SIGNAL DEFINITION AND ITS CLASSIFICATION

SIGNAL.

A signal is a function which contains some information

SYSTEM

A system is a interconnection of devices or components that converts signal from one form to another form

CLASSIFICATION OF SIGNAL.

1. CONTINUOUS AND DISCRETE

a) Continuous Time Signal

A signal is c/d continuous, if time axis is continuous in nature.

b) Discrete Time Signal

A signal is c/d discrete, if time axis is integral in nature.

→ continuous & discrete terms are related to x-axis or time-axis

2. ANALOG AND DIGITAL

a) Analog Signal

A signal is c/d analog, if it can take any value on y-axis or magnitude axis.

b) Digital signal

A signal is c/d digital, if it can take only finite values on y-axis or magnitude axis.

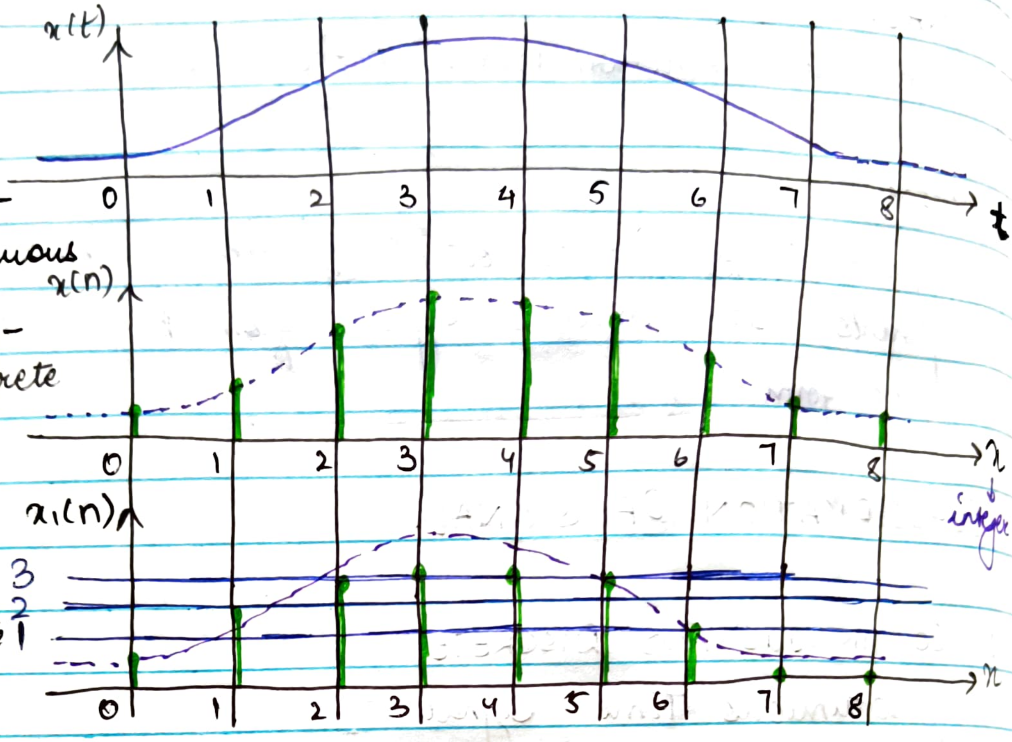
→ Analog & digital terms are related to y-axis or magnitude axis.

eg.

Analog - continuous

Analog - discrete

digital discrete



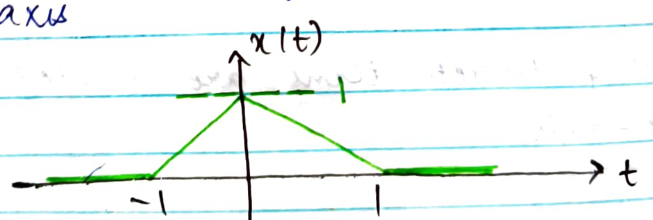
3. EVEN AND ODD SIGNAL.

a) Even signal. Eg - cos t, $\sin^2 t$

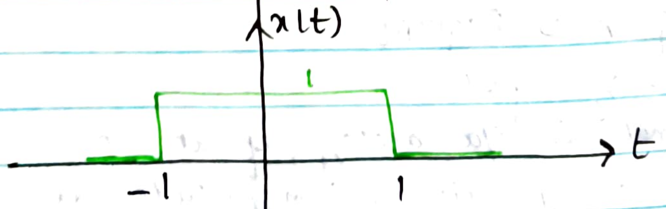
$$\boxed{x(t) = x(-t)}$$

→ Signal will be symmetric / mirror-image about Y-axis

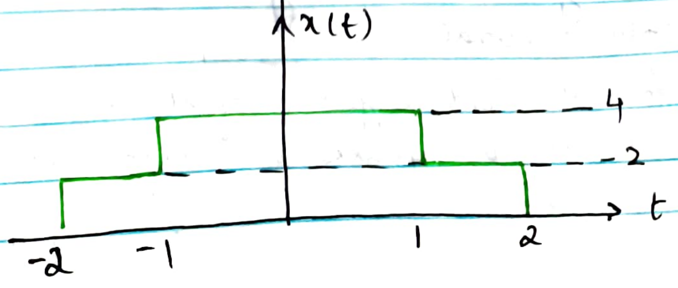
eg. 1.



