CHAPTER 3

CIRCUIT ANALYSIS BY CLASSICAL METHOD

3.1. INTRODUCTION

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transition period during which the branch currents and element voltages change from their former values to new ones. This period is called the *transient*. After the transient has passed, the circuit is said to be in the *steady state.*

Now, the linear differential equation that describes the circuit will have two parts to its solution, the complementary function corresponds to the transient and the particular solution corresponds to the steady-state.

The v-i relation for an inductor or a capacitor is a differential. A circuit containing an inductor L or a capacitor C, and resistors will have current and voltage variables given by differential equations of the same form. It is a linear first order differential equation, with constant coefficients when the values of R, L and C are constant. L and C are storage elements. Circuits have two storage elements like one L and one C are referred to as second order circuits.

Therefore, the series or parallel combinations of R and L or R and C are first order circuits, and RLC in series and RLC in parallel are typical second order circuits.

The circuit changes are assumed to occur at time $t = 0$, and represented by a switch. The switch may be supposed to close/$_{\text{on}}$ and open/$_{\text{off}}$ at $t = 0$ as shown in figure 3.1 (a) or (b) respectively, for convenience, it is defined that :

$t = 0^-$, the instant prior to $t = 0$ and

$t = 0^+$, the instant immediately after switching.

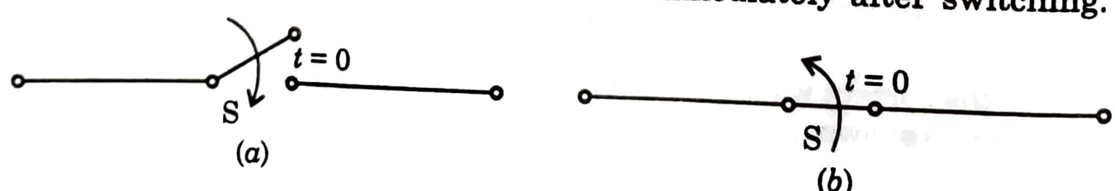


Fig. 3.1. Switch S is (a) closed at t = 0, (b) opened at t = 0.

Switching on or off an element or source in a circuit at $t = 0$ will not disturb the storage element so that $i_L(0^-) = i_L(0^+)$ and $v_c(0^-) = v_c(0^+)$. This provides a basis for constructing an equivalent circuit for a charged capacitor voltage (V_0) and current (I_0) carrying inductor.

The voltage-current relationships of the three circuit elements R, L and C are given in Table 3.1.

Table 3.1. *Relationship for the Parameters*

Parameter	Basic Relationship	Voltage-current Relationships	Energy
R $\left(G = \frac{1}{R}\right)$	$v(t) = R\,i(t)$	$v_R(t) = R \cdot i_R(t)$ $i_R(t) = G\,v_R(t)$	$W_R(t) = \int_{-\infty}^{t} v_R(t)\, i_R(t)\, dt$
L	$\Psi(t) = Li(t)$	$v_L(t) = L\frac{di_L(t)}{dt}$ $i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\, dt$ or $i_L(t) = \frac{1}{L}\int_{0}^{t} v_L(t)\, dt + i_L(0^+)$	$W_L(t) = \frac{1}{2}Li_L^2(t)$
C	$q(t) = Cv(t)$	$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\, dt$ or $v_c(t) = \frac{1}{C}\int_{0}^{t} i_c(t)\, dt + v_c(0^+)$ $i_c(t) = C\frac{dv_c(t)}{dt}$	$W_C(t) = \frac{1}{2}Cv_c^2(t)$

3.2. DIFFERENTIAL EQUATIONS

To study the transients in electric circuits, it is necessary to be familiar with the mathematical concept of differential equations and the solution techniques.

The order of the differential equation represents the highest derivative involved and is equal to the number of energy storing elements. As the differential equation contains no partial derivatives, it is considered as ordinary.

Hence, in this book, the differential equations that are formed for transient analysis will be linear, ordinary, differential equations (LODE) with constant coefficients.

Type I (First Order Homogeneous Differential Equation)

$$\frac{dy(t)}{dt} + Py(t) = 0 \quad \Rightarrow y(t) = Ke^{-Pt}.$$

(where P is any constant.)

$$\frac{dy(t)}{y(t)} = -P\,dt$$

On integrating,

$$ln\ y(t) = -Pt + K'$$

Take $\quad K' = ln\ K$

$$ln\ y(t) = ln\ e^{-Pt} + ln\ K$$

or $\quad ln\ y(t) = ln(K\,e^{-Pt})$

with the equation in this form, the antilogarithm may be taken to give a general solution

$$y(t) = K e^{-Pt}$$

where K is a constant.

If the constant K is evaluated, the solution is a particular solution.

Type II (First Order Non-homogeneous Differential Equation)

$$\frac{dy(t)}{dt} + Py(t) = Q$$

where P is a constant and Q may be a function of independent variable t or a constant.

The equation is not altered if every term is multiplied by the e^{Pt}. *i.e.*

$$e^{Pt}.\frac{dy(t)}{dt} + e^{Pt}.Py(t) = Q e^{Pt}$$

since $d(x \cdot y) = x\,dy + y\,dx$, so left hand side of above equation is equal to $\frac{d}{dt}[y(t) \cdot e^{Pt}]$. Thus we have

$$\frac{d}{dt}[y(t) \cdot e^{Pt}] = Q e^{Pt}$$

Thus equation may be integrated to give

$$y(t) \cdot e^{Pt} = \int Q e^{Pt} dt + K$$

or

$$\boxed{y(t) = e^{-Pt} \int Q.e^{Pt} dt + K.e^{-Pt}}$$

where K is a constant.

The first term of above solution is known as the *particular integral*; the second is known as the *complementary function*. Note that the particular integral does not contain the arbitrary constant, and the complementary function does not depend on the forcing function Q.

If Q is a constant, then

$$y(t) = e^{-Pt}.Q.\frac{e^{Pt}}{P} + K e^{-Pt})$$

$$\boxed{y(t) = \frac{Q}{P} + K e^{-Pt}}$$

Type III (Second Order Differential Equation)

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + Cy(t) = 0$$

The general solution of the above second order differential equation is

$$y(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

where K_1 and K_2 are constants.

And, p_1 and p_2 are the roots of the quadratic equation $Ap^2 + Bp + C = 0$, and are given by

$$p_1, p_2 = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC}$$

If $p_1 = p_2$, *i.e.*, roots of the quadratic equation are repeated, then the general solution of the given second order differential equation is given as

$$y(t) = K_1 \cdot e^{p_1 t} + K_2 \cdot t \cdot e^{p_1 t}$$

3.3. INITIAL CONDITIONS IN CIRCUITS

We need initial (or boundary) conditions to evaluate the arbitrary constants in the general solution of differential equations. The number of initial conditions required is equal to the order of the differential equation for a unique solution.

Procedure to Evaluate the Initial Conditions

There is no set procedure to follow in determining the initial conditions. First step is to draw the equivalent circuit at $t = 0^+$ based on the element given in Table 3.2. The next step is to evaluate the initial values of voltages and currents of all the branches. After this, the derivatives at $t = 0^+$ are evaluated.

Initial values of current and voltage may be found directly from a study of the network schematic. For each element in the network, we must determine just what will happen when the switching action takes place. From this analysis, a new schematic of an equivalent network for $t = 0^+$ may be constructed according to these rules :

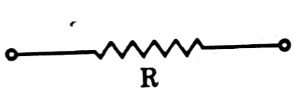
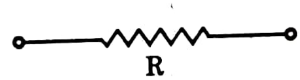
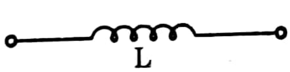
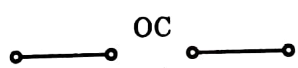
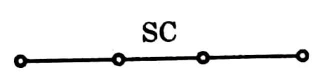
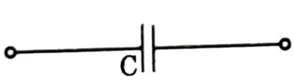
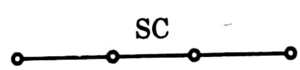
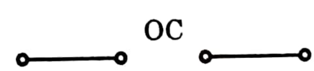
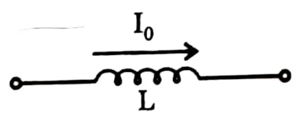
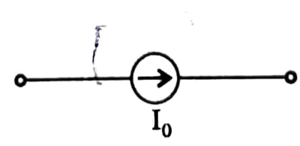
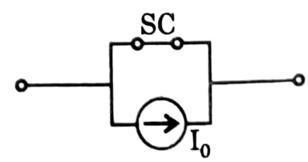
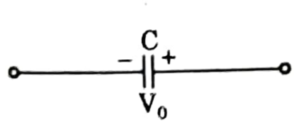
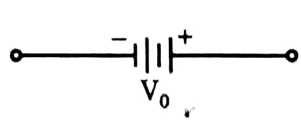
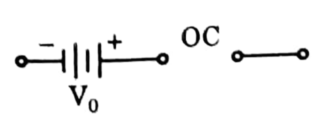
(i) Replace all inductors with open circuits or with current generators having the value of current flowing at $t = 0^+$.

