Assignment No:- 02

0901CS213D06
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Q2 Prove that in a distributive lattice, if an element has a complement then this complement is unique.

Solution:

let $(L, \leq, \wedge, \vee)$ be a bounded distributed lattice.

let b and c be two complement of $a \in L$, then

$a \vee b = 1,$

$a \wedge b = 0$

$a \vee c = 1,$

$a \wedge c = 0$

we have $b = b \wedge 1 \quad (a \vee c = 1)$

$b = b \wedge (a \vee c)$

$= (b \wedge a) \vee (b \wedge c)$

$= 0 \vee (b \wedge c) \quad \because \{0 = a \wedge c\}$

$= (a \wedge c) \vee (b \wedge c)$

$= (a \vee b) \wedge c$

$= 1 \wedge c$

$= \underline{c}$ Prove

Q2 Prove that every chain is a Distributed lattice

Solution: we know that by distribute of chain is a linearly ordered set (or totally ordered set) Every pair of element is this set (i.e in chain)

Poses a least upper bound and a greatest lower bound in the set such ~~the~~ a chain is a lattice let $(L, \cap, \cup)$ be a chain ~~is~~ ~~a~~ ~~lattice~~ ~~set~~ and a, b, c be any three element of L. so there arises two possible cases.

Case 1! $a \leq b$ or $a \leq c$ are case segment $b \leq a$ and $c \leq a$ in case (i) by defination

$$a \cap (b \cup c) = a \quad \text{and}$$
$$(a \cap b) \cup (a \cap c) = a.$$

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \quad \cdots (1)$$

in case (ii) by definition we have $a \cap (b \cup c)$

$= b \cup c$ and $(a \cap b) \cup (b \cap c) = b \cup c$

$a \cap (b \cup c) = (a \cap b) \cup (b \cap c)$ - (2)

from Equation (1) and (2) we get

$a \cup (b \cup c) = a \cup b$

$a \cup (b \cup c) = (a \cap b) \cup (a \cup c)$

both cases are distributed law holds

similarly the second distributed law

$a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$ also

holds by the principle of dual hence

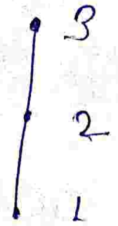
chain is a distributed lattice.

Q3 Define Hasse diagram? Let $A = \{a, b, c, d\}$ and $P(A)$ its power set. Draw Hasse diagram of $(P(A), \subseteq)$

Hasse diagram : Hasse diagram is an a diagrammatic representation of a finite partial order on a set. In this diagram, the element are shown as vertices (or dots) two related vertices in the Hasse diagram of a partial order are connected by a line if and only if they are related.

Let $(P, \leq)$ be a po-set. an element $b \in P$ is said to be cover $a \in P$ if $a \subset b$ and if there does not exist any element $c \in P$ such that $a \subseteq c$ and $a \subseteq b$. If 'b covers a' then a line is drawn between the elements a and b in the Hasse diagram.

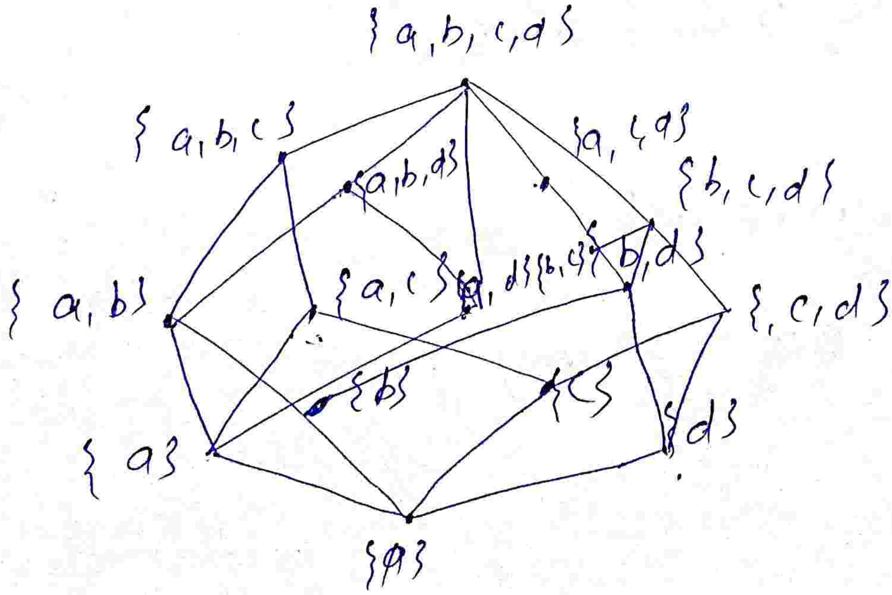
Eg let $A = \{1, 2, 3\}$ and $\leq$ be relation "less than or equal to" on A. then the Hasse diagram of po-set $(A, \leq)$ is



Draw Hasse diagram:

$A = \{a, b, c; d\}$.

