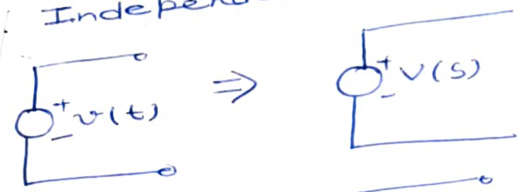
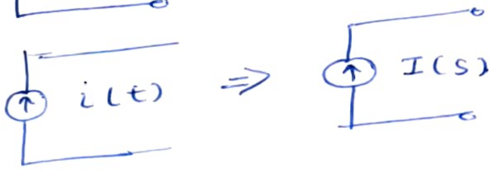
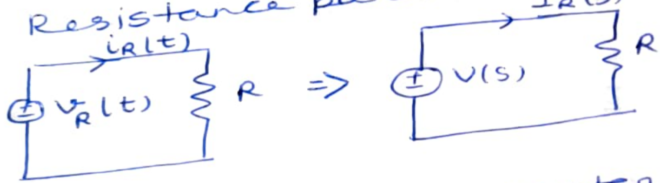
Transformed ckt. component Representation →

Independent Sources →





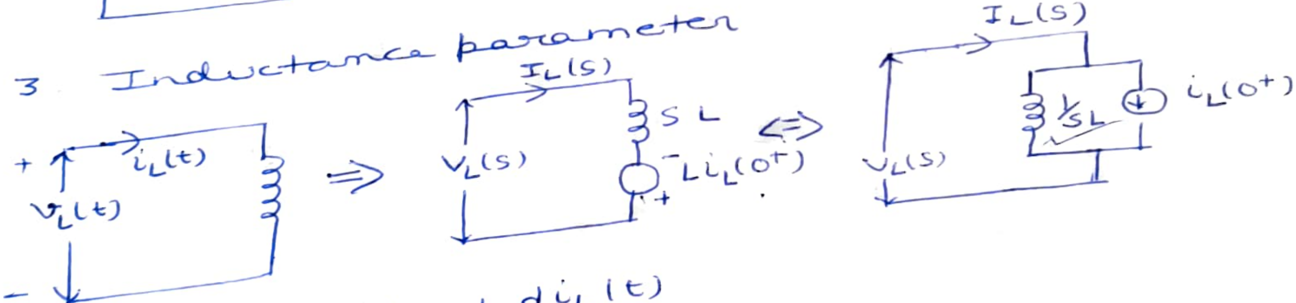
2. Resistance parameter



$= v_R(t) = R\, i_R(t)$

$\Rightarrow V(S) = R\, I(S)$

3. Inductance parameter



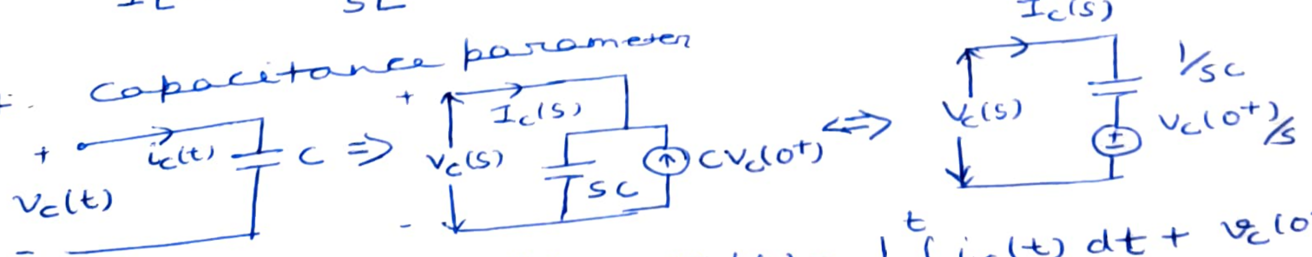
$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + i_L(0^+)$$

$$V_L(S) = LS\, I(S) - L i_L(0^+)$$

$$I_L(S) = \frac{1}{SL}V(S) + \frac{i_L(0^+)}{S}$$

4. Capacitance parameter



$$i_c(t) = \frac{C\,dv_c(t)}{dt} \qquad v_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt + v_c(0^+)$$

$$I_c(S) = SC\,V_c(S) - C v_c(0^+) \qquad V_c(S) = \frac{1}{CS}I_c(S) + \frac{v_c(0^+)}{S}$$

$$I(s) = \frac{k_1}{s+1} + \frac{k_2}{s+2+j2} + \frac{k_3}{s+2-j2}$$

$$k_1 = (s+1)\,I(s)\Big|_{s=-1} = \frac{s^2+5s+9}{s^2+4s+8} = \frac{5}{5} = 1$$

$$K_2 = (s+2+j2)\,I(s)\Big|_{s=-(2+j2)}$$

$$= \frac{s^2+5s+9}{(s+1)(s+2-j2)}\Bigg|_{s=-(2+j2)}$$

$$= \frac{4-4+8j-10-10j+9}{(-2-j2+1)(-2-j2+2-j2)} = \frac{\cancel{-1-2j}}{\cancel{(-1-j2)}(-j4)} = \frac{-}{j}$$

$$K_3 = K_2^{*} = \frac{1}{j4}$$

$$I(s) = \frac{1}{s+1} + \frac{-1/j4}{s+2+j2} + \frac{1/j4}{s+2-j2}$$

$$= e^{-t} - \frac{1}{j4}e^{-(2+j2)t} + \frac{1}{j4}e^{-(2-j2)t}$$

$$= e^{-t} - \frac{1}{j4}e^{-2t}\left[e^{-j2t} - e^{j2t}\right]$$

$$= e^{-t} + \frac{1}{2}e^{-2t}\sin 2t.$$

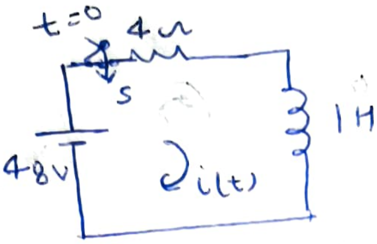
alternatively →

$$I(s) = \frac{s^2+5s+9}{(s+1)(s^2+4s+8)}$$

$$= \frac{1}{s+1} + \frac{1}{s^2+4s+8}$$

$$= \frac{1}{s+1} + \frac{1}{2}\left[\frac{2}{(s+2)^2+2^2}\right]$$

$$i(t) = e^{-t} + \frac{1}{2}e^{-2t}\sin 2t$$



Assume initial ct. through inductor is 3 A. Using L.T. determine ct. i(t).

By KVL →

$$48 = 4i(t) + 1\frac{di(t)}{dt}$$

Taking L.T.

$$4I(s) + 1[sI(s) - i_L(0^+)] = 48/s$$

$$4I(s) + sI(s) - 3 = 48/s$$

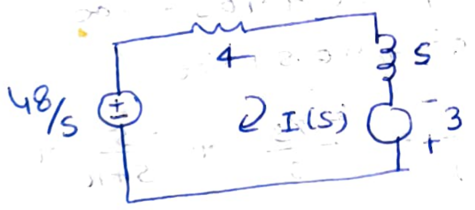
$$I(s) = \frac{3s+48}{s(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+4}$$

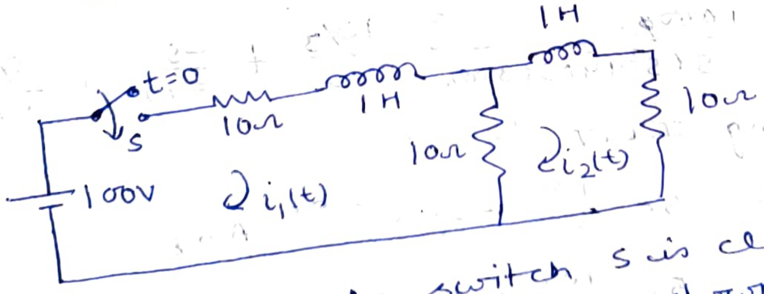
$$k_1 = sI(s)|_{s=0} = \frac{3s+48}{s+4}\Big|_{s=0} = 12$$

$$k_2 = (s+4)I(s)|_{s=-4} = \frac{3s+48}{s}\Big|_{s=-4} = -9$$

$$I(s) = \frac{12}{s} - \frac{9}{s+4}$$

$$i(t) = 12 - 9e^{-4t}$$



2) 

In the N/w, the switch S is closed at t = 0. with the N/w parameter value shown, find the expression for $i_1(t)$ & $i_2(t)$, if the N/w is unenergized before the switch S is closed.

