Strength of Materials - GATE

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Contents

Chapter 1

Properties of Materials

1.1 Basics

- **Longitudinal Axis:** The Line passing through the center of all the planes along the longest dimension of the member is called as Longitudinal Axis.
- **Cross Sectional Area (CS):** The plane normal to the longitudinal axis of the object
- **Prismatic Bar:** A member with constant cross-sectional area along its whole length

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1.2 Stress(σ)

- **Stress(σ):** The internal resisting force which resists deformation in object when a force in acting on it. $\sigma = \dfrac{F}{A}$
  - Stress is developed only when the motion due to the force is restricted
  - Pressure and Stress are not the same. Pressure is external Normal force over a surface.
  - **Normal Stress(σ):** Stress acting perpendicular to CS area.

    Sign Convention:
    * Tensile stress ($\leftarrow \boxed{\dots} \rightarrow$) = Positive (+ve)
    * Compressive stress ($\rightarrow \boxed{\dots} \leftarrow$) = Negative (-ve)
  - **Shear Stress(τ):** Stress acting tangential(parallel) to CS area.

- **Engineering (or) Nominal Stress**

  → $\boxed{\sigma_{Engg} = \dfrac{F}{A_0}}$

  → A_0 = Original CS Area. Its called Original because, when an object is developing stress, there will be deformation to the object will change the cross sectional area. But, if we are using the initial cross sectional area to calculate stress, then its called Engineering stress (or) Nominal stress (or) **Average stress**

- **True stress (or) Actual stress**

  → $\boxed{\sigma_{actual} = \dfrac{F}{A_a}}$

  → $A_a = A_0 + \Delta A$
    * ΔA = +ve for Compression, as Area(↑) during compression
    * ΔA = -ve for Tension, as Area(↓) during Tension

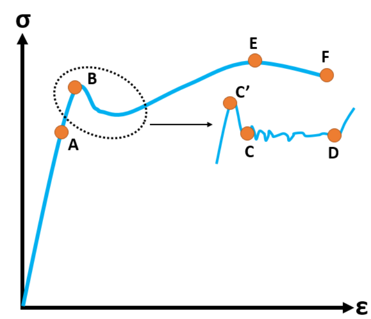
$\Longrightarrow$ In Tension $\boxed{\sigma_{True} > \sigma_{actual}}$ $\Longrightarrow$ In Compression $\boxed{\sigma_{True} < \sigma_{actual}}$

1.3 Strain($\in$)

- $\boxed{\in = \dfrac{Change\ in\ dimension}{Original\ dimension} = \dfrac{\Delta L}{L}} \implies \boxed{\in_{Engg} = \dfrac{\Delta L}{L_0}} \implies \boxed{\in_{actual} = \dfrac{\Delta L}{L_a}} \Longleftarrow \boxed{L_a = L_0 \pm \Delta L}$
- $L_a = L_0 + \Delta L$ for Tension $\qquad L_a = L_0 - \Delta L$ for Compression
- **Relationship between σ_{Engg} & σ_{actual} :** $\boxed{\sigma_{actual} = \sigma_{Engg}(1 \pm \in_{Engg})} \Longleftarrow +ve(Tension)\ \&\ -ve(Compression)$

1.4 Stress-Strain curve

- The Mechanical properties of materials used in engineering are determined using experiments performed on small specimens
  - **ATSM -** American Society for Testing and Materials
  - **UTM -** Universal Testing Machine (for Tension test)
  - **Specimen spec -** Must be a cylindrical rod with L/D = 4
- **Stress-Strain curve for Tension**



  - <u>A = PROPORTIONAL LIMIT</u>
    - **Hooke's Law:** stress $\propto$ strain
    - Hooke's law is valid upto this point i.e., Linear variation of stress and strain upto A
  - <u>B = ELASTIC LIMIT</u>
    - Maximum stress upto which material can retain its original dimension upon load removal
    - Material behaves perfectly elastic up until B
    - Only Elastic or Elastoplastic deformation (Elastoplastic = both plastic and elastic deformation)
  - <u>C' = UPPER YIELD POINT</u>
    - Depends on CS area, shape of specimen and type of the test equipment.
    - Has no practical significance
  - <u>C = LOWER YIELD POINT</u>
    - Also called as actual yield point and stress at C is the Yield stress(σ_y)
    - The yielding begins from this point
  - <u>CD = PERFECTLY PLASTIC REGION</u>
    - Strain occurring without any increase in stress
  - <u>DE = STRAIN HARDENING REGION</u>
    - strain increases with faster rate in this region
    - material undergoes change in the crystalline structure
  - <u>E = ULTIMATE YIELD POINT</u>
    - Stress corresponding to this point is called Ultimate stres (σ_U)
  - <u>F = FRACTURE POINT</u>
    - Stress corresponding to this point is called Ultimate (σ_F)
    - Region between EF is called the **Necking region**, where the CS area is drastically reduced.
- **Plastic Strain:** Strain before Yield point
- **Elastic Strain:** Strain After Yield point
- Fracture Strain($\in_F$) depends on Carbon content %. If carbon($\uparrow$), fracture strain decreases

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1.5 Properties of Materials

1.5.1 Ductility

- Large deformations are possible in ductile material before fracture
- These materials have post-elastic strain greater than 5%

1.5.2 Brittleness

- These materials have post-elastic strain less than 5%
- Fracture takes place immediately after elastic limit
- Fracture and ultimate points are the same

1.5.3 Malleability

- The property that tells if a metal can be converted into thin sheet by pressing it.
- This property is great use in operations like forging, hot rolling, stamping, etc.,

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1.5.4 Hardness

- Resistance to scratch or abrasion
- Two methods of Hardness measurement:
  - Mohr's test
  - Indentation hardness - Brinell, Rockwell, Vickers, Knoop

1.5.5 Toughness

- Property which enables material to absorb energy without fracture.
- If a material is tough it has ability to store large amount of strain energy before fracture.
- Ductile materials are tough and brittle materials are hard
- **Modulus of Toughness:** Total strain energy per unit volume up until fracture
- Modulus of Toughness = $\boxed{\frac{\sigma_y + \sigma_U}{2} * \in_F}$

1.6 Creep

- Permanent deformation in a material under constant loading after a long period of time
- Factors affecting creep: Load magnitude, type of loading, age of loading, Temperature
- **Homologous temperature:** Half of melting point, creep becomes appreciable at this temperature

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1.7 Stress relaxation

- The reason why electric wires sag after a long period of time.
- The stress gradually diminishes and reaches a constant value after a period of time.

1.8 Elasticity

- The property by which original dimensions can be recovered upon unloading is called elasticity
- within elastic limit, the curve can be both linear and non-linear.

1.9 Resilience

- The total strain energy which can be stored in the given volume of the metal and can be released after unloading is called resilience.
- Resilience = Area under stress-strain curve within elastic limit
- **Modulus of resilience(U_r):** Maximum elastic energy per unit volume. This occurs when elastic limit coincides with yield point.
- $\boxed{U_r = \frac{1}{2} * \sigma_y * \in_y = \frac{\sigma_y^2}{2E}} \Longleftarrow \boxed{\in_y = \frac{\sigma_y}{E}}$

1.10 Proof stress

- Some materials do not show clear yield point on the stress-strain curve.
- For such materials the yield point is calculated by offset method.
- A line parallel to the curve until the elastic limit is drawn starting from 0.2% of the strain. This line meets the curve at a point and the corresponding stress at this point is called Proof stress.

1.11 Elasto-Plastic behaviour

- during unloading if only part of the original dimension was recovered, then the remaining unrecoverable strain energy is called **Inelastic strain energy**
- Beyond elastic limit, if a material undergoes continuous loading and unloading, then yield limit of material increases continuously

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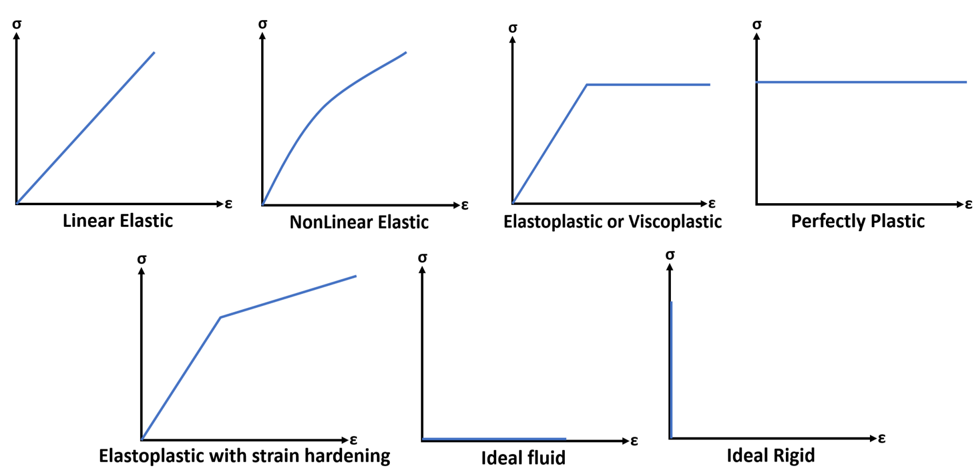
1.12 Types of Material Behaviour



1.13 Fatigue

- Materials behave differently under static and dynamic loading
- Factors affecting fatigue: Loading, Temperature, Loading frequency, Corrosion, Stress concentration
- **Fatigue Initiation life:** The number of load cycles required to initiate a surface crack
- **Fatigue Propagation life:** The number additional load cycle required to propagate surface crack
- **Endurance limit:** The stress below which material has no probability of cracking even with infinite load cycles. Endurance limit exists between elastic limit and yield point.

1.14 Failure of Materials in Tension and Compression

1.14.1 Ductile Metals in Tension

- Ductile materials are weak in shear
- Cup and cone failure
- failure plane angle is 45°

1.14.2 Brittle Metals in Tension

- Brittle materials are weak in tension
- Failure plane angle is 90°to load

1.14.3 Ductile Metals in Compression

- Failure plane anlge is 90°to load

1.14.4 Brittle Metals in Compression

- Brittle materials fail in shear
- Failure plane angle is 45°

Chapter 2

Stress, Strain and Elastic Constants

2.1 Normal Stress

- Stress acting perpendicular to the CS area
- They are of two types: Direct axial stress, Bending stress

2.1.1 Direct axial stress

- These stress are produced when axial force is acting at CG of cross section

2.1.2 Bending stress

- Produced due to Bending moments. Bending stresses vary linearly from 0 at Neutral axis to Maximum at farthest fibre from Neutral Axis

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2.2 Shear (or) Tangential stress(τ)

- Shear stress(τ) = $\boxed{\frac{ShearForce}{Area} = \frac{S}{A}}$
- They are of two types: Direct shear, Torsional shear

2.2.1 Direct shear stress

- Due to direct shear force acting on the surface

2.2.2 Torsional shear stress

- Produced when a member is subjected to torsional moment (Twisting)

2.3 Matrix representation of stress and strain

- Stress and strain are called **tensor** quantities as are they are defined with respect to an area. They are 2nd order Tensors
- In a 3D body, stress or strain at a point has 9 components (3 Normal and 6 shear)
- In a 3D, there are 3 mutually perpendicular planes(xy,yz,xz). Each plane has 1 normal component and 2 shear component.

$$\boxed{stress = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}} \qquad \boxed{strain = \begin{bmatrix} \in_{xx} & \phi_{xy}/2 & \phi_{xz}/2 \\ \phi_{yx}/2 & \in_{yy} & \phi_{yz}/2 \\ \phi_{zx}/2 & \phi_{zy}/2 & \in_{zz} \end{bmatrix}}$$

- Shear stresses in two mutually perpendicular directions are equal: $\tau_{yx} = \tau_{xy}$ $\tau_{zx} = \tau_{xz}$ $\tau_{yz} = \tau_{zy}$
- Total shear strain in x-y plane: $\dfrac{\phi_{xy}}{2} + \dfrac{\phi_{yx}}{2} = \phi_{xy}$
- Total shear strain in y-z plane: $\dfrac{\phi_{zy}}{2} + \dfrac{\phi_{yz}}{2} = \phi_{yz}$
- Total shear strain in x-z plane: $\dfrac{\phi_{xz}}{2} + \dfrac{\phi_{zx}}{2} = \phi_{xz}$
- $\in_{xx}, \in_{yy}, \in_{zz}$ are linear strains in x, y, z directions respectively
- Under pure normal stress = Volume changes, shape remains same
- Under pure shear stress = Volume remains same, shape changes.

