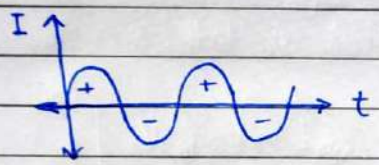
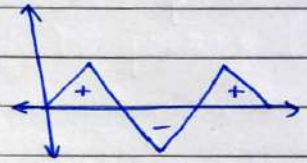
Unit – II
AC Circuits

AC (Alternating Current) → Current which changes its direction (may change magnitude also.)

(AC source)

ex :-





□ **Standard Terms related to AC.** (for sinusoidal waveforms)

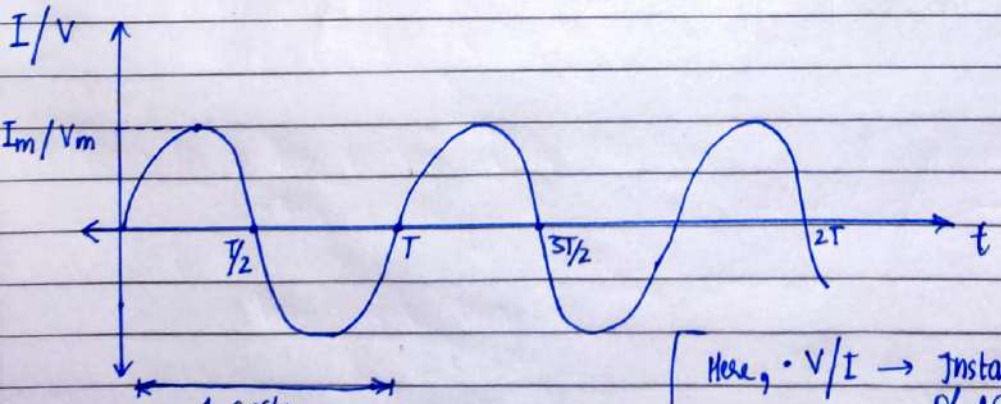
1.) **Instantaneous Value** → The value of an alternating quantity at a particular instant.

Alternating Voltage

$$V = V_m \sin\theta$$
$$= V_m \sin\omega t$$

Alternating Current

$$I = I_m \sin\theta$$
$$= I_m \sin\omega t$$



1 cycle
Time Period (T)

Here,
- V/I → Instantaneous values of AC voltage & current
- V_m/I_m → Peak / Max. value of AC voltage & current
- ω → Angular frequency.

2.) **Average Value** → The average of all the instantaneous values of Alternating Quantities over one complete cycle.

$$I_{avg} = \frac{\int I \cdot dt}{\int dt} \quad \text{Or} \quad I_{avg} = \frac{\int I \cdot d\theta}{\int d\theta}$$

for full cycle :

$$I_{avg} = \frac{\int_0^T I \cdot dt}{\int_0^T dt} \quad \text{Or} \quad I_{avg} = \frac{\int_0^{2\pi} I \cdot d\theta}{\int_0^{2\pi} d\theta}$$

$$\Rightarrow I_{avg} = \frac{\int_0^T I_m \sin\omega t}{T} \quad \Rightarrow I_{avg} = \frac{\int_0^{2\pi} I_m \sin\theta}{2\pi}$$

$$\Rightarrow \boxed{I_{avg} = 0} \quad \Rightarrow \boxed{I_{avg} = 0}$$

for Half cycle :

$$I_{avg} = \frac{\int_0^{T/2} I \cdot dt}{\int_0^{T/2} dt} \quad \text{Or} \quad I_{avg} = \frac{\int_0^{\pi} I \cdot d\theta}{\int_0^{\pi} d\theta}$$

$$\Rightarrow I_{avg} = \frac{\int_0^{T/2} I_m \sin\omega t}{T/2} \quad \Rightarrow I_{avg} = \frac{\int_0^{\pi} I_m \sin\theta}{\pi}$$

$$\Rightarrow \boxed{I_{avg} = \frac{2I_m}{\pi}} \quad \Rightarrow \boxed{I_{avg} = \frac{2I_m}{\pi}}$$

Similarly,

$$\boxed{V_{avg} = 0} \quad \boxed{V_{avg} = \frac{2V_m}{\pi}}$$

(for full cycle) (for Half Cycle)

3.) **RMS Value (Root Mean Square)**

{ Square ⟶ find average ⟶ Put root }

$I_{rms} = \sqrt{\langle I^2 \rangle}$

(where, $I = I_m \sin\omega t$)

$\rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

$V_{rms} = \sqrt{\langle V^2 \rangle}$

(where, $V = V_m \sin\omega t$)

$\rightarrow \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$

→ The significance of introducing the concept of RMS value is that, the relation b/w AC rms current & AC rms voltage is the same as that in the case of DC quantities (Ohm's Law).

$$\boxed{V_{RMS} = I_{RMS} R}$$

→ In fact, the RMS current & RMS voltage are the equivalent DC current & voltage that would produce the same average power loss.

$$\boxed{P = V_{RMS} I_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R}$$

→ Also, in everyday life, AC voltages and currents are always given RMS values because this allows a sensible comparison to be made with steady DC voltages & currents such as from a battery.

<u>for ex</u> :- The household line voltage of 220V in India, is an RMS value with a peak voltage of :—

$$V_m = \sqrt{2}\, V_{RMS}$$
$$= \sqrt{2} \cdot 220$$
$$= 311\,V$$

4) Waveform → The graph of instantaneous values of an alternating quantity plotted against time.

5) frequency → Number of cycles completed by an alternating quantity per second.

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$ (unit → Hz)

(Note : In India, standard frequency of Alternating voltages and currents is 50 Hz.)

6) Angular frequency (ω) → Rate of change of waveform per sec.

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$ (Unit → rad/sec)

7) Amplitude (I_m / V_m) → Maximum value attained by an alternating qty. during +ve or −ve half cycle.

8) Peak to Peak Value → Value of an alternating qty. from its +ve to −ve peak.

$$\boxed{\text{Peak to Peak Value} = 2 \times \text{Amplitude}}$$

9) form factor (K_f)

$$K_f = \frac{\text{RMS value}}{\text{Avg. Value}}$$

RMS value $= \frac{e_m}{\sqrt{2}} = \frac{e_m}{1.414} = 0.707\, e_m$

Avg value $= \frac{2 e_m}{\pi} = 0.637\, e_m$

Here, e → Alternating qty.

$\Rightarrow K_f = \frac{0.707\, e_m}{0.637\, e_m}$

$\Rightarrow K_f = 1.11$ (for sin waves only)

10.) Crest factor / Peak factor (K_p)

$K_p = \frac{\text{Peak / max value}}{\text{RMS value}}$

$\Rightarrow K_p = \frac{e_m}{0.707\, e_m}$

$\Rightarrow K_p = 1.414$ (for sin waves only)

Ques.1.) A sinusoidally varying alternating current of frequency 60 Hz has a maximum value of 15 A.

(i.) Write down the equation for instantaneous value.

(ii.) Find the value of current after 1/200 sec.

(iii.) Find the time to reach 10 A for the first time, and

(iv.) Find its average value.

Ans.1.) (i.) $I = I_m \sin \omega t$

$\Rightarrow I = 15 \sin(2\pi \times 60 t)$

(ii.) At $t = 1/200$ sec.

$I = 15 \sin(2 \times 3.14 \times 60 \times 1/200)$

$\Rightarrow I = 15 \sin(1.884)$

$\Rightarrow I = 15 \times 0.95135$

$\Rightarrow I = 14.27$ A

(iii) $10 = 15 \sin(2 \times 3.14 \times 60 \times t)$

$\Rightarrow 376.8 \times t = \sin^{-1}\left(\frac{10}{15}\right)$

$\Rightarrow t = \cancel{\frac{0.61837}{376.8}} \quad \frac{0.729}{376.8}$

$\Rightarrow t = 0.001934$ sec

(iv) $I_{avg} = 0.637\, I_m$

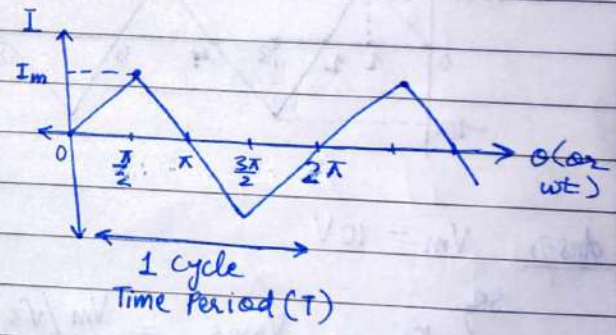
$= 0.637 \times 15$

$= 9.555$ A

- RMS value, Average value, form factor & Peak factor of a Triangular Waveform.

- Instantaneous Current :

$$\boxed{I = \frac{2}{\pi} I_m \theta} \quad (0 \le \theta < \pi/2)$$



- RMS value :

$$I_{rms}^2 = \frac{1}{\pi/2} \int_0^{\pi/2} I^2 \, d\theta = \frac{2}{\pi} \int_0^{\pi/2} \frac{4 I_m^2 \theta^2}{\pi^2} \cdot d\theta$$

$$\Rightarrow I_{rms} = \sqrt{\frac{8 I_m^2}{\pi^3} \int_0^{\pi/2} \theta^2 \cdot d\theta} = \sqrt{\frac{8 I_m^2}{\pi^3} \left[\frac{\theta^3}{3}\right]_0^{\pi/2}}$$

$$\Rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{3}}}$$

- Average value :

$$I_{avg} = \frac{1}{\pi/2} \int_0^{\pi/2} I \cdot d\theta = \frac{2}{\pi} \int_0^{\pi/2} \frac{2 I_m \theta}{\pi} \cdot d\theta$$

$$\rightarrow \boxed{I_{avg} = \frac{I_m}{2}}$$

- form factor (K_f)

$$K_f = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{3}}{I_m/2}$$

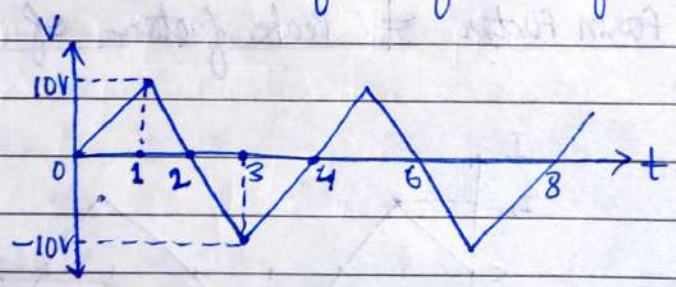
$$\rightarrow \boxed{K_f = \frac{2}{\sqrt{3}} = 1.155}$$

- Peak factor (K_p)

$$K_p = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{3}}$$

$$\rightarrow \boxed{K_p = \sqrt{3} = 1.732}$$

Ques.) Determine the form factor of a voltage waveform shown below.



Ans.) $V_m = 10V$

So,

$$K_f = \frac{V_{rms}}{V_{avg}} = \frac{V_m/\sqrt{3}}{V_m/2} = \frac{10/\sqrt{3}}{10/2}$$

$$\rightarrow K_f = 1.155$$

□ PHASORS

→ The phase of an alternating quantity is defined as the fraction of time period or cycle through which the AC qty. leads or lags from a zero position of reference.

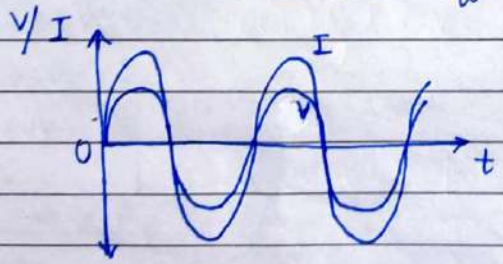
→ AC Quantities can be represented graphically by a straight line with an arrow in the phasor representation method.

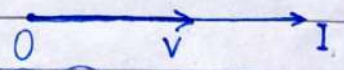
length of line → magnitude.
Arrow → direction.

• **Phase Difference** : The phase diff. b/w alternating quantities having same time period is defined as the fractional part of a period through which one wave leads or lags w.r.t other.

CASE – I : Zero Phase Diff.

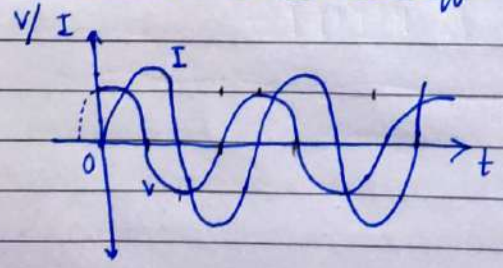
(If current (I) is taken as reference)

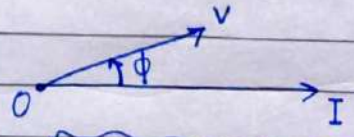




$I = I_m \sin \omega t$
$V = V_m \sin \omega t$

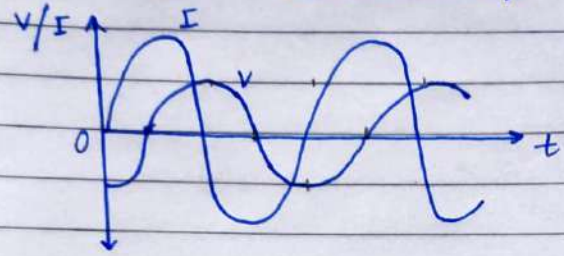
CASE – II : +ve Phase Diff.

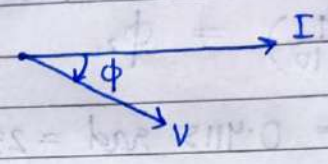




$I = I_m \sin \omega t$
$V = V_m \sin(\omega t + \phi)$

CASE-III : -ve Phase Diff.





$$I = I_m \sin\omega t$$
$$V = V_m \sin(\omega t - \phi)$$

Ques.) In a certain circuit supplied from 50Hz mains, the potential difference has a maximum value of 500 V & the current has a maximum value of 10A. At the instant, $t=0$, the instantaneous values of potential difference and current are 400 V and 4 A resp. both increasing in +ve direction. State expressions for instantaneous values of potential diff. and current at time 't'. Calculate the instantaneous values at time 't' = 0.015 sec. find phase angle b/w potential diff. & current.

