

# Thermodynamics GATE

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## 1. Basic Concepts & Zeroth Law of Thermodynamics

### Macroscopic Approach

- Average molecular behaviour considered
- **Continuum** concept should be valid
  - Matter is continuously distributed
  - In case of gases, the **Mean Free Path** (average distance travelled by molecule between two successive collisions) should be less than system dimensions.

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Quasistatic process - Infinitely slow process.

- All reversible processes are quasistatic, but not all quasistatic processes are reversible.
- Frictionless quasistatic process is reversible
- Properties of a system are Exact differentials

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### Intensive properties

- Independent of mass
- All specific properties like, specific volume, specific heat are Intensive properties.
- Density is also an intensive property.
- Intensive properties when divided will still be the same.

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### Extensive properties

- Dependent on mass
- When divided will be half as before.

→ Converting Degree Celcius to Fahrenheit  $\implies t_f = \frac{9}{5}t_c + 32 \implies \text{Note: } \Delta t_F = 1.8\Delta t_C$

→ Mixture of Refrigerants which behave as pure substance is called **Azeotrope**

→ Mixture of Air and Liquid air is not a pure substance.

→ **Extensive properties are additive**, meaning extensive property's whole is the sum of parts of the extensive property.

## 2. Energy and Energy Interactions

→ The molecular weight of air is approximately taken as 29. Air is approx 77% of  $N_2$  and 23%  $O_2$  by weight. So,

$$0.77 * 28 + 0.23 * 32 = 28.92 \approx 29$$

### Ideal Gas Equation

- $PV = mRT$  Where,
  - P = Pressure
  - V = Volume
  - m = Mass
  - R = Characteristic Gas Constant
  - T = Temperature
- Also  $PV = n\bar{R}T$  Where,
  - n = Number of Kilo moles
  - $\bar{R}$  = Universal Gas Constant =  $8.314 \text{ KJ/KmolK}$
- Now, R for Air is calculated as:  $R_{Air} = \frac{\bar{R}}{m_{Air}} = \frac{8.314}{29} = 0.287 \text{ KJ/KmolK}$

→  $\boxed{\text{mass}(m) = \text{number of moles}(n) * \text{Molecular mass}(M)}$

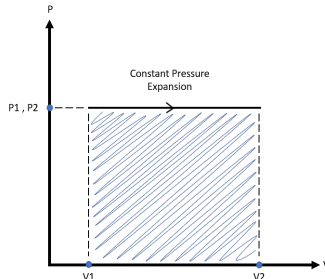
**Work** - (+ve = Done by system, -ve = Done on system)

- The work done by a **reversible process or closed system process** is the area under the curve in a P-V diagram.
- The work done in a **reversible cycle or closed system cycle** is the area enclosed by the curve in a P-V diagram.

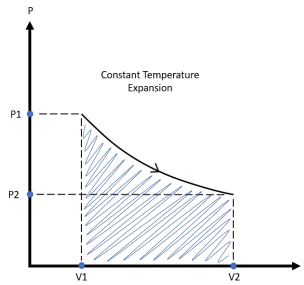
### Work in various Quasi-static processes

1. Constant Volume - Isochoric - Isometric process  $\Rightarrow \boxed{W_{1-2} = 0}$

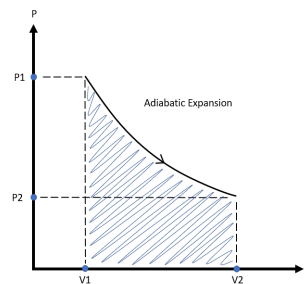
2. Constant Pressure - Isobaric - Isopiestic process  $\Rightarrow \boxed{W_{1-2} = \int_1^2 P dV = P(V_2 - V_1)}$



3. Constant Temperature - Isothermal - Hyperbolic process  $\Rightarrow \boxed{W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{P_1}{P_2}}$



4. Adiabatic process - 
$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \quad \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma}{\gamma - 1}} = \left( \frac{V_1}{V_2} \right)^{\gamma - 1}$$



5. Polytropic process - 
$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{n - 1} \quad \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n}{n - 1}} = \left( \frac{V_1}{V_2} \right)^{n - 1}$$

**Heat** - (+ve = Transferred to system, -ve = Rejected from system)

- The Heat transfer in a **reversible process** is the area under the curve in a T-S diagram
- The Heat transfer in a **reversible cycle** is the area enclosed by the curve in a T-S diagram.
- The specific heats ( $C_p$  and  $C_v$ ) of monoatomic gas is constant with temperature. They vary for Di and tri atomic gases

• We know  $\frac{C_P}{C_V} = \gamma$ . But  $\gamma$  can also be calculate in another way: 
$$\gamma = 1 + \frac{2}{n}$$

- $n$  = Degree of freedom, T = Translational, R = Rotational
- $n = 3$  for mono atomic gases [3T]
- $n = 5$  for di atomic gases [3T+2R]
- $n = 6$  for tri atomic gases [3T+3R]

- Heat capacity of the substance = Mass \* Specific Heat

- LATENT HEAT

- It is the Amount of Heat required to cause phase change in unit mass of any substance. It is of three types:
  - **Latent heat of fusion** ( $L_f$ ) → Not much affected by pressure
  - **Latent heat of vapourization** ( $L_v$ ) → Highly sensitive to pressure.
  - **Latent heat of sublimation**
  - **Very important thing to note is that, in these phase change scenarios, even when heat transfer is taking place, the temperature of the system remains constant**

– consider the scenario where a solid at temperature  $T_1$  is to be converted into vapour at temperature  $T_2$ . The freezing temperature is  $T_f$  and the boiling temperature is  $T_V$ . The total heat required for this conversion to occur is calculated as follows: Here  $(T_1 < T_f)$ ,  $(T_2 > T_V)$

\* Heat added to raise temp from  $T_1$  to  $T_f$ :  $Q_1 = mC_1(T_f - T_1)$

\* Heat added for phase change from Solid to Liquid:  $Q_2 = mL_f$

\* Heat added to raise temp from  $T_f$  to  $T_V$ :  $Q_3 = mC_2(T_V - T_f)$

\* Heat added for phase change from Liquid to Vapour:  $Q_4 = mL_V$

\* Heat added to raise temp from  $T_V$  to  $T_2$ :  $Q_5 = mC_3(T_2 - T_V)$

\* Here,  $C_1, C_2, C_3$  are specific heats of the substance in Solid, Liquid and Gaseous phases respectively.