Sol. apply KVL, loop eq^n are

$$\frac{di_1}{dt} + 10i_1(t) + 10[i_1(t) - i_2(t)] = 100$$

$$\frac{di_1(t)}{dt} + 20i_1(t) - 10i_2(t) = 100$$

and $\frac{di_2(t)}{dt} + 20i_2(t) - 10i_1(t) = 0$ —— (2)

L.T. of eq'n (1) & (2)

$$I_1(s)[s+20] - I_2(s)(10) = 100/s$$

and

$$-10 I_1(s) + I_2(s)[20+s] = 0$$

In Matrix form

$$\begin{bmatrix} s+20 & -10 \\ -10 & 20+s \end{bmatrix} = \begin{bmatrix} 100/s \\ 0 \end{bmatrix}$$

By cramer rule

$$I_1(s) = \frac{\Delta_1}{\Delta} \qquad I_2(s) = \frac{\Delta_2}{\Delta}$$

$$\Delta_1 = \begin{bmatrix} 100/s & -10 \\ 0 & s+20 \end{bmatrix} \qquad \Delta_2 = \begin{bmatrix} s+20 & 100/s \\ -10 & 0 \end{bmatrix}$$

$$= (s+20)\left(\frac{100}{s}\right) + 10 \times 0 \qquad \Delta_2 = 10 \times \frac{100}{s}$$

$$= \frac{(s+20)100}{s} \qquad = \frac{1000}{s}$$

$$\Delta = \begin{bmatrix} s+20 & -10 \\ -10 & 20+s \end{bmatrix} = s^2 + 400 + 40s - 100$$

$$= s^2 + 40s + 300$$

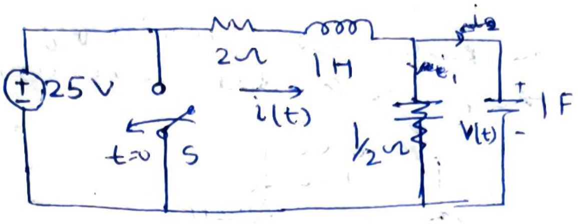
$$I_1(s) = \frac{(s+20)100}{s(s^2+40s+300)} = \frac{20/3}{s} + \frac{-5}{s+10} + \frac{-5/3}{s+30}$$

$$I_2(s) = \frac{1000}{s(s^2+40s+300)} = \frac{10/3}{s} + \frac{-5}{s+10} + \frac{5/3}{s+30}$$

On taking I.L.T.

$$i_1(t) = \frac{20}{3} - 5e^{-10t} - \frac{5}{3}e^{-30t} \text{ Amp}$$

$$i_2(t) = \frac{10}{3} - 5e^{-10t} + \frac{5}{3}e^{+30t} \text{ Amp}$$



S.S. is reached with S open. S is closed at $t=0$. Determine $i(t)$, $v(t)$ for $t>0$

Sol. at $t=0^-$ S.S. is reached i.e. L is S.C.
C is O.C.

$$i(0^-) = \frac{25}{2+1/2} = 10\,A$$

$$25 - 2\,i(0^-) - v(0^-) = 0$$

$$v(0^-) = 25 - 2\times 10 = 5\,V$$

now S is closed, Apply KVL

$$2i(t) + \frac{di(t)}{dt} + v(t) = 0$$

Taking L.T.

$$2I(s) + LsI(s) - i(0^+) + V(s) = 0$$

$$(2+s)I(s) - 10 + V(s) = 0$$

as $i(0^+) = i(0^-) = 10A$

$$(2+s)I(s) + V(s) = 10 \quad \text{—— (1)}$$

also

$$i(t) = \frac{v(t)}{1/2} + 1.\frac{dv(t)}{dt}$$

Taking L.T.

$$I(s) = 2V(s) + sV(s) - v(0^+)$$

$$I(s) = 2V(s) + sV(s) - 5 \quad \text{—— (2)}$$

$$I(s) = 2V(s) + sV(s) - I(s) = 0$$

$$5 = V(s)(2+s) - I(s) = 0$$

On solving (1) & (2)

$$\cancel{(2+s)I(s)} + V(s) = 10$$

$$-\cancel{(2+s)}I(s) + (2+s)^2V(s) = 5(2+s)$$

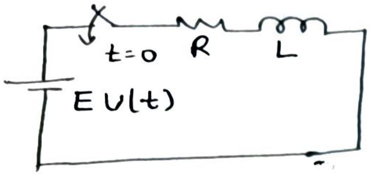
$$V(s)[1+(2+s)^2] = 10 + [10+5s]$$

$$V(s) = \frac{20+5s}{s^2+4s+5} = \frac{\cancel{100}+\cancel{50}s}{s^2\cancel{+5s-s}+5}$$

$$= \frac{20+5s}{(s+2)^2+(1)^2} = \frac{5(s+2)}{(s+2)^2+1} + \frac{10}{(s+2)^2+1}$$

$$v(t) = 5e^{-2t}\cos t + 10e^{-2t}\sin t\ V$$

Step Response of R-L Series circuit



at $t=0$ switch is closed

so $i(0^+) = i(0^-) = 0$

By KVL →

$$E\,u(t) = R\,i(t) + L\frac{di(t)}{dt} \quad \text{— (1)}$$

Input is unit step signal.

On taking Laplace transform of eqn (1), we have

$$\frac{E}{S} = R\,I(S) + L[S\,I(S) - i(0^+)]$$

$$i(0^+) = 0$$

$$\frac{E}{S} = R\,I(S) + LS\,I(S)$$

$$\Rightarrow \quad I(S) = \frac{E/L}{S(S+R/L)} = \frac{A}{S} + \frac{B}{S+R/L}$$

$$A = \left.\frac{E/L}{S+R/L}\right|_{S=0} = \frac{E}{R}$$

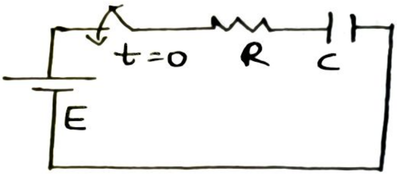
$$B = \left.\frac{E/L}{S}\right|_{S=-R/L} = -\frac{E}{R}$$

Thus $$I(S) = \frac{E/R}{S} - \frac{E/R}{S+R/L}$$

On taking Inverse laplace transform

$$i(t) = E/R - \frac{E}{R}e^{-R/L\,t}$$

Step Response of RC Series Circuit



at $t=0$ switch is closed.

By KVL →

$$E\,u(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt \quad — (1)$$

On taking L.T. of eqn (1)

$$\frac{E}{S} = R\,I(S) + \frac{1}{C}\left[\frac{I(S)}{S} + \frac{i(0^+)}{S}\right]$$

at $t=0$ capacitor act as S.C. so $i(0^+) = \frac{E}{R}$

$$\frac{E}{S} = R\,I(S) + \frac{1}{CS}I(S) + \frac{1}{CS}\cdot\frac{E}{R}$$

$$\Rightarrow \frac{E}{S} - \frac{1}{RCS}E = I(S)\left[\frac{RCS+1}{CS}\right]$$

$$\Rightarrow \frac{E}{S}\left[\frac{RC-1}{RC}\right] = I(S)\left[\frac{RCS+1}{CS}\right]$$

$$\Rightarrow I(S) = \frac{E}{R}\left(\frac{RC-1}{1+RCS}\right)$$

$$= \frac{E}{R}\cdot\frac{RC\,(1-1/RC)}{RC\,(S+1/RC)}$$

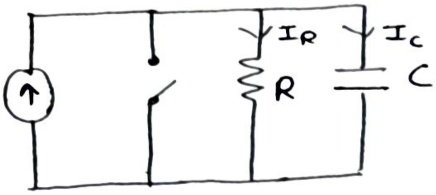
$$= \frac{E}{R}\left\{(1-\frac{1}{RC})/(S+1/RC)\right]$$