Ans.) $f = 50Hz$
$V_m = 500V$
$I_m = 10A$

A/q

At $t = 0$:

$$V = V_m \sin(\omega t + \phi_1)$$
$$400 = 500 \sin(2 \times 3.14 \times 50 \times (0) + \phi_1)$$
$$\Rightarrow \sin^{-1}\left(\frac{4}{5}\right) = \phi_1$$
$$\Rightarrow \phi_1 = 0.9272 \text{ rad} = 53.130^\circ$$

and,

$I = I_m \sin(\omega t + \phi_2)$

$4 = 10 \sin(\omega(0) + \phi_2)$

$\Rightarrow \sin^{-1}\left(\frac{4}{10}\right) = \phi_2$

$\Rightarrow \phi_2 = 0.4115 \text{ rad} = 23.578^\circ$

so,

$\Delta\phi = \phi_1 - \phi_2$

$\Rightarrow \Delta\phi = 53.130^\circ - 23.578^\circ$

$\Rightarrow \Delta\phi = 29.552^\circ$

$\Rightarrow \Delta\phi = 0.5157 \text{ rad}$ (voltage leads current)

So,

$V = V_m \sin(\omega t + \phi_1)$

$\Rightarrow V = 500 \sin(2 \times 3.14 \times 50 \times 0.015 + 0.9272)$ (At $t = 0.015$ sec)

$\Rightarrow V = 500 \sin(5.6372)$

$\Rightarrow V = 500 \times -0.60198$

$\Rightarrow V = -300.99273 \text{ V}$

and,

$I = I_m \sin(\omega t + \phi_2)$

$\Rightarrow I = 10 \sin(2 \times 3.14 \times 50 \times 0.015 + 0.4115)$ (At $t = 0.015$ sec)

$\Rightarrow I = 10 \sin(5.1215)$

$\Rightarrow I = 10 \times -0.917474$

$\Rightarrow I = -9.17474 \text{ A}$

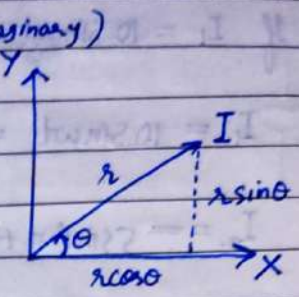
Forms of Phasor Representation

- **Rectangular Form**

$$I = r\cos\theta \pm j\, r\sin\theta$$

- **Polar Form**

$$I = r\angle \pm\theta$$



If,

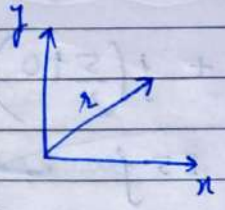
$$I = x + jy$$

So,

$$\boxed{r = \sqrt{x^2+y^2}}$$

and,

$$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$



* For Addition / Subtraction → Use Rectangular form.
* For multiplication / division → Use Polar form.

If,

$$I_1 = x_1\angle\theta_1 = x_1\cos\theta_1 + j\,x_1\sin\theta_1$$
$$I_2 = x_2\angle\theta_2 = x_2\cos\theta_2 + j\,x_2\sin\theta_2$$

(where, x_1, x_2 → rms value)

So,

$$I_1 + I_2 = (x_1\cos\theta_1 + x_2\cos\theta_2) + j(x_1\sin\theta_1 + x_2\sin\theta_2)$$

$$I_1 - I_2 = (x_1\cos\theta_1 - x_2\cos\theta_2) + j(x_1\sin\theta_1 - x_2\sin\theta_2)$$

$$I_1 \times I_2 = (x_1 \times x_2)\angle(\theta_1+\theta_2)$$

$$\frac{I_1}{I_2} = \frac{x_1}{x_2}\angle(\theta_1-\theta_2)$$

Dt.:
Pg.:

Delta

Ques.) If $I_1 = 10\sin\omega t$ & $I_2 = 5\sin(\omega t + 45^\circ)$, find $I_1 + I_2$.

Ans.) $I_1 = 10\sin\omega t = \frac{10}{\sqrt{2}}\angle 0^\circ = \frac{10}{\sqrt{2}}\cos 0^\circ + j\frac{10}{\sqrt{2}}\sin 0^\circ = \frac{10}{\sqrt{2}} + 0j$

$I_2 = 5\sin(\omega t + 45^\circ) = \frac{5}{\sqrt{2}}\angle 45^\circ = \frac{5}{\sqrt{2}}\cos 45^\circ + j\frac{5}{\sqrt{2}}\sin 45^\circ$

$= \frac{5}{2} + j\frac{5}{2}$

So,

$I_1 + I_2 = \left(\frac{10}{\sqrt{2}} + \frac{5}{2}\right) + j\left(\frac{5}{2} + 0\right)$

$\Rightarrow I_1 + I_2 = 9.57 + 2.5j$

Ques.) If $A = 4 + 7j$, $B = 8 + 9j$, $C = 5 - 6j$. find:

(i) $\frac{A \times B}{C}$

(ii) $\frac{A+B}{B+C}$

Ans.) (i) $A = 4 + 7j = \sqrt{16+49}\angle\tan^{-1}\left(\frac{7}{4}\right) = 8.06\angle 60.25^\circ$

$B = 8 + 9j = \sqrt{64+81}\angle\tan^{-1}(9/8) = 12.04\angle 48.36^\circ$

$C = 5 - 6j = \sqrt{25+36}\angle\tan^{-1}(-6/5) = 7.81\angle -50.19^\circ$

So,

$\frac{A \times B}{C} = \frac{8.06 \times 12.04\angle 60.25 + 48.36^\circ}{7.81\angle -50.19^\circ} = \frac{97.0424}{7.81}\angle 108.61^\circ - (-50.19^\circ)$

$\Rightarrow \frac{A \times B}{C} = 12.425\angle 158.19^\circ$

Ans.) (ii) $A + B = 12 + 16j = \sqrt{144+256}\angle\tan^{-1}(16/12) = 20\angle 53.1301^\circ$

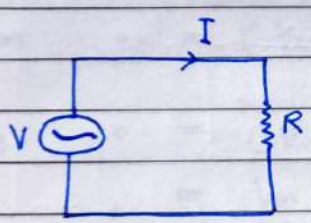
& $B + C = 13 + 3j = \sqrt{169+9}\angle\tan^{-1}(3/13) = 13.34\angle 12.994^\circ$

Sol, $\frac{A+B}{B+C} = \frac{20\angle 53.1301^\circ}{13.34\angle 12.994^\circ} = \frac{20}{13.34} \angle 53.1301^\circ - 12.994^\circ$

$$\frac{A+B}{B+C} = 1.4992\angle 40.1361^\circ$$

□ Alternating Quantities through

- **Pure Resistive Circuit**



$V = V_m \sin\omega t$

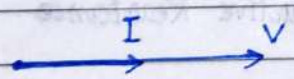
So,

$$I = \frac{V}{R} = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$

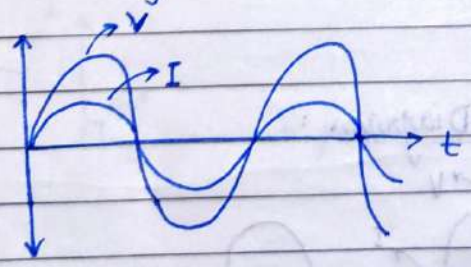
$$\boxed{I = I_m \sin\omega t}$$

($\phi = 0^\circ$ (The voltage & current are in same phase)

Phasor Diagram :—

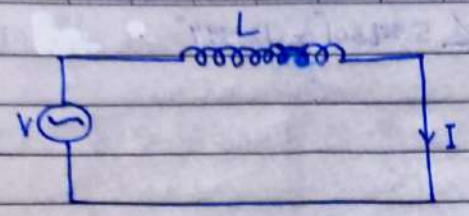


Wave Diagram :—



- **Pure Inductive Circuit**

Please Turn Over →



$V = V_m \sin\omega t$

we know :—

$$e = -L\frac{dI}{dt}$$

so,

$$V = -e = L\frac{dI}{dt}$$ (a −ve sign because of lenz's law.)

$$\Rightarrow \frac{V_m \sin\omega t}{L} = \frac{dI}{dt}$$

$$\Rightarrow \int dI = \int \frac{V_m}{L}\sin\omega t \cdot dt$$

$$\Rightarrow I = \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

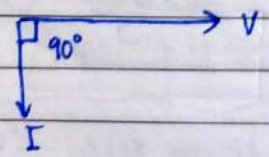
$$\Rightarrow I = -\frac{V_m}{\omega L}\cos\omega t$$

$$\Rightarrow I = -\frac{V_m}{\omega L}\left(\sin\left(\frac{\pi}{2} - \omega t\right)\right)$$ ($X_L = \omega L$) → Inductive Reactance.

