Keeping an Eye on Hidden Markov Models in Gaze Data Classification: Supplementary

Material Simulation Study

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Approximate Ranges for Response Variables

The approximate ranges that were used to define true parameter ranges and default values are displayed in Table 1. Since we would also like to test the algorithm on extreme data distributions, we extended the ranges beyond those found in typical eye movement data.

Starting Values and Missing Data

In the third part of the simulation study, we increased the variation in the starting values used for parameter estimation. The model parameters were set to their default values. For the shape, scale, and concentration parameters, we simultaneously increased the scale parameters of the starting value gamma distributions: For $k \in \{2, 3, 4\}$ states and $\beta_{start} = (\psi_{true}/2)\tau_{start}$ with $\tau_{start} \in \{1, 2, 3\}$, 100 data sets with $N = 2500$ samples were generated each ($100 \times 3 \times 3 = 900$ recoveries).

In the last part, we set intervals of the generated data to be missing. The model parameters were set to their default values. For $k \in \{2, 3, 4\}$ states and $m \in \{1, 3, 5\}$ intervals, 100 data sets with $N = 2500$ samples were generated ($100 \times 3 \times 3 = 900$ recoveries). The length of the missing data interval $l \in [1, 200]$ samples varied in equidistant steps between the data sets.

Results

Addendum Part One

Figure 1 displays the RMdSPD between true and estimated parameters depending on which parameter has been manipulated in the HMM. The RMdSPD was below 0.1 for all

estimated and manipulated parameters, indicating good recovery. The regressions between manipulated true and estimated parameters are shown in Figures 2 and 3. With one outlier at parameter $\alpha_{acc;1}$, the estimated parameters matched the true parameters very closely. The deviation seemed to increase slightly with parameter magnitude. Thus, parameter change was capture well and the estimation almost unbiased. Considering accuracy, Figure 4 displays Cohen's kappa between true and estimated hidden state sequences. With two exceptions, kappa values were almost one, suggesting nearly perfect agreement. In sum, the HMM with two states recovered parameters and hidden states very well and outliers only occurred rarely.

Addendum Part Two

This section describes how we exploratorily examined the recovery of parameters depending on the amount of noise added to the data. Figure 5 shows the trends in absolute proportional deviation between estimated and true parameters (i.e., $\text{abs}((\hat{\theta}_i - \theta_i)/\theta_i)$) for the two-state HMM. We observed that the that the deviation is low for most parameters and does not change with increased noise. Exceptions were the scale parameters of the gamma distributions in both states, which started around 0.6 and almost linearly increased (with a plateau at the end) until 2.5 absolute proportional deviation when the noise level rose. In light of the previously adopted scale for RMdSPD, we conclude that these parameters are badly recovered across all noise levels (i.e., they are higher than 0.5). Conversely, for the parameters of the von Mises distribution in state two, the deviation exponentially declined with increasing noise, starting from 1 and ending at almost 0. The exponential decline occurs because we manipulated the noise for values generated by the von Mises distribution on the inverse scale. Thus, for high noise the von Mises parameters are badly recovered, whereas for lower levels of noise they are well recovered.

Figure 6 shows a similar pattern for the three-state HMM. In contrast to the previous model, the mean parameter of the von Mises distribution in state two exhibited increased

deviation for lower noise levels. Moreover, the deviations for transition probabilities from state two to state three increased slightly with higher noise.

The trends for the HMM with four states are shown in Figure 7. States two and three behave similarly as in the previous model, except that the probabilities to transition to state one are initially high and decrease with higher levels of noise. For state one, the trends are almost indistinguishable from each other and do not visibly change with the noise level. State 4 still shows an increasing trend for the scale parameters and decreasing trends for transition probabilities towards states one and three.

In conclusion, this exploratory analysis showed that the scale parameters of the gamma distributions were already badly disturbed by small amounts of noise. Parameters of the von Mises distributions were susceptible to deviations through large noise levels but recovered well for lower noise levels.

Part Three

The third part of the simulation investigated how increasing the variation in generating starting values affected parameter recover and accuracy of the HMM. For the two-state HMM, all parameters displayed RMdSPDs lower than 0.1 (see Figure 8), suggesting good recovery. Cohen's kappa values were slightly lower than 1, indicating almost perfect accuracy (see Figure 9). For the three-state HMM, RMdSPDs were lower than 0.1 except for a_{12} and a_{31}, which were below 0.5 (see Figure 10), which means they were moderately well recovered. As in previous parts, Cohen's kappa values were mostly above 0.95 (almost perfect accuracy) with exceptions clustering around zero and -0.33 (see Figure 9). Regarding the four state-HMM, RMdSPDs of transition probabilities for states one and four were clustered above 0.1, other estimated parameters in the model showed values below 0.1 (see Figure 11). The HMM showed substantial to almost perfect accuracy, as Cohen's kappa values were mostly above 0.8 with a few values clustering around 0.6, 0.25, and zero (see Figure 9). Taken together, increasing the variation in starting values for

parameter estimation did neither affect the parameter recovery nor the accuracy.

Part Four

In this part, we explored post-hoc whether clusters of low Cohen's kappa values were due to label switching. According to Visser and Speekenbrink (2019), label switching occurs when exchanging the order of the states leads to equally likely models. Thus, the model is accurate but has oppositely estimated states compared to the true states. In this case, an HMM with three states that is perfectly accurate but has two labels switched would have a Cohen's kappa of 0. To test if label switching occurred, we manually switched one or two labels post-hoc (see Figure 12). It can be seen that this approach resolved low accuracy clusters for three and four states.

In the last part, data intervals of varying length were set to be missing. Regarding parameter recovery, RMdSPDs were almost exactly mirroring those in the previous part for two, three, and four states (see Figures 13, 14, and 15, respectively). Looking at accuracies, Figure 16 illustrates an interaction between the length of and amount of missing data intervals. Cohen's kappa values linearly decreased with the length of missing data intervals and the decrease became linearly steeper when more intervals were included. For two and three state models, kappas started at almost one (almost perfect) and decreased to 0.6 (substantial accuracy) for five missing intervals. For the four-state model, kappas started around 0.85 (almost perfect) and decreased to 0.5 (substantial accuracy) for five missing intervals. As in previous parts, several kappa values clustered around zero and -0.33 for the three-state model and around 0.6, 0.25, and zero for the four-state model. In total, missing data did not affect the parameter recovery but linearly decreased the accuracy of the model from a nearly perfect to a moderate extent.

References

Larsson, L., Nystrom, M., & Stridh, M. (2013). Detection of saccades and

    postsaccadic oscillations in the presence of smooth pursuit. *IEEE Transactions*

    on Biomedical Engineering, 60(9), 2484–2493.

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Visser, I., & Speekenbrink, M. (2019). *Mixture and hidden Markov models with R*

    including latent class models. Manuscript in preparation.

Table 1

Approximate Ranges of Response Variables Used to Generate Parameter Values in the Simulation

Event	Resp.variable	Range
Fixation	Velocity	0-50
Fixation	Acceleration	0-50
Fixation	Angle	uniform
Saccade	Velocity	50-1000
Saccade	Acceleration	50-500
Saccade	Angle	~0
PSO	Velocity	20-100
PSO	Acceleration	10-90
PSO	Angle	~pi
Smooth pursuit	Velocity	20-100
Smooth pursuit	Acceleration	0-30
Smooth pursuit	Angle	~0

