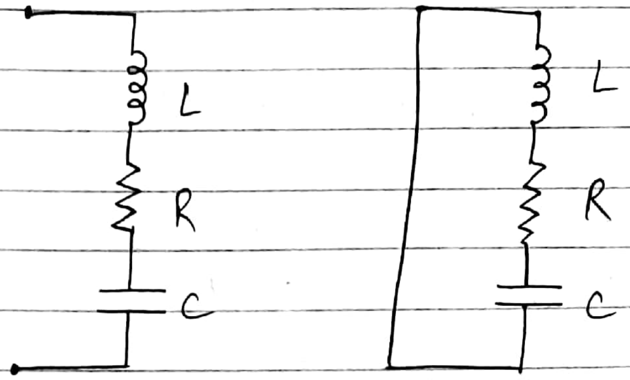
10/08/17

CIRCUITS & SYSTEMS

An electric network is any possible interconnection of electric elements ~~by~~ while an electric circuit is a closed network.

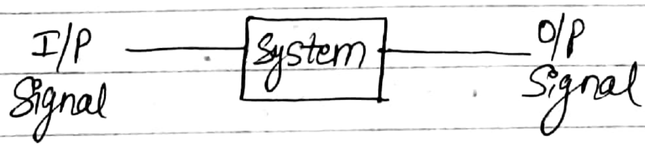


Network Circuit

∴ All circuits are networks but all networks are not circuits.

System : A collection of physical components which together perform an objective.

A system takes an input & gives the output.



Signal is a quantity having information associated with it.

Eg. - We deal with electric signals, i.e. voltage & current, both are functions of time.

- Audio Signal (20 Hz - 20 kHz)
- Video Signal (0 - 5MHz)
- Data Signal (Based on application)

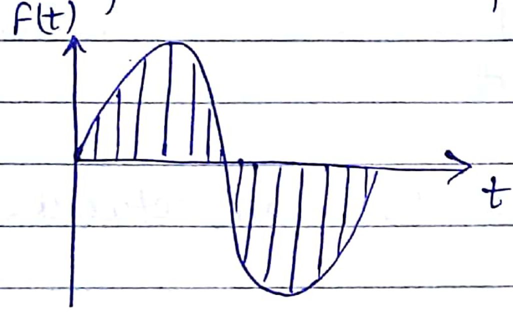
Single variable signal → Eg.- $f(t)$

Multi variable signal → Eg.- $f(t_1, t_2)$

Classification of Signals

1. Continuous Time Signal & Discrete Time Signal

A continuous time signal is one where signal value is defined for all values of time instant.



Discrete signals are sampled version of continuous time signals & ------.

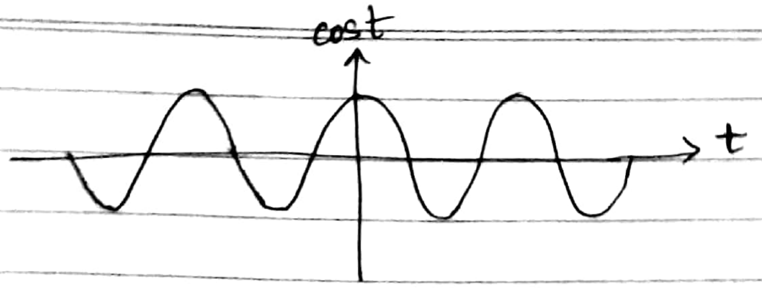
2. Even & Odd Signal

Another set of useful properties of signals related to their symmetry under time traversal.

A signal $x(t)$ is referred to as an even signal if it is identical to its time reverse counter part.

$$x(-t) = x(t) \Rightarrow \text{Even Signal}$$

Eg.- $\cos t$, t^n where n is even.



$f(t) = \cos t$

A signal is referred to as an odd signal if it is negative of its reflection.

Eg- $\sin t$, t^n where n is odd.

3. <u>Periodic & Unperiodic Signal</u>

A signal $x(t)$ is periodic if & only if $x(t+t_0) = x(t)$ where constant t_0 is called period of $x(t)$.
The smallest value of 't_0' such that equation is satisfied is referred as time period.

Eg- $\sin(t+2\pi) = \sin t$

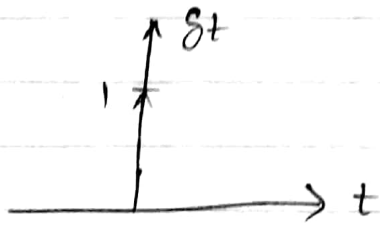
— $\cos t$

Unperiodic → e.g.- e^t, t^2, t.

Standard Signals/ Test Signals

1. <u>Unit Impulse Signals</u>

$$\delta(t) = \begin{cases} 0 & ; t \neq 0 \\ 1 & ; t = 0 \end{cases}$$



$$\int_{-\infty}^{0} 0 + \int_{0}^{\infty} e^{-st}\, dt$$

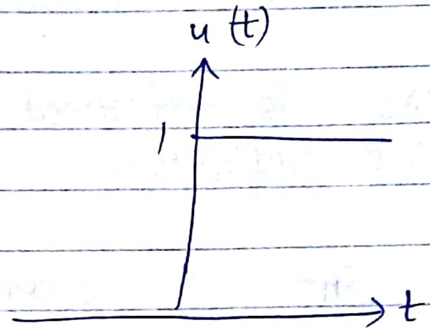
$$-\frac{1}{s}\left[e^{-st}\right]_0^\infty \qquad L[f(t)] = \int_{-\infty}^{\infty} e^{-st} f(t)\, dt$$

$$-\frac{1}{s}\left[0 - 1\right] = \frac{1}{s}$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

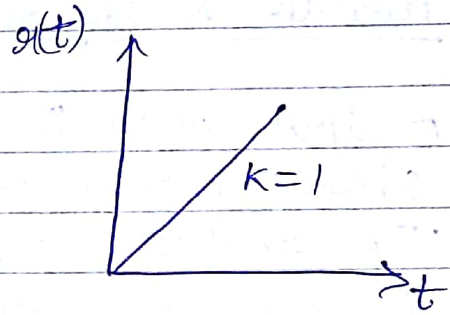
2. <u>Unit - Step Signals</u>

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



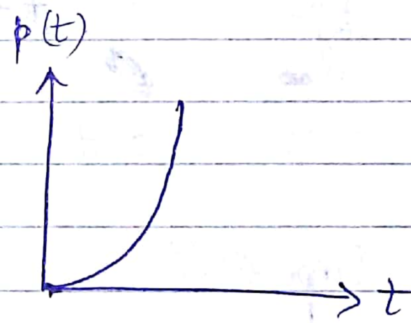
3. <u>Unit ramp Signal</u>

$$r(t) = \begin{cases} t & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



4. <u>Unit parabolic signal</u>

$$p(t) = \begin{cases} t^2/2 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$



