Algorithmic aspects of game theory. Homework 3

Deadline. 20 May by the end of the day.

Grading. **1.5 points**. Team work is accepted but the grade will be distributed among the authors.

In case of a team work, each author is asked to give a short description of her/his own contribution.

Solution may be written in English or in Polish.

Easy game

Let G be a parity game, and let $W_\exists$ be the *winning region* of Eve, i.e., the set of positions from where Eve has a winning strategy.

We say that a parity game G is *easy* for Eve if any function $f : W_\exists \cap Pos_\exists \to Pos$ satisfying

$$(\forall v \in W_\exists \cap Pos_\exists)\ f(v) \in W_\exists$$

is a winning positional strategy for Eve.

Informally speaking, staying in the winning region is enough for Eve to win.

Tasks

Give examples of games which do have, and which don't have this property.

Show that the following computational problems are polynomially equivalent, i.e., reducible to each other in polynomial time[1].

- Given a finite parity game, decide if it is easy for Eve.
- Given a finite parity game, find the winning regions (i.e., solve the game).

[1] This means that there is a polynomial-time algorithm solving problem A, using problem B as oracle, cf. https://en.wikipedia.org/wiki/Polynomial-time_reduction.