

# **Image Operations**

# Point Operations

- Point operations are simple image enhancement techniques
- $q = T(p)$
- Let  $f(x,y) = p$  and  $g(x,y) = q$ ; transformation  $T$   
e.g.  $[0, 255] \xleftarrow{T} [0, 255]$

$T \rightarrow$

- Linear function
- Non-linear function
- Clipping
- Window function

# Linear function

- $q = s * p + o;$
- where,  $s$  is the slope of the line and  $o$  is the offset from the origin.
- particularly useful for enhancing white of gray details embedded in the dark regions of an image.

# Non-linear function

e.g.  $q = c * \log(1+p)$

- $\log$  is nonlinear function;  $c$  is a constant
- This function converts a narrow range of low gray-level values of  $p$  in to a wider range of  $q$  values or vice versa
- This type of transformation is useful in expanding values of dark pixels while compressing the higher values

# Clipping

- The value of output pixel is related to the input pixel value in piece-wise linear fashion
- complex and require user input but useful in many practical situations

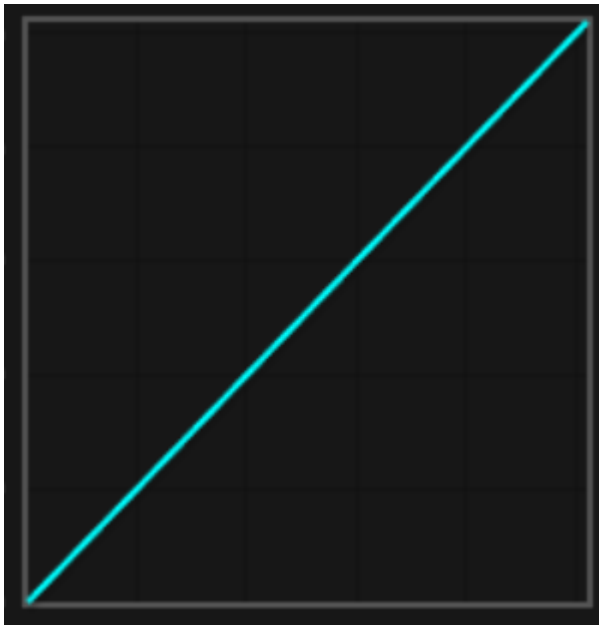
$$q = \begin{cases} 0 & p < a \\ s * p + o & a \leq p \leq b \\ L & p > b \end{cases}$$

- where, a and b are constants that define input pixel value range in which linear function with a slope s and offset o is applied
- e.g. gray level image; L = 255

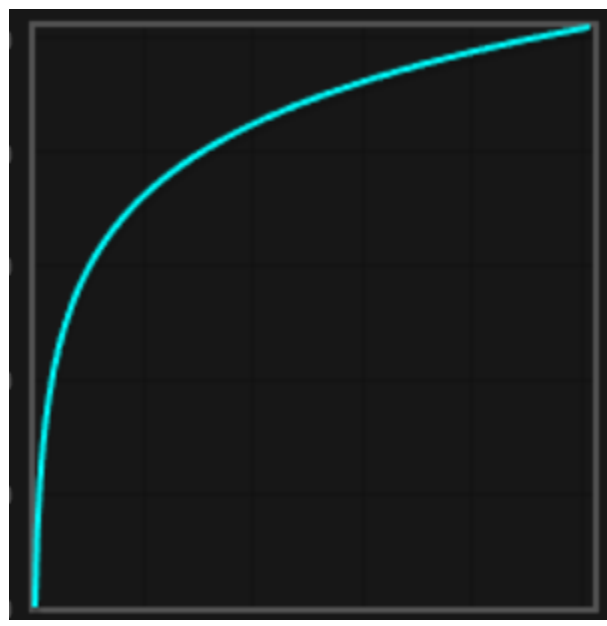
# Window

- similar to the clipper
- However, only a desired range of gray levels is preserved in the output image

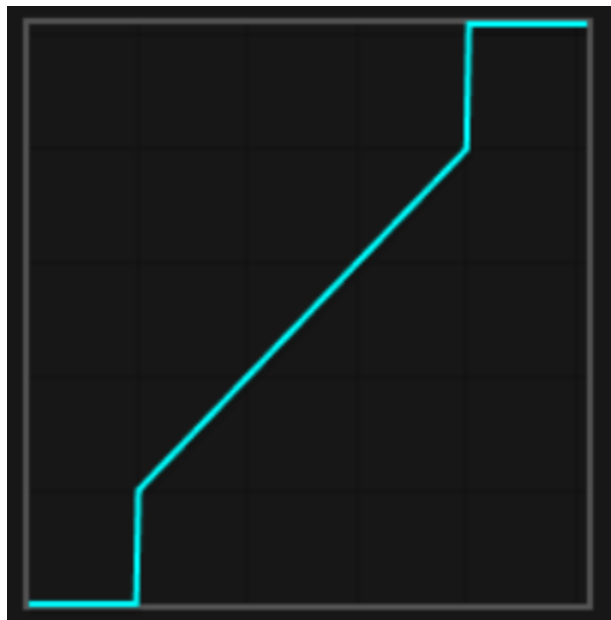
$$q = \left\{ \begin{array}{ll} 0 & p < a \\ s * p + o & a \leq p \leq b \\ 0 & p > b \end{array} \right\}$$



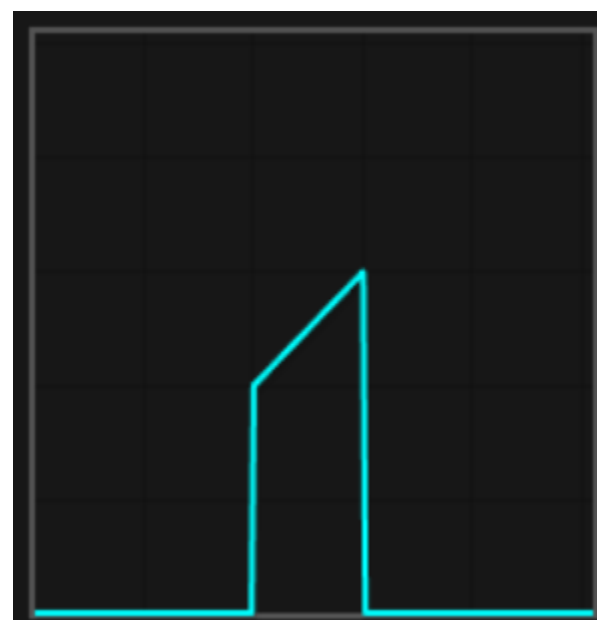
**Linear T**



**Non-Linear T**



**Clipping T**



**Windowing T**

# Image Interpolation

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# Spatial Interpolation

## Point Interpolation

## Areal Interpolation

### Exact

Weighting

Kriging

Splines

Interpolating  
Polynomials

Finite  
Difference

### Approximate

Power Series Trend

Fourier Series

Least-squares  
Fitting with Splines

Distance-weighted  
Least-squares

### Non-Volume- Preserving (Point-based)

(see both exact  
and approximate  
methods)

### Volume- Preserving (Area-based)

Overlay

Pycnophylactic



# Types of Spatial Interpolation

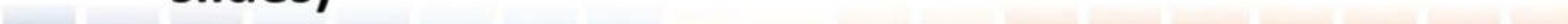
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- Global or Local
  - Global-use every known points to estimate unknown value.
  - Local – use a sample of known points to estimate unknown value.
- Exact or inexact interpolation
  - Exact – predict a value at the point location that is the same as its known value.
  - Inexact (approximate) – predicts a value at the point location that differs from its known value.
- Deterministic or stochastic interpolation
  - Deterministic – provides no assessment of errors with predicted values
  - Stochastic interpolation – offers assessment of prediction errors with estimated variances.

