Chapter 1

Introductory Concepts

Contents in this Chapter

1.1. NATURE OF ELECTRICITY

It is comparatively easy to describe electricity by its action than by its nature. Electricity is convenient form of energy and have got numerous applications, such as for lighting, transportation, communication, heating, elevators, and for driving of various types of machine tools owing to its inherent numerous advantages.

Although no one knows precisely what electricity is, it has been possible to develop theories about electricity through experiments and observations of its behaviour. As a result, it is now believed that all matters are essentially electrical in nature.

In order to appreciate the nature of electricity, it is necessary to consider the structure of matter. Matter has been defined any thing that occupies space and has weight such as copper, iron, water, air, smoke, aluminium, silver etc. Matter is made up of extremely small particles known as *molecules*. A molecule is the smallest particle into which a given substance may be divided without changing its identity. Molecules can further be subdivided chemically into still minute particles known as *atoms*. An atom may be regarded as the smallest particle of an element that can exist. The substances, whose molecules consist of similar atoms, are called the *elements* whereas the substances whose molecules consist of dissimilar atoms are called the *compounds*.

According to modern electron theory, the atoms of all elements are composed of two parts, namely, the central *nucleus* and the surrounding or *orbital electrons* (Fig. 1.1).

The electrons each have a negative electric charge of 1.602×10^{-19} coulomb and some particles within the nucleus have a positive charge of the same magnitude. Since opposite charges attract each other, a force of attraction exists between the oppositely charged electrons and nucleus. Compared to the mass of nucleus, electrons are relatively tiny particles of almost negligible mass (each electron weighing 9.107×10^{-31} kg).

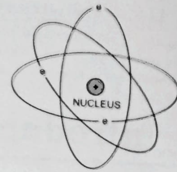


Fig. 1.1 *Rutherford's Nuclear Model of Atomic Structure*

The nucleus of an atom is largely a cluster of two types of particles, called the *protons* and *neutrons*. Protons each have a positive electrical charge, equal in magnitude (but opposite in polarity) to the negative charge of an electron. A neutron has no charge at all. Protons and neutrons, each have mass about 1,800 times the mass of an electron. For a given atom, the number of protons in the nucleus normally is equal to the number of orbiting electrons. Thus, an atom is electrically neutral. If an atom loses an electron, it loses the negative charge and becomes a *positive ion*. Similarly if an atom gains an additional electron, it becomes a *negative ion*. However, nearly all atoms have some electrons which are loosely bound to their nuclei. These electrons are known as *free electrons* and may be dislodged by one means or another and transferred from one atom to another. The body, that contains unequal number of electrons and protons is said to be *electrically charged*. If a body contains electrons more than its normal number then the body is said to be – *vely charged*. Similarly a body containing electrons less than its normal number is said to be + *vely charged*.

The atoms of some materials such as silver, copper, aluminium and zinc, known as *conductors*, have many free electrons. These free electrons of a metal move about haphazardly in all directions from atom to atom but when a certain electrical pressure or potential is applied to such metals at the two ends, the electrons move only in one direction. The drift of electrons in a conductor in one direction is known as the *electric current*, *i.e.*, the flow of electric current takes place by the movement of the electrons in a conductor. Since electrons are negatively charged, the direction of their motion is opposite to the direction of conventional current which is from higher potential to the lower potential. In non-metallic materials, such as glass, mica, slate and porcelain, the electrons are very closely bound to the nucleus and it is difficult to remove the electrons from the atoms. Such materials are known as *non-conductors or insulators.*

1.2. ELECTRIC CURRENT

Electric current may be defined as the time rate of net motion of an electric charge across a cross-sectional boundary (Fig. 1.2). A random motion of electrons in a metal does not constitute a current unless there is a net transfer of charge with time

i.e., electric current, i = Rate of transfer of electric charge

$$= \frac{\text{Quantity of electric charge transferred during a given time duration}}{\text{Time duration}}$$

$$= \frac{dQ}{dt} \qquad \text{...(1.1)}$$

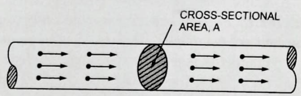


Fig. 1.2

Coulomb is the practical as well as SI unit for measurement of electric charge. One coulomb is approximately equal to 624×10^{16} electrons.

Since current is the rate of flow of electric charge through a conductor and coulomb is the unit of electric charge, the current may be specified in coulombs per second. In practice, the term coulomb per second is seldom used, a shorter term, *ampere* is used instead.

1.3. ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

Electromotive force (emf) is the force that causes an electric current to flow in an electric circuit while the *potential difference* (pd) between two points in an electric circuit is that difference in their electrical state which tends to cause flow of electric current between them.

Volt is a unit of electromotive force as well as potential difference in practical as well as in SI system of units.

The volt is defined as that potential difference between two points of a conductor carrying a current of one ampere when the power dissipated between these points is equal to one watt.

1.4. RESISTANCE

Resistance may be defined as that property of a substance which opposes (or restricts) the flow of an electric current (or electrons) through it.

The practical as well as mks (or SI) unit of resistance is ohm (Ω), which is defined as that resistance between two points of a conductor when a potential difference of one volt, applied between these points, produces in this conductor a current of one ampere, the conductor not being a source of any emf.

For insulators having high resistance, much bigger units kilo ohm or kΩ (10^3 ohm) and mega ohm or MΩ (10^6 ohm) are used. In case of very small resistances smaller units like milli ohm (10^{-3} ohm) or micro ohm (10^{-6} ohm) are employed.

1.5. OHM'S LAW

As the rate of flow of water through a pipe is directly proportional to the effective pressure (*i.e.*, difference of pressure at two ends) and inversely proportional to the frictional resistance, similarly the current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor and inversely proportional to the conductor resistance. This relation was discovered by German physicist Georg Simon Ohm and so it is known as Ohm's law.

If I is the current flowing through a conductor of resistance R across which a potential difference V is applied then according to Ohm's law

$$I \propto V \quad \text{and} \quad I \propto \frac{1}{R} \quad \text{or} \quad I \propto \frac{V}{R} \quad \text{or} \quad I = \frac{V}{R} \qquad \text{...(1.2)}$$

where V is in volts, R is in ohms and I is in amperes.

Ohm's law may be defined as follows:

Physical state i.e., temperature etc. remaining the same, the current flowing through a conductor is directly proportional to the potential difference applied across its ends.

Or

The ratio of potential difference applied across a conductor and current flowing through it remains constant provided physical state i.e., temperature etc. of the conductor remains unchanged.

$$i.e., \quad \frac{V}{I} = \text{Constant} = R \qquad \text{...(1.3)}$$

where R is known as the resistance of the conductor.

Ohm's law may be alternatively expressed as

$$V = IR \qquad \text{...(1.4)}$$

Equations (1.2), (1.3) and (1.4) give Ohm's law in three forms with which the student should be familiar.

Ohm's law cannot be applied to circuits consisting of electronic tubes or transistors because such elements are not bilateral *i.e.*, they behave in different way when the direction of flow of current is reversed as in case of a diode. Ohm's law also cannot be applied to circuits consisting of nonlinear elements such as powdered carbon, thyrite, electric arc etc. For example, for silicon carbide, the relationship between applied voltage (or potential difference) V and current flowing I is given as $V = K I^m$ where K and m are constants and m is less than unity.

1.6. LAWS OF RESISTANCE

The resistance of a wire depends upon its length, area of x-section, type of material, purity and hardness of material of which it is made of and the operating temperature

Resistance of a wire is

(*a*) directly proportional to its length, l *i.e.*, $R \propto l$

(*b*) inversely proportional to its area of x-section, a *i.e.*, $R \propto 1/a$

Combining above two facts we have $R \propto l/a$

$$\text{or } R = \rho \frac{l}{a} \qquad ...(1.5)$$

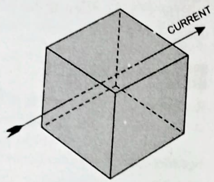


Fig. 1.3

where ρ (rho) is a constant depending upon the nature of the material and is known as the *specific resistance or resistivity* of the material of the wire.

To determine the nature of the constant ρ let us imagine a conductor of unit length and unit cross-sectional area, for example, a cube whose edges are each of length one unit, and let the current flow into the cube at right angles to one face and out at the other face. Then putting $l = 1$ and $a = 1$ in Eq. (1.5) we have R = ρ. *Hence resistance of a material of unit length having unit cross-sectional area is defined as the resistivity or specific resistance of the material.*

Specific resistance or resistivity of a material is also defined as the resistance between opposite faces of a unit cube of that material.

Resistivity is measured in ohm-metres (Ω-m) or ohms per metre cube in mks (or SI) system and ohm-cm (Ω-cm) or ohm per cm cube in cgs system.

$$1\ \Omega\text{-m} = 100\ \Omega\text{-cm}$$

1.7. CONDUCTANCE AND CONDUCTIVITY

The reciprocal of resistance *i.e.*, 1/R is called the *conductance* and is denoted by English letter G. It is defined as the inducement offered by the conductor to the flow of current and is measured in siemens (S).* Earlier, the unit of conductance was mho (℧).

1 siemen = 1 mho

From Eq. (1.5)

$$G = \frac{1}{R} = \frac{1}{\rho \frac{l}{a}} = \frac{1}{\rho} \cdot \frac{a}{l} = \sigma \frac{a}{l} \qquad ...(1.6)$$

where $\sigma = \frac{1}{\rho}$ and is known as *specific conductance* or *conductivity* of the material. Hence *conductivity* is the reciprocal of the resistivity and is defined as the conductance between the two opposite faces of a unit cube. The unit of conductivity is siemens/metre (S/m).

Example 1.1

A coil consists of 2,000 turns of copper wire having a cross-sectional area of 0.8 mm². The mean length per turn is 80 cm, and the resistivity of copper is 0.02 μ Ω-m. Find the resistance of the coil.

Solution: Length of the coil, l = Number of turns × mean length per turn = 2,000 × 0.8 = 1,600 m

* Named after German electrical engineer Werner von Siemens

Cross-sectional area of wire, $a = 0.8\ \text{mm}^2 = 0.8 \times 10^{-6}\ \text{m}^2$

Resistivity of copper, $\rho = 0.02\ \mu\Omega\text{-m} = 2 \times 10^{-2} \times 10^{-6}\ \Omega\text{-m} = 2 \times 10^{-8}\ \Omega\text{-m}$

Resistance of the coil, $R = \rho \frac{l}{a} = 2 \times 10^{-8} \times \frac{1{,}600}{0.8 \times 10^{-6}} = 40\ \Omega$ **Ans.**

Example 1.2

The resistance of a conductor 1 mm² in cross section and 20 m long is 0.346 Ω. Determine the specific resistance of the conductor material. [*Bangalore Univ. Elec. Circuits-I, 1991*]

Solution: Resistance of conductor, R = 0.346 Ω

Length of conductor, l = 20 m

Cross-sectional area of conductor, $a = 1\ \text{mm}^2 = 1 \times 10^{-6}\ \text{m}^2$

Specific resistance of conductor material, $\rho = \frac{R \times a}{l} = \frac{0.346 \times 1 \times 10^{-6}}{20} = 1.73 \times 10^{-8}\ \Omega\text{-m}$ **Ans.**

Example 1.3

A wire of length 1 m has a resistance of 2 Ω. Obtain the resistance if specific resistance is doubled, diameter is doubled and the length is made three times of the first.

[*Pb. Technical Univ. Basic Electrical and Electronics Engineering Second Semester 2004-05*]

Solution: New specific resistance, $\rho_2 = 2\rho_1$

New length of wire, $l_2 = 3\,l_1$

New cross-sectional area, $a_2 = \frac{\pi}{4}(d_2^2) = \frac{\pi}{4}(2d_1)^2 = 4\,a_1$

New resistance, $R_2 = \frac{\rho_2 l_2}{a_2} = \frac{2\rho_1 \times 3l_1}{4a_1} = 1.5\frac{\rho_1 l_1}{a_1} = 1.5\,R_1 = 1.5 \times 2 = 3\ \Omega$ **Ans.**

1.8. TYPES OF SUPPLY

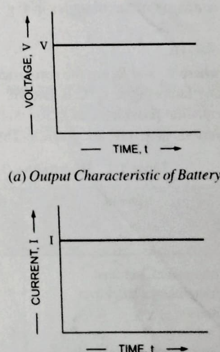
There are two types of supply viz dc and ac.

The voltage or current available from batteries or solar cells is *direct* in the sense that the polarity remains the same. Such sources are called direct current sources. A plot of output of such sources with respect to time is a straight line parallel to time axis as illustrated in Fig. 1.4. This shows that the voltage / current output of a dc source is constant with respect to time, unless the chemicals in the battery are exhausted or the light incident on solar cell varies.

Power obtainable from batteries or solar cells is very small as compared to the total power we need. The dc power in bulk can be generated by using dc generators, and will be discussed in detail later on. Standard dc voltages are 230 and 460 V.

A current (or voltage) is called *alternating* if it periodically changes its direction and magnitude whereas direct currents are steady and in one direction. In more restricted sense, alternating current is a periodically varying current, the average value of which, over a period, is zero.

At present a large percentage of electrical energy (nearly all) being used for domestic, commercial and industrial purposes is generated as alternating current because of technical and economical reasons (refer to Art. 3.1).



(*a*) *Output Characteristic of Battery*

(*b*) *Output Characteristic of Solar Cell*

Fig. 1.4

For industrial applications of direct current like electrolytic processes, welding processes and variable speed motor drives, the present trend is to generate ac and then convert it into dc by converters. For conversion of ac into dc the dc generator as a part of motor (ac)- generator (dc) set has to compete the SCR rectifiers and other various power controlled devices which usually are cheaper in cost, compact in size, relatively noise-free in operation and require minimum maintenance, though they suffer from the disadvantages of having poor power factor, harmonic generation, poor braking etc.

In our country the standard voltage for low tension (*lt*) distribution is 240 / 415 V (240 V between phase and neutral wires and 415 V from line-to-line). This is a reasonably safe voltage. Higher the voltage used, lower is the installation cost *i.e.*, economy is achieved but increase of voltage means more hazard to human life and property. Hence, a compromise is made. All over the world the standard voltage used for *lt* distribution is in the range of 200 / 350 V to 250 / 440 V except in American continent where 110 /190 V is the distribution voltage. Generation, transmission and *ht* distribution voltages are governed by economy and operation.

1.9. RESISTANCE VARIATION WITH TEMPERATURE

The resistance of all pure metallic conductors increases with the increase in temperature but the resistance of the insulators and non-metallic materials generally decreases with the increase in temperature.

If the resistance of any pure metal is plotted on a temperature base, it is found that over the range of temperature from 0 to 100 °C the graph is practically a straight line, as illustrated in Fig. 1.5. If this straight line is extended, it cuts the temperature axis at some temperature, $-t_0$ °C, known as *inferred zero resistance temperature*. This does not mean that the resistance of the metal is actually zero at that temperature, but $-t_0$ °C is the temperature at which the resistance would be zero if the rate of decrease between 100 and 0°C were maintained constant at all temperatures. From the similarity of the triangles in Fig. 1.5

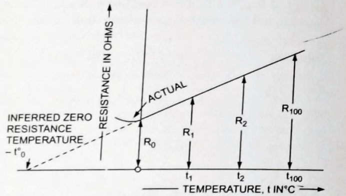


Fig. 1.5

$$\frac{R_2}{R_1} = \frac{t_0 + t_2}{t_0 + t_1} \qquad ...(1.7)$$

where R_1 and R_2 are the resistances at temperatures t_1 °C and t_2 °C respectively. Thus, if the resistance R_1 for any temperature t_1 °C is known, then resistance for any other temperature t_2 can be computed from above equation provided that t_0 for that particular material is known. Inferred zero resistance temperatures, t_0 for various materials are given in Table 1.1.

Table 1.1 Resistivities, Temperature Coefficients and Inferred Zero Resistance Temperatures

Material	*Resistivity in Ω-m at 20 °C (× 10^{-8})*	*Temperature Coefficient of Resistance in ohms per ohm per °C rise in temperature at 20 °C*	*Inferred Zero Resistance Temperature in °C*
Aluminium, annealed	2.82	0.0039	– 236
Aluminium, hard drawn	2.92	0.0038	– 243
Brass	6 – 8	0.002	– 480
Carbon	3,000 – 7,000	– 0.0005	—
Constantan or Eureka (60% copper and 40% nickel)	49	0.000008	– 125,000
Copper, annealed	1.724	0.00393	– 234.5

Contd...

Material	*Resistivity in Ω-m at 20 °C (× 10^{-8})*	*Temperature Coefficient of Resistance in ohms per ohm per °C rise in temperature at 20 °C*	*Inferred Zero Resistance Temperature in °C*
Copper, hard drawn	1.79	0.00382	– 242
German Silver (80% Cu, 12% Ni, 4% Zn)	33	0.0004	– 2,480
Gold	2.44	0.0034	– 274
Iron	12	0.005	– 180
Lead	22	0.0039	– 236
Manganin (84% Cu, 12% Mn, 4% Ni)	42 to 74	0.00003	– 33,000
Mercury	96	0.00089	– 1,100
Molybdenum	5.7	0.0033	– 280
Nichrome (60% Cu, 25% Fe,15 % Cr)	100	0.00044	– 2,250
Nickel	7.8	0.006	– 147
Phosphor, Bronze	5–10	0.0035	– 266
Platinum	10	0.003	– 310
Platinum–iridium	24.6	0.0012	– 814
Silver (99.98% pure)	1.6	0.0038	– 243
Tungsten	5.51	0.0045	– 200
Zinc	6.3	0.004	– 230

Note: *It has been found that the resistivity of the metallic conductors is significantly affected by variations in temperature but it is also affected by other factors like tension and pressure to some extent. For most metals except lithium and calcium resistivity decreases with the increase in pressure. However, resistivity increases with increase in tension.*

The variation of resistance with temperature is often utilized in determining temperature variations. For example, in testing of an electric machine, the resistance of the coil is measured both before and after the test run, and the increase in resistance is a measure of the rise in temperature. For computation of temperature rise Eq. (1.7) may be transposed to the following form

$$t_2 - t_1 = \frac{R_2 - R_1}{R_1}(t_0 + t_1) \qquad ...(1.8)$$

A temperature of 20°C has been adopted as the *standard reference temperature* for measurement of resistance, and the handbooks give the resistance of the various materials at that temperature. Consequently, when a designer is computing the resistance of any conductor from its dimensions, the initial temperature *t*, at which the resistance is known, is generally 20°C.

1.9.1. Temperature Coefficient of Resistance. Let a metallic conductor having a resistance of R_0 at 0°C be heated to t°C and let its resistance at this temperature be R_t. From Eq. (1.7)

$$\frac{R_t}{R_0} = \frac{t_0 + t}{t_0 + 0}$$

$$\text{or } R_t = R_0 + \frac{1}{t_0} R_0\, t$$

$$\text{or change in resistance, } \Delta R = R_t - R_0 = \frac{1}{t_0} R_0\, t = \alpha_0 R_0\, t \qquad ...(1.9)$$

where $\alpha_0 = \dfrac{1}{t_0}$ and is called the temperature coefficient of resistance of the material at 0°C

From Eq. (1.9) it may be concluded that change in resistance due to change in temperature

(*a*) varies directly as its initial resistance,

(*b*) varies directly as rise in temperature and

(*c*) depends on the nature of the material of the conductor.

The Eq. (1.9) may be rewritten as

$$\alpha_0 = \frac{\Delta R}{R_0 t} \quad ...(1.10)$$

So the temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree rise of temperature to the original resistance.

If R_0 is the resistance of any conductor at 0°C and α_0 is the temperature coefficient of resistance at 0°C, then resistance at t°C is given as

$$R_t = \text{Original resistance} + \text{increase in resistance} = R_0 + R_0 \alpha_0 t = R_0 (1 + \alpha_0 t) \quad ...(1.11)$$

The above expression holds good for both increase as well as decrease in temperature.

It is to be noted that

(*i*) temperature coefficient of resistance for all pure metallic conductors is positive *i.e.*, the resistance of all pure metallic conductors increases with the increase in temperature, that of non-metallic materials such as of carbon is negative *i.e.*, the resistance of non-metallic materials such as of carbon decreases with the increase in temperature. The temperature coefficient of resistance of alloys like constantan and manganin is negligible.

(*ii*) temperature coefficient of resistance is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then the temperature coefficient of resistance has the value of α_0. At any other temperature t°C, value of temperature coefficient of resistance is α_t and so on. For any material the temperature coefficient of resistance at 0°C *i.e.*, α_0 has the maximum value.

The temperature coefficient of resistance at any temperature t_1 is given as

$$\alpha_1 = \frac{1}{\frac{1}{\alpha_0} + t_1} \quad ...(1.12)$$

The temperature coefficient of resistance at temperature t_2 in terms of temperature coefficient of resistance at temperature t_1 is given as

$$\alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)} \quad ...(1.13)$$

If R_1 is the resistance of any conductor at t_1 °C and α_1 is the temperature coefficient of resistance at t_1°C, then resistance of the conductor at t_2°C is given

$$R_2 = R_1 \,[1 + \alpha_1 (t_2 - t_1)] \quad ...(1.14)$$

(*iii*) As the resistance of the material changes with the change in temperature so it is obvious that resistivity of the material depends on temperature.

Knowledge of temperature coefficient of resistance is utilized in determining the temperature rise of electrical machines, as already described in Art. 1.9.

Values of resistivity, temperature coefficient of resistance at 20°C and inferred zero resistance temperature for various materials are given in Table 1.1.

1.10. SI SYSTEM OF UNITS

SI is the latest form of metric system and absorbs in it the rationalized MKSA system. SI stands for "Systeme International d' Unites" in French. This abbreviation is now adopted by the International Standardising Organisation as the abbreviated name of this new system of units in all languages.

The SI system is, in fact, simply the RMKSA* system expanded by adding the degree Kelvin, Candela and mole as basic units of temperature, luminous intensity and amount of substance respectively. The SI system is a comprehensive, logical and coherent system, designed for use in all branches of science, engineering and technology.

This system derives all the units from the following seven base units.

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Intensity of electric current	ampere	A
Thermodynamic temperature	Kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

The SI system, besides seven base units, has following supplementary units.

Quantity	Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

Recommended prefixes for formation of multiples and submultiples of units are given below.

Multiple	Prefix	Symbol	Fraction	Prefix	Symbol
10	deca	*da**	0.1	deci	*d**
100	hecto	*h**	0.01	centi	*c**
1000	kilo	k	0.001	milli	*m*
1000 000	mega	M	0.000 001	micro	μ
1000 000 000	giga	G	0.000 000 001	nano	*n*
1000 000 000 000	tera	T	0.000 000 000 001	pico	*p*
			0.000 000 000 000 001	fento	*f*
			0.000 000 000 000 000 001	atto	*a*

*Should be restricted as much as possible.

1.11. WORK, POWER AND ENERGY

Work is said to be done by or against a force, when its point of application moves in or opposite to the direction of the force and is measured by the product of the force and the displacement of the point of application in the direction of force.

i.e., Work done, W = Force [F] × distance [*d*]

The SI or MKS unit of work is the joule, which is defined as the work done when a force of one newton acts through a distance of one metre in the direction of the force. Hence, if a force F acts through distance *d* in its own direction,

W = F [newtons] × *d* [metres] = F*d* joules

Power is defined as the rate of doing work or the amount of work done in unit time.

The MKS or SI unit of power is the joule/second or watt. In practice, the watt is often found to be inconveniently small and so a bigger unit, the *kilowatt* is frequently used.

1 kilowatt = 1,000 watts

* RMKSA stands for rationalized metre, kilogram, second, ampere.

The bigger unit of power, most commonly used in engineering practice (not at all in SI system) is horse power defined as below :

Metric Horse Power : It is the practical unit of power in MKS system (not in SI system) which according to ISI specifications is equal to 75 kgf-m of work done per second.

Energy is defined as the capacity of doing work. Its units are same as those of work, mentioned above.

If a body having mass m, in kg, is moving with velocity v, in metres/second,

$$\text{Kinetic energy} = \frac{1}{2}\, mv^2 \text{ joules}$$

If a body having mass m, in kg, is lifted vertically through height h, in metres, and if g is the gravitational acceleration, in metres/second^2 in that region, potential energy acquired by the body .

$$= \text{Work done in lifting the body} = mgh \text{ joules} = 9.81\, mh \text{ joules}$$

As already stated, in SI system the unit of energy of all forms is joule. Bigger unit of energy is mega joules (MJ) where 1 MJ = 10^6 J.

The thermal units of energy, calorie (gm calorie) and kilocalorie (kilogram calorie), are defined below:

Calorie : It is the amount of heat required to raise the temperature of one gram of water through 1°C.

$$1 \text{ calorie} = 4.18 \text{ J} \simeq 4.2 \text{ J}$$

Kilocalorie : It is the amount of heat required to raise the temperature of 1 kg of water through 1°C.

$$1 \text{ k. calorie} = 1{,}000 \text{ calories} = 4{,}180 \text{ joules} \simeq 4{,}200 \text{ J}$$

1.12. ELECTRICAL UNITS OF WORK, POWER AND ENERGY

The unit of work done and of energy expended is joule. It is equal to the energy expended in passing 1 coulomb of charge through a resistance of 1 ohm *i.e.*, the energy expended in passing one ampere current for 1 second through a resistance of one ohm is taken as one joule. It may also be expressed as 1 watt-second *i.e.*, one watt of power consumed for one second.

$$i.e., \ 1 \text{ joule} = 1 \text{ watt-second}$$

The unit of energy, joule or watt-second is too small for practical purposes, so a bigger unit Mega joule (MJ) or kilowatt-hour (kWh) is used in electrical engineering.

$$1 \text{ kWh} = 1{,}000 \text{ watt-hours} = 1{,}000 \times 3{,}600 \text{ watt-seconds or joules} = 3.6 \text{ MJ}$$

The kWh, also called the *Board of Trade* (BOT) *unit*, is the energy absorbed by supplying a load of 1 kW or 1,000 watts for the period of one hour. This is legal unit on which charges for electrical energy are made, and, therefore, it is called the Board of Trade (BOT) unit.

Watt: It is defined as the power expended when there is an unvarying current of one ampere between two points having a potential difference of one volt. As already stated the bigger unit of power is kW or Megawatt.

$$1 \text{ kW} = 1{,}000 \text{ watts}$$

$$1 \text{ MW} = 1{,}000 \text{ kW} = 1 \times 10^6 \text{ watts}$$

1.13. CONVERSION OF ELECTRICAL UNITS INTO MECHANICAL AND THERMAL UNITS OR VICE VERSA

$$1 \text{ watt} = 1 \text{ joule/second} = 1 \text{ N-m/s}$$

$$1 \text{ kW} = 1{,}000 \text{ watts or J/s or N-m/s} = \frac{1{,}000}{735.5} \ i.e., \ 1.36 \text{ hp (metric)}$$

$$1 \text{ kWh} = 1{,}000 \text{ watt-hours} = 3{,}600{,}000 \text{ watt-seconds or joules} = 1.36 \text{ hp-hour (metric)}$$

$$1 \text{ calorie} = 4.18 \text{ J or watt-seconds}$$

$$1 \text{ k cal} = 4{,}180 \text{ J or watt-seconds} = \frac{4{,}180}{3{,}600{,}000} \ i.e., \ \frac{1}{860} \text{ kWh}$$

$$1 \text{ kWh} = 36 \times 10^5 \text{ watt-seconds} = \frac{36 \times 10^5}{4{,}180} \ i.e., \ 860 \text{ kcals}$$

1.14. HEATING EFFECT OF ELECTRIC CURRENT

When an electric current flows through a conductor, electrical energy is expended in overcoming the frictional resistance between the electrons and the molecules of the wire.

If potential difference of V volts is applied across a conductor and current of I amperes flows through it for time of t seconds, then energy expended will be equal to VIt watt-seconds or joules.

If R is the resistance of the conductor through which a current of I amperes flows and V is the potential difference applied across its ends then by Ohm's law

$$V = IR$$

$$\text{and energy expended, } W = VIt = IR \times I \times t = I^2 Rt \text{ joules} \quad ...(1.15)$$

$$\text{or W also} = V \times \frac{V}{R} t = \frac{V^2}{R} t \text{ joules} \quad ...(1.16)$$

According to the *law of conservation of energy* this electrical energy expended must be converted in some other form of energy and that other form is heat *i.e.*, electrical energy expended is converted into heat energy and conversion of electrical energy into heat energy is called the heating or *thermal effect of electric current*.

1.15. JOULE'S LAW OF ELECTRIC HEATING

From Eq. (1.15) the energy expended or heat generated in joules when a current of I amperes flows through a resistance of R ohms for t seconds is given as

$$H = I^2 Rt \text{ joules}$$

The above expression is known as *Joule's law*, which states that the amount of heat produced in an electric circuit is

(*i*) proportional to the square of the current *i.e.*, $H \propto I^2$

(*ii*) proportional to the resistance of the circuit *i.e.*, $H \propto R$, and

(*iii*) proportional to the time duration for which the current flows through the circuit *i.e.*, $H \propto t$

$$\text{Heat produced in kcals, } H = \frac{I^2 Rt}{4{,}180} \simeq \frac{I^2 Rt}{4{,}200} \qquad [\because 1 \text{ kcal} = 4{,}180 \text{ J} \simeq 4{,}200 \text{ J}]$$

1.16. SERIES CIRCUITS

When the resistors are connected end to end, so that they form only one path for the flow of current, then resistors are said to be connected in series and such circuits are known as *series circuits*.

Let resistors R_1, R_2 and R_3 be connected in series, as shown in Fig. 1.6, and the potential difference of V volts be applied between extreme ends A and D to cause flow of current of I amperes through all the resistors R_1, R_2 and R_3.

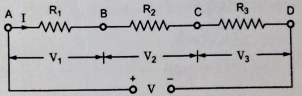


Fig. 1.6 *Series Circuit*

Now according to Ohm's law

Voltage drop across resistor R_1, $V_1 = I R_1$

Voltage drop across resistor R_2, $V_2 = I R_2$

Voltage drop across resistor R_3, $V_3 = I R_3$

Voltage drop across whole circuit,

$$V = \text{Voltage drop across resistor } R_1 + \text{voltage drop across resistor } R_2 + \text{voltage drop across resistor } R_3$$

$$i.e., \; V = I R_1 + I R_2 + I R_3 = I (R_1 + R_2 + R_3)$$

$$\text{or} \quad \frac{V}{I} = R_1 + R_2 + R_3 \qquad ...(1.17)$$

And according to Ohm's law $\frac{V}{I}$ gives the whole circuit resistance, say R

∴ Effective resistance of the series circuit,

$$R = R_1 + R_2 + R_3 \qquad ...(1.18)$$

Thus, when a number of resistors are connected in series, the equivalent resistance is given by the arithmetic sum of their individual resistances.

$$i.e., \; R = R_1 + R_2 + R_3 + ... R_n \qquad ...(1.19)$$

From the above discussions for a series circuit we conclude that

1. same current flows through all parts of the circuit,
2. applied voltage is equal to the sum of voltage drops across the different parts of the circuit,
3. different resistors have their individual voltage drops,
4. voltage drop across individual resistor is directly proportional to its resistance, current being the same in each resistor,
5. voltage drops are additive,
6. resistances are additive,
7. powers are additive.

Series circuits are common in electrical equipment. The tube filaments in small radios are usually in series. Current controlling devices are wired in series with the controlled equipment. Fuses are in series with the equipment they protect. A thermostat switch is in series with the heating element in an electric iron. Automatic house-heating equipment has a thermostat, electromagnet coils, and safety cut-outs in series with a voltage source. Rheostats are placed in series with the coils in large motors for motor current control.

Example 1.4

Three resistors are connected in series across a 12 V battery. The first resistor has the value of 1 ohm, second has a voltage drop of 4 V and third has a power dissipation of 12 W. Calculate the value of each resistance and circuit current.

Solution: Let the three resistors be of R_1 (= 1 Ω), R_2 and R_3 ohms, current flowing through the three resistors R_1, R_2 and R_3 be of I amperes and voltage drops across resistors R_1, R_2 and R_3 be of V_1, V_2 and V_3 volts respectively. The circuit is shown in Fig. 1.7.

Now $V_1 = I R_1 = I$ volts

$V_2 = I R_2 = 4$ volts

$$V_3 = \frac{\text{Power dissipation}}{I} = \frac{12}{I} \text{ volts}$$

Since $V = V_1 + V_2 + V_3$

So $12 = I + 4 + \dfrac{12}{I}$

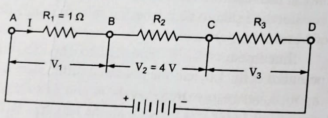


Fig. 1.7

$$\text{or } I^2 - 8I + 12 = 0$$

$$\text{or } (I - 6)(I - 2) = 0$$

$$\text{or } I = 6 \text{ or } 2 \text{ A } \textbf{Ans.}$$

When I = 6 A;
$$R_2 = \frac{V_2}{I} = \frac{4}{6} = \frac{2}{3}\,\Omega$$

$$\text{and } R_3 = \frac{P}{I^2} = \frac{12}{6^2} = \frac{1}{3}\,\Omega$$

$$i.e., \; R_1 = 1\,\Omega; \; R_2 = \frac{2}{3}\,\Omega \text{ and } R_3 = \frac{1}{3}\,\Omega \textbf{ Ans.}$$

When I = 2 A;
$$R_2 = \frac{V_2}{I} = \frac{4}{2} = 2\,\Omega$$

$$\text{and } R_3 = \frac{P}{I^2} = \frac{12}{2^2} = 3\,\Omega$$

$$i.e., \; R_1 = 1\,\Omega; \; R_2 = 2\,\Omega \text{ and } R_3 = 3\,\Omega \textbf{ Ans.}$$

Example 1.5

A 100 V, 60 watt bulb is to be operated from a 220 V supply. What is the resistance to be connected in series with the bulb to glow normally ?

Solution: Rated power of lamp, P = 60 W

Rated voltage of lamp, V = 100 V

Current drawn by the lamp, when operated on rated voltage, *i.e.*,

$$\text{Rated current, } I = \frac{P}{V} = \frac{60}{100} = 0.6 \text{ A}$$

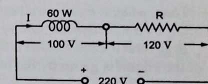


Fig. 1.8

Lamp will operate normally on 220 V also if the current flowing through the lamp remains the rated current *i.e.*, 0.6 A.

Let the resistance connected in series with the lamp to make it glow normally on 220 V be of R ohms, as shown in Fig. 1.8.

Now since the resistance R is in series with the lamp, the same current will flow through the resistance R, as in the lamp *i.e.*, 0.6 A and voltage drop across series resistance R will be equal to supply voltage less voltage drop across the lamp (*i.e.*, rated voltage of the lamp)

or Voltage drop across the series resistance, I R = Supply voltage – rated voltage of the lamp

$$= 220 - 100 = 120 \text{ V}$$

$$\text{or } R = \frac{120}{I} = \frac{120}{0.6} = 200\,\Omega \textbf{ Ans.}$$

1.17. PARALLEL CIRCUITS

When a number of resistors are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point, as shown in Fig. 1.9, then resistors are said to be connected in parallel and such circuits are known as *parallel circuits*. In these circuits current is divided into as many paths as the number of resistances.

Let the resistors R_1, R_2 and R_3 be connected in parallel, as shown in Fig. 1.9, and the potential difference of V volts be applied across the circuit.

Since potential difference across each resistor is same and equal to potential difference applied to the circuit *i.e.*, V

∴ According to Ohm's law

$$\text{Current in resistor } R_1, \; I_1 = \frac{V}{R_1} \qquad ....(1.20)$$

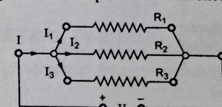


Fig. 1.9 *Parallel Circuit*

Current in resistor R_2, $I_2 = \frac{V}{R_2}$ ...(1.21)

Current in resistor R_3, $I_3 = \frac{V}{R_3}$ ...(1.22)

Adding Eqs. (1.20), (1.21) and (1.22), we have

$$I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad ...(1.23)$$

and since $I_1 + I_2 + I_3 = I$, the total current flowing through the circuit

$$\text{so } I = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\text{or } \frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

and since $\frac{I}{V} = \frac{1}{R}$ where R is the equivalent resistance of the whole circuit.

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad ...(1.24)$$

Thus, when a number of resistors are connected in parallel, the reciprocal of the equivalent resistance is given by the arithmetic sum of the reciprocals of their individual resistances.

In general if n resistors of resistances $R_1, R_2, R_3, ... R_n$ are connected in parallel, then equivalent resistance R of the circuit is given by the expression

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n} \quad ...(1.25)$$

$$\text{Also } G = G_1 + G_2 + G_3 +, ... G_n \quad ...(1.26)$$

where $G = \frac{1}{R}$, $G_1 = \frac{1}{R_1}$, $G_2 = \frac{1}{R_2}$, $G_3 = \frac{1}{R_3}$ and so on.

From the above discussions for a parallel circuit we conclude that

1. same voltage acts across all branches of the circuit,
2. different resistors (or branches) have their individual currents,
3. total circuit current is equal to the sum of individual currents through the various resistors (or branches),
4. branch currents are additive,
5. conductances are additive,
6. powers are additive,
7. the reciprocal of the equivalent or combined resistance is equal to the sum of the reciprocals of the resistances of the individual branches.

Parallel circuits are very common in use. Various lamps and appliances in a house are connected in parallel, so that each one can be operated independently. A series circuit is an "all or none" circuit, in which either every thing operates or nothing operates. For individual control, devices are wired in parallel.

1.18. CURRENT DISTRIBUTION IN PARALLEL CIRCUITS

Let two resistors of resistances R_1 and R_2 be connected in parallel across a pd of V volts. According to Ohm's law

Current flowing through resistor R_1, $I_1 = \frac{V}{R_1}$ ...(1.27)

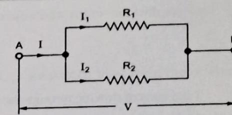


Fig. 1.10

Current flowing through resistor R_2, $I_2 = \frac{V}{R_2}$ ...(1.28)

Dividing Eq. (1.27) by Eq. (1.28), we have

$$\frac{I_1}{I_2} = \frac{V/R_1}{V/R_2} = \frac{R_2}{R_1} \quad ...(1.29)$$

Hence, current flowing through each resistor, when connected in parallel; is inversely proportional to their respective resistances.

Since conductance is reciprocal of resistance and if G_1 and G_2 are the respective conductances of resistors R_1 and R_2 then

$$\frac{1}{G_1} = R_1 \text{ and } \frac{1}{G_2} = R_2$$

Substituting the above values of R_1 and R_2 in Eq. (1.29), we have

$$\frac{I_1}{I_2} = \frac{1/G_2}{1/G_1} = \frac{G_1}{G_2} \quad ...(1.30)$$

Adding 1 on both sides of the above equation, we have

$$\frac{I_1}{I_2} + 1 = \frac{G_1}{G_2} + 1$$

$$\text{or } \frac{I_1 + I_2}{I_2} = \frac{G_1 + G_2}{G_2}$$

$$\text{or } I_2 = \frac{I_1 + I_2}{G_1 + G_2} \times G_2$$

$$\text{Since } I_1 + I_2 = I \text{ and } G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} = G$$

$$\therefore I_2 = I \times \frac{G_2}{G} \quad ...(1.31)$$

$$\text{and } I_1 = I \times \frac{G_1}{G} \quad ...(1.32)$$

Hence, current in any branch of a parallel circuit is directly proportional to its respective conductance and is equal to the total current flowing through the circuit multiplied by the ratio of the conductance of the branch to that of the circuit.

The same relation holds good for parallel circuit consisting of more than two resistors and is very useful for its solution.

Let us consider a circuit consisting of resistances R_1, R_2, R_3 and R_4 ohms respectively connected in parallel across a potential difference of V volts, as shown in Fig. 1.11.

Conductance of first resistor, $G_1 = \frac{1}{R_1}$

Conductance of second resistor, $G_2 = \frac{1}{R_2}$

Conductance of third resistor, $G_3 = \frac{1}{R_3}$

Conductance of fourth resistor, $G_4 = \frac{1}{R_4}$

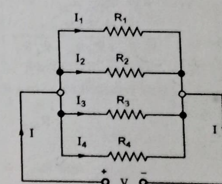


Fig. 1.11

Total conductance of the whole circuit is given by

$$G = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$
$$= G_1 + G_2 + G_3 + G_4$$

Total current flowing through the circuit, $I = \frac{V}{R} = V \times G$

$$\text{or } V = \frac{I}{G} \quad ...(1.33)$$

Current flowing through resistance R_1, $I_1 = \frac{V}{R_1} = \frac{I}{G} \times G_1 = \frac{G_1}{G} I$

Similarly current flowing through resistance R_2, $I_2 = V \times G_2 = I \times \frac{G_2}{G}$

Current flowing through resistance R_3, $I_3 = V \times G_3 = I \times \frac{G_3}{G}$

and current flowing through resistance R_4, $I_4 = V \times G_4 = I \times \frac{G_4}{G}$.

1.19. SERIES-PARALLEL CIRCUITS

So far, only simple series and simple parallel circuits have been considered. Practical electric circuits very often consist of combinations of series and parallel resistances. Such circuits may be solved by the proper application of Ohm's law and the rules for series and parallel circuits to the various parts of the complex circuit. There is no definite procedure to be followed in solving complex circuits, the solution depends on the known facts concerning the circuit and the quantities which one desires to find. One simple rule may usually be followed, however—reduce the parallel branches to an equivalent series branch and then solve the circuit as a simple series circuit.

For example, consider a series-parallel circuit shown in Fig. 1.12 for solution.

First of all equivalent resistances of all parallel branches are determined separately *e.g.*, of branches AB and CD by the law of parallel circuits discussed in Art. 1.17.

Equivalent resistance of parallel branch AB,

$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

and equivalent resistance of parallel branch CD,

$$R_{CD} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}}$$
$$= \frac{R_4 R_5 R_6}{R_5 R_6 + R_4 R_6 + R_4 R_5}$$

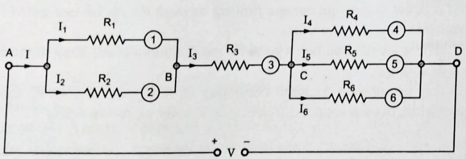


Fig. 1.12

Now the circuit shown in Fig. 1.12 gets reduced to a simple series circuit shown in Fig. 1.13 consisting of three resistances,

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}, \quad R_{BC} = R_3$$

$$\text{and } R_{CD} = \frac{R_4 R_5 R_6}{R_5 R_6 + R_4 R_6 + R_4 R_5}$$

Total resistance of circuit, $R_T = R_{AB} + R_{BC} + R_{CD}$

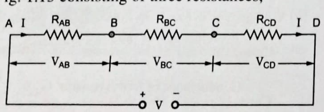


Fig. 1.13

Now circuit current may be determined from the relation

$$I = \frac{V}{R_T}$$

After knowing I, potential differences across branches AB, BC and CD are determined from the relations

PD across branch AB, $V_{AB} = I.R_{AB} = \frac{V}{R_T} R_{AB}$

PD across branch BC, $V_{BC} = I.R_{BC} = \frac{V}{R_T} R_{BC}$

and PD across branch CD, $V_{CD} = IR_{CD} = \frac{V}{R_T} R_{CD}$.

After determination of potential difference across each parallel branch, the currents in the various resistances are determined from the relations

Current in resistance $R_1 = I_1 = \frac{V_{AB}}{R_1}$

Current in resistance $R_2 = I_2 = \frac{V_{AB}}{R_2}$

Current in resistance $R_3 = I_3 = I$

Current in resistance $R_4 = I_4 = \frac{V_{CD}}{R_4}$

Current in resistance $R_5 = I_5 = \frac{V_{CD}}{R_5}$

Current in resistance $R_6 = I_6 = \frac{V_{CD}}{R_6}$

Thus, equivalent resistance of the whole circuit, voltage drop across each branch and currents in the various resistors may be determined.

Example 1.6

Two resistances of 20 Ω and 30 Ω respectively are connected in parallel. These two parallel resistances are further connected in series with a resistance of 15 Ω. If the current through the 15 Ω resistance is 3 A find (*a*) the currents through the 20 Ω and 30 Ω resistances respectively (*b*) the voltage across the whole circuit (*c*) the total power consumed.

Solution: Equivalent resistance of branch AB, $R_{AB} = \frac{1}{\frac{1}{20} + \frac{1}{30}} = 12\ \Omega$

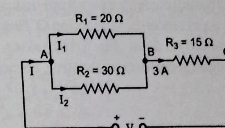


Fig. 1.14

Effective resistance of the circuit, $R_{eff} = R_{AB} + R_{BC} = 12 + 15 = 27\ \Omega$

Circuit current, I = Current through 15 Ω resistance
= 3 A

(*b*) Voltage across the whole circuit, $V = I R_{eff} = 3 \times 27 = 81$ V **Ans.**

(*c*) Total power consumed, $P = VI = 81 \times 3 = 243$ watts **Ans.**

(*a*) Voltage drop across branch AB, $V_{AB} = I R_{AB} = 3 \times 12 = 36$ V

Current through 20 Ω resistance, $I_1 = \frac{V_{AB}}{R_1} = \frac{36}{20} = 1.8$ A **Ans.**

Current through 30 Ω resistance, $I_2 = \frac{V_{AB}}{R_2} = \frac{36}{30} = 1.2$ A **Ans.**

Example 1.7

For the circuit shown in Fig. 1.15, using the method of series-parallel combination, find V_1 and I_2.

Solution: Total resistance of the circuit,

$$R_{eq} = 25 + 2.5 + \text{equivalent resistance of parallel combination of resistors of 10, 50 and 20 }\Omega$$

$$= 25 + 2.5 + \frac{1}{\frac{1}{10}+\frac{1}{50}+\frac{1}{20}}$$

$$= 25 + 2.5 + \frac{100}{10+2+5} = \frac{1,135}{34}\ \Omega$$

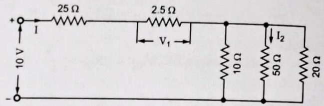


Fig. 1.15

Applied voltage, V = 10 V

$$\text{Current drawn from the supply, } I = \frac{V}{R_{eq}} = \frac{10}{1,135/34} = \frac{10\times 34}{1,135} = \frac{68}{227}\ \text{A}$$

$$\text{Voltage drop across resistor of 2.5 }\Omega,\ V_1 = I \times 2.5 = \frac{68}{227}\times 2.5 = \frac{170}{227}\ \text{V} \quad \textbf{Ans.}$$

$$\text{Voltage drop across parallel combination, } V_2 = \frac{68}{227}\times\frac{100}{17} = \frac{400}{227}\ \text{V}$$

$$\text{Current through 50 }\Omega\text{ resistor, } I_2 = \frac{V_2}{50} = \frac{400}{227\times 50} = \frac{8}{227}\ \text{A} \quad \textbf{Ans.}$$

1.20. KIRCHHOFF'S LAWS

The basic laws, that electric circuits follow rationally from the nature of the electrical quantities, have already been defined. They lead directly to methods for the systematic analysis of electric circuits. These laws are known as *Kirchhoff's laws*, and they describe the relationships among circuit voltages and circuit currents that must be satisfied. These laws are very helpful in determining the equivalent resistance (or impedance) of a complex network and the current flowing in the various branches of the network.

