softmax

January 31, 2023

0.1 Loading Data

[ ]:
```
import os

# Scientific and vector computation for python
import numpy as np

# Plotting library
import matplotlib.pyplot as plt

import pandas as pd
```

[ ]:
```
SAVE_DIR = 'Plots'
```

[ ]:
```
Data_Dir = 'DATA'
df_train = pd.read_csv(os.path.join(Data_Dir, 'fashion-mnist_train.csv'))
df_test = pd.read_csv(os.path.join(Data_Dir, 'fashion-mnist_test.csv'))
```

[ ]:
```
label_map = {0: 'T-shirt/top', 1: 'Trouser', 2: 'Pullover', 3: 'Dress', 4:
 ↪'Coat', 5: 'Sandal', 6: 'Shirt', 7: 'Sneaker', 8: 'Bag', 9: 'Ankle boot'}
```

[ ]:
```
def displayData_fashion(X,y,y_pred=None , save_img_dir=None):
    """
    Displays the data from X
    """
    import random
    # Create figure
    fig, ax = plt.subplots(nrows=10, ncols=10, sharex=True, sharey=True,
 ↪figsize=(10, 12))
    for r in range(10):
        for c in range(10):
            res = random.sample(range(1, 5000), 1)
            ax[r, c].matshow(X[res][0].reshape((28,28)), cmap='binary')
            if y_pred is not None:
                if y[res][0] == y_pred[res][0]:
                    ax[r,c].title.set_color('green')
                    ax[r,c].title.set_text(label_map[y[res][0]])
                else:
```

```
                ax[r,c].title.set_color('red')
                ax[r,c].title.set_text(label_map[y_pred[res][0]])
        else:
            ax[r,c].title.set_text(label_map[y[res][0]])
        plt.xticks(np.array([]))
        plt.yticks(np.array([]))
    plt.tight_layout()
    if save_img_dir is not None:

        plt.savefig(os.path.join(save_img_dir))
plt.show();
```

```
[ ]: displayData_fashion(df_train.iloc[:,1:].values,df_train.iloc[:,0].
     ↪values,save_img_dir=os.path.join(SAVE_DIR,'0601.png'))
```

0.2 Preprocesing

The following preprocessing steps are performed:

1. Normalize the data
2. One-hot encode the labels
3. Train-test split

0.2.1 Normalize the data

[ ]:
```
X = df_train.iloc[:,1:].values
y = df_train.iloc[:,0].values
X = np.insert(X, 0, 1, axis=1)

test_X = df_test.iloc[:,1:].values
test_X = np.insert(test_X, 0, 1, axis=1)
test_y = df_test.iloc[:,0].values

print(X.shape)
```

```
(10000, 785)
```

[ ]:
```
X = X/255
test_X = test_X/255
```

0.2.2 One-hot encode the labels

[ ]:
```
def one_hot(y,c):
    y_hot = np.zeros((y.shape[0],c))
    for i in range(y.shape[0]):
        y_hot[i,y[i]] = 1
    return y_hot
```

[ ]:
```
y_one_hot = one_hot(y,10)
print(y_one_hot.shape)
```

```
(10000, 10)
```

0.2.3 Train-test split

[ ]:
```
#train test split
ids = np.random.permutation(X.shape[0])
train_ids = ids[:int(X.shape[0]*0.8)]
test_ids = ids[int(X.shape[0]*0.8):]

X_train = X[train_ids]
y_train = y_one_hot[train_ids]

X_test = X[test_ids]
y_test = y_one_hot[test_ids]

assert X_train.shape[0] == y_train.shape[0]
assert X_train.shape[0]+X_test.shape[0] == X.shape[0]
```

