Properties of complex No.

$i = \sqrt{-1}$,
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Product $(x+iy_1)(x_2+iy_2) = x_1x_2 + x_1 \times iy_2 + iy_1x_2 + iy_1 \times iy_2$

$= x_1x_2 + iy_2x_1 + iy_1x_2 + iy_1 \times iy_2$

$$\boxed{x_1x_2 - y_1y_2 + i(y_1x_2 + y_2x_1)}$$

Quotient $\frac{x_1+iy_1}{x_2+iy_2} = \frac{(x_1+iy_1)\times(x_2-iy_2)}{(x_2+iy_2)\times(x_2-iy_2)}$ → rationalization

$$\boxed{\frac{x_1x_2 + y_1y_2}{x_2^2+y_2^2} + i\left(\frac{x_2y_1 - x_1y_2}{x_2^2+y_2^2}\right)}$$

$r(\cos\theta + i\sin\theta)$ $R(x+iy) \Rightarrow x = r\cos\theta$
$Im(x+iy) \Rightarrow y = r\sin\theta$

$$\boxed{\begin{gathered} r = \sqrt{x^2+y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{gathered}}$$

Frequency domain analysis

- Relationship between magnitude and phase of sinusoidal I/p and steady state o/p of a system.

~~H~~ we use it because it is simple and accurate to use.

② apparatus required are simple

③ can be applied to systems which do not have rational transfer funcn.

polar plot

plot of magnitude $|G(j\omega)|$ v/s the phase angle $\angle G(j\omega)$ on polar co-ordinates as 'ω' varies from '0' to ∞.

- polar plot ~~also~~ also called Nyquist plot.

Advantage :- depicts the frequency response characteristics of a system over the entire frequency range in a single plot.

Disadvantage :- does not indicate the contributions of each individual factor of open loop transfer funcn.

Step1

Step2

Step3

Ste

S

Procedure:-

Step 1: Determine transfer func^n of $G(s)$ of the system.

Step 2: Put $s = j\omega$

$a + ib$

Step 3 Calculate magnitude

$$\boxed{\begin{array}{l} \text{At } \omega = 0 \quad \lim_{\omega \to 0} |G(j\omega)| \\ \text{at} \qquad \& \\ \omega = \infty \quad \lim_{\omega \to \infty} |G(j\omega)| \end{array}}$$

where $|G(j\omega)| = \sqrt{a^2 + b^2}$

Step 4 Calculate phase angle $\angle G(j\omega)$

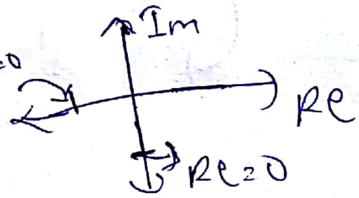
at $\omega = 0$ $\lim_{\omega \to 0} \angle G(j\omega)$

where $\angle G(j\omega) = \tan^{-1}\frac{b}{a}$

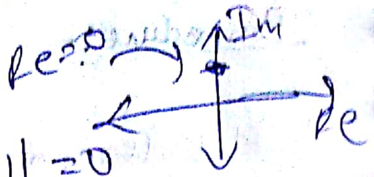
$\omega = \infty$ $\lim_{\omega \to \infty} \angle G(j\omega)$

Step 5 Rationalize the func^n $G(j\omega)$ & seperate Real and Im part.

Step 6 Put Im(part) $= 0$ ($Im|G(j\omega)|$ $Im = 0$) and find ω at which plot Intersect real axis and put it in $G(j\omega)$ to find value.



Step 7



Similarly equate Real part $Re|G(j\omega)| = 0$ and find ω, at which it cuts Im, axis and calculate value of $G(j\omega)$ By putting ω, in rationalized exp. of $G(j\omega)$

Step 8

Sketch polar plot.

General values and exp for types

$\lim_{\omega \to 0} |G(j\omega)| = K$

$\lim_{\omega \to \infty} |G(j\omega)| = 0$

$\lim_{\omega \to 0} \angle G(j\omega) = 0^\circ$

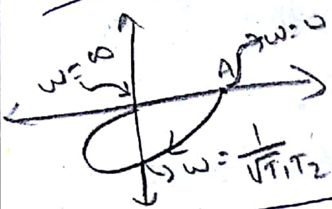
$\lim_{\omega \to \infty} \angle G(j\omega) = -180^\circ$

