

unit 1 Quantum mechanics

unit 2 Quantum statistics

unit 3 Crystal Structure

unit 4 Band theory of Solids

wave - particle Duality - Interference, diffraction and polarisation can be explained by

the wave - theory of light but wave theory of light couldn't explain the photoelectric effect, Compton effect etc.

Due to this physicists assumed the particle nature of light. All the above phenomenon can be explained using the quantum hypothesis.

Acc. to quantum hypothesis, electro-magnetic radiation of light propagates in the form of wave packets or bundles. These packets or bundles are known as photons. It means electro-magnetic radiation or light behaves as a particle as well as wave. This characteristics of electro-magnetic radiation is known as wave - particle Duality.

De-Broglie - wavelength.

De-Broglie derived an eqn between the properties of particle and wave. Acc. to electro-magnetic theory

→ The energy of electro-magnetic radiation is given by

$$E = h\nu$$

$$P = \frac{E}{C}$$

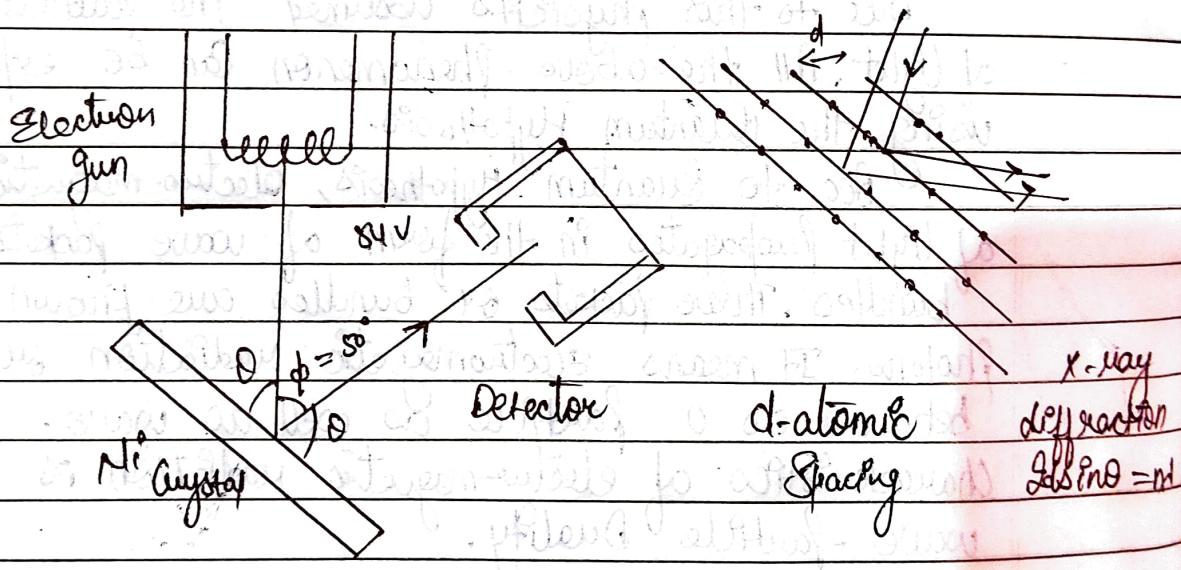
$$P = \frac{h\nu}{C}$$

$$P = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{P}$$

De-Broglie hypothesis: De Broglie suggested that the light is dual nature can be extended to the dual nature of material particles ($e/p/n$). It means when a material particle moves then it behaves as a particle as well as wave. The wave associated with material particle is known as matter wave or de-Broglie wave.

Dawson-German Experiment



Acc. to de Broglie Hypothesis, every material particle ($e/p/n$) in motion behaves as a particle as well as wave.

Dawson-German experiment is used to prove the above De-Broglie hypothesis.

when a charged particle is accelerated $\rightarrow eV = \frac{1}{2}mv^2$
we know that $\lambda = \frac{h}{mv}$

$$v = \sqrt{\frac{2eV}{m}}$$

80,

$$\lambda = h = \frac{h}{mv} = \frac{h}{\sqrt{2eUm}}$$

for electron = 6.6×10^{-84}

$$\sqrt{2} \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-34} \times 1.34$$

$$d(\text{electron}) = 1.67 \text{ \AA}$$

In - X-ray diffraction $\rightarrow d(\text{furan}) = 0.91 \text{ \AA}$

$$\frac{\lambda}{(\text{electron})} = \frac{2ds \sin \theta}{n} = \frac{2 \times 0.91 \times \sin(65)}{1} = 1.65 \text{ \AA}$$

Definition of wave function: In water wave height values periodically and in

em wave electric & magnetic field vary periodically.

