

SYLLABUS ACADEMIC SESSION: 2021-2022 ONWARDS

APPLIED PHYSICS-II (BS 106)

Marking Scheme:

- (a) Teacher Continuous Evaluation: 25 marks
(b) Term End Theory Examination : 75 marks

UNIT I

Quantum Mechanics: Introduction: Wave particle duality, de Broglie waves, the experiment of Davisson and Germer, electron diffraction, physical interpretation of the wave function, properties, the wave packet, group and phase velocity, the uncertainty principle . The Schrodinger wave equation (1D), Eigen values and Eigen functions, expectation values, simple Eigen value problems - solutions of the Schrodinger's equations for the free particle, the infinite well, the finite well, tunneling effect, the scanning electron microscope, the quantum simple harmonic oscillator (qualitative), zero point energy.

[T1][T2] [Hrs. 8]

UNIT II

Quantum Statistics: The need for statistics , statistical distributions: Maxwell Boltzmann, Bose-Einstein and Fermi-Dirac statistics, their comparisons, Fermions and Bosons, Applications of quantum statistics: 1. Molecular speed and energies in an ideal gas; 2. The Black body spectrum, the failure of classical statistics to give the correct explanations - Bose-Einstein statistics applied to the Black Body radiation spectrum; Fermi-Dirac distribution, free electron theory, electronic specific heats, Fermi energy and average energy; Dying stars.

[T1][T2] [II Hrs. 8]

UNIT III

Crystal Structure: Types of solids, Unit cell, Types of crystals, Translation vectors, Lattice planes, Miller indices, Simple crystal structures, Interplaner spacing. Crystal structure analysis: Bragg's law, Laue method, Point defects: Schottky and Frankel defects.

[T1][T2] [Hrs. 8]

UNIT IV

Band Theory of Solids: Origin of energy bands in solids, motion of electrons in a periodic potential - the Kronig-Penny model (Qualitative). Brillouin zones, effective mass, metals, semi-conductors and insulators and their energy band structures. Extrinsic and intrinsic semiconductors, doping - Fermi energy for doped and undoped semiconductors, the p-n junction (energy band diagrams with Fermi energy), the unbiased diode, forward and reverse biased diodes - tunnel diodes, zener diode, photo diode its characteristics, LED.

[T1][T2][II Hrs. 8]

SYLLABUS

ACADEMIC SESSION: 2015-2016

APPLIED PHYSICS-II (ETPHI-104)

Marking Scheme:

- (a) Teacher Continuous Evaluation: 25 marks
- (b) Term End Theory Examination : 75 marks

UNIT I

Electromagnetic Theory: Gradient, Divergence, Curl, Gauss' law, Ampere's Law, Continuity equation, Maxwell's equations (differential and integral forms), Significance of Maxwell's equations, Poynting Theorem, Electromagnetic wave propagation in dielectrics and conductors.

[T1][T2] [No. of Hrs. 8]

UNIT II

Statistical Physics: Black body radiation, Planck's radiation formula, Wien's and Rayleigh-Jeans Laws. Distribution laws: Qualitative features of Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics & their comparison (without derivation).

Quantum Mechanics: Postulates of Quantum mechanics, de-Broglie hypothesis, Davisson Germer experiment, Wave function and its physical significance, Wave Packet, Phase and group velocities, Uncertainty principle, Schrodinger equation for free particle, Time dependent Schrodinger equation, Particle in a box (1 -D)

[T1][T2] [No. of Hrs. 8]

UNIT III

Crystal Structure: Types of solids, Unit cell, Types of crystals, Translation vectors, Lattice planes, Miller indices, Simple crystal structures, Interplaner spacing, Crystal structure analysis: Bragg's law, Laue method, Point defects: Schottky and Frankel defects.

[T1][T2] [No. of Hrs. 8]

UNIT IV

Band Theory of Solids: Introduction, Kronig-Penney model: E-k diagram, Effective mass of an electron, Intrinsic semiconductors: Electron concentration in conduction band, Hole concentration in valence band, Extrinsic semiconductor: p-type and n-type semiconductors, Fermi level, Hall Effect: Hall voltage and Hall coefficient.

[T1][T2][No. of Hrs. 8]

**NEW TOPICS ADDED FROM ACADEMIC SESSION
2021-22 ONWARDS
SECOND SEMESTER
APPLIED PHYSICS-II (BS-1061)
UNIT - I**

Q.1. If a beam of electrons impinges on an energy barrier of height 0.030 eV and of infinite width, find the fraction of electrons reflected and transmitted at the barrier if the energy of the impinging electron is

- (a) 0.040 eV (b) 0.025 eV (c) 0.030 eV

Ans. (a) Given $E = 0.040 \text{ eV} = 0.040 \times 16 \times 10^{-19} \text{ joule} = 0.064 \times 10^{-19} \text{ joule}$

and

$$\begin{aligned} V_0 &= 0.030 \text{ eV} = 0.030 \times 1.6 \times 10^{-19} \text{ joule} \\ &= 0.048 \times 10^{-19} \text{ joule} \end{aligned}$$

i.e.,

$$E > V_0$$

Then the fraction of electrons reflected i.e., the reflectance is given by

$$R = \frac{(p_1 - p_2)^2}{(p_1 + p_2)^2}$$

$$\text{The fraction of electrons transmitted } T = \frac{4p_1 p_2}{(p_1 + p_2)^2}$$

$$\text{We know } p_1 = \sqrt{2mE} = \sqrt{(2 \times 9 \times 10^{-31} \times 0.064 \times 10^{-19})} \quad (\text{since } m = \text{mass of the electron} = 9 \times 10^{-31} \text{ kg})$$

$$= 1.07 \times 10^{-25} \text{ kg-m/sec}$$

$$\begin{aligned} p_2 &= \sqrt{[2m(E - V_0)]} = \sqrt{[2 \times 9 \times 10^{-31} \times (0.040 - 0.030) \times 1.6 \times 10^{-19}]} \\ &= \sqrt{(2 \times 9 \times 10^{-31} \times 0.10 \times 1.6 \times 10^{-19})} = 5.37 \times 10^{-26} \text{ kg-m/sec} \end{aligned}$$

$$\text{Reflectance} = \frac{(p_1 - p_2)^2}{(p_1 + p_2)^2} = \frac{(1.07 \times 10^{-25} - 5.37 \times 10^{-26})^2}{(1.07 \times 10^{-25} + 5.37 \times 10^{-26})^2} \approx 0.1$$

i.e., fraction of electrons reflected = 0.1

The fraction electrons transmitted

$$\begin{aligned} &= \frac{4p_1 p_2}{(p_1 + p_2)^2} \\ &= \frac{4 \times 1.07 \times 10^{-25} \times 5.37 \times 10^{-26}}{(1.07 \times 10^{-25} + 5.37 \times 10^{-26})^2} \approx 0.9 \end{aligned}$$

(b) In this case $E < V_0$.

∴ The transmission coefficient = 0.

i.e., The fraction of electrons transmitted = 0

and The reflection coefficient i.e., the fraction of electrons reflected = 1.

(c) In this case $E = V_0$.

$$p_1 = \sqrt{2mE} = \sqrt{(2 \times 9 \times 10^{-31} \times 0.030 \times 1.6 \times 10^{-19})} \text{ kg-m/sec}$$

$$p_2 = [\sqrt{2m(E - V_0)}] = 0 \text{ (since } E = V_0\text{).}$$

$$\therefore \text{The fraction of electrons reflected} = \frac{(p_1 - p_2)^2}{(p_1 + p_2)^2} = 1.$$

The fraction of electrons transmitted = $\frac{4p_1 p_2}{(p_1 + p_2)^2} = 0$ since $p_2 \approx 0$

are different for different elements and the exponential is very sensitive to these quantises.

Q.2. Calculate the probability of transmission of α -particle through the rectangular barrier indicated below:
 $V_0 = 2\text{ eV}$, $E = 1\text{ eV}$ and barrier width = 1\AA , mass of α -particle = $6.4 \times 10^{-27}\text{ kg}$

Ans. The probability of transmission is given by

$$T = \frac{16E(V_0 - E)}{V_0^2} e^{[-2\sqrt{2m(V_0 - E)/\hbar^2} a/\hbar]}$$

Here $V_0 = 2\text{ eV} = 2 \times 1.6 \times 10^{-19}\text{ joule}$, $E = 1\text{ eV} = 1.6 \times 10^{-19}\text{ joule}$.
 $m = 6.4 \times 10^{-27}\text{ kg}$, $a = 1\text{\AA} = 10^{-10}\text{ m}$; $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\text{ Joule-sec}$

$$T = \frac{1.6 \times 1.6 \times 10^{-19} \times 1 \times 1.6 \times 10^{-19}}{(2 \times 1.6 \times 10^{-19})^2} \times \exp \left[\frac{-2\sqrt{(2 \times 6.4 \times 10^{-27} \times 1 \times 1.6 \times 10^{-19}) \times 10^{-10}}}{1.05 \times 10^{-34}} \right]$$

$$T = 4e^{-86}$$

UNIT - II

Q.3. Evaluate the Fermi function for an energy $K_B T$ above the Fermi energy

Ans. The Fermi function is given by

$$F(E) = \frac{1}{1 + \exp \left[\frac{E - E_F}{K_B T} \right]}$$

Given that, $E - E_F = K_B T$

$$\therefore F(E) = \frac{1}{1 + \exp(1)} = \frac{1}{1 + 2.78}$$

$$= \frac{1}{3.78} = 0.269$$

Q.4. In a solid consider the energy level lying 0.01 eV below Fermi level. What is the probability of this level not being occupied by an electron? Given $(E - E_F) = 0.01\text{ eV}$, $K_B T = 0.026\text{ eV}$ at $T = 300\text{ K}$.

Ans. The probability of an energy level E not being occupied by an electron is given by $(1 - F(E))$. Therefore,

$$1 - F(E) = 1 - \frac{1}{1 + \exp \left[\frac{E - E_F}{K_B T} \right]}$$

$$= \frac{1 + \exp \left[\frac{E - E_F}{K_B T} \right] - 1}{1 + \exp \left[\frac{E - E_F}{K_B T} \right]}$$

$$\begin{aligned} &= \frac{1}{\exp\left(\frac{E_F - E}{K_B T}\right) + 1} = \frac{1}{\exp\left(\frac{0.01}{0.026}\right) + 1} \\ &= \frac{1}{\exp(0.303) + 1} = \frac{1}{1.47} \approx 0.683 \end{aligned}$$

Q.8. At what temperature we can expect a 10% probability that electrons in silver have an energy which is 1% above Fermi energy? The Fermi energy of silver is 5.5 eV

Ans. We know that

$$F(E) = \frac{1}{1 + \exp[(E - E_F)/K_B T]}$$

Given that, $F(E) = 10\% = 0.1$ and $E_F = 5.5$ eV

$$E = E_F + \frac{1}{100} E_F = (5.5 + 0.055) = 5.555 \text{ eV}$$

$$\begin{aligned} E - E_F &= 5.555 - 5.5 = 0.055 \text{ eV} \\ &= 0.055 \times (1.6 \times 10^{-19}) \text{ J} \end{aligned}$$

Substituting these values, we get

$$0.1 = \frac{1}{1 + \exp\left[\frac{0.055 \times (1.6 \times 10^{-19})}{1.38 \times 10^{-23} \times T}\right]}$$

$$0.1 = \frac{1}{1 + \exp\left(\frac{637.7}{T}\right)}$$

$$\exp\left(\frac{637.7}{T}\right) + 1 = 10 \text{ or } \exp\left(\frac{637.7}{T}\right) = 9$$

$$\frac{637.7}{T} = \log_e(9) \text{ or } T = \frac{637.7}{\log_e 9}$$

$$T = \frac{637.7}{2.197} = 290.2 \text{ K}$$

Q.6. Show that the occupation probability at $E = E_F + \Delta E$ is equal to the non-occupation probability at $E = E_F - \Delta E$.

Ans. The occupation probability is given by

$$F(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{K_B T}\right]} \quad \dots(1)$$

At $E = E_F + \Delta E$, the occupation probability is given by

$$[F(E)]_{E_F + \Delta E} = \frac{1}{1 + \exp\left[\frac{\Delta E}{K_B T}\right]} \quad \dots(2)$$

The non-occupation probability is given by

$$1 - F(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{K_B T}\right)}$$

$$\text{Non-occupation probability} = \frac{1}{\exp\left(\frac{E_F - E}{K_B T}\right) + 1}$$

At $E = E_F - \Delta E$,

$$\text{Non-occupation probability} = \frac{1}{\exp\left(\frac{\Delta E}{K_B T}\right) + 1}$$

Comparing eqs. (2) and (3), we see that occupation probability for $E = (E_F + \Delta E)$ is equal to non-occupation probability for $E = (E_F - \Delta E)$.

Q.7. Calculate the number of energy states available for the electrons in a cubical box of side 1 cm lying below an energy of 1 eV.

Ans. We know that

$$Z(E)dE = \frac{\pi}{2} \left(\frac{8mL^2}{h^2} \right)^{3/2} E^{1/2} dE$$

$$\text{or } Z(E)dE = V \left(\frac{\pi}{2} \right) \left(\frac{8m}{h^2} \right)^{3/2} E^{1/2} dE$$

The number of energy states below one electron volt is given by

$$\begin{aligned} \int_b^{1\text{eV}} Z(E)dE &= V \left(\frac{\pi}{2} \right) \left(\frac{8m}{h^2} \right)^{3/2} \int_b^{1\text{eV}} E^{1/2} dE \\ &= V \left(\frac{\pi}{2} \right) \left(\frac{8m}{h^2} \right)^{3/2} \left[\frac{2}{3} E^{3/2} \right]_b^{1\text{eV}} \end{aligned}$$

Substituting the values, we get

$$\begin{aligned} \int_b^{1\text{eV}} Z(E)dE &= (10^{-6}) \left(\frac{3.14}{2} \right) \left[\frac{8 \times (9.1 \times 10^{-31})}{(6.62 \times 10^{-34})^2} \right]^{3/2} \left(\frac{2}{3} \right) \times [(1 \times 1.6 \times 10^{-19})]^{3/2} \\ &= 4.5 \times 10^{21} \end{aligned}$$

Q.8. Fermi velocity of electron in Cs metal is 0.73×10^6 m/s. Calculate its Fermi energy in eV.

Also compute its Fermi temperature.

Ans.

$$\text{K.E.} = \frac{1}{2} m v_F^2 = E_F$$

$$\begin{aligned} E_F &= \frac{1}{2} (9.1 \times 10^{-31}) \times (0.73 \times 10^6)^2 \\ &= 2.42 \times 10^{-19} \text{ joule} \\ &= \frac{2.42 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV} \end{aligned}$$

Further

$$K_B T_F = E_F$$

$$T_F = \frac{E_F}{K_B} = \frac{2.42 \times 10^{-19}}{1.38 \times 10^{-23}} = 1.75 \times 10^4 \text{ K}$$

Q.9. For lithium, the Fermi energy is 4.70 eV and density of electrons is $6 \times 10^{28}/\text{m}^3$. Find the electron density for a metal with Fermi energy 2.35 eV.

Ans. We know that

$$n = \left(\frac{2m}{h^2} \right)^{3/2} \left(\frac{8\pi}{3} \right) (E_F(0))^{3/2}$$

or

$$\begin{aligned} N_{\text{metal}} &= n_{\text{lithium}} \times \left(\frac{2.35}{4.70} \right)^{3/2} \\ &= (4.6 \times 10^{28}) \times (0.2128)^{3/2} \\ &= 1.6 \times 10^{28}/\text{m}^3 \end{aligned}$$

Q.10. Calculate the Fermi energy in eV for a metal at 0 K where density is 0500 kg/m^3 , atomic weight is 107.9 and it has one conduction electron per atom. (Given $1 \text{ J} = 6.24 \times 10^{18} \text{ eV}$ and $N_A = 6.025 \times 10^{26}/\text{k mole}$)

Ans. The electron concentration is given by

$$\begin{aligned} n &= \frac{\text{Number of free electrons per atom} \times N_A \times d}{A} \\ &= \frac{1 \times (6.025 \times 10^{26}) \times 10500}{107.9} = 5.863 \times 10^{28} \end{aligned}$$

The Fermi energy at 0 K is given by

$$\begin{aligned} E_{F(0)} &= \left(\frac{h^2}{8m} \right) \left(\frac{3}{\pi} \right)^{2/3} n^{2/3} \\ &= \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31}} \times \left(\frac{3}{3.14} \right)^{2/3} \times (5.863 \times 10^{28})^{2/3} \\ &= (5.85 \times 10^{-28}) \times 1.5091 \times 10^{19} \\ &= 8.83 \times 10^{-19} \text{ J} \\ &= (8.83 \times 10^{-19}) \times (6.24 \times 10^{18}) \text{ eV} \\ &= 5.5 \text{ eV} \end{aligned}$$

UNIT - IV

Q.11. The forward current through a silicon diode is 10 mA at room temperature (27° C). The corresponding forward voltage is 0.75 V. Calculate the reverse saturation current.

Ans. Given, $I = 10 \text{ mA}$, $V = 0.75 \text{ Volt}$ and $T = 27^\circ \text{C}$

$$V_T = \frac{T}{11,600} = \frac{300}{11,600} = 26 \text{ mV}$$

Now

$$I = I_0 \left[\left(e^{\frac{V}{nV_T}} \right) - 1 \right]$$

$$\text{or } 10 \times 10^{-3} = I_0 \left[\exp \left(\frac{0.75}{2 \times (26 \times 10^{-3})} \right) - 1 \right]$$

Solving we get, $I_n = 5.446 \text{ nA}$

Q.12. A germanium P-N junction diode have reverse saturation current of $4 \times 10^{-6} \text{ A}$ at 25°C . If a sine wave having peak amplitude of 0.15 V is applied across the junction, what is rectification ratio (ratio of forward to reverse peak currents)?

Ans. Given, $I_n = 4 \times 10^{-6} \text{ A}$, Forward Voltage, $V = 0.15 \text{ V}$

Now

$$V_T = \frac{T}{11600} = \frac{273 + 25}{11600} = \frac{298}{11600}$$

$\eta = 1 \text{ for Ge}$

$$I_F = 4 \times 10^{-6} \left[\exp\left(\frac{0.15 \times 11600}{298}\right) - 1 \right]$$

Solving we get

$$I_F = 1.32 \times 10^{-3} \text{ A}$$

For negative half cycle, $V = -0.15 \text{ V}$. Therefore, reverse current

$$I_R = 4 \times 10^{-6} \left[\exp\left(\frac{-0.15 \times 11600}{298}\right) - 1 \right]$$

$$= -4 \times 10^{-6} \text{ A}$$

$$\text{Ratio} = \frac{|I_R|}{|I_F|} = \frac{1.32 \times 10^{-3}}{4 \times 10^{-6}} = 330$$

Q.13. For what voltage, will the reverse current in P-N junction Ge diode reach 90% of its saturation value at room temperature?

Ans. Here, $T = 300 \text{ K}$ (room temperature)

$$V_T = \frac{300}{11600}, (\eta = 1 \text{ for Ge})$$

Now

$$I = I_0 \left[\exp\left(\frac{V}{\eta V_T}\right) - 1 \right]$$

$$-0.9 I_0 = I_0 \left[\exp\left(\frac{V \times 11600}{300}\right) - 1 \right]$$

$$\text{or } \exp\left(\frac{V \times 11600}{300}\right) = 0.1$$

Taking log of both sides, we get

$$\frac{V \times 11600}{300} = \log(0.1) = -2.3 \Rightarrow V = -0.06 \text{ Volt}$$

Q.14. Write short notes on:

- (i) Photodiode, and
- (ii) LEDs.

Ans. (i) Photodiode

A built-in-voltage exists across a p-n junction, but this voltage cannot deliver current in an external circuit. However, if light is shown on the junction, there is current in the circuit. This effect is called the photovoltaic effect, and the junction is known as photodiode.

The principle underlying the effect, let the junction be illuminated. Under this conditions many excess electron-hole pairs are generated in the regions on the either side of the junction via absorption of photons. As the field within the junction is from n-side to p-side, the excess minority carriers thus generated diffuse to the junction where they are carried across and become majority carriers, the holes generated on n-side move towards p-side and the electron generated on p-side move towards n-side. If the junction (+ ve charge on p-side and -ve charge on n-side) tending to lower the built-in-voltage from V_B to some value $V_B - V_o$, as shown in Fig.1. This change in built-in voltage appears as a measurable potential difference across the junction, which thereby behaves as if a source of voltage V_o .

If the external circuit is closed, the current will therefore, flow therein. This current will continue so long as there is diffusion of excess electron from n-side and of excess holes from p-side. This means that the current will flow so long as the semiconductor regions are illuminated. This explains how the incident light sets up the current flow in the external circuit.

Photodiodes can be used as useful electrical generators employing solar energy that strikes the earth, i.e., as solar cells. Such a cell has indeed been built and operated, but its efficiency is not as much as one would wish. The problem is mainly connected with difficulties in fabricating high-quality junctions of large areas necessary to intercept the maximum amount of incident light.

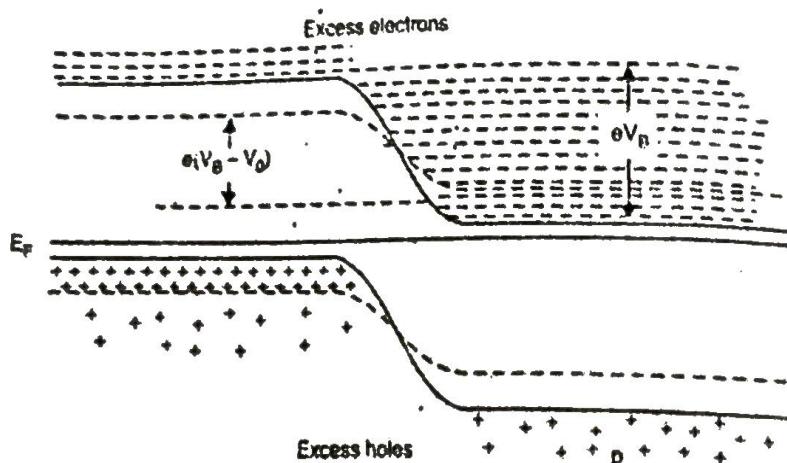


Fig 1. Band model of a photodiode. Band edges shown by solid line for darkness and by dotted line for illuminated.

Another major use of photodiode is in radiation detection. The radiation to be detected is allowed to fall on a suitable p-n junction and the voltage produced across the junction is measured. This voltage is proportional to the intensity of incident radiation. A typically photodetector is CdS.

(ii) Light-Emitting Diode (LED)

Light - emitting diodes are $p-n$ junction diodes which emit electromagnetic radiation mostly in the infrared or visible regions of em spectrum when an electric current flows through these. LEDs are generally made up of III-V compound semiconductors, such as GaAs, which have a direct band gap. They worked on the principle that the flow of electric current through a forward biased p-n diode causes the injection of minority carriers into the regions where they can recombine with majority carriers thus producing electromagnetic radiation. The effect is known as injection electroluminescence. In indirect band gap semiconductor, such as Si or Ge, where the electrons in the conduction band have a momentum different from holes in the valence band, this type of radiati

recombination is unlikely to occur, generally, the recombination releases heat to the lattice in such cases. The wavelength of emitted radiations is given by

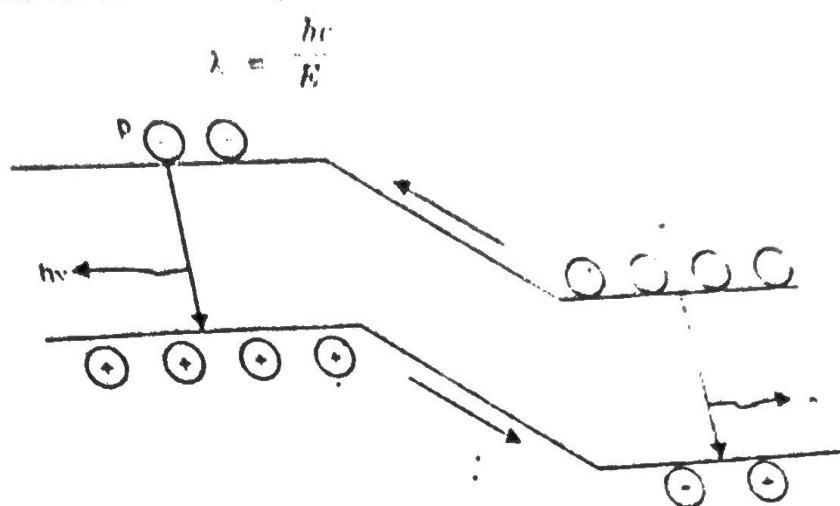


Fig. 2

where h is Planck's constant, c the velocity of light and E_g , the band gap energy. The transition of electrons and then emission of photons are shown in Fig. 2. Most of the radiations are emitted within a distance comparable to the diffusion length of the carriers from the junction. Substituting the values of h and c , and expressing E_g in eV in the above expression, we get

$$\lambda = \frac{1.24}{E_g} \mu\text{m}$$

Diodes built with $\text{GaAs}_{1-x}\text{P}_x$ are commercially variable because of lower cost and ease of fabrication. The brightness of such diodes is a maximum at $x = 0.4$ when $E_g = 1.9 \text{ eV}$. The corresponding radiation is red and the wavelength is about 6500\AA . By changing x it is possible to obtain orange, yellow and green emissions but reduced brightness and efficiency. $\text{GaAs}_{1-x}\text{P}_x$ light-emitting diode are used in the manufacture of signal lamps and displays. The infrared LED is a potential source for optical-fibre communication.

Q.15. In a given n-type Si material, the donor concentration is 1 atom per $2 \times 10^8 \text{ Si atoms}$. Assuming that the effective mass of the electron is equal to the true mass. Find the value of the temperature at which the Fermi level coincides with the edge of the conduction band. Given density of atoms in Si = $5 \times 10^{28} \text{ atoms/m}^3$.

Ans. Given the density of atoms in Si = $5 \times 10^{28} \text{ atoms/m}^3$

$$N_d = \frac{5 \times 10^{28}}{2 \times 10^8} = 2.5 \times 10^{20} \text{ per m}^3$$

The Fermi level is equal to conduction band only if $N_d = N_e$
We know that

$$N_e = 2 \left[\frac{2\pi m_e K_B T}{h^2} \right]^{3/2}$$

or

$$2.5 \times 10^{20} = 2 \left[\frac{2\pi m_e K_B}{h^2} \right]^{3/2} T^{3/2} = 4.82 \times 10^{21} T^{3/2} \text{ per m}^3$$

Which gives $T = 0.14 \text{ K}$

(if m_e = mass of electron)

Q.16. For the circuit shown in the Fig. 3. Find: (i) output voltage (ii) drop across series resistance (iii) current through Zener diode.

Ans.

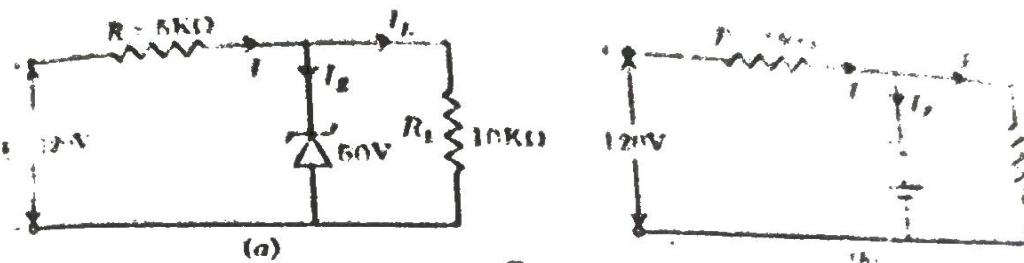


Fig. 3

If the zener diode is removed, then open circuit voltage is

$$V = \frac{R_L E_i}{R + R_L} = \frac{10 \times 120}{5 + 10} = 80 \text{ V}$$

Since 80 V is greater than V_z (50 V), so the Zener is in ON state

(i) Referring to their Fig. 3 (b)

$$\text{output voltage} = V_z = 50 \text{ V}$$

$$\text{(ii) Voltage drop across } R = 120 - 50 = 70 \text{ V}$$

$$\text{(iii) Load current, } I_L = V_z/R_L = \frac{50 \text{ V}}{10 \text{ K}\Omega} = 5 \text{ mA}$$

$$\text{Current through } R, \quad I = \frac{70 \text{ V}}{5 \text{ K}\Omega} = 14 \text{ mA}$$

Applying Kirchoff's first law,

$$I = I_L + I_z$$

$$\therefore \text{Zener current, } I_z = I - I_L = 14 - 5 \\ = 9 \text{ mA}$$

Q.17. A 10 V Zener diode is used to regulate the voltage across a variable load resistor. The input voltage varies between 13V and 16V and the load current varies between 10 and 85 mA. The minimum Zener current is 15mA. Calculate the value of series resistance R.

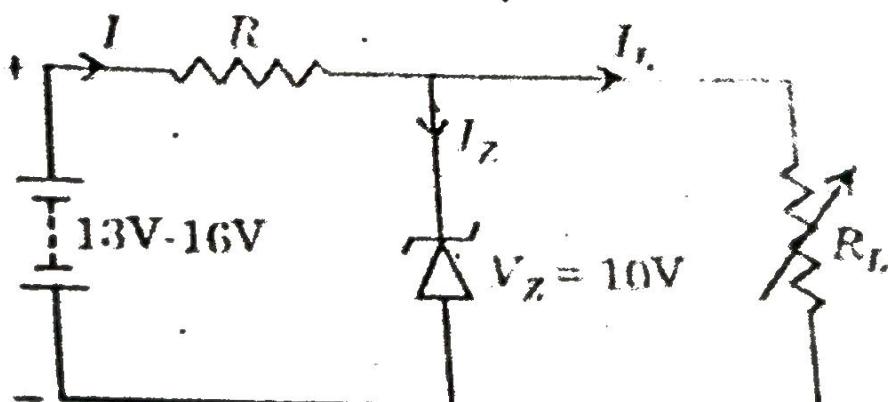


Fig. 4

Ans. The Zener will conduct minimum current (i.e., 15 mA) when input voltage is minimum (i.e., 13V),

$$R = \frac{E_i - E_o}{(I_Z)_{\min} + (I_L)_{\max}}$$

$$= \frac{3V}{100mA} = 30 \Omega$$

Q.18. Over what range of input voltage will the Zener circuit shown in Fig. maintain 30V across 2000 Ω load; assuming that series resistance $R = 200\Omega$ and zener current rating is 25 mA?

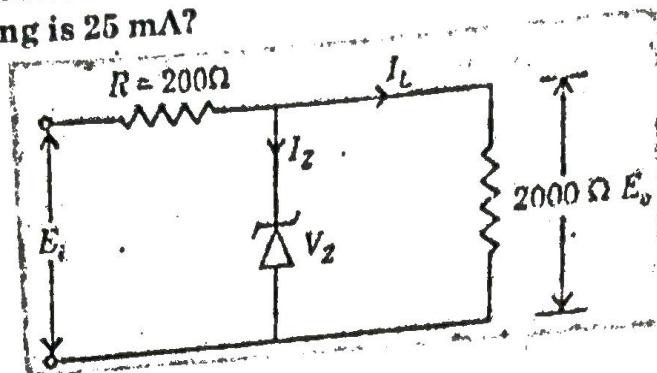


Fig. 5

Ans. The minimum input voltage required will be when $I_z = 0$, Then

$$I_L = I = \frac{30V}{2000\Omega} = 15 \text{ mA}$$

$$\begin{aligned}\therefore \text{Minimum input voltage} &= 30 + IR \\ &= 30 + 15 \text{ mA} \times 200\Omega \\ &= 30 + 3 = 33 \text{ V}\end{aligned}$$

The maximum input voltage required will be when

$$I_z = 25 \text{ mA}$$

Under such condition

$$\begin{aligned}I &= I_L + I_z \\ &= 15 + 25 = 40 \text{ mA}\end{aligned}$$

$$\begin{aligned}\therefore \text{Max input voltage} &= 30 + I_R \\ &= 30 + 40 \text{ mA} \times 200\Omega \\ &= 30 + 8 = 38 \text{ V}\end{aligned}$$

So the input voltage range is 33 - 38 V.

**PAST EXAMINATION QUESTIONS AS PER NEW
SYLLABUS EFFECTIVE FROM 2021-22 ONWARDS**

UNIT I

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FIRST TERM EXAMINATION [FEB.-2016]

SECOND SEMESTER [B.TECH]

APPLIED PHYSICS-II [ETPH-104]

Time: 1 hrs.

M.M. : 30

Note: Q. No. 1 is compulsory and attempt any two more questions from the rest.

Q.1. (a) State Gauss Law in magnetostatics. Write its integral form and mention its physical interpretation. (2)

Ans. The net outward magnetic flux through any closed surface is zero. In integral form.

$$\int_{\text{S}} \vec{B} \cdot d\vec{s} = 0$$

where \vec{B} is the magnetic induction and measured in weber/m².

This equation signifies that "the outward flux of magnetic induction \vec{B} through any closed surface is equal to zero". There exists no magnetic monopoles.

Q.1. (b) Compare the salient features of MB, BE and FD statistics with example. (2)

Ans. Comparison of M.B, B.E. and F.D. Statistics are given below

Maxwell-Boltzmann statistics (Classical statistics)	Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum (statistics))
1. Apply to systems of distinguishable particles, such as gases.	Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature	Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)
2. The distribution law is $n_i = \frac{g_i}{e^{\alpha} e^{E_i/k_B T}}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i/k_B T} - 1)}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha} e^{E_i/k_B T} + 1)}$
3. There is no restriction on the number of particles in the given state.	There is no restriction on the number of particles in a given state.	Only one particle in a given quantum state is allowed.
4. In the phase space the volume of the phase cell is not fixed.	The volume of phase cell is of order of h^3 .	The volume of phase cell is of the order of h^3
5. The behaviour of distribution function $f(E_i)$ against E_i is exponential.	For $E_i \gg k_B T$, exponential $E_i \ll k_B T$, lies above M.B.	For $E_i \gg k_B T$, exponential where $E_i > E_F$. If $E_F \gg k_B T$ decreases abruptly near E_F .
6. The energy of M.B. system, such as in ideal monoatomic gas, may be zero at absolute zero.	The energy may be zero at absolute zero.	The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

Q.1. (c) Write the equation of continuity and explain displacement current. (2)

Ans. Equation of continuity is $A_1 J_1 + A_2 J_2 = 0$

Displacement Current

The displacement current does not have significance like current in which the charge actually move. To demonstrate the displacement current, consider a simple circuit (Fig. 1) where the parallel plates of a capacitor are joined by a conducting wire. The current flowing in the circuit is equal to rate of change of charge on the plates i.e.

$$I = \frac{dq}{dt}$$

where q is the charge on the positive plate of the capacitor and is related to the field E in the capacitor as

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

where A is the area of the plates, σ and ϵ_0 are the surface charge density on the plate and the permittivity of medium between (air) the plates respectively. Thus

$$I = \frac{dq}{dt} = \epsilon_0 A \frac{\partial E}{\partial t} = A \frac{\partial D}{\partial t}$$

($\because \epsilon_0 E = D$)

But

$$\frac{\partial D}{\partial t} = J'$$

Thus

$$\frac{I}{A} = \frac{\partial D}{\partial t} = J'$$

Therefore, the displacement current in the gap between the capacitor
 $= JA = I$

It means the displacement current directly passes through the capacitor. Thus the quantity $\partial D / \partial t$ can be interpreted as the density of some current which flows in space between a pair of plates of a capacitor, when charged plates are connected by wires to complete the circuit, thus completing the conduction current. This shows that concept of displacement current satisfies the basic principle that the current is continuous.

Q.1. (d) What is de-Broglie wavelength associated with an electron accelerated through a potential difference of 1000V? (2)

Ans. Given

$$V = 1000V$$

De-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where

$$h = 6.62 \times 10^{-34} J-S$$

$$m = 9.1 \times 10^{-31} kg$$

$$q = 1.6 \times 10^{-19}$$

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1000}} = 0.389 A^\circ$$

Q.1. (e) Using Uncertainty principle explain the non-existence of electrons inside the nucleus. (2)

Ans. According to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x = \frac{h}{2\pi} \quad (1)$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty Δx in the position of electron is the same as the diameter of the nucleus.

$$\Delta x = 10^{-14} \text{m}$$

The minimum uncertainty in the momentum is given by

$$\begin{aligned} \Delta p_x &= \frac{h}{2\pi\Delta x} \\ &= \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-14}} = \frac{6.63 \times 10^{-20}}{2\pi} = 1.055 \times 10^{-20} \text{ kg m/sec.} \end{aligned}$$

It means that if electron exists in the nucleus, its minimum momentum must be

$$p_{\min} = 1.055 \times 10^{-20} \text{ kg m/s}$$

For the electron of the minimum momentum, the minimum energy is given by

$$\begin{aligned} E_{\min}^2 &= p_{\min}^2 c^2 + m_0^2 c^4 \\ &= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4 \\ &= (3 \times 10^8)^2 [1.113 \times 10^{-40} + 7.4692 \times 10^{-44}] \end{aligned}$$

Since the second term in the bracket is much smaller than the first, it can be neglected, then,

$$\begin{aligned} E_{\min} &= 3 \times 10^8 \times \sqrt{1.113 \times 10^{-40}} \text{ J} \\ &= 3 \times 10^8 \times 1.055 \times 10^{-20} \text{ J} = 3.1649 \times 10^{-12} \text{ J} \end{aligned}$$

or $E_{\min} = \frac{3.1649 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} = 20 \text{ MeV.}$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β -particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons can not be present within the nucleus.

Q.2. (a) Show that the velocity of electromagnetic wave in isotropic dielectric is always less than the velocity in free space. (5)

Ans. A non conducting medium which has same properties in all directions is called an isotropic dielectric.

The fundamental Maxwell's equations for the time varying fields are

$$\bar{\nabla} \cdot \bar{D} = \rho; \quad \bar{\nabla} \cdot \bar{B} = 0$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}; \quad \bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

In an isotropic dielectric (or non-conducting) medium

$$\bar{J} = \sigma \bar{E}, \bar{B} = \mu \bar{H} \text{ and } \bar{D} = \epsilon \bar{E}$$

where ϵ and μ are the permittivity and permeability of the medium. For a non-conducting medium (i) the conductivity σ would be zero, so that $\bar{J} = \sigma \bar{E} = 0$ (ii) the medium is taken as isotropic, so there is no volume distribution.

With these conditions, Maxwell's equations for non-conducting medium are

$$\left. \begin{array}{l} \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \vec{E} = 0 \quad (a) \\ \vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \vec{H} = 0 \quad (b) \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (c) \\ \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad (d) \end{array} \right\}$$

Wave equation for electric vector \vec{E} and magnetic vector \vec{H}

Taking curl of eqn. (1), we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \left(\vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} \right)$$

or
$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

using eqns 1(a) and 1(d) we get

$$-\vec{\nabla}^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

or
$$\vec{\nabla}^2 \vec{E} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Similarly taking curl of eqn. 1(d), we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \epsilon \vec{\nabla} \times \frac{\partial \vec{E}}{\partial t}$$

or
$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 \vec{H} = \epsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

Putting value of $\vec{\nabla} \cdot \vec{H}$ and $\vec{\nabla} \times \vec{E}$ from eqns. 1(b) and 1(c) respectively, we get

$$-\vec{\nabla}^2 \vec{H} = \epsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial \vec{H}}{\partial t} \right)$$

or
$$\vec{\nabla}^2 \vec{H} = \epsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2}$$

or
$$\vec{\nabla}^2 \vec{H} - \epsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

Comparing eqns. (2) and (3) with general wave equation

$$\Delta^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

we find that electric and magnetic field vectors \vec{E} and \vec{H} propagate as wave in non-conducting medium with a velocity

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

Thus, we can write eqns. (2) and (3) as

$$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad ... (3)$$

and

$$\nabla^2 \vec{H} - \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad ... (4)$$

these are the wave equations for the electromagnetic waves in non-conducting medium, propagating with velocity

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \quad ... (5)$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$ is the velocity of electromagnetic waves in free space. μ_r and ϵ_r are relative permeability and permittivity respectively and greater than unity. Thus $v < c$, i.e. velocity of e.m waves in free space.

Q.2. (b) Prove the orthogonality of \vec{E} , \vec{H} and \vec{k} vectors. ... (6)

Ans. Wave equation:

$$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad ... (1)$$

$$\nabla^2 \vec{H} - \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad ... (2)$$

have plane wave solutions

$$\vec{E}(r,t) = \vec{E}_0 e^{i\vec{k} \cdot \vec{r} - i\omega t} \quad ... (3)$$

$$\vec{H}(r,t) = \vec{H}_0 e^{i\vec{k} \cdot \vec{r} - i\omega t} \quad ... (4)$$

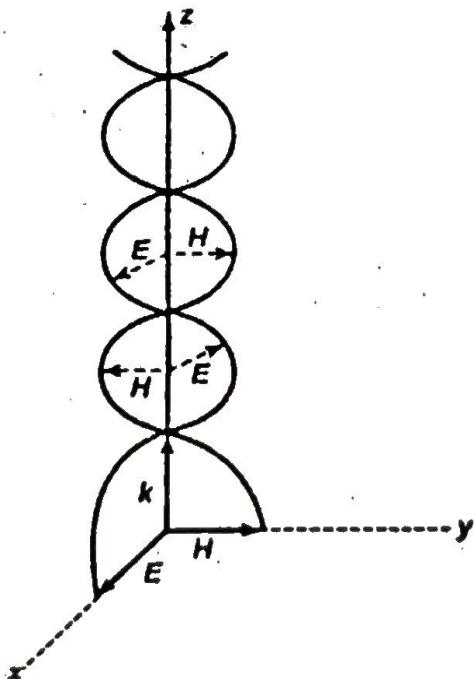


Fig.2. Propagation of electromagnetic wave.

where E_0 and H_0 are complex amplitudes which are constant in space and time, while k is wave propagation vector given by

$$\vec{k} = \vec{h} \times \vec{n} = \frac{2\pi}{\lambda} \vec{n} \approx \frac{\omega}{c} \vec{n}$$

Here \vec{n} is a unit vector in the direction of wave propagation.

$$\text{curl } \vec{E} = \frac{\partial \vec{H}}{\partial t} \text{ and curl } \vec{H} = \frac{\partial \vec{E}}{\partial t}$$

using eqns. (3 and 4), the above equations yield

$$\vec{k} \times \vec{E} = \mu_0 \vec{H}$$

and

$$\vec{k} \times \vec{H} = -\epsilon_0 \vec{E}$$

From these equations it is obvious that field vectors \vec{E} and \vec{H} are mutually perpendicular and also they are perpendicular to the direction of propagation vector \vec{k} . This in turn implied that a plane electromagnetic wave in isotropic dielectric, vector $(\vec{E}, \vec{H}, \vec{k})$ form a set of orthogonal vectors which form a right handed coordinate system in a given order (Fig. 2)

Q.2. (c) Calculate the skin depth at a frequency of 1.6 MHz in aluminium

where $\sigma = 38.2 \text{ MS/m}$ and $\mu_r = 1$.

Ans. Given

$$\omega = 2\pi f \Rightarrow \text{But } f = 1.6 \times 10^6 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 1.6 \times 10^6$$

$$\mu_r = 1 \text{ and } \mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1$$

$$\sigma = 38.2 \times 10^6 \text{ S/m}$$

and

Skin depth P_s given by

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} \quad \delta = \sqrt{\frac{2}{4\pi \times 10^{-7} \times 38.2 \times 10^6 \times 1.6 \times 10^6 \times 2\pi}}$$

$$\delta = \sqrt{\frac{2 \times 10^{-6}}{482.095 \times 10^5}} = \sqrt{\frac{2 \times 10^{-6}}{482.095}} = \sqrt{0.00415 \times 10^{-3}}$$

$$\delta = 0.0644 \times 10^{-3} = 64.4 \times 10^{-6} \text{ m}$$

Q.3. (a) Derive Planck's formula for distribution of energy in the spectrum of blackbody. Hence deduce Wien's displacement law from it.

Ans. Keeping in view the discrepancies of Rayleigh-Jeans and Wien's expression with observed spectra of black bodies, Planck tackled the problem with entirely different concept, called Planck's hypothesis.

In 1901 Planck was able to derive a theoretical expression for energy distribution of a black body on the basis of quantum theory of heat radiations. He made following assumptions:

(i) A black-body chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators of molecular dimensions (called as Planck's oscillators or resonators) which can vibrate with all possible frequencies. The vibration of the resonator entails one degree of freedom only.

(ii) The classical principle of equipartition of energy is not applicable to black body oscillators because this assumption would lead to the Rayleigh-Jean's expression. Instead it is assumed that the oscillator of the black body cannot have any amount of energy but has a discrete energy equal to the integral multiple of some minimum

$$\epsilon = nh\nu$$

where ν is the frequency of an oscillator and n an integer and h is Planck's constant and its value is $6.626 \times 10^{-34} \text{ J.s}$

(iii) The oscillators can not radiate or absorb energy continuously. However, the oscillators can exchange energy (emission or absorption) with its surroundings in discrete values viz. $0, h\nu, 2h\nu, 3h\nu, \dots nh\nu$, i.e., the multiples of some unit called the quantum. This quantum of radiation is called photon.

In order to derive Planck's radiation law, we shall first derive the number of resonators per unit volume lying in the frequency range ν and $\nu + d\nu$ and the average energy of Planck's resonator.