Types of system

Type 0 e.g

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

$1 = s^0$

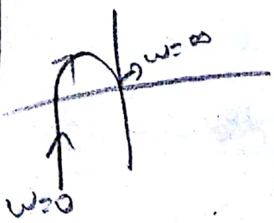


$\omega = \pm\infty$ & $\omega^2 = \frac{1}{\sqrt{T_1 T_2}}$

Type 1

$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

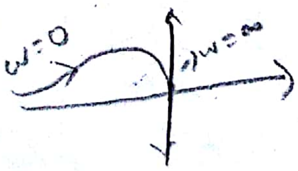
$s = s^1$



Type 2

$$G(s) = \frac{K}{s^2(1+sT_1)}$$

s^2



e.g Sketch polar plot

$$G(s) = \frac{20}{s(s+1)(s+2)}$$

Step 1 Determine G(s) (Given here)

$$G(s) = \frac{20}{s(s+1)(s+2)}$$

Step 2 put $s = j\omega$

$$G(j\omega) = \frac{20}{j\omega(1+j\omega)(2+j\omega)} \quad \text{--- (1)} \qquad a+ib$$

$$|G(j\omega)| = \frac{20}{|0+j\omega(1+j\omega)(2+j\omega)|} \Rightarrow \frac{20}{\sqrt{(0^2+\omega^2)(1^2+\omega^2)(2^2+\omega^2)}}$$

a b a b a b

$(\because \sqrt{a}\sqrt{b}\sqrt{c} = \sqrt{abc})$

$$\Rightarrow \frac{20}{\sqrt{0^2+\omega^2}\sqrt{(1^2+\omega^2)}\sqrt{(2^2+\omega^2)}} \Rightarrow \frac{20}{\sqrt{\omega}\sqrt{(1+\omega^2)}\sqrt{(4+\omega^2)}} \quad \text{--- (III)}$$

$$\boxed{|G(j\omega)| = \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}} \quad \text{--- (2)}$$

for log cos $\left(\frac{\theta_1}{\theta_2}\right) = \theta_1 - \theta_2$

$$\angle G(j\omega) = \tan^{-1}\frac{b}{a} \Rightarrow \frac{\tan^{-1}\left(\frac{0}{20}\right)}{\tan^{-1}\frac{\omega}{0} + \tan^{-1}\frac{\omega}{1} + \tan^{-1}\frac{\omega}{2}}$$

$$\Rightarrow \frac{\tan^{-1}(0)}{\tan^{-1}(\infty) + \tan^{-1}(\omega) + \tan^{-1}(\omega/2)}$$

$(\tan^{-1}(0) = 0)$

$\tan^{-1}(\infty) = \frac{\pi}{2} = 90^\circ$

$$= \frac{0^\circ}{90 + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2}} \Rightarrow -(90 + \tan^{-1}\omega + \tan^{-1}(\omega/2))$$

$\because \left(\frac{\theta_1}{\theta_2} = \theta_1 - \theta_2\right)$

$$= \boxed{-90 - \tan^{-1}\omega - \tan^{-1}(\omega/2)}$$

$\left(\theta_1 - \theta_2 = \frac{\theta_1}{\theta_2}\right)$

Scanned by CamScanner

Step 3 Calculate

$\omega \to 0$ $\lim_{\omega \to 0} |G(j\omega)| \Rightarrow \lim_{\omega \to 0} \frac{20}{\omega \sqrt{1+\omega^2}\sqrt{4+\omega^2}} = \frac{20}{0}$, $\left(\frac{1}{0} = \infty\right)$

$= \infty$ $\boxed{\lim_{\omega \to 0} |G(j\omega)| = \infty}$ $\left(\frac{1}{\infty} = 0\right)$

and $\omega = \infty$

$\lim_{\omega \to \infty} |G(j\omega)| = \lim_{\omega \to \infty} \frac{20}{\omega \sqrt{1+\omega^2}\sqrt{4+\omega^2}} = \frac{20}{\infty} = 0$

0 $\boxed{\lim_{\omega \to \infty} |G(j\omega)| = 0}$

Step 4 Calculate

$\lim_{\omega \to 0} \angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}(\omega/2)$

$= -90^\circ - \tan^{-1}(0) - \tan^{-1}(0/2)$ $\because \tan^{-1}(0) = 0$

$= -90^\circ$ $\boxed{\lim_{\omega \to 0} \angle G(j\omega) = -90^\circ}$

also $\lim_{\omega \to \infty} \angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$

$= -90^\circ - \tan^{-1}(\infty) - \tan^{-1}(\infty)$ $\left(\tan^{-1}(\infty) = \frac{\pi}{2} = 90^\circ\right)$