On taking I.L.T

$$= \frac{E}{R}(1-1/RC)\,e^{-t/RC}$$

-> Current Response of RC parallel Circuit ->

at $t = 0$ switch is opened



By KVL ->

$$I = I_R + I_C \quad \text{--- (1)}$$

also for parallel circuit

$$V_R = V_C$$

$$R I_R = \frac{1}{C}\int I_C \, dt$$

Multiply eqn (1) both side by R

$$R I = R I_R + R I_C$$

$$R I = \frac{1}{C}\int I_C \, dt + R I_C$$

On taking Laplace Transform

$$\frac{R I(s)}{s} = \frac{1}{Cs} I_C(s) + R I_C(s)$$

$$= I_C(s)\left[R + \frac{1}{Cs}\right]$$

$$I_C(s) = \frac{I(s) R}{\left(R + \frac{1}{Cs}\right) s}$$

$$= I(s)\left[\frac{1}{s + 1/RC}\right]$$

On taking I. L. T.

$$i_C(t) = i(t)\left[e^{-1/RC \cdot t}\right]$$

Step Current Response of a R-L parallel Cir

By KCL

$$I = I_R + I_L \quad \text{--- (1)}$$

for parallel Circuit

$$V_R = V_L$$

i.e. $$R I_R = L\frac{dI_L}{dt} \quad \text{--- (2)}$$

On multiplying eq'n (1) by R, we get

$$RI = RI_R + RI_L$$

$$RI = L\frac{dI_L}{dt} + RI_L$$

On taking L.T.

$$R.\frac{I(s)}{s} = (Ls + R)\, I_L(s)$$

as all initial conditions are zero

$$I_L(s) = I(s).\frac{R}{s(R+Ls)}$$

$$= I(s)\frac{R/L}{s(s+R/L)} = I(s)\left[\frac{A}{s} + \frac{B}{s+R/L}\right]$$

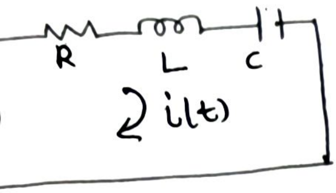
$$A = \left.\frac{R/L}{s+R/L}\right|_{s=0} = 1$$

$$B = \left.\frac{R/L}{s}\right|_{s=-R/L} = -1$$

$$I_L(s) = I(s)\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

$$i_L(t) = i(t)\left[1 - e^{-R/L\, t}\right]$$

Series Circuit with step Input →



Let all initial conditions are zero.

By KVL

$$V\,u(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

On taking laplace Transform

$$\frac{V}{S} = RI(S) + LS\,I(S) + \frac{1}{CS}I(S)$$

$$I(S) = \frac{V/L}{S^2 + \frac{R}{L}S + \frac{1}{LC}} \quad \text{—— (1)}$$

Roots of denominator polynomial of eqn (1) is

$$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

$$S_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$S_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$ → natural freq. of oscillation

$\xi\omega_0 = \frac{R}{2L}$ → damping factor, i.e. $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

$$S_1 = -\xi\omega_0 + \omega_0\sqrt{\xi^2-1}$$

$$S_2 = -\xi\omega_0 - \omega_0\sqrt{\xi^2-1}$$

$$I(S) = \frac{V/L}{(S-S_1)(S-S_2)} = \frac{A}{S-S_1} + \frac{B}{S-S_2}$$

$$A = \left.\frac{V/L}{S-S_2}\right|_{S=S_1} = \frac{V}{2\xi\omega_0 L\sqrt{\xi^2-1}}$$

$$B = \left.\frac{V/L}{S-S_1}\right|_{S=S_2} = \frac{-V}{2\omega_0 L\sqrt{\xi^2-1}}$$

$$I(s) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

On taking I.L.T. we get

$$i(t) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[e^{s_1 t} - e^{s_2 t}\right]$$

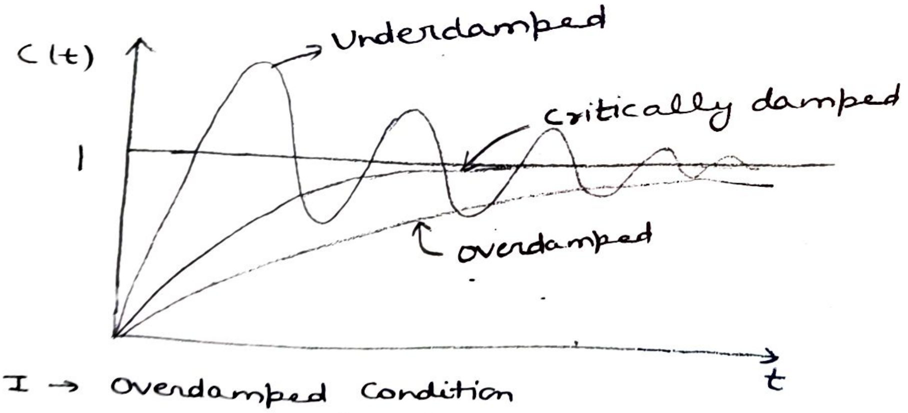
$$= \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\, e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

Depending upon the value of R, L, C three cases may appear

a) $\frac{R}{2L} > \frac{1}{\sqrt{LC}} \Rightarrow$ Overdamped condition

b) $\frac{R}{2L} < \frac{1}{\sqrt{LC}} \Rightarrow$ Underdamped Condition

c) $\frac{R}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow$ critically damped condition



Case I → Overdamped Condition

$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$ i.e $\xi > 1$

$$i(t) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\, e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

$$= \frac{V}{\omega_0 L\sqrt{\xi^2-1}}\, e^{-\xi\omega_0 t}\sinh(\omega_0\sqrt{\xi^2-1})\,t$$

Critically damped condition $(\xi = 1)$

$$I(s) = \frac{V/L}{s^2 + 2\omega_0 s + \omega_0^2}$$

$$= \frac{V}{L}\left(\frac{1}{(s+\omega_0)^2}\right)$$

On taking I. L. T.

$$i(t) = \frac{V}{L} t e^{-\omega_0 t}$$

III : Underdamped condition $(\xi > 1)$

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

$$i(t) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

$$= \frac{V}{\omega_0 L\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[\frac{e^{(j\omega_0\sqrt{1-\xi^2})t} - e^{-(j\omega_0\sqrt{\xi^2-1})t}}{2j}\right]$$

$$= \frac{V}{\omega_0 L\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\sin(\omega_0\sqrt{1-\xi^2})t$$

R-L Series Circuit with impulse Input

By KVL

$$V\delta(t) = Ri(t) + L\frac{di(t)}{dt}$$

On taking L.T.

$$V = RI(s) + LS\,I(s)$$

$$I(s) = \frac{V}{R+LS}$$

$$= \frac{V}{L}\left(\frac{1}{s+R/L}\right)$$

On taking I.L.T.

$$i(t) = V/L\, e^{-R/L\,t}$$

$$V_R = Ri(t) = \frac{VR}{L}e^{-R/L\,t}$$ (Vg. across Resistor)

$$V_L = L\frac{di(t)}{dt} = L\cdot\frac{d}{dt}\left(V/L\, e^{-R/L\,t}\right)$$

$$= L\cdot\frac{V}{L}\cdot\left(-R/L\right)e^{-R/L\,t}$$

$$V_L = -\frac{VR}{L}e^{-R/L\,t}$$ (Vg. across inductor)

R-C Series Circuit with impulse Input

By KVL →

$$V\delta(t) = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

$$V = RI(s) + \frac{1}{CS}I(s)$$

$$I(s) = \frac{V}{R+1/CS} = \frac{V}{R}\left(\frac{s}{s+1/RC}\right)$$

$$= \frac{V}{R}\left(1 - \frac{1/RC}{s+1/RC}\right)$$

taking I.L.T.

$$i(t) = \frac{V}{R}\left[\delta(t) - \frac{1}{RC} e^{-t/RC}\right]$$

$$V_R = R\,i(t) = V\left[\delta(t) - \frac{1}{RC} e^{-t/RC}\right]$$

$$V_C = \left[V\delta(t) - V_R\right] = \frac{V}{RC} e^{-t/RC}$$

$\frac{N_1(S)}{M_2(S)}$

$\frac{I_1(S)}{J_2(S)}$

R-L-C Series circuit with Impulse Input →

By KVL

$$V\delta(t) = R\,i(t) + L\frac{d}{dt}i(t) + \frac{1}{C}\int i(t)\,dt$$

On taking L.T.

