

Sampling and Quantization



Symbolic
Image

Rendering

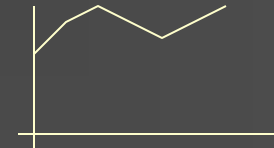
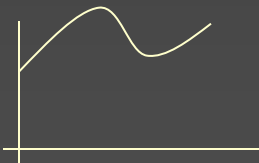


Digital
Image

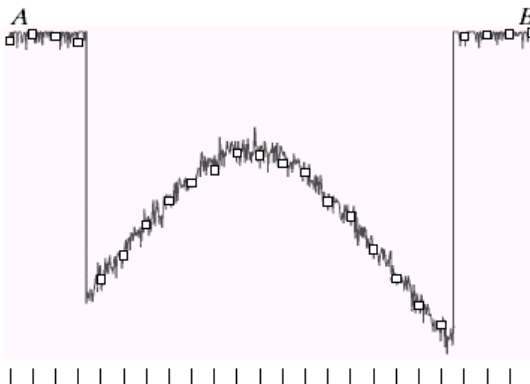
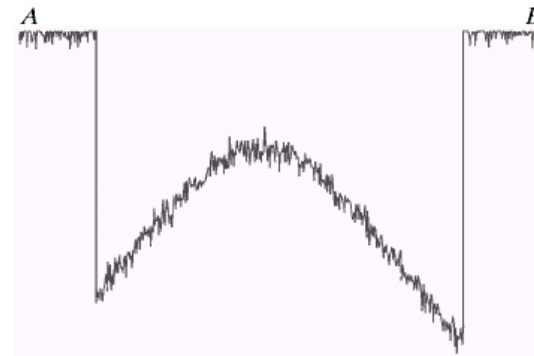
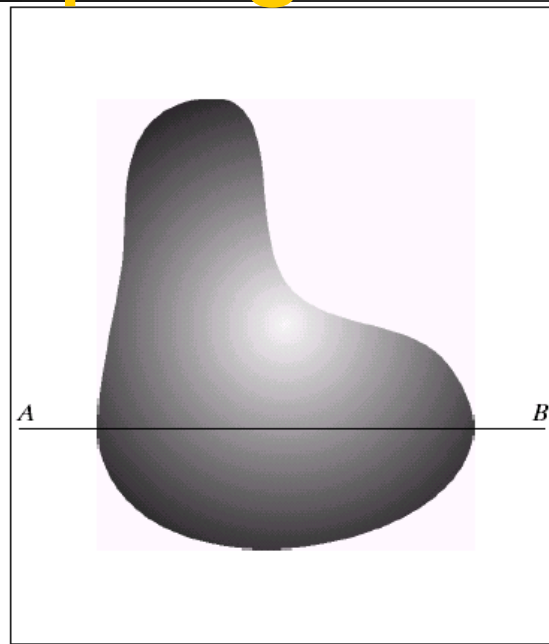
Display