\* Total heat required for the aforementioned conversion is the sum of all the above heats from  $Q_1$  to  $Q_5$

- Increasing of order of slopes for the following processes in P-V diagram:

$$\text{Isothermal} < \text{Polytropic} < \text{Adiabatic}$$

### 3. First Law of Thermodynamics

**First Law :-**

- $dE = dU + dK.E. + dP.E.$
- $\delta W = \delta W_{PdV} + \delta W_{shaft} + \delta W_{elec}$
- $\delta Q = dE + \delta W$  for process
- $\Rightarrow \delta Q = dU + \delta W$  for stationary process
- $\delta Q = \delta W_{impliedby}$  for cycle
- $U(\text{Internal energy}), H(\text{Enthalpy}) = f(T)$  only for an ideal gas.

**Heat Transfer in Various processes :-**

CONSTANT VOLUME

- from first law,  $\delta Q = dU + \delta W \Rightarrow dU + PdV \Rightarrow dU + 0$
- $\Rightarrow \delta Q = dU = mC_V \Delta T$

CONSTANT PRESSURE

- **Enthalpy(H)** =  $U + PV$
- In constant pressure process, from First law,  $\delta Q = dU + \delta W \Rightarrow dU + PdV \Rightarrow dU + dPV \Rightarrow d(U + PV) = dH$
- $\Rightarrow \delta Q = dH = mC_p \Delta T \Leftarrow$  applicable in cases of constant pressure process or Ideal gas involving any process

CONSTANT TEMPERATURE

- In case of constant temperature,  $dU = 0$ . So,  $\delta Q = \delta W$
- **NOTE:** The above is not possible due the limitations of second law.

ADIABATIC HEAT TRANSFER

- Since, its adiabatic,  $\delta Q = 0 \Rightarrow W = -dU = U_1 - U_2$

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## POLYTROPIC HEAT TRANSFER

- $Q_{poly} = W_{poly} * \left( \frac{\gamma - n}{\gamma - 1} \right) \Rightarrow \boxed{\frac{P_1 V_1 - P_2 V_2}{n - 1} * \left( \frac{\gamma - n}{\gamma - 1} \right)}$
- Specific heat for polytropic process,  $\boxed{C_{poly} = -C_V * \left( \frac{\gamma - n}{n - 1} \right)}$
- the above is derived using the first law and Meyer's equation:  $C_P - C_V = R$ . This relation is derived from the definition of enthalpy as  $H = U + PV \Rightarrow mC_P dT = mC_V dT + mRdT$   
 $\Rightarrow C_V = \frac{R}{\gamma - 1}$  and  $C_P = \frac{\gamma R}{\gamma - 1}$
- Polytropic specific heat being negative implies that even though heat is added to the gas, its temperature decreases because the work done by the gas exceeds the heat supplied to the gas and the additional work is done at the expense of the internal energy
- Generally  $1 < n < \gamma$

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## Free Expansion of gases

- The free expansion process is **highly irreversible**
- When partition is removed the internal energy of the fluid gets used as Kinetic energy
- When the particles hit the walls of the container and reaches the equilibrium, the Kinetic energy gets converted back into Internal energy
- Even though the Initial and final internal energy is same, meaning Initial and final temperature is same, the process is not isothermal, because when the fluid is expanding with Kinetic energy, the temperature varies.
- **Note:** Instead of partition, if there is movable frictionless piston, it is **not the case of free expansion**, as the fluid is doing work on the piston. here also the final and initial temperature will be same.

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## 4. Open System Analysis by First Law

$$\boxed{\text{SFEE} \Rightarrow U_1 + P_1 V_1 + \frac{C_1^2}{2} + gZ_1 + Q = U_2 + P_2 V_2 + \frac{C_2^2}{2} + gZ_2 + W_{CV}}$$

### Steady Flow systems

- Fluid properties can change from point to point within the control volume, but they don't change with respect to time
- The volume, mass and total energy of the control volume remain constant. As a result, the displacement work is zero as V is constant.
- Mass balance equation: Mass incoming = Mass outgoing. That is  $\boxed{\sum_{inlet} \dot{m}_i = \sum_{outlet} \dot{m}_e}$
- **NOTE:** In case of Air as the fluid, the continuity equation is  $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$
- **NOTE:** But in case of Liquids(Incompressible fluids), the continuity equation is  $\boxed{A_1 V_1 = A_2 V_2}$

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## Nozzle & Diffusers

- Assumptions:
  - $\Delta PE = 0$  i.e  $Z_1 - Z_2 = 0$

- Adiabatic process (Nozzle is insulated)  $\implies Q=0$ ;
- $W_{CV} = 0$

- So, SFEE =  $\boxed{h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}}$

- **NOTE:** Generally the enthalpies in the sum will be given in KJ/kg which has to be converted to J/Kg to get the velocities in m/s.

### Turbine and Compressors

- Assumptions:
  - Insulated  $\implies Q = 0$ ;
  - Flow velocities are negligible  $\implies C_1 = C_2 = 0$ ;
  - $Z_2 - Z_1$  also negligible  $\Delta PE = 0$
- So, SFEE =  $\boxed{h_1 = h_2 + W_{CV}}$
- For Turbines,  $W_{CV}$  is positive. For compressor, its negative.