Gustov Robert Kirchhoff derived two basic laws governing networks, one commonly known as *first law* or *current law* (KCL) or *point law*, whereas the second is called *second law* or *voltage law* (KVL) or *mesh law*.

1. Kirchhoff's First Law or Current Law (KCL) or Point Law. According to this law in any network of wires carrying currents, the algebraic sum of all currents meeting at a point (or junction) is zero or the sum of incoming currents towards any point is equal to the sum of outgoing currents away from that point. If I_1, I_2, I_3, I_4, I_5, and I_6 are the currents meeting at junction O, flowing in the directions of arrowheads marked on them (Fig. 1.16), taking incoming currents as positive and outgoing currents as negative, according to Kirchhoff's first law (KCL)

$$I_1 - I_2 - I_3 + I_4 + I_5 - I_6 = 0$$

$$\text{or } I_1 + I_4 + I_5 = I_2 + I_3 + I_6$$

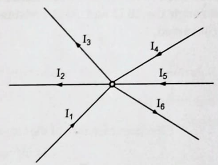


Fig. 1.16

2. Kirchhoff's Second Law or Voltage Law (KVL) or Mesh Law. According to this law in any closed circuit or mesh the algebraic sum of emfs acting in that circuit or mesh is equal to the algebraic sum of the products of the currents and resistances of each part of the circuit.

If the circuit shown in Fig. 1.17 is considered, then according to Kirchhoff's second law (or KVL)

In mesh AFCBA,

$$E_1 = R_1(I_1 + I_2) + R_2(I_1 + I_2) + R_5 I_1$$

$$\text{or } E_1 = (R_1 + R_2)(I_1 + I_2) + R_5 I_1$$

In mesh FEDCF,

$$-E_2 = -R_3 I_2 - R_4 I_2 - R_2(I_1 + I_2) - R_1(I_1 + I_2)$$

$$\text{or } E_2 = R_3 I_2 + R_4 I_2 + (R_1 + R_2)(I_1 + I_2)$$

and in mesh AFEDCBA,

$$E_1 - E_2 = -R_3 I_2 - R_4 I_2 + R_5 I_1$$

$$\text{or } E_1 - E_2 = R_5 I_1 - (R_3 + R_4) I_2$$

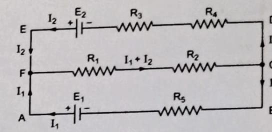


Fig. 1.17

1.20.1. Application of Kirchhoff's Laws To Circuits. First of all the current distribution in various branches of the circuit is made with directions of their flow complying with first law of Kirchhoff. Then Kirchhoff's second law is applied to each mesh (one by one) separately and algebraic equations are obtained by equating the algebraic sum of emfs acting in a mesh equal to the algebraic sum of respective drops in the same mesh. By solving the equations so obtained unknown quantities can be determined. While applying Kirchhoff's second law, the question of algebraic signs may be troublesome and is a frequent source of error. If, however, the following rules are kept in mind, no difficulty should occur.

The resistive drops in a mesh due to current flowing in clockwise direction must be taken positive drops.

The resistive drops in a mesh due to current flowing in counter-clockwise direction must be taken as negative drops.

Similarly the battery emf causing current to flow in clockwise direction in a mesh must be taken as positive emf and the battery emf causing current to flow in counter-clockwise direction in a mesh must be taken as negative emf.

For example, for the circuit shown in Fig. 1.17 let the current distribution be made as shown, which satisfy Kirchhoff's first law fully.

Taking first, mesh AFCBA for the application of Kirchhoff's second law, we see that there is only one emf acting in the mesh (E_1) and since it tries to send current in clockwise direction so E_1 be taken as positive, similarly all the resistive drops *i.e.*, $R_1(I_1 + I_2)$, $R_2(I_1 + I_2)$ and $R_5 I_1$ are clockwise, so these must be taken as positive.

∴ According to Kirchhoff's second law in mesh AFCBA

$$E_1 = R_1(I_1 + I_2) + R_2(I_1 + I_2) + R_5 I_1$$

In mesh FEDCF, there is only one emf acting in the mesh (E_2) and since it tries to send current in counter-clockwise direction through the mesh under consideration, it may be taken as negative. Since all of the resistive drops $R_3 I_2$, $R_4 I_2$, $R_2(I_1 + I_2)$ and $R_1(I_1 + I_2)$ are counter-clockwise, these may be taken as negative.

Hence according to Kirchhoff's second law in mesh FEDCF, we get,

$$-E_2 = -R_3 I_2 - R_4 I_2 - R_2(I_1 + I_2) - R_1(I_1 + I_2)$$

$$\text{or } E_2 = (R_3 + R_4) I_2 + (R_1 + R_2)(I_1 + I_2)$$

In mesh AFEDCBA, emf E_1 tries to cause current in clockwise direction so be taken as positive and emf E_2 tries to cause current in counter-clockwise direction so be taken as negative. Similarly resistive drop $R_5 I_1$ being clockwise be taken as positive and resistive drops $R_3 I_2$ and $R_4 I_2$ being counter-clockwise be taken as negative.

$$\text{Hence } E_1 - E_2 = -R_3 I_2 - R_4 I_2 + R_5 I_1 = R_5 I_1 - (R_3 + R_4) I_2$$

Example 1.8

With reference to Fig. 1.18, given $i_1 = -\frac{1}{2}e^{-2t}$, $V_3 = 2e^{-2t}$, $V_4 = 2e^{-2t}$. Find V_2.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007]

Solution: Applying Kirchhoff's current law at junction O, we have

$$i_1 - i_2 + i_3 - 6i_1 + i_4 = 0$$

$$\text{or } i_2 = -5i_1 + i_3 + i_4 \qquad ...(i)$$

Now current $i_1 = -\frac{1}{2}e^{-2t}$

Current $i_3 = C\frac{dV_3}{dt} = \frac{1}{2}\frac{d}{dt}(2e^{-2t}) = -2e^{-2t}$

$i_4 = \frac{V_4}{R} = \frac{2e^{-2t}}{10} = 0.2e^{-2t}$

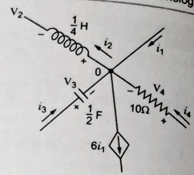


Fig. 1.18

Substituting the values of i_1, i_3 and i_4 in Eq. (*i*), we have

$$i_2 = -5\times\left(-\frac{1}{2}e^{-2t}\right)+(-2e^{-2t})+0.2e^{-2t} = 0.7e^{-2t}$$

The voltage V_2 developed across the coil is

$$V_2 = L\frac{di_2}{dt} = \frac{1}{4}\frac{d}{dt}(0.7e^{-2t}) = \frac{1}{4}\times 0.7\times(-2)e^{-2t} = -0.35e^{-2t} \quad \textbf{Ans.}$$

Example 1.9

In the network given in Fig. 1.19, find i_4 if $V_1 = 6$ V, $V_2 = 2\sin 4t$ and $i_3 = \frac{1}{2}e^{-2t}$.

Solution: Applying Kirchhoff's voltage law (KVL) to the closed mesh ADCBA, we have

$$-V_4 + V_3 + V_2 - V_1 = 0 \quad \text{...(i)}$$

Now $V_3 = L\frac{di_3}{dt} = 4\frac{d}{dt}\left(\frac{1}{2}e^{-2t}\right) = 4\times\frac{1}{2}\times(-2)e^{-2t} = -4e^{-2t}$

Substituting values of V_1, V_2 and V_3 in Eq. (*i*), we have

$$-V_4 + (-4e^{-2t}) + 2\sin 4t - 6 = 0$$

or $V_4 = 2\sin 4t - 4e^{-2t} - 6$

Now current $i_4 = C\frac{dV_4}{dt}$

$= 6\frac{d}{dt}[2\sin 4t - 4e^{-2t} - 6]$

$= 6[2\times 4\cos 4t - 4\times(-2)e^{-2t}]$

$= 48\cos 4t + 48e^{-2t}$ **Ans.**

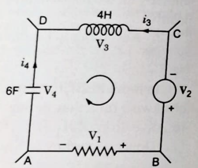


Fig. 1.19

Example 1.10

Obtain I_R in terms of I_s in Fig. 1.20. [*G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007*]

Solution: Applying Kirchhoff's current law (KCL) to node A, we have

$$I_s = i_r + I_R \quad \text{...(i)}$$

Applying KVL to closed mesh ABCD, we have

$I_R R - i_r r + \alpha v = 0$

or $I_R R - i_r r + \alpha i_r r = 0 \quad \because v = i_r r$

or $I_R R = r i_r (1-\alpha)$

$= r(I_s - I_R)(1-\alpha)$

or $I_R = \frac{r(1-\alpha)}{R + r(1-\alpha)}\cdot I_s$ **Ans.**

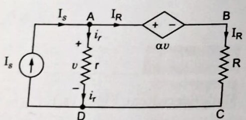


Fig. 1.20

$\because$ From Eq. (*i*) $I_r = I_s - I_R$

Example 1.11

Find the current I in the circuit given in Fig. 1.21(*a*). [*U.P. Technical Univ. Electrical Engineering Second Semester 2006-07*]

Solution: Applying Kirchhoff's current law at junction B, we have

Current supplied by 24 V battery, $I_1 = (I + 4)$ amperes

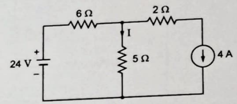
Applying Kirchhoff's voltage law to mesh ABEFA, we have

$6I_1 + 5I = 24$

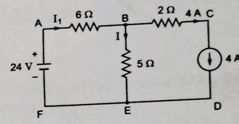
or $6(I+4) + 5I = 24$

or $11I + 24 = 24$

or $I = 0$ **Ans.**



(*a*)



(*b*)

Fig. 1.21

Example 1.12

Fig. 1.22 shows two batteries connected in parallel each represented by an emf along with its internal resistance. A load resistance of 6 Ω is connected across the ends of the batteries. Calculate the current through each battery and the load.

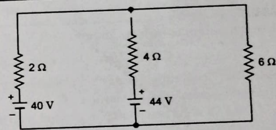


Fig. 1.22

Solution: Let the current distribution in the network be as shown in Fig. 1.23.

Applying KVL to meshes ABEFA and BCDEB respectively, we have

$2I_1 - 4I_2 = 40 - 44$

or $2I_2 - I_1 = 2 \quad \text{...(i)}$

and $4I_2 + 6(I_1 + I_2) = 44$

or $5I_2 + 3I_1 = 22 \quad \text{...(ii)}$

Solving equations (*i*) and (*ii*), we have

$I_1 = \frac{34}{11}$ A **Ans.**

$I_2 = \frac{28}{11}$ A **Ans.**

Current through load, $I_L = I_1 + I_2 = \frac{34}{11} + \frac{28}{11} = \frac{62}{11}$ A **Ans.**

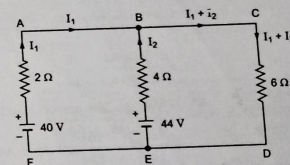


Fig. 1.23

Example 1.13

In Fig. 1.24 the potential of point A = – 30 V. Using Kirchhoff's laws find (*a*) value of V and (*b*) power dissipated by 5 Ω resistance.

Solution: The potential of point A is – 30 V and potential of point G is zero, being grounded. So the *pd* across 12 Ω resistor connected between terminals A and H is 30 V and current flowing through 12 Ω resistor is 30/12 *i.e.*, 2.5 A from terminal H to A.

Let the current distribution in the other branches of the circuit be, as shown in Fig. 1.25. Applying Kirchhoff's current law (KCL) to node H, we have

$$I_2 + I_3 = 2.5 \quad ...(i)$$

Applying Kirchhoff's voltage law to meshes CHGDC and AFEGHA respectively, we have

$$6 I_2 - 3(I_1 - I_2) = 0$$

$$\text{or } I_2 = \frac{I_1}{3} \quad ...(ii)$$

$$\text{and } 4(I_1 - I_2 - I_3) - 12 \times 2.5 = 0$$

$$\text{or } I_1 - (I_2 + I_3) = 7.5$$

Substituting $I_2 + I_3 = 2.5$ from Eq. (i) in the above equation, we have

$$I_1 - 2.5 = 7.5$$

$$\text{or } I_1 = 10 \text{ A}$$

$$\text{and } I_2 = \frac{I_1}{3} = \frac{10}{3} \text{ A}$$

Applying KVL to mesh ABCHA, we have

$$5 I_1 + 6 I_2 + 12 \times 2.5 = V$$

(a) $$\text{or } V = 5 I_1 + 6 I_2 + 30$$

$$= 5 \times 10 + 6 \times \frac{10}{3} + 30 = 100 \text{ V } \textbf{Ans.}$$

(b) Power dissipated in 5 Ω resistor $= I_1^2 \times 5 = 10^2 \times 5 = 500$ W **Ans.**

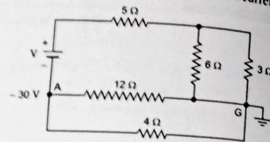


Fig. 1.24

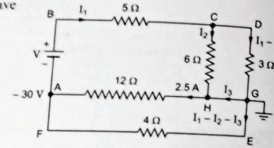


Fig. 1.25

Example 1.14

Use the source transformation to find V_1 of Fig. 1.26(a).

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]

Solution: Converting 4 A current source in parallel with a resistance of 10 Ω into equivalent voltage source of 40 V in series with a resistance of 10 Ω and 15 A current source in parallel with a parallel combination of resistances of 4 Ω and (3 + 1) Ω into equivalent voltage source of 15 × [4 ‖ (3 + 1)] *i.e.*, 15 × 2 or 30 V in series with a resistance of 2 Ω we have the circuit, that is shown in Fig. 1.26(b).

From equivalent circuit shown in Fig. 1.26(b) we have current flowing through the circuit,

$$I = \frac{30 + 40}{2 + 10 + 1} = \frac{70}{13} \text{ A}$$

V_1 = Voltage drop across resistance of 1 Ω

$$= \frac{70}{13} \times 1 = \frac{70}{13} \text{ or } 5.38 \text{ V } \textbf{Ans.}$$

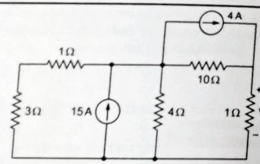


Fig. 1.26(a)

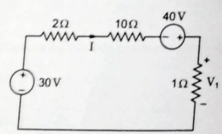


Fig. 1.26(b)

Example 1.15

Find the value of resistance R and the current I_R in Fig. 1.27 when the branch AD carries no current.

Solution: Let the current through the branches AB and DB of the circuit be I_1 and I_2 amperes respectively. Applying KCL to junctions B, A and D respectively, we have

Current supplied by battery or current in branch BC = $I_1 + I_2$

Current through branch CA = I_1 ∵ Current through branch AD is zero

Current through branch CD = I_2

Now applying Kirchhoff's voltage law to meshes ABDA, CADC and BCDB respectively, we have

$$3 I_1 - 12 I_2 + 0 = 0 \quad ...(i)$$

$$\text{or } I_1 = 4 I_2$$

$$2 I_1 + 0 - R I_2 = 0$$

$$\text{or } R = \frac{2 I_1}{I_2} = 2 \times 4 = 8 \ \Omega \textbf{ Ans.}$$

$$\because \text{ from Eq. } (i) \ \frac{I_1}{I_2} = 4$$

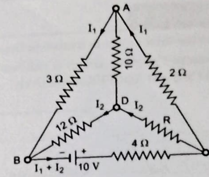


Fig. 1.27

$$\text{and } 4(I_1 + I_2) + R I_2 + 12 I_2 = 10$$

Substituting $I_1 = 4 I_2$ from Eq. (i) and R = 8 Ω, as determined above in the above equation, we have

$$4(4 I_2 + I_2) + 8 I_2 + 12 I_2 = 10$$

$$\text{or } I_2 = \frac{10}{40} = 0.25 \text{ A } \textbf{Ans.}$$

Example 1.16

Determine the current through the 5Ω resistor in Fig. 1.28. [G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007]

Solution: Applying KCL at junction A respectively, we have

$$I = 0.5i + i = 1.5i \quad ...(i)$$

Applying KVL to outer closed loop, we have

$$2I + 1 \times i + 5i = 6$$

$$\text{or } 2 \times 1.5i + 6i = 6$$

$$\because \text{ From Eq. } (i) \ I = 1.5i$$

$$\text{or } i = \frac{6}{9} = \frac{2}{3} \text{ A } \textbf{ Ans.}$$

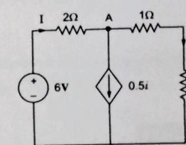


Fig. 1.28

Example 1.17

Applying Kirchhoff's current law, determine current I_s in the electric circuit of Fig. 1.29. Take V_0 = 16 V.

Solution : Let the current flowing through the various branches of the circuit, be as shown in the Fig. 1.29.

Now applying Kirchhoff's current law to nodes A and B respectively, we have

$$I_1 = I_2 + I_s \quad ...(i)$$

$$\text{and } I_2 + I_3 = \frac{V_1}{4} \quad ...(ii)$$

Also voltage of node B = V_0 = 16 V

$$\text{and, therefore, } 4 I_2 + V_1 = 16 \quad ...(iii)$$

$$I_1 = \frac{V_1}{6} \quad ...(iv)$$

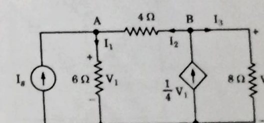


Fig. 1.29

Solving Eqs. (i), (ii), (iii) and (iv), we have

$$V_1 = 12 \text{ V}; I_1 = 2 \text{ A}; I_2 = 1 \text{ A and } I_s = I_1 - I_2 = 2 - 1 = 1 \text{ A}$$

$$\text{So } I_s = 1 \text{ A } \textbf{Ans.}$$

Example 1.18

Find the total power delivered to the circuit by two sources in Fig. 1.30(*a*).

Solution: The given circuit is redrawn with assumed distribution of current in Fig. 1.30(*b*). Applying KVL to meshes AEFCBA and ADCFEA respectively, we have

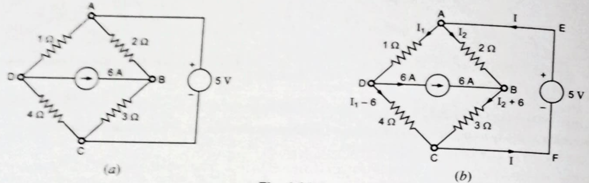


Fig. 1.30

$$2\,I_2 + (I_2 + 6) \times 3 = 5$$

$$\text{or } I_2 = \frac{-13}{5} = -2.6 \text{ A}$$

$$\text{and } I_1 + 4\,(I_1 - 6) = 5$$

$$\text{or } I_1 = \frac{29}{5} = 5.8 \text{ A}$$

Total power delivered by the two sources = Total power absorbed by the circuit

$$= I_2^{\,2} \times 2 + (I_2 + 6)^2 \times 3 + (I_1 - 6)^2 \times 4 + I_1^{\,2} \times 1$$

$$= (-2.6)^2 \times 2 + (-2.6 + 6)^2 \times 3 + (5.8 - 6)^2 \times 4 + (5.8)^2 \times 1 = 82 \text{ W } \textbf{Ans.}$$

Example 1.19

Calculate the current through the galvanometer in the following bridge (Fig. 1.31).

Solution: Assume current distribution in the bridge network as shown in Fig. 1.32.

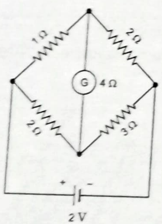


Fig. 1.31

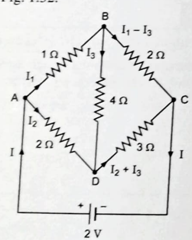


Fig. 1.32

Applying Kirchhoff's second law to meshes ABDA, BCDB and ABCA respectively, we have

$$I_1 + 4\,I_3 - 2\,I_2 = 0 \qquad ...(i)$$

$$2(I_1 - I_3) - 3\,(I_2 + I_3) - 4\,I_3 = 0$$

$$\text{or } 2\,I_1 - 3\,I_2 - 9\,I_3 = 0 \qquad ...(ii)$$

$$\text{and } I_1 + 2\,(I_1 - I_3) = 2$$

$$\text{or } 3\,I_1 - 2\,I_3 = 2 \qquad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*), we get

$$I_3 = \frac{1}{44}\text{ A} ; I_2 = \frac{17}{44} \text{ A and } I_1 = \frac{30}{44} \text{ A}$$

$$\text{So current through galvanometer } = I_3 = \frac{1}{44} \text{ A } \textbf{Ans.}$$

HIGHLIGHTS

1. All matters are made of electrons, protons and neutrons.
2. The controlled movement of electrons (or drift) through a substance is called the electric current. Current is expressed in amperes.
3. Electromotive force (emf) or potential difference is that which causes the current to flow in a closed circuit. The unit of potential difference in SI system is volt.
4. Resistance may be defined as that property of substance which opposes (or restricts) the flow of an electric current (or electrons) through it. The unit of resistance in SI system is ohm (Ω).
5. Ohm's law may be given as
$$\frac{V}{I} = \text{Constant} = R \qquad \text{for no change in physical state}$$
6. Laws of resistance are summed up as
$$R = \rho\,\frac{l}{a}$$
where ρ is the specific resistance or resistivity of the material of wire, which may be defined as the resistance between opposite faces of a unit cube of that material.
Resistivity is measured in ohm-metres (Ω-m) or ohms per metre cube.
7. Conductance G is reciprocal of resistance *i.e.*, $G = \frac{1}{R}$. The unit of conductance is siemens (S).
8. Conductivity σ is reciprocal of resistivity ρ *i.e.*, $\sigma = \frac{1}{\rho}$. The unit of conductivity is siemens per metre.
9. A current (or voltage) is called alternating if it periodically changes its direction and magnitude whereas direct currents are steady and flow in one direction only.
10. In case of good conductors, especially metals, the variations in resistance with temperature (within normal range) are given by
$$R_t = R_0\,(1 + \alpha_0 t)$$
Similarly, resistivity of given material varies as
$$\rho_t = \rho_0\,(1 + \alpha_0 t)$$
11. Temperature coefficient of resistance is not constant but depends upon the initial temperature on which the increment in resistance is based. The values of temperature coefficient for different temperatures can be had from the relations
$$\alpha_1 = \frac{1}{\frac{1}{\alpha_0} + t_1} \quad \text{and} \quad \alpha_2 = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$
12. *Power* is defined as the rate of doing work or the amount of work done in unit time. It is expressed in watts or kilowatts.
$$1 \text{ kW} = 1{,}000 \text{ watts}$$
Power dissipated in a resistor of R ohms is given as
$$P = VI = I^2R = \frac{V^2}{R} \text{ watts}$$

13. Energy is defined as the capacity of doing work. Its units are joules, newton-metres, watt-seconds or kilowatt-hours (kWh)

$$1 \text{ kWh} = 3{,}600{,}000 \text{ watt-seconds or joules}$$

Energy expended or heat generated in a conductor of resistance R ohms when a current of I amperes flows through it for t seconds is given as

$$H = I^2Rt = VIt = \frac{V^2}{R}t \text{ watt-seconds or joules}$$

14. When a number of resistors are connected in series, the equivalent resistance is given by the arithmetic sum of their individual resistances

i.e., $R = R_1 + R_2 + R_3 + ... + R_n$

15. When a number of resistors are connected in parallel, the reciprocal of the equivalent resistance is given by the arithmetic sum of the reciprocals of their individual resistances

$$i.e., \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

16. When two resistances R_1 and R_2 are in parallel and carry a total current I, then

$$\text{current through resistance } R_1, I_1 = \frac{R_2}{R_1 + R_2} I$$

$$\text{and current through resistance } R_2, I_2 = \frac{R_1}{R_1 + R_2} I$$

17. When n resistors, each of R ohms, are connected in parallel, then total resistance is given by $R_T = \frac{R}{n}$ ohms.

18. Current in any branch of a parallel circuit is directly proportional to its conductance and is equal to the total current flowing through the circuit multiplied by the ratio of the conductance of the branch to that of the circuit.

19. According to *Kirchhoff's first law* (*or current law*), the algebraic sum of currents in two or more conductors at a point (junction) is always zero

$$i.e., \Sigma I = 0$$

While applying above law, incoming currents are taken as positive and outgoing currents as negative.

According to *Kirchhoff's second law* (*or voltage law*), the algebraic sum of emfs acting in any closed circuit or mesh is equal to the algebraic sum of the products of currents and resistances of each part of that closed circuit or mesh *i.e.*,

$$\Sigma IR = \Sigma \text{ emf}$$

EXERCISES

1. What is ohm's law ? State its limitations.
[Pb. Technical Univ. Basic Electrical and Electronics Engineering December 2006]
2. State laws of resistance and hence define specific resistance.
3. On what factors does the resistance offered by a conductor depend ?
4. What is the effect of temperature on the resistance of copper, aluminium, porcelain and carbon ? Define temperature coefficient of resistance. How does its value change with temperature ?
5. How is energy related to power ? State the SI units of work, power and energy. What are the practical units of electrical energy and mechanical power ?
6. Explain the terms watt, joule, watt-hour and kWh.
7. State and explain Joule's law of electric heating.
8. Compute equivalent resistance of three resistances R_1, R_2 and R_3 connected in (*i*) series (*ii*) parallel.
9. State the Kirchhoff's laws as applied in electric circuits. **[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]**
10. State Kirchhoff's current and voltage laws. Explain with suitable examples.
[M.D. Univ. Rohtak Electrical Technology 2006-07]

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. Do conventional current and electron current flow in the same direction in an electric circuit ?
Ans. No, the direction of flow of electrons is opposite to that of conventional current.

Q. 2. What is drift current ?
Ans. The steady flow of electrons in one direction caused by the applied electric field constitutes electric current, called the drift current.

Q. 3. What is difference between emf and potential difference ?
Ans. EMF (electromotive force) is the force that causes a current to flow in an electric circuit while the potential difference between any two points in an electric circuit is that difference in their electrical state which tends to cause flow of electricity from one point to the other.

Q. 4. What is relation between voltage, current and resistance of an electric circuit ?
Ans. $\frac{V}{I} = R$

Q. 5. What are limitations of Ohm's law ? **[G.G.S.I.P. Univ. Delhi 1st Term Exam Feb./March 2011]**
Ans. Ohm's law cannot be applied to the circuits consisting of (*i*) electronic tubes or transistors (because such elements are not bilateral) and (*ii*) nonlinear elements such as powdered iron, electric arc etc.

Q. 6. On what factors does the resistance of a conductor depend ?
Ans. The resistance of a conductor depends upon its length, area of x-section and type of material, purity and hardness of material of which it is made of and the operating temperature.

Q. 7. How does the resistance of a homogeneous material having constant cross section vary with its length ?
Ans. Resistance of a homogeneous material having constant cross section varies linearly with its length *i.e.*, $R \propto l$.

Q. 8. How does the resistance of a homogeneous material having constant length vary with the changing cross-sectional area ?
Ans. Resistance of a homogeneous material having constant length is inversely proportional to its area of x-section *i.e.*, $R \alpha \frac{1}{a}$.

Q. 9. If the length of a wire of resistance R is uniformly stretched to n times its original value, what will be its new resistance ?
Ans. $\text{New resistance} = \text{Old resistance} \times \frac{\text{new length}}{\text{old length}} \times \frac{\text{old cross section}}{\text{new cross section}} = R \times \frac{nl}{l} \times \frac{a}{a/n} = n^2R$

Q. 10. A network consists linear resistors and ideal voltage source. If the value of the resistors are doubled, then voltage across each resistor is
(*i*) Halved (*ii*) Doubled (*iii*) Increased by 4 times (*iv*) Not changed.
[G.G.S.I.P. Univ. Delhi Electrical Science May 2005]
Ans. The magnitude of each linear resistance will be doubled, the equivalent resistance of the network will be doubled, hence the current flowing through each resistance will be halved and the voltage drop across each resistor, being the product of the double the resistance and half the current, will not be changed.

Q. 11. What is meant by conductivity and how is it related with resistivity ?
Ans. Conductivity is defined as the conductance between the two opposite faces of a unit cube. It is reciprocal of resistivity.

Q. 12. How does increase in temperature affect the resistance of (*i*) conductors and (*ii*) insulators ?
Ans. The resistance of the conductors (except of carbon) increases with the increase in temperature, therefore, on high temperature good conductors become bad conductors even.

The insulators lose their insulating power with the increase in temperature and some may even become good conductors if they are strongly heated.

Q. 13. Define temperature coefficient of resistance. **[G.G.S.I.P. Univ. Delhi 1st Term Exam February 2008]**
Ans. The temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree rise of temperature to the original resistance. It is expressed in Ω/Ω/°C.

Q. 14. Give relation between kWh, MJ and kilocalorie.
Ans. 1 kWh = 3.6 MJ = 860 kcals.

Q. 15. Express Joule's law of electric heating.

Ans. According to Joule's law of electric heating, the energy expended or heat generated in joules is given as

$$\text{Work done or heat generated} = I^2Rt = \frac{V^2}{R}t = VIt = Wt \text{ joules.}$$

PROBLEMS

1. A piece of silver wire has a resistance of 1 Ω. What will be the resistance of the manganin wire of one-third length and one-third the diameter of silver wire, if the specific resistance of manganin is 30 times of silver ? **[Ans. 90 Ω]**
2. The resistivity of a ferric-chromium-aluminium alloy is 51×10^{-8} Ω-m. A sheet of material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine the resistance between (*a*) opposite ends and (*b*) opposite sides.
   [Allahabad Univ. Elec. Circuits 1983]
   [Ans. (*a*) 0.0091 Ω (*b*) 79.3×10^{-10} Ω]
3. The equivalent resistance of four resistors joined in parallel is 30 Ω. The currents flowing through them are 4, 2, $\frac{4}{3}$ and $\frac{2}{3}$ amperes. Determine the value of each resistor. **[Ans.** 60 Ω ; 120 Ω ; 180 Ω and 360 Ω]
4. A current of 78 A is divided into three branches. The lengths of the wire in the three branches are in the ratio of 8, 12 and 16. The wires are of same material and cross-sectional area. Calculate the current in each branch.
   [Ans. 36 A, 24 A and 18 A]
5. How many 60 W lamps may be safely run on a 230 V circuit fitted with a 5 A fuse ? **[Ans.** 19]
6. Twenty lamps of 60 watts each are run for 5 hours from 240 V supply. Calculate (a) the units consumed. (b) the current through the mains, and (c) resistance of each lamp. **[Ans.** (*a*) 6 units; (*b*) 5 A; (*c*) 960 Ω]
7. Determine current flowing through 5 Ω resistor in the circuit shown in Fig. 1. Use source transformation technique. **[Ans.** 2.667 A]
8. Two batteries A and B are connected in parallel and a load of 10 Ω is connected across their terminals. A has an emf of 12 V and an internal resistance of 2 Ω and B has an emf of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the magnitude of currents and also the directions in each of the batteries. Also determine the pd across external resistance.

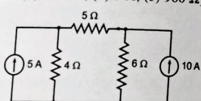


Fig. 1

[Ans. 1.625 A, – 0.75 A and 8.75 V]

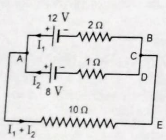


Fig. 2

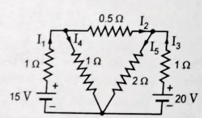


Fig. 3

9. Find the various branch currents in the passive elements of the network, as shown in Fig. 3.
   [Ans. $I_1 = 5.75$ A, $I_2 = -3.5$ A; $I_3 = 9$ A; $I_4 = 9.25$ A, $I_5 = 5.5$ A]
10. A Wheatstone bridge consists of AB = 400 Ω, BC = 220 Ω, CD = 225 Ω and DA = 100 Ω. A 2 V cell is connected between A and C and a galvanometer of 200 Ω resistance between B and D. Estimate the current through the galvanometer by applying Kirchhoff's laws.
    [Pb. Univ. Electrical Engineering-I Nov., 1983]
    [Ans. 2/703 A from D to B]

❑❑❑

UNIT-I

DC CIRCUITS

CONTENTS IN THIS UNIT

Chapter

2

DC Circuits

Contents in this Chapter

2.1. CONCEPT OF NETWORK

An electric circuit (or network) is an interconnection of physical electrical devices. The purpose of electric networks is to distribute and convert electrical energy into some other forms. Accordingly, the basic circuit components are an energy source (or sources), an energy converter (or converters), and conductors connecting them.

An *energy source* (or source), such as a primary or secondary cell, a generator, and the like, is a device that converts chemical, mechanical, thermal or some other form of energy into electrical energy.

An *energy convertor,* also called the *load*, (such as lamp, heating appliance, or an electric motor) converts electrical energy into light, heat, mechanical work and so on.

Events in an electrical circuit may be defined in terms of emf (or voltage) and current.

When electrical energy is generated, transmitted and converted under conditions such that the currents and voltages involved remain constant with time, the electric circuit is identified as *direct current* (dc) *circuit*. If the currents and voltages do change with time, the circuit is defined as *alternating current (ac) circuit*.

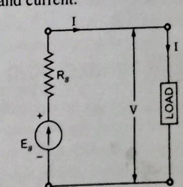


Fig. 2.1

A graphic representation of an electric circuit is called a *circuit diagram* (Fig. 2.1). Such a diagram consists of interconnected symbols called *circuit elements or circuit parameters.* Two elements are necessary to represent processes in a dc circuit. These are source of emf E_s and of internal (or source) resistance R_s and the load resistance (which includes the resistance of the conductors) R.

In any electric circuit the energy convertor (or load) and the conductors connecting it to the source make up the *external circuit* in which current flows from the +ve side to the –ve side of the source whereas inside the source, current flows in the opposite direction, *i.e.*, from the –ve side to the +ve side. The source emf is directed from the terminal at a lower potential to that at a higher one. In diagrams this is shown by arrows.

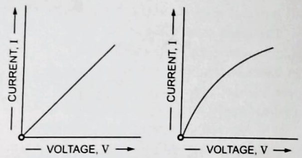
The *source emf* (or open-circuit voltage) is the voltage that appears across the source when no load is connected across it.

When a load is connected to the source terminals and the circuit is closed, an electric current starts flowing through the circuit. Now voltage across source terminals (called the *terminal voltage*) is not equal to source emf. It is due to voltage drop inside the source, *i.e.*, across the source resistance.

Voltage drop inside the source $= I R_s$

The relationship between the current through a resistance and the voltage across the same resistance is called its *volt-ampere (or voltage-current) characteristic*. When represented graphically, voltages are laid off as abscissae and currents as ordinates.

There are two types of volt-ampere characteristics– straight line and nonlinear (curve), as shown in Figs. 2.2 (a) and 2.2 (b) respectively.



(a) (b)

Fig. 2.2

Resistive elements for which the volt-ampere characteristic is a straight line [Fig. 2.2 (a)] are called *linear*, and the electric circuits containing only linear resistances are called *linear circuits*.

Resistive elements for which the volt-ampere characteristic is other than a straight line are termed *non-linear*, and so the electric circuits containing them are called *nonlinear circuits*. Examples of nonlinear elements are tungsten lamps, vacuum tubes and transistors, etc.

An electric circuit, whose characteristics or properties are same in either direction (*e.g.*, a distribution or transmission line), is called the *bilateral circuit*. The distribution or transmission line can be made to perform its function equally well in either direction.

An electric circuit, whose characteristics or properties change with the direction of its operation (*e.g.* a diode rectifier), is called the *unilateral circuit*. A diode rectifier cannot perform rectification in both directions.

A network is said to be *passive* if it contains no source of emf in it. The *equivalent resistance* between any two terminals of a passive network is the ratio of potential difference across the two terminals to the current flowing into (or out of) the network. When a network contains one or more sources of emf and/or current, it is said to be *active*.

In case, a branch is removed from an electric network, the remainder of the network is left with a pair of terminals. The part of the network, which is considered with respect to the removed branch or terminal pair or port is termed as *one-port network*. When two branches are removed so that the network is left with four terminals or two pairs of terminals, the remainder network is called the *two-port network*. Usually one port accepts a source and the other port is coupled to a load, so that there is an input port and an output port in any two-port system.

2.2. SINGLE- AND MULTI-MESH NETWORK

Electric network may provide a single closed path (known as a *mesh or loop*) or several closed paths for the flow of current. An elementary *single-mesh* network, in which all the elements carry the same current, is shown in Fig. 2.1. An elementary multi-mesh network is shown in Fig. 2.3. It has 6 nodes, seven branches, three loops and two meshes. A *junction (or node*)* is a point in a network where two or more branches meet. A *branch* is

* Nodes are considered to be resistanceless and infinitesimal.

any section of a network which joins two nodes directly, that is, without passing through a third node. A *loop* is a closed path in a network formed by a number of connected branches. Mesh is a loop that contains no other loop within it.

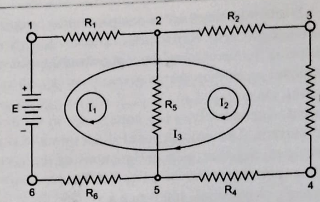
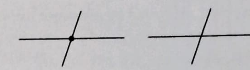


Fig. 2.3

If in the circuit diagram, as in Fig. 2.4 (*a*), there is a bold dot at the intersection of two branches, these branches are electrically connected and have a common node. Otherwise they simply cross, as in Fig. 2.4 (*b*), and are not connected electrically.

2.3. ACTIVE AND PASSIVE ELEMENTS



(*a*) (*b*)

Fig. 2.4

Network elements may be classified into two categories viz. active elements and passive elements.

The elements which supply energy to the network are known as *active elements*. The voltage sources like batteries, dc generators, ac generators and current sources like photoelectric cells, metadyne generators fall under the category of active elements. Most of the semiconductor devices like transistors are treated as current sources.

The components which dissipate or store energy are known as *passive components*. Resistors, inductors and capacitors fall under the category of passive elements. The resistor is the only component which dissipates electrical energy. The inductor and capacitors are the components which store energy, the inductor stores energy by virtue of a current passing through it whereas the capacitor stores energy by virtue of potential difference across it.

2.4. RESISTANCE

1. Physical Phenomenon. The passage of electrons through a material is not accomplished without collisions of the electrons with other atomic particles. Moreover, these collisions are not elastic, and energy is lost in each collision. The loss in energy per unit charge is interpreted as a drop in potential across the material. The amount of energy lost by the electrons is related to the physical properties of a particular substance.

Thus, the resistance is a *dissipative* element, which converts electrical energy into heat, when the current flows through it in any direction. This process of energy conversion is *irreversible*.

2. Field Interpretation. The German physicist Georg Simon Ohm found experimentally that there is a relationship between the current in a substance and the potential drop. In terms of the field concept, the change in energy per unit charge causes a force per unit charge or electric field. This effect may be interpreted in terms of a field in the direction of current through the conducting substance. Ohm's experiment may be stated in terms of this field and the current per unit cross-sectional area as

$$J = \sigma E \qquad \text{...(2.1)}$$

where, in SI units, J is the current density in amperes per square metre (A/m^2), E is the field along the conducting substance in volts per metre (V/m), and σ is the conductivity of the substance, which is a constant for a particular material.

3. Circuit Interpretation. The circuit element used to represent energy dissipation is most commonly described by requiring the voltage across the element be directly proportional to the current through it. Mathematically, the voltage is

$$v = Ri \text{ volts} \qquad \text{...(2.2)}$$

where i is the current in amperes. The constant of proportionality R is the resistance of the element and is measured in ohms (abbreviated Ω). The voltage-current relation expressed by Eq. (2.2) is known as Ohm's law. A physical device whose principal electrical characteristic is resistance is known as a *resistor*.

Electrical resistance is comparable to pipe friction in the hydraulic analog and also to friction in a mechanical system. Resistance or friction directly opposes the current, water flow or motion and the energy dissipated in overcoming this opposition appears as heat.

Since an electric charge gives up energy when passing through a resistor, the voltage v in Eq. (2.2) is a voltage drop in the direction of current. Alternatively, v is a voltage rise in the direction opposite to the current. The conventional diagrammatic representation of a resistance, together with designations of the current direction and voltage polarity, is shown in Fig. 2.5. The plus and minus signs denote decrease of potential, and hence a voltage drop, from left to right (or plus to minus). The element has two terminals (also called nodes).

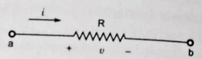


Fig. 2.5 *Schematic Representation of Resistance*

The power dissipated by resistance may be given by expression

$$p = vi = i^2R = \frac{v^2}{R} \text{ watts} \quad ...(2.3)$$

Eq. (2.2) gives the voltage across a resistor in terms of its current. A reciprocal relationship providing the current in terms of voltage is often of equal or greater value in a particular case. As a result, Ohm's law is often expressed as

$$i = Gv \text{ amperes} \quad ...(2.4)$$

where

$$G = \frac{1}{R} \quad ...(2.5)$$

Reciprocal of resistance R *i.e.*, G is called *conductance* and is measured in mhos or siemens (SI unit of conductance is siemens, but mho is more frequently used).

Power dissipated can then be expressed in the alternative form

$$p = vi = v(Gv) = v^2G = i \times \frac{i}{G} = \frac{i^2}{G} \text{ watts} \quad ...(2.6)$$

Resistance value also vary with environmental conditions, with temperature being most significant. This has already been explained in Art 1.9.

Each resistor has two main characteristics *i.e.*, its resistance value in ohms and its power dissipating capacity in watts. Resistors are employed for many purposes such as electric heaters, telephone equipment, electric and electronic circuit elements, and current limiting devices. As such their resistance values and tolerances and their power rating vary widely. Resistors of 0.1 Ω to many mega ohms are manufactured. Acceptable tolerances may range from ± 20 % (resistors serving as heating elements) to ± 0.001 per cent (precision resistors in sensitive measuring instruments). The power rating may be as low as 1/10 W and as high as several hundred watts. Since no single resistor material or type can be made to encompass all the required ranges and tolerances, many different designs are available.

The value of R is selected to have a desired current I or permissible voltage drop IR. At the same time wattage of the resistor is selected so that it can dissipate the heat losses without getting itself overheated. Too much heat may burn the resistor.

2.5 CAPACITANCE

1. Physical Phenomenon. The presence of charge on two spatially separated substances – for example, those shown in Fig. 2.6 causes an "action at a distance" in the form of a force between the two substances. This phenomenon is considered as a property of nature, a basic experimental fact. Coulomb found that this force was of such nature that "like charges repel" and "unlike charges attract" and that the force varied according to the relation

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2} \quad ...(2.7)$$

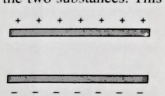


Fig. 2.6

where F is the force in newtons directed from point charge to point charge, r is the separation of the point charges in metres, ε is the permittivity of the medium, having free-space value of 8.854×10^{-12} farad per metre (F/m) in SI system, and q_1 and q_2 are the charges measured in coulombs.

2. Field Interpretation. This phenomenon can be stated in terms of a force on a unit charge placed between the two charged bodies. This force per unit charge, a vector quantity since force is a vector quantity, is called an electric field of value

$$E = \frac{F}{q} \quad ...(2.8)$$

As a conceptual aid, this field may be represented by lines drawn in the direction of the force that would be exerted on the unit positive exploring charge at that point. Such lines are illustrated in Fig. 2.7.

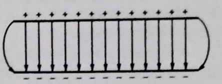


Fig. 2.7 *Electric Field Lines Or Lines of Force Between Two Charged Conductors*

3. Circuit Interpretation. Capacitance is a two-terminal element that has the capability of charge storage and, consequently, energy storage. The stored energy can be fully retrieved.

The current through the capacitor is proportional to the derivative of voltage across it and is given by expression

$$i = C\frac{dv}{dt} \quad ...(2.9)$$

where C has the unit of farads, the practical unit being a microfarad (μ F) because a farad is physically a large unit. Integrating above Eq. (2.9), we have

$$v = \frac{1}{C}\int_0^t i\, dt + v_c(0) \quad ...(2.10)$$

where $v_c(0)$ = Capacitance voltage at $t = 0$

For an initially uncharged capacitor $v_c(0) = 0$, so that

$$v = \frac{1}{C}\int_0^t i\, dt = \frac{q}{C} \quad \text{or} \quad C = \frac{q}{v} \quad \text{or} \quad q = Cv \quad ...(2.11)$$

The proportionality constant C expresses the charge-storing property of the element and is called the *capacitance*. With q in coulombs and v in volts, the capacitance C is in farads (abbreviated F). A capacitor is a physical element which exhibits the property of capacitance.

The schematic representation of capacitance, in which current and voltage reference directions are indicated, is depicted in Fig. 2.8. In this figure and in Eqs. (2.9) and (2.11), a voltage drop exists in the direction of flow of current. Charge flow from a higher potential to a lower potential *i.e.*, from plus to minus, signifies that energy can be removed from the circuit and stored.

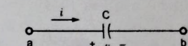


Fig. 2.8 *Schematic Representation of Capacitance*

The power associated with a capacitance is

$$p = vi = Cv\frac{dv}{dt} \text{ watts} \quad ...(2.12)$$

Energy stored in the capacitance may be had by integrating above Eq. (2.12) as

$$W = \int p\, dt = \int Cv\frac{dv}{dt} = \frac{1}{2}Cv^2 \text{ joules} \quad ...(2.13)$$

The value of the energy stored in the capacitance, given by Eq. (2.13), is dependent only on the magnitude of voltage and not on the manner of attaining the magnitude. The stored energy is returned to the circuit as the voltage is reduced to zero. For example, if a capacitance is discharged by placing a resistance across it, a current persists in the resistance until the stored energy is dissipated as heat in the resistance. When a *short circuit*, *i.e.*, an element of zero resistance, is placed across the capacitance terminals, the stored energy is radiated to the surrounding environment. Practically speaking, however, no zero resistance element exists, so that energy is converted into heat or any spark produced.

