

Image Operations

Point Operations

- Point operations are simple image enhancement techniques
- $q = T(p)$
- Let $f(x,y) = p$ and $g(x,y) = q$; transformation T
e.g. $[0, 255] \xleftarrow{T} [0, 255]$

$T \rightarrow$

- Linear function
- Non-linear function
- Clipping
- Window function

Linear function

- $q = s * p + o;$
- where, s is the slope of the line and o is the offset from the origin.
- particularly useful for enhancing white of gray details embedded in the dark regions of an image.

Non-linear function

e.g. $q = c * \log(1+p)$

- \log is nonlinear function; c is a constant
- This function converts a narrow range of low gray-level values of p in to a wider range of q values or vice versa
- This type of transformation is useful in expanding values of dark pixels while compressing the higher values

Clipping

- The value of output pixel is related to the input pixel value in piece-wise linear fashion
- complex and require user input but useful in many practical situations

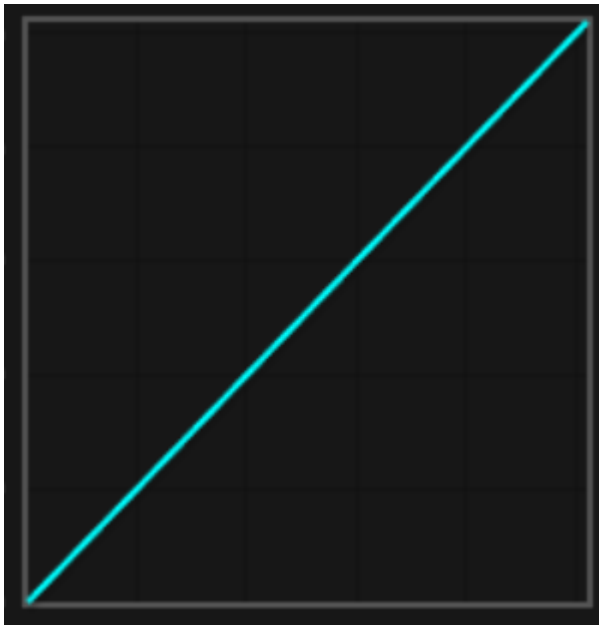
$$q = \begin{cases} 0 & p < a \\ s * p + o & a \leq p \leq b \\ L & p > b \end{cases}$$

- where, a and b are constants that define input pixel value range in which linear function with a slope s and offset o is applied
- e.g. gray level image; L = 255

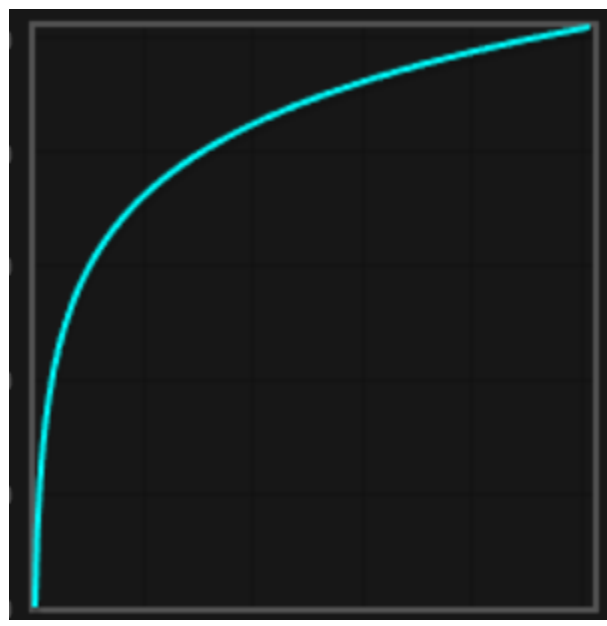
Window

- similar to the clipper
- However, only a desired range of gray levels is preserved in the output image

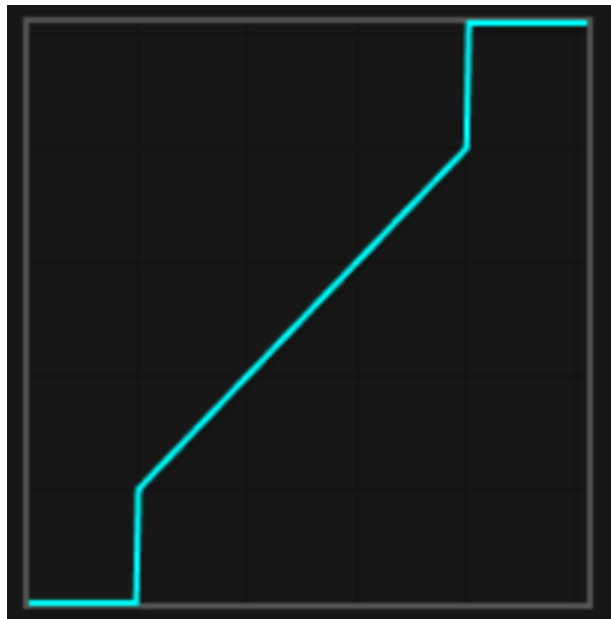
$$q = \left\{ \begin{array}{ll} 0 & p < a \\ s * p + o & a \leq p \leq b \\ 0 & p > b \end{array} \right\}$$



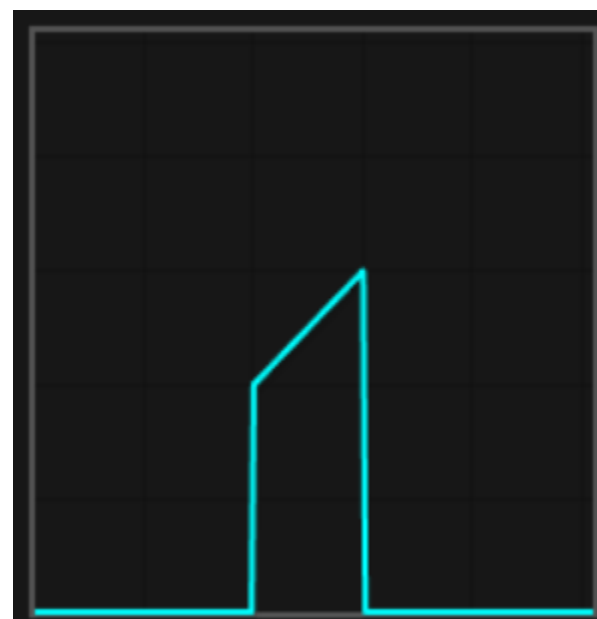
Linear T



Non-Linear T



Clipping T



Windowing T

Image Interpolation

Spatial Interpolation

Point Interpolation

Areal Interpolation

Exact

Weighting

Kriging

Splines

Interpolating
Polynomials

Finite
Difference

Approximate

Power Series Trend

Fourier Series

Least-squares
Fitting with Splines

Distance-weighted
Least-squares

Non-Volume- Preserving (Point-based)

(see both exact
and approximate
methods)

Volume- Preserving (Area-based)

Overlay

Pycnophylactic



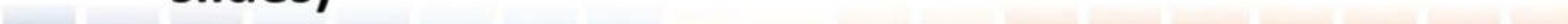
Types of Spatial Interpolation

- Global or Local
 - Global-use every known points to estimate unknown value.
 - Local – use a sample of known points to estimate unknown value.
- Exact or inexact interpolation
 - Exact – predict a value at the point location that is the same as its known value.
 - Inexact (approximate) – predicts a value at the point location that differs from its known value.
- Deterministic or stochastic interpolation
 - Deterministic – provides no assessment of errors with predicted values
 - Stochastic interpolation – offers assessment of prediction errors with estimated variances.

