

①

Principal component Analysis (PCA)

X	Y
2.5	2.4
0.5	0.7
2.2	2.9
1.9	2.2
3.1	3.0
2.3	2.7
2	1.6
1	1.1
1.5	1.6
1.1	0.9

$$\bar{X} = 1.81 \quad \bar{Y} = 1.91$$

$$C = \begin{bmatrix} \text{Cov}(X, X) & \text{Cov}(X, Y) \\ \text{Cov}(Y, X) & \text{Cov}(Y, Y) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix}$$

$$\text{Cov}(X, Y) = \frac{N}{n-1} \sum_{i=1}^N \frac{(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\text{Cov}(X, X) =$$

X	X - \bar{X}	(X - \bar{X})(X - \bar{X})
2.5	0.69	0.4761
0.5	-1.31	1.7161
...

$$\text{Sum} = 5.5490$$

$$\text{Cov}(X, X) = \frac{5.5490}{9} = 0.6165$$

$$\text{Cov}(Y, Y) =$$

Y	Y - \bar{Y}	(Y - \bar{Y})(Y - \bar{Y})
2.4	0.49	0.2401
0.7	-1.21	1.4641
...

$$\text{Sum} = 6.449$$

$$\text{Cov}(Y, Y) = \frac{6.449}{9} = 0.7165$$

$$\text{Cov}(X, Y) = \text{Cov}(Y, X)$$

X	Y	X - \bar{X}	Y - \bar{Y}	(X - \bar{X})(Y - \bar{Y})
2.5	2.4	0.69	0.49	0.3381
0.5	0.7	-1.31	-1.21	1.5851
...

$$\text{Sum} = 5.5390$$

$$\text{Cov}(X, Y) = \frac{5.5390}{9} = 0.6154$$

$$\det (C - \lambda I) = 0$$

λ - eigen value (a constant)

I - Identity matrix

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.6154 \\ 0.6154 & 0.7165 - \lambda \end{bmatrix} = 0$$

$$\lambda_1 = 0.4908 \quad \lambda_2 = 1.2840$$

$$\begin{bmatrix} 0.1257 & 0.6154 \\ 0.6154 & 0.2257 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0$$

$$0.1257x_1 + 0.6154y_1 = 0$$

$$0.6154x_1 + 0.2257y_1 = 0$$

$$\begin{bmatrix} -0.6675 & 0.6154 \\ 0.6154 & -0.5675 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0$$

$$-0.6675x_2 + 0.6154y_2 = 0$$

$$0.6154x_2 - 0.5675y_2 = 0$$

$$\lambda_1 \quad x_1 \begin{bmatrix} -0.735 & -0.677 \\ -0.678 & -0.73 \end{bmatrix} \quad \text{Eigenvector}$$

$$\lambda_2 > \lambda_1$$

$$1.248 > 0.4908$$

$$\begin{bmatrix} -0.678 & -0.73 \end{bmatrix}$$

Principal component
Analysis

PCA

Method

Step.1 Get some data

Step.2 Subtract the mean

(This produces a dataset whose mean is zero)

Step.3 calculate the covariance matrix

Step.4 calculate the eigenvectors and eigenvalues of covariance matrix

Step.5 choosing components & forming a feature vector

Step.6 Deriving new data set .

$$\text{Final set} = \text{Row feature vector} \times \text{Row data Adjust}$$