can't be zero}} \quad (x=0, y \to 0 \text{ to } b)$$

$$0 = C_2 \sin Ba \quad ; \quad \sin Ba = 0 = \sin m\pi$$

$$Ba = m\pi$$

$$\boxed{B = \frac{m\pi}{a}} \rightarrow (9d)$$

Now, we get

$$\boxed{E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z} \cdot e^{j\omega t}}$$

where; $e^{-\gamma z}$ = propagation along z' direction

$e^{j\omega t}$ = Sinusoidal variation w.r.t 't'.

Let $C_2 C_4 = C$ (other constant)

$$\boxed{E_z = C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z} \cdot e^{j\omega t}}$$

From the previous general eqn to propagate any wave.

- From the wave propagation in RWG general eqn;

i.e. $E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

So, for TM wave, $H_z = 0$

$$E_z = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

To differentiate E_z;

$$\Rightarrow \boxed{E_x = \frac{-\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}}$$

$$\rightarrow E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \quad (TM;\ H_z = 0)$$

$$\Rightarrow \boxed{E_y = \frac{-\gamma}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}}$$

$$\rightarrow H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow \boxed{H_x = \frac{j\omega\varepsilon}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}}$$

$$\rightarrow H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow \boxed{H_y = \frac{j\omega\varepsilon}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}}$$

• **Dominant, Degenerated and Evanscent Modes :-**

→ **Dominant mode :-** Having lowest cut off freq. Known as dominant mode.

$$f_c = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}$$

for TE Modes :- ① TE_{01} (m, n) ② TE_{10} (m, n) ③ TE_{11} (m, n)

$$f_c = \frac{c}{2b} \qquad f_c = \frac{c}{2a} \qquad f_c = \frac{c}{2}\left[\frac{1}{a^2} + \frac{1}{b^2}\right]^{1/2}$$

① st and ② nd cut off freq. is lowest than TE_{11}.

→ Depending on the dimension of 'a' and 'b'

if $a > b$ then TE_{10} (dominant mode)

if $a < b$ then TE_{01} (dominant mode)

But in general, TE_{10} is considered as "Dominant Mode of RWG"

For TM modes :- ① TM_{11} ② TM_{12} ③ TM_{21}

The lowest cutoff freq. in TM modes is of TM_{11}, So the dominant mode is TM_{11}.

→ **Degenerated mode :-** When two different modes having same cutoff freq. is known as Degenerated mode.

Ex for TE_{32} and TM_{32}, cutoff freq. is same ie. $\frac{c}{2}\left[\frac{9}{a^2}+\frac{4}{b^2}\right]^{1/2}$

∴ TE_{32} and TM_{32} → degenerated mode

TE_{12} and TM_{12} → " " etc.

• **Evanescent Modes :-** When the freq. of a mode is less than the cut off freq., then wave will not propagate. This mode where no propagation of wave will occur is called "Evanescent Modes."

Cutoff frequency of RWG :-

We know that; $h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

Propagation constant → $\boxed{\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}} = \underset{\text{Attenuation constant}}{\alpha} + j\underset{\text{phase constant}}{\beta}$

⇒ At lower freq;

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

then γ = +ve only + real value

So $\gamma = \alpha$ ie. all components will be attenuated, not propagated.

- At higher freq;

$$\omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

then γ = imaginary

So, $\gamma = \beta$ ie. propagation just start.

→ "freq. at which γ just become zero is defined as Cut off freq. (threshold freq.) 'f_c'.

→ As RWG passes high freq. and stop low frequency. So it acts as HPF.

→ At exact cutoff freq, the wave oscillates b/w the guide walls, is not propagate forward. It is resonating b/w the conducting walls.

