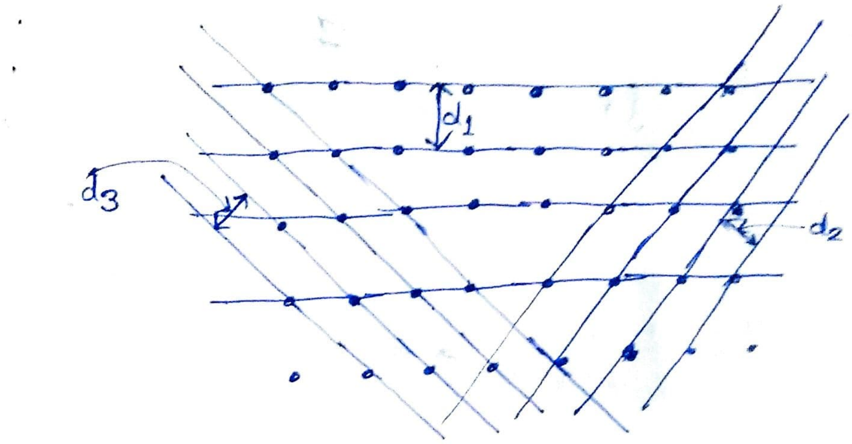
Lattice Planes of Crystal:→ A crystal lattice may be considered as an aggregate of a set of parallel, equally-spaced planes passing through the lattice points. The planes are called 'lattice Planes'.

The perpendicular distance between adjacent planes is called interplaner spacing. It is denoted by d.



A given space lattice may have an infinite sets of lattice planes, each set has its characteristics interplanar spacing d_1, d_2 and d_3. (Figure) Each covers the all the lattice points.

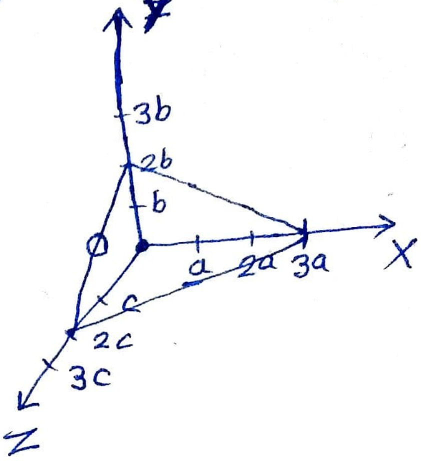
Out of these sets of lattice planes, only those which have high density of lattice points are significant and show diffraction of X-rays. They are known as Bragg planes or cleavage planes.

Miller Indices:→ The position and orientation of a lattice plane in a crystal is determined by three smallest whole numbers which have same ratios with one another as the reciprocals of the intercepts of the plane on three crystal axis. Miller indices are denoted by h, k, l of particular plane (or of any plane parallel to it) and the plane is specified as (hkl).

Steps to find miller Indices of a lattice plane →

There are five steps, which are followed to find miller Indicies of a lattice plane -

Step I → Take a point or atom as origin & construct three coordinate axis, Find the intercepts of the plane along three coordinate axis of the plane along three coordinate axis.



x	y	z
3a	2b	2c

Step II → Express intercepts as multiples of $\vec{a}, \vec{b}, c$.

3 2 2

Step III - Take reciprocals of intercepts

$\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{2}$

Step IV → Reduce Reciprocals of intercepts into smallest set of integers in the same ratio by multiplying with these LCM.

$\frac{1}{3} \times 6^{2}$ $\frac{1}{2} \times 6^{3}$ $\frac{1}{2} \times 6^{3}$

2 3 1

Step V → Enclose the smallest set of integers in the paranthes & we will get miller Indicis.

So miller Indicis are $(2\,3\,1) = (hkl)$

Important features of miller Indices:→

(i) In cubic crystal the direction perpendicular to (hkl) is represented by [hkl].
for Exp- The direction perpendicular to (233) is [233]

(ii) No Commas are introduced in between the indices of single digit i.e. (233), but when miller Indices of plane are of double digit, these are separated by commas-

(10, 5, 15)

(iii) The miller Indices (266), (133) represents the same plane.

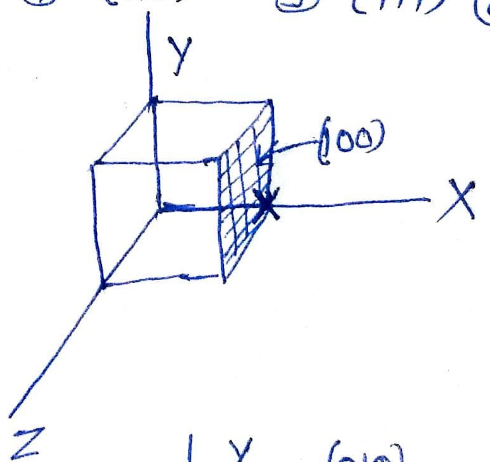
Sketching of Lattice Planes in a cubic crystal→

Ex..

