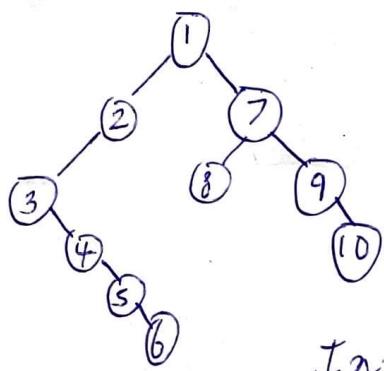


Iterative Postorder Traversal using 1 stack →



- 1) We'll go left until $\text{left}! = \text{NULL}$
- 2) Then we'll go right, then we will again go left until $\text{left}! = \text{NULL}$
- 3) Then again right, and so on

In this way, we will perform the traversal.

We are directly writing the code w/o writing alg.

$\text{cur} = \text{root}$

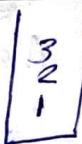
curr

Take a stack

```

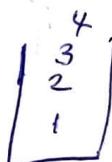
while (cur != NULL || !st.empty())
{
    if (cur == NULL)
        st.push (cur)
        cur = cur->left
    else
        temp = st.top() ->right
        if (temp == NULL)
            temp = st.top()
            st.pop()
            post.add (temp);
        while (!st.empty() && temp == st.top() ->right)
            temp = st.top();
            st.pop();
            post.add (temp->val);
        else
            cur = temp;
}
  
```

$\text{cur} = X \neq \text{NULL}$



Now $\text{cur} = \text{NULL}$

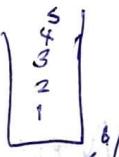
$\text{temp} = 3 \rightarrow \text{right} = 4$



As $\text{temp}! = \text{NULL}$, $\text{cur} = X \neq \text{NULL}$

null

$\text{temp} = 4 \rightarrow \text{right} = 5$



Now $\text{cur} = \text{NULL}$,

$\text{temp} = 5 \rightarrow \text{right} = 6$

null

$\text{cur} = X \neq \text{NULL}$



Now, $\text{cur} = \text{NULL}$,

$\text{temp} = 6 \rightarrow \text{right} = \text{NULL}$

As $\text{temp} = \text{NULL}$,

$\text{cur} = X \neq \text{NULL}$



$\text{temp} = 6$, $\text{pop } 6$

Print 6, while ($\text{temp} == 5 \rightarrow \text{right}$) Yes it is true

$\text{temp} = 5$, $\text{pop } 5$

Print 5

This loop will print 6 5 4 3

Now also, $\text{cur} = \text{NULL}$, $\text{temp} = 2 \rightarrow \text{right}$

$\text{temp} == \text{NULL}$, print 2 & pop



6 5 4 3 2

Again we will comeback, $\text{cur} = \text{NULL}$, $\text{temp} = 1 \rightarrow \text{right} = 7$

$\text{cur} = 7$



Now, ~~if~~ cur=7, push ?

& cur = 8

cur=8, push 8

& cur=NULL



Now, cur=null

temp = 8 \rightarrow right = null

temp==null, yes

temp = 8 - pop & print 8

~~temp~~ 8 == 7 \rightarrow right, No,

Now, it comes back & again cur=null

temp = 7 \rightarrow right = null

if (temp == null) {

temp = 7, pop & print

while (7 == 1 \rightarrow right)

temp = 1, pop & print



Now, both cur=null & st is empty.

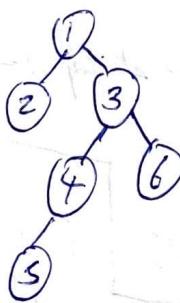
TC - $O(2N)$

SC - $O(N)$

Maximum Depth of Binary Tree

We have 2 approaches,

i.e., Recursive / Level Order Traversal

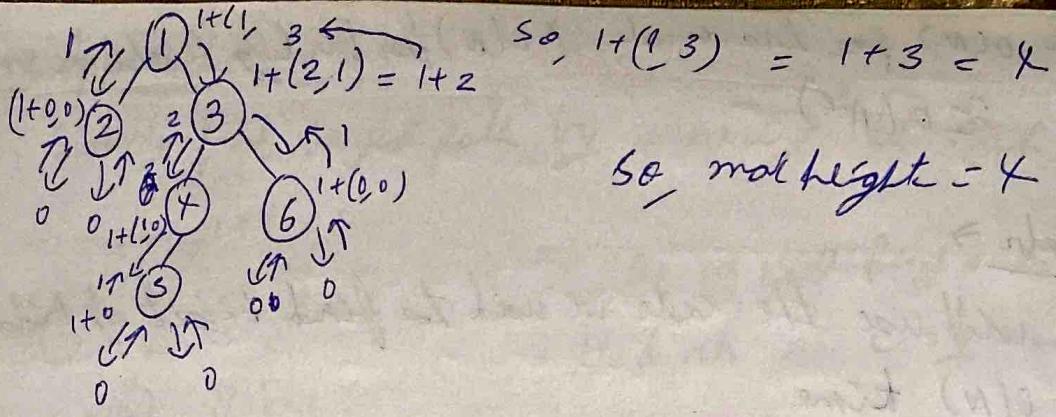


height = 4

Here, we will be using recursive approach

If we are standing at any given node, then height will be
1 + max (height(node->left), height(node->right))

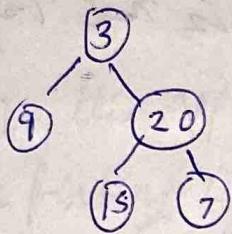
$$1 + \max(l, r)$$



Check for Balanced Binary Tree \rightarrow

For every node, $\text{height}(\text{left}) - \text{height}(\text{right}) \leq 1$

E.g.



For ③,

left height = 1

right height = 2

diff = 1

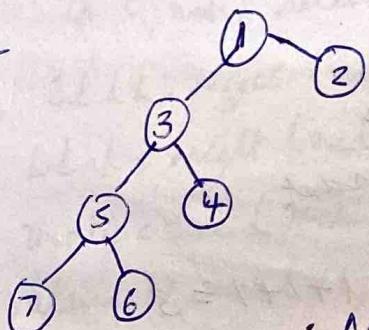
For ⑨, no left & right subtree

For ⑩, left height = 1 & right height = 1
diff = 0

For ⑯ & ⑰, no subtrees.

\therefore This tree is a balanced binary tree

E.g. 2



For ①

left height = 3

right height = 1

diff = 2

\therefore Not a balanced binary tree

Naive approach

Go to every node & figure out left height & right height

& compare difference, if it is > 1 return false

Pseudo code \rightarrow

if (node == null) ~~return true~~
return true

Lh = height (node \rightarrow left)
Rh = height (node \rightarrow right)

if (abs(Lh - Rh) > 1) return false;

bool left = check (node \rightarrow left);

bool right = check (node \rightarrow right);

if (!left || !right) return false;

$T.C = O(N)$ for traversal $\propto O(N)$ for finding height
 $\approx O(N^2)$

Optimal soln \Rightarrow

We will use the code we used to find height of binary tree in $O(N)$ time.

But we want the answer in true/false.

So, we will write a function, which will return height of tree if our tree is balanced,

Otherwise it will return -1, if it is not balanced.

int check(node) {

if (!node) return 0;

lh = check (node->left);

rh = check (node->right); \leftarrow if ($lh == rh == -1$) return -1;

if (abs(lh-rh)>1) return -1;

return max(lh,rh)+1;

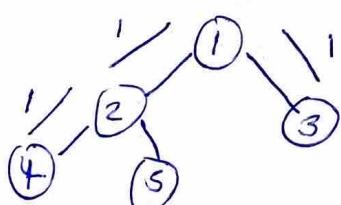
}

Diameter of Binary Tree \Rightarrow

\rightarrow Longest path b/w any 2 nodes

\rightarrow path does not need to pass via root

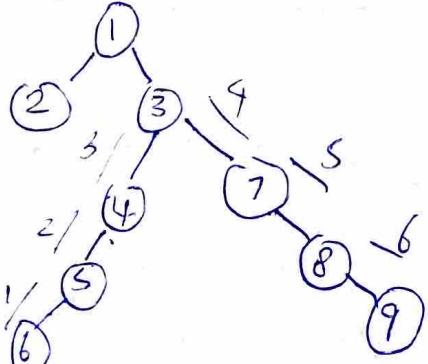
E.g -



$$1+1+1=3$$

So, diameter = 3

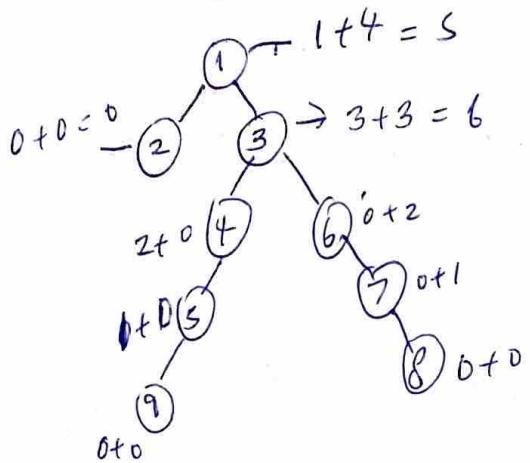
Eg -



Diameter = 6

Naive Approach -

Simply find largest path by summing lh + rh



So, we will find sum of lh & rh at every node

& This approach will take $O(N^2)$ TC

Optimal -

In this approach ~~also~~, we will modify the code for finding height of tree which will take $O(N)$ Time complexity

We will calculate sum of left height & right height after making recursive calls for node \rightarrow left & node \rightarrow right, and compare the sum with max variable.

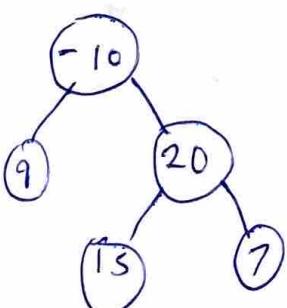
```

int find(node, maxi)
{
    if (!node) return 0;
    int lh = height(node->left);
    int rh = height(node->right);
    maxi = max(maxi, lh+rh);
    return 1+max(lh,rh);
}
  
```

TC - $O(N)$ recursion
SC - $O(N)$ per stack

3

Maximum Path Sum \rightarrow



$$15 + 20 + 7 = 42 \leftarrow \text{Max}$$

$$9 + (-10) + 20 + 15 = 34$$

Brute Force -

Trying every possible pair in the tree of Node A & B, where A can be anything & B can be anything.

But this solution is complicated, & for N nodes we will have to try N^2 combinations.

Better -

We will use the same approach we used for finding max height of binary tree & finding max width of binary tree.

If we are standing at any node
maxLeft } O } maxRight , we compute maxLeft &
maxRight which represent
max path sum in left & right nodes of
curr node

So, ~~and return the~~

ans for this node = val + (maxL + maxR)

And we will also maintain a max variable, which will store max path sum

```
int maxSum (node, max) {  
    if (!node) return 0;  
    lSum = maxSum (node->left);  
    rSum = maxSum (node->right);  
    maxi = max (max, node->val + lSum + rSum);  
    return node->val + max (lSum, rSum);
```

Check if two trees are identical or not



Two trees are same if they are structurally identical, and the nodes have same value.

Approach:

We can do traversal of both the trees, and the traversal of both the trees should be equal.

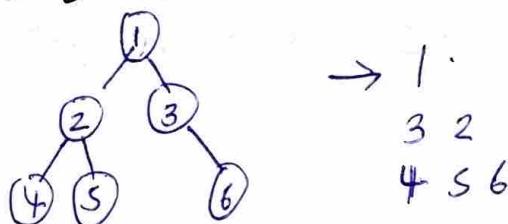
{

if ($p == \text{NULL}$ || $q == \text{NULL}$) { return ($p == q$); }

return ($p->\text{val} == q->\text{val}$) && isSameTree ($p->\text{left}$, $q->\text{left}$)
&& isSameTree ($p->\text{right}$, $q->\text{right}$);

}

Zigzag Level Order Traversal \rightarrow



We will follow same technique, which we used in Level Order Traversal.

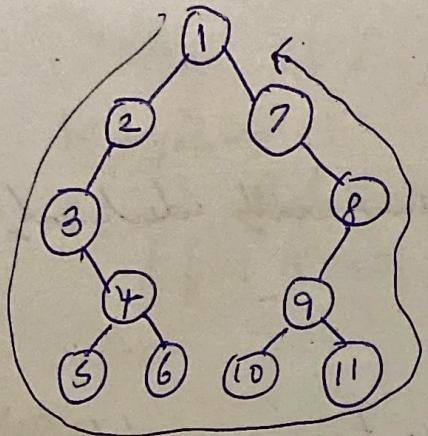
Take a <queue>, start root

Introduce a flag variable, $\text{flag} = 0$, which tells direction is $L \rightarrow R$

$\text{flag} = 1$ tells, direction is $R \rightarrow L$

Whenever we insert a level in an array, then level-1 toggle level variable i.e. Change its 0 to 1 alternately

Boundary Traversal →



1 2 3 4 5 6 10 11 9 8 9 7

Anti clockwise

Approach

First we will take left boundary excluding leaf nodes

Then we will take leaf nodes

Then we will take right boundary in the reverse direction, excluding leaf & root node.

Initially store the root in ans array

ds [1]

Then start from left boundary i.e. root → left

Again we will check its left, if it exists then move left, otherwise move right

ds [1 2 3 4]

Now 4's left & right both are leaf nodes, we will stop here.

Now, we have stored the left boundary, we'll move towards leaf nodes.

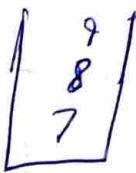
To store leaf nodes we will do inorder traversal.

Then our ds will be

[1 2 3 4 5 6 10 11]

Now, right boundary in reverse excluding leaf.
start from root \rightarrow right.

We'll use a temp stack to store values now
We'll keep storing until it reaches leaf nodes

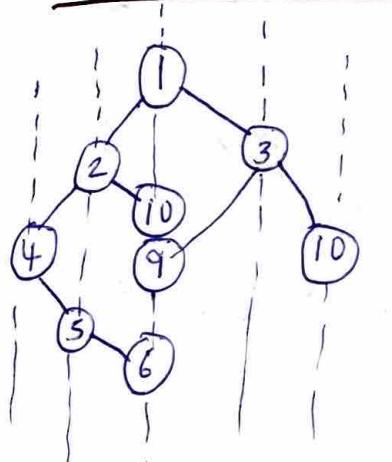


Now store the stack in final ans array

ds [1 2 3 4 5 6 10 11 9 8 7]

This is the final ans.

Vertical Order Traversal



Vertical order traversal is \rightarrow

~~4 2 5 1 9 10 6 3~~
4
2 5
1 9 10 6
3
10

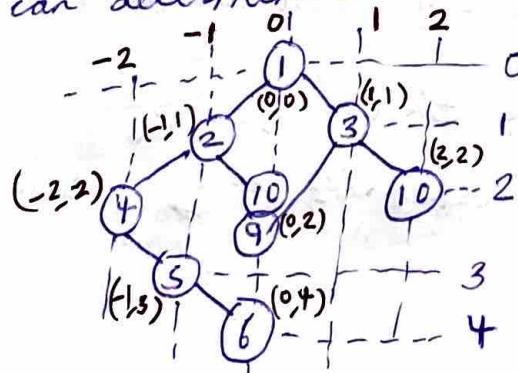
9 & 10 were on same level, so we will write smaller one first.