2.



3.



4. $x(t) = \cos \omega_0 t \longrightarrow$ Even s/g.

$$\Big\downarrow t = -t$$

$$x(-t) = \cos \omega_0 (-t)$$
$$= \cos(-\omega_0 t)$$
$$= \cos \omega_0 t$$

$\Rightarrow x(-t) = x(t)$.

$\Rightarrow$ cosine function is an even s/g.

5. $x(t) = t^2 \longrightarrow$ Even s/g.

$$\Big\downarrow t = -t$$

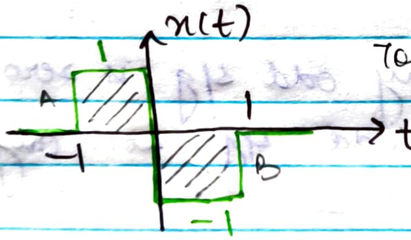
$$x(-t) = (-t)^2$$
$$= t^2$$

$x(-t) = x(t)$.

b) Odd Signal.

$$\boxed{\begin{array}{c} x(-t) = -x(t) \\ \text{or} \\ x(t) = -x(-t) \end{array}}$$

eg. 1.

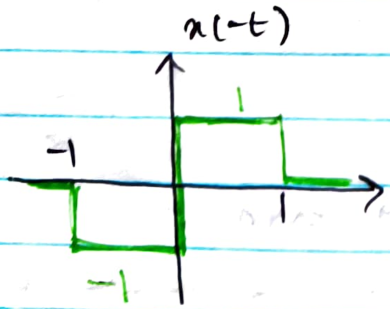


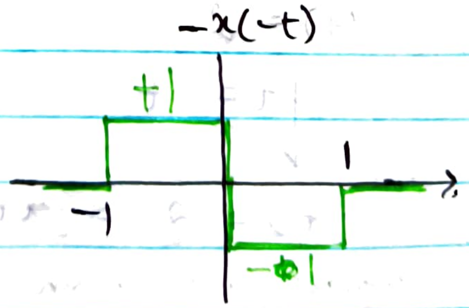
Total area = 0

$\because$ area of A = $-$(area of B)

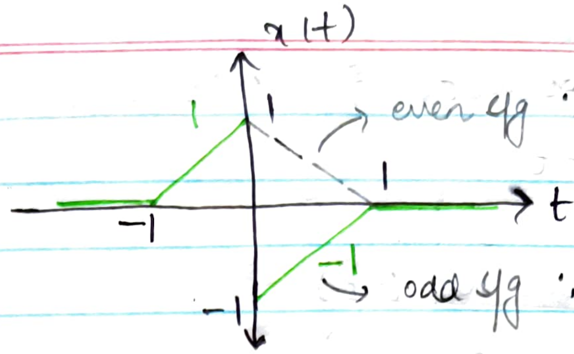
$\therefore$ 0

$\therefore$ avg value = 0.





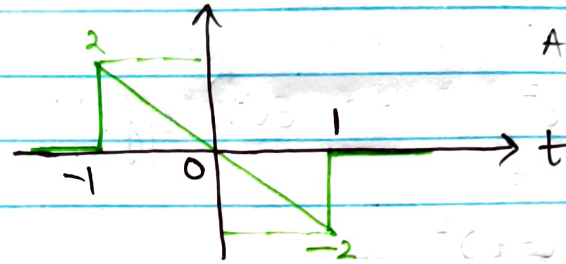
both the s/g waveforms are equal $\therefore$ odd s/g.

2. 

→ even sig ∵ symmetrical about x-axis

→ odd sig ∵ anti-symmetrical

→ Odd sig are anti-symmetrical about y-axis.

3. 

A sig is discontinuous when the value of sig just before & after origin is different

here, before zero = 2
after zero = -2

4. $x(t) = \sin w_0 t$ → odd signal.

$\downarrow t = -t$

$x(-t) = \sin(-w_0 t)$

$= -\sin w_0 t$

$x(-t) = -x(t)$

⇒ sine function is an odd sig.

→ Average value of any odd sig is zero.

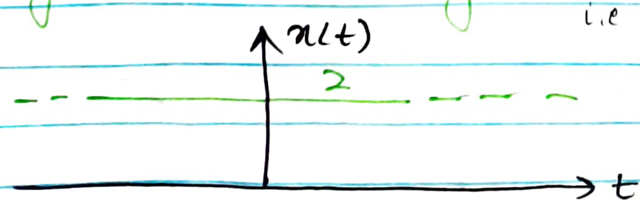
→ The value of any odd sig at origin is zero.

