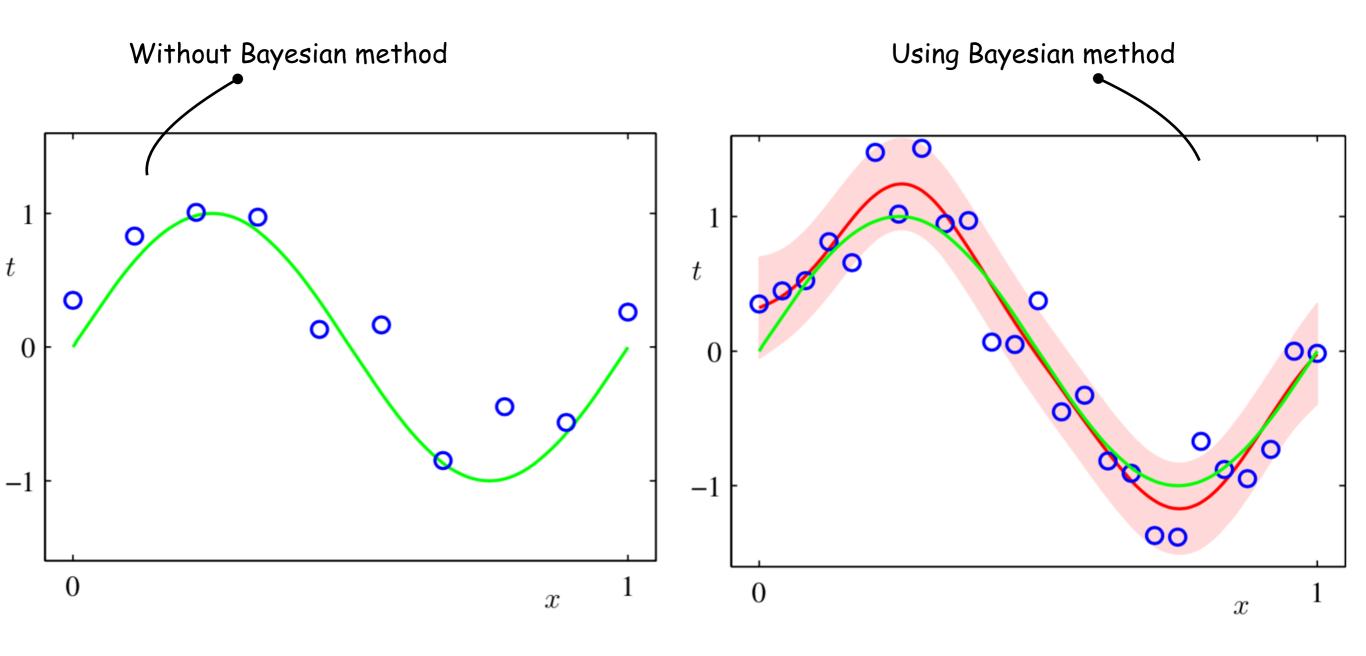
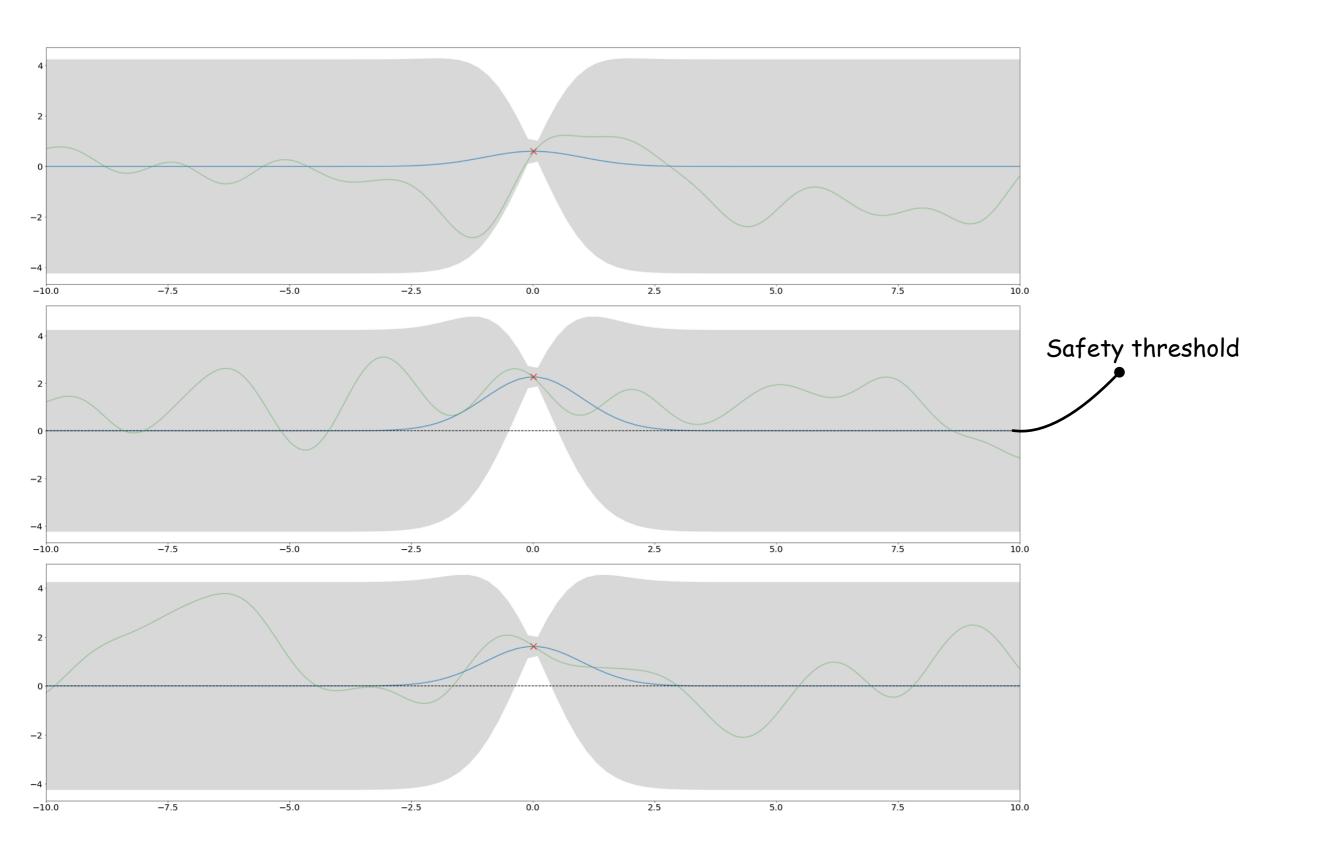
