Contents: Reduction Techniques

- *Block Diagram Reduction*
- *Signal flow graph*

Control Systems

Apply KCL at node 2.

$$\frac{1}{L_1}\int(V_2(t)-V_1(t)) + \frac{1}{R_1}[V_2(t)-V_1(t)) + C_2\frac{dV_2}{dt} + \frac{1}{R_2}(V_2(t)-V_3(t)) = 0$$

$$\Rightarrow \frac{1}{L_1 S}[V_2(S)-V_1(S)) + \frac{1}{R_1}[V_2(S)-V_1(S)] + C_2 S\, V_2[S] + \frac{1}{R_2}V_2[S] - \frac{1}{R_2}V_3(S)$$

$$- V_1(S)\left[\frac{1}{L_1 S}+\frac{1}{R_1}\right] + V_2(S)\left[\frac{1}{L_1 S}+\frac{1}{R_1}+C_2 S+\frac{1}{R_2}\right] - V_3(S)\frac{1}{R_2} = 0$$

$$\Rightarrow -\phi_1(S)\left[\frac{S}{R_1}+\frac{1}{L_1}\right] + \phi_2(S)\left[C_2 S^2 + S\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{L_1}\right] - \frac{S}{R_2}\phi_3(S) = 0. \rightarrow (8)$$

Apply KCL at node-3.

$$\frac{1}{L_2}\int V_3(t)dt + C_3\frac{dV_3}{dt} + \frac{1}{R_2}[V_3(t)-V_2(t)) = 0$$

$$\Rightarrow \frac{1}{L_2 S}V_3(S) + C_3 S\, V_3(S) + \frac{1}{R_2}V_3(S) - \frac{1}{R_2}V_2(S) = 0$$

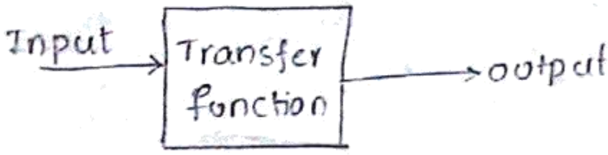
$$\Rightarrow -\frac{S}{R_2}\phi_2(S) + \phi_3(S)\left[C_3 S^2 + \frac{S}{R_2} + \frac{1}{L_2}\right] = 0 \rightarrow (9)$$

and comparing Eqns (7), (8) & (9) with (1), (2) & (3) they are same.

* Block Diagram Representation of Systems :

A block diagram of a system is a pictorial Representation of the system functions performed by each component and representation of the signal flow. The elements of a block diagram are Block, Branch point and summing point.

Blocks :

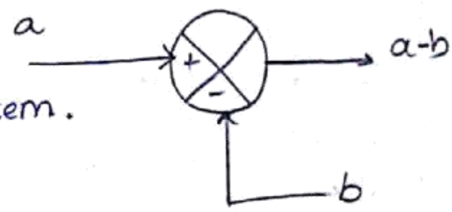


In a block diagram all system variables are linked to each other through functional blocks. The arrow head pointing towards the block indicates the input and the arrow head leading away from the block represents the output.

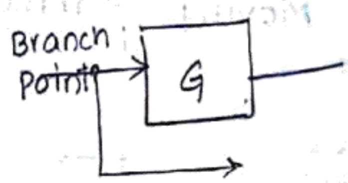
The transfer functions of the components are usually entered in the corresponding blocks.

Summing Point :

Summing points are used to add two or more signals in the system.



Branch Point :

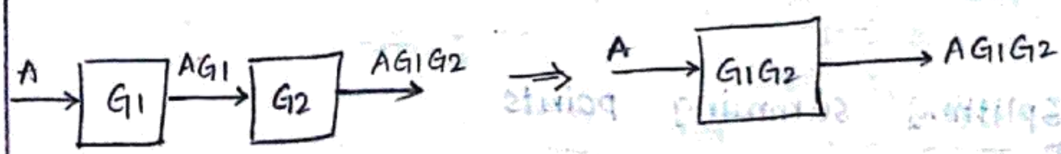


* Block Diagram Algebra :

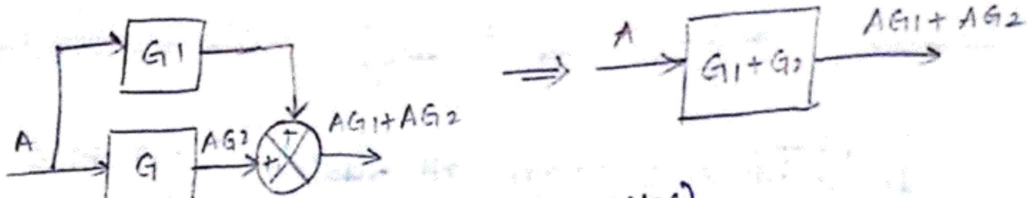
Block Diagram can be used to find overall transfer function of the system. Rules are framed such that any modification made on the diagram does not alter the input - output relation.

Rules :

1. Combining the blocks in cascade

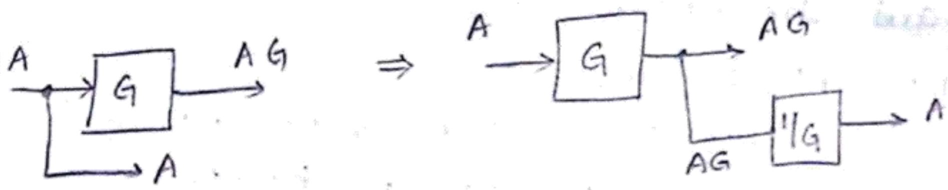


2. Combining blocks in parallel :

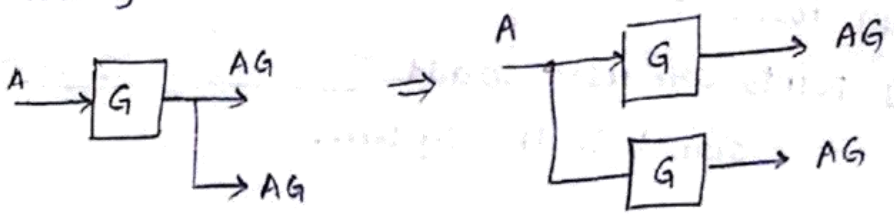


(combining feed forward paths)

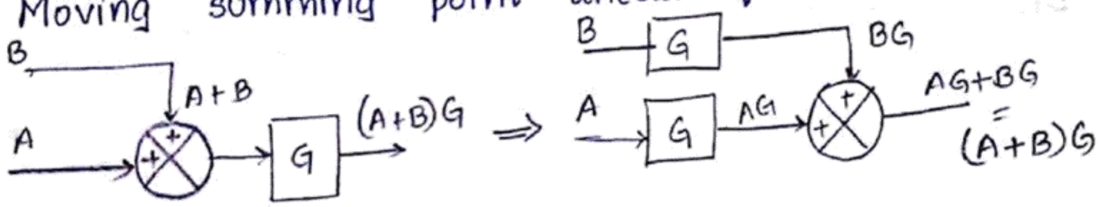
3. Moving the branch point ahead of the block.



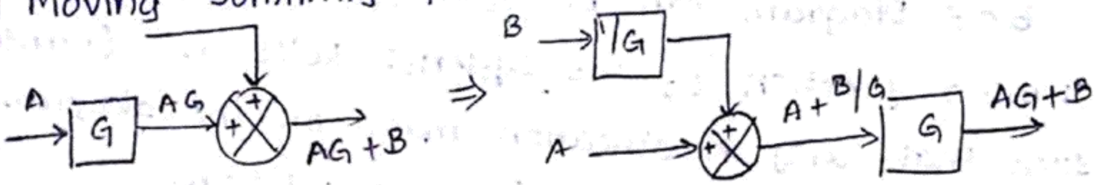
4. Moving the branch point before the block



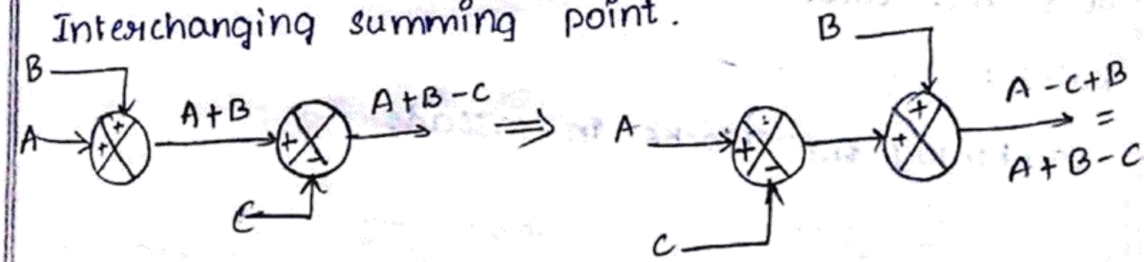
5. Moving summing point ahead of the block.



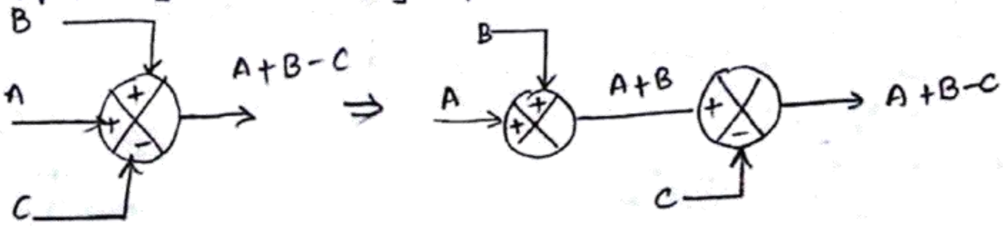
6. Moving summing point before the block.



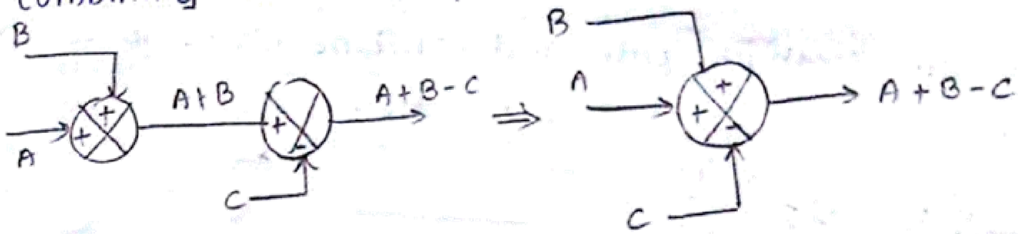
7. Interchanging summing point.