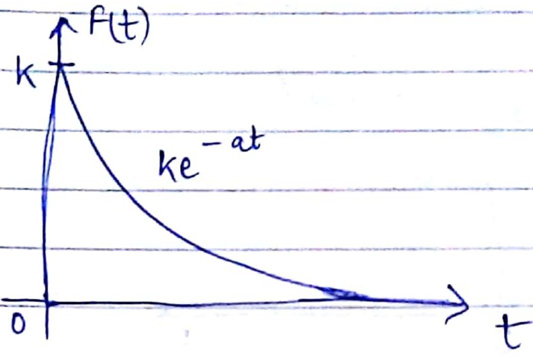
$$\cancel{\int_{-\infty}^{t} r(t)\, dt = \int_{-\infty}^{0} r(t)\, dt + \int_{0}^{t} r(t)\, dt = \int_{-\infty}^{0} 0 . dt + \int_{0}^{\infty} t\, dt}$$

$$\cancel{= \frac{1}{2}\left[t^2\right]_0^\infty = \infty}$$

11/08/17

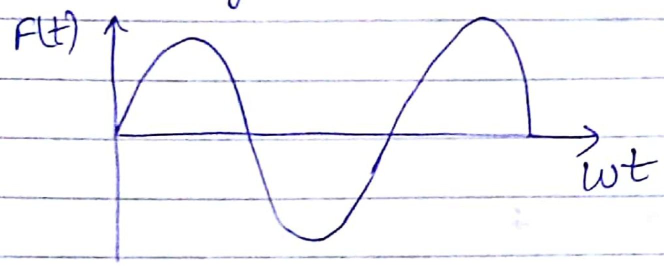
Other Basic Signals

Exponential Signal



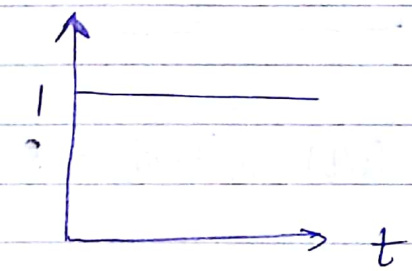
$$f(t) = \begin{cases} ke^{-at} & ; \; t \geq 0 \\ 0 & ; \; t < 0 \end{cases}$$

Sinusoidal Signal

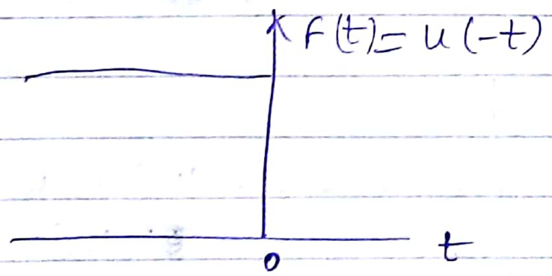


$$f(t) = \begin{cases} V_m \sin \omega t & ; \; t \geq 0 \\ 0 & ; \; t < 0 \end{cases}$$

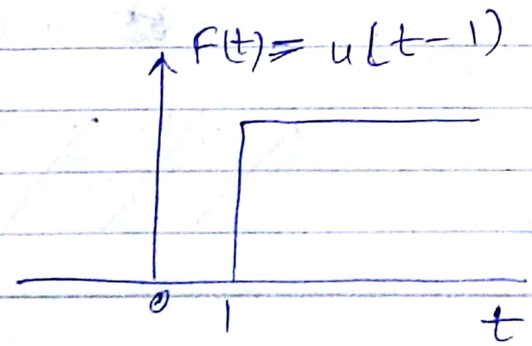
$$u(t) = \begin{cases} 1 & ; \; t \geq 0 \\ 0 & ; \; t < 0 \end{cases}$$



$u(-t) =$



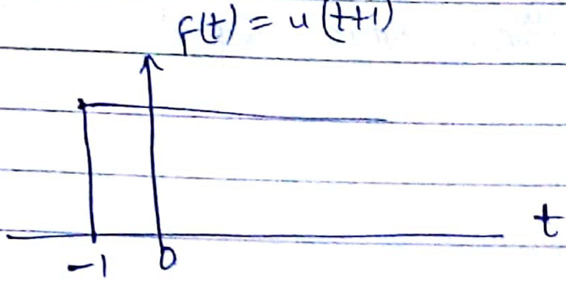
$u(t-1) =$



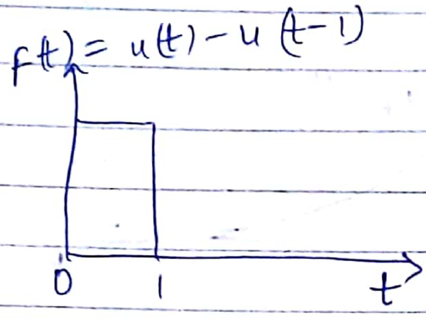
~~75 76 74 75~~ - - -

X ~~80 78 72 70~~ - - - -

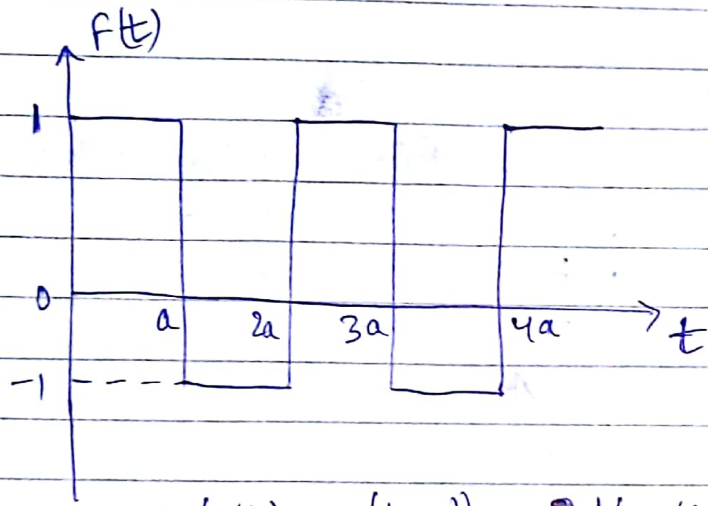
$u(t+1) =$



$u(t) - u(t-1) =$

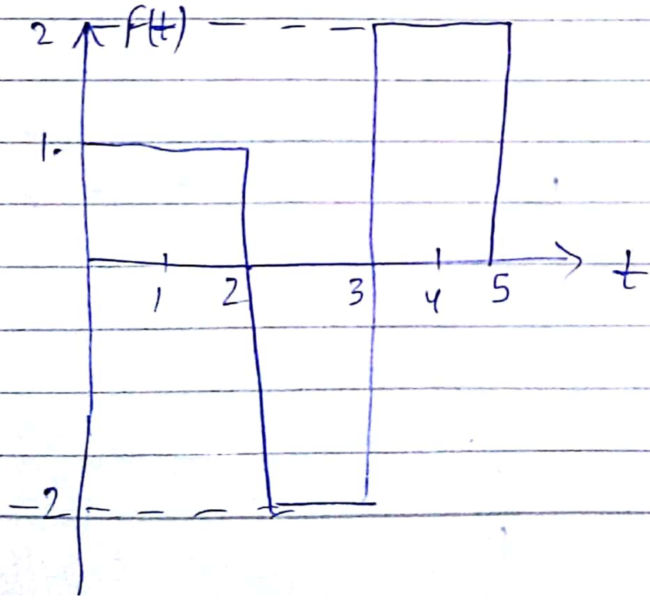


→



$= 1\,(u(t) - u(t-a)) - 1(u(t-a) - u(t-2a)) + 1(u(t-2a) - u(t-3a)) - 1(u(t-3a) - u(t-4a)) + \dots$

→



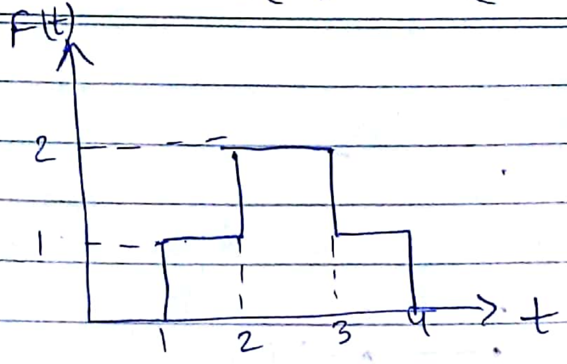
$= 1.(u(t) - u(t-2)) - 2(u(t-2) - u(t-3)) + 2(u(t-3) - u(t-5))$

$\cancel{= u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)}$

$1(u(t-1) - u(t-2)) + 2(u(t-2) - u(t-3)) - 1(u(t-3) - u(t-4))$

$1(u(t-1) - u(t-2)) + 1.(u(t-2) - u(t-3)) + 1(u(t-3) - u(t-4))$ $(3a,1)$ $(4a,0)$

