Laplace Transform

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4.1. INTRODUCTION

The Laplace transform is one of the mathematical tools used for the solution of linear ordinary integro-differential equations. (Mostly continuous-time systems are described by integro-differential equations). In comparison (as shown in figure 4.1.) with the classical method of solving linear integro-differential equations, the Laplace transform method has the following two attractive features :

(i) The homogeneous equation and the particular integral of the solution are obtained in one operation.

(ii) The Laplace transform converts the integro-differential equation into an algebraic equation in s (Laplace operator). It is then possible to manipulate the algebraic equation by simple algebraic rules to obtain the expression in suitable forms. The final solution is obtained by taking the inverse Laplace transform.

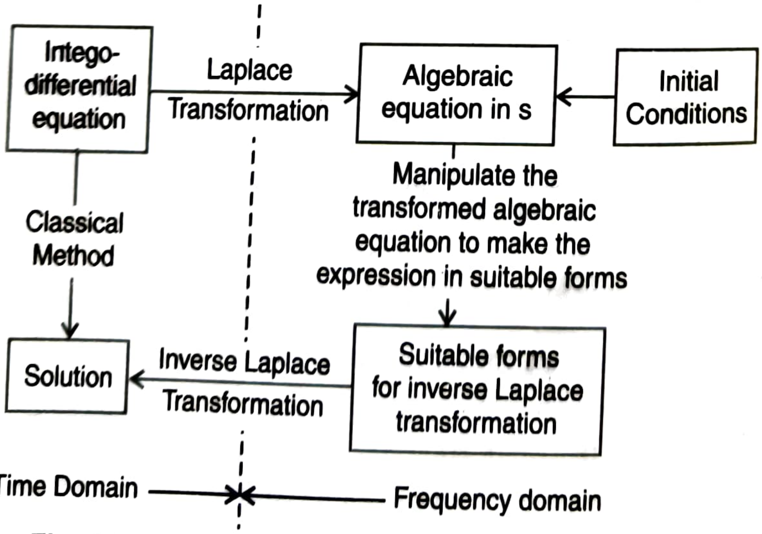

Fig. 4.1. Comparison of Classical method and Laplace transform method

From the 2nd feature of Laplace transform over classical method, Laplace transformation is somewhat similar to logarithmic operation. To find the product or quotient of two number, we find

(*i*) logarithm of two numbers

(*ii*) add or subtract

(*iii*) take antilogarithm to get product or quotient

4.2. DEFINITION OF THE LAPLACE TRANSFORM

The Laplace transform method is a powerful technique for solving circuit problems. We define a Laplace transform as follows :

For the time function $f(t)$ ~~which is zero for~~ $t < 0$ and that satisfy the condition

$$\int_0^\infty |f(t)e^{-\sigma t}|\, dt < \infty$$

for some real and positive σ, the Laplace transform of $f(t)$ is defined as

$$F(s) = \pounds[f(t)] = \int_0^\infty f(t)e^{-st}\, dt$$

The variable s is referred to as the *Laplace operator*, which is complex variable, *i.e.*, $s = \sigma + j\omega$. And the functions $f(t)$ and $F(s)$ are known as *Laplace transform pair.*

4.3. INVERSE LAPLACE TRANSFORMATION

Given the Laplace transform $F(s)$, the operation of obtaining $f(t)$ is termed the *inverse Laplace transformation* and is denoted by

$$f(t) = \pounds^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st}\, ds$$

Though one could evaluate the inverse transform of a function F(s) by using above equation, normally the transform table is used to obtain the inverse transformation.

8. Initial Value Theorem

If the function $f(t)$ and its first derivative $\frac{df(t)}{dt}$ are both Laplace trans-formable, then

$$f(0^+) = \lim_{t \to 0} f(t) = \lim_{s \to \infty}[s \cdot F(s)]$$

Proof : Using time-differentiation property,

$$s.F(s) - f(0^+) = \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt$$

or

$$s.F(s) = f(0^+) + \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt$$

$$\lim_{s \to \infty} [s.F(s)] = f(0^+) + \lim_{s \to \infty} \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt$$

$$\lim_{s \to \infty} [s.F(s)] = f(0^+) + \int_0^\infty \frac{df(t)}{dt} \left(\lim_{s \to \infty} e^{-st} \right) dt = f(0^+)$$

9. Final Value Theorem

The final value of a function $f(t)$ is given as

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0}[s \cdot F(s)]$$

Proof : Using time-differentiation property, and we let $s \to 0$,

$$\lim_{s \to 0} [s.F(s) - f(0^+)] = \lim_{s \to 0} \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt = \int_0^\infty \frac{df(t)}{dt} \left(\lim_{s \to 0} e^{-st} \right) dt$$

$$= \int_0^\infty \frac{df(t)}{dt}\, dt = f(t)\big|_0^\infty = \lim_{t \to \infty} [f(t) - f(0)]$$

or

$$\lim_{s \to 0} [s.F(s)] = \lim_{t \to \infty} f(t) \qquad \text{(since } f(0^+) = f(0)\text{)}$$

10. Theorem for Periodic Functions

The Laplace transform of a periodic function (wave) with period T is equal to $\frac{1}{1-e^{-Ts}}$ times the Laplace transform of the first cycle of that function (wave).

Proof:

Let $f_1(t), f_2(t), f_3(t)$,..... be the functions describing the first, second, third, ...cycles of a periodic function $f(t)$ whose time period is T. Then

$$f(t) = f_1(t) + f_2(t) + f_3(t) + \ldots\ldots$$

$$= f_1(t) + f_1(t-T)\, U(t-T) + f_1(t-2T)\, U(t-2T) + \ldots\ldots$$

Now, let $\pounds[f_1(t)] = F_1(s)$

Therefore, by shifting theorem (property 7[*a*] of L.T.), we get

$$\pounds[f(t)] = F_1(s) + e^{-Ts}.F_1(s) + e^{-2Ts} F_1(s) + \ldots\ldots$$

$$= F_1(s)\, [1 + e^{-Ts} + e^{-2Ts} + \ldots\ldots]$$

$$= \frac{1}{1-e^{-Ts}} \cdot F_1(s)$$

Table 8.1 Summary of properties of LT.

S.No	Property	Time function $x(t)$	Frequency function $X(s)$
1.	Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2.	Time shifting	$x(t - t_0)$	$X(s)e^{-st_0}$
3.	Frequency shifting	$x(t)e^{at}$	$X(s - a)$
4.	Time scaling	$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
5.	Frequency scaling	$\frac{1}{a}x\left(\frac{t}{a}\right)$	$X(as)$
6.	Time differentiation	$\frac{d}{dt}(x(t))$	$sX(s) - x(0^-)$
		$\frac{d^2}{dt^2}(x(t))$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
		$\frac{d^n}{dt^n}(x(t))$	$s^nX(s) - \sum_{k=1}^{n} s^n x^{(k-1)}(0^-)$
7.	Time integration	$\int_0^t x(\tau)d\tau$	$\frac{X(s)}{s}$
8.	Time convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
9.	Complex frequency differentiation	$-tx(t)$	$\frac{d}{ds}(X(s))$
		$t^nx(t)$	$(-1)^n\frac{d^n}{ds^n}X(s)$
10.	Complex frequency shifting	$e^{-at}x(t)$	$X(s + a)$
11.	Conjugation	$x^*(t)$	$X^*(-s)$
12.	Initial value theorem	$\underset{t\to 0}{Lt}\ x(t)$	$\underset{s\to\infty}{Lt}\ sX(s)$
13.	Final value theorem	$\underset{t\to\infty}{Lt}\ x(t)$	$\underset{s\to 0}{Lt}\ sX(s)$
14.	Shift theorem	$x(t - a)$	$X(s)e^{-as}$

The following examples illustrate the method of determining LT.

■ Example 8.8

Determine the LT of unit impulse function $\delta(t)$ shown in Figure 8.10.

t

Figure 8.16 $x(t) = u(t - 3)$.

Table 8.2 Laplace transform tables

S.No	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{\angle n}{s^{n+1}}$
5	$e^{at}u(t)$	$\frac{1}{(s-a)}$
6	$e^{-at}u(t)$	$\frac{1}{(s+a)}$
7	$\cos at\, u(t)$	$\frac{s}{(s^2+a^2)}$
8	$\sin at\, u(t)$	$\frac{a}{(s^2+a^2)}$
9	$e^{-bt}\cos at\, u(t)$	$\frac{(s+b)}{(s+b)^2+a^2}$
10	$e^{-bt}\sin at\, u(t)$	$\frac{a}{(s+b)^2+a^2}$
11	$\delta(t-a)$	e^{-as}
12	$u(t-a)$	$\frac{e^{-as}}{s}$
13	$t \sin at\, u(t)$	$\frac{2as}{(s^2+a^2)^2}$
14	$\sin h\, at$	$\frac{a}{(s^2+a^2)}$
15	$\cos h\, at$	$\frac{s}{s^2+a^2}$
16	$\sin(at+\theta)$	$\frac{s\sin\theta + a\cos\theta}{(s^2+a^2)}$
17	$\cos(at+\theta)$	$\frac{s\cos\theta - a\sin\theta}{(s^2+a^2)}$

EXAMPLE 4.1 **Find the Laplace transform of the following standard signals (functions)**

(a) The unit step function $U(t)$.

(b) The delayed step function $KU(t-a)$.

(c) The ramp function $Kr(t)$ or $Kt\,U(t)$.

(d) The delayed unit ramp function $r(t-a)$.

(e) The unit impulse function $\delta(t)$.

(f) The unit doublet function $\delta'(t)$.

Solution :

(a)
$$f(t) = U(t)$$
$$F(s) = \int_0^\infty U(t) \cdot e^{-st}\, dt$$

By the definition of $U(t)$ given in chapter 2, we have

$$= \int_0^\infty e^{-st}\, dt = \left.\frac{e^{-st}}{-s}\right|_0^\infty = \frac{1}{s}$$

(b)
$$f(t) = KU(t-a)$$
$$F(s) = \int_0^\infty KU(t-a)e^{-st}\, dt$$

By the definition of $U(t-a)$ given in chapter 2, we have

$$= \int_a^\infty K e^{-st}\, dt = K\left.\frac{e^{-st}}{-s}\right|_a^\infty = K\frac{e^{-as}}{s}$$

(Alternatively we can find directly using property 7[a])

(c)
$$f(t) = Kr(t) = Kt\ U(t)$$
$$F(s) = \int_0^\infty KtU(t)e^{-st}\, dt = \int_0^\infty Kt\, e^{-st}\, dt = K\left[t\cdot\left.\frac{e^{-st}}{-s}\right|_0^\infty - \int_0^\infty 1\cdot\frac{e^{-st}}{-s}\, dt\right]$$
$$= K[0-0] + \frac{K}{s}\int_0^\infty e^{-st}\, dt = -\left.\frac{K}{s^2}e^{-st}\right|_0^\infty = \frac{K}{s^2}$$

(d)
$$f(t) = r(t-a) = (t-a)\ U(t-a)$$
$$F(s) = \int_0^\infty (t-a)U(t-a)e^{-st}\, dt$$
$$= \int_a^\infty (t-a)e^{-st}\, dt = (t-a)\left.\frac{e^{-st}}{-s}\right|_a^\infty - \int_a^\infty 1\cdot\frac{e^{-st}}{-s}\, dt$$
$$= 0 - 0 + \frac{1}{s}\int_a^\infty e^{-st}\, dt = \frac{1}{s}\cdot\left.\frac{e^{-st}}{-s}\right|_a^\infty = \frac{e^{-as}}{s^2}$$

