Numerical Optimization - Handin 5

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10.1

It's given that:
J is an m x n matrix, with $m \leq n$,
vector $y \in R^m$

In order to show that J has full column rank iff $J^T J$ is nonsingular, the goal is to show that

$$\text{"J has full column rank"} \implies \text{"}J^T J\text{ is nonsingular"}$$
$$\text{"}J^T J\text{ is nonsingular"} \implies \text{"J has full column rank"}$$

A full-rank matrix is also linearly independent, so from *"J has full column rank"*

$$Jx = 0 \implies x = 0 \tag{1}$$

$J^T J$ being nonsingular means that

$$J^T Jx = 0 \implies x = 0 \tag{2}$$

a)

Now the goal is to prove that they each imply each other.

$$J^T Jx = 0 \implies$$
$$x^T J^T Jx = x^T * 0 \implies x^T J^T Jx = 0 \implies (xJ)^T Jx = 0 \implies (Jx)(Jx) = 0 \implies$$
$$||Jx||^2 = 0 \implies$$
$$Jx = 0 \implies x = 0$$

Which is what was desired. Next

$$Jx = 0 \implies J^T Jx = J^T * 0 \implies$$
$$J^T Jx = 0 \implies x = 0$$

10.2

To prove that $f(x) = \frac{1}{2}||Jx - y||^2$ is convex, it's sufficient to prove that the second derivative is positive for all values of x the interval in question, which is R.

$$f(x) = \frac{1}{2}||Jx - y||^2$$
$$f'(x) = J^2x - Jy$$
$$f''(x) = J^2$$

Which is clearly positive for any value of x. Hence the function is convex.

10.3

b

If $\Pi = I$, $J\Pi = J = Q_1R$. Using this J on the form of 10.15 gives

$$J^TJ = (Q_1R)^T(Q_1R) = R^TR$$

A Cholesky factorization is suppose to be unique if the upper triangular has positive diagonal elements, which is the case for both R and $\hat{R}$. Hence for this to be the case here $R = \hat{R}$

Programming