Contents

Introduction / Motivation (3 pgs)

liquid helium discovery, 1908, Heike Onnes, liquid state at 4.2K, superfluid state below 2.17K, full phase diagram:

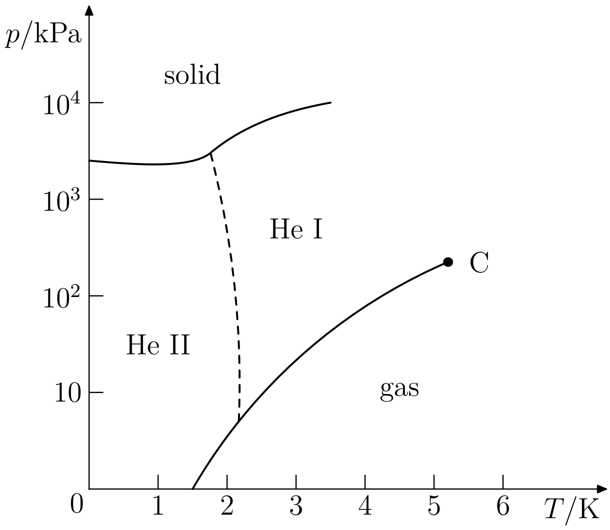

Figure 1: p-T diagram

labelling He-I, He-II, no solid state at 0K (weak van der Waals), only at 2.5MPa

strange properties, thermal conductivity, negligible viscosity through capillaries

Landau, Tisza: phenomenology, two-fluid model, proved bz rotating discs:

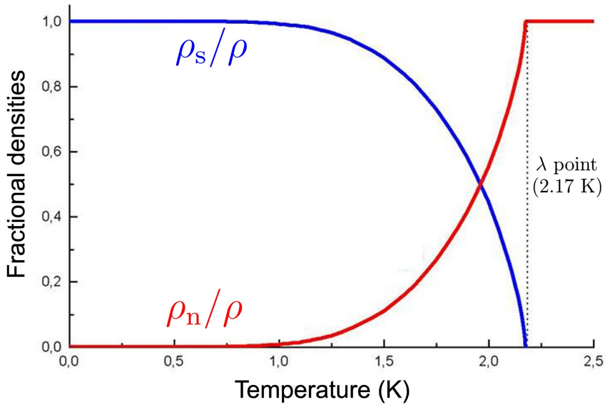

Figure 2: temperature dependence of densities

London: similarity of superfluid component with orbiting electorns, macroscopic wave func

irrotational fluid, quantum vortices, tangle:

CT experiments: transition to turbulence, drag coeffs

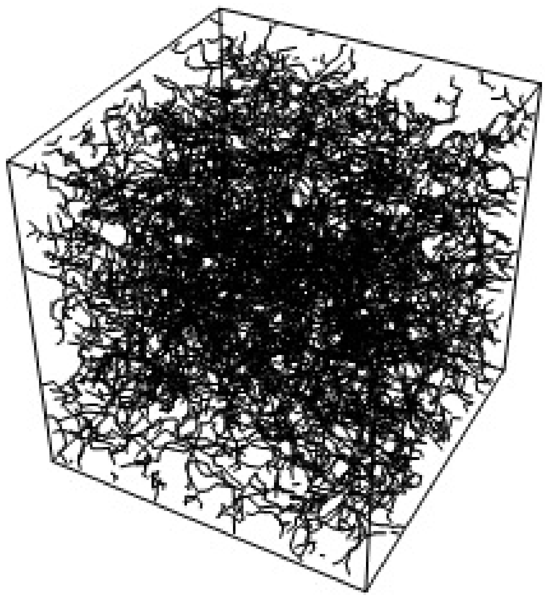

Figure 3: Quantum Turbulence

QT experiments: coflow, counterflow, second sound

QT vs CT: complicated N-S equations, critical velocity or Reynolds number, QT has probably more critical velocities

Simulations: filament model, boundaries

Motivations: investigate critical velocities and vortex density, create numeric model

Goals: measure hydrodynamic profiles for more temperatures with oscillating object, transition from CT to QT, investifate numerically vortex rings

1. Theoretical Background (15 pgs)

The theoretical part of this Thesis is composed of two chapters:

1. Mesoscopic view - theoretically cover London's theory, creation and numerical modelling of quantum vortex, vortex dynamics.

