SAFEOPT & STAGEOPT

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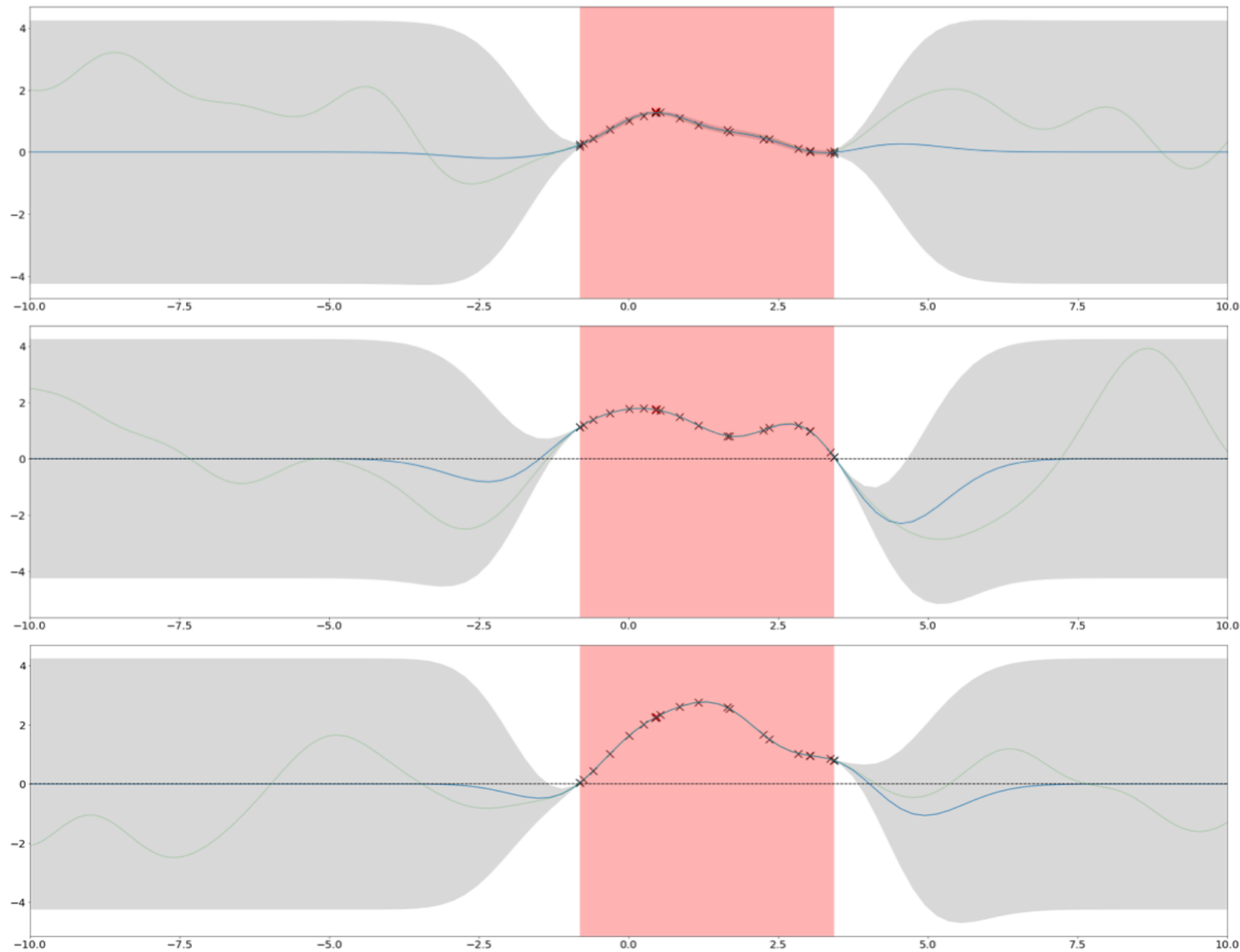
2019.07.31

- **Fit theory**
  - Gaussian Processes
- **Bayesian optimization**
  - Noise interference
  - Safety restrictions



- **SafeOpt**
  - $Q_t(x) := [\mu_{t-1}(x) \pm \beta_t^{1/2}\sigma_{t-1}(x)]$
  - $g_t(x) := |\{x' \in D/S_t \,|\, u_t(x) - Ld(x, x') \geq h\}|$

