Paranormal Beliefs: Latent Class Analysis

Šimon Kucharský & Ingmar Visser

Questions

(1) Does the current dataset provide support for a 4-factor structure identified in the 1985 study (based on items 1-24 of the paranormal beliefs scale)?

In principle, we cannot say for sure. Latent Class Analysis (LCA) searches for clusters of *respondents* that have something in common, whereas factors are clusters of *items* that have something in common. As such, LCA is not suitable to provide evidence for a factor structure of a questionnaire, as it answers a completely orthogonal question to that (*cf.* Oberski, 2016).

That said, we found some evidence that informing the LCA model with constraints that allow grouping the items into clusters based on the original CFA factors improves the fit of the model and finds additional groups of participants in the data; most prominently, a group of participants that are relatively skeptical towards everyday superstitions but less skeptical towards other types of paranormal phenomena. As such it is plausible that a 4-factor model would fit the data, even though it is likely that the factors would be highly correlated, raising the question of whether it is practical to distinguish between the four factors in the first place. In fact, the interest in these data may lie more in the possibility to distinguish groups of people with various levels of skepticism, than in determining the commonalities between items.

(2) What is the most important conclusion/finding based on these data and the application of technique Latent Class Analysis?

About half (47%, 95%CI[45%, 49%]) of the population are extremely skeptical towards any kind of paranormal phenomena, uniformly across all item types. About a quarter of the population (25%, 95%CI[23%, 28%]) are less skeptical towards any and all paranormal phenomena. About a tenth of the population (10%, 95CI[8%, 11%]) are extremely skeptical towards everyday superstitions, but less skeptical towards other types of paranormal phenomena.

Model details

We used Latent Class models for the analysis. Response y_{pi} of participant $p \in \{1, ..., P\}$ with a survey weight w_p on item $i \in \{1, ..., 24\}$ is modeled as a mixture of $G \in \mathbb{Z}^+$ classes with mixture proportions $\pi \in \mathbb{R}^G$; $0 < \pi_g < 1$, $\sum_{g=1}^{G} \pi_g = 1$, resulting in the following general likelihood:

$$\mathcal{L}(y|\pi, \theta) = \prod_p \left(\sum_g \pi_g \prod_i p(y_{pi}|\theta_{gi}) \right)^{w_p},$$

As the response scale for all items was a 5-point Likert scale, we used the Ordinal Logistic likelihood to model the data. Specifically, with $K = 5$ response categories, we have cutpoints $c \in \mathbb{R}^4$ such that $c_i < c_{i+1}$ for $i \in \{1, 2, 3\}$ and that $\sum_i^4 c_i = 0$, and we have the latent agreement with the item $\eta \in \mathbb{R}$, then the observed response $k \in \{1, 2, 3, 4, 5\}$ is modelled by

$$p(y = k \mid c, \eta) = \begin{cases} 1 - \text{logit}^{-1}(\eta - c_1) & \text{if } k = 1 \\ \text{logit}^{-1}(\eta - c_{k-1}) - \text{logit}^{-1}(\eta - c_k) & \text{if } 1 < k < 5, \text{ and} \\ \text{logit}^{-1}(\eta - c_5) & \text{if } k = 5 \end{cases}$$

We will use the probability sampling statements to allow easy navigation between the models. Specifically, we use the following notation to express that the data follow the ordered logistic distribution, and that the parameters of the ordered logistic distribution depend on the latent class z_p to which a participants p belongs to:

$$\begin{aligned} y_{pi} &\sim \text{OrderedLogistic}(\eta_{z_p i}, c_i) \\ z_p &\sim \text{Categorical}(\pi), \end{aligned}$$

Note that in all models we let the cutpoints c estimate freely for each item (as long as they satisfied the sum to zero constraint: $\sum_i^4 c_i = 1$), assuming that each item has different "difficulty". However, the cutpoints were set equal between all classes within each item, assuming that the latent classes do not differ in terms of item functioning; in other words this is akin to assuming measurement invariance of the individual items between classes. This way the latent class components are distinguished only by the values of η.

Different models were defined by 1) varying the number of latent classes, and 2) imposing equality constraints on η parameters between items.

Uninformed models

As a baseline, we considered models that are not informed by the original CFA results.

Constrained model

The baseline model is such that

$$
\begin{aligned}
y_{pi} &\sim \text{OrdinalLogistic}(\eta_{z_p}, c_i) \\
z_p &\sim \text{Categorical}(\pi),
\end{aligned}
$$

the parameter η is fixed equal across all items within each class, representing a constant belief in paranormal phenoment across all items, see Figure 1 for an example.

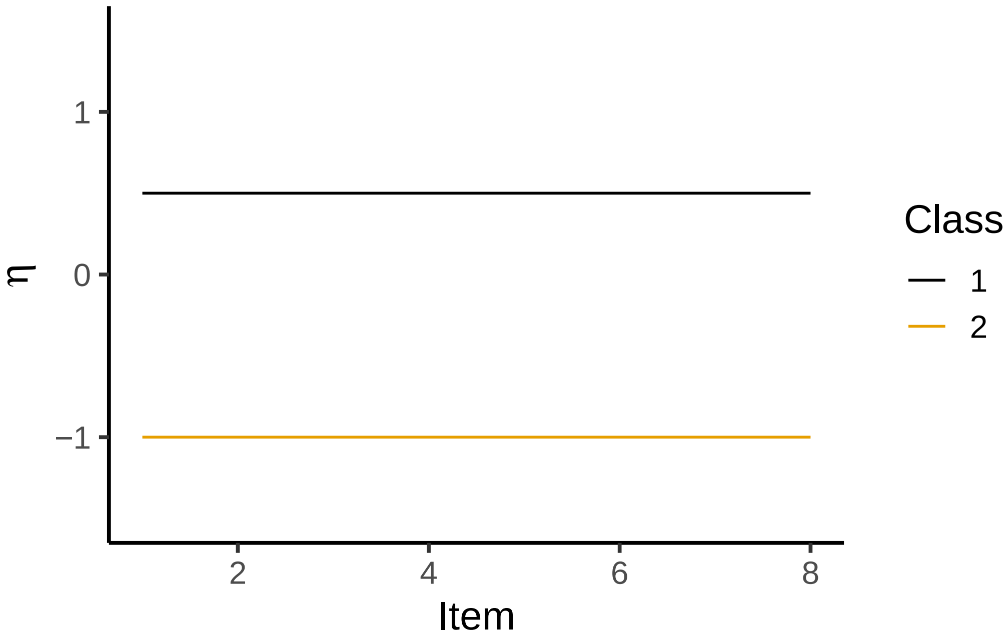

Figure 1: Illustration of the constrained model with 2 classes. Each class is assigned a separate level of η value that is held constrant across all items.

Unconstrained model

The unconstrained model estimates the η parameters freely per item:

$$
\begin{aligned}
y_{pi} &\sim \text{OrdinalLogistic}(\eta_{z_p i}, c_i) \\
z_p &\sim \text{Categorical}(\pi),
\end{aligned}
$$

the parameter η is set free across items and for each class, see Figure 2 for an example.