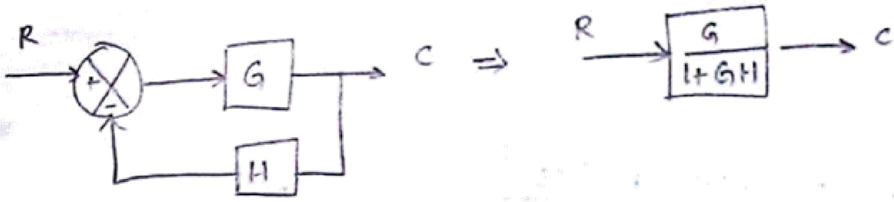
8. Splitting summing points



9. Combining summing points :



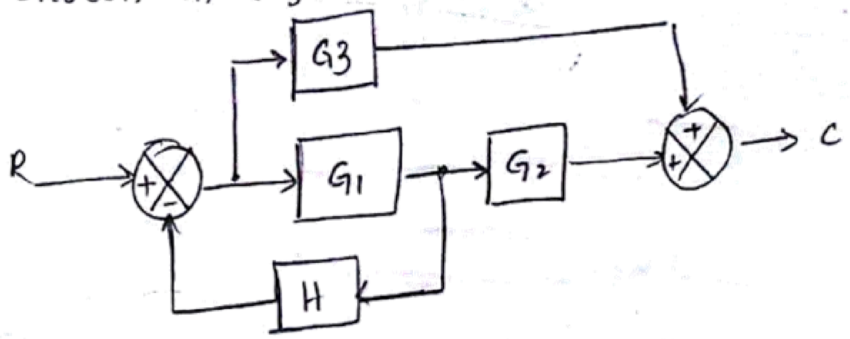
10 Elimination of Feedback loop :



Steps to Reduce Block diagram :

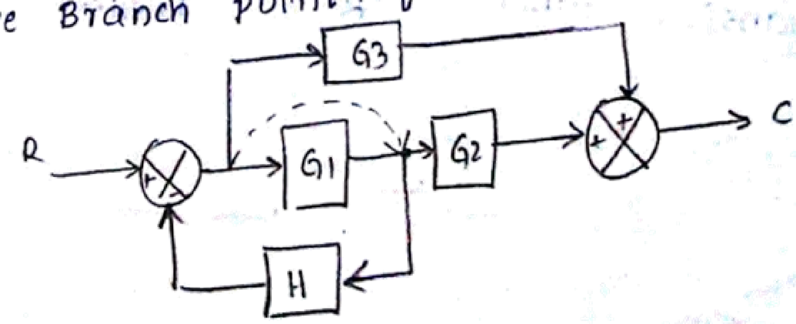
1. Reduce series blocks
2. Reduce parallel blocks
3. Reduce minor feedback loops
4. Shift summing point to the left and take off point to the right as far as possible.
5. Repeat steps 1 to 4 until final transfer function is obtained.

1. Using Block diagram Reduction technique find the transfer function of the system whose block diagram is shown in figure below.



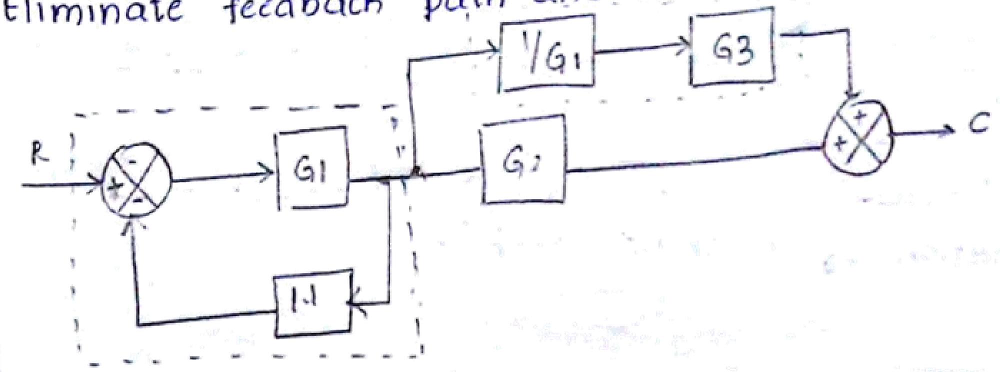
Sol:- Step-1 :

Move Branch point after the block.



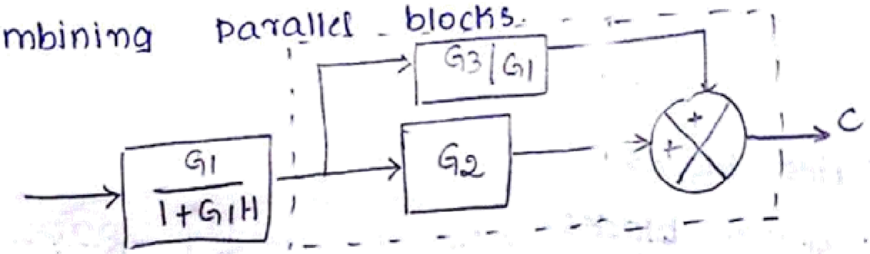
Step-2 :

Eliminate feedback path and combine blocks in cascade



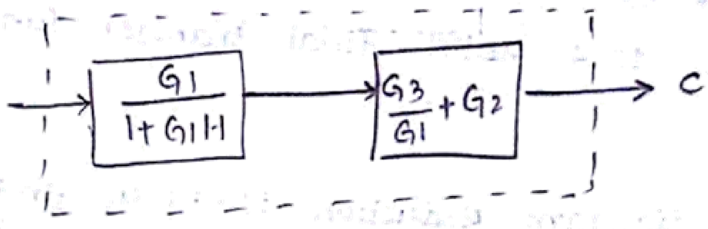
Step-3 :

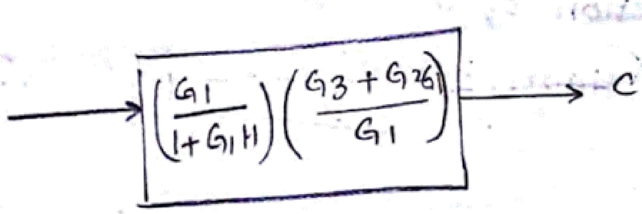
Combining parallel blocks.

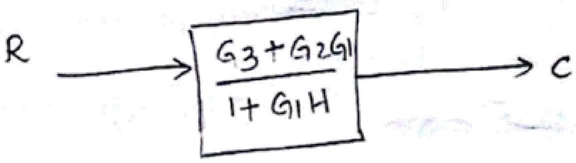


Step-4 :

Combining blocks in cascade.

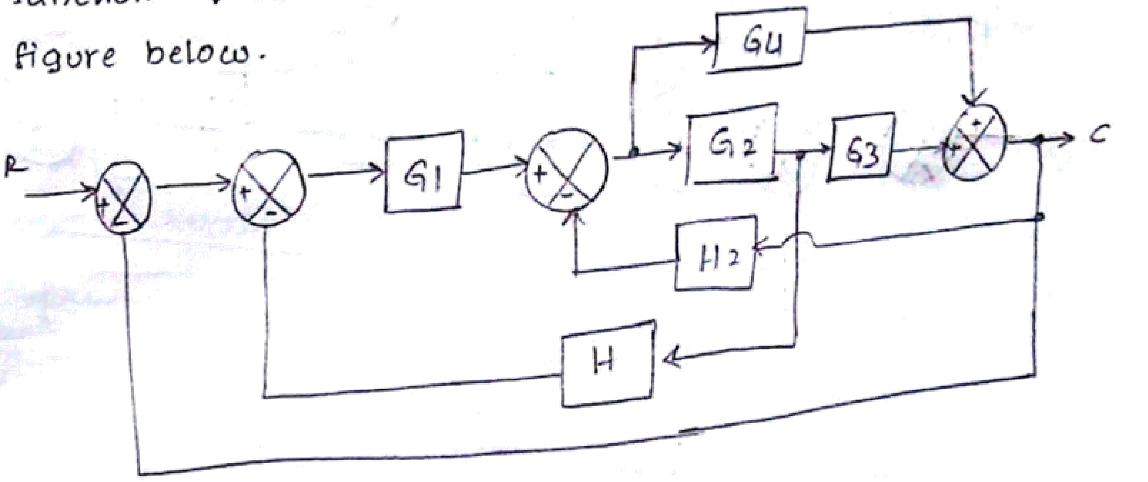






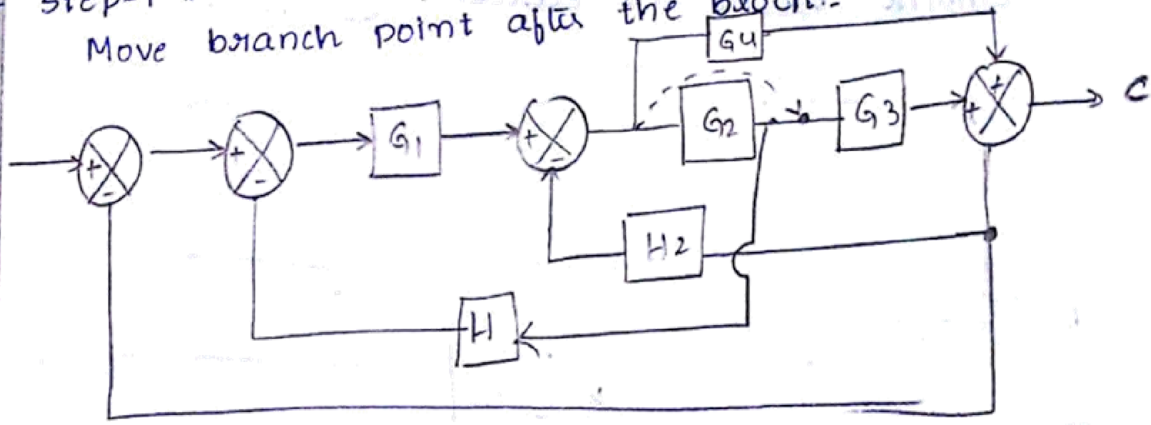
$\Rightarrow$ Transfer function $\frac{C}{R} = \frac{G_3+G_2G_1}{1+G_1H}$

2. Using block diagram Reduction Technique, find transfer function of the system whose block diagram is shown in figure below.



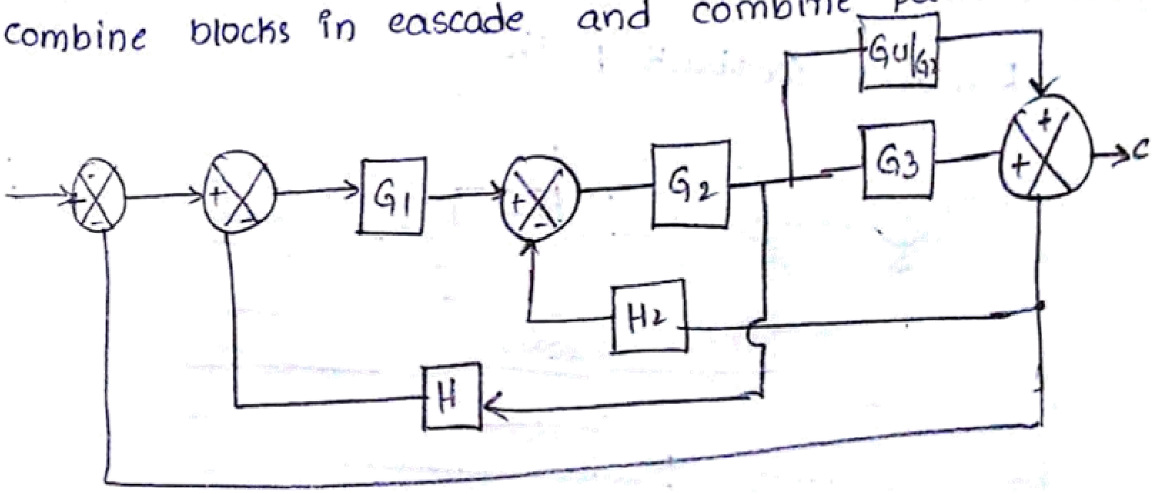
Sol:- Step-1 :

Move branch point after the block.