# Spatial Data Perspectives

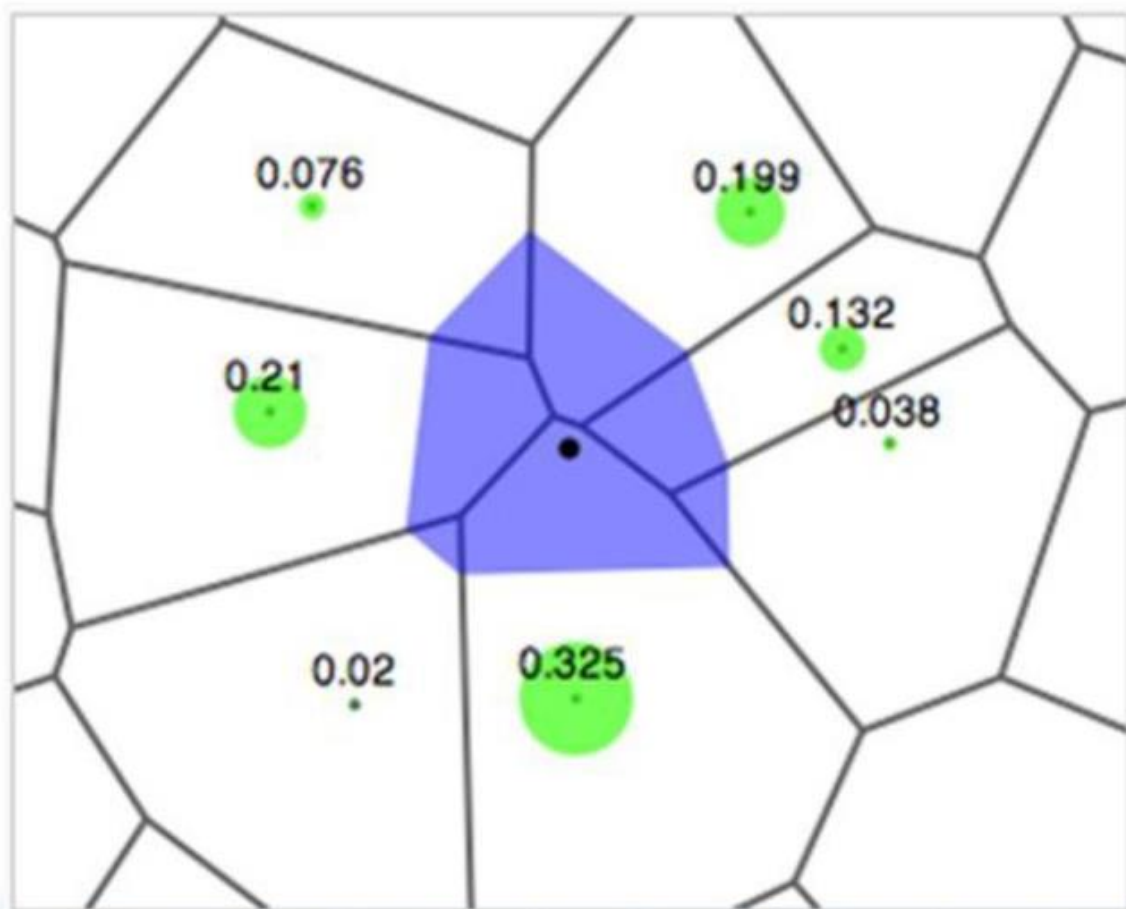
- **Deterministic perspective:** if we understand the system, we can predict the outcome with certainty
- **Stochastic perspective:** uncertainty remains in the best-designed model or experiment

# Types of Interpolation Methods

- ***Deterministic perspective:***
    - Inverse Distance Weighted: points weighted by distance
    - Spline: passes exactly through points with constraining equations
    - Polynomial/trend analysis (nearest/natural neighbor)
  - ***Stochastic perspective:***
    - Kriging: weighted by fitted semivariogram (next slides)
- 

# Natural Neighbor

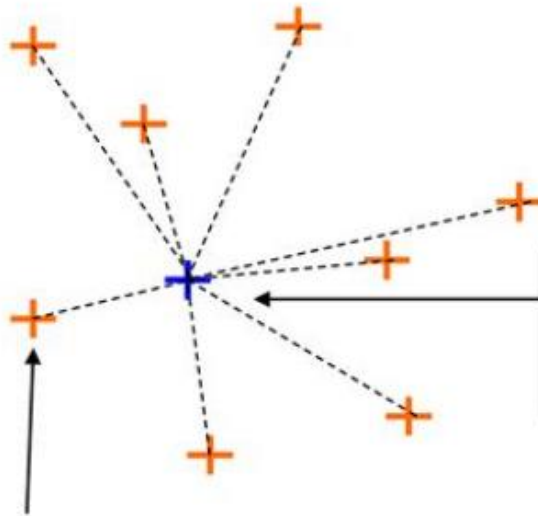
The colored circles, which represent the interpolating weights, are generated using the ratio of the shaded area to that of the cell area of the surrounding points. The shaded area is due to the insertion of the point to be interpolated into the Voronoi tessellation (Theissen polygons).





# Inverse Distance Weighted

The estimate is a weighted average



unknown value (to be  
interpolated)  
location  $x$

point  $i$   
known value  $z_i$   
location  $x_i$   
weight  $w_i$  distance  $d_i$

$$z(\mathbf{x}) = \frac{\sum_i w_i z_i}{\sum_i w_i}$$

$$w_i = 1/d_i^n$$

# Trend Surface/Polynomial

- Flat but TILTED plane to fit data  
(1<sup>st</sup> order polynomial)

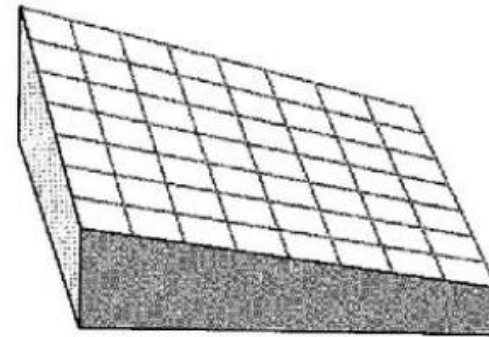
$$Z = a + bx + cy$$

- Tilted but WARPED plane to fit data  
( 2<sup>nd</sup> order polynomial )

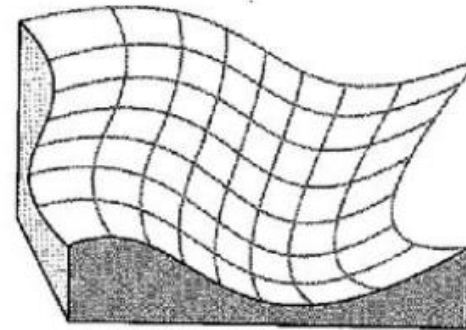
$$Z = a + bx + cy + dx^2 + exy + fy^2$$

# Trend Surfaces

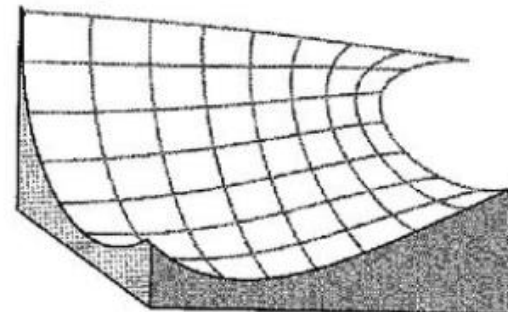
- **Simplifies the surface representation to allow visualization of general trends.**
- **Higher order polynomials can be used**
- **Robust regression methods can be used**



1st degree trend surface



2nd degree trend surface



3rd degree trend surface






# Kriging

- Spatial prediction of variable  $Z$  at location  $x$

$$Z(x) = m(x) + \gamma(h) + \varepsilon$$

- Three components:
    - *structural (constant mean),*
    - *random spatially correlated component*
    - *residual error*
- 