Note. Units are deg/s (velocity), deg/s (acceleration), and radians (angle). ~ indicates that the distribution has a peak at this value. Velocity ranges are based on event velocities reported in Larsson, Nystrom, and Stridh (2013).
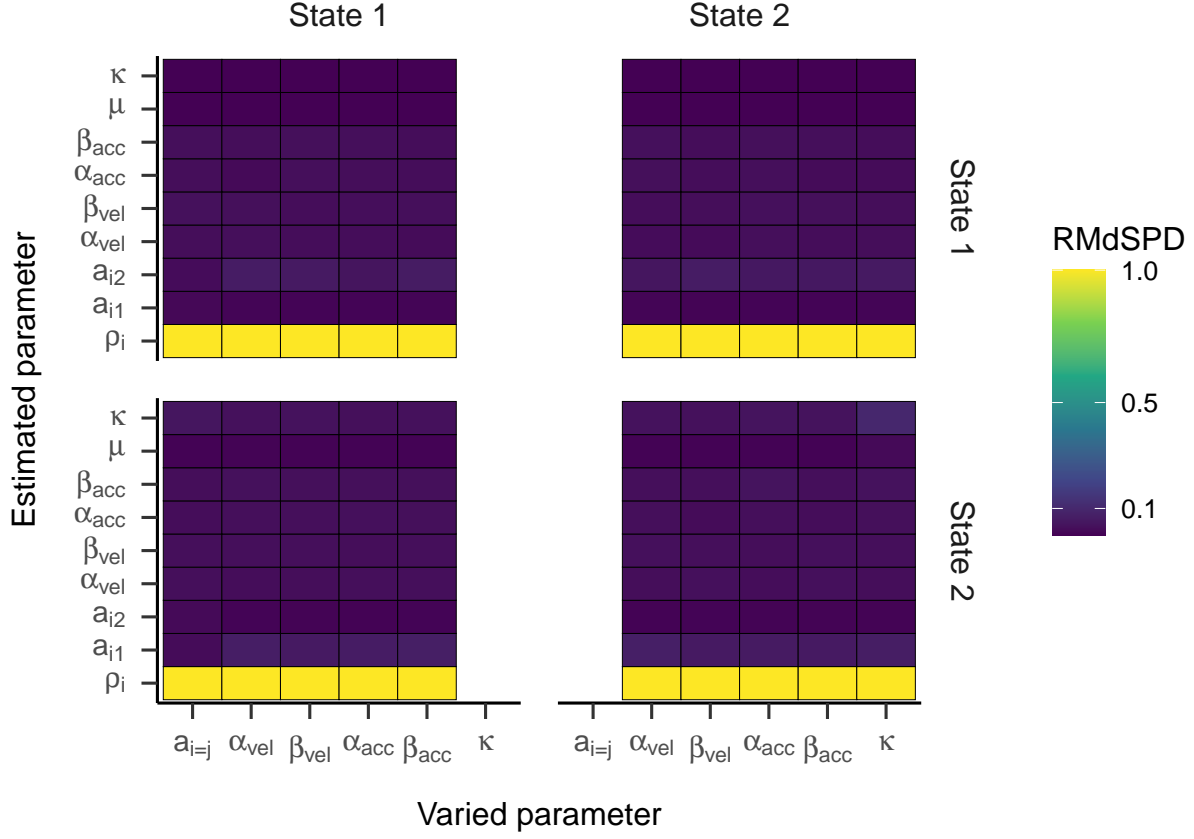
Figure 1. RMdSPD between true and estimated parameters of the two-state HMM in part two of the simulation. Labels on the x-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
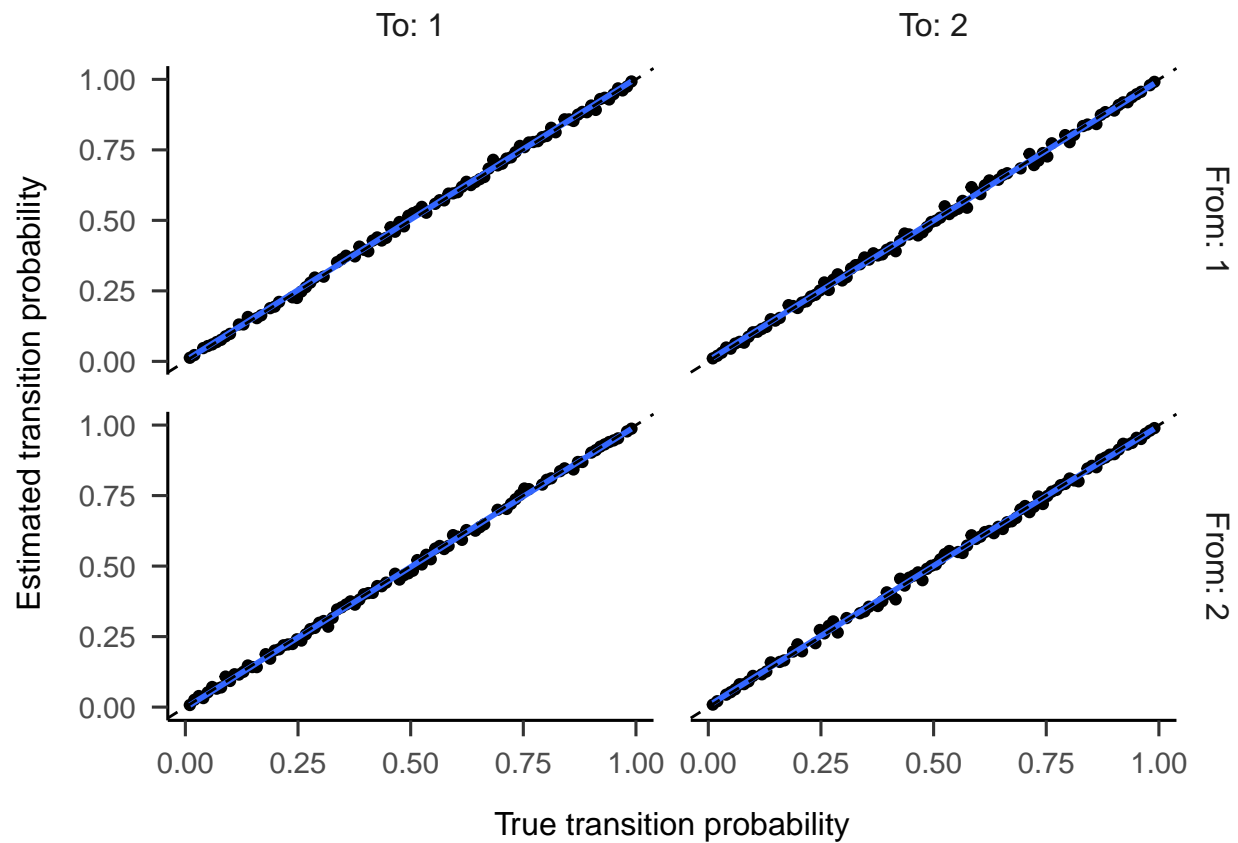
Figure 2. Regression lines between true and estimated transition probabilities for the two-state HMM in part one. Top facet labels show to and right facet labels show from which state the HMM is moving. Dashed lines refer to perfect recovery.
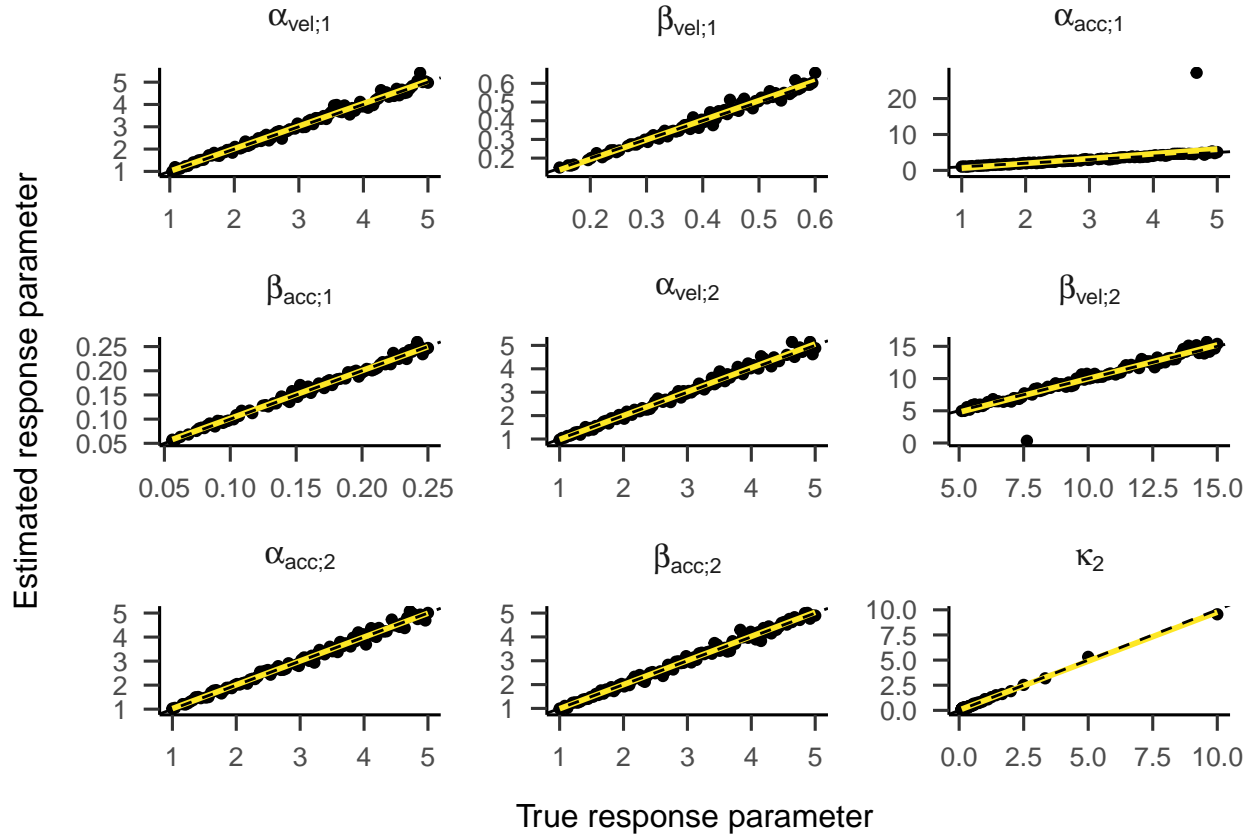