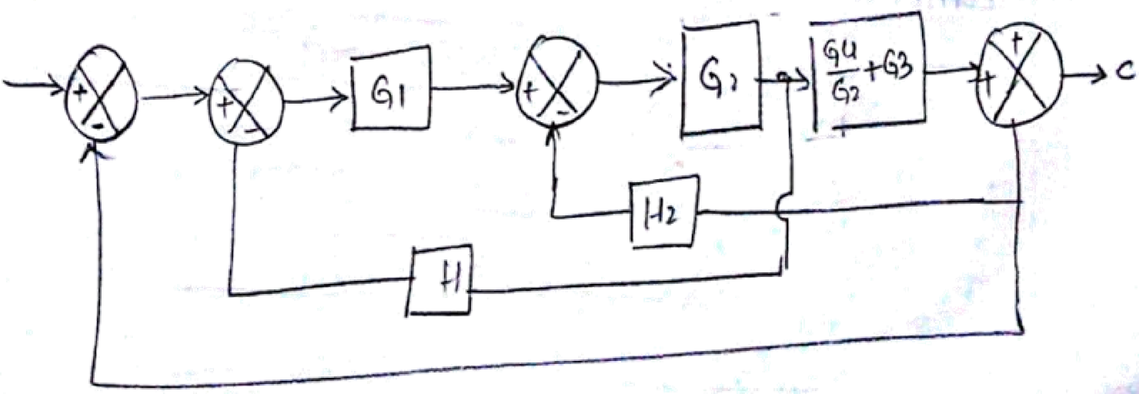


Step-2 :

Combine blocks in cascade and combine parallel blocks.

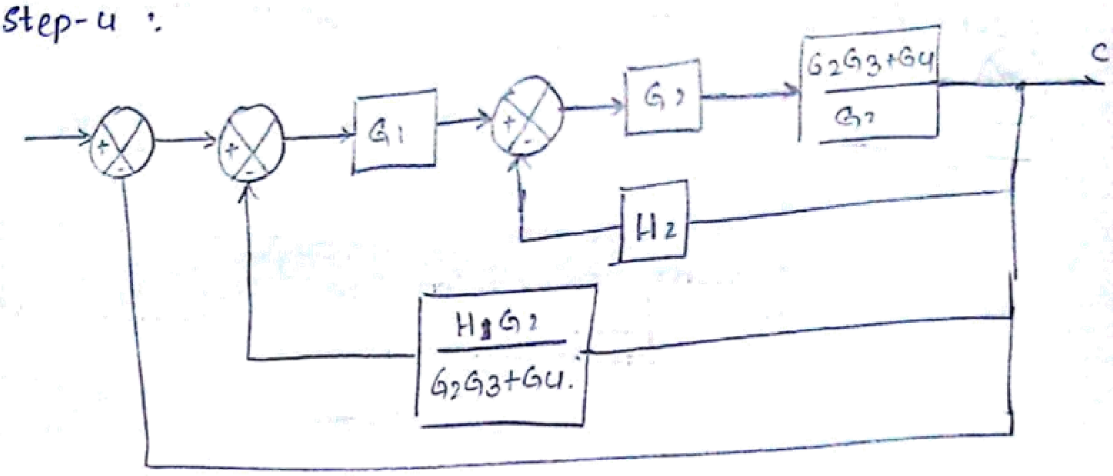


Step-3 :

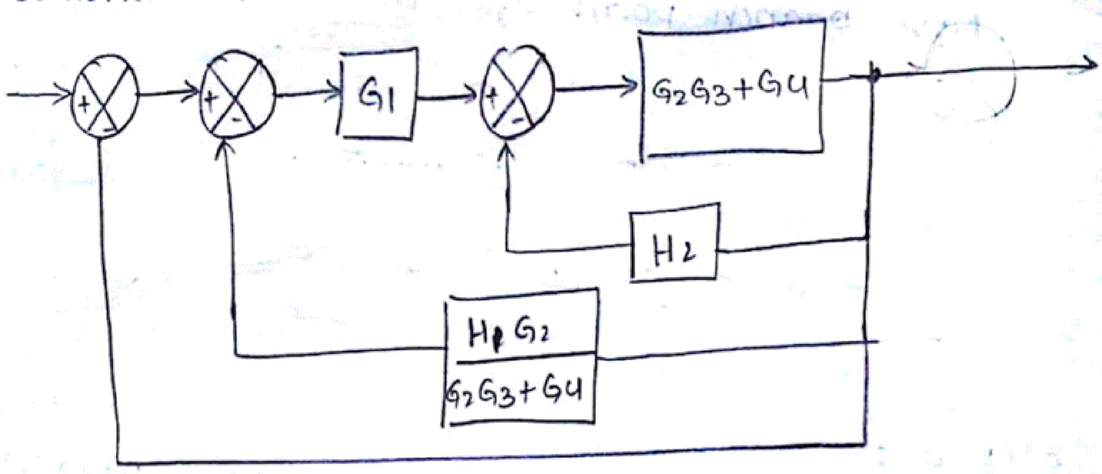


Moving take off point ahead of the block :

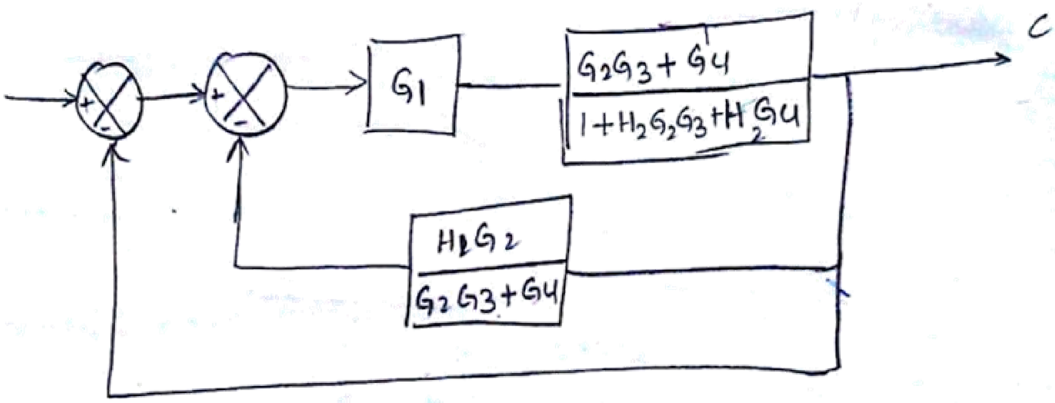
Step-4 :



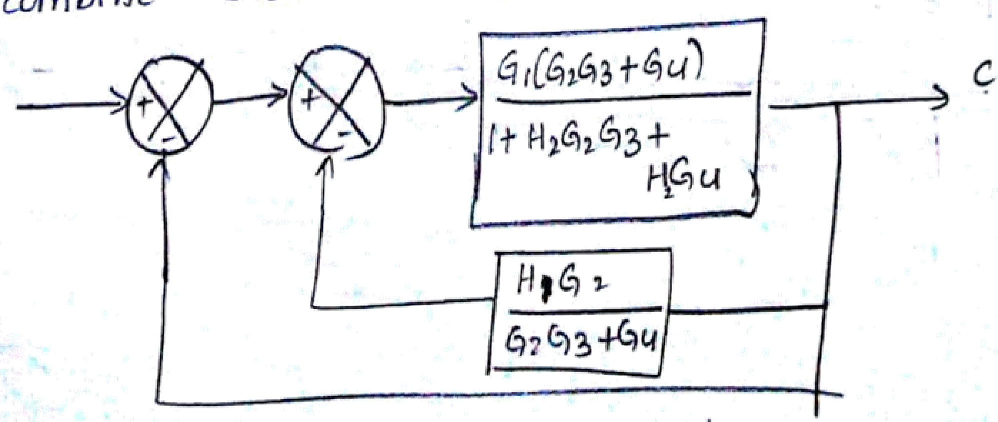
Combine blocks in cascade.



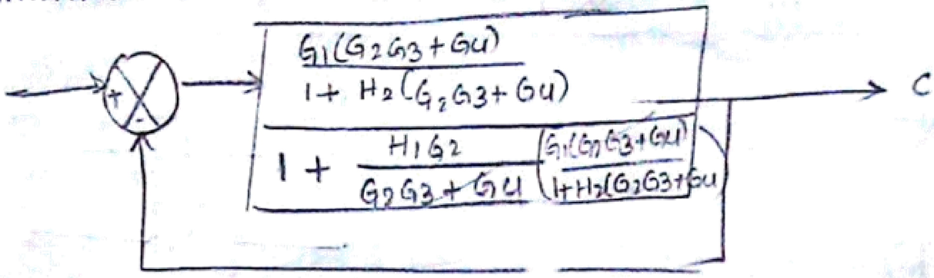
Remove . feedback path

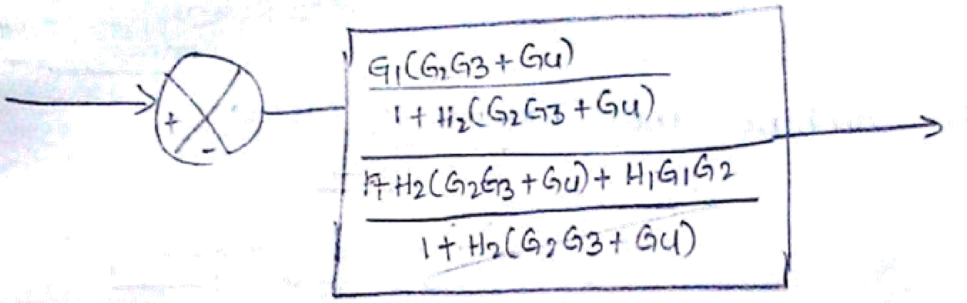


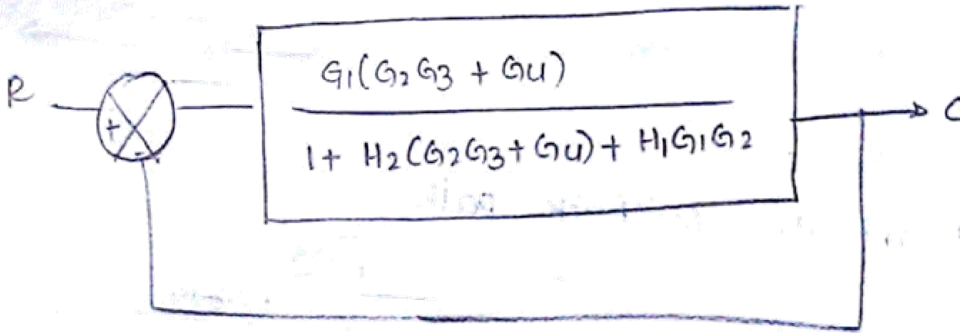
combine blocks in cascade.