Spatial Data Perspectives

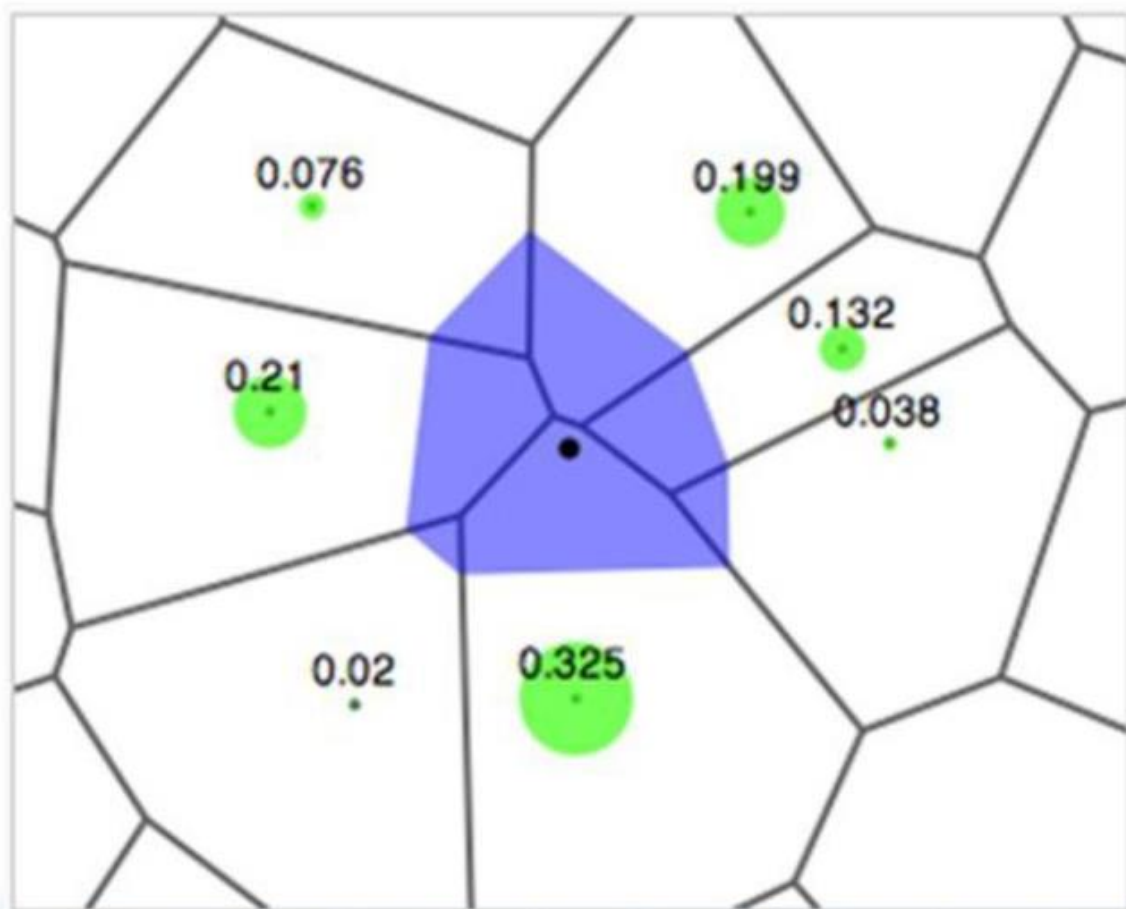
- **Deterministic perspective:** if we understand the system, we can predict the outcome with certainty
- **Stochastic perspective:** uncertainty remains in the best-designed model or experiment

Types of Interpolation Methods

- ***Deterministic perspective:***
 - Inverse Distance Weighted: points weighted by distance
 - Spline: passes exactly through points with constraining equations
 - Polynomial/trend analysis (nearest/natural neighbor)
 - ***Stochastic perspective:***
 - Kriging: weighted by fitted semivariogram (next slides)
- 

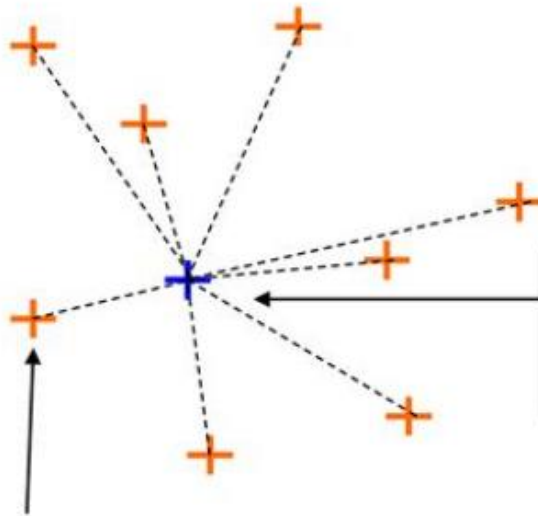
Natural Neighbor

The colored circles, which represent the interpolating weights, are generated using the ratio of the shaded area to that of the cell area of the surrounding points. The shaded area is due to the insertion of the point to be interpolated into the Voronoi tessellation (Theissen polygons).



Inverse Distance Weighted

The estimate is a weighted average



unknown value (to be
interpolated)
location x

point i
known value z_i
location x_i
weight w_i distance d_i

$$z(\mathbf{x}) = \frac{\sum_i w_i z_i}{\sum_i w_i}$$

$$w_i = 1/d_i^n$$

Trend Surface/Polynomial

- Flat but TILTED plane to fit data
(1st order polynomial)

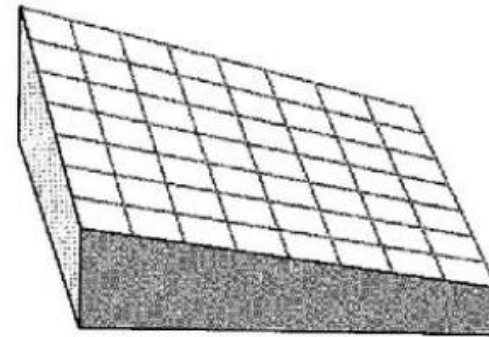
$$Z = a + bx + cy$$

- Tilted but WARPED plane to fit data
(2nd order polynomial)

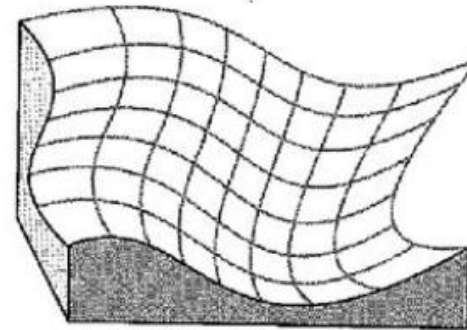
$$Z = a + bx + cy + dx^2 + exy + fy^2$$

Trend Surfaces

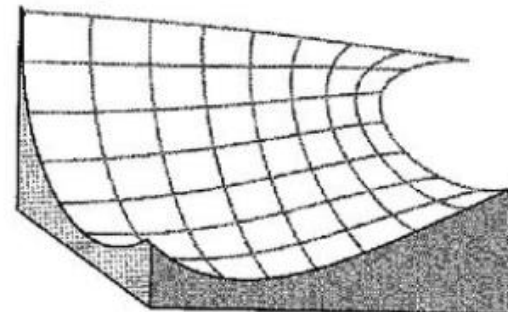
- Simplifies the surface representation to allow visualization of general trends.
- Higher order polynomials can be used
- Robust regression methods can be used



1st degree trend surface



2nd degree trend surface




3rd degree trend surface



Kriging

- Spatial prediction of variable Z at location x

$$Z(x) = m(x) + \gamma(h) + \varepsilon$$

- Three components:
 - *structural (constant mean),*
 - *random spatially correlated component*
 - *residual error*
- 

Types of Kriging

- **Simple Kriging**: assumes mean is constant and known
- **Ordinary Kriging**: assumes mean is constant but unknown (MapWindow)
- **Universal Kriging**: assumes mean is varying and unknown
 - Modeled by a constant, linear, second or third order equation