0.3 Mathematical Formulation

The hypothesis function, for sigmoid regression has a form

$$h(\theta) = \sigma(\mathbf{w}^T X) = y$$

Here, y is just a scalar. Suppose there are k classes, then, in softmax, the hypothesis function itself returns a vector with $\hat{y} \in \mathbb{R}^k$. That is,

$$\mathbf{h}(\ ) = \text{softmax}(W^T X) = \hat{y}$$

Note that I've written upper case W instead of bold lower case $\mathbf{w}$ becuase in case of softmax, the weight is a matrix with dimension of $k \times n$. The mathematical form of softmax is:

$$\phi(\mathbf{z}) = \frac{e^{\mathbf{z}}}{\sum_{j=1}^{k} e^{\mathbf{z}_j}}$$

where

$$\mathbf{z} = W^T X$$

The loss function, used for softmax is the cross-entropy, defined as:

$$J = -\sum_{c=1}^{k} y_c \log(\hat{y}_c)/(2m)$$

The update equation is the same:

$$W_j := W_j - \alpha \frac{\partial J}{\partial W_j}$$

The update equation is similar to that in binary classification:

$$W := W - \alpha \frac{1}{m} \sum_{i=1}^{m} (\hat{y}^{(i)} - \mathbf{y^{(i)}})\mathbf{x}^{(i)}$$

0.4 Implementation

[ ]:
```
class SoftmaxBGD:
    def __init__(self,lr=0.1,n_iter=1000,see_loss=False, tol=None):
        self.lr = lr
        self.n_iter = n_iter
        self.w = None
        self.b = None
        self.loss = []
        self.tol = tol
```

```
        self.see_loss = see_loss
        self.loss_old = 1


    def softmax(self,z):

        return np.exp(z) / np.sum(np.exp(z),axis=1,keepdims=True)


    def loss_fun(self,y,y_softmax):
        return  -np.sum(y*np.log(y_softmax))*(1/(2*y.shape[0]))


    def grad(self,X,y,y_softmax):
        return np.dot(X.T,y_softmax-y )


    def fit(self,X,y):
        self.w = np.zeros((X.shape[1],y.shape[1]))
        for i in range(self.n_iter):
            z = np.dot(X,self.w)
            y_softmax = self.softmax(z)
            loss = self.loss_fun(y,y_softmax)
            self.loss.append(loss)
            self.w = self.w - (self.lr/X.shape[0])*self.grad(X,y,y_softmax)
            if self.see_loss:
                if (i+1)%1000 == 0:
                    print("Epoch: {}, Loss: {}".format(i+1,loss))
            if self.tol is not None:
                if (abs(self.loss_old-loss)/self.loss_old) < self.tol:
                    print("Converged at epoch: {}".format(i+1))
                    break
                self.loss_old = loss


    def predict(self,X):
        pred = self.softmax(np.dot(X,self.w))
        return np.argmax(pred,axis=1)

    def accuracy(self,y,y_pred):
        return np.mean(y_pred == y)

    def plot_loss(self, file_name):
        plt.plot(self.loss)
        plt.xlabel("Epoch")
        plt.ylabel("Loss")
        plt.title("Loss vs Epoch")
```

```
        plt.savefig(os.path.join(SAVE_DIR,file_name))
```

[ ]:
```
bgd =  SoftmaxBGD(lr=0.1,n_iter=10000,see_loss=True)
bgd.fit(X_train,y_train)
```

```
Epoch: 1000, Loss: 0.23716906189038822
Epoch: 2000, Loss: 0.21359095533081615
Epoch: 3000, Loss: 0.20075306438501256
Epoch: 4000, Loss: 0.19189882598172678
Epoch: 5000, Loss: 0.18512175803620953
Epoch: 6000, Loss: 0.17961993277953503
Epoch: 7000, Loss: 0.17498213168979185
Epoch: 8000, Loss: 0.17096974558439584
Epoch: 9000, Loss: 0.1674318763241986
Epoch: 10000, Loss: 0.16426683351094293
```

