Gaussian Elimination and Gauss-Jordan Method

Assessment

Create a function that will solve for the given system of linear equations using Gaussian elimination and another function for the Gauss Jordan method. The two functions should print the following:

- ➔ Augmented Coefficient Matrix (using the previous exercise)
- ➔ For each iteration:
  - ◆ Matrix after pivoting
  - ◆ Normalization (for Gauss-Jordan only)
  - ◆ Current pivot row
  - ◆ Pivot element
  - ◆ Value to be eliminated
  - ◆ Vector to be subtracted
  - ◆ Resulting matrix
- ➔ Backward Substitution (for Gaussian elimination only)

The two functions should return the values of the unknowns (vector)

Note: If you did not finish the previous exercise for getting the augmented coefficient matrix, the function will accept the augmented coefficient matrix instead of the list of equations.

Sample Run:

```
> E1 <- function(x0, x1, x2) 1 * x0 + 0 * x1 + 1 * x2 + -4
> E2 <- function(x0, x1, x2) 8 * x0 + -3 * x1 + 4 * x2 + -12
> E3 <- function(x0, x1, x2) 6 * x0 + -3 * x1 + 1 * x2 + -8
> system <- list(E1, E2, E3)
> Gaussian(system)
Augmented Coefficient:
  x0 x1 x2 RHS
1  1  0  1   4
2  8 -3  4  12
3  6 -3  1   8

Result of Pivot
  x0 x1 x2 RHS
1  8 -3  4  12
2  1  0  1   4
3  6 -3  1   8

Current Pivot Row: [ 8 -3 4 12 ]
Pivot Element:  8
Value to be eliminated:  1
Vector: [ 1 -0.375 0.5 1.5 ]

Resulting Matrix
  x0     x1  x2  RHS
1  8 -3.000 4.0 12.0
2  0  0.375 0.5  2.5
```

```
3  6 -3.000 1.0  8.0

Current Pivot Row: [ 8 -3 4 12 ]
Pivot Element:  8
Value to be eliminated:  6
Vector: [ 6 -2.25 3 9 ]

Resulting Matrix
  x0     x1   x2  RHS
1  8 -3.000  4.0 12.0
2  0  0.375  0.5  2.5
3  0 -0.750 -2.0 -1.0

Result of Pivot
  x0     x1   x2  RHS
1  8 -3.000  4.0 12.0
2  0 -0.750 -2.0 -1.0
3  0  0.375  0.5  2.5

Current Pivot Row: [ 0 -0.75 -2 -1 ]
Pivot Element:  -0.75
Value to be eliminated:  0.375
Vector: [ 0 0.375 1 0.5 ]

Resulting Matrix
  x0    x1   x2 RHS
1  8 -3.00  4.0  12
2  0 -0.75 -2.0  -1
3  0  0.00 -0.5   2

Backward Substitution

-0.5 * x2 = 2
-0.5 * x2 = 2
x2 = -4

-0.75 * x1 + -2 * 0 = -1
-0.75 * x1 = -9
x1 = 12

8 * x0 + -3 * 0 + 4 * 0 = 12
8 * x0 = 64
x0 = 8


[1]  8 12 -4
```

