Artificial Intelligence and Machine Learning

Assignment 01

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Solution 1 : Fisher's Linear Discriminant

- **Estimation (1.1):** Given RGB image dataset i build a four class classifier using Fisher's Linear Discriminant.

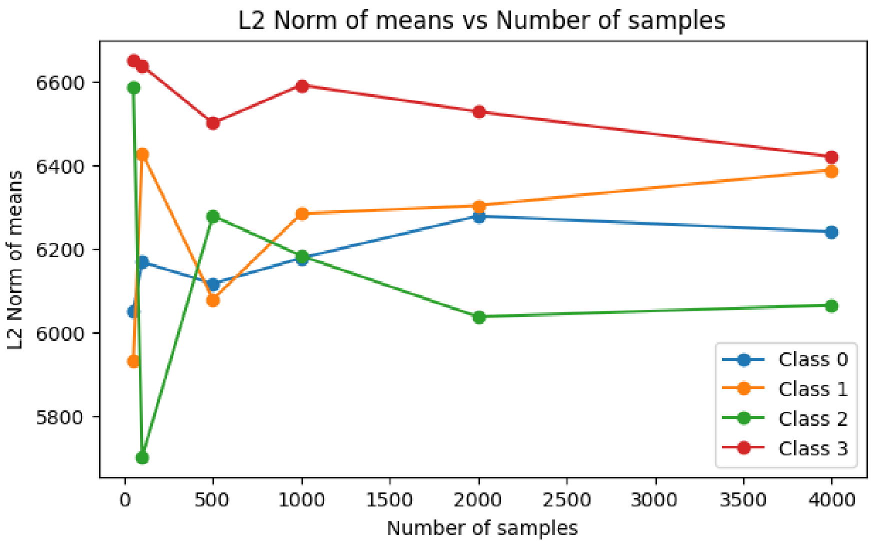
  From oracle using my SR Number i got a tuple with 5 elements as follows

  - Tuple with two elements representing attributes of the dataset *('Mouth_Slightly_Open', 'Smiling')*
  - Train Image dataset ($dim = 20000 \times 3 \times 32 \times 32$)
  - Train Label dataset ($dim = 20000 \times 1$)
  - Test Image dataset ($dim = 1000 \times 3 \times 32 \times 32$)
  - Test Label dataset ($dim = 1000 \times 1$)

  After this i caclulated L_2 norm of mean and $Frobenius\ Norm$ of covarience matrices for each class taking subsamples of sizes $n = 50, 100, 500, 1000, 2000, 4000$ also the plot of L_2 norm of mean and $Frobenius\ Norm$ of covarience matrices for each class is shown below.

  if the mean vector is $x = [x_1, x_2 \ldots x_d]$ and the covarience matrix is $C \in \mathbb{R}^d$ with C_{ij} being element corrosponding to i^{th} row and j^{th} column of covarience matrix then

$$L_2\ Norm = \sqrt{\sum_{i=1}^{d}(x_i)^2} \qquad Frobenius\ Norm = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d}(C_{ij})^2}$$



- **FLD Implementation(1.2.a) :** For each of 4 class with number of subsamples $n = 2500, 3500, 4000, 4500, 5000$

- **Theory :** Fisher's Linear Discriminant is a dimensionality reduction technique that finds the linear combination of features that maximizes the separation between multiple classes. It does this by maximizing the between-class variance while minimizing the within-class variance.

