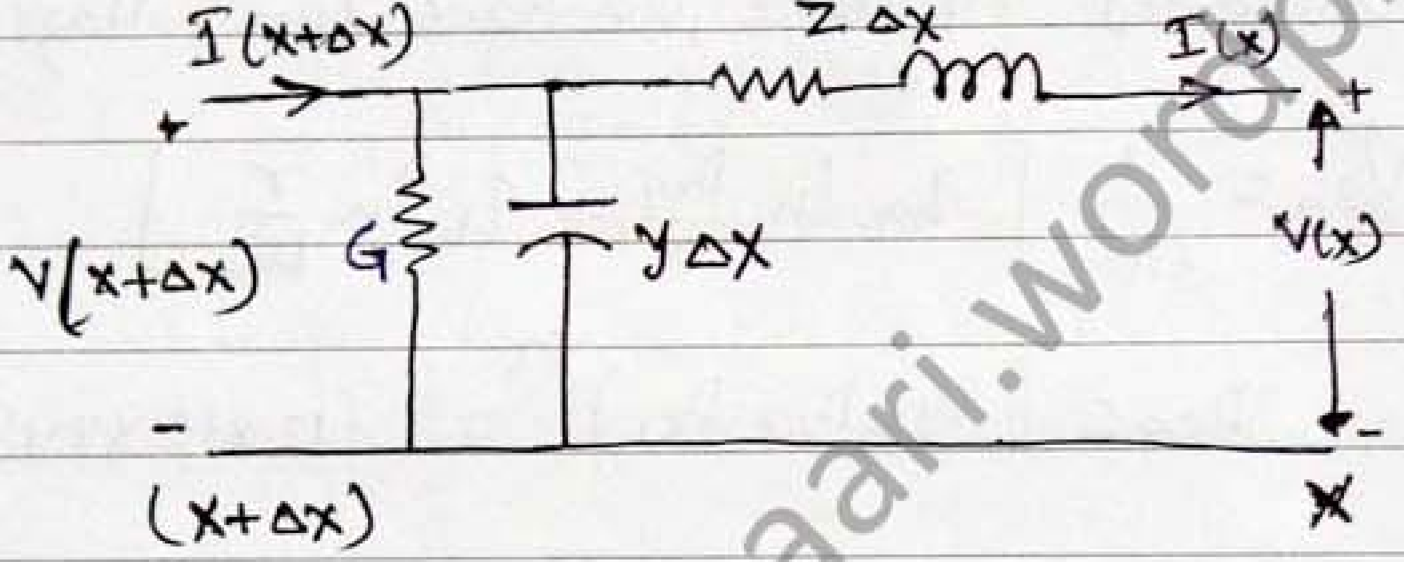
Lecture : 07

TRANSMISSION LINE MODELING

Lesson Summary:

1. Distributed Vs Lumped parameter model
2. Short line Model
3. Medium line Model
4. Long line Model
5. Voltage Regulation



A transmission line is cutted with Length Δx.
Let $Z =$ Impedance/unit Length.
For 60Hz line G is negligible. So, we neglect conductance.

Lumped Parameter model:

For Sinusoidal waves on overhead lines

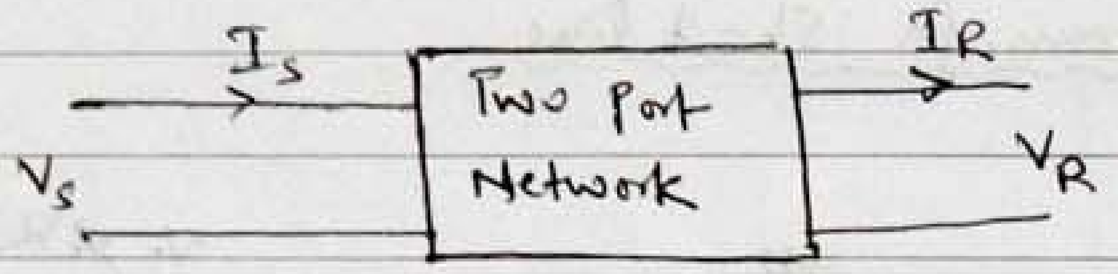
$$\lambda = \frac{c}{f} \quad \text{where } c = 3\times10^8 \text{ m/s}.$$

for 50Hz frequency $\lambda = 6000$KM.

