Q9 Show that in a PD with unit mean, deviation about mean is $\frac{2}{e}$.

A. mean, $m = 1$

PD, $\quad P(x) = \dfrac{e^{-m} m^x}{x!}, \quad x = 0, 1, \ldots$

$$= \frac{e^{-1}}{x!}$$

mean deviation, $E(|x-1|) = \displaystyle\sum_{x=0}^{\infty} |x-1| \cdot \frac{e^{-1}}{x!}$

$$= e^{-1} + 0 + \sum_{x=2}^{\infty} (x-1) \frac{e^{-1}}{x!}$$

$$= e^{-1} + e^{-1} \sum_{x=2}^{\infty} \frac{(x-1)}{x!}$$

$$= e^{-1} \left[1 + \sum_{x=2}^{\infty} \left(\frac{x}{x!} - \frac{1}{x!} \right) \right]$$

$$= e^{-1} \left[1 + \sum_{x=2}^{\infty} \left(\frac{1}{(x-1)!} - \frac{1}{x!} \right) \right]$$

$$= e^{-1} \left[1 + \frac{1}{1!} \right]$$

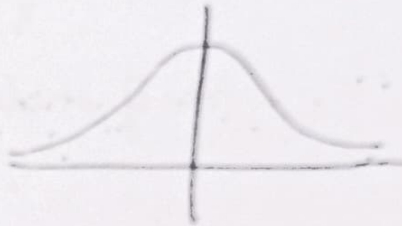
$$= \frac{2}{e}$$

In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

(i) how many students score b/w 12 & 15?

(ii) how many students score above 18?

$n = 1000, \quad \mu = 14, \quad \sigma = 2.5$



(i) $z_1 = \dfrac{x-\mu}{\sigma} = \dfrac{12-14}{2.5} = -0.8$

$z_2 = \dfrac{15-14}{2.5} = 0.4$

$$\begin{aligned} P(12 \le X \le 15) &= P\left(-0.8 \le \frac{X-\mu}{\sigma} \le 0.4\right) \\ &= P(-0.8 \le Z \le 0.4) \\ &= P(-0.8 \le Z \le 0) + P(0 \le Z \le 0.4) \\ &= P(0 \le Z \le 0.8) + P(0 \le Z \le 0.4) \\ &= 0.2881 + 0.1554 \\ &= \boxed{0.4435} \end{aligned}$$

$\therefore$ no of students $= 1000 \times 0.4435$

$= 443.5$

$\simeq \boxed{444}$

(ii) $z = \dfrac{18-14}{2.5} = \dfrac{4}{2.5} = 1.6$

$P(X>18) = P(Z>1.6)$ (only ...

$= 0.5 - P(0 \le Z \le 1.6)$

(∵ normal curve symmetrical & total Probability is 1 ∴ total Probability on the right hand side is 0.5)

$= 0.5 - 0.4452$

$= 0.0548$

∴ no. of students $= 1000 \times 0.0548$

$= 54.8$

$\simeq \boxed{55}$

Q11 Fit a binomial distribution to the following data:

x	0	1	2	3	4
f	30	62	46	10	2

$n = 4$

$N = \sum_{i=0}^{4} f_i = 150$

∴ mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{192}{150} = np$

$\Rightarrow p = \frac{192}{600} = 0.32$

Binomial frequency distribution is $F(x) = N\,{}^4C_x\,(0.32)^x\,(0.68)^{4-x}$

x	0	1	2	3	4
$F(x)$	32	60	43	13	2

Q12 Find the area under normal curve.

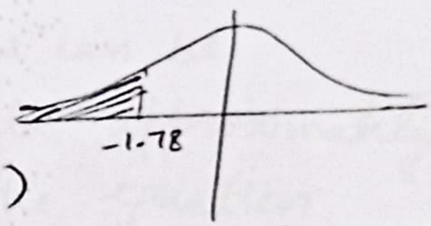
a) to the left of $z = -1.78$

b) to the left of $z = 0.56$

c) to the right of $z = -1.45$

a) $P(z < -1.78)$

$= 0.5 - \text{Area of } (0 \text{ to } -1.78)$



$= 0.5 - P(-1.78 \leq z \leq 0)$ (∵ half curve is 0.5 area under)

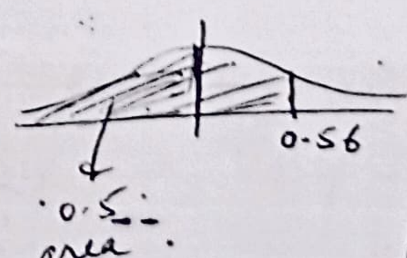
$= 0.5 - P(0 \leq z \leq 1.78)$ (curve is symmetrical)

$= 0.5 - 0.4625$ (from table).

$= \boxed{0.0375}$

b) $P(z < 0.56) = 0.5 + P(0 \leq z \leq 0.56)$

$= 0.5 + 0.2123$ (from table)



$= \boxed{0.7123}$

Q13 Find first three moments of binomial distribution.

[Find μ_1', μ_2', μ_3']

Q14 Find first three central moments of binomial distribution.

[Find $\mu_1, \mu_2,$