

## ECON 381 HOMEWORK 4

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**1.** A linked, weighted graph's MST is a subgraph that joins all of its vertices together, with no cycles and the smallest feasible total edge weight.

Iteratively adding the least expensive edge that doesn't form a cycle is how Kruskal's method operates.

The steps for the provided graph are as follows:

A. We sort each edge by weight

- A-B (3)

- B-C (3)

- A-D (4)

- C-E (4)

- D-E (7)

- B-E (9)

B. We begin with a blank graph.

C. We include edges in a sorted manner, except those that form loops

Add A to B (3), B to C (3), A to D (4), and C to E (4).

The edges A-B, B-C, A-D, C-E make up the MST, which has a total weight of 14.

**2.** If every edge weight is different, then an MST will be unique. There may be more than one MST if there are several edges with the same weight. The edges A-D and C-E in the above graph have the same weight (4), as do edges A-B and B-C (3). Consequently, there is no assurance that the MST is unique. To obtain another MST with the same total weight, for instance, we may have selected B-E rather than A-D.

**3.** Dijkstra's Algorithm for shortest paths:

The shortest routes between a single source vertex and every other vertex in a graph are determined by Dijkstra's algorithm.

Steps	Current Node	Visited Nodes	Distances
0	A	(A)	A:0, B:3, C:infinity, D:4, E: infinity
1	B	(A,B)	A:0, B:3, C:6, D:4, E: infinity
2	D	(A,B,D)	A:0, B:3, C:6, D:4, E:11
3	C	(A,B,D,C)	A:0, B:3, C:6, D:4, E:10
4	E	(A,B,C,D,E)	A:0, B:3, C:6, D:4, E:10

**4.** An edge that, when removed, increases the number of connected components in the graph is known as a critical edge. Put otherwise, the graph becomes disconnected if a vital edge is removed.

A-D is a key edge in this graph. Nodes A and D will be in different components if A-D is removed.

**5.** A cut vertex, is a vertex whose removal expands the graph's linked components.

Node B is an articulation point in this graph. Nodes A and D will be isolated from C and E if B is eliminated.

**6.** Yes. The graph won't disconnect if a single node (such as C) is removed if there are no essential edges or articulation points. A different route from B to E must thus exist.

**7.** The capacity of a graph to remain connected and functional even after nodes or edges are removed is known as graph resilience. Two important makers of a graph's vulnerability are critical edges and articulation points. A graph that has a lot of articulation points and crucial edges is not as strong.