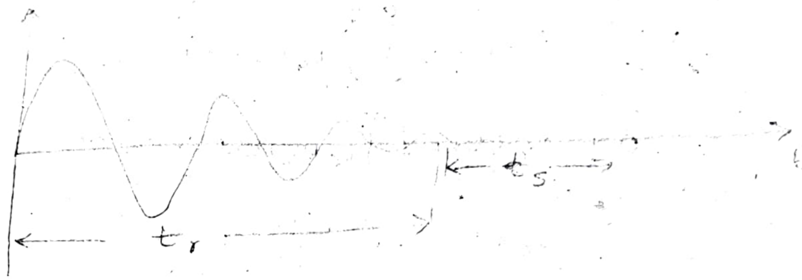
Circuit Analysis by classical Method.

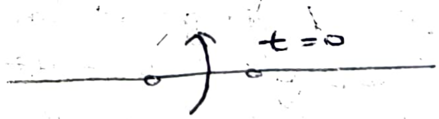
Whenever a ckt. is switched from one condition to another either by a change in the applied Vg or a change in ckt. element, there is a transition period during which branch current and element Voltages change from their former value to new ones. This period is called <u>Transient</u>. After the transient has passed, the ckt is said to be in <u>steady state</u>



→ The linear diff. eqn that describes the ckt will have two parts to its solution.
(i) Complementary fn. corresponds to transient
(ii) Particular solution correspond to steady state.

→ The ckt. changes are assumed to occur at time $t=0$ and represented by a switch.

Closed switch
(on)



open switch.
(off)

$t = 0^-$ → Instant prior to $t=0$
$t = 0^+$ → Instant immediately after switching

→ Switching on or off an element or source in a ckt at $t=0$ will not disturb the storage element so that $i_L(0^-) = i_L(0^+)$, $V_c(0^-) = V_c(0^+)$

Relationship for parameters :-

Parameter	Basic Relation	V-I Relation	Energy
1. R. $G = 1/R$.	$v(t) = R\,i(t)$	$V_R(t) = R\,i_R(t)$ $i_R(t) = G\,V_R(t)$	$W_R(t) = \int_{-\infty}^{t} V_R(t)\, i_R(t)$
2. L	$\psi(t) = L\,i(t)$	$V_L(t) = L\frac{d i_L(t)}{dt}$ $i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$ or $i_L(t) = \frac{1}{L}\int_{0}^{t} V_L(t)\,dt + i_L(0^+)$	$W_L(t) = \frac{1}{2} L\, i_L^2(t)$
3. C	$q(t) = C\,V(t)$	$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt$ or $V_c(t) = \frac{1}{C}\int_{0}^{t} i_c(t)\,dt + V_c(0^+)$ $i_c(t) = C\frac{dV_c}{dt}$	$W_c(t) = \frac{1}{2} C V_c$

first or dy

$p \rightarrow$

Differential equation →

1. I order Homogeneous diff. eqn →

$$\frac{dy(t)}{dt} + P\,y(t) = 0$$

$P \rightarrow$ const.

$$\frac{dy(t)}{y(t)} = -P\,d(t)$$

On Integration,

$$\ln y(t) = -Pt + k'$$

let $k' = \ln k$

$$\ln y(t) = -Pt + \ln k$$

$$= \ln(k e^{-Pt})$$

$$y(t) = k e^{-Pt}$$

If k is evaluated, the soln is a particular soln.

⇒ first order non homogeneous diff. eqn

$$\frac{dy(t)}{dt} + P y(t) = Q$$

$P \rightarrow$ const.

