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Assignment 2 - Calibration and DLT

1. Introduction

In this assignment you will study camera calibration and the DLT method. You will solve the resection and triangulation problems using DLT and compute inner parameters using RQ factorization. In addition try out SIFT for feature detection/matching.

2. Calibrated vs. Uncalibrated Reconstruction

Exercise 1.

$$\lambda x = P X$$

$$\tilde{X} = T^{-1} X$$

$$\lambda x = P T T^{-1} X = P T \tilde{X}$$

Computer Exercise 1.

Plot the 3D points of the reconstruction. Does this look like a reasonable reconstruction?

- Yes

Plot the image, the projected points, and the image points in the same figure. Do the projections appear to be close to the corresponding image points?

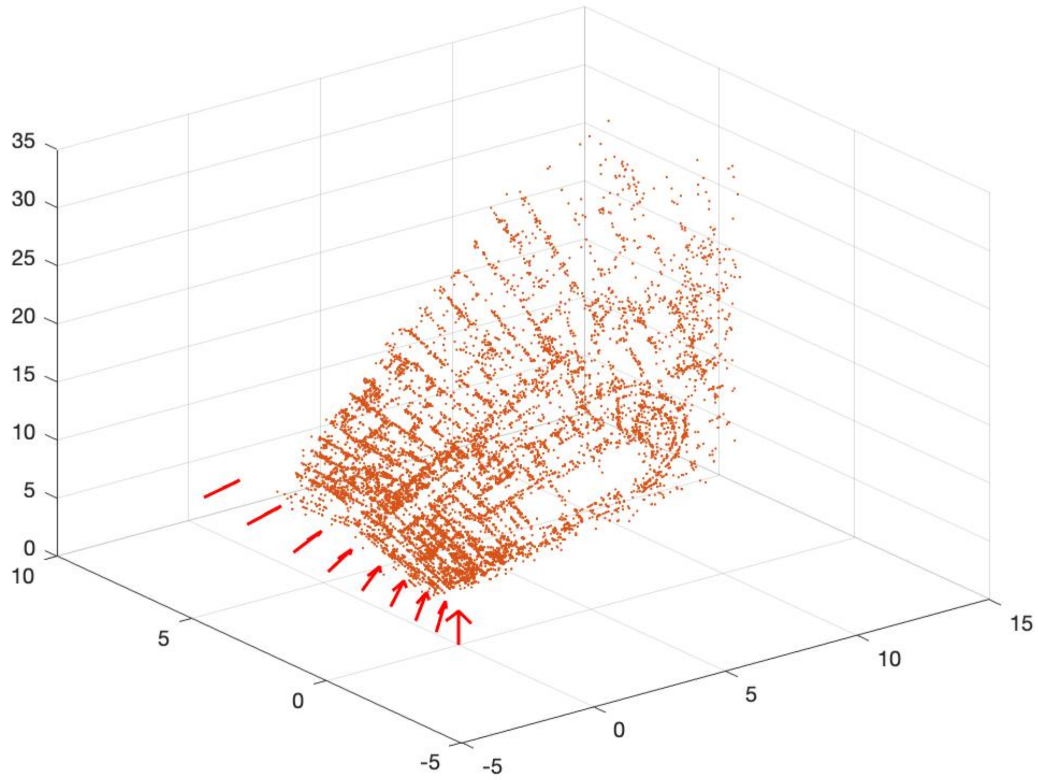
- Yes but slighty off when zoomed in

Plot the 3D points and cameras in the same figure for each of the solutions. Do any of them seem reasonable?

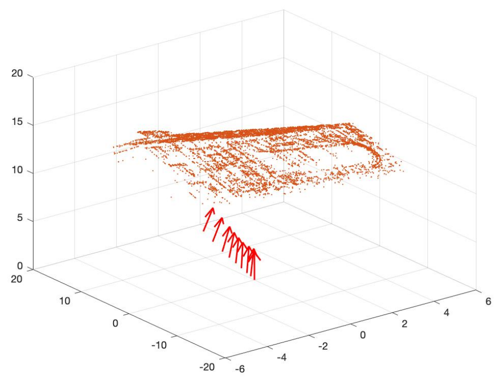
- T2 looks okay, especially after dividing with the fourth coordinate..

