problem_14

May 31, 2023

[ ]:
```python
%matplotlib inline

import matplotlib.pyplot as plt
from matplotlib import cm, colors
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
from blcommon.timer import Timer

plt.style.use('scandic')
```

[ ]:
```python
def polar_to_cartesian(theta, phi):
    '''
    Convert spherical coordinates to cartesian coordinates.

    Parameters
    ----------
    theta : array_like
        Array of theta values.
    phi : array_like
       Array of phi values.

    Returns
    -------
    x : array_like
        Array of x values.
    y : array_like
        Array of y values.
    z : array_like
        Array of z values.
    '''
    x = np.sin(theta) * np.cos(phi)
    y = np.sin(theta) * np.sin(phi)
    z = np.cos(theta)
    return x, y, z

def plot_on_sphere(x, y, z, ax=None):
    '''
```

```
    Plot points on a sphere. Returns a matplotlib figure.

    Parameters
    ----------
    x : array_like
        Array of x values.
    y : array_like
        Array of y values.
    z : array_like
        Array of z values.

    Returns
    -------
    fig : matplotlib figure
        Figure containing the plot.

    '''

    # Create a sphere
    r = 1
    pi = np.pi
    cos = np.cos
    sin = np.sin
    phi, theta = np.mgrid[0.0:pi:100j, 0.0:2.0*pi:100j]
    x_sphere = r*sin(phi)*cos(theta)
    y_sphere = r*sin(phi)*sin(theta)
    z_sphere = r*cos(phi)

    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')

    print(ax.get_proj())

    ax.plot_surface(
        x_sphere, y_sphere, z_sphere,  rstride=1, cstride=1, color='C0',
 ↪alpha=0.3, linewidth=0)

    ax.scatter(x,y,z,color="k",s=20)

    ax.set_xlim([-1,1])
    ax.set_ylim([-1,1])
    ax.set_zlim([-1,1])
    ax.set_aspect("equal")
    return fig

def cartesian_to_polar(x, y, z):
    '''
```

```
    Convert cartesian coordinates to spherical coordinates.

    Parameters
    ----------
    x : array_like
        Array of x values.
    y : array_like
        Array of y values.
    z : array_like
        Array of z values.

    Returns
    -------
    theta : array_like
        Array of theta values.
    phi : array_like
        Array of phi values.
    '''
    r = np.sqrt(x**2 + y**2 + z**2)
    theta = np.arccos(z/r)
    phi = np.arctan2(y,x)
    return theta, phi

def plot_eval(x, y, z, title = None):
    fig = plt.figure(figsize=(7, 7))

    ax_0 = plt.subplot(2, 2, 1, projection='3d')
    ax_1 = plt.subplot(2, 2, 2, projection=None)
    ax_2 = plt.subplot(2, 2, 3, projection=None)
    ax_3 = plt.subplot(2, 2, 4, projection=None)

    # plotting on the sphere
    r = 1
    pi = np.pi
    cos = np.cos
    sin = np.sin
    phi, theta = np.mgrid[0.0:pi:100j, 0.0:2.0*pi:100j]
    x_sphere = r*sin(phi)*cos(theta)
    y_sphere = r*sin(phi)*sin(theta)
    z_sphere = r*cos(phi)

    ax_0.plot_surface(
        x_sphere, y_sphere, z_sphere,  rstride=1, cstride=1, color='C1',␣
 ↪alpha=0.3, linewidth=0)

    ax_0.scatter(x,y,z,color="k",s=1)
```

```python
    ax_0.set_xlim([-1.1,1.1])
    ax_0.set_ylim([-1.1,1.1])
    ax_0.set_zlim([-1.1,1.1])

    ax_0.set_aspect("equal")

    ax_0.set_xlabel('x')
    ax_0.set_ylabel('y')
    ax_0.set_zlabel('z')

    # plotting histograms
    nums, bins = np.histogram(x, bins=11)
    bins = (bins[:-1] + bins[1:])/2
    ax_1.bar(bins, nums, width=2/13)
    ax_1.set_ylabel('Number of points')
    ax_1.set_xlabel('x position')
    ax_1.set_xlim([-1,1])


    nums, _ = np.histogram(y, bins=11)
    ax_2.bar(bins, nums, width=2/13)
    ax_2.set_ylabel('Number of points')
    ax_2.set_xlabel('y position')
    ax_2.set_xlim([-1,1])


    nums, _ = np.histogram(z, bins=11)
    # print(bins)
    ax_3.bar(bins, nums, width=2/13)
    ax_3.set_ylabel('Number of points')
    ax_3.set_xlabel('z position')
    ax_3.set_xlim([-1,1])

    if title is not None:
        fig.suptitle(title)

    fig.tight_layout()
    return fig
```

1 Generating random points on sphere

1.1 What is uniform and how to visualise it?

Lets say I would use the spherical coordinates $\phi \in \langle 0, 2\pi \rangle$, $\theta \in \langle 0, \pi \rangle$. And lets say I would generate one point every 15^{o} of anfle one point. What does this lead to?