$$V = R\,I(S) + LS\,I(S) + \frac{1}{CS} I(S)$$

$$I(S) = \frac{V/L\, S}{S^2 + R/L\, S + 1/LC}$$

Roots of denominator polynomial are

$$S^2 + \frac{R}{L}S + 1/LC = 0$$

$$S_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$S_2 = \frac{-R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \xi\omega_0 = \frac{R}{2L}$ $\quad$ i.e $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$ = damping Ratio

$$S_1 = -\xi\omega_0 + \omega_0\sqrt{\xi^2-1}$$

$$S_2 = -\xi\omega_0 - \omega_0\sqrt{\xi^2-1}$$

$$I(S) = \frac{(V/L)S}{(S-S_1)(S-S_2)} = \frac{A}{S-S_1} + \frac{B}{S-S_2}$$

$$A = \left.\frac{(V/L)S}{S-S_2}\right|_{S=S_1} = \frac{VS_1}{S_1-S_2} = \frac{VS_1}{2\omega_0 L\sqrt{\xi^2-1}}$$

$$B = \frac{\left(\frac{V}{L}\right) s}{(s-s_1)}\Bigg|_{s=s_2} = \frac{\frac{V}{L} s_2}{\cancel{2\omega_0} (s_2 - s_1)}$$

$$= \frac{V s_2}{2\omega_0 L\sqrt{\xi^2-1}}$$

$$I(s) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[\frac{s_1}{s-s_1} - \frac{s_2}{s-s_2}\right]$$

$$= \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[s_1 e^{s_1 t} - s_2 e^{s_2 t}\right]$$

$$= \frac{V}{2\omega_0 L\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[s_1 e^{(\omega_0\sqrt{\xi^2-1})t} - s_2 e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

$$= \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[e^{-\xi\omega_0 t}\left\{(-\xi\omega_0 + \omega_0\sqrt{\xi^2-1})e^{(\omega_0\sqrt{\xi^2-1})t} - (-\xi\omega_0 - \omega_0\sqrt{\xi^2-1})e^{-(\omega_0\sqrt{\xi^2-1})t}\right\}\right]$$

Case I : overdamped condition $(\xi > 1)$

$$i(t) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\left[e^{-\xi\omega_0 t}\left[(-\xi\omega_0 + \omega_0\sqrt{\xi^2-1}) e^{(\omega_0\sqrt{\xi^2-1})t} - (-\xi\omega_0 - \omega_0\sqrt{\xi^2-1}) e^{-(\omega_0\sqrt{\xi^2-1})t}\right]\right]$$

$$= \frac{V}{L\sqrt{\xi^2-1}}\left[\sqrt{\xi^2-1}\cosh(\omega_0\sqrt{\xi^2-1})t - \xi\sinh(\omega_0\sqrt{\xi^2-1})t\right]$$

Case II : Critically Damped Condition $(\xi = 1)$

$$I(s) = \frac{V/L\, s}{s^2 + 2\omega_0 s + \omega_0^2} = \frac{V}{L}\left(\frac{s}{(s+\omega_0)^2}\right)$$

$$= \frac{V}{L}\left[\frac{A}{(s+\omega_0)^2} + \frac{B}{(s+\omega_0)^2}\right]$$

$$\frac{s}{(s+\omega_0)^2}\Bigg|_{s=-\omega_0} = -\omega_0$$

$$\frac{d}{ds}\left[(s+\omega_0)^2 \cdot \frac{s}{(s+\omega_0)^2}\right]_{s=-\omega_0} = 1$$

$$I(s) = \frac{V}{L}\left[\frac{1}{s+\omega_0} - \frac{\omega_0}{(s+\omega_0)^2}\right]$$

On taking I.L.T.

$$i(t) = \frac{V}{L}(1-\omega_0 t)\, e^{-\omega_0 t}$$

Case III: Underdamped Condition $\xi < 1$

$$i(t) = \frac{V}{2\omega_0 L\sqrt{\xi^2-1}}\, e^{-\xi\omega_0 t}\Big[\Big(\omega_0\sqrt{\xi^2-1}\Big\{e^{(\omega_0\sqrt{\xi^2-1})t} + e^{-(\omega_0\sqrt{\xi^2-1})t}\Big\} - \xi\omega_0\Big\{e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\Big\}\Big]$$

$$= \frac{V}{2\omega_0 L j\sqrt{1-\xi^2}}\, e^{-\xi\omega_0 t}\Big[\Big(j\omega_0\sqrt{1-\xi^2}\Big\{e^{(j\omega_0(1-\xi^2)t)} + e^{-(j\omega_0\sqrt{1-\xi^2})t}\Big\} - \xi\omega_0\Big\{e^{(j\omega_0\sqrt{1-\xi^2})t} - e^{-(j\omega_0\sqrt{1-\xi^2})t}\Big\}\Big]$$

$$= \frac{V}{L\sqrt{1-\xi^2}}\, e^{-\xi\omega_0 t}\Big[\sqrt{1-\xi^2}\cos(\omega_0\sqrt{1-\xi^2})t - \xi\sin(\omega_0\sqrt{1-\xi^2})t$$

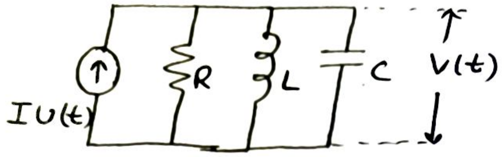
$$= \frac{V}{L\sqrt{1-\xi^2}}\, e^{-\xi\omega_0 t}\cos\Big\{(\omega_0\sqrt{1-\xi^2})t + \theta\Big]$$

where $\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

$\frac{N_1(s)}{M_2(s)}$

$\frac{I_1(s)}{V_2(s)}$

R-L-C parallel circuit with step input



By KCL

$$I\,u(t) = \frac{V(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int v$$

On taking Laplace Transform

$$\frac{V(s)}{R} + sC\,V(s) + \frac{1}{sL}V(s) = \frac{I}{s}$$

$$V(s) = \frac{I/C}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Roots of denominator equation are

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$ $\qquad \xi\omega_0 = \frac{1}{2RC}$

i.e. $\xi = \frac{1}{2R}\sqrt{L/C}$ = damping Ratio

$$s_1 = -\xi\omega_0 + \omega_0\sqrt{\xi^2 - 1}$$

$$s_2 = -\xi\omega_0 - \omega_0\sqrt{\xi^2 - 1}$$

$$V(s) = \frac{I/C}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

$$A = \left.\frac{I/C}{s-s_2}\right|_{s=s_1} = \frac{I/C}{s_1 - s_2} = \frac{I}{2\omega_0 C\sqrt{\xi^2 - 1}}$$

$$B = \left.\frac{I/C}{s-s_2}\right|_{s=s_1} = \frac{I/C}{s_2 - s_1} = \frac{-I}{2\omega_0 C\sqrt{\xi^2 - 1}}$$

$$= \frac{I}{2\omega_0 C\sqrt{\xi^2-1}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

aking I. L. T.

$$) = \frac{I}{2\omega_0 C\sqrt{\xi^2-1}}\left[e^{s_1 t} - e^{s_2 t}\right]$$

$$\frac{I}{2\omega_0 C\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

pending upon the value of R, L, C three

ases may appear

se I Overdamped condition $(\xi > 1)$

$$v(t) = \frac{I}{2\omega_0 C\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

$$= \frac{I}{\omega_0 C\sqrt{\xi^2-1}}\left(\sinh(\omega_0\sqrt{\xi^2-1})\,t\right)$$

ase II Criticall damped Condition $(\xi = 1)$

$$V(s) = \frac{I/c}{s^2 + 2\omega_0 s + \omega_0^2}$$

$$= \frac{I/c}{(s+\omega_0)^2}$$

$$v(t) = I/c \; t \, e^{-\omega_0 t}$$

Case III Underdamped condition $(\xi < 1)$

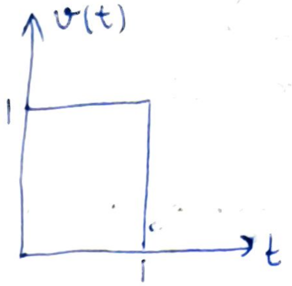
$$v(t) = \frac{I}{2\omega_0 C\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[e^{(\omega_0\sqrt{\xi^2-1})t} - e^{-(\omega_0\sqrt{\xi^2-1})t}\right]$$

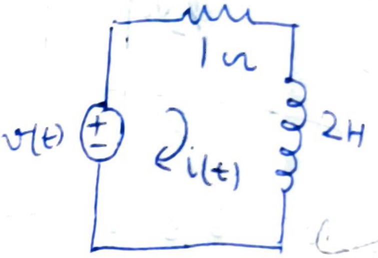
$$= \frac{I}{\omega_0 C\sqrt{1-\xi^2}} e^{-\xi\omega_0 t}\left[\frac{e^{(j\omega_0\sqrt{1-\xi^2})t} - e^{-(j\omega_0\sqrt{1-\xi^2})}}{2j}\right]$$

$$= \frac{I}{\omega_0 C\sqrt{1-\xi^2}} e^{-\xi\omega_0 t}\sin(\omega_0\sqrt{1-\xi^2})t$$