Approach If we consider these lines as points on the x-axis



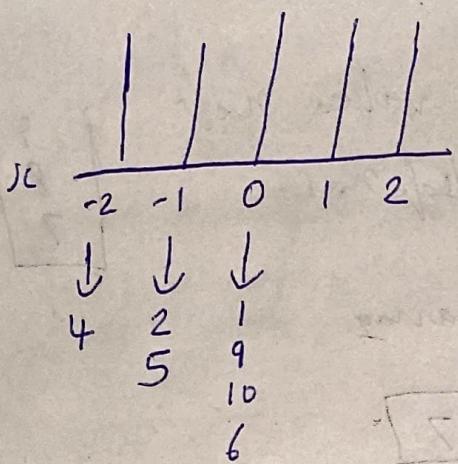
And if we consider levels as points on y-axis

Then we can determine the coordinates of node.



So, coordinates of a node are written in (x, y) format.

We can write this more simply now



This way, we can get every vertical,

Suppose, we have x , then we iterate in the ascending order of x , and similarly in y , we iterate in ascending order of y .

So, our first step will be to assign (x, y) to every node.

We can do this task with any traversal method.

Here, we will use Level order traversal to assign ~~and~~ coordinates.

→ We will take a Queue which will store

(node, x, y) x - Vertical position
 y - Level position

→ We will use another data structure map,

$\text{map} < \text{int}, \text{map} < \text{int}, \text{multiset} < \text{int} >>$

This will store, x y values

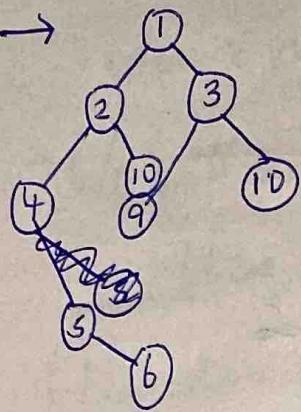
$x \rightarrow y \rightarrow$ values in multiset

We are using multiset because, we can have same values.
And if we use set, then we will only have unique values.

So, our map will store the values, whereas
Queue will store coordinates along with node

~~After \leftarrow~~

Now, we will start the process →



Initially while starting, store
 $(1, 0, 0)$ in Queue before
beginning loop.

Then we will take out this
 $q.front()$

$$\text{node} = 1, (0, 0)$$

Enter this in map,

$$0 \rightarrow 0 \rightarrow \{1\}$$

For $\text{node} = 1$, do level order traversal

i.e. Take left node $\Rightarrow \text{left} = 2$,

now if we are moving left, the our x will decrement
& ~~if~~ if we are moving downward, y will increment
so, we will get $(2, -1, 1)$ & store in queue

$$(2, -1, 1)$$

Now, Take right & increment y

We get $(3, 1, 1)$ & store in queue

$$\begin{cases} (3, 1, 1) \\ (2, -1, 1) \end{cases}$$

Now go to next iteration, get $q.front()$

which is $(2, -1, 1)$

$\text{node} = 2, x = -1 \& y = 1$

$$\begin{cases} (3, 1, 1) \\ (2, -1, 1) \end{cases}$$

Store this in map.

$$-1 \rightarrow 1 \rightarrow \{2\}$$

& perform Level order Traversal on this

We get $(4, -2, 2)$ & $(10, 0, 2)$ store in q

$$\begin{cases} (10, 0, 2) \\ (4, -2, 2) \\ (3, 1, 1) \end{cases}$$

Now, get $q.front() \rightarrow (3, 1, 1)$ - perform
same steps

$$-1 \rightarrow 1 \rightarrow \{3\}$$

& perform Level order traversal, we get

$$(9, 0, 2)$$

$$(10, 2, 2) \text{ store in } q$$

$$\begin{cases} (10, 2, 2) \\ (9, 0, 2) \\ (10, 0, 2) \\ (4, -2, 2) \end{cases}$$

This way we will iterate over the queue & store it in map.

Note We can easily store in map using $\text{mp}[x][y]$, $\text{insert}(\text{node} \rightarrow \text{val})$;

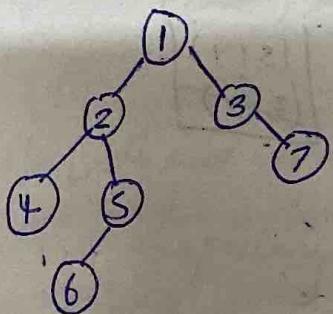
Now to store data from map in vector, code is:

```
for (auto p : mp) {
    vector<int> val;
    for (auto q : p.second) {
        sl.insert(sl.end(), q.second.begin(), q.second.end());
    }
    ans.push_back(val);
}
```

$T \leftarrow O(N) * O(\log N)$

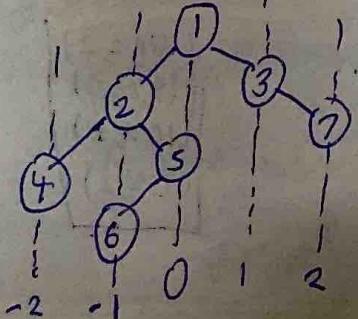
$S \leftarrow O(N)$

Top View of Binary Tree



Top View \rightarrow 4 2 1 3 7

Approach - We will use Level order traversal for this.
We will also use the vertical line concept from Vertical order traversal.



On every line, whatever is the 1st node from top will be our answer

- We will use a queue to store (Node, x)
 where x represent vertical pos. $\boxed{(1,0)}_Q$
- We will use a map data structure to store
 $(\text{Line}, \text{node})$ i.e. (x, node)
 (Map store everything in a sorted fashion of keys)
- Initially insert $(1,0)$ in q $\boxed{(1,0)}_Q$
- Take out $(1,0)$
 $\text{node} = 1 \quad \text{x} = 0$
- We will check whether on line 0, do we have anything in map, i.e. $\text{mp}[0]$ exists in map or not.
 If no, then insert in map
- $0 \rightarrow 1$
- Now, goes for level order traversal of node 1,
 i.e. goes left & right
 On left we have $\text{node} = 2$ & x will decrement,
 it will become $\text{x} = -1$
- Push it in queue
- Similarly, on right push $(3,1)$ in q $\boxed{(3,1)}_Q \quad \boxed{0 \rightarrow 1}_{\text{mp}}$
- Now take, $\text{node} = 2, \quad \text{x} = -1$
- check if $\text{mp}[\text{x}]$ exist in map, if no, then create entry $\boxed{(3,1)}_{\text{mp}} \quad \boxed{-1 \rightarrow 2}_{\text{mp}} \quad \boxed{0 \rightarrow 1}_{\text{mp}}$
- start its left right in q .
 i.e. $(4,-2)$ & $(5,0)$ $\boxed{(5,0)}_Q \quad \boxed{(4,-2)}_Q \quad \boxed{(3,1)}_Q \quad \boxed{+1 \rightarrow 2}_{\text{mp}} \quad \boxed{0 \rightarrow 1}_{\text{mp}}$

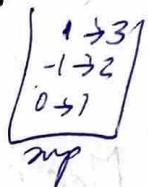
Now, node = 3, xc = 1

Check whether $mp[1]$ exist in map.

If not create entry

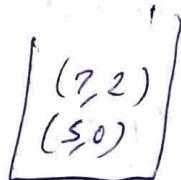
& push its children in q

i.e., (7, 2)



Now, node = 4, xc = -2

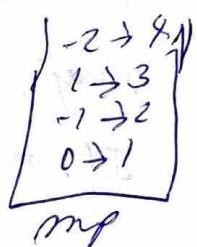
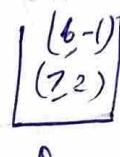
Put it in map & add its
children in q



Now, node = 5, xc = 0

$mp[0]$ already exist in map, so we will not
insert at this node 5 in map

Push 5's children



Similarly store all nodes in map.

This method will store only first occurrence/ root of
a vertical line in map.

This will give us top-view of tree.

Map will automatically sort keys, so answer will be in
correct order.

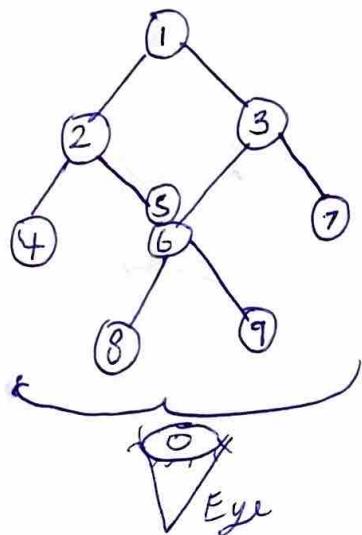
TC - O(n)

SC - O(n)

Can we use recursive traversal to solve this?

No because if we use recursive traversal, it will store
high nodes view first, and we will not be able to get
top view.

Bottom View of Binary Tree



Bottom view will be

4 8 6 9 7

5 & 6 both are visible from bottom & both lie on same vertical line,
So, in this case we will take the right ~~left~~ node, i.e. 6

Approach: We will use the concept of vertical line, just like we used in Vertical order traversal

1 | | | 1
-2 -1 0 1 2

On every line, the last node, we will store that

Take a Queue $\langle \text{TreeNode}^*, \text{int} \rangle$ which will store (Node, rl) where rl represents vertical line

We will also use a map to store $(\text{rl}, \text{node} \rightarrow \text{val})$

Initially start with $(1, 0)$

Node = 1, rl = 0

We will insert $\text{mp}[\text{rl}] = \text{Node} \rightarrow \text{val}$
& insert its children in the queue

i.e., $(2, -1) \& (3, 1)$

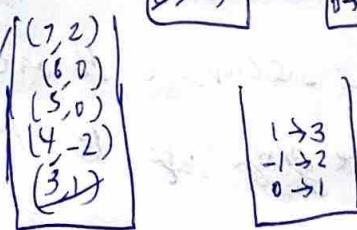
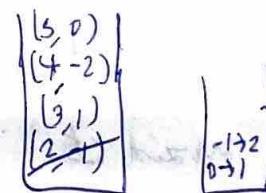
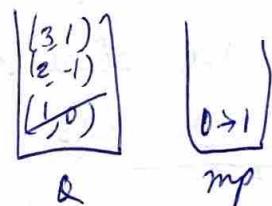
Now, Node = 2, rl = -1

Insert in map, push its children

Now, Node = 3, rl = 1

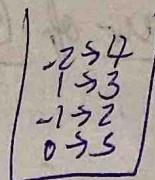
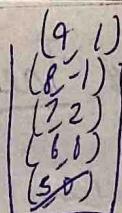
Perform same task

Now, Node = 4, rl = -2



Now, node = 5, n = 0

Update mp[n]



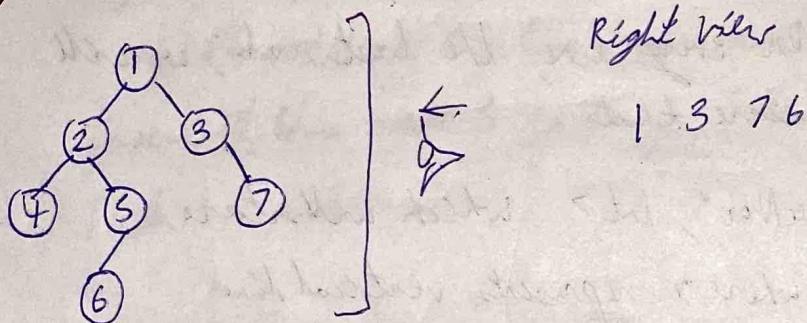
Similarly in this way we will proceed, we will overwrite previous entry if it exists, which will give us the better view in the end.

TC - O(N)

SC - O(N)

Here, also we cannot use recursion

Right / Left Side View of Binary Tree



Approach - We have to traverse in such a way that we store the last node of every level will be

Right View

We can use any traversal i.e. recursive / iterative

Level order traversal will use more space, so we will avoid it.

But, in recursive traversal, then space complexity will be ~~given by~~ ~~to~~ height of tree

TC - O(N)

SC - O(H)

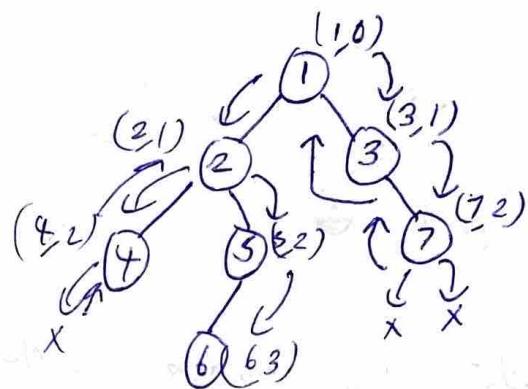
Here, we will use recursive approach, because code is short

Here we will use Reverse Preorder Traversal, i.e.

Root Right Left

```
f(node, level)
{
    if (!node)
        return;
    if (level == ds.size())
        ds.add(node);
    f(node->right, level+1);
    f(node->left, level+1);
```

do me answer store kro,



ds

3

We will only store 1 element at each level, as we will check whether ($level == ds.size()$)

Suppose, $level = 0$ & $size = 0$, we'll add

Now, $level = 1$ & $size = 1$, we'll add

$level = 2$ & $size = 2$, we'll add

Now, it will go back to root, $level$ became 0

Now, $root \rightarrow left$, $level = 1$, & $size = 3$, we'll not add

$root \rightarrow right$, $level = 2$, $size = 3$, we'll not add

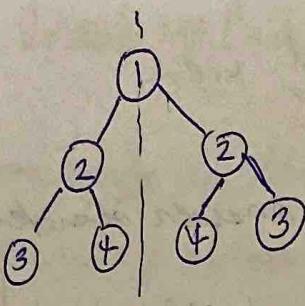
$root \rightarrow left$, $level = 3$, $size = 3$, then only we'll add

TC - O(N) SC - O(H)

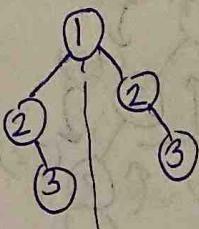
For left view, we'll go left first then right

Check for Symmetric Binary Tree

If form a mirror of itself around
the center



This is a symmetric binary tree



This is not symmetric

Approach In a minor Left \rightarrow Right & Right \rightarrow Left
Left will start from

If we ignore the root, left subtree will start from
 $\text{root} \rightarrow \text{left}$ & its mirror will be $\text{root} \rightarrow \text{right}$ subtree
 \Rightarrow $(P \rightarrow \text{right})$

If we are doing
Fiborder traversal
I.e.