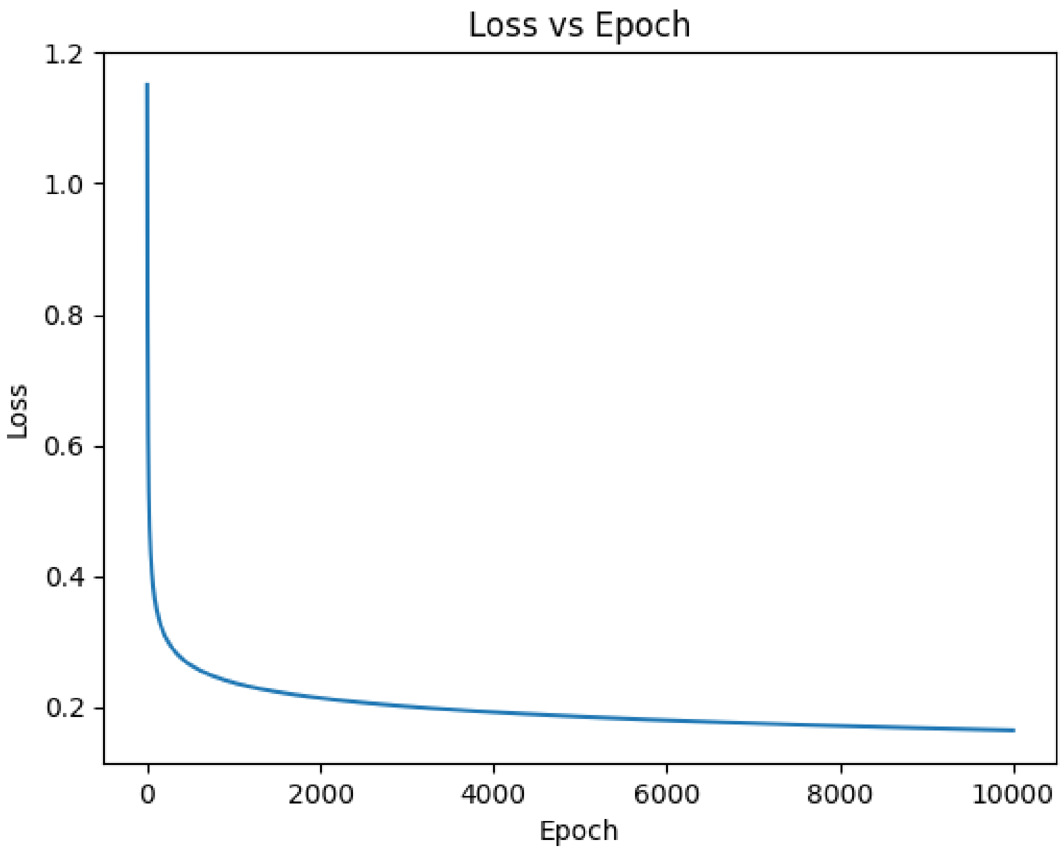
[ ]:
```
y_pred_train= bgd.predict(X_train)
y_pred_test = bgd.predict(X_test)
```

[ ]:
```
train_acc = bgd.accuracy(np.argmax(y_train,axis=1),y_pred_train)
test_acc = bgd.accuracy(np.argmax(y_test,axis=1),y_pred_test)

print("Train Accuracy: {:.2f}%".format(train_acc*100))
print("Test Accuracy: {:.2f}%".format(test_acc*100))
```

```
Train Accuracy: 89.66%
Test Accuracy: 83.45%
```

[ ]:
```
fig = bgd.plot_loss("0602.png")
```



[ ]:
```
y_train.shape
```

[ ]: (8000, 10)

[ ]:
```
displayData_fashion(X_train[:, 1:],np.argmax(y_train, axis=1), y_pred_train, os.
    ↪path.join(SAVE_DIR, '0603.png'))
```


Sandal Pullover Sneaker Dress Bag Sandal Ankle boot Dress Sneaker Dress
T-shirt/top Trouser Bag Sneaker Pullover Trouser Dress Ankle boot Pullover Sneaker
Ankle boot T-shirt/top Bag Trouser Trouser T-shirt/top Dress Pullover T-shirt/top Coat
Dress Trouser T-shirt/top Trouser Ankle boot Sandal T-shirt/top Ankle boot T-shirt/top Trouser
Shirt Sandal T-shirt/top Bag Trouser Sneaker Sandal Sandal Sandal Pullover
T-shirt/top Bag T-shirt/top Coat Bag Pullover Coat Bag Dress Coat
Trouser Trouser Dress Coat Pullover Coat Bag Ankle boot Ankle boot Shirt
Bag Sandal Trouser Sandal Pullover Bag Ankle boot Pullover Dress Sneaker
Dress Coat Shirt Sandal Sandal Sneaker T-shirt/top Sneaker Sandal Shirt
Sneaker Sandal Coat Dress Ankle boot T-shirt/top Sandal Pullover Sandal Coat