Contents

Introduction / Motivation (3 pgs)

liquid helium discovery, 1908, Heike Onnes, liquid state at 4.2K, superfluid state below 2.17K, full phase diagram:

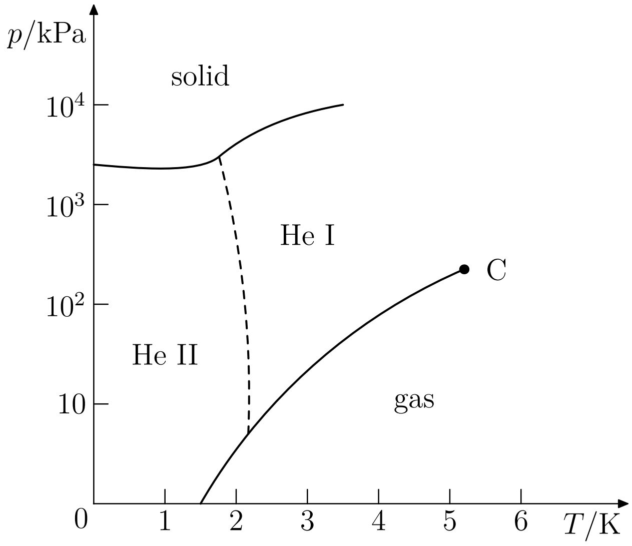

Figure 1: p-T diagram

labelling He-I, He-II, no solid state at 0K (weak van der Waals), only at 2.5MPa

strange properties, thermal conductivity, negligible viscosity through capillaries

Landau, Tisza: phenomenology, two-fluid model, proved bz rotating discs:

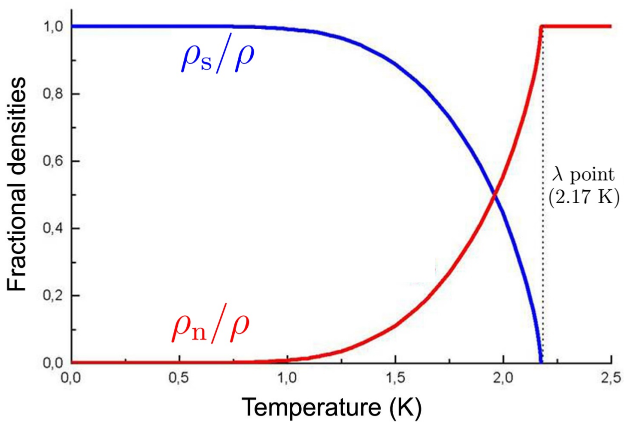

Figure 2: temperature dependence of densities

London: similarity of superfluid component with orbiting electorns, macroscopic wave func

irrotational fluid, quantum vortices, tangle:

CT experiments: transition to turbulence, drag coeffs

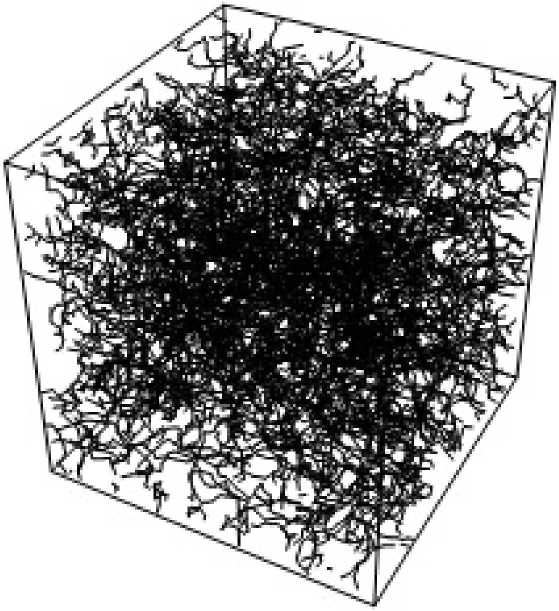

Figure 3: Quantum Turbulence

QT experiments: coflow, counterflow, second sound

QT vs CT: complicated N-S equations, critical velocity or Reynolds number, QT has probably more critical velocities

Simulations: filament model, boundaries

Motivations: investigate critical velocities and vortex density, create numeric model

Goals: measure hydrodynamic profiles for more temperatures with oscillating object, transition from CT to QT, investifate numerically vortex rings

1. Theoretical Background (15 pgs)

The theoretical part of this Thesis is composed of two chapters:

1. Mesoscopic view - theoretically cover London's theory, creation and numerical modelling of quantum vortex, vortex dynamics.
3. Macroscopic view - hydrodynamics of two-fluid model, oscillatory motion in such fluid, creation of QT, existence and usage of second sound

Many of this is covered in textbooks and papers.