Q. $x(t) = 2.$

$\downarrow t = -t$

$x(-t) = 2 = x(t)$ → even.

→ DC signals are even signals (because no time reversal) i.e time independent



IMPORTANT POINTS.

Sum of even & odd fn = neither even nor odd

Sum of even fn = even

• Sum of odd fn = odd

even x odd fn = ~~even~~ odd

odd fn x odd fn = odd

even x even = even

1. $\boxed{E \times E = E}$

$t^2 \times t^4 = t^6$

2. $\boxed{O \times O = E}$

$t^3 \times t^5 = t^8$

3. $\boxed{E \times O = O}$

$t^2 \times t^3 = t^5$

Multiplication and division rules are same.

4. $\boxed{\underset{\text{even}}{DC} + \text{even} = \text{even}}$

5. $\boxed{\underset{\text{even}}{DC} + \text{odd} = \text{neither even nor odd}}$

eg. $x(t) = \overset{dc}{2} + \overset{odd}{t^3}$

$t = -t$

$x(-t) = 2 - t^3$

$\neq x(t) \rightarrow$ not even

$\neq -x(t) \rightarrow$ not odd.

6. $$\boxed{\begin{aligned} \frac{d}{dt}[\text{even}] &= \text{odd sig} \\ \frac{d}{dt}[\text{odd}] &= \text{even sig} \end{aligned}}$$

7. $\boxed{\begin{array}{l}\int \text{odd } s/g \; dt = \text{even } s/g \\ \int \text{even } dt = \text{odd } s/g\end{array}}$

It should not be definite integral as they always give a const i.e. dc (even)

eg: $\int_{-2}^{2} t^2 dt = \left[\frac{t^3}{3}\right]_{-2}^{2}$

→ Any s/g can be represented as a sum of even or odd parts i.e.

$$\boxed{x(t) = x_e(t) + x_o(t)}$$

where,

$$x_e(t) = \text{even part of } x(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \text{odd part of } x(t) = \frac{x(t) - x(-t)}{2}$$

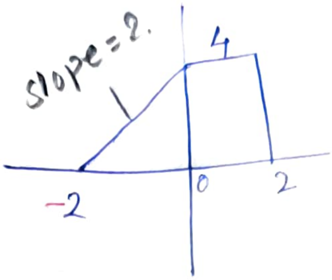
Q. find even part $x_e(t)$ & odd part $x_o(t)$ of $x(t)$

$$x(t) = \underbrace{2t^2}_{E} + \underbrace{\frac{t}{\cos t}}_{\frac{O}{E}=O} - \underbrace{t^3 \cdot \sin t}_{O \times O = E} - \underbrace{\frac{\sin^2 t}{t^5}}_{\frac{E}{O}=O} + \underbrace{t^4 \cdot \sin^3 t}_{E \times O = O} + \underbrace{\frac{t^3}{\cos^2 t}}_{O/E = O}$$

$$x_e(t) = 2t^2 + \cancel{\frac{t}{\cos t}} + \cancel{\frac{t^3}{\cos^2 t}} - t^3 \sin t$$

$$x_o(t) = +\frac{t}{\cos t} - \frac{\sin^2 t}{t^5} + t^4 \sin^3 t + \frac{t^3}{\cos^2 t}$$

$x(t)$



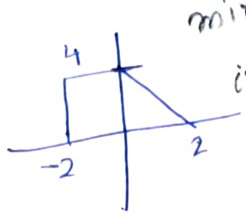
$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}$$

only TR

AR TR.

dc + linear = downword or upword with same slope.

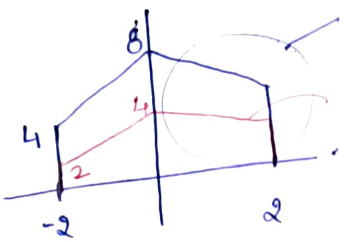
$x(-t)$



mirror or inverted signal about x axis.

be carefull about discontinuity.

$x_{e_1}(t) = x(t) + x(-t)$.

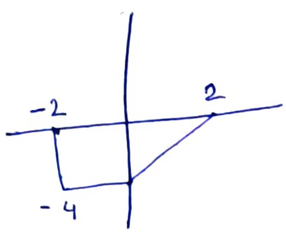


mirror image.

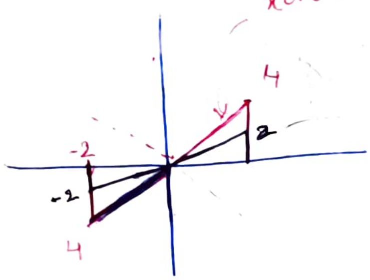
$x_e(t) = \frac{x_{e_1}(t)}{2}$.

for odd signal perform AR of $x(-t)$ i.e. $-x(-t)$

$-x(-t)$



$x_{o_1}(t)$ → $\overset{x}{\text{MI}}$ of $\overset{y}{\text{MI}}$ of RHS.



