APPLICATIONS OF DIFFERENTIAL EQUATIONS OF FIRST ORDER:

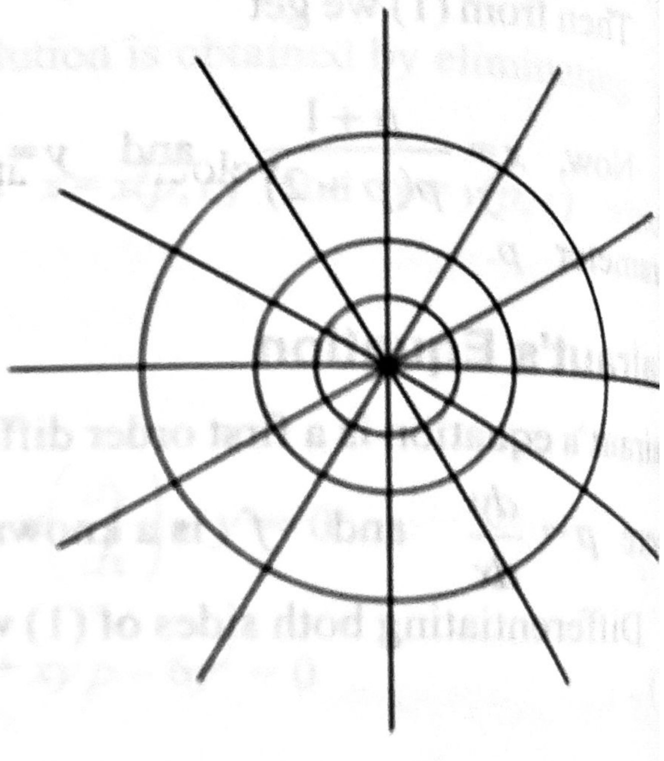
Let us consider here only those physical and geometrical problems which lead to a differential equation of first order and first degree.

Applications to Geometrical Problems:

ORTHOGONAL TRAJECTORIES: Two curves intersecting at a point P are said to be **orthogonal** if their tangents at P are perpendicular to each other. Two families of curves (or trajectories) are orthogonal if each curve of the first family is orthogonal to each curve of the second family, wherever an intersection takes place.

Orthogonal families are formed in many situations. On the globe the parallels and meridians (also called latitudes and longitudes) are orthogonal families. Also the equipotential lines and the electrical lines of force are mutually orthogonal.

In the early days when calculus was being developed Newton the great, was occupied with the problem of finding the family of orthogonal trajectories of a given family of curves. In other words, suppose we are given a family S of curves in the plane and we want to construct another family T of curves so that every curve in S is orthogonal to every curve in T provided they intersect each other. As a simple example, suppose S consists of all circles about the origin. Then T consists of all straight lines through the origin as shown in the adjacent figure. Clearly each straight line is orthogonal to each circle whenever they intersect.



In general, suppose we are given a family S of curves given by the equation $F(x, y, k) = 0$, giving different curve for each choice of the constant k. Think of these curves as integral curves of a differential equation $\frac{dy}{dx} = f(x, y)$ which we determine from the equation $F(x, y, k) = 0$ by differentiation. At a point (x_0, y_0) the slope of the curve C in S through this point must have slope $-1/f(x_0, y_0)$.

Then the family T of orthogonal trajectories of S, consists of the integral curves of the differential equation $\frac{dy}{dx} = -\frac{1}{f(x, y)}$.