Project the new 3D points into one of the cameras. Plot the image, the projected points, and the image points in the same figure. Do the projections appear to have changed?

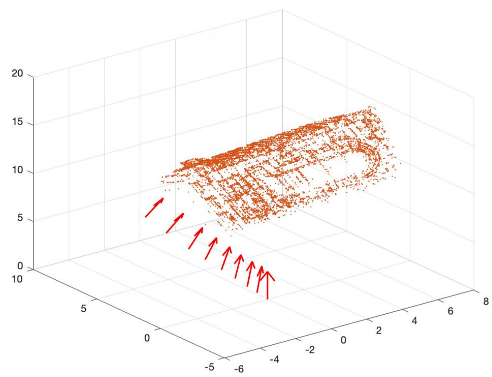
- Yes, requires more zoom to differ now



Original



Transform 1



Transform 2

Exercise 2.

Explain why we can not get the same projective distortions as in Computer Exercise 1 when we use calibrated cameras.

- The inner parameters K are known. Which transforms the point between the 3D coordinates and 2D

What is the corresponding statement for calibrated cameras to that of Exercise 1?

$$P = K[R\,t]$$

$$\tilde{x} = K^{-1}x$$

$$\tilde{x} = K^{-1}K[R\,t]X = [R\,t]\mathbf{X}$$

3. Camera Calibration

Exercise 3.

What is the geometric interpretation of the the transformations A and B?

- A, to rescale the the first two columns / rows
- B, move the first two columns/rows by x0 and y0

What is the interpretation of this operation?

- A, to rescale the images between pixels and meters
- B, to translate the points of the image

Where does the principal point (x0 , y0) end up?

- In the middle of the image

And where does a point with distance f to the principal point end up?

- At the coordinate divided by the focal length f

Normalize the points (0, 240), (640, 240).

What is the angle between the viewing rays projecting to these points?

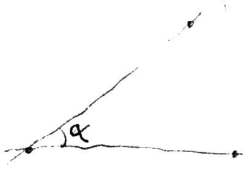
Show that the camera K[R t] and the corresponding normalized version [R t] have the same camera center and principal axis.

$$K = \begin{pmatrix} 320 & 0 & 320 \\ 0 & 320 & 240 \\ 0 & 0 & 1 \end{pmatrix}, \quad K^{-1} = \begin{pmatrix} 0{,}0031 & 0 & -1 \\ 0 & 0{,}0031 & -0{,}75 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{x}_1 = K^{-1}x = \begin{pmatrix} 0{,}0031 & 0 & -1 \\ 0 & 0{,}0031 & -0{,}75 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\tilde{x}_2 = \begin{pmatrix} 0{,}0031 & 0 & -1 \\ 0 & 0{,}0031 & -0{,}75 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 640 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$u = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = v$$

$$uv = |u||v|\cos\alpha \Rightarrow \cos\alpha = \frac{uv}{|u||v|}$$

$$\cos\alpha = \frac{-1+0+1}{|u||v|} = 0$$

$$\alpha = 90^\circ$$

Camera $K[R\ t]$, normalized $[R\ t]$

$$0 = [R\ t]\begin{bmatrix} C \\ 1 \end{bmatrix} \Leftrightarrow K[R\ t]\begin{bmatrix} C \\ 1 \end{bmatrix}$$

$$0 = [KR\ Kt]\begin{bmatrix} C \\ 1 \end{bmatrix} = KRC + Kt = 0 \Rightarrow RC + t = 0$$

$$P_{1:3}C + P_4 = 0 \Rightarrow C = -P_{1:3}^{-1}P_4$$

Exercise 4.