2.3.1 Strain Types

Axial Strain($\in$)

- Strain the direction of the applied force. Also called as **Linear strain**
- $\boxed{\in = \dfrac{\textit{Change in dimension}}{\textit{original dimension}} = \dfrac{\Delta L}{L}}$

Lateral Strain($\in_L$)

- Strain in the direction perpendicular to the applied force.
- Eg. When an object is stretched, its length increases, but its width and height decreases. This decrease in width and height is called Lateral strain. The increase in length is called Linear strain

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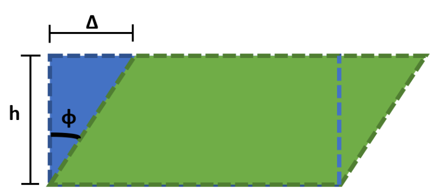
Shear strain(ϕ)



- Angular deformation caused by shearing force, $\boxed{\phi = \dfrac{\Delta}{h}}$

2.3.2 Differential form of Strains

- Consider a point P(x,y,z). A force acting on P, shifts its position to (u,v,w)
- Then linear strains and shear strains are given by:

$\Longrightarrow$ $\boxed{\in_{xx} = \dfrac{\delta u}{\delta x}}$ $\boxed{\in_{yy} = \dfrac{\delta v}{\delta y}}$ $\boxed{\in_{zz} = \dfrac{\delta w}{\delta z}}$ $\boxed{(\phi_{xy} = \phi_{yx}) = \dfrac{\delta u}{\delta y} + \dfrac{\delta v}{\delta x}}$ $\boxed{(\phi_{xz} = \phi_{zx}) = \dfrac{\delta w}{\delta x} + \dfrac{\delta u}{\delta z}}$ $\boxed{(\phi_{yz} = \phi_{zy}) = \dfrac{\delta w}{\delta y} + \dfrac{\delta v}{\delta z}}$

2.4 Allowable stresses

- **Strength:** The ability of a structure to resist loading is called strength. For safety reasons, materials should have higher strength than what is required due to loading.
- **Factor of Safety** $= \dfrac{Actual\ strength}{Strength\ required}$
- **Allowable Stress** $(\sigma_A) = \boxed{\sigma_{A(ductile)} \dfrac{Yield\ stress}{FOS}} = \boxed{\sigma_{A(brittle)} \dfrac{Ultimate\ stress}{FOS}}$
- A term called **Margin of safety** is used for aircrafts. Margin of safety = FOS - 1

2.5 Saint Venant's principle

- This principle states that the stress distribution in a prismatic bar is uniform except in the region of extreme ends.
- b = width of the prismatic bar
- Section(1-1): $\dfrac{b}{2}$ distance from the extreme ends
- Section(2-2): $\dfrac{b}{2} + \dfrac{b}{2}$ distance from the extreme ends
- $\boxed{\sigma_{1-1} = 1.387\sigma_{avg}}$ $\boxed{\sigma_{2-2} = 1.027\sigma_{avg}}$ $\boxed{\sigma_{3-3} = \sigma_{avg}}$

2.6 Hooke's law

- Assumptions: Homogeneous (made of same material), Isotropic (properties are same in all directions), elastic
- Stress $\propto$ Strain $\implies \boxed{\dfrac{\sigma}{\in} = E} \impliedby$ (Valid upto Proportional limit)
- E = Modulus of elasticity = slope of stress strain curve under proportional limit

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2.7 Elastic constants

2.7.1 Young's modulus(E)

- $\boxed{E = \dfrac{Direct\ stress}{Direct\ strain} = \dfrac{\sigma}{\in}}$

2.7.2 Shear modulus (or) Rigidity modulus(G)

- $\boxed{G = \dfrac{Shear\ Stress}{Shear\ strain} = \dfrac{\tau}{\phi}}$

2.7.3 Bulk modulus(k)

- $\boxed{k = \dfrac{Volumetric\ stress}{Volumetric\ strain} = \dfrac{\sigma_{vol}}{\in_V}}$ $\boxed{\in_V = \dfrac{\Delta Volume}{Volume}}$

2.7.4 Poisson's ratio(μ)

- $\boxed{\mu = \frac{-Lateral\ strain}{Linear\ strain}}$ $\Longleftarrow$ (defined in elastic region)
- μ = 0.05 - 1 (glass), 0.1 - 0.2 (concrete),0.25 - 0.42 (metals), 0.5 (pure rubber, perfectly plastic)
- For non-elastic region, it is called **Contraction ratio**

2.7.5 Relationship between elastic constants

- $\boxed{E = 3K(1-2\mu)}$ $\boxed{E = 2G(1+\mu)}$ $\boxed{E = \frac{9KG}{3K+G}}$ $\boxed{\mu = \frac{3K-2G}{6K+2G}}$
- **Orthotropic material** = 9 elastic constants, **Anisotropic material** = 21 elastic constants

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2.8 Applications of Hooke's law

2.8.1 Effect of Uniaxial Loading

- Consider a rectangular prismatic bar with tensile force acting along x-axis

$\boxed{\in_{xx}= \frac{\sigma_x}{E}}$ $\Longrightarrow$ $\boxed{\in_{yy}= -\mu\frac{\sigma_x}{E}}$ $\boxed{\in_{zz}= -\mu\frac{\sigma_x}{E}}$ $\Longleftarrow \in_{yy}$ & $\in_{zz}$ are Lateral strains

2.8.2 Effect of Triaxial Loading

- Consider: σ_x acting along x-direction, σ_y acting along x-direction, σ_z acting along x-direction. (All are Tensile)
- $\boxed{\in_{xx}= \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}}$ $\boxed{\in_{yy}= \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}}$ $\boxed{\in_{zz}= \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}}$

2.9 Volumetric Strain($\in_V$)

- How to derive?
- compute the volume of the member. take that as V.
- Now find ΔV, that is differentiate V wrt to each factor.
- Then use: $\boxed{\in_V=\in_x + \in_y + \in_z}$

2.9.1 For Rectangular prismatic member

- $\boxed{\frac{\sigma_x + \sigma_y + \sigma_z}{E} * (1-2\mu)}$

2.9.2 For Cylindrical rod

- $\boxed{\in_V=\in_L +2 \in_d}$ $\Longleftarrow$ ($\in_d$ = Diametrical strain)

2.9.3 For Spherical body

- $\boxed{\in_V= 3 \in_d}$

2.10 Elongation in axially loaded members

- How to derive?
- consider an element and find its elongation using $\frac{PL}{AE}$, Then integrate that δL for the whole length

2.10.1 Axially loaded prismatic bar

- In this case Elemental elongation is $\frac{P_x d_x}{A_x E_x} \Longleftarrow d_x =$ Length of the element.
- $\boxed{\Delta = \frac{PL}{AE}}$ $\quad (AE =$ Axial rigidity) and ($\frac{AE}{L} =$ Axial stiffness)

2.10.2 Axially loaded Circular Tapered bar

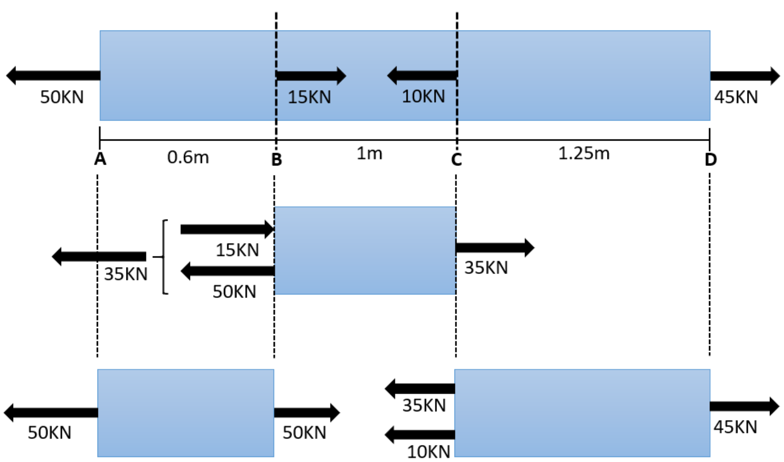
- Here Diamater of the element $= D_x = D_1 + \frac{D_2 - D_1}{L} x$
- $\boxed{\Delta = \frac{4PL}{\pi E D_1 D_2}} \Longleftarrow D_1 =$ smaller diameter and $D_2 =$ Larger diameter

2.10.3 Axially loaded Rectangular Tapered bar

- The thickness of the bar is uniform
- $\boxed{\Delta = \frac{PL \log_e \left(\frac{B_2}{B_1}\right)}{(B_2 - B_1) t E}} \Longleftarrow (B_1 =$ Smaller height and $B_2 =$ Larger height)

2.11 Principle of Superposition

- If a member is subjected to various loadings then the resultant deformation will be equal to the algebraic sum of the deformation caused by the individual forces acting on the member.
- Consider the following example:

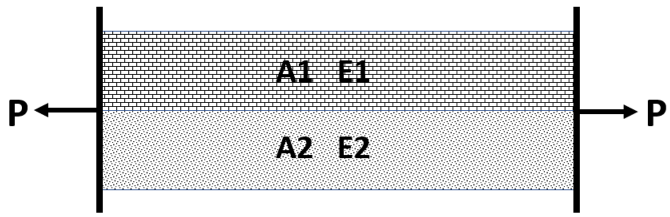


- Total elongation of the element(Δ) $= \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$

2.12 Elongation in Composite members

- Composite structures are those that are made of more than one material

2.12.1 Elongation in composite rectangular member



- Condition of Equilibrium: $P = P_1 + P_2$
- As two materials are joined firmly: $\Delta_1 = \Delta_2 \implies \dfrac{P_1 L_1}{A_1 E_1} = \dfrac{P_2 L_2}{A_2 E_2}$
- $\boxed{\Delta = \dfrac{PL}{A_1 E_1 + A_2 E_2}}$ and $E_{eq} = \dfrac{A_1 E_1 + A_2 E_2}{A_1 + A_2} \implies \Delta = \dfrac{PL}{(A_1 + A_2) E_{eq}}$

2.13 Elongation due to Self-Weight

- How to derive?
- Find the elemental deflection due to the self weight and then Integrate for the whole length.

2.13.1 Rectangular Prismatic bar

- $\gamma = Unit\ weight = \dfrac{Weight}{unit\ volume}$
- $W_x = \gamma(A * x) \impliedby W_x =$ Self weight acting at an element x distance away from the bottom of the bar
- $\boxed{\Delta = \dfrac{\gamma L^2}{2E}} \implies \boxed{\dfrac{WL}{2AE}}$

2.13.2 Conical Bar

- $\boxed{W_x = \gamma * \left(\dfrac{1}{3}\right) \pi \left(\dfrac{D_x}{2}\right)^2 x}$ $\boxed{\Delta = \dfrac{\gamma L^2}{6E}}$

2.13.3 Bar of Uniform strength

- It is possible to maintain uniform stress at all the sections by increasing the area from the lower level to Higher level.
- $\boxed{A_1 = A_2 e^{\left(\dfrac{\rho g L}{\sigma}\right)}}$

2.14 Statically Indeterminate Axial Loaded structures

- Those structures which cannot be solved using static equilibrium equations alone are called Statically Indeterminate structures
- They can be solved using flexibility approach or stiffness approach
- Flexibility approach involves equations using the relations in Slope, deflection and rotation and are called **compatibility equation**

2.15 Thermal stresses

- **NOTE:** Thermal stresses do not depend on the area of cross section
- Thermal Strain $\boxed{\in_{Th} = \alpha T}$ $\Longleftarrow$ (α = Coefficient of thermal expansion)
- Consider a rectangular prismatic bar of length L between fixed supports. If the temperature of the bar is raised, then $\Delta = 0$ (As the bar is fixed)
- But due to rise in temperature of T°C, the bar will try to expand and so will be under compression
- $\Delta = 0 = L\alpha T - \dfrac{\sigma_{Th} L}{E} \implies \boxed{\sigma_{Th} = E\alpha T}$ $\Longleftarrow$ (Thermal stresses are independent of member dimensions)

2.15.1 Thermal stresses in Composite bars

- Mostly we use three sets of relations
- $P_1 = P_2$ (under no other external load) $\implies \sigma_1 A_1 = \sigma_2 A_2$
- $\Delta_1 = \Delta_2$
- The material with larger α value, will tend to deform more under thermal load
- Consider a composite cantilever bar made of copper and steel and both the materials are rigidly fixed to each other. Copper has higher coefficient of thermal expansion and so upon temperature rise will tend to expand more than steel.
- As a result, copper part will be under compression as steel is not letting it to expand and steel part will be under tension as copper is pulling it.
- As they are affixed to each other rigidly, both their end deformation value will be the same. Using that relation we can write: Copper free expansion + contraction = steel free expansion + tension $\implies \boxed{L\alpha_c T - \dfrac{\sigma_c L}{E_c} = L\alpha_s T + \dfrac{\sigma_s L}{E_s}}$