Similarly, the quantity which may vary in the matter wave is known as wave function.

wave function is represented by ' ψ '

$\psi(x, t)$ \rightarrow one dimension

$\psi(x, y, z, t)$ \rightarrow three dimension

Significance of wave function and its properties:

y

+

-

$$(\psi)^2 = \psi^* \psi$$

defined probability of finding a particle

$\downarrow \psi$

$\downarrow \psi$

$-\psi$

$$\int_{-2}^{+2} |\psi|^2 dx = 1$$

\therefore means Prob. of finding a particle b/w -2 to 2 is 100%

Physical Significance: ' ψ ' have any physical significance but $|\psi|^2 = \psi^* \psi$ has its physical significance. It represents the probability of finding a particle at a given place and at a given time.

Imp. Note: 1. ψ - "Single valued", finite value, continuous.

2. $\frac{d\psi}{dx}, \frac{d\psi}{dy}, \frac{d\psi}{dz}$ - Single valued, finite value, continuous.

properties of wave function ψ are, probability

1. ' ψ ' must be Single valued, finite & Continuous.

2. $\frac{d\psi}{dx}, \frac{d\psi}{dy}, \frac{d\psi}{dz}$ must be Single valued, finite & Continuous.

3. ' ψ ' must be normalisable

Normalisation Cond'n: $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

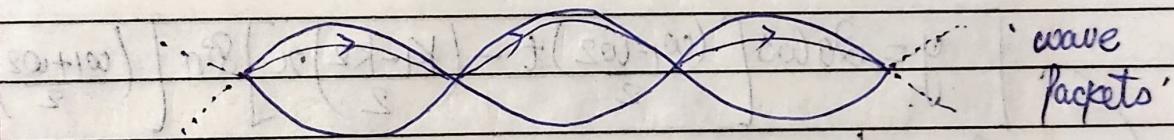
Phase Velocity / wave velocity $'v_p' = \omega + K$ ω - angular freq.

when a monochromatic wave travels in a medium, then its velocity of advancement in the medium is known as phase velocity or wave velocity.

Represented by v_p' and $v_p = \frac{\omega}{K}$

wave packet & group velocity: $v_g = \frac{d\omega}{dK}$

when plane waves having slightly different wavelengths travels in a straight line simultaneously in a dispersive medium then the groups of the waves are formed. These groups of waves are known as wave packets. The velocity with which the wave particle travels in a medium is known as group velocity ' v_g '.



Note: Every single particle moves as 'bunch of waves' / not a single wave.

Expression for group velocity ' $v_g = d\omega/dK$

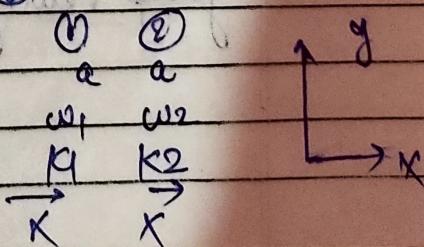
Let us consider two plane waves having same amplitudes, slightly different frequencies ' ω_1 ' & ' ω_2 ' & different propagation constants ' k_1 ' & ' k_2 ' respectively.

The displacement due to wave ① is given by

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

The displacement due to wave ② is given by

$$y_2 = a \sin(\omega_2 t - k_2 x)$$



Acc to Superposition principle, the resultant displacement due to wave ① & ② is, $y = (y_1 + y_2)$

$$y = a \sin(\omega_1 t - k_1 x) + a \sin(\omega_2 t - k_2 x)$$

$$y = a [\sin(\omega_1 t - k_1 x)] t [\sin(\omega_2 t - k_2 x)]$$

$$\therefore 8\pi c + 8\pi D = 2 \sin \left(\frac{c+d}{2} \right) \cos \left(\frac{c-d}{2} \right)$$

$$y = 2a \left[\sin \left(\frac{\omega_1 t - k_1 x + \omega_2 t - k_2 x}{2} \right) \right] \cos \left(\frac{\omega_1 t - k_1 x - \omega_2 t + k_2 x}{2} \right)$$

$$y = 2a \left[\sin \left(\frac{\omega_1 + \omega_2}{2} t - \frac{(k_1 + k_2)}{2} x \right) \right] \cos \left[\frac{(\omega_1 - \omega_2)}{2} t - \frac{(k_1 - k_2)}{2} x \right]$$

$$y = 2a \cos \left[\frac{(\omega_1 - \omega_2)}{2} t - \frac{(k_1 - k_2)}{2} x \right] \sin \left[\frac{(\omega_1 + \omega_2)}{2} t - \frac{(k_1 + k_2)}{2} x \right]$$