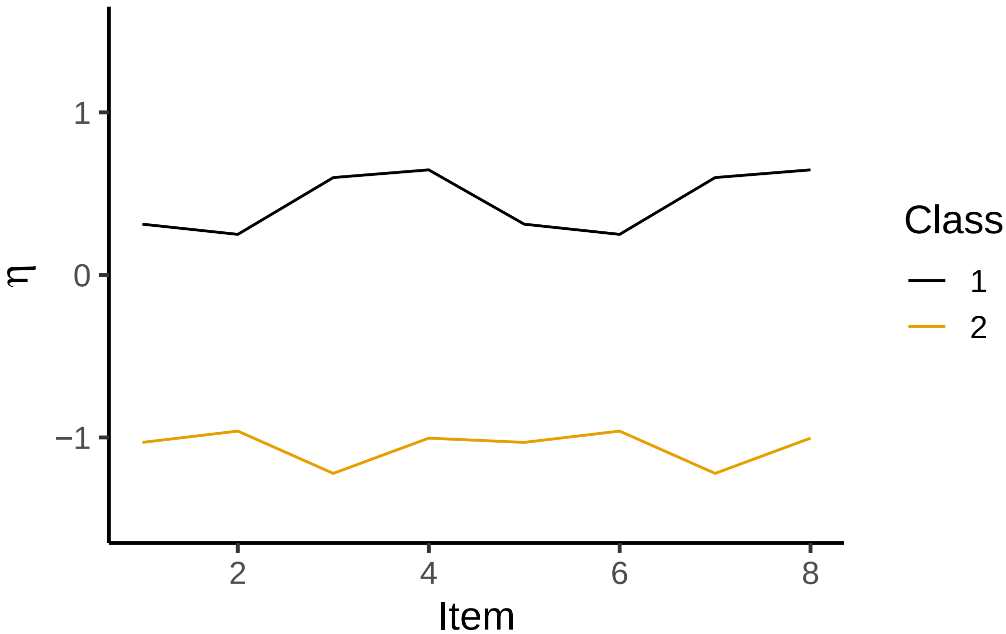

Figure 2: Illustration of the unconstrained model with 2 classes. Each class and item is assigned a separate level of η value.

Models informed by the original CFA

Next to the mixture models defined above, we also fitted models that are informed by the original CFA results. Specifically, we constrain the parameters η for items that belong to one of the four factors identified by the original CFA results. The general idea is that each item is modeled by two η parameters, corresponding to "low" and high "belief" in the statement. Correspondingly, this allows us to specify all combinations of "low" and "high" belief in any of the four factors, resulting in $2^4 = 16$ latent classes - representing all combinations of beliefs in individual factors:

Class	Extraordinary human abilities	Supernatural reality	Unearthly beings	Everyday superstition
1	low	low	low	low
2	low	low	low	high
3	low	low	high	low
4	low	low	high	high
5	low	high	low	low
6	low	high	low	high
7	low	high	high	low
8	low	high	high	high
9	high	low	low	low

Class	Extraordinary human abilities	Supernatural reality	Unearthly beings	Everyday superstition
10	high	low	low	high
11	high	low	high	low
12	high	low	high	high
13	high	high	low	low
14	high	high	low	high
15	high	high	high	low
16	high	high	high	high

Item	Label	Factor
par_5	Healing by laying on of hands	Extraordinary human abilities
par_7	Seeing into the future	Extraordinary human abilities
par_8	Life course description	Extraordinary human abilities
par_9	Mind-reading	Extraordinary human abilities
par_10	Seeing events happening	Extraordinary human abilities
par_11	Graphology	Extraordinary human abilities
par_13	Dowsing	Extraordinary human abilities
par_15	Causing events ("willpower")	Extraordinary human abilities
par_1	Predictive dreams	Supernatural reality
par_2	Contact with deceased	Supernatural reality
par_3	Astrology	Supernatural reality
par_6	Palmistry	Supernatural reality
par_14	Telekinesis	Supernatural reality
par_17	Reincarnation	Supernatural reality
par_23	Recounting (previous life)	Supernatural reality
par_4	Spirits and ghosts	Unearthly beings
par_12	Extraterrestrial beings	Unearthly beings
par_16	Gnomes and elves	Unearthly beings
par_18	Sorcery, black magic	Unearthly beings
par_19	Haunted places	Unearthly beings
par_24	Devil possesion	Unearthly beings
par_20	Lucky numbers	Everyday superstition
par_21	Unlucky numbers	Everyday superstition
par_22	Walking under a ladder	Everyday superstition

Constrained

The most constrained model is such that we estimate only two values of η_{low}, η_{high}, holding the values fixed across classes and items.

$$
\begin{aligned}
y_{pi} &\sim \text{OrdinalLogistic}(\eta_{q_{pi}}, c_i) \\
q_{pi} &= \begin{cases} \text{low} & \text{if } k = 1 \\ \text{high} & \text{if } 1 < k < 5\text{, and} \end{cases} \\
z_p &\sim \text{Categorical}(\pi),
\end{aligned}
$$

This model is equivalent to the Constrained model of two classes from the previous section in terms of the structure of estimated parameters η and c; however, it adds flexibility in allowing to cluster participants into 16 groups instead of 2, see Figure 3.

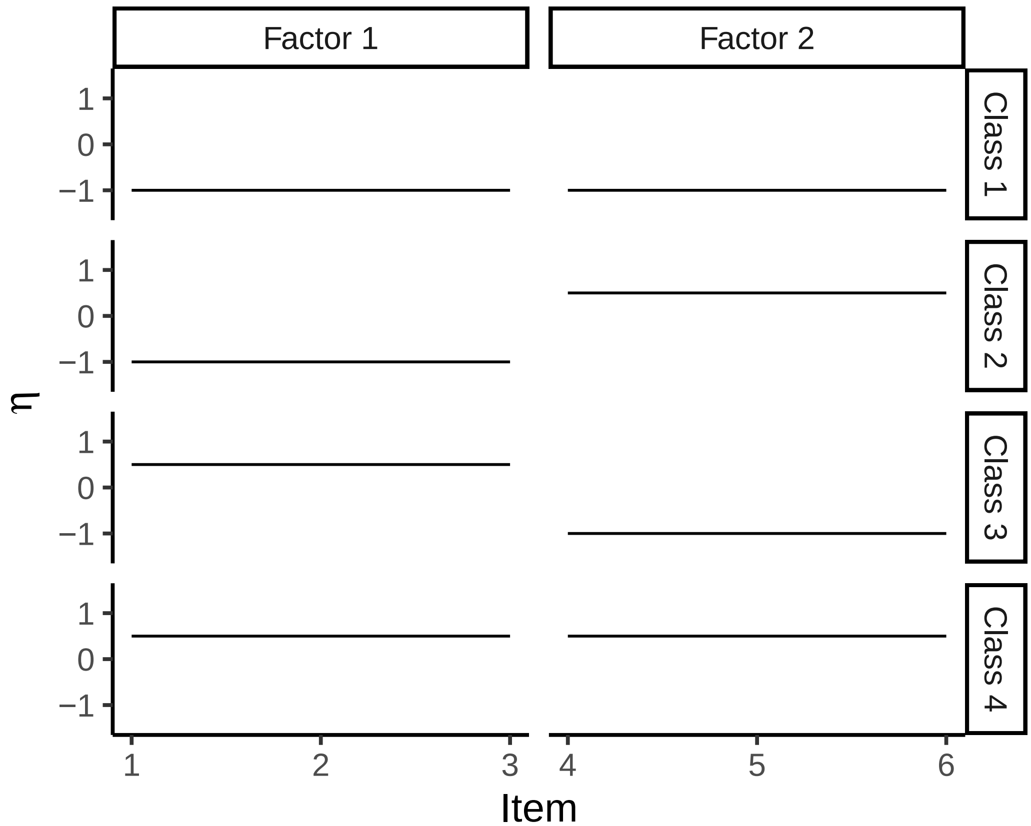

Figure 3: Illustration of the CFA-constrained model. For each item two levels of η values are estimated - these values are fixed equal across items. Each class represents a combination of "factors"; Class 1 is fixed on the low level for all items, Class 2 is fixed on the low level for items in factor 1 but on high level for items in factor 2, etc.

