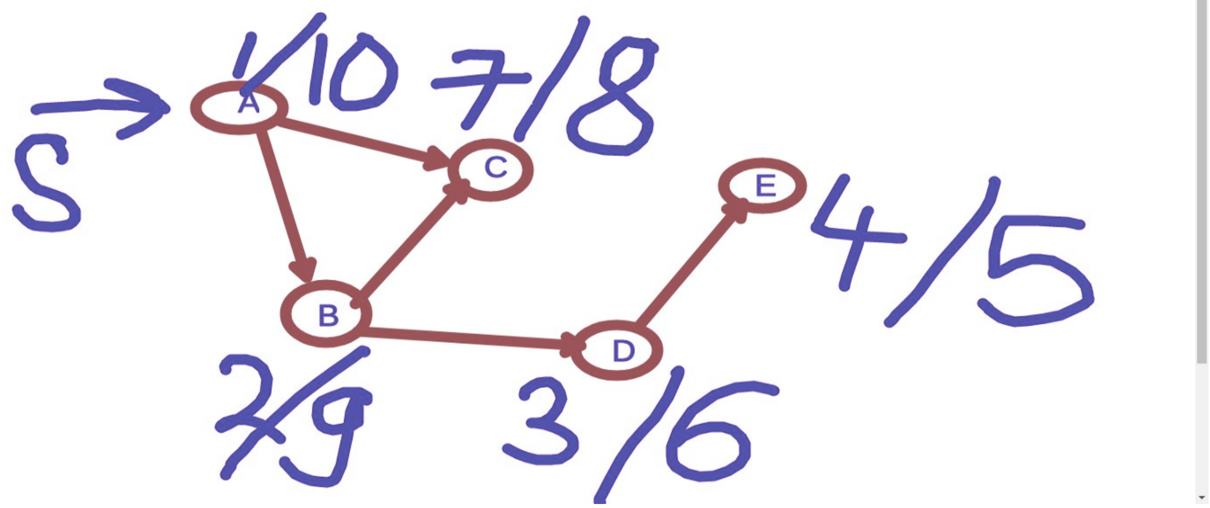
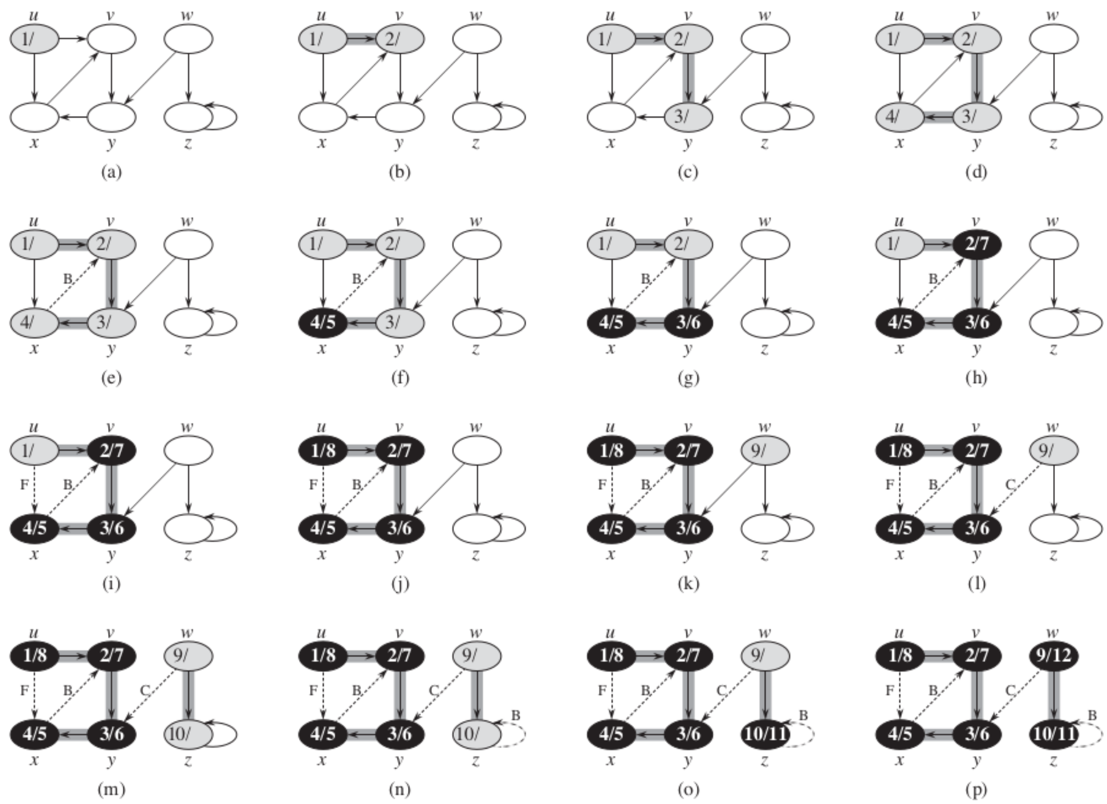
DFS (Depth first search)