(Alternatively we can find directly using property 7[a])

(e)
$$f(t) = \delta(t)$$
$$F(s) = \int_0^\infty \delta(t)e^{-st}\, dt$$

By the definition of $\delta(t)$ given in chapter 2, we have

$$F(s) = \left.e^{-st}\right|_{t=0} = 1 \qquad \text{(Since } \delta(t) = 1 \text{ only at } t = 0\text{)}$$

(f) $f(t) = \delta'(t)$ $\qquad F(s) = s.\mathcal{L}\,[\delta(t)] = s$

EXAMPLE 4.2 **Find the Laplace transform of the following functions :**

(a) **The exponential decay function Ke^{-at}**
(b) **The sinusoidal function $\sin \omega t$.**
(c) **The cosine function $\cos \omega t$.**
(d) **$e^{-at} \sin \omega t$**
(e) **$e^{-at} \cos \omega t$**
(f) **$e^{-at}\, t\, U(t)$**
(g) **$\sin h\ at$**
(h) **$\cos h\ at$**

Solution : (a) $\qquad f(t) = Ke^{-at}$

$$\pounds\,(f_1(t)] = -\frac{2K}{T}\left[\frac{1}{s^2}-\frac{e^{-Ts}}{s^2}-\frac{T}{2}\left\{\frac{1}{s}+\frac{e^{-Ts}}{s}\right\}\right]$$

or
$$F_1(s) = \frac{2K}{Ts}\left\{\frac{T}{2}(1+e^{-Ts})-\frac{1}{s}(1-e^{-Ts})\right\}$$

Therefore, the Laplace transform of the periodic waveform $f(t)$ of period T is given by

$$F(s) = \frac{1}{1-e^{-Ts}}\cdot F_1(s) = \frac{1}{1-e^{-Ts}}\cdot\frac{2K}{Ts}\left\{\frac{T}{2}(1+e^{-Ts})-\frac{1}{s}(1-e^{-Ts})\right\}$$

$$= \frac{2K}{Ts}\left[\frac{T}{2}\left(\frac{1+e^{-Ts}}{1-e^{-Ts}}\right)-\frac{1}{s}\right] = \frac{2K}{Ts}\left[\frac{T}{2}\left(\frac{e^{Ts/2}+e^{-Ts/2}}{e^{Ts/2}-e^{-Ts/2}}\right)-\frac{1}{s}\right]$$

$$= \frac{2K}{Ts}\left[\frac{T}{2}\coth h\frac{Ts}{2}-\frac{1}{s}\right]$$

EXAMPLE 4.12 **Determine the initial value $f(0^+)$, if**

$$F(s) = \frac{2(s+1)}{s^2+2s+5}.$$

Solution : $f(0^+) = \lim_{s\to\infty}[s.F(s)]$

$$= \lim_{s\to\infty}\left[\frac{2s(s+1)}{s^2+2s+s}\right] = \lim_{s\to\infty}\left[\frac{2+\frac{2}{s}}{1+\frac{2}{s}+\frac{5}{s^2}}\right] = 2$$

EXAMPLE 4.13 **For the current $i(t) = 5\,U(t) - 3e^{-2t}$, find $I(s)$ and hence to determine the value of $i(0^+)$ and $i(\infty)$.**

Solution :
$$i(t) = 5U(t) - 3e^{-2t}$$

$$I(s) = \frac{5}{s}-\frac{3}{s+2} = \frac{2s+10}{s(s+2)}$$

Therefore,
$$i(0^+) = \lim_{s\to\infty}[s.I(s)] = \lim_{s\to\infty}\left[\frac{2s+10}{s+2}\right]$$

$$= \lim_{s\to\infty}\left[\frac{2+\frac{10}{s}}{1+\frac{2}{s}}\right] = 2$$

And
$$i(\infty) = \lim_{s\to 0}[s.I(s)] = \lim_{s\to 0}\left[\frac{2s+10}{s+2}\right] = 5$$

EXAMPLE 4.14 **Find the initial value of the function, $f(t) = 9 - 2\,e^{-5\,t}$.**

Solution :

$$f(0^+) = \lim_{t\to 0} f(t) = \lim_{t\to 0}(9-2\,e^{-5\,t}) = 9-2\,e^0 = 7$$

EXAMPLE 4.15 **Given the function $F(s) = \dfrac{5s+3}{s(s+1)}$. Find the initial value $f(0^+)$, final value $f(\infty)$, and the corresponding time function $f(t)$.** *(I.P. Univ., 2001)*

Solution : Initial value, $f(0^+) = \lim_{s\to\infty}[s.F(s)] = \lim_{s\to\infty}\left[\frac{5s+3}{s+1}\right]$

$$= \lim_{s\to\infty}\left[\frac{5+\frac{3}{s}}{1+\frac{1}{s}}\right] = 5$$

Final value, $f(\infty) = \lim_{s\to 0}[s.F(s)] = \lim_{s\to 0}\left[\frac{5s+3}{s+1}\right] = 3$

And, $F(s) = \frac{5s+3}{s(s+1)} = \frac{3}{s} + \frac{2}{s+1}$ (Using partial fraction expansion)

Therefore, $f(t) = \pounds^{-1}[F(s)] = (3 + 2e^{-t})\, U(t)$

Alternative ways :

$$f(0^+) = \lim_{t\to 0} f(t) = \lim_{t\to 0}(3+2e^{-t}) = 3 + 2 = 5$$

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{t\to\infty}(3+2e^{-t}) = 3 + 0 = 3$$

EXAMPLE 4.16 Without finding the inverse Laplace transform of $F(s)$, determine $f(0^+)$ and $f(\infty)$ for each of the following functions : *(U.P.T.U., 2001)*

(i) $\frac{4e^{-2s}(s+50)}{s}$ *(ii)* $\frac{s^2+6}{s^2+7}$

Solution : Since we know that

$$f(0^+) = \lim_{s\to\infty} s.F(s) \quad \text{and} \quad f(\infty) = \lim_{s\to 0} s.F(s)$$

(i) $$f(0^+) = \lim_{s\to\infty} s\cdot\frac{4e^{-2s}(s+50)}{s} = \lim_{s\to\infty} 4e^{-2s}(s+50) = \mathbf{0}$$

$$f(\infty) = \lim_{s\to 0} 4e^{-2s}(s+50) = 4.(50) = \mathbf{200}$$

(ii) $$f(0^+) = \lim_{s\to\infty} s\cdot\left(\frac{s^2+6}{s^2+7}\right) = \lim_{s\to\infty}\frac{1+\frac{6}{s^2}}{\frac{1}{s}+\frac{7}{s^3}} = \infty$$

$$f(\infty) = \lim_{s\to 0}\frac{s(s^2+6)}{(s^2+7)} = \mathbf{0}$$

EXAMPLE 4.17 Find the Laplace transform of the time function shown in figure 4.6. *(U.P.T.U., 2001)*

Solution : $f(t) = \frac{5}{2}(t-2)\,G_{2,4}(t)$

$$= \frac{5}{2}(t-2)[U(t-2) - U(t-4)]$$

$$= \frac{5}{2}(t-2)U(t-2) - \frac{5}{2}(t-2)U(t-4)$$

$$= \frac{5}{2}r(t-2) - \frac{5}{2}(t-4+2)U(t-4)$$

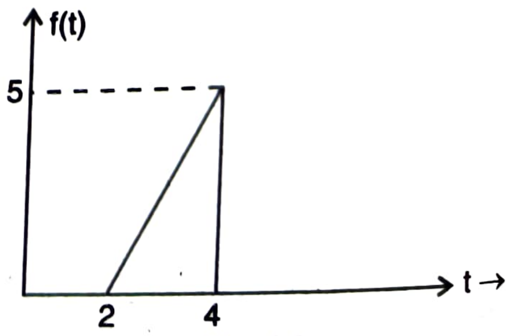

Fig. 4.6.

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$$f(t) = \frac{5}{2}r(t-2) - \frac{5}{2}r(t-4) - 5U(t-4)$$

So,

$$F(s) = \pounds\,[f(t)]$$

$$= \frac{5}{2}\frac{e^{-2s}}{s^2} - \frac{5}{2}\frac{e^{-4s}}{s^2} - \frac{5e^{-4s}}{s}$$

$$= \frac{5e^{-2s}}{2s^2}\left[1 - e^{-2s} - 2s\,e^{-2s}\right]$$

EXAMPLE 4.18 **Without finding inverse Laplace Transform of $F(s)$, determine $f(0^+)$) and $f(\infty)$ for the following function :**

$$F(s) = \frac{5s^3 - 1600}{s(s^3 + 18s^2 + 90s + 800)}$$ *(U.P.T.U., 2002)*

Solution : $f(0^+) = \lim\limits_{s \to \infty} s.F(s)$

$$= \lim_{s \to \infty} \cdot \frac{5s^3 - 1600}{s^3 + 18s^2 + 90s + 800} = \lim_{s \to \infty} \cdot \frac{5 - \frac{1600}{s^3}}{1 + \frac{18}{s} + \frac{90}{s^2} + \frac{800}{s^3}} = 5$$

$$f(\infty) = \lim_{s \to 0} \cdot s \cdot F(s) = -\frac{1600}{800} = -2$$

EXAMPLE 4.19 **Determine Laplace Transform of the following wave shown in figure 4.7.** *(U.P.T.U., 2002)*

Solution : From the wave form shown in figure 4.7,

$$f(t) = 4G_{0,3}(t) + (-2t + 10)\,G_{3,5}(t)$$
$$= 4\,[U(t) - U(t-3)] - 2\,(t-5)\,[U(t-3) - U(t-5)]$$
$$= 4\,U(t) - 4U(t-3) - 2(t - 5 + 2 - 2)\,U(t-3) + 2(t-5)\,U(t-5)$$
$$= 4\,U(t) - 2(t-3)\,U(t-3) + 2(t-5)\,U(t-5)$$

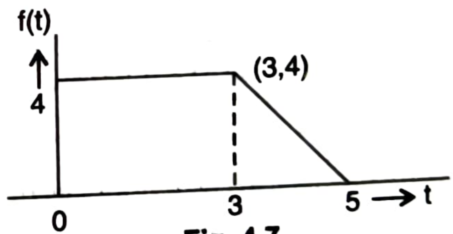


Fig. 4.7.

$4U(t) - 4U(t-3) + 4U(t-3)$
$-2(t-3)U(t-3) + 2(t-5)$
$U(t-5)$

Therefore, Laplace transform of the waveform is

$$F(s) = \pounds\,[f(t)]$$

$$= \frac{4}{s} - \frac{2e^{-3s}}{s^2} + \frac{2\,e^{-5s}}{s^2}$$

EXAMPLE 4.20 **Find the Laplace transforms of the following functions :**

(a) $e^{-at}\,U(t)$

(b) $e^{-at}\,U(t-b)$

(c) $e^{-a(t-b)}\,U(t-b) = e^{ab}\,.\,e^{-at}\,U(t-b)$

(d) $e^{-a(t-b)}\,U(t-c) = e^{ab}\,.e^{-at}\,U(t-c)$.

