

Approach → First thing which comes to mind is Take / Not take

So, 3 main parameters will be
 $f(index, target, ds[])$

$[2, 3, 6, 7]$

target = 7

When we are picking we will not increment index, as we can again pick it.

$f(0, 7, [])$

Not Pick

$f(1, 7, [])$

Pick

$f(0, 3, [2])$

Pick \nearrow Not \searrow

$f(1, 5, [2])$

$f(0, 1, [2])$ $f(1, 3, [2])$

// NotPick \nearrow Pick \nearrow

X $f(1, 1, [2])$ $f(1, 0, [2])$

X $f(2, 1, [2])$

X $f(2, 0, [2])$

X $f(3, 0, [2])$

(It will not be able to pick here because $target < arr[index]$)

if ($ind == n$)

target == 0, ~~return~~.

push

Recursive calls

$f(ind, target, ds)$

Pick

$f(ind, target - arr[ind], ds)$

$f(ind+1, target, ds)$

$if (arr[ind] \leq target)$

Base Case

else

$f(ind == n)$
if ($target == 0$) $ds \rightarrow []$
else
return;

where t = target
k = arr length

TC = $2^t \times k$

Because each of t inter has 2 options pick / nonpick

Combination Sum II →

Find all unique combinations in arr where the sum == target

Each no. in arr may only be used once in the combination.

E.g. candidates = [10, 1, 2, 7, 6, 1, 5] target = 8

O/P → [[1, 1, 6], [1, 7], [2, 6], [1, 2, 5]]

E.g. candidates = [2, 5, 2, 1, 2] target = 5

O/P → [[2, 2, 1], [5]]

combinations should be in lexicographically sorted order.

D. (P.S: above combinations are not in correct order)

Correct order is →

O/P₍₁₎ → [[1, 1, 6], [1, 2, 5], [1, 7], [2, 6]]

O/P₍₂₎ → [[2, 2], [5]]

Brute Force → (Inspired from previous ques approach)

Generate all possible sequences

In this approach, we will change f(index+1, ...) when we include an element in the data structure.

i.e. We will move to the next index after picking an element, because in this ques duplicates are not allowed.

We will store the ans in Set to store unique only

$$TC \rightarrow (2^n) \times K \log N$$

$$SC \rightarrow O(N) \text{ for set}$$

Optimal →

This time instead of pick / not pick, we are gonna try to pick subsequences.

$$f(\text{index}, \text{target}, \sqcup)$$

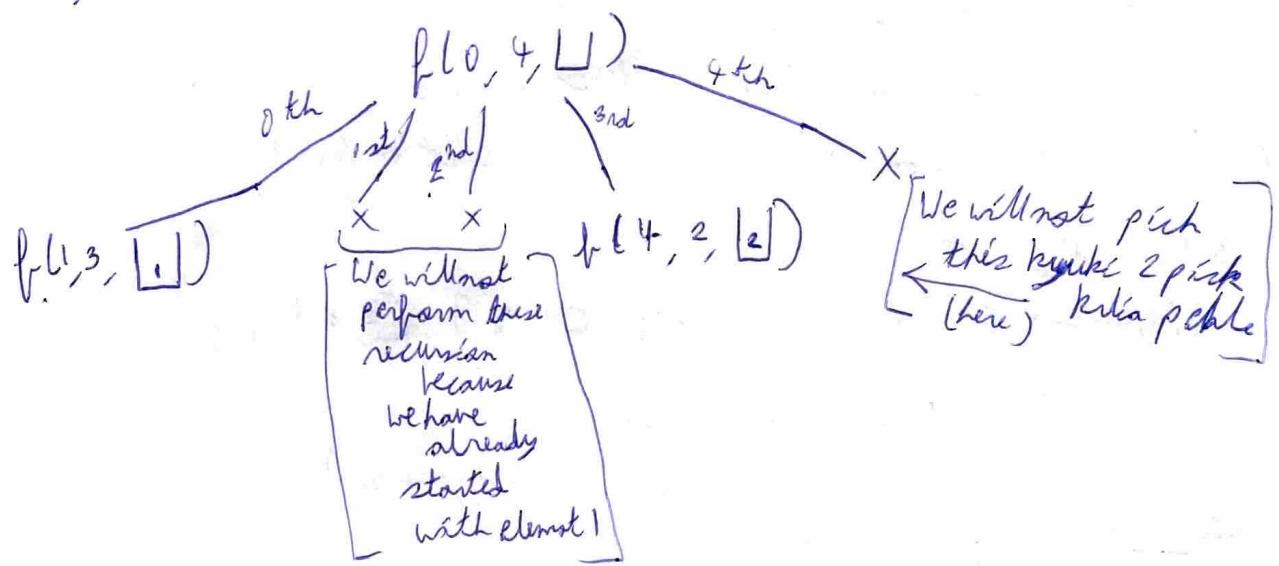
$$\text{arr}[] = [1, 1, 1, 2, 2]$$

$$f(0, 4, \boxed{\quad})$$

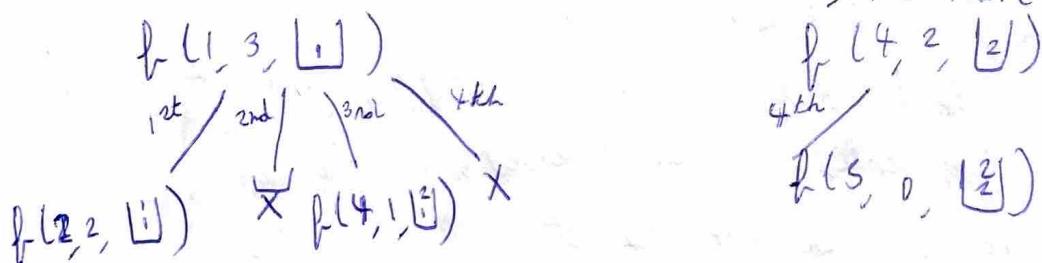
We can start from any index, which means we can pick element from any index.

Eg - Pick 1 from index 0,
Pick 2 from index 3
Pick 2 from index 4

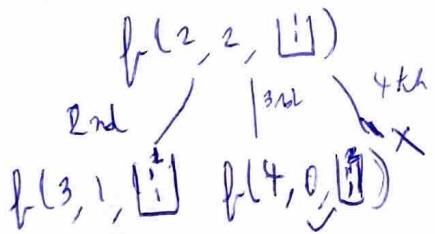
So, in this case we have 5 possibilities of choosing inde.



Now to choose 2nd element of combination, we have 4 possibilities



Now to choose 3rd element, we have 3 possibilities



So this is how the recursion tree proceeds, when elements are similar in the array, it will only make one fix call for that index, instead of making call again & again.

$\begin{pmatrix} 1, 1, 2 \\ 2, 2 \end{pmatrix}$

Will be the answer

~~VJMP *~~

$f(\text{ind}, \text{target}, ds, ans)$

Now, initially when we are at 0th index, we are trying can we pick index 0, index 1, ..., 2, 3, 4 (any index)

We try from 0 till the end.

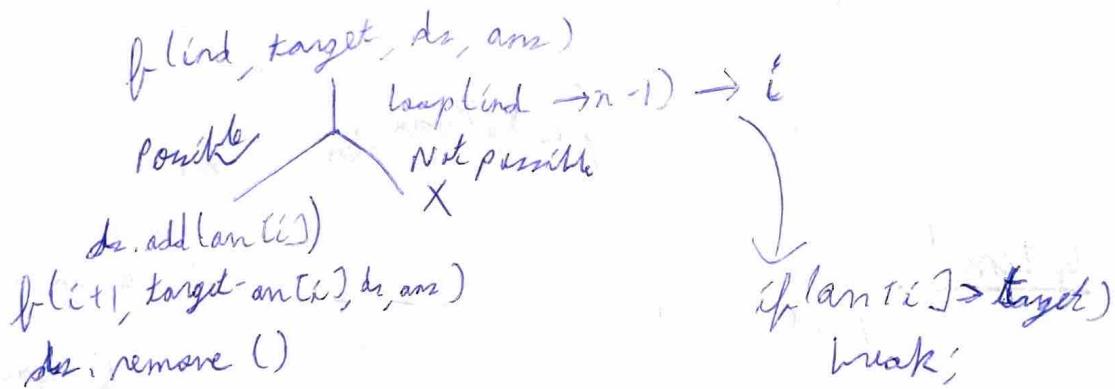
Then we move to index 1,

we try from 1 till the end.

Similarly, we move to index 2 & try from 2 till the end

Basically, we are looping from index to n-1

Whenever it is possible we make a recursion call, otherwise no recursion call.



Base case \rightarrow if ($\text{target} == 0$)
 $\text{ans. add } (\text{ds})$

TC $\rightarrow 2^n \times k$

SC $\rightarrow k \times n$

k is avg length of every combination
 n is no. of combinations

Note: Array must be sorted beforehand.

$\{ f(\)$

$\{ \text{if } (\text{target} == 0) \{$
 $\text{ans.push_back } (\text{temp}); \text{return};$

$\}$
 $\} \text{for (int } i = \text{index}, i < \text{arr.size()}, i++)$

$\{ \text{if } (i > \text{index} \text{ and } \text{ans}[i] == \text{arr}[i - 1]) \text{ continue};$

$\{ \text{if } (\text{arr}[i] > \text{target}) \text{ break};$

$\}$

Books code is pta hinhak

Subset Sums →

Given an arr of N integers, print sums of all subsets in it.

$$N=2 \quad \text{arr}[] = \{2, 3\}$$

O/P → 0 2 3 5

$$N=3 \quad \text{arr}[] = \{5, 2, 1\}$$

O/P → 0 5 2 7 1 3 6 8

0 1 2 3 5 6 7 8

(Ascending order)

For $n=3$, there are 2^3 subsets, i.e., 8

So, SC $\rightarrow 2^n$ always

Brute Force

Generate all subsets using Power Set,

Power set algo takes $2^n \times N$ Time to generate all the subsets,

It uses bit manipulation

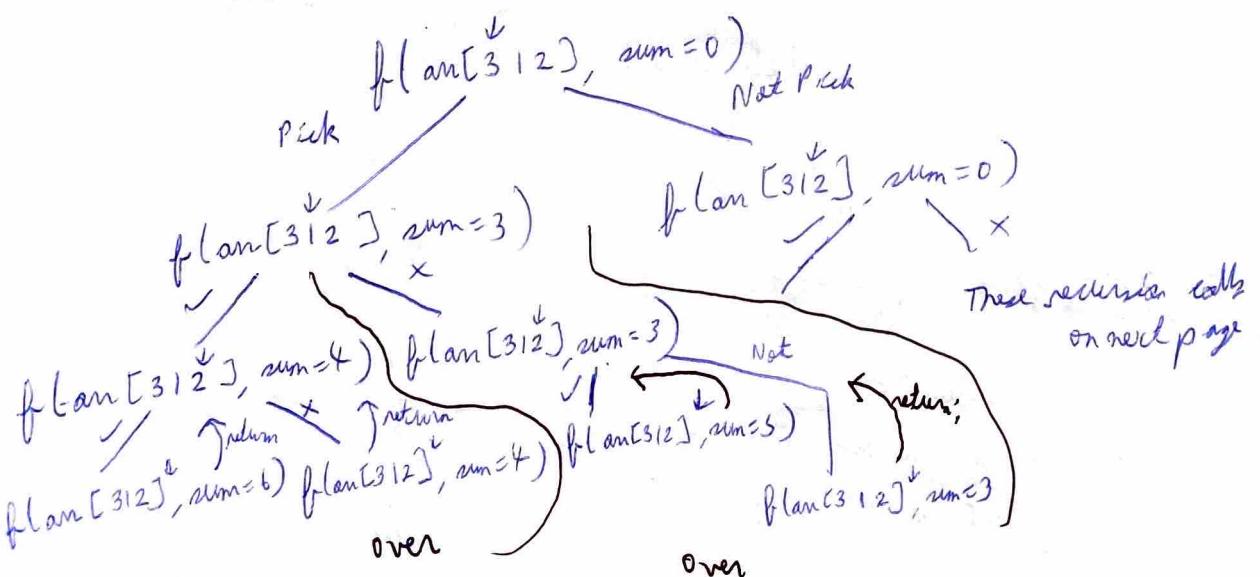
Optimal

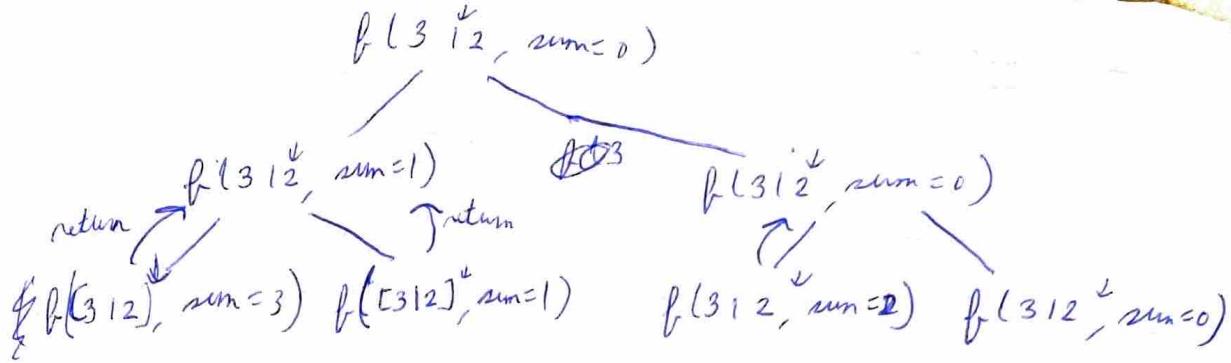
We have 3 indices $\overline{0 \ 1 \ 2}$, if we decide

which index to choose in subset, and which not to select

$$\text{e.g. } \begin{matrix} \checkmark & \times & \checkmark \\ 0 & 1 & 2 \end{matrix} \rightarrow [5, 1]$$

$$\text{e.g. } \begin{matrix} \checkmark & \checkmark & \times \\ 0 & 1 & 2 \end{matrix} \rightarrow [5, 2]$$

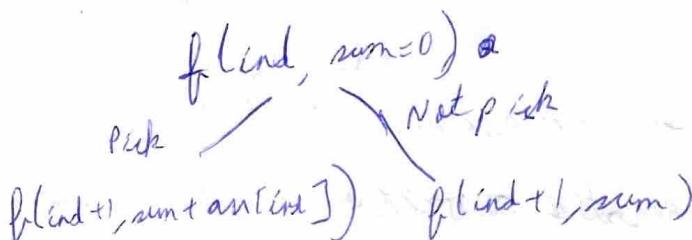




Now, all 8 subset sums got i.e.

$[6, 4, 3, 3, 1, 2, 0]$

Sort in increasing before returning



Base Case if ($\text{ind} == n$)

Push sum in ans array

T.C - For every index we have 2 ways, $\approx 2^n$

Subsets II

Given an integer array `nums` that may contain duplicates, return all possible subsets (the power set).

The solution set must not contain duplicate subsets.
Return the solution in any order,

E.g 1 - $[1, 2, 2]$

O/P $\Rightarrow [[], [1], [2], [1, 2], [2, 2], [1, 2, 2]]$

E.g 2 - $\text{nums} = [0]$

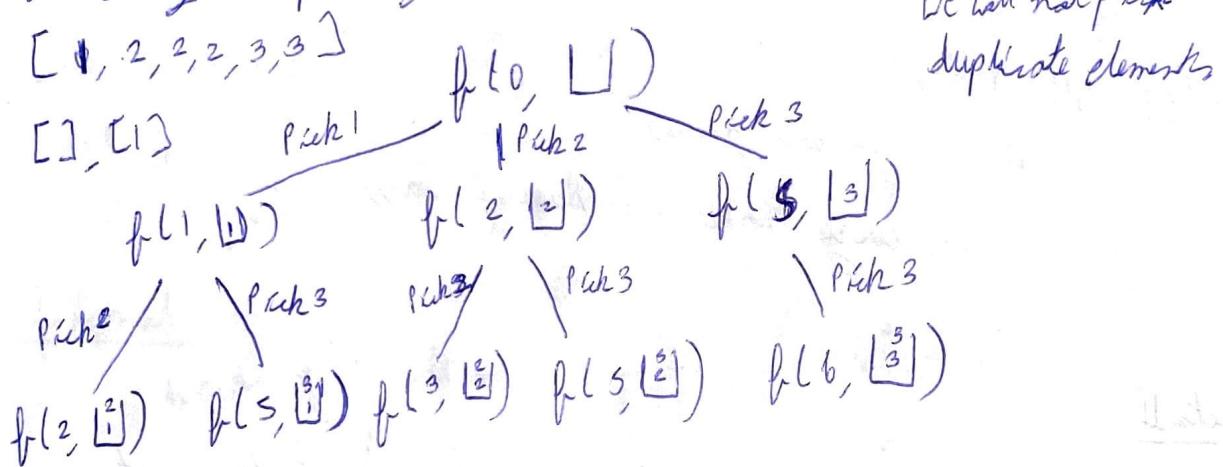
O/P $\Rightarrow [[], [0]]$

Brute Force → We will use the pick & don't pick technique to generate 2^n subsets.
 To remove duplicates we will put the subsets in a set first. Then later store in vector <vector>. Extra time and space used to convert set to vector<vector>.

$$TC = m \times \log m \quad \text{where } m = 2^n$$

Optimal →

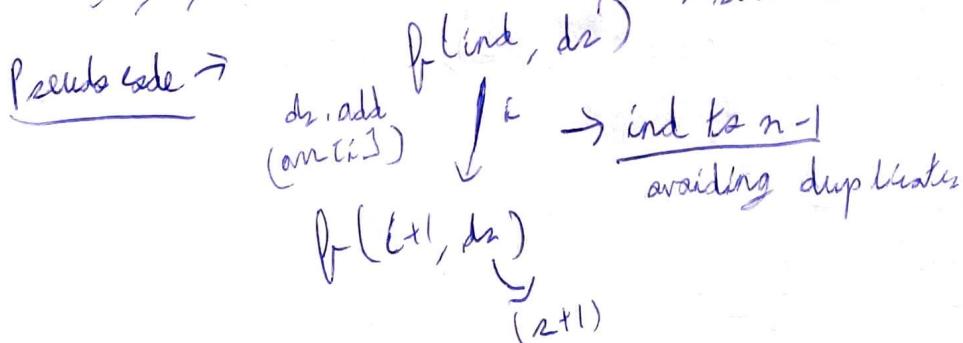
We have to write recursion in such a way such that at every step it generates a vector.