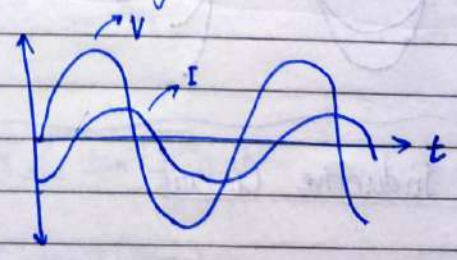
$$\Rightarrow I = -\frac{V_m}{X_L}\left(-\sin\left(\omega t - \frac{\pi}{2}\right)\right)$$

$$\Rightarrow \boxed{I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$ ($\phi = -\pi/2$ rad (current lags by 90°))

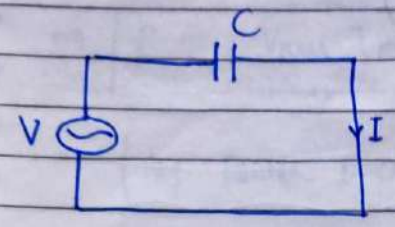
Phasor Diagram :



Wave Diagram :



- **Pure Capacitive Circuit**



$V = V_m \sin\omega t$

we know : $q = CV$

and,

$$I = \frac{dq}{dt}$$

$$\Rightarrow \quad " = \frac{d(CV_m \sin\omega t)}{dt}$$

$$\Rightarrow \quad " = CV_m \frac{d}{dt}(\sin\omega t)$$

$$\Rightarrow \quad " = CV_m\, \omega \cos\omega t$$

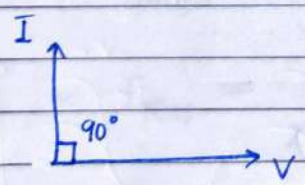
$$\Rightarrow \quad " = \frac{V_m}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \boxed{I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

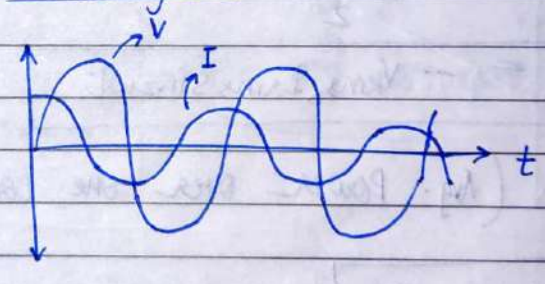
($X_C = 1/\omega C$ ↓ (Capacitive reactance))

($\phi = \pi/2$ rad (current leads by 90°))

Phase Diagram :



Wave Diagram :



- **Power Consumed in :**

- **Pure Resistive Circuit**

$P = VI = V_m \sin\omega t \cdot I_m \sin\omega t$

$\Rightarrow P = V_m I_m \sin^2\omega t = V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$

$\Rightarrow P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$

$\Rightarrow \boxed{P = V_{RMS} I_{RMS} - \frac{V_m I_m}{2}\cos 2\omega t}$

(Avg. Power over one complete cycle i.e. for 'T' sec)

$\Rightarrow P_{avg} = V_{RMS} I_{RMS} - \cancel{\frac{V_m I_m}{2}\cos 2\omega t}^{\,0}$

$\Rightarrow \boxed{P_{avg} = V_{RMS} I_{RMS}}$

- Pure Inductive circuit -

$P = VI$

$\Rightarrow P = V_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2)$

$\Rightarrow P = -V_m I_m \frac{\sin\omega t \cos\omega t}{2} \times 2$

$\Rightarrow P = -\frac{V_m I_m}{2}\sin 2\omega t$

$\Rightarrow \boxed{P = -V_{RMS} I_{RMS} \sin 2\omega t}$

(Avg. Power over one complete cycle)

$\Rightarrow \boxed{P_{avg} = 0}$

- Pure Capacitive Circuit

$P = VI = V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$

$\Rightarrow P = \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$

$\Rightarrow \boxed{P = V_{RMS} I_{RMS} \sin 2\omega t}$

(Avg. Power over complete one cycle)

$\Rightarrow \boxed{P_{avg.} = 0}$

Ques.) A voltage $V = 141 \sin(314t + \pi/3)$ is applied to :-
(i.) resistor of $20\,\Omega$.
(ii.) inductance of 0.1 H.
(iii.) capacitance of 100 μF.
Find in each case RMS value of current & power dissipated. Draw the phasor diagram in each case.

Ans.) (i.) $V = 141 \sin(314t + \pi/3)$

So, $V_m = 141$

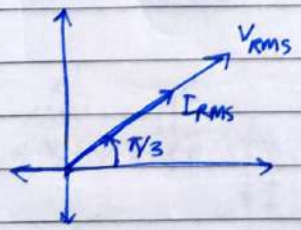
$R = 20\,\Omega$

So, $I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{V_m/R}{\sqrt{2}}$

$\Rightarrow I_{RMS} = \frac{141}{20 \times \sqrt{2}} = 4.985\ A$

So, $P = V_{RMS} I_{RMS} = \frac{141}{\sqrt{2}} \times 4.985$

$\Rightarrow P = 497.089$ watt



$I = I_{RMS} \sqrt{2} \sin\omega t$

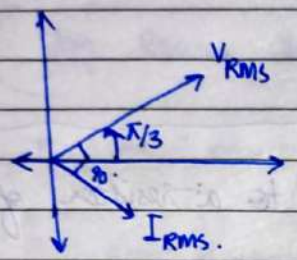
$I = 7.05 \sin(314t + \pi/3$

(ii) $I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{V_m / X_L}{\sqrt{2}}$

$\Rightarrow I_{RMS} = \frac{V_m}{\omega L \sqrt{2}} = \frac{141}{314 \times 0.1 \times 1.414}$

$\Rightarrow I_{RMS} = 3.175\ A$

So, $P = 0$



$I = I_{RMS}\sqrt{2}\sin(\omega t + \pi/3 - \pi/2)$

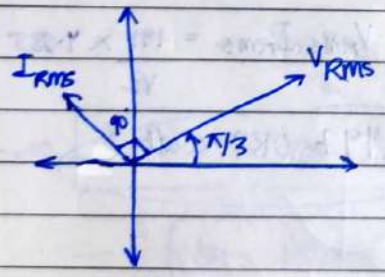
$\Rightarrow I = 4.49 \sin(314t - \pi/6)$

(iii) $I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{V_m / 1/\omega C}{\sqrt{2}}$

$\Rightarrow I_{RMS} = \frac{V_m \omega C}{\sqrt{2}} = \frac{141 \times 314 \times 100 \times 10^{-6}}{1.414}$

$\Rightarrow I_{RMS} = 3.1311\ A$

So, $P = 0$



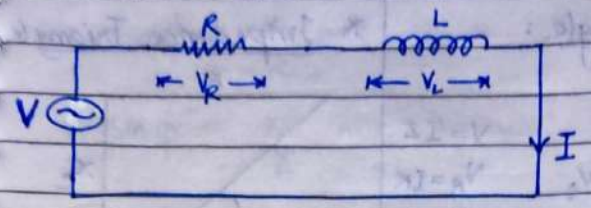
$I = I_{RMS}\sqrt{2}(\sin\omega t + \pi/3 + \pi/2)$

$\Rightarrow I = 4.4274 \sin(314t + 5\pi/6)$

□ Alternating Quantities through

◉ R–L Circuit

(for series circuit, I (current) is the reference.)