In one sense, the behaviour of a capacitance can be compared to that of a mechanical spring. Whether it is elongated or compressed, the potential energy stored in a spring is a function of position only. This energy can reappear when the spring returns to its equilibrium condition: The analogy that may be drawn is then between charge and position, energy in the electric field and potential energy.

A capacitor offers low impedance to ac but very high impedance to dc. So capacitors are used to couple alternating voltage from one circuit to another circuit and at the same time to block dc voltage from reaching the next circuit. It is also employed as a *bypass capacitor* where it passes the ac through it without letting the dc to go through the circuit across which it is connected. A capacitor forms a *tuned circuit* in series or in parallel with an inductor.

A capacitor consists of two conducting plates, separated by an insulating material, called the *dielectric*. Capacitors, like resistors and inductors, can either be fixed or variable. Some of the commonly used fixed capacitors are mica, ceramic, paper, plastic-film and electrolytic capacitors.

2.6. INDUCTANCE

1. Physical Phenomenon. In 1819 it was discovered by a Danish physicist, Hans Oersted that an electric current is always accompanied by certain magnetic effects. In the same year Ampere measured the force caused by the current and expressed the relationship in equation form. This magnetic effect is an "action at a distance" just as in the case of the force between two charged bodies. This action at a distance is a basic observational fact; it is not deduced from other knowledge.

2. Field Interpretation. The phenomenon described above can be interpreted in terms of force per unit magnetic pole at all points in space. Oersted discovered that this force was directed at right angles to the current carrying conductor. In terms of the geometry of Fig. 2.9. Ampere described a magnetic field density B, the force per unit magnetic pole, of value

$$dB = \frac{\mu i \cos \alpha \, dl}{4 \pi r^2} \qquad ...(2.14)$$

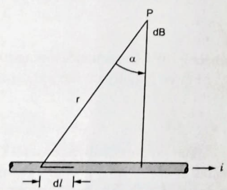


Fig. 2.9

where μ is the magnetic permeability, which is a function of medium in which the magnetic field exists, i is the current in amperes, and other quantities are indicated on the figure. Figure 2.10 depicts the cross section of a current carrying conductor. By Eq. (2.14), the magnetic flux density will be constant at a constant distance from the conductor. Continuous lines with arrows may be drawn to indicate the direction of B as a conceptual aid. These are magnetic field density lines or "lines of force".

Faraday experimented with two inductive circuits in spatial approximity. He found that a changing magnetic field produced by one circuit induced a voltage in the other circuit. The changing magnetic field could be caused by (*i*) a conductor moving in space, or (*ii*) a current changing with time.

Faraday did not envisage this method of inducing voltage in terms of "action at a distance" but in terms of changes in flux linkages. A conductor moving in a magnetic field (as in case of a generator) is thought of as "cutting of flux and hence reducing the flux linkages"; the voltage induced in a stationary conductor (as in a transformer) is thought of as caused by "changing flux linkages" with time. Faraday law is

$$v \propto \frac{d\psi}{dt}$$

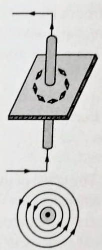


Fig. 2.10

$$\text{or} \quad v = K\frac{d\psi}{dt} \qquad ...(2.15)$$

where K is a proportionality constant. In SI system, the units are selected to make K unity. When ψ is in weber-turns, t is in seconds, and K = 1, then v is in volts.

3. Circuit Interpretation. The circuit element used to represent the energy stored in a magnetic field is defined by the relation

$$v = L\frac{di}{dt} \qquad ...(2.16)$$

The above expression describes a situation in which the voltage across the element is proportional to the time rate of change of current through it. The constant of proportionality L is the *self-inductance* or simply the *inductance* of the element, and is measured in *henrys* (abbreviated H).

The voltage v in Eq. (2.16) is a voltage drop in the direction of current and can be considered to oppose an increase in current. Figure 2.11 depicts the schematic representation of an inductance and its associated reference direction for current and voltage polarity.

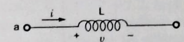


Fig. 2.11 *Schematic Representation of Inductance*

Integrating Eq. (2.16), we have

$$i = \frac{1}{L}\int_0^t v \, dt + i(0) \qquad ...(2.17)$$

where $i(0)$ = Inductance current at $t = 0$.

According to Eq. (2.17) *current through an inductance* cannot change instantly (compared with capacitance voltage) as it would require infinite voltage.

Because the effect of inductance is to oppose the change in the magnitude of current, inductance is analogous to mass or inertia in a mechanical system and to the mass of liquid in hydraulics. Inductance prevents the current from changing instantly as it requires infinite voltage to cause an instantaneous change in current, just as the mass of an automobile prevents it from stopping or starting instantaneously.

The power associated with the inductive effect in a circuit is

$$p = vi = Li\frac{di}{dt} \text{ watts} \qquad ...(2.18)$$

and the energy stored is

$$W = \int p \, dt = \int Li\frac{di}{dt}dt \int Li \, di = \frac{1}{2} Li^2 \text{ joules} \qquad ...(2.19)$$

unlike the resistive energy, which is transformed into heat, the inductive energy is stored in the same sense that kinetic energy is stored in a moving mass. Eq. (2.19) reveals that the magnitude of stored energy depends on the magnitude of current and not in the manner of attaining that magnitude. The stored inductive energy reappears in the circuit as the current is reduced to zero. For example, if a switch is opened in a current carrying inductive circuit, the current decays rapidly, but not instantaneously. In accordance with Eq. (2.16), a relatively high voltage appears across the separating contacts of the switch, and an arc may form. The arc makes it possible for the stored energy to be dissipated as heat in the arc and the circuit resistances.

In case of an inductor current does not change instantaneously. It offers high impedance to ac but very low impedance to dc *i.e.*, it blocks ac signal but passes dc signal.

A piece of wire, or a conductor of any type, has inductance *i.e.*, a property of opposing the change of current through it. By coiling the wire the inductance is increased as the square of the number of turns. The inductance is represented by English capital letter L and measured in henrys.

Specially made components consisting of coiled copper wire are called the *inductors*. Inductors are of two types viz. *air-core* (wound on non-ferrous materials) and *iron-core* (wound on ferrite cores). Inductors range in value from the tiny (few turn air-core coils of 0.1 μH used in high frequency systems) to iron-core choke coils of 50 H or more for low frequency applications.

2.7. SUMMARY OF RELATIONSHIPS FOR THE PARAMETERS

Some of the relationships discussed so far in this chapter are summarised in tabular form (Table 2.1). These equations are encountered so frequently in the study of electrical engineering that they should be memorized.

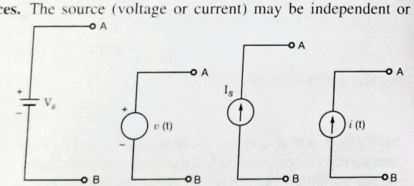
Table 2.1 Summary of Relationships For The Parameters

Parameter	*Basic Relationship*	*Voltage-Current Relationships*		*Energy*
Resistance, R	$v = Ri$	$v_R = R i_R$	$i_R = G v_R$	$W_R = \int_{-\infty}^{t} v_R i_R \, dt$
$G = \frac{1}{R}$				
L (or M)	$\psi = Li$	$v_L = L \frac{di_L}{dt}$	$i_L = \frac{1}{L} \int_{-\infty}^{t} v_L \, dt$	$W_L = \frac{1}{2} LI^2$
C	$q = Cv$	$v_C = \frac{1}{C} \int_{-\infty}^{t} i_C \, dt$	$i_C = C \frac{dv_C}{dt}$	$W_C = \frac{1}{2} CV^2$

2.8. VOLTAGE AND CURRENT SOURCES

Most of the sources encountered in everyday life (such as batteries, dynamos, alternators etc.) are voltage sources but some current sources do exist. Some examples are: photoelectric cells as used in light meters, metadyne generators as used in military gun controls. Other devices may be regarded as current sources, such as the collector circuits of transistors and the anode circuits of pentode thermionic tubes.

2.8.1. Independent and Dependent Sources. The source (voltage or current) may be independent or dependent. A source is said to be independent when it does not depend on any other quantity in the circuit. Figure 2.12 (*a*) shows an independent dc voltage source whereas Fig. 2.12 (*b*) depicts a time varying voltage source. The positive sign indicates that terminal A is positive with respect to terminal B *i.e.*, the potential of terminal A is V volts higher than that of terminal B.

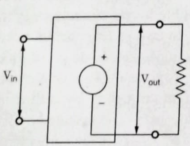


(*a*) (*b*) (*c*) (*d*)

Fig. 2.12

Similarly an ideal constant current source is shown in Fig. 2.12 (*c*) whereas time varying current source is shown in Fig. 2.12 (*d*). The arrow indicates the direction of current flow at any moment under consideration.

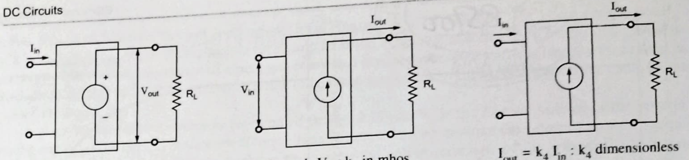
A *dependent source* is one which depends on some other quantity in the circuit which may be either a voltage or a current. In electronic circuits, we very often find that the current through an element, (say collector current through a bipolar junction transistor) is dependent on a current through some other element or in a MOSFET it is dependent on the voltage across some other element. Such a source is called a *dependent source*. In a dependent source the output voltage (or current) depends on another voltage (or current). The relationship may be linear or nonlinear. There are four possible dependent sources as are represented in Fig. 2.13.



$V_{out} = k_1 V_{in}$; k_1 dimensionless

(*a*) *Voltage-Dependent Voltage Source*

Fig. 2.13



$V_{out} = k_2 I_{in}$; k_2 in ohms

(*b*) *Current-Dependent Voltage Source*

$I_{out} = k_3 V_{in}$; k_3 in mhos

(*c*) *Voltage-Dependent Current Source*

$I_{out} = k_4 I_{in}$: k_4 dimensionless

(*d*) *Current-Dependent Current Source*

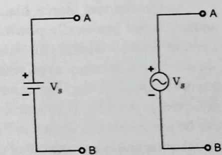
Fig. 2.13 *Dependent Sources*

Such sources can also be either constant sources or time varying sources.

Independent sources actually exist as physical entities such as an accumulator, a dc generator and an alternator. But dependent sources are parts of *models* that are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.

Another major difference is that four terminals are required to define a controlled source; whereas only two are required for an independent source. Of the four dependent source terminals, one pair provides the control and the second pair exhibits the properties of the source.

2.8.2. Ideal Voltage Source. A constant voltage source is an ideal source element capable of supplying any current at a given voltage. If the internal resistance of a voltage source is zero, the terminal voltage (voltage across the load) is equal to the voltage across the source (the source emf), and is independent of the amount of load current, or in other words the voltage of an ideal voltage source is independent of load current supplied by it. For example, if the terminals are connected together the source will supply an infinite current. The symbolic representations of dc and ac ideal voltage sources are given in Figs. 2.14 (*a*) and 2.14 (*b*) respectively.



(*a*) (*b*)

Fig. 2.14

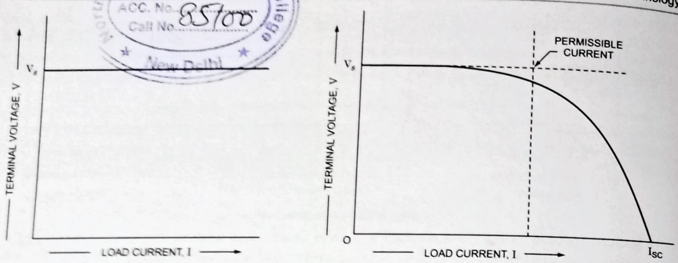
There are two noteworthy points regarding ideal voltage sources. First, ideal voltage source cannot be short circuited (because this will be contrary to the definition of the ideal voltage source itself). Secondly (and for the same reason) two ideal voltage sources of unequal output voltages cannot be placed in parallel.

An ideal voltage source is not practically possible. There is no voltage source which can maintain its terminal voltage constant even when its terminals are short circuited.

A lead-acid battery or a dry cell are examples of a constant voltage source when the current drawn is below a certain limit. However, a practical source always shows a drop in its terminal voltage which increases with load current. A dc generator or a rectifier operating on mains supply is a voltage source (the output voltage is contaminated with ripples) and exhibit a voltage drop which is load dependent.

The volt-ampere (V-I) characteristics of an ideal voltage source are depicted in Fig. 2.15 (*a*) in comparison to that of a practical voltage source shown in Fig. 2.15 (*b*). Dotted line is that of an ideal voltage source for differentiation. Within a permissible range of current a practical dc voltage source maintains the terminal (output) voltage within a narrow range of its nominal voltage. Beyond this value of current, called the *rated current*; the voltage drops rapidly till the short circuit current I_{sc}, where the terminal voltage drops to zero. In contrast, an ideal dc voltage source would have maintained the voltage even up to infinitely large current.

2.8.3. Ideal Current Source. Like a constant voltage source, there may be a constant current source—a source that supplies a constant current to a load even if its impedance varies. Ideally, the current supplied by such a source should remain constant irrespective of the load impedance. A symbolic representation of such an ideal constant current source is shown in Fig. 2.12 (*c*). The arrow inside the circle indicates the direction of flow of current in the circuit, when a load is connected to the source.



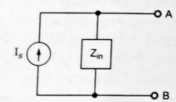
(a) For Ideal Voltage Source (b) For a Practical Voltage Source

Fig. 2.15 *V-I Characteristics of Voltage Sources*

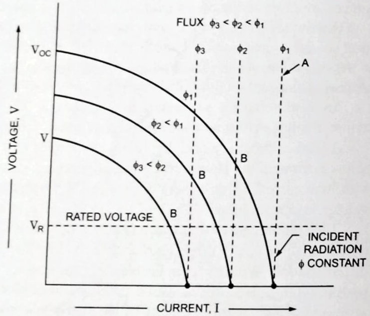
There are two noteworthy points regarding ideal current sources. First, an ideal current source cannot be open circuited (because this will be contrary to the definition of the constant current source itself). Again, for the same reason, two ideal current sources of different output currents cannot be placed in series.

An ideal current source, like an ideal voltage source, is not practically possible. There is no current source which can maintain current supplied by it constant even when its terminals are open circuited. An ideal current source does not exist in practice. A practical current source can be represented as shown in Fig. 2.16 (*a*).

A solar cell is an example of *current source*. It provides constant current to a resistance within a specified range of output voltage. The value of the current delivered by a current source depends on the flux incident on the cell. An ideal current source provides a constant current to a resistor of resistance R ohms ($0 < R < \infty$). V-I characteristics of a practical current source is compared with that of an ideal current source (dotted vertical lines) in Fig. 2.16 (*b*). The limit V_R up to which a current source maintains the current is drawn by horizontal dotted line. Beyond this value of output voltage (hence load resistance) the current falls. A current source can also be had by connecting a high resistance R_{se} in series with a high voltage dc source. It will behave as a current source in a range R_L is less than 5 to 10% of series resistance R_{se}. In that case, the current is primarily decided by R_{se}. The current through load resistance R_L is nearly constant unless R_L is significant in comparison to R_{se}.



(*a*) *Practical Current Source*

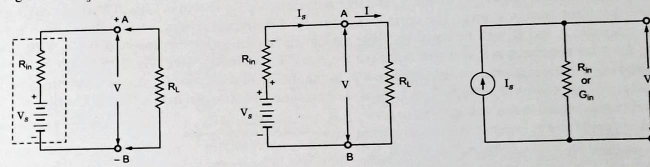


(*b*) *Comparison of V-I Characteristics of an Ideal Current Source (A) With That of a Practical Current Source (B) For Different Values of Radiation Intensity*

Fig. 2.16

2.8.4. Practical Sources. In most applications, ideal sources are approximation. The internal resistance R_{in} of a voltage source, which is responsible for a drop in terminal voltage of a source on load is quite small in comparison to the load resistance of a network connected across the voltage source. Hence, voltage drop is considered negligible. Internal resistance needs to be taken into account when it is significant in comparison to load resistance R_L.

A practical dc voltage source is represented in Fig. 2.17 (*a*). V_s is the internal voltage of the voltage source and R_{in} is the internal resistance of the voltage source. When we measure the voltage across the source terminals without any load (no resistance connected to the output terminals of the source) the terminal voltage is $V = V_s$ as there is no current through R_{in} to cause a voltage drop.



(*a*) *A Practical Voltage Source* (*b*) *Voltage Drop in R_{in}* (*c*) *Practical Current Source*

Fig. 2.17 *Practical Sources*

Assuming V_s to be constant, the terminal voltage falls on loading by an external or load resistance R_L. The resulting current,

$$I = \frac{V_s}{R_{in} + R_L} \quad ...(2.20)$$

and the terminal voltage is then $V = IR_L = \dfrac{V_s}{R_{in} + R_L} \times R_L$ ...(2.21)

As the load resistance R_L reduces (load on the source increases), terminal voltage V falls linearly, if R_{in} and V_s are assumed constant.

When the terminals of a practical voltage source are short circuited by a thick wire of zero resistance resulting in a short circuit, source current I_s is given as

$$I_s = I_{sc} = \frac{V_s}{R_{in}} \quad ...(2.22)$$ In Fig. 2.17 (*b*), R_L being zero.

$$\text{Hence, } R_{in} = \frac{V_s}{I_{sc}} = \frac{\text{Open-circuit voltage of the source, } V_{oc}}{\text{Short-circuit current, } I_{sc}} \quad ...(2.23)$$

Similarly an ideal current source must produce infinite voltage on open circuit. A practical current source will have a finite output voltage. A practical current source is represented as shown in Fig. 2.17 (*c*). In this case short circuit current,

$$I_{sc} = I_s \quad ...(2.24)$$

and the open-circuit voltage, $V_{oc} = I_s R_{in}$ ...(2.25)

$$\text{Hence, } R_{in} = \frac{V_{oc}}{I_{sc}} = \frac{\text{Open-circuit voltage}}{\text{Short-circuit current}} \quad ...(2.26)$$

$$\text{or } G_{in} = \frac{1}{R_{in}} = \frac{I_{sc}}{V_{oc}} = \frac{\text{Short-circuit current}}{\text{Open-circuit voltage}} \quad ...(2.27)$$

2.9. SOURCE EQUIVALENCE OR TRANSFORMATION

Practically, a voltage source is not different from a current source. In fact, a source can either operate as a current source or as a voltage source. It merely depends upon its operating conditions. If load impedance is very large in comparison to internal impedance of the source, it will be advantageous to treat the source as a voltage source. On the other hand, if the load impedance is very small in comparison to the internal impedance of the source, it is better to represent the source as a current source. From the circuit point of view it does not matter at all whether the source is treated as a voltage source or a current source. In fact, it is possible to convert a voltage source into a current source and vice versa.

Consider a voltage source of voltage V_s and internal resistance R_{in} shown in Fig. 2.17 (*a*) for conversion into an equivalent current source. The current supplied by this voltage source, when a short circuit is put across terminals A and B, will be equal to V_s/R_{in}. A current source supplying this current $I_s = V_s/R_{in}$ and having the same resistance across it will represent the equivalent current source [Fig. 2.17 (*c*)].

Similarly a current source of output current I_s in parallel with resistance R_{in} can be converted into an equivalent voltage source of voltage $V_s = I_s R_{in}$ and a resistance R_{in} in series with it [Fig. 2.17 (*a*)].

It should be noted that a voltage source series resistance combination is equivalent to a current source–parallel resistance combination if, and only if their respective open-circuit voltages are equal, and their respective short-circuit currents are equal.

For example, a voltage source branch consisting of a 10 V source in series with a resistance of 2.5 Ω may be replaced by a current source branch consisting of a 4 A source in parallel with a 2.5 Ω resistance and vice versa, as shown in Figs. 2.18 (*a*) and 2.18 (*b*) respectively.

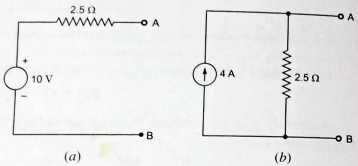


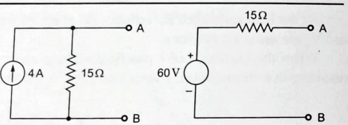
Fig. 2.18

Example 2.1

Convert 4A source with its parallel resistance of 15 Ω into its equivalent voltage source.

[*G.G.S.I.P. Univ. Delhi Ist Term Exam. February/March 2011*]

Solution: 4 A current source with its parallel resistance of 15 Ω will be equivalent of a voltage source of 4 × 15 *i.e.*, 60 V in series with a resistance of 15 Ω as shown in Fig. 2.19 (*b*).



(*a*) *Current Source* (*b*) *Equivalent Voltage Source*

Fig. 2.19

Example 2.2

Determine the current in 28 Ω resistor in the circuit shown in Fig. 2.20.

Solution: The two current sources cannot be combined together because of the 28 Ω resistance presence between points A and C. However, this hurdle can be removed by converting 10 A current source into the equivalent voltage source of 40 V, as shown in Fig. 2.21 (*a*). Similarly 5 A current source can be converted into the equivalent voltage source of 40 V, as shown in Fig. 2.21 (*b*).

The circuit shown in Fig. 2.21 (*b*) is a simple series circuit having total voltage of 80 V and total resistance of 4 + 28 + 8 *i.e.*, 40 Ω. So the current flowing through resistance of 28 Ω is 2 A **Ans.**

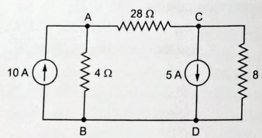


Fig. 2.20

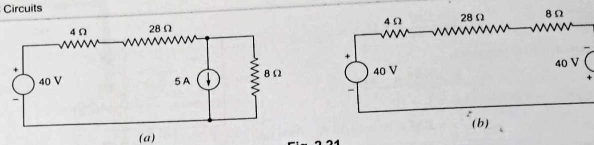


Fig. 2.21

2.10. NETWORK THEOREMS

The circuit variables, we are interested in determining, generally are (*i*) current through or (*ii*) voltage across a resistance of interest or a group of resistances. Sometimes it is required to work out the necessary source voltage or source current which will cause a specific current through or voltage across a given resistor or result into a specified power dissipation by it.

The current flowing through a circuit is governed by basic law called the *Ohm's law*, already discussed in chapter 1. Distribution of current or the voltage over a circuit is governed by Kirchhoff's laws, already discussed in the Art. 1.20.

The circuit to be analysed may be simple or quite complex. In case of complex networks the solution procedure may be too tedious and time consuming. Certain techniques for solution of such networks have been developed which reduces the networks to simpler form for quick solution. This may be accomplished through the use of what are called as network theorems. Few of these network theorems, which are relevant at this stage, will be discussed here.

2.11. MAXWELL CIRCULATING CURRENT THEOREM

If a network with several sources has more than two nodes the current in it may be determined by this theorem. This is one of the most universal methods for solving networks.

In a number of cases, a network may be considered as consisting of a set of adjoining loops (two in Fig. 2.3), each of which forms a polygon made up of several branches of the network (without any diagonals). Some branches of the network are common to two adjacent loops, while others form an external circuit where each branch occurs in one loop only.

This theorem involves representing a current that is assumed to circulate around a closed loop by a curved arrow and labelling the arrow with its identifying current symbol I with a subscript. By this theorem the current flowing through the branch common to two meshes will be equal to the algebraic sum of the two loop currents flowing through it. The direction of any loop current may be taken either as clockwise or counter-clockwise but for systematic solution the directions of all loop currents are assumed to be the same (say clockwise). Then Kirchhoff's second law is applied to each mesh and algebraic equations are obtained. The total number of independent equations is equal to the number of meshes (*i.e.*, there are fewer equations than in a purely Kirchhoffian solution). Therefore, they can be solved as simultaneous equations to give the circulating currents and then the branch currents. Thus, this method eliminates a great deal of tedious calculation work involved in the branch current method, discussed in Art. 1.20.

Application of Maxwell circulating current theorem will be more clear from the following illustrations.

Example 2.3

Find the currents in all the resistive branches of the circuit shown in Fig. 2.22 by KVL.

[*U.P. Technical Univ. Electrical Engineering Second Semester 2007-08*]

Solution: Applying KVL to loop 1, we have

$$10\,I_1 + 5\,(I_1 + 10) = 100$$

$$\text{or} \quad 15\,I_1 + 50 = 100$$

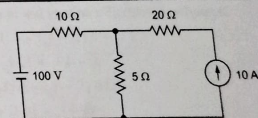


Fig. 2.22

$$\text{or } I_1 = \frac{50}{15} = 3.33 \text{ A}$$

Thus current in 10 Ω resistance,

$$I_1 = 3.33 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current in 5 Ω resistance} = I_1 + 10 = 3.33 + 10 = 13.33 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current in 20 Ω resistance} = 10 \text{ A} \quad \textbf{Ans.}$$

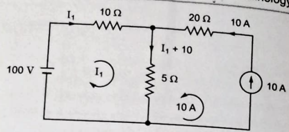


Fig. 2.23

Example 2.4

Using loop current method find the current I_1 and I_2 shown in Fig. 2.24.

[U.P. Technical Univ. Electrical Engineering Second Semester 2005-06]

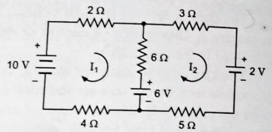


Fig. 2.24

Solution: The network is redrawn, as illustrated in Fig. 2.25. There are two independent loops. The loop currents have been taken clockwise, as marked in the given circuit. The individual branch currents along with their directions of flow are also shown in the circuit diagram. Applying Kirchhoff's voltage law to meshes I and II respectively, we have

$$2 I_1 + 6 (I_1 - I_2) + 4 I_1 = 10 - 6$$
$$\text{or } 12 I_1 - 6 I_2 = 4$$
$$\text{or } 6 I_1 - 3 I_2 = 2 \quad \ldots(i)$$
$$\text{and } 3 I_2 + 5 I_2 - 6 (I_1 - I_2) = 6 - 2$$
$$\text{or } - 6 I_1 + 14 I_2 = 4 \quad \ldots(ii)$$

Adding expressions (*i*) and (*ii*), we have

$$11 I_2 = 6$$
$$\text{or } I_2 = \frac{6}{11} \text{ A } \textbf{Ans.}$$

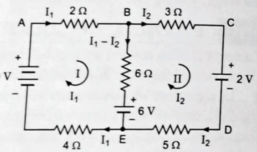


Fig. 2.25

Substituting $I_2 = \frac{6}{11}$ in equation (*i*), we have

$$I_1 = \frac{2 + 3I_2}{6} = \frac{2 + 3 \times (6/11)}{6} = \frac{40}{66} = \frac{20}{33} \text{ A } \textbf{Ans.}$$

Example 2.5

Solve the circuit shown in Fig. 2.26 using the mesh method of analysis and determine the mesh currents I_1, I_2 and I_3. Evaluate the power developed in the 10 V voltage source. *[GATE 1999]*

Solution: The circuit is redrawn with branch currents with the flow directions marked, as illustrated in Fig. 2.27.

Applying Kirchhoff's current law at node N, we have

$$I_1 - I_2 + 2 + I_2 - I_3 = 0$$
$$\text{or } I_3 - I_1 = 2 \quad \ldots(i)$$

Applying Kirchhoff's voltage law to meshes ABCDNA and ANDEFGA respectively, we have

$$2 I_2 + 6 (I_2 - I_3) - 4 (I_1 - I_2) = 2$$
$$\text{or } - 4 I_1 + 12 I_2 - 6 I_3 = 2 \quad \ldots(ii)$$
$$\text{and } 4 (I_1 - I_2) - 6 (I_2 - I_3) = 12 - 10$$
$$\text{or } 4 I_1 - 10 I_2 + 6 I_3 = 2 \quad \ldots(iii)$$

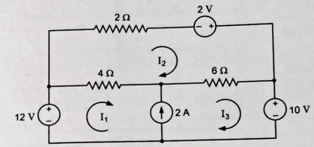


Fig. 2.26

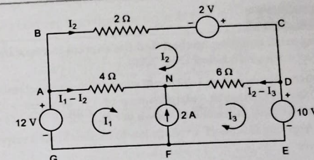


Fig. 2.27

Solving Eqs. (*i*), (*ii*) and (*iii*), we have

$$I_1 = 1 \text{ A}; I_2 = 2 \text{ A and } I_3 = 3 \text{ A } \textbf{Ans.}$$

$$\text{Power developed in 10 V source} = VI_3 = 10 \times 3 = 30 \text{ W } \textbf{Ans.}$$

Example 2.6

Find mesh currents i_1 and i_2 in the electric circuit of Fig. 2.28.

Solution: Applying Kirchhoff's voltage law to the outer mesh, we have

$$i_1 \times 1 + 2 \times i_2 = 4 - 3$$
$$\text{or } i_1 + 2 i_2 = 1 \quad \ldots(i)$$

Applying Kirchhoff's voltage law to the mesh I, we have

$$i_1 + (i_1 - i_2) \times 1 + 3 i_2 = 4$$
$$\text{or } i_1 + i_2 = 2 \quad \ldots(ii)$$

Solving Eqs. (*i*) and (*ii*), we have

$$i_1 = 3 \text{ A } \textbf{Ans.}$$
$$i_2 = -1 \text{ A } \textbf{Ans.}$$

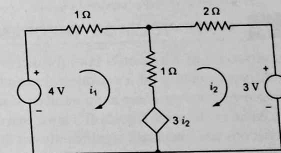


Fig. 2.28

Example 2.7

Using mesh current method, determine current I_x in the following circuit (Fig. 2.29).

[U.P. Technical Univ. Electrical Engineering First Semester 2005-06]

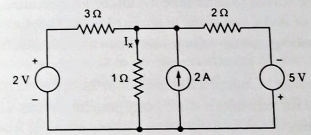


Fig. 2.29

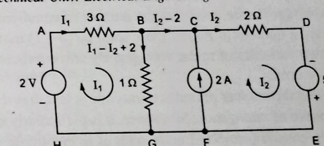


Fig. 2.30

Solution: Current distribution in different branches of the circuit satisfying Kirchhoff's first law is shown in the Fig. 2.30.

Applying Kirchhoff's second law to meshes I and II respectively, we have

$$3I_1 + (I_1 - I_2 + 2) = 2 \quad \ldots(i)$$
$$\text{and } 2I_2 - (I_1 - I_2 + 2) = 5 \quad \ldots(ii)$$
$$\text{or } 4I_1 - I_2 = 0$$
$$\text{or } I_2 = 4I_1 \quad \ldots(iii)$$
$$\text{and } 3I_2 - I_1 = 7 \quad \ldots(iv)$$

$$\text{Solving Eqs. (iii) and (iv), we have } I_1 = \frac{7}{11} \text{ A and } I_2 = \frac{28}{11} \text{ A}$$

$$\text{Current } I_x = \text{Current through branch BG} = I_1 - I_2 + 2 = \frac{7}{11} - \frac{28}{11} + 2 = \frac{1}{11} \text{ A An}$$

Example 2.8

Using mesh equation method find the current in resistance R_1 of the network given below (Fig. 2.31).

Solution: By converting 1 A current source with a parallel resistance of 5 Ω into an equivalent voltage source of 5 V in series with a 5 Ω resistor we have the circuit as shown in Fig. 2.32.

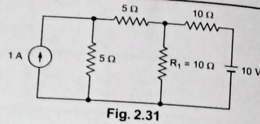


Fig. 2.31

Applying Kirchhoff's voltage law to meshes I and II respectively, we have

$$10\ I_1 + 10\ (I_1 + I_2) = 5\text{ V}$$

$$\text{or } 4I_1 + 2I_2 = 1 \qquad ...(i)$$

$$\text{and } 10\ I_2 + 10\ (I_1 + I_2) = 10\text{ V}$$

$$\text{or } I_1 + 2I_2 = 1 \qquad ...(ii)$$

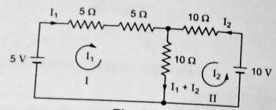


Fig. 2.32

Solving Eqs. (*i*) and (*ii*) we have $I_1 = 0$ and $I_2 = 0.5$ A

So current flowing through 10 Ω resistance

$$= I_1 + I_2 = 0 + 0.5 = 0.5\text{ A } \textbf{Ans.}$$

2.12. NODE–VOLTAGE THEOREM OR NODAL ANALYSIS

The direct use of Kirchhoff's laws for determining the currents in complicated networks necessitates the simultaneous solution of a considerable number of equations, making the computations extremely time-consuming. However, there are a number of methods (such as loop-current method, node voltage method etc.), based on the same Kirchhoff's laws, that obviate the solving of a set of equations or reduce the number of equations and, therefore, significantly cut the computation work.

For application of node voltage theorem one of the node is taken as *reference* or *zero potential* or *datum node* and the potential difference between each of the other nodes and the reference node is expressed in terms of an unknown voltage (symbolized as V_1, V_2 or V_A, V_B or V_x, V_y etc.) and at every node Kirchhoff's first (or current) law is applied assuming the possible directions of branch currents. This assumption does not change the statement of problem, since the branch currents are determined by the potential difference between respective nodes and not by absolute values of node potentials. Like Maxwell's circulating current theorem, node-voltage theorem reduces the number of equations to be solved to determine the unknown quantities. If there are *n* number of nodes, there shall be $(n - 1)$ number of nodal equations in terms of $(n - 1)$ number of unknown variables of nodal voltages. By solving these equations, nodal voltages are known to compute the branch currents.

When the number of nodes minus one is less than the number of independent meshes in the network, it is, n fact more advantageous. Moreover, it is particularly suited for networks having many parallel circuits with ommon ground connected node such as in electronic circuits.

lustration. Consider, for example, a two node network, as illustrated in Fig. 2.33.

Node C has been taken as reference node. Let and V_B be the voltages of nodes A and B pectively with respect to node C. Let the current tribution be as shown on the circuit diagram . 2.33) arbitrarily. Now let us get independent ations for these two nodes.

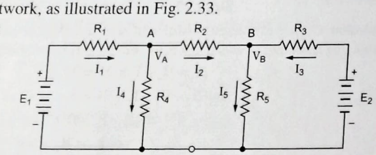


Fig. 2.33

Node A is the junction of resistors R_1, R_2 and R_4. urrent equation for node A is

$$I_1 = I_2 + I_4$$

$$\text{or } \frac{E_1 - V_A}{R_1} = \frac{V_A - V_B}{R_2} + \frac{V_A}{R_4} \quad \text{or} \quad V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} = \frac{E_1}{R_1} \qquad ...(2.28)$$

Node B is the junction of resistors R_2, R_3 and R_5. So current equation for node B is

$$I_5 = I_2 + I_3$$

$$\text{or } \frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \quad \text{or} \quad \frac{-V_A}{R_2} + V_B\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right] = \frac{E_2}{R_3} \qquad ...(2.29)$$

The Eqs. (2.28) and (2.29) can now be solved to get the values of V_A and V_B and then the values of currents I_1, I_2, I_3, I_4 and I_5 can be computed easily.

Example 2.9

Find the currents in all the resistive branches of the circuit shown in Fig. 2.34 by KCL.

[U.P. Technical Univ. Electrical Engineering Second Semester 2007-08]

Solution: Let the given circuit be redrawn as with node E as the reference node and arbitrarily assumed distribution of current as shown in Fig. 2.34.

Applying Kirchhoff's current law (KCL) to nodes C and B respectively we have

$$I_3 = 10\text{ A} \qquad ...(i)$$

$$\text{and } I_1 + I_3 = I_2$$

$$\text{or } \frac{V_A - V_B}{10} + 10 = \frac{V_B}{5}$$

$$\text{or } \frac{100 - V_B}{10} + 10 = \frac{V_B}{5}$$

$$\text{or } 100 - V_B + 100 = 2\ V_B$$

$$\text{or } 3\ V_B = 200$$

$$\text{or } V_B = \frac{200}{3}\text{ V}$$

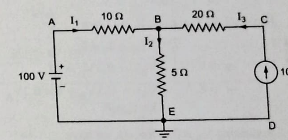


Fig. 2.34

$$\text{Thus current in 10 Ω resistor, } I_1 = \frac{V_A - V_B}{10} = \frac{100 - \frac{200}{3}}{10} = \frac{10}{3}\text{ A } \textbf{Ans.}$$

$$\text{Current in 5 Ω resistor, } I_2 = \frac{V_B}{5} = \frac{200}{3 \times 5} = \frac{40}{3}\text{ A } \textbf{Ans.}$$

$$\text{Current in 20 Ω resistor } = I_3 = 10\text{ A } \textbf{Ans.}$$

Example 2.10

Calculate currents in all the resistances of the circuit shown in Fig. 2.35 using node analysis method.

[U.P. Technical Univ. Electrical Engineering First Semester 2006-07]

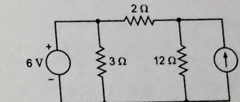


Fig. 2.35

Solution: Let the given circuit be redrawn with terminal G as the reference node and arbitrarily assumed distribution of currents, as shown in Fig. 2.36.

Applying Kirchhoff's current law to node B, we have

$$I_4 = I_3 + I_5$$

$$\text{or } \frac{V_B}{12} = \frac{V_A - V_B}{2} + 4$$

$$\text{or } \frac{V_B}{12} = \frac{6 - V_B}{2} + 4 \qquad \because V_A = 6\text{ V}$$

$$\text{or } V_B = 36 - 6\ V_B + 48$$

$$\text{or } 7\ V_B = 84$$

$$\text{or } V_B = 12\text{ V}$$

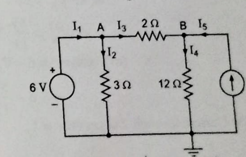


Fig. 2.36

So, we have

$$\text{Current in 3 Ω resistance, } I_2 = \frac{V_A}{3} = \frac{6}{3} = 2\text{ A } \textbf{Ans.}$$

Current in 2 Ω resistance, $I_3 = \dfrac{V_A - V_B}{2} = \dfrac{6-12}{2} = -3$ A **Ans.** *i.e.*, 3 A from node B to node A **Ans.**

Current in 12 Ω resistance, $I_4 = \dfrac{V_B}{12} = \dfrac{12}{12} = 1$ A **Ans.**

Example 2.11

Determine the voltage across 0.8 mho using nodal analysis in Fig. 2.37.
[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007]

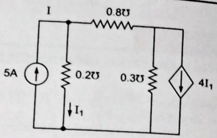


Fig. 2.37

Solution: Let the given circuit be redrawn as with node C as the reference node and arbitrarily assumed distribution of current as shown in Fig. 2.38.

Applying KCL at node A, we have

$$I = I_1 + I_2$$

$$\text{or } 5 = 0.2V_A + 0.8(V_A - V_B) \qquad \because I = 5A$$

$$\text{or } V_A - 0.8V_B = 5 \qquad ...(i)$$

Applying KCL at node B, we have

$$I_2 = I_3 + I_4$$

$$\text{or } 0.8(V_A - V_B) = 0.3V_B + 4I_1$$

$$\text{or } 0.8(V_A - V_B) = 0.3V_B + 4 \times 0.2V_A \qquad \because I_1 = 0.2V_A$$

$$\text{or } V_B = 0$$

Substituting $V_B = 0$ in Eq. (*i*), we have

$$V_A = 5\text{ V}$$

and current in 0.8 ℧ resistor $= I_2 = (V_A - V_B) \times 0.8 = (5-0) \times 0.8 = 4$A **Ans.**

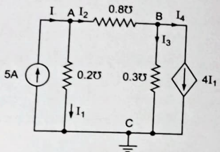


Fig. 2.38

Example 2.12

For the circuit shown in Fig. 2.39 find (*a*) voltage *v* (*b*) current through the 2 Ω resistor using nodal method.

Solution: Let the circuit be redrawn with node C as the reference node and arbitrarily assumed distribution of current, as shown in Fig. 2.40. From circuit diagram shown in Fig. 2.40, we have

$$V_B = -8\text{ V} \qquad \because V_C = 0$$

$$\text{and } v = I_1 \times 1 = I_1 = \frac{6 - V_A}{1} = (6 - V_A)$$

Applying KCL to node A, we have

$$\text{or } \frac{6-V_A}{1} + \frac{5v - V_A}{2} = \frac{V_A - V_B}{3}$$

$$\text{or } (6 - V_A) + \frac{5(6-V_A) - V_A}{2} = \frac{V_A + 8}{3}$$

$$\text{or } 13V_A = 55$$

$$\text{or } V_A = \frac{55}{13}\text{ V}$$

(*a*) Voltage $v = 6 - V_A$

$$= 6 - \frac{55}{13} = \frac{23}{13}\text{ V } \textbf{Ans.}$$

(*b*) Current through 2 Ω resistor $= I_2$

$$= \frac{5v - V_A}{2} = \frac{5 \times \frac{23}{13} - \frac{55}{13}}{2} = \frac{30}{13}\text{ A } \textbf{Ans.}$$

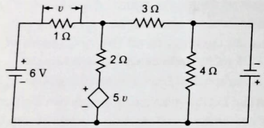


Fig. 2.39

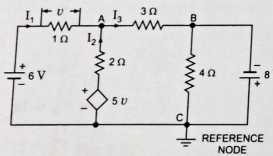


Fig. 2.40

Example 2.13

For the circuit shown in Fig. 2.41, find voltages of nodes B and C, and determine current in 8 Ω resistor.

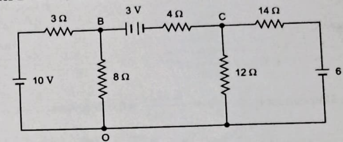


Fig. 2.41

Solution: Let the circuit be redrawn with terminal O as the reference node and arbitrarily assumed distribution of current, as shown in Fig. 2.42.

Applying KCL to nodes B and C respectively, we have

$$I_1 = I_2 + I_3$$

$$\text{or } \frac{10 - V_B}{3} = \frac{V_B}{8} + \frac{V_B - V_C - 3}{4} \qquad ...(i)$$

$$\text{and } I_4 = I_5 + I_3$$

$$\text{or } \frac{V_C}{12} = \frac{6 - V_C}{14} + \frac{V_B - V_C - 3}{4} \qquad ...(ii)$$

$$\text{or } 17V_B - 6V_C = 98 \qquad ...(iii)$$

$$\text{and } 21V_B - 34V_C = 27 \qquad ...(iv)$$

Solving above Eqs. (*iii*) and (*iv*), we have

Voltage of node B, $V_B = 7.0133$ V **Ans.**

Voltage of node of C, $V_C = 3.5376$ V **Ans.**

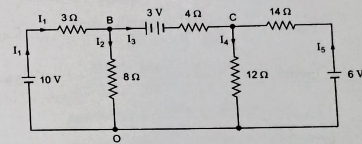


Fig. 2.42

Current in 8 Ω resistor $= I_2 = \dfrac{V_B}{8} = \dfrac{7.0133}{8} = 0.87666$ A **Ans.**

Example 2.14

Use nodal analysis to find the currents in various resistors of the circuit shown (Fig. 2.43).
[U.P. Technical Univ. Electrical Engineering Second Semester 2005-06]

Solution: Let the circuit be redrawn with its different nodes marked A, B, C and D, the last one being taken as reference or datum node (Fig. 2.44). Applying KCL to nodes A, B and C respectively, we have

$$I_1 + I_2 + I_3 = I$$

$$\text{or } \frac{V_A}{2} + \frac{V_A - V_B}{3} + \frac{V_A - V_C}{5} = 10$$

$$\text{or } 15V_A + 10V_A - 10V_B + 6V_A - 6V_C = 300$$

$$\text{or } 31V_A - 10V_B - 6V_C = 300 \qquad ...(i)$$

$$I_2 = I_4 + I_5$$

$$\text{or } \frac{V_A - V_B}{3} = \frac{V_B - V_C}{1} + \frac{V_B}{5}$$

$$\text{or } 5V_A - 5V_B = 15V_B - 15V_C + 3V_B$$

$$\text{or } 5V_A - 23V_B + 15V_C = 0 \qquad ...(ii)$$

$$\text{and } I_3 + I_4 - I_6 = 2$$

$$\text{or } \frac{V_A - V_C}{5} + \frac{V_B - V_C}{1} - \frac{V_C}{4} = 2$$

$$\text{or } 4V_A - 4V_C + 20V_B - 20V_C - 5V_C = 40$$

$$\text{or } 4V_A + 20V_B - 29V_C = 40 \qquad ...(iii)$$

Solving Eqs. (*i*), (*ii*) and (*iii*), we have

$$V_A = \frac{6.572}{545} \text{ V}$$

$$V_B = \frac{556}{109} \text{ V}$$

$$V_C = \frac{2{,}072}{545} \text{ V}$$

Thus current $I_1 = \frac{V_A}{2} = \frac{6.572}{2 \times 545} = 6.029$ A **Ans.**

$$\text{Current } I_2 = \frac{V_A - V_B}{3} = \frac{6.572 - 2.780}{3 \times 545} = 2.319 \text{ A } \textbf{Ans.}$$

$$\text{Current, } I_3 = \frac{V_A - V_C}{5} = \frac{6.572 - 2.072}{5 \times 545} = 1.651 \text{ A } \textbf{Ans.}$$

$$\text{Current, } I_4 = \frac{V_B - V_C}{1} = \frac{556}{109} - \frac{2.072}{545} = 1.299 \text{ A } \textbf{Ans.}$$

$$\text{Current, } I_5 = \frac{V_B}{5} = \frac{556}{109 \times 5} = 1.02 \text{ A } \textbf{Ans.}$$

$$\text{Current, } I_6 = \frac{V_C}{4} = \frac{2.072}{4 \times 545} = 0.95 \text{ A } \textbf{Ans.}$$

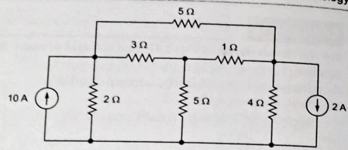


Fig. 2.43

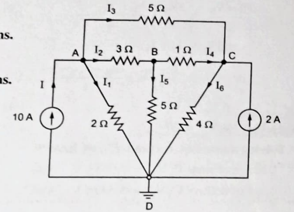


Fig. 2.44

Example 2.15

Using nodal analysis, determine current in each branch of the network shown in Fig. 2.45. Also find total power loss in the network.

Solution: Let the circuit given in Fig. 2.45 be redrawn with terminal G as the reference node and arbitrarily assumed distribution of current, shown in Fig. 2.46.

