Computer Vision Assignment 1

by Ludwig Jakobsson

dat14lja

Points in Homogeneous Coordinates

Exercise 1

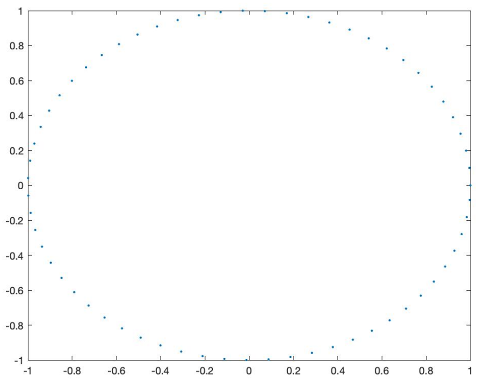
What are the 2D Cartesian coordinates of the points with homogeneous coordinates

$$x_1 = (2, -1) \qquad x_2 = (-3, -2) \qquad x_3 = (2, -1)$$

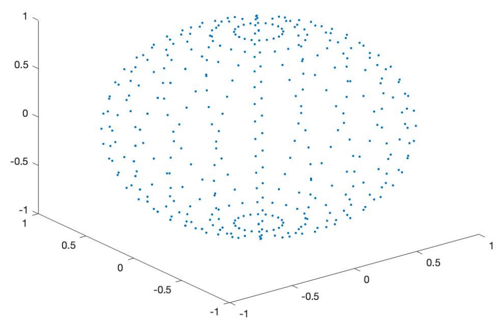
What is the interpretation of the point with homogeneous coordinates (4, -2, 0)?

- The point is not in the picture, its parallell to the image

Computer Exercise 1



x2D plot



x3D plot

Lines

Exercise 2

Compute the homogeneous coordinates of the intersection

What is the corresponding point in R2?

Compute the intersection (in P2) of the lines

What is the geometric interpretation in R2?

homogeneous coord of intersection

$$l_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad l_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0 \\ 3x + 2y + z = 0 \end{cases} \Leftrightarrow \begin{cases} x + y + z = 0 \\ -y - 2z = 0 \end{cases} \Leftrightarrow \begin{cases} x = t \\ y = -2t \\ z = t \end{cases}$$

$$\boxed{X \sim (1, -2, 1) \in \mathbb{P}^2 \text{ , } (1, -2) \text{ in } \mathbb{R}^2}$$

intersection of

$$l_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ , } l_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2y + 3z = 0 \\ x + 2y + z = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2y + 3z = 0 \\ -2z = 0 \end{cases} \Leftrightarrow \begin{cases} x = -2t \\ y = t \\ z = 0 \end{cases}$$

$$\boxed{X \sim (-2, 1, 0) \in \mathbb{P}^2 \text{ , not in the } \mathbb{R}^2 \text{ image}}$$

Compute the line that goes through the points with Cartesian coordinates line through the points

$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$y - 1 = \frac{2-1}{3-1}(x-1)$$

$$\boxed{y = \frac{x+1}{2}}$$

Exercise 3

Explain why the intersection point (in homogeneous coordinates) of l1 and l2 (from Exercise 2) is in the null space of the matrix

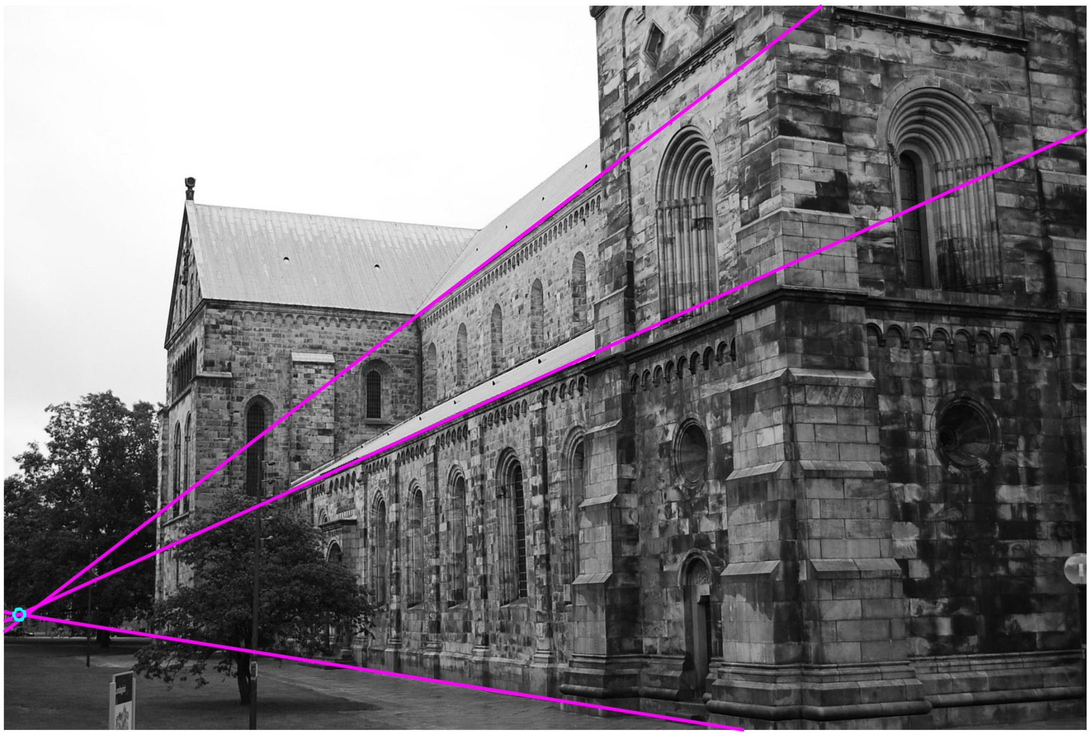
- The intersection point won't move when applying the m transformation