$= -90 - 90 - 90$

$= -270^\circ$

$\boxed{\lim_{\omega \to \infty} \angle G(j\omega) = -270^\circ}$

Now from eqn (1), we have

$$G(j\omega) = \frac{20}{j\omega(1+j\omega)(2+j\omega)}$$

$$G(j\omega) = \frac{20}{(j\omega + j^2\omega^2)(2 + j\omega)}$$

$$= \frac{20}{2j\omega + j^2\omega^2 + 2j^2\omega^2 + j^3\omega^3}$$

$$= \frac{20}{2j\omega + 3j^2\omega^2 + j^3\omega^3} = \frac{20}{2j\omega - 3\omega^2 - j\omega^3} \quad \left(\because j^2 = -1,\ j^3 = -j\right)$$

$$G(j\omega) = \frac{20}{-3\omega^2 + 2j\omega - j\omega^3} = \frac{20}{-3\omega^2 + j(2\omega - \omega^3)}$$

Now steps Rationalization

$$G(j\omega) = \frac{20}{-3\omega^2 + j(2\omega - \omega^3)} \times \frac{-3\omega^2 - j(2\omega - \omega^3)}{-3\omega^2 - j(2\omega - \omega^3)}$$

$$= \frac{-60\omega^2 - 20j(2\omega - \omega^3)}{(-3\omega)^2 - (2j\omega - j\omega^3)^2} \quad \left(\because (a+b)(a-b) = a^2 - b^2\right)$$

$$= \frac{-60\omega^2}{9\omega^4 - (2j\omega - j\omega^3)^2} + j\frac{20(\omega^3 - 2\omega)}{9\omega^4 - (2j\omega - j\omega^3)^2}$$

$$= \frac{-60\omega^2}{9\omega^4 - [\underbrace{(2j\omega)^2}_{a^2} + \underbrace{(j\omega^3)^2}_{b^2} - \underbrace{2(2j\omega)(-j\omega^3)}_{2ab}]} + j\frac{20(\omega^3 - 2\omega)}{9\omega^4 - [(2j\omega)^2 + (j\omega^3)^2 - 2(2j\omega)(-j\omega^3)}$$

~~$(a^2+b)^2 = a^2 + b^2$~~

$$\because (a-b)^2 = a^2 + b^2 - 2ab.$$

$$\frac{-60\omega^2}{9\omega^4 - [4j^2\omega^2 + j^2\omega^6 - 4j^2\omega^4]} + j\frac{20(\omega^3 - 2\omega)}{9\omega^4 - [4j^2\omega^2 + j^2\omega^6 - 4j^2\omega^4]}$$

$(2 \times 2j\omega \times j\omega^3$
$= 4j\omega \times j\omega^3$
$= 4j^2\omega^4)$

$$\Rightarrow \frac{-60\omega^2}{9\omega^4 - 4j^2\omega^2 - j^2\omega^6 + 4j^2\omega^4} + j\frac{20(\omega^3 - 2\omega)}{9\omega^4 - 4j^2\omega^2 - j^2\omega^6 + 4j^2\omega^4}$$

$(\because j^2 = -1)$

$$\Rightarrow \frac{-60\omega^2}{9\omega^4 + 4\omega^2 + \omega^6 - 4\omega^4} + j\frac{20(\omega^3 - 2\omega)}{9\omega^4 + 4\omega^2 + \omega^6 - 4\omega^4}$$

Now $\dfrac{-60\omega^2}{\cancel{9\omega}5\omega^4 + 4\omega^2 + \omega^6} + j\dfrac{20(\omega^3 - 2\omega)}{5\omega^4 + 4\omega^2 + \omega^6}$

$$\frac{-60\omega^2}{\omega^6 + 5\omega^4 + 4\omega^2} + j\frac{20(\omega^3 - 2\omega)}{\omega^6 + 5\omega^4 + 4\omega^2}$$

Addition

$$\boxed{\begin{aligned}\omega^6 + 5\omega^4 + 4\omega^2 &= \omega^6 + 4\omega^4 + \omega^4 + 4\omega^2 \\ &\quad \omega^6 + \omega^4 + 4\omega^4 + 4\omega^2 \\ &= \omega^2(\omega^4 + \omega^2) + 4(\omega^4 + \omega^2) \\ &= (\omega^4 + \omega^2)(\omega^2 + 4)\end{aligned}}$$

$$\frac{-60\omega^2}{(\omega^4 + \omega^2)(4 + \omega^2)} + j\frac{20(\omega^3 - 2\omega)}{(\omega^4 + \omega^2)(4 + \omega^2)} \quad \text{--- (3)}$$