Advantages of Kriging

- It handles (embraces) spatial autocorrelation
- Less sensitive to preferential sampling in specific areas
- Allows uncertainty to be estimated (Kriging error)

Content

1 Introduction

2 Nearest neighbor

3 Bilinear interpolation

4 Bicubic

5 Matlab

Outline

1 Introduction

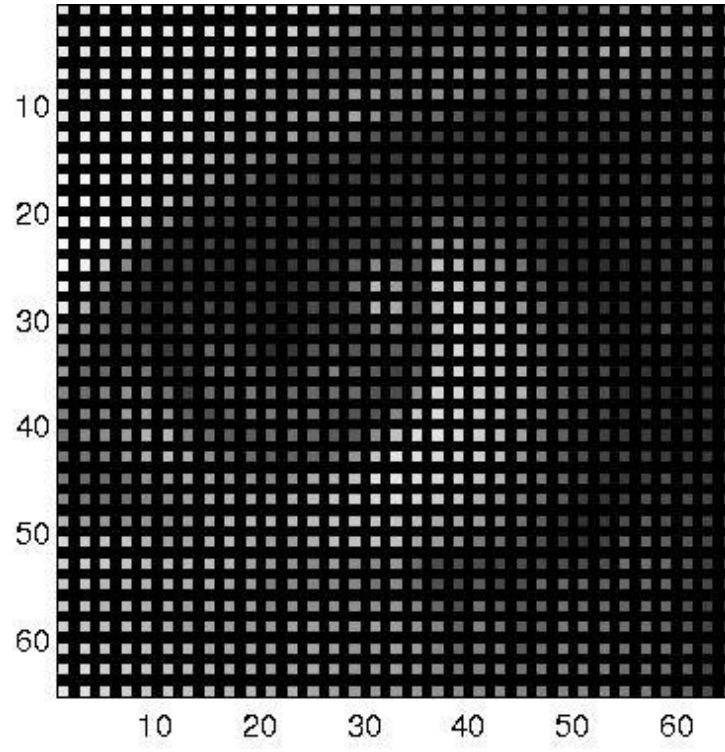
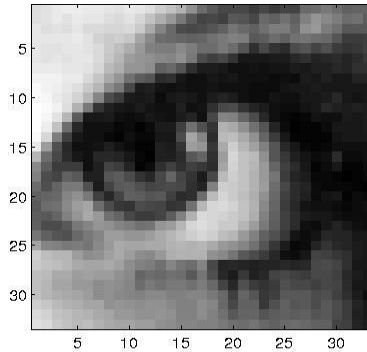
2 Nearest neighbor

3 Bilinear interpolation

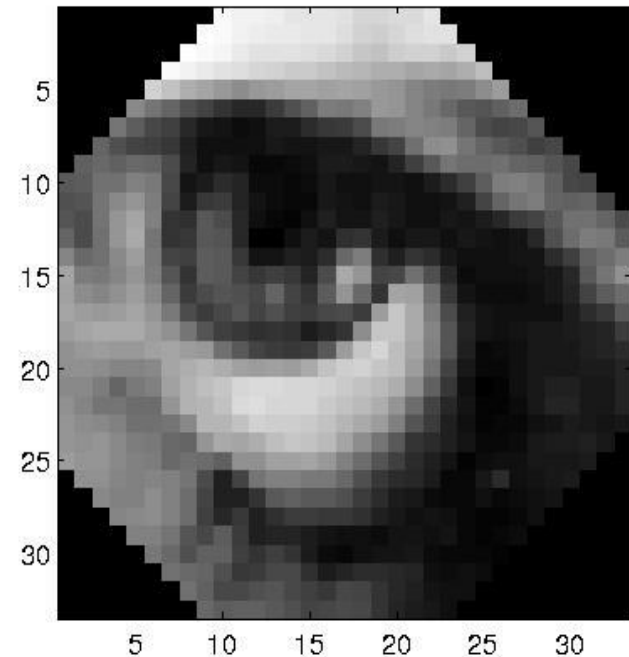
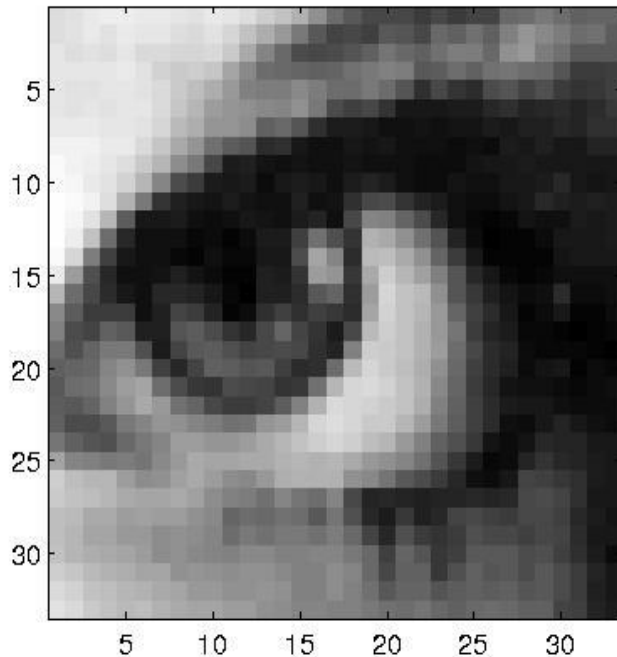
4 Bicubic

5 Matlab

Resizing (resampling)



Remapping (geometrical transformations-rotation, change of perspective,...)



Inpainting (restauration of *holes*)