So, for 6000KM long transmission line distributed effect is considered

→ When the line Length is <250KM then one can neglect the distributed effect and in this case distributed Network can be considered as Lumped N/W i.e. We can make the Lumped Network of distributed N/W when transmission line <250KM otherwise not.

→ Once we make Lumped parameter N/W then ~~it will be~~ we can consider this transmission line like Two Port Network.



$$V_s = AV_R + BI_R \quad \text{Volts}$$
$$I_s = CV_R + DI_R \quad \text{Ampere}$$

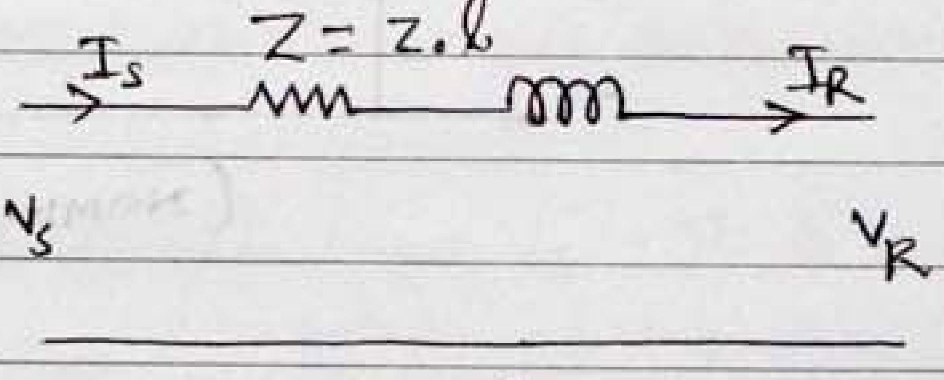
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad \boxed{AD - BC = 1}$$

→ For transmission line < 80km long, we neglect the effect of Capacitance bcoz the changing current is not very Large. Also these lines are medium/low voltage lines so Capacitive effect can be neglected.

SHORT LINE MODEL

- Line Length < 80KM
- Generally Medium/Low Voltage lines
- Capacitance can be neglected.

$Z = (R + j\omega) \cdot l$



From this circuit, $I_s = I_R$.

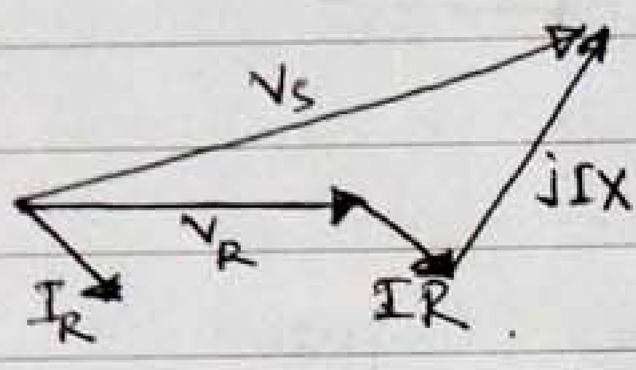
$V_s = V_R + Z I_R$.
$I_s = I_R$.

l = Length of T.L.
z = Impedance/length.

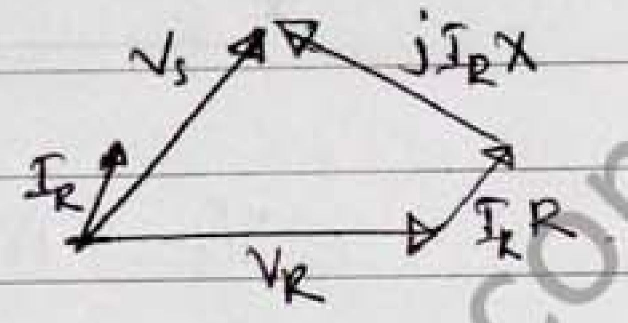
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\boxed{\begin{array}{l} A = D = 1 \text{ per unit} \\ B = Z\ \Omega \\ C = 0\ \Omega^{-1} \end{array}}$$

Phasor diagram for Short line



(Lagging P.f)



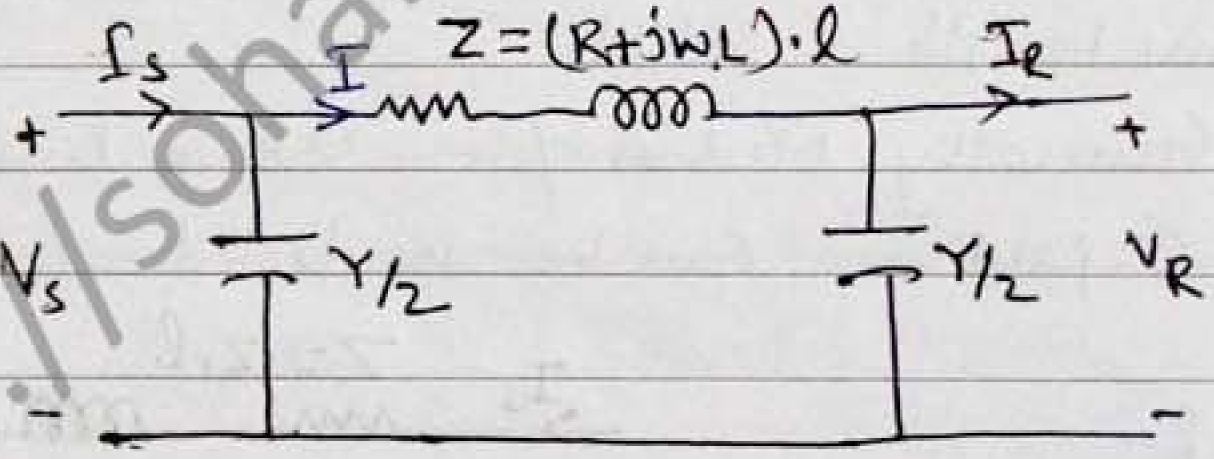
(Leading P.f.)

→ In case of Leading P.f. the sending end voltage magnitude can be lower than receiving end voltage magnitude.

→ For short line $V_s = V_R$ at No Load. (bcoz $I_R = 0$)

MEDIUM LINE MODEL

- 80KM < Length < 250KM
- Capacitance can't be Neglected
- EHV or HV lines.



(NOMINAL π Circuit)

$$V_S = V_R + IZ = V_R + \left(I_R + V_R Y/2\right) Z.$$

$$V_s = \left(1 + \frac{ZY}{2}\right) V_R + Z I_R.$$

$$I_s = I_R + V_R \frac{Y}{2} + V_S \frac{Y}{2}$$

$$= I_R + V_R \frac{Y}{2} + \left(\left(1 + \frac{YZ}{2}\right) V_R + I_R\right) \frac{Y}{2}$$

$$I_s = Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1+YZ/2 & Z \\ Y\left(1+\frac{YZ}{4}\right) & \left(1+\frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\boxed{\begin{aligned} &A = D = 1+YZ/2 \quad \text{Per unit} \\ &B = Z\ \Omega \\ &C = Y\left(1+\frac{YZ}{4}\right)\ S \text{ or } \Omega^{-1} \end{aligned}}$$

VOLTAGE REGULATION

It tells us how much the voltage drop in the transmission line in terms of rated voltage or sending end voltage.

$$\%VR = \frac{|V_{R(noload)}| - |V_{R(fullload)}|}{|V_{R(Fullload)}|} \times 100$$

For Short line $V_{R(noload)} = V_s$.

For Medium line $V_{R(noload)} = AV_s$

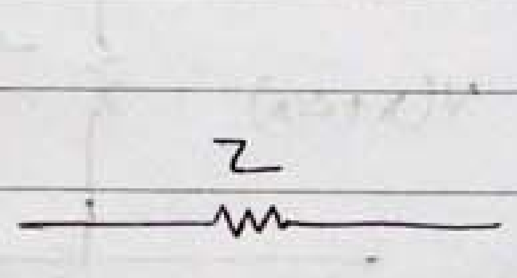
→ While designing a transmission line we must take care of VR i.e. VR must be less than 8-10%.

→ For Leading P.f., V_R may be greater than V_s. If it becomes very high then it may damage the insulator of transmission line.