$P(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\},$
$\{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\},$
$\{b,c,d\}, \{a,b,c,d\} \}$



Q Let $L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation '|' where $x | y$ means 'x' divides y. Show that D_{24} the set of all divisors of integer 24 of L is a sub-lattice of lattice $(L, |)$

Solution : The given lattice

$L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ which is partially ordered by the relation $|$

the set of all divisors of 24 $\in L$ in the set D_{24}

$D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$x \wedge y = (\text{H.C.F of } x, y)$

$x \vee y = (\text{L.C.M of } x, y)$

H.C.F

$\wedge$	1	2	3	4	6	8	12	24
1	1	1	1	1	1	1	1	1
2	1	2	1	2	2	2	2	2
3	1	1	3	1	3	1	3	3
4	1	2	1	4	2	4	4	4
6	1	2	3	2	6	2	6	6
8	1	2	1	4	2	8	4	8
12	1	2	3	4	6	4	12	12
24	1	2	3	4	6	8	12	24

L.C.M.

$\vee$	1	2	3	4	6	8	12	24
1	1	2	3	4	6	8	12	24
2	2	2	6	4	6	8	12	24
3	3	6	3	12	6	24	12	24
4	4	4	12	4	12	8	12	24
6	6	6	6	12	6	24	12	24
8	8	8	24	8	24	8	24	24
12	12	12	12	12	12	24	12	24
24	24	24	24	24	24	24	24	24

Since all Entry of Composition tables of meet $\wedge$ and join $\vee$ are Element D_{24} so for Each pair of Element $x, y \in D_{24}$ we have $x \wedge y$, $x \vee y \in D_{24}$.

The Binary operation meet ($\wedge$) and join ($\vee$) are closed in D_{24}

Hence (D_{24} , '|') is Sublattice of the lattice (L, '|')

Proved

Q Define: lattice, sub-lattice, Distributive lattice, chain, Complete lattice, complemented lattice, Complemented Complete lattice, supremum Infimum.

Lattice:. Let L be a non-Empty set closed under two binary operations called meet and join denoted by $\wedge$ and $\vee$ then L is called lattice, if the following rules hold where a, b, c any element.

[L1] Commutative law

ⓐ $a \wedge b = b \wedge a$

ⓑ $a \vee b = b \vee a$.

$\forall$ $a, b \in L$

(L2) absorption laws;

(d) $a \vee (a \wedge b) = a$

(d') $a \wedge (a \vee b) = a$.

[L3] Idempotent laws:

(a) $a \vee a = a$

(a') $a \wedge a = a$

[L4] association laws:

(c) $a \vee (b \vee c) = (a \vee b) \vee c$

(c') $a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

Sub lattice :- Let $(L, \wedge, \vee)$ be a lattice. A subset M of L is set to be sub lattice of L if M is closed with respect to Meet $(\wedge)$ and Join $(\vee)$ Each point of element $x, y \in M$, $x \wedge y$ and $x \vee y$ are contained in M.

Distributed lattice : let $L(L, \wedge, \vee)$ be a lattice then L is set to be distributed if for any elements.

$a, b, c \in L$ we have the following law.

1) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

2) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Complete Latice : Let $(L, \wedge, \vee)$ be a lattice then L is said to be complete if every subset A (finite infinite) of ~~LAA~~ ~~and~~ L $\wedge A$ and $\vee A$ Exist in L thus in every complete latices $(L, \wedge, \vee)$ there Exist there a greatest element b and a least element l.

Complemented Lattice :- let $(L, \wedge, \vee)$ be a lattice with universal bound 0 and 1 the lattice L is said to be Complemented Lattice if every element in L has a Complement that is $a \vee 1 = 1$,

$a \wedge 1 = a$,

$a \wedge 0 = 0$

$a \vee 0 = a$

$a \wedge a' = 0$

$a \vee a' = 1$

Where $a \leq 0 \leq 1$

$\forall\ 0 \in L$ so a' is the Complement $a \in L$.

Complement Complete Lattice :- Let $(L, \wedge, \vee)$ be a Complete lattice with greatest and lowest element 'g' and L respectively then 'L' is called Complemented Complete lattice if for Each Element $a \in L$;

Infimum: In mathematics, the infimum (abbreviated, inf, plural infima) of a subset S of a partially ordered set P is greatest element in P the is less than or equal to all elements of s, such an element exists.

Superymum: The supermum (abberviated sup; plural suprema) of a subset S of a partially ordered set P is the least element in P that is greater than or Equal to all elements of S, if such an Element exists.