Solution : (a) $$F(s) = \int_0^\infty f(t)\,e^{-st}dt = \int_0^\infty e^{-at}.U(t)\,e^{-st}dt = \frac{1}{s+a}$$

(Alternatively we can find directly using properly 7 [b])

EXAMPLE 5.1 Solve the Differential Equation
$x'' + 3x' + 2x = 0, \; x(0^+) = 2, \; x'(0^+) = -3.$

Solution : Taking the Laplace transform,

$$s^2X(s) - sx(0^+) - x'(0^+) + 3sX(s) - 3x(0^+) + 2X(s) = 0$$

$$(s^2 + 3s + 2)X(s) = sx(0^+) + x'(0^+) + 3x(0^+)$$

$$(s^2 + 3s + 2)X(s) = 2s + 3 \quad \text{[By putting initial conditions]}$$

or

$$X(s) = \frac{2s+3}{s^2+3s+2} = \frac{2s+3}{(s+1)(s+2)}$$

Hence, all roots of denominator polynomial are simple. Then by partial fraction expansion,

$$X(s) = \frac{2s+3}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

Where

$$K_1 = (s+1)\cdot X(s)\Big|_{s=-1}$$

$$= \frac{2s+3}{s+2}\Bigg|_{s=-1} = \frac{-2+3}{1+2} = 1$$

and

$$K_2 = (s+2)\cdot X(s)\,\Big|_{s=-2} = \frac{2s+3}{s+1}\Bigg|_{s=2} = 1$$

The result of the partial fraction expansion is thus,

$$X(s) = \frac{2s+3}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{1}{s+2}$$

Therefore, the solution of given differential equation is

$$x(t) = \pounds^{-1}[X(s)] = \pounds^{-1}\left[\frac{1}{s+1} + \frac{1}{s+2}\right]$$

or

$$\mathbf{x(t) = e^{-t} + e^{-2t}}$$

EXAMPLE 5.2 **Find $i(t)$, if $I(s) = \dfrac{1}{s(s+1)^2(s+2)}$.**

Solution :

$$I(s) = \frac{1}{s(s+1)^2(s+2)} = \frac{K_1}{s} + \frac{K_{21}}{s+1} + \frac{K_{22}}{(s+1)^2} + \frac{K_3}{s+2}$$

Where

$$K_1 = sI(s)\,|_{s=0} = \frac{1}{2}, \quad K_3 = (s+2)\,I(s)\,|_{s=-2} = -\frac{1}{2}$$

$$K_{22} = (s+1)^2\, I(s)\big|_{s=-1} = -1$$

$$K_{21} = \frac{d}{ds}[(s+1)^2\, I(s)]\,|_{s=-1}$$

$$= \frac{d}{ds}\left[\frac{1}{s(s+2)}\right]\Bigg|_{s=-1} = \frac{-(2s+2)}{s^2(s+2)^2}\Bigg|_{s=-1} = 0$$

The complete expansion is

$$I(s) = \frac{1}{2s} - \frac{1}{(s+1)^2} - \frac{1}{2(s+2)}$$

Therefore,

$$i(t) = \frac{1}{2} - te^{-t} - \frac{1}{2}e^{-2t}\ A$$

EXAMPLE 5.3 **If $I(s) = \dfrac{s^2+5s+9}{s^3+5s^2+12s+8}$; find $i(t)$.**

Solution : $$I(s) = \frac{s^2+5s+9}{s^3+5s^2+12s+8} = \frac{s^2+5s+9}{(s+1)(s^2+4s+8)}$$

$$I(s) = \frac{s^2+5s+9}{(s+1)\left[(s+2)^2-(j2)^2\right]} = \frac{[s^2+5s+9]}{(s+1)(s+2+j2)(s+2-j2)}$$

or $$I(s) = \frac{K_1}{s+1} + \frac{K_2}{(s+2+j2)} + \frac{K_2^*}{(s+2-j2)}$$

$$K_1 = (s+1)\, I(s)\,|_{s=1} = \left.\frac{s^2+5s+9}{s^2+4s+8}\right|_{s=-1} = 1$$

$$K_2 = (s+2+j2)\, I(s)\,|_{-(2+j2)} = -\frac{1}{j4}$$

∴ $$K_2^* = \frac{1}{j4}$$

Therefore, the complete expansion is

$$I(s) = \frac{1}{s+1} + \frac{-\frac{1}{j4}}{(s+2+j2)} + \frac{\frac{1}{j4}}{(s+2-j2)}$$

Hence, $$i(t) = e^{-t} - \left(\frac{1}{j4}\right)e^{-(2+j2)t} + \left(\frac{1}{j4}\right)e^{-(2-j2)t} = e^{-t} - \frac{1}{j4}e^{-2t[e^{-j2t}-e^{j2t}]}$$

or $$\boxed{i(t) = e^{-t} + \frac{1}{2}e^{-2t}\sin 2t\ A}$$

Alternatively, $$I(s) = \frac{s^2+5s+9}{(s+1)(s^2+4s+8)} = \frac{1}{s+1} + \frac{1}{s^2+4s+8} = \frac{1}{s+1} + \frac{1}{2}\left[\frac{2}{(s+2)^2+(2)^2}\right]$$

Therefore, $$i(t) = £^{-1}[I(s)] = e^{-t} + \frac{1}{2}e^{-2t}\sin 2t\ A$$

as $$£^{-1}\left[\frac{\omega}{(s+a)^2+\omega^2}\right] = e^{-at}\sin \omega t$$

5.3. TRANSFORMED CIRCUIT COMPONENTS REPRESENTATION

5.3.1. Independent Sources

The sources $v(t)$ and $i(t)$ may be represented by their transformations, namely $V(s)$ and $I(s)$ respectively as shown in figure 5.1.

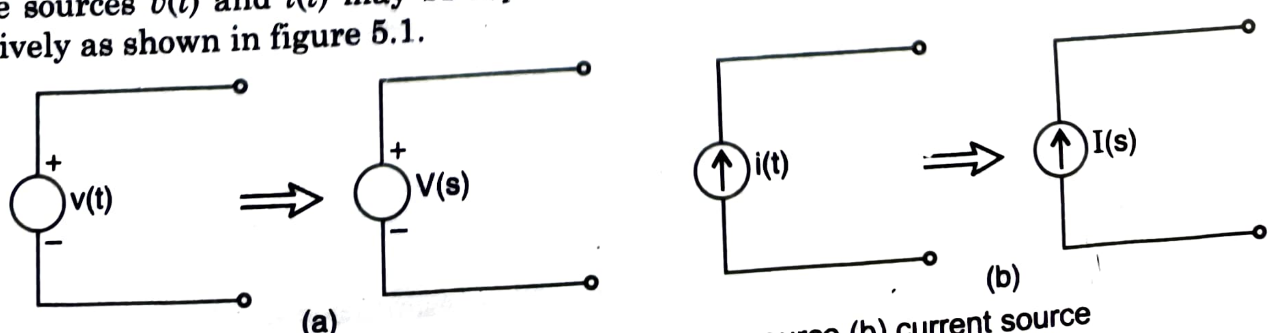


Fig. 5.1. Representation of (a) voltage source (b) current source

5.3.2. Resistance Parameter

By Ohm's law, the v–i relationship for a resistor in t-domain is

$$v_R(t) = Ri_R(t)$$

In the complex-frequency domain (s-domain), above equation becomes

$$V_R(s) = RI_R(s)$$

From above two equations, we observe that the representation of a resistor in t-domain and s-domain are one and the same as shown in figure 5.2.

5.3.3. Inductance Parameter

The v-i relationship for an inductor is

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{0^+}^{t} v_L(t)\, d(t) + i_L(0^+)$$

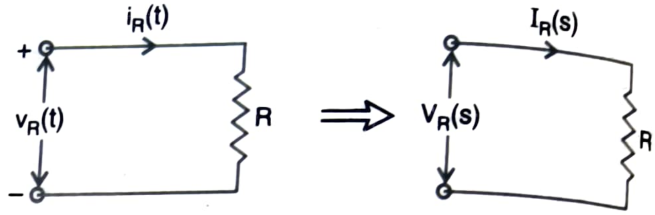


Fig. 5.2. Representation of a resistor

The corresponding Laplace transforms are

$$V_L(s) = sL\, I_L(s) - Li_L(0^+)$$

or $$I_L(s) = \frac{1}{sL}V(s) + \frac{i_L(0^+)}{s}$$

From above equations, we get the transformed circuit representation for the inductor as shown in figure 5.3.

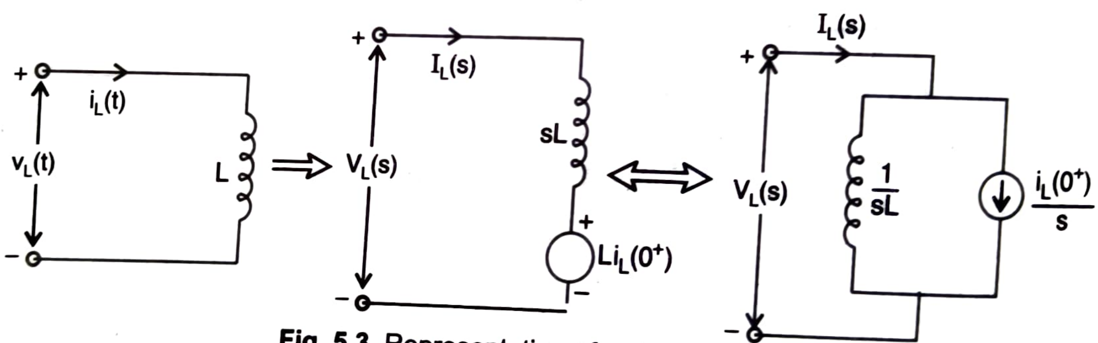


Fig. 5.3. Representation of an inductor

5.3.4. Capacitance Parameter

For a capacitor, the v–i relationship is

$$i_c(t) = C\frac{dv_c(t)}{dt} \quad \text{or} \quad v_c(t) = \frac{1}{C}\int_{0^+}^{t} i_c(t)\, dt + v_c(0^+)$$

The corresponding Laplace Transform are

$$I_c(s) = s\,C\,V_c(s) - C\,v_c(0^+)$$

or $$V_c(s) = \frac{1}{sC}I_c(s) + \frac{v_c(0^+)}{s}$$

From above equations, we get the transformed circuit representation for the capacitor as shown in figure 5.4.

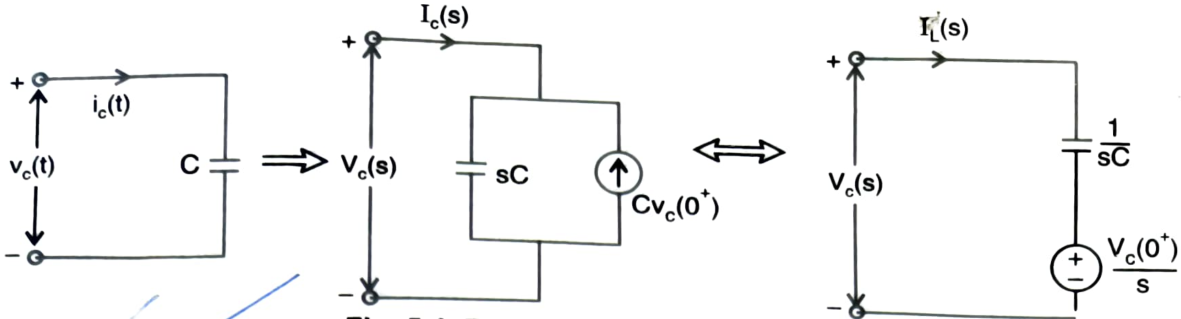


Fig. 5.4. Representation of a capacitor

EXAMPLE 5.4 Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = 5\,U(t)$$

Where $U(t)$ is unit-step function. The initial conditions are $y(0^+) = -1$ and $\frac{dy}{dt}(0^+) = 2$. Determine $y(t)$ for $t \geq 0$.