2.16 Stresses in Nuts and Bolts

- **Effect of Tightening of Nut**
- P = pitch of screw on the bolt, D_b = Bolt diameter
- A_s = Area of steel bolt = $\dfrac{\pi D_b^2}{4}$
- D_i = Inner diameter of copper tube, D_O = Outer diameter of copper tube
- θ = Nut is rotated by $\theta°$, n = Number of turns = $\dfrac{\theta}{360}$
- np = Axial movement of nut
- Total tensile for in bolt = total compressive force in copper tube $\implies \sigma_s A_s = \sigma_c A_c$
- $\boxed{np = \left(\dfrac{\sigma_s L}{E_s}\right) + \left(\dfrac{\sigma_c L}{E_c}\right)}$ $\Longleftarrow$ Using the above relation, solve for σ_s and σ_c

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2.17 Strain Energy(U)

- The strain energy is equal to the work done by the load provided no energy is added or subtracted in the form of heat.
- **NOTE:** Strain energy due to more than one load can be found by applying Superposition principle. i.e. Total strain energy due to multiple loads = Sum of individual strain energy developed due to individual loads
- $\boxed{U = \frac{1}{2}P\Delta}$ $\Longleftarrow$ Δ = Axial deflection

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2.17.1 Strain energy in Prismatic bar

- $\boxed{U = \frac{P^2L}{2AE}}$

2.17.2 Strain energy in Prismatic bar of varying cross section

- $\boxed{U = \frac{P^2}{2E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + ...\right]}$

2.17.3 Strain energy due to shear force

- $\boxed{U = \int \frac{S_x^2 dx}{2A_r G}}$ $\Longrightarrow$ (A_r = Reduced area)

2.17.4 Strain energy in terms of principal stresses

- $\boxed{U = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2)}$ $\Longleftarrow$ (2D) $\boxed{U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]}$ $\Longleftarrow$ (3D)

2.17.5 Strain energy due to Bending moment

- $\boxed{U = \int \frac{M_x^2 ds}{2EI}}$ $\Longleftarrow$ (I = MOI about NA)

2.17.6 Strain energy due to Torque

- $\boxed{U = \int \frac{T_x^2 ds}{2GI_P}}$ $\Longleftarrow$ (I_P = Polar MOI)

Chapter 3

Shear force and Bending Moment

3.1 Types of Loading

- Point load
- Uniformly Distributed Load
- Uniformly Varying Load
- Couple - Concentrated moment at any point is called couple

3.2 Types of supports

3.2.1 Types of 2D Supports

- Fixed support - R_x, R_y, M_z
- Hinged Support - R_x, R_y
- Roller support - R_y
- Double roller support - R_y, M_z

3.2.2 Types of 3D Supports

- Fixed support - $R_x, R_y, R_z, M_x, M_y.M_z$
- Hinged support - R_x, R_y, R_z
- Roller support - R

3.3 Types of Beams

- Simply supported beam
- Cantilever beam
- Propped cantilever beam
- Fixed beam
- Continuous beam
- Overhanging beam
- **NOTE: If an internal hinge is present in a beam, then Bending moment at hinge is Zero. Hinges carry only Axial load.**

3.4 Statically determinate structure

- A structure is said to be statically determinate if all the reaction forces can be found using the static equilibrium equations (2D): $\boxed{\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0}$
- If R denotes the number of reactions and E denotes the number of equilibrium equations available, then if (R = E) the structure is determinate, if (R > E) then the structure is indeterminate

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3.5 Shear Force

- Clockwise shear = +ve and Anticlockwise shear = -ve

3.6 Bending Moment

- Sagging Bending moment = +ve and Hogging Bending moment = -ve

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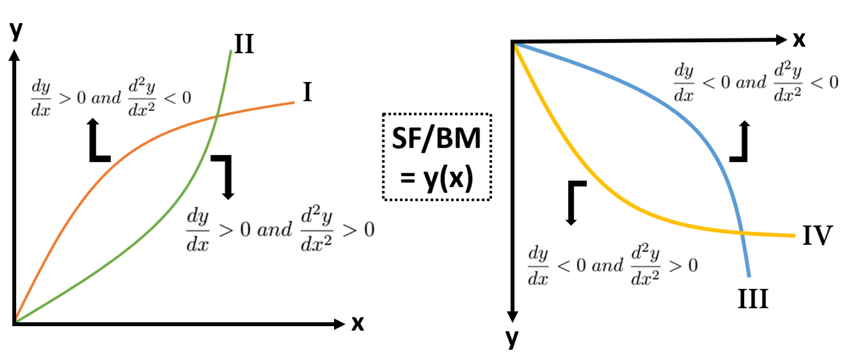
3.7 Important points about SFD and BMD

- SFD is one degree higher than Loading diagram and BMD is one degree higher than SFD
- If a point load is present, then the SFD will change by that load's magnitude
- If a concentrated moment is present, then the BMD will change by that moment's magnitude
- Bending will be maximum/minimum at the point where Shear force is zero, but the inverse is not true
- **Point of Contraflexure -** The point where Bending moment changes sign
- **Focal length -** The distance between two adjacent points of contraflexure
- **Shear span -** the portion of beam where shear force is constant.
- Relation between Loading rate and Shear force: $\boxed{\dfrac{dS}{dx} = -W}$
- Relation between Shear force and bending moment: $\boxed{\dfrac{dM}{dx} = S_x}$ $\Longrightarrow$ Slope of bending moment = shear force.
- Relation between Bending moment and Loading rate: $\boxed{\dfrac{d^2M}{dx^2} = -w}$

3.8 Curve Tracing for SFD and BMD



3.9 SFD and BMD by Integration

- Use the relations between Loading rate, shear force and Bending moment:

$$\implies \boxed{\frac{d^2M}{dx^2} = -w} \text{ ...(i) } \boxed{\frac{dM}{dx} = S_x = -Wx + C_1} \text{ ...(ii) } \boxed{M_x = \frac{-Wx^2}{2} + C_1x + C_2} \text{ ...(iii)}$$

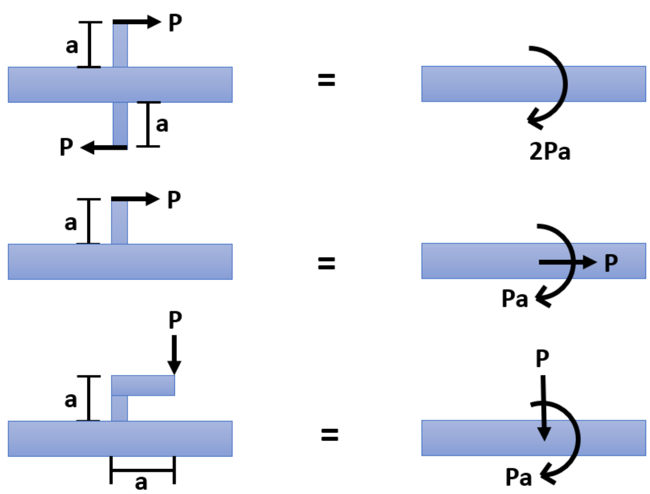
  - **NOTE:** Eqn(i)'s RHS will change according the loading. For eg. in case of Uniformly variying load,

$$\boxed{\frac{d^2M}{dx^2} = -W_x = \frac{-Wx}{L}}$$

- The constants C_1 and C_2 can be found by boundary conditions
- From (ii) SFD can be plotted, from (iii) BMD can be plotted

3.10 Effect of Concentrated moment on SFD and BMD

- At the point of concentrated moment, discontinuity occurs in BMD. The BMD ordinate changes by the magnitude of the moment. For SFD, concentrated moment induces support reactions only.



3.11 Loading diagram and BMD from SFD

- In SFD, the portion where SF is constant, it means no load is acting in that portion
- In SFD, Vertical lines represent loads (Generally upward loads denote reactions)
- Inclined line in SFD denotes Uniformly distributed load. Intensity of UDL = slope of line in SFD
- Parabolic curve in SFD denotes Uniformly varying load.
- If $\sum M$ at supports is zero, then the support is hinged or roller. If its not zero, then it is a fixed support.

3.12 Loading diagram from BMD

- In BMD, the portion where there is an inclined line, that means no load is acting in that region
- Parabolic curve in BMD denotes Unifromly distributed load.
- Cubic curve in BMD denotes Uniformly varying load.
- The point where BMD changes its ordinate suddenly, a concentrated moment is acting at that point.
- BMD is one degree higher than SFD and two degree higher than Loading diagram

---

Chapter 4

Centroid and Moment of Inertia

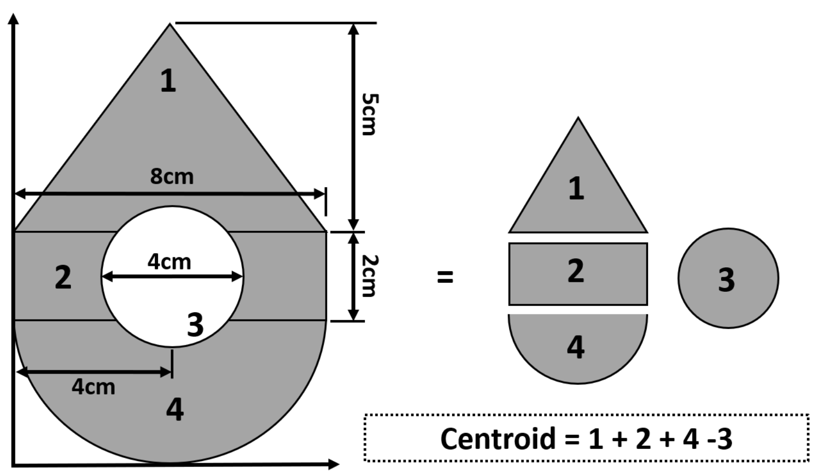
4.1 Centroid($\bar{x}$,$\bar{y}$)

- $\boxed{\bar{x} = \dfrac{A_1\bar{x_1} + A_2\bar{x_2} + ... + A_n\bar{x_n}}{A_1 + A_2 + ... + A_n} = \sum_{i=1}^{n} \dfrac{A_i\bar{x_i}}{A_i} = \dfrac{\int x dA}{A}}$ $\boxed{\bar{y} = \dfrac{A_1\bar{y_1} + A_2\bar{y_2} + ... + A_n\bar{y_n}}{A_1 + A_2 + ... + A_n} = \sum_{i=1}^{n} \dfrac{A_i\bar{y_i}}{A_i} = \dfrac{\int y dA}{A}}$
- It is the center of a shape
  - If the shape is Bi-Symmetrical, then the centroid will lie on the point of intersection of the two symmetrical axis. Eg. Rectangle, Square, Circle, etc.
  - If the shape is symmetrical about only one axis, then centroid will be anywhere on that symmetrical axis. Eg. Triangle

---

4.1.1 Finding centroid for a composite shape

- A composite shape is a shape that is made of many regular shapes like: semicircle, circle, rectangle, square etc,
- How to find Centroid for a composite shape? An example of composite shape is shown below



- Break down the composite shapes into its regular shapes. Then form a table with the following columns:

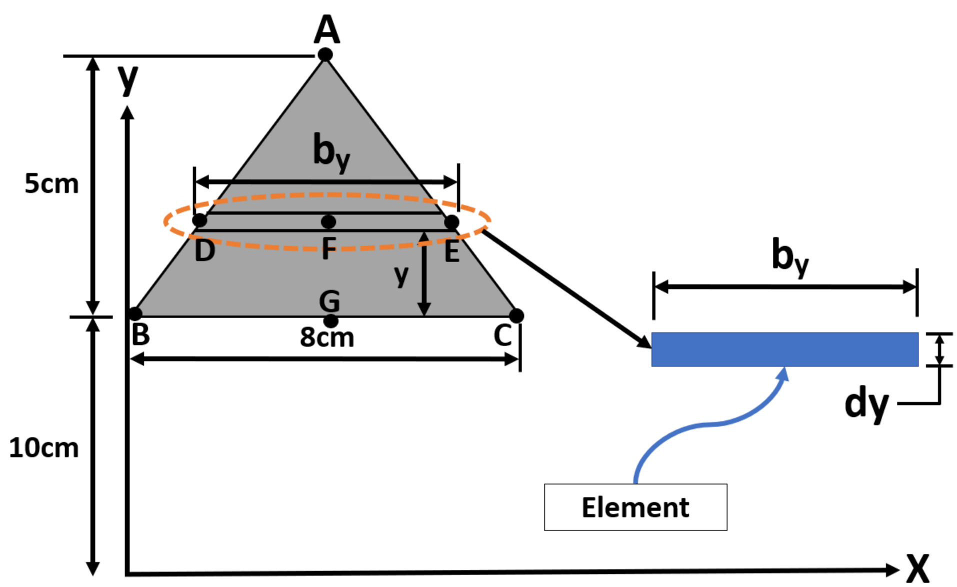
$$shape, \;\; Area(A), \;\; \bar{x}, \;\; \bar{y}, \;\; A\bar{x}, \;\; A\bar{y}$$

- Then find: $\sum A$, $\sum A\bar{x}$, $\sum A\bar{y}$ and then use, $\bar{x} = \dfrac{\sum A\bar{x}}{\sum A}$ and $\bar{y} = \dfrac{\sum A\bar{y}}{\sum A}$
- For example, the above shape can be broken down into 4 parts as shown above: Triangle, Rectangle, Circle and Semi-circle
- Now, we need to know 3 things about each of the shape above to compute the centroid of the whole shape.