### Throttling (Pressure drop)

- Assumptions
  - Adiabatic  $\implies Q = 0$ ;
  - No workdone  $\implies W_{CV} = 0$
  - K.E. = 0 & P.E. = 0
- So, SFEE  $\implies \boxed{h_1 = h_2}$  - Since the enthalpy at inlet and outlet are same, the throttling valves are referred to as **Isenthalpic valves**
- Dryness friction(x)
  - SFEE  $\implies \boxed{h_1 = h_f + x(h_g - h_f)}$

### Heat Exchangers

- Assumptions
  - Heat exchange is within the CV. So, no heat interactions with the surroundings.  $\implies Q = 0$ ;
  - No workdone  $\implies W_{CV} = 0$
  - K.E. = 0 & P.E. = 0
- So, SFEE for (non-contact type - hot and cold fluid do not mix)

$$\implies \boxed{\dot{m}_c h_1 + \dot{m}_h h_2 = \dot{m}_c h_3 + \dot{m}_h h_4}$$

- So, SFEE for (contact type - hot and cold fluid mix)  $\implies \boxed{\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3}$

### Comparison of SFEE with Euler and Bernoulli's equation

$$\begin{aligned} \text{SFEE} \implies U_1 + P_1 V_1 + \frac{C_1^2}{2} + gZ_1 + Q &= U_2 + P_2 V_2 + \frac{C_2^2}{2} + gZ_2 + W_{CV} \\ \implies H_1 + \frac{C_1^2}{2} + gZ_1 + Q &= H_2 + \frac{C_2^2}{2} + gZ_2 + W_{CV} \\ \implies h_1 + \frac{C_1^2}{2} + gZ_1 + \delta q &= h_2 + \frac{C_2^2}{2} + gZ_2 + \delta w_{CV} \end{aligned}$$

where  $h_1, h_2$   $\delta q$  and  $\delta w_{CV}$  are specific quantities, i.e., they are divided by mass(m)

$$\Rightarrow \delta q = -h_1 + -\frac{C_1^2}{2} + -gZ_1 + h_2 + \frac{C_2^2}{2} + gZ_2 + \delta w_{CV}$$

$$\Rightarrow \delta q = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1) + \delta w_{CV}$$

- $(h_2 - h_1) = dH$

- $(z_2 - z_1) = dz$

- $\frac{C_2^2 - C_1^2}{2} = CdC$

- **Assumptions:**

- Insulated.  $\Rightarrow \delta q = 0$

- No Work done  $\Rightarrow \delta w_{CV} = 0$

- Temperature remains constant  $\Rightarrow du = 0$

- Fluid is incompressible  $\Rightarrow dv = 0$

$$\Rightarrow 0 = dh + CdC + gdZ + 0$$

$$= du + Pdv + vdP + CdC + gdZ$$

$$= 0 + 0 + \boxed{vdP + CdC + gdZ} - - - \text{Euler's form of SFEE}$$

Integration of the above between two section will give the Bernoulli's form of SFEE:

$$\boxed{\frac{P_1}{\rho} + \frac{C_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{C_2^2}{2} + gZ_2}$$

### Unsteady flow

- The rate at which mass of fluid within the control volume is accumulated is equal to the net rate of mass flow across the control surface  $\Rightarrow \frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$

- Similarly  $\Rightarrow \frac{dE_{cv}}{dt} = \dot{E}_i - \dot{E}_e$

- So,  $\boxed{\frac{dE_{cv}}{dt} = \left\{ \dot{m}_i \left( h_i + \frac{C_i^2}{2} + gZ_i \right) + \dot{Q} \right\} - \left\{ \dot{m}_e \left( h_e + \frac{C_e^2}{2} + gZ_e \right) + \dot{W}_{CV} \right\}}$

- The above is called the **First law of thermodynamics for an open system with Unsteady flow**

- If the above system is stationary, then  $\Rightarrow \boxed{\frac{dE_{cv}}{dt} = \frac{dU_{cv}}{dt} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{CV}}$

### Charging of tank

- a supply line carrying a fluid in connected to a **rigid, isolated and evacuated** tank.
- No changes in K.E. and P.E.
- No Work done
- No heat interactions (as Insulated)
- Since there is no exit, the rate of change of mass inside the tank is given by:

$$\Rightarrow \frac{dm_{CV}}{dt} = \dot{m}_i \quad \Rightarrow \frac{dU_{CV}}{dt} = \dot{m}_i h_i$$

- Assuming  $h_i$  as constant and integrating will give us:

$$\implies (U_f - U_i)_{CV} = h_i(m_f)_{CV} \quad (U_i = 0, U_f = u_f * m_f)$$

$$\text{So, } \boxed{u_f = h_i} \quad (u_f = c_v * T_f, \quad h_i = c_p * T_f)$$

$$\implies \boxed{T_f = \gamma * T_i}$$

## 5. Second Law of Thermodynamics

### Thermal Energy Reservoir (TER)

- A hypothetical body with large heat capacity (mass \* SpecificHeat) that can supply or absorb large amounts of heat with not much change in its temperature. Example: The air in a room with a TV as a heating source can be considered as a TER

### Heat engines

- Rotating a paddle shaft in water will increase the internal energy of the water, which will later leave the water as heat. So, work gets converted into heat naturally.
- But the reverse requires usage of special devices called **Heat engines** which takes in heat as input and produce work.
- There are different types of heat engines, but they have common characteristics like, they absorb heat from a source, **convert a part of it as work and reject remaining** to the sink
- Example:** Steam powerplant is a heat engine. It has five major components: Furnace, Boiler, Turbine, Condenser, Pump. The boiler takes in heat from the furnace, heats up the water to convert it to steam, the steam runs the turbine and generate work, then gets condensed back to water by the condenser and the pump pushes the water back into the boiler.

### Thermal Efficiency ( $\eta$ )

- The ratio of fraction of heat that gets converted as work, to the total heat input to the boiler is called as the Thermal efficiency of the Heat engine.

$$\bullet \quad \boxed{\eta = \frac{W_{net}}{Q_{supplied}}}$$

- $\delta Q_{net}$  = Amount of heat supplied to boiler( $Q_1$ ) - Amount of heat rejected from condenser( $Q_2$ )
- $\delta W_{net}$  = Amount of Work produced by Turbine( $W_1$ ) - Amount of work required by pump to get water back to boiler pressure( $W_2$ )
- In a cyclic process,  $\Delta U = 0$ . So, according to first law,  $\delta Q = \delta W \implies \delta Q_{net} = \delta W_{net}$
- So,  $\eta = \frac{W_{net}}{Q_1} \implies \frac{Q_{net}}{Q_1} \implies \frac{Q_1 - Q_2}{Q_1} \implies \boxed{1 - \frac{Q_2}{Q_1}}$
- Thermal efficiency of 100 percent is impossible in reality. For  $\eta = 100\%$   $Q_2$  has to be Zero. I.e., There must be zero heat loss and all of heat input should be converted to work.