Figure 3. Regression lines between true and estimated response parameters of the two-state HMM in part one. Top facet labels indicate response parameters. Dashed lines refer to perfect recovery.
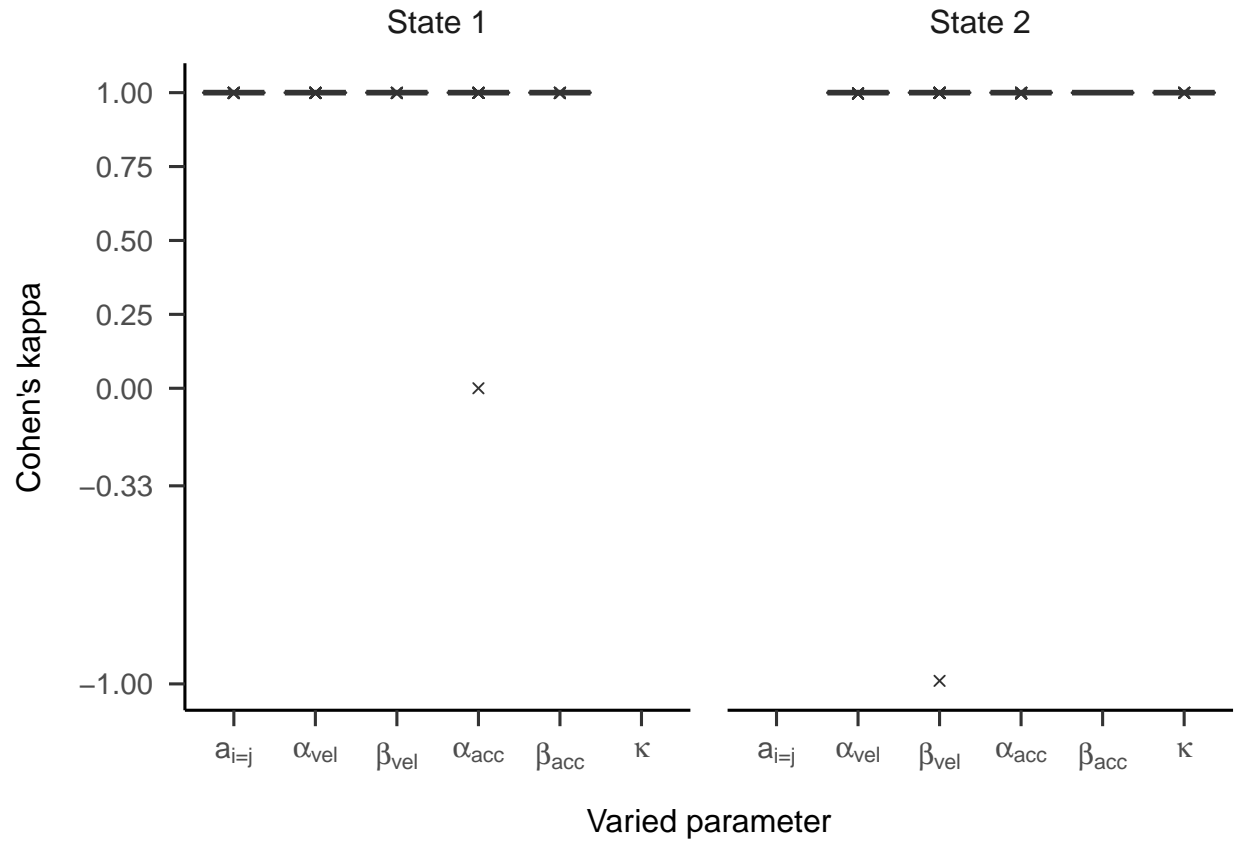
Figure 4. Boxplots displaying Cohen's kappa depending on which parameter of the two-state HMM has been manipulated in part one. Top facet labels indicate for which state parameters have been manipulated. Black solid lines symbolize medians. Crosses represent outliers (distance to first/third quartile higher than 1.5 times the inter-quartile range [IQR]).
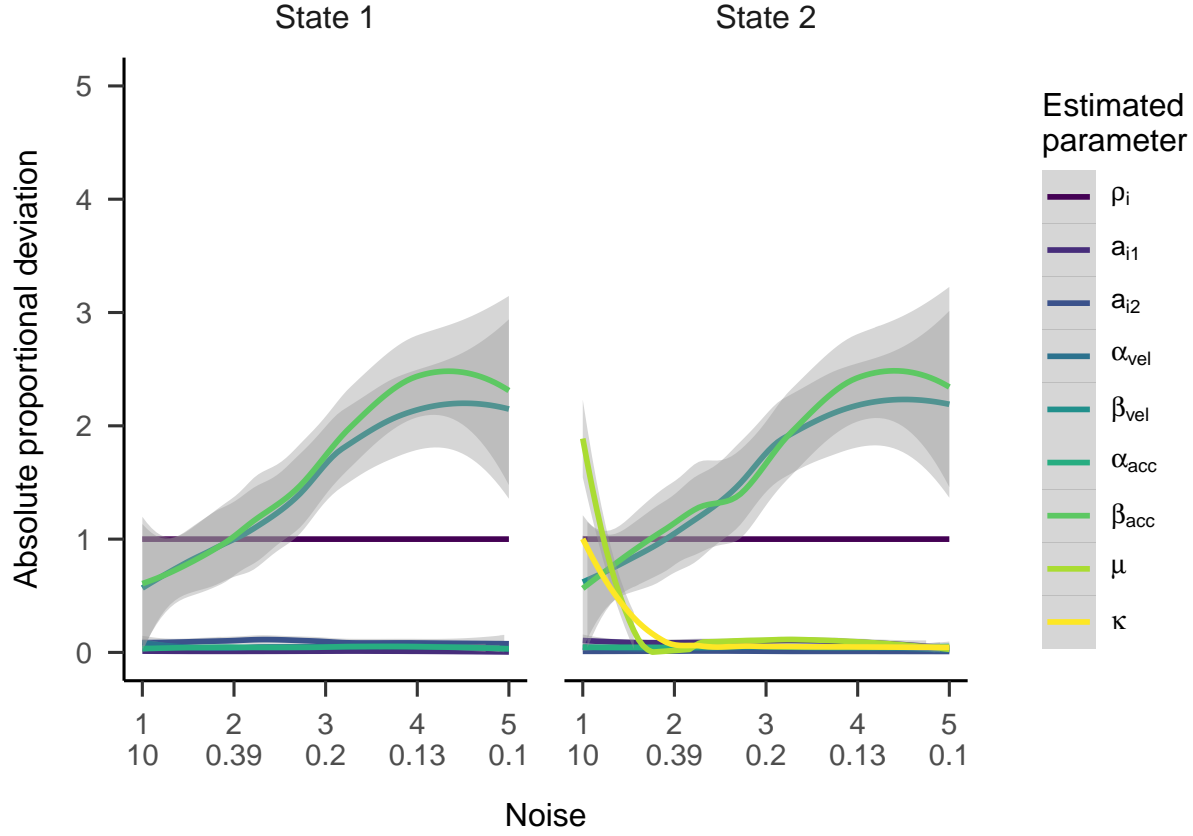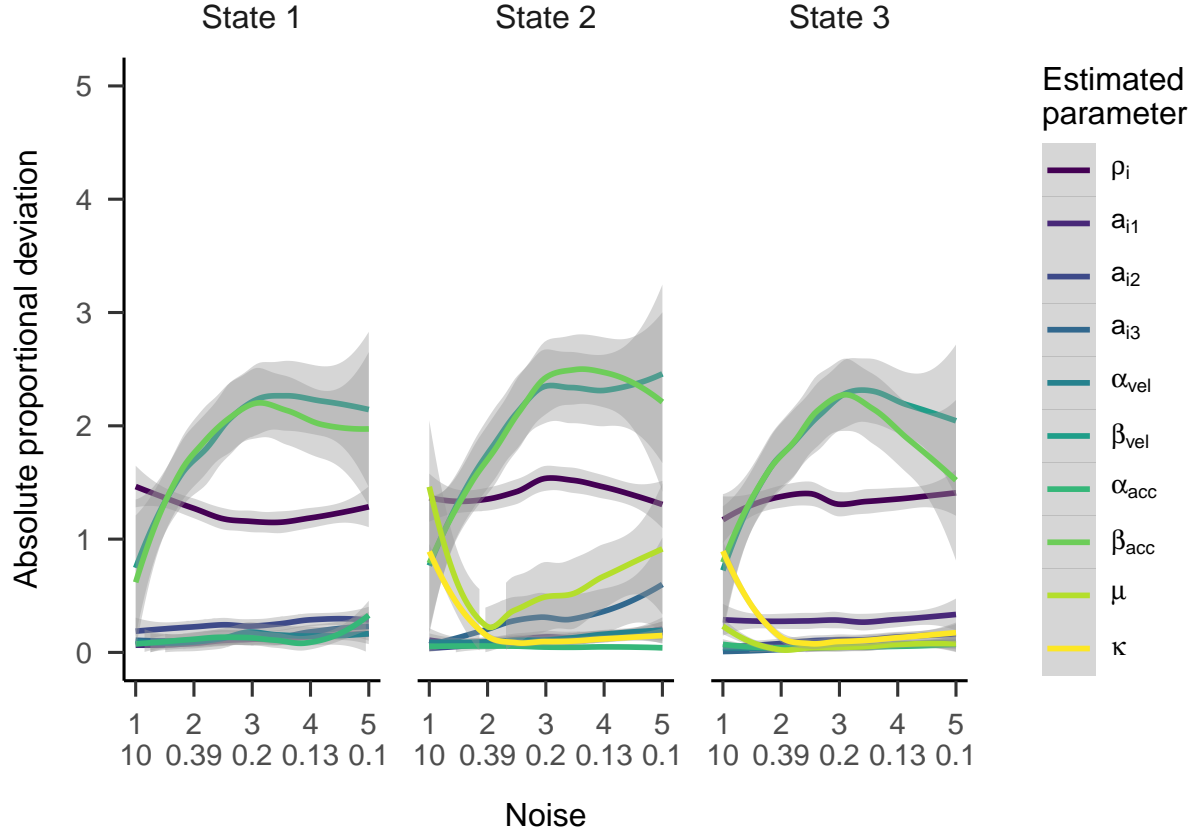