Applying Kirchhoff 's current law to nodes A, B and C respectively, we have

$I_1 = I_2 + I_3$ at node A

or $I = \frac{V_A}{10} + \frac{V_A - V_B}{10}$

or $2V_A - V_B = 10$ ...(*i*)

$I_3 + I_5 = I_4$ at node B

$$\frac{V_A - V_B}{10} + \frac{V_C - V_B}{20} = \frac{V_B + 10}{20}$$

or $2V_A - 4V_B + V_C = 10$ ...(*ii*)

and $I_5 + I_6 = I_7$ at node C

or $\frac{V_C - V_B}{20} + \frac{V_C}{20} = 0.5$

or $2V_C - V_B = 10$ ...(*iii*)

Solving Eqs. (*i*), (*ii*) and (*iii*), we have

$V_A = 6$ V, $V_B = 2$ V and $V_C = 6$ V

Currents in different resistors are given below:

Current from current source, $I_1 = 1$ A

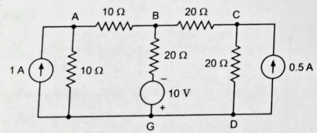


Fig. 2.45

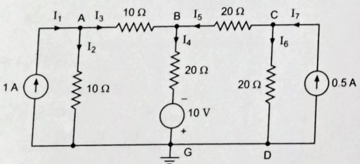


Fig. 2.46

Current through 10 Ω resistor connected between terminals A and G,

$$I_2 = \frac{V_A}{10} = \frac{6}{10} = 0.6 \text{ A } \textbf{Ans.}$$

Current through branch AB (10 Ω resistor), $I_3 = \frac{V_A - V_B}{10} = \frac{6-2}{10} = 0.4$ A **Ans.**

Current through branch BG (20 Ω resistor), $I_4 = \frac{V_B + 10}{20} = \frac{2+10}{20} = 0.6$ A **Ans.**

Current through branch BC (20 Ω resistor), $I_5 = \frac{V_C - V_B}{20} = \frac{6-2}{20} = 0.2$ A **Ans.**

Current through branch CD (20 Ω resistor), $I_6 = \frac{V_C}{20} = \frac{6}{20} = 0.3$ A **Ans.**

$$I_7 = 0.5 \text{ A}$$

$$\text{Total power loss} = (0.6)^2 \times 10 + (0.4)^2 \times 10 + (0.6)^2 \times 20 + (0.2)^2 \times 20 + (0.3)^2 \times 20$$

$$= 3.6 + 1.6 + 7.2 + 0.8 + 1.8 = 15 \text{ W } \textbf{Ans.}$$

Example 2.16

Use the nodal analysis to find I_1 to I_6 in various branches of circuit shown in Fig. 2.47.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2010]

Solution : Let the given circuit be redrawn with terminal G as the reference node and arbitrarily assumed the current in 50 Ω to be I_6 in the direction shown in Fig. 2.48. Assume the resistor of arm AG to be 10 Ω (missing data). Let V_A, V_B and V_C be the voltages of nodes A, B and C respectively.

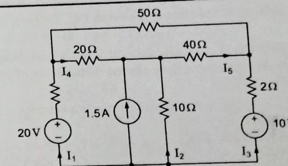


Fig. 2.47

Applying KCL to nodes A, B and C respectively, we have

$$I_1 = I_4 + I_6$$

or $\frac{20 - V_A}{10} = \frac{V_A - V_B}{20} + \frac{V_A - V_C}{50}$

or $200 - 10V_A = 5V_A - 5V_B + 2V_A - 2V_C$

or $17V_A - 5V_B - 2V_C = 200$ ...(*i*)

$$I_4 + 1.5 + I_2 = I_5$$

or $\frac{V_A - V_B}{20} + 1.5 + \frac{0 - V_B}{10} = \frac{V_B - V_C}{40}$

or $2V_A - 2V_B + 60 - 4V_B = V_B - V_C$

or $2V_A - 7V_B + V_C = -60$ ...(*ii*)

and $I_5 + I_6 + I_3 = 0$

or $\frac{V_B - V_C}{40} + \frac{V_A - V_C}{50} + \frac{10 - V_C}{2} = 0$

or $5V_B - 5V_C + 4V_A - 4V_C + 1{,}000 - 100V_C = 0$

or $4V_A + 5V_B - 109V_C = -1{,}000$ ...(*iii*)

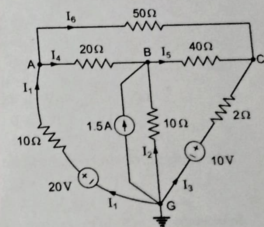


Fig. 2.48

Solving Eqs. (*i*), (*ii*) and (*iii*), we have

$$V_A = 17.43 \text{ V}$$

$$V_B = 15.05 \text{ V}$$

$$V_C = 10.5 \text{ V}$$

$$\text{Current } I_1 = \frac{20 - V_A}{10} = \frac{20 - 17.43}{10} = 0.257 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current } I_2 = \frac{0 - V_B}{10} = \frac{0 - 15.05}{10} = -1.505 \text{ A} \quad \textit{i.e.,} \quad 1.505 \text{ A from node B to node G} \quad \textbf{Ans.}$$

$$\text{Current } I_3 = \frac{10 - V_C}{2} = \frac{10 - 10.5}{2} = -0.25 \text{ A} \quad \textit{i.e.,} \quad 0.25 \text{ A from node C to node G} \quad \textbf{Ans.}$$

$$\text{Current } I_4 = \frac{V_A - V_B}{20} = \frac{17.43 - 15.05}{20} = 0.119 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current } I_5 = \frac{V_B - V_C}{40} = \frac{15.05 - 10.5}{40} = 0.11375 \text{ A} \quad \textbf{Ans.}$$

$$\text{Current } I_6 = \frac{V_A - V_C}{50} = \frac{17.43 - 10.5}{50} = 0.1386 \text{ A} \quad \textbf{Ans.}$$

Example 2.17

Using nodal analysis method find voltage across the terminals '*c*' and '*d*' of the circuit shown in Fig. 2.49 (*a*).

[G.G.S.I.P. Univ. Delhi Electrical Science 2004-05]

Solution : Let the given circuit be redrawn with node *d* as the reference node and arbitrarily assumed distribution of current, as shown in Fig. 2.49 (*b*).

Applying Kirchhoff's current law to nodes *a*, *b* and *c* respectively, we have

$$I_1 + I_2 = 2 - I_6 \quad ...(i)$$

$$I_1 = I_3 + I_4 \quad ...(ii)$$

$$\text{and } I_2 + I_3 = I_5 \quad ...(iii)$$

$$\text{or} \quad \frac{V_a - V_b}{8} + \frac{V_a - V_c}{4} = 2 - \frac{V_a}{12} \quad ...(iv)$$

$$\frac{V_a - V_b}{8} = \frac{V_b - V_c}{6} + \frac{V_b}{20} \quad ...(v)$$

$$\text{and} \quad \frac{V_a - V_c}{4} + \frac{V_b - V_c}{6} = \frac{V_c}{10} \quad ...(vi)$$

$$\text{or} \quad 11V_a - 3V_b - 6V_c = 48 \quad ...(vii)$$

$$15V_a - 41V_b + 20V_c = 0 \quad ...(viii)$$

$$15V_a + 10V_b - 31V_c = 0 \quad ...(ix)$$

Subtracting Eq. (*ix*) from Eq. (*viii*), we have

$$-51V_b + 51V_c = 0$$

$$\text{or } V_b = V_c$$

Substituting $V_b = V_c$ in Eq. (*viii*) or (*ix*), we have

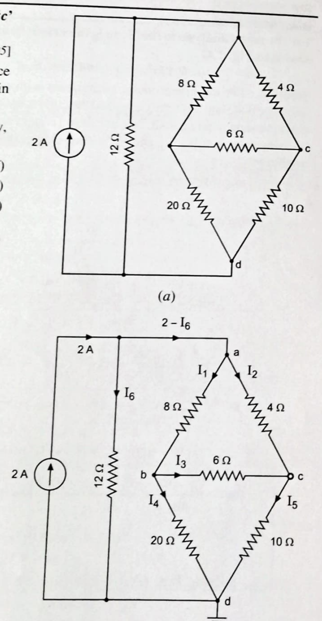
$$V_a = \frac{21}{15} V_c = 1.4 V_c$$

Substituting $V_a = 1.4\, V_c$ and $V_b = V_c$ in Eq. (*vii*), we have

$$15.4\, V_c - 3V_c - 6V_c = 48$$

$$\text{or } V_c = \frac{48}{6.4} = 7.5 \text{ V}$$

It means potential difference across the terminals *c* and *d* is 7.5 V **Ans.**



(*a*)

(*b*)

Fig. 2.49

Example 2.18

Using the concept of supernode, find node voltage V_A, V_B, V_C and V_D of Fig. 2.50.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]

Solution : Nodes A and B form one supernode and nodes C and D form another supernode [Fig. 2.51 (*a*)].

For supernodes A and B in Fig. 2.51 (*b*), we have

$$I_1 + I_2 = I_3 + I_4$$

$$\text{or} \quad \frac{V_A - V_D}{3} + \frac{V_A - 0}{2} = \frac{V_C - V_B}{6} + \frac{0 - V_B}{10}$$

$$\text{or} \quad \frac{V_C - V_B}{6} - \frac{V_B}{10} - \frac{V_A - V_D}{3} - \frac{V_A}{2} = 0$$

$$\text{or } 5\,V_C - 5\,V_B - 3\,V_B - 10\,V_A + 10\,V_D - 15\,V_A = 0$$

$$\text{or } 25\,V_A + 8\,V_B - 5\,V_C - 10\,V_D = 0 \quad ...(i)$$

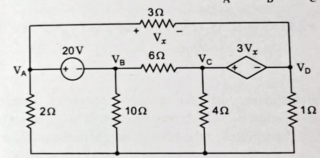


Fig. 2.50

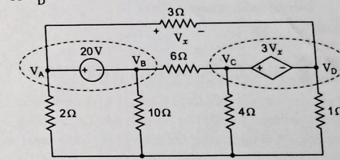


Fig. 2.51(*a*)

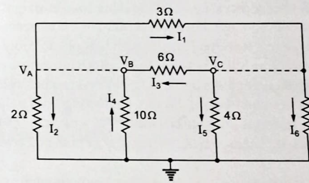


Fig. 2.51(*b*)

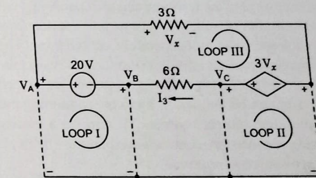


Fig. 2.51(*c*)

For supernodes C and D, we have

$$I_1 = I_3 + I_5 + I_6$$

$$\text{or} \quad \frac{V_A - V_D}{3} = \frac{V_C - V_B}{6} + \frac{V_C}{4} + \frac{V_D}{1}$$

$$\text{or } 4\,V_A + 2\,V_B - 5\,V_C - 16\,V_D = 0 \quad ...(ii)$$

Now applying KVL to various loops, we have

$$\text{For loop I} \quad V_A - V_B = 20 \quad ...(iii)$$

$$\text{For loop II} \quad -V_C + 3\,V_x + V_D = 0$$

$$\text{But} \quad V_x = V_A - V_D$$

So, we have

$$3\,V_A - V_C - 2\,V_D = 0 \quad ...(iv$$

$$\text{and for loop III} \quad -2\,V_x + 6\,I_3 - 20 = 0 \quad ...($$

$$\text{But} \quad 6\,I_3 = V_C - V_B$$

Substituting $V_x = V_A - V_D$ and $6I_3 = V_C - V_B$ in Eq. (*v*), we have

$$-2(V_A - V_D) + V_C - V_B - 20 = 0$$

$$\text{or } -2V_A - V_B + V_C + 2V_D = 20 \qquad \text{...(vi)}$$

Substituting $V_A - V_B = 20$ or $V_B = V_A - 20$ in Eqs. (*i*) and (*ii*) respectively, we have

$$25V_A + 8(V_A - 20) - 5V_C - 10V_D = 0$$

$$\text{and } 4V_A + 2(V_A - 20) - 5V_C - 16V_D = 0$$

$$\text{or } 33V_A - 5V_C - 10V_D = 160 \qquad \text{...(vii)}$$

$$\text{and } 6V_A - 5V_C - 16V_D = 40 \qquad \text{...(viii)}$$

Writing Eqs. (*iv*), (*vii*) and (*viii*) in matrix form, we have

$$\begin{bmatrix} 3 & -1 & -2 \\ 33 & -5 & -10 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} V_A \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 0 \\ 160 \\ 40 \end{bmatrix}$$

Solving above matrix, we have

$$V_A = 8.88\text{ V}, V_B = -11.12\text{ V}, V_C = 66.6\text{ V and } V_D = -20\text{ V} \quad \textbf{Ans.}$$

2.13. SUPERPOSITION THEOREM

This theorem is applied when we are to determine the current in one particular branch of a network containing several voltage sources or current sources or both voltage sources and current sources. This scheme is to determine how much current each of the individual source contributes to the branch in question, and then add algebraically these component currents.

If there are several sources of emfs acting simultaneously in an electric circuit, then according to this theorem emf of each source acts independently of those of other sources, *i.e.*, as if the other sources of emf did not exist and current in any branch or conductor of a network is equal to the algebraic sum of the currents due to each source of emf separately, all other emfs being taken equal to zero. This theorem is applicable only in linear circuits, *i.e.*, circuits consisting of resistances in which Ohm's law is valid. In circuits having nonlinear resistances such as thermionic valves and metal rectifiers, this theorem is not applicable. However, superposition theorem can be applied to a circuit containing current sources and even to circuits having both voltage sources and current sources. To remove a current source from the circuit, circuit of the source is opened leaving in place any conductance that may be in parallel with it, just as series resistance is kept in place when voltage source is removed.

Though the application of the above theorem requires a little more work than other methods such as the circulating current method but it avoids the solution of two or more simultaneous equations. After a little practice with this method, equations can be written directly from the original circuit diagram and labour in drawing extra diagrams is saved.

The superposition theorem can be stated as below :

In a linear resistive network containing two or more voltage sources, the current through any element (resistance or source) may be determined by adding together algebraically the currents produced by each source acting alone, when all other voltage sources are replaced by their internal resistances. If a voltage source has no internal resistance, the terminals to which it was connected are joined together. If there are current sources present they are removed and the network terminals to which they were connected are left open.

The procedure for applying superposition theorem is as follows:

1. Replace all but one of the sources of supply by their internal resistances. If the internal resistance of any source is very small as compared to other resistances existing in the network, the source is replaced by a short circuit. In case of a current source open the circuit leaving in place any conductance that may be in parallel with it.

2. Determine the currents in various branches using Ohm's law.
3. Repeat the process using each of the emfs turn-by-turn as the sole emf each time. Now the total current in any branch of the circuit is the algebraic sum of currents due to each source.

The details of the above procedure can best be understood by examining its application to the following solved examples.

Example 2.19

Calculate the current in each branch of Fig. 2.52 by superposition theorem.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]

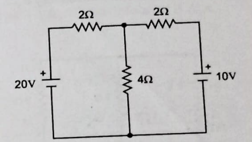


Fig. 2.52

Solution: Replacing 10 V source by a short circuit, the circuit is reduced to a simple circuit shown in Fig. 2.53(*a*).

Equivalent resistance of the circuit shown in Fig. 2.53(*a*)

$$R' = R_{AB} + R_{BC} \| R_{BD}$$

$$= 2 + 2 \| 4 = 2 + \frac{1}{\frac{1}{2}+\frac{1}{4}} = 2 + \frac{4}{3} = \frac{10}{3}\ \Omega$$

Current supplied by 20 V source, $I'_1 = \frac{20}{R'} = \frac{20}{10/3} = 6\text{A}$

Current in resistance BC, $I'_2 = I'_1 \times \frac{R_{BD}}{R_{BC} + R_{BD}}$

By current division rule

$$= 6 \times \frac{4}{2+4} = 4\text{A}$$

Current in resistance BD, $I'_3 = I'_1 \times \frac{R_{BC}}{R_{BC} + R_{BD}} = 6 \times \frac{2}{6} = 2\text{A}$

Similarly on replacement of 20 V source by a short circuit, the given circuit is reduced to a simple circuit shown in Fig. 2.53 (*b*).

Equivalent resistance of the circuit shown in Fig. 2.53 (*b*),

$$R'' = R_{CB} + R_{BD} \| R_{BA}$$

$$= 2 + 4 \| 2 = 2 + \frac{1}{\frac{1}{4}+\frac{1}{2}} = \frac{10}{3}\ \Omega$$

Current supplied by 10 V source, $I''_2 = \frac{10}{R''} = \frac{10}{10/3} = 3\text{A}$

Current in resistance BD, $I''_3 = I''_2 \times \frac{R_{BA}}{R_{BA} + R_{BD}} = 3 \times \frac{2}{2+4} = 1\text{A}$

Current in resistance BA, $I''_1 = I''_2 \times \frac{R_{BD}}{R_{BA} + R_{BD}} = 3 \times \frac{4}{2+4} = 2\text{A}$

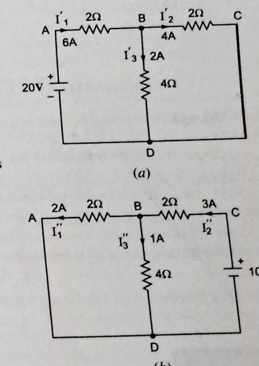


Fig. 2.53

Current in branch AB is equal to the algebraic sum of I'_1 and I''_1

$$\textit{i.e.},\ I_1 = I'_1 + I''_1 = 6 + (-2) = 4\text{A} \quad \textbf{Ans.}$$

$$\text{Similarly } I_2 = I'_2 + I''_2 = 4 + (-3) = 1\text{A} \quad \textbf{Ans.}$$

$$\text{and } I_3 = I'_3 + I''_3 = 2 + 1 = 3\text{A} \quad \textbf{Ans.}$$

Example 2.20

Find the current through 3 Ω resistor using superposition theorem for the circuit shown in Fig. 2.54.

[*G.G.S.I.P. Univ. Delhi Electrical Science 2005-06*]

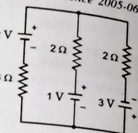


Fig. 2.54

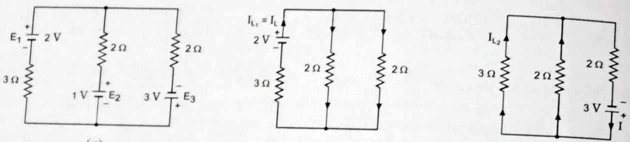
Solution: Taking $E_2 = 0$ and $E_3 = 0$ we have the circuit shown in Fig. 2.55 (*b*).

$$\text{Current drawn from battery } E_1,\ I = \frac{2}{3 + 2\,||\,2} = \frac{2}{4} = 0.5 \text{ A}$$

Current through 3 Ω resistor, $I_{L_1} = I = 0.5$ A.

Taking each of emfs E_1 and E_2 equal to zero, the circuit becomes as shown in Fig. 2.55 (*c*).

$$\text{Current drawn from battery } E_3,\ I = \frac{3}{2 + 3\,||\,2} = \frac{3}{2 + \dfrac{3 \times 2}{3 + 2}} = \frac{3}{3.2} \text{ A}$$



(*a*) (*b*) (*c*)

Fig. 2.55

$$\text{Current through 3 Ω resistor, } I_{L_2} = I \times \frac{2}{2 + 3} = \frac{3}{3.2} \times \frac{2}{5} = 0.375 \text{ A}$$

Taking both of E_1 and E_3 equal to zero, the circuit becomes as shown in Fig. 2.55 (*d*).

$$\text{Current drawn from battery } E_2,\ I = \frac{1}{2 + 3\,||\,2} = \frac{1}{2 + \dfrac{3 \times 2}{3 + 2}} = \frac{1}{3.2} \text{ A}$$

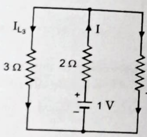


Fig. 2.55 (*d*)

$$\text{Current in 3 Ω resistor, } I_{L_3} = I \times \frac{2}{3 + 2} = \frac{1}{3.2} \times \frac{2}{5} = 0.125 \text{ A}$$

Since the direction of I_{L_3} is opposite to that of currents I_{L_1} and I_{L_2},

$$I_{L_3} = -0.125 \text{ A}$$

Total current through 3 Ω resistor when all the three batteries exist in the circuit,

$$I_L = I_{L_1} + I_{L_2} + I_{L_3} = 0.5 + 0.375 - 0.125 = 0.75 \text{ A} \quad \textbf{Ans.}$$

Example 2.21

For Fig. 2.56, find the current which flows through the 15 Ω resistor by using the mesh method. Check your solution by using superposition theorem.

[*G.G.S.I.P. Univ. Delhi Electrical Science May 2011*]

Solution: The network is redrawn as illustrated in Fig. 2.57.

There are two independent loops. The loop currents have been taken clockwise, as marked. The individual branch currents along with their direction of flow are also shown in the circuit diagram. Apply KVL to meshes I and II respectively, we have

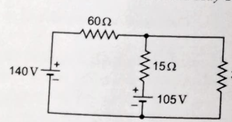


Fig. 2.56

$$60\,I_1 + 15\,(I_1 - I_2) = 140 - 105$$

$$\text{and } 30\,I_2 - 15\,(I_1 - I_2) = 105$$

$$\text{or } 15\,I_1 - 3\,I_2 = 7 \quad \ldots(i)$$

$$\text{and } -15\,I_1 + 45\,I_2 = 105 \quad \ldots(ii)$$

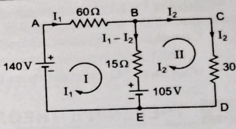


Fig. 2.57

Adding Eqs (*i*) and (*ii*), we have

$$42\,I_2 = 112$$

$$\text{or } I_2 = \frac{8}{3} \text{ A}$$

$$\text{and } I_1 = \frac{7 + 3I_2}{15} = \frac{7 + 8}{15} = 1 \text{ A}$$

$$\text{Current through 15 Ω resistor} = I_1 - I_2 = 1 - \frac{8}{3} = \frac{-5}{3} \text{ A or 1.667 A from node E to node B } \textbf{Ans.}$$

Current through 15 Ω resistor due to 105 V source alone

$$I_1 = \frac{105}{15 + 60\,||\,30} = \frac{105}{15 + \dfrac{60 \times 30}{60 + 30}} = \frac{105}{35} = 3 \text{ A upward}$$

Current supplied by 140 V source alone

$$I = \frac{140}{60 + 15\,||\,30} = \frac{140}{60 + \dfrac{15 \times 30}{15 + 30}} = \frac{140}{70} = 2 \text{ A}$$

Current through 15 Ω resistance due to 140 V alone

$$I_2 = \frac{2}{15 + 30} \times 30 = 1.333 \text{ A downward}$$

So current flowing through 15 Ω resistance = 3 – 1.333 = 1.667 A **Ans.**

Thus, the current through 15 Ω resistance is same by mesh method as well as by superposition theorem.

Example 2.22

Using superposition theorem, determine currents in all the resistors of the following network (Fig. 2.58).

[*U.P. Technical Univ. Electrical Engineering, First Semester 2005-06*]

Solution: Replacing 10 V source by a short circuit, the circuit is reduced to a simple circuit as shown in Fig. 2.59 (*a*).

The current of 2 A is divided into resistances of 5 Ω and 5 Ω equally as shown in Fig. 2.59 (*a*). Replacing 2 A source by an open circuit, the circuit is reduced to a simple circuit as shown in Fig. 2.59 (*b*).

$$\text{Equivalent resistance of the circuit, } R = (5 + 5)\,||\,10 = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

$$\text{Current supplied by the voltage source, } I = \frac{10}{5} = 2 \text{ A}$$

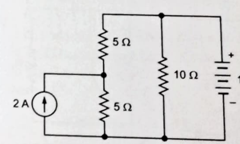
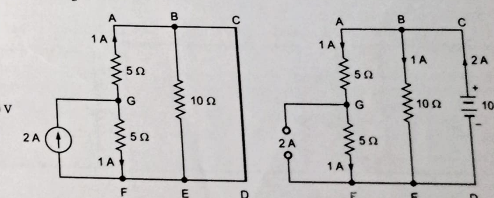


Fig. 2.58



(*a*) (*b*)

Fig. 2.59

Current of 2 A is divided between resistance arms BE and AGF equally because their resistances are equal, so current distribution will be as shown in Fig. 2.59 (*b*). From Figs. 2.59 (*a*) and (*b*), we have

Current in 10 Ω resistance (branch BE) = 0 + 1 = 1 A **Ans.**

Current in 5 Ω resistance (branch AG) = 1 − 1 = 0 A **Ans.**

Current in 5 Ω resistance (branch GF) = 1 + 1 = 2 A **Ans.**

2.14. THEVENIN'S THEOREM

This theorem provides a mathematical technique for replacing a two-terminal network by a voltage source V_T, and resistance R_T connected in series. The voltage source V_T (called the *Thevenin's equivalent voltage*) is the open-circuit voltage that appears across the load terminals when the load is removed or disconnected and resistance R_T, called the *Thevenin's equivalent resistance*, is equal to the resistance of the network looking back into the load terminals. A Thevenin's equivalent circuit is shown in Fig. 2.60. The steady-state current will be given as

$$I = \frac{V_T}{R_T + R_L} \quad \text{in case of a dc network}$$

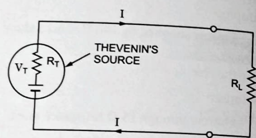


Fig. 2.60

For visualizing the application of Thevenin's theorem, let us consider a circuit shown in Fig. 2.61 (*a*) which consists of a source of emf E volts and internal resistance *r* ohms connected to an external circuit consisting of resistances R_1 and R_2 ohms in series.

So far as terminals AB across which a resistance of R_2 ohms is connected the network acts as source of open-circuit voltage V_{OC} (also called the Thevenin's equivalent voltage V_T) and internal resistance R_{in} (also called the Thevenin's resistance R_T).

For determination of open-circuit voltage V_{OC} (or V_T), disconnect the load resistance R_L from the terminals A and B to provide open circuit [Fig. 2.61 (*b*)].

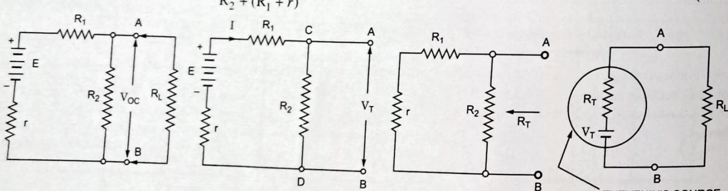
Now current through resistance R_2, $I = \dfrac{E}{R_1 + R_2 + r}$

and open-circuit voltage, V_{OC} or V_T = Voltage across terminals AB

$$= \text{Voltage drop across resistance } R_2 = IR_2 = \frac{ER_2}{R_1 + R_2 + r} \quad ...(2.30)$$

For determination of internal resistance R_{in} (or R_T) of the network under consideration remove the voltage source from the circuit, leaving behind only its internal resistance *r*, as illustrated in Fig. 2.61 (*c*). Now view the circuit inwards from the open terminals A and B. It is found that the circuit [Fig. 2.61 (c)] now consists of two parallel paths—one consisting of resistance R_2 only, and the other consisting of resistance R_1 and *r* in series. Thus, the equivalent resistance (R_T), as viewed from the open terminals A and B, is given as

$$R_T = \frac{R_2 (R_1 + r)}{R_2 + (R_1 + r)} \quad ...(2.31)$$



(*a*) (*b*) (*c*) (*d*)

Fig. 2.61 *Illustration of Application of Thevenin's Theorem*

Now when load resistance R_L is connected across terminals A and B, the network behaves as a source of voltage V_T and internal resistance R_T [Fig. 2.61 (*d*)] and current flowing through the load resistance R_L is given as

$$I_L = \frac{V_T}{R_T + R_L} = \frac{ER_2/[R_1 + R_2 + r]}{\dfrac{R_2(R_1 + r)}{R_1 + r + R_2} + R_L} = \frac{ER_2}{R_2(R_1 + r) + R_L(R_1 + R_2 + r)} \quad ...(2.32)$$

The Thevenin's theorem can be stated as follows:

The current in any *passive circuit element* (which may be called R_L) in a network is the same as would be obtained if R_L were supplied with a voltage source V_{OC} (or V_T) in series with an equivalent resistance R_{in} (or R_T), V_{OC} being the open-circuit voltage at the terminals from which R_L has been removed and R_{in} (or R_T) being the resistance that would be measured at these terminals after all sources have been removed and each has been replaced by its internal resistance.

This theorem is advantageous when we are to determine the current in a particular element of a linear bilateral network particularly when it is desired to find the current which flows through a resistor for its different values. It makes the solution of the complicated networks (particularly electronic networks) quite simple.

Example 2.23

For the circuit given in Fig. 2.62, determine the Thevenin's voltage and Thevenin's resistance as seen at 'ab'.

Solution: The equivalent resistance of the network (with voltage source short circuited) with reference to terminals *a* and *b* (Fig. 2.62)

$$R_T = 5 + 10 \,||\, 10 = 5 + \frac{1}{\frac{1}{10} + \frac{1}{10}} = 10\ \Omega \ \textbf{Ans.}$$

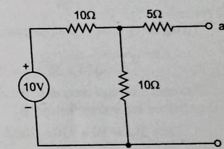


Fig. 2.62

With terminals "*ab*" opened, the current through the closed mesh formed by voltage source and two resistors, each of 10 Ω is,

$$I = \frac{10}{10 + 10} = 0.5\text{A}$$

So open-circuit voltage, V_T = Voltage drop across 10 Ω resistor

$= 0.5 \times 10 = 5$ V **Ans.**

Example 2.24

For the circuit shown in Fig. 2.63, find the current in 12 Ω resistance using Thevenin's theorem. [*G.G.S.I.P. Univ. 1st Term Exam. February 2008*]

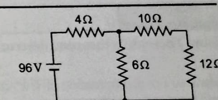
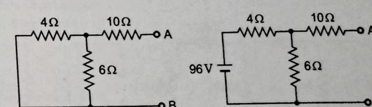


Fig. 2.63

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B, the voltage source is short circuited, as shown in Fig. 2.64(*a*).

$$\text{Thevenin's equivalent resistance, } R_T = 10 + 6 \,||\, 4 = 10 + \frac{6 \times 4}{6 + 4} = 12.4\ \Omega$$

For determination of open-circuit voltage, the resistance of 12 Ω between terminals A and B is removed, as shown in Fig. 2.64 (*b*). Current flowing through closed mesh, we have



(*a*) (*b*)

Fig. 2.64

$$I = \frac{96}{4 + 6} = 9.6 \text{ A}$$

Thevenin's voltage, V_T = Voltage drop across 6 Ω resistance

$= 6\,I = 6 \times 9.6 = 57.6$ V

Current through load resistance of 12 Ω

$$I_L = \frac{V_T}{R_T + R_L} = \frac{57.6}{12.4 + 12} = \frac{57.6}{24.4} = 2.36 \text{ A } \textbf{Ans.}$$

Example 2.25

By Thevenin's theorem calculate the load current in load resistance of 150 Ω in Fig. 2.65.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]

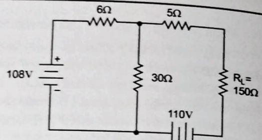


Fig. 2.65

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B, the voltage sources are short circuited, as shown in Fig. 2.66 (*a*).

Thevenin's equivalent resistance,

$$R_T = 5 + 6 \,||\, 30 = 5 + \frac{1}{\frac{1}{6} + \frac{1}{30}} = 5 + 5 = 10\ \Omega$$

For determination of open-circuit voltage, the resistance of 150 Ω between terminals A and B is removed, the circuit becomes as shown in Fig. 2.66 (*b*).

Current flowing through the closed mesh, we have

$$I = \frac{108}{6+30} = 3\text{A}$$

Voltage drop across 30 Ω resistance

$$= 30\,I = 30 \times 3 = 90 \text{ V}$$

Thevenin's voltage is equal to the algebraic sum of voltage drop across 30 Ω resistance and voltage battery B_2

Thus $V_T = 90 + 110 = 200$ V

Current through load resistance of 150 Ω.

$$I_L = \frac{V_T}{R_T + R_L} = \frac{200}{10 + 150} = 1.25\text{A } \textbf{Ans.}$$

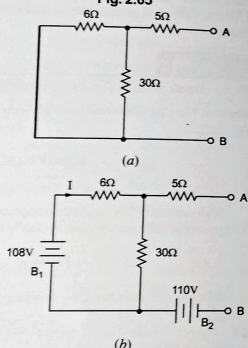


Fig. 2.66

Example 2.26

Using Thevenin's theorem, determine current and voltage in 2 ohm resistance in the circuit shown in Fig. 2.67.

[U.P. Technical Univ. Electrical Engineering First Semester 2006-07]

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals A and B, the voltage source is short circuited and current source is open circuited, as shown in Fig. 2.68(*a*).

$$\text{Thevenin's equivalent resistance, } R_T = 5 + 10 \,||\, 6 = 5 + \frac{10 \times 6}{10 + 6} = 8.75\ \Omega$$

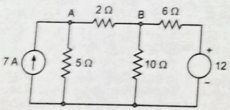


Fig. 2.67

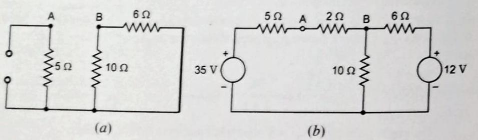


Fig. 2.68

Converting current source into equivalent voltage source, the circuit becomes as shown in Fig. 2.68 (*b*).

When terminals A and B are open,

$$\text{Thevenin's voltage, } V_T = V_A - V_B = 35 - \frac{12}{16} \times 10 = 27.5 \text{ V}$$

Current through 2 Ω resistance connected across AB,

$$I_L = \frac{V_T}{R_T + R} = \frac{27.5}{8.75 + 2} = 2.558 \text{ A } \textbf{Ans.}$$

Example 2.27

Find the Thevenin's equivalent of the circuit of Fig. 2.69.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]

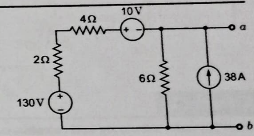


Fig. 2.69

Solution: For determination of Thevenin's equivalent resistance of the circuit w.r.t. terminals *a* and *b*, the voltage source is short circuited and current source is open circuited, as shown in Fig. 2.70 (*a*).

Thevenin's equivalent resistance, $R_T = 6 \,||\, (4 + 2) = 3\ \Omega$

For determination of Thevenin's voltage. The given circuit is redrawn as shown in Fig. 2.70 (*b*) and let the current flowing through various branches of the circuit be as shown in the circuit diagram.

Applying KVL to the mesh I, we have

$$2\,I + 4\,I + 6\,(I + 38) = 130 - 10$$

$$\text{or } I = \frac{120 - 228}{12} = \frac{-108}{12} = -9 \text{ A}$$

Thevenin's voltage, V_T = Voltage drop 6 Ω resistor $= 6 \times (I + 38) = 6\,(-9 + 38) = 174$ V

Thevenin equivalent circuit is shown in Fig. 2.70 (*c*).

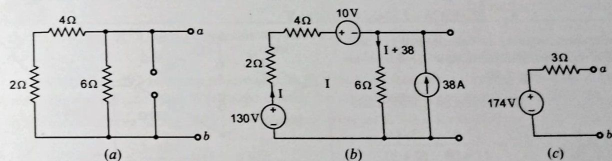


Fig. 2.70

Example 2.28

Find Thevenin's equivalent circuit across AB shown in Fig. 2.71.

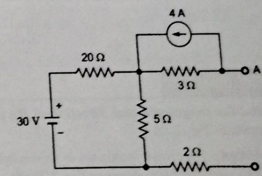


Fig. 2.71

Solution: For determining the Thevenin's equivalent resistance of the circuit w.r.t. terminals AB the voltage source is short circuited and current source is open circuited, as shown in Fig. 2.72 (*a*).

$$R_T = 3 + 2 + 5 \,||\, 20 = 3 + 2 + 4 = 9\ \Omega$$

Converting current source into the equivalent voltage source the circuit becomes as shown in Fig. 2.72 (*b*).

Since there is no current through resistors of 3 Ω and 2 Ω, there is no voltage drop across them.

Thevenin's voltage V_{Th} equals algebraic sum of battery voltage and drop across 5 Ω resistor. Hence

$$V_{Th} = 30 \times \frac{5}{20+5} - 12 = -6 \text{ V}$$

Thevenin's equivalent circuit is shown in Fig. 2.72 (c).

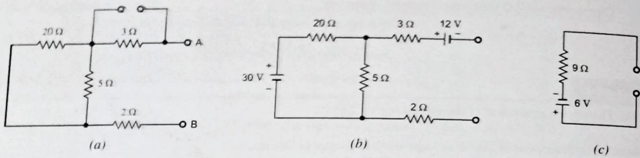


Fig. 2.72

Example 2.29

State Thevenin's theorem and calculate current in a 1,000 Ω resistor connected between terminals A and B, as shown in Fig. 2.73.

Solution: Now in the circuit shown in Fig. 2.73,

Potential of point B w.r.t. terminal D, $V_{BD} = \dfrac{5 \times 880}{1{,}000 + 880} = 2.340426$ V

Potential of point A w.r.t. to terminal D, $V_{AD} = \dfrac{5 - 0.05}{100 + 85} \times 85 + 0.05 = 2.324324$ V

PD between terminals B and A,

$$V_T = 2.340426 - 2.324324 = 0.0161 \text{ V}$$

After short circuiting the batteries in the circuit, equivalent resistance of the network with reference to terminals A and B (Fig. 2.74),

$$R_T = 100 \parallel 85 + 1{,}000 \parallel 880$$

$$= \frac{1700}{17+20} + \frac{22{,}000}{22+25}$$

$$= 514 \ \Omega$$

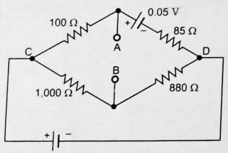


Fig. 2.73

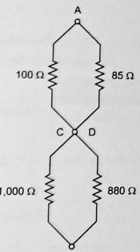


Fig. 2.74

Current in 1,000 Ω resistor connected between terminals A and B,

$$I = \frac{V_T}{R_T + R_L} = \frac{0.0161}{1{,}000 + 514} = 10.625 \ \mu\text{A from terminals B to A } \textbf{Ans.}$$

Example 2.30

Find the magnitude and direction of the current in the 2 Ω resistor by using Thevenin's theorem for the circuit shown in Fig. 2.75.

Solution : The circuit is redrawn as shown in Fig. 2.76 (*a*). After removing the 2 Ω resistor between terminals B and D, the circuit takes the form shown in Fig. 2.76 (*b*).

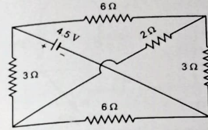


Fig. 2.75

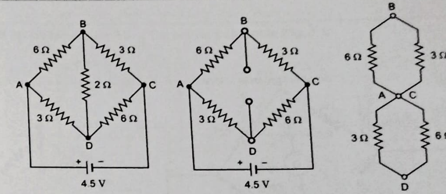


Fig. 2.76

Now in the circuit shown in Fig. 2.76 (*b*), potential of terminal B w.r.t. terminal C,

$$V_{BC} = 4.5 \times \frac{3}{6+3} = 1.5 \text{ V}$$

Similarly,

$$V_{DC} = 4.5 \times \frac{6}{3+6} = 3 \text{ V}$$

PD between terminals D and B, $V_T = V_{DC} - V_{BC} = 3 - 1.5 = 1.5$ V

After removing 4.5 V battery from the circuit (4.5 V battery being short circuited) equivalent resistance of the network with reference to terminals B and D. [Fig. 2.76 (*c*)]

$$R_T = 6 \| 3 + 3 \| 6 = 2 + 2 = 4 \ \Omega$$

Current in 2 Ω resistor, $I = \dfrac{V_T}{R_T + R_L} = \dfrac{1.5}{4+2} = 0.25$ A from terminal D to B **Ans.**

Example 2.31

Calculate the current flowing in the galvanometer shown in Fig. 2.77.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2010]

Solution : Let the voltage applied across the given circuit be 10 V (missing data) as shown in Fig. 2.78 (*a*).

After removing the galvanometer G from the circuit, the circuit becomes as shown in Fig. 2.78 (*b*).

Now in the circuit shown in Fig. 2.78 (*b*), potential of terminal B w.r.t. terminal C,

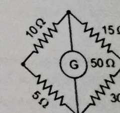


Fig. 2.77

$$V_{BC} = \frac{10}{10+15} \times 15 = 6 \text{ V}$$

$$\text{Similarly } V_{DC} = \frac{10}{5+30} \times 30 = \frac{60}{7} \text{ V}$$

PD between terminals D and B,

$$V_T = V_{DC} - V_{BC} = \frac{60}{7} - 6 = \frac{18}{7} \text{ V}$$

After removing 10 V source from the circuit (10 V source being short circuited) equivalent resistance of the network with reference to terminals B and D [Fig. 2.78 (*c*)]

$$R_T = 10 \parallel 15 + 5 \parallel 30 = \frac{10 \times 15}{10+15} + \frac{5 \times 30}{5+30} = 6 + \frac{30}{7} = \frac{72}{7} \ \Omega$$

Current in galvanometer G (50 Ω resistor),

$$I = \frac{V_T}{R_T + R_G} = \frac{18/7}{\frac{72}{7} + 50} = \frac{18}{422} \text{ A} = 42.654 \text{ mA} \quad \textbf{Ans.}$$

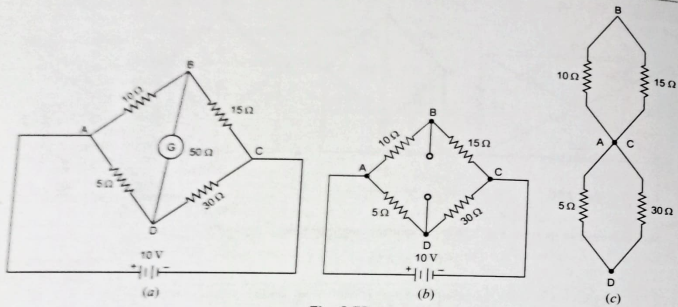


Fig. 2.78

Example 2.32

Determine the current flowing through 5 Ω resistor in the network shown in Fig. 2.79 using Thevenin's theorem.

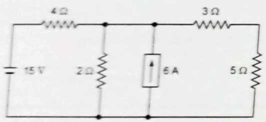


Fig. 2.79

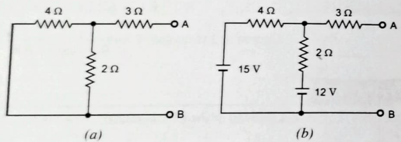


Fig. 2.80

Solution: The equivalent resistance of the network with voltage source short circuited and current source open circuited with reference to terminals A and B [Fig. 2.80 (*a*)].

$$R_T = 3 + 4 \,||\, 2 = 3 + \frac{4}{2+1} = 3 + \frac{4}{3} = \frac{13}{3}\,\Omega$$

Converting current source of 6 A connected across 2 Ω resistance into equivalent voltage source of 6 × 2 *i.e.*, 12 V in series with resistor of 2 Ω and removing the 5 Ω resistor, the given circuit is converted into a circuit shown in Fig. 2.80 (*b*).

The current flowing through the mesh formed by voltage sources and resistors of 4 Ω and 2 Ω is given as

$$I = \frac{15-12}{4+2} = 0.5 \text{ A}$$

Voltage across terminals A and B, $V_{AB} = 15 - 4 \times 0.5 = 13$ V

Also $V_{AB} = 12 + 2 \times 0.5 = 13$ V

Current flowing through resistor of 5 Ω $= \dfrac{V_{AB}}{R_T + R_L} = \dfrac{13}{\frac{13}{3}+5} = \dfrac{39}{28}$ A **Ans.**

Example 2.33

Find the Thevenin's equivalent circuit for terminal pair AB of the network shown in Fig. 2.81.

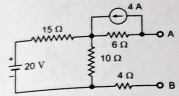


Fig. 2.81

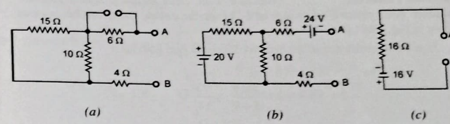


Fig. 2.82

Solution: For determining the Thevenin's equivalent resistance of the circuit with reference to terminals AB the voltage source is short circuited and current source is open circuited, as shown in Fig. 2.82 (*a*).

$$R_T = 6 + 4 + (15 \,||\, 10) = 16\,\Omega$$

Converting current source into the equivalent voltage source, the circuit becomes as shown in Fig. 2.82 (*b*).

Since there is no current through resistors of 4 Ω and 6 Ω (hence no voltage drop across them), Thevenin's voltage V_{Th} equals algebraic sum of battery voltage and drop across 10 Ω resistor. Hence

$$V_{Th} = 20 \times \frac{10}{10+15} - 24 = -16 \text{ V}$$

Thevenin's equivalent circuit is shown in Fig. 2.82 (*c*).

Example 2.34

Determine the current in the 1 Ω resistor connected across AB of the network of Fig. 2.83 using Thevenin's theorem.

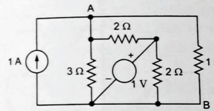


Fig. 2.83

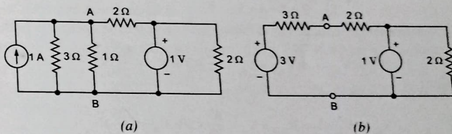


Fig. 2.84

Solution: The given circuit may be redrawn, as illustrated in Fig. 2.84 (*a*). Now the current source is converted into its equivalent voltage source, the circuit, with terminals AB kept open, becomes as shown in Fig. 2.84 (*b*).

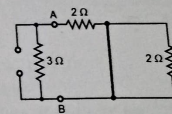
Open-circuited voltage across terminals AB,

$$V_T = 3 - \frac{3-1}{3+2} \times 3 = 1.8 \text{ V}$$

The equivalent resistance of the network (with voltage source short circuited and current source open circuited) with reference to terminals A-B, [Fig. 2.84 (*c*)]

$$R_T = 3 \,||\, 2 = \frac{3 \times 2}{3+2} = 1.2\,\Omega$$

Current in 1 Ω resistor $= \dfrac{V_T}{R_T + 1} = \dfrac{1.8}{1.2+1} = 0.82$ A **Ans.**



(*c*)

Fig. 2.84

Example 2.35

Calculate the current in 6 Ω resistor in the circuit shown in Fig. 2.85 using Thevenin's theorem. [G.G.S.I.P. Univ. Delhi Electrical Science 2005-06]

Solution: After removal of resistor of 6 Ω from the circuit, the circuit becomes as shown in Fig. 2.86 (*a*).