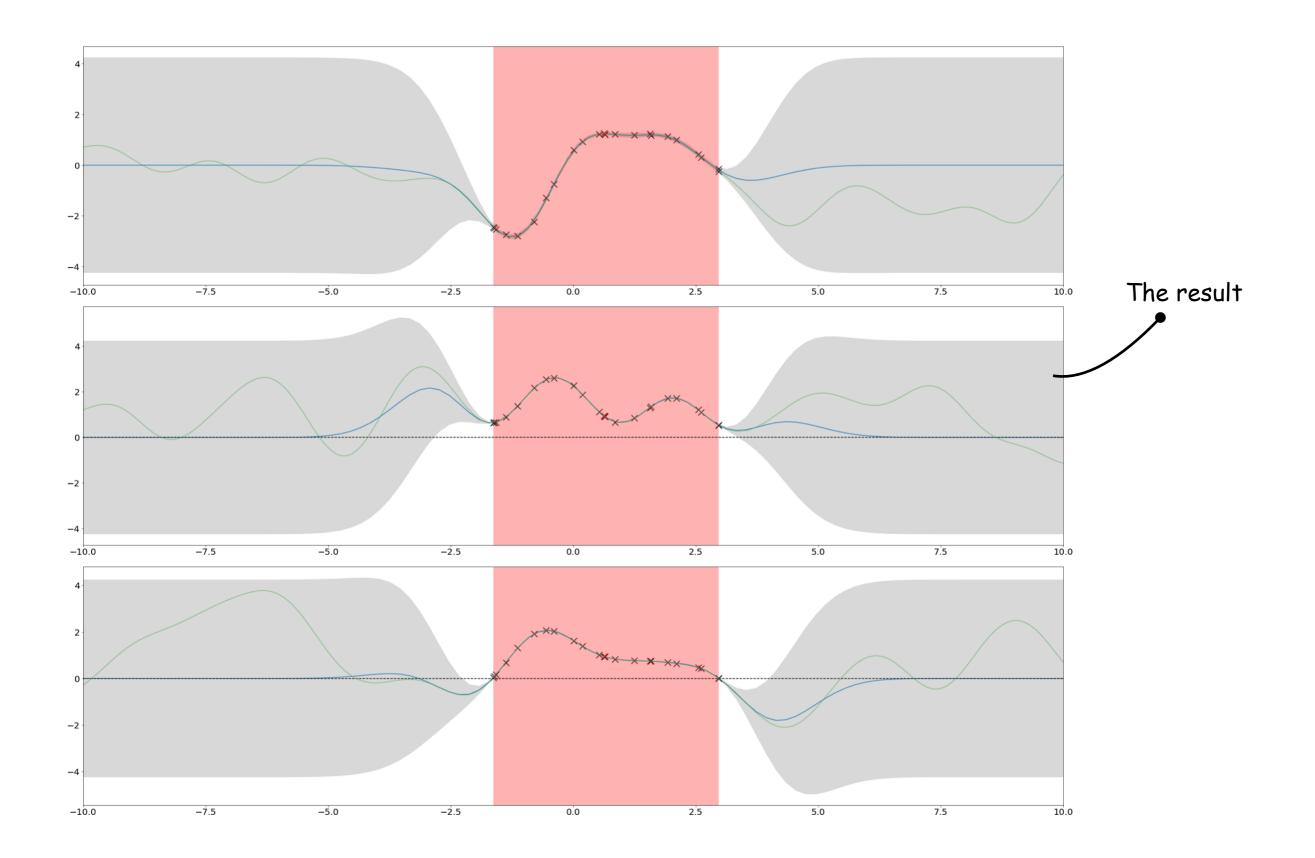
SAFEOPT & STAGEOPT

