INTERFERENCE OF LIGHT

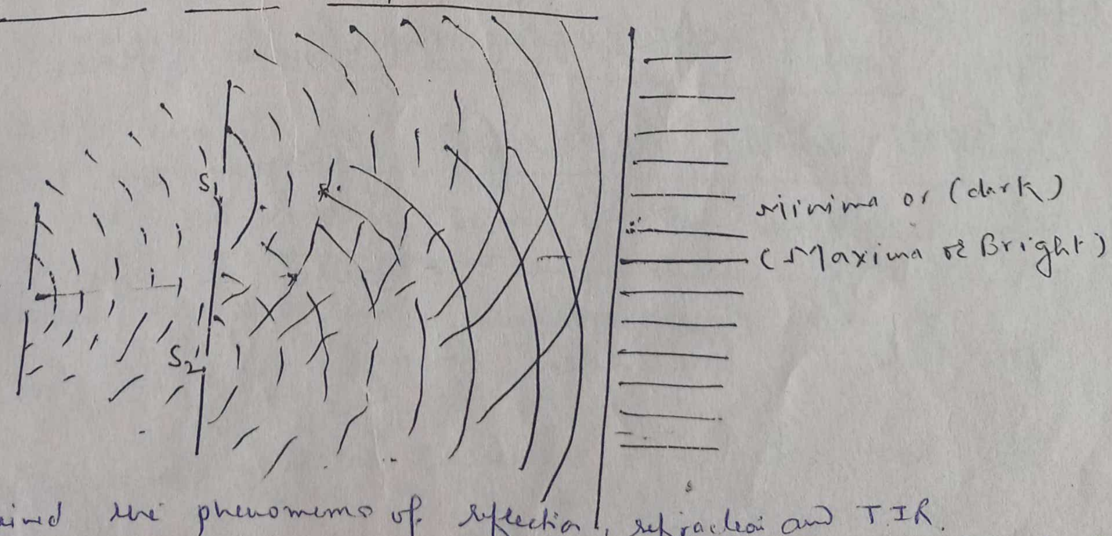
"The phenomenon of modification in intensity due to two light waves (or more) having same frequency, nearly equal amplitude and constt phase difference in the region of superposition is called the interference."

1. Constructive Interference:- At certain points, the waves superimpose in such a way that the resultant intensity is greater than the sum of the intensities due to individual waves. The interference produce at these points is called "Constructive Interference."

2. Destructive Interference:- At certain points, the waves superimpose in such a way that the resultant intensity is less than the sum of the intensities due to individual waves. The interference produced at these points is called "Destructive Interference."

The phenomenon of Interference of light has proved the validity of wave theory of light. wave Theory of light was first put forwards by Huygens in 1678

YOUNG'S Double Slits Experiments:-



On this, he explained the phenomena of reflection, refraction and T.I.R. According to this a luminous body is a source of disturbance in a hypothetical medium called ether which propagate in the form of wave through space and energy is distributed equally in all direction.

wavefront → Continuous ... which are vibrating

In 1801, Thomas Young demonstrated the phenomenon of interference of light. Arrangement is shown in figure. S be a monochromatic source of light, and S_1 S_2 be the two narrow slits placed equidistance from source S. When light passes through the slits. The cylindrical waves spread out from S. According to Huygen's principle, each point of a wavefront become the source of secondary waves and spread out in all directions. Hence, the waves also spread out from closely spaced holes S_1 and S_2, and they superimpose on each other. At points, where a crest falls over a crest or trough over a trough, the vibrations are large and the resultant amplitude is greater and hence result in constructive interference. At some points, where a crest falls over trough of the other, the intensity is minimum. This is the case of destructive interference. Hence, we get alternate dark and bright fringes on the screen.

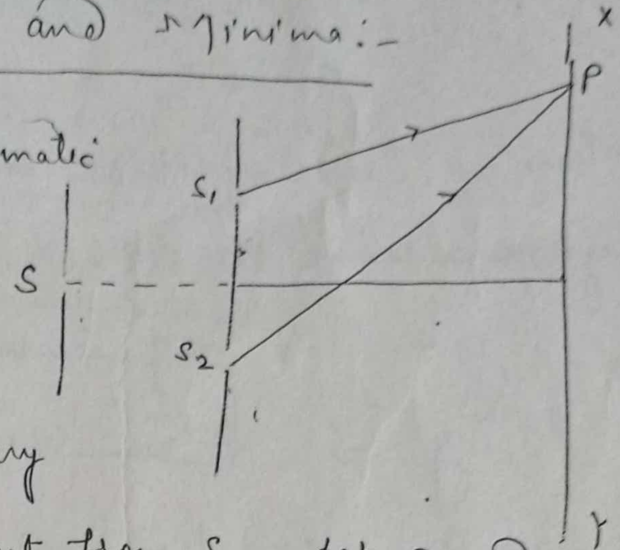
→ Sunlight was first allowed to pass through a pinhole S and then through two pin holes S_1 & S_2. The two sets of spherical waves emerging from S_1 and S_2 interfered with each other and a few coloured fringes of varying intensity were seen on the screen.

Improvement of the original arrangement. (3)

the pinholes S_1 & S_2 are replaced by narrow slits and sunlight by monochromatic light.

Theory of Interference:-

Condition of Maxima and Minima:-



Let us consider a monochromatic source of light S emitting waves of wavelength λ. S_1 and S_2 are the two similar parallel slits of very close together and equidistant from S. Let a_1 and a_2 be the amplitudes of the waves from S_1 and S_2 respectively.

The displacement due to one wave is

$$y_1 = a_1 \sin \omega t \quad \text{------} \quad (1)$$

and the displacement due to second wave is

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{------} \quad (11)$$

where, δ, is the phase difference between the two waves reaching at P at instant t.

Now, according to <u>principle of superposition</u>, the resultant displacement at any point due to a number of waves, is the algebraic sum of the displacements of the ~~individual~~ individual ~~displacement~~. waves.

$$\therefore \quad Y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$= a_1 \sin\omega t + a_2 (\sin\omega t \cos\delta + \cos\omega t \sin\delta)$ (4)

$= (a_1 + a_2 \cos\delta) \sin\omega t + (a_2 \sin\delta) \cos\omega t$

Let- $a_1 + a_2 \cos\delta = A \cos\phi$ ---- (3)

$a_2 \sin\delta = A \sin\phi$ ---- (4)

where A, is amplitude and ϕ is phase difference.

using equations (3), (4)

$$y = A \cos\phi \sin\omega t + A \sin\phi \cos\omega t$$

$$y = A \sin(\omega t + \phi) \quad \text{---} \quad (5)$$

This is the equation of the resultant wave.

Now, squaring eqn (4) and (5) in both side and then adding

$$(a_1 + a_2 \cos\delta)^2 + (a_2 \sin\delta)^2 = A^2(\cos^2\phi + \sin^2\phi)$$

$$A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos\delta. \quad \text{———} \quad (6)$$

Dividing equ. (4) by (3)

$$\tan\phi = \frac{a_2 \sin\delta}{(a_1 + a_2 \cos\delta)} \quad \text{———} \quad (7)$$

$\because$ Intensity $\propto$ (amplitude)^2

$$\therefore I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$$

$$\therefore \boxed{I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta}$$

—— (8)

This is the resultant intensity at any point 'p'

Condition for Maxima and Minima.

for maxima. $\cos\delta = +1$

$$\therefore \boxed{\delta = 2n\pi} \quad ----(8)$$

where $n = 0, 1, 2, 3, \ldots\ldots$

$$\therefore I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$

$$\boxed{I_{max} = (a_1 + a_2)^2} \quad ----(9)$$

or $$\boxed{I_{max} > I_1 + I_2} \quad ----(10)$$

$\because$ path diff $= \frac{\lambda}{2\pi} \times$ phase diff.

$$= \frac{\lambda}{2\pi} \times 2\pi n$$

$$= n\lambda$$

or $$\boxed{\Delta = (2n)\lambda/2} \quad ----(11)$$

$\Rightarrow$ Intensity is maximum when phase difference is even multiple of 'π' or the path difference is even multiple of half wavelength ($\lambda/2$).

for minima. $\cos\delta = -1$

$$\boxed{\delta = (2n-1)\pi} \quad \text{where } n = 1, 2, 3 \ldots \quad ----(12)$$

$$I_{min} = a_1^2 + a_2^2 + 2a_1a_2(-1)$$

$$\boxed{I_{min} = (a_1 - a_2)^2} \quad ----(13)$$

$$\therefore \quad \Delta = \frac{\lambda}{2\pi} \times \delta$$

$$\Delta = \frac{\lambda}{2\pi} \times (2n-1)\pi$$

$$\boxed{\Delta = (2n-1)\lambda/2} \quad --- \quad (12)$$

where $n = 1, 2, 3 - - -$

or $\boxed{I_{min} < a_1^2 + a_2^2 \quad \text{or} < I_1 + I_2}$ --- (13)

If $(2n+1)$, then $n = 0, 1, 2, 3 - - -$

$\Rightarrow$ The intensity is minimum when phase difference is odd multiple of 'π' or path difference is odd multiple of half wavelength $(\lambda/2)$.

$$\therefore \quad \boxed{\begin{array}{l} I_{max} > I_1 + I_2 \\ I_{min} < I_1 + I_2 \end{array}}$$

If $a_1 = a_2 = a$.