3. Macroscopic view - hydrodynamics of two-fluid model, oscillatory motion in such fluid, creation of QT, existence and usage of second sound

Many of this is covered in textbooks and papers.

He properties, total spin, Bose gas, critical temperature, heat capacity

Mesoscopic view

1.1 London's theory

Gross-Pitaevskii NLSE:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - m\varepsilon\psi + V_0|\psi|^2\psi \tag{1.1}$$

Macroscopic wave function:

$$\psi(\mathbf{r},t) = \sqrt{\frac{\rho_\mathrm{s}}{m_4}}\, e^{i\phi(\mathbf{r},t)}\,, \tag{1.2}$$

London's idea, properties of ψ, zero vorticity

Circulation:

$$\Gamma = \oint_{\mathcal{C}} \mathbf{v}_\mathrm{s}\cdot \mathrm{d}\boldsymbol{\ell} \tag{1.3}$$

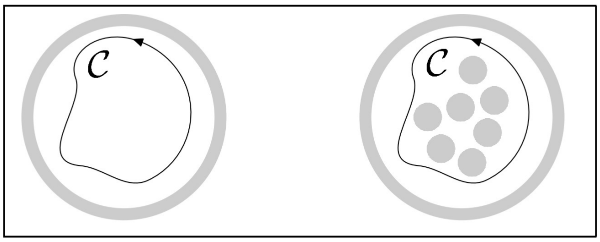

Figure 1.1: singularities within fluid

1.2 Quantum vortex

- definition
- induced velocity
- energy
- quantized circulation
- quantum turbulence

tiny vortex of size 1A, created with energetic excitation and ψ collapse, core contains pure normal component, superfluid is circulating around, limitation by Landau critical velocity

Quantized circulation:

$$\Gamma = \frac{\hbar}{m_4} 2\pi n =\equiv n\kappa \tag{1.4}$$

Ordering of vortices in rotating container:

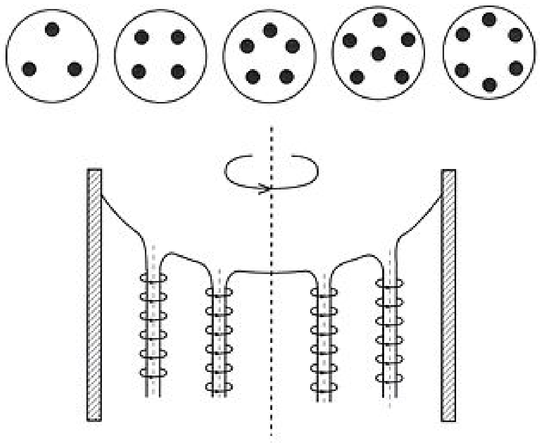

Figure 1.2: Quantized vortices in rotating container

Superfluid velocity distirbution around vortex core:

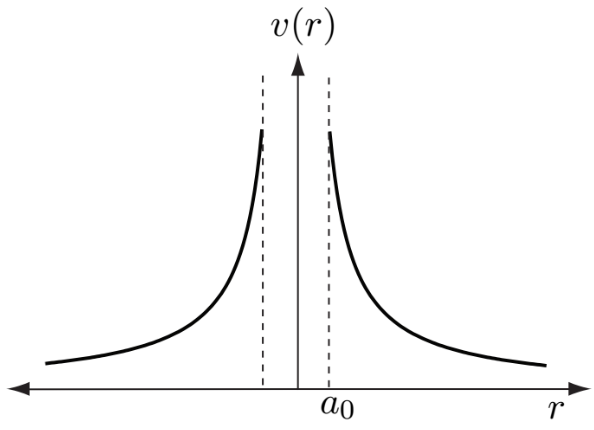

Figure 1.3: decrease of rotational velocity around vortex

1.3 Vortex filament model

each segment know its position and neighbours, defining vectors $\mathbf{s}$, $\mathbf{s}'$, $\mathbf{s}''$ + physical meaning, state definition

Associated vectors with segment:

