Mathematics - GATE

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February 2, 2023

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Chapter 1

Linear Algebra

Chapter 2

Differential and Integral Calculus

Chapter 3

Vector Calculus

Chapter 4

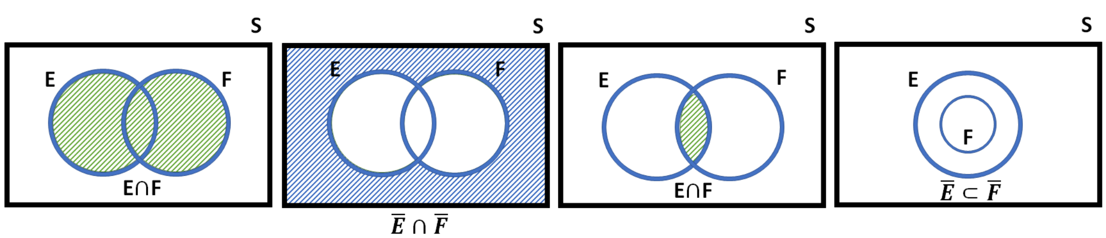
Differential Equations

Chapter 5

Probability

5.1 Introduction and Basics

- **Random Experiment** : An experiment in which the result is unpredictable, but the possibilities of the result is known. Eg. When you throw a coin, you don't know its gonna be head or tail. But you know it will be either one of them.
- **Sample Space(S)** : A set of all possible outcomes of a random experiment is called its sample space. For the above mentioned coin toss example the sample space is s:{H,T} $\boxed{P(s) = 1}$
- **Event** : A subset of the sample space is called as event. For the above mentioned example, the event can be in which the coin lands as H, or the event can be in which the coin lands as tail. Another example: Throwing a fair dice, S={1,2,3,4,5,6}. An event can be in which the number on the top of the dice will be even, So $\implies$ {2,4,6}. $\boxed{0 \leq P(E) \leq 1}$
  - **Simple event:** Only one element is possible in the event. Eg. coin toss $\implies$ {H} or {T}
  - **Compound event:** More than one element is possible in the event. Eg. Even number on the dice top.
- **Union($\cup$)** Let S={1,2,3,4,5,6,7,8,9,10} If three events are described,say:
  - A = should be a multiple of 2 = {2,4,6,8,10}
  - B = should be a multiple of 3 = {3,6,9}
  - C = Either a multiple of 2 or 3 = {2,3,4,6,8,9,10}
  - The event C can be written as $(\boldsymbol{A} \cup \boldsymbol{B})$ as it contains all the elements in A as well as all the elements in B
- **Intersection($\cap$)** For the same sample space mentioned above, let there be a fourth event D such that D = should be a multiple of both 2 and 3 = {6}. Then D can be written as $(\boldsymbol{A} \cap \boldsymbol{B})$



- **Permutations(^nP_r)**
  - Permutation means **Arrangement** $\boxed{^nP_r = \dfrac{n!}{(n-r)!}}$ $\Longleftarrow$ (Ways to arrange r things out of n things)
- **Combinations(^nC_r)**
  - Combination means **Selection** $\boxed{^nC_r = \dfrac{n!}{(n-r)!r!} = \dfrac{^nP_r}{r!}}$ $\Longleftarrow$ (Ways to select r things out of n things)

5.2 Types of Events

5.2.1 Complementary events

- If S={1,2,3,4,5} and A = even numbers - {2,4}, then the complimentary even of A is denoted by $\bar{A}$={1,3,5}

5.2.2 Equally likely events

- Two events are equally likely events when the probability for both the events are same. For Eg. Getting a head or a tail on a coin toss is 50/50

5.2.3 Mutually Exclusive events

- Two events are mutually exclusive if when one event occurs, the other cannot occur simultaneously. For eg. you can get only a head or a tail but not both on a coin toss on a single try.
- $\boxed{p(E_1 \cup E_2) = p(E_1) + p(E_2)} \Longleftarrow$ (E_1 and E_2 are mutually exclusive) $\Longrightarrow \boxed{p(E_1 \cap E_2) = 0}$

5.2.4 Collectively Exhaustive events

- If the union of two events give the sample space, then they are called collectively exhaustive events

5.2.5 Independent events

- When the occurrence of one event doesn't affect the occurrence of another event. For eg. In a bag of 5 red balls and 3 black balls, if two balls are drawn at random one at a time **with replacement**, then the probability of getting say 2 red balls is not affected by whether you got red ball on the first pick or not.
- If it had been **without replacement**, then the probability will change
- If two events are independent, then conditional probability becomes marginal probability
- $\Longrightarrow p(A/B) = p(A), \;\; p(B/A) = p(B) \;\; \boxed{p(A \cap B) = p(A) * p(B)}$

