* $\sqrt{-7} = \pm i\sqrt{7}$

because

$\sqrt{-7} = \sqrt{i^2 7} = i\sqrt{7}$

Example 4.2.6

$z^2 + 2z + 6 = 0 \quad z \in \mathbb{C}$

complete square

$(z+1)^2 + 5 = 0$

$z+1 = \pm\sqrt{-5}$

$\therefore z+1 = \pm\sqrt{i^2 5}$

$z+1 = \pm\sqrt{5}\, i$

$\therefore z = -1 \pm \sqrt{5}\, i$

Quadratic formula

$$z = \frac{-2 \pm \sqrt{4-24}}{2}$$

$$= -1 \pm \frac{\sqrt{-20}}{2}$$

$$= -1 \pm \frac{i\sqrt{20}}{2}$$

$$\therefore = -1 \pm \frac{i\sqrt{5 \times 4}}{2}$$

$$= -1 + \frac{i\sqrt{5} \cdot 2}{2}$$

$$\therefore z = -1 \pm \sqrt{5}\, i$$

Example 4.3.1

1. $z = 1 + i \quad (1,1)$

$|z| = \sqrt{2}$ and $\tan\theta = \frac{1}{1} =$ $\arctan(1) = \frac{\pi}{4}$

$\therefore \arg z = \theta = \frac{\pi}{4}$

$\therefore 1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right) = \sqrt{2}\, e^{i\frac{\pi}{4}}$

2. $z = -1 + i$

$|z| = \sqrt{2} \quad \tan\theta = \frac{1}{-1} = -1$

$\frac{2\pi}{3} - \frac{\pi}{3}$

$\frac{6\pi}{3}$

$\frac{\pi}{3} - \pi$

$\operatorname{Arg} z = \theta = \frac{-\pi}{4} + \pi = \frac{3\pi}{4}$

$-1+i = \sqrt{2}\left(\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4} i\right) = \sqrt{2}\, e^{i\frac{3\pi}{4}}$

3. $z = -1 - \sqrt{3} i \quad (-1, -\sqrt{3})$

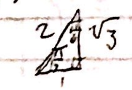
$|z| = \sqrt{1+3} = 2$

$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\operatorname{Arg} z = \theta = \arctan(\sqrt{3}) - \pi = \frac{\pi}{3} - \pi = \frac{-2\pi}{3}$

$\therefore -1 - \sqrt{3} i = 2\left(\cos\left(-\frac{2\pi}{3}\right) + \sin\left(-\frac{2\pi}{3}\right) i\right) = 2e^{-i\frac{2\pi}{3}}$

Example 4.3.2



$z = 2e^{i\frac{2\pi}{3}} \quad \frac{\pi}{3} + \frac{\pi}{3}$

$= 2\left(\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right) i\right)$

$2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) \quad -1 + \sqrt{3} i$

the line of intersection between the next 2 pairs is given by

$$\begin{vmatrix} 1 & 1 & 4 & 5 \\ 2 & 5 & 1 & 3 \end{vmatrix} \quad \begin{array}{c} P_2 \\ P_1 \end{array}\left|\begin{array}{ccc|c} 1 & 1 & 4 & 5 \\ 0 & 1 & 1 & 1 \end{array}\right| \xrightarrow{-R_2+R_1} \left|\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 1 & 1 \end{array}\right|$$

thus

$x = 4-3t,\ y = 1-t,\ z = t$

which gives $\underline{r} = \langle 4,1,0\rangle + t\langle -3,-1,1\rangle$

and finally

$$\begin{array}{c} P_3 \\ P_1 \end{array}\left|\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 2 & -5 & 1 & 3 \end{array}\right| \xrightarrow{-2R_1+R_2} \left|\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 0 & -11 & -11 & 1 \end{array}\right| \xrightarrow[-\frac{1}{11}R_2]{\frac{3}{11}R_2+R_1} \left|\begin{array}{ccc|c} 1 & 0 & 3 & \frac{14}{11} \\ 0 & 1 & 1 & \frac{-1}{11} \end{array}\right|$$

gives

$x = \frac{14}{11} - 3t,\ y = \frac{-1}{11} - t,\ z = t$ thus

$\underline{r} = \langle \frac{14}{11}, -\frac{1}{11}, 0\rangle + t\langle -3,-1,1\rangle$

b.) the normal vectors are the same as in a.) so question a therefore planes are not parallel

$$\left|\begin{array}{ccc|c} 1 & 1 & 4 & 5 \\ 1 & 3 & 6 & 7 \\ 2 & -5 & 1 & 3 \end{array}\right| \rightarrow \left|\begin{array}{ccc|c} 1 & 1 & 4 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right| \xrightarrow[-R_2+R_3]{-R_2+R_1} \left|\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