Solution : Taking Laplace transform on both sides of given differential equation :

$$s^2Y(s) - sy(0^+) - y'(0^+) + 3sY(s) - 3y(0^+) + 2Y(s) = \frac{5}{s}$$

Substituting the values of initial conditions and solving for $Y(s)$, we get

$$s^2Y(s) + s - 2 + 3sY(s) + 3 + 2Y(s) = \frac{5}{s}$$

or $$Y(s) = \frac{-s^2 - s + 5}{s(s^2 + 3s + 2)} = \frac{-s^2 - s + 5}{s(s+1)(s+2)}$$

Expanded by partial fraction expansion,

$$Y(s) = \frac{-s^2 - s + 5}{s(s+1)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = sY(s)\,|_{s=0} = \frac{5}{2}$$

$$K_2 = (s+1)\,Y(s)\,|_{s=-1} = \frac{-1+1+5}{(-1)(1)} = -5$$

$$K_3 = (s+2)\,Y(s)\,|_{s=-2} = \frac{-4+2+5}{(-2)(-1)} = \frac{3}{2}$$

Therefore, $$Y(s) = \frac{5}{2s} - \frac{5}{s+1} + \frac{3}{2(s+2)}$$

Hence, taking the inverse Laplace transform, we get the complete solution as

$$y(t) = \frac{5}{2} - 5e^{-t} + \frac{3}{2}e^{-2t} \quad ; t \geq 0$$

EXAMPLE 5.5 Consider the R–L circuit with $R = 4\ \Omega$ and $L = 1$H excited by a 48V d.c. source as shown in figure 5.5(a). Assume the initial current through the inductor is 3A. Using the Laplace transform determine the current $i\,(t)$; $t \geq 0$. Also draw the s-domain representation of the circuit.

Solution : Applying *KVL*,

$$R(i) + L\frac{di(t)}{dt} = 48$$

Taking Laplace transform

$$RI(s) + L[sI(s) - i_L(0^+)] = \frac{48}{s}$$

$$4I(s) + 1.[sI(s) - 3] = \frac{48}{s}$$

or $$I(s) = \frac{3s + 48}{s(s+4)}$$

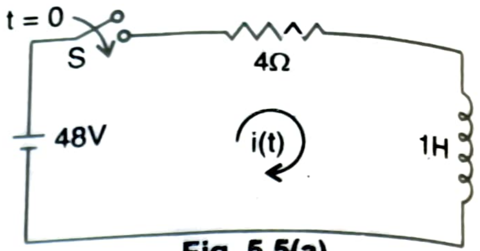


Fig. 5.5(a).

Applying the partial fraction expansion, we get

$$I(s) = \frac{3s+48}{s(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+4}$$

where $$K_1 = s.I(s)\,|_{s=0} = \left.\frac{3s+48}{s+4}\right|_{s=0} = 12$$

and $$K_2 = (s+4).I(s)\,|_{s=-4}$$

$$= \left.\frac{(3s+48)}{s}\right|_{s=-4} = -9$$

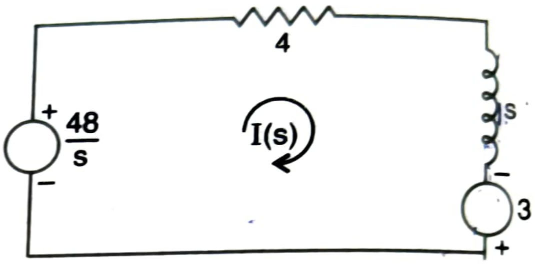


Fig. 5.5(b).

Then, $$I(s) = \frac{12}{s} - \frac{9}{s+4}$$

or $$i(t) = \mathcal{L}^{-1}[\,I(s)] = 12 - 9e^{-4t} \text{ A}$$

And the *s*-domain representation is shown in figure 5.5(*b*).

EXAMPLE 5.6 Consider a series R-L-C circuit with the capacitor initially charged to voltage of 1 V as indicated in figure 5.6(*a*). Find the expression for *i*(*t*). Also draw the s-domain representation of the circuit.

Solution : The differential equation for the current *i*(*t*) is

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt + v_c(0^+) = 0$$

and the corresponding transform equation is

$$L[sI(s) - i(0^+)] + RI(s) + \frac{1}{Cs}I(s) + \frac{v_c(0^+)}{s} = 0$$

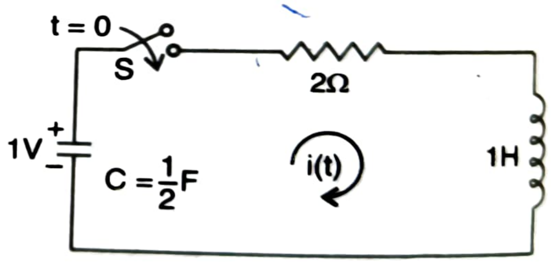


Fig. 5.6(a).

The parameters have been specified as $C = \frac{1}{2}F$, $R = 2\Omega$, $L = 1H$, and $v_c(0^+) = -1\,V$ (with the given polarity). The initial current $i(0^+) = 0$, because initially inductor behaves as a open circuit. The transform equation $I(s)$ then becomes

$$sI(s) + 2I(s) + \frac{2}{s}I(s) - \frac{1}{s} = 0$$

or $$I(s) = \frac{1}{s^2 + 2s + 2}$$

or, Completing the square,

$$I(s) = \frac{1}{(s+1)^2 + 1}$$

Therefore, $$i(t) = \mathcal{L}^{-1}[I(s)]$$

or $$i(t) = e^{-t}\sin t \text{ A}$$

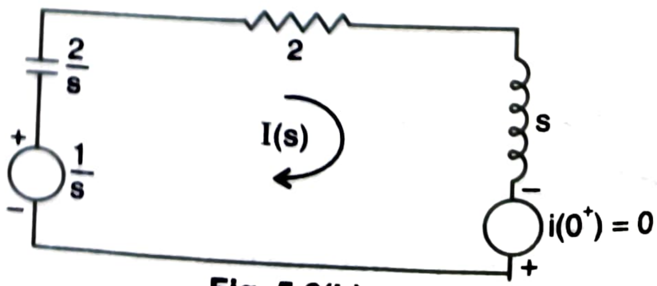


Fig. 5.6(b).

The *s*–domain representation is shown in figure 5.6(*b*).

EXAMPLE 5.7 **Consider the *R-C* parallel circuit as shown in figure 5.7(*a*) with $R = 0.5\Omega$ and $C = 4F$ excited by d.c. current source of 10 A. Determine the voltage across the capacitor by applying Laplace transformation. Assume the initial voltage across the capacitor as 2V. Also draw the s-domain representation of the circuit.**

Solution : Applying *KCL*,

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 10$$

Taking Laplace transform

$$\frac{V(s)}{R} + C[sV(s) - v(0^+)] = \frac{10}{s}$$

$$2V(s) + 4[sV(s) - 2] = \frac{10}{s}$$

or
$$V(s) = \frac{8s+10}{s(4s+2)}$$

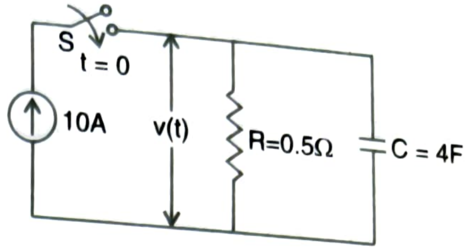


Fig. 5.7(a).

Applying the partial fraction expansion,

$$V(s) = \frac{2s+2.5}{s(s+0.5)} = \frac{K_1}{s} + \frac{K_2}{s+0.5}$$

$$K_1 = sV(s)|_{s=0} = 5$$

and
$$K_2 = (s+0.5)V(s)|_{s=-0.5} = -3$$

or
$$V(s) = \frac{5}{s} + \frac{-3}{s+0.5}$$

Therefore, $v(t) = \pounds^{-1}[V(s)] = 5 - 3e^{-0.5t}$ V

The *s*-domain representation of the circuit is shown in figure 5.7 (*b*).

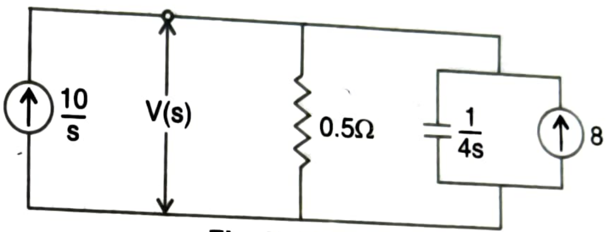


Fig. 5.7(b).

EXAMPLE 5.8 **In the network shown in figure 5.8, the switch *S* is closed at *t* = 0. With the network parameter values shown, find the expressions for $i_1(t)$ and $i_2(t)$, if the network is unenergized before the switch is closed.**

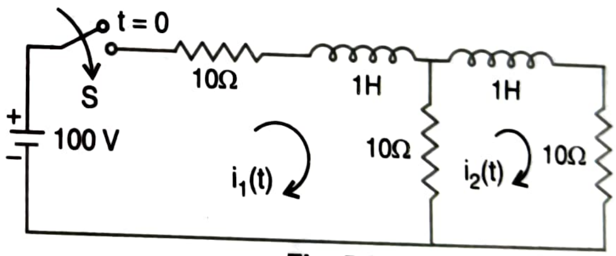


Fig. 5.8.

Solution : Applying *KVL*, Loop equations are

$$\frac{di_1(t)}{dt} + 10i_1(t) + 10[i_1(t) - i_2(t)] = 100$$

or
$$\frac{di_1(t)}{dt} + 20i_1(t) - 10i_2(t) = 100 \quad ...(i)$$

And,
$$\frac{di_2(t)}{dt} + 10i_2(t) + 10[i_2(t) - i_1(t)] = 0$$

or
$$\frac{di_2(t)}{dt} + 20i_2(t) - 10i_1(t) = 0 \quad ...(ii)$$

The transform equations of (*i*) and (*ii*) may be written as (keeping initial conditions are zero, as given) :

$$(s+20)\,I_1(s) - 10I_2(s) = \frac{100}{s}$$

and
$$-10I_1(s) + (s+20)\,I_2(s) = 0$$

Writing in matrix form,

$$\begin{bmatrix} s+20 & -10 \\ -10 & s+20 \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{100}{s} \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$I_1(s) = \frac{\Delta_1}{\Delta} \text{ and } I_2(s) = \frac{\Delta_2}{\Delta}$$

Where, $\Delta_1 = \begin{vmatrix} \frac{100}{s} & -10 \\ 0 & s+20 \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} s+20 & \frac{100}{s} \\ -10 & 0 \end{vmatrix}$; and $\Delta = \begin{vmatrix} s+20 & -10 \\ -10 & s+20 \end{vmatrix}$

Therefore, $\Delta_1 = \frac{100}{s}(s+20)$; $\Delta_2 = \frac{1000}{s}$

and $\Delta = (s+20)^2 - 100 = s^2 + 40s + 300$

So, $$I_1(s) = \frac{100(s+20)}{s(s^2+40s+300)}$$

and $$I_2(s) = \frac{1000}{s(s^2+40s+300)}$$

The partial fraction expansion of above expressions $I_1(s)$ and $I_2(s)$ are

$$I_1(s) = \frac{20/3}{s} + \frac{-5}{s+10} + \frac{-5/3}{s+30}$$

and $$I_2(s) = \frac{10/3}{s} + \frac{-5}{s+10} + \frac{5/3}{s+30}$$

The inverse Laplace transformation give $i_1(t)$ and $i_2(t)$ as

$$i_1(t) = \frac{20}{3} - 5e^{-10t} - \frac{5}{3}e^{-30t} \text{ A}$$

$$i_2(t) = \frac{10}{3} - 5e^{-10t} + \frac{5}{3}e^{-30t} \text{ A}$$

Which is the required solution.