Semi-constrained

In a semi-constrained model we let η_{low}, η_{high} also vary between factors, but holding them fixed across items within a factor.

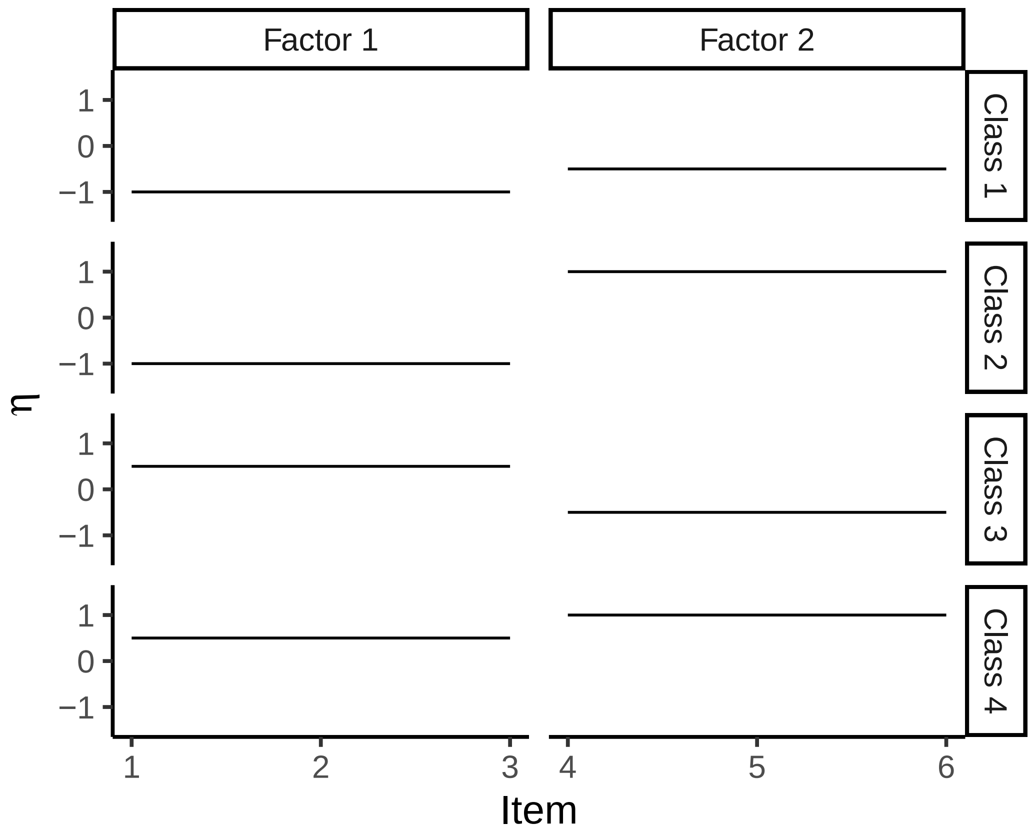

Figure 4: Illustration of the CFA-semi-constrained model. For each item two levels of η values are estimated - these values are fixed equal across items within a factor but vary between factors. Each class represents a combination of "factors"; Class 1 is fixed on the low level for all items, Class 2 is fixed on the low level for items in factor 1 but on high level for items in factor 2, etc.

Unconstrained

In an unconstrained model we let η_{low}, η_{high} vary between all items.

This model is equivalent to the Unconstrained model of two classes from the previous section in terms of the structure of estimated parameters η and c; however, it adds flexibility in allowing to cluster participants into 16 groups instead of 2. See Figure 5 for an example.

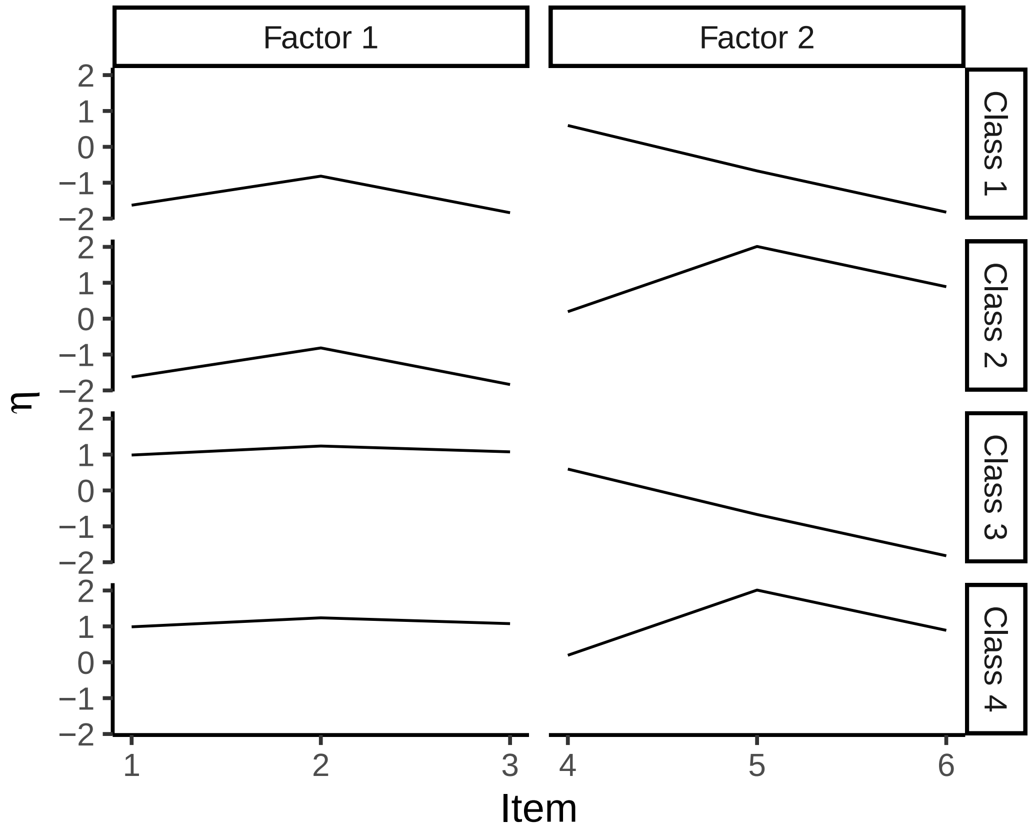

Figure 5: Illustration of the CFA-unconstrained model. For each item two levels of η values are estimated - these values vary between items. Each class represents a combination of "factors"; Class 1 is fixed on the low level for all items, Class 2 is fixed on the low level for items in factor 1 but on high level for items in factor 2, etc.

Results

The main analysis consisted of fitting the constrained model with one and two classes, the unconstrained model with one and two classes, and the three CFA-informed models (constrained,

Table 3: Overview of the models fitted to the data.