(a) Obtain the L.T of the pulse shown

(b) In fig (b) If $v(t)$ is the pulse (a) determ





$v(t) = G_{0,1}(t) = U(t) - U(t-1)$

$V(s) = 1/s - 1/s \, e^{-s} = \frac{1}{s}(1 - e^{-s})$

in fig (b) →

$v(t) = 1.i(t) + 2\frac{di(t)}{dt}$

$V(s) = (1 + 2s) I(s)$

$I(s) = \frac{V(s)}{1+2s}$

If $v(t)$ is $U(t)$ then

$I_1(s) = \frac{1}{s.2(1/2 + s)} = \frac{1}{s} - \frac{1}{s+1/2}$

$i_1(t) = \left(1 - e^{-1/2 t}\right) U(t)$

If $v(t)$ is $U(t-1)$ then

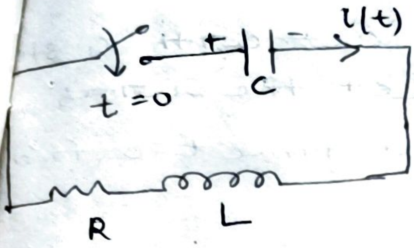
$I_2(s) = \frac{1}{s} e^{-s} \left(\frac{1}{2(s+1/2)}\right)$

$= e^{-s}\left[\frac{1}{s} - \frac{1}{s+1/2}\right]$

$i_2(t) = 1 - e^{-(t-1/2)} \, U(t-1)$

$i(t) = i_1(t) - i_2(t)$

(a)



$L = 2H$, $R = 12\Omega$, $C = 62.5\,mF$.
Initial condition are $v_c(0^+) = 100V$
$i_L(0^+) = 1A$. The switch is closed
at $t=0$. find $i(t)$

Sol. ~~Apply~~ KVL

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(t)dt + v_c(0^+) = 0$$

On taking L.T.

$$L[sI(s) - i_L(0^+)] + RI(s) + \frac{1}{Cs}I(s) + \frac{v_c(0^+)}{s} = 0$$

$$2[sI(s) - 1] + 12I(s) + \frac{1}{(62.5\times10^{-3})s}I(s) + \frac{100}{s} = 0$$

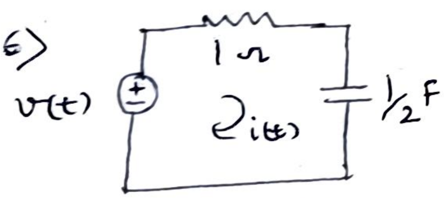
$$I(s)\left[2s + 12 + \frac{16}{s}\right] = -\frac{100}{s} + 2$$

$$I(s) = \frac{-100+2s}{2s^2+12+16} = \frac{-50+s}{s^2+6s+8} = \frac{s-50}{(s+4)(s+2)}$$

$$I(s) = \frac{27}{s+4} - \frac{26}{s+2}$$

$$i(t) = 27e^{-4t} - 26e^{-2t}$$

6)



If $v(t) = 2e^{-.5t}u(t)$, $v_c(0^+) = 0$
find $i(t)$

Sol. $$2e^{-.5t}u(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

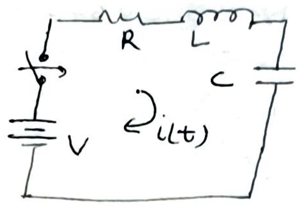
$$2e^{-.5t}u(t) = i(t) + 2\int_0^t i(t)dt + v_c(0^+)$$

$$\frac{2}{s+.5} = I(s) + \frac{2}{s}I(s) + 0$$

$$I(s) = \frac{2s}{(s+2)(s+.5)} = \frac{8/3}{s+2} - \frac{2/3}{s+.5}$$

$$i(t) = \frac{8}{3}e^{-2t} - \frac{2}{3}e^{-.5t}$$

$$i(t) = \frac{2}{3}(4e^{-2t} - e^{-.5t})u(t)\,A$$

7) 

Series RLC ckt with Input Vg. Let the Sw be closed at time t=0

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + v_c(0^+) = VU(t)$$

$$RI(s) + L[sI(s) - i_L(0^+)] + \frac{1}{Cs}I(s) + \frac{v_c(0^+)}{s} = V/s$$

at $t=0$ there is no stored energy in the ckt hence $i_L(0+)=0$ $v_c(0+)=0$. eq'n (2) reduces to

$$\left(R + sL + \frac{1}{Cs}\right)I(s) = V/s$$

$$I(s) = \frac{V/L}{s^2 + R/L\,s + 1/LC}$$

The roots of denominator on R.H.S.

$$s_2, s_1 = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$

$\xi\omega_0 = \frac{R}{2L}$

$s_1 = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$

$s_2 = -\xi\omega_0 - \omega_0\sqrt{\xi^2 - 1}$

$$I(s) = \frac{V/L}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

$A = \frac{V/L}{s_1 - s_2}$

A is found by multiplying throug $s-s_1$ and putting $s = s_1$

$B = \frac{V/L}{s_2 - s_1}$

$$A = \frac{V/L}{2\omega_0\sqrt{\xi^2 - 1}} = -B$$

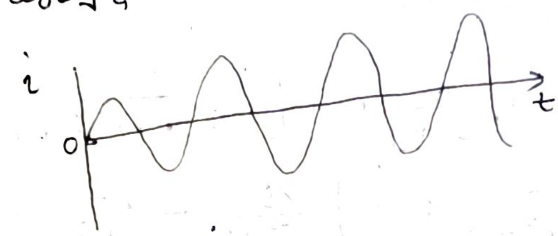
$$I(s) = \frac{V}{2L\omega_0\sqrt{\xi^2-1}}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

$$i(t) = \frac{V}{2L\omega_0\sqrt{\xi^2-1}} e^{-\xi\omega_0 t}\left[e^{\omega_0\sqrt{\xi^2-1}\,t} - e^{-\omega_0\sqrt{\xi^2-1}\,t}\right]$$

There may be 3 cases depending on value of ξ

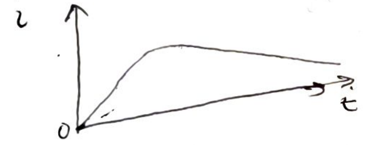
Case 1 when $\xi > 1 \Rightarrow \frac{R^2}{4L^2} > 1/LC \Rightarrow$ Overdamped case

$$i(t) = \frac{V}{\omega_0 L\sqrt{\xi^2-1}} e^{-\xi\omega_0 t} \sinh \omega_0 \sqrt{\xi^2-1}\, t$$



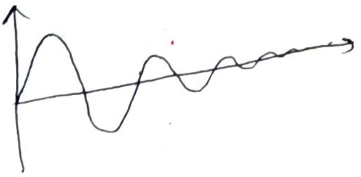
Case 2 when $\xi = 1 \Rightarrow \frac{R^2}{4L^2} = 1/LC$ = critically damped

$$i(t) = \frac{V}{L} t e^{-\omega_0 t}$$



Case 3 when $\xi \not> 1 \Rightarrow \frac{R^2}{4L^2} < \frac{1}{LC}$, underdamped case

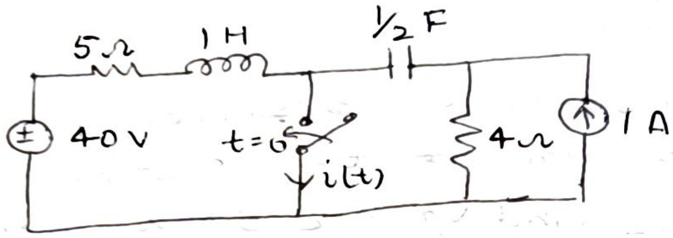
$$i(t) = \frac{V}{\omega_0 L\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin \omega_0\sqrt{1-\xi^2}\, t$$