$Q \rightarrow$ fn. of independent variable t or const.

$$e^{Pt}\frac{dy(t)}{dt} + P e^{Pt} y(t) = Q e^{Pt}$$

$\because \quad d(x.y) = x\,dy + y\,dx$

$$\Rightarrow \frac{d}{dt}(y(t)e^{Pt}) = Q e^{Pt}$$

On Integrating

$$y(t)\, e^{Pt} = \int Q e^{Pt} dt + k$$

$$y(t) = \underbrace{e^{-Pt}\int Q e^{Pt} dt}_{P.I.} + \underbrace{k e^{-Pt}}_{C.F.}$$

- If Q is a const

$$y(t) = e^{-Pt}\frac{e^{Pt}}{P}.Q + k e^{-Pt}$$

$$= \underbrace{\frac{Q}{P}}_{P.I.} + \underbrace{k e^{-Pt}}_{C.F.}$$

3) Second order differential eqn. →

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + C y(t) = 0$$

$$\boxed{y(t) = k_1 e^{P_1 t} + k_2 e^{P_2 t}} \rightarrow \text{general soln}$$

$k_1, k_2 \rightarrow$ const.

$P_1, P_2 \rightarrow$ roots of quadratic eqn. $AP^2 + BP + C = 0$

$$P_1, P_2 = -\frac{B}{2A} \pm \frac{1}{2A}\sqrt{B^2 - 4AC}$$

If $P_1 = P_2$ then

$$y(t) = k_1 e^{P_1 t} + k_2 . t e^{P_1 t}$$

Initial conditions in circuits →

→ Initial conditions are required to eval. arbitary const. in general sol n of d. eq'n.

→ no. of initial conditions required is equal to the order of diff. eq n.

Steps to evaluate Initial conditions

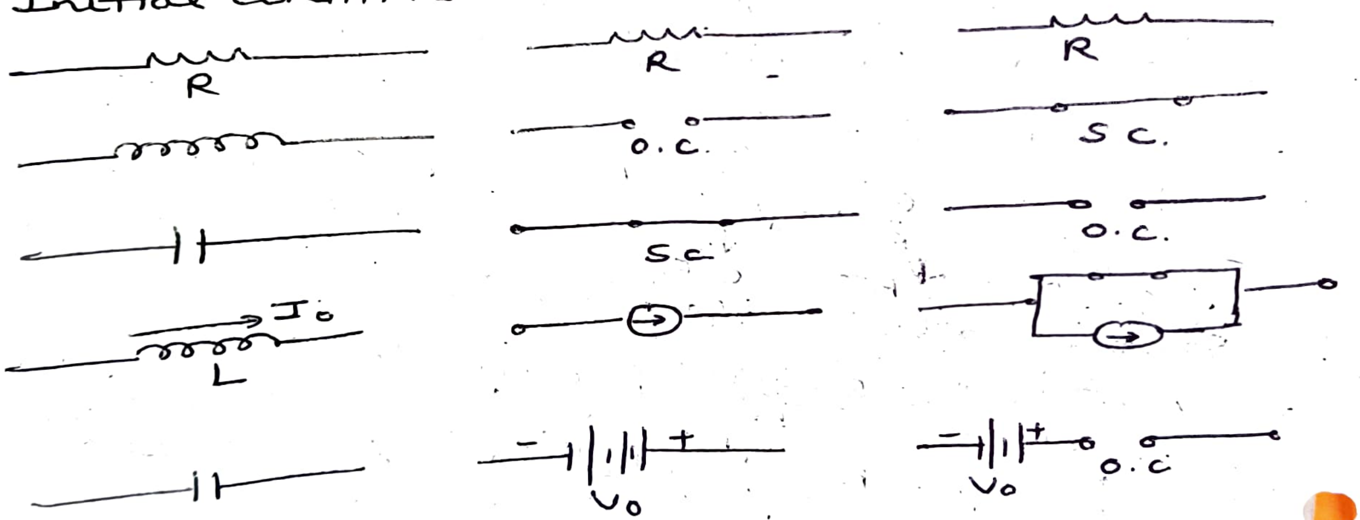
1> Draw the equivalent ckt. at t= 0+

2> evaluate the initial values of Vg. & Ct. of all the branches.