Suppose N be the total number of Planck's oscillators and E their total energy. Then the energy per oscillator is given as follows:

$$\bar{\epsilon} = \frac{E}{N}$$

If $N_0, N_1, N_2, \dots, N_r, \dots$ etc. are the number of oscillators having energies $0, \epsilon, 2\epsilon, r\epsilon, \dots$ etc respectively, then we get

$$N = N_0 + N_1 + N_2 + \dots + N_r + \dots \quad (1)$$

$$E = 0 + \epsilon N_1 + 2\epsilon N_2 + \dots + r\epsilon N_r + \dots \quad (2)$$

From Maxwell's distribution formula it follows that the number of oscillators having energy $r\epsilon$ will be

$$\left. \begin{aligned} N_r &= N_0 e^{-r\epsilon/kT} \\ N_1 &= N_0 e^{-\epsilon/kT} \\ N_2 &= N_0 e^{-2\epsilon/kT} \end{aligned} \right\} \quad \dots(3)$$

On substituting from equation (3), the values of N_1, N_2, \dots etc. in equations (1) and (2), we obtain

$$\begin{aligned} N &= N_0 + N_0 e^{-\epsilon/kT} + N_0 e^{-2\epsilon/kT} + \dots + N_0 e^{-r\epsilon/kT} + \dots \\ &= N_0 [1 + e^{-\epsilon/kT} + e^{-2\epsilon/kT} + \dots + e^{-r\epsilon/kT} \dots] \\ N &= \frac{N_0}{1 - e^{-\epsilon/kT}} \quad \dots(4) \end{aligned}$$

and the total of Planck's resonators will be

$$\begin{aligned} E &= 0 + \epsilon N_0 e^{-\epsilon/kT} + 2\epsilon N_0 e^{-2\epsilon/kT} + \dots + r\epsilon N_0 e^{-r\epsilon/kT} + \dots \\ &= N_0 \epsilon [e^{-\epsilon/kT} + 2e^{-2\epsilon/kT} + \dots + re^{-r\epsilon/kT} + \dots] \\ &= N_0 \epsilon e^{-\epsilon/kT} [1 + 2e^{-2\epsilon/kT} + \dots + re^{-r\epsilon/kT} + \dots] \\ E &= \frac{N_0 \epsilon e^{-\epsilon/kT}}{(1 - e^{-\epsilon/kT})^2} \left[\because \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots \right] \quad \dots(5) \end{aligned}$$

The average energy of resonator or oscillator is given as follows:

$$\bar{\epsilon} = \frac{E}{N} = \frac{\frac{N_0 \epsilon e^{-\epsilon/kT}}{(1 - e^{-\epsilon/kT})^2}}{\frac{N_0}{1 - e^{-\epsilon/kT}}} = \frac{N_0 \epsilon e^{-\epsilon/kT}}{(1 - e^{-\epsilon/kT})^2}$$

$$\bar{e} = \frac{e^{hv/kT}}{(1-e^{-hv/kT})} = \frac{e^{hv/kT}}{e^{hv/kT}-1}$$

According to Planck's hypothesis of quantum theory $\epsilon = hv$; therefore the average energy of Planck's oscillator is given by

$$\bar{e} = \frac{hv}{e^{hv/kT}-1}$$

It is also known that the number of oscillators per unit volume in frequency range v and $v + dv$ will be given by

$$N = \frac{8\pi v^2}{c^3} dv$$

The average energy of the oscillator on multiplying by the number of oscillators per unit volume in the frequency range v and $v + dv$, will give the energy density (E_d) belonging to the range dv . Thus, energy density is as follows:

$$E_d dv = \frac{8\pi v^2}{c^3} dv \times \frac{hv}{e^{hv/kT}-1}$$

$$E_d = \frac{8\pi h v^3}{c^3} \times \frac{1}{e^{hv/kT}-1} dv$$

This is known as Planck's radiation law in terms of frequency.
This law in terms of wavelength may be represented as follows:

$$v = \frac{c}{\lambda} \text{ and } |dv| = \left| -\frac{c}{\lambda^2} d\lambda \right|$$

$$E_d d\lambda = \frac{8\pi h}{c^3} \left(\frac{c^3}{\lambda^5} \right) \frac{1}{e^{hc/\lambda kT}-1} \left(\frac{c}{\lambda^2} \right) d\lambda$$

$$E_d d\lambda = \frac{8\pi h c}{c^3} \frac{1}{e^{hc/\lambda kT}-1} d\lambda$$

This is Planck's radiation law in terms of wavelength.

When Planck's formula is differentiated with respect to λ and equating it to zero, the Wien's first displacement law ($\lambda T = \text{a const.}$) is obtained for wavelength which corresponds to maximum energy emission for a given value of T . thus

$$\frac{d}{d\lambda} E_\lambda = 8\pi ch \frac{d}{d\lambda} \left\{ \frac{\lambda^5}{e^{ch/\lambda kT}-1} \right\} = 0$$

This on simplification becomes as follows:

$$e^{ch/\lambda kT} \left(1 - \frac{ch}{5\lambda kT} \right) = 1$$

It is the form of

$$e^x \left(1 - \frac{x}{5} \right) = 1, \text{ where } x = \frac{ch}{\lambda kT}$$

The roots are

$$x = 0 \text{ and } \frac{x}{5} = 1 - \frac{1}{e^x} \text{ or } x = 4.965$$

But $x = 0$ is not possible. Therefore, we have

$$\frac{ch}{\lambda k T} = 4.965 \text{ or } \lambda T = \frac{ch}{4.965k} = \text{a const.}$$

Q.3. (b) If the average distance between the sun and the earth is $1.5 \times 10^{11} \text{ m}$ and the power radiated by the sun is $3.8 \times 10^{26} \text{ watt}$, show that the average Solar energy incident on earth is $2 \text{ cal/cm}^2 \cdot \text{min}$. (2)

Ans. Poynting vector = $\frac{\text{Power}}{\text{Area}}$

$$\text{Area} = 4\pi r^2$$

where, r = distance between sun and earth

$$\vec{S} = \frac{\vec{P}}{4\pi r^2}$$

where,

$$P = 3.8 \times 10^{26} \text{ watt} \text{ (given)} \quad r = 1.5 \times 10^{11} \text{ m} \text{ (given)}$$

$$\bar{S} = \frac{3.8 \times 10^{26}}{4 \times 3.14 \times (1.5 \times 10^{11})^2} \frac{\text{watt}}{\text{m}^2}$$

$$= \frac{3.8 \times 10^{26} \times 60}{4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 4.2 \times 10^4} \frac{\text{Cal}}{\text{cm}^2 \cdot \text{min}} = 2 \text{ cal/cm}^2 \cdot \text{min}$$

As,

$$1 \text{ Cal} = 4.2 \text{ J and} \\ 1 \text{ watt} = 1 \text{ J/E}$$

Q.4. (a) Derive an expression for energy Eigen values of a particle of mass "m" confined to a 1-D box of length L. State whether the permitted energy levels are quantized or continuous. (8)

Ans. Suppose a particle of mass m is in motion along the x -axis. Suppose no force is acting on the particles so that the potential energy of the particle is constant. For convenience, the constant potential energy is taken to be zero i.e., $V = 0$.

Hence Schrodinger equation becomes,

$$9 \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \dots(1)$$

$$\text{or,} \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{\hbar^2} E \psi = 0 \quad \dots(2)$$

Multiplying by $\frac{\hbar^2}{8\pi^2 m}$ throughout, we get $8\pi^2 m$.

$$\frac{\hbar^2}{8\pi^2 m} \cdot \frac{\partial^2 \psi}{\partial x^2} + E \psi = 0 \quad \text{or} \quad \frac{-\hbar^2}{8\pi^2 m} \frac{\partial^2 \psi}{\partial x^2} = E \psi \quad \dots(3)$$

Since the particle is moving freely with zero potential energy its total energy E is the kinetic energy, given by

$$E = \frac{p_x^2}{2m} \quad \dots(4)$$

where p_x is the momentum of the particle.

Rewriting Eq. (1),

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{\hbar^2} E \psi = 0 \quad \text{or} \quad \frac{\partial^2 \psi}{\partial x^2} K^2 \psi = 0 \quad \dots(5)$$

where

$$K_2 = \frac{8\pi^2 m E}{\hbar^2}$$

So that,

$$E = \frac{K^2 \hbar^2}{8\pi^2 m}$$

In this case energy E is not quantised. Hence when a particle is not bound in system it does not have quantised energy state.

Particle in box

Consider a particle moving inside a box along the x -direction. The particle is bouncing back and forth between the walls of the box a is the width of the box.

The potential energy V of the particle is infinite on both sides of the box.

$$V = 0 \text{ for } 0 < x < a$$

$$V = \infty \text{ for } x \leq 0 \text{ and } x \geq a$$

The particle can not exist outside the box, so its wave function ψ is 0 for $x \leq 0$ and $x \geq a$.

Within the box, the Schrodinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad [\because V \text{ is 0 for the free particle}] \quad \dots(7)$$

$$\frac{d^2\psi}{dx^2} + K^2 \psi = 0 \quad \text{where } K = \frac{\sqrt{2mE}}{\hbar} \quad \dots(8)$$

The general solution of this equation is

$$\psi = A \sin Kx + B \cos Kx$$

Using boundary conditions

$$\psi = 0 \text{ at } x = 0$$

$$0 = A \sin 0 + B$$

$$B = 0$$

$$\psi = 0 \text{ at } x = a$$

$$0 = A \sin Ka$$

$$\sin n\pi = \sin Ka$$

$$K = \frac{n\pi}{a}$$

Wave function,

$$\psi_n(x) = A \sin \frac{n\pi x}{a} \quad \text{where } n = 1, 2, 3, \dots \quad \dots(9)$$

Energy level,

$$E_n = \frac{K^2 \hbar^2}{2m} \left[\hbar = \frac{\hbar}{2\pi} \right]$$

\therefore Eigen value

$$E_n = \left(\frac{n\pi}{a} \right)^2 \left(\frac{\hbar}{2\pi} \right)^2 \frac{1}{2m}$$

$$E_n = \boxed{E_n = \frac{n^2 \hbar^2}{8ma^2}} \quad \text{where } n = 1, 2, 3, 4, \dots \quad \dots(10)$$

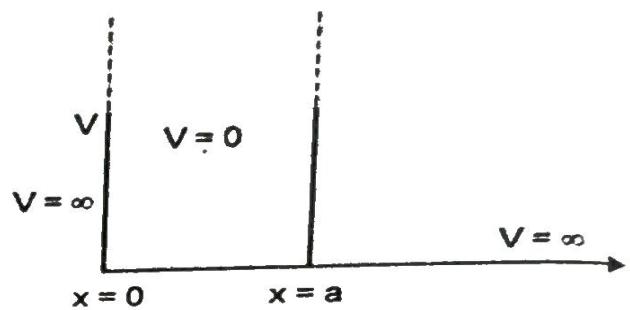


Fig.3. Particle in box

Therefore, it is clear from Eq. (10) that inside an infinitely deep potential well (or in an infinite square well), the particle cannot have an arbitrary energy, but can have only certain discrete energy corresponding to $n = 1, 2, 3, \dots$. Each permitted energy level energy is called *eigen-value* of the particle and constitutes the energy level of the system. The wave function ψ corresponding to each eigen value are called *eigen functions*.

Q.4. (b) Describe the implication of Davisson and Germer experiment. (2)

Ans. De-Broglie suggested that the dual nature of electromagnetic radiation may be extended to material particles such as electrons, protons, neutrons etc. It means that a moving particle have wave properties associated with it. The waves associated with these particles are known as matter waves or De-Broglie waves. The Davisson and Germer experiment on electron diffraction demonstrated these waves. According to this

$$\Rightarrow \lambda = \frac{h}{p} \text{ where, } p\text{-momentum of the particle.}$$

M.M.

END TERM EXAMINATION [MAY-2016]
SIXTH SEMESTER [B.TECH]
APPLIED PHYSICS-II [ETPH-104]

Time : 3 Hrs.

Note: Attempt any five questions including Q. No. 1 which is compulsory. Select one question from each unit.

Q.1 Attempt all the parts.

(a) Find the gradient of a vector -

$$A = (x^2 - xy + z) \hat{i} + (x^3 - xz + x) \hat{j} + (y^2 - y + z) \hat{k}$$

$$\bar{A} = (x^2 - xy + z) \hat{i} + (x^3 - z + x) \hat{j} + (y^2 - y + z) \hat{k}$$

Ans.

Taking A as scalar.

Gradient

$$\bar{A} = \bar{\nabla}A = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)]$$

$$= \frac{\partial}{\partial x} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{i} +$$

$$+ \frac{\partial}{\partial y} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{j} +$$

$$+ \frac{\partial}{\partial z} [(x^2 - xy + z) + (x^3 - xz + x) + (y^2 - y + z)] \hat{k}$$

$$= [2x - x + 3x^2 - x + 1 + 0] \hat{i} + [-x + 0 + 2y - 1] \hat{j} +$$

$$[1 - x + 1] \hat{k}$$

$$= [3x^2 + 1] \hat{i} + [2y - x - 1] \hat{j} + [2 - x] \hat{k}$$

At point (2, 1, 1)

$$\bar{\nabla}A = [3 \times 4 + 1] \hat{i} + [2 \times 1 - 2 - 1] \hat{j} + [2 - 2] \hat{k}$$

$$\boxed{\bar{\nabla}A = 13 \hat{i} - \hat{j}}$$

Q.1. (b) Write the expression of gradient in spherical coordinates.

Ans. In spherical co-ordinate gradient is given by following-

$$x = r \sin \theta \cos \phi; y = r \sin \theta \sin \phi; z = r \cos \theta$$

$$\bar{\nabla}f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{\partial f}{r \partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

Q.1. (c) Give the expression of Gauss's law for magnetic field in differential and integral form.

Ans. Gauss's law for magnetic field in differential form is-

$$\bar{\nabla} \cdot \bar{B} = 0$$

Gauss's law for magnetic field in integral form is-

$$\int \bar{B} d\bar{S} = 0$$

Q.1. (d) How will the equation for Poynting theorem change for static field?

Ans. Poynting theorem for static field is $\int_s (\bar{E} \times \bar{H}) d\bar{s} = 0$

Q.1. (e) Write the expression for general wave equation show how it changes in case of dielectric medium. (2)

Ans. The general wave equation is - $\nabla^2 \phi - \frac{1}{V^2} \frac{\partial^2 \phi}{\partial t^2} = 0$

For dielectric medium, the wave equation is - $\nabla^2 \phi - \mu \epsilon \frac{\partial^2 \phi}{\partial t^2} = 0$ (2)

Q.1. (f) Calculate the value of wave impedance in vacuum.

Ans.

$$Z_0 = \left| \frac{E}{H} \right| = \left| \frac{E_0}{H_0} \right| = \mu_0 c \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \left[c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right]$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 376.6 \text{ Ohms}$$

where the units of Z_0 are most easily seen from the fact that it measures a ratio of E in V/m to H in A/m and these must equal V/A or Ohms. Because the units of E/H are the same as those of impedance, the value of Z_0 is often referred to as the wave impedance of free space. Further, since the ratio $Z_0 = \left| \frac{E}{H} \right|$ is real and positive; this implies that field vectors E and H are in the same phase i.e., they have same relative magnitude at all points at all times.

Q.1. (g) State the significance of normalization of wave function. (2)

Ans. The normalized wave equation is - $\int_{-\infty}^{+\infty} \psi^* \psi dv = 1$

This normalized wave gives the physically applicable wave function or probability amplitude. All wave function, representing the real particles must be normalized. This help in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval. This integral cannot be infinite. The integral must be real quantity greater than or equal to zero if ψ has to describe a real body properly. Also the wave function should have single value and its first derivative and second derivate should be finite and continuous.

Q.1. (h) Are energy levels of a particle in a box equally spaced? Show with appropriate equation. (2)

Ans. The energy of particle in a box is given by following equation-

$$E_n = \frac{n^2 h^2}{8ml^2}$$

$$n = 1, 2, 3, 4, \dots$$

where

and l is the length of the box

Now, for $n = 1$

$$E_1 = \frac{h^2}{8ml^2}$$

for $n = 2$

$$E_2 = \frac{4h^2}{8ml^2}$$

for $n = 3$

$$E_3 = \frac{9h^2}{8ml^2}$$

As, $E_n \propto n^2$, the energy levels are not equally spaced.

Q.1. (i) What thermodynamic statistics do electrons follow, Bose-Einstein, Fermi-Dirac?

Ans. Electrons follow Fermi-Dirac statistics as electrons are indistinguishable and have half integral spin. Also, it has anti-symmetric wave function and obey Pauli exclusion principle.

Q.1. (j) Draw the plane in a unit cube represented by the Miller index (110).

Ans.

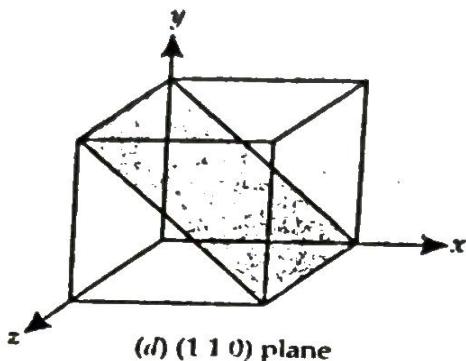


Fig. 1.

Q.1. (k) A p-type semiconductor has a Hall coefficient equal to $0.0125 \text{ m}^3/\text{C}$. Find the density of charge carrier in it.

Ans. Given

$$R_H = 0.0125 \text{ m}^3/\text{C}$$

$$R_H = \frac{1}{Ne}$$

$$N = \frac{1}{R_H e}$$

$$= \frac{1}{0.0125 \times 1.6 \times 10^{-19}} = 50 \times 10^{19} \text{ per m}^3$$

$$\boxed{N = 50 \times 10^{20} \text{ per m}^3}$$

Q.1. (l) State Bloch Theorem.

Ans. The Bloch theorem is a mathematical statement regarding the form of the electron wave functions for a perfectly periodic potential.

UNIT-I

Q.2. (a) Show that the trajectory of motion of a charged particle in crossed electric and magnetic fields (constant) is a cycloid.

Ans. If a charged particle (mass = m , charge = q) enters with velocity v in such a region where an electric field E and a magnetic field B simultaneously acts, then the force acting on the particle will be:

$$F = q\vec{E} + q\vec{v} \times \vec{B} - q[\vec{E} + ((\vec{v} \times \vec{B})]$$

Now if electric field E and magnetic field B are in mutually perpendicular Y and Z directions respectively, then

$$\vec{E} = E\hat{j} \text{ and } \vec{B} = B\hat{k}$$

Then force

$$F = q[E\hat{j} + (v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times B\hat{k}] \\ = qE\hat{j} + qB(v_y\hat{i} - v_x\hat{j})$$

But

$$F = m \left(\hat{i} \frac{dv_x}{dt} + \hat{j} \frac{dv_y}{dt} + \hat{k} \frac{dv_z}{dt} \right)$$

Hence the cartesian components of acceleration of the particle will be

$$\frac{dv_x}{dt} = \frac{qBv_z}{m}, \frac{dv_z}{dt} = \frac{qE}{m} - \frac{qBv_x}{m}$$

$$\frac{dv_y}{dt} = 0$$

and

It is clear that the electric and magnetic fields have no effect on the component of velocity of the particle along the magnetic field (i.e., in Z-direction). It initially $v_z = 0$, then the motion of particle will be in X-Y plane.

From Eq. (3), we have

$$v_x = \frac{E}{B} - \frac{m}{qB} \frac{dv_y}{dt}$$

$$\frac{d}{dt} \left[\frac{E}{B} - \frac{m}{qB} \frac{dv_y}{dt} \right] = \frac{qBv_y}{m}$$

$$-\frac{m}{qB} \frac{d^2v_y}{dt^2} = \frac{qB}{m} v_y \quad \text{or} \quad \frac{d^2v_y}{dt^2} + \left(\frac{qB}{m} \right)^2 v_y = 0$$

or

Assuming

$$\left(\frac{qB}{m} \right)^2 = \omega^2, \text{ then we get}$$

$$\frac{d^2v_y}{dt^2} + \omega^2 v_y = 0$$

...(4)

The above expression (4) represents the simple harmonic motion of the particle, whose solution is

$$v_y = A \sin(\omega t + \phi)$$

where ω = angular frequency = $\frac{qB}{m}$

Again from Eq. (3), we have

$$v_x = \frac{E}{B} - \frac{m}{qB} \frac{d}{dt} [A \sin(\omega t + \phi)]$$

$$= \frac{E}{B} - A \cos(\omega t + \phi) \quad \dots(5)$$

Now for calculating the constant A and ϕ , let the particle be initially stationary i.e.

$t = 0, v_x = 0$ and $v_y = 0$

Then

$$\phi = 0 \text{ and } A = \frac{E}{B}$$

$$\text{Hence } v_y = \frac{dy}{dt} = \frac{E}{B} \sin \omega t, v_x = \frac{dx}{dt} = \frac{E}{B} [1 - \cos \omega t] \text{ and } v_z = \frac{dz}{dt} = 0$$

Integrating above equations, we get

$$x = \frac{E}{B} \left[t - \frac{\sin \omega t}{\omega} \right] + C_1, y = -\frac{E}{B} \cos \omega t + C_2 \text{ and } z = C_3$$

Now if $t = 0, x = y = z = 0$ i.e., the particle at origin, then

$$C_1 = 0, C_2 = \frac{E}{B\omega} \text{ and } C_3 = 0$$

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Then

$$x = \frac{E}{B\omega} (\omega t - \sin \omega t)$$

$$y = \frac{E}{B\omega} (1 - \cos \omega t)$$

$$z = 0$$

and
It is clear from Eq. (6) that the path of motion of the particle is cycloidal as shown in Fig. 2.

By putting

$$R = \frac{E}{\omega B}$$

We can get

$$(x - R\omega t)^2 + (y - R)^2 = R^2$$

This is the equation of a circle having radius R and centre which is moving in direction with the constant speed.

$v = \omega R = \frac{E}{B}$

Suppose the charged particle is at rest at the origin. The magnetic force is zero; the electric force accelerates the particle in Y-direction. When it starts moving, a magnetic force is developed. This magnetic force pulls the particle in a circular path in downward direction. The motion is now against the electric force, therefore, it slows down. The magnetic force also goes on decreasing. The particle comes at rest at point (Fig. 2). The entire process starts again which takes the particle to point 2 and it continues. This trajectory is called as Cycloid. The particle moves as if a point on the rim of wheel rolling down along x-axis.

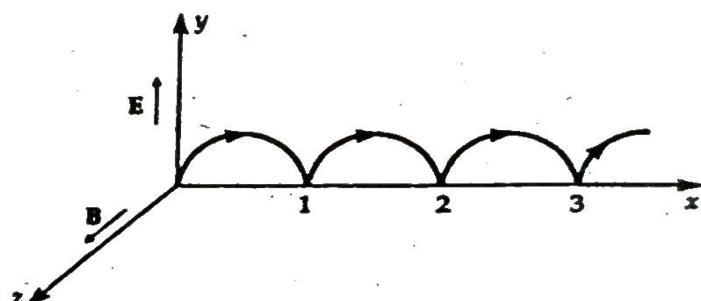


Fig. 2. Cycloid motion of charged particle in mutual perpendicular E and B fields.

Q.2. (b) What is skin depth in electromagnetic? Does it depend upon the frequency of the electromagnetic radiation? (2)

Ans. The distance an electromagnetic wave takes to reduce its amplitude by a factor

$\frac{1}{e} = 0.369$ i.e., about one third its value at the surface, is called skin depth and is denoted by d . The skin depth also called penetration depth, is a measure of the distance penetration of an electromagnetic wave into a conductor. It is expressed as

$$d = \frac{1}{\beta} = \frac{1}{\omega \sqrt{\mu \epsilon}} \left[\frac{-1 + \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2}}{2} \right]^{-1/2}$$

(i) A material is called poor conductor if $\sigma \ll \omega \epsilon$ or $\sigma/\omega \epsilon \ll 1$, then

$$\alpha = \sqrt{\frac{\mu \epsilon}{2}} \omega \left[1 + \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} \right]^{1/2}$$

becomes

$$\alpha \approx \omega \sqrt{\mu \epsilon}$$

and

$$\alpha = \sigma \sqrt{\frac{\mu_0}{2}} \left[1 + \sqrt{1 + \left(\frac{\omega}{\omega_0} \right)^2} \right]^2$$

becomes

$$\beta = \frac{\alpha}{2} \sqrt{\mu_0 \epsilon_0}$$

Thus skin depth

$$d = \frac{1}{\beta} = \frac{2}{\sigma} \sqrt{\mu_0 \epsilon_0}$$

Here the skin depth is independent of frequency

(ii) For a good conductor, $\sigma \gg \omega \epsilon_0$ or $\sigma / \omega \epsilon_0 \gg 1$. In this case, α and β are approximately equal i.e.

$$\alpha \approx \beta = \sqrt{\frac{\mu_0 \epsilon_0}{2}}$$

thus the skin depth

$$d = \frac{1}{\beta} \approx \sqrt{\frac{2}{\mu_0 \epsilon_0 \omega}}$$

It is shows that the skin depth decreases with increasing frequency. This is why in high frequency circuits, the current flows only on the surface of the conductor and a thin sheet of conducting material can act as a low pass filter for electromagnetic waves. The major importance of skin depth is that it measures the depth to which an electromagnetic wave can penetrate a conducting medium. Thus the same substance can be a good conductor at low frequency and poor conductor at high frequency. The conducting sheets which are used as electromagnetic shields must be thicker than the skin depth.

Q.2. (c) Write the Maxwell equations in differential form and state their significance. (4)

Ans. Maxwell's First Equation:

$$(\vec{\nabla} \cdot \vec{D} = \rho)$$

⇒ The net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume.

2. Maxwell's Second Equation: $(\vec{\nabla} \cdot \vec{B} = 0)$

⇒ The net outward flux of magnetic induction \vec{B} through any closed surface is equal to zero.

3. Maxwell's Third Equation: $(\vec{\nabla} \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t})$

⇒ The electromotive force ($emf = e = \int_C \vec{E} \cdot d\vec{l}$) around a closed path is equal to negative rate of change of magnetic flux with the path (\because magnetic flux $\phi = \int_S \vec{B} \cdot d\vec{s}$).

4. Maxwell's Fourth Equation: $(\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t})$

⇒ The magnetomotive force ($m.m.f = \oint H \cdot d\vec{l}$) around a closed path is equal to the conduction current plus displacement current through any surface bounded by the path.

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Q.3. (a) An electric field in a region is given by $\vec{E} = 3\hat{i} + 4\hat{j} - 5\hat{k}$. Calculate the electric flux through the source. $\vec{S} = 2.0 \times 10^{-6} \text{ m}^2$

Ans.

$$\begin{aligned}\phi_E &= \vec{E} \cdot \vec{S} = (3\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (2.0 \times 10^{-6} \hat{k}) \\ &= 0 + 0 - 10 \times 10^{-6} = 1.0 \times 10^{-4} \text{ Vm.}\end{aligned}$$

$$\boxed{\phi_E = 1.0 \times 10^{-4} \text{ Vm}}$$

Q.3. (b) Discuss the continuity equation. Distinguish between conduction current and displacement current.

Ans. An equation, which expresses the equality of incoming and outgoing charges, a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density J and the charge density ρ are related at each point through differential equation. This relation is based on the fact that electric charge can neither be created nor be destroyed and rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.

$$I = \int_S \mathbf{J} d\mathbf{S}$$

Again, considering charge leaving a volume V per second

$$I = -\frac{\partial}{\partial t} \int_V \rho dV$$

The negative sign comes here because the current is positive when the net charge from the outside of V to within.

Since, we are dealing with a fixed volume V , hence we may write,

$$-\frac{\partial}{\partial t} \int_V \rho dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

Again, using divergence theorem, we can write

$$\int_S \mathbf{J} d\mathbf{S} = -\int_V (\nabla \cdot \mathbf{J}) dV$$

Therefore, from Eqs. (2), (3) and (4), we have

$$I = \int_V (\nabla \cdot \mathbf{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

$$\int_V (\nabla \cdot \mathbf{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

or

$$\int_V \left((\nabla \cdot \mathbf{J}) + \frac{\partial \rho}{\partial t} \right) dV = 0$$

This integral must be zero for any arbitrary volume. It is only possible when integral is zero, i.e.,

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial \rho}{\partial t} = 0$ and hence for steady current, we have

$$\nabla \cdot \mathbf{J} = 0$$

S.No.	Conduction current	Difference between conduction current and displacement current is given below
1.	Actually flows through in conductivity medium and obeys Ohm's law.	Displacement current
2.	$J_c = \sigma E, I = \frac{V}{R}$	Set up in a dielectric medium due to changing electric field across the dielectric, which leads to variation of induced displacement of charge
3.	For constant E, $J_c \neq 0$	$J_d = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$ For constant E, $J_d = 0$

Q.3. (c) If the earth receives 20 cal/min/sq. cm. solar energy, what are the amplitudes of electric and magnetic fields of radiation.

Ans. Poynting vector is—

(4)

$$\begin{aligned}\vec{S} &= \vec{E} \times \vec{H} = EH \sin 90^\circ = EH \\ \text{Solar energy} &= 20 \text{ Cal/min/cm}^2, \\ &= \frac{20 \times 4.18 \times 10^4}{60} \text{ Jm}^{-2}\text{s}^{-1} \\ \therefore EH &= \frac{20 \times 4.18 \times 10^4}{60} = 14000\end{aligned}$$

But

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377$$

$$EH \times \frac{E}{H} = 14000 \times 377$$

$$E^2 = \sqrt{14000 \times 377} = \sqrt{5278000} = 2297.4 \frac{V}{m}$$

and

$$H = \frac{E}{377} = \frac{2297.4}{377} = 6.094 \approx 6.1 \frac{A}{m}$$

Amplitudes of electric and magnetic fields of radiation are-

$$E_0 = E \sqrt{2} = 2297.4 \sqrt{2} = 3249.01 V/m$$

$$H_0 = H \sqrt{2} = 6.1 \sqrt{2} = 8.63 \frac{A}{m}$$

Q.3.(d) A 2 KW laser beam is concentrated by a lens into cross-sectional area about 10^{-6} cm^2 . Find the poynting vector.

(2)

Ans. Poynting vector is $\frac{\text{Power}}{\text{Area}}$

$$\therefore \vec{S} = \frac{P}{\text{Area}}$$

Given

$$\begin{aligned}P &= 2 \text{ kw} = 2 \times 10^3 \text{ w} \\ \text{Area} &= 10^{-6} \text{ cm}^2 = 10^{-10} \text{ m}^2\end{aligned}$$

$$\therefore \vec{S} = \frac{2 \times 10^3}{10^{-10}} = 2 \times 10^{13} \frac{w}{m^2}$$

UNIT-II

Q.4. (a) Using the uncertainty principle show that an electron does exist inside a nucleus.

Ans. According to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

The diameter of the nucleus is of the order of 10^{-14} m. If the electron exists in nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty Δx in the position of electron is the same as the diameter of the nucleus, i.e. $\Delta x = 10^{-14}$ m

The minimum uncertainty in the momentum is given by

$$\begin{aligned}\Delta p_x &= \frac{h}{2\pi\Delta x} \\ &= \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-14}} = \frac{6.63 \times 10^{-20}}{2\pi} \\ &= 1.055 \times 10^{-20} \text{ kg m/sec.}\end{aligned}$$

It means that if electron exists in the nucleus, its minimum momentum must be $p_{\min} = 1.055 \times 10^{-20}$ kg m/s

For the electron of the minimum momentum, the minimum energy is given by

$$\begin{aligned}E^2_{\min} &= p^2 + m^2 c^4 \\ &= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4 \\ &= (3 \times 10^8)^2 [1.113 \times 10^{-40} + 7.4692 \times 10^{-44}]\end{aligned}$$

Since the second term in the bracket is much smaller than the first, it can be neglected, then,

$$E_{\min} = 3 \times 10^8 \sqrt{1.113 \times 10^{-40} \text{ J}} = 3 \times 10^8 \times 1.055 \times 10^{-20} \text{ J} = 3.1649 \times 10^{-12} \text{ J}$$

$$E_{\min} = \frac{3.1649 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \approx 20 \text{ MeV.}$$

or

Thus, if a free electron exists in the nucleus it must have a minimum energy about 20 MeV.

The maximum K.E. which a β -particle, emitted from radioactive nuclei is of order of 4 MeV. Therefore, this clearly shows that electrons can not be present within nucleus.

Q.4. (b) Describe Davisson-Germer experiment. Find the lowest energy eV, for an electron in one dimensional box of length $a = 0.2$ nm.

Ans. The first proof of the existence of "matter waves" was obtained in 1927 by Davisson and Germer the two American physicists. They succeeded in measuring the de Broglie wavelength for slow electrons, accelerated by a low potential difference by diffraction methods. The experimental arrangement is shown in the Fig. 3. The electron beam is produced from an electron gun consisting of tungsten filament F heated by low tension battery. The electrons excited by the filament are accelerated in an electric field of known potential difference from a high tension battery. The electrons are collimated to a fine beam and made to strike a Nickel target

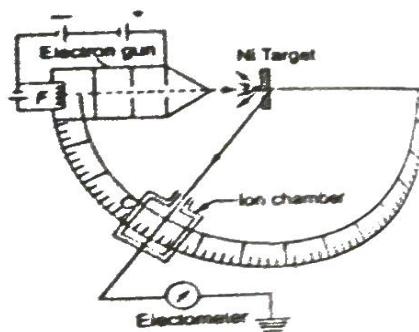


Fig.3. Davisson and Germer's Diffraction apparatus

which is capable of rotation about an axis parallel to the axis of the incident beam. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a given direction is measured by allowing it to enter in a Faraday cylinder called collector C which can be moved along a graduated scale of scale S, so that it is able to receive the reflected electrons at all angles between 2θ and 90° .

Davisson and Germer noticed that the strongest of the scattered electron beams corresponded accurately to diffraction maxima that would be expected in the diffraction of X-rays by the same crystal. The angular distribution of scattered electrons was analogous to optical diffraction patterns from a plane diffraction grating whose lines consisted of the rows of nickel atoms in the surface of the target crystal. The wavelength associated with the diffraction pattern can be obtained according to Bragg's law:

$$n\lambda = 2d \sin \theta$$

where d is the distance between the rows of atoms.

It was observed by Davisson and Germer that when an electron beam accelerated by a potential of 54 volts was directed upon a Nickel target, a sharp diffraction maxima appeared in the electron currents. The incident and the scattered beams in this case make an angle of 65° with the family of Bragg's planes. The spacing of planes in this family which can be determined by X-rays diffraction is 0.91 \AA . From above Bragg's equation, taking $n = 1$, we have,

$$2 \times 0.91 \times \sin 65^\circ = 1 \cdot \lambda \text{ or } \lambda = 1.65 \text{ \AA}$$

The wavelength of electrons accelerated through potential of 54 volts can be calculated as

$$\frac{1}{2}mv^2 = eV,$$

m is the mass of the electron.

$$m^2v^2 = 2meV$$

or

$$mv = \sqrt{2meV}$$

The de-Broglie wavelength of the electron will be given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

Putting $h = 6.6 \times 10^{-34}$ joule sec, $m = 9 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ coulomb, we obtain

$$\lambda = \frac{12.28}{\sqrt{V}}, \text{ with } V = 54 \text{ volts}$$

$$\lambda = 1.66 \text{ \AA}$$

There is an excellent agreement between the two results. Thus Davisson Germer experiment is a definite evidence that the electron beams do behave as wave and the wavelength of these beams is thus given by the de-Broglie equation.

Energy of particle in a box is given by

$$E_n = \frac{h^2 n^2}{8ml^2}$$

Given,

$$n = 1, h = 6.62 \times 10^{-34} \text{ J-S}, m = 9.1 \times 10^{-31} \text{ kg}$$

$$l = 0.2 \times 10^{-9} \text{ m}$$

Lowest energy will be

$$\begin{aligned} E &= \frac{1 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.2 \times 10^{-9})^2} \\ &= \frac{43.83}{2.912} \times 10^{-68} \times 10^{+31} \times 10^{+18} = 15.05 \times 10^{-19} \text{ J} \end{aligned}$$

$$E = 15.05 \times 10^{-19} J$$

$$E = \frac{15.05 \times 10^{-19} J}{1.6 \times 10^{-19}} = 9.4 \text{ eV}$$

Q.4. (c) The eigen function of an operator $\frac{d^2}{dx^2}\psi(x) = e^{ax}$. Find the corresponding eigen value.

Ans. The operator and eigen value relation is

$$\hat{O}\psi = 0\psi$$

where \hat{O} is operator and 0 is eigen value

Here,

$$\hat{O} = \frac{d^2}{dx^2} \text{ and } \psi = e^{ax}$$

$$\hat{O}\psi = \frac{d^2}{dx^2}e^{ax}$$

$$\hat{O}\psi = a^2e^{ax}$$

We can write this as-

Such that

$\therefore a^2$ is eigen value.

Q.5. (a) What type of statistics shall be applicable for a gas of photon? Justify your answer.

Ans. Bose-Einstein statistics shall be applicable for a gas of photons. Bose-Einstein statistics is obeyed by indistinguishable particles of integral spin quantum number that have symmetric wave function and does not obey Pauli exclusion principle.

Q.5. (b) Compare the qualitative features of Maxwell Boltzmann, Bose Einstein and Fermi-Dirac statistics on the basis of their functions.

Ans. Refer Q.1.(b) First Term Examination 2016.

Q.5(c) Show that Bose-Einstein, Fermi-Dirac statistics reduces to Maxwell-Boltzmann statistics at high temperature.

Ans. The distribution laws of three statistics are given below

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/k_B T} \quad \dots(1) \text{[For M-E]}$$

$$\frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} - 1 \quad \dots(2) \text{[For B-any of them]}$$

$$\text{and} \quad \frac{g_i}{n_i} = e^{\alpha} e^{E_i/K_B T} + 1 \quad \dots(3) \text{[For F-fundamental lattice]}$$

If $\frac{g_i}{n_i} \gg 1$ then $\frac{g_i}{n_i} \approx \left(\frac{g_i}{n_i} + 1\right) \approx \left(\frac{g_i}{n_i} - 1\right)$. In this limit both B.E. and F.D. distributions are identical with M.B. distribution. This limit $\left(\frac{g_i}{n_i} \gg 1\right)$ occurs when the temperature is not too low and pressure (or density) is not too high.

UNIT-III

Q.6. (a) Chromium has structure. It has atomic radius 0.125 nm. Calculate the free volume/unit cell.

Ans. Given-

$$r = 0.125 \text{ nm} = 1.25 \times 10^{-10} \text{ m}$$

Since chromium has bcc unit cell, so

$$a = \frac{4}{\sqrt{3}}r = \frac{4}{\sqrt{3}} \times 1.25 \times 10^{-10} = 2.885 \times 10^{-10} \text{ m}$$

Volume of unit cell.

$$V = a^3 = (2.885 \times 10^{-10})^3 = 2.402 \times 10^{-29} \text{ m}^3$$

No. of atoms in unit cell = 2

$$\text{Volume of atoms in unit cell} = v = \frac{4}{3}\pi r^3 \times 2$$

$$V = \frac{4}{3} \times 3.14 \times (1.25 \times 10^{-10})^3 \times 2 = 1.633 \times 10^{-29} \text{ m}^3$$

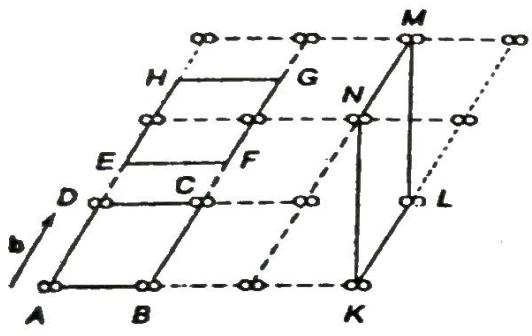
$$\text{Free volume per cell} = V - v = 2.402 \times 10^{-29} - 1.633 \times 10^{-29} = 7.67 \times 10^{-30} \text{ m}^3$$

Q.6. (b) Describe with proper diagram the following terms.

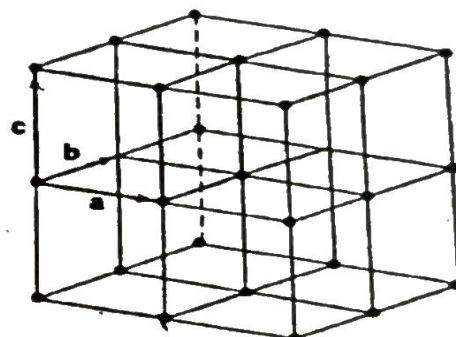
(a) Unit cell (b) Packing factor (c) Coordination number (d) fcc (4)

Ans. (a) Unit cell: In a space lattice an array of lines divides the space into identical volumes. These volumes fill the space completely by translation along the respective sets of axes and are known as unit cells.

Consider part of a two dimensional crystal, the atoms of which are arranged in a pattern shown in Fig. 4. (a). Each cluster of atoms consists of a dot two open circles and called pattern unit. It is observed that when a parallelogram as ABCD is repeatedly translated by vector \mathbf{a} and \mathbf{b} corresponding to AB and AD respectively, the pattern may be reproduced. (the dotted parallelograms). The region ABCD is called



(a) Unit cells in two dimensional lattice



(b) Primitive cells of the space lattice in three dimensions

Fig.4.

a unit cell and the vectors \mathbf{a} and \mathbf{b} as basis vectors. Unit cells can also be defined in any of the possible ways $EFGH$ or $KLMN$. The unit cell may thus be defined as "a region of the crystal that will cover all space under action of the translation operations defined by fundamental translation vectors." In two dimensions \mathbf{a} , \mathbf{b} are translation vectors of the lattice and area of unit cell is $a \cdot b$ and in three dimensions volume of the unit cell is $|a \times b \cdot c|$.

(b) Packing factor – It is the ratio of volume occupied by the atoms molecules or ions in a unit cell (v) by the total volume of unit cell (V).

$$\begin{aligned} \text{PF} &= \frac{\text{Volume occupied by the atoms in an unit cell}}{\text{Volume of unit cell}} \\ &= \frac{(\text{Number of atoms present}) \times (\text{Volume of an atom})}{\text{Volume of unit cell}} \end{aligned}$$

$$\text{PF} = \frac{v}{V}$$

(c) Coordination Number – It is the number of nearest neighbouring atoms, molecules or ions to a particular atom.

(d) Face Centered Cubic (fcc) structure—A face centered cubic cell contains eight corner atoms and six face centered atoms. A face centered cubic unit cell is shown in Fig. 4. (c)

The atom in a fcc unit cell touches along the face diagonal. Each and every corner atom is shared by eight adjacent unit cells. Therefore each and every corner atoms contributes $1/8$ of its part to one unit cell.

So, the number of atoms contributed by

$$\text{the corner atoms} = \frac{1}{8} \times 8 = 1.$$

Two unit cells share each and every face centered atom. Therefore, the contribution of a face centered atom to unit cell is $1/2$. So the total number of atoms contributed by face centered atoms

$$= \frac{1}{2} \times 6 = 3.$$

∴ Total number of atoms present in a fcc unit cell = 1 + 3 = 4.

Q.6. (c) Deduce the Miller indices of a plane which cuts off intercepts in the ratio $1a:3b:-2c$ along the three axis.

Ans. From the law of rational indices

$$\therefore \frac{1}{h} : \frac{1}{k} : \frac{1}{l} = 1 : 3 : -2; \quad 1a : 3b : -2c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

$$\therefore h:k:l = 1:\frac{1}{3}:\frac{1}{2} = 6:2:-3$$

Thus

$$h=6, k=2, l=-3$$

Hence the plane is $(62\bar{3})$

Q.6. (d) What is the difference between (111) and $\langle 111 \rangle$ for Miller indices?

Ans. (111)-It represent $h = 1, k = 1, l = 1$ - miller indices. It represent family planes orthogonal to $ha + kb + lc$ where a, b, c are the basis of reciprocal lattice vector
 $\langle 111 \rangle$ -It represent the set of all directions that are equivalent to [111] by symmetry
[111] denotes a direction in the basis of the direct lattice vectors instead of the reciprocal lattice.

Q.7. (a) Define the following:

(c) Primitive Lattice (d) Bravais Lattice

Ans. (a) Unit cell- A unit cell is defined as a fundamental building block of crystal structure which can generate the complete crystal by repeating its own dimensions in various directions.

(b) **Primitive cell**- It is the smallest unit cell in volume constructed by primitive It consists of only one full atom.

(c) **Primitive lattice**-It is a lattice having lattice point only at the corners of the unit cell.

(d) Bravis lattice— It is a three dimensional lattice. A bravis lattice tiles space without any gaps or holes. There are 14 ways in which this can be accomplished. Lattices are characterized by translational symmetry. Bravis lattices contain seven crystal systems and four lattice centering types.

Q.7. (b) Germanium crystallizes in diamond form structure with 8 atoms per unit cell. If lattice constant is 5.62 Angstrom, calculate the density of germanium. (2)

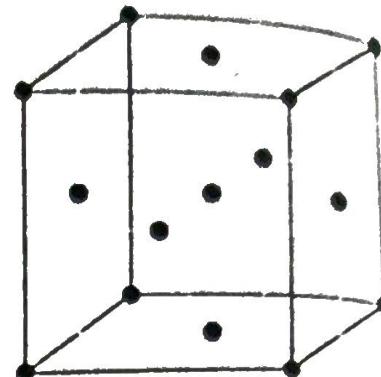


Fig. 4. (c) Face centered cubic

$$a^3 = \frac{nM}{\rho N}$$

or

$$\rho = \frac{nM}{N_A^2} = (5.62 \times 10^{22})^2 \times 6.023 \times 10^{23}$$

$$\rho = 54.34 \text{ g/cm}^3 = 5.434 \text{ kg/m}^3$$

Q.7. (c) Write short notes on-

(a) Point defect (b) Schottky defect (c) Frenkel defect

(b)

Ans. (a) Point defect: A lattice defect which spreads out very little in all the three dimensions is called a point defect. There are following types -

(i) Interstitial atom (ii) Vacancies (iii) Impurity atom.

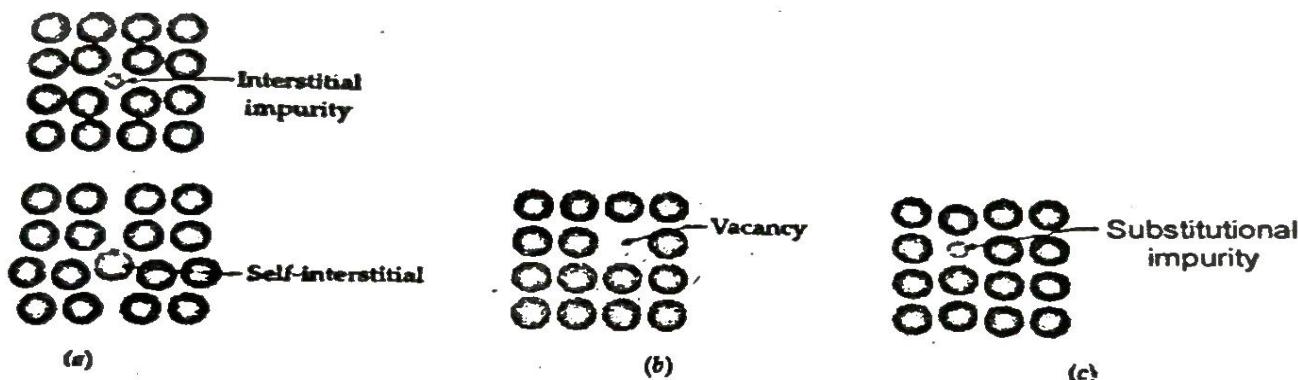
(i) Interstitial atom: This is an atom inserted into the voids (called interstice of the lattice) between the regularly occupied sites. Thus such an atom does not occupy regular lattice sites. This extra atom may be an impurity atom or an atom of the same type as on the regular sites. Fig. 5(a).

(ii) Vacancies: These are the lattice sites from which the atoms are missing [Fig. 5(b).] Such a vacancy is also called Schottky defect.

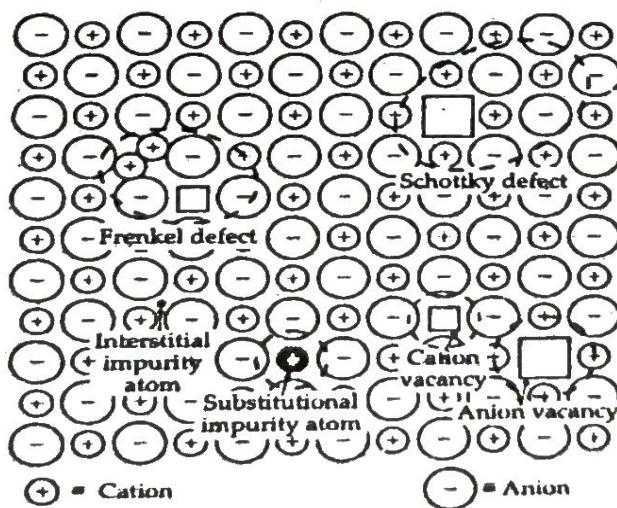
But if vacancy is created by transferring an atom from a regular lattice site to an interstitial position then it is called Frenkel defect.