→ At $f = f_c$; $\gamma = 0$ or $\omega = \omega_c = 2\pi f_c$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon \longrightarrow \omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \qquad \left(\because \omega_c = 2\pi f_c\right)$$

Cutoff freq. of RWG.

$$\boxed{f_c = \frac{c}{2} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}} \qquad \left[\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right]$$

$\therefore$

$$\boxed{\lambda_c = \frac{c}{f_c} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}}$$

Cutoff wavelength of RWG

Phase Velocity of RWG (V_p) :- It is defined as the rate at which the wave changes its phase in terms of the guide wavelength.

i.e. $$V_p = \frac{\lambda g}{\text{unit time}} = \lambda g \cdot f = \frac{\lambda g \cdot 2\pi f}{2\pi} = \frac{2\pi f}{2\pi/\lambda g}$$

$$\Downarrow$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

where $\beta = \frac{2\pi}{\lambda g}$ (λg = guide wavelength)

Expression for V_p :- We know,

For wave propagation, $\gamma = j\beta$ ($\because$ attenuation $\alpha = 0$)

$$\therefore \quad \gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon \quad - (1)$$

at $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$\therefore \quad \omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad - (2)$$

put value of (2) in (1), we get

$$\gamma^2 = (j\beta)^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\gamma^2 = \beta^2 = \omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon$$

or $$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$\beta = \sqrt{\mu\varepsilon\,(\omega^2 - \omega_c^2)} = \sqrt{\mu\varepsilon}\,\sqrt{\omega^2 - \omega_c^2}$$

$$\therefore \quad V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\,\sqrt{\omega^2 - \omega_c^2}}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \frac{1}{\sqrt{1 - (\omega_c/\omega)^2}}$$

i.e. Phase velocity
$$\boxed{V_p = \frac{c}{\sqrt{1 - (\omega_c/\omega)^2}} \text{ or } \frac{c}{\sqrt{1 - (f_c/f)^2}}}$$

In terms of wavelength ;

$$\boxed{V_p = \frac{c}{\sqrt{1-(\lambda/\lambda_c)^2}}}$$

* **Group Velocity of RWG (Vg) :-** If there is modulation in the carrier, the modulation envelope actually travels at velocity slower than that of carrier alone which is slower than speed of light.

The velocity of modulation envelope is called group velocity (Vg). This occurs when a modulated signal travels in a waveguide.

ie. $V_g = \frac{d\omega}{d\beta}$.

Expression of Vg :- we know , $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$

$$\frac{d\beta}{d\omega} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\omega\mu\varepsilon}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$\frac{d\beta}{d\omega} = \frac{\cancel{\omega}\mu\varepsilon/\sqrt{\mu\varepsilon}}{\cancel{\omega}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

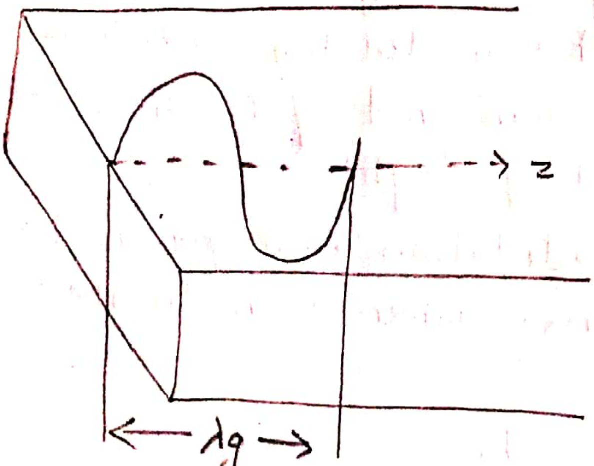
$$\frac{d\beta}{d\omega} = \frac{1}{c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

But $V_g = \frac{d\omega}{d\beta}$ $\Rightarrow \boxed{V_g = c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \text{ or } c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

Relation b/w Vp and Vg :-

$\cancel{w.r.t}$ $\boxed{V_p \cdot V_g = c^2}$

Guide Wavelength (λg) → It is defined as the distance travelled by the wave in order to undergo a phase shift of 2π radians.



It is related to the phase constant by the relation i.e.

$$\lambda g = \frac{2\pi}{\beta}$$

→ The wavelength in the wavelength is related to free space wavelength λ_0 and cutoff wavelength λ_c

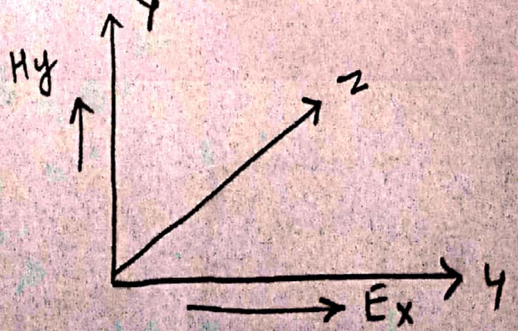
i.e. $$\frac{1}{\lambda g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

or $$\boxed{\lambda g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

→ This equation is valid for any mode in a waveguide of any cross-section.