Morphing, nonlinear transformations



Outline

1 Introduction

2 Nearest neighbor

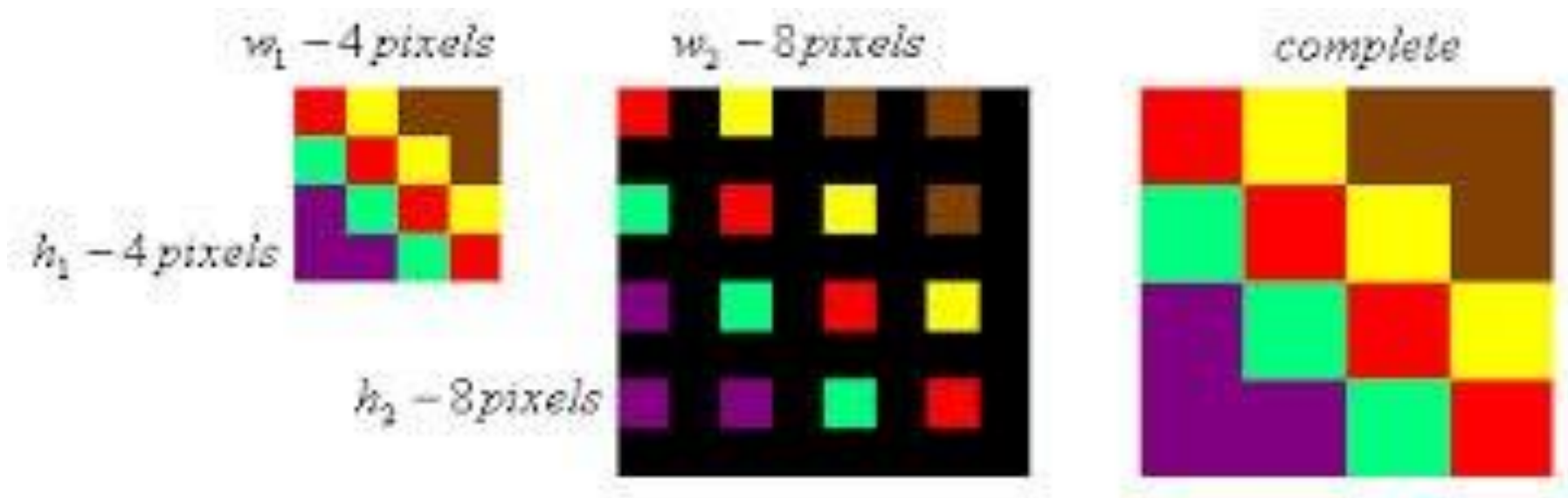
3 Bilinear interpolation

4 Bicubic

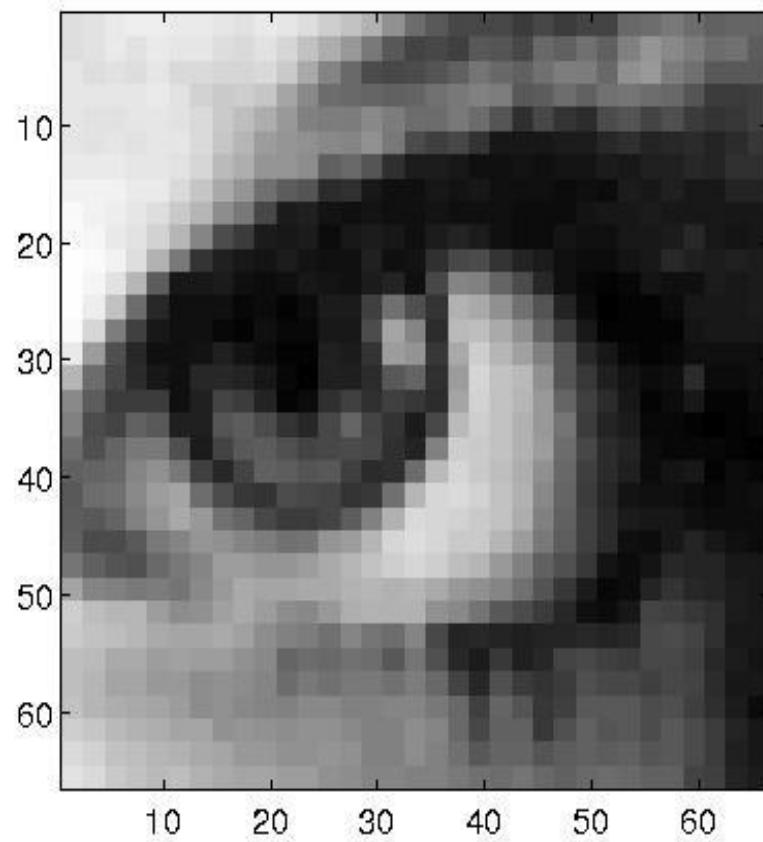
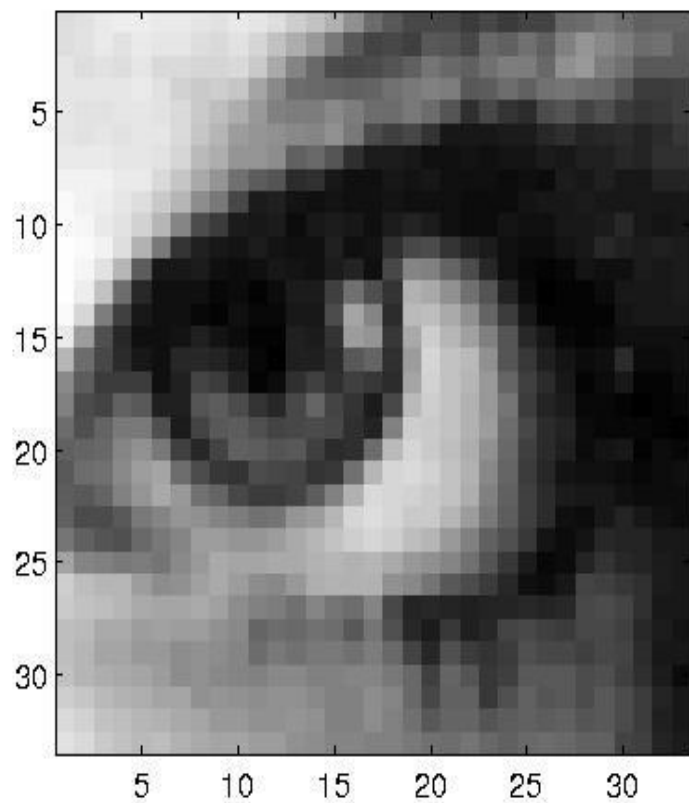
5 Matlab

Nearest neighbor

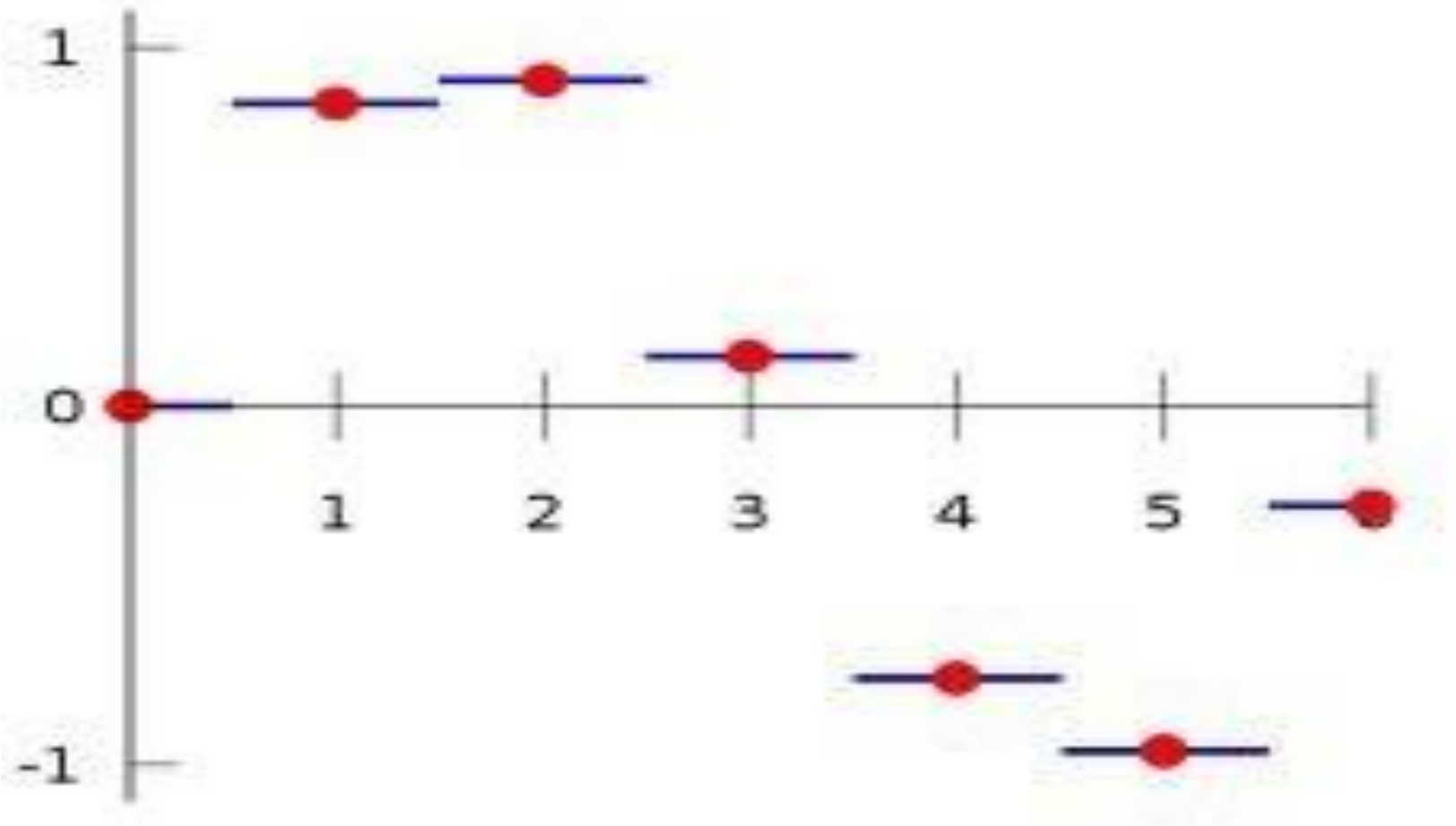
- Most basic method
- Requires the least processing time
- Only considers one pixel: the closest one to the interpolated point has the effect of simply making each pixel bigger



Nearest Neighbor



Relationship with 1D interpolation



Outline

- 1 Introduction
- 2 Nearest neighbor
- 3 Bilinear interpolation**
- 4 Bicubic
- 5 Matlab

Bilinear

- Considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixels
- Takes a weighted average of these 4 pixels to arrive at the final interpolated values
- Results in smoother looking images than nearest neighborhood
- Needs of more processing time

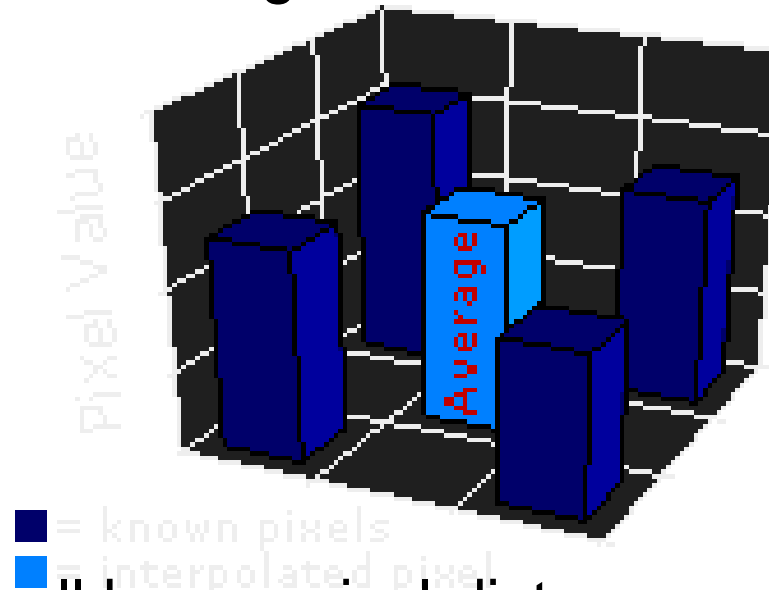
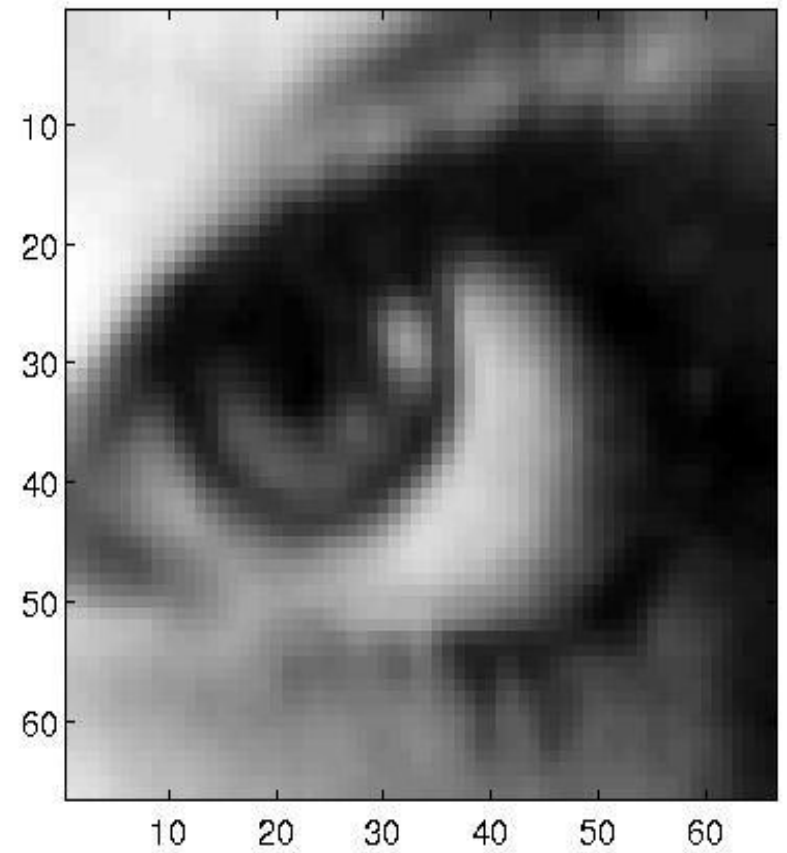
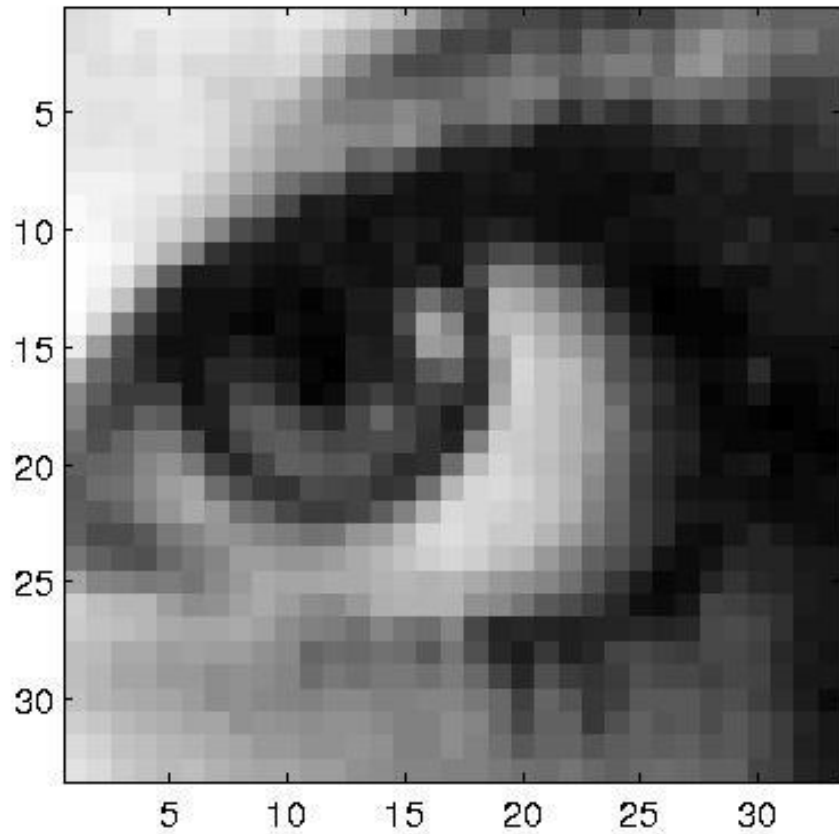
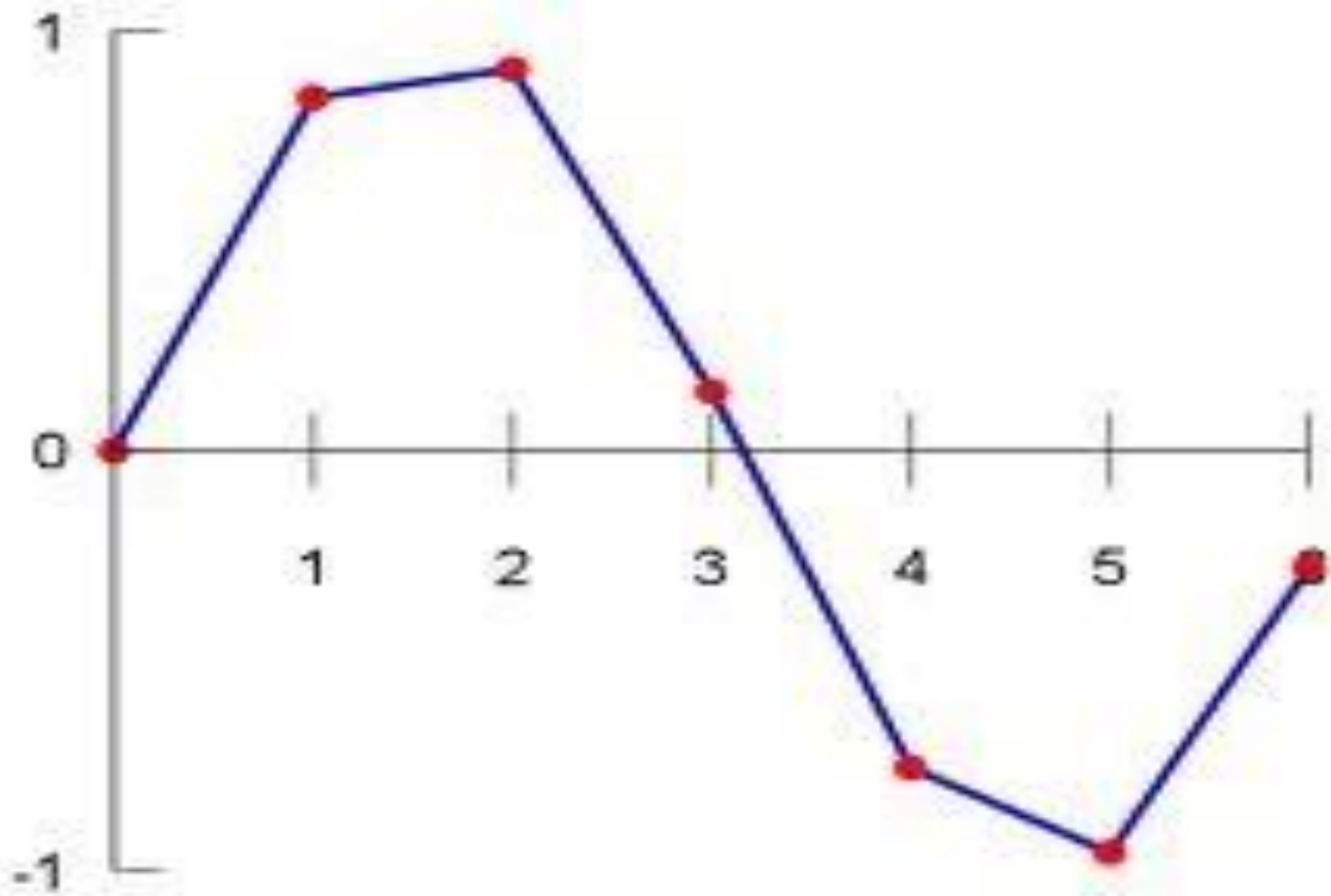