3> Derivatives at t=0+ are evaluated.

Equivalent ckt. of the paramer.

Element with Initial conditions | Equivalent ckt. at t = 0+ | Equivalent ckt at t=∞



Imp. char. of Inductor / Capacitor →

1. There is no Vg./ct. across/through the L/C if the i/v through/across it is not changing with time, so L/C is short/open ckt. to d.c.

2. A finite amount of energy can be stored in the L/C even if the V/i across/through L/C is zero such as when i/v across/through it is const.

3. The L/C never dissipates energy but only store it. It is true only for mathematical model but not for physical model.

Various Responses →

1. <u>Transient Response</u> → (means short lived)

→ The Value of Vg. & ct. during transient period are known as transient response.
→ It is also defined as part of total response that goes to zero as time becomes large.
→ It depend upon N/w elements and independent of forcing fn. (source).
→ C.F. is the sol'n of diff. eq'n with forcing fn. set to zero.

2. <u>Steady state response</u> →

→ The Value of Vg. & ct after the transient has died out are known as steady state response.
→ It is also defined as part of total time response which remains after transient has passed.
→ It depends on both N/w element and source.
→ P.I. represent the forced or steady state response

$$\boxed{\text{Total time Response} = T_t + T_{s.s}}$$

3. <u>Zero Input Response</u> →

→ Value of Vg. & ct. that result from initial conditions when the I/P or forcing fn is zero.

4. <u>Zero state response</u> →

→ Value of Vg. & ct. for an excitation which is applied when all initial conditions are zero.

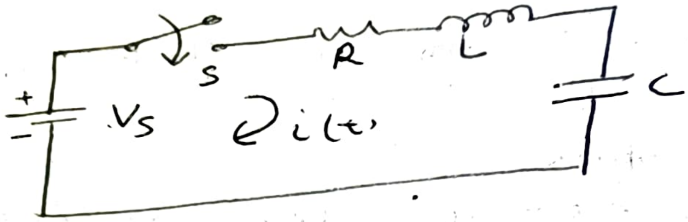
Transient Response of Series RLC ckt

d.c excitation →

Let us consider an eg.

$V_s = 2V$ $R = 6\Omega$ $L = 2H$ $C = .25F$

Determine $i(0^+)$, $\frac{di}{dt}(0^+)$, $\frac{d^2i}{dt^2}(0^+)$ and $i(t)$



By KVL

$$V_s = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \quad \text{——— (1)}$$

On differentiating

$$0 = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t)$$

$$0 = 6\frac{di(t)}{dt} + 2\frac{d^2i(t)}{dt^2} + \frac{1}{.25}i(t)$$

$$0 = \frac{d^2i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) \quad \text{——— (2)}$$

Let $\frac{d^2i(t)}{dt^2} = p^2$ $\frac{di(t)}{dt} = p$

$p^2 + 3p + 2 = 0$

$\Rightarrow$ $P_1 = -1$. $P_2 = -2$

$$i(t) = K_1 e^{-t} + k_2 e^{-2t} \quad \text{——— (3)}$$

K_1, K_2 can be evaluated for a specific problem by the knowledge of initial conditions

If the switch is closed at $t=0$ then

$$\boxed{i(0^+) = 0} \quad \text{——— (4)}$$

[$\because$ ~~Inductor~~ ct. can't change instantaneously in inductor and L behave as o.c.]