Figure 5. Absoluted proportional deviation between estimated and true parameters of the two-state HMM depending on noise level in part two of the simulation. The upper labels on the x-axis indicate τ_{noise} and the lower labels κ_{noise}. Colours indicate for which estimated parameter and top facet labels for which state the deviation is displayed. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
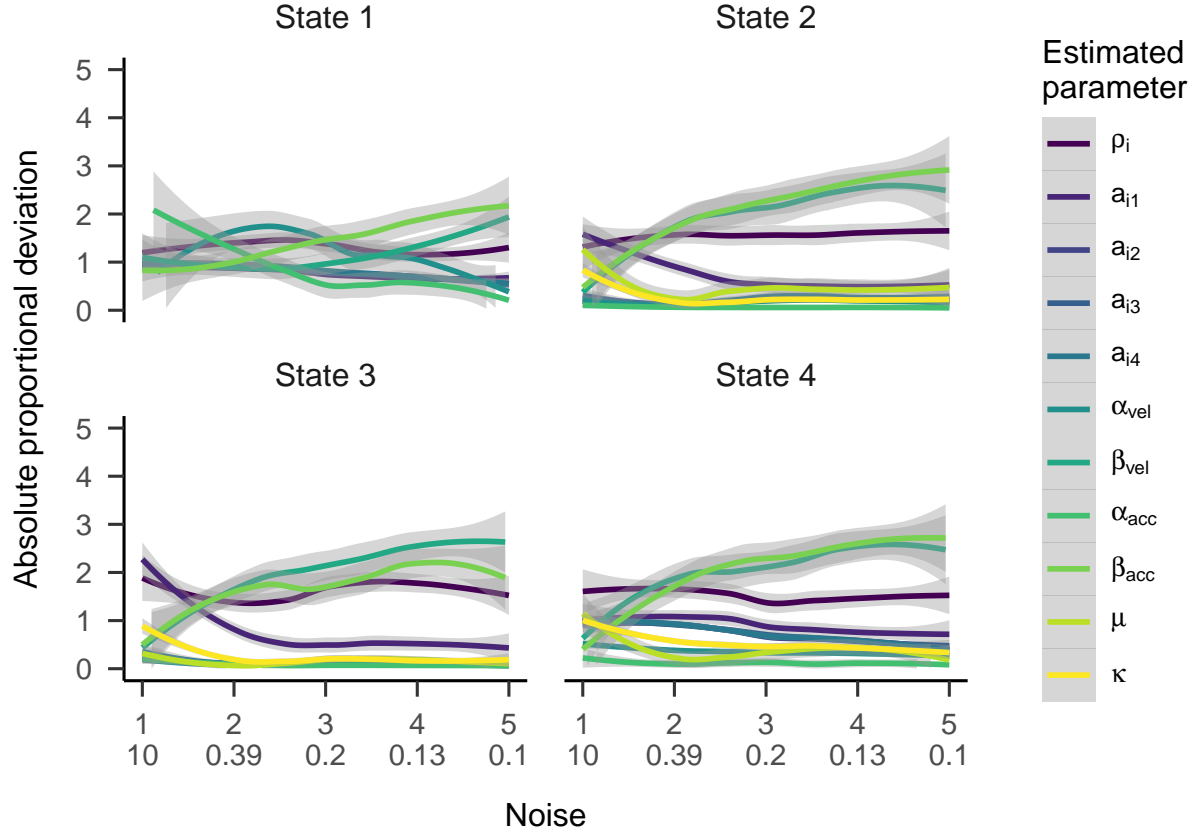
Figure 6. Absoluted proportional deviation between estimated and true parameters of the three-state HMM depending on noise level in part two of the simulation. The upper labels on the x-axis indicate τ_{noise} and the lower labels κ_{noise}. Colours indicate for which estimated parameter and top facet labels for which state the deviation is displayed. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.

Figure 7. Absoluted proportional deviation between estimated and true parameters of the four-state HMM depending on noise level in part two of the simulation. The upper labels on the x-axis indicate τ_{noise} and the lower labels κ_{noise}. Colours indicate for which estimated parameter and top facet labels for which state the deviation is displayed. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
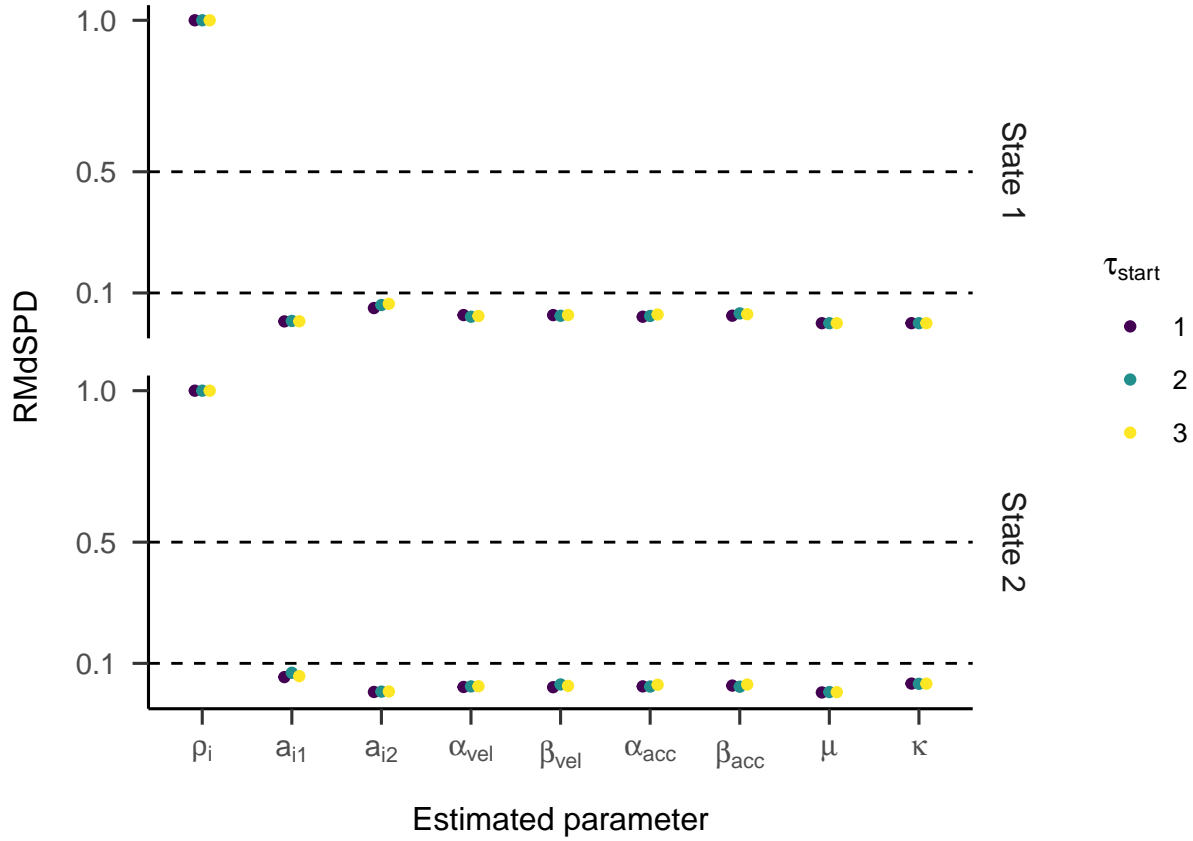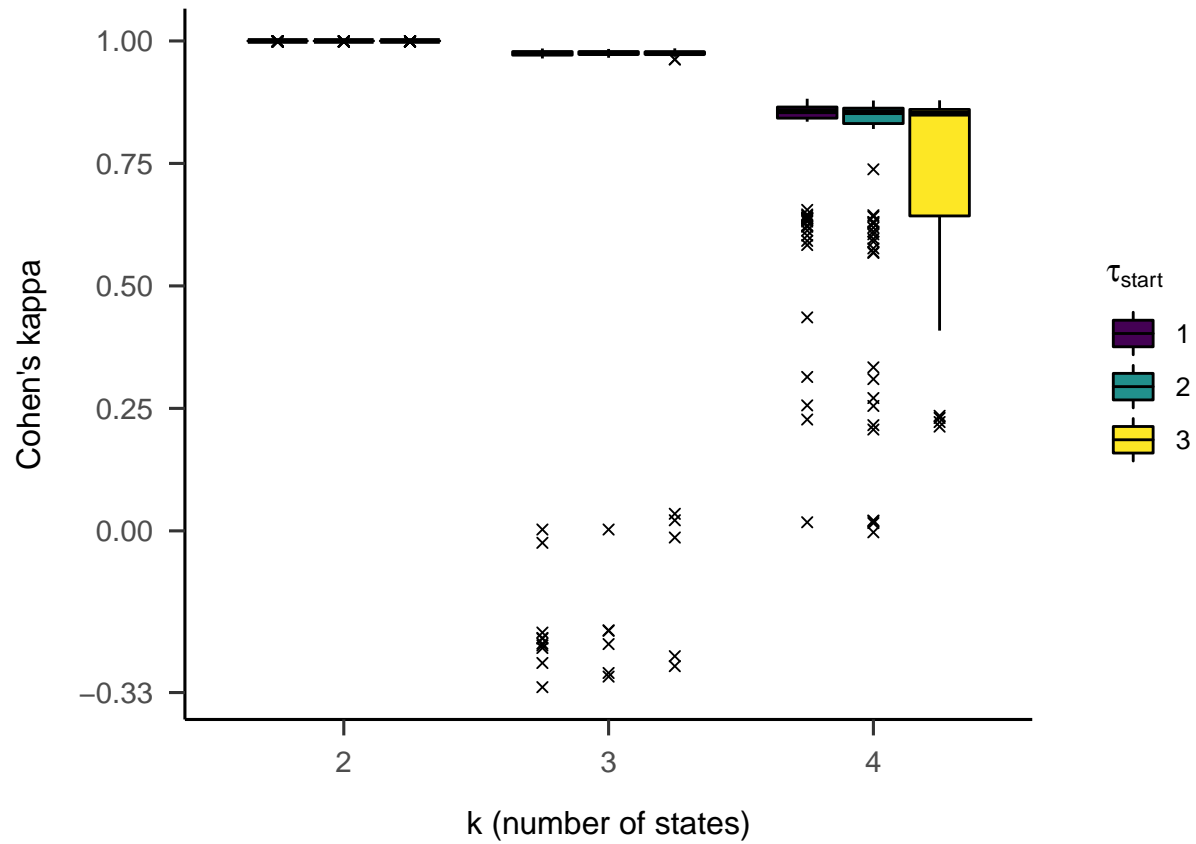
Figure 8. RMdSPD between true and estimated parameters of the two-state HMM in part three of the simulation. Colours indicate the variation in starting values used to estimate parameters. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.