# Types of Kriging

- **Simple Kriging**: assumes mean is constant and known
- **Ordinary Kriging**: assumes mean is constant but unknown (MapWindow)
- **Universal Kriging**: assumes mean is varying and unknown
  - Modeled by a constant, linear, second or third order equation

# Advantages of Kriging

- It handles (embraces) spatial autocorrelation
- Less sensitive to preferential sampling in specific areas
- Allows uncertainty to be estimated (Kriging error)

# Content

**1 Introduction**

**2 Nearest neighbor**

**3 Bilinear interpolation**

**4 Bicubic**

**5 Matlab**

# Outline

**1 Introduction**

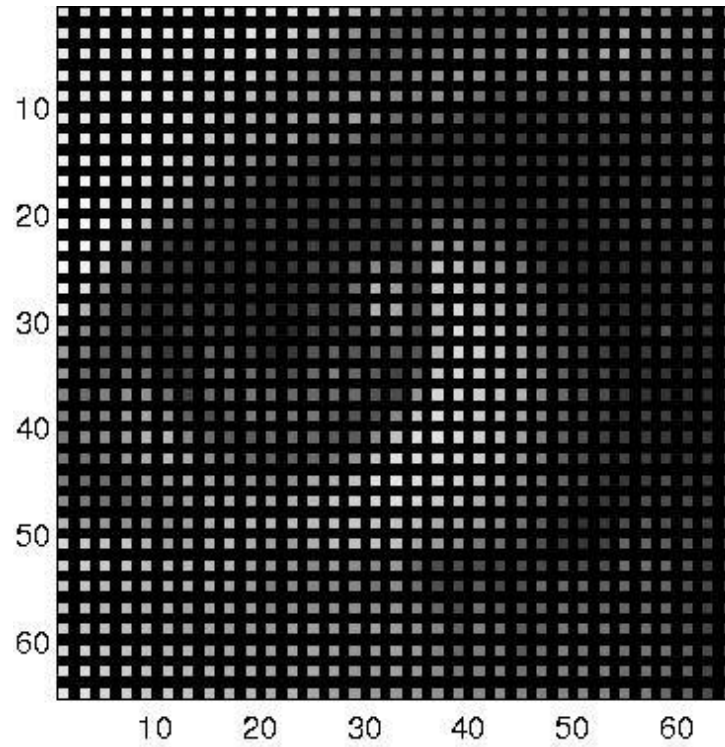
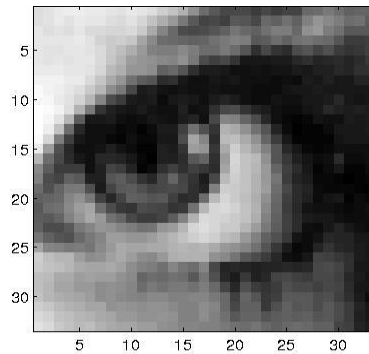
2 Nearest neighbor

3 Bilinear interpolation

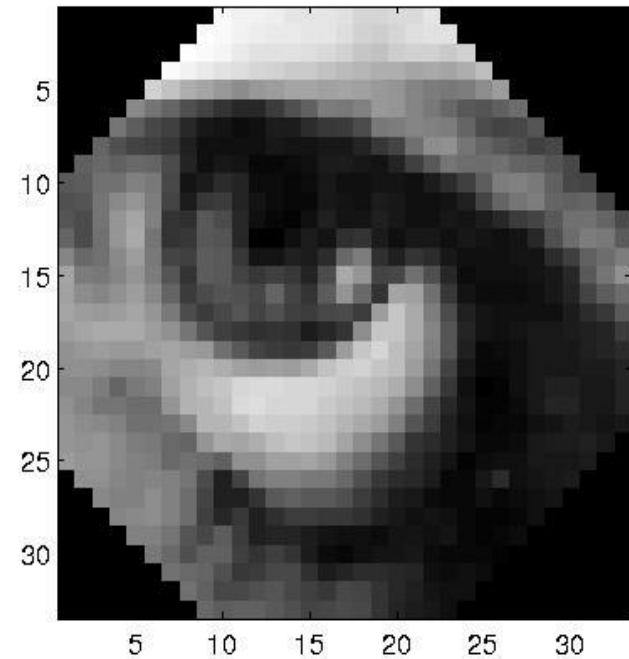
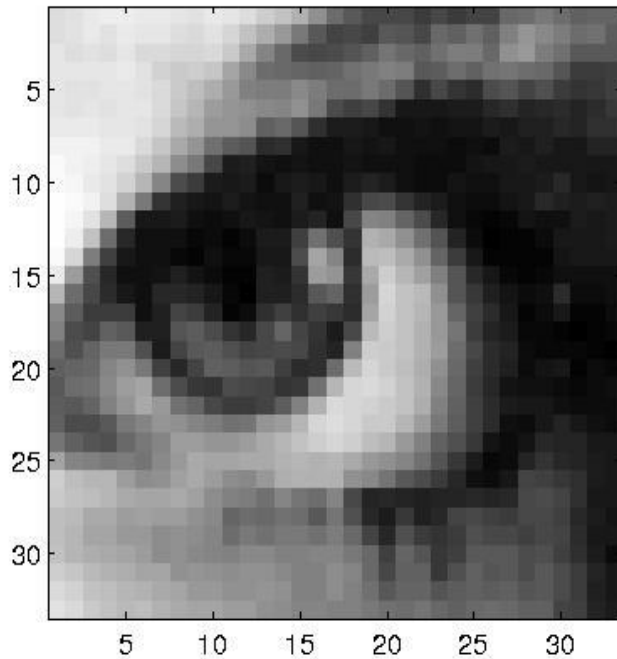
4 Bicubic