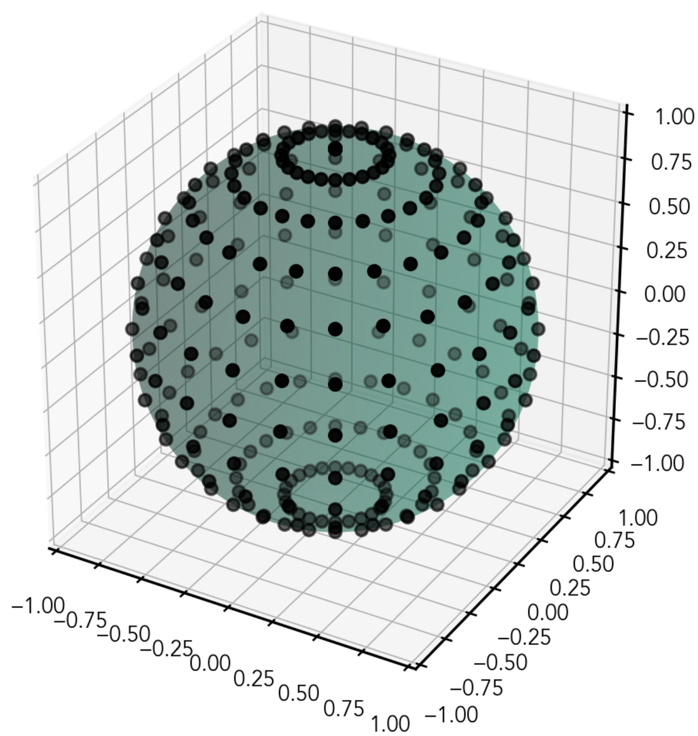
Around the ‘poles’ ($ = 0,, = 2)$therewouldbeseveral'circles'ofpoints.Ontheotherhandifwetakepointonthe'equat$
 $), in the same suroundings of this point there will be less points. However there should not really matter on the orientation of the sphere, if we want the points to be uniformly distributed. Wherever we take a point, there should be the same number of points in the same surroundings.

Outcome:

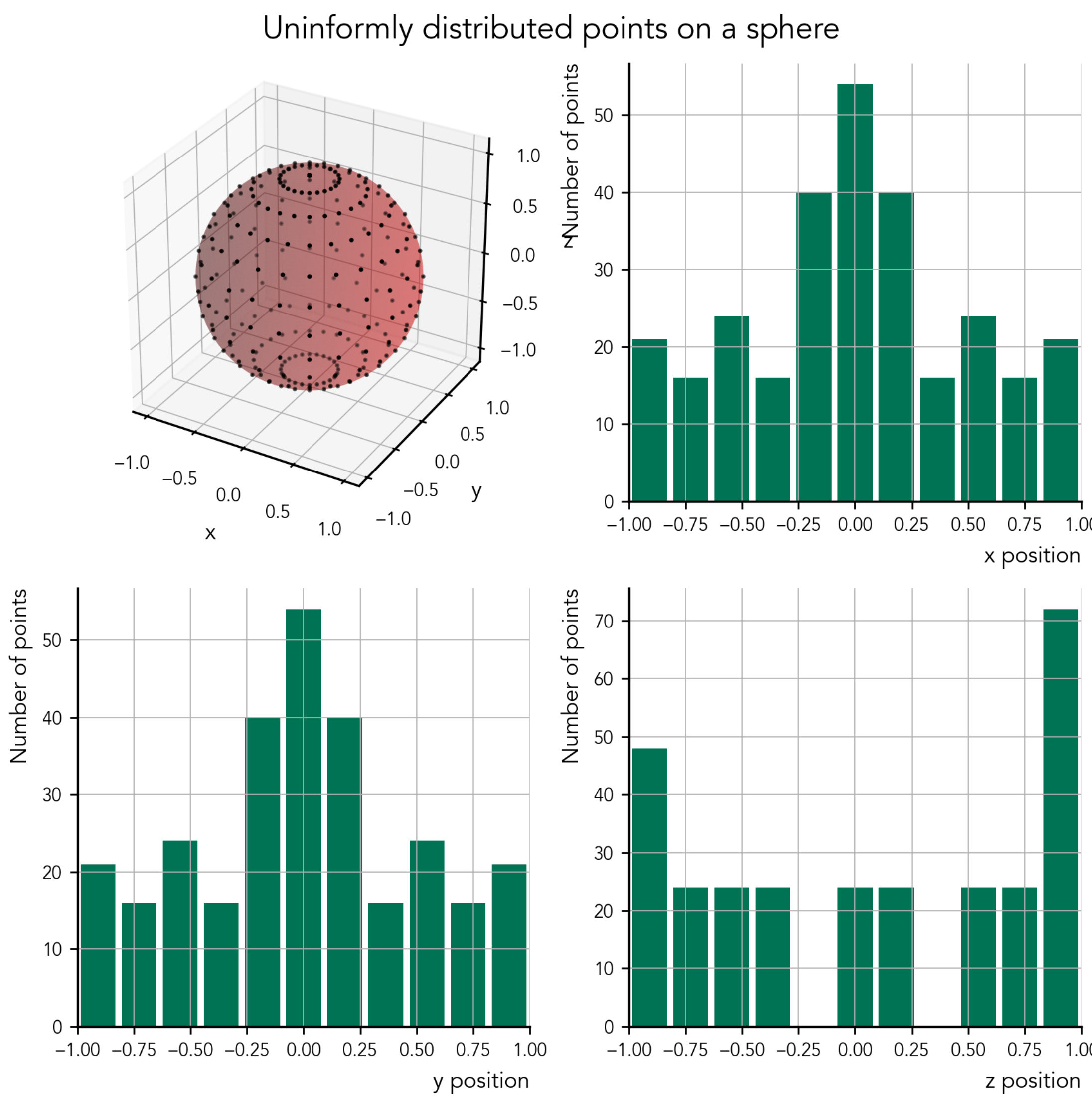
The distribuion of points as well as the visualisation method should be independent of the orientation of the sphere.