### Kelvin-Planck Statement

- " It is impossible for a system to operate in a thermodynamic cycle and deliver a net amount of energy by work to its surroundings while receiving energy by heat transfer from a single reservoir "
- In simple terms, Thermal efficiency can never be 100%.



## Perpetual Motion Machine

- **PMM - I:** A machine that violates first law of thermodynamics - Producing work without Heat.  $\implies \delta Q \neq \delta W$
- **PMM - II:** A machine that violates second law of thermodynamics (Kelvin-planck) - 100% Thermal efficiency
- **PMM - III:** A machine that works continuously without any heat or work interaction with the surrounding. This is possible only in the complete absence of all forms of irreversibilities.

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## Refrigerators and Heat pumps

- Heat transfer from high temp to low temp can occur naturally. But for the reverse to occur, special devices like **Refrigerator or Heat pump** is required.

### – REFRIGERATOR

\* Like Heat engine which uses water as the working fluid, refrigerators use refrigerants as working fluid. **Vapor compression refrigeration** is one of the most commonly used process.

\* It has four main components: Compressor, condensor, expansion valve (capillary tube) and evaporator.  
\* The refrigerant enters the compressor (the compressor requires external work) and gets its pressure increased. Then it enters the condensor where its temperature decreases by rejected heat to the surrounding. Then it goes through an expansion valve, where due to throttling its pressure and temperature drop significantly. Then the refrigerant enters the evaporator where it absorbs heat from the surrounding and re-enters the compressor.

\*  $Q_L$  - Amount of heat removed from refrigerated space which is at temperature  $T_L$

\*  $Q_H$  - Amount of heat rejected from condensor to surrounding which is at a temperature of  $T_H$

\*  $W_{net}$  - Work input to the compressor.

\*  $Q_H - Q_L = W_{net} \implies \boxed{Q_H = W_{net} + Q_L}$

\*  $\boxed{COP_R = \frac{\text{Desired effect}}{\text{Work input}} \implies \frac{Q_L}{W_{net}} \implies \frac{Q_L}{Q_H - Q_L}}$

### – HEAT PUMPS

\* Heat pump is another device that transfers heat from low temp to high temp. But the operation objective of heat pump is different from that of refrigerator as it is used to make the place warmer than surroundings whereas refrigerator makes a space colder than surroundings.

\*  $\boxed{COP_H = \frac{Q_H}{W_{net}} \implies \frac{Q_H}{Q_H - Q_L}}$  **NOTE:**  $\boxed{COP_H = COP_R + 1}$

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## Clausius Statement :-

- Heat pumps and refrigerators are based on clausius statement of second law of thermodynamics
- " *It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher temperature body* "
- In simple terms, Transfer of heat from low temperature to high temperature without the aid of an external agency is impossible.
- According to Clausius statement, **NOTE:**  $COP_H$  and  $COP_R$  can never be  $\infty$

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## Reversible process :-

- They do not occur in nature due to second law of thermodynamics.
- They need to be carried out infinitely slowly (Quasi-static)
- Since they don't occur in nature, devices imagined to be performing reversible processes have the best efficiency compared to the ones performing Irreversible processes. It applies to both work-producing as well as work-consuming devices.
- concept of reversibility leads to **Second Law efficiency** for actual processes which tells by what percent the process is like a reversible process.
- **Internally Reversible:** If in a process, the system can be brought to its original state, then its called Internally Reversible

- **Externally Reversible:** If in a process, the surrounding can be brought to its original state, then its called Externally reversible

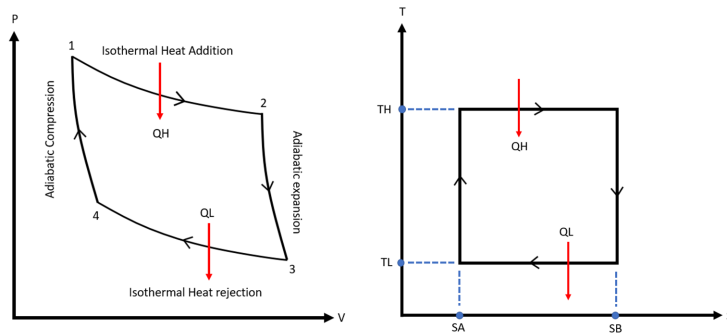
### Irreversible process :-

- Factors like friction, free expansions, inelastic deformation, etc., create irreversibilities in processes.
- **Friction** - a brick is in contact with the floor. Moving the brick requires force which should be greater than the friction force between the brick and the floor. When applied force is higher than the frictional force, the brick will start moving. But since there is friction, heat will be generated due to motion, and this in turn will raise the temperature of the body. Now if we try to bring back the body to its original position, still some more amount of heat will be generated since frictional force is always opposite to the direction of motion. Those heats that were generated will get dissipated into the surrounding (considering the brick to be the system).

### Carnot Cycle :-

- Carnot cycle is composed of an **adiabatic piston cylinder with gas inside**
- Processes:

- (1-2) Reversible isothermal heat addition:  $Q_H = (U_2 - U_1) + W_{1-2}$
- (2-3) Reversible adiabatic expansion:  $0 = (U_3 - U_2) + W_{2-3}$
- (3-4) Reversible isothermal heat rejection:  $-Q_L = (U_4 - U_3) - W_{3-4}$
- (4-1) Reversible adiabatic compression:  $0 = (U_1 - U_4) - W_{4-1}$



- Carnot cycle cannot be used in actual devices because it becomes impractical to have a reversible isothermal heat transfer process, cause the heat transfer has to occur between infinitesimally small temperature difference and infinitely slowly.
- A practical and slightly modified version of the carnot cycle is the **Rankine cycle** used in Steam plants.