$x_o(t) = \frac{x_{o_1}(t)}{2}$

■ Example 1.38

Sketch the even and odd components of the triangular wave shown in Figure 1.60(a).

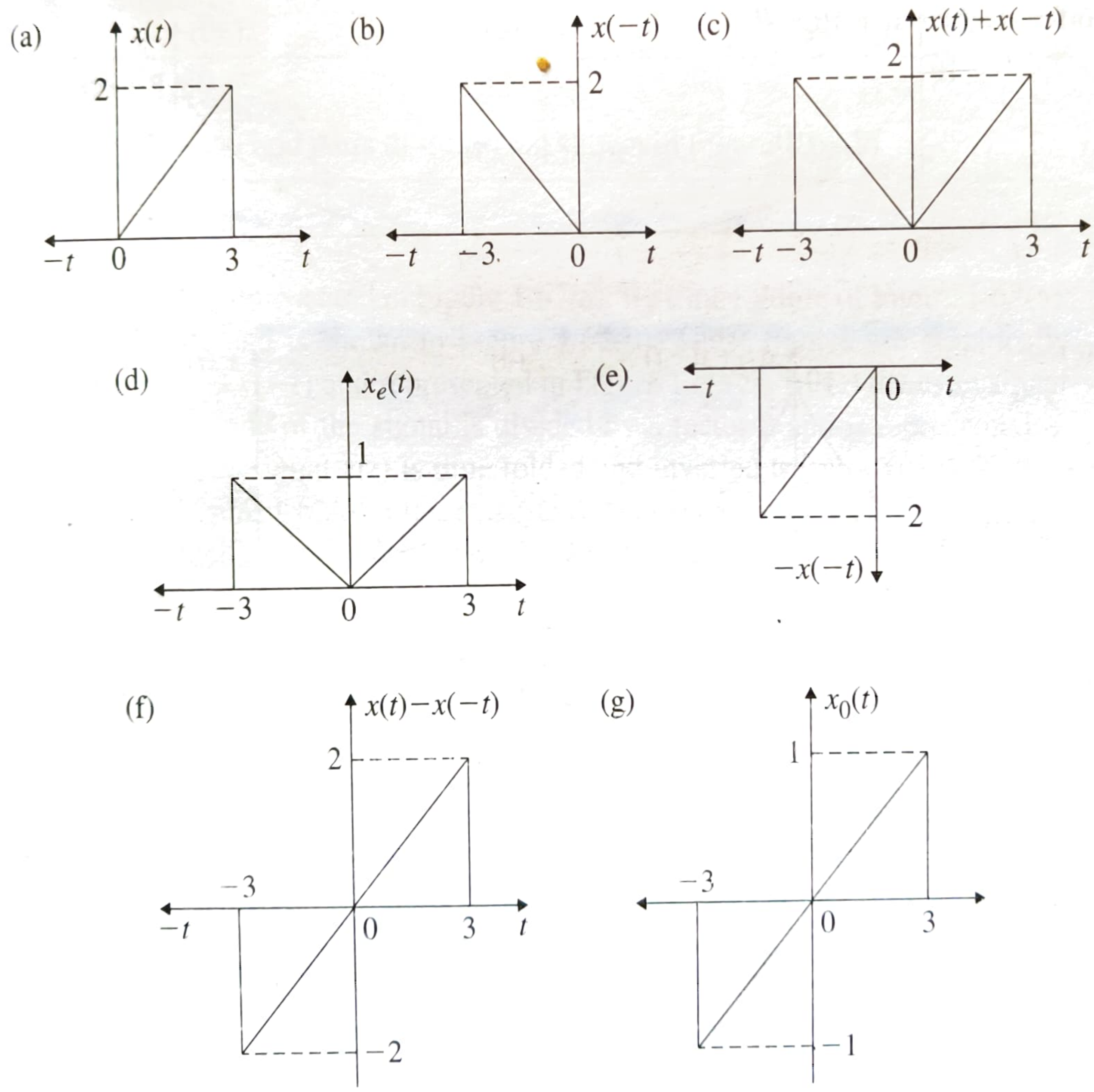


Figure 1.60 Even and odd components of a triangular wave.

odd signal.

Example 1.34

Show that the sum of the two even functions is an even function and the sum of the two odd functions is an odd function.