$V - V_R - V_L = 0$

$\rightarrow \boxed{V = V_R + V_L}$

$\Rightarrow \boxed{V = IR + IX_L}$

Impedance in this circuit :—

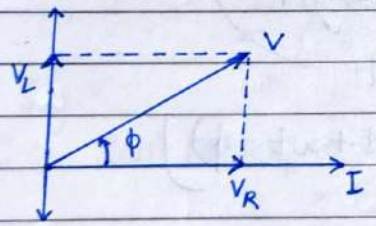
$\boxed{Z = R + jX_L}$ (Rectangular form)

so,

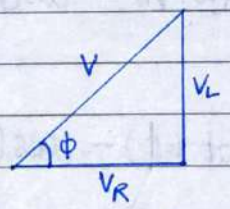
$\boxed{Z = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)}$ (Polar form)

Here,
$Z \rightarrow$ Impedance
$X_L \rightarrow$ Inductive Reactance
$\boxed{X_L = \omega L = 2\pi f L}$

* Phasor Diagram :



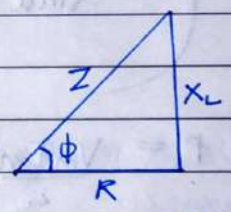
* Voltage Triangle :



$V = IZ$
$V_R = IR$
$V_L = IX_L$

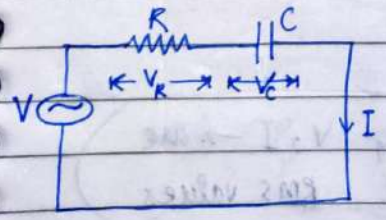
$(Z = \sqrt{R^2 + X_L^2})$

* Impedance Triangle :



$\boxed{Z = \sqrt{R^2 + X_L^2}}$

- R–C Circuit

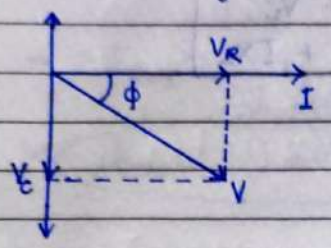


$\boxed{\begin{aligned} V &= V_R + V_C \\ V &= IR + IX_C \end{aligned}}$

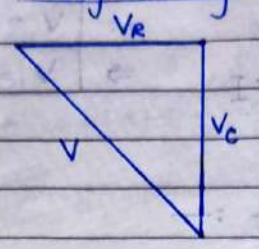
Here,
$X_C \rightarrow$ Capacitive Reactance
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\boxed{\begin{aligned} Z &= R + jX_C \\ Z &= \sqrt{R^2 + X_C^2} \angle \tan^{-1}\left(\frac{X_C}{R}\right) \end{aligned}}$

* **Phasor Diagram:**

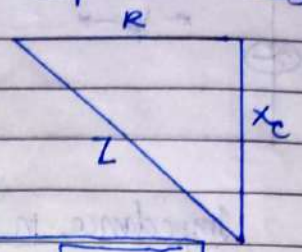


* **Voltage Triangle:**



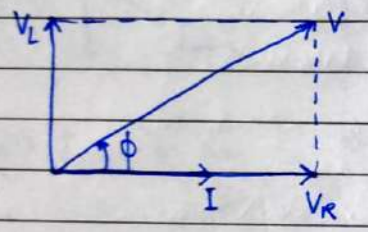
$V = IZ$
$V_R = IR$
$V_C = IX_C$

* **Impedance Triangle:**



$$\boxed{Z = \sqrt{R^2 + X_C^2}}$$

- **Power Consumption**



$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$ (reference)

say

$P = VI$

$\Rightarrow P = V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

(we know:
$\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$)

$\Rightarrow P = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$

$\Rightarrow P = V_{RMS} I_{RMS} \cos\phi - \underbrace{V_{RMS} I_{RMS} \cos(2\omega t - \phi)}_{\downarrow\, 0}$

$\Rightarrow \boxed{P = V_{RMS} I_{RMS} \cos\phi}$ (where, $\cos\phi \rightarrow$ Power factor)

(Real Power)

Apparent Power $= VI$ (KVA)
Real Power $= VI\cos\phi$ (watt)
Reactive Power $= VI\sin\phi$ (VAR)

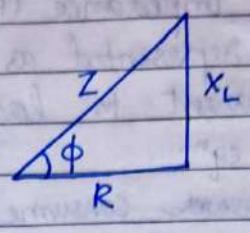
(Here, $V, I \rightarrow$ are RMS values)

and,

$$\tan\phi = \frac{X_L}{R}$$

$$\sin\phi = \frac{X_L}{Z}$$

$$\cos\phi = \frac{R}{Z}$$



Ques.) If load draws a current of 10A at 0.8 power factor lagging when connected to 100V supply. Calculate the values of real, reactive & apparent power. Also find out the value of resistance of load.

Ans.) $I_{rms} = 10A$

$\cos\phi = 0.8$

$V_{rms} = 100\,V$

So,

$$\sin\phi = \sqrt{1-\cos^2\phi}$$
$$= \sqrt{1-(0.8)^2}$$
$$= 0.6$$

So,

Real Power $= V_{rms} I_{rms} \cos\phi$
$= 100 \times 10 \times 0.8$
$= 800$ watt

Apparent Power $= V_{rms} I_{rms}$
$= 100 \times 10$
$= 1000$ VA

Reactive Power $= V_{rms} I_{rms} \sin\phi$
$= 100 \times 10 \times 0.6$
$= 600$ VA

So, $Z = V_{rms}/I_{rms}$
$= 100/10$
$= 10\,\Omega$

So,

$$\cos\phi = \frac{R}{Z} \Longrightarrow 0.8 = \frac{R}{10}$$

$\Rightarrow R = 80\,\Omega$

Ques.) A load having impedance $(1+1j)\Omega$ is connected to an AC voltage, represented as $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$.

(i) find the current in load expressed in the form of its standard eqn.

(ii) find real power consumed by the load.

Ans.) $Z = (1+1j)\Omega$

$= \sqrt{2}\angle 45^\circ$

$V = 20\sqrt{2}\cos(\omega t + 10^\circ)$

$= 20\sqrt{2}\sin(90^\circ + \omega t + 10^\circ)$

$= 20\sqrt{2}\sin(\omega t + 100^\circ)$

(from here, we get :—)

$V_m = 20\sqrt{2}\ V$

$V_{rms} = 20\ V$

So,

V (in polar form) $= 20\angle 100^\circ$

So,

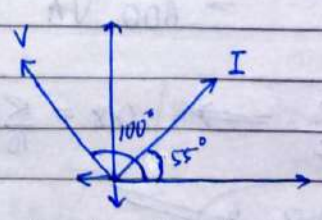
$I = \frac{V}{Z} = \frac{20\angle 100^\circ}{\sqrt{2}\angle 45^\circ} = \frac{20}{\sqrt{2}}\angle 100^\circ - 45^\circ$

$\Rightarrow I = \frac{20}{\sqrt{2}}\angle 55^\circ$

So,

$I = \frac{20}{\sqrt{2}} \times \sqrt{2}\sin(\omega t + 55^\circ)$

$I = 20\sin(\omega t + 55^\circ)$



So,

$\phi = 100^\circ - 55^\circ$

$\Rightarrow \phi = 45^\circ$

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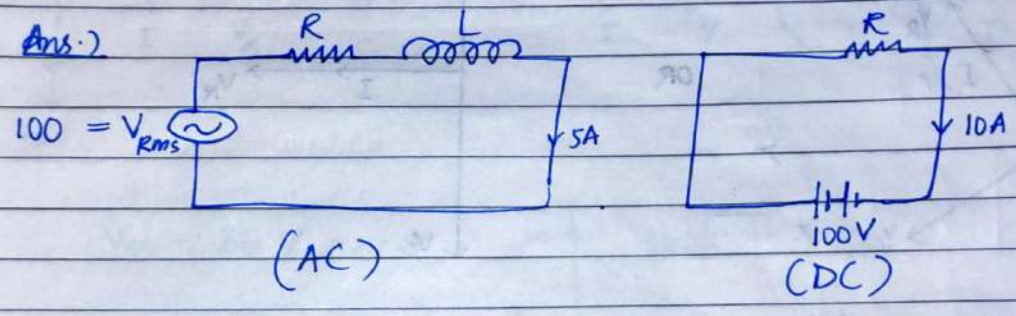
So, $P = V_{Rms} I_{RMS} \cos 45°$

$\Rightarrow P = 20 \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow P = 200$ watt

Ques.) A coil connected to 100V DC supply draws 10A and the same coil when connected to 100V AC voltage of frequency 50Hz draws 5A. Calculate the parameters of the coil and the power factor.