Type	Variant	dim(π)	dim(η)	dim(c)	n(par)	Log-likelihood	AIC	BIC
Uninformed	Constrained(1)	1	1	4×24	73	-77356.74	154859.5	155287.7
Uninformed	Constrained(2)	2	2	4×24	75	-66900.35	133950.7	134390.7
Uninformed	Unconstrained(1)	1	24	4×24	96	-75925.45	152042.9	152606.0
Uninformed	Unconstrained(2)	2	2×24	4×24	121	-64673.29	129588.6	130298.4
Informed by CFA	Constrained	16	2	4×24	89	-65477.59	131133.2	131655.3
Informed by CFA	Semi-constrained	16	2×4	4×24	95	-64888.66	129967.3	130524.6
Informed by CFA	Unconstrained	16	2×24	4×24	135	-63703.61	127677.2	128469.1

semi-constrained, and unconstrained), and selecting the best model based on AIC/BIC for interpretation.

An exploratory analysis consisted of fitting the unconstrained model with varying numbers of classes to potentially uncover patterns in the data not captured by the main analysis, and provides a robustness check for the conclusions made during the main analysis.

We fitted all models using maximum likelihood estimation using Stan (Carpenter et al., 2017). Where applicable, we show 95% confidence intervals.

Model comparison

We considered the following models in the main model comparison:

As shown in Table 3 and Figure 6, both AIC and BIC overwhelmingly supported the unconstrained CFA model, followed by the uninformed unconstrained model with two classes, the semi-constrained CFA model, and the constrained CFA model.

These results suggest the following:

1. There is a substantial variability between items which is not captured properly if we constrain η across items.
2. Allowing the model to classify participants into 16 groups instead of 2, while only allowing "low"/"high" responses on each item substantially improves fit to the data; this suggests that there are subgroups that score high on items associated with one factor but low on items associated with other factor(s).

Results from the best model

In this section we interpret the results from the best fitting model (unconstrained model informed by the CFA).

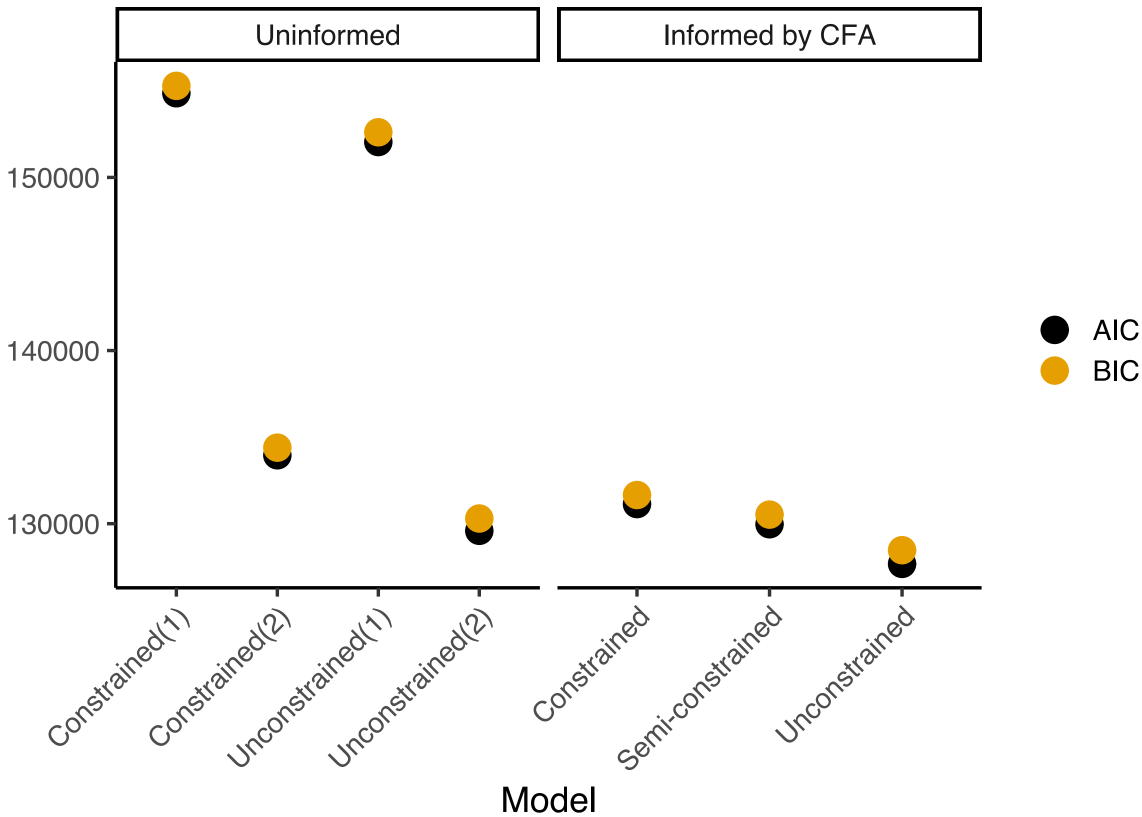

Figure 6: Model comparison of the seven models under consideration.

Figure 7 shows that model captures the observed response distribution (both observed proportions as well as proportions weighted by the survey weights) well on all items; there are no visible patterns of misfit.

Figure 8 and Figure 9 show the estimated parameters η and c for each item. Figure 8 shows that whereas there is relatively low variability between items with respect to the cutpoints c, there is a large variability between items with respect to η; this might suggest that some items are in general more believable than others for the entire population. Figure 9 shows the η parameters grouped based on which factor an item belongs to (based on the original CFA results). Notably, items 20-22 associated with "Everyday superstitions" ("Lucky numbers", "Unlucky numbers", and "Walking under a ladder") are found on average less believable in contrast to other items. Further, the item 16 - "Gnomes and elves" is also rated with considerable skepticism across the whole sample. These observations are also apparent when inspecting the observed response distributions in Figure 7.

These results suggest that the interpretation of the "low" and "high" values of η is not straightforward: If a person is classified into a group that scores "low" on some group of items, it does not necessarily mean they are complete skeptics, and on the other hand scoring "high" does not automatically mean they have a high confidence in their belief. In particular, since the cutpoints c are centered around zero, values of η around zero may correspond to a whole range of responses centered around the neutral "Maybe" response category. As Figure 8 shows even "high" η values range somewhere between -2 and 0.5, suggesting that being classified into a group that scores "high" on a particular item does not necessarily correspond to believing in the given statement uncritically.

More concretely, Figure 10 shows the actual response probabilities under the "low" and "high" values of η, for each item separately. Whereas "low" level of belief (η) leads to generally majority of responses in the "Definitely not"/"Probably not" categories, the "high" levels of η leads to a range of responses, most commonly ranging between "Probably not"/"Maybe"/"Probably". As such, we cannot associate "high" values of η with high beliefs in the item, although we can perhaps associate "low" values of η with a skeptical view towards the item.

Table 4 shows the estimated proportions of the 16 classes in the population and the counts of the number of participants in the sample classified into the class using the maximum aposteriori principle; The observed counts are not exactly proportional to the estimated proportions because the model was weighted by the survey weights. The largest group of participants (47%, 95%CI [45%, 49%]) is skeptical towards all items in the questionnaire, followed by a smaller group of "believers" (27%, 95%CI [23%, 28%]) who are not particularly skeptical to any items in the questionnaire. There also appears to be a group of participants (10%, 95%CI [8%, 11%]) that shows relatively low levels of skepticism for items about extraordinary human abilities, supernatural reality, or unearthly beings, but still being relatively skeptical about everyday superstitions, followed by a smaller group still (4%, 95%CI [3%, 5%]) of respondents who are not particularly skeptical towards extraordinary human abilities and supernatural reality, but are relatively skeptical about unearthly beings and everyday superstitions.