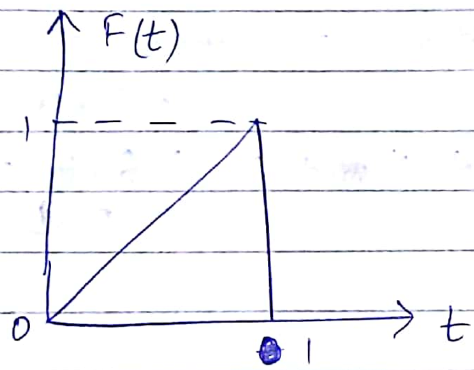
$u(t-1) - 2u(t-3) + u(t-4)$

→ 

$\cancel{u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)}$

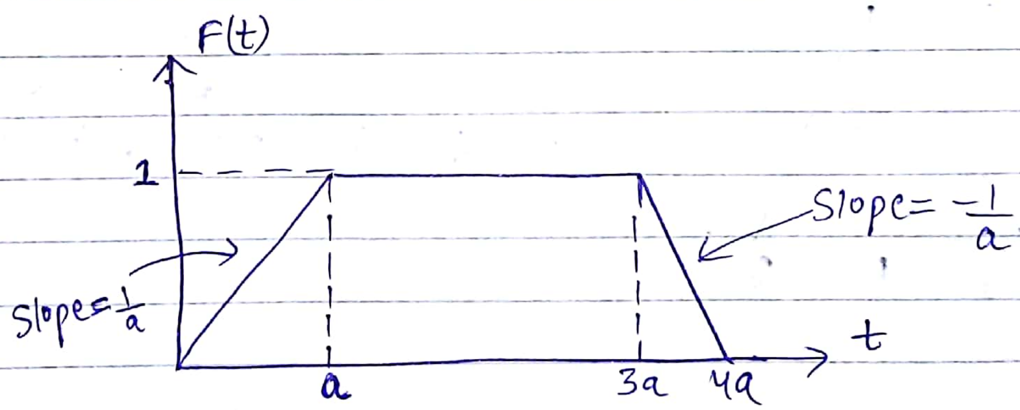
$= 1(u(t-1) - u(t-2)) + 2(u(t-2) - u(t-3)) + 1(u(t-3) - u(t-4))$

$= u(t-1) + u(t-2) - u(t-3) - u(t-4)$

→ 

$= t(u(t) - u(t-1))$

17/08/17

→ 

$= \frac{1}{a} t (u(t) - u(t-a)) + 1 (u(t-a) - u(t-3a))$
$- \frac{1}{a}(t-4a)(u(t-3a) - u(t-4a))$

System Properties

1. Static & Dynamic System : If the output of the system depends only on present value of input at each & every instant of time, then system is called static. These systems are also known as memory-less systems.

If output of the system depends on the past or future value of input at any instant of time, then system is called dynamic system. It is also known as system with memory.

$x(t) \longrightarrow$ Present
$x(t-1) \longrightarrow$ Past
$x(t+1) \longrightarrow$ future

2. Linear & Non-Linear System

Linear System : System in which output varies linearly with input & it satisfies superposition principle.

Non-Linear System : System in which output does not vary linearly with input & does not satisfy superposition principle.

3. Time-Invariant & Time Variant System

Time Invariant : Output of the system is independent of time at which input is applied.

Time Variant : Output varies depending on the time at which input is applied.

$y = f(x) = x \quad \infty$

4. **Causal & non-causal System**

Causal : System in which Output of the system depends on present & past values of the input.

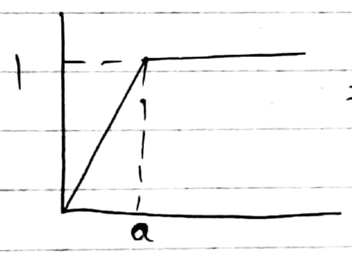
Non-Causal : System in which output depends on present, past or future values of the input.

5. **Stable & unstable System**

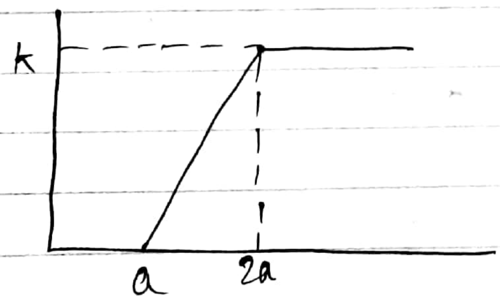
Stable system : If the output of the system is finite or bounded in amplitude for bounded input, then the system will be stable.

Unstable System : If the output of the system is unbounded for bounded input, then the system will be unstable.

---

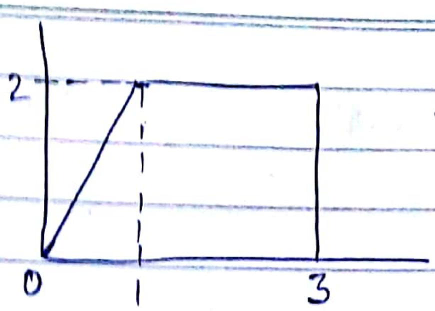
① 

$= \frac{1}{a} r(t) - \frac{1}{a} r(t-a)$

② 

$= \frac{k}{a} r(t-a) - \frac{k}{a} r(t-2a)$

(3).



$= 2r(t) - 2r(t-1) - 2u(t-3)$ ✓

~~$= 2tU(t) - 2(t-1)\{U(t-1)\} - 2u(t-3)$~~

~~$2t[U(t) - U(t-1)] + 2[U(t-1) - U(t-3)]$~~ ✓

(1). $u(t) = x(t+1) + x(t-1)$ Dynamic

$\underbrace{(t+1)}_{\text{future}}$

(2). $x(t)$, $x(t-1)$, $x(t+1)$

$y(t) = x(2t)$
$y(0) = x(0)$
$y(1) = x(2)$

Dynamic

(3). $e^{-t+1} \cdot x(t)$ Static

(1). $y(t) = x(2t) \rightarrow$ Depends on future. $y(0) = x(0)$, $y(1) = x(2)$ $\therefore$ Non-causal

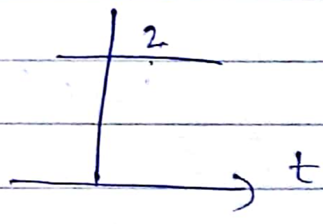
(2). $y(t) = x(-t) \rightarrow y(-1) = x(1)$, $y(0) = x(0)$, $y(1) = x(-1)$ $\therefore$ Non-causal

(3). $y(t) = \sin(t+2)\, x(t-1) \rightarrow$ Depends on past $\therefore$ Causal

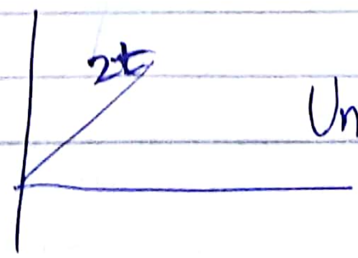
(4). $y(t) = \cos x(t) \rightarrow$ Causal

Q. Tell whether stable or unstable : (1). $y(t) = t \times x(t)$

When $x(t) =$



~~Unstable~~.



Unstable

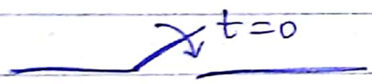
(2). $y(t) = \sin x(t)$ → Stable

22/08/17

Transient

Transients are present in circuit when the circuit is subjected to any changes, either by changing source magnitude or any circuit element provided the circuit consist of energy storage element because inductors doesn't allow sudden change in current & capacitors doesn't allow sudden change in voltage.