Figure: Case when all known pixel distances are equal. Interpolated value is simply their sum divided by four.

Bilinear



Relationship with 1D interpolation

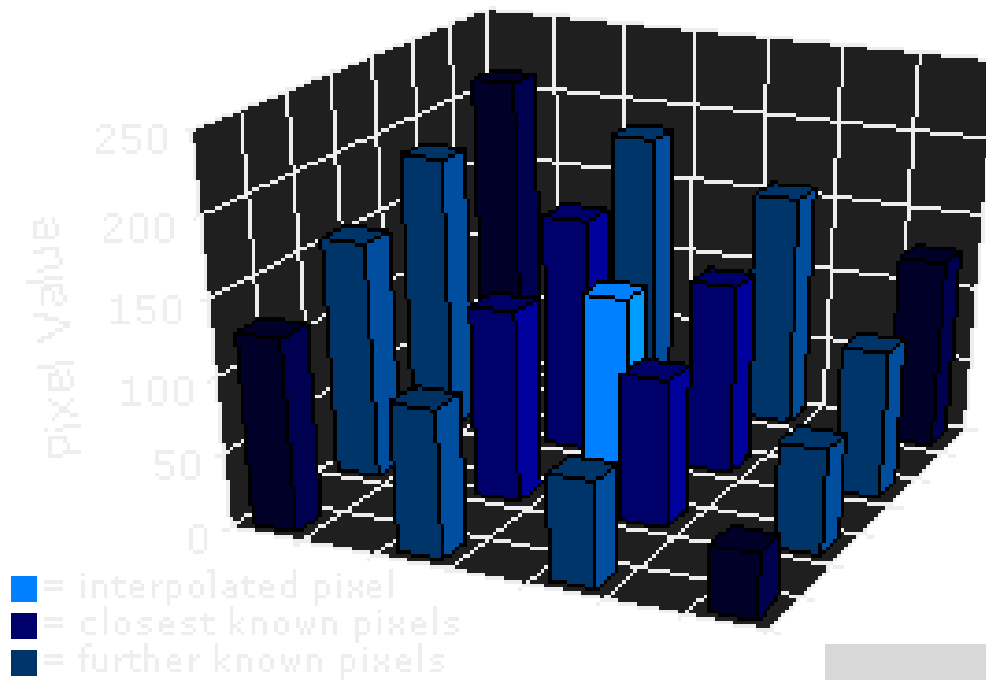


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- 1 Introduction
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- 3 Bilinear interpolation
- 4 Bicubic**
- 5 Matlab

Bicubic

- One step beyond bilinear by considering the closest 4x4 neighborhood of known pixels, for a total of 16 pixels
- Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation
- Produces sharper images than the previous two methods.
- Good compromise between processing time and output quality
- Standard in many image editing programs, printer drivers and in-camera interpolation