Normalize the corners of the image

(0, 0) → (-0.5, -0.5)

(0, 1000) → (-0.5, 0.5)

(1000, 0) → (0.5, -0.5)

(1000, 1000) → (0.5, 0.5)

(1000, 500) → (0, 0)

4. RQ Factorization and Computation of K

Exercise 5.

e3.

$$K = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \quad KR = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix}, \quad R = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix}$$

$$A = KR = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix} = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \\ -\frac{700}{\sqrt{2}} & 1400 & \frac{700}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$fR_3^T = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \|A_3\| = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}} = 1 = f \Rightarrow R_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A_2 = dR_2^T + eR_3^T$$

$$e = A_2^T R_3 = \begin{pmatrix} -\frac{700}{\sqrt{2}} \\ 1400 \\ \frac{700}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} = 700$$

$$dR_2^T = A_2 - eR_3 = \begin{pmatrix} -\frac{700}{\sqrt{2}} \\ 1400 \\ \frac{700}{\sqrt{2}} \end{pmatrix} - 700 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1400 \\ 0 \end{pmatrix} = 1400 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{matrix} d = 1400 \\ R_2 = (0 \ 1 \ 0) \end{matrix}$$

$$A_1 R_2 = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T = 0 = b$$

$$A_1 R_3 = \begin{pmatrix} \frac{800}{\sqrt{2}} & 0 & \frac{2400}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}^T = -400 + 0 + 1200 = 800 = c$$

$$A_1 = aR_1^T + bR_2^T + cR_3^T \Rightarrow aR_1^T = A_1 - bR_2^T - cR_3^T$$

$$= \begin{pmatrix} \frac{800}{\sqrt{2}} \\ 0 \\ \frac{2400}{\sqrt{2}} \end{pmatrix} - 0 - 800 \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1600}{\sqrt{2}} \\ 0 \\ \frac{1600}{\sqrt{2}} \end{pmatrix} = \frac{1600}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{matrix} a = \frac{1600}{\sqrt{2}} \\ R_1^T = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \end{matrix}$$

$a = \frac{1600}{\sqrt{2}}$

$b = 0$

$c = 800$

$d = 1400$

$e = 700$

$R_1 = (1 \ 0 \ 1)$

$R_2 = (0 \ 1 \ 0)$

$R_3 = \left(\frac{-1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}}\right)$

$f = 1$

$$AR = \begin{pmatrix} \frac{1600}{\sqrt{2}} & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$f = 1400$

$s = 0$

$AR = \frac{4\sqrt{2}}{7}$

$x_0 = 800$

$y_0 = 700$

Computer Exercise 2.

5 Direct Linear Transformation DLT

Exercise 7.

