Number Theory 3

You are given an array of n integers a. Print the largest gcd of any pair among these integers. Constraints n<= 1e5 and 1<= a[i] <= 1e6.

https://cses.fi/problemset/task/1081

```
#include <bits/stdc++.h>
using namespace std;

int main(){
    int n;
    cin>>n;
    vector<int> a(n);
    int mx = 0;
    for(int i=0;i<n;i++){
        cin>>a[i];
        mx = max(mx, a[i]);
    }
    mx++;
    vector<int> m(mx);
    for(int i=0;i<n;i++){
        m[a[i]]++;
    }
    int ans= 1;
    for(int i=2;i< mx;i++){
        int cnt = 0;
        for(int j= i;j< mx;j+=i){
            cnt += m[j];
        }
        if(cnt >=2){
            ans = max(ans, i);
        }
    }
    cout<<ans<<endl;
```

```
}
```

In the previous question count the number of coprimes pairs in the array.

https://cses.fi/problemset/task/2417

a = [4, 6, 8, 12]

2 => 4, 6, 8, 12

3 => 6, 12

6 => 6, 12

Euler totient function (Phi function)

```
f(x) = y

The number of integers in the range [1 , x] which are coprime
to x.

phi(1) = 1
For prime p, phi(p) = p-1

P, 2p, 3p, 4p, …, p^k = p^k/p

For prime p, phi(p^k) = p^k - p^k/p = p^k(1 - 1/p)

phi(a.b) = phi(a).phi(b) when a and b are coprime.


N = p1^a1 p2^a2 … pn^an

phi(N) = phi(p1^a1) phi(p2^a2) … phi(pn^an)

phi(N) = p1^a1(1 - 1/p1)  p2^a2(1-1/p2) …  pn^an(1-1/pn)
```

phi(N) = N (1 - 1/p1) (1 - 1/p2) … (1 - 1/pn)

```
int etf(int n){
    if(n==1) return 1;

    int phi = n;

    for(int i=2;i*i<=n;i++){
        if(n%i==0){
            while(n%i==0) n/=i;
            phi = phi - phi/i;
        }
    }

    if(n>1) phi = phi - phi/n;

    return phi;
}
```

```
int N;
vector<int> phi(N);
for(int i=0;i<N;i++) phi[i]=i;

for(int i=2;i<N;i++){
    if(phi[i]==i)
    for(int j=i;j<N;j+=i)
        phi[j]-=phi[j]/i;
}

// Complexity => O(nloglogn)
```

Q. You are given a single integer N. Find the number of different straight lines given by the equation ax+by=0 such that a and b are integers and satisfy 1 <= a,b <= N.

Ans: 2*summation(etf) -1

Generalised Fermat’s Little Theorem

If p is prime
a^(p-1) = 1 mod p

a^phi(p) = 1 mod p

a^(phi(p)-1) * a = 1 mod p
inverse(a) mod p = a^(phi(p)-1)

a^(c/d) mod p = a^((c/d)mod(p-1)) mod p
condition to (c/d)mod (p-1) exist(d and p-1 are coprime) and p is prime

NOTE: Summation over all divisors d of n (phi(d)) = n

For example,[1, 2, 3, 6 are divisors of 6] hence
6 = phi(1)+ phi(2) + phi(3) + phi(6)
= 1 + 1 + 2 + 2
= 6