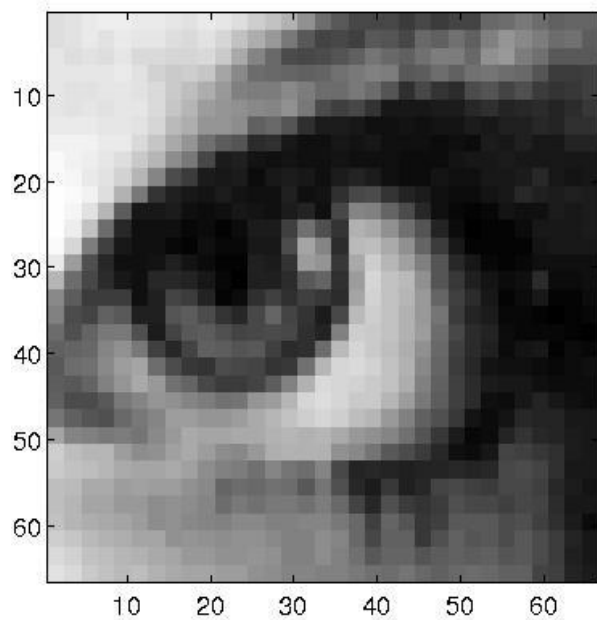


Figure: Nearest

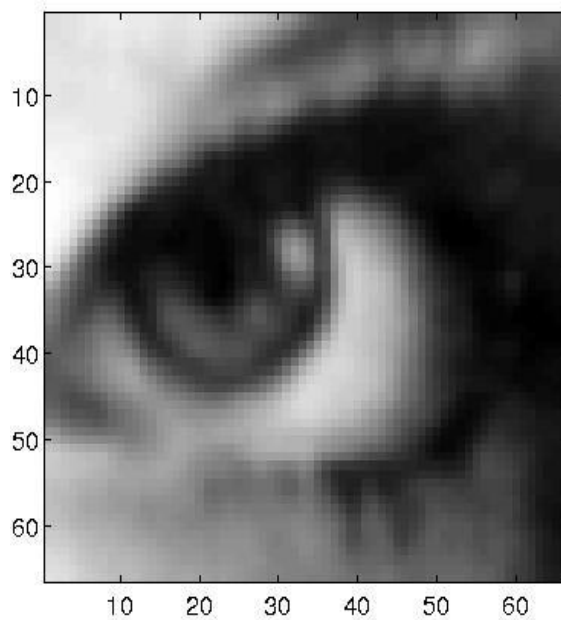


Figure: Bilinear

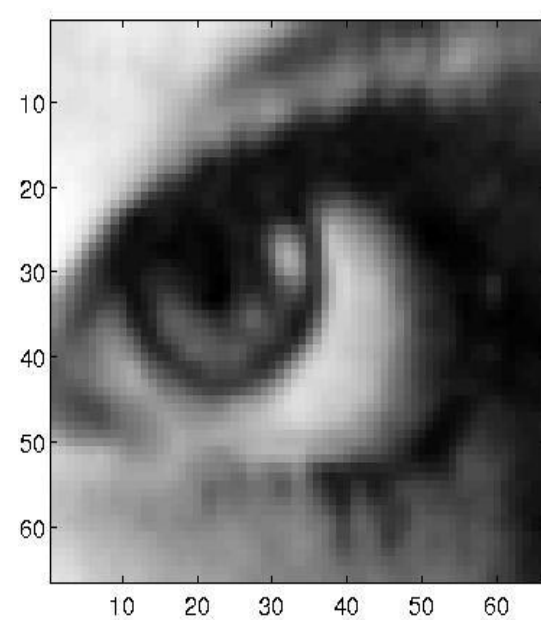
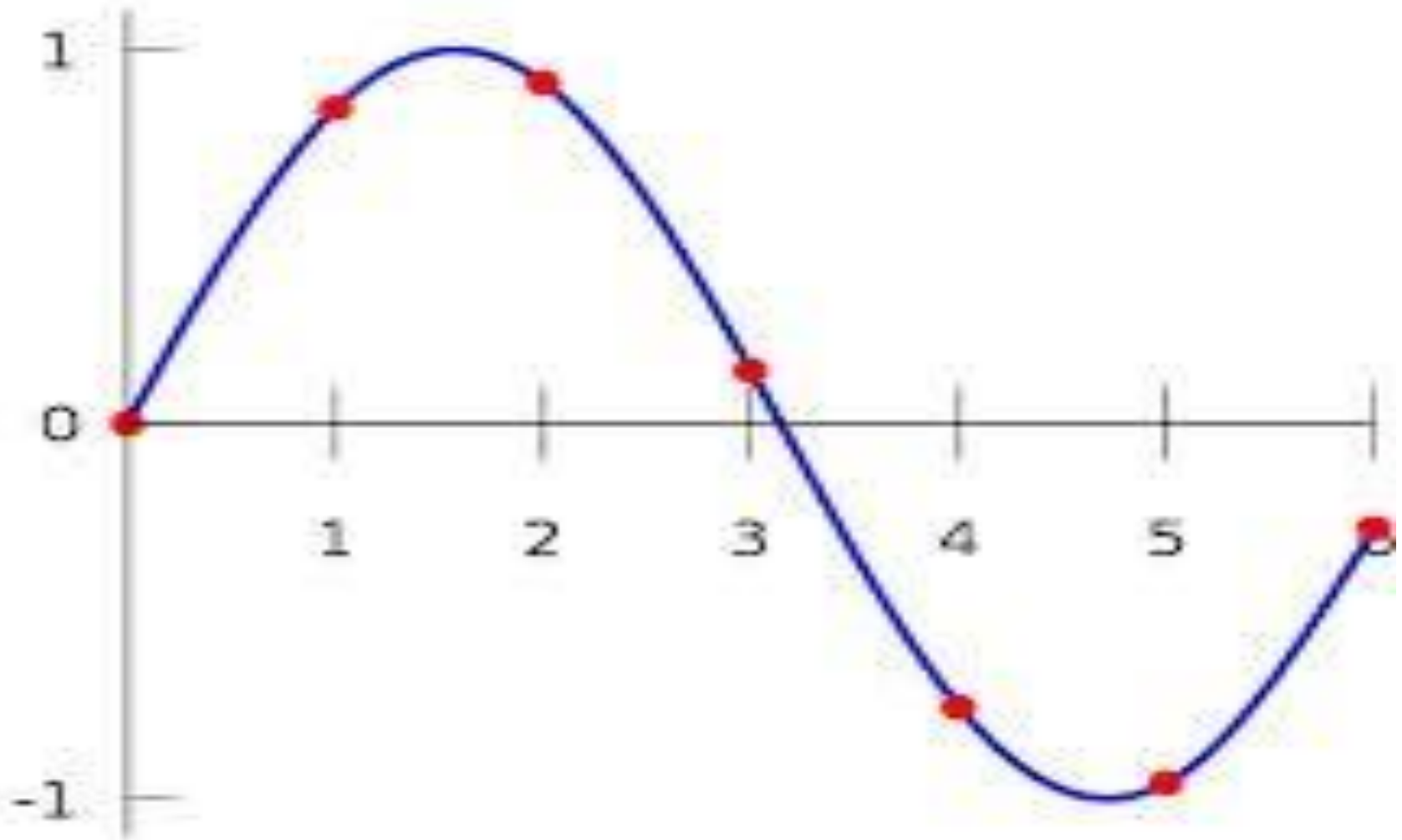


Figure: Bicubic

Relationship with 1D interpolation



Another example (wiki)

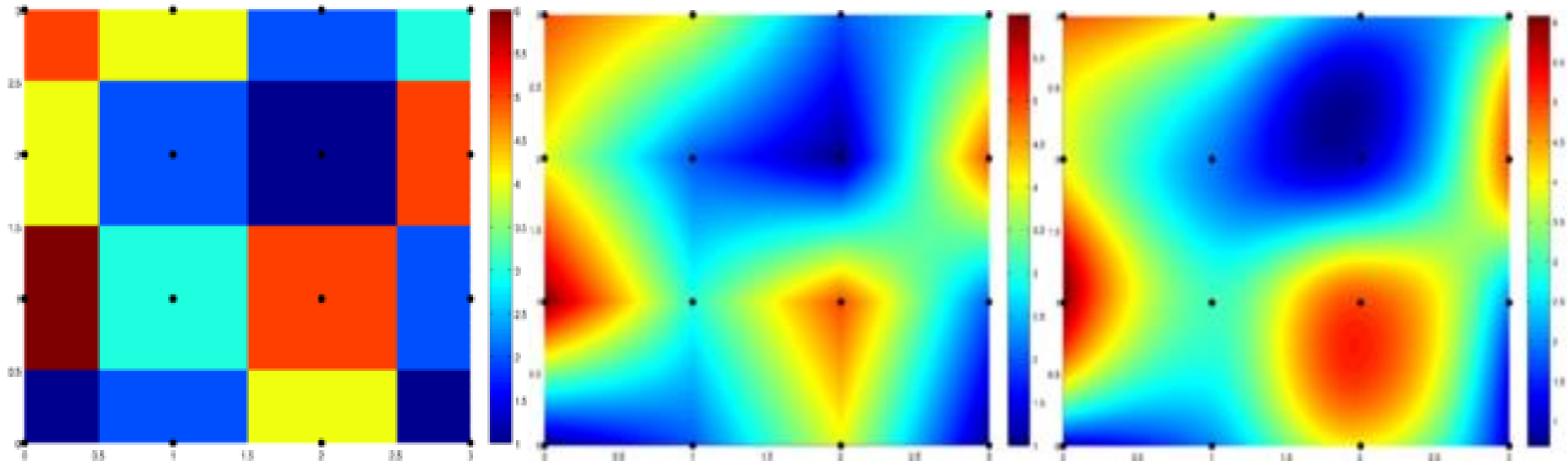
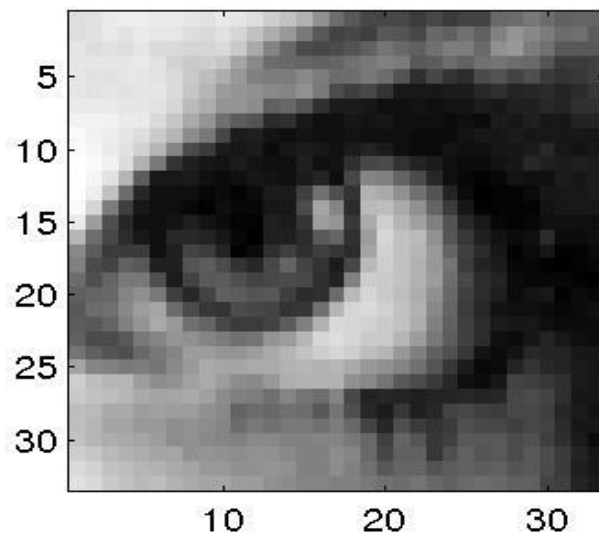
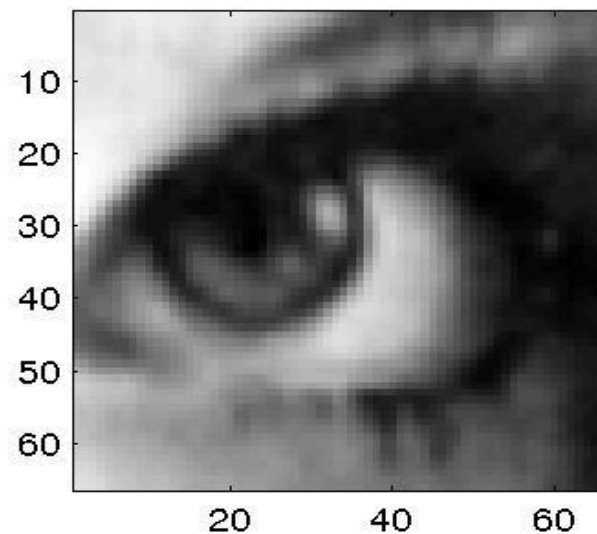