1. Area of the shape 2. $\bar{x}$ 3. $\bar{y}$

- For triangle:
  - $\rightarrow$ Area: $\left(\frac{1}{2} * Base * Height\right) = \frac{1}{2} * 8 * 5 = 20cm^2$
  - $\rightarrow$ $\bar{x}$ = Base/2 (as the triangle is symmetrical about the y-axis) = 4cm
  - $\rightarrow$ $\bar{y}$ = 10 + h/3 = (10+1.6667cm)
    - $*$ 10 is added because, the triangle is at a height of 10cm from x-axis
    - $*$ Why $\bar{y}$ = h/3 ? Derived below:

4.1.2 Finding $\bar{y}$ for a triangle



- We know that $\boxed{\bar{y} = \frac{A_1\bar{y_1} + A_2\bar{y_2} + ... + A_n\bar{y_n}}{A_1 + A_2 + ... + A_n} = \sum_{i=1}^{n} \frac{A_i\bar{y_i}}{A_i} = \frac{\sum A_i\bar{y_i}}{A}}$
- Here, Area = $20cm^2$
- Consider a horizontal element of thickness (dy) at a height of y from origin.
- The width of the element can be found using **Law of similar triangles** concept. Based on that:

$$\frac{AG}{BC} = \frac{AF}{DE} \implies \frac{5}{8} = \frac{5-y}{b_y} \implies b_y = \frac{8(5-y)}{5}$$

- Now, Area of this element is $A_i = b_y dy = \frac{8(5-y)}{5} dy$ and $\bar{y_i} = 10 + y + \frac{dy}{2} \Longleftarrow$ But already dy is very small. So dy/2 is negligble $\implies \bar{y_i} = 10 + y$
- Applying the formula of $\bar{y}$ we get,

$$\bar{y} = \frac{\sum \frac{8(5-y)}{5} dy(10+y)}{20} = \frac{\int_0^5 \frac{8(5-y)}{5}(10+y)dy}{20}$$

$$= \frac{1}{100}\int_0^5 (40-8y)(10+y)dy = \frac{1}{100}\int_0^5 400 + 40y - 80y - 8y^2 = \frac{1}{100}\int_0^5 400 - 40y - 8y^2$$

$$= \frac{8}{100}\int_0^5 50 - 5y - y^2 = 0.08 * \left[50y - 5\frac{y^2}{2} - \frac{y^3}{3}\right]_0^5 = 0.08 * \left[50(5) - 5(12.5) - \frac{125}{3}\right]$$

$$= 0.08 * (145.833) = 11.667 \implies 10 + 1.667 \implies 10 + 5/3 \implies 10 + h/3$$

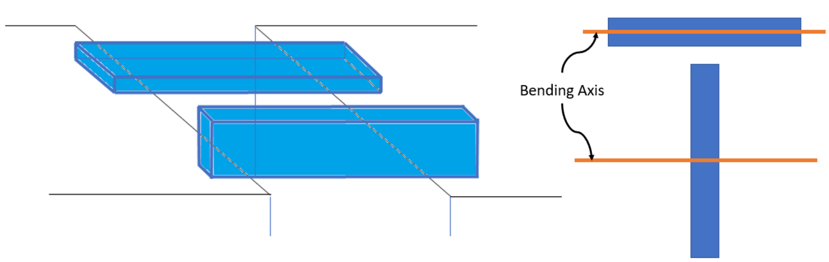
- Hence, $\bar{y}$ for a triangle is h/3

4.1.3 Finding centroid for area under a given curve

- Consider a vertical strip of thickness(dx) at a distance of x from origin and height(y)=f(x)
- The area of this element will be dA = ydx = f(x)dx
- Integrate this small area over the entire width of the curve and find Total area(A).
- Use the formula, $\bar{x} = \dfrac{\int xdA}{A}$ and $\bar{y} = \dfrac{\int ydA}{A}$

4.2 Moment of Inertia (I)

- Normally Inertia means, from Newton's first law, it is the resistance to change of state.
- Moment of Inertia are of various types.
  - Area Moment of Inertia - Resistance to Bending
  - Mass Moment of Inertia - Resistance to Rotation (based on mass distribution)
  - Polar Moment of Inertia - Resistance to Twisting (based on Perpendicular axis theorem)
  - Product of Inertia - Resistance to Rotation (based on two perpendicular axes)
- Here we are concerned with Area MOI and Polar MOI
- Area MOI is also called as 2nd moment of Area.
- Area MOI is always Non-zero and Positive
- Area MOI about x-axis = $\boxed{I_x = \int y^2 dA}$ Area MOI about y-axis = $\boxed{I_y = \int x^2 dA}$
- The further a material is spread from its bending axis, the more resistant it is towards bending. See the figure below for better understanding.
- The Area MOI of a section is lowest if it is taken about an axis passing through the centroid of the section



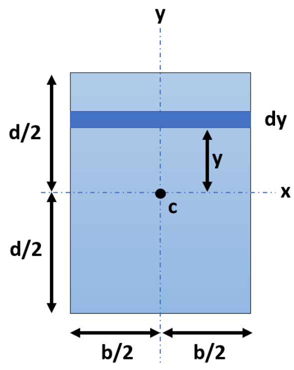
- In the figure above, the first bar is flatly laid. When a load is acting at its center, its more prone to bending and hence breaking compared to the same bar that is laid in a different position and subjected to the same load.
- Because, in the first bar, the material is less spread from the bending axis, compared to the second bar
- This demonstrates why computing MOI is very important to consider the safety of structures.

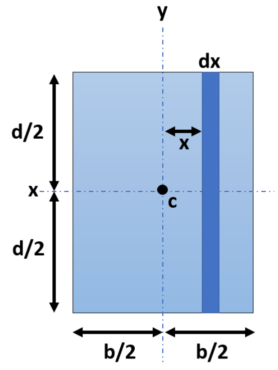
4.2.1 Steps to find Area MOI for any area

- Consider elementary strip (horizontal strip for I_x and vertical strip for I_y) and find it's area (dA)
- Use the above two formula with appropriate limits

4.2.2 Finding Area MOI for a rectangle about its centroidal axis

- Remember: ($\int_{-a}^{a}(even\ function) = 2\int_{0}^{a}(even\ function)$)

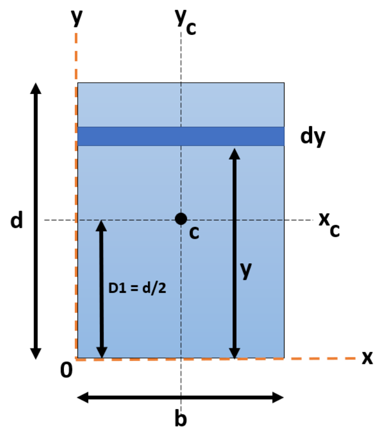


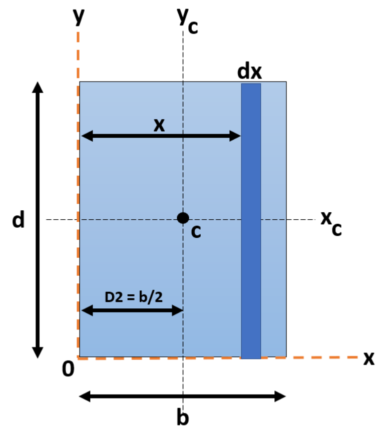


$$dA = b(dy)$$
$$We\ know\ I_x = \int y^2 dA$$
$$I_x = \int_{-d/2}^{d/2} y^2(bdy) = 2b\int_0^{d/2} y^2 dy$$
$$I_x = 2b\left(\frac{y^3}{3}\right)_0^{d/2} = 2b\left(\frac{d^3}{8*3}\right)$$
$$\boxed{I_x = \frac{bd^3}{12}}$$

$$dA = d(dx)$$
$$We\ know\ I_y = \int x^2 dA$$
$$I_y = \int_{-b/2}^{b/2} x^2(d(dx)) = 2d\int_0^{b/2} x^2 dx$$
$$I_y = 2d\left(\frac{x^3}{3}\right)_0^{b/2} = 2d\left(\frac{b^3}{8*3}\right)$$
$$\boxed{I_y = \frac{db^3}{12}}$$

4.2.3 Finding Area MOI for a rectangle with origin at bottom left corner





$$I_x = \int_0^d y^2(bdy) = b\int_0^d y^2 dy$$
$$I_x = b\left[\frac{y^3}{3}\right]_0^d = \frac{bd^3}{3}$$

$$I_y = \int_0^b x^2(d(dx)) = d\int_0^b x^2 dx$$
$$I_y = d\left[\frac{x^3}{3}\right]_0^b = \frac{db^3}{3}$$

- The above can also be solved using Parallel axis theorem, since the axis I_x is parallel to the centroidal axis I_{x_c} and the axis I_y is parallel to the centroidal axis I_{y_c}

4.3 Parallel axis theorem

- $\boxed{I_x = I_{xc} + Ad_1^2}$ $\boxed{I_y = I_{yc} + Ad_2^2}$ $\boxed{I_{xy} = I_{x_c y_c} + Ad_1 d_2}$
- d_1 = Distance of between x-axis and centroidal x-axis
- d_2 = Distance between y-axis and centroidal y-axis

4.3.1 Finding Area MOI of rectangle with origin at bottom right corner using Parallel axis theorem

- We have derived that Area MOI of the rectangle(b*d) about its centroidal axis is:
- $\boxed{I_{x_c} = \frac{bd^3}{12}}$ $\boxed{I_{y_c} = \frac{db^3}{12}}$
- Now using the above result, we can find I_x and I_y that is parallel to their respective centroidal axis as follows:

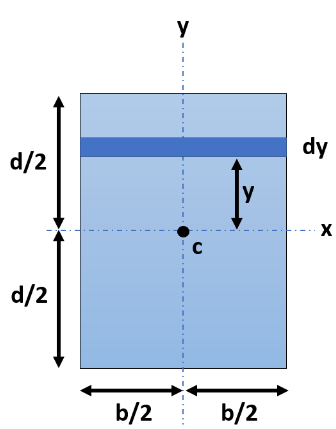
$$
\begin{aligned}
I_x &= I_{x_c} + Ad_1^2 \Longleftarrow \quad (Here\ d_1 = d/2) \\
I_x &= \frac{bd^3}{12} + (b*d)\left(\frac{d}{2}\right)^2 \\
I_x &= \frac{bd^3}{12} + \frac{bd^3}{4} \\
\Longrightarrow &\boxed{I_x = \frac{bd^3}{3}}
\end{aligned}
$$

$$
\begin{aligned}
I_y &= I_{y_c} + Ad_2^2 \Longleftarrow \quad (Here\ d_2 = b/2) \\
I_y &= \frac{db^3}{12} + (b*d)\left(\frac{b}{2}\right)^2 \\
I_y &= \frac{db^3}{12} + \frac{db^3}{4} \\
\Longrightarrow &\boxed{I_y = \frac{db^3}{3}}
\end{aligned}
$$

- From the above, it is evident how necessary and useful it is to remember the Area MOI of regular shapes about their centroidal axis as it can greatly help in deriving their Area MOI about any axis that is parallel to their centroidal axis.