> GaussJordan(system)

```
Augmented Coefficient:
  x0 x1 x2 RHS
```

```
1  1  0  1   4
2  8 -3  4  12
3  6 -3  1   8

Result of Pivot
  x0 x1 x2 RHS
1  8 -3  4  12
2  1  0  1   4
3  6 -3  1   8

Result of Normalization
  x0     x1  x2 RHS
1  1 -0.375 0.5 1.5
2  1  0.000 1.0 4.0
3  6 -3.000 1.0 8.0

Current Pivot Row: [ 1 -0.375 0.5 1.5 ]
Pivot Element:  1
Value to be eliminated:  1
Vector: [ 1 -0.375 0.5 1.5 ]

Resulting Matrix
  x0     x1  x2 RHS
1  1 -0.375 0.5 1.5
2  0  0.375 0.5 2.5
3  6 -3.000 1.0 8.0

Current Pivot Row: [ 1 -0.375 0.5 1.5 ]
Pivot Element:  1
Value to be eliminated:  6
Vector: [ 6 -2.25 3 9 ]

Resulting Matrix
  x0     x1   x2  RHS
1  1 -0.375  0.5  1.5
2  0  0.375  0.5  2.5
3  0 -0.750 -2.0 -1.0

Result of Pivot
  x0     x1   x2  RHS
1  1 -0.375  0.5  1.5
2  0 -0.750 -2.0 -1.0
3  0  0.375  0.5  2.5

Result of Normalization
  x0     x1       x2      RHS
1  1 -0.375 0.500000 1.500000
2  0  1.000 2.666667 1.333333
3  0  0.375 0.500000 2.500000
```

```
Current Pivot Row: [ 0 1 2.666667 1.333333 ]
Pivot Element:  1
Value to be eliminated:  -0.375
Vector: [ 0 -0.375 -1 -0.5 ]

Resulting Matrix
  x0    x1       x2      RHS
1  1 0.000 1.500000 2.000000
2  0 1.000 2.666667 1.333333
3  0 0.375 0.500000 2.500000

Current Pivot Row: [ 0 1 2.666667 1.333333 ]
Pivot Element:  1
Value to be eliminated:  0.375
Vector: [ 0 0.375 1 0.5 ]

Resulting Matrix
  x0 x1        x2      RHS
1  1  0  1.500000 2.000000
2  0  1  2.666667 1.333333
3  0  0 -0.500000 2.000000

Result of Pivot
  x0 x1        x2      RHS
1  1  0  1.500000 2.000000
2  0  1  2.666667 1.333333
3  0  0 -0.500000 2.000000

Result of Normalization
  x0 x1       x2       RHS
1  1  0 1.500000  2.000000
2  0  1 2.666667  1.333333
3  0  0 1.000000 -4.000000

Current Pivot Row: [ 0 0 1 -4 ]
Pivot Element:  1
Value to be eliminated:  1.5
Vector: [ 0 0 1.5 -6 ]

Resulting Matrix
  x0 x1       x2       RHS
1  1  0 0.000000  8.000000
2  0  1 2.666667  1.333333
3  0  0 1.000000 -4.000000

Current Pivot Row: [ 0 0 1 -4 ]
Pivot Element:  1
Value to be eliminated:  2.666667
Vector: [ 0 0 2.666667 -10.66667 ]
```

```
Resulting Matrix
  x0 x1 x2 RHS
1  1  0  0   8
2  0  1  0  12
3  0  0  1  -4

[1]  8 12 -4
```

Word Problems

Answer the following problems in a sheet of yellow paper.

1. A small school has 100 students who occupy three classrooms: A, B, and C. After the first period of the school day, half the students in room A move to room B, one-fifth of the students in room B move to room C, and one-third of the students in room C move to room A. Nevertheless, the total number of students in each room is the same for both periods. How many students occupy each room?
    a. Set up the system of linear equations
    b. Set up the augmented coefficient matrix
    c. Solve the system using Gaussian Elimination
2. A man invests his savings in two accounts, one paying 6% and the other paying 10% simple interest per year. He puts twice as much in the lower-yielding account because it is less risky. His annual interest is $3520. How much did he invest at each rate?
    a. Set up the system of linear equations
    b. Set up the augmented coefficient matrix
    c. Solve the system using Gauss Jordan Method
3. A forester wants to know how to source wood from different planting sites. These are to be used by a construction company. There are six sites, and six species of wood that is needed. Some species are not applicable to some sites because of the soil composition.

    The table below shows the timber density for each site.

Planting Site	**Volume per hectare (m^3 / ha)**	**Yakal (%)**	**Falcata (%)**	**Gimelina (%)**	**Apitong (%)**	**Ipil (%)**	**Nato (%)**
Abra	350	70	10	5	5	0	10
Zambales	270	20	60	10	0	5	5
Biliran	300	10	0	75	5	5	5
Leyte	290	10	10	10	60	10	0
Bukidnon	325	5	5	0	5	75	10
Davao City	360	0	10	5	5	20	60

    The construction company needs to be provided with the following wood types, in cubic meters.

Wood	**Volume (m^3)**	**Wood**	**Volume (m^3)**	**Wood**	**Volume (m^3)**

Yakal	1000	Gimelina	1300	Ipil	1500
Falcata	700	Apitong	900	Yato	1200

How many hectares should be logged in each planting site, to deliver the required volume of logs?

a. Set up the augmented coefficient matrix