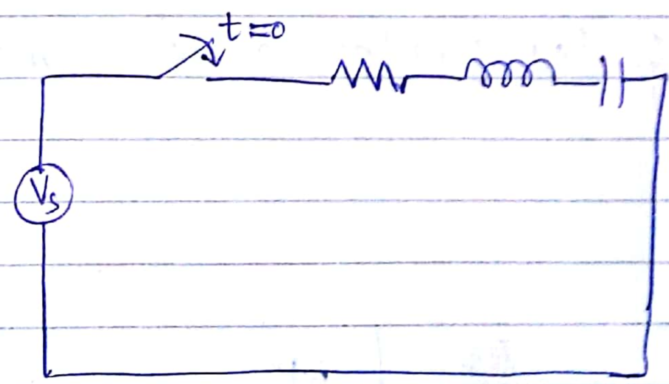
When the circuit is having only resistive element, then there will be no transient in the circuit because resistor allow sudden change in voltage & current.



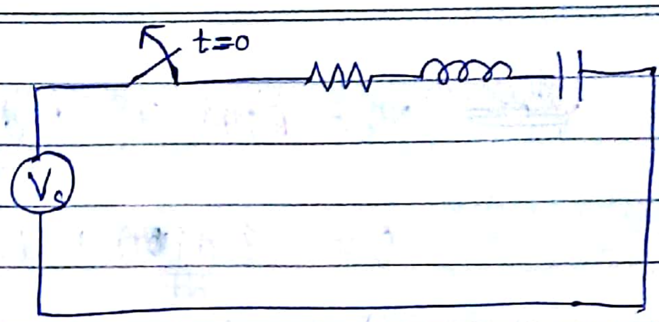
$t(0^-)$ = Just before the switch operation
$t(0^+)$ = Just after " " "
$t(\infty)$ = Steady State

$i_L(0^-) = i_L(0^+) = 0$

$V_c(0^-) = V_c(0^+) = 0$



$i_L(0^-) = i_L(0^+) \neq 0$



~~At t~~

Inductor will act as open circuit at $t=0^+$ when initial condition is not given.
Inductor will act as short circuit at $t=\infty$.

Capacitor will act as short circuit at $t=0^+$
" " " " open " " $t=\infty$.

Steady State Response

Response of network with source present in it is called forced response & it gives steady state output.

This response is independent of the nature of passive elements. It only depends on the type of input. This part of the response is found out by solving a particular integral part of differential equation.

Transient Response

The response of the network without a source in it is called natural response & it depends on the nature of passive elements & is independent of type of input. This part of the response is found out by solving complimentary function of differential equation.

$\cancel{V(s)}$ $V_s = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt$ → First Order

$0 = R\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{L} i(t)$ → Second Order

1. **Ist order non-homogeneous**

$$\frac{dy(t)}{dt} + P\,y(t) = 0$$

$$\boxed{y(t) = ke^{-Pt}}$$

2. **Ist order homogeneous**

$$\frac{dy(t)}{dt} + P\,y(t) = \varphi$$

$$\boxed{y(t) = ke^{-Pt} + e^{-Pt}\int \varphi e^{Pt}\,dt}$$

3. **2nd order non-homogeneous**

$$\frac{d^2y(t)}{dt} + \frac{dy(t)}{dt} + y(t) = 0$$

Roots = P_1 & P_2

$$\boxed{y(t) = k_1 e^{P_1 t} + k_2 e^{P_2 t}}$$

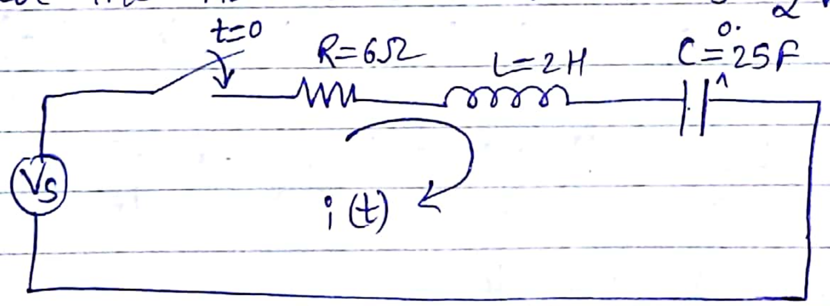
When $P_1 = P_2 = P$,

$$\boxed{y(t) = k_1 e^{pt} + k_2 t e^{pt}}$$

1. **Zero Input Response :**
The value of voltage & current that results from initial condition when the excitation or forcing function is zero, are known as zero input response.

2. **Zero State Response :**
The value of voltage & current for an excitation which is applied when all initial conditions are zero, are known as zero state response.

Q. Consider the RLC circuit when $V_s = 9V$,



Find 1. $i(0^+)$

2. $\frac{di(0^+)}{dt}$

3. $i(t)$

Ans.

$$V_s = Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)$$

$$0 = R\frac{di(0^+)}{dt} + L\frac{d^2 i(0^+)}{dt^2} + \frac{1}{L}\, i(0^+)$$

at $t = 0^+$.

$$\frac{d^2 i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2\,i(t) = 0$$

$$D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

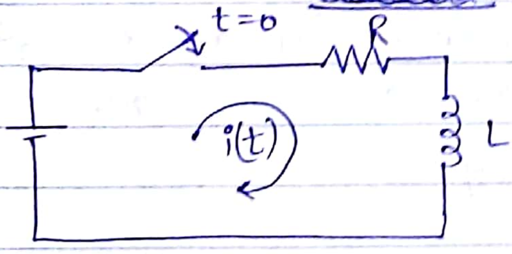
$$\cancel{\Rightarrow D = -1, -2}$$

Roots are $P_1 = -1$ & $P_2 = -2$

$$\Rightarrow \boxed{i(t) = k_1 e^{-t} + k_2 e^{-2t}}$$

24/08/17

Q.



Find ①. V_R
②. V_L
③. $i(t)$

Ans. $V_R = iR$, $V_L = L\frac{di}{dt}$ $\Rightarrow$ $V = V_R + V_L = iR + L\frac{di}{dt}$

$\Rightarrow V - iR = L\frac{di}{dt}$ $\Rightarrow$ $\cancel{\frac{di}{V-iR} = \frac{dt}{L}}$ $\Rightarrow$ $\frac{di}{dt} + i\frac{R}{L} = \frac{V}{L}$ —— ①.

~~Integrating b/s~~ $\cancel{\Rightarrow -\frac{1}{R}\log V}$

This differential eq^n is of the form $\frac{dy(t)}{dt} + Py(t) = Q$.

The solution of this eq^n is $\boxed{y(t) = ke^{-pt} + \frac{Q}{P}}$

$$\Rightarrow \boxed{i(t) = ke^{-\frac{R}{L}t} + \frac{V}{R}}$$

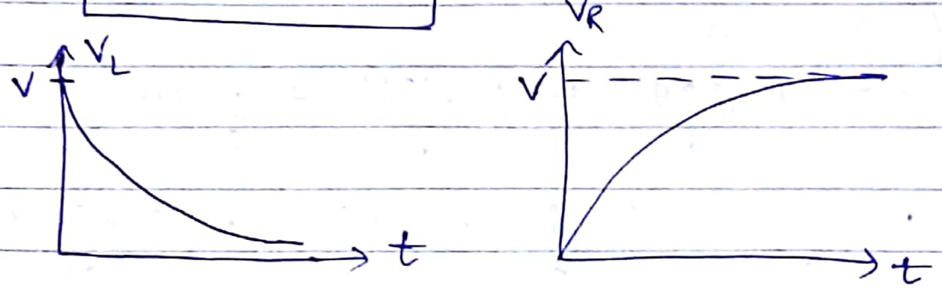
At $t=0^+$, inductor acts as open circuit. $\therefore i(t)=0$.