First recursion call generated all subsets of size 1, i.e. $[], [1], [2], [3]$
 Second generated subsets of size 2 i.e. $[1, 2], [1, 3], [2, 3]$

Further more recursion calls will generate subsets of size

3, 4, 5 & 6



We will skip recursion calls for duplicate elements
if ($i > \text{end}$ && $\text{arr}[i] == \text{arr}[i-1]$) continue;

$$TC \rightarrow 2^n \times n$$

$$SC \rightarrow 2^n \times k \quad - (\text{avg length of subsets}) \\ + O(n)$$

Palindrome Partitioning

Given a string s . Partition s , such that every substring of the partition is a palindrome. Return all possible palindrome partitioning of s .

E.g 1 $s = "aab"$

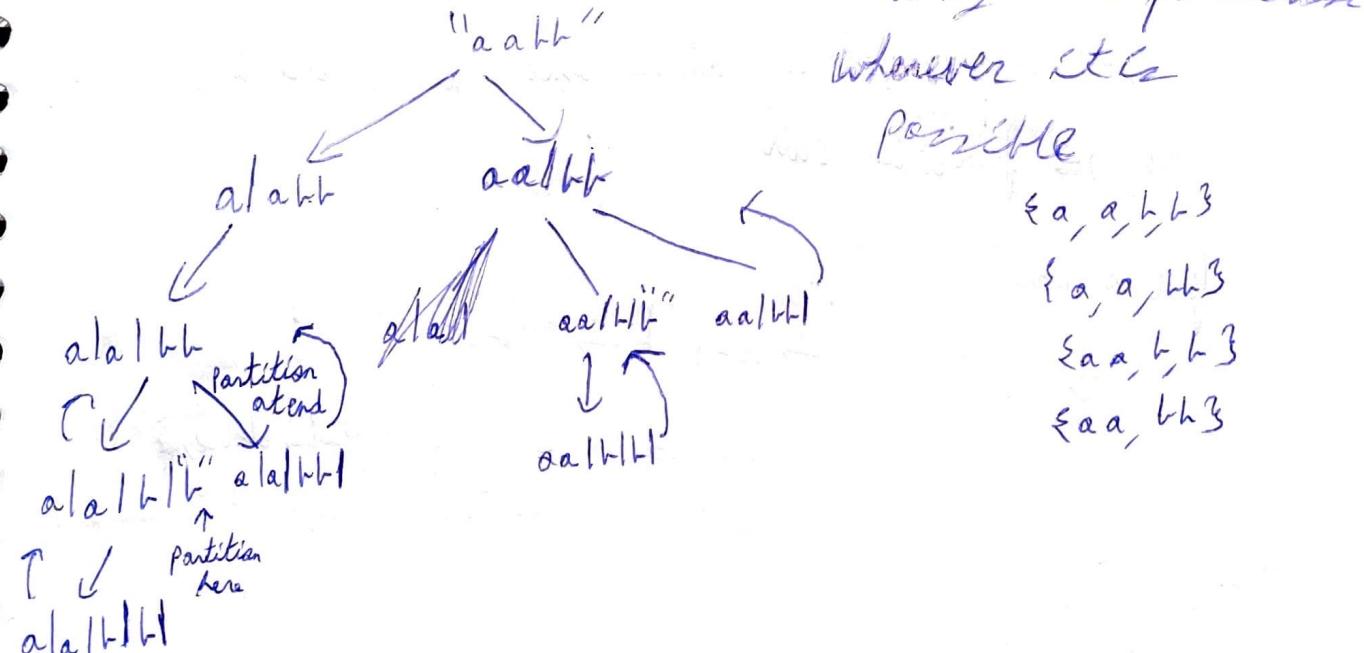
O/P $\rightarrow [[["a"], "a", "b"], ["aa", "b"]]]$

Eg 2 $s = "a"$

O/P $\rightarrow [[["a"]]]$

Approach: "aab"

Whenever we do a partition, every substring has to be a palindrome
We try to do partition wherever it is possible



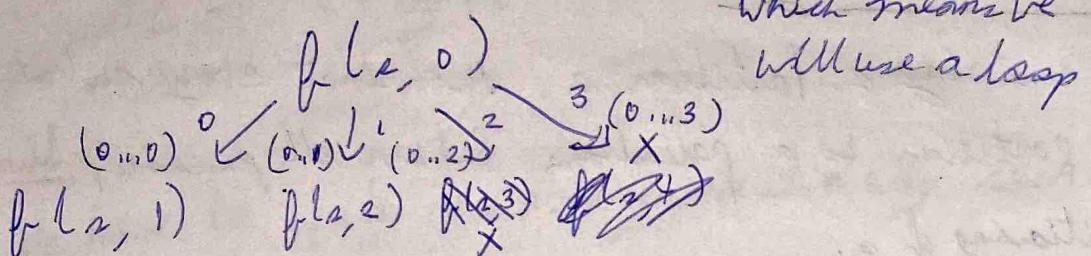
i.e. $\{a, a, b, b\}$ is a partition where every substring is a palindrome

Whenever we are able to ~~get~~ partition in the end of substring, we will know that we have generated palindromic substrings.

So, basically whenever pointer reaches end, we return.

"a[a] b[b]"

We are trying to partition at all these positions initially.



Before ~~go~~ making a function call we will check whether it is a palindrome or not.

E.g. - We will check $(0\dots 0)$ is a palindrome or not

Yes it is, so make a function call (a)

$(0\dots 1)$ is also a palindrome i.e. (aa)

$(0\dots 2)$ is not a palindrome i.e. (aah)

So, no function call

$(0\dots 3)$ is also not a palindrome i.e., (aabb)

So, no function call

$f(n, \text{ind})$

$\downarrow \text{loop } \text{ind} \rightarrow n-1 \rightarrow \text{using } i \text{ variable}$

$f(\text{ind} \dots i)$ is a palindrome
then & generate f^* 'n call

Base Case \Rightarrow

if ($\text{ind} == n$)
return;

base is Palindrome (string s, int start, int end) {

 while (start <= end) {

 if (s[start + 1] != s[end - 1]) {
 return false;

 }

 return true;

}

f()

{

// base case

for (int i = index; i < n.size(); i++)

 if (isPalindrome(s, index, i)) {

 dr.push_back(s.substr(index, i - index + 1));

 // del(i+1, ...)

 dr.pop_back();

}

g

Kth Permutation Sequence

Set $\{1, 2, 3, \dots, n\}$ contains a total of $n!$ unique permutations.

For $n=3$, permutations in order are:

"123" "132" "213" "231" "312" "321"
1 2 3 4 5 6

Given n & k . Return k^{th} permutation sequence

Ex. 2 - $n=3$, $k=3$

O/P \rightarrow "213"

Ex. 2 - $n=4$, $k=9$

O/P \rightarrow "2314"

"1234"

Brute Force - $n=4$, $k=14$

Use recursion to generate all the permutations

Store it in a vector, then sort the vector

k^{th} permutation will be at the $dr[k-1]$ index

$T.C = n!$ to generate all permutation

Θn^2 to store copy of permutation

$\Theta n \log N$ to sort

Optimal Solution →

$$n=4, k=18$$

For $n=4$, 2^4 permutations are possible

$$4 \rightarrow (1, 2, 3, 4)$$

Starting permutation with $1 \rightarrow 1 + (2, 3, 4)]^6$

" " " " $2 \rightarrow 2 + (1, 3, 4)]^6$

" " " " $3 \rightarrow 3 + (1, 2, 4)]^6$

" " " " $4 \rightarrow 4 + (1, 2, 3)]^6$

$\underline{24}$

No. of permutations that start with 1 will be 6, \Rightarrow this is same for other no.s also.

1st permutation $\rightarrow 1 \ 2 \ 3 \times \ (0^{\text{th}})$

↓

↓

↓

↓

Last permutation $\rightarrow 4 \ 3 \ 2 \ 1 \ (23^{\text{rd}})$

So, we can say that,

Permutations starting with 1 $\rightarrow 6$ permutations i.e. (0-5)

" " " " $2 \rightarrow 6$ " i.e. (6-11)

" " " " $3 \rightarrow 6$ " i.e. (12-17)

" " " " $4 \rightarrow 6$ " i.e. (18-23)

So, as $k=18$ is asked in question, we'll be looking for the 16^{th} permutation.

As $\frac{16}{6}$ th permutation lies b/w (12-17), meaning it is starting with 3 - - -

We can also determine this by $\frac{\text{index}}{6}$ = $\frac{16}{6}$ th permutation / 6 permutations
= 2nd index of initial arr, (k/fact)

i.e., $\underbrace{[1, 2, 3, 4]}_{0 \ 1 \ 2 \ 3}$
↑

This will be the starting digit, i.e., 3

Now from the 6 permutations in (12-17) range we have to find $16 \% 6 = 4^{\text{th}}$ permutation (k%ofact)

→ Now, we have to figure out which is the 4th permutation in {1, 2, 4, 3}, K = 4 now

As we can see we are at the initial condition again (earlier we were finding 1, 2, 3, 4 & K = 16th)

We will follow the same procedure as above

$$\{1, 2, 4, 3\}, K = 4$$

Now, Permutation starting with 1 $\rightarrow 1 - -$ [2 permutations of 0-1]

$$\begin{array}{lll} " & " & 2 \rightarrow 2 - - \left[" " \right. \text{i.e., } 2-3 \\ " & " & 4 \rightarrow 4 - - \left[" " \right. \text{i.e., } 4-5 \end{array}$$

Total = 6 permutations

Now, we can see that K = 4 lies in 4-5th permutation i.e., starting with 4.

We will calculate it using this $\rightarrow \frac{\text{index}}{2} = \frac{4}{2}$ permutations = 2nd index in $\underbrace{\{1, 2, 4, 3\}}_{0 \ 1 \ 2}$

Now, we have figured out 2 digits

$$3 \ 4 - -$$

Now, value of k left $\Rightarrow \frac{4}{4} \% 2 = 0$
($k=k \% \text{fact}$)
→ Now we have a set of $\{1, 2, 3\}$ — we have to find 0^{th} permutation
 $\{1, 2, 3\} \quad k=0$

Permutation starting with 1 $\rightarrow 1 -]^1$ permutation i.e., 1

Permutation starting with 2 $\rightarrow 2 -]^1$ permutation i.e., 1

We can see, permutation will begin with 1.

We will calculate index = $\frac{k}{\text{no.of permutations}} = \frac{0}{1} = 0$
(fact)

Now, $k=0 \% 1 = 0$

→ Now we have to find $\frac{3}{3} \frac{4}{4} \frac{1}{1} -$
Again follow the same process.

3 4 1 2

TC $\rightarrow O(N) \times O(N)$

SC $\rightarrow O(N)$

Code \Rightarrow Find factorial of n & store 1 to n in an array;

Decrement k string ans = "";

while(true) {

 ans = ans + to_string (arr [numbers / fact]);

 arr.erase (arr.begin () + n / fact);

 if (arr.empty ()) break;

 k = k % fact;

 fact = fact / arr.size ();

}

Permutations

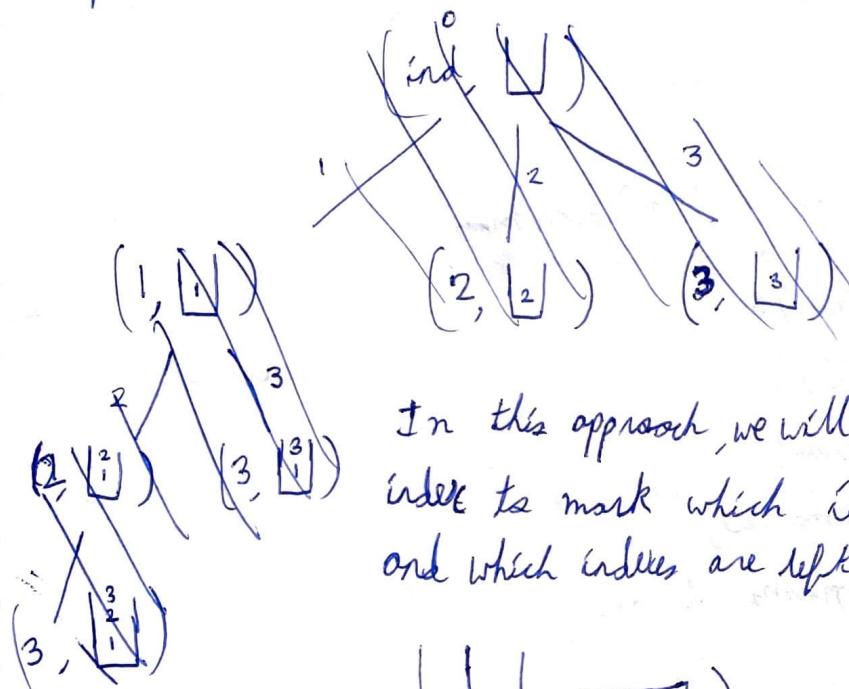
- Approach I

Given an array & num of distinct integers, return all the possible permutations. You can return the answer in any order.

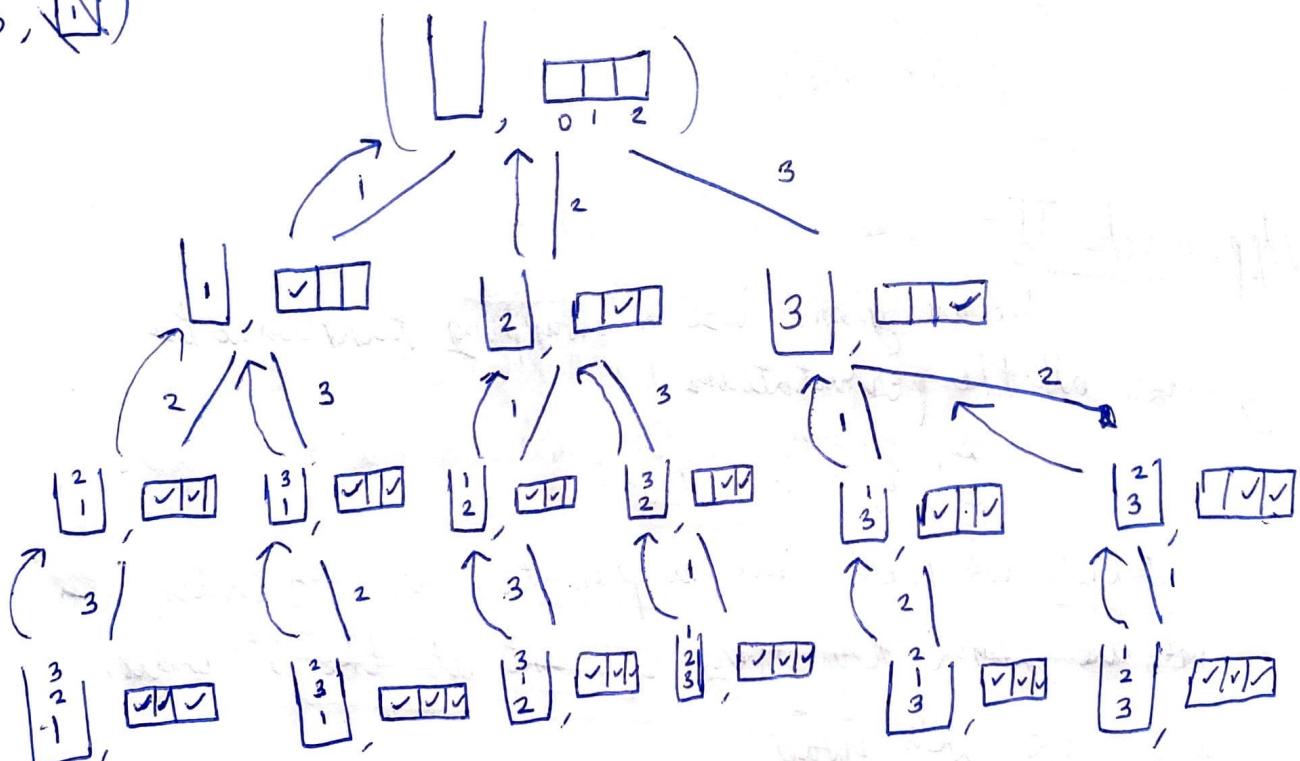
E.g - [1, 2, 3]

O/P $\rightarrow [1, 2, 3], [1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]$

- A First approach which comes in mind is to generate all permutations



In this approach, we will use a map instead of index to mark which index has been taken and which indices are left



When we return back pop the element from ds and then also unmark the index in the map.