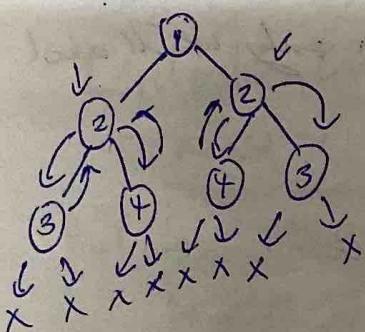
Fast Left Right

Rest Right Left

Better will
be the men

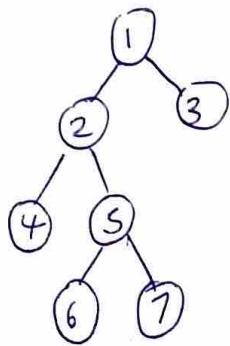
We will do both these traversals simultaneously on both left & right subtrees

We will just move opposite
in both subtrees



If everyone was matching then it
is symmetric

Print Root to Node Path in Binary Tree



Node = 7

From ①, we have to find path to all ⑦

O/P $\rightarrow \{1, 2, 5, 7\}$

Approach - We will use Inorder Traversal here

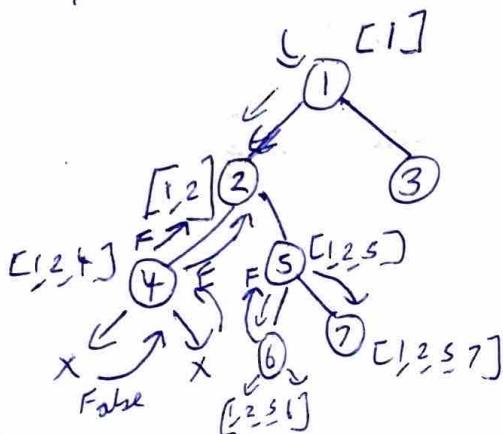
Preorder & Postorder can also be used but they are complicated.

1) Initially we will start with the root, when we ~~start~~ are at root, we'll have a data structure, which will be empty initially.

If $\text{node}! = \text{null}$ & ~~node == target~~

push it into ds

First left recursion
then right recursion



When a node returns false it means that ⑦ is not present in this path, so we will go back & pop that element from our data structure.

Here ④ returned a false to ②, so it was removed from ds.

Now ② goes to right

Now when we reach node ⑦, our ds has $\{1, 2, 5, 7\}$ as we have got our required node, we will return true with array as it is.

Code → ~~ans~~ arr.push_back(root->val);

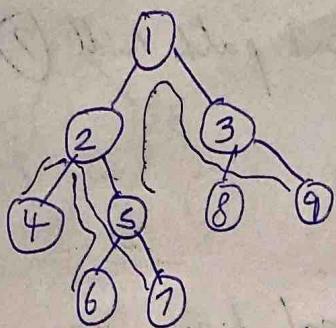
if ($\text{root} \rightarrow \text{val} == \text{x}$) return true;

if ($\text{getPath}(\text{root} \rightarrow \text{left}, \text{arr}, \text{x}) \text{ || } \text{getPath}(\text{root} \rightarrow \text{right}, \text{arr}, \text{x})$)
return true;

arr.pop_back();

return false;

Lowest Common Ancestor in Binary Tree



$$\text{lca}(4, 7) = 2$$

$$\text{lca}(5, 4) = 1$$

$$\text{lca}(2, 6) = 2$$

The ancestor that exist at
deepest node

Brute Force

We can store path of both the nodes and then compare paths.

$$\text{e.g. } \text{node} = 4 \quad (1 \ 2 \ 4)$$

$$\text{lca}(4, 7) \text{ node} = 7 \quad (1 \ 2 \ 5 \ 7)$$

We can see both paths match till ②. So, ② will be the LCA

$$TC - O(N) + O(N)$$

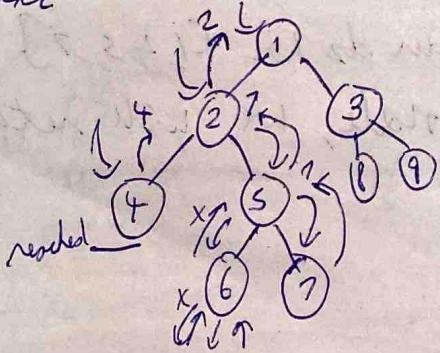
$$SC - O(N) + O(N)$$

Better

We will optimise space in this approach.

First we will go left, when we reach one of the node we are looking for, we'll return that node to previous func

$$\text{lca}(4, 7)$$



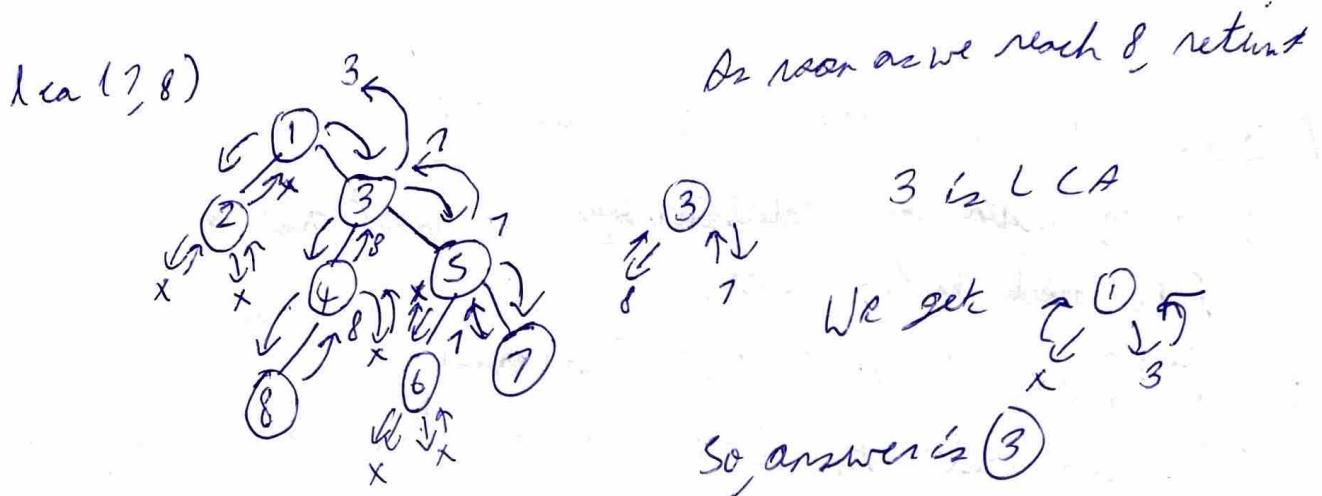
Now at ② we are getting both ④ & ⑦.

Which means 2 is LCA

When we returned ④, we will make a recursion call in the right to check its right children.

Node ② will return 2, now ① will go in right & look for nodes again it will get back NULL.

$2 \neq 1 \Rightarrow X$ so, answer will be 2



Code ↳

```
if (!root || root == p || root == q) return root;
```

```
TreeNode *l = solve (root->left)
```

```
n = solve (root->right)
```

```
If (l != 1) return n;
```

```
else if (!n) return l;
```

```
else return root;
```

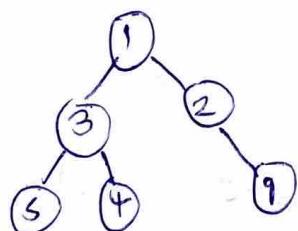
TC - O(N)

SC - O(N)

3

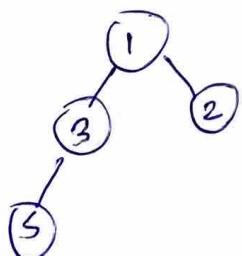
Maximum Width of a Binary Tree →

Width means $\underset{\text{in a level}}{\text{No. of nodes b/w any 2 nodes}}$



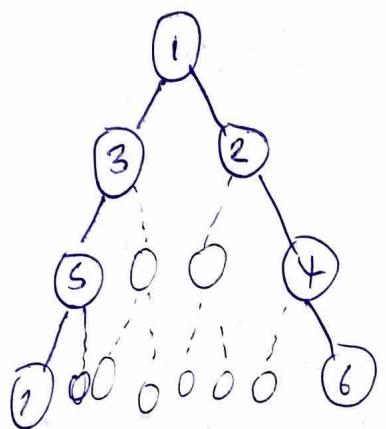
Max width of this tree = 4

because if 2's left exist, then it would have been 4.



Here max width = 2

We cannot consider last level here because our task is to find max width b/w any 2 nodes & there is no node in right side of last level.



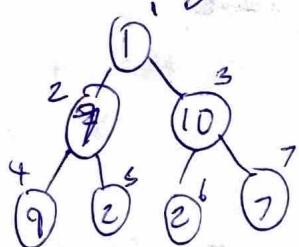
Max width = 8

Approach - Width is dependent on the level

We can say, width can be calculated only b/w first node in that level & last node in that level,

We should follow Level Order Traversal.

If we can index the tree based on their position, then task will be very simple

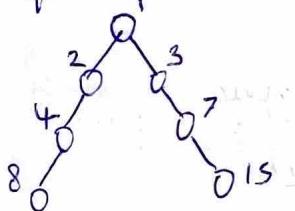


Then we'll have to go to each level.
Pick up the first node under pick up the last node under subtract it and add +1.

$$\text{e.g. } 7 - 4 + 1 = 8 \leftarrow \text{max width}$$

Even if there are no nodes in b/w, we'll index in proper format.

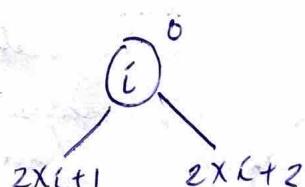
Now, our main task is How to index the nodes?



If we consider 0 based indexing, then

$$l\text{child} = 2 \times i + 1$$

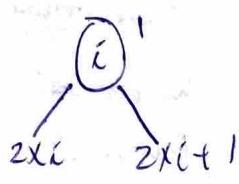
$$r\text{child} = 2 \times i + 2$$



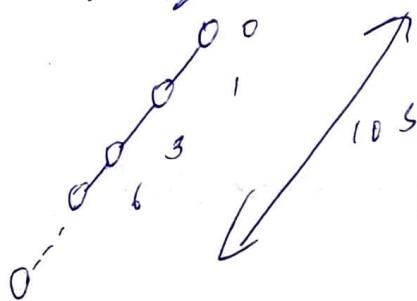
If we consider 1 based indexing, then

$$l\text{child} = 2 \times i$$

$$r\text{child} = 2 \times i + 1$$

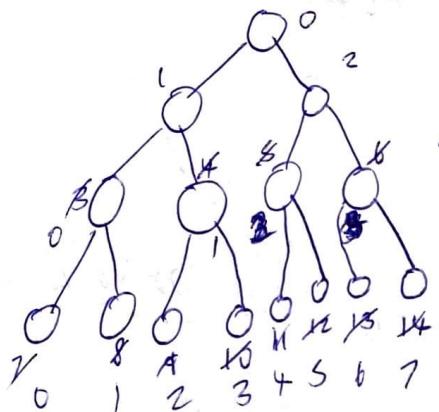


But this method also has one issue, suppose if we have a skewed tree of 10^5 nodes. then



then integer will overflow
because products increase by three

So, in order to prevent overflow



instead of 3 4 5 6 indices
we'll make it 0 1 2 3
in this level

← In this level we'll change
7 8 9 10 11 12 13 14 to
0 1 2 3 4 5 6 7

Q. How will we do this?

Soli-

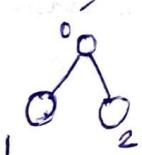
In this note



$$\begin{aligned} i &= 1 \\ \text{l child} &= 3 \\ \text{n child} &= 4 \end{aligned}$$

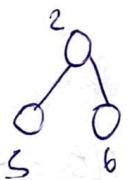
We'll discuss
formula of closed
embedding

What if we do $i-1$, i.e. make it $i=0$



Then 1 child = 1
2 child = 2

E.g 2



$$\begin{array}{l} i = 2 \\ \lambda_{child} = 5 \\ \gamma_{child} = 6 \end{array}$$

We'll do $i = i - 1 = 1$, then



Now we have got

1 2 3 4 in level 3

We'll perform same steps in calculating index of level 4



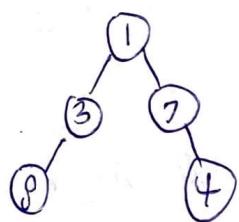
$$i = i - \min$$

$$\begin{array}{c} / \\ 2i+1 \end{array} \quad \begin{array}{c} \backslash \\ 2i+2 \end{array}$$

Then On every level take the min & max index, ~~then~~
subtract & store +1.

We'll follow Level Order Traversal,

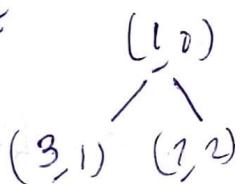
We'll store (node, index) in Queue.



max width = 1

Pop it from ~~queue~~ node = 1, ind = 0

Now add its child

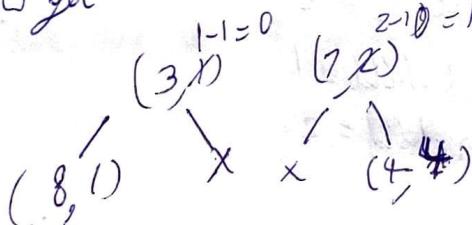


$2^*i+1 \rightarrow$ l child
 $2^*i+2 \rightarrow$ r child

Push in queue

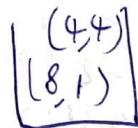


Now get

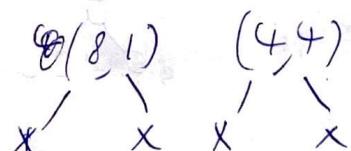


max width = $2-1+1$
= 2

Pop them from ~~queue~~



Now,



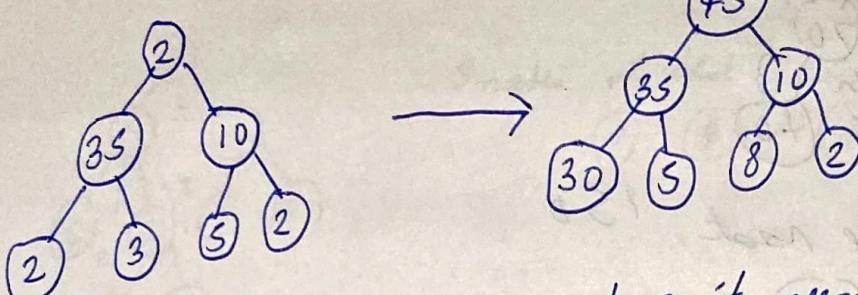
max width = $4-1+1$
= 4

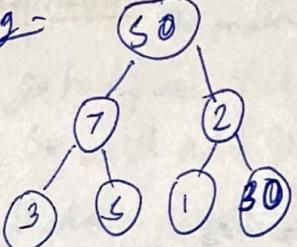
Code \Rightarrow

Children Sum Property

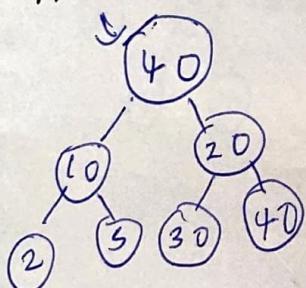
According to this property, ~~Node val~~
 $\text{Node} \rightarrow \text{val} = \text{Lnode} \rightarrow \text{val} + \text{Rnode} \rightarrow \text{val}$

If binary tree does not follow this property, then we can increment any node by +1, any no. of times To make it follow this property
Also, we cannot change structure of tree.