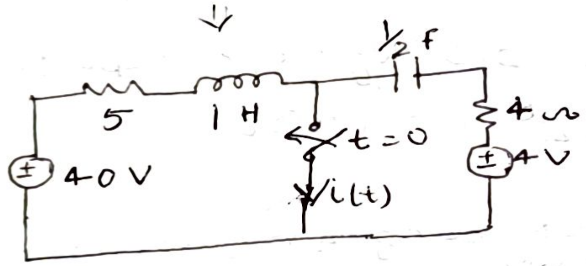
ansfer functions →

$$H(S) = \frac{Y(S)}{X(S)} \Bigg|_{\text{all initial conditions are zero.}}$$

9)



⇓



at S.S. inductor behave as a S.C. and Capacitor as a O.C. So $i_L(0^-) = 0$ $v_c(0^-) = 40 - 4 = 36$

$i_1(t) \rightarrow$ ct. due to 40 V source

$i_2(t) \rightarrow$ " " " 4 V source

$$i(t) = i_1(t) + i_2(t)$$

apply KVL

$$40 = 5i_1(t) + \frac{di_1(t)}{dt}$$

$$\frac{40}{s} = (5 + s)\, I_1(s)$$

$$I_1(s) = \frac{40}{s(s+5)} = 8\left[\frac{1}{s} - \frac{1}{s+5}\right]$$

$$i_1(t) = 8(1 - e^{-5t})$$

$$4 = 4i_2(t) + 2\int_0^t i_2(t)\, dt - 36$$

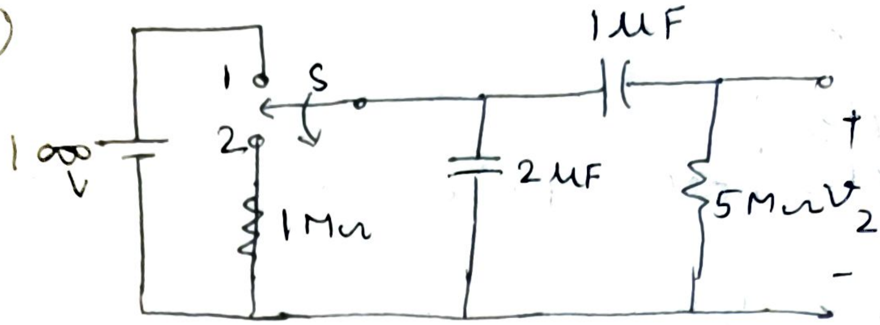
$$\frac{40}{s} = \left(4 + \frac{2}{s}\right) I_2(s)$$

$$I_2(s) = \frac{40}{4s+2} = \frac{10}{s+.5}$$

$$i_2(t) = 10e^{-.5t}$$

$$i(t) = 8(1 - e^{-5t}) + 10e^{-.5t}$$

(13)

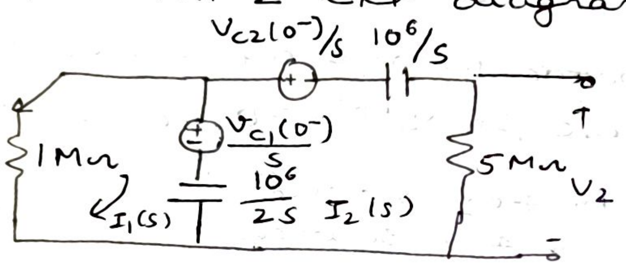


When switch s is in position 1, ckt attain equilibrium, at t=0 switch is moved to position 2. Find Vg a/c 5 MΩ resistor

Sol. at position 1 s.s. Vg a/c capacitor

$$v_{c1}(0^-) = v_{c2}(0^-) = 1000V$$

at position 2 ckt diagram is as follow



$$10^6 I_1(s) + \frac{1000}{s} + \frac{10^6}{2s}[I_1(s) - I_2(s)] = 0 \quad \text{—(1)}$$

~~$I_1(s) = (10s+3)I_2(s)$~~

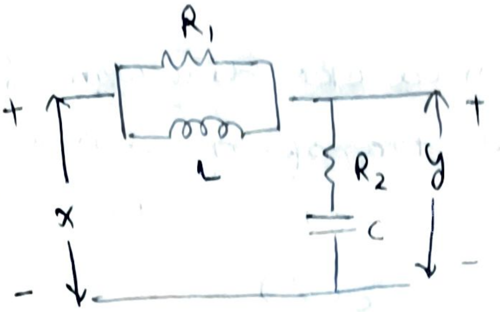
$$\frac{1000}{s} = \frac{1000}{s} + \frac{10^6}{s} I_2(s) + 5\times10^6 I_2(s) + \frac{10^6}{2s}[I_2(s) - I_1(s)]$$

from (1) & (2)

$$I_2(s) = -\frac{1000}{10^6[10s^2+8s+1]}$$

$$v_2(s) = 5\times10^6 I_2(s).$$

$$v_2(t) = 1020(e^{-.645t} - e^{-.155t})V.$$



Obtain T.F

Draw ~~the block~~ diagram ~~representation~~ of fig.

$x \rightarrow$ I/P Variable

$y \rightarrow$ O/P Variable

$$X(s) - Y(s) = I(s)\,[R_1 \| Ls]$$

$$X(s) - Y(s) = I(s) \cdot \frac{R_1 s}{s + R_1/L}$$

$$Y(s) = I(s)\left[R_2 + 1/Cs\right]$$

$$= I(s) \cdot \frac{s + 1/R_2 C}{1/R_2\, s}$$

$$H(s) = \frac{Y(s)}{X(s)}$$

11. $$\frac{C(s)}{R(s)} = \frac{(s+1)(s+3)}{(s+2)(s^2+8s+32)}$$

Determine time response c(t) for unit step I/P

Sol. $R(s) = 1/s$

$$C(s) = \frac{(s+1)(s+3)}{s(s+2)(s^2+8s+32)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{Cs + D}{s^2+8s+32}$$

$A = 3/64 \quad B = 1/40 \quad C = \frac{-23}{230} \quad D = \frac{19}{40}$

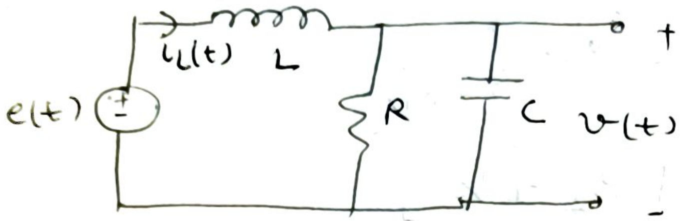
$$C(s) = \frac{3}{64}\cdot\frac{1}{s} + \frac{1}{40(s+20)} - \frac{\frac{23}{320}s + \frac{19}{40}}{(s+4)^2 + 4^2}$$

$$\frac{3}{64} + \frac{1}{40}e^{-2t} - \frac{23}{320}e^{-4t}\cos 4t + \frac{61}{320}e^{-4t}\sin 4t$$

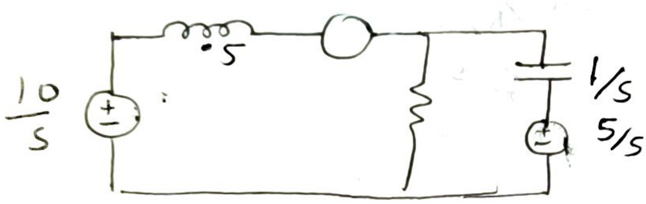
12> Draw the transformed ckt. diagram and obtain appropriate transformed no equation. Give $R = \frac{1}{3}\,\Omega$ $L = .5\,H$ $C = 1\,F$

$e(t) = 10V$, $i_L(0^-) = 15\,A$ $v_C(0^-) = 5\,V$

Solve for $v(t)$

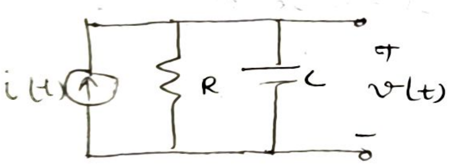


$\Downarrow$



13> If $R = 1\,\Omega$ $C = 1\,F$, $v_c(0^-) = 0V$

find response $v(t)$ when $i(t)$ is (a) Unit impulse fn (b) Unit step fn



$$V(s) = I(s)\left[1 \| \tfrac{1}{s}\right]$$

$$= \frac{I(s)}{s+1}$$

$i(t) = \delta(t)$

$I(s) = 1$

$$V(s) = \frac{1}{s+1} \qquad v(t) = e^{-t}u(t)$$

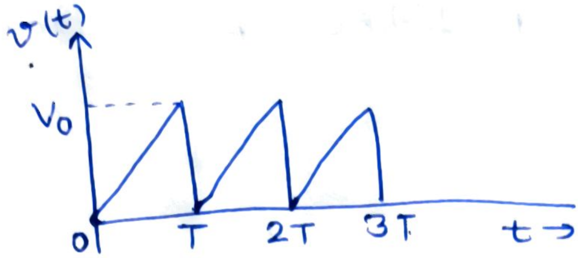
If $i(t) = u(t)$ i.e.

