Designing and Tuning a Fuzzy Inference System

Creation of the Final Model

In this section I will describe the process I went through creating and tweaking a model until I was happy with the final result.

Identifying key Elements

Before producing any models a few key elements were identified. These elements would be used a starting point for all models created. Some of these decisions came from specification points for the model and others from a mix of common sense/research.

Temperature Universe of Discourse

While both Severity of Headache and Urgency are given a defined universe of discourse in the specification ([0, 10] and [0, 100] respectively), the Temperature linguistic variable doesn't have a defined range. It is noted however that this system is meant to aid a GP, so it would only be used in a situation where a doctor would need advice. It was decided that the minimum and maximum of the universe of discourse should be based upon when an extreme temperature becomes a severe condition with the idea that a GP should notice when the condition is this severe through other symptoms. When looking on the Wikipedia article for hypothermia[1] a lower bound of 28°C. A similar approach was used for an upper bound by looking into hyperthermia[2] getting a value of 40°C. Therefore the universe of discourse for temperature is between 28 and 40°C.

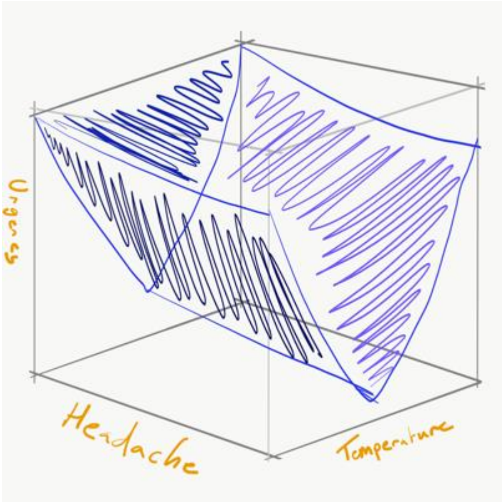
Control Surface

Before creating any models, it seemed sensible to have a rough idea of what the final control surface would look like. A level of intuition would allow a model to be viewed at a glance to see if it is suitable for further investigation. The following ideas needed to hold for the control surface to make sense:

1. As Severity of Headache increases the overall urgency shouldn't decrease for the same temperature.
2. There is a normal range of temperature where the urgency should be the lowest
3. As temperature deviates from the normal range the urgency shouldn't decrease for the same severity of headache.

[1] wikipedia.org/wiki/Hypothermia
[2] wikipedia.org/wiki/Hyperthermia

4. Maximum urgency should be when severity of headache is at its highest and the temperature is at an extreme, Minimum urgency should be when severity of headache is at its lowest and temperature is normal.



Taking all of these points into account we can create a rough sketch of what the final control surface should look like, which is shown in the sketch to the right. This gives a rough valley shape with the bottom of the valley perpendicular to the temperature axis. While the sketch shows slopes with a constant gradient, this doesn't have to be the case in the final control surface. The important property is that at no point on any slope does the gradient go the other way, more dangerous conditions shouldn't make the overall urgency lower.

Initial Model

To begin with a simple model was produced with the hope that it could outline and significant pros or cons with the system. Simple systems are also preferred as Occam's razor suggests that a simple explanation is stronger than a complex one. Starting with a simple system will also help ensure that the final system is explainable, if this system were to be introduced into real medical situations the ability to explain its decision making process would allow professionals to validate the safety and then approve the system.

All of the linguistic terms were made up of simple shapes (trapeziums and triangles) with the overlap being a linear crossover. This is to help create a sensible control surface. Apart from where specified the default values were used in the FuzzyR toolkit.

Membership functions

Temperature was difficult to split up as creating equal sections would mean certain ranges wouldn't make sense. 5 terms were used to allow for a normal range and 2 terms on either side. Having 2 terms on either side allowed for a mild and severe option for both high and low temperatures. The ranges for terms were decided upon using the Wikipedia pages for hyper/hypothermia[1, 2] and a research paper on normal human temperature[3]. Once all of the terms and ranges were decided upon a simple overlap of 1°C was created.

Headache was divided into 3 linguistic terms as 5 seemed to have too much crossover to be useful. The categories and ranges were taken from a chart[4] used to describe headache severity on the same scale. As the scale is discrete the 3 categories were given a crossover over each boundary, to successfully create a fuzzy membership function.