Analog
Image



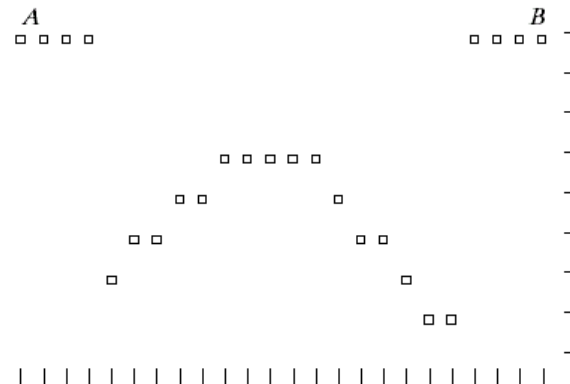
Sampling and Quantization



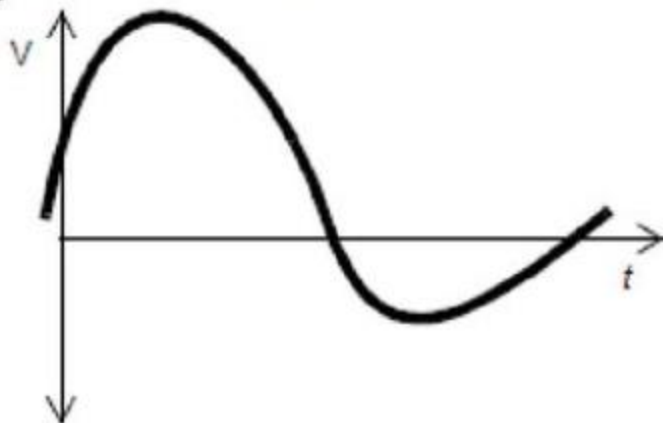
Sampling



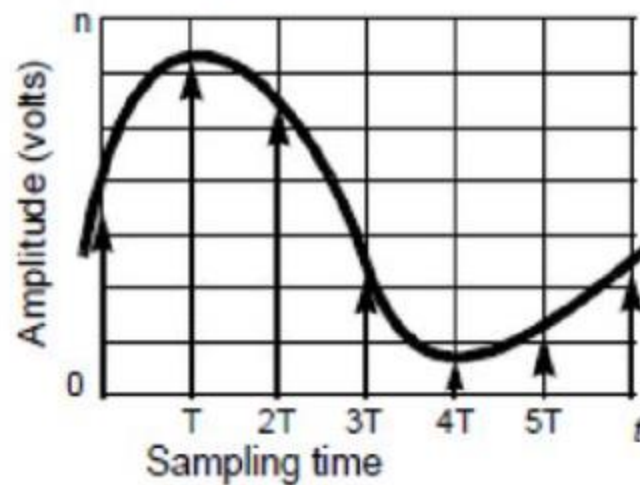
Quantization



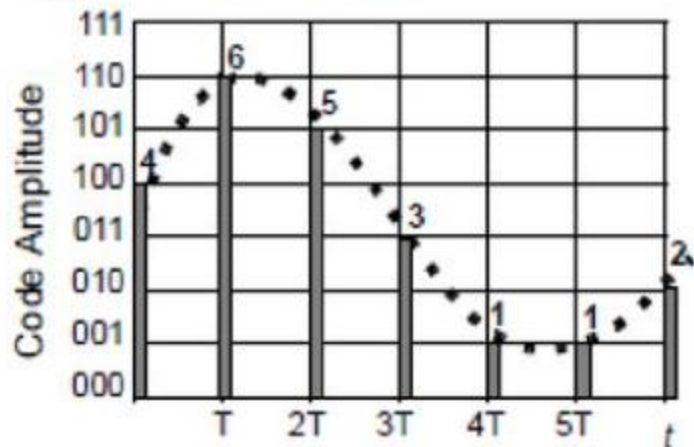
0 Analog Signal



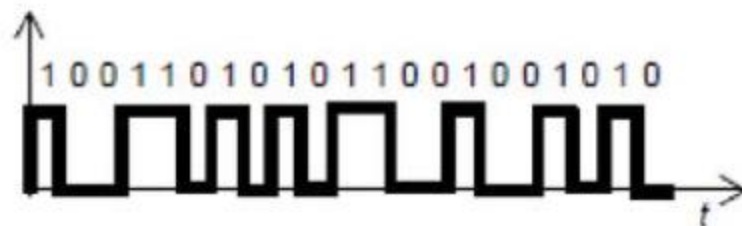
1 Sampling



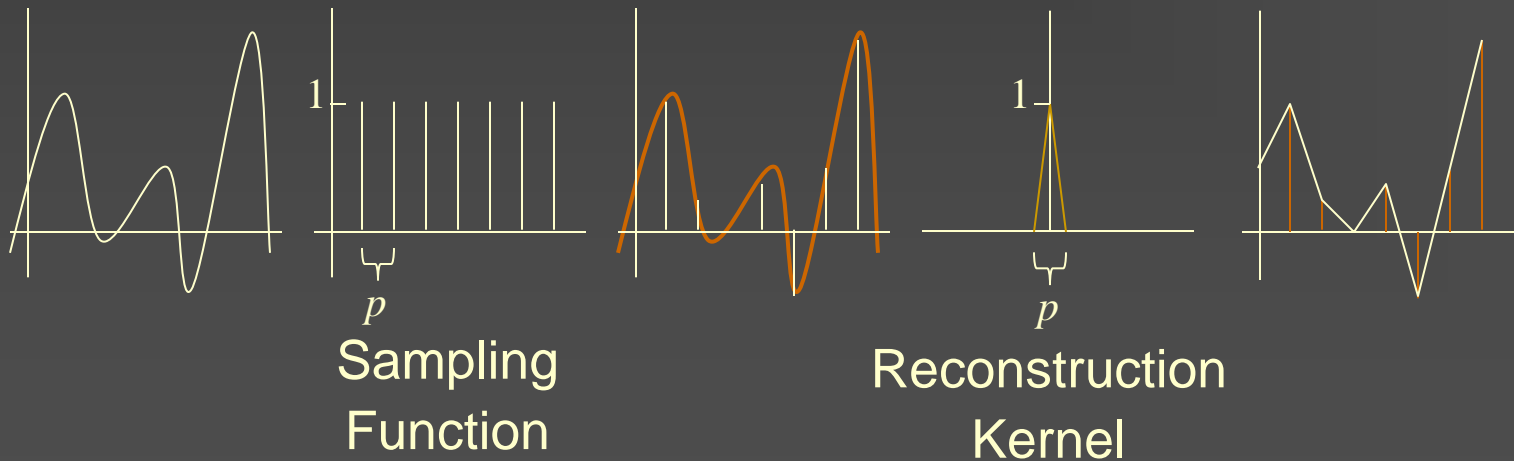
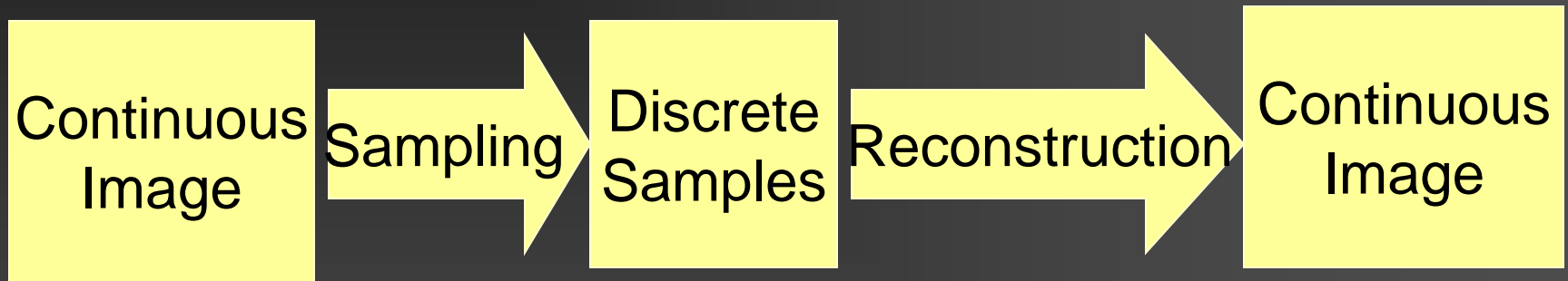
2 Quantization



3 Coding



Sampling and Reconstruction

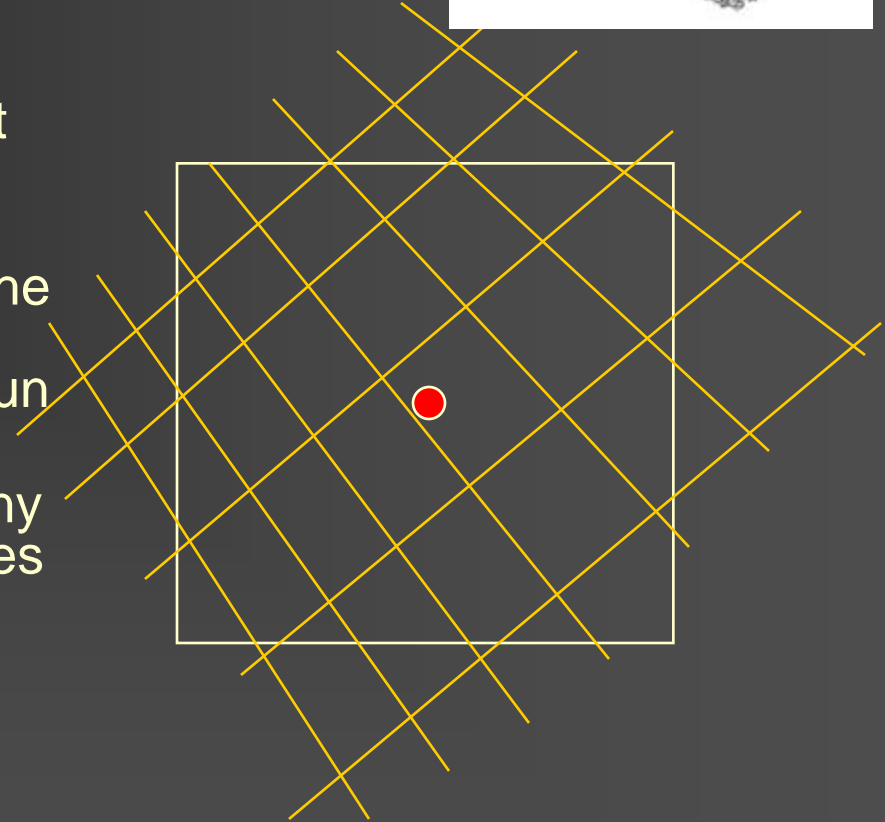
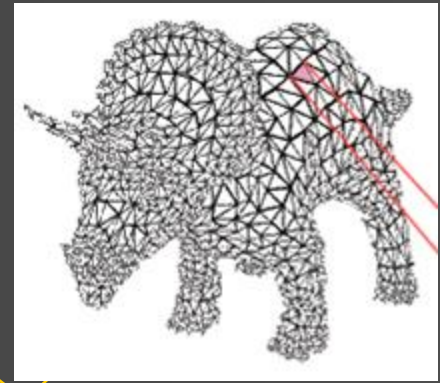


