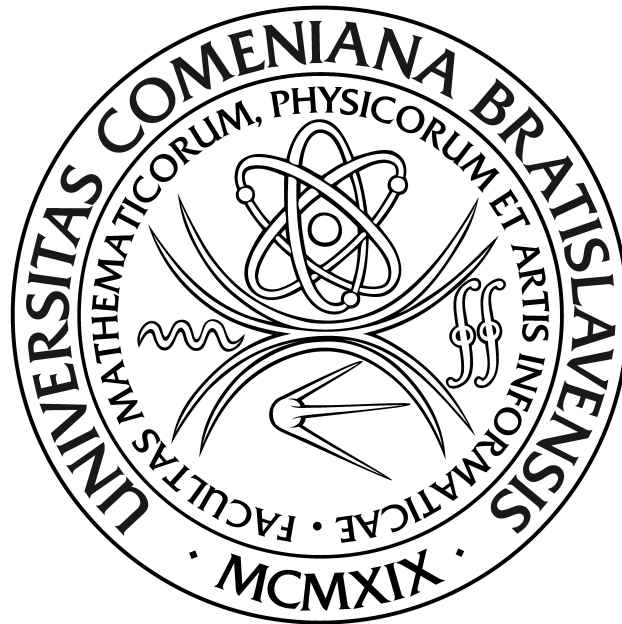


Comenius University

Faculty of Mathematics, Physics and Informatics

# DIPLOMA THESIS



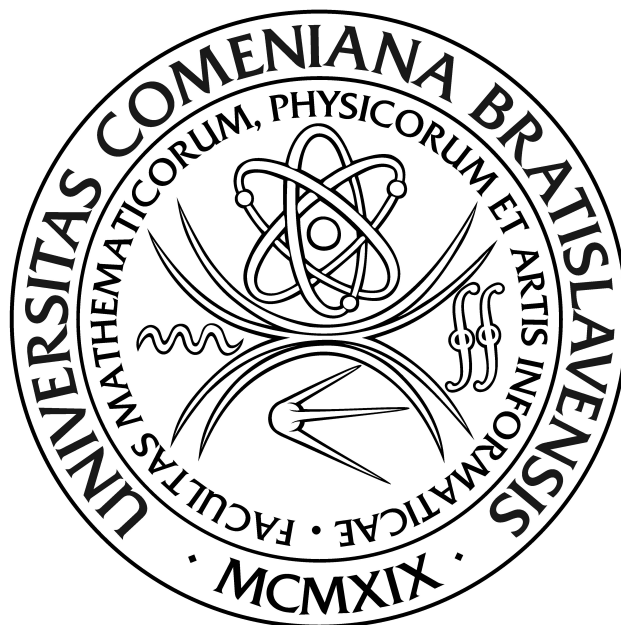
Jakub Bahyl

## **Quantum Turbulence Measurements in Superfluid Helium and Numerical Simulations**

Bratislava, 2018

Comenius University  
Faculty of Mathematics, Physics and Informatics

## DIPLOMA THESIS



Jakub Bahyl

## **Quantum Turbulence Measurements in Superfluid Helium and Numerical Simulations**

Department of Low Temperature Physics, Charles University in Prague

Study programme: Condensed Matter Physics

Study programme number: mFTL

Supervisor of the thesis: RNDr. David Schmoranzer, PhD.

Consultant of the thesis: Mgr. Emil Varga

Bratislava, 2018



Comenius University in Bratislava  
Faculty of Mathematics, Physics and Informatics