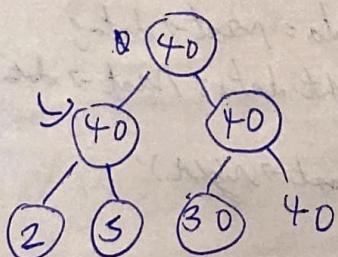
This problem is not as simple as it seems to be
e.g. 
First thing which comes to our mind is that we make $7 \rightarrow 8$ & $2 \rightarrow 31$
but we can't just change this, because above those 2 nodes, their children sum property will be disturbed.

Approach- We'll use recursive traversal



$$10 + 20 = 30 < 40$$

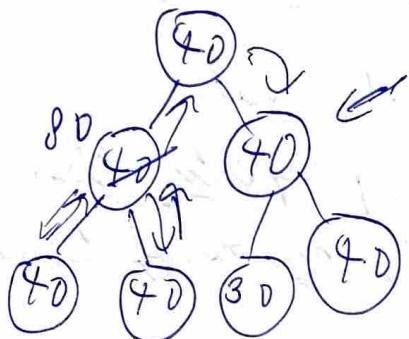
Now, if $\text{sum} < 40$, then we replace both 10 & 20 by 40



$$\text{New } 2 + 5 = 7 < 40$$

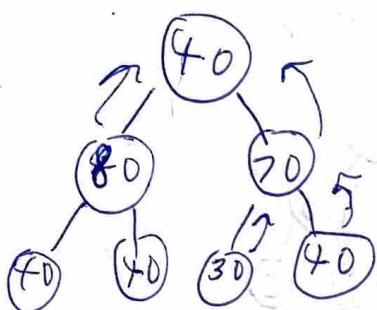
Now, again if $\text{sum} < 40$

We'll change 2 & 3 to 80 & when we go back
make 40 to 80 in their parent
Now

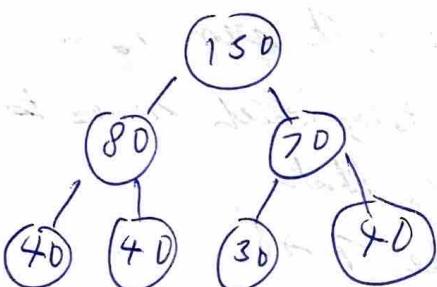


$$30 + 40 = 70 \geq 40$$

Now if sum > 40, then
make root 70



Now make root 150



So while going down increase the node values
& while coming back up, change node values
accordingly.

Code →

```

if (!root) return;
int child = 0;
if (root->left) child += root->left->data;
if (root->right) child += root->right->data
if (child >= root->data) root->data = child;
else {
    if (root->left) root->left->data = root->data;
    else if (root->right) root->right->data = root->data;
}
  
```

3 solved root->left), solve (root->right);
int tot = 0;
if (root->left) tot += root->left->data;

if (root \rightarrow right)) total += root \rightarrow right \rightarrow data;

if (l root \rightarrow left || root \rightarrow right) root \rightarrow data = total;

3

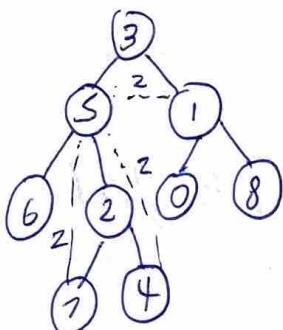
TC - O(N)

SC - O(1)

Nodes at a distance K

K=2, target = 5

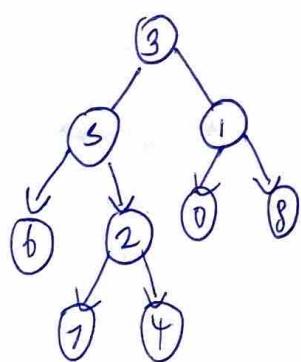
Print all the nodes at a dist. 2 from node ③



3 nodes at dist 2 are

⑦ ⑨ & ④

Approach The main issue is solving this is that we cannot go back from child node to parent node.
So, we'll add parent ~~Node~~ pointer to all nodes, i.e.
we'll store parent of all nodes in a map or array
We'll use a BFS to perform this task



Initially take a queue
Push root in it
Now start iterating over queue's elements



Node = 3
Left / Right
S I



store
We'll push 1 & 5 in q & update both's parents as ③

We can store it in a map.

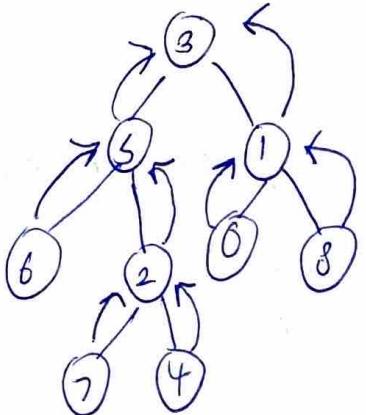
Now, nodes = 5



Pushing up & store parent node of both

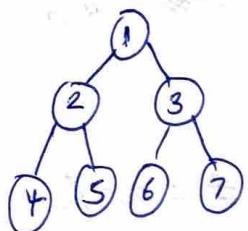
Similarly we'll process & store parents of all nodes

$K=2$, target = 5



Now, if target is given, we have to find other nodes at dist = 2

Count total Nodes in a Complete Binary Tree



cnt = 7

We can simply count by traversing the tree.

This is Naive approach.

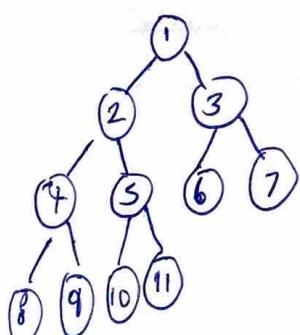
T - O(N) SC - O(1)

Optimal approach-

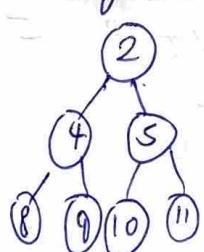
If height of tree = 3

The no. of nodes = $2^3 - 1 = 7$

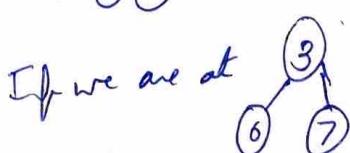
This is a property of Complete Binary Tree.
But what if some nodes are NULL in last level



If we are standing at
then height of subtree = 3



& no. of nodes = 7



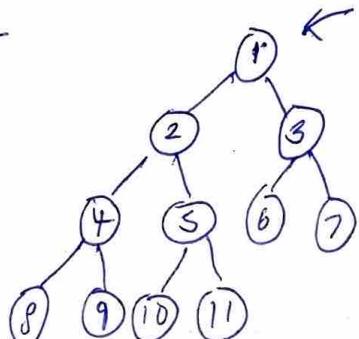
then height = 2
& no. of nodes = $2^2 - 1 = 3$

Now, if we are at root, then

$$\text{total nodes} = 1 + 7 + 3 = 11$$

So, we'll check for every subtree in this way.

Dry Run



at root, calculate
left height & right height

$$\text{lh} = 4$$
$$\text{rh} = 3$$

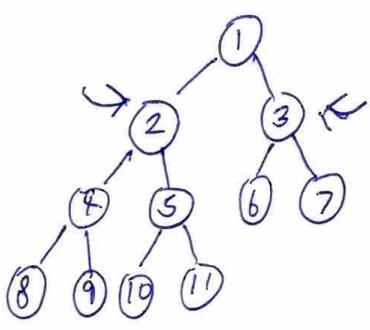
We can see $\text{lh} \neq \text{rh}$

Then we cannot use this formula

here.

So, we'll add $1 + \text{left rec} + \text{right rec}$.

We'll move left & right ^{subtrees}, we'll check whether $\text{lh} == \text{rh}$ in their subtrees.



In ② $\text{lh} = 3$
 $\text{rh} = 3$
 ~~$\text{lh} = \text{rh}$~~

We can use the formula
 $\text{no. of nodes} = 2^3 - 1$
 $= 7$

In ③ $\text{lh} = 2$ $\text{lh} == \text{rh}$. We can use the formula
 $\text{rh} = 2$,
 $\text{no. of nodes} = 2^2 - 1 = 3$

So, we get the answer $1 + 7 + 3 = 11$

$T.C - O(\log N^3)$ $S.L - O(\log N)$

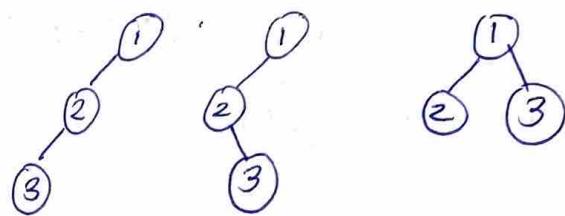
Requirements needed to construct a Unique Binary Tree

a) Preorder & Postorder -

PreOrder \rightarrow 1 2 3

PostOrder \rightarrow 3 2 1

Many trees are possible



So, if we are given a preorder & postorder traversal then we cannot construct a unique binary tree

b) Inorder & Preorder -

Preorder \rightarrow 3 9 20 15 7

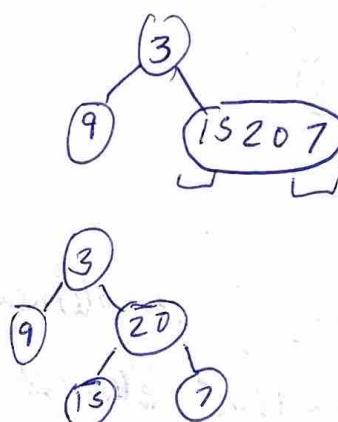
Inorder \rightarrow 9 3 15 20 7
Left Right

③ is root of tree, then

Now ⑨ becomes root

Now ②0 becomes root

We are determining root by checking how elements are coming in pre-order traversal



c) Inorder & Postorder can also be used

Construct a Binary Tree from Inorder & Preorder

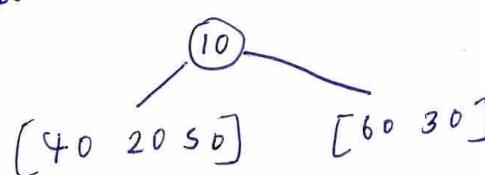
inorder \rightarrow [40 20 50 10 60 30]

preorder \rightarrow [10 20 40 50 30 60]
root

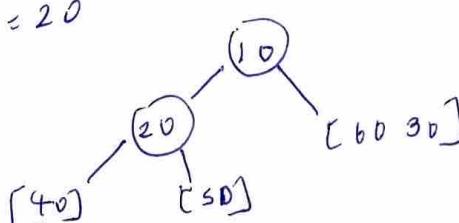
Left Root Right

Root Left Right

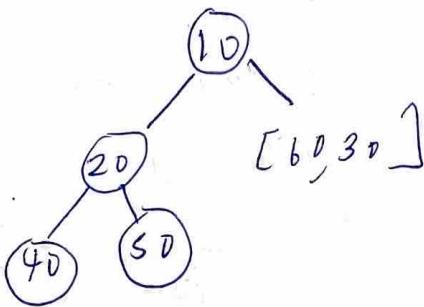
We split using inorder traversal



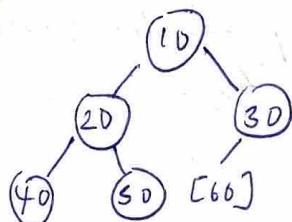
Now, root = 20



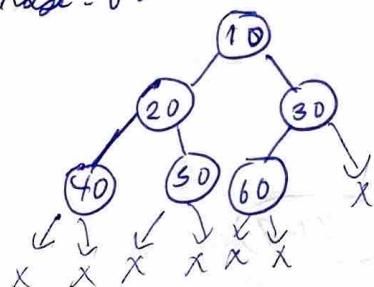
Now root = 40, then so



Now root = 30



Now root = 60



Code Approach -

We are choosing first element of pre-order as root,
then we search for that element in inorder traversal

Then we do cut off

inorder \rightarrow [(Inorder left) 0 (Inorder right)]

pre-order \rightarrow [(2) | |]
 Pre-order left Pre-order right

Take a recursion call with left & right subarrays

TC - O(N)

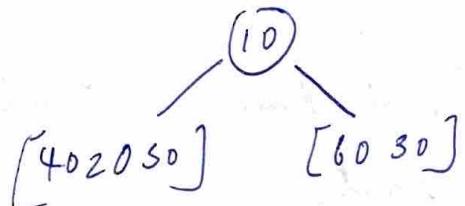
SC - O(N)

Construct Binary Tree from PostOrder & Inorder

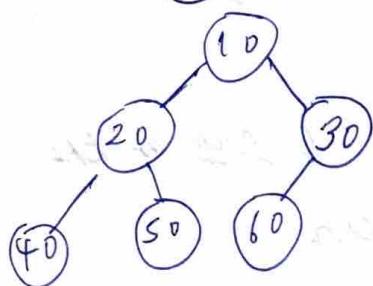
Inorder $\rightarrow [40 \ 20 \ 50 \ 10 \ 60 \ 30]$

Postorder $\rightarrow [40 \ 50 \ 20 \ 60 \ 30 \ 10]$

Last element is root in postorder



Here, 20 will be root



This will be final tree

Map

40 - 0

inlast = 3

20 - 1

num_left = 3 - 0 = 3

50 - 2

10 - 3

Find index using map

60 - 4

30 - 5

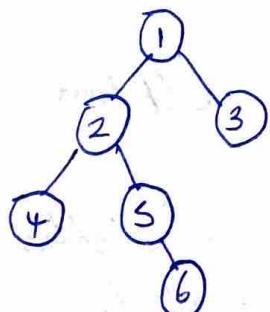
Same code as previous

Morris Traversal

TC - O(N)

SC - O(1)

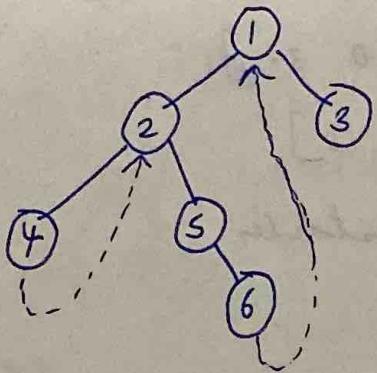
- It uses constant space
- It uses Threaded Binary Tree



Inorder - 4 2 5 6 1 3

Preorder - 1 2 4 5 6 3

Suppose, we had a backward pointer, like



We can observe a pattern, that from last node w.r.t a subtree we go back to root.

Case 1-

If $\text{left} == \text{null}$, then print() & go right

Case 2-

Before moving left, rightmost guy on the left subtree → cur
Connect it.

Then after that $\text{cur} = \text{cur} \rightarrow \text{left};$

Q. Now, we have a doubt, that when we keep going left, then our threaded binary tree will again come back to root eventually. In this case, how will we determine that we don't have to go left again, we have to move right?