Step 6 equate Imaginary part $= 0$ of eqn (3)

$$\frac{20(\omega^3 - 2\omega)}{(\omega^4 + \omega^2)(4 + \omega^2)} = 0$$

$20(\omega^3 - 2\omega) = 0$
$(\omega^3 - 2\omega) = 0$
$\omega(\omega^2 - 2) = 0$
$\omega = 0$ | $\omega^2 = 2$
$\omega = \pm\sqrt{2}$

$$\frac{1}{(\omega^4 + \omega^2)(4 + \omega^2)} = 0$$

$\left(\frac{1}{\infty} = 0\right)$

$\omega = \pm\infty$

Intersection at real axis.

from eqn (2)

$$G(j\omega) = \frac{20}{\omega\sqrt{1+\omega^2}\sqrt{2+\omega^2}}$$

here we have three values of ω,

$\omega = 0$

$$G(j\omega)_{\omega=0} = \frac{20}{0\sqrt{1+0}\sqrt{2+0}}$$

$$G(j\omega) = \frac{20}{0} = \infty$$

$\omega = \sqrt{2}$

$$G(j\omega)_{\omega=\sqrt{2}} = \frac{20}{\sqrt{2}\sqrt{1+(\sqrt{2})^2}\,\sqrt{4+(\sqrt{2})^2}}$$

$$= \frac{20}{\sqrt{2}\sqrt{1+2}\sqrt{4+2}}$$

$$= \frac{20}{\sqrt{2}\sqrt{3}\sqrt{6}} = \frac{20}{\sqrt{36}}$$

$$= \frac{\cancel{20}}{\cancel{6}} = \frac{10}{3}$$

$\omega = \infty$

$$G(j\omega)_{\omega=\infty} = \frac{20}{\infty}$$

$$G(j\omega) = 0$$

Note:- here we are ~~no~~ taking only modulus values of $|G(j\omega)|$ at +ve ω because ~~plot~~ will be b/w $|G(j\omega)|$ and phase (if ω = -ve then $G(j\omega)$ = -Imag)

Step 7 Now equate real part of eqn (3) = 0 and find intersection on angle.

$$\frac{-60\omega^2}{(\omega^4+\omega^2)(4+\omega^2)} = 0$$

$-60\omega^2 = 0$

$\omega = 0$

$$\angle G(j\omega)_{\omega=0} = -90 - \tan(0) - \tan(0/2)$$

$= -90$

$$\frac{1}{(\omega^4+\omega^2)(4+\omega^2)} = 0$$

$\omega = \pm\infty$

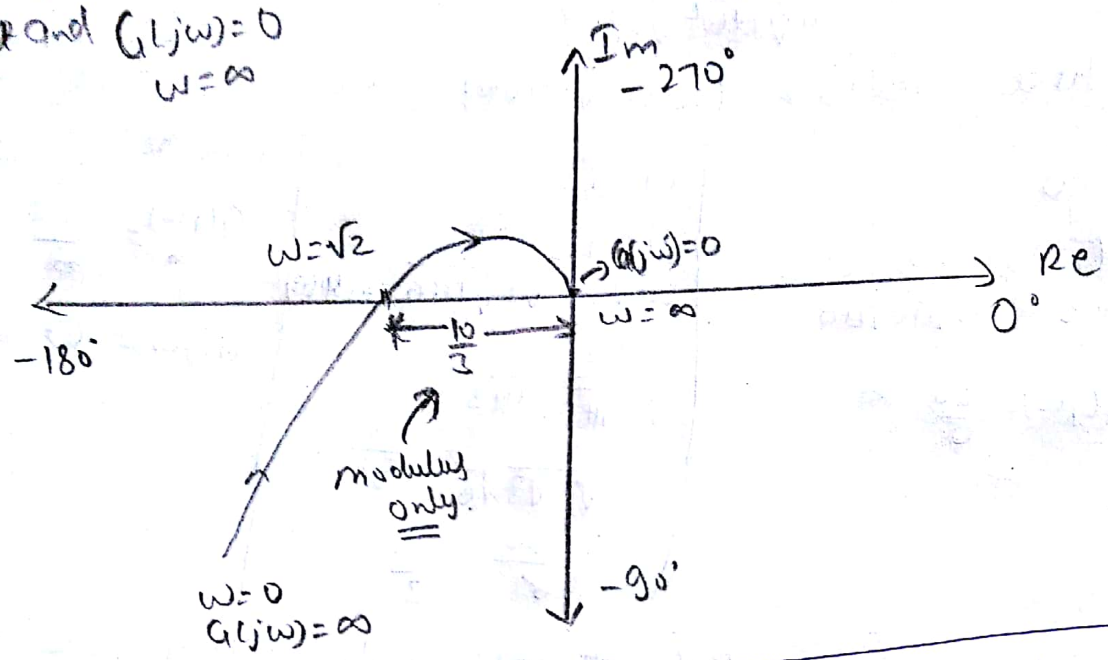
$$\angle G(j\omega)_{\omega=\infty} = -90 - \tan(\infty) - \tan(\infty/2)$$

