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$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

- $T(n) = 2T\left(\frac{n}{4}\right) + 1$

$$a=2, b=4, f(n)=1$$

$$n^{\log_4 2} \text{ vs. } 1$$

$$n^{\frac{1}{2}} \text{ vs. } 1$$

case 1: $f(n) = O(n^{\frac{1}{2}-\epsilon})$ $\epsilon = \frac{1}{2}$

$$\therefore T(n) = \Theta(n^{\frac{1}{2}})$$

- $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$$a=2, b=4, f(n)=\sqrt{n}$$

$$n^{\log_4 2} \text{ vs. } \sqrt{n}$$

$$n^{\frac{1}{2}} \text{ vs. } n^{\frac{1}{2}}$$

case 2: $f(n) = \Theta(n^{\frac{1}{2}} (\log n)^0)$

$$\therefore T(n) = \Theta(n^{\frac{1}{2}} \log n) = \Theta(\sqrt{n} \log n)$$

- $T(n) = 2T\left(\frac{n}{4}\right) + n$

$$a=2, b=4, f(n)=n$$

$$n^{\log_4 2} \text{ vs. } n$$

$$n^{\frac{1}{2}} \text{ vs. } n$$

$$\frac{1}{2} + \epsilon \geq 1$$

case 3: $f(n) = \Omega(n^{\frac{1}{2}+\epsilon})$ $\epsilon = \frac{1}{2}$ $\epsilon \geq \frac{1}{2}$

and

$$2\left(\frac{n}{4}\right) \leq cn \quad c = \frac{1}{2}$$

$$\therefore T(n) = \Theta(n)$$

$$\bullet \quad T(n) = 2T\left(\frac{n}{4}\right) + n^2$$

$$n^{\log_4 2} \quad \text{vs.} \quad n^2 \quad a=2, \quad b=4, \quad f(n)=n^2$$

$$n^{\frac{1}{2}} \quad \text{vs.} \quad n^2$$

$$\text{case 3: } f(n) = \Omega(n^{\frac{1}{2} + \epsilon}) \quad \epsilon = \frac{1}{2}$$

and

$$2\left(\frac{n}{4}\right)^2 \leq cn^2 \quad c = \frac{1}{8}$$

$$\therefore \underline{T(n) = \Theta(n^2)}$$