5 Matlab

# Resizing (resampling)



# Remapping (geometrical transformations- rotation, change of perspective,...)



# Inpainting (restauration of *holes*)





# Morphing, nonlinear transformations



# Outline

1 Introduction

**2 Nearest neighbor**

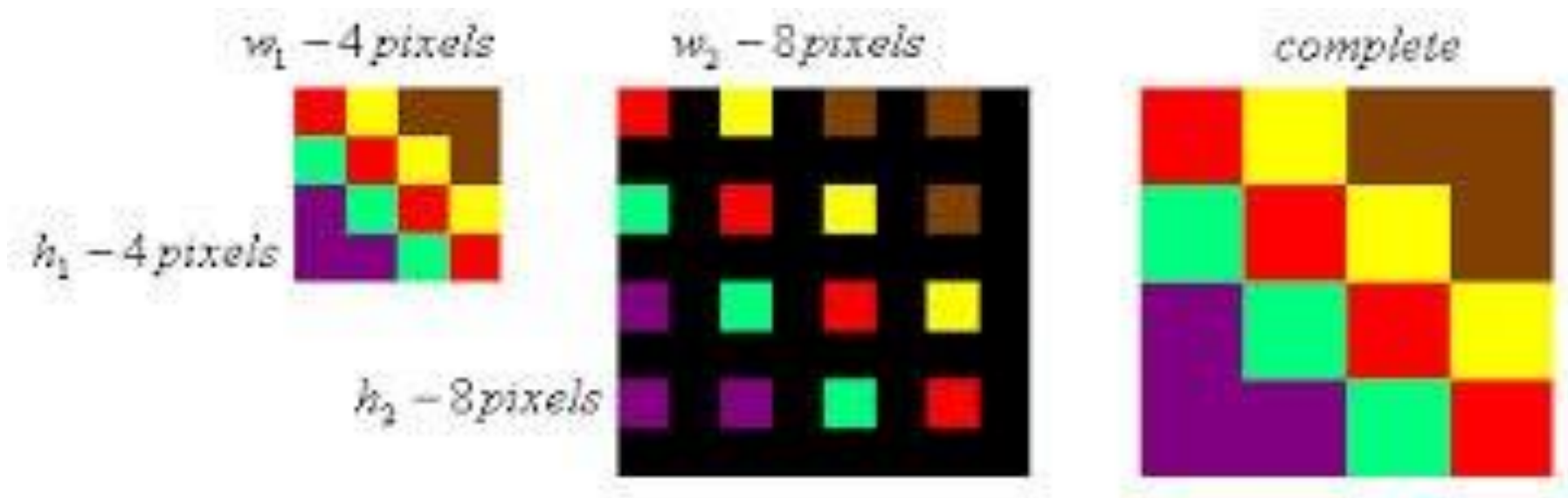
3 Bilinear interpolation

4 Bicubic

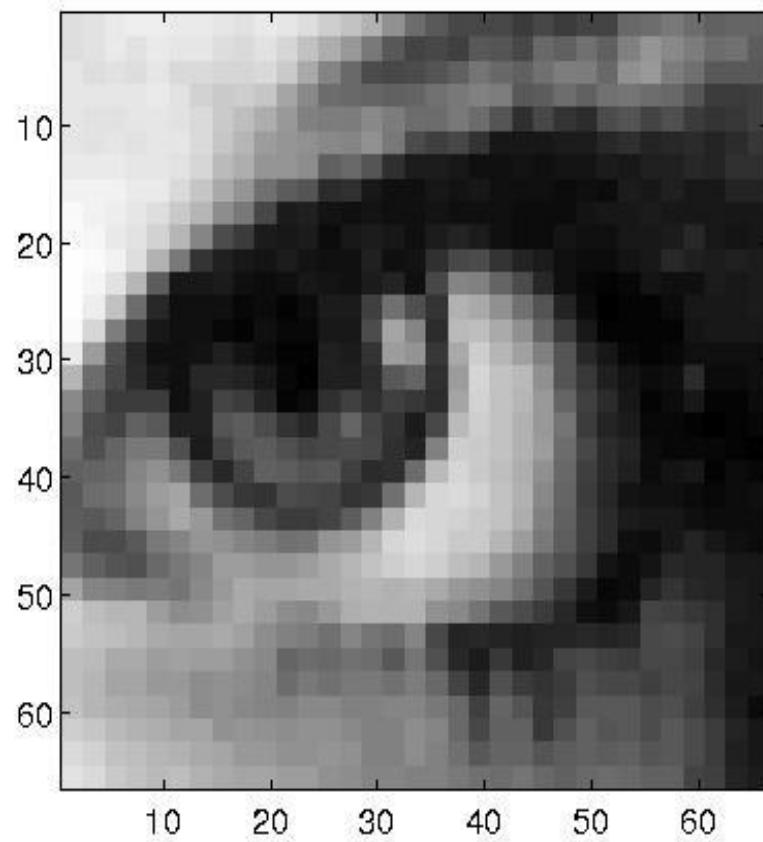
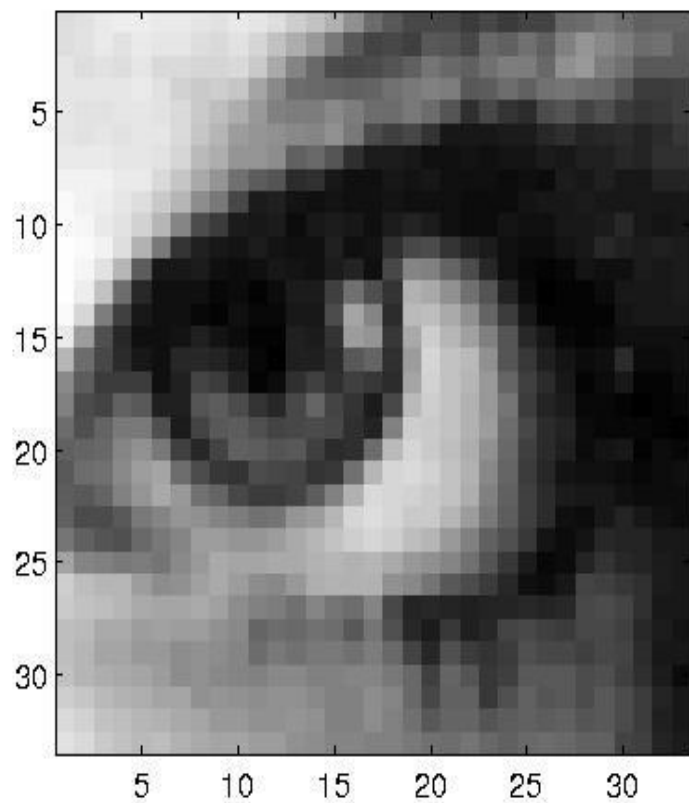
5 Matlab

# Nearest neighbor

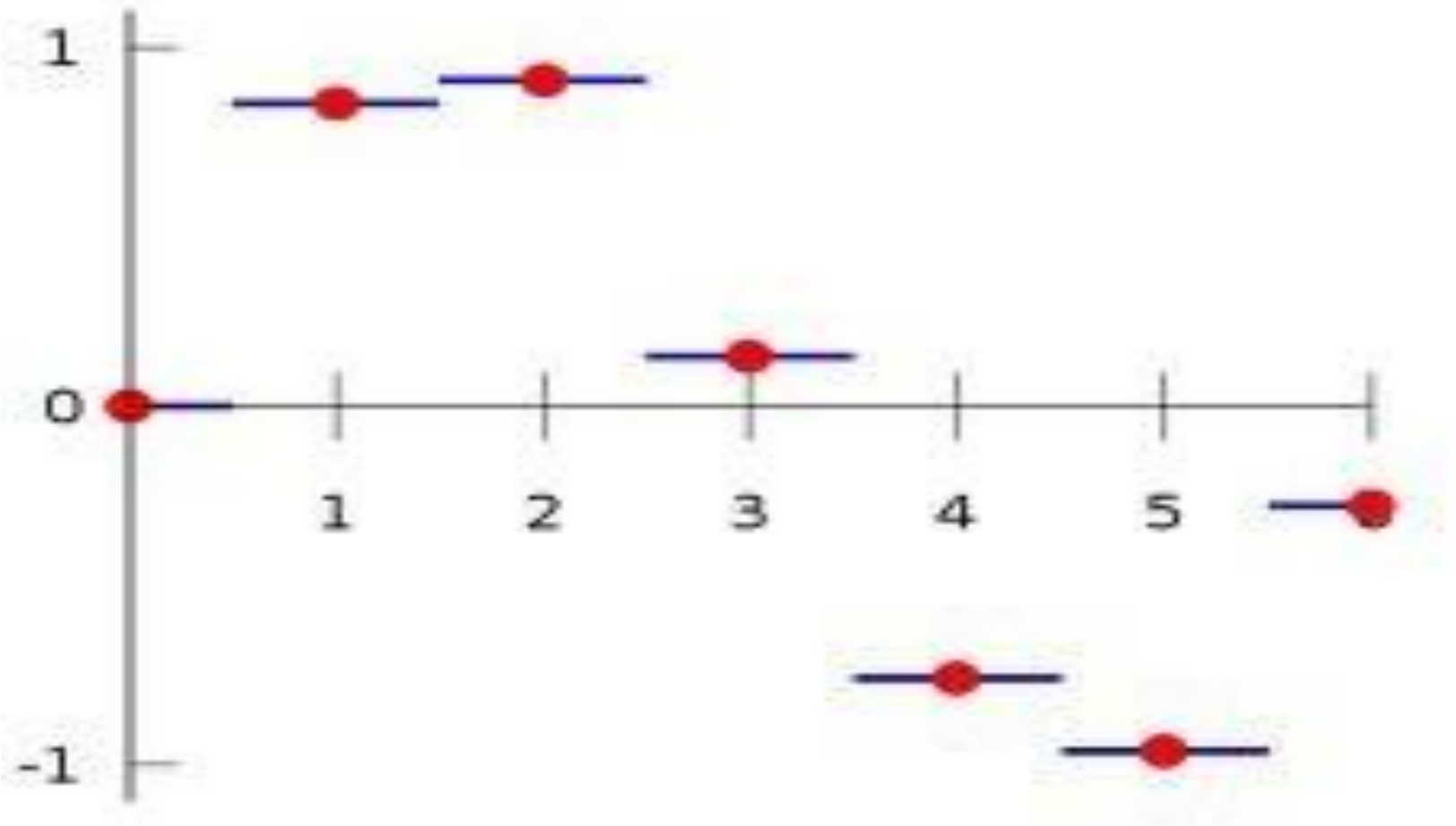
- Most basic method
- Requires the least processing time
- Only considers one pixel: the closest one to the interpolated point has the effect of simply making each pixel bigger



