$$9.\ f(x)=\begin{cases}3x-1, & x\in[0,1)\\ 0, & x\in[1,2]\end{cases}$$

$$\exists\ x=\frac{1}{\pi}t+1;\quad f(t)=\begin{cases}\frac{3}{\pi}t+2, & t\in[-\pi,0)\\ 0 & t\in[0;\pi]\end{cases}$$

$$a_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\cos kt\,dt=\int_{-\pi}^{0}\left(\frac{3}{\pi}t+2\right)\cos kt\,dt=$$

$$=\frac{1}{\pi}\left(\frac{3}{\pi}\int_{-\pi}^{0}t\cos kt\,dt+2\int_{-\pi}^{0}\cos kt\,dt\right)=\frac{2\sin(\pi k)}{\pi k}-\frac{3(\pi k\sin(\pi k)+\cos(\pi k)-1)}{\pi^2k^2}=$$

$$=0-\frac{3(\pi k\cdot 0+(-1)^k-1)}{\pi^2k^2}=\frac{3(1-(-1)^k)}{\pi^2k^2}$$

$$b_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\sin kt\,dt=\frac{1}{\pi}\int_{-\pi}^{0}\left(\frac{3}{\pi}t+2\right)\sin kt\,dt=$$

$$=\frac{3\sin\pi k-\pi k\cos(\pi k)-2\pi k}{\pi^2k^2}=\frac{-(-1)^k-2}{\pi k}=$$

$$=\frac{(-1)^{k+1}-2}{\pi k}\qquad a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)\,dt=\frac{1}{2\pi}\int_{-\pi}^{0}\frac{3}{\pi}t+2\,dt=\frac{1}{4}$$

Тогда $S_f(t)=\frac{1}{4}+\sum_{k=1}^{\infty}\left(\frac{3(1-(-1)^k)}{\pi^2k^2}\cos kt+\frac{(-1)^{k+1}-2}{\pi k}\sin kt\right)$

$$\frac{1}{\pi}t+1=x;\quad \frac{1}{\pi}t=x-1\quad t=\pi(x-1)$$

$$S_f(x)=\frac{1}{4}+\sum_{k=1}^{\infty}\left(\frac{3(1-(-1)^k)}{\pi^2k^2}\cos(\pi k(x-1))+\frac{(-1)^{k+1}-2}{\pi k}\sin(\pi k(x-1))\right)-$$

~~Общий тригонометрический ряд~~

По следствию признака Дини $S_f(x)=\begin{cases}3x-1, & 0<x<1\\ 0, & 1<x<2\\ 1, & x=1\\ -\frac{1}{2}, & x\in\{0;2\}\end{cases}$

$f(0+)=-1;\ f(0-)=f(2-)=0\quad S_f(0)=\frac{f(0+)+f(0-)}{2}=-\frac{1}{2}=$

$=S_f(2);\quad S_f(1)=\frac{f(1+)+f(1-)}{2}=\frac{2+0}{2}=1$

Ряд по косинусам

$x = \frac{2}{\pi}t \qquad f(t) = \begin{cases} \frac{6}{\pi}t - 1 & x \in [0, \frac{\pi}{2}) \\ 0 & x \in [\frac{\pi}{2}; \pi] \end{cases}$

Продолжим $f(t)$ до $\tilde{f}(t) = \begin{cases} 0; & t \in [-\pi; -\frac{\pi}{2}) \\ -1 - \frac{6}{\pi}t, & t \in [-\frac{\pi}{2}; 0) \\ \frac{6}{\pi}t - 1, & t \in [0; \frac{\pi}{2}) \\ 0, & t \in [\frac{\pi}{2}; \pi] \end{cases}$

$\tilde{f}(t)$ четная, зн. $b_k = 0$

$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \tilde{f}(t)\cos kt\,dt = \frac{1}{\pi}\left(\int_{-\frac{\pi}{2}}^{0} (-1 - \frac{6}{\pi}t)\cos kt\,dt + \int_{0}^{\frac{\pi}{2}} (\frac{6}{\pi}t - 1)\cos kt\,dt\right)$

$= \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}} (\frac{6}{\pi}t - 1)\cos kt\,dt = \frac{4(2\pi k \sin(\frac{\pi k}{2}) + 3\cos\frac{\pi k}{2} - 3)}{\pi^2 k^2}$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \tilde{f}(t)\,dt = \frac{1}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{6}{\pi}t - 1\,dt = \frac{1}{4}$

$S_{\tilde{f}}(t) = \frac{1}{4} + \sum_{k=1}^{\infty} \frac{4}{\pi^2 k^2}(\pi k \sin\frac{\pi k}{2} + 3\cos\frac{\pi k}{2} - 3)\cos kt$

$S_{\tilde{f}}(x) = \frac{1}{4} + \sum_{k=1}^{\infty} -''- \cos\frac{\pi k}{2}x \ ; \quad S_{\tilde{f}}(x) = \begin{cases} 0; & x \in [-2, -1) \\ -1 - 3x, & x \in (-1, 0) \\ 3x - 1, & x \in (0; 1) \\ 0; & x \in (1; 2] \\ 1, & x = \pm 1 \end{cases}$

$S_{\tilde{f}}(1) = \frac{f(1+) + f(1-)}{2} = \frac{2+0}{2} = 1$

$S_{\tilde{f}}(-1) = \frac{f(-1+) + f(-1-)}{2} = \frac{2+0}{2} = 1$

Ряд по синусам:

$x = \frac{2}{\pi}t \qquad \tilde{f}(t) = \begin{cases} 0, & t \in [-\pi; -\frac{\pi}{2}) \\ \frac{6}{\pi}t + 1, & t \in [-\frac{\pi}{2}; 0) \\ \frac{6}{\pi}t - 1, & t \in [0, \frac{\pi}{2}) \\ 0, & t \in [\frac{\pi}{2}; \pi] \end{cases}$

$\tilde{f}(t)$ - нечетная, зн. $a_n = 0$

$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} \tilde{f}(t)\sin kt\,dt = \frac{1}{\pi}\left(\int_{-\frac{\pi}{2}}^{0} (\frac{6}{\pi}t + 1)\sin kt\,dt + \int_{0}^{\frac{\pi}{2}} (\frac{6}{\pi}t - 1)\sin kt\,dt\right)$

$= \frac{2(6\sin\frac{\pi k}{2} - \pi k - 2\pi k\cos\frac{\pi k}{2})}{\pi^2 k^2}$

$$S_{\tilde f}(t) = 2\sum_{k=1}^{\infty} \frac{6\sin\frac{\pi k}{2} - \pi k - 2\pi k\cos\frac{\pi k}{2}}{\pi^2 k^2} \sin kt$$

$$S_{\tilde f}(x) = 2\sum_{k=1}^{\infty} \text{—//—} \sin\left(\frac{\pi}{2} k x\right)$$

$$S_{\tilde f}(x) = \begin{cases} 0, & x \in [-2, -1) \\ 3x+1, & x \in (-1, 0) \\ 3x-1, & x \in (0, 1) \\ 0, & x \in (1, 2] \\ -1, & x = -1 \\ 1, & x = 1 \\ 0, & x = 0 \end{cases}$$

(рассуждения аналогичны предыдущим пунктам)