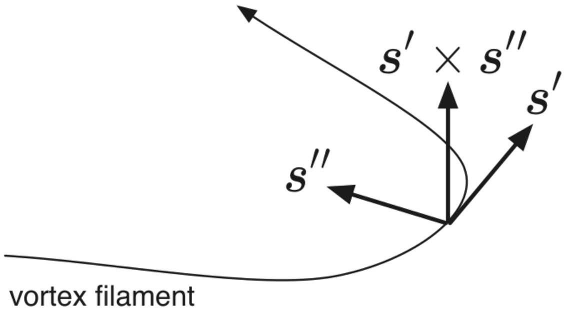

Figure 1.4: vortex filament

Incompressibility:

$$\nabla \cdot \mathbf{v}_s = 0 \tag{1.5}$$

Biot-Savart law:

$$\mathbf{v}_i(\mathbf{s}) = \frac{\Gamma}{4\pi} \int_{\mathcal{L}} \frac{(\mathbf{r} - \mathbf{s}) \times \mathrm{d}\mathbf{r}}{|\mathbf{r} - \mathbf{s}|^3} \tag{1.6}$$

Problems if $\mathbf{s}$ lies on the line

LIA approximation:

$$\mathbf{v}_i(\mathbf{s}) = \beta \mathbf{s}' \times \mathbf{s}'' + \frac{\Gamma}{4\pi} \int_{\mathcal{L}'} \frac{(\mathbf{r} - \mathbf{s}) \times \mathrm{d}\mathbf{r}}{|\mathbf{r} - \mathbf{s}|^3} \tag{1.7}$$

LIA with substitued beta:

$$\mathbf{v}_i \approx \frac{\Gamma}{4\pi} \ln\left(\frac{R}{a_0}\right) \hat{\mathbf{b}} \tag{1.8}$$

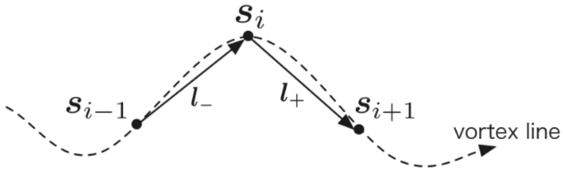

Figure 1.5: vortex filament 2

1.4 Vortex dynamics

- magnus force
- mutual friction
- Schwarz's equation
- special case - quantum ring (properties)
- Kelvis waves (?)

Magnus force:

$$\mathbf{f}_M = \rho_s \Gamma \mathbf{s}' \times (\mathbf{v}_i - \mathbf{v}_{tot}) \tag{1.9}$$

Total velocty:

$$\mathbf{v}_{tot} = \mathbf{v}_s + \mathbf{v}_i \tag{1.10}$$

Drive force (mutual friction):

$$\mathbf{f}_D = -\alpha \rho_s \Gamma \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_i - \mathbf{v}_{tot})] - \alpha' \Gamma \mathbf{s}' \times (\mathbf{v}_i - \mathbf{v}_{tot}) \tag{1.11}$$

Assuming no mass of vortex core:

$$\mathbf{f}_D + \mathbf{f}_M = 0 \tag{1.12}$$

Shwarz's equation:

$$\frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} = \mathbf{v}_s + \mathbf{v}_i + \alpha \mathbf{s}' \times (\mathbf{v}_{ns} - \mathbf{v}_i) - \alpha' \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_{ns} - \mathbf{v}_i)] \tag{1.13}$$

Quantized vortex rings, its dynamics, velocity, life expectancy

Center motion:

$$v_{\text{center}} = \frac{\Gamma}{4\pi R}(\eta - 1/4) \tag{1.14}$$

$$\eta = \ln(8R/a) \tag{1.15}$$

Energy:

$$E = \frac{\rho \Gamma^2 R}{2}(\eta - 7/4) \tag{1.16}$$

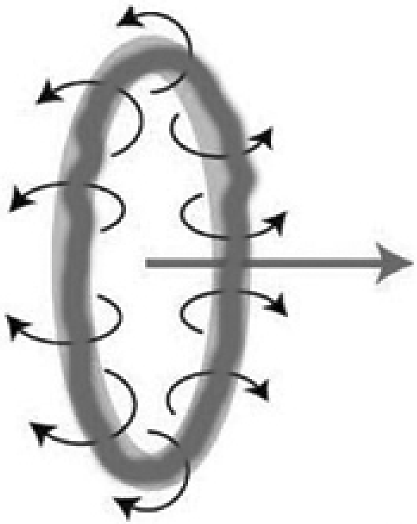

Figure 1.6: dynamics of quantum vortex ring

Kelvin waves (?)