$f(ds, map)$

↓
 $\text{loop } (0 - n-1)$

$ds.\text{add}(\text{arr}[i])$
 $\text{map}[i] = 1$

↓

$f(ds, map)$

Base case → $f(ds, \text{size} == n)$

$T.C. - n! \times n$

$S.C. \rightarrow O(n) \text{ for } ds + O(n) \text{ for } map$

Code →

```

for (i = 0 to n-1) {
    if (!map[i]) {
        mp[i] = true;
        ds.push(num[i]);
        solve(mp, num, ds, ans);
        ds.pop();
        mp[i] = false;
    }
}

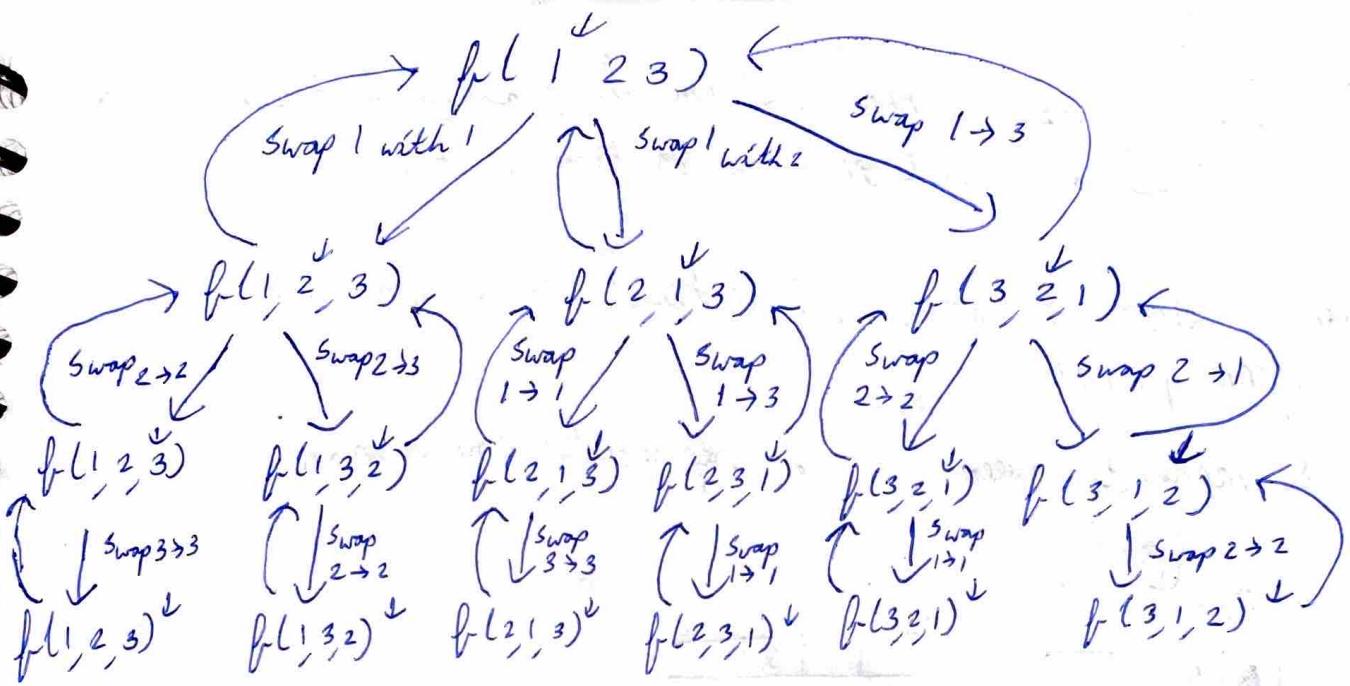
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Approach II -

We are gonna use a swapping technique to generate all the permutations.

$[1, 2, 3] \quad n=3$

First we will have a pointer at 0th index,
we are gonna try every element at the 0th index,
i.e. we will swap,



When we go back ~~to~~ swap ka reverse kar denge

Pseudo code →

```

f(ind, arr)
    loop i → (ind to n-1)
        Swap(a[ind], a[i]);
        f(ind+1, arr)
    Base case → if(ind == n)

```

TC → $n!$ permutations + $O(n)$ loop we are running
~~& n time to push permutation into ds.~~

So, $O(n!) \times O(n)$

SC → ~~$O(n^2)$~~ Auxiliary space for recursion depth

N-Queens →

n-queens puzzle is the problem of placing n queens on an $n \times n$ chessboard such that no two queens attack each other.

Given an integer n, return all distinct solutions to the n-queens puzzle.

→ 'Q' represents queen '.' represents empty space

Eg: $n=4$

Q	Q	.	.
.	.	Q	.
Q	.	.	Q
.	Q	.	.

.	Q	.	.
Q	.	.	.
.	.	Q	.
.	.	.	Q

O/P → $\left[\begin{bmatrix} \cdot & Q & \cdot & \cdot \\ \cdot & \cdot & Q & \cdot \\ Q & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & Q \end{bmatrix}, \begin{bmatrix} \cdot & \cdot & Q & \cdot \\ Q & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & Q \\ \cdot & \cdot & Q & \cdot \end{bmatrix} \right]$

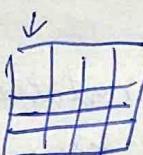
Approach:

Rules → 1) Every row $\rightarrow 1 Q$

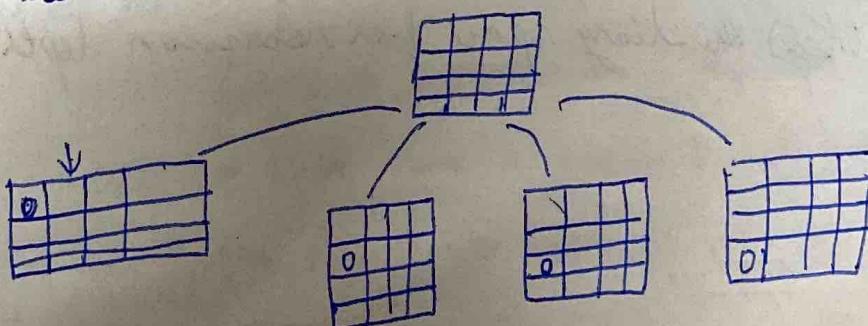
2) Every col $\rightarrow 1 Q$

3) None of the Q should attack each other

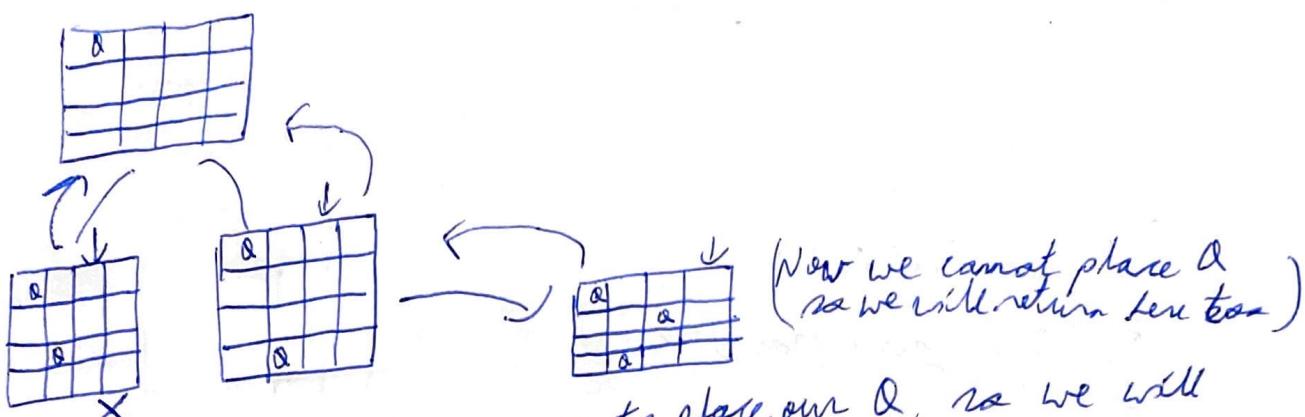
We will write a algo which will try to place Q in all the places

Starting from 1st column  We have 4 positions where we can

place Q in the first column.



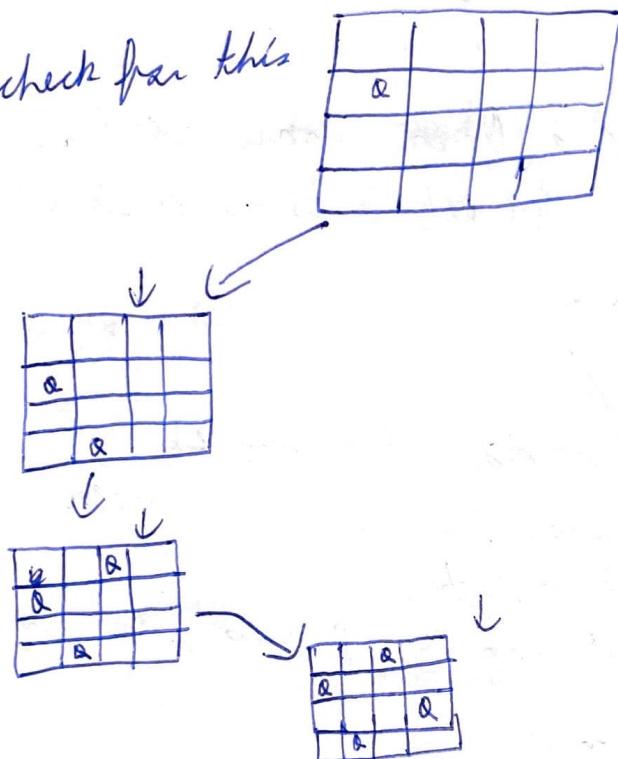
Now we move to col 2, and we will try possibilities. In first 2nd row Q cannot be placed, because Q in col 1 will attack Q at these locations. So, now we have 2 possibilities.



Now in col 3 we don't have any way to place our Q, so we will return back.

When we go back, remember to remove this Q from that index.

Now, we will check for this



Now the moment we see our pointer goes out, we have successfully placed Q at every position.

Store this answer in a ds, and ~~return~~ return.
While returning remove all Q's.

Similarly, we will get the other answer also.

Thought process → We are trying to fill up columns
So, we will have f(col) parameter

In $f(\text{col})$, we are checking every row from 0 to $n-1$, whether a Q can be placed or not.

f(col)
for (i=0 → n-1)

If (i can fill a) {

Fill a at given [row][col] = "a"

}

// Move to next col

f(i+1)

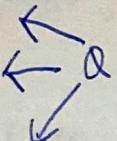
// When it comes back

remove a at given [row][col] = ""

}

Before placing a 'a' in board, we don't need to check in all directions, we only need to check its left.

i.e.



only these 3 directions will be checked

because in right we know there will be no queen as in recursion

we are moving left → right

and in col only one 'a' can be placed, so no need to check in the col also.

Sudoku Solver →

Write a program to solve a Sudoku puzzle by filling the empty cells.

A sudoku solution must satisfy these rules:

1. Each of the digits 1-9 must occur exactly one in each row

2. Each of the digits 1-9 " " " " " " " " " " col.

3, " " " " " " " " " " " " of the

9 3x3 sub-boxes of the grid.

The '.' character indicates empty cells.

E.g -

5	3		7					
6		1	9	5				
.	9	8			6			
8			6			3		
.	4		8	3			1	
7			2				6	
	6			2	8			
		4	1	9			5	
			8			7	9	

Approach:

Trying every possible way in empty index

- As we start traversing the board, we find an empty cell $[arr[0][2]]$. We first try to fill "1" in that cell. Before filling we check its entire row, entire col and 3×3 grid to which that cell belongs whether 1 exists ~~exists~~ anywhere or not. If it doesn't exist we will fill 1. Another possibilities which we can fill are 4, 2, *. So, we will fill each and then make separate recursion calls for each.
- Now, we go to the first recursion call, find next empty cell, consider all the possibilities. Second empty cell is $[arr[0][3]]$, nos. which we can put at this place are 2 & 6. We will put both separately and make a recursion call.

Then we go to next lower recursion call, i.e., a next recursion call per range. In this way, we will proceed.

Pseudocode →

```
for (i = 1 → n)
  {
    if (possible)
      {
        f() → recursion
      }
  }
```

```

    {
        for (i = 0 to 9) {
            {
                for (j = 0 to 9) {
                    if (b[i][j] == '.') {
                        for (char c = '1'; c <='9'; c++) {
                            if (isValid(b, i, j, c)) {
                                b[i][j] = c;
                                if (solve(b) == true)
                                    return true;
                                else
                                    board[i][j] = '.';
                            }
                        }
                        return false;
                    }
                }
            }
        }
        return true;
    }

```

```

bool isValid ( b , int row, int col, char c ) {
    for (i = 0 to 9) {
        if (b[row][i] == c) return false;
        if (b[i][col] == c) return false;
        if (b[3*(row/3)+i/3][3*(col/3)+i%3] == c)
            return false;
    }
    return true;
}

```

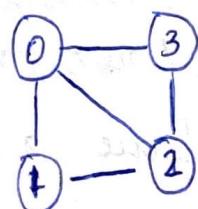
M - Coloring Problem →

Given an undirected graph and an integer M . The task is to determine if the graph can be colored with at most M colors such that no 2 adjacent vertices of the graph are colored with the same color. Here the coloring of a graph means the assignment of colors to all vertices. Print 1 if it is possible to color vertices and 0 otherwise.

$$\text{Eg: } N=4 \quad M=3 \quad E=5$$

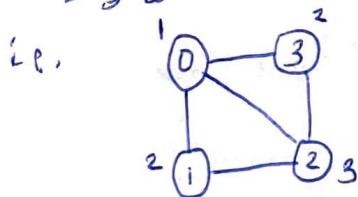
$$\text{Edges}[] = \{(0,1), (1,2), (2,3), (3,0), (0,2)\}$$

$$O/P \rightarrow 1$$



We have to check whether this graph can be colored with 3 colors?

We assign 1 to 0th node, 2 to 3rd node, 2 to 1st node & 3 to 2nd node



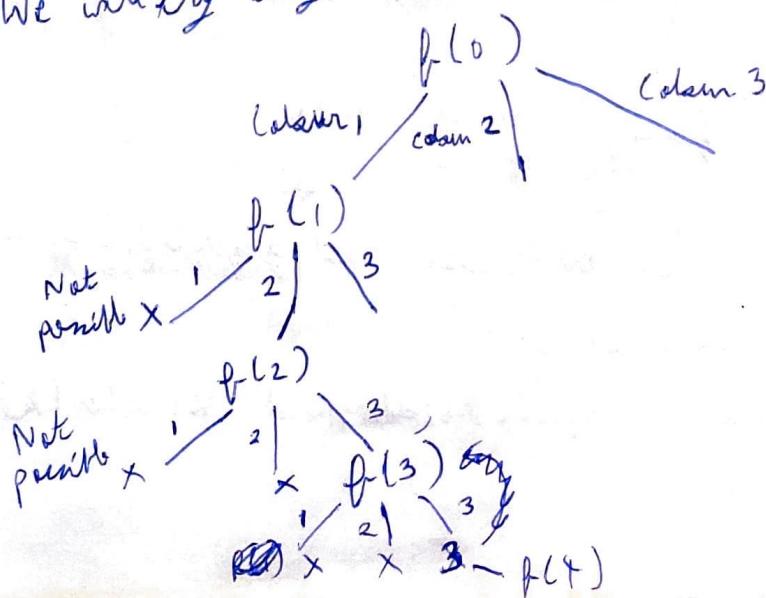
This way all nodes/vertices are colored and no adjacent vertices have same color.

Approach:

Ques states whether we can color a graph with at most M colors.

So, we will try every possible way.

i.e. We will try every color on every node.



- No. of nodes were n , whenever we reach n in recursion call, it will be our base case.
- Before assigning a colour to any vertex, we will check its adjacent vertices whether they have same colour or not.
- If they don't have same colour then we will colour the node and then move to next vertex.
- Whenever at each function call we will check for every possibility of 1 to M colours, so we are using a loop for this purpose.
- Whenever we reach base case we will return true. And whenever get our first true, we will not make further function calls and return true as the ans.

Pseudo code →

$$TC \rightarrow N^M$$

$$SC \rightarrow O(M) + O(N)$$