4.4 Product of Inertia (I_{xy})

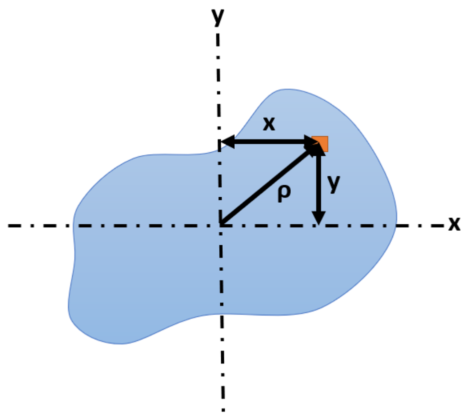
- If $I_{xy} = 0$, then that axis is called **Principal axis**
- Unlike Area MOI, Product of Inertia can be +ve, 0, -ve
- Product of Inertia about any symmetrical axis will be zero
- $\boxed{I_{xy} = \int xy dA}$



$$
\begin{aligned}
I_{x_c y_c} &= \int \bar{x}\bar{y} dA = \int_{-d/2}^{d/2} \left(\frac{b}{2}\right) y(bdy) \\
I_{x_c y_c} &= b^2 \left[\frac{y^2}{2}\right]_{-d/2}^{d/2} \\
I_{x_c y_c} &= b^2(0) = 0 \quad (As\ x_c\ and\ y_c\ are\ Symmetrical\ axis)
\end{aligned}
$$

4.5 Perpendicular axis theorem

- The Polar moment of Inertia (I_z) is based on the perpendicular axis theorem which defines an Area MOI wrt to an axis that is perpendicular to the cross-sectional area
- $\boxed{I_z = I_x + I_y} \implies I_z = \int \rho^2 dA = \int (x^2 + y^2) dA = \int x^2 dA + \int y^2 dA = I_y + I_x$
- It is based on Pythagorean theorem



- This can be used to calculate indirectly the Area MOI of a circle of radius R about its centroidal axes

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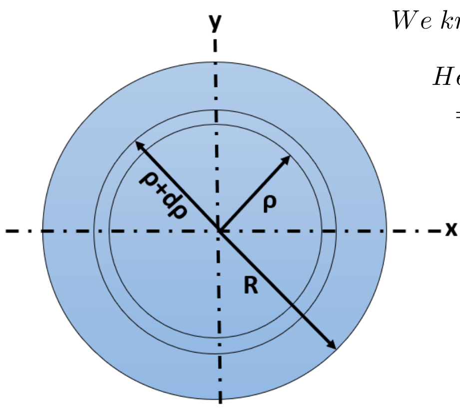
4.5.1 Finding Area MOI of a circle about its centroidal axes using I_z



$$We\ know,\ I_z = \int \rho^2 dA$$

$$Here,\ dA = \pi(\rho + d\rho)^2 - \pi\rho^2 = \pi(\rho^2 + (d\rho)^2 + 2\rho d\rho - \rho^2)$$

$$\implies dA = \pi 2\rho d\rho$$

$$I_z = \int_0^R \rho^2(\pi 2\rho d\rho) = 2\pi \int_0^R \rho^3 d\rho$$

$$I_z = 2\pi \left[\frac{\rho^4}{4}\right]_0^R = \frac{\pi R^4}{2}$$

$$I_z = I_x + I_y \implies \frac{\pi R^4}{2} = \frac{\pi R^4}{4} + \frac{\pi R^4}{4}$$

$$So,\ \boxed{I_x = I_y = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}}$$

4.6 Rotation of Axes

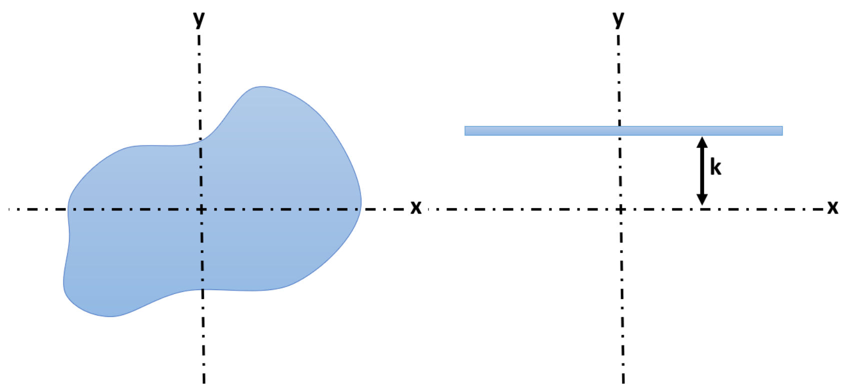
- $\boxed{I_{x'} = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right) cos2\theta - I_{xy} sin2\theta}$ $\boxed{I_{y'} = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right) cos2\theta + I_{xy} sin2\theta}$
- $\boxed{I_{x'y'} = \left(\frac{I_x - I_y}{2}\right) sin2\theta + I_{xy} cos2\theta}$
- The above three formulae help us to find the Area MOI of any shape about an axis that is tilted at an anticlockwise angle of $\theta^\circ$from one of the centroidal axis.

4.6.1 Principal Moment of Inertia(I_1, I_2)

- $\boxed{I_1, I_2 = \left(\frac{I_x + I_y}{2}\right) \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}}$
- Principal MOI denote the maximum(I_1) and minimum(I_2) Area MOI of a planar section
- The position of these principal Area MOI can be found using:
- $\boxed{\theta_{P_1} = \frac{tan^{-1}\left(\frac{-2I_{xy}}{I_x - I_y}\right)}{2}}$ $\boxed{\theta_{P_2} = \theta_{P_1} + 90\,\mathrm{deg}}$
- If an area has axis of symmetry, then product of inertia about those axes will be zero. Hence such axis must be principal axes. **Symmetrical axis is always principal, but the reverse is not true**

4.7 Radius of Gyration(k)

- It is the theoretical distance at which we can condense the entire area of cross section into a narrow strip and have the same Area MOI.
- $\boxed{k = \sqrt{\frac{I_x}{A}}}$

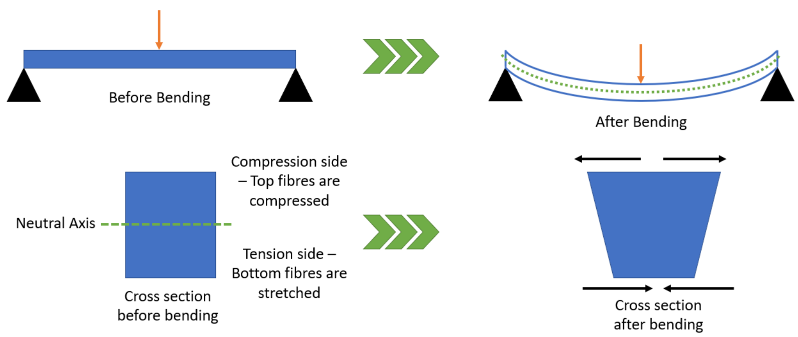


Chapter 5

Bending stresses in Beams

5.1 Effect of Bending

- **Neutral Axis** : The transverse axis about which the CS area rotates is called Neutral Axis. In pure bending, the NA always passes through the centroid. In plastic bending, NA passes through equal area axis.



- Hence, a rectangular section before bending will be trapezoidal section after bending. But such deformations in CS area (called transverse deformations since they are perpendicular to the applied) are very small and so negligible. So for all practical purposes, the CS area can be considered to be rectangular even after bending.

5.2 Simple bending or Pure bending

- If the bending moment is constant along a portion of the beam, then that portion is said to be under pure bending. $\implies \frac{dM}{dx} = 0 \implies SF = 0$

5.2.1 Assumptions in Theory of pure bending

- Material is **Homogeneous, Isotropic and linearly elastic**
- The beam is **Prismatic** and **straight** before loading
- The **Young's modulus(E) is same for tension and compression**
- Plane section before bending remains plane section after bending $\implies$ Longitudinal strain vary linearly from zero at NA to max at the surface. $\boxed{Longitudinal\ strain \propto Distance\ of\ the\ fibre\ from\ NA}$
- The section of the beam is symmetrical in the loading plane. If asymmetrical, then twisting and warping may occur along with bending.

5.3 Equation of Pure bending

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

- R = Radius of curvature
- I = Area MOI of CS about NA
- M = Bending Moment
- σ = Bending stress in a fibre at a distance y from NA
- E = Modulus of Elasticity
- **Limitation of Pure bending eqn:** Applicable only in case of pure bending

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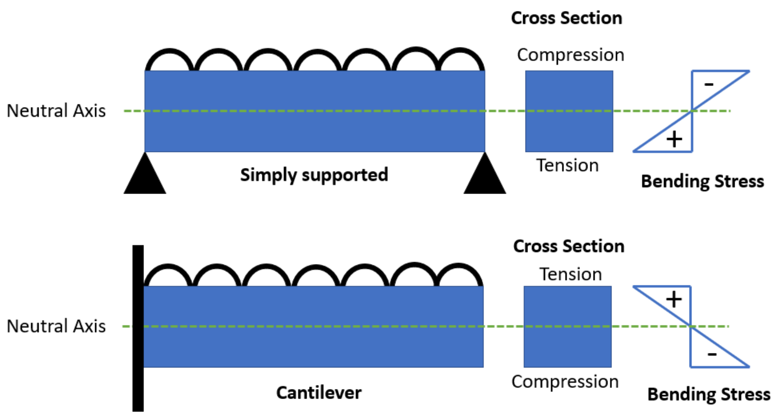
5.3.1 Nature of Bending stress



- $\boxed{\text{Total compressive force = Average stress * Area under compression}}$
- $\boxed{\text{Total Tensile force = Average stress * Area under Tension}}$
- Average stress is nothing but the stress calculated using the pure bending equation. $\boxed{\sigma = \frac{M}{I}y}$

5.4 Section Modulus(Z)

- Represents the strength of the section
- It is the ratio of Area MOI of the beam's Cross section about NA to the distance of the extreme fibre from NA.
- $\implies \boxed{Z = \frac{I}{y_{max}}}$ Unit: mm^3
- If Section modulus is known, then Max Bending stress: $\boxed{\sigma_{max} = \frac{M}{Z}}$

5.5 Moment of Resistance (M_R)

- Represents the Max Bending moment that can be resisted by the section without failure
- $\boxed{M_R = \sigma_P Z}$ $\Longleftarrow$ (Valid only for Symmetrical section about NA, σ_P = Permissible stres)
- If section is asymmetrical (Eg. T section), $\boxed{M_{R_1} = \sigma_c Z_c}$ and $\boxed{M_{R_2} = \sigma_t Z_t}$. The M_R will be the lesser of the two values.

$\Longrightarrow$ σ_c, σ_t are permissible bending stress in compression and Tension respectively

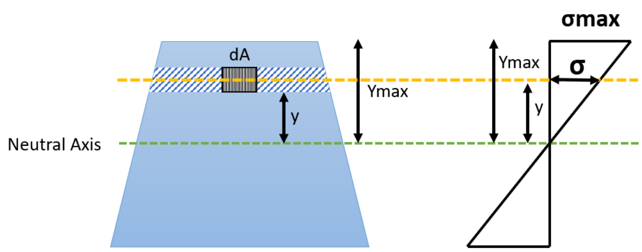
---

5.6 Bending Stresses in Axially loaded beams

- Direct Axial Load: $\boxed{\sigma = \pm\frac{M}{I}y \pm \frac{P}{A}}$ $\Longleftarrow$ (P = Axial load)
- Eccentric Axial Load: $\boxed{\sigma = \pm\frac{M}{I}y \pm \frac{P}{A} \pm \frac{Pe}{I}y}$ $\Longleftarrow$ (e = Eccentricity)

5.7 Forces on Partial Area of a Section

- The force on a partial area of a cross section can be found by using similar triangles concept on the stress diagram when Max bending stress(σ_{max}) is known



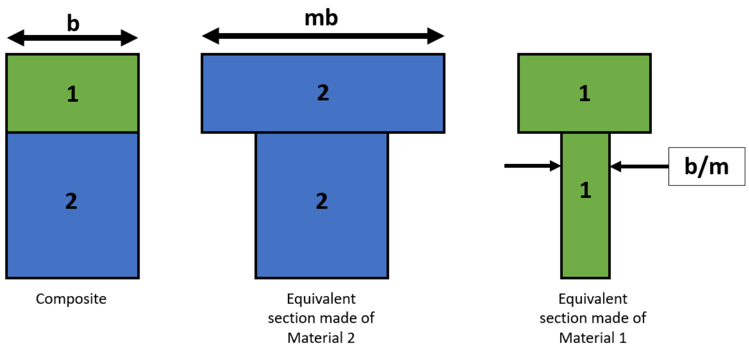
- From the above diagram, $\boxed{\sigma = \frac{\sigma_{max}}{y_{max}}y}$
- We know, $Stress = \frac{Force}{Area}$. So, Force = Stress * Area
- So, $dF = \sigma dA = \frac{\sigma_{max}}{y_{max}}ydA \implies F = \frac{\sigma_{max}}{y_{max}}\int ydA = \boxed{\frac{\sigma_{max}}{y_{max}}A\bar{y}}$

5.8 Bending stress distribution in Composite Beam

- If the materials are joined firmly with each other, there will be a common NA
- If they are not joined together, then both beam will bend independently about their own NA. Eg. Leaf springs. The Total M_R is this case will be the sum of the two.
- In case of composite beams, the position of NA will not be the centroid of the section as they are made of different materials and hence don't satisfy the condition of pure bending
- The composite section can be converted into an equivalent section made of either of the two materials. For that equivalent section, the centroid will coincide with the NA
- In composite beams, the bending strain diagram is linear, hence strain in the two materials at a given distance from NA will be the same $\Longleftarrow$ **Strain compatibility condition**

5.8.1 Equivalent section

- **Modular ratio(m):** $= \dfrac{E_1}{E_2}$



5.9 Flitched Beam

- Wooden section strengthened by metal plate either provided at top-and-bottom or at left-and-right symmetrically
- Flitched beams are provided when a homogeneous beam of wood will require large cross-sectional area for same moemnt of resistance.
- Generally top-and-bottom flitched beam is 3-5 times stronger than side flitched beam