EXAMPLE 5.9 **In the series R-L-C circuit shown in figure 5.9, There is no initial charge on the capacitor. If the switch S is closed at t = 0, determine the resulting current.**

Solution : The time-domain equation of the given circuit is

$$2i(t) + 1.\frac{di(t)}{dt} + \frac{1}{0.5}\int i(t)\,dt = 50$$

or $$2i(t) + \frac{di(t)}{dt} + 2\int i(t)\,dt = 50 \qquad (\because v_c(0^-) = 0)$$

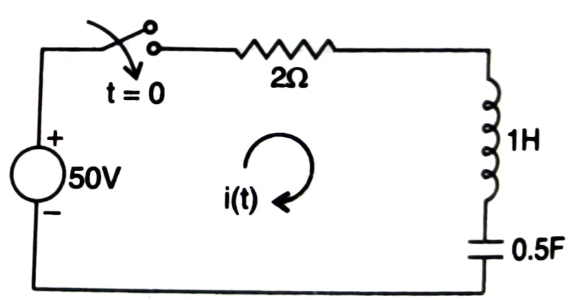


Fig. 5.9.

Taking Laplace transform,

$$2I(s) + sI(s) - i(0^+) + 2\frac{I(s)}{s} = \frac{50}{5}$$

Because $i(0^+) = 0$, therefore,

$$I(s) = \frac{50}{s^2+2s+2} = \frac{50}{(s+1)^2+1}$$

Therefore,

$$i(t) = £^{-1}[I(s)]$$

$$i(t) = 50\, e^{-t} \sin t \text{ A}$$

EXAMPLE 5.10 **Repeat Example 3.8 as shown in figure 3.11 using Laplace transform.**

Solution : Before the switching action takes place,

$$V = i'(t)(R_1 + R_2) + L\frac{di'(t)}{dt}$$

Taking Laplace transform,

$$\frac{V}{s} = I'(s)\,(R_1 + R_2) + L[s{\cdot}I'(s) - i'(0^+)]$$

Since $i'(0^+) = 0$, therefore,

$$I'(s) = \frac{V}{s(R_1 + R_2 + Ls)} = \frac{\frac{V}{L}}{s\left(s + \frac{R_1 + R_2}{L}\right)} = \frac{V}{R_1 + R_2}\left[\frac{1}{s} - \frac{1}{s + \frac{R_1 + R_2}{L}}\right]$$

$$i'(t) = \pounds^{-1}\,[I'(s)] = \frac{V}{R_1 + R_2}\left(1 - e^{-\frac{R_1 + R_2}{L}t}\right)$$

Therefore, $$i'(\infty) = \frac{V}{R_1 + R_2}$$

When switch is closed : R_2 is short circuited. Then

$$V = L\frac{di(t)}{dt} + R_1\,i(t)$$

Taking Laplace transform,

$$\frac{V}{s} = L[sI(s) - i(0^+)] + R_1 I(s)$$

$$i(0^+) = i'(\infty) = \frac{V}{R_1 + R_2}$$

$$\frac{V}{s} = (Ls + R_1)\,I(s) - \frac{LV}{R_1 + R_2}$$

$$(Ls + R_1)\,I(s) = \frac{V}{s} + \frac{LV}{R_1 + R_2}$$

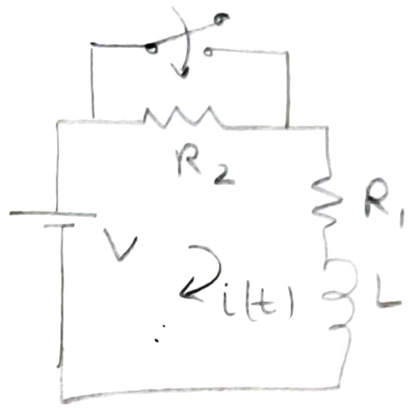
$$I(s) = \frac{V}{s(Ls + R_1)} + \frac{LV}{(R_1 + R_2)(Ls + R_1)}$$

$$I(s) = \frac{V}{R_1}\left[\frac{1}{s} - \frac{1}{s + \frac{R_1}{L}}\right] + \frac{V}{R_1 + R_2}\cdot\frac{1}{s + \frac{R_1}{L}}$$

$$= \frac{V}{R_1}\cdot\frac{1}{s} - \frac{1}{s + \frac{R_1}{L}}\left\{\frac{V}{R_1} - \frac{V}{R_1 + R_2}\right\} = \frac{V}{R_1}\cdot\frac{1}{s} - \frac{VR_2}{R_1(R_1 + R_2)}\cdot\frac{1}{s + \frac{R_1}{L}}$$

Therefore, $$i(t) = \pounds^{-1}\,[I(s)] = \frac{V}{R_1} - \frac{VR_2}{R_1(R_1 + R_2)}e^{-\frac{R_1}{L}t}$$

or $$i(t) = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1 + R_2}e^{-\frac{R_1}{L}t}\right) A$$



Switch S is closed at t = 0, a steady state current having previously been attained. Solve for i(t)

EXAMPLE 5.11 **Repeat Example 3.10 as shown in figure 3.13 using Laplace transform.**

Solution : Applying *KVL*,

$$1 = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt + v_c(0^+)$$

Taking Laplace transform and putting $R = 4$, $C = \dfrac{1}{16}$ and $v_c(0^+) = 9V$, we have

$$\frac{1}{s} = 4I(s) + \frac{16}{s}I(s) + \frac{9}{s}$$

$$-\frac{8}{s} = \left(4 + \frac{16}{s}\right)I(s)$$

$$I(s) = \frac{8}{(4s+16)} = -\frac{2}{s+4}$$

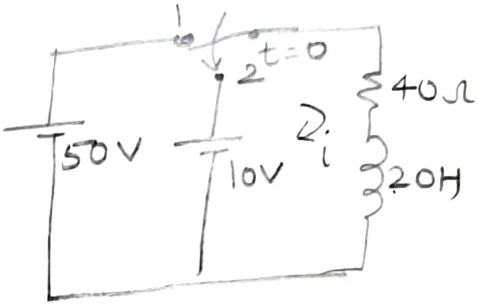
$$i(t) = \pounds^{-1}\,[I(s)] = -\,2e^{-4t}$$

Therefore, $$v_c(t) = 16\int_0^t i(t)\,dt + 9 = 16(-2)\int_0^t e^{-4t}\,dt + 9 = 8\cdot\left[e^{-4t}\right]_0^t + 9$$

or $$v_c(t) = 1 + 8e^{-4t} \text{ V}$$

EXAMPLE 5.12 **Repeat Example 3.13 as shown in figure 3.16 using Laplace transform.**

Solutions : With the switch on 1,



The switch has been in position 1 for a long time. It is moved to 2 at t = 0 obtain i for t > 0

$$50 = 40i'(t) + 20\frac{di'(t)}{dt}$$

Taking Laplace transform,

$$\frac{50}{s} = 40\,I'(s) + 20[sI'(s) - i'(0^+)]$$

Since $i'(0^+) = 0$, therefore,

$$I'(s) = \frac{50}{s(40+20s)} = \frac{2.5}{s(s+2)}$$

Using partial fraction expansion,

$$I'(s) = \left(\frac{2.5}{2}\right)\cdot\frac{1}{s} + \left(\frac{2.5}{-2}\right)\cdot\frac{1}{s+2} = 1.25\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

Therefore, $$i'(t) = \pounds^{-1}[I'(s)] = 1.25(1 - e^{-2t})$$

as $t \to \infty$

$$i'(\infty) = 1.25 \text{ A}$$

With the switch on 2,

$$10 = 40\,i(t) + 20\frac{di(t)}{dt}$$

Taking Laplace transform,

$$\frac{10}{s} = 40\,I(s) + 20[sI(s) - i(0^+)]$$

As $$i(0^+) = i'(\infty) = 1.25$$

Therefore, $$\frac{10}{s} = (40 + 20s)\,I(s) - 20 \times 1.25$$

$$I(s) = \frac{\left(\dfrac{10}{s} + 25\right)}{(40+20s)} = \frac{10+25s}{s(40+20s)}$$

or $$I(s) = \frac{25(s+0.4)}{20s(s+2)} = 1.25\left[\frac{s+0.4}{s(s+2)}\right]$$

$$v_c(t) = \frac{IR}{2}\left(1 - e^{-\frac{2}{RC}t}\right)$$

And $\quad i_c(t) = C\frac{d}{dt}[v_c(t)] = C\cdot\frac{IR}{2}\left[0 - \left(\frac{-2}{RC}\right)e^{-\frac{2}{RC}t}\right]$

or $\quad \boldsymbol{i_c(t) = I.e^{-\frac{2}{RC}t}\ A}$

EXAMPLE 5.15 **In the circuit of figure 5.12, S_1 is closed at $t = 0$, and S_2 is opened at $t = 4$ msec. Determine $i(t)$ for $t > 0$.**

(Assume inductor is initially de-energised)

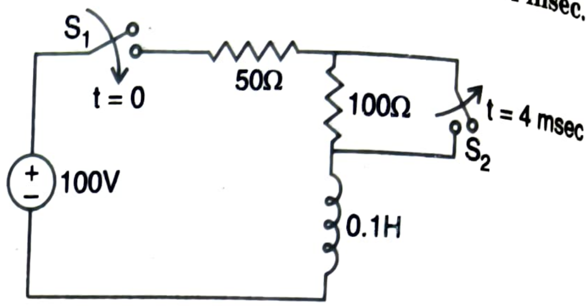


Fig. 5.12.

Solution : (*i*) for $0 \le t \le 4$ msec : (S_1 is closed and also S_2 is closed)

Loop equation becomes

$$100 = 50i(t) + 0.1\frac{di(t)}{dt}$$

Taking Laplace transform,

$$\frac{100}{s} = 50\,I(s) + 0.1\,[sI(s) - i(0^+)]$$

Since $\quad i(0^+) = 0$ (given).