Daoran Jing 2019.07.31

Fit theory & Bayesian methods







SafeOpt

•
$$Q_t(x) := [\mu_{t-1}(x) \pm \beta_t^{1/2} \sigma_{t-1}(x)]$$
 4: $Q_0(x) \leftarrow \mathbb{R}$, for all $x \in D$

•
$$g_t(x) := |\{x' \in D/S_t | u_t(x) - Ld(x, x') \ge h\}|$$

Algorithm 1 SAFEOPT

```
1: Input: sample set D,
              GP prior (\mu_0, k, \sigma_0),
              Lipschitz constant L,
              seed set S_0,
              safety threshold h
2: C_0(\boldsymbol{x}) \leftarrow [h, \infty), for all \boldsymbol{x} \in S_0
```

- 3: $C_0(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D \setminus S_0$
- 5: **for** t = 1, ... **do**
- $C_t(\boldsymbol{x}) \leftarrow C_{t-1}(\boldsymbol{x}) \cap Q_{t-1}(\boldsymbol{x})$
- 7: $S_t \leftarrow \bigcup_{\boldsymbol{x} \in S_{t-1}} \left\{ \boldsymbol{x}' \in D \mid \ell_t(\boldsymbol{x}) Ld(\boldsymbol{x}, \boldsymbol{x}') \geq h \right\}$
- $G_t \leftarrow \{ \boldsymbol{x} \in S_t \mid g_t(\boldsymbol{x}) > 0 \}$
- $M_t \leftarrow \{ \boldsymbol{x} \in S_t \mid u_t(\boldsymbol{x}) \ge \max_{\boldsymbol{x}' \in S_t} \ell_t(\boldsymbol{x}') \}$
- $\boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in G_t \cup M_t}(w_t(\boldsymbol{x}))$
- 11: $y_t \leftarrow f(\boldsymbol{x}_t) + n_t$
- Compute $Q_t(\boldsymbol{x})$, for all $\boldsymbol{x} \in S_t$
- **13: end for**