Figure 9. Boxplots displaying Cohen's kappa depending on the number of states in the HMM in part three. Colours indicate the variation in starting values used to estimate parameters. Solid vertical lines symbolize medians and hinges the first and third quartile. Whiskers range from hinges to lowest/highest value within 1.5 times the IQR. Crosses represent outliers.
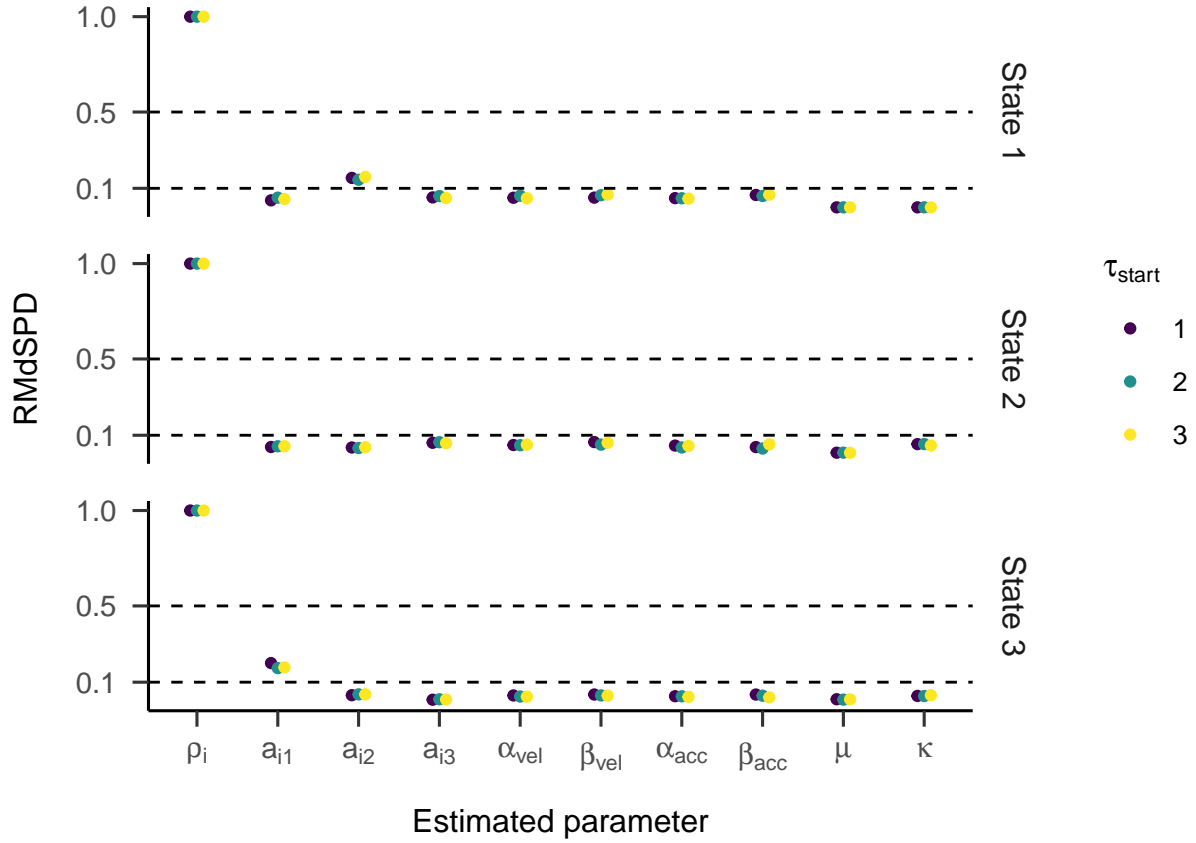
Figure 10. RMdSPD between true and estimated parameters of the three-state HMM in part three of the simulation. Colours indicate the variation in starting values used to estimate parameters. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
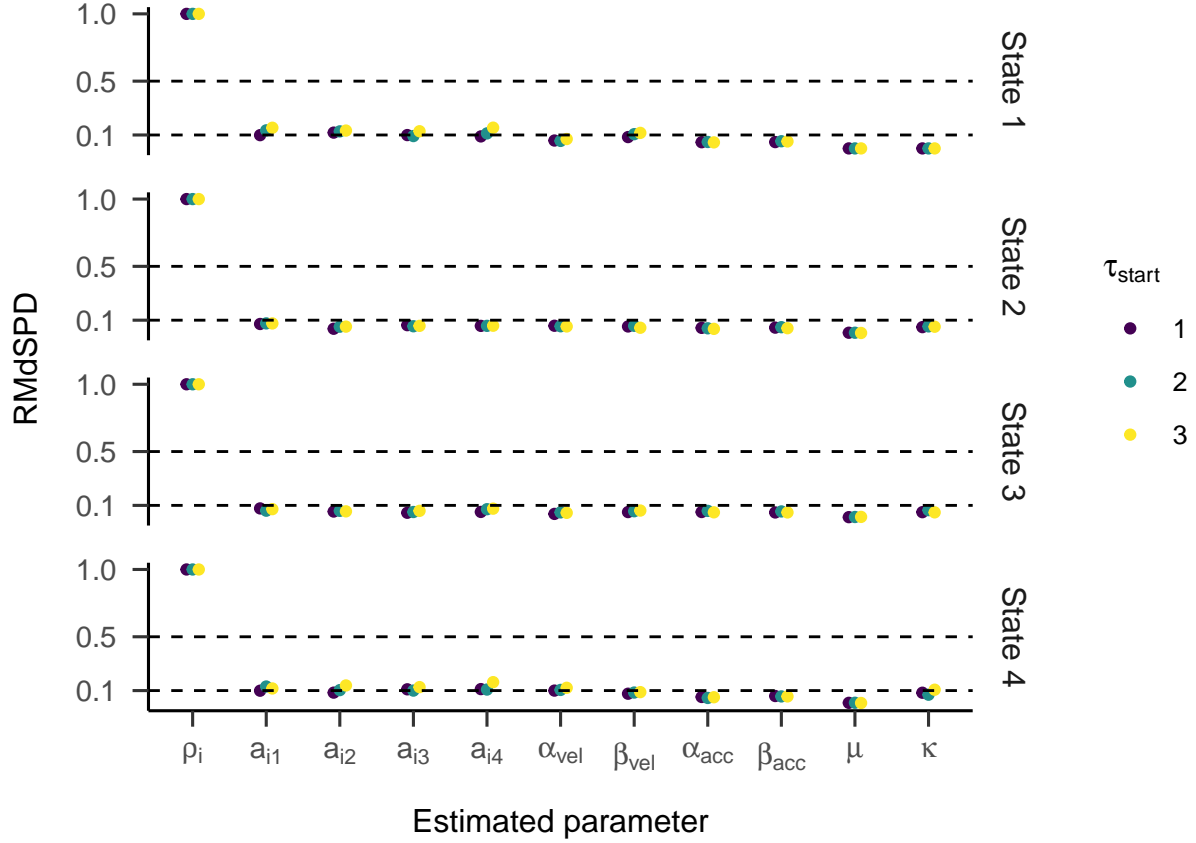
Figure 11. RMdSPD between true and estimated parameters of the four-state HMM in part three of the simulation. Colours indicate the variation in starting values used to estimate parameters. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