In this case, therefore, two imperfections are created vacancy as well as an interstitial atom.

(iii) Impurity atom: This is a defect in which a foreign atom occupies a regular lattice site (Fig. 5 (c))

**Fig.5. Three common point defects. (a) interstitial, (b) vacancy, (c) impurity.**

A point imperfection is localised near a point or atom in the structure or in other words point defects are lattice errors at isolated lattice points. Point defects are lattice errors at isolated lattice points. Point defects occurs in metallic as well as ionic lattices. These types of points defects are shown in Fig. 5 (d)

**Fig. 5(d): Illustrating a number of possible point defects in a two-dimensional ionic crystal.**

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(b) Schottky defect

In real crystals, there are irregularities of the atomic array in which atoms are missing at some lattice points. Such a point is called a vacancy (also called Schottky defect). This is shown in Fig 5. (e).

In spite of care taken in the preparation of crystals, vacancies are always present in all crystals. In fact, as a result of thermal fluctuations, vacancies are produced and destroyed constantly in the crystal. Formally such a defect might be produced by plucking an interior atom out of its regular lattice site and placing it on the surface. This act requires energy. Moreover, the disorder increases resulting in an increase in the entropy. In thermal equilibrium, a certain number of lattice vacancies are always present which can be estimated as follows :

If E_v is the energy required to take an atom from a lattice site inside the crystal to a lattice site on the surface, $n E_v$ is the increase in internal energy associated with the production of n isolated vacant sites. The total number of ways in which we can pick n atoms from the crystal consisting N atoms is

$$\omega = \frac{N!}{(N-n)!n!}$$

Since disorder increases due to creation of n vacancies the corresponding increase in entropy is given by

$$S = K_B \ln \omega$$

$$= K_B \ln \frac{N!}{(N-n)!n!}$$

This in turn produces a change in free energy F

$$F = U - TS$$

\Rightarrow

$$F = nE_v - K_B T \ln \frac{N!}{(N-n)!n!}$$

The second term on right hand side can be simplified by the use of Stirling's approximation.

$$\ln x! = x \ln x - x$$

$$F = nE_v - K_B T [N \ln N - N - (N-n) \ln (N-n) + N - n - n \ln n + n]$$

$$F = nE_v - K_B T [N \ln N - (N-n) \ln (N-n) - n \ln n]$$

Free energy in thermal equilibrium at constant volume must be minimum with respect to changes in n i.e.

$$\left(\frac{\partial F}{\partial n} \right)_T = 0 = E_v - K_B T \ln \frac{N-n}{n}$$

or

$$0 = E_v + K_B T \ln \frac{n}{N-n}$$

or

$$n = (N-n) \exp \left(\frac{-E_v}{K_B T} \right)$$

If $n \ll N$, we can neglect n in $(N-n)$ so that

$$n \approx N \exp \left(\frac{-E_v}{K_B T} \right)$$

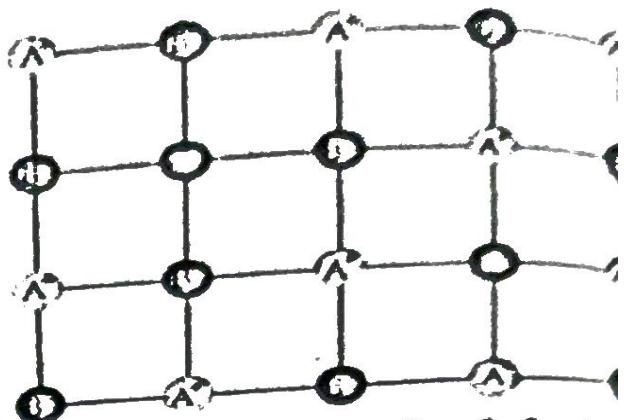


Fig. 5 (e) Schottky defect

If $E_p = 1 \text{ eV}$ and $T = 1000 \text{ K}$, then

$$n = e^{-12} = 10^{-4} / \text{cm}^3$$

The equilibrium concentration of vacancies decreases as the temperature decreases.

Figure 5(i) shows an alkali halide crystal with vacancies. Fig. 5(ii) displays two separate vacancies and a vacancy pair in alkali halide crystal. The pairs are formed because the cation and anion vacancies in alkali halides are electrically charged and they attract each other electrostatically.

Due to formation of the pair a dipole is also produced.

In ionic crystals, formation of paired vacancies is most favoured, i.e., in ionic crystals, equal number of positive and negative ion vacancies are produced. The formation of pairs makes it possible to keep the surface of the crystal electrostatically neutral. The number of pairs can be related to the total number of atoms present in the crystal on following the same procedure as adopted above.

The different ways in which n separated pairs can be formed are:

$$\omega = \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(11)$$

So that increase in entropy will be

$$S = K_B \ln \omega \\ = K_B \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(12)$$

with corresponding change in free energy

$$F = U - TS \\ \Rightarrow F = nE_p - K_B T \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(13)$$

where E_p = Energy of formation of a pair.

Applying Stirling's approximation to simplify the factorial terms i.e.,

$$\ln \left[\frac{N!}{(N-n)!n!} \right]^2 \approx 2[\ln N! - \ln(N-n)! - \ln n!] \\ = 2[N \ln N - N(N-n) \ln(N-n) + (N-n) - n \ln n + n] \\ 2[N \ln N - (N-n) \ln(N-n) - n \ln n] \quad \dots(14)$$

Putting it in Eq. (13), we get

$$F = nE_p - 2K_B T [N \ln N - (N-n) \ln(N-n) - n \ln n]$$

Differentiating the above equation with respect to n , we get

$$\left(\frac{\partial F}{\partial n} \right)_T = E_p - 2k_B T [0 + \ln(N-n) + 1 - \ln n - 1] \\ = E_p - 2k_B T \ln \frac{(N-n)}{n} \quad \dots(15)$$

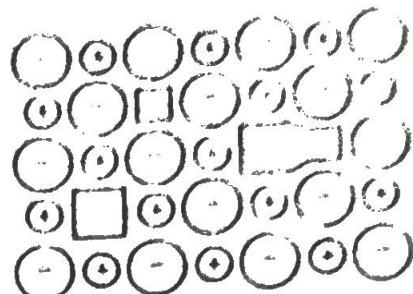


Fig. 5 (i) A plane of a pure alkali halide crystal, showing a vacant positive ion site, a vacant negative ion site, and a coupled pair of vacant sites of opposite sign (vacancies in alkali halide)

Free energy in thermal equilibrium attained at temperature T is constant.

$$E_p - 2K_B T \ln \frac{(N-n)}{n} = 0$$

$$\ln \frac{N-n}{n} = \frac{E_p}{2K_B T}$$

or

$$\frac{N-n}{n} = \exp \left[\frac{E_p}{2K_B T} \right]$$

or

$$n = N \exp \left(\frac{-E_p}{2K_B T} \right)$$

or

provided $n \ll N$. In NaCl crystal $E_p = 2.02 \text{ eV}$ and at room temperature $n = 10^6 \text{ cm}^{-3}$

(c) FRENKEL DEFECT

We know that interstitial atoms are the extra atoms that are squeezed in between normal atom sites. When an interstitial is caused by transferring an atom from lattice site to an interstitial position, a vacancy is created. The associated vacancy-interstitial atom is called *Frenkel defect*. This is shown in Fig. 5(g)

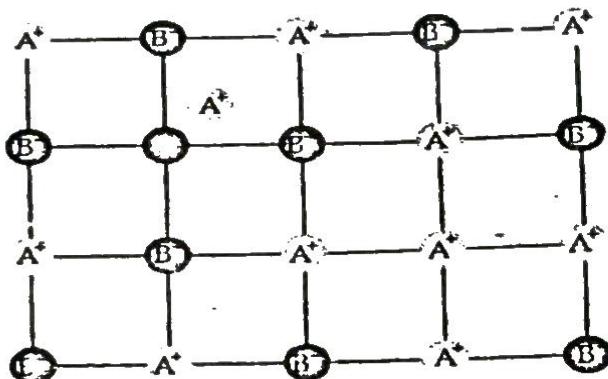


Fig. 5(g) Frenkel defect

Proceeding in the same way as in the case of schottky defect, we can calculate number of Frenkel in equilibrium at a temperature T .

Let in a perfect crystal

E_i = the energy required to displace an atom from regular lattice site to interstitial position;

N_i = the interstitial atoms and

N = the atoms

Then the total number of ways in which n Frenkel defects can be formed, will be given by

$$\omega = \frac{N!}{(N-n)!n!} \cdot \frac{N_i!}{(N_i-n)!n!}$$

The corresponding increase in entropy due to creation of Frenkel defect is

$$S = K_B \ln \omega = K_B T \ln \left[\frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \right]$$

which in turn produces a change in free energy

$$F = U - TS$$

$$= nE_i - K_B T \ln \left[\frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \right] \quad \dots(20)$$

Using the Stirling's approximation for logarithmic term, we get

$$\ln \left[\frac{N!}{(N-n)!n!} \times \frac{N_i!}{(N_i-n)!n!} \right] = \ln \frac{N!}{(N-n)!n!} + \ln \frac{N_i!}{(N_i-n)!n!}$$

$$\approx N \ln N + N_i \ln N_i - (N-n) \ln(N-n) - (N_i-n) \ln(N_i-n) - 2n \ln n \quad \dots(21)$$

Putting this value of logarithmic term in the expression for free energy and then differentiating with respect to n , we get

$$\left(\frac{\partial F}{\partial n} \right)_T = E_i - K_B T \ln \frac{(N-n)(N_i-n)}{n^2} \quad \dots(22)$$

At equilibrium, the energy is constant, so that

$$\left(\frac{\partial F}{\partial n} \right)_T = 0$$

$$E_i = K_B T \ln \frac{(N-n)(N_i-n)}{n^2}$$

$$\cong K_B T \ln \frac{NN_i}{n^2} \quad \dots(23)$$

Taking $N \gg n$ and $N_i \gg n$

$$\text{Thus, } E_i = K_B T [\ln(NN_i) - 2 \ln n]$$

$$\text{or } 2 \ln n - \ln NN_i = \frac{E_i}{K_B T}$$

$$\text{or } \ln \frac{NN_i}{n^2} = + \frac{E_i}{K_B T}$$

$$n_2 = NN_i \exp \left(-\frac{E_i}{K_B T} \right)$$

$$n = (NN_i)^{1/2} \exp \left(-\frac{E_i}{2K_B T} \right) \quad \dots(24)$$

Showing that n should be proportional to $(NN_i)^{1/2}$

In an *ionic crystal* Schottky and Frenkel defects are shown in Fig. 5(h). The arrows indicate the direction of the displacement of the ions. In Schottky defect the ion ends up on the surface of the crystal, whereas in *Frenkel defect* ion is removed to an interstitial position. From the study ionic conductivity and the density measurements, it is concluded that in pure alkali halides Schottky vacancies are most common, whereas in pure silver halides Frenkel vacancies are most common. Due to Schottky defects, volume of the crystal is increased without any change in the mass and consequently, production of this defect, lowers the density of the crystal. On the other hand, the production of

Frenkel defects does not change the volume of the crystal so that the density of crystal remains constant.

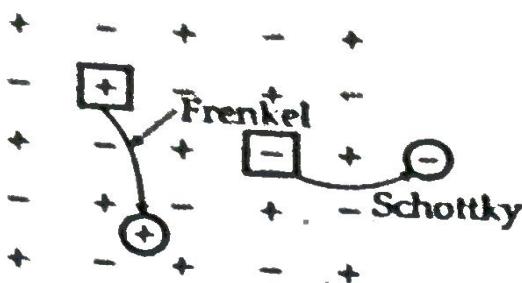


Fig.5.(h) Schottky and Frenkel defects in an ionic crystal. The arrows indicate the direction of displacement of the ions.

In alkali halide crystal doped with divalent impurities like Ca^{2+} , each impurity replaces two host cations to keep the crystal electrically neutral e.g., if KCl crystal taken then impurity ion Ca^{++} will replace two K^+ ions, one K^+ ion place will be filled by Ca^{++} ion and the second will remain vacant (Fig. 5(i)). Though Ca^{++} ion is heavier than K^+ ion but due to production of vacancies, Density of KCl crystal decreases as predicted by experimental results.

K^+	Cl^-	K^+	Cl^-	K^+
Cl^-	Ca^{++}	Cl^-	K^+	Cl^-
K^+	Cl^-	\square	Cl^-	K^+
Cl^-	K^+	Cl^-	K^+	Cl^-

Fig.5(i) Production of cation lattice vacancies by the solution of CaCl_2 in KCl : to ensure electrical neutrality a cation vacancy is introduced into the lattice with each divalent cation Ca^{++} .

UNIT-IV

Q.8. (a) Describe Kronig - Penney Model.

Ans. The Kronig-Penney model is a simple, idealized quantum mechanical model that consists of an infinite array of rectangular potential barriers.

Kronig and Penney in 1931 solved the Schrodinger's equation for electrons in a simple idealized periodic field. The model is of considerable importance because it interprets the main features of the band structure of metals. The periodic field is assumed in the form of potential wells of zero potential energy of width a separated by rectangular barriers of width b and height V_0 as shown in Fig. 6. For the potential barrier, the Schrodinger equation has to be solved in two regions:

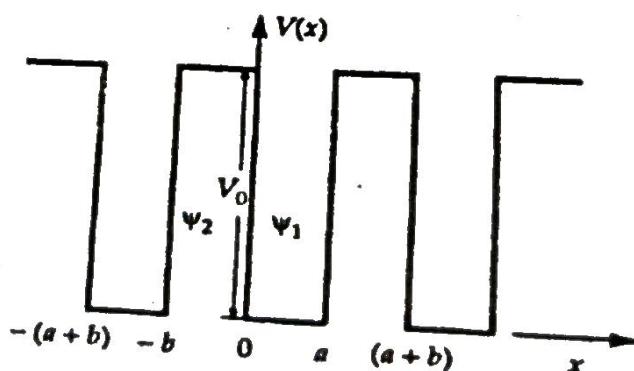


Fig. 6. One dimensional chain of potential wells.

- (a) Region I, $0 < x < a$, potential well region in which $V = 0$ and $\psi \neq 0$
- (b) Region II, $-b < x < 0$, barrier region in which $V = V_0$ and $\psi = 0$

In order to find the allowed energies, we solve the Schrodinger's equation for the two regions and apply the appropriate boundary conditions.

In region I, the Schrödinger's equation is

or $\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0$

$$\text{or } \frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0 \quad [\text{Here, } \alpha^2 = \frac{2m(E - V_0)}{\hbar^2}]$$

Solution of this equation is

$$\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x} \quad \text{where A and B are constants.}$$

In region II, the Schrödinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

$$\text{or } \frac{d^2\psi_2}{dx^2} - \alpha^2\psi_2 = 0 \quad \left[\text{Here, } \alpha^2 = \frac{2m(E - V_0)}{\hbar^2}\right] \quad \dots(3)$$

Solution of this equation, for $E < V_0$ is

$$\psi_2 = Ce^{i\alpha x} + De^{-i\alpha x} \quad \dots(4)$$

where C and D are constants.

For continuity at $x = 0$

$$\psi_1(0) = \psi_2(0) \quad \dots(5)$$

$$\left. \frac{\partial\psi_1}{\partial x} \right|_{x=0} = \left. \frac{\partial\psi_2}{\partial x} \right|_{x=0} \quad \dots(6)$$

In a periodic lattice $V(x+a) = V(x)$ it is expected that the wavefunction will also exhibit this periodicity. Since the wave function must be a Bloch wave function, therefore Eq. (1) gives

$$\psi_k(x+a+b) = e^{-ik(a+b)}\psi_k(x) \quad \dots(7)$$

We incorporate this requirement at $x = -b$ and $x = a$, then we obtain the continuity boundary condition as

$$\psi_2 \Big|_{x=-b} = e^{-ik(a+b)}\psi_1 \Big|_{x=a} \quad \dots(8)$$

$$\left. \frac{\partial\psi_1}{\partial x} \right|_{x=-b} = e^{-ik(a+b)} \left. \frac{\partial\psi_1}{\partial x} \right|_{x=a} \quad \dots(9)$$

When we apply the boundary conditions, Eqn. (5), (6), (7) and (8), we obtain four equations for four unknowns

$$A + B = C + D \quad \dots(10)$$

$$i\beta(A - B) = \alpha(C - D) \quad \dots(11)$$

$$Ce^{ab} + De^{ab} = e^{-ik(a+b)} [Ae^{i\beta a} + Be^{-i\beta a}] \quad \dots(12)$$

$$aCe^{ab} - \alpha De^{ab} = e^{-ik(a+b)} i\beta [Ae^{i\beta a} - Be^{-i\beta a}] \quad \dots(13)$$

By solving these equations simultaneously e.g. by requiring that the determinants of the coefficients of A, B, C and D vanish, we get

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\beta & -i\beta & -\alpha & \alpha \\ -e^{-ik(a+b)}(ba) & -e^{-ik(a+b)}(ba) & e^{-ab} & e^{ab} \\ -i\beta e^{-ik(a+b)}(ba) & i\beta e^{-ik(a+b)}(ba) & ae^{-ab} & -ae^{ab} \end{vmatrix} = 0$$

On solving the determinant Eq. (14) and after simplification, we get

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} \sin h ab \sin \beta a + \cosh ab \cos \beta a = \cos(a + b) k$$

To simplify Eq. (15), Kronig and Penney considered the case when $V_0 \rightarrow \infty$, but product $V_0 b$ has a finite value i.e., the potential barriers become *delta function*.

Under these conditions, the model is modified in such a way that represent of well separated by infinitely thin potential barriers of infinitely large potential limiting value of $V_0 b$ for $V_0 \rightarrow \infty$ and $b \rightarrow 0$ is known as *barrier strength*.

Also from Eqs. (9) and (3)

$$\alpha^2 = \frac{2m(E - V_0)}{\hbar^2} \text{ and } \beta^2 = \frac{2mE}{\hbar^2}$$

Then

$$\beta^2 - \alpha^2 = \frac{2mV_0}{\hbar^2}$$

or

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta} = \frac{mV_0}{\alpha\beta\hbar^2}$$

Putting the value $\frac{\beta^2 - \alpha^2}{2\alpha\beta}$ from Eq. (16) in Eq. (15) we get

$$\frac{mV_0}{\alpha\beta\hbar^2} ab \sin \beta a + \cos \beta a = \cos ka$$

$$\frac{mV_0 b}{\beta\hbar^2} \sin \beta a + \cos \beta a = \cos ka$$

Let us define a quantity $p = \frac{mV_0 ba}{\hbar^2}$, which is measure of the area $V_0 b$ potential barrier,

then

$$p \frac{\sin \beta a}{\beta a} + \cos \beta a = \cos ka$$

The physical significance of the quantity p is that if p is increased and the vector is bound more strongly to a particular potential well.

When $p \rightarrow 0$, then potential barrier becomes very weak which means that electrons are free electrons. In this, we obtain from Eq. (18),

$$\beta a = ka \quad \text{or} \quad \beta = k$$

Hence

$$\beta^2 = \frac{2\pi^2}{l^2} - k^2$$

This result is similar as obtained by free electron theory. Equations (1) also gives the condition, which must be satisfied so that all terms of the wave equation are present. Since $\cos \beta a$ can have values between +1 and -1. Such values of βa , therefore, represent wave like solutions of the form

$$V(x) = e^{i\beta x} U_0(x)$$

and are allowed values. The other values of βa are not allowed.

If we plot a graph between $\left(\frac{p \sin \beta a}{\beta a} + \cos \beta a \right)$ and βa for the value of $p = \frac{3\pi}{2}$, we get the curve as shown in Fig. 7.

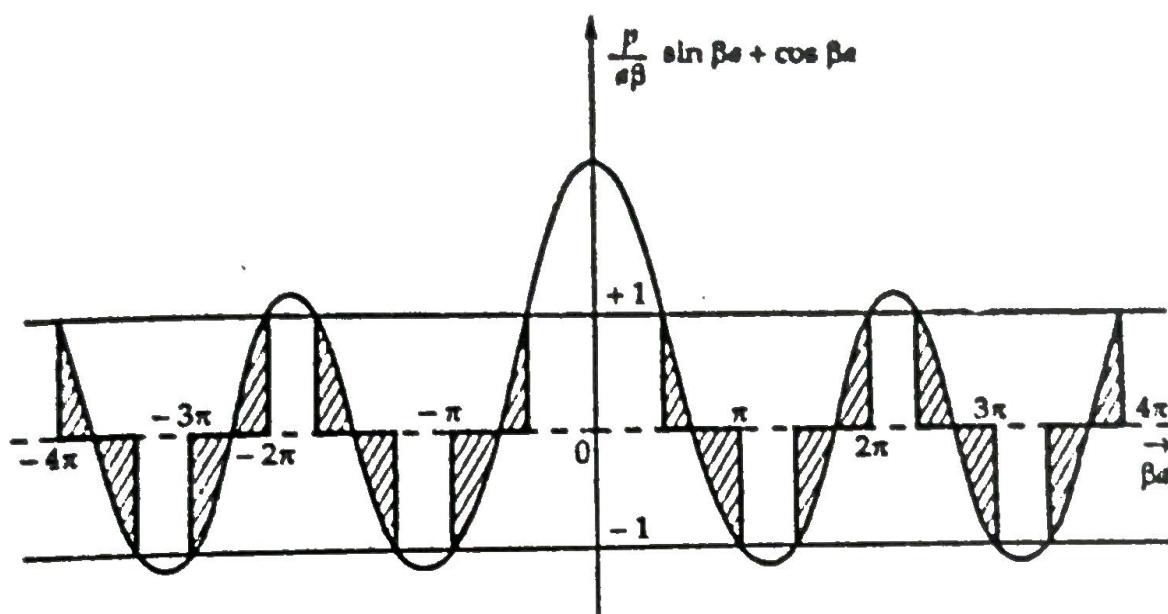


Fig. 7. Plot of $\frac{p}{\beta a} \sin \beta a + \cos \beta a$ vs. βa to show allowed and forbidden band for the K.P. approximation.

Q.8. (b) Define law of mass action for a semiconductor.

(2.5)

Ans. The law of mass action states that the product of the electron concentration in the conduction band and hole concentration in the valence band is equal to the square of intrinsic carrier concentration of the material. Let n_i , n_e and n_h are the intrinsic concentration of the material, the electron concentration in the conduction band and the hole concentration in the valence band respectively, then

$$n_i^2 = n_e n_h$$

This equation is known as *law of mass action*.

Q.8. (c) How does the location of Fermi level change with doping of a semiconductor?

Ans. In an intrinsic semiconductor the number of electrons is equal to number of holes ($n_i = p_i$). But in N-type extrinsic semiconductor number of electrons are increased

due to doping of pentavalent atom and number of holes are decreased than which would be available for intrinsic semiconductor. The number of holes are decreased because of the large number of electrons present which increase the rate of recombination of electrons and holes. With similar argument it can be shown that in P-type extrinsic semiconductor the number of holes are increased due to doping of trivalent impurity atom ($n_h > n_e$) and number of electrons decreased ($n_e < n_i$) than which would be available for intrinsic semiconductor. Now since the Fermi level E_F is a measure of the probability of occupancy of the allowed energy states by the electron, so for an intrinsic semiconductor when $n_i = p_i$, Fermi level is at the centre of the forbidden gap [Fig. 11.(a)]. But for N-type semiconductor, since $n_e > n_h$, it is clear that E_F must move closer to conduction band [Fig. 11. (b)] to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band. Similarly, for P-type semiconductor since $n_h > n_e$ so E_F must move from the centre of the forbidden gap closer to the valence band [Fig. (11.1 (c))]

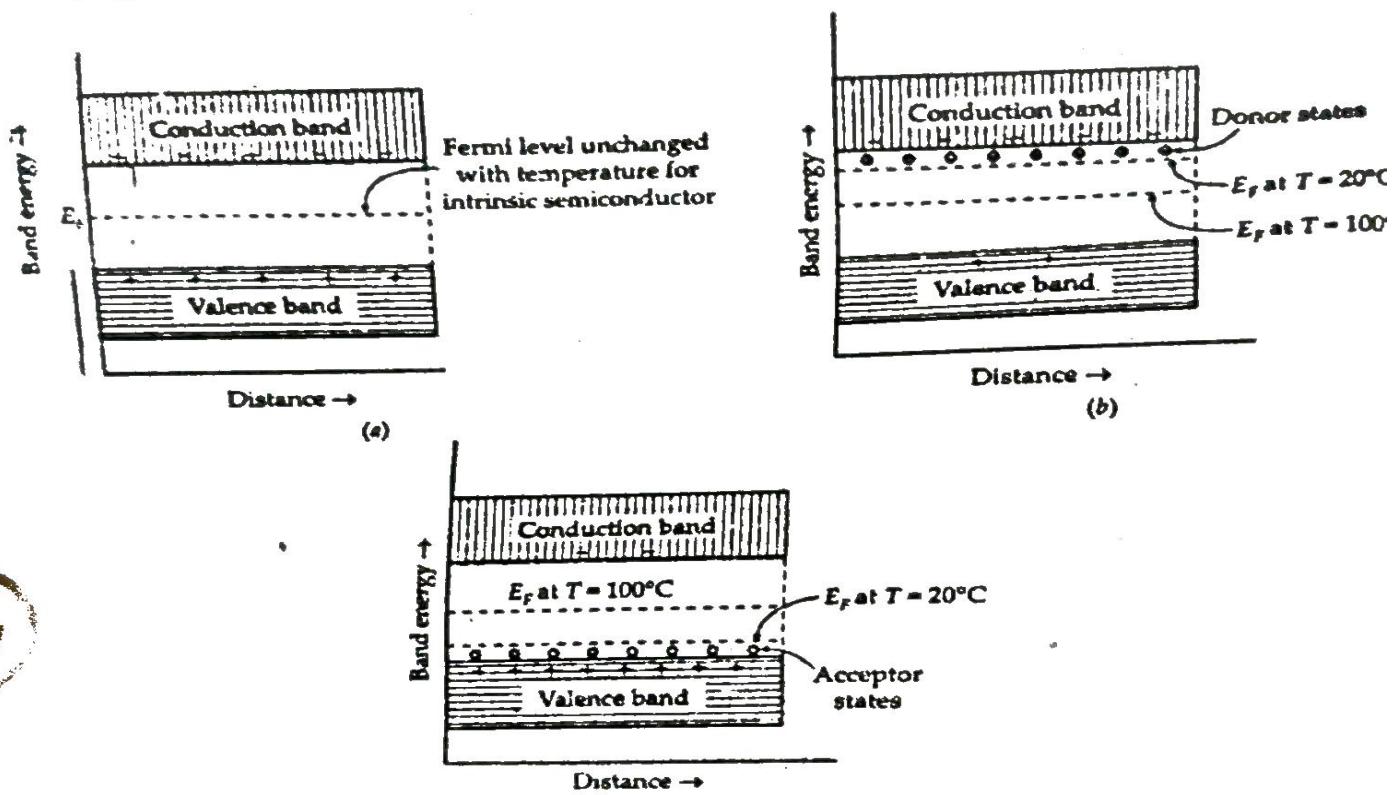


Fig. 11. Position of Fermi level (E_F) for (a) intrinsic semiconductor; (b) N-type semiconductor; (c) P-type semiconductor.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor Fermi level is independent of temperature [Fig. 11.(a)].

But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor states which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature is raised. So proportion of the total number of conduction electrons will

increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the gap (Fig. 11.(b)).

Similarly, for P-type semiconductor as temperature increases the material also becomes increasingly intrinsic and the Fermi level rises until it approaches the centre of the gap as in the intrinsic semiconductor [Fig. 11(c)]. Thus both N-type and P-type materials become more and more like intrinsic material at high temperature. This places a limit on the operating temperature of an extrinsic semiconductor device.

Q.8. (a) Derive the expression for hall coefficient.

(5)

Ans. Consider a specimen in the form of a rectangular cross-section carrying current I_x in the x direction A uniform magnetic field B_z is applied along z-axis as shown in Fig. 12(a).

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction

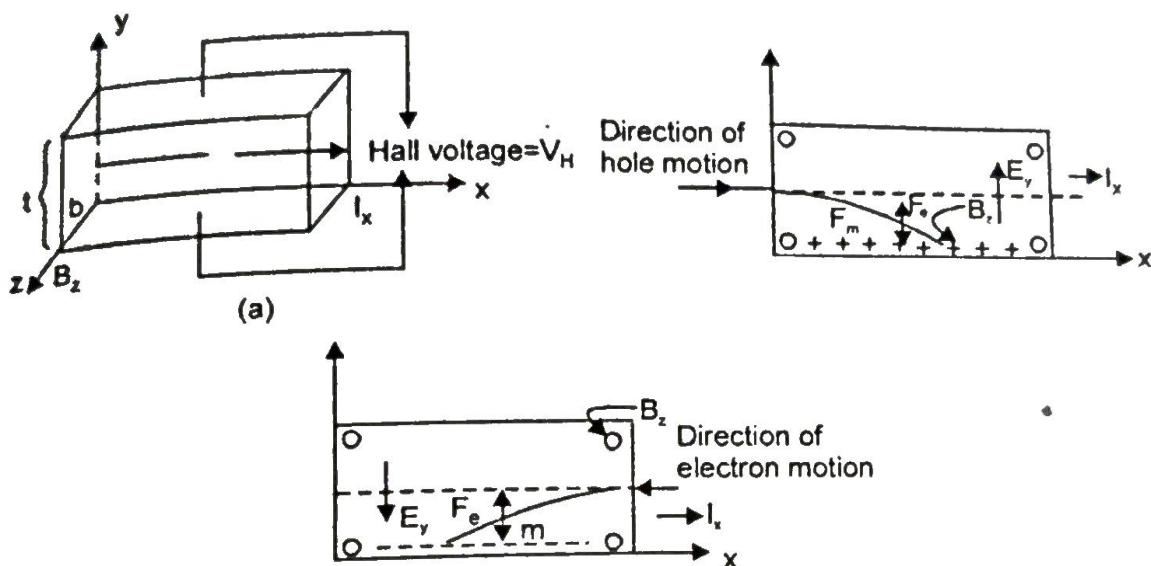


Fig.(12)

Fig. 12.(a) Illustration of Hall effect 12(b) motion of hole in p-type semiconductor specimen (c) motion of electron of N-type semiconductor or metal specimen of Hall experiment.

Force $ev_x B_z$ [Fig. 12. (a) and (c)] This shift of charges produces a potential difference or an electric field E_y along y direction. The produced electric field also applies a force eE_y on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium,

$$F_e = F_m$$

$$eE_y = ev_x B_z$$

$$E_y = v_x B_z$$

... (1)

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \quad ... (2)$$

Here n is density of charge carrier (electron or hole or both) and A is area, thickness is the cross sectional area of specimen perpendicular to direction of J_x .

From Eqs. (1) and (2), we can write

$$E_y = \frac{B_z J_x}{ne}$$

This is the expression for Hallfield. The Hall effect is described by means of Hall coefficient R_H defined in terms of J_x by the following relation:

$$R_H = \frac{E_y}{J_x B_z}$$

From Eqs. (3) and (4) we have

$$R_H = \frac{1}{ne}$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{ne}$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e}$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Q.9. (b) What are Brillouin zones?

Ans. If Schrodinger wave equation for electron energies is solved with a periodic function $u(k)$ to give the energies of an electrons in a solid, the solutions falls in permitted energy bands. If the solutions are plotted in the reciprocal lattice of the crystal being considered, the zones enclosing the solutions for $k = 1, 2, 3, \dots, n$ are called Brillouin zones.

Q.9. (c) Show that the Fermi energy lies midway between the conduction band and valence band for an intrinsic semiconductor.

Ans. From the law of mass action $n_i^2 = n_e \cdot n_h$

we put the value of n_e and n_h .

$$n_i^2 = N_C N_V \exp \left[\frac{E_F - E_C}{k_B T} \right] \exp \left[\frac{E_V - E_F}{k_B T} \right]$$

$$n_i^2 = N_C N_V \exp \left[\frac{-(E_C - E_V)}{k_B T} \right]$$

or

Since $E_C - E_V = E_g$, the band gap of the semiconducting material, Eq. (1) may be written as

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{k_B T}\right)$$

Substituting the value of N_c and N_v in Eq. (2) we get

$$\begin{aligned} n_i^2 &= 2 \left[\frac{2\pi n_e^* k_B T}{h^2} \right]^{3/2} \times 2 \left[\frac{2\pi n_h^* k_B T}{h^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \\ &= 4 \left[\frac{2\pi k_B T}{h^2} \right]^3 \times [m_e^* m_h^*]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \end{aligned} \quad \dots(3)$$

Multiplying the numerator and denominator of Eq. (3) by m^3 , we get

$$n_i^2 = 4 \left[\frac{2\pi k_B m T}{h^2} \right]^3 \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \quad \dots(4)$$

The value of n_i can be written as

$$\begin{aligned} n_i &= 2 \left[\frac{2\pi k_B m T}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) \\ &= 2 \left[\frac{2\pi k_B m}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4} T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \end{aligned} \quad \dots(5)$$

Equation (5) may also be written as

$$n_i = A T^{3/2} \exp\left[\frac{-E_g}{2k_B T}\right] \quad \dots(6)$$

where

$$A = 2 \left[\frac{2\pi k_B m}{h^2} \right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2} \right] = 4.82 \times 10^{21} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{3/4}$$

Equation (5) gives the intrinsic carrier concentration in semiconductor.

Value of Fermi energy

In intrinsic semiconductor $n_e = n_h$

$$\begin{aligned} \therefore 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{(E_F - E_C)k_B T} &= 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \times e^{(E_V - E_F)k_B T} \\ \Rightarrow (m_e^*)^{3/2} \times e^{(E_F - E_C)k_B T} &= (m_h^*)^{3/2} \times e^{(E_V - E_F)k_B T} \\ \Rightarrow e^{(2E_F - EC - EV)k_B T} &= \left(\frac{m_h^*}{m_e^*} \right)^{3/2} \end{aligned} \quad \dots(7)$$

$$\Rightarrow \left\{ \frac{2E_F - E_C - E_V}{k_B T} \right\} = \frac{3}{2} \log \left(\frac{m_h^*}{m_e^*} \right)$$

$$\Rightarrow E_F = \frac{E_C + E_V}{2} + \frac{3}{4} k_B T \log \left(\frac{m_h^*}{m_e^*} \right)$$

$$\left[\text{for } m_e^* = m_h^* \cdot \log \frac{m_h^*}{m_e^*} = \log 2 \right]$$

$$\Rightarrow E_F = \frac{E_C + E_V}{2}$$

So Fermi level lies exactly in the middle of the top valence band and the bottom conduction band.

FEW IMPORTANT QUESTIONS

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Q.1.(a) Define the divergence of a vector and its significance.

Ans. The divergence of a vector field is a scalar quantity which measures the rate at which the fluid is diverging or converging through the surface enclosing unit volume.

Physical significance of divergence: If A represents the velocity of a fluid at any point P then $\operatorname{div} A$ gives the rate at which the fluid is diverging per unit volume from point P .

If $\operatorname{div} A = 0$ at any point P , then either the fluid is expanding or its density is decreasing with time or, the point P is source of the fluid. If $\operatorname{div} A < 0$ then either the fluid is contracting or its density is increasing at P or, the point is a sink.

If $\operatorname{div} A \neq 0$, then flux of A entering any element of space is exactly balanced by the flux leaving it. Then there is no source or sink in the field nor its density is changing.

Q.1.(b) Set up a relationship between group and phase velocity.

(2)

Ans. Phase velocity \Rightarrow

$$V_p = \frac{w}{k}$$

$$\text{Group Velocity} \Rightarrow V_g = \frac{dw}{dx}$$

Where, w is the angular frequency of the wave and k is the wave vector.

But

$$w = v_p k$$

$$V_g = \frac{d}{dk}(v_p k)$$

$$V_g = v_p + k \frac{dV_p}{dx}$$

$$V_g = V_p + k \frac{dV_p}{d\lambda} \frac{d\lambda}{dx}$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{d\lambda}{dk} = \frac{d}{dx} \left(\frac{2\pi}{\lambda} \right) = \frac{-2\pi}{k^2}$$

$$\text{But } V_g = V_p + x \frac{dV_p}{d\lambda} \left(\frac{-2\pi}{k^2} \right)$$

$$V_g = V_p - \frac{2\pi}{k} \frac{dv_p}{d\lambda}$$

$$V_g = V_p - \frac{\lambda dV_p}{d\lambda}$$

Q.1.(c) Explain Laue Method of crystal structure.

Ans. Laue Method of crystal structure determination

A single crystal is mounted on a goniometer, which enables the crystal to be rotated through known angles in two perpendicular planes, and maintained stationary in a beam of X-rays ranging in wavelength from about 0.2 to 2.0 Å. The crystal selects out

FEW IMPORTANT QUESTIONS

2016

and diffracts those values of λ , for which planes exist, of spacing d and glancing angle θ , satisfying the Bragg's equation. A flat photographic film is placed to record either the transmitted diffracted beam or the reflected diffracted beams as illustrated in Fig. 13.

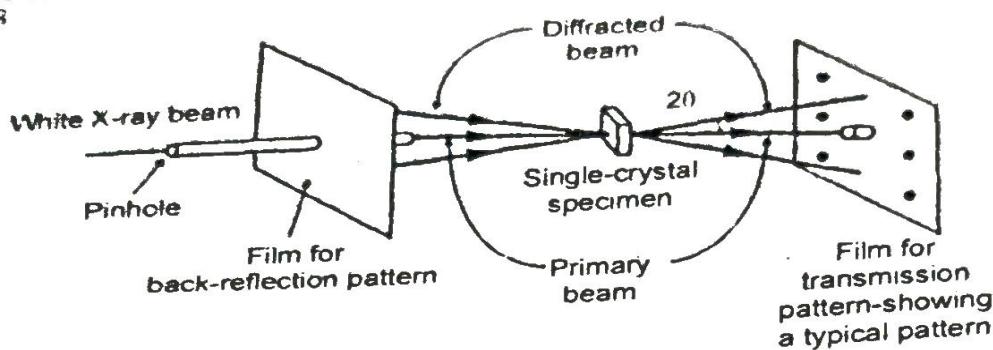


Fig. 13. Illustration for Laue technique.

As shown in Fig. 3 the resulting Laue pattern consists of a series of spots. Well-defined spots on the film are good evidence of a perfect crystal structure, while diffuse, broken or extended spots indicate lattice distortion, defects or other departures from the perfect crystal lattice.

The Laue pattern reveals the symmetry of the crystal structure in the orientation used; for example, if a cubic crystal is oriented with a cube edge, e.g., [100] axis, parallel to the incident beam, the Laue pattern will show the four fold symmetry appropriate to this axis.

Q.1. (d) If $\vec{A} = 2x\hat{i} + 2y\hat{j} + 3z\hat{k}$. Find $\text{Curl } \vec{A}$

$$\begin{aligned}
 \text{Ans. } \vec{\nabla} \times \vec{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 2y & 3z \end{vmatrix} \\
 &= \hat{i} \left(\frac{\partial}{\partial y} 3z - \frac{\partial}{\partial z} 2y \right) - \hat{j} \left(\frac{\partial}{\partial x} 3z - \frac{\partial}{\partial z} 2x \right) + \hat{k} \left(\frac{\partial}{\partial x} 2y - \frac{\partial}{\partial y} 2x \right) \\
 &= 0 + 0 + 0 \\
 &\boxed{\vec{\nabla} \times \vec{A} = 0}
 \end{aligned}$$

FIRST TERM EXAMINATION [FEB. 2017]
SECOND SEMESTER [B.TECH.]
APPLIED PHYSICS-II
(ETPH-104)

M.M.: 30

Time : 1.5 hrs.
 Note: Q.No. 1 is compulsory and attempt any two questions from the rest.

Q.1. Do any five.

Q.1. (a) Prove that vector $\vec{A} = 3Y^2Z^2\hat{i} + 3X^2Z^2\hat{j} + 3X^2Y^2\hat{k}$ is solenoidal.

(2)

Ans. For solenoidal vector.

$$\nabla \cdot \vec{A} = 0$$

$$\begin{aligned}\nabla \cdot \vec{A} &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (3Y^2Z^2\hat{i} + 3X^2Z^2\hat{j} + 3X^2Y^2\hat{k}) \\ &= 0 + 0 + 0\end{aligned}$$

$$\boxed{\nabla \cdot \vec{A} = 0}$$

Q.1. (b) Distinguish between conduction current and displacement current. (2)

Ans. The difference between conduction and displacement current is as following-

Conduction Current	Displacement Current
(i) Actually flows through in conductive medium and obeys ohm's law	Set up in a dielectric medium due to changing electric field across the dielectric which leads to variation of induced displacement of charge
(ii) $\bar{J}_c = \sigma \bar{E}$	$J_d = \frac{\partial \bar{D}}{\partial t} = \frac{\epsilon \partial \bar{E}}{\partial t}$
I = V/R (iii) For Constant E $\Rightarrow J_c \neq 0$	For constant E $\Rightarrow J_d = 0$

Q.1. (c) Which type of the statistics shall be applicable for a gas of photons? (2)
 Justify your answer.

Ans. BE statistic is applicable to photons as these particles have symmetric wave function and the particles are identical and indistinguishable with integral spin.

Q.1. (d) Explain how matter waves differ from light and sound waves? (2)

Ans. Light travels as transverse waves and can travel through a vacuum. Sound travels as longitudinal waves and needs to travel through a solid, liquid or gas. It cannot travel through a vacuum. Matter waves are the waves associated with matter particles and these waves are not electromagnetic waves but are pilot waves which guide the particle.

Q.1. (e) Find phase velocity and group velocity of an electron whose speed is 0.88c. (2)

Ans.

$$V_p V_g = c^2$$

and

$$V_s = 0.88c \Rightarrow V_s = 2.64 \times 10^8 \frac{m}{s}$$

$$V_p = \frac{c^2}{V_s}$$

$$V_p = \frac{3 \times 10^8 C}{0.88C} = 3.4 \times 10^8 \frac{m}{s}$$

$$V_p = 3.4 \times 10^8 \frac{m}{s}$$

Q.1. (f) A central spectral line has wavelength 4000 Å. Calculate the energy of the photon in eV.

Ans.

$$E = \frac{hc}{\lambda}$$

where,

$$h = 6.63 \times 10^{-34} \text{ J.S}$$

$$C = 3 \times 10^8 \text{ m/s}$$

$$\lambda = 4000 \text{ Å} = 4000 \times 10^{-10} \text{ m}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} = 4.97 \times 10^{-19} \text{ J}$$

$$= \frac{4.97 \times 10^{-19}}{1.69 \times 10^{-19}} \text{ eV}$$

$$E = 3.1 \text{ eV}$$

Q.2. (a) Write Maxwell's equations in differential form and also derive equations in conducting medium?

Ans. The Maxwell's equations in differential form are as following-

$$\vec{\nabla} \cdot \vec{D} = \rho \text{ or } \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Plane Electromagnetic Waves in conducting Medium
Maxwell's equations are:

$$\text{div } \vec{D} = \nabla \cdot \vec{D} = \rho \quad \dots(a)$$

$$\text{div } \vec{B} = \nabla \cdot \vec{B} = 0 \quad \dots(b)$$

$$\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \dots(c)$$

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \dots(d)$$

Let us assume that medium is linear and isotropic and its character can be determined by permittivity ϵ and permeability μ and conductivity σ , but no any charge or current density. Then we can write $D = \epsilon E$, $B = \mu H$, $J = \sigma E$ and $\rho = 0$. So that Maxwell's Eqs. (1) in this case take the form

$$\begin{aligned} \text{div } E &= 0 & \dots(a) \\ \text{div } H &= 0 & \dots(b) \\ \text{curl } E &= -\frac{\partial H}{\partial t} & \dots(c) \\ \text{curl } H &= \sigma E + \epsilon \frac{\partial E}{\partial t} & \dots(d) \end{aligned}$$

}

Taking curl of Eq. (3), we get

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t} (\text{curl } H)$$

Substituting curl H from Eq. 3. in above equation we get

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t} \left(\sigma E + \epsilon \frac{\partial E}{\partial t} \right)$$

$$\text{curl curl } E = -\sigma \mu \frac{\partial E}{\partial t} - \epsilon \mu \frac{\partial^2 E}{\partial t^2} \quad \dots(4)$$

Similarly, if we take the curl of Eq. 3 (d) and substitute curl E from Eq. 3. (c) we obtain

$$\text{curl curl } H = -\sigma \mu \frac{\partial H}{\partial t} - \epsilon \mu \frac{\partial^2 H}{\partial t^2} \quad \dots(5)$$

Now, using vector identity

$$\text{curl curl } A = \text{grad div } A - \nabla^2 A$$

Keeping in view Eqs. 3.(a) and 3.(b) [i.e., $\text{div } E = 0$ and $\text{div } H = 0$]

Equation (4) and (5) take the form—

$$\nabla^2 E - \sigma \mu \frac{\partial E}{\partial t} - \epsilon \mu \frac{\partial^2 E}{\partial t^2} = 0 \quad \dots(6)$$

$$\nabla^2 H - \sigma \mu \frac{\partial H}{\partial t} - \epsilon \mu \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots(7)$$

These equations represent wave equations governing electromagnetic fields E and H in a homogeneous conducting medium of conductivity σ . It is apparent that these equations are vector equations of E and H separately, satisfies the same scalar wave equation of the form.

$$\nabla^2 \phi - \sigma \mu \frac{\partial \phi}{\partial t} - \epsilon \mu \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(8)$$

where ϕ is a scalar and can stand for any one of components E and H.

Q.2. (b) Show that for a good conductor the magnetic field lags the electric field by 45° .

