

# Longest Increasing Subsequence

0 1 2 3 4  
→ 2, 1, 3, 4, 5  
3 2 4 1 5 3  
dp[index] = Max. length of increasing subseq which ends at 'index'

2, 1, 3, 3, 4, 5

1....(i-1) i

----->

$dp[i] = \max(1, 1 + dp[j])$  where  $a_j \leq a_i$  and  $j < i$

$O(n \cdot n)$

$dp[0] = 1;$

$dp[1] = 1$

$dp[2] = 2$

$dp[3] = 3$

$dp[4] = 4$

**Code:**

```
#include<bits/stdc++.h>
using namespace std;
// 12 10 9 1
// 0 1 2 3
int LIS(vector<int> v) {
    int n=v.size();
    int dp[n+1];

    for(int i=0;i<n;i++){
```

```

        dp[i]=1;
        for(int j=i-1;j>=0;j--){
            if(v[i]>=v[j])
                dp[i]=max(dp[i],1+dp[j]);
        }
    }
    int ans=*max_element(dp,dp+n);
    return ans;
}

int main(){
    int n;
    cin>>n;
    vector<int>v(n);

    for(int i=0;i<n;i++)
        cin>>v[i];

    cout<<LIS(v);

    return 0;
}

```

**Time Complexity:  $O(n^2)$**

## Longest Strictly Increasing Subsequence:

Hint: Just change  $v[i] \geq v[j]$  to  $v[i] > v[j]$  in LIS function.

## Longest Increasing Subarray/Substring:

**Note:** A substring needs to be continuous.

```

#include<bits/stdc++.h>
using namespace std;
// 12 10 9 1
// 0 1 2 3
int LIS(vector<int> v){
    int n=v.size();
    int dp[n+1];

    for(int i=0;i<n;i++){

```

```

        dp[i]=1;
        if(i>0&&v[i]>=v[i-1])
            dp[i]=1+dp[i-1];
    }
    int ans=*max_element(dp,dp+n);
    return ans;
}

int main() {
    int n;
    cin>>n;
    vector<int>v(n);

    for(int i=0;i<n;i++)
        cin>>v[i];

    cout<<LIS(v);

    return 0;
}

```

**Time Complexity:  $O(n)$**

**Note:** Can be solved with two pointers as well in linear time.

## Longest Common Subsequence (LCS)

LCS:

$dp[i][j]$ :

// base case

if( $a[i]=b[j]$ )

$dp[i][j]=1+dp[i-1][j-1]$  //  $dp[1][0]=1$

else

$dp[i][j]=\max(dp[i][j-1],dp[i-1][j]);$  //

**Code:**

```

#include<bits/stdc++.h>
using namespace std;

int LCS(vector<int> a,vector<int> b) {
    int n=a.size();
    int m=b.size();

    int dp[n+1][m+1]; // 1 based indexing

    // int dp[n]; dp[0] -> [0.....n-1]
    // int dp[n+1] -> 0.1,2...n

    dp[0][0]=0;

    // a: 1...i
    // b: empty
    for(int i=1;i<=n;i++){
        dp[i][0]=0;
    }

    // a: empty
    // b: 1...j
    for(int j=1;j<=m;j++){
        dp[0][j]=0;
    }

    for(int i=1;i<=n;i++){
        for(int j=1;j<=m;j++){
            if(a[i-1]==b[j-1])
                dp[i][j]=1+dp[i-1][j-1];
            else
                dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        }
    }

    return dp[n][m];
}

int main() {
    int n;

```

```

cin>>n;

vector<int>a(n);

for(int i=0;i<n;i++)
cin>>a[i];

int m;
cin>>m;

vector<int>b(m);

for(int i=0;i<m;i++)
cin>>b[i];

cout<<LCS(a,b);
return 0;
}

```

Time Complexity:  **$O(n*m)$**

## Coin change

Watch this video if you have doubt ->

<https://www.youtube.com/watch?v=ZI17bgz07EE>

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
typedef long long ll;
```

```
ll dp[1005][1005];
```

```
ll get_length(ll amount, ll pos, vector<ll> &coins) {
```

```

if(pos >= coins.size() || amount<0){
    if(amount==0){
        return 0;
    }
    return INT_MAX; // we will not be counting invalid case this way as answer will
//always be less than infinity
}

```

```

if(dp[amount][pos]!=-1){
    return dp[amount][pos];
}

```

```

// unvisited state-> -1
// visited state -> 0-infinity

```

```

ll ans= min({get_length(amount, pos+1, coins),
            1+get_length(amount-coins[pos], pos, coins),
            1+get_length(amount-coins[pos], pos+1, coins)});

```

```

// three choices at every state
// 1) we don't take the coin and move to next position
// 2) we take the coin and don't move to next position
// 3) we take the coin and move to next position

```

```

// dp[amount][pos]=ans;
// return dp[amount][pos];
return dp[amount][pos]=ans;
}

```

```

int main(){
    memset(dp,-1,sizeof(dp)); // making every element of dp array -1

    ll amount,n; cin>>amount>>n;
    vector<ll>coins(n);
    for(ll i=0;i<n;i++){
        cin>>coins[i];
    }
}

```

```
}  
    cout<<get_length(amount,0,coins);  
}
```

Complexity-> amount \* size of coin's array

**Home work problem-**

[https://www.hackerrank.com/challenges/ctci-coin-change/problem?h\\_r=internal-search](https://www.hackerrank.com/challenges/ctci-coin-change/problem?h_r=internal-search)