## THESIS ASSIGNMENT

<b>Name and Surname:</b>	Jakub Bahyl
<b>Study programme:</b>	Physics (Single degree study, bachelor I. deg., full time form)
<b>Field of Study:</b>	Physics
<b>Type of Thesis:</b>	Bachelor's thesis
<b>Language of Thesis:</b>	English
<b>Secondary language:</b>	Slovak
<b>Title:</b>	Measurement of Quantum Turbulence in Superfluid 4He Using Second Sound Attenuation
<b>Aim:</b>	During the work on this Thesis, the student will become familiar with the theoretical foundations of the description of quantum fluids and superfluidity. The student will learn about several experimental methods used for characterization of the flows of superfluid helium, such as second sound attenuation, which differs significantly from any techniques used in classical fluids. Given the focus of the Thesis, classical hydrodynamics will be used as a stepping-stone to help in the understanding of quantum fluids. On the practical side, the student will learn to design and run complex cryogenic experiments. This work will cover a wide range of skills, starting with the understanding of safety procedures for handling cryoliquids, of specific measures needed for operation of experiments with liquid helium at temperatures between 1.2 K and 4.2 K, and including work with sensitive electronic detectors such as quartz tuning forks, as well as a fundamental understanding of modern automated data acquisition and processing.
<b>Literature:</b>	<p>[1] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Pergamon Books, 1987</p> <p>[2] L. Skrbek a kol., Fyzika nízkých teplot, Matfyzpress, 2011</p> <p>[3] D.R. Tilley, J. Tilley, Superfluidity and Superconductivity, Adam Hilger, 1986</p> <p>[4] R.J. Donnelly: Quantized vortices in helium II, Cambridge University Press, 2005</p> <p>[5] E. Varga, S. Babuin, L. Skrbek, Second-sound studies of coflow and counterflow of superfluid He-4 in channels, Phys. Fluids 27(6), 065101 (2015)</p> <p>[6] Vinen, W. F. Mutual friction in a heat current in liquid helium II. II. Experiments on transient effects. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 240, 1220 (1957).</p> <p>[7] R. Blaauwgeers, M. Blažková, M. Človečko, V.B. Eltsov, R. de Graaf, J. Hosio, M. Krusius, D. Schmoranz, W. Schoepe, L. Skrbek, P. Skyba, R.E. Solntsev, D.E. Zmeev: Quartz Tuning Fork: Thermometer, Pressure- and Viscometer for Helium Liquids, J. Low Temp. Phys. 146, 537-562 (2007)</p> <p>[8] S.L. Ahlstrom, D.I. Bradley, M. Človečko, S.N. Fisher, A.M. Guénault, E.A. Guise, R.P. Haley, O. Kolosov, P.V.E. McClintock, G.R. Pickett, M. Poole, V. Tsepelin, and A.J. Woods, Phys. Rev. B 89, 014515 (2014).</p>
<b>Annotation:</b>	The main focus of this Thesis will be study of oscillatory flows of superfluid helium at low temperatures. Specifically, we will characterize the transition to



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## ZADANIE ZÁVEREČNEJ PRÁCE

- Meno a priezvisko študenta:** Jakub Bahyl  
**Študijný program:** fyzika (Jednoodborové štúdium, bakalársky I. st., denná forma)  
**Študijný odbor:** 4.1.1. fyzika  
**Typ záverečnej práce:** bakalárska  
**Jazyk záverečnej práce:** slovenský  
**Sekundárny jazyk:** anglický
- Názov:** Meranie intenzity kvantovej turbulencie v supratekutom 4He pomocou tlmenia druhého zvuku  
*Measurement of Quantum Turbulence in Superfluid 4He Using Second Sound Attenuation*
- Cieľ:** V rámci tejto práce sa študent po teoretickej stránke zoznámí so základmi popisu kvantových kvapalín, ich supratekutosti a spôsobmi, akými sa dajú charakterizovať supratekuté prúdenia, ktoré sa veľakrát odlišujú od správania sa bežných kvapalín. S ohľadom na zvolenú problematiku budú predmetom štúdia aj vybrané kapitoly klasickej dynamiky tekutín. Prakticky sa študent naučí vykonávať a obsluhovať komplexný kryogénny experiment počínajúc prácou s kryokvapalinami cez nastavovanie špecifických opatrení pre experimenty s kvapalným héliom pri teplotách 1,2 až 4,2 K až po počítačové riadenie experimentu a jeho vyhodnotenie.
- Literatúra:** [1] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Pergamon Books, 1987  
[2] L. Skrbek a kol., Fyzika nízkých teplot, Matfyzpress, 2011  
[3] D.R. Tilley, J. Tilley, Superfluidity and Superconductivity, Adam Hilger, 1986  
[4] R.J. Donnelly: Quantized vortices in helium II, Cambridge University Press, 2005  
[5] E. Varga, S. Babuin, L. Skrbek, Second-sound studies of coflow and counterflow of superfluid He-4 in channels, Phys. Fluids 27(6), 065101 (2015)  
[6] Vinen, W. F. Mutual friction in a heat current in liquid helium II. II. Experiments on transient effects. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 240, 1220 (1957).
- Anotácia:** Náplňou tejto bakalárskej práce bude štúdium prúdenia supratekutého hélia pri nízkych teplotách. Konkrétne sa budeme zaujímať o meranie hustoty kvantových vírov v supratekutom héliu metódou tlmenia druhého zvuku. Budú študované rôzne druhy prúdenia supratekutého hélia vrátane tzv. tepelného protiprúdu, v ktorom sa študent zameria najmä na vznik turbulentného prúdenia v prúdovom kanáli. Jedným z hlavných cieľov bude súbežným meraním na viacerých senzoroch druhého zvuku zistiť, či kvantové víry vznikajú v celom objeme tekutiny súčasne, alebo či sa šíri postupne z miesta prvých nestabilít prúdení. Táto práca bude teda priamo nadväzovať na výskum tejto problematiky, ktorý sa robí v Laboratóriách supratekutosti na MFF UK v Prahe [5].

