Assignment 1

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

We can find gradient of function using numerical Methods:

1. Forward Difference
2. Backward Difference
3. Central Difference

In [ ]:
```
def fd(f,x1,x2,epsilon):
    gradfx1 = (f(x1+epsilon,x2)-f(x1,x2))/epsilon
    gradfx2 = (f(x1,x2+epsilon)-f(x1,x2))/epsilon
    return np.array([gradfx1,gradfx2])
```

In [ ]:
```
def bd(f,x1,x2,epsilon):
    gradfx1 = (f(x1,x2)-f(x1-epsilon,x2))/epsilon
    gradfx2 = (f(x1,x2)-f(x1,x2-epsilon))/epsilon
    return np.array([gradfx1,gradfx2])
```

In [ ]:
```
def cd(f,x1,x2,epsilon):
    gradfx1 = (f(x1+0.5*epsilon,x2)-f(x1-0.5*epsilon,x2))/epsilon
    gradfx2 = (f(x1,x2+0.5*epsilon)-f(x1,x2-0.5*epsilon))/epsilon
    return np.array([gradfx1,gradfx2])
```

Problem 1

The Given function is:

$$f(x_1,x_2) = 12.069x_1^2 + 21.504x_2^2 - 1.7321x_1 - x_2$$

Gradient of the function is:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \\ \frac{\partial f}{\partial x_2} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} 24.138x_1 - 1.7321 \\ \\ 43.008x_2 - 1 \end{bmatrix}$$

At given point:

$$x_0 = \begin{bmatrix} 5 \\ \\ 6 \end{bmatrix}$$

Analytically the gradient of the function is:

$$\nabla f(x_0) = \begin{bmatrix} 118.9579 \\ 257.048 \end{bmatrix}$$

```
Using Forward Difference:
```

In [ ]:
```
def f(x1,x2):
    return 12.069*x1**2 + 21.504*x2**2-1.7321*x1-x2
```

In [ ]:
```
fd(f,5,6,0.001)
```

Out[ ]:
```
array([118.969969, 257.069504])
```

```
Using Backward Difference:
```

In [ ]:
```
bd(f,5,6,0.001)
```

Out[ ]:
```
array([118.945831, 257.026496])
```

```
Using Central Difference:
```

In [ ]:
```
cd(f,5,6,0.001)
```

Out[ ]:
```
array([118.9579, 257.048 ])
```

Analytically gradient of function 1 is:

In [ ]:
```
def gradf1(x1,x2):
    gradx1= 24.138*x1 - 1.7321
    gradx2 = 43.008*x2-1
    return np.array([gradx1,gradx2])
```

In [ ]:
```
gradf1(5,6)
```

Out[ ]:
```
array([118.9579, 257.048 ])
```

Calculating Error

Next, we ned to calculate errors of different ϵ s. For this, we are using the norm of the difference between the gradient of the analytical solution and the calculated error.

In [ ]:
```
def err(epsilon):
    err_fd = np.linalg.norm(gradf1(5,6)-fd(f,5,6,epsilon))
    err_bd = np.linalg.norm(gradf1(5,6)-bd(f,5,6,epsilon))
    err_cd = np.linalg.norm(gradf1(5,6)-cd(f,5,6,epsilon))
    return np.array([err_fd,err_bd,err_cd])
```