(ii) Replace all capacitors with short circuits or with voltage sources having the value $V_0 = \frac{q_0}{C}$ if there is an initial charge, q_0.

(iii) Resistors are left in the network without change.

The equivalent circuit for the three parameters (R, L and C) at $t = 0^+$ and $t = \infty$ are shown in Table 3.2.

Table 3.2. The Equivalent circuit of the parameter

Element with initial conditions	Equivalent circuit at $t = 0^+$	Equivalent circuit at $t = \infty$
 R	 R	 R
 L	 OC	 SC
 C	 SC	 OC
 I_0, L	 I_0	 SC, I_0
 C, V_0	 V_0	 V_0, OC
OC - Open circuit		SC - Short circuit

3.5. TRANSIENT RESPONSE OF SERIES R-L-C CIRCUIT HAVING DC EXCITATION

Example 3.1. Consider the RLC series circuit shown in figure 3.2. V_s = 2V; $R = 6\Omega$; $L = 2H$; $C = 0.25F$. Determine $i(0^+)$; $\frac{di}{dt}(0^+)$, $\frac{d^2i}{dt^2}(0^+)$ and $i(t)$.

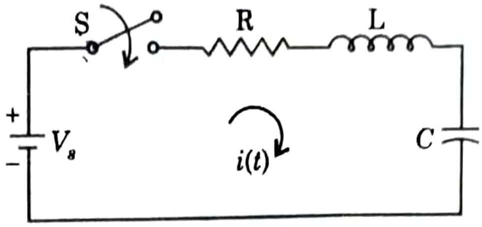


Fig. 3.2.

Solution : The differential equation for the current in the circuit of figure 3.2 is given by Kirchhoff's law as

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)\,dt = V_s \qquad ...(1)$$

Differentiating and using numerical values for R, L and C gives

$$2\frac{d^2i(t)}{dt^2} + 6\frac{di(t)}{dt} + \frac{1}{0.25}i(t) = 0$$

or

$$\frac{d^2i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 0 \qquad ...(2)$$

Substituting p^2 for $\frac{d^2i(t)}{dt^2}$ and p for $\frac{di(t)}{dt}$; thus

$$p^2 + 3p + 2 = 0$$

This equation has the roots $p_1 = -1$ and $p_2 = -2$, so that the general solution is

$$i(t) = K_1 e^{-t} + K_2 e^{-2t} \qquad ...(3)$$

The constants K_1 and K_2 can be evaluated for a specific problem by a knowledge of the initial conditions.

If the switch S is closed at $t = 0$, then

$$i(0^+) = 0 \qquad ...(4)$$

(because current cannot change instantaneously in the inductor or inductor behaves as a open circuit at $t = 0^+$)

In equation (1), the second and third voltage terms are zero at the instant of switching, $Ri(0^+)$ being zero becaue $i(0^+) = 0$ and $\frac{1}{C}\int_{-\infty}^{0^+} i\,dt$ being zero because it is the initial voltage across the capacitor. Hence

$$\frac{di}{dt}(0^+) = \frac{V_s}{L} = \frac{2}{2}$$

or

$$\frac{di}{dt}(0^+) = 1 \text{ A/sec} \qquad ...(5)$$

From equation (2),

$$\frac{d^2i}{dt^2}(0^+) + 3\frac{di}{dt}(0^+) + 2i(0^+) = 0$$

i.e.

$$\frac{d^2i}{dt^2}(0^+) = -3 \times 1 - 2 \times 0$$

or

$$\frac{d^2i}{dt^2}(0^+) = -3 \text{ A/sec}^2 \qquad ...(6)$$

The above two initial conditions (4) and (5), substituted into the general solution, equation (3), give the equations

$$K_1 + K_2 = 0 \quad \text{and} \quad -K_1 - 2K_2 = 1$$

The solution of these equations is $K_1 = 1$, and $K_2 = -1$, hence the particular solution is

$$i(t) = e^{-t} - e^{-2t} \text{ A}$$

A plot of the separate parts and their combination is shown in figure 3.3.

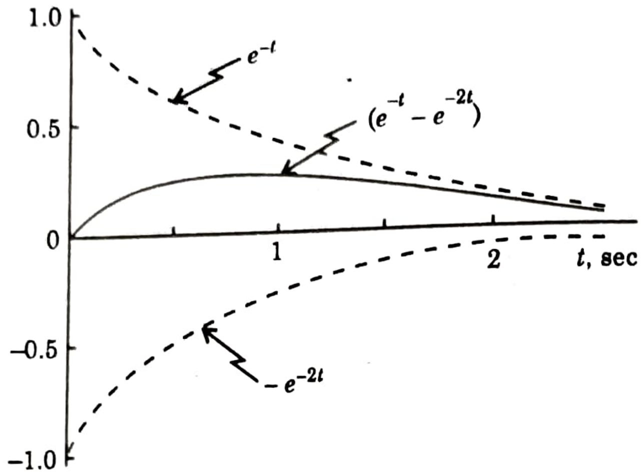


Fig. 3.3. Response including the two component parts that sum to give the response.

Example 3.2. Consider the RLC parallel circuit shown in figure 3.4. $I_s = 2$A; $R = \frac{1}{16}\Omega$; $L = \frac{1}{16}$H ; C = 4F. Determine $v(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2v}{dt^2}(0^+)$ and $v(t)$.

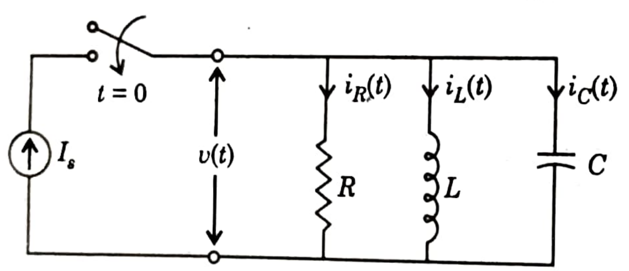


Fig. 3.4.

Solution : The circuit equation by *KCL* is

$$I_s = i_R(t) + i_L(t) + i_C(t)$$

or
$$I_s = \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + C\frac{dv(t)}{dt} \qquad ...(1)$$

Differentiating and using numerical values for *R*, *L* and *C* gives

$$0 = 16\frac{dv(t)}{dt} + 16v(t) + 4\frac{d^2v(t)}{dt^2}$$

or
$$\frac{d^2v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = 0 \qquad ...(2)$$

Substituting p^2 for $\frac{d^2v(t)}{dt^2}$ and p for $\frac{dv(t)}{dt}$; thus

$$p^2 + 4p + 4 = 0$$

This equation has the repeated roots $p_{1,2} = -2$

Thus, the general solution to our problem with repeated roots

$$v(t) = K_1 e^{-2t} + K_2 t e^{-2t} \quad ...(3)$$

To obtain a particular solution for this problem will require knowledge of two initial conditions. From the circuit of figure 3.4, $v(0^+)$ must equal zero, since the capacitor acts as a short circuit at the initial instant. *i.e.*,

$$v(0^+) = 0 \quad ...(4)$$

In equation (1), the first and second current terms are zero at the instant of switching, because $v(0^+) = 0$ and there is no current in the inductor at the initial instant. Hence

$$\frac{dv}{dt}(0^+) = \frac{I_s}{C} = \frac{2}{4}$$

or
$$\frac{dv}{dt}(0^+) = \frac{1}{2} \text{ V/sec} \quad ...(5)$$

From equation (2),

$$\frac{d^2v}{dt^2}(0^+)\, 4.\frac{1}{2} + 4.0 = 0$$

or
$$\frac{d^2v}{dt^2}(0^+) = -\,2 \text{ V/sec}^2 \quad ...(6)$$

The above two initial conditions (4) and (5), substituted into equation (3), give the equations

$$K_1 = 0 \quad \text{and} \quad -2K_1 + K_2 = \frac{1}{2} \quad \text{or} \quad K_2 = \frac{1}{2}$$

Therefore, the particular solution of our problem is

$$v(t) = \frac{1}{2}\, t \cdot e^{-2t} \text{ V} \quad ...(7)$$

A plot of this solution is shown in figure 3.5.

