Assignment No- 02

0901CS213D06
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Q2 Prove that in a distributive lattice, if an element has a complement then this complement is unique.

Solution:

let $(L, \leq, \wedge, \vee)$ be a bounded distributed lattice.

let b and c be two complement of $a \in L$, then

$a \vee b = 1,$
$a \wedge b = 0$
$a \vee c = 1,$
$a \wedge c = 0$

we have $b = b \wedge 1 \quad (a \vee c = 1)$
$b = b \wedge (a \vee c)$
$= (b \wedge a) \vee (b \wedge c)$
$= 0 \vee (b \wedge c) \quad \because \{0 = a \wedge c\}$
$= (a \wedge c) \vee (b \wedge c)$
$= (a \vee b) \wedge c$
$= 1 \wedge c$
$= c$ Prove

Q2 Prove that every chain is a Distributed lattice

Solution: we know that by distribute of chain is a linearly ordered set (or totally ordered set) Every pair of element in this set (i.e in chain)

Pases a least upper bound and a greatest lower bound in the set such the a chain is a lattice let $(L, \cap, \cup)$ be a chain is a lattice set and a, b, c be any three element of L. so there arises two possible cases.

case 1! $a \leq b$ or $a \leq c$ are case segment $b \leq a$ and $c \leq a$ in case (i) by definition

$$a \cap (b \cup c) = a \text{ and}$$

$$(a \cap b) \cup (a \cap c) = a.$$

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \quad \cdot (1)$$

in case (ii) by defination we have $a \cap (b \cup c) = b \cup c$ and $(a \cap b) \cup (b \cap c) = b \cup c$

$a \cap (b \cup c) = (a \cap b) \cup (b \cap c)$ - (2)

from Equation (1) and (2) we get

$a \cup (b \cup c) = a \cup b$

$a \cup (b \cup c) = (a \cap b) \cup (a \cup c)$

both cases ane distributed law holds

Similarly the second distributed law

$a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$ also

holds by the principle of dual hence

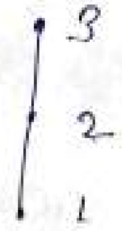
chain is a distributed lattice.

Q3 Define Hasse diagram? Let $A = \{a, b, c, d\}$ and $P(A)$ is power set. Draw Hasse diagram of $(P(A), \subseteq)$

Hasse diagram : Hasse diagram is an a diagrammatic representation of a finite partial order on a set. In this diagram, the element are shown as vertices (or dots) two related vertices in the Hasse diagram of a partial order are connected by a line if and only if they are related.

Let $(P, \leq)$ be a poset. an element $b \in P$ is said to be cover $a \in P$ if $a < b$ and if there does not exist any element $c \in P$ such that $a \leq c$ and $a \leq b$. If 'b covers a' then a line is drawn between the elements a and b in the Hasse diagram

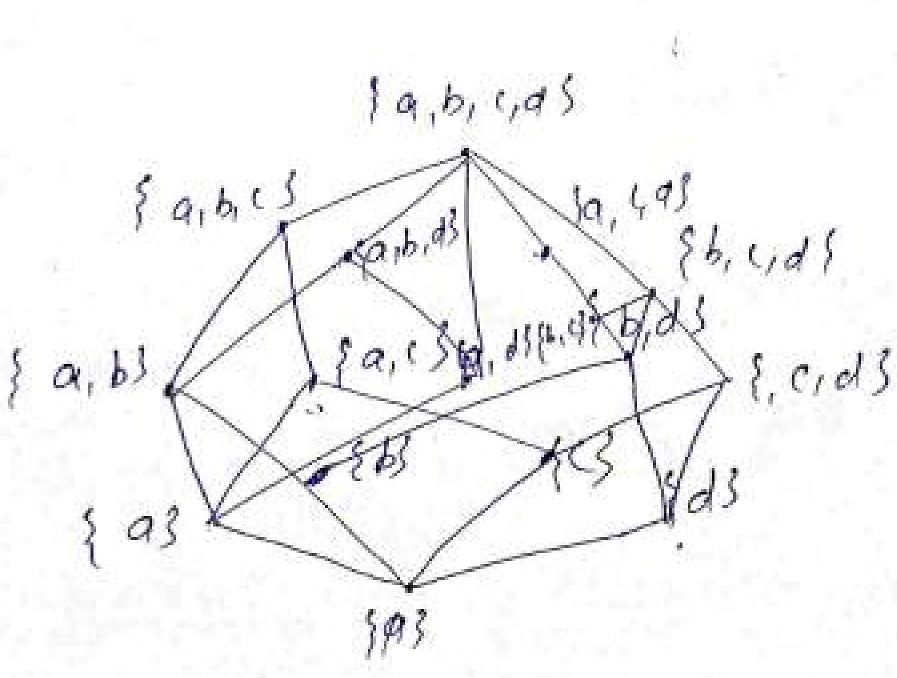
Eg Let $A = \{1, 2, 3\}$ and $\leq$ be relation "less than or equal to" on A. then the Hasse diagram of po-set $(A, \leq)$ is



Draw Hasse diagram:

$A = \{a, b, c, d\}$.

$P(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\},$
$\{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\},$
$\{b,c,d\}, \{a,b,c,d\} \}$



Q Let $L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be

order by the relation '|' where $x | y$

means 'x' divides y show that D_{24} the

set of all divisors of integer 24 of L

is a sub lattice of lattice $(L, |)$

Solution : The given lattice

$L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ which is

Partially ordered by the relation $|$

the set of all divisors of $24 \in L$

in the set D_{24}

$D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$x \cap y = (\text{H.C.F of } x, y)$

$x \cup y = (\text{L.C.M of } x, y)$