Algorithmic aspects of game theory. Homework 6

Deadline. 20 May by the end of the day.

Grading. **1 point**. Team work is accepted but the grade will be distributed among the authors.

In case of a team work, each author is asked to give a short description of her/his own contribution.

Solution may be written in English or in Polish.

Universal graphs

Let $n, h \in \mathbb{N}$. We consider ordered trees of height h which are *leveled*, meaning that each leaf has a path of length h from the root. We say that a tree T embeds a tree T' if T' can be obtained by removing nodes of T. The size of a tree is its number of leaves. A tree is (n, h)-universal if it has height h and embeds all trees with n leaves.

Recall from the tutorial that there for all n, h, there are (n, h)-universal trees of quasipolynomial size

$$2n\binom{\lfloor \log n \rfloor + h - 1}{h - 1}.$$

Task

Show that a (n, h)-universal tree has size at least

$$\binom{\lfloor \log n \rfloor + h - 1}{h - 1}.$$

Remark. *It is open whether we can close the linear gap between upper and lower bound.*