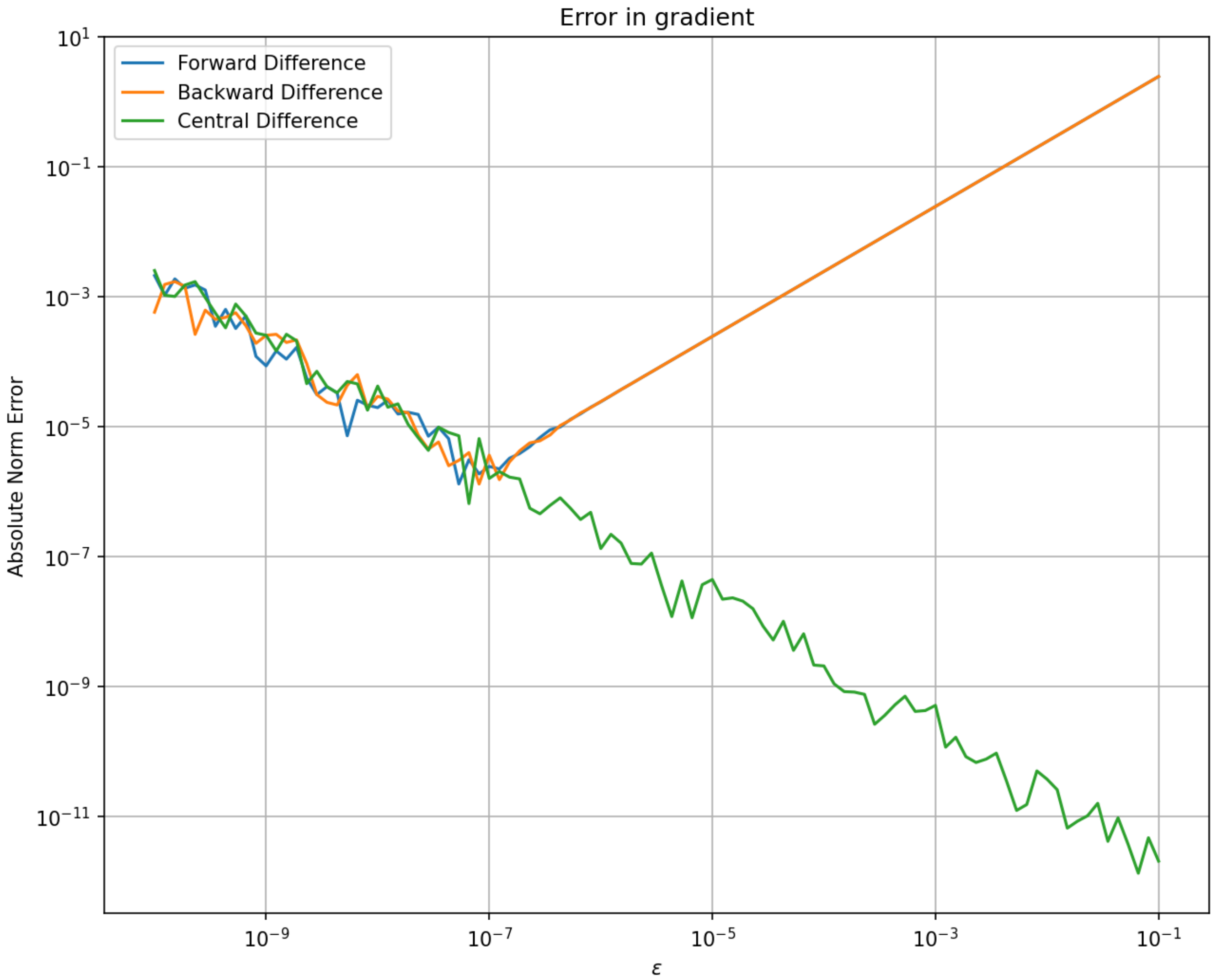
We are using log scale to calculate ϵ

In [ ]:
```
epsilon = np.logspace(-10,-1,100)
```

In [ ]:
```
abserr = np.zeros((100,3))
for i in range(epsilon.shape[0]):
    abserr[i] = err(epsilon[i])
```

In [ ]:
```
plt.figure(figsize=(10,8),dpi=150)
plt.loglog(epsilon,abserr[:,0],label='Forward Difference')
plt.loglog(epsilon,abserr[:,1],label='Backward Difference')
plt.loglog(epsilon,abserr[:,2],label='Central Difference')
plt.title('Error in gradient')
plt.xlabel("$\epsilon$")
plt.ylabel('Absolute Norm Error')
plt.legend()
```

```
plt.grid()
plt.savefig("Error Problem 1")
```