then $I_{max} = a^2 + a^2 + 2a^2$

$I_{max} = 4a^2$

$I_{min} = a^2 + a^2 - 2a^2$

$I_{min} = 0$

Conservation of Energy in Interference :-

Average Intensity :- The average energy at any point in the region of superposition may be shown analytically by averaging the energy corresponding to one wave is

$$I_{av} = \frac{\int_0^{2\pi} I \, d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2 \cos\delta)\, d\delta}{\int_0^{2\pi} d\delta}$$

$$= \frac{\left[a_1^2\delta + a_2^2\delta + 2a_1a_2 \sin\delta\right]_0^{2\pi}}{\left[\delta\right]_0^{2\pi}}$$

$$= \frac{\left[a_1^2 + a_2^2\right] 2\pi}{2\pi}$$

$$I_{av} = a_1^2 + a_2^2$$

$$I_{av} = I_1 + I_2$$

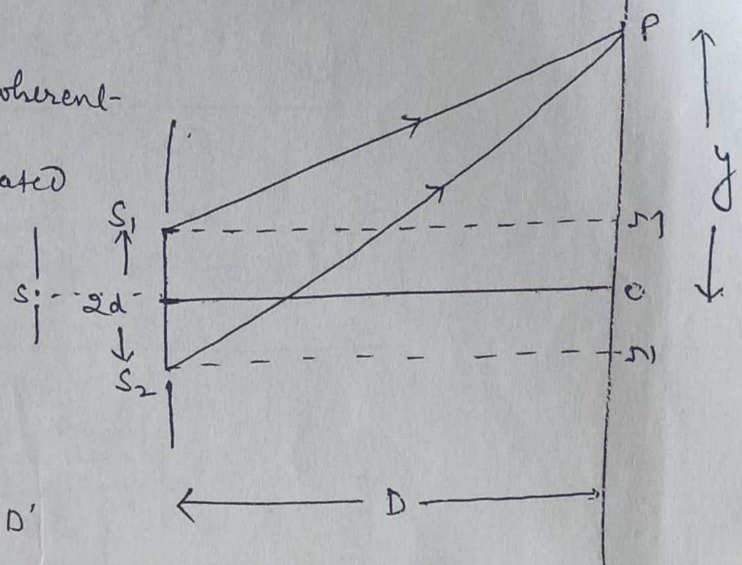
If $a_1 = a_2$

$$I_{av} = 2a^2$$

⇒ The average intensity is equal to the sum of the separate intensities. ie, the energy disappears at the minima is actually present at maxima. (This simply means that in interference pattern the energy $2a_1a_2$ is simply transferred from minima to maxima.) Thus, the phenomenon of interference is based on conservation of energy.

Theory of Interference fringes:-

Let us consider two coherent sources S_1 and S_2 separated by a distance $2d$



Now consider a point P on the screen (which is placed at a distance 'D' from the sources) at a vertical distance 'y' from 'O' at which the conditions of bright or dark fringes are to be determined.

$$PM = y-d \quad , \quad PN = y+d$$

In ΔS_2PN,

$$(S_2P)^2 = (S_2N)^2 + (NP)^2$$

$$S_2P = \left[(D)^2 + (y+d)^2 \right]^{1/2}$$

$$= D\left[1 + \left(\frac{y+d}{D}\right)^2 \right]^{1/2}$$

using binomial theorem and neglecting the higher terms

$\because D >> y$ or d

$$S_2P = D\left[1 + \left(\frac{(y+d)^2}{2D^2}\right) \right]$$

$$S_2P = D + \frac{(y+d)^2}{2D} \quad --- \quad (1)$$

Similarly, in ΔS_1MP

$$S_1P = \left[(S_1M)^2 + (MP)^2 \right]^{1/2}$$

$$= D + \frac{(y-d)^2}{2D} \quad --- \quad (2)$$

Path diff $\Delta = S_2P - S_1P$

$$= \left[D + \frac{(y+d)^2}{2D}\right] - \left[D + \frac{(y-d)^2}{2D}\right]$$

$$= \frac{1}{2D}\left[\cancel{y^2} + \cancel{d^2} + 2yd - \cancel{y^2} - \cancel{d^2} + 2yd\right]$$

$$\Delta = \frac{2yd}{D} \quad \text{———} \quad (3)$$

$$\delta = \frac{2\pi}{\lambda} \cdot \Delta$$

$$\delta = \frac{2\pi}{\lambda} \cdot \left(\frac{2yd}{D}\right) \quad \text{———} \quad (4)$$

(i) Position of Bright fringes:- for maxima or bright fringe, path diff must be even multiple of $\lambda/2$

$$\frac{2yd}{D} = 2n \cdot \frac{\lambda}{2}$$

$$\Rightarrow y = \frac{nD\lambda}{2d}.$$

This equation gives the distance of n^{th} bright fringe from 'O', i.e. $y = y_n$

$$\boxed{y_n = \frac{nD\lambda}{2d}} \quad \text{where } n = 0, 1, 2, 3 \ldots$$

(ii) Position of Dark fringe:- for minimum intensity or dark fringe, the path diff must be an odd multiple of $\lambda/2$.

$$\frac{2yd}{D} = (2n-1)\lambda/2$$

$$y = \frac{(2n-1)\lambda D}{4d}$$

$$y = \left(n-\frac{1}{2}\right)\frac{D\lambda}{2d}$$

let. $y = y_n$, for position of n^{th} dark fringe

$$\boxed{y_n = \left(n-\frac{1}{2}\right)\frac{D\lambda}{2d}} \quad ----$$

where $n = 1, 2, 3 ....$

(iii) Fringe width (w or β) :- The distance between any two consecutive bright fringe or dark fringes is called fringe width.

(a) Dark fringe width:- If y_n and y_{n+1} denot the distances of two consecutive bright- fringes. from 'o' then

$$\beta = [y_{n+1} - y_n]_{bright}$$

$$= (n+1)\frac{D\lambda}{2d} - \frac{nD\lambda}{2d}$$

$$\boxed{\beta = \frac{D\lambda}{2d}}$$

For ~~dark~~ bright fringe:-

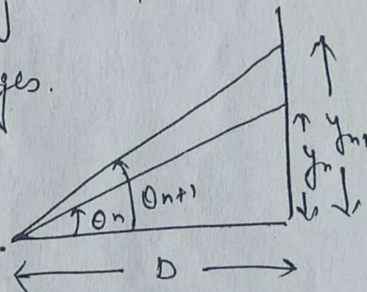
$$\beta = \left[y_{n+1} - y_n\right]_{dark}$$

$$= \left(n - \frac{1}{2} + 1\right)\frac{D\lambda}{2d} - \left(n - \frac{1}{2}\right)\frac{D\lambda}{2d}$$

$$\boxed{\beta = \frac{D\lambda}{2d}}$$

$\Rightarrow$ fringe width is independent of 'n'. Hence the spacing between any two bright or any two dark fringes is same.

<u>Angular fringe width (ω_θ)</u> :- Angular separation between consecutive bright or dark fringes.



$$\omega_\theta = \theta_{n+1} - \theta_n$$

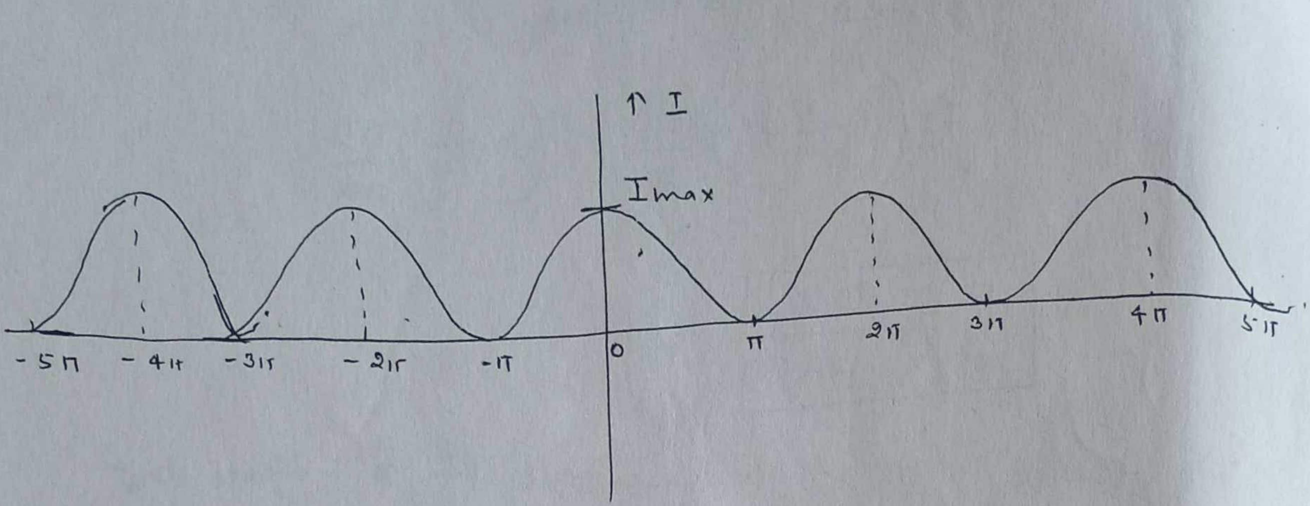
$$= \frac{y_{n+1}}{D} - \frac{y_n}{D}$$

$$= \frac{\omega \text{ or } \beta}{D}$$

$$= \frac{D\lambda}{2d} \cdot \frac{1}{D}$$

$$\boxed{\omega_\theta = \frac{\lambda}{2d}}$$

Energy Distribution



Coherent Sources:- Two sources which have a constant phase difference between them are called coherent sources.

Q. Why two independent sources of light of same wave length cannot show interference?

Ans. Sustained interference can never be obtained with two independent sources of light, such as two bulbs or two candles. Actually a beam of light is built up of waves radiated from millions of excited atoms or molecules, whose vibrations are completely independent of each other. Each atom emitting light for about 10^{-8} sec. i.e light emitted by an atom is essentially a pulse lasting for only 10^{-8} sec. So light coming out from the two independent sources will have a fixed phase relationship for a period of 10^{-8} sec. So Interference pattern will change after 10^{-8} sec. and we observe a uniform intensity.

for a period of about 10^{-8} sec., hence interference pattern will change after 10^{-8} sec. The eye cannot notice this, and we observe a uniform intensity over the screen.