StageOpt

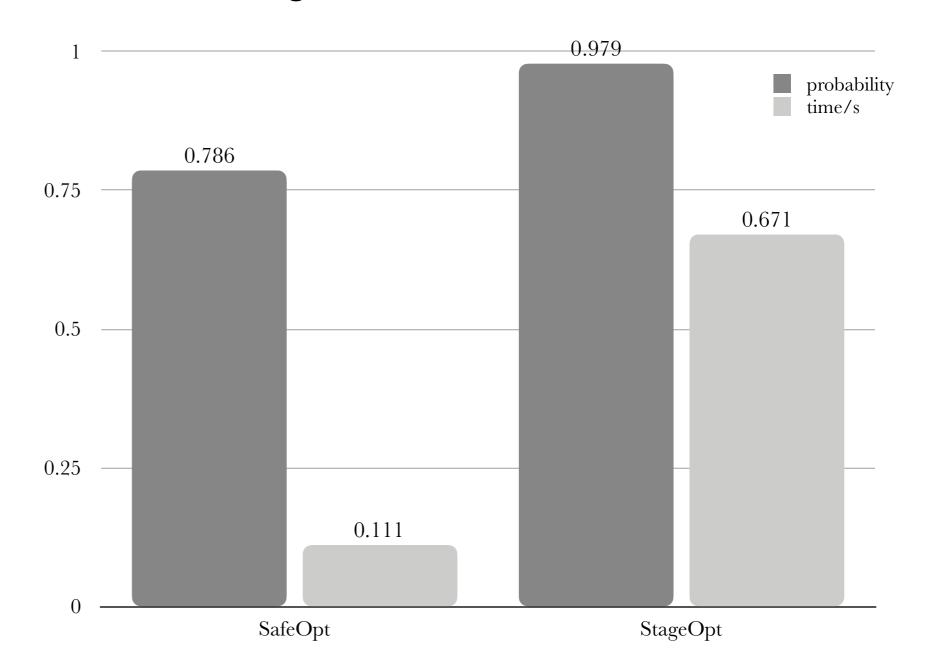
- Safe region expansion
- Optimization

Algorithm 1 STAGEOPT

```
1: Input: sample set D, i \in \{1, ..., n\},\
                        GP prior for utility function f,
                        GP priors for safety functions g_i,
                       Lipschitz constants L_i for g_i,
                        safe seed set S_0.
                        safety threshold h_i,
                        accuracies \epsilon (for expansion), \zeta (for optimization).
  2: C_0^i(\boldsymbol{x}) \leftarrow [h_i, \infty), for all \boldsymbol{x} \in S_0
  3: C_0^i(\boldsymbol{x}) \leftarrow \mathbb{R}, for all \boldsymbol{x} \in D \setminus S_0
  4: Q_0^i(\boldsymbol{x}) \leftarrow \mathbb{R}, for all \boldsymbol{x} \in D
  5: C_0^f(\boldsymbol{x}) \leftarrow \mathbb{R}, for all \boldsymbol{x} \in D
 6: Q_0^f(\boldsymbol{x}) \leftarrow \mathbb{R}, for all \boldsymbol{x} \in D
 7: for t = 1, ..., T_0 do
  8: C_t^i(\boldsymbol{x}) \leftarrow C_{t-1}^i(\boldsymbol{x}) \cap Q_{t-1}^i(\boldsymbol{x})
 9: C_t^f(\boldsymbol{x}) \leftarrow C_{t-1}^f(\boldsymbol{x}) \cap Q_{t-1}^f(\boldsymbol{x})
             S_t \leftarrow \bigcap_i \bigcup_{\boldsymbol{x} \in S_{t-1}} \left\{ \boldsymbol{x}' \in D \mid \ell_t^i(\boldsymbol{x}) - L_i d(\boldsymbol{x}, \boldsymbol{x}') \ge h_i \right\}
             G_t \leftarrow \{ \boldsymbol{x} \in S_t \mid e_t(\boldsymbol{x}) > 0 \}
11:
             if \forall i, \epsilon_t^i < \epsilon then
                   \boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in G_t, i \in \{1, \dots, n\}} w_t^i(\boldsymbol{x})
13:
14:
                   \boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in S_t} \mu_{t-1}^f(\boldsymbol{x}) + \beta_t \sigma_{t-1}^f(\boldsymbol{x})
15:
16.
              end if
17:
              y_{f,t} \leftarrow f(x_t) + i \epsilon_{f,t}
18:
              y_{i,t} \leftarrow q_i(\boldsymbol{x}_t) + n_{i,t}
              Compute Q_{f,t}(\boldsymbol{x}) and Q_{i,t}(\boldsymbol{x}), for all \boldsymbol{x} \in S_t
20: end for
21: for t = T_0 + 1, \dots, T do
             C_t^f(\boldsymbol{x}) \leftarrow C_{t-1}^f(\boldsymbol{x}) \cap Q_{t-1}^f(\boldsymbol{x})
23: \boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in S_t} \mu_{t-1}^f(\boldsymbol{x}) + \beta_t \sigma_{t-1}^f(\boldsymbol{x})
             y_{f,t} \leftarrow f(\boldsymbol{x}_t) + n_{f,t}
25:
             y_{i,t} \leftarrow q_i(\boldsymbol{x}_t) + n_{i,t}
             Compute Q_{f,t}(\boldsymbol{x}) and Q_{i,t}(\boldsymbol{x}), for all \boldsymbol{x} \in S_t
26:
27: end for
```