Amplitude

This above eqn represents the resultant wave having

→ Frequency : $\frac{\omega_1 + \omega_2}{2}$, Propagation Const. : $\frac{k_1 + k_2}{2}$

& amplitude : $2a \cos \left[\frac{(\omega_1 - \omega_2)}{2} t - \frac{(k_1 - k_2)}{2} x \right]$

group velocity : $v_g = \frac{(\omega_1 - \omega_2)/2}{(k_1 - k_2)/2} = \frac{d\omega}{dk}$

wave velocity: $V_p = \frac{\omega}{k}$

Relation between group Velocity (v_g) & phase Velocity (v_p)

we know the phase Velocity (v_p) = ω $\Rightarrow \omega = k v_p$ - ①

we know the group Velocity (v_g) $\left[\frac{d\omega}{dk} \right]_{\text{dis}}$

using eqn ① we get $v_g = \frac{d(k v_p)}{dk} = \frac{k d v_p}{dk} + v_p$ $\therefore k = \frac{2\pi}{\lambda}$

$$v_g = v_p + \frac{2\pi}{\lambda} \frac{d v_p}{d(\lambda)} = v_p + \frac{2\pi}{\lambda} \frac{d v_p}{1 \cdot \frac{d\lambda}{d(\lambda)}} = v_p + \frac{2\pi}{\lambda} \frac{d v_p}{d(\lambda)}$$

$$v_g = v_p + \frac{1}{\lambda} \frac{d v_p}{(-1/\lambda^2) d\lambda} = v_p - \frac{1}{\lambda} \frac{d v_p}{d\lambda}$$

$$v_g = v_p - \frac{1}{\lambda} \frac{d v_p}{d\lambda}$$

i) dispersive medium $\rightarrow v_p$ depends on ' λ '

$$v_g = v_p - \frac{1}{\lambda} \frac{d v_p}{d\lambda}$$

ii) Non-dispersive medium $\rightarrow v_p$ does not depend on ' λ '

$$\frac{d v_p}{d\lambda} = 0$$

$$v_g = v_p$$

Heisenberg uncertainty principle

Acc. to Heisenberg uncertainty principle, it is impossible to determine simultaneously the exact position and momentum of a moving small particle like e-, photon, proton etc.

mathematical form is

$$\Delta x \cdot \Delta p \approx \hbar, \quad \hbar = h/2\pi$$

Proof: we know $|\psi(x,t)|^2$ represents the probability of finding the particle at position x and at given time t

$\rightarrow |\psi(x,t)|^2 \Delta x$: represents the probability of finding the particle in given region between x & $x + \Delta x$.

$$\Delta x \propto \Delta k, \quad K = \frac{2\pi}{\lambda}, \quad \Delta K = \frac{2\pi}{\Delta x}$$

$$\text{So, } \Delta x \propto \frac{1}{\Delta K} \quad \therefore \Delta x \propto \frac{1}{\Delta K} \quad \xrightarrow{\Delta K} \quad x \quad x + \Delta x$$

$$\lambda = h/p, \quad K = 2\pi/\lambda, \quad \lambda = 2\pi/K$$

$$\frac{2\pi}{K} = \frac{h}{p}, \quad K = \frac{2\pi p}{h}, \quad \boxed{\Delta K = \frac{\Delta p}{\hbar}}$$

$$\Delta x \cdot \frac{\Delta p}{\hbar} = 1$$

$$\boxed{\Delta x \cdot \Delta p \approx \hbar} \rightarrow \text{minimum uncertainty}$$

$$\boxed{\Delta x \cdot \Delta p \geq \hbar}$$

→ uncertainty principle in Energy & time form

$$\Delta E \Delta t \geq \frac{\hbar}{4}$$

→ uncertainty principle in angular momentum & angle form.

$$\Delta L \Delta \theta \geq \frac{\hbar}{2}$$

Applications of Heisenberg uncertainty principle

Non-existence of electron in nucleus:

The radius of nucleus is of the order of 10^{-14} m .
Diameter of nucleus will be of order of $2 \times 10^{-14} \text{ m}$.
If e- exists in nucleus, then the uncertainty in the location of e- in the nucleus will be $\Delta x = 2 \times 10^{-14} \text{ m}$.