# Nearest Neighbor



# Relationship with 1D interpolation

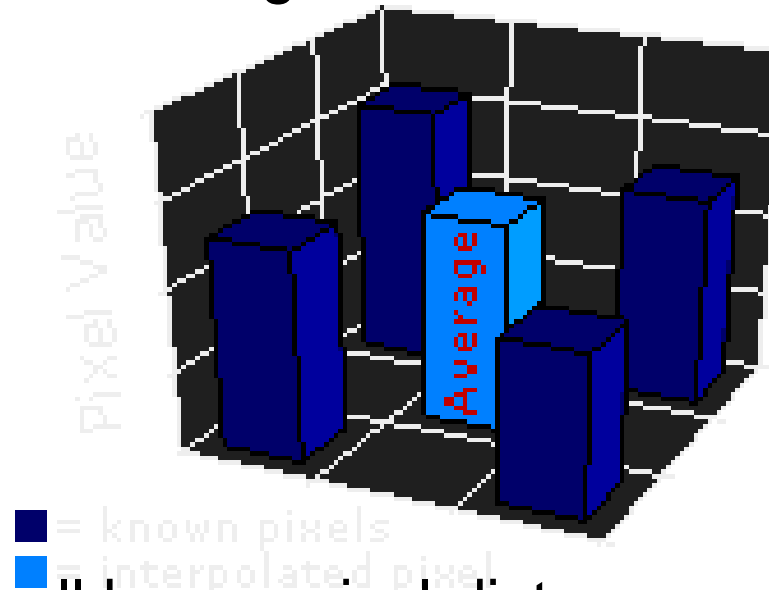


# Outline

- 1 Introduction
- 2 Nearest neighbor
- 3 Bilinear interpolation**
- 4 Bicubic
- 5 Matlab

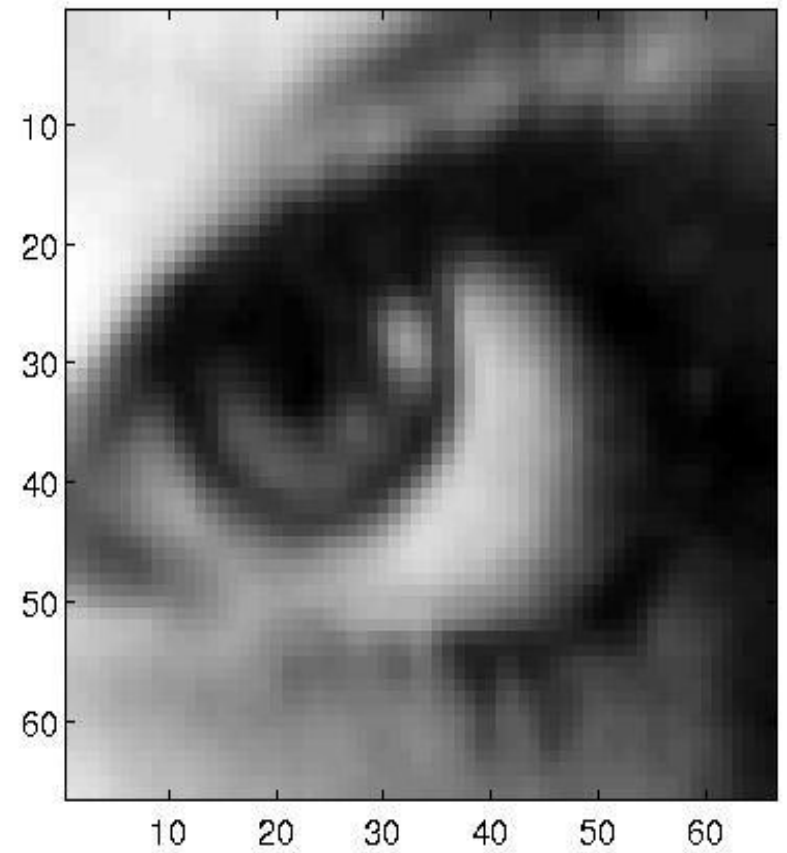
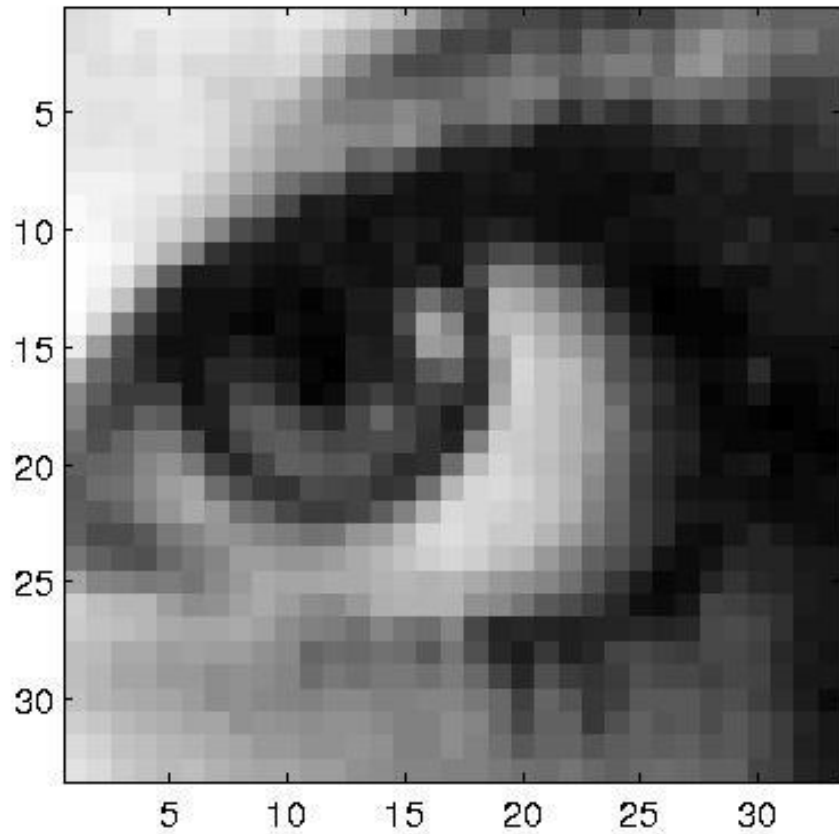
# Bilinear

- Considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixels
- Takes a weighted average of these 4 pixels to arrive at the final interpolated values
- Results in smoother looking images than nearest neighborhood
- Needs of more processing time