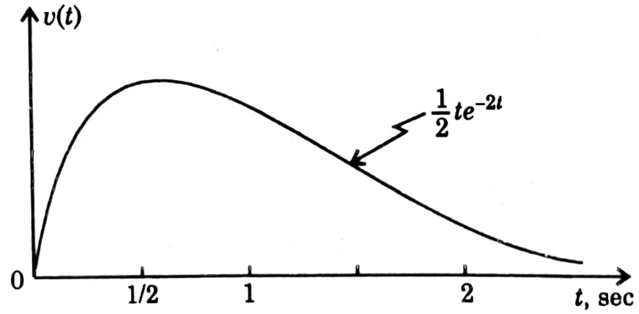


Fig. 3.5. Voltage response of network of Figure 3.4 as given by equation (7).

3.6. TRANSIENT RESPONSE OF SERIES R-L CIRCUIT HAVING DC EXCITATION

Example 3.3. Consider a series *R-L* circuit, as shown in figure 3.6. The switch S is closed at time $t = 0$. Find the current $i(t)$ through and voltage across the resistor and inductor.

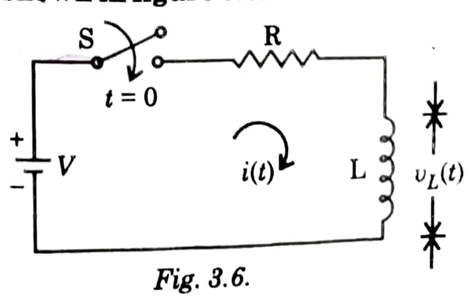


Fig. 3.6.

Solution : Applying *KVL*,

$$L\frac{di(t)}{dt} + Ri(t) = V$$

or
$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{V}{L}$$

General solution of this differential equation is given as

$$i(t) = \frac{V}{R} + K\, e^{-\frac{R}{L}t}$$

Since inductor behaves as a open circuit at switching

$$\therefore \qquad i(0^+) = 0$$

$$0 = \frac{V}{R} + K$$

or
$$K = -\frac{V}{R}$$

Therefore,
$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t} = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

And voltage across the resistor and inductor are given as

$$v_R(t) = i(t)\cdot R$$

or
$$v_R(t) = V\left(1 - e^{-\frac{R}{L}t}\right)$$

$$v_L(t) = L\frac{di(t)}{dt} \qquad [\text{or } V - v_R(t)]$$

$$= L.\frac{V}{R}\left[0 - \left(\frac{-R}{L}\right) e^{-\frac{R}{L}t}\right]$$

or
$$v_L(t) = Ve^{-\frac{R}{L}t}$$

at $t = 0$; $i(t) = 0$ and $v_L(t) = V$, $v_R(t) = 0$

at $t = \infty$; $i(t) = \frac{V}{R}$ and $v_L(t) = 0$, $v_R(t) = V$

at $t = \frac{L}{R} = \tau$; $i(t) = \frac{V}{R}(1 - e^{-1}) = 0.632\frac{V}{R}$

and $v_L(t) = V.e^{-1} = 0.368\ V$, $v_R(t) = 0.632\ V$

These $i(t)$ and $v_R(t)$ and $v_L(t)$ are plotted in figure 3.7(*a*) and (*b*).

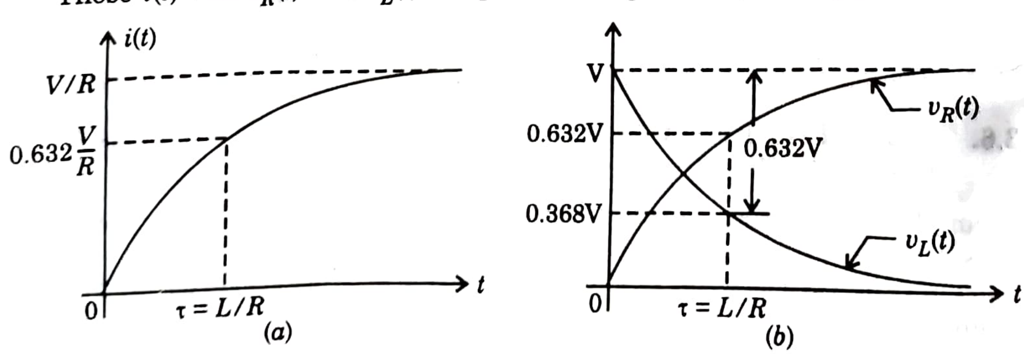


Fig. 3.7(a) for i(t) and (b) for $v_R(t)$ and $v_L(t)$.

Note : $\tau = \frac{L}{R}$ is known as the time constant of the circuit and is defined as the interval after which current or voltage changes 63.2% of its total change.

Let us now analyse another transient condition of the *R-L* circuit as the circuit reaches at steady state (at $t = \infty$) and suddenly the voltage is withdrawn by opening the switch *S* and throwing it to *S'* as shown in figure 3.7 (c) at $t = 0$.

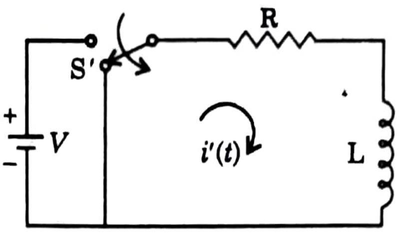


Fig. 3.7(c).

Then

$$L\frac{di'(t)}{dt} + Ri'(t) = 0$$

General solution of this differential equation is given as

$$i'(t) = Ke^{-\frac{R}{L}t}$$

However, at $t = 0^+$ the inductor keep the steady state current $i'(0^+) = i(\infty) = \frac{V}{R}$

or $$\frac{V}{R} = K'e^0 \text{ or } K' = \frac{V}{R}$$

Therefore, $$i'(t) = \frac{V}{R}e^{-\frac{R}{L}t}$$

The corresponding voltages across the resistor and inductor are then given as

$$v'_R(t) = i'(t)\cdot R = Ve^{-\frac{R}{L}t}$$

and $$v'_L(t) = L\frac{di'(t)}{dt} = -Ve^{-\frac{R}{L}t}$$

[As $v'_R(t) + v'_L(t) = 0$]

These $i'(t)$ and $v'_R(t)$ and $v'_L(t)$ are plotted in figure 3.7 (*d*) and (*e*).

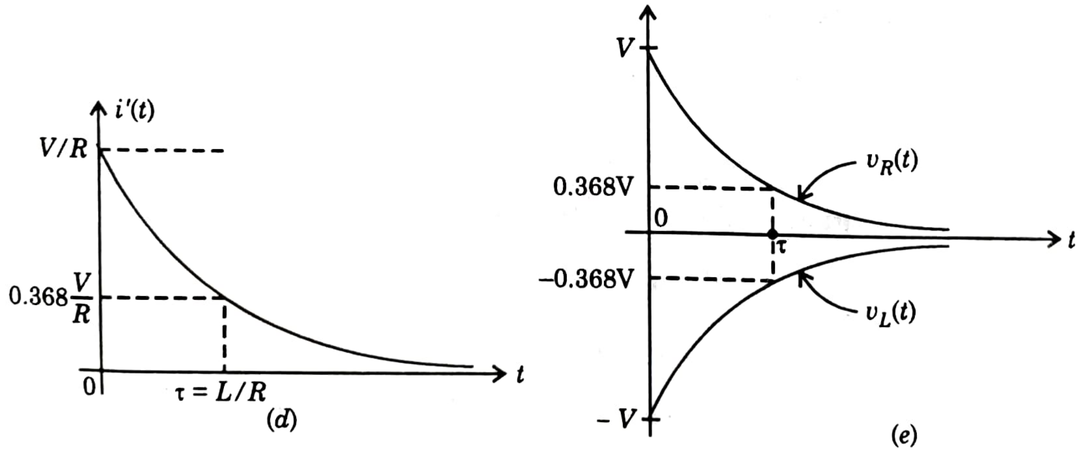


Fig. 3.7 (d) for $i'(t)$ and (e) for $v'_R(t)$ and $v'_L(t)$.

Example 3.4. A d.c. voltage of 200 V is suddenly applied to a series L-R circuit having $R = 20\Omega$ and inductance 0.2 H. Determine the voltage drop across the inductor at the instant of switching on and 0.02 sec later.

Solution : Since inductor initially behaves as a open circuit at $t = 0^+$, then $i(0^+) = 0$

Hence, at instant of switching, the voltage drop across inductor is **200 V**.