Algorithm 1 SAFEOPT

Input: sample set D,
GP prior (μ_0, k, σ_0),
Lipschitz constant L,
seed set S_0,
safety threshold h
$C_0(\boldsymbol{x}) \leftarrow [h, \infty)$, for all $\boldsymbol{x} \in S_0$
$C_0(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D \setminus S_0$
$Q_0(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D$
for $t = 1, \ldots$ **do**
  $C_t(\boldsymbol{x}) \leftarrow C_{t-1}(\boldsymbol{x}) \cap Q_{t-1}(\boldsymbol{x})$
  $S_t \leftarrow \bigcup_{\boldsymbol{x} \in S_{t-1}} \left\{\boldsymbol{x}' \in D \,\middle|\, \ell_t(\boldsymbol{x}) - Ld(\boldsymbol{x}, \boldsymbol{x}') \geq h\right\}$
  $G_t \leftarrow \left\{\boldsymbol{x} \in S_t \,\middle|\, g_t(\boldsymbol{x}) > 0\right\}$
  $M_t \leftarrow \left\{\boldsymbol{x} \in S_t \,\middle|\, u_t(\boldsymbol{x}) \geq \max_{\boldsymbol{x}' \in S_t} \ell_t(\boldsymbol{x}')\right\}$
  $\boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in G_t \cup M_t}(w_t(\boldsymbol{x}))$
  $y_t \leftarrow f(\boldsymbol{x}_t) + n_t$
  Compute $Q_t(\boldsymbol{x})$, for all $\boldsymbol{x} \in S_t$
end for

- **StageOpt**
  - Safe region expansion
  - Optimization

Algorithm 1 StageOpt

Input: sample set D, $i \in \{1, \ldots, n\}$,
GP prior for utility function f,
GP priors for safety functions g_i,
Lipschitz constants L_i for g_i,
safe seed set S_0,
safety threshold h_i,
accuracies ϵ (for expansion), ζ (for optimization).
$C_0^i(\boldsymbol{x}) \leftarrow [h_i, \infty)$, for all $\boldsymbol{x} \in S_0$
$C_0^i(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D \setminus S_0$
$Q_0^i(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D$
$C_0^f(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D$
$Q_0^f(\boldsymbol{x}) \leftarrow \mathbb{R}$, for all $\boldsymbol{x} \in D$
for $t = 1, \ldots T_0$ **do**
  $C_t^i(\boldsymbol{x}) \leftarrow C_{t-1}^i(\boldsymbol{x}) \cap Q_{t-1}^i(\boldsymbol{x})$
  $C_t^f(\boldsymbol{x}) \leftarrow C_{t-1}^f(\boldsymbol{x}) \cap Q_{t-1}^f(\boldsymbol{x})$
  $S_t \leftarrow \bigcap_i \bigcup_{\boldsymbol{x} \in S_{t-1}} \left\{ \boldsymbol{x}' \in D \;\middle|\; \ell_t^i(\boldsymbol{x}) - L_i d(\boldsymbol{x}, \boldsymbol{x}') \geq h_i \right\}$
  $G_t \leftarrow \left\{ \boldsymbol{x} \in S_t \;\middle|\; e_t(\boldsymbol{x}) > 0 \right\}$
  if $\forall i, \epsilon_t^i < \epsilon$ **then**
    $\boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in G_t, i \in \{1, \ldots, n\}} w_t^i(\boldsymbol{x})$
  else
    $\boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in S_t} \mu_{t-1}^f(\boldsymbol{x}) + \beta_t \sigma_{t-1}^f(\boldsymbol{x})$
  end if
  $y_{f,t} \leftarrow f(\boldsymbol{x}_t) + n_{f,t}$
  $y_{i,t} \leftarrow g_i(\boldsymbol{x}_t) + n_{i,t}$
  Compute $Q_{f,t}(\boldsymbol{x})$ and $Q_{i,t}(\boldsymbol{x})$, for all $\boldsymbol{x} \in S_t$
end for
for $t = T_0 + 1, \ldots, T$ **do**
  $C_t^f(\boldsymbol{x}) \leftarrow C_{t-1}^f(\boldsymbol{x}) \cap Q_{t-1}^f(\boldsymbol{x})$
  $\boldsymbol{x}_t \leftarrow \operatorname{argmax}_{\boldsymbol{x} \in S_t} \mu_{t-1}^f(\boldsymbol{x}) + \beta_t \sigma_{t-1}^f(\boldsymbol{x})$
  $y_{f,t} \leftarrow f(\boldsymbol{x}_t) + n_{f,t}$
  $y_{i,t} \leftarrow g_i(\boldsymbol{x}_t) + n_{i,t}$
  Compute $Q_{f,t}(\boldsymbol{x})$ and $Q_{i,t}(\boldsymbol{x})$, for all $\boldsymbol{x} \in S_t$
end for

