Constant term: coefficient of x^0

$\therefore 3r - 12 = 0$ i.e. $r = 4$

$$\binom{12}{4} 2^4 (-1)^8 3^{-8} = \frac{12.11.10.9.8!}{4!\,8!} \cdot \frac{16 \times (+1) \times 1}{3^8}$$

$$= \frac{11.10.9 \times 8}{3^8}$$

coefficient of x^5?

$3r - 12 = 5$

$3r = 17$

$r = 17$

But $r \in \mathbb{N}_0$

has to be real number including zero

$\therefore$ there is no term with x^5 in the expansion

Eg 3.6.17 Find the coefficient of x^5 in the expansion of $(2+x)(3-5x)^8$

$$= (2+x) \sum_{r=0}^{8} \binom{8}{r} (3)^{8-r} (-5x)^r$$

$$= 2 \sum_{r=0}^{8} \binom{8}{r} 3^{8-r} (-1)^r (5)^r x^r + x \sum \binom{8}{r} 3^{8-r} (-1)^r (5)^r x^r$$

$$= 2 \sum_{r=0}^{8} \binom{8}{r} \cdot 3^{8-r} (-1)^r 5^r x^r + \sum_{r=0}^{8} \binom{8}{r} 3^{8-r} (-1)^r (5)^r x^{r+1}$$

Coefficient of x^5

in the first term $r = 5$

in the second term $r + 1 = 5 \therefore r = 4$

$$2 \times \binom{8}{5} 3^3 (-1)^5 5^5 + \binom{8}{4} 3^4 (-1)^4 5^4$$

$$= -2 \cdot \frac{8.7.6.5!}{5!\,3!} \times 3^3 5^5 + \frac{8.7.6.5.4!}{4!\,.\,4!} 3^4 5^4$$

$$\frac{-2.8.7.6.3^3.5^5}{3!} + \frac{8.7.6.5.3^4.5^4}{4!}$$

$$= \frac{-2.4.8.7.6.3^3.5^5 + 8.7.6.5.3^4.5^4}{4.3!}$$

$$= \frac{8.7.6\left(-2^3.3^3.5^5 + 5^5.3^4\right)}{4!}$$

$$= \frac{8.7.6.5^5.3^3(3-8)}{4!} = \frac{-8.7.6.5^6.3^3}{4!}$$

minus $x+($

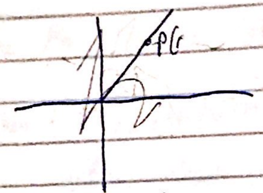
$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}$$

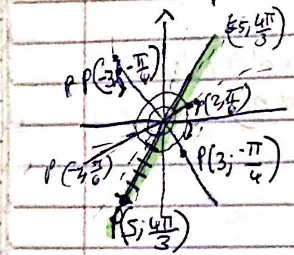
$$(x-y)^n = (x+(-y))^n = \sum_{r=0}^{n} \binom{n}{r} x^r (-y)^{n-r}$$

$(r;\theta) = 2;\frac{\pi}{6}$

2.5.1

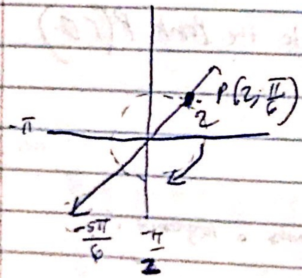
eg



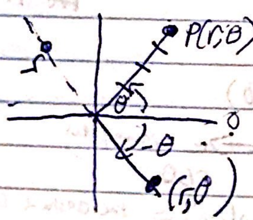


2.5.4) Find all the polar co-ordinates of $P(2;\frac{\pi}{6})$

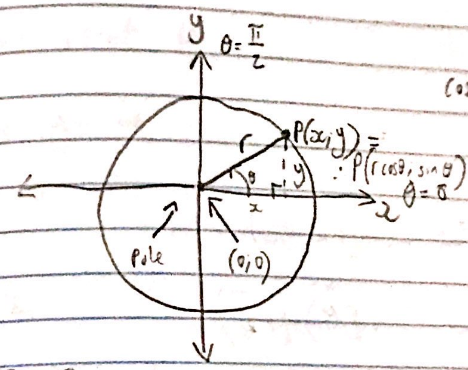
$(r,\theta) = (2;\frac{\pi}{6} + 2n\pi)$