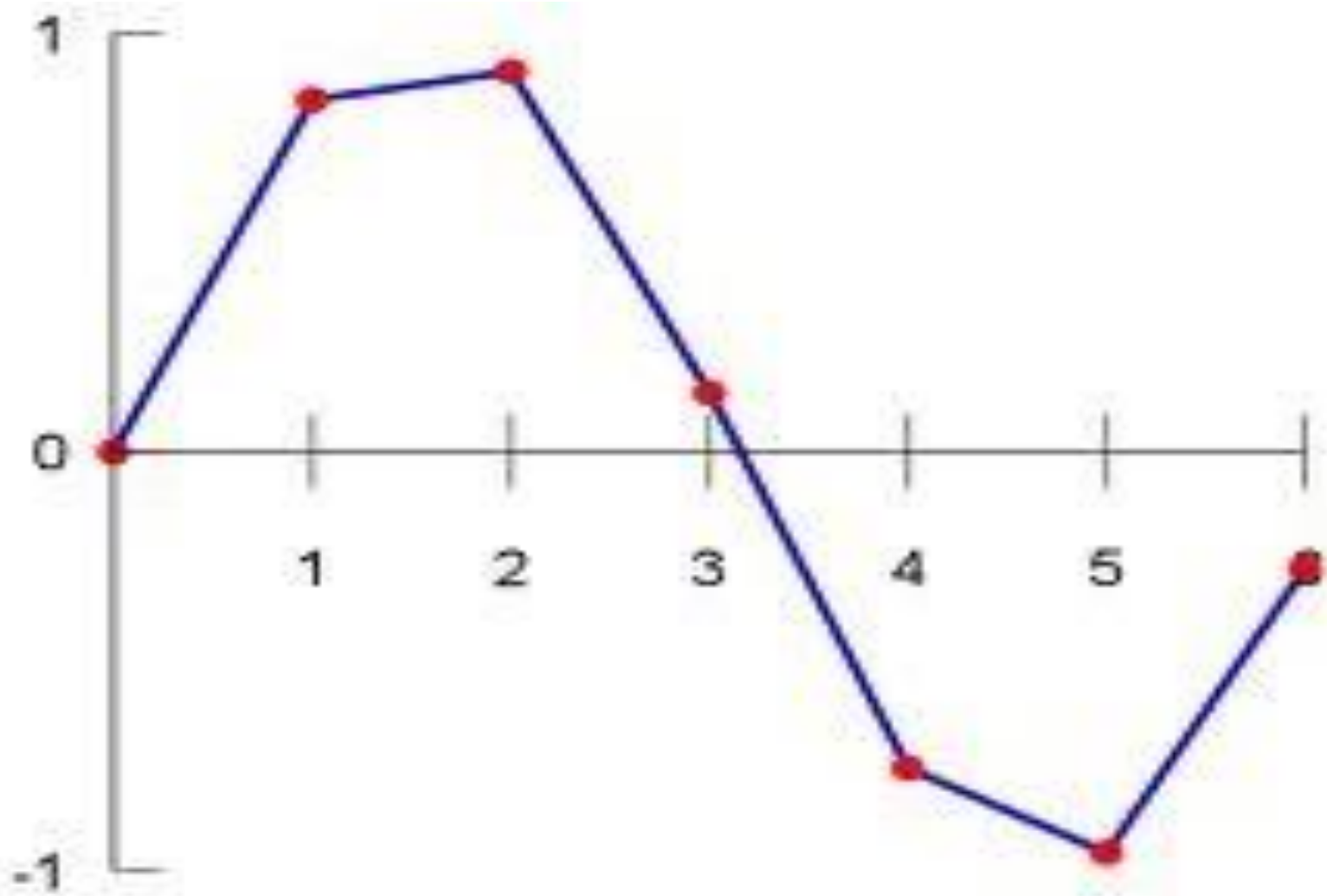
**Figure:** Case when all known pixel distances are equal. Interpolated value is simply their sum divided by four.

# Bilinear





# Relationship with 1D interpolation

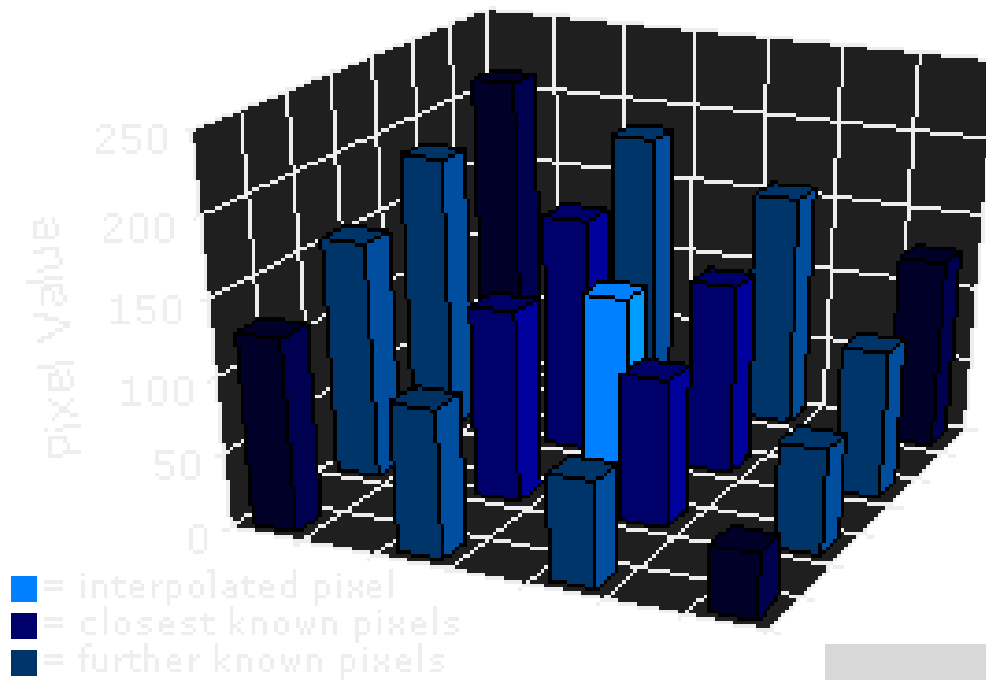


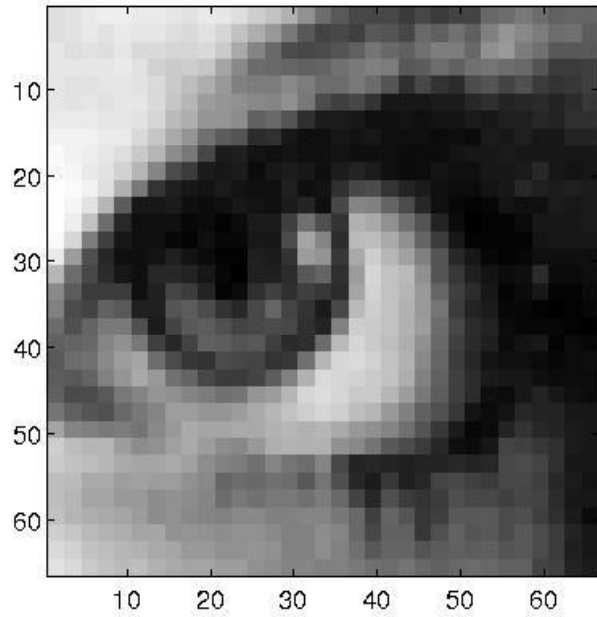
# Outline

- 1 Introduction
- 2 Nearest neighbor
- 3 Bilinear interpolation
- 4 Bicubic**
- 5 Matlab

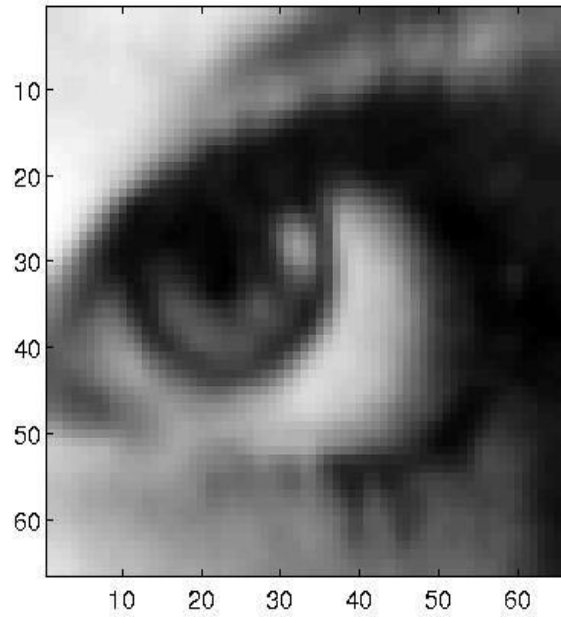
# Bicubic

- One step beyond bilinear by considering the closest 4x4 neighborhood of known pixels, for a total of 16 pixels
- Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation
- Produces sharper images than the previous two methods.
- Good compromise between processing time and output quality
- Standard in many image editing programs, printer drivers and in-camera interpolation