Samples

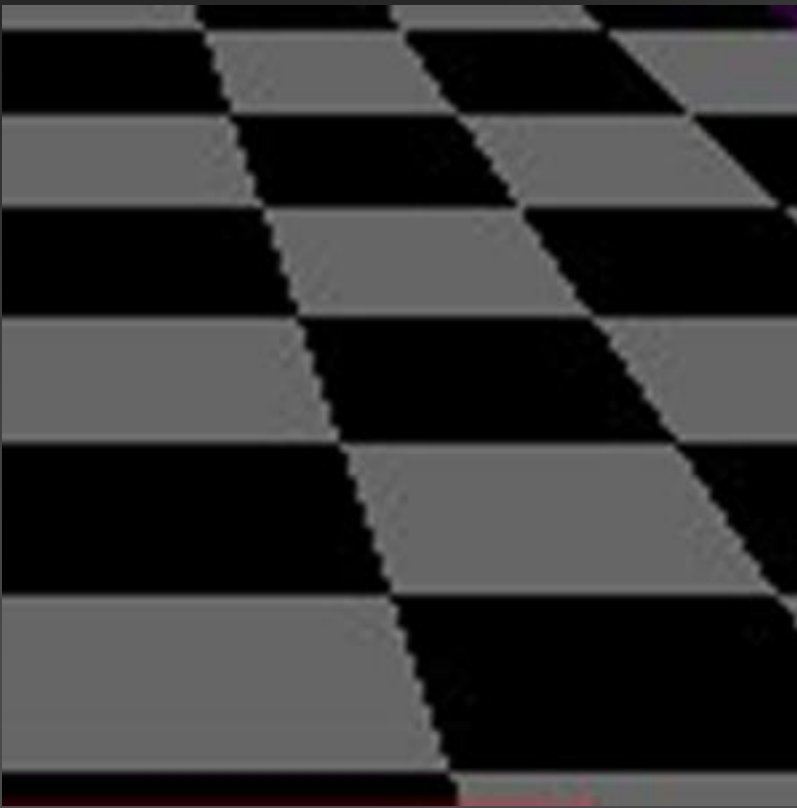
- most things in the real world are **continuous**
- everything in a computer is **discrete**
- the process of mapping a continuous function to a discrete one is called **sampling**
- the process of mapping a discrete function to a continuous one is called **reconstruction**
- the process of mapping a continuous variable to a discrete one is called **discretization**
- rendering an image requires **sampling and discretization**
- displaying an image involves **reconstruction**

Texture Minification

- Consider a texture mapped triangle
- Assume that we *point sample* our texture so that we use the nearest texel to the center of the pixel to get our color
- If we are far enough away from the triangle so that individual texels in the texture end up being smaller than a single pixel in the framebuffer, we run into a potential problem
- If the object (or camera) moves a tiny amount, we may see drastic changes in the pixel color, as different texels will rapidly pass in front of the pixel center
- This causes a flickering problem known as **shimmering** or **buzzing**
- Texture buzzing is an example of **aliasing**



- **Aliasing:** distortion artifacts produced when representing a high-resolution signal at a lower resolution.
- **Antialiasing:** techniques to remove aliasing



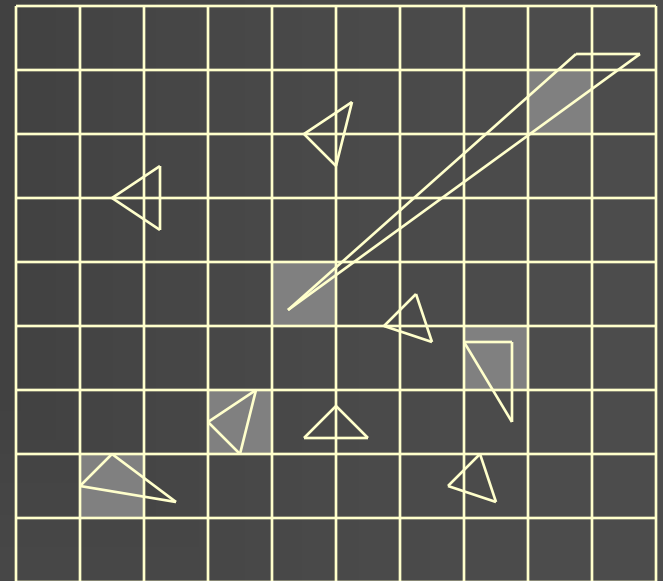
**Aliased polygons
(jagged edges)**



Antialiased polygons

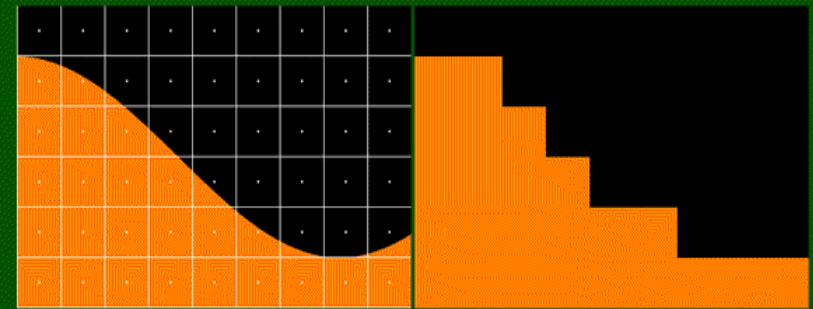
Small Triangles

- A similar problem happens with very small triangles
- Scan conversion is usually designed to *point sample* triangles by coloring the pixel according to the triangle that hits the center of the pixel
- This has the potential to miss small triangles
- If we have small, moving triangles, they may cause pixels to flicker on and off as they cross the pixel centers
- A related problem can be seen when very thin triangles cause pixel gaps
- These are more examples of **aliasing** problems



Stairstepping

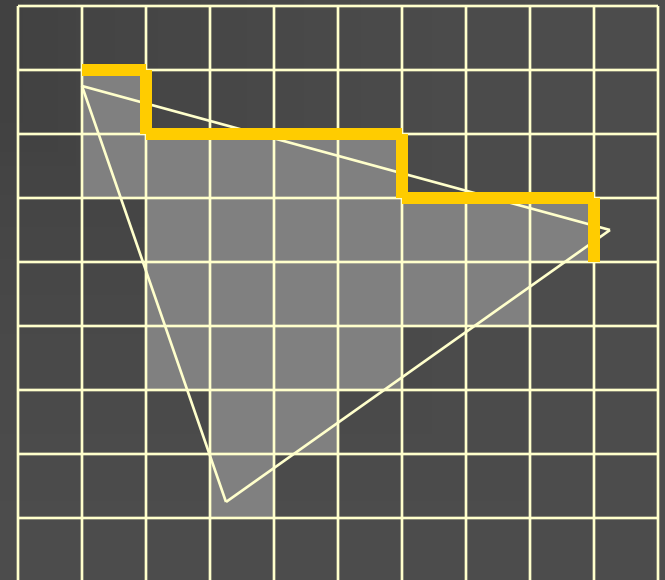
- What about the jagged right angle patterns we see at the edges of triangles?
- This is known as the **stairstepping** problem, also affectionately known as “**the jaggies**”
- These can be visually distracting, especially for high contrast edges near horizontal or vertical
- Stairstepping is another form of **aliasing**