After the instant of switching the current is given by

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{200}{20}\left(1 - e^{-\frac{20}{0.2}t}\right) = 10\,(1 - e^{-100t}).$$

at $$t = 0.02 \text{ sec},\ i(t) = 10(1 - e^{-100 \times 0.02}) = 8.646\text{A}$$

$$Ri(t) + L\frac{di(t)}{dt} = 200$$

or $$L\frac{di(t)}{dt} = 200 - Ri(t)$$

The voltage across the inductor after lapse of 0.02 sec from switching is given as

$$L\frac{di(0.02)}{dt} = 200 - 20 \times 8.646 = 27\text{V}$$

3.7. TRANSIENT RESPONSE OF SERIES R-C CIRCUIT HAVING DC EXCITATION

Example 3.5. Consider a series R-C circuit, as shown in figure 3.8. The switch S is closed at time $t = 0$. Find the current $i(t)$ through and voltage across the resistor and capacitor.

Solution : By Kirchhoff's voltage law,

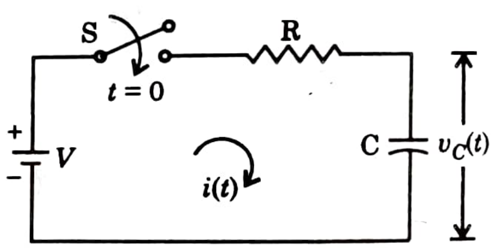


Fig. 3.8.

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = V$$

or $$Ri(t) + \frac{1}{C}\int_{0}^{t} i(t)dt + v_C(0^+) = V$$

Assume, Initially, capacitor was uncharged, *i.e.*,

∴ $$v_C(0^+) = 0$$

Differentiating, we get

$$R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$$

or $$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0$$

General solution of this differential equation is

$$i(t) = K \cdot e^{-\frac{1}{RC}t}$$

at $$t = 0^+,\ i(0^+) = \frac{V}{R}$$

(Since capacitor behaves as a short circuit at switching)

or $$\frac{V}{R} = K \cdot e^0 = K$$

Therefore, $$i(t) = \frac{V}{R}e^{-\frac{1}{RC}t}$$

And voltage across the resistor and capacitor are

$$v_R(t) = i(t) \cdot R$$

or $$v_R(t) = Ve^{-\frac{1}{RC}t}$$

$$v_C(t) = \frac{1}{C}\int_0^t i(t)dt \qquad [\text{or } V - v_R(t)]$$

$$= \frac{1}{C}\cdot\frac{V}{R}\cdot\left(e^{-\frac{1}{RC}t}\right)\Bigg|_0^t (-RC) = -V\cdot e^{-\frac{1}{RC}t}\Bigg|_0^t = -V\left(e^{-\frac{1}{RC}t} - 1\right)$$

or $$v_C(t) = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

at $t = 0$; $i(t) = \frac{V}{R}$ and $v_C(t) = 0$, $v_R(t) = V$

at $t = \infty$; $i(t) = 0$ and $v_C(t) = V$, $v_R(t) = 0$

at $t = RC = \tau$; $i(t) = \frac{V}{R}e^{-1} = 0.368\frac{V}{R}$

and $v_C(t) = V(1 - e^{-1}) = 0.632\ V$, $v_R(t) = 0.368\ V$

These $i(t)$ and $v_R(t)$ and $v_C(t)$ are plotted in figure 3.9 (*a*) and (*b*).

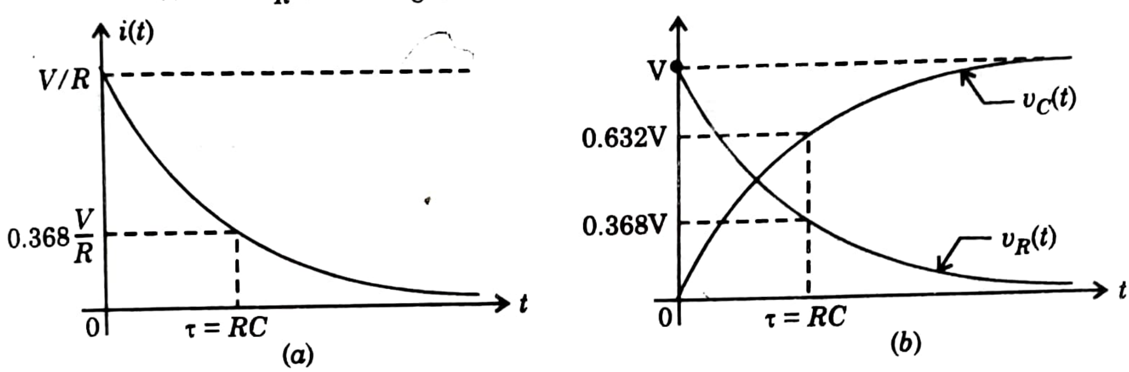


Fig. 3.9 (a) for i(t), and (b) for $v_R(t)$ and $v_C(t)$.

Note. Here $\tau = RC$ is known as time constant of the circuit.

Let us now analyse another transient condition of the *R*-*C* circuit as the circuit reaches at steady state (at $t = \infty$) and suddenly the voltage is withdrawn by opening the switch *S* and throwing it to *S'* as shown in figure 3.9 (c) at $= 0$.

Then by *KVL*,

$$Ri'(t) + \frac{1}{C}\int_0^t i'(t) + v_C(0^+) = 0$$

Differentiating, we get

$$R\frac{di'(t)}{dt} + \frac{1}{C}i'(t) = 0$$

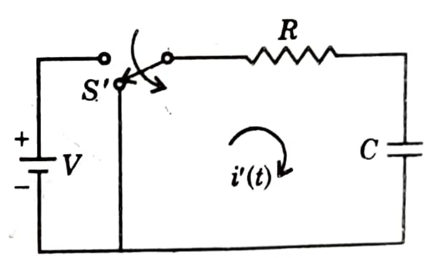


Fig. 3.9(c).

Its solution is $$i'(t) = K'e^{-\frac{1}{RC}t}$$

However, at $t = 0^+$, the capacitor keeps the steady state voltage $v_C(0^+) = V$. And the direction of $i'(t)$ during discharge is negative. Thus

$$i'(0^+) = -\frac{V}{R}$$

or $$-\frac{V}{R} = K'e^0 \quad \text{or} \quad K' = -\frac{V}{R}$$

Therefore, $$i'(t) = -\frac{V}{R}e^{-\frac{1}{RC}t}$$

The corresponding transient voltages across the resistor and capacitor are given by

$$v'_R(t) = i'(t) \cdot R$$

or $$v'_R(t) = -Ve^{-\frac{1}{RC}t}$$

and $$v'_C(t) = \frac{1}{C}\int i'(t)\,dt$$

or $$v'_C(t) = Ve^{-\frac{1}{RC}t}$$

[Obviously $v'_R(t) + v'_C(t) = 0$]

These $i'(t)$ and $v'_R(t)$ and $v'_C(t)$ are plotted in figure 3.9 (*d*) and (*e*).

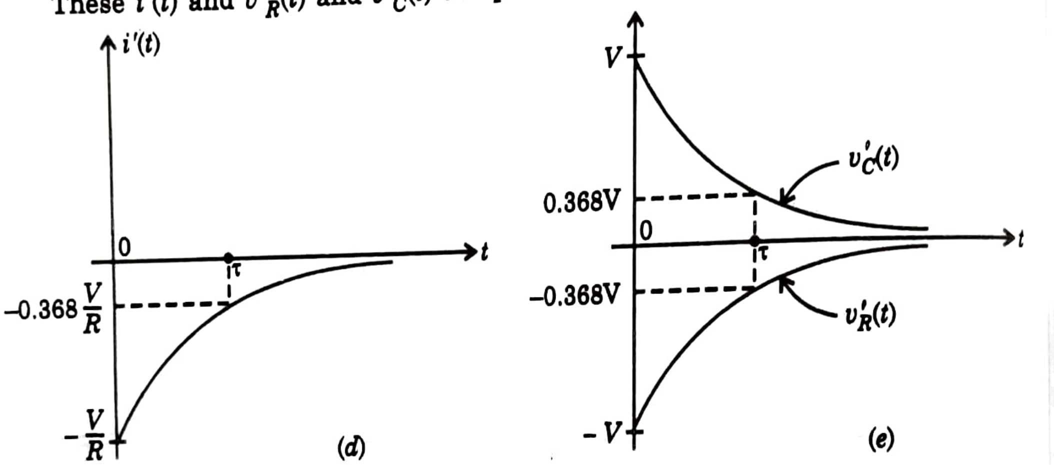


Fig. 3.9(d) for i' (t) and (e) for $v'_R(t)$ and $v'_C(t)$.