Ans.)



In DC:

$R = V/I = 100/10$

$\Rightarrow R = 10\,\Omega$

In AC:

$Z = V_{Rms}/I_{Rms} = 100/5$

$\Rightarrow Z = 20\,\Omega$

So,

$Z^2 = R^2 + X_L^2$

$\Rightarrow 20^2 = 10^2 + X_L^2$

$\Rightarrow X_L^2 = 400 - 100$

$\Rightarrow X_L = 10\sqrt{3}\,\Omega$

$\Rightarrow X_L = 17.32\,\Omega$

and,

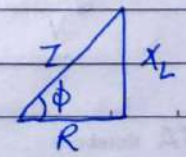
$X_L = \omega L = 2\pi f L$

$\Rightarrow 17.32 = 2 \times 3.14 \times 50 \times L$

$\Rightarrow L = \frac{17.32}{314}$

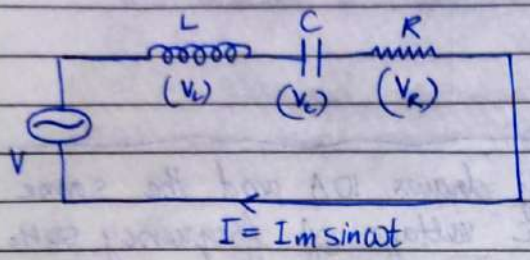
$\Rightarrow L = 0.05516$ H

Power factor $(\cos\phi) = \frac{R}{Z} = \frac{10}{20}$



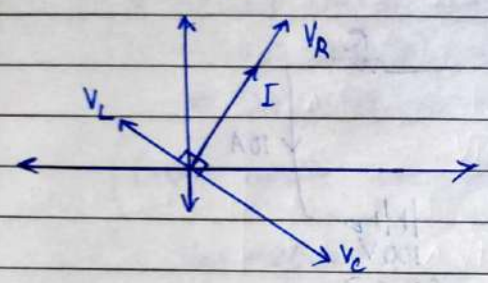
$\Rightarrow \cos\phi = 0.5$

Series LCR Circuit

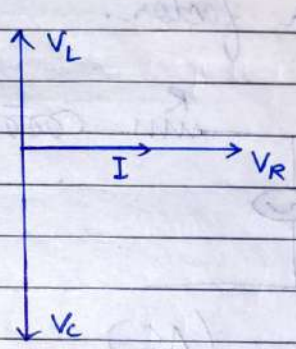


$$V_R = (V_m)_R \sin\omega t$$
$$V_L = (V_m)_L \sin(\omega t + \pi/2)$$
$$V_C = (V_m)_C \sin(\omega t - \pi/2)$$

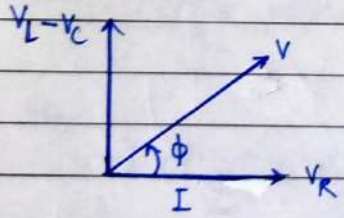
* Phasor Diagram :



OR



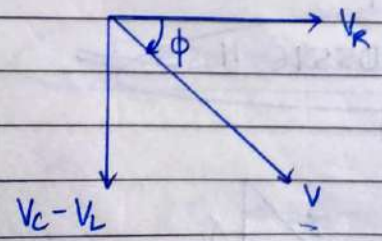
- CASE :- $X_L > X_C$ (dominantly inductive)



$$\boxed{\begin{aligned} V_m &= I_m Z \\ \Rightarrow V_m &= I_m \sqrt{R^2 + (X_L - X_C)^2} \end{aligned}}$$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

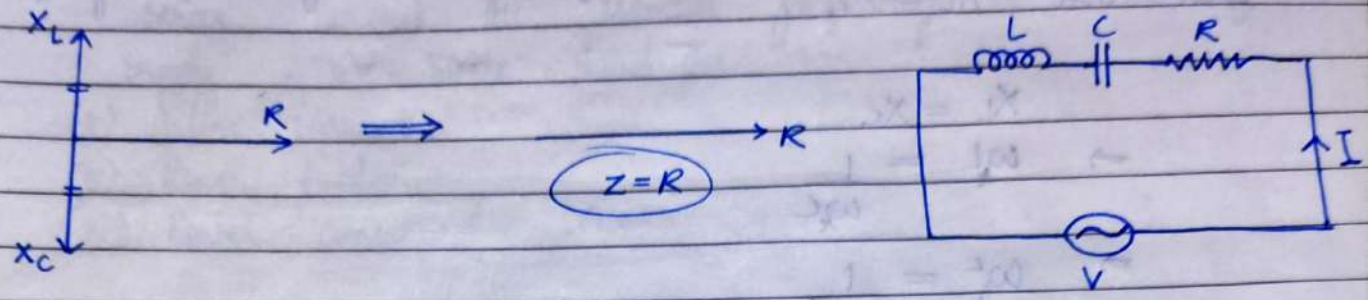
- CASE :- $X_C > X_L$ (dominantly capacitive)

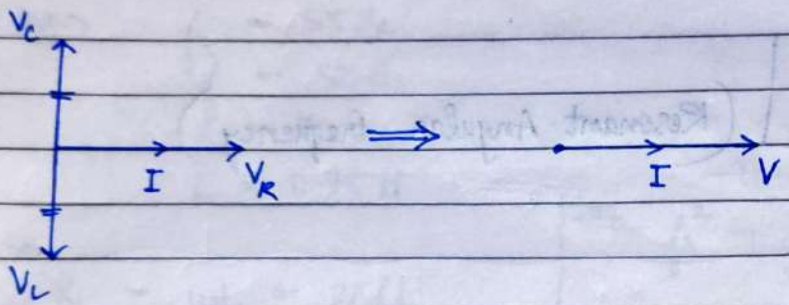


$$\boxed{\begin{aligned} V_m &= I_m Z \\ \Rightarrow V_m &= I_m \sqrt{R^2 + (X_C - X_L)^2} \end{aligned}}$$

$$\boxed{\tan\phi = \frac{X_C - X_L}{R}}$$

• CASE : $X_L = X_C$ (RESONANCE)





(No phase diff. b/w V & I.)

$V_m = I_m Z$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow Z = \sqrt{R^2 + 0^2}$

$\Rightarrow Z = R$

$\Rightarrow \boxed{V_m = I_m R}$

$\tan\phi = \frac{X_L - X_C}{R}$

$\Rightarrow \tan\phi = \frac{0}{R}$

$\Rightarrow \phi = 0°$ (No phase diff.)

so,

$$\boxed{\begin{aligned} I &= I_m \sin\omega t \\ V &= V_m \sin\omega t \end{aligned}}$$

→ At resonance, LCR circuit behaves as pure resistive circuit.