[ ]:
```
theta = np.arange(0, np.pi, 15/180*np.pi)
phi = np.arange(0, 2*np.pi, 15/180*np.pi)
theta, phi = np.meshgrid(theta, phi)

# plotting
x, y, z = polar_to_cartesian(theta, phi)
fig = plot_on_sphere(x, y, z)
plt.show()
```

```
[[ 0.98974332   0.57142857   0.          -0.78058595]
 [-0.28571429   0.49487166   0.74230749  -0.47573243]
 [ 0.           0.           0.         -10.        ]
 [-0.49487166   0.85714286  -0.42857143  10.03315012]]
```



```
[ ]: theta = np.arange(0, np.pi, 15/180*np.pi)
     phi = np.arange(0, 2*np.pi, 15/180*np.pi)
     theta, phi = np.meshgrid(theta, phi)

     # plotting
     x, y, z = polar_to_cartesian(theta, phi)
     fig = plot_eval(x, y, z, title='Uninformly distributed points on a sphere')
```

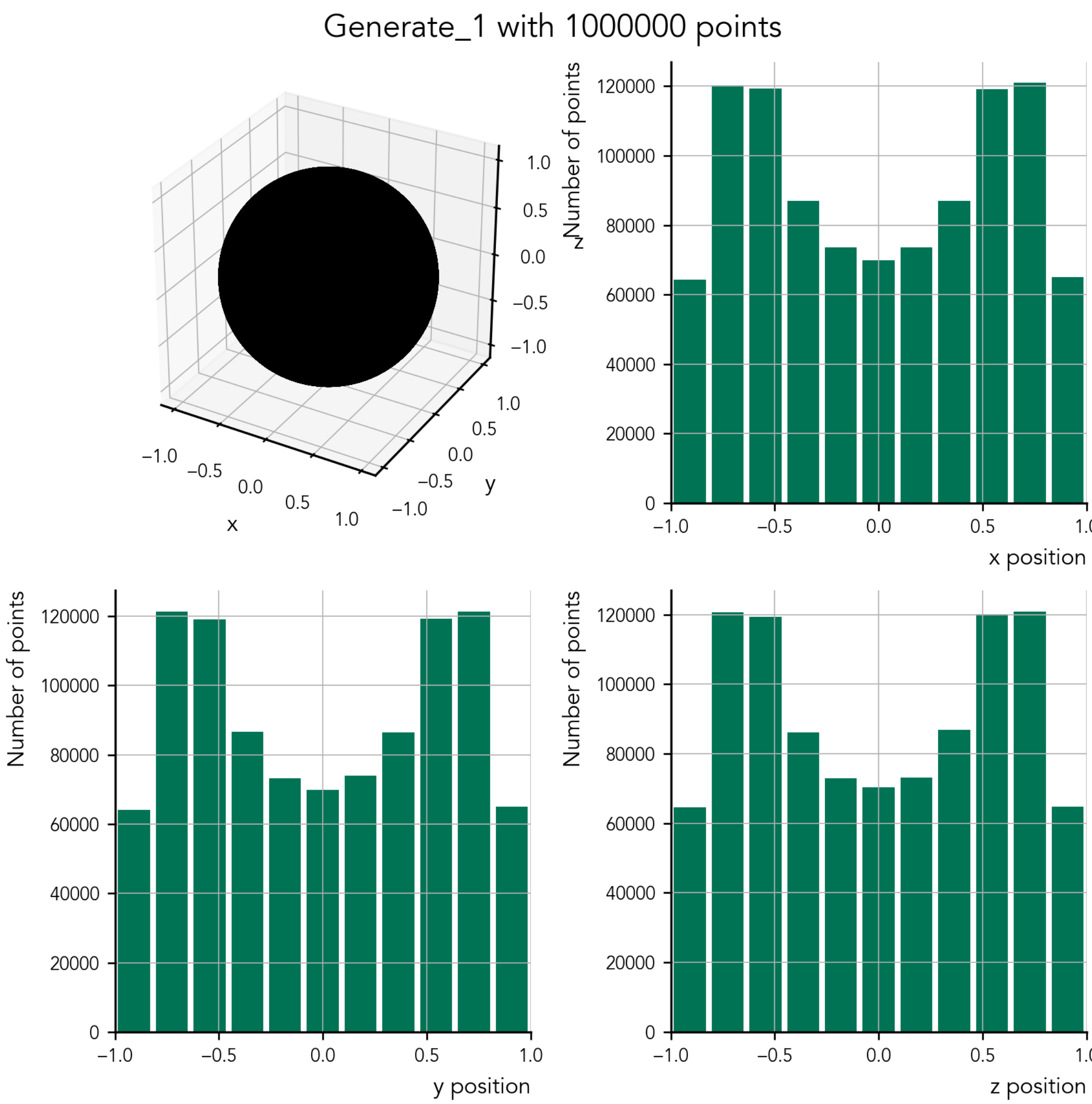



```
[ ]: @Timer("generate_1")
     def generate_1(no_of_points):
         '''
         Generate points on a sphere using the method described in the problem␣
      ↪statement.
```

```
    Parameters
    ----------
    no_of_points : int
        Number of points to generate.

    Returns
    -------
    x : array_like
        Array of x values.
    y : array_like
        Array of y values.
    z : array_like
        Array of z values.
    '''
    #generates n random 3D vectors between -1 and 1
    x = np.random.uniform(-1,1,no_of_points)
    y = np.random.uniform(-1,1,no_of_points)
    z = np.random.uniform(-1,1,no_of_points)
    r = np.array([x,y,z])
    #normalizes the vectors
    r_norm = np.linalg.norm(r,axis=0)
    r_norm = np.tile(r_norm,(3,1))
    r = r/r_norm
    return r[0],r[1],r[2]
```