5.3 De Morgan's Law

- $\boxed{\overline{(E_1 \cup E_2)} = (\bar{E_1} \cap \bar{E_2})}$ $\boxed{\overline{(E_1 \cap E_2)} = (\bar{E_1} \cup \bar{E_2})}$ $\boxed{p(\bar{E_1} \cap \bar{E_2}) \; i.e., (\; Neither \; E_1 \; nor \; E_2) = 1 - p(E_1 \cup E_2)}$

5.4 Approaches to Probability

5.4.1 Classical Approach

- $\boxed{p(E) = \frac{n(E)}{n(S)}} \Longleftarrow$ (Classical approach assumes that all outcomes are equally likely)

5.5 Rules of Probability

1. $\boxed{p(A \cup B) = p(A) + p(B) - p(A \cap B)} \Longleftarrow$ **(Inclusion-Exclusion rule)**
2. $\boxed{p(A \cup B) = p(A) * p(B/A) = p(B) * p(A/B) \;\; \Longleftarrow}$ **(Conditional Probability)**
   - Here, p(A) and p(B) are called Marginal probabilities
   - p(A/B) and p(B/A) are called Conditional probabilities
   - p(A/B) = Probability of occurrence of A when B has already occurred
   - p(B/A) = Probability of occurrence of B when A has already occurred

- Conditions for three events A,B,C to be independent:
  - $\boxed{\text{p(ABC) = p(A)p(B)p(C)}}$ $\boxed{\text{p(AB) = p(A)p(B)}}$ $\boxed{\text{p(BC) = p(B)p(C)}}$ $\boxed{\text{p(AC) = p(A)(C)}}$

3. $\boxed{p(A) = 1 - p(\bar{A})}$ $\boxed{p(A) + p(\bar{A}) = 1}$ $\Longleftarrow$ **(Complimentary probability)**

4. $\boxed{p(A/B) = \dfrac{p(A \cap B)}{p(B)}}$ $\boxed{p(B/A) = \dfrac{p(A \cup B)}{p(A)}}$ $\Longleftarrow$ **(Conditional probability - multiplication rule)**

5. $\boxed{p(E) = p(A \cap E) + p(B \cap E) = p(A) * (p(E/A) + p(B) * p(E/B))}$ $\Longleftarrow$ **(Total probability)**
   - If an event E can occur in two ways A and B, then the total probability for the occurrence of E is the sum the probability of it happening by A and the probability of E happening by B.

6. $\boxed{p\left(E_i/A\right) = \dfrac{p(E_i \cap A)}{A} = \dfrac{p(E_i) * p(E_i/A)}{\sum_{k=1}^{n} p(E_k).p(A/E_k)}}$ $\Longleftarrow$ **(Baye's Theorem)**

Chapter 6

Statistics

6.1 Basics and Introduction

- A branch of mathematics that gives us the means to work with large datasets and derive meaningful results from it
- Descriptive measures:
  - Measure of Central tendency: indicates the avg value of data
  - Measure of dispersion: denotes about the extent to which data items deviate from the central tendency value. In other words, "It quantifies the variation in data"

6.2 Arithmetic Mean($\bar{x}$)

- $\boxed{\bar{x} = \frac{\sum x}{n}} \Longleftarrow$ (for raw data) $\boxed{\bar{x} = \frac{\sum (f_i x_i)}{\sum f_i}} \Longleftarrow$ (for Grouped data)

6.3 Median

6.3.1 Median for Raw data

- First arrange the elements in Ascending order
- $\boxed{\left(\frac{n+1}{2}\right)^{th} term} \Longleftarrow$ (n=odd) $\boxed{\frac{\left(\frac{n}{2}\right)^{th} term + \left(\frac{n}{2}+1\right)^{th} term}{2}} \Longleftarrow$ (n=even)

6.3.2 Median for Grouped data

$$\text{Median} = \boxed{L + \left[\frac{\left(\frac{N+1}{2}\right) - (F+1)}{f_m}\right] * h}$$

L = Lower limit of median class

N = Total number of data items = $\sum F$

F = cumulative frequency of the class immediately preceding the median class

f_m = Frequency of the median class

h = width off the median class

- Consider the following example for calculation of Median in grouped data:

Mark Range	f (No. of students)	Cumulative frequency
(00-20)	02	02
(21-40)	03	05
(41-60)	10	15
(61-80)	15	30
(81-100)	20	50

- In the above example, $\left(\frac{N+1}{2}\right) = \frac{(2+3+10+15+20)+1}{2} = 25.5$
- Class (61-80) is the median class as $30 > 25.5$
- Here, $\boxed{\text{L=61}}$ $\boxed{\text{h=20}}$ $\boxed{\text{F=15}}$ $\boxed{f_m = 15}$ So, Median $= 60 + \frac{25.5-(15+1)}{15} * 20 = 69.66 \approx 69.7$

6.4 Mode

- Mode is the value that occurs most frequently in the data. If more than one data is occuring most frequently, then both of them are mode.