Example 3.6. A resistance *R* and 5 μF capacitor are connected in series across a 100 V d.c. supply. Calculate the value of *R* such that the voltage across the capacitor becomes 50 V in 5 sec after the circuit is switched on.

Solution : In case of charging, the voltage at any time across the capacitor is given as

$$v_C(t) = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

or $$50 = 100\left(1 - e^{-\frac{5}{R \times 5 \times 10^{-6}}}\right)$$

or $$0.5 = \left(1 - e^{-\frac{10^6}{R}}\right)$$

or $$e^{-\frac{10^6}{R}} = 0.5$$

∴ $$R = 1.45 \times 10^6\ \Omega$$

Example 3.7. In the given circuit as shown in figure 3.10, switch S is changed from position 'a' to 'b' at $t = 0$. Find values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

Solution : At position 'a' :

The steady state value of current $i(0^-)$

$$= \frac{100}{1000} = 0.1 \text{ A}$$

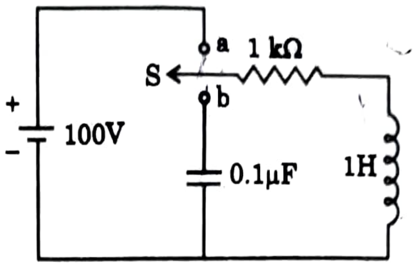


Fig. 3.10.

At position 'b' :

$$i(0^+) = i(0^-) = \mathbf{0.1\ A}$$

Applying *KVL*, we have

$$1000\, i(t) + 1.\frac{di(t)}{dt} + \frac{1}{0.1\times10^{-6}}\int_{-\infty}^{t} i(t)\, dt = 0 \qquad ...(1)$$

Since initially capacitor is uncharged and at switching instant capacitor behaves as a short circuit, *i.e.* last term of left hand side of above equation is zero. Then from equation (1) ;

$$1000\, i(t) + \frac{di(t)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = -1000\, i(0^+) = \mathbf{-100\ A/sec}$$

On differentiating, equation (1) becomes ;

$$\frac{d^2i(t)}{dt^2} = -1000\frac{di(t)}{dt} - \frac{1}{0.1\times10^{-6}}\, i(t)$$

or
$$\frac{d^2i(0^+)}{dt^2} = -1000\times(-100) - 10^7\times(0.1)$$

$$= -(-10^5 + 10^6) = \mathbf{-9\times10^5\ A/sec^2}$$

Example 3.8. In the network shown in figure 3.11, switch *S* is closed at *t* = 0, a steady state current having previously been attained. Solve for the current as a function of time.

Solution : Steady state current (before the switching action takes place)

$$i(0^-) = \frac{V}{R_1 + R_2}$$

(Since inductor behaves as a short circuit at $t = \infty$)

When switch is closed : R_2 is short-circuited

Applying *KVL*

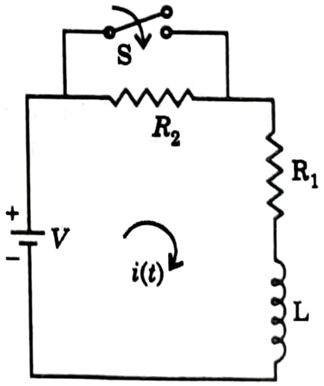


Fig. 3.11.

$$V = L\frac{di(t)}{dt} + R_1\, i(t)$$

or
$$\frac{di(t)}{dt} + \frac{R_1}{L} i(t) = \frac{V}{L}$$

The general solution of the above V equation is given as differential

$$i(t) = \frac{V}{R_1} + Ke^{-\frac{R_1}{L}t}$$

and
$$i(0^+) = i(0^-) = \frac{V}{R_1 + R_2}$$

or
$$\frac{V}{R_1 + R_2} = \frac{V}{R_1} + K$$

Therefore;
$$K = -V.\frac{R_2}{R_1(R_1+R_2)}$$

Hence, Solution of our problem is

$$i(t) = \frac{V}{R_1} - \frac{VR_2}{R_1(R_1+R_2)}.e^{-\frac{R_1}{L}t}$$

or
$$i(t) = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1+R_2}e^{-\frac{R_1}{L}t}\right) A$$

Example 3.9. The circuit shown in figure 3.12, is in the steady state with the switch S closed. The switch is opened at $t = 0$.

Determine voltage across the switch v_s and $\frac{dv_s}{dt}$ at $t = 0^+$.

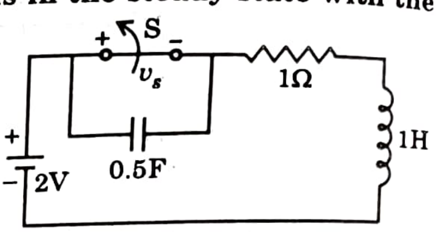


Fig. 3.12.

Solution : When, Circuit is in the steady state with the switch S closed, capacitor is short circuited *i.e.* voltage across capacitor is zero.

or
$$v_s(0^-) = 0, \text{ and steady state current } i(0^-) = \frac{2}{1} = 2\text{A}.$$

And, when switch is opened at $t = 0$, the capacitor behaves as a short circuit,

so
$$v_s \text{ at } t = 0^+ \quad \text{or} \quad v_s(0^+) = 0.$$
$$i(0^+) = i(0^-) = 2 \text{ A}$$

$$C\frac{dv_s}{dt} = i(t) \quad \text{or} \quad \frac{dv_s}{dt} = \frac{1}{C}i(t)$$

Hence
$$\frac{dv_s}{dt}(0^+) = \frac{1}{C}i(0^+) = \frac{1}{0.5}.2$$

or
$$\frac{dv_s}{dt}(0^+) = 4 \text{ V/sec}$$

Example 3.10. Using classical method of solution of differential equations, find the value of $v_C(t)$ for $t > 0$ in the circuit shown in figure 3.13. Assume initial condition $v_C(0^-) = 9$V.

Solution : Let the current in the circuit is $i(t)$. Applying KVL in the circuit; we have

$$1 = 4i(t) + v_C(t)$$

and
$$\boxed{v_C(t) = 9 + \frac{1}{C}\int_0^t i(t)\,dt}$$

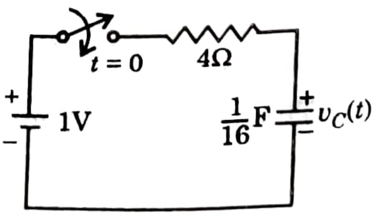


Fig. 3.13.

Therefore
$$1 = 4i(t) + 9 + 16\int_0^t i(t)\,dt$$

or
$$4i(t) + 16\int_0^t i\,dt = -8$$

or
$$i(t) + 4\int_0^t i(t)\,dt = -2$$

On differentiating,

$$\frac{di(t)}{dt} + 4i(t) = 0$$

The general solution of the above differential equation is

$$i(t) = Ke^{-4t}$$

Since initial voltage across capacitor is 9V, therefore, initial current

$$i(0^+) = \frac{1-9}{4} = -2 = K{\cdot}e^0$$

$$\therefore \quad K = -2$$

So, The value of the current ; $i(t) = -2e^{-4t}$

$$\text{Then } v_C(t) = 9 + \frac{1}{C}\int_0^t i(t)\,dt = 9 + 16.\left(\frac{-2e^{-4t}}{-4}\right)\Bigg|_0^t = 9 + 8e^{-4t}\Big|_0^t = 9 + (8e^{-4t} - 8)$$

or $\quad v_C(t) = 1 + 8\,e^{-4t}$ V

Example 3.11. The circuit shown in figure 3.14, is in the steady state with the switch S closed. The switch is opened at $t = 0$. Determine $i_L(t)$ in the circuit.

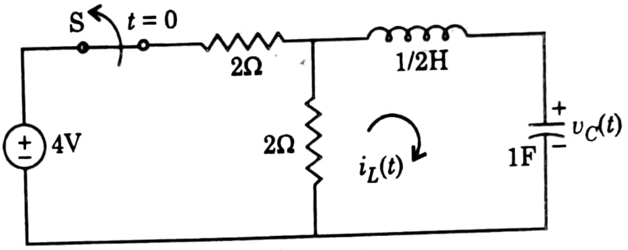


Fig. 3.14.

Solution : At steady state with the switch S closed. The capacitor behaves as a open circuit.

