APRIORI Algorithm

The Apriori Algorithm: Basics

The Apriori Algorithm is an influential algorithm for mining frequent itemsets for boolean association rules.

Key Concepts :

- Frequent Itemsets: The sets of item which has maximum support (denoted by L_i for i^{th}-Itemset).
- Apriori Property: Any subset of frequent itemset must be frequent.
- Join Operation: To find L_k , a set of candidate k-itemsets is generated by joining L_{k-1} with itself.

The Apriori Algorithm

- Find the *frequent itemsets*: the sets of items that have minimum support
  - A subset of a frequent itemset must also be a frequent itemset
    - i.e., if {*AB*} is a frequent itemset, both {*A*} and {*B*} should be a frequent itemset
  - Iteratively find frequent itemsets with cardinality from 1 to *k (k*-itemset*)*
- Use the frequent itemsets to generate association rules.

The Apriori Algorithm : Pseudo code

- Join Step: C_k is generated by joining $L_{k-1}$with itself
- Prune Step: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset
- Pseudo-code:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

The Apriori Algorithm: Example

TID	List of Items
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2 ,I3, I5
T900	I1, I2, I3

- Consider a database, D , consisting of 9 transactions.
- Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %)
- Let minimum confidence required is 70%.
- We have to first find out the frequent itemset using Apriori algorithm.
- Then, Association rules will be generated using min. support & min. confidence.

Step 1: Generating 1-itemset Frequent Pattern

Scan D for count of each candidate

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

C_1

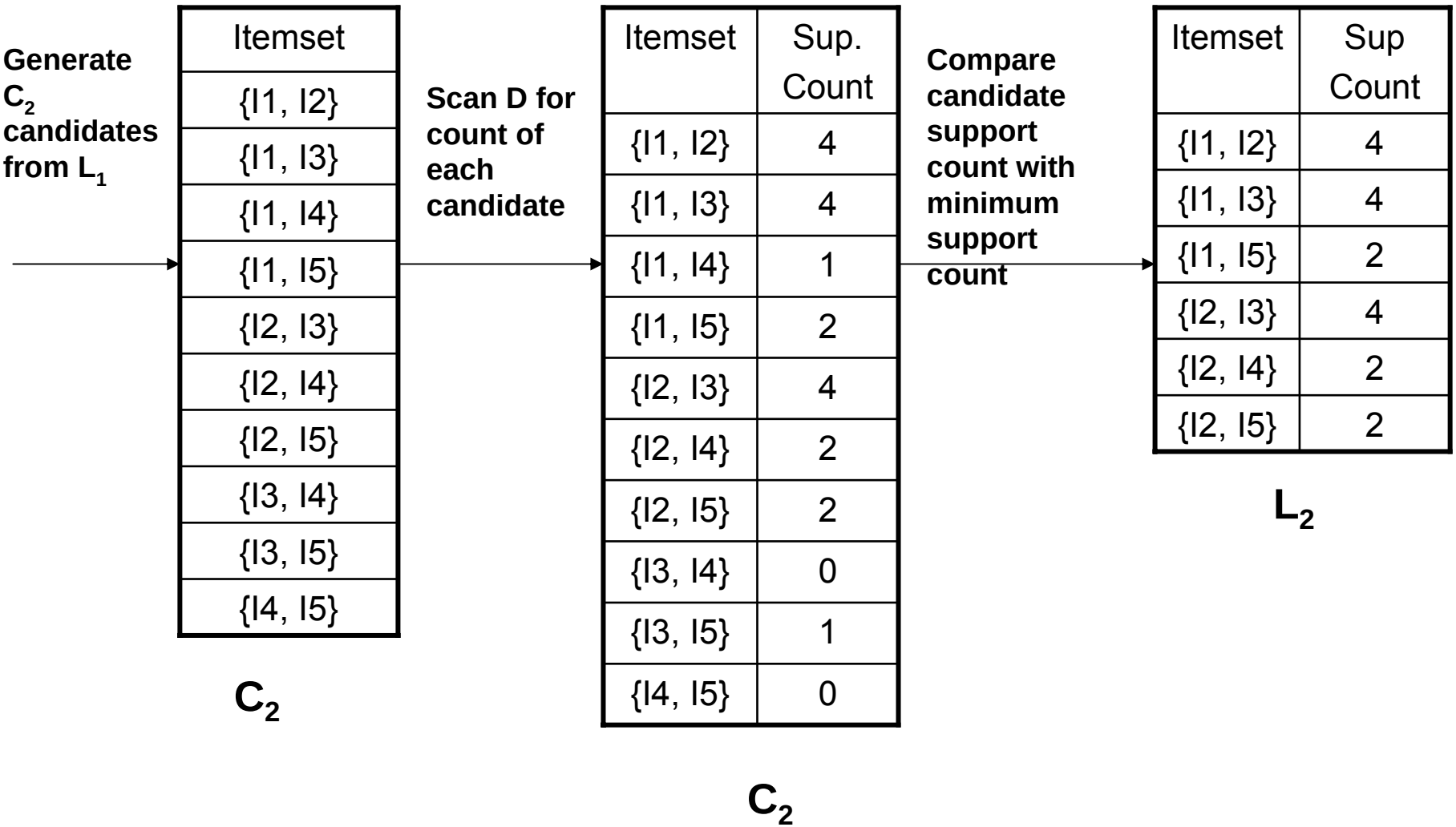
Compare candidate support count with minimum support count