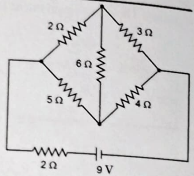


Fig. 2.85

Equivalent resistance of the network shown in Fig. 2.86 (*a*) is

$$R_{eq} = 2 + (2+3) \,||\, (5+4)$$

$$= 2 + 5 \,||\, 9 = 2 + \frac{5 \times 9}{5+9} = \frac{73}{14}\ \Omega$$

$$\text{Current drawn from battery, } I = \frac{9V}{\frac{73}{14}\Omega} = \frac{126}{73}\ \text{A}$$

This current I is divided between two parallel paths ABC and ADC.

Current in path ABC,

$$I_1 = I \times \frac{\text{Resistance of path ADC}}{\text{Total resistance of the loop ABCD}}$$

$$= \frac{126}{73} \times \frac{5+4}{2+3+4+5} = \frac{81}{73}\ \text{A}$$

Current in path ADC,

$$I_2 = I \times \frac{\text{Resistance of path ABC}}{\text{Total resistance of the loop ABCD}}$$

$$= \frac{126}{73} \times \frac{2+3}{2+3+4+5} = \frac{45}{73}\ \text{A}$$

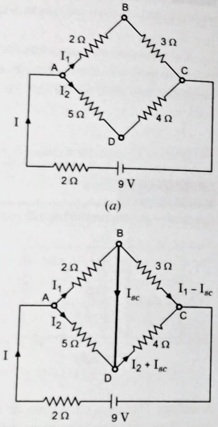
Potential of terminal B,

$$V_B = V_A - I_1 \times R_{AB} = V_A - \frac{81}{73} \times 2 = V_A - \frac{162}{73}\ \text{V}$$

Potential of terminal D,

$$V_D = V_A - I_2 \times R_{AD} = V_A - \frac{45}{73} \times 5 = V_A - \frac{225}{73}\ \text{V}$$

$$\text{Hence } V_T = V_B - V_D = \left(V_A - \frac{162}{73}\right) - \left(V_A - \frac{225}{73}\right) = \frac{63}{73}\ \text{V}$$



(*a*)

(*b*)

Fig. 2.86

On short circuiting the terminals B and D the circuit becomes as shown in Fig. 2.86 (*b*). For determination of short circuit current I_{sc} let us solve the circuit shown in Fig. 2.86 (*b*) by applying Kirchhoff's laws. The current distribution, according to Kirchhoff's first law, is made as shown in Fig. 2.86 (*b*). Applying Kirchhoff's second law to meshes ABD, BCD and ADCA, respectively, we have

$$2I_1 - 5I_2 = 0 \qquad \text{...(i)}$$

$$3(I_1 - I_{sc}) - 4(I_2 + I_{sc}) = 0$$

$$\text{or } 3I_1 - 4I_2 = 7I_{sc} \qquad \text{...(ii)}$$

$$\text{and } 2(I_1 + I_2) + 5I_2 + 4(I_2 + I_{sc}) = 9 \qquad \text{...(iii)} \quad \because I = I_1 + I_2$$

From Eq. (*i*) $\quad I_1 = 2.5\, I_2$

Substituting the value of $I_1 = 2.5\, I_2$ in Eq. (*ii*), we have

$$I_2 = 2\, I_{sc}$$

$$\text{and } I_1 = 2.5\, I_2 = 5\, I_{sc}$$

Substituting the values of $I_1 = 5\, I_{sc}$ and $I_2 = 2I_{sc}$ in Eq. (*iii*), we have

$$2 \times (5I_{sc} + 2I_{sc}) + 5 \times 2I_{sc} + 4(2I_{sc} + I_{sc}) = 9$$

$$\text{or } I_{sc} = \frac{9}{36} = \frac{1}{4}\ \text{A}$$

$$\text{Thevenin's equivalent resistance, } R_T = \frac{V_T}{I_{sc}} = \frac{63}{73} \div \frac{1}{4} = \frac{252}{73}\ \Omega$$

$$\text{Current in 6 Ω resistor, } I_L = \frac{V_T}{R_T + R_L} = \frac{63/73}{\frac{252}{73} + 6} = \frac{63}{690}\ \text{A} = \frac{21}{230}\ \text{A} \quad \textbf{Ans.}$$

2.15. NORTON'S THEOREM

This theorem is in fact, an alternative to the Thevenin's theorem. Whereas by Thevenin's theorem a complex two-terminal network may be simplified for solution by reducing it into a simple circuit in which the so called open-circuit voltage and looking-back resistance are connected in series with the load resistance, by Norton's theorem network is reduced into a simple circuit in which a parallel combination of constant current source and looking-back resistance feeds the load resistance.

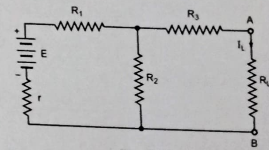
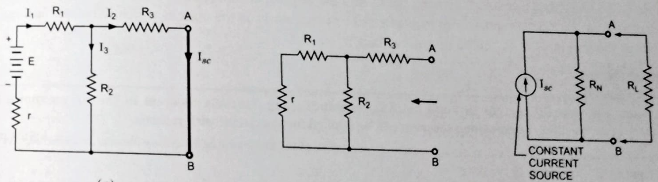


Fig. 2.87

In both theorems use of resistance looking back into the network from the load terminals, with all sources removed leaving their internal resistances in the circuit is made. However, while solving circuit by Thevenin's theorem, the open-circuit voltage is determined at the load terminals with the load removed whereas in Norton's method use of a fictitious constant current source is made, the constant current delivered being equal to the current that would pass into a short circuit connected across the output terminals of the given network.

Now for understanding this theorem let us consider a circuit shown in Fig. 2.87 in which load current I_L is to be determined.

For *determination of short-circuit current* I_{sc}, terminals A and B are short circuited by zero resistance thick wire, as illustrated in Fig. 2.88 (*a*).



(*a*)

(*b*)

(*c*)

Fig. 2.88

$$\text{Equivalent resistance of the network, } R = (R_1 + r) + R_2 \,||\, R_3 = (R_1 + r) + \frac{R_2 R_3}{R_2 + R_3} \qquad \text{...(2.33)}$$

$$\text{Current supplied by battery, } I_1 = \frac{E}{R} = \frac{E}{R_1 + r + \frac{R_2 R_3}{R_2 + R_3}} \qquad \text{...(2.34)}$$

Short-circuit current, I_{sc} = Current flowing through resistance R_3,

$$I_2 = I_1 \times \frac{R_2}{R_2 + R_3} \qquad \text{By current division rule}$$

$$= \frac{E}{R_1 + r + \dfrac{R_2 R_1}{R_2 + R_3}} \times \frac{R_2}{R_2 + R_3} = \frac{E R_2}{(r + R_1)(R_2 + R_3) + R_2 R_3} \qquad ...(2.35)$$

For determination of internal resistance R_{in} (or R_N) of the network under consideration, remove the load resistance R_L from terminals A and B and also remove voltage source from the circuit leaving behind only its internal resistance, as illustrated in Fig. 2.88 (*b*).

Equivalent resistance (R_N), as viewed from the open terminals A and B is given as

$$R_N = R_3 + R_2 \,||\, (R_1 + r)$$

$$= R_3 + \frac{R_2 (R_1 + r)}{R_2 + R_1 + r} = \frac{R_2 R_3 + (R_1 + r)(R_2 + R_3)}{R_1 + R_2 + r} \qquad ...(2.36)$$

Now when load resistance R_L is connected across terminals A and B, the network behaves as constant current source of current I_{sc} in parallel with a resistance R_N, as shown in Fig. 2.88 (*c*) and current flowing through the load resistance is given as

$$I_L = \frac{I_{sc} R_N}{R_N + R_L} \qquad \text{By current division rule}$$

$$= \frac{\dfrac{E R_2}{(r + R_1)(R_2 + R_3) + R_2 R_3} \times \dfrac{R_2 R_3 + (R_1 + r)(R_2 + R_3)}{R_1 + R_2 + r}}{\dfrac{R_2 R_3 + (R_1 + r)(R_2 + R_3)}{R_1 + R_2 + r} + R_L}$$

$$= \frac{E R_2}{R_2 R_3 + (R_1 + r)(R_2 + R_3) + R_L (R_1 + R_2 + r)} \qquad ...(2.37)$$

Norton's equivalent circuit is illustrated in Fig. 2.88 (*c*).

Norton's theorem can be stated as follows:

The current in any *passive circuit element* (which may be called R_L) in a network is the same as would flow in it if it were connected in parallel with R_N and the parallel pair were supplied with a constant current I_{sc}. R_N is the resistance measured "looking back" into the original circuit after R_L has been disconnected and all the sources have been replaced by their internal resistances: I_{sc} is the current which will flow in a short placed at the terminals of R_L in the original circuit.

Example 2.36

Determine the value of current through the 5 Ω resistance using Norton's theorem in the circuit shown in Fig. 2.89. State whether superposition theorem can be applied for the circuit with reasons.

[U.P. Technical Univ. Electrical Engineering Second Semester 2007-08]

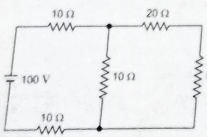


Fig. 2.89

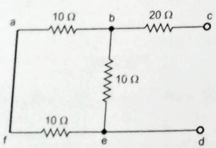


Fig. 2.90(*a*)

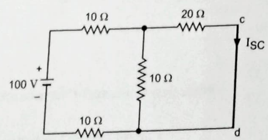


Fig. 2.90(*b*)

Solution: Equivalent resistance of the network when viewed from terminals *c* and *d* keeping the 100 V source short circuited,

$$R_N = 20 + 10 \,||\, (10 + 10) = 20 + \frac{10 \times 20}{10 + 20} = 26.667\ \Omega$$

For determination of short-circuit current, I_{sc} *i.e.*, current in zero resistance connected between terminals *c* and *d*, 5 Ω resistance is replaced by zero resistance, as shown in Fig. 2.90 (*b*), current supplied by 100 V source

$$I = \frac{100}{10 + (10 \,||\, 20) + 10} = \frac{100}{20 + \dfrac{10 \times 20}{10 + 20}} = \frac{100}{80/3} = 3.75 \text{ A}$$

Short-circuit current, I_{sc} = Current through 20 Ω resistance

$$= I \times \frac{10}{10 + 20} = 3.75 \times \frac{10}{30} = 1.25 \text{ A} \qquad \text{By current division rule}$$

Current through a resistance of 5 Ω when connected between terminals *c* and *d*

$$I = \frac{I_{sc}}{R_N + R_L} \times R_N = \frac{1.25 \times 26.667}{26.667 + 5} = \frac{20}{19} \text{ or } 1.053 \text{ A} \quad \textbf{Ans.}$$

Superposition theorem cannot be applied for the given circuit because the network consists only one supply source.

Example 2.37

Determine current in 10 Ω resistance using Norton's theorem in the following network (Fig. 2.91).

[G.G.S.I.P. Univ. Delhi 1st Term Exam. February/March 2011]

Solution: Norton's equivalent resistance of the given network (*i.e.*, equivalent resistance of the network after terminals *a* and *b* are open circuited and the two batteries are short circuited) as viewed from terminals *a* and *b* shown in Fig. 2.92 (*a*),

$$R_N = 4 + (6 \,||\, 8) = 4 + \frac{6 \times 8}{6 + 8} = \frac{52}{7}\ \Omega$$

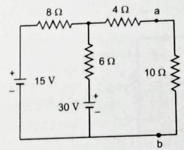
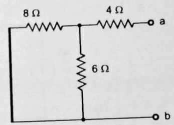
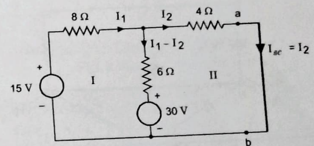


Fig. 2.91



(*a*)



(*b*)

Fig. 2.92

For determining short-circuit current I_{sc} *i.e.*, current in zero resistance connected across terminals *a* and *b* [10 Ω resistance is replaced by a zero resistance as shown in Fig. 2.92 (*b*)] Applying Kirchhoff's second law to loop I and II respectively, we have

$$8I_1 + 6(I_1 - I_2) = 15 - 30 \qquad ...(i)$$

and $$4I_2 - 6(I_1 - I_2) = 30 \qquad ...(ii)$$

or $$6I_2 - 14I_1 = 15 \qquad ...(iii)$$

and $$10I_2 - 6I_1 = 30 \qquad ...(iv)$$

Multiplying Eq. (*iii*) by 3 and Eq. (*iv*) by 7, we have

$$18I_2 - 42I_1 = 45 \qquad ...(v)$$

$$70I_2 - 42I_1 = 210 \qquad ...(vi)$$

Subtracting Eq. (*v*) from Eq. (*vi*), we have

$$I_2 = \frac{165}{52} \text{ A } i.e., \text{ short-circuit current } I_{sc} = \frac{165}{52} \text{ A}$$

Current through 10 Ω resistance, $I = \dfrac{I_{sc}}{R_N + R_L} \times R_N = \dfrac{165/52}{\frac{52}{7} + 10} \times \dfrac{52}{7} = \dfrac{165}{122}$ A **Ans.**

Example 2.38

Using Norton's theorem, find current which would flow in a 25 Ω resistor connected between points N and O in Fig. 2.93.

Solution: Equivalent resistance of network when viewed from terminals N and O, keeping all the voltage short circuited, Fig. 2.94(*a*).

$$R_N = 5 \,||\, 10 \,||\, 20 = \frac{1}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = \frac{20}{7}\ \Omega$$

Short-circuit current *i.e.*, the current in zero resistance conductor connected across terminals ON Fig. 2.94(*b*).

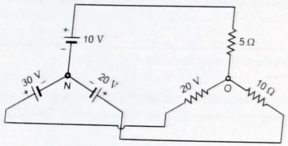


Fig. 2.93

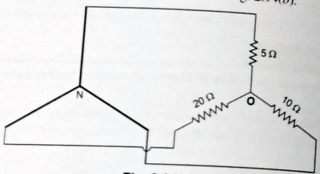


Fig. 2.94(*a*)

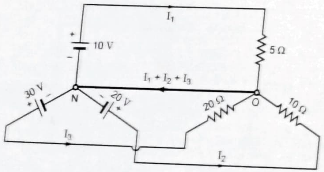


Fig. 2.94(*b*)

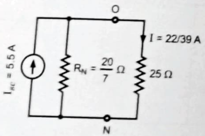


Fig. 2.94(*c*)

$$I_{sc} = I_1 + I_2 + I_3 = \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5 \text{ A}$$

Current through a resistance of 25 Ω connected between points O and N,

$$I = \frac{I_{sc}}{R_N + R_L} \times R_N = \frac{5.5 \times \frac{20}{7}}{\frac{20}{7} + 25} = \frac{22}{39} \text{ A } \textbf{Ans.}$$

Example 2.39

Draw the Norton's equivalent circuit across AB, and determine current flowing through 12 Ω resistor for the network shown in Fig. 2.95.

Solution: As illustrated in Fig. 2.96 (*a*), terminals A and B have been shorted after removing 12 Ω resistor.

Now short circuit current is determined by making use of superposition theorem.

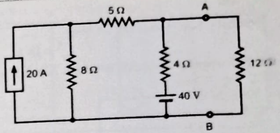


Fig. 2.95

(*i*) *When only current source is present* [Fig. 2.96 (*b*)]

In this case 40 V battery is replaced by a short circuit. The 20 A current divides at point C between parallel combination of 8 Ω and 5 Ω (no current will flow through 4 Ω resistor being short circuited at terminals A and B).

I_{sc_1} = Current through 5 Ω resistor due to current source alone

$$= 20 \times \frac{8}{8+5} = \frac{160}{13} \text{ A}$$

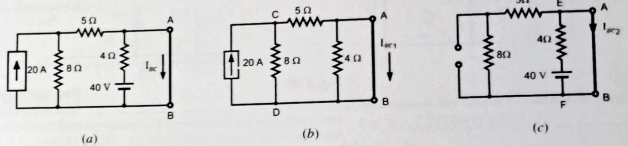


Fig. 2.96

(*ii*) *When only voltage source is present* [Fig. 2.96 (*c*)]

In this case, current source is replaced by an open circuit. Voltage across terminals EF is equal to voltage across terminals A and B *i.e.*, zero. So short-circuit current,

I_{sc_2} = Current through 4 Ω resistor

$$= \frac{40}{4} = 10 \text{ A}$$

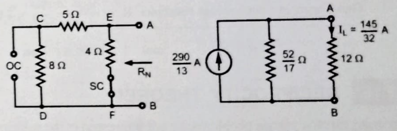


Fig. 2.96

Short-circuit current,

$$I_{sc} = I_{sc_1} + I_{sc_2} = \frac{160}{13} + 10 = \frac{290}{13} \text{ A}$$

As seen from Fig. 2.96 (*d*), Norton's equivalent resistance of the network,

$$R_N = 4 \,||\, (5+8) = 4 \,||\, 13 = \frac{1}{\frac{1}{4} + \frac{1}{13}} = \frac{52}{17}\ \Omega$$

Figure 2.96 (*e*) shows the Norton's equivalent circuit with the load resistor of 12 Ω.

$$\text{Load current, } I_L = I_{sc} \times \frac{R_N}{R_N + R_L} = \frac{290}{13} \times \frac{52/17}{\frac{52}{17} + 12} = \frac{1{,}160}{256} = 4.53125 \text{ A } \textbf{Ans.}$$

Example 2.40

Determine the values of I and R in the circuit shown in Fig. 2.97.

[*U.P.S.C. I.E.S. Electrical Engineering-I, 2003*]

Solution: Short circuiting the terminals X and Y we get the circuit as shown in Fig. 2.98 (*a*).

Applying Kirchhoff's voltage law to meshes (1) and (2) respectively, we have

$$10\,(i + 1) + 3\,(1 - i - I_{sc}) - 8\,i = 0$$

and $\quad -4\,I_{sc} + 3\,(1 - i - I_{sc}) = 0$

or $\quad i + 3\,I_{sc} = 13 \qquad ...(i)$

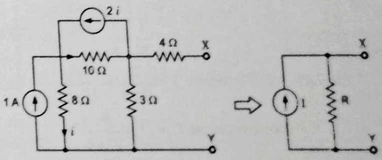


Fig. 2.97

and $3i + 7I_{sc} = 3$ ...(ii)

Solving Eqs. (i) and (ii), we have

$$I_{sc} = 18\ \text{A}$$

Now opening the circuit terminals X and Y we have circuit as shown in Fig. 2.98 (b).

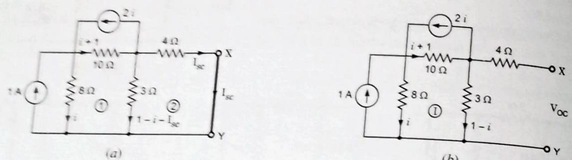


Fig. 2.98

Applying Kirchhoff's voltage law to mesh I, we have

$$10(i+1) + 3(1-i) - 8i = 0$$

or $i = 13$ A

Open-circuit voltage, V_{OC} = Voltage across 3 Ω resistor $= (1-i) \times 3 = (1-13) \times 3 = -36$ V

Thus Thevenin's equivalence resistance, $R = \dfrac{V_{OC}}{I_{sc}} = \dfrac{36}{18} = 2\ \Omega$ **Ans.**

and $I = -18$ A **Ans.**

2.16. RECIPROCITY THEOREM

In many electric circuit problems we are interested in the relationship between an impressed source in one part of the circuit and a response to it in some other part of the same circuit. This is where the *property of reciprocity* comes in useful. The circuits having this property are called *reciprocal* ones, and obey the *reciprocity theorem*, which is stated below:

According to this theorem if the source voltage and zero-resistance ammeter are interchanged, the magnitude of the current through the ammeter will be the same, no matter how complicated the network. Indeed this principle states that the directional characteristics of a receiving antenna are the same as the directional characteristics of the same antenna when used for transmission. This is a highly useful relation.

In other words, *in a linear passive network*, supply voltage V and current I are mutually transferable. The ratio of V and I is called the *transfer resistance* (or transfer impedance in an ac system).

Example 2.41

For the two circuits given in Fig. 2.99, calculate the current A.

Solution: Equivalent resistance of the network shown in Fig. 2.100 (a).

$$R' = 5 + 10 \,||\, 8 = 5 + \frac{1}{\frac{1}{10}+\frac{1}{8}} = \frac{85}{9}\ \Omega$$

Current supplied by battery, $I_1 = \dfrac{25}{85/9} = \dfrac{45}{17}$ A

Ammeter current, $I' = I_1 \times \dfrac{10}{10+8}$

$$= \frac{45}{17} \times \frac{10}{18} = \frac{25}{17}\ \text{A Ans.}$$

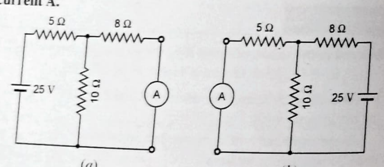


Fig. 2.99

Equivalent resistance of network shown in Fig. 2.100 (b)

$$R'' = 8 + \frac{1}{\frac{1}{10}+\frac{1}{5}} = \frac{34}{3}\ \Omega$$

Current supplied by battery,

$$I_2 = \frac{25}{34/3} = \frac{75}{34}\ \text{A}$$

Ammeter current,

$$I'' = I_2 \times \frac{10}{10+5} = \frac{75}{34} \times \frac{10}{15} = \frac{25}{17}\ \text{A Ans.}$$

The above two results signify reciprocity theorem.

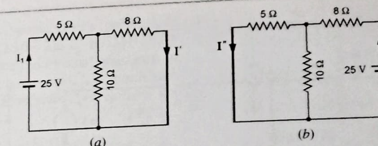


Fig. 2.100

Example 2.42

Verify the reciprocity theorem of the network shown in Fig. 2.101.

[G.G.S.I.P. Univ. Dehli Electrical Science May-June 2006, May-June 2007]

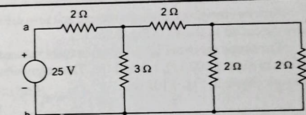


Fig. 2.101

Solution: The circuit given in Fig. 2.101 can be redrawn as shown in Fig. 2.102 (a) and reduced in steps to circuit shown in Figs. 2.102 (b), (c), (d) respectively, using series-parallel reduction technique.

Thus equivalent resistance of the circuit,

$$R_{eq} = (2 + 1.5)\ \Omega = 3.5\ \Omega$$

Current supplied by battery, $I_1 = \dfrac{V}{R_{eq}} = \dfrac{25}{3.5} = \dfrac{50}{7}$ A

Now the branch currents can be determined by restoring the circuit step-by-step to its original form in the reverse order by using current distribution rule in parallel circuits.

Current in branches ae' and $e'f'$ = Current supplied by battery, $I = \dfrac{50}{7}$ A

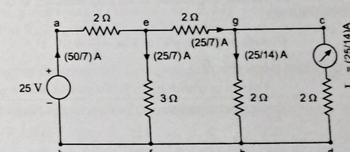


Fig. 2.102 (a)

Current of $\dfrac{50}{7}$ A is equally divided in branches ef and $g''h'$ because they have equal resistances, so current in each of branches ef and $g''h' = \dfrac{1}{2} \times \dfrac{50}{7} = \dfrac{25}{7}$ A

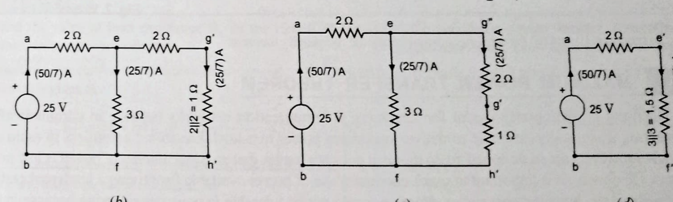
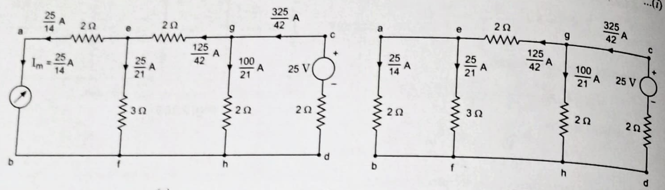


Fig. 2.102

Now current flowing in branches $g'h'$ is divided into branches gh and cd equally due to their equal resistances.

Thus ammeter current, $I_m = \frac{1}{2} \times \frac{25}{7} A = \frac{25}{14} A$ ...(*i*)



(*e*) (*f*)

Fig. 2.102

Current distribution thus obtained is shown in Fig. 2.102 (*a*).

Now by changing the source voltage and zero resistance ammeter, we have the circuit as shown in Fig. 2.102 (*e*).

The circuit given in Fig. 2.102 (*e*) is again reduced in steps to circuits shown in Figs. 2.102 (*f*), (*g*) and (*h*). Thus equivalent resistance of the network shown in Fig. 2.102 (*e*),

$$R'_{eq} = \left(\frac{16}{13} + 2\right)\Omega = \frac{42}{13}\Omega$$

$$\text{Current supplied by battery, } I' = \frac{V}{R'_{eq}} = \frac{25}{42/13} A = \frac{325}{42} A$$

Now the current distribution in various branches is determined as discussed above

$$\text{Current in branch } g'h' = I' = \frac{325}{42} A$$

Current in branch $e'f'$

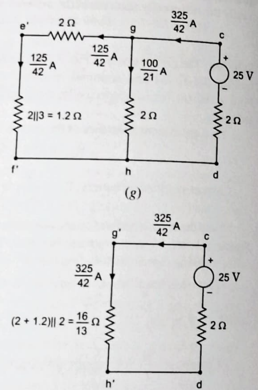
$$= \frac{\text{Current in branch } g'h' \times \text{Resistance of branch } gh}{\text{Resistance of branche } ge'f' + \text{Resistance of branch } gh}$$

$$= \frac{325}{42} \times \frac{2}{(1.2+2)+2} = \frac{125}{42} A$$

Current in ammeter, I'_m = Current in 2 Ω resistor

$$= \text{Current in branch } e'f' \times \frac{3}{2+3}$$

$$= \frac{125}{42} \times \frac{3}{5} = \frac{25}{14} A \quad ...(ii)$$



(*g*)

(*h*)

Fig. 2.102

Expressions (*i*) and (*ii*) for ammeter current verify the reciprocity theorem.

2.17. MAXIMUM POWER TRANSFER THEOREM

This theorem is particularly useful for analyzing communication networks because in communication engineering it is usually desirable to deliver maximum power to a load. Electronic amplifiers in radio and television receivers are so designed to do this as it does not matter that an equal amount of power is lost in the process. Obviously, it is important to make maximum use of power available for driving a loudspeaker. One can readily agree, however, that such a situation would not be tolerable in power engineering because in this situation efficiency of transmission will be very poor whereas in this case goal is high efficiency and not maximum power transfer.

Consider a generator supplying electrical power over a transmission line to a load resistor R_L, as shown in Fig. 2.103. Let E_g be the voltage generated inside the generator, and let R_0 be the internal resistance of generator plus the resistance of the two line conductors. It is required to determine the value of R_L that will draw maximum power from the source.

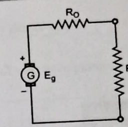


Fig. 2.103

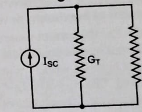


Fig. 2.104

The current delivered to load resistance, $I = E_g/(R_0 + R_L)$

$$\text{Power delivered to load resistance, } P = I^2 R_L = \frac{E_g^2 R_L}{(R_0 + R_L)^2}$$

To find the maximum value of power, differentiate the above expression with respect to R_L and equate to zero. Thus,

$$\frac{dP}{dR_L} = \frac{E_g^2 (R_0 + R_L)^2 - 2R_L (R_0 + R_L) E_g^2}{(R_0 + R_L)^4} = 0$$

or $R_L = R_0$

Thus, the condition for maximum power transfer is that the load resistance R_L shall be equal to R_0.

$$\text{The value of the maximum power transferred is seen to be } P_{max} = \frac{E_g^2}{4R_0} \quad ...(2.38)$$

As applied to dc networks, this theorem may be stated as follows:

A resistive load, served through a resistive network, will abstract maximum power when the load resistance value is the same as the resistance "viewed by the load as it looks back into the network". This is also called the output resistance of the network or the resistance presented to the output terminals by the network. It is the resistance we called R_T in explaining Thevenin's theorem (Art. 2.14). The load resistance R_L will draw maximum power when it will be equal to R_T i.e., $R_L = R_T$ and the maximum power drawn will

be equal to $\frac{V_{oc}^2}{4R_L}$ where V_{oc} is the open-circuit voltage at the terminals from which R_L has been disconnected.

Similarly for a current source, as illustrated in Fig. 2.104, the power transferred will be maximum when load conductance is equal to the source conductance *i.e.*,

$$G_L = G_T$$

$$\text{and the maximum power transferred is } P_{max} = \frac{I_{sc}^2}{4G_L} \quad ...(2.39)$$

Example 2.43

Find the value of load resistance R_L in the circuit of Fig. 2.105 for maximum power transfer. Determine the maximum power transferred. [*G.G.S.I.P. Univ. Delhi Electrical Science 2005-06*]

Solution: The equivalent resistance of the network, when viewed from terminals A and B after short circuiting the voltage source, is given as

$$R_T = 38 + 20 \,||\, 30 = 38 + \frac{60}{3+2} = 38 + 12 = 50\ \Omega$$

Maximum power will be transferred to load resistance R_L when it is equivalent to Thevenin's equivalent resistance, R_T

i.e., $R_L = R_T = 50\ \Omega$ **Ans.**

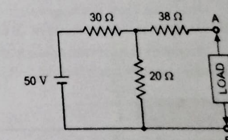


Fig. 2.105

$$\text{Open-circuit voltage, } V_T = \text{Voltage across } 20\ \Omega \text{ resistor} = 50 \times \frac{20}{20+30} = 20 \text{ V}$$

$$\text{Maximum power transferred, } P_{max} = \frac{V_T^2}{4R_L} = \frac{20^2}{4 \times 50} = 2 \text{ watts } \textbf{Ans.}$$

Example 2.44

Find the value of resistance 'R' to have maximum power transfer in the circuit as shown in Fig. 2.106. Also obtain the amount of maximum power.

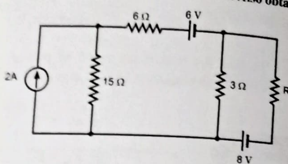


Fig. 2.106

Solution: Let the current source of 2 A shunted by 15 Ω resistor be converted into its equivalent voltage source of 2 × 15 = 30 V connected in series with a resistance of 15 Ω and load resistor R be removed. The circuit is shown in Fig. 2.107 (*a*).

The equivalent resistance of the network with voltage sources short circuited when viewed from terminals A-B [Fig. 2.107 (*b*)].

$$R_T = 3 \,||\, (6+15) = 3 \,||\, 21 = \frac{1}{\frac{1}{3}+\frac{1}{21}} = \frac{21}{8}\ \Omega$$

Current flowing through a resistance of 3 Ω in the circuit shown in Fig. 2.107 (*a*).

$$I = \frac{30-6}{15+6+3} = 1 \text{ A}$$

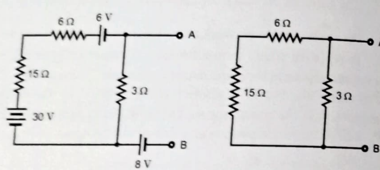
Open-circuit voltage,

$$V_T = \text{Voltage across } 3\ \Omega \text{ resistor} + 8 \text{ V}$$
$$= 1 \times 3 + 8 = 11 \text{ V}$$

According to maximum power transfer theorem, R will absorb maximum power when it is equal to R_T *i.e.*, $\frac{21}{8}\ \Omega$ **Ans.**

Maximum power transferred,

$$P_{max} = \frac{V_T^2}{4R_L} = \frac{11^2}{4 \times \frac{21}{8}} = 11.524 \text{ W } \textbf{Ans.}$$



(*a*) (*b*)

Fig. 2.107

Example 2.45

For the circuit shown in Fig. 2.108, find the Thevenin's equivalent voltage source. Also give the value of load resistance to be connected across terminals A and B to get maximum power delivered.

[U.P. Technical Univ. Electrical Engineering First Semester 2004-05]

Solution:

$$R_T = 15 + 10 \,||\, 10 = 15 + 5 = 20\ \Omega$$

$$V_T = \frac{100 \times 10}{10+10} = 50 \text{ V}$$

Hence the Thevenin's equivalent voltage source is of open-circuit voltage 50 V in series with a resistance of 20 Ω **Ans.**

According to maximum power transfer theorem, load resistance will absorb maximum power when it is equal to R_T *i.e.*, 20 Ω **Ans.**

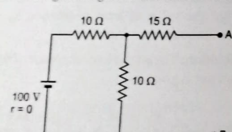


Fig. 2.108

Example 2.46

Find the value of R in the circuit of Fig. 2.109 such that the maximum power transfer takes place.

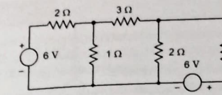


Fig. 2.109

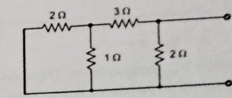


Fig. 2.110

Solution: The equivalent resistance of the network when viewed from terminals A and B after removing resistor R from the circuit and short circuiting the voltage (Fig. 2.110) sources, we have

$$R_T = 2 \,||\, [3 + 2 \,||\, 1]\ \Omega = 2 \,||\, \left[3 + \frac{2}{3}\right]\ \Omega = \left(2 \,||\, \frac{11}{3}\right)\ \Omega = \frac{22}{17}\ \Omega$$

Maximum power will be transferred to the resistor R when it is equal to Thevenin's equivalent resistance R_T

i.e., $R = R_T = \frac{22}{17} = 1.29\ \Omega$ **Ans.**

Example 2.47

Explain maximum power transfer theorem. Using this theorem find the value of load resistance R_L for maximum power flow through it in Fig. 2.111. *[U.P. Technical Univ. Electrical Engineering Second Semester 2006-07]*

Solution: For determining the Thevenin's equivalent resistance of the circuit with respect to terminals AB the voltage source is shorted and current source is open circuited, as shown in Fig. 2.112.

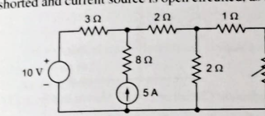


Fig. 2.111

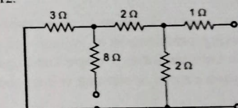


Fig. 2.112

$$R_T = 1\ \Omega + 2\ \Omega \,||\, (2\ \Omega + 3\ \Omega) = 1 + \frac{1}{\frac{1}{2}+\frac{1}{5}} = 1 + \frac{10}{7} = \frac{17}{7}\ \Omega$$

Hence the value of load resistance R_L for maximum power flow through it in Fig. 2.112

= Output resistance or Thevenin's equivalent resistance, $R_T = 17/7 = 2.4286\ \Omega$ **Ans.**

2.18. MILLMAN'S THEOREM

In practice, cases frequently arise where a network has only two terminal points between which any number of parallel branches may be connected. Their calculations can be greatly simplified by the use of Millman's theorem, which is combination of Thevenin's and Norton's theorem. This theorem enables a number of voltage (or current) sources to be combined into a single voltage (or current) source.

Consider a network (shown in Fig. 2.113) with two common terminals A and B between which 3 voltage sources E_1, E_2 and E_3 of internal resistances R_1, R_2 and R_3 respectively are connected.

According to this theorem, these voltage sources between terminals A and B can be replaced by a single voltage source of V_{AB} volts in series with an equivalent resistance R_{AB} where

$$V_{AB} = \frac{\frac{E_1}{R_1}+\frac{E_2}{R_2}+\frac{E_3}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{E_1 G_1 + E_2 G_2 + E_3 G_3}{G_1 + G_2 + G_3} = \frac{\Sigma E G}{\Sigma G} \qquad \text{...(2.40)}$$

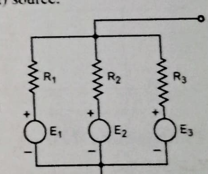


Fig. 2.113

and $R_{AB} = \dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \dfrac{1}{G_1+G_2+G_3} = \dfrac{1}{\Sigma G}$ ...(2.41)

The voltage V_{AB} represents the Thevenin's voltage V_T and R_{AB} represents Thevenin's equivalent resistance R_T. If there is a load resistance R_L across the terminals A – B, then load current is given as

$$I_L = \frac{V_T}{R_T + R_L} \quad ...(2.42)$$

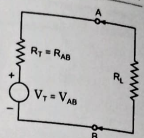
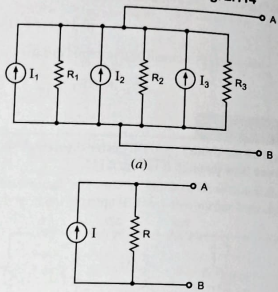


Fig. 2.114

If in the circuit, a branch does not contain any voltage source, the same procedure is employed except that value of voltage for that branch is equated to zero as illustrated in solved Examples 2.48 and 2.49.

The above theorem is also applicable to current sources. According to this theorem any number of current sources in parallel may be replaced by a single current source, whose current is the algebraic sum of individual source currents and source resistance is the parallel combination of individual source resistances [Fig. 2.115].

The above network theorem can be applied to a network having combination of voltage and current sources that are reduced to a single final equivalent source (constant current or constant voltage source).



(b) Equivalent Circuit of Circuit Shown in Fig. 2.115 (a)

Fig. 2.115

Example 2.48

Calculate the current I shown in Fig. 2.116 using Millman's theorem.

[G.G.S.I.P. Univ. Delhi Electrical Science 2005-06]

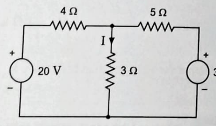


Fig. 2.116

Solution: The circuit shown in Fig. 2.116 can be redrawn as shown in Fig. 2.117.

The common terminal voltage,

$$V = \frac{\dfrac{E_1}{R_1}+\dfrac{E_2}{R_2}+\dfrac{E_3}{R_3}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}$$

$$= \frac{\dfrac{20}{4}+\dfrac{0}{3}+\dfrac{30}{5}}{\dfrac{1}{4}+\dfrac{1}{3}+\dfrac{1}{5}} = \frac{660}{47}\text{ V}$$

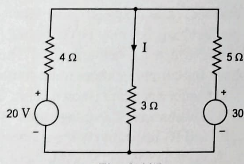


Fig. 2.117

Current in resistor of 3 Ω $= \dfrac{V}{3} = \dfrac{660/47}{3} = \dfrac{220}{47}$ A **Ans.**

Example 2.49

Determine the common terminal voltage across terminals A and B and the load current I_L in the network shown in Fig. 2.118 using Millman's theorem.

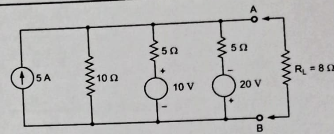


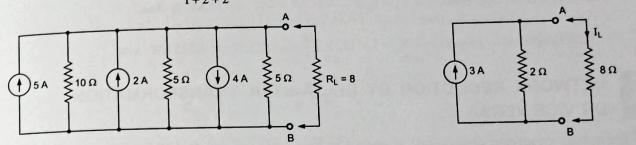
Fig. 2.118

Solution: Converting the given voltage sources into equivalent current sources, we get the circuit that shown in Fig. 2.119 *(a)*

The algebraic sum of the currents = 5 + 2 – 4 = 3 A

$$\text{The equivalence resistance} = \frac{1}{\dfrac{1}{10}+\dfrac{1}{5}+\dfrac{1}{5}}$$

$$= \frac{10}{1+2+2} = 2\ \Omega$$



The simplified equivalent circuit is shown as current source in Fig. 2.119 *(b)* and as voltage source in Fig. 2.119 *(c)*.

As seen from Fig. 2.119 *(b)*

$$I_L = 3\times\frac{2}{2+8} = 0.6\text{ A} \quad \textbf{Ans.}$$

As seen from Fig. 2.119 *(c)*

$$I_L = \frac{6}{2+8} = 0.6\text{ A} \quad \textbf{Ans.}\text{ (the same as above)}$$

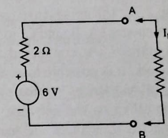


Fig. 2.119 (c)

Example 2.50

Figure 2.120 represents a circuit in which a dc generator is connected to two storage batteries in parallel so that the latter may be charged. The open-circuit voltages of the batteries are $E_A = 108$ V and $E_B = 110$ V. Several resistances are indicated on the diagram. Calculate I_A, I_B and I, and power delivered by the generator.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]

Solution: Given circuit can be redrawn as shown in Fig. 2.121.

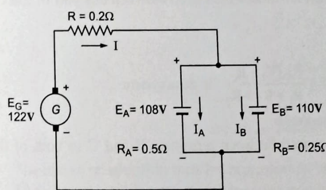


Fig. 2.120

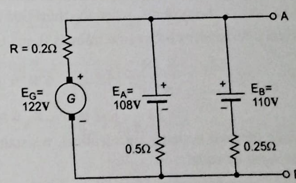


Fig. 2.121

According to Millmans' theorem, voltage across terminals A and B is

$$V_{AB} = \frac{\frac{E_C}{R}+\frac{E_A}{R_A}+\frac{E_B}{R_B}}{\frac{1}{R}+\frac{1}{R_A}+\frac{1}{R_B}} = \frac{\frac{122}{0.2}+\frac{108}{0.5}+\frac{110}{0.25}}{\frac{1}{0.2}+\frac{1}{0.5}+\frac{1}{0.25}} = \frac{610+216+440}{5+2+4} = \frac{1{,}266}{11}\ \text{V}$$

$$\text{Current, } I_A = \frac{V_{AB}-E_A}{R_A} = \frac{\frac{1{,}266}{11}-108}{0.5} = \frac{156}{11}\ \text{A or } 14.182\ \text{A}\ \textbf{Ans.}$$

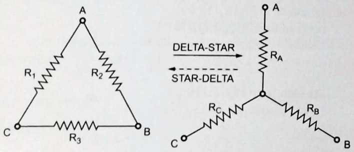
$$\text{Current, } I_B = \frac{V_{AB}-E_B}{R_B} = \frac{\frac{1{,}266}{11}-110}{0.25} = \frac{224}{11}\ \text{A or } 20.364\ \text{A}\ \textbf{Ans.}$$

$$\text{Current delivered by generator, } I = I_A + I_B = \frac{156}{11}+\frac{224}{11} = \frac{380}{11}\ \text{or } 34.546\ \text{A}\ \textbf{Ans.}$$

$$\text{Power delivered by generator} = V_a \times I = 122 \times \frac{380}{11} = 4{,}215\ \text{W or } 4.215\ \text{kW}\ \textbf{Ans.}$$

2.19. NETWORK REDUCTION BY DELTA-STAR TRANSFORMATION OR VICE VERSA

Three resistances connected nose-to-tail as shown in Fig. 2.122 (*a*), are said to be *delta*- (or Δ) or *mesh-connected* (they form a mesh). Three resistances connected together at a common point O, as shown in Fig. 2.122 (*b*), are said to *be star* – (or Y) *connected*. If the nodes (A, B, and C) to which the two sets of resistances are connected are part of a larger network, it is possible to assign values to the two sets of resistances so that they have exactly the same effect on the network. If, therefore, delta-connected resistances are part of a network it is possible to substitute them by the star-connected ones and vice versa. The obvious advantages are that a delta-star transformation eliminates a mesh which reduces by one the variables and equations necessary to solve a network by mesh analysis whereas star-delta transformation eliminates a node (node O) which reduces by one the variables and equations necessary to solve a network by node analysis.



(*a*) (*b*)

Fig. 2.122 *Delta-Star Transformation*

2.19.1. Delta-Star Transformation. *The replacement of delta or mesh by equivalent star system is known as delta-star transformation.*

The two systems will be equivalent or identical if the resistances measured between any pair of lines is same in both of the systems, when the third line is open.

Hence resistances between terminals B and C,

$$R_{BC} = R_3 \parallel (R_1+R_2) = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \quad \text{in delta system}$$

$$\text{and } R_{BC} = R_B + R_C \quad \text{in star system}$$

Since the two systems are identical, resistances measured between terminals B and C in both of the systems must be equal.

$$\text{So } R_B + R_C = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \qquad \text{...(2.43)}$$

Similarly resistances between terminals C and A being equal in the two systems

$$R_C + R_A = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \qquad \text{...(2.44)}$$

And resistance between terminals A and B

$$R_A + R_B = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3} \qquad \text{...(2.45)}$$

Adding Eqs. (2.43), (2.44) and (2.45) we have

$$2(R_A+R_B+R_C) = \frac{2(R_1R_3+R_2R_3+R_1R_2)}{R_1+R_2+R_3}$$

$$\text{or } R_A+R_B+R_C = \frac{R_1R_3+R_2R_3+R_1R_2}{R_1+R_2+R_3} \qquad \text{...(2.46)}$$

Subtracting Eqs. (2.43), (2.44) and (2.45) from Eq. (2.46) we have respectively

$$R_A = \frac{R_1R_2}{R_1+R_2+R_3} \qquad \text{...(2.47)}$$

$$R_B = \frac{R_2R_3}{R_1+R_2+R_3} \qquad \text{...(2.48)}$$

$$R_C = \frac{R_3R_1}{R_1+R_2+R_3} \qquad \text{...(2.49)}$$

These relationships may be expressed as follows:

The equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta-connected resistances.

If the three delta-connected resistances have the same value R_D, the three resistances in the equivalent star for identical systems will be

$$R_s = \frac{R_D R_D}{R_D+R_D+R_D} = \frac{R_D}{3} \qquad \text{...(2.50)}$$

2.19.2. Star-Delta Transformation. Multiplying Eqs. (2.47) and (2.48), (2.48) and (2.49) and (2.49) and (2.47) and then adding them, we get

$$R_AR_B+R_BR_C+R_CR_A = \frac{R_1R_2^2R_3+R_1R_3^2R_2+R_1^2R_2R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1R_2R_3(R_2+R_3+R_1)}{(R_1+R_2+R_3)^2} = \frac{R_1R_2R_3}{R_1+R_2+R_3} \qquad \text{...(2.51)}$$

Dividing Eq. (2.51) by Eqs. (2.47), (2.48) and (2.49) separately, we have

$$R_3 = \frac{R_AR_B+R_BR_C+R_CR_A}{R_A} = R_B+R_C+\frac{R_BR_C}{R_A} \qquad \text{...(2.52)}$$

$$R_1 = \frac{R_AR_B+R_BR_C+R_CR_A}{R_B} = R_A+R_C+\frac{R_AR_C}{R_B} \qquad \text{...(2.53)}$$

$$\text{and } R_2 = \frac{R_AR_B+R_BR_C+R_CR_A}{R_C} = R_A+R_B+\frac{R_AR_B}{R_C} \qquad \text{...(2.54)}$$

The above relationship may be expressed as below:

The equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same divided by the third star resistance.