Q. How the Coherent Sources are obtained in practice?

Ans. In general there are two methods of obtaining Coherent sources

(I.) Division of wavefront:- In this case, the wave front originating from a common source is divided into two parts by using mirrors, prism or lenses. This class of interference requires essentially a point source or a narrow slit source. The instruments used to obtain interference by division of wavefront are fresnel Biprism Lloyd's mirror, Laser etc.

(II.) Division of amplitude:- In this case, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction. This class of interference requires broad light source, so that interference effects are in general of greater intensity than the division of wave front. The effect resulting from the superposition of two beams ~~are~~ is refered to as two beam interference, and from the superposition of more beams ~~are~~ is referred to as multiple beam interference.

Ex. Newton's Rings, Michelson Interferometer (2 beams), Fabry - Perot interferometer etc (multi-beam).

Conditions for Sustained Interference of Light :-

To obtaind well defined interference pattern, the intensity at points corresponding to destructive interference must be zero, and corresponding to constructive interference must be maximum. following condition must be satisfied:

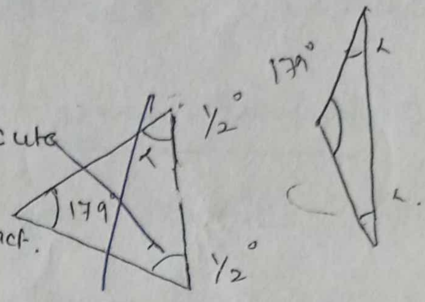
1. The two interfering sources must have same frequency and they must be coherent. If this condition is not satisfied, the phase difference between the interfering waves will vary continuously and position of maxima and minima will change with time.

2. Must have equal amplitude. If $a_1 \neq a_2$, then mini intensity $\neq 0$ and there will very poor contrast between maxima and minima.

3. The two sources must be narrow. Because a broad source is equivalent to a large number of fine sources Each pair of fine sources will give its own interference pattern. That will overlap and we get a uniform illumination on the screen.

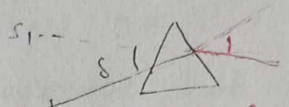
4. The separation between the coherent source (2d) must be small as possible. $\because \beta \propto \frac{1}{2d}$. So, if '2d' is not small, the fringe width will be very small. As a result bright and dark fringes will be very close to each other and will not be visible separately.

FRESNEL'S BI-PRISM :- fresnel used a biprism to obtain two coherent sources for producing interference fringes.

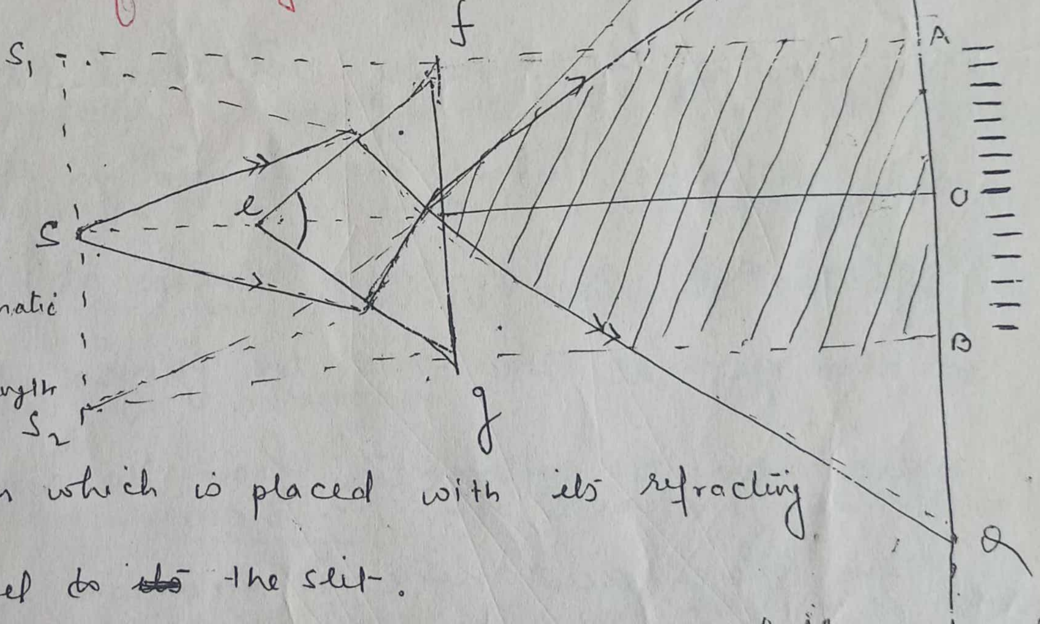


Construction :- It consists of two acute angled prism with their bases in contact.

It is actually a simple prism ~~with base~~ of obtuse angle of about 179°. The ~~actute~~ acute angle α on both sides is about 30'' (or $\frac{1}{2}$°).



Working :- (In practice, a thin glass plate is taken and its faces is grownd and polished till a prism)



S → A narrow vertical slit illuminated by monochromatic light of wavelength λ.

efg → prism which is placed with its refracting edge parallel to ~~the~~ the slit.

The light from the slit S is allowed to fall symmetrically on the refracting edge of biprism. when light falls on the lower half of the biprism, it is bent upwards and appears to come from source S_2. Similarly, when the light falls on the upper half of the biprism, it is

(22)

bent in downwards and appears to come from the source S_1. Thus, each half of the biprism produces a virtual image. The virtual images S_1 & S_2 act as two coherent sources, because -

* The light rays coming from two sources S_1 and S_2 are of same wavelength, i.e. both the sources are monochromatic.
* The phases of the two sources are in the same phase because both the images are of the same source.

The cones of light AS_1Q and BS_2P are superimposed and interference fringes are formed in the overlapping region AB on a screen placed at O. Along with we also get at the ends the fringes of unequal width due to diffraction from the vertex of the prism which acts like a straight edge.

Theory :- The point 'O' is equidistance from the sources S_1 and S_2. Therefore at point O, the intensity is maximum. On both sides of point O alternatively dark and bright fringes are produced.

The fringe width $\beta = \frac{D\lambda}{2d}$ - - - (i)

Distance of n^{th} bright fringe from 'O'

$$y_n = \frac{nD\lambda}{2d} \quad \text{- - - - - -} \quad \text{where } n = 0,1,2 \ldots \quad (ii)$$

and the distance of n^{th} dark fringe from 'O'

$$y_n = \left(n - \frac{1}{2}\right)\frac{D\lambda}{2d} \quad \text{- - - -} \quad (iii)$$

where $n = 1, 2, 3 \ldots$

from eqn. (i)

$$\lambda = \beta \cdot \frac{2d}{D} \quad \text{- - - -} \quad (iv)$$

Fresnel's Biprism is used to determine the wavelength of a monochromatic light. Hence, to the wavelength of light λ, the fringes are obtained and the value of β D and 2d are measured.

Experiment Arrangement :- It consists of an optical ~~arrangement~~ bench of about one and half metre long. This bench is in millimetres and carries four uprights stands supporting an adjustable slit, biprism, a lens and a micrometer eye-piece. Each of the uprights can be easily moved vertically and adjusted to any desired height. The slits and biprism can be made to move and along $\perp$ to the length of the bench

The slit is made narrow and is illuminated with a monochromatic light whose wavelength is determined.

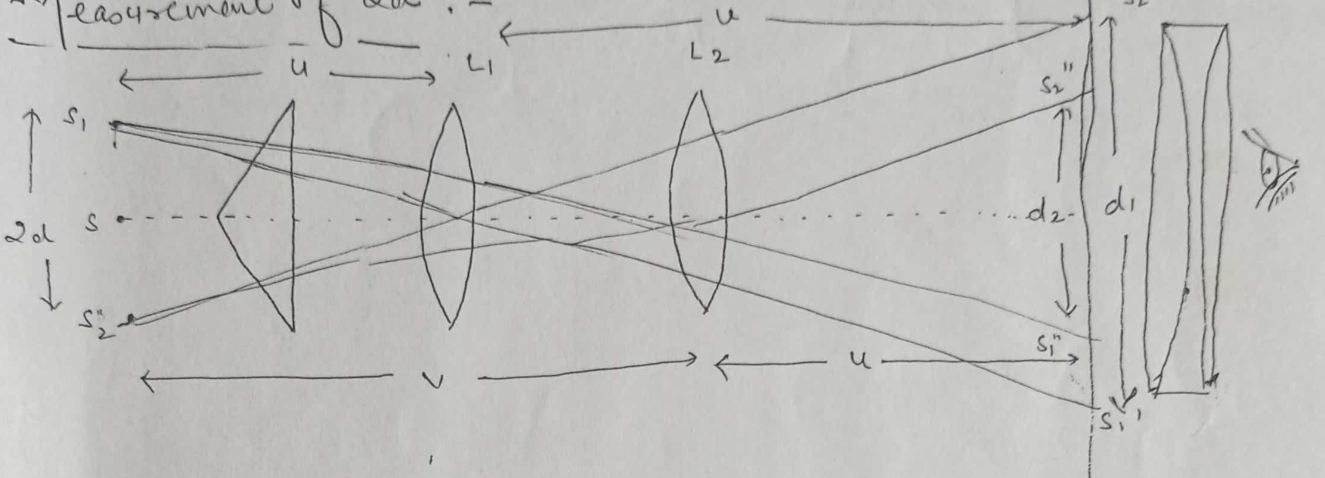
Measurements:-

(1) **Measurement of fringe width (β):-** The stand carrying the eye-piece is fixed at such a distance from the biprism the bright and distinct fringes are visible in the field of view of eye-piece. Now the vertical cross-wire of the eye-piece is set on a bright fringe on one side of the interference pattern. The reading of micrometer screw is noted. Then the eye-piece is moved and fixed the cross-wire at every bright fringe and readings are noted and measure the fringe width given by

$$\beta = \frac{\text{Distance moved}}{\text{No. of fringes passed}}$$

Measurement of D :- The distance between slit and eye-piece is directly measured on the scale of the optical bench.