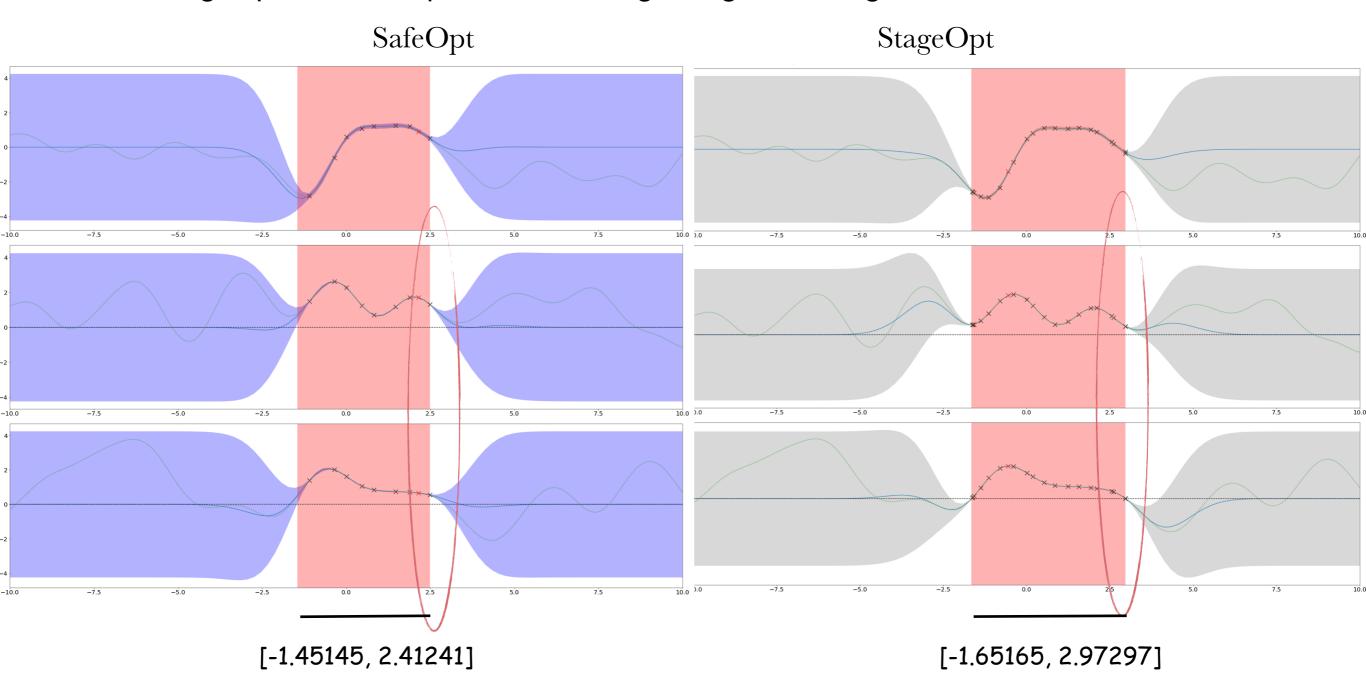
Probability of guaranteeing safety

- Two safety function [f1, f2], with noise[1e-5, 1e-3]
- repeat 1000 times and get results:



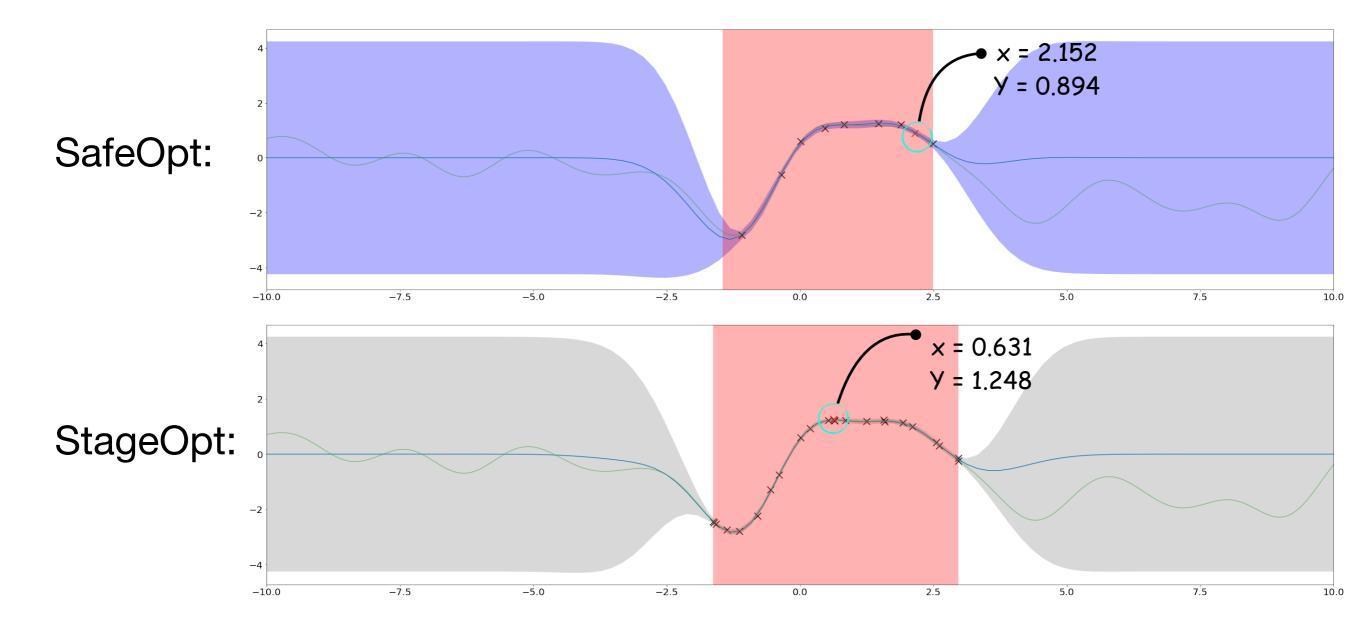
• Safe region result

- Two safety function [f1, f2], with noise[1e-5, 1e-3], number of iterations = 20
- StageOpt has more probabilities to get larger safe region.



Optimization result

- Two safety function [f1, f2], with noise[1e-5, 1e-3], number of iterations = 25
- StageOpt has more probabilities to get greater optimization result and more controllable



Compare StageOpt and SafeOpt

Strategy Name	Guarantee safety	Safe region	Optimization result	Controllabilit y	Numbers of iterations
StageOp t	Higher probability	Larger	Greater	Controllable	Smaller
SafeOpt	High probability	Smaller	Smaller	Uncontrollabl e	Greater

Future Work

- How to deal with a smaller signal to noise ratio?
- How to apply these works to continuous intervals?
- If the utility function is changing?