If the three star-connected resistances have the same value, say R_s, the three resistances of the equivalent delta for identical systems will be

$$R_D = R_s + R_s + \frac{R_s R_s}{R_s} = 3R_s \qquad ...(2.55)$$

The advantage of delta-star transformation may be shown by reference to network of Fig. 2.123. Fig. 2.123 (*a*) illustrates the network before conversion, where the dotted lines are drawn around the delta connection to be transformed into a star. Figure. 2.123 (*b*) illustrates the same network after transformation. The currents in the transformed form [Fig. 2.123 (*b*)] are much simpler to determine.

The advantage of star-delta transformation can be illustrated by reference to network of Fig. 2.124. Figure 2.124 (*a*) illustrates the network prior to transformation, with the dotted lines around the star to be transformed. Figure 2.124 (*b*) illustrates the same network after transformation. The original network is now reduced to a simple series parallel connection of resistances.

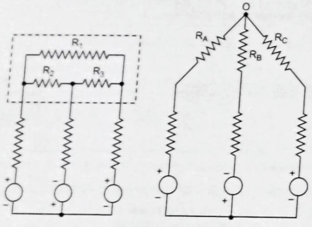


Fig. 2.123

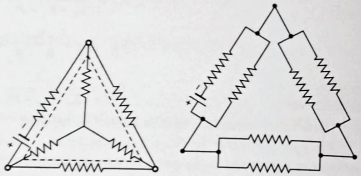


Fig. 2.124

Example 2.51

Three resistors each of R Ω are connected in delta. If they are transferred to Y-connection, what will be the resistance of each resistor?

[G.G.S.I.P. Univ. Delhi 1st Term Exam. February/March 2011]

Solution: Three resistances in the equivalent star will be given as

$$R_s = \frac{R_D}{3} = \frac{R}{3}\ \Omega \ \textbf{Ans.} \qquad \text{Refer to Eq. (2.50)}$$

Example 2.52

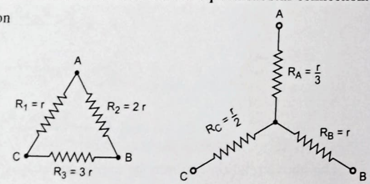
Three resistances *r*, 2 *r*, and 3 *r* are connected in delta. Determine the resistances for an equivalent star connection.

Solution: Resistances for the equivalent star-connection [Fig. 2.125(*b*)] are worked out as below:

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{r \times 2r}{r + 2r + 3r} = \frac{r}{3} \quad \textbf{Ans.}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2r \times 3r}{r + 2r + 3r} = r \quad \textbf{Ans.}$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3} = \frac{3r \times r}{r + 2r + 3r} = \frac{r}{2} \quad \textbf{Ans.}$$



(*a*) Delta-Connected Resistors (*b*) Equivalent Star-Connected Resistors

Fig. 2.125

Example 2.53

Find the star equivalent of the given delta configuration in Fig. 2.126 (*a*).

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2006]

Solution: From Eq. (2.50)

$$Z_{star} = \frac{1}{3} Z_{delta} = \frac{1}{3} \times (5 + j10) = (1.667 + j\,3.333)\ \Omega$$

Star equivalent of the given delta configuration is shown in Fig. 2.126 (*b*).

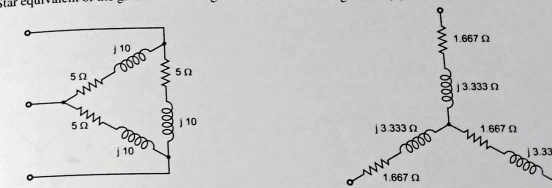


Fig. 2.126

Example 2.54

Use the technique of V-Δ transformation to find the Thevenin equivalent resistance of circuit of Fig. 2.127.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]

Solution: Converting ΔABC into star equivalent as shown in Fig. 2.128 (*a*),

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{3 \times 1}{3 + 1 + 4} = \frac{3}{8}\ \Omega$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{1 \times 4}{3 + 1 + 4} = \frac{1}{2}\ \Omega$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3} = \frac{4 \times 3}{3 + 1 + 4} = 1\frac{1}{2}\ \Omega$$

(✓)

The circuit now becomes as shown in Fig. 2.128 (*b*),

Equivalent resistance of the network shown in Fig. 2.128 (*b*) with reference to terminals B and D

2.72151 Ω

$$R_T = \frac{1}{2} + \left(\frac{3}{8} + 3\right) \| \left(1\frac{1}{2} + 5\right) = \frac{1}{2} + \frac{27}{8} \| \frac{13}{2} = \frac{1}{2} + \frac{1}{\frac{8}{27} + \frac{2}{13}} = \frac{1}{2} + \frac{27 \times 13}{104 + 54} = 2.2215\ \Omega \ \textbf{Ans.}$$

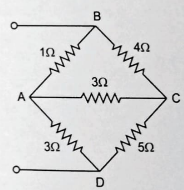


Fig. 2.127

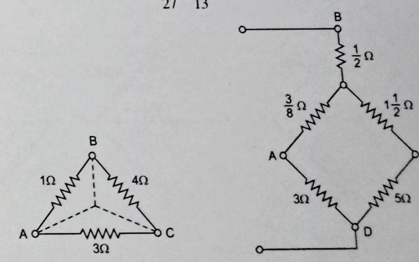


Fig. 2.128

Example 2.55

Compute the resistance measured between terminals A and B of the circuit shown in Fig. 2.129.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2007]

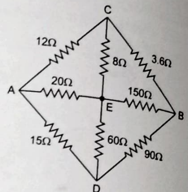


Fig. 2.129

Solution: Converting ΔACE and ΔBED into equivalent star, as shown in Fig. 2.130 (*a*)

$$R_{AF} = \frac{20 \times 12}{20+12+8} = \frac{240}{40} = 6\ \Omega$$

$$R_{CF} = \frac{12 \times 8}{20+12+8} = \frac{96}{40} = 2.4\ \Omega$$

$$R_{EF} = \frac{8 \times 20}{20+12+8} = \frac{160}{40} = 4\ \Omega$$

$$R_{BG} = \frac{150 \times 90}{150+90+60} = \frac{13{,}500}{300} = 45\ \Omega$$

$$R_{DG} = \frac{60 \times 90}{300} = 18\ \Omega$$

$$R_{EG} = \frac{60 \times 150}{300} = 30\ \Omega$$

The circuit shown in Fig. 2.130 (*a*) is simplified to reduce it, as shown in Fig. 2.130 (*b*) and then 2.130 (*c*).

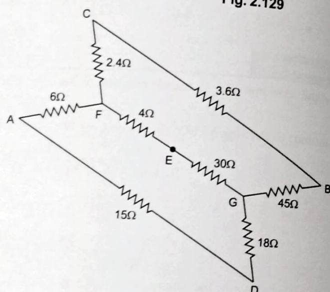
Converting ΔBFG into star equivalent as shown in Fig. 2.130 (*d*)

$$R_B = \frac{6 \times 45}{6+45+34} = \frac{270}{85} = \frac{54}{17}\ \Omega$$

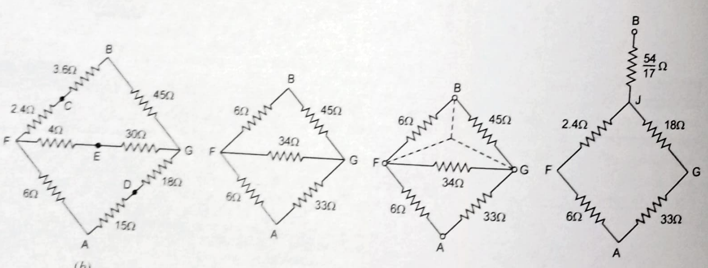
$$R_F = \frac{6 \times 34}{85} = \frac{12}{5} = 2.4\ \Omega$$

$$R_G = \frac{45 \times 34}{85} = 18\ \Omega$$

Equivalent resistance of the network shown in Fig. 2.130 (*e*) with reference to terminals A and B.



(*a*)



(*b*) (*c*) (*d*) (*e*)

Fig. 2.130

$$\left[(33+18) \| (6+2.4) + \frac{54}{17}\right]\Omega = 51 \| 8.4 + \frac{54}{17} = \frac{1}{\frac{1}{51}+\frac{1}{8.4}} + \frac{54}{17} = \frac{238}{33} + \frac{54}{17} = 10.39\ \Omega \quad \textbf{Ans.}$$

Example 2.56

Using delta to star transformation determine the resistance between terminals *a-b* and the total power drawn from the supply in the circuit shown in Fig. 2.131. *[G.G.S.I.P. Univ. Delhi 1st Term Exam. Feb./March 2011]*

Solution: By transforming delta formed by resistors of 8 Ω, 7 Ω and 3 Ω into star, we have

$$R_A = \frac{R_{AC} \times R_{AB}}{R_{AC}+R_{AB}+R_{BC}} = \frac{3 \times 8}{3+8+7} = \frac{4}{3}\ \Omega$$

Similarly $R_B = \frac{8 \times 7}{18} = \frac{28}{9}\ \Omega$

and $R_C = \frac{3 \times 7}{18} = \frac{7}{6}\ \Omega$

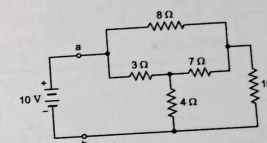


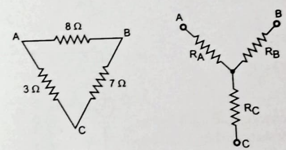
Fig. 2.131

Now the given circuit is reduced to that as shown in Fig. 2.133.

Resistance between terminals *a* and *b*,

$$R = \frac{4}{3} + \left(\frac{7}{6}+4\right) \| \left(\frac{28}{9}+10\right) = \frac{4}{3} + \frac{31}{7} \| \frac{118}{9} = 1.3333 + 3.7062 = 5.04\ \Omega \quad \textbf{Ans.}$$

$$\text{Total power drawn, } P = \frac{V^2}{R} = \frac{10^2}{5.04} = 19.84 \text{ watts} \quad \textbf{Ans.}$$



(*a*) (*b*)

Fig. 2.132

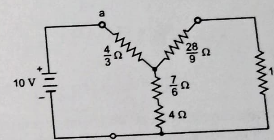


Fig. 2.133

Example 2.57

In the network shown in Fig. 2.134, find (i) the Norton's equivalent circuit at terminals A-B (ii) the maximum power that can be provided to a resistor R connected to terminals A-B.

Solution: Converting star-connected resistors of 4 Ω, 8 Ω and 2 Ω in Fig. 2.134 into an equivalent delta-connected resistors and then redrawing the circuit we have the circuit, as shown in Fig. 2.135 (*a*).

$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c} = 4 + 8 + \frac{4 \times 8}{2} = 28\ \Omega$$

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a} = 8 + 2 + \frac{8 \times 2}{4} = 14\ \Omega$$

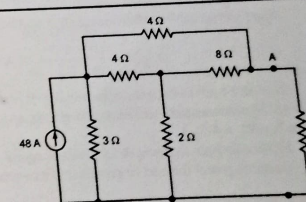
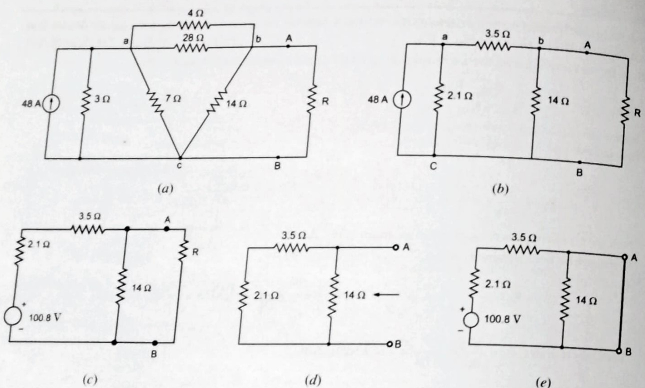


Fig. 2.134

$$R_{ca} = R_c + R_a + \frac{R_c\,R_a}{R_b} = 2 + 4 + \frac{2\times 4}{8} = 7\ \Omega$$

Circuit shown in Fig. 2.135 *(a)* is further simplified by determining equivalent resistances of parallel combination of 28 Ω and 4 Ω connected across terminals *a* and *b* and of parallel combination of 7 Ω and 3 Ω connected across terminals *a* and *c*. The simplified circuit is shown in Fig. 2.135 *(b)*.



$$R_{ab} = 28 \,||\, 4\ \Omega = \frac{28\times 4}{28+4} = 3.5\ \Omega$$

$$R_{ac} = 3 \,||\, 7\ \Omega = \frac{3\times 7}{7+3} = 2.1\ \Omega$$

Converting current source of 48 A having a resistor of 2.1 Ω connected across it into an equivalent voltage source and redrawing the circuit, we get the circuit shown in Fig. 2.135 *(c)*.

Voltage of equivalent voltage source, V = 48 × 2.1 = 100.8 V

Equivalent resistance of the network when viewed from terminals A and B after removing resistor R from the circuit and short circuiting the voltage source [Fig. 2.135 *(d)*]

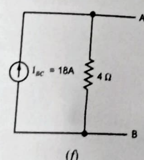


Fig. 2.135

$$\text{i.e., } R_N = 14 \,||\, (3.5 + 2.1) = 14 \,||\, 5.6 = \frac{14\times 5.6}{19.6} = 4\ \Omega$$

Short-circuit current when terminals A and B are short circuited,

$$I_{sc} = \frac{100.8}{2.1+3.5} = 18\ \text{A}$$

Norton's equivalent circuit at terminal A-B is shown in Fig. 2.135 *(f)*.

(ii) Maximum power will be provided to external resistor R when it is equal to Norton's equivalence resistance R_N *i.e.*, when $R = R_N = 4\ \Omega$.

In that case source current of 18 A will be equally divided among R_N and R *i.e.*, current flowing through R = 18/2 = 9 A and maximum power that can be provided to a resistance R connected to terminals A-B = $9^2 \times 4$ = 324 watts **Ans.**

Example 2.58

With the help of star/delta transformation, obtain the value of current supplied by battery in the circuit shown in Fig. 2.136.

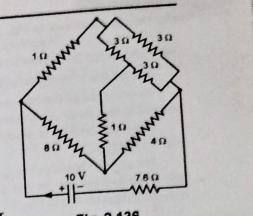


Fig. 2.136

Solution: Converting the star illustrated in Fig. 2.137 *(a)* into equivalent delta, we have

$$R_{AB} = R_A + R_B \times \frac{R_A\,R_B}{R_C} = 3 + 3 + \frac{3\times 3}{1} = 15\ \Omega$$

$$R_{BC} = 3 + 1 + \frac{3\times 1}{3} = 5\ \Omega$$

$$R_{CA} = 1 + 3 + \frac{1\times 3}{3} = 5\ \Omega$$

The equivalent circuit then becomes that shown in Fig. 2.137 *(c)*, which when further simplified, reduces to that shown in Fig. 2.137 *(d)*.

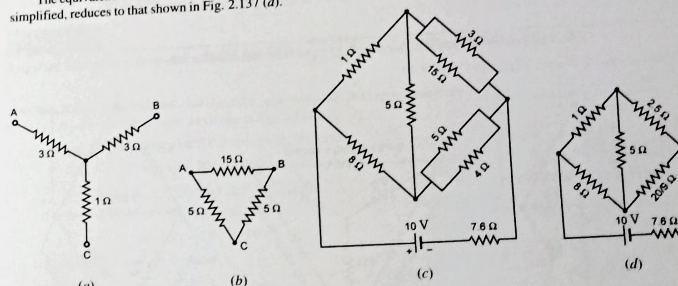


Fig. 2.137

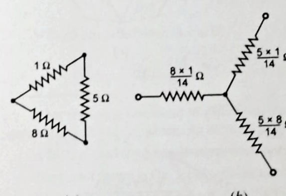


Fig. 2.138

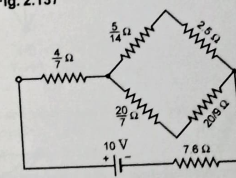


Fig. 2.139

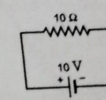


Fig. 2.140

Converting again the delta of Fig. 2.138 *(a)* into equivalent star [Fig. 2.138 *(b)*], the circuit finally reduces to that shown in Fig. 2.139.

On simplification, the circuit shown in Fig. 2.139 reduces to a single resistor of 10 ohms, as illustrated in Fig. 2.140.

$$R_{eq} = \frac{4}{7} + \left[\frac{5}{14} + 2.5\right] \| \left[\frac{20}{7} + \frac{20}{9}\right] + 7.6 = \frac{4}{7} + \frac{20}{7} \| \frac{320}{63} + 7.6 = 10\Omega$$

So current supplied by the battery, $I = \dfrac{V}{R_{eq}} = \dfrac{10}{10} = 1$ A **Ans.**

Example 2.59

For the circuit shown in Fig. 2.141, determine the resistance between points A and B.

[G.S.S.I.P. Delhi Univ. Electrical Science 2004-05]

Solution: Given circuit can be redrawn as shown in Fig. 2.142 (*a*).

Let the star in centre be reduced to delta as shown in Fig. 2.142 (*b*).

Now the circuit becomes as shown in Fig. 2.142 (*c*), which is further reduced to the circuit as shown in Fig. 2.142 (*d*).

Now the Δ CDB is converted into equivalent star as shown in Fig. 2.142 (*e*).

The circuit shown in Fig. 2.142 (*d*) is reduced, by replacing Δ CDB into its equivalent star, to that shown in Fig. 2.142 (*f*) which is further reduced to that shown in Fig. 2.142 (*g*).

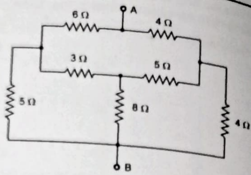


Fig. 2.141

The resistance between terminals A and B shown in Fig. 2.142 (*g*) is given by

$$R = (6 + 2.187) \,\|\, (4 + 2) + 0.77 = \frac{8.187 \times 6}{8.187 + 6} + 0.77 = 4.232\ \Omega \quad \textbf{Ans.}$$

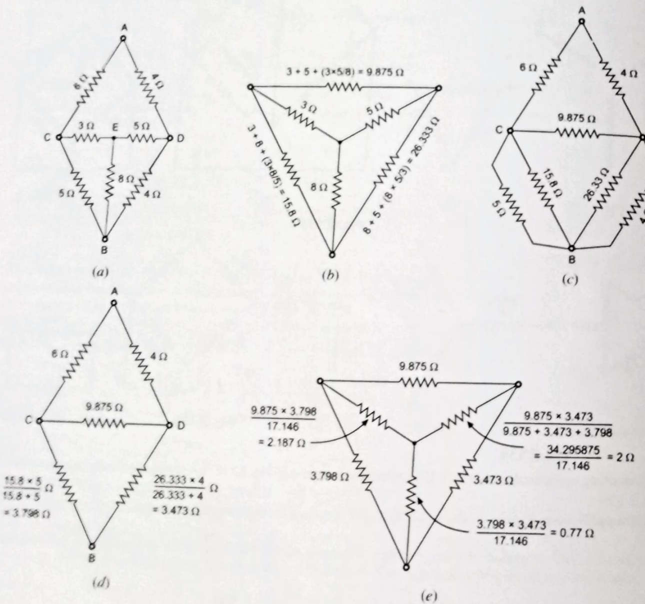


Fig. 2.142

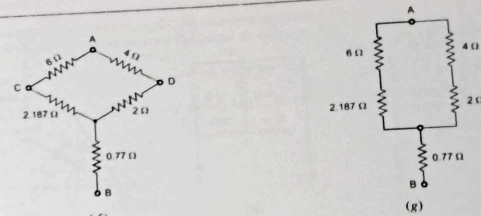


Fig. 2.142

Example 2.60

Applying Thevenin's theorem, calculate the current in 13 Ω resistor of circuit given in Fig. 2.143.

[G.S.S.I.P. Univ. Delhi Electrical Science 2004-05]

Solution: Keeping the terminals AB open (*i.e.*, removing the 13 Ω resistance from the arm AB), the circuit becomes as shown in Fig. 2.144 (*a*).

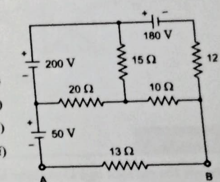


Fig. 2.143

Applying KVL to meshes I and II respectively, we have

$$15 (I_1 - I_2) + 20I_1 = 200 \qquad \text{...}(i)$$

$$12I_2 + 10I_2 + 15(I_2 - I_1) = -180 \qquad \text{...}(ii)$$

$$\text{or } 35I_1 - 15I_2 = 200 \qquad \text{...}(i)$$

$$\text{and } 15I_1 - 37I_2 = 180 \qquad \text{...}(ii)$$

Solving Eqs. (*i*) and (*ii*), we have

$$I_1 = \frac{470}{107} \text{A} \quad \text{and} \quad I_2 = -\frac{330}{107} \text{A}$$

Voltage across terminals A and B,

$$V_T = 50 + \frac{470}{107} \times 20 - \frac{330}{107} \times 10 = 50 + \frac{9{,}400 - 3{,}300}{107} = 107 \text{ V}$$

After removing resistance of 13 Ω from the circuit and short circuiting all the voltage sources we have the circuit as shown in Fig. 2.144 (*b*) which can be redrawn as shown in Fig. 2.144 (*c*).

Δ BCD is converted into its equivalent star, as shown in Fig. 2.144 (*d*).

The circuit shown in Fig. 2.144 (*c*) is reduced, by replacing Δ BCD into its equivalent star, to that shown in Fig. 2.144 (*e*).

Equivalent resistance of circuit shown in Fig. 2.144 (*e*) with reference to terminals A and B, we have

$$R_T = \left(20 + \frac{150}{37}\right) \Big\| \frac{180}{37} + \frac{120}{37} = \frac{\frac{890}{37} \times \frac{180}{37}}{\frac{890}{37} + \frac{180}{37}} + \frac{120}{37} = 7.29\ \Omega$$

$$\text{Current through 13 } \Omega \text{ resistor, } I = \frac{V_T}{R_T + R} = \frac{107}{7.29 + 13} + \frac{107}{20.29} = 5.274 \text{ A} \quad \textbf{Ans.}$$

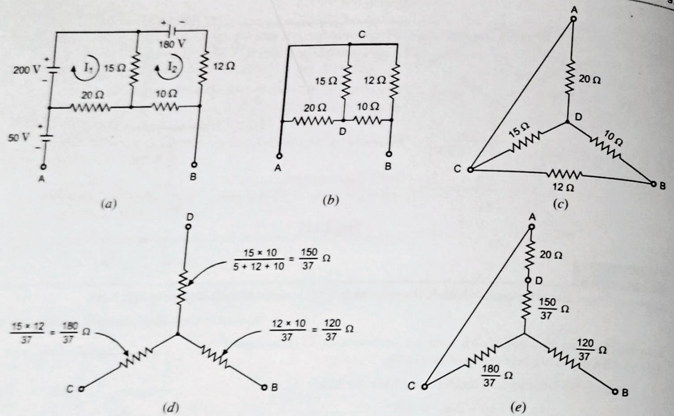


Fig. 2.144

Example 2.61

Calculate the current in 6 Ω resistor in Fig. 2.145 using Norton's theorem.

[G.S.S.I.P. Univ. Delhi Electrical Science 2004-05]

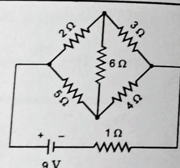


Fig. 2.145

Solution: The given circuit with 6 Ω resistor removed and 9 V battery short circuted becomes as shown in Fig. 2.146 (*a*).

Converting ΔABC into star equivalent as shown in Fig. 2.146 (*b*).

$$R_A = \frac{R_1 \times R_2}{R_1+R_2+R_3} = \frac{2\times 1}{2+1+3} = \frac{1}{3}\ \Omega$$

$$R_B = \frac{R_2 R_3}{R_1+R_2+R_3} = \frac{2\times 3}{2+1+3} = 1\ \Omega$$

$$R_C = \frac{R_3 R_1}{R_1+R_2+R_3} = \frac{3\times 1}{2+1+3} = \frac{1}{2}\ \Omega$$

The circuit now becomes as shown in Fig. 2.146 (*c*).

Equivalent resistance of the network shown in Fig. 2.146 (*c*) with reference to terminals B and D,

$$R_N = 1 + \left(\frac{1}{3}+5\right) || \left(\frac{1}{2}+4\right)$$

$$= 1 + \frac{16}{3} || \frac{9}{2} = 1 + \frac{\frac{16}{3}\times\frac{9}{2}}{\frac{16}{3}+\frac{9}{2}} = 1 + \frac{144}{59} = \frac{203}{59}\ \Omega$$

For determining short-circuit current I_{sc} *i.e.*, current in zero resistance connected in place of 6 Ω resistor we have the circuit as shown in Fig. 2.146 (*d*).

Let the current distribution satisfying Kirchhoff's first law be as shown in Fig. 2.146 (*d*).

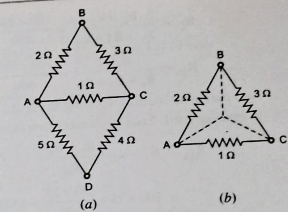
Applying Kirchhoff's second law to closed meshes ABDA, BCDB and ABCA, we have

$$2I_1 - 5I_2 = 0 \quad ...(i)$$

$$3(I_1 - I_{sc}) - 4(I_2 + I_{sc}) = 0 \quad ...(ii)$$

$$\text{and } 2I_1 + 3(I_1 - I_{sc}) + 1 \times (I_1 + I_2) = 9 \quad ...(iii)$$

$$I_1 = 2.5\, I_2$$



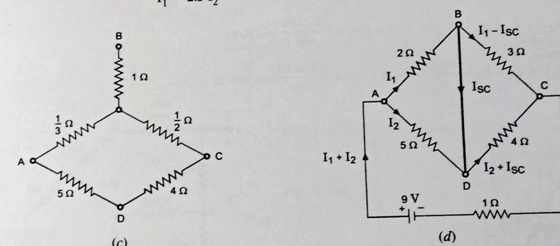


Fig. 2.146

Substituting $I_1 = 2.5I_2$ from Eq. (*i*) in Eq. (*ii*), we have

$$3(2.5I_2 - I_{sc}) - 4(I_2 + I_{sc}) = 0$$

$$\text{or } I_2 = \frac{7}{3.5} I_{sc} = 2\, I_{sc}$$

$$\text{Thus } I_1 = 2.5I_2 = 2.5 \times 2I_{sc} = 5I_{sc}$$

Substituting $I_1 = 5I_{sc}$ and $I_2 = 2I_{sc}$ in Eq. (*iii*), we have

$$2 \times 5I_{sc} + 3(5I_{sc} - I_{sc}) + 5I_{sc} + 2I_{sc} = 9$$

$$\text{or } I_{sc} = \frac{9}{29}\ A$$

$$\text{Current through 6 }\Omega\text{ resistor, } I = \frac{I_{sc} \times R_N}{R_N + R_L} = \frac{\frac{9}{29}\times\frac{203}{59}}{\frac{203}{59}+6} = \frac{63}{557}\ A \text{ Ans.}$$

Example 2.62

Use star-delta transformation method to solve the network of Fig. 2.147.

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]

Solution: Converting Δ BCA into equivalent star, we have

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{100 \times 200}{100 + 200 + 100} = 50\ \Omega$$

$$R_C = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} = \frac{100 \times 200}{400} = 50\ \Omega$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{100 \times 100}{400} = 25\ \Omega$$

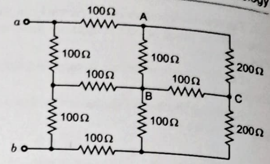


Fig. 2.147

Now the circuit becomes as shown in Fig. 2.148 (c).

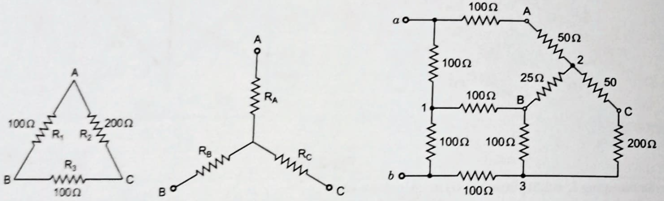
Converting star 1, 2 and 3 into equivalent delta, we have

$$R_{12} = R_1 + R_2 + \frac{R_1 + R_2}{R_3}$$

$$= 100 + 25 + \frac{100 \times 25}{100} = 150\ \Omega$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} = 25 + 100 + \frac{25 \times 100}{100} = 150\ \Omega$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} = 100 + 100 + \frac{100 \times 100}{25} = 600\ \Omega$$



(a) (b) (c)

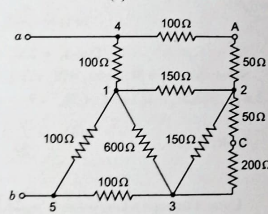
After conversion of star 1, 2, 3 into equivalent delta the circuit becomes as shown in Fig. 2.148 (d).

Converting delta 1, 2, 4 into equivalent star, we have

$$R_1 = \frac{100 \times 150}{100 + 150 + 150} = \frac{100 \times 150}{400} = 37.5\ \Omega$$

$$R_2 = \frac{150 \times 150}{400} = 56.25\ \Omega$$

$$R_4 = \frac{150 \times 100}{400} = 37.5\ \Omega$$



(d)

Fig. 2.148

Converting delta 1, 3, 5 into equivalent star, we have

$$R_1 = \frac{600 \times 100}{600 + 100 + 100} = 75\ \Omega$$

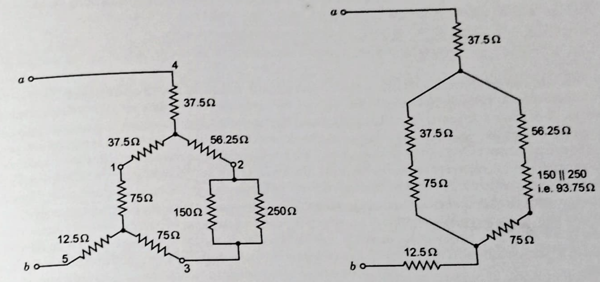
$$R_3 = \frac{100 \times 600}{800} = 75\ \Omega$$

$$R_5 = \frac{100 \times 100}{800} = 12.5\ \Omega$$

Now the circuit shown in Fig. 2.148 (d) becomes as shown in Fig. 2.148 (e). The circuit may redrawn as shown in Fig. 2.148 (f).

The equivalent resistance of the circuit shown in Fig. 2.148 (f)

$$= 37.5 + [(37.5 + 75) \parallel (56.25 + 93.75 + 75)] + 12.5 = 50 + \frac{112.5 \times 225}{112.5 + 225} = 125\ \Omega \text{ Ans.}$$



(e) (f)

Fig. 2.148

Example 2.63

Find the current in 10 Ω resistor in the network shown by star/delta transformation in Fig. 2.149.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2011]

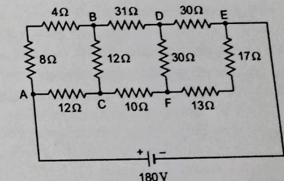


Fig. 2.149

Solution: Converting delta ABC and delta DEF into equivalent star, we have

$$R_A = R_B = R_C = \frac{12}{3} = 4\ \Omega$$

$$R_D = R_E = R_F = \frac{30}{3} = 10\ \Omega$$

The equivalent circuit becomes as shown in Fig. 2.150.

The equivalent resistance of the circuit shown in Fig. 2.150 is given as

$$R = 4 + [(4 + 31 + 10) \parallel (4 + 10 + 10)] + 10$$

$$= 14 + \frac{45 \times 24}{45 + 24} = 29.65\ \Omega$$

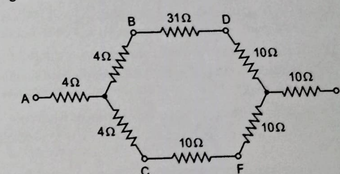


Fig. 2.150

$$\text{Current drawn, } I = \frac{180}{29.65} = 6.07\ \text{A}$$

Current through 10 Ω resistor

$$= I \times \frac{(4 + 31 + 10)}{(4 + 31 + 10) + (4 + 10 + 10)}$$

$$= 6.07 \times \frac{45}{69} = 3.96\ \text{A Ans.}$$

HIGHLIGHTS

1. An *electric circuit* (or network) is an interconnection of physical electrical devices such as an *energy source* (or sources), an *energy convertor* or convertors (load or loads), and *conductors* that connect them.
2. A *junction* (or *node*) is a point in a network where two or more branches meet.
3. A *loop* is a closed path in a network formed by a number of connected branches. *Mesh* is a loop that contains no other loop within it.
4. Network elements may be classified into two categories viz. active elements and passive elements.

   The elements which supply energy to the network are known as *active elements*. The voltage sources like batteries, dc generators, ac generators and current sources like photoelectric cells, metadyne generators fall under the category of active elements. Most of the semiconductor devices like transistors are treated as current sources.

   The components which dissipate or store energy are known as *passive components*. Resistors, inductors and capacitors fall under the category of passive elements. The resistor is the only component which dissipates electrical energy. The inductors and capacitors are the components which store energy, the inductor stores energy by virtue of a current passing through it whereas the capacitor stores energy by virtue of potential difference across it.
5. A voltage source of voltage V_s and internal resistance R_{in} can be converted into an equivalent current source of current $I_s = V_s/R_{in}$ and a resistance R_{in} across it. Similarly a current source of output current I_s in parallel with resistance R_{in} can be converted into an equivalent voltage source of voltage $V_s = I_s R_{in}$ and a resistance R_{in} in series with it.

   A voltage source-series resistance combination is equivalent to a current source-parallel resistance combination if, and only if their respective open-circuit voltages are equal, and their respective short-circuit currents are equal.
6. In *loop method of analysis*, independent mesh currents are taken and the network is solved by framing equations according to Kirchhoff's second or voltage law (KVL).
7. In *nodal analysis* independent nodes are considered and voltages are assumed at these nodes w.r.t. one reference node, called the datum node. The equations are framed according to Kirchhoff's current law (KCL) which reveal the desired results after their solution.
8. According to Superposition theorem if there are a number of voltage and current sources acting simultaneously in any linear bilateral network, then each source can be considered acting independently of the others.
9. Thevenin's theorem may be stated as follows:

   The current in any passive circuit element (which may be called R_L) in a network is the same as would be obtained if R_L were supplied with a source voltage V_{OC} or V_T in series with an equivalent resistance R_{in} or R_T; V_{OC} being the open-circuit voltage at the terminals from which R_L has been removed and R_T being the resistance that would be measured at these terminals after all sources have been removed and each source has been replaced by its internal resistance.
10. *Norton's theorem* is an alternative to Thevenin's theorem. According to this theorem, any two-terminal active network, when viewed from output terminals is equivalent to a constant current source in parallel with a resistance.
11. According to Reciprocity Theorem, if the source voltage and zero-resistance ammeter are interchanged, the magnitude of the current through the ammeter will be the same, no matter how complicated the network.
12. *Maximum power transfer theorem* states that a resistive load, served through a resistive network, will abstract maximum power when the load resistance value is the same as the resistance "viewed by the load as it looks back into the network".
13. Millman's theorem is a combination of Thevenin's and Norton's theorem and enables a number of voltage (or current) sources to be combined in a single voltage (or current) source.

   According to this theorem $V_{OC} = \dfrac{\sum EG}{\sum G}$ and $R_T = \dfrac{1}{\sum G}$.
14. *In delta-star transformation*, the equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta-connected resistances.
15. *In star-delta transformation*, the equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same divided by the third star resistance.

EXERCISES

1. Explain voltage and current sources with examples. **[R.G.P.V. Bhopal Basic Electrical Engineering Second Semester 2004-05]**
2. Explain, what do you understand by the voltage-dependent current source and current-dependent current source. **[U.P.S.C. I.E.S. Electrical Engineering-I, 2004]**
3. Explain the loop current method of solving a network. **[R.G.P.V. Bhopal Basic Electrical Engineering June 2007]**
4. State and explain the nodal method of analysis. **[M.D. Univ. Rohtak Electrical Technology July 2001]**
5. Explain the difference between nodal analysis and mesh analysis. **[G.G.S.I.P. Univ. Delhi Electrical Science May 2010]**
6. State and explain superposition theorem. Mention its limitations. **[U.P. Technical Univ. Electrical Engineering First Semester 2006-07]**
7. State Thevenin's theorem. Explain its application to a dc electric circuit. **[G.G.S.I.P. Univ. Delhi 1st Term Exam. Feb./March 2011]**
8. State and explain Norton's theorem. **[M.D. Univ. Rohtak Electrical Technology May 2002]**
9. Show that Thevenin's and Norton's theorems are dual to each other. **[U.P.S.C. I.E.S. Electrical Engineering I, 2004]**
10. Define the following:
    (*i*) Thevenin's theorem (*ii*) Superposition theorem and
    (*iii*) Norton's theorem. **[M.D. Univ. Rohtak Electrical Technology May 2001]**
11. State the reciprocity theorem. **[G.G.S.I.P. Univ. Delhi Electrical Science 2005-06]**
12. State maximum power transfer theorem. For which type of circuits is it generally applied. Derive condition for maximum power transfer to a purely resistive load. **[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2010]**
13. State and prove maximum power transfer theorem. **[G.G.S.I.P. Univ. Delhi Electrical Science 2004-05]**
14. What is the use of delta-star and star-delta transformation in the circuit ? Explain the method of transforming a star network into delta network. **[M.D. Univ. Rohtak Electrical Technology May 2002]**
15. Derive expression for converting a delta network to a star network. **[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2010]**
16. Derive expression for converting a star network to a delta equivalent network. **[G.G.S.I.P. Univ. Delhi 1st Term Exam. Feb./March 2011]**

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. What is an electric network ?

Ans. An electric network is an interconnection of physical electrical devices such as an energy source (or sources), an energy convertor or convertors (load or loads), and conductors that connect them.

Q. 2. What is an energy source ?

Ans. An energy source (or source), such as primary or secondary cell, a generator, and the like, is a device that converts chemical, mechanical, thermal or some other form of energy into the electrical energy.

Q. 3. What is an energy convertor ?

Ans. An energy convertor, also called the load, such as lamp, heating appliance, or electric motor, converts electrical energy into light, heat, mechanical work etc.

Q. 4. What is an active element ? **[G.G.S.I.P. 1st Term Exam. Feb./March 2011]**

Ans. The elements which supply energy to the network are known as *active elements*. The voltage sources like batteries, dc generators, ac generators and current sources like photoelectric cells, metadyne generators fall under the category of active elements. Most of the semiconductor devices like transistors are treated as current sources.

Q. 5. What is meant by 'node' ?

Ans. A junction or node is a point in a network where two or more branches meet.

Q. 6. Define mesh. **[G.G.S.I.P. 1st Term Exam. Feb./March 2011]**

Ans. Mesh is a loop that contains no other loop within it.

Q. 7. Distinguish between mesh and loop of a network. **[G.G.S.I.P 1st Term Exam. February 2010]**

Ans. A loop is a closed path in a network formed by a number of connected branches. Mesh is a loop that contains no other loop within it.

Q. 8. Distinguish among a mesh current, branch current and loop current.

[G.G.S.I.P. Univ. Delhi Electrical Science May 2011]

Ans. The current flowing through any section of a network, which joins the two nodes directly, (without passing through a third network) is known as branch current.

The current flowing through a loop/mesh, defined as a closed path in a network formed by a number of connected branches, is known as loop/mesh current.

Q. 9. State/define superposition theorem. **[G.G.S.I.P. Univ. 1st Term Exam. February 2008; Feb./March 2011]**

Ans. In a linear resistive network containing two or more voltage sources, the current through any element (resistance or source) may be determined by adding together algebraically the currents produced by each source acting alone, when all other voltage sources are replaced by their internal resistances. If a voltage source has no internal resistance, the terminals to which it was connected are joined together. If there are current sources present they are removed and the network terminals to which they were connected are left open.

Q. 10. What is the utility of superposition theorem ?

Ans. This theorem is applied when we are to determine the current in one particular branch of a network containing several voltage sources or current sources or both voltage sources and current sources.

Q. 11. What is Thevenin theorem ?

Ans. Thevenin's theorem may be stated as follows:

The current in any passive circuit element (which may be called R_L) in a network is the same as would be obtained if R_L were supplied with a source voltage V_{OC} or V_T in series with an equivalent resistance R_{in} or R_T; V_{OC} being the open-circuit voltage at the terminals from which R_L has been removed and R_T being the resistance that would be measured at these terminals after all sources have been removed and each source has been replaced by its internal resistance.

Q. 12. What is the utility of Thevenin theorem ?

Ans. Thevenin's theorem is advantageous when we are to determine the current in a particular element of a linear bilateral network particularly when it is desired to find the current which flows through a resistor for its different values. It makes the solution of the complicated networks (particularly electronic networks) quite simple.

Q. 13. In what respect is Norton's theorem similar to Thevenin's theorem ? In what respect do they differ ?

[G.G.S.I.P. Univ. Delhi Electrical Science May 2008]

Ans. Norton's theorem is in fact, an alternative to the Thevenin's theorem. Whereas by Thevenin's theorem a complex two-terminal network may be simplified for solution by reducing it into a simple circuit in which the so called open-circuit voltage and looking-back resistance are connected in series with the load resistance, by Norton's theorem network is reduced into a simple circuit in which a parallel combination of constant current source and looking-back resistance feeds the load resistance.

In both theorems use of resistance looking back into the network from the load terminals, with all sources removed leaving their internal resistances in the circuit is made. However, while solving circuit by Thevenin's theorem, the open-circuit voltage is determined at the load terminals with the load removed whereas in Norton's method use of a fictitious constant current source is made, the constant current delivered being equal to the current that would pass into a short circuit connected across the output terminals of the given network.

Q. 14. Define Thevenin's theorem. How is the Norton's equivalent circuit related to Thevenin's equivalent circuit ?

[G.G.S.I.P. Univ. Delhi Electrical Science May 2011]

Ans. For definition of Thevenin's theorem refer to Answer of Q 11.

Thevenin's and Norton's theorems are dual of each other.

According to the Norton's theorem statement Norton's current source equals the short-circuit current I_{sc} that flows through a short across the output terminals *i.e.*, $I_{sc} = \dfrac{V_T}{R_T}$ while the Thevenin's equivalent voltage source V_T is the voltage on open circuit and is given as V_{oc} or $V_T = I_{sc} R_T$.

Q. 15. State the reciprocity theorem. **[G.G.S.I.P. Univ. Delhi May-June 2006]**

Ans. According to this theorem if the source voltage and zero-resistance ammeter are interchanged, the magnitude of the current through the ammeter will be the same, no matter how complicated the network. Indeed this principle states that the directional characteristics of a receiving antenna are the same as the directional characteristics of the same antenna when used for transmission. This is a highly useful relation.

In other words, *in a linear passive network*, supply voltage V and current I are mutually transferable. The ratio of V and I is called the *transfer resistance* (or transfer impedance in an ac system).

Q. 16. State maximum power transfer theorem ? **[G.G.S.I.P. Univ. Delhi May 2008]**

Ans. A resistive load, served through a resistive network, will abstract maximum power when the load resistance value is the same as the resistance "viewed by the load as it looks back into the network".

Q. 17. State Millman's theorem. **[G.G.S.I.P. Univ. 1st Term Feb./March 2011]**

Ans. Millman's theorem is a combination of Thevenin's and Norton's theorem and enables a number of voltage (or current) sources to be combined in a single voltage (or current) source.

According to this theorem $V_{OC} = \dfrac{\sum EG}{\sum G}$ and $R_T = \dfrac{1}{\sum G}$.

Q. 18. Where does the Millman theorem find utility ?

Ans. In practice, cases frequently arise where a network has only two terminal points between which any number of parallel branches may be connected. Their calculations can be greatly simplified by the use of Millman's theorem, which is combination of Thevenin's and Norton's theorem. This theorem enables a number of voltage (or current) sources to be combined into a single voltage (or current) source.

Q. 19. Give the relationship between resistances connected in delta and equivalent star systems ?

Ans. The equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta connected resistances.

Q. 20. Give the relationship between resistances connected in star and equivalent delta systems ?

Ans. The equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same divided by the third star resistance.

PROBLEMS

1. Convert 4 A source with its parallel resistance of 15 Ω into its equivalent voltage source.

   [Ans. Voltage source of 60 V in series with a resistance of 15 Ω**]**

2. Convert 240 V source with a series resistance of 40 Ω into its equivalent current source.

   [Ans. Current source of 6 A is parallel with a resistor of 40 Ω**]**

3. Determine the value of current in 8 Ω resistor of the network shown in Fig. 1, using mesh current method.

   [M.D. Univ. Rohtak Electrical Technology First Semester 2006-07]

   [Ans. 0.31945 A]

4. Using mesh current analysis, find current through galvanometer G shown below in Fig. 2. Internal resistance of galvanometer G is 50 Ω. **[M.D. Univ. Rohtak Electrical Technology First Semester 2006-07]**

   [Ans. 0.04875 A upward]

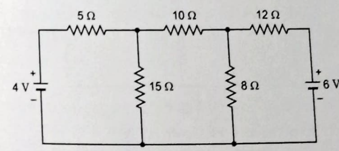


Fig. 1

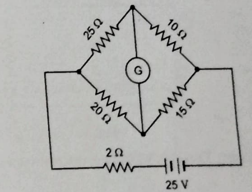


Fig. 2

5. Using nodal analysis, find the current through 8 Ω resistor in the circuit shown in Fig. 3.

[M.D. Univ. Electrical Technology First Semester 2006-07]

[Ans. 0.82934 A]

6. Use nodal analysis to determine the voltage across BC and the current in the 12 V source. (Fig. 4)

[R.G.P.V. Bhopal Basic Electrical Engineering Second Semester 2006-07]

[Ans. 5.75 V, $\frac{19}{16}$ A]

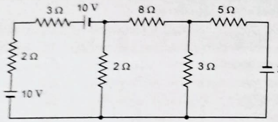


Fig. 3

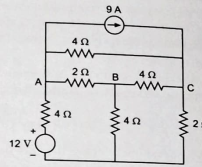


Fig. 4

7. Use the node voltage method to solve the mesh currents in the network shown in Fig. 5.

[U.P. Technical Univ. Electrical Engineering June 2001]

[Ans. – 5.0373 A, – 2.0522 A and – 9.7015 A]

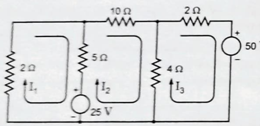


Fig. 5

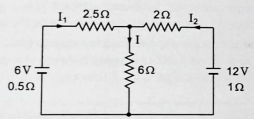


Fig. 6

8. Using superposition theorem, find I_1, I_2 and I. Also find voltage across 6 Ω resistor shown in Fig. 6.

[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2010]

[Ans. I_1 = – 0.4 A, I_2 = 1.6 A, I = 1.2 A, 7.2 V]

9. Determine current I in the circuit of Fig. 7 using superposition theorem.

[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2008]

[Ans. 0.75 A]

10. State Thevenin's theorem. Find current in resistor R_3 of the network shown below (Fig. 8), using Thevenin's theorem.