- **Probability of guaranteeing safety**
  - Two safety function [f1, f2], with noise[1e-5, 1e-3]
  - repeat 1000 times and get results:
    - SafeOpt : 786 / 1000
    - StageOpt : 979 / 1000

- **Safe region and the optimization result**

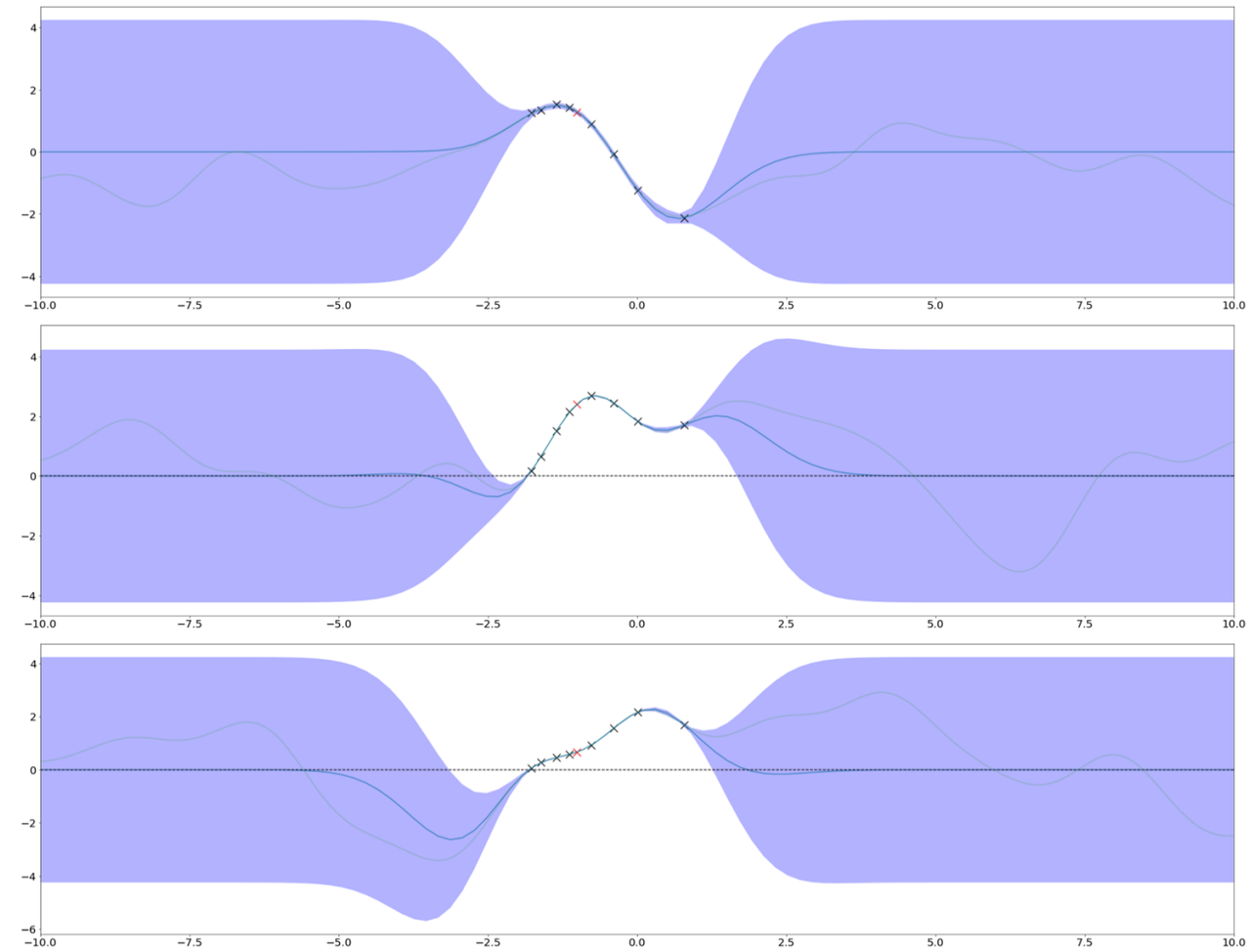
  - Two safety function [f1, f2], with noise[1e-5, 1e-3]

