PUTNAM TRAINING PROBLEMS 2001.3
Par For The Course

1. Is the matrix
$$\begin{bmatrix} 1235 & 2344 & 1234 & 1990 \\ 2124 & 4123 & 1990 & 3026 \\ 1230 & 1234 & 9095 & 1230 \\ 1262 & 2312 & 2324 & 3907 \end{bmatrix}$$
   invertible?

2. Show that a finite group with an even number of elements must contain an element x or order 2. (That is $x^2 = e$, where e is the identity.)

3. Prove that there is no polynomial $p(x)$ with integer coefficients such that $p(1) = 4$ and $p(3) = 5$.

4. Suppose that 32 dominoes tile an 8×8 chessboard. Show that the number of tiles with a vertical orientation must be an even number.

5. Place a knight on each square of a 7×7 chessboard. Is it possible for each knight to simultaneously make a legal move?

6. Place a knight on a $4 \times n$ chessboard. For which values of n is it possible, in $4n$ consecutive knight moves, to visit each square of the board and return to the original square?

7. Find the smallest value of n such that it is possible, in n^2 consecutive knight moves, to visit each square of an $n \times n$ chessboard and return to the original square.

8. Suppose that 13 stones of integer weights have the property that any 12 of them can be divided into two groups of 6 stones with each group having equal weight. Prove that all 13 stones weigh the same.

9. Show that $x^2 - y^2 = a^3$ always has integral solutions for x and y whenever a is a positive integer.