

For every function  $S: \mathbb{N} \rightarrow \mathbb{N}$ , the time complexity class  $\text{NSPACE}(S)$  is set of all languages that can be decided by multiple tape nondeterministic TM with worst case bounded above by  $O(S)$ .

We can use these complexity classes to define the non-deterministic analogue of  $\text{SPACE}$ .

$$\text{NPSPACE} = \bigcup_{k \geq 1} \text{NSPACE}(n^k)$$

Savitch theorem that any nondeterministic TM can be simulated by a deterministic Turing machine with at most a quadratic increase in the amount of space required.

Savitch theorem for every  $S: \mathbb{N} \rightarrow \mathbb{N}$ .

$$\text{NSPACE}(S(n)) \subseteq \text{SPACE}(S(n)^2)$$

$$\text{PSPACE} = \text{NPSPACE}$$

## Savitch's Theorem

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- We can consider the space complexity analogue of the P vs NP problem by considering the space cost of non-deterministic Turing machines.

### Non-deterministic Space Complexity

The space cost of non-deterministic TM  $M$  on i/p  $x$  is the  $\max^m$  no. of distinct tape cells that are visited by  $M$ 's tape head before  $M$  halts, with the  $\max^m$  taken over all possible computational paths.

The (worst-case) space cost of non-deterministic TM  $M$  is the function  $S: N \rightarrow N$  where  $S(n)$  is maximum space cost of  $M$  on any i/p  $x$  of length  $|x| = n$ .