$$\tilde{X} \sim N X$$

$$\tilde{x} \sim \tilde{P} x$$

solve P, $x \sim PX$ from $\tilde{P}$

$$\tilde{X} \sim N x$$

$$\tilde{x} \sim N P X$$

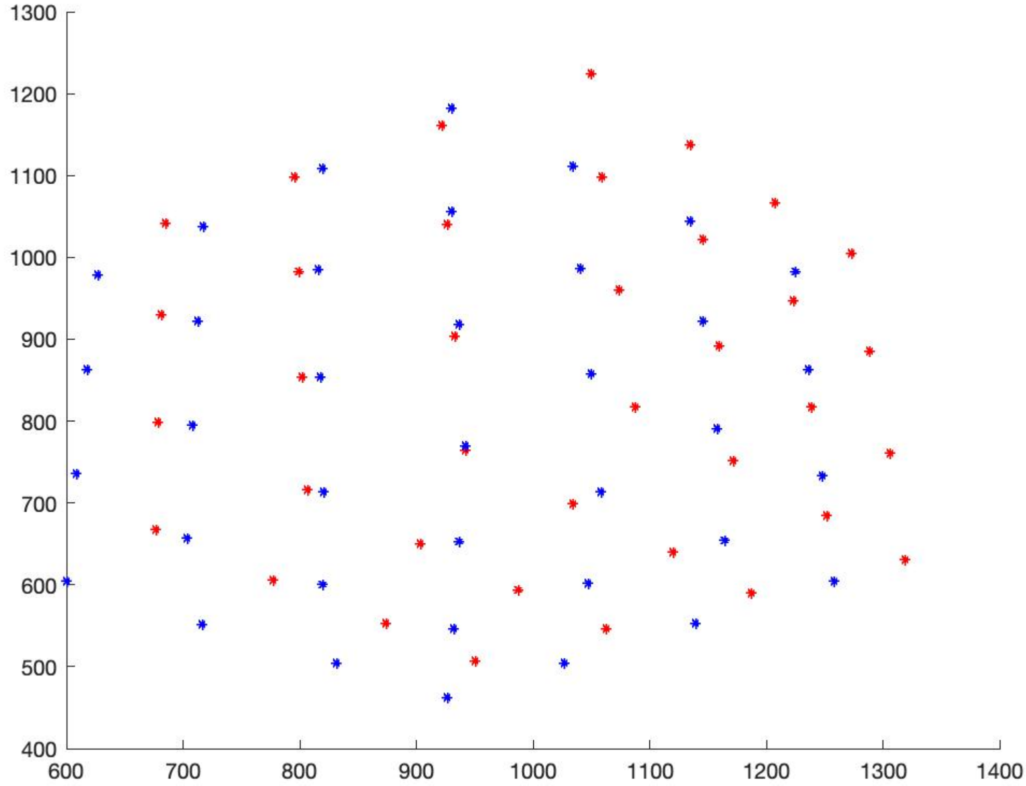
$$\tilde{P} X \sim N P X$$

$$\tilde{P} \sim N P$$

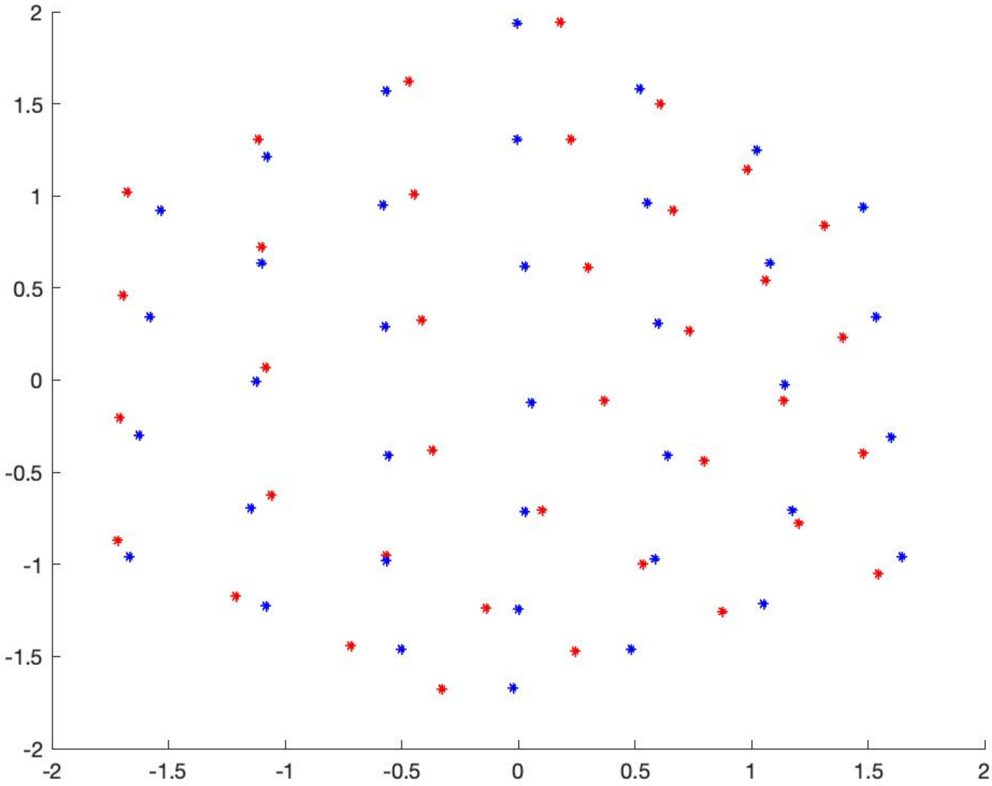
$$N^{-1} \tilde{P} \sim P$$

Computer Exercise 3.