[M.D. Univ. Electrical Technology Ist Semester, 2006-07]

[Ans. 2.25 A]

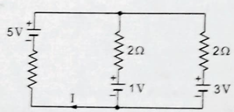


Fig. 7

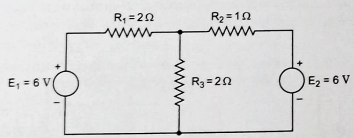


Fig. 8

11. Using Thevenin's theorem, determine the current in branch BD of the network shown in Fig. 9.

[Bombay Univ. Basic Electricity October 1974]

[Ans. 2.7 A from B to D]

12. Use Thevenin's theorem to find the current flowing through the 6 Ω resistor of the network shown in Fig. 10.

[Nagpur Univ. Network Theory 1992]

[Ans. 1 A from terminal A to terminal B]

13. Determine current through 6-Ω resistance connected across A-B terminals in the electric circuit of Fig. 11 using Thevenin's Theorem.

[U.P. Technical Univ. Electrical Engineering February 2001]

[Ans. 1 A]

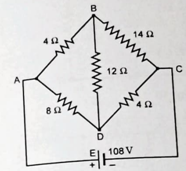


Fig. 9

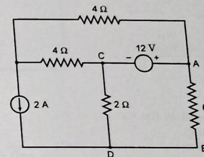


Fig. 10

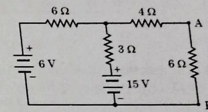


Fig. 11

14. Give the limitations of Thevenin's theorem. Find the current I using this theorem in circuit in Fig. 12.

[U.P. Technical Univ. Electrical Engineering 2006-07]

[Ans. I = 1.5 A]

15. Explain Thevenin's theorem. Draw the Thevenin's equivalent of the circuit across AB shown in Fig. 13.

[Ans. $\frac{45}{11}$ V, $\frac{10}{11}$ Ω]

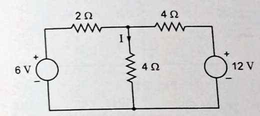


Fig. 12

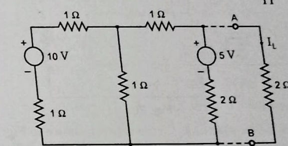


Fig. 13

16. Find the current in 1 Ω resistor of network shown in Fig. 14 using Thevenin's theorem.

[G.G.S.I.P. Univ. 1st Term Exam. February 2009]

[Ans. 0.3246 A]

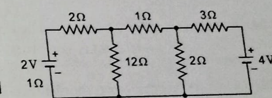


Fig. 14

17. Find Thevenin's equivalent circuit at terminals BC of Fig. 15. Hence determine current through the resistor R = 1 Ω.

[Ans. $\frac{76}{7}$ V, $\frac{32}{7}$ Ω, $\frac{76}{39}$ A]

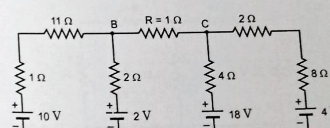


Fig. 15

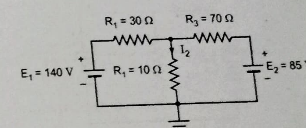


Fig. 16

18. In the circuit of Fig. 16, find the branch current I_2 which flows through R_2, when R_2 has the following values: 5 Ω, 15 Ω and 50 Ω. **[Ans.** 4.75 A, 3.43 A, 1.74 A]

19. For the circuit of Fig. 17, obtain Norton current and equivalent resistance seen from '*ab*'. **[Ans.** 1.2 A, 10 Ω]

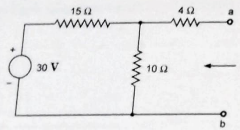


Fig. 17

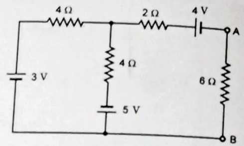


Fig. 18

20. For the circuit shown in Fig. 18, calculate the current in the 6 Ω - resistance by using Norton's theorem.
[Osmania Univ. Electrical Technology 1992]
[Ans. 0.5 A from B to A]

21. Find current through 10 Ω resistor by Norton's theorem. Shown in Fig. 19. **[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2010]**
[Ans. 0.68 A]

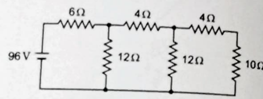


Fig. 19

22. Show that reciprocity theorem holds good for the following circuit (Fig. 20)

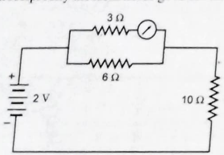


Fig. 20

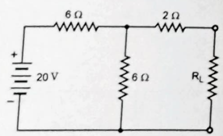


Fig. 21

23. Determine the value of R_L in the network shown in Fig. 21 to have power transferred to it maximum. Determine also the output power of the battery and the power lost in the three star-connected resistances.
[Ans. 5 Ω, 5 W, 38.33 W, 43.33 W]

24. In the circuit of Fig. 22, find R_L for maximum power in it.
[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2008]
[Ans. 10 Ω]

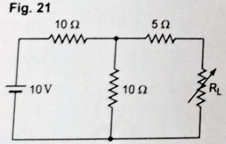


Fig. 22

25. Determine branch currents shown in Fig. 23 using Millman's theorem. **[Ans.** 0.4 A, 1 A, 0.6 A]

26. In the network shown in Fig. 24, determine :
(*i*) The value of load resistance R_L to give maximum power transfer, and
(*ii*) The power delivered to the load. **[U.P. Technical Univ. Elec. Engineering Second Semester 2002-03]**
[Ans. 9 Ω, 2.778 W]

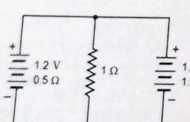


Fig. 23

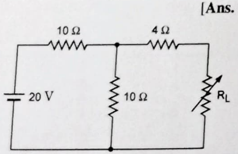


Fig. 24

27. Three resistances of 20 Ω each are connected in star. Find the equivalent delta resistance. If a source of emf of 120 V is connected across any two terminals of the equivalent delta-connected resistance find the current supplied by the source. **[Calcutta Univ. Elec. Engineering 1980]**
[Ans. 60 Ω; 3 A]

28. Three resistances of 20, 25 and 30 ohms are connected in delta. Calculate the corresponding resistances in an equivalent star connection. **[Ans.** 8 Ω, 6.67 Ω and 10 Ω]

29. Find the resistance at the A–B terminals in the electric circuit of Fig. 25 using Δ–Y transformation.
[U.P. Technical Univ. Elec. Engineering February 2001]
[Ans. 36 Ω]

30. Find the current drawn I from the source voltage of 100 V in the circuit shown in Fig. 26.
[U.P. Technical Univ. Elec. Engineering First Semester 2004-05]
[Ans. 2.174 A]

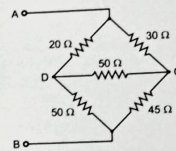


Fig. 25

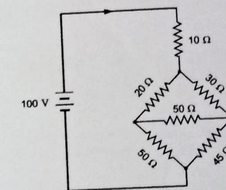


Fig. 26

31. A network of resistances is shown in Fig. 27. Compute the equivalent network resistance measured between (*i*) A and B (*ii*) B and C and (*iii*) C and A.

[Ans. $\frac{4914}{2679}$ Ω, $\frac{2160}{2679}$ Ω, $\frac{3726}{2669}$ Ω]

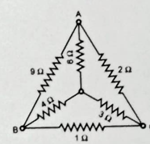


Fig. 27

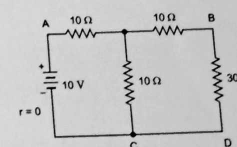


Fig. 28

32. In the circuit shown in Fig. 28 transform the star ABC to delta and then apply Thevenin's theorem to find the voltage across 30 Ω resistance. **[M.D. Univ. Rohtak Electrical Technology, May 2002]**
[Ans. 3.33 V]

❑❑❑

CONTENTS IN THIS UNIT

AC CIRCUITS: AC fundamentals; Phasor representation; Steady state response of series and parallel R-L, R-C and R-L-C circuits using *j* notation; Series and parallel resonance of R-L-C circuits; Quality factor; Bandwidth; Complex power; Introduction to balanced 3-phase circuits with star-delta connections.

Chapter 3

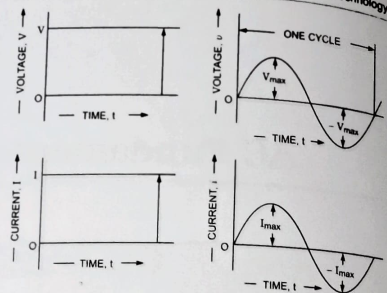
AC Fundamentals

Contents in this Chapter

3.1. INTRODUCTION

A current (or voltage) is called *alternating* if it reverses periodically in direction, and its magnitude undergoes a definite cycle of changes in definite intervals of time. Each cycle of alternating current (or voltage) consists of two half cycles, during one of which the current (or voltage) acts in one direction; while during the other in opposite direction. In more restricted sense, alternating current is a periodically varying current, the average value of which, over a period, is zero. The direct current always flows in one direction, and its magnitude remains unaltered. In order to produce an alternating current through an electric circuit, a source capable of reversing the emf periodically (ac generator) is required while for generating dc in an electric circuit, a source capable of developing a constant emf is required such as a battery or dc generator. The graphical representations of direct current and alternating current are given in Figs. 3.1(*a*) and 3.1 (*b*) respectively.

At present a large percentage of the electrical energy (nearly all) being used for domestic and commercial purposes is generated as alternating current. In fact, almost the whole of the vast amount of electrical energy used throughout the world for every imaginable purpose is generated by alternating current generators. This is not due to any superiority of alternating current over direct current in the sphere of applicability to industrial and domestic use. In fact there are certain types of works for which alternating current is unsuitable and, therefore, direct current is absolutely necessary such as for electroplating, charging of storage batteries, refining of copper, refining of aluminium, electrotyping, production of industrial gases by electrolysis, municipal traction etc. In some power applications, the ac motor is unsatisfactory such as for metal rolling mills, paper making machines, high speed gearless elevators, automatic machine tools and high-speed printing presses. Direct current required for these applications is nowadays derived from an ac supply by the use of suitable convertors or rectifiers. For lighting and heating dc and ac are equally useful. The reasons for generation of electrical energy in the form of alternating current are given below:



(a) DC Voltage and Current *(b) AC Voltage and Current*

Fig. 3.1

1. AC generators have no commutator and can, therefore, be built in very large units to run at high speeds producing high voltages (as high as 33,000 volts), so that the construction and operating cost per kW is low, whereas dc generator capacities and voltages are limited to comparatively low values.
2. Alternating current can be generated at comparatively high voltages and can be raised and lowered readily by a static machine called the *transformer*, which makes the transmission and distribution of electrical energy economical. In direct current use of transformers is not possible.
3. AC induction motor is cheaper in initial cost and in maintenance since it has got no commutator and is more efficient than dc motor for constant speed work, so it is desirable to generate power as alternating current.
4. The high transmission efficiency in ac makes the generation of electrical energy economical by generating it in large quantities in a single station and distributing over a large territory.
5. The switchgear (*e.g.*, switches, circuit breakers etc.) for ac system is simpler than that required in a dc system.
6. The maintenance cost of ac equipment is less.

3.2. GENERATION OF ALTERNATING EMF

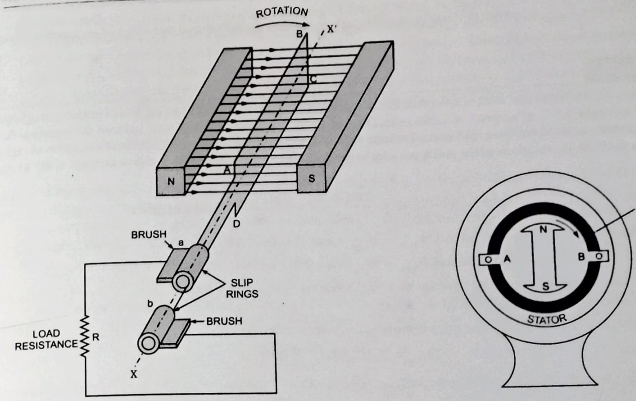
We know that an alternating emf can be generated either by rotating a coil within a stationary magnetic field, as illustrated in Fig. 3.2 (*a*) or by rotating a magnetic field within a stationary coil, as illustrated in Fig. 3.2 (*b*). The emf generated, in either case, will be of sinusoidal waveform. The magnitude of emf generated in the coil depends upon the number of turns on the coil, the strength of magnetic field and the speed at which the coil or magnetic field rotates. The former method is employed in case of small ac generators while the later one is employed for large sized ac generators.

Now consider a rectangular coil of N turns rotating in counter-clockwise direction with angular velocity of ω radians per second in a uniform magnetic field, as illustrated in Fig. 3.3.

Let the time be measured from the instant of coincidence of the plane of the coil with the X-axis. At this instant maximum flux, Φ_{max} links with the coil. Let the coil assume the position, as shown in Fig. 3.3, after moving in counter-clockwise direction for t seconds.

The angle θ through which the coil has rotated in t seconds = ωt

In this position, the component of flux along perpendicular to the plane of coil = $\Phi_{max} \cos \omega t$.



(*a*) (*b*)

Fig. 3.2 *Generation of Alternating EMF*

Hence, flux linkages of the coil at this instant

= Number of turns on coil × linking flux

i.e; instantaneous flux linkages = $N \Phi_{max} \cos \omega t$

Since emf induced in a coil is equal to the rate of change of flux linkages with minus sign,

EMF induced at any instant,

$$e = -\frac{d}{dt}[N \Phi_{max} \cos \omega t]$$

$$= \Phi_{max} N \frac{d}{dt}[-\cos \omega t] = \Phi_{max} N \omega \sin \omega t \qquad ...(3.1)$$

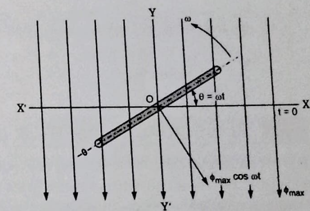


Fig. 3.3

when $\omega t = 0$, $\sin \omega t = 0$, therefore, induced emf is zero, when $\omega t = \frac{\pi}{2}$, $\sin \frac{\pi}{2} = 1$, therefore, induced emf is maximum, which is denoted by E_{max} and is equal to $\Phi_{max} N\omega$

Substituting $\Phi_{max} N\omega = E_{max}$ in Eq. (*i*), we have

Instantaneous emf, $e = E_{max} \sin \omega t$ ...(3.2)

So the emf induced varies as the sine function of the time angle ωt, and if emf induced is plotted against time, a curve of sine waveshape is obtained as illustrated in Fig. 3.4. Such an emf is called the *sinusoidal* emf. The sine curve is completed when the coil rotates through an angle of 2π radians. The induced emf e will have maximum value,

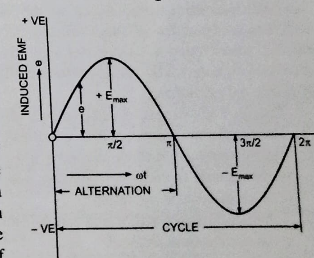


Fig. 3.4

represented by E_{max}, when the coil has turned through $\frac{\pi}{2}$ radians (or 90°) in counter-clockwise direction from the reference axis (*i.e.*, OX axis).

Example 3.1

A coil having 200 turns and area of x-section 250 cm² is rotated about its axis at right angle to a uniform magnetic field of strength 0.5 T at a speed of 1,200 rpm. Determine (*i*) maximum value of emf induced (*ii*) equation for instantaneous induced emf and (*iii*) instantaneous values of induced emf when (*a*) the plane of the coil is at right angle to the field. (*b*) the plane of the coil is parallel to the field and (*c*) the plane of the coil is at an angle of 60° to the field.

Solution: Angular velocity, $\omega = \frac{2\pi \times RPM}{60} = \frac{2\pi \times 1,200}{60} = 40\pi$ radians/s

Maximum flux linking with the coil, $\Phi_{max} = B_{max} \times$ area of coil $= 0.5 \times 250 \times 10^{-4} = 0.0125$ Wb

(*i*) Maximum value of induced emf, $E_{max} = \Phi_{max} N\omega = 0.0125 \times 200 \times 40\pi = 314$ V **Ans.**

(*ii*) Equation for instantaneous induced emf $= E_{max} \sin \omega t = 314 \sin 40\pi t$ **Ans.**

(*iii*) Instantaneous values of induced emf when

(*a*) the plane of the coil is at right angle to the field *i.e.*, when $\theta = \omega t = 0$

$$\therefore \quad e_0 = 314 \sin 0° = 0 \text{ Ans.}$$

(*b*) the plane of the coil is parallel to the field, *i.e.*, when $\theta = \omega t = \frac{\pi}{2}$ radians

$$\therefore \quad e' = 314 \sin \frac{\pi}{2} = 314 \text{ V Ans.}$$

(*c*) the plane of the coil makes an angle of 60° to the field *i.e.*, when $\theta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ radians

$$\therefore \quad e'' = 314 \sin \frac{\pi}{6} = 314 \times \frac{1}{2} = 157 \text{ V Ans.}$$

3.3. SINUSOIDAL QUANTITIES (EMF, VOLTAGE OR CURRENT)

It is not an accident that the bulk of electric power generated in electric power stations throughout the world and distributed to the consumers appears in the form of sinusoidal variations of voltage and current. There are many technical and economical advantages associated with the use of sinusoidal voltages and currents. For example, it will be learned that the use of sinusoidal voltages applied to appropriately designed coils results in a revolving magnetic field which has the capacity to do work. As a matter of fact it is this principle which underlies the operation of almost all the electric motors found in home appliances and about 90% of all electric motors found in commercial and industrial applications. Although other waveforms can be used in such devices, none leads to an operation which is as efficient and economical as that achieved through the use of sinusoidal quantities.

The other advantages of using sinusoidal voltages and currents are:

1. The waveform from generation to utilization remains the same if a sinusoidal waveform is generated.
2. Electromagnetic torque developed in three phase machines (generators and motors) with balanced three-phase currents is uniform (constant), and therefore, there are no oscillations in developed torque and absence of noise in operation.
3. Non-sinusoidal voltages which contain harmonic frequencies, according to Fourier analysis, are harmful to the system on account of
   (*i*) increased losses in generators, motors, transformers, and transmission and distribution systems,
   (*ii*) more interference (noise) to nearby communication circuits,

   (*iii*) resonance may result in overvoltages or overcurrents at many pockets on the way from generating station to consumer's premises which may damage the equipment and increase losses.
   (*iv*) increased current through power factor improvement capacitors.

In practical electrical engineering it is assumed that the alternating voltages and currents are sinusoidal, though they may slightly deviate from it. The advantage of this assumption is that calculations become simple. *It may be noted that alternating voltage and current mean sinusoidal voltage and current unless stated otherwise.*

Alternating emf following sine law (*i.e.*, sinusoidal emf) is illustrated in Fig. 3.4 and is expressed in the form

$$e = E_{max} \sin \omega t \quad ...(3.3)$$

where e is the instantaneous value of alternating emf (or voltage), E_{max} is the maximum value of the alternating emf (or voltage) and ω is angular velocity of the coil.

The rotating coil moves through an angle of 2π radians in one cycle, so angular velocity $\omega = 2\pi f$ where f is the number of cycles completed per second.

Substituting $\omega = 2\pi f$ in Eq. (3.3), we have

$$e = E_{max} \sin 2\pi f t \quad ...(3.4)$$

If the alternating emf (or voltage) given by Eq. (3.3) is applied across a load, alternating current flows through the circuit which would also vary sinusoidally *i.e.*, following a sine law. The expression for alternating current is given as

$$i = I_{max} \sin \omega t = I_{max} \sin 2\pi f t \quad ...(3.5)$$

provided the load is pure resistive.*

3.4. TERMINOLOGY

An *alternating quantity* (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time. It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again, as illustrated in Fig. 3.4. The emf (or voltage) and current repeat the procedure.

The important ac terms are defined below:

3.4.1. Waveform. The shape of the curve of the voltage or current when plotted against time as abcissa (base) is called the *waveform*. The waveform of an alternating voltage varying sinusoidally is shown in Fig. 3.4. The waveform of the induced emf in an alternator differs slightly from that of sine wave but for calculation purposes it is treated as such.

3.4.2. Instantaneous Value. The value of alternating quantity (emf, voltage or current) at any particular instant is called the *instantaneous value* and is designated by a small italic letter (e for emf, v for voltage and i for current). The instantaneous values of an alternating quantity can be determined either from the curve or from an equation of the alternating quantity. For example, the instantaneous values of emf represented by the curve shown in Fig. 3.4 at 0, $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$ are zero, $+E_{max}$, zero and $-E_{max}$ respectively.

3.4.3. Alternation and Cycle. When a periodic wave, such as sinusoidal wave, goes through one complete set of positive or negative values, it completes one *alternation* and when it goes through one complete set of positive and negative values it is said to have completed one *cycle*.

Alternation and cycle can also be defined in terms of angular measure. One alternation corresponds to 180°(or π radians) while one cycle corresponds to 360° (or 2π radians).

3.4.4. Time Period and Frequency. The time taken in seconds by an alternating quantity to complete one cycle is known as *time period* or *periodic time* and is denoted by T.

* The load may be resistive, inductive or capacitive. It will be shown in chapter 4 that if the load is inductive or capacitive the current equation differs in time angle.

The number of cycles completed per second by an alternating quantity is known as *frequency* and is denoted by f. In SI system the frequency is expressed in hertz (pronounced as hurts). One hertz (or Hz) is equal to one cycle per second.

The number of cycles completed per second = f

Time period, T = Time taken in completing one cycle = $1/f$

or $f = 1/T$ ...(3.6)

The commercial ac power is generated at frequency of 50 Hz or 60 Hz.* The reasons of suitability of frequency of this range are:

1. The output of the equipment increases with the increase in frequency. For a given output, smaller size machines are required as compared to those for lower frequency output. Because of high power-weight ratio, relative cost of the equipment is also reduced.
2. Lower regulation, lower skin effect resulting in lower ohmic losses, lower magnetic and dielectric losses resulting in higher efficiency, lower corona loss and higher power transmission line capacity as compared to those at higher frequency.

3.4.5. Angular Velocity and Frequency. A glance at Fig. 3.4 indicates that each cycle spans 2π radians. Hence, if this quantity is divided by the time period, *angular velocity* of the sine function is obtained. It is denoted by ω and is expressed in radians per second.

$$\therefore \text{ Angular velocity, } \omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T} = \frac{2\pi}{1/f} = 2\pi f \text{ radians per second} \quad ...(3.7)$$

3.4.6. Electrical Time Degrees and Mechanical Degrees. It is seen that the coil must revolve past a pair of poles in order to carry the generated emf through one complete cycle. In circuit work, one complete cycle of voltage or current is designated as 360 electrical degrees or 2π electrical radians. To correlate with this designation the arc through which a coil of dynamo must rotate in order to generate one cycle of emf is called 360 electrical degrees. In a 2-pole machine one complete revolution of coil produces one cycle of emf. But in a multipolar machine, such as four, six or eight pole machine, the emf completes one cycle or 360 electrical degrees or 2π electrical radians as soon as the coil passes a pair of poles and a mechanical degree will be equal to as many electrical degrees as there are pairs of poles in the dynamo structure. In a multipolar machine, the number of cycles completed per second by generated emf,

$$f = \text{Pair of poles} \times \text{number of revolutions made per second} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \quad ...(3.8)$$

where N is the speed of rotation of the coil in rpm.

3.4.7. Amplitude. The maximum value, positive or negative, which an alternating quantity attains during one cycle is called the *amplitude* of the alternating quantity. The amplitude of an alternating emf (or voltage) and current is designated by E_{max} (or V_{max}) and I_{max} respectively.

Example 3.2

What is the time period of the wave produced by a 6-pole alternator which is driven at 1,000 rpm ?

[Pb Technical Univ. Basic Electrical Engineering, June 2003]

Solution: Number of poles on alternator, P = 6

Speed of alternator, N = 1,000 rpm

$$\text{Number of cycles completed per second by generated emf, } f = \frac{PN}{120} = \frac{6 \times 1{,}000}{120} = 50$$

$$\text{Time period, } T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ second } \textbf{Ans.}$$

* In India the standard frequency for power supply is 50 Hz.

3.5. DETERMINATION OF MAXIMUM VALUE AND FREQUENCY FROM EMF OR CURRENT EQUATIONS

From Eq. (3.4)

$$e = E_{max} \sin 2\pi f t$$

From above expression we observe that

(*i*) *the maximum value of an alternating emf is given by the coefficient of the sine of the time angle.*

(*ii*) *the frequency is given by coefficient of time t divided by 2π*

$$\text{i.e., } f = \frac{\text{Coefficient of time, } t}{2\pi} \quad ...(3.9)$$

Similarly we can also find the maximum value and frequency of the current from the equation of instantaneous values of current.

Example 3.3

An ac voltage of 50 Hz frequency has a peak value of 220 V. Write down the expression for the instantaneous value of this voltage. *[Pb. Technical Univ. Electrical Engineering January 2000]*

Solution: Supply frequency, f = 50 Hz

Peak value of ac voltage, V_{max} = 220 V

Expression for instantaneous value of ac voltage (assumed sinusoidal) with θ as zero when time is zero, is given as

$$v = V_{max} \sin \omega t = V_{max} \sin 2\pi ft = 220 \sin 2\pi \times 50\, t = 220 \sin 314t \textbf{ Ans.}$$

Example 3.4

An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate (*a*) its value $\frac{1}{600}$ second after the instant the current is zero and its value decreasing there afterwards (*b*) how many seconds after the instant the current is zero (increasing there afterwards) will the current attain the value of 86.6 A ?

[Allahabad Univ. Elec. Technology 1991]

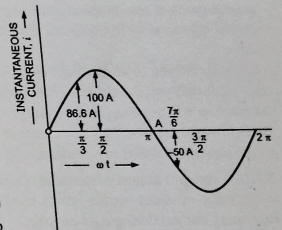


Fig. 3.5

Solution: The current wave form is shown in Fig. 3.5.

The equation of the alternating current (assumed sinusoidal) with respect to the origin O is

$$i = 100 \sin 100\pi t \quad ...(i)$$

(*a*) Since the current is measured from the instant the current is zero and is decreasing there afterwards (*i.e.*, from point A in Fig. 3.5), the equation for the alternating current with respect to the point A becomes

$$i = 100 \sin(100\pi t + \pi) = -100 \sin 100\pi t \quad ...(ii)$$

Substituting $t = \frac{1}{600}$ second in above equation we get the instantaneous value of current $\frac{1}{600}$ second after the instant the current is zero and decreasing there afterward.

$$\text{So } i = -100 \sin 100\pi \times \frac{1}{600} = -100 \sin \frac{\pi}{6} = -50 \text{ A } \textbf{Ans.}$$

(*b*) Let the current attain the value of 86.6 A, t seconds after the zero value of the current. Now substituting i = 86.6 A in Eq. (*i*), we get

$$86.6 = 100 \sin 100\pi t$$

$$\text{or } t = \frac{1}{100\pi} \sin^{-1} \frac{86.6}{100} = \frac{1}{100\pi} \times \frac{\pi}{3} = \frac{1}{300} \text{ second } \textbf{Ans.}$$

3.6. PLOTTING OF SINE WAVEFORM

Sine curve may be graphically drawn, as illustrated in Fig. 3.6. Draw a circle of radius equal to the maximum value of sinusoidal quantity. Divide the circumference of the circle drawn so into any number of equal parts, say 12, and draw a horizontal line AB (the base on which the sine wave is to be drawn) passing through the centre of the circle. Divide the line AB into the same number of equal parts *i.e.*, 12 and number the points correspondingly. Draw perpendicular ordinates from each point. Project the points on the circle horizontally to meet the perpendicular ordinates having corresponding numbers. Draw smooth curve through these points. Curve so drawn will be of sine waveform.

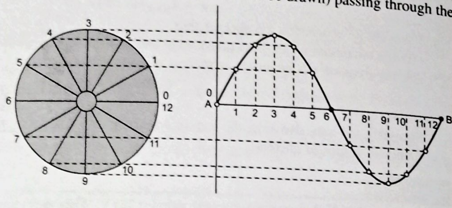


Fig. 3.6 *Simple Method of Plotting Sine Wave*

3.7. AVERAGE AND EFFECTIVE (RMS) VALUES OF ALTERNATING VOLTAGE AND CURRENT

In a dc system, the voltage and current are constant and, therefore, there is no problem in specifying their magnitude. But in case of ac system, an alternating voltage or current varies from instant to instant and so poses a problem how to specify the magnitude of an alternating voltage or current. An alternating voltage or current may possibly be expressed in terms of peak (maximum) value, average (mean) value or effective (rms) value.

In specifying an alternating voltage or current, its peak or maximum value is rarely used because it has that value only twice each cycle. Furthermore, the average or mean value cannot be used because it is positive as much as it is negative, so the average value is zero. Although the average value over half cycle might be used, it would not be as logical a choice as what we shall find *effective (virtual or rms) value* which is related to the power developed in a resistance by the alternating voltage or current.

3.7.1. Average Value of Alternating Current. The average (or mean) value of an alternating current is equal to the value of direct current which transfers across any circuit the same charge as is transferred by that alternating current during a given time.

Since in the case of a symmetrical alternating current (*i.e.*, one whose two half cycles are exactly similar, whether sinusoidal or non-sinusoidal) the average or mean value over a complete cycle is zero hence for such alternating quantities average or mean value means the value determined by taking the average of instantaneous values during half cycle or one alternation only. However, for unsymmetrical alternating current, as half wave rectified current, the average value means the value determined by taking the mean of instantaneous values over the whole cycle.

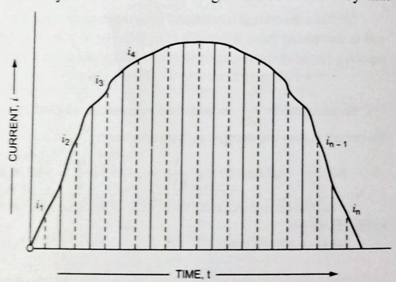


Fig. 3.7

The average value is determined by measuring the lengths of a number of equidistant ordinates and then taking their mean *i.e.*, of $i_1, i_2, i_3 \ldots i_n$ etc. which are mid-ordinates.

$$\because \text{ Average value of alternating current, } I_{av} = \frac{i_1 + i_2 + i_3 + i_4 \ldots + i_n}{n}$$

$$= \frac{\text{Area of one alternation (or half cycle)}}{\text{Length of base over one alternation (or half cycle)}}$$

Using the integral calculus the average (or mean) value of a function $f(t)$ over a specific interval of time between t_1 and t_2 is given by

$$F_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f(t)\,dt \qquad \ldots(3.10)$$

Any function whose cycle is repeated continuously, irrespective of its waveshape, is termed as *periodic function*, such as sinusoidal function, and its average value is given by

$$F_{av} = \frac{1}{T}\int_0^T f(t)\,dt \qquad \ldots(3.11)$$

where T is time period of periodic function.

In case of a symmetrical alternating current, whether sinusoidal or non-sinusoidal the average value is determined by taking average of one half cycle or one alternation only.

$$\text{i.e., for symmetrical waveforms, } F_{av} = \frac{2}{T}\int_0^{T/2} f(t)\,dt \qquad \ldots(3.12)$$

3.7.2. RMS Value or Effective Value of Alternating Current. The rms or effective value of an alternating current or voltage is given by that steady current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

Consider an alternating current of waveform shown in Fig. 3.8 flowing through a resistor of R ohms. Divide the base of one alternation into n equal parts and let the mid-ordinates be $i_1, i_2, i_3 \ldots i_n$ etc.

$$\text{Heat produced during 1st interval} = i_1^2 R \times \frac{T}{n} \text{ joules}$$

$$\text{Heat produced during 2nd interval} = i_2^2 R \times \frac{T}{n} \text{ joules}$$

$$\text{Heat produced during 3rd interval} = i_3^2 R \times \frac{T}{n} \text{ joules}$$

$$\text{Heat produced during } n\text{th interval} = i_n^2 R \times \frac{T}{n} \text{ joules}$$

Total heat produced in time T

$$= RT\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots i_n^2}{n}\right) \text{ joules}$$

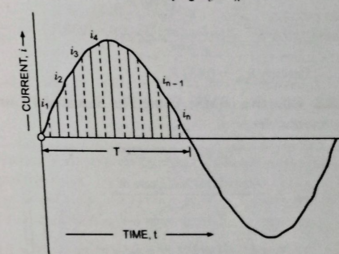


Fig. 3.8

Now if I_{eff} is the effective current, then heat produced by this current in time T = I_{eff}^2 RT joules. By definition these two expressions are equal

$$\therefore\ I_{eff}^2\, RT = RT\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots i_n^2}{n}\right)$$

$$\text{or } I_{eff}^2 = \frac{i_1^2 + i_2^2 + i_3^2 + \ldots i_n^2}{n}$$

$$\text{or } I_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots i_n^2}{n}}$$

Hence the *effective or virtual value* of alternating current or voltage is equal to the square root of the mean of the squares of successive ordinates and that is why it is known as *root mean square (rms) value.*

Using the integral calculus the root mean square (rms) or effective value of an alternating quantity over a time period is given by

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} \quad \ldots(3.13)$$

3.8. AVERAGE AND EFFECTIVE (RMS) VALUES OF SINUSOIDAL CURRENT AND VOLTAGE

3.8.1. Average Value For Sinusoidal Current or Voltage. The average value of a sine wave over a complete cycle is zero. Therefore, the half cycle average value is intended.

Instantaneous value of sinusoidal current is given by

$$i = I_{max} \sin \omega t$$

Considering first half cycle *i.e.*, when ωt varies from 0 to π, we get

$$I_{av} = \frac{\text{Area of first half cycle}}{\pi}$$

$$= \frac{1}{\pi}\int_0^\pi i\, d(\omega t) = \frac{1}{\pi}\int_0^\pi I_{max} \sin \omega t \; d(\omega t)$$

$$\text{or } I_{av} = \frac{I_{max}}{\pi}\left[-\cos \omega t\right]_0^\pi$$

$$= \frac{2}{\pi} I_{max} = 0.637\, I_{max} \quad \ldots(3.14)$$

Similarly $E_{av} = 0.637\, E_{max}$

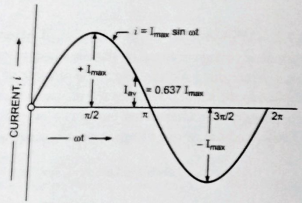


Fig. 3.9

3.8.2. Effective (RMS) Value For Sinusoidal Current or Voltage. A sinusoidal alternating current is represented by

$$i = I_{max} \sin \omega t$$

$$I_{rms}^2 = \frac{\text{Area of first half cycle of } i^2}{\pi}$$

$$= \frac{1}{\pi}\int_0^\pi i^2\, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^\pi I_{max}^2 \sin^2 \omega t\, d(\omega t)$$

$$= \frac{I_{max}^2}{2\pi}\int_0^\pi (1 - \cos 2\omega t)\, d(\omega t)$$

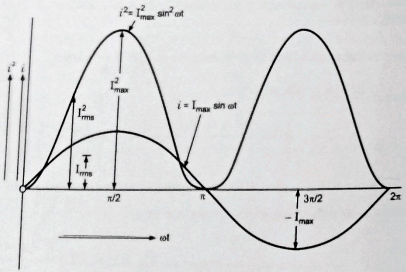


Fig. 3.10

$$= \frac{I_{max}^2}{2\pi}\left[\omega t - \frac{1}{2}\sin 2\omega t\right]_0^\pi = \frac{I_{max}^2}{2\pi} \times \pi = \frac{I_{max}^2}{2}$$

$$\text{or } I_{rms} = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}} \quad \ldots(3.15)$$

$$\text{Similarly, } E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

Example 3.5

A sinusoidally varying alternating current of frequency 60 Hz has a maximum value of 15 amperes.

(i) Write down the equation for instantaneous value,

(ii) Find the value of current after 1/200 second,

(iii) Find the time taken to reach 10 amperes for the first time, and

(iv) Find its average value.

Solution: Maximum value of current, $I_{max} = 15$ A

Frequency of alternating current, $f = 60$ Hz

(i) The equation for instantaneous value of sinusoidal alternating current with θ as zero when time is zero, is given as

$$I = I_{max} \sin 2\pi f t = 15 \sin 2\pi \times 60\, t = 15 \sin 120\pi t \textbf{ Ans.}$$

(ii) Substituting $t = \frac{1}{200}$ second in the expression $I = 15 \sin 120\pi t$ we have value of current after $\frac{1}{200}$ second

$$= 15 \sin 120\,\pi \times \frac{1}{200} = 15 \sin 0.6\,\pi = 15 \times 0.951 = 14.266 \text{ A } \textbf{Ans.}$$

(iii) Let the instantaneous value of current be 10 A for the first time t seconds after the instant the current is zero and becoming positive

$$\therefore \quad 10 = 15 \sin 120\,\pi\, t$$

$$\text{or } t = \frac{1}{120\pi}\sin^{-1}\frac{10}{15} = \frac{1}{120\pi}\sin^{-1} 0.6667 = 0.001936 \text{ second } \textbf{Ans.}$$

(iv) Average value, $I_{av} = \frac{2}{\pi} I_{max} = \frac{2}{\pi} \times 15 = 9.55$ A **Ans.**

Example 3.6

An ac sinusoidal current has rms value of 40 A at 50 Hz frequency. Write expression of instantaneous current and obtain its value 0.002 sec after passing through maximum positive value.

[Pb Technical Univ. Basic Electrical and Electronics Engineering First Semester 2004-05]

Solution: Expression for instantaneous value of an ac sinusoidal current with θ as zero when time is zero is given as

$$i = I_{max} \sin \omega t = I_{max} \sin 2\pi f t = 40\sqrt{2} \sin 2\pi \times 50t = 56.57 \sin 314t \textbf{ Ans.}$$

When time is measured from the positive maximum value, the above equation is modified into

$$v = 40\sqrt{2} \sin\left(314t + \frac{\pi}{2}\right) = 40\sqrt{2} \cos 314t = 40\sqrt{2} \cos 314 \times 0.002$$

$$= 40\sqrt{2} \cos 0.628$$

$$= 40\sqrt{2} \cos \frac{\pi}{5} = 45.77 \text{ A } \textbf{Ans.}$$

Example 3.7

An alternating current when passed through a resistance immersed in water for 5 minutes, just raised the temperature of water to boiling point. When a direct current of 4 amperes was passed through the same resistance under identical conditions it took 8 minutes to boil the water. Find the rms value of the alternating current.

Solution : Let the rms value of alternating current passed through the resistance be I_{rms} amperes.

Heat produced when an alternating current of I_{rms} amperes is passed through a resistance R immersed in water for 5 minutes

$$= (I_{rms})^2 \times R \times 5 \times 60 = 300\ I_{rms}^2\ R \text{ joules} \quad ...(i)$$

Heat produced when a direct current of 4 A is passed through the same resistance R immersed in the water for 8 minutes

$$= (4)^2 \times R \times 8 \times 60 = 7{,}680 \text{ R joules} \quad ...(ii)$$

Since heat produced in both of the cases is same, equating Eqs. (*i*) and (*ii*), we get

$$300\ I_{rms}^2\ R = 7{,}680\ R$$

$$\text{or } I_{rms} = \sqrt{\frac{7{,}680}{300}} = \sqrt{25.6} = 5.06 \text{ A } \textbf{Ans.}$$

Example 3.8

An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 20 A. Write down the equation for the instantaneous value and find this value (*a*) 0.0025 second (*b*) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A ? *[Allahabad Univ. Electrical Science 1992]*

Solution : The current waveform is shown in Fig. 3.11.

Peak value of current,

$$I_{max} = \sqrt{2}\ I_{rms} = \sqrt{2} \times 20 = 28.28 \text{ A}$$

The equation of alternating current with respect to origin O is

$$i = I_{max} \sin 2\pi f t = 28.28 \sin 2\pi \times 50\, t$$
$$= 28.28 \sin 100\pi t$$

The above equation is valid when time is measured from the instant the current is zero and increasing in positive direction. Since the time is measured from the positive maximum value (point A in Fig. 3.11), the above equation is modified to

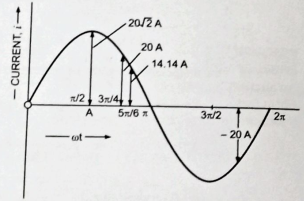


Fig. 3.11

$$i = 28.28 \sin\left(100\pi t + \frac{\pi}{2}\right) = 28.28 \cos 100\pi t \textbf{ Ans.}$$

(*a*) When $t = 0.0025$ s, the instantaneous value of current,

$$i_1 = 28.28 \cos 100\pi \times 0.0025 = 28.28 \cos\frac{\pi}{4} = 28.28 \times \frac{1}{\sqrt{2}} = 20 \text{ A } \textbf{Ans.}$$

(*b*) When $t = 0.0125$ s, the instantaneous value of current,

$$i_2 = 28.28 \cos 100\pi \times 0.0125 = 28.28 \cos\frac{5\pi}{4} = 28.28\left(\frac{-1}{\sqrt{2}}\right) = -20 \text{ A } \textbf{Ans.}$$

(*c*) Substituting $i = 14.14$ A in expression $i = 28.28 \cos 100\pi t$, we have

$$14.14 = 28.28 \cos 100\pi t$$

$$\text{or } t = \frac{1}{100\pi}\cos^{-1}\frac{14.14}{28.28} = \frac{1}{100\pi}\cos^{-1} 0.5 = \frac{1}{100\pi} \times \frac{\pi}{3} = \frac{1}{300} \text{ second } \textbf{Ans.}$$

3.9. FORM FACTOR AND PEAK FACTOR

3.9.1. Form Factor. In certain cases it is convenient to have calculations at first upon the mean value of the emf over half a period, therefore, it becomes essential to have some means of connecting this mean value with the effective or rms value. The knowledge of form factor, which is defined as the ratio of effective value to the average or mean value of periodic wave is, therefore, necessary.

Mathematically form factor is given by the relation

$$\text{Form factor} = \frac{\text{Effective value}}{\text{Average value}}$$

$$\text{Form factor for sinusoidal wave, } K_f = \frac{E_{rms}}{E_{av}} = \frac{\frac{E_{max}}{\sqrt{2}}}{\frac{E_{max}}{\pi/2}} = 1.11$$

3.9.2. Peak Factor. Knowledge of peak factor of an alternating voltage is very essential in connection with determining the dielectric strength since the dielectric stress developed in any insulating material is proportional to the maximum value of the voltage applied to it.

Peak or crest or amplitude factor of a periodic wave is defined as the ratio of maximum or peak to the effective or rms value of the wave

$$\textit{i.e.}, \text{ Peak factor, } K_p = \frac{\text{Maximum value}}{\text{Effective value}}$$

$$\text{Peak factor for sinusoidal wave, } K_p = \frac{E_{max}}{E_{rms}} = \frac{E_{max}}{E_{max}/\sqrt{2}} = 1.414$$

Example 3.9

An alternating voltage is $v = 100 \sin 100t$. Find (*i*) Amplitude (*ii*) Time period and frequency (*iii*) Angular velocity (*iv*) Form factor (*v*) Peak factor. *[U.P. Technical Univ. Electrical Engineering Second Semester 2007-08]*

Solution: Instantaneous value of alternating voltage is given by equation

$$v = 100 \sin 100\, t$$

(*i*) Amplitude of alternating voltage is given by the coefficient of the sine of the time angle, so

$$\text{Amplitude of given wave} = V_{max} = 100 \text{ V } \textbf{Ans.}$$

(*ii*) Frequency is given by coefficient of time, *t* divided by 2π

$$\therefore \text{Frequency}, f = \frac{100}{2\pi} = 15.9 \text{ Hz } \textbf{Ans.}$$

$$\text{Time period, T} = \frac{1}{f} = \frac{1}{15.9} = 0.063 \text{ second or 63 ms } \textbf{Ans.}$$

(*iii*) Angular velocity, $\omega = 2\pi f = 2\pi \times 15.9 = 100$ radians/second **Ans.**

$$\text{RMS value, } V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71 \text{ V}$$

$$\text{Average value } V_{av} = 0.637\, V_{max} = 0.637 \times 100 = 63.7 \text{ V}$$

$$(iv) \text{ Form factor, } K_f = \frac{V_{rms}}{V_{av}} = \frac{70.71}{63.7} = 1.11 \textbf{ Ans.}$$

$$(v) \text{ Peak factor, } K_p = \frac{V_{max}}{V_{rms}} = \frac{100}{70.71} = 1.4142 \textbf{ Ans.}$$

Example 3.10

Calculate the rms value, the form factor and peak factor of a periodic voltage having the following values for equal time intervals, changing suddenly from one value to the next 0, 10, 20, 30, 40, 100, 120, 100, 40, 30, 20, 10, 0, −10, −20, −40 etc. *[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]*

Solution: The waveform of the given alternating voltage is shown below in Fig. 3.12. Since wave is symmetrical, considering one half cycle only.

$$\text{Average value, } V_{av} = \frac{v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 + v_9 + v_{10} + v_{11} + v_{12}}{12}$$

$$= \frac{0 + 10 + 20 + 30 + 40 + 100 + 120 + 100 + 40 + 30 + 20 + 10}{12} = \frac{520}{12} = 43.33 \text{ V}$$

$$\text{RMS value, } V_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2 + v_7^2 + v_8^2 + v_9^2 + v_{10}^2 + v_{11}^2 + v_{12}^2}{12}}$$

$$= \sqrt{\frac{0^2 + 10^2 + 20^2 + 30^2 + 40^2 + 100^2 + 120^2 + 100^2 + 40^2 + 30^2 + 20^2 + 10^2}{12}} = 58 \text{ V } \textbf{Ans.}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{58}{43.33} = 1.3386 \textbf{ Ans.}$$

$$\text{Peak factor} = \frac{V_{max}}{V_{rms}} = \frac{120}{58} = 2.07 \textbf{ Ans.}$$

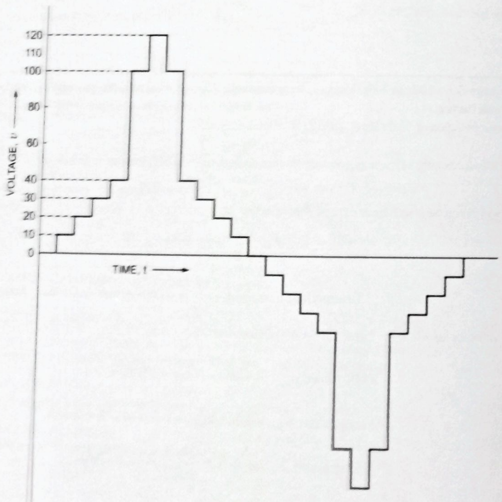


Fig. 3.12

3.10. RMS VALUE, AVERAGE VALUE, PEAK FACTOR AND FORM FACTOR OF HALF WAVE RECTIFIED ALTERNATING CURRENT

Half wave rectified alternating current is one whose one-half cycle has been suppressed *i.e.*, one which flows for half the time during one cycle. It is illustrated in Fig. 3.13 where suppressed half cycle is shown dotted.