# Acknowledgment

!!TODO!!

BACHELOR:

My special thanks go to my supervisor, David Schmoranzer, who gave me a lot of experiences, knowledge and much more of his time, without which I definitely could not write this Thesis. Together with consultant Martin Jackson, they guided me during the whole experiment and helped me to understand all the important aspects included in this work. Moreover, I am thankful to Ladislav Skrbek and Patrik Švančara for giving me such a great opportunity to find out how astounding the experimental physics can be and to work with Superfluidity group within Charles University in Prague.

In the end, I would like to hugely thank my friends and family for all their „*Good luck*” and „*Keep going*” motivational quotes and other sort of supports, which I deeply appreciated.

DIPLOMA:

Thanks.

I declare that I carried out this master thesis independently, and only with the cited sources, literature and other professional sources.

In ..... date .....

Signature of the author:

!!TODO!!

## Názov práce

Kvantová turbulencia v supratekutom héliu v teplotnej limite absolútnej nuly

## Autor

Jakub Bahyl

## Školiteľ bakalárskej práce

RNDr. David Schmoranzer, Ph.D.

## Abstrakt

V tejto práci prezentujeme meranie kvantovej turbulencie generovanej oscilujúcim 6.5 kHz kremenným oscilátorom, ponoreným v supratekutom  $^4\text{He}$  pri viacerých teplotách pod kritickou  $T_\lambda = 2.17\text{ K}$ . Pozorovaná nelineárna odporová sila pôsobiaca na oscilátor je kvalitatívne spôsobená prítomnosťou klasickej turbulencie a kvantovaných vírov. Odporové sily a množstvo vytvorených kvantovaných vírov (hustota vírov čiar  $L$ ) sú nepriamo určené z útlmu druhého zvuku a mechanicko-elektrických vlastností oscilátora. Výsledky, ktoré prezentujeme, kvantitatívne charakterizujú tvorbu klasickej aj kvantovej turbulencie. Ako prví ukazujeme, že obidve turbulencie vedia vzniknúť samostatne a nezávisle. Tento fakt je prezentovaný, v prvom priblížení, pomocou *prúdového fázového diagramu*, v ktorom pre danú teplotu vieme predpokladať, kedy a ktorá turbulencia vzniká.

## Kľúčové slová

supratekutosť  $^4\text{He}$  • kvantový vír a turbulencia

---

## Title

Quantum turbulence in superfluid helium down to the zero temperature limit

## Author

Jakub Bahyl

## Supervisor

RNDr. David Schmoranzer, Ph.D.

## Abstract

In this work we present measurements of quantum turbulence generated by a 6.5 kHz oscillating quartz tuning fork submerged in superfluid  $^4\text{He}$  at various temperatures below  $T_\lambda = 2.17\text{ K}$ . The observed non-linear drag acting on the tuning fork is qualitatively described by a presence of classical turbulence and quantized vortices. Drag forces and the amount of produced quantized vortices (the vortex line density  $L$ ) are determined indirectly by the attenuation of second sound and by the measurement of mechanical and electrical properties of the tuning fork. We present results which characterize quantitatively the formation of classical and quantum turbulence. For the first time, we show that both turbulences can arise separately. This is presented, in first approximation, via a *flow phase diagram*, where for a given temperature, one can predict when each type of turbulence forms.

