UMC 202
PROBLEM SET 6

(1) Use Taylor's series method of order 2 to approximate the solution for each of the following initial value problems and compare the results to the actual values

(a) $y' = \frac{y}{x} - \frac{y^2}{x^2}, \;\; x \in [1, 1.2], \;\; y(1) = 1$ with $h = 0.1, ;$
actual solution $y(x) = \frac{x}{1+ln(x)}$

(b) $y' = \frac{(y^2+y)}{x}, \;\; x \in [1, 3], \;\; y(1) = -2,$ with $h = 0.5, ;$
actual solution $y(x) = \frac{2\,x}{1-2x};$

(c) $y' = -x\,y + \frac{4x}{y}, \;\; x \in [0, 1], \;\; y(0) = 1,$ with $h = 0.25;$
actual solution $y(x) = \sqrt{4 - 3e^{-x^2}}.$

(2) Redo Problem 1 using the Runge Kutta method of order 2.

(3) Redo Problem 1 using the Trapezoidal Method.

(4) Using the Taylor's series method of order 2, Runge Kutta method of order 2 and the Trapezoidal Method to approximate the solutions of the following initial value problems and compare the results

(a) $y' = xe^{3x} - 2y, \;\; x \in [0, 1], \; y(0) = 0$ with $h = 0.5;$

actual solution $y(x) = \frac{1}{5}xe^{3x} - \frac{1}{25}e^{3x} + \frac{1}{25}e^{-2x}.$

(b) $y' = 1 + (x - y)^2, \;\; x \in [2, 3], \; y(2) = 1,$ with $h = 0.5;$

actual solution $y(x) = x + \frac{1}{(1-x)}.$

(c) $y' = 1 + \frac{y}{x}, \;\; x \in [1, 2], \; y(1) = 2$ with $h = 0.25;$

actual solution $y(x) = x\,ln\,x + 2x.$