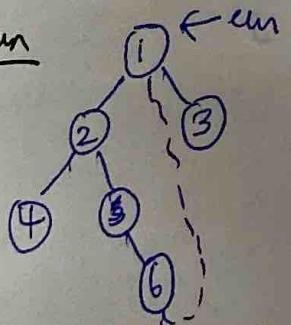
Sol: Case 3-

If threaded binary tree already exist then we'll remove thread,

then we'll move right

i.e. $\text{cur} = \text{cur} \rightarrow \text{right}$

Dry Run

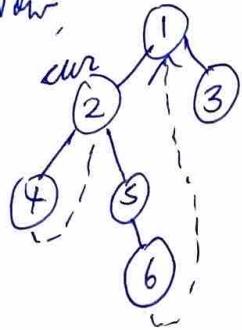


Now, for inorder traversal, we'll have to go to left.

Before going left, we'll make the rightmost ptr in the left subtree & point it to cur.

Once the thread is created, move left

Now,



Again, create a thread from last guy in left subtree to cur node

remove cur = cur \rightarrow left

Now, ④ don't have a left, then this guy will be the root itself.

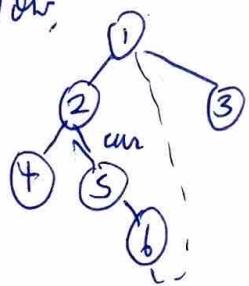
In this case, print 4

Now, we'll go back to the root via the thread we created.

By the moment we go back to root, it has to be printed. We'll print it.

Now, this is the scenario where we will check whether a thread exists or not in the left subtree. If it exists, then we'll go left remove thread & once thread is removed, we'll move cur = cur \rightarrow right.

Now,

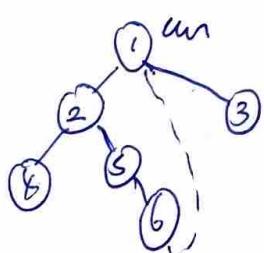


cur is at ⑤

It has no left, which means this node itself is a root
so, print 5 & go to right

Now, cur at ⑥. Print it,

Now, go to root via thread



Print root.

Now, check whether left thread exists, if yes, then we'll remove the thread then move right & print it.

Code → cur = root;

while (cur != NULL) {

if (cur->left == NULL) {

ans.add (cur->val);

cur = cur->right;

}

else {

Tree Node^{*} thread = cur->left;

while (thread->right && thread->right->val == cur) {

thread = thread->right;

}

if (thread->right == NULL) {

thread->right = cur;

cur = cur->left;

}

else if (thread->right == cur) {

thread->right = NULL;

ans.add (cur->val);

cur = cur->right;

3

3

return ans;

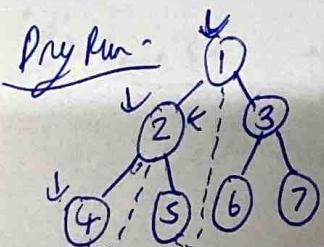
Inorder - 4 2 5 1 6 3 7

cur = ①, thread = ②

②->right = ①, cur = ②

cur = ②, thread = ①

①->right = ②, cur = ④

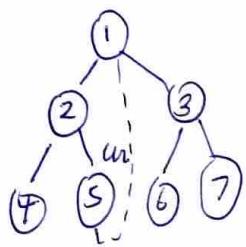


④->left == NULL, print 4

cur = ②

thread = ④, thread->right == NULL, print ②

cur = ⑤



⑤ cur \rightarrow left == NULL

print ⑤, move cur to ①

Now, cur = ①

cur \rightarrow left != NULL

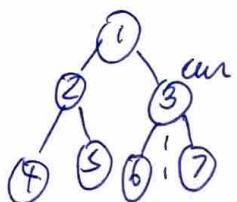
thread = ②, ~~thread = thread \rightarrow right~~

thread = ②, now

thread \rightarrow right == cur, i.e. ③ \rightarrow right == ①

\Rightarrow ③ \rightarrow right = NULL, print, cur = cur \rightarrow right

Now, cur = ③

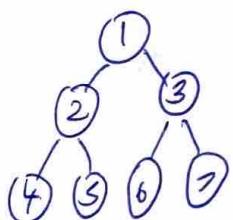


Again follow same procedure.

This will get us the inorder traversal

T_C - O(N) SC - O(1)

Now, for Preorder traversal \Rightarrow

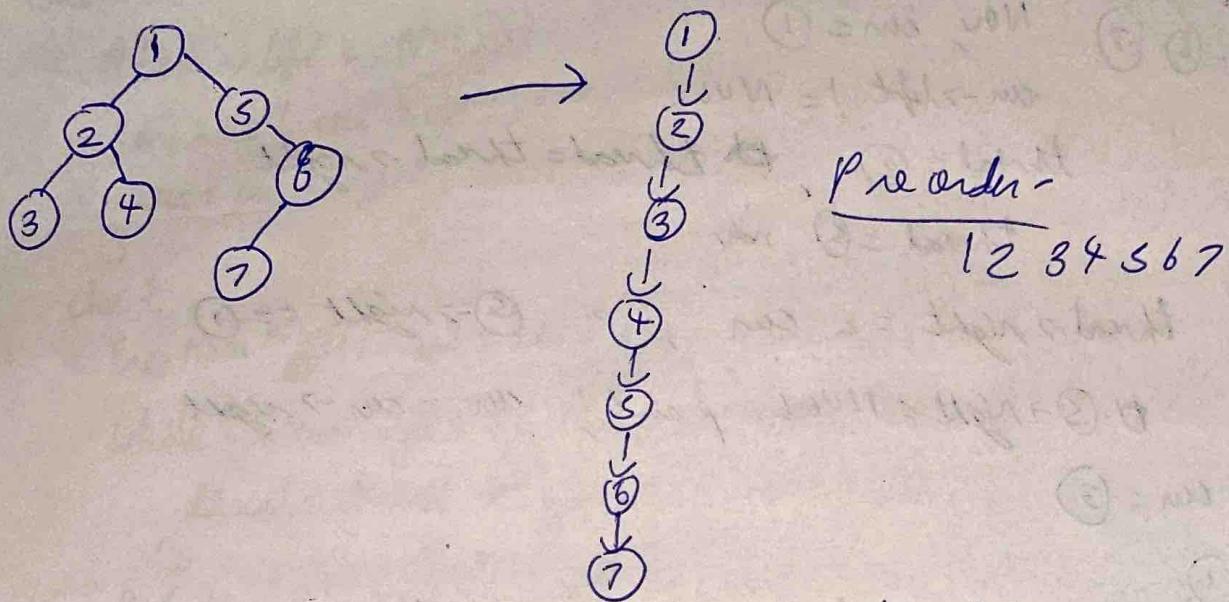


Preorder \Rightarrow 1 2 4 5 3 6 7

Only thing we need to change in code is that when we are pointing thread \rightarrow right = cur, then only we'll print the cur value.

Then move cur = cur \neq left.

Flatten a Binary Tree to Linked List



We don't have to make a separate LL

We have to rearrange the nodes such that

$(1 \rightarrow \text{right}) = 2$, and so on

Recursive Approach -

Right Left Root

We'll arrange right subtree first, then left subtree
then root

```

prev = null
flatten(node)
  { if (node == null) return;
    flatten(node->right);
    flatten(node->left);
    node->right = prev;
    node->left = null;
    p_new = node
  }

```

TC - O(N)
SC - O(N)

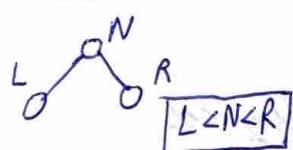
Iterative Approach

We'll use stack

```

    st.push(root)
    while(!st.empty()){
        cur = st.top(), st.pop();
        if(cur->right) st.push(cur->right);
        if(cur->left) st.push(cur->left);
        if(!st.empty()) cur->right = st.top();
        cur->left = null;
    }
  
```

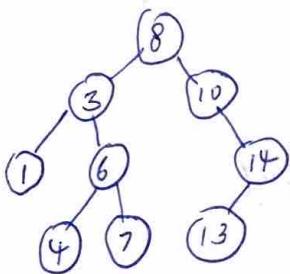
Binary Search Tree -



Left subtree \rightarrow BST

Right subtree \rightarrow BST

E.g.



a. Are duplicates allowed?

Ans - No

But if someone still insists on inserting duplicates, then

we'll modify this condition of

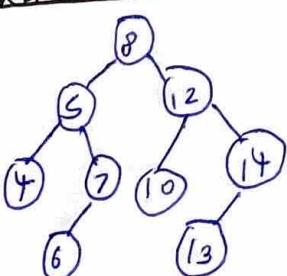
BST, I.P. $L \leq N < R$

or we can store (node, freq) together

Why BST?

Ans - In most BST, height is always $\log_2 N$

Search in a BST



node = 10

start at 8, $8 < 10$, so move

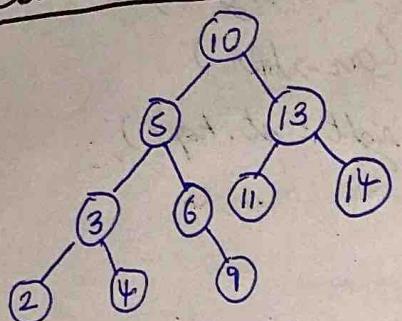
cur \rightarrow right

Now $12 > 10$, so move cur \rightarrow left

node = 10, return true

We are not traversing the entire tree to search for an element, so we can say max Time Complexity to search is $O(\log N)$

Search in a BST



Suppose we are given a key = 8
We have to find node val \geq key
But that val should be smallest.
E.g. Here it is 9

Approach-

Initialise a variable cell = -1

key = 8

Start with cur = 10.

Now cur > key, change cell = 10

& move left

New cur = 3

Now cur = key, don't modify cell

& move right

New cur = 6, again go right

Now cur = 9, cur > key, update cell
go left

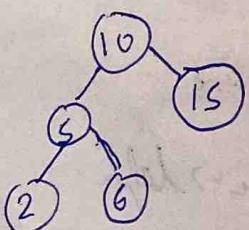
Now, cur = NULL, we can say 9 is the answer

Return cell

$T.C - O(\log N)$

Find in a BST

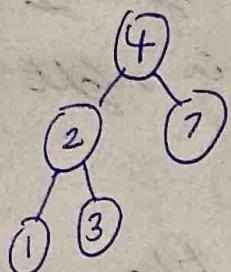
key = 7, We have to find greatest value which is ~~small~~ ≤ 7



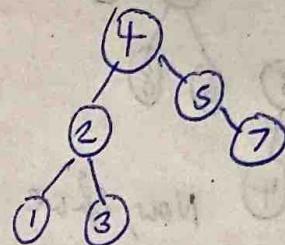
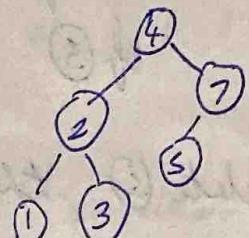
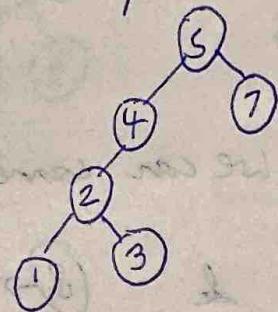
Ans - 6

Same approach as above

Insert a given node in BST



node = 5,
Multiple trees are possible



Approach -

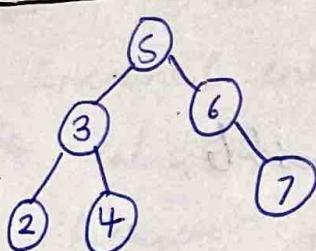
We'll insert 5, at whatever leaf is possible.

Find where it can be inserted then insert

We'll try to find a leaf position, because it's easier
to insert in leaf.

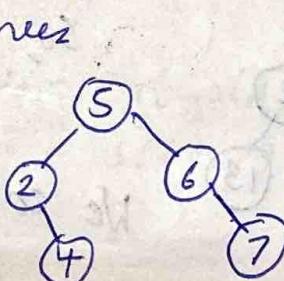
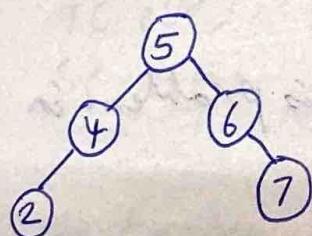
TC - $O(\log_2 N)$

Delete a Node in BST



node = 3

If we delete 3, then we have
2 possible trees

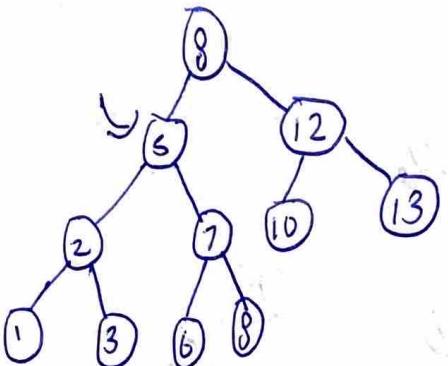


There can be multiple answers

Approach - We have to do deletion in parts, i.e.,

- First, we will search whether mentioned node exist in
the tree or not. (SEARCH)

- Then we will delete

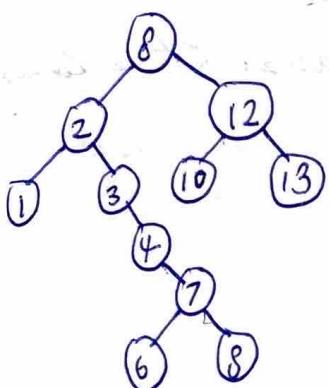


node = 5

Now, we can observe that everything in left of 5 < everything in right of 5

Now, if we delete 5, then we can connect the subtree to 4 → right & 8 → left should become 2

The tree should look like this,

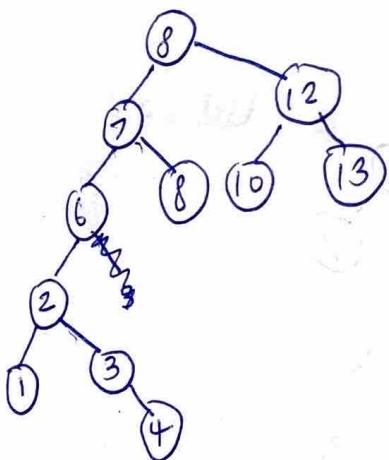


Now, other way of solving, if we delete 5 & put 8 → left of 5 = 7 & its subtree.

& in the left of 6 we can attach 2

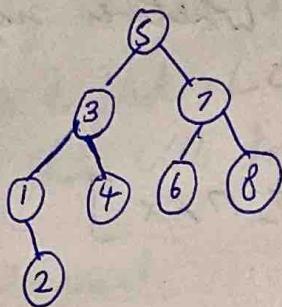
The tree should look like this

So, 2 ways are possible.



We can do this problem in whatever way

K^{th} Smallest Element in BST



$k = 3$

1 2 3 4 5 6 7 8



This is 3rd smallest.

Naive

Approach -

\$Do a recursive DFS traversal, and store all the nodes in a vector. Then we can easily find kth smallest element.