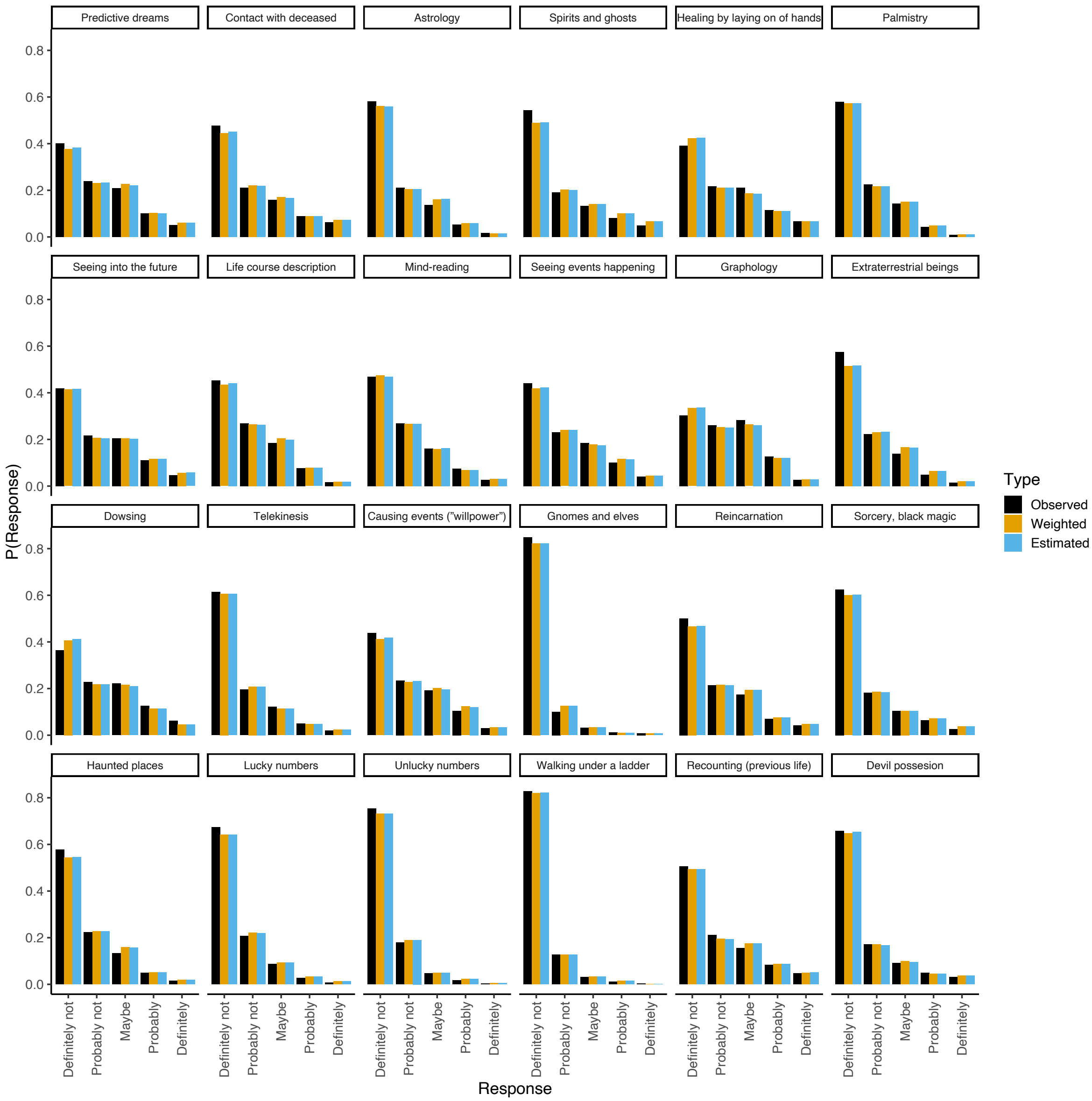

Figure 7: Estimates of the marginal response probability for each item

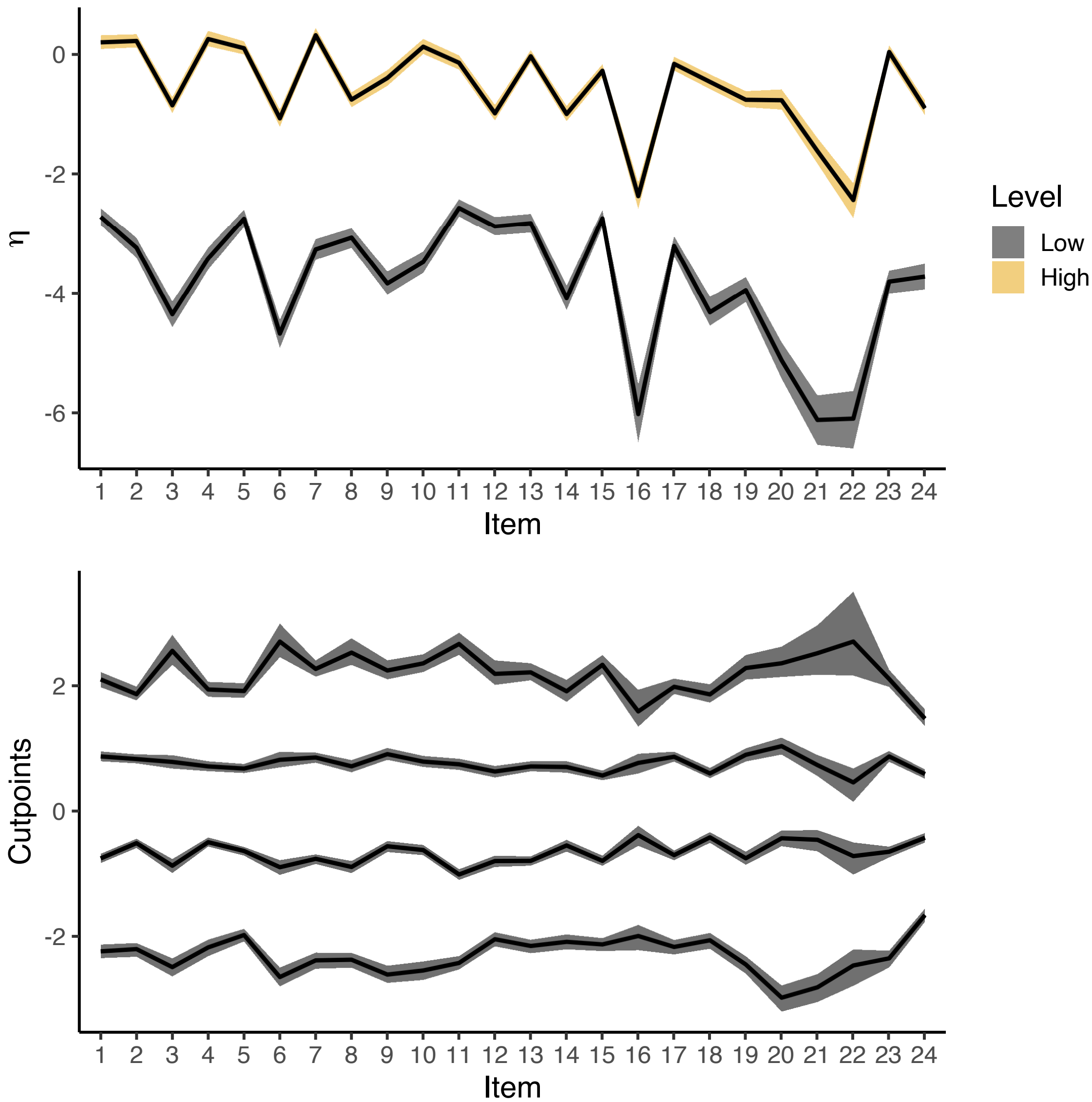

Figure 8: Estimates of the parameter η and the cutpoints for each item.

