Resonance in series RLC

It is condition in RLC circuit at which applied voltage & current are in same phase

In this, impedance of circuit become minimum & current become max.

(At resonance $X_L = X_C$)

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$Z = \sqrt{0 + R^2}$ $\Rightarrow \boxed{Z = R}$

Z is min so, current is max

Resonant frequency is frequency at which resonance occurs.

(At resonance $X_L = X_C$)

$\omega L = 1/\omega C$

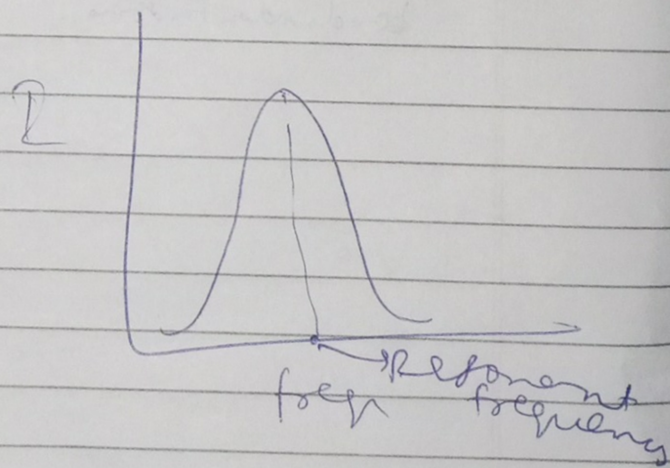
$\omega \times \omega = \frac{1}{LC}$

$2\pi f \times 2\pi f = \frac{1}{LC}$

$4\pi^2 f^2 = \frac{1}{LC}$

$f^2 = \frac{1}{4\pi^2 LC}$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$



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Quality factor is ratio of potential drop across L or C to potential drop across R.

$$Q = \frac{V_C \text{ or } V_L}{V_R}$$

$$Q = \frac{I X_L}{I R} \Rightarrow Q = \frac{\omega L}{R}$$

$$\cancel{Q = \frac{2\pi f L}{R}} \Rightarrow Q = \frac{2\pi f L}{R}$$

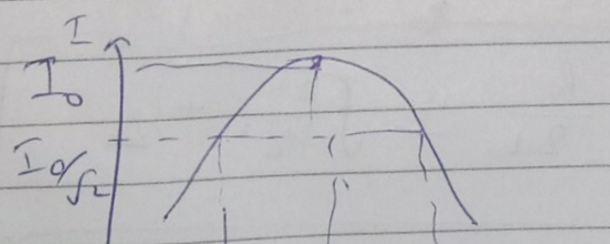
But $f = \frac{1}{2\pi\sqrt{LC}}$

$$Q = \frac{\cancel{2\pi} 1 L}{\cancel{2\pi}\sqrt{LC} R}$$

$$Q = \frac{\sqrt{L^2}}{R\sqrt{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Bandwidth
The difference between upper half power frequency & lower half power frequency.



$$\Delta\omega = \omega_2 - \omega_1$$

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$I_{max} = V/R$

At half power

$$I = \frac{I_{max}}{\sqrt{2}} = \frac{V}{\sqrt{2}R}$$

⇒ Impedance at half power

$Z = \sqrt{2}R$

⇒ Now lets find ω_1 & ω_2

$Z = \sqrt{2}R$

$$\Rightarrow R + j\left(\omega L - \frac{1}{\omega C}\right) = R \pm iR$$

$$\Rightarrow \omega L - 1/\omega C = \pm R$$

$$\Rightarrow \omega L - 1/\omega C = R \text{ (for inductive)}$$

$$\Rightarrow \omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\Rightarrow \omega_2^2 LC - 1 = \omega_2 CR$$

$$\Rightarrow \omega_2^2 LC - \omega_2 CR - 1 = 0$$

$$\Rightarrow \omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\Rightarrow \omega_2 = \frac{\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$\Rightarrow \boxed{\omega_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}}$$

$$\Rightarrow \omega L - 1/\omega C = -R \text{ (Capacitive)}$$

$$\Rightarrow \omega_1 L - \frac{1}{\omega_1 C} = -R$$

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$\Rightarrow \omega_1^2 LC - 1 = -RC\omega_1$

$\Rightarrow \omega_1^2 LC + RC\omega_1 - 1 = 0$

$\Rightarrow \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$

$$\Rightarrow \omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$\Rightarrow \omega_1 = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$\Rightarrow$ Bandwidth $\Rightarrow \Delta\omega = \omega_2 - \omega_1$

$$\Rightarrow \Delta\omega = \left[\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right] - \left[-\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right]$$

$$\Rightarrow \boxed{\Delta\omega = \frac{R}{L}}$$