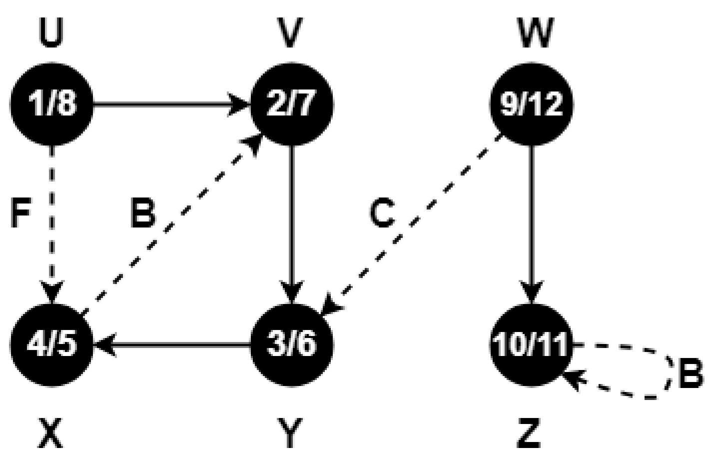


Visiting Time
Finishing Time

Colour codes in DFS

White: Unvisited node
Gray: Visited but not finished node
(Visited but all its neighbours are not visited)
Black: Finished node
(Visited and all its neighbours are also visited)
Try running DFS on below graph and calculating visiting and finishing time of each node:





Taking graph input as adjacency list

```
int n; // no. of nodes
vector<vector<int> > adj; // adjacency list
```

```cpp
int main()
{
cin>>n;
int e; // no. of edges;
cin>>e;
adj.resize(n);
while (e--)
{
int u,v;
cin>>u>>v;
adj[u].push_back(v);
adj[v].push_back(u);  // for undirected graph
}
}
```

Code for normal DFS

```cpp
int n; // no. of nodes
vector<vector<int> > adj; // adjacency list
vector<bool> vis; // initialise all values
with false

void dfs(int node)
{
vis[node]=true;
```

```
for(auto child: adj[node])
{

      if ( ! vis[child] )
     {
          dfs(child);
     }
}
}
```

Code for DFS when you need to calculate colour of each node at every instant

```
vector<int> colour;
// 0 - white (Use constants or #define for colours)
// 1 - gray
// 2 - black

void dfs(int node)
{
colour[node]=gray;
for(auto child: adj[node])
{
      if ( colour[child] ==0 )
     {
```

```
            dfs(child);
      }
}
colour[node]=black;
}
```

