CSCE 3110.002

ASSIGNMENT 2

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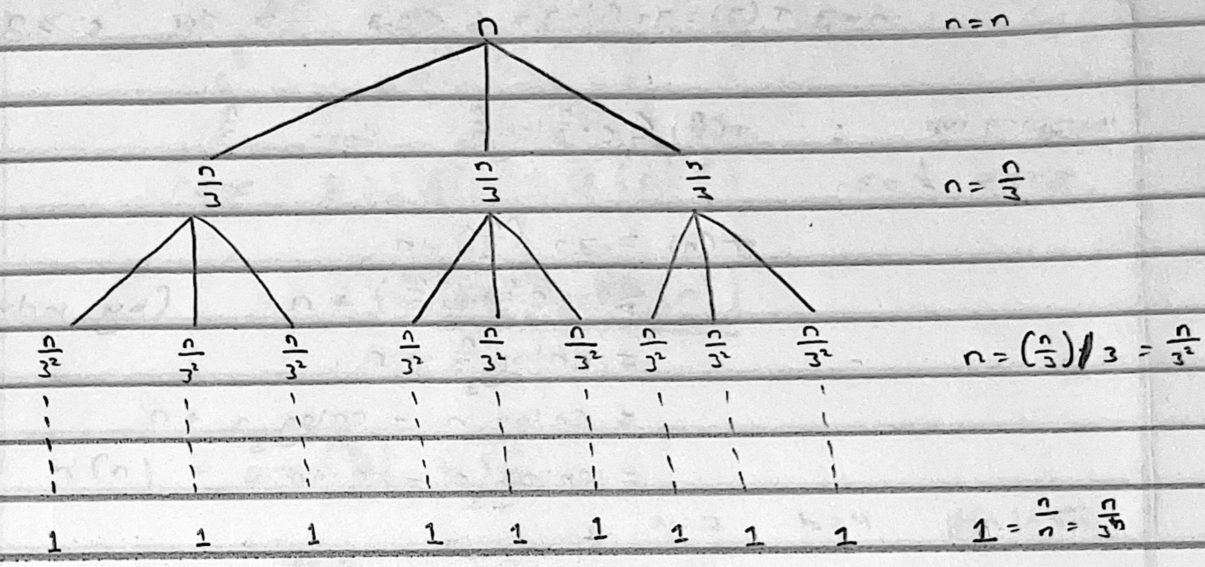
KYL0029.

1/ Recursion tree: $T(n) = 3T\left(\frac{n}{3}\right) + n$



CALCULATING HEIGHT OF TREE:

$$\frac{n}{3^h} = 1$$

$$3^h = n$$

$$\log_3 n = h \qquad \therefore \text{height} = 1 + \log_3 n$$

$T(n) \leq (n + n \ldots + n) \cdot (\log_3 n + 1)$

$T(n) \leq n\log_3 n + n$

$T(n) = O(n\log_3 n)$

Verify using substitution method.

GUESS: $T(n) \leq cn\log_3 n$ $\quad n > n_0$

BASIS: $n = 1$ | $T(1) = 1 < c.1\log_3 n$ $\Rightarrow$ not true

$n = 3$ | $T(3) = 3T(1)+3 = 6 < c.3$ $\Rightarrow$ for $c \geq 2$

INDUCTIVE STEP : $T(\frac{n}{3}) \leq c \cdot \frac{n}{3}\log\frac{n}{3}$ for $\frac{n}{3}$.

$$\begin{aligned} T(n) &= 3T\left(\tfrac{n}{3}\right) + n \\ &\leq 3\left(c\tfrac{n}{3}\log\tfrac{n}{3}\right) + n \quad \text{(by inductive step)} \\ &= cn\log_3\tfrac{n}{3} + n \\ &= cn\log_3 n - cn\log_3 3 + n \\ &= cn\log_3 n \underbrace{- cn + n} \end{aligned}$$

$\therefore T(n) \leq cn\log_3 n$ for $c \geq 1$

→ For $c \geq 2$, $T(n) \leq cn\log_3 n$ has proved to be true

hence $T(n) = O(n\log_3 n)$