Macroscopic view

1.5 Hydrodynamics of two-fluid

- Landau's assumptions
- two densities, velocities (+pic)
- updated hydrodynamical equations - HVBK
- dynamical similarity
- Reynolds number

HVBK equations:

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\frac{1}{\rho}\nabla P - \frac{\rho_s}{\rho_n} S\nabla T + \nu_n \nabla^2 \mathbf{v}_n + \mathbf{F}_{ns}\,, \tag{1.17}$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s = -\frac{1}{\rho}\nabla P + S\nabla T + \mathbf{T} - \frac{\rho_n}{\rho}\mathbf{F}_{ns}\,, \tag{1.18}$$

, where we have defined:

$$\mathbf{\Omega}_s = \nabla \times \mathbf{v}_s\,, \tag{1.19}$$

$$\mathbf{F} = \frac{B}{2}\hat{\mathbf{\Omega}} \times [\hat{\mathbf{\Omega}}_s \times (\mathbf{v}_n - \mathbf{v}_s - \nu_s \nabla \times \hat{\mathbf{\Omega}})] + \frac{B'}{2}\mathbf{\Omega}_s \times (\mathbf{v}_n - \mathbf{v}_s - \nu_s \nabla \times \hat{\mathbf{\Omega}}_s)\,, \tag{1.20}$$

$$\hat{\mathbf{\Omega}}_s = \mathbf{\Omega}_s / |\mathbf{\Omega}_s|\,, \tag{1.21}$$

$$\mathbf{T} = -\nu_s \mathbf{\Omega}_s \times (\nabla \times \hat{\mathbf{\Omega}}_s) \tag{1.22}$$

$$\nu_s = \frac{\Gamma}{4\pi}\log(b_0/a_0) \tag{1.23}$$

Drag coeff:

$$C_D \propto v^{\alpha}, \qquad \text{where} \begin{cases} \alpha = -1 & \text{for Re} \in (0 - 10) \\ \alpha = 0 & \text{for Re} \in (10^3 - 10^5) \end{cases} .$$

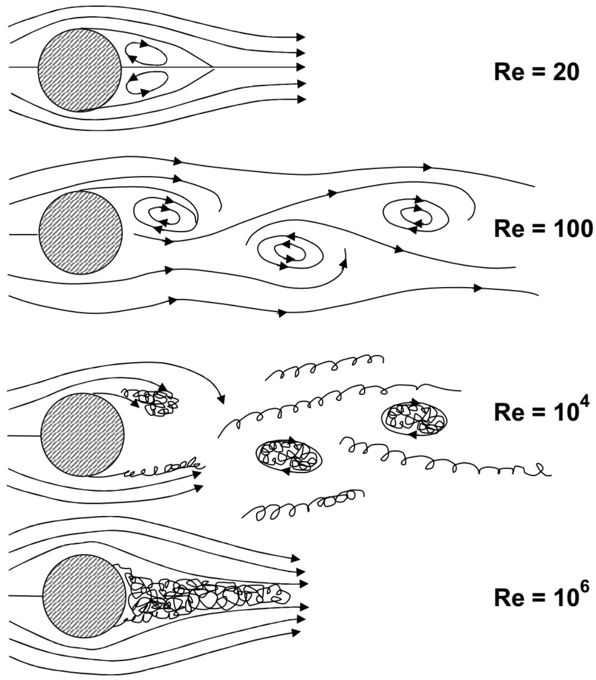

Figure 1.7: transition from laminar to turbulent flow

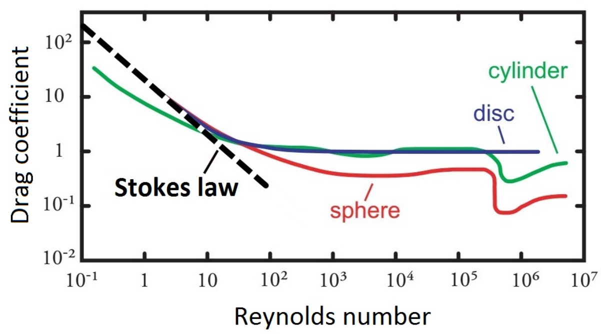

Figure 1.8: drag coeffs of different objects

1.6 Oscillatory motion in superfluid

- penetration depth
- Re for oscillations
- defining depth and Re separately for normal and superfluid components

Attenuated wave:

$$\mathbf{v} \propto e^{-|\mathbf{r}|/\delta}\hat{\mathbf{e}}_\mathbf{r}(\omega, t)\,, \tag{1.24}$$