Wave Impedance in TM and TE waves → Z_z or Z_g

→ It is the ratio of EF in one transverse direction to the strength of the MF along the other transverse direction.



$$E_z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

or $$E_z = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

For TM wave in RWG;

$$Z_z = Z_{TM} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

For TM, $H_z = 0$ and $\gamma = j\beta$

$$Z_{TM} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}}{-\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon} = \frac{\beta}{\omega\varepsilon}$$

$$Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon} \quad \left(\because \beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}\right)$$

$$Z_{TM} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$ where η = intrinsic impedance $= \sqrt{\frac{\mu}{\varepsilon}}$

$$\therefore \boxed{Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Similarly, For TE wave

$$\boxed{Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

$$Z_{TM} \cdot Z_{TE} = \eta^2$$

Power transmission in RWG :- The power transmission through a waveguide and power loss in the guide walls can be calculated by means of complex poynting theorem.

The power transmitted P_{tr}, through a waveguide given by,

$$P_{tr} = \oint P \cdot ds = \oint \frac{1}{2}(E \times H^*) \cdot ds$$

For a lossless dielectric, the time average power flows through a rectangular waveguide is

$$P_{tr} = \frac{1}{2Z_z}\int_a |E|^2 da = \frac{Z_z}{2}\int_a |H|^2 da$$

where $Z_z = \frac{E_x}{E_y} = \frac{-E_y}{H_x}$

and $|E|^2 = |E_x|^2 + |E_y|^2$

$|H|^2 = |H_x|^2 + |H_y|^2$

- For TM_{mn} mode, average power transmitted through a RWG is. (dimension a and b)

$$P_{tr} = \frac{1}{2\eta\sqrt{(1-\lambda_0/\lambda_c)^2}}\int_0^b\int_0^a |E_x|^2 + |E_y|^2 \, dx\, dy$$

For TE_{mn} mode,

$$P_{tr} = \frac{\sqrt{1-(\lambda_0/\lambda_c)^2}}{2\eta}\int_0^b\int_0^a |E_x|^2 + |E_y|^2 \, dx\, dy.$$

- Power losses in waveguide :-

We know, $\beta = \frac{2\pi}{\lambda g}$ where $\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$$\therefore \beta = \frac{2\pi}{\lambda}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\beta = j\frac{2\pi}{\lambda}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

$$\beta = j\frac{2\pi}{\lambda}\cdot\frac{f_c}{f}\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$$\beta = j\frac{2\pi f_c}{c}\sqrt{1-(f/f_c)^2}$$

As instead of propagation wave will be attenuated
($\therefore \beta = j\alpha$)

$$\therefore \alpha = \frac{2\pi f_c}{c}\sqrt{1-(f/f_c)^2}$$

In dB ; $\boxed{\alpha = \frac{54.6}{\lambda_c}\sqrt{1-(f/f_c)^2}}$ dB/ length

→ Actually this α works as stop band attenuation of the waveguide high pass filter.

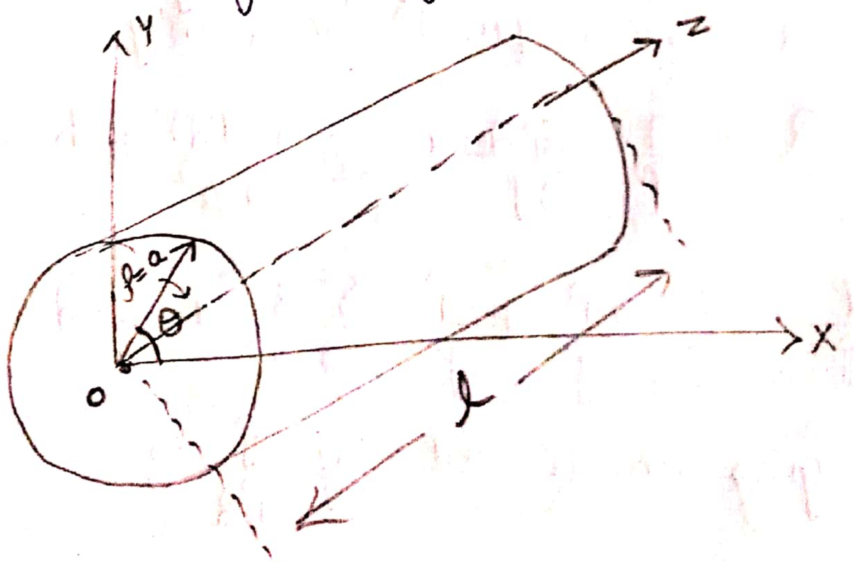
for $f > f_c$; very low loss (α)

$f < f_c$; $\alpha \uparrow\uparrow$; full reflection known as <u>Reflection loss</u>.