Solution: Let $x(t)$ be expressed as the sum of two functions $x_1(t)$ and $x_2(t)$.

$$x(t) = x_1(t) + x_2(t)$$

Substituting $t = -t$ in the above equation we get,

$$x(-t) = x_1(-t) + x_2(-t)$$

If $x_1(t)$ and $x_2(t)$ are even functions, the above equation is written as

$$\begin{aligned} x(-t) &= x_1(t) + x_2(t) \\ &= x(t) \end{aligned}$$

This shows that $x(t)$ which is the sum of two even functions is an even function. If $x_1(t)$ and $x_2(t)$ are odd functions, equation (a) can be written as

$$\begin{aligned} x(-t) &= x_1(-t) + x_2(-t) \\ &= -(x_1(t) + x_2(t)) \\ &= -x(t) \end{aligned}$$

Thus, $x(t)$ which is the sum of two odd functions is an odd function.

Example 1.35

Find whether the following signals are odd or even. Find the odd and even components.

(a) $x(t) = t^2 - 5t + 10$

(b) $x(t) = t^4 + 4t^2 + 6$

(c) $x(t) = t^3 + 3t$

(d) $x(t) = 10\sin\left(10\pi t + \frac{\pi}{4}\right)$

(e) $x(t) = e^{j10t}$

Solution:

(a) $\boldsymbol{x(t) = t^2 - 5t + 10}$
Put $t = -t$

$$\begin{aligned} x(-t) &= t^2 + 5t + 10 \\ &\neq x(t) \\ &\neq -x(t) \end{aligned}$$

The function is neither even nor odd.

$$\begin{aligned} x_e(t) &= \frac{1}{2}[x(t) + x(-t)] \\ &= \frac{1}{2}[t^2 - 5t + 10 + t^2 + 5t + 10] \end{aligned}$$

$$\boxed{x_e(t) = (t^2 + 10)}$$

$$\begin{aligned} x_0(t) &= \frac{1}{2}[x(t) - x(-t)] \\ &= \frac{1}{2}[t^2 - 5t + 10 - t^2 - 5t - 10] \end{aligned}$$

$$\boxed{x_0(t) = -5t}$$

(b) $\boldsymbol{x(t) = t^4 + 4t^2 + 6}$
Put $t = -t$

$$\begin{aligned} x(-t) &= t^4 + 4t^2 + 6 = x(t) \\ x(t) &= x(-t) \end{aligned}$$

The function is even. The odd part should be zero which can be verified as

$$\begin{aligned} x_0(t) &= \frac{1}{2}[x(t) - x(-t)] \\ &= \frac{1}{2}[t^4 + 4t^2 + 6 - t^4 - 4t^2 - 6] \\ &= 0 \end{aligned}$$

$$\boxed{x_e(t) = x(t) = t^4 + 4t^2 + 6}$$

(c) $\boldsymbol{x(t) = t^3 + 3t}$
Put $t = -t$

$$x(-t) = -(t^3 + 3t) = -x(t)$$

The function is odd. The even component is zero.

$$\boxed{\begin{array}{l} x_0(t) = t^3 + 3t \\ x_e(t) = 0 \end{array}}$$

(d) $\mathbf{x(t) = 10\sin\left(10\pi t + \frac{\pi}{4}\right)}$
Put $t = -t$

$$\begin{aligned} x(-t) &= 10\sin\left(-10\pi t + \frac{\pi}{4}\right) \\ &= -10\sin\left(10\pi t - \frac{\pi}{4}\right) \\ &= -10\left[\sin 10\pi t \cos\frac{\pi}{4} - \cos 10\pi t \sin\frac{\pi}{4}\right] \\ &= \frac{-10}{\sqrt{2}}[\sin 10\pi t - \cos 10\pi t] \\ &\neq x(t) \\ &\neq -x(t) \end{aligned}$$

The above signal is neither even nor odd.

$$\begin{aligned} x(t) &= 10\left[\sin 10\pi t \cos\frac{\pi}{4} + \cos 10\pi t \sin\frac{\pi}{4}\right] \\ &= \frac{10}{\sqrt{2}}[\sin 10\pi t + \cos 10\pi t] \end{aligned}$$

$$\begin{aligned} x_e(t) &= \frac{1}{2}[x(t) + x(-t)] \\ &= \frac{10}{2\sqrt{2}}[\sin 10\pi t + \cos 10\pi t - \sin 10\pi t + \cos 10\pi t] \end{aligned}$$

$$\boxed{x_e(t) = \frac{10}{\sqrt{2}}\cos 10\pi t}$$

$$\begin{aligned} x_0(t) &= \frac{1}{2}[x(t) - x(-t)] \\ &= \frac{10}{2\sqrt{2}}[\sin 10\pi t + \cos 10\pi t + \sin 10\pi t - \cos 10\pi t] \end{aligned}$$

$$\boxed{x_0(t) = \frac{10}{\sqrt{2}}\sin 10\pi t}$$

(e) $x(t) = e^{j10t}$

$$x(-t) = e^{-j10t}$$
$$x(t) \neq x(-t)$$
$$x(t) \neq -x(-t)$$

The signal is neither odd nor even.

$$x_e(t) = \frac{1}{2}[x(t) - x(-t)]$$
$$= \frac{1}{2}[e^{j10t} + e^{-j10t}]$$

$$\boxed{x_e(t) = \cos 10t}$$

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$$
$$= \frac{1}{2}[e^{j10t} - e^{-j10t}]$$

$$\boxed{x_0(t) = j \sin 10t}$$

Note: In all the above cases $x_0(t)$ passes through the origin at $t = 0$.