Measurement of 2d :- (1) Displacement Method :-



A convex lens is placed between biprism and the eye-piece. The upright carrying the lens is brought very close to the biprism so as to get the position L_1 of the lens such that the sharp and real images S_1' and S_2' are obtained in the field of eye-piece. The distance (say d_1) between these images measured with the help of micrometer eye-piece.

Now keeping the position of the eye-piece and biprism fixed, the upright carrying lens is moved towards the eye-piece to obtain the position L_2 of the lens such that the images of S_1 and S_2 are obtained in field of eye-piece. The distance (say d_2) between these images is again measured and then using the lens magnification formula -

distance of eye piece to slit becomes > 4f

(20)

$$m = \frac{I}{O} = \frac{v}{u} = \frac{S_1' S_2'}{S_1 S_2}$$

$$\frac{d_1}{2d} = \frac{v}{u}$$

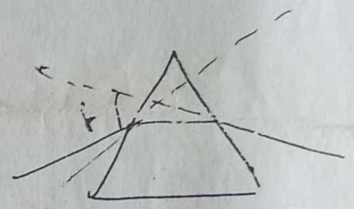
$$2d = \frac{u}{v} d_1 \qquad \text{——— (i)}$$

As the two positions of the lens are conjugate so for lens L_2

$$\frac{u}{v} = \frac{d_2}{2d} \qquad \text{——— (ii)} \Rightarrow 2d = \frac{v}{u} d_2$$

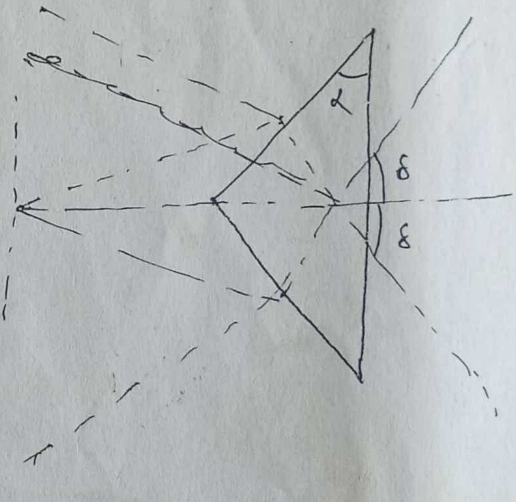
$$\cancel{\frac{u}{v}} \times \cancel{\frac{v}{u}} \times d_1 d_2 = 4d^2$$

$$\boxed{2d = \sqrt{d_1 d_2}}$$



(II) Deviation Method :-

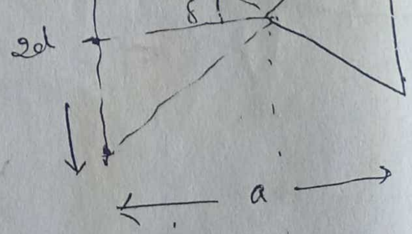
The small deviation δ_1 produced in a ray of light by a prism of very small refractive angle α is given by



$$\delta = (\mu - 1)\alpha$$

where μ - refractive index

$$\therefore \delta = \frac{d}{a}$$



$$\therefore \frac{d}{a} = (\mu - 1)\alpha$$

$$\boxed{2d = 2a(\mu - 1)\alpha}$$

$D = a + b$

$a \rightarrow$ slit to biprism

$b \rightarrow$ prism to screen

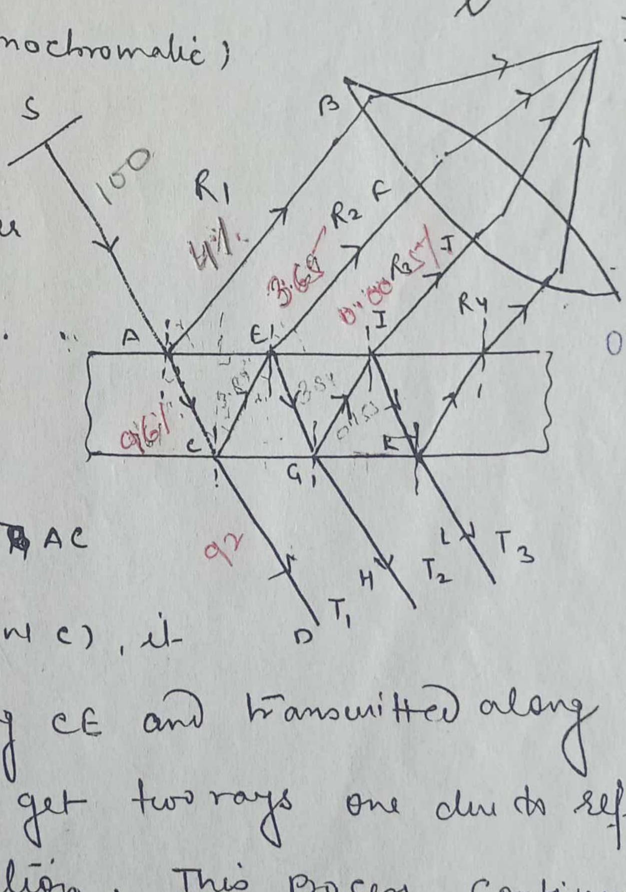
The refractive index μ and refractive angle α are found with the help of spectrometer.

INTERFERENCE BY DIVISION OF AMPLITUDE

Interference In a Thin film :- when monochromatic light falls on a thin transparent film (such as glass film oil film, soap film etc) we observe either formation of maxima or minima or fringes. The interference in thin film is based on the division of amplitude phenomenon.

Principle of Interference in Thin films :- $t \sim$ 0.5 — 10 μm

Let a ray of light (monochromatic) from an extended source S is incident on a upper surface of a thin film. Then it will be partly reflected along AB and partly ~~transmitted~~ refracted along AC



0.5 × 10⁻⁶

$\frac{0.5}{1000000}$

.0000005 m

5000 Å

At the lower surface (point C), it will again reflected along CE and transmitted along CD. At point E, we again get two rays one due to reflection and other due to refraction. This process continues at different points of the film and as a result of multiple reflections and transmission in the film

(28)

we get the following two systems of rays.

1. Reflected System -

2. Transmitted system.

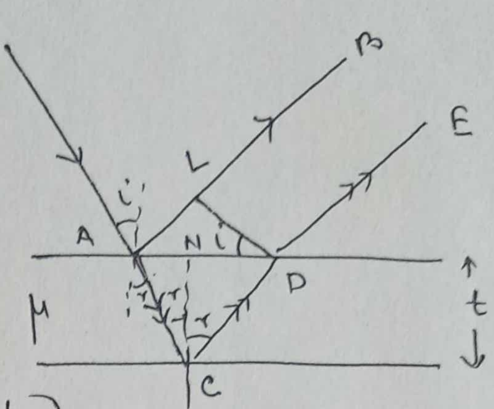
If the rays of any system are focussed by a convex lens, we get the effect of interference. As shown in fig. the different rays reaching I will travel different paths. The path difference between any two rays may be such that they produce either maximum or minimum at the point.

It should be remembered that interference pattern will not be perfect because the intensities of rays will not be the same and their amplitudes are different. The amplitudes will depend on the amount of light reflected and transmitted through the film. It has been found that for normal incident, about 4% of the incident light is reflected and 96% is transmitted. Therefore the intensity never vanishes completely and perfectly dark fringes will not be observed. But in case of multiple reflection, the intensity of the minima will be zero.

* A thin film is an optical medium whose thickness is comparable to the wavelength of light in visible region. film $\simeq$ 0.5 μm to 10 μm may be considered as a thin film.

(1) Interference Due to Reflected Light :-

Consider a transparent film of thickness t and refractive index μ. A ray SA is incident on the upper surface of film is partly reflected along AB and partly refracted



along AC. At point C, the ray is again reflected along CD and finally emerged out along DE.

The first two rays of reflected system AB and DE have nearly the same intensity, while the intensities of the other reflected rays go on decreasing. Thus, we study the effect of interference between first two rays and hence, calculate the path difference between first two rays only. Since, they are obtained from the same parent ray SA, hence they are coherent.

Draw a perpendicular DL and CN to AB and AD respectively

The paths of the rays AB and DE beyond DL are equal.

The path diff between them -

$$P = \text{path } ACD \text{ in film} - \text{Path } AL \text{ in air}$$

$$= \mu(AC + CD) - AL \quad -(1) \qquad \mu = \frac{c}{v}$$

(30)

In ΔACN . $\cos r = \frac{CN}{AC}$

$\Rightarrow AC = t/\cos r$ —— (II)

In ΔNCD , $\cos r = \frac{CN}{CD}$

$\Rightarrow CD = t/\cos r$ —— (III)

In ΔADL $\sin i = \frac{AL}{AD}$

$AL = AD \sin i = (AN + ND)\sin i$

$= (t\tan r + t\tan r)\sin i$

$= 2t\tan r \sin i$

$\because \sin i = \mu \sin r$

$= 2t \frac{\sin r}{\cos r} \mu \sin r$

$AL = 2\mu t \frac{\sin^2 r}{\cos r}$ —— (IV)

$\therefore P = \mu\left[\frac{t}{\cos r} + \frac{t}{\cos r}\right] - 2\mu t \frac{\sin^2 r}{\cos r}$

$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r}\cos^2 r$

$= 2\mu t \cos r$ —— (V)

$\because$ from the stoke's treatment -
If the ray is reflected from the surface of the denser medium then occurs a phase change of π or path difference of $\lambda/2$.