```

f(node)
{
    if (node == N)
        return true; // Base Case

    for (col = 1 → M)
        if (possible → v)
            color [node] = col;
            if (f(node+1) == T)
                return T;
            color [node] = 0;
    }
    return false;
}

```

```

bool possible (int node, int []color, bool graph [][][], int n, int col)
{
    for (k = 0; k < n; k++)
        if (k != node && graph [k][node] == 1 && color [k] == col)
            return false;
    return true;
}

```

Rat in a Maze Problem -1

Rat placed at $(0,0)$ in a square matrix of order $N \times N$.
 It has to reach destination $(N-1, N-1)$.
 Find all possible paths that rat can take to reach from Source to Destination.

Directions \rightarrow U D L R

D means blocked cell

I means not blocked

Note: - No cell can be visited more than once.

- If same cell is 0, the rat cannot move to any other cell.

E.g - $N=4$

$$m[4][4] = \{ \{1, 0, 0, 0\}, \\ \{1, 1, 0, 1\}, \\ \{1, 1, 0, 0\} \\ \{0, 1, 1, 1\} \}$$

1	0	0	0
1	0	1	
1	1	0	0
0	1	1	1

O/P \Rightarrow

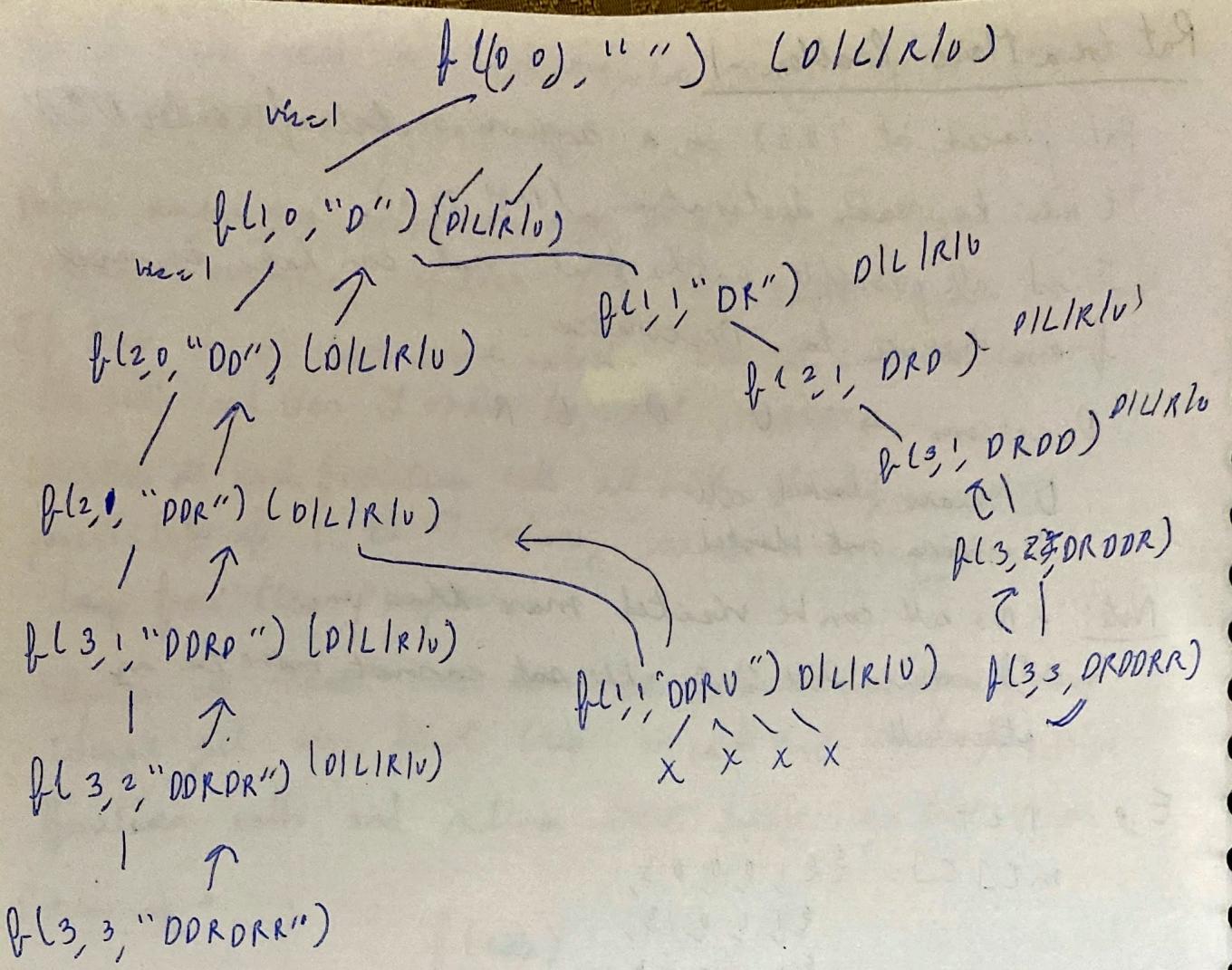
DRDPRR
DDRDRR

DDRDRR
PRDDRA

(Answer should be in Lexicographical order)

Approach -

- We start from Source i.e. $(0,0)$ and we try every possible path that we can go.
- Since answer should be in Lexicographical order, then we will also try directions in this order only i.e. D L R U
- We will also use a visited array. We will mark visited as 1 whenever we leave that cell and move to next recursion call. This is to ensure that we do not visit same cell twice.



We will print DDRDRR, wherever we reach dest.

Then we will return & unmark visited i.e. visit = 0.

Pseudo code →

```

f()
{
    down
    f()
    Left
    f()
    Right
    f()
    Up
    f()
}
  
```

T.C - $4^{n \times m}$ because at every cell we are trying 4 possibilities
 S.C - Depth of recursion tree

Selection Sort

As the name suggests, we select minimum

- 1) Traverse the entire array, determine the min element
- 2) Place the min in the first index

↓	13	46	24	52	20	9
	0	1	2	3	4	5

We find 9 is minimum, place 9 at index 0 & place
13 in 9's index.

(select min &
swap)

↓	9	46	24	52	20	13
	0	1	2	3	4	5

Now in 1 - 5 part of the array find min.

13 is the minimum, swap it with first index in this
array

↓	9	13	24	52	20	46
	0	1	2	3	4	5

This part of array is still unsorted, so
we will perform the same steps as above.

20 is min., swap with first index element

↓	9	13	20	52	24	46
	0	1	2	3	4	5

24 is min. now

↓	9	13	20	24	52	46
	0	1	2	3	4	5

46 is min. now

↓	9	13	20	24	46	52
	0	1	2	3	4	5

So, in 5 steps our array got sorted.
6 elements → 5 steps

So, basically what we are doing is →

- In the entire array figure out the minimum
- On whichever index the min appeared, swap it with the first index

Pseudo code →

```
for (i = 0 → n - 1) {  
    min = i;  
    for (j = 0 → n) {  
        if (a[j] < a[min]) {  
            min = j;  
        }  
        if (i == min)  
            swap (a[i], a[min]);  
    }  
}
```

TC → $O(n^2)$ → Best
→ Worst
→ Avg.

Bubble Sort

- * It pushes the maximum to the last, which is opposite of what selection sort was doing
 - Selection sort was pushing minimum to the front,
 - So, this pushes maximum to the last
- It uses adjacent swapping

[13, 46], 24, 52, 20, 9

(compares first 2 elements, whether $13 < 46$?)

Yes it is ✓

So, we do not do anything
Go to next two elements

13, [46, 24], 52, 20, 9

$46 < 24$ } , No, so swap them and move to next 2 elements

13, 24, [46, 52], 20, 9

13, 24, 46, 52, 20, 9
Swap

1st pass

13, 24, 46, 20, 52, 9

13, 24, 46, 20, 9, 52 This is sorted

After adjacent swaps, we can see that max elements reached the last "

Now, perform the same steps

2nd pass

13, 24, 46, 20, 9, 52

13, 24, 46, 20, 9, 52 → 13, 24, 46, 20, 9, 52
Swap

13, 24, 20, 46, 9, 52 Swap → 13, 24, 20, 9, 46, 52

This is sorted now

Again perform same steps

13, 24, 20, 9, 46, 52 → 13, 24, 20, 9, 46, 52 → 13, 20, 24, 9, 46, 52
Swap Swap

3rd pass

13, 20, 9, 24, 46, 52 This is sorted now

Again perform same steps

13, 20, 9, 24, 46, 52 → 13, 20, 9, 24, 46, 52 → 13, 9, 20, 24, 46, 52
Swap Swap

4th pass

This is sorted now

Again perform same steps

13, 9, 20, 24, 46, 52 → 9, 13, 20, 24, 46, 52
Swap Full array sorted now

5th pass

In 1st pass → 0 - n - 1

So, j loop runs from

2nd pass → 0 - n - 2

0 to $n-i-1$

3rd pass → 0 - n - 3

4th pass → 0 - n - 4

```

for (i = 0; i < n; i++) {
    for (j = 0; j < n - i - 1; j++) {
        if (arr[j] > arr[j + 1]) {
            swap(arr[j], arr[j + 1]);
        }
    }
}

```

TC - $O(n^2)$ - Worst complexity
Avg complexity

$O(N)$ - Best case if we do some optimizations

Optimized →

We will check whether swap was performed or not,

e.g. - 1 2 3 4 5 is sorted,

No swap will be performed in further iterations also so, we can maintain a bool swap = false earlier

Whenever we do swap, we do swap = true

And after getting out of inner loop, we check

if (swap == false) break;

Means no swap performed.

Insertion Sort →

→ Takes an element & places it in its correct position.

- The algorithm starts with looking at the first element as an array.

14	9	15	12	6	8	13
----	---	----	----	---	---	----

We will check whether 14 is at correct position in this array.

Yes it is, so we include next element.

14	9	15	12	6	8	13
----	---	----	----	---	---	----

Now, is 9 at correct position in this array?

No, so we swap

9 14 15 12 6 8 13

Now add 15 in array

9 14 15 12 6 8 13

15 is at correct pos, so no swap

Add 12 in array

9 14 15 12 6 8 13

12 is not at correct position

Right shift 14 & 15 & insert 12 after 9

9 12 14 15 6 8 13

Include 6

9 12 14 15 6 8 13

→ 6 9 12 14 15 8 13

6 9 12 14 15 8

→ 6 8 9 12 14 15 13

6 8 9 12 14 15 13

→ 6 8 9 12 13 14 15

; The array is sorted

- We are doing right shift and then inserting the element

in sorted array

T(-) O(n²) - Worst case
O(N) - Avg case

O(N) - Best case

SLT

for (j = 0; j < n-1; j++)

{

if (a[j] >= a[j+1])

while (j > 0 && a[j-1] > a[j])

{

swap (a[j-1], a[j])

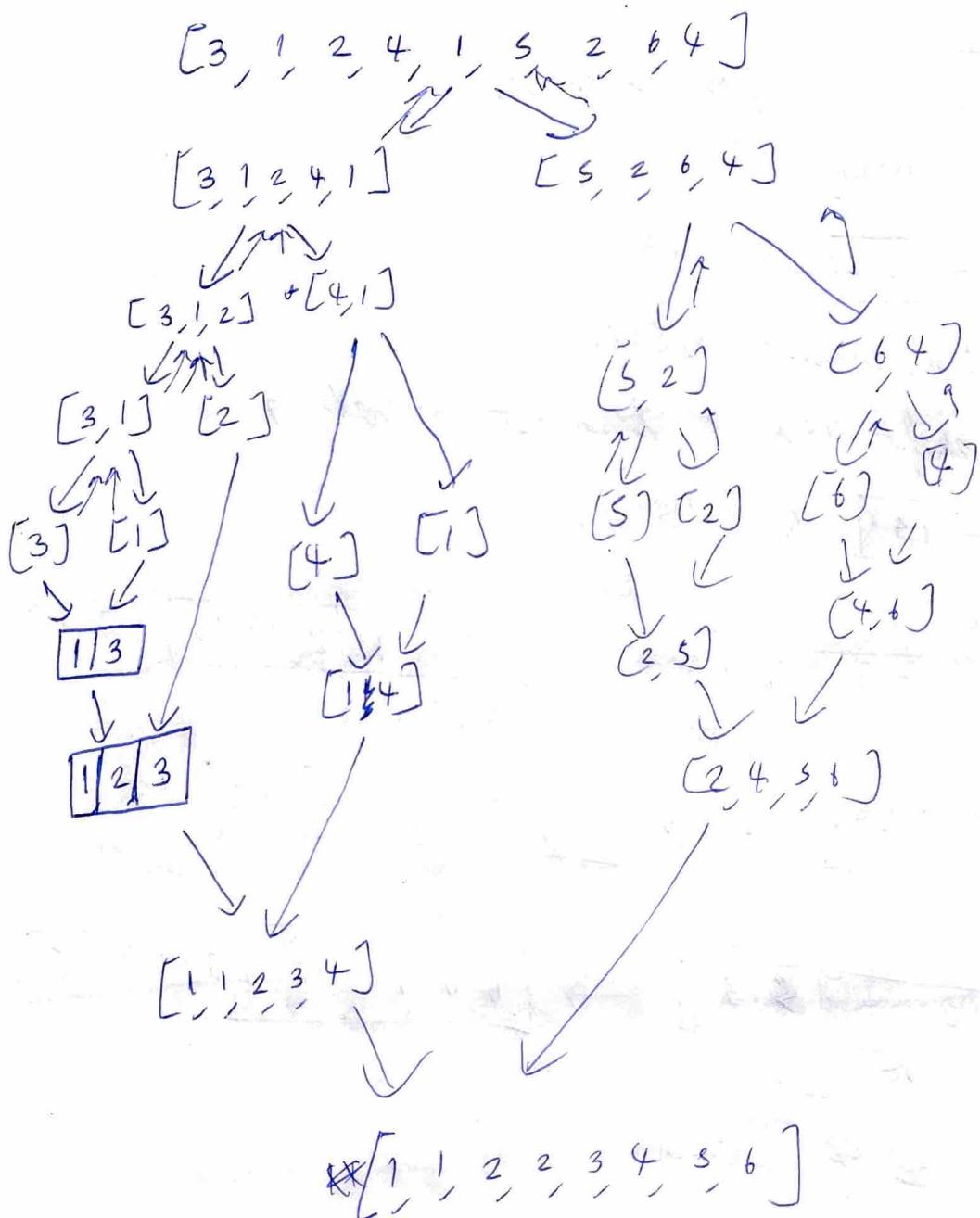
j = j - 1

}

3

Merge Sort

- Much more optimized sorting algs
 → Divide & Merge



Pseudocode →

