Q Let $L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be

order by the relation '|' where $x|y$

means 'x' divides y show that D_{24} the

set of all divisors of integer 24 of L

is a sub lattice of lattice $(L, |)$

Solution: The given lattice

$L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ which is

Partially ordered by the relation $|$

the set of all divisor of $24 \in L$

in the set D_{24}

$D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$x \cap y = $ (H.C.F of x, y)

$x \cup y = $ (L.C.M of x, y)

H.C.F

$\wedge$	1	2	3	4	6	8	12	24
1	1	1	1	1	1	1	1	1
2	1	2	1	2	2	2	2	2
3	1	1	3	1	3	1	3	3
4	1	2	1	4	2	4	4	4
6	1	2	3	2	6	2	6	6
8	1	2	1	4	2	8	4	8
12	1	2	3	4	6	4	12	12
24	1	2	3	4	6	8	12	24

L.C.M.

$\vee$	1	2	3	4	6	8	12	24
1	1	2	3	4	6	8	12	24
2	2	2	6	4	6	8	12	24
3	3	6	3	12	6	24	12	24
4	4	4	12	4	12	8	12	24
6	6	6	6	12	6	24	12	24
8	8	8	24	8	24	8	24	24
12	12	12	12	12	12	24	12	24
24	24	24	24	24	24	24	24	24

Since all entry of Composition tables of meet $\wedge$ and join $\vee$ are element D_{24} so for each pair of element $x, y \in D_{24}$ we have $x \wedge y$, $x \vee y \in D_{24}$.

The Binary Operation meet $(\wedge)$ and join $(\vee)$ are closed in D_{24}

Hence $(D_{24}, '|')$ is sublattice of the lattice $(L, '|')$

Answer

Q Define: lattice, sub-lattice, Distributive lattice, chain, Complete lattice, complemented lattice, Complemented Complete lattice, supremum Infimum.

<u>Lattice</u>:. Let L be a non-Empty set closed under two binary operations called meet and join denoted by $\wedge$ and $\vee$ then L is called lattice, if the following rules hold where a, b, c any element.

[L1] Commulative low

(a) $a \wedge b = b \wedge a$
(b) $a \vee b = b \vee a$.

$\forall$ $a, b \in L$

(L2) absorption laws;

(d) $a \vee (a \wedge b) = a$

(d') $a \wedge (a \vee b) = a$.

[L3] Idempotent laws:

(a) $a \vee a = a$

(a') $a \wedge a = a$

[L4] association laws:

(c) $a \vee (b \vee c) = (a \vee b) \vee c$

(c') $a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

Sub-lattice :- Let $(L, \wedge, \vee)$ be a lattice. A subset M of L is set to be sub lattice of L if M is closed with respect to Meet $(\wedge)$ and Join $(\vee)$ Each point of element $x, y \in M$, $x \wedge y$ and $x \vee y$ are contained in M.

Distributed Lattice : let L (L, ∧, ∨) be a lattice then L is set to be distributed if for any elements.

a, b, c ∈ L we have the following law.

1) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

2) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Complete Lattice : let (L, ∧, ∨) be a lattice then L is said to be complete if every subset A (finite infinite) of L A and L ∧A and ∨A Exist in L thus in every complete lattices (L, ∧, ∨) there Exist there a greatest element 1 and a least element 0.

Complemented Lattice :- let $(L, \wedge, \vee)$ be a lattice with universal bound 0 and L the lattice L is said to be complemented lattice if every element in L has a complement that is $a \vee 1 = 1$,

$a \wedge 1 = a$;

$a \wedge 0 = 0$

$a \vee 0 = a$

$a \wedge a' = 0$

$a \vee a' = 1$

where $a \leq 0 \leq 1$

$\forall\ a \in L$ so a' is the complement $a \in L$.

Complement Complete Lattice :- Let $(L, \wedge, \vee)$ be a complete lattice with greatest and lowest element 'g' and l respectively then 'L' is called complemented complete lattice if for each element $a \in L$;

Infimum: In mathematics, the infimum(abbreviated, inf, plural infima) of a subset S of a partially ordered set P is greatest element in P the is less than or equal to all elements of S, such an element exists.

Supremum: The supremum (abbreviated sup; plural suprema) of a subset S of a partially ordered set P is the least element in P that is greater than or equal to all elements of S, if such an element exists.