→ infinitely many solutions

$x = 4-3t,\ y = 1-t$ and $z = t$

∴ planes intersect in a line given by $\underline{r} = \langle 4,1,0\rangle + t\langle -3,-1,1\rangle$

note that $\langle -3,-1,1\rangle$ is perpendicular to the normals of all three planes.

c.) Answer P_1, P_2 and P_3 are not parallel

$n_1 = \langle 1,1,4\rangle$
$\underline{n}_2 = \langle 1,3,6\rangle$
$\underline{n}_3 = \langle 2,-5,1\rangle$

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 3 & 4 & 20 \\ 1 & -1 & 1 & 2 \end{array}\right] \xrightarrow[-R_1+R_3]{-2R_1+R_2} \left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 14 \\ 0 & -2 & 0 & -4 \end{array}\right| \xrightarrow[2R_2+R_3]{} \left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 4 & 12 \end{array}\right|$$

unique solution given by

$z = 3$
$y = 8-6 = 2$
$x = 6-3-2 = 1$

the planes intersect in the point $(x,y,z) = (1,2,3)$

d.)

$P_1 \quad x+y+z = 6 \qquad \underline{n}_1 = \langle 1,1,1\rangle$
$P_2 \quad -2x-2y-2z = 0 \qquad \underline{n}_2 = \langle -2,-2,-2\rangle$
$P_3 \quad x-y+z = 4 \qquad \underline{n}_3 = \langle +1,-1,1\rangle$

$$\begin{array}{c} P_1 \\ P_3 \end{array}\left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 4 \end{array}\right| \xrightarrow{-R_1+R_3} \left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -2 \end{array}\right|$$

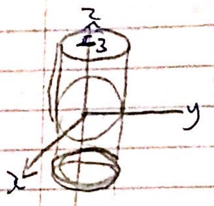
∴ $z = t$
$y = 1$
$x = 6-t-1 = 5-t$

line of intersection for P_1 and P_3

$\underline{r} = \langle 5,1,0\rangle + t\langle -1,0,1\rangle$

Example 3.1.3

1) $x^2+y^2=1$ and $z=3$ circle



2) cylinder - with radius 1 symmetrical about the z-axis.

Definition the distance $|P_1P_2|$ between two points $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$

$\therefore |P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

the formula is extend from distance between two points $(\mathbb{R}^2)$

$\therefore$ being exuding $(\mathbb{R}^3)$

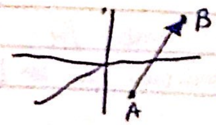
Example. 3.1.4

a) $C(2,4,4)$

b) $A(-4,0,-1)$

* Given two points $A(x_1,y_1,z_1)$ & $B(x_2,y_2,z_2)$ and let $\underline{AB}$

$\underline{AB} = (x_2-x_1)(y_2-y_1,(z_2-z_1))$



$\underline{AB} = OB - OA$

$OA = (x_1, y_1, z_1)$

$OB = (x_2, y_2, z_2)$

$\underline{a} = \langle a_1, a_2, a_3 \rangle$ $\underline{b} = \langle b_1, b_2, b_3 \rangle$

$\underline{a}+\underline{b} = \langle a_1+b_1, a_2+b_2, a_3+b_3 \rangle$

$\underline{a}-\underline{b} = \langle a_1-b_1, a_2-b_2, a_3-b_3 \rangle$

$\lambda \underline{a} = \langle \lambda a_1, \lambda a_2, \lambda a_3 \rangle$

- The properties held in $\mathbb{R}^2$ ~~and~~ are held $\mathbb{R}^3$ (vector addition and subtraction)

$|OA| = \sqrt{a_1^2+a_2^2+a_3^2}$ = the length of the position vector $(\mathbb{R}^3)$