$\therefore \Delta = 2\mu t \cos r - \lambda/2$ —— (VI)

for maximum intensity $\Delta = n\lambda$

$2\mu t \cos r - \lambda/2 = n\lambda$

$$2\mu t \cos r = n\lambda + \lambda/2$$

$$\boxed{2\mu t \cos r = (2n+1)\lambda/2}$$

where $n = 0, 1, 2, 3 \ldots$

for minimum intensity

$$\Delta = (2n-1)\lambda/2$$

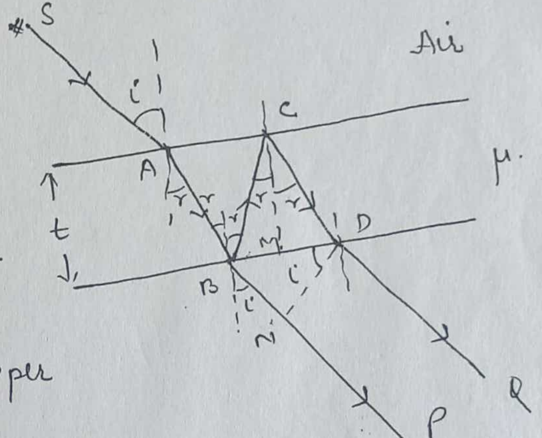
$$2\mu t \cos r - \lambda/2 = 2n \times \frac{\lambda}{2} - \lambda/2$$

$$\boxed{2\mu \cos r = n\lambda}$$

where $n = 0, 1, 2 \ldots$

Interference Due to Transmitted light :-

Consider a thin transparent film of thickness 't' and refractive index μ. Let a ray of light of wavelength λ be incident on the upper



surface of a thin transparent film. This ray SA is refracted along AB at an angle r. The refracted part AB is partly reflected along BC and partly refracted along BP at an angle of i. Reflected part BC is again reflected from C along CD and finally emerges out along DQ. The rays BP and DQ are derived from the same incident ray and hence they are coherent.

(3.2)

To evaluate the path difference between BP and ~~and~~ DQ the perpendiculars DN and CM are drawn on BP and BD respectively. As the paths of the rays BP and DQ beyond DN are equal, the optical path diff.

$$\Delta = \text{path BCD in film} - \text{path BN in air}$$

$$= \mu\,(BC + CD) - BN \quad \text{— (1)}$$

In ΔBCM, $\cos r = \dfrac{MC}{BC} \Rightarrow BC = t/\cos r$ — (11)

In ΔMCD, $\cos r = \dfrac{MC}{CD} \Rightarrow CD = t/\cos r$ — (111)

In ΔBDN, $\sin i = \dfrac{BN}{BD} \Rightarrow BN = BD \sin i$

$$= (BM + MD)\sin i \quad \text{— (14)}$$

In ΔBCM, $BM = CM \tan r = t \tan r$

In ΔCDM, $MD = CM \tan r = t \cdot \tan r$

$$\therefore \quad BN = 2t \tan r \sin i$$

$$= 2t \frac{\sin r}{\cos r} \cdot \mu \sin r$$

$$= 2t\mu \frac{\sin^2 r}{\cos r} \quad \text{— (V)}$$

$$\therefore \quad \Delta = \mu\left[\frac{t}{\cos r} + \frac{t}{\cos r}\right] - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$$\Delta = 2\mu t \cos r \quad \text{——— (vi)}$$

for maximum Intensity

$$\Delta = n\lambda$$

$$\boxed{2\mu t \cos r = n\lambda} \quad \text{where } n = 0, 1, 2, 3 \ldots$$

for minimum Intensity –

$$\Delta = (2n-1)\lambda/2$$

or $\boxed{2\mu t \cos r = (2n-1)\lambda/2}$ where $n = 1, 2, 3 \ldots$

or $\boxed{2\mu t \cos r = (2n+1)\lambda/2}$ where $n = 0, 1, 2 \ldots$

These conditions of maxima and minima in transmitted light are just opposite to those for reflected light. Hence, the point of the film which appears bright in reflected light, appears dark in transmitted light. Hence, the interference patterns of reflected and transmitted monochromatic light are complementary.

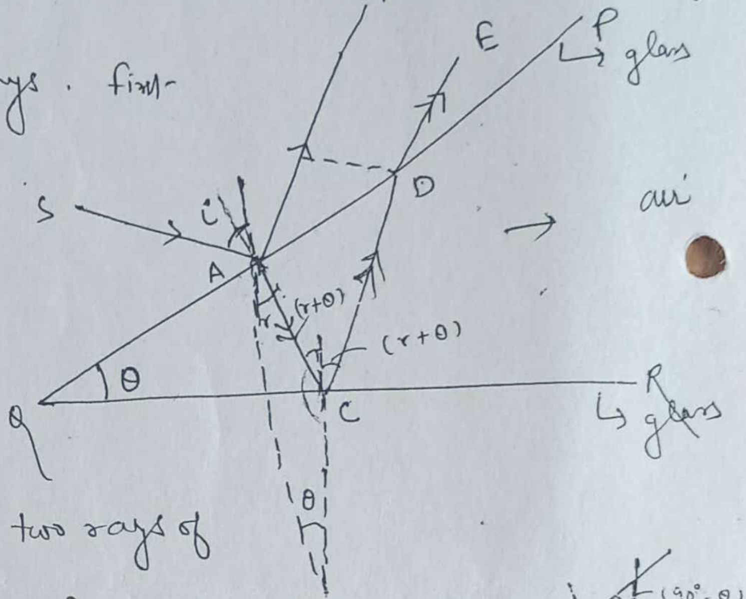
$$\Delta = 2\mu t \cos r - \lambda/2 \Rightarrow \text{Constt}$$

⇒ Locus of the points with same thickness is constt over a circle, hence the fringes are circular.

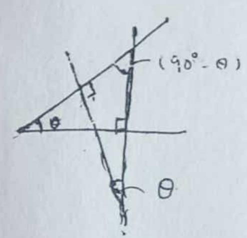
(34)

Interfuence in Wedge shaped film:-

we have seen that when a monochromatic ray of light is incident on a thin film then due to multiple reflections and refractions on the outer and inner surface of the film we get two system of rays. first the reflected system and second the transmitted system.



we know that the path difference between first two rays of reflected system of thin film of uniform thickness is given by



$$P = 2\mu t \cos r - \lambda/2 \quad \text{--- (1)}$$

where, r is angle of refraction for the incident ray

$\lambda/2$ is the extra path of the first ray which is introduced in the first ray due to its reflection at the optically denser medium.

A wedge shaped film is a thin transparent film of gradually varying thickness as shown in fig..

Here, the wedge shaped transparent film lies between two inclined surfaces QP and QR. The point Q is

* The thickness of the glass plate is large compared with the wavelength of the incident

called wedge of the film.

When a ray of monochromatic light SA is incident on the upper surface of the film, it gets splited up into two rays. One reflected ray AR_1 and other refracted ray AB. The ray AB further gets reflected giving the ray BC. On refraction at C we get the ray CR_2. Thus, we get two systems of rays.

The angle of incidence, at the lower surface $= (r+\theta)$

Mathematically, it can be proved the optical path diff. between the first two rays of reflected system is given

$$P = 2\mu t \cos(r+\theta) - \lambda/2 \quad \text{—— (II)}$$

where $\theta \rightarrow$ wedge angle of the film

$t \rightarrow$ thickness of the film at point A

$\mu \rightarrow$ refractive index

for air film, $\mu = 1$

$$P = 2t \cos(r+\theta) - \lambda/2 \quad \text{—— (III)}$$

for maximum intensity

$$P = n\lambda$$

$$2t \cos(r+\theta) - \lambda/2 = n\lambda$$

$$\boxed{2t \cos(r+\theta) = (2n+1)\lambda/2} \quad \text{—— (IV)}$$

where $n = 0, 1, 2 - -$

(36)

for minimum intensity

$$2t\cos(r+\theta) - \lambda/2 = (2n-1)\lambda/2$$

$$\boxed{2t\cos(r+\theta) = n\lambda} \quad \text{—} \quad (v)$$

where $n = 0, 1, 2 \ldots$

If light is incident normally on the film

$$r = 0$$

$$\boxed{\begin{aligned} 2t\cos\theta &= (2n+1)\lambda/2 \\ 2t\cos\theta &= n\lambda \end{aligned}}$$

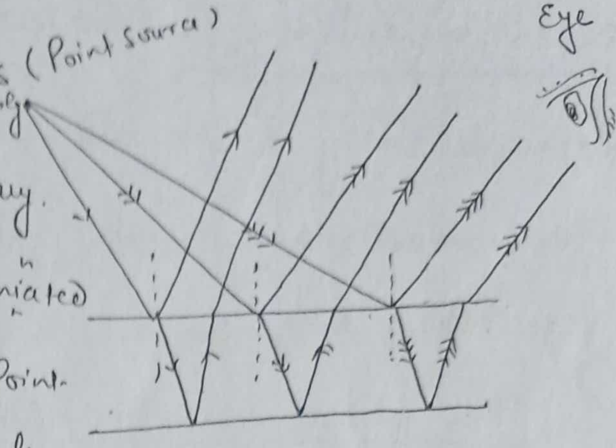
Nature of Interference fringes:- Since, the thickness of the wedge shaped film varies from point to point, the value of $2t\cos\theta$ ie Path difference also varies accordingly. for any ~~value of~~ bright or dark fringe the value of path difference should remain constant. At a particular point, thickness is constant so we get a bright or dark fringe at that point due to constant path difference. In a wedge shaped film the locus of the points with same thickness of the film are straight lines parallel to the edge of the film, hence, in this case we get bright and dark straight fringes parallel to the edge of the film

~~Newton's Rings:-~~

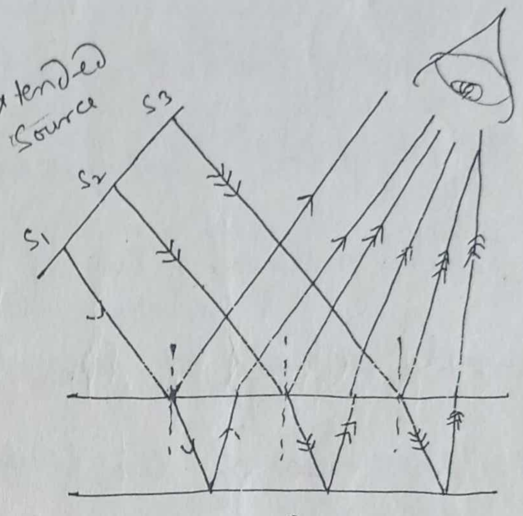
Necessity of an Extended Source:-

To see the interference effects over the entire film simultaneously a broad source of light is necessary.