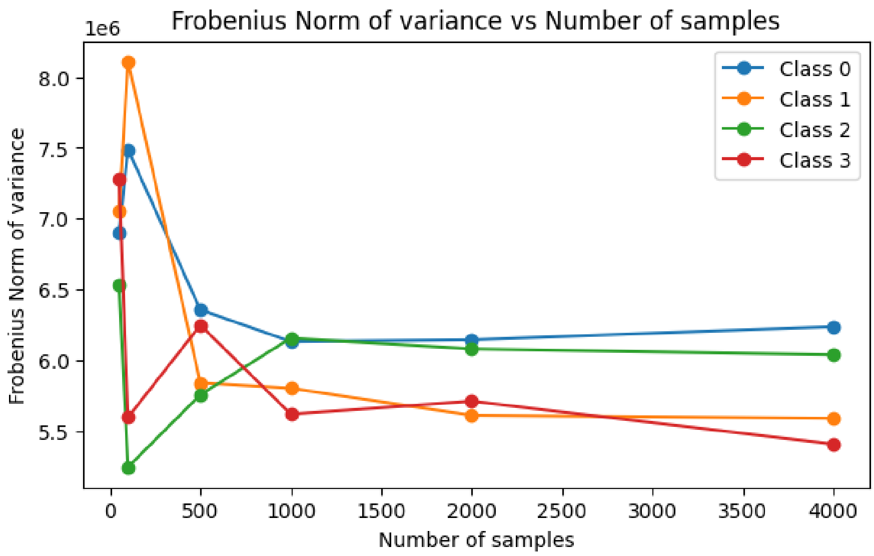
  Suppose we have C classes and d features & the within-class scatter matrix S_W is defined as follows

$$S_W = \sum_{i=1}^{C} S_i$$

  where S_i is the scatter matrix for i^{th} class

$$S_i = \sum_{x \in X_i}(x - m_i)(x - m_i)^T$$

  where X_i is the set of samples for i^{th} class and m_i is the mean vector for i^{th} class.



and between-class scatter matrix S_B is defined as follows

$$S_B = \sum_{i=1}^{C} N_i(m_i - m)(m_i - m)^T$$

where m is the overall mean vector and N_i is the number of samples in i^{th} class.

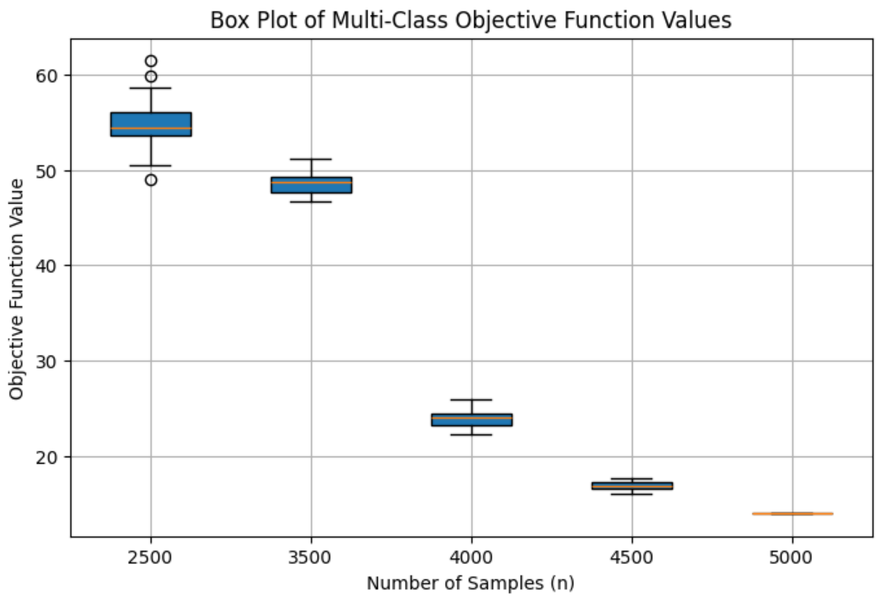
The Fisher's Linear Discriminant is defined as the vector w that maximizes the ratio of between-class scatter to within-class scatter.

