Algorithmic aspects of game theory. Homework 1

Deadline. 30 April by the end of the day.

Grading. **1.5 point**. Team work is accepted but the grade will be distributed among the authors. In case of a team work, each author is asked to give a short description of her/his own contribution. Solution may be written in English or in Polish.

Fair–merge game

We consider a game over a finite colored arena (cf. lecture of 15 March)

$$G = \langle Pos_{\exists}, Pos_{\forall}, Move, C, rank, \mathbf{C}_{\exists}, \mathbf{C}_{\forall} \rangle,$$

where the winning objective for Eve is

$$\begin{aligned} \mathbf{C}_{\exists} &= \{u \in C^{\omega} : \forall c \in C, c \in \mathit{Inf}\,(u)\} \\ \mathbf{C}_{\forall} &= C^{\omega} - \mathbf{C}_{\exists}, \end{aligned}$$

where $\mathit{Inf}\,(u) = \{c : |u^{-1}(\{c\})| = \omega\}$. In words: Eve wants to see *every* color infinitely often.

Tasks

1. Design an algorithm that, for a finite arena, decides the winner of each position.
   Note. Try to achieve polynomial time.
2. Is it true that, whenever Eve has a winning strategy, she can use a *positional* winning strategy ?
3. Is it true that, whenever Adam has a winning strategy, he can use a *positional* winning strategy ?

Please note that the answers may depend on the cardinality of C.