$\therefore |OB| = \sqrt{b_1^2+b_2^2+b_3^2}$

while $|AB| = |OB| - |OA|$

$|\lambda OA| = |\lambda||OA|$

$\rightarrow$ we have 3 basic vectors

$i = \langle 1,0,0 \rangle$ $j = \langle 0,1,0 \rangle$ $k = \langle 0,0,1 \rangle$

Example 3.1.5

Find two ~~vect~~ unit vectors parallel to $\langle 3,4,-1 \rangle = \underline{v}$

$|\underline{v}| = |\langle 3,4,-1 \rangle| = \sqrt{9+16+1} = \sqrt{26}$

$\pm \frac{1}{\sqrt{26}} \langle 3,4,-1 \rangle$

Example 3.1.7

$(x-a)^2+(y-b)^2+(z-c)^2 = k^2$

and vector equation is

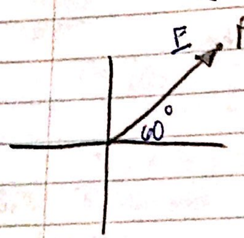
$k = |OP - OA| = |\langle x,y,z \rangle - \langle a,b,c \rangle|$

and

$\text{comp}_{\underline{a}} \underline{b} = \frac{\underline{b}\cdot\underline{a}}{|\underline{a}|} = \frac{\langle 2,-1,1\rangle\cdot\langle 1,2,-4\rangle}{\sqrt{1+4+16}} = \frac{-4}{\sqrt{21}}$

$\underline{F} = (5\cos 60)i + (5\cos 30)j = \langle 5\cos 60, 5\cos 30\rangle$

$\mathbb{R}^2$ $= \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2}\right\rangle$ $\underline{F} = 5$



$\underline{F} = \underline{OP}$

$\cos 30^\circ = \frac{\underline{F}\cdot\underline{j}}{|F||\underline{j}|} = \frac{y}{5}$ $\cos 60^\circ = \frac{\underline{F}\cdot\underline{i}}{|F||i|} = \frac{x}{5}$

$y = 5\cos 30^\circ$ $\therefore$ $x = 5\cos 60$

Example 3.2.4

$\text{Comp}_{\underline{v}} \underline{u} = \frac{\underline{u}\cdot\underline{v}}{|\underline{v}|} = \frac{6-6-4}{\sqrt{1+4+4}} = \frac{-4}{\sqrt{9}} = \frac{-4}{3}$

$\text{Proj}_{\underline{v}} \underline{u} = \left(\frac{\underline{u}\cdot\underline{v}}{|\underline{v}|}\right)\frac{\underline{v}}{|\underline{v}|} = \frac{-4}{3}\cdot\frac{\langle 1,-2,-2\rangle}{3} = \frac{-4}{9}$

$= \left\langle \frac{-4}{9}, \frac{8}{9}, \frac{8}{9}\right\rangle$

3.2.2 Direction cosines

$\cos\alpha = \frac{x}{|\underline{r}|}$ $\cos\beta = \frac{y}{|\underline{r}|}$ $\cos\gamma = \frac{z}{|\underline{r}|}$

$\underline{r} = \langle r_1, r_2, r_3\rangle = \langle |\underline{r}|\cos\alpha, |\underline{r}|\cos\beta, |\underline{r}|\cos\gamma\rangle$

$\frac{\underline{r}}{|\underline{r}|} = \langle\cos\alpha, \cos\beta, \cos\gamma\rangle$

Example 3.2.5

$\underline{AB} = \langle 0-1, -1-1, 2-0\rangle$
$= \langle -1, -2, 2\rangle$

$|\underline{AB}| = \sqrt{1+4+4}$
$= \sqrt{9} = 3$

$\underline{AB} = \langle -1,-2,2\rangle = 3\left\langle -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right\rangle$

$\cos\alpha = \frac{-1}{3}$ $\cos\beta = \frac{-2}{3}$ $\cos\gamma = \frac{2}{3}$

$\therefore \alpha = \arccos\left(\frac{-1}{3}\right)$ $\beta = \arccos\left(\frac{-2}{3}\right)$ $\gamma = \arccos\left(\frac{2}{3}\right)$

Example 3.2.6

$\frac{\underline{v}}{|\underline{v}|} = \langle\cos\alpha, \cos\beta, \cos\gamma\rangle$; $\cos\alpha = \frac{4}{5}$ $\cos\beta = 0$ $\cos\gamma = \frac{3}{5}$