When a thin film is illuminated with monochromatic light from a point source and is viewed with a lens of small aperture, the light reflected from ~~a point source~~ Extended Source and all corresponding points on the film does not reach the eye simultaneously as shown in fig (a). Thus, only a small portion of the film will be visible. To see the whole film, the eye will have to be moved from one position to the other. If we employ an extended source, the light reflected by every point of film reaches the eye. Hence, the entire film can be viewed simultaneously by keeping the eye at one place only.



(a)



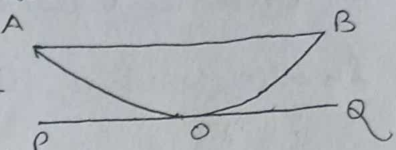
(b)

Hence, an extended source of light is necessary to view a film simultaneously.

(40)

Newton's Ring :- Newton's was the first to obtain experimentally bright and dark interference rings due to interference in a wedge shaped film, which are called Newton's Ring.

The phenonmea of Newton's ring is a special case of interference in air film of variable thickness.



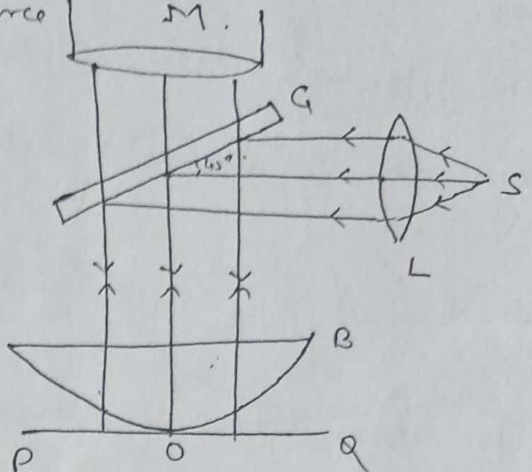
If we place a planoconvex lens on a plane glass surface, a thin film of air is formed between the curved surface of lens AOB and plane glass plate POQ. The thickness of air film is zero at point 'O' and increase as move away from the point of contact. Since the curved side of the lens is a spherical surface, the thickness of the air film be constant over a circle (whose centre will be at O). When mono-chromatic light falls normally on such a film, we get inner dark spot surrounded by alternatively bright and dark circular rings, when seen by reflected and vice-versa by transmitted light. These rings are known as Newton's Rings.

When white light is used, a few coloured circular rings are seen around the point of contact with violet [illegible] and red colour [illegible] visible in both reflected and transmit[illegible]

2t -

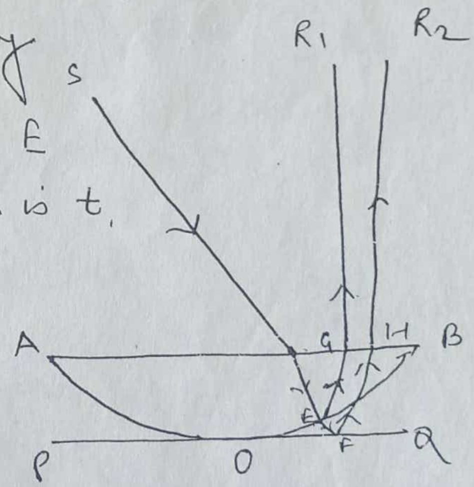
Experimental Arrangement :-

S → An extended monochromatic source
L → Convex lens
G → Glass plate inclined at 45°



Light rays from an extended source are made parallel by lens L. These horizontal parallel rays fall on a glass plate G at 45° and are partly reflected from it. These reflected rays fall normally on the lens placed on the glass plate. Interference occurs between the rays reflected from the upper and lower surfaces of the film. The interference rings are viewed with a low-power microscope M focussed on the air film where the rings are formed.

Theory :- If a monochromatic ray of light falls on air film at E where the thickness of the film is t. this ray gets reflected at E and give GR_1. The other ray EF at E is due to refraction which get reflected at F and finally comes on the side of ~~HR_2~~ GR_1 as the ray HR_2



These two rays are coherent, producing bright

(42)

The effective path difference between the interfering rays in reflected light is given by

$$\Delta = 2\mu t \cos(r+\theta) \pm \lambda/2$$

for normal incidence, $r = 0$, for air film $\mu = 1$

$$\cos(r+\theta) = \cos\theta$$

for very small angle $\cos\theta \simeq 1$

$$\therefore \Delta \doteq 2\mu t \pm \lambda/2 \quad \text{--- (1)}$$

At point of contact, $t = 0$

$\Delta = \lambda/2$ (which is the condition for darkness)

Hence, the central ring or spot is dark

for bright ring $\Delta = 2t \pm \lambda/2 = n\lambda$

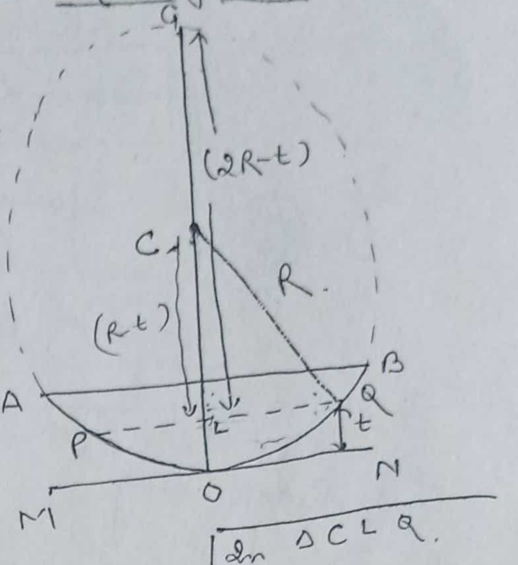
$$\Delta = 2t = (2n \pm 1)\lambda/2 \quad \text{--- (II)}$$

for dark ring $\Delta = 2t \pm \lambda/2 = (2n-1)\lambda/2$

$$2t = n\lambda \quad \text{--- (III)}$$

Diameters of Bright and Dark Rings :-

To evaluate the diameters of Bright and dark rings, Consider a plano - convex lens AOB placed on a plane glass plate MON. Let 't' be the thickness of the film at any point Q and R be the radius of curved surface AOB



In ΔCLQ.

$CL = (R-t)$

$LQ = r$.

$CQ = R$

$R^2 = (R-t)^2 + r^2$

$R^2 = \cancel{R^2} + t^2 + r^2 - 2Rt$

$t^2 = 2Rt - r^2$

$r^2 = 2Rt - t^2$

from the property of circle

$$PL \times QL = OL \times GL \quad \text{--- (I)}$$

$$r \times r = t \; (2R - t)$$

$$r^2 = 2Rt - t^2$$

$\because t <<< R$, t^2 may be neglected

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \quad \text{--- (II)}$$

for bright fringe $2t = (2n+1)\lambda/2$

substitute the value of 't' from eqn. (II)

$$\frac{r^2}{R} = (2n+1)\lambda/2$$

If this point 'Q' gives the n^{th} bright ring

$$r_n^2 = \frac{(2n+1)\lambda R}{2}$$

If $D_n \rightarrow$ Diameter of n^{th} bright ring

$$D_n^2 = 2(2n+1)\lambda R$$

or $$\boxed{D_n = \sqrt{2(2n+1)\lambda R}} \quad \text{--- (III)}$$

where $n = 0, 1, 2, 3 ...$

(44)

$$\boxed{D_n \propto \sqrt{2n+1}}$$

If 'n' is starting from '1'

then. $$\boxed{D_n \propto \sqrt{2(2n-1)\lambda R}} \quad \text{---(iv)}$$

for Dark ring.

$$2t = n\lambda$$

$$\frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$\boxed{D_n = \sqrt{4n\lambda R}} \quad \text{---(v)}$$

If D_n and D_{n+1} are the diameters of n^{th} and $(n+1)^{th}$ dark rings then the spacing between the consecutive rings -

$$D_{n+1} - D_n = \sqrt{4(n+1)\lambda R} - \sqrt{4n\lambda R}$$

$$= \sqrt{4\lambda R}\left[\sqrt{n+1} - \sqrt{n}\right]$$

putting $n = 1, 2, 3, 4 - - -$

$$D_2 - D_1 = \sqrt{4\lambda R}\left[\sqrt{2} - \sqrt{1}\right] = 0.414\sqrt{4\lambda R}$$

$$D_3 - D_2 = \sqrt{4\lambda R}\left[\sqrt{3} - \sqrt{2}\right] = 0.318\sqrt{4\lambda R}$$

$$D_4 - D_3 = \sqrt{4\lambda R}\left[\sqrt{4} - \sqrt{3}\right] = 0.268\sqrt{4\lambda R}$$

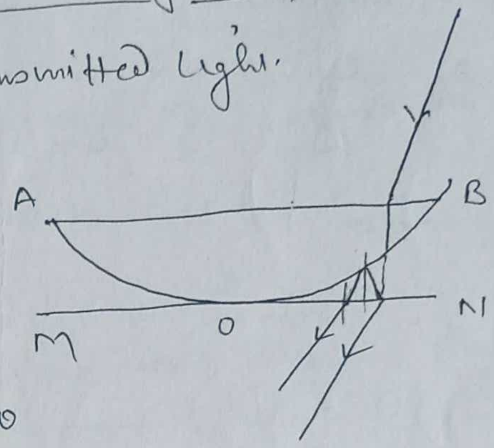
$\vdots \qquad \vdots \qquad \vdots$

Hence, it is clear from the above values that (ii)
the spacing between the consecutive rings decrease
with increasing the order of the ring.