→ At resonance, impedance of the circuit is minimum and so, the current is maximum.

and, (Power factor) $\cos\phi = \cos 0°$

$= 1$

→ We can attain resonance by setting the value of frequency (f) such that $X_L = X_C$.

$$X_L = X_C$$

$$\rightarrow \omega_r L = \frac{1}{\omega_r C}$$

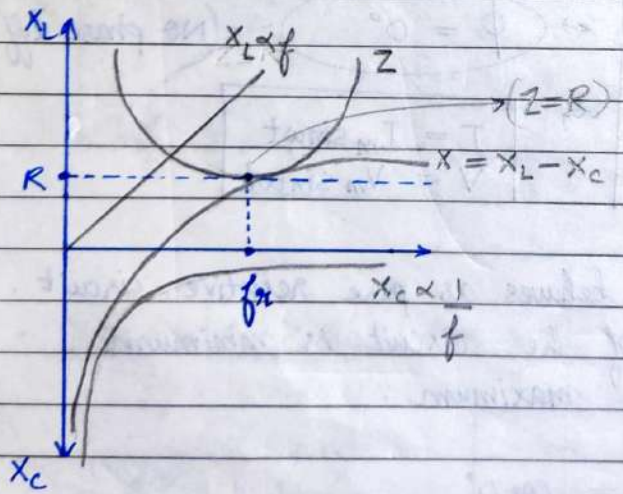
$$\Rightarrow \omega_r^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\omega_r = \frac{1}{\sqrt{LC}}} \quad \text{(Resonant Angular frequency)}$$

$$\Rightarrow 2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \quad \text{(Resonant frequency)}$$

- <u>Resonance Curve</u>



$$X_L = \omega L = 2\pi f L \quad (X_L \propto f)$$
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \left(X_C \propto \frac{1}{f}\right)$$

- <u>Power consumption</u> (under Resonance)

$$P = V_{RMS} I_{RMS} \cos\phi \qquad (\cos\phi = \cos 0^\circ = 1)$$

$$\Rightarrow \boxed{P = V_{RMS} I_{RMS}}$$

$$\Rightarrow \boxed{P = (I_{RMS})^2 R}$$

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(f)

Ques.) A coil of resistance 40 Ω & inductance of 0.75 H are in a series circuit. The resonant frequency is 55 Hz. If supply 250V, 50Hz. Find the :-

(i) Line Current.
(ii) Power factor.
(iii) Power consumed.

Ans.)

$f_r = 55\,Hz$
$f = 50\,Hz$
$R = 40\,\Omega$
$L = 0.75\,H$

So,

$X_L = \omega L = 2\pi f L$

$\Rightarrow X_L = 2 \times 3.14 \times 50 \times 0.75$

$\Rightarrow X_L = 235.5\,\Omega$

So,

$f_r^2 = \frac{1}{4\pi^2 LC}$

$\Rightarrow C = \frac{1}{4\pi^2 L f_r^2} = \frac{1}{4 \times (3.14)^2 \times 0.75 \times 55^2}$

$\Rightarrow C = \frac{1}{89475.87}$

$\Rightarrow C = 1.117 \times 10^{-5}\,F$

So,

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\Rightarrow X_C = \frac{1}{2 \times 3.14 \times 50 \times 1.117 \times 10^{-5}}$

$\Rightarrow X_C = 285.113\,\Omega$

So,

$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\Rightarrow Z = \sqrt{40^2 + (285.113 - 235.5)^2}$

$\Rightarrow Z = \sqrt{1600 + 2461.46}$

$\Rightarrow Z = \sqrt{4061.46}$

$\Rightarrow Z = 63.729\,\Omega$

So,

$I_{RMS} = \frac{V_{RMS}}{Z}$

$\Rightarrow I_{RMS} = \frac{250}{63.729}$

$\Rightarrow I_{RMS} = 3.9228\,A$

and,

(Power factor) $\cos\phi = \frac{R}{Z}$

$\Rightarrow \cos\phi = \frac{40}{63.729}$

$\Rightarrow \cos\phi = 0.6276$

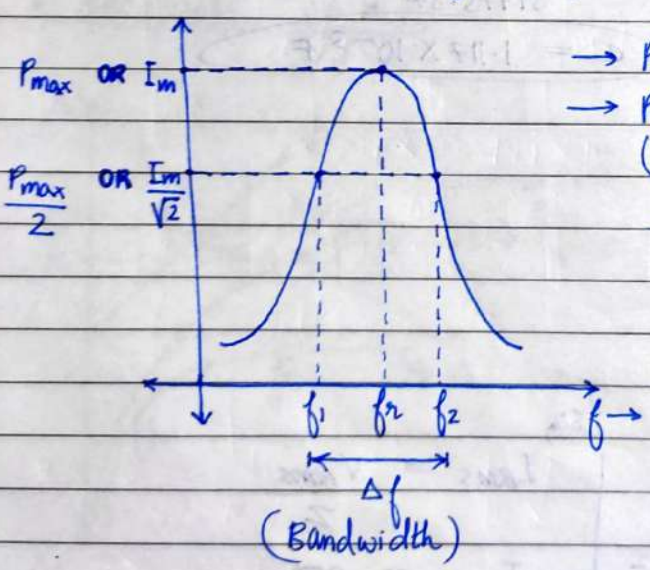
and,

$P = V_{RMS} I_{RMS} \cos\phi$

$\Rightarrow P = 250 \times 3.9928 \times 0.6276$

$\Rightarrow P = 626.5279$ watt

□ **Bandwidth** : It is defined as the difference b/w the upper and lower cut-off frequencies of a signal generated of an AC source.



→ Power will be max. at f_r.

→ Power will be half of max. power ($P_{max}/2$) at f_1 & f_2, that's why f_1 & f_2 are called Half-Power frequencies.

(Also,
$f_1 \rightarrow$ lower cut-off frequency.
$f_2 \rightarrow$ Upper cut-off frequency.)

(Bandwidth) $\Rightarrow$ $\boxed{\begin{aligned} \Delta f &= f_2 - f_1 \\ \Delta f &= \frac{R}{2\pi L} \end{aligned}}$

Voltage Magnification / Quality Factor (Q)

The Quality factor of RLC circuit (series) is the voltage magnification in the circuit at resonance.
And, so, we can say :—

$$\boxed{Q \text{ Factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Supply Voltage}}}$$

At resonance :—

$$I = \frac{V}{Z} = \frac{V}{R}$$

so, $V_L = IX_L = \frac{V}{R}(\omega_r L)$
(voltage across inductor)

$$\Rightarrow Q = \frac{V_L}{V} = \frac{\cancel{V}/R(\omega_r L)}{\cancel{V}}$$

$$\Rightarrow Q = \frac{\omega_r L}{R} \qquad \left(\because \omega_r = \frac{1}{\sqrt{LC}}\right)$$

$$\Rightarrow Q = \frac{L}{R\sqrt{LC}}$$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ The significance of Q-factor can be stated as, it indicates the selectivity or sharpness of the tuning of the series RLC circuit.

→ It gives the correct indication of the selectivity of such series RLC circuits which are used in many radiostations.

For Good Tuning $\Rightarrow$ Sharp Resonance $\rightarrow$ Small Bandwidth
$\rightarrow$ High Q-Factor i.e. $R \rightarrow$ small
$L \rightarrow$ Large.
$C \rightarrow$ small

Ques.) A series RLC circuit has $R = 10\,\Omega$, $L = 0.1\,H$ and $C = 8\,\mu F$.
Determine :—
(i) Resonant frequency.
(ii) Q-factor of the circuit at Resonance.
(iii) The half-power frequencies.