## Keywords

superfluidity of  $^4\text{He}$  • quantum vortex and turbulence

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# Introduction / Motivation (3 pgs)

- history
- phase diagram (+pic)
- superfluid transition
- unconventional properties
- theories from Landau, London
- quantum vortices
- QT (+pic)
- QT vs CT
- overview on experiments with QT
- overview on numerical simulations of QT
- motivations
- goals - CT to QT transition, simulations

# 1. Theoretical Background (15 pgs)

The theoretical part of this Thesis is composed of two chapters:

1. Mesoscopic view
3. Macroscopic view

Content:

- Bose-Einstein gas
- critical temperature

# Mesoscopic view

## London's theory

- London's theory
- NLSE (Schr eq)
- macroscopic wave function
- no vorticity
- quantized circulation

## Quantum vortex

- definition
- induced velocity
- energy
- quantized circulation
- quantum turbulence

## Vortex filament model

- graph model
- state definition
- curve coordinates
- derivatives
- self-induced velocity
- LIA approximation

## Vortex dynamics

- magnus force
- mutual friction
- Schwarz's equation
- special case - quantum ring (properties)
- Kelvis waves (?)

# Macroscopic view

## Hydrodynamics of two-fluid

- Landau's assumptions
- two densities, velocities (+pic)
- updated hydrodynamical equations - HVBK
- dynamical similarity
- Reynolds number

## Oscillatory motion in superfluid

- penetration depth
- Re for oscillations
- defining depth and Re separately for normal and superfluid components

## Quantum turbulence

- critical velocity according to Landau
- critical velocity scaling in oscillatory case
- T dependence of critical velocities (Bc. results)

## Second sound

- what it is
- velocity of second sound
- attenuation
- vortex line density estimate

## 2. Experimental Approach (10 pgs)

### Apparatus

- cryostat
- cooling system
- insert
- resonator

### QT Generators

- quartz tuning fork
- other oscillators

### QT Detection

- Second sound generating
- SS detection

### Measurement methods and Processing

- fork modes - fund, overtone
- SS modes - working with ??-th mode
- frequency sweeps
- amp sweeps
- constant drives with SS on/off

# 3. Simulations (10 pgs)

## Finite differences

- FD order, radius
- Vandermonde vs analytical method
- comp complexities of LIA and Biot-Savart
- coords and velocities updating

## Integration

- Euler vs. RK4 step
- time stepping
- stability

## Resegmentation

- adding and removing segments
- local spline

## Vortex ring

- initialisation
- movement, decreasing radius
- comparison with theory
- Kelvin waves (?)

## 4. Results (15 pgs)

### Vortex line density

- steps made for achieving results
- graphs for fundamental and overtone

### Drag force graphs

- velocity vs force
- $C$  vs velocity
- $C_n$  vs Reynolds

### Universal Scaling

- Donnelly number
- universal scaling

### Correlations

- compare vortex generating with drag force graphs
- fund and overtone

### Simulations

- compare rings with various radii
- theoretical vs simulation velocity / range
- stability tests
- Kelvin waves(?)



## 5. Conclusions (2 pgs)

- summarize mainly what have we done
- repeat motivations and goals
- list of achievements
- list of failures
- list of improvements
- last words

# Bibliography

- [1] KAPITSA, P. *The Super-Fluidity of Liquid Helium II*. Nature **141**, 74 (1938)
- [2] TONG, D. *Statistical physics*. University of Cambridge Part II
- [3] TISZA, L. *The viscosity of liquid helium and the Bose-Einstein statistics*. Comptes Rendus Acad. Sciences, **207**:1186-1189 (1952)
- [4] LANDAU, L.D. and LIFSHITZ, E.M. *Fluid Mechanics*, Second English Edition. Pergamon Books Ltd., (1987). ISBN 0-08-033933-6
- [5] LANDAU, L.D. *The theory of superfluidity of helium II*. J. Phys. USSR, Vol. **11**, 91 (1947)
- [6] ANDRONIKASHVILI, E.L. J. Phys. USSR, **10**, 201 (1946)
- [7] SKRBK, L. et al. *Fyzika nízkých teplot*, 1. vydání. Praha: MatfyzPress, (2011). ISBN 978-80-7378-168-2.