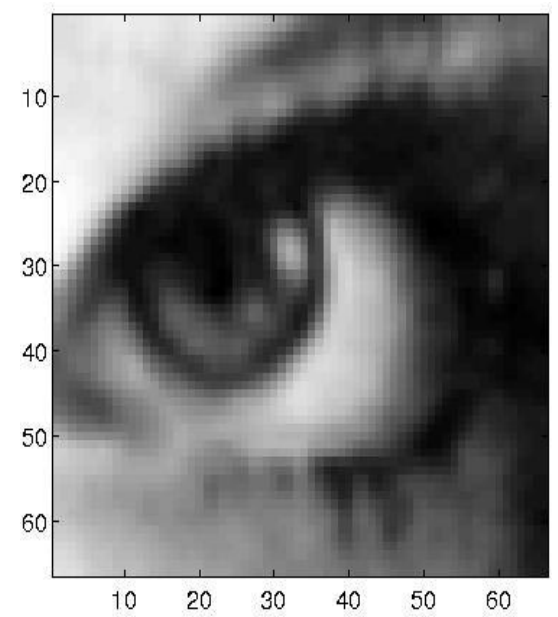




**Figure:** Nearest

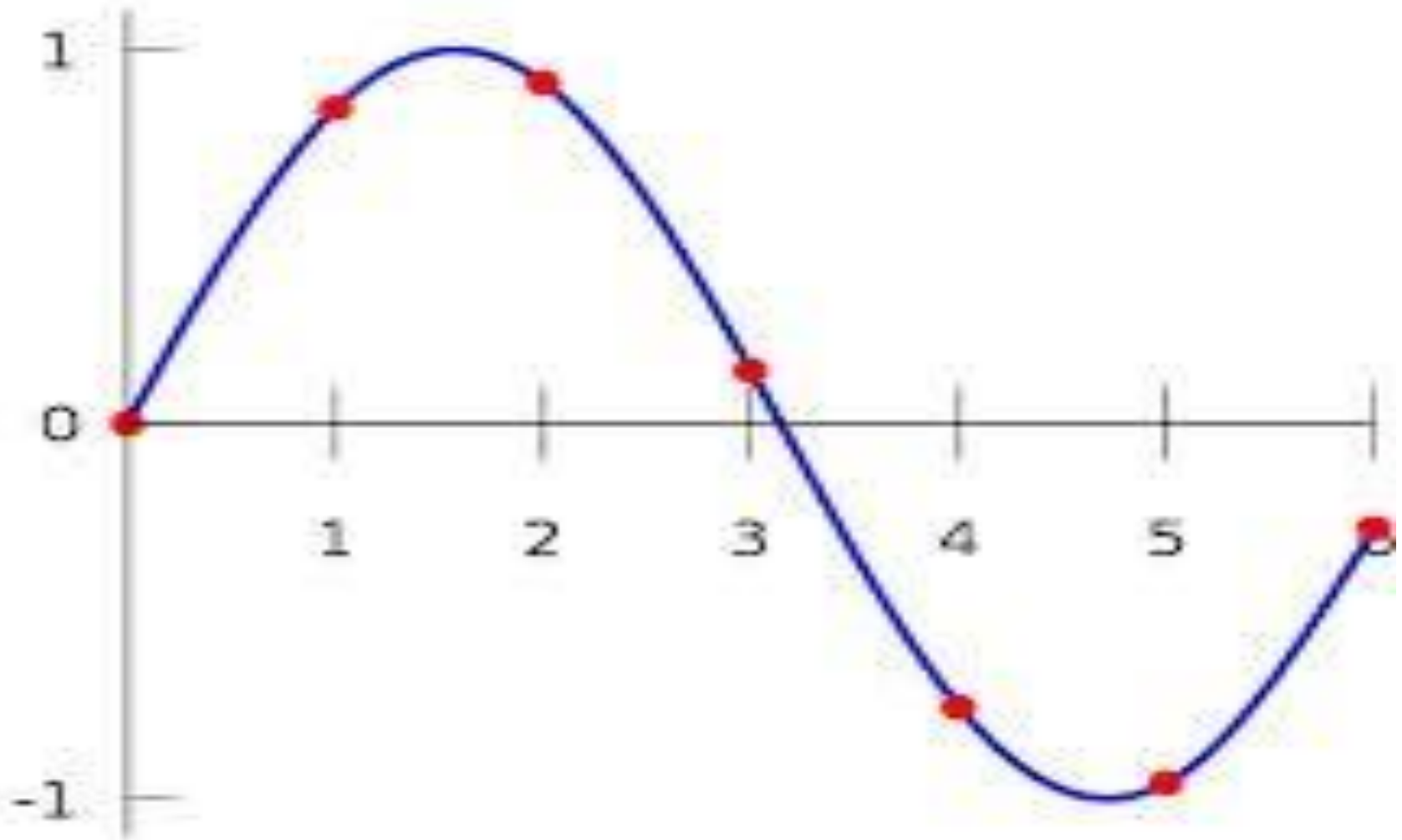


**Figure:** Bilinear

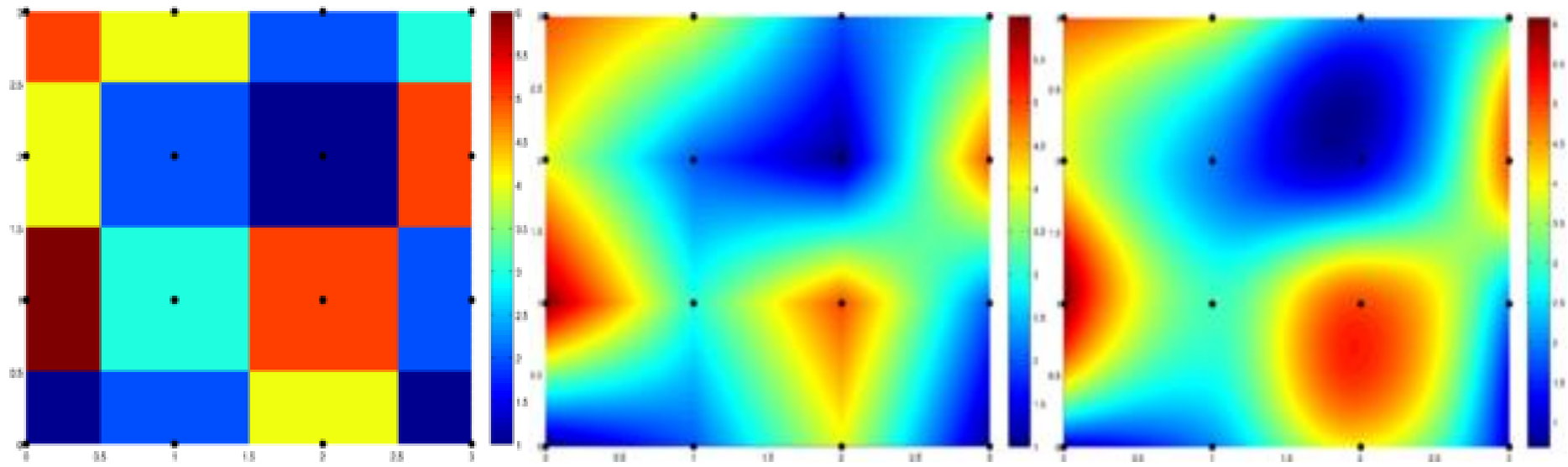


**Figure:** Bicubic

## Relationship with 1D interpolation

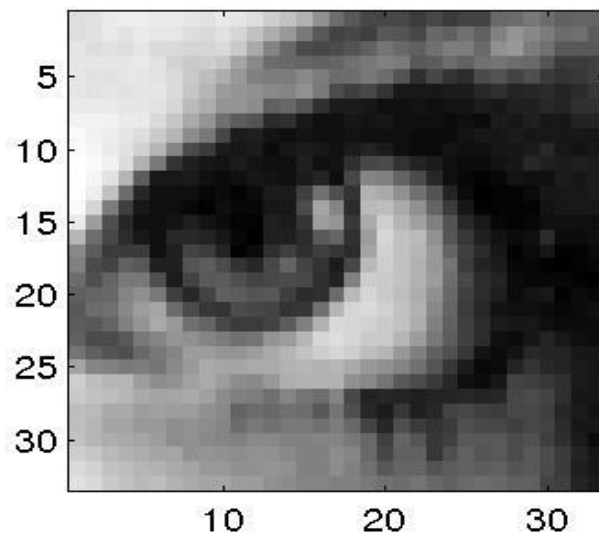


## Another example (wiki)

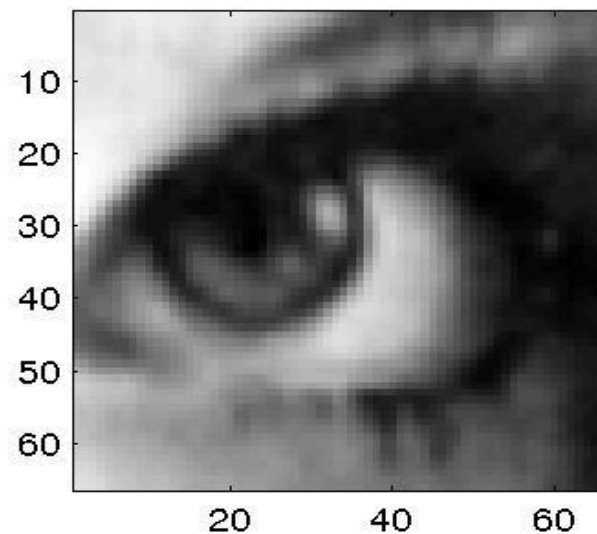