Apply Heisenberg uncertainty principle, $\Delta x \Delta p = \hbar$
putting $\Delta x = 2 \times 10^{-14} \text{ m}$

$$\text{So, } \Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar}{2 \times 10^{-14}} = \frac{6.626 \times 10^{-34}}{2 \times 10^{-14}} = 3.313 \times 10^{-20} \text{ kg m/sec}$$

$$\text{K.E.} = \frac{(\Delta p)^2}{2m} = \frac{(3.313 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31} \times 2} = 96 \text{ mev}$$

∴ Theoretical max. K.E of e- = 4 mev

If e- exist in nucleus it should have energy at 96 mev but experimentally it's observed that the max. K.E of e- is 4 mev. It shows that e- doesn't exist in nucleus.

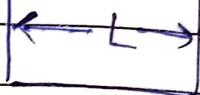
Energy of Particle in a box or in infinitely potential

Let us consider a particle having mass 'm' confined in a box having width 'L' and potential being infinite and infinite height.

Then uncertainty in location of position of particle will be, $\Delta x = L$

from uncertainty principle, $\Delta x \cdot \Delta p = \hbar$

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar}{L}$$



$$K.E = (\Delta p)^2 = \frac{\hbar^2}{2m} = \frac{h^2}{8\pi^2 L^2 m}$$

$$K.E = \frac{h^2}{8\pi^2 L^2 m}$$

Note: In this particle is losing energy while colliding with walls of container, so, $K.E$ is decreasing.

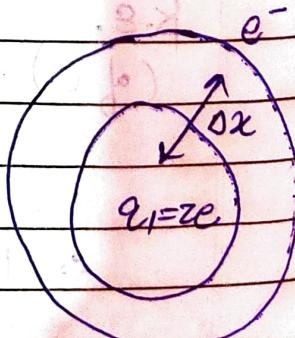
Radius of Bohr's first orbit.

If Δx & Δp are the uncertainty in the position and momentum of e^- in the orbit then using uncertainty principle.

$$\Delta x \cdot \Delta p = \hbar$$

$$\Delta p = \frac{\hbar}{\Delta x}$$

$$(K.E) = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{2m(\Delta x)^2}$$



$$E = T + V$$

$$\Delta E = \frac{\hbar^2}{2m(\Delta x)^2} - \frac{ze^2}{4\pi\epsilon_0 \Delta x} = \frac{\partial}{\partial \Delta x} \left(\frac{\Delta E}{\Delta x} \right) = 0$$

$$\frac{\partial E}{\partial \Delta x} = \frac{e \hbar^2}{\pi m e^2} \quad \boxed{\gamma = \frac{e \hbar^2}{\pi m e^2}}$$

Schrodinger wave equation

Schrodinger eqn is the fundamental equation of quantum mechanics. In the same sense as the Newton's Second Law of motion in classical mechanics.

Schrodinger eqn is used to describe the motion of material particles like electron (e^-), proton (p^+), neutron

Schrodinger eqn are of 2 types:

1. Time independent Schrodinger eqn.
2. Time dependent Schrodinger eqn.

1. Time independent Schrodinger eqn.

General wave eqn: $\frac{d^2y}{dx^2} = -\frac{1}{m} \frac{d^2y}{dt^2}$

ψ is wave function

for matter wave $y = \psi(x, t)$

$$= \frac{\partial^2 \psi(x, t)}{\partial x^2} = -\frac{1}{m} \frac{\partial^2 \psi(x, t)}{\partial t^2} \quad \text{--- (1)}$$

The solution of eqn ① may be written as:

$$\psi(x, t) = \psi_0(x) e^{-i\omega t}$$

Differentiating w.r.t t,

$$\frac{\partial \psi(x, t)}{\partial t} = \psi_0(x) (-i\omega) e^{-i\omega t}$$

Again differentiating w.r.t t,

$$\frac{\partial^2 \psi(x, t)}{\partial t^2} = \psi_0(x) (-i\omega)^2 e^{-i\omega t}$$

$$\frac{\partial^2 \psi(x, t)}{\partial t^2} = \psi_0(x) (-i\omega)^2 e^{-i\omega t}$$

$$\frac{\partial^2 \psi(x, t)}{\partial t^2} = (\omega)^2 \psi_0(x) e^{-i\omega t}$$

$$\frac{\partial^2 \psi(x, t)}{\partial t^2} = -\omega^2 \psi(x, t)$$

Putting the value of $\frac{\partial^2 \psi(x, t)}{\partial t^2}$ in eqn ②

$$\left| \begin{array}{c} \frac{\partial^2 \psi(x, t)}{\partial x^2} + \omega^2 \psi(x, t) \\ \hline \end{array} \right|$$

$$\frac{\partial^2 \psi(x, t)}{\partial x^2} + \omega^2 \psi(x, t) = 0$$

$$(t, x) \psi = u \text{ since million } u$$

$$\textcircled{1} - (t, x) \psi = (t, x) \psi - u$$

operator, Eigen function and Eigen Value \rightarrow

operator is anything which is capable of doing something to the wave function, when operator operates on a wave function it must give observable quantity multiples by wave function.

$$\text{operator } A\psi = a\psi \quad \text{observable} \quad \rightarrow \text{①}$$

Quantity or. Eigen Value.