### Reversed Carnot Cycle :-

- If all the process in carnot cycle is reversed, we obtain what is called a reversed carnot cycle or **Carnot Refrigeration cycle** which can be used for Heat pumps and refrigerators

### Carnot Principles :-

- **Principle I:**  $\eta_{Rev} > \eta_{Irr}$  for operation between the same temperature range
- **Principle II:** Efficiencies of all reversible engines working between the same temperature range are equal

### Thermodynamic Temperature scale :-

- A temperature scale independent of the properties of the substances that are used to measure the temperature.
- From carnot's principle II, the efficiency is dependent on the operating temperature range only, i.e.,  $\eta_{REV} = f(T_H, T_L)$
- We know  $\eta = 1 - \frac{Q_L}{Q_H}$ . This can be written as:  $\eta = 1 - \frac{\Phi(T_L)}{\Phi(T_H)}$ . Kelvin suggested,  $\Phi(T) = T$ .
- So,  $\eta = 1 - \frac{T_L}{T_H}$
- And since  $\eta$  cannot be 1, the lowest possible temperature for  $T_L$  is Zero. Hence the Kelvin scale is called as absolute scale.
- **NOTE:** We now know  $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$  So, inorder to reach absolute zero, we would need heat rejected to the sink to be zero which would violate the Kelvin-planck statement of second law.

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### Maximum performance measures for cycles operating between two reservoirs :-

- EFFECT OF TEMPERATURE ON PERFORMANCE OF **reversible** DEVICES

- **Heat engine**  $\Rightarrow \eta = 1 - \frac{T_L}{T_H} \Rightarrow \eta \uparrow$  for  $T_L \downarrow$  (or)  $T_H \uparrow$ .
- Decreasing the  $T_L \downarrow$  is more effective than increasing the  $T_H \uparrow$
- **Refrigerator**  $\Rightarrow COP_R = \frac{T_L}{T_H - T_L} = \frac{1}{\left(\frac{T_H}{T_L} - 1\right)} \Rightarrow COP_R \uparrow$  for  $T_L \uparrow$  (or)  $T_H \downarrow$
- **Heat Pump**  $\Rightarrow COP_H = \frac{T_H}{T_H - T_L} = \frac{1}{\left(1 - \frac{T_L}{T_H}\right)} \Rightarrow COP_H \uparrow$  for  $T_L \uparrow$  (or)  $T_H \downarrow$
- For Refrigerator and Heat pump, increasing the  $T_L \uparrow$  is more effective rather than decreasing the  $T_H \downarrow$

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## 6. Entropy

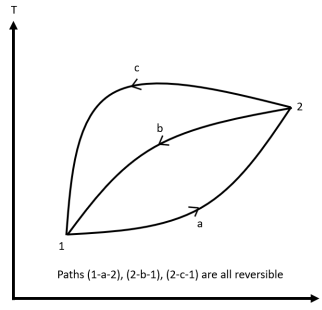
### Clausius Inequality :-

- Entropy is a measure of Irreversibility
- Clausius inequality =  $\oint \frac{\delta Q}{T} \leq 0 \Rightarrow \begin{matrix} \oint \frac{\delta Q}{T} = 0 & \oint \frac{\delta Q}{T} < 0 & \oint \frac{\delta Q}{T} > 0 \\ \text{Reversible} & \text{Irreversible} & \text{Impossible} \end{matrix}$

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### The property of Entropy :-

- Just like how energy was defined using first law, entropy can be defined using clausius inequality.



- So,  $\int_{2b1} \frac{\delta Q}{T} = \int_{2c1} \frac{\delta Q}{T}$ , which means  $\left(\frac{\delta Q}{T}\right)_{rev}$  must represent a property, which we are defining as Entropy(S).
- $S_2 - S_1 = \Delta S = \int_1^2 dS = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$  (Only for Reversible processes)
- **NOTE:** We have defined change in entropy instead of entropoy just like how we defined change in energy instead of energy itself.
- Entropy is **Extensive** property

### Entropy Increase :-

- If in the above diagram, **had the path C been irreversible**, Then
- $\oint_{1a2b1} \frac{\delta Q}{T} = 0$  but,  $\oint_{1a2c1} \frac{\delta Q}{T} < 0$  as it is irreversible.
- $\int_{2b1} \frac{\delta Q}{T} > \int_{2c1} \frac{\delta Q}{T} \Rightarrow \int_1^2 dS > \int_{2c1} \frac{\delta Q}{T}$
- Which can be generalized as:  $dS \geq \frac{\delta Q}{T}$
- The entropy change is equal to heat transfer in case of reversible process and in case of irreversible process, it is greater than it, which implies that **some entropy is generated during an irreversible process**.
- So,  $\Delta S_{sys} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$   $S_{gen}$  is always  $\geq 0$ .  $S_{gen}$  is a measure of magnitude of irreversibilities
- **NOTE:** Unlike entropy, which doesn't depend on the path, the  $S_{gen}$  is a path function so it depends on the path.
- In case of an **Isolated system**,  $\Delta S_{sys} = S_{gen}$  and since  $S_{gen}$  is always positive, the entropy of the universe always increases or incase of the never naturally occuring reversible process, it could remain constant, **but it could never decrease**. This is called **Increasing entropy principle**
- **NOTE:** The above principle does not mean that change in entropy cannot be negative. It only means that entropy generation can never be negative.
- **NOTE:** Since entropy is an extensive property, total entropy = sum of parts of entropies of the system.

$$\Rightarrow S_{gen} = \Delta S_{univ} = \Delta S_{isolated} = \Delta S_{total} = (\Delta S_{Sys} + \Delta S_{Surr}) \geq 0$$

### T-S diagram :-

- We know  $dS = \left(\frac{\delta Q}{T}\right)_{rev}$ . So,  $\delta Q_{rev} = TdS \Leftarrow$  **Very important!!**
  - $\Rightarrow Q_{rev} = \int_1^2 Tds$ . Therefore, we can say area under the curve on a T-S diagram represent the heat transfer during a reversible process.
  - In case of an adiabatic process, there is no heat transfer, so
    - i. Reversible adiabatic  $\Rightarrow dS = 0$ .  $\Leftarrow$  Isentropic process
    - ii. Irreversible adiabatic  $\Rightarrow dS = S_{gen}$
  - From first law,  $\delta Q = dU + \delta W$  and now we know, for a reversible process,  $\delta Q = TdS$  &  $\delta W = PdV$ . So,  **$TdS = dU + PdV$**
  - And for a reversible cyclic process,  **$TdS = PdV$**  as  $dU$  is zero in a cyclic process. So, for a cyclic process, **net heat interaction = net work interaction**
- 