$\Rightarrow 0 = ke^0 + \frac{V}{R} \Rightarrow k = -\frac{V}{R}$

$$\Rightarrow \boxed{i(t) = \frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)}$$

$$\Rightarrow \boxed{V_R = V\left(1-e^{-(R/L)t}\right)} \quad \& \quad V_L = L\frac{di(t)}{dt} = L\frac{V}{R}\left(0 + \frac{R}{L}e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \boxed{V_L = Ve^{-(R/L)t}}$$



24/08/17

Tutorial

$\rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \rightarrow$ It depends on past ($\because$ of $-\infty$).
$\therefore$ Dynamic.

Imp:
(Whenever there is integration or differentiation, the system is always dynamic.)

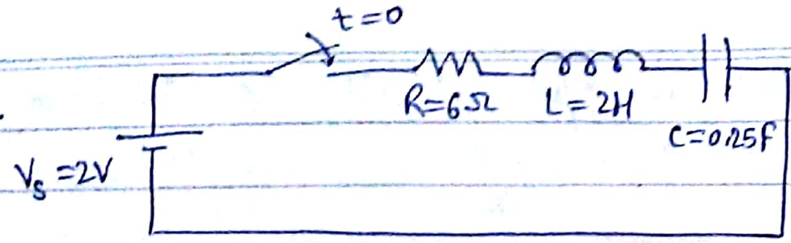
$\rightarrow y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau \rightarrow$ Non causal ($\because$ It depends on future too).

$\rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \rightarrow$ Causal ($\because$ It depends on past & present only).

$\rightarrow y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ where $x(t) = u(t) \rightarrow$ Unstable.

Note: If any non-linear operator (sin, cos, tan, cot, exponential, log, modulus, square, cube, higher order power, root, real, imaginary, conjugate) operates on either x or y, then the system will be non-linea

Q.



Find $i(t)$, $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$

Ans. $i(0^+)$ = Current just after the switching operation
$= 0$ ($\because$ Inductor acts as open circuit at $t=0^+$).

Using KVL, $V_s = i(t).R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$ — (1).

Diff. b/s w.r.t. t.

$\Rightarrow 0 = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t)$

$\Rightarrow 2\frac{d^2i(t)}{dt^2} + 6\frac{di(t)}{dt} + 4i(t) = 0 \Rightarrow \left(\frac{di(t)}{dt}+2\right)\left(\frac{di(t)}{dt}+1\right) = 0$

Two roots are $P_1 = -2$ & $P_2 = -1$.

$\Rightarrow i(t) = k_1 e^{-t} + k_2 e^{-2t}$ — (2).

(Now, we have to find the value of k_1 & k_2).

We know that $i(0^+) = 0$.

$\Rightarrow 0 = k_1 e^0 + k_2 e^0 \Rightarrow \boxed{k_1 = -k_2}$

We know that capacitor doesn't allow sudden change in voltage.

$\therefore$ From eq^n (1), $V_s = 0 + L\frac{di(t)}{dt} + 0$

$\Rightarrow \frac{di(t)}{dt} = \frac{V_s}{L} = 1\ A/s$ — (3).

Diff eq^n (2) w.r.t t. $\Rightarrow \frac{di(t)}{dt} = -k_1 e^{-t} - 2k_2 e^{-2t}$

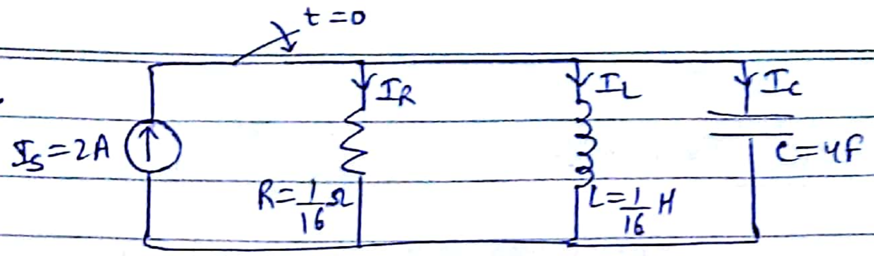
$\Rightarrow 1 = -k_1 e^0 - 2k_2 e^0$ (From (3), at $t=0^+$).

$\Rightarrow k_1 + 2k_2 = -1 \Rightarrow -k_2 + 2k_2 = -1 \Rightarrow \boxed{k_2 = -1}$

$\Rightarrow \boxed{k_1 = 1}$

$\Rightarrow \boxed{i(t) = e^{-t} - e^{-2t}}$ (Now, find other required values).

9.



Determine $V(t)$, $V(0^+)$, $\frac{dV(0^+)}{dt}$, $\frac{d^2V(0^+)}{dt^2}$.

Ans. $I_s = I_L + I_R + I_c \Rightarrow I_s = \frac{1}{L}\int V(t)\,dt + \frac{V(t)}{R} + c\frac{dV(t)}{dt}$ —— ①.

Diff b/s w.r.t. t.

$$\Rightarrow 0 = \frac{V(t)}{L} + \frac{1}{R}\frac{dV(t)}{dt} + c\frac{d^2V(t)}{dt^2}$$

$$\Rightarrow \left(\frac{dV}{dt} + 2\right)^2 = 0 \Rightarrow \text{Two roots are } P_1 = P_2 = -2.$$

$$\Rightarrow V(t) = K_1e^{-2t} + K_2te^{-2t} \text{ —— ②.}$$

$V(0^+) = 0 \Rightarrow 0 = K_1e^0 + 0.K_2e^0 \Rightarrow \boxed{K_1 = 0}$

From eqn ①, $I_s = 0 + 0 + c\frac{dV(t)}{dt}$ ~~(at t = 0⁺).~~

$$\Rightarrow \frac{dV(t)}{dt} = \frac{I_s}{c} = \frac{1}{2}\ V/s$$

Diff. eqn ② w.r.t. t. $\Rightarrow \frac{dV(t)}{dt} = K_2[-2te^{-2t} + e^{-2t}]$

At $t = 0^+$, $\frac{1}{2} = K_2(-2.0.e^0 + e^0) \Rightarrow \boxed{K_2 = 1/2}$

$$\Rightarrow \boxed{V(t) = \frac{1}{2}te^{-2t}}$$

$$\Rightarrow \frac{dV(t)}{dt} = \frac{1}{2}[1.e^{-2t} - 2te^{-2t}]$$

$$\Rightarrow \frac{d^2V(t)}{dt^2} = \frac{1}{2}[-2e^{-2t} - 2(e^{-2t} - 2te^{-2t})]$$

$$\Rightarrow \frac{d^2V(0^+)}{dt^2} = \frac{1}{2}[-2 - 2(1 - 0)] = -2$$

$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) + \dots - f^{n-1}(0)$

29/08/17

Laplace Transform

The Laplace transform is one of the mathematical tools used for solution of linear ordinary integral of differential eq^n.

In comparison with classical method of solving linear integral of diff. eq^n, the laplace transform has two attractive features:

1. The homogeneous equations & particular integral of the solution are always operated in one operation.

2. The Laplace transform converts integral of diff. eq^n into an algebraic equation S (laplace operator). It is then possible to manipulate the algebraic equation by simple algebraic rules. The final solution is obtained by taking inverse laplace transform.

$$F(s) = L[f(t)] = \int_0^\infty f(t) e^{-st} dt$$

Properties of laplace transform

1. Multiplication by a constant

$$L[kf(t)] = kF(s)$$

2. $$L[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$$

3. $$L\left[\frac{d f(t)}{dt}\right] = sF(s) - F(0^+)$$

Scanned by CamScanner

$$(-1)^n \frac{d^n}{ds^n} F(s)$$

4. $$L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

5. $$L[t f(t)] = -\frac{d F(s)}{ds}$$

6. $$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$$

7. $$L[e^{at} f(t)] = F(s-a)$$

8. Initial Value Theorem

$$f(0^+) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

9. Final Value Theorem

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

10. $$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

11. Theorem for Periodic Functions

The Laplace transform of periodic f^n with period "T" is $\frac{1}{1-e^{-Ts}}$ times the Laplace transform of 1st cycle of that function.

$$\boxed{L[f(t)] = \frac{1}{1-e^{-Ts}} (F_1(s))}$$

$\frac{1}{s^2}$ $t^n f(t)$ $t^n = \frac{\Gamma n+1}{s^{n+1}}$ $\Gamma n+1$ $L[f(t)] = \int_0^\infty f(t) e^{-st} dt$

$\frac{1-3e^{-1}+2e^{-2}}{s(1-e^{-2s})}$

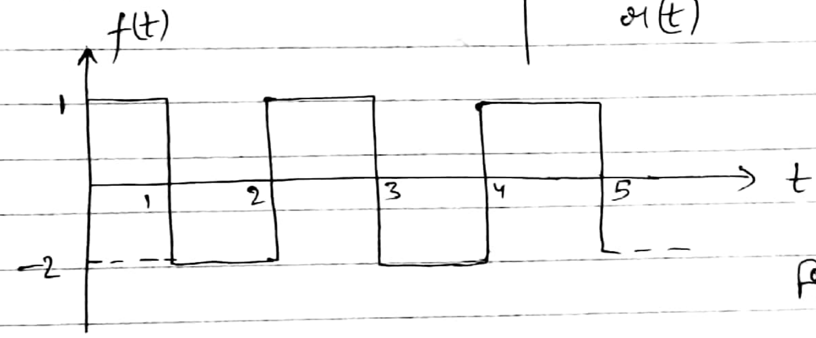
31/08/17

Lecture

$f(t)$	$F(s)$	$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\delta(t)$	1	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
e^{at}	$\frac{1}{s-a}$	$f(t-t_0)$	$e^{-t_0 s} F(s)$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	te^{at}	$\frac{1}{(s-a)^2}$
		$r(t)$	$\frac{1}{s^2}$

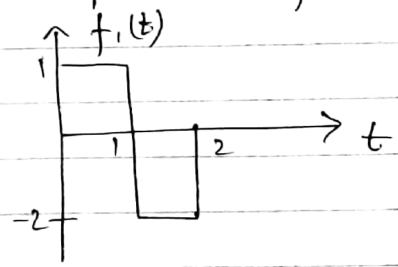
LAPLACE TRANSFORM

Q.



→ Periodic function with period $T=2$.

Find the Laplace transform.



Ans. $L[f(t)] = \frac{1}{1-e^{-2s}} \cdot L[f_1(t)]$ —— ①.

$\left[f_1(t) = u(t) - 3u(t-1) + 2u(t-2)\right]$

$\Rightarrow L[f_1(t)] = L[u(t)] - 3L[u(t-1)] + 2L[u(t-2)]$

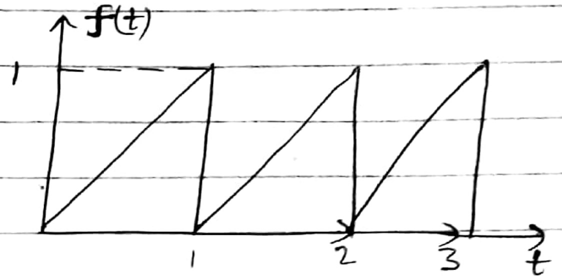
$= \frac{1}{s} - 3\frac{e^{-s}}{s} + 2\frac{e^{-2s}}{s}$ —— ②.

Put ② in ①.

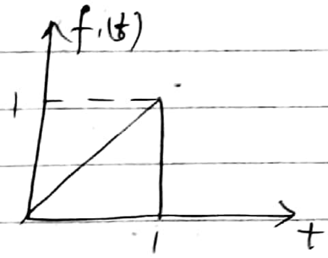
$$t\,u(t) - (t-1)\,u(t-1) - u(t-1)$$
$$t\,u(t) - (t - \cancel{1} + \cancel{1})\,u(t-1))$$

$$\Rightarrow \boxed{L[f(t)] = \frac{1 - 3e^{-s} + 2e^{-2s}}{s(1-e^{-2s})}}$$

Q. Find the Laplace transform of



Ans. $L[f(t)] = \dfrac{1}{1-e^{-s}}\, L[f_1(t)]$ (Period, T=1)

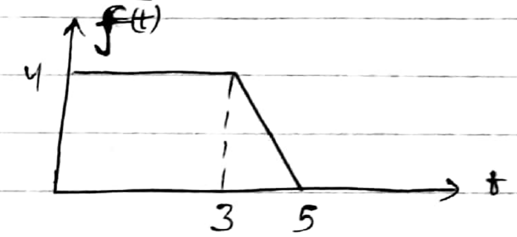


$$f_1(t) = \cancel{u(t)}\; t\,(u(t) - u(t-1))$$

$$\Rightarrow L[f_1(t)] = L[t\,u(t)] - L[t\,u(t-1)]$$
$$= -\frac{d}{ds}\left(\frac{1}{s}\right) + \frac{d}{ds}\left(e^{-s}\cdot\frac{1}{s}\right)$$
$$= \frac{1}{s^2} + \frac{s(-e^{-s}) - 1\cdot e^{-s}}{s^2} = \frac{1-e^{-s}-se^{-s}}{s^2}$$
$$= \frac{1-e^{-s}(1+s)}{s^2}$$

$$\Rightarrow \boxed{L[f(t)] = \frac{1}{1-e^{-s}}\left(\frac{1-e^{-s}(1+s)}{s^2}\right)}$$

Q. Find the Laplace transform of



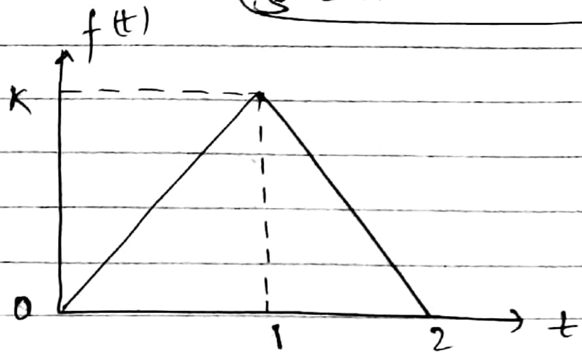
Ans. $f(t) = 4\,u(t) - 2r(t-3) + 2r(t-5)$

$$\Rightarrow L[f(t)] = \frac{4}{s} - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-5s}$$

$t^n f(t)$ $\quad L[t^{-1}\sin t]$ $\quad \int_0^\infty e^{-st} \frac{\sin t}{t} dt$ $\quad \int_0^\infty \frac{\sin t}{t} dt$ $\quad (-1)^n \frac{d^n}{ds^n} F(s)$ $\quad \frac{-se^{-s} - e^{-s}}{s^2}$ $\quad k$ $\quad t \operatorname{cosec} t$ $\quad -k$

$\int_s^\infty \frac{1}{s^2+1} ds$ $\quad \tan^{-1}\infty - \tan^{-1}s$ $\quad \frac{\pi}{2} - \tan^{-1}s$ $\quad \frac{\pi}{2} - 0$

Q.



find F(s).

Ans. $f(t) = k\,r(t) - 2k\,r(t-1) + k\,r(t-2)$

~~$= k[t\,u(t) - 2$~~

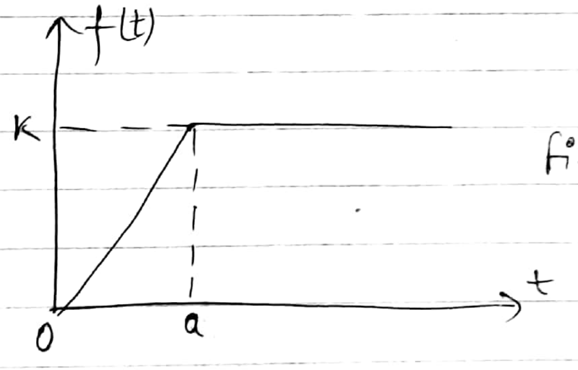
$= k\,[r(t) - 2\,r(t-1) + r(t-2)]$

$\Rightarrow L[f(t)] = F(s)$

$= k\left[\frac{1}{s^2} - 2\frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\right]$

$= \frac{k}{s^2}(1 - 2e^{-s} + e^{-2s})$

Q.



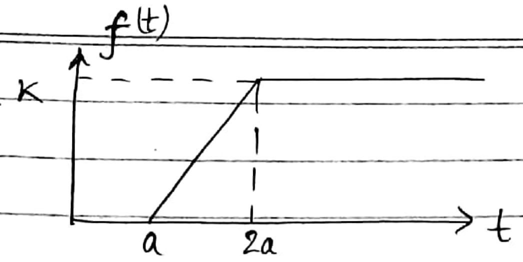
find F(s).

Ans. $f(t) = \frac{k}{a} r(t) - \frac{k}{a} r(t-a) = \frac{k}{a}\left(r(t) - r(t-a)\right)$

$\Rightarrow F(s) = \frac{k}{a}\left[\frac{1}{s^2} - \frac{e^{-as}}{s^2}\right] = \frac{k}{as^2}(1 - e^{-as})$

$\frac{-2 \pm \sqrt{4-20}}{2}$ $\qquad s^2+2s+1+4$ $\qquad s^2+a^2$

Q. Find F(s).



Ans. $f(t) = \frac{k}{a} r(t-a) - \frac{k}{a} r(t-2a)$

$\Rightarrow F(t) = \frac{k}{a}\left[\frac{e^{-as}}{s^2} - \frac{e^{-2as}}{s^2}\right] = \frac{k}{as^2}(e^{-as} - e^{-2as})$

Q. Find $f(0^+)$ & $f(\infty)$.

$F(s) = \frac{2s+1}{s^2+2s+5}$

Ans. $F(s) = 2\frac{s}{s^2+2s+5} + \frac{1}{s^2+2s+5}$

$= 2\frac{(s+1-1)}{(s+1)^2+2^2} + \frac{1}{(s+1)^2+2^2} = \frac{2(s+1)}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+2^2}$

$f(t) = L^{-1}[F(s)] = 2e^{-t}\cos 2t - e^{-t}\sin 2t$

$\Rightarrow f(0^+) = 2e^0\cos 0 - e^0 \sin 0 = 2$

$f(\infty) = 2e^{-\infty}\cos\infty - e^{-\infty}\sin\infty = 0$

OR

Using Initial & Final Value Theorem,

$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2s^2+s}{s^2+2s+5} = \lim_{s\to\infty} \frac{2+\frac{1}{s}}{1+\frac{2}{s}+\frac{5}{s^2}}$

$= 2$

$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2s^2+s}{s^2+2s+5} = \frac{0}{5} = 0$

Q. Find $f(0^+)$ & $f(\infty)$.

$$f(t) = 5\,u(t) - 3e^{-2t}.$$

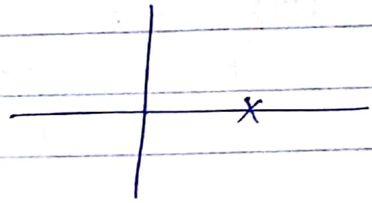
Ans.

$$f(0^+) = 5 \times 1 - 3e^{0} = 2$$

$$f(\infty) = 5 \times 1 - 3e^{-\infty} = 5$$

Note :- We cannot apply final value theorem when pole is on the right side of s-plane.

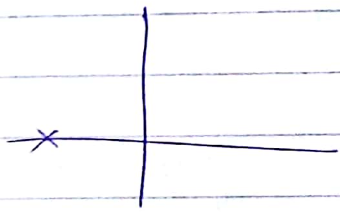
For eg.- ①. $\frac{1}{s-a} \rightarrow$ Pole $\rightarrow s = a$



$\Rightarrow$ Right side of s-plane.

$\therefore$ F.V.T. cannot be applied.

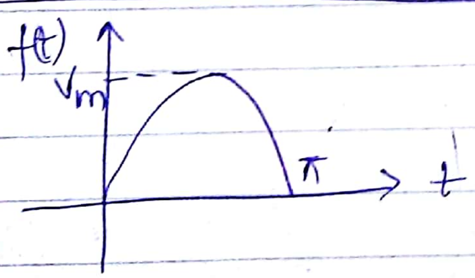
②. $\frac{1}{s+a} \rightarrow$ Pole $\rightarrow s = -a$



$\Rightarrow$ Left side of s-plane.

$\therefore$ F.V.T. can be applied.

Q.



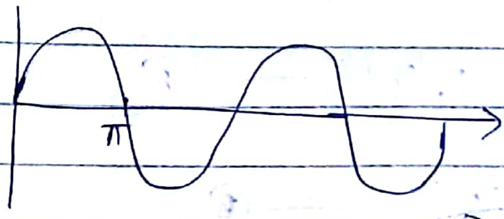
Find $F(s)$.

~~Ans.~~ ~~This is the first cycle of function / Vm sint /~~ .

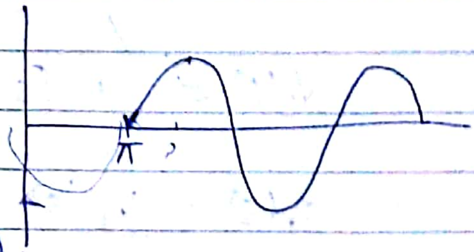
$$V_m|\sin t| = \begin{cases} V_m \sin t \ ; & 0 < t < \pi/2 \\ -V_m \sin t \ ; & \pi/2 < t < \pi \end{cases}$$

$\frac{2\pi}{2}$

Ans.



$+$



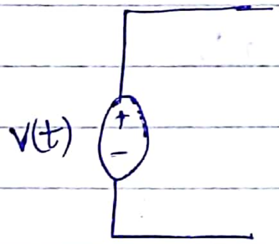
$$\cancel{\Rightarrow f(t) = V_m[\sin t + \sin(t+\pi)] = V_m[\sin t - \sin(\pi - t)]}$$
$$\cancel{\Rightarrow 2V_m \sin t}$$

08/09/17

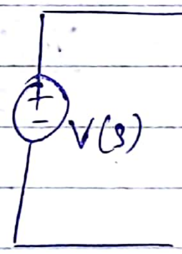
Transform Circuit Component Representation

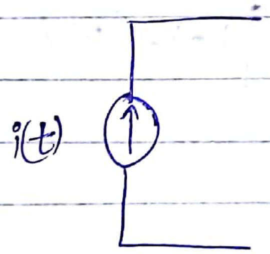
1. Independent Sources

The sources $v(t)$ & $i(t)$ may be represented by a transformation $V(s)$ & $I(s)$

