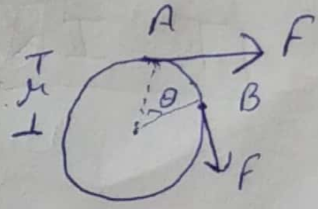
Kinetics of rigid body

Torque - Turning moment of a force on a body on which it acts.

$$\boxed{T = F \cdot r}$$



Work done by force F

$W = F \cdot$ length of arc AB

$= F \cdot r\theta$

$$\boxed{W = T\theta}$$

In one rotation $\theta = 2\pi$

If the body turns N revolution per minute, then the angular displacement per seconds is $\frac{2\pi N}{60}$

(Power) $\boxed{P = T \cdot \frac{2\pi N}{60}}$ [work done per sec]

Angular momentum: The product of mass M.O.I. and the angular velocity of a rotating body is known as angular momentum or moment of momentum.

$$\boxed{L = I\omega}$$

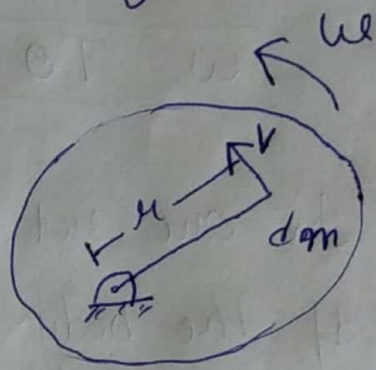
Also

$$T = \frac{d}{dt}(I\omega) \quad \text{[Second law of rotary motion]}$$

$$\boxed{T = I\alpha}$$

K.E. due to rotation

Consider an elemental mass of a body rotating with velocity V



K.E. of elementary mass $= \frac{1}{2} \cdot dm \cdot V^2$

$$= \frac{1}{2} dm\, r^2 \omega^2$$

K.E. of whole body $= \int \frac{1}{2} dm\, r^2 \omega^2$

$$= \frac{1}{2} \omega^2 \int dm\, r^2$$

$$\boxed{K.E = \frac{I\omega^2}{2}}$$

Acc. to work energy principle

$$\boxed{W = T \cdot \theta = \frac{I}{2}\left[\omega_2^2 - \omega_1^2\right]}$$

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when a body is rolling without slipping, It has K.E due to translation as well as rotation.

$$\boxed{\text{Total K.E. of body} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

Also $I = mK^2$ [K = radius of gyration]

$\omega = \frac{V}{r}$

$\therefore$ Total K.E $= \frac{1}{2}mV^2 + \frac{1}{2}mK^2\left(\frac{V}{r}\right)^2$

$= \frac{mV^2}{2}\left[1 + \frac{K^2}{r^2}\right]$

work energy principle for general plane motion (translation + rotation)

$W_{(translation)} + W_{(rotation)} = K.E_{(Translation)} + K.E_{(rotation)}$

$$\boxed{F.s + T.\theta = \frac{1}{2}m(V_2^2 - V_1^2) + \frac{1}{2}I(\omega_2^2 - \omega_1^2)}$$

M.O.I of

Circular lamina $= \frac{mr^2}{2}$

Solid sphere $= \frac{2}{5}mr^2$

Uniform bar $= \frac{ml^2}{12}$

hoop $= mr^2$

[about C.G]

Q A flywheel of mass 4000 kg and radius of gyration 0.8m loses its speed from 360 rpm to 250 rpm in 2 minutes. Make calculations for

i) the torque acting on the wheel

ii) change in K.E.

iii) change in angular momentum of the wheel during the given period.

Sol:
$$\omega_0 = \frac{2\pi N_0}{60} = 37.68 \text{ rad/s}$$

$$\omega = \frac{2\pi N}{60} = 26.17 \text{ rad/s}$$

from kinematic relation

$$\omega = \omega_0 + \alpha t$$

$$\alpha = -\cancel{245.5}\ -0.097 \text{ rad/s}^2 \qquad [t = 120 \text{ sec}]$$

