DHARMSINH DESAI UNIVERSITY, NADIAD
FACULTY OF TECHNOLOGY
B.TECH. SEMESTER-I [CE, IT, EC]
SUBJECT: (BS 101) MATHEMATICS-I

Examination	: Third sessional	Seat No.	: CE 32
Date	: 16/01/2023	Day	: Monday
Time	: 3:45 to 5:00 pm	Max. Marks	: 36

INSTRUCTIONS:
1. Figures to the right indicate maximum marks for that question.
2. The symbols used carry their usual meanings.
3. Assume suitable data, if required & mention them clearly.
4. Draw neat sketches wherever necessary.

Q.1 Do as directed. [12]

CO2 A (a) If $\vec{F} = 2xyz^2\hat{\imath} + [x^2z^2 + z\cos(yz)]\hat{\jmath} + [2x^2yz + y\cos(yz)]\hat{k}$, then find $curl\vec{F}$. If $\vec{F}$ is irrotational then find the scalar potential function. [4]

CO5 U (b) If $w = x + 2y + z^2$, where $x = \frac{u}{v}$, $y = u^2 + e^v$, $z = 2u$ then show that $u\frac{\partial w}{\partial u} + v\frac{\partial w}{\partial v} = 12u^2 + 2ve^v$. [4]

CO2 A (c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. [4]

Q.2 Attempt any *TWO* from the following [12]

CO5 E (a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$ [6]

CO2 E (b) If the directional derivative of $\emptyset = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the surface $x\log z - y^2 + 4 = 0$ at $(-1, 2, 1)$. [6]

CO5 E (c) If $x^2 = au + bv$, $y^2 = au - bv$ then check whether [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u \text{ or not?}$$

Q.3

CO3 C (a) Expand $\log_e x$ in powers of (x – 1) by Taylor's theorem and hence find the value of $\log_e(1.1)$ correct up to four decimal places. [6]

CO5 N (b) Find the extreme values of $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$. [6]

OR

CO3 C (a) Prove that $\cot\left(\frac{\pi}{4} + x\right) = 1 - 2x + 2x^2 - \frac{8x^3}{3} \ldots$ [6]

CO5 N (b) Find the maximum value of $f = x^2y^3z^4$ subject to the condition $x + y + z = 5$ using Lagrange's method of multipliers. [6]