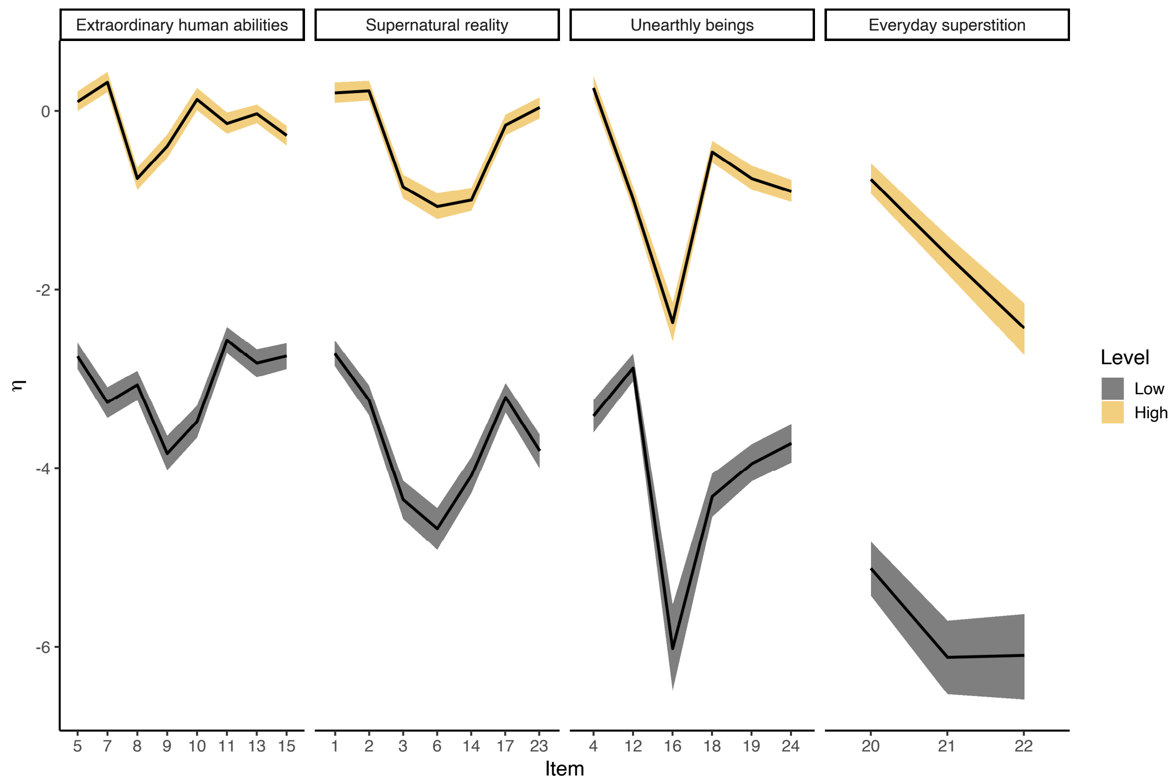

Figure 9: Estimates of the parameter η for each item - grouped by the factors from the original CFA results.

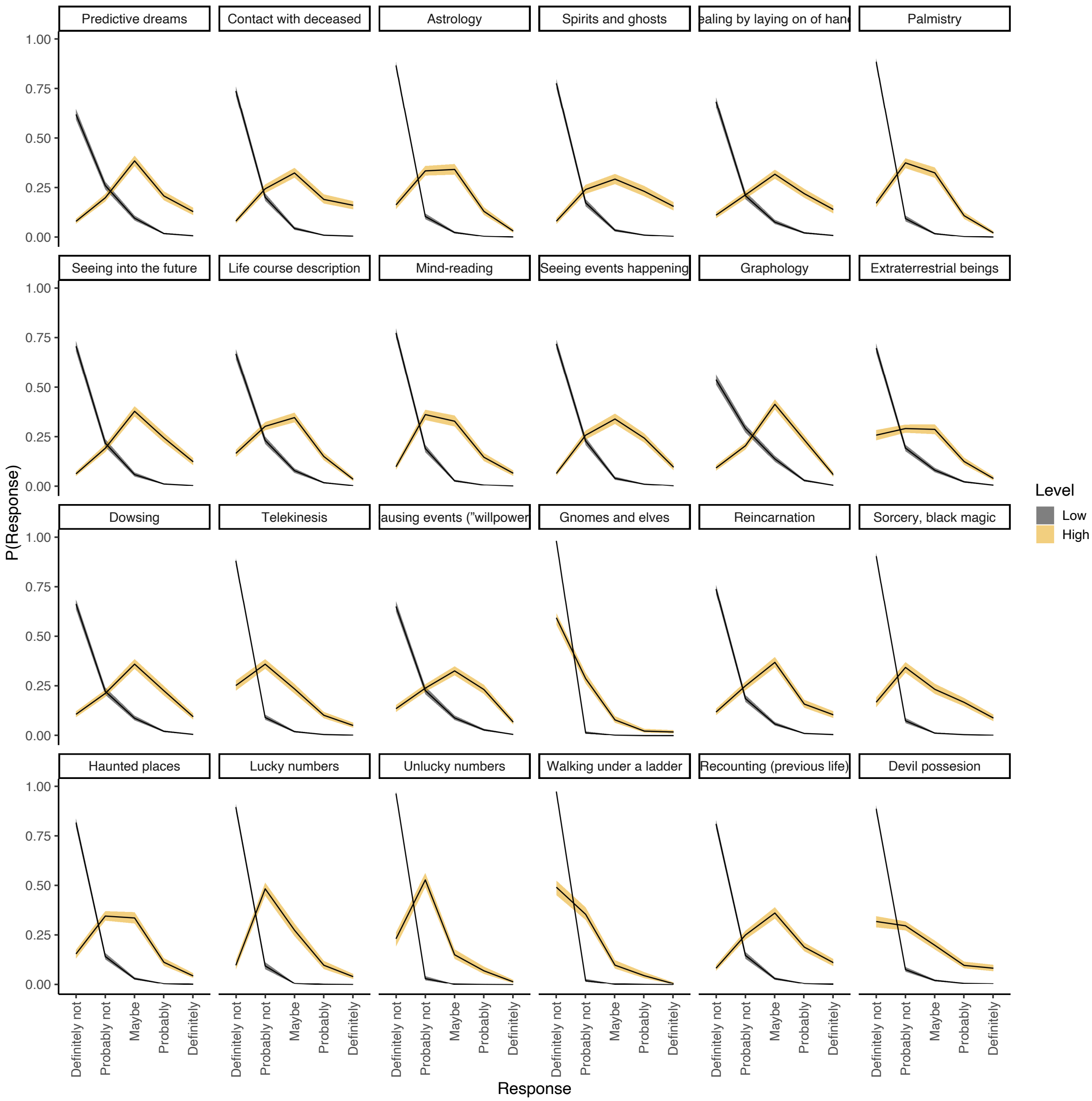

Figure 10: Estimates of the response probabilities under low/high η for each item.

Table 4: Proportions of the 16 classes estimated by the model.