Therefore $v_C(0^-) = 2$ V and $i_L(0^-) = 0$

Now, switch S is opened, then applying KVL with $v_C(0^+) = v_C(0^-)$ and

$$i_L(0^+) = i_L(0^-)$$

$$2i_L(t) + \frac{1}{2}\frac{di_L(t)}{dt} + \frac{1}{1}\int_0^t i_L(t) + v_C(0^-) = 0$$

On Differentiating, we have

$$2\frac{di_L(t)}{dt} + \frac{1}{2}\frac{d^2 i_L(t)}{dt^2} + i_L(t) = 0$$

or

$$\frac{d^2 i_L(t)}{dt^2} + 4\frac{di_L(t)}{dt} + 2i_L(t) = 0$$

Characteristic equation is

$$p^2 + 4p + 2 = 0$$

having the roots $\quad p_1 = -3.414,\ p_2 = -0.586$

Thus $\quad i_L(t) = K_1\,e^{-3.414t} + K_2\,e^{-0.586\,t}$

$i_L(0^+) = 0$ requires that $K_1 + K_2 = 0$

$$\text{Now } v_L(0^+) = -v_c(0^+) = -2 = L\frac{di_L(0^+)}{dt}$$

or

$$\frac{di_L(0^+)}{dt} = \frac{-2}{L} = \frac{-2}{\frac{1}{2}} = -4$$

$$-3.414\,K_1 - 0.586\,K_2 = -4$$

Solving for K_1 and K_2 yields

$$K_1 = 1.414$$

and $$K_2 = -1.414$$

finally, $$i_L(t) = 1.414(e^{-3.414t} - e^{-0.586\,t})\ \text{A}$$

Example 3.12. The 12 V battery in figure 3.15 is disconnected (opened) at $t = 0$. Find the inductor current and voltage as a function of time.

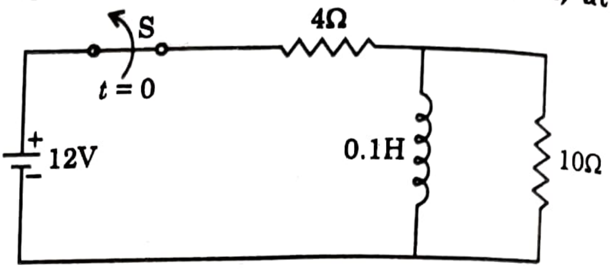


Fig. 3.15.

Solution : Assume the switch S has been closed for a long time before $t = 0$. The inductor behaves as a short circuit. *i.e.*

$$v_L(0^-) = 0$$

and $$i_L(0^-) = \frac{12}{4} = 3\text{A}$$

After the battery is disconnected, at $t > 0$, Applying *KVL*

$$0.1\frac{di_L(t)}{dt} + 10i_L(t) = 0$$

$$\frac{di_L(t)}{dt} + 100i_L(t) = 0$$

This gives $$i_L(t) = K e^{-100\,t}$$

Since $$i_L(0^+) = i_L(0^-) = 3 = K\cdot e^0.$$

or $$K = 3$$

Therefore, $$i_L(t) = 3e^{-100\,t}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = 0.1 \times 3.\ (-100)\ e^{-100\,t}$$

$$= -30\ e^{-100\,t}\ \text{V}$$

Example 3.13. The switch in figure 3.16 has been in position 1 for a long time; it is moved to 2 at $t = 0$. Obtain the expression for i, for $t > 0$.

Solution : With the switch on 1,

Fig. 3.16.

$$i\,(0^-) = \frac{50}{40} = 1.25\ \text{A}$$

When switch at 2,

$$i(0^+) = i(0^-) = 1.25\ \text{A}$$

Applying *KVL*,

$$10 = 40\,i(t) + 20\frac{di(t)}{dt}$$

or $$\frac{di(t)}{dt} + 2i\,(t) = 0.5$$

or $$i(t) = \frac{0.5}{2} + Ke^{-2t}$$

Putting $i(0^+) = 1.25$ in above equation

$$1.25 = 0.25 + K$$

or $$K = 1.00$$

Therefore, $$i(t) = 0.25 + e^{-2t}\ \text{A}$$

Example 3.14. **In figure 3.17, the switch S is closed. Find the time when the current from the battery reaches to 500 mA.**

Solution : Let the current through 50 Ω be I_1 and through 70 Ω (or 100 μF) be I_2 after the switch S is closed.

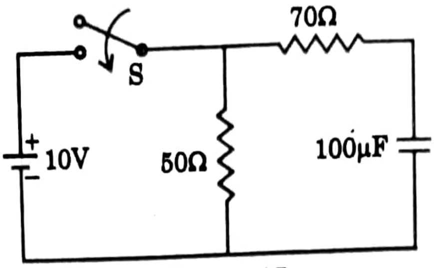


Fig. 3.17.

$$I_1 = \frac{10}{50} = 0.2\ A$$
$$= 200 \text{ mA}$$

However,

$$I = I_1 + I_2 \quad \text{[I being the current from the supply]}$$

or
$$500 = 200 + I_2$$
∴
$$I_2 = 300 \text{ m A}$$

This I_2 equal to $\frac{10}{70} e^{-\frac{t}{70\times100\times10^{-6}}}$ $\quad [\therefore\ i_C(t) = \frac{V}{R} e^{-t/RC}]$

This gives t = **5.2 msec.**

Example 3.15. The circuit of figure 3.18 was under steady state before the switch was opened. If $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $C = 0.167$ F, determine $v_c(0^-)$ and $v_c(0^+)$. Also find $i(0^+)$.

Solution : Since at steady state capacitor behaves as a open circuit or the voltage can not change instantaneously in the capacitor, we have

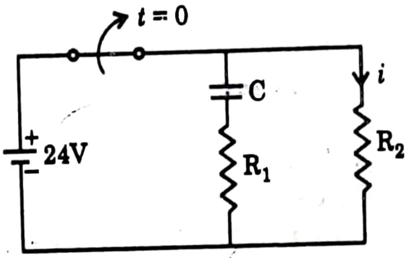


Fig. 3.18.

$$v_c(0^-) = v_c(0^+) = 24 \text{ V}$$

After the switch is opened at $t = 0^+$,

$$v_c = v_{R_1} + v_{R_2}$$
or
$$24 = i(1 + 2)$$
Hence,
$$i(0^+) = 8\text{A}$$

Example 3.16. Consider the first order *R-L* series circuit shown in figure 3.19 with $R = 5\ \Omega$, $L = 1$H, $V_s = 48$ V.

Determine

(a) the expression $i(t)$, $V_R(t)$, $V_L(t)$, and $\frac{di}{dt}$ for $t \geq 0$;

(b) $\frac{di}{dt}$ at $t = 0^+$;

(c) the time at which $V_R = V_L$;

(d) the resistance is decreased from 5 to 4 Ω at t = 0.5 sec, determine $i(t)$.

Assume $i_L(0^-) = 0$

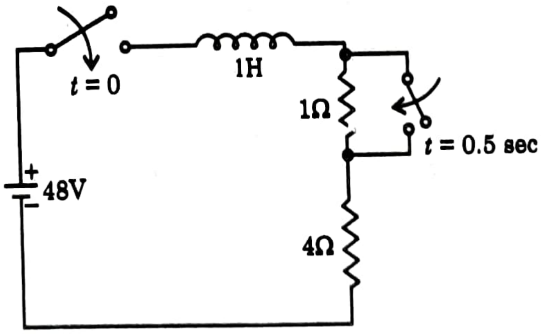


Fig. 3.19.

Solution : (*a*) Applying *KVL*,

$$48 = 1.\frac{di(t)}{dt} + 5i\,(t)$$

This gives,
$$i(t) = 9.6\ (1-e^{-5t})\ \text{A} \quad (\text{since } i_L(0^-) = 0)$$

Then
$$V_L(t) = L\frac{di}{dt} = 48\ e^{-5t}\ \text{V}$$

And,
$$V_R(t) = Ri\,(t) = 48\ (1 - e^{-5t})\ \text{V}$$

$$\frac{di}{dt} = 48e^{-5t} \text{ A/sec.}$$

(b) $$\frac{di}{dt}(\text{at } t = 0^+) = 48e^{-5t}\Big|_{t=0^+} = 48 \text{ A/sec.}$$

(c) Say at time $t = t_1$ at which $V_R = V_L$, *i.e.*

$$48(1 - e^{-5t1}) = 48e^{-5t1}$$

Solving, we get $t_1 = 0.1386$ *sec*

(d) at $t = t_2 = 0.5$ sec

$$i(t_2) = 9.6\,[\,1 - e^{-5 \times 0.5}] = 8.812 \text{ A}$$

Now, let resistance is decreased from 5 to 4Ω at $t = 0$. Then $i(0^-) = 8.812$ A.