Then

$$I(s) = \frac{100}{s(50 + 0.1s)} = \frac{1000}{s(s+500)}$$

Therefore, $\quad i(t) = \mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}\left[\frac{2}{s} - \frac{2}{s+500}\right] = \mathbf{2(1-e^{-500\,t})\ A}$

Thus $\quad i(4\times10^{-3}) = \mathbf{1.729\ A}$

(*ii*) For 4 msec $\le t < \infty$: If $t' = t - 4\times10^{-3}$, then $0 \le t' < \infty$

Loop equation becomes

$$100 = 150\,i(t) + 0.1\frac{di(t')}{dt}$$

Taking Laplace transform,

$$\frac{100}{s} = 150I(s) + 0.1[sI(s) - i(4\times10^{-3})]$$

$$\frac{100}{s} = (150 + 0.1s)\,I(s) - 0.1\times1.729$$

$$I(s) = \frac{100 + 0.1729s}{s(0.1s + 150)} = \frac{1.729\,s + 1000}{s(s+1500)}$$

$$I(s) = \frac{0.667}{s} + \frac{1.062}{s+1500} \qquad \text{(By partial fraction expansion)}$$

Therefore, $\quad i(t') = 0.667 + 1.062\cdot e^{-1500t'}$

or $\quad i(t) = 0.667 + 1.062\cdot e^{-1500(t-4\times10^{-3})}$

$\quad = 0.667 + 1.062\,e^{6}\cdot e^{-1500t}$

or $\quad \boldsymbol{i(t) = 0.667 + 428.4\,e^{-1500\,t}\ A}$

$$= 5\, e^{-2t} \text{ Cos } t + 10\, e^{-2t} \text{ Sin } t \quad \text{A}$$

EXAMPLE 5.18 **(a) Obtain the Laplace transform of the pulse shown in figure 5.15(a).**
(b) In figure 5.15(b) if $v(t)$ is the pulse (a), determine $i(t)$.

Solution :

$$v(t) = G_{0,1}(t) = U(t) - U(t-1)$$

$$V(s) = \frac{1}{s} - \frac{1}{s} e^{-s} = \frac{1}{s}(1 - e^{-s})$$

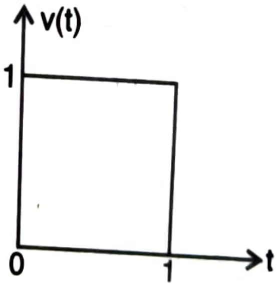


Fig. 5.15(a).

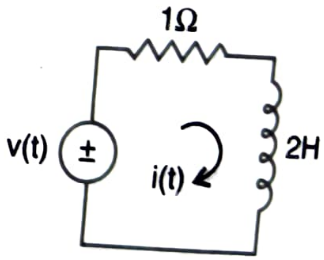


Fig. 5.15(b).

For the circuit in figure (b), applying KVL,

$$v(t) = 1.i(t) + 2\frac{di(t)}{dt}$$

Taking Laplace transform, (Assuming inductor is initially denergised)

$$V(s) = (1 + 2s)\, I(s)$$

or

$$I(s) = \frac{V(s)}{(2s+1)}$$

If $v(t)$ is $U(t)$, then

$$I_1(s) = \frac{1}{s}\cdot\frac{1}{2\left(s+\frac{1}{2}\right)} = \frac{1}{s} - \frac{1}{s+\frac{1}{2}}$$

$$i_1(t) = \left(1 - e^{-\frac{t}{2}}\right)U(t)$$

Similarly, if $v(t)$ is $U(t-1)$, then

$$I_2(s) = \frac{1}{s}e^{-s}\cdot\left(\frac{1}{2\left(s+\frac{1}{2}\right)}\right) = e^{-s}\left[\frac{1}{s} - \frac{1}{s+\frac{1}{2}}\right]$$

$$i_2(t) = \left(1 - e^{-\left(\frac{t-1}{2}\right)}\right)U(t-1)$$

Therefore, $$i(t) = i_1(t) - i_2(t) = \left(1 - e^{-\frac{t}{2}}\right)U(t) - \left(1 - e^{-\frac{(t-1)}{2}}\right)U(t-1)\ A$$

EXAMPLE 5.19 **In the network shown in figure 5.16, find current $i_2(t)$ in thecircuit.**

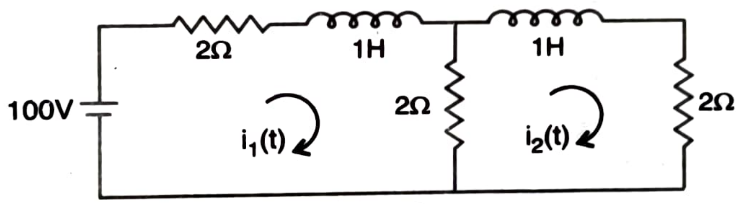


Fig. 5.16.

Solution : Applying *KVL*,

Loop 1: $$100 = 2i_1(t) + 1\cdot\frac{di_1(t)}{dt} + 2\,[i_1(t) - i_2(t)]$$

Taking Laplace transform, with $i_1(0^+) = 0$

$$\frac{100}{s} = 2I_1(s) + sI_1(s) + 2\,[I_1(s) - I_2(s)]$$

or $$\frac{100}{s} = (4+s)\,I_1(s) - 2I_2(s) \qquad \ldots(1)$$

Loop 2 : $$2[i_2(t) - i_1(t)] + 1\cdot\frac{di_2(t)}{dt} + 2i_2(t) = 0$$

Taking Laplace transform, with $i_2(0^+) = 0$

$$2[I_2(s) - I_1(s)] + sI_2(s) + 2I_2(s) = 0$$

$$(4+s)\,I_2(s) = 2I_1(s) \qquad \ldots(2)$$

From equation (2), putting the value of $I_1(s)$ in equation (1), we have

$$\frac{100}{s} = \left[(4+s)\cdot\frac{(4+s)}{2} - 2\right]I_2(s)$$

$$I_2(s) = \frac{200}{s(s^2+8s+12)} = \frac{200}{s(s+2)(s+6)}$$

Using partial fraction expansion

$$I_2(s) = \frac{50/3}{s} - \frac{25}{s+2} + \frac{25/3}{s+6}$$

Therefore, $$i_2(t) = £^{-1}[I_2(s)] = \left[\frac{50}{3} - 25e^{-2t} + \frac{25}{3}e^{-6t}\right]U(t)\ A$$

EXAMPLE 5.20 **In the circuit of figure 5.17, L = 2H, R = 12Ω and C = 62.5 mF. The initial conditions are $v_c(0^+)$ = 100V and $i_L(0^+)$ = 1.0 A. The switch is closed at t = 0. Find $i(t)$.**

Solution : Applying *KVL*,

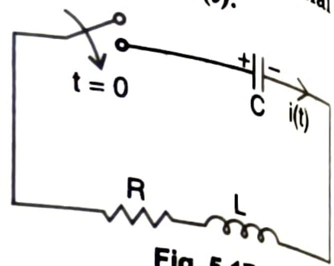


Fig. 5.17.

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)dt + v_c(0^+) = 0$$

Taking Laplace transform

$$L[sI(s) - i_L(0^+)] + RI(s) + \frac{1}{Cs}I(s) + \frac{v_c(0^+)}{s} = 0$$

$$2[sI(s) - 1] + 12I(s) + \frac{1}{(62.5\times10^{-3})}I(s) + \frac{100}{s} = 0$$

$$I(s).\left[2s+12+\frac{16}{s}\right] = -\frac{100}{s}+2$$

$$I(s) = \frac{-100+2s}{2s^2+12s+16} = \frac{s-50}{s^2+6s+8} = \frac{s-50}{(s+4)(s+2)}$$

Using partial fraction expansion

$$I(s) = \frac{27}{s+4} - \frac{26}{s+2}$$

Therefore, $$i(t) = £^{-1}[I(s)] = [27\,e^{-4t} - 26\,e^{-2t}]\,U(t)$$

EXAMPLE 5.21 **Find the current $i(t)$ for the network shown in figure 5.18, if the voltage source $v(t) = 2\,e^{-0.5\,t}\,U(t)$ and $v_c(0^+) = 0$.**

Solution : Applying *KVL*,

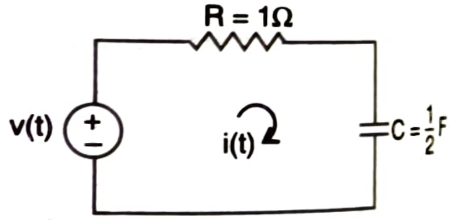


Fig. 5.18.

$$2\,e^{-0.5t}\,U(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

$$2\,e^{-0.5t}\,U(t) = Ri(t) + \frac{1}{C}\int_0^t i(t)dt + v_c(0^+)$$

Taking Laplace transform, with $R = 1\Omega$, $C = \frac{1}{2}$ F and $v_c\,(0^+) = 0$.

$$\frac{2}{s+0.5} = I(s) + \frac{2}{s}I(s) + 0$$

$$I(s) = \frac{2s}{(s+2)(s+0.5)}$$

$$1.\ \frac{di_L(t)}{dt} + \frac{1}{20\times10^{-6}}\int_0^t i_L(t)\,dt = 0\ ;\ [\text{with } i_L(0^+) = i_L(0^-) = 10 \text{ A}]$$

Taking Laplace transform, we have

$$sI_L(s) - i_L(0^+) + \frac{1}{20\times10^{-6}}\frac{I_L(s)}{s} = 0$$

$$\left[s + \frac{5\times10^4}{s}\right]I_L(s) = i_L(0^+) = 10$$

or

$$I_L(s) = \frac{10\,s}{s^2 + 5\times10^4} = \frac{10\,s}{s^2 + (223.6)^2}$$

Taking inverse Laplace transform, we have

$$i_L(t) = \mathbf{10\ cos\ 223.6}t \quad \mathbf{A}$$

EXAMPLE 5.34 **In the circuit of the figure 5.28, the switch S is closed at $t = 0$ with the capacitor initially unenergised. For the numerical values given, find $i(t)$.**

Solution : Applying KVL,

$$10\,e^{-t}\sin t = 10\,i(t) + \left\{\frac{1}{10\times10^{-6}}\int_0^t i(t)\,dt + 0\right\}$$

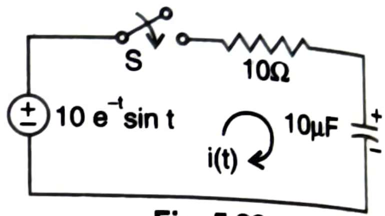


Fig. 5.28.

Taking laplace transform,

$$\frac{10}{(s+1)^2+1} = 10\,I(s) + 10\times10^4\,\frac{I(s)}{s}$$

$$I(s) = \frac{10\,s}{\{(s+1)^2+1\}(10\,s + 10\times10^4)} = \frac{s}{\{(s+1)^2+1\}(s+10^4)}$$

Using partial fraction expansion, we have

$$I(s) = \frac{As+B}{(s+1)^2+1} + \frac{C}{s+10^4} \cong \frac{10^{-4}\,s}{(s+1)^2+1} - \frac{10^{-4}}{s+10^4}$$

$$I(s) = 10^{-4}\left[\frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}\right] - 10^{-4}\,\frac{1}{s+10^4}$$

Taking inverse Laplace transform, we have

$$\mathbf{i(t) = 10^{-4}\,e^{-t}\,(cos\ t - sin\ t) - 10^{-4}\,e^{-10^4 t}}.$$

EXAMPLE 5.35 **Draw the transformed circuit diagram of the circuit in figure 5.29(a) and obtain the appropriate transformed nodal equation. Given $R = 1/2\ \Omega$, L ...**

$$1.\ \frac{di_L(t)}{dt} + \frac{1}{20 \times 10^{-6}} \int_0^t i_L(t)\,dt = 0\ ;\ [\text{with } i_L(0^+) = i_L(0^-) = 10\ \text{A}]$$