[ ]:
```
# trying the function
x,y,z = generate_1(1000000)
fig = plot_eval(x, y, z, title=f'Generate_1 with {x.shape[0]} points')
plt.show()
```

```
generate_1 > Elapsed time: 0.0678 seconds
```

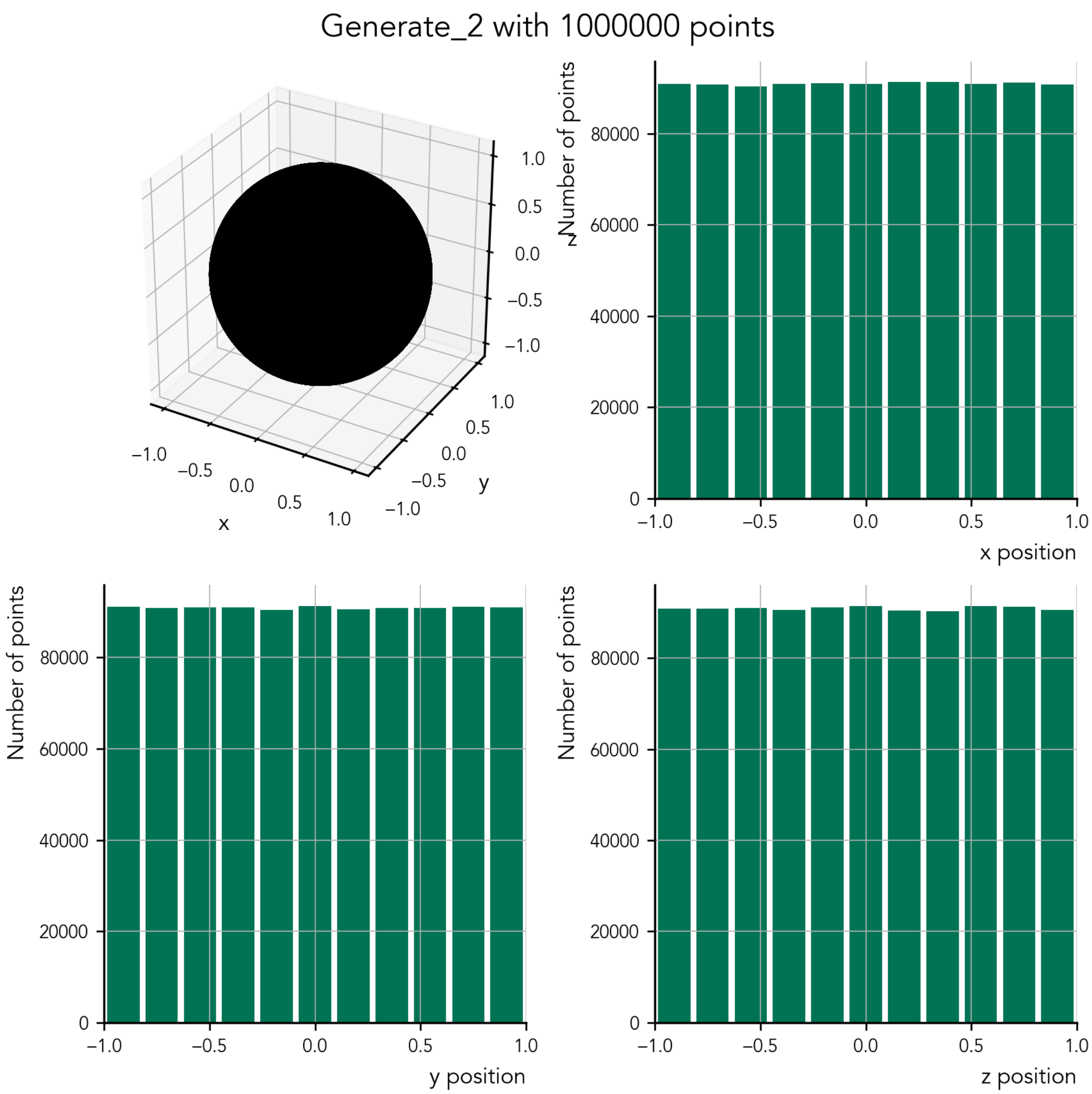


[ ]:
```python
@Timer("generate_2")
def generate_2(no_of_points):
    '''
    Choose 3 random numbers between -1 and 1 and interpret them as a vector, v.␣
↪Discard all vectors with |v| > 1 and normalize the rest, as above. Repeat␣
↪until you have n vectors.
    '''
    x = np.random.uniform(-1,1,int(no_of_points*np.sqrt(5)))
    y = np.random.uniform(-1,1,int(no_of_points*np.sqrt(5)))
    z = np.random.uniform(-1,1,int(no_of_points*np.sqrt(5)))
    r = np.array([x,y,z])
    # print(r.shape)
    r_norm = np.linalg.norm(r,axis=0)
```

```python
    # print(r_norm.shape)
    r_norm = np.tile(r_norm,(3,1))
    # print(r_norm.shape)
    r = r[r_norm<=1]
    # print(r.shape)
    r_norm = r_norm[r_norm<=1]
    r = r/r_norm
    #reshaping the array
    r = r.reshape(3,-1)
    r = r[:,:no_of_points]
    return r[0],r[1],r[2]
```

