Fun Template 2

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Contents

Conventions

$\mathbb{F}$ denotes either $\mathbb{R}$ or $\mathbb{C}$.
$\mathbb{N}$ denotes the set $\{1, 2, 3, \ldots\}$ of natural numbers (excluding 0).
Inner products are taken to be linear in the first argument and conjugate linear in the second.
The Einstein summation convention is used for tensors unless otherwise specified.

Foreword

1 Introduction

1.1 Scale, Speed, Units

1.2 Galiliean Relativity, Speed of Light, Postulates of SR

** Galiliean relativity is NOT applicable in higher speeds, near speed of light..

1.3 Phenomena of SR

1.3.1 Time Dilation

1.3.2 Length Contraction

1.3.3 Relativity of Simultaneity

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2 Foundations of Special Relativity

2.1 Relativity of Simultaneity

2.2 Time Dilation

This subsection is from *Time in motion* to *Time dilation: experimental evidence.*

2.2.1 Moving mirrors

*What is a clock? It is any physical system that undergoes cyclical repetitive motion in a uniform way.

For example, a light clock is a two mirrors facing each other and a light bulb going up and down (tic toc). This light clock reveals how motion affects passage of time. Draw the figure in page 6, on Kumail's notes. It shows how that motion slows down time.

Does the observer in motion feel ro recognize that his/her clock is running slow? The answer is No, he/she will NOT feel the *uniform* motion.

Another question, why don't we notice time running slow in everyday life? The answer is that because the difference is very very small to notice.

2.2.2 Derivation

2.3 Length Contraction

2.4 Paradoxes of Simultaneity

2.5 Lorentz Transformations

2.5.1 Properties

2.5.2 Addition of velocities

This subsubsection is from *Combining velocities* to *Combining velocities: Example in 3D.*

The expectation of adding velocities:
If you run towards(away) a light beam with speed v, then the relative light speed to you with is $c + v(c - v)$. And this is wrong!

It will be like this:

$$w - v \rightarrow \frac{w - v}{1 - vw/c^2}$$
$$w + v \rightarrow \frac{w + v}{1 + vw/c^2}$$

The proof is: the cheeta example.. See it later..

Platform:-
The cheeta (t, wt); You (t, vt)
So, what is the speed of cheeta from your perspective?

$$t' = \gamma(t - vx/c^2) = \gamma(t - v/c^2 \bullet wt) = \gamma t(t - vw/c^2)$$
$$x' = \gamma(x - vt) = \gamma(wt - vt) = \gamma t(w - v)$$

Thus, the cheeta speed from your perspective is $\frac{x'}{t'}$:

$$= \frac{\gamma t(w - v)}{\gamma t(t - vw/c^2)} = \frac{(w - v)}{(t - vw/c^2)}$$

What we have done is for one dimensino, what if the moving object moves in 3D? For instance, if the train frame has a velocity v in direction x, and the object moves in the platform with $\mathbf{w}$, (w_x, w_y, w_z). Our goal now is to find the velocity of the object from the perspective of the train?

Platform: $(t, w_x t, w_y t, w_z t)$ Train: $(t', x', y', z') = (\gamma(t - vw_x t/c^2), \gamma(w_x t - vt), w_y t, w_z t)$

$$w'_x = \frac{\gamma(w_x t - vt)}{\gamma(t - vw_x t/c^2)} = \frac{(w_x - v)}{(t - vw_x/c^2)}$$
$$w'_y = \frac{w_y t}{\gamma(t - vw_y t/c^2)} = \frac{w_y}{\gamma(t - vw_y/c^2)}$$
$$w'_z = \frac{w_z t}{\gamma(t - vw_z t/c^2)} = \frac{w_z}{\gamma(t - vw_z/c^2)}$$

Example: 1D
Graice runs by George at $0.8c$ and yells "tag, your it". He then runs after her at $0.7c$. From his perspective, how quickly is she getting away? Draw a schematic figure.

Solution:
Graice's velocity: $w = 0.8c$
George's velocity: $v = 0.7c$

$$\frac{w-v}{1-wv/c^2} = \frac{0.8c-0.7c}{1-0.56} = 0.23c$$

Example: 3D

Graice turns a sharp corner, and now runs at $0.8c$ north, while George still runs at $0.7c$ due east. From George's perspective, what is Graice's velocity now? Draw a schematic figure.

Solution:

Graice's velocity: $\overrightarrow{w} = (0, 0.8c, 0)$
George's velocity: $\overrightarrow{v} = (0.7c, 0, 0)$

$$\begin{aligned}
&(\frac{(w_x - v)}{(t - vw_x/c^2)}, \frac{w_y}{\gamma(t - vw_y/c^2)}, \frac{w_z}{\gamma(t - vw_z/c^2)}) \\
&= (\frac{0 - 0.7c}{1 - 0}, \frac{0.8c}{1.4 * (1 - 0)}, 0) \\
&= (-0.7c, 0.57c, 0)
\end{aligned}$$

3 Spacetime Diagrams

3.1 Introduction

3.1.1 Speed of light invariants, rest, constant velocity

3.2 Cause and Effect

3.3 Invariants

3.4 Examples

3.4.1 Addition of velocities

3.4.2 Algebric treatment - as in David Tong's Notes

3.4.3 Lorentz transformations as an "exotic" rotation

4 Paradoxes of SR

4.1 Pole in Barn

Discuss the two approaches, the spacetime diagrams and the issue of locking the doors (simultaneity)

4.2 Twin Paradox

3 approaches: Without acceleration, relativistic Doppler effect and communication between twins (Spacetime diagram)

4.3 Dr. Jawad's Train Tunnel paradox

It is the same as pole in barn paradox, but here the new this is a bomb is fixed on the train and it will explode if the train is in the tunnel.

5 Relativistic Kinematics

5.1 Relativistic Mass

5.2 Momentum

5.3 Force and Energy

5.4 Solved Example: Compton Scattering

Afterword

Maxwell's equations in SR, GR, the loose ends.

A Appendix A

This appendix about the 4-vectors (Momentum, forces, energy). Check David Tong's and Joe's notes.

References

[1] Joseph L. Taylor. *Complex Variables*. AMS, 2011. ISBN: 978-0-8218-6901-7.