Taking Laplace transform, we have

$$sI_L(s) - i_L(0^+) + \frac{1}{20 \times 10^{-6}} \frac{I_L(s)}{s} = 0$$

$$\left[s + \frac{5 \times 10^4}{s}\right] I_L(s) = i_L(0^+) = 10$$

or

$$I_L(s) = \frac{10\,s}{s^2 + 5 \times 10^4} = \frac{10\,s}{s^2 + (223.6)^2}$$

Taking inverse Laplace transform, we have

$$i_L(t) = \mathbf{10 \cos 223.6t}\ \ \text{A}$$

EXAMPLE 5.34 **In the circuit of the figure 5.28, the switch S is closed at $t = 0$ with the capacitor initially unenergised. For the numerical values given, find $i(t)$.**

Solution : Applying KVL,

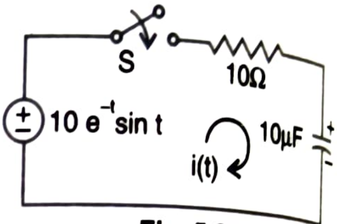


Fig. 5.28.

$$10\,e^{-t} \sin t = 10\,i(t) + \left\{\frac{1}{10 \times 10^{-6}} \int_0^t i(t)\,dt + 0\right\}$$

Taking laplace transform,

$$\frac{10}{(s+1)^2 + 1} = 10\,I(s) + 10 \times 10^4 \frac{I(s)}{s}$$

$$I(s) = \frac{10\,s}{\{(s+1)^2 + 1\}(10\,s + 10 \times 10^4)} = \frac{s}{\{(s+1)^2 + 1\}(s + 10^4)}$$

Using partial fraction expansion, we have

$$I(s) = \frac{As + B}{(s+1)^2 + 1} + \frac{C}{s + 10^4} \cong \frac{10^{-4}\,s}{(s+1)^2 + 1} - \frac{10^{-4}}{s + 10^4}$$

$$I(s) = 10^{-4}\left[\frac{s+1}{(s+1)^2 + 1} - \frac{1}{(s+1)^2 + 1}\right] - 10^{-4} \frac{1}{s + 10^4}$$

Taking inverse Laplace transform, we have

$$i(t) = \mathbf{10^{-4}\,e^{-t}\,(\cos t - \sin t) - 10^{-4}\,e^{-10^4 t}}.$$

EXAMPLE 5.35 **Draw the transformed circuit diagram of the circuit in figure 5.29(a) and obtain the appropriate transformed nodal equation. Given $R = 1/3\ \Omega$, $L = 0.5\ H$, $C = 1F$, $e(t) = 10$ V, $i_L(0^-) = 15$ A and $v_c(0^-) = 5$V. Than solve for $v(t)$.**

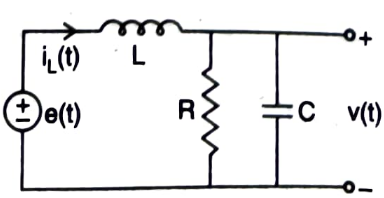


Fig. 5.29(a).

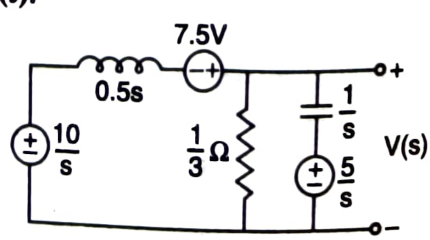


Fig. 5.29(b).

Solution : Transformed circuit diagram of the circuit of figure 5.29(a) is drawn in figure 5.29(b). Applying KCL, Nodal equation is given as,

$$\frac{\frac{10}{s}+7.5-V(s)}{0.5s} = \frac{V(s)}{1/3}+\frac{V(s)-\frac{5}{s}}{\frac{1}{s}}$$

or
$$\frac{20}{s^2}+\frac{15}{s} = \frac{2}{s}V(s)+3V(s)+sV(s)-5$$
$$(20+15s+5s^2) = (2s+3s^2+s^3)\,V(s)$$

or
$$V(s) = \frac{5\left(s^2+3s+4\right)}{s(s+1)(s+2)}$$

Using partial fraction expansion, we have

$$V(s) = \frac{10}{s}-\frac{10}{s+1}+\frac{5}{s+2}$$

Therefore,
$$v(t) = 10-10\,e^{-t}+5\,e^{-2t}\ \text{V}$$

EXAMPLE 5.36 **For the circuit in figure 5.30(a) with $R = 1\Omega$, $C = 1$F and $V_c(0^-) = 0$V, determine output (response) $v(t)$ when input $i(t)$ is (a) unit- impulse function and (b) unit-step function.**

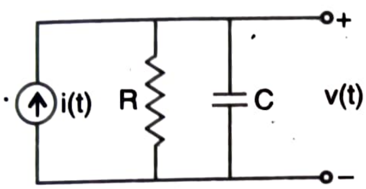


Fig. 5.30(a).

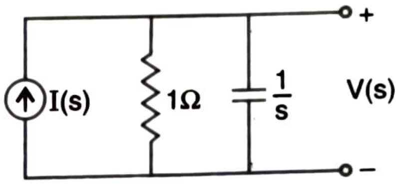


Fig. 5.30(b).

Solution : Using transformed circuit diagram as shown in figure 5.30(b), we have

$$V(s) = I(s).\left[1 \,\Big\|\, \frac{1}{s}\right] = \frac{I(s)}{s+1}$$

(a)
$$i(t) = \delta(t)\ ;\ \textit{i.e.},\ I(s) = 1$$

Then
$$V(s) = \frac{1}{s+1} \quad \text{or} \quad v(t) = e^{-t}\,U(t)$$

(b)
$$i(t) = U(t)\ ;\ \textit{i.e.},\ I(s) = \frac{1}{s}$$

Then
$$V(s) = \frac{1}{s(s+1)} = \frac{1}{s}-\frac{1}{s+1}$$

or
$$v(t) = (1-e^{-t})\,U(t)$$

EXAMPLE 5.37 **In the circuit shown in figure 5.31, C_1 is initially charged to a voltage V_0. At time $t = 0$, switch S is closed. Obtain the expressions for (i) current, (ii) voltage across R, (iii) charge across C_1, (iv) voltage across C_2 as a function of time, if $C_1 = C_2 = 1\mu$ F, $V_0 = 10$V and $R = 10\ \Omega$. Also calculate the values of (i) to (iv) after $t = 10\ \mu$ sec.**

Solution : Applying KVL,

or $$v(t) = 1 + 4e^{-10t}\text{ V}$$

EXAMPLE 5.41 **In the circuit shown in figure 5.34(*a*), the switch *S* is closed at $t = 0$ connecting a source e^{-t} to the RC circuit. At $t = 0$, it is observed that the capacitor voltage has the value $v_c(0) = 0.5$ V. For the element values given, determine $v_2(t)$.**

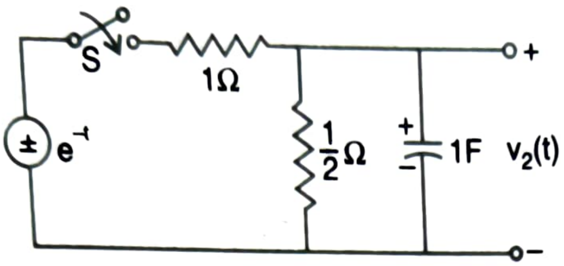


Fig. 5.34(a).

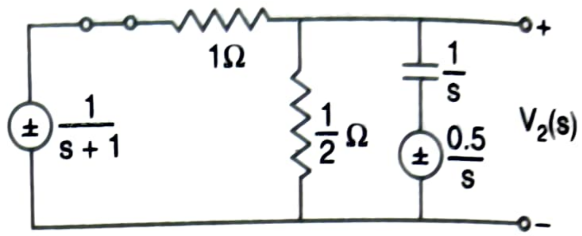


Fig. 5.34(b).

Solution : The transformed circuit diagram of the circuit of figure 5.34(*a*) at $t = 0^+$ is shown in figure 5.34(*b*).

Applying KCL,

$$\frac{\frac{1}{s+1} - V_2(s)}{1} = \frac{V_2(s)}{\frac{1}{2}} + \frac{V_2(s) - \frac{0.5}{s}}{\frac{1}{s}}$$

or $$\frac{1}{s+1} + 0.5 = (3 + s)\, V_2(s)$$

or $$V_2(s) = \frac{1 + 0.5(s+1)}{(s+3)\,(s+1)} = \frac{0.5\,(s+3)}{(s+3)\,(s+1)} = \frac{0.5}{s+1} = \mathbf{0.5\, e^{-t}\, V}$$

EXAMPLE 5.42 **In the circuit shown in figure 5.35, the switch *S* is moved from position 1 to position 2 at $t = 0$ (a steady state existing in position 1 before $t = 0$). Solve for the current $i_L(t)$.**

Solution : At position 1, the steady state current

$$i_L(0^-) = \frac{V}{R}$$

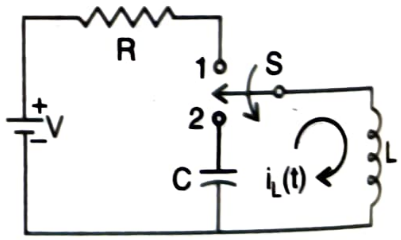


Fig. 5.35.

Now at position 2, applying KVL,

$$L\frac{di_L(t)}{dt} + \frac{1}{C}\int_0^t i_L(t)\, dt = 0$$

Taking Laplace transform, we have

$$L\left[sI_L(s) - i_L(0^+)\right] + \frac{1}{C}\frac{I_L(s)}{s} = 0$$

$$\left(Ls + \frac{1}{Cs}\right) I_L(s) = L.\frac{V}{R}$$

$$I_L(s) = \frac{LV}{R}\,\frac{Cs}{LCs^2 + 1} = \frac{V}{R}\cdot\frac{s}{s^2 + \frac{1}{LC}}$$

$$i(t) = 4e^{-4(t-1)} - e^{-(t-1)} \text{ A}$$

EXAMPLE 5.45 **A step voltage $3U(t-3)$ is applied to a series RLC circuit comprising resistor $R = 5\Omega$, inductor $L = 1$H and capacitor $C = \frac{1}{4}F$. Find the expression for current $i(t)$ in the circuit.**

(U.P.T.U., 2003 C.O.)

Solution : Applying KVL, (from figure 5.84)

$$3U(t-3) = 5i(t) + 1.\frac{di(t)}{dt} + 4\int_0^t i(t)\,dt$$

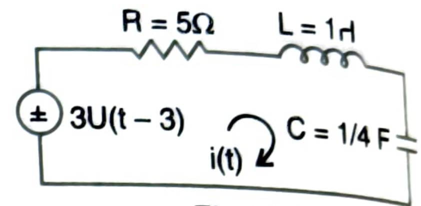


Fig. 5.38.

Taking Laplace transform,

$$\frac{3e^{-3s}}{s} = \left[5 + s + \frac{4}{s}\right] I(s) - i(0^+)$$

or
$$I(s) = \left(\frac{3e^{-3s}}{s^2 + 5s + 4}\right) \qquad \text{(Since } i(0^+) = 0\text{)}$$

Using partial fraction expansion, we have

$$I(s) = e^{-3s}\left[\frac{1}{s+1} - \frac{1}{s+4}\right]$$

or
$$i(t) = [e^{-(t-3)} - e^{-4(t-3)}]\, U(t-3)$$

EXAMPLE 5.46 **Using Laplace transformation, solve the following differential equation :**

$$\frac{d^2i}{dt^2} + 4\frac{di}{dt} + 8i = 8U(t)$$

Given that $i(0^+) = 3$ and $\frac{di}{dt}(0^+) = -4$.