Then applying *KVL*,

$$48 = L\frac{di(t)}{dt} + 4\,i(t)$$

$$\frac{di(t)}{dt} + 4\,i(t) = 48$$

$$i(t) = 12 - Ke^{-4t}$$

at $t = 0^-$, $i(t) = 8.812$

This gives $K = 3.188$

Therefore, $i(t) = 12 - 3.188\,e^{-4t}$ A

Example 3.17. For the network shown in figure 3.20. Find i_1, i_2 and V_1 at

(a) $t = 0^-$ (b) $t = 0^+$

(c) $t = \infty$ (d) $t = 50$ m sec.

(I.P. Univ., 2001)

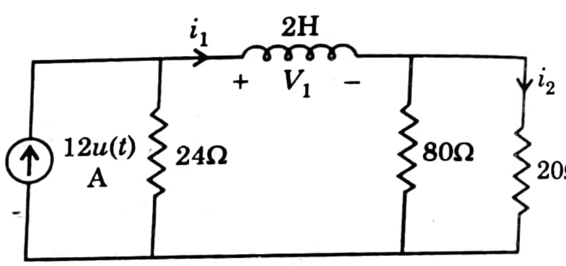


Fig. 3.20.

Solution : (a) at $t = 0^-$: By the Definition of step signal.

at $t = 0^-$, There is no energy source.

Therefore, $i_1(0^-) = 0$

$i_2(0^-) = 0$

$V_1(0^-) = 0$

(b) at $t = 0^+$: First we convert current source of $12u(t)$ in an equivalent voltage source $= 12\,u(t) \times 24 = 288\,u(t)$ as shown in figure 3.21.

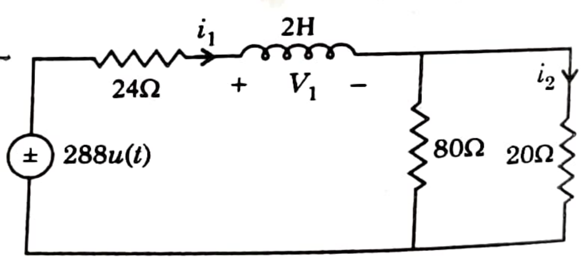


Fig. 3.21.

Applying *KVL*,

$$288\,u(t) = 24i_1 + 2\frac{di_1}{dt} + 80\,(i_1 - i_2) \quad \text{...(1)}$$

and $$i_2 = i_1 \cdot \frac{80}{20+80} \quad \text{(By current division rule)}$$

$$i_2 = 0.8\,i_1 \quad \text{...(2)}$$

From equations (1) and (2), we have

$$288\,u(t) = 24\,i_1 + 2\frac{di_1}{dt} + 80(i_1 - .08i_1) = 40\,i_1 + 2\frac{di_1}{dt}$$

or $$\frac{di_1}{dt} + 20\,i_1 = 144\,u(t)$$

The general solution of above differential equation is given as

$$i_1(t) = \frac{144}{20} + Ke^{-20t}$$

at $t = 0^+$, inductor behaves as a open circuit. *i.e.*

$$i_1(0^+) = 0, \text{ This gives } K = -\frac{144}{20}$$

Therefore, $$i_1(t) = \frac{144}{20}(1 - e^{-20t}) = 7.2\ (1 - e^{-20\ t}) \quad ...(3)$$

and $$i_2(t) = 0.8 i_1(t) = 5.76\ (1 - e^{-20\ t}) \quad ...(4)$$

so $$i_1(0^+) = \mathbf{0}$$
$$i_2(0^+) = \mathbf{0}$$
$$V_1(0^+) = \mathbf{288\ V}$$

(*c*) **at $t = \infty$:** From equations (3) and (4),

$$i_1(\infty) = \mathbf{7.2\ A}$$
$$i_2(\infty) = \mathbf{5.76\ A}$$

and $$V_1(\infty) = L\frac{di_1(\infty)}{dt} = 2 \times 0 = \mathbf{0}$$

(*Alternatively:* since inductor behaves as a short circuit at $t \to \infty$ so $V_1(\infty) = 0$)

(*d*) **at $t = 50$ msec :**

$$i_1 = 7.2\ (1 - e^{-20 \times 50 \times 10^{-3}}) = 7.2(1 - e^{-1}) = \mathbf{4.55A}$$
$$i_2 = 5.76\ (1 - e^{-1}) = \mathbf{3.64A}$$

(or from equation (2) $i_2 = 0.8\ i_1 = 0.8 \times 4.55 = 3.64$ A)

$$V_1 = L\left.\frac{di_1}{dt}\right|_{\text{at 50 msec}} = 2 \times (7.2 \times 20\ e^{-20 \times 50 \times 10^{-3}})$$
$$= \mathbf{105.95\ V}$$

3.8. TRANSIENT RESPONSE OF SERIES R-L CIRCUIT HAVING SINUSOIDAL EXCITATION

Example 3.18. Consider a series R-L circuit excited by a sinusoidal voltage source as shown in figure 3.22. The switch S is closed at time $t = 0$. Find the response (current) $i(t)$.

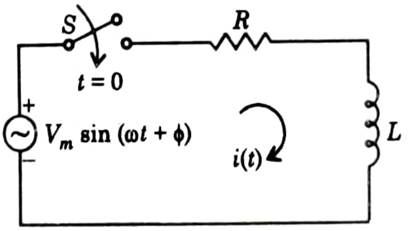


Fig. 3.22.

Solution : Applying KVL,

$$L.\frac{di(t)}{dt} + R.i(t) = V_m \sin(\omega t + \phi)$$

or $$\frac{di(t)}{dt} + \frac{R}{L}.i(t) = \frac{V_m}{L}\sin(\omega t + \phi) \quad ...(1)$$

This is a non-homogeneous equation. The current $i(t)$ consists of the sum of complementary function $i_c(t)$ and particular integral $i_p(t)$, *i.e.*

$$i(t) = i_c(t) + i_p(t)$$

The complementary function of equation (1) is

$$\boxed{i_c(t) = K.e^{-\frac{R}{L}t}}$$

And the particular integral of equation (1) is

$$i_p(t) = e^{-\frac{R}{L}t}\int V_m \sin(\omega t + \phi).e^{\frac{R}{L}t}\,dt$$

$$= \frac{V_m . e^{-\frac{R}{L}t}}{2j} \int \left\{ e^{j(\omega t + \phi)} - e^{-j(\omega t + \phi)} \right\} e^{\frac{R}{L}t}\, dt$$

$$= \frac{V_m . e^{-\frac{R}{L}t}}{2j} . \left[\frac{e^{j(\omega t + \phi) + \frac{R}{L}t}}{j\omega + \frac{R}{L}} - \frac{e^{-j(\omega t + \phi) + \frac{R}{L}t}}{-j\omega + \frac{R}{L}} \right]$$

$$= \frac{V_m}{2j} \left[\frac{e^{j(\omega t + \phi)}\left(-j\omega + \frac{R}{L}\right) - e^{-j(\omega t + \phi)}\left(j\omega + \frac{R}{L}\right)}{\left(j\omega + \frac{R}{L}\right)\left(-j\omega + \frac{R}{L}\right)} \right]$$

$$= \frac{V_m}{L\left(\omega^2 + \frac{R^2}{L^2}\right)} \left[\frac{R}{L} \sin(\omega t + \phi) - \omega \cos(\omega t + \phi) \right]$$

$$= \frac{V_m}{R^2 + \omega^2 L^2} \left[R \sin(\omega t + \phi) - \omega L \cos(\omega t + \phi) \right]$$

This can be reduced to a single sinusoid in the form

$$i_p(t) = \frac{V_m}{R^2 + \omega^2 L^2} \left[C \sin(\omega t + \phi + \theta) \right]$$

where C and θ can be determined as

$$C \cos \theta = R \quad \text{and} \quad C \sin \theta = -\omega L$$

i.e., $$C = \sqrt{R^2 + \omega^2 L^2} \quad \text{and} \quad \theta = -\tan^{-1} \frac{\omega L}{R}$$

Substituting C and θ,

$$\boxed{i_p(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R} \right)}$$

Therefore,

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \phi - \tan^{-1} \frac{\omega L}{R} \right) + K e^{-\frac{R}{L}t}$$

Since inductor behaves as a open circuit at switching

$\therefore$ $$i(0^+) = 0$$

or $$0 = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\phi - \tan^{-1} \frac{\omega L}{R} \right) + K$$

or $$K = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\phi - \tan^{-1} \frac{\omega L}{R} \right)$$

$\therefore$ $$i(t) = \frac{V_m}{Z} \left(\sin(\omega t + \phi + \theta) - \sin(\phi + \theta) e^{-\frac{R}{L}t} \right)$$

where, $$\theta = -\tan^{-1} \frac{\omega L}{R}$$

and $\boxed{Z = \sqrt{R^2 + \omega^2 L^2}}$; impedance of R-L circuit.