Eigen func : If any wave func. satisfy eqn ① then the wave func. is known as Eigen function. And the corresponding observable quantity is known as Eigen value.

App. of Schrodinger
particle in rigid one dimensional box or infinite potential well.

$$V(x) = 0 \quad 0 \leq x \leq L \quad \rightarrow \text{①}$$

$$V(x) = \infty \quad L < x < 0 \quad \rightarrow \text{②}$$

$$V(u) = 0$$

$$V = \infty$$

$$x=0$$

$$x=L$$

let us consider a particle having mass 'm' confined in a rigid box having width 'L' and height $\frac{1}{2}h$. Particle is free to move in a box but restricted to move outside the box.

using time independent Schrodinger eqn in one-dimension,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (\epsilon - V) \psi = 0$$

→ apply wave func. boundary condition

Applying eqn 1, putting $V = 0$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \text{(3)}$$

$$\text{let } \frac{2mE}{\hbar^2} = k^2 \rightarrow (2)$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \rightarrow (5)$$

The general form of eqn (5) may be written as

$$\psi = A \sin kx + B \cos kx \rightarrow (6)$$

Now we write two boundary condition using this system.

$$\psi = 0, \text{ at } x=0 \rightarrow (7)$$

$$\psi = 0, \text{ at } x=L \rightarrow (8)$$

using eqn (7) in eqn. (6)

$$0 = A \sin 0 + B \cos 0$$

$$\boxed{B=0}$$

using eqn (8) in eqn. (6)

$$0 = A \sin KL + 0$$

$$A \sin KL = 0$$

either; $A=0$ or $\sin KL=0$

$$A \neq 0, \text{ so } \sin KL = 0$$

$$KL = n\pi$$

$$\therefore n = 1, 2, 3, \dots$$

$$K = \frac{n\pi}{L}$$

Putting $K = \frac{n\pi}{L}$ in eqn (4)

$$\frac{\partial E}{\partial t^2} = \frac{n^2 \pi^2}{L^2} \Rightarrow E = \frac{n^2 \pi^2 \hbar^2}{2m L^2}$$

$$E \rightarrow E_n$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m L^2}$$

- (9)

Eqn (9) represents that
the energy of particle in
quantum mechanic system

is in discrete form not in
continuous form.

The value of this energy is also known as Eigen
Value of the system.

$$\psi = A \sin kx$$

$$\psi = A \sin \frac{n\pi}{L} x$$

using normalization condn: $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

for this System

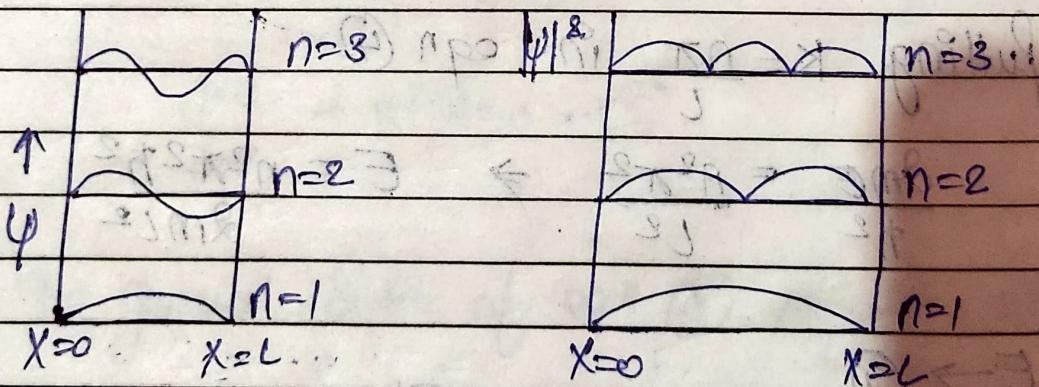
$$\int_0^L |\psi|^2 dx = 1$$

$$\int_0^L A \sin \frac{n\pi}{L} x dx = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\psi = \sqrt{\frac{2}{L}} n \sin \frac{n\pi}{L} x$$

Eigen func. of this System.