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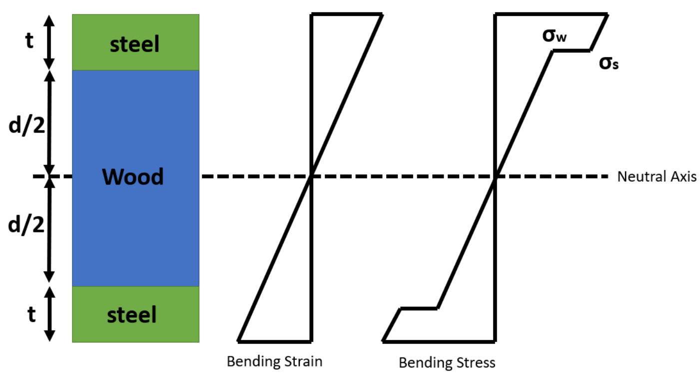
5.9.1 Finding M_R of Top and Bottom Flitched beam using Strain compatibility method

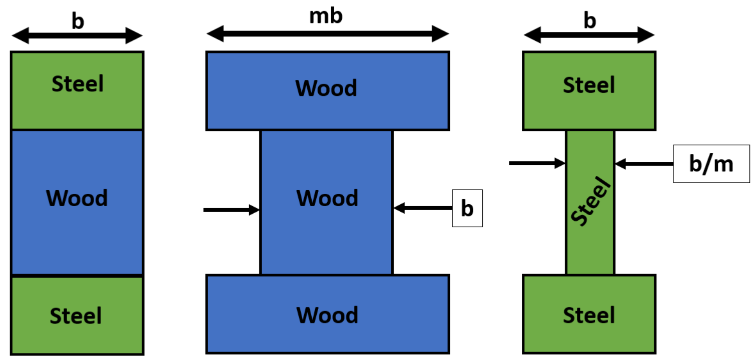


- For composite beams, by strain compatibility condition: $\boxed{\text{Strain in steel} = \text{Strain in Wood}}$ $\implies \epsilon_s{=}\epsilon_w$
- $\implies \dfrac{\sigma_s}{E_s} = \dfrac{\sigma_w}{E_w} \implies \dfrac{\sigma_s}{\sigma_w} = \dfrac{E_s}{E_w} = m$

$\implies$ At the same distance from NA, if σ_s is the bending stress in steel, then σ_w at that level will be σ_s/m

- Let permissible stress in wood be σ_w. Then Total $M_R = M_{R_W} + M_{R_S}$
- $M_{R_W} = \sigma_W Z = \sigma_W \left(\dfrac{I}{y_{max}}\right) \implies \sigma_w \left(\dfrac{bd^3/12}{d/2}\right) = \sigma_W \dfrac{bd^2}{6}$
- $M_{R_S} = M_R$ of Steel of size(b*(d+2t)) - M_R of Steel of size(b*d)
- $M_{R_S} = \sigma_{S_{max}} \left(\dfrac{b(d+2t)^3/12}{d/2+t}\right) - \sigma_S \left(\dfrac{bd^2}{6}\right)$

5.9.2 Finding M_R of Top and Bottom Flitched beam using Equivalent section method

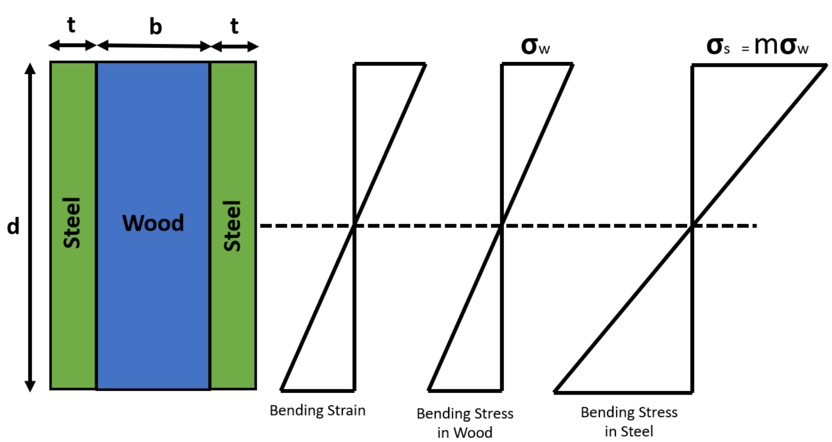


M_R for Equivalent Wood section

$$M_R = \sigma_W \left(\frac{I}{y_{max}}\right)$$
$$= \sigma_W ....$$

- M_R (In terms of wood) $= \boxed{\sigma_W \left[\frac{mb(d+2t)^2}{6} - \frac{(mb-b)d^2}{6}\right]}$
- M_R (In terms of steel) $= \boxed{\sigma_s \left[\frac{b(d+2t)^2}{6} - \frac{b - \frac{b}{m}d^2}{6}\right]}$

5.9.3 Side Flitched Beam



- $\boxed{M_{R_W} = \sigma_W \left(\frac{bd^2}{6}\right)}$ $\boxed{M_{R_S} = \sigma_S \left(\frac{2td^2}{6}\right)}$

$\Longrightarrow$ $\boxed{M_R = (M_{R_W} + M_{R_S}) = \sigma_W \frac{d^2}{6}[b + m2t] = \sigma_S \frac{d^2}{6}[\frac{b}{m} + 2t]}$

5.10 Beam of Uniform Strength

- For an economical design, the section of the beam may be reduced towards the support, as bending moment decreases towards the support.

5.10.1 Beam of constant width

- Consider a Simply supported beam AB with cross section (b*d). Section Modulus (Z) of the beam is : $\frac{bd^2}{6}$
- But for Uniform strength, lets consider that the beam has **constant width and varying depth** along its length. $\implies Z = \frac{bd_x^2}{6} \impliedby$ (d_x = depth of the beam at a distance x from support A). $M_{R_x} = \sigma \frac{bd_x^2}{6}$
- Just like how we calculate the moment at a distance x (M_x) in BMD, M_x will be $\frac{Wx}{2}$ for a point load W acting at the center of the beam
- $(M_{R_x} = M_x) \implies \sigma \frac{bd_x^2}{6} = \frac{Wx}{2} \implies \boxed{d_x = \sqrt{\frac{3W}{b\sigma}}\sqrt{x}} \impliedby$ (Parabolic)

5.10.2 Beam of constant depth

- Similarly let's consider instead of depth, the width is varying. So, $(M_{R_x} = M_x) \implies \sigma \frac{b_x d^2}{6} = \frac{Wx}{2}$

$\implies \boxed{b_x = \frac{3Wx}{\sigma d^2}} \impliedby$ (Linear)

5.11 Biaxial Bending

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Chapter 6

Shear stresses in Beams

6.1 Shear stress distribution in Beams

- **Assumptions:**
  - Material is Homogeneous, elastic
  - Obeys Hooke's Law
  - Shear stress(τ) is assumed constant along width and variation along depth
- $\boxed{\tau = \dfrac{SA\bar{y}}{Ib}}$
- S = Shear force at any section
- A$\bar{y}$ = Area MOI of section above y about NA
- $\bar{y}$ = Distance from centroid of area above y to NA
- I = Area MOI of the whole section about NA
- b = Width of the section at a distance y from NA

6.2 Shear Stress distribution

STEPS TO FIND THE SHEAR STRESS DISTRIBUTION FOR ANY CROSS SECTION

- Find the terms in the formula $\boxed{\dfrac{SA\bar{y}}{Ib}}$ for the given cross sectionf
- Equate $\dfrac{d\tau}{dy} = 0$ to find the value of y for which τ is max.
- Then see if τ_{max} can be equated to τ_{avg} which is $\dfrac{S}{Area\ of\ whole\ section}$
- Equate τ with τ_{avg} to find the value of y where τ_{avg} occurs

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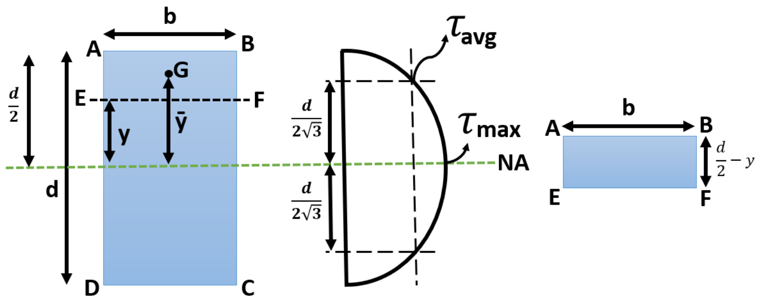
6.2.1 Shear stress distribution in Rectangular section



- We know, $\boxed{\tau = \frac{SA\bar{y}}{Ib}}$
- Here, Area of ABEF (A) $= b\left(\frac{d}{2} - y\right)$
- $\bar{y} = y + \frac{(\frac{d}{2} - y)}{2} = \frac{2y + \frac{d}{2} - y}{2} = \frac{4y + 2 - 2y}{4} = \frac{2y + d}{4} = \frac{2(y + \frac{d}{2})}{2 * 2} \implies \boxed{\bar{y} = \frac{\left(y + \frac{d}{2}\right)}{2}}$
- Area MOI of whole section about NA (I) $= \frac{bd^3}{12}$

$$\implies \tau = \frac{S * \left(b\left(\frac{d}{2} - y\right)\right)\left(\frac{d}{2} + y\right)}{2\left(\frac{bd^3}{12}\right)b} \implies \boxed{\tau = \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)} \impliedby \text{(Parabolic variation)}$$

- From the above result, we can infer that $\tau = 0$ when y $= \pm\frac{d}{2}$ and for τ_{max}, $\frac{d\tau}{dy} = 0$
- $\frac{d\tau}{dy} = 0 \implies \frac{d}{dy}\left[\frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)\right] = 0 \implies \frac{-12Sy}{bd^3} = 0 \implies \boxed{\tau_{max}\ at\ (y = 0)}$
- $\implies \boxed{\tau_{max} = \frac{3S}{2bd}} \impliedby$ (Here, $\frac{S}{bd} = \frac{S}{A} = \tau_{avg}$) $\implies \boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$
- Now, to find the distance at which τ_{avg} occurs, let $\tau = \tau_{avg}$

$\implies \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right) = \frac{S}{bd}$ Solving this, will give $\boxed{y = \pm\frac{d}{2\sqrt{3}}}$

6.2.2 Shear Stress distribution in Triangular section

- $\boxed{\tau_{max} = \frac{3S}{bh}}$ $\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$
- Shear stress at centroid is $\frac{4}{3}$ times the Avg Shear stress
- τ_{max} occurs at h/2. but NA is at h/3. So unlike the rectangular section for which the max shear occurs at NA, triangular section is different.
- τ_{avg} occurs at ???

6.2.3 Shear Stress distribution in Circular section

- $\boxed{\tau_{max} = \dfrac{4S}{3\pi R^2}}$ $\boxed{\tau_{max} = \dfrac{4}{3}\tau_{avg}}$
- Shear stress at centroid is $\dfrac{4}{3}$ times the Avg Shear stress
- τ_{max} occurs at NA
- τ_{avg} occurs at $\pm\dfrac{R}{2}$

6.2.4 Shear Stress distribution in I-section

Shear stress distribution in Flange

-

Shear stress distribution in Web

-

6.3 Shear stresses in composite sections

- Convert the given section into its equivalent section of any one material using the **modular ratio (m)** and then apply the formula $\dfrac{SA\bar{y}}{Ib}$

6.4 Shear Center

- For transverse load acting on the plane of symmetry of a a symmetrical cross section beam, the beam will bend without twisting.
- But that's not the case with asymmetrical beam. For symmetrical beam to bend without twisting, the load must act at a point called **Shear center**

6.5 Shear stress distribution in Thin walled open cross sections

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6.5.1 Shear stresses in wide flange beams

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6.5.2 Shear stresses in Channel section

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6.5.3 Shear stresses in Angle section

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6.5.4 Shear stresses in Z-Section

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Chapter 7

Principal Stress-Strain & Failure Theories

7.1 Principal stress and Principal planes

- **Principal Plane:** A plane in which only Normal stresses are acting and Shear stresses are zero
- **Principal stress:** Normal stresses acting on mutually perpendicular planes on which shear stresses are zero
  - **Major Principal stress(σ_1):** Max value of such normal stress
  - **Major Principal stress(σ_2):** Min value of such normal stress

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7.2 Methods to find Principal planes and Principal stresses

1. Analytical method
2. Graphical method (Mohr's circle)

7.3 Four cases of Stress combinations

1. Member subjected to Uni-axial stress
2. Member subjected to Bi-axial stress in mutually perpendicular directions
3. Member subjected to pure shear stresses
4. Member subjected to Bi-axial stresses in mutually perpendicular directions as well as shear stresses