Original

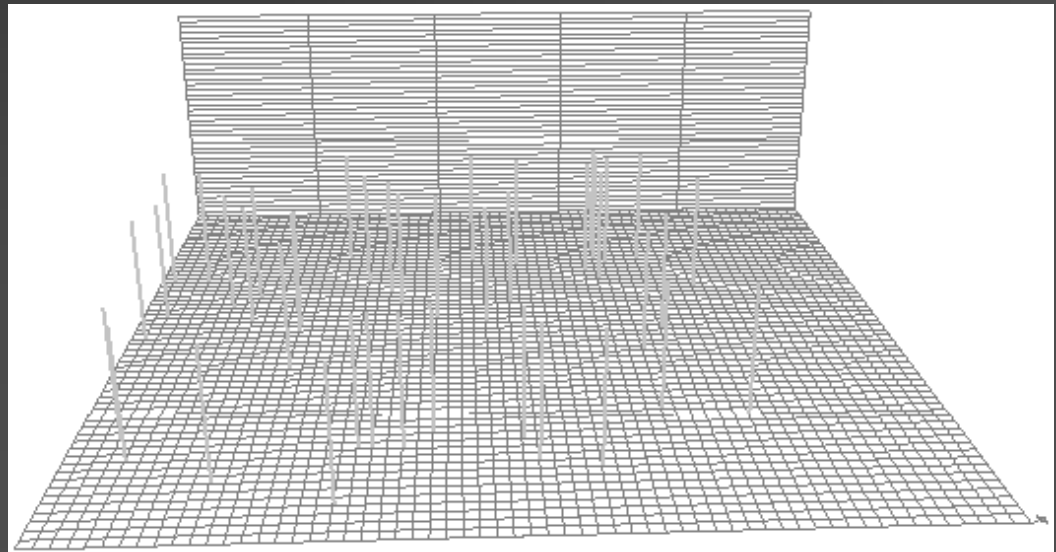
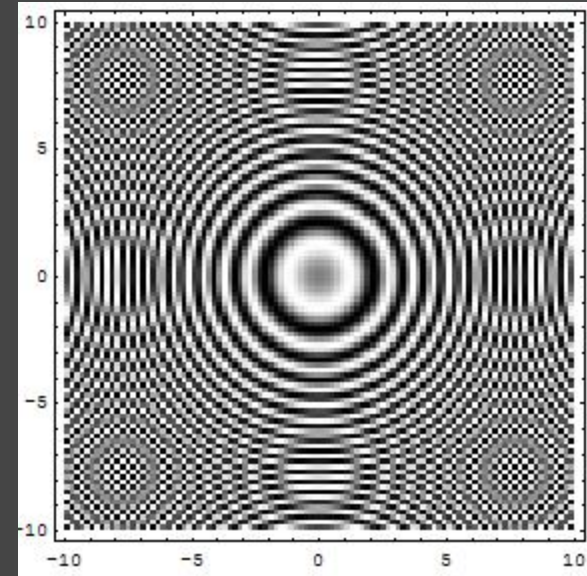
Rendered

Jagged profiles



Moiré Patterns

- When we try to render high detail patterns with a lot of regularity (like a grid), we occasionally see strange concentric curve patterns forming
- These are known as **Moiré patterns** and are another form of **aliasing**
- You can actually see these in real life if you hold two window screens in front of each other



The Propeller Problem

- Consider an animation of a spinning propeller, that is rendering at 30 frames per second
- If the propeller is spinning at 1 rotation per second, then each image shows the propeller rotated an additional 12 degrees, resulting in the appearance of correct motion
- If the propeller is now spinning at 30 rotations per second, each image shows the propeller rotated an additional 360 degrees from the previous image, resulting in the appearance of the propeller sitting still!
- If it is spinning at 29 rotations per second, it will actually look like it is slowly turning backwards
- These are known as **strob ing problems** and are another form of **aliasing**

Aliasing

- These examples cover a wide range of problems, but they all result from essentially the same thing
- In each situation, we are starting with a **continuous signal**
- We then **sample** the signal at **discrete points**
- Those samples are then used to **reconstruct** a new signal, that is intended to represent the original signal
- However, the reconstructed signals are a false representation of the original signals
- In the English language, when a person uses a false name, that is known as an **alias**, and so it was adapted in *signal analysis* to apply to falsely represented signals
- Aliasing in computer graphics usually results in **visually distracting artifacts**, and a lot of effort goes into trying to stop it. This is known as **antialiasing**

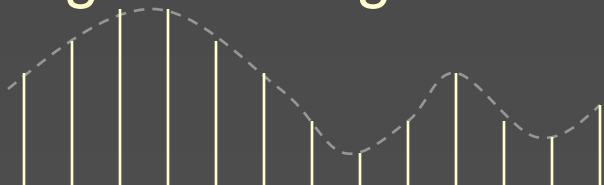
Signals

- The term *signal* is pretty abstract, and has been borrowed from the science of *signal analysis*
- Signal analysis is very important to several areas of engineering, especially electrical, audio, and communications
- Signal analysis includes a variety of mathematical methods for examining signals such as Fourier analysis, filters, sampling theory, digital signal processing (DSP), and more
- In electronics, a one dimensional signal can refer to a voltage changing over time. In audio, it can refer to the sound pressure changing over time
- In computer graphics, a **one dimensional signal could refer to a horizontal or vertical line** in our image. Notice that in this case, the signal doesn't have to change over time, instead it varies over space (the x or y coordinate)
- Often signals are treated as functions of one variable and examples are given in the 1D case, however the concepts of signal analysis extend to multidimensional signals as well, and so we can think of our entire 2D image as a signal



Sampling

- If we think of our image as a bunch of perfect triangles in continuous (floating point) device space, then we are thinking of our image as a *continuous signal*
- This continuous signal can have essentially infinite resolution if necessary, as the edges of triangles are perfect straight lines
- To render this image onto a regular grid of pixels, we must employ some sort of **discrete sampling** technique
- In essence, we take our original continuous image and sample it onto a **finite resolution grid of pixels**
- If our signal represents the red intensity of our virtual scene along some horizontal line, then the sampled version consists of a row of discrete 8 bit red values
- This is similar to what happens when a continuous analog sound signal is digitally sampled onto a CD