■ Example 1.36

Sketch the even and odd components of a step signal shown in Figure 1.58(a).

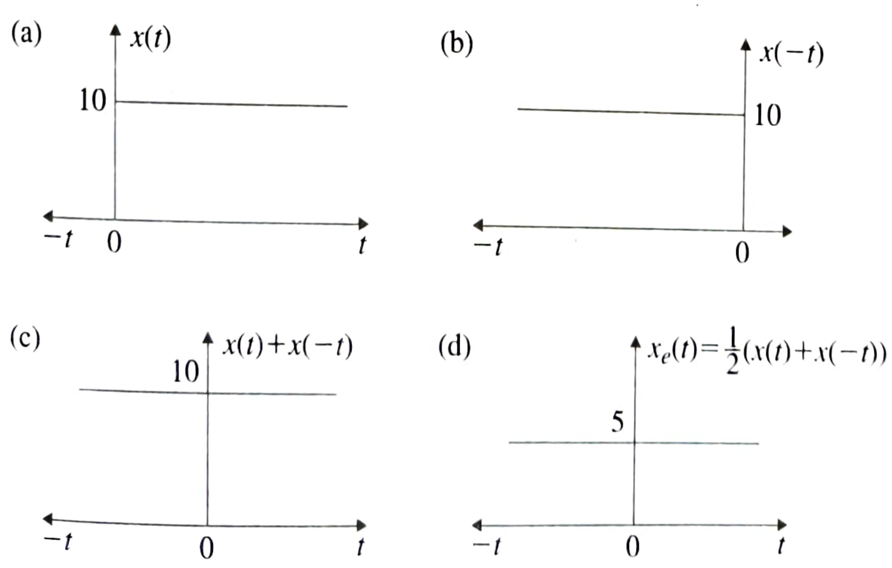


Figure 1.58

of the ramp signals as given below

$$f(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$$

Fig. 1.23

Problem 1.11 ***Determine the even and odd components of the following signals:***

(a) Unit step signal;

(a)

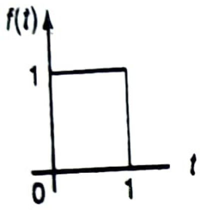


Fig. 1.24 (a)

(b)

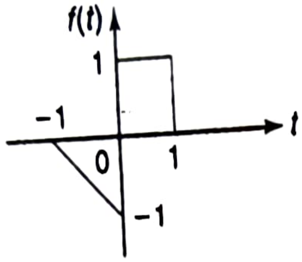


Fig. 1.24 (b)

(c)

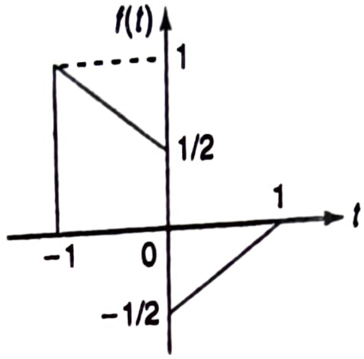


Fig. 1.24 (c)

(d)

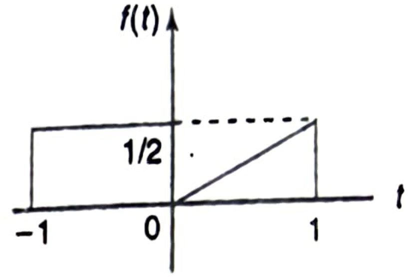


Fig. 1.24 (d)

(f) $f(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$

Solution

(a) To find the even and odd components of a unit step signal, we need to find the folded signal, i.e. $u(-t)$, as shown in the figure below.

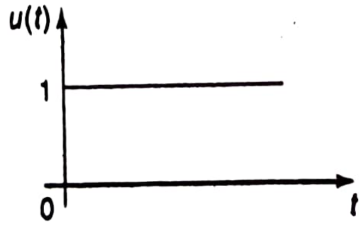


Fig. 1.25 (a) *Unit step signal*

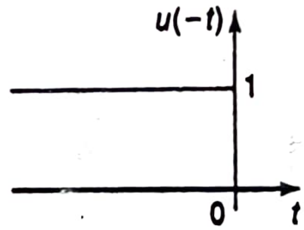


Fig. 1.25 (b) *Folded signal*

Now,

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)] \quad f_0(t) = \frac{1}{2}[f(t) - f(-t)]$$

By point-by-point addition and subtraction of the signals of Fig. 1.25 (a) and Fig. 1.25 (b), we get the even and odd components, respectively, as shown in Fig. 1.25 (c) and Fig. 1.25 (d) below.

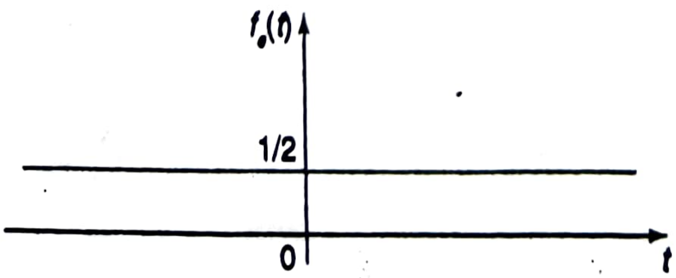


Fig. 1.25 (c) *Even component of unit step signal*

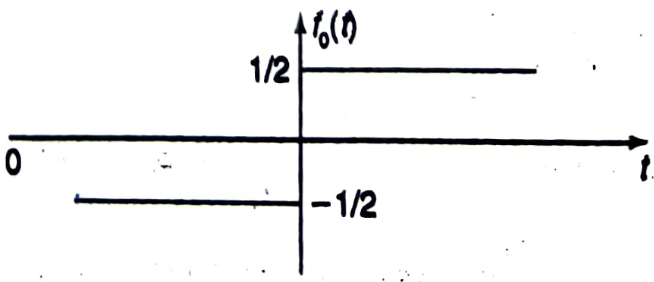


Fig. 1.25 (d) *Odd component of unit step singal*

$= 0.6\,[1-e^{-2t}]$
step-response

... the even and odd components, we need the folded signal, i.e. $f(-t)$, as shown in the ... 1.26 (b)

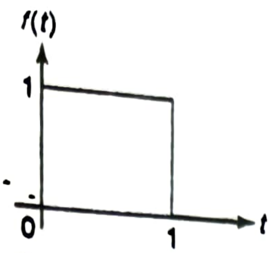


Fig. 1.26 (a) *Signal*

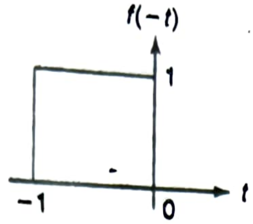


Fig. 1.26 (b) *Folded signal*

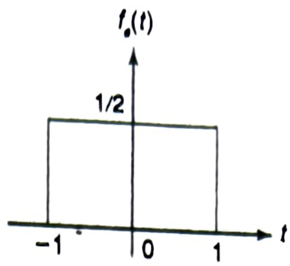


Fig. 1.26 (c) *Even component of signal*

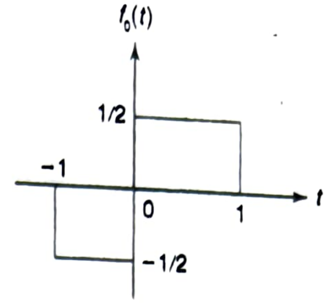


Fig. 1.26 (d) *Odd component of signal*

By point-by-point addition and subtraction, we get the even and odd components as shown in Fig. 1.26 (c) and Fig. 1.26 (d).

(c) The procedure is followed as mentioned below.

By point-by-point addition and subtraction, we get the even and odd components as shown in Fig. 1.27 (c) and Fig. 1.27 (d).

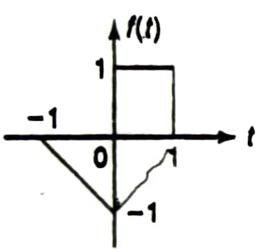


Fig. 1.27 (a) *Signal*

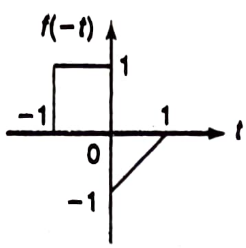


Fig. 1.27 (b) *Folded signal*

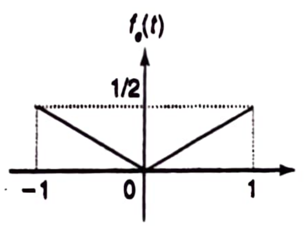


Fig. 1.27 (c) *Even component of the signal*

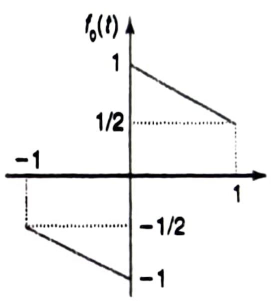


Fig. 1.27 (d) *Odd component of the signal*

(d) To find the even and odd components we need the folded signal, i.e. $f(-t)$, as shown in Fig. 1.28 (b).

By point-by-point addition and subtraction, we get the even and odd components as shown in Fig. 1.28 (c) and Fig. 1.28 (d).