(U.P.T.U., 2003)

Solution : $\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 8\,i(t) = 8U(t)$

Taking Laplace transform, we have

$$s^2I(s) - si(0^+) - i'(0^+) + 4[s\,I(s) - i(0^+)] + 8I(s) = \frac{8}{s}$$

$$(s^2 + 4s + 8)\, I(s) = \frac{8}{s} + (s+4)\, i(0^+) + i'(0^+)$$

$$= \frac{8}{s} + 3(s+4) - 4 \qquad \text{(Since } i(0^+) = 3 \text{ and } i'(0^+) = -4\text{)}$$

or $$I(s) = \frac{3s^2 + 3s + 8}{s(s^2 + 4s + 8)}$$

Using partial fraction expansion, we have

$$I(s) = \frac{1}{s} + 2\frac{(s+2)}{(s+2)^2 + (2)^2}$$

Therefore, $$i(t) = [1 + 2\,e^{-2t}\cos 2t]\ U(t)$$

EXAMPLE 5.47 **Find $i(t)$ for $t > 0$ in the circuit shown in figure 5.39. Switch is opened at $t = 0$.** *(U.P.T.U., 2003 C.O.)*

Solution : The steady state current in the inductor (with switch closed).

$$i_L(0^-) = \frac{12}{2} + \frac{16}{4} = 10\text{ A}$$

Now switch is opened, applying KVL,

$$12 = 2i(t) + 1\cdot\frac{di(t)}{dt}$$

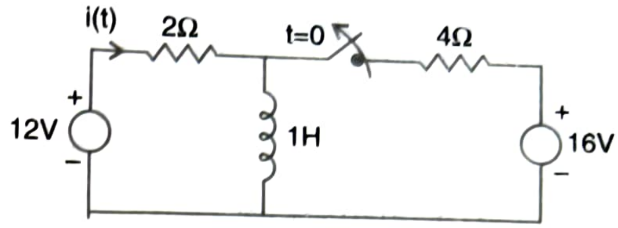


Fig. 5.39.

Taking Laplace transform,

$$\frac{12}{s} = 2\,I(s) + sI(s) - i_L(0^-)$$

or $$I(s) = \frac{\frac{12}{s} + 10}{s+2} = \frac{10s + 12}{s(s+2)}$$

Using partial fraction expansion, we have

$$I(s) = \frac{6}{s} + \frac{4}{s+2}$$

Therefore, $$i(t) = \mathbf{2(3 + 2e^{-2t})\ U(t)}$$

EXAMPLE 5.48 **In the network shown in Figure 5.40, the switch K is in position 1 long enough to establish steady state condition. At time $t = 0$, the switch K is moved from position 1 to 2. Find expression for the current $i(t)$.** *(U.P.T.U., 2003 C.O.)*

Solution : At position 1, the steady state current

$$i_L(0^-) = \frac{V_0}{R}$$

Now at position 2, applying KVL

$$\frac{1}{C}\int_0^t i(t) + L\frac{di(t)}{dt} = 0$$

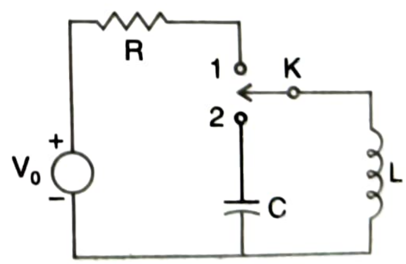


Fig. 5.40.

Taking Laplace transform,

$$\frac{1}{Cs}I(s) + L[s\,I(s) - i_L(0^+)] = 0$$

$$\left(\frac{1}{Cs} + Ls\right)I(s) = L.\frac{V_0}{R} \qquad \left(\text{Since } i_L(0^+) = i_L(0^-) = \frac{V_0}{R}\right)$$

or $$I(s) = \frac{LV_0/R}{(1 + LCs^2)/Cs} = \frac{V_0}{R}\left(\frac{s}{s^2 + \frac{1}{LC}}\right)$$

Therefore, $$i(t) = \frac{V_0}{R}\cos\left(\frac{t}{\sqrt{LC}}\right)U(t)$$

EXAMPLE 5.49. **Find out impulse response of the network shown in figure 5.41.**

(U.P.T.U., 2003 C.O.)

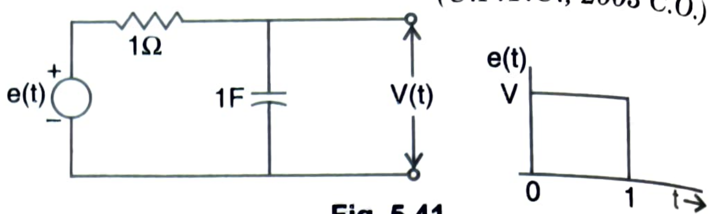


Fig. 5.41.

Solution : $$e(t) = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt$$

where $$e(t) = V[U(t) - U(t-1)]$$

Taking Laplace transform,

$$E(s) = \frac{V}{s}\left[1 - e^{-s}\right] = \left(1 + \frac{1}{s}\right)I(s)$$

[Since $R = 1\Omega$, $C = 1F$ and $v_c(0^+) = 0$]

$$I(s) = \frac{V[1 - e^{-s}]}{s+1}$$

And $$V(s) = \frac{1}{s}.I(s) = \frac{V[1 - e^{-s}]}{s(s+1)}$$

So, $$\frac{V(s)}{E(s)} = H(s) = \frac{1}{s+1}$$

Therefore, impulse response,

$$h(t) = £^{-1}[H(s)] = e^{-t}\,U(t)$$

EXAMPLE 5.50 **A voltage pulse of magnitude 8 volts and duration 2 seconds extending from $t = 2$ seconds to $t = 4$ seconds is applied to a series RL circuit as shown in figure 5.42(a). Obtain the expression for the current $i(t)$.**

(U.P.T.U., 2003 C.O.)

Solution : The voltage pulse is given as [as shown in figure 5.59(b)]

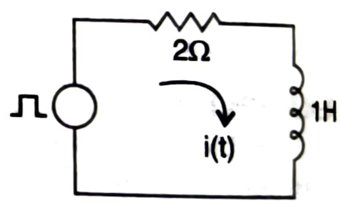


Fig. 5.42(a).

$$v(t) = 8[U(t-2) - U(t-4)]$$

or $$V(s) = \frac{8}{s}(e^{-2s} - e^{-4s})$$

Applying KVL in the circuit of figure 5.89(a), we have

$$v(t) = 2i(t) + 1.\frac{di(t)}{dt}$$

Taking Laplace transform,

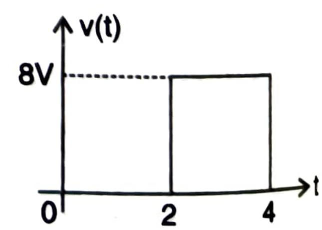


Fig. 5.42(b).

$$V(s) = 2I(s) + sI(s)$$

or $$I(s) = \frac{V(s)}{s+2} = \frac{8(e^{-2s} - e^{-4s})}{s(s+2)}$$

Using partial fraction expansion,

$$I(s) = e^{-2s}\left[\frac{4}{s} - \frac{4}{s+2}\right] - e^{-4s}\left[\frac{4}{s} - \frac{4}{s+2}\right]$$

or $$i(t) = 4(1 - e^{-2(t-2)})\,U(t-2) - 4\,(1 - e^{-2(t-4)})\,U(t-4)$$

EXAMPLE 5.51 **Determine the steady-state mesh currents i_1 and i_2 in the circuit of figure 5.43. There is no initial energy stored in the circuit.**

(U.P.T.U., 2004)

5.4. TRANSFER FUNCTION

The concept of a transfer function is of great importance in system studies. It provides a method of completely specifying the behaviour of a system subjected to arbitrary inputs. We define the transfer function of a fixed, linear system as the ratio of the Laplace transform of the system output to the Laplace transform of the system input, when all initial conditions are zero. For a perticular system, this transfer function is also called as system function $H(s)$, may be expressed as,

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{all initial Conditions are zero.}}$$

Where $Y(s)$ is the Laplace transform of the system output $y(t)$ and $X(s)$ is the Laplace transform of the system input $x(t)$.

This relation must hold true for any input. Suppose that $x(t) = \delta(t)$. Then it is obvious that $H(s)$ is the Laplace transform of the unit impulse response of the system, since $X(s) = 1$. Therefore, inverse Laplace transform of $H(s)$ is the unit impulse response of the system.

Note :

Unit impulse response of the system (also called as weighting function of the system) is the inverse Laplace transform of the system function.

Now, $$Y(s) = H(s) \cdot X(s)$$

Taking inverse Laplace transform, we have

$$y(t) = \pounds^{-1}[H(s) \cdot (X(s)]$$

$$= \int_0^t x(\tau).h(t-\tau)\,d\tau = \int_0^t x(t-\tau).h(\tau)\,d\tau$$

EXAMPLE 5.27 **Find the response of the system whose system function**

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1} \text{ for the input}$$

(i) $x(t) = \delta(t)$ (i.e. impulse response)
(ii) $x(t) = e^{-2t}$

Solution : $$H(s) = \frac{1}{s+1} \text{ or } h(t) = e^{-t}$$

(i) $$x(t) = \delta(t) \text{ (impulse function)}$$
$$X(s) = 1$$

Therefore, $$Y(s) = \frac{1}{s+1}$$

Hence, $$y(t) = e^{-t}$$

(ii) $$x(t) = e^{-2t}$$

or $$X(s) = \frac{1}{s+2}$$

$$y(t) = \int_0^t x(t-\tau).\,h(\tau)\,d\tau = \int_0^t e^{-2(t-\tau)}e^{-\tau}\,d\tau = e^{-2t}\int_0^t e^{\tau}\,d\tau = e^{-2t}\,[e^{t}-1]$$

or $$y(t) = e^{-t} - e^{-2t}$$

Alternatively : $$Y(s) = H(s)\cdot X(s) = \frac{1}{(s+2)\,(s+1)}$$

Using partial fraction expansion, we have

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

or $$y(t) = e^{-t} - e^{-2t}$$

EXAMPLE 5.28 **Find the response of a network if**

$$H(s) = \frac{s^2+3s+5}{(s+1)(s+2)} \text{ and excitation } x(t) = e^{-3t}.$$

Solution : $$x(t) = e^{-3t}$$

or $$X(s) = \frac{1}{s+3}$$

$$Y(s) = H(s)\cdot X(s)$$

or $$Y(s) = \frac{s^2+3s+5}{(s+1)(s+2)(s+3)}$$

Using partial fraction expansion, we get

$$Y(s) = \frac{A}{s+1} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$A = \left.\frac{s^2+3s+5}{(s+2)(s+3)}\right|_{s=-1} = \frac{1-3+5}{(1)(2)} = \frac{3}{2}$$

$$B = \left.\frac{s^2+3s+5}{(s+1)(s+3)}\right|_{s=-2} = \frac{4-6+5}{(-1).(1)} = -3$$

$$C = \left.\frac{s^2+3s+5}{(s+1)(s+2)}\right|_{s=-3} = \frac{9-9+5}{(-2)(-1)} = \frac{5}{2}$$

Now, $$Y(s) = \frac{\frac{3}{2}}{s+1} - \frac{3}{s+2} + \frac{\frac{5}{2}}{s+3}$$

Therefore, the response of the network is given by

$$y(t) = \pounds^{-1}[Y(s)] = \frac{3}{2}e^{-t} - 3e^{-2t} + \frac{5}{2}e^{-3t}$$