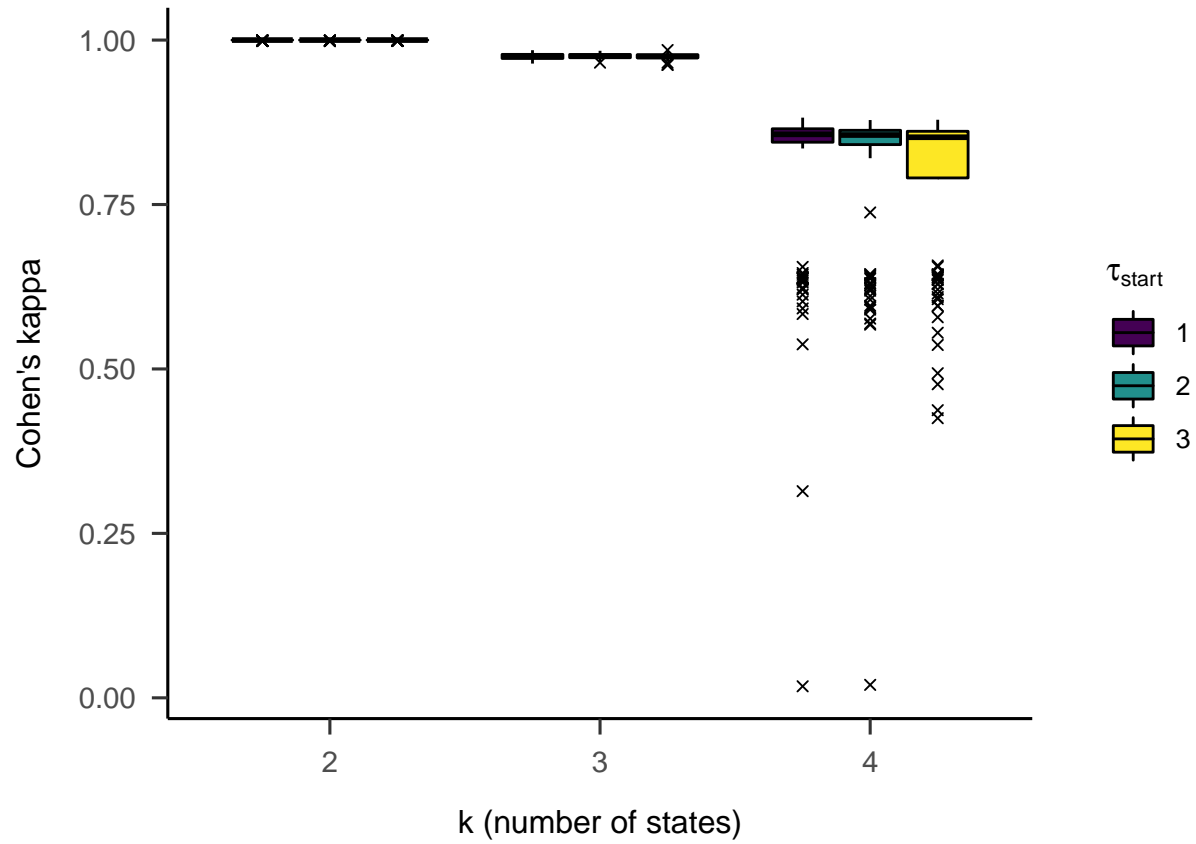
Figure 12. Boxplots displaying Cohen's kappa depending on the number of states in the HMM in part three after state labels were switched post-hoc (exploratory analysis). Colours indicate the variation in starting values used to estimate parameters. Solid lines symbolize medians and hinges the first and third quartile. Whiskers range from hinges to lowest/highest value within 1.5 times the IQR. Crosses represent outliers.
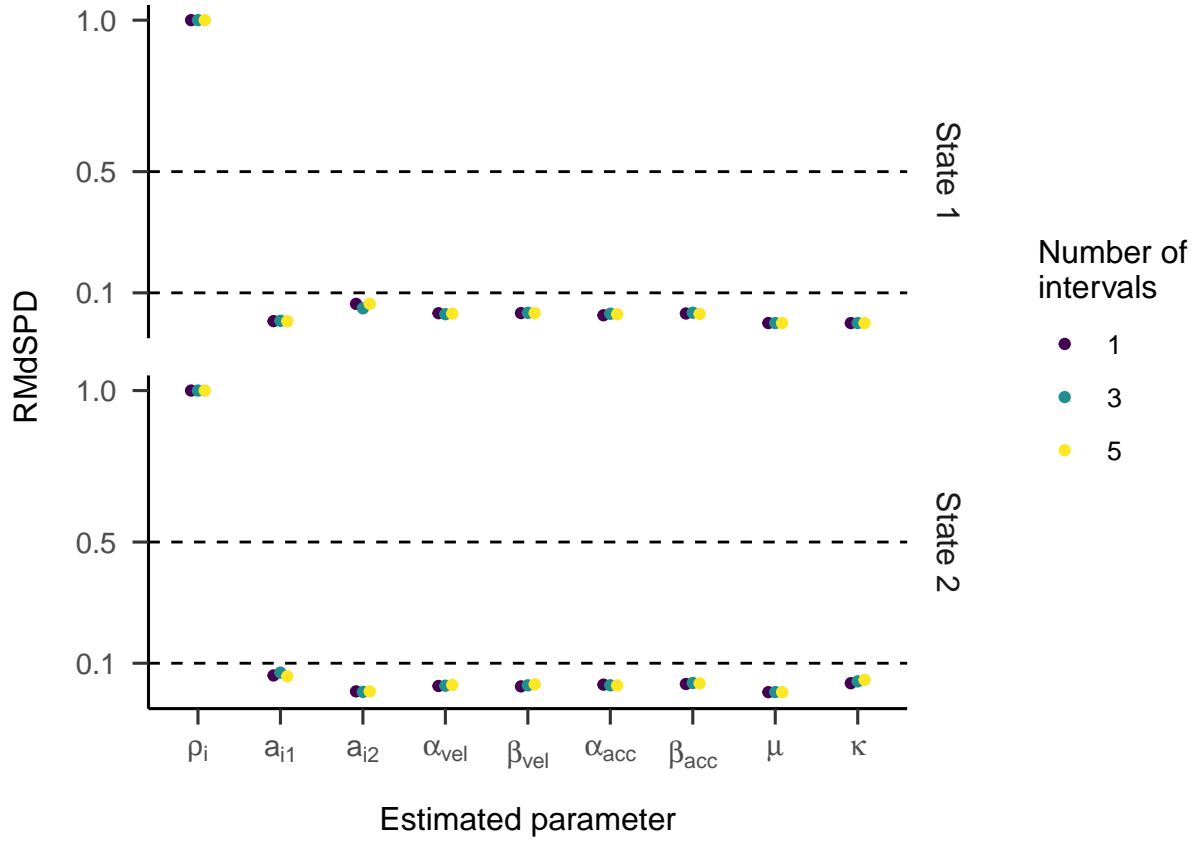
Figure 13. RRMdSPD between true and estimated parameters of the two-state HMM in part four of the simulation. Colours indicate the number of missing data intervals. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Right facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
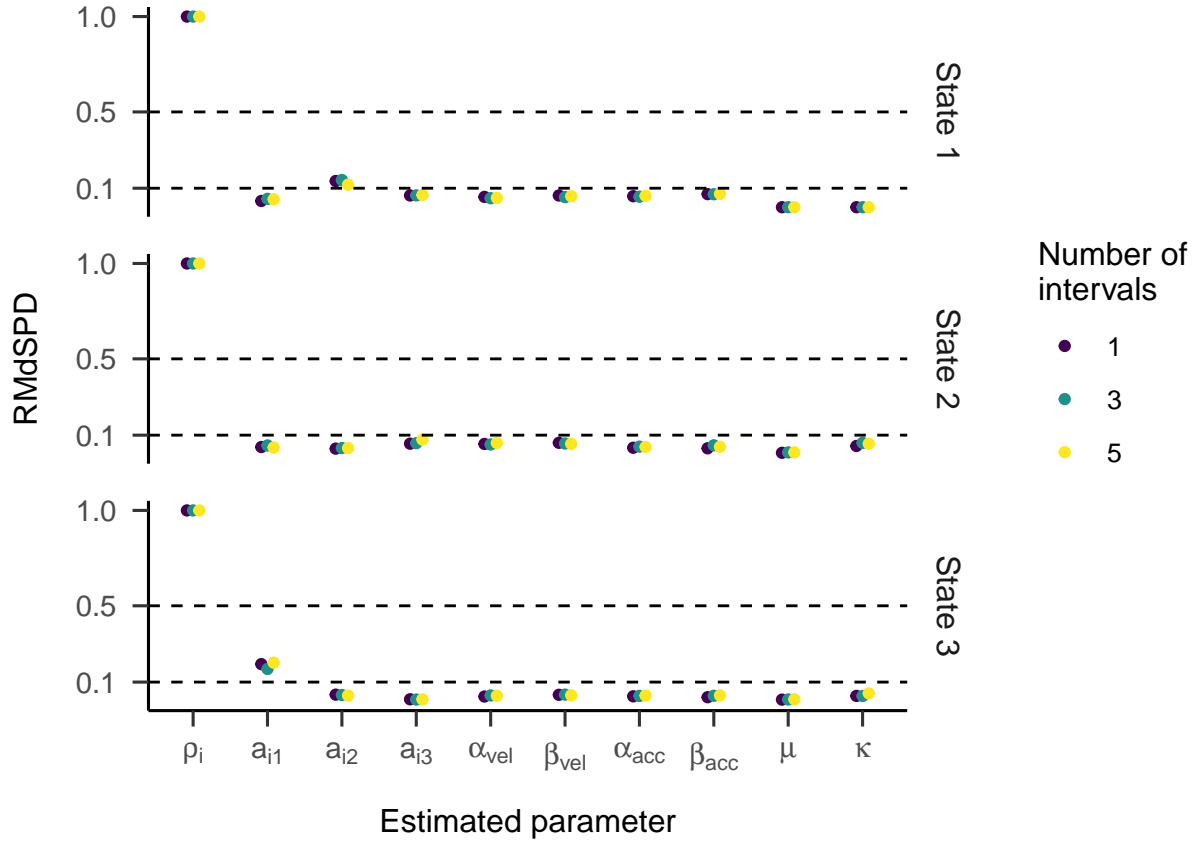
Figure 14. RMdSPD between true and estimated parameters of the three-state HMM in part four of the simulation. Colours indicate the number of missing data intervals. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
