DHARMSINH DESAI UNIVERSITY, NADIAD
FACULTY OF TECHNOLOGY
B.TECH. SEMESTER - I [CE /IT /EC]
SUBJECT: (BS101) NAME: MATHEMATICS - I

Examination	: Regular	Seat No	: 032
Date	: 13/02/2023	Day	: Monday
Time	: 2:30 to 5:30pm	Max. Marks	: 60

INSTRUCTIONS:

1. Answer each section in separate answer book.
2. Figures to the right indicate maximum marks for that question.
3. The symbols used carry their usual meanings.
4. Assume suitable data, if required & mention them clearly.
5. Draw neat sketches wherever necessary.

SECTION – I

Q.1 Do as directed. [10]

CO4 A (a) Using Cramer's rule solve the system of equations: $4x + 3y = 12$, $2x + 5y = -8$. [2]

CO1 C (b) Find the value of $m(> 0)$ given that $\beta(m, 2) = \frac{1}{12}$. [2]

CO3 A (c) Using Rolle's theorem, find the value of c for the function $f(x) = x(x + 3)e^{-x/2}$ in $-3 \le x \le 0$. [2]

CO4 A (d) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -4 & 5 \\ 2 & 4 & k \end{bmatrix}$ then find the value of k for which rank(A) = 2. [2]

CO3 C (e) Prove that $\frac{1}{x-1} = 1 - (x-2) + (x-2)^2 - (x-2)^3 + \cdots$ [2]

Q.2 Attempt *Any TWO* from the following questions. [10]

CO1 E (a) Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma function and hence evaluate $\int_0^1 x^5 (1 - x^3)^{10} dx$. [5]

CO1 E (b) State and prove relation between Beta and Gamma function. [5]

CO1 E (c) Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1-x^4}}$ using Beta and Gamma function. [5]

Q.3
CO4 N (a) Investigate for what values of λ and μ, the system of simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) unique solution and (iii) infinite number of solutions. [5]

CO3 A (b) Using Lagrange's mean value theorem, prove that $\left(1 - \frac{a}{b}\right) < \log\frac{b}{a} < \left(\frac{b}{a} - 1\right)$ where $f(x) = log x, x \in [a, b]$ and $0 < a < b$. Hence deduce that $\frac{1}{2} < log 2 < 1$. [5]

OR

Q.3
CO4 N (a) Examine whether the vectors $(1, -1, 1)$, $(2, 1, 1)$, $(3, 0, 2)$ are linearly dependent or independent? If linearly dependent, then find relation between them. [5]

CO3 A (b) Find the expansion of $y = \tan^{-1} x$ in powers of x and hence find the expansion of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ up to the terms of x^5. [5]

SECTION – II

Q.4 Do as directed. [10]

CO2 A (a) If $\vec{F} = (x + 3y^2)\hat{\imath} + (2y + 2z)\hat{\jmath} + (x^2 + az)\hat{k}$ then determine the value of constant a such that $\vec{F}$ is solenoidal. [2]

CO6 C (b) If $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ then find the eigenvalues of A^{-1}, A^{10}. [2]

CO5	A	(c)	If $u = x^2 y$ and $x^2 + xy + y^2 = 1$ then find $\frac{du}{dx}$.	[2]
CO2	A	(d)	Find the gradient of $\emptyset(x, y, z) = e^z + \log(x^2 + y^2)$ at the point (1, 1, 0).	[2]
CO6	N	(e)	Check whether $A = \frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$ is orthogonal matrix or not? If so, find A^{-1}.	[2]
Q.5			Attempt ***Any TWO*** from the following questions.	[10]
CO5	E	(a)	If $u = r^m$ where $r^2 = x^2 + y^2 + z^2$ then prove that $u_{xx} + u_{yy} + u_{zz} = m(m + 1)r^{m-2}$	[5]
CO5	E	(b)	Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$	[5]
CO5	E	(c)	If $w = x + 2y + z^2$, where $x = \frac{u}{v}, y = u^2 + e^v, z = 2u$ then show that $u\frac{\partial w}{\partial u} + v\frac{\partial w}{\partial v} = 12u^2 + 2ve^v$.	[5]
Q.6 CO6	A	(a)	Find the eigenvalue and eigenvector for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.	[5]
CO2	A	(b)	Find the directional derivative of $x^2y^4 + z^2y^4 + x^2z^4$ at the point (2, 0, 3) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1).	[5]

OR

Q.6 CO6	A	(a)	Diagonalize the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ and hence find the matrix A^5.	[5]
CO2	A	(b)	Show that $\vec{F} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational and hence find the scalar function $\emptyset$, such that $\vec{F} = \nabla\emptyset$.	[5]