(i) Torque = $I\alpha = mk^2\alpha$

$$= -245.5 \text{ Nm}$$

(ii) $$\Delta K.E = \frac{1}{2} mk^2 [\omega^2 - \omega_0^2]$$

$$= -9.407 \times 10^5 \text{ Nm}$$

(iii) change in angular momentum $= mk^2(\omega - \omega_0)$

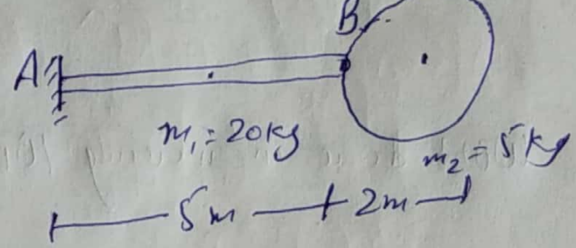
$$= -29466 \text{ Nm/s}$$

Equation of motion for rigid body in plane motion

$$\left.\begin{aligned}\sum F_x &= ma_x \\ \sum F_y &= ma_y\end{aligned}\right\}$$ Translatory motion

$$\sum M_G = I_G \alpha \}$$ Rotary motion

Q Consider a composite system consisting of a rod and sphere as shown in fig. Determine the angular acc. of the system at the instant when it is released from the horizontal position. mass of rod = 20 kg
" " sphere = 5 kg



Sol:

M.O.I of rod about A = $\frac{ml^2}{3}$

$I_1 = 166.67$ kg m²

M.O.I of sphere about A

$I_2 = I_{G_2} + mh^2$ (Parallel axis theorem

$I_2 = \frac{2}{5}mr^2 + mh^2 = 182$ kg m² [h = 6 m]

Total M.O.I = 348.67 kg m²

For rigid body undergoing rotary motion

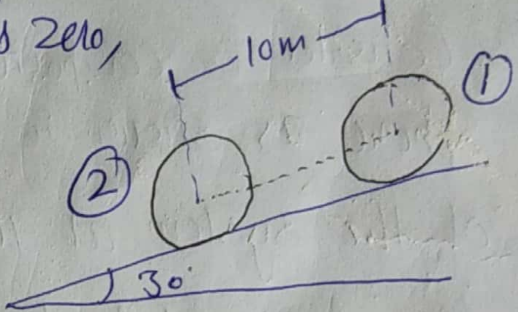
$$\sum M = I\alpha$$

$M_a = I_a \cdot \alpha$; $[M_a = (20 \times 9.81 \times 2.5 + 5 \times 9.81 \times 6)]$

$\alpha = 2.251$ rad/s

Q. A homogeneous cylinder of mass 50 kg and 0.5 m in radius is having initial velocity of 6 m/s down a 30° plane. Calculate the velocity of cylinder when it has reached 10 m down the plane from starting point. It may be presumed that the cylinder rolls without slipping.

Sol. The velocity at point of contact is zero, so, No work done by frictional force in rolling motion.



Component of weight along plane $= mg\sin 30$
$= 245.25\,N$

Work done in moving 10m down the plane $= 245.25 \times 10$
$= 2452.5\ Nm$

$$\Delta K.E. = \frac{1}{2} m \left(V_2^2 - V_1^2 \right) + \frac{1}{2} I \left(\omega_2^2 - \omega_1^2 \right)$$

$$I = \frac{mr^2}{2} \; ; \; V_1 = \omega_1 r \; ; \; V_2 = \omega_2 r$$

$$\Delta K.E = 25\left(V_2^2 - 36\right) + 3.125\left(4V_2^2 - 144\right) = 2452.5$$

[work energy principle]

$$\boxed{V_2 = 10.07\ m/s}$$

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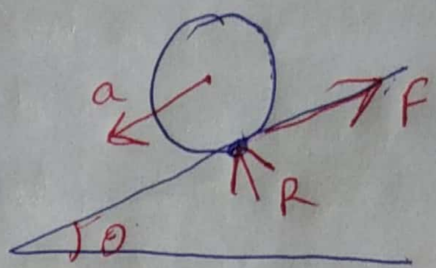
II method



$\sum F_x = ma_x$

$mg\sin\theta - f = ma$ — (i)

$\sum F_y = ma_y$

$-mg\cos\theta + R = 0$ — (ii)

$\sum M_G = I_G\alpha$

$f(r) = I_G\alpha$

$f \cdot r = \frac{mr^2}{2} \cdot \frac{a}{r}$

$f = \frac{ma}{2}$ — (ii)

from (i) and (iii)

