Step-by-Step Integral Evaluation

```
x, y, z = symbols('x,y,z')
f = Symbol('f(x,y,z)')

# Define limits of integration
x_llim = 0
x_ulim = 2
y_llim = 0
y_ulim = 3
z_llim = 0
z_ulim = 4

print(r'\begin{align*}')

# Notice how I define f as a symbol, then later as an actual function
left = Integral(f, (x, x_llim, x_ulim), (y, y_llim, y_ulim), (z, z_llim, z_ulim))
f = x*y + y*sin(z) + cos(x+y)
right = Integral(f, (x, x_llim, x_ulim), (y, y_llim, y_ulim), (z, z_llim, z_ulim))
print(latex(left) + '&=' + latex(right) + r'\\')

# For each step, I move limits from an outer integral to an inner, evaluated
# integral until the outer integral is no longer needed
right = Integral(Integral(f, (z, z_llim, z_ulim)).doit(), (x, x_llim, x_ulim),
                 (y, y_llim, y_ulim))
print('&=' + latex(right) + r'\\')

right = Integral(Integral(f, (z, z_llim, z_ulim), (y, y_llim, y_ulim)).doit(),
                 (x, x_llim, x_ulim))
print('&=' + latex(right) + r'\\')

right = Integral(f, (z, z_llim, z_ulim), (y, y_llim, y_ulim),
                 (x, x_llim, x_ulim)).doit()
print('&=' + latex(right) + r'\\')

print('&=' + latex(N(right)) + r'\\')

print(r'\end{align*}')
```

$$\begin{aligned}
\int_0^4 \int_0^3 \int_0^2 f(x,y,z)\,dx\,dy\,dz &= \int_0^4 \int_0^3 \int_0^2 xy + y\sin(z) + \cos(x+y)\,dx\,dy\,dz \\
&= \int_0^3 \int_0^2 4xy - y\cos(4) + y + 4\cos(x+y)\,dx\,dy \\
&= \int_0^2 18x - 4\sin(x) + 4\sin(x+3) - \frac{9}{2}\cos(4) + \frac{9}{2}\,dx \\
&= 4\cos(3) + 4\cos(2) - 4\cos(5) - 9\cos(4) + 41 \\
&= 40.1235865133293
\end{aligned}$$