Examination	: SECOND INTERNAL	Seat No.	: 032
Date	: 12/12/2022	Day	: MONDAY
Time	: 1 Hr. 15 Min.	Max. Marks	: 36

INSTRUCTIONS:
1. Figures to the right indicate maximum marks for that question.
2. The symbols used carry their usual meanings.
3. Assume suitable data, if required & mention them clearly.
4. Draw neat sketches wherever necessary.

Q-1   **Do as directed.**

CO3   A   (a)   Verify Roll's theorem for $f(x) = \log\left[\dfrac{x^2 + ab}{(a+b)x}\right]$; $a > 0, b > 0$; $x \in [a, b]$.   [4]

CO1   A   (b)   Evaluate, $\displaystyle\int_0^1 x^m \left[\log\frac{1}{x}\right]^n dx$.   [4]

CO3   A   (c)   Expand $y = \log(x + \sqrt{x^2 + 1})$ up to first four terms by Maclaurin's series.   [4]

Q-2   **Attempt any three.**

CO3   A   (a)   Show that by Lagrange's mean value theorem, $\dfrac{2x}{1-x^2} > \log\left(\dfrac{1+x}{1-x}\right) > 2x$; $0 < x < 1$ [4]

where, $f(x) = \log\left[\dfrac{1+x}{1-x}\right]$; $(o, x]$.

CO3   A   (b)   Verify Cauchy mean value theorem for $f(x) = \sin x$ and $g(x) = \cos x$ in $\left[0, \dfrac{\pi}{2}\right]$.   [4]

CO3   A   (c)   Expand $y = \tan^{-1} x$ in powers of x by Maclaurin's expansion and hence Find series [4] expansion of $y = \tan^{-1}\left\{\dfrac{2x}{1-x^2}\right\}$.

CO3   A   (d)   Expand $y = e^{x \sin x}$ in powers of x up to the term contains x^6.   [4]

Q-3

CO1   A   (a)   State and prove Beta-Gamma relation.   [4]

CO1   A   (b)   Evaluate, $\displaystyle\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx$, in terms of Gamma function.   [4]

CO1   A   (c)   Evaluate, $\displaystyle\int_0^\infty e^{-k^2 x^2}\,dx$.   [4]

OR

Q-3

CO1   A   (a)   Prove that , $\overline{)n}\,\overline{\left)n + \dfrac{1}{2}\right.} = \dfrac{\sqrt{\pi}\,\overline{)2n}}{2^{2n-1}}$.   [4]

CO1   A   (b)   Show that , $\displaystyle\int_0^\infty \dfrac{x^2}{(1+x^4)^3}\,dx = \dfrac{5\pi\sqrt{2}}{128}$.   [4]

CO1   A   (c)   Evaluate, $\displaystyle\int_0^2 x^7 (16 - x^4)^{10}\,dx$.   [4]