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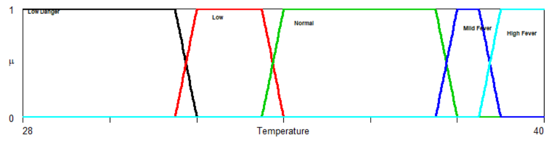
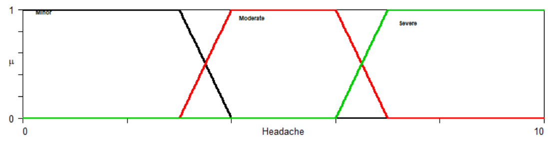
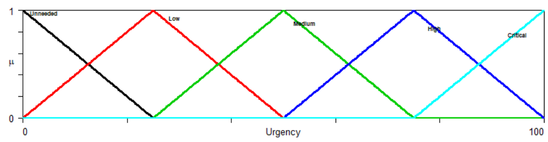
[3] Sund-Levander, M., Forsberg, C. and Wahren, L.K., 2002. Normal oral, rectal, tympanic and axillary body temperature in adult men and women: a systematic literature review. Scandinavian journal of caring sciences, 16(2), pp.122-128.
[4] https://i.pinimg.com/originals/d2/35/47/d235474d21b0f5901420c17d92ef3e2a.png

For Urgency 5 terms seemed to be a suitable amount so the universe of discourse was divided equally across 5 terms making sure that there were 2 shoulders at either side.

The membership functions were defined a graph showing all of the membership functions could be created which is shown below.







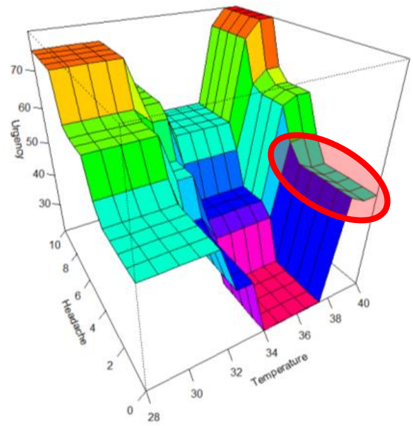
Rules

With 5 temperature terms and 3 headache terms there becomes 15 different possibilities for rule combinations, although a small number of rules in practice this method isn't tractable for larger models. To decrease this combination, a rule was made for each of the linguistic terms, that didn't contain the other linguistic variable. This lead to 8 simple relationships. These 8 rules can be combined with the defuzzification method to create a control surface that spans the entire universe of discourse. Using the following rule table a control surface can be generated.

Linguistic Variable	Linguistic Term	Urgency
Temperature	Low Danger	High
	Low	Medium
	Normal	Unneeded
	Mild Fever	High
	High Fever	Critical
Headache	Minor	Low
	Moderate	Medium
	Severe	High

Control Surface

While I wasn't expecting strong results from this model, the final control surface is quite promising. I have highlight a region with a red circle that is problematic, in this region while the temperature goes up to more dangerous levels the urgency decreases. Otherwise the control surface follows the general idea that was set out initially. Some key problems with this output is how extreme temperatures with low headaches have quite low urgency ratings. There are a few ways to try and improve this changing the defuzzification method, changing the weights of the current rules



or changing/adding rules. Another issue is the steep sides of the valley, these drastic changes make a fraction of a degree influence the urgency by a drastic amount. These small changes are often in the operational inaccuracy of the instruments being used to measure these values.

Adaptations

Changing Defuzzification Method

In the FuzzyR package there are 5 different defuzzification methods, centroid, bisector, middle of maximum, largest of maximum, and smallest of maximum. The default option is centroid and upon comparing to the other methods, it produced the most sensible control surface. Many of the other defuzzification methods produced strange control surfaces that weren't suitable, so ultimately centroid was kept.

Adding More Rules

As suggested when describing the model there is a total of 15 possible combinations of linguistic terms for the rules. Despite this being in tractable for larger problems it could be a solution for this problem. Unfortunately creating a rule for all 15 combinations didn't create the level of control that I was looking for, so the initial 8 rules were kept.

Adjusting Weights

Adjusting the weights of each rule is a good way of changing the control surface. Headaches are a common symptom of many minor issues so the rules involving headaches should have decreased importance. All of the headache rules were given a weight of 0.1, and the temperature rules kept at 1. This produced a control surface that better reflected the significance of an extreme temperature. This still had the problem of steep slopes on either side of the normal range. This is due to the underlying membership function, especially for the temperature linguistic variable.

Changing Membership Functions

Changing the membership function could help provide less steep edges to the slopes of the valley shape. A strong candidate for this was the Gaussian membership function. One issue with changing the membership functions is that we want them to cover the same range of values while defining them by differing parameters.