In eq'n (1) 1 & 3 vg. terms are zero at the instant of switching. $Ri(0^+)$ being 0 b/c $i(0^+)=0$ & $\frac{1}{C}\int_{-\infty}^{0^+} id(t)$ b/c it is the initial vg. a/c capacitor

hence $\frac{di}{d}$ from e

hence $\frac{di}{dt}(0^+) = \frac{V_s}{L} - \frac{2}{2} = 1\ A/sec$ —— (5)

from eq'n (2)

$$\frac{d^2 i}{dt^2}(0^+) + 3\frac{di}{dt}(0^+) + 2i(0^+) = 0$$

$$\frac{d^2 i}{dt^2}(0^+) = -3 \times 1 - 2 \times 0$$

$$= -3\ A/sec^2 \quad —— (6)$$

The above two initial conditions (4) & (5) put into the general sol'n, eq'n (3) gives

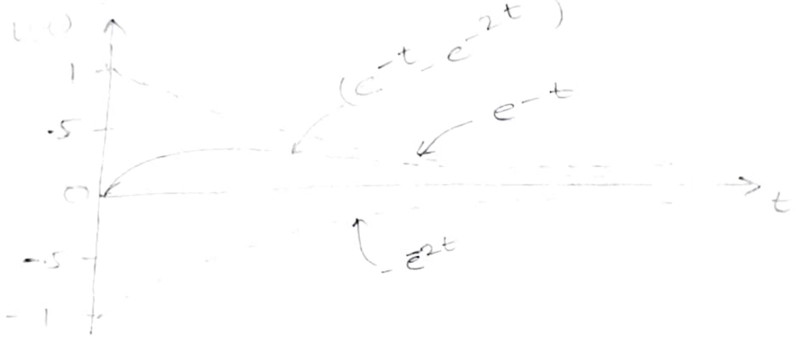
$$k_1 + k_2 = 0$$

$$-k_1 - 2k_2 = 1$$

$$\Rightarrow \quad k_1 = 1, \quad k_2 = -1$$

Hence Particular sol'n is

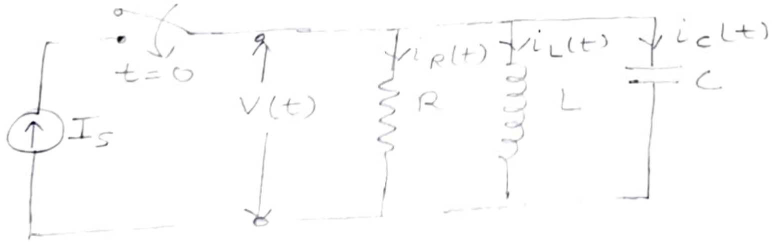
$$i(t) = e^{t} - e^{-2t}\ A$$



RLC parallel circuit

Let $R = \frac{1}{16}\ \Omega$ $\quad L = \frac{1}{16}\ H$ $\quad C = 4F$ $\quad I_s = 2A$

determine $v(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2v}{dt^2}(0^+)$; $V(t)$



By KCL

$$I_s = i_R(t) + i_L(t) + i_C(t)$$

$$= \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt + C\frac{dv(t)}{dt} \quad —— (1)$$

differentiate & using numerical value of RLC

$$0 = 16\frac{dv(t)}{dt} + 16\,v(t) + 4\frac{d^2 v(t)}{dt^2}$$

$$\Rightarrow \frac{d^2 v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = 0 \quad \text{—— (2)}$$

$$\Rightarrow p^2 + 4p + 4 = 0$$

On solving $p = -2, -2$

$$\boxed{v(t) = K_1 e^{-2t} + K_2 t e^{-2t}} \rightarrow \text{general sol}^n \quad (3)$$

To obtain a particular sol'n for this problem will require knowledge of two initial conditions.
from ckt. $V(0^+)$ must equal zero, since capacitor acts as a s.c at initial instant i.e.