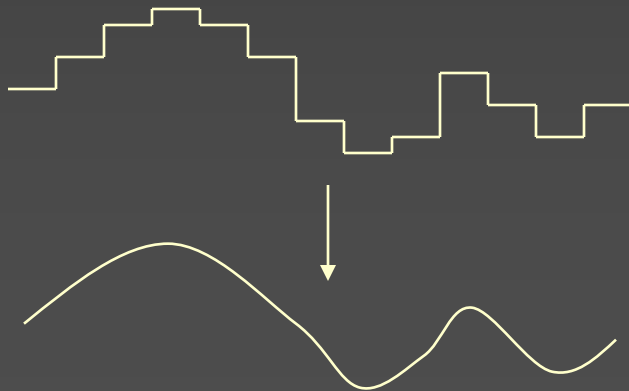
Reconstruction

- Once we have our sampled signal, we then *reconstruct* it
- In the case of computer graphics, this reconstruction takes place as a bunch of colored pixels on a monitor
- In the case of CD audio, the reconstruction happens in a DAC (digital to analog converter) and then finally in the physical movements of the speaker itself



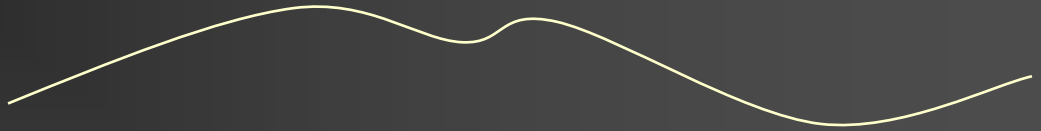
Reconstruction Filters

- Normally, there is some sort of additional **filtration** that happens at the reconstruction phase
- In other words, the actual pixels on the monitor are not perfect squares of uniform color. Instead they will have some sort of color distribution
- Additional filtration happens in the human eye so that the grid of pixels appears to be a continuous image
- In audio, the perfect digital signal is filtered first by the analog electronic circuitry and then by the physical limitations of the speaker movement

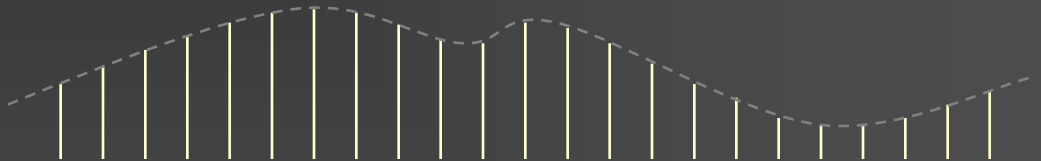


Low Frequency Signals

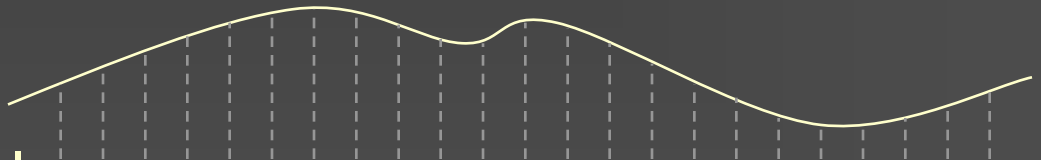
- Original signal



- Point sampled at relatively high frequency

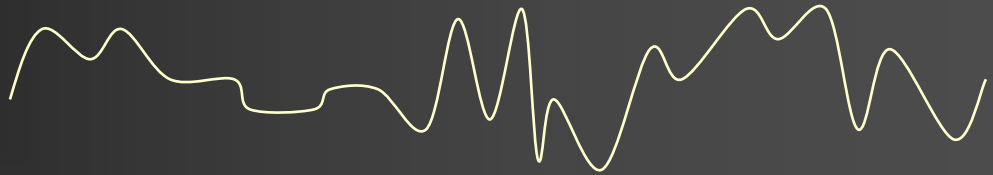


- Reconstructed signal

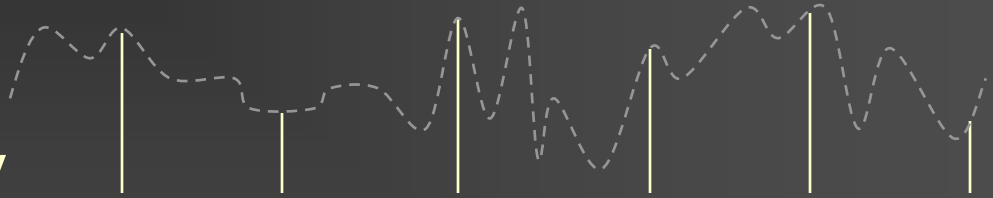


High Frequency Signals

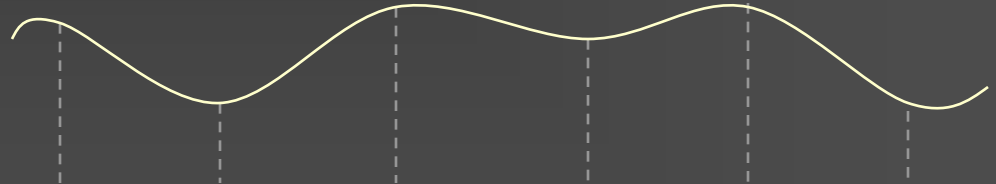
- Original signal



- Point sampled at relatively low frequency

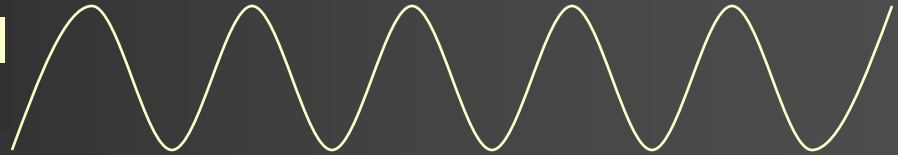


- Reconstructed signal

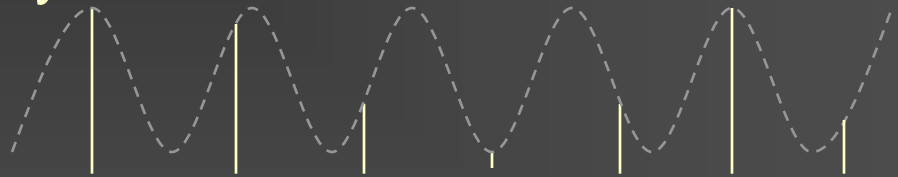


Regular Signals

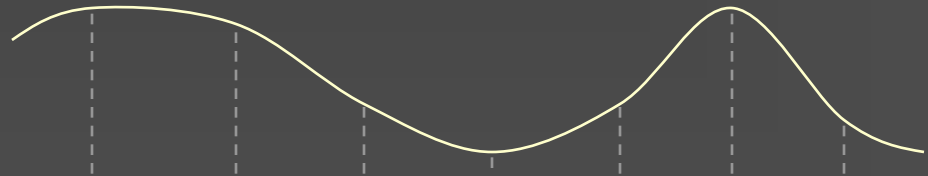
- Original repeating signal



- Point sampled at relatively low frequency



- Reconstructed signal repeating at incorrect frequency



Nyquist Frequency

- Theoretically, in order to adequately reconstruct a signal of frequency x , the original signal must be sampled with a frequency of **greater than $2x$**
- This is known as the *Nyquist frequency* or *Nyquist limit*
- However, this is assuming that we are doing a somewhat idealized sampling and reconstruction
- In practice, it's probably a better idea to sample signals at a minimum of $4x$

Aliasing Problems

- **Shimmering / Buzzing:**

Rapid pixel color changes (flickering) caused by high detail textures or high detail geometry. Ultimately due to point sampling of high frequency color changes at low frequency pixel intervals

- **Stairstepping / Jaggies:**

Noticeable stairstep edges on high contrast edges that are nearly horizontal or vertical. Due to point sampling of effectively infinite frequency color changes (step gradient at edge of triangle). This also causes incorrect rendering of fine details of the object.

