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Introduction / Motivation (3 pgs)

liquid helium discovery, 1908, Heike Onnes, liquid state at 4.2K, superfluid state below 2.17K, full phase diagram:

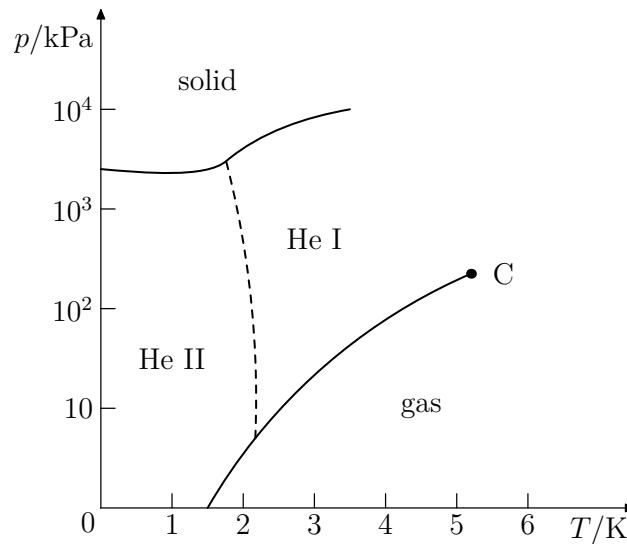


Figure 1: p-T diagram

labelling He-I, He-II, no solid state at 0K (weak van der Waals), only at 2.5MPa
 strange properties, thermal conductivity, negligible viscosity through capillaries
 Landau, Tisza: phenomenology, two-fluid model, proved bz rotating discs:

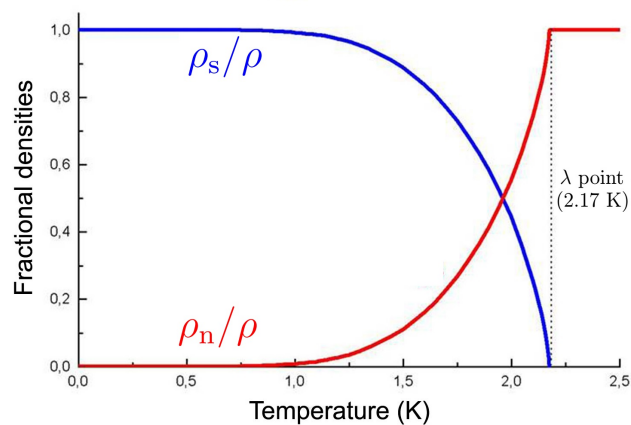


Figure 2: temperature dependence of densities

London: similarity of superfluid component with orbiting electrons, macroscopic wave function
 irrotational fluid, quantum vortices, tangle:

CT experiments: transition to turbulence, drag coeffs

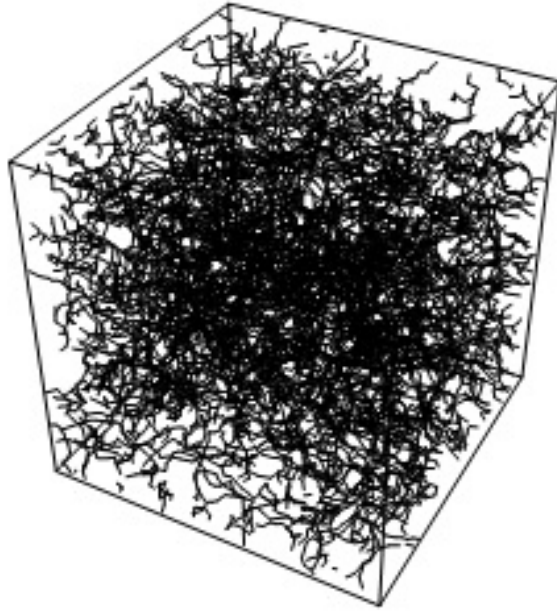


Figure 3: Quantum Turbulence

QT experiments: coflow, counterflow, second sound

QT vs CT: complicated N-S equations, critical velocity or Reynolds number, QT has probably more critical velocities

Simulations: filament model, boundaries

Motivations: investigate critical velocities and vortex density, create numeric model

Goals: measure hydrodynamic profiles for more temperatures with oscillating object, transition from CT to QT, investigate numerically vortex rings

1. Theoretical Background (15 pgs)

The theoretical part of this Thesis is composed of two chapters:

1. Mesoscopic view - theoretically cover London's theory, creation and numerical modelling of quantum vortex, vortex dynamics.
3. Macroscopic view - hydrodynamics of two-fluid model, oscillatory motion in such fluid, creation of QT, existence and usage of second sound

Many of this is covered in textbooks and papers.

He properties, total spin, Bose gas, critical temperature, heat capacity

Mesoscopic view

London's theory

- London's theory
- NLSE (Schr eq)
- macroscopic wave function
- no vorticity
- quantized circulation

Quantum vortex

- definition
- induced velocity
- energy
- quantized circulation
- quantum turbulence

Vortex filament model

- graph model
- state definition
- curve coordinates
- derivatives
- self-induced velocity
- LIA approximation

Vortex dynamics

- magnus force
- mutual friction
- Schwarz's equation
- special case - quantum ring (properties)
- Kelvis waves (?)

Macroscopic view

Hydrodynamics of two-fluid

- Landau's assumptions
- two densities, velocities (+pic)
- updated hydrodynamical equations - HVBK
- dynamical similarity
- Reynolds number

Oscillatory motion in superfluid

- penetration depth
- Re for oscillations
- defining depth and Re separately for normal and superfluid components

Quantum turbulence

- critical velocity according to Landau
- critical velocity scaling in oscillatory case
- T dependence of critical velocities (Bc. results)

Second sound

- what it is
- velocity of second sound
- attenuation
- vortex line density estimate

2. Experimental Approach (10 pgs)

Apparatus

- cryostat
- cooling system
- insert
- resonator

QT Generators

- quartz tuning fork
- other oscillators

QT Detection

- Second sound generating
- SS detection

Measurement methods and Processing

- fork modes - fund, overtone
- SS modes - working with ??-th mode
- frequency sweeps
- amp sweeps
- constant drives with SS on/off

3. Simulations (10 pgs)

Finite differences

- FD order, radius
- Vandermonde vs analytical method
- comp complexities of LIA and Biot-Savart
- coords and velocities updating

Integration

- Euler vs. RK4 step
- time stepping
- stability

Resegmentation

- adding and removing segments
- local spline

Vortex ring

- initialisation
- movement, decreasing radius
- comparison with theory
- Kelvin waves (?)

4. Results (15 pgs)

Vortex line density

- steps made for achieving results
- graphs for fundamental and overtone

Drag force graphs

- velocity vs force
- C vs velocity
- C_n vs Reynolds

Universal Scaling

- Donnelly number
- universal scaling

Correlations

- compare vortex generating with drag force graphs
- fund and overtone

Simulations

- compare rings with various radii
- theoretical vs simulation velocity / range
- stability tests
- Kelvin waves(?)

5. Conclusions (2 pgs)

- summarize mainly what have we done
- repeat motivations and goals
- list of achievements
- list of failures
- list of improvements
- last words

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