Figure: Nearest, bilinear and bicubic interpolations

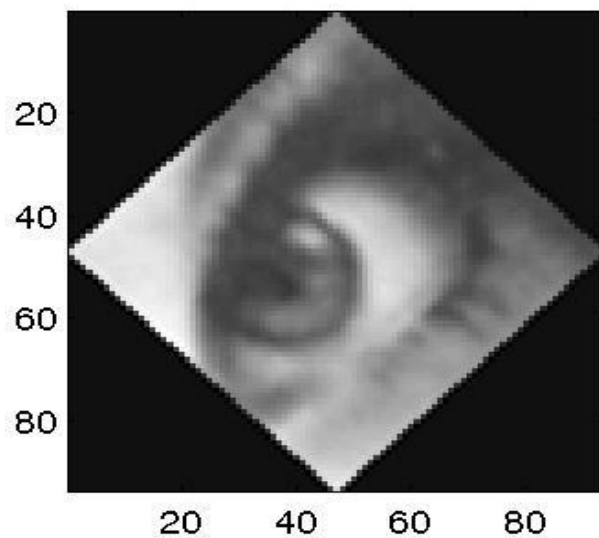
Original



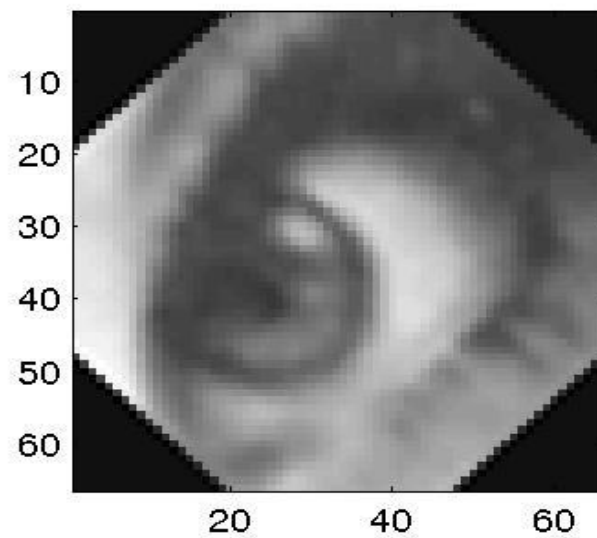
Resize



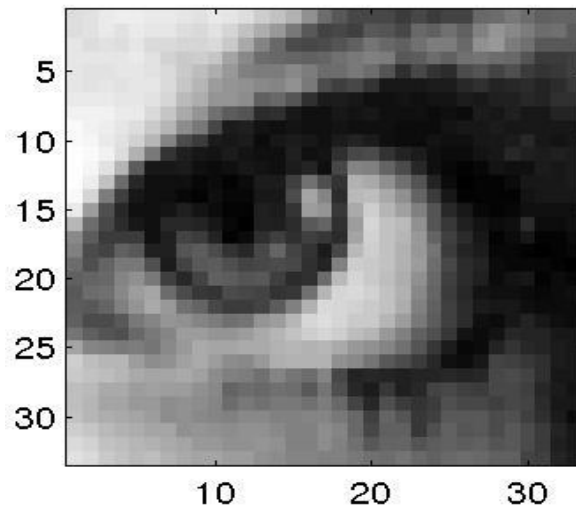
Rotate resized



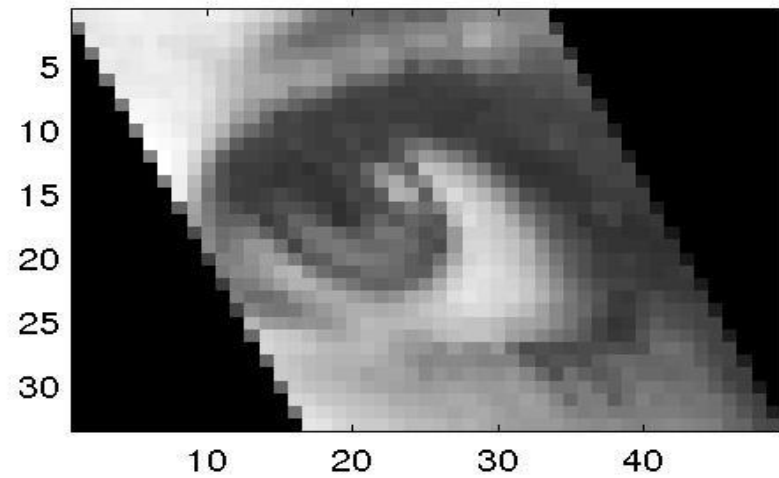
...and cropped



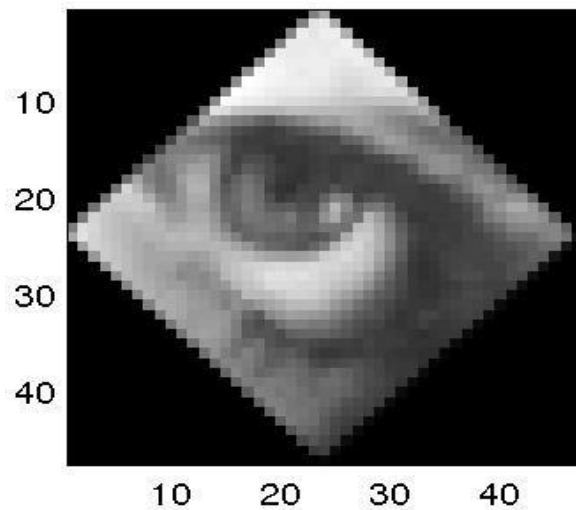
Original



Horizontal shear



Rotate



composition