$$\boxed{V(0^+) = 0} \quad \text{—— (4)}$$

In eq'n (1) the I & II ct. terms are zero at the instant of switching, b/c $v(0^+) = 0$ and there is no ct. in the inductor at initial instant. Hence

$$\frac{dv}{dt}(0^+) = \frac{I_s}{C} = \frac{2}{4} = \frac{1}{2} \text{ V/sec.} \quad \text{—— (5)}$$

from eq'n (2)

$$\frac{d^2v}{dt^2}(0^+) + 4\cdot\frac{1}{2} + 4\cdot 0 = 0$$

$$\frac{d^2v}{dt^2}(0^+) = -2 \text{ V/sec}^2 \quad \text{—— (6)}$$

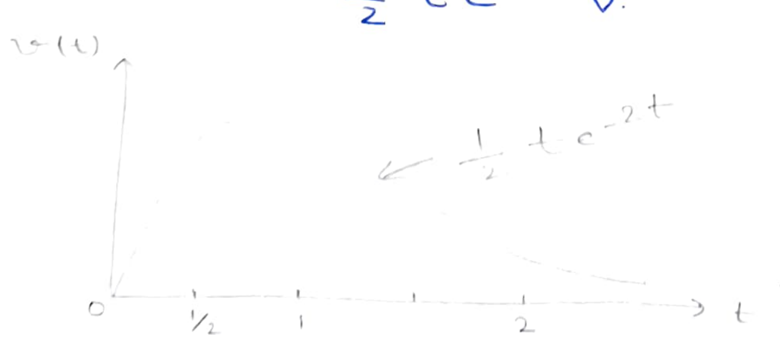
above two initial conditions (4) & (5) are put into eq'n (3)

$$K_1 = 0 \quad \& \quad -2K_1 + K_2 = 1/2$$

$$\Rightarrow K_2 = 1/2$$

So particular sol'n is

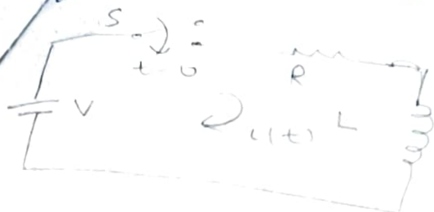
$$v(t) = \frac{1}{2} t e^{-2t} \text{ V.}$$



$t = 0 \rightarrow v(t) = 0$

$t = 1/2 \rightarrow v(t) =$

Transient Response of Series RL Ckt. having (13)
d.c. excitation →



at the instant when switch is closed at $t=0$

$i(t) = \frac{V}{R}(1 - e^{-R/L\, t})$

$v_R(t) = i(t)\, R$

$v_L(t) = L\frac{di(t)}{dt}$

$= Ve^{-R/L\, t}$

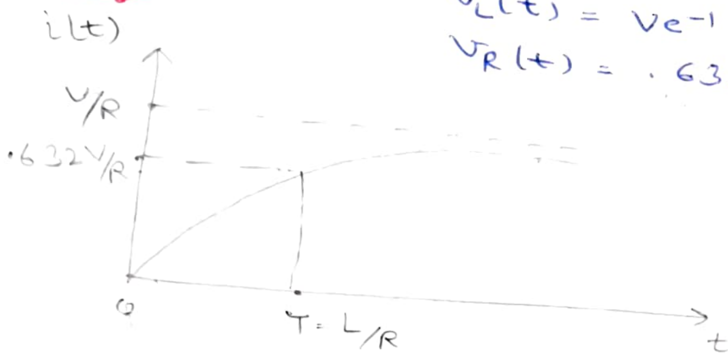
at $t=0 \Rightarrow i(t) = 0$ $\quad V_L(t) = V \quad v_R(t) = 0$

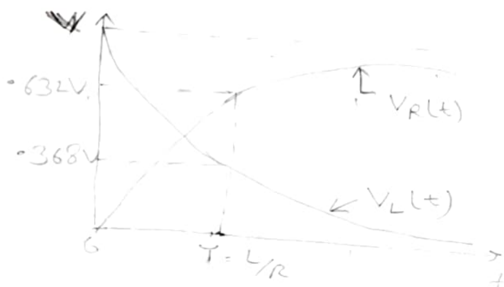
at $t=\infty \Rightarrow i(t) = V/R$ $\quad V_L(t) = 0 \quad V_R(t) = V$

at $t = \frac{L}{R} = \tau \Rightarrow i(t) = \frac{V}{R}(1-e^{-1}) = .632\, V/R$

$V_L(t) = Ve^{-1} = .368\, V$

$V_R(t) = .632\, V$





When RL ckt. reaches at steady state ($t=\infty$) and suddenly Vg. is withdrawn by opening the switch S and throwing it to S′

$L\frac{di'(t)}{dt} + Ri'(t) = 0$

$i'(t) = Ke^{-R/L\, t}$

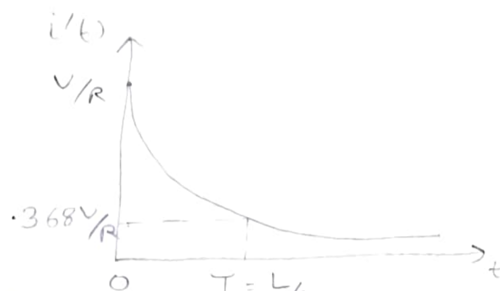
at $t=0^+$ inductor keep the S.S. ct. $i(0^+) = i(\infty) = V/R$

$\frac{V}{R} = Ke^{0} \Rightarrow K' = V/R$

$$\boxed{i'(t) = \frac{V}{R}e^{-R/L\, t}}$$

$v_R'(t) = i'(t)\, R = Ve^{-R/L\, t}$

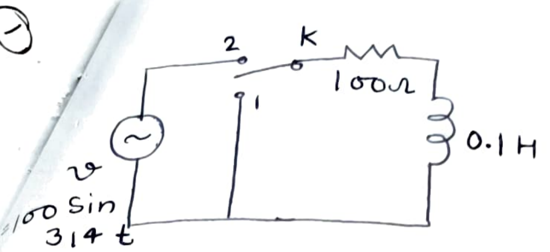
$V_L'(t) = L\frac{di'(t)}{dt} = -Ve^{-R/L\, t}$



→ $\tau = L/R$ is known as the time constant of the circuit

It is defined as the time taken for the current to reach 63% of its ~~value~~ final value. Thus it is a measure of rapidity with which steady state is reached

1



find the current at $t>0$ if a.c vg. is applied when switch k is moved to 2 from 1 at $t=0$. Assume a steady state current of 1 A in R-L circuit when the switch was at position 1.

Sol. at $t=0^-$ $\quad i(0^-) = 1\,A$

also $\quad i(0^-) = i(0^+) = 1$

$$Z = R + jX_L = 100 + j(2\pi \times 50 \times .1)$$
$$= 104.8 \angle 17.47^\circ \ \Omega$$

by KVL in R-L circuit

$$v = Ri + L\frac{di}{dt}$$

$$100 \sin 314t = 100 i + 0.1 \frac{di}{dt}$$

$$\frac{di}{dt} + 1000 i = 1000 \sin 314 t$$

$$(P + 10^3) i = 10^3 \sin 314 t$$

$$i_c = Ke^{-R/L\,t}$$
$$= Ke^{-\frac{100}{.1}t}$$

$$i_p = \frac{V_m}{Z} \sin(\omega t + \phi - \tan^{-1}\frac{\omega L}{R})$$

$$= \frac{100\sqrt{2}}{104.8} \sin(314t - 17.47^\circ) \qquad [\because \phi = 0^\circ]$$

$$= 1.345 \sin(314t - 0.304)$$

$$i = i_c + i_p$$

$$= Ke^{-R/L\,t} + 1.345 \sin(314t - 0.304)$$

at $t=0^+$ $\quad i(0^+) = 1$

$$1 = K + 1.345 \sin(-.304)$$
$$1 = K + (-.3)$$
$$K = 1.3$$

$$i = 1.3 e^{-1000t} + 1.345 \sin(314t - 0.304)$$