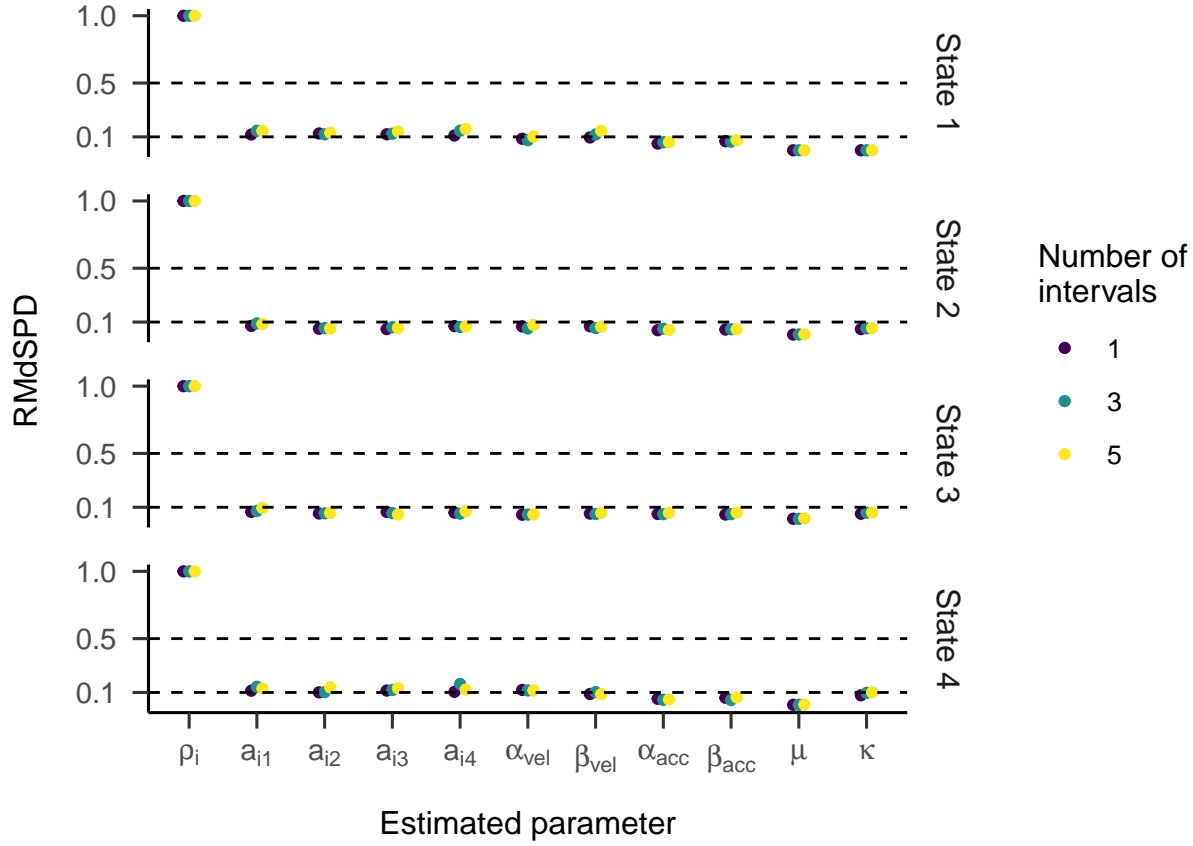
Figure 15. RMdSPD between true and estimated parameters of the four-state HMM in part four of the simulation. Colours indicate the number of missing data intervals. Labels on the y-axis indicate for which estimated parameter the RMdSPD is displayed. Top facet labels denote to which state estimated parameters belong. ρ_i is the initial probability for state i (indicated by the top facet label), a_{ij} is the probability to transition from state i to state j, α and β are the shape and scale parameters of the gamma distributions, and μ and κ are the mean and concentration parameter of the von Mises distribution.
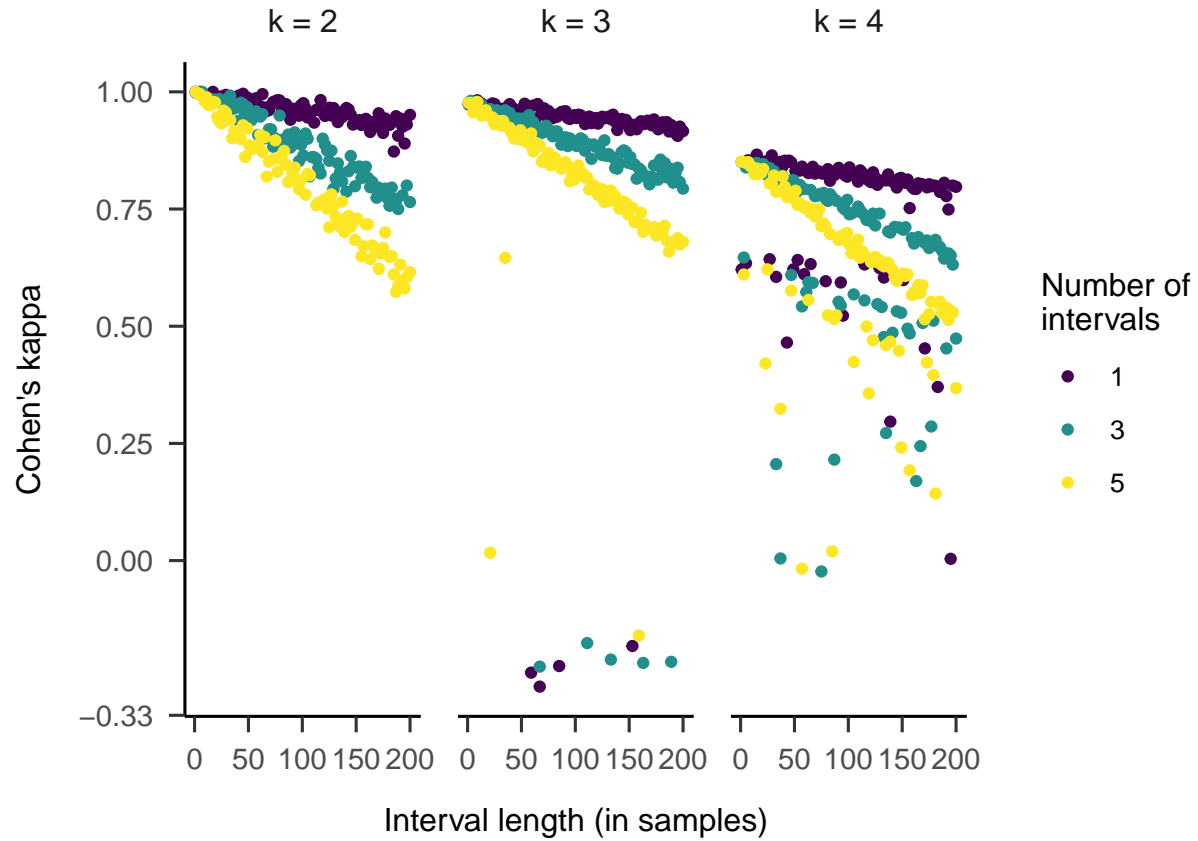
Figure 16. Cohen's kappa depending on the length of missing data intervals. Colours indicate the number of missing data intervals. Top facet labels indicate the number of states in the HMM.