Circular Waveguide

:- A circular waveguide is basically a tubular, circular conductor.

→ Circular waveguide basically depends on 3 variables and circular co-ordinate geometry (ρ, ϕ, z).



Here ϕ varies from 0 to 2π, ρ varies 0 to a and l varies along z-axis.

· Propagation of TE waves in CWG:-

For TE wave $E_z = 0$ and $H_z \neq 0$

Expanding ∇ in circular co-ordinates;

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z$$

As we know; $\frac{\partial^2}{\partial z^2} = \gamma^2$ (an operator)

$$\therefore \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

$$\because (\gamma^2 + \omega^2 \mu \varepsilon) = h^2$$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + h^2 H_z = 0$$

. This differential eq^n can be solve by 'variable's separation' method.

So, Let's consider $H_z = P \cdot Q$

where P = is a funct^n of ρ only.
Q = is a funct^n of ϕ only.

$$\text{So;} \quad \frac{\partial^2 (PQ)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial (PQ)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 (PQ)}{\partial \phi^2} + h^2 PQ = 0$$

$$Q\frac{\partial^2 P}{\partial \rho^2} + \frac{Q}{\rho}\frac{\partial P}{\partial \rho} + \frac{P}{\rho^2}\frac{\partial^2 Q}{\partial \phi^2} + h^2 PQ = 0$$

Multiplying throughout by $\frac{\rho^2}{PQ}$;

$$\rightarrow \underbrace{\frac{\rho^2}{P}\frac{\partial^2 P}{\partial \rho^2}}_{(1)} + \underbrace{\frac{\rho}{P}\frac{\partial P}{\partial \rho}}_{(2)} + \underbrace{\frac{1}{Q}\frac{\partial^2 Q}{\partial \phi^2}}_{(3)} + \underbrace{h^2\rho^2}_{(4)} = 0$$

All these (1) (2) (3) and (4) parts are the pure function of ρ and ϕ only.

So, let $\frac{1}{Q} \cdot \frac{\partial^2 Q}{\partial \phi^2} = -n^2$ (pure function of ϕ)

Divide by P :- (we will get)

$$\boxed{\rho^2 \frac{\partial^2 P}{\partial \rho^2} + \rho \frac{\partial P}{\partial \rho} + (\rho^2 h^2 - n^2) P = 0}$$

By using Bessel function, we solve the value of P and Q ;

$$P = C_n J_n(\rho h)$$

$$Q = \underbrace{\sqrt{A_n^2 + B_n^2}}_{C_n'} \cos \underbrace{\left(n\phi + \tan^{-1}\frac{A_n}{B_n}\right)}_{n\phi}$$

Now, $H_z = P \cdot Q = C_n J_n(\rho h)\, C_1' \cos n\phi$

$$H_z = C_0 J_n(\rho h) \cos(n\phi) \qquad [\because C_0 = C_n C_n']$$

If we consider a sinusoidal variation along 'z'

$$\boxed{H_z = C_0 J_n(\rho h) \cos(n\phi)\, e^{-\gamma z}} \quad \text{— (1)}$$

<u>Boundary Condition</u> → We know all along the surface area of the circular W/G at $\rho = a$

$E_\phi = 0$ for all values of ϕ varying b/w 0 to 2π.

i.e. $\left.\frac{\partial H_z}{\partial \rho}\right|_{\rho = a} = 0$

→ From ① eqⁿ: $C_0 J_n(\rho h) \cos n\phi \cdot e^{-\gamma z}\Big|_{\rho=a} = 0$

$$J_n'(ah) = 0$$

→ Bessel function $J_n'(ah)$ denotes prime differentiation with respect to ah, so m^{th} root of this eqⁿ is denoted by P'_{nm}, which are the eigen values;

$$\{ P'_{nm} = ah \} \text{ for } TE_{nm}$$

$$h = \frac{P'_{nm}}{a}$$ → This represent all possible solⁿ of H_z for TE_{nm} in CWG.

here n represents → no. of full cycles of field variation in one revolution through 2π radians of ϕ.

m represents → No. of zeroes of $E\phi$ i.e. $J_n'(ah)$ along the radial of the waveguide but the zero on the axis is excluded if it exists.