Are there any other points in the null space besides the intersection point?

- Reduced echelon form gives rank 2 which gives a null dimension of 1 with 3 coloumns so only the zero vector.

Computer Exercise 2

Compute the line going through the points



Compute the distance between the first line and the the intersection point.

$$d = 8.2695$$

Is it close to zero? Why/why not?

- Vanishing Point

Projective Transformations

Exercise 4

Compute the transformations y1 ~ Hx1 and y2 ~ Hx2

$$y_1^T = (1, 0, 0) \qquad y_2^T = (1, 1, 1)$$

Compute the lines l1, l2 containing x1,x2 and y1,y2 respectively.

$$l_1^T = (-1, -1, 1) \qquad l_2^T = (0, -1, 1)$$

Compute (H−1)T l1 and compare to l2.

$$(H^1)^T l_1 = (0, -1, 1)$$

Show that projective transformations preserve lines

$l_1^T x = 0$ then $l_1^T H^{-1} H x = 0$

$y \sim Hx$

$0 = l_1^T x = l_1^T H^{-1} H x \sim l_1^T H^{-1} y = ((H^{-1})^T l_1)^T y = l_2 y$

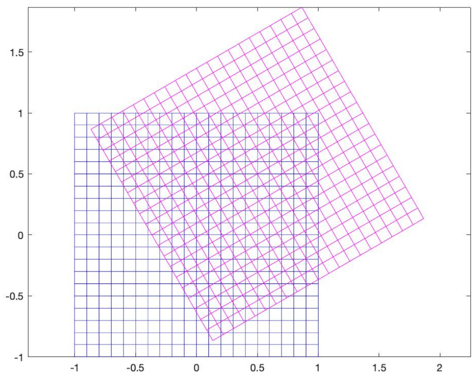
Computer Exercise 3

Which of the transformations preserve lengths between points?

Which preserve angles between lines?

Which maps parallel lines to parallel lines?

Classify the transformations into euclidean, similarity, affine and projective transformations.



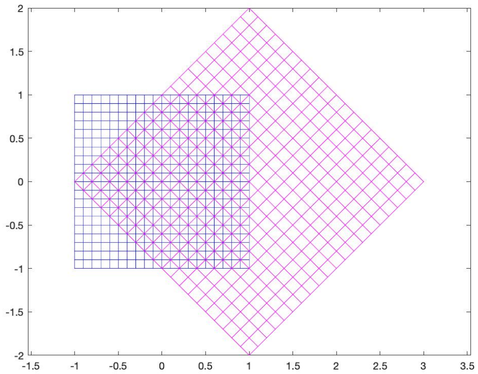
H1 transform

Similarity transformation

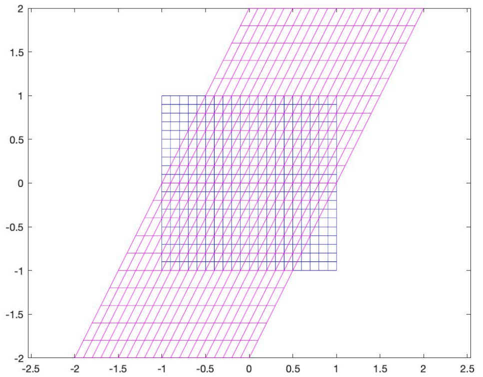
- preserves angles
- parallel lines

H2 transform

Euclidean



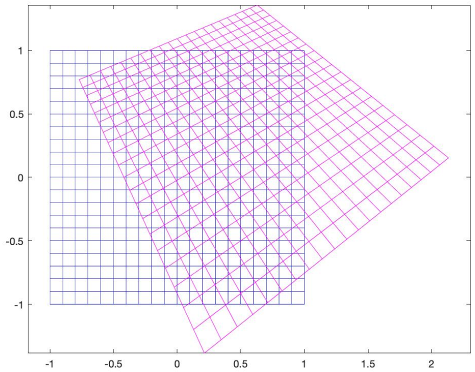
- preserves lengths between points
- preserves angles
- parallel lines