### T-dS Relations :-

1.  $TdS = dU + PdV$
  2.  $TdS = dH - VdP \Leftarrow (H = U + PV \Rightarrow dH = dU + PdV + VdP \Rightarrow dH = TdS + VdP)$
- **NOTE:** Even though the above relation is developed based on reversibility, since entropy is a point function, the above relations hold good for irreversible processes as well.
  - They can be applied to both closed / open systems as well.
- 

### Slopes of Isobaric and Isochoric processes in T-S diagram :-

- For an Isochoric / Constant volume process,  $V = \text{Constant}$ . So,  $TdS = dU + PdV \Rightarrow TdS = dU + 0 \Rightarrow TdS = mC_V dT$   
 $\Rightarrow \frac{dT}{dS} = \frac{T}{C_V} \Leftarrow \left(\frac{dT}{dS} = \text{Slope of a curve in TS diagram.}\right)$
  - For an Isobaric / Constant pressure process,  $P = \text{Constant}$ , So,  $dH = TdS + VdP \Rightarrow dH = TdS + 0 \Rightarrow mC_P dT = TdS$   
 $\Rightarrow \frac{dT}{dS} = \frac{T}{C_P}$
  - Since,  $C_P > C_V \Rightarrow \frac{T}{C_P} < \frac{T}{C_V} \Rightarrow$  Slope of Constant Pressure < Slope of Constant Volume in TS diagram.
- 

### Entropy Change for an Ideal gas :-

- $S_2 - S_1 = C_V \ln \frac{P_2}{P_1} + C_P \ln \frac{V_2}{V_1}$
  - Derived using TDS equations,  $dH = mC_P dT$ ,  $dU = mC_V dT$  and  $PV = mRT$
- 

### Entropy Change for an incompressible substance :-

- For most liquids and solids the density does not change and hence  $dv \approx 0$

- On the above pretext,  $Tds = du + Pdv = du = 0$
- $du = cdT$  where, for incompressible substance,  $c_p = c_v = c$
- So,  $\boxed{Tds = cdT} \implies ds = \frac{cdT}{T} \implies s_2 - s_1 = \int_1^2 ds = \int_1^2 \frac{cdT}{T} \implies c \ln \frac{T_2}{T_1}$
- Total Change in Entropy( $dS$ ) =  $m(s_2 - s_1) = \boxed{S_2 - S_1 = mc \ln \left( \frac{T_2}{T_1} \right)}$
- From the above, **NOTE:** The change in entropy for an incompressible substance is zero in case of an isothermal process.

### Finite Body analysis :-

#### MIXING OF TWO INCOMPRESSIBLE FLUIDS OR SOLIDS BROUGHT INTO CONTACT

- An adiabatic enclosure with two subsystems each with an incompressible fluid separated by a partition.
  - Subsystem 1:  $m_1, C_1, T_1$
  - Subsystem 2:  $m_2, C_2, T_2$
- Partition removed, the two fluids mix and attain a final temperature( $T_f$ )
- Applying first law to the whole system:  $Q = \Delta U + W$ . But here,  $Q=0$  and  $W=0$ . As the system is adiabatic and no boundary work.
 
$$\implies \Delta U = 0 \implies \Delta U_1 + \Delta U_2 = 0 \implies m_1 C_1 (T_f - T_1) + m_2 C_2 (T_f - T_2) = 0$$

$$\implies \boxed{T_f = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}}$$
- and Since, the system is adiabatic,  $S_{gen} = \Delta S_{Sys} + \Delta S_{Surr} \implies \Delta S_{Sys} + 0$  (as no heat goes to the surrounding)
- $S_{gen} = \Delta S_{univ} = \Delta S_1 + \Delta S_2$  and since they are assumed incompressible  $\implies \boxed{m_1 C_1 \ln \left( \frac{T_f}{T_1} \right) + m_2 C_2 \ln \left( \frac{T_f}{T_2} \right)}$
- **Special Case:**  $\boxed{m_1 = m_2 = m}$  and  $\boxed{C_1 = C_2 = C}$ 

$$\implies \Delta S_{Sys} = mC \ln \left( \frac{T_f^2}{T_1 T_2} \right) = 2mC \ln \left( \frac{T_f}{\sqrt{T_1 T_2}} \right) = 2mC \ln \left( \frac{A.M}{G.M} \right) = nmC \ln \left( \frac{A.M}{G.M} \right)$$

$$\implies \text{A.M} = \boxed{T_f = \frac{T_1 + T_2 + \dots T_n}{n}} \text{ and G.M.} = \boxed{\sqrt[n]{T_1 T_2 \dots T_n}}$$

#### MAXIMUM WORK OBTAINABLE FROM TWO FINITE BODIES OPERATING BETWEEN $T_1$ AND $T_2$ :-

- **Body 1:**  $m_1, C_1, T_1$  and **Body 2:**  $m_2, C_2, T_2 (T_2 < T_1)$
- Heat exchange through a reversible heat engine and heat exchange only till they both attain  $T_f$  - common Final Temperature
- $Q_1$  - Heat rejected from body 1 and  $Q_2$  - Heat gained by body 2  $\boxed{W = Q_1 - Q_2}$
- From first law,  $Q = dU + W$  and  $W = 0$  (no boundary work)
- $Q_1 = m_1 C_1 (T_1 - T_f)$  and  $Q_2 = m_2 C_2 (T_f - T_2) \implies W = m_1 C_1 (T_1 - T_f) - m_2 C_2 (T_f - T_2) \dots \text{Eqn(1)}$
- Here  $\Delta S_{Surr} = 0$  since no heat interaction with the surrounding (But work interaction is present)
- So,  $\Delta S_{Univ} = \Delta S_1 + \Delta S_2 + \Delta S_{Engine} \leftarrow (\Delta S_{engine} = 0 \text{ as it is a cyclic device})$
- $\Delta S_{Univ} \implies \boxed{m_1 C_1 \ln \left( \frac{T_f}{T_1} \right) + m_2 C_2 \ln \left( \frac{T_f}{T_2} \right)}$  and for  $W_{max} \implies \Delta S_{Univ} = 0$