[ ]:
```python
x,y,z = generate_2(1000000)
fig = plot_eval(x, y, z, title=f'Generate_2 with {x.shape[0]} points')
plt.show()
```

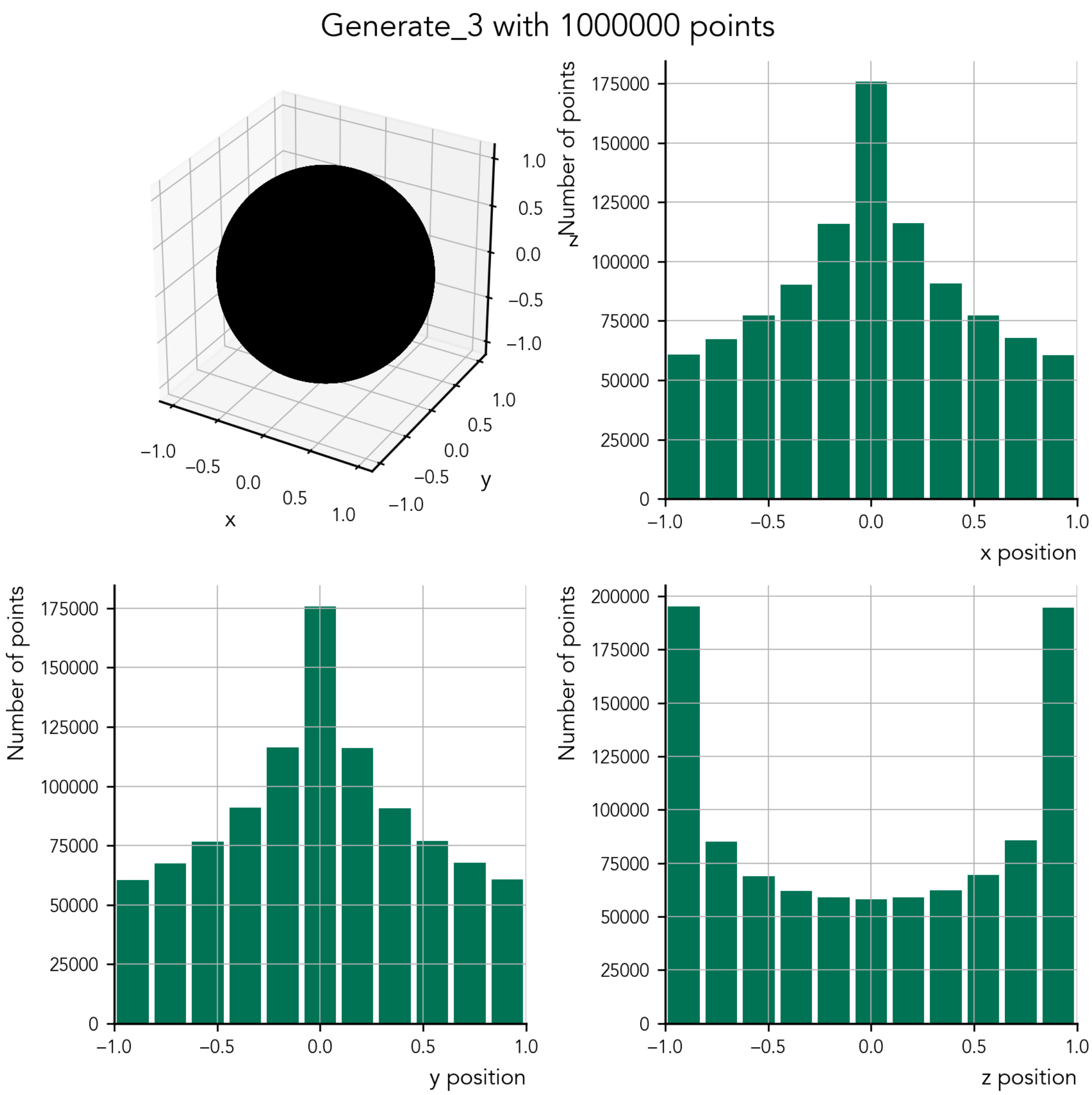
```
generate_2 > Elapsed time: 0.1844 seconds
```




```
[ ]: @Timer("generate_3")
     def generate_3(no_of_points):
         '''
         Choose 2 random numbers    [0, 2 ) and    [0, ] and interpret ( ,   ) as␣
       ↪the spherical
         coordinates of the vector.
         '''
         theta = np.random.uniform(0,np.pi,no_of_points)
         phi = np.random.uniform(0,2*np.pi,no_of_points)
         x,y,z = polar_to_cartesian(theta,phi)
         return x,y,z
```