- **Moiré patterns:**

Strange concentric curve features that show up on regular patterns. Due to sampling of regular patterns on a regular pixel grid

- **Strobing:**

Incorrect or discontinuous motion in fast moving animated objects. Due to low frequency sampling of regular motion in regular time intervals

Spatial / Temporal Aliasing

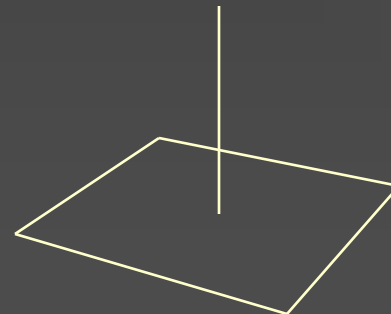
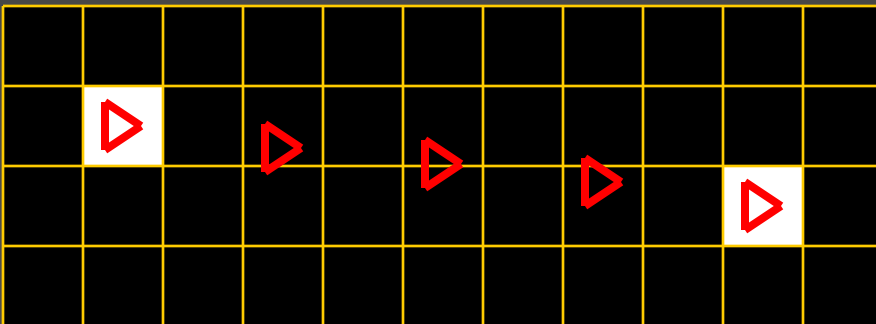
- Aliasing shows up in a variety of forms, but usually those can be separated into either *spatial* or *temporal* aliasing
- ***Spatial aliasing*** refers to aliasing problems based on regular sampling in space. This usually implies device space.
- ***Temporal aliasing*** refers to aliasing problems based on regular sampling in time
- The antialiasing techniques used to fix these two things tend to be very different, although they are based on the same fundamental principles

Antialiasing Methods

- Increase the sampling rate
 - Supersampling
 - Calculate at higher resolution & display at actual screen resolution
- Treat the pixel as finite area rather than a point

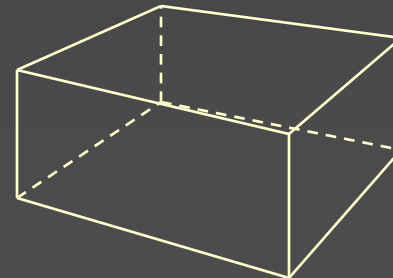
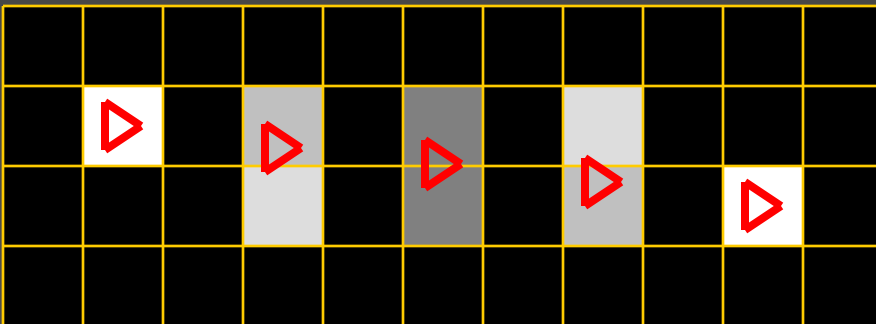
Point Sampling

- The aliasing problems we've seen are due to low frequency **point sampling** of high frequency information
- With point sampling, we sample the original signal at precise points (pixel centers, etc.)
- Is there a better way to sample continuous signals?



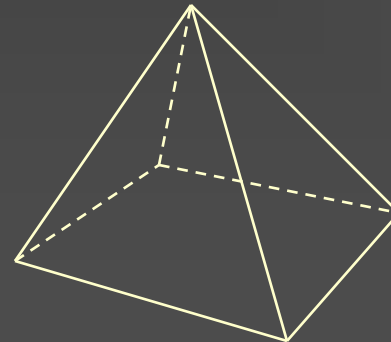
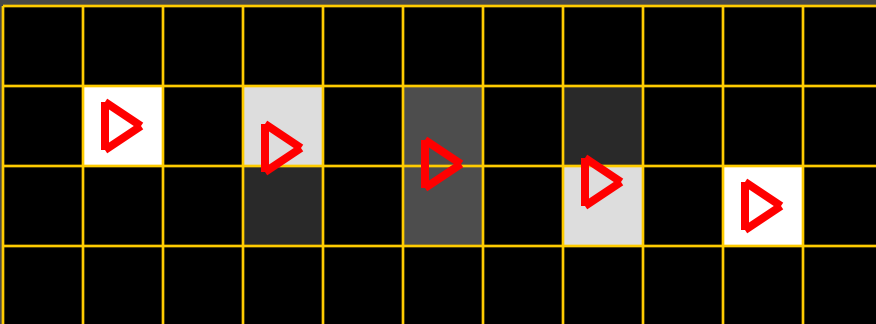
Box Sampling

- We could also do a hypothetical *box sampling (or box filter)* of our image
- In this method, each triangle contributes to the pixel color based on the area of the triangle within the pixel
- The area is equally weighted across the pixel



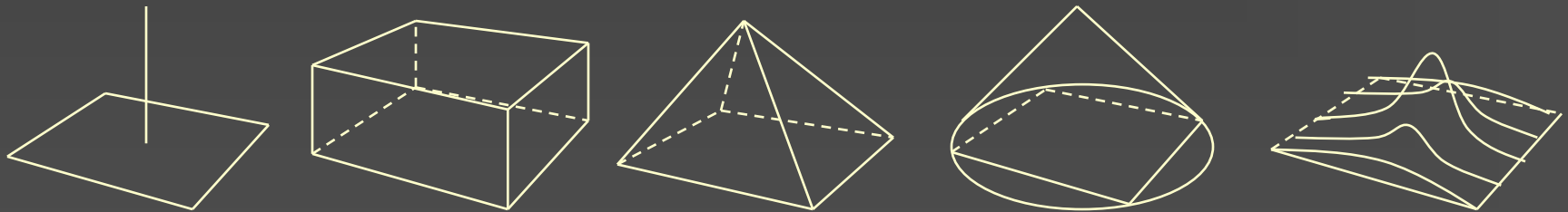
Pyramid Sampling

- Alternately, we could use a weighted sampling filter such as a **pyramid filter**
- The pyramid filter considers the area of triangles in the pixel, but weights them according to how close they are to the center of the pixel