The methods of designing the plane if miller Indices are given as follows:- If miller Indices are given as-
① (100), ② (010), ③ (001), ④ (110) ⑤ (111) ⑥ $(\bar{1}00)$

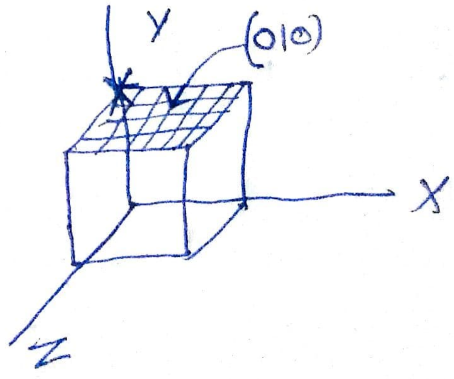
① (100) ⇒

$$\begin{array}{ccc} h & k & l \\ 1 & 0 & 0 \\ x & y & z \\ \frac{1}{1} & \frac{1}{0} & \frac{1}{0} \\ =1 & \infty & \infty \end{array}$$



② (010) ⇒

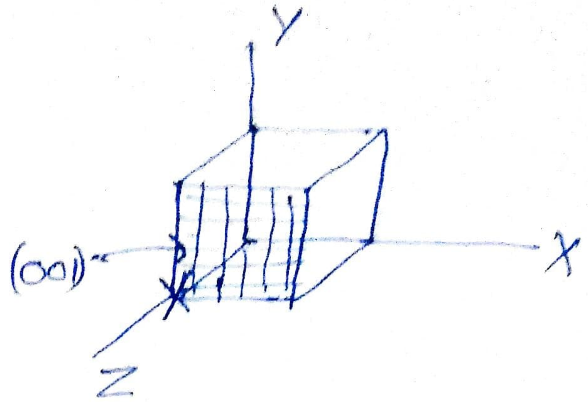
$$\begin{array}{ccc} h & k & l \\ 0 & 1 & 0 \\ x & y & z \\ \frac{1}{0} & \frac{1}{1} & \frac{1}{0} \\ \infty & 1 & \infty \end{array}$$



③ $(001) \Rightarrow$

$$\begin{array}{ccc} h & k & l \\ 0 & 0 & 1 \\ x & y & z \\ \frac{1}{0} & \frac{1}{0} & \frac{1}{1} \end{array}$$

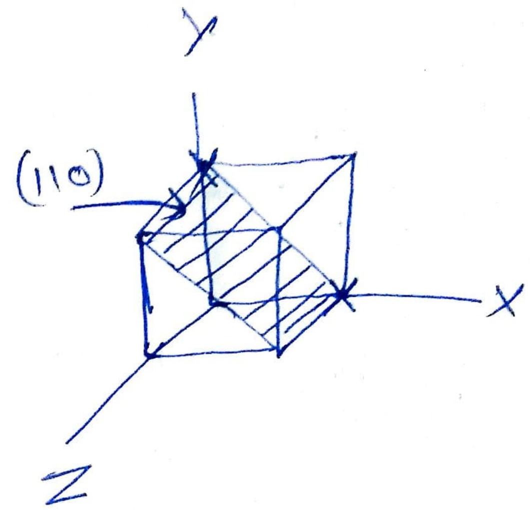
$$= \infty \quad \infty \quad 1$$



④ $(110) \Rightarrow$

$$\begin{array}{ccc} h & k & l \\ 1 & 1 & 0 \\ x & y & z \\ \frac{1}{1} & \frac{1}{1} & \frac{1}{0} \end{array}$$

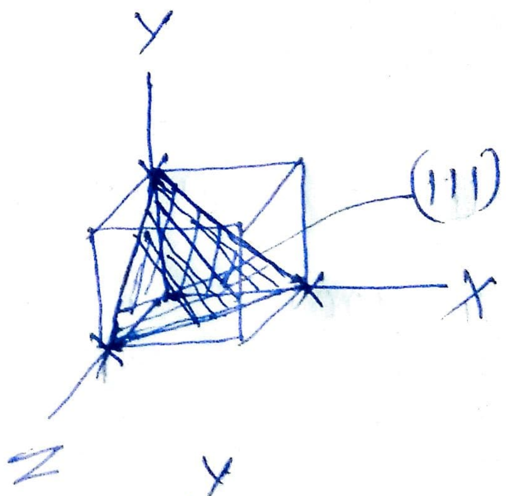
$$= 1 \quad 1 \quad \infty$$



⑤ $(111) \Rightarrow$

$$\begin{array}{ccc} h & k & l \\ 1 & 1 & 1 \\ x & y & z \\ \frac{1}{1} & \frac{1}{1} & \frac{1}{1} \end{array}$$