As mentioned earlier, for determining rms and average values of such an alternating current summation would be carried over the period for which current actually flows *i.e.*, 0 to π but would be averaged for the whole cycle *i.e.*, from 0 to 2π.

∴ RMS value of current,

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_{max}^2 \sin^2\theta\, d\theta}$$

$$= \frac{I_{max}}{\sqrt{2\pi}}\sqrt{\int_0^{\pi}\sin^2\theta\, d\theta}$$

$$= \frac{I_{max}}{\sqrt{4\pi}}\sqrt{\int_0^{\pi}(1-\cos 2\theta)\, d\theta} = \frac{I_{max}}{\sqrt{4\pi}}\sqrt{\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}} = \frac{I_{max}}{\sqrt{4\pi}} \times \sqrt{\pi} = \frac{I_{max}}{2}$$

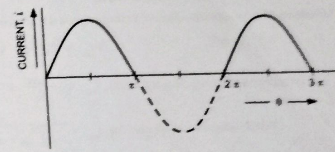


Fig. 3.13

Average value of current,

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} i\, d\theta = \frac{1}{2\pi}\int_0^{\pi} I_{max}\sin\theta\, d\theta = \frac{I_{max}}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_{max}}{\pi}$$

$$\text{Peak Factor} = \frac{I_{max}}{I_{rms}} = \frac{I_{max}}{I_{max}/2} = 2$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{I_{max}/2}{I_{max}/\pi} = \frac{\pi}{2} = 1.57$$

Example 3.11

An alternating voltage $e = 200 \sin 314\,t$ is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction, while preventing the flow of current in opposite direction. Calculate rms value, average value and form factor for the current over one cycle. *[Nagpur Univ. Elec. Engineering 1992]*

Solution: The instantaneous voltage applied to the rectifying device is given by the equation

$$e = 200 \sin 314\,t$$

Maximum value of applied voltage, E_{max} = Coefficient of the sine of time angle = 200 volts

Resistance of rectifying device, R = 20 Ω

Maximum value of half-wave rectified alternating current,

$$I_{max} = \frac{E_{max}}{R} = \frac{200}{20} = 10 \text{ amperes}$$

RMS value of half-wave rectified alternating current,

$$I_{rms} = \frac{I_{max}}{2} = \frac{10}{2} = 5 \text{ amperes } \textbf{Ans.}$$

Average value of the half-wave rectified alternating current,

$$I_{av} = \frac{I_{max}}{\pi} = \frac{10}{\pi} = 3.18 \text{ amperes } \textbf{Ans.}$$

Form factor of the half-wave rectified alternating current

$$= \frac{I_{rms}}{I_{av}} = \frac{5}{3.18} = 1.57 \textbf{ Ans.}$$

Example 3.12

A voltage $v(t) = 220\sqrt{2}$ sin 100t is applied to the circuit shown. What is the rms value of current through the resistor R of 100 Ω ? Derive the formula used.

[Pb. Technical Univ. Electrical Engineering May 2002]

Solution: The maximum value of voltage applied to the circuit

$$V_{max} = 220\sqrt{2} \text{ V}$$

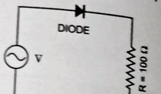


Fig. 3.14

Maximum value of circuit current, $I_{max} = \frac{V_{max}}{R} = \frac{220\sqrt{2}}{100} = 3.11$ A

RMS value of circuit current, $I_{rms} = \frac{I_{max}}{2} = \frac{3.11}{2} = 1.55$ A **Ans.**

Example 3.13

Define average value of alternating current. Find this value for the waveform shown in Fig. 3.15.

[Pb Technical Univ. Basic Electrical Engineering December 2002]

Solution: Maximum value of alternating current, $I_{max} = 10$ A

Average value, $I_{av} = \frac{I_{max}}{\pi} = \frac{10}{\pi} = 3.183$ A **Ans.**

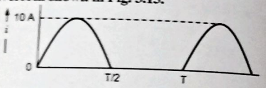


Fig. 3.15

Example 3.14

Find the average and effective values of voltage for sinusoidal waveform shown in Fig. 3.16.

[Allahabad Univ. Electrical Science-I, 1991]

Solution: The equation of the given sinusoidal waveform is $v = 100 \sin \theta$

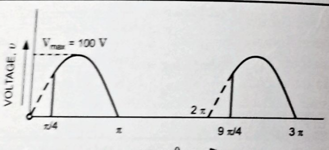


Fig. 3.16

$$V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100 \sin \theta \, d\theta$$

$$= \frac{100}{2\pi}\left[-\cos\theta\right]_{\pi/4}^{\pi} = 27.17 \text{ V } \textbf{Ans.}$$

$$V_{rms}^2 = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100^2 \sin^2\theta \, d\theta = \frac{10{,}000}{4\pi}\int_{\pi/4}^{\pi}(1-\cos 2\theta)\,d\theta = \frac{2{,}500}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi} = \frac{2{,}500}{\pi}\left[\pi - \frac{\pi}{4} + \frac{1}{2}\right] = 2{,}273$$

or $V_{rms} = \sqrt{2{,}273} = 47.67$ V **Ans.**

3.11. RMS VALUE, AVERAGE VALUE, FORM FACTOR AND PEAK FACTOR OF A TRIANGULAR WAVEFORM

Let the maximum value of the current be I_{max} amperes.

Since $i = I_{max}$ when $\theta = \pi$. Hence, expression for the instantaneous current can be written as

$$i = \frac{I_{max}}{\pi}\theta \quad \text{for } 0 < \theta < \pi$$

RMS value of the current wave,

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\frac{I_{max}^2}{\pi^2}\theta^2 \, d\theta} = \frac{I_{max}}{\sqrt{3}}$$

Average value of the current wave,

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i \, d\theta = \frac{1}{\pi}\int_0^{\pi}\frac{I_{max}}{\pi}\theta \, d\theta = \frac{I_{max}}{2}.$$

Form factor, $K_f = \frac{I_{rms}}{I_{av}} = \frac{I_{max}/\sqrt{3}}{I_{max}/2} = \frac{2}{\sqrt{3}} = 1.155$

Peak factor, $K_p = \frac{I_{max}}{I_{rms}} = \frac{I_{max}}{I_{max}/\sqrt{3}} = \sqrt{3}$

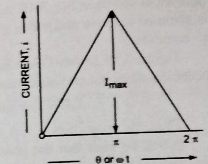


Fig. 3.17

Example 3.15

Find average and rms values of the waveform $v(t)$ shown in Fig. 3.18.

[G.G.S.I.P. Univ. Delhi 1st Term Exam. February 2008]

Solution: The given wave is triangular wave of peak value of 10 V.

Hence average value, $V_{av} = \frac{V_{max}}{2} = \frac{10}{2} = 5$ V **Ans.**

RMS value, $V_{rms} = \frac{V_{max}}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.7735$ V **Ans.**

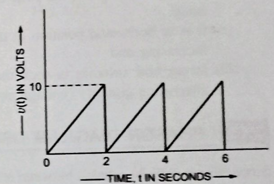


Fig. 3.18

3.12. GRAPHICAL OR PHASOR REPRESENTATION OF ALTERNATING (SINUSOIDAL) QUANTITIES (VOLTAGES AND CURRENTS)

It has already been pointed out that it is assumed that alternating voltages and currents follow sine law and generators are designed to give emfs having sine waveform. The above said assumption makes the calculations simple. The method of representing alternating quantities by waveform or by the equations giving instantaneous values is quite cumbersome. For solution of ac problems it is advantageous to represent a sinusoidal quantity (voltage or current) by a line of definite length rotating in counter-clockwise* direction with the same angular velocity as that of the sinusoidal quantity. Such a rotating line is called the *phasor*.**

Consider a line OA (or phasor as it is called) representing to scale the maximum value of an alternating quantity, say emf *i.e.*, OA = E_{max} and rotating in counter-clockwise direction at an angular velocity ω radians/second about the

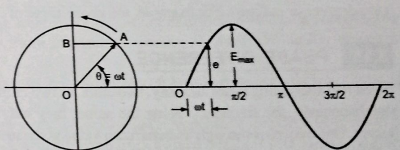


Fig. 3.19 *Phasor Representation of An Alternating Quantity*

* It is a standard convention that the phasor is rotated in counter-clockwise direction—a convention that is in harmony with the general use of polar coordinates.

** Strictly speaking, sinusoidal quantities are scalar quantities varying periodically with time and, according to the definition of vector quantities, they are not true vectors. Voltage is simply energy or work per coulomb and cannot be classified as a vector. Current is also not a vector quantity because it is merely the flow of electrons through a wire. But keeping in mind that any sinusoidal quantity at a given frequency is completely specified by its amplitude and phase angle, its similarity to a vector quantity is evident, since the amplitude may be considered as the magnitude and the phase angle as the direction of a vector. To account for the difference the term phasor has been adopted, instead of the term vector, for representing graphically the magnitude and phase of a sinusoidal current or voltage.

point O, as shown in Fig. 3.19. An arrow head is put at the outer end of the phasor, partly to indicate which end is assumed to move and partly to indicate the precise length of the phasor when two or more phasors happen to coincide.

Figure 3.19 shows OA when it has rotated through an angle θ, being equal to ωt, from the position occupied when the emf was passing through its zero value. The projection of OA on Y-axis,

$$OB = OA \sin\theta = E_{max} \sin \omega t = e, \text{ the value of the emf at that instant}$$

Thus, the projection of OA on the vertical axis represents to scale the instantaneous value of emf.

It will be seen that the phasor OA rotating in counter-clockwise direction will represent a sinusoidal quantity (voltage or current) if

(i) its length is equal to the peak or maximum value of the sinusoidal voltage or current to a suitable scale.

(ii) it is in horizontal position at the instant the alternating quantity (voltage or current) is zero and increasing and

(iii) its angular velocity is such that it completes one revolution in the same time as taken by the alternating quantity (voltage or current) to complete one cycle.

3.13. PHASOR DIAGRAM USING RMS VALUES

Since there is definite relation between maximum value and rms value ($E_{max} = \sqrt{2}\ E_{rms}$), the length of phasor OA can be taken equal to rms value if desired. But it should be noted that in such cases, the projection of the rotating phasor on the vertical axis will not give the instantaneous value of that alternating quantity.

The phasor diagram drawn in rms values of the alternating quantities helps in understanding the behaviour of the ac machines under different loading conditions.

3.14. PHASE AND PHASE ANGLE

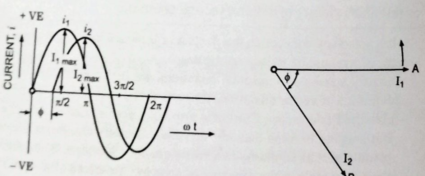
By phase of an alternating current is meant the fraction of the time period of that alternating current that has elapsed since the current last passed through the zero position of reference. The phase angle of any quantity means the angle the phasor representing the quantity makes with the reference line (which is taken to be at zero degrees or radians). For example, the phase angle of current I_2 in Fig. 3.20 is (– Φ).

3.15. PHASE DIFFERENCE

When two alternating quantities, say, two emfs, or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant. One may pass through its maximum value at the instant when the other passes through the value other than its maximum one. These two quantities are said to have a *phase difference*. Phase difference is always given either in degrees or in radians.

The phase difference is measured by the angular distance between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity while the second quantity is said to lag behind the first one. In Fig. 3.20 (*b*) current I_1 represented by phasor OA leads the current I_2 represented by phasor OB by Φ or current I_2 lags behind the current I_1 by Φ. The leading current I_1 goes through its zero and maximum values first and the current I_2 goes through its zero and maximum values after time angle Φ. The two waves representing these two currents are shown in Fig. 3.20 (*a*). If I_1 is taken as reference phasor, the two currents can be expressed as



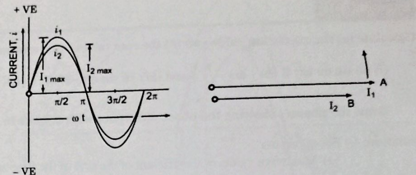
(*a*) (*b*)

Fig. 3.20

$$i_1 = I_{1\,max} \sin \omega t$$

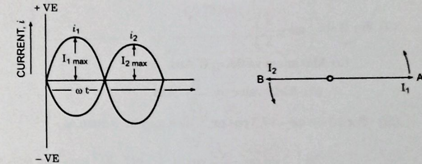
$$\text{and } i_2 = I_2 \sin(\omega t - \Phi)$$

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig. 3.21. But the two quantities passing through zero values at the same instant but rising in opposite directions, as shown in Fig. 3.22 are said to be in phase opposition, *i.e.*, phase difference is 180°. When the two alternating quantities have a phase difference of 90° or $\frac{\pi}{2}$ radians they are said to be in quadrature.



(*a*) (*b*)

Fig. 3.21



(*a*) (*b*)

Fig. 3.22

3.16. CONVENTIONS FOR DRAWING PHASOR DIAGRAMS

As already mentioned, the alternating quantities (voltages and currents) in practice are represented by straight lines having definite direction and length. Such lines are called the *phasors* and the diagrams in which phasors represent currents, voltages and their phase difference are known as *phasor diagrams*.

Though phasor diagrams can be drawn to represent either maximum or effective values of voltages and currents but since effective values are of much more importance, phasor diagrams are mostly drawn to represent effective values.

In order to achieve consistent and accurate results it is essential to follow certain conventions. Some of the common conventions in this regard are enlisted below:

1. Counter-clockwise direction of rotation of phasors is usually taken as positive direction of rotation of phasors *i.e.*, a phasor rotated in a counter-clockwise direction from a given phasor is said to lead the given phasor while a phasor rotated in clockwise direction is said to lag the given phasor.
2. For series circuits, in which the current is common to all parts of the circuit, the current phasor is usually taken as reference phasor for other phasors in the same diagram and drawn on horizontal line.
3. In parallel circuits in which the voltage is common to all branches of the circuit, the voltage phasor is usually taken as reference phasor and drawn on the horizontal line. Other phasors are referred to the common voltage phasor.
4. It is not necessary that current and voltage phasors are drawn to the same scale; in fact it is often desirable to draw the current phasor to a larger scale than the voltage phasor when the values of currents being represented are small. However, if several voltage phasors are to be used in the same phasor diagram, they should all be drawn to the same scale. Likewise all current phasors in the same diagram should be drawn to the same scale.

Example 3.16

Calculate (a) the maximum value and (b) the root mean square values of the following quantities:

(i) 40 sin ωt (ii) B sin $\left(\omega t - \frac{\pi}{2}\right)$ and (iii) 10 sin ωt −17.3 cos ωt.

Draw the phasors showing the phase difference with respect to A sin $\left(\omega t - \frac{\pi}{6}\right)$.

Solution: (i) For 40 sin ω t

(a) Maximum value = Coefficient of the sine of the time angle = 40 **Ans.**

(b) RMS value = $\frac{\text{Maximum value}}{\sqrt{2}} = \frac{40}{\sqrt{2}}$ = 28.28 **Ans.**

(ii) For B sin $\left(\omega t - \frac{\pi}{2}\right)$

(a) Maximum value = B **Ans.**

(b) RMS value = $\frac{B}{\sqrt{2}}$ **Ans.**

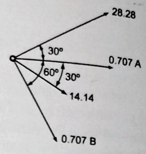


Fig. 3.23

(iii) For 10 sin ωt − 17.3 cos ωt, which may be written as

$$20\left(\frac{1}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos\omega t\right) \quad \text{or} \quad 20\left(\cos\frac{\pi}{3}\sin\omega t - \sin\frac{\pi}{3}\cos\omega t\right) \quad \text{or} \quad 20\sin\left(\omega t - \frac{\pi}{3}\right)$$

(a) Maximum value = 20

(b) RMS value = $\frac{20}{\sqrt{2}}$ = 14.14 **Ans.**

Phasors showing the phase difference with respect to A sin $\left(\omega t - \frac{\pi}{6}\right)$ are shown in Fig. 3.23.

3.17. ADDITION AND SUBTRACTION OF ALTERNATING QUANTITIES

In most of the ac circuits, it is necessary to consider the combined action of several emfs or voltages acting in a series circuit and action of several currents flowing through the different branches of a parallel circuit.

Let it be required to add two currents given by the equations

$$i_1 = I_{1\,max}\sin\omega t \text{ and } i_2 = I_{2\,max}\sin(\omega t - \Phi)$$

The resultant sum may be expressed as

$$i_r = i_1 + i_2 = I_{1\,max}\sin\omega t + I_{2\,max}\sin(\omega t - \Phi)$$

but it is too awkward and gives no idea of the peak value and phase angle of the resultant current.

The currents may be added graphically by plotting their curves in the same system of coordinates and then adding the ordinates of i_1 and i_2 point by point, according to the equation $i_r = i_1 + i_2$. Evidently this method is also too cumbersome and unwieldy to be practical. This is particularly so when situation arise where more than two sinusoidal quantities are to be added.

A simpler and more direct method consists in adding the sinusoidal quantities as phasors.

Consider the phasors $I_{1\,max}$ and $I_{2\,max}$ that would generate the two curves i_1 and i_2 and let them be in a position, as shown in Fig. 3.24 at one particular instant of time. If we now add $I_{1\,max}$ and $I_{2\,max}$ by completing the parallelogram as shown, the diagonal $I_{r\,max}$ will, when rotated, generate a third sine curve. It remains to be shown that this third sine curve coincides with the waveform of i_r obtained by adding i_1 and i_2 point by point.

Now the vertical component of $I_{r\,max}$ is the sum of the vertical components of $I_{1\,max}$ and $I_{2\,max}$. Therefore, the waveform of i_r is the graph generated by rotating $I_{r\,max}$ in counter-clockwise direction.

It follows, therefore, that two or more alternating quantities may be added in the same way as forces are added, namely by constructing parallelograms or closed polygons and either measuring or calculating the lengths of the diagonals or closing sides and the magnitude of the phase angles.

Illustrations: The way in which the two given currents i_1 and i_2 can be added by the parallelogram rule of phasor addition is illustrated in Fig. 3.25 where the currents are shown as phasor drawn from the origin O of the system of coordinates. The resultant phasor is the diagonal of the parallelogram formed by the phasors $I_{1\,max}$ and $I_{2\,max}$.

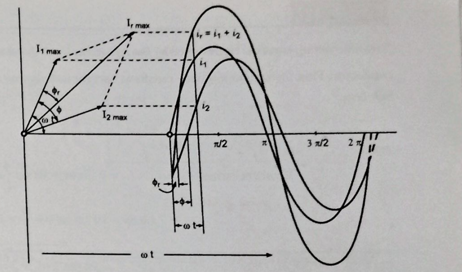


Fig. 3.24

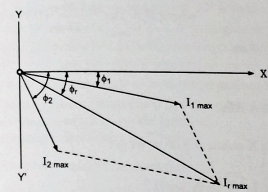


Fig. 3.25

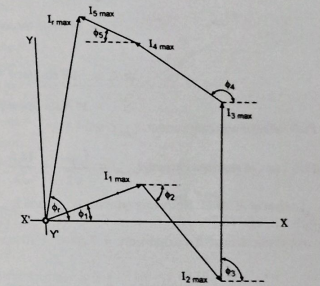


Fig. 3.26

This method is more convenient when more than two phasors are to be added, as shown in Fig. 3.26. From the end point of $I_{1\,max}$ a phasor is constructed parallel to $I_{2\,max}$ of the same magnitude and direction as the latter; then from the end point of $I_{2\,max}$ a phasor is constructed parallel to $I_{3\,max}$ and so on. Phasor $I_{r\,max}$ from the origin of the first phasor ($I_{1\,max}$) to the end point of the last phasor ($I_{5\,max}$) represents the sum of all the phasors.

Phasors may also be subtracted by the above method. For example, if phasors $I_{2\,max}$ and $I_{3\,max}$ are to be subtracted from phasor $I_{1\,max}$ each of the two phasors be reversed in direction and then added as explained above [Fig. 3.27]. This time, the phasor drawn from the origin of the first phasor $I_{1\,max}$ to the terminal point of the last phasor $I_{3\,max}$ gives the difference of phasors $I_{2\,max}$ and $I_{3\,max}$ from phasor $I_{1\,max}$.

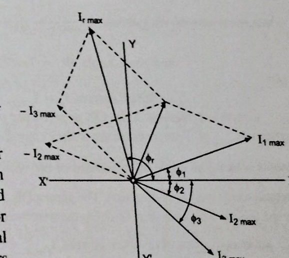


Fig. 3.27

Example 3.17

Two alternating currents represented by the equations $i_1 = 7 \sin \omega t$ and $i_2 = 10 \sin\left(\omega t + \frac{\pi}{3}\right)$ are fed into a common conductor. Find the equation for the resultant current and its rms value.

Solution:

$$i_1 = 7 \sin \omega t$$

$$i_2 = 10 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\text{Resultant current, } i_r = i_1 + i_2 = 7 \sin \omega t + 10 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= 7 \sin \omega t + 10 \sin \omega t \cos\frac{\pi}{3} + 10 \cos \omega t \sin\frac{\pi}{3} = 12 \sin \omega t + 8.66 \cos \omega t$$

Multiplying and dividing RHS of the above expression by $\sqrt{(12)^2 + (8.66)^2}$ *i.e.* 14.8, we get

$$i_r = 14.8\left(\sin \omega t \times \frac{12}{14.8} + \cos \omega t \times \frac{8.66}{14.8}\right)$$

$$= 14.8 (\sin \omega t \cos \Phi_r + \cos \omega t \sin \Phi_r) \quad \text{where } \cos \Phi_r = \frac{12}{14.8} \text{ and } \sin \Phi_r = \frac{8.66}{14.8}$$

$$\text{or } i_r = 14.8 \sin(\omega t + \Phi_r) \qquad \text{where } \Phi_r = \text{Tan}^{-1}\frac{8.66}{12} = 0.199\,\pi \text{ radians}$$

or $i_r = 14.8 \sin(\omega t + 0.199\,\pi)$ **Ans.**

Peak value of resultant current, $I_{r\,max} = 14.8$ A

RMS value of resultant current, $I_{r\,rms} = \frac{I_{r\,max}}{\sqrt{2}} = \frac{14.8}{\sqrt{2}} = 10.46$ A **Ans.**

Analytical Method: Resolving currents $I_{1\,max}$ and $I_{2\,max}$ along X-axis and Y-axis, we get

Algebraic sum of X-components $= 7 \cos 0° + 10 \cos\frac{\pi}{3} = 7 + 5 = 12$ A

Algebraic sum of Y-components $= 7 \sin 0° + 10 \sin\frac{\pi}{3} = 8.66$ A

Maximum value of resultant current, $I_{r\,max} = \sqrt{(\text{X-components})^2 + (\text{Y-components})^2} = \sqrt{12^2 + (8.66)^2} = 14.8$ A

and phase angle, $\Phi_r = \text{Tan}^{-1}\frac{\text{Y-components}}{\text{X-components}} = \text{Tan}^{-1}\frac{8.66}{12} = 0.199\,\pi$

$\therefore$ Resultant current, $i_r = I_{r\,max} \sin(\omega t + \Phi_r) = 14.8 \sin(\omega t + 0.199\,\pi)$ **Ans.**

Graphical Method. Take OX as reference phasor. Draw phasor OA 3.5 cm in length along OX so as to represent current $I_{1\,max}$ of 7 amperes (to the scale 1 cm = 2 A) in magnitude as well as in direction. Then draw phasor AB = 5 cm in length making an angle of $\frac{\pi}{3}$ radians or 60° with the reference phasor so as to represent current $I_{2\,max}$ of 10 amperes in magnitude as well as in direction. The phasor OB, obtained by joining points O and B, will represent the resultant current in magnitude as well as in direction. On measurement from Fig. 3.28(*b*).

Maximum value of resultant current, $I_{r\,max} = \text{OB} \times 2 = 7.4 \times 2 = 14.8$ A

and phase angle, $\Phi_r = 35.8°$ or $0.199\,\pi$ radians

Hence the resultant current may be expressed as

$$i_r = 14.8 \sin(\omega t + 0.199\,\pi) \textbf{ Ans.}$$

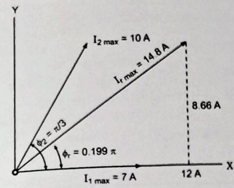


Fig. 3.28 (*a*)

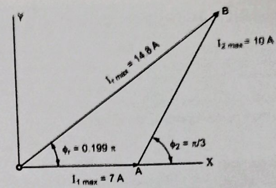


Fig. 3.28 (*b*)

Example 3.18

Three sinusoidal currents flow into a junction $i_1 = 3\sqrt{2} \sin \omega t$, $i_2 = 5\sqrt{2} \sin(\omega t + 30°)$ and $i_3 = 6\sqrt{2} \sin(\omega t - 120°)$. Find the time expression for the resultant sinusoidal current which leaves the junction (use phasor diagram).

[G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009]

Solution: Take OX as reference phasor. Draw phasor OA 3 cm in length along OX so as to represent $I_{1\,max}$ of $3\sqrt{2}$ A (to the scale of 1 cm = $\sqrt{2}$ A) in magnitude as well as in direction. Then draw line AB = 5 cm in length making an angle of 30° with the reference phasor so as to represent $I_{2\,max} = 5\sqrt{2}$ A in magnitude as well as in direction. Then draw line BC = 6 cm in length making an angle of −120° with the reference phasor so as to represent $I_{3\,max} = 6\sqrt{2}$ A in magnitude as well as in direction. The phasor CO obtained by joining points C and O, will represent the resultant sinusoidal current which leaves the junction.

On measurement from Fig. 3.29.

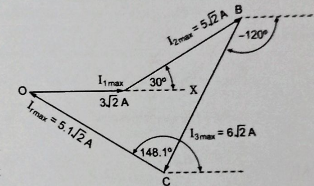


Fig. 3.29

Maximum value of resultant current, $I_{r\,max} = \text{OC} \times \sqrt{2} = 5.1\sqrt{2}$ A

Phase angle $\Phi_r = 148.1°$

Hence resultant current leaving the junction is given by expression $i_r = 5.1\sqrt{2} \sin(\omega t + 148.1°)$.

Example 3.19

Two ac voltages are represented by

$$v_1(t) = 30 \sin(314\,t + 45°)$$

$$v_2(t) = 60 \sin(314\,t + 60°)$$

Calculate the resultant voltage $v(t)$ and express in the form —

$$v(t) = V_m \sin(314\,t + \phi)$$

Solution:

$$v_1(t) = 30 \sin(314\,t + 45°)$$

$$v_2(t) = 60 \sin(314\,t + 60°)$$

Resolving voltages $V_{1\,max}$ and $V_{2\,max}$ along X-axis and Y-axis, we have

Algebraic sum of X-components $= 30 \cos 45° + 60 \cos 60° = 30 \times \frac{1}{\sqrt{2}} + 60 \times \frac{1}{2} = 51.21$ V

Algebraic sum of Y-component $= 30 \sin 45° + 60 \sin 60° = 30 \times \frac{1}{\sqrt{2}} + 60 \times \frac{\sqrt{3}}{2} = 73.17$ V

Maximum value of resultant voltage, $V_{r\,max} = \sqrt{(\text{X-components})^2 + (\text{Y-components})^2}$

$$= \sqrt{(51.21)^2 + (73.17)^2} = 89.31 \text{ V}$$

Phase angle, $\phi_r = \text{Tan}^{-1} \dfrac{\text{Y-component}}{\text{X-component}} = \text{Tan}^{-1} \dfrac{73.17}{51.21} = 55°$

So expression for resultant voltage is

$$v(t) = V_{max} \sin(\omega t + \phi_r) = 89.31 \sin(314\,t + 55°) \textbf{ Ans.}$$

Example 3.20

Draw a phasor diagram showing the following voltages:

$$v_1 = 100 \sin 500\,t \qquad v_2 = 200 \sin\left(500t + \frac{\pi}{3}\right)$$

$$v_3 = -50 \cos 500\,t \qquad v_4 = 150 \sin\left(500t - \frac{\pi}{4}\right)$$

Find rms value of resultant voltage.

[U.P. Technical Univ. Elec. Engineering Second Semester 2005-06]

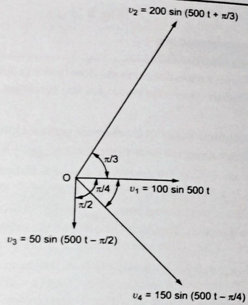


Fig. 3.30 *Phasor Diagram*

Solution : $v_1 = 100 \sin 500\,t$

$$v_2 = 200 \sin\left(500t + \frac{\pi}{3}\right)$$

$$v_3 = -50 \cos 500\,t$$

$$= -50 \sin\left(500t + \frac{\pi}{2}\right) = 50 \sin\left(500t - \frac{\pi}{2}\right)$$

$$v_4 = 150 \sin\left(500t - \frac{\pi}{4}\right)$$

The phasor diagram is shown in Fig. 3.30.

Resolving voltages V_1, V_2, V_3 and V_4 along X-axis and Y-axis, we have

$$\text{Algebraic sum of X-components} = 100 \cos 0° + 200 \cos\frac{\pi}{3} + 50 \cos\left(-\frac{\pi}{2}\right) + 150 \cos\left(-\frac{\pi}{4}\right)$$

$$= 100 + 200 \times 0.5 + 0 + \frac{150}{\sqrt{2}} = 306 \text{ V}$$

$$\text{Algebraic sum of Y-components} = 100 \sin 0° + 200 \sin\frac{\pi}{3} + 50 \sin\left(-\frac{\pi}{2}\right) + 150 \sin\left(-\frac{\pi}{4}\right)$$

$$= 0 + 200 \times 0.866 - 50 - 106 = 17.2 \text{ V}$$

Maximum value of resultant voltage, $V_{r\,max} = \sqrt{(306)^2 + (17.2)^2} = 306.5$ V

RMS value of resultant voltage, $V_{r\,rms} = \dfrac{V_{r\,max}}{\sqrt{2}} = \dfrac{306.5}{\sqrt{2}} = 216.72$ V **Ans.**

HIGHLIGHTS

1. An *alternating quantity* (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time.

   An alternating quantity that varies sinusoidally (*i.e.*, according to the sin of angle θ) is called the *sinusoidal quantity*. Sinusoidal quantities are expressed as

   $$e = E_{max} \sin\theta = E_{max} \sin\omega t = E_{max} \sin 2\pi f t$$

   $$\text{and } i = I_{max} \sin\theta = I_{max} \sin\omega t = I_{max} \sin 2\pi f t$$

2. The shape of the curve of the voltage or current when plotted against time as abscissa (base) is called the *waveform*.

3. When a periodic wave, such as sinusoidal wave, goes through one complete set of positive and negative values it is said to have completed one *cycle*. One cycle corresponds to 360° or 2 π radians.

   Alternation is one half of cycle and corresponds to 180° or π radians.

4. The time taken in seconds by an alternating quantity to complete one cycle is known as *time period* or *periodic time* (T), while the number of cycles completed per second by an alternating quantity is known as *frequency* (*f*).

   $$f = \frac{1}{T}$$

   In India the standard frequency for power supply is 50 Hz (cycles per second).

5. Angular velocity, $\omega = 2\pi f$ radians per second.

6. In a multipolar machine, the number of cycles completed per second by generated emf.

   $$f = \text{Pairs of poles} \times \text{number of revolutions made per second} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120}$$

7. The maximum value, positive or negative, which an alternating quantity attains during one cycle is called the *amplitude* of the alternating quantity.

8. The general expression for an alternating voltage and current are given as

   $$v = V_{max} \sin(\omega t \pm \phi)\,; \quad i = I_{max} \sin(\omega t \pm \phi)$$

   where φ is the phase angle in degrees or radians.

   The coefficient of the sine of the time angle gives maximum or peak value of the alternating quantity (emf, voltage or current)

   Coefficient of time *t* divided by 2 π gives the frequency of the periodic wave.

9. The *average* (*or mean*) *value* of an alternating current is expressed by that steady current (dc) which transfers across any circuit the same charge as transferred by that alternating current during a given time.

   Average or mean value of alternating current is given as

   $$I_{av} = \frac{i_1 + i_2 + i_3 + \ldots + i_n}{n} = \frac{\text{Area of one alternation}}{\text{Length of base over one alternation}}$$

   For sinusoidal current $I_{av} = \dfrac{2}{\pi} I_{max} = 0.637\, I_{max}$.

10. *Effective* or *virtual value* of alternating current or voltage is equal to the square root of the mean of the squares of successive ordinates and that is why it is known as *root mean square* (rms) value.

    $$i.e.,\ I_{eff} \text{ or } I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots + i_n^2}{n}}$$

    The *rms* or *effective value* of an alternating current or voltage is given by that steady current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage flowing or applied to the same resistance for the same time.

    For sinusoidal current, $I_{rms} = \dfrac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$

11. The ratio of rms value to the average value of a waveform is known as *form factor*. For sinusoidal wave its value is 1.11.
12. The ratio of maximum value to the rms value of a waveform is known as peak factor. For sinusoidal wave its value is $\sqrt{2}$ or 1.414.
13. The effective and average values of different waves are given below in tabular form.

Waveform	*Sinusoidal*	*Half Wave Rectified AC*	*Square*	*Triangular*
Effective Value	$0.707\ I_{max}$	$0.5\ I_{max}$	I_{max}	$\frac{1}{\sqrt{3}} I_{max}$
Average Value	$0.637\ I_{max}$	$0.318\ I_{max}$	I_{max}	$0.5\ I_{max}$

14. By *phase* of an alternating quantity is meant the fraction of time period of that alternating quantity that has elapsed since the quantity last passed through zero position of reference.

The angular displacement between the maximum positive values of two alternating quantities having the same frequency is called the phase difference between them.

An alternating quantity that attains its positive maximum value prior to the other is called the *leading quantity.*

An alternating quantity that attains its positive maximum value after the other is called the *lagging quantity.*

15. The resultant current of two currents repersented by

$$i_1 = I_{max_1} \sin \omega t \text{ and } i_2 = I_{max_2} \sin (\omega t + \theta)$$

flowing through a common conductor is given as

$$i_r = I_{r\,max} \sin (\omega t \pm \phi)$$

where
$$I_{r\,max} = \sqrt{I_{max_1}^2 + I_{max_2}^2 + 2 I_{max_1} I_{max_2} \cos \theta}$$

$$\text{and } \phi = \text{Tan}^{-1} \frac{I_{max_2} \sin \theta}{I_{max_1} + I_{max_2} \cos \theta}.$$

The sign of the phase angle ϕ of the resultant current will depend upon its position w.r.t. the reference phasor.

EXERCISES

1. Explain why a large percentage of electrical energy used for commercial purposes is generated as ac.
2. With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field. **[V.T.U. Belgaum Karnataka Univ., Summer 2003]**
3. Why sinusoidal waveshape is preferred in ac electric power ? **[G.G.S.I.P. Univ. 1st Term Exam. Feb./March 2011]**
4. Derive an expression for the instantaneous value of alternating sinusoidal emf in terms of its maximum value, angular frequency and time. **[Gujarat Univ. June/July 2003]**
5. Define and explain the following:
   (*i*) Time period (*ii*) Amplitude (*iii*) Phase difference.
   [Gujarat Univ. June/July 2003; G.G.S.I.P. Univ. 1st Term Exam. Feb./March 2011]
6. Define and explain the following:
   (*i*) Frequency (*ii*) Time period (*iii*) Instantaneous value (*iv*) Average value (*v*) RMS value with respect to alternating quantities (*vi*) Amplitude. **[R.G.P.V. Bhopal Basic Electrical Engineering 2002]**
7. Define the terms: Instantaneous value, Average value, RMS value, frequency and phase.
   [Pb. Technical Univ. Electrical Engineering June 2000; December 2000; May 2001]
8. What is an rms value of an alternating quantity ? Obtain expression for the rms value of a sinusoidal current in terms of its maximum value. **[V.T.U. Belgaum Univ. February 2002; G.G.S.I.P. Univ. 1st Term Exam. Feb./March 2011]**

9. Derive expressions for average value and rms value of a sinusoidally varying ac voltage.
   [V.T.U. Belgaum Univ. Summer 2003]
10. Define rms value, average value and form factor of a sinusoidal alternating voltage.
   [Pb. Technical Univ. Basic Electrical and Electronics Engineering First Semester 2004-05]
11. Explain the terms: rms value, average value and form factor w.r.t. alternating quantity. Deduce the value of form factor and ratio of peak to average value of a sinusoidal voltage. **[G.G.S.I.P. Univ. Delhi Electrical Science May 2011]**
12. Find average value, rms value and form factor of half wave rectified alternating current.
   [U.P. Technical Univ. Electrical Engineering Second Semester 2006-07]
13. Calculate the average value, effective value and form factor of the output voltage wave of a half wave rectifier as shown in Fig. 1. **[U.P. Technical Univ. Electrical Engineering Second Semester 2003-04]**

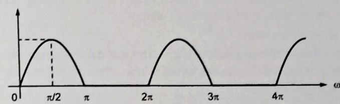


Fig. 1

14. Explain the need and method of representing alternating quantities as phasor quantity.
15. What do you understand by phasor representation of alternating quantity ? What is an alternating quantity ? When is an alternating quantity said to be lagging or leading with respect to another quantity ?
16. Explain with diagrams what do you understand by (*i*) in-phase (*ii*) lagging and (*iii*) leading as applied to sinusoidal ac quantities.
17. Explain the difference between ac and dc quantities. Draw the waveform and phasor diagram of two alternating quantities having a phase difference of 90°.
18. Explain the component method for the addition and subtraction of alternating quantities.

SHORT ANSWER TYPE QUESTIONS WITH ANSWERS

Q. 1. Why sinusoidal waveshape is insisted for voltages and currents while generating, transmitting and utilising ac electric power ? **[P.T.U. January 2000]**

Ans. Sinusoidal waveshape is insisted for voltages and currents while generating, transmitting and utilising ac electrical power because it has following advantages.

(*i*) Sinusoidal voltages and currents produce minimum disturbance in the electrical circuits during operation.

(*ii*) Sinusoidal voltages and currents cause less interference (noise) to nearby communication circuits (telephone lines etc.).

(*iii*) Sinusoidal voltages and currents result in low iron as well as low copper losses in transformers, and ac rotating machines for a given output. So ac machines with sinusoidal voltages and currents operate at higher efficiency.

Q. 2. What do you understand by ω ? **[G.G.S.I.P. Univ. Delhi 1st Term Exam. Feb./March 2011]**

Ans. Each cycle of a sinusoidal wave spans 2π radians. Hence, if this quantity is divided by the time period, angular velocity of the sinusoidal wave is obtained. It is denoted by ω and is expressed in radians per second.

Q. 3. What is difference between time period and frequency of a periodic wave. How are they related ?

Ans. The time taken in seconds by an alternating quantity to complete one cycle is known as *time period* or *periodic time* (T), while the number of cycles completed per second by an alternating quantity is known as *frequency* (*f*).

$$f = \frac{1}{T}$$

In India the standard frequency for power supply is 50 Hz (cycles per second).

Q. 4. What is the average value of an alternating current ? **[G.G.S.I.P. Univ. Delhi 1st Term Exam. Feb./March 2011]**

Ans. The *average (or mean) value* of an alternating current is expressed by that steady current (dc) which transfers across any circuit the same charge as transferred by that alternating current during a given time.

Average or mean value of alternating current is given as

$$I_{av} = \frac{i_1 + i_2 + i_3 + ... + i_n}{n} = \frac{\text{Area of one alternation}}{\text{Length of base over one alternation}}$$

For sinusoidal current $I_{av} = \frac{2}{\pi} I_{max} = 0.637 I_{max}$.

Q. 5. Why the rms value of an alternating current or voltage is used to denote its amplitude ?

Ans. RMS value of an alternating current or voltage is used to denote its amplitude because it is related to the power developed in a resistance by the alternating current or voltage.

Q. 6. What is rms value of an alternating current ? **[P.T.U. December 2003]**

Ans. The effective or rms value of an alternating current is given by that steady current which when flows through a given resistance for a given time produces the same amount of heat as when the alternating current is flowing through the same resistance for the same time duration.

Q. 7. Do waveshapes other than sine wave have effective value ? Explain. **[P.T.U. December 2000]**

Ans. Yes, all the waveshapes including the sine wave have effective value since in each half cycle of the wave, work is being done. Actually the effective value of an alternating wave (sinusoidal or non-sinusoidal) is defined as below:

The rms or effective value of an alternating current or voltage is given by that steady current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

Q. 8. Differentiate between form factor and peak factor. **[P.T.U. June 2003]**

Ans. Form factor is defined as the ratio of the effective value to the average or mean value of the periodic wave while peak factor is defined as the ratio of peak or maximum value to the effective or rms value of the periodic wave.

i.e., $K_f = \frac{\text{RMS value}}{\text{Average value}}$ while $K_p = \frac{\text{Peak value}}{\text{RMS value}}$

Q. 9. What is significance of form factor ?

Ans. Form factor is a means of relating the mean value with the effective or rms value of alternating quantity and it is useful in determination of effective or rms values of the alternating quantities whose mean or average values over half a period can be determined conveniently.

Q. 10. What is the significance of peak factor ?

Ans. Knowledge of peak factor of an alternating voltage is very essential in connection with determining the dielectric strength since the dielectric stress developed in an insulating material is proportional to the peak value of the voltage applied to it.

Q. 11. What is the significance of the phasor representation of an alternating quantity ?

Ans. The phasor representation of an alternating quantity enables us to understand its magnitude and position with respect to reference line.

Q. 12. What do you mean by phase and phase difference ?

Ans. The phase of an alternating quantity (voltage or current) at any instant is defined as the fractional part of a cycle through which the quantity has advanced while the phase difference may be defined as the angular displacement between the maximum positive values of the two phasors representing the two quantities having the same frequency.

PROBLEMS

1. For ac single phase 230 V, 50 Hz, assuming sinusoidal waveform find (*i*) maximum value (*ii*) rms value (*iii*) frequency (*iv*) instantaneous value voltage equation (*v*) voltage 0.003 second after starting from zero and rising in + ve direction. **[Ans.** (*i*) 325.3 V (*ii*) 230 V (*iii*) 50 Hz (*iv*) 325.3 sin 314 *t* (*v*) 263.17 V**]**

2. $e = 300 \sin 628\, t$
Find maximum value, rms value, average value and frequency. Find the instantaneous value of voltage at $t = 0.001$ second after e is zero and rising in positive direction. **[Ans.** 300 V, 212.13 V, 190.98 V, 100 Hz, 176.26 V**]**

3. An alternating current is represented by $i = 400 \sin\left(157t + \frac{\pi}{6}\right)$.
Determine: (*i*) Peak value (*ii*) Average value (*iii*) RMS value (*iv*) Frequency (*v*) Time period (*vi*) Form factor (*vii*) Peak factor, and (*viii*) Phase angle of the current represented.
[Ans. (*i*) 400 A (*ii*) 254.65 A (*iii*) 282.84 A, (*iv*) 25 Hz (*v*) 0.04 second (*vi*) 1.11 (*vii*) 1.414 (*viii*) 30°**]**

4. An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate (i) its value $\frac{1}{600}$ second after the instant the current is zero (ii) in how many seconds after the zero value the current attains the value of 86.6 A ? **[Kanpur Univ. 1977]**
[Ans. (*i*) 50 A (*ii*) $\frac{1}{300}$ second**]**

5. The ac supply at a house is 230 V, 50 Hz. Find maximum, average and rms values of voltage. **[Ans.** 325.27 V, 207.2 V, 230 V**]**

6. The equation of an alternating current $i = 42.42 \sin 628\, t$. Determine (*i*) its maximum value (*ii*) frequency (*iii*) rms value (*iv*) average value (*v*) form factor. **[Pb. Univ. Elements of Electrical Engineering December 1988]**
[Ans. (*i*) 42.42 A (*ii*) 100 Hz (*iii*) 30 A (*iv*) 27 A (*v*) 1.11**]**

7. Calculate the rms value, the form factor and the peak factor of a periodic voltage having the following values for equal time intervals, changing suddenly from one value to the next:
0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0 –5, –10 V etc.
What would be the rms value of the sine wave having the same peak value? **[Ans.** 31.064 V; 1.35; 1.9315; 42.43 V**]**

8. The currents flowing in the three branches of an ac parallel circuit are :
$i_1 = 20 \sin \omega t$; $i_2 = 25 \sin\left(\omega t - \frac{\pi}{6}\right)$; $i_3 = 15 \sin\left(\omega t + \frac{\pi}{6}\right)$. Determine the total current drawn from the supply.
[Ans. 54.87 sin ($\omega t - 0.029\pi$)**]**

9. Three circuits in parallel take currents which can be represented by
$i_1 = 10 \sin 314\, t$; $i_2 = 7.5 \sin\left(314t - \frac{\pi}{3}\right)$ and $i_3 = 12 \sin\left(314t - \frac{\pi}{4}\right)$
Sketch a phasor diagram to represent the three currents and their resultant. Express the resultant in the same form as the three individual currents expressed above. What is the rms value and the frequency of the resultant current ?
[Ans. 26.35 sin (314 *t* – 0.584); 18.63 A; 50 Hz**]**

10. Three voltages represented by $e_1 = 20 \sin \omega t$; $e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right)$ and $e_3 = 40 \cos\left(\omega t + \frac{\pi}{6}\right)$ act together in a circuit.
Find an expression for the resultant voltage. Represent them by appropriate phasors.
[Madras Univ. Electrotechnics 1981; Nagpur Univ. Elec. Circuits 1991]
[Ans. 25.1 sin ($\omega t + 0.18\pi$)**]**