H3 transform

Affine transformation

- parallel lines



H4 transform

Projective transformation

The Pinhole Camera

Exercise 5

Compute the projections of the 3D points

$$X_1 projection = (1/4, 1/2)$$

$$X_2 projection = (1/2, 1/2)$$

$$X_3 projection = (1/0, 1/0)$$

What is the geometric interpretation of the projection of X3?

- It's not in the image

Compute the camera center (position) of the camera and the principal axis (viewing direction).

$$C = (0, 0, 1)$$

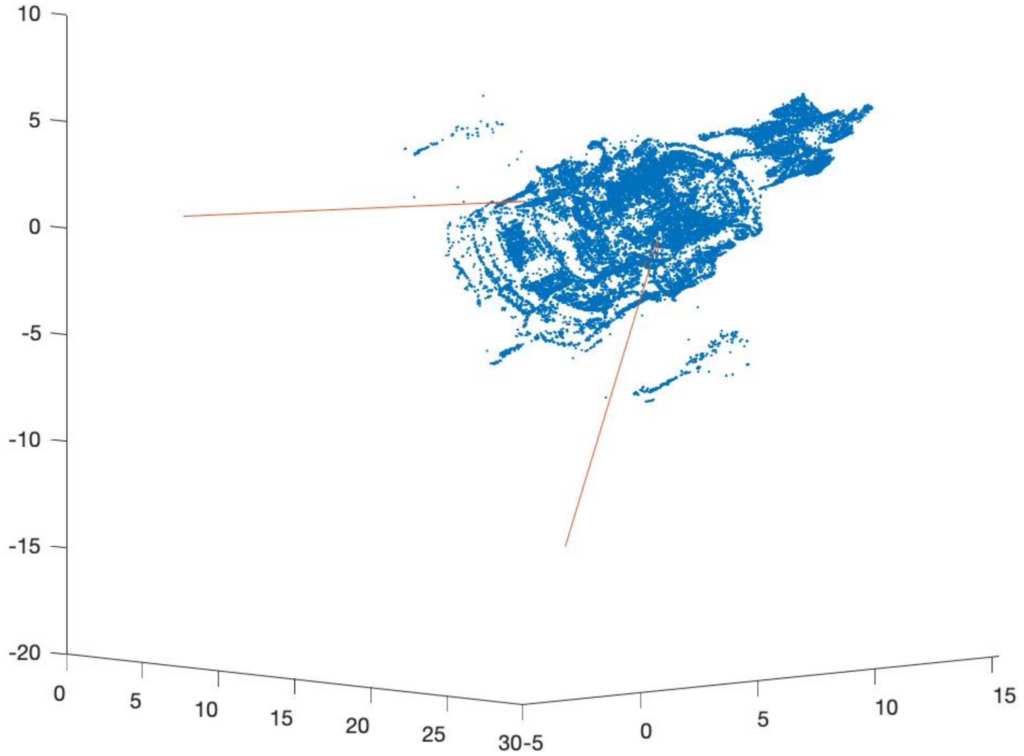
Computer Exercise 4

Compute the camera centers and principal axes of the cameras.

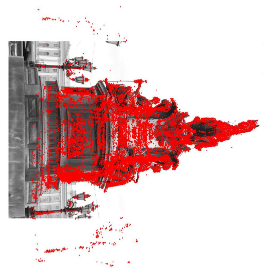
$$C_1 = (0, 0, 0) \qquad C_2 = (6.6352, 14.8460, -15.0691)$$

$$v_1 = (0.3129, 0.9461, 0.0837) \qquad v_2 = (0.0319, 0.3402, 0.9398)$$

Plot the 3D-points in U and the camera centers in the same 3D plot. In addition plot a vector in the direction of the principal axes.



Project the points in U into the cameras P1 and P2



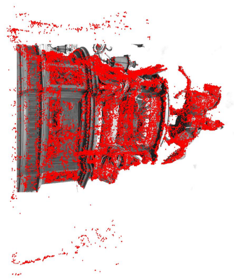


Image 1

Image 2