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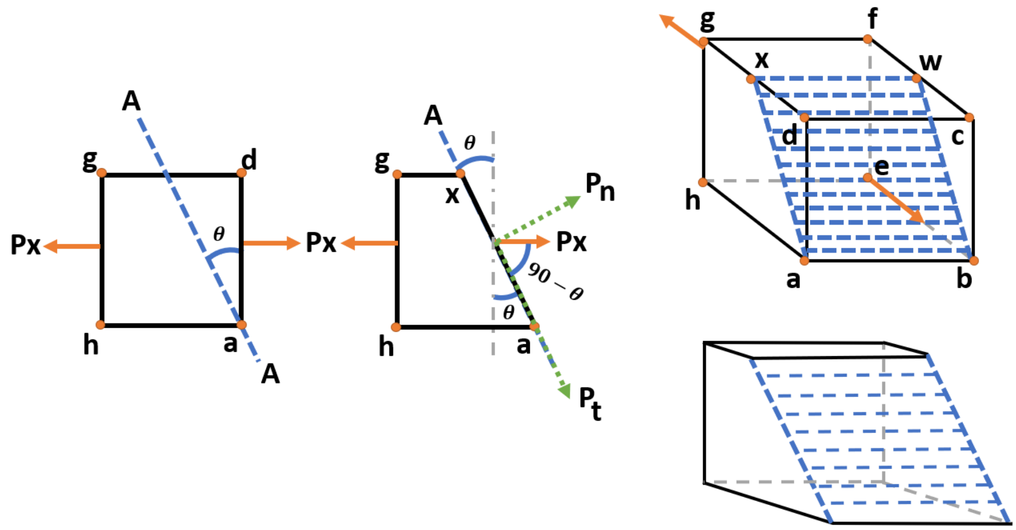
7.4 Analytical method to solve for Principal stresses & Principal planes

7.4.1 Stress analysis in Member subjected to Uni-axial stress



- Normal stress due to the force P_x acting on $A_{abcd} = \sigma_x = \dfrac{Px}{A_{abcd}}$
- But what is the normal stress acting on an oblique plane like (abwx) ?
- Let the normal stress acting on the oblique plane (abwx) be $\sigma_{ob} \implies \sigma_{ob} = \dfrac{P_n}{A_{abwx}}$
- The force P_x is resolved into two components. P_n is the normal force acting on the oblique plane (abwx) and P_t is the tangential force acting along the oblique plane (abwx)
- By resolution of P_x, $\boxed{P_n = P_x \sin(90 - \theta) = P_x \cos\theta}$ $\boxed{P_t = P_x \cos(90 - \theta) = P_x \sin\theta}$
- $\cos\theta = \dfrac{A_{abcd}}{A_{abwx}} \implies \boxed{A_{abwx} = \dfrac{A_{abcd}}{Cos\theta}}$
- So, The normal stress (σ_{ob}) acting on the plane (abwx) is given by:

$$\boxed{\sigma_{ob} = \frac{P_n}{A_{abwx}} = \frac{P_x \cos\theta}{A_{abcd}/\cos\theta} = \frac{P_x \cos^2\theta}{A_{abcd}} \implies \boxed{\sigma_{ob} = \sigma_x \cos^2\theta}}$$

- σ_{ob} is max when θ is 0°, 180°... $\Longleftarrow$ (Because at these angles Cos is max)

$\implies$ $\boxed{\textbf{Principal stresses: } \pm\sigma_x \implies \boxed{+\sigma_x = \sigma_1} \;\&\; \boxed{-\sigma_x = \sigma_2}}$

- Similarly, Tangential stress (τ_{ob}) acting along the plane (abwx) is given by:

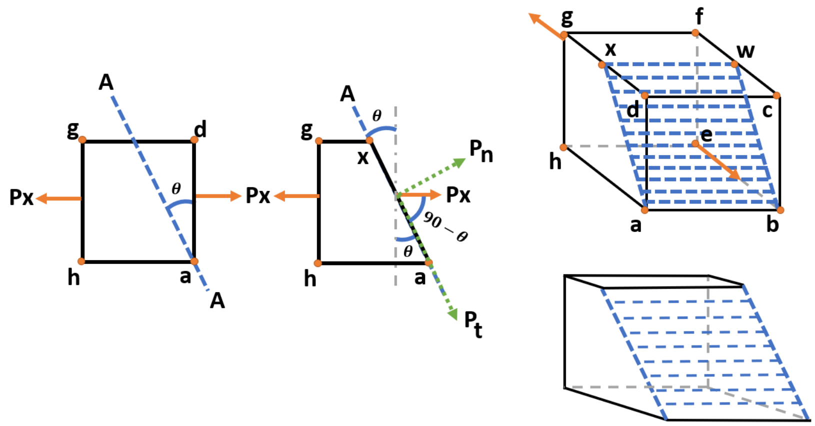
$$\boxed{\tau_{ob} = \frac{P_t}{A_{abwx}} = \frac{P_x \sin\theta}{A_{abcd}/\cos\theta} = \frac{P_x \sin\theta\cos\theta}{A_{abcd}} \implies \boxed{\tau_{ob} = \frac{\sigma_x}{2}\sin 2\theta}}$$

- τ_{ob} is max when θ is 45°, 135°... $\Longleftarrow$ (Because at these angles $\sin 2\theta$ is max) $\implies \boxed{\tau_{max} = \pm\dfrac{\sigma_x}{2}}$
- Value of Normal stresses in the plane of Maximum shear stress:

$$\sigma_{ob} = \sigma_x \cos^2(45°\text{or } 135°) \implies \sigma_x \left(\frac{1}{\sqrt{2}}\right)^2 \text{ or } \sigma_x \left(\frac{-1}{\sqrt{2}}\right)^2 \implies \boxed{\sigma_{ob} = \frac{\sigma_x}{2}}$$

$\implies$ **Magnitude of normal and shear stress is same in the plane of Max shear stress under Uni-axial loading**

7.4.2 Strain analysis of element subjected to Uni-axial stress

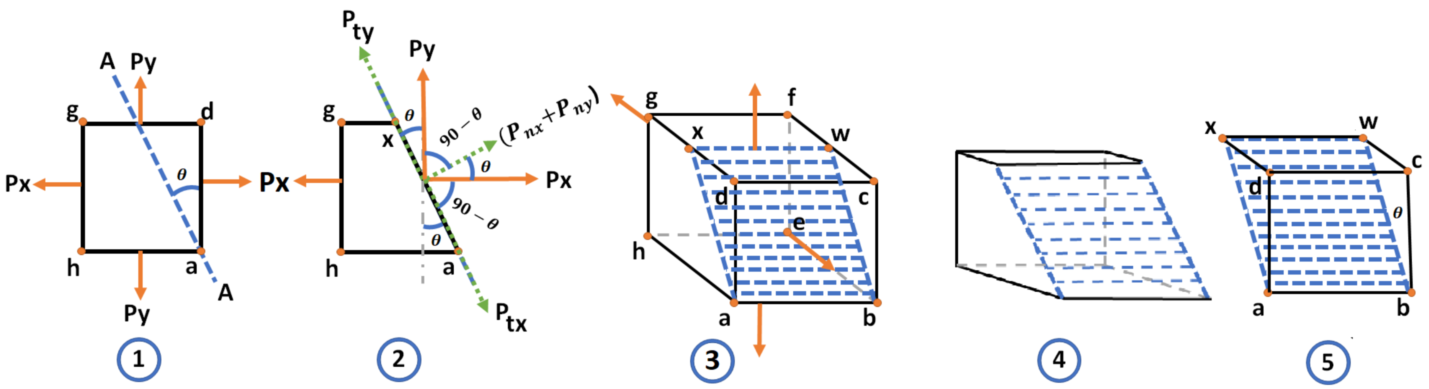


- We know $\boxed{\sigma_{ob} = \sigma_x \cos^2\theta}$ and $\boxed{\tau_{ob} = \frac{\sigma_x}{2}\sin 2\theta}$
- For principal stress: $\cos^2\theta$ should be max. So θ will can 0°, 180°. If θ is either of these two values, then τ_{ob} will be zero. $\implies$ **Shear stresses will be zero in principal planes for Uni-axial loading**

$\implies$ $\boxed{\text{Principal stresses: } \pm\sigma_x \implies \boxed{+\sigma_x = \sigma_1} \ \& \ \boxed{-\sigma_x = \sigma_2}}$

- **strain due to to the principal stress**$(\sigma_x, -\sigma_x)$ **will be the Principal strain**$(\in_1, \in_2)$
- $\boxed{\textbf{Principal strains} \implies \boxed{\in_1 = \frac{\sigma_1}{E}} \text{ and } \boxed{\in_2 = \frac{-\mu\sigma_1}{E}}}$ $\Longleftarrow$ (where μ is Poisson's ratio of the material)
- Let $\in_{ob}$ be the normal strain for any plane and ϕ_{ob} be the shear strain for any plane.
- $\boxed{\in_{ob} = \frac{\sigma_{ob}}{E} = \frac{\sigma_x \cos^2\theta}{E} = \in_x \cos^2\theta}$ and $\boxed{\frac{\phi_{ob}}{2} = \frac{\in_x}{2}\sin 2\theta}$
- The strain formulae can be easily recovered from stress formulae. Just change, $\sigma \rightarrow \in$ and $\tau \rightarrow \frac{\phi}{2}$

7.4.3 Stress analysis of member subjected to Bi-axial stress in mutually perpendicular direction



- From Diagram(5) above, $\boxed{\sin\theta = \dfrac{A_{dcwx}}{A_{abwx}}}_{(1)}$ $\boxed{\cos\theta = \dfrac{A_{abcd}}{A_{abwx}}}_{(2)}$

- $\boxed{\sigma_x = \dfrac{P_x}{A_{abcd}}}_{(3)}$ $\boxed{\sigma_y = \dfrac{P_y}{A_{cdwx}}}_{(4)}$ $\boxed{\tau = \dfrac{P_y}{A_{abcd}}}_{(5)}$ $\implies$ $\boxed{P_x = \sigma_x A_{abcd}}_{(6)}$ $\boxed{P_y = \sigma_y A_{dcwx}}_{(7)}$

- From Diagram(2):
  - Resolving P_x into P_{nx} and P_{tx} $\implies$ $\boxed{P_{nx} = P_x\sin(90-\theta) = P_x\cos\theta}$ $\boxed{P_{tx} = P_x\cos(90-\theta) = P_x\sin\theta}$
  - Resolving P_y into P_{ny} and P_{ty} $\implies$ $\boxed{P_{ny} = P_y\cos(90-\theta) = P_y\sin\theta}$ $\boxed{P_{ty} = P_y\sin(90-\theta) = P_y\cos\theta}$

- $\sigma_{ob} = \dfrac{P_x\cos\theta + P_y\sin\theta}{A_{abwx}} = \dfrac{\sigma_x A_{abcd}\cos\theta + \sigma_y A_{dcwx}\sin\theta}{A_{abwx}} = \dfrac{\sigma_x A_{abcd}\cos\theta}{A_{abwx}} + \dfrac{\sigma_y A_{dcwx}\sin\theta}{A_{abwx}}$ (Using (6) & (7))

$\implies$ (Using (1) & (2)) $\sigma_x\cos^2\theta + \sigma_y\sin^2\theta = \sigma_x\left[\dfrac{1+\cos 2\theta}{2}\right] + \sigma_y\left[\dfrac{1-\cos 2\theta}{2}\right] = \dfrac{\sigma_x}{2} + \dfrac{\sigma_x\cos 2\theta}{2} + \dfrac{\sigma_y}{2} - \dfrac{\sigma_y\cos 2\theta}{2}$

$$\implies \boxed{\boldsymbol{\sigma_{ob} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta}} \ldots Eqn(1)$$

- Eqn(1) above is maximum for when $\theta = 0^\circ, 90^\circ$. So, $\boxed{\textbf{Principal stress: } \boldsymbol{\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \left(\frac{\sigma_x-\sigma_y}{2}\right)}}$

- $\tau_{ob} = \dfrac{P_y\cos\theta - P_x\sin\theta}{A_{abwx}} = \dfrac{\sigma_y A_{dcwx}\cos\theta}{A_{abwx}} - \dfrac{\sigma_x A_{abcd}\sin\theta}{A_{abwx}} = \sigma_y\sin\theta\cos\theta - \sigma_x\sin\theta\cos\theta$ (Using (1) & (2))

$$\implies \boxed{\tau_{ob} = (\sigma_x-\sigma_y)\sin\theta\cos\theta = \boldsymbol{\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta}} \ldots Eqn(2)$$

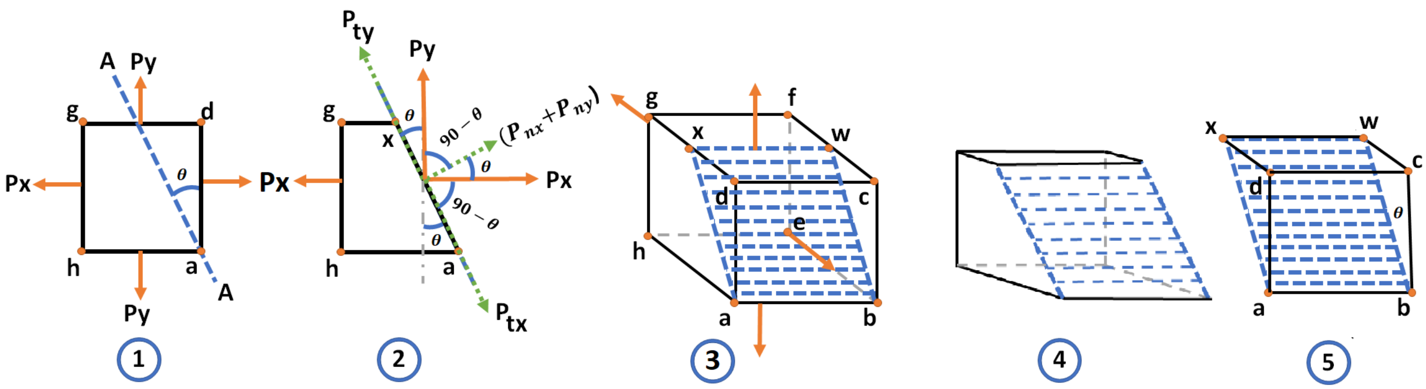
- Eqn(2) above is maximum for when θ=45°or 135°So, Maximum shear stress: $\boxed{\boldsymbol{\tau_{max} = \left(\frac{\sigma_x-\sigma_y}{2}\right)}}$

- Value of normal stress in the plane of τ_{max} is $\boxed{\sigma_{ob} = \left(\dfrac{\sigma_x+\sigma_y}{2}\right)}$ $\implies$ **Normal stresses on the plane of $\boldsymbol{\tau_{max}}$ are equal and alike**. Meaning they are the same for both θ=45°,135°. As $\cos 2\theta$ is zero for both these angles.