Newton's Rings by Transmitted Light:-

The effective path diff in transmitted light.

$\Delta = 2\mu t \cos(r+\theta)$



for air film $\mu = 1$

normal incidence $r = 0$

$\cos(r+\theta) \simeq \cos\theta$

when $\theta \sim$ very small $\cos\theta = 1$

$\therefore \Delta = 2t$

for bright ring $\Delta = n\lambda$ where

$\boxed{2t = n\lambda}$ $n = 0, 1, 2, 3 \ldots$

for dark $\Delta = (2n-1)\lambda/2$

$\boxed{2t = (2n-1)\lambda/2}$ $n = 1, 2, 3 \ldots$

On comparing these equations with equations obtained in reflected system, it is obvious that the rings are observed in transmitted light are complementary to the rings observed in reflected light.

(46) (1) Experimental Determination of wavelength of Light By Newton's Rings :-

The experimental arrangement used is the same as we have discussed in the begining.

If D_n is the diameter of n^{th} dark ring, then

$$D_n^2 = 4n\lambda R \quad \text{— (i)}$$

D_{n+m}^2 is the diameter of $(n+m)^{th}$ order dark ring

$$D_{n+m}^2 = 4(n+m)\lambda R \quad \text{— (ii)}$$

(ii) − (i)

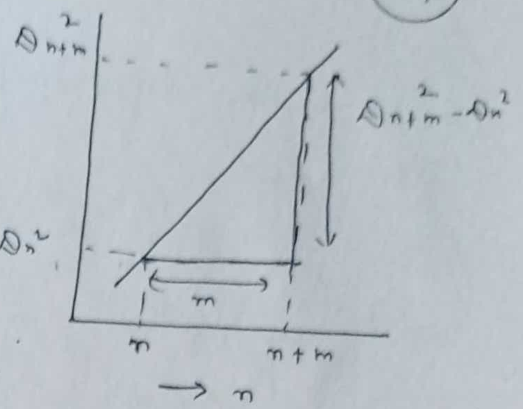
$$D_{n+m}^2 - D_n^2 = 4m\lambda R$$

$$\boxed{\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}} \quad \text{— (iii)}$$

The cross-wire of eye-piece is focussed on the any dark ring (say 10^{th}) in left hand side and reading of the microscope is noted. The microscope is now moved to right-hand ~~right~~-side and reading of each dark rings are noted. On reaching the centre of Newton's ring the microscope is further moved to the right side and the reading is recorded till we reach again 10^{th} order dark ring. ~~A graph is plotted~~ The diameters of successive rings are measured and a graph between n & D^2 (ie order and square of diameter) is plotted.

The slope of this graph gives $\frac{D_{n+m}^2 - D_n^2}{m}$

$\therefore$ Slope $= \dfrac{D_{n+m}^2 - D_n^2}{m}$



R → radius of curvature is determined with the help of spherometer, and using formula (iii), we can measure λ i.e wavelength of light..

(II) Refractive Index of a liquid using Newton's Rings:-

The liquid whose refractive index μ is to be determined is introduced between the lens and the glass plate.

$\therefore$ Diameter of n^{th} dark ring $D_n^2 = \dfrac{4\lambda n R}{\mu}$ - (i)

Similarly for $(n+m)^{th}$ dark ring $D_{n+m}^2 = \dfrac{4(n+m)\lambda R}{\mu}$ - (ii)

(ii) - (i)

$$\left[D_{n+m}^2 - D_n^2\right]_{liquid} = \frac{4m\lambda R}{\mu} \quad \text{- (iii)}$$

for air film $\mu = 1$

$$\left[D_{n+m}^2 - D_n^2\right]_{air} = 4m\lambda R \quad \text{- (iv)}$$

Dividing (iv) by (iii), we get

$$\boxed{\mu = \frac{\left[D_{n+m}^2 - D_n^2\right] air}{\left[D_{n+m}^2 - D_n^2\right] liquid}} \quad \text{- (v)}$$

This relation also holds for bright rings

(48) With liquid film the diameter of n^{th} dark ring is

$$(D_n)^2_{liquid} = \frac{4n\lambda R}{\mu} \quad \text{—(I)}$$

with air film

$$(D_n)^2_{air} = 4n\lambda R \quad \text{—(II)}$$

$$\frac{(D_n)^2_{liquid}}{(D_n)^2_{air}} = \frac{1}{\mu} \Rightarrow \frac{(D_n)_{liquid}}{(D_n)_{air}} = \frac{1}{\sqrt{\mu}}$$

$\because \mu > 1$

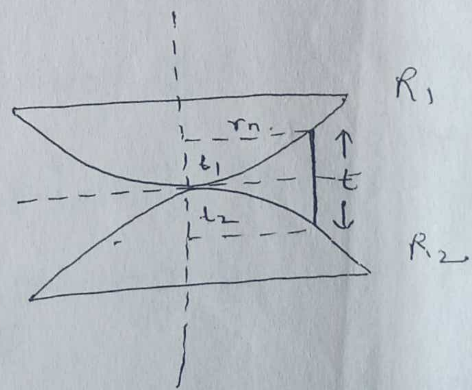
$$(D_n)_{liquid} < (D)_{air}$$

Thus, when the liquid is introduced between the lens and the plate the diameters of the ring decrease.

<u>Newton's Rings with Both curved Surfaces:-</u>

<u>case-1. when the lower surface is convex</u>

Let the radii of curvature of the convex faces of the above & lower lenses be R_1 & R_2 respectively.



from fig $\quad t = t_1 + t_2 \quad$ —(I)

$$= \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2}$$

$$2t = r_n^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad \text{—(II)}$$

where r_n is the radius of n^{th} ring corresponding to thickness 't'.

The condition for dark ring

$$2t = n\lambda \quad \text{——— (iii)}$$

$$r_n^2\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = n\lambda$$

$$r_n^2 = \frac{n\lambda}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}$$

$$\boxed{D_n^2 = \frac{4n\lambda}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}} \quad \text{——— (iv)}$$

where $D_n \rightarrow$ diameter of n^{th} dark ring.

where $n = 0, 1, 2, 3 \ldots$

The condition for bright ring.

$$2t = (2n+1)\lambda/2$$

$$r_n^2\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = (2n+1)\lambda/2$$

$$r_n^2 = \frac{(2n+1)\lambda}{2\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}$$

$$\boxed{D_n^2 = \frac{2(2n+1)\lambda}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}}$$

where $n = 0, 1, 2, 3 \ldots$

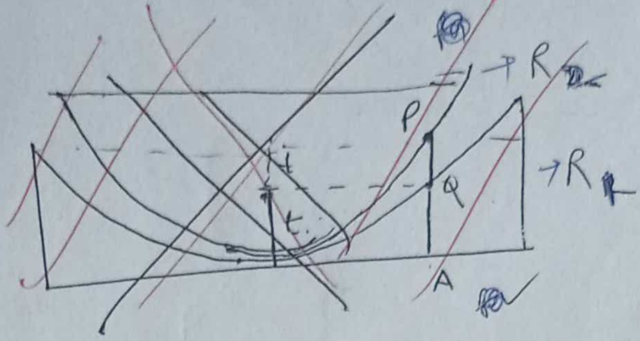
If $D_n^2 = \dfrac{2(2n-1)\lambda}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}$, $n = 1, 2, 3 \ldots$

50

Case. II when the lower surface is concave:- $(R_2 > R_1)$

$PQ = t$, ~~$PA = t_1$, $QA = t_2$~~

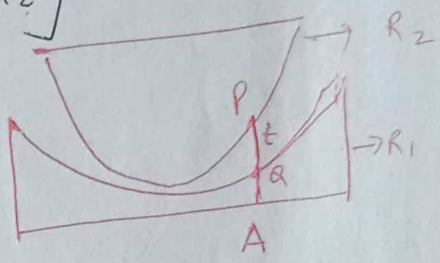
$t = PA - AQ$.



$$PA = \frac{r_n^2}{2R_1}$$

$$QA = \frac{r_n^2}{2R_2}$$

$$t = \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2} = \frac{r_n^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$