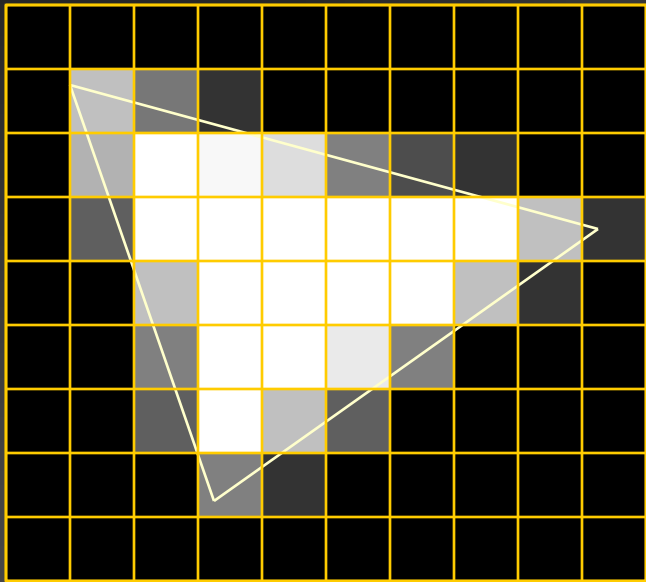


Sampling Filters

- We could potentially use any one of several different sampling filters
- Common options include the point, box, pyramid, cone, and Gaussian filters
- Different filters will perform differently in different situations, but the best all around sampling filters tend to be Gaussian in shape
- The filters aren't necessarily limited to cover only pixel. It is possible, and not uncommon to use filters that extend slightly outside of the pixel, thus overlapping with the neighboring pixels. Filters that cover less than the square pixel, however, tend to suffer from similar problems as point sampling



Edge Antialiasing



Coverage

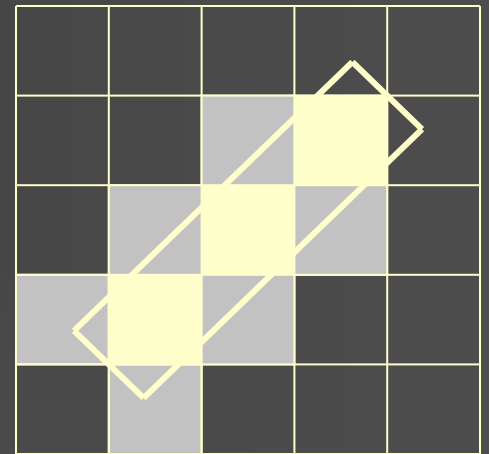
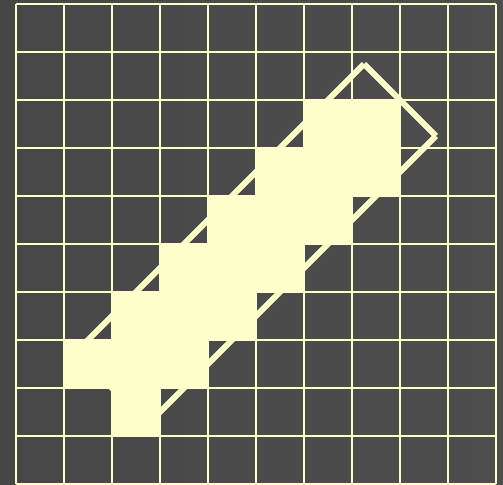
- There have been several edge antialiasing algorithms proposed that attempt to base the final pixel color on the exact area that a particular triangle covers
- However, without storing a lot of additional information per pixel, this is very difficult, if not impossible to do correctly, in cases where several triangle edges cross the pixel
- Making a coverage based scheme that is compatible with z_buffering and can handle triangles drawn in any order has proven to be a pretty impractical approach
- Some schemes work reasonably well if triangles are sorted from back to front (distant to near), but even these methods are not 100% reliable

Supersampling

- A more popular method (although less elegant) is *supersampling*
- With supersampling, we point sample the pixel at several locations and combine the results into the final pixel color
- For high quality rendering, it is not uncommon to use 16 or more samples per pixel, thus requiring the framebuffer and z_buffer to store 16 times as much data, and requiring potentially 16 times the work to generate the final image
- This is definitely a brute force approach, but is straightforward to implement and very powerful

Supersampling

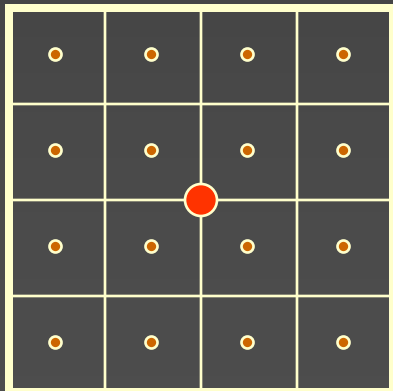
- Sample at a higher resolution than required for display, and filter image down
- Issues of which samples to take, and how to average them
- 4 to 16 samples per pixel is typical
- Samples might be on a uniform grid, or randomly positioned, or other variants
- Number of samples can be adapted



Uniform Sampling

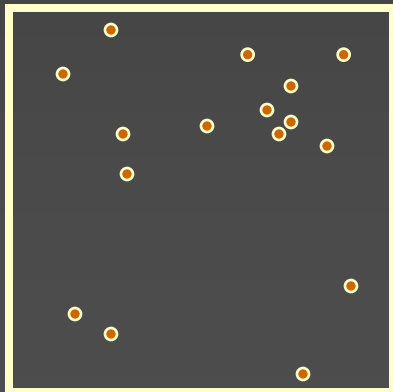
(Ref: Fig 2.62, *Procedural Elements for Comp. Graphics*, Rogers, TMH p.p. 151)

- With *uniform sampling*, the pixel is divided into a uniform grid of *subpixels*
- Uniform supersampling should certainly generate better quality images than single point sampling
- It will filter out some high frequency information, but may still suffer from Moiré problems with highly repetitive signals



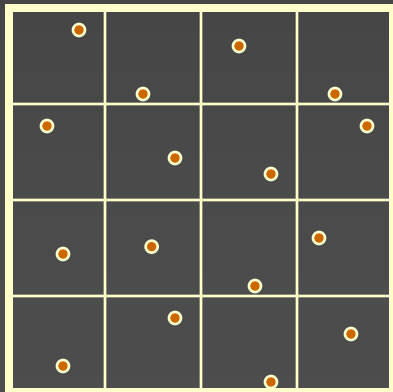
Random Sampling

- With *random sampling*, the pixel is supersampled at several randomly located points
- Random sampling has the advantage of breaking up repeating signals, and so can completely eliminate Moiré patterns. It does, however, trade the regular patterns with random *noise* in the image, which tends to be less annoying to the viewer
- It also suffers from potential clustering and gaps of the samples