ABCD Matrix

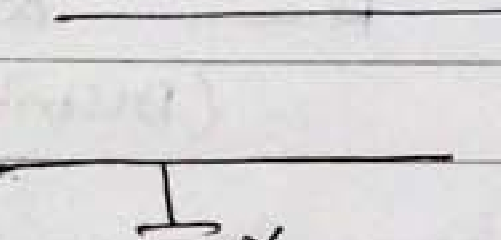
(a) Series Impedance

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

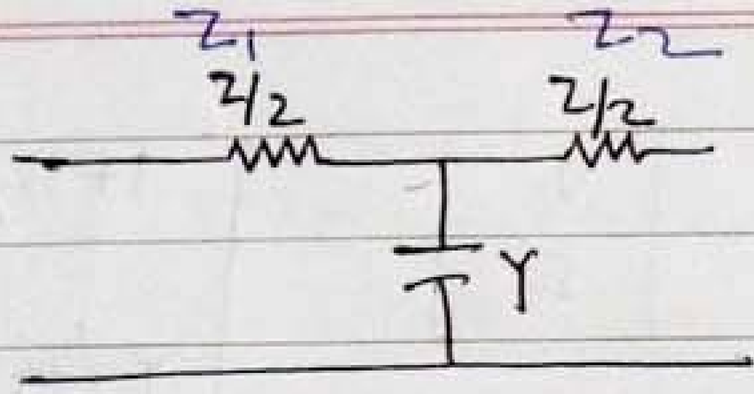


(b) Shunt Admittance

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

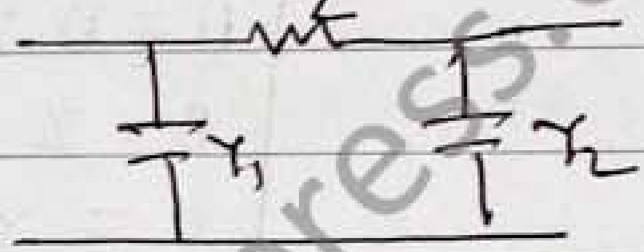


$$\begin{bmatrix} 1+YZ_1 & Z_1+Z_2+YZ_1Z_2 \\ Y & 1+YZ_2 \end{bmatrix}$$



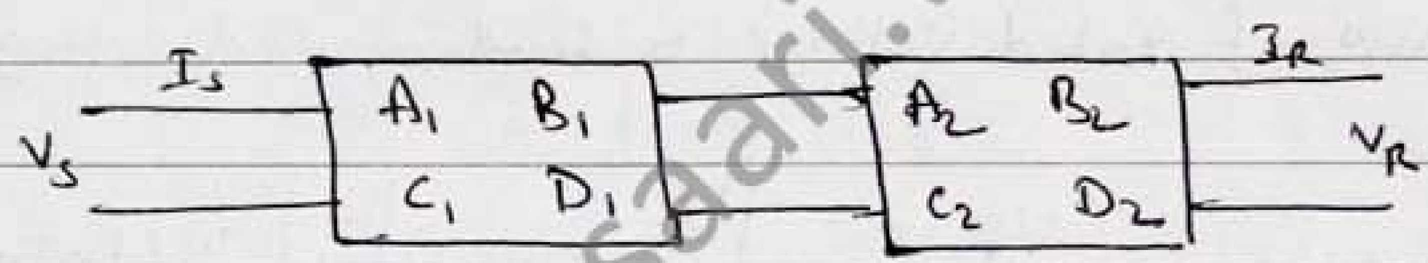
(T-Model)

$$\begin{bmatrix} 1+Y_2Z & Z \\ Y_1+Y_2+Y_1Y_2Z & 1+Y_1Z \end{bmatrix}$$



(π-Model)