TC - $O(N) + O(N \log N)$

SC - $O(N)$

Efficient Approach -

Generally, inorder traversal of BST is in sorted order. We will do inorder traversal, maintain $cnt = 0$.

Increment cnt whenever we encounter a node.

When $cnt = k$, return that node

Recursive - TC - $O(n)$

SC - $O(n)$

orris - TC - $O(n)$

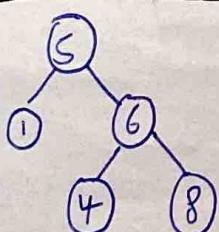
SC - $O(1)$

K^{th} largest \rightarrow Do one traversal to count no. of nodes in BST.

Then K^{th} largest = ($N - K^{th}$) smallest

So, 2 traversals required here.

Validate a BST



This is not a valid BST, because $4 < 5$

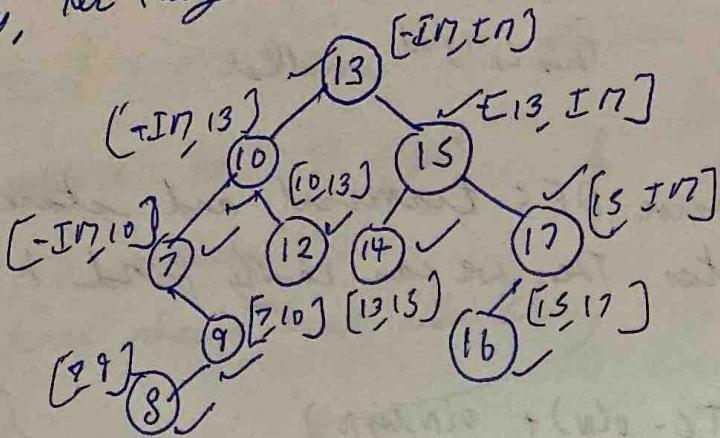
Approach -

For a node to be valid, it has to be
 $< \text{left child}$ & $> \text{right child}$. So, we will

provide a range to particular node,

e.g. [5, 6], then we can see whether node lies in particular range or not.

Initially, let range be [INT_MIN, INT_MAX]



We will check whether $\textcircled{13}$ lies in range or not. If yes, then it is a valid node. Update range and move left & right.

For left, range will be $[-\text{INT_MAX}, 13]$ for left child
& $[13, \text{INT_MAX}]$ for right child -

Now $\textcircled{10}$ is lying in range

Hence, this tree is valid

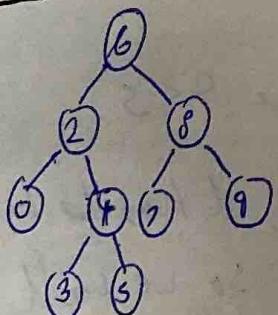
TC - $O(N)$ SC - $O(1)$

To initialize long int min & max int no \rightarrow

long int minValue = numeric_limits<long int>::min();

,, maxValue = numeric_limits<long int>::max();

LCA in BST



$$\text{LCA}(5, 0) = \textcircled{2}$$

$$\text{LCA}(2, 5) = \textcircled{2}$$

Naive Approach

* Simply traverse do the same approach, as we did to find LCA in binary tree.
But this will take $O(n)$ time complexity.

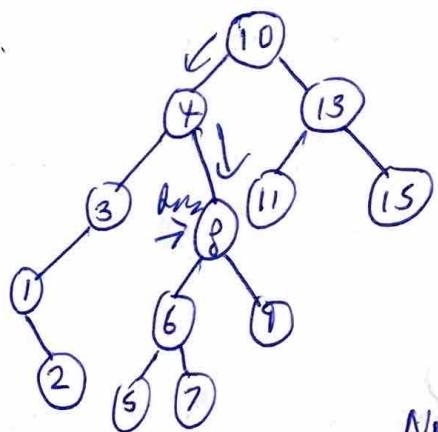
Better Approach

There are 3 possibilities for 2 nodes to exist that are,

- Both on left
- Both on Right
- One left & One right

In last case, we can say that the point where they split is our LCA.

E.g.



LCA(5, 9)

Start from root

(5 & 9) < 10, move left

(5, 9) > 4, move right

Nw, 5 < 8 & 8 < 9 which means

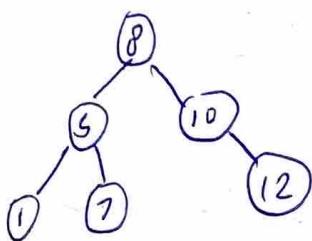
This is our LCA.

So, the point where we split, is our LCA

$$TC - O(H) \quad SC - O(1)$$

Construct BST from preorder traversal

preorder $\rightarrow \{8 \ 5 \ 1 \ 7 \ 10 \ 12\}$

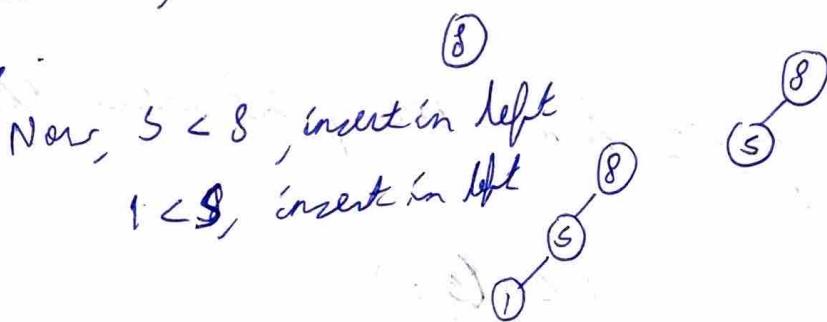


Naive approach

Naively create Binary tree

As we know 8 is root, because first element is root, so, insert root

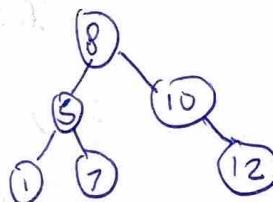
Now



Now, $5 < 8$, insert in left
 $1 < 5$, insert in left



And similarly we get



TC - $O(N) \times O(N)$

Better Approach

Sort the given preorder traversal, which will eventually become inorder traversal of BST

Inorder $\rightarrow \{1 \ 5 \ 7 \ 8 \ 10 \ 12\}$

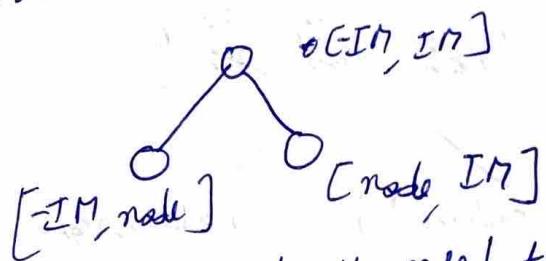
Now, with inorder & preorder, we can create a BST, just like we created a Binary tree

TC - $O(N \log N) + O(N)$

SC - $O(N)$

Optimal approach

We'll use the same approach we used to validate a BST, i.e., we'll use INT-MIN & INT-MAX



But, in this approach, we don't need the Lower Bound Range. We can just solve with INT-MAX

$$\text{Upper-Bound} \rightarrow ub = INT_{MAX}$$

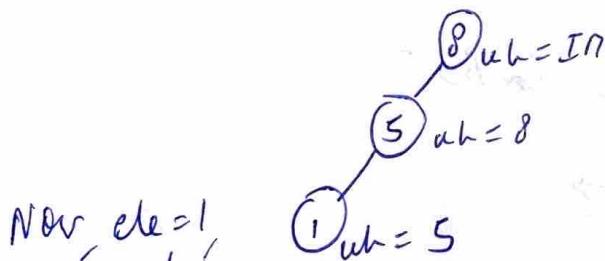
Initially,

preorder $\rightarrow [8 \downarrow S171012]$

node = 8, is $8 < ub$, Yes it is
Now, insert 8.

Now, go left & update $ub = 8$, we can observe that S17 are < 8 .

Now, ele = 5, $5 < ub$, yes we can insert, update $ub = 5$



Now, ele = 1,
insert

update $ub = 1$, Now again go left

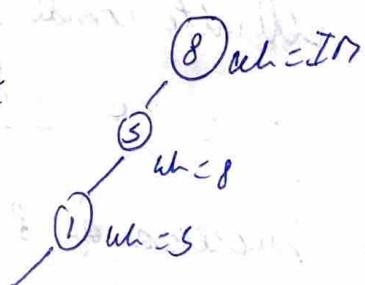
Now, ele = 7, it is not less than ($ub = 1$)

so return NULL.

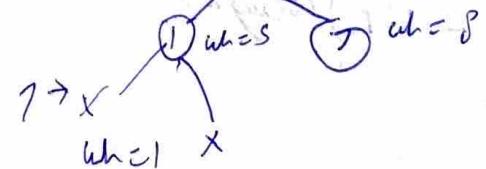
now it goes back to ① & $ub = 5$ here,

it cannot be inserted here also,

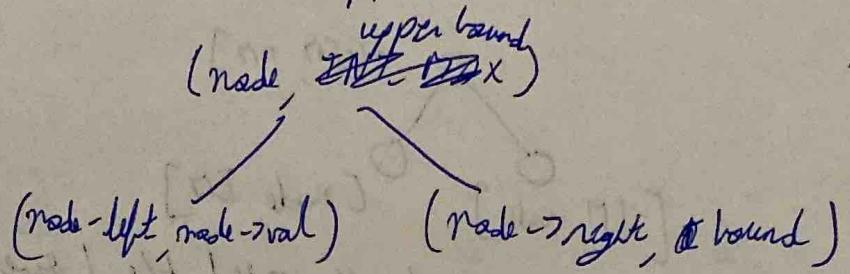
so return NULL



X $ub = 1$



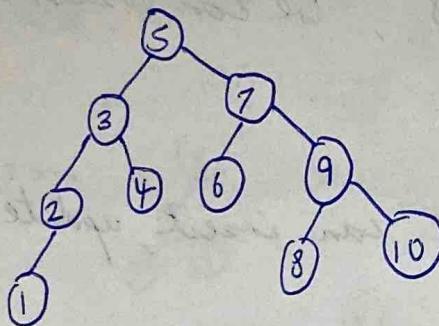
This way we'll keep inserting all the elements.
We just have to maintain upper bound,



We'll also have to maintain at which element we are in the array, then return.

$$TC - O(3N) \approx O(N)$$
$$SC - O(1)$$

Inorder Successor in BST



val = 8

We have to find inorder successor in BST

First we will write inorder traversal \rightarrow

1 2 3 4 5 6 7 8 9 10

Inorder successor of 8 is 9 here

Brute Force

Store inorder in vector & then find successor

If no successor then return null.

$$TC - O(N) + O(N)/O(\log N)$$

$$SC - O(N)$$

Optimal approach

Do inorder traversal, and the first value which we encounter > 8 , is our answer.

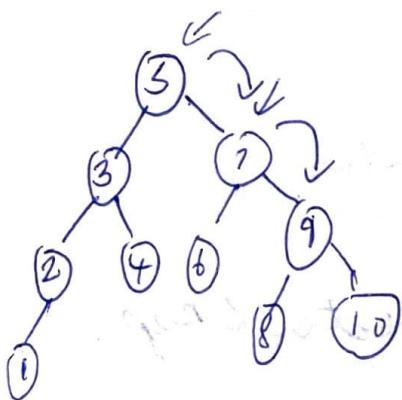
TC - O(N)
SC - O(1)

We will optimise Time
to O(H)

Optimal approach

Initialise a variable successor = NULL.

First node is 5, which is < 8 , so our task can't be success & $8 > 5$, so move right.



Again on ?, $8 > ?$, move right & successor is still NULL.

Now, node is 9, $9 > 8$, successor = 9 & $8 < 9$, move left.

Now, node is 8, when we reach the node itself we'll go to its right, if its null then return successor.

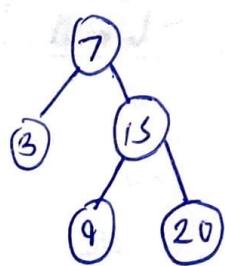
Inorder - 3 7 9 15 20

TC - O(H) SC - O(1)

BST Iterator

BST Iterator (7) — This will be root

next — Initially nextptr points to NULL. The moment we ask next it should return its next node in inorder traversal.



Inorder - 3 7 9 15 20
next ↑
(initially) next

When next is called it will return 3 & move to 3

hasnext - return True or False whether next exists or not.

BST iterator (?) - represents root

next \rightarrow 3

next \rightarrow 7

hasnext \rightarrow True

next \rightarrow 9

hasnext \rightarrow True

next \rightarrow 15

hasnext \rightarrow True

next \rightarrow 20

hasnext \rightarrow False

3 7 9 15 20

So we have to design this BST iterator

Naive
Approach

Store the inorder traversal in a vector & keep next pointer at arr[0].

TC - O(1) + O(N)

SC - O(N)

Better

Inorder is \rightarrow Left Root Right

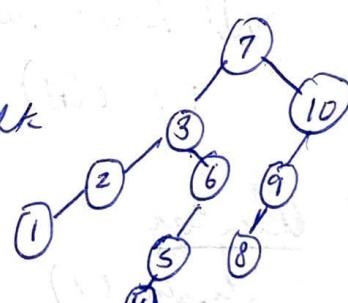
TC - O(H)

TC - O(1)

We will have to create a constructor BST iterator (?), which will be initially the root

We will take a stack.

Go to the extreme left & put everything in stack

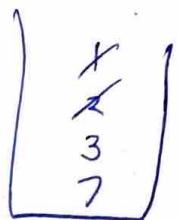


next - return st.top(), this will be our next, (1 gets popped)

After popping go to its right & push elements

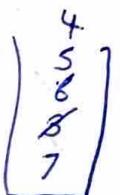
next - returns 2

1 & 2 has no right pointer, so we didn't push anything



next - Returns 3

Now, 3 has right, go to right & put everything in stack



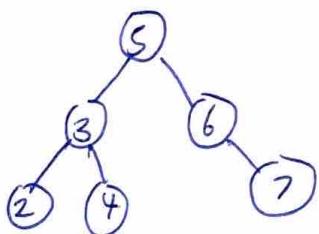
We'll follow the same procedure.

~~TC~~

TC - O(1)

SC - O(H)

Two Sum in BST



$$K = 9$$

$$5 + 4 = 9$$

$$3 + 6 = 9$$

Yes there exist 2 nodes whose sum is 9

$$2 + 7 = 9$$

Brute Force -

Write in-order traversal in vector. Then perform normal 2 sum approach using 2 pointers

TC - O(N) + O(N)

SC - O(N)

Optimal -

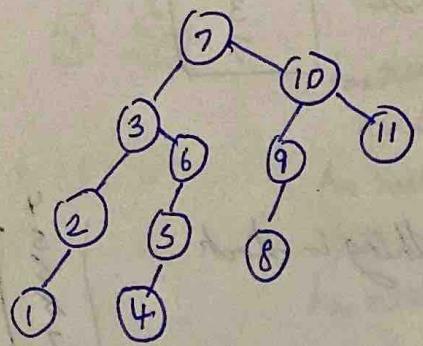
We'll follow some approach we used in BST iterator.