Code for DFS when you need to calculate visiting time and finishing time of nodes

```
int timerCode=1;
vector<int> visTime;
vector<int> finishTime;
void dfs(int node)
{
vis[node]=true;
visTime[node]=timerCode;
timerCode++;
for(auto child: adj[node])
{
      if ( ! vis[child] )
     {
           dfs(child);
     }
}
finishTime[node]=timerCode
timerCode++;
}
```

Calling DFS in main()

```
for(int i=0; i<n; i++)
{
```

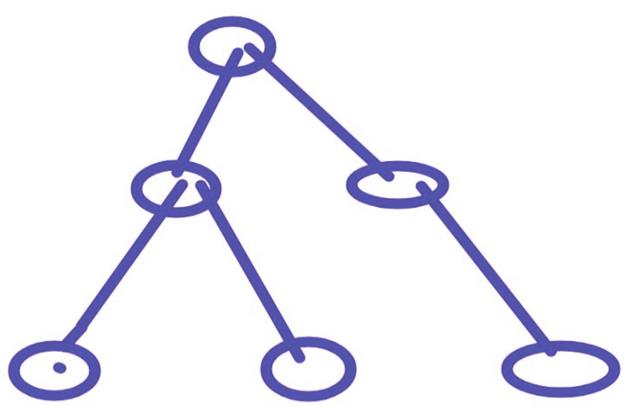
```
  if (!vis[i])
   {
      dfs(i);
   }
}
```

Tree

Tree is an undirected connected graph without any cycle

Properties:

- If there are n nodes in a tree, then it will have n-1 edges



- If you are given a connected graph with n nodes and n-1 edges, then the given graph is a tree

- If you are given a connected graph with n nodes and n edges, then the given graph will contain exactly 1 cycle
- Each node has a single parent in tree

You have been given a tree of n nodes with root at 0. Find level of each node using DFS

```
vector<int> level;
// level[0] =0;
void dfs(int node)
{
vis[node]=true;
for(auto child: adj[node])
{
      if ( ! vis[child] )
     {
   level[child]=level[node]+1;
          dfs(child);
     }
}
}
// in main, call dfs(0); // Because 0 is root of tree
```

DFS on trees can also be implemented as:

```
// call dfs(0, -1) in main()
void dfs(int node, int par)
```

```
{
for(auto child: adj[node])
{
      if (child != par)
     {
   level[child]=level[node]+1;
          dfs(child);
     }
}
}
```

Q. Checking whether given graph is bipartite graph

Link: https://cses.fi/problemset/task/1668

It can be done using DFS

```
vector<int> teamNum;
bool dfs(int node, int currTeam)
{
vis[node]=true;
teamNum[node]=currTeam;
for(auto child: adj[node])
{
    if ( teamNum[child] == teamNum[node])
         return false;
      if ( ! vis[child] )
```

```
     {
          bool temp=dfs(child, 3 - currTeam);
          if(temp==false)
               return false;
     }
}
return true;
}

int main()
{



for(int i=0; i<n; i++)
{
  if(!vis[i])
  {
   bool temp=dfs(i,1);
   if(!temp)
   {
    cout<<"IMPOSSIBLE";
    return 0;
   }
  }
}
// Print the team values if division is possible
```

```
}
```

Detecting cycle in a graph

no of nodes=n

```
int colour[n];
```

Return true if cycle present else false.

```
bool dfs(int node, int parent)
{
colour[node]=1;
for(auto child: adj[node])
{
      if ( child!=parent && colour[child] == 1 )
     {
    // You can remove the condition for
child!=parent, when you want to detect even the
2 node cycles in directed graph like 1->2, 2->1
          return true;
     }
else if( child!=parent && colour[child] == 0 )
```