- Solving for  $T_f$  in the above and plugging it in Eqn(1) will give  $W_{max}$

- **Special Case:**  $m_1 = m_2 = m$  and  $C_1 = C_2 = C \Rightarrow T_f = \sqrt{T_1 T_2} \Rightarrow W_{max} = mC (T_1 + T_2 - 2\sqrt{T_1 T_2})$

→ CASE A: Two incompressible body exchanging energy and no work extracted

→ CASE B: Two incompressible body exchanging energy and max work extracted.

\* The  $(T_f)_A > (T_f)_B$  and if there was another CASE C: in which due to irregularities only a part of the maximum work was extracted then  $(T_f)_C$  will be between that of A and B

### Reversible Steady Flow Work :-

- We know that the differential form of SFEE is  $\delta q = dh + C dC + g dZ + \delta W_{CV}$
- For a reversible process, from clausius inequality and entropy definition,  $\delta q = T ds$  and from TdS relations,  $T ds = dh - v dP \Rightarrow \delta q = dh - v dP$
- $\Rightarrow dh - v dP = dh + C dC + g dZ + \delta W_{CV} \Rightarrow -v dP = C dC + g dZ + \delta W_{CV}$
- if  $\Delta K.E = \Delta P.E = 0 \Rightarrow \delta W_{CV} = -v dP \Rightarrow W_{CV} = -\int v dP$
- The term  $-\int v dP$  represents **the area under between the P-axis and the curve on a P-V diagram** and gives us the value of reversible adiabatic work in a steady flow system for both work producing and work consuming devices

### Open system work for various reversible processes:-

- CONSTANT VOLUME - ISOCHORIC - ISOMETRIC

$$\Rightarrow W_{CV} = v(P_2 - P_1)$$

- CONSTANT PRESSURE - ISOBARIC - ISOPIESTIC

$$\Rightarrow W_{CV} = 0 \text{ since, } dP \text{ is } 0.$$

- CONSTANT TEMPERATURE - ISOTHERMAL - HYPERBOLIC

$$\Rightarrow W_{CV} = P_1 v_1 \ln \frac{P_2}{P_1}$$

- ADIABATIC

$$\Rightarrow W_{CV} = \frac{\gamma(P_1 v_1 - P_2 v_2)}{\gamma - 1} = \gamma * W_{closed} \Leftarrow W_{CV} = -\int_1^2 \left( \frac{P_1 v_1}{P} \right)^{1/\gamma} dP \Leftarrow P_1 v_1^\gamma = P v^\gamma$$

- POLYTROPIC

$$\Rightarrow W_{CV} = \frac{n(P_1 v_1 - P_2 v_2)}{n - 1} = n * W_{closed} \Leftarrow W_{CV} = -\int_1^2 \left( \frac{P_1 v_1}{P} \right)^{1/n} dP \Leftarrow P_1 v_1^n = P v^n$$

### Second Law analysis of a control volume :-

- $\left( \frac{dS}{dt} \right)_{CV} = \dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen}$

### Steady-state steady flow process :-

- The above equation can be simplified to  $\left( \frac{dS}{dt} \right)_{CV} = 0$  in case of steady flow.

$$\bullet \Rightarrow 0 = \left( \dot{m}s_i + \frac{\dot{Q}}{T} \right) - \dot{m}s_e + \dot{S}_{gen} \Rightarrow \boxed{\dot{m}(s_e - s_i) = \frac{\dot{Q}}{T} + \dot{S}_{gen}}$$

$$\bullet \text{ For more than one inlet and exit: } \boxed{\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}}$$

### Available energy (Exergy) :-

- **Availability** - The maximum useful work that can be obtained.
- **Dead state** - Thermodynamic equilibrium with the surrounding
- If a reversible process is carried out that takes the system to a dead state, then the work output / required of that process is maximum / minimum.
- So the available energy is a function of:
  - i. Total energy of the system
  - ii. State of the system
  - iii. State of the surrounding
- CLASSIFICATION OF SOURCE OF ENERGY
  - **High Grade:** High grade energy is totally available for work and is exempt from the limitation of second law. Eg. Work
  - **Low Grade:** Only a part of low grade energy can be converted to useful work due to the limitation of second law and this is called availability. Eg. Heat
- Exergy( $W$ ) =  $Q\eta = Q \left[ 1 - \frac{T_0}{T} \right] = Q - T_0\Delta S = \boxed{(T - T_0)\Delta S} \Leftarrow$  carnot engine operating between  $T$  and  $T_0$

### Heat transfer through a finite temperature difference :-

- Exergy decreases when heat transfers through a finite temperature difference
- Consider the following:
  - **CASE 1:** A carnot engine operating between two reservoirs at temp  $T_1$  and  $T_2$ . ( $T_1 > T_2$ )
 
