

## 05/06 高等数学标准答案

一、 1.  $\frac{1}{3}$ ; 2.  $\frac{1}{2}$ ; 3.  $(0, -2)$ ; 4.  $\frac{\pi}{2}$  5.  $y = \frac{1}{x} e^{cx}$ ;

6.  $y = -x^2 + c_1 e^x + c_2 x + c_3$

二、 1D ; 2.B 3.D; 4.B;

三、 1.  $f'(x) = 3x^2 + 2ax + b$  (2 分)

$$f'(1) = 3 + 2a + b = 0 \quad (3 \text{ 分})$$

$$f(1) = 3 + a + b = -2 \quad (4 \text{ 分})$$

$$a = 0, b = -3 \quad (6 \text{ 分})$$

2.  $= \int \frac{1+x}{\sqrt{1-x^2}} dx \quad (2 \text{ 分})$

$$= \int \frac{1}{\sqrt{1-x^2}} dx - \frac{1}{2} \int \frac{d(1-x^2)}{\sqrt{1-x^2}} dx \quad (4 \text{ 分})$$

$$= \arcsin x - \sqrt{1-x^2} + c \quad (6 \text{ 分})$$

3.  $= \lim_{x \rightarrow +\infty} \frac{1}{\xi} \cos \ln(\xi) \quad (\xi \text{ 介于 } x \text{ 与 } x+1 \text{ 之间}) \quad (2 \text{ 分})$

$$= \lim_{\xi \rightarrow +\infty} \frac{1}{\xi} \cos \ln(\xi) \quad (4 \text{ 分})$$

$$= 0 \quad (6 \text{ 分})$$

四、1.  $f(x) = \begin{cases} \frac{1}{2} + x - \frac{x^2}{2}, & x > 0 \\ \frac{1}{2} + x + \frac{x^2}{2}, & x \leq 0 \end{cases}$  (2 分)

交点为  $(-1,0), (1+\sqrt{2},0)$  (4 分)

$$\text{面积} = \int_{-1}^0 \left( \frac{1}{2} + x + \frac{x^2}{2} \right) dx + \int_0^{1+\sqrt{2}} \left( \frac{1}{2} + x - \frac{x^2}{2} \right) dx \quad (6 \text{ 分})$$

$$= \frac{1}{6} + \frac{1}{2}(1+\sqrt{2}) + \frac{1}{2}(1+\sqrt{2})^2 - \frac{1}{6}(1+\sqrt{2})^3 \quad (7 \text{ 分})$$

2. 与  $x$  轴交点为  $(0,0)$  和  $(4, 0)$  (2 分)

$$= \int_0^4 2\pi x \sqrt{4-(2-x)^2} dx \quad (5 \text{ 分})$$

$$= 8\pi^2 \quad (7 \text{ 分})$$

五、  $f'(x) = \cos x - \int_0^x f(t) dt$  (1 分)

$$f''(x) = -\sin x - f(x) \quad (2 \text{ 分})$$

$$f(x) = c_1 \cos x + c_2 \sin x + \frac{1}{2} x \cos x \quad (4 \text{ 分})$$

$$f(0) = 0, f'(0) = 1, \quad (6 \text{ 分})$$

$$f(x) = \frac{1}{2} \sin x + \frac{1}{2} x \cos x \quad (7 \text{ 分})$$

六、  $f(x) = f(0+x) = f(0)f(x) \Rightarrow f(0) = 1$  (1 分)

$$\lim_{\Delta x \rightarrow 0} f(x+\Delta x) = \lim_{\Delta x \rightarrow 0} f(x)f(\Delta x) = f(x)f(0) = f(x) \Rightarrow f(x) \text{ 处处连续} \quad (3 \text{ 分})$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x)f(\Delta x) - f(x)}{\Delta x}$$

$$= f(x) \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x) - f(0)}{\Delta x} = f(x)f'(0) \Rightarrow f(x) \text{ 可导.} \quad (6 \text{ 分})$$

$$f'(x) = f(x)f'(0) \Rightarrow f(x) = ce^{f'(0)x} \quad (7 \text{ 分})$$

$$f(0) = 1 \Rightarrow f(x) = e^{f'(0)x} \quad (8 \text{ 分})$$

七、  $f(x) = f(0) + f'(0)x + f''(\xi)\frac{x^2}{2}$  (1 分)

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{f(x)}{x} x = 0 \Rightarrow f(0) = 0 \quad (2 \text{ 分})$$

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} \Rightarrow f'(0) = 1 \quad (3 \text{ 分})$$

$$f(x) = x + f''(\xi)\frac{x^2}{2} \Rightarrow f(x) > 0 \quad (4 \text{ 分})$$

八、 (1)  $= \int_l^0 |\sin x| dx + \int_0^\pi |\sin x| dx + \int_\pi^{l+\pi} |\sin x| dx$  (2 分)

$$\text{令 } t = x + \pi \text{ 有 } \int_0^\pi |\sin x| dx = - \int_\pi^{l+\pi} |\sin x| dx \quad (4 \text{ 分})$$

$$(2) \quad \int_l^{l+n\pi} |\sin x| dx = \int_l^{l+\pi} |\sin x| dx + \int_{l+\pi}^{l+2\pi} |\sin x| dx + \cdots + \int_{l+(n-1)\pi}^{l+n\pi} |\sin x| dx = 2n \quad (7 \text{ 分})$$

$$(3) \quad 2n = \int_0^{n\pi} |\sin x| dx \leq \int_0^x |\sin x| dx \leq \int_\pi^{(n+1)\pi} |\sin x| dx = 2(n+1) \quad (8 \text{ 分})$$

$$\frac{2n}{n+1} \leq \frac{\int_0^x |\sin x| dx}{x} \leq \frac{2(n+1)}{n}$$

由夹逼定理得原式=2 (9 分)