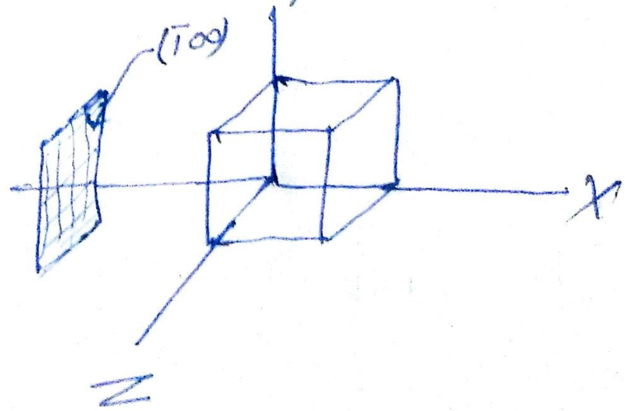
$$= 1 \quad 1 \quad 1$$



⑥ $(\bar{1}00) \Rightarrow$

$$\begin{array}{ccc} h & k & l \\ -1 & 0 & 0 \\ x & y & z \\ -\frac{1}{1} & \frac{1}{0} & \frac{1}{0} \end{array}$$

$$\Rightarrow -1 \quad \infty \quad \infty$$



$$d_{h,k,l} = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

Interplanar Spacing : The separation between successive lattice planes of cubic, tetragonal and orthorhombic crystals, for which $\alpha = \beta = \gamma = 90°$, can be deduced as follows :

Let OX, OY and OZ be three axes parallel to the crystal axes (Fig. 30) Let ABC be one of a series of parallel lattice planes in the crystal. Let the plane ABC have intercepts

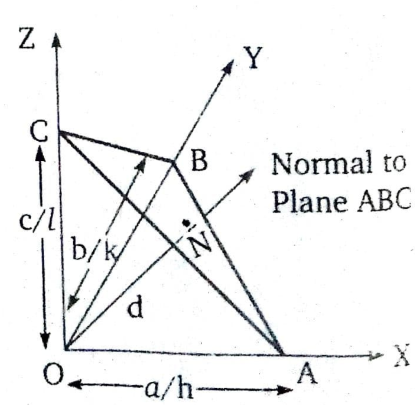


Fig. 30

$OA = a/h, OB = b/k$ and $OC = c/l$. Let us suppose that the origin O lies in the plane adjacent to ABC. Then, ON, the length of the normal from the origin to the plane ABC, is equal to the interplanar distance d. Let θ_a, θ_b and θ_c be the angles which ON makes with the three crystallographic axex respectively. Then, the direction-cosines of ON are

$$\cos\theta_a = \frac{ON}{OA} = \frac{d}{a/h}, \quad \cos\theta_b = \frac{ON}{OB} = \frac{d}{b/k} \text{ and } \cos\theta_c = \frac{ON}{OC} = \frac{d}{c/l}$$

We know that the sum of the squares of the direction-cosines of a line is equal to unity ($\cos^2\theta_a + \cos^2\theta_b + \cos^2\theta_c = 1$). Therefore

$$\left(\frac{d}{a/h}\right)^2 + \left(\frac{d}{b/k}\right)^2 + \left(\frac{d}{c/l}\right)^2 = 1$$

or
$$d^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$$

or
$$\boxed{d = \frac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}}}$$

For a cubic crystal, the lengths of the sides of a unit cell are equal, that is, $a = b = c$.

$$\therefore \quad \boxed{d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}}$$

Interplanar Separation Ratios : The interplanar separation ratios $d_{100} : d_{110} : d_{111}$ for the sc lattices may be found as follows :

Simple Cubic (sc) Lattice : For the (100) planes, we have $h = 1$, $k = 0$ and $l = 0$.

$$\therefore \quad d_{100} = a$$

For the (110) planes, we have $h = 1$, $k = 1$ and $l = 0$

$$\therefore \quad d_{110} = a/\sqrt{2}$$

For the (111) planes, we have $h = 1$, $k = 1$, and $l = 1$.

$$\therefore \quad d_{111} = a/\sqrt{3}$$