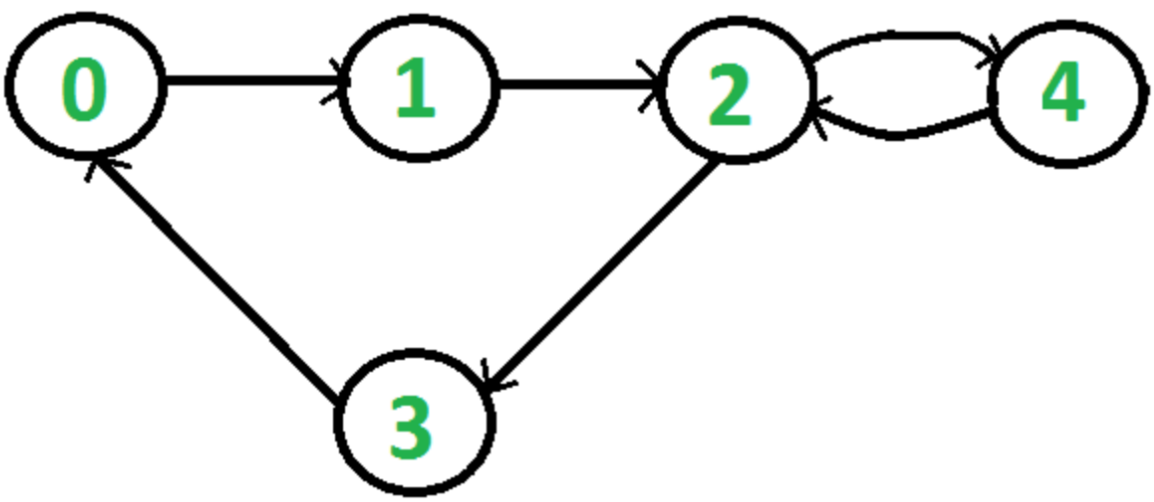
```
     {
          bool temp = dfs(child, node);
          if( temp == true) return true;
     }
}
colour[node]=2;
return false;
}
```

```
int main()
{
    // Take graph input
// .....
   for(int i=0;i<n;i++)
      colour[i]=0;
   // Mark colour of all nodes as WHITE
(unvisited)

  for(int i=0; i<n; i++)
{
   if(colour[i] == 0)
   {
      bool cycle = dfs(i,-1);
      if(cycle)
      {
```

```
            cout<<"Cycle Present";
            return 0;
        }
    }
}
cout<<"No cycle found";
return 0;
}
```

You can try running this code over below example:

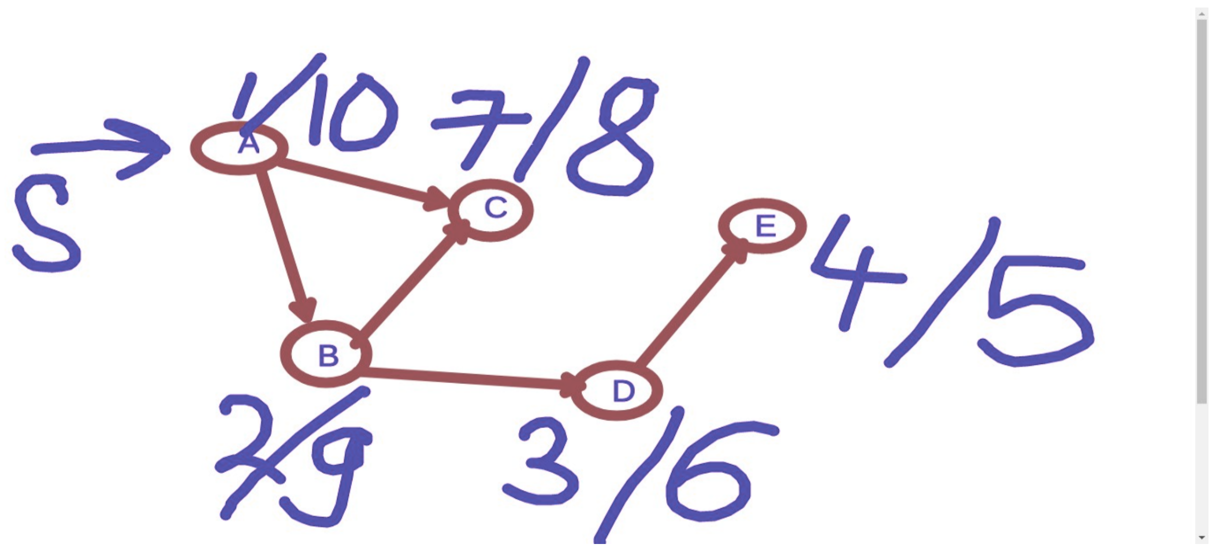


*dfs(0)
col[0]=1
0->child==1
**dfs(1,0(parent))
col[1]=1
***dfs(2)
col[2]=1
****dfs(3)

col[3]==0
  col[3]=1
3->
child=0
color of child==1-->cycle
col[0]==1
return true;

****dfs(4)

col[3]=2
col[2]=2
col[1]=2
col[0]=2

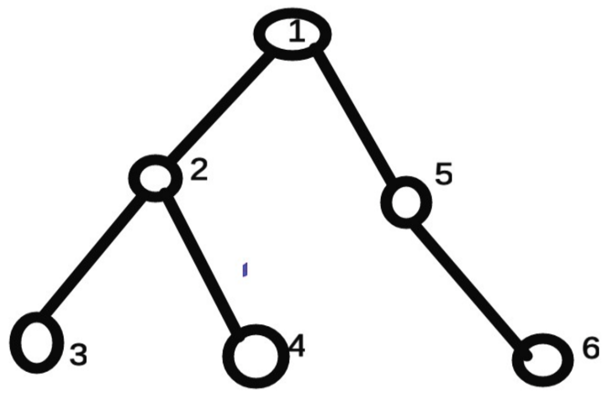


dfs(A)
col[A]=1
A->child:B,C

```
  *dfs(B)
   child:C,D
     **dfs(C)
       col[C]=1;
       child C-NULL
      col[C]=2
     false ret

**dfs(D)
 col[D]=1
  D->CHILD:E
    ***dfs(E)
         col[E]=1
        col  [E]=2
        ret false;
   col[D]=2
   ret false

*dfs(C)
```



Given a tree, you need to find the gcd of all the values of nodes in subtree of every node. 0 is root node
Example, in above graph, ans[5] = gcd(5,6)
ans[2]= gcd(2,3,4)

No. of nodes, n <= 10^5

Try to solve this problem using DFS

vector<int> ans(n);

```
// call dfs(0, -1) in main()
void dfs(int node, int par)
{
ans[node]=val[node];
for(auto child: adj[node])
{
```

```
        if (child != par)
       {
            dfs(child);
            ans[node] = __gcd(ans[node], ans[child]);
       }
}
}
```

Topological Sorting:

(Kanh's Algo)

```
int indeg[n+1]={0};

for(int i=0;i<m;i++){
    int u,v;
    cin>>u>>v;
    // u -> v
    indeg[v]++;
}

vector<int>topl;

queue<int>q;

for(int i=1;i<=n;i++)
if(indeg[i]==0)
q.push(i);

while(!q.empty()){
    auto fst=q.front();
    q.pop();

    topl.push_back(fst);
    for(auto child:adj[fst])
    {
```

```
        indeg[child]--;

        if(indeg[child]==0)
        q.push(child);
    }
}

for(auto e:topl)
cout<<e<<" ";
```

DFS Method:
Arrange in decreasing order of exit times!

```
int n; // number of vertices
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> ans;

void dfs(int v) {
    visited[v] = true;
    for (int u : adj[v]) {
        if (!visited[u])
            dfs(u);
    }
    ans.push_back(v);
}

void topological_sort() {
    visited.assign(n, false);
    ans.clear();
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs(i);
    }
    reverse(ans.begin(), ans.end());
}
```

Practice:
https://cses.fi/problemset/task/1679
[Link to Rough]()