• As in RWG, in CWG; we can calculate all field components which are analogous to;

$E\rho$	$E\phi$	E_z	$H\rho$	$H\phi$	H_z
↓	↓	↓	↓	↓	↓
E_x	E_y	0	H_x	H_y	0
		↓			↓
		TE mode			TM mode

In CWG, field components are given by

$$E_\rho = \frac{-j\omega\mu}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$E_z = 0$$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$H_z = C_0 \cdot J_n(\rho h)\cos n\phi\, e^{-\gamma z}$$

- Substituting for H_z in above equations with $h = P'_{nm}/a$, Complete field equations for TE$_{nm}$ modes in circular WG are given by ;

$$E_\rho = C_{0\rho} J_n\left(\frac{P'_{nm}}{a}\rho\right)\sin n\phi\, e^{-\gamma z}$$

$$E_\phi = C_{0\phi} J'_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

$$E_z = 0$$

$$H_\rho = -\frac{C_{0\phi}}{Z_z} J'_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

$$H_\phi = \frac{C_{0\phi}}{Z_z} J_n\left(\frac{P'_{nm}}{a}\rho\right)\sin n\phi\, e^{-\gamma z}$$

$$H_z = C_0 J_n\left(\frac{P'_{nm}}{a}\rho\right)\cos n\phi\, e^{-\gamma z}$$

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• Dominant mode in CWG → TE_{11} is dominant mode in CWG

⇒ TM_{01} is dominant mode in CWG

• Cutoff wavelength in CWG →

For TE ; $\lambda_c = \dfrac{2\pi a}{P'_{nm}}$

For TM ; $\lambda_c = \dfrac{2\pi a}{P_{nm}}$

⊙ INTRODUCTION TO PLANAR TRANSMISSION LINES :-

• The conventional open wire T.L are not suitable for μwave transmission because of radiation loss.

• Due to bulky structure of co-axial.

• At μwave freq, the following TL are employed

① Single conductor lines (ie. waveguide)

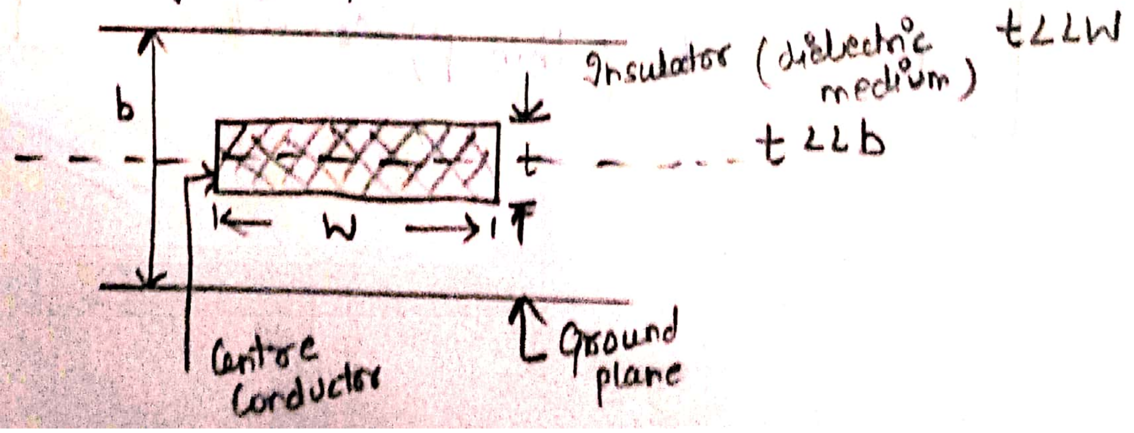
a) RWG   b) CWG   c) Coplanar lines etc.