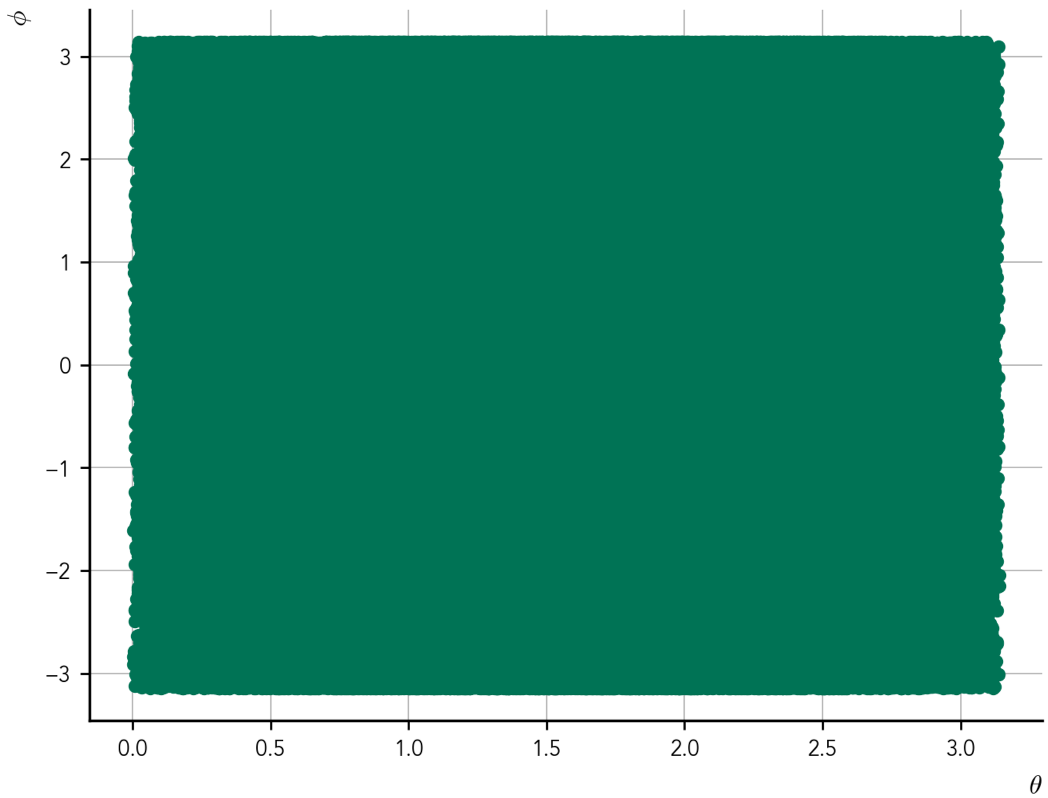
[ ]:
```
x,y,z = generate_3(1000000)
fig = plot_eval(x, y, z, title=f'Generate_3 with {x.shape[0]} points')
plt.show()
```

```
generate_3 > Elapsed time: 0.0775 seconds
```



[ ]:
```
plt.plot(cartesian_to_polar(x,y,z)[0],cartesian_to_polar(x,y,z)[1],'o')
plt.xlabel(r'$\theta$')
plt.ylabel(r'$\phi$')
```

[ ]:
```
Text(0, 1, '$\\phi$')
```



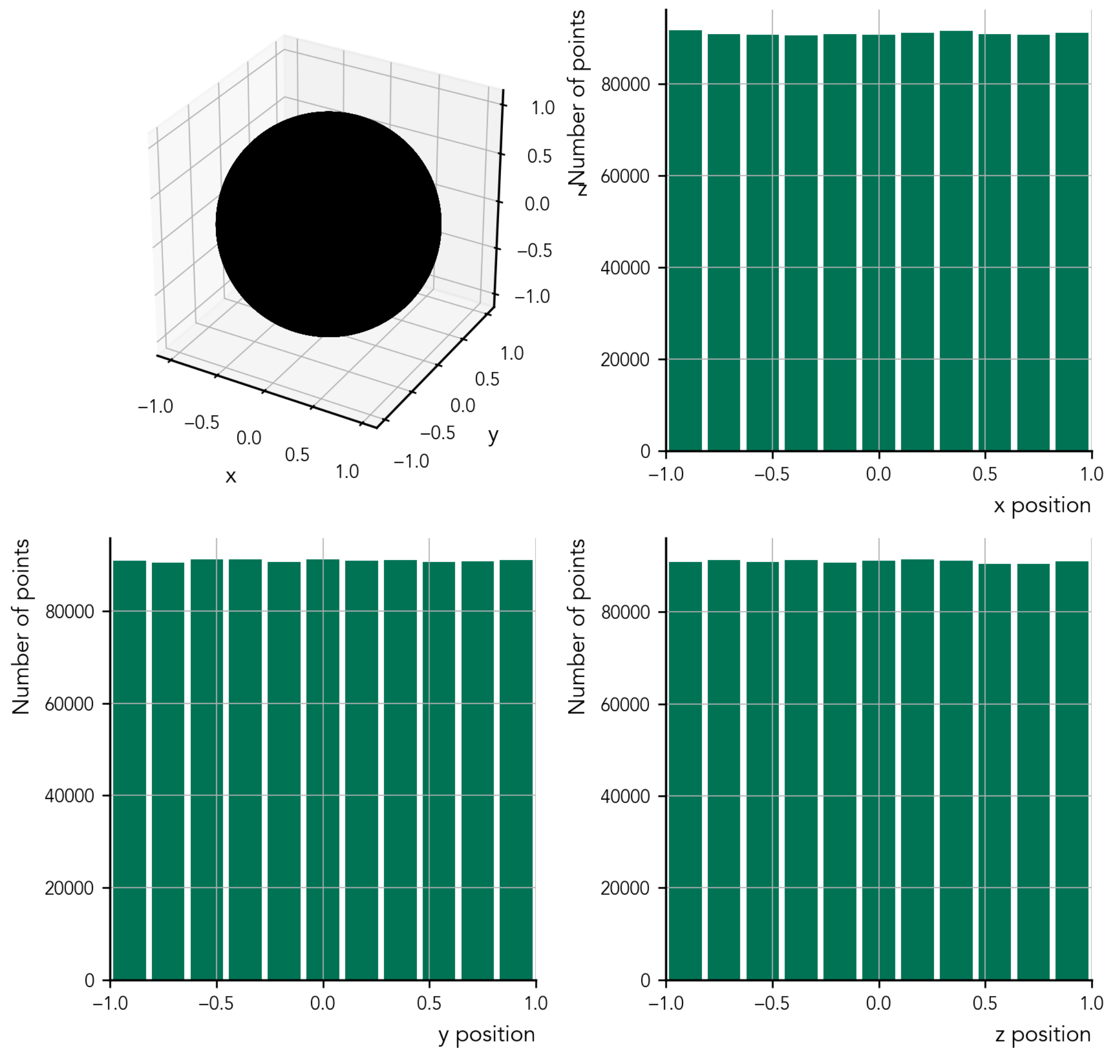
idea s kulovym vrchlikem !!