Eliminate feedback path





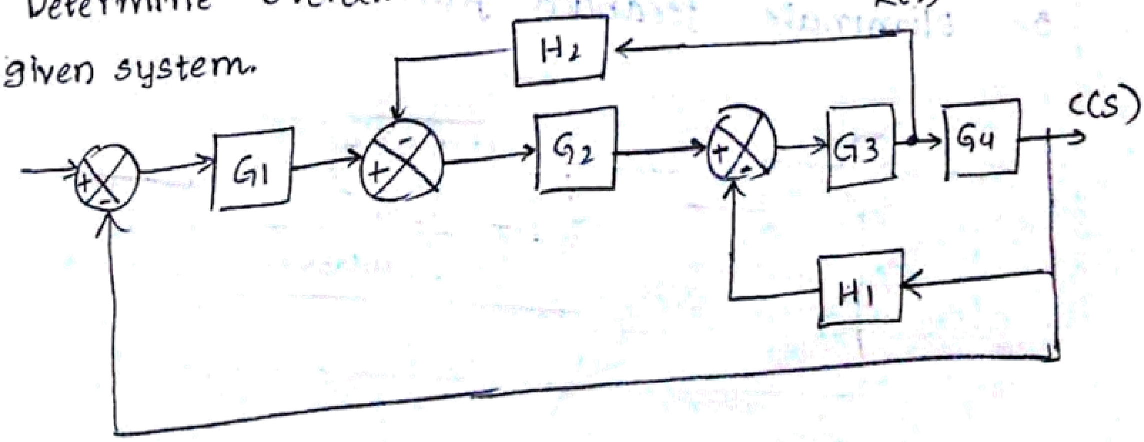


Again Eliminate feedback path.

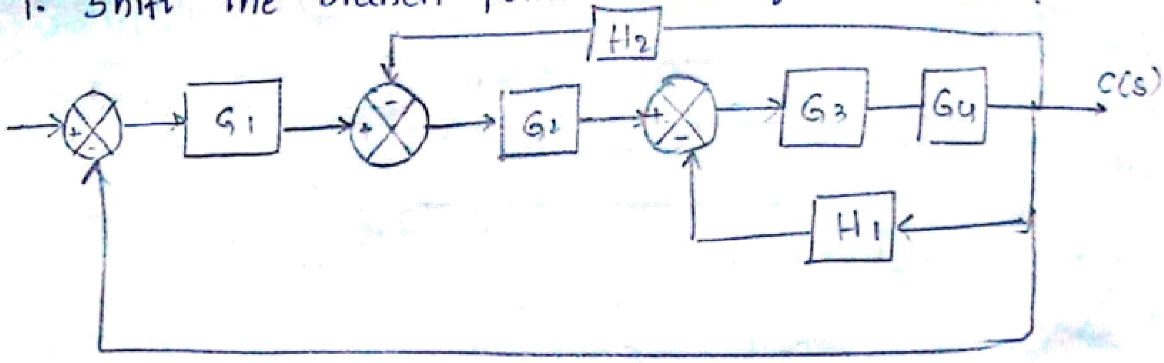
$$\frac{C}{R}=\frac{G}{1+G}=\frac{\dfrac{G_1(G_2G_3+G_4)}{1+H_2(G_2G_3+G_4)+H_1G_1G_2}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+H_2(G_2G_3+G_4)+H_1G_1G_2}}.$$

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1G_4}{1+H_2G_2G_3+H_2G_4+H_1\cancel{H_2}G_1G_2+G_1G_2G_3+G_1G_4}$$

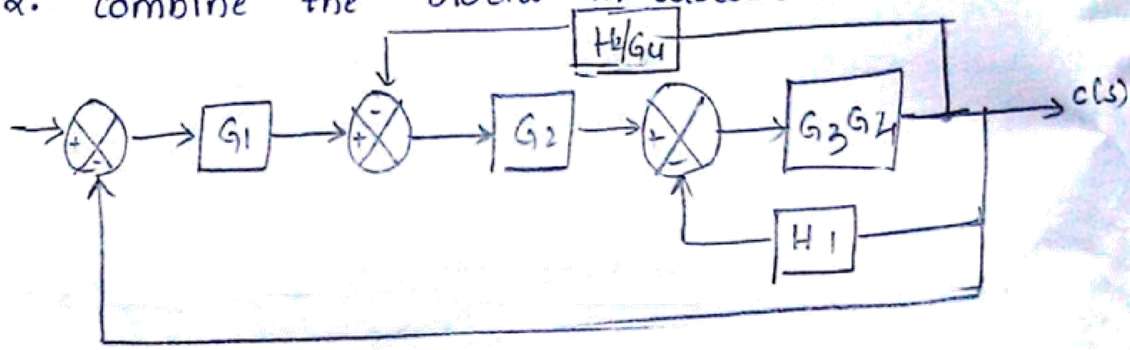
3. Determine overall transfer function $\frac{C(s)}{R(s)}$ for the given system.



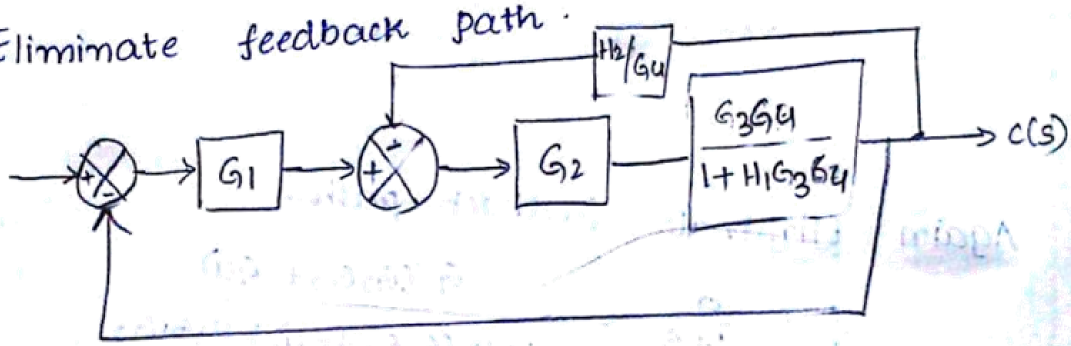
Sol:- 1. Shift the branch point ahead of the block.



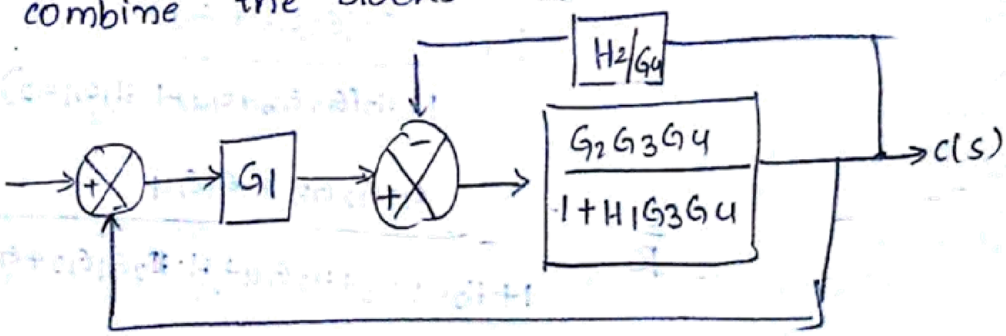
2. Combine the blocks in cascade.



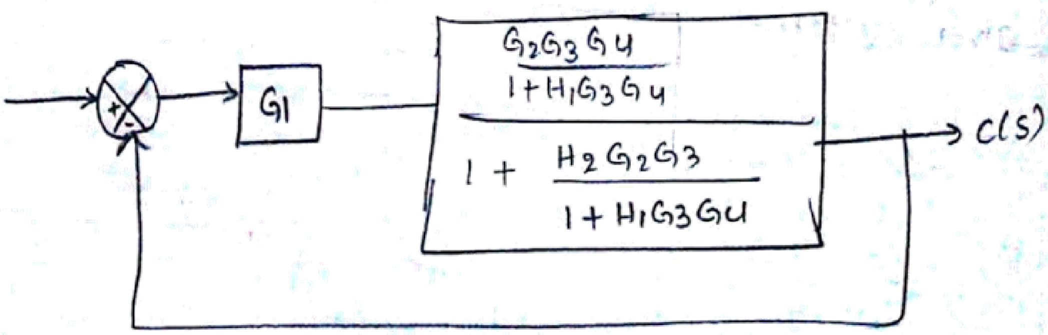
3. Eliminate feedback path.



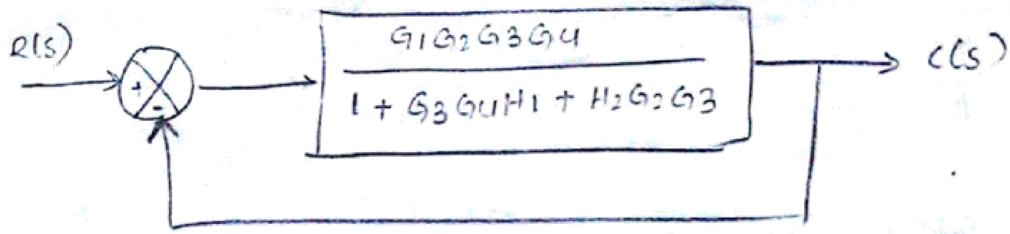
4. Combine the blocks in cascade.



5. Eliminate feedback path.



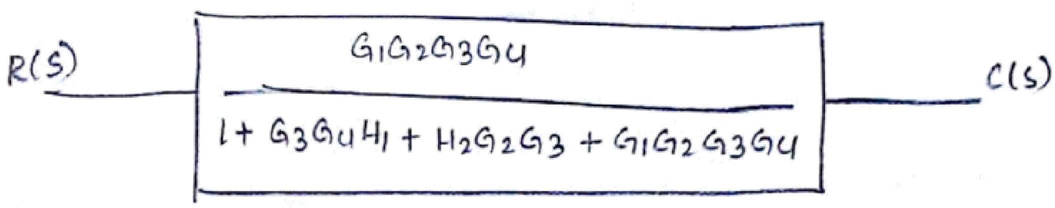
6. combine blocks in cascade



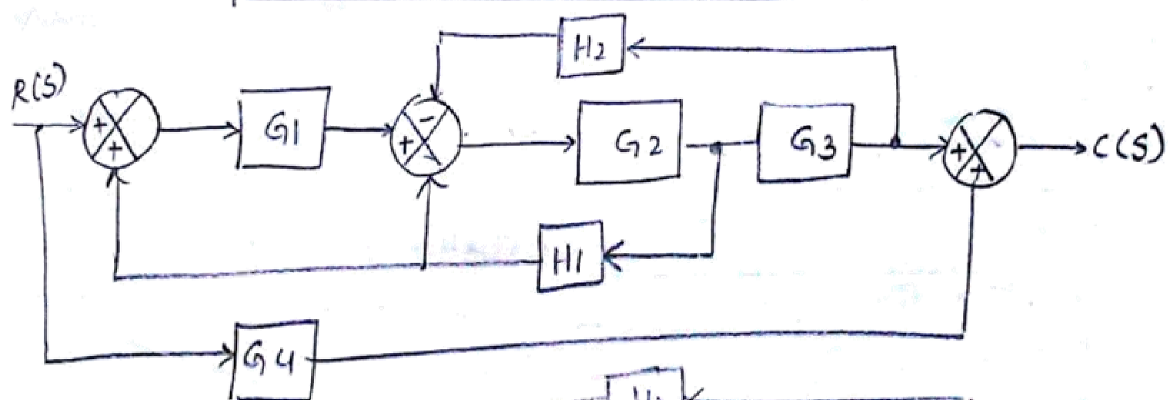
7. Eliminate feedback path.