We will take 2 stacks to determine next() & before()

element, i = next()

j = before()

We will determine before () using opposite approach, we used to determine next().
 i.e. We will keep going node->right & push it in stack.
 Now, $i = \text{next}()$ $\& j = \text{before}()$



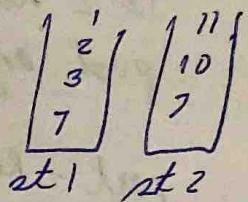
In this case,

$$i=1 \& j=11$$

$$i+j = 12 < 16,$$

so we have to increase
the value

$$k=16$$



To increase the value we'll increase the next()
guy.

$$\text{Now, } i=2 \& j=11 < 16$$

$$i=3 \& j=11 < 16$$

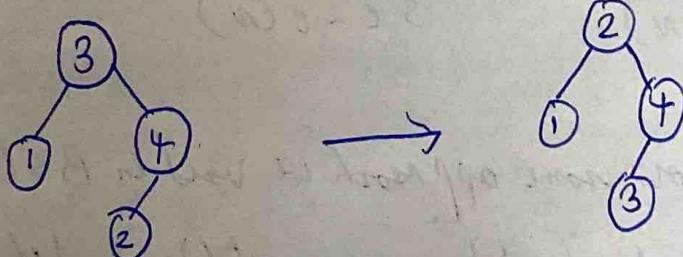
$$\text{Again } i=4 \& j=11 < 16,$$

$$\text{Again } i=5 \& j=11 < 16. \quad \text{Return true}$$

$$+ C - O(N) \quad SC - 2 \times O(H)$$

Recover BST

Two nodes are swapped in given BST, we have to
remap them to recover our BST,



Brute Force

Do in-order traversal & store it in vector

Then sort the traversal. ~~Empty the tree~~

Now simply traverse with the tree again & compare nodes values with values inside tree.

If not same then change

$$TC = 2N + N \log N$$
$$SC = O(N)$$

Better Soln

Write the inorder of the given incorrect BST.

This will take $O(N)$ time

Now, as we know 2 nodes are swapped in the sorted array.

So, we have to figure out the swapped elements

Case I - Swapped nodes are not adjacent

8 2 2 3

3 2 5 7 8 10 15 20 5

Case II - Swapped nodes are adjacent

3 5 8 7 10 15 20 2 5

We don't need to store the inorder traversal in any array, we just have to figure out what nodes are swapped.

Now, in case I; start traversing

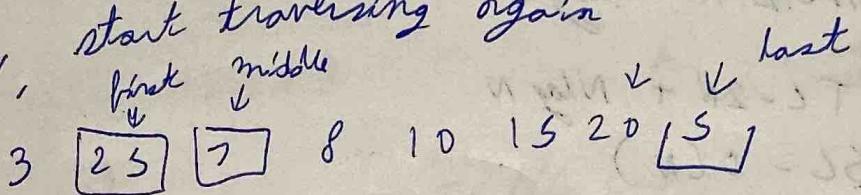
↓ ↓ ↓
3 [2 5] 7 8 10 15 20 5

Now as soon as we see that $2s > 7$, or $7 \leq$ not greater than prev element.

So, we can mark 2s as the first node which needs to be swapped.

We'll store 7 as second element which is violated/mapped.

Now, start traversing again

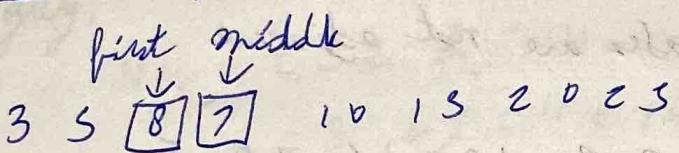


We get 5 is < 20, this is second violation.

We'll store 5.

Now, we'll simply swap 25 & 5

Now Case II - If there is no second violation



We'll not find any 2nd violation in this case.

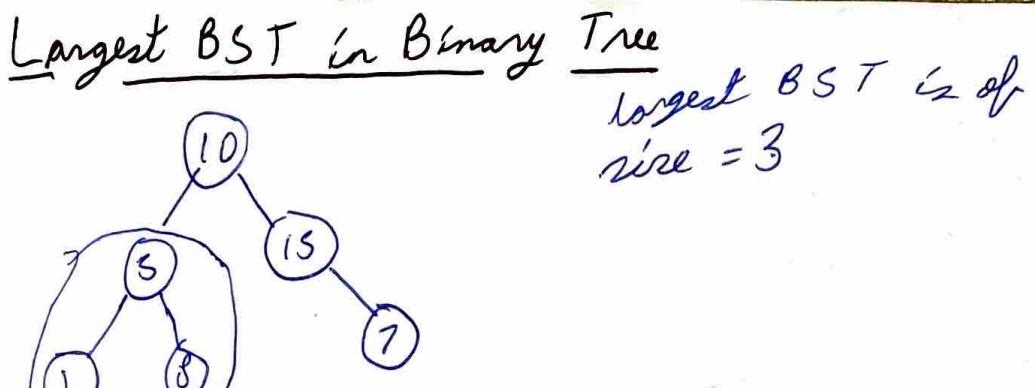
So, we'll simply swap 7 & 8.

We only swap first & last if both are found.

If not found, we only swap first & middle.

TC - $O(N)$

SC - $O(1)$



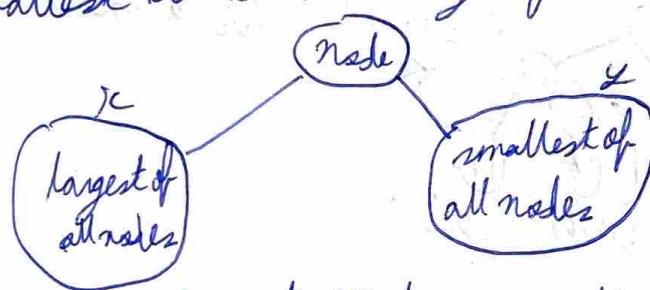
Brute Force

We apply the same approach we used to validate a BST.

 $T.C = O(N^2)$
 $S.C = O(N)$

Better approach -

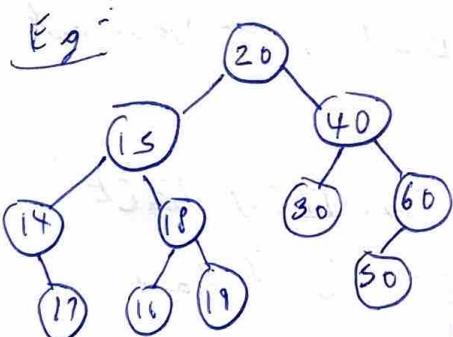
If we can find largest ele in the left of node & smallest ele in the right of node



$\text{largest} < \text{node} < \text{smallest}$, then it is a Valid BST.

And for size we say, left subtree has x elements & right has y elements. Then total size = $1 + x + y$.

(largest, smallest, size)



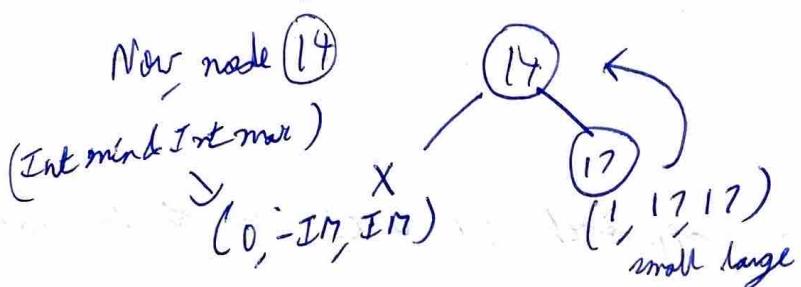
We'll follow Postorder traversal to go left, right & then compute root.

For a leaf node, we can say it is definitely a BST & its size = 1.

We go left & reach 14, $(14) \rightarrow \text{left} == \text{NULL}$, so, we go right

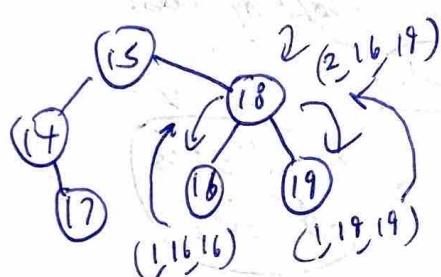
Did we reach node 17
we update our values

(17, 17, 1)
large small size

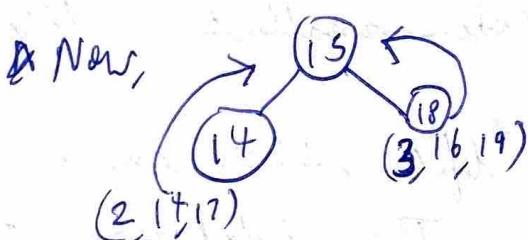


This will return to 14.

Now, left of 15 is computed, we'll go right.

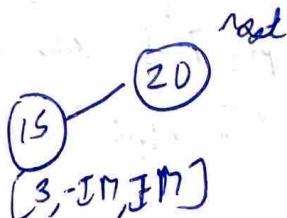


We'll move to 18

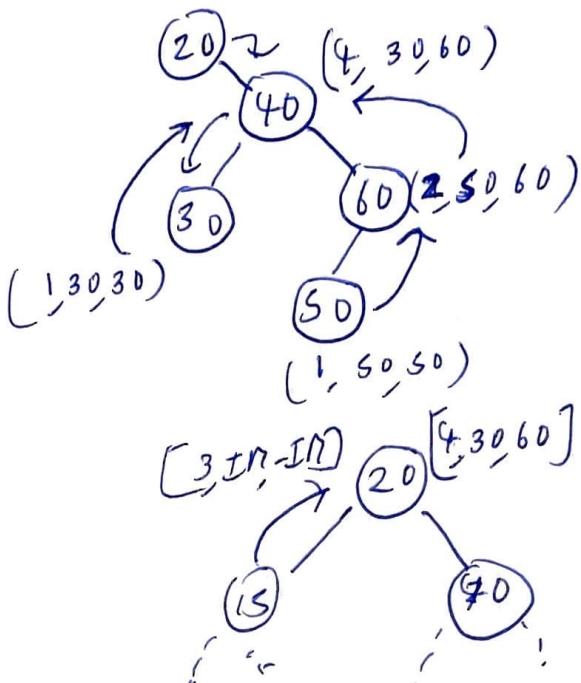


As largest in left is 17, which is not less than 15, so, this is not a valid BST, so we'll not update size, we'll not do size+1.

And we'll pass ~~INT MAX & INT MIN~~ to it as $\textcircled{2D} \rightarrow$ left is not a valid BST. We'll have to look in right only.



Now we'll compute
 $\textcircled{2D} \rightarrow$ right



Yes it is a valid
BST & its size = 4

20 gets 2 values

So, max size = 4

So, if valid bst, size = left + right + 1

if not, size = max(left, right)

& update ~~the~~ largest & smallest.

TC - $O(N)$ SC - $O(1)$

Strings

Remove Outermost Parenthesis

$$((())()) \rightarrow ()()()$$

$$((())((())(())) \rightarrow ()()()()()$$

$$()() \rightarrow - \text{ empty}$$

Approach

We will maintain a `cnt` variable while traversing the string, whenever we get '`(`' we increment `cnt` & whenever we get '`)`' we decrement `cnt`.

And in each loop we will check whether `cnt == 0` then

`ans += str[i];`

`cnt = 0`

`ans = ""`

E.g:-

\downarrow $\&$ $\&$

`((())())` $cnt = 1$ `ans = " "`

z

`ans = "(`

1

`ans = "()"`

2

`()(`

1

`)()`

0

`((())()`

1

2

1

0

Reverse words in a string →

"hello world!" → "world! hello"

Approach -

- Start Using 2 pointer approach.
- For leading and trailing blank spaces we'll use a inner while loop which will keep incrementing until i does not encounter a non-space character.
- As soon as i encounters non space character, we will initialise $j = i + 1$. And move j until j again encounters a space character. This space will embark end of 1 word.
- Now we will store string word = s.substr(i, j-i)
- * substr takes 2 parameters (i, length of substring)

And we will add this result to the front of result/an

ans = word + " " + ans; ~~ans~~

i = 0 res = ""

Dry Run

" --- hello-world! --- "

- Blank character
(space)

i will keep incrementing until it encounters first alphabet

" --- hello-world! --- " Now, initialise $j = i + 1$
 & increment j until space

" --- hello-world! --- " word = "hello"
 res = word + res = "Hello"

Move i to $j + 1$

~~Again~~ Again follow same procedure

TC - O(N) SC - O(N) i.e. Auxiliary space for answer & substring

In the previous approach, we will get Memory Limit Exceeded error, because we have to solve this in $O(1)$ constant space.

Approach II - This approach also takes extra space i.e. $O(N)$ space. We'll use a stack in this approach.

E.g. "i like program"

Push all the words in stack in one traversal

Program
like
i

Then in second traversal pop the strings from stack.

Optimal $O(1)$ approach -

- 1) ~~Store the words in vector <string>.~~
- 2) ~~Traverse backward in the vector and store the words in string arr with extra space appended to each string except the first string i.e. $i = 0$)~~

Store the answer in original string itself.

Initially before entering the loop do $s += " "$;

Then in loop do $s += s.substr(i, j-i)$

After loop

$s = s.substr(len+1, s.size() - (len+2))$;

return s

$T(-O(N)) \quad SC-O(1)$

Largest Odd len No in String - O/P

num = "52" → "5"

num = "4206" → "

num = "35427" → "35427"

Approach-

Last digit in string decides whether the given string is odd or even.

e.g. - "11740" is even

"700895" is odd

So, we'll traverse the string from Right to Left

Q. 2 -

11740

↑↑↑
i i i

If even decrement i

till current i position.

Q. 2 -

700895

↑
i

odd digit then return whole string

Now we don't need to convert char to int to check whether a digit is odd or even.

We can check with ASCII value also.

ASCII value of even digit is even.

Longest Common Prefix

I/P → ["apple", "appz", "apt"]

O/P → "ap"

Brute Force

Approach -

We will compare 2 strings at once. And then what substring is common in them. We will compare that substring with remaining strings.

Compare (s1, s2)

T.C - O(N) * O(min string length)
S.C - O(1)

Isomorphic strings -

① Two strings s & t are isomorphic if the characters in s can be replaced to get t .

Eg- $s = "egg"$, $t = "add"$

O/P - true

$s = "foo"$, $t = "bar"$

O/P - false

2 characters may not map to same character, but a character may map to itself.

Approach -

If all characters in both strings are One-to-One mapped, then these strings are isomorphic.

We will use 2 maps, one will map char - char & other will map char - base

There are 2 cases in which our soln. can fail.

Case 1

foo

bar

o is mapped with 2 char

i.e. a & r

Case 2

for

bot.

r is mapped with 2 char i.e. o & t

For case 2, we will use a second map which will check whether a char in t string has already been mapped or not.

Find if one string is rotation of another string -

$$s = "abcde" \quad goal = "cdelab"$$

O/P \rightarrow true

Approach- If left rotation has been done by k places, then by right rotation of $n-k$ places, we can get original string.

