Problem set

Kubi H

May 30, 2025

2.26

Verify the formal identities

$$\zeta(s)^{-1} = \sum_{n=1}^{\infty} \mu(n)/n^s$$

$$\zeta(s)^2 = \sum_{n=1}^{\infty} \nu(n)/n^s$$

$$\zeta(s)\zeta(s-1) = \sum_{n=1}^{\infty} \sigma(n)/n^s$$

,

2.27

Show that $\sum 1/n$, the sum being over square free integers, diverges. Conclude that $\prod_{p<N}(1+1/p) \to \infty$ as $N \to \infty$. Since $e^x > 1 + x$, conclude that $\sum_{p<N} 1/p \to \infty$.

3.4

Show that the equation $3x^2 + 2 = y^2$ has no solution in integers

3.5

Show that the equation $7x^3 + 2 = y^3$ has no solution in integers.

3.9

Use Exercise 7 to prove that $(p-1)! \equiv -1(p)$. This is known as Wilson's theorem. Well, I've done this problem before.

3.10

If n is not a prime. show that $(n-1)! \equiv 0(n)$, except when $n = 4$.

3.16

Use the proof of the Chinese Remainder Theorem to solve the system $x \equiv 1(7), x \equiv 4(9), x \equiv 3(5)$.

3.19

If p is an odd prime, show that 1 and -1 are the only solutions to $x^2 \equiv 1(p^a)$.