- Resultant stress on the plane of τ_{max} is $\sigma_r = \sqrt{\sigma_{ob}^2 + \tau_{max}^2} = \sqrt{\left(\dfrac{\sigma_x+\sigma_y}{2}\right)^2 + \left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2} = \sqrt{\dfrac{\sigma_x^2+\sigma_y^2}{2}}$

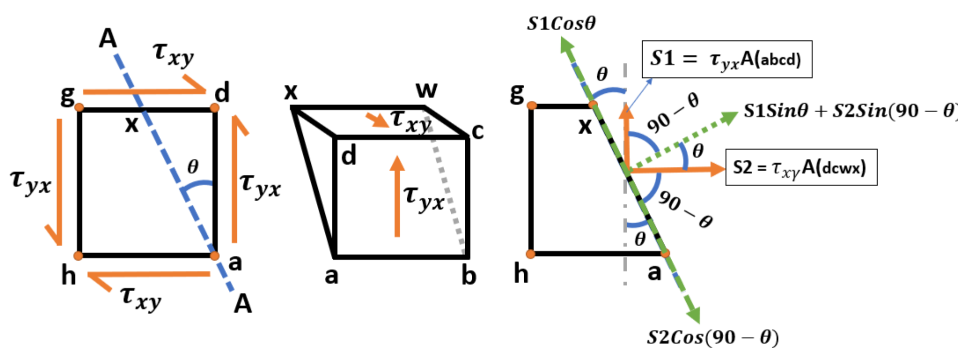
- **Angle of obliquity(β)** : The angle between σ_{ob} and σ_r $\implies$ $\boxed{\tan\beta = \frac{\tau_{max}}{\sigma_{ob}} = \frac{\sigma_x - \sigma_y}{\sigma_x + \sigma_y}}$

7.4.4 Strain analysis of member subjected to Bi-axial stress in mutually perpendicular direction



- We know Principal Stress: $\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left(\frac{\sigma_x - \sigma_y}{2}\right)$.
- So, $\boxed{\textbf{Principal Strain } (\in_{1,2}) = \left(\frac{\in_x + \in_y}{2}\right) \pm \left(\frac{\in_x - \in_y}{2}\right)}$
- Similarly, Maximum Shear stress: $\tau_{max} = \left(\frac{\sigma_x - \sigma_y}{2}\right)$ So, $\boxed{\text{Maximum Shear strain: } \phi_{ob} = (\in_x - \in_y)}$ $(\tau = \phi/2)$

7.4.5 Stress analysis of member subjected to Pure shear stresses



- $\boxed{\sigma_{ob} = \tau_{xy}\sin 2\theta}$ $\Longleftarrow$ Maximum for when θ=45°, 135° $\Longrightarrow$ $\boxed{\textbf{Principal Stress: } \sigma_{1,2} = \pm\tau_{xy}}$
- $\boxed{\tau_{ob} = \tau_{xy}\cos 2\theta}$ $\Longleftarrow$ Maximum for when θ=0°, 90° $\Longrightarrow$ $\boxed{\tau_{max} = \pm\tau_{xy}}$
- Normal stress in the plane of Max shear stress will be Equal and Unlike $\sigma_{ob} = \pm\tau_{xy}$

7.4.6 Strain analysis of member subjected to pure shear

- $\boxed{\in_{ob}= \dfrac{\tau_{xy}\sin 2\theta}{E}}$ $\boxed{\text{Principal strain: } \in_{1,2}= \dfrac{\pm\tau_{xy}}{E}}$
- $\boxed{\dfrac{\phi_{ob}}{2} = \dfrac{\tau_{xy}\cos 2\theta}{G}}$ $\boxed{\text{Maximum Shear strain: } \dfrac{\phi_{max}}{2} = \dfrac{\pm\tau_{xy}}{G}}$ Where G = Shear modulus

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7.4.7 Stress analysis of member subjected to Bi-axial stress in mutually perpendicular direction as well as shear stresses

- $\boxed{\sigma_{ob} = \left(\dfrac{\sigma_x+\sigma_y}{2}\right) \pm \left(\dfrac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta}$ $\boxed{\tau_{ob} = -\left(\dfrac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta}$
- To find principal plane, Equate $\tau_{ob} = 0 \implies \boxed{\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x-\sigma_y}}$
- $\boxed{\textbf{Principal stresses: } \boldsymbol{\sigma_1, \sigma_2 = \left(\dfrac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}}$

7.4.8 Strain analysis of member subjected to Bi-axial stress in mutually perpendicular direction as well as shear stresses

- $\boxed{\textbf{Principal Strain: } \in_{1,2}= \left(\dfrac{\in_x+\in_y}{2}\right) \pm \sqrt{\left(\dfrac{\in_x-\in_y}{2}\right)^2 + \tau_{xy}^2}}$
- $\boxed{\textbf{Maximum Shear strain: } \dfrac{\phi_{max}}{2} = \left(\dfrac{\in_x-\in_y}{2}\right)}$

7.5 Graphical Method - Mohr's circle

7.5.1 Construction of Mohr's circle

- Horizontal axis represents Normal stress and Vertical axis represents shear stress

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7.5.2 Properties of Mohr's circle

- Center of Mohr's circle always lies on x-axis $\implies$ Mohr's circle is always symmetrical about σ-axis
- Radius of Mohr circle = Max Shear stress (τ_{max})
- The center coordinate of Mohr circle represent the Normal stress on the plane of τ_{max}
- The circle cuts the σ-axis at two points. Those points represent the Principal stresses
- If two point on the circumference of Mohr's circle subtend an angle 2θ wrt to center, then the angle between those planes will be θ
- Mohr's circle will be symmetrical about both the axis if stress element is subjected to equal and unlike normal stresses
- **Special case: Pure shear**

$\implies$ Center of Mohr's circle will be (0,0) $\implies$ Mohr's circle will be symmetrical about both the axis
$\implies$ Radius of Mohr's circle = $\tau \implies \tau_{max} = \tau$

$\implies$ Sum of principal stresses will be zero

- **Special case: Hydrostatic stress**
  - Hydrostatic stresses: Equal and alike stresses in mutually perpendicular directions without any shear
  - Mohr's circle will reduce to a point

7.6 Properties of Strain Mohr's circle

- Horizontal axis : $\in$-axis　　Vertical axis : $\dfrac{\phi}{2}$-axis
- Diameter of Mohr's circle represents Max shear strain
- The center coordinate of Mohr's circle $\left(\dfrac{\in_x + \in_y}{2}\right)$ represent normal strain in the direction of ϕ_{max}

7.7 Strain Rosettes

- It is an arrangement of 3 linear strain gauges using which linear strains in 3 directions are measured

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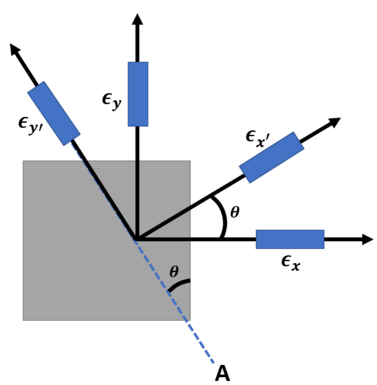
7.7.1 Rectangular rosette

- Of the 3 strain gauges, if two among them are mutually perpendicular to each other, then it is called as **rectangular rosette**



7.7.2 Delta Δ rosette

- If all the 3 gauges are at equal angular distance from one another, then its called **star rosette** or **delta rosette**

7.7.3 Star rosette

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7.8 Theories of Elastic failure

- Generally there are two modes of failure:
  1. Yielding or Ductile failure
  2. Fracture or Brittle failure

7.8.1 Maximum Principal Stress theory - Rankine's Theory

- Suitable for Brittle materials
- Material will fail if Max.Principal stress(σ_1) = σ_y in Tension
- Material will fail if Min.Principal stress(σ_3) = σ_y in Compression
- For no failure, $\sigma_1 \leq \sigma_y$ and For design, $\sigma_1 \leq \dfrac{\sigma_y}{FOS}$

7.8.2 Maximum Principal strain theory - St.Venant's Theory

- **Applicability and Limitations:**
- applicable for both ductile and brittle materials, but results are not accurate in both the cases
- In case of pure shear, results are still unsafe but better than Rankine's theory
- Material will fail if Max.Principal strain($\in_1$)=$\in_y$ under Uni-axial loading where $\boxed{\in_1 = \dfrac{\sigma_1}{E} - \dfrac{\mu\sigma_2}{E} - \dfrac{\mu\sigma_3}{E}}$ and $\in_y = \dfrac{\sigma_y}{E}$
- For no failure, $\in_1 \leq \in_y$ and for design $\in_1 \leq \left(\dfrac{\sigma_P}{E} = \dfrac{\sigma_y}{FOS * E}\right)$

7.8.3 Maximum Shear stress theory - Guest & Tresca's Theory

- **Applicability and Limitations:**
  - suitable for ductile materials and for pure shear case, it is over safe
  - Not suitable for hydrostatic loading $\Longleftarrow$ It says shear stress will be zero and so material will never fail, which not possible in reality.
  - Not applicable to brittle materials as they have different yield stress in tension and compression
- Material will fail when Max. Shear stress(τ_{max}) = $\dfrac{\sigma_y}{2}$ under uniaxial loading
- For no failure, $\tau_{max} \leq \dfrac{\sigma_y}{2}$ and for design $\tau_{max} \leq \dfrac{\sigma_y/FOS}{2}$
- Max shear stress in Bi-axial loading will be $\dfrac{\sigma_1 - \sigma_2}{2}$
- Max shear stress in Tri-axial loading will be $\max\left(\dfrac{\sigma_1 - \sigma_2}{2}, \dfrac{\sigma_2 - \sigma_3}{2}, \dfrac{\sigma_3 - \sigma_1}{2}\right)$

7.8.4 Maximum Strain Energy theory - Haigh and Beltrami's Theory

- **Applicability and Limitations:**
  - suitable for ductile materials. Not suitable for brittle materials as elastic limit stress in tension and compression are quite different
  - In case of pure shear, results are still unsafe for ductile materials
- Material will fail when Max. Strain energy(u) = Strain energy developed at yield stress under Uni-axial loading
  $\implies \boxed{u = \frac{\sigma_y^2}{2E}}$
- for no failure, $u \leq \frac{\sigma_y^2}{2E}$ and for design, $u \leq \frac{\left(\sigma_y/FOS\right)^2}{2E}$
- $\boxed{u = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2)} \Longleftarrow$ (2D) $\boxed{u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]} \Longleftarrow$ (3D)

7.8.5 Distortion energy theory - Mises-Henky Theory

- **Applicability and Limitations:**
  - suitable for ductile materials. For the case of pure shear, theoretical results is same as actual results
  - Not applicable for brittle and not applicable under hydrostatic loading
- Material will fail when $u_s = u_{ys} \Longleftarrow$ Max.Shear strain energy = shear strain energy stored at yield stress in Uni-axial loading
- Total strain energy(u) = Volumetric strain energy(u_v) + Shear strain energy(u_s)
- $\boxed{u_v = \frac{1}{2} * \sigma_{avg} * \in_v}$
- $\boxed{\sigma_{avg} = \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right)}$ $\boxed{u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]} \Longleftarrow$ (3D)
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7.8.6 Octahedral Shear stress theory

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