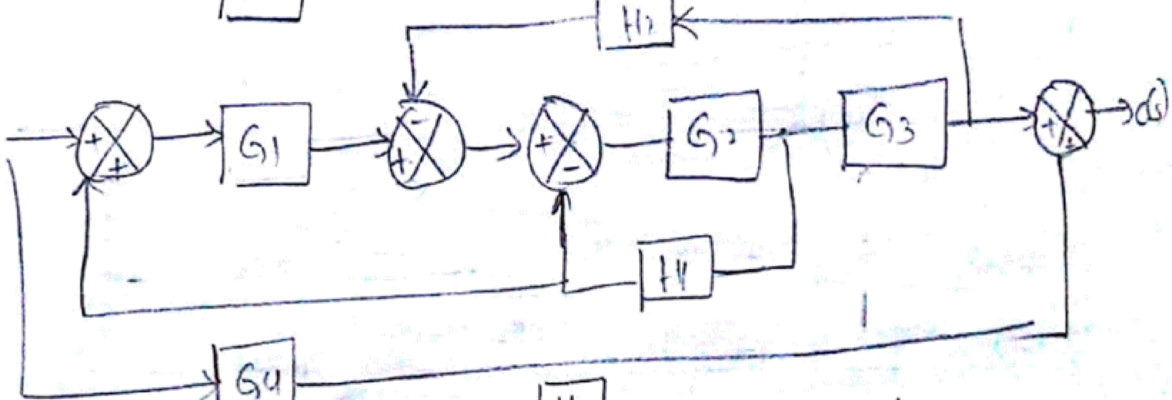
$$\frac{C(s)}{R(s)} = \frac{G}{1+G} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_1+H_2G_2G_3}}{1+\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_1+H_2G_2G_3}}$$

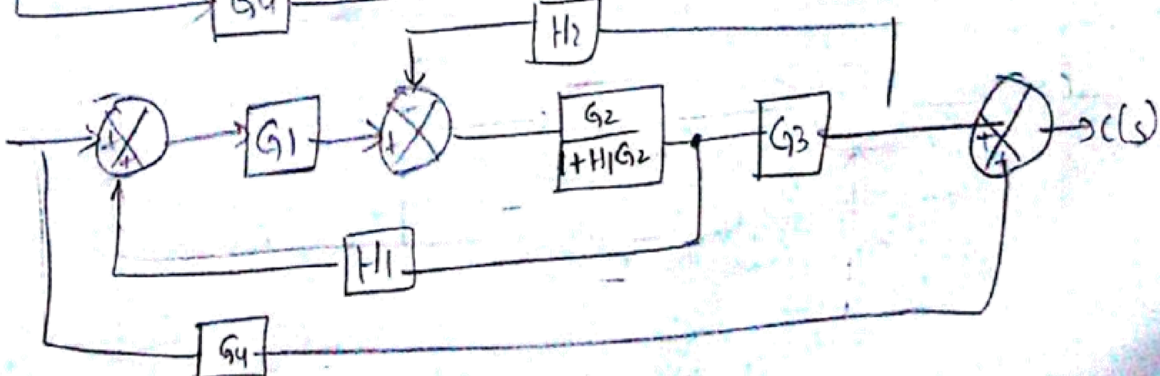
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+H_2G_2G_3+G_1G_2G_3G_4}$$

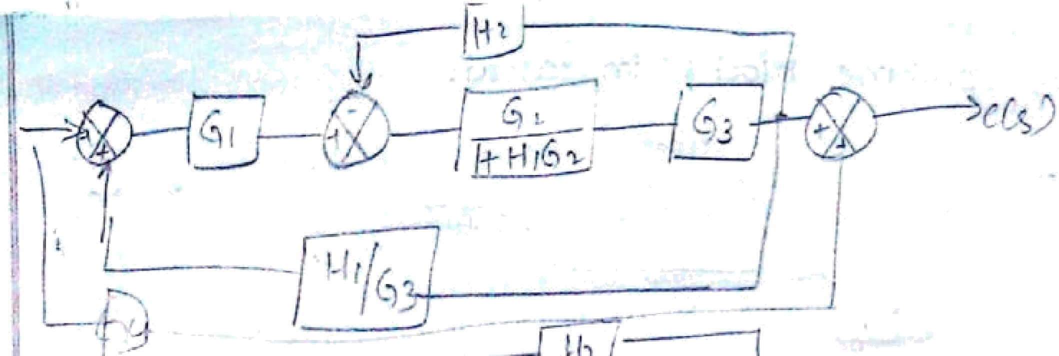


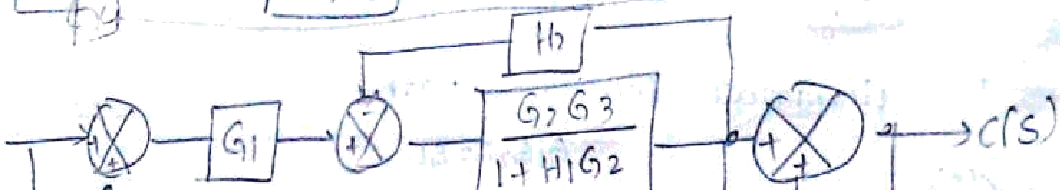
4.



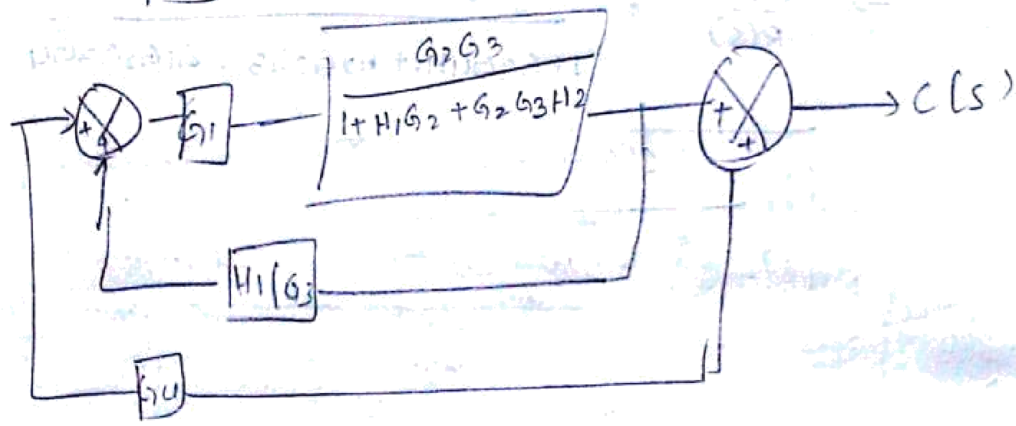


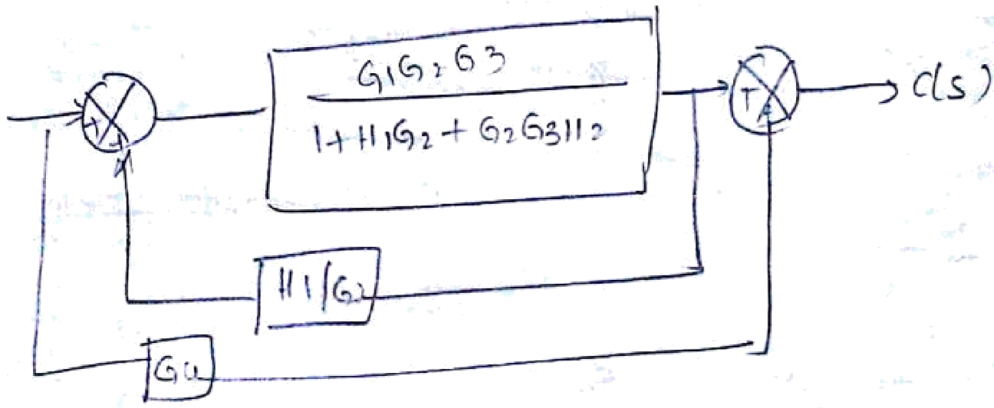


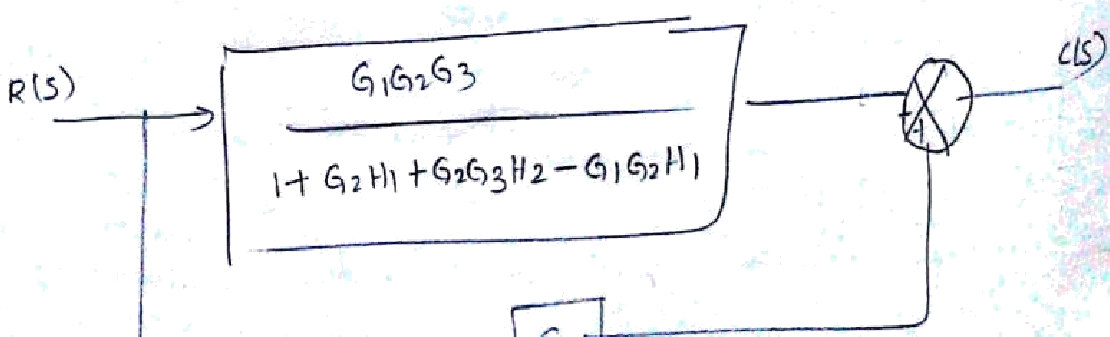




$$\frac{G_2G_3/(1+H_1G_2)}{1+\frac{H_2G_2G_3}{1+H_1G_2}} = \frac{G_2G_3}{1+H_1G_2+G_2G_3H_2}$$



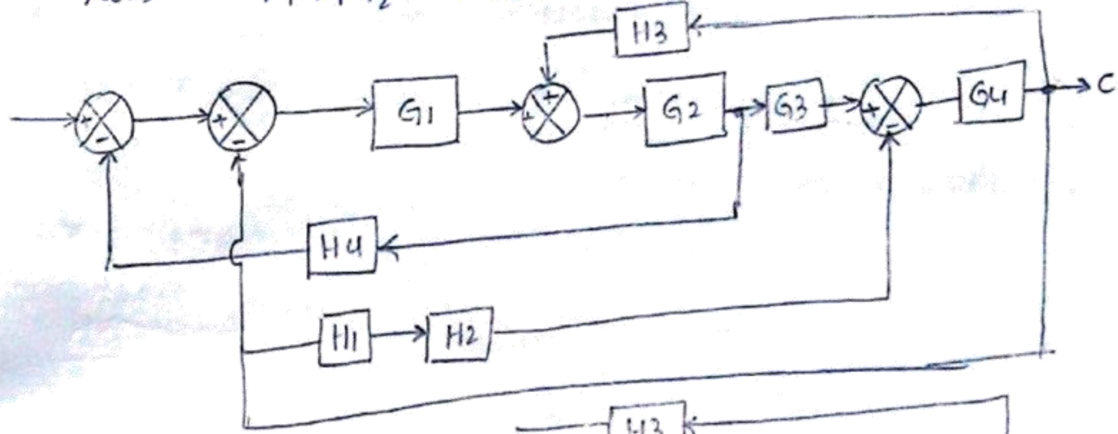


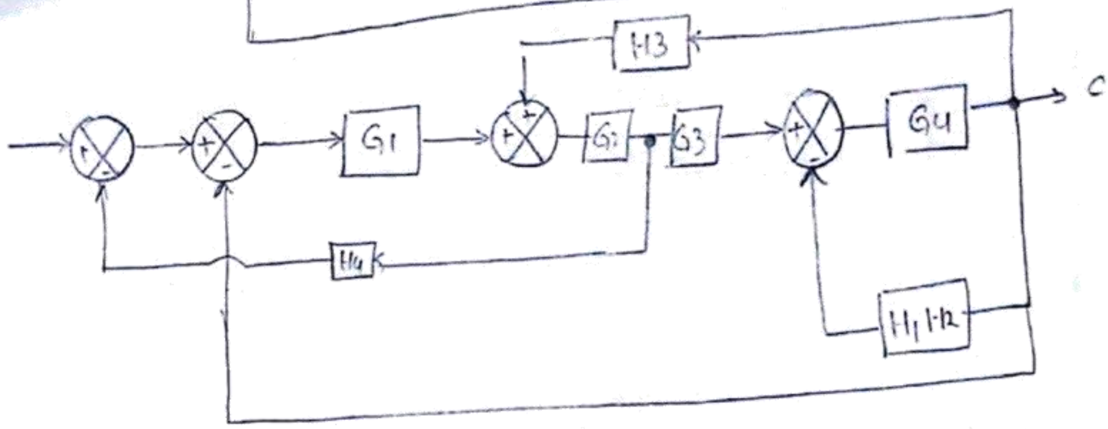


$$\frac{G_1G_2G_3/(1+H_1G_2+G_2G_3H_2)}{1-\frac{G_1G_2H_1}{1+H_1G_2+G_2G_3H_2}} = \frac{G_1G_2G_3}{1+H_1G_2+G_2G_3H_2-G_1G_2H_1}$$

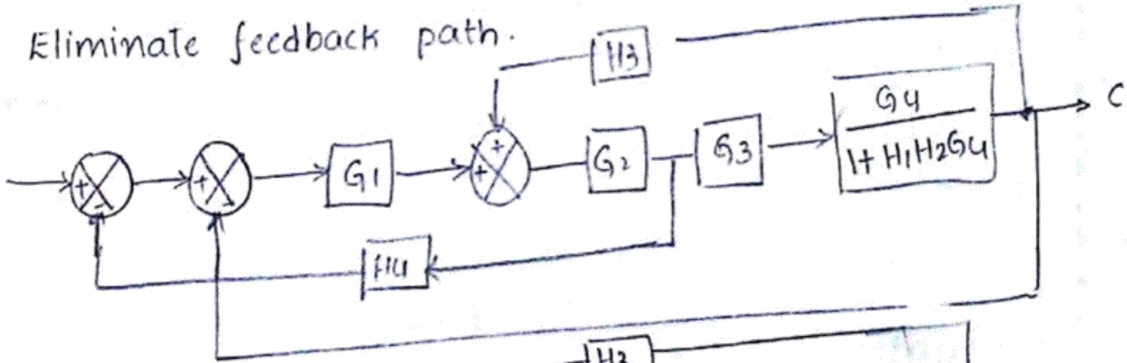
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 + H_1G_2 + G_2G_3H_2 - G_1G_2H_1}$$

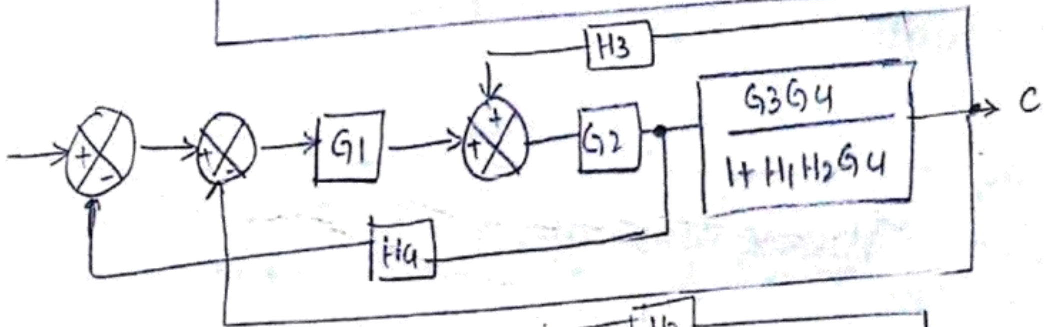
5.

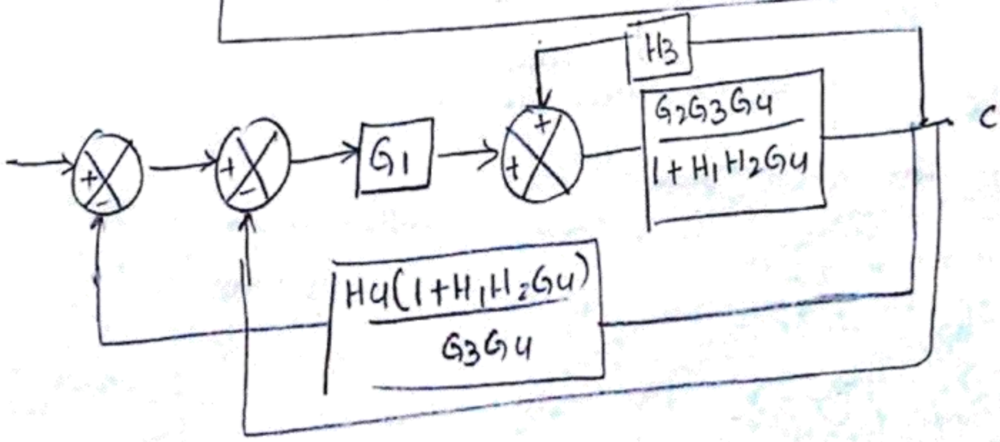


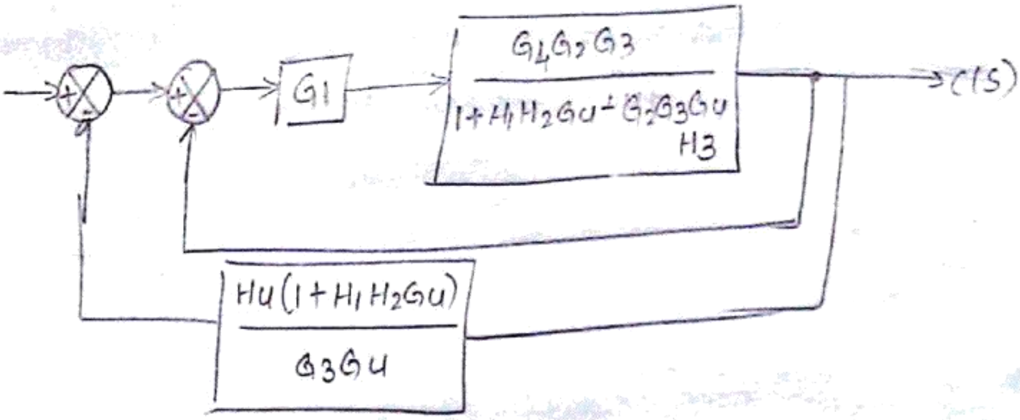


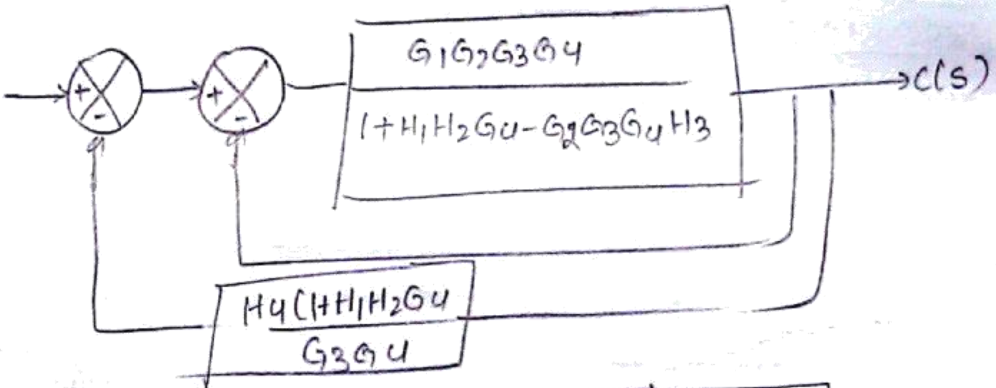
Eliminate feedback path.

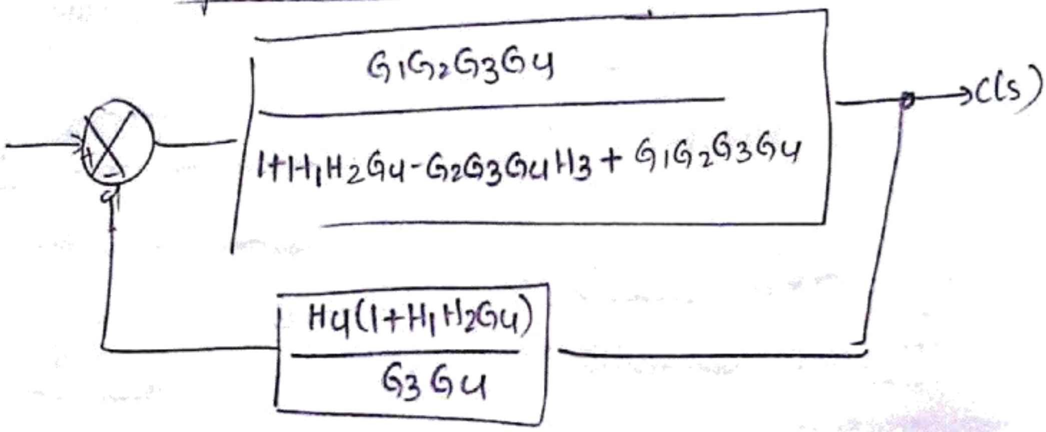












$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+H_1H_2G_4 - G_2G_3G_4H_3 + G_1G_2G_3G_4 + H_4(1+H_1H_2G_4)G_1G_2}$$

Properties of Block diagram

Advantages "

Disadvantages "

* Representation of control systems by signal flow Graph Method :

Signal Flow Graph Method is used to represent the control system graphically and it was developed by S.J. Mason

Terms used in signal Flow Graph :

1. Node :
A node is a point representing a variable (or) signal.

2. Branch :
A Branch is directed line segment joining two nodes.

3. Transmittance :
The gain acquired by the signal when it travels from one node to another is called transmittance. The transmittance can be real (or) complex.

4. Input Node (or) source :
It is a node that has only outgoing branches.

5. Output Node (or) sink :
It is a node that has only incoming branches.

6. Mixed Node :
It is a node which has both incoming and outgoing branches.

7. Path :
A path is a traversal of connected branches in the direction of branch arrows. The path should

not cross a node more than once.

8. Open Path :
A open path starts at a node and ends at another node.

9. Closed Path :
A closed path starts and ends at a same node.

10. Forward Path :
It is a path from an input node to an output node that does not cross any node more than once.

11 Forward Path Gain :
It is the product of Branch gains of a forward path.

12. Individual Loop :
It is a closed path starting from a node and after passing through a certain part of a graph arrives at the same node without crossing any node more than once.

13. Loop Gain :
It is the product of branch gains of a loop.

14. Non-touching Loops :
If loops does not have a common node then they are said to be Non-touching loops.

* Advantages of Block Diagram :

1. It is easy to construct block diagram of complicated system.
2. The transfer function of individual components can be shown in block diagram.
3. Individual as well as the overall system performance can be studied.
4. The closed loop transfer function of a system can be obtained.

Disadvantages of Block Diagram :

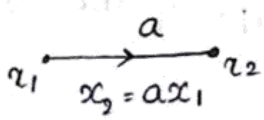
1. In block diagram physical working is not included
2. The source of energy cannot be represented in block diagram.

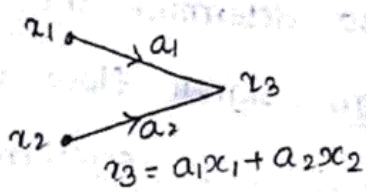
* Signal flow graph Algebra :

Signal flow graph for a system can be reduced to obtain transfer function of the system using rules.

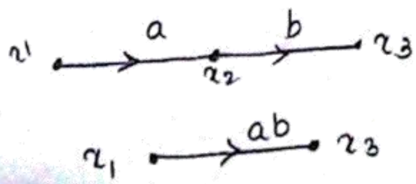
Rules :

1. Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.



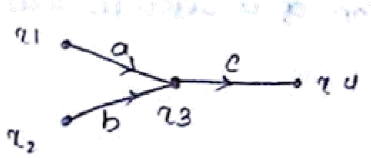


2. Cascaded branches can be obtained combined to give a single branch whose gain is equal to product of individual branch gains.



3. Parallel branches may be represented by single branch whose gain is the sum of individual branch gains

4 A mixed node can be eliminated by multiplying gain of outgoing branch with the gain of all incoming branches.



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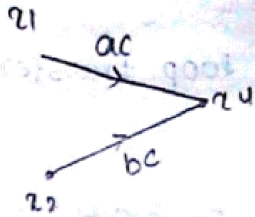
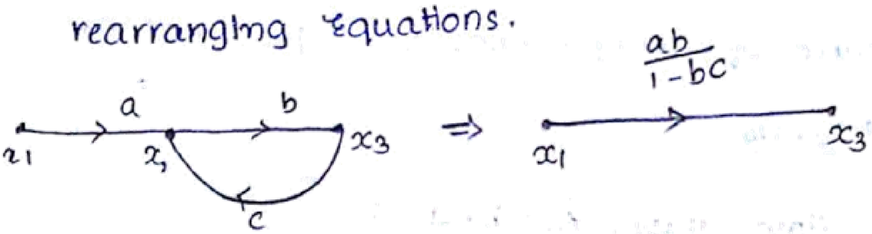


5. A loop may be eliminated by writing and rearranging equations.



$x_2 = ax_1 + cx_3$

where $x_3 = bx_2$ $\Rightarrow x_3 = b(ax_1 + cx_3)$

$x_2 = ax_1 + bcx_2$ $\Rightarrow x_3(1-bc) = abx_1$

$x_2(1-bc) = ax_1$ $\quad \frac{x_3}{x_1} = \frac{ab}{1-bc}$

* Masons Gain Formula :

It is used to determine the transfer function of the system from signal flow graph of the system.

Let $R(s)$ = input to the system.

$C(s)$ = output of the system.

Transfer function of the system $T(s) = \frac{C(s)}{R(s)}$

Mason's Gain Formula states. 'the overall gain of the system

Overall Gain, $T = \frac{1}{\Delta}\sum_{K} P_K \Delta_K$

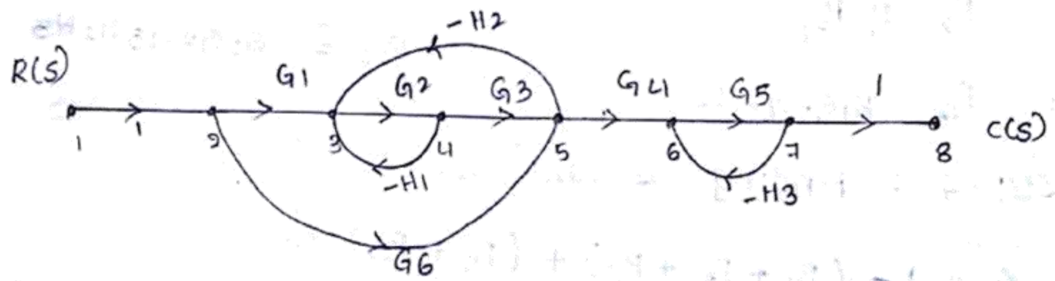
where $T = T(s)$ = transfer function of the system.

P_K = forward path gain of K^{th} forward path

Δ = 1 − (sum of individual loop gains) + (sum of gain products of all possible combinations of two non-touching loops) − (sum of gain products of all possible combinations of three non-touching loops) + (. . . . .

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path.

1. Find overall transfer function of the system whose signal flow graph is shown in figure below.

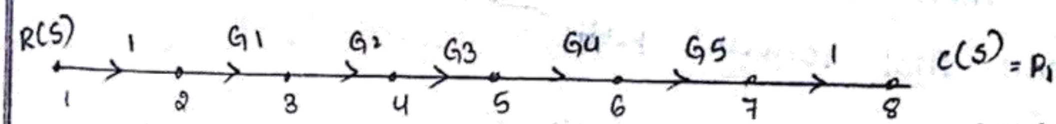


step-1 :

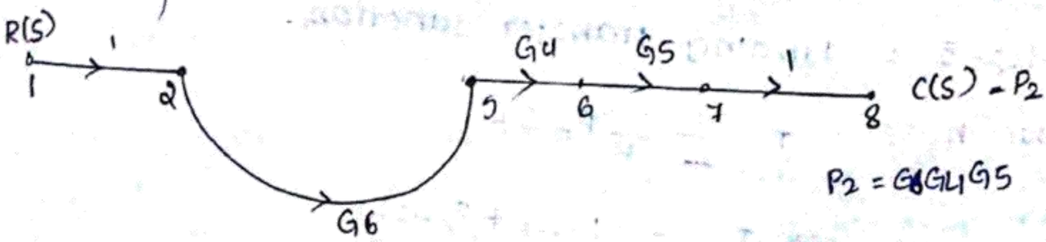
Finding forward path gains:

There are 2 forward paths ∴ K = 2.

Let the forward paths be P_1 and P_2.



$P_1 = G_1G_2G_3G_4G_5$

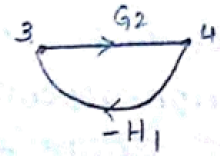


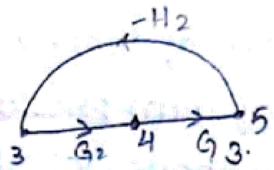
$P_2 = G_6G_4G_5$

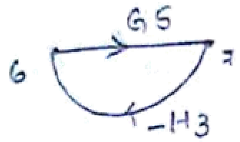
step-2 : Finding individual loop gains.

There are 3 individual loops.

Let individual loop gains be P_{11}, P_{21} & P_{31}





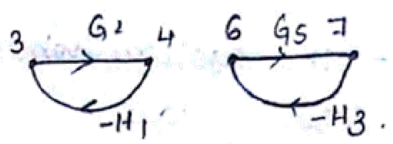


$P_{11} = -H_1G_2$ $\qquad$ $P_{21} = -G_2G_3H_2$ $\qquad$ $P_{31} = -G_5H_3$

step-3 : Finding gain product of 2 non-touching loops

There are 2 combinations of 2 non-touching loops.

Let gain products of 2 non-touching loops be P_{12} and P_{22}



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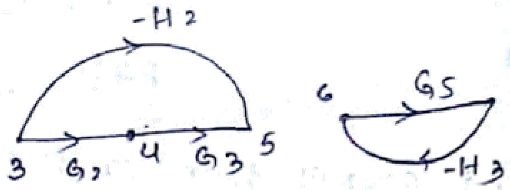


$P_{12} = P_{11}P_{31}$ $\qquad$ $P_{22} = P_{21} \cdot P_{31}$

$P_{12} = H_1G_2G_5H_3$ $\qquad$ $P_{22} = G_2G_3G_5H_2H_3$

Step-4 : Finding Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$\Delta = 1 + [H_1G_2 + G_2G_3H_2 + G_5H_3] + [H_1G_2G_5H_3] + [G_2G_3G_5H_2H_3]$$

$\Delta_1 = 1 - (0) = 1$

$\because$ there is no part of graph which is not touching with first forward path.

$\Delta_2 = 1 - P_{11} = 1 + G_2H_1$

Step-5 : Finding transfer function.

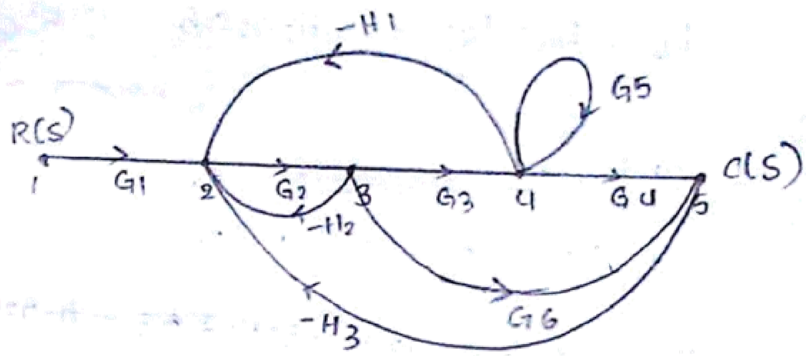
we have $\quad T = \frac{1}{\Delta} \sum_K P_K \Delta_K$

$\rightarrow T = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$

$$= \frac{1}{1+H_1G_2+G_2G_3H_2+G_5H_3+H_1G_2G_5H_3+G_2G_3G_5H_2H_3} \times \left[G_1G_2G_3G_4G_5(1) + G_4G_5G_6(1+G_2H_1)\right]$$

$$T(s) = \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+H_1G_2+G_2G_3H_2+G_5H_3+H_1G_2G_5H_3+G_2G_3G_5H_2H_3}$$

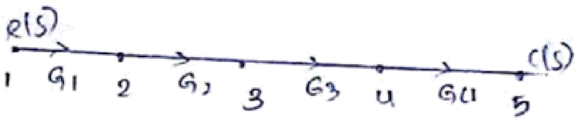
2.



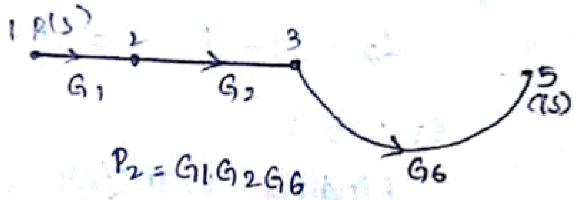
Step-1 : Finding forward path Gains

There are 2 forward paths i.e., $K=2$.

Let the forward paths be P_1 and P_2.



$P_1 = G_1G_2G_3G_4$

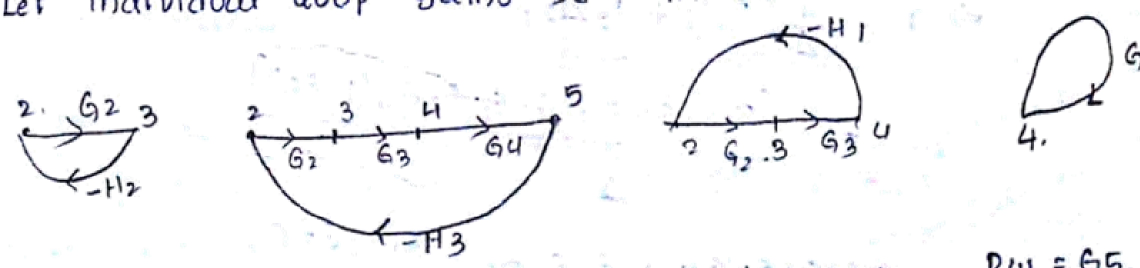


$P_2 = G_1G_2G_6$

Step-2 : Finding individual loop gains

There are 4 individual loops.

