

## ASSIGNMENT-2 REPORT

As multiple local minima are possible in polynomial regression, multiple optimal solutions to minimizing the cost function are observed based on the initialization of the weights.

All the data was initially normalized and new features were generated according to the degree of the polynomial to be fit. An  $N$ -th degree polynomial has  $(N+1)C2$  coefficients.

Learning rate:  $(1e-6)$

Stopping criteria:  $(E-E' \leq 5e-2)$

Maximum iterations: 50000

### Gradient Descent:

| DEG | TRAIN ERR   | AVG TEST ERR | R2 ERROR     | RMSE      | WEIGHTS(np.random.randn())  |
|-----|-------------|--------------|--------------|-----------|---|
| 1   | 2504.969060 | 0.0082859983 | 2.5010214167 | 0.1287322 | [ 0.20848472 0.09536773 -0.09982884]  |
| 2   | 2416.236659 | 0.008031471  | 6.212487647  | 0.1267396 | [ 0.1423993 0.52602894 -0.47462574 -<br>0.17928082 -0.03033481 0.13114884]  |
| 3   | 2243.467785 | 0.007348374  | 12.65263576  | 0.1212301 | [ 0.22956625 -0.13999825 1.79395354 -<br>2.04201335 -0.53483783 -1.52421099<br>1.46343568 1.93033192 0.42746262 -<br>1.5781994 ]  |
| 4   | 2165.918867 | 0.007224321  | 15.68624651  | 0.1202025 | [ 0.20091943 -0.00290968 1.27206105 -<br>0.40164413 -1.72933962 -0.21291626<br>-1.98774206 1.08462712 1.66735796<br>0.57902535 0.02645383 0.27884248<br>1.56978226 -0.93956409 -1.40090874]   |
| 5   | 2124.145589 | 0.00695314   | 17.6060985   | 0.1179249 | [ 0.17442318 0.00980305 1.31990218 -<br>0.28355097 -1.4693503 -0.94444191<br>0.08125487 -1.98970689 0.69908159<br>1.32451212 1.63676402 -0.2194968<br>-0.29283898 0.28029615 0.39492675<br>1.56601267 -0.10389244 -0.71290387<br>0.67861609 -1.26979099 -0.87991535]  |
| 6   | 2110.53817  | 0.00695059   | 17.77905469  | 0.1179032 | [ 0.17048744 -0.08962674 1.50332933 -<br>0.02050638 -1.53132001 -1.52887083<br>-0.15819974 0.19751607 -1.95866819<br>0.57652114 0.99373625 1.036232<br>1.56734689 -0.37704709 -0.29113032<br>0.29275535 0.31045194 0.60387746<br>1.19626403 0.06177757 -0.36096084 -<br>0.31454571 0.73898155 -0.59345753<br>-0.86432248 -0.03954899 -0.88730104 -<br>0.15256575] |

From the above data:

- We can observe that the model has converged to some local minima.
- As higher degree polynomials are fit to the data, the training error decreases while the R2 error increases, indicating some overfitting.
- Over fit: degree 6 polynomial
- Best fit: degree 4 polynomial

### With L1 Regularization:

| DEG | VAL ERR   | AVG TEST ERR | R2 ERROR   | RMSE       | REG. COEFF. | WEIGHTS   |
|-----|-----------|--------------|------------|------------|-------------|---|
| 6   | 645.36448 | 0.007294469  | 14.2374103 | 0.12078467 | 0.875       | [ 0.18457507 -0.07398244 1.71922973 -<br>1.7966707 0.44542585 -0.806041<br>-0.581372 -0.11608601 -0.81777302 -<br>0.09801393 -0.87337241 0.40464632<br>2.5543963 -0.12994462 1.07814396<br>0.33470576 -0.42566046 -0.09357612<br>0.6889279 -0.36274973 0.72563591 -<br>0.44936528 0.10844836 -0.27020638<br>0.10681549 -0.79221959 -0.63772207<br>0.25556698] |

#### After L1-regularization:

- Regularization coeff = 0.875. The accuracy after regularization is close to that of degree 4 polynomial.

### With L2 Regularization:

| DEG | VAL ERR   | AVG TEST ERR | R2 ERROR   | RMSE       | REG. COEFF. | WEIGHTS   |
|-----|-----------|--------------|------------|------------|-------------|---|
| 6   | 648.47769 | 0.007259853  | 11.9812831 | 0.12049774 | 0.0         | [ 0.16959764 0.18025695 0.65518339 -<br>0.06978974 -0.58611338 -0.57898445 -<br>-0.16013752 -0.31185269 -0.4559507 -<br>0.43000127 -0.33544481 0.07765144<br>0.75703779 0.55313306 -0.15820445 -<br>0.19200157 0.24284709 1.00197346<br>0.63461067 -0.08260375 0.11207165<br>0.82501614 0.15354349 -0.17163259<br>0.36499707 -0.42624223 -0.25293941 -<br>0.89574433] |

#### After L2-regularization:

- Regularization coeff = 0, indicating no regularization in this case will yield a better answer.

*With L2 Regularization:*

| <u>DEG</u> | <u>TRAIN<br/>ERR</u> | <u>AVG TEST<br/>ERR</u> | <u>R2 ERROR</u> | <u>RMSE</u> | <u>REG.<br/>COEFF.</u> | <u>WEIGHTS</u> |
|------------|----------------------|-------------------------|-----------------|-------------|------------------------|----------------|
| 1          |                      |                         |                 |             |                        |                |
| 2          |                      |                         |                 |             |                        |                |
| 3          |                      |                         |                 |             |                        |                |
| 4          |                      |                         |                 |             |                        |                |
| 5          |                      |                         |                 |             |                        |                |
| 6          |                      |                         |                 |             |                        |                |