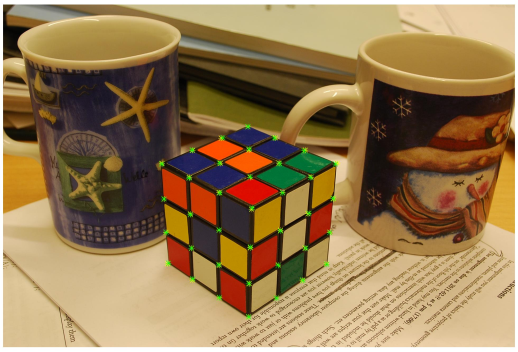
Plotting the measured projections of the model points with red being the first image and blue the second.



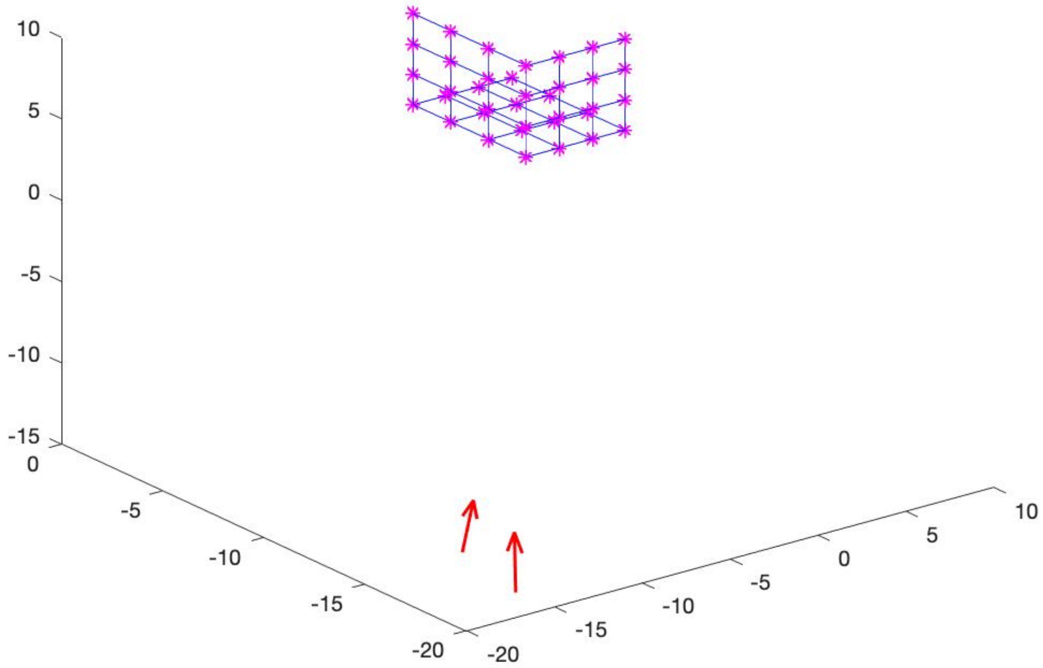
After normalization.



Project the model points onto the images.



Plotting the camera centers and viewing directions.



Inner parameters

P1

```
K1 =

   12.7600   -0.0938    5.0043
         0   12.7513    3.5222
         0         0    0.0052


R1 =

    0.5584   -0.0434   -0.8285    1.1145
   -0.3753    0.8774   -0.2989    6.4414
    0.7399    0.4778    0.4736   28.7007
```

P2

```
K2 =

   12.7751   -0.1314    4.3536
         0   12.8416    4.2316
         0         0    0.0053
```

```
R2 =

    0.7182    0.0115   -0.6957    1.4018
   -0.3747    0.8488   -0.3729    4.5831
    0.5863    0.5285    0.6139   28.1415
```

6 Feature Extraction and Matching using SIFT

Computer Exercise 5.

Image 1 with the projected points being pink and measured projection points being green

Image 2

Projected model with cameras