$$w = argmax \left(\frac{w^T S_B w}{w^T S_W w} \right)$$

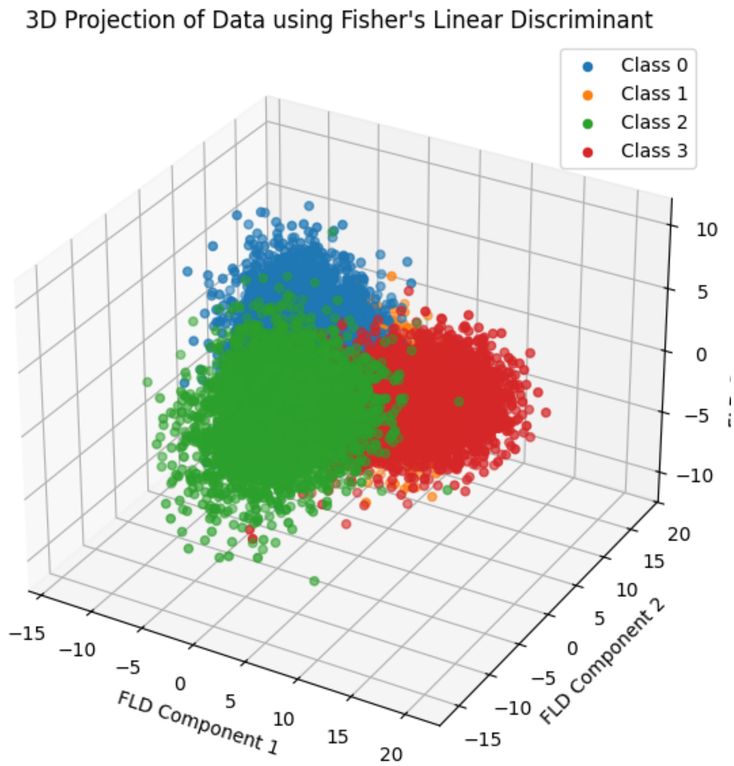
Objective function value of fisher's linear discriminant is defined as follows

$$J(w) = \frac{|w^T S_B w|}{|w^T S_W w|} \quad \text{where } |X| \text{ represents determinant of } X$$

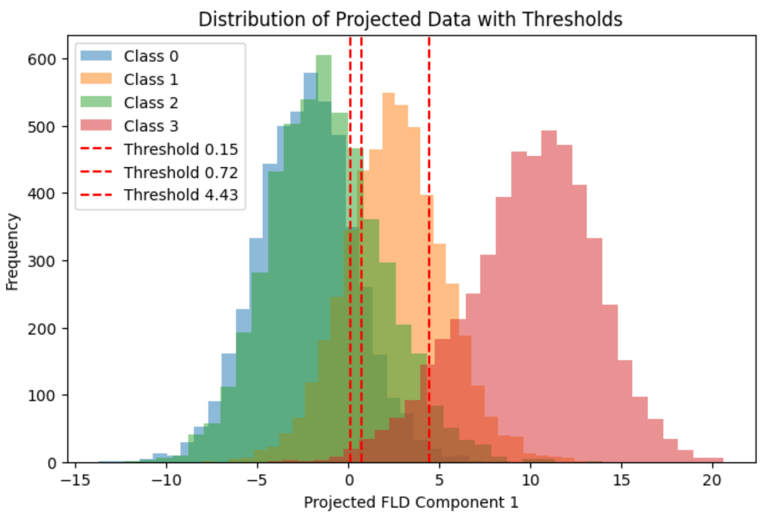
- **Box Plot** for the multiclass objective value of the Fisher's Linear Discriminant for different n.



- **Projections** of 5000 points from training dataset in $3D$.



- **Threshold Report** for the given dataset is as follows



- **Accuracy on Test Data(1.2.b) :** 74.20% (Please refer Code)

Solution 2 : Bayes Classification

- **Theory :** Bayes Classifier is a probabilistic model that makes a decision based on the probability of an event. It is based on Bayes' theorem.