Let individual loop gains be P_{11}, P_{21}, P_{31} & P_{41}

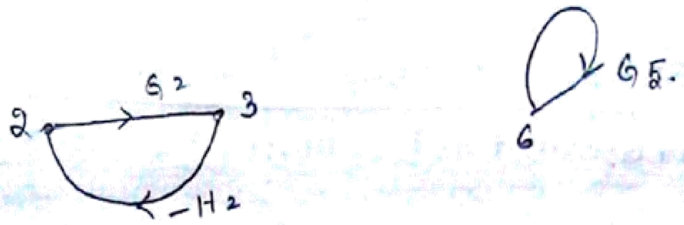


$P_{11} = -H_2G_2$ $\quad P_{21} = -G_2G_3G_4H_3$ $\quad P_{31} = -G_2G_3H_1$ $\quad P_{41} = G_5$.

Step-3 : Finding gain product of 2 non-touching loops.

There is only 1 combination of Non-touching loops.

Let the gain product of non-touching loop be P_{12}.



$$P_{12} = P_{11} \times P_{41} = -H_2G_2G_5$$

step-4 :

Finding Δ and ΔK

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12})$$

$$= 1 + H_2G_2 + G_2G_3G_4H_3 + G_2G_3H_1 + G_5 - H_2G_2G_5.$$

$$\Delta_1 = 1 - (0) = 1$$

$\because$ there is no part of graph which is not touching the first forward path.

$$\Delta_2 = 1 - P_{41} = 1 - G_5$$

Step-5 :

Finding transfer function.

We have $T = \frac{1}{\Delta} \sum_K P_K \Delta_K$

$$\Rightarrow T = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{[G_1G_2G_3G_4 + G_1G_2G_6(1-G_5)]}{\Delta}$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + H_2G_2 + G_2G_3G_4H_3 + G_2G_3H_1 + G_5 - H_2G_2G_5}.$$

* Comparision of block diagram & Signal flow Graph Methods:

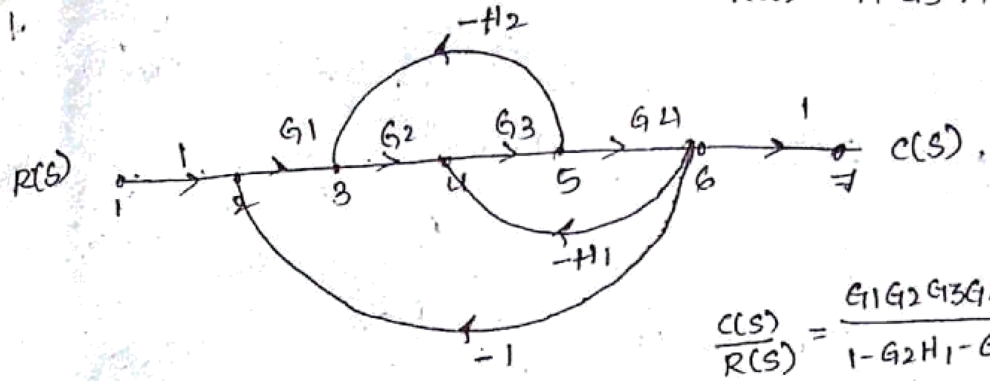
Block Diagram Method	Signal flow Graph.
1. It is pictorial representation of functions performed by each component and of the flow of signals.	1. It is graphical representation of relationships b/w variables of a set of linear algebraic eqns written in form of cause-to-effect relations.
2. It can be used to represent linear as well as non-linear systems.	2. It can be used to represent only linear systems.
3. No direct formula is available to find the over-all transfer function of the system.	3. Mason's Gain formula is available to find overall transfer function of system.
4. Step-by-step procedure is to be followed to find transfer function.	4. Transfer function can be obtained in one step itself
5. It is not a systematic method.	5. It is a systematic method.
6. It indicates more realistically signal flows of system than original system itself.	6. It is constrained by more rigid mathematical rules than a block diagram.
7. For a given system, the block diagram is not unique. Many dissimilar and and unrelated Systems can be represented by same block diagram.	7. For a given system, the signal flow graph is not unique.

* Properties of Signal flow Graph :

1. Signal flow graph applies only to linear systems.
2. The equations for which a signal flow graph is drawn must be algebraic equations in the form of cause and effect.
3. Nodes are used to represent variables. Normally the nodes are arranged from left to right, from the input to the output following a succession of cause-and-effect relations through the system.
4. Signals travel along branches only in the direction described by the arrows of branches.
5. The branch directing from node ~~x_k~~ x_k to x_j represents the dependence of x_j upon x_k but not the reverse.
6. A signal x_k travelling along a branch between x_k and x_j is multiplied by the gain of the branch a_{kj} so that a signal $a_{kj} x_k$ is delivered at x_j.
7. For a given system, a signal flow graph is not unique. Many different signal flow graphs can be drawn for a given system by writing the system equations differently.

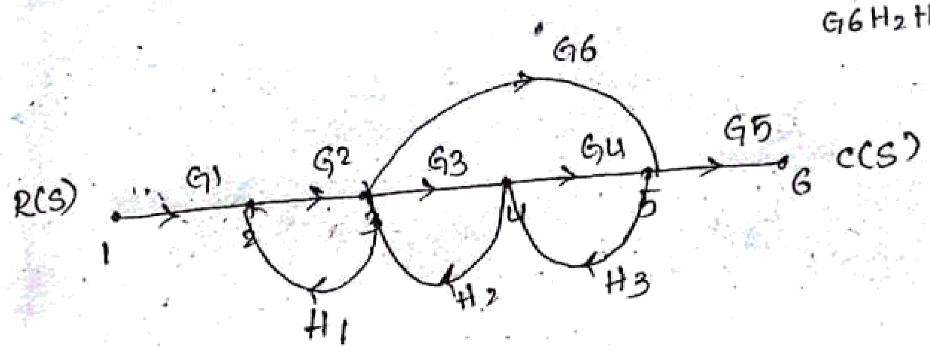
$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

1.



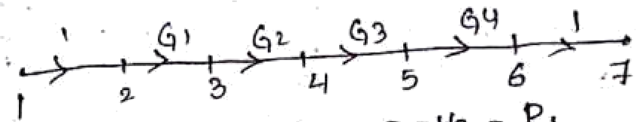
$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4G_5 - G_1G_2G_3G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_2H_3}.$$

2.



①. Step-1 : Finding no. of forward paths & their Gains

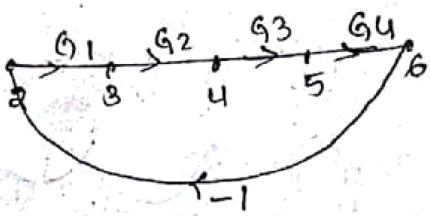
No. of forward paths = 1

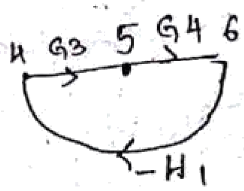


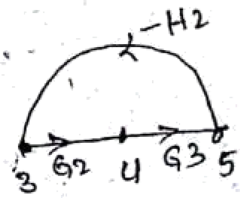
$P_1 = G_1G_2G_3G_4$.

Let the forward path = P_1

Step-2 : Finding Individual loop gains.







$P_{11} = -G_1G_2G_3G_4$. $P_{12} = -H_1G_3G_4$ $P_{13} = -H_2G_2G_3$.

Step-3 : Find gain product of 2 non-touching loops

Since there are no non-touching loops gain = 0

Step-4 : Finding Δ & ΔK

$$\Delta = 1-(-G_1G_2G_3G_4 - H_1G_3G_4 - H_2G_2G_3)$$

$$\Delta = 1+G_1G_2G_3G_4+H_1G_3G_4+H_2G_2G_3.$$

$\Delta_K = 1 - [0]$

Since there is no part of graph non-touching the forward path.

$\Delta_K = 1$.

Step - 5 : Finding Transfer function

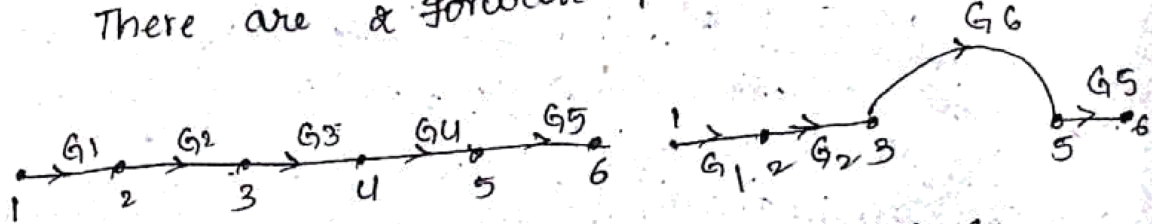
$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum P_K \Delta_K$$

$$= \frac{1}{\Delta}[P_1 \Delta_1]$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4}$$

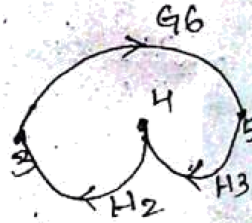
2. 1. Finding forward path gains.

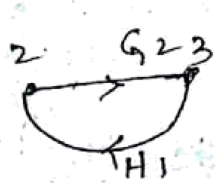
There are 2 forward paths. Let those be P_1 & P_2

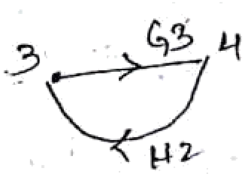


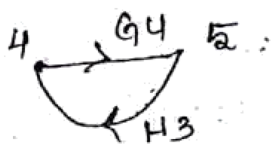
$P_1 = G_1G_2G_3G_4G_5$ $\qquad$ $P_2 = G_1G_2G_6G_5$.

2. Finding individual loop gains :









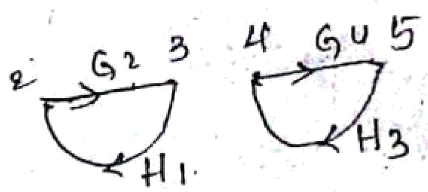
$P_4 = H_2H_3G_6$

$P_{31} = H_3G_4$

$P_{21} = H_2G_3$

$P_{11} = H_1G_2$

3. Finding gain of 2 non-touching loops.



$P_{12} = P_{11} \cdot P_{31}$

$= H_1G_2H_3G_4$.

4. Finding Δ & ΔK

$$\Delta = 1 - [H_1G_2 + H_2G_3 + H_3G_4 + H_2H_3G_6) + H_1G_2H_3G_4.$$

$$\Delta_K = \cancel{1 - [H_1G_2H_3G_4]} \quad 1 - [0].$$

Since there is no part of graph non touching

$$\Rightarrow \Delta_1 = 1 \text{ \& } \Delta_2 = 1.$$

5. Finding Transfer function.

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta} \Sigma P_K \Delta_K$$

$$= \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2)$$

$$= \frac{1}{\Delta_1} [G_1G_2G_3G_4G_5 + G_1G_2G_6G_5)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_6G_5}{1 - H_1G_2 - H_2G_3 - H_3G_4 - H_2H_3G_6 + H_1H_3G_2G_4}$$