1 Système

$$\left\{\begin{array}{rcl} m_s\ddot{z_s}+u\left(\dot{z_s}-\dot{z_u}\right)+k_s\left(z_s-z_u\right) & = & -F_f \\ m_u\ddot{z_u}-u\left(\dot{z_s}-\dot{z_u}\right)-k_s\left(z_s-z_u\right)+k_t\left(z_u-z_r\right) & = & F_f \\ F_f & = & C_f\tanh\left(\gamma_f\dot{z_s}\right) \end{array}\right.$$

2 Sortie plate

$$S_p = m_u z_u + m_s z_s$$

$$S_p^{(1)} = m_u\dot{z_u} + m_s\dot{z_s}$$

$$\begin{array}{rcl} S_p^{(2)} & = & m_u\ddot{z_u}+m_s\ddot{z_s} \\ & = & -k_t\left(z_u-z_r\right) \end{array}$$

$$S_p^{(3)} = -k_t\left(\dot{z_u}-\dot{z_r}\right)$$

$$\begin{array}{rcl} S_p^{(4)} & = & k_t\ddot{z_r}-k_t\ddot{z_u} \\ & = & k_t\ddot{z_r}+\dfrac{k_t}{m_u}\left[u\left(\dot{z_s}-\dot{z_u}\right)+k_s\left(z_s-z_u\right)-k_t\left(z_u-z_r\right)+F_f\right] \end{array}$$

On obitent alors

$$z_u = z_r - \frac{1}{k_t}S_p^{(2)}$$

et

$$z_s = \frac{1}{m_s}S_p - \frac{m_u}{m_s}z_r + \frac{m_u}{k_t m_s}Sp^{(2)}$$

2.1 Linéarisation

$$\left\{\begin{array}{rcl} S_p^{(4)} & = & v \\ v & = & S_{pr}^{(4)}-\lambda_0 e_{sp}-\lambda_1 e_{sp}^{(1)}-\lambda_2 e_{sp}^{(2)}-\lambda_3 e_{sp}^{(3)} \end{array}\right.$$

$$e_{sp} = m_u z_u + m_s z_s - S_{pr}$$

$$e_{sp}^{(1)} = m_u\dot{z_u} + m_s\dot{z_s} - S_{pr}^{(1)}$$

$$e_{sp}^{(2)} = k_t z_r - k_t z_u - S_{pr}^{(2)}$$

$$e_{sp}^{(3)} = k_t\dot{z_r} - k_t\dot{z_u} - S_{pr}^{(3)}$$

On obtient la commande :

$$u\left(\dot{z_s}-\dot{z_u}\right)=\frac{m_u}{k_t}v-m_u\ddot{z_r}-k_s\left(z_s-z_u\right)+k_t\left(z_u-z_r\right)-F_f$$