Therefore, the required expression of current is given by

$$i(t) = 9.49\, e^{-25t} + 9.52 \sin (314t - 1.49)$$

At $t = 0.01$ sec,

$$i_c(t) = 9.49\, e^{-25 \times 0.01} = 7.39 \text{ A}$$

$$i_p(t) = 9.52 \sin (314 \times 0.01 - 1.49) = 9.49 \text{ A}$$

$$i(t) = i_c(t) + i_p(t) = 16.88 \text{ A}$$

3.9. TRANSIENT RESPONSE OF SERIES R-C CIRCUIT HAVING SINUSOIDAL EXCITATION

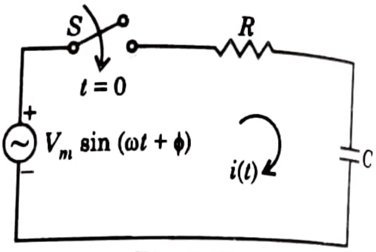


Fig. 3.24.

Example 3.20. Consider a series R-C circuit excited by a sinusoidal voltage source as shown in figure 3.24. The switch S is closed at time $t = 0$. Find the current $i(t)$.

Solution : Applying *KVL*,

$$R.i(t) + \frac{1}{C}\int_0^t i(t)\, dt + v_c(0^+) = V_m \sin (\omega t + \phi)$$

On differentiating, we get

$$\frac{di(t)}{dt} + \frac{1}{RC} i(t) = \frac{V_m \omega}{R} \sin (\omega t + \phi)$$

General solution of this differential equation is

$$i(t) = i_c(t) + i_p(t)$$

$$= K e^{-t/RC} + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

[The particular integral $i_p(t)$ has been obtained in a similar way as has been done for the case of R-L circuit of Article 3.7.]

Since capacitor behaves as a short circuit at switching.

$$\therefore \quad i(0^+) = \frac{V_m \sin \phi}{R}$$

or

$$\frac{V_m \sin \phi}{R} = K + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

or

$$K = \frac{V_m \sin \phi}{R} - \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

$$\therefore \quad i(t) = \left[\frac{V_m \sin \phi}{R} - \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)\right] e^{-\frac{t}{RC}}$$

$$+ \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

or $$i(t) = \left[\frac{V_m \sin\phi}{R} - \frac{V_m}{Z}\sin(\phi+\theta)\right]e^{-t/RC} + \frac{V_m}{Z}\sin(\omega t+\phi+\theta)$$

where $$\theta = \tan^{-1}\frac{1}{\omega CR}$$

and $$\boxed{Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\text{; impedance of } R\text{-}C \text{ circuit.}$$

The first term if $i(t)$ is the transient current with decay factor $e^{-t/RC}$, which dies out with time constant RC, and the second term is the steady state current which leads the applied voltage by $\theta = \tan^{-1}\frac{1}{\omega CR}$.

Example 3.21. A voltage $v = 300 \sin 314t$ is applied at $t = 2.14$ m sec to a series R-C circuit having resistance 10Ω and capacitance 200 μF. Find an expression for current. Also, find the value of current 1 msec after switching on.

Solution : It may be noted that the voltage is not applied at $t = 0$, but at ϕ where $\phi = 2.14 \text{ msec} = 2.14 \times 10^{-3} \times 314$

or $$\phi = 0.672 \text{ rad}$$

Impedance of R-C circuit

$$Z = \sqrt{(10)^2 + \left(\frac{1}{314\times 200\times 10^{-6}}\right)^2} = 18.8\,\Omega$$

Transient current ;

$$\boxed{i_c(t) = Ke^{-\frac{t}{RC}}} = Ke^{-\frac{t}{10\times 200\times 10^{-6}}}$$

or $$i_c(t) = Ke^{-500t}$$

Steady state current ;

$$\boxed{i_p(t) = \frac{V_m}{Z}\sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)}$$

$$= \frac{300}{18.8}\sin\left[314t + 0.672 + \tan^{-1}\left(\frac{1}{314\times 200\times 10^{-6}\times 10}\right)\right]$$

$$= 15.96 \sin(314t + 0.672 + 1.59)$$

Therefore, $$i(t) = i_c(t) + i_p(t)$$
$$= Ke^{-500t} + 15.96 \sin(314t + 0.672 + 1.59)$$

Since capacitor behaves as a short-circuit at switching.

$$\therefore\ i(2.14 \text{ msec}) = \frac{300 \sin(314\times 2.14\times 10^{-3})}{10} = 18.67 \text{ A}$$

Hence, we get·

$$18.67 = K(1) + 15.96 \sin(0.672 + 1.59)$$

or $$K = 18.67 - 12.29 = 6.38$$

Therefore, the required expression of current is given by

$$i(t) = 6.38\, e^{-500t} + 15.96 \sin(314t + 2.262)$$

After 1 msec the current becomes

$$i = 6.38\,e^{-500\times 10^{-3}} + 15.96 \sin(314\times 10^{-3} + 2.262)$$
$$= 3.87 + 8.55 = \mathbf{12.42\ A}$$

From equations (1) and (2), we have

$$v(t) = 4\left[\frac{d\,i_L(t)}{dt} + \frac{d^2\,i_L(t)}{dt^2} + i_L(t)\right] + i_L(t) + \frac{d\,i_L(t)}{dt}$$

or
$$v(t) = 4\frac{d^2\,i_L(t)}{dt^2} + 5\frac{d\,i_L(t)}{dt} + 5\,i_L(t)$$

Example 3.24. In the circuit of figure 3.29, at time t_0 after the switch S was closed, it is found that $v_2 = +5$V. We are required to determine the values of $i_2(t_0)$ and $\frac{di_2(t_0)}{dt}$.

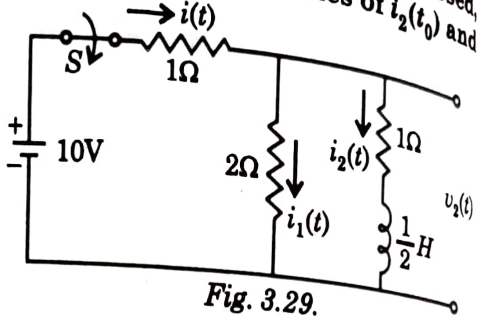


Fig. 3.29.

Solution : Applying KCL,

$$i(t) = i_1(t) + i_2(t) \qquad \text{....}(i)$$

where $i_1(t_0) = \frac{v_2}{2} = \frac{5}{2}A$

Now, applying KVL,

$$10 = 1.\ i(t) + 2i_1(t) \qquad \text{....}(ii)$$

From equations (*i*) and (*ii*), we have

$$10 = i_1(t) + i_2(t) + 2i_1(t)$$

or
$$i_2(t) = 10 - 3i_1(t)$$

At
$$t = t_0, \quad i_2(t_0) = 10 - 3\ i_1(t_0) = \mathbf{2.5A}$$

And
$$v_2(t) = 1.i_2(t) + \frac{1}{2}\frac{di_2(t)}{dt}$$

At
$$t = t_0,$$

$$5 = 1.(2.5) + \frac{1}{2}\frac{di_2(t_0)}{dt}$$

or
$$\frac{di_2(t_0)}{dt} = 5 \text{ A/sec}$$

Example 3.25. In the circuit of figure 3.30, it is given that $v_2(t_0) = 2$ V, and $\frac{dv_2(t_0)}{dt} = -10$V/sec, where t_0 is the time after the the switch S was closed. Determine the value of C.

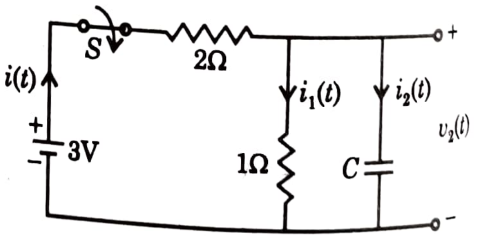


Fig. 3.30.

Solution : From figure 3.30, using two Kirchhoff's Laws,

We have,

$$i(t) = i_1(t) + i_2(t) \qquad \text{...}(i)$$

and
$$3 = 2i(t) + i_1(t) \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$3 = 3i_1(t) + 2i_2(t) \qquad \text{...}(iii)$$

At
$$t = t_0, \quad i_1(t_0) = \frac{v_2(t_0)}{1} = 2$$

Now, from equation (*iii*),

$$3 = 3.2 + 2\ i_2(t_0)$$