```

mergeSort (arr, low, high)
{
    if (low >= high) return;
    mid = (low + high) / 2;
    mergeSort (arr, low, mid);
    mergeSort (arr, mid + 1, high);
    merge (arr, low, mid, high);
}
  
```

$T(n) = O(n \log n)$

Best Avg Worst

Quick Sort →

arr[] = [4 6 2 5 7 9 13]

T - O(N log N)

SC - O(1)

Step 1:

- Pick a pivot from the array & place it in its correct place in the sorted array.

Pivot can be: - 1st element in the array

Last element in the array

Median of the array

Random element of the array

Pivot

↓
[4] 6 2 5 7 9 13

Place it its correct place in sorted array

- Smaller on the left and larger on the right

[4] 6 2 5 7 9 13

2 1 3 4 5 6 7 9

We put 4 in its place, then we check its further elements if they are smaller we place them in left otherwise right
~~else~~ we place them according to how they occur in the array.

So after 1 step 1 element is at correct place in the array.
left & right portion ~~are~~ still unsorted

Now repeat these steps

[2 13] 4 [5 6 7 9]

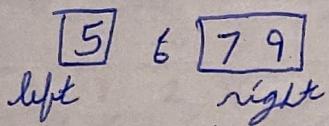
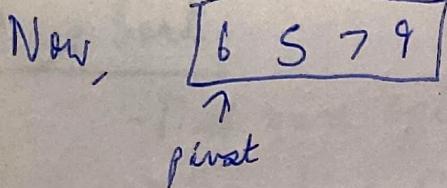
Let's take left portion first

↓
[2] 1 3

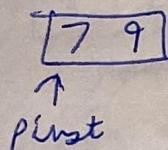
[1] 2 [3]
left right

Now again two smaller elements, now array have one element we don't need to do anything.

[1 2 3] is sorted.



S is a single element, so we don't need to do anything



Place it in correct position



Now, at the end

1 2 3 4 5 6 7 9

Pseudo code →

$low = 0$ & $high = n - 1$

\downarrow low	i	\downarrow $high$
4	6 2 5 7 9 1 3	j

$pivot = a[low]$

Now to place pivot at its correct position, take $i = low$ & $j = high$

Now, let's find 1st element which is greater than pivot

$if (a[0] > pivot) \rightarrow No$

$if (a[1] > pivot) \rightarrow Yes \quad i.e., 6 > 4$

i pointer stops here.

Now, we'll find first element smaller than pivot using j

$if (a[j] < pivot) \rightarrow Yes \quad i.e., 3 < 4$

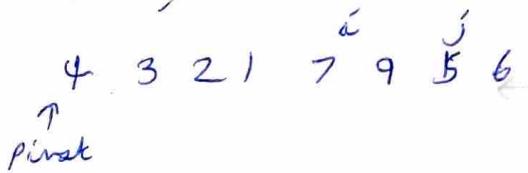
Now, swap both of them

$\overset{pivot}{\downarrow} \quad i$
4 3 2 5 7 9 1 $\overset{j}{\downarrow}$ 6

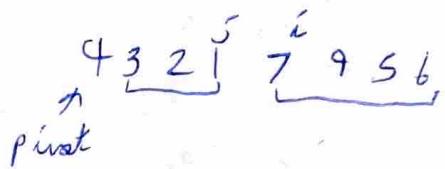
Now, again figure out the 1st element $>$ pivot & $<$ pivot.
Swap them

$\overset{i}{\downarrow} \quad \overset{j}{\downarrow}$
4 3 2 + 7 9 $\overset{j}{\downarrow}$ 5 6

Now, again find $a[i] > \text{pivot}$
 $j > i$, so i reaches?



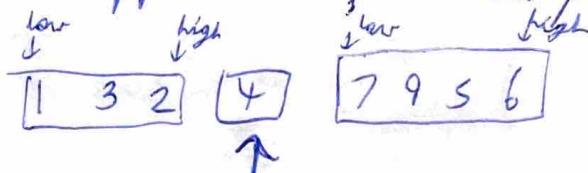
Now find $a[j] < \text{pivot}$



But here $j > i$, we'll stop we do not swap

Now j is in the territory where everything is smaller than pivot & i is " " " greater.

Since we stopped at i , we swap with pivot



This is the partition index

~~No high partition~~,

so, one array (low, partition - 1)

(partition + 1, high)

Perform quicksort again on this

Pseudo code \Rightarrow qsort (arr, low, high)

{
if (low < high)
{

partition = f (arr, low, high)

qsort (arr, low, partition - 1);

qsort (arr, partition + 1, high);

}

}

f (arr, low, high)

{
pivot = arr [low];
i = low, j = high

while (i < j)

{
while (arr [i] <= arr [pivot])
 && (i <= high) i++;

while (arr [j] > arr [pivot])
 && (j >= low) j--;

if (i < j) swap (arr [i], arr [j]);

}

Radix Sort

- Linear sorting algo
- We do digit by digit sorting

Implement Queue using Arrays →
front & rear ptn

```
push(x) {
    if (cnt == n) return -1;
    a[rear % n] = x;
    rear++;
}
```

```
top() {
    if (front == 0) return -1;
    return a[front % n];
}
```

Implement Queue using stack
We need min 2 stacks for this purpose

Push(n)

- $S_1 \approx S_2$
- $n \rightarrow S_1$
- $S_2 \rightarrow S_1$

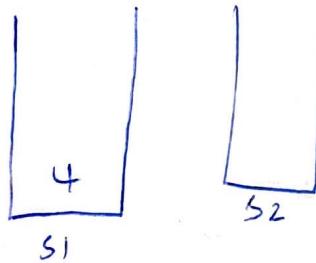
~~push(4)~~

~~push(3)~~

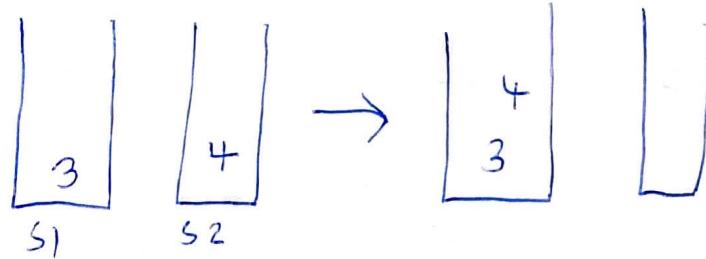
~~push(2)~~

pop(x)

$S_1, pop()$

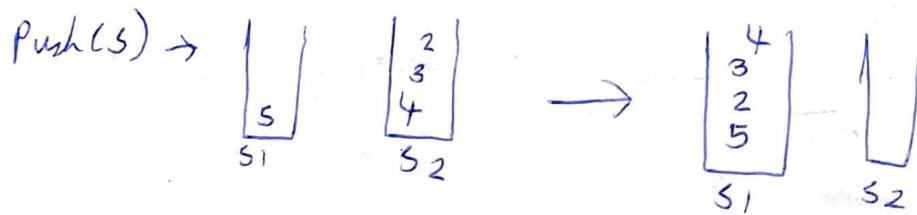
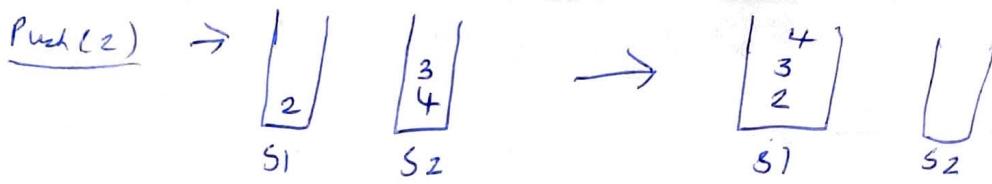


push(3)



pop(x) {

```
if (cnt == 0) return;
a[front % n] = -1;
front++;
cnt--;
}
```



top() \rightarrow 4 (Follows LIFO now)

pop() \rightarrow 4 gets popped

top() \rightarrow 3

TC $\rightarrow O(N)$

SC $\rightarrow O(2N)$

Optimal Soln:

In order to decrease TC of push operation,
we will take 2 stacks named Input & Output

push(x)

• Add x \rightarrow input

pop()

if (output not empty)
output.pop()

else

input \rightarrow output
output.pop()

top()

if (output not empty)
return output.top

else

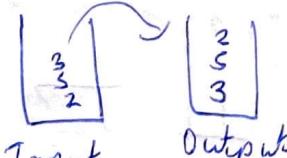
input \rightarrow output
output.top

Push(2)

Push(3)

Push(5)

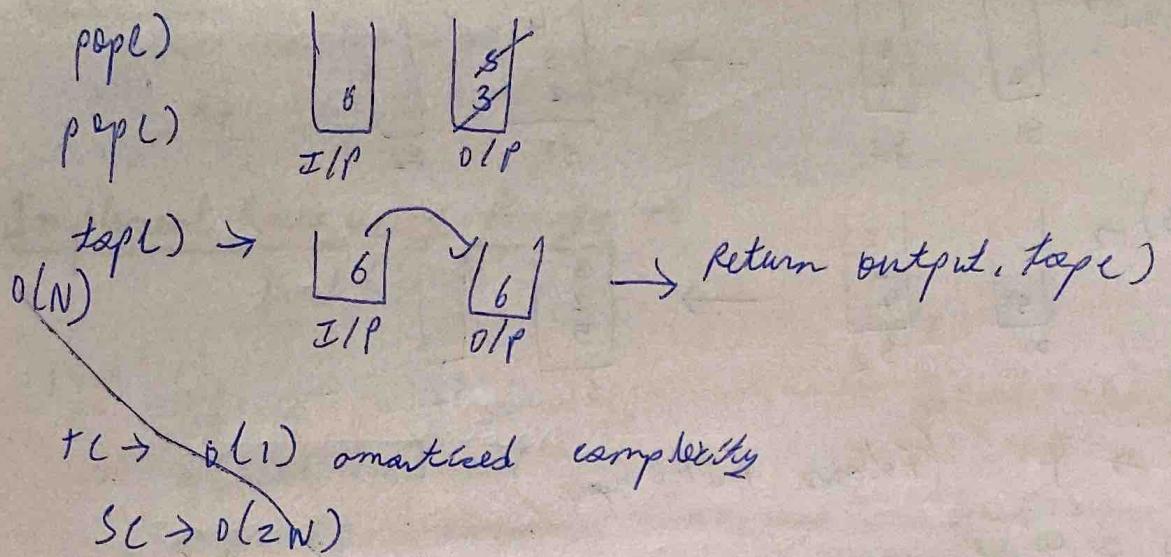


O(N) top() \rightarrow  \rightarrow return output.top()

pop() \rightarrow Output not empty, so pop it, O(1)

push(b)





Implement Stack Using Queue \rightarrow

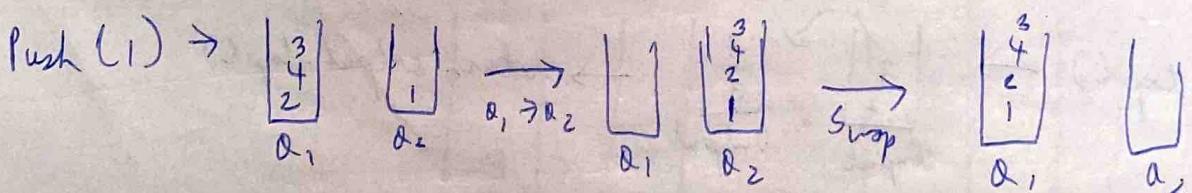
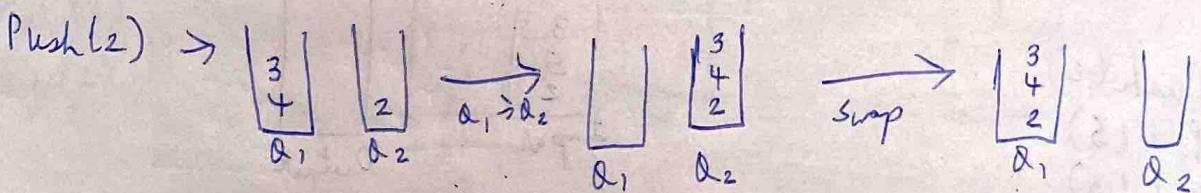
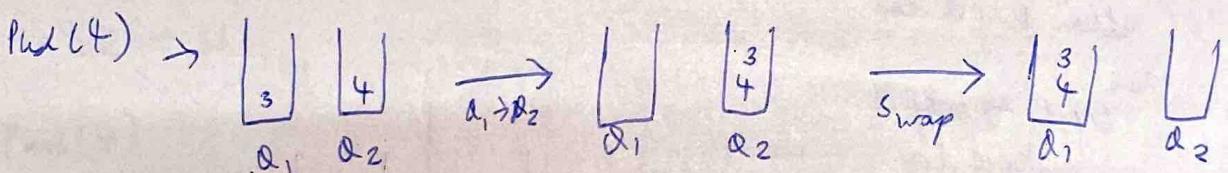
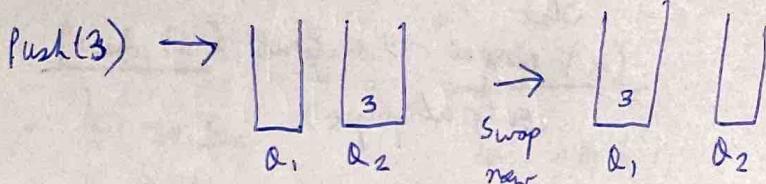
Take 2 queues

Steps \rightarrow

- $\text{push}(x)$
- Add $x \rightarrow Q_2$
- $Q_1 \rightarrow Q_2$ (element by element)
- Swap ($Q_1 \leftrightarrow Q_2$)

pop()

• Remove top of Q_1 ,



$\text{top}() \rightarrow$ 1 i.p. LIFO

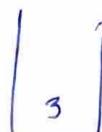
$\text{pop}() \rightarrow$ $\boxed{3}$ $\boxed{4}$ $\boxed{2}$
 Q_1 Q_2

$\text{top}() \rightarrow 2$

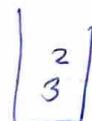
$T.C \rightarrow O(N)$
$S.C \rightarrow O(N)$

Optimal - Using Queue (Single)

Push(3) →



Push(2) →



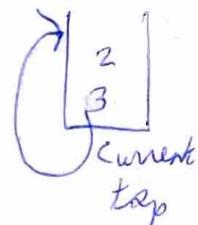
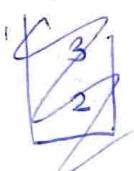
size = 2

Q1

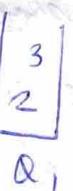
Q1

Now, as size = 2, one lesser than that, i.e. for 1 time
Take the top most element from queue & push it again.

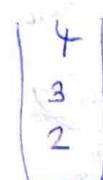
i.e.



→

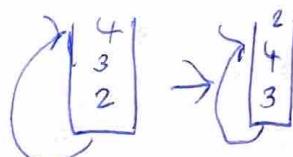


Push(4) →

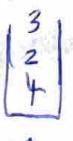


size = 3

So we will do above operation
2 times i.e. size - 1 times



→

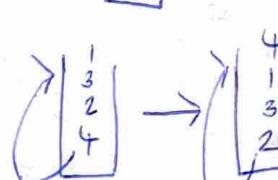


Q1

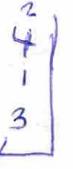
Now

size = 4, do 3 times

Push(1) →



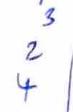
→



Q1

top() → top is pointing 1, i.e. it is following LIFO

pop() →



top() → 4

Valid Parentheses

$$s = () \{ \} () \quad \checkmark$$

$$s =] () \{ \} \quad \times$$

$$s = [) (] \quad \times$$

Approach:

- We will use stack

- Whenever we encounter an opening bracket, push it into the stack $\{ \}$

- Whenever we encounter a closing bracket, check if stack is not empty.

~~If it is not empty, then for the closing tag, bracket, there might be a opening bracket in stack~~

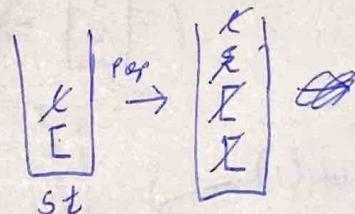
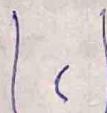
- Take the top element out, if they are corresponding, then pop it otherwise return false.

e.g - $[() \{ \} () \}]$

↑ ↑ ↑ ↑ ↑ ↑

if (st.empty ()) return true;

$() \{ \} \{ \}$



$Tc - O(N)$

$Sc - O(N)$

Next Greater Element

In an array of N integers, we have to figure out for every element which is the next greater element

3	10	4	2	1	2	6	1	7	2	9
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	-1	6	6	2	6	7	7	9	9	10

$$N = 11$$

$$\text{nge}[i] \geq 10 - 1 \quad 6 \quad 6 \quad 2 \quad 6 \quad 7 \quad 7 \quad 9 \quad 9 \quad 10$$

For last element i.e., 9, we will look from starting for its next greater

Brute Force-

Using 2 loops for $i = 0 \text{ to } n-1$
for $j = i+1 \text{ to } n-1$

if $a[i] > a[j]$

$\text{nge}[i] = a[j]$

else break;

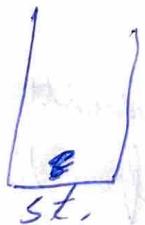
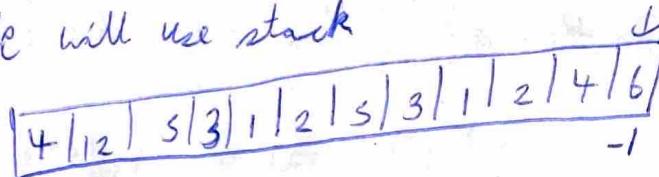
$\text{nge}[i] = -1$

~~TC~~ $= O(N^2)$

~~TC~~ -

Optimal Approach -

We will use stack



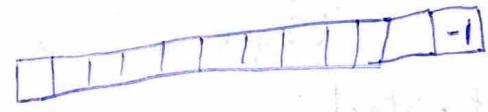
We will start from the last element.

- We will see whether there are elements $< b$ in the stack
- Right now, no element $< b$, so we push 6 in stack

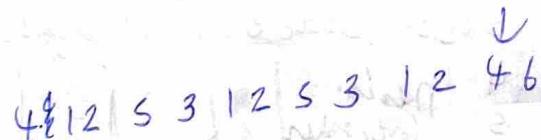
and put $\text{nge}[6] = -1$

Then do $i--$;

nge

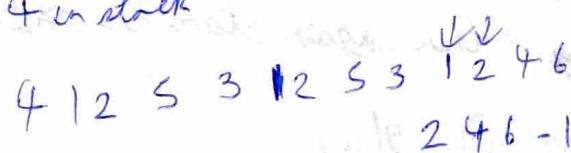


Now ele = 4.



* elements < 4 in stack, No

Since there are no lesser elements in the stack, take the topmost guy, which will be our next greater element
Then push 4 in stack

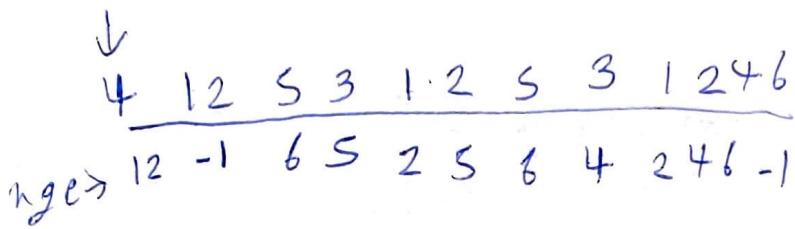


$\text{nge} \Rightarrow$

Now ele = 3, there are elements < 3 in stack

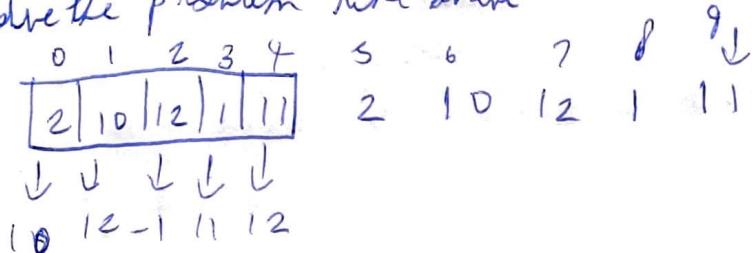
pop them until st.top < 3 .

Similarly, we will proceed in this way



Now for circular array →

We copy paste the portion once again, and then we will just solve the problem like above



We will not append another array, but we will use $i \% n$ for this purpose.

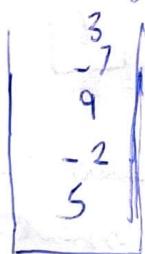
$$TC = O(2n + 2n)$$

$$\approx O(N)$$

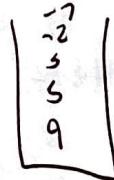
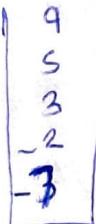
$$SC = O(N)$$

Sort a Stack

N integers



Sort stack in decreasing order



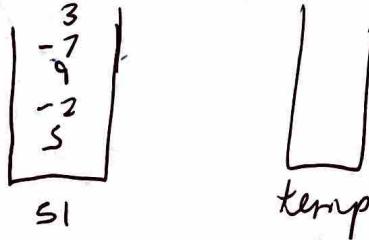
Brute Force → We copy the contents of stack in an array

then sort the array, then again store the ~~array~~ array in stack

$$TC = O(N) + O(N \log N) + O(N)$$

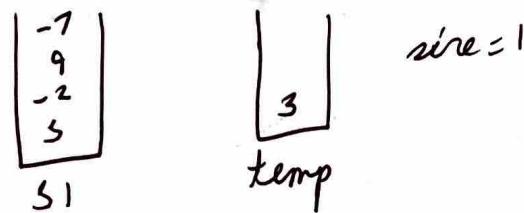
$$SC = O(N)$$

Better - what if we use two stacks
We will create a temp stack



If $\text{temp.empty}() \rightarrow \text{Push } S1.\text{top}() \text{ & } zl.\text{pop}$

In the temp array we will sort elements in ~~increasing~~ increasing order by doing a sorted insert
Before inserting we will compare $st.\text{top}()$

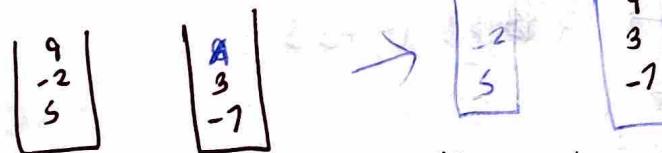


If $st.\text{top}() < \text{temp.top}()$

{
 while (size){
 ~~if~~ store elements in ^{another} temp array
 ~~or~~ ~~store elements in~~ temp array
 size--;
 temp.pop();

Now push $st.\text{top}()$ in temp & do $zl.\text{pop}()$

Now our stacks are



Follow above steps

on
9 3

Now again $-2 < 9$

Implement LRU Cache

Design a data structure that follows the constraints of a LRU cache.

- LRU Cache (int capacity) initialize the LRU cache with positive size capacity.
- int get (int key) Return the value of the key if key exists, otherwise return -1.
- void put (int key, int value) Update the value of the key if the key exists.

Otherwise, add the key-value pair to the cache. If no. of keys exceeds the capacity from this operation, evict the least recently used key.

E.g:-

size = 2

put(1, 1)

{ {1, 1} }

put(2, 2)

{ {1, 1}, {2, 2} }

get(1) → 1

put(3, 3)

→ But here our capacity is full, so we will remove Least Recently Used pair and insert this.

i.e., {2, 2}

{ {1, 1}, {3, 3} }

get(2) → -1

put(4, 4) → { {2, 2}, {4, 4} }

get(1) → -1

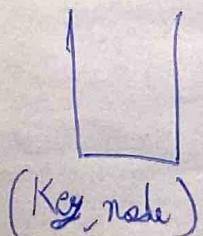
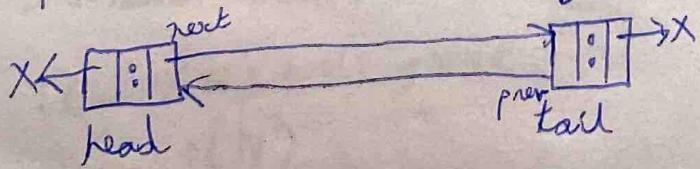
get(3) → 3

get(4) → 4

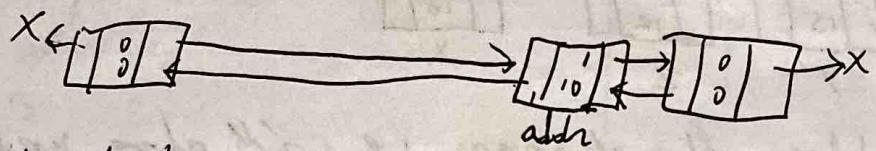
Do these functions in O(1) complexity

Approach

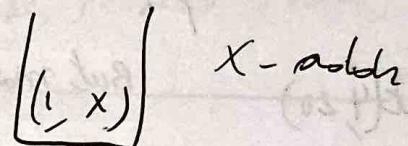
For this we need a doubly linked list & a hashmap



- $\text{Put}(1, 10)$ We will check whether $(\text{key}, \text{node})$ exists in hash map or not, If it doesn't then we will insert it right after head.
- Here, we are using doubly linked list to maintain order of how elements are coming, with this we can determine which is LRV element.

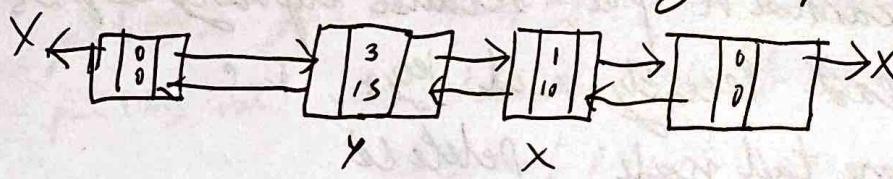


Take the key, and its addr. and store it in hash map

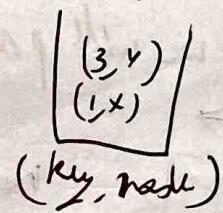


$\text{Put}(3, 15)$

Perform same steps as above. If it ~~exists~~ does not exist in map, then insert it right after head



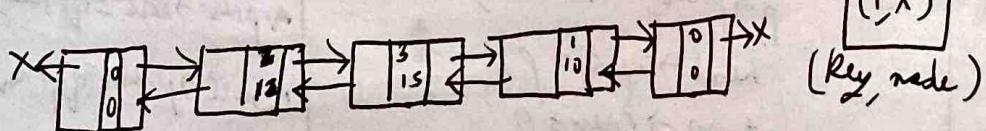
Store key, add in map



Similarly, now

$\text{Put}(2, 12)$

Perform same steps & insert at addr. 2



As we can see, elements are being inserted in the way they were used, i.e. least recently or most recently

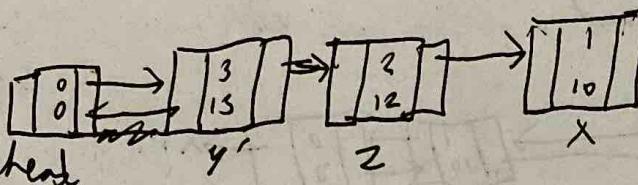
Now, get(3). We will check whether it exists in hash map or not.

Yes, it exists, we will go to the corresponding address and fetch.

Now, as we have used 3 most recently, we will also have to move it to a linked list. To move it

- Delete the node first, then
- Insert it right after head.

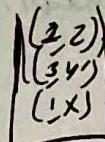
Now,



As its address also changes, we will also update in hashmap.

Put(4, 20)

But max size = 3



Step 1 - Check in hashmap if it exists or not. No it doesn't.

Step 2 - Check the size & current size cache, i.e., 3 & max capacity also 3.

So (4, 20) cannot be put, because capacity is full.

Step 3 - Remove Least Recently Used key, i.e., right before tail node. Delete it & do tail \rightarrow prev = tail \rightarrow prev \rightarrow prev

Code \Rightarrow Similarly we will proceed

Make a doubly LL \Rightarrow

