Tutorial - 4

Kulmeet Singh
13-F
2016827

Q1, Solve using Master's theorem :-

1) $T(n) = 3T(n/2) + n^2$

$T(n) \Rightarrow a\,T(n/b) + f(n)$ where $a \geq 1,\ b > 1$

On comparing, $a = 3,\ b = 2,\ f(n) = n^2$

Now, $c = \log_b a$ i.e. $\log_2 3$ i.e. 1.584

and n^c i.e. $n^{1.584} < n^2$

$\therefore f(n) > n^c$ $\therefore$ $\boxed{T(n) = \theta(n^c)}$

2) $T(n) = 4T(n/2) + n^2$

$a \geq 1,\ b > 1$ and $a = 4,\ b = 2$, $f(n) = n^2$

$c = \log_2 4$ i.e. 2

$n^c = n^2 \Rightarrow f(n) = n^2$

$\therefore$ $\boxed{T(n) = \theta(n^2 \log n)}$

3) $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2,\ f(n) = 2^n$

$c = \log_b a \Rightarrow \log_2 1$ i.e. 0

Now, n^c i.e. $n^0 \Rightarrow 1$ and $f(n) > n^c$

$\boxed{T(n) = \theta(2^n)}$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$, $f(n) = n^n$

$c = \log_2 2^n \Rightarrow n$ and $n^c = n^n$

$f(n) = n^c$

$T(n) = \theta(n^n \log n)$

Q5. $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$, $f(n) = n$

$c = \log_4 16 \Rightarrow \log_4 (4)^2$ i.e. $2 \log_4 4$ i.e. 2

Now, $n^c = n^2$ and $f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Q6. $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$, $f(n) = n \log n$

$c = \log_2 2$ i.e. 1 and $n^c \Rightarrow n^1 \Rightarrow n$

Since, $n \log n > n$ i.e. $f(n) > n^c$

So, $T(n) = \theta(n \log n)$

Q7. $T(n) = 2T(n/2) + \frac{n}{\log n}$

So $a = 2$, $b = 2$, $f(n) = \frac{n}{\log n}$

$c = \log_2 2$ i.e. 1 and $n^c \Rightarrow n^1$

Since $\frac{n}{\log n} < n$ $\therefore f(n) < n^c$

$\therefore$ $\boxed{T(n) = \theta(n)}$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2$, $b = 4$, $f(n) = n^{0.51}$

$c = \log_b a$ i.e. $\log_4 2$ i.e. 0.5.

and $n^c = n^{0.5}$

So, $n^{0.5} < n^{0.51}$ i.e. $f(n) > n^c$

$\therefore$ $\boxed{T(n) = \theta(n^{0.51})}$

9) $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5$, $b = 2$, $f(n) = 1/n$

Since, according to Master's theorem, $a \geq 1$ but here a is 0.5 so we can't apply Masters theorem.

10) $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$c = \log_4 16$ i.e. 2

Now, $n^c = n^2$

As $n! > n^2$ $\therefore$ $\boxed{T(n) = \theta(n!)}$

11) $4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_2 4$ i.e. 2 $\qquad$ Now, $n^c = n^2$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore \boxed{f(n) = \theta(n^2)}$

12) $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$a = \sqrt{n},\ b = 2,\ f(n) = \log n$

Now, $c = \log_2 \sqrt{n}$ i.e. $\frac{1}{2} \log_2 n$

Now, $\frac{1}{2} \log_2 n < n \log n$ $\qquad \therefore f(n) < n^c$

So, $T(n) = \theta(f(n))$ i.e. $\theta(\log(n))$

13) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

Now,

$c = \log_2 3$ i.e. 1.5849

Now, $n^c = n^{1.5849}$

So, $n < n^{1.5849}$

i.e. $f(n) < n^c$

$\therefore \boxed{T(n) = \theta(n^{1.5849})}$

~~$T(n) = 3T(n/2) + n$~~

~~$a=3,\ b=2,\ f$~~

14) $T(n) = 3T(n/3) + sqrt(n)$

$a=3,\ b=3,\ f(n) = \sqrt{n}$

Now, $c = \log_3 3$ i.e. 1 $\therefore n^c = n^1$

as $\sqrt{n} < n$ so, $f(n) < n$

$\therefore T(n) = \theta(n)$

15) $T(n) = 4T(n/2) + c^n$

$a=4,\ b=2,\ f(n) = c^n$

Now, $c = \log_2 4$ i.e. 2 so, $n^c = n^2$

$\therefore c^n < n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore \boxed{T(n) = \theta(n^2)}$

16) $T(n) = 3T(n/4) + n\log n$

$a=3,\ b=4,\ f(n) = n\log n$

Now, $c = \log_4 3$ i.e. 0.792, $n^c \Rightarrow n^{0.792}$

Now, $n^{0.792} < n^{\log n}$

$\therefore \boxed{T(n) = \theta(n\log n)}$

17. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6;\ b = 3,\ f(n) = n^2 \log n$

Now, $c = \log_3 6 \Rightarrow 1.63903$

Now, $n^c \Rightarrow n^{1.63}$

So, $n^{1.63} < n^2 \log n$

So, $\boxed{T(n) = \theta(n^2 \log n)}$

18. $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

So,

$c = \log_3 3$ i.e, 1,

Now, $n^c \Rightarrow n^1$

$\therefore n^c > n/2$ i.e. $f(n) < n^c$

So, $T(n) = \theta(n)$

19. $T(n) = 4T(n/2) + \frac{n}{\log n}$

$\rightarrow a = 4, b = 2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\boxed{T(n) = \theta(n^2)}$

$20.$ $T(n) = 64T(n/8) - n^2 \log n$

$a = 64 \quad b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$\therefore n^2 \log n > n^2$

$\boxed{T(n) = \theta(n^2 \log n)}$

21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$\boxed{T(n) = \theta(n^2)}$

22. $T(n) = T(n/2) + n(2 - \cos k)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos k) > n^c$

$T(n)\ \theta(n(2 - \cos k))$