He properties, total spin, Bose gas, critical temperature, heat capacity

Mesoscopic view

London's theory

- London's theory
- NLSE (Schr eq)
- macroscopic wave function
- no vorticity
- quantized circulation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - m\varepsilon\psi + V_0\psi|\psi|^2$$

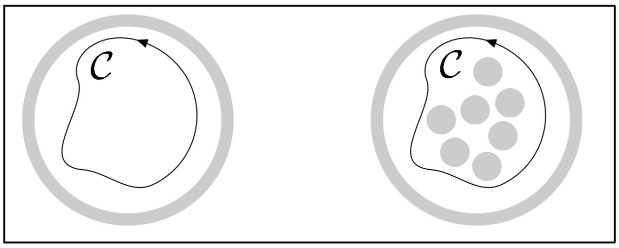

Figure 1.1: singularities within fluid

Quantum vortex

- definition
- induced velocity
- energy
- quantized circulation
- quantum turbulence

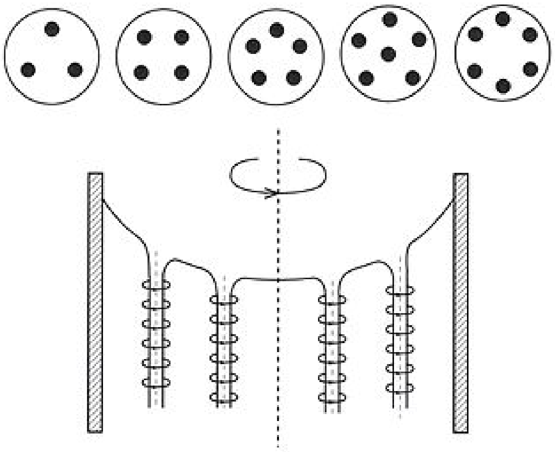

Figure 1.2: Quantized vortices in rotating container

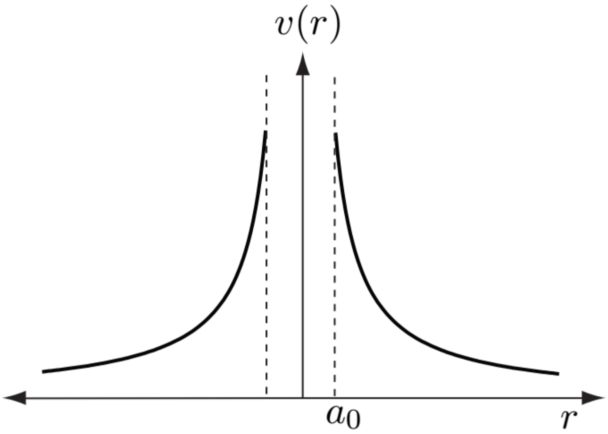

Figure 1.3: decrease of rotational velocity around vortex

Vortex filament model

- graph model
- state definition
- curve coordinates
- derivatives
- self-induced velocity
- LIA approximation

$$\mathbf{f}_M = \rho_s \Gamma \mathbf{s}' \times (\mathbf{v}_L - \mathbf{v}_{tot})$$

$$\mathbf{v}_{tot} = (v)_s + \mathbf{v}_i$$

$$\mathbf{v}_i(\mathbf{s}) = \frac{\Gamma}{4\pi} \int_{\text{line}} \frac{(\mathbf{r} - \mathbf{s}) \times \mathrm{d}\mathbf{r}}{|\mathbf{r} - \mathbf{s}|^3}$$

Vortex dynamics

- magnus force
- mutual friction
- Schwarz's equation
- special case - quantum ring (properties)
- Kelvis waves (?)

Macroscopic view

Hydrodynamics of two-fluid

- Landau's assumptions
- two densities, velocities (+pic)
- updated hydrodynamical equations - HVBK
- dynamical similarity
- Reynolds number

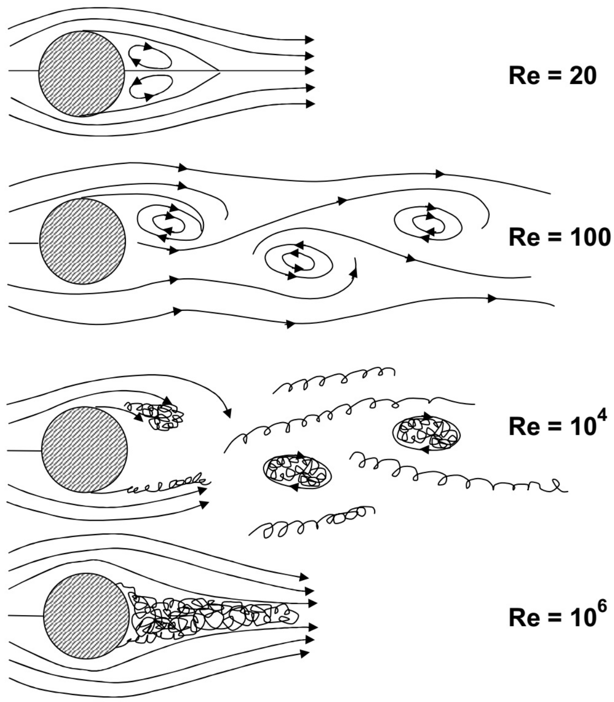

Figure 1.4: transition from laminar to turbulent flow

Oscillatory motion in superfluid

- penetration depth
- Re for oscillations
- defining depth and Re separately for normal and superfluid components