```
class Node {  
    int key, val;  
    Node *next, *prev;  
    Node(int k, int v){  
        key = k;  
        val = v;  
    }  
};
```

```
void addNode(Node *p){  
    Node *temp = head  $\rightarrow$  next;  
    p  $\rightarrow$  next = temp;  
    p  $\rightarrow$  prev = head;  
    head  $\rightarrow$  next = p;  
    temp  $\rightarrow$  prev = p;  
}
```

```
void deleteNode(Node *p){  
    Node *p or prev = q  $\rightarrow$  prev;  
    Node *q  $\rightarrow$  next = q  $\rightarrow$  next;  
    q  $\rightarrow$  next  $\rightarrow$  next = q  $\rightarrow$  next;  
    q  $\rightarrow$  next  $\rightarrow$  prev = q  $\rightarrow$  prev;  
}
```

```

int get(int key) {
    if(mp.find(key) != mp.end())
    {
        Node *resNode = mp[key];
        int res = resNode->val;
        mp.erase(key);
        deleteNode(resNode);
        addNode(resNode);
        mp[key] = head->next;
        return res;
    }
    return -1;
}

```

```

void put(int key, int val)
{
    if(mp.find(key) != mp.end())
    {
        Node *existing = mp[key];
        mp.erase(key);
        deleteNode(existing);
    }
    if(mp.size() == cap)
    {
        mp.erase(tail->prev->key);
        deleteNode(tail->prev);
    }
    addNode(newNode(key, val));
    mp[key] = head->next;
}

```

LFU Cache (Least Frequently Used) Cache

We have to design a data structure with 2 functionalities i.e.,
get() & put().

get(key) → gets the value of key if it exists, else -1.

put(key, val) → update the value of key if present, or insert key if not present.

When the cache is full, it removes LFU one & if tie, then it removes LRU one.

Design in O(1) complexity

E.g. size=2

put(1, 10)

freq

1 → (1, 10) (2, 20)

put(2, 20)

1 → (2, 20)
2 → (1, 10)

get(1) → 10

→ (1, 10) & (2, 20) has
freq=1 for now.

→ As get(1) is called, its
frequency increases

put(3, 30)

Now our cache is full, as size=2, so we will remove the LFU guy. Here, LFU element is with freq=1.
i.e., we will remove (2, 20)

Now,

1 → (3, 30)
2 → (1, 10)

$\text{get}(2) \rightarrow -1$ $\rightarrow 2$ is not there in our cache
 $\text{get}(3) \rightarrow 30$
 ① $\rightarrow x$ \rightarrow freq of 30 will increase
 2 $\rightarrow (1, 10) (3, 30)$

~~put(4, 4) \rightarrow~~ Now cache size = 2, we will have to remove the LFU.

Here, LFU is 2 in this case, and with freq = 2, we have 2 elements.

So, here's a tie, ~~so~~ now we will remove the LRU.

$(1, 10)$ is LRU, so we will remove it

1 $\rightarrow (4, 4)$

2 $\rightarrow (3, 30)$

$\text{get}(1) \rightarrow -1$
 $\text{get}(3) \rightarrow 30$
 $\text{get}(4) \rightarrow 4$

1 $\rightarrow (4, 4)$

3 $\rightarrow (3, 30)$

2 $\rightarrow (4, 4)$

3 $\rightarrow (3, 30)$

Implementation \rightarrow In order to implement LFU, we will use 2 hashmaps

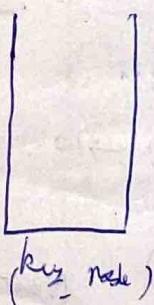
and a doubly linked list

$\text{map} < \text{freq}, \text{List}^{\rightarrow \text{DLL}} >$

$\text{map} < \text{key}, \text{Node}^* >$

capacity = 0

freq = 0



freqList maintains freq of list

keyNode will store the key & its address

freq variable tracks which is the least frequency guy
of now.

size = 3

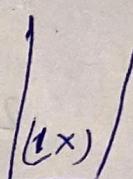
Put(1,10)

- Step:
- 1) Check whether it exists previously in keyNode map
 - 2) Check is there space in cache
 - 3) Now put it in cache

$1 \rightarrow \boxed{(1, 10)}$ DLL just like LRU

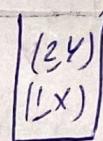
- 4) Update least frequency, i.e. freq = 1
& capacity = 1

5) Store (key, addr) in keyNode map



Put(2, 20) perform same steps

$1 \rightarrow \boxed{(2, 20)} \boxed{(1, 10)}$

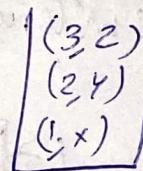


capacity = 2 & least freq = 1



Put(3, 30) perform same steps

$1 \rightarrow \boxed{(3, 30)} \boxed{(2, 20)} \boxed{(1, 10)}$



capacity = 3 & freq = 1

Put(4, 40) Now capacity == size

So, we will have to remove LFU

We will check what is the least freq using freq variable.

Here, freq = 1, which has 3 elements

So, we will remove LRU guy, which is the same way we did in previous ques. E.g. remove tail's prev.

*

$1 \rightarrow (3, 30) \quad (2, 20) \quad (\cancel{1, 10})$

Now put(4, 40)

1 \rightarrow $\boxed{(4, 40)(3, 30)(2, 20)}$

w z y

$(4, 4)$
 $(3, 2)$
 $(2, 4)$

get(3) \rightarrow 30
Yes it exists in key node map
update its freq

1 \rightarrow $\boxed{(4, 40)(2, 20)}$

w y

2 \rightarrow $\boxed{(3, 30)}$ Its address will make change

2

change freq

2 \rightarrow $\boxed{(2, 20)(3, 30)}$

y z

get(4) \rightarrow 40

2 \rightarrow $\boxed{(4, 40)(2, 20)(3, 30)}$

Now our freq $\rightarrow 1$ list becomes empty

We will update freq = 2

Put(5, 50)

1 \rightarrow $\boxed{(5, 50)}$

freq = 1 again

2 \rightarrow $\boxed{(4, 40)(2, 20)}$

put(2, 25)

1 \rightarrow $\boxed{(5, 50)}$

2 \rightarrow $\boxed{(4, 40)}$

3 \rightarrow $\boxed{(2, 25)}$

Code implementation is in LC 460. LRU Cache

Largest Rectangle in Histogram

Given an array of integers heights representing the histogram bars height where the width of each bar is 1, return the area of the largest rectangle in the histogram.

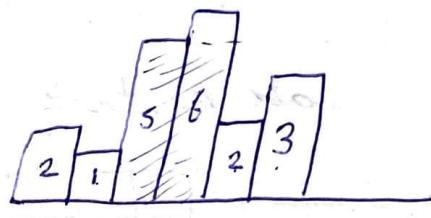
E.g) heights = [2, 1, 5, 6, 2, 3]

Output → 10

Largest rectangle is highlighted

height's width = 2

∴ area = $5 \times 2 = 10$



Brute Force -

We will try every histogram block as a particular rectangle.

If we take 2:  we cannot extend its width towards

right, as $i < 2$, so we will take width = 1 & height = 2

Area = 2, we will maintain this in max variable

If we take 1 as height, we will try to extend its width in both left & right direction.



As we can see max width we can achieve with height = 1, is 6,

so, $6 \times 1 \Rightarrow \text{Area} = 6$

To check till where we can extend width we will check if $\text{height}[j] \leq \text{height}[i] \rightarrow \text{width}++$

where j is index of elements to i's left & right.

Similarly, we will proceed and calculate max area

We will use 2 loops to find leftSmaller & rightSmaller

width = ~~rightSmallerIndex - LeftSmallerIndex + 1~~
rightSmallerIndex - LeftSmallerIndex + 1;

TC - $O(n^2)$

SC - $O(1)$

Better

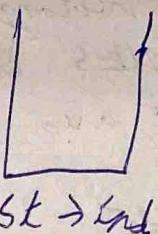
Now to ~~calculate~~ avoid finding left smaller & right smaller for each index, we will create a leftmax of size n .

Also create a stack which will store indices

leftdn \rightarrow

0	1	2	3	4	5	6

arr $\rightarrow [2, 1, 5, 6, 2, 3, 1]$

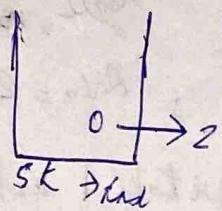


st \rightarrow ind

Start iterating from 0^{th} index, check in stack is there any greater element. Since, there is not, put index in stack & leftdn.

leftdn \rightarrow

0	1					
0	1	2	3	4	5	6



Now, ind = 1,

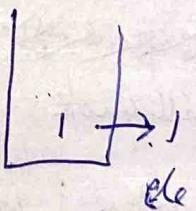
element \neq value in stack.

We don't want to store larger elements in stack, so we will pop out greater value, and empty our stack.

Now, on the left we don't have any smaller element < 1 , so push this in stack & update leftdn

leftdn \rightarrow

0	0	1				
0	1	2	3	4	5	6



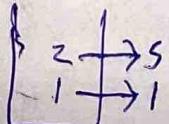
Now, ind = 2, value = 5

Check in stack, yes $de \leq value$, so we will add 1 to the index in stack and ~~not~~ store it in leftdn

~~Not 1st step~~

Our leftdn is representing boundary

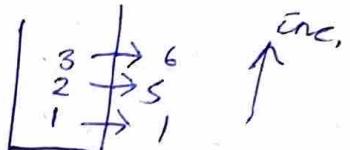
0	0	2				
0	1	2	3	4	5	6



Now, $\text{ind} = 3$, smaller will become s , which is on its left

left small \rightarrow

0	1	0	2	3			
0	1	2	3	4	5	6	



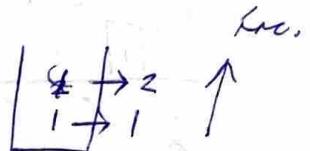
Now, $\text{ind} = 4$, $\text{val} = 2$, we will find next smaller by comparing in stack

Now, the element in stack
is smaller than curr val

So, update leftArr & stack

leftArr \rightarrow

0	1	0	2	3	2			
0	1	2	3	4	5	6		



Similarly,

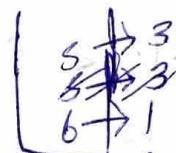
leftArr will become

0	1	0	2	3	2		s		2
0	1	2	3	4	5	6			

We will do similar thing to compute rightSmaller Array

rightArr \rightarrow

				s	s		6
0	1	2	3	4	5	6	



st \rightarrow val

We will start $\text{ind} - 1$ in rightArr &

compare whether elements in stack are greater or lesser,
if greater we will pop out. Because we have to find
smaller elements in right. The moment we find smaller
element, we will store its $\text{ind} - 1$ in rightArr.

So, our arr will become

rightSmall \rightarrow

0	1	6		3		3		s		6
0	1	2	3	4	5	6				

Now, calculate area for each index, and maintain a large element.

$$T \rightarrow O(N) + O(N) \approx O(N)$$
$$O(N) + O(N)$$

$$S \rightarrow O(N) \times 3$$

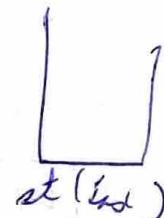
Optimal

We will convert previous 2 pass soln to 1 pass soln in this ~~other~~ method.

We were using 2 separate passes to calculate leftSmaller & rightSmaller.



$$\text{maxA} = 0$$



At $\text{ind}=0$, $\text{ele}=3$, st is empty, so store it in stack



At $\text{ind}=1$, $\text{ele}=1$, st is not empty,

compare it with element in stack.

$\text{ele} <$ element in stack

So, for 3, right smaller is index 1

Pop 3 from stack.

For $\text{index}=0$, right Smaller=1



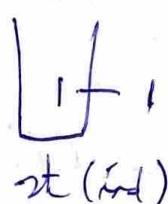
leftSmaller will ~~also~~ be 0, st(ind))

because we don't have anything on left.

So, whatever is rightSmaller, the total will become width.

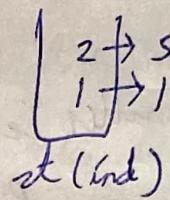
$$\text{So, } \text{maxA} = 1 - 0 * 3 = 3$$

put $\text{ind}=1$ in stack



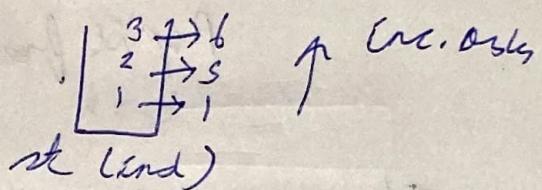
st.ind = 2, ele = 5

Compare with ele in stack, it is greater, so put it in stack



At ind = 3, ele = 6

Compare with ele in stack, it is greater, so put in stack



st.ind = 4, ele = 2

which is smaller than top of stack; in that case

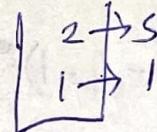
take the ~~go~~ index 3, and say its rightsmall = 4
& leftsmall is the element below top which is ind = 2

$$\text{So, maxArea} = 4 - 2 \cancel{+} 1 = 1 \times 6 = 6$$

Remove 6,

Compare a[ind] with top of stack again

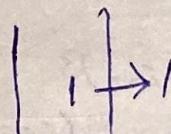
top is greater, which means for ind = 2, its rightsmaller = 4
leftsmaller = 1



$$\text{maxA} = \begin{matrix} \text{RS} & \text{LS} \\ 4 & -1 \\ = 2 & * 5 = 10 \end{matrix} \quad * \text{height} =$$

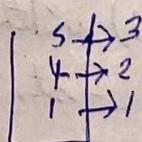
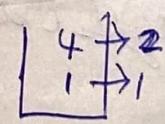
Pop it from stack

New, compare it again with ind = 4,
top is lesser, so push in stack



At ind = 5, ele = 3

top is lesser, so push in stack



Now, we will do one more iteration with $ind = 6$

We will take top of stack,

[3]

(5)

its right smaller is 6
itself

& left smaller is the element below top.

so, R.S = 6 & L.S = 4

$$\text{area} = 6 - 4 - 1 = 1 * 3 = 3$$

Pop it from stack

[4, 2]

Its right small is also 6 & left small = 1

$$\text{area} = 6 - 1 - 1 * 2 = 8$$

Pop it from stack

It's

[1, 2]

Its right small is also 6 & left small is 0

$$(6 - 0 - 1) * 1 = 5$$

TC = $O(n) + O(n)$

SC = $O(n)$

Code \Rightarrow

```
stack<int> st;
int maxArea = 0;
for (i = 0; i < n; i++) {
    while (!st.empty() && (i == n || heights[st.top()] > heights[i])) {
        int h = heights[st.top()];
        st.pop();
        int width;
        if (st.empty()) width = i;
        else width = i - st.top() - 1;
        maxArea = max(maxArea, width * h);
    }
    st.push(i);
}
```

Sliding Window Maximum

array of integers \rightarrow nums sliding window of size $\rightarrow k$
 which is moving from the very left of the array to the
 very right. You can see only k numbers in the window.
 Each time the sliding window moves right by one position.
 Return the max sliding window.

E.g -

[1 3 -1 3 5 3 6 7]	k=3	
<u>Window</u>	<u>Max</u>	
[1 3 -1]	3	
[3 -1 3]	3	
[-1 3 5]	5	
[3 5 3]	5	
[5 3 6]	6	
[3 6 7]	7	
o/p \rightarrow [3, 3, 5, 5, 6, 7]		

Brute force -

Using 2 loops -

for ($i = 0$ to $n - k + 1$)

{
 max = $a[i]$ }

 for ($j = i \rightarrow i + k - 1$)

{
 max = $\max(a[j], \text{max})$

}

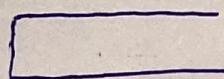
TC - $O(N * K)$

Optimal - Here, also we will use the concept of nge
i.e. next greater element.

But here we will store elements in decreasing order
In order to do so, we will use degree

1	3	-1	-3	5	3	6	7
0	1	2	3	4	5	6	7

$$k = 3$$



We are storing
order in degree



At $ind = 0$, degree is empty, so push it in degree

0

degree

At $ind = 1$, $val = 3$, as we have got 3, we will remove 0th index
from degree & push 1 index, because we need to store max
element. So storing 1 is useless, so pop it

degree

At $ind = 2$, $val = -1$

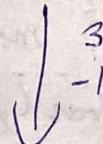
New, $val <$ element in degree
so, push it in degree

degree

At $ind = 3$, $val = -3$ which is less than -1
so, push it in degree

degree

Value is in decreasing



decreasing order,
is maintained

Now, everytime we reach $k=3$, we will keep storing max by looking at the front of the queue.

So, till now maxdn $\boxed{3 \ 3 \ | \ | \ |}$

Now, ind = 4, val = 5,

If we insert it in degree, it will disturb the decreasing order of our degree.

Also, in the boundary

2 to 4, index 1 should not be stored in queue as we can pop it

2 3

this is our degree with elements $\begin{matrix} -1 \\ -3 \end{matrix}$ dec.

Now, to not disturb the decreasing order, we will pop ind 2 & 3 from degree at push ind = 4

degree 4

~~3 deg~~ \downarrow 5 \downarrow dec.

start in maxdn $\rightarrow 3 \ 3 \ 5$

Now, ind = 5, val = 3

4 5

$\begin{matrix} 5 \\ 3 \end{matrix} \downarrow$ dec. (value)

decreasing order is maintained

start in maxdn $\rightarrow 3 \ 3 \ 5 \ 5$

Now, ind = 6, val = 6

If we insert it decreasing order will not be maintained, so pop elements from degree then insert.

6

maxdn $\rightarrow 3 \ 3 \ 5 \ 5 \ 6$

\downarrow 6 dec.

Now, ind = 7, val = 7

• Pop ind = 6, then push 7

7

maxdn $\rightarrow 3 \ 3 \ 5 \ 5 \ 6 \ 7$

\downarrow 7 dec.

Instruktion - We keep storing max elements in the beginning of the deque, and maintain decreasing order in the deque. If anywhere order is disturbed by a new element, we will pop previous elements in deque.

$$TC \rightarrow O(N) + O(N) \approx O(N)$$

$$SC \rightarrow O(K)$$

Code →