Jittered Sampling

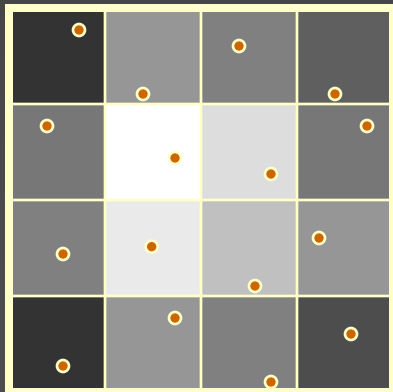
- With *jittered* or *stratified sampling*, the pixel is divided into a grid of *subpixels*, but the subpixels themselves are sampled at a random location within the subpixel
- This combines the advantages of both uniform and random sampling



Weighted Sampling

(Ref: Fig 2.62, *Procedural Elements for Comp. Graphics*, Rogers, TMH p.p. 151)

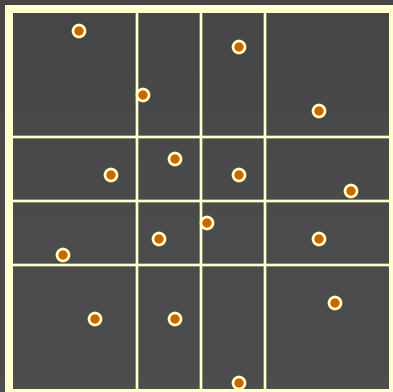
- If we average all of the samples equally to get the final pixel color, we are essentially performing a box filter on the samples
- We can also perform a weighted average of the samples to achieve other shaped filters
- For example, we can weight the samples according to a box, cone, pyramid, or Gaussian shape if desired
- We can apply weighting to uniform, random, or jittered supersamples with little additional work



1	2	1
2	4	2
1	2	1

Weighted Distribution

- By combining supersampling, jittering, and Gaussian weighting, we make good progress against aliasing problems
- However, if we look at the 16 samples in the previous image, we see that some are much more important than others, yet they all have the same computational cost
- In other words, the 4 samples in the center of the grid might have more total weight than the other 12 samples around the perimeter
- By adjusting our distribution so there are more samples in the higher valued areas, we can achieve the benefits of jittered and weighted sampling while maintaining efficiency by treating all samples equally

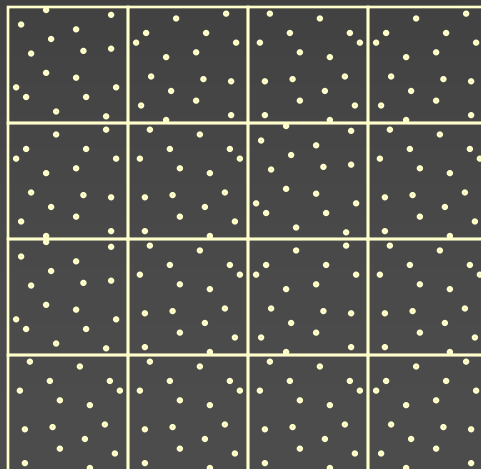
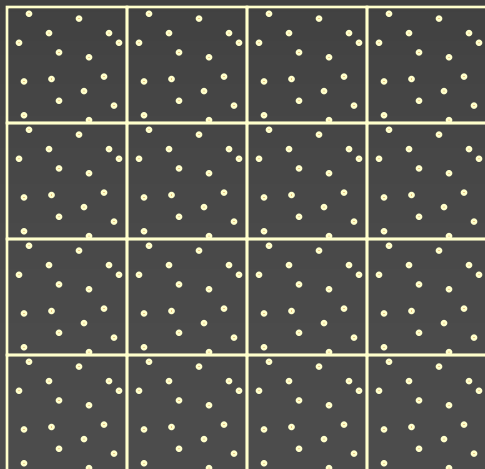


Adaptive Sampling

- An even more sophisticated option is to perform *adaptive sampling*
- With this scheme, we start with a small number of samples and analyze their statistical variation
- If the colors are all similar, we accept that we have an accurate sampling
- If we find that the colors have a large variation, we continue to take further samples until we have reduced the statistical error to an acceptable tolerance

Semi-Jittered Sampling

- We can apply a unique jittering pattern for each pixel (*fully jittered*) or we can re-use the pattern for all of the pixels (*semi-jittered*)
- Both are used in practice
- Semi-jittering has potential performance advantages and has the other advantage that straight edges look cleaner
- Semi-jittering, however, will potentially allow subtle Moire patterns due to the semi-regularity of the sampling grid



Spatial Aliasing

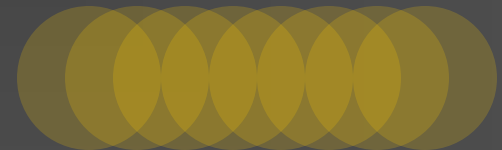
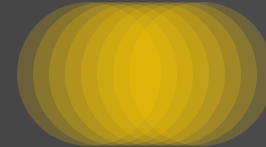
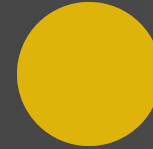
- Many of the spatial aliasing problems we've seen so far happen because of the regular grid of pixels
- Using antialiasing techniques such as Gaussian weighted jittered supersampling can significantly reduce these problems
- However, they can definitely add a large cost, depending on the exact implementation

Temporal Aliasing

- Properly tuned supersampling techniques address the spatial aliasing problems pretty well
- We still may run into temporal aliasing or strobing problems when we are generating animations
- Just as the spatial antialiasing techniques apply a certain blurring at the pixel level, temporal antialiasing techniques apply blurring at the frame level
- In other words, the approach to temporal antialiasing is to add ***motion blur*** to the image

Motion Blur

- Motion blur can be a tricky subject, and several different approaches exist to address the issue
- The simplest, brute force approach is supersampling in time
- Just as pixel antialiasing involves increasing the spatial resolution and blurring the results, motion blur involves increasing the temporal resolution and blurring the results
- In other words, if we want to apply motion blur over a $1/30^{\text{th}}$ second interval, we render several images spaced in time and combine them into a final image
- If an object moves 16 pixels in one frame, then 16 supersample images (or even less) should be adequate to blur it effectively



Combining Antialiasing Techniques

- We can even combine the two techniques without exponentially increasing the work (without requiring 16×16 times the work)
- This can be done by rendering 16 supersamples total, each one spread in time and jittered at the pixel level
- This overall approach offers a powerful foundation for other blurry effects such as: soft shadows (penumbrae), lens focus (depth of field), color separation (dispersion), glossy reflections, diffuse interreflections, etc.