6.4.1 Mode of Raw data

- Found using observation

6.4.2 Mode of Grouped data

$$\boxed{Mode = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} * h}$$

L = Lower limit of the modal class
f_0 = Modal frequency i.e. Largest frequency
f_1=Frequency in the class preceding the modal class
f_2 = Frequency in the class succeeding the modal class
h = width of the class

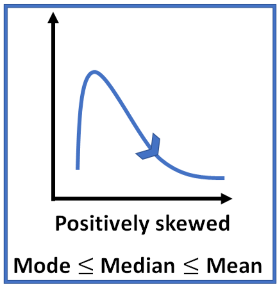
- Consider the following example for calculation of Mode in grouped data:

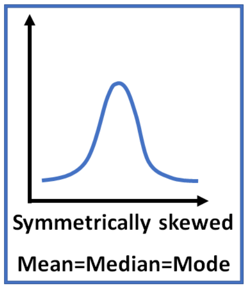
Height(in ft)	**Number of students**
3.0 - 3.5	12
3.6 - 4.0	37
4.1 - 4.5	79
4.6 - 5.0	152
5.1 - 5.5	65
5.6 - 6.0	7
Total	352

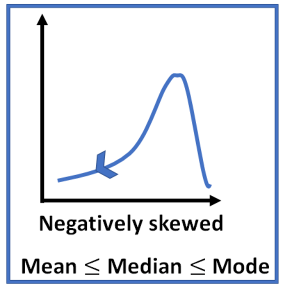
- Since, 152 is the largest frequency, the modal class is (4.6 - 5.0)
- Thus, $\boxed{\text{L=4.6}}$ $\boxed{f_0 = 152}$ $\boxed{f_1 = 79}$ $\boxed{f_2 = 65}$ $\boxed{\text{h = 0.5}}$ Mode $= 4.5 * \frac{152-79}{2(152)-79-65} * 0.5 \approx 4.73$

6.5 Properties relating Mean, Median and Mode

- When an approx value of mode is required: $\boxed{Emprical\ mode = 3 * (Median) - 2 * (Mean)}$







6.6 Standard Deviation and Variance

6.6.1 Standard Deviation for RAW data

- It is a measure of variation among the data
- Consider there are five values: x_1, x_2, x_3, x_4, x_5 The mean $\bar{x} = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$.
- The individual deviations are $(x_1 - \bar{x}),\ (x_2 - \bar{x}),\ (x_3 - \bar{x}),\ (x_4 - \bar{x}),\ (x_5 - \bar{x})$
- **Variance(σ^2)** $=$ Mean of Square of each deviation of data $=$ $\boxed{\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$
- **Standard deviation(σ)** $=$ $\boxed{\sqrt{Variance} = +\sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$

6.6.2 Standard Deviation for Grouped data

- $\boxed{Arithmetic\ Mean(\bar{x}) = \dfrac{\sum(f_i x_i)}{\sum f_i}}$ $\boxed{Variance(\sigma^2) = \sum_{i=1}^{n}\left(\dfrac{f_i(x_i - \bar{x})^2}{f_i}\right)}$

6.7 Coefficient of Variation

- The standard deviation is an absolute measure of variation and hence cannot be used for comparison of variation between 2 different data sets with different mean.
- For the above purpose a relative measure of variation called, **Coefficient of Variation(CV)** is used. $\boxed{CV = \dfrac{\sigma}{\bar{x}}}$.

  CV is often represented as a percentage

6.8 Probability Distribution

6.8.1 Random Variable

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6.8.2 Discrete Random Variable

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6.8.3 Continuous Random Variable

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6.8.4 Probability Density Function

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6.8.5 Probability Mass Function

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6.9 Distributions

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6.9.1 Properties of Discrete Distribution

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6.9.2 Properties of Continuous Distribution

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6.10 Binomial Distribution

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6.11 Hypergeometric Distribution

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6.12 Geometric Distribution

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6.13 Poisson Distribution

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6.14 General Continuous Distribution

•

6.15 Uniform Distribution

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6.16 Exponential Distribution

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6.17 Normal Distribution

•

6.18 Standard Normal Distribution

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Chapter 7

Numerical Methods

Chapter 8

Complex Numbers

Chapter 9

Transform Theory