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

$$\eta(x) = \frac{P(Y = 1 \mid X = x) \times P(Y = 1)}{P(X = x)}$$

$$1 - \eta(x) = \frac{P(Y = 0 \mid X = x) \times P(Y = 0)}{P(X = x)}$$

and hence we classify x as

$$\text{Classifier} = \begin{cases} 1 & \text{if } \eta(x) \geq \frac{1}{2} + \epsilon \\ 0 & \text{if } \eta(x) \leq \frac{1}{2} - \epsilon \\ \text{Reject} & \text{otherwise} \end{cases}$$

Equivalently we can say classify x as 1 if

$$\frac{\eta(x)}{1 - \eta(x)} \geq \frac{\frac{1}{2} + \epsilon}{\frac{1}{2} - \epsilon} \quad \text{or} \quad log\left(\frac{\eta(x)}{1 - \eta(x)}\right) \leq log\left(\frac{\frac{1}{2} + \epsilon}{\frac{1}{2} - \epsilon}\right)$$

and classify x as 0 if

$$\frac{\eta(x)}{1 - \eta(x)} \leq \frac{\frac{1}{2} - \epsilon}{\frac{1}{2} + \epsilon} \quad \text{or} \quad log\left(\frac{\eta(x)}{1 - \eta(x)}\right) \geq log\left(\frac{\frac{1}{2} - \epsilon}{\frac{1}{2} + \epsilon}\right)$$

Rejce the sample if none of the above conditions are satisfied & $log\left(\frac{\eta(x)}{1-\eta(x)}\right)$ can be calculate as follows

$$log\left(\frac{\eta(x)}{1 - \eta(x)}\right) = log\left(\frac{P(Y = 1 \mid X = x) \times P(Y = 1)}{P(Y = 0 \mid X = x) \times P(Y = 0)}\right)$$

$$= -\frac{1}{2}(x - \mu_1)^T C_1^{-1}(x - \mu_1) + \frac{1}{2}(x - \mu_2)^T C_2^{-1}(x - \mu_2) - \frac{1}{2}log|C_1| + \frac{1}{2}log|C_2|$$

- **Modified Bayes Classifier(2.1 , 2.2) :** From oracle for question 2 i got two element tuples as follows
  - Training dataset ($dim = 4800 \times 785$)
  - Test dataset ($dim = 800 \times 785$)

  where the first column is labels corrosponding to each of 784 features.

  we are aksed to implement a modified bayes classifier as follows

  $$h_\epsilon(x) = \begin{cases} 1 & \text{if } \eta(x) \geq \frac{1}{2} + \epsilon \\ 0 & \text{if } \eta(x) \leq \frac{1}{2} - \epsilon \\ \text{Reject} & \text{otherwise} \end{cases}$$

  where $\eta(x) = P(Y = 1|X = x)$ and $\epsilon \in (0, 0.5)$ is a threshold parameter.

  for each $\epsilon \in \{0.01, 0.1, 0.25, 0.4\}$ we have to find the following on the dataset
  - Misclassification Loss among the non-rejected samples
  - Number of rejected samples

  In next part we have to do the same for but with different split of training (60-40 , 80-20 , 90-10 , 99-1).