$= -90 - 90 - 90$

$= -270°$

the required plot is

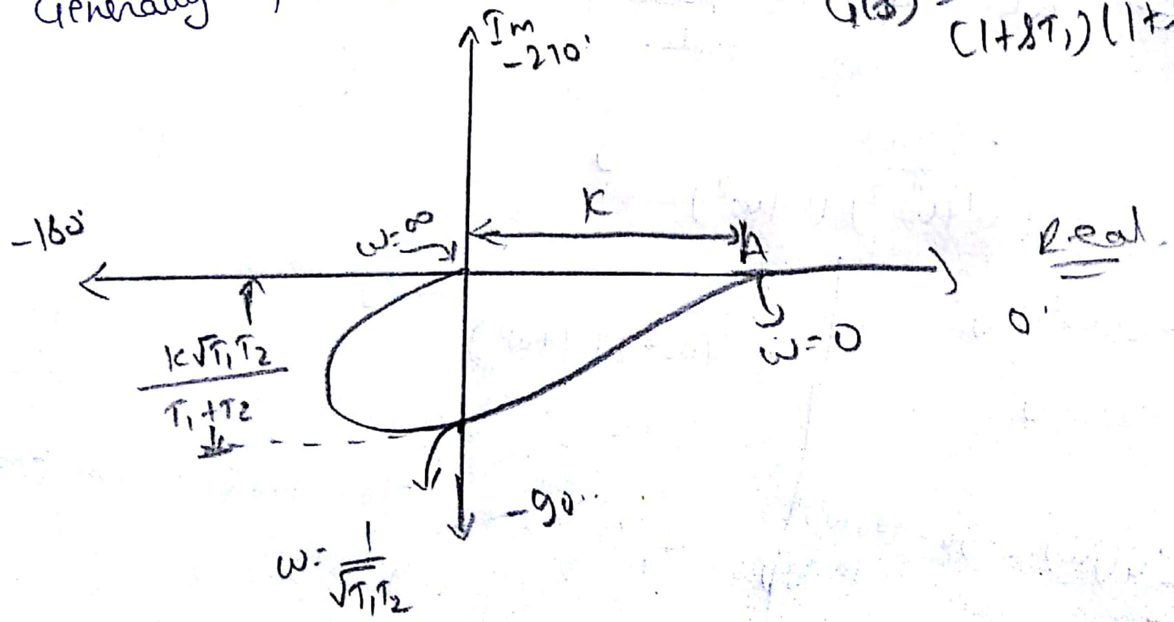
at $\omega = 0$
$G(j\omega) = \infty$
at and $G(j\omega) = 0$
$\omega = \infty$



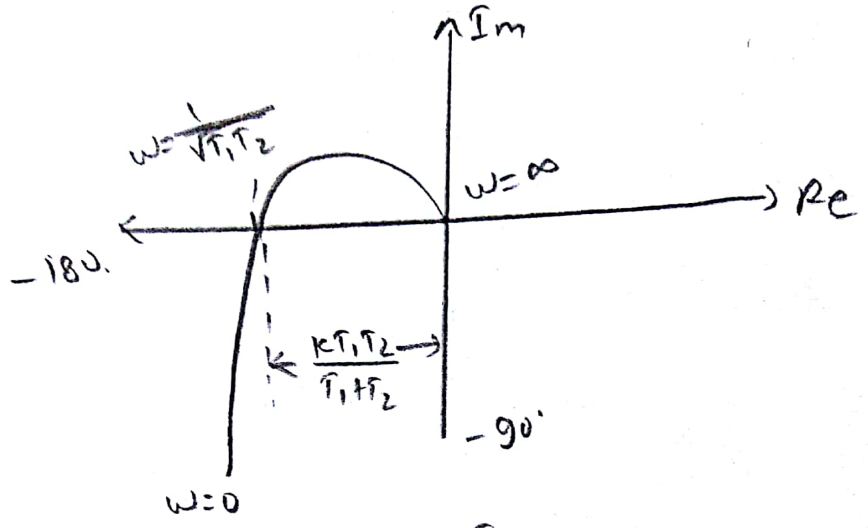
Imp. polar plot of diff. types:-

Generally for type 0 plot is kind of when

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$



for type 1 $G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$



for type 2 $G(s) = \frac{K}{s^2(1+sT_1)}$