$I(s) = 1/s$

$$V(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

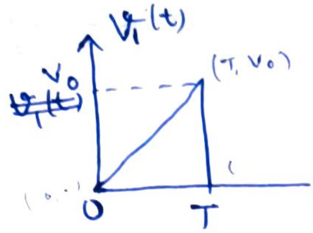
$$v(t) = (1 - e^{-t})\,u(t)$$

Practice Questions



find the Laplace Transform

Sol.



Let $v_1(t)$ is the first cycle of given w/f.

$$v_1(t) = \frac{V_0}{T} t \, G_{0,T}(t)$$

$$= \frac{V_0}{T} t \left[u(t) - u(t-T)\right]$$

$$= \frac{V_0}{T} t \, u(t) - \frac{V_0}{T}(t-T)\, u(t-T) - V_0 u(t-T)$$

$$\cancel{V_1(s) = \frac{V_0}{T} s^2 - \frac{V_0}{T} s^2}$$

$$V_1(s) = \frac{V_0}{T} \cdot \frac{1}{s^2} - \frac{V_0}{T} \cdot \frac{1}{s^2} e^{-Ts} - \frac{V_0 e^{-Ts}}{s}$$

$$= \frac{V_0}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2}\right] - \frac{V_0 e^{-Ts}}{s}$$

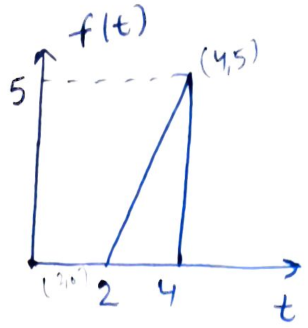
By using theorem for periodic function

$$V(s) = \frac{1}{1-e^{-Ts}} V_1(s) =$$

$$= \frac{1}{1-e^{-Ts}} \left[\frac{V_0}{Ts^2}\left[1-e^{-Ts}\right] - \frac{V_0}{s} e^{-Ts}\right]$$

$$= \frac{V_0}{Ts^2} - \frac{V_0 e^{-Ts}}{s(1-e^{-Ts})}$$

2)



find the Laplace tr

Sol. $f(t) = \frac{5-0}{4-2}(t-2)\,[u(t-2) - u(t-4)]$

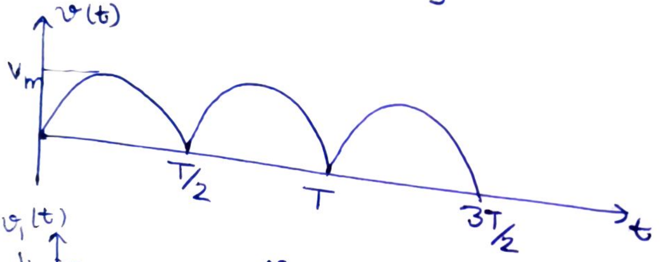
$= \frac{5}{2}(t-2)\,[u(t-2) - u(t-4)]$

$= \frac{5}{2}(t-2)\,u(t-2) - \frac{5}{2}(t-4+2)\,[u(t-4)]$

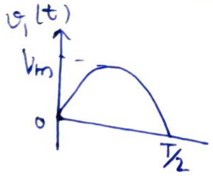
$= \frac{5}{2}\,r(t-2) - \frac{5}{2}(t-4)\,u(t-4) - 5\,u(t-4)$

$= \frac{5}{2s^2}e^{-2s} - \frac{5}{2s^2}e^{-4s} - \frac{5e^{-4s}}{s}$

3)



Sol.



$v_1(t) = V_m \sin\omega t\,[u(t) - u(t-T/2)]$

$= V_m\,[\sin\omega t\,u(t) - \sin\omega t\,u(t-T/2)]$

$= V_m\,[\sin\omega t \cdot u(t) - \sin\frac{2\pi}{T}(t - T/2 + T/2)\,u(t-T/2)]$

$= V_m\,[\sin\omega t\,u(t) - \sin\{\frac{2\pi}{T}(t-T/2) + \pi\}\,u(t-T/2)]$

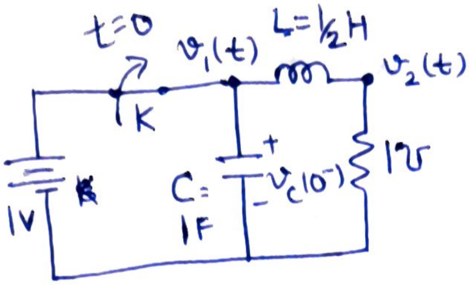
$= V_m\,[\sin\omega t\,u(t) + \sin\{\frac{2\pi}{T}(t-T/2)\}\,u(t-T/2)]$

$= V_m\,[\sin\omega t\,u(t) + \sin\omega(t-T/2)\,u(t-T/2)]$

$V_1(s) = V_m\left[\frac{\omega}{s^2+\omega^2} + e^{-T/2\,s}\,\frac{\omega}{s^2+\omega^2}\right]$

By using theorem for periodic function

$V(s) = \frac{1}{1-e^{-T/2\,s}} \cdot \frac{V_m\,\omega}{s^2+\omega^2}\left(1 + e^{-T/2\,s}\right)$

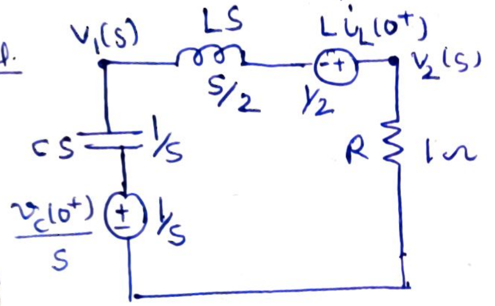


Switch K is opened at $t=0$. The circuit was in steady state before the switch is opened

find the node voltage $v_1(t)$ and $v_2(t)$. Also draw the transformed circuit.

Initial conditions are $v_C(0^-) = 1V$ $\quad i_L(0^-) = 1A$

Sol.



By KVL

$$\frac{1}{S} + \frac{1}{2} = I(S)\left[\frac{1}{S} + \frac{S}{2} + 1\right]$$

$$I(S) = \frac{S+2}{S^2+2S+2}$$

$$V_1(S) = \frac{1}{S} - \frac{1}{S}I(S)$$

$$= \frac{1}{S}\left[1 - \frac{S+2}{S^2+2S+2}\right] = \frac{S^2+S}{S(S^2+2S+2)}$$

$$\Rightarrow \frac{S+1}{S^2+2S+2} = \frac{S+1}{(S+1)^2+(1)^2}$$

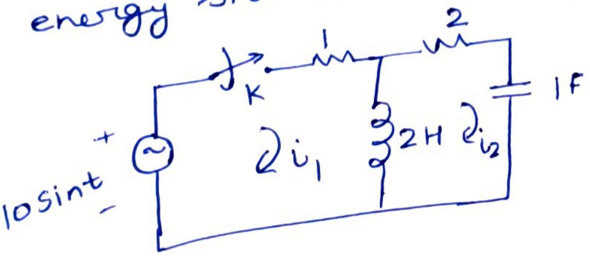
$$v_1(t) = e^{-t}\cos t$$

$$V_2(S) = 1 \cdot I(S) = \frac{S+2}{S^2+2S+2} = \frac{(S+1)+1}{(S+1)^2+(1)^2}$$

$$= e^{-t}\cos t + e^{-t}\sin t$$

$$= e^{-t}[\cos t + \sin t]$$

eg. Determine the steady state mesh current i_1 and i_2 in the given fig. There is no initial energy stored in the circuit.