```
for (i = 0 → n-1) {  
    if (!dq.empty() && dq.front() == i-k) dq.pop_front();  
    while (!dq.empty() && dq.back() < num[i]) {  
        dq.pop_back();  
    }  
    dq.push_back(i);  
    if (i >= k-1) ans.push_back(dq.front());  
}  
return ans;
```

Min Stack

Design a stack that supports push, pop, top, and retrieving the min element in constant time.

Minstack() - initializes the stack object

void push(val) - pushes val onto stack

void pop() - removes element on top of stack

int top() - get top element of stack

int getMin() - retrieve the min element in the stack.

$$TC - O(1)$$

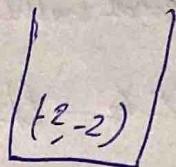
Approach 1 - We will store pair in the stack

where (val, min)

val represents current value & min represents minimum element till now

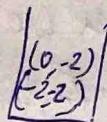
Push (-2)

Stack is empty so directly push



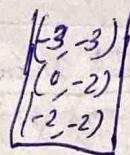
Push (0)

Push 0, but min remains same



Push (-3)

Push -3, min gets updated



get min() → return top(). second

pop() - pop out top

get min() return top(). second

T(-O(1))

S(-O(2N))

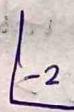
Approach 2

We will not store pair in stack

Maintain a min variable = INT_MAX

Push (-2) → Stack is empty, push -2

mini = -2



Push (0) → 0 > -2, don't do anything

push 0



mini = -2

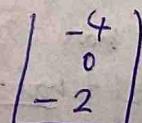
Push (-3) → -3 < -2, so now instead of pushing original

value, we will push (2 * val - prev mini) *

$$\text{L.e. } (2 \times -3 - (-2)) = -4$$

push -4 into stack

mini = -3 now



`getmin()` → return min element
which is -3

`top()` → -4 is not original value, this
is modified value, so to retrieve $\begin{bmatrix} -4 \\ 0 \\ -2 \end{bmatrix}$
original value we will check if $-4 < \text{min}$ i.e., $-4 < -3$
Yes, it is then we will simply return -3 as top.

`pop()` → Now, when we pop, min should go back to -2
In order to that, ~~we have~~ update min

$$\begin{aligned}\text{min} &= (2 \times \text{min}) - (\text{st. top}()) \\ &= 2 \times -3 - (-4) \\ &= -6 + 4 \\ &= -2\end{aligned}$$

min returns to -2

We will have to update min before popping

Before doing this we will check if -4 i.e. modified
value $< \text{min}$, then yes it confirms that it is a
modified value.

`top()` → 0, we know this is not $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$
modified value, because
 $0 > -2$ i.e. min

Intuition - Why is modified value always $< \text{min}$?

We know the value we were trying to insert before
modifying was -3, which was < -2

So, we are inserting $2 \times \text{val} - \text{min}$

$$\text{val} < \text{min}$$

$$\text{val} - \text{min} < 0$$

Add val to both sides

$$\text{val} + \text{val} - \text{min} < 0 + \text{val}$$

$$\text{Modified value } \leftarrow (2 \text{val} - \text{min}) < \text{val}$$

$\therefore \text{Modified value} < \text{mini}$

Intuition \rightarrow Why does $\text{mini} = 2 \times \text{mini} - \text{st. top}()$ works?

We inserted $(2 \times \text{val} - \text{mini})$ in stack

So, $\text{mini} = 2 \times \text{mini} - \text{st. top}()$ can be written as
 $= 2 \times \text{mini} - (2 \times \text{val} - \text{prev mini})$

We know that $\text{val} == \text{mini}$

$$= 2 \times \text{val} - (2 \times \text{val} + \text{prev mini}) \\ = \text{prev mini}$$

\therefore This reverts back to previous min

Rotten Oranges

max grid

0 represents empty cell

1 represents fresh orange

2 represents rotten orange

Every minute, any fresh that is 4-directionally adjacent to a rotten orange becomes rotten.

Return the min no. of minutes that must elapse until no cell has a fresh orange.

If it is impossible return -1.

$$\text{Ex - grid} = \begin{bmatrix} [2, 1, 1], \\ [1, 1, 0], \\ [0, 1, 1] \end{bmatrix} \quad 0/P \rightarrow 4$$

1 minute	2 minute	3 minute	4 minute
$\begin{bmatrix} 2, 2, 1 \\ 2, 1, 0 \\ 0, 1, 1 \end{bmatrix}$	$\begin{bmatrix} 2, 2, 2 \\ 2, 2, 0 \\ 0, 1, 1 \end{bmatrix}$	$\begin{bmatrix} 2, 2, 2 \\ 2, 2, 0 \\ 0, 2, 1 \end{bmatrix}$	$\begin{bmatrix} 2, 2, 2 \\ 2, 2, 0 \\ 0, 2, 2 \end{bmatrix}$

$$E_2 \rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$



This orange can never become rotten

$$\text{So, O/P} \rightarrow -1$$

Approach-

We will use a BFS technique

- 1) We will start counting, what is the no. of oranges we are currently having

$$\begin{bmatrix} 2 & 0 & 0 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 4 \quad \text{cnt} = 8$$

Initially store the rotten orange index in the queue.

$$\text{cnt} = 0$$

In first iteration, we perform BFS on elements in queue.

* Try to go in all 4 directions

$$x \leftarrow (0, 0) \rightarrow \begin{matrix} \times & \text{empty} \\ \uparrow & \end{matrix}$$

$$(1, 0) \downarrow \quad \text{put it in queue}$$

$$\begin{matrix} \times & \text{empty} \\ \uparrow & \end{matrix} \quad x \leftarrow (0, 3) \rightarrow \begin{matrix} \times & \text{empty} \\ \uparrow & \end{matrix}$$

$$(1, 3) \downarrow \quad \text{put it in queue}$$

$$\text{minutes} = \text{minutes} + 1;$$

$$\begin{bmatrix} (0, 3) \\ (0, 0) \end{bmatrix}$$

$$(i, j)$$

&

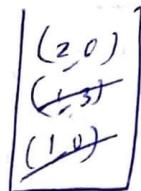
$$\begin{bmatrix} (1, 3) \\ (1, 0) \\ (0, 3) \\ (0, 0) \end{bmatrix}$$

Now take

$$x \leftarrow (1, 0) \rightarrow x$$



(2, 0) Put in queue



$$x \leftarrow (1, 3) \rightarrow x$$



(2, 3) Put in queue



if (!q.empty())

minute++;

Now take,

$$x \leftarrow (2, 0) \rightarrow (2, 1)$$

Put in q

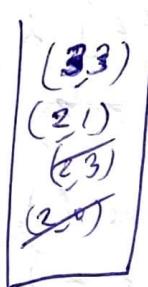


$$x \leftarrow (2, 3) \rightarrow x$$



(2, 3) Put in q

minute++;



Now take

$$x \leftarrow (2, 1) \rightarrow x$$



$$x \leftarrow (3, 3) \rightarrow x$$



X minute will remain same



minute = 3

We will also keep a track of no. of elements we inserted in queue. Here, we put 8 oranges in queue, which means we have made all oranges rot.

TC - $O(N^N) \times 4$

SC - $O(N \times N)$

Online Stock Span

Collect daily price quotes for some stock and return the span of that stock's price for the current day.

Span of stock's price in 1 day is max no. of consecutive days (starting from that day and going backwards) for which stock price \leq curr price

Eg:- In $[7, 2, 1, 2]$ & price of stock today = 2

Then span = 4, because starting from today

→ total 4 days stock price \leq curr price

In $[7, 3, 4, 1, 2]$ & price of today = 8

Then span = 3, because $Opt[2] < 8$

StockSpanner() - Initialization object of the class
 next(price) - Returns the span of the stock, given today's price

Approach:

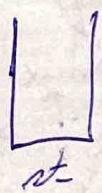
~~Method 1~~ We will use a stack & maintain a span variable

Initially stack is empty & span = 1

next(100)

As stack is empty, push 100

& return 1



next(80)

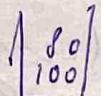
We will compare with top of stack, if top $<$ curr price



then we will pop & increment span

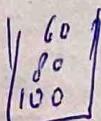
Here, span will remain 1

push 80



next(60)

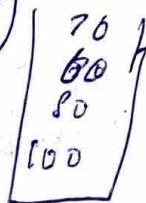
push 60 return 1



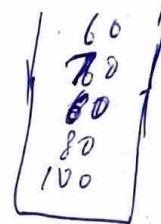
next(70)

$st.top() < curr \text{ price}$ (Remember we don't have to pop increment span
keep doing this until $st.top() \geq curr$)
At last span = 2 & push 10

& return span



next(60) initialize span = 1,
return span, push 60



next(75)
span = 4

This soln will give TLE because when we will keep taking $curr < ele$, then we will pop & store it in same temp stack.

We will have to use another loop to push all the elements again.

$$TC - O(N) + O(N)$$

$$SC - O(N) + O(N)$$

Optimal-

We will store price, span in stack

Eg $\text{next } \text{push}(100)$ span = 1

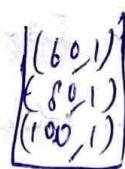


$\text{push}^{next}(80)$ span = 1

compare it with top just like prev approach



$\text{next } \text{push}(60)$ span = 1



$\text{next}(70)$ Now here span = 1

$st.top() < curr \text{ price}$, So, add $st.top()$ span with curr span. Span becomes 2

Pop (60, 1) & Push (70, 2)

(70, 2) represents there were 2 days at which price

$\begin{bmatrix} (70, 2) \\ (80, 1) \\ (100, 1) \end{bmatrix}$

$$c = 70,$$

new
~~stk(60, 1)~~

$\begin{bmatrix} (60, 1) \\ (70, 2) \\ (80, 1) \\ (100, 1) \end{bmatrix}$

new
~~stk(75)~~ span = 1

st.top() < curr.

Pop elements from stack until st.top <= curr
& add span

Stack (75, 4) in stack

SC - $O(2N)$

TC - $O(N)$

Celebrity Problem

Given N , which represents N people in a party

Each person has been assigned a unique ID b/w 0 to $N-1$

Identify the celebrity.

A celebrity is a person whom everyone knows

But celebrity does not know everyone

Note: There is a known A, B func which tells whether
A knows B, returns true / false,

A MXN matrix is given which tells whom knows whom

Eg: $N=3$ $\text{NEIGHBORS} = \{ \{ 010 \} \}$

$\begin{matrix} \{ 000 \} \\ \{ 010 \} \end{matrix}$

$011 \rightarrow 1$

Explanation: 0 & 2nd person both know 1, so 1 is celebrity

Brute Force Approach

As we can see celebrity knows no one, so celebrity now will have all 0's

i.e., $\begin{matrix} 0 & 0 & 0 \end{matrix}$

And all celebrity columns will have all 1's except its same row, because $(1,1) = 0$.

for ($i=0$ to $N-1$)

$\begin{array}{c} \boxed{\text{Row loop}} \quad O(N) \\ \downarrow \\ \boxed{\text{Col loop}} \quad O(N^+) \end{array}$

$+ O(N^2)$

$O(1)$

Optimal Approach - We will use a stack

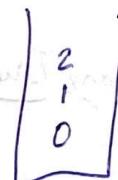
Stack will have celebrity element in the end.

Algorithm 1) Put all elements in stack

2) While (st.size() != 1)

$A \rightarrow \text{st.pop()} \rightarrow \text{st.pop()}$

$B \rightarrow \text{st.pop()} \rightarrow \text{st.pop()}$



if (A knows B) \Rightarrow

~~if~~ true, then A is not a celebrity for sure,
we will discard A & push B

$O(n) \rightarrow$ if (B knows A)

B discard, A \rightarrow push in stack

We will keep doing this, i.e. we will take 2 elements whether either of them know each other, then we will pop accordingly. At last, we will be only left with celebrity.

But, we still cannot say it is a celebrity.

So, to confirm, we will check its row & col

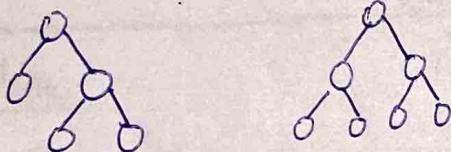
TC - $O(N)$

SC - $O(1)$

Binary Trees

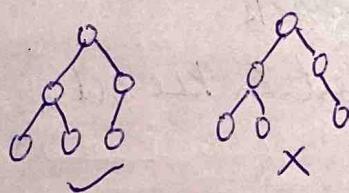
Types of Binary Tree \Rightarrow

1) Full Binary Tree - either has 0 or 2 children

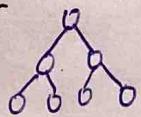


2) Complete Binary Tree - All levels are completely filled except the last level.

The last level has all nodes as left as possible



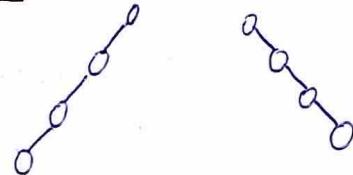
3) Perfect Binary Tree - all leaf nodes are at same level



4) Balanced Binary Tree - Height of tree at most $\log N$
where N is no. of nodes

$$n=8, \log_2 8 = 3$$

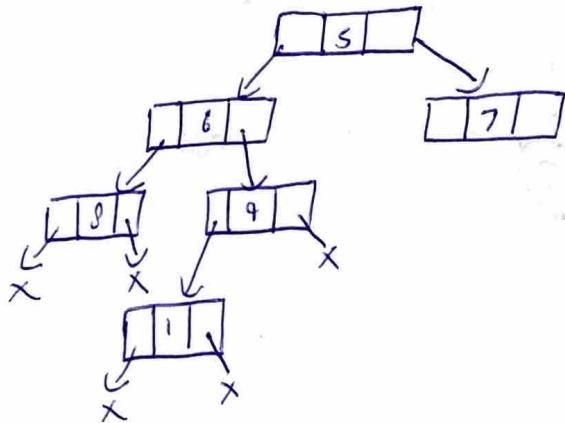
5) Degenerate Trees - If $n=4$, then these trees will be like



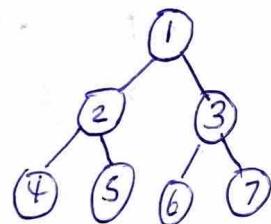
Every node has a single children
(skewed trees)

Binary Tree Representation

We will create a struct Node with val, left & right ptr



Traversal Techniques (BFS | DFS)



→ Inorder Traversal (Left Root Right)

4 2 5 1 6 3 7

→ Pre-Order Traversal (Root Left Right)

1 2 4 5 3 6 7

→ Post-Order Traversal (Left Right Root)

4 5 2 6 7 3 1

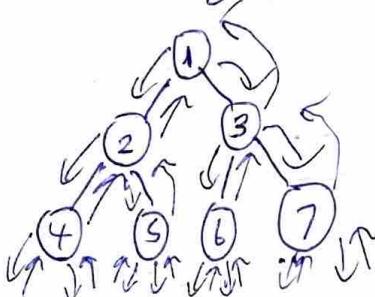
→ BFS → Goer Level wise

1 2 3 4 5 6 7

Pre-order Traversal \Rightarrow

```
void preorder(node) {
    if (node == NULL) return;
    print (node->data)
    preorder (node->left)
    preorder (node->right)
```

{}

 $T(-O(N))$ $S(-O(N))$

↑ worst case

1 2 4 5 3 6 7

Inorder Traversal \Rightarrow

```
void inorder(node) {
    if (node == NULL) return;
    inorder (node->left)
    print (node->data)
    inorder (node->right)
```

{}

 $T(-O(N))$ $S(-O(N))$

↑ worst case

Postorder Traversal \Rightarrow

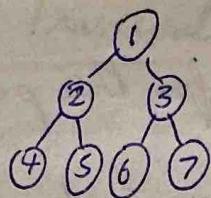
```
void postorder(node) {
    if (node == NULL) return;
    postorder (node->left)
    postorder (node->right)
    print (node->data)
```

{}

 $T(-O(N))$ $S(-O(N))$

↑ worst case

Level Order Traversal



~~1 2 3 4 5 6 7~~

We need a queue data structure, which will be initially having the root of the tree.

In order to store the traversal level wise we will be requiring same data structure.

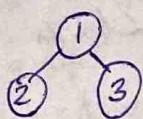
Let's take vector <vector<int>>

Initially 1 is present in queue

Q []

Take this 1 out

Check whether its left / right exists, if yes then put them in queue



[2 3]

Now, we have done it for 1, since there was only 1 element in this level, what we will put this into a vector.

Now, in next iteration pop out elements from the queue & perform same task as above.

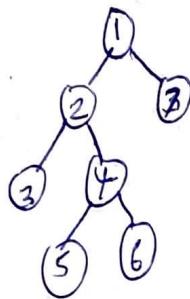
Code →

```

q.push(root);
while(!q.empty()){
    int sz = q.size();
    vector<int> level;
    for(i=0; i<sz; i++){
        TreeNode *node = q.front();
        q.pop();
        if(node->left) q.push(node->left);
        if(node->right) q.push(node->right);
        level.push_back(node->val);
    }
    ans.push_back(level);
}
  
```

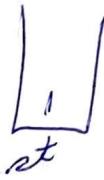
Iterative Preorder Traversal \rightarrow

We will be using stack for this purpose



1) We will take a stack, push the root in stack

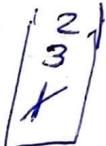
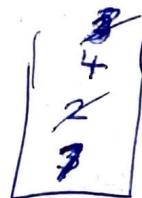
1



2) Pop out 1, check for its right & left children.

First insert right children in stack
then push left children

1 2



Now pop out 2, ~~then~~ print it

& then push its right & left children



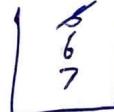
Now pop out 3, print it & insert its children

1 2 3



Then 4, print it & insert children

a 1 2 3 4



Then 5, print & check children

Then 6, " " "



Then 7, " " "

1 2 3 4 5 6 7

TC - O(N)

SC - O(N)

Iterative Inorder Traversal

In this approach also, we will use stack.

i) Push root into stack

2) Initially traverse in complete

left until left == NULL

& keep pushing left children in stack

Now ④ \rightarrow left == NULL,

Now ~~check~~ print ④, now move to 4's right, which is also NULL

Now come back from next iteration

Now go to ②'s right i.e. 5

Now move to 5's left which is 6,

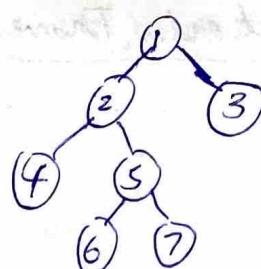
push in stack

Move to 6's left, which is NULL

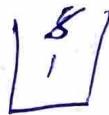
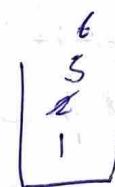
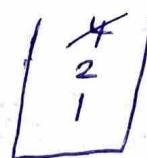
if (node == NULL), then take top of stack, print it
and go to top's right, 6 \rightarrow right is also null

~~So pop to Node state & because~~

Now also if (node == NULL) then take top of stack,
print it & go for its children



4 2 6 5 7 1 3



So, whenever we see node == NULL, we print st.top()

Tree node * node = root;

while (true) {

if (node == NULL) {

st.push(node);

node = node \rightarrow left;

} else {

if (!st.empty()) break;

node = st.top();

st.pop();

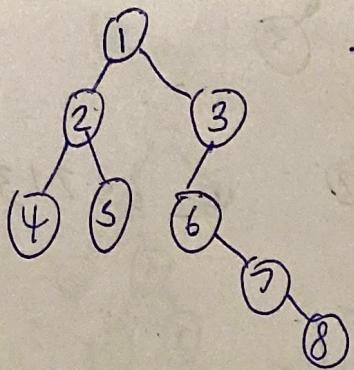
ans.push_back(node \rightarrow val);

node = node \rightarrow right;

}

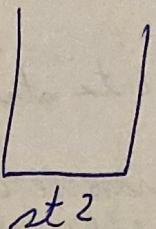
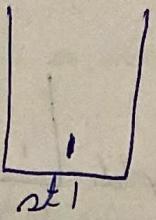
} return ans;

Iterative Postorder Traversal using 2 stack

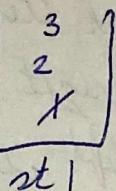


$\rightarrow 4 \ 5 \ 2 \ 8 \ 7 \ 6 \ 3 \ 1$

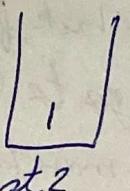
TC - $O(2N)$
SC - $O(N)$



- 1) Take the rest of tree and insert in st1.
- 2) Take st1, top() out and insert in st2.
And if top has left & right then insert it in st1, ~~top~~.
Insert left first then Right.
- 3) Take st1, top(), put it in
st2, push left & right in st1

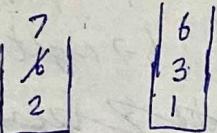


st1



st2

- 4) Now 6,



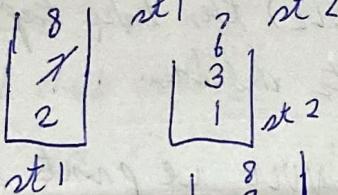
st1

st1

st2

st2

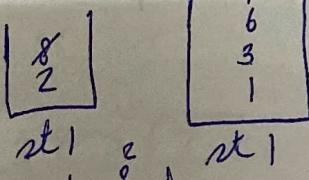
- 5) Now 7,



st1

st2

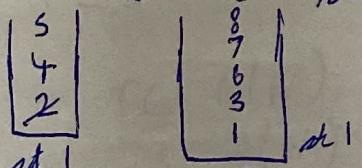
- 6) Now 8,



st1

st1

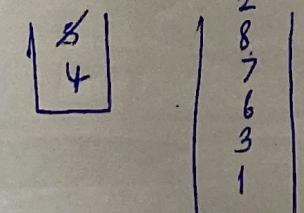
- 7) Now 2,



st1

st1

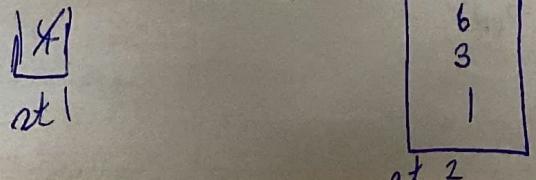
- 8) Now 5,



st1

st1

- 9) Now 4,



st1

st1

~~St2 stores the~~ St2 stores the
postorder traversal