		95% CI		
Class	Proportion	Lower	Upper	Count
"Skeptics"	0.47	0.45	0.49	1302
"Believers"	0.26	0.23	0.28	668
EHA + SR + UB	0.10	0.08	0.11	212
EHA + SR	0.04	0.03	0.05	145
UB	0.02	0.02	0.03	41
EHA	0.02	0.01	0.03	66
ES	0.02	0.01	0.03	54
EHA + ES	0.01	0.01	0.02	27
EHA + UB	0.01	0.01	0.02	25
SR + UB + ES	0.01	0.00	0.02	5
EHA + SR + ES	0.01	0.00	0.02	21
SR + UB	0.01	0.00	0.01	11
SR	0.01	0.00	0.01	15
SR + ES	0.00	0.00	0.01	9
ES + UB	0.00	0.00	0.01	7
EHA + UB + ES	0.00	0.00	0.03	0

* EHA - Belief in extraordinary human abilities;
SR - Belief in supernatural reality;
UB - Belief in unearthly beings;
ES - Belief in everyday superstition.

The rest of the classes are relatively minor and amount to single percentage points from the total population.

Finally, Table 5 shows the estimated proportions of the population that are not entirely skeptical towards the four groups (~ factors) of items; around 40-50% of the population are not skeptical towards extraordinary human abilities, supernatural reality, or unearthly beings. About a thirds of the population is not entirely skeptical towards everyday superstitions.

Exploratory analysis

As an exploratory analysis, we also fitted the unconstrained mixture of ordered logistic items with the number of classes ranging between 1 and 16. This analysis serves as a check of whether the observed patterns found using the constrained models can be also recovered if the models are uninformed by the results of the original factor analysis. Models with nine and more classes failed to converge to the same optimum, suggesting that the results are dependent

Table 5: Proportion of people who are not entirely sceptical towards one of the four groups of items.

		95% CI	
Factor	Proportion	Lower	Upper
Extraordinary human abilities	0.45	0.42	0.47
Supernatural reality	0.44	0.40	0.45
Unearthly beings	0.41	0.39	0.43
Everyday superstition	0.32	0.30	0.34

on the starting values. As such, we omit the results of models 9-16 from this analysis as we do not have a high confidence that we found a global optimum.

Figure 11 shows the estimated η parameters under each of the unconstrained models. It is apparent that models with 1-5 classes resulted in *parallel* classes across items, suggesting that these classes differ in terms of the overall belief in paranormal phenomena but there is relatively little (co-)variation across different item types.

Figure 12 shows the estimated parameters η under the 6-class model, and reveals that the class 3 is the only one that crosses with the others between items. Averaging the estimates within the factor items groups results in Figure 13 and shows that class 3 in this model corresponds to the group of participants that are on average not that skeptical towards paranormal phenomena, except for everyday superstitions; cross-classification with the CFA-constrained model revealed that about 50% of cases that classified in this class are also classified in the EHA + SR + UB class in the CFA-constrained model, with the rest classified in the EHA +SR class, EHA + UB class, UB class, EHA class, and some in "Believers" class. Although the overlap is not perfect, this class thus maps onto the third largest EHA + SR + UB class found using the constrained model.

Figure 14 shows the averaged estimates of η within the four item factor groups from the 8-class unconstrained model (which was favored by both AIC and BIC out of all models). However, this model did not uncover any additional subgroup patterns and more or less adds additional *parallel* classes. Thus, uncovering additional subgroup patterns using the unconstrained models was unsuccessful.

The results of the exploratory analysis suggest that more than "Skeptics" and "Believers", there are people who more or less belief in paranormal activities across all items (as suggested by recovering parallel classes). The only exception is the group of participants that are skeptical towards everyday superstitions but not so skeptical towards other types of paranormal phenomena.

Taken together, it is possible that the 4-factor model would fit the data; however, it is quite likely that the factors are highly correlated and it remains a question if it is practical to distinguish between the item types, perhaps with the exception of everyday superstitions.

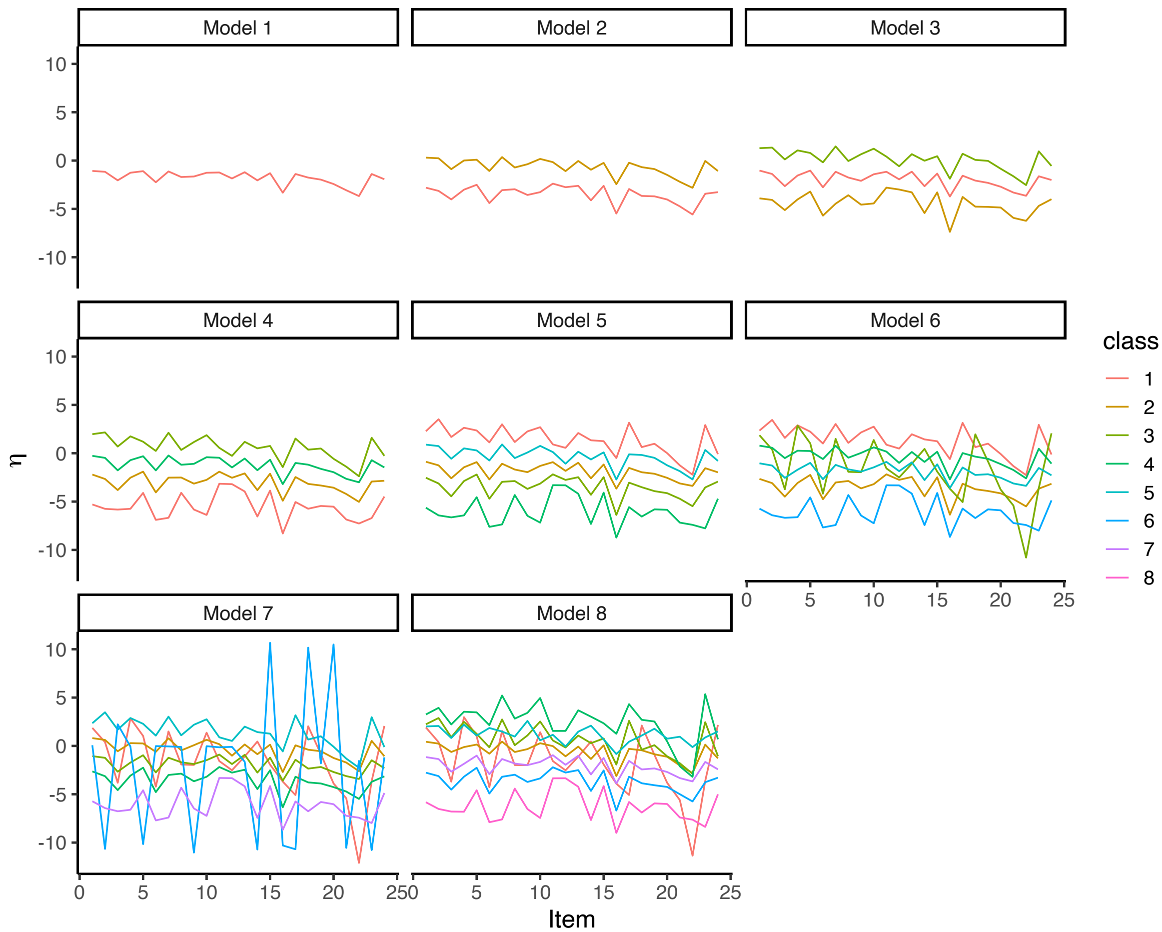

Figure 11: Estimates of the parameter η for each item, class, under the 8 unconstrained models that converged.