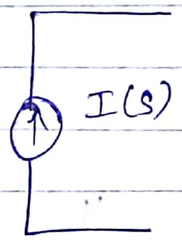


$\Rightarrow$

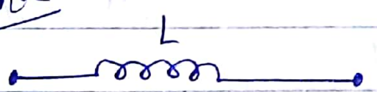




$\Rightarrow$



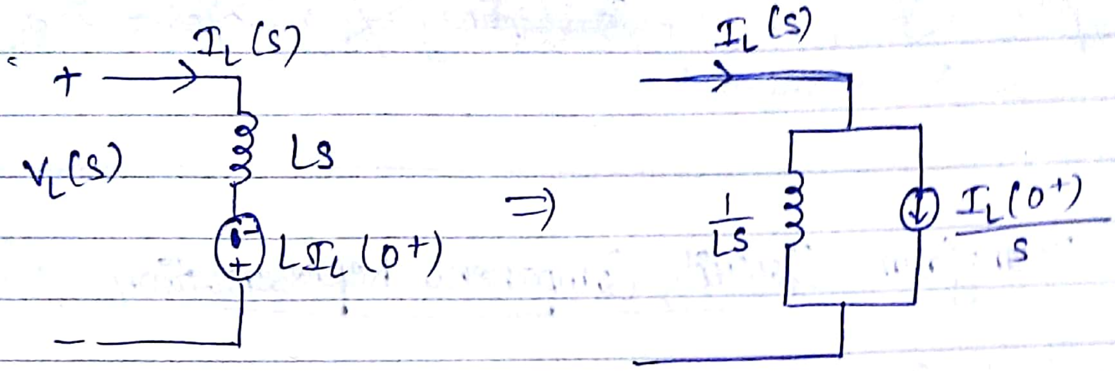
Inductance



$$v_L(t) = L\frac{di_L}{dt}$$

$$V_L(s) = L\left[s\,I_L(s) - I_L(0^+)\right] \quad \text{—— (A)}$$

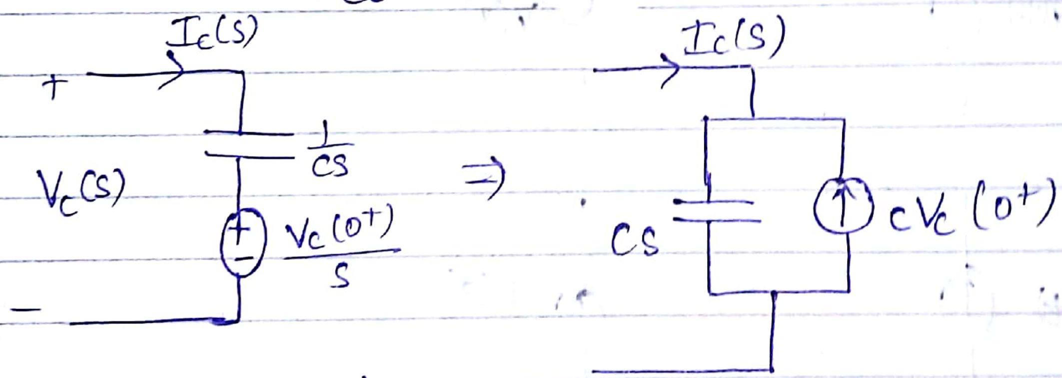
$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{I_L(0^+)}{s} \quad \text{—— (B).}$$



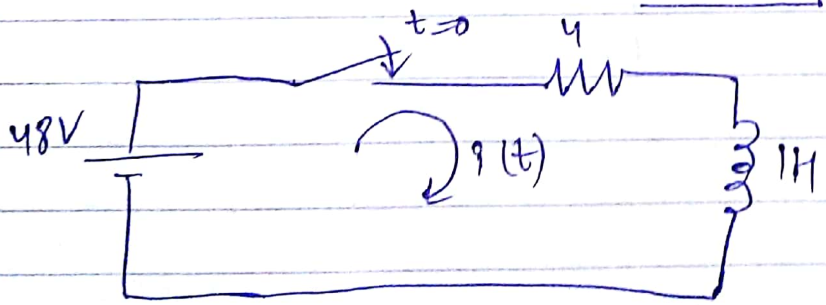
$$i_c(t) = c\,\frac{dV_c(t)}{dt}$$

$$I_c(s) = c\left[s\,V_c(s) - V_c(0^+)\right] \quad \text{—— (A).}$$

$$V_c(s) = \frac{I_c(s)}{cs} + \frac{V_c(0^+)}{s} \quad \text{—— (B).}$$



Q.



$i(0^+) = 3A$

Find $i(t)$ using Laplace transform.

$v = \frac{1}{c}\frac{dI}{dt}$ $\quad q = cv$, $\frac{dq}{dt} = cv$, $I = c$ $\quad 10 = \frac{V(t)}{0.5} + c\int v(t)\,dt$

Ans. Using KVL.

$$48 = L\frac{di(t)}{dt} + Ri(t) \quad \text{— (A)}$$

$$\frac{48}{s} = \left(sI(s) - I(0^+)\right) + 4I(s) \quad \text{— (B)}.$$ (Took Laplace transform)

$$\frac{1}{s+4}\left(\frac{48}{s} + I(0^+)\right) = I(s)$$

$$\Rightarrow I(s) = \frac{3s+48}{s(s+4)} \quad (\because i(0^+) = 3A)$$

let $\frac{3s+48}{s(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+4}$

$$3s+48 = k_1(s+4) + k_2 s$$

$\Rightarrow k_1 + k_2 = 3$ & $4k_1 = 48$

$\boxed{k_1 = 12}$

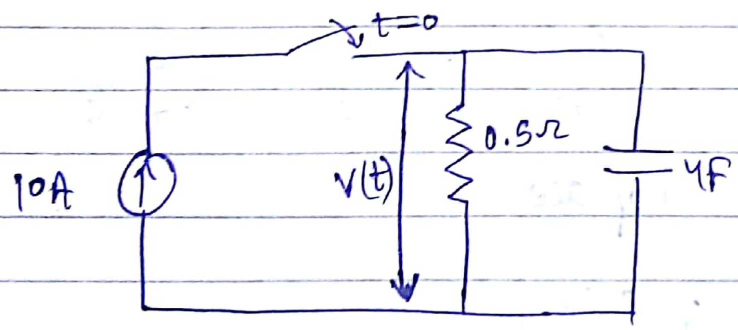
$\Rightarrow \boxed{k_2 = -9}$

$$\Rightarrow I(s) = \frac{12}{s} - \frac{9}{s+4}$$

$\Rightarrow$ ~~i(t)~~ Taking inverse laplace transform,

$\Rightarrow \boxed{i(t) = 12 - 9e^{-4t}}$

Q.



$V_c(0^+) = 2$
Find $V(t)$ using Laplace transform.

$\cancel{q = CV}$ $\cancel{\frac{dI}{dt} = CV}$ $\cancel{I = C\int V dt}$

Ans. Using KCL,

$$10 = \frac{V(t)}{R} + \frac{C\,dV(t)}{dt}$$

$$10 = \frac{V(t)}{0.5} + 4\frac{dV(t)}{dt}$$

Taking Laplace Transform,

$$\frac{10}{s} = 2V(s) + 4(sV(s) - V_c(0^+))$$

$$\Rightarrow \frac{10}{s} = 2V(s) + 4sV(s) - 4V_c(0^+)$$

$$\Rightarrow V(s) = \left(\frac{10}{s} + 8\right) \times \frac{1}{(2+4s)}$$

$$\Rightarrow V(s) = \frac{8s+10}{s(4s+2)} = \frac{4s+5}{s(2s+1)}$$

let

$$\frac{4s+5}{s(2s+1)} = \frac{k_1}{s} + \frac{k_2}{2s+1}$$

$$4s+5 = 2k_1s + k_1 + k_2s$$

$$\Rightarrow 2k_1 + k_2 = 4 \quad \& \quad \boxed{k_1 = 5}$$

$$\Rightarrow \boxed{k_2 = -6}$$

$$\Rightarrow V(s) = \frac{5}{s} - \frac{6}{2s+1} = \frac{5}{s} - \frac{3}{s+0.5}$$

Taking Inverse Laplace,

$$\boxed{V(t) = 5 - 3e^{-0.5t}}$$