**Figure:** Nearest, bilinear and bicubic interpolations

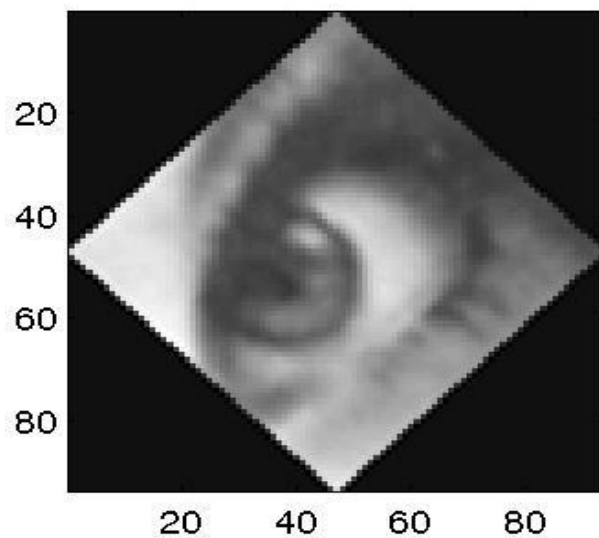
Original



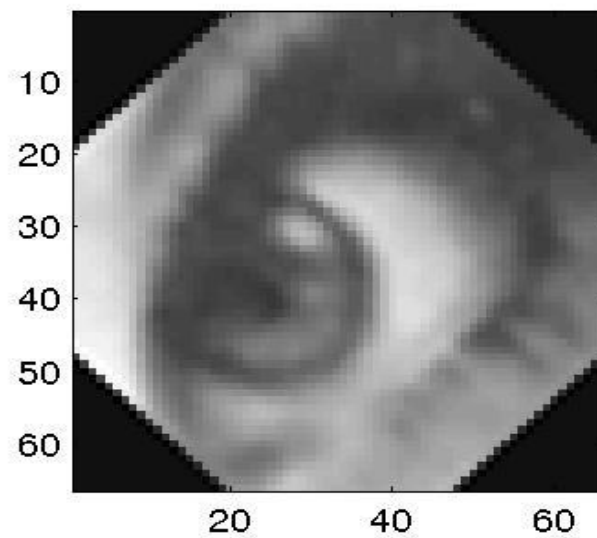
Resize



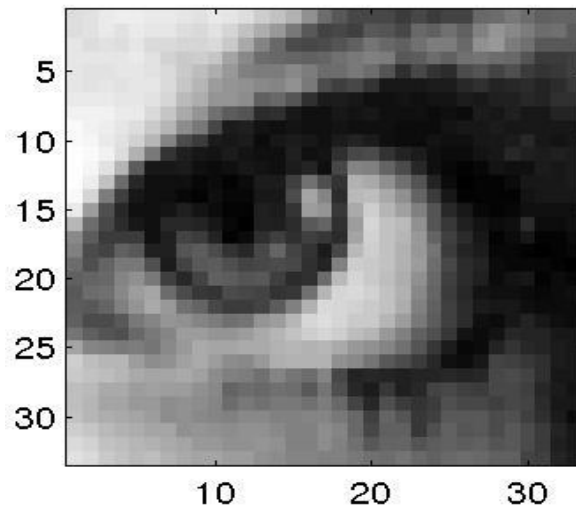
Rotate resized



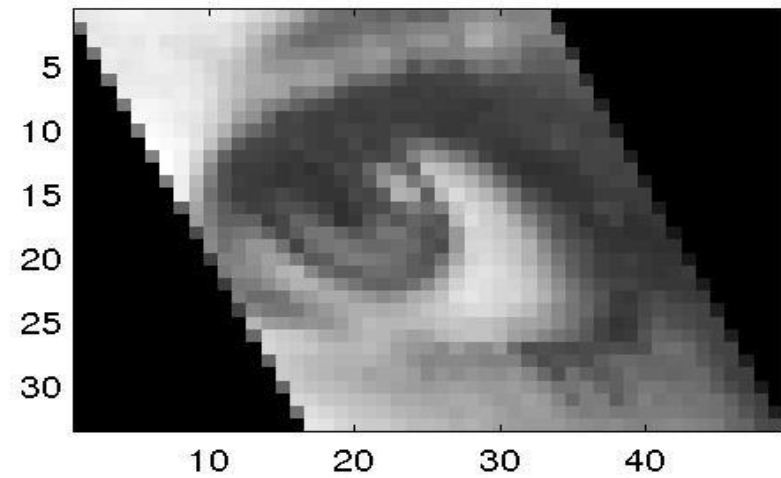
...and cropped



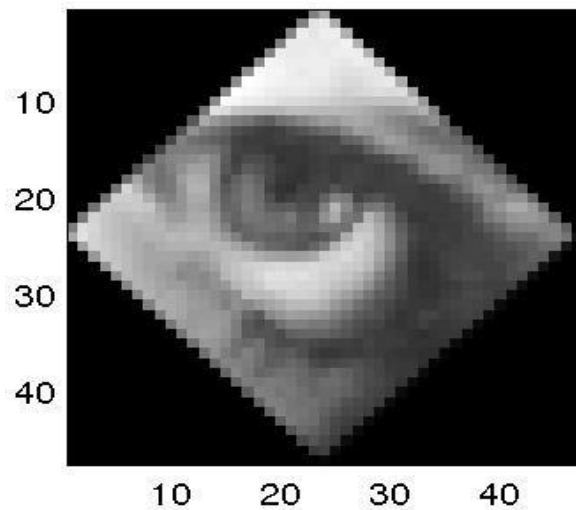
Original



Horizontal shear



Rotate



composition