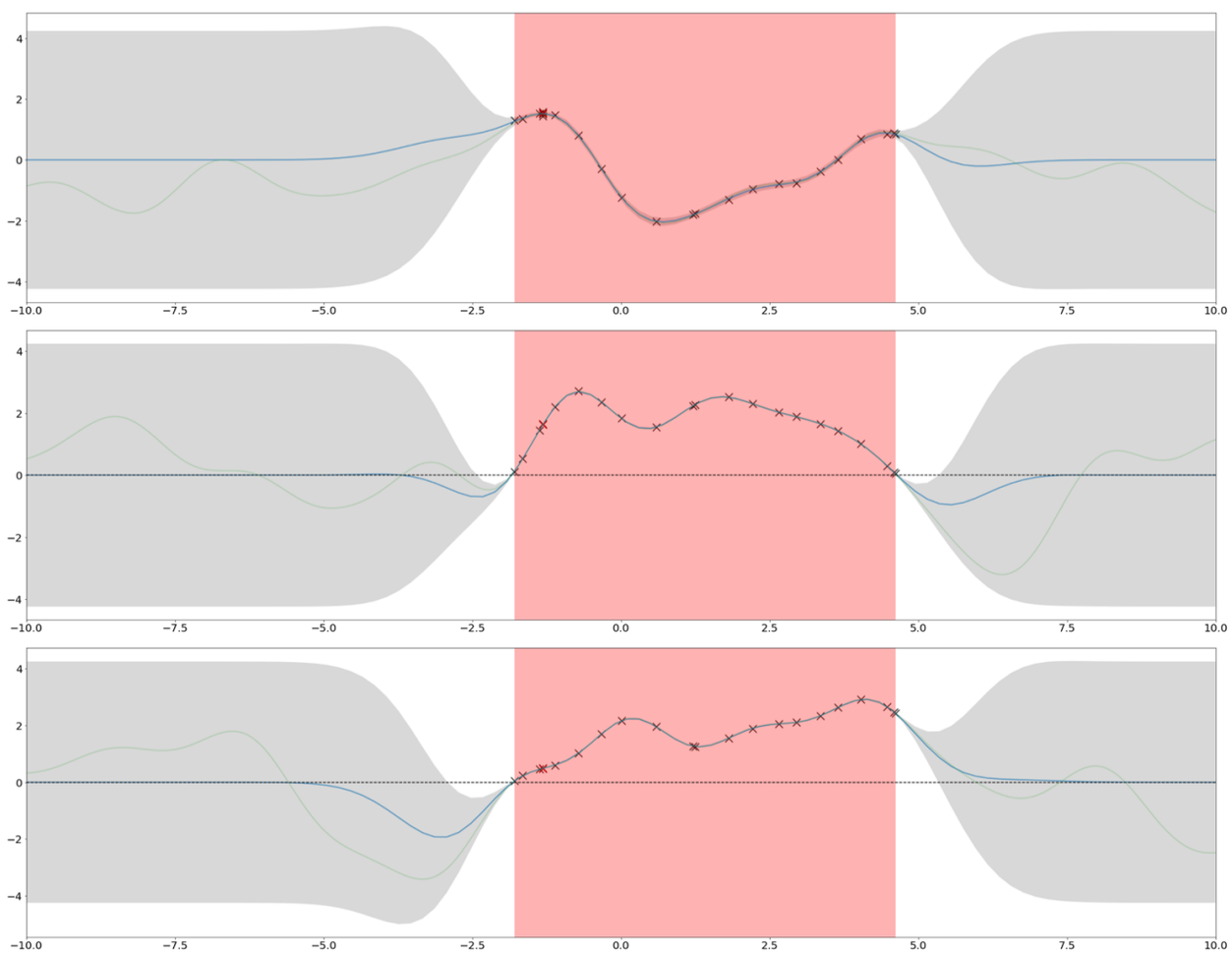
Strategy Name	**Safe Region** SafeOpt t = 20, StageOpt t_exp = 20	**F_utility value** SafeOpt t = 25, StageOpt t_exp = 20 t_opt = 5
StageOpt	[-1.81181, 4.61461]	1.55066
SafeOpt	[-1.79179, 1.25125]	1.27593

- **Safe region and the optimization result**

SafeOpt

StageOpt





- **Compare StageOpt and SafeOpt**

    - StageOpt is more probabilities to get larger safe region.
    - StageOpt needs fewer iterations to converge.
    - StageOpt is more controllable.
    - SafeOpt spends less time.

- **Future Work**

  - How to deal with a smaller signal to noise ratio?

  - How to apply these works to continuous intervals?

  - If the utility function is changing?