[ ]:
```
@Timer("generate_4")
def generate_4(no_of_points):
    '''Choose 2 random numbers    [0, 2 ) and cos( )   [-1, 1] and interpret (␣
 ↪,   ) as the
    spherical coordinates of the vector.
    '''
    theta = np.arccos(np.random.uniform(-1,1,no_of_points))
    phi = np.random.uniform(0,2*np.pi,no_of_points)
    x,y,z = polar_to_cartesian(theta,phi)
    return x,y,z
```

[ ]:
```
x,y,z = generate_4(1000000)
fig = plot_eval(x, y, z, title=f'Generate_4 with {x.shape[0]} points')
plt.show()
```

```
generate_4 > Elapsed time: 0.0896 seconds
```

Generate_4 with 1000000 points

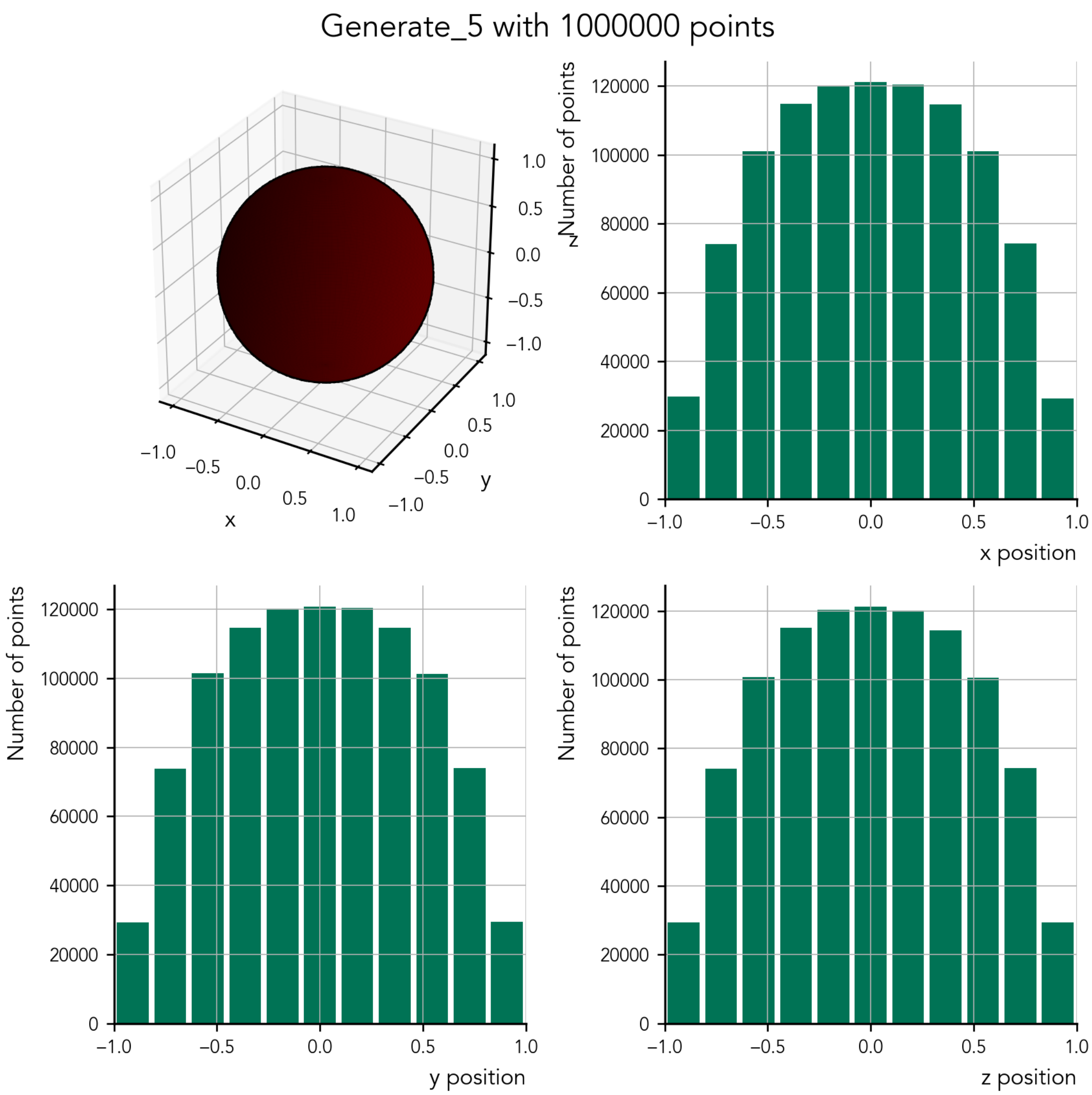



```
[ ]: @Timer("generate_5")
     def generate_5(no_of_points):
         v = np.random.uniform(-1,1,(4, int(no_of_points*3.5)))
         norms = np.linalg.norm(v, axis=0)
         v = v[:, norms<=1]
         norms = norms[norms<=1]

         # take only the first no_of_points
         v = v[:,:no_of_points]
         norms = norms[:no_of_points]

         x = 2*(v[1]*v[3] + v[0]*v[2])/norms
         y = 2*(v[2]*v[3] - v[0]*v[1])/norms
```

```
    z = (v[0]**2 + v[3]**2 - v[1]**2 - v[2]**2)/norms
    return x,y,z
```