The 2 parameters defining a Gaussian mf are mean and standard deviation. As we want to keep similar ranges to the previous membership functions we need to calculate these parameters. Calculating the mean is trivial as the trapezium mf that were being used are symmetrical, the midpoint of the trapezium base will be the mean of the Gaussian function, for triangular functions the midpoint is the mean. The standard deviation is a more tricky value to define, ultimately we wish for the membership function to cover a similar range to the previous mf. To do this we can use a well-known rule for Gaussian distributions, the 68-95-99.7 rule, sometimes referred to as the 3-sigma rule[5]. This rule says that 99.7% of a Gaussian distribution is contained within $\mu \pm 3\sigma$ (where μ is the mean, and σ the standard deviation), using this rule we can take the range our original mf and then divide it by 3 to get the standard deviation.

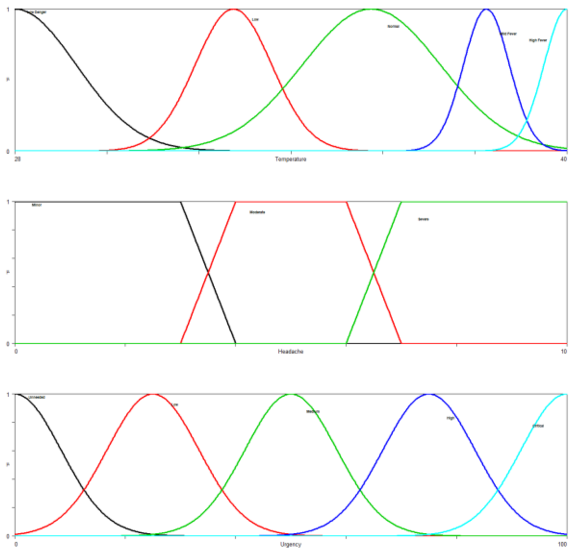
[5] https://en.wikipedia.org/wiki/68%E2%80%9395%E2%80%9399.7_rule

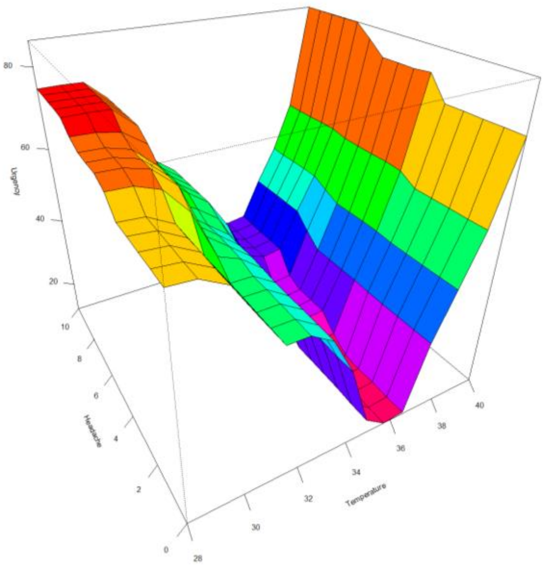
Detail of Final Fuzzy Model

Ultimately as my model was built via an iterative process all the major elements and decisions behind them have been described in the previous section. This Section aims to show some key visualizations and how to approach explaining the model to a layman in terms of fuzzy.

Visualizing the Model

In this section I simply present the plots for the final membership functions and the control surface. The control surface presented looks like it follows the pattern that was shown in the expert consultation phase of designing the fuzzy system, and it tackles the key issues that were described from the very initial model.





Explaining the Model

A key problem in the current application of any AI system is the explainability of the system and how it comes to its final decision. This would be critical if the system was to get any approval by a medical authority, as it would have to be trusted by medical professionals. This model was built upon the idea that it would have to be simple to explain to try and combat this issue in the application of AI.