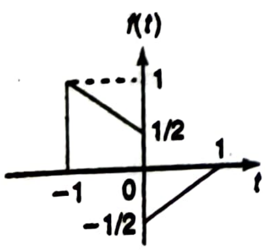


Fig. 1.28 (a) *Signal*

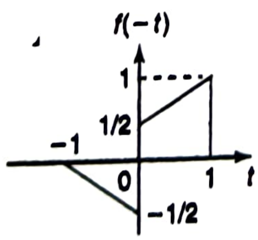


Fig. 1.28 (b) *Folded signal*

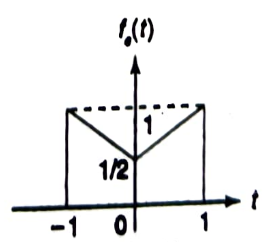


Fig. 1.28 (c) *Even component of the signal*

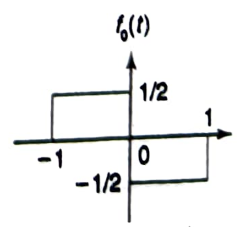


Fig. 1.28 (d) *Odd component of the signal*

(e) To find the even and odd components the signal and the folded signal, i.e. $f(-t)$ are shown in Fig. 1.29 (a) and Fig. 1.29 (b), respectively.

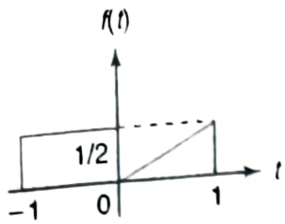


Fig. 1.29 (a) *Signal*

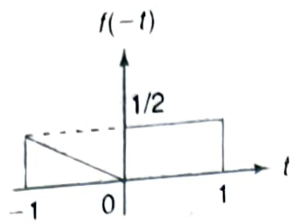


Fig. 1.29 (b) *Folded signal*

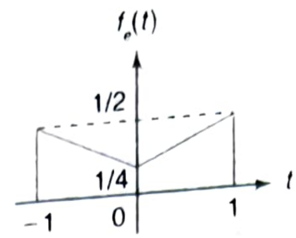


Fig. 1.29 (c) *Even component of the signal*

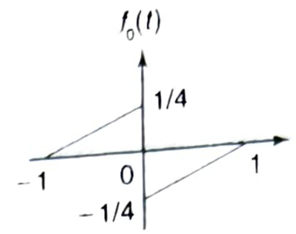


Fig. 1.29 (d) *Odd component of the signal*

By point-by-point addition and subtraction, we get the even and odd components as shown in Fig. 1.29 (c) and Fig. 1.29 (d).

(f) $f(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$

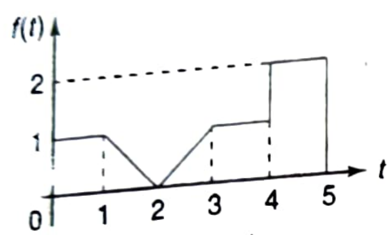


Fig. 1.30 (a) *Signal*

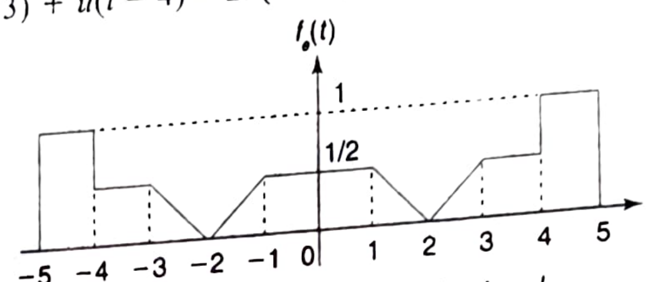


Fig. 1.30 (c) *Even component of the signal*

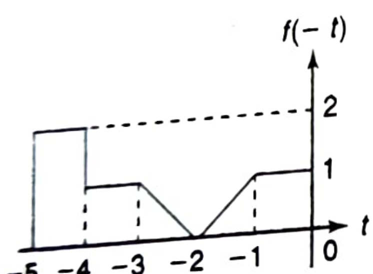


Fig. 1.30 (b) *Folded signal*

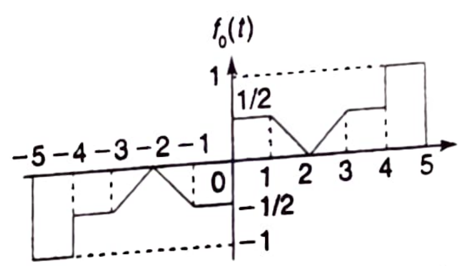


Fig. 1.30 (d) *Odd component of the signal*

Here, the signal is drawn as shown in Fig. 1.30 (a). The folded signal is shown in Fig. 1.30 (b).

The even component and the odd components of the signal are obtained by point-by-point addition and subtraction of signals of Fig. 1.30 (a) and Fig. 1.30 (b), respectively. These are shown in Fig. 1.30 (c) and Fig. 1.30 (d).

Summary

...ified from different points of view,