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SOLVED EXAMPLES

1. A certain volume of air at 300 K expands adiabatically until its volume is doubled. Find the resulting fall in temperature given that γ is 1.4 for air.

Solution:

For an adiabatic change $TV^{\gamma-1}$ = Constant

Initial temperature $= T_1 = 300$ K

Final volume $= V_2 = 2V_1$

= Twice the initial volume

If therefore the final temperature is T_2

then $300\,V_1^{1.4-1} = T_2\,(2V)^{1.4-1}$

Therefore $T_2 = \dfrac{300}{2^{0.4}} = 227.3$ K

Therefore fall in temperature = 300 − 227.3 = 72.7 K.

2. Calculate the rise in temperature when a gas initially at 300 K is compressed adiabatically to 8 times its initial pressure ($\gamma = 1.4$)

Solution:

Initial temperature $= T_1 = 300$ K

Let the final temperature be T_2

Let P_1 and P_2 be the initial and final pressures

The $P_2 = 8P_1$

For an adiabatic process $T^{\gamma}P^{1-\gamma}$ = constant

Therefore $300^{1.4} \times P_1^{1-1.4} = T_2^{1.4} \times (8P_1)^{1-1.4}$

Therefore $T_2^{1.4} = 300^{1.4} \times 8^{0.4}$

Therefore, Final temperature $= T_2 = 300 \times 8^{0.4/1.4}$

$= 543.3$ K

3. A quantity of dry air at 21° C is compressed (i) slowly and (ii) suddenly to 1/3 of its volume. Find the change in temperature in each case, assuming γ to be 1.4 for dry air.

Solution:

(i) When the process is slow, the temperature of the system remains constant. Therefore there is no change in temperature.

ii) When the compression is sudden, the process is adiabatic

Hence $V_1 = V$ and $V_2 = 1/3$ V

$T_1 = 300$ K and $T_2 = ?$

$$T_2\,(V_2)^{\gamma-1} = T_1\,(V_1)^{\gamma-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\left(\frac{3V}{V}\right)^{\gamma-1}$$

$$= 300\,(3)^{1.4-1}$$

$$= 465.5\text{ K}$$

4. Calculate the work done when one litre of a monoatomic perfect gas at N.T.P is compressed adiabatically till the temperature is increased to 100°C.

Solution:

Gas constant = $8.313\text{ J K}^{-1}\text{ mol}^{-1}$

At N.T.P 22.4 litres of the gas has a mass of 1 gram mole and hence a gas constant of $8.313\text{ J K}^{-1}\text{ mol}^{-1}$

Therefore 1 litre of gas will have a gas constant $= \dfrac{8.313}{22.4}\text{ J K}^{-1}$

Work done $= W = \dfrac{R}{1-\gamma}(T_2 - T_1)$

$$= \frac{8.313}{22.4}\,\frac{(373-273)}{(1-1.67)}$$

$$= -55.39\text{ Joules.}$$

(–ve sign indicates that the work is done on the gas)

5. Calculate the efficiency of a reversible engine working between the temperature 440 K and 330 K.

Solution:

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1}$$

$$\text{Here } T_1 = 440\text{ K}$$

$$T_2 = 330\,K$$

$$\text{Efficiency} = \frac{440-330}{440} = 0.25$$

6. The efficiency of a Carnot engine was found to increase from 25% to 40% on increasing the temperature of the source alone through 100 K. Find the temperature of the sink.

Solution:

$$\text{Initially Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{25}{100} = \frac{1}{4}$$

Simplifying we get $3\,T_1 = 4\,T_2$

After the temperature of the source alone is rised

$$\frac{T_1 + 100 - T_2}{T_1 + 100} = \frac{40}{100} = \frac{2}{5}$$

Cross multiplying and simplifying

$$3\,T_1 - 5\,T_2 + 300 = 0$$

Solving the two equations we get

T_2 = Temperature of the sink = 300 K

7. The efficiency of a Carnot's engine is 25% when the temperature of the sink is 300 K. By how much should the temperature of the source be raised for the efficiency to become 50% ?