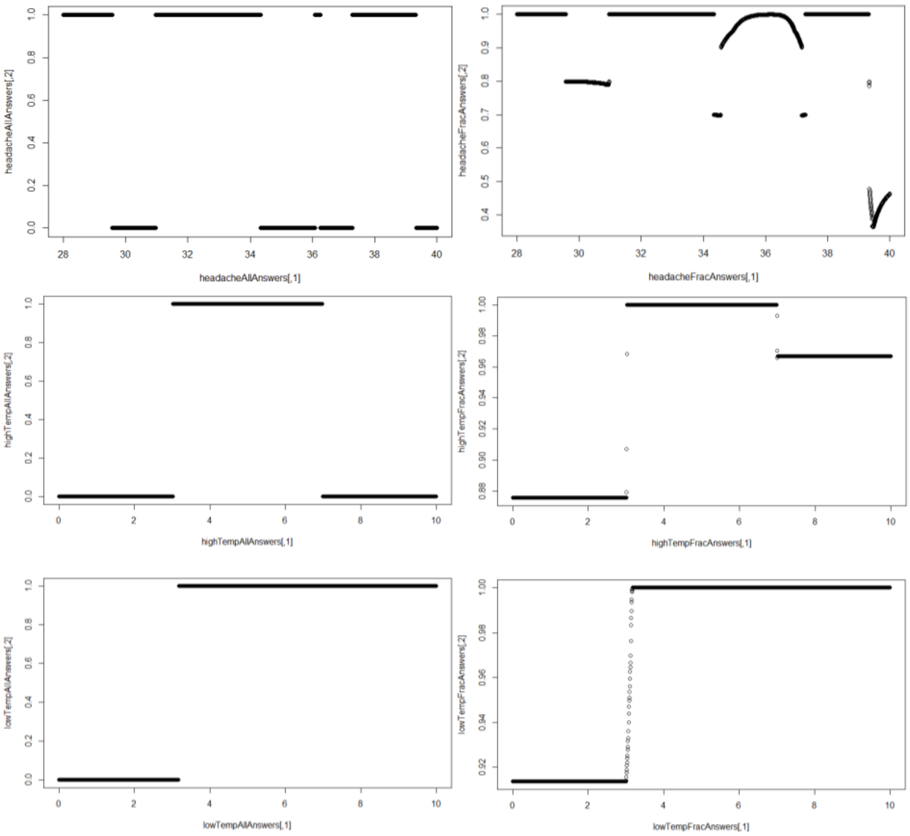
If I were to explain this to a medical expert I would show them the membership functions and the simple rules, stating that final control surface comes from a combination of these rules. It can be seen that as temperature increases to either extreme the urgency increases, a similar thing also happens for the headache severity. The visualization of the control surface would also be critical in showing how the system will react for a variety of input values.

Discussion of Final Fuzzy Model

Evaluating the Model

One key feature that was discussed in the beginning of the report was how the final control surface should behave when fixing one input and changing the other. While the final control surface looks like it has followed this behavior we can test to see if it is true.

Code to analyze these properties was created (can be found in the appendix). This code can be used to plot which values don't follow these properties and how much of the control surface doesn't follow it. This can be useful to test if the behavior we want is being shown by the system, and if not what percentage of that value does the behavior not hold for. This is done by generating a series of values to pass to evalfis and then analyzing the results. The following graph shows this analysis.



While the graph does show strong performance by these metrics, it also shows that some parts of the model won't follow these properties – meaning they don't make logical sense. That being said the model is performing fairly well. One thing this analysis doesn't take into account is by how much the values deviate, just whether they go in an unexpected direction. By looking at the control surface we can tell that although parts can be seen that do move in an unexpected direction the values don't deviate too much so it wouldn't have much real world impact.

If I was being more serious about this I would have implemented this analysis (as well as others) earlier in the process and would have kept to it more strictly. While 100% performance by these metrics is unrealistic a lower bound could be decided alongside the client.

Appendix

Full Evaluation Code

```
stepSize <- 0.005
# Test that urgency increase as headache increases for "all"
temperatures
headacheAllAnswers <- c()
headacheFracAnswers <- c()
for (temp in seq(28, 40, stepSize)){
  a = seq(0, 10, stepSize)
  b = cbind(rep(temp, length(a)), a)
  x = evalfis(b, fis)
  z = x == cummax(x)

  headacheAllAnswers <- rbind(headacheAllAnswers, c(temp, all(z)))
  headacheFracAnswers <- rbind(headacheFracAnswers, c(temp,
                                              (sum(z)/length(z))))
}

# Test that urgency decreases as low Temp decreases for "all"
headaches
lowTempAllAnswers <- c()
lowTempFracAnswers <- c()
for (headache in seq(0, 10, stepSize)){
  a = seq(28, 35.75, stepSize)
  b = cbind(a, rep(headache, length(a)))
  x = evalfis(b, fis)
  z = x == cummin(x)

  lowTempAllAnswers <- rbind(lowTempAllAnswers, c(headache, all(z)))
  lowTempFracAnswers <- rbind(lowTempFracAnswers, c(headache,
                                                (sum(z)/length(z))))
}

# Test that urgency increases as high Temp increases for "all"
headaches
highTempAllAnswers <- c()
highTempFracAnswers <- c()
for (headache in seq(0, 10, stepSize)){
  a = seq(35.75, 40, stepSize)
  b = cbind(a, rep(headache, length(a)))
  x = evalfis(b, fis)
  z = x == cummax(x)

  highTempAllAnswers <- rbind(highTempAllAnswers, c(headache,
                                                      all(z)))
  highTempFracAnswers <- rbind(highTempFracAnswers, c(headache,
                                               (sum(z)/length(z))))
}
```