Ans.) (i) $R = 10\,\Omega$
$L = 0.1\,H$
$C = 8\,\mu F = 8 \times 10^{-6}\,F$

So,

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{0.1 \times 8 \times 10^{-6}}}$$

$$\Rightarrow f_r = \frac{1}{2 \times 3.14 \times 0.8944 \times 10^{-3}}$$

$$\Rightarrow f_r = \frac{10^3}{5.616832}$$

$$\Rightarrow f_r = 178.036\ Hz$$

(ii) $$Q = \frac{\sqrt{L}}{R\sqrt{C}} = \frac{1}{10}\sqrt{\frac{0.1}{8 \times 10^{-6}}} = \frac{1}{10}\sqrt{12500}$$

$$\Rightarrow Q = 11.1803$$

(iii) (BW) $$\Delta f = \frac{R}{2\pi L} = \frac{10}{2 \times 3.14 \times 0.1}$$

$$\Delta f = 15.923\ Hz$$

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So,

$$\frac{\Delta f}{2} = f_r - f_1 = f_2 - f_r$$

So,

$$\frac{\Delta f}{2} = f_r - f_1$$

$$\Rightarrow f_1 = f_r - \frac{\Delta f}{2}$$

$$\Rightarrow f_1 = 178.036 - 7.961$$

$$\Rightarrow f_1 = 170.074 \text{ Hz}$$

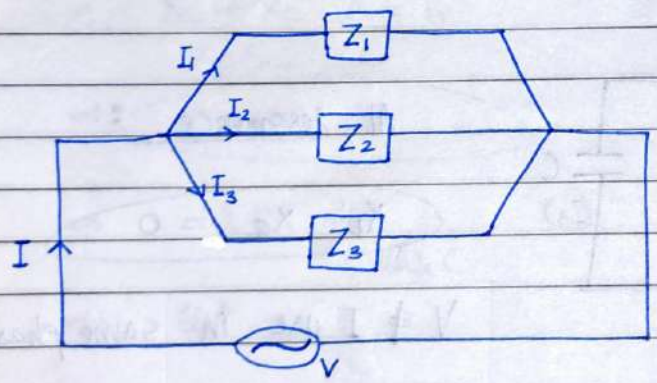
$$\frac{\Delta f}{2} = f_2 - f_r$$

$$\Rightarrow f_2 = f_r + \frac{\Delta f}{2}$$

$$\Rightarrow f_2 = 178.036 + 7.961$$

$$\Rightarrow f_2 = 185.997 \text{ Hz}$$

PARALLEL AC CIRCUIT



$$\boxed{Z = R + jX = Z\angle\phi}$$

where,

$$\boxed{\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

($Z_1, Z_2, Z_3 \rightarrow$ Impedances)

($X \rightarrow$ Reactance ($X_L - X_C$))

(This is one method)

- Another method :—

$$Z = R + jX$$

and,

$$\boxed{Y = G + jB}$$

where,

$Y \rightarrow$ Admittance ($1/Z$)

$G \rightarrow$ Conductance ($1/R$)

$B \rightarrow$ Susceptance ($1/X$)

So, $Y = \frac{1}{Z} = \frac{1}{R+jX} \times \left(\frac{R-jX}{R-jX}\right)$

$\Rightarrow Y = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$

$\Rightarrow \boxed{Y = \frac{R}{Z^2} - j\frac{X}{Z^2}}$

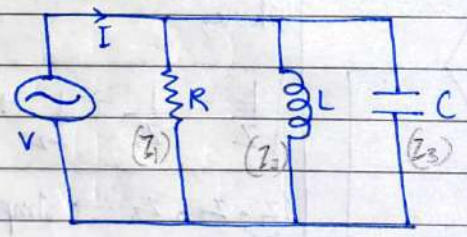
So, we get :—

$\boxed{G = \frac{R}{Z^2}}$ and $\boxed{B = \frac{X}{Z^2}}$

Note : The susceptance is said to be inductive if its sign is (-ve) while it is said to be capacitive if its sign is (+ve)

Parallel Resonance

CASE - I :



At resonance :

$X_L - X_C = 0$

V & I are in same phase.

So,

$\boxed{\cos\phi = 1}$

$Z = R + j\underbrace{X}_{0}$ and, $Y = G + j\underbrace{B}_{0}$

So, where :—

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R} + \frac{j}{j^2 X_L} - \frac{j}{j^2 X_C}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$\left(\begin{array}{l} Y = G + jB \\ \text{and, At resonance, } B = 0 \end{array} \right)$$

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

$$\Rightarrow \frac{X_L - X_C}{X_C X_L} = 0$$

$$\Rightarrow X_L = X_C$$

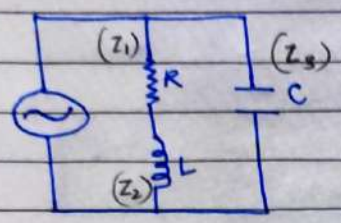
$$\Rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\Rightarrow \omega_r^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

CASE - II :



$$\frac{1}{Z} = \frac{1}{Z_1+Z_2} + \frac{1}{Z_3}$$

$$\Rightarrow \frac{1}{Z} = \frac{1}{R+jX_L} - \frac{1}{jX_C}$$

$$\Rightarrow \frac{1}{Z} = \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$\Rightarrow \frac{1}{Z} = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right)$$

(At Resonance : $B=0$)

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\Rightarrow R^2 + X_L^2 - X_L X_C = 0$$

$$\Rightarrow X_L X_C = R^2 + X_L^2$$

$$\Rightarrow \cancel{\omega_r} L \times \frac{1}{\cancel{\omega_r} C} = R^2 + \omega_r^2 L^2$$

$$\Rightarrow \omega_r^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right)$$

$$\Rightarrow \boxed{\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

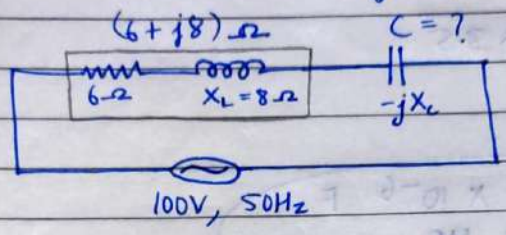
$$\Rightarrow \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Ques.) An inductive coil $6+j8\,\Omega$ impedance is connected to 100V, 50Hz AC Supply. It is desired to improve the power factor of supply current to 0.8 lagging by connecting a capacitor :—

(i) In series to the coil.
(ii) In parallel to the coil.

So, determine the value of capacitance in each case.

Ans.) (i)



$$\cos\phi = \frac{R}{Z} = \frac{6}{\sqrt{6^2+8^2}} = \frac{6}{10} = 0.6$$

<u>After improvement</u> :-

$$Z = 6 + j(8 - X_c)$$

then,

$$\cos\phi = \frac{R}{Z} = \frac{6}{\sqrt{6^2+(8-X_c)^2}}$$

$$\Rightarrow \frac{6}{0.8} = \sqrt{36 + (8-X_c)^2}$$

$$\Rightarrow (8-X_c)^2 + 36 = \left(\frac{6}{0.8}\right)^2$$

$$\Rightarrow 8 - X_c = \sqrt{\frac{36}{0.64} - 36}$$

$$\Rightarrow X_c = 8 - \sqrt{20.25}$$

$$\Rightarrow X_c = 8 - 4.5$$

$$\Rightarrow X_c = 3.5\ \Omega$$

Dt.:
Pg.:

so, $X_c = \frac{1}{\omega C}$

$\Rightarrow C = \frac{1}{\omega X_c} = \frac{1}{2\pi f X_c}$

$\Rightarrow C = \frac{1}{2 \times 3.14 \times 50 \times 3.5}$

$\Rightarrow C = \frac{1}{1099}$

$\Rightarrow C = 909.918 \times 10^{-6}$ F

$= 909.918$ MF