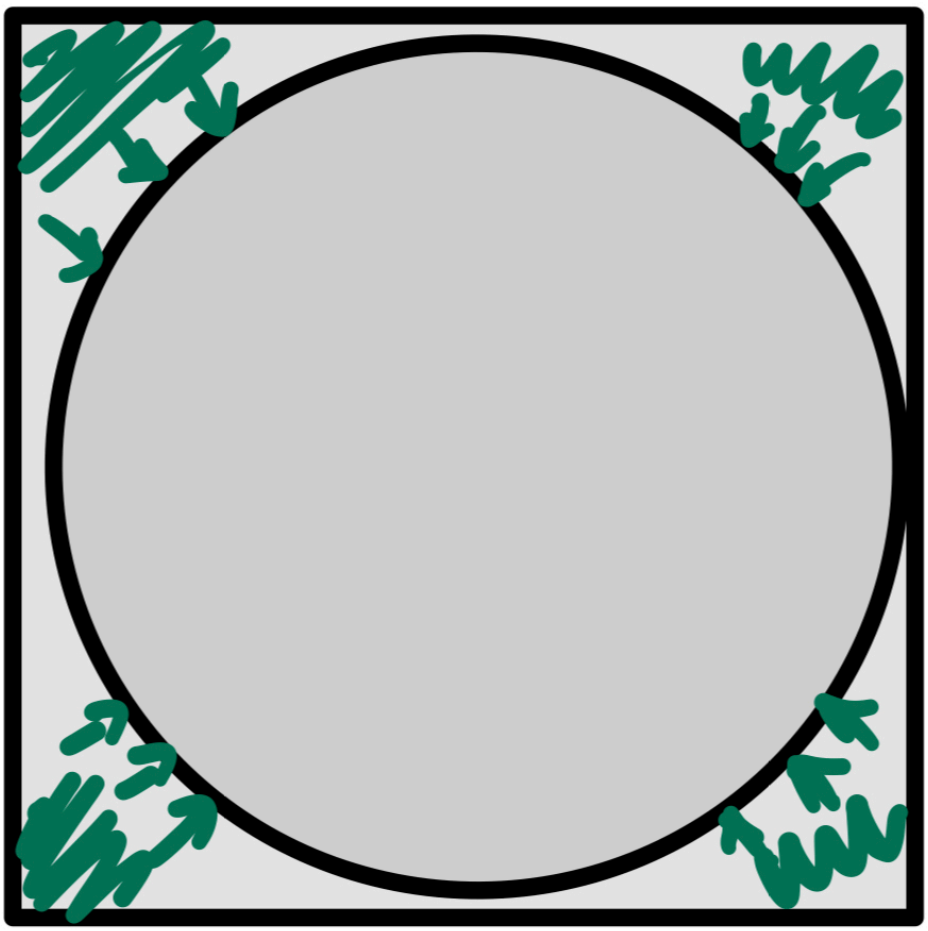
[ ]:
```
x,y,z = generate_5(1000000)
fig = plot_eval(x, y, z, title=f'Generate_5 with {x.shape[0]} points')
plt.show()
```

```
generate_5 > Elapsed time: 0.2663 seconds
```



1.2 summary of speed and uniformity

method	speed (sec/1e6 points)	uniformity
1	0.0678	more around +/- 0.5 in every axis *bcs we are transforming cube into the sphere, see the figure*
2	0.1844	perfectly uniform
3	0.0775	center of x and y, edges of z
4	0.0896	uniform
5	0.2663	more to the center of each axes



2 answers to the questions

b. Which methods work? Which methods do not—and why?

It seems that the method 2 is the best. It is perfectly uniform and it is not that slow. The method 5 is the worst. It is not uniform and it is the slowest. The method 1 is not uniform, but it is the fastest. The method 3 is uniform, but it is not that fast. The method 4 is uniform and it is faster than the method 3.

c. Between methods iii and iv, argue which of these methods is correct.

Methods 2 and 4 as they generate uniform samples.

d. Which method would you choose to be as computationally efficient as possible?

Method 4 as it is fast and uniform.