② Multiconductor lines (planar TL)

a) Coaxial lines   b) Strip lines   c) Microstrip lines

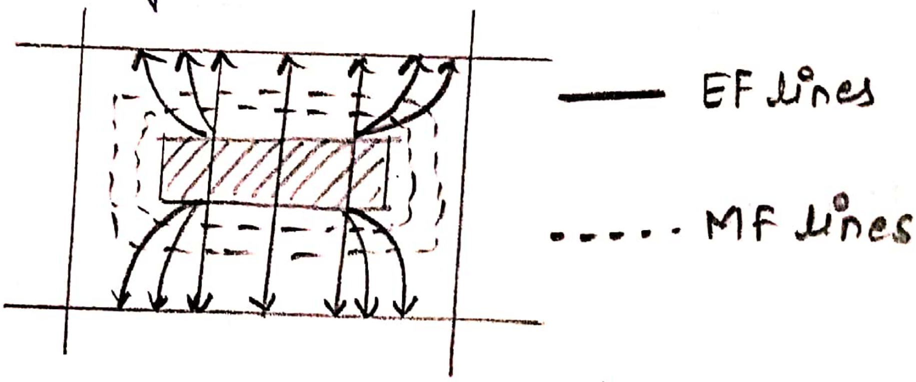
d) Parallel strip lines   e) coplanar lines   f) Coupled striplines.

• Strip lines → It is a 3 conductor T.L used fat freq. b/w 100 MHz to 100 GHz.



Strip line consist of a central thin conducting strip of width (W) and thickness t. There are 2 ground plates. There is a homogenous low loss dielectric b/w the two ground planes. The centric conductor is embedded in dielectric medium. (17)

- Strip configuration may be either symmetrical or asymmetrical. The symmetrical structure is called Strip line while the asymmetrical arrangement is denoded as Microstrip.
- The voltage is applied b/w centre strip and pair of ground planes. The current flows down the centre strip and returns via ground planes. Although the structure is open at both end, even it is a non radiating TL.
- Dominant mode for SL = TEM mode.



- The field lines are concentrated near the centre strip and decay exponentially towards ground planes. which is at zero potential.

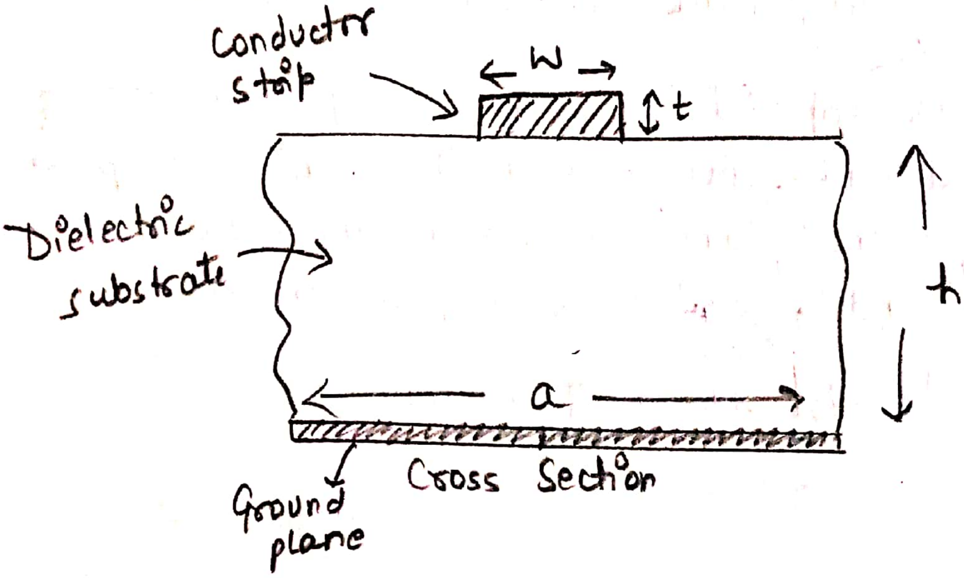
→ Phase velocity $V_p = \frac{V_c}{\sqrt{\varepsilon_r}}$ → velocity of EM wave in free space