Application of Schrodinger Time independent equation

- Particle in finite potential
- Box or Non-Rigid Box

Let us consider a particle $v(x) = U$ having mass m confined in a finite potential well in a non-rigid box having width 'L' and height finite.

let us consider a particle $v(x) = U$ having mass m confined in a finite potential well in a non-rigid box	$E < U$	$v(x) = U$
	I Region	II Region
	$v(x) = 0$	III Region
having width 'L' and height finite.	$-x \quad 0 \quad L \quad +x$	
	$v(x) = U$ in I & III region. $v(x) = 0$ in II region.	

Acc. to Classical mechanics particle having energy $E < U$ will move in II region and will strike the walls but cannot go to the region I and III.

Acc. to Quantum mechanics particle having energy $E < U$ will move in II region and will strike at the walls of the potential well but has certain probability of penetrating region I & III, due to wave particle duality property of material particle.

→ we know Schrodinger time independent equation-

$$\frac{d^2\psi}{dx^2} + \frac{2m(\epsilon - V)}{\hbar^2} \psi = 0$$

In I-region $V = U$, $E < U$

$$\frac{d^2\psi_I}{dx^2} + \frac{2m(\epsilon - U)}{\hbar^2} \psi_I = 0$$

$$\frac{\partial^2 \psi_I}{\partial x^2} - 2m(E - \varepsilon) \psi_I = 0 \quad \text{in region I}$$

$$\text{Let } \frac{\partial^2 \psi_I}{\partial x^2} = \frac{k^2}{h^2}$$

$$\text{let } 2m(E - \varepsilon) = \alpha^2$$

$$\frac{k^2}{h^2}$$

$$\frac{\partial^2 \psi_I}{\partial x^2} - \alpha^2 \psi_I = 0 \quad \text{--- (1)}$$

* In III region

$$\text{Similarly in III region, } \frac{\partial^2 \psi_{III}}{\partial x^2} - \alpha^2 \psi_{III} = 0 \quad \text{--- (II)}$$

* In II region

$$V=0, \text{ and } \frac{\partial^2 \psi_{II}}{\partial x^2} + 2mE \psi_{II} = 0$$

$$\frac{\partial^2 \psi_{II}}{\partial x^2} + K^2 \psi_{II} = 0 \quad \text{--- (3)}$$

$$\frac{\partial^2 \psi_{II}}{\partial x^2} + K^2 \psi_{II} = 0 \quad \text{--- (3)}$$

$$\frac{\partial^2 \psi_I}{\partial x^2} - \alpha^2 \psi_I = 0 \quad \text{--- (1)}$$

$$\frac{\partial^2 \psi_{III}}{\partial x^2} - \alpha^2 \psi_{III} = 0 \quad \text{--- (II)}$$

$$\frac{\partial^2 \psi_{III}}{\partial x^2} + K^2 \psi_{III} = 0 \quad \text{--- (III)}$$

The general Solution of eqn ④, ⑤ and ⑥

$$\Psi_I = Ae^{\alpha x} + Be^{-\alpha x} \quad \text{--- } ④$$

$$\Psi_{III} = Ce^{\alpha x} + De^{-\alpha x} \quad \text{--- } ⑤$$

$$\Psi_{II} = Esinkx + Fcoskx \quad \text{--- } ⑥$$

Now we will check the general solutions i.e eq. 4, 5, 6

$$\rightarrow \Psi_I = Ae^{\alpha x} + Be^{-\alpha x} \quad \text{--- } ④$$

when $x \rightarrow -\infty$, $Be^{-\alpha(-\infty)} = Be^{\infty} = \infty$, so

$\Psi_I = \infty$, so to get the finite value of Ψ_I , we put $B=0$.

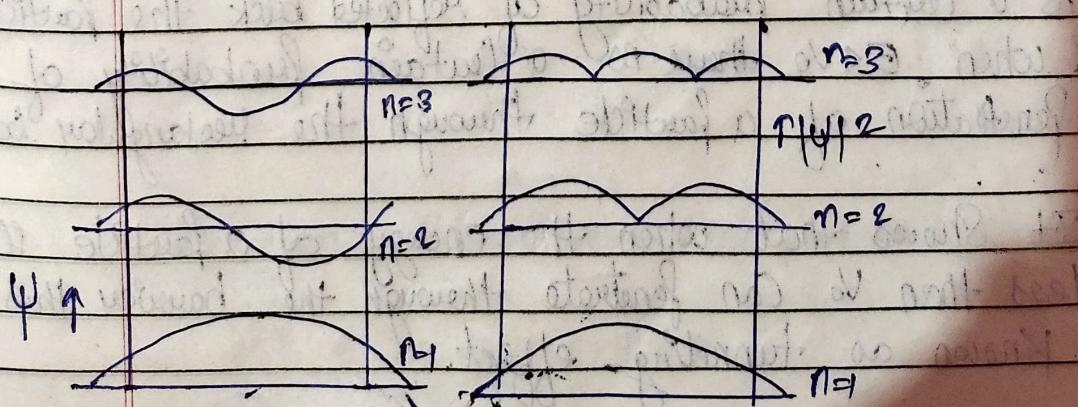
$$[\Psi_I = Ae^{\alpha x}] \quad \text{--- } ⑦$$