$$\Rightarrow \text{Available work } (W_1) : Q_1 - Q_{R1} = (T_1 - T_2) * \Delta S_1$$
    - \*  $Q_1$  = Amount of heat absorbed from reservoir at  $T_1$
    - \*  $Q_{R1}$  = Amount of heat rejected to reservoir at  $T_2$
  - **CASE 2:** Now same carnot engine operating between two reservoirs at temp  $T_2$  and  $T_3$  ( $T_2 > T_3$ ) and the reservoir at  $T_2$  is receiving heat( $Q_1$ ) from another reservoir at a temperature  $T_1$  ( $T_1 > T_2$ )
    - \* Still the heat input the engine is  $Q_1$
    - \*  $Q_1 = T_1\Delta S_1 = T_2\Delta S_2 \Leftarrow$  but  $T_1 > T_2$
    - \* So,  $\Delta S_2 > \Delta S_1$ . Also,  $Q_{R1} = T_0\Delta S_1$  and  $Q_{R2} = T_0\Delta S_2 \Rightarrow \boxed{Q_{R2} > Q_{R1}}$
    - \*  $W_2 = Q_1 - Q_{R2}$  and  $W_1 = Q_1 - Q_{R1} \Rightarrow \boxed{W_2 < W_1}$
- Thus due to irreversibilities in heat transfer between finite temperature difference, exergy decreases
- Loss of A.E. =  $W_1 - W_2 = Q_{R2} - Q_{R1} = T_0(\Delta S_2 - \Delta S_1) \Leftarrow$  here,  $\boxed{(\Delta S_2 - \Delta S_1) = S_{gen}}$
- $\boxed{\text{Loss of A.E. or U.A.E} = T_0(S_{gen})}$
- According to First law, Thermal energy at higher temperature is same as compared to thermal energy at lower temperature. That's what this means:  $Q_1 = T_1\Delta S_1 = T_2\Delta S_2$ . Hence the first law is called **Quantitative law**
- But, According to Second law, Thermal energy at higher temperature is more significant than thermal energy at lower temperature. This is evident from  $Q_{R1}$  being lesser than  $Q_{R2}$  as we want the heat rejection to be as minimal as possible. Hence second law is called **Qualitative law**



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### Availability and Availability Function :-

#### AVAILABILITY FUNCTION FOR A NON-FLOW PROCESS OR CLOSED SYSTEM

- Consider:
  - Ambient Pressure ( $P_0$ )
  - Ambient Temperature ( $T_0$ )
  - Initial Volume of system ( $V_1$ )
  - Final Volume of system ( $V_0$ )
  - Final Pressure of system ( $P_0$ )
- Available work:  $W_{max} = P_0(V_0 - V_1)$
- From first law,  $\delta Q = dU + \delta W \implies \delta W = TdS - dU \iff$  since  $\boxed{\frac{\delta Q}{T} = dS}$  for reversible process
- $W_{max} = T_0(S_0 - S_1) - (U_0 - U_1) \implies T_0S_0 - T_0S_1 - U_0 + U_1 \implies \boxed{W_{max} = (U_1 - T_0S_1) - (U_0 - T_0S_0)}$
- Available work:  $(U_1 - T_0S_1) - (U_0 - T_0S_0) - P_0V_0 + P_0V_1 \implies (U_1 + P_0V_1 - T_0S_1) - (U_0 + P_0V_0 - T_0S_0)$
- Available work or Availability:  $\Phi_1 - \Phi_0 \iff \boxed{\Phi = (U + P_0V - T_0S)}$
- Change in Availability =  $\Phi_1 - \Phi_2$  for a system that undergoes a state change from 1 to 2.

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#### AVAILABILITY FUNCTION FOR A FLOW PROCESS OR OPEN SYSTEM

- We know from SFEE,  $\boxed{\dot{m} \left( h_i + \frac{C_i^2}{2} + gZ_i \right) + \dot{Q} = \dot{m} \left( h_e + \frac{C_e^2}{2} + gZ_e \right) + \dot{W}_{CV}}$
- $\implies \dot{W}_{CV} = \dot{m} \left( h_i + \frac{C_i^2}{2} + gZ_i \right) + \dot{Q} - \dot{m} \left( h_e + \frac{C_e^2}{2} + gZ_e \right)$
- $S_{gen} = 0 = \dot{m}(s_e - s_i) - \frac{\dot{Q}}{T} \implies \dot{Q} = \dot{m}T_0(s_e - s_i)$
- $\implies \dot{W}_{CV} = \dot{m} \left( h_i + \frac{C_i^2}{2} + gZ_i \right) - \dot{m} \left( h_e + \frac{C_e^2}{2} + gZ_e \right) - \dot{m}T_0(s_i - s_e)$
- $\dot{W}_{CV} = \dot{m} \left[ (h_i - h_e) + \frac{C_i^2 - C_e^2}{2} + g(Z_i - Z_e) - T_0(s_i - s_e) \right]$
- Here, Availability function  $(\Phi) = h + \frac{C^2}{2} + gZ - T_0s \implies \dot{W}_{CV_{max}} = \dot{m}(\Phi_i - \Phi_e)$
- **NOTE:** For open system, Maximum work = Maximum useful work
- **NOTE:** For closed system, Maximum work = Maximum useful work + Atmospheric work

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### Irreversibility :-

- $I = W_{rev} - W_{actual} = W_{max} - W_{actual}$
- For non-flow processes,  $I = (\Phi_1 - \Phi_2) - W_{actual}$
- Upon deriving we obtain, for both flow and non-flow processes:  $\boxed{I = T_0(\Delta S_{Univ}) = T_0S_{gen}}$  and  $I \geq 0$
- **Gouy-Stodola Theorem:**  $\dot{I} = T_0(\dot{S}_{gen}) \implies$  rate of loss of availability is proportional to the rate of entropy generation

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## Second Law Efficiency :-

- The performance of Heat engines and Refrigerators or Heatpumps have been measured using the Efficiency and Coefficient of performance quantities. They are also called **First law efficiencies**
- But they make no reference to how best the machine is performing vs how the machine is possible to perform as best.
- Consider: Heat engine operating between 500K and 450K, withdrawing 100KJ and producing work output of 9.5KJ. Now,  $\eta = \frac{W}{Q_{in}} = \frac{9.5}{100} = 9.5\% \Leftarrow$  This may seem very low. But,
- $\eta_{max} = 1 - \frac{T_l}{T_H} = 1 - \frac{450}{500} = 10\% \Leftarrow$  Best efficiency possible for the given temperature range.
- Now, The second law efficiency  $\eta_{II} = \frac{\eta_{actual}}{\eta_{max}} = \frac{9.5}{10} = 95\% \Leftarrow$  as you can see the engine is performing quite the best.
- similarly, for work consuming devices,  $\eta_{II} = \frac{\eta_{rev}}{\eta_{actual}}$
- For Heat pumps and refrigerators,  $\eta_{II} = \frac{COP_{actual}}{COP_{rev}}$