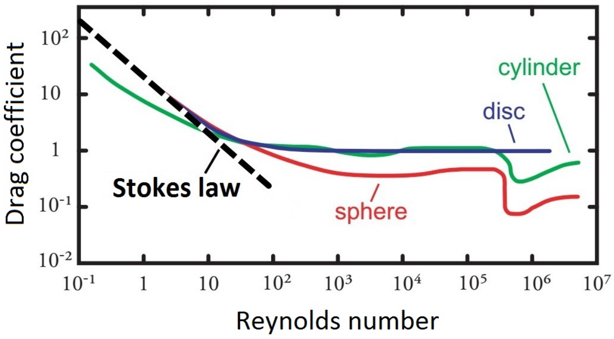

Figure 1.5: drag coeffs of different objects

Quantum turbulence

- critical velocity according to landau
- critical velocity scaling in oscillatory case
- T dependence of critical velocities (Bc. results)

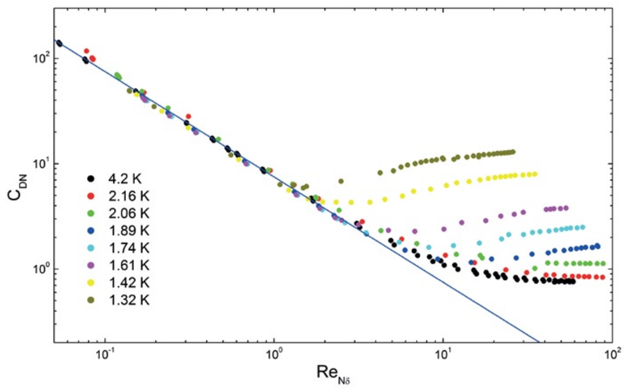

Figure 1.6: drag coeffs vs Reynolds number of normal component

Second sound

- what it is
- velocity of second sound
- attenuation
- vortex line density estimate

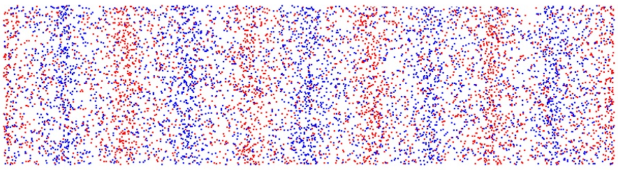

Figure 1.7: first mode of second sound

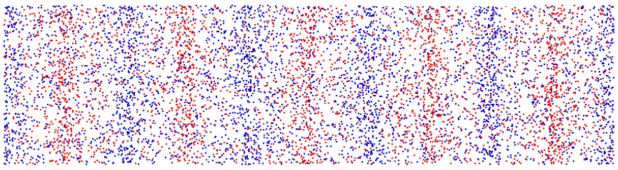

Figure 1.8: second mode of second sound

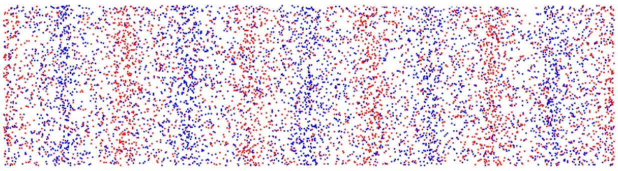

Figure 1.9: first mode of second sound

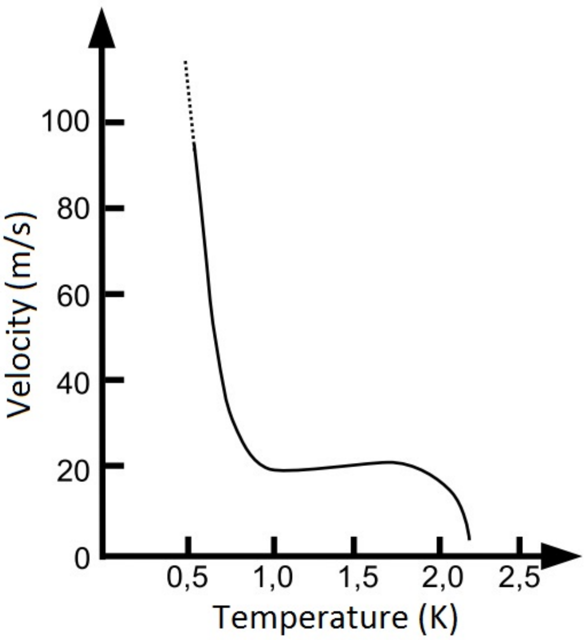

Figure 1.10: velocity of ss with temperature