$\frac{-6\pi}{6} + \frac{\pi}{6} = \frac{-5\pi}{6}$



2.5.1



$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$

$\therefore x = r\cos\theta$, $r\sin\theta = y$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

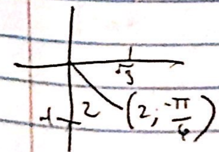
two-coordinate system, related by the following equation

$$\boxed{x = r\cos\theta, \quad y = r\sin\theta \quad r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}}$$

Example 2.5.5.

$x = 2\cos(\frac{-\pi}{6}) = 2.\frac{\sqrt{3}}{2} = \sqrt{3}$

$y = 2\sin(\frac{-\pi}{6}) = 2(\frac{-1}{2}) = -1$

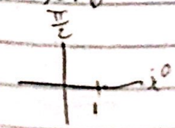


P $P(\sqrt{3}; -1)$

$(r;\theta) = 2, \frac{-\pi}{6}$

$\leftarrow (x,y) = (\sqrt{3}, -1)$

Ex 2.5.6



$(x;y) = (1,0)$

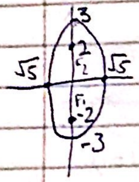
corresp $(r,t) = (1,0)$

$x = 1 \cdot \cos 0° = 1$

$y = 1 \cdot \sin 0° = 0$

Eg. 4.1.10 Eqn of the ellipse

y-int $(0, \pm 3)$ foci $(0, \pm 2)$



$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{9} + \frac{x^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

$$4 = 9 - b^2$$

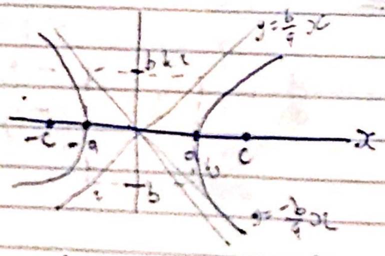
$$b^2 = 5$$

$$\therefore \frac{y^2}{9} + \frac{x^2}{5} = 1$$

Canonical form (hyperbola)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Foci $c^2 = a^2 + b^2$



Asymptotes $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

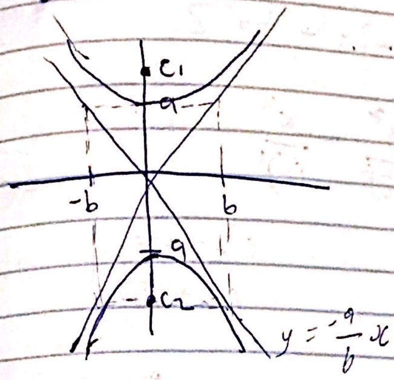
$$\frac{y^2}{b^2} = \frac{x^2}{a^2}$$

$$y^2 = \frac{b^2 x^2}{a^2}$$

$$\boxed{y = \pm \frac{b}{a} x}$$

$|k_1 - k_2| = |k_3 - k_4|$ = constant

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



$$c^2 = a^2 + b^2$$

$\frac{y^2}{a^2} + \frac{x^2}{b^2} \Rightarrow$ Asymptotes $y = \pm\frac{a}{b}x$

or

Plot a rectangle using $\pm a$ and $\pm b$ on the y and x-axes respectively

Join the corners

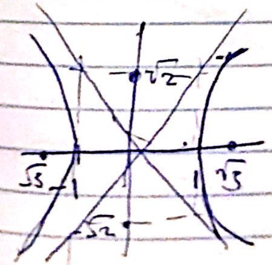
Eg. 4.1.12 Sketch $2x^2 - y^2 = 2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{(1)} - \frac{y^2}{(\sqrt{2})^2} = 1$$

Foci $c^2 = a^2 + b^2$

$(\pm c; 0)$



$$\frac{y^2}{2} + \frac{x^2}{1}$$

$$y = \pm\sqrt{2}x$$