Method 1- We do

$$str_temp = s + goal;$$

Step 2:

Concat string with original string

$$\text{i.e. } s = abcdeabcde$$

Then start traversing this string and look for goal.

If found, then return true.

We can search using

```
if (temp.find(goal) != string::npos)
```

return true;

else

return false;

Valid Anagram -

Given 2 strings s & t, return true if t is an anagram of s, and false otherwise.

e.g. $s = "anagram", t = "nagaram" \quad O/P \rightarrow \text{TRUE}$

$s = "rat", t = "car" \quad O/P \rightarrow \text{FALSE}$

Sort characters by Frequency

$s = \text{true}$
 $O/P \rightarrow \text{ee t n / eent}$

$s < A \& t < b$
 $\text{re} = bbaA / bbaA$

Approach -

We will use Hashmap & then sort the hashmap according to values.

$$TC - \log(s) + s^2 \log s^2$$

(s^2 for s^2 alphabets
uppercase & lowercase)

use a vector <pair<int, char>> hash ('z' + '0', 0, 033)

Insert frequency & then sort in ascending order.

Roman to Integer

Use a map to map roman characters to their corresponding values.
A special case we need to take care of is that if smaller value precedes larger value then we subtract smaller value from ans. Not add it.

$$\text{Eg} - IV \rightarrow -1 + 5 = 4$$

 $TC - O(N) \quad SC - O(2^7) \approx O(1)$

Integer to Roman

$$9 \rightarrow "IX"$$

We will map all values E.g. I, II, III, IV, V, etc to their corresponding values.

Start traversing the map

While $\text{num} \geq \text{mp}.first$

$\text{ans} += \text{mp}[\text{i}], \text{second}$

$\text{num} -= \text{mp}[\text{i}], \text{first}$

string ans = ""

Count Number of Substrings with Exactly K distinct characters

→

Input : abc $k = 2$

O/P ≥ 2

Substrings are { "ab", "bc" } with 2 distinct characters

Input → aba $k = 2$

O/P ≥ 3 Substrings are { "ab", "ba", "aba" }

Input → aa $k = 1$

O/P ≥ 3 Substrings are { "a", "a", "aa" }

Brute Force Approach -

Generate all substrings which will take $O(n^2)$ time

Then for each substring count distinct characters in it.

Total TC - $O(n^3)$.

Better Approach -

We'll maintain a hash table while generating substring and check the number of unique characters using that hash table.

Optimal Approach -

We will use 2 hashmaps m1 & m2

"a b c a b d a b b c f a" $k = 3$

m1 hashmap - chara hashmap
(till k-1 characters)

m2 hashmap - bade hashmap
(all k characters)

↑ - adding in m1 hashmap

↖ - adding in m2 hashmap

↑ - release from hashmap
pop

Here, $k=3$, $m1$ hashmap will go till $k-1$ i.e. 2 characters, it means it will store only 2 characters at max.

Now, let's do dry run

$\downarrow \downarrow \downarrow \downarrow$
a b c a b d a b b c f a

$\begin{bmatrix} b-1 \\ a-1 \end{bmatrix}$
 $m1$

$\begin{bmatrix} c-1 \\ b-2 \\ a-2 \end{bmatrix}$
 $m2$

We will keep pushing characters in $m1$ until its size! = $k-1$

& we will keep pushing in $m2$ until size! = k

Here, size of hashmap denotes how many distinct characters it has in it.

Now we have 3 substrings with k unique characters in window!

that are, "abc" "abca" "abcabc"

Now, we'll release first character from both maps, i.e.

delete it

a b c a b d a b b c f a

$\begin{bmatrix} b-1 \end{bmatrix}$
 $m1$

$\begin{bmatrix} c-1 \\ b-2 \\ a-1 \end{bmatrix}$
 $m2$

↑
Now our $m1$ is invalid as its size! < $k-1$.

So, insert next character in $m1$

$\begin{bmatrix} c-1 \\ b-1 \end{bmatrix}$
 $m1$

$\begin{bmatrix} c-1 \\ b-2 \\ a-1 \end{bmatrix}$
 $m2$

Now, we get 2 strings, "bca" & "beat" $m1$
Now again move character pointer to next i.e. pop b from both

a b c a b d a b b c f a

$\begin{bmatrix} c-1 \\ b-1 \end{bmatrix}$
 $m1$

$\begin{bmatrix} c-1 \\ b-1 \\ a-1 \end{bmatrix}$
 $m2$

$m1$ size again < $k-1$, So, push next char

Now strings are "cat"

Now release c from both maps.

$\begin{bmatrix} a-1 \end{bmatrix}$
 $m1$

$\begin{bmatrix} b-1 \\ a-1 \end{bmatrix}$
 $m2$

Both maps got invalid now

push next characters until size = k in $m2$

a b c a b d a b b c b a

$\left[\begin{matrix} L-1 \\ a-1 \end{matrix} \right]_{m1}$ $\left[\begin{matrix} d-1 \\ b-3 \\ a-2 \end{matrix} \right]_{m2}$

Now we get 4 substrings, i.e., $b-2=4$
in this window

That are "abdab" "abdah" "abd'a", "abd"

Now move a

X

Longest Palindromic Substring -

I/P \rightarrow "babad"

O/P \rightarrow "bab" / "aba"

Brute Force - generate all possible strings, and check whether it is a palindrome or not.
 $T.C = O(n^3)$

Better - Earlier we used to determine palindromic string by starting from edge characters, if they match then we move inwards, e.g.

b a b

But here we will do it opposite way, i.e., $b \xrightarrow{i} \xleftarrow{j} L$

We'll start from single character which is obviously a palindrome, then we'll move outer until we find equal characters on both sides.

Dry Run - "ba b a d" $\text{longest} = ""$

First "b" as center, we cannot move left as this is largest palindrome & increment is its next char

"

$\text{longest} = "b"$

Now 'a' is center, move left & right until characters match

Now, "ba b" is largest $\text{longest} = "ba b"$

Now, "ba b a d" "aba" is a palindrome

So, we can update longest or keep it same it's on us.

But, this method only calculates palindrome of odd length.
What if our palindrome is of even length?

Eg - "cb b d" We'll modify our code a little bit to
I/P → "b b" handle this edge case

To handle this edge case, we will initialize $\text{left} = \text{right} = \ell + 1$
Whereas to check for odd length, we do $\text{left} = \text{right} - 1$
Let's do a dry run

① "c b b d" $\text{left} = 0$ $\text{right} = 1$ $\text{longest} = "c"$

"c b b d" $\text{left} = 1$ $\text{right} = 2$ $\text{longest} = "b b"$

$T C - O(n^2)$
 $\rightarrow S C - O(1)$

Sum of Beauty of all Substrings -

Beauty of a string is = Most frequent character frequency
 - Least frequent character frequency

$$\text{E.g.} \text{ Beauty of "abaaacc"} = 3 - 1 = 2$$

So, given a string s , return sum of beauty of all of its substrings

I/P \rightarrow "aabccb"

$$\text{Beauty of substring aab} = 2 - 1 = 1$$

$$\text{" " " aabc} = 2 - 1 = 1$$

$$\text{" " " aabcb} = 2 - 1 = 1$$

$$\text{" " " abccb} = 2 - 1 = 1$$

$$\text{" " " bcb} = 2 - 1 = 1$$

$$\text{Sum} = 1 + 1 + 1 + 1 + 1 = 5 \leftarrow \text{O/P}$$

Approach -

Generate all substrings and insert the characters in map. And then add a third loop to traverse the map which will determine max & least frequency

$$TC - O(n^2) * O(26) \approx O(n^2)$$

$$SC - O(26) \approx O(1)$$

for ($i = 0 \rightarrow n$) {

 map <char, int> mp

 for ($j = i \rightarrow n$) {

 mp[s[j]]++; int mf = 0, lf = INT_MAX;

 for (auto i : mp) {

 mf = max(mf, i.second)

 lf = min(lf, i.second)

 }

 ans += (mf - lf);

}

Better approach -

We can use a multiset instead of using map because we only need to know frequency, not character. Map would have failed if the question was of mapping integers.

multiset<int> ms;

(Sorted & can store duplicates)

First ele of multiset - lowest frequency
Last " " " - highest "

(Frequency can be same of 2 characters so we take multiset)

So, we won't have to traverse whole set to find max & min frequencies.

```
for (i = 0 → n) {
    map<char, int> m;
    multiset<int> st;
    for (j = i → n) {
        if (m.find(s[j]) != m.end())
            st.erase(st.find(m[s[j]]));
        m[s[j]]++;
        st.insert(m[s[j]]);
    }
    ans += (*st.rbegin()) - *st.begin();
}
```

We will delete previous frequency of a char if it has already been inserted in set.

Because we have to update new frequency in set

Rabin Karp Algorithm

If a string is given, and a pattern is given. We have to determine whether pattern exists in string or not.

Text: a a a a a b

Pattern: a a b

Approach: We'll not compare each & every alphabet in this algo. In pattern, we see 3 alphabets, instead of checking single character each time, we'll make it as a single value. We'll sum it up with ASCII values, but for the sake of understanding this algo easily, we'll make our own values

Pattern: a a b
 $\underbrace{1+1+2}_{n(p)} = 4$

This is hash f^x_n

$n(p)$

\uparrow
 This is hashcode

| | |
|-------|--------|
| a - 1 | b - 7 |
| c - 2 | d - 8 |
| e - 3 | f - 9 |
| g - 4 | h - 10 |
| i - 5 | j - 6 |

We use this hash f^x_n to find pattern inside this text.

Text: $\underbrace{a a a}_{n=3} a a b$
 $1+1+1=3$

Depending on size of pattern ($m=3$), we'll take 3 letters in text.

Pattern: a a b
 $m=3 \quad 1+1+2=4$

Both values are not matching, using constant time we found value for above 3 characters. Then we'll move further until we find 4 in Text.

$\frac{\overbrace{a a a}^3 \overbrace{a a b}^3}{3} \rightarrow 4$

This sliding is
Rolling Hash Fxn.

Eg'

$$\begin{array}{r} \text{Text: a b c d a b c e} \\ n=8 \quad \underline{\quad 6 \quad} \\ 6-1+4=9 \\ \underline{9-2+1=8} \\ 8-3+2=7 \\ \underline{6} \\ 10 \end{array}$$

Pattern: b c e

$$m=3 \quad 2+3+5 = 10$$

When we slide our window further, subtract removed character & add next character ASCII value in the total.

Drawback - We were going through the main string only once & we have checked for pattern only once,

$$\text{So, } TC = O(n-m+1) \text{ This is average case}$$

But, in worst case scenario like below

$$\begin{array}{r} \text{Text: c c a c c a a c d h a} \\ n=11 \quad \underline{\quad 3+3+1=7 \quad} \\ \underline{3+1+3} \end{array}$$

pattern: d h a

$$m=3 \quad 4+2+1 = 7$$

Both values match, we'll compare characters, characters do not match as we move our window.
Again values match, we'll again compare characters so if values same same, we'll have to compare pattern everytime.

These are called as Spurious Hits.

$$\text{And } MC\text{Time} = O(m^2)$$

This is happening because of very simple hash fn.

Now we'll study hash fn by Robin Karp.

Main idea -

We define a better hash f^{*n} .

pattern: d b a,

$$4 \times 10^2 + 2 \times 10^1 + 1 \times 10^0 = 400 + 20 + 1$$

Here, we have taken 10 because we are considering alphabets from a-z i.e. 1-10.

It is completely dependent on us, what value we take as base to define hash f^{*n} .

Text: c c a c c a a e d h a

$$3 \times 10^2 + 3 \times 10^1 + 1 \times 10^0$$

$$300 + 30 + 1 = 331$$

Now to move the window forward

$$[3 \times 10^2 + 3 \times 10^1 + 1 \times 10^0] - \cancel{3 \times 10^2}$$

$$[3 \times 10^1 + 1 \times 10^0] \times 10 \rightarrow 3 \times 10^2 + 1 \times 10^1$$

$$3 \times 10^2 + 1 \times 10^1 + 3 \times 10^0$$

Similarly on moving forward we'll find 'dha' pattern

Now $T.C. = O(n-m+1)$ avg case

$T.C. = O(mn)$ worst case

But now chances of worst case are very less

How to code?

Hash at next shift must be efficiently computable ($O(1)$) from the current hash value and next character in text

Rolling hash f^{*n} :

$$\text{hash}(\text{txt}[s+1 \dots s+m]) = d(\text{hash}(\text{txt}[s \dots s+m-1]) - \text{txt}[s]^*h) + \text{txt}[s+m] \% q$$

Hash($\text{txt}[s+1]$) - next char

$\text{txt}[s]$ - curr char

d - No. of characters in the alphabet

q - A prime no.

h - $d^{(m-1)}$

m - length of pattern.

Simply written

$$\text{hash}(s[i+1]) = d(\text{hash}(s[i]) - s[i]^*h) + s[i+m] \quad \% q$$

$d \rightarrow 256$

\Rightarrow prime no. (here 11)

int $p = 0$; // hash value for pattern

int $t = 0$; // " " " text

int $h = 1$;

Calculate h - for ($i=0 \rightarrow n-1$) $h = (h * d) \% q$;

Calculate hash value - for ($i=0 \rightarrow n$)

for both strings

$p = d * p + pat[i] \% q$;

$t = d * t + txt[i] \% q$;

Now we'll search in main string

for ($i=0 \rightarrow N-m$)

if ($p == t$) {

for ($j=0 \rightarrow m$) {

if ($txt[i+j] != pat[j]$) break;

}

if ($j == m$) Found

}

if ($i < N-m$) {

$t = (d * (t - txt[i]^*h) + txt[i+m]) \% q$

if ($t < 0$) $t = (t + q)$;

Naive String Matching Algorithm

String: a b c d e f g h

pattern: d e f

We have to search for the pattern in the string.

Sliding window of size m , where m is length of pattern.

If it matches then return index, otherwise shift the window towards right

Eg 2: str = abc d abc abc df
 str[1] 2 3 4 5 6 7 8 9 10 11 12

We'll use i & j here

pat = abc d f
 pat[1] 2 3 4 5

$str[i] = pat[j]$
 $i++$, $j++$

j j j j At index 4, there is a mismatch

so, now j will go at index 1, i.e. starting
 But i will go at index 2, i.e. starting +1

Now, $\xrightarrow{\text{mismatch}}$ (from here it starts matching)
 a b c d a b c a b c d f

So, this i going back to
 index 2 is a waste of
 time. This is a drawback
 of Naive alg.

a b c d f
 j j j j

Again $pat[j] \neq str[i]$
 i will move to index 6, and j will move to start
 of pattern.

i will move to that index +1, where both characters matched
 for the first time.

so, drawback of this alg is that same characters are
 compared repeatedly

TC - O(m * n)

KMP Algorithm (Knuth-Morris-Pratt)

Pattern: a b c d a b c

We'll find prefix
 suffix of a pattern

prefix: a, ab, abc, abcd, ...

suffix: c, bc, abc, dabc, ...