Solution:

Let the temperature of the source initially be T_1 then

$$\frac{T_1 - 300}{T_1} = \frac{25}{100} = \frac{1}{4}$$

Cross multiplying and simplifying, we get

$$T_1 = 400\text{ K}$$

Let the temperature of the source be increased by T, then

$$\frac{400 + T - 300}{400 + T} = \frac{50}{100} = \frac{1}{2}$$

Cross multiplying and simplifying we get

$$T = 200\text{ K}$$

8. An inventor claims to have developed an engine working between 600 K and 300 K capable of having an efficiency of 52%. Comment on his claim.

Solution:

The efficiency of a Carnot engine working between 600 K and 300 K

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{300}{600} = 0.5 = 50\%$$

The efficiency claimed is 52%

It means that the efficiency of the engine is more than the efficiency of a Carnot engine working between the same two temperatures. But no engine can have an efficiency more than a Carnot engine. So his claim is invalid.

9. A Carnot engine working as a refrigerator between 260 K and 300 K receives 500 calories of heat from the reservoir at the low temperature.

Calculate the amount of heat rejected to the reservoir at the higher temperature. Calculate also the amount of work done in each cycle to operate the refrigerator.

Solution:

$Q_1 = ?$ $\qquad Q_2 = 500$ cols

$T_1 = 300$ K $\qquad T_2 = 260$ K

We have $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

Therefore $$Q_1 = Q_2 \cdot \left(\frac{T_1}{T_2}\right)$$

$$= \frac{500 \times 300}{260} = 576.92 \text{ Calories}$$

$$W = Q_1 - Q_2 = 76.92 \text{ Cals.}$$

10. Calculate the change in entropy when 10^{-3} kg water at 273 K is heated to 373 K assuming the specific heat capacity of water to be constant.

Solution:

If a body of mass m and specific heat capacity 'c' is heated through dT the heat absorbed by it is $mcdT$. Hence its increase in entropy when heated from T_1 to T_2 is

$$\int_{T_1}^{T_2} \frac{mcdT}{T} = mc \ln \frac{T_2}{T_1}$$

In the present case $m = 10^{-3}$ Kg, c = 4200 Kg K^{-1}. $T_1 = 273$ K and $T_2 = 373$ K

Therefore increase in entropy

$$= 10^{-3} \times 4200 \times \ln\left(\frac{373}{273}\right)$$

$$= 10^{-3} \times 2.303 \times \log_{10}\left(\frac{373}{273}\right) \times 4200$$

$$= 1.31 \text{ J K}^{-1}$$

11. One mole of perfect gas is expanded isothermally to twice its initial volume. Calculate the change in entropy. $R = 8.313$ J K^{-1} mol^{-1}, if a quantity of heat dQ is given to the gas,

Solution:

$$dQ = dU + p \cdot dV$$

If the change is isothermal $dU = 0$

$$\therefore \qquad dQ = pdV = \frac{RT}{V} \cdot dV$$

Therefore change in entropy $= \dfrac{dQ}{T} = \dfrac{RdV}{V}$

In the present case

$$\text{Change in entropy} = \int_{v}^{2v} \frac{2dV}{V}$$

$$= R \times 2.303 \times \log_{10}^{2}$$

$$= 8.313 \times 0.3010 \times 2.303 \text{ J K}^{-1}$$

$$= 5.763 \text{ J K}^{-1}$$

12. Calculate the change in entropy when 20×10^{-3} Kg of ice at 273 K melts into water at 313 K. ($L = 336 \times 10^3$ JKg^{-1} specific heat of water $= 4.2 \times 10^3$ J Kg^{-1} K^{-1})

Solution:

Increase in entropy of 20×10^{-3} Kg of ice when it melts into water without change of temperature

$$= \frac{20 \times 10^{-3} \times 336 \times 10^3}{273}$$

$$= 24.62 \text{ J K}^{-1}$$

Increase in entropy of 20×10^{-3} Kg of water at 273 K when its temperature rises to 313 K

$$= 20 \times 10^{-3} \times 4200 \times 2.303 \times \log \frac{313}{273}$$

$$= 11.49 \text{ J K}^{-1}$$

Therefore total increase in entropy $= 24.62 + 11.49$

$$= 36.11 \text{ J K}^{-1}$$