Ans. Phase of E and H

We know that,

$$\begin{aligned} H &= \frac{1}{\mu\omega} (\mathbf{k} \times \mathbf{E}) = \frac{1}{\mu\omega} \mathbf{k} (\hat{n} \times \mathbf{E}) \\ &= \frac{1}{\mu\omega} (\alpha + i\beta) (\hat{n} \times \mathbf{E}) \end{aligned}$$

This implies that

$$\left| \frac{H}{E} \right| = \frac{H_0}{E_0} = \frac{\alpha + i\beta}{\mu\omega} = \text{complex quantity}$$

i.e., field vectors H and E are out of phase in a conductor.
The magnitude and phase of complex k is written as $k = |k| e^{i\phi}$, may be defined.

$$|k| = |\alpha + i\beta| = \sqrt{(\alpha^2 + \beta^2)} = \omega\sqrt{\mu\epsilon} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2}$$

$$\phi = \tan^{-1} \frac{\beta}{\alpha} = \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\omega\epsilon} \right)$$

and

So, Eq. (1) may be expressed as

$$\begin{aligned} H &= \frac{1}{\mu\omega} \omega\sqrt{\mu\epsilon} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2} e^{i\phi} (\hat{n} \times \mathbf{E}) \\ &= \sqrt{\frac{\epsilon}{\mu} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2}} e^{-i(-\phi)} (\hat{n} \times \mathbf{E}) \end{aligned}$$

This interpretation of this equation is that H lags behind E in time by phase ϕ given by Eq. (4) and has a relative magnitude

$$\left| \frac{H}{E} \right| = \frac{H_0}{E_0} = \sqrt{\frac{\epsilon}{\mu} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2}}$$

Impedance of a conducting medium
The impedance of conducting medium is complex quantity and is given by

$$Z = \frac{E}{H} = \frac{E}{\left| \frac{1}{\mu\omega} \mathbf{k} \times \mathbf{E} \right|} = \frac{\mu\omega}{\mathbf{k}} = \frac{\mu\omega k}{k^2} = \frac{\mu\omega(\alpha + i\beta)}{\alpha^2 + \beta^2}$$

The Poynting vector is given by

$$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$$

the time average of Poynting vector may be expressed as

$$\langle S \rangle = \frac{1}{2} \text{Real part of } (\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2} \text{Re} (\mathbf{E} \times \mathbf{H}^*)$$

where \mathbf{H}^* (denotes complex conjugate of H and Re denotes real part)

$$\begin{aligned} \langle S \rangle &= \frac{1}{2} \text{Re} \left[\mathbf{E} \times \left\{ \sqrt{\frac{\epsilon}{\mu} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2}} e^{-i\phi} \hat{n} \times \mathbf{E}^* \right\} \right] \\ &= \frac{1}{2} \sqrt{\frac{\epsilon}{\mu} \left[1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right]^{1/2}} \text{Re} \{ \mathbf{E} \times (\hat{n} \times \mathbf{E}^*) e^{-i\phi} \} \end{aligned}$$

$$= \frac{1}{2} \sqrt{\mu} \left[1 + \left(\frac{\sigma}{\omega \epsilon_0} \right)^2 \right]^{1/2} \operatorname{Re} \left\{ (E \cdot E^*)_{\text{eff}} - (E \cdot n) E^*_{\text{parallel}} \right\}$$

$$= \frac{1}{2} \sqrt{\mu} \left[1 + \left(\frac{\sigma}{\omega \epsilon_0} \right)^2 \right]^{1/2} E_0^2 e^{-2M_r \frac{\sigma}{\omega \epsilon_0}}$$

$$\left[\because E \cdot E^* = E_0^2 e^{-2M_r} \text{ and } \operatorname{Re}(e^{-M_r}) > \text{const} \right]$$

for good conductor $\frac{\sigma}{\omega \epsilon_0} \gg 1$ so that $\phi = \frac{\pi}{4}$ and also $E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$ hence

$$\langle S \rangle = \sqrt{\frac{\sigma}{2\mu\omega}} E_{\text{rms}}^2 e^{-2M_r r_n} \quad \dots (19)$$

So, we can see that in good conductor \vec{H} lags behind \vec{E} by $\frac{\pi}{4} = 45^\circ$.

Q.2. (c) A charge of $1500 \mu\text{C}$ is distributed over a very large sheet having surface area of 300 m^2 . Calculate the electric field at a distance of 25 cm. (2.5)

$$\sigma = \frac{1500 \mu\text{C}}{300 \text{ m}^2}$$

Ans.

$$= \frac{1500 \times 10^{-6}}{300} = 5 \times 10^{-6} \frac{\text{C}}{\text{m}^2}$$

$$\boxed{\sigma = 5 \times 10^{-6} \frac{\text{C}}{\text{m}^2}}$$

Q.3. (a) How Davission and Germer demonstrated matter waves in laboratory through experiment? (3)

Ans. Demonstration of Matter Waves: Davisson-Germer Experiment on Electron Diffraction

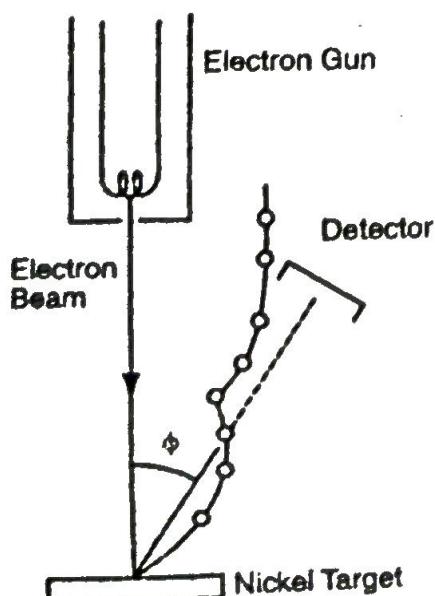


Fig. 1.

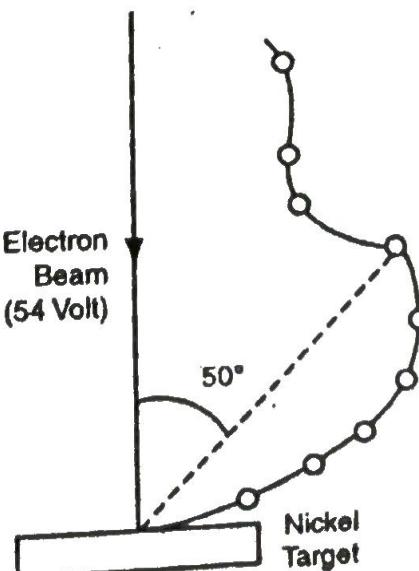


Fig. 2.

The Davisson Germer experiment was conducted in 1927. The experimental setup shown in Fig. 1. Here electrons from a heated filamentary electron gun were accelerated by voltage V and allowed to fall on surface of nickel target. Davisson Germer measured the intensity of scattered electrons as a function of angle ϕ and plotted it in the polar diagram. Fig. 2 shows results from the accelerating voltage of 54 V. For this case there is an intense scattering or a prominent peak at an angle of $\phi = 50^\circ$. Such deflection can be explained by assuming that the electron beam have wave associated with it. This situation is similar to the Bragg deflection. So the waves associated with the electron beam were satisfying Bragg's law caused a diffraction peak. From Fig. 3 it is seen that atomic planes crystal having $\theta = 50^\circ$, $\phi = (180 - 50)/2$, $d = 0.91\text{ \AA}$ and $n = 1$. From Bragg's law $n\lambda = 2d \sin \theta$ gives

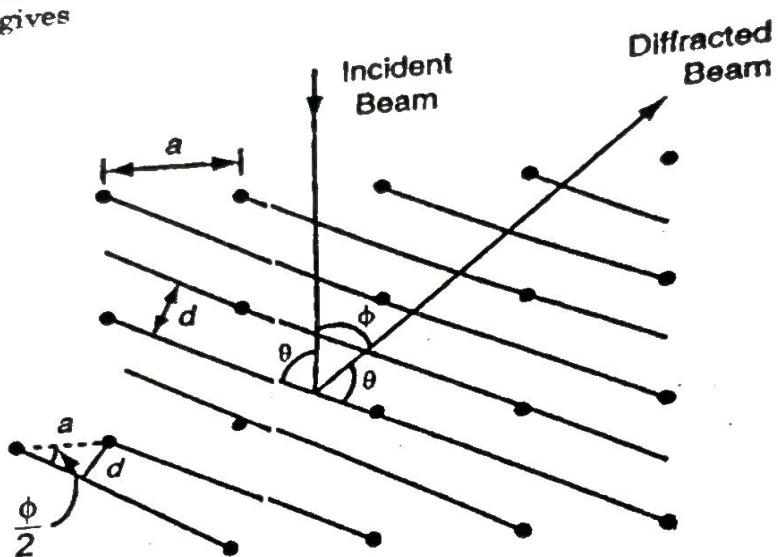


Fig. 3.

$$\lambda = 2d \sin \theta = 2 \times 0.91 \times \sin 65^\circ = 1.65 \text{ \AA}$$

Since Bragg's law basically talks about the diffraction of X rays, this experiment enables us to treat the electrons as waves and the wavelength associated with electrons should be 1.65 \AA when they are scattered at $\phi = 50^\circ$.

Now we apply deBroglie's hypothesis. Since the electron of mass m gain the velocity v when it gets accelerated through a potential difference of V , we write the following relation for the energy for the nonrelativistic motion of the electron

$$\frac{1}{2}mv^2 = eV$$

So the deBroglie wavelength associated with the electron is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

or

$$\lambda = \sqrt{\frac{150}{V}} \text{ \AA}$$

Therefore, deBroglie wavelength associated with the electron that is accelerated through V is given as

$$\lambda = \sqrt{\frac{150}{V}} \text{ \AA} = \sqrt{\frac{150}{54}} \text{ \AA} = 1.67 \text{ \AA}$$

A comparison of Eq. (i) with Eq. (ii) shows that the value of the wavelength λ is the same in both the cases. It means there is a wave called deBroglie wave associated with the electrons. Therefore, this confirms the deBroglie hypothesis.

Q.8. (a) A particle of mass 'm' bounces back and forth in a container box with perfectly elastic sides of length 'l'. Show that using Schrodinger's time independent wave equation that the particle possesses discrete energy given by

$$E_n = n^2 \hbar^2 / 8ml^2$$

(Can you observe the energy states for a ball of mass 10g moving in a box of length 10 cm?)

Ans. Consider a particle moving inside a box along the x-direction. The particle is bouncing back and forth between the walls of the box. l is the width of the box as shown in fig 4.

The potential energy V of the particle is infinite on both sides of the box.

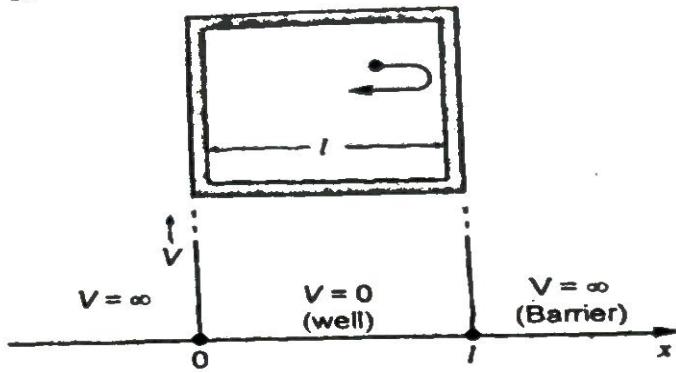


Fig. 4. A particle confined to a box of width a .

$$V = 0 \text{ for } 0 < x < l$$

$$V = \infty \text{ for } x \leq 0 \text{ and } x \geq l$$

The particle cannot exist outside the box, so its wave function Ψ is 0 for $x \leq 0$ and $x \geq l$. Within the box, the Schrodinger's equation becomes

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} E \Psi = 0 \quad [\because V \text{ is 0 for the free particle}] \quad \dots(1)$$

$$\Rightarrow \frac{d^2\Psi}{dx^2} + K^2 \Psi = 0 \text{ where } K = \sqrt{\frac{2mE}{\hbar^2}} \quad \dots(2)$$

The general solution of this equation is—

$$\Psi = A \sin Kx + B \cos Kx$$

Using boundary conditions

$$\Psi = 0 \text{ at } x = 0 \text{ and } \Psi = 0 \text{ at } x = l$$

$$0 = A \sin 0 + B \quad 0 = A \sin Kl$$

$$B = 0$$

$$A \neq 0 \sin kl = 0$$

$$\sin n\pi = \sin Kl$$

$$K = \frac{n\pi}{l}$$

Wave function,

$$\Psi_n(x) = A \sin \frac{n\pi x}{l} \text{ where } n = 1, 2, 3, \dots$$

Energy level,

$$E_n = \frac{k^2 h^3}{2m} \left[n = \frac{h}{2\pi} \right]$$

Eigenvalue

$$E_n = \left(\frac{n\pi}{l} \right)^2 \left(\frac{h}{2\pi} \right)^2 \frac{1}{2m}$$

$$E_n = \frac{n^2 h^2}{8ml^2} \text{ where } n = 1, 2, 3, 4, \dots$$

As,

$$E_n = \frac{n^2 h^2}{8ml^2}$$

Given,

$$m = 10 \text{ gm}, l = 10 \text{ cm}$$

for $n = 1$

$$E_1 = \frac{6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{8 \times 10 \times 10^{-3} \times 10 \times 10^{-2} \times 10 \times 10^{-2}} \\ = 5.49 \times 10^{-64} \text{ J}$$

for $n = 2$

$$E_2 = \frac{(2)^2 \times 6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{8 \times 10 \times 10^{-3} \times 10 \times 10^{-2} \times 10 \times 10^{-2}} \\ = 21.96 \times 10^{-64} \text{ J}$$

Q.4. (a) State and prove Poynting theorem.

Ans. Poynting theorem states that the net power flowing out of a given volume equal to the time rate of decrease in energy stored within V minus the ohmic loss.

i.e.,

$$\oint \operatorname{div}(\vec{E} \times \vec{H}) dV = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] dV - \int_V \sigma E^2 dt$$

Total Power leaving the volume = Rate of decrease in energy stored in electric and magnetic fields

- Ohmic power dissipated

Since we know that,

Electrostatic potential energy

$$U_e = \frac{1}{2} \int_V \vec{E} \cdot \vec{D} dV$$

and energy stored in magnetic field

$$U_m = \frac{1}{2} \int_V \vec{H} \cdot \vec{B} dV$$

Maxwell's equations in differential form are:

$$\begin{aligned} \bar{\nabla} \cdot \bar{D} &= \rho & \text{or} & \quad \operatorname{div} \bar{D} = \rho & \dots (a) \\ \bar{\nabla} \cdot \bar{B} &= 0 & \text{or} & \quad \operatorname{div} \bar{B} = 0 & \dots (b) \\ \bar{\nabla} \times \bar{E} &= -\frac{\partial \bar{B}}{\partial t} & \text{or} & \quad \operatorname{curl} \bar{E} = -\frac{\partial \bar{B}}{\partial t} & \dots (c) \\ \bar{\nabla} \times \bar{H} &= \bar{J} + \frac{\partial \bar{D}}{\partial t} & \text{or} & \quad \operatorname{curl} \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} & \dots (d) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

Taking scalar product of Eq. (5) with \mathbf{H} and Eq. (6) with \mathbf{E} , we get

$$\mathbf{H} \cdot \operatorname{curl} \mathbf{E} = -\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \quad \dots(5)$$

and

$$\mathbf{E} \cdot \operatorname{curl} \mathbf{H} = \mathbf{E} \cdot \mathbf{J} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \quad \dots(6)$$

Using vector identity,

$$\operatorname{div}(\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \operatorname{curl} \mathbf{E} - \mathbf{E} \cdot \operatorname{curl} \mathbf{H} \quad \dots(7)$$

Now, subtracting Eq. (5) from Eq. (6), we get

$$\mathbf{H} \cdot \operatorname{curl} \mathbf{E} - \mathbf{E} \cdot \operatorname{curl} \mathbf{H} = -\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} - \mathbf{E} \cdot \mathbf{J} \quad \dots(7)$$

From Eqs. (6) and (7), we get

$$\operatorname{div}(\mathbf{E} \times \mathbf{H}) = -\left(\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}\right) - \mathbf{E} \cdot \mathbf{J} \quad \dots(8)$$

Now if medium is linear so that the relation,

$$\bar{\mathbf{B}} = \mu \bar{\mathbf{H}} \text{ and } \bar{\mathbf{D}} = \epsilon \bar{\mathbf{E}} \quad \dots(9)$$

Apply, then we may write,

$$\bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{D}}}{\partial t} = \bar{\mathbf{E}} \cdot \frac{\partial}{\partial t}(\epsilon \bar{\mathbf{E}}) = \frac{1}{2} \epsilon \frac{\partial}{\partial t}(\bar{\mathbf{E}})^2 = \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{\mathbf{E}} \cdot \bar{\mathbf{D}}\right)$$

$$\bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{B}}}{\partial t} = \bar{\mathbf{H}} \cdot \frac{\partial}{\partial t}(\mu \bar{\mathbf{H}}) = \frac{1}{2} \mu \frac{\partial}{\partial t}(\bar{\mathbf{H}})^2 \frac{\partial}{\partial t}\left(\frac{1}{2} \bar{\mathbf{H}} \cdot \bar{\mathbf{B}}\right)$$

Using these relationships, Eq. (8) takes the form—

$$\operatorname{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = \frac{\partial}{\partial t}\left[\frac{1}{2}(\bar{\mathbf{E}} \cdot \bar{\mathbf{D}} + \bar{\mathbf{H}} \cdot \bar{\mathbf{B}})\right] - \bar{\mathbf{J}} \cdot \bar{\mathbf{E}} \quad \dots(10)$$

Each term in Eq. (10) has certain physical significance which may be seen by integrating eq. (10) over a volume V bounded by surface S . Thus,

$$\int_V \operatorname{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) dV = - \int_V \left\{ \frac{\partial}{\partial t} \frac{1}{2} (\bar{\mathbf{E}} \cdot \bar{\mathbf{D}} + \bar{\mathbf{H}} \cdot \bar{\mathbf{B}}) \right\} dV - \int_V \bar{\mathbf{J}} \cdot \bar{\mathbf{E}} dV \quad \dots(11)$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\oint_S (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) \cdot dS = - \frac{\partial}{\partial t} \int_V \frac{1}{2} (\bar{\mathbf{E}} \cdot \bar{\mathbf{D}} + \bar{\mathbf{H}} \cdot \bar{\mathbf{B}}) dV - \int_V \bar{\mathbf{J}} \cdot \bar{\mathbf{E}} dV \quad \dots(12)$$

Rearranging this equation, we get

$$-\int_V \bar{\mathbf{J}} \cdot \bar{\mathbf{E}} dV = \frac{\partial}{\partial t} \int_V \frac{1}{2} (\bar{\mathbf{E}} \cdot \bar{\mathbf{D}} + \bar{\mathbf{H}} \cdot \bar{\mathbf{B}}) dV + \oint_S (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) \cdot dS \quad \dots(13)$$

Equation (11) may also be written as

$$\int_V \operatorname{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) dV = - \frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \epsilon \bar{\mathbf{E}}^2 + \frac{1}{2} \mu \bar{\mathbf{H}}^2 \right) dV - \int_V \sigma \bar{\mathbf{E}}^2 dV \quad \dots(14)$$

Equations (13) and (14) represent the expression for poynting theorem.

10-2017

Second Semester, Applied Physics-II

Q.4. (b) Derive Planck's radiation formula for the distribution of energy in the spectrum of black body.

Ans. If N be the total number of Planck's oscillators and E is their energy, then average energy per Planck's oscillator is given by

$$\bar{E} = \frac{E}{N}$$

Let, there be $N_0, N_1, N_2, N_3, \dots, N_r$ oscillators having energy $0, E, 2E, \dots, rE$ respectively. Now, we have

$$N = N_0 + N_1 + N_2 + N_3 + \dots + N_r$$

and

$$E = 0 + EN_1 + 2EN_2 + 3EN_3 + \dots + rEN_r$$

According to Maxwell's distribution formula, the number of oscillators having energy rE is given by

$$N_r = N_0 e^{-rE/k_B T} = N_0 \exp\left[-\frac{rE}{k_B T}\right]$$

where k_B = Boltzmann's constant [$= 1.3807 \times 10^{-23} \text{ J/K}$].

The total energy E is given by

$$\begin{aligned} E &= N_0 \times 0 + EN_0 \exp\left(\frac{-E}{k_B T}\right) + 2EN_0 \exp\left(\frac{-2E}{k_B T}\right) + 3E \exp\left(\frac{-3E}{k_B T}\right) + \dots + rN_0 E \exp\left(\frac{-rE}{k_B T}\right) \\ &= N_0 E \exp\left(\frac{-E}{k_B T}\right) \left[1 + 2 \exp\left(\frac{-E}{k_B T}\right) + 3 \exp\left(\frac{-2E}{k_B T}\right) + \dots + r \exp\left(\frac{-(r-1)E}{k_B T}\right) \right] \end{aligned}$$

Again assume $x = \exp\left(\frac{-E}{k_B T}\right)$

Then $E = N_0 E \exp\left(-\frac{E}{k_B T}\right) [1 + 2x + 3x^2 + \dots + rx^{r-1}]$

$$= N_0 E \exp\left(-\frac{E}{k_B T}\right) \left[\frac{1}{(1-x)^2} \right] = \frac{N_0 E \exp\left(-\frac{E}{k_B T}\right)}{\left[1 - \exp\left(-\frac{E}{k_B T}\right) \right]^2}$$

Now the average energy of oscillator is given by

$$\langle E \rangle = \bar{E} = \frac{E}{N}$$

$$= \frac{N_0 E \exp\left(-\frac{E}{k_B T}\right) / \left[1 - \exp\left(-\frac{E}{k_B T}\right) \right]^2}{N_0 / (1 - \exp(-E/k_B T))}$$

$$= \frac{E \exp\left(-\frac{E}{k_B T}\right)}{1 - \exp\left(-\frac{E}{k_B T}\right)} = \frac{E}{\exp\left(\frac{E}{k_B T}\right) - 1}$$

$$E = \frac{hv}{\exp\left(\frac{hv}{k_B T}\right) - 1} \quad (7)$$

Thus we see that the average energy of Planck's oscillator given by Eq (7) is different than the energy $k_B T$ of a classical oscillator.
We know that the number of oscillators per unit volume in frequency range v and Δv is given by

$$N = N(v) dv = \frac{8\pi v^2}{c^3} dv \quad (8)$$

Multiply Eq. (8) with average energy of oscillators given by Eq. (7), we get the total E per unit volume belonging to the range dv or the density belonging to range dv as

$$E_v dv = \frac{8\pi v^2}{c^3} dv \times \frac{hv}{\left(\exp\frac{hv}{k_B T} - 1\right)} \quad (9)$$

$$E_v dv = \frac{8hv^3}{c^3} \times \frac{1}{\left(\exp\frac{hv}{k_B T} - 1\right)} dv \quad (9)$$

Equation (9) is said to be Planck's radiation law.
Q.4 (c) If the magnitude of \bar{H} in a plane wave is 1.0 Amp/m. Find the magnitude of \bar{E} for a plane wave in free space. (2)

Ans. In free space

$$Z = \left| \frac{E_0}{H_0} \right| = \mu_0 c = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.720 \text{ ohm}$$

$$H = 1 \frac{\text{Amp}}{\text{m}}$$

$$E = ZH_0$$

$E = 376.72 \frac{\text{V}}{\text{m}}$

Given

Date _____

END TERM EXAMINATION [MAY-JUNE 2017]

SECOND SEMESTER [B.TECH.]

APPLIED PHYSICS-II

(ETPH-104)

Time : 3 hrs.

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q.1. Attempt each one of the following:

Q.1. (a) Define divergence and curl of a vector.

Ans. Divergence: The divergence of a vector field at any point is defined as the amount of flux per unit volume diverging from that point.

Curl: The curl of a vector field is defined as the maximum line integral of the field per unit area.

Q.1. (b) Distinguish between displacement current and conventional current.

Ans. Refer Q.1(b) of First Term Examination Feb. 2017.

Q.1. (c) Find constant a , b and c so that

$\mathbf{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

Ans. For irrotational vector-

$$\bar{\nabla} \times \bar{A} = 0$$

$$\bar{\nabla} \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

Here,

$$A_x = x + 2y + az$$

$$A_y = bx - 3y - z$$

$$A_z = 4x + cy + 2z$$

Solving for $\bar{\nabla} \times \bar{A}$

$$\begin{aligned} \bar{\nabla} \times \bar{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x + 2y + az & bx - 3y - z & 4x + cy + 2z \end{vmatrix} \\ &= \hat{i} \left[\frac{\partial}{\partial y} (4x + cy + 2z) - \frac{\partial}{\partial z} (bx - 3y - z) \right] + \\ &\quad \hat{j} \left[\frac{\partial}{\partial z} (x + 2y + az) - \frac{\partial}{\partial x} (4x + cy + 2z) \right] + \\ &\quad \hat{k} \left[\frac{\partial}{\partial x} (bx - 3y - z) - \frac{\partial}{\partial y} (x + 2y + az) \right] \\ &= \hat{i}(c + 1) + \hat{j}(a - 4) + \hat{k}(b - 2) = 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.1. (a) Describe the implication of Davisson-Germer experiment. (2.5)
 Ans. Davisson and Germer in 1927 designed an apparatus to determine the wavelength associated with electrons to confirm the dual nature of matter.

Q.1. (b) Why is the energy of a particle trapped inside a box quantized? (2.5)

Ans. Energy of a particle trapped inside a box is given by the following equation:-

$$E_n = \frac{n^2 h^2}{8ml^2}$$

$$n = 1, 2, 3, 4, \dots$$

where,
 It is clear from the equation that particle cannot possess an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3, \dots$

Q.1. (c) An electron is confined to move between two rigid walls separated by 2×10^{-9} m. Find the deBroglie wavelengths representing the first three allowed energy states of the electron. (2.5)

Ans.

$$E_n = \frac{n^2 h^2}{8ml^2}$$

$$l = 2 \times 10^{-9} \text{ m}$$

$$m = 9 \times 10^{-31} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

For $n = 1$ -

$$E_1 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}}$$

and

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_1} \times \frac{1}{1.6 \times 10^{-19}}$$

Solving for λ_1 -

$$\lambda_1 = 1.31 \times 10^{-5} \text{ m}$$

For $n = 2$ -

$$E_2 = \frac{(2)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

and

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} \times \frac{1}{1.6 \times 10^{-19}}$$

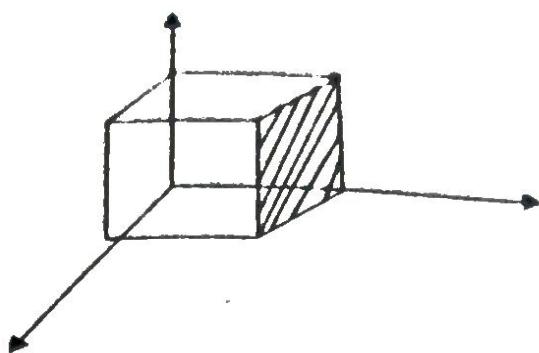
Solving for λ_2

$$\lambda_2 = 3.2 \times 10^{-6} \text{ m}$$

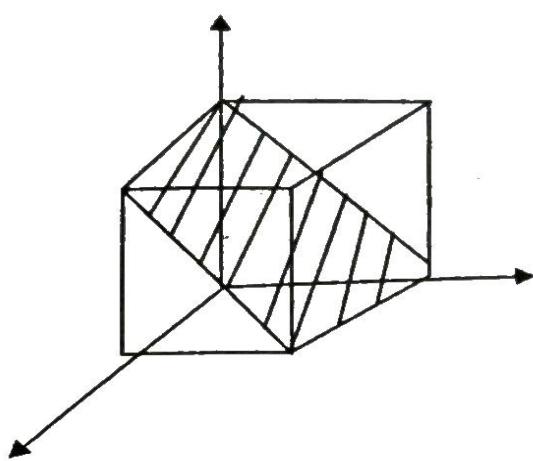
For $n = 3$

$$E_3 = \frac{(3)^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

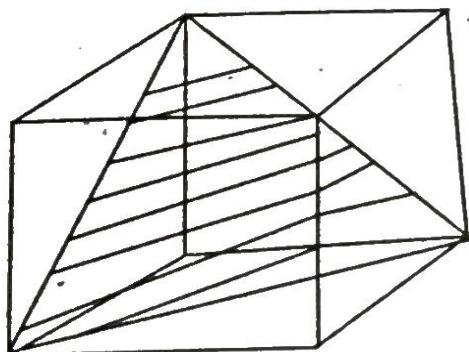
Q.1. (i) Draw the planes (100), (110) and (111)
 Ans. (100) plane

**Fig. 1.**

(110) plane-

**Fig. 2.**

(111) plane-

**Fig. 3.**

Q.1. (j) Define Fermi energy. What is its physical significance? (2.5)

Ans. Fermi energy is a quantum phenomenon which translates as the difference in energy state occupied by the lowest level electron to the highest level. It can also be measured at the top of the collection of electron energy level at absolute zero. The minimum energy that electrons may possess at 0 kelvin is the Fermi energy.

UNIT-I

Q. 1. (a) Define Maxwell's equation and express it in integral form. State physical significance of each.

Ans. 1. Maxwell's first equation: $\nabla \cdot D = \rho$

Integrating this over an arbitrary volume V, we get

$$\int_V D \cdot dV = \rho V$$

Changing volume integral into surface integral in L.H.S. of above equation by Gauss divergence theorem, we get

$$\boxed{\int_S D \cdot dS = \int_V \rho dV}$$

Where S is the surface, which bounds volume V. Eq. (1) represents Maxwell's first equation $\nabla \cdot D = \rho$ in integral form. Since $\int_V \rho dV = q$, the net charge contained in volume V, therefore, Maxwell's first equation signifies that:

The net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume.

2. Maxwell's second equation: $\nabla \cdot B = 0$

Integrating this over an arbitrary volume V, we get

$$\int_V \nabla \cdot B dV = 0$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\boxed{\int_S B \cdot d\bar{S} = 0}$$

Where S is the surface, which bounds volume V. Equation (2) represents Maxwell's second equation in integral form and signifies that:

The net outward flux of magnetic induction B through any closed surface is equal to zero.

3. Maxwell's Third equation: $\nabla \times E = -\frac{\partial B}{\partial t}$

Integrating above equation over a surface S bounded by a curve C, we get

$$\int_S (\nabla \times E) \cdot d\bar{S} = - \int_S \frac{\partial B}{\partial t} \cdot d\bar{S}$$

Using Stoke's theorem to convert surface integral on L.H.S. of above equation into line integral along the boundary C, we get

$$\boxed{\int_C E \cdot d\vec{l} = - \frac{\partial}{\partial t} \int_S B \cdot d\bar{S}}$$

The electromotive force $(\text{emf}) = \int_C \vec{E} \cdot d\vec{l}$ around a close path is equal to the rate of change of magnetic flux linked with the path. Maxwell's fourth equation:

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking surface integral over surface S bounded by curve C, we obtain

$$\int_S (\nabla \times \vec{H}) d\vec{S} = \int \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$$

Using stoke's theorem to convert surface integral on L.H.S of above equation into line integral, we get

$$\oint \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$$

Equation (4) represents Maxwell's 4th equation in integral form and signifies that.

The magnetomotive force $m.mf = \oint \vec{H} \cdot d\vec{l}$ around a close path is equal to the conduction current plus displacement current through any surface bounded by the path.

Q.2. (b) Write down the equation of continuity and mention its physical significance. (2)

Ans. An equation, which expresses the equality of incoming and outgoing charges in a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density J and the charge density ρ are related at each point through a differential equation. This relation is based on the fact that electric charge can neither be created nor be destroyed and rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.

$$I = \int_S \vec{J} \cdot d\vec{S} \quad \dots(1)$$

Again, considering charge leaving a volume V per second

$$I = -\frac{\partial}{\partial t} \int_V \rho dV \quad \dots(2)$$

The negative sign comes here because the current is positive when the net charge is from the outside of V to within.

Since, we are dealing with a fixed volume V , hence we may write,

$$\frac{\partial}{\partial t} \int_V p dV = \int_V \frac{\partial p}{\partial t} dV$$

Again, using divergence theorem, we can write

$$\int_S J \cdot dS = - \int_V (\nabla \cdot J) dV$$

Therefore, from Eqs. (2), (3) and (4), we have

$$I = \int_V (\nabla \cdot J) dV = - \int_V \frac{\partial p}{\partial t} dV$$

$$\int_V (\nabla \cdot J) dV = - \int_V \frac{\partial p}{\partial t} dV$$

or

$$\int \left\{ (\nabla \cdot J) + \frac{\partial p}{\partial t} \right\} dV = 0$$

This integral must be zero for any arbitrary volume. It is only possible when integral is zero, i.e.,

$$\nabla \cdot J + \frac{\partial p}{\partial t} = 0$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial p}{\partial t} = 0$ and hence for steady current, we have

$$\nabla \cdot J = 0$$

Q.2. (c) A plane electromagnetic wave is travelling in an unbounded lossless dielectric medium with relative permeability $\mu_r = 1$ and permittivity $\epsilon_r = 3$. Find the velocity of the wave and the impedance of the medium.

Ans. Velocity of the wave is-

$$V = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$$

Given

$$\mu_r = 1, \epsilon_r = 3 \text{ and } c = 3 \times 10^8 \frac{m}{s}$$

$$V = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 = 1.732 \times 10^8 \frac{m}{s}$$

$$V = 1.732 \times 10^8 \frac{m}{s}$$

Impedance of the wave is-

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}}$$

$$Z = \sqrt{\frac{1 \times 4\pi \times 10^{-7}}{3 \times 8.86 \times 10^{-12}}} = 2.17 \times 10^2 \Omega$$

$$[Z = 2.17 \times 10^2 \Omega]$$

Q.3. (a) Derive Pointing theorem from Maxwell's equations and give its interpretation.

Ans. The electromagnetic waves carry energy when they propagate and there is an energy density associated with both electric and magnetic field. The amount of energy flowing through unit area, perpendicular to the direction of energy propagation per unit time, i.e. the rate of energy transport per unit area is called poynting vector. It is also called instantaneous energy flux density and is represented by $\vec{S} = \vec{E} \times \vec{H}$

Pointing theorem:

When em wave propagates through space from source to receiver there exist a simple and direct relationship between power transferred and amplitude of electric and magnetic field strength. The relationship may be obtained through maxwell equation.

from Maxwell IVth eqn.

$$\vec{\nabla} \times \vec{B} = \mu \left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

But, as

$$\vec{B} = \mu \vec{H} \Rightarrow \frac{\vec{B}}{\mu} = \vec{H}$$

and

$$\epsilon \vec{E} = \vec{D}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \epsilon \frac{d \vec{E}}{dt}$$

$$\vec{J} = \vec{\nabla} \times \vec{H} - \epsilon \frac{\partial \vec{E}}{\partial t}$$

This is equation of current density, when multiplied by E, this will result in a relation between the quantities which have the dimension of power.

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot (\vec{\nabla} \times \vec{H}) - \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t}$$

from vector identity-

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\vec{E} \cdot \vec{J} = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

From Maxwell's 111th equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} - \mu \frac{\partial \vec{H}}{\partial t}$$

we have

$$= \vec{E} \cdot \vec{J} = -\mu \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) - \epsilon \left(\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} H^2$$

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} E^2$$

$$\vec{E} \cdot \vec{J} = -\frac{\mu}{2} \frac{\partial}{\partial t} H^2 - \frac{\epsilon}{2} \frac{\partial}{\partial t} E^2 - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

Integrating over volume, v -

$$\int_v \vec{E} \cdot \vec{J} dv = -\frac{\partial}{\partial t} \int_v \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv - \int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv$$

Using divergence theorem.

$$\int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv = \oint_s (\vec{E} \times \vec{H}) \cdot ds$$

$$\boxed{\oint_s (\vec{E} \times \vec{H}) \cdot ds = -\frac{\partial}{\partial t} \int_s \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) ds - \int_v \vec{E} \cdot \vec{J} dv}$$

or

$$\frac{-\partial}{\partial t} \left\{ \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) \right\} dv = \int_s (\vec{E} \times \vec{H}) \cdot ds + \int_v \vec{E} \cdot \vec{J} dv$$

i.e. Rate of energy flow = rate of decrease of stored energy + total instantaneous ohmic power dissipated within the volume.

This is called the "Poynting theorem" and

$\vec{S} = \vec{E} \times \vec{H}$ is called "poynting vector".

$-\int_v \vec{E} \cdot \vec{J} dv$ - Rate of energy transferred into em field through the motion of charge in Volume V.

$-\frac{\partial}{\partial t} \int_v \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv$ - Rate of decrease of em energy stored in volume V.

$\oint_s (\vec{E} \times \vec{H}) \cdot ds$ - Amount of em energy crossing the closed surface per second.

Physical meaning of equation- The time rate of change of em energy with certain plus time rate of energy flowing out through the boundary surface is equal to transferred into em field.

Q.2. (b) Show that in free space the direction of flow of electromagnetic energy is along the direction of wave propagation. (4)

Ans. The Poynting vector (i.e., energy flow per unit area per unit time) for a plane electromagnetic wave is given by

$$\begin{aligned}
 \vec{S} &= \vec{E} \times \vec{H} = \vec{E} \times \frac{\hat{n} \times \vec{E}}{\mu_0 c} \\
 &= \frac{1}{\mu_0 c} \vec{E} \times (\hat{n} \times \vec{E}) \\
 &= \frac{1}{\mu_0 c} E^2 \hat{n} \quad [\vec{E} \cdot \hat{n} = 0; E \text{ being } \perp \text{ to } \hat{n}] \\
 &= \frac{E^2}{Z_0} \hat{n}
 \end{aligned}$$

For a plane electromagnetic wave of angular frequency ω , the average value of S in a complete cycle is given by

$$\begin{aligned}
 \langle S \rangle &= \frac{1}{Z_0} \langle E^2 \rangle \hat{n} = \frac{1}{Z_0} \left\langle (E_0 e^{ik.r - i\omega t})^2 \right\rangle_{real} \hat{n} \\
 &= \frac{1}{Z_0} E_0^2 \langle \cos^2(k.r - \omega t) \rangle \hat{n} \\
 &= \frac{1}{Z_0} \frac{E_0^2}{2} \hat{n} \quad [\text{Average value of } \cos^2(k.r - \omega t) = \frac{1}{2}] \\
 &= \frac{1}{Z_0} E_{rms}^2 \hat{n} \quad [\because E_{rms} = E_0 / \sqrt{2}]
 \end{aligned}$$

It is obvious that the direction of Poynting vector is along the direction of propagation of electromagnetic wave. This means that the flow of energy in a plane electromagnetic wave in free space is along the direction of wave.

Q.3. (c) Find the skin depth at a frequency of 90 Hz in aluminium where $\mu = 3.54 \times 10^7$ mho/m. Also find the wave velocity. (2.5)

Ans. Skin depth is-

Given-

$$\mu = \mu_0 \mu_r$$

$$\mu = 4\pi \times 10^{-7} \text{ (taking } \mu_r = 1)$$

$$w = 2\pi f$$

$$f = 90 \text{ Hz}$$

$$\delta = \sqrt{\frac{2}{\omega \mu \alpha}} = \sqrt{\frac{2}{2\pi \times 90 \times 4\pi \times 10^{-7} \times 3.54 \times 10^7}}$$

$$\delta = 8.92 \times 10^{-3} \text{ m}$$

Wave velocity is-

$$V = \frac{w}{\alpha} = \frac{w}{\beta}$$

($\alpha = \beta$ in given case)

$$\text{But } \delta = \frac{1}{\beta}$$

$$\therefore V = \omega \delta = 2\pi \times 90 \times 8.92 \times 10^{-3}$$

$$V = 5.041 \text{ m/s}$$

UNIT-II

Q.4. (a) Distinguish between a boson and a fermion. Give one example.

Ans.

Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum statistics)
<p>Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature</p> <p>The distribution law is</p> $n_i = \frac{g_i}{(e^{\alpha E_i/k_B T} - 1)}$ <p>There is no restriction on the number of particles in a given state.</p> <p>The volume of phase cell is of order of h^3.</p> <p>For $E_i \gg k_B T$, exponential $E_i \ll k_B T$, lies above M.B.</p> <p>The energy may be zero at absolute zero.</p>	<p>Apply to systems of indistinguishable particles obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)</p> <p>The distribution law is</p> $n_i = \frac{g_i}{(e^{\alpha E_i/k_B T} + 1)}$ <p>Only one particle in a given quantum state is allowed.</p> <p>The volume of phase cell is of the order of h^3.</p> <p>For $E_i \gg k_B T$, exponential where $E_i \gg k_B T$. If $E_F \gg k_B T$ decreases abruptly near E_F.</p> <p>The energy at absolute zero, cannot be zero because all particles cannot drop down to ground state due to Pauli exclusion principle.</p>

Q.4. (b) Write down the Planck's formula for the distribution of energy spectrum of blackbody. Show that Rayleigh-Jean's law and Wein's law are special cases of Planck's radiation law.

Ans. Refer to Q.4. (c) End Term Examination April 2017.

As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy

of these Planck's oscillators were calculated and finally Planck's radiation formula was derived.

Wein's law is deduced from Planck's radiation formula under the condition when the wavelength λ , and temperature T are very small. However, Planck's radiation formula under the condition of high temperature T and wavelength λ , takes the form of Rayleigh-Jeans law.

Planck radiation law is as following.

$$\mu \lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

When λ and T are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\therefore \mu \lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting

$$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

we get

$$\boxed{\mu \lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda}$$

This is Wein's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

$$\mu \lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-\frac{hc}{\lambda KT}} d\lambda$$

By substituting

$$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

we get

$$\boxed{\mu \lambda d\lambda = \frac{A}{\lambda^5} e^{-\frac{B}{\lambda T}} d\lambda}$$

This is Wein's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu \lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left\{ \left[1 + \frac{hc}{\lambda KT} \right]^{-1} \right\}} d\lambda$$

$$\boxed{\mu \lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength.

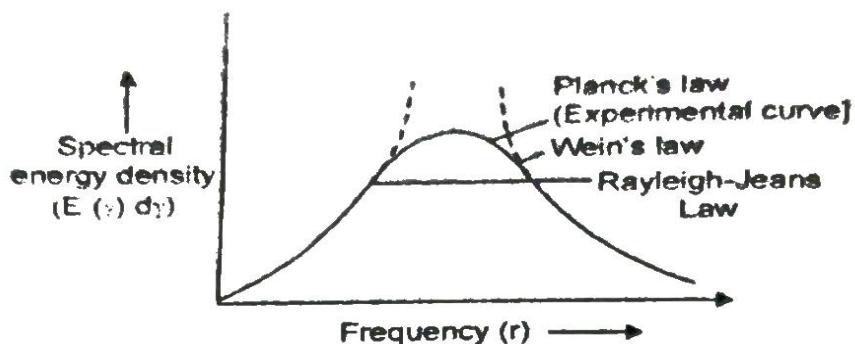


Fig. 4.

As shown in Fig. 4 the spectral energy density decreases with frequency at high frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the complete energy distribution for blackbody radiation, while Planck's law explains it completely.

Q.4. (c) Draw a neat diagram showing the energy distribution spectra of blackbody radiation. Explain how classical theory fails to explain the spectral distribution of energy. (3.5)

Ans. The postulates of quantum theory of radiation are as follows-

(i) A radiation has energy. As light and heat are radiations, they are also associated with energy.

(ii) Radiant energy is not emitted or observed continuously but discontinuously in the form of small packets called photons. Photon is not a material body but is considered to be a massless packet of energy.

(iii) The energy E of a photon is related to the frequency of radiation, ν , the two being related as $E = h\nu$, where h is Planck's constant.

(iv) Whenever a body emits or absorbs energy it does so in whole number multiples by photons, i.e. $n h\nu$, where $n = 1, 2, 3, 4\dots$

Planck's radiation formula-

Let dn be the number of photons distributed in frequency interval ν to $(\nu + d\nu)$.

$$dn = n(\nu)d\nu = g(\nu)f(\nu)d\nu \quad \dots(1)$$

$$g(\nu)d\nu = \frac{8\pi h\nu^2}{c^3}d\nu \quad \dots(2)$$

and

$$f(\nu) = \frac{1}{(e^{h\nu/k_B T} - 1)} \quad [\text{as } \alpha = 0 \text{ and } E = h\nu] \quad \dots(3)$$

Putting these values in Eq.(1), we get

$$dn = \frac{8\pi h\nu^2}{c^3} \cdot \frac{1}{(e^{h\nu/k_B T} - 1)} d\nu \quad \dots(4)$$

Let dE be the energy distributed in frequency interval ν and $(\nu + d\nu)$

$$dE = Edn = h\nu dn \quad \dots(5)$$

From Eqs. (4) and (5), we obtain

$$dE = E(v)dv = \frac{8\pi h v^3}{c^3} \left(e^{hv/k_B T} - 1 \right)^{-1} dv \quad (6)$$

Equation (6) is known as Planck's radiation formula for the spectral energy density of blackbody radiation, which agrees with experimental curve (Fig. 4) for low frequency or high wavelength.

$$(e^{hv/k_B T} - 1) = 1 + \frac{hv}{k_B T} - 1 = \frac{hv}{k_B T}$$

Equation (6) becomes as

$$dE = E(v)dv = \frac{8\pi v^2 k_B T}{c^3} dv \quad (7)$$

Equation (7) is termed as Rayleigh-Jeans law. This law holds good for low frequencies and high wavelengths. Experimental agreement is shown in Fig. 5 for high frequency low frequency:

Equation (6) can be expressed in terms of wavelength of radiation as follows:

$$E_\lambda d\lambda = \frac{8\pi c h}{\lambda^5} \left(\frac{1}{c[(hc/\pi k_B T) - 1]} \right) d\lambda$$

$$E_\lambda = 8\pi h c (\lambda^{-5}) \left[\exp \left(\frac{hc}{\lambda k_B T} - 1 \right) \right]^{-1}$$

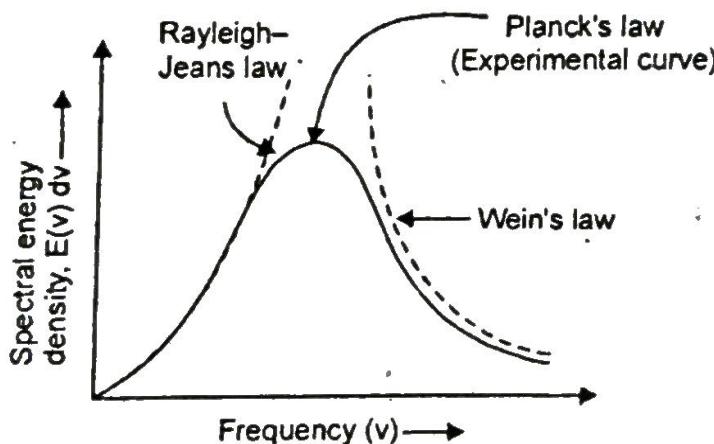


Fig. 5. v vs $E(v) dv$ -curve

Let λ_{\max} be the wavelength whose energy density is the greatest.

Then, $\frac{dE_\lambda}{d\lambda} = 0$

Hence, $\lambda_{\max} T = \frac{hc}{4.965 K} = 2.898 \times 10^{-3} mK$

This is Wein's displacement law.

The peak in the black body spectrum shifts to progressively shorter wavelength (higher frequencies) as temperature is increased.

Q.5. (a) Derive Schrödinger time dependent wave equation for a relativistic particle. Give the physical interpretation of Hamilton operator.

Ans. Let us assume that Ψ for a particle moving freely in positive x -direction

$$\Psi = Ae^{-\frac{im}{\hbar} \left(t - \frac{x}{v} \right)}$$

as

$$m = 2\pi v \quad v = \lambda$$

$$\Psi = Ae^{-\frac{2\pi i}{\hbar} \left(vt - \frac{x}{\lambda} \right)}$$

$$\text{As } E = hv = 2\pi\hbar v \text{ and } \lambda = \frac{\hbar}{p} = \frac{2\pi\hbar}{p}$$

∴ For a free particle wave equation becomes

$$\Psi = Ae^{-\frac{i}{\hbar} (Et - px)}$$

But

$$E\Psi = i\hbar \frac{\partial \Psi}{\partial t} \text{ and } p\Psi = \frac{\hbar}{i} \frac{\partial \Psi}{\partial x} \dots (a)$$

As total energy, $E = \text{Kinetic energy (K)} + \text{Potential energy (V)}$

Now,

$$\text{K.E.} = \frac{p^2}{2m}$$

∴ Equation (4) in term of wave function Ψ can be written as

$$E\Psi = \left(\frac{p^2}{2m} \right) \Psi + V\Psi$$

Putting the values of $E\Psi$ and $p\Psi$ from Eq. (a) in Eq. (5) we have

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 \frac{1}{2m} \Psi + V\Psi$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Equation (6) is Schrödinger's time dependent wave equation in one-dimensional form.
The time dependent Schrödinger's equation in three-dimensional form,

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \left[\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right] + V\Psi$$

or

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi$$

$$\left[\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right]$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{d\psi}{dt}$$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V \right] \psi = i\hbar \frac{d\psi}{dt}$$

Equation (10) contains time and hence is called time dependent Schrodinger equation.