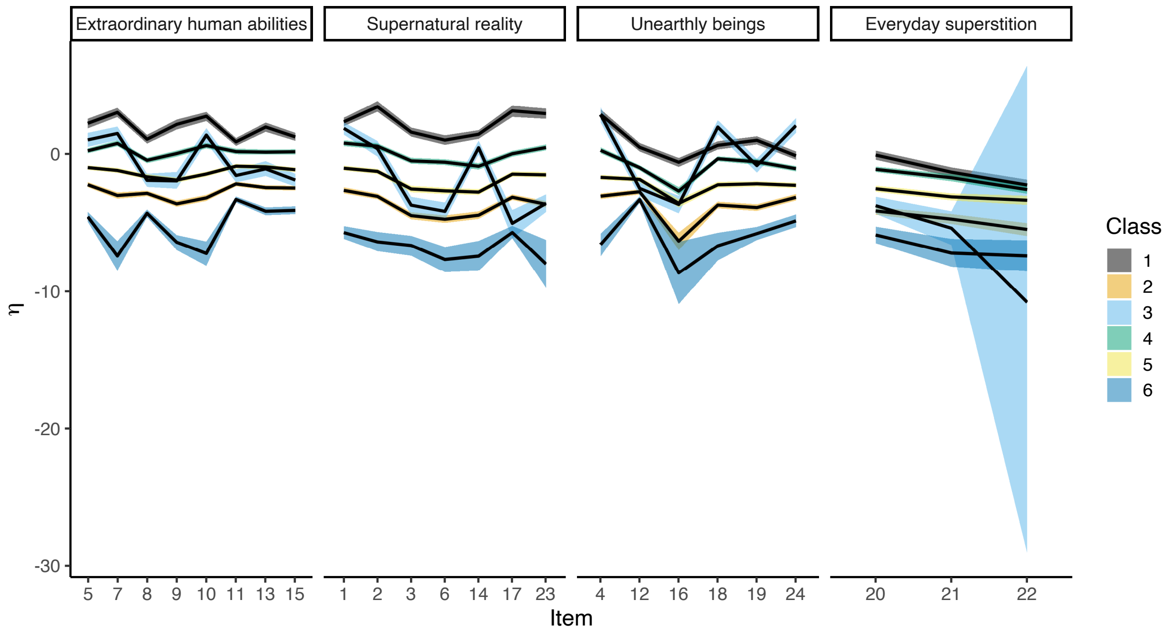

Figure 12: Estimates of the parameter η for each item, class, under 6-class unconstrained model, grouped into the items factor groups.

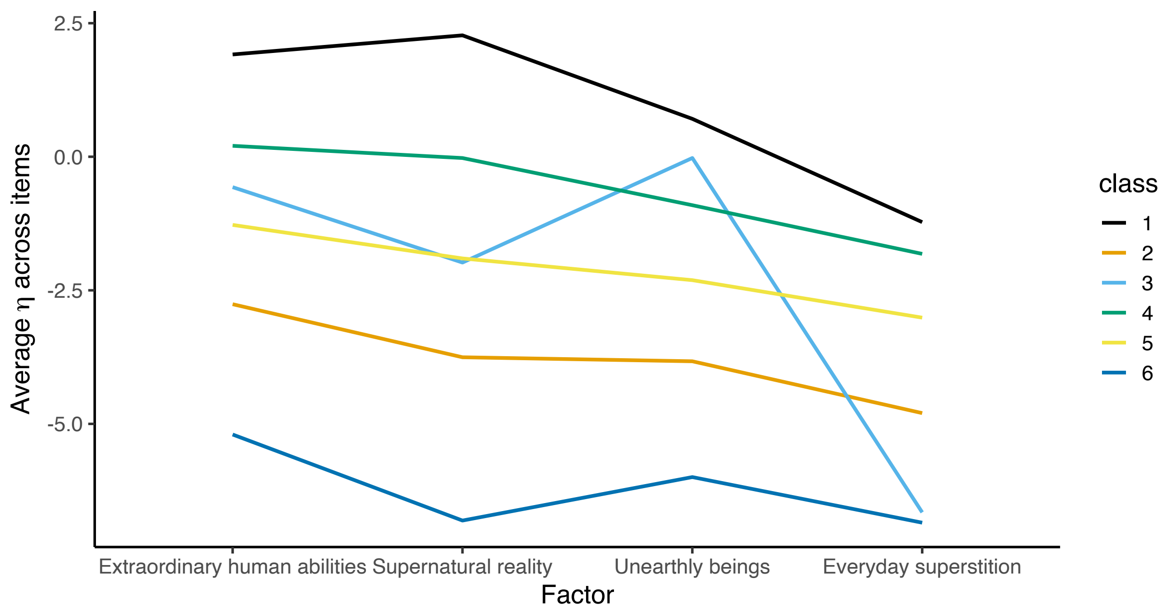

Figure 13: Average η per item factor group for each class under the 6-class model.

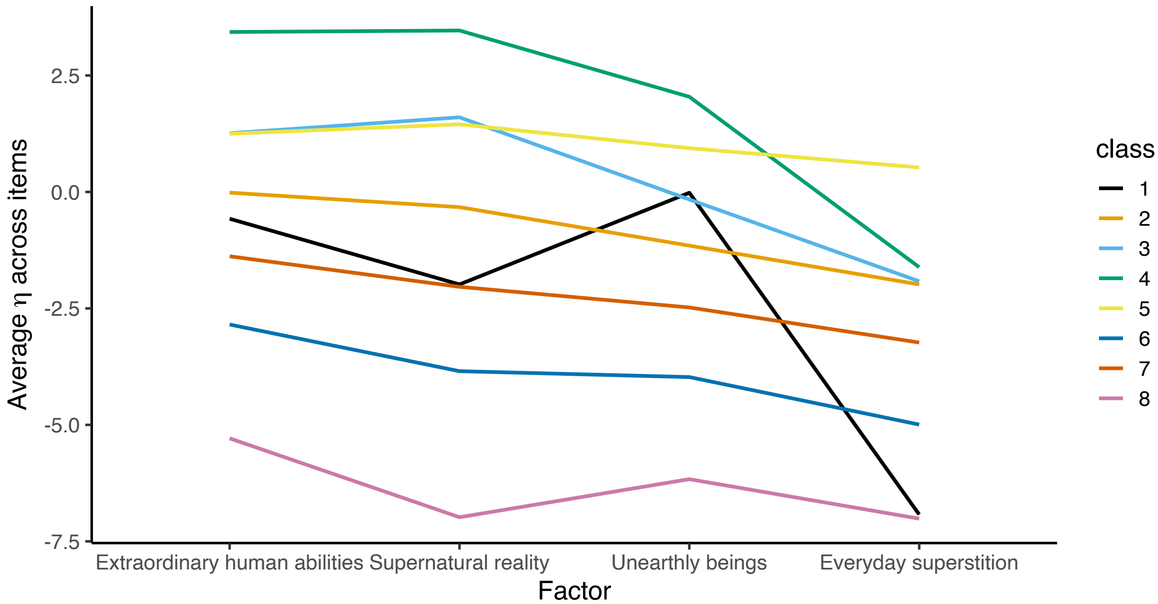

Figure 14: Average η per item factor group for each class under the 8-class model.

References

Carpenter, B., Gelman, A., Hoffman, M. D., Lee, D., Goodrich, B., Betancourt, M., Brubaker, M. A., Guo, J., Li, P., & Riddell, A. (2017). Stan: A probabilistic programming language. *Journal of Statistical Software*, *76*.

Oberski, D. (2016). Mixture models: Latent profile and latent class analysis. *Modern Statistical Methods for HCI*, 275–287.