Numerical Optimization - Handin 3

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3.2

When $0 < c_2 < c_1 < 1$, it means that there will be a *Line of Sufficient Decreaase* which is steeper than the *desired slope* line. In figure 1, I've placed the Line of Sufficient Decrease in such a way, that the interval of the function which upholds the Line of Sufficient Decrease is too does not intersect with the desired slope of the function.

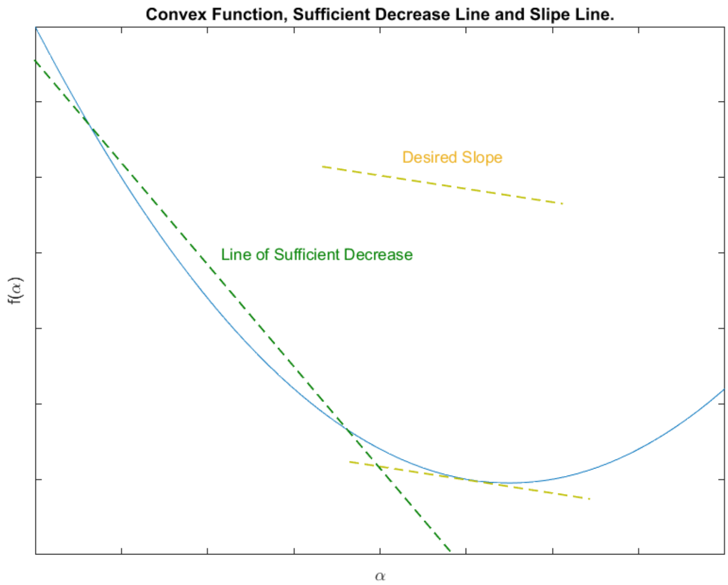

Figure 1: Function where Wolfe Conditions doesn't hold.

This results in no stepline being able to be chosen for the function in figure 1.

3.3

The strongly convex quadratic function has form

$$f(x) = \frac{1}{2}x^T Qx - b^T x$$

From page 42 in the book it's given that

$$\nabla f(x) = Qx - b \tag{1}$$

The descent direction can be defined as

$$\phi(\alpha) = f(x + \alpha p)$$

Which has the derivative (page 33 in the book)

$$\phi'(\alpha) = \nabla f(x_k + \alpha_k p_k)^T p_k$$

Which means the minimizer must fulfil the following:

$$\nabla f(x_k + \alpha_k p_k)^T p_k = 0 \tag{2}$$

Combining these two ∇'s into a singular expression, gives

$$\begin{aligned}
(Q(x + \alpha p) - b)^T p = 0 \\
(Qx + Q\alpha p - b)^T p = 0 \\
(Qx - b)^T p + \alpha p^T Qp = 0 \\
\alpha = -\frac{(Qx - b)^T p}{p^T Qp}
\end{aligned}$$

And remembering (1):

$$\alpha_k = -\frac{\nabla {f_k}^T p_k}{{p_k}^T Q p_k} \tag{3}$$

3.1

Steepest Descent

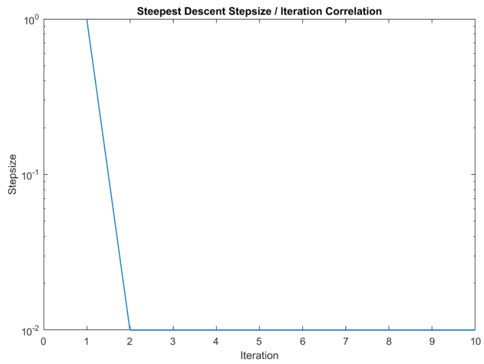

Figure 2: Stepsize α for Steepest Descent on Rosenbrock starting in $[1.2, 1.2]^T$

Figure 2 shows my stepsize, α, after having implemented Backtracking - Steepest Descent in matlab for the rosenbrock function. The implementaiton ended having to break its search loop due to max iterations reached, but ended at a coordinate of $x^* = (0.9949, 0.9781)$ which is close, in my opinion, to the real minimum of $(1, 1)$. The stepsizes reduce in size rapidly within the first iterations, after which my implementation settles with the same stepsize.

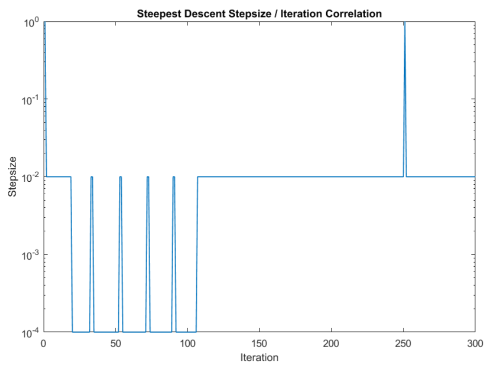

Figure 3: Stepsize α for Steepest Descent on Rosenbrock starting in $[-1.2, 1]^T$

Looking at Figure. 3, as mentioned in the exercise the second start location was a bit harder for my implementation to handle. Approximating a minimum at $x^* = (0.9945, 0.9780)$, it did come close, but when looking at the stepsizes they vary quite a bit in the first 100 iterations, clearly having problems with the rosenbrock function, after which the stepsize remains steady, until it suddenly jumps to a stepsize of 1 before returning to the previous stepsize.

Newton Method

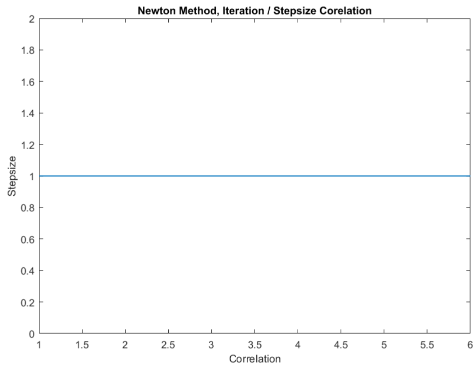

Figure 4: Stepsize α for Newton Method on Rosenbrock starting in $[1.2, 1.2]^T$

Within 5 iterations my Newton Method found the minimum, within a certainty of 10^-7, but the stepsize remained 1 through all the iterations.

Newton Method

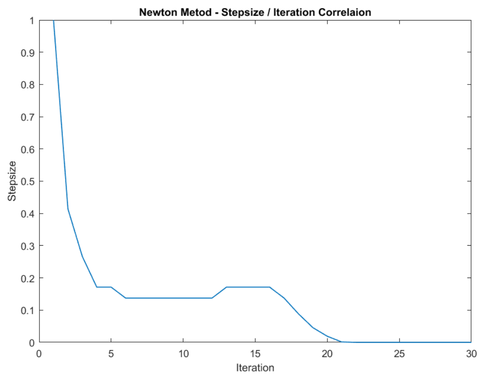

Figure 5: Stepsize α for Newton Method on Rosenbrock starting in $[-1.2, 1]^T$

This time my Newton Method failed to find the right answer. Only getting to other locations such as $(-0.8254, 0.6584)$, depending on the initial values for c and p.