$$\rightarrow \text{Eqn. 5}, \Psi_{III} = Ce^{\alpha x} + De^{-\alpha x}$$

$x \rightarrow \infty$, $\Psi_{III} = \infty$, so to get the finite value of Ψ_{III} , we put $C=0$

$$[\Psi_{III} = De^{-\alpha x}] \quad \text{--- } 8$$

$$[\Psi_{II} = Esinkx + Fcoskx] \quad \text{--- } 9$$



It is clear from the graph that the wave function exist in region I & III. It means there is a certain probability of penetrating the particle from region II into I & III. This is due to the wave-particle duality property of material particle.

Tunneling effect:

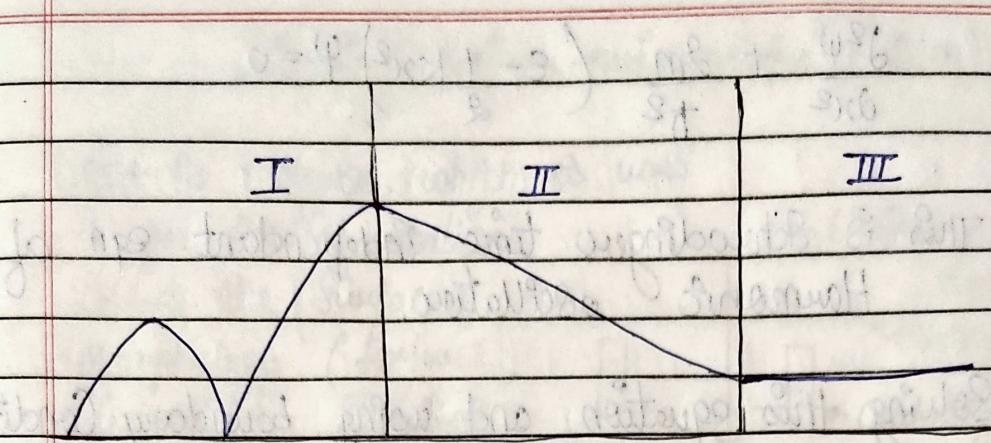
Let us consider a particle moving from left to right meeting to a rectangular barrier having height V_0 . when acc. to classical mechanics when energy of particle is less than ($E < V_0$)

then the particle will not cross rectangular barrier but will always reflected back to the region - I. when Energy of the particle is greater than V_0 ($E > V_0$) then particle will always cross the barrier.

But acc. to quantum mechanics even when $E > V_0$ there is a certain probability of reflected back the particle & when $E < V_0$ there is a certain probability of penetration of a particle through the rectangular barrier.

It shows that when the energy of a particle is less than V_0 can penetrate through the barrier this is known as tunneling effect.

I II III



→ Applications of tunneling effect

1. Tunnel diode

→ used in switches

Simple Harmonic oscillation

In solid atoms vibrate in their equilibrium equation positions so atoms can be treated as harmonic oscillators.

Let us consider a harmonic oscillator when it is displaced through a distance 'x' then a restoring force acts through a harmonic oscillator.

And the restoring force, $F = -Kx$, $K \rightarrow \text{const.}$

Potential, $V = -\int F dx$

$$V = \int_0^x Kx dx , \quad V = \frac{1}{2} Kx^2$$

We know Schrodinger time independent equation.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (\varepsilon - V) \psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} \left(\frac{\epsilon - 1}{2} kx^2 \right) \psi = 0$$

This is Schrodinger's time independent eqn of Harmonic oscillation.

Solving this equation and using boundary conditions-

we get energy of Harmonic oscillator

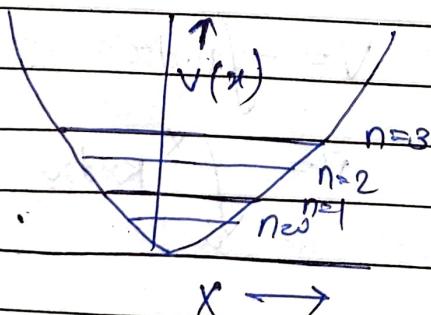
$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega, \quad n=0, 1, 2, \dots$$