↓ Relative Permittivity

→ Impedance, $Z_0 = \frac{1}{V_p \cdot C}$

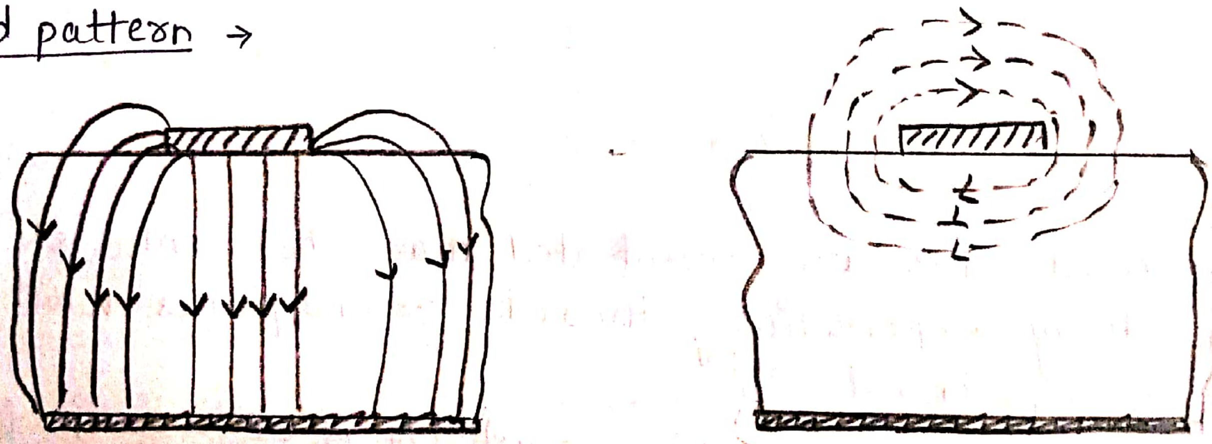
↓ Shunt Capacitance of unit length of line.

• Microstrip Lines → The μstrip line is also called as open strip line. Basically, it is an unsymmetric strip line. The bottom face is a ~~metallic~~ metallic ground. The top face has a thin conducting strip of width 'w' and thickness 't'. They are separated by a low loss dielectric material. So, the upper ground plane is not present in this microstrip as compared to strip line.



Cross Section

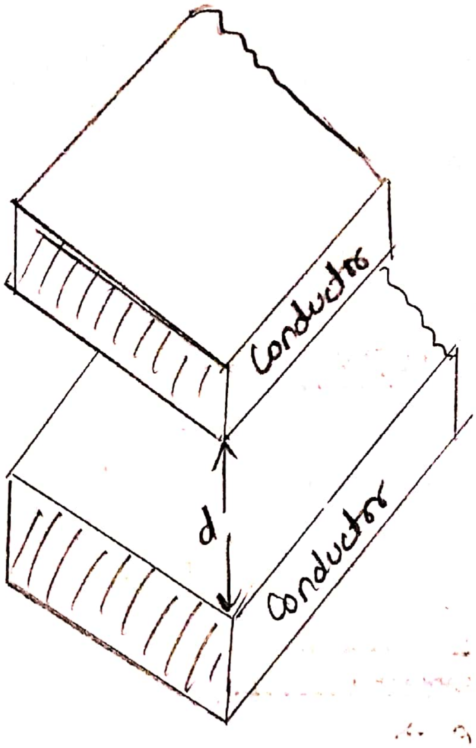
• Field pattern →


E Fields   H Fields

• Types of μStrip lines ; 1) Parallel SL
2) Coplanar SL

Note :-
* The EM field lines remains partially in air and partially in dielectric substrate. Therefore, Microstrip lines does not support pure TEM mode. It supports "quasi TEM mode" of propagation.

5. **Parallel Strip line** → It consist of 2 perfectly parallel strips separated by a perfect dielectric slab of uniform thickness (d). The plate width is 'W' and relative dielectric constant of slab is 'ε_r'.