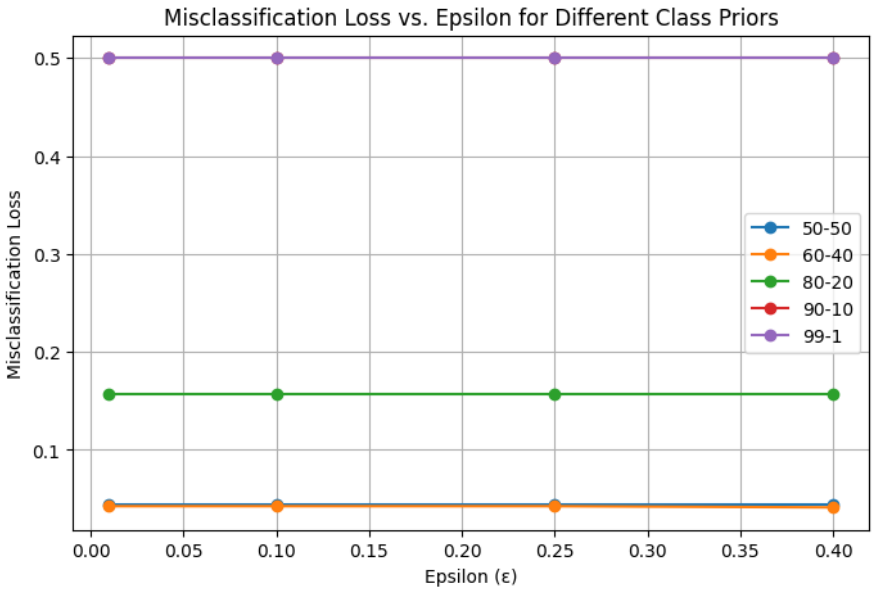
  I have implemented both simulteneously and got the following results.
  - **Misclassification Loss**

split/ϵ	**0.01**	**0.1**	**0.25**	**0.4**
50-50	0.04375	0.04375	0.04375	0.04375
60-40	0.0425	0.0425	0.0425	0.04130162703379224
80-20	0.1575	0.1575	0.1575	0.1575
90-10	0.5	0.5	0.5	0.5
99-1	0.5	0.5	0.5	0.5

  - **Number of Rejected Samples**

- **K-Fold Cross Validation (2.3.a , 2.3.b) :** In this part we have to implement K-Fold Cross Validation for the given dataset and we are allowed to use the inbuilt function *K-Fold* from *sklearn.model_selection*.

  You can refer the code for the implementation of K-Fold Cross Validation in the code file. I got the following results after implementing K-Fold Cross Validation.



split/ϵ	**0.01**	**0.1**	**0.25**	**0.4**
50-50	0	0	0	0
60-40	0	0	0	0
80-20	0	0	0	0
90-10	0	0	0	0
99-1	0	0	0	0

- Recall : 0.9300339683788492
- Precision : 0.9510064802802078
- F1 Score : 0.9403905714487368
- Accuracy : 0.9410416666666667
- Confusion Matrix

$$CM = \begin{bmatrix} TN & FP \\ FN & TP \end{bmatrix} = \begin{bmatrix} 451 & 25 \\ 33 & 451 \end{bmatrix}$$

- Misclassification Loss : 0.05895833333333332
- Number of Rejected Samples : 0

Solution 3 : Decision Trees

- **Visualization (3.1) :** You can zoom the following decision tree and see the details.
- **Report (3.2) :** from oracle for question 3 i got (*log_loss, best, 6*) as hyperparameters for Decision Tree Classifier where *log_loss* is the criterion, *best* is the splitter and *6* is the max_depth in decision tree.

  Suppose TP, TN, FP, FN represent True Positive, True Negative, False Positive and False Negative respectively then the following formulas are used to calculate the performance of the model.