$mg\sin\theta - \frac{ma}{2} = ma$

$mg\sin\theta = \frac{3}{2}ma$

$$\boxed{a = \frac{2}{3}g\sin\theta}$$

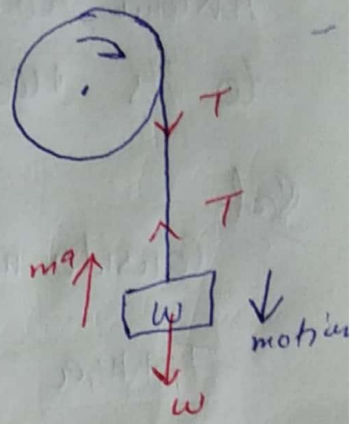
$a = 3.27\ m/s^2$

$v^2 - u^2 = 2as$

$v^2 - 6^2 = 2 \times 3.27 \times 10$

$v = 10.07\ m/s$

Q A body of weight 8 N is suspended by a light rope wound round a pulley of weight 60N and radius 30 cm. The other end of the rope is fixed to the periphery of the pulley. If the weight is moving downward, make calculations for the acc. of 8N weight and tension in the string.



Sol Consider block

Applying D'Alembert principle

$$T + ma = W$$

$$W - T = \frac{W}{g} \cdot a \quad -(i)$$

Consider motion of Pully

Torque on Pully = $I\alpha$

Torque = $T \cdot r$

$$T = I \cdot \frac{a}{r} \cdot r = I \cdot \frac{a}{r^2} \quad -(ii)$$

If pully is considered as solid disc

$$I = \frac{m_0 r^2}{2} = \frac{W_0 r^2}{2g} \quad -(iii)$$

from (ii) and (iii)

$$T = \frac{W_0}{2g} a \quad -(iv)$$

from (i) and (iv)

$$W - \frac{W_0}{2g} a = \frac{W}{g} a$$

$$a = \frac{gW}{W + \frac{W_0}{2}}$$

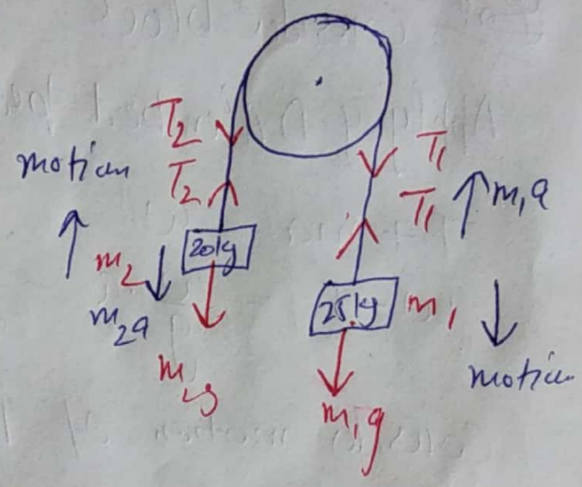
$$a = 2.065 \; m/s^2$$

from (i)

$$T = 6.31 N$$

Q) Two blocks of masses $m_1 = 25$ kg and $m_2 = 20$ kg are connected by a light inextensible string which passes over a 25 cm diameter pulley of 2.5 kg mass. Neglecting friction, work out the acc. of the system and tension in the string when the masses are released from rest. Assume radius of gyration of the pulley to be equal to its radius

Sol.



Consider motion of masses

$T_1 + m_1 a = m_1 g$

$25g - T_1 = 25a$ — (i)

$T_2 - 20g = 20a$ (ii)

Consider Pulley

Resultant torque $= (T_1 - T_2) r$

$= (T_1 - T_2) \times 0.125$ — (iii)

$M.O.I = mK^2$

$= 0.0391 \text{ kg m}^2$

$\alpha = \frac{a}{r} = \frac{a}{0.125}$ — (iv)

From Newton's second Law

$T = I\alpha$

$(T_1 - T_2) \times 0.125 = 0.0391 \times \frac{a}{0.125}$

$T_1 - T_2 = 2.5a$ — (v)

Using (i), (ii) and (v), we get

$\boxed{a = 1.033 \text{ m/s}^2}$

from (i)

$T_1 = 219.42$ N

$T_2 = 216.86$ N

Ans