ω - angular frequency

It is clear from the above expression that the energy of harmonic oscillator is in discrete form. The energy at $n=0$, that is $E = \frac{1}{2} \hbar \omega$

P8 Known as Zero Point Energy of Harmonic oscillator

Ground State Energy



$$n=0, \quad E = \frac{1}{2} \hbar \omega$$

$$n=1, \quad E = \frac{5}{2} \hbar \omega$$

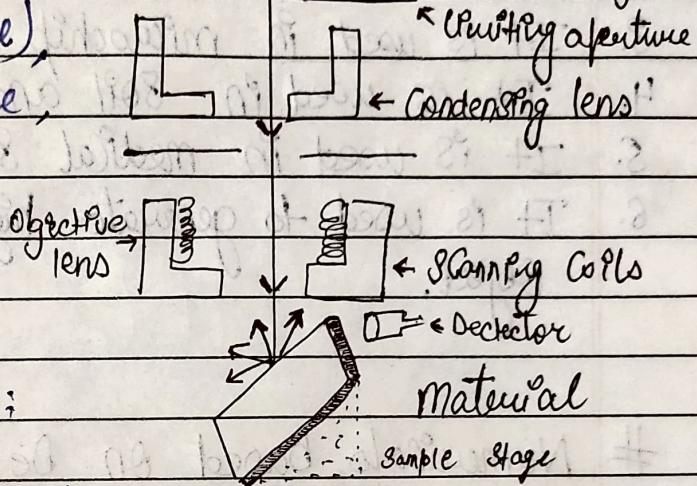
$$n=2, \quad E = \frac{9}{2} \hbar \omega$$

$$n=3, \quad E = \frac{13}{2} \hbar \omega$$

Scanning electron microscope (SEM)

It is a device which is used to get the information about the surface morphology (texture), Crystalline Structure, Chemical Composition.

Construction



Main parts of SEM :

- Electron Source or electron gun, Converging aperture, Condensing lens, Scanning coils, detector, material and Sample Stage.

Working :

Electrons are emitted from the electron source or electron gun. These emitted electrons are accelerated and focussed using the condenser lenses. The beam of electrons is then moved across the object or material by the scanning coils, this electron beam generates secondary electrons from the material. The secondary electrons are detected using detector. The signal obtained from the detector is sent to the computer system and we get the images in the computer. After analysing the images we get the information about the surface morphology, Crystalline Structure, Chemical Composition of the material.

Applications.

1. It is used in material Science for research, quality control and analysis of structures.
2. It is used in Semiconductor inspection.
3. It is used in microchip design and production.
4. It is used in Soil and rock Sampling.
5. It is used in medical Science to identify the veins.
6. It is used to generate high resolution images of the object.

Numericals based on De brague wave

Ques. Calculate the wavelength of a photon of Energy $5 \times 10^{-19} \text{ J}$.

Soln: $E = h\nu$, $E = \frac{hc}{\lambda}$, $5 \times 10^{-19} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$

$\lambda = 3.96 \times 10^{-7} \text{ m}$

Ques. A photon is moving with a Speed $v = 2 \times 10^8 \text{ m/s}$. Find the wavelength of matter wave associated with heat.

Soln: $\lambda = \frac{v}{f} = \frac{2 \times 10^8}{1.67 \times 10^{27}} \text{ m}$

$f_{max} = 1.67 \times 10^{27} \text{ Hz}$

$$\lambda = \frac{h}{P} = \frac{6.6 \times 10^{-34}}{1.67 \times 10^{-27} \times 2 \times 10^8} = \frac{6.6 \times 10^{-34}}{3.34 \times 10^{-19}}$$

$$\lambda = 1.98 \times 10^{-84+89}$$

$$\lambda = 1.98 \times 10^{-15} \text{ meters}$$

Ques. The De-Broglie wavelength associated with an electron is 0.1 Å° . Find the potential difference by which electron is accelerated.

$$V = \sqrt{\frac{2eV}{m}}, \quad eV = \frac{1}{2}mv^2; \quad \lambda = \frac{h}{\sqrt{2meV}}$$

$$1.6 \times 10^{-19} \times 5 \times 10^{-10} = \frac{1}{2} \times$$

$$0.1 \times 10^{-10} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\sqrt{29.12 \times 10^{-80} V} = \frac{6.6 \times 10^{-34}}{0.1 \times 10^{-10}}$$

$$\sqrt{29.12 \times 10^{-80} V} = 66 \times 10^{-24}$$

$$5.396 \times 10^{-25} V = 66 \times 10^{-24}$$

$$V = \frac{66 \times 10^{-24}}{5.396 \times 10^{-25}}$$

$$V = 12.28 \times 10$$