- As 2 conducting in middle of dielectric. So, it behaves like capacitor.

$$\boxed{C = \frac{\varepsilon_d W}{d}}$$

- The inductance along the 2 conducting strips will be;

$$\boxed{L = \frac{\mu d}{W}}$$

μ → permeability of conductor

- Characteristic Impedance;

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

- Attenuation Constant;

$$\boxed{\alpha = \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]}$$

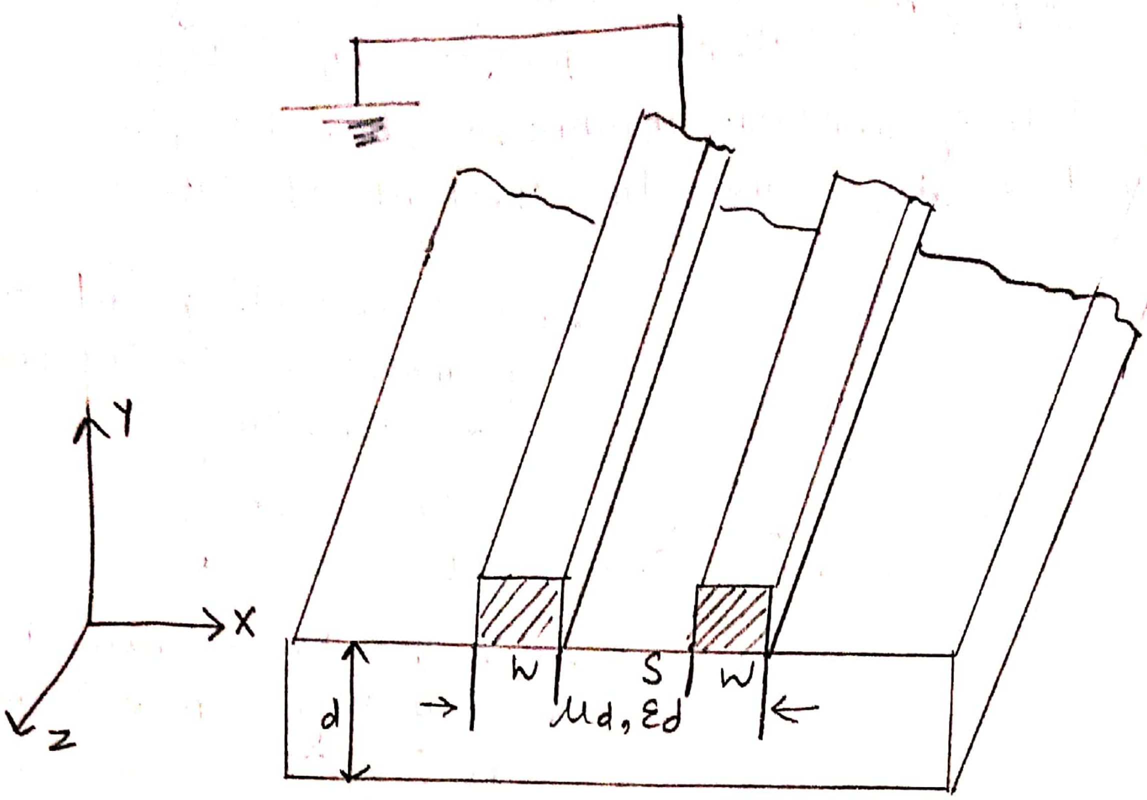
(where $G = \frac{1}{R}$ ↓ transconductance)

- Phase Constant;

$$\boxed{\beta = \omega\sqrt{LC}}$$

2) **Coplanar SL** → It consists of 2 conducting strips on one surface (⇓ substrate surface) with one strip grounded.

- It has advantage over parallel S.L because its 2 strips are on the same substrate surface for convenient connections.



• Coplanar Strip line

• Losses in Microstrip line →

① Dielectric loss : Dielectric materials have some conductivity, of course, this is very small. But due to this small conductivity, a dielectric attenuation α_d will be produced.

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

② Ohmic loss : Due to non perfect conductors.

→ Ohmic losses arise due to the current on the strip.

→ Conducting attenuation constant is defined as;

$$\boxed{\alpha_c = \frac{8.68 R_s}{Z_0 W}}$$

R_s → surface skin R.

W → width of ustrip

Z_0 → charac. impedance

Radiation Loss : Due to thin open structure and any current discontinuities in strip conductor. Radiation loss depends on its geometry and dielectric constant.

• Advantages of Microstrip lines

1. Better connection features.
2. Compact in size, light weight, low cost
3. Use of thin, high dielectric materials reduces the radiation loss.
4. Easier fabrication because the entire pattern can be deposited on single dielectric substrate.

• Disadvantages of Microstrip Lines

1. As structure is open, so may be higher radiation loss or interference due to nearby conductors.
2. There is always discontinuity in EF and MF.
3. Higher attenuation.
4. Pure TEM doesn't exist.