Penetration depth:

$$\delta = \sqrt{\frac{2\nu}{\omega}} \,. \tag{1.25}$$

Oscillatory Reynolds number:

$$\mathrm{Re}_\delta = \frac{v_0 \delta}{\nu} = \frac{v_0}{\sqrt{\nu \pi f}} \,. \tag{1.26}$$

Depth and Re for normal component:

$$\delta_\mathrm{n} = \sqrt{\frac{2\eta}{\rho_\mathrm{n}\omega}} \,, \qquad \mathrm{Re}_\mathrm{n} = \frac{v_0 \delta_\mathrm{n} \rho_\mathrm{n}}{\eta} \,. \tag{1.27}$$

1.7 Quantum turbulence

- critical velocity according to landau
- critical velocity scaling in oscillatory case
- T dependence of critical velocities (Bc. results)

Critical velocity scaling:

$$v_\mathrm{crit} \propto \sqrt{\omega} \tag{1.28}$$

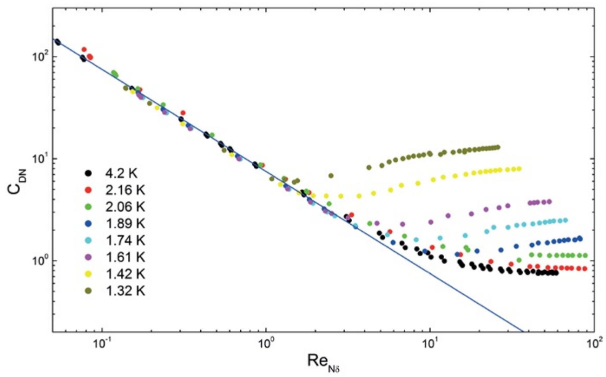

Figure 1.9: drag coeffs vs Reynolds number of normal component

1.8 Second sound

- what it is

- velocity of second sound
- attenuation
- vortex line density estimate

$$\mathbf{v}_{\mathrm{ns}} \propto e^{-\alpha z} \hat{\mathbf{e}}_{\mathbf{r}}(\mathbf{k}, \mathbf{r}, \omega, t)\,, \tag{1.29}$$

$$\alpha^* = \frac{B\kappa L}{6c_2}\,. \tag{1.30}$$

First and second sounds:

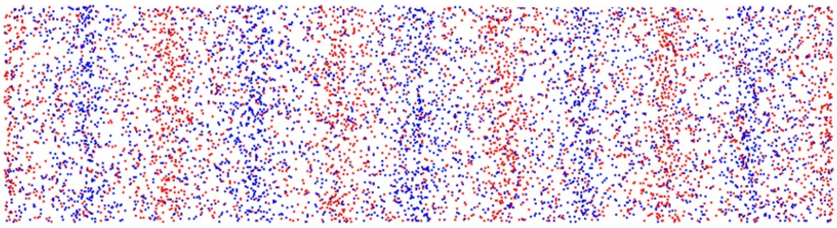

Figure 1.10: first mode of second sound

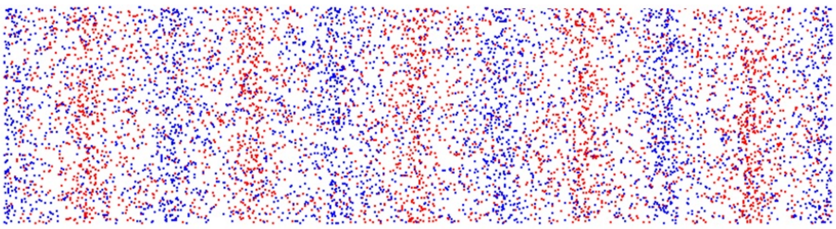

Figure 1.11: second mode of second sound

Velocity of second sound:

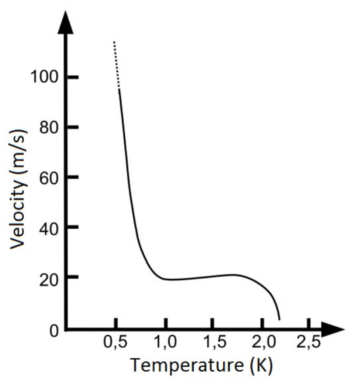

Figure 1.12: velocity of ss with temperature