

definition using this or observe when find roots for equation

$$-a \leq x \leq a$$

domain $x \in (-\infty, \infty)$ and range $(0, \infty)$

$$2 = |2x-7| = 2x-7,$$

$$2x-7 \geq 0$$

$$x \geq \frac{7}{2}$$

restriction

$$2 = 2x-7$$

$$9 = 2x$$

$$x = \frac{9}{2}$$

or \rightarrow

$$2x-7 < 0$$

$$x < \frac{7}{2}$$

$$2 = -(2x-7)$$

$$2 = -2x+7$$

$$-5 = -2x$$

$$x = \frac{5}{2}$$

equation satisfied.

Example 1.2.5

$$① \quad |x-1| < 2$$

$$x-1 < 2$$

$$x < 3$$

$$x-1 \geq 0$$

$$x \geq 1$$

$$\therefore 1 \leq x < 3$$

$$\text{and}$$

$$-1 < x < 1$$

$$② \quad |x-1| \leq 2$$

$$-(x-1) \leq 2$$

$$-x+1 \leq 2$$

$$-1 \leq x$$

$$-(x-1)$$

$$x-1 \leq 0$$

$$x \leq 1$$

1.3 composition

A function with in a function.

$$(f \circ g)(x) = f(g(x))$$

$x \in D_g \rightarrow$ Domain of g

$$y = g(x)$$

y value is domain of f

Example 1.3.2

$$2\left(\frac{1}{x-3}\right) + 1$$

$$g(1) = \frac{1}{1-3} = -\frac{1}{2}$$

$$f(g(1)) = 6 \cdot 2\left(-\frac{1}{2}\right) + 1 = 0$$

$$g(f(2))$$

$$f(2) = 2(2) + 1 = 5$$

$$g(f(2)) = \frac{1}{5-3} = \frac{1}{2}$$

$$f(g(x))$$

$$f(g(x)) = 2\left(\frac{1}{x-3}\right) + 1$$

$$= \frac{2}{x-3} + 1$$

$$\frac{2 + (x-3)}{x-3}$$

$$= \frac{-1+x}{x-3}$$

$$= \frac{x-1}{x-3}$$

$$g(f(x)) = \frac{1}{(2x+1)-3}$$

$$= \frac{1}{2x-2}$$

$$f(f(x)) = 2x +$$

$$2x$$

$$= 2(2x+1) + 1$$

$$= 4x+2+1$$

$$= 4x+3$$

if

$$\frac{dy}{dx} = (3x^2 + 3) \sin x + (x^3 + 3x) \cos x$$

3.1.5

$$y = u(x) \cdot v(x) \cdot w(x)$$

$$\frac{dy}{dx} = \frac{d(uv)}{dx} w + uv \frac{dw}{dx}$$

$$\text{but } \frac{d(uv)}{dx} = \frac{dy}{dx} v + u \frac{du}{dx}$$

$$\frac{dy}{dx} = \left(\frac{du}{dx} v + u \frac{dv}{dx} \right) w + uv \frac{dw}{dx}$$

$$= \frac{dy}{dx} vw + uv \frac{dv}{dx} + uv \frac{dw}{dx}$$

product of 3 functions

3.1.6

$$y = x^2(x+1) \cos x$$

$$x^2(x+1) = x^3 + x^2$$

$$\frac{dy}{dx} = 2x(x+1) \cos x + x^2 \cos x (1) + x^2(x+1) (-\sin x)$$

derivative

derivative of (x+1)

derivative of cos x

derivative of product

eg 3.1.7

the derivative of quotient of two functions

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

3.2.2

$$\frac{dy}{dx} = \frac{[-6x](5x^4 + 2) - (1 - 3x^2)[20x^3]}{(5x^4 + 2)^2}$$

ex 3.2.3

$$\frac{dy}{dt} = \frac{-\sin t (\sin t - t^2) - \cos t (\cos t - 2t)}{(\sin t - t^2)^2}$$

Example 3.2.4

$$y = \tan x$$

$$= \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x \cdot \cos x - \sin x \cdot [-\sin x]}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

derivative of tan x
tan x = sec^2 x

Homework

$$\frac{\cos x}{\sin x}$$

$$\frac{1}{\cos x}$$

$$\frac{1}{\sin x}$$

Example 4.4.3

$$\begin{aligned} \text{Area} &= \int_1^3 -2(x-3) dx \\ &= \left[-\frac{1}{3}x^3 + \frac{3}{2}x^2 \right]_1^3 = -\frac{1}{3}(3)^3 + \frac{3}{2}(3)^2 - \left(-\frac{1}{3}(1)^3 + \frac{3}{2}(1)^2 \right) \\ &= -9 + \frac{27}{2} + \frac{1}{3} - \frac{3}{2} = 3\frac{1}{3} \end{aligned}$$

Example 4.4.4

$$\begin{aligned} \text{Area} &= \int_{-2}^1 (x-1)(x+1)(x+2) dx + \int_{-1}^1 -(x+1)(x+1)(x+2) dx + \\ &\quad \int_1^2 (2-1)(x+1)(x+2) dx \end{aligned}$$

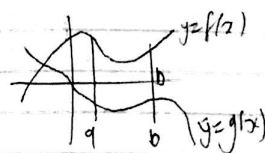
we can multiply all the way through to this
is no product rule for integration

$$\begin{aligned} &= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2x \right]_{-2}^1 - \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2x \right]_{-1}^1 \\ &\quad + \left[\frac{1}{6}x^3 + \frac{2}{3}x^2 - \frac{1}{2}x - 2 \right]_1^2 \end{aligned}$$

= 8

4.4.1 Area between curves

$f(x)$ and $g(x)$ $f(x) \geq g(x)$



$$\text{Area} = \int_a^b (f(x)+c) dx - \int_a^b (g(x)+c) dx \quad (\text{top curve} - \text{bottom curve})$$

$$\begin{aligned} &= \int_a^b (f(x)+c - g(x)-c) dx \\ &= \int_a^b (f(x) - g(x)) dx \end{aligned}$$

$$= \int_1^2 [e^x - (2x-1)] dx$$

Example 4.4.5

$$= \int_1^2 [e^x - (2x-1)] dx$$

$$\text{Area} = \int_1^2 e^x - 2x + 1 \cdot dx$$

$$= \left[e^x - \frac{2}{2}x^2 + x \right]_1^2$$

$$= e^2 - (2)^2 + 2 - [e^1 - (1)^2 + 1] = e^2 - e^1 - 2$$