$\boxed{AD-BC=1}$

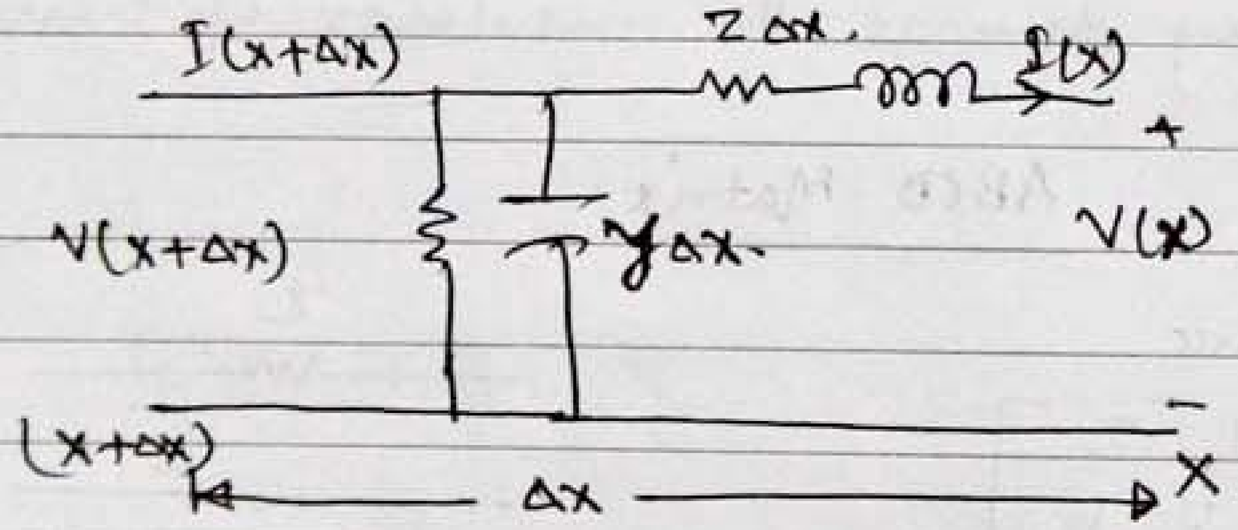


Cascaded n/w.

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ (C_1A_2+D_1C_2) & (C_1B_2+D_1D_2) \end{bmatrix}$$

LONG LINE MODEL

- Distributed Effect can't be Neglected bcoz T.L.>250KM.
- We can't use Lumped Model



(Distributed Parameter Model)

→ We have taken a section of transmission line for operation.

→ Conductance is neglected for 50-60Hz transmission line

Let voltage at a distance x from receiving end = V(x)
Let voltage at a distance (x+Δx) from receiving end = V(x+Δx)

By using KVL.

$$V(x+\Delta x) = V(x) + I(x) Z \Delta x.$$

$$Z I(x) = \frac{V(x+\Delta x) - V(x)}{\Delta x} = \frac{dV(x)}{dx}$$

$$Z I(x) = \frac{dV(x)}{dx} \quad \text{—— (I)}$$

By KCL

$$I(x+\Delta x) = I(x) + V(x+\Delta x)\, y\, \Delta x.$$

$$V(x+\Delta x)\, y = \frac{I(x+\Delta x) - I(x)}{\Delta x} = \frac{dI(x)}{dx}.$$

Taking $\lim_{\Delta x \to 0}$, we have $V(x+\Delta x) = V(x)$.

$$\therefore \; y V(x) = \frac{dI(x)}{dx} \quad \text{—— (II)}$$

~~For~~ Differentiating (I) w.r.t x.

$$\frac{d^2V(x)}{dx^2} = Z \frac{dI(x)}{dx} = Zy\, V(x).$$

$$\boxed{\frac{d^2V(x)}{dx^2} - Zy\, V(x) = 0.}$$

$$\boxed{V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \text{ volts}}$$

$$\gamma = \sqrt{Zy} \;.\; m^{-1}.$$

$$\frac{dV(x)}{dx} = A_1 \gamma e^{\gamma x} - \gamma A_2 e^{-\gamma x} = Z I(x).$$

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{(Z/\gamma)}$$

$$\boxed{I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z_c}}$$

where $Z_c = Z/\gamma$.
characteristic impedance

characteristic Impedance $= \frac{Z}{\gamma} = \frac{Z}{\sqrt{Zy}} = \sqrt{\frac{Z}{y}}\ \Omega$

$\gamma \rightarrow$ Gamma $y \rightarrow$ Admittance

$$\boxed{Z_c = \sqrt{\frac{Z}{y}}\ \Omega}$$

→ Using Boundary condition.

At $x=0$ $V(0) = V_R$.

$I(0) = I_R$.

$\therefore V_R = A_1 + A_2$

$I_R = \frac{A_1 - A_2}{Z_c}$

$\therefore A_1 = \frac{V_R + Z_c I_R}{2}, A_2 = \frac{V_R - Z_c I_R}{2}.$

$\therefore V(x) = \left(\frac{V_R + Z_c I_R}{2}\right) e^{\gamma x} + \left(\frac{V_R - Z_c I_R}{2}\right) e^{-\gamma x}$. (γ → Gamma)

$V(x) = \left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right) V_R + Z_c \left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right) I_R$

$$\boxed{V(x) = \cosh(\gamma x) V_R + Z_c \sinh(\gamma x) I_R}$$

$$\boxed{I(x) = \frac{1}{Z_c} \sinh(\gamma x) V_R + \cosh(\gamma x) I_R}$$ (γ = Gamma)

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \\ C(x) & D(x) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

At sending end $x = l$.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$A = D = \cosh(\gamma l)$ Perunit (γ → Gamma)

$B = Z_c \sinh(\gamma l)\ \Omega$

$C = \frac{1}{Z_c} \sinh(\gamma l)\ S$