$$\text{Precision} = \frac{TP}{TP + FP}$$

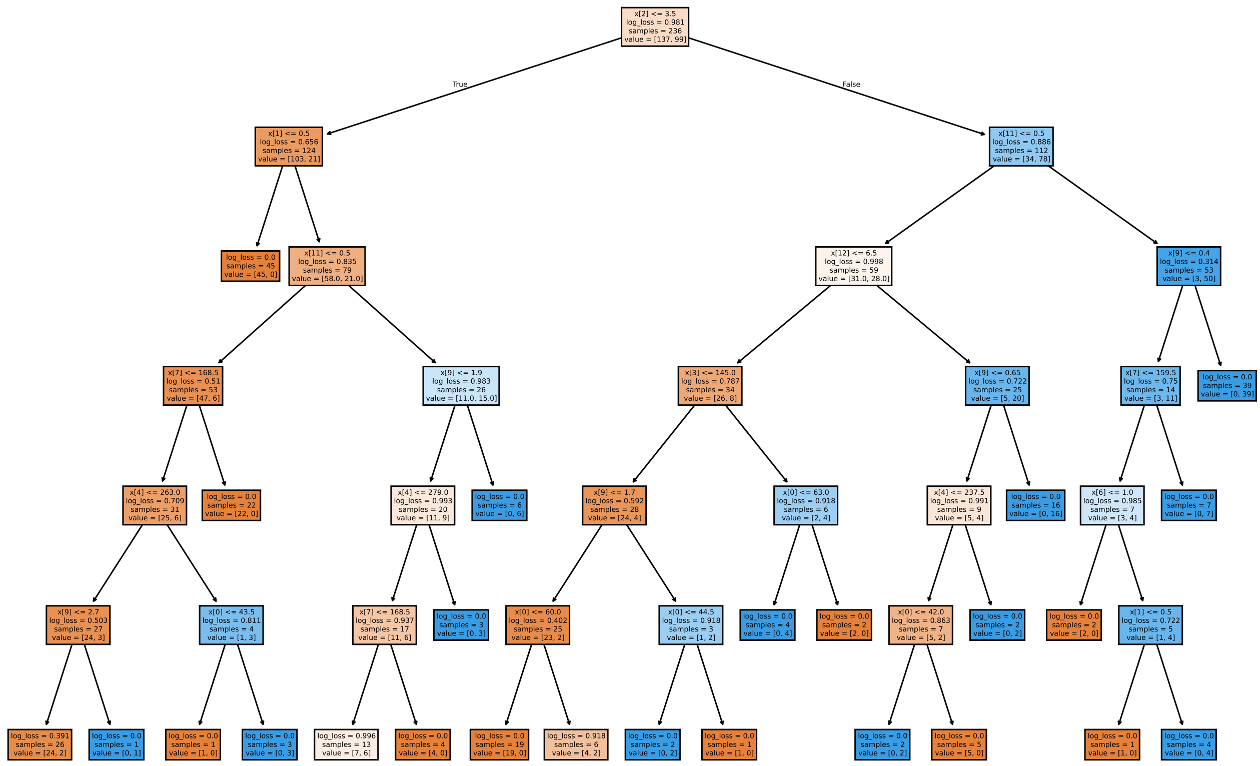
$$\text{Accuracy} = \frac{TP + FN}{TP + FP + TN + FN}$$

$$\text{Recall} = \frac{TP}{TP + FN}$$

$$\text{F1 Score} = \frac{2 * Precision * Recall}{Precision + Recall}$$

after training the model with these hyperparameters i got the following results on test data.

$$TN = 17\ ,\ FP = 5\ ,\ FN = 9\ ,\ TP = 29$$



- Precision : 0.8484848484848485
- Accuracy : 0.75
- Recall : 0.7368421052631579
- F1 Score : 0.7887323943661972
- Training Data Accuracy
  - Before Pruning : 1.0000000000000000
  - After Pruning : 0.9576271186440678
- Confusion Matrix : Suppose C denotes the confusion matrix then

$$C = \begin{bmatrix} TN & FP \\ FN & TP \end{bmatrix} = \begin{bmatrix} 17 & 5 \\ 9 & 29 \end{bmatrix}$$

- **Best Feature (3.3) :** The best feature in a Decision Tree is the one that provides the most informative split, meaning it reduces impurity (Gini or Entropy or log_loss) the most. we can deduce the best feature using feature importance scores and tree structure analysis.

  Feature 2 is the root node in the decision tree which means the decision tree chose it as the first splitting feature because it provides the most information gain hence the most important feature is **Feature 2**.

  Using Inbuilt Python Function $\boxed{\textit{feature\_importances\_}}$ I got the following order of importance of features.

$$2 > 11 > 9 > 0 > 1 > 7 > 4 > 12 > 3 > 6 > 10 > 8$$

  Note : It is 0 Index order! (we can add 1 to each index to get the actual feature number)

Thank You!