Sol. Switch k is closed at $t = 0$

By KVL

$$10 \sin t = i_1(t) + 2\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right]$$

$$0 = 2 i_2(t) + \frac{1}{1}\int_{-\infty}^{t} i_2(t)\,dt + 2\left[\frac{di_2}{dt} - \frac{di_1}{dt}\right]$$

as there is no initial energy stored in the

By L. T.

$$\frac{10}{s^2+1} = I_1(s) + 2s\,I_1(s) - 2s\,I_2(s)$$

$$= I_1(s)\,[1 + 2s] - 2s\,I_2(s) \quad \text{--- (1)}$$

$$0 = 2 I_2(s) + \frac{I_2(s)}{s} + 2s\,I_2(s) - 2s\,I_1(s)$$

$$= -2s\,I_1(s) + I_2(s)\left[2 + \frac{1}{s} + 2s\right] \quad \text{--- (2)}$$

On solving (1) & (2) we have

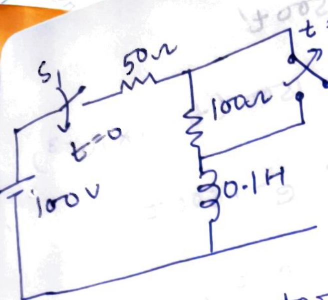
$$I_1(s) = \frac{10\,(s^2 + 2s + 1)}{(s^2+1)\,(6s^2 + 4s + 1)}$$

$$I_2(s) = \frac{20 s^2}{(s^2+1)\,(6s^2 + 4s + 1)}$$

S.S. value of i_1 and i_2 is given as

$$i_1(\infty) = \lim_{s \to 0} s I_1(s) = 0$$

$$i_2(\infty) = \lim_{s \to 0} s I_2(s) = 0$$



In the given fig. switch S_1 is closed at $t=0$ and S_2 is opened at $t=4$ msec. Determine $i(t)$ for $t>0$

(Assume inductor is initially de-energised)

Sol. for $0 \leq t \leq 4$ msec ; (S_1 is closed & S_2 is closed)

By KVL

$$100 = 50\,i(t) + 0.1\frac{di(t)}{dt}$$

by L.T.

$$\frac{100}{s} = 50\,I(s) + 0.1\, s\,I(s)$$

$$i(0+) = 0$$

$$I(s) = \frac{100}{s(50+0.1s)} = \frac{1000}{s(s+500)}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}\left[\frac{2}{s} - \frac{2}{s+500}\right]$$

$$= 2(1-e^{-500t})\,A$$

for $t = 4\times10^{-3}$ sec

$$i(4\times10^{-3}) = 1.729\,A$$

for $4\,msec \leq t < \infty$ If $t' = t - 4\times10^{-3}$

then $0 \leq t' < \infty$

By KVL →

$$100 = 150\,i(t) + 0.1\frac{di(t')}{dt}$$

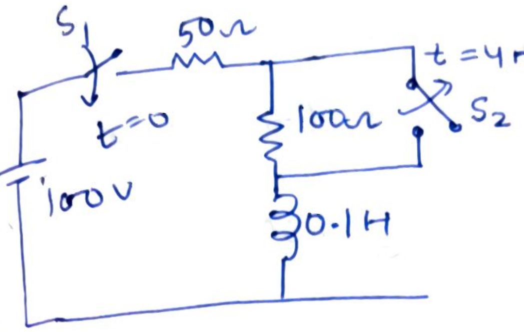
By L.T.

$$\frac{100}{s} = 150\,I(s) + 0.1\left[s\,I(s) - i(4\times10^{-3})\right]$$

$$\Rightarrow \frac{100}{s} = (150+0.1s)\,I(s) - .1\times1.729$$

$$I(s) = \frac{100 + .1729s}{s(0.1s+150)} = \frac{1.729s+1000}{s(s+1500)}$$

$$I(s) = \frac{.667}{s} + \frac{1.062}{\ldots}$$



In the given fig. switch S_1 is closed at $t = 0$ and S_2 is opened at $t = 4$ msec. Determine $i(t)$ for $t > 0$

(Assume inductor is initially de-energised)

Sol. for $0 \leq t \leq 4$ msec. ; (S_1 is closed & S_2 is closed)

By KVL

$$100 = 50\, i(t) + 0.1 \frac{di(t)}{dt}$$

by L.T.

$$\frac{100}{s} = 50\, I(s) + 0.1\, s\, I(s)$$

$$i(0^+) = 0$$

$$I(s) = \frac{100}{s(50 + 0.1 s)} = \frac{1000}{s(s+500)}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}\left[\frac{2}{s} - \frac{2}{s+500}\right]$$

$$= 2(1 - e^{-500t})\ A$$

for $t = 4 \times 10^{-3}$ sec

$$i(4 \times 10^{-3}) = 1.729\ A$$

for $4\ \text{msec} \leq t < \infty$ If $t' = t - 4 \times 10^{-3}$

then $0 \leq t' < \infty$

By KVL →

$$100 = 150\, i(t) + 0.1 \frac{di(t')}{dt}$$

By L.T.

$$\frac{100}{s} = 150\, I(s) + 0.1\,[s\, I(s) - i(4 \times 10^{-3})]$$

$$\Rightarrow \frac{100}{s} = (150 + 0.1 s)\, I(s) - .1 \times 1.729$$

$$I(s) = \frac{100 + .1729 s}{s(0.1 s + 150)} = \frac{1.729 s + 1000}{s(s + 1500)}$$

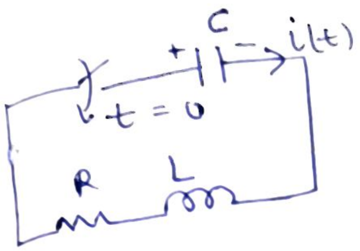
$$I(s) = \frac{.667}{s} + \frac{1.062}{s + 1500}$$

$$i(t') = .667 + 1.062\, e^{-1500t'}$$

$$i(t) = .667 + 1.062\, e^{-1500(t - 4\times10^{-3})}$$

$$= .667 + 1.062 \cdot e^{6}\, e^{-1500t}$$

$$= .667 + 428.4\, e^{-1500t} \text{ A}$$



$L = 2H \quad R = 12\,\Omega \quad C = 62.5\ mF$

Initial conditions are

$v_C(0^+) = 100V \quad i_L(0^+) = 1A$

Switch is closed at $t=0$. find $i(t)$

Sol. By KVL

$$Ri(t) + L\frac{di}{dt}(t) + \frac{1}{C}\int i(t)\,dt + v_C(0^+) = 0$$

On taking L.T.

$$RI(S) + L[SI(s) - i_L(0^+)] + \frac{1}{CS}I(S) + \frac{v_C(0^+)}{S} = 0$$

$$12\,I(S) + 2\,S\,I(S) - 2\times 1 + \frac{1}{62.5\times 10^{-3}\,S}I(S) + \frac{100}{S} = 0$$

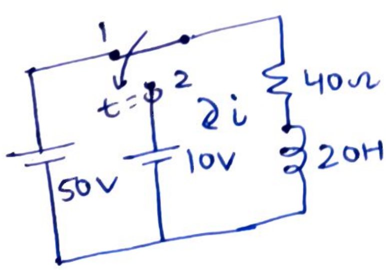
$$I(S)\left[2S + 12 + \frac{16}{S}\right] = \frac{-100}{S} + 2$$

$$I(S) = \frac{-100 + 2S}{2S^2 + 12S + 16}$$

$$= \frac{S - 50}{S^2 + 6S + 8} = \frac{S-50}{(S+4)(S+2)}$$

$$I(S) = \frac{27}{S+4} - \frac{26}{S+2}$$

$$i(t) = 27e^{-4t} - 26e^{-2t}$$

3) 

The switch has been in position 1 for a long time it is moved to 2 at $t=0$. obtain expression for i for $t>0$ using L.T.

Sol. when switch was on 1

$$50 = 40\,i'(t) + 20\frac{di'}{dt}$$

$$\frac{50}{S} = 40\,I'(S) + 20\,[S\,I'(S) - i(0^+)]$$

$$i(0^+) = 0$$

$$I'(s) = \frac{50}{s(40+20s)}$$

$$= \frac{2.5}{s(s+2)}$$

$$i'(t) = 1.25\,(1-e^{-2t})$$

as $t \to \infty$

$$i'(\infty) = 1.25$$

when switch is on 2

$$10 = 40\,i(t) + 20\frac{di}{dt}$$

$$\frac{10}{s} = 40\,I(s) + 20\,[s\,I(s) - i(0^+)]$$

$$\frac{10}{s} = (40+20s)\,I(s) - 20 \times 1.25$$

$$I(s) = \frac{10+25s}{s(40+20s)}$$

$$= \frac{1.25\,(s+0.4)}{s(s+2)}$$

$$i(t) = (.25 + e^{-2t})\,A$$