The operator $\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right)$ is called Hamiltonian and is represented by H , while operator $i\hbar \frac{d}{dt}$, operated on ψ , given E which may be seen by Eq. (7) Eq. (10) may be written as

$$H\psi = E\psi$$

The above forms of the Schrodinger's equation describe the motion of a non-relativistic material particle.

Q.5. (b) State the properties of wave function in a quantum mechanical system. Give one example each for an acceptable and non-acceptable wave function. (4)

Ans. Properties of Wave Function are—

- $\psi(x)$ must be single valued, finite and continuous for all values of x .

- $\frac{d\psi(x)}{dt}$ must be finite and continuous for all values of x , except at those point

where $V \rightarrow \infty$. At these points $\frac{d\psi(x)}{dt}$ has a finite discontinuity but ψ remains continuous.

- For bound state and probability of finding the particle between x and $(x + dx)$, i.e., $|\psi|^2 dx$ must vanish as $|x| \rightarrow \infty$. Hence $|\psi(x)| \rightarrow 0$ as $|x| \rightarrow \infty$ i.e., $\psi(x)$ is a square integrable wave function.

Examples of not acceptable wave functions:

(i) $\psi = \frac{1}{x}$ as $x \rightarrow 0$, $\psi \rightarrow \infty$ (ii) $\psi = x^n$ as $a \rightarrow \pm \infty$, $\psi \rightarrow \pm \infty$

Examples of acceptable wave functions:

(i) $\psi = \frac{1}{(1+x^2)}$ as $x \rightarrow \infty$, $\psi \rightarrow 0$ (ii) $\psi = \sin x$ as ψ oscillates between +1 and -1.

Q.5. (c) An electron has a speed of $2 \times 10^4 \frac{m}{s}$ within the accuracy of 0.01%. (2.5)

Calculate the uncertainty in the position of the electron.

Ans. Given-

$$\Delta V = \frac{0.01}{100} \times 2 \times 10^4 = \frac{2m}{s}$$

Second Semester, Applied Physics - II

28-2017

From uncertainty principle

$$\Delta x \Delta p_x \geq \frac{\hbar}{2\pi}$$

Uncertainty Δx position-

$$\Delta x = \frac{\hbar}{2\pi p_x} = \frac{\hbar}{2\pi m \Delta V}$$

$$\begin{aligned}\Delta x &= \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 2} \\ &= \frac{6.63 \times 10^{-34}}{114.296 \times 10^{-31}} \\ &= 0.058 \times 10^{-3} \text{ m}\end{aligned}$$

$$\boxed{\Delta x = 58 \times 10^{-6} \text{ m}}$$

UNIT - III

Q.6. (a) What do you understand by packing factor in crystals? Calculate the packing factor in the case of bcc and fcc lattices. (4.5)

Ans. Atomic packing factor (APF) or density of packing: It is the ratio of volume occupied by the atoms, molecules or ions in a unit cell (v) to the total value of the unit cell (V).

$$APF = \frac{\text{Volume occupied by the atoms in an unit cell}}{\text{Volume of the unit cell}}$$

$$\Rightarrow APF = \frac{\text{Number of atoms present in an unit cell} \times \text{Volume of an atom}}{\text{Volume of the unit cell}}$$

i.e.,

$$\boxed{APF = \frac{v}{V}}$$

BCC-

$$\boxed{APF = \frac{v}{V}}$$

The number of atoms present in an unit cell = 2

$$v = 2 \times \frac{4}{3} \times \pi r^3; \quad V = a^3$$

$$APF = \frac{2 \times \frac{4}{3} \times \pi r^3}{a^3}$$

Substituting $r = \frac{\sqrt{3}a}{4}$, we get .

$$APF = \frac{2 \times \frac{4}{3} \pi r^3}{a^3}$$

$$\boxed{APF = \frac{\frac{32}{3} \pi r^3}{a^3} = 0.68}$$

Thus, 68% of the volume of the bcc unit cell is occupied by atoms and remaining 32% volume of the unit cell is vacant or void space

FCC-

Atomic Packing Factor (APF)/ Packing Density

$$APF = \frac{v}{V}$$

$$v = 4 \times \frac{4}{3} \pi r^3; \quad V = a^3$$

$$APF = \frac{4 \times 4 \pi r^3}{3a^3}$$

Substituting $r = \frac{\sqrt{2}a}{4}$, we get

$$\boxed{APF = \frac{\pi}{3\sqrt{2}} = 0.74}$$

Thus 74% of the volume of the fcc unit cell is occupied by atoms and the remaining 26% volume of the unit cell is vacant or void space.

Q.6. (b) What do you mean by lattice constant? Obtain a relation between the lattice constant of a cubic crystal and the density of the crystal material.

(4)

Ans. In a lattice a unit cell has the length of the edges and the angles between the edges are of definite values. Each edge of the unit cell is a distance between the atoms of the same kind and is known as the lattice constant.

Let us consider a cubic lattice constant a . If ρ is the density of the crystal under consideration, then

$$\text{Volume of unit cell} = a^3$$

$$\text{Mass in each unit cell} = a^3 \rho \quad \dots(1)$$

If M is the molecular weight and N the Avogadro's number (i.e., the number of molecules per kg mole of the substance), then

$$\text{Mass of each molecule} = \frac{M}{N}$$

Further if n is the number of molecules (lattice points) per unit cell, then

$$\text{the mass in each unit cell} = n \frac{M}{N} \quad \dots(2)$$

From Eqs. (1) and (2)

$$a^3 V = \frac{nM}{N}$$

$$a = \left(\frac{nM}{Np} \right)^{1/3}$$

From the relation the values of the lattice constant 'a' can be calculated.

Q.6. (c) Define Miller indices. If a, b and c are principle vectors of the unit cell a plane of miller indices (310) cut the crystal lattice, find the intercepts, the plane along the three axes.

Ans. Miller indices is defined as the reciprocals of the intercepts made by the plane on the three axes.

Given- Miller indices - (3 1 0)

The formula is-

$$a:b:c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

$$a, b, c \Rightarrow (3 1 0)$$

3a, 1b, 0c are the intercepts

\therefore 3a and 1b are the intercepts

Q.7. (a) What are Schottky and Frenkel defects? Obtain an expression of concentration of schottky defects in crystal.

Ans. Refer End Term Examination Q.7. (c) May 2016.

Q.7. (b) Why do we use X-rays to study crystal diffraction? Is it possible observe diffraction pattern if radiation of wavelength 2×10^{-6} m is incident on crystal with interplanas separation of 10^{-8} cm. Justify.

Ans. A crystal is nothing but a three dimensional grating of periodic array of atoms. The typical interatomic spacing in crystal is of the order of 1 \AA . Therefore to observe diffraction in crystal, wavelength of electromagnetic radiation used should be of the order of interatomic spacing. Since this wavelength range corresponds to X-rays, so rays are used for diffraction studied in crystals.

Given

$$\lambda = 2 \times 10^{-6} \text{ m}$$

$$d = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

For $n = 1 \Rightarrow$ from Bragg's law-

$$2d \sin \theta = n\lambda$$

$$\sin \theta = \frac{1 \times 2 \times 10^{-6}}{2 \times 10^{-10}}$$

$$\sin \theta = 10^4$$

But $\sin \theta > 1$ which is not possible.

So, it is not possible to observe diffraction pattern with $\lambda = 2 \times 10^{-6}$ m.

Q.7. (c) Lead has fcc structure and its body diagonal is 0.86 nm. When X-ray of wavelength 0.7 Å undergoes diffraction from (110) plane to produce second order maxima, calculate the glancing angle.

Ans. We know that-

$$2d \sin \theta = n\lambda$$

and

Given

$$d_{\text{AB}} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$h = 1, k = 1, l = 0$$

$$n = 2, \lambda = 0.71 \times 10^{-10} \text{ m}$$

$$d_{110} = \frac{a}{\sqrt{1+1+0}} = \frac{a}{\sqrt{2}}$$

$$\frac{2a}{\sqrt{2}} \sin \theta = 2 \times 0.71 \times 10^{-10}$$

For FCC-

Also,

$$a = 2 \times \sqrt{2} \times 0.86 \times 10^{-9} \text{ m}$$

$$a = 2.98 \times 10^{-9} \text{ m}$$

$$\frac{2 \times 2.98 \times 10^{-9}}{\sqrt{2}} \sin \theta = 2 \times 0.71 \times 10^{-10}$$

$$3.44 \times 10^{-9} \sin \theta = 1.42 \times 10^{-10}$$

$$\sin \theta = \frac{1.42 \times 10^{-10}}{3.44 \times 10^{-9}}$$

$$= 0.413 \times 10^{-1}$$

$$\sin \theta = 0.0413$$

$$\theta = \sin^{-1}(0.0413)$$

$$\boxed{\theta = 2.37^\circ}$$

UNIT-IV

Q.8. (a) What are Brillouin zones? How are they related to the energy levels of an electron in a metal?

(4)

Ans. If Schrodinger wave equation for electron energies is solved with a periodic function $u(k)$ to give the energies of electrons in a solid, the solutions fall into permitted energy bands. If the solutions are plotted in the reciprocal lattice of the crystal being considered, the zones enclosing the solutions for $k = 1, 2, 3, \dots, n$ are called Brillouin zones.

BRILLOUIN ZONES

Linear Lattice-

According to Kronig and Penney, the discontinuities in a one dimensional lattice

occur when $k = \frac{n\pi}{a}$, where n is a positive or negative integer. In a one dimensional monoatomic lattice a line representing the value of k is divided by energy discontinuities into segments of length as shown in Fig. 6. These line segments are known as Brillouin zones.

The first reflection and the first energy gap occurs at $k = \pm \frac{\pi}{a}$. The reflection at $k = \pm \frac{\pi}{a}$ arises because the wave reflected from one atom in a linear lattice interacts constructively with the wave from a nearest neighbour atom, the phase difference between the two reflected waves for the values of $k = \pm \frac{\pi}{a}$ and $k = -\frac{\pi}{a}$ being $\frac{2\pi}{a}$. The region in k-space between $-\frac{\pi}{a}$ and $+\frac{\pi}{a}$, i.e., the segment $-\frac{\pi}{a} \leq k \leq +\frac{\pi}{a}$ is called the Brillouin Zone.

The second Brillouin zone contains electrons with $k > \frac{\pi}{a}$, i.e., electrons having k-values between $\frac{\pi}{a}$ and $\frac{2\pi}{a}$ for electrons moving in the $\pm x$ direction. The second reflection and second energy gap occurs at $k = \pm \frac{2\pi}{a}$ and therefore, the region in the k-space between $-\frac{2\pi}{a}$ to $-\frac{\pi}{a}$ and $+\frac{\pi}{a}$ to $+\frac{2\pi}{a}$, i.e., the segment $-\frac{2\pi}{a} \leq k \leq -\frac{\pi}{a}$ and $+\frac{\pi}{a} \leq k \leq +\frac{2\pi}{a}$ forms the second Brillouin zone of the lattice.

Similarly, we can discuss the formation of third Brillouin zone; ...etc.

The first and second Brillouin zones and the energy gaps are shown in Fig. 6 plotting value of E in eV corresponding to various values to k.

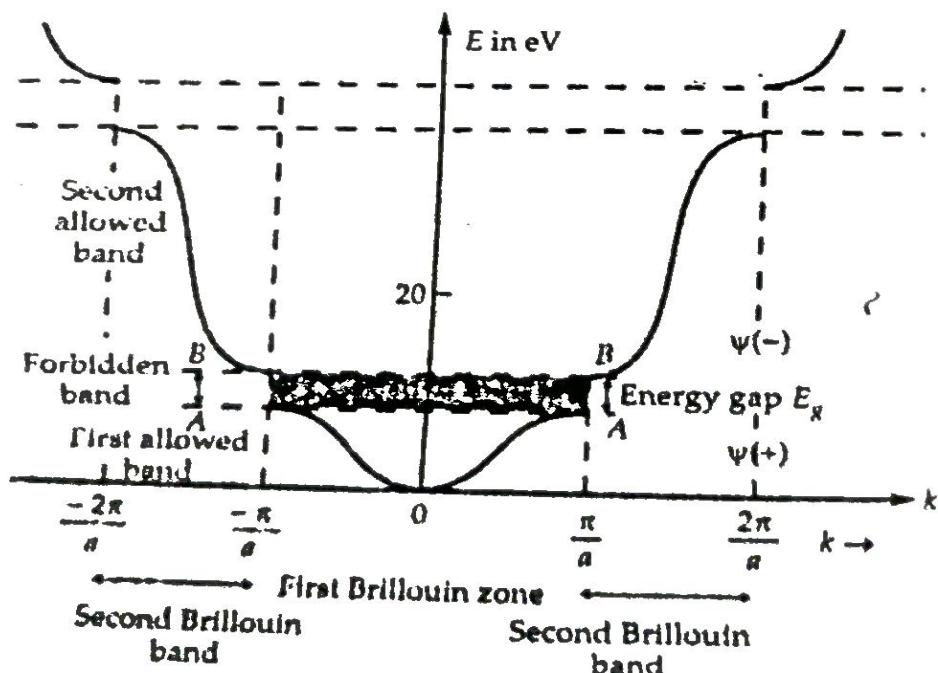


Fig. 6. The first and second Brillouin Zones and energy gap

Two Dimensional Lattice: Proceeding as in the case of a linear lattice, the first zone for a two dimensional lattice in the X-Y plane is the square ABCD the boundary, of which are given by the relation

$$k_x = \pm \frac{\pi}{a} \text{ and } k_y = \frac{\pi}{a} \text{ and } k_y = \pm \frac{\pi}{a} \text{ and}$$

$$k_y = -\frac{\pi}{a} \text{ as shown in Fig.}$$

The second Brillouin zone for a two dimensional lattice in the X-Y plane is the square EFGH the boundary of which are given by the relation

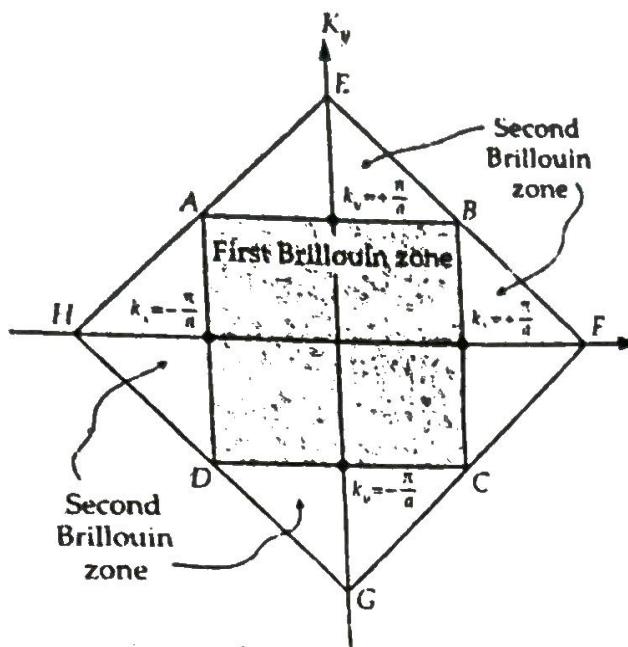


Fig. 7. First and Second Brillouin Zones in 2D lattice

$$k_x = \pm \frac{2\pi}{a}; \quad k_y = \pm \frac{2\pi}{a}$$

Q.8. (b) Define effective mass of electron. Obtain an expression for effective mass of an electron moving in a periodic potential (5)

Ans. The electrons in a crystal are not completely free but interact with the periodic potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alternate value of particle mass or effective mass after taking into account the effect of the crystal lattice so that the electron (and the hole) can be treated as almost free carriers of charge for the calculations.

The motion of an electron in a crystal in the presence of an external applied electric field ξ . Suppose the electron is initially in a state k , when the external electric field is applied the magnitude of the force acting on the electron is $e\xi$.

If the electron moves a distance dx in a small time dt , under the action of this force, Then

$$\text{Work done} = \text{Energy gained } dE = e\xi dx \quad \dots(1)$$

If v is the velocity gained by the electron, then

$$v = \frac{dx}{dt} \text{ or } dx = v dt$$

$$dE = e\xi v dt$$

$$v = \frac{1}{\hbar} \frac{dE}{dk}$$

Now

$$dE = \frac{e\xi}{\hbar} \frac{dE}{dk} dt$$

or

Now $\hbar k = p$ and

$$\hbar \frac{dk}{dt} = \frac{dp}{dt} = F$$

$$\hbar \frac{dk}{dt} = e\xi = F$$

Differentiating Eq. (3) with respect to t , we get

$$\frac{dv}{dt} = \frac{1}{\hbar} \frac{d^2E}{dk dt} = \frac{1}{\hbar} \frac{d^2E}{dk^2} \frac{dk}{dt}$$

From Eq. (6)

$$\frac{dk}{dt} = \frac{F}{\hbar}$$

Also

$$\frac{dv}{dt} = a \text{ (the acceleration)}$$

From Eq. (7)

$$a = \frac{1}{\hbar} \frac{d^2E}{dk^2} \cdot \frac{F}{\hbar}$$

or

$$\frac{a}{F} = \frac{1}{\hbar^2} \frac{d^2E}{dk^2}$$

Taking $m^* = \frac{F}{a}$ as the effective mass of electron in the crystal, we have

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

In some cases, a factor f_k is also used. It gives the ratio of the rest mass of an electron m to its effective mass in the crystal in the k -state

$$f_k = \frac{m}{m^*} = \frac{m}{\hbar^2} \left(\frac{d^2E}{dk^2} \right) \quad \dots$$

This expression determines the extent to which the electron in the k -state behaves as a free electron.

If $f_k = 1$, $m^* = m$, the electron in the crystal in the k -state behaves as a free electron.

Q.8. (c) Where does Fermi level lie in an n-type semiconductor? Discuss the effect of temperature and doping concentration on the position of the Fermi level in an n-type semiconductor.

Ans. For N-type semiconductor, since $n_i > n_s$, E_F must move closer to conduction band to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band.

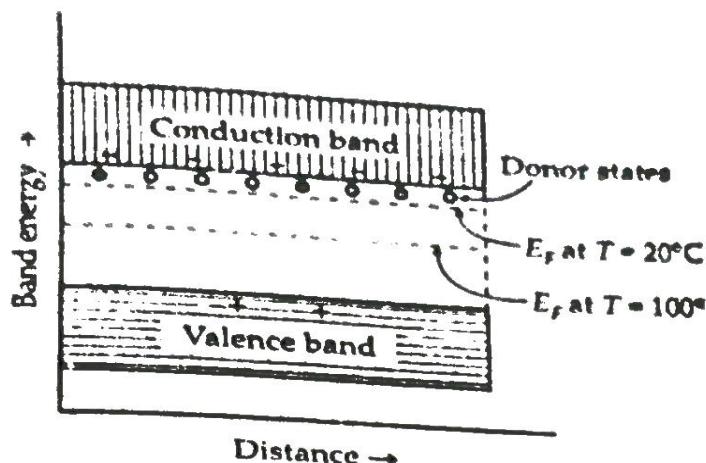


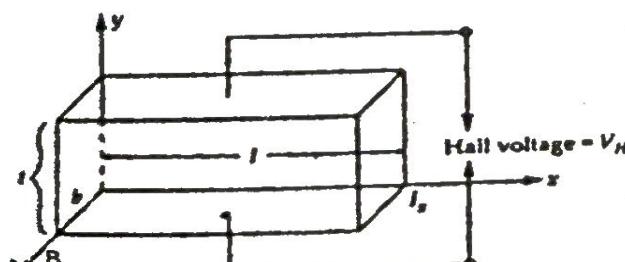
Fig. 8.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor fermi level is independent of temperature.

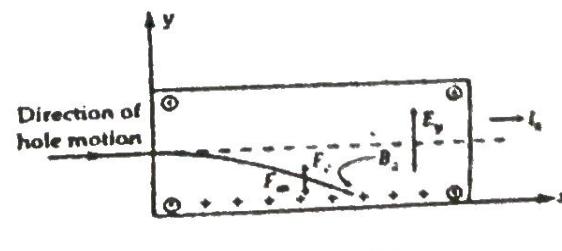
But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap.

Q.9. (a) What is Hall effect? Derive an expression for Hall coefficient. Explain how the measurement of hall coefficient helps to determine the mobility of electrons in metals.

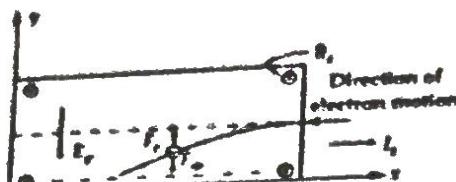
Ans. The phenomenon of Hall Effect is a consequence of motion of charge in both magnetic and electric fields. If a piece of conductor (metal or semiconductor) carrying current is placed in a transverse magnetic field, then an electric field or a potential difference is produced inside the conductor in a direction normal to both current and magnetic field. This phenomenon is known as *Hall effect*. The developed electric field and potential are called *Hall field* and *Hall voltage*.



9.(a)



9.(b)



9.(c)

Fig. 9. (a) Illustration of Hall effect, 9.(b) Motion of hole in P-type semiconductor specimen 9.(c) Motion of electron of N-type semiconductor

Consider a specimen in the form of a rectangular cross-section carrying current J_x in the x -direction. A uniform magnetic field B_z is applied along z -axis as shown in Fig. 9. If specimen is of conducting material or N-type semiconductor, then current flows due to motion of electron, but in P-type semiconductor and intrinsic semiconductor, current carried by motion of holes and electrons-holes respectively.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction due to magnetic force $e v_x B_z$, Fig. (a) & (c). This shift of charges produces a potential difference or an electric field E_y along y -direction. The produced electric field also applies a force E_y on charge. When both the forces become equal, the equilibrium occurs, i.e., upto that condition charges shift along y direction.

In equilibrium

$$\begin{aligned} F_e &= F_m \\ eE_y &= ev_x B_z \\ E_y &= v_x B_z \end{aligned}$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \Rightarrow V_x = \frac{J_x}{ne}$$

Here n is density of charge carrier (electron or hole or both) and A (breadth \times thickness) is the cross-sectional area of specimen perpendicular to direction of J_x . From Eqs. (1) and (2) we can write.

$$E_y = \frac{B_z J_x}{ne}$$

This is the expression for Hall field. The Hall effect is described by means of Hall coefficient R_H , defined in terms of J_x by the following relation:

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(3)$$

From Eqs. (3) and (4), we have-

$$R_H = \frac{1}{ne} \quad \dots(4)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write.

$$R_H = -\frac{E_y}{J_x B_z} = \frac{1}{ne} \quad (6)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e} \quad (7)$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Application of Hall Effect

1. The sign of Hall coefficient can be used to determine whether a given semiconductor is N-type or P-type.
2. The carrier concentration can be evaluated when R_H is known (as $n = 1/R_H e$)
3. Since $u = R_H \sigma$, thus mobility of charge carrier can also be well determined with the help of R_H .
4. Since Hall voltage is proportional to magnetic flux density B_z and current I_x through a sample or specimen. The Hall effect can be used as the basis for design of a magnetic flux density meter.
5. Hall effect can be used to determine the power flow in an electromagnetic wave because V_H is proportional to product of E_y and B_z , i.e., magnitude of Poynting vector in wave.

Q.9. (b) The Hall voltage for the metal sodium is 0.001 mV, measured at

100 mA, $B = \frac{2\omega b}{m^2}$ and width of specimen is 0.05 mm. Calculate the number of carriers per cubic meter in sodium. (5)

Ans. Given

$$V_H = 0.001 \text{ mV} = 0.001 \times 10^{-3} \text{ V}$$

$$I = 100 \text{ mA} = 100 \times 10^{-3} \text{ A}$$

$$B = 2\omega b/m^2$$

$$d = 0.05 \text{ mm} = 0.05 \times 10^{-3} \text{ m}$$

The number of carriers per cubic meter in sodium is as following-

$$n = \frac{IB}{V_H d}$$

$$n = \frac{100 \times 10^{-3} \times 2}{0.001 \times 10^{-3} \times 0.05 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

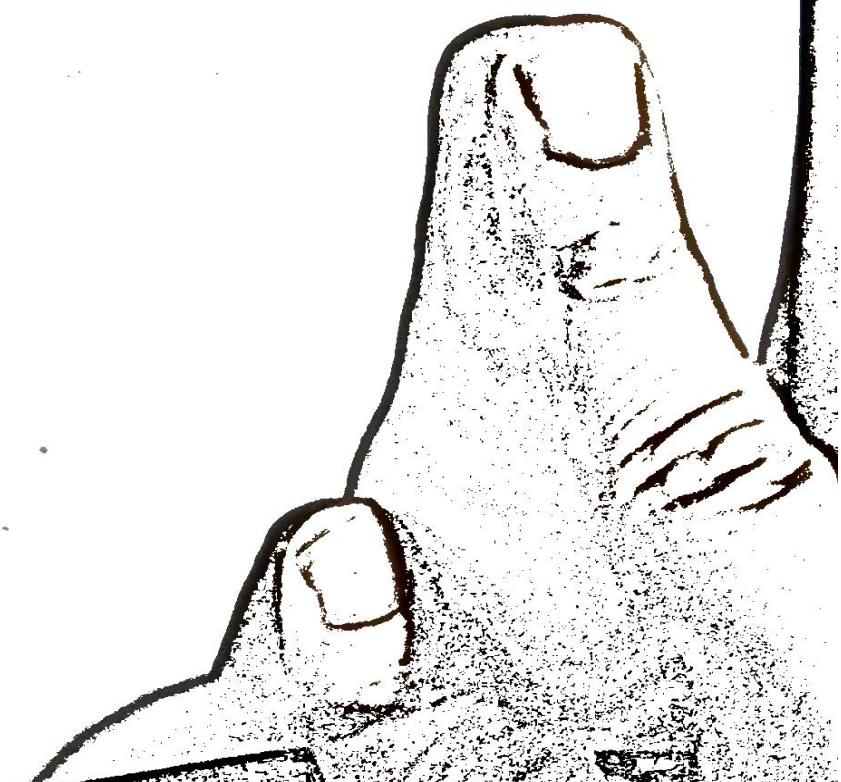
$$= \frac{0.2}{0.8 \times 10^{-29}} = 0.25 \times 10^{29}$$

$n = 2.5 \times 10^{28} / \text{m}^3$

Q.9. (c) Distinguish between intrinsic and extrinsic semiconductors. (2)

Ans. The difference between intrinsic and extrinsic semiconductors are as following:

Intrinsic Semiconductor	Extrinsic Semiconductor
<ol style="list-style-type: none"> 1. It is a semiconductor in its pure form. 2. These semiconductors have low electrical conductivity. 3. The operating temperature is low for this type of semiconductors. 4. Charge carriers are produced due to thermal excitation. Examples : Ge, Si 	<p>It is semiconductor doped with impurities.</p> <p>These semiconductors have high electrical conductivity.</p> <p>The operating temperature is high for this type of semiconductors.</p> <p>Additional charge carriers are also produced due to added impurities. Examples: Ge and Si doped with P, As, Bi, Sb etc.</p>



FIRST TERM EXAMINATION [FEB. 2018]
SECOND SEMESTER (B.TECH)
APPLIED PHYSICS-II [ETPH-104]

Time : 1.5 hrs.
 Note: Question No. 1 is compulsory and attempt any two from the rest

M.M. : 30

Q. 1. Attempt any five.

(a) Consider a vector field $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Is this field solenoidal? (2)

Ans. Solving for $\nabla \cdot \vec{A}$

$$\begin{aligned}\nabla \cdot \vec{A} &= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) (x^2\hat{i} + y^2\hat{j} + z^2\hat{k}) \\ &= 2x + 2y + 2z \neq 0\end{aligned}$$

As, $\nabla \cdot \vec{A} \neq 0$, this vector is not solenoidal.

Q. 1. (b) Distinguish between displacement and conventional current. (2)

Ans. Difference between displacement and conventional current is as following.

Conventional Current	Displacement Current
(i) Flows in conductive medium and obeys Ohm's Law	(i) Set up in dielectric medium due to changing field across the dielectric which leads to variation of induced displacement current.
(ii) $J_c = \sigma E, I = \frac{V}{R}$	(ii) $J_d = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$
(iii) For constant $E, J_c \neq 0$	(iii) For constant $E, J_d = 0$

Q. 1. (c) State Ampere's Law. Discuss the inconsistency in Ampere's Law. (2)

Ans. According to Ampere's circuital law, the line integral of the magnetic field B along any closed loop C is proportional to the current I passing through the closed-loop, i.e.,

$$\oint B \cdot dl = \mu_0 I \quad \dots(1)$$

In 1864, Maxwell showed that Eq. (1) is logically inconsistent. To prove this inconsistency, we consider a parallel plates capacitor being charged by a battery as shown in Fig 1. (a). As the charging continues, a current I flows through the connecting wires, which of course changes with time. This current produces a magnetic field around the capacitor. Consider two planar loops C_1 and C_2 , C_1 just left of the capacitor and C_2 in between the capacitor plates, with their planes parallel to these plates.

Now the current I flows across the area bounded by loop C_1 because connecting wires pass through it. Hence from Ampere's law, we have

$$\oint B \cdot dl = \mu_0 I \quad \dots(2)$$

But the area bounded by C_2 lies in the region between the capacitor plates, so no current flows across it.

$$\oint B \cdot dI = 0$$

Imagine the loops C_1 and C_2 to be infinitesimally close to each other, as shown.

Fig. 1

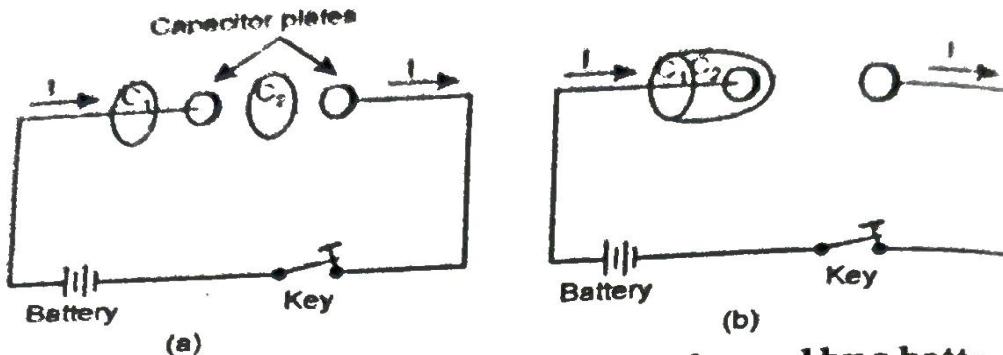


Fig. 1. A parallel plate capacitor being charged by a battery.

Then we must have

$$\oint B \cdot dI = \oint B \cdot dI$$

This result is inconsistent with Eqs. (3) and (4) so a need for modifying Ampere's law was felt by Maxwell.

Maxwell's Modification of Ampere's Law: Displacement Current: To modify Ampere's law, Maxwell followed a symmetry consideration. By Faraday's law, a changing magnetic field induces an electric field, hence a changing electric field must induce a magnetic field. As currents are the usual sources of magnetic fields, a changing electric field must be associated with a current. Maxwell called this current as the 'displacement current' to distinguish it from the usual current caused by the drift of electrons.

Displacement current is that current, which comes into existence, in addition to the conduction current, whenever the electric field and hence the electric flux changes with time.

To maintain the dimensional consistency, the displacement current is given by the form.

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

where ϕ_E = electric field \times area = ES , is the electric flux across the loop.

\therefore Total current across the closed loop.

$$= I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}$$

Hence the modified form of Ampere's law is

$$\oint B \cdot dI = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right]$$

Unlike the conduction current, the displacement current exists whenever the electric field and hence the electric flux is changing with time. Thus according to Maxwell, source of magnetic field is not just conduction electric current due to flowing charges but also the time varying electric field. Hence the total current I is the sum of conduction current I_c and displacement current I_d i.e.,

$$I = I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}$$

Q. 1. (d) Find the phase velocity and group velocity of the de-Broglie wave of an electron whose speed is 0.9C.

(2)

Ans. Group velocity is-

$$V_g = 0.9C$$

Phase velocity is given by

$$V_p V_g = C^2$$

$$V_p = \frac{C^2}{V_g} = \frac{C^2}{0.9C} = 1.1C$$

$$V_p = 3.33 \times 10^8 \frac{m}{s}$$

Q. 1. (e) Write Maxwell's equations in integral form.

Ans. Refer Q. 2. (a) End Term Exam. May-June 2017.

Q. 1. (f) The de-Broglie wavelength associated with an electron is 0.1 Å. Find the potential difference by which the electron is accelerated.

(2)

Ans. de-Broglie wave length is given by

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$V = \frac{h^2}{\lambda^2 2me}$$

Given $\lambda = 0.1 \text{ Å} = 10^{-11}$; $h = 6.63 \times 10^{-34} \text{ J. sec}$; $m = 9.1 \times 10^{-31} \text{ kg}$; $e = 1.6 \times 10^{-19} \text{ J}$

$$V = \frac{6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{10^{-11} \times 10^{-11} \times 2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} = 1.51 \times 10^4 \text{ V}$$

Q. 2. (a) Define poynting vector. State and prove poynting theorem. (7)

Ans. The electromagnetic waves carry energy when they propagate and there is an energy density associated with both electric and magnetic field. The amount of energy flowing through unit area, perpendicular to the direction of energy propagation per unit time, i.e. the rate of energy transport per unit area is called poynting vector. It is also called instantaneous energy flux density and is represented by. $\vec{S} = \vec{E} \times \vec{H}$

Poynting theorem:

When em wave propagates through space from source to receiver there exist a simple and direct relationship between power transferred and amplitude of electric and magnetic field strength. The relationship may be obtained through maxwell equation.

from Maxwell IVth eqn.

$$\vec{\nabla} \times \vec{B} = \mu \left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\vec{B} = \mu \vec{H} \Rightarrow \frac{\vec{B}}{\mu} = \vec{H}$$

But, as

$$\epsilon \vec{E} = \vec{D}$$

and

$$\vec{v} \cdot \vec{H} = J + \epsilon \frac{\partial \vec{B}}{\partial t}$$

$$\vec{v} \times \vec{H} = J + \epsilon \frac{d\vec{E}}{dt}$$

$$\vec{J} = \vec{v} \times \vec{H} - \epsilon \frac{\partial \vec{E}}{\partial t}$$

This is equation of current density, when multiplied by \vec{E} , this will result relation between the quantities which have the dimension of power.

$$\vec{E} \cdot \vec{J} = \vec{E} \cdot (\vec{v} \times \vec{H}) - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

from vector identity-

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H})$$

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\vec{E} \cdot \vec{J} = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) - \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

From Maxwell's IIIrd equation

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

we have,

$$= \vec{E} \cdot \vec{J} = -\mu \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) - \epsilon \left(\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right) - \vec{\nabla} \cdot (\vec{E}$$

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} H^2$$

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} E^2$$

$$\vec{E} \cdot \vec{J} = -\mu \frac{\partial}{\partial t} H^2 - \frac{\epsilon}{2} \frac{\partial}{\partial t} E^2 - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

Integrating over volume, v -

$$\int_v \vec{E} \cdot \vec{J} dv = -\frac{\partial}{\partial t} \int_v \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv - \int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv$$

Using divergence theorem-

$$\int_v \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dv = \oint_s (\vec{E} \times \vec{H}) \cdot ds$$

$$\left[\oint (\vec{E} \times \vec{H}) \cdot ds = - \frac{\partial}{\partial t} \left\{ \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv - \int \vec{E} \cdot \vec{J} dv \right\} \right]$$

$$\frac{\partial}{\partial t} \left\{ \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) \right\} dv = \oint (\vec{E} \times \vec{H}) \cdot ds + \int \vec{E} \cdot \vec{J} dv$$

Rate of energy flow = rate of decrease of stored energy + total instantaneous ohmic power dissipated within the volume.
This is called the "Poynting theorem" and
 $\vec{S} = \vec{E} \times \vec{H}$ is called "poynting vector".

$\int \vec{E} \cdot \vec{J} dv$ - Rate of energy transferred into em field through the motion of free charge in Volume V.

$\frac{\partial}{\partial t} \int \left(\frac{\mu}{2} H^2 + \frac{\epsilon}{2} E^2 \right) dv$ - Rate of decrease of em energy stored in volume V.

$\oint (\vec{E} \times \vec{H}) \cdot ds$ - Amount of em energy crossing the closed surface per second.

Physical meaning of equation- The time rate of change of em energy with certain time plus time rate of energy flowing out through the boundary surface is equal to the energy transferred into em field.

Q. 2. (b) The constitution parameter of Aluminium are given by $\mu_r = 1$, $\epsilon_r = 1$ and $\sigma = 3.54 \times 10^7$ mho/m. Find the frequency for which the skin depth of aluminium is 0.01 mm. (3)

Ans. Skin depth is given by

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} = \sqrt{\frac{2}{\mu \sigma 2\pi f}}$$

$$f = \frac{1}{\delta^2 \mu \sigma \pi}$$

$$\delta = 0.01 \text{ mm} = 0.01 \times 10^{-3} \text{ m}$$

$$\mu = \mu_r = 1$$

$$\sigma = 3.54 \times 10^7 \frac{\text{mho}}{\text{m}}$$

$$f = \frac{1}{(0.01 \times 10^{-3})^2 \times 1 \times 3.54 \times 10^7 \times 3.14} \quad f = 7.16 \times 10^7 \text{ Hz}$$

Q. 3. (a) Derive electromagnetic wave equation in dielectric medium and show that the velocity of the electromagnetic wave in dielectric medium is always less than the velocity in free space. (5)

Ans. Maxwell's equations are:

$$\operatorname{div} \vec{D} = \vec{\nabla} \cdot \vec{D} = \rho \quad \dots(a)$$

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$$\operatorname{div} \mathbf{B} = \nabla \cdot \mathbf{B} = 0 \quad \dots (b)$$

$$\operatorname{curl} \mathbf{E} = \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \dots (c)$$

$$\operatorname{curl} \mathbf{H} = \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \dots (d)$$

and

In an isotropic dielectric (or non-conducting isotropic medium)

$$\mathbf{D} = \epsilon \mathbf{E}, \mathbf{B} = \mu \mathbf{H}, \mathbf{J} = \sigma \mathbf{E} = 0 \text{ and } \rho = 0$$

Therefore, Maxwell's equations in this case take the form

$$\operatorname{div} \mathbf{E} = \nabla \cdot \mathbf{E} = 0 \quad \dots (a)$$

$$\operatorname{div} \mathbf{B} = \nabla \cdot \mathbf{B} = 0 \quad \dots (b)$$

$$\operatorname{curl} \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad \dots (c)$$

and

$$\operatorname{curl} \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad \dots (d)$$

Taking curl of Eq. 3 (c), we get

$$\operatorname{curl} \operatorname{curl} \mathbf{E} = -\mu \frac{\partial}{\partial t} (\operatorname{curl} \mathbf{H})$$

Substituting curl H from Eq. 3. (d) in above equation.

$$\operatorname{curl} \operatorname{curl} \mathbf{E} = -\mu \frac{\partial}{\partial t} \left(\epsilon \frac{\partial \mathbf{E}}{\partial t} \right)$$

i.e.,

$$\operatorname{curl} \operatorname{curl} \mathbf{E} = -\mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Similarly, if we take curl of Eq. 3 (d) and substitute curl \mathbf{E} from Eq. 3. (c) we

$$\operatorname{curl} \operatorname{curl} \mathbf{H} = -\mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

and keeping in mind Eqs. 3. (a) and 3. (b) i.e., $\operatorname{div} \mathbf{E} = 0$ and $\operatorname{div} \mathbf{H} = 0$ Eqs. (4.5) give

$$\nabla^2 \mathbf{E} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

and

$$\nabla^2 \mathbf{H} - \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

These equations are vector equations of identical form which means that each of the components of \vec{E} and \vec{H} separately satisfies the same scalar wave equation of

$$\nabla^2 \phi - \mu \epsilon \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(8)$$

where ϕ is a scalar and stand for any one of the components of \vec{E} and \vec{H} . It is obvious that Eqs. (6) and (7) resemble with general wave equation

$$\nabla^2 \phi - \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(9)$$

where v = speed of wave.

This means that the field vectors E and H are propagated in isotropic dielectric as waves with speed v given by

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} \quad \dots(10)$$

where μ_r is relative permeability of medium and ϵ_r is relative permittivity (or electric constant) of the medium,

As $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$, speed of electromagnetic waves in free space.

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}} \quad \dots(11)$$

Since $\mu_r > 1$ and $\epsilon_r > 1$, thereby indicating that the speed of electromagnetic waves in an isotropic dielectric is than the speed of electromagnetic in free space.

Q. 3. (b) Give the experimental verification of uncertainty principle by diffraction of electron beam by a single slit.

Ans. Diffraction of electron beam at a narrow slit. When an electron beam passes through a narrow slit a diffraction pattern is obtained and from the Bragg's equation,

$$2d \sin \theta = n\lambda$$

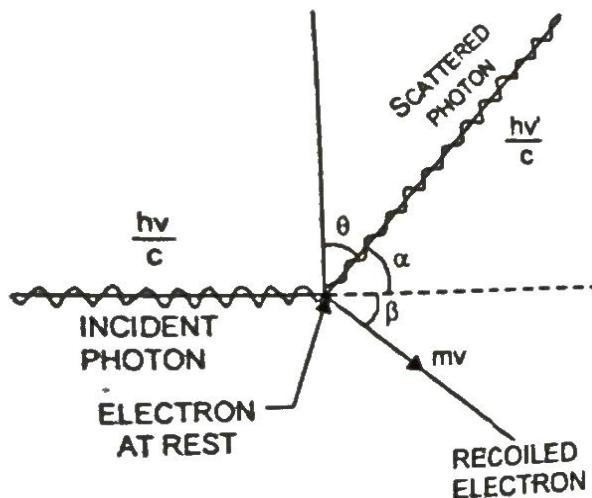


Fig. 1.

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where $2d$ is the width of the slit and θ is the angle of deviation, for the first minimum we have, $\Delta y \sin \theta = \lambda$

$$\Delta y = \frac{\lambda}{\sin \theta}$$

or

Since the component of momentum of electron along y -axis may lie between $+P$ and $-P \sin \theta$, the uncertainty in the y -component of momentum of the electron is

$$\Delta p_y = 2P \sin \theta = 2 \frac{h}{\lambda} \sin \theta$$

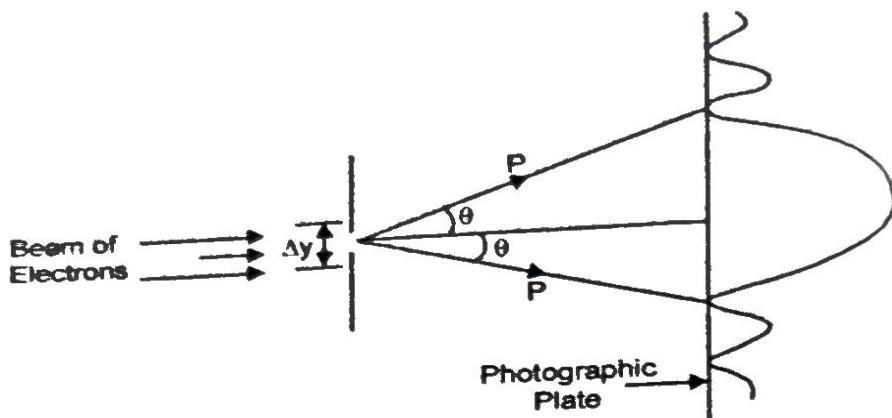


Fig. 2.

From Eqns. (1) and (2), $\Delta y \cdot \Delta p_y = 2h \geq h/4\pi$.

Q. 3. (c) If uncertainty in the location of the particle is equal to de-Broglie wavelength, then prove that the uncertainty in its velocity is equal to its velocity.

Ans. Given
we know that

$$\Delta x = \lambda$$

$$\Delta x \cdot \Delta p_x = h$$

$$\lambda \cdot \Delta p_x = h$$

$$\Delta p_x = \frac{h}{\lambda} = \frac{h}{\lambda} = p \quad (\text{As by de-Broglie theorem, } \lambda = \frac{h}{p})$$

$$\therefore \Delta p_x = p$$

$$\text{Or} \quad m \Delta V_x = m V_x$$

$$\boxed{\Delta V_x = V_x}$$

Q. 4. (a) Derive Planck's radiation formula for the energy distribution in the spectrum of black body. Deduce from it Wein's Law and Rayleigh Jean's Law.

Ans. Refer Q. 4. (b) + Q. 4. (c) End Term Exam May-June 2017.

Q. 4. (b) If the filament of a 60 W bulb has an area 0.03 m^2 and behave as a perfect black body. Find the wavelength corresponding to the maximum in its energy distribution. The Stefan's constant (σ) is $5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$ and Wein's constant (c) is $2.89 \times 10^{-3} \text{ mK}$.

Ans We know that

$$\frac{E}{A} = \sigma T^4$$

$$T = \left(\frac{E}{\sigma A} \right)^{\frac{1}{4}}$$

Given $E = 60 \text{ W}$; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$; $A = 0.03 \text{ m}^2$

Also,

$$T = \left(\frac{60}{5.67 \times 10^{-8} \times 0.03} \right)^{\frac{1}{4}} = 433.4 \text{ K}$$

$$\lambda_m T = 2.89 \times 10^{-3}$$

$$\lambda_m = \frac{2.89 \times 10^{-3}}{433.4} = 6.67 \times 10^{-8} \text{ m}$$

M.M. : 75

END TERM EXAMINATION [MAY-JUNE 2018]
SECOND SEMESTER [B.TECH]
APPLIED PHYSICS-II [ETPH-104]

Time : 3 hrs.

Note: Attempt five questions in all including Q. No. 1 which is compulsory. Select one question from each unit. Symbols have their usual meaning.

Q. 1. (a) Show that curl of gradient of a scalar function is always zero.

Ans. Let the scalar function be $\phi(x, y, z)$

Solving for $\bar{\nabla} \times \bar{\nabla}\phi$

$$\bar{\nabla} \times \bar{\nabla}\phi = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \times \left(\hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \hat{i} \left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial z \partial y} \right) + \hat{j} \left(\frac{\partial^2 \phi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial x \partial z} \right) + \hat{k} \left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} \right)$$

$$= 0$$

$\left[\because \frac{\partial^2 \phi}{\partial y \partial z} = \frac{\partial^2 \phi}{\partial z \partial y}$ and so on

Q. 1. (b) Distinguish between conduction current density and displacement current density.

Ans. Refer Q. 1. (b) First Term Examination, Feb, 2018.

Q. 1. (c) Find the skin depth at a frequency of 10^5 Hz for silver. Given

$$\sigma = 2 \times 10^7 \frac{S}{m} \text{ and } \mu = 4 \times 10^{-7} \frac{H}{m}$$

Ans. Skin depth is given by $\delta = \sqrt{\frac{2}{\mu \sigma \omega}}$

Given, $\mu = 4 \times 10^{-7}$; $\sigma = 2 \times 10^7$; $\omega = 2\pi \times 10^5$

$$\delta = \sqrt{\frac{2}{4 \times 10^{-7} \times 2 \times 10^7 \times 2 \times 3.14 \times 10^5}} \\ = \sqrt{\frac{10^{-6}}{2.512}} = 0.63087 \times 10^{-3} \text{ m} \\ \boxed{\delta = 630.87 \mu\text{m}}$$

Q. 1. (d) Write down the normalization condition of a wave function. Why it is necessary for a wave function to be normalized?

Ans. ψ cannot be interpreted in terms of an experiment. The probability something be in certain place at a given time must lie between 0 (the object is definitely there) and 1 (the object is definitely there). An intermediate probability 0.2 means, there is a 20% chance of finding the object. But the amplitude of wave can be negative as well as positive and negative probability is meaningless. Hence ψ by itself cannot be an observed quantity.

The probability of experimentally finding the body located at the point (x, y, z) at the time t is proportional to the value $|\psi|^2$ where ψ is the wave function. It means the strong possibility of the body's presence, while a small value of $|\psi|^2$ means the strong possibility of the body's absence.

A wave function is a mathematical tool in quantum mechanics describing the quantum state of a particle or system of particles. Mathematically, it is a function from a space that maps the possible states of the system. The laws of quantum mechanics described how the wave function evolves over the time.

In quantum mechanics, probability amplitude is a complex number whose modulus squared represents a probability density. The principal use of probability amplitudes is in the physical meaning of the wave function.

Before going through actual calculation of ψ , it must satisfy certain requirements in quantum mechanics, wave function describing real particle must be normalizable, i.e., the integral of $|\psi|^2$ over all space must be finite after all the body is located somewhere. Mathematically, it is expressed as:

$$\int_{-\infty}^{\infty} \psi^* \psi dV = 1$$

$$\text{or } \int_{-\infty}^{\infty} |\psi|^2 dV = 1$$

A wave function that obeys above equation is said to be normalized. The normalization of wave function is done to obtain the physically applicable wave function of probability amplitudes. All wave function representing real particles must be normalisable. This helps in discarding solutions of Schrodinger's equation which do not have a finite integral in a given interval.

Q. 1. (e) Why is the wave nature of matter not more apparent in our daily observation?

Ans. Since de-Broglie wavelength is given by $\lambda = \frac{h}{mv}$ and the value of Planck's constant is very small, so the wavelength associated with ordinary object is small and difficult to observe. In our daily observations we deal with the objects having larger mass and smaller velocity, that is why the wave nature of such objects is not more apparent in our daily life. But for smaller objects like electrons and neutrons the wave behaviour of particle is dominant as the mass of electron and neutron are smaller than the mass of an ordinary tennis ball.

Q. 1. (f) What is energy quantization? How is it possible for particle in a rigid box?

Ans. Energy quantization means that the system or an atom can have only certain energies and not a continuum of energies. This discretization of energy values is known as quantization, meaning that the allowed energies are separated by discrete energy differences known as "quanta".

For particle in a box of length "l" has following energy.

$$E_n = \frac{n^2 h^2}{8ml^2}$$

where $n = 1, 2, 3, \dots$; m = mass of particle; h = Planck's constant

Particle cannot possess an arbitrary energy but can have only certain discrete energy corresponding to $n = 1, 2, 3$. Each permitted energy level energy is called eigen value of particle and constitute the energy level of the system.

Q. 1. (g) Based on Fermi-Dirac statistics, state the nature of Fermi distribution function. How does it vary with temperatures?

Ans. The Fermi-Dirac distribution is plotted in Fig. 1. for three different values of T and α . In the distribution the occupation index never goes above 1. This signifies that we cannot have more than 1 particle per quantum state as required by Pauli's exclusion principle which applies in this case.

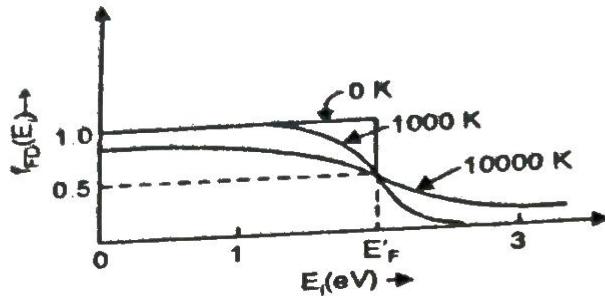


Fig. 1. F.D. distribution curve.

Further, in this distribution the parameter α is strongly dependent on temperature T , and we write

$$\alpha = \frac{-E_F}{k_B T}$$

So that the Fermi-Dirac occupation index becomes.

$$f_{FD}(E_i) = \frac{1}{(e^{(E_i - E_F)k_B T} + 1)}$$

where E_F is called Fermi energy.

Let us consider the situation at the absolute zero of temperature.

At $T = 0$, $(E_i - E_F)/k_B T = -\infty$ (for $E_i < E_F$) and $(E_i - E_F)/k_B T = +\infty$ (for $E_i > E_F$).

Therefore

$$\text{for } E_i < E_F \quad f_{FD}(E_i) = \frac{1}{e^{-\infty} + 1} = 1 \quad (\because e^{-\infty} = 0)$$

$$\text{and for } E_i > E_F \quad f_{FD}(E_i) = \frac{1}{e^{\infty} + 1} = 0 \quad (\because e^{\infty} = \infty)$$

Thus at $T = 0$, all energy states from $E_i = 0$ to $E_i = E_F$ are occupied because $f_{FD}(E_i) = 1$ while all states above E_F are vacant.

As the temperature rises, some of the states just below E_F become vacant, while some just above E_F are occupied. The higher the temperature, the more in the spread in $f_{FD}(E_i)$.

At

$$E_i = E_F, \text{ we have}$$

$$f_{FD}(E_i) = \frac{1}{e^0 + 1} = \frac{1}{2}, \text{ at all temperatures.}$$

That is, the average number of particles per quantum state is exactly $\frac{1}{2}$. In other words, the probability of finding an electron with energy equal to the Fermi energy in a metal is $\frac{1}{2}$ at any temperature.

Q. 1. (a) What is ultraviolet catastrophe?

(2)

Ans. It is observed that Wien's fifth power law ($E(\lambda) \propto \frac{1}{\lambda^5}$) agrees with experimental results only in the low wavelength ($\lambda \rightarrow 0$) region but breaks down in large wavelength ($\lambda \rightarrow \infty$) region as shown in Fig. 2.

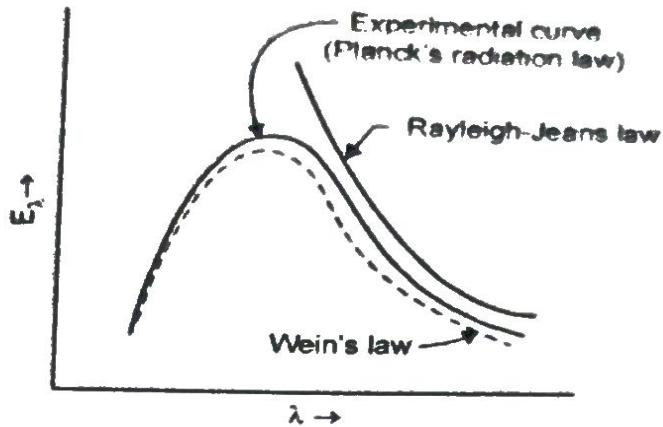


Fig. 2. Comparison of Wien's law and Rayleigh-Jeans law with the experimental curve of blackbody radiation.

Rayleigh-Jeans law $[E_\lambda \propto \frac{1}{\lambda^4}]$ on the other hand can account for the distribution of energy in the long wavelength region and fails completely in short wavelength. This result is said to be ultraviolet catastrophe.

Ultraviolet Catastrophe: According to Rayleigh-Jeans law the energy density in blackbody radiation between wavelength λ and $(\lambda + d\lambda)$ at temperature T is given as

$$E_\lambda d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda \quad \dots(1)$$

Therefore, the total energy radiation per unit volume of the enclosure for all wavelengths from $\lambda = 0$ to $\lambda = \infty$ is given by

$$E = \int_0^\infty E_\lambda d\lambda = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda = 8\pi k_B T \left[-\frac{1}{3\lambda^3} \right]_0^\infty \quad \dots(2)$$

$$= 8\pi k_B T \left[\frac{1}{0} - \frac{1}{\infty} \right] = \infty$$

This law leads to the fact that, the energy density $E_\lambda \rightarrow \infty$ as $\lambda \rightarrow 0$, whereas experimental results show that $E_\lambda \rightarrow 0$ as $\lambda \rightarrow 0$. This implies that for a given quantity of radiant energy is finally confined in the short wavelength (ultraviolet) range.

This is serious discrepancy between theory and experiment and is known as ultraviolet catastrophe or jeans paradox.

Q. 1. (i) Sodium Crystallies is bcc structure. If the radius of the sodium atom is 1.55 nm, compute the spacing between (111) planes. (2)

Ans. Interplanar spacing is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Given

$$a = \frac{4r}{\sqrt{3}} = \frac{4 \times 1.53 \times 10^{-9}}{\sqrt{3}} = 1.193 \times 10^{-9} \text{ m}$$

$$h = 1, k = 1, l = 1$$

$$\sqrt{h^2 + k^2 + l^2} = \sqrt{3}$$

$$d = \frac{4 \times 1.53 \times 10^{-9}}{\sqrt{3} \times 3} = \frac{6.2 \times 10^{-9}}{5.196}$$

$$d = 1.193 \times 10^{-9} \text{ m}$$

$$d = 1.193 \text{ nm}$$

Q. 1. (j) Which type of cubic crystal structure has closest packing of atoms? How many nearest neighbours does an atom in this type of crystal have?

Ans. FCC has closest packing of atoms, that is 74%. Nearest neighbours are given by its coordination Number as—

$$CN = 4 + 4 + 4 = 12$$

Therefore it has 12 nearest neighbours.

Q. 1. (k) What do you understand by effective mass of an electron? Explain its significance.

Ans. The electrons in a crystal are not completely free but interact with the potential of the crystal lattice. As a result their wave particle motion is different from that of an electron in free space. Thus while using the equations of electrodynamics for charge carriers in a crystal, we must use alter value of particle mass of effective mass after taking into account the effect of the crystal lattice so that the electron (and hole) can be treated as almost free carriers of charge for the calculations.

Let us now consider the motion of an electron in a crystal in the presence of an applied electric field E . Suppose the electron is initially in a state k , when the electric field E is applied the magnitude of the force acting on the electron is eE .

If the electron moves a distance dx in a small time dt , under the action of this force then

$$\text{Work done} = \text{Energy gained} \quad dE = eEdx$$

If v is the velocity gained by the electron, then

$$\therefore v = \frac{dx}{dt} \text{ or } dx = vdt$$

$$dE = eEv dt$$

Now

$$v = \frac{1}{h} \frac{dE}{dk}$$

$$\therefore dE = \frac{eE}{h} \frac{dE}{dk} dt$$

or

$$\frac{dk}{dt} = \frac{eE}{h}$$

Now

$$hk = p \text{ and } h \frac{dk}{dt} = \frac{dp}{dt} = F$$

$$h \frac{dk}{dt} = eE = F$$

Differentiating Eq. (3) with respect to t , we get

$$\frac{dv}{dt} = \frac{1}{h} \frac{d^2 E}{dk dt} = \frac{1}{h} \frac{d^2 E}{dk^2} \frac{dk}{dt} \quad \dots(7)$$

From Eq. (6),

$$\frac{dk}{dt} = \frac{F}{h}$$

Also

$$\frac{dv}{dt} = a \text{ (the acceleration)}$$

From Eq. (7),

$$a = \frac{1}{h} \frac{d^2 E}{dk^2} \frac{F}{h}$$

or

$$\frac{a}{F} = \frac{1}{h^2} \frac{d^2 E}{dk} \quad \dots(8)$$

Taking $m^* = \frac{F}{a}$ as the effective mass of electron in the crystal, we have

$$f_k = \frac{h^2}{d^2 E / dk^2} \quad \dots(9)$$

In some cases, a factor f_k is also used. It gives the ratio of the rest mass of a free electron m to its effective mass in the crystal in the k -state

$$m^* = \frac{m}{m^*} = \frac{m}{h^2} \left(\frac{d^2 E}{dk^2} \right) \quad \dots(10)$$

This expression determines the extent to which the electron in the k -state behaves like a free electron.

If $f_k = 1$, $m^* = m$, the electron in the crystal in the k -state behaves as a free electron.

UNIT-I

Q.2 (a) A conducting sphere of radius 5cm has an unknown charge. If the electric field 10 cm from the centre of the sphere is $1.5 \times 10^3 \frac{N}{C}$ and points towards, what is the charge on the sphere? (2.5)

Ans. For conducting sphere at a point outside the charged sphere is given by

$$E = \frac{q}{4\pi r^2 \epsilon_0 \epsilon_r}$$

Given:

$$\text{Radius of sphere } = a = 5 \text{ cm} ; E = 1.5 \times 10^3 \frac{N}{C} ; r = 10 \text{ cm} = 10^{-1} \text{ m} = 0.1 \text{ m}$$

$$\text{For air } \epsilon_r = 1$$

$$\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$$

$$1.5 \times 10^3 = \frac{q}{4 \times 3.14 \times (10^{-1})^2 \times 1 \times 8.854 \times 10^{-12}}$$

$$q = 1.5 \times 10^3 \times 4 \times 3.14 \times 10^{-2} \times 8.854 \times 10^{-12} \approx 166.80 \times 10^{-12}$$

$$[q = 0.1668 \times 10^{-12} C]$$

Q. 2. (b) Show that Ampere's Law is inconsistent in time varying fields.

Ans. Refer Q. 1. (c) of First Term Examination 2018.

Q. 2. (c) Write down the significance of Pointing vector. Derive an expression for Pointing Theorem and give interpretation of each term. (2 + 5 = 7)

Ans. Refer Q. 2. (a) First Term Examination 2018.

Q. 3. (a) Deduce the equation for propagation of electromagnetic waves in free space and obtain an expression for the velocity. Show that electric and magnetic field vectors are normal to each other and to the direction of propagation of waves.

Ans. Maxwell's equations are:

$$\text{div } D = \nabla \cdot D = \rho$$

$$\text{div } B = \nabla \cdot B = 0$$

$$\text{curl } E = -\frac{\partial B}{\partial t}$$

and

$$\text{curl } H = J + \frac{\partial D}{\partial t}$$

Free space is characterised by,

$$\rho = 0, \sigma = 0, \mu = \mu_0 \text{ and } \epsilon = \epsilon_0$$

Therefore, Maxwell's Eqs. (1) reduce to

$$\text{div } E = 0$$

$$\text{div } H = 0$$

$$\text{curl } E = -\mu_0 \frac{\partial H}{\partial t}$$

and

$$\text{curl } H = \epsilon_0 \frac{\partial E}{\partial t}$$

Taking curl of Eq. 3. (c), we get

$$\text{curl curl } E = -\mu_0 \frac{\partial}{\partial t} (\text{curl } H)$$

Substituting curl H from Eq. 3. (d) we get

$$\text{curl curl } E = -\mu_0 \frac{\partial}{\partial t} \left(\epsilon_0 \frac{\partial E}{\partial t} \right)$$

i.e.,

$$\text{curl curl } E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Now, $\text{curl curl } E = \text{grad div } E - \nabla^2 E = -\nabla^2 E$

[$\because \text{div } E = 0$ from Eq. 3. (a)]

Making substitution Eq. (4), becomes

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \dots(5)$$

Similarly, on taking curl of Eq. 3. (d) and putting the value of curl E from Eq. 3. (c)

$$\begin{aligned} \text{curl curl } H &= \epsilon_0 \frac{\partial}{\partial t} \left(-\mu_0 \frac{\partial H}{\partial t} \right) \\ &= -\mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \end{aligned} \quad \dots(6)$$

Again using identity curl curl H = grad div H - \nabla^2 H and noting that div H = 0 from Eq. 3. (b), we obtain

$$\text{curl curl } H = -\nabla^2 H$$

Making this substitution in Eq. (6), we get

$$\nabla^2 H - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots(7)$$

Equations (5) and (7) represent wave equation governing electric and magnetic fields (E and H) in free space. It may be noted that these equations may be obtained by using Eq. 3. (a). Eqs. (5) and (7) are vector equations of identical form, which means that each of six components of E and H separately satisfies the same scalar wave equation of the form

$$\nabla^2 \phi - \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \dots(8)$$

where ϕ is a scalar and cannot stand for one of the components of E and H. It is seen that Eq. (8) resembles with general wave equation

$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} \quad \dots(9)$$

where v is the velocity of wave.

Comparing Eqs. (8) and (9), we see that the field vectors E and H are propagated in space as waves at a speed equal to

$$\begin{aligned} v &= \frac{1}{\sqrt{\mu_0 \epsilon_0}} \\ &= \sqrt{\frac{4\pi}{\mu_0 4\pi \epsilon_0}} = \sqrt{\frac{4\pi}{4\pi \times 10^{-7}} \times 9 \times 10^9} \\ &= 3 \times 10^8 \text{ m/s} \\ &= c = \text{speed of light in vaccum.} \end{aligned} \quad \dots(10)$$

Therefore, it is reasonable to write c the speed of light in place of $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$, so Eqs. (5) and (7) take the form

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad \dots(11)$$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad \dots(12)$$

and

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0$$

The plane wave solution of above equations in well known form many be written,

$$E(r, t) = E_0 e^{ik_r r - i\omega t}$$

$$H(r, t) = H_0 e^{ik_r r - i\omega t}$$

$$\phi(r, t) = \phi_0 e^{ik_r r - i\omega t}$$

where E_0 , H_0 and ϕ_0 are complex amplitudes which are constant in space and time while k is a wave propagation vector denoted as

$$k = k\hat{n} = \frac{2\pi}{\lambda} \hat{n} = \frac{2\pi v}{c} \hat{n} = \frac{\omega}{c} \hat{n}$$

Here \hat{n} is a unit vector in the direction of wave propagation. Now in order to satisfy the condition $\nabla \cdot E = 0$ and $\nabla \cdot H = 0$. Let us first find $\nabla \cdot E$ and $\nabla \cdot H$.

$$\nabla \cdot E = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot E_0 e^{ik_r r - i\omega t}$$

$$= \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot [(i E_{0x} + j E_{0y} + k E_{0z}) e^{i(k_r x + k_y y + k_z z) - i\omega t}]$$

$$[\because \hat{k} \cdot \hat{r} = (ik_x + jk_y + kk_z)(ix + jy + kz) = k_x x + k_y y + k_z z]$$

$$\nabla \cdot E = (E_{0x} i \hat{k}_x + E_{0y} j \hat{k}_y + E_{0z} k \hat{k}_z) e^{ik_r r - i\omega t}$$

$$= i(k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{ik_r r - i\omega t}$$

$$= i(i k_x + j k_y + k k_z) \cdot (i E_{0x} + j E_{0y} + k E_{0z}) e^{ik_r r - i\omega t}$$

$$= i k \cdot E_0 e^{ik_r r - i\omega t} = i k \cdot E$$

$$\text{Similarly, } \nabla \cdot H = i k \cdot H$$

Thus the requirement $\nabla \cdot E = 0$ and $\nabla \cdot H = 0$ demands that

$$k \cdot E = 0 \text{ and } k \cdot H = 0$$

This means that electromagnetic field vectors E and H are both perpendicular to the direction of propagation vector k . This implies that electromagnetic waves are transverse in character.

Further restrictions are provided by curl. From Eqs. 3. (c) and 3. (d).

$$\text{curl } E = -\mu_0 \frac{\partial H}{\partial t} \text{ and } \text{curl } H = \epsilon_0 \frac{\partial E}{\partial t}$$

Using Eqs. (14), (15) and (17), above equation yields

$$ik \cdot E = -\mu_0 \cdot (-i\omega H) \text{ or } k \cdot E = \mu_0 \omega H$$

and

$$ik \cdot H = \epsilon_0 \cdot (-i\omega E) \text{ or } k \cdot H = -\epsilon_0 \omega E$$

From Eq. (17), it is obvious that field vector H is perpendicular to both k and E according to Eq. (18), E perpendicular to both k and H . This simply means that the vectors E and H are mutually perpendicular and they are also perpendicular to the direction of propagation of wave.

Q. 3. (b) If the earth receives 4 cal/min/cm² solar energy. What are the amplitudes of electric and magnetic fields of radiation?
Ans. Here, solar energy which the earth receives is 2 cal/min/cm²

$$\frac{E_0}{H_0} = \sqrt{\frac{n_0}{\epsilon_0}} = 377$$

printing vector

$$\vec{S} = \vec{E} \times \vec{H} = EH \text{ Jm}^{-2} \text{ sec}^{-1}$$

$$EH = \frac{4 \times 42}{60 \times 10^4} = 2800 \text{ Jm}^{-2} \text{ sec}^{-1}$$

$$\frac{E_0}{H_0} = 377 = \frac{E}{H}$$

Also, Multiply eqn. (1) and (2)

$$E^2 = 2800 \times 377 = 1055600$$

$$E = 1027.42 \frac{\text{V}}{\text{m}}$$

$$H = \frac{E}{377} = \frac{1027.42}{377} = 2.73 \frac{\text{A}}{\text{m}}$$

Amplitudes of electric and magnetic field vectors are

$$E_0 = E\sqrt{2} = 1.452.78 \text{ V/m}$$

$$H_0 = H\sqrt{2} = 3.85 \text{ A/m}$$

UNIT-II

Q. 4. (a) State and explain Rayleigh-Jeans law. Show how its drawbacks can be overcome using Planck's radiation law. (3.5)

Ans. As per Planck's quantum hypothesis, the atoms of the wall of a blackbody behave as oscillators and each has a characteristic frequency of oscillation. Then average energy of these Planck's oscillators were calculated and finally Planck's radiation formula was derived.

Wien's law is deduced from Planck's radiation formula under the condition when wavelength λ and temperature T are very small. However, Planck's radiation formula for the condition of high temperature T and wavelength λ takes the form of Rayleigh-Jeans law.

Planck radiation law is as following.

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

When λ and T are very small then $e^{\left(\frac{hc}{\lambda KT}\right)} \gg 1$

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$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{\frac{hc}{\lambda KT}} d\lambda$$

$$8\pi hc = A \text{ and } \frac{hc}{K} = B$$

By substituting

we get

$$\boxed{\mu_\lambda d\lambda = \frac{B}{\lambda^5} e^{\frac{B}{\lambda T}} d\lambda}$$

This is Wien's law valid at low temperature and small wavelength.

When λ and T are large then $e^{\frac{hc}{\lambda KT}} \approx 1 + \frac{hc}{\lambda KT}$

$$\mu_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left\{ \left[1 + \frac{hc}{\lambda KT} \right]^{-1} \right\}} d\lambda$$

$$\boxed{\mu_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh-Jeans law valid at high temperature and large wavelength.

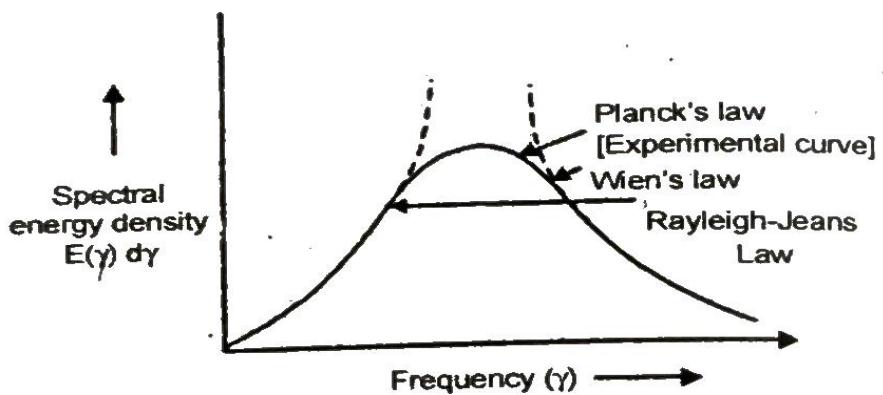


Fig. 3

As shown in Fig. 3, the spectral energy density decreases with frequency. Hence Rayleigh-Jeans and Wien's law are unable to explain the energy distribution for blackbody radiation, while Planck's law explains it correctly.

Q. 4. (b) Distinguish between Maxwell-Boltzmann (MB), Bose-Einstein and Fermi-Dirac (FD) statistics on the basis of their distribution functions. Show that the BE and FD distributions at very high temperature go to the same distribution.

Ans. Difference between Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics and as following—

Boltzmann statistics	Bose-Einstein statistics (Quantum statistics)	Fermi-Dirac statistics (Quantum statistics)
for systems of distinguishable particles, such as	Apply to systems of indistinguishable particles not obeying Pauli's exclusion principle, such as photons, phonons and liquid helium at low temperature	Apply to systems of indistinguishable particle obeying Pauli exclusion principle, such as free electrons in metal electrons in a star (white dwarf star)
distribution law is S.	The distribution law is $n_i = \frac{g_i}{(e^{\alpha E_i/k_B T}) - 1}$	The distribution law is $n_i = \frac{g_i}{(e^{\alpha E_i/k_B T}) + 1}$
no restriction on number of particles, in given state.	There is no restriction on the number of particles in a given state.	Only one particle in a given quantum state is allowed.
in phase space the volume of the phase cell is fixed.	The volume of phase cell is of order of \hbar^3 .	The volume of phase cell is of the order of \hbar^3
behaviour of distribution function $f(E_i)$ is exponential.	For $E_i \gg k_B T$, exponential $E_i \ll k_B T$, lies above M.B.	For $E_i \gg k_B T$; exponential where $E_i \gg E_F$. If $E_F \gg k_B T$ decreases abruptly near E_F .
energy of M.B. such as in ideal atomic gas, may be at absolute zero.	The energy may be zero at absolute zero.	The energy at absolute zero, cannot be zero because all particles cannot come down to ground state due to Pauli's exclusion principle.

In N-type semiconductor, since $n_e > n_h$, E_F must move closer to conduction band to that many of the energy states in the band are filled by the donor electrons, and holes exist in the valence band.

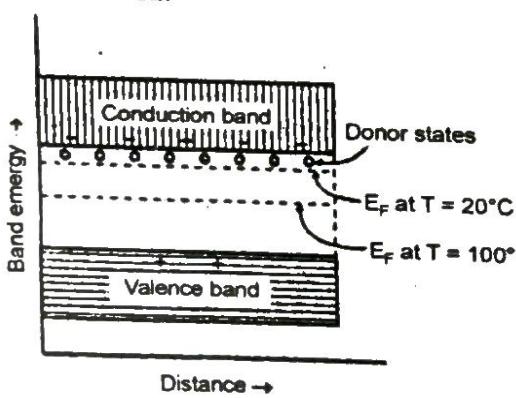


Fig.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i increase. Thus the Fermi level (E_F) will remain approximately at the centre of the禁带 gap. Thus for intrinsic semiconductor fermi level is independent of temperature. But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as the temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap.

Q. 5. Differentiate between Bosons and Fermions. Give one example.

Ans. Refer Q. 4 (a) End Term Examination May-June 2017.

Q. 5. (a) Derive Schrodinger's time independent wave equation.

Ans. Consider a system of stationary waves associated with a moving particle. The position coordinates of the particle are (x, y, z) and Ψ be the periodic displacement wave by a differential equation as follows.

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}$$

where v is the velocity of wave associated with the particle. The solution given Ψ as a periodic displacement in terms of time i.e.

$$\Psi(x, y, z, t) = \Psi_0(x, y, z)e^{-i\omega t}$$

where Ψ_0 is the amplitude of the particle wave at the point (x, y, z) independent of time (t). It is a function of (x, y, z) . i.e., the position r and not Here.

$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

Eq. (2) may be expressed as

$$\Psi(r, t) = \Psi_0(r)e^{-i\omega t}$$

Differentiating Eq. (4) twice with respect to t , we get

$$\frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi_0(r)e^{-i\omega t}$$

$$\text{or} \quad \frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi$$

Substituting the value of $\frac{\partial^2 \Psi}{\partial t^2}$ from this equation in Eq. (1), we get

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{\omega^2}{v^2} \Psi = 0$$

where

$$\omega = 2\pi v = 2\pi(v/\lambda)$$

so that

$$\frac{\omega}{v} = \frac{2\pi}{\lambda}$$

$$\text{Also } \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \nabla^2 \Psi$$

where ∇^2 is known as Laplacian operator. Using Eqs. (6), (7) and (8), we

$$\nabla^2 \Psi + \frac{4\pi^2}{\lambda^2} \Psi = 0$$

Also from the de-Broglie wave concept

$$\lambda = \frac{h}{mv}$$

Using this relation in Eq. (9) gives-

$$\nabla^2\psi + \frac{4\pi^2m^2v^2}{h^2}\psi = 0 \quad \dots(10)$$

If E and V are respectively the total energy and potential energy of the particle then kinetic energy is given by

$$\begin{aligned} \frac{1}{2}mv^2 &= E - V \\ m^2v^2 &= 2m(E - V) \end{aligned} \quad \dots(11)$$

The use of Eq. (11) in Eq. (10) gives

$$\begin{aligned} \nabla^2\psi + \frac{8\pi^2m}{h^2}(E - V)\psi &= 0 \\ \nabla^2\psi + \frac{2m}{h^2}(E - V)\psi &= 0 \end{aligned} \quad \dots(12)$$

This is the time independent Schroedinger equation, where the quantity ψ is known as wave function.

For a freely moving or free particle $V = 0$. Therefore, Eq. (12) becomes

$$\nabla^2\psi + \frac{2mE}{h^2}\psi = 0 \quad \dots(13)$$

This is called time independent Schroedinger equation for a free particle.

Q. 5. (b) What is a wave packet? Show that phase velocity of de-Broglie wave is greater than the velocity of light. (1 + 3)

Ans. When plane waves of slightly different wavelength travel simultaneously in the same direction along a straight line, through a dispersive medium then successive groups of waves are produced. These waves are called wave packets. Each wave group travels with a velocity called group velocity.

$$\text{Phase velocity} = \frac{w}{K} \quad \dots(1)$$

$$\text{Also, phase velocity} = V_p = \gamma\lambda \dots(2)$$

For an electromagnetic wave

$$\begin{aligned} E &= h\nu \\ \gamma &= \frac{E}{h} \end{aligned} \quad \dots(3)$$

or

According to de-Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots(4)$$

From eqn. (2), (3) and (4), we get

$$\begin{aligned} V_p &= \gamma\lambda \\ &= \frac{E}{h} \times \frac{h}{mv} = \frac{E}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v} \\ V_p &= \frac{C^2}{V} \end{aligned} \quad \dots(5)$$

Since $C \gg v$, Eqn (5) implies that the phase velocity of de-Broglie wave is greater than the velocity of light.

Q. 5. (c) A particle is confined to an one-dimensional infinite potential well of width 0.2×10^{-9} m. It is found that when the energy of the particle is 230 eV, its eigenfunction has 5 antinodes. Find the mass of the particle and show that it can never have energy equal to 1 KeV.

Ans. Energy of particle of mass "m" in a box of length "l" is given by

$$E_n = \frac{n^2 h^2}{8ml^2}$$

Given

$$l = 0.2 \times 10^{-9} \text{ m} = 2 \times 10^{-10} \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ J-S}$$

$$n = 5$$

$$E = 230 \text{ eV} = 368 \times 10^{-19} \text{ J}$$

$$m = ?$$

$$368 \times 10^{-19} = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times m \times (2 \times 10^{-10})^2}$$

$$m = \frac{25 \times (6.63 \times 10^{-34})^2}{8 \times (2 \times 10^{-10})^2 \times 368 \times 10^{-19}}$$

$$= \frac{1098.93 \times 10^{-68}}{11776 \times 10^{-39}}$$

$$m = 0.0933 \times 10^{-29} \text{ kg} = 9.33 \times 10^{-31} \text{ kg}$$

$$\boxed{m = 9.33 \times 10^{-31} \text{ kg}}$$

Solving for energy at $n = 1$

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.33 \times 10^{-31} \times (2 \times 10^{-10})^2} = 0.147 \times 10^{-17} \text{ J}$$

$$E_1 = \frac{0.147 \times 10^{-17}}{1.6 \times 10^{-19}} = 9.187 \text{ eV}$$

For

$$n = 2 \Rightarrow E_2 = 36.75 \text{ eV}$$

For

$$n = 3 \Rightarrow E_3 = 82.683 \text{ eV}$$

For

$$n = 4 \Rightarrow E_4 = 146.992 \text{ eV}$$

For

$$n = 5 \Rightarrow E_5 = 229.675 \text{ eV}$$

$$E_n < 1 \text{ KeV} \text{ Or } E_n < 1000 \text{ eV}$$

UNIT-III

Q. 6. (a) Establish the relation between the lattice constant of a cubic crystal and the density of the crystal material.

Ans. In a lattice a unit cell has the length of the edges and the angles between edges are of definite values. Each edge of the unit cell is a distance between the atoms of the same kind and is known as the lattice constant.

Let us consider a cubic lattice constant a . If ρ is the density of the crystal under consideration, then

$$\text{Volume of unit cell} = a^3$$

$$\text{Mass in each unit cell} = a^3 \rho$$

If M is the molecular weight and N the Avogadro's number (i.e., the number of molecules per kg mole of the substance), then (1)

$$\text{Mass of each molecule} = \frac{M}{N}$$

Further if n is the number of molecules (lattice points) per unit cell, then

$$\text{the mass in each unit cell} = n \frac{M}{N}$$

From Eqs. (1) and (2)

$$a^3 \rho = \frac{nM}{N}$$

$$a = \left(\frac{nM}{N\rho} \right)^{1/3} \quad \dots(3)$$

From the relation the values of the lattice constant 'a' can be calculated.

Q. 6. (b) Copper has fcc structure and its atomic radius is 0.1278 nm. If the atomic weight of copper is 63.5. Calculate its density. (2.5)

Ans. Density is given by

$$\rho = \frac{nM}{a^3 N}$$

Given that

$$n = 4; M = 63.5; N = 6.02 \times 10^{23}$$

$$a = \frac{4r}{\sqrt{2}} = \frac{4 \times 0.1278 \times 10^{-9}}{\sqrt{2}}$$

$$\rho = \frac{4 \times 63.5 \times (\sqrt{2})^3}{(4 \times 0.1278 \times 10^{-9})^3 \times 6.02 \times 10^{23}} = 8.98 \frac{\text{gm}}{\text{CC}}$$

$$\boxed{\rho = 8.98 \frac{\text{gm}}{\text{CC}}}$$

Q. 6. (c) What are the different types of point defect in crystal lattice? Show that the number of Schottky defects at a given temperature is proportional to the number of atoms present in the crystal. (7.5)

Ans. (a) Point defect: A lattice defect which spreads out very little in all the three dimensions is called a point defect. There are following types :

(i) Interstitial atom (ii) Vacancies (iii) Impurity atom.

(i) Interstitial atom: This is an atom inserted into the voids (called interstices of lattice) between the regularly occupied sites. Thus such an atom does not occupy regular lattice sites. This extra atom may be an impurity atom or an atom of the same element as on the regular sites. Fig. 5(a).

(ii) Vacancies: These are the lattice sites from which the atoms are missing. Such a vacancy is also called Schottky defect.

If a vacancy is created by transferring an atom from a regular lattice site to an interstitial position then it is called Frenkel defect.

In this case, therefore, two imperfections are created vacancy as well as an interstitial

(iii) Impurity atom: This is a defect in which a foreign atom occupies a lattice site (Fig. 5(c)).

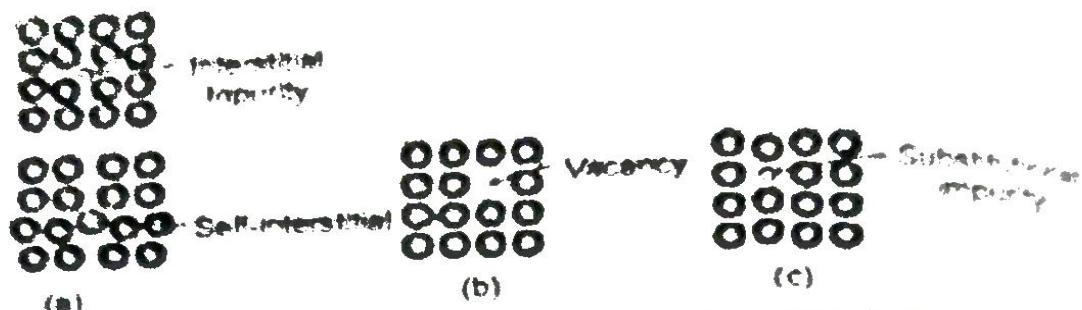


Fig. 5. Three common point defects, (a) interstitial, (b) vacancy, (c) impurity.

A point imperfection is localised near a point or atom in the structure or in other words point defects are lattice errors at isolated lattice points. Point defects are errors at isolated lattice points. Point defects occurs in metallic as well as ionic lattices. These types of point defects are shown in Fig. 5 (d).

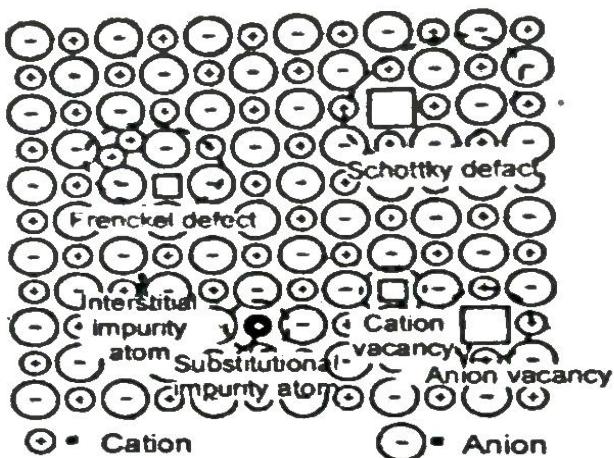


Fig. 5. (d): Illustrating a number of possible point defects in a two-dimensional ionic crystal.

(b) Schottky defect

In real crystals, there are irregularities of the atomic array in which atoms are missing at some lattice points. Such a point is called a **vacancy** (also called Schottky defect). This is shown in Fig 5. (e).

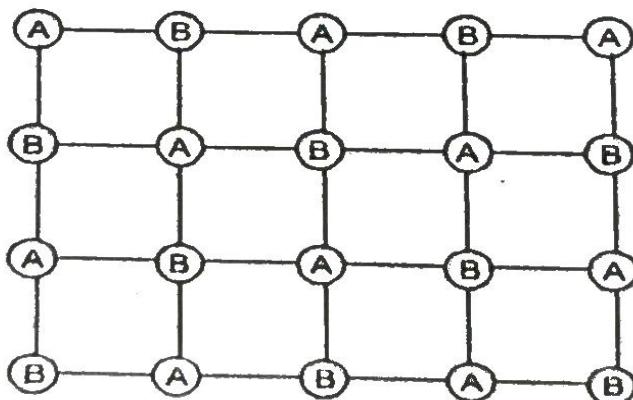


Fig. 5(e) Schottky defect

In spite of care taken in the preparation of crystals, vacancies are always present in all crystals. In fact, as a result of thermal fluctuations, vacancies are produced and destroyed constantly in the crystal. Formally such a defect might be produced by plucking an interior atom out of its regular lattice site and placing it on the surface. This act requires energy. Moreover, the disorder increases resulting in an increase in the entropy. In thermal equilibrium, a certain number of lattice vacancies are always present which can be estimated as follows:

If E_v is the energy required to take an atom from a lattice site inside the crystal to lattice site on the surface, $n E_v$ is the increase in internal energy associated with the production of n isolated vacant sites. The total number of ways in which we can pick up n atoms from the crystal consisting N atoms is

$$\omega = \frac{N!}{(N-n)!n!}$$

Since disorder increases due to creation of n vacancies the corresponding increase in entropy is given by

$$S = K_B \ln \omega$$

$$= K_B \ln \frac{N!}{(N-n)!n!}$$

This in turn produces a change in free energy F

$$F = U - TS$$

\Rightarrow

$$F = nE_v - K_B T \ln \frac{N!}{(N-n)!n!}$$

The second term on right hand side can be simplified by the use of Stirling approximation.

$$\ln x_i \approx x \ln x - x \quad \dots(5)$$

$$F = nE_v - K_B T [N \ln N - N - (N-n) \ln (N-n) + N - n - n \ln n + n]$$

$$F = nE_v - k_B T [N \ln N - (N-n) \ln (N-n) - n \ln n] \quad \dots(6)$$

Free energy in thermal equilibrium at constant volume must be minimum with respect to changes in n i.e.

$$\left(\frac{\partial F}{\partial n} \right)_T = 0 = E_v - k_B T \ln \frac{N-n}{n} \quad \dots(7)$$

$$0 = E_v + k_B T \ln \frac{n}{N-n}$$

$$n = (N-n) \exp \left(\frac{-E_v}{K_B T} \right)$$

If $n \ll N$, we can neglect n in $(N-n)$ so that

$$n \approx N \exp \left(\frac{-E_v}{K_B T} \right)$$

$$E_v = 1 \text{ eV} \text{ and } T = 1000 \text{ K, then}$$

$$n \approx e^{-12} \approx 10^{-5} / \text{cm}^3$$

The equilibrium concentration of vacancies (decreases as the temperature decreases).

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Figure 5(I) shows an alkali halide crystal with f vacancies. Fig. 5(II) displays two separate vacancies and a vacancy pair in alkali halide crystal. The pairs are formed because the cation and anion vacancies in alkali halides are electrically charged and they attract each other electrostatically. Due to formation of the pair a dipole is also produced.

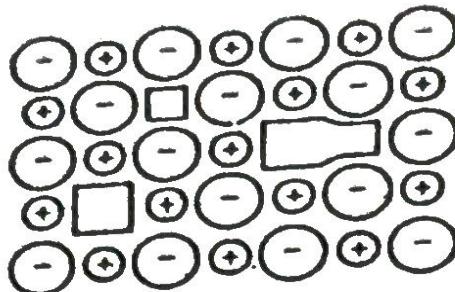


Fig. 5 (I) A plane of a pure alkali halide crystal, showing a vacant positive ion site, a vacant negative ion site, and a coupled pair of vacant sites of opposite sign (vacancies in alkali halide)

In ionic crystals, formation of paired vacancies is most favoured, i.e., in ionic crystals, equal number of positive and negative ion vacancies are produced. The formation of pairs makes it possible to keep the surface of the crystal electrostatically neutral. The number of pairs can be related to the total number of atoms present in the crystal on following the same procedure as adopted above.

The different ways in which n separated pairs can be formed are.

$$\omega = \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(11)$$

So that increase in entropy will be

$$S = k_B \ln \omega \\ = k_B \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(12)$$

with corresponding change in free energy

$$F = U - TS$$

$$\Rightarrow F = nE_p - K_B T \ln \left[\frac{N!}{(N-n)!n!} \right]^2 \quad \dots(13)$$

where E_p = Energy of formation of a pair.

Applying Stirling's approximation to simplify the factorial terms i.e.,

$$\ln \left[\frac{N!}{(N-n)!n!} \right]^2 \approx 2[\ln N! - \ln(N-n)! - \ln n!]$$

Putting it in Eq. (13), we get

$$2[N \ln N - (N-n) \ln(N-n) - n \ln n] \quad \dots(14)$$

Differentiating the above equation with respect to n , we get

$$\left(\frac{\partial P}{\partial n}\right)_T = E_p - 2k_B T \ln(N-n) + k_B T = 0$$

$$= E_p - 2k_B T \ln \frac{N-n}{n}$$

Free energy in thermal equilibrium attained at temperature T is constant, we have

$$E_p - 2K_B T \ln \frac{(N-n)}{n} = 0$$

or

$$\ln \frac{N-n}{n} = \frac{E_p}{2K_B T}$$

or

$$\frac{N-n}{n} = \exp \left[\frac{E_p}{2K_B T} \right]$$

or

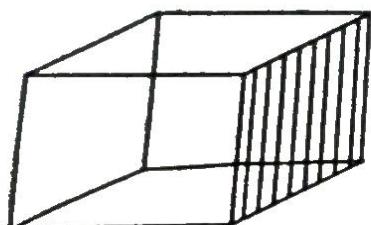
$$n = N \exp \left(\frac{-E_p}{2K_B T} \right)$$

provided $n \ll N$. In NaCl crystal $E_p = 2.02 \text{ eV}$ and at room temperature $n = 10^6 \text{ cm}^{-3}$

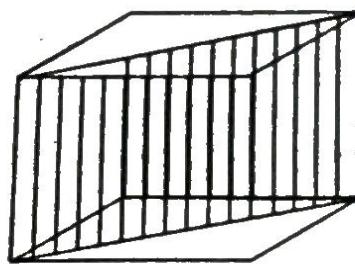
Q. 7. (a) Draw (010), (110) and (222) planes in a cubic crystal.

(4.5)

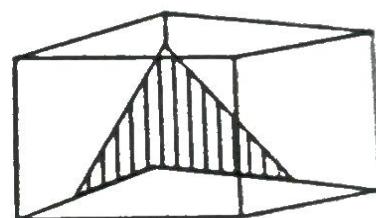
Ans.



(010) Plane
(a)



(110) Plane
(b)



(222) Plane
(c)

Fig. (6)

Q. 7. (b) A certain crystal has lattice constant of 4.24 \AA , 10 \AA and 3.66 \AA on the x, y, z axis respectively. Determine the miller indices of the plane of this crystal having 4.24 \AA , 5 \AA and 1.83 \AA as its x, y, z intercepts respectively. (3)

Ans. Given

$$a = 4.24 \text{ \AA}; b = 10 \text{ \AA}; c = 3.66 \text{ \AA}$$

Intercepts at 4.24 \AA , 5 \AA and 1.83 \AA respectively

\therefore we can say that intercepts are in the ratio $a : \frac{b}{2} : \frac{c}{2}$

From the law of rational indices

$$a : \frac{b}{2} : \frac{c}{2} = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

where h, k, l are the miller indices.

$$\begin{aligned} \frac{1}{h} : \frac{1}{k} : \frac{1}{l} &= \frac{1}{2} : \frac{1}{2} \\ h : k : l &= 1 : 2 : 2 \\ h &= 1, k = 2, l = 2 \end{aligned}$$

Thus

Hence, the plane is (122)

Q. 7. (c) Describe Bragg's spectrometer and explain with the help of an example how it is used to study the crystal structure? (3)

Ans. Bragg's X-ray Spectrometer

Bragg devised an apparatus used to study the glancing angle θ and the intensities of diffracted X-rays for a given sample. The essential parts of Bragg's spectrometer are shown in Fig. 7.

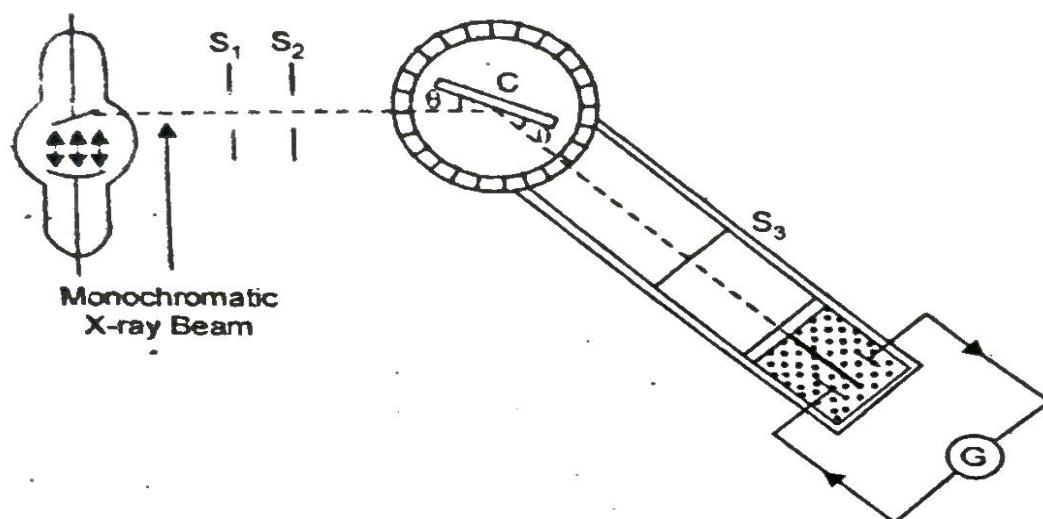


Fig. 7.

A monochromatic X-ray beam from the source is passed through two slits, S_1 and S_2 , which collimate it into a fine narrow beam. This X-ray beam is then allowed to fall on the crystal sample C mounted at the centre of a turn table. A movable arm is also attached to the turn table for detecting the reflected X-ray beam from the crystal C. This turn table is capable of rotation about a vertical axis and the angle of rotation can be measured on the circular scale. The rates of rotation of the turn table and the detector (ionisation chamber) arm are such that the ionisation chamber always receives the diffracted beam. When the turn table rotates through an angle θ , the ionisation chamber arm automatically rotates through an angle 2θ with the direction of incident ray. This way the measurements of different diffracted X-ray beams' intensities and angles are recorded. Then using Bragg's condition, we get the interplanar spacing and the structure of the crystal.

UNIT-IV

Q. 8. (a) Derive an expression for the density of holes in an intrinsic semiconductor. (6)

Ans. Since a hole signifies a vacancy created by removal of an electron i.e. an empty energy level, the Fermi function for a hole is $1 - f(E)$. Here $f(E)$ represents the probability that the level is occupied by an electron. Along with the expression of $f(E)$, the factor $1 - f(E)$ becomes

$$\begin{aligned}
 1 - f(E) &= 1 - \frac{1}{1 + e^{(E - E_F)/kT}} \\
 &= 1 - [1 + e^{(E - E_F)/kT}]^{-1} \\
 &= 1 - [1 - e^{(E - E_F)/kT}] \quad \text{for } (E - E_F)/kT \ll 1 \\
 &\approx 1 - 1 + e^{(E - E_F)/kT} \\
 1 - f(E) &= e^{(E - E_F)/kT}
 \end{aligned}$$

For the top of the valence band (the maximum energy), the density of states is given by

$$N(E) = \frac{4\pi}{h^3} (2m_h)^{3/2} (E_V - E)^{1/2}$$

Here m_h is the effective mass of holes near the top of the valence band, where the energy is E_V . With the above relation, the density of holes in the valence band is calculated

$$\begin{aligned}
 n_h &= \frac{4\pi}{h^3} \int_{-\infty}^{E_V} N(E)[1 - f(E)]dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{(E - E_F)/kT} dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} e^{(E_F - E)/kT} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{(E - E_V)/kT} dE \\
 &= \frac{4\pi}{h^3} (2m_h)^{3/2} e^{(E_V - E_F)/kT} \int_{-\infty}^0 x^{1/2} (kT)^{1/2} e^{-x/kT} dx
 \end{aligned}$$

here we have substituted $\frac{E_V - E}{kT} = x$ so that $dE = -kT dx$. Now

$$n_h = \frac{4\pi}{h^3} (2m_h)^{3/2} (kT)^{3/2} e^{(E_V - E_F)/kT} \int_{-\infty}^0 x^{1/2} e^{-x} dx.$$

$$n_h = 2 \left[\frac{2\pi m_h k T}{h^2} \right]^{3/2} e^{(E_V - E_F)/kT}$$

(b) Show that the Fermi Energy level in an intrinsic semiconductor lies approximately half way between the top of the valence band and the bottom of conduction band.

From the law of mass action $n_i^2 = n_e \cdot n_h$

put the value of n_e and n_h .

$$n_i^2 = N_C N_V \exp\left(\frac{E_F - E_C}{k_B T}\right) \exp\left[\frac{E_V - E_F}{k_B T}\right] \quad \dots(1)$$

or

$$n_i^2 = N_C N_V \exp\left[\frac{-(E_C - E_V)}{k_B T}\right]$$

Since $E_C - E_V = E_g$, the band gap of the semiconducting material, Eq. (1) may be written as

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{k_B T}\right)$$

Substituting the value of N_C and N_V in Eq. (2) we get

$$\begin{aligned} n_i^2 &= 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{3/2} \times 2\left[\frac{2\pi m_h^* k_B T}{h^2}\right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \\ &= 4\left[\frac{2\pi k_B T}{h^2}\right]^3 \times \left[m_e^* m_h^*\right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right) \end{aligned}$$

Multiplying the numerator and denominator of Eq. (3) by m^3 , we get

$$n_i^2 = 4\left[\frac{2\pi k_B m T}{h^2}\right]^3 \times \left[\frac{m_e^* m_h^*}{m^2}\right]^{3/2} \exp\left(\frac{-E_g}{k_B T}\right)$$

The value of n_i can be written as

$$\begin{aligned} n_i &= 2\left[\frac{2\pi k_B m T}{h^2}\right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2}\right]^{3/4} \exp\left(\frac{-E_g}{2k_B T}\right) \\ &= 2\left[\frac{2\pi k_B m T}{h^2}\right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2}\right]^{3/4} T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \end{aligned}$$

Equation (5) may also be written as

$$n_i = A T^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right)$$

where

$$A = 2\left[\frac{2\pi k_B m}{h^2}\right]^{3/2} \times \left[\frac{m_e^* m_h^*}{m^2}\right]^{3/4} = 4.82 \times 10^{21} \times \left[\frac{m_e^* m_h^*}{m^2}\right]^{3/4}$$

Equation (5) gives the intrinsic carrier concentration in semiconductor.

Value of Fermi energy-

In intrinsic semiconductor $n_e = n_h$

$$\begin{aligned} \therefore 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \times e^{(E_F - E_C)k_B T} &= 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} \times e^{(E_V - E_F)k_B T} \\ \Rightarrow &= (m_e^*)^{3/2} \times e^{(E_F - E_C)k_B T} = (m_h^*)^{3/2} \times e^{(E_v - E_F)k_B T} \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow e^{(2E_F - E_C - E_V)/k_B T} = \left(\frac{m_h^*}{m_e^*} \right)^{3/2} \quad \dots(7) \\
 & \Rightarrow \frac{2E_F - E_C - E_V}{k_B T} = \frac{3}{2} \log \left(\frac{m_h^*}{m_e^*} \right) \\
 & \Rightarrow E_F = \frac{E_C + E_V}{2} + \frac{3}{4} k_B T \log \left(\frac{m_h^*}{m_e^*} \right) \\
 & \qquad \qquad \qquad \left[\text{for } m_e^* = m_h^*, \log \frac{m_h^*}{m_e^*} = \log 1 = 0 \right] \\
 & \Rightarrow E_F = \frac{E_C + E_V}{2}
 \end{aligned}$$

So Fermi level lies exactly in the middle of the top valence band and the bottom of the conduction band.

Q. 8. (c) Hall voltage of 0.001 mV is found to be developed when a sample carrying a current of 100 mA is placed in a transverse magnetic field of 2T. Calculate the charge carrier concentration of the sample, given the thickness of the sample along the direction of magnetic field is 0.05 mm. (2.5)

Ans. Hall coefficient

$$R_H = \frac{V_H d}{I_x B}$$

Also,

$$R_H = \frac{1}{ne}$$

$$n = \frac{I_x B}{V_H d e}$$

Given that: $V_H = 0.001 \times 10^{-3} \text{ V}$; $d = 0.05 \times 10^{-3} \text{ m}$; $e = 1.6 \times 10^{-19}$; $I_x = 100 \times 10^{-3} \text{ A}$

$$B = 2 \text{ T}$$

$$\begin{aligned}
 n &= \frac{100 \times 10^{-3} \times 2}{(0.001 \times 10^{-3}) \times (0.05 \times 10^{-3}) \times 1.6 \times 10^{-19}} \\
 &= 25 \times 10^{27} \text{ m}^{-3}
 \end{aligned}$$

$$n = 2.5 \times 10^{28} \text{ m}^{-3}$$

Q. 1. (a) What is the effect of periodic potential of the energy of electrons in metal? Explain it on the basis of Kronig-Penny model and also explain the formation of energy bands. (8.5)

b. Free electron model ignore the effects those arise when the electrons interact with lattice. However, now we consider this by making some general remarks about effect of the periodic variation in the potential. Due to this periodicity in

The electron of not too high energy is practically bound within one of the wells that are deep and widely spaced. So the lower energy eigen value are those of a single well. However, for the wells those are closer together the eigen function can penetrate the potential barriers more easily. Because of this, spreading of previously single energy level into a band energy levels takes place. The band becomes wider with the decrease in the separation of the wells. Under the limit of zero barrier thickness, we obtain an infinitely wide single well in which all energies are allowed. So the present case is reduced to the free electron model. The comparison between the allowed energies of a single well and an array of wells (Kronig-penney model) is shown in the Fig. 9. In this figure, we have assumed $b = a/16$ and the well strength as $2m V_0 a^2/h^2$. It is clear from the figure that each band corresponds to a single energy level of the single well. The forbidden bands appear even for energies $E > V_0$.

Here we will solve the Schrodinger wave equation for electron for Kronig-Penny potential under the condition that ψ and $\frac{d\psi}{dx}$ are continuous at the boundaries of the wells. A complicated expression for the allowed energies in terms of k shows that gaps in energy are obtained at values such that

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$$

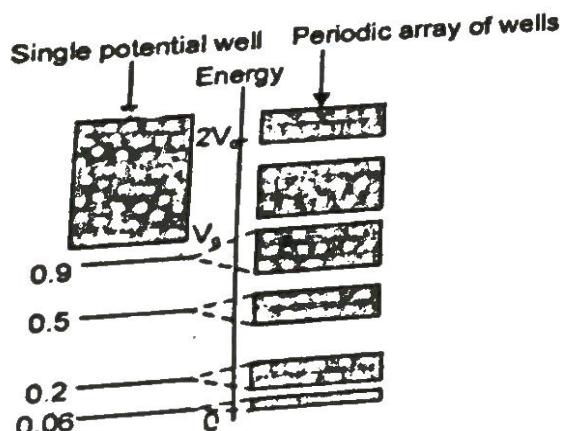


Fig. 9

The solution of the Schrödinger wave equation for free-electrons results in the energy levels given by

$$E = \frac{h^2 k^2}{8\pi^2 m} = \frac{h^2 k^2}{2m} \quad \dots(vi)$$

Energy E Versus wave Number k Diagram

From Eq. (v), it is clear the relation between E and k is parabolic. The parabolic nature between E and k , valid in case of free electrons, is therefore, interrupted at certain values of k , as shown in Fig. 10. It means the energies corresponding to the values given by Eq. (iv) are not permitted for electrons in the crystal. Thus, the energies of the bands are divided into forbidden and allowed bands (Fig. 9). The occurrence of the gaps can be understood on the basis of Bragg's condition for reflection, given as

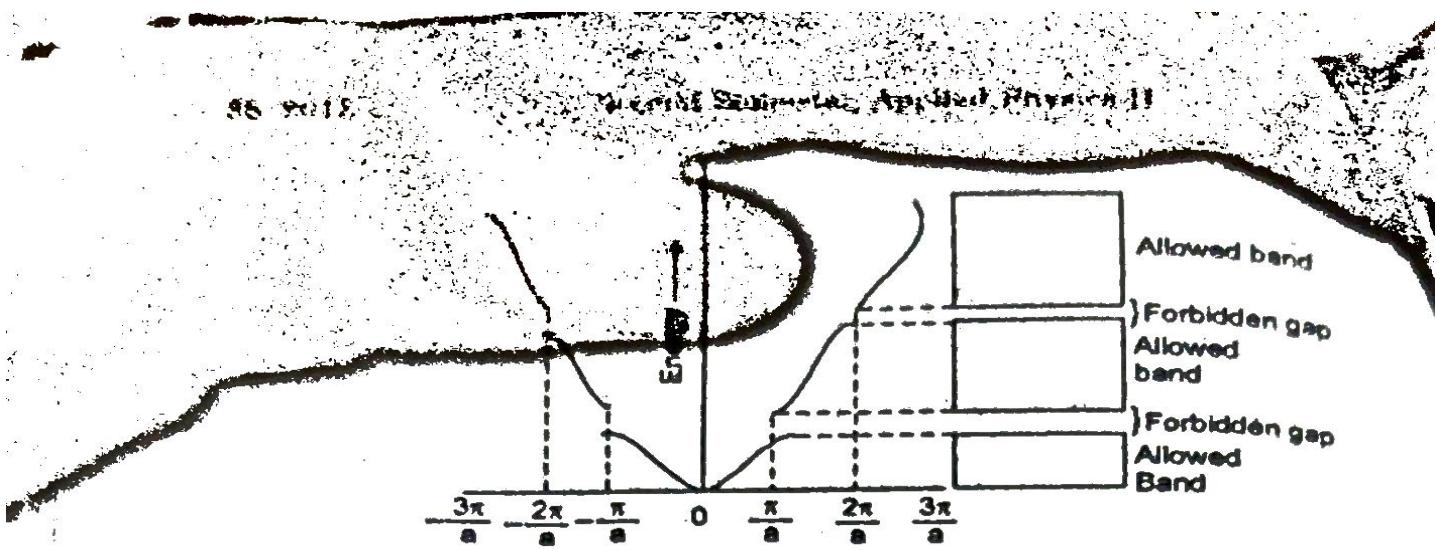


Fig. 10

$$2a \sin \theta = n\lambda \quad (\text{where, } n = 1, 2, 3, \dots)$$

Where a is spacing between the ions of the lattice and θ is the angle of incidence
 \therefore

$$2a = n\lambda \quad (\text{for } \theta = 90^\circ)$$

or

$$2a = n \frac{2\pi}{k}$$

or

$$k = \frac{n\pi}{a}$$

or

$$k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$$

We have put \pm signs because the incident wave can travel along $+x$ -axis as well as along $-x$ -axis. At all these values of k the gaps in energy occur, as shown in fig.

The waves corresponding to value of k satisfying the Bragg's condition are reflected and resulted in standing waves. On each subsequent Bragg reflection, the direction in which the wave is travelling is reversed again. The eigen function and corresponding reflected waves for $k = \pm n \frac{\pi}{a}$ are therefore $e^{i(\pi/a)x}$ and $e^{-i(\pi/a)x}$. These two eigen functions can be combined in two different ways to give total eigen function

$$\omega_1 = e^{i(\pi/a)x} + e^{-i(\pi/a)x} = \cos(\pi/a)x$$

$$\omega_2 = e^{i(\pi/a)x} - e^{-i(\pi/a)x} = \sin(\pi/a)x$$

or

$$\psi_1 \propto \cos \frac{\pi}{a} x$$

and

$$\psi_2 \propto \sin \frac{\pi}{a} x$$

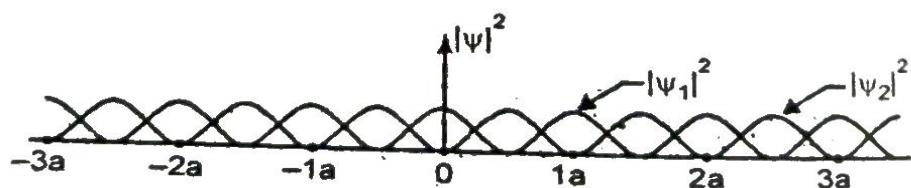


Fig. 11

Hence, the two standing waves are obtained. The probability density curve for these two stationary waves, i.e. $|\psi_1|^2$ and $|\psi_2|^2$, are shown in fig. 11. From this figure and Eq. 8 it is clear that the value of $|\psi_1|^2$, is maximum in between the positive of positive ions and minimum at the positions of the ions. So an electron can have two different value of energies, i.e. E_1 and E_2 for $k = \frac{\pi}{a}$ corresponding to the standing waves ψ_1 and ψ_2 .

Hence, no electron can have any energy between E_1 and E_2 . This phenomenon creates difference in energy ($E_1 - E_2$) which is known as energy gap.

Q. 9. (b) Show that the Hall coefficient is independent of the applied magnetic field and is inversely proportional to the current density and electronic charge. (4)

Ans. Consider a specimen in the form of a rectangular cross-section carrying current in the x direction. A Uniform magnetic field B_z is applied along z -axis as shown in Fig. 12.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction.

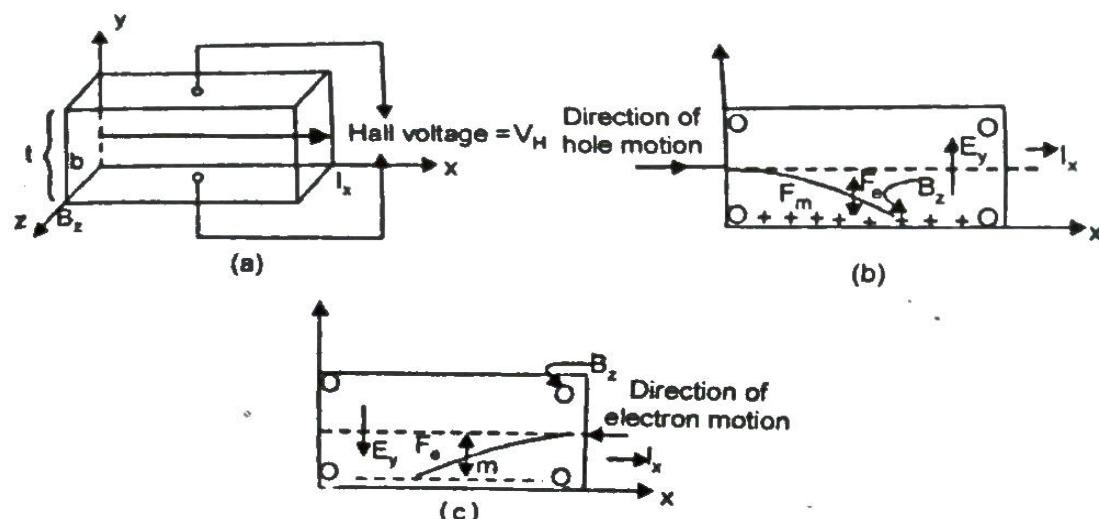


Fig. 12

Fig. 12. (a) Illustration of Hall effect (b) motion of hole in p-type semiconductor specimen (c) motion of electron of N-type semiconductor or metal specimen of Hall experiment.

Force $ev_x B_z$ [Fig. 12. (a) and (c)] This shift of charges produces a potential difference an electric field E_y along y direction. The produced electric field also applies a force eE_y on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium,

$$\begin{aligned} F_e &= F_m \\ eE_y &= ev_x B_z \\ E_y &= v_x B_z \end{aligned} \quad \dots(1)$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x$$

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Here n is density of charge carrier (electron or hole or both) and A (thickness) is the cross-sectional area of specimen perpendicular to direction of J_x .
From Eqs. (1) and (2), we can write

$$E_y = \frac{B_z J_x}{ne}$$

This is the expression for Hallfield. The Hall effect is described by means of coefficient R_H defined in terms of J_x by the following relation :

$$R_H = \frac{E_y}{J_x B_z}$$

From Eqs. (3) and (4) we have

$$R_H = \frac{1}{ne}$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{ne}$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = -\frac{1}{n_h e}$$

Here n_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

MID TERM EXAMINATION [FEB. 2019]
SECOND SEMESTER [B.TECH]
APPLIED PHYSICS-II [ETPH-104]

Time : 1.30 hrs.

M.M. : 30

Note : Attempt any three questions including Q 1. which is compulsory.

Q.1. (a) For position vector \vec{r} prove that $\nabla r^n = nr^{n-1} \hat{r}$.

Ans. Solving for ∇r^n

Where,

$$\vec{r} = xi + yj + zk$$

$$\nabla r^n = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (x^2 + y^2 + z^2)^{n/2}$$

as

$$r = (x^2 + y^2 + z^2)^{1/2}$$

$$\begin{aligned} \nabla r^n &= \left[\frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2x \right] \hat{i} + \left[\frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2y \right] \hat{j} \\ &\quad + \left[\frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} 2z \right] \hat{k} \\ &= n (x^2 + y^2 + z^2)^{\frac{n}{2}-1} (xi + yj + zk) \\ &= nr^{n-2} \vec{r} \\ &= nr^{n-2} \hat{r} \cdot r \quad \left(\text{as } \hat{r} = \frac{\vec{r}}{r} \right) \end{aligned}$$

$$\nabla r^n = nr^{n-1} \hat{r} \quad \text{proved}$$

Q.1. (c) A radio transmitter operates at a frequency of 880 kHz and power of 10 kW. How many photons per second will it emit?

(2)

Ans. Given,

$$\gamma = 880 \text{ KHz} = 880 \times 10^3 \text{ Hz}$$

$$P = 10 \text{ KW} = 10^4 \frac{\text{J}}{\text{S}}$$

We know that,

$$E = h\gamma$$

$$E = 6.62 \times 10^{-34} \times 880 \times 10^3$$

$$E = 5.83 \times 10^{-28}$$

Photons emitted per second will be

$$n = \frac{P}{E}$$

$$n = \frac{10^4}{5.83 \times 10^{-28}} = 0.1716 \times 10^{32}$$

$n = 1.716 \times 10^{31}$

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Q.1. (b) Define Poynting vector and explain its significance.

Ans. Refer Q.2. (a) First Term Examination 2018. (page 3-2018)

Q.1. (d) An electron and a proton have the same de-Broglie wavelength. Show that the energy of the electron is greater.

Ans. de-Broglie wavelength is given by-

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For electron

$$\lambda_e = \frac{h}{\sqrt{2m_e E_e}}$$

For proton

$$\lambda_p = \frac{h}{\sqrt{2m_p E_p}}$$

But

$$\lambda_e = \lambda_p \frac{h}{\sqrt{2m_e E_e}} = \frac{h}{\sqrt{2m_p E_p}}$$

$$m_p E_p = m_e E_e$$

$$\frac{E_e}{E_p} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1 \times 10^{-31} \text{ kg}} = 1837$$

$$E_e = 1837 E_p$$

$$E_e > E_p$$

Q.1. (e) Establish a relation between group velocity and phase velocity for non-relativistic free particle.

Ans. According to de-Broglie theorem

$$\lambda = \frac{h}{mv_g}$$

Total Energy is-

$$E = \frac{1}{2}mv_g^2$$

Also,

$$E = h\gamma$$

$$\gamma = \frac{E}{h} = \frac{mv_g^2}{2h}$$

Phase velocity is-

$$v_p = \gamma\lambda$$

Put equation (1) and (2) in equation (3)-

$$v_p = \frac{h}{mv_g} \times \frac{mv_g^2}{2h}$$

$$v_p = \frac{v_g}{2}$$

Hence, for a non-relativistic free particle, the phase velocity is half of the group velocity.

Q.2. (a) Show that equation of continuity is a consequence of Maxwell's equations.

Ans. An equation, which expresses the equality of incoming and outgoing charges in a system and follows the law of conservation of charge, is known as the equation of continuity.

The current density \bar{J} and the charge density ρ are related at each point through a differential equation. This relation is based on the fact that electric charge can neither be created nor be destroyed and rate of increase of the total charge inside any arbitrary volume must be equal to the net flow of charge into this volume.

$$I = \int_S \bar{J} \cdot d\bar{S} \quad \dots(1)$$

Again, considering charge leaving a volume V per second

$$I = -\frac{\partial}{\partial t} \int_V \rho dV \quad \dots(2)$$

The negative sign comes here because the current is positive when the net charge is from the outside of V to within.

Since, we are dealing with a fixed volume V , hence we may write,

$$-\frac{\partial}{\partial t} \int_V \rho dV = -\int_V \frac{\partial \rho}{\partial t} dV \quad \dots(3)$$

Again, using divergence theorem, we can write

$$\int_S \bar{J} \cdot d\bar{S} = -\int_V (\bar{\nabla} \cdot \bar{J}) dV \quad \dots(4)$$

Therefore, from Eqs. (2), (3) and (4), we have

$$I = \int_V (\bar{\nabla} \cdot \bar{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV \quad \dots(5)$$

$$\int_V (\bar{\nabla} \cdot \bar{J}) dV = -\int_V \frac{\partial \rho}{\partial t} dV$$

$$\int_V \left\{ (\bar{\nabla} \cdot \bar{J}) + \frac{\partial \rho}{\partial t} \right\} dV = 0 \quad \dots(6)$$

This integral must be zero for any arbitrary volume. It is only possible when integrand is zero, i.e.,

$$\bar{\nabla} \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0 \quad \dots(7)$$

This differential equation is known as the equation of continuity. If the region does not contain a source or sink of current, $\frac{\partial \rho}{\partial t} = 0$ and hence for steady current, we have

$$\bar{\nabla} \cdot \bar{J} = 0$$

Q.2. (b) Calculate the penetration depth for 2 MHz electromagnetic wave in copper if $\sigma = 5.8 \times 10^7 \text{ S/m}$.

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Ans. Given

$$f = 2 \text{ MHz} = 2 \times 10^6 \text{ Hz}$$

$$\text{Also, } \omega = 2\pi f, \mu = \mu_0 = 4\pi \times 10^{-7}$$

$$\sigma = 5.8 \times 10^7 \text{ S/m}$$

Penetration depth is given by-

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} = \sqrt{\frac{2}{4\pi \times 10^{-7} \times 5.8 \times 10^7 \times 2\pi \times 2 \times 10^6}}$$

$$\delta = 46.7 \mu\text{m}$$

Q.2. (c) Show that the velocity of e.m. waves in isotropic medium is always less than their velocity in free space.

Ans. Refer to Q.3. (a) First Term Examination 2018. (Pg. No. - 5 - 2018)

Q.3. (a) Derive plank's radiation law and show that Rayleigh's Jeans Law is a special case of Plank's radiation law.

Ans. Refer to Q.4. (b) End Term Examination 2019.

Q.3. (b) Plot Fermi Dirac distribution function $F(E)$ as a function of energy for the temperatures $T = 0\text{K}$ and $T >> 0\text{K}$.

Ans. Refer to Q.1. (g) End Term Examination 2018. (Pg. No. - 12 - 2018)

Q.3. (c) What is displacement current? Mention the cause responsible for its production.

Ans. Displacement Current: Set up in a dielectric medium due to changing electric field across the dielectric which leads to variation of induced displacement of charge.

$$J_d = \frac{\partial \bar{D}}{\partial t} = \frac{\epsilon \partial \bar{E}}{\partial t}$$

For constant

$$E \Rightarrow J_d = 0$$

Q.4. (a) How does Heisenberg's uncertainty principle help to decide whether electron exists in the nucleus or not?

Ans. According to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

The diameter of the nucleus is of the order of 10^{-14} m . If the electron exists in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore, the minimum uncertainty Δx in the position of electron is the same as the diameter of the nucleus, i.e.

$$\Delta x = 10^{-14}\text{ m}$$

∴ The minimum uncertainty in the momentum is given by

$$\Delta p_x = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-14}} = \frac{6.63 \times 10^{-20}}{2\pi}$$

$$= 1.055 \times 10^{-20} \text{ kg m/sec.}$$

It means that if electron exists in the nucleus, its minimum momentum

$$p_{\min} = 1.055 \times 10^{-20} \text{ kg m/s}$$

for the electron of the minimum momentum, the minimum energy is given by

$$\begin{aligned} E_{\min}^2 &= p_{\min}^2 c^2 + m_0^2 c^4 \\ &= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4 \\ &= (3 \times 10^8)^2 [1.113 \times 10^{-40} + 7.4692 \times 10^{-44}] \end{aligned}$$

Since the second term in the bracket is much smaller than the first, it can be neglected, then,

$$\begin{aligned} E_{\min} &= 3 \times 10^8 \sqrt{1.113 \times 10^{-40}} \text{ J} \\ &= 3 \times 10^8 \times 1.055 \times 10^{-20} \text{ J} = 3.1649 \times 10^{-12} \text{ J} \end{aligned}$$

or

$$E_{\min} = \frac{3.1649 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \approx 20 \text{ MeV.}$$

Thus, if a free electron exists in the nucleus it must have a minimum energy of about 20 MeV.

The maximum K.E. which a β -particle, emitted from radioactive nuclei is of the order of 4 MeV. Therefore, this clearly shows that electrons can not be present within the nucleus.

Q.4. (b) Derive time dependent one-dimensional Schrodinger wave equation for a non-relativistic free particle. Also explain the significance of wave function.

Ans. Schrodinger's Time dependent Wave Equation

(7)

Let us assume that ψ for particle moving freely in positive x -direction is

$$\psi = A e^{-i\omega(t-\frac{x}{v})} \quad \dots(1)$$

$$\omega = 2\pi\nu, \quad v = \nu\lambda \quad \dots(2)$$

$$\psi = A e^{-2\pi i(\nu t - \frac{x}{\lambda})}$$

As

$$E = h\nu = 2\pi\hbar\nu \quad \text{and} \quad \lambda = \frac{\hbar}{p} = \frac{2\pi\hbar}{p}$$

\therefore For a free particle wave equation becomes

$$\psi = A e^{-\frac{i}{\hbar}(Et - px)} \quad \dots(3)$$

But

$$E\psi = i\hbar \frac{\partial\psi}{\partial t} \quad \text{and} \quad p\psi = \frac{\hbar}{i} \frac{\partial\psi}{\partial x} \quad \dots(a)$$

As total energy, $E = \text{Kinetic energy (K)} + \text{Potential energy (V)}$

(4)

$$\text{K.E.} = \frac{p^2}{2m}$$

Now,

\therefore Equation (4) in terms of wave function ψ can be written as

$$E\psi = \frac{p^2}{2m}\psi + V\psi$$

Putting the values of $E\psi$ and $p\psi$ from Eq. (4) in Eq. (5), we have

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 \frac{1}{2m} \psi + V\psi$$

or

$$i\hbar \frac{\partial \psi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$
(6)

Equation (6) is Schrodinger's time dependent wave equation in one-dimension.
For non-relativistic free particle, $V = 0$

\therefore Schrodinger time dependent wave equation will be—

$$i\hbar \frac{\partial \psi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

Physical Significance of wave equation- The Schrodinger wave equation provides a way to calculate the wave function of a system and how it changes dynamically on time. Schrodinger time dependent equation describe the energies of the particle. It is used to find the allowed energy levels of quantum mechanical systems. The associated wave function gives the probability of finding the particle at a certain position.

END TERM EXAMINATION [MAY 2019]

SECOND SEMESTER [B.TECH]

APPLIED PHYSICS-II [ETPH-104]

Time : 3 hrs.

M.M. : 75

Note : Attempt any five questions in all including question Q 1. which is compulsory. Select one question from each unit.

Q.1. Attempt each one of the following.

[$2.5 \times 10 = 25$]

Q.1. (a) State Gauss's law in magnetostatics. Write its integral form and mention its physical interpretation.

Ans. The net outward magnetic flux through any closed surface is zero. In integral form.

$$\int \vec{B} \cdot d\vec{s} = 0$$

where \vec{B} is the magnetic induction and measured in weber/m².

This equation signifies that "the outward flux of magnetic induction \vec{B} through any closed surface is equal to zero". There exists no magnetic monopoles.

Q.1. (b) Using exchange symmetry of the wave function show that Bosons do not obey Pauli's exclusion principle.

Ans. Bosons have integral spin angular momenta as multiple of \hbar , i.e., they have integral spin $0, \hbar, 2\hbar, 3\hbar$. In this statistics, all quantum states appear with equal probability and each quantum state can have one or more particles in it. The wave function of bosons are symmetric, i.e., interchange of any pair of particles among its arguments, leaves the wave function unchanged.

$$\hat{P}_{1,2}\psi_s(1, 2) = +\psi_s(1, 2)$$

Here, pauli exclusion principle is not obeyed as there can be more than one boson in one quantum stat. Example is photons, α -particles.

Q.1. (c) Calculate the phase velocity and group velocity of the de Broglie waves of an electron whose speed is 0.9c.

Ans. We know that-

and given that -

or,

and

$$V_p \cdot V_g = c^2$$

$$V_g = 0.9c$$

$$V_g = 2.7 \times 10^8 \text{ m/s}$$

$$V_p = \frac{c^2}{V_g}$$

$$V_p = \frac{3 \times 10^8 \times 3 \times 10^8}{0.9 \times 3 \times 10^8}$$

$$V_p = 3.33 \times 10^8 \text{ m/s}$$

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Q.1. (d) What type of statistics shall be applicable for a gas of photons? Justify your answer.

Ans. Bose-Einstein statistics shall be applicable for a gas of photons. Bose-Einstein statistics is obeyed by indistinguishable particles of integral spin quantum number that have symmetric wave function and does not obey Pauli exclusion principle.

Q.1. (e) Calculate the packing fraction for chromium metal having BCC structure. If its density is 5.96 gm/cc and atomic weight = 50.

Ans. We know that –

$$a = \left(\frac{nM}{N\rho} \right)^{1/3}$$

In BCC, number of atoms per unit cell $\Rightarrow n = 2$

Given, $\rho = 5.96 \text{ gm/cc}$; $M = 50$ and $N = 6.02 \times 10^{23}$

$$a = \left(\frac{2 \times 50}{6.02 \times 10^{23} \times 5.96} \right)^{1/3}$$

$$= 3.032 \times 10^{-8} \text{ cm}$$

Also,

$$r = \frac{a\sqrt{3}}{4} = 1.31 \times 10^{-8} \text{ cm}$$

Packing fraction

$$= \frac{2 \times \frac{4}{3} \pi r^3}{a^3} = \frac{8 \times 3.14 \times (1.31 \times 10^{-8})^3}{3 \times (3.032 \times 10^{-8})^3}$$

$$= \frac{56.47}{27.87 \times 3} = 0.6852 = 0.68$$

Packing fraction is 68%.

Q.1. (f) Mark the Fermi level for an intrinsic semiconductor, n type semiconductor and p type semiconductor.

Ans. In an intrinsic semiconductor the number of electrons is equal to number of holes ($n_e = p_h$). But in n-type extrinsic semiconductor number of electrons are increased due to doping of pentavalent atom and number of holes are decreased than which would be available for intrinsic semiconductor. The number of holes are decreased because of the large number of electrons present which increase the rate of recombination of electrons and holes. With similar argument it can be shown that in P-type extrinsic semiconductor the number of holes are increased due to doping of trivalent impurity atom ($n_h > n_e$) and number of electrons decreased ($n_e < n_h$) than which would be available for intrinsic semiconductor. Now since the Fermi level E_F is a measure of the probability of occupancy of the allowed energy states by the electron, so for an intrinsic semiconductor when $n_e = p_h$, Fermi level is at the centre of the forbidden gap [Fig. 1.(a)]. But for N-type semiconductor, since $n_e > n_h$, it is clear that E_F must move closer to conduction band [Fig. 1. (b)] to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band. Similarly, for P-type semiconductor since $n_h > n_e$ so E_F must move from the centre of the forbidden gap closer to the valence band [Fig. 1. (c)]

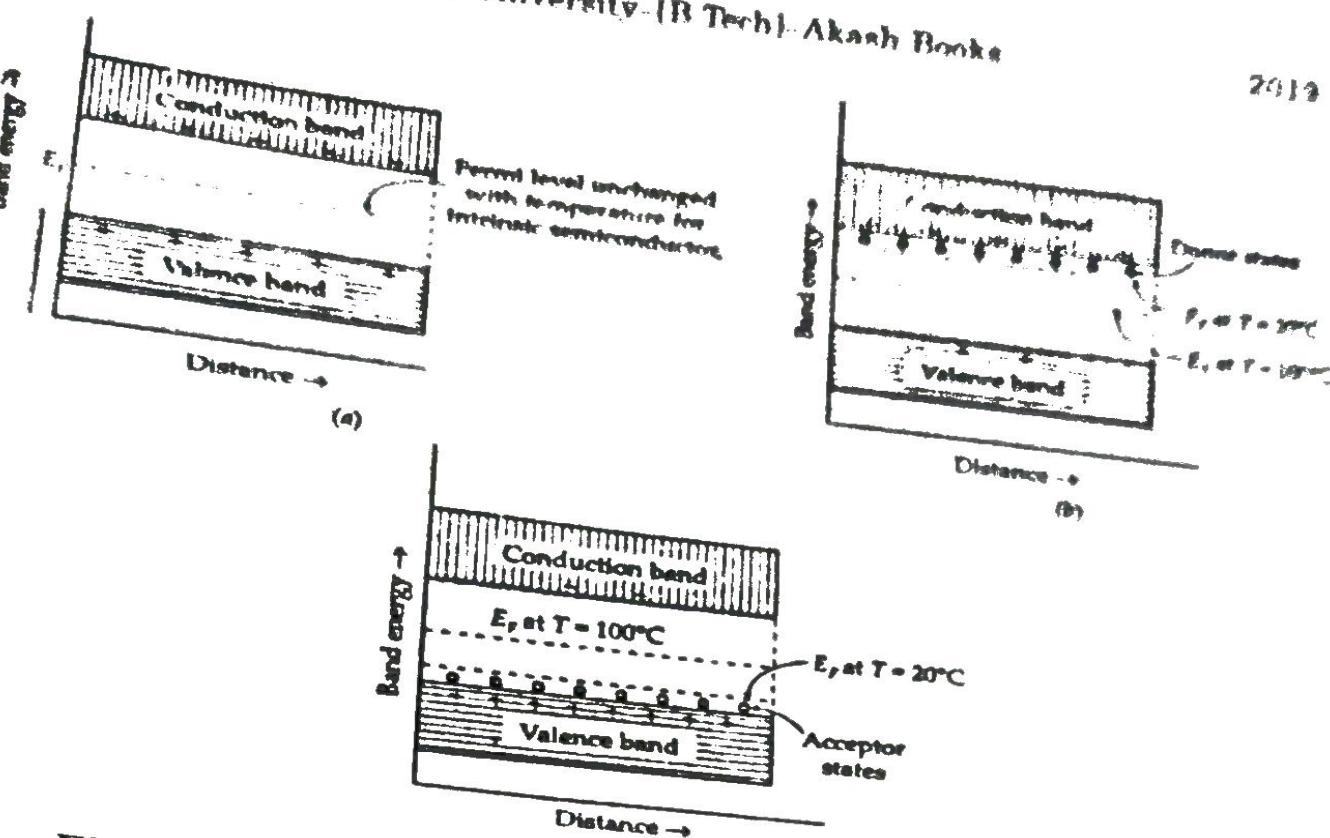


Fig. 1. Position of Fermi level (E_F) for (a) intrinsic semiconductor; (b) N-type semiconductor; (c) P-type semiconductor.

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor Fermi level is independent of temperature [Fig. 1.(a)].

But in an extrinsic semiconductor it is different. Say for n-type material the electrons coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature increased. So proportion of the total number of conduction electrons will increase from valence band. Thus as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre the forbidden gap is reduced.

Similarly, for P-type semiconductor as temperature increases the material also becomes increasingly intrinsic and the Fermi-level rises until it approaches the centre of the gap as in the intrinsic semiconductor [Fig. 1(c)]. Thus both n-type and p-type materials become more and more like intrinsic material at high temperature. This is a limit on the operating temperature of an extrinsic semiconductor device.

**Q1 (g) Deduce the Miller indices for the plane having intercepts a, b and c
Ans. -2. Also draw the plane**

Ans. From the law of rational indices, we have

$$-2a : \alpha b : -2c = \frac{a}{h} : \frac{b}{k} : \frac{c}{l}$$

Hence, the plane is $(\bar{1}0\bar{1})$

Q.1. (h) Determine the constant "a" so that the vector
 $\vec{A} = (x + 3y)\hat{i} + (2y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoid.

Ans. For solenoid $\nabla \cdot \vec{A} = 0$

$$= \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) ((x + 3y)\hat{i} + (2y - 2z)\hat{j} + (x + az)\hat{k})$$

$$= 1 + 2 + a = 0$$

$$\therefore a = -3$$

Q.1. (i) A proton and a deuteron have same kinetic energy. Which of the two has longer de-Broglie Wave Length?

Ans. Given-

$$E_p = E_d$$

de-Broglie wave for proton is-

$$\lambda_p = \frac{h}{\sqrt{2m_p E_p}}$$

de-Broglie wave for deuteron is-

$$\lambda_d = \frac{h}{\sqrt{2m_d E_d}}$$

$$\frac{\lambda_p}{\lambda_d} = \frac{\frac{h}{\sqrt{2m_p E_p}}}{\frac{h}{\sqrt{2m_d E_d}}} = \sqrt{\frac{m_d}{m_p}}$$

But mass of proton $= m_p = 1.67 \times 10^{-27}$ Kg

and mass of deuteron $= m_d = 3.34 \times 10^{-27}$ Kg

$$\therefore \frac{\lambda_p}{\lambda_d} = \sqrt{\frac{3.34 \times 10^{-27}}{1.67 \times 10^{-27}}} = \sqrt{2} = 1.414$$

$$\lambda_p = 1.414 \lambda_d \Rightarrow \lambda_p > \lambda_d$$

\therefore Proton has longer de-Broglie wavelength.

Q.1. (i) Give the physical significance of wave function. List two important conditions that a wave functions must obey.

Ans. In classical mechanics, the square of a wave amplitude associated with electromagnetic radiation is interpreted as a measure of radiation intensity. This suggests that we should make a similar interpretation for de Broglie waves associated with electrons or any particle. Hence, if we consider a system of electrons, and if ψ is the wave function associated with the system, then $|\psi|^2$ may be regarded as a measure of density of electrons. Also, if τ is a volume inside which, an electron is known to be present as shown in fig. 1, but where exactly the electron is situated inside volume τ is not known, and if ψ is the wave function associated with the electron, then the probability of finding the electron in a certain element of volume $d\tau'$ is given equal to the $|\psi|^2$ is called the probability function. This interpretation was first given by Max Born in 1926.

Since the electron must be somewhere inside the volume τ , the integration of $|\psi|^2$ over the whole volume τ must be unity, so that

$$\int |\psi|^2 d\tau = 1$$

in quantum mechanics it is postulated that the state of a system is completely specified by a wave function.

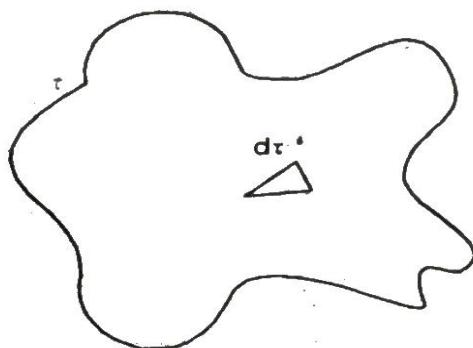


Fig. 1: Presence of electron in a certain region

Properties of Wave Function are—

- $\psi(x)$ must be single valued, finite and continuous for all values of x .
- $\frac{d\psi(x)}{dt}$ must be finite and continuous for all values of x , except at those point where $V \rightarrow \infty$. At these points $\frac{d\psi(x)}{dt}$ has a finite discontinuity but ψ remains continuous.
- For bound state and probability of finding the particle between x and $(x + dx)$, i.e., $|\psi|^2 dx$ must vanish as $|x| \rightarrow \infty$. Hence $|\psi(x)| \rightarrow 0$ as $|x| \rightarrow \infty$ i.e., $\psi(x)$ is a square integrable wave function.

Examples of not acceptable wave functions:

- (i) $\psi = \frac{1}{x}$ as $x \rightarrow 0$, $\psi \rightarrow \infty$
- (ii) $\psi = x^n$ as $a \rightarrow \pm \infty$, $\psi \rightarrow \pm \infty$

Examples of acceptable wave functions:

- (i) $\psi = \frac{1}{(1+x^2)}$ as $x \rightarrow \infty$, $\psi \rightarrow 0$
- (ii) $\psi = \sin x$ as ψ oscillates between +1 and -1.

UNIT-I

Q.2. (a) Set up the electromagnetic wave equation using Maxwell's equation in an isotropic dielectric medium and show that the speed of the wave is less than the speed of the wave in vacuum?

Ans. Refer Q.3. (a) of First Term Examination 2018. (Page 16 - 2018)

Q.2. (b) State Poynting theorem and why is it also called energy conservation law in electromagnetism?

Ans. $\vec{S} = \vec{E} \times \vec{H}$, is known as Poynting vector and is interpreted as the power flux i.e., amount of energy crossing unit area placed perpendicular to the vector, per unit time. The conception of energy of the electromagnetic field as residing in the medium is very fundamental one and has great advantage in the development of the theory. Maxwell thought of the medium as resembling as elastic solid the electrical energy representing the potential energy of strain of the medium, the magnetic energy the kinetic energy of motion. Though such a mechanical view no longer exists, still the energy is regarded as being localised in space and as travelling in the manner indicated by poynting vector. In a light wave there is certain energy per unit volume, proportional to the square of the amplitude (E or H). This energy travels along it and poynting vector is the vector that measures the rate of flow or the intensity of the wave. In a plane electromagnetic wave E and H are at right angle to each other and at right angle to the direction of flow. thus

$\vec{E} \times \vec{H}$ must be along the direction of flow. In more complicated waves as well, poynting vector point along the directions of flow of radiation. For example, if we have a source of light and we wish to find at what rate it is emitting energy, we surround it by a close surface and integrate the normal component of poynting vector over the surface. The whole conception of energy being transported in the medium is fundamental to the electromagnetic theory of light.

"Poynting theorem states that the net power flowing out of a given volume V is equal to the time rate of decrease in energy stored within V minus the ohmic losses."

i.e

$$\int_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = - \frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] dV - \int_V \sigma E^2 dV$$

Total power leaving the volume = Rate of decrease in energy stored in electric and magnetic field - Ohmin power dissipated

Q.2. (c) Determine the conduction and displacement current densities in material having conductivity of 10^{-4} S/m and relative permittivity $\epsilon_r = 2.25$. The electric field in the material is $E = 5 \times 10^{-6} \sin(9 \times 10^9 t)$ V/m.

Ans. We know that,

$$J_c = \sigma E$$

Where,

$$\sigma = 10^{-4} \text{ S/m} \text{ and } E = 5 \times 10^{-6} \sin(9 \times 10^9 t) \text{ V/m}$$

∴

$$J_c = 10^{-4} \times 5 \times 10^{-6} \times \sin(9 \times 10^9 t) \frac{A}{m^2}$$

$$\sigma = 10^{-4} \text{ S/m} \text{ and } E = 5 \times 10^{-6} \sin(9 \times 10^9 t) \text{ V/m}$$

Electric displacement vector = $\vec{D} = \epsilon \cdot \vec{E}$ or. $\vec{D} = \epsilon_r \vec{E}$

Also,

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

Displacement current density = $\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \epsilon_r \frac{\partial \vec{E}}{\partial t}$

Given, $\epsilon_r = 2.25$ and $\epsilon_0 = 8.854 \times 10^{-12}$

$$\begin{aligned}\vec{J}_d &= 8.85 \times 10^{-12} \times 2.25 \times \frac{\partial}{\partial t} [5 \times 10^{-6} \times \sin(9 \times 10^9 t)] \\ &= 19.92 \times 10^{-12} \times 5 \times 10^{-6} \times 9 \times 10^9 \cos(9 \times 10^9 t) \\ &= 896.4 \times 10^{-9} \cos(9 \times 10^9 t)\end{aligned}$$

$$\boxed{\vec{J}_d = 8.964 \times 10^{-7} \cos(9 \times 10^9 t) \frac{A}{m^2}}$$

Q.3. (a) What is the field due to a uniformly charged spherical shell of radius R, carrying a charge Q, at a distance r from the centre of the sphere? (5)

Ans. Electric field due to uniformly charged Sphere at a distance "r" from the centre is solved below.

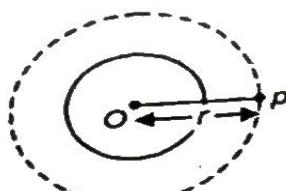


Fig.2

We consider a Gaussian surface, an imaginary sphere of radius r concentric with the given sphere. The flux, ϕ_E across the imaginary sphere is:-

$$\begin{aligned}\phi_E &= \int \vec{E} \cdot d\vec{s} = \int E ds \cos 0^\circ \\ &= E \int ds = E(4\pi r^2)\end{aligned}$$

The charge enclosed by the imaginary sphere is the total charge Q on the given sphere. Using Gauss's Law, we obtain.

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi r^2 \epsilon_0}}$$

This is the field at a distance r from the centre of the uniformly charged sphere.

Q.1 (b) Show that Ampere's law is inconsistent in time varying fields and deduce the modified form of Ampere's law. (5)

Ans. Refer To Q.1. (c) First Term Examination 2018. (Page 1-2018)

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Q.3. (a) Calculate the penetration depth for 2MHz electromagnetic wave through copper for conductivity $5.8 \times 10^7 \text{ S/m}$ and permeability $4\pi \times 10^{-7} \text{ Vs/A}$

Ans. Penetration depth is given by

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}}$$

$$\mu = 4\pi \times 10^{-7}; \sigma = 5.8 \times 10^7$$

$$\omega = 2\pi f = 2 \times 3.14 \times 2 \times 10^6 = 12.56 \times 10^6$$

Given-

$$\delta = \sqrt{\frac{2}{4 \times 3.14 \times 10^{-7} \times 5.8 \times 10^7 \times 12.56 \times 10^6}}$$

$$= \sqrt{\frac{2}{914.97 \times 10^6}} = \sqrt{0.00218} \times 10^{-3}$$

$$= 0.04669 \times 10^{-3}$$

$$\delta = 46.69 \times 10^{-6} \text{ m}$$

$$\boxed{\delta = 46.65 \mu\text{m}}$$

or,

UNIT-II

Q.4. (a) Show that Bose-Einstein and Fermi Dirac statistics reduce to Maxwell Boltzmann statistics at high temperature.

Ans. The distribution laws of three statistics are given below

$$\frac{g_i}{n_i} = e^{\alpha E_i / k_B T} \quad \dots(1) [\text{For M-B}]$$

$$\frac{g_i}{n_i} = e^{\alpha E_i / K_B T} - 1 \quad \dots(2) [\text{For B-E}]$$

and

$$\frac{g_i}{n_i} = e^{\alpha E_i / K_B T} + 1 \quad \dots(3) [\text{For F-D}]$$

If $\frac{g_i}{n_i} \gg 1$ then $\frac{g_i}{n_i} \approx \left(\frac{g_i}{n_i} + 1 \right) \approx \left(\frac{g_i}{n_i} - 1 \right)$. In this limit both B.E. and F.D.

distributions are identical with M.B. distribution. This limit $\left(\frac{g_i}{n_i} \gg 1 \right)$ occurs when the temperature is not too low and pressure (or density) is not too high.

Q.4. (b) Derive Plank's radiation law of energy distribution in thermal spectrum.

Ans. The postulates of quantum theory of radiation are as follows-

(i) A radiation has energy. As light and heat are radiations, they are also associated with energy.

(i) Radiant energy is not emitted or observed continuously but discontinuously in the form of small packets called photons. Photon is not a material body but is considered to be a massless packet of energy.

(ii) The energy E of a photon is related to the frequency of radiation, γ , the two being related as $E = h\gamma$, where h is Planck's constant.

(iii) Whenever a body emits or absorbs energy it does so in whole number multiples of photons, i.e. $n\gamma$, where $n = 1, 2, 3, 4, \dots$

Planck's radiation formula-

Let dn be the number of photons distributed in frequency interval v to $(v + dv)$.

$$dn = n(v)dv = g(v)f(v)dv \quad \dots(1)$$

$$g(v)dv = \frac{8\pi h v^2}{c^3} dv \quad \dots(2)$$

and

$$f(v) = \frac{1}{(e^{hv/k_B T} - 1)} \quad [\text{as } \alpha = 0 \text{ and } E = hv] \quad \dots(3)$$

Putting these values in Eq.(1), we get

$$dn = \frac{8\pi h v^2}{c^3} \cdot \frac{1}{(e^{hv/k_B T} - 1)} dv \quad \dots(4)$$

Let dE be the energy distributed in frequency interval v and $(v + dv)$

$$dE = Edn = hv dn \quad \dots(5)$$

From Eqs. (4) and (5), we obtain

$$dE = E(v)dv = \frac{8\pi h v^3}{c^3} \cdot \frac{1}{(e^{hv/k_B T} - 1)} dv \quad \dots(6)$$

Equation (6) is known as *Planck's radiation formula* for the spectral energy density of blackbody radiation, which agrees with experimental curve (Fig.)

For low frequency or high wavelength

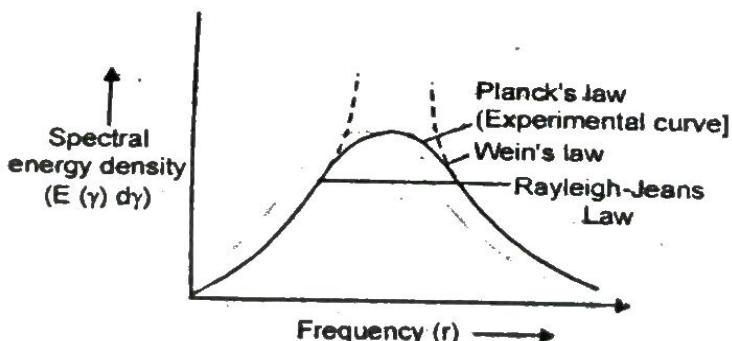


Fig.

$$(e^{hv/k_B T} - 1) = 1 + \frac{hv}{k_B T} - 1 = \frac{hv}{k_B T}$$

Equation (6) becomes as

$$dE = E(v) dv = \frac{8\pi v^2 k_B T}{c} dv$$

Equation (7) is termed as Rayleigh-Jeans law. This law is valid at low frequencies and high wavelengths. Experimental agreement is shown in Fig. 5.1.

Equation (6) can be expressed in terms of wavelength of radiation as follows:

$$E_\lambda d\lambda = \frac{8\pi c h}{\lambda^5} \left(\frac{1}{c(hc/\pi k_B T) - 1} \right) d\lambda$$

$$E_\lambda = \frac{8\pi hc(\lambda^{-5})}{\left[\exp\left(\frac{hc}{\lambda k_B T}\right) - 1 \right]}^1$$

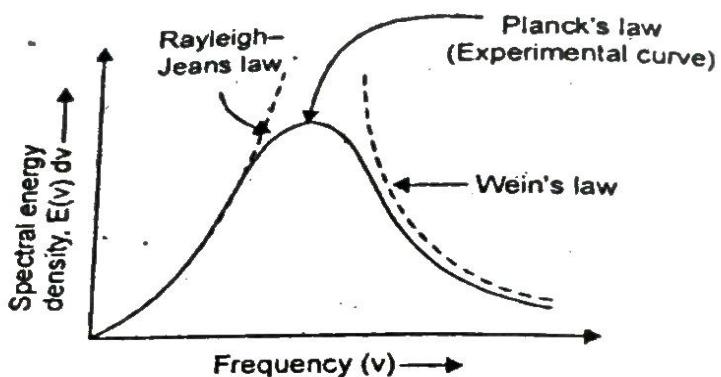


Fig. v vs $E(v) dv$ - curve

Let λ_{\max} be the wavelength whose energy density is the greatest.

$$\text{Then, } \frac{dE_\lambda}{d\lambda} = 0$$

$$\text{Hence, } \lambda_{\max} T = \frac{hc}{4.965 K} = 2.898 \times 10^{-3} mK$$

This is Wein's displacement law.

The peak in the black body spectrum shifts to progressively shorter wavelengths (higher frequencies) as temperature is increased.

Q.4. (c) A black body at $1373^\circ C$ has λ_m , the wavelength corresponding to the maximum emission equal to 1.78 micron. Find the temperature of the moon. λ_m for the moon is the 14 micron. Assume the moon to be a black body.

Ans. We know that,

$$\lambda_m T = C$$

For blackbody -

$$\lambda_{mb} T_b = C$$

For moon

$$\lambda_{mmoon} T_{moon} = C$$

Given,

$$\lambda_{mb} = 1.78 \times 10^{-6} m$$

$$T_b = 1373^\circ C$$

and
since,

$$\lambda_{mmoon} = 14 \times 10^{-6} \text{ m}$$

$$\lambda_{mb} T_b = \lambda_{mmoon} T_{moon}$$

$$T_{moon} = \frac{\lambda_{mb} T_b}{\lambda_{mmoon}} = \frac{1.78 \times 10^{-6} \times 1373}{14 \times 10^{-6}}$$

$$T_{moon} = 174.57^\circ C$$

Q.5. (a) Derive an expression for the energy eigenvalue of a particle of mass m confined to a 1-D box of length L . State whether the permitted energy levels are continuous or discrete.

Ans. Suppose a particle of mass m is in motion along the x -axis. Suppose no force acting on the particles so that the potential energy of the particle is constant. For convenience, the constant potential energy is taken to be zero i.e., $V=0$.

Hence Schrodinger equation becomes,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{h^2} E \psi = 0 \quad \dots(1)$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} E \psi = 0 \quad \dots(2)$$

Multiplying by $\frac{h^2}{8\pi^2 m}$ throughout, we get $8\pi^2 m$.

$$\frac{h^2}{8\pi^2 m} \cdot \frac{\partial^2 \psi}{\partial x^2} + E \psi = 0 \quad \text{or} \quad \frac{-h^2}{8\pi^2 m} \frac{\partial^2 \psi}{\partial x^2} = E \psi \quad \dots(3)$$

Since the particle is moving freely with zero potential energy its total energy E is kinetic energy, given by

$$E = \frac{p_x^2}{2m} \quad \dots(4)$$

where p_x is the momentum of the particle,

Rewriting Eq. (1),

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} E \psi = 0 \quad \text{or} \quad \frac{\partial^2 \Psi}{\partial x^2} K^2 \psi = 0 \quad \dots(5)$$

where

$$K^2 = \frac{8\pi^2 m E}{h^2}$$

So that,

$$E = \frac{K^2 h^2}{8\pi^2 m} \quad \dots(6)$$

In this case energy E is not quantised. Hence when a particle is not bound in a box it does not have quantised energy state.

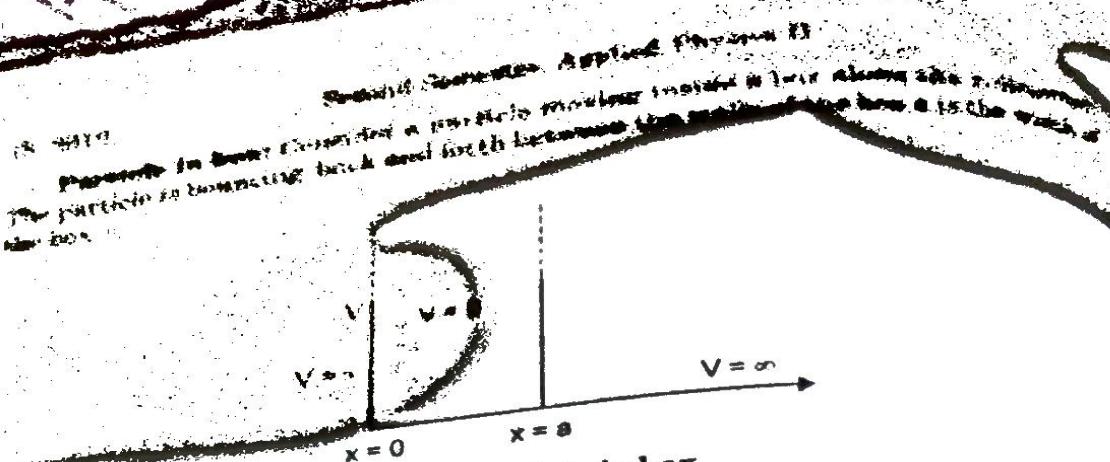


Fig. Particle in box

The potential energy V of the particle is infinite on both sides of the box.
 $V = 0$ for $0 < x < a$
 $V = \infty$ for $x \leq 0$ and $x \geq a$

The particle can not exist outside the box, so its wave function ψ is 0 for $x \leq 0$ and $x \geq a$.
 Within the box, the Schrodinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad [\because V is 0 for the free particle] \dots (7)$$

$$\frac{d^2\psi}{dx^2} + K^2 \psi = 0 \quad \text{where } K = \frac{\sqrt{2mE}}{\hbar} \dots (8)$$

The general solution of this equation is

$$\psi = A \sin Kx + B \cos Kx$$

Using boundary conditions

$$\psi = 0 \text{ at } x = 0$$

$$0 = A \sin 0 + B$$

$$B = 0$$

$$\psi = 0 \text{ at } x = a$$

$$0 = A \sin Ka$$

$$\sin n\pi = \sin Ka$$

$$K = \frac{n\pi}{a}$$

Wave function,

$$\psi_n(x) = A \sin \frac{n\pi x}{a} \quad \text{where } n = 1, 2, 3, \dots \quad (9)$$

Energy level,

$$E_n = \frac{K^2 \hbar^2}{2m} \left[\hbar = \frac{\hbar}{2\pi} \right]$$

\therefore Eigen value

$$E_n = \left(\frac{n\pi}{a} \right)^2 \left(\frac{\hbar}{2\pi} \right)^2 \frac{1}{2m}$$

$$E_n = \boxed{E_n = \frac{n^2 \hbar^2}{8ma^2}} \quad \text{where } n = 1, 2, 3, 4, \dots \quad (10)$$

Therefore, it is clear from Eq. (10) that inside an infinitely deep potential well (or a finite square well), the particle cannot have an arbitrary energy, but can have certain discrete energy corresponding to $n = 1, 2, 3, \dots$. Each permitted energy level is called *eigen-value* of the particle and constitutes the energy level of the system. The wave function ψ corresponding to each eigen value are called *eigen functions*.

Q.5. (b) Describe an experiment to confirm the wave nature of electrons. (4.5)

Ans. Davisson and germer experiment proves the wave nature of the electrons. The arrangement is described below.

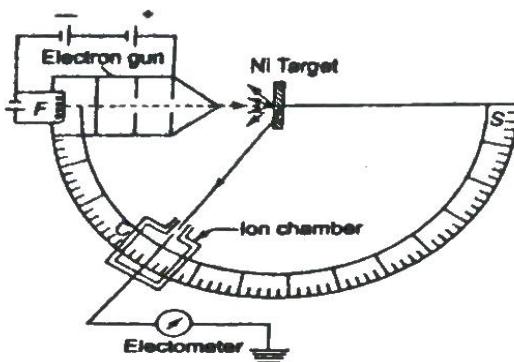


Fig. Davisson and Germer electron Diffraction apparatus

The first proof of the existence of "matter waves" was obtained in 1927 by Davisson and Germer the two American physicists. They succeeded in measuring the de Broglie wavelength for slow electrons, accelerated by a low potential difference by diffraction methods. The experimental arrangement is shown in the Fig.. The electron beam is produced from an electron gun consisting of tungsten filament F heated by low tension battery. The electrons excited by the filament are accelerated in an electric field of known potential difference from a high tension battery. The electrons are collimated to a fine beam and made to strike a Nickel target which is capable of rotation about an axis parallel to the axis of the incident beam. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a given direction is measured by allowing it to enter in a Faraday cylinder called collector C which can be moved along a graduated circular scale S , so that it is able to receive the reflected electrons at all angles between 20° and 90° .

Davisson and Germer noticed that the strongest of the scattered electron beams corresponded accurately to diffraction maxima that would be expected in the diffraction of X-rays by the same crystal. The angular distribution of scattered electrons was analogous to optical diffraction patterns from a plane diffraction grating whose lines consisted of the rows of nickel atoms in the surface of the target crystal. The wavelength associated with the diffraction pattern can be obtained according to Bragg's law:

$$n\lambda = 2d \sin \theta$$

where d is the distance between the rows of atoms.

It was observed by Davisson and Germer that when an electron beam accelerated by a potential of 54 volts was directed upon a Nickel target, a sharp diffraction maxima appeared in the electron currents. The incident and the scattered beams in this case

make an angle of 68° with the family of Bragg's planes. The spacing of planes in this family which can be determined by X-ray diffraction is 0.91 \AA . From above Bragg's equation, taking $n = 1$, we have,

$$2 \times 0.91 \times \sin 68^\circ = 1 \text{ \AA} \text{ or } \lambda = 1.66 \text{ \AA}$$

The wavelength of electrons accelerated through potential of 54 Volts can be calculated as

$$\frac{1}{2}mv^2 = eV$$

m is the mass of the electron.

$$m^2v^2 = 2meV$$

or

$$mv = \sqrt{2meV}$$

The de-Broglie wavelength of the electron will be given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

Putting $h = 6.6 \times 10^{-34}$ joule sec, $m = 9 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ coulomb, we obtain

$$\lambda = \frac{12.28}{\sqrt{V}}, \text{ with } V = 54 \text{ volts}$$

$$\lambda = 1.66 \text{ \AA}$$

There is an excellent agreement between the two results. Thus Davisson Germer experiment is a definite evidence that the electron beams do behave as wave and the wavelength of these beams is thus given by the de-Broglie equation.

Energy of particle in a box is given by

$$E_n = \frac{n^2 h^2}{8ml^2}$$

Q.5. (c) Using uncertainty principle explains the non existence of electrons inside the atomic nucleus.

Ans. Refer Q.4. (a) of First Term Examination 2019.

UNIT-III

Q.6. (a) What do you mean by inter planar distance? Show that in a cubic lattice the distance between successive planes of indices (hkl) is given by (6)

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Ans. Let us consider a cube with edge a and a plane IJK as shown in Fig.5.

Let the Miller indices of the plane be $(h k l)$. Now, draw HL perpendicular from origin H of the cube to the plane IJK .

Let a be the lattice parameter i.e., cube edge and d be the perpendicular distance between the origin H and the plane i.e., HL .

H , HJ and HK be the intercepts of the plane along x , y and z -axis respectively.

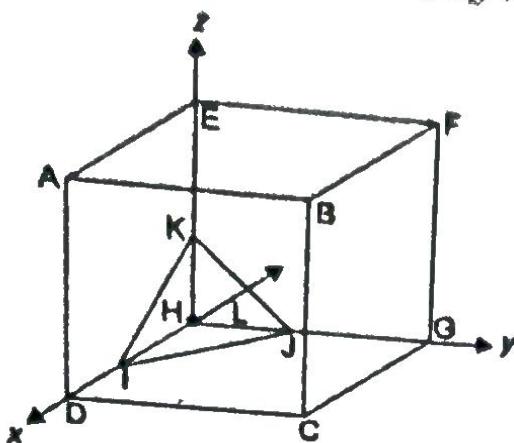


Fig. Spacing of the plane IJK

We know that the miller indices of a plane are the smallest integers of the reciprocals of its intercepts. Therefore the intercepts may also be expressed as reciprocals of Miller indices or in other words,

$$HI : HJ : HK = \frac{1}{h} : \frac{1}{k} : \frac{1}{l} = \frac{a}{h} : \frac{a}{k} : \frac{a}{l}$$

From the geometry of Fig. (right angles HIL, HJL and HKL) we know that

$$\cos \alpha = \frac{HL}{HI} = \frac{hd}{a} \quad \dots(1)$$

$$\cos \beta = \frac{HL}{HJ} = \frac{kd}{a} \quad \dots(2)$$

$$\cos \gamma = \frac{HL}{HK} = \frac{ld}{a} \quad \dots(3)$$

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \quad \dots(4)$$

Using Eqs. (1), (2) and (3) and Eq. (4),

$$\frac{d^2}{a^2} (h^2 + k^2 + l^2) = 1$$

or

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \text{proved}$$

Q.6. (b) Calculate the glancing angle of the (110) plane of simple cubic crystal with lattice constant 2.814 \AA° ; for second order diffraction maxima for X-rays of wavelength 0.710 \AA° . (3.5)

Ans. From Bragg's law we get-

$$2d \sin \theta = n\lambda$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

and

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$$a = 2.814 \text{ \AA}, h = 1, k = 1, l = 0$$

Given,

$$d_{110} = \frac{2.814}{\sqrt{1^2 + 1^2 + 0}} = \frac{2.814}{\sqrt{2}} = 1.989 \text{ \AA}$$

$$n = 2.3 = 0.710 \text{ \AA}$$

Also,

Bragg's law becomes-

$$2 \times 1.989 \sin \theta = 2 \times 0.710$$

$$\sin \theta = \frac{0.710}{1.989} = 0.3569$$

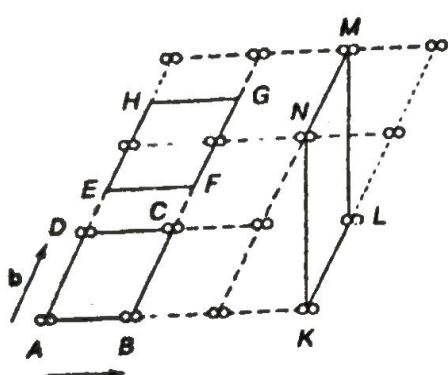
$$\theta = \sin^{-1}(0.3569)$$

$$\boxed{\theta = 20^\circ 55'}$$

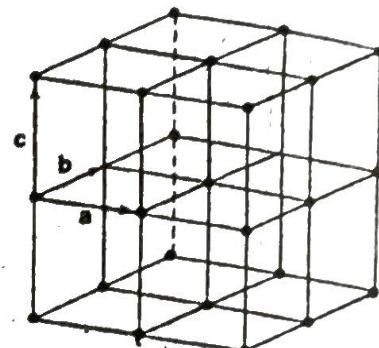
Q.6. (c) Explain the following terms:- (i) Unit Cell (ii) Translation Vector (iii) Bravais Lattice.

Ans. (i) Unit cell: In a space lattice an array of lines divides the space into identical volumes. These volumes fill the space completely by translation along the respective sets of axes and are known as unit cells.

Consider part of a two dimensional crystal, the atoms of which are arranged in a pattern shown in Fig. (a). Each cluster of atoms consists of a dot two open circles and called pattern unit. It is observed that when a parallelogram as ABCD is repeatedly translated by vector \mathbf{a} and \mathbf{b} corresponding to AB and AD respectively, the pattern may be reproduced. (the dotted parallelograms). The region ABCD is called



(a) Unit cells in two dimensional lattice



(b) Primitive tive cells of the space lattice in three dimensions

Fig.

A **unit cell** and the vectors \mathbf{a} and \mathbf{b} as basis vectors. Unit cells can also be defined in any of the possible ways $EFGH$ or $KLMN$. The unit cell may thus be defined as "a region of the crystal that will cover all space under action of the translation operations defined by fundamental translation vectors." In two dimensions \mathbf{a} , \mathbf{b} are translation vectors of the lattice and area of unit cell is $a \cdot b$ and in three dimensions volume of the unit cell is $a \times b \cdot c$.

(ii) Translation Vector: For any type of lattice, there exist three fundamental translational vectors \mathbf{a} , \mathbf{b} and \mathbf{c} , not lying in a plane, in terms of which any lattice point can be specified. A translational vector T is defined as

$$T = n_1 a + n_2 b + n_3 c$$

where n_1, n_2, n_3 be integers. Thus it is meant that by applying the operation T to a point r , we reach another point r' , which has the same environment as r .

$$r' = r + T = r + n_1 a + n_2 b + n_3 c$$

The set of points r' , given by above equation for all possible values of the integers n_1, n_2 and n_3 defines the space lattice.

(iii) Bravais Lattice: It is a three dimensional lattice. A bravais lattice tiles space without any gaps or holes. There are 14 ways in which this can be accomplished. Lattices are characterized by translational symmetry. Bravis lattices contain seven crystal systems and four lattice centering types.

Q.7. (a) Derive Bragg's law for diffraction of X-rays by crystal. Explain how the crystal structure can be determined by laue' method? (8)

Ans. X-rays are electromagnetic waves of wavelength ($= 1\text{ \AA}$) much shorter than the wavelength of visible light. It is known that wav shows differaction effects if the wavelength of waves in of the order of the dimensions of diffracting substance. When the wave nature of X-rays was confirmed attempts were made to measure the wavelength of X-rays. Therefore in order to diffract X-rays, grating with much finer rulings, having distance between lines comparable with the wavelength of X-rays, are required.

It is impossible to construct such a grating artificially Van Laue in 1913 pointed the crystals have geometrically regular and periodic arrangements of atoms in lattice space such that the spacing between atoms is of the order of wavelength rays.

Hence crystals may act as a three dimensional natural grating for X-rays. Lane's students actually obtained a diffraction pattern by passing X-rays through a crystal of rock-salt. The diffraction pattern is known as Laue's pattern. Bragg studied X-rays diffraction in detail and used the crystal as reflection diffraction grating.

Bragg's Law: First of all Schuster pointed out that X-rays are electromagnetic waves with wavelengths thousand times smaller than the visible light. In order to measure the wavelengths, a grating of corresponding dimensions is required and hence simple grating cannot be used. Moreover, it is rather impossible to prepare artificially a grating of such fine dimension. Laue and his collaborators showed that atoms in crystals are arranged in a regular manner and the spacing between them is comparable to the wavelength of X-rays and hence the crystal could act as suitable natural grating for diffracting the X-rays. W.H. Bragg and W.L. Bragg presented a simple explanation of the observed angle of the diffracted beams from a crystal. Consider a series of parallel row in which the atoms are arranged in a given plane of the crystal. Suppose parallel beam of X-rays is incident in a direction making a glancing angle θ_0 with the surfaces of the planes. Each atom acts as a centre of disturbance and sends spherical wavefronts by Huygen construction. As X-rays are much more penetrating than ordinary light, it is necessary to consider the rays reflected from a single layer but from several layers together. There will, be however, no diffracted beam unless the waves reflected from different planes are exactly in phase. They will reinforce themselves and an intense reflected beam will result. The

condition that the reflected wavefronts be in same phase, must be an exact wavelength or the integral multiple of it. If there is a smallest disagreement in phase between the beams reflected from successive planes, it causes destructive interference.

Let us consider two parallel rays LMN and PQR which are reflected by two atoms M and Q as shown in Fig. 3. The atom Q is vertically below M . The length of the path PQR is greater than the length of the path LMN . The path difference is $(AQ + QB)$ and according to the condition of reflection, we have.

$$(AQ + QB) = n\lambda \quad \dots(1)$$

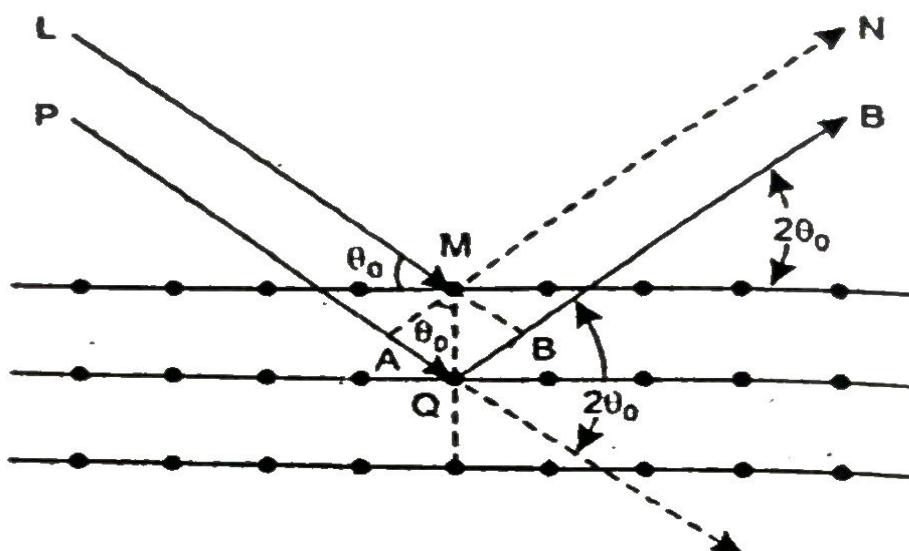


Fig. Illustration for the derivation of the Bragg's law.

But from Fig.

$$AQ = BQ$$

$$= d \sin \theta_0$$

$$2d \sin \theta_0 = n\lambda \quad \dots(2)$$

Equation (2) is known as Bragg's Law and gives the condition for reflection of X-rays from series of atomic layers in a given plane. For a given set up with monochromatic radiation, the wavelength λ is fixed and hence, Eq. (2) has only a particular set of solutions namely,

$$n = 1 \quad \theta_0 \sin^{-1} \left(\frac{\lambda}{2d} \right)$$

$$n = 2; \quad \theta_0 \sin^{-1} 2 \left(\frac{\lambda}{2d} \right)$$

$$n = 3; \quad \theta_0 = \sin^{-1} 3 \left(\frac{\lambda}{2d} \right) \dots \text{etc}$$

... (3)

These are known as the first, second third etc, reflection according to n is 1, 2, 3, ... etc. This shows that crystal cannot give rise to reflections at any angle but only at those discrete angles indicated by Eq. (3). Bragg's reflection can occur only for wavelength $\lambda \leq 2d$. Due to this fact, the visible light wavelength cannot be used in diffraction.

Laue Method of crystal structure determination: A single crystal is mounted as a goniometer, which enables the crystal to be rotated through known angles in two perpendicular planes, and maintained stationary in a beam of X-rays ranging in wavelength from about 0.2 to 2.0 Å. The crystal selects out and diffracts those values of d for which planes exist, of spacing d and glancing angle θ , satisfying the Bragg's equation. A flat photographic film is placed to receive either the transmitted diffracted beam or the reflected diffracted beams as illustrated in Fig.

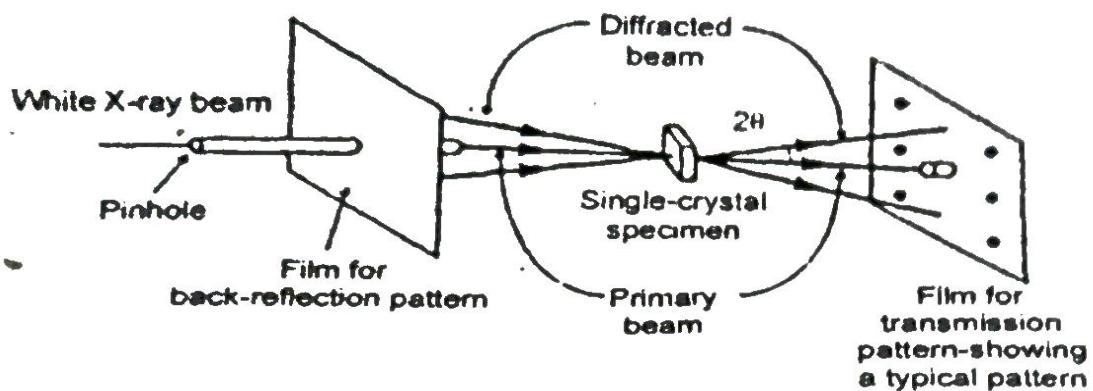


Fig. Illustration for Laue technique.

As shown in Fig. the resulting Laue pattern consists of a series of spots. Sharp well-defined spots on the film are good evidence of a perfect crystal structure, whereas diffuse, broken or extended spots indicate lattice distortion, defects or other departures from the perfect crystal lattice.

The Laue pattern reveals the symmetry of the crystal structure in the orientation and for example, if a cubic crystal is oriented with a cube edge, e.g., [100] axis, parallel to the incident beam, the Laue pattern will show the four fold symmetry appropriate to this axis.

Q.7. (b) How Frankel's defects are different from Schottky defects? Derive the necessary relation to show that the Schottky defects in ionic crystal depend on the temperature. (6.5)

Ans. Refer to Q.6. (c) of End Term Examination 2018. (Page No. 25-2018)

UNIT-IV

Q.8. (a) What is Hall effect? Show that Hall coefficient is independent of the applied magnetic field and is inversely proportional to the current density and electronic charge. (9)

Ans. The phenomenon of Hall Effect is a consequence of motion of charge in both magnetic and electric fields. If a piece of conductor (metal or semiconductor) carrying current is placed in a transverse magnetic field, then an electric field or a potential difference is produced inside the conductor in a direction normal to both current and magnetic field. This phenomenon is known as *Hall effect*. The developed electric field and potential are called *Hall field* and *Hall voltage*.

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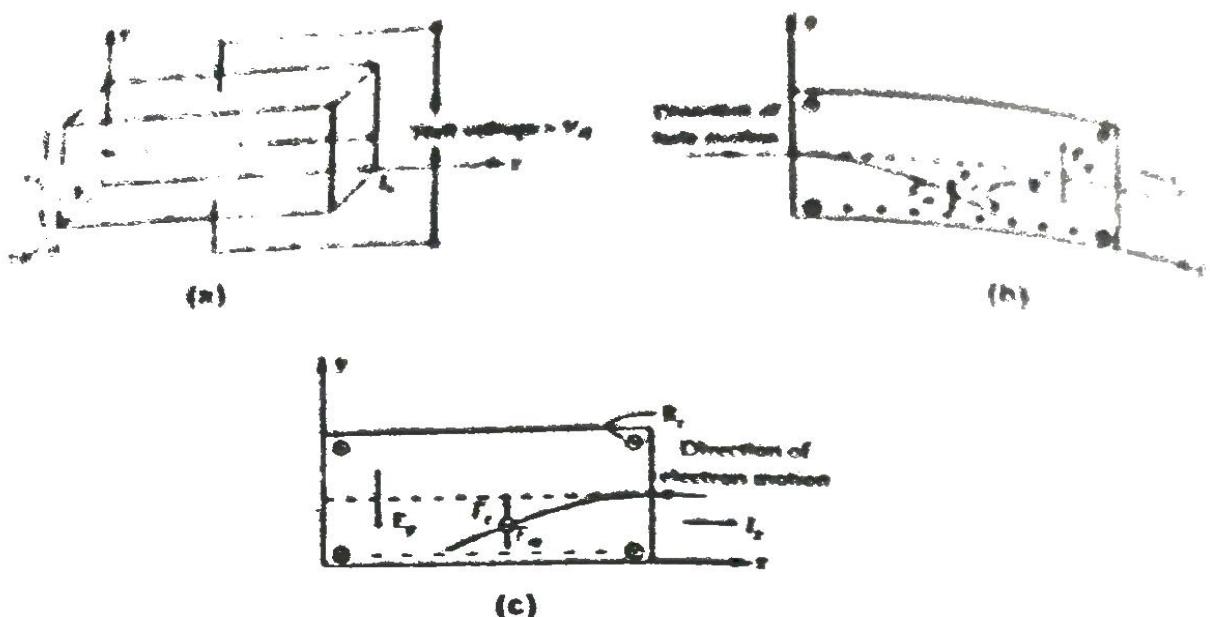


Fig. (a) Illustration of Hall effect, (b) Motion of hole in P-type semiconductor specimen (c) Motion of electron of N-type semiconductor

Consider a specimen in the form of a rectangular cross-section carrying current J_x in the x -direction. A uniform magnetic field B_z is applied along z -axis as shown in Fig. If specimen is of conducting material or N-type semiconductor, then current flows due to motion of electron, but in P-type semiconductor and intrinsic semiconductor, current is carried by motion of holes and electrons-holes respectively.

A moving charge in transverse magnetic field experiences a force normal to both current and magnetic field. Thus moving charges slightly shift towards y direction due to magnetic force $ev_x B_z$, Fig. (a) & (c). This shift of charges produces a potential difference or an electric field E_y along y -direction. The produced electric field also applies a force, E_y , on charge. When both the forces become equal, the equilibrium occurs, i.e., upto this condition charges shift along y direction.

In equilibrium

$$F_e = F_m$$

$$eE_y = ev_x B_z$$

$$E_y = v_x B_z$$

If J_x is the current density in x direction, then

$$J_x = \frac{I_x}{A} = nev_x \Rightarrow V_x = \frac{J_x}{ne}$$

Here n is density of charge carrier (electron or hole or both) and A (breadth \times thickness) is the cross-sectional area of specimen perpendicular to direction of J_x . From Eqs. (1) and (2) we can write,

$$E_y = \frac{B_z J_x}{ne}$$

This is the expression for Hall field. The Hall effect is described by means of Hall coefficient R_H defined in terms of J_x by the following relation:

$$R_H = \frac{E_y}{J_x B_z} \quad \dots(4)$$

From Eqs. (3) and (4), we have-

$$R_H = \frac{1}{ne} \quad \dots(5)$$

For the conductor or N-type semiconductor, the electric field is developed in negative y direction. Hence from Eqs. (4) and (5), we can write.

$$R_H = -\frac{E_y}{J_x B_z} = -\frac{1}{ne} \quad \dots(6)$$

In P-type semiconductor, the produced Hall field is along y direction, thus we have

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{n_h e} \quad \dots(7)$$

Here R_h is the hole density. Equations (6) and (7) indicate that the sign of R_H is same as the sign of carrier. Most of the metals have negative Hall coefficient or resistance, but some metals have positive Hall coefficient (i.e., charge carriers are holes). If both holes and electrons contribute to current, then R_H can be positive or negative depending upon the relative density and mobility of carriers.

Application of Hall Effect

1. The sign of Hall coefficient can be used to determine whether a given semiconductor is N or P-type.
2. The carrier concentration can be evaluated when R_H is known (as $n = 1/R_H e$)
3. Since $u = R_H \sigma$, thus mobility of charge carrier can also be well determined with the help of R_H .
4. Since Hall voltage is proportional to magnetic flux density B_z and current I_x through a sample or specimen. The Hall effect can be used as the basis for design of a magnetic flux density meter.
5. Hall effect can be used to determine the power flow in an electromagnetic wave because V_H is proportional to product of E_y and B_z , i.e., magnitude of Poynting vector in a.m. wave,

Q.8. (b) The resistivity of semiconductor material is $0.00912 \Omega \cdot m$ at room temperature. The flux density in the Hall model is 0.48 b/m^2 . Calculate Hall angle or a Hall coefficient of $3.55 \times 10^{-4} \text{ m}^2/\text{coulomb}$. (3.5)

Ans. Hall Angle is given by-

$$\Theta_H = \tan^{-1} \frac{E_H}{E_x}$$

After putting the value of E_H and E_x . We get-

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$$\theta_H = \tan^{-1} \alpha BR_H = \tan^{-1} \frac{BR_H}{\rho}$$

$$\rho = 0.00912; B = 0.48; R_H = 3.55 \times 10^{-4}$$

(Given)

$$\theta_H = \tan^{-1} \left(\frac{0.48 \times 3.55 \times 10^{-4}}{0.00912} \right)$$

$$\theta_H = \tan^{-1}(0.0187)$$

$$\boxed{\theta_H = 1.071^\circ}$$

Q.9. (a) What are Brillouin zones? Explain using E-k diagram.

Ans. If Schrodinger wave equation for electron energies is solved with a periodic function $u(k)$ to give the energies of electrons in a solid, the solutions fall into permitted energy bands. If the solutions are plotted in the reciprocal lattice of the crystal being considered, the zones enclosing the solutions for $k = 1, 2, 3, \dots, n$ are called *Brillouin zones*. (6)

BRILLOUIN ZONES

Linear Lattice: According to Kronig and Penney, the discontinuities in a one dimensional lattice occur when $k = \frac{n\pi}{a}$, where n is a positive or negative integer. In a one dimensional mono-atomic lattice a line representing the value of k is divided by energy discontinuities into segments of length as shown in Fig. These line segments are known as Brillouin Zones.

The first reflection and the first energy gap occurs at $k = \pm \frac{\pi}{a}$. The reflection at $k = \pm \frac{\pi}{a}$ arises because the wave reflected from one atom in a linear lattice interfaces constructively with the wave from a nearest neighbour atom, the phase difference between the two reflected waves for the values of $k = \pm \frac{\pi}{a}$ and $k = -\frac{\pi}{a}$ being $\frac{2\pi}{a}$. The region in k -space between $\frac{-\pi}{a}$ and $\frac{\pi}{a}$, i.e., the segment $\frac{-\pi}{a} \leq k \leq \frac{\pi}{a}$ is called the first Brillouin Zone.

The second Brillouin zone contains electrons with $k > \frac{\pi}{a}$, i.e., electrons having k -values between $\frac{\pi}{a}$ and $\frac{2\pi}{a}$ for electrons moving in the $\pm x$ direction. The second reflection and second energy gap occurs at $k = \pm \frac{2\pi}{a}$ and therefore, the region in the k space between $-\frac{2\pi}{a}$ to $\frac{-\pi}{a}$ and $\frac{\pi}{a}$ to $\frac{2\pi}{a}$, i.e., the segment $-\frac{2\pi}{a} \leq k \leq -\frac{\pi}{a}$ and $\frac{\pi}{a} \leq k \leq \frac{2\pi}{a}$ forms the second Brillouin zone of the lattice.

Similarly, we can discuss the formation of third Brillouin zone; ... etc.

The first and second Brillouin zones and the energy gaps are shown in Fig. by plotting value of E in eV corresponding to various values to k .

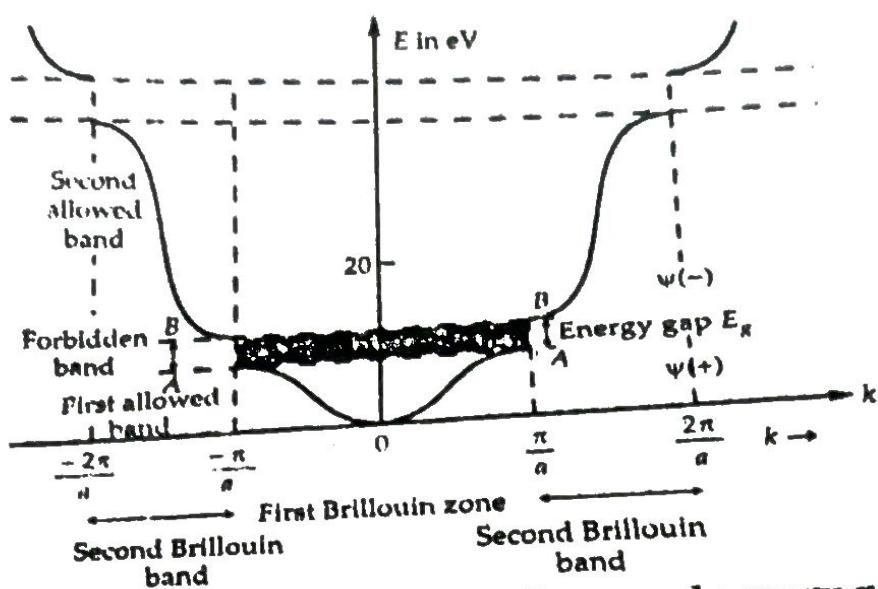


Fig. The first and second Brillouin Zones and energy gap

Two Dimensional Lattice: Proceeding as in the case of a linear lattice, the first zone for a two dimensional lattice in the X-Y plane is the square ABCD the boundaries of which are given by the relation

$$k_x = +\frac{\pi}{a} \text{ and } k_x = -\frac{\pi}{a} \text{ and } k_y = +\frac{\pi}{a} \text{ and }$$

$$k_y = -\frac{\pi}{a} \text{ as shown in Fig.}$$

The second Brillouin zone for a two dimensional lattice in the X-Y plane is the square EFGH the boundary of which are given by the relation

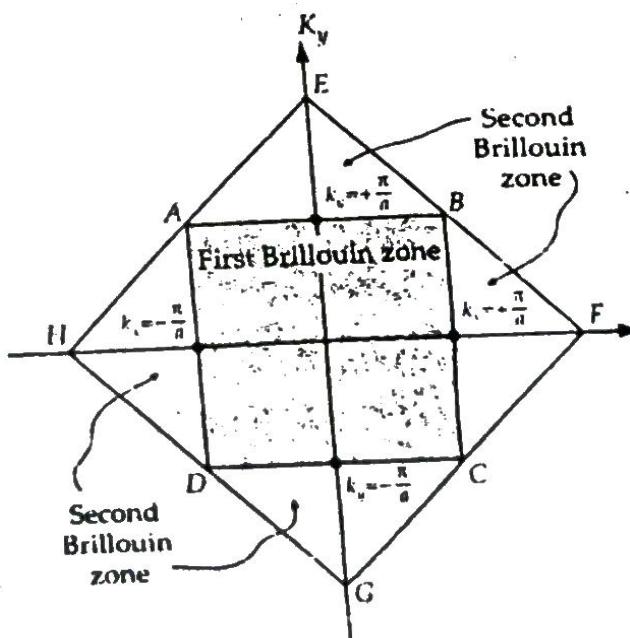


Fig. First and Second Brillouin Zones in 2D lattice

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$$k_x = \pm \frac{2\pi}{a}; \quad k_y = \pm \frac{2\pi}{a}$$

Refer to Q. 9. (a) of End Term Examination 2018. (Page No. 33-2018)
 Q. 9. (b) Define the effective mass of an electron. Briefly discuss the variation
 of effective mass as a function of wave number. (4)

Ans. Refer to Q. 1. (k) of End Term Examination 2018. (Page. No. 14-2018)

Q. 9. (c) For an intrinsic semiconductor having a band gap $E_g = 0.7$ eV, calculate
 the density of holes and electrons at room temperature ($27^\circ C$). (2.5)

Ans. Given

In intrinsic semiconductor, the concentration of electrons and holes are same. So

$$n_e = n_h = 2 \left[\frac{2\pi k T m}{h^2} \right]^{3/2} e^{(E_F - E_C - kT)}$$

The fermi level lies exactly in the middle of conduction and valence band.

$$\text{i.e. } E_F = \frac{E_C + E_V}{2}$$

$$\therefore E_F - E_C = \frac{E_C + E_V}{2} - E_C = - \frac{(E_C - E_V)}{2} = - \frac{E_g}{2}$$

$$n_e = n_h = 2 \left[\frac{2\pi k T m}{h^2} \right]^{3/2} e^{-E_g/2kT}$$

$$= 2 \times \left[\frac{2 \times 3.14 \times 1.38 \times 10^{-23} \times 300 \times 9.1 \times 10^{-31}}{6.62 \times 10^{-34}} \right] e^{\left[\frac{0.7}{2 \times 0.026} \right]}$$

$$= 3.6 \times 10^{19} \text{ per } m^3$$