$PQ = t$

$PA = \frac{r_n^2}{2R_1}$

$QA = \frac{r_n^2}{2R_2}$

$$2t = r_n^2\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

condition for dark ring

$$2t = n\lambda$$

$$r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = n\lambda$$

$$\boxed{D_n^2 = \frac{4n\lambda}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$ where $n = 0, 1, 2 ..$

condition for bright ring

$$2t = (2n-1)\lambda/_2$$

$$r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (2n-1)\lambda/_2$$

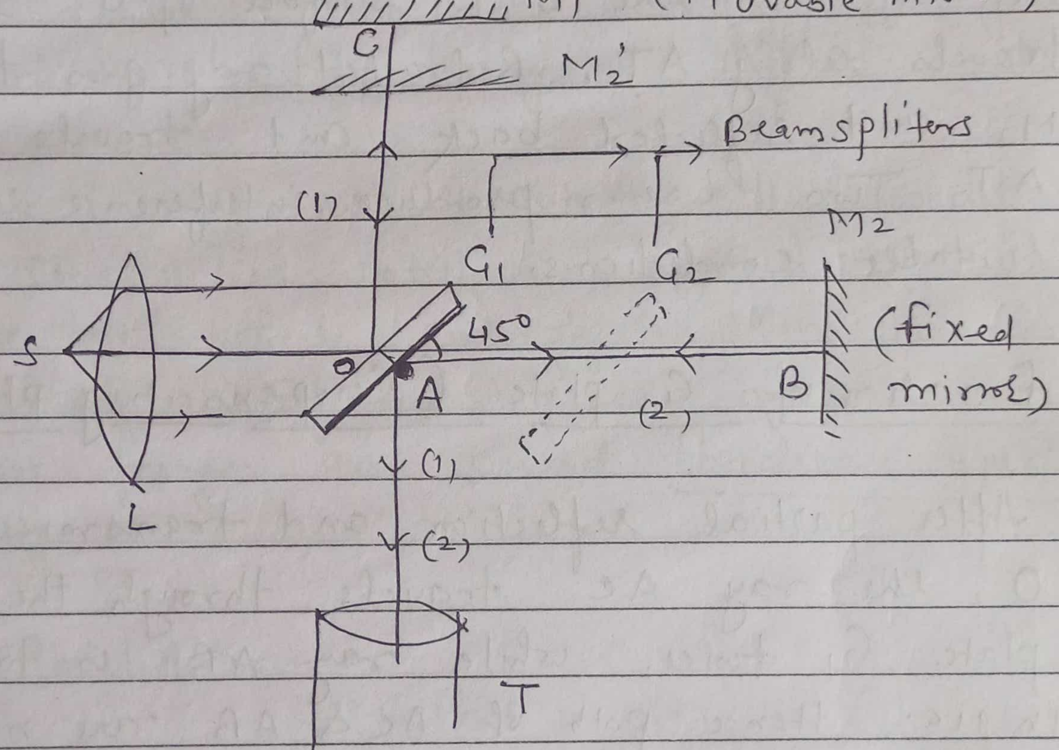
$$r_n^2 = \frac{(2n-1)\lambda}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)2} \Rightarrow \boxed{D_n^2 = \frac{2(2n-1)\lambda}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}}$$

$n = 1, 2, 3 - \rightarrow$

The Michelson Interferrometer

Principle:- The amplitude of light beam is divided into two parts of nearly equal intensity by partial reflection and refraction.

Construction:-



M_1 M_2 — Two highly polished front silvered plane mirrors.

G_1 G_2 - Two parallel plates of same thickness

G_1 - Semisilvered on the backside and function as a beam splitter.

G_2 - inclined at an angle of 45° to the incident beam. The glass plate G_2 is called the Compensating plate.

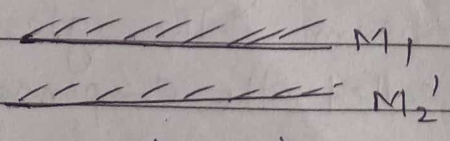
Working:- Light from an extended source S rendered nearly parallel by a lens L, falls on G_1, which is inclined at an angle 45^o to the incident beam. A ray of light incident on G_1 is partially reflected and partially transmitted as AC and AB. AC moves towards M_1 and reflected back along the same path. After reflection at the back surface of G_1 it travels along AT. Refracted ray goes towards M_2 and reflected back and travels along AT. Two beams produce interference under suitable conditions.

Function of G_2 plate (Compensating plate):-

After partial reflection and transmission at O, the ray AC travels through the glass plate G_1 twice, while ray AB is totally in air. Hence path of AC & AB are not equal. To equalize these path a glass plate G_2 (having same thickness) is placed parallel to G_1

Formation of fringes:-

M_2' – Image of M_2



M_1 and M_2' will treat as two surfaces of a thin film giving reflected beam of s.

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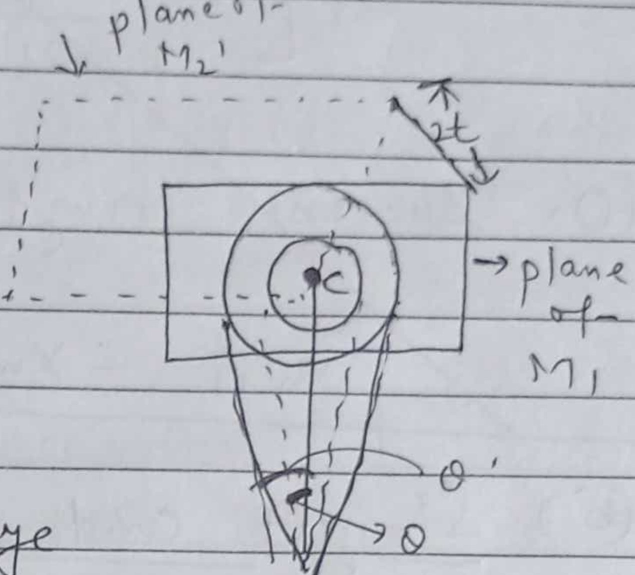
to interference.

(a) <u>Circular fringes</u>:- When M_1 and M_2' are parallel. If distances of mirror M_2 and M_1 from G_1 differ by distance t, then separation between $2t$. The path difference along a circle with centre C as the foot of the $\perp$ from the eye is constant.

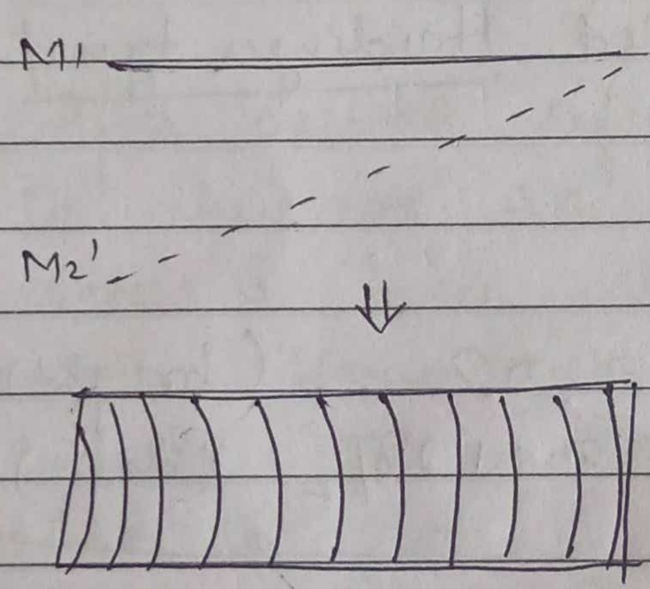


- Hence a ring which satisfies the condition for constructive interference appears bright and one which satisfy the condition for destructive interference appear dark.

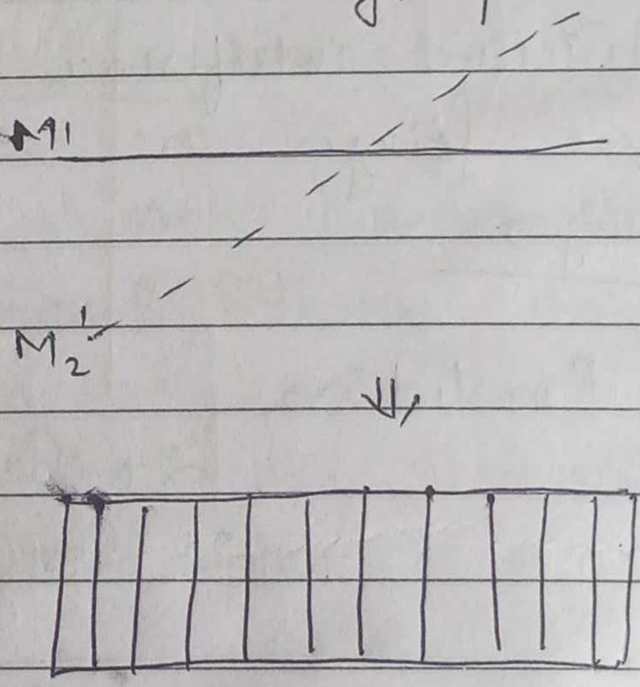
These fringes are called <u>Haidinger type of fringes</u>

(b) Straight and Curved fringes:-

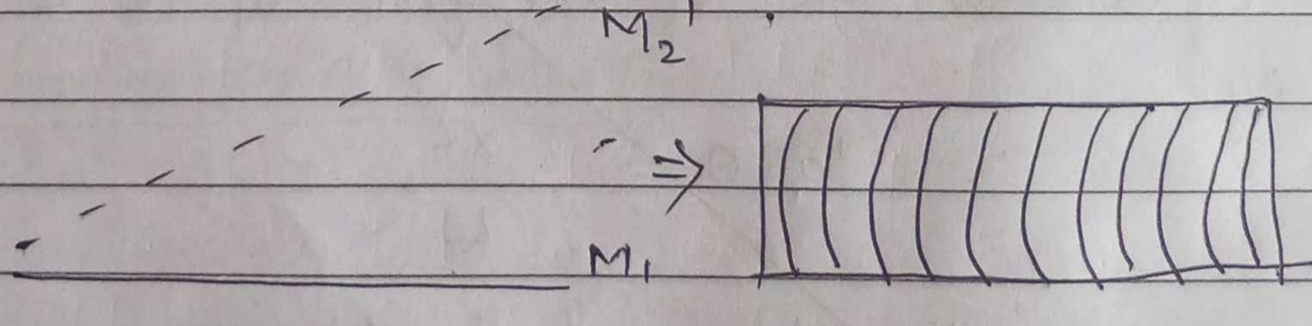
When M_1 is not $\perp M_2$ i.e M_1 and M_2' (image of M_2) are inclined, the air film between M_1 and M_2' is wedge shaped.



(a)



(b)



(c)

In general, the fringes are curved and always convex towards thin edges of wedge ((a) & (c)) and straight (b).