Problem 2

$$f(x_1,x_2) = \frac{4x_2^2 - x_1x_2}{10000(x_2x_1^3 + x_1^4)}$$

Analytically, the gradient is:

$$\nabla f(x_1,x_2) = \begin{bmatrix} -\dfrac{3x_2\cdot\left(x_1^2 - 6x_2x_1 + 4x_2^2\right)}{10000x_1^4\cdot(x_1-x_2)^2} \\ \dfrac{4x_2^2 - 8x_1x_2 + x_1^2}{10000x_1^3\cdot(x_2-x_1)^2} \end{bmatrix}$$

Find the gradient of the function using numerical methods at the point:

$$x_0 = \begin{bmatrix} 0.5 \\ \\ 1.5 \end{bmatrix}$$

The Function is:

In [ ]:
```
def f2(x1,x2):
    num = 4*x2*x2 -x1*x2
```

```
    den = 10000*(x2*x1**3-x1**4)
    return num/den
```

In [ ]:
```
f2(5,6)
```

Out[ ]:
```
9.12e-05
```

In [ ]:
```
fd(f2,0.5,1.5,0.001)
```

Out[ ]:
```
array([-0.03406902,  0.0026006 ])
```

In [ ]:
```
bd(f2,0.5,1.5,0.001)
```

Out[ ]:
```
array([-0.03433183,  0.0025994 ])
```

In [ ]:
```
cd(f2,0.5,1.5,0.00001)
```

Out[ ]:
```
array([-0.0342,  0.0026])
```

In [ ]:
```
def gradf2(x1,x2):
    dfx1n = -(3 * x2 * (x1**2 - 6 * x2 * x1 + 4 * x2**2))
    dfx1d = 10000 * x1**4 * (x1 - x2) ** 2
    dfx1 = dfx1n / (dfx1d)

    dfx2n = 4 * x2**2 - 8 * x1 * x2 + x1**2
    dfx2d = 10000 * x1**3 * (x2 - x1) ** 2
    dfx2 = dfx2n / (dfx2d)

    return np.array([dfx1, dfx2])
```

In [ ]:
```
gradf2(0.5,1.5)
```

Out[ ]:
```
array([-0.0342,  0.0026])
```

In [ ]:
```
np.linalg.norm(gradf2(0.5,1.5)-bd(f2,0.5,1.5,0.001))
```

Out[ ]:
```
0.00013182802623465544
```

In [ ]:
```
def err(epsilon):
    err_fd = np.linalg.norm(gradf2(0.5,1.5)-fd(f2,0.5,1.5,epsilon))
    err_bd = np.linalg.norm(gradf2(0.5,1.5)-bd(f2,0.5,1.5,epsilon))
    err_cd = np.linalg.norm(gradf2(0.5,1.5)-cd(f2,0.5,1.5,epsilon))
    return np.array([err_fd,err_bd,err_cd])
```

In [ ]:
```
err(0.001)
```

Out[ ]:
```
array([1.30977223e-04, 1.31828026e-04, 1.06350322e-07])
```

In [ ]:
```
epsilon = np.logspace(-10,-1,100)
```

In [ ]:
```
abserr = np.zeros((100,3))
for i in range(epsilon.shape[0]):
    abserr[i] = err(epsilon[i])
```

In [ ]:
```
plt.figure(figsize=(10,8),dpi=150)
plt.loglog(epsilon,abserr[:,0],label='Forward Difference')
plt.loglog(epsilon,abserr[:,1],label='Backward Difference')
plt.loglog(epsilon,abserr[:,2],label='Central Difference')
plt.title('Error in gradient')
plt.xlabel("$\epsilon$")
plt.ylabel('Absolute Norm Error')
plt.legend()
```

```
plt.grid()
plt.savefig("Error Problem 2")
```