Itemset	Sup.Count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

L_1

- The set of frequent 1-itemsets, L_1 , consists of the candidate 1-itemsets satisfying minimum support.
- In the first iteration of the algorithm, each item is a member of the set of candidate.

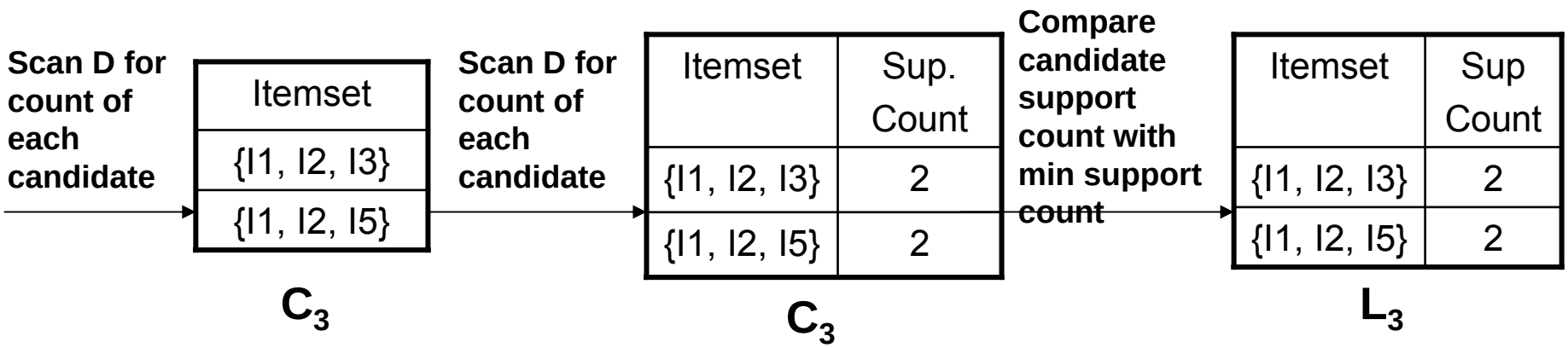
Step 2: Generating 2-itemset Frequent Pattern



Step 2: Generating 2-itemset Frequent Pattern

- To discover the set of frequent 2-itemsets, L_2 , the algorithm uses L_1 *Join* L_1 to generate a candidate set of 2-itemsets, C_2.
- Next, the transactions in D are scanned and the support count for each candidate itemset in C_2 is accumulated (as shown in the middle table).
- The set of frequent 2-itemsets, L_2 , is then determined, consisting of those candidate 2-itemsets in C_2 having minimum support.
- Note: We haven't used Apriori Property yet.

Step 3: Generating 3-itemset Frequent Pattern



- The generation of the set of candidate 3-itemsets, C_3 , involves use of the Apriori Property.
- In order to find C_3, we compute L_2 *Join* L_2.
- C_3 = L2 *Join* L2 = {{I1, I2, I3}, {I1, I2, I5}, {I1, I3, I5}, {I2, I3, I4}, {I2, I3, I5}, {I2, I4, I5}}.
- Now, Join step is complete and Prune step will be used to reduce the size of C_3. Prune step helps to avoid heavy computation due to large C_k.

Step 3: Generating 3-itemset Frequent Pattern

- Based on the Apriori property that all subsets of a frequent itemset must also be frequent, we can determine that four latter candidates cannot possibly be frequent. How ?
- For example , lets take {I1, I2, I3}. The 2-item subsets of it are {I1, I2}, {I1, I3} & {I2, I3}. Since all 2-item subsets of {I1, I2, I3} are members of L_2, We will keep {I1, I2, I3} in C_3.
- Lets take another example of {I2, I3, I5} which shows how the pruning is performed. The 2-item subsets are {I2, I3}, {I2, I5} & {I3,I5}.
- BUT, {I3, I5} is not a member of L_2 and hence it is not frequent violating Apriori Property. Thus We will have to remove {I2, I3, I5} from C_3.
- Therefore, C_3 = {{I1, I2, I3}, {I1, I2, I5}} after checking for all members of result of Join operation for Pruning.
- Now, the transactions in D are scanned in order to determine L_3, consisting of those candidates 3-itemsets in C_3 having minimum support.

Step 4: Generating 4-itemset Frequent Pattern

- The algorithm uses L_3 *Join* L_3 to generate a candidate set of 4-itemsets, C_4. Although the join results in {{I1, I2, I3, I5}}, this itemset is pruned since its subset {{I2, I3, I5}} is not frequent.
- Thus, $C_4 = \varphi$, and algorithm terminates, having found all of the frequent items. This completes our Apriori Algorithm.
- What's Next ?

  These frequent itemsets will be used to generate strong association rules (where strong association rules satisfy both minimum support & minimum confidence).

Step 5: Generating Association Rules from Frequent Itemsets

- Procedure:
  - For each frequent itemset ***"l",*** generate all nonempty subsets of ***l.***
  - For every nonempty subset ***s*** of ***l***, output the rule **"s → (l-s)"** if **support_count(l) / support_count(s) >= min_conf** where min_conf is minimum confidence threshold.

- Back To Example:

  We had L = {{I1}, {I2}, {I3}, {I4}, {I5}, {I1,I2}, {I1,I3}, {I1,I5}, {I2,I3}, {I2,I4}, {I2,I5}, {I1,I2,I3}, {I1,I2,I5}}.
  - Lets take ***l*** = {I1,I2,I5}.
  - Its all nonempty subsets are {I1,I2}, {I1,I5}, {I2,I5}, {I1}, {I2}, {I5}.

Step 5: Generating Association Rules from Frequent Itemsets

- Let minimum confidence threshold is , say 70%.
- The resulting association rules are shown below, each listed with its confidence.
  - R1: I1 ^ I2 → I5
    - Confidence = sc{I1,I2,I5}/sc{I1,I2} = 2/4 = 50%
    - R1 is Rejected.
  - R2: I1 ^ I5 → I2
    - Confidence = sc{I1,I2,I5}/sc{I1,I5} = 2/2 = 100%
    - R2 is Selected.
  - R3: I2 ^ I5 → I1
    - Confidence = sc{I1,I2,I5}/sc{I2,I5} = 2/2 = 100%
    - R3 is Selected.

Step 5: Generating Association Rules from Frequent Itemsets

- R4: I1 → I2 ^ I5
  - Confidence = sc{I1,I2,I5}/sc{I1} = 2/6 = 33%
  - R4 is Rejected.
- R5: I2 → I1 ^ I5
  - Confidence = sc{I1,I2,I5}/{I2} = 2/7 = 29%
  - R5 is Rejected.
- R6: I5 → I1 ^ I2
  - Confidence = sc{I1,I2,I5}/ {I5} = 2/2 = 100%
  - R6 is Selected.

    In this way, We have found three strong association rules.

Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent.
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans.
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB.
- Sampling: mining on a subset of given data, lower support threshold + a method to determine the completeness.
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent.