1 Système

$$\left\{\begin{array}{rcl} m_s\ddot{z_s} + u\left(\dot{z_s} - \dot{z_u}\right) + k_s\left(z_s - z_u\right) &=& -F_f \\ m_u\ddot{z_u} - u\left(\dot{z_s} - \dot{z_u}\right) - k_s\left(z_s - z_u\right) + k_t\left(z_u - z_r\right) &=& F_f \\ F_f &=& C_f \tanh\left(\gamma_f \dot{z_s}\right) \end{array}\right.$$

2 Sortie plate

$$S_p = m_u z_u + m_s z_s$$

$$S_p^{(1)} = m_u \dot{z_u} + m_s \dot{z_s}$$

$$\begin{array}{rcl} S_p^{(2)} &=& m_u\ddot{z_u} + m_s\ddot{z_s} \\ &=& -k_t\left(z_u - z_r\right) \end{array}$$

$$S_p^{(3)} = -k_t\left(\dot{z_u} - \dot{z_r}\right)$$

$$\begin{array}{rcl} S_p^{(4)} &=& k_t\ddot{z_r} - k_t\ddot{z_u} \\ &=& k_t\ddot{z_r} + \dfrac{k_t}{m_u}\left[u\left(\dot{z_s} - \dot{z_u}\right) + k_s\left(z_s - z_u\right) - k_t\left(z_u - z_r\right) + F_f\right] \end{array}$$

On obitent alors

$$z_u = z_r - \frac{1}{k_t} S_p^{(2)}$$

et

$$z_s = \frac{1}{m_s} S_p - \frac{m_u}{m_s} z_r + \frac{m_u}{k_t m_s} Sp^{(2)}$$

2.1 Linéarisation

$$\left\{\begin{array}{rcl} S_p^{(4)} &=& v \\ v &=& S_{pr}^{(4)} - \lambda_0 e_{sp} - \lambda_1 e_{sp}^{(1)} - \lambda_2 e_{sp}^{(2)} - \lambda_3 e_{sp}^{(3)} \end{array}\right.$$

$$\begin{array}{rcl} e_{sp} &=& m_u z_u + m_s z_s - S_{pr} \\ e_{sp}^{(1)} &=& m_u \dot{z_u} + m_s \dot{z_s} - S_{pr}^{(1)} \\ e_{sp}^{(2)} &=& k_t z_r - k_t z_u - S_{pr}^{(2)} \\ e_{sp}^{(3)} &=& k_t \dot{z_r} - k_t \dot{z_u} - S_{pr}^{(3)} \end{array}$$

On obtient la commande :

$$u\left(\dot{z_s} - \dot{z_u}\right) = \frac{m_u}{k_t} v - m_u\ddot{z_r} - k_s\left(z_s - z_u\right) + k_t\left(z_u - z_r\right) - F_f$$

3 Limiter le jerk

Fixons

$$S_{pr}(t) = \alpha \tanh(\gamma t)$$

On a alors

$$\begin{aligned}
S_{pr} & & & \in & [-\alpha, \alpha] \\
S_{pr}^{(1)} &= \alpha\gamma - \frac{\gamma}{\alpha} S_{pr}^2 & & \in & [0, \alpha\gamma] \\
S_{pr}^{(2)} &= -2\frac{\gamma}{\alpha} S_{pr} S_{pr}^{(1)} & & \in & \left[-2\alpha\gamma^2, 2\alpha\gamma^2\right] \\
S_{pr}^{(3)} &= -2\frac{\gamma}{\alpha}\left[\left(S_{pr}^{(1)}\right)^2 + S_{pr} S_{pr}^{(2)}\right] & & \in & \left[-6\alpha\gamma^3, 4\alpha\gamma^3\right] \\
S_{pr}^{(4)} &= -2\frac{\gamma}{\alpha}\left[3 S_{pr}^{(1)} S_{pr}^{(2)} + S_{pr} S_{pr}^{(3)}\right] & & \in & \left[-20\alpha\gamma^4, 24\alpha\gamma^4\right] \\
S_{pr}^{(5)} &= -2\frac{\gamma}{\alpha}\left[3\left(S_{pr}^{(2)}\right)^2 + 4 S_{pr}^{(1)} S_{pr}^{(3)} + S_{pr} S_{pr}^{(4)}\right